

TWINNING TWO MATHEMATICS TEACHERS
TEACHING GRADE 11 ALGEBRA:
A STRATEGY FOR CHANGE IN PRACTICE

by

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Submitted in fulfilment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

in Mathematics, Science and Technology Education

at the

UNIVERSITY OF THE NORTH-WEST

Mafikeng Campus

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September 2016

DECLARATION

I, Sello William Makgaka, hereby declare that this thesis, submitted for the qualification of Doctor Educationis in Mathematics, Science and Technology Education at the University of the North-West has not previously been submitted to this or any other university. I further declare that it is my own work, and that all the sources that I have used have been recognised by complete references.

Signature: -----

Date: September 2016

DEDICATION

This thesis is dedicated to my children, Hlogi, Nakedi and Karabo who brought love, peace and happiness to my family and in my life. May God be with you for the rest of your life.

Without your presence in my life, it would have not been possible for me to cope.

Not forgetting my mother, Raesetja Makgakga for bringing me up and the primary education she gave me. My guardian, Noko Semenya, for being supportive, motivation and understanding me throughout my PhD journey.

May God bless you all.

ACKNOWLEDGEMENTS

Firstly, I would like to thank God for giving me the wisdom and strength to complete this study. The completion of this thesis has been a unique experience in which many people contributed in a unique way, and are worth mentioning:

My deepest appreciation and gratitude goes to my supervisor, Prof. Percy Sepeng, for his invaluable guidance, support, scholarly insight, encouragement, and guidance throughout this study.

A special word of thanks goes to the following persons: Dr. Maupi Letsoalo of the Department of Statistics at Tshwane University of Technology (TUT), Ms Genevieve Wood (language editor), the school principals, the mathematics teachers and the learners who participated in this study.

I further acknowledge the support that I received from colleagues in Brown Bag at the University of South Africa, Prof. Mankoko Ramorola, Prof. Elias Mathipa, and all who have contributed to the ongoing dialogue that has helped me shape my thinking.

I also thank My librarians, Ms Cathy Lekganyane and Ms Busi Ramasodi assisted in literature search for my study, thanks for your support.

Lastly, I would like to thank my parents, especially my mother, Mrs. Raesetja Makgakga, for raising me, and my guardian, Mrs. Noko Semenya, who encouraged me throughout my educational career. You are my true heroines.

ABSTRACT

Twinning is a strategy that is used to bring two or more schools together to share teaching expertise, experiences and resources. This strategy was implemented in two secondary schools in the Polokwane District of Limpopo Province, South Africa. The purpose of this study was to explore the effectiveness of twinning two mathematics teachers teaching Grade 11 algebra. Furthermore, the study intended to concentrate on the development of the poorly-performing school in terms of their academic performance in Grade 11 algebra, and also, to change the poorly-performing teacher's practices by exposure to the new practices gained during the twinning process. The study followed a pre-test-intervention-post-test mixed methods design, utilising both quantitative and qualitative data. The data collection techniques used to respond to the research questions of this study included interviews (N=2) before the intervention, as well as an interview (N=1) with the teacher in the experimental group during and after the intervention. In addition, tests were administered in the two experimental (N=42) and control (N=42) groups, before and after the intervention. Classroom observations were conducted in the experimental group before and after the intervention, and also during the intervention. The study was underpinned by observational learning theory, which proposes that when a teacher in the experimental group observes a teacher from the control group, his/her teaching practices might improve. Again, observational learning would motivate the teacher in the experimental group, with the result that the learners' performance in Grade 11 Algebra would improve.

The analysis of the data generated from the pre- and post-tests and the classroom observations suggest that the intervention strategy improved the learners' academic performance. The statistical results of the experimental group indicated that they performed significantly better, with a rank-sum score of 2639.5 in the post-test, as compared to the pre-test's rank-sum score of 1101.5 ($p = 0.0018$). The data gained from the experimental group suggests that the interventional strategy had a positive influence on the conceptual and procedural understanding of the learners when solving algebra problems. Furthermore, the intervention strategy had a positive impact, in improving the learners' participation during the teaching and learning of Grade 11 Algebra.

An analysis of the classroom observations and interviews with the teachers indicated that the intervention strategy had changed the teacher's own practices in the experimental group by being exposed to the new practices of the teacher from the control group. The benefits of the twinning process in the experimental group were obvious, where the teacher in the experimental group used the expertise, experience and resources after the intervention. Moreover, the learners in the experimental group were encouraged to participate actively during the teaching and learning of Grade 11 Algebra, even after the intervention. Overall, the findings of this study show that the intervention in the experimental group was directly related to the teacher's change in practice, and by the improvement of the learners' academic performance in Grade 11 Algebra.

KEYWORDS: Twinning; Algebra; Learner; Teacher; Teaching and Learning

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EDITING CERTIFICATE

LANGUAGE EDITING SERVICES

Date: 2016/10/19

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TEACHING GRADE 11 ALGEBRA:
A STRATEGY FOR CHANGE IN PRACTICE**

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ABBREVIATIONS

ANA - Annual National Assessment

CCK - Common Content Knowledge

CAPS - Curriculum and Assessment Policy

DBE - Department of Basic Education

EE₁ - Exponential Expression

EE₂ - Exponential Equations

EFAL – English First Additional Language

EF - Exponential Function(s)

FM - Financial Mathematics

HCK - Horizon Content Knowledge

HF - hyperbolic function(s)

ICT - Information and Communication Technology

LTSM - Learner Teacher Support Materials

LI - Linear Inequality

LoLT - Language of Learning and Teaching

NCTM - National Council of the Teachers of Mathematics

NMAPR - National Mathematics Advisory Panel Report

NEEDU - National Education Evaluation and Development Unit

NRC - National Research Council

NGOs - Non-governmental Organisations

NP - Number Patterns

PF - Parabolic Function(s)

PCK - Pedagogical Content Knowledge

QE - Quadratic Equations

SACMEQ - Southern Africa Consortium for the Measurement of Educational Quality

SCL - Social Cognitive Learning Theory

SCK - Specialised Content Knowledge

SE - Simultaneous Equation

STAD - Student Teams-achievement Divisions

SQP - Student Questionnaire on Performance

UNESCO - United Nations Educational, Scientific and Cultural Organization

TPD - Teacher Professional Development

TGT - Teams-Games-Tournament

WP – Word-problem

ZPD - Zone of Proximal Development

CHAPTER 1

INTRODUCTION AND GENERAL ORIENTATION

1.1 INTRODUCTION

According to the Global Economic Report of 2015, the education system in South Africa is generally problematic (The World Economic Forum, 2015:15). South Africa was one of 140 selected countries to participate in the Global Economic Competition in September 2015. The purpose of the competition was to measure how the education system of each country meets the needs of their respective economies. The results of the competition revealed that South Africa ranked 138th out of 140 countries.

Especially as far as Mathematics is concerned, South African learners performed dismally in this subject, where South Africa ranked last. Armstrong (2014: 18) admits to the poor performance of South African learners in Mathematics as compared to other countries economically poorer than South Africa. According to the Southern Africa Consortium for the Measurement of Educational Quality (SACMEQ) in 2007, South Africa was positioned number 10 out of 15 selected countries, which also performed badly in Mathematics (Spaull, 2011: 2).

Certain factors contribute towards the learners' poor achievement in Mathematics (Gitaari, Nyaga, Muthaa, & Reche, 2013: 6), such as inadequate teaching, the students' absenteeism, poor entry marks from primary school to secondary school, and poor assessment techniques and teaching methods. In particular, Mbugua, Kibet, Muthaa and Nkonke (2012: 90) add that poor academic achievement in Mathematics is affected by student factors, such as entry behaviour (e.g., motivation and attitude), socio-economic factors (e.g. the parents' education level and economic status) and school-based factors (e.g., availability and use of teaching and learning facilities). Some studies (e.g. those of Rice, 2003: 4; Wilson & Floden, 2003: 12; and Betts, Zau & Rice, 2003: 8) have focused on the following variables that affect learners' learning of Mathematics results, namely the teachers' qualifications, their subject majors, their teaching experience and professional development.

The root cause of the poor results in Mathematics may be ineffective teaching approaches, as suggested by Luneta (2008: 386). The resources, the language of teaching and learning, and the behaviour or attitude of the learners towards mathematics might also have played a fundamental role in the poor achievement in this discipline in schools. The problem may stem from the teachers' instructional approaches, or possibly the classroom assessment of the learners, socio-economic factors, overcrowded classrooms, a lack of resources, motivation and attitude, absenteeism or mathematics anxiety (Mbugua et al., 2012: 89). Some other factors that may affect the learners' poor performance in mathematics are the teachers' workload, school discipline, and time management, as suggested by Musasia, Nakhunu, & Wekesa (2012: 56).

In South Africa, particularly in Limpopo Province, the learners' academic performance in Mathematics is a serious concern (Department of Basic Education, 2011: 3). The Grade Three Mathematics learners in Limpopo Province participated in the Annual National Assessment (ANA) of 2014 with the other eight provinces in South Africa (Ndebele, 2016: 16). The ANA tests in mathematics are administered in order to identify learners with difficulties and also to inform the teachers, parents and district officials in planning, improving and supporting the learners in the schools (DBE, 2014: 9). The 2014 Grade Three ANA results showed that Limpopo Province ranked 9th out of nine provinces that participated in the assessment, revealing the learners in Limpopo Province to perform worse than learners in other parts of the country.

Moreover, the poor performance in Mathematics in Limpopo Province is also found in the matric results for 2014 and 2015, respectively, in the National Senior Certificate Examination (DBE, 2014:9). From the 2014 results, Limpopo Province ranked 5th out of nine provinces in South Africa with 53.5% in Grade 12, which showed that this subject needs close attention in order to improve the performance. Again, this province was ranked 7th, with 52.1% in Grade 12 Mathematics, which shows a decline of 1.4 percent. The decline of the results, according to the percentage given (1.4%), denotes a significant number of learners who failed the subject.

In its strategic plan, the Department intends to introduce a *Twinning Model* (DBE, 2011: 18), which is similar to the intention of this study, which aims to explore the effectiveness of twinning two teachers who will be teaching Grade 11 algebraic topics. In their endeavour, the Department planned to twin performing schools and the poor-performing schools in mathematics and science.

However, some schools are not participants in the twinning project and continue to perform poorly in Mathematics. This study resolved to twin the two schools which were not part of the twinning implemented by the Department.

The DBE chose performing schools to participate in the Dinaledi project in all the subjects. This project was established in 2001 for disadvantaged learners in the National Senior Certificate Mathematics and Physical Sciences (DBE, 2011: 11). The Dinaledi schools can demonstrate their potential for increasing the learners' participation in these two disciplines. The DBE planned to twin schools that are in Dinaledi project and the poor-performing schools that were not participating in the same project. In the context of this study, the two schools participated in twinning were not part of Dinaledi schools. They are well-resourced and adequately supported to improve teaching and learning in the subjects or discipline (*subject* and *discipline* are used interchangeably in this study). Since the DBE in Limpopo Province intended to twin schools, the study explores this model in teachers' teaching and learning of Grade 11 algebraic topics. The experience of teaching algebraic topics and assessing learners in mathematics by the teachers both at the well-performing school (Dinaledi school) and the poor-performing school will be presented.

Despite the government's professional development initiatives in Limpopo Province (DBE, 2011: 9), some schools are still characterised by poor academic achievement in mathematics; other schools are involved in remedial developments, such as twinning or school clustering, and the outsourcing of the expertise of teachers or professional development. Many schools are not twinned, even though they perform badly in Mathematics, where the schools are seen to be marginalised in participating in the twinning strategy.

Some schools in the Polokwane District, Limpopo Province performed dismally in Mathematics between the academic years 2010 and 2013, respectively, while others consistently performed well (DBE, 2014: 14). The vastly different mathematics performance of two schools located in the same circuit in the Polokwane District drew the researcher's attention with regard to an investigation of the problem, and suggested an inquiry in which twinning will be used in teaching Grade 11 algebraic topics. The problem was diagnosed through studying the final Grade 12 mathematics results in these two schools from 2010 to 2013. The performing school's results that were obtained from 2010, 2011, 2012 and 2013 were found to be 78.2%, 89.6%, 91.1% and 88.5%, and those of the poor-performing school were found to be 31.5%, 35.2%, 43.1% and 48.6%, respectively.

Learners in the poor-performing school appear to experience difficulties when solving algebraic problems. The poor academic achievement was discovered by doing an analysis of the learners' academic performances in a neighbouring village in Mathematics, where I identified two schools, one with a good performance in Mathematics over a four-year period and another with the converse over the same period in the same circuit.

The poor results of the latter school had implications for the number of school-leavers with a minimum university entrance requirement. This situation presented a question as to why has one school performed poorly in Mathematics since 2010, where the other performed well, while both were located in the same vicinity, serving the same community.

Against this background, the primary research question and the secondary research questions are indicated as follows:

The primary research question:

What is the impact on teacher practice of twinning two teachers as a strategy to improve learners' academic achievements in teaching Grade 11 Algebra?

The research sub-questions:

- What are the pedagogical approaches used by the two teachers in the poor-performing school to teach Grade 11 Algebra?
- What is needed for a successful twinning process (if any)?
- What are the possible barriers for implementation?
- What are the benefits of using twinning as a strategy in a Grade 11 Mathematics classroom?
- How can the effective twinning model be developed?

There may also be some factors that contribute towards the betterment of academic achievement in the performing school. This study sought to investigate those factors and the reason why the well-performing and poor-performing schools performed differently in Mathematics. In this research, the poor-performing school is the experimental group, while the performing school is the control group. This led me to propose twinning the teachers teaching Grade 11 algebraic topics in the latter two schools, in an attempt to improve the performance of the low-performing school in algebra. Algebraic topics in Grade 11 appear to comprise more marks and learners experience challenges in learning this topic, and also the same learners should have background to deal with tertiary mathematics, which is why I

resolved to focus on and investigate the way in which the poor-performing school's teacher taught learners, before the intervention.

1.2 PURPOSE OF THE STUDY

The purpose of this study was to explore the effectiveness of twinning two Mathematics teachers teaching Grade 11 algebraic topics. This exploration enabled me to understand the sharing of practices during the twinning strategy; the focus was on teachers' teaching Grade 11 algebraic topics sharing their new knowledge and expertise. The study concentrated on the development of the low-performing school in terms of academic achievement and the change in the low-performing teacher's own practices. This study also intended to concentrate on the practices that the two well-performing and poor-performing Grade 11 Mathematics teachers will share during the twinning strategy, such as improving the learners' mathematical reasoning through discussion. Sepeng (2014:756) asserts that problem-solving develops learners' mathematical reasoning skills in solving mathematical problems through discussion. It has the potential to improve the learners' drawing of graphs, their interpretation of graphs and their understanding how graphs transform, solve equations, number patterns, exponents, financial mathematics and word problems.

The study comprised two phases (Phase 1 and 2). Phase 1 investigated the low-performing school teacher's teaching approaches in teaching Grade 11 algebraic topics prior to the twinning strategy. In the investigation the researcher focused on the teachers' teaching approaches, teacher-learner and learner-learner interaction and learner assessment, such as questioning style during the lesson. The assessment also focused on the types of questions the two teachers set during mathematics classroom activities, such as homework, classwork or assignments. The conceptual knowledge of the teachers and their strategies to engage learners' curiosity was investigated. The type of teaching materials and their use were also considered in teaching algebraic topics in the two schools.

This research analysed the effectiveness of twinning in the low-performing school in respect of Grade 11 algebraic topics. The poor-performing school in this study is the experimental group, and the well-performing school is the control group, as mentioned earlier. In the twinning process, meetings were held with the teachers to facilitate changes in their teaching practice. These meetings were conducted before the intervention, with others after observing the two teachers teaching Grade 11 algebraic topics. I was present when the teachers conducted their general planning, and the actions to implement the plan, monitoring the

learning and reflecting on the effects of the actions. The study follows a mixed method design, where the data will be analysed both qualitatively and quantitatively.

The sub-aims of this study are as follows:

- to make sense of how the two teachers teach Grade 11 Algebra;
- to investigate the key elements of the successful twinning model;
- to explore possible barriers in implementation;
- to understand the benefits and disadvantages in a context of twinning; and
- to develop a framework for the successful implementation of the twinning model.

1.3 RESEARCH DESIGN AND METHODOLOGY

The study follows a pragmatic worldview, as the intention was to use the different approaches available to understand the research problem, where a mixed methods approaches intertwines both qualitative and quantitative methods at once (Litchman, 2010: 17; McMillan & Schumacher, 2014: 354). The research involves a case study conducted according to qualitative and quantitative research paradigms (Rule & John, 2011:10), following an explanatory, sequential, mixed-methods design (Creswell, 2014:224). The collected data was triangulated in order to make a stronger case in terms of the explanatory quality of this study (McMillan & Schumacher, 2014:354), since triangulation is held to provide a better argument (Creswell, 2008:765), and produces a better understanding and verification (Creswell & Garrett, 2008:322). Qualitative and quantitative methods, when mixed or combined, complement one another and allow for a more robust analysis, by taking advantage of the strength of each method (Creswell, 2013:47). The data are triangulated in order to allow a greater validity of the study, by seeking corroboration between qualitative and the quantitative data. The mixed-method approach helped in answering research questions that cannot be answered either by the quantitative or qualitative methods alone.

1.3.1 QUANTITATIVE APPROACH

The case study included a pre-test-intervention-post-test design (Tashakkori & Teddlie, 2010:237). Being a true quasi-experimental design, it takes on the form of a pre-test-post-test control group design. Pre-tests and post-tests were written by both the good-performing school (control group) as well as the low-performing school (experimental group). An intervention in this study was implemented only in the experimental group (the low-performing school) after the pre-test was administered, while no intervention was implemented in the control group (the good-performing school). The purpose of this design

was to investigate the situation in terms of performance in the pre-test of the Grade 11 mathematics class, before twinning and the administration of the post-test. The study was divided into two phases: the first phase of the study was conducted before twinning and the second phase of the study took place during the implementation of the twinning strategy.

1.3.1.1 POPULATION

Polokwane District comprises 31 secondary schools, where the participating schools were sampled. This case study was conducted in the Polokwane District of the Limpopo Province in two rural secondary schools with two teachers teaching Grade 11 Algebra. The two Grade 11 Mathematics classes had participated in this study before, during and after the twinning strategy, when the two teachers were teaching Algebra.

1.3.1.2 MEASURING INSTRUMENTS

The pre-tests and post-tests were designed the researcher, using the Grade 10 Mathematics examination bank question papers, for the purpose of validity and reliability, in testing Grade 11 learners' knowledge of Algebra. The test instruments focused on quadratic equations, quadratic inequalities, simultaneous equations, word problems, number patterns, financial mathematics and algebraic functions, which are all included in the algebra topics.

1.3.1.3 DATA-COLLECTION PROCEDURE

The pre-test was administered in the two schools in the Grade 11 Mathematics class for the purpose of confirming the assumptions made from the Grade 12 results from 2010-2013, on the experimental and control schools, before twinning. The experimental and control schools' teachers' semi-structured interviews were conducted and thereafter the experimental group teacher four classroom observations was conducted before twinning process. Since this quantitative aspect is quasi-experimental, the twinning strategy was introduced in the low-performing school (experimental group). The classroom observations were conducted when the experimental and control schools' teachers engaged in twinning to observe what they would be sharing during the process. After twinning, observations were conducted with the experimental group's teacher to check if the teacher was applying the skills and knowledge gained during the twinning process. Then, the semi-structured interviews were conducted to determine whether twinning would have a positive impact on learners' learning of algebra, and improve teacher's own practices. The post-test was then administered in both the experimental and the control schools to measure the effectiveness of the twinning strategy, by comparing the mean differences between and within the two groups.

1.3.1.4 DATA ANALYSIS AND STATISTICAL TECHNIQUES

The data collected from the pre-tests and post-tests were initially categorised into correct and incorrect responses and blanks (Didis & Erbas, 2015:1141). To obtain a general view of the learners' performance in algebra, the percentage of the categories were calculated using the absolute numbers of the learners, not by the mean percentages. In other words, all the responses not completed, or even if used correctly were categorised incomplete. The blank category would be regarded as no response at all, or no attempt(s) made to solve the algebra problem.

Statistical techniques were also used to analyse the collected data from the pre-tests and post-tests by using the mean, median and standard deviation, namely the Wilcoxon-Rank Sum Test (Field, 2013:217). The purpose of the statistical test was to describe statistical significance, confidence intervals, and the effect sizes (Creswell, 2014:165). The statistical significance summarised the difference in the scores between the experimental and control groups in the pre-test and post-test results. The confidence intervals measure the range of values describing the level of uncertainties in the test score. The effect sizes identify the strengths of the conclusion about the group differences.

The statistical data were also examined and organised according to categories, using the Wilcoxon-Rank Sum Test, in order to obtain descriptive statistics of the mean, median, mode and standard deviations (Field, 2013:18). The minimum and maximum values were presented in this part of the statistics for the experimental (poor-performing school) and the control (well-performing school) groups involved in the study.

The statistical analysis was done on the difference of the mean scores, in order to assess if the marks of the one school improved significantly more or less than the marks of the other school during the twinning strategy. The statistical data were analysed using the Wilcoxon-Rank Sum Test to determine the statistical significant difference and the improvement of the experimental and the control groups; a p-value < 0.01 is regarded as highly significant at 99% level of confidence, and a p-value < 0.05 is regarded as significant at 95% level of confidence.

1.3.2 QUALITATIVE APPROACH

As noted earlier, this study used a qualitative approach to collect data by means of which to answer the research questions. The following research components forms part of the methodology in this study, namely the selection of the site, the selection of the participants,

data-collection strategies, data-analysis, the trustworthiness of the study and the role of the researcher.

1.3.2.1 SITE SELECTION

This study was conducted at two public rural secondary schools in the Limpopo Province in the Polokwane District. The two schools are situated in one area, meaning they serve the same community with the same socio-economic status and education status of the parents. The two schools have qualified Mathematics teachers, with more than fifteen years teaching experience.

The sample consisted of the two Grade 11 Mathematics classes and their two Mathematics teachers. All these schools are situated in the Polokwane District. The experimental group is the poor-performing school, while the control group is the well-performing one. In addition, the schools have similar characteristics in their approach to teaching and learning, are public schools, and previously marginalised. The schools draw learners enjoying a low socio-economic status. Sepedi is the mother tongue of the learners and the teachers, i.e. the language that the learners use at home, and when they play and communicate informally at school.

1.3.2.2 SELECTION OF PARTICIPANTS

This study used purposive sampling to select the participants, because the participants should all share the same characteristics, roles, opinions, knowledge, ideas or experiences that may be particularly relevant to this research (Cohen, Manion, & Morrison, 2010:122). The twinned teachers shared teaching practices with the aim of changing their own practices as exposed to them, and gained by them, during the twinning process. As mentioned earlier, the learners in the experimental group came from the low-performing school, and those from the control group were from the performing school.

1.3.2.3 DATA COLLECTION STRATEGIES

As noted earlier, a pre-test of the Grade 10 syllabus was administered to Grade 11 learners. The questions were divided into five sections with a total of six questions (or tasks), namely quadratic equations and linear inequalities, number patterns, financial mathematics, exponential expressions, exponential equations and exponential functions, and algebraic functions. The questions in the pre-test were piloted first, before they were given to the learners in the experimental and control groups. All the questions are drawn from the examination bank and previous examination questions.

The study consisted of two phases (Phase 1 and 2). Phase 1 investigated the low-performing school teacher's teaching approaches in teaching Grade 11 algebraic topics prior to the twinning strategy. Phase 2 of the study investigated the effect of twinning two mathematics teachers teaching Grade 11 Algebra in the low-performing school. In the investigation, the study focused on the teachers' teaching approaches, teacher-learner and learner-learner interaction and learner assessment, such as questioning style. The assessment also focused on the types of questions the two teachers set during classroom activities such as homework, classwork or assignments. The conceptual knowledge of the teachers and their strategies to engage the learners' curiosity were also investigated. The types of teaching materials and their use were also considered.

Twinning was then introduced in the low-performing school, with the intention of the teacher in the experimental group observing the teacher from the well-performing school teaching Grade 11 Algebra. The teacher from the control school convened a meeting with the teacher from the experimental group on how to implement twinning so as to improve the teaching and learning of algebra in the experimental group. The two teachers chose the topics and teaching and learning resources they liked to share during the implementation of twinning.

The semi-structured interview sessions were conducted in two phases: the first phase was for the teacher in the experimental group (poor-performing school) and one in the control (well-performing) school. At this stage, there were no measurable constructs, and therefore, the researcher made use of interview responses to inductively analyse the emerging constructs before the twinning process. The second phase of semi-structured interviews with the teacher of the poor-performing school took place during and after the twinning process, in order to reflect on this intervention.

Qualitative observations are those where the researcher takes field-notes of the behaviour and activities of individuals at the research site (Creswell, 2013:47). In this study, 12 classroom (or lesson) observations were held and field notes-were also taken. The study adapted Sepeng's (2010:4) classroom observation schedule during lesson observations in the twinned schools, before and during twinning process.

The study gathered up-close information by actually engaging or interacting with and talking directly to the teachers, and seeing them behaving and acting within their context, which Creswell (2013:47) refers to as a major characteristic of qualitative research. In the natural setting, the researcher had face-to-face interaction with the teachers over time without interfering in the lessons, observing how they taught mathematics to the learners and how

they interacted with those learners. Notes were taken during the classroom observations. The audio-taped lessons would be stored in a safe-locked cabinet.

1.3.2.4 DATA ANALYSIS

The analysis of the qualitative data was done concurrently with gathering the data, making the interpretations and writing the report (Creswell, 2013:51). In this research study, the analysis of the qualitative data involved gathering the data, based on asking the teachers both general and specific open-ended questions during the interviews. The qualitative analysis was developed from the information supplied by the two teachers before and during the twinning. The data collected from the semi-structured interviews and the classroom observations were organised, coded, themes were formed, and interrelated in order to interpret the meaning of those themes.

The quantitative statistical data generated from the pre- and post-tests (N= 89) and the instruments of the classroom observations (N=12), were captured on a Microsoft Office Excel spreadsheet and subjected to the analysis of variance (ANOVA) techniques to provide both descriptive and inferential statistics. Where necessary, the statistical technique of Matched-Pairs t-test was computed in order to compare the mean scores of the comparison and the experimental groups.

1.3.2.5 TRUSTWORTHINESS

In order to assess the accuracy of the findings and to be convincing, the researcher incorporated the use of multiple validity strategies (Creswell, 2013:51). Data is triangulated using various data sources by testing evidence from the sources and use it to build a strong case. Member-checking was also used to determine the accuracy of the qualitative findings through specific rich descriptions and themes (only polished transcriptions) by referring back to the teacher(s) through follow-up interviews, and giving them the opportunity to comment on the findings. A long time was spent at the research site, and repeated observations were made to further develop an in-depth understanding of the phenomenon under scrutiny (Tashakkori & Teddlie, 2010:239).

In this study, the data-collection followed a simple pre-test-intervention-post-test design, aimed at the experimental group, and a pre- and post-test design, administered to the control group using the same test items. A possible threat to the validity of this design was that the participants may remember their responses in the post-test from the pre-test (Sepeng, 2010: 78). However, because of the time gap between the two tests, it would probably not be the case. Peer-debriefing took place, with a senior peer who was not part of the study, but who

had an understanding of the study in order for him or her to review the perceptions, insight and analyses (Babbie & Mouton, 2006:129).

1.3.2.6 ROLE OF THE RESEARCHER

The researcher was a participant observer during the classroom observations, taking down notes and video-taping or audio taping (Cohen et al., 2010:461). Verbal interaction only took place during the interview sessions, in order to avoid interfering during the lessons. The researcher established a rapport with the participants by introducing himself, and explaining the study and its purpose, indicating the reasons why the participants were to be interviewed. The purpose of this was to motivate the participants and to ensure their sincerity in order to collect the accurate data (Cohen et al., 2010:461).

1.3.2.7 ETHICAL CONSIDERATIONS

An Ethical Clearance Certificate was received from the University of South Africa's Ethics Committee to access the schools. The informed consent of the teachers was requested after prior permission to conduct this research. The researcher approached the Limpopo Department of Basic Education in Limpopo Province for permission to visit the schools. Thereafter the researcher approached the principals and the Grade 11 Mathematics teachers of the two schools. The roles of the participants, their rights to choose to be participants and to participate or not in this study were explained to them (Tashakkori & Teddlie, 2010:239). They were assured of confidentiality, that their participation was voluntary, that they could withdraw from the study at any time, and that no personal details would be disclosed. The confidentiality of the information collected at the schools was ensured, together with the guarantee that no part of the data would be used for any other purpose than for this research. The consent forms were then signed by both the researcher and the two teachers for the consolidation of the agreement.

1.4 RATIONALE AND CONTRIBUTION

This research study could play a fundamental role in transforming the teachers' own practices by exposure to effective practices during the twinning process. It was ascertained that the two mathematics teachers in the schools appeared to have gained additional knowledge on the use of resources and classroom teaching through the shared experiences (Rees & Woodward, 1998:28). In so doing, the teaching and learning of algebraic topics also appeared to have improved through this collaborative teaching. Moreover, the teachers understood how to teach algebraic topics using new approaches acquired during the twinning process. The algebraic topics seemed to have become easier for the learners to solve, and their confidence

seemed to have increased. The learners were now able to solve algebraic equations, number patterns, financial mathematics, exponents and their equations, to draw graphs, and to interpret the different functions and their transformations during twinning. Teachers from other learning areas may also have gained knowledge through observing the process of twinning between the two participating teachers, and developed an interest in implementing a similar process to improve the teaching and learning in their disciplines.

These teachers had received support on how to use the twinning model. The principals of the two schools developed an interest in school twinning or 'buddying.' They may in future want to twin both schools in their entirety to develop each other in teaching practices and improve the overall results of the respective schools. The sharing of resources advanced the exchange of ideas for change to take place in the teaching and learning (Nachtigal, 1992:8) of algebraic topics in the context of the twinning process. Rees (2003:24) forwards that twinning is the sharing of resources to promote school improvement, and therefore it can be anticipated that their schools may improve in their academic achievement in other disciplines, not only in mathematics.

In the researcher's ten years of teaching experience, school clustering had been implemented in this district, but it was unfortunately not effective. In most circuits, schools were clustered but this did not serve its purpose, and was consequently ineffective. The aim of school twinning is to share effective practices in the Mathematics classroom, with a view to change (Cheah & Yau, 2011:2). The well-performing school, which formed part of the Dinaledi Project, would share their experiences with the poor-performing school (Department of Basic Education, 2011: 16). This study will follow a mixed method case approach (Creswell, 2013: 47) of two schools, in which one will be the well-performing school, and the other one will be poor-performing.

1.5 PRELIMINARY STRUCTURE/CHAPTER DIVISION

This section outlined the structure of the dissertation which was as follows:

Chapter 1

Chapter 1 provided the introduction and background to the study. The research problem was discussed, together with how the problem was identified. The research questions that assisted in addressing the research problem were outlined. The purpose of the study was to focus on the effective practices shared by the two teachers. The significance of the study was also

discussed, where the teachers would gain knowledge during the implementation of the strategy.

Chapter 2

Chapter 2 presents the theoretical framework of the study. The framework focuses on the social learning theory following observational learning. It explains what constitutes this framework, why the framework is relevant to this study, and how it will play a role in the twinning strategy.

Chapter 3

The third chapter presents a literature review that will explain the work done by other researchers on the topic. The researcher discusses the literature review, why it is important to this study and how it will be used to guide the research. The important aspects of the study to be discussed in this chapter are the teachers' teaching approaches and strategies, twinning in general, teacher collaboration, the teachers' subject-content knowledge of mathematics, teachers' pedagogical content knowledge, and the way in which the learners learn mathematics, teaching aids, the types of mathematics textbooks, assessment techniques, assessment methods, types of group discussions, assessment tasks (what they are, why they are important, and how they are used in the mathematics class), remedial support (what remedial support is, why it is important and how it is used to help the learners), enrichment activities, teaching resources, such as human/personal resources such as teachers and material/physical resources such as textbooks, the teachers' qualifications and experience, the teachers' professional development, school leadership, case study and qualitative and quantitative research.

Chapter 4

In this chapter, the researcher discusses the research design and methodology. As this case study takes place in the form of mixed method research, it follows a pre-test and post-test design. The data-collection techniques are discussed, such as classroom observations, interviews with the teachers, the pre-test and post-test questionnaires. A discussion of the ethical considerations and how permission was accessed from the University, the Department of Basic Education, the school principals and the mathematics teachers, are presented.

Chapter 5

This chapter presents the qualitative and quantitative analysis of the data of the learners' pre-tests and post-tests, the classroom observations and the interviews with the teachers before and after the twinning. The teachers' and learners' holistic classroom participation that included the learners' group discussions, their engagement with their teachers in the mathematics classroom before and during the twinning, are both discussed.

Chapter 6

Chapter 6 explicates the main findings of this study. The classroom observations prior to twinning and during twinning will also be compared and contrasted. The classroom practices, including teaching strategies, teaching materials, assessment techniques and methods and the teachers' questioning styles are compared and contrasted with the literature review. The data from the questionnaires, the interviews and focus group discussions are discussed. The overall findings are compared and contrasted with the literature review. This chapter also examined the data either confirmed or refuting the literature review used for this study.

Chapter 7

The final chapter presents the conclusion and recommendations, and the twinning framework of the study. The areas for future research on the twinning model are discussed.

CHAPTER 2

THEORETICAL AND CONCEPTUAL FRAMEWORK

2.1 INTRODUCTION

Chapter 1 presented the background to the study which included the problem statement, the reason for the study, the rationale for the study, the significance of the study, and an overview of the chapters. The researcher provided a summary of how the twinning process was implemented in the two schools, and briefly explained how this chapter would frame the study.

This chapter presents the theories that informed the study within the contexts of twinning as a technique and as a strategy. The theory that is presented in this chapter is the Social Cognitive Learning Theory (SCL) by Bandura (1986:459), supported by Vygotsky's (1978: 86) Zone of Proximal Development (ZPD) and scaffolding.

This chapter provides an outline of the constructs used, such as the teachers' experiences and expertise, their teaching practices at the level of the individual as well as during collaboration. This is explained as implemented during twinning, with an explication of the learners' academic achievement, a good- and poor-performing school, subject content and pedagogical content knowledge. These concepts are discussed within the contexts of the theories used to underpin this study. Furthermore, the researcher discusses the rationale for using the Social Cognitive Theory, as well as the significance of adopting ZPD and scaffolding in order to understand the phenomenon under investigation.

The Social Cognitive Theory focuses on learning that occurs in a social context or environment (Bandura, 1986:459). This theory was initially named the Social Learning Theory and was later renamed the Social Cognitive Theory, after Bandura realised that certain factors were not considered, such as vicarious learning. The researcher used this theory as framework for this study in order to give it scope and to provide parameters within which to understand the broader issues of teaching and learning in the two schools, with the aim of improving the learners' achievement in mathematics in the poor-performing school.

Against this background, the chapter begins by presenting the theories used in general, in an educational context, as well as within the specific context of this research. It demonstrates the

importance of using theories in educational and research practices. The early roots of the theories and their general history are discussed within the context of school twinning.

2.2 THE NOTION OF GENERAL THEORIES

A general theory is defined as “a set of statements or principles devised to explain a group of facts or phenomena, especially one that has been repeatedly tested or is widely accepted” (American Heritage Dictionary, 2006:1429). According to Tracey and Morrow (2012:3), general theories are regarded as explanations that are grounded in belief systems usually supported by extensive research, and are often held by a large number of people. The abovementioned researchers argue that all the theories that could be considered and used in the education system are often studied, tested and debated over a long period of time.

These definitions provide a deeper understanding of the chosen theories which guide the twinning process, in order to grasp the way in which the two teachers sought to share their teaching practices. The theories used in this study have been tested, studied, debated for a long period of time, and also used to frame a significant proportion of studies in the field of education. Educational theories are often called “lenses”, through which researchers can view the research settings (Tracey & Morrow, 2012:3). Researchers acquire the opportunity to use theories to frame their studies (Mertens, 2010:6). This also determined my choice of theoretical approach in this study.

In educational research the concept of *theory* is frequently used in mathematics education as in other scientific fields. The term *theory* in educational practice refers to an explanation of a phenomenon related to teaching and learning (Tracey & Morrow, 2012:3). In addition, theories are used to explain motivation, memory, achievement and intelligence. Theories assist in understanding phenomena (e.g., when a learner experiences difficulty in reading or writing), by providing explanations of cognitive problems. Tracey and Morrow (2012:3) further argue that theories used to explain reading and writing problems include theories of motivation, language, behaviour and/or social differences. Such explanations were expected to be of assistance during the observation sessions of the two teachers teaching Grade 11 Algebra in the two schools.

2.3 THE ROLE OF THEORIES IN EDUCATIONAL RESEARCH

Tracey and Morrow (2012:3) argue that researchers are more grounded if they are supported by theoretical foundations in the field of education. The knowledge of educational theories is a fundament to both educational practitioners and researchers. Linking research with relevant

educational theories is a hallmark of high-quality, scientifically-based research (National Research Council [NRC], 2002:28). Thus, this researcher relies on two theories in order to make sense of the twinning process of the two teachers from two different school environments. Creswell (2002:137) argues that a study that is theoretically linked to another makes a more substantial contribution towards extending the knowledge base, than one which is not linked. Any study that is not linked to theory is likely to be of low quality.

The two theories used in this study were chosen in order to predict, explain and control the research process during the twinning of the two teachers, and made a contribution towards making sense of the phenomenon under investigation. Gay, Mills and Airasian (2006:4) argue in this regard:

The goal of all scientific endeavours is to explain, predict, and/or control the phenomena. This goal is based on the assumptions that all behaviours and events are orderly and that these are effects that have discovered causes. Progress towards the goal involves acquiring knowledge and developing and testing theories. The existence of a viable theory greatly facilitates scientific progress by simultaneously explaining many phenomena.

The goals of the educational theories in this study are to provide a broader understanding of the teachers' teaching approaches used during the twinning process. They would illuminate how effective the teaching approaches are in the two schools, and how the teachers assist the learners to acquire new knowledge of algebra.

The variables are generated and evaluated in quantitative studies by educational theories underpinning research (Best & Kahn, 2003:162). In this study, the attempt at manipulating the variables under research-site conditions and according to the characteristics of the participants with a view to effective controlled observation appeared to be successful. Creswell (2014:9) indicates that in any mixed-methods research, the variables are identified and investigated so that possible relationships between them may be studied. Similarly, in this study, both dependent and independent variables were expected to be identified and the relationship(s) that emerged between them to be understood within the context of a twinning process. As a consequence, the relationships between these variables were expected to be explained, described and presented using the theories that underpinned this study. In other words, a theory plays the role of a bridge, which connects the dependent and independent variables. According to Creswell (2014:9), the investigators locate a theory in the literature review,

examine the predicted relationships among the variables and test the relationships with new participants, or at a new site.

Theories are also important in qualitative research. Theories are likely to serve a number of important purposes for qualitative researchers in educational research (Gay et al., 2006:4). The first important role of a theory is to provide a way for researchers to link their work to broader issues. Secondly, it allows the researchers to search for increased abstraction and to move beyond a purely descriptive account. Lastly, they can provide a rationale or sense of meaning to the work researchers do (Gay et al., 2006:4).

2.4 SOCIAL COGNITIVE THEORY

The Social cognitive theory concerns an internal mental process that may not be reflected in immediate behavioural change (Bandura, 1986:459). Bandura further explains this theory as relating to psychosocial functioning in terms of triadic reciprocal causation. The model, involving behaviour, cognitive, other personal factors and environmental events all operate as interacting determinants that influence each other in a bidirectional way during the causation (Bandura, 1988:275). This study sought to understand teachers' contribution during the twinning process, by observing each other's teaching approaches. Focus was also placed on how the twinning process could develop the competencies of teachers in teaching algebra through mastery modelling, strengthening the teachers' beliefs in the capabilities to use the strategies learned, and enhancing self-motivation by setting goals.

2.4.1 SOCIAL LEARNING PERSPECTIVES

Tracey and Morrow (2012:122) report that social learning perspectives incorporate different theories that focus on the role of social interaction in the development of knowledge and skills. They further argue that the social learning perspective emphasises the importance of social interaction and social influences, which also play a pivotal role in the effectiveness of twinning.

Social interaction is the relationship between two people sharing experiences, whereas social influence is the effect that people have upon the beliefs or behaviours of others (Aronson, 2004:223). The social learning perspective enabled this researcher to make sense of how the twinned teachers from the well-performing and the poor-performing schools worked collaboratively during the twinning process in sharing expertise, resources, skills and knowledge.

Social constructivism, just like social learning perspectives, emphasises aspects of the learners' learning as a result of social interaction (Vygotsky, 1978: 86; 1986:13). Davidson (2010:872) argues that social constructivism involves cognitive development that places the emphasis on the interaction with the people in the learners' world. Furthermore, Davidson (2010: 872) reports that the learners' knowledge, attitudes, and values develop by means of interaction in a social situation. The knowledge, attitudes and values also emerged during the twinning process, when the teachers shared their teaching practices in Grade 11 Algebra. Moreover, the teachers' experiences and expertise were shared during their social interaction.

Vygotsky (1979:30) also mentioned another key aspect in his theory, namely scaffolding, which supports social cognitive learning, as was also found in this study. *Scaffolding* refers to the assistance to a learner provided by adults or more competent peers during the learning experience. He advises that a learner should experience the use of higher-order mental functioning in a social context before he or she internalises it and independently uses it. Au (1997:183) labels this a *bridge* or *transition* from the inter-psychological plane (between people) and the intra-psychological plane (within an individual). This study incorporated the two key aspects of Vygotsky's theory (the Zone of Proximal Development (ZPD) and scaffolding). These two key aspects of Vygotsky's theory are discussed at length at a later stage in this chapter.

Rotter (1954:416) developed a social learning theory where he examined the integration of learning and personality. Rotter's social learning focused on people's deep-seated instinctual motives as determining behaviour. According to Rotter's theory (1954:416), people mostly consider the likely consequences of their actions in any situation and take actions according to their own beliefs. His social learning theory highlighted four major variables, namely behaviour potential, expectancy, reinforcement value, and psychological situations (Rotter, 1954:416).

Bandura's (1986:467) theory highlights the notion that a person's learning occurs in a social context or environment, as in the theories outlined above. Bandura (1986:467) argued that learning occurs by observing others' behaviours and the outcomes of those behaviours. This social learning theory was developed directly in response to behaviourism, which emphasised behavioural change, but omitted observing other persons' behaviour. In this study, the two participating teachers observed each other during the twinning process in order to share skills, resources and knowledge in teaching Grade 11 Algebra. They required social learning opportunity to share their teaching approaches and experiences during the process. Bandura

(1986:467) built his theory by identifying three areas of weaknesses in behaviourism. Those weaknesses are outlined as follows, namely:

- the limited range of behaviours for research in laboratory-type settings;
- the inability of the theories to account for the acquisition of new responses to the learning situation; and
- the fact that the theory dealt with only one type of learning, that is, direct learning, where the learner performs a response and experiences the consequences.

According to Bandura (1986:467), indirect learning involves delayed matching, where the learner observes reinforced behaviour and later enacts the same type of behaviour. In this study, the teacher in the poor-performing school observed the lessons of the teacher from the well-performing school teaching algebra during twinning.

2.4.2 THE SIGNIFICANCE OF SOCIAL COGNITIVE THEORY

The purpose of using SCL as a framework for this study is to explore twinning as it complements teaching and learning Grade 11 Algebra, and hence the learners' mathematics achievement. It also focuses on how teachers develop competencies through observing each other during the twinning process. In addition, a focus was placed on how to strengthen the two teachers' beliefs in their capabilities to teach algebra effectively, and to enhance their self-motivation in following their set goals. According to this theory, the study sought to focus on observational learning, where both teachers observed each other's teaching practices.

This theory also helps to guide focus to the variables that emerge during the study, namely the sharing of new knowledge through teaching practices, the learners' academic achievement, mathematical reasoning, subject content knowledge, and pedagogical content knowledge. It helped to direct, channel and shape the study within its context with the view to meeting the research aims and addressing the research problem and questions. This framework assisted in defining the framework used to analyse and interpret the collected data before and after twinning.

The SCT as the framework for this study provided a broader understanding of twinning and how well the two teachers shared their teaching practices in the teaching and learning of algebra. This theory incorporates the idea that a learner observes a role-model's behaviour and does not only imitate the behaviour of that role-model, but also interprets the role-model's responses (Tracey & Morrow, 2012:124). In this study, the teacher in the well-performing school was seen as the model, demonstrating teaching practices of algebra

concepts, where the teacher in the poor-performing school was the observer during their interaction in the twinning process. However, at times these roles were reversed, where the teacher in the poor-performing school served as a model and the teacher in the well-performing school served as an observer.

2.5 THE NOTION OF OBSERVATIONAL LEARNING

Observational learning refers to continuous, interactive learning which mediates between cognitive, behavioural and environmental influences (Bandura, 1986:198). Richardson (1967:107) defines it as the ability to learn skills and knowledge through observing others in a particular setting, and further explains that it is a symbolic rehearsal of a physical activity in the absence of any gross muscular movement. Galef (2003: 137) asserts that observational learning facilitates the transmission of information from one person to another. In other words, the learner observes the teacher demonstrating skills in and knowledge of a particular topic, and then the same learner practises what s/he learned during the observation process. Observational learning takes learning as the interaction between behavioural, environmental and cognitive influences (Groenendijk, Janssen, Rijlaarsdam, & Van den Bergh, 2011:2).

The teacher in the poor-performing school sought an opportunity to observe the expert teacher teaching Grade 11 Algebra or vice versa, as twinning is a two-way process. Groenendijk et al. (2011:2) explain that the apprentice observes the expert demonstrating his/her behaviour. Furthermore, the observer observes direct contingencies received by another and subsequently emits the target behaviour observed (Greer & Ross, 2007: 20; Greer, Singer-Dudek & Gautreaux, 2006:486).

Observational learning proved to be an effective learning activity in various domains, such as mathematics (Schunk & Hanson, 1985:72) and reading (Couzijn & Rijlaarsdam, 2004:243). A number of studies focused on argumentative writing during observational learning (Rijlaarsdam, Braaksma, Couzijn, Janssen, Raedts, Van Steendam, Toorenaar & Van den Berg, 2008:55; Van Steendam, Rijlaarsdam, Sercu, & Van den Bergh, 2010:318). Observational learning, in this study, played a role in assisting the two teachers to observe each other's expertise and experiences in the classroom. This study also focused on how this type of learning could yield benefits to the teachers. Caspers, Zilles, Laird and Eickhoff (2010:1148) concur that observational learning has gained increased attention, due to evidence in both human and non-human primates. This type of learning is demonstrated in this study by the behaviour of human primates, in which one teacher is seen to be a model (a model-teacher) and the other teacher an observer (the teacher-observer). The teacher-

observer was expected to learn by watching, interpreting and evaluating the model-teacher carrying out a task during the twinning process.

2.5.1 MODELLING PROCESSES

In the SCT, modelling is a process whereby a model displays the behaviours or performs an action for the observer to learn from, and a model is an expert who performs the actions (Tracey & Morrow, 2012:130). Haston (2007:27) defines *modelling* as a technique that can help learners learn effectively in many situations. According to Bandura (1986:470), *modelling* is a general term that refers to behavioural, cognitive and affective changes derived from observing models. Furthermore, modelling has been acknowledged as a powerful means in transmitting values, attitudes, and patterns of thought and behaviour to the observer. In this study, the model-teacher sought to share his skills and expertise of subject content knowledge and pedagogical content knowledge of Grade 11 Algebra with the teacher-observer with a view to developing his competencies in teaching algebra. During modelling, in the twinning process, the teacher-observer needs guidance in order to develop new skills, subject and pedagogical content knowledge. Some studies say that modelling succeeds due to the observer's participation in the construction of knowledge and the observer's knowledge of the modelling process is promoted (Mendonça & Justi, 2011:480 Souza & Justi, 2012:386).

Modelling is the first step in developing competencies during observational learning (Bandura, 1988:278). Whenever a teacher demonstrates a concept to a learner, that teacher is a model (Haston, 2007:27). In other words, the model-teacher needs to break down complex skills into sub-skills to enhance the teacher-observer's competencies in a social context.

The sub-skills are modelled (Haston, 2007:27) so that the teacher-observer can master the steps during interaction with the teacher-model during observational learning. Furthermore, after the sub-skills are learned through modelling, those sub-skills are combined into complex strategies to serve different purposes.

2.5.1.1 THE CHARACTERISTICS OF MODELS

It is essential for the teacher-observer to understand the characteristics of the model-teacher, because they play an important role in determining the degree to which attention is being paid to the model by the observer (Bandura, 1986:470). In other words, the two teachers need to understand each other, recognise each other's characteristics and set ground rules. The ground rules shape the twinning process and encourage the powerful influence of modelling that can simultaneously change the observer's behaviour, thoughts, patterns, emotional reactions, and evaluations (Rosenthal & Bandura, 1978:622). Bandura (1986:470) states that

the response of the observer to the modelling behaviour is largely determined by three sets of factors, namely:

- the particular attributes of the model, such as relevance and credibility for the learner;
- the prestige of the model; and
- the satisfaction already present in the situation where the behaviour is being modelled.

In this study, the three modelling behaviours stated above provided the teacher-observer an opportunity to understand the model-teacher's status in terms of expertise and experience. The teacher-observer also understood how reliable the model-teacher was in terms of subject content knowledge and pedagogical content knowledge.

2.5.1.2 THE ROLES OF MODELLING

The action performed by others can serve as social prompts for previously learned behaviour that observers can perform, as mentioned earlier (Mendonça & Justi, 2011: ; Souza & Justi, 2012:390). Similarly, the teacher-observer should observe how the model-teacher teaches algebra, especially when it comes to concepts that are difficult for the learners to master in the poor-performing school. Moreover, the observational learning during the twinning process can inspire motivation in the teacher-observer to model the modelled actions. Models have the potential to strengthen existing behaviours, such as knowledge and skills, in the teaching and learning of Grade 11 Algebra. Such behaviours can be strengthened during the sharing of expertise. In the modelling process, the teachers can also learn new behaviours that they did not know prior to observing them during twinning.

The key mechanism in observational learning is the information conveyed by the model-teacher, which may cultivate new skills in the teacher-observer in teaching Grade 11 Algebra.

This mechanism is shown when models exhibit novel patterns of thought or behaviour which the observer did not possess, but which followed the observed behaviour; thus, they can produce similar forms of behaviour (Bandura, 1971:47; Rosenthal & Zimmerman, 1978:64). Modelling influences the teaching of component skills, which are organised into new structures of behaviour, and which further develop existing knowledge and cognitive skills. Moreover, modelling give mathematics teachers the opportunity to understand how to structure what they have learned in order to apply them in their teaching of Grade 11 algebra and other mathematical concepts.

2.5.2 THE PROCESSES OF OBSERVATIONAL LEARNING

Bandura (1986:210) has argued that learning is an information-processing activity about the structure of environmental events that is transformed into symbolic representation that serves as a guide for action to be taken. Bandura's (1977:200) theory assumes that modelling influences learning, principally through its informative functions, and in a process whereby the learners acquire symbolic representations of modelled behaviour, rather than specific stimulus-response associations. In this formulation, modelling phenomena in observational learning is governed by four interrelated processes that are used to guide actions that may be taken into consideration during the twinning process. The interrelated processes, which are summarised in Figure 2.1, and discussed within the context of this study in the ensuing subsections, are: the attention process, the retention process, the production process and the motivation process.

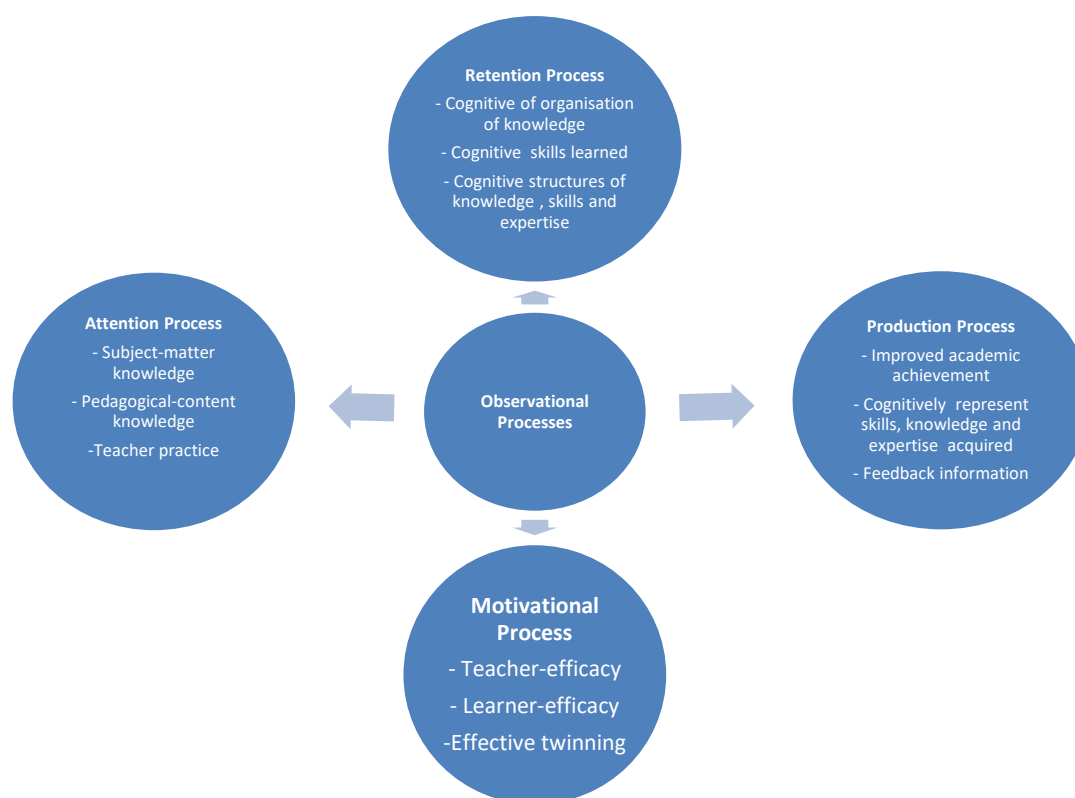


Figure 2.1: Observational processes (Bandura, 1986:210)

2.5.2.1 THE ATTENTION PROCESS

An attention process is a process where the observer absorbs information by paying attention to the behaviours modelled, and accurately perceives those actions during observations (Bandura, 1986:210). In other words, the observer is required to pay attention to the expertise, knowledge and strategies that the model-teacher can share with him/her in the twinning process with the hope that it opens new avenues to teach Grade 11 Algebra

effectively. The teachers can select topics that are challenging to the learners and give them additional attention. Bandura (1986:210) argues selective attention to be one of the crucial sub-functions in observational learning.

The attention process is not simply a matter of absorbing information modelled during the observation process, but it involves the self-directed exploration of the environment and the constructions of meaningful perceptions (Bandura, 1986:210). Thus, the model-teacher is expected to possess the appropriate skills and knowledge to share with the teacher-observer during the twinning process. The teacher-observer should understand the learners' background and the content knowledge shared during the twinning process.

However, the learning of the teacher-observer during twinning should not be controlled; the teacher-observer should extract what is important for him to learn, and how he should interpret the knowledge learned from the model-teacher. Habits and perceptions are selected behaviours in the attention processes, reflecting the level of the psychological development of the learner. The teacher-observer's capabilities for processing the modelled information limits the amount of observational learning that can be achieved from the exposure to the new pedagogical practices. If the expertise modelled is at a higher rate or level of complexity, overburdening the teacher-observer's cognitive skills, the observational learning becomes fragmentary (Bandura, 1986:201). Modelling influences appear to bring about changes rapidly and reliably when they are adjusted to the cognitive capabilities of the observers (Bullock, 1983:98). The pedagogical practices that are shared during the twinning process should meet the teacher-observer's needs and not cause confusion. If the skills and knowledge which are shared are clearly unpacked to the teacher-observer, he/she will have the opportunity to learn and apply them effectively in teaching Grade 11 Algebra and other mathematical concepts.

The modelled strategies in the attention process vary in their usefulness for dealing with the challenges faced by the teacher in the poor-performing school. The environment for twinning should be conducive for both the teacher-observer and the model-teacher, where both ought to share a common goal in the twinning process. The anticipated benefits of modelled skills, knowledge and strategies provide incentives for paying attention to the way in which others model themselves to achieve a particular goal. When events compete for attention, people who expect to perform similar activities pay greater attention to modelled conduct and learn it better than if they consider the modelled behaviours to be personally irrelevant (Kanfer, Duerfeldt, Martin & Dorsey, 1971:216). The two teachers were required to pay attention to

both modelled and observed activities in order to gain knowledge of how to teach algebra in the Grade 11 classroom more effectively.

The attention process in an on-going modelled event seems to be affected by the consequences experienced by others, as well as by the consequences one experiences directly and/or indirectly. The teacher-observer is expected to pay attention to modelled conduct that produces rewarding outcomes, but to disregard those having no noticeable effects during the sharing of expertise (Bandura, 1986:204). The experiences of others may operate through paying attention to the modelled behaviour, so as to enhance observational learning (Yussen, 1974:95). The teacher-observer observes the experiences that appeared to be rewarding at that time, and then pays attention to them with the aim of improving subject content knowledge and pedagogical content knowledge. The more often and longer the learners attends to the model's activities, the higher the level of observational learning. In other words, when the teacher-observer pays more attention to knowledge that appears beneficial to him, there will be knowledge gains of teaching practices of the teacher from the well-performing school teaching algebra.

The models that are interesting to learners and which are rewarding, tend to cause a sway of attraction, and this makes it easy for the observer to pay attention to the models, where those that lack attractiveness, or do not make sense to the learners, are rejected. The model-teacher is knowledgeable about the selected topics to be discussed during the twinning process and can present them effectively to the teacher-observer. The model-teacher should also use different resources that might increase the attractiveness of his presentation, and use those resources effectively to attract the attention of the teacher-observer. Bandura (1986:209) argues that the kind of modelling that attracts the attention of learners of all ages for an extended period is televised modelling, which may be a resource to be used in the twinning process. Bandura, Grusec and Menlove (1966:504) concur that televised models are effective in holding the attention, and consequently, that viewers learn the behaviour depicted, regardless of whether or not they are given extra incentives to do so.

2.5.2.2 THE RETENTION PROCESS

The retention process takes place when the observer is able to transform the knowledge gained during observations into symbolic forms, and organise that knowledge into easily remembered structures (Bandura, 1986:216). It is the second sub-function that governs observational learning, in which knowledge is retained about the events that have been modelled in one way or another. The knowledge and skills that are shared by the twinned teachers in teaching that algebra concepts are expected to be retained and later demonstrated

in the absence of the model-teacher by the teacher-observer. Moreover, the knowledge and skills acquired by the two teachers should be retained by using three modes of the retention, namely symbolic transformation, representation and rehearsal. These modes are briefly outlined in the light of their pivotal role in this research.

As a process, retention involves the active transformation and restructuring of information that has been learned about the behaviours. The teacher-observer ought to understand how to cognitively transform the skills and knowledge shared during modelling in twinning in his own teaching of algebra and other mathematical concepts. The learners must transform the subject content knowledge and pedagogical content knowledge that they observe during the twinning process into symbols, and thereby capture the essential features and structures of the modelled skills and knowledge. The observational learning and retention processes are aided by symbolic transformations, because they carry a great deal of information in an easily remembered form (Bandura, 1986:220). When a learner is ready to convert the information that is modelled into images and is ready to utilise these symbols verbally, then those conceptions guide subsequent action.

The retention process relies mainly upon two representational systems, involving verbal constructions. The information modelled by the model-teacher in some of the activities ought to be largely represented in verbal coding. The symbolic coding can enhance observational learning, as shown by studies conducted both with children (Bandura et al., 1966:503; Coates & Hartup, 1969:559) and with adults (Bandura & Jeffery, 1971: ; Gerst, 1971:12). This is demonstrated when the teacher-observer is able to model activities that he observed in his own words or through concise labels or vivid imagery. He will be able to retain the cognitive organisation and skills better than if he observed closely, where retention will be negatively affected if he was mentally preoccupied with other matters while watching the presentation of the model-teacher.

Representational system guides play an influential role in the early phase of the learning activities of models. After the model-teacher has represented the knowledge and skills on how well to present a particular lesson of algebra, it eventually becomes a routine that can be enacted smoothly and automatically, without requiring representational guidance. This was observed as an element of Bandura's theory, which was particularly useful in the conceptual grounding of the twinning process.

In addition to symbolic coding, rehearsal serves as an important memory aid. Learners who cognitively rehearse or actually perform modelled patterns of behaviour are less likely to

forget than those learners who do not think about modelled behaviour or practise what they have observed (Bandura, 1986:220). The teacher-observer needs to implement the knowledge and skills about the subject content knowledge and pedagogical content knowledge of algebra to avoid losing information learned during the twinning process. It is essential to distinguish between the contribution of rehearsal to both acquisition and the improvement of behaviour patterns. The observer should rehearse the behaviour learned during observation so as to prevent the information becoming vulnerable or lost from the memory; delayed rehearsal is of little value (Bandura & Jeffery, 1973:10). The modelled activities should be cognitively reinstated from time to time, after they were first observed, rather than after a long period of time. Bandura, Jeffery and Bachicha (1974:299) postulated that the modelled patterns can be strengthened in one way or another, especially by implementing them immediately after they were learned.

Bandura (1986:210) explains that repetition may contribute to retention by increasing the strength or number of memory traces. However, the facilitative effects of rehearsal on long-term retention derive more from applying memory strategies to the modelled information than from sheer repetition (Bandura et al., 1974:299; Bower, 1972:60; Rosenthal & Zimmerman, 1978:45). Similar processes contribute to the benefits of enactive rehearsal as well, especially when intermixed with modelling (Swanson, Henderson & Williams, 1979: 91). Enactments provide opportunities to organise and verify what one knows and to heighten the attentiveness to problematic aspects in subsequent modelling. By enhancing and channelling the attention, enactive rehearsal can refine the symbolic representation of the activity (Carroll & Bandura, 1985:583). Mental rehearsal of modelled activities can increase retention (Bandura & Jeffery, 1971:18; Michael & Maccoby, 1961:277).

2.5.2.3 THE PRODUCTION PROCESS

The production process is the third modelling system that involves converting symbolic conceptions into appropriate actions (Bandura, 1989:222). Bandura (1989:222) has indicated that conceptions that are in a symbolic mode have to be transformed into a corresponding action mode in the production process. Furthermore, the actions of the observer require the development of transformational skills in the intermodal guidance of the learned behaviour. Most of the modelled behaviours are abstractly represented as conceptions and rules of actions, which specify what has to be done.

The production process requires the observer to organise action sequences, monitor those actions, and compare them against the symbolic model (Carroll & Bandura, 1987:586). If the

teacher-observer depends primarily on the model-teacher information, he would not detect and correct mismatches.

2.5.2.4 THE MOTIVATIONAL PROCESS

The motivational process is an internal condition that can activate, provide energy and direct the behaviour (Endler, Rey & Butz, 2012:1124). Several studies correlate motivation with learning performance. For example, Schiefele, Krapp and Winteler (1992:184) concluded from meta-analysis that measured interest can predict the learners' academic achievement. Song and Keller (2001:8) showed that a motivationally adaptive strategy promises a significantly higher performance and efficiency in the learning of genetics when compared to a motivationally minimised strategy among students. Keeley, Zayak, and Correia (2008:8) indicated a statistically significant, non-linear relationship between anxiety and the test performance of students enrolled in an introductory statistics course.

Motivation has three interrelated aspects of behaviour, "the choice of a particular action, persistence with it, and efforts extended on it" (Dornyei, 2000:2). In addition to these aspects, motivation is responsible for why people embark on a particular activity, how long they will sustain it, and how hard they will pursue it. In other words, motivation can drive teachers to produce better results in their teaching of algebra.

Herzberg (1966:218) identified the factors that affect human motivation, but pointed out that these factors may not ensure motivation of any higher level. The factors identified were achievement, responsibility or autonomy, recognition, and opportunities. Andrews (2011:60) asserts recognition to be a positive strategy, which can produce improved teacher motivation and respect in the teaching fraternity. Moreover, he (2011:60) argues that motivation keeps teachers of quality in the schools. If these factors are incorporated in the twinning process, the twinned teachers would be more motivated to engage in the process.

Maslow (1954:194) presents human needs in a hierarchy (as cited in Andrews, 2011:60). The more the basic needs are fully satisfied, the more higher level needs can be realised. The teacher's basic needs in the poor-performing school ought to be considered in order for them to be satisfied and this will thus increase his motivation during the twinning process. The higher-level needs identified are striving for excellence and self-actualisation. These higher-level needs that are identified will be pursued, and met, only if the basic needs of the person are satisfied. The twinning process will only be successful if the twinned teachers' basic needs are satisfied during the process.

Researchers have identified four principal challenges in the history of motivation research during the 20th century (Dornyei, 2000:2). Dornyei states that these principal challenges play an important part in promoting or preventing consensus. These challenges include: (a) consciousness vs unconsciousness (i.e., distinguishing conscious and unconscious influences on human behaviour (Sorrentino, 1996:119); (b) cognition vs. affect (i.e. explaining both the cognitive and the affective/emotional influences on human behaviour in a unified framework, Weiner, 1992:10); (c) context (i.e. explaining the interrelationship of the individual organism, the individual's immediate environment and the broader socio-cultural context, Wentzel, 1999:85); and (d) time (i.e. accounting for the diachronic nature of motivation, that is, conceptualising a motivation construct with a prominent temporal axis, Heckhausen, 1991:212; Husman & Lens, 1999:120). These challenges should be considered during data-collection in this study on twinning.

2.5.2.4.1 TEACHER EFFICACY

Teacher efficacy is defined as a teacher's "judgement of his or her capacities to bring about desired outcomes of students' engagement and learning, even among those students who may be difficult or unmotivated" (Tschannen-Moran & Woolfolk Hoy, 2001:783). Hoy (2000:480) defines *teacher efficacy* as the teacher's confidence in his/her ability to promote the learners' learning in the classroom. Teacher efficacy has to do with the beliefs that determine how much teachers perceive environmental opportunities and impediments (Bandura, 2006a:6). Furthermore, it affects the choice of activities planned, how much of an effort is extended on the activity and how long people will persevere when confronted with challenges (Pajares, 1997:11). Thus, the beliefs teachers harbour in relation to their own effectiveness are known as teacher-efficacy, and underlie their instructional decisions, which ultimately shape the learners' educational experiences, and in turn affect the learners' academic performance (Woodcock, 2011:84).

Teacher efficacy consistently relates to the teachers' positive behaviours, and hence the learners' achievements (Cakiroglu, Cakiroglu & Boone, 2005:). Some scholars assert that teacher efficacy is related to the learners' success (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998) and to the classroom environment (Raudenbush, Rowan, & Cheong, 1992:153). Teacher efficacy results in a better teacher, who will influence a good learner's achievement in a positive way (Perry & Ball, 2005:180). Schunk and Meece (2006:8) concur that teacher efficacy affects one's goal and behaviours and can also be influenced by actions and conditions in the classroom.

In this study, the researcher established the level of teacher efficacy in the well-performing and poor-performing schools during classroom observations, and in interviews with the twinned teachers. Teacher efficacy has been characterised into two independent dimensions in certain studies (e.g. Tschannen-Moran & Woolfolk Hoy, 2001:783; Woolfolk & Hoy, 1990:82). The first dimension is termed the teachers' *harboured beliefs* about their own personal abilities to influence their learners' learning and performance; this is known as 'personal teacher efficacy' (PTE).

The second dimension is a teachers' belief concerning the extent to which teaching can overcome external influences on the learner; this is known as 'general teacher efficacy' (GTE). Some research postulates that the two independent dimensions may relate differently to pre-service and in-service teachers' beliefs about control, management, and motivation in the classroom (e.g. Woolfolk & Hoy, 1990:82; Woolfolk, Rosoff, & Hoy, 1990:138; Woolfolk-Hoy & Burke-Spero, 2005:343). Woodcock (2011:28) argues that a teacher might possess a high level of PTE, but a lower GTE, if he/she believes that the external factors beyond his/her control have a greater impact on the learners' learning than the teacher does.

Those teachers who believe in external factors are mostly influential in their own skills, and cannot effect much change in their teaching and learning, especially with low-achieving learners (Durgunoglu & Hughes, 2010:35). In contrast, teachers with high levels of teacher efficacy tend to be motivated and persevere through challenges (Allinder, 1995:249; Stripling, Ricketts, Roberts, & Harlin, 2008:126). Wolters and Daugherty (2007:182) found that a high level of teacher-*efficacy* creates positive teacher practice, and contributes to constructive policies that are implemented in the classroom. In summary, teacher-*efficacy* is a key feature, which determines their success or failure.

2.6 SCAFFOLDING AS A TEACHING STRATEGY

Researchers use the concept of *scaffolding* as a metaphor to describe and explain the role of teachers, or more capable peers, in guiding the learners' learning and mental development (Hammond, 2002:2). According to Vygotsky (1978:86), *scaffolding instruction* is defined as the role of the teacher, and a more capable peer, in providing support to a learner's development in learning new materials. In addition, scaffolding provides structures by means of which to move a learner to the next level or stage during learning and instruction. The concept of scaffolding thus plays a role in effective learning, by allowing the teachers to use the knowledge gained to assist each individual learner in developing his/her own knowledge and thinking. Scaffolding as a teaching strategy facilitates a learner's ability to build on prior

knowledge, and to internalise new information through the support of the teacher (Van der Stuyf, 2002:5). The activities provided in scaffolding instruction are just beyond the level of what the learner can do alone (Olson & Pratt, 2000:175). When one incorporates scaffolding in teaching and learning, one becomes more of a mentor and facilitator of knowledge rather than the dominant content expert.

Vygotsky (1978:86) defines *scaffolding instruction* as the “role of teachers and others in supporting the learner’s development and providing support structures to get to that next stage or level” (Raymond, 2000:176). However, the effect of scaffolding is not permanent during the learning of new material; it is temporary (Van der Stuyf, 2002:5). Therefore, the goal of the educator when using scaffolding as a teaching strategy is for the learner to become independent, self-regulating and a problem-solver (Hartman, 2002:26). This implies that when a learner’s competencies increase independently, or his/her level of understanding is increasing, scaffolding should be gradually decreased, and ultimately withdrawn (Ellis, Worthington & Larkin, n.d.:15). At that moment, a learner should be able to solve problems independently, or have mastered the concepts (Chang, Cheng, & Sung, 2002:19). Twinned teachers who use scaffolding as a teaching strategy aim to enable and assist individual learners who struggle to solve Grade 11 algebraic problems, to improve.

Scaffolding could occur in the form of tasks. Bransford, Brown and Cocking (2000:155) describe these as follows:

- motivating the learners’ interests in relation to the task given to those learners;
- manageable, simplified, and achievable tasks for the learners;
- providing the learners with a direction in order for them to achieve their set goal;
- clearly indicating the differences between the learners’ work and the standard or desired solution;
- reducing the learners’ frustrations in learning algebraic problems; and
- modelling and clearly defining the expectations of the task that has to be performed.

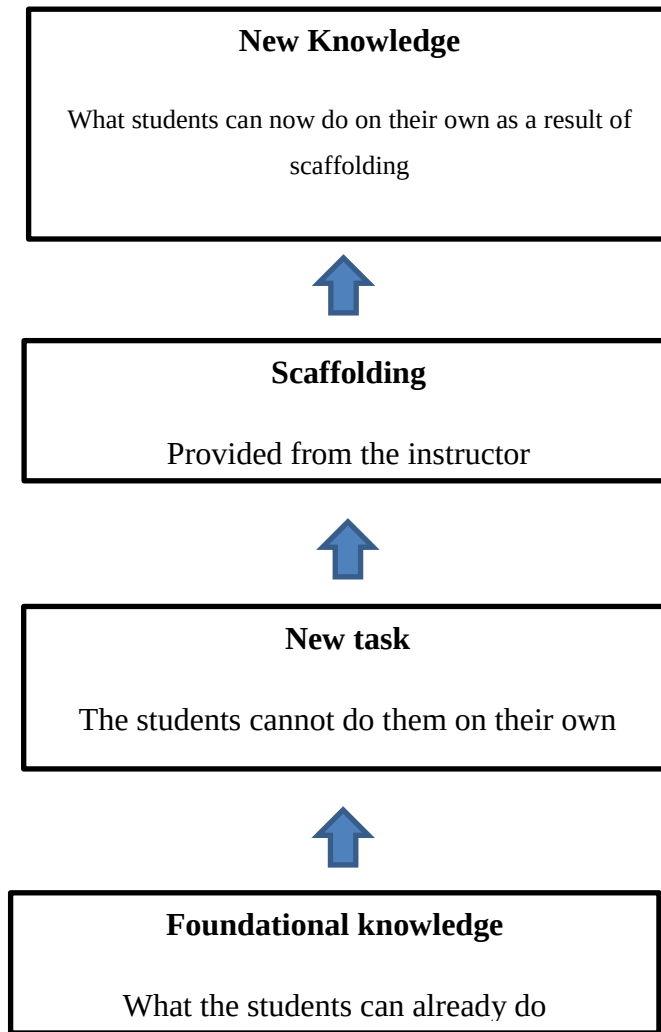


Figure 2.2: Illustrative Model of Scaffolding (Adapted from Hogan and Pressley, 1997:75)

Scaffolding is effective when the teachers set tasks that will effectively engage the learners during classroom instruction. Tasks that are clear, manageable and directive enable the learners to take ownership of their own learning or to encourage learner-centeredness. The teacher, in this regard, facilitates learning and is not acting as an expert in the subject. In this study, scaffolding instruction can be effective, where the teacher in the poor-performing school observes his counterpart's support of learners through learning activities in algebra.

Scaffolding instructions are suitable to be employed in problem-based learning situations aimed at the learners' mental development (Ngeow & Yoon, 2001:1). Problem-based learning is an educational approach that challenges learners to "learn to learn" (Ngeow and Yoon, 2001:1).

In this PBL, the teacher has to provide the learners with activities that they can solve independently, and know what the learners have to learn to complete a given new task. The teacher then “designs activities which offer just enough of a scaffold for students to overcome this gap in knowledge and skills” (Ngeow and Yoon, 2001:2). In this study, activities were based on an understanding of the level of the learners’ mastery of Grade 11 Algebra, in order to overcome the gap between the knowledge and skills by means of scaffolding.

The concept of scaffolding has its own advantages, such as engaging the learners in classroom instruction (Van der Stuyf, 2002:6). When using scaffolding during instruction, the learners are fully engaged in a lesson; the learners are not regarded as passive recipients of information. The teacher prompts the learners to build on prior knowledge, in order to form new knowledge during classroom instruction. In the educational setting the scaffolds may include models, cues, prompts, hints, partial solutions, think-aloud modelling and direct instruction (Hartman, 2002:26; Alibali, 2006:155). Scaffolding develops the learners with a low self-esteem and learning disabilities, through the positive feedback provided by the teacher (Van der Stuyf, 2002:6). In the twinning process, the researcher observed how the twinned teachers scaffold their slow learners during classroom instruction in the first phase of the study. The researcher also investigated the way in which they shared their experiences of scaffolding, slow learners during the twinning process in the second phase of the study.

Disadvantages also exist in respect of scaffolding instruction: planning for and implementing scaffolds is time-consuming and demanding (Hogan & Presley, 1997:75; Van der Stuyf, 2002:6). To select appropriate scaffolds and match the diverse learning and communication styles of learners can be challenging for teachers (Hogan & Presley, 1997:75). This type of instruction requires that teachers give up some of their responsibilities or classroom control to allow the learners to make errors (Van der Stuyf, 2002:6). Although there are some challenges in scaffolding instruction, learning through dialogue, feedback and shared responsibility can be promoted. With this strategy, twinned teachers should learn, during the twinning process, how to help their learners become life-long and independent learners, especially those who are in poor-performing schools.

2.7 THE ZONE OF PROXIMAL DEVELOPMENT

There is consensus among researchers that socio-cultural theory and the theory of the ZPD are at the heart of the concept of scaffolding (Wells, 1999:127; Krause, Bochner, & Dechesne, 2003:63; Vygotsky, 1978:81). Wells (1999:127) refers to *scaffolding* as “a way of operationalizing [sic] Vygotsky’s (1987:211) concept of working in a zone of proximal

development”. Scaffolding cannot work effectively without the ZPD. Vygotsky (1978:81) defines the ZPD as a distance between what learners can perform independently, and what they can perform with competent assistance. In other words, the more capable peer provides the scaffolds, so that the learner can perform a task that he/she cannot perform, thus helping the learner through the ZPD (Bransford et al., 2000:378; Van der Stuyf, 2002:9). The ZPD is described by Vygotsky to be “the distance between the actual development levels as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers” (1978:86), or, “what the child is able to do in collaboration today he will be able to do independently tomorrow” (1998:202).

The ZPD originated from the socio-cultural theory of Vygotsky (1978:86). Cognitive processes have social origins, according to this theory. Moreover, the individual’s cognitive development is interrelated with social, cultural and historical understanding (Kargozari & Tafazoli, 2013:164). Higher mental functions have their origins in social interactions with more experienced adults or peers (Vygotsky, 1978:86). Vygotsky has proposed a difference between what learners are able to do autonomously, and what they can perform with the more knowledgeable peers or adults. Thus, this concept constitutes the relationship between the two types of problem-solving behaviours of the learners (Kargozari & Tafazoli, 2013:164). The first behaviour is when the learners can solve a problem in social interaction, which is called the potential level of development. The second behaviour is when the learners can solve a problem by themselves, which is called the actual level of development.

Bassot (2012:34) points out that various studies have inevitably interpreted the ZPD and some of Vygotsky’s ideas in ways that differ. As a result, three groups of interpretations of the ZPD can be found in the literature (Lave & Wenger, 1991:49). The first interpretation of the ZPD is the scaffolding interpretation, which focuses on the relationship between a learner and a more experienced person in the social construction of knowledge (Wood, 1988:26). The second is cultural interpretation, which focuses on the difference between knowledge imparted during instruction, and knowledge generated by everyday experience (Davydov & Markova, 1982:60). Lastly, there is the collectivist interpretation, which views the ZPD as the difference between what an individual can achieve by means of everyday actions, and what can be achieved by learners within collectives, and the collectives themselves. In this regard, the ZPD plays a fundamental role in the broader understanding of how these interpretations might surface in the first phase of this study, as well as the second phase (the twinning process).

The term 'ZPD' is probably the most widely used and well-known ideas associated with Vygotsky's scientific work. The ZPD is mostly used in educational research pertaining to teaching and learning in many subject-areas, such as reading, writing, mathematics, science, second-language learning (Dunn & Lantolf, 1998:421; Lantolf & Parlenko, 1995:115) and moral education (Tappan, 1997:85). Furthermore, it is used with diverse kinds of learners, including the disadvantaged, the learning-disabled, the retarded, and gifted learners, as well as pre-school learners (Smith, 1993:56) and adults (Kilgore, 1999:196). Lastly, it is used in information technology and computer-mediated communication (Hung, 2001:35), and with the learners' use of libraries (McKennie, 1997:68).

The concept of ZPD may be understood in context, and as part of Vygotsky's theory on the whole. Tudge has noted that "in fact, failure to see the connections between the zone and the theory as a whole means that it is difficult to differentiate Vygotsky's concept from any instructional technique that systematically leads children with the help of an adult, through a number of steps in the process of learning some set of skills" (1992:156). The twinned teachers in this study gave attention to their learners' ZPD in order to identify the best type of instruction to teach Grade 11 Algebra and other mathematical concepts. Teachers ought to know their learners' prior knowledge, before introducing new knowledge in order to reach their goals during learning instruction.

Social interactions are crucial for the learners' development during learning experiences (Vygotsky, 1978:86). Vygotsky (1978:86) asserts that any higher mental function go through an external social stage in their development before becoming an internal, truly mental function. The development of the mental functioning of a learner is initially social, and can undergo a process which has to become an internal function, known as 'internalisation' (Vygotsky, 1978:86). Thus, the teachers interact with the learners after understanding their learners' prior knowledge, before the learners internalise the content. The role of social interaction in the learners' activities has been strongly emphasised by Engeström (1996:137). The establishment of shared understandings between the teacher and the learners in the concept of social mediation is called 'inter-subjectivity' (Dixon-Krauss, 1996:86). Inter-subjectivity is essential in the process of internalisation, where the teacher is required to gradually reduce their support, leaving the learners to accomplish the task independently during twinning, so as to improve their academic performance.

In short, the learners learn new concepts best when they are within their ZPD (or proximal to them) rather than outside the zone (or at a distance). In addition, learners learning through interaction with others are able to perform much better, or achieve more, than when they

perform tasks independently without the support of more capable peers (Bassot, 2012:36). The ZPD and scaffolding instructions complement each other; scaffolding takes place more effectively in the presence of ZPD.

2.8 CONCLUSION

The notion of general theories in this chapter in educational contexts and research studies have been discussed to provide an understanding of what theories are and how they underpin research. The role of general theories in educational and research contexts was explained, so as to gain a deeper understanding of how such theories function in this study and in the classroom. The Social Cognitive Theory of Bandura (1986:459) was introduced as the main theory that framed this study. The purpose and basic principles of the Social Cognitive Theory were explained in this chapter, and the rationale given for its selection from amongst other social learning theories. As the study used observational learning central to social cognitive theory, the attention process, the retention process, the production process, and the motivational process were discussed in order to guide the study. The ZPD and scaffolding were introduced and incorporated in the study so as to support the idea of observational learning in social cognitive theory.

CHAPTER 3

LITERATURE REVIEW

3.1 INTRODUCTION

This chapter provides an overview of the literature review on the issue of the twinning of schools within the context of a case of two mathematics teachers. As noted in Chapter 2 of this study, the study is framed by social cognitive theory, the zone of proximal development (ZPD) and scaffolding (Bandura, 1986:459; Vygotsky, 1978:86).

This chapter begins with a discussion of the twinning process within the context of this study, which incorporates the background and definitions of the twinning process, followed by the role of twinning in schools. The advantages and disadvantages or challenges of the twinning process are outlined according to the literature review. The kind of information shared by the twinned teachers during the twinning process is also discussed.

3.2 TWINNING

3.2.1 BACKGROUND, CONTEXT AND DEFINITION

School twinning is described as an important programme that brings two schools together to share their experiences and expertise (The Ministry of Education and Sports, the Republic of Serbia Vocational Education and Training Reform Program, 2006:4). Oxfam (2007:6) calls the twinning process a partnership between schools, in terms of the learners' learning and the character of the schools participating in the programme. According to Berliner (1990:5), *school twinning* refers to the joint commitment of two schools sharing resources for the sake of mutual benefit, and in particular, to promote better school results. Other researchers refer to school twinning as the clustering of schools (see for example, Rees and Woodward, 1998:26). Lock (2011:5) agrees that school twinning is the clustering together of schools to advance their improvements, that is, working together for peer support with an external colleague such as the teacher from another school, to present lessons. For the purposes of this study, the researcher used these definitions of the twinning of schools to frame all the explanations that relate to the two participating teachers' sharing of resources, experiences and expertise.

School clustering or twinning in the United Kingdom is implemented not only for the purpose of fiscal change, but also in response to a nationally-imposed demand for the increased knowledge of teachers (Galton & Hargreaves, 1995:179). According to Hargreaves (1996:12), *school clustering* in the United Kingdom context, is referred to as "...a group of two or more schools (defined as small schools with less than 100 pupils) which have agreed to cooperate with each other... for children's social development, the sharing of resources, or a combination of these". One reason for clustering in the United Kingdom appears to be in order to sustain small schools. The schools work together for the sake of sharing resources and also for their learners to work cooperatively.

Lock (2011:5) gave as another reason for the twinning or clustering of schools, namely to combat school isolation in rural areas. In combatting the isolation of schools, bonding and bridging are encouraged among the teachers (Collaborne & West-Burnham, 2008:28). They furthermore indicate that for communities to succeed, it is better that they work jointly in such a way that each individual is involved, interdependent and heterogeneous. West-Burnham (2009:53) describes *bridging* as a network, a cluster or a partnership. Lock (2011:13) asserts that clustering develops and supports schools with stimuli used for the improvement of the teachers' change in practice, through working collaboratively.

Hill (2007:12) has argued that the twinning of schools allows for collaboration in order to improve the quality of teaching and learning. He (2007:14) further asserts that the good ideas and practices during the process of twinning or clustering bring about the transformation of the teachers in terms of teaching and learning. The ideas and practices are not only trapped on location, but rather shared among the teachers. The best ideas and practices are not only applied in one school, but seem to be shared amongst the teachers in other schools as well, in order to avoid fruitless competition.

Smith (2009:14) asserts that open-mindedness is the key to the sustainability of teachers working collaboratively to improve the quality of education. Furthermore, open-mindedness appears to play a major role in the benefits and potential development of teachers working collaboratively in pursuing the needs of their schools. The sustainability of the twinning process is sought by improving the learners' academic achievement in solving algebraic problems through the sharing of practices, knowledge and ideas.

3.2.2 BENEFITS

The twinning of schools has been studied by researchers, including the way in which it affected good professional practice within the European context. It is widely documented that the benefits of twinning are economic, educational, social and political in nature (Galto, 1993:16; Lock, 2011:9; Nachtigal, 1992:8; Rees & Woodward, 1998:28; Rees, 2003:9). Nachtigal (1992:8) argues that most small schools do not have good financial backing, and therefore, require support. The notion of financial constraint is a central reason why schools are twinned or clustered in European countries. Rees (2003:9) argues that the twinning of schools reduces their salary costs by using one principal to run two schools concurrently so as to avoid the closure of the smaller rural school. Rees and Woodward (1998:28) concur that costs are reduced in terms of redundancy efforts, as well as the fact that the duplication of equipment and services is eliminated. Other cost savings found in the literature are due to a reduction of the schools' administration requirements, the sharing of speciality materials and equipment, and the sharing of professional development (Lock, 2011:14; Rees & Woodward, 1998:29; Rees, 2003:25). However, as noted earlier, the purpose of twinning in the study reported here is to improve collaboration between the two participating teachers' own practices.

The benefits of twinning were also identified within an educational context, which appeared mostly in guiding this study, as it focused on the benefits that twinning can have, and also consider whether it can change the learners' performance in the poor-performing school. The advantages found in the literature mainly examined the learners' learning and the teachers' teaching methods (Lock, 2011:9; Nachtigal, 1992:10; Rees & Woodward, 1998:26; Rees, 2003:24). Rees (2003:24) argues that learners benefit more when given the opportunity to interact with other learners from other schools during twinning, further indicating that parental involvement increased, namely in supporting their children by transporting them to and from different schools wherever there is a twinning project.

The teachers also seem to benefit from the twinning process. Rees (2003:24) indicates the benefits of the teachers in the twinning process as "a regular exchange of ideas, expertise and new knowledge among staff, a renewed teaching staff, and support for creating and testing a restructured/alternative delivery system". Other researchers agreed that teachers working collaboratively share experiences and ideas during the clustering or twinning process (for example, Lock, 2011:15; Rees, & Woodward, 1998:32). The teachers in the two schools shared their experiences, knowledge, ideas and resources in order to see if this might improve

the academic performance of learners during twinning. Its purpose was to expose the teacher in the non-performing school to the new practices gained during the process. Galton and Hargreaves (1995:25) asserted the fact that the teachers have the opportunity to be engaged in the joint planning of activities, where the one teacher can act as a specialist. Hargreaves (2010:13) indicated that the teachers who work collaboratively are able to transfer professional knowledge more readily and become more efficient in the use of resources. Rees (2003:26) supported the idea that the smaller schools may access a variety of facilities and materials for development and increase the teachers' expertise during the twinning process.

Teachers working collaboratively enjoy learning in a social environment (Galton & Hargreaves, 1995:179; Lock, 2011:15; Nachtigal, 1992:8; Rees & Woodward, 1998:32; Rees, 2003:28). Even during twinning the twinned teachers can enjoy working together. The abovementioned researchers argue that the learners in the twinned schools may now participate in extra-curricular activities that they could not do previously. Moreover, the researchers say that smaller schools in the rural areas come to realise that they are part of a larger community of learners. In other words, the twinned teachers can benefit both professionally and socially as they participate within a social context, sharing experiences about teaching and learning. This implies that the teachers work collaboratively in the social environment to expose one another to new classroom practices.

The climate of cooperation for the mutual benefit of the twinned schools is encouraged in order to limit the spirit of competition and control (Nachtigal, 1992:10), where the author furthermore indicated that the benefits of school twinning in a political dimension are educational equity, providing opportunities for reciprocal relationships among schools, and to form political alliances by means of which to prevent school closures. The schools will thereby gain from long-term stability through working in collaboration during twinning, rather than merely gaining from a short-term improvement (Pipho, 1987:6). It appears to be important for this study that the schools gain long-term stability when working together, and sharing experiences, expertise and knowledge during the twinning process.

3.2.3 CHALLENGES

It has already been indicated that the twinning process has its own challenges (Rees & Woodward, 1998:32; Rees, 2003:45). A number of other researchers concur that schools entering in a twinning arrangements indeed face certain challenges (De Young & Howley, 1990:65; De Young & Lawrence, 1995:108; Haller and Monk, 1988:476). Streifel, Foldesy and Holman (1991:16) identified some challenges faced by twinned or clustered schools. Those challenges are learners' travelling costs from one school to another, work overload of

twinning teachers, vandalism by learners during twinning, exchanging of resources, changing of school culture etcetera. One of these challenges is the cost of travelling for the teacher from the one school to the other school in order to perform twinning activities. The time factor is another drawback in the twinning process for the staff members who have to plan meetings and discussions regarding their joint activities (Hill, 1993:8; Klein, 1988:19; Nachtigal, 1990:9). Bochar (1997: 30) cited the challenges of twinned schools as being learners' learning time when travelling from one school to another, staff development and supervision, school safety and liability, maintaining and enhancing community relations, fulfilling contractual obligations to educators and support staff, and working conditions. School twinning has repercussions for the teachers by giving them additional responsibilities and work-related stress (Rees, 2003:28). These drawbacks seem to have provided reasons for what has to be taken into account to make the twinning process successful.

3.3 SUBJECT CONTENT KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE WITHIN THE TWINNING CONTEXT

Shulman (1987:8) recommended seven domains that constitute a knowledge base for teachers, as indicated below:

General pedagogical content knowledge, with special reference to those broad principles and strategies of classroom management and organisation that appear to transcend the subject matter, knowledge of learners and their characteristics, knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and culture, knowledge of educational ends, purposes and values, and their philosophical and historical grounds, content knowledge, curriculum knowledge, with particular grasp of the materials and programmes that serves as 'tools of the trade' for teachers, pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding.

These knowledge domains were taken up seriously by other studies, such as those by Gess-Newsome (1999:4), Verloop, Van Driel and Meijer (2001:441). Many other studies have been conducted on teacher education, specifically with the emphasis on pedagogical content

knowledge (PCK). The content and pedagogy are seen as the dominant knowledge domains amongst the seven knowledge domains.

The seven knowledge domains identified by Shulman (1987:8) are broadly grouped into two kinds for mathematics teachers. One kind of domain is mainly subject-specific, which is required by teachers to teach mathematics effectively in the classroom (Tsang & Rowland, 2005:11). This knowledge domain is referred to as *content knowledge*. Extensive research has been conducted focusing on content knowledge (Faulkner & Cain, 2013:117; Shulman, 1986:8; Thanheiser, Browning, Moss, Watanabe & Garza-Kling, 2010:3). These studies on the content knowledge of the teachers focused on its relationship with effective teaching, according to Stylianides and Stylianides (2006:206). The other knowledge domain is not subject-specific, but generic in nature, in a way that it provides an opportunity for the teachers to function professionally in the field of education (Tsang & Rowland, 2005:11). This non-subject-specific knowledge domain is called pedagogical knowledge.

3.3.1 THE ROLE OF THE CONTENT KNOWLEDGE OF MATHEMATICS TEACHERS

Sepeng (2014:756) asserts that the effective teaching and learning of school mathematics requires that teachers have a good command of subject content knowledge. According to Hill (2010:515), a teacher's knowledge of mathematics is related to classroom teaching and is bound to influence the learners' academic achievement. The mathematics content knowledge of teachers is described as complex, and consists of many facets (Chick, Baker, Pham, & Cheng, 2006:298). Shulman (1988) defined *content knowledge* as "the amount and organisation of knowledge in the mind of the teacher" (p. 9). He (1988:9) further described *content knowledge* as that knowledge including facts and concepts. Turnunklu and Yesildere (2007:2) regard the content knowledge of mathematics as knowledge of mathematics, and the representational knowledge of mathematics in the classroom. It is accompanied by an understanding of what the teachers know, how much they know and what they should know (Leavit, 2008:38).

The teachers' content knowledge of mathematics is the thorough understanding of mathematics in depth, breadth, connectedness and thoroughness (Ma, 1999:124). Ma (1999:124) referred to the mathematical content knowledge of teachers as a profound understanding of fundamental mathematics. This type of knowledge is the one that mathematics teachers ought to have in order to teach the learners. Schoenfeld and Kilpatrick (2008:323) concur that the teachers ought to have mathematical content knowledge in order to teach learners. Livy and Vale (2011:23) also support the fact that proficient mathematics

teachers ought to display multiple methods, and a deep knowledge of mathematics. Two teachers' working collaboratively during twinning shared their experiences, and seemed have to developed their knowledge in teaching algebra. Livy and Vale (2011:40) furthermore said that proficient mathematics teachers ought to have knowledge of the curriculum, and to know how ideas develop from conceptual knowledge.

Ball, Bass and Hill (2004:52) see the teachers' mathematical content knowledge as specialised content knowledge (SCK). SCK is one of the sub-categories of mathematical content knowledge (Livy & Vale, 2011:23). In their model, Ball, Thames and Phelps (2008) identified the three sub-categories of content knowledge as common content knowledge (CCK), specialised content knowledge (SCK), and horizon content knowledge (HCK). They also indicated that, "common content knowledge is held by an adult who can use a method to solve a mathematical problem; whereas specialised content knowledge is mathematical content that is unique to teaching" (p. 399). According to these researchers, effective mathematics teachers need a better background of mathematical knowledge than the average adult in order to teach learners.

The Australian Association of Mathematics Teachers (2006:5) and Shulman (1986:14) have argued that good mathematics teachers ought to demonstrate the mathematics appropriate to the grade level. The teachers' knowledge of mathematics can be demonstrated in various ways (Livy & Vale, 2011:23). The teachers can use their knowledge in teaching the students how to identify a range of solutions and mathematical connections in planning lessons and in evaluating the students' work (Ball, Thames, Bass, Sleep, Lewis & Phelps, 2009:96; Ball et al., 2008:399; Chick, Baker, Pham & Cheng, 2006:140; Schoenfeld & Kilpatrick, 2008:323; Stylianides & Stylianides, 2006:206). Other researchers have argued that effective teaching can have a range of mathematical knowledge, such as procedural knowledge, procedural fluency, conceptual knowledge, and mathematical connections (Ball & Bass, 2003: 5; Kilpatrick et al., 2001:117; The National Curriculum Board, 2009:22).

Teachers with good content knowledge usually display an understanding of mathematical horizons whereby they can demonstrate the idea of mathematical connections (Livy & Vale, 2011:23). Mathematical ideas can be connected during teaching and learning by teachers and learners (Ball et al., 2009:96; Ball et al., 2008:399). Mathematical connections during teaching are important during twinning, when the two teachers share their expertise, experiences and mathematical knowledge so as to address the issue of poor performance in the non-performing school. Teachers who have a good understanding of mathematical horizons has peripheral vision and basic knowledge of the types of questions used to prompt

the students' understanding of mathematical proofs. Furthermore, they know when to assist learning, as well as when to be patient, thereby allowing the students to solve mathematical problems independently (Ball & Bass, 2009:18; Ball et al., 2009:96; Ball et al., 2008:399).

Ball et al. (2008:399) stated that those teachers who lack a sound knowledge of mathematics are unable to address the students' difficulties in learning the content. The ideas of Ball et al. (2009:399) helped this researcher become familiar with the situations in the non-performing school during the classroom observations in the first phase of this study. On the other hand, knowing the subject well may not be enough for teaching mathematical concepts. Teachers should make sense of their students' work and choose ways to make mathematical ideas more meaningful to them (Ball et al., 2008:399). The National Mathematics Advisory Panel Report (2008:9) indicated that elementary teachers of mathematics course qualifications cannot predict their students' achievement gains. What matters most is having mathematical knowledge, and to consider how to use it in a meaningful way in the field of mathematics teaching.

Livy and Vale (2011:27) conducted a study on the first year pre-service teachers' mathematical content knowledge of the methods of solutions to ratio questions. They explored two items about ratio from mathematical competency: skills and knowledge. In addition, their study also reported on pre-service teachers' thinking strategies, as well as common errors and misconceptions in their responses. The results suggested that pre-service teachers had difficulties in interpreting a worded, multi-step, ratio (scale) question, with errors relating to knowledge of ratio and/or the conversion of measurement. Errors and misconceptions in algebra could have been a major problem that can be developed from a teacher's teaching methods and needed attention to be addressed so as to improve the learners' performance. These challenges could be due to the teachers lacking sufficient knowledge of the structure of mathematics and mathematical connections. Moreover, the teachers appeared to struggle with deconstructing the key components of mathematical problems.

3.3.2 THE ROLE OF PEDAGIGICAL CONTENT KNOWLEDGE

Pedagogical content knowledge (PCK) is one of the knowledge domains introduced and identified by Shulman (1986: 14, 1987:9) as necessary for teachers to possess. Extensive research was conducted on PCK (e.g., by Kleickmann, Richter, Kunter, Elsner, Besser, Krauss & Baumert, 2013:91; Harr, Eichler & Renkl, 2014:6; and Kwong, Joseph, Eric & Khoh, 2007:32). The debate about PCK has long existed and continues into the 21st century (Ball et al., 2008:399; Blömeke & Delaney, 2012:2; Shulman, 1986:14; Kaino & Moalosi,

2013:189). According to Ball et al. (2008:399), this knowledge domain is regarded as being highly influential in mathematics. It requires teachers to be competent in PCK in order to enhance their quality of teaching and learner performance in mathematics. This knowledge domain has aided policy-makers, researchers and curriculum coordinators (Margerum-Leys & Marx, 2004:426; Shulman, 1986: 4; Shulman, 1997:15; Veal & Makinster, 1999:18).

PCK is described as the type of knowledge required by teachers to teach the subject, the knowledge of understanding learners' ways of thinking, the teachers' ability to diagnose the learners' errors and the sources of those errors, and knowledge of various alternative ways of representing specific topics (Shulman, 1986:16; Tirosh, Even & Robinson, 1998:55). PCK has also been described as the knowledge of what made the learning of specific topics easy or difficult, and as the knowledge that comprised subject matter knowledge and pedagogical knowledge (Shuman, 2007:6). Other studies described PCK as the subject of knowledge to be taught, knowledge of different teaching strategies and assessment strategies and context knowledge, which includes knowledge about the background of the learners, the organisational culture of the school, etc. (Banks, Leach & Moon, 2005:333; Zeidler, Walker, Ackett & Simmons, 2002:344). Veal and MaKinster (1999:18) described PCK as the way in which expert teachers describe the knowledge they possess.

Shulman (2007:14) further described PCK as the knowledge of explaining and representing, and the importance of knowledge of subject-related learner cognitions (conceptions, preconceptions, misconceptions). Some studies documented that pre-service and in-service teachers often have misconceptions and fragmented knowledge that limit them in their response to their learners' conceptions, or possibly their ability to create cognitively challenging concepts (Van Driel & Berry, 2010:26).

Turnuklu and Yesildere (2007:2) relate PCK to the knowledge of teachers and learners. Other studies, such as those by Zeidler et al. (2002:344), and Banks et al. (2005:333) view PCK as the knowledge of the subject that needed to be taught, general pedagogical content knowledge (knowledge of teaching strategies, classroom management strategies, assessment strategies, etc.), and content knowledge (knowing the background of the learners and knowing the organisational culture of the school). Additionally, PCK refers to the ability of the teachers to transform content into forms that are pedagogically powerful and yet adaptive to variations in the ability and background mathematical knowledge presented by the learners (Shulman, 1987:5).

An, Kulm and Wu (2004:345) have identified the three main categories of PCK as knowledge of content, knowledge of the curriculum, and knowledge of teaching. The three categories are very important in the teaching and learning of mathematics, and hence of algebra in this study. During the observations that were conducted in the two schools, these three categories were necessary to assess how well those teachers articulated them in teaching Algebra. After an extensive literature review, Bukova-Güzel (2010:1875) developed a comprehensive PCK framework, consisting of the three main categories and their components, as shown in Table 3.1.

Table 2.1: Framework of PCK (adapted from Bukova-Güzel, 2010:1875)

Knowledge of teaching strategies and multiple representations	Knowledge of the learner	Knowledge of the curriculum
- Using appropriate activities in instruction	- Having knowledge of the learners' prior knowledge	- Being aware of the elements of the mathematics curriculum (its conceptions, purposes, etc.)
- Using real-life examples and analogies in instruction	- Having knowledge of the possible difficulties the learners may experience during learning	- Being aware of the variety of instructional tools presented in the mathematics curriculum and how to use them
- Utilising different instructional strategies in the presentations	- Having knowledge of possible learner misconceptions	- Being aware of the instruments to assess the learners' learning and how to use them
- Making use of different representations in instruction (graphics, tables, formulas, etc.)	- Having knowledge of learner differences	- Having both horizontal and vertical programme knowledge of a topic

This framework seemed to address issues of the first phase, where each teacher was observed individually, and later in the second phase, where the observation focused on the twinning process. This type of knowledge helped the researcher to gain an understanding of the teachers' PCK in the two schools, and also of the knowledge that the teachers possess in order to be able to teach the learners algebra. Research has previously been conducted to explore in-service and the pre-service Mathematics teachers' PCK (Botha & Reddy, 2011:257; Kaino & Moalosi, 2013:189; Livy & Vale, 2011:27; Marshman & Porter, 2013:474; Ramma, Bhoola, Watts & Ramasawmy, 2014:30). The researchers explored Mathematics teachers' PCK by analysing how the teachers taught mathematics, and their responses to the questions asked by the learners on the same topic. Their findings showed that the teachers did not have adequate PCK by means of which to understand the characteristics of the problems tackled, of the solutions obtained by different methods, and a lack of understanding of the learners' problems. Some researchers found that the pre-service teachers are good in certain knowledge domains, and less knowledgeable in others (Botha & Reddy, 2011:257). Ramma et al. (2014:34) found that there was no distinction between the pre-service and the in-service teachers' mathematical knowledge, where all of them displayed difficulties in demonstrating CK and PCK.

On the other hand, Chick (2011:10) explored the primary teachers' PCK, using the framework of PCK to underpin their study. Questionnaires and interviews were used as instruments to measure the primary school teachers' PCK about mathematics and mathematics teaching. The findings of the study differed in many ways, including the connectedness of the knowledge and the specificity with which they discussed the mathematics involved. In this study, the PCK appeared to be a problem for the teachers, and more attention was required for those teachers to be developed in this knowledge domain.

Turnuklu and Yesildere (2007:2) determined the pre-service primary mathematics teachers' competency of PCK in the subject. The teachers who participated in the study were given four open-ended questions by means of which to measure their competency in PCK. Pre-determined criteria were used to analyse the findings. It was found that having a thorough understanding of mathematics does not guarantee good teaching. Turnuklu and Yesildere (2007:2) suggested that teachers of mathematics be educated, both in respect of mathematical knowledge, and PCK.

A study by Kwong et al. (2007:27) also focused on pre-service teachers' PCK in mathematics. They developed a 16-item instrument to measure the understanding of the knowledge domains of PCK in mathematics of the pre-service teachers. The instrument was administered to the Postgraduate Diploma in Education for pre-service teachers. The performances of the pre-service teachers were analysed, and it was found that their PCK was generally weak in mathematics, and needed improvement. Van Driel and Berry (2010:26) supported the fact that the pre-service teachers do really lack PCK. These studies revealed that either the pre-service or the in-service teachers possess a low PCK. The teachers' lack of PCK could have had an impact on the teacher's mathematical knowledge in the low-performing school.

During the late 1980s, Shulman created activities for teachers to learn good teaching practices (Shulman, 1986a:3). These activities were comprehension, transformation, instruction, evaluation, and reflection of mathematical knowledge, and finally, mathematical knowledge constructed by learners. The activity of comprehension deals with the teachers' understanding of the purpose of the subject, the structures of knowledge, and ideas within that particular subject. Moreover, the teachers need to understand what they teach, when and how to teach mathematics, using different methods. The teachers' comprehension of the subject helps the learners to understand the subject, enabling them to use different methods to solve mathematical problems and enjoy their learning experiences in the Mathematics classroom.

Transformation is the second activity in which the teachers ought to have an understanding to convert the content knowledge into pedagogical knowledge, which can contribute towards the learners' abilities and backgrounds in learning mathematics (Shulman, 1992:15). Transformation requires a combination of the preparation of the text materials, the representation of the teachers' ideas in the form of new analogies and metaphors, and of presenting the materials. Shulman (1986a:7, 1986b:6) stated that teaching instructions comprise teacher activities such as classroom management, lessons presentations, interaction with the learners through questioning, forming and observing group-work, and maintaining classroom discipline.

Evaluation is another stage of pedagogical reasoning amongst Shulman's activities. It is the extension of the teaching instructions where the teachers assess their learners. The learners, in this process, are assessed by checking their level of understanding concepts and the challenges they encounter in the learning of mathematics. Evaluation enables the teachers to examine and judge their own performance and adjust their teaching methods accordingly (Shulman, 1986a:7, 1986b:6).

Another important stage of the teaching cycle is reflection, which is often neglected, according to Shulman (1992b:6). The teachers can review their teaching strategies used construct new knowledge, enact new knowledge, and critically analyse their own teaching. Reflection is important to learners as well, as they can reconstruct, re-enact and recapture classroom events and accomplishments (Ornstein, Thomas & Lasley, 2000:14).

3.3.3 BEST PRACTICE WITHIN THE TWINNING CONTEXT

Teaching instructions should be effective so as to cater for all learners, including those with learning difficulties in mathematics. Extensive research has been conducted on effective teaching instructions for learners with learning difficulties (e.g. Adams & Carnine, 2003:404; The National Mathematics Advisory Panel Report, 2008:8). For the purposes of this study, this researcher will discuss the teaching instructions relevant to the phenomenon under investigation.

3.3.3.1 EXPLICIT AND SYSTEMATIC INSTRUCTIONS

Explicit instructions refer to the instructional practice that constructs interactions between the teacher and the learner (Steedly, Dragoo, Arafeh & Luke, 2008:4). In other words, the teachers are expected to state their teaching objectives to the learners so as to understand the outcomes of lessons and follow the defined instructional sequence (Steedly et al., 2008:4). The prior knowledge of the learners is also essential in order to tailor subsequent teaching instructions. Kroesbergen and Van Luit (2003:98) asserted that explicit instructions have been found to be successful when a learner has difficulties in learning a specific or isolated skill.

These scholars from a study by Kroesbergen et al. (2003:99) revealing that small group discussions provide support to learners with difficulties. Moreover, the researchers recommended that the teachers have to provide the learners with the learning objectives of the lessons and give the necessary remediation when learners make mistakes using small group discussion.

Anthony and Walshaw (2009:150) regard the classroom community as having an ethic of care. The researchers argued that classroom community should focus on a mathematical goal that helps the learners to develop their mathematical identities and proficiencies. Noddings (1995:695) asserts that effective teachers facilitate learning by truly caring about their learners' engagement. The teachers should develop an interrelationship with the learners that could create opportunities for the learners to enhance their mathematical identities.

Boaler (2008:167) and Ingram (2008:282) reported that most learners want to learn in small groups classroom environment. Angier and Pavey (1999:147) argue that effective teaching has to promote relationships that allow the learners to think for themselves, to ask questions, and to take intellectual risks. This type of relationship promotes discussion that is used as a tool to increase reasoning that has gained emphasis in classrooms worldwide (for example, Yore, Bisanz & Hand, 2003:689).

Explicit instruction is also effective when communication is promoted among learners. Classroom communication is essential in developing dialogue that focuses on mathematical argumentation. Walshaw and Antony (2008:152) have asserted that teaching ways of communicating mathematically demands skilful work on the teacher's part. Hunter (2005:415) indicated that teachers ought to use explicit strategies in teaching mathematics, such as telling the learners how to communicate during teaching and learning process.

Forman and Ansell (2001:114) argue that teachers can also use a re-voicing technique in mathematics communication; they repeat, rephrase or expand on the learners' discussions. Anthony and Walshaw (2009:152) asserted that teachers can use the re-voicing technique in many ways, namely:

- to highlight ideas that have come directly from the learners;
- to help the development of the learners' understandings implicit in those ideas; and
- to negotiate meaning with their learners, and to add new ideas or move to the next development of learning.

Re-voicing is seen as a useful technique in the teaching and learning of mathematics, as the learners can share the meaning of concepts during the process. This re-voicing assists learning, by accommodating new knowledge, and also by making meaning of what they will be discussing.

Hall (2002:12) cites that the Centre for Applied Technology (CAST) offered a helpful snapshot of an explicit instructional delivery component episode, namely communication between the teacher and learners, that ought to be consistent in creating a foundation for the instructional process. The instructional episodes of lessons involve pacing a lesson appropriately, allowing adequate processing and feedback time, encouraging frequent learners' responses, and listening and monitoring throughout the lessons.

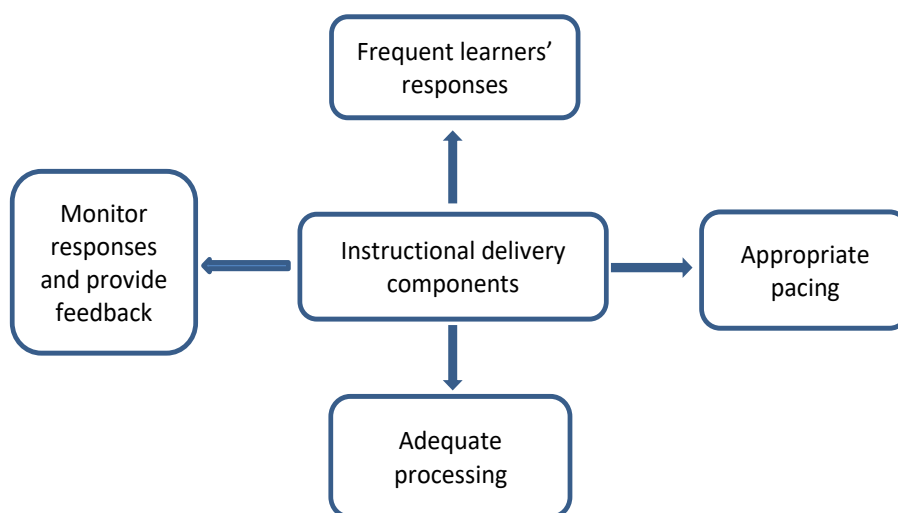


Figure 3.1: Standard instructional delivery components essential to all explicit instructional episodes (Hall, 2002:12)

Systematic instruction works jointly with explicit instruction in the teaching and learning of mathematics. *Systematic instruction* refers to strategies the learners learn that help them integrate new information with what is already known, in a way that makes sense and enables the learner to recall the information or skills later, even in a different situation (Steedly et al., 2008:4). The National Mathematics Advisory Panel (2008:8) found that this instruction is effective primarily in computation (i.e. basic mathematical operations, but not as effective for higher-order problem-solving skills). Systematic instruction has a great impact when combined with explicit instruction, through the breaking-down of steps, working with small groups, questioning learners directly, and promoting on-going practice and feedback amongst the learners.

3.3.3.2 COOPERATIVE LEARNING IN THE TWINNING CONTEXT

Cooperative learning is another approach that was seen to be effective in classroom teaching and was included in the twinning process. Miheso (2012:81) and Opolot-Okurot (2005:168) argue that poor performance in mathematics is due to inadequate teaching and learning, the negative attitude of teachers and learners, and ineffective teaching methods. This study sought to use cooperative learning as one of the approaches used in the literature for guidance, in order to equip the teacher to understand how to use it in the non-performing school during the twinning process.

Cooperative learning is a teaching strategy where a small number of learners with different abilities (heterogeneous group), who use different learning activities in order to understand the subject matter, are grouped together (David & Rodger, 2002:100). Naomi and Githua (2012:177) argue that during cooperative learning, each member of a group ought to have a

responsibility to assist others to learn, which provides an atmosphere of achievement. Zacharia and Iksan (2007:36) concurred that this teaching strategy is grounded in the belief that learning is effective when the learners share ideas and work together in a group to complete a given mathematical task. Furthermore, this approach encourages the interaction of learners and communication with others in a group in harmony, by respecting each other's work (Zacharia, Solfitri, Daud & Abidin, 2013:98).

Cooperative learning requires from the teacher to have a good coterie of mathematical activities in order for it to be successful. It also has a number of essential elements or requirements (Cohen, 1992:10), namely: a clear set of the specific learners' learning objectives; a clear and complete set of task-completion direction such as giving examples or instructions; heterogeneous groups; equal opportunity for success; positive interdependence; positive social interaction behaviours and attitude of learners; opportunities to complete the required information-processing tasks; sufficient time spent during learning; individual accountability; public recognition and rewards for the group's academic success; and post-group reflection (or debriefing) on group behaviours.

Johnson, Johnson and Holubec (1994:36) summarised the five elements of this teaching strategy. These elements are briefly discussed below in order to give a broader understanding of each element and to know how effectively they could be used in the process. These elements include:

- Positive interdependence: the success of one learner is dependent on the success of the other learner. The learners learn more when they share knowledge in a group.
- Promotive interactive: individuals can achieve promotive interaction by helping each other, exchanging resources, challenging each other's conclusions, providing feedback, and encouraging and striving for mutual benefits.
- Individual accountability: the teachers should assess the amount of effort that each member is contributing. This could be done by giving an individual test to each learner and randomly calling learners to present their group's work.
- Interpersonal and small-group skills: the teachers are required to provide opportunities for the group members to get to know each other, accept and support each other, communicate accurately, and resolve differences constructively.
- Group-processing: the teachers are also required to provide the learners with opportunities to assess the group's progress. Group-processing enables the groups

to focus on good working relationships and to facilitate the learning of cooperative skills, and ensures that members receive constructive feedback.

This teaching strategy essentially represents a shift in the educational paradigm from the conventional way of teaching to learner-centred teaching in small groups. Effandi (2005:63) agreed that this teaching strategy can create an excellent opportunity for the learners to engage in problem-solving with the assistance of their peers in their respective groups.

Extensive research has been conducted on cooperative learning that supports problem solving abilities of learners, which has a positive impact on learner achievement (e.g., Naomi & Githua, 2012:178; Zakaria et al., 2013:99; Zakaria & Iksan, 2007:36). These researchers examined the findings of the effect of using cooperative learning in the mathematics classroom, and found that this teaching strategy promotes the learners' academic achievement in the subject. Zakaria, Chin and Daud (2010:272) summarised that cooperative learning has a positive impact in improving the learners' academic achievements in mathematics. Shimazoe and Aldrich (2010:52) have argued that this strategy promotes the learning of materials, and helps the learners to achieve better grades. This strategy provided benefits not only to the learners, but also to the teachers. Shimazoe and Aldrich (2010:53) focused on three models of group information and development, the components of a successful group process, the way in which these components respond to typical student challenges, and the instructors' roles in groups, and how these roles can best be carried out.

They recommended that instructors ought to explain the rationale for cooperative learning on the first day of class and tell the learners what to expect. In other words, the teachers have to tell the learners about the chosen method, rather than about other teaching methods. Cooperative learning can be used as instructional variations in the teaching and learning of mathematics. Those instructional variations are Student Teams-Achievement Divisions (STAD), Teams-Games-Tournament (TGT), and jigsaw (Slavin, 1995:36). In STAD the learners are grouped according to their heterogeneity (sex and ethnicity) (Zakaria & Iksan, 2007:37). In TGT, quizzes are replaced by tournaments, and the learners compete against learners from other teams, who are equal to them in terms of past performance (Awofala, Fatade & Ola-Oluwa, 2012:7; Zakaria & Iksan, 2007:37). The jigsaw method allows learners to be responsible for teaching each other (Slavin, 1995:38).

Nunnery (1997:11) investigated the effects of cooperative learning using the STAD method on learner achievement and attitude in mathematics. Two different groups were used. Nunnery (1997:11) used quasi-experimental design in the study on the experimental and

control groups, and the results revealed that the experimental group performed significantly better than the control group. These studies suggested that teachers can use STAD/TGT as variants of cooperative learning, to complement the teaching of mathematics in secondary schools. Awofala et al. (2012:8) concurred that this cooperative learning and individualistic goal structure had a significant positive impact on junior secondary mathematics. Instructional variations were used, which are STAD/TGT variants in cooperative learning. The study found a significant gain difference in learner performance between the cooperative and individualistic goal-structured groups in favour of the cooperative group.

Nunnery (1997:44) argued in their study that STAD/TGT is useful when the teachers require from the learners to focus on clearly defined skills and content material. Heterogeneity should be maintained by including different groups (racial/ethnic), and both sexes for STAD and TGT, to be effective in teaching and learning mathematics (Sherman & Thomas, 1986:169). The study found that cooperative learning, using these two variants, had a positive gain on learners' performances.

Unlike STAD and TGT in cooperative learning, the jigsaw method aimed to include all the fast, medium and slow learners when using cooperative learning. Jigsaw is one of the instructional variations of cooperative learning, as suggested earlier. It is an instructional variation in which each learner is assigned one aspect of the learning area that he or she specialises to demonstrate either into an expert group or home group (Naomi & Guthua, 2013:182; Zakaria & Iksan, 2007:38). Experts of the same group meet and discuss the various aspects of their learning unit. Each expert in the learning unit returns to the home group to teach it (Zakaria & Iksan, 2007:38). Neer (1987:154) also found that learners who tutor each other develop a clear idea of the concept they are presenting, and are orally communicating to their partners. These researchers, Zakaria and Iksan (2007:38) and Naomi and Guthua (2013:182), agree that the jigsaw learning strategy returns a positive gain on learner performance if used effectively in the teaching and learning of mathematics.

Hänze and Berger (2007:34) discussed the impact of the jigsaw cooperative learning strategy. They found that the jigsaw strategy was effective when used in classroom teaching, as it improved the learners' academic achievements in mathematics, compared to that of the learners who were taught with the conventional teaching methods. Burns (1984:18) supported Hänze and Berger (2007:34), noting that the jigsaw cooperative strategy produced better academic achievements, as learners were engaged in challenging tasks in their expert groups. Baird and White (1984:32) assert that the jigsaw cooperative learning strategy makes learning interesting, is highly interactive, that the learners learn actively, and that those learners

involved in the cooperative learning are encouraged to take the responsibility for their own learning.

Moreover, Naomi and Githua (2012:177) discussed the effects of the jigsaw cooperative learning strategy on the learners' academic achievements in secondary school mathematics. They (2012:177) found that learners who were taught by means of this strategy performed significantly better than those learners who were taught using the conventional teaching/learning method.

Cooperative learning enhances the social interaction of learners to meet their needs and to maintain trust among themselves during the teaching and learning of mathematics (Goodwin, 1999:29; Slavin, Leavy & Madden, 1989:44). The learners who are in social interaction in this regard are expected to work with enthusiasm in their expert groups where they solve one common mathematical problem so they could report back to their home group, where every learner has to report his or her problem in the previous group (Goodwin, 1999:29). The learners involved in cooperative learning are expected to assist one another, for all the learners to master the concepts learnt in their expert groups. The high, medium, and low achievers worked cooperatively, as their activities required them to work as a team.

3.3.3.3 PROBLEM-SOLVING IN THE TWINNING CONTEXT

Problem-solving is another approach seen to be important in the twinning process during which the two teachers share their ideas and experiences in teaching algebra. Much research has been done on the problem-solving approach in mathematics, and much of the research attempts to clarify the meaning of this approach to the teaching of mathematics (e.g., Sepeng, 2010:217). The emphasis in this approach has been shifted from teaching problem-solving to teaching via problem-solving (Lester, Masingila, Mau, Lambdin, Dos Santon & Raymond, 1994:156; Schoenfeld, 2014:1). The Principles and Standards for School Mathematics is one of these documents (National Council of the Teachers of Mathematics, 2000:52). The use of this approach to teaching would seem to be beneficial to the teacher and learners alike, when used in conjunction with an explicit and systematic approach and cooperative learning.

Problem-solving has been defined by many researchers in mathematics. NCTM (2000) defined it as follows, "problem-solving is an integral part of all learning, and so it should not be an isolated part of the mathematics problem" (p. 52). Schoenfeld (1992:340) refers to *problem-solving* as a process where the learners encounter a problem which does not have an immediate solution or an algorithm to the direct answer. Kantowski (1977:162), and Lester (1980: 286) give as a common element of problem-solving that there is no known algorithm

to solve a given problem. Kilpatrick (1985:4) refers to *problem-solving* as an initial goal in a problem situation. Polya (1957: 8) the ‘father’ of problem-solving, does not refer explicitly to the goals, but identified two categories of problem:

- problems to find, the principal parts of which are unknown, the data, and the condition; and
- problems to prove, which comprise a hypothesis and a conclusion.

These definitions provided by the researchers refer to the same concept to be used in opening avenues for a broader understanding of this approach, in order to know how effectively it could be used in cooperation with learners, when instructions are explicit and systematic.

Lester et al. (1994:155) characterised *problem-solving* to be when the teacher helps the learners in solving mathematical problems either by creating, conjecturing, exploring, testing and verifying. Some research outlines specific characteristics of the problem-solving approach, which include:

- interaction between students/students and teacher/students (Van Zoest, Jones & Thornton, 1994:37);
- mathematical dialogue and consensus between the students (ibid. 1994:37);
- the teachers providing just enough information to establish the background/intent of the problem, and the learners clarifying, interpreting, and attempting to construct one or more solution processes (Cobb, Wood & Yackel, 1991:161);
- the teachers accepting right/wrong answers in a non-evaluative way (ibid., 1991:161);
- the teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems (Lester et al., 1994:155);
- the teachers knowing when it is appropriate to intervene, and when to step back and let the learners make their own way (ibid., 1994:155); and
- a further characteristic is that a problem-solving approach can be used to encourage the learners to make generalisations about the rules and concepts – a process central to mathematics (Evan & Lappin, 1994:131).

This approach enables the learners to reorganise their existing ideas, as well as the new ones that emerged as they work in their small groups with the assistance of the teacher, who asks questions that enable the learners to review their knowledge and construct new connections.

This simplified process was first summarised in the book of Polya (1957:12), and has since inspired much research.

Several research studies have been conducted on the problem-solving approach which found that it had a positive effect when learners worked in small groups in solving mathematical problems. For example, Sepeng and Webb (2012:61) explored whether the learners' discussions as a teaching strategy in mathematics could assist in improving their problem-solving performance, as well as in making sense of real-world problems. Their study used a pre-test intervention or a post-test mixed-methods design, which delivered qualitative and quantitative data. Two different groups were used in the study, namely an experimental and a control group. The data generated from the study suggested that the intervention significantly improved the experimental groups' problem-solving skills, and sense-making performance. The discussion technique seemed to have been successfully implemented, as the learners' problem-solving skills were improved through the results of the post-test in the experimental group, and could also assist during the twinning process.

Hardin (2002: 12) argued that problem-solving has a positive effect when the learners learn in small groups. Hardin (2002:12) described prior findings regarding the expert-novice teachers' differences in solving problems of various kinds. The study analysed chess players in which mathematical skills were required to engage in the game. The expert teachers, which have an experience in teaching mathematics, were able to group the relevant information, while the novice teachers envisioned single 'pieces' of information (Bruner, 1983:9).

However, developing successful problem-solvers is a complex task, that should allow for a range of skills and dispositions (Stacey, 2005:342). The mathematics learners have to display knowledge of mathematics and a general reasoning ability as well as heuristic strategies for solving non-routine problems. Problem-solving skills are effective if they are coupled with good communication skills and the ability to work in cooperative groups.

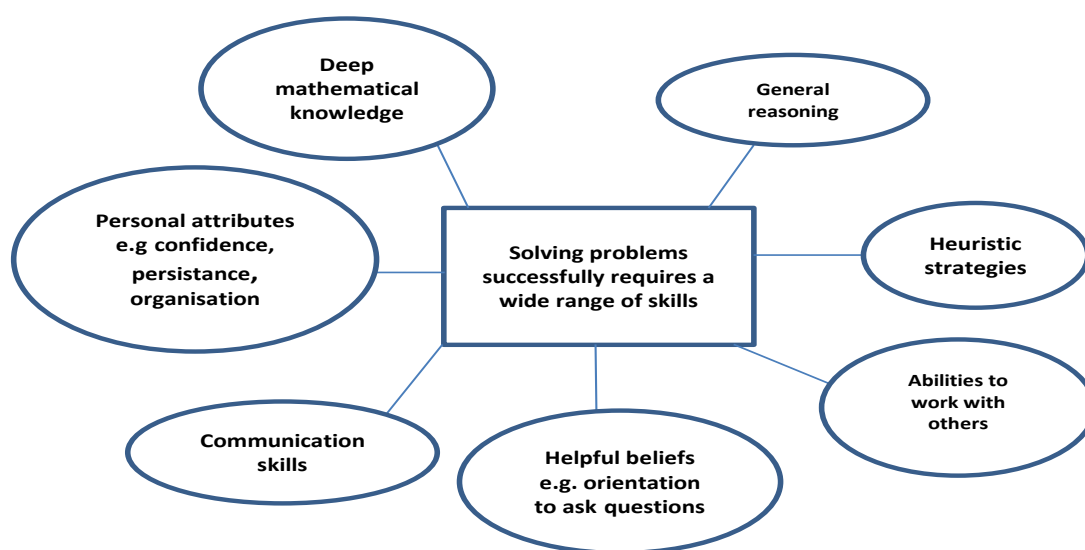


Figure 3.2: Factors contributing to successful problem-solving (Stacey, 2005:342)

Problem-solving is recognised as an important skill involving a range of processes that include analysing, interpreting, reasoning, predicting, evaluating and reflecting (Anderson, 2009:3). It can either be an overarching goal or a fundamental component of the school mathematics curriculum. Cai (2003:241) has argued that teachers have the opportunities to build knowledge about teaching problem-solving and using problems as a focus of learning in mathematics. The pre-service and in-service programmes were put in place in order to change teaching practices from the more traditional approaches to contemporary or reform methods, where the teachers use non-routine problems and problem-centred tasks (Anderson & Bobis, 2005:65).

3.3.3.4 VISUAL REPRESENTATION IN THE TWINNING CONTEXT

Visual representation can be useful if incorporated as another strategy to be used with the other three strategies discussed earlier on, namely explicit and systematic instruction, cooperative learning and problem-solving. This strategy received increasing attention when it was added to a new standard of education (NCTM, 2000:52). Visualisation has a long tradition in mathematics and there is a long list of mathematics teachers using or explicitly advocating for it (Rosken & Rolka, 2006:8).

Visual representation has a role to play in mathematics learning and has been the subject of much research (e.g. English, 1997:77; Staliyanon & Silver, 2004:353). Arcavi (2003) defined *visualisation* as “... the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams [sic], in our minds, on paper or with

technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding” (p. 217). Zimmerman and Cunningham (1991:1) defined *visualisation* as a skill, a product and a way of creativity and interpretation, a reflection of a diagram [sic], depictions, and images in our minds. Some research debated whether the learning of mathematics is aided by visual or pictorial images in real life (e.g., Ball & Ball, 2007:4; Hadamard, 1973:10; Naidoo, 2007:8; Singh, 2007:10). Mudaly (2010:68) averred, in a study constructing knowledge using diagrammes, that it is common knowledge that a large part of communication in our daily lives depends on visual imagery.

Rosken and Rolka (2006:8) emphasise the fact that visualisation is a powerful tool to explore mathematical problems and to give meaning to mathematical concepts and the relationships between them. Furthermore, they argued that this strategy of using diagrammes reduces the complexity of mathematical problems when dealing with a multiplicity of information. Rosken and Rolka (2006:9) found visualisation to be important in problem-solving, where it is seen as a useful tool for working on mathematical problems. Mudaly and Rampersad (2010:39) argued that visualisation assists the learners in seeing with the mind’s eye, in other words, the learners’ minds will be capable of creating new information or the transformation of old knowledge.

Guler and Ciltas (2011:148) studied the relationship between the visual representation levels of mathematics teachers and learners, in solving verbal problems. A Mathematical Processing Instrument was used to determine the relationship in their study, together with a Likert-type test, which consisted of 14 items. Guler and Ciltas (2011:148) found that the relationship between the visual representation usage level of the teachers and learners was positive when solving problems. Moreover, those learners who used visual representations were frequently more successful at problem-solving and had positive beliefs about the use of visual representations in solving mathematical problems. Mudaly (2010:68) used geometry software as a visual tool to develop analogical reasoning, which had a positive gain in learners’ performance.

Lowrie and Clement (2001:7) explored the influence of visual representations in mathematical problem-solving and numeracy performance. His study examined the relationships between different forms of problem-representations and learner performance on a number of mathematical measures, including problem-solving, spatial ability and numeracy sense. The test was also used to assess the learners’ performances while solving mathematical problems in a visual or non-visual mode. The results revealed that those learners who solved

mathematical problems using diagrammes performed significantly better on these measures. Linneweber-Lammerskitten, Schäfer and Samson (2010:29) investigated the application of cell phone technology in the teaching and learning of mathematics, which they found had a positive outcome, mainly for the teachers in the rural settings, where the access to mathematics resources is very limited.

Cai and Lester (2005:225), and Cai (2004:136), revealed that the teachers' use of visual representations bring to light important ideas about which representations ought to be used and how they can be used effectively in the teaching and learning of mathematics. In some cases, the teachers used symbolic representations, while on the other hand, other teachers used verbal explanations and pictorial representations, according to Cai and Lester (2005:225). Cai and Lester (2005:255) asserted that the visual representations that the teachers use in teaching mathematics influence the representations that the learners choose to use in problem-solving. It was found that the learners who used symbolic representations produced better results in terms of performance when compared with those who used pictorial representations.

A number of studies made use of schematic spatial representations. It was found that schematic spatial representations were associated with positive results in terms of performance in mathematical problem-solving, whereas pictorial representations were not (e.g. Hegarty & Kozhevnikor, 1999:684; Van Garderen & Montaque, 2003:246). Hegarty and Kozhevnikor (1999:684) distinguished two types of visual-spatial representations when learners solve mathematical problem.

Those types of visual-spatial representations were schematic representations that encode the spatial relations described in the problem, and pictorial representations, which encoded persons, places or things described in the problem. The two visual-spatial representations revealed that schematic representations and pictorial representations had differing impact, in terms of the learners' academic achievements. Schematic representations appeared to have been effective in the learners' academic achievements in comparison with pictorial representations. Van Garderen and Montaque (2003:246) argued that schematic representations was significantly correlated with the better performance of the learners when solving mathematical problems.

However, the limitations and difficulties in respect of visualisation, and even the learners' reluctance to visualise, have also been indicated (Arcavi, 2003:238; Stylianou & Silver, 2004:353). Mudaly and Rampersad (2010:39) indicated in their study that the learners

displayed weak visual representations in solving problems, and depended on the diagrammes the teachers used in the classroom. Mudaly and Rampersad (2010:39) explored the role of visualisation in the learners' conceptual understanding of graphical functional relationships of problems. Visualisations of graphs were interpreted by the learners when solving problems, and explored the relationship between graphs and the learners' understanding of those graphs. In their study it was found that the learners displayed a weak visual understanding, and were dependent on the diagrammes the teachers used in the classroom.

3.4 THE TRANSFER OF KNOWLEDGE DURING TEACHING PRACTICE

The knowledge of mathematics is often difficult to acquire (Kaminski, Sloutski & Heekler, 2009:151). This knowledge is acquired through the transfer of information, and depends on how it is imparted during classroom practice. The transfer of knowledge is a multifaceted problem which is at the heart of learning. Mathematics teachers use different approaches to transfer the knowledge during classroom teaching (Martin, 2009:4; McNeil & Uttal, 2009:137; Kaminski et al., 2009:151). These approaches are used in classroom teaching in order for the learners to become proficient in the subject. These proficiencies were discussed by Kilpatrick, Swafford and Findell (2001:116), who indicated five strands of mathematical proficiency.

These strands will be discussed in this chapter, as to how they play a role during the twinning process. These strands are not independent, but represent different aspects of a whole (Kilpatrick et al., 2001:116).

They are interwoven and interdependent in the development of proficiency in mathematics, which appeared to be important in the teaching and learning of algebra during the twinning process.

The five strands are indicated in the figure below (Kilpatrick et al., 2001:116).

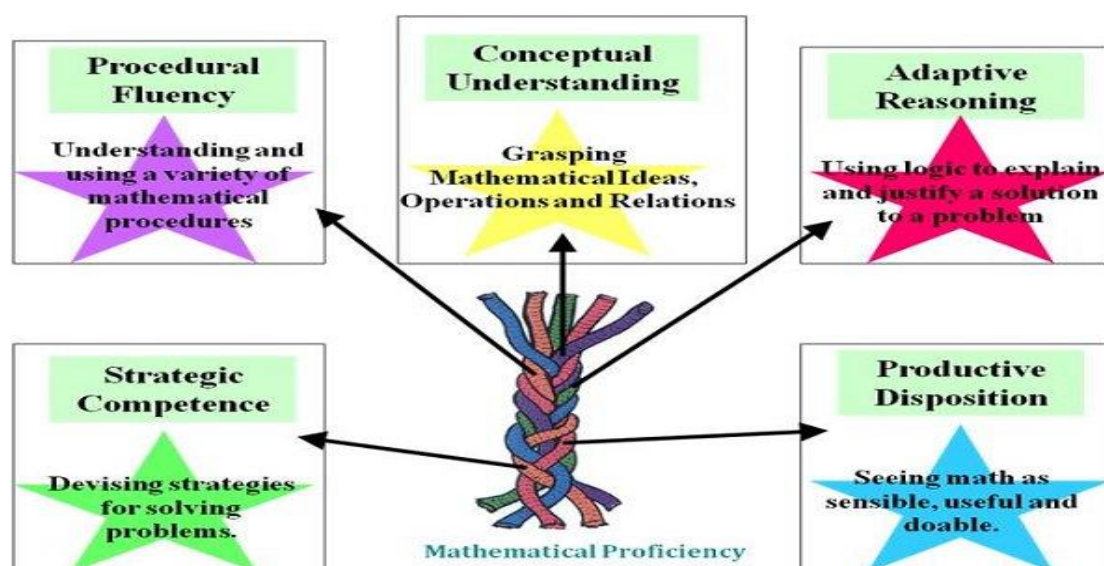


Figure 3.3: Adding up describes five strands of mathematical proficiency

The five strands are interwoven and interconnected (National Council of the Teachers of Mathematics, 2001:79). They provide a framework for discussing the knowledge, skills, abilities, and beliefs that constitute proficiency in mathematics. They must all work together for the learners to be mathematically proficient. It is imperative for the teachers to reflect on these interrelated components in the teaching and learning of mathematics.

The abovementioned strands are discussed in order to outline their importance in the teaching and learning of mathematics.

3.4.1 CONCEPTUAL UNDERSTANDING IN THE TWINNING CONTEXT

Conceptual understanding is described as a learner's comprehension of mathematical concepts, operations and relations (Kilpatrick et al., 2001:116; Watson & Sullivan, 2008:110). Skemp (1976:22) argued that it is not enough for the learners to merely understand how to perform mathematical tasks (instrumental understanding), but also to know why a particular task is performed (relational understanding). Bransford et al. (1999:70) and Carpenter and Lehrer (1999:25) argue that learners who are taught how to develop a conceptual understanding can organise knowledge into a coherent whole, and connect new and old ideas. In this regard the learners would be able to represent their mathematical situations in different ways and know how those representations can be useful for different purposes. The learners could therefore connect mathematical ideas to various representations, and see their similarities and differences in solving algebraic problems, and hence mathematical problems.

Kilpatrick et al. (2001) added that, “connections are most useful when they link related concepts and methods in appropriate ways” (p. 141). This is unlike mnemonic techniques, that are learned through rote learning (Hiebert & Wearne, 1986:201; Kilpatrick, 1985:4), and which allows the learners to make connections to perform mathematical operations, but does not promote a better understanding of concepts. Kilpatrick et al. (2001:141) argue that these kinds of connections are not in a better position to promote the acquisition of mathematical proficiency. Skemp (1986:26) supported Kilpatrick et al. (2001: 118) on mathematical connections that when well-constructed, knowledge is interconnected, so then, when one network of ideas is recalled, it may lead to the another network of ideas to be recalled. In contrast, learners who acquire knowledge with this understanding can generate new knowledge and can solve new and unfamiliar problems (Bransford et al., 1999:71). It would also appear to the learners that they can connect concepts and procedures and can justify their mathematical ideas, facts and methods. Kilpatrick et al. (2001:118) further argued that those learners who have an understanding of concepts avoid critical errors in solving mathematical problems.

3.4.2 PROCEDURAL FLUENCY IN THE TWINNING CONTEXT

Kilpatrick et al. (2001:121) and Watson and Sullivan (2008:115) refer to *procedural fluency* as the knowledge of procedures, of when and how to use them appropriately, and skills in performing them flexibly, accurately, and efficiently. Watson and Sullivan (2008:115) preferred the term ‘mathematical fluency’. Pegg (2010:35) presented a clear and cogent argument for the importance of developing fluency in the learners. Pegg (2010:36) explained that initial processing of information in a working memory, which is of limited capacity, can cause learners to use incorrect procedures to solve problems. Pegg (2010:36) focused on the need of teachers developing fluency on procedural knowledge in calculation as a way of reducing the load of working memory, and allowing greater capacity in other mathematical activities as arithmetic and algebra. An example of the way this works is in mathematical language and definitions of mathematical terms (Sullivan, 2011:6).

Sullivan (2011:7) further argued that if the learners cannot define mathematical terms such as parallel, right angle, index etc., the instructions for using those terms will be confusing and ineffective, because many learners will be trying to gather clues of the terminologies in mathematics. On the other hand, if the learners can readily recall key definitions and facts, these facts can facilitate problem-solving and actions.

Procedural fluency seems to be essential if it is coupled with conceptual understanding; Kilpatrick et al. (2001:22) gave the example of place value, and the meaning of rational

numbers, to consolidate the procedural fluency of learners. The National Council of the Teachers of Mathematics (2000:7, 2014:15) supported Kilpatrick et al. (2001:22) on procedural fluency, noting that it should build on a foundation of conceptual understanding, strategic reasoning and problem-solving. The differences and similarities between the different methods of calculation could be used when appropriate procedures are learnt when working with rational numbers and place values.

Research suggests that once the learners have memorised and practiced the procedures that are difficult for them to understand, they may be lead to less of an understanding of the meaning and reasoning behind those procedures (e.g., Hierbert, 1999:6). Hence, the development of conceptual understanding should precede and also coincide with instructions on procedures. Even though conceptual knowledge is an essential foundation for learners acquiring procedural knowledge, it is important in its own right. The learners need to have a deeper and more flexible knowledge of the variety of procedures, along with the ability to make critical judgements about which procedures are appropriate for use in particular situations (National Research Council, 2012:9; Star, 2005:405).

The NCTM (2014:9) argued that procedural fluency builds its knowledge from initial exploration and discussion of number concepts, to use informal reasoning and the properties to solve problems. It is further argued that effective teaching approaches provide experiences that can help the learners to connect the learned procedures with the underlying concepts and also to provide the learners with the opportunity to practice those teaching strategies in order to justify those procedures (NCTM, 2014:9). Therefore, those teaching practices need to be brief, purposeful and engaging (Rohrer, 2009:6). The learners' procedures when solving mathematical problems ought to be analysed in order to reveal the insights and misunderstanding underlying the concepts taught. The analysis of learners' procedures used to solve mathematical problems can help the teachers to plan the type of instructions to be used in teaching mathematics. Booth, Lange, Koedinger and Newton (2013:4) asserted that worked examples in mathematics can serve as a valuable instructional tool to enable teachers to understand how the learners analyse problems, why certain procedures work or do not work, and also to consider what procedure is most useful in a particular situation.

3.4.3 STRATEGIC COMPETENCE IN THE TWINNING CONTEXT

Kilpatrick et al. (2001:124), the NRC (2001:119) and Watson and Sullivan (2008:125) refer to *strategic competence* as the ability to formulate mathematical problems, represent them, and effectively solve them. Turner (2010:59) defined the devising of strategies as "...a set of critical control process that guides an individual to effectively recognise, formulate and solve

problems. This skill is characterised by selecting or devising a plan or strategy to use mathematics to solve problems arising from a task or context as well as guiding its implementation". Problem-solving has been the focus of research, and of the curriculum and teaching for some time (Sullivan, 2011:7). Teachers are supposed to be familiar with problem-solving's meaning and resources that can be used to support learning of mathematics.

Strategic competence of the learners is seen as problem-solving and problem formulation in the literature of mathematics education and cognitive science (Mayer & Wittrock, 1996:227). Strategic competence is viewed as being interdependent with other strands of mathematical proficiency, and represents different aspects of a different kind of mathematics teaching and learning (Ostler, 2011:17). The teachers ought to use a variety of strategies to allow them to monitor their learners' capacity in using different strategies when solving problems in order to analyse a mathematical situation (NCTM, 2000:8). Moreover, the learners should be able to frame and solve problems and make better sense of the procedures taught. Ostler (2011:17) recommended these methods and strategies that teachers use in teaching that provide the insight into the strategic competence of learners.

The ability of the learners to formulate a mathematical problem ought to enable them to represent the problem mathematically in some fashion, either numerically, systematically, verbally or graphically. Kilpatrick et al. (2001:125) argue that for the learners to represent a problem mathematically, they need to build a mental image of its essential components. Mathematical tasks are often dynamic, and depend on the mathematical preparedness of the learners (Ostler, Grandgenett & Mitchell, 2008:34). These researchers further argued that mathematical exercises that are given to learners can emphasise the nature and necessity of strategic thinking.

There exists mutually supportive relationships between strategic competence and both procedural fluency and conceptual understanding in routine and non-routine problems. The various strategies learned by the learners depend on what they learned before and the relationships, which include fluency in solving routine problems. Kilpatrick et al. (2001:127) argue that for the learners, developing mathematical competence in solving non-routine problems provides a context and motivation for learning to solve routine problems and for understanding concepts such as given, unknown, conditions, and solutions. Ostler et al. (2008:35) furthermore indicated that the learners should select and develop appropriate mathematical methods when solving problems based on their level of competency.

3.4.4 ADAPTIVE REASONING IN THE TWINNING CONTEXT

Kilpatrick et al. (2001:129) refer to *adaptive reasoning* as the logical thinking-capacity of a learner regarding conceptual and situational relationships. Adaptive reasoning is when a learners is able to give reasons and justify why the solutions are appropriate within the context of the problems that are large in scope NRC (2001:119). Adaptive reasoning glues everything together to guide learning, which is to assist the teacher in using the new teaching strategies gained during teaching to improve learner performance by promoting learners' reasoning capacity. Stacey (2010:18) argued that mathematical reasoning has been underemphasised, and that there is a need for resources and teacher learning to be reinforced to support mathematical reasoning.

Mathematical reasoning is usually confined to formal proofs in geometry and other forms of deductive reasoning. Adaptive reasoning is much broader than procedural knowledge only, according to Kilpatrick et al. (2001:127), in that it does not only include explanations and justifications, but also intuitive and inductive reasoning based on patterns, analogy, and metaphors. This deductive and inductive reasoning could serve as a powerful mechanism for the learners in the low-performing school to improve their academic achievement in algebra. English (1997:4) has postulated that for learners to have the ability to give analogy to the mathematical concepts learned, those analogies can serve as a powerful reasoning mechanism in mathematics problem-solving. English (1997) furthermore says that analogical reasoning, metaphors, and mental and physical representations, are “tools to think with”, often serving as sources of hypothesis, problem-solving operations, and techniques and aids to learning and transfer (p. 4).

The ability to incorporate adaptive reasoning in mathematics teaching is challenging (Ostler, 2011:17). Research on lesson planning suggested that reform-based curricula in mathematics present contemporary challenges in mathematics that are much different from those lesson development challenges within traditional reforms (Superfine, 2008:18). Thus, the development of instructional tasks requires learners adaptive reasoning can be challenging to deal with (Ostler, 2011:17). Nevertheless, adaptive reasoning remains a critical element of the coherent understanding of mathematics. The learners can build a coherent understanding of mathematics with the appropriate guidance of the teachers (Romberg, 2000:7). In addition, the learners can understand how the symbolic processes of mathematics can evolve into increasingly abstract and scientific reasoning. This can suggest that mathematical problems ought to encourage learners' reasoning, which require higher levels of comprehension than problems focusing on procedural understanding.

3.4.5 PRODUCTIVE DISPOSITION IN THE TWINNING CONTEXT

A productive disposition is one of the five interdependent strands that play a pivotal role in the teaching and learning of mathematics in schools. Resnick (1987:171) and the NCTM (2001:119) refer to a *productive disposition* as a tendency to see sense in mathematics content, to perceive it as both useful and worthwhile, to believe that a steady effort in the learning of mathematics pays off, and to see oneself as an effective learner and doer of mathematics. Watson and Sullivan (2008:130) described a *productive disposition* as a habitual inclination to see mathematics as sensible, useful and worthwhile, and can also develop learners' confidence in learning mathematics. The beliefs that learners may have included confidence, the teacher as the holder of knowledge, success determined by self-confidence, that hard work leads to success, and that the understanding of mathematics takes time (NCTM, 2001:119). These beliefs bring forth the interests, purpose and general attitude of the learners towards learning.

Kilpatrick et al. (2001:131) have argued that a productive disposition develops well when it is supported by other strands, which help each other in developing learners' learning. Their arguments about the strands that support each other are supported by the example of the learners who builds strategic competence in solving routine problems, and learners can develop positive attitudes and beliefs as mathematics doers. The more algebraic topics the learners understood during the twinning process, the more they would make sense of those algebra topics and other mathematical concepts. In contrast, those learners who are seldom given challenging problems to solve, tend to use memorisation, rather than sense-making, which opens avenues to the learning of mathematics (Schoenfield, 1989:342). McLeod (1992:19) has asserted that once learners adopt memorising mathematical content, it would be extremely difficult for them to make sense of mathematics.

This productive strand of disposition is not directly related to mathematical knowledge, but instead, to the attitudes and beliefs the learners have towards mathematics (Siegfried, 2012:10). Sullivan (2011:7) suggested that productive disposition does not involve greater action of learners in mathematics than other strands do, but remains one of the key issues to teaching mathematics. Boaler (2002:8) has argued that the learners are not merely learning mathematics, but are learning about what it means to be a learner and a doer of the subject. Learners with productive dispositions are confident in their knowledge and abilities in mathematics (NCTM, 2001:5). The learners with productive dispositions can continue increasing their knowledge and abilities by steady effort, and the beginning of new mathematical experiences (Jansen, 2012:39). The learners need to be supported and

encouraged to develop productive dispositions in order to develop a positive attitude towards mathematics. The learners lacking this productive disposition may not see themselves as learners and doers of mathematics, and cannot conceive that mathematics may make sense.

3.5 RESOURCES IN THE MATHEMATICS CLASSROOM

Resources can be essential in the teaching and learning of mathematics and are regarded as human resources and material resources (Tachie & Chireshe, 2013:67). Institutions and organisations, including schools, are made up of human resources and non-human resources. Human and non-human resources can play a role in developing the institutions and organisations. Yara and Uganda (2010:126) indicated that the availability of teaching and learning resources can enhance the effectiveness of schools and can also have a positive impact on the learners' academic achievement. Mbugua (2011:113) mentioned that teaching and learning resources in schools, such as the chalkboard, mathematics textbooks and charts are material resources that can be used to teach.

The material resources in schools ought to yield good results if the teachers' activities are learner-centred, and based on experiments and improvisation where necessary (Oguntuase, Awe & Ajayi, 2013:2). A mathematics laboratory is a place where learners learn and explore various mathematical concepts, and verify facts and theories, using a variety of activities and materials (Igbokwe, 2000:242; Okigbo & Osuafor, 2008:558). In other words, the teachers ought to use the classroom as a mathematics laboratory for the learners in the low-performing school to synchronise theory and practical work in the teaching and learning of algebra. Yadar (2007:9) and Yara and Otieno (2010:130) argued that no course in science or mathematics can be considered complete without some practical work through performing experiments. This argument is based on the findings by UNESCO (2008:9), Yadar (2007:9), and Yara and Otieno (2010:130), namely that learning through performing experiments can have more benefits than traditional learning.

3.5.1 THE TEXTBOOK AS A TEACHING AND LEARNING RESOURCE

The mathematics textbook is seen as a powerful tool in teaching and learning (Mbugua, 2011:114). There are no recommended mathematics textbooks and mathematics teachers are free to choose their own textbooks for their learners. Studies have been conducted on the availability of mathematics textbooks in schools, and it indicated that most of the primary and secondary schools in Kenya have a shortage of textbooks (Oguntuase et al., 2013:6; Mbugua, 2011:114; Yara & Uganda, 2010:126). These shortages of textbooks can derail the progress in the teaching and learning of mathematics.

The school curriculum provides mathematics teachers with a list of topics to be covered in the syllabus. Textbooks play an important role in determining the sequence of teaching and in providing exercises for the mastery of concepts in mathematics (Mbugua, 2011:114). A variety of mathematics textbooks exist. Some provide a lot of information for the learners to drill and practice, while others provide little information of little help in understanding the mathematical concepts (Reys, Lindquist, Lambdin, Smith & Suydam, 2001:16). Malloy and Jones (1998:114) argue that many textbooks have a range of problems but some do not include the challenges learners need to grow. The twinned teachers could have an opportunity shared the types of textbooks they use in their respective schools teach algebraic topics during twinning to enhance learning of Algebra.

Mathematical concepts have logical relationships which can make teaching and learning effective. Thus, the mathematics teachers and learners must have textbooks in order to refer to concepts that have to be taught (Mbugua, 2011:114). Ndirangu, Kathuri and Mungai (2003:80) were of the opinion that the curriculum and examinations may be prepared without considering the availability or the variations of teaching materials. Otunga (2000:146) also found that resources that can be used to prepare curriculum implementation should adequately be addressed in schools. The issue of the shortage of textbooks and textbooks with little information and insufficient exercises can have a negative impact on the learners' academic achievement in mathematics. The twinned teachers could have improvised other materials that had to complement textbooks they used to teach algebraic topics.

3.5.2 INFORMATION AND COMMUNICATION TECHNOLOGY (ICT) AS A TEACHING RESOURCE

ICT has been encouraged, mostly in the teaching and learning of mathematics, in many countries of the world (Ndlovu, Wessels & De Villiers, 2011:1). South Africa is one of the countries that encourages teachers to incorporate available technological resources when teaching mathematics (Department of Basic Education, 2011:12). ICT is a form of multimedia used to teach mathematics (and other disciplines), such as calculators, computers, projectors, sketch-pads, geogebra (Gayeski, 1999:4).

The use of ICT resources is necessary to integrate into the process of teaching and education (La Velle & Nichol, 2000, Lever-Duffy, McDonald & Mizell, 2003:15). This integration of ICT in the classroom may not only bring about opportunities for learning and teaching, but it is strongly encouraged from within the mathematical and scientific community (Ndlovu et al., 2011:4). Hamdane, Khaldi and Bouzihab (2013:321) postulated that ICT was found to be important in the teaching and learning of mathematics. Hamdane et al. (2013:321)

furthermore said that the integration of ICT into teaching can make learning fun, interesting and more effective. The integration of ICT in teaching and learning could be of importance during twinning, as the twinned would have had an opportunity to share ICT tools in teaching algebra.

However, the effective integration of ICT into mathematics teaching is a challenge to mathematics teachers (Kilicman, Hassan & Husain-Said, 2010:613). Hamdane et al. (2013:321) identified the reasons for this to be resistance, due to a lack of knowledge or skills in the use of these new technologies, and a difficulty in accepting new working methods, and sometimes, asserting the ineffectiveness of these technologies. Ndlovu et al. (2011:4) agreed that teachers integrating new technologies find it to be a challenge, and simultaneously find ways to empower or enhance the learners' mathematical learning. Studies showed that the integration of technologies complicates the lives of the teachers' practices in the classroom (e.g. Robert & Rogalski, 2005:280). Moreover, the availability of technology amplifies the complexity of mathematical content, and hence, challenges the stability of teaching practices (Lagrange & Monaghan, 2009:1609).

3.5.3 THE TEACHER AS PROBLEM-SOLVER

Teachers in the mathematics classroom are seen as human resources in achieving learning objectives (Oguntuase et al., 2013:3). They are responsible for facilitating learning in schools, especially during problem-solving. Hansen (1996:318) argues that teachers are important in the classroom to deliver mathematics content, which involves highly skilled labour resources that are combined with educational materials. They can be seen as problem-solvers in the classroom, who improve teaching and learning of mathematics. Akudu (2007:75) asserts that their effectiveness can be vital, as it may affect the future development of learners' learning of mathematics.

Ukeje (1979:88) saw teachers as hubs in mathematics classroom. Mathematics teachers ought to act as rolemodels to learners, and be the custodian of mathematical knowledge during teaching and learning, by demonstrating their knowledge of these. Balogun (1995:25) has argued that teachers' jobs cannot only involve teaching efficiently and effectively, but a teacher should also be a role-model to his learners. The learners may look up to him/her both as a mirror and as an instrument of learning, not only doing what he says, but can also follow what he/she does. The teacher as a role-model ought to play a fundamental role in developing and enhancing learners' problem-solving abilities. The teacher should model the mathematical content to the learners and encourage problem solving during teaching. Oguntuase et al. (2013:1) argued that the outcomes of the teachers' efforts in the teaching and

learning of mathematics are typically represented by learners' grades. Furthermore, Oguntuase et al. (2013:1) say that well-trained teachers of mathematics secondary schools can produce learners who will perform well in mathematics. This study sought to understand how they developed and enhanced learners' problem solving abilities in the poor performing school during twinning.

Mathematics is a pillar of science and technology and its functional roles in the development of technology are so multi-dimensional that no area of science, technology and business enterprise escapes its application (Okereke, 2006:254; Okigbo & Osuafor, 2008:258). Knowledge of mathematics is seen as important to enable learners to apply it in other subjects, or to understand other subjects, or it can be applied when doing calculations across other science disciplines.

However, in most developing countries insufficient mathematics teachers are produced by the universities and colleges (World Bank Report, 2007:20). Olatunde (2010:33) said that most mathematics teachers do not have the expertise in teaching mathematics and other related subjects. Most learners at the universities and colleges are not encouraged to pursue these mathematics teaching courses, especially in the field of mathematics education, in order to fill the gap in mathematics streams. Few learners who passed pursue mathematics courses at a tertiary level, where the issue of learners shying away from mathematics has led to a great shortage of mathematics teachers. This study twinned two teachers whose learners performed differently to alleviate the challenges learners faced in the poor performing school.

The other factor that affects poor performance is a low teacher-learner ratio, which is affected by the shortage of teachers, especially in public schools (Olatunde, 2010:34). This low teacher-learner ration may contribute to the poor performance of learners in mathematics in most schools. The lack of trained mathematics teachers may negatively affect the teaching and learning of mathematics (Yara et al., 2010:127). Teacher training, especially in mathematics, needs to be given attention in order to increase the number of mathematics teachers in schools.

3.6 TEACHING AND LEARNING RESOURCES VERSUS ACADEMIC ACHIEVEMENT IN MATHEMATICS

Teaching and learning resources are important in classroom teaching as they appear to play an important role in the mathematics performance of learners. Research has been conducted on the teaching and learning resources in mathematics versus learners' academic achievements (Oguntuase et al., 2013:1; Yadar, 2007:12; UNESCO, 2008:9; Mbugua, 2011:

112; Yara & Uganda, 2010:126; Ruthven & Hennesy, 2003:21; Sun & Pyzdrowski, 2009:38). Research done previously was based on resources such as textbooks, chalkboards, three dimensional figures, charts etc., where some focused on the use of ICT to teach mathematics. Yadar (2007:12) and UNESCO (2008:13) have indicated that teaching and learning materials such as textbooks, teaching aids (chalk-boards, ruler and protractor, and stationary) have a positive effect on the learners' academic achievements in mathematics. Yara & Uganda (2010:126) has meanwhile argued that teaching materials such as textbooks, exercise books, and teaching aids strengthen the teaching and learning in mathematics and other subjects. The effects of teaching and learning resources on learners' academic achievement in secondary school mathematics were examined by Yara and Uganda (2010:127). A descriptive survey research design was used with secondary school learners.

The Student Questionnaire on Performance (SQP) was used as a validated research instrument to measure the learners' performances. The findings revealed the positive correlation of the teaching and learning resources and learners' academic achievements in secondary schools. This indicated that the schools that lack teaching and learning resources in mathematics are likely to display poor performance, and that this may have implications in a poor-performing school. Yara and Uganda (2010:127) proposed that a review of the curriculum, in-service training of teachers, the recruiting of more competent teachers, the motivation of the learners, improved government support of education, good teaching methods, improved learner-book ratio, and the better remuneration of teachers, to be factors that the government and all school stakeholders ought to pay more attention to, in order to improve the learners' academic achievements in mathematics.

The effects of teaching and learning resources on the learners' academic achievements were also studied by Oguntuase et al. (2013:1). The results were dissimilar to the ones in Yara and Uganda's (2010:127) study. The findings were that all the independent variables could not account for the learners' academic achievements in mathematics. The SQP instrument was also administered among the other secondary schools mathematics learners when it comes to their academic achievement. Their findings showed that there was a significant difference between the effects of the availability of mathematics textbooks, a mathematics laboratory, and the availability of teaching resources on learners' academic achievements in mathematics. This study revealed that mathematics teaching and learning resources do not always have a positive correlation with the learners' academic achievements. This may depend on how the teachers use those resources to effectively teach mathematics.

3.7 THE LEARNERS' ACADEMIC ACHIEVEMENT IN MATHEMATICS

The poor performance of South African schools in mathematics has been established at primary school level in mathematics and reading (Howie, Venter, Van Staden, Zimmerman, Long, Scherman & Archer, 2007:2; Makgato & Mji, 2006:253) and at secondary school level in mathematics and science (Howie, 2003:89; Reddy, 2006:18; Taylor, 2008:1). The poor performance of the learners in mathematics was found in the Southern and Eastern African Consortium for Monitoring Education Quality (SACMEQ) scores when South Africa's Grade Six learners obtained 9th place out of 14 countries. The poor performance in mathematics may be due to various factors, as mentioned previously, such as poverty (Taylor, 2008:4); learners' attitude towards mathematics (Maat & Zakaria, 2010:272; Manoah, Indoshi & Othuon, 2011:965; Mohamed & Waheed, 2011:277; Mohd, Mahmood & Ismail, 2011:49; Tahar, Ismail, Zamani, & Adnan, 2010:476; Tezer & Karasel, 2010:5808); and teachers' motivation (Adeyinka, Asabi & Adedotum, 2013:37, Makgato & Mji, 2006:253). The factors that affect poor performance in mathematics enabled this researcher to determine the challenges the teacher and the learners might face in low-performing school during observations, semi-structured interviews and pre-and post-test.

Howie (2003:89) reported in his study on a number of factors that contribute to the South African schools poor academic achievement in mathematics, which included: the subject matter knowledge of teachers; the inadequate communication between teachers and learners in the language of learning and teaching (LoLT); the lack of instructional materials; problems that the teachers encounter in managing classroom activities effectively, pressure to complete an examination-driven syllabus, heavy teaching loads, over-crowded classrooms, poor communication between policy-makers and practitioners, and the lack of support due to the lack of professional staff in the Ministry of Education. These findings are also reflected in the work of Passos (2009:268), who carried out a comparative study of teacher competence and its effects on Grade Six learners' performances in upper primary schools in Mozambique and other SACMEQ countries. Passos (2009:268) contended furthermore that the teachers' and the learners' competence in mathematics and reading is influenced by cognitive, affective and behavioural factors. Makgato and Mji (2006:260), in their study, identified teaching strategies, content knowledge, motivation, laboratory usage by the teachers, and the non-completion of the syllabus, as contributing factors towards the learners' poor performance in mathematics and science. Makola (2005:8) argued about the class size in terms of the teacher-learner ratio, and the formal contact hours for the teaching and learning of mathematics, as contributing factors towards the learners' performance.

3.7.4 CLASSROOM ASSESSMENT

Assessment is a way for a teacher to understand a learner, in order to take an informed decision about that learners' performance so as to determine the type of teaching instruction to be used (Sattler, 2008:5). Van de Walle, Karp and Bay-Williams (2013:78) defined *assessment* as a tool used by teachers to enhance the learners' learning, either formally or informally, and also to assist those teachers in making instructional decisions. Assessment is a process of collecting data, as evidence about a learner's knowledge, skills and disposition to develop learners' learning towards mathematics and other subjects and makes an inference from that evidence for a particular purpose (National Council of Teachers of Mathematics, 1995:78). The NCTM (1995:78) further indicates that the notion of collecting evidence is necessary within a context of tests or quizzes, but ought to be an on-going process, as an integral part of instruction. Hattie (2009:260) postulated that if assessment is restricted to tests or quizzes, additional information may be missed regarding a learner's knowledge.

The validity of assessment is referred to teachers' measuring what it says it is measuring, knowledge, understanding, application, subject content, skills, information, etc. (Tracey, 2010:336). Reynolds, Livingston, Willson, & Willson (2010:123) indicates *validity* to be a measure of the accuracy of assessment. It is the most important criterion for the quality of a test. If the validity of a test is poor, then it is highly possible that it cannot measure what it is supposed to. There will be no justification for using the test results for the intended purpose of the assessment. Tracey (2010:336) argued that teachers should use their professional judgement to determine whether their assessment is valid or not, through quality assurance of the questions set.

The reliability of a test is another important element of the quality of assessment. *Reliability* refers to the consistency of assessment outcomes (Reynolds et al., 2010:119). *Consistency* refers to the same judgements being made in the same or similar context each time a particular assessment for specified stated intentions is administered. A reliable test assessment is likely to yield the same results, even after being administered for a second time (Tracey, 2010:338). On the other hand, the assessment with poor reliability might result in different scores across the administration of the two tests. If the assessment yields inconsistent results, it may be unethical to take substantial action on the basis of the test.

Classroom assessment comprises two major categories, namely summative and formative assessment (Dodge, 2009:3). Summative assessments are mainly used to gather evidence that provides a single score, such as standardised tests or end-of-year examinations (Van de Walle et al., 2013:78). They (2013:78) indicated that this type of assessment is like a digital

snapshot, which means that it is a once-off type of assessment. Summative assessment takes place at the end of a term or year in which a learner's knowledge is measured, in order that the learner may be promoted to the next grade, or retained (Dodge, 2009:8). Although these once-off scores are important to both the teachers and the learners, they do not provide the teachers with teaching instructions or decisions on particular topics.

On the other hand, formative assessment works quite differently. This type of assessment is described as an evaluation that monitors the progress of a learner. (Hattie (2009:260), Popham (2008:13) and William (2007:41) described *formative assessment* as a planned process, diagnosing the learners' understanding of concepts during classroom instruction. Bennet (2011:10) view *formative assessment* as ongoing assessment, observations, summaries and reviews that inform the teacher's instruction and provide feedback to learners on a daily basis. In other words, if the formative assessment is well-planned and implemented, it can provide constructive feedback to learners (William, 2007:193; Wilson & Kenney, 2003:58).

Feedback has a powerful influences on learners' performance, as errors and misconception underlying in mathematics can be identified in the learning process (Hattie, 2009:260). The evidence of learners' work collected during a feedback session may provide the teacher with the information about what has to take place in the next progression. Williams (2010:10) indicated that formative assessment ought to provide the teacher with future action. It is difficult for teachers to give feedback without any kind of assessment, so as to understand what learners display (Björklund Boistrup, 2010a:165), and no future action may be planned or taken.

However, assessment is not only used for feedback to plan for the next action of teaching. The NCTM (1995:79) indicated four purposes of classroom assessment, namely that it is for: monitoring the learners' progress; making instructional decisions; evaluating the learners' achievement; and evaluating programmes. Monitoring the learners' progress provides the teachers and learners with on-going feedback towards the learning objectives and long-term goals (Van de Walle et al., 2013:79). Each teacher and learner will be informed by the work of the learners about the learners' problem-solving abilities and growth towards the understanding of mathematical concepts, practices, and procedural fluency.

On the other hand, teachers are able to make instructional decisions, which are one of the purposes that provides the teacher with an understanding of how learners think, and the naïve ideas they use. Problem-solving skills and discussions provide a richer and more useful array

of data that can be gathered to make informed decisions. During the discussion process, the teacher can actually formulate plans on how to assist the learners to develop new ideas about mathematical concepts and make changes in those concepts.

Moreover, evaluating the learners' achievement assists the teachers in collecting evidence from learners to make an informed judgement. The NCTM (1995:79) defined *evaluation* as the process of determining the worth of something on the basis of careful examination and judgement. The teachers' judgements of the learners' achievements play a pivotal role in the classroom, such as in respect of instructional planning, screening, placement, referrals and communication with the parents (Gittma & Koster, 1999:14; Stiggins & Conklin, 1992:31). Judgement can also influence the learners' study patterns, self-perceptions, attitudes, efforts and motivation (Black & William, 1998:2; Rodriguez, 2004:5). A wide variety of sources and types of information gathered can serve as evidence that can be taken into account during instruction (Van de Walle et al., 2013:81) in order to make judgement. Most importantly when it comes to evaluation, is reflecting performance criteria about what learners know and can do, rather than comparing learners' performances.

Again evaluating programmes focus on the objectives of the lessons that can be achieved at the end of a unit. Van de Walle et al. (2013:81) have indicated that when evaluating programmes, assessment data should be used to answer the question: 'how well did this lesson or unit of study achieve my goals?' The teacher can evaluate whether his/her learning objectives are achieved during teaching and learning. The classroom tasks can be selected, or problems for learners, sequence of activities, kinds of questions developed during instructions, and how models or representations are used.

3.8 PERFORMING SCHOOLS

Performing schools are schools that are generally effective in respect of the learners' academic achievements. The principals, teachers and learners in these schools set and meet high standards or expectations for all their learners (Calman, 2010:2). In addition, these schools enhance aspects of the learners' achievements and development. A learner-centred approach is advocated as effective learning for all the learners (Black, 2006:3).

Those schools that mostly focus on a learner-centred approach in teaching and learning may demonstrate:

- an integrated approach to change that takes place in teaching, the curriculum, assessment, school organisation and school culture;

- effective and supportive school leadership;
- high expectations amongst the school leadership and by the teachers together with respectful and caring relationships among the adults, the learners and between the adults and the learners;
- collaborative decision-making between the leadership and the staff members and a cooperative culture among the teachers;
- a high quality of teachers and policies, structures and resources that support continued teacher development; and
- relationships with the parents and the wider community that support the families and enrich learning (Cole, 2001:6; Mitchel, Gale, Edwards & Zyngier, 2004:7).

A learner-centred approach plays a pivotal role in the teaching and learning of mathematics, as it involves all the stakeholders, such as teachers, learners and parents. The learners existing knowledge is taken into consideration during teaching and learning.

The well-performing schools mostly possess strong instructional leadership (Calman, 2010:2). The principals in these well-performing schools provide a strong and effective leadership by motivating teachers and learners (Teddle & Stringfield, 2007:136). Leithwood, Harris and Hopkins (2008:27) found that school principals' leadership plays a pivotal role in classroom teaching and learning; it has an influence on the learners' learning. The principal has an impact on the learners' learning through a positive influence on the staff's beliefs, values, motivation, skills and knowledge, and by ensuring an environment that is conducive to working in the school (Calman, 2010:2). This strong instructional leadership is regarded as *transformational* (Ofsted, 2009:8). Transformational leadership is fundamental, because school leaders are able to face difficult situations, put learners first, and have faith in what learners can do and what teachers can also do in schools. The principals in these well-performing schools also share their leadership responsibilities with teachers, which promotes delegation empowerment (Moos & Huber, 2007:785).

The well-performing schools are committed to teacher learning (Black, 2006:4). These schools become professional learning communities, where the teachers can examine each other's classroom practice, and use research to improve classroom practice (Elmore, 2006:43; Hill & Russel, 1999:167; Instance, 2006:65). This is important in classroom teaching, as teachers are encouraged to work together to improve their teaching practices. Professional learning communities improve the learners' learning in disadvantaged schools (Grant, Badger, Wilkinson, Rogers & Munt, 2003:63). Black (2006:6) argued that teachers meet on a

weekly basis to share ideas, planning and practice, and to act as informal coaches for one another. Moreover, the schools bring their whole staff members together in regular professional learning forms. The teachers in that community create a shared vision, which can be associated with shared values and goals, by means of which to develop a collaborative learning community (Calman, 2010:3).

Well-performing schools are, for the most part, able to create an orderly and supportive environment (Calman, 2010:4). Schools that lack order, discipline and classroom control would be difficult for teachers to attain high level of learner attention and interaction within classroom practice (Reynolds & Teddlie, 2000:26). Shulman (2010:9) has postulated that the well-performing schools do not merely create an orderly space in which to learn, but foster a supportive learning environment. These well-performing schools may provide stability and a purposeful and structured experience in teaching and learning mathematics, as well as other subjects (Ofsted, 2009:8). In other words, the supportive, caring attitudes of the staff members toward the learners are brought together (affection), in the orderly environment (stability) and the focus on academics (the purposeful, structured experience). Shulman (2010:9) has argued that effective schools act as hubs to support the learners, and to instill self-esteem and confidence, ensuring the safety and wellbeing of their learners. They acknowledge, honour and respect each individual learner's differences.

Effective schools can maximise school learning time for their learners by starting the classrooms' lessons on time, and finishing them on time. Calman (2010:3) argues that some studies highlighted the need to maximise learning time at school level. Calman (2010:6) further postulated that increasing allocated time does not translate into increased learning time. Aronson, Zimmerman and Carlos (2005:12) pointed out that what is critical is maximising the time for the learners, focusing on and engaging in the classroom activities. Cotten (2000:18) indicated that performing schools ensure the adequate allocation of time for core subjects, and implement appropriate policies to deter lateness, absenteeism and disruptive behaviour.

3.9 POOR-PERFORMING SCHOOLS

Non-performing schools are schools that perform below the expected average percentage, according to the Department of Education (2003:25). These schools that are not performing well academically, are mostly the rural ones that lack sanitation, electricity, running water and have dilapidated (mud) buildings (Skelton, 2013:3). Skelton (2013:3) further found that these schools have inadequate seats and desks for the number of learners attending them. The

roofs of the buildings are often constructed of corrugated iron, and rusted through, causing the teachers, learners and furniture to get wet when it rains. The school programmes may not be conducted, due to leaking roofs during these rainy days, and the learners fall behind in their schoolwork.

However, the Department of Basic Education (2010:12) has a policy that requires the schools to be maintained in a condition that makes teaching and learning possible, however, many schools remain dilapidated despite this. The needs of the schools have been identified by the DBE, namely that there are 510 inappropriate structures, 2401 schools lack running water, 3544 schools lack electricity and 913 schools have no ablution facilities.

The infrastructure may well play a role in the academic performance of learners as classrooms are likely to be overcrowded, creating difficulty for teachers to control the learners. However, some teachers can produce good results from learners who learn in dilapidated schools (Skelton, 2013:5). It should, however, be borne in mind that the extremely poor infrastructure may have a negative effect on both the teachers and the learners. On the other hand, Skelton (2013:5) further indicated that results of good quality are unlikely in these kinds of schools, with no electricity, no toilets, and no running water. These circumstances are generally significant in learners' academic achievement (Branson & Zuze, 2012:70; Van den Berg, 2008:145). Spaul (2013a:437) claimed that if the Post-2015 Development Agenda requires no one to be left behind, the inequality between the learning environment offered by dilapidated (mud) schools and other public schools in South Africa is unacceptable.

Teacher absenteeism and the teachers' mathematical content knowledge should be treated seriously, because they affect the learners' academic achievement in the schools (Armstrong, 2014:18; NEEDU, 2014:9). Spaul (2013b:436) indicated that the two binding constraints contribute to the quality throughput in South Africa's basic education, namely teacher absenteeism and teacher content knowledge. Mason (2013:407) has argued that issues related to teachers such as the two binding constraints in schools have a greater impact on learner performance. Teacher absenteeism and mathematical content knowledge should be given attention to improve teaching and learning in schools.

Jungic, Kent and Menz (2006:1) indicated the challenges experienced by the teachers in large classes. Specific concerns have been raised in large mathematics classes, which include issues of preparation of materials for learners, the organisation of classes during teaching, the administration of courses, teaching instructions, the use of technology, student management

and learners' grading that are common to large classes in other subject areas (Jungic et al., 2006:3). Studies have been conducted aiming to improve the teaching and learning in a large class environment, which found no resource specific to teach a large mathematics class (Gadalof, 2002:18; Gibbs & Jenkins, 1992:32; Stanley, 2002:12). It is still a challenge to control over-crowded classes, to provide feedback to the learners and to create an atmosphere conducive to learning and teaching.

The learning environment at home may impeded the learners' learning (Lupton, Noden, Brady & West, 2013:2), incorporating different aspects such as: attitudes towards learning; the educational resources at home; and the actual support from the parents with their children's learning (Branson & Zuze, 2014:70; Hartas, 2011:72). Parental income and educational background may also play a role in the learners' school performance, by being unable to buy extra resources, and assisting learners with homework and assignments. The low socio-economic backgrounds of the learners and schools with large numbers of these learners perform less well when compared to the higher socio-economic backgrounds of the learners (Black, 2006:6). Black (2006:7) furthermore argued that poor learners are mostly clustered in economically-depressing locations (regions), with low educational outcomes. The educational disadvantage of the learners has been studied before, and is linked to the geographical location of their schools (Keating & Lamb, 2004:39; Teese, 2000:49; Teese & Polesel, 2003:63, Thomson, 2002:19). Lupton et al. (2013:4) argued that the parents in those geographical locations, such as rural areas, need to help their children with their schoolwork, but that they cannot, due to their low educational background.

3.10 CHAPTER SUMMARY

This chapter presented the literature review and a discussion of the concept of twinning with its advantages and disadvantages. Drawing from the literature, twinning was implemented in other countries, but little has been attention in South Africa. Attention was given to the twinning of principals of schools in those countries, like in the State of Carlifornia in the USA. Since the study is conducted at teacher level when teaching Algebra, the teachers' content knowledge and pedagogical content knowledge were reviewed in order to understand the position of the twinned teachers during the twinning process. The issues of teaching practices and resources were also reviewed, to gain a broader picture of how they impact on the performances of the learners in mathematics. The chapter also focused on well-performing and the poor-performing schools so as to understand the factors playing a role in these two kinds of schools.

CHAPTER 4

RESEARCH METHODOLOGY

4.1 INTRODUCTION

This chapter discusses the methodological tools employed in a case study of two schools, which followed a mixed-method approach. The researcher discusses research paradigms which are interpretive, positivistic and post-positivistic and pragmatic, followed by a description of the use of qualitative, quantitative and mixed methods. The data-collection instruments are discussed, followed by an explanation of the data-analysis techniques used. Sampling, validity and reliability issues are considered and the instruments used are described and motivated. Issues of ethics are considered, as well as the ethical stance taken in this study.

4.2 RESEARCH PARADIGM

Research is described as a basic, systematic inquiry employed to revise the current body of knowledge by collecting (empirical) data to develop new knowledge through analysis and interpretation (McMillan & Schumacher, 2014:354). Sepeng (2010:46) indicates that research can be motivated by the researcher's mental views, which are referred to as a 'paradigm'. The term *paradigm* is defined as a perspective of the researcher's view about the world he lives in, and is based on a set of shared assumptions, concepts, values and practices (Johnson & Christensen, 2012:32). A paradigm constitutes what has to be studied in a particular setting, what research questions should be answered in the study, and the way in which those questions will be analysed and interpreted through the collected data. Creswell (2013:17) viewed a *paradigm* as a set of beliefs that guide actions regarding ontology, epistemology, axiology and methodology. These four aspects of philosophy are seen to be related to as a paradigm and to themselves as well which assist in defining the delimitation or boundaries of the research. The boundaries of the study are established in order to inform how the researcher ought to behave or what he ought to focus on in conducting successful research. The definitions mentioned above suggest that paradigms can either consciously or subconsciously set the rationale and expectations of the study.

4.2.1 POSITIVIST AND POST-POSITIVIST PARADIGMS

A *positivist paradigm* is described as the study of human behaviour and actions, and assumes that science is quantitatively measure facts about a single apprehensible reality (Healy & Perry, 2000:118). Positivists believe that reality is stable, and is described from an objective viewpoint (Levin, 1988:186). In other words, the researcher does not interfere with the phenomenon that is being studied. Positivists imply a particular stance as an observer of social reality (Cohen, Manion & Morrison, 2013:25). Positivists test hypotheses that are based on what has been observed in the experiment previously, and explain realities and their inter-relationships. Krauss (2005:764) indicates that positivists believe that the objective of knowledge is to describe the phenomena under study, where observations and measurements are the core of scientific endeavour.

In addition, Coolican (2004:363) indicated that positivists in their research form a hypothesis to test the theory, and hence conduct research to test that hypothesis in order to support or adjust the theory. Positivists in their studies, observe empirical events in order to analyse them, with the researcher detached from the field of study. Positivist researchers believe that the universe or the world conforms to permanent and unchanging laws and rules of causation and occurrences (Aliyn, Bello, Kasim & Martin, 2014:89). The methodologies frequently used by positivist researchers are confirmatory analysis, nomothetic experiments, quantitative analysis, laboratory experiments and deductions (Olesen, 2004:260). This study used a nomothetic experiment and quantitative analysis in order to measure the impact of twinning in the experimental post-test results.

The post-positivists traditionally developed from the 19th century writers tradition comes from 19th century writers, such as Comte, Mill, Durkheim, Newton and Locke Smith (1983:6), and more recently, from writers such as Phillips and Burbules (2000, as cited in Creswell, 2014:7). Mertens (2005:17) postulated that positivism was superseded by post-positivism after the Second World War. Post-positivists hold a deterministic stance, in which the causes of the events determine their outcomes or effects. Creswell (2014:7) indicated that post-positivists believe in identifying the causes of events, and also assess those causes that influence the outcomes or the effects of the experiments in the research fields. Moreover, Creswell (2014:7) has emphasised that post-positivism starts the research with theory, travelling to the research site to collect data, in order to support or refute a theory, and then revises the process, so as to conduct additional tests. This means that the laws or theories that govern the world need to be tested and verified, so that they can be refined to understand the world. Post-positivists do not believe in a single reality of the study, but view research as

logical steps and also believe in multiple realities of participants. Both quantitative and qualitative methodologies are seen to be important in post-positivism when conducting research (Sepeng, 2010:47).

4.2.2 INTERPRETIVIST/CONSTRUCTIVIST PARADIGMS

According to Sepeng (2010:47), “while the positivist approach relies on a single objective reality that is orderly and predictable, researchers who work within an interpretivist paradigm believe that each individual constructs their own view of the world based on experiences and perceptions.” The interpretivist paradigm is described as constructivism (Denzin & Lincoln, 2011:12; Mertens, 2010:59). Constructivism is typically viewed as qualitative research, because the research question(s) are broad, or general, for the participants to construct their meaning about the situation through the interactions with others (Creswell, 2014:8). Qualitative researchers use open-ended questions to interview the participants to share their views about their world in order to gain an in-depth understanding of the data. The construction of meaning is mainly generated in a social setting, through interaction with the people in the community.

4.2.3 PRAGMATIC PARADIGM

Pragmatic researchers focus on what works in practice, and what promotes social justice which is important and valid in research (Johnson & Christensen, 2012:221). According to Creswell (2003:216), pragmatic researchers focus on the what and how of the research problem. Pragmatic researchers are against positivist paradigm, where the truth about the event is accessed through a single method of collecting data in the research site (Mertens, 2005:18). Creswell (2014:10) concurs that pragmatism arises from the way in which the researcher view of world through participants’ actions, situations and consequences of the inquiry rather than antecedent condition. Tashakkori and Teddlie (2010:112) describe the pragmatic researcher as the one who decides what one wants to research and what the researcher think is important to study. Pragmatism is a philosophical underpinning for mixed methods studies which pays attention to the research problem and uses multiple approaches to answer it (Morgan, 2007:49; Tashakkori & Teddlie, 2010:112).

In positivist paradigms, the results of the study can be used to make generalisations, making statistical tools for analysis in the form of research (Popper, 1968:70). Although a study that is underpinned by a positivist paradigm uses statistical data in the analysis, such a the study does not solely rely on the positivist paradigm. The objective of the current study is to explore the effectiveness of the twinning of two teachers teaching Grade 11 Algebra, and to change their own practices by exposure to the new practices during the process. Pre-test-post-

test, classroom observations and semi-structured interviews were used to assess the outcomes of the two tests administered in the schools and also of the use the strategies shared during twinning process. The study used both the quantitative and qualitative data to answer the research questions. Therefore, both the positivist and interpretivist paradigms seem to be appropriate framework within which to show the rationale and expectations of the study.

However, the study is pragmatic in nature, in its approach and attempts to shed light on how research methods can be mixed, in order to answer to the research questions (Hoshmand, 2003: 43; Johnson & Onwuegbuzie, 2004:16). This study acknowledges Creswell's (2014:10) opinion that pragmatism opens doors for researchers to use multiple methods, different worldviews, different assumptions, including the different forms of data-collection and analysis which allow for mixed methods research. This study used three methods to collect data, such as pre-test and post-test, semi-structured interviews, and classroom observations.

4.3 QUALITATIVE METHODS

Qualitative research may be described as being naturalistic, where the researcher enters the world of the participants as it exists (McMillan & Schumacher, 2014:354). Creswell (2013:193) described *qualitative research* as that research where a researcher spends time at the research site collecting extensive data, establishing a rapport, and getting participants' perspectives about the study. According to McMillan and Schumacher (2010:114, 2014:354), qualitative research is based on what they call a naturalistic phenomenological philosophy. Furthermore, these researchers argued that there are multiple realities that are constructed in a social context through individuals and collective definitions of situations. However, there are no manipulations or control of behaviour or setting, nor any external imposed constraints, in qualitative research (Johnson & Christensen, 2012:35; McMillan & Schumacher, 2010:114).

The classroom in this study is a natural setting, where human behaviour and events occur (Creswell, 2009:176). The Grade 11 Mathematics teachers explained how their learners performed and the challenges they face in teaching and learning, their teaching strategies, resources and experiences, before and during the twinning process. As such, this study incorporates concepts of field research, naturalistic enquiry, as well as case study research, in the natural setting of the teachers in the rural schools. The information from the twinned teachers and learners is gathered through semi-structured interviews and observations, so as to gain a broader understanding of the twinned teachers who participated in this study.

Case study research is qualitative research that examines a bounded system over a period of time in detail, employing multiple data sources found in research site (McMillan &

Schumacher, 2014:354; Gay, Mills & Airasian, 2011:252). Although Stake (2005:450) stated that case study research is not a methodology, rather it is a choice of what has to be studied in the particular area for a certain period of time. Other researchers present case study as a strategy of inquiry, a methodology or a comprehensive research strategy (Denzin & Lincoln, 2005:11; Meriam, 1998:26; Yin, 2009:48). This case study aimed to gather in-depth of twinning two teachers teaching Grade 11 Algebra if could improve learners' performance and also improving teacher's teaching practices in the low-performing school.

Case study also accepts many variables operating in a single case, which requires more than one tool of data-collection and many sources of evidence (Cohen et al., 2011:316). Yin (2009:48) espoused that case study can incorporate both qualitative and quantitative approaches to case study development and discussed explanatory, exploratory and descriptive case studies in order for a researcher to understand the type of case he or she might research. Cohen et al. (2011:316) concur that some case studies are able to blend both qualitative data and quantitative data in a single case, and they are prototypical instances of mixed-methods research. Although case studies are mostly dominant in qualitative research, they are also prevalent in quantitative research. This study intended to incorporate both qualitative and quantitative research, which is a mixed-method in its design, in order to respond to the research questions, using interviews, observations and documents.

Qualitative researchers hopes to understand how participants interact in a social context and why they interact the way they do in a particular situation. This study followed an interactional approach that allowed the researcher to understand and to discover the nature of the interaction of the twinned teachers in teaching Grade 11 algebra during twinning process. Gay, Mills and Airasian (2011:252) argued that the qualitative researcher seeks an in-depth understanding of phenomena as they occur naturally and that no attempt is made to manipulate the situation. In this study, the researcher visited the low-performing school, where twinning of two teachers took place for a period of time, so as to observe how those twinned shared their teaching practices.

.3.1 THE INTERPRETIVE APPROACH

The object of this study was to understand the situation of the twinned schools individually in terms of performance in the pre-test before twinning, the performing and the non-performing schools. Furthermore, this study necessitates engaging the twinned teachers to share their teaching practices, experiences, expertise and resources to understand the pedagogical approaches used in in the low-performing school. The principle of inclusion serves to put bias under control to ensure that the population is represented (Howie, 2009:45). But for the

purpose of this study, it provides a room for a democratic dimension in attempting to ensure that all voices are heard for both teachers during twinning process.

In this study, the twinned teachers interacted and shared their teaching experiences during the twinning process. This made it possible for the participants to truthfully give their teaching expertise and experiences on the algebra topics they shared during the twinning process. Howie (2003:37) and Cohen et al. (2010:374) have suggested that research methods such as participants' observations, semi-structured interviews, focus groups, and others are suitable to promote dialogue in the research settings. This study enabled the researcher to interact with the twinned teachers so as to get to the bottom of the information to gain a richer and better understanding of their teaching experiences through semi-structured interviews.

4.3.2 QUALITATIVE RESEARCH

Qualitative research mostly provides the researchers with an in-depth description and understanding of the human experience in a social setting (Lichtman, 2010:17; McMillan & Schumacher, 2014:354). The purpose of this type of research is to describe and understand human phenomena, human interactions or human discourse in the field of study (Cohen et al., 2011:316; Johnson & Christensen, 2012:221). Furthermore, these researchers stressed that many studies believe that the researcher is obliged to generate the understanding, interpretation and meaning to descriptions (Lichtman, 2010:17).

The purpose of qualitative research is to investigate, unearth and uncover more about a specific phenomenon, and then provide detailed, comprehensive and rich descriptions of it (Creswell, 2013:47; Gay et al., 2011:253). This study investigated the experiences of the twinned teachers in the classroom setting, thereby gaining an in-depth understanding of their perspectives in teaching and learning Grade 11 Algebra. The research provided the researcher with an understanding of the situation in the twinned schools individually before twinning, and then subsequently, of the impact of the twinning of the two teachers teaching 11th Grade Algebra. The descriptions assisted the researcher in understanding the strategies the twinned teachers used in their classrooms before the twinning process and how their practices changed by means of their exposure to new practices gained during the twinning process.

4.3.3 QUALITATIVE DATA

Qualitative researchers visit the field of study to collect data so as to answer the research problem (McMillan & Schumacher, 2010:343; 2014:354). In this study, I attempted to collect close information by actually interacting directly with the twinned teachers, which Creswell (2009:173) refers to as a major characteristic of qualitative research. In the natural setting of

this study, I had face-to-face interaction with the twinned teachers twice for semi-structured interviews, and observed the teacher in the experimental group four times, so as to understand how he taught Algebra. Thereafter, the twinned were observed during twinning process in the experimental group for ten lessons. During the study the researcher paid attention to learning about the teachers' sharing of teaching practices, resources and experiences during the twinning process.

In qualitative studies, the data are collected through examining documents, observing behaviours, or interviewing the participants in a research setting (Creswell, 2009:173; Cohen et al., 2011:316). In this study, in order to get meaningful results (Le Voi, 2002:1457; Patton, 2002:1457), the researcher attempted to sensitise himself to participant prejudice, of which they might not have been aware, such as stereotypes, expectations and privileges (Ponterotto, 2002:398). Although qualitative researchers may follow procedures in the study, they ought to develop instruments for data collection so as to guide them on the type of data to be collected (Creswell, 2009:173; McMillan & Schumacher, 2010:343, 2014:354; Gay et al., 2011:252), in order to solve the research problem.

The interviews conducted in this research describe the experiences of the participants, and the interviewers also reflect on the descriptions of the interview data (Cohen et al., 2011:316; De Vos, et al., 2005:334; Tashakkori & Teddlie, 2010:253). Babbie (2007:113) and McMillan and Schumacher (2014:364) define an *interview* as a conversation between the researcher and the research participants, the purpose of which is to gather information. Interviews were conducted in this study to make sense of how the twinned teachers viewed this twinning, and how it made an impact on their teaching practices, as well as on the learners' performance in the teaching and learning of Grade 11 Algebra. Interviews are interactions between the interviewer and the interviewees, not merely conversations, according to Denscombe (2001:275) and Kumar (2012:790). In this study, the interviews were planned, so as to know and understand what type of data had to be collected during the interview sessions.

Qualitative observations include the researcher visiting the research site in person, so as to observe and take notes on the behaviour and activities of the participants (Creswell, 2013:47; Cohen et al., 2010:316; Gay et al., 2011:252). Gibson and Brown (2009:207) have argued that observational research can be conducted for many reasons, but is very often a part of general interest in understanding, for one reason or another, what people do and why. In this study, classroom (lesson) observations were conducted by the researcher, and field-notes were kept. Multiple forms of qualitative data were gathered, rather than relying on a single

data source. The data-collection instruments used will be discussed in greater detail in the research design section of this chapter.

4.4 QUANTITATIVE METHODS

Quantitative approaches follow a positivist paradigm, where science quantitatively measures independent facts about a single apprehensible reality (Healy & Perry, 2000:122; Tashakkori & Teddlie, 2010:345). Quantitative approaches constitute observations, measurements and quantity of facts that are objectively observed in a study (Johnson & Christensen, 2012:225; Seers & Critelton, 2001:495). Quantitative methods mostly allow for deductive reasoning, the scientific testing of hypotheses and standardised data-collection, usually from a large number of respondents amenable to statistical analysis (Johnson & Christensen, 2012:225; Johnson & Onwuegbuzie, 2004:18; Tashakkori & Teddlie, 2010:345).

Although this study employed some quantitative techniques, it shared none of the positivists' assumptions described above. The object of the quantitative aspects of the study is to provide data for triangulation from the qualitative data generated, in order to attempt to answer the following sub-questions:

- What is the background situation of the selected schools in mathematics achievement?

4.5 MIXED-METHODS

The rationale for mixing both kinds of methods within one study is grounded in the belief that often, neither qualitative nor quantitative methods are sufficient by themselves to compare the trends and details of a particular situation (Ivankova, Creswell & Stick, 2006:8; Gay et al., 2011:252). Researchers (Caracelli & Grane, 1993:200; Miles & Huberman, 1994:41; Green & Caracelli, 1997:7; Tashakkori & Teddlie, 2010:559; Gay et al., 2011:252) agree that when qualitative and quantitative methods are used in combination, both methods complement one another, and allow for a more robust analysis, taking advantage of the strength of each method. Johnson and Christensen (2012:225) postulated that when combining two methods with different strengths and weaknesses in a research study, one can make less mistakes in the process of the research.

There are many reasons that can be provided for conducting a mixed-methods study. For the purpose of this study and its design, the main rationale for undertaking a mixed-methods study, adapted from Bryman (2006:100), are as follows:

- triangulation: this allows for a greater validity in a study by seeking corroboration between qualitative and quantitative data;
- completeness: using a combination of research approaches provides a more complete and a comprehensive picture of the study phenomenon;
- answering different research questions: Creswell and Plano Clark (2007:8) argue that mixed-methods research helps to answer research questions that cannot be answered by either the qualitative or quantitative methods alone, and provides a greater repertoire of tools to meet the aims and objectives of the study;
- explanation of the findings: mixed methods studies can use one type of research, either the qualitative or the quantitative method to explain the data generated from a study using the other research approach. For example, findings from a quantitative survey can be followed up and explained by conducting interviews using the sample of those surveyed to gain an understanding of the findings obtained;
- illustration of data: using a qualitative research approach to illustrate quantitative findings can help paint a better picture of the phenomenon under investigation. Bryman (2006:100) uses the analogy that this is similar to putting ‘meat on the bones’ of ‘dry’ quantitative data; and
- instrument development and testing: a qualitative research study may generate items for inclusion in a questionnaire to be used in a quantitative phase of a study. These points identify the usefulness that a mixed methods research approach can have in answering (a) particular research question(s) and providing a rationale for using mixed methods in this study.

4.5.1 MIXED-METHODS APPROACHES

The purpose of this study was to explore the effectiveness of twinning two mathematics teachers teaching Grade 11 Algebra, with the aim of changing the teacher’s own practices through new practices gained during the process. This exploration of the effectiveness of this study enabled the researcher to understand the sharing of practices, and new knowledge and expertise during the twinning process. To address the above aim of this study, a mixed methods approach was used, that is, by gathering quantitative and qualitative data using tests, semi-structured interviews and classroom observations. This data was triangulated in order to make a stronger case in terms of the explanatory quality of this study (McMillan & Schumacher, 2014:364); to provide a better argument (Creswell, 2008:765; Wilkins & Woodgate, 2008:25); and to produce a better understanding and verification of the data

(Creswell & Garrett, 2008:322; Creswell & Plano Clark, 2007:27; Wilkins & Woodgate, 2008:25).

Mixed methods approaches intertwine both qualitative and quantitative methods (Gay et al., 2011:252; Litchman, 2010:17; McMillan & Schumacher, 2014:365). The two methods are not parallel, but are an attempt to mix the best of qualitative and quantitative research design (Litchman, 2010:17). Gilbert (2006:305) suggested that a mixed-methods approach intensifies the effects and enriches the adaptability of the research design.

It is proposed by some researchers that mixed methods approaches may be the third paradigm capable of bridging the gap between qualitative and quantitative approaches (Johnson & Onwuegbuzie, 2004:19; McMillan & Schumacher, 2014:364). These researchers feel that the field of mixed-methods research not only moves beyond sterile, qualitative versus quantitative arguments, but makes explicit the usefulness of fusing competing paradigms, and helps to identify how these approaches can be used together, in a single study, so as to maximise the strengths and minimise the weaknesses of the two approaches (Johnson & Onwuegbuzie, 2004:20; McMillan & Schumacher, 2014:364).

4.5.1.1 TRIANGULATION

Concurrent triangulation design (Creswell, Plano Clark, Gutmann & Hanson, 2003:212; McMillan & Schumacher, 2010:407; 2014:365) implies that qualitative and quantitative data occur at the same time, with both methods usually given equal weight (Doyle, Brady & Byrne, 2009:176), and being of equal status in design.

In other words, researchers use both qualitative and quantitative approaches equally, so as to understand the phenomenon under study (Teshakkori & Teddlie, 2010:559; Gay et al., 2011:252). Johnson & Christensen (2012:230) have argued that researchers seek convergence and corroboration of results from different methods studying the same phenomenon. Triangulation is used when the strengths of one method offset the weakness of the other, so that together they provide a more comprehensive data set (McMillan & Schumacher, 2010:407; 2014:364). Creswell and Plano Clark (2007:6) postulated that data are collected simultaneously from two separate approaches, and they are then merged by either combining the data in the analysis stage, or by bringing the separate results into the interpretations.

Doyle et al. (2009:176) explained that within the data transformation model, qualitative and quantitative data are gathered concurrently. After the initial analysis, the data are transformed, either by quantifying the qualitative data or qualifying the quantitative data. In this study, commitment to equal representation of qualitative and quantitative approaches is

not guaranteed. Instead of attempting to distribute the qualitative and quantitative contribution artificially to the mixed-methods design chosen for this study, a position or stance of equal value is adopted (Morse, 1991:21). From this stance, neither approach inherently overrides the other, because the contributing methodologies are equally valued throughout the research process.

4.5.1.2 EXPLANATORY DESIGN

The *explanatory design* is described by Creswell (2014:224) as a ‘sequential explanatory design’, which involves two distinct phases, in which the researcher starts to collect quantitative data in the first phase, analysing the data thereafter, and then using the results to build on qualitative data (Creswell & Plano Clark, 2007:7; McMillan & Schumacher, 2010:401, 2014:365; Teshakkori & Teddlie, 2010:339). The qualitative (text) data help to explain in detail the initial quantitative results in the first phase (Ivankova, Creswell & Stick, 2006:12). The intent of this approach is that the quantitative data and its subsequent analysis provide a general understanding of the research problem and also alert the researcher to the types of questions they might ask when collecting qualitative data.

The quantitative and qualitative databases are analysed separately in the explanatory sequential design (Creswell, 2014:224). Creswell and Plano Clark (2007:7) concur that the data are collected and analysed quantitatively in the first phase, followed by the process whereby data is collected and analysed qualitatively in the second phase (McMillan & Schumacher, 2010:401, 2014:364). Although the explanatory design is considered to be the easiest method to implement, it requires a longer implementation time, due to its sequential nature. Because of this limitation, the explanatory design was not used in this study to collect and analyse the data.

4.5.1.3 QUASI-EXPERIMENTAL DESIGN

In the quasi-experimental design the researchers make use of experimental and control groups but do not assign participants to groups (Cresswell, 2009:7; Johnson & Christensen, 2012:31; McMillan & Schumacher, 2010:258, 2014:364, Gay et al., 2011:255). Both the experimental and control groups write a pre-test and a post-test, but only the experimental group receive the treatment. In this study, the twinned teachers met to plan how the twinning could be implemented in the experimental group, and how they could share their experiences, skills and expertise in teaching Grade 11 Algebra. That is, the teacher from the well-performing school visited the teacher at the poor-performing school for lesson presentations. The twinned teachers were introduced to twinning as a strategy for change in practice by the exposure to the new practices gained during the process. The strategies were introduced in order for the

teacher in the poor-performing school to employ those strategies learned during twinning in teaching Grade 11 Algebra. Twinning was mainly used through the twinned teachers' discussion during their meetings and collaborative teaching at the poor-performing school. The teacher from the well-performing school presented lessons at the poor-performing school in the presence of the other teacher observing him.

4.6 RESEARCH DESIGN

In this study, an intervention strategy was introduced that explored the effectiveness of twinning two teachers teaching Grade 11 Algebra. The intervention strategy focused on understanding the sharing of teaching practices, that is, sharing new knowledge and expertise of those teachers. The purpose of introducing twinning as a strategy was to change the teachers' own practices with the exposure to the new practices gained during the process in the non-performing school. In the twinning process, the twinned teachers were expected to share their experiences in teaching and learning 11th Grade Algebra.

On the other hand, the twinning process focused on the algebra topics with which teachers were uncomfortable, especially the teacher in the non-performing school. Those topics were, namely, the drawing of graphs, the interpretation of graphs, and the transformation of those graphs, solving equations, number patterns, financial mathematics and word problems. The teachers' subject content knowledge; pedagogical content knowledge; their new knowledge gained; their exchange of ideas and the opportunities for the teachers to be engaged in joint planning in the twinning process, were considered.

Furthermore, the teaching resources the twinned teachers shared during the process of twinning for the change of practice by means of the new exposure to teaching were discussed. The issue of teaching resources enabled the researcher to understand it as one of the factors that could have played a role in the poor performance of the learners in the non-performing school. It was understood that resources also play a fundamental role in the learners' performance in the well-performing school. This study focused on these resources in the well-performing school before the twinning process, in understanding the factors that assist in the good performance of the learners in mathematics.

Prior to twinning, the teachers' teaching approaches used to teach the Grade 11 Algebra was investigated in the experimental group through observations. The strategies used by those teachers during the teaching and learning of algebra, how they interacted with the learners, the type of remedial support provided to the learners, feedback to the learners during teaching and learning, and how their learners are assessed, were all observed. The study observed the

type of teaching materials used by the twinned teachers, and the way in which they used them to teach algebra.

4.6.1 DESIGN TYPE

A *research design* is a plan or a road map that indicates how the researcher will address the research problem. This study used a pre-test-intervention-post-test design (Johnson & Christensen, 2012:31; Tashakkori & Teddlie, 2010:353). Firstly, the researcher investigated the situation in terms of learners' performance in the pre-test of the Grade 11 English First Additional learners solving algebra problems. The study was divided into two phases, namely Phase 1, before the twinning process, and Phase 2, during the twinning process. Since this study was at a teacher level, the twinned teachers were interviewed individually after the pre-test, where the interview questions informed by the results. The purpose was to ascertain the challenges their learners faced in solving algebra problems and why the learners have those problems.

i. Pre-testing and post-testing

A pre-test is an instrument providing a measure on some attribute or characteristic that one assesses for participants in an experiment before a treatment, whereas a post-test is a measure on an attribute that is assessed for participants in an experiment after a certain treatment (Gay et al., 2011:318; Johnson & Christensen, 2012:34). Barbie (2007:223) and Johnson and Christensen (2012:34) explained *pre-testing* as a measurement of a dependent variable among subjects, and *post-testing* as a measurement of a dependent variable among subjects, after they have been exposed to an independent variable. This study administered a pre-test to measure the performance of the learners in algebra before the intervention to understand the challenges the learners faced. Subsequently, a post-test was administered to measure the performance of the learners after the implementation of the twinning process.

In this study, the pre-test and post-test were administered on the same day in the same order in the twinned schools so that there was uniformity in its application. Both the two mathematics teachers in the well-performing and the poor-performing school assisted in invigilating the learners in their respective schools during the administration of the pre-test and the post-test. The administration of the pre-test and the post-test on the same day assisted the researcher in collecting data that appeared to be valid and reliable, as the results were appeared to be free from contamination. All the learners in the well-performing and the poor-performing schools wrote the tests and submitted their scripts at the same time.

The two groups of learners played different roles in the study, where the learners in the good-performing school were used as a control group, and the learners in the non-performing school were the experimental group. The teachers in all the schools were on site to clarify any emerging questions and/or problems encountered in the algebra content.

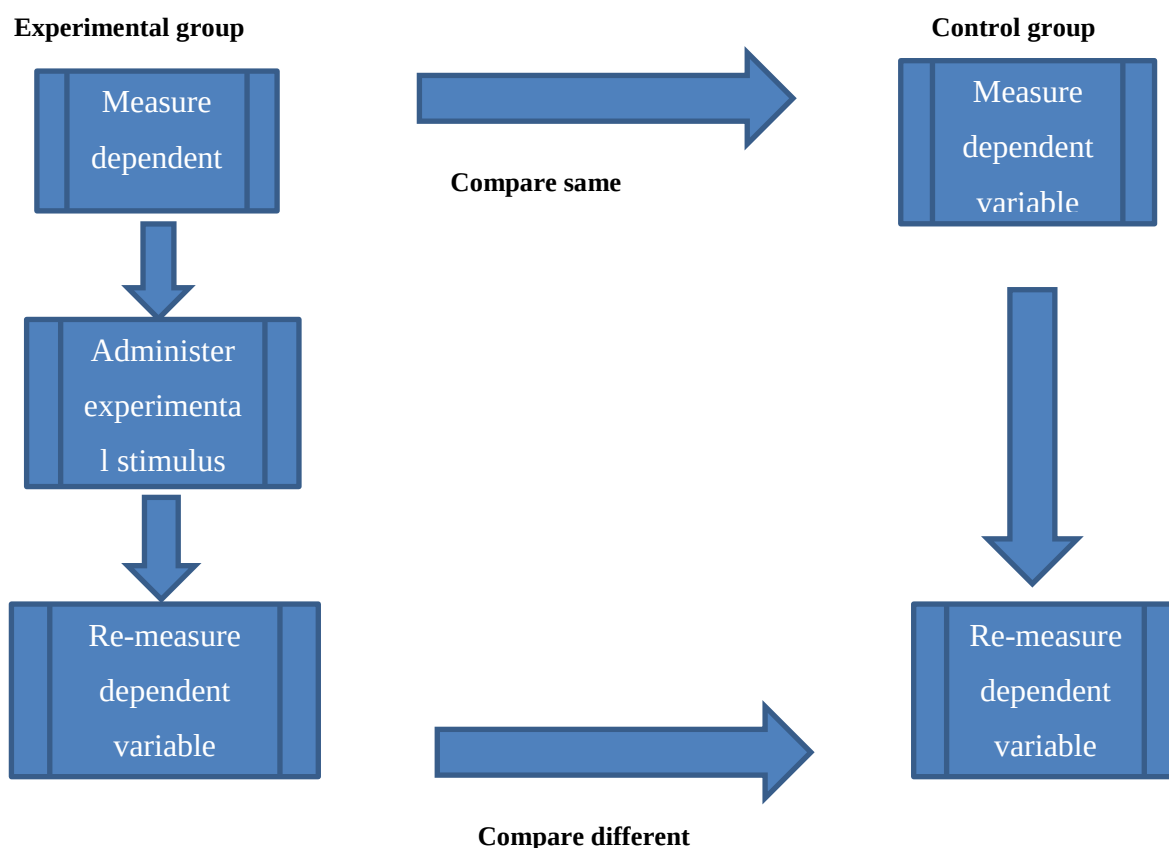


Figure 4.1: Diagramme of the basic experimental comparison design. Adapted from Barbie (2007:223)

This diagramme provides a better view of how the administration of the pre-test and the post-test works in the field of research. The experimental group and the control group's pre-test and post-test sessions were administered at the same time, without interventions. Consequently, the post-test was administered in both schools simultaneously after the experimental group had received treatment or intervention. The two groups were initially given a pre-test simultaneously, without any intervention, where later, the experimental group received a treatment or had an intervention during the twinning process. Both groups wrote a post-test.

Table 4.1: The structure of the design of the pre-test and the post-test

Items	Meanings
1. Quadratic equations	Finding factors of equations either by factoring, completing a square or using quadratic formula
2. Simultaneous equations	Solving two equations at the same time by making one of them the subject of the formula and subsequently substitute one of the chosen variables in another equation
3. Linear inequalities	Finding a point lying between two intervals
4. Word problems	Translating words into mathematical equations
5. Number patterns	Following the sequence of numbers to develop a general term
6. Financial mathematics	Calculating monthly instalments and investments
7. Functions	Interpreting graphs with given points and equations, finding equations, ranges and domains

The pre-test and post-test consisted of six sections of the Grade 11 Algebra with six questions, in total 28 questions. Those sections, amongst others, included quadratic equations (QE) with two items; linear inequality (LI) with one item; simultaneous equation(s) [SE] with one item; word problem(s) (WP) with one item all of which were sub-sections of Section 1; number patterns (NP) with four items in Section 2; financial mathematics (FM) with two items in Section 3; exponential expression(s) [EE₁] with one item; exponential equations [EE₂] tasks with two items were sub-sections of Section 4; hyperbolic function(s) [HF] were in Section 5; and exponential function(s) [EF] was a sub-section of the Section 4 task with five items, and the parabolic function(s) [PF] task with nine items was(were) in Section 6.

ii. Semi-structured interviews

As noted earlier, interviews were conducted in this study in order to understand the participants' experiences and the meaning they make of those experiences through their descriptions and their reflection on the descriptions (De Vos et al., 2005:301; Cohen et al., 2010:122; Kumar, 2012:791). The face-to-face interviews with the twinned teachers were

conducted twice in the first phase of data-collection, and twice in the second phase. The interviews consisted of semi-structured and a few open-ended questions that intend to access more information from the participants (Creswell, 2009:8; Gay et al., 2011:318; Kumar, 2012:791; Tashakkori & Teddlie, 2010:253). The interviews were recorded with the permission of the twinning teachers and the recorded clips were saved, with back-ups on a memory stick. Field-notes were taken in order to compare them with the transcriptions, to maintain a level of accuracy. The interview process was flexible with regard to the sequence of the topics that were addressed (Denscombe, 2008:278).

iii. Classroom observations

Observational research may be conducted for various reasons. It is very often part of a general interest in understanding, for one reason or another, what people do, and the reason why (Creswell, 2013:47; Gibson & Brown, 2009:207; Johnson & Christenson, 2012:34). Johnson and Christensen (2012:34) defined *observation* as a systematic and accurate collection of data that lead to informed judgement, in order to make necessary changes in practice. In other words, observation in research displays several features, such as the collection of evidence, the analysis or examination of evidence, and the formation of significant judgement, based on the evidence and the subsequent implications, such as changes in and the improvement of practices. This study used the adapted classroom observation schedule from Sepeng (2010:64) to collect the data in the twinned schools (Appendix G). This schedule was initially checked and studied to see if it could pick up the required form of data. Gibson and Brown (2009:207) argued that the classroom observation schedule ought to satisfy the following requirements:

- check that they are sensitive and pick up the required form of data;
- check that there are no issues that may be irrelevant and that are not included in the schedule; and
- ensure that it can easily be interpreted and followed by the researcher.

In conducting classroom observations, the researcher needed to know how the teachers in the twinned schools taught the material, namely their styles, approaches, methods and rapport with the learners. It was also important to understand how receptive the learners were and what was going on in the classrooms during the researcher's stay at the two schools. Such understanding is essential to the refinement of each individual teacher's identity, and to the development of principles for the design of an effective learning environment, guided by the data collected in the process. Researchers and practitioners generally agree that the most

effective use of classroom observations is for the teacher's professional development (Montgenery, 2002:2; Lasagabaster & Sierra, 2011:456).

The teachers in the twinned schools were assured that the classroom observations were being conducted solely for the purpose of the research. In addition, these observations were not meant to evaluate them, but to observe the events in their classrooms. The teachers were informed that the observations would be conducted over a certain time frame. Furthermore, a congenial relationship was established with the teachers in the twinned schools as seen as a basic step, as observations has to be built on a foundation of trust. The two teachers' voices were prioritised, in order to boost and facilitate their participation in the observation activities, by unravelling the conditions that may lead to mutual confidence between the teachers and the researcher before the twinning process, and between the two teachers during the twinning process. Aubusson, Steele, Dinham and Brady (2007:139) stated that it is necessary to build a climate of trust during classroom observations.

However, it has been show (Aubusson et al., 2007:139; Borich, 2008:55) that many teachers, even experienced teachers, dislike and even fear being observed, and they find classroom observations stressful and intimidating (Lasagabaster & Sierra, 2011:456). Lasagabaster and Sierra (2011:456) furthermore noted that many professionals express their anxiety and worry when it comes to classroom observations, although formal observations and feedback are integral to improving teaching and practice (Johnson, 2008:24). Borich (2008:55) further argued that both in-service and pre-service teachers are not accustomed to being observed when teaching, and that mere observation provokes uneasiness, nervousness and tension amongst them, in the belief that their professional competence may be questioned or judged. These challenges were considered during the twinning process. The teachers in the twinned schools were made to feel comfortable psychologically, trust being the fundamental objective before the benefits of twinning could be reaped.

The baseline observations were conducted just before the beginning of the twinning process in the first week of February 2015 after schools re-opened, with the object of understanding the nature of the instructions in the 11th Grade mathematics classrooms of the experimental group. The researcher also wanted to understand how the teacher in the experimental group interacted with his learners, questioning styles, how he introduced lessons, assessment, classroom activities and remedial support to under-achieving learners. The data collected in the experimental group from the classroom observations advised me and informed the planning and the implementation of the twinning process in this study, by making the process

conducive to the participants. The classroom observations were only conducted in the experimental group as the intervention was meant for that group.

The observations were conducted with the purpose of measuring the teachers' implementation of the new strategies gained during the twinning process. The teacher from the experimental group had a meeting with the teacher from the control group, sharing their teaching strategies and resources to teach algebra. These two teachers also observed each other during the process of twinning gain new knowledge and skills of teaching the Grade 11 Algebra. The aim of sharing new teaching practices and resources was to develop and improve the approaches of algebra of learners, especially in the experimental group.

The focus on the new strategies gained by the two teachers from the twinned schools was on how they teach, and how they interact with the learners. The study also aimed to source the teacher from the control group to share his teaching strategies with the teacher of the experimental group, namely with regards to how he maintains the good performance of the learners in mathematics. The main factors in sharing teaching strategies were based on the classroom environment and instructions. They are outlined briefly in table 4.2 below.

Table 4.2: Classroom environment and instructions (adapted from Sepeng, 2010)

Classroom environment	Instructions
- Creating an environment of respect and rapport	- Communicating with the learners
- Establishing a culture of learning	- Using questioning and discussion techniques
- Managing classroom procedures	- Engaging the learners in learning
- Managing the learners' behaviour	- Using assessments in learning
- Organising a physical place	- Demonstrating flexibility and responsiveness

These domains are essential for the teachers to share during teaching in order to improve their own practices through the exposure of the new practices gained during the twinning process. The classroom environment domain sought to enable the teacher in the experimental group to learn how the teacher in the control group creates, manages and organises his class. On the other hand, the teacher from the experimental group also learned how the teacher in the control group communicates, how he questions and discusses with the learners, and how he engaged and assesses the learners during classroom instructions.

iv. Sampling

Sampling refers, in broad terms, to the points of data-collection or cases to be included within the research project (Creswell, 2013:48; Gibson & Brown, 2009:210; Kumar, 2012:792). These points of data may be a person, a document, an institution, a setting, or any instance of information or gathering of data. In quantitative studies, it refers to the selection of people to participate in a research project, usually with the goal of being able to use these people to make inferences about a large group of individuals (Creswell, 2009:145; Johnson & Christensen, 2012:12; Tashakkori & Teddlie, 2010:353).

A sample is a sub-set of the population (Cohen et al., 2011:122; Gay et al., 2011:318; Howell, 2004:175), drawn from a setting or research site. A *population* is the entire group of people in a setting in which a researcher may take interest (Johnson & Christensen, 2012:12; Howell, 2004:78; McMillan & Schumacher, 2014:365; Tashakkori & Teddlie, 2010:353). The aim of sampling in quantitative research is to select people that can represent the population. A *random sample* is a sample where every individual of the population stands a chance to be selected in the sampling process (Gay et al., 2011:318). Gibson and Brown (2009:218) argued that where the generalisation of findings is a key concern, then the issue of representativity can become important. Random sampling was not used in this study, as there was no intention to generalise the results.

This researcher made use of purposive sampling, because they characterise roles, opinions, knowledge, ideas or experiences that may be particularly relevant in this research (Cohen et al., 2011:124; Johnson & Christensen, 2012:12; Gibson & Brown, 2009:212; McMillan & Schumacher, 2014:365). The sample consisted of the two Grade 11 classes and their mathematics teachers from rural secondary schools, one an experimental group and the other a control group. Both these schools are situated in the district of Polokwane. The experimental group is the poor-performing school, whereas the control group is the well-performing one. In addition, the schools are both public and previously marginalised schools. The schools draw learners from a low socio-economic status. Sepedi is the mother tongue of the learners, viz. the language that the learners use at home, as well as when they play and communicate informally at school.

4.6.2 THE DATA-GENERATING INSTRUMENT

The data-collection instruments described below were used in an attempt to address the primary research question. Quantitative data were gathered by means of pencil and paper, with learners written (pre-test and post-test, appendices H). A classroom observation schedule (Appendix G), adapted from Sepeng (2010) as mentioned earlier, was used to

generate both qualitative and quantitative data. The qualitative data were collected by means of a semi-structured teacher interview schedule, developed by the researcher after their lesson presentations. The secondary research questions, which underpin the primary research question, were also addressed by these instruments, which are discussed in the next sections.

4.6.2.1 PRE-TEST AND POST-TEST

As noted earlier, the pre-test was divided into five sections with a total of six questions (or tasks), namely quadratic equations (QA) and linear inequalities (LI), number patterns (NP), financial mathematics (FM), exponential expressions (EE₁), exponential equations (EE₂), exponential functions (EF), and algebraic functions (AF). The questions used in the pre-test were piloted first before they could be given to the learners in the experimental and control groups. These question items satisfy the assessment standards as reflected in the South African Curriculum and Assessment Policy Statement (CAPS, 2012).

The purpose of this pre-test was to help the researcher to measure the learners' test performance when solving algebraic problems. The problems the learners experienced in solving algebraic problems were also considered. The problems found in the learners' work subsequently assisted the researcher in identifying the problems they may have mathematically (how they go about solving the problems) and situationally, in response to tasks with real life problems. The post-test investigated whether there was any changes in terms of learners' performance after the twinning process on the experimental group, and possibly why there were changes.

4.6.2.2 SEMI-STRUCTURED INTERVIEWS

The interview sessions were conducted in two phases: the first phase was for the teacher in the experimental group and the one in the control group, who were purposively selected to participate in this study. At this stage, there were no measurable constructs, and therefore, the interview responses were used to analyse inductively the emerging constructs before the twinning process. The open-ended, unstructured and semi-structured interviews, which are described below, were then used to clarify and understand the issues emerging from the pre-test results. The two teachers from the twinned schools were interviewed after capturing the results of the pre-test. The following questions were asked of the teachers in the experimental group:

1. What do you think can be a challenge for the poor performance of the learners? Why?
2. How can these challenges be addressed?
3. What kind of support do you give learners with difficulties?

4. What kind of support do you get from your school?
5. How can you explain your classroom environment in general?
6. How do you establish a culture of learning in your mathematics classroom?
7. How do you engage your learners during teaching and learning?
8. What type of assessment do you use in your mathematics classroom?
9. How often do you assess your learners?
10. How is the communication between you and the learners, and between the learners?

These interview questions were used to understand the situation in each school. These open-ended interview questions were also used to measure the extent to which the teacher's professional development (TPD) in each school influenced their current practices regarding the teaching and learning of mathematics. The questions were as follows:

1. Is there any teacher professional development (TPD) in your school? If yes, what kind of TPD and how does it help you?
2. What are the areas of the greatest development need?
3. What is the relation between the support received and the levels of participation?
4. What is the impact of TPD in your school?
5. What sorts of barriers are experienced in the TPD?
6. What do you think can be done to alleviate those barriers?

The interviews were conducted with the aim of distinguishing between and comparing the perceptions of the two teachers from the twinned schools so that the results of the survey could guide and influence the planning and implementation of the twinning process in the experimental group. The interview responses were sorted into themes that are discussed in the qualitative results section of this study. The reasons provided by the teachers about the teaching and learning of Grade 11 Algebra were analysed qualitatively. A classroom observation schedule was used to triangulate the qualitative data gathered from interviews, and is described in the next section.

4.6.2.3 CLASSROOM-OBSERVATION SCHEDULE

Qualitative observations are where the researcher takes field-notes of the behaviour and activities of individuals at the research site (Creswell, 2009:148; Johnson & Christensen, 2012:13). In this study, classroom (or lesson) observations were done by the researcher and field-notes were taken. This study adapted Sepeng's (2010) classroom observation schedule, as indicated earlier during the lesson observations in the twinned schools before and after the twinning process.

Qualitative researchers collect data at the site where the participants experience challenges, issues or problems (Cohen et al., 2011:124; Creswell, 2009:148; Kumar, 2012:791). This study gathered information by actually interacting with the teachers, talking directly to them, and seeing them behave and act within their context, which Creswell (2009:148) refers to as major characteristics of qualitative research. The researcher had face-to-face interaction with teachers in the natural setting over time, observing how they taught mathematics and how they interacted with the learners.

In the entire process of this study, the researcher kept focusing on learning the meaning that the participants held about the teaching and learning of algebra, not the meaning that he brought to the research, or that the writers express in the literature (Creswell & Plano Clark, 2007:72; Marshal & Rossman, 2006:97). The classroom observation schedule, as adapted from Sepeng (2010), focused on the following:

- observing classroom practice (the teaching and learning of algebra, the classroom arrangement, the teaching approaches, methodology, dealing with the correct and incorrect responses of the learners);
- observing learners' practical skills (includes, amongst other things, solving problems, defining and describing algebraic terms, the use of technology, decision-making in solving problems);
- evaluating skills (the learners evaluating their own work, self-assessment, identifying the errors committed, the use of alternative ways to solve problems);
- reflection (reflecting on errors committed in solving problems, selecting and using appropriate methods in solving problems, reflecting on decision-making);
- teacher promoting learners' small group discussion (collaborative task – paired activities, group presentations, argumentations); and
- learner responses (individual, group, paired, hands-up, at the chalk board, verbal, in writing, the negotiation of meaning, etc.).

A four-point scale was used in this study in designing the instrument, the classroom observation schedule adapted from Sepeng (2010). The instrument has spaces available for the observer to record the name of the school, the physical and postal address of the school, the telephone and fax numbers, the name and gender of the principal, the name and gender of the teacher, the grade observed, and the number of learners in that grade. The classroom observation schedule was designed according to the points discussed above for collecting the data in the twinned schools.

4.6.3 ANALYSIS OF DATA

The analysis of data implies when the researcher transforms raw data into meaningful data, or the researcher makes sense of the data (Cohen et al., 2010:461, 2011:219; Creswell, 2009:173). The data that is collected in the field requires the skill to depict the understanding of data in writing (Gay et al., 2011:264; Johnson & Christensen, 2012:44). In other words, the data are analysed and interpreted by a process which involves preparing the data for analysis, conducting different analyses, moving deeper and deeper into understanding the data, representing the data, and making interpretations of the larger meaning of the data (Sepeng, 2010:75). In this study the data were represented, prepared by transcribing the data and interpreted them to make more sense of two tests administered, teachers' semi-structured interviews and classroom observations.

4.6.3.1 ANALYSIS OF QUALITATIVE DATA

The analysis of the qualitative data can be done concurrently with the gathering of the data, making interpretations and writing reports, according to Creswell (2009:9). This study followed Creswell's steps in analysing the qualitative data from specific to general, involving multiple levels of analysis. Figure 4.2 shows a linear, hierarchical approach, building the organisation of data from the bottom to the top. Creswell (2009:9) views this approach as being interactive in practice, interrelated and not always visited in the order presented. In this research, the analysis of the qualitative data involved gathering open-ended data collected from semi-structured interviews and classroom observations, based on asking the teachers general and specific questions. Qualitative data was analysed by using the information supplied by the two teachers before and during the twinning process.

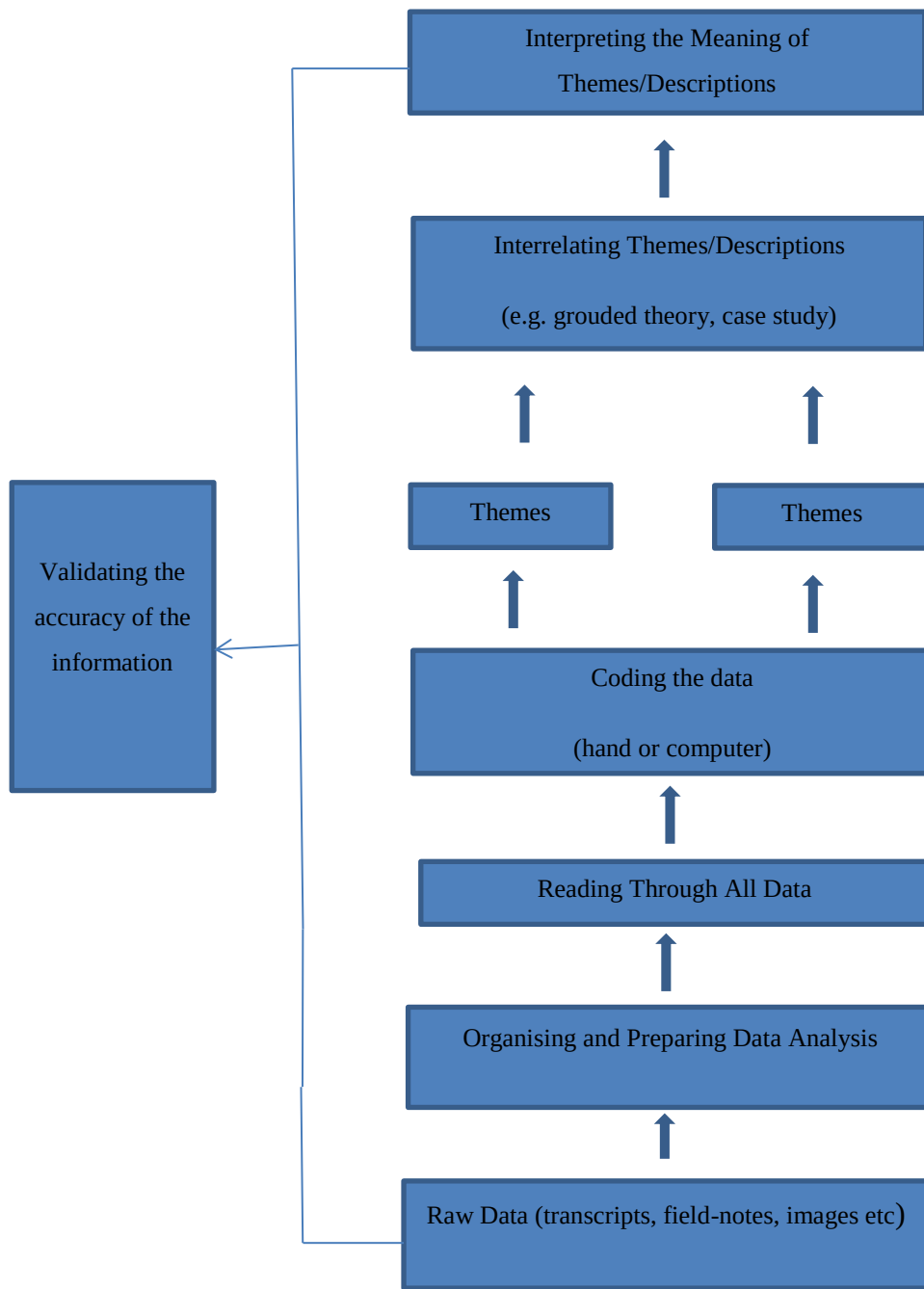


Figure 4.2: Data analysis in qualitative research (Creswell, 2014:197)

In order to reduce and organise the content of the qualitative data to become more manageable and meaningful, the researcher was guided by the following processes, as cited in Creswell (2014:197):

- The researcher organised and prepared the data for analysis. This involved optically scanning all the observations, and the field-notes, and transcribing the interviews with the teachers.

- He then read through all the data in order to obtain a general sense of all the information and to reflect on its own overall meaning.
- He chose one outstanding and interesting interview form the interviews with the teachers, read through it carefully and in the process gaining knowledge about its meaning.
- After repeating the procedure in 3 above for all participants (or transcriptions), he then drew up themes, with similar themes grouped together.
- The themes were arranged and then coded next to the appropriate segments of the text. This was done in order to determine the possible emergence of new categories.
- He then identified the most descriptive wording for his themes and placed them into categories.
- He then decided on the numbering and abbreviations of all the categories.
- The data for each category was put together in one place and the preliminary analysis was done.
- Extant data was re-coded where necessary.

According to Creswell (2009:197), these steps engage a researcher in a systematic process of analysing textual data. The coding of the data followed Creswell's (2014:197) encouragement to qualitative researchers to analyse their data for material that can address the following:

- Codes on topics that the readers would expect to find, grounded on previous or existing literature.
- Surprising codes that were not predicted at the beginning of the study.
- Codes that cover a prominent theoretical perspective in the research.

For the purpose of coding in this study, a combination of pre-determined and emerging codes was used with the guidance of the supervisor.

4.6.3.2 ANALYSIS OF THE QUANTITATIVE DATA

The quantitative data were initially examined and organised according to categories in the test items, using the learners' written responses to each test item first coded as correct response (CR), incorrect response (IR), incomplete response (InR), and blank response (BR) (Didis & Erbas, 2015:1141). The general view of the performance, the percentage of the coding, was calculated using the absolute numbers or frequencies. Didis and Erbas (2015:1141) argued that responses that are not totally completed, including the ones used to follow the mathematically correct procedures, are categorised as incomplete responses. Furthermore, these researchers argued that those learners who set up correct equations to

solve mathematical problems and who could not solve the problems were also seen as giving incomplete responses. The rationale for categorising the learners' responses was to have a descriptive picture of the learners' solutions to the problems in Grade 11 Algebra.

The descriptive statistics generated from the pre-test and the post-test data are discussed in the light of the research objectives of the study. The statistical data are analysed, using the Wilcoxon Rank-Sum (Mann-Whitney) test to test the statistical significance, *using p -value* < 0.05 regarded as significant at 95% confidence limit. The statistical software package that was used to analyse the collected data is Stata V13. The Mann-Whitney U test (also called the Mann-Whitney-Wilcoxon (MWW), Wilcoxon rank-sum test, or Wilcoxon-Mann-Whitney test) is defined as a non-parametric of the null hypothesis, that two samples come from the same population against an alternative hypothesis, especially that a particular population tends to have larger values than the other. Therefore, we used Wilcoxon rank-sum test to assess the nett effect of twinning, and the interpretation of the results was performed at $\alpha = 0.05$ error rate. Thus, the results were declared significant if $p < 0.05$ Wilcoxon Rank-Sum (Mann-Whitney) test (Field, 2013:228). This study compared two different groups that performed differently in the pre-test and post-test, and also gave different results. The statistical results did not use ANOVA, as the data were not normally distributed between the two study groups, experimental and control groups, and were thus skewed. The statistical results were analysed between the two study groups and within each study group.

The quantitative statistical data generated from the pre- and post-tests ($N = 85$) and the classroom observation ($N=20$) instruments in this study were captured on a Micro-Soft Office Excel spreadsheet, and were not subjected to an analysis of variance (ANOVA) to provide both descriptive and inferential statistics. The statistical technique of Matched-Pairs t -test was also not computed for comparing the rank scores of the control and the experimental groups, as suggested during the process of this study.

4.7 VALIDITY AND RELIABILITY

Qualitative and quantitative methods rely on different degrees of validity and are subject to different threats (Sepeng, 2010:78). Creswell (2005:599) defines *threats* as the problems that threaten our ability to draw the correct and effect inferences that arise because of the experimental procedures or the experiences of the participants. In this study both qualitative and quantitative techniques were used, and both notions of validity and reliability came into play.

4.7.1 PRE-TEST AND POST-TEST

The reliability of the quantitative (pre- and post-test) results can be gauged according to whether a test returns the same results repeatedly. The test was piloted to a school to measure learners' performance and the results were similar as the ones in the poor-performing school. The reliability of the results of this study was measured by using the Wilcoxon Rank-Sum (Mann-Whitney) test, with the data captured through Microsoft Excel Office.

In this study, the collection of the data followed a simple pre-test-intervention-post-test design, aimed at the experimental group, and a pre- and post-test design administered on the control group. The control group were taught the same content by their teacher as followed pace setter provided by DBE. A possible threat to the validity of this design was that the participants might have remembered the responses on the post-test from the pre-test (Sepeng, 2010:82). However, because of the time-gap between the two tests, it is probable that this situation was not applicable to the experimental and control groups.

However, this possibility was unlikely to affect the results of the twinning process, as merely passing-on information would not be sufficient (as compared to the treatment given to the experimental group) to threaten the validity of the exercise. Furthermore, the learners from the control group were not merely part of the research project school linked to this study, as they were not part of the intervention. Subsequently, none of the control group mathematics learners received any treatment during the twinning process, such as the teacher from the other school teaching, with the other teacher observing the lesson.

4.7.2 INTERVIEWS WITH THE TEACHERS

Qualitative researcher(s) perceive 'validity' (e.g. truth value, credibility, dependability, trustworthiness, generalisability, legitimation, authenticity) as being an unclear and ambiguous concept (Dellinger & Leech, 2007:312; Johnson & Christensen, 2012:35). At least seventeen different terminologies for 'validity in quantitative research' have been documented (Guba & Lincoln, 2005:200; Maxwell, 1992:191), but no agreed-upon definition exists (Dellinger & Leech, 2007:312). However, according to Maxwell (1992:291), *validity* refers to the degree to which the findings described by the researcher are the real presentation of the data gathered.

Creswell (2009:191) and Oppenheim (1992:147) agree that *reliability* concerns the consistency of the measurement or the degree to which an instrument measures the same way each time it is used under the same circumstances. This study used semi-structured teacher interviews, based on the interview protocol developed by Inoue (2005:116). During the

interview sessions, the researcher probed unclear responses to gain an in-depth understanding of the teachers' interpretation, as suggested by Ginsberg (1997:604). Judgement as to the degree of validity and reliability attained in this study are based on Creswell's (2009:191) perspectives.

4.7.3 THE CLASSROOM-OBSERVATION SCHEDULE

As mentioned earlier, the researcher adapted the classroom observation schedule from Sepeng (2010). It was piloted in two different mathematics classroom settings in order to satisfy the requirements suggested by Gibson and Brown (2009:539).

4.7.4 OVERVIEW OF RELIABILITY AND VALIDITY

In order to assess the accuracy of the findings and convince the readers of such accuracy, the researcher incorporated the use of multiple validity strategies, as recommended by Creswell (2009:191):

- The researcher triangulated various data sources of information by testing evidence from the sources and using these to build a strong justification of the themes. This process added value to the validity of this study, because the themes were established based on converging several sources of data and perspectives from the participants.
- He used member-checking to determine the accuracy of the qualitative findings through specific rich descriptions and themes (only polished products of raw data and all transcriptions) by referring back to the participants to find out if they agreed with the accuracy of the findings. This procedure was done through follow-up interviews with the teachers in the study, giving them an opportunity to comment on the findings.
- A prolonged time was spent at the research site and repeated observations were made to further develop an in-depth understanding of the phenomena under study. The procedure enabled the researcher to have more experience with the participants in their natural setting, which gave him more accurate or valid findings.
- The researcher clarified bias that was brought to the qualitative phase of the study. He previously commented on how the interpretation of the findings is shaped and/or influenced by his background, such as culture, history, socio-economic origin, etc.

The primary strategy utilised in this study to ensure external validity was the provision of thick, rich and detailed descriptions, so that anyone interested in its transferability would have a solid framework for comparison (Merriam, 1998:19). Nixon and Power (2007:75) point out that the warranting of claims must fulfil the criteria of trustworthiness, soundness, coherence, plausibility and fruitfulness. *Trustworthiness* refers to the quality of the qualitative

data collected (Anastas, 2004:62), and in the essence of neutrality in the findings or discussions of the study (Guba & Lincoln, 2005:4).

Reliability is the degree to which the instrument measures whatever it is measuring consistently (Best & Kahn, 2003:12). According to Silverman (2000:177), *reliability* refers to the degree of consistency with which instances are assigned to the same category by different observers, or by the same observer on different occasions. Neuman (2003:179) suggested that *reliability* concerns dependability. The dependability of data in this study was established by capturing all the interviews and classroom observations on a tape and transcribing it both manually in writing and using computer software. Attempts were made in the process to reduce the interview scripts as accurately as possible, so as to eliminate possible threats to the reliability of the instruments (Sepeng, 2010:83).

The researcher followed the following procedures, as suggested by Gibbs (2007:235), in order to ensure the reliability of this study:

- he checked all the transcripts for possible mistakes that may have been made during the initial transcriptions;
- during the process of coding, he ensured that the definition and the meaning of the codes were consistent throughout the entire analysis of the qualitative data by regularly comparing the data with the codes and by writing memos about the codes and their definitions; and
- he cross-checked the codes developed within the existing literature by comparing the results that were independently derived.

Attempts were made to include these procedures as evidence, that strove to obtain consistent results in this study.

4.8 ETHICAL ISSUES

According to Barbie (2013a:92) and Barbie and Mouton (2010:81), researchers have a duty and obligation to abide by the code of conduct that governs most professions. Neuman (2003:179) has argued that researchers have a moral and professional obligation to be ethical, even when the research subjects are unaware of or not concerned about ethics. When conducting research, social scientists enter into the private lives of their participants (Berg, 2001:318; Creswell, 2013:53). Researchers have to make sure that the privacy, the rights and the welfare of their participants are guaranteed (Gay et al., 2011:266; Johnson & Christensen, 2012:31; Kumar, 2011:155; McMillan & Schumacher, 2014:364).

In this study, informed consent from the teachers was requested after prior permission to conduct this research. After ethics clearance, the researcher approached the Limpopo Department of Basic Education for permission to visit the schools. After being granted permission and given a permission letter to submit to the school principals, the researcher approached the principals and the Grade 11 mathematics teachers of the participating schools. The roles of the participants, their rights to choose to be participants and to participate or not to in this study were explained to them (Cohen et al., 2011:340; Johnson & Christensen, 2012:31). They were assured of confidentiality, and it was indicated to them that participation was voluntary, and they were given permission to withdraw from the study at any stage, and told that no personal details would be disclosed. The confidentiality of the information collected at the schools was also ensured, and it was indicated that no portion of the data would be used for any purpose other than this research. The consent form was then signed by both the researcher and the two teachers, for the consolidation of the agreement.

Teacher A was approached about twinning due to low performance in mathematics in his school. Learners' performance was discussed with teacher A and he indicated a point that the control group in the vicinity had consistently performed well in mathematics. This intervention was suggested with him if it was possible for him to twin with the teacher from the control group to share teaching practices. Teacher A was made aware that the research focused on his teaching practices and the teaching practices during and after twinning. After data collection, teacher A was consulted to discuss the findings of the study and agreed with those findings that his teaching practices had improved and learned a lot from the teacher B.

4.9 SUMMARY

In this chapter, the researcher discussed the research paradigms of the study, the qualitative, quantitative and mixed method approaches, the research design, the instruments and strategies of gathering the data and the detailed research process used in this study. The methods used are informed and guided by Creswell (2009:191), and explanations of concurrent mixed methods design is followed in addition to the research questions.

The treatment of the data, issues pertaining to validity and reliability, as well as the ethical issues that guided the process were also described, discussed and explained.

The next chapter focusses on the discussion of both qualitative and quantitative data of the study. The data collected from qualitative and quantitative data will then be mixed for triangulation purposes.

CHAPTER 5

RESULTS

5.1 INTRODUCTION

In the previous chapter the researcher discussed the methodology used in this study, namely the mixed-methods approach. This chapter reports on the quantitative and qualitative data generated from the pre- and post-tests, the lesson observations and the interviews.

This chapter is divided into five sub-sections: the analysis of the interviews with the teachers where the interviews are described from initial stage, through the planning twinning stage, during the twinning stage and after the twinning stage; the classroom observations during the twinning stage; the pre-tests and post-tests (the quantitative data); the statistical analysis, and the conclusion of the chapter.

As noted earlier, this study intended to answer the following research questions (RQ):

RQ1: What pedagogical approaches are used by teachers to teach Grade 11 Algebra?

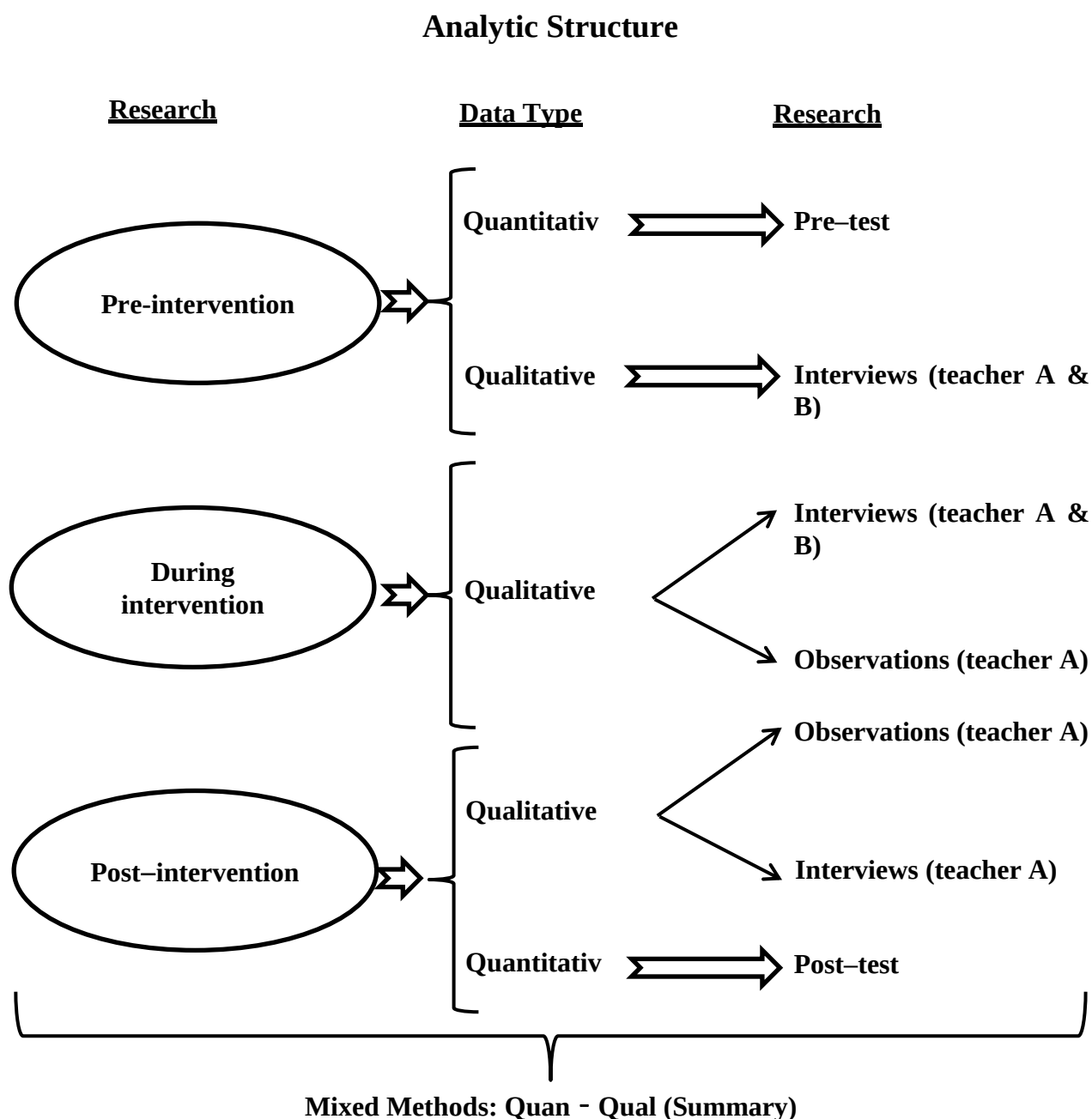
RQ2: What is needed for a successful twinning process and the possible barriers (if any) for implementation?

RQ3: What are the benefits of using twinning as a strategy in a Grade 11 Mathematics classroom?

RQ4: How can effective twinning be developed?

The data-analysis in this chapter followed the following analytic structure designed for the purposes of data-analyses of qualitative and quantitative data. Furthermore, the analytic structure reflects the research stages that match the research methods used in this study, which are mapped against the data set type formed from the qualitative and quantitative phases linked to the research question(s) likely to be answered by the corresponding data set(s).

Figure 5.1: Analytic structure



5.2 PRE-TEST RESULTS QUANTITATIVE ANALYSIS

The quantitative analysis of the pre-test results used Table 5.1-5.6 representing the learners' performances, which depict the data generated from the experimental and control groups on Question 1 to Question 6 (see Appendix O). As indicated earlier, the participating group at the Motlokwa Secondary School is taken as experimental group, whereas the participating group at Mahlong Secondary School is taken as the control group.

The names of schools are pseudonyms. The data generated from the pre-test followed Didis and Erbas's (2014:1141) categories of learners' responses for analysis: *Correct Responses (CR)*, *Incomplete Responses (InC)*, *Incorrect Responses (IR)*, and *Blank Responses (BL)*. As noted in Chapter 4, to obtain the general view of the performance of the learners, the percentages were calculated using the absolute numbers (Didis & Erbas, 2014:1141). This researcher suggested that all the responses that were not totally complete, which included those that followed the correct mathematical responses, were categorised as incomplete responses. In addition, the *incorrect responses* were considered as computationally incorrect answers given by the learners, and the *blank responses* were those that the learners left unanswered. The rationale for using Didis and Erbas's (2014:1141) categories was to get the overall descriptive picture of the learners' responses in the pre-test in both the experimental and the control groups. The pre-test was used to measure the Grade 11 learners' performance in Algebra before the intervention.

5.2.1 LEARNER RESPONSES TO QUESTION 1 ITEMS

Table 5.1 below indicates that the experimental group performed poorly when compared to the control group in response to Q1 question items on equations. The highest percentage for the control group in CR was found to be 69.0%, and that of the experimental group was found to be 11.9 percent. Question 1 consisted of five items, which were coded as quadratic equations 1 and 2 (QE₁ and QE₂), quadratic inequalities (QI), simultaneous equation (SE), and word-problems (WP).

Table 5.1: Distribution of learners' percentages (and absolute numbers) of Correct Responses (CR), Incorrect Responses (IC), Incomplete Responses (InR) and Blank Responses (BR) on Q1 question items for the experimental and the control groups.

Items	Group	Pre-test			
		CR	InR	IR	BR
QE ₁	Experimental	9.5% (4)	19.1% (8)	23.8% (10)	42.5% (18)
	Control	76.2% (32)	9.5% (4)	14.3% (6)	0.0% (0)
QE ₂	Experimental	2.4% (1)	45.2% (19)	52.4% (22)	0.0% (0)
	Control	69.0% (29)	9.5% (4)	21.4% (9)	0.0% (0)
QI	Experimental	11.9% (5)	38.1% (16)	40.5% (17)	9.5% (4)
	Control	54.8% (23)	19.1% (8)	14.3% (6)	11.9% (5)
SE	Experimental	2.4% (1)	42.9% (18)	40.5% (17)	14.3% (6)
	Control	54.8% (23)	9.5% (4)	23.8% (10)	11.9% (5)
WP	Experimental	0.0% (0)	31.0% (13)	28.6% (12)	40.5% (17)
	Control	9.5% (4)	28.6% (12)	23.8% (10)	38.1% (16)
TOTAL	Experimental	5.3% (11)	35.6% (74)	37.2% (78)	21.9% (45)
	Control	52.9% (111)	15.2% (32)	19.2% (41)	12.4% (26)

The data in Table 5.1 indicates that 8.9% of the learners (N=42) in the experimental group solved the question items (TI) for QE correctly and the control group obtained 47.3% of CR. As Watson and Sullivan (2008:115) suggested, the learners should be able to connect concepts and methods when solving the mathematical problems. This suggested that learners should have showed an understanding of finding factors of quadratic equations, solved the value of x and y , analysing and solving the word problem. The learners were unable to factorise QE₁ and QE₂, could not interpret inequality signs for QI, and were unsuccessful in responding to SE and WO. The data showed that the percentage of CR, which ranged between 2.4% to 11.9% of the QE₁, QE₂ and SE₄, respectively for the experimental group, appeared to suggest that the learners could not connect the concepts and the methods when solving Question 1 (see Table 5.1).

The percentage of the control group ranged from 9.5% to 76.2% of CR in QE₁ and WP respectively, which seems to suggest that the results were consistent with the learners' comprehension of the connection of new and old mathematical ideas (Kilpatrick et al., 2001:323). For TI₅ in Question 1 in the experimental group, no learner gave CR, which revealed that neither of the learners could organise the knowledge into a coherent whole, nor connect new and old ideas in the question items. However, the control group produced a low percentage of 9.5% of CR in TI₅, where 90.5% of the learners failed to reach the correct answer, suggesting a lack of logical thinking and an ability to reason the word-problems given to them to solve. The methods the learners used to solve the problems were inappropriate, where a percentage of the learners were found in InR, IR, and BR's 42.5%, 52.4%, and 47.6%, respectively. The results suggest that the learners in the experimental group lacked problem-solving skills when solving simple quadratic equations, quadratic inequalities, simultaneous equation and word-problems.

5.2.2 LEARNER RESPONSES TO QUESTION 2 ITEMS

Table 5.2 below depict the results of Q2 question items coded as number patterns (NP₁-NP₄) which revealed that the experimental group performed poorly in solving number patterns. The results revealed a high BR of 59.5% and 50.0% for the experimental and the control groups.

Table 5.2: Percentages (and absolute numbers) of Correct Responses (CR), Incorrect Responses (IR), Incomplete Responses (InR) and Blank Responses (BR) on Q2 question items for the experimental and control groups.

Items	Group	Pre-test			
		CR	InR	IR	BR
NP ₁	Experimental	64.3% (27)	0.0% (0)	11.9% (5)	23.8% (10)
	Control	81.0% (34)	0.0% (0)	16.7% (7)	2.4% (1)
NP ₂	Experimental	35.7% (15)	7.1% (3)	21.4% (9)	35.7% (15)
	Control	42.9% (18)	11.9% (5)	28.6% (12)	16.7% (7)
NP ₃	Experimental	11.9% (5)	7.3% (3)	40.5% (17)	40.5% (17)
	Control	28.6% (12)	16.7% (7)	23.8% (10)	31.0% (13)
NP ₄	Experimental	2.4% (1)	11.9% (5)	26.2% (11)	59.5% (25)
	Control	14.3% (6)	23.8% (10)	11.9% (5)	50.0% (21)
Total	Experimental	28.6% (48)	6.6% (11)	25.0% (42)	39.9% (67)
	Control	41.7% (70)	13.1% (22)	20.3% (44)	25.0% (42)

The results in the table above revealed that the experimental group's CR category percentage ranged from 2.4% to 64.3%, which showed that the learners were able to determine the next term of the sequence for NP₁, and had difficulties with the number of terms in a sequence. The quantitative results suggest that the experimental group performed better in CR as compared to Q1 question items, suggested that learners displayed correct procedures to determine the next term using the second difference sequence. However, some of the responses found in the InR and IR categories suggested that learners had inadequate knowledge of the procedures, as interpreted with reference to Kilpatrick et al. (2001:323). Most of the learners who gave InR and IR used incorrect procedures to determine the n^{th} and number of terms of the sequence of question items NP₃ and NP₄. NP₄ produced a high percentage (59.5%) of blank responses for the experimental group who did not determine the number of terms than the control group with 50.0%, indicating a lack of formulating, representing and effectively solving problems (Watson & Sullivan, 2008:115).

5.2.3 LEARNER RESPONSES TO QUESTION 3 ITEMS

Table 5.3 below shows the data generated from Q3 question items were coded as financial mathematics: FM₁ and FM₂. The data revealed that learners produced the mathematically incorrect responses (IR) in both the experimental and control groups.

Table 5.3: Percentages (and absolute numbers) of Correct Responses (CR), Incorrect Responses (IR), Incomplete Responses (InR) and Blank Responses (BR) on Q3 question items for the experimental and the control groups.

Items	Group	Pre-test			
		CR	InR	IR	NR
FM ₁	Experimental	7.1% (3)	28.6% (12)	61.9% (26)	2.4% (1)
	Control	19.1% (8)	19.1% (8)	57.1% (24)	4.8% (2)
FM ₂	Experimental	7.1% (3)	21.4% (9)	66.7% (28)	4.8% (2)
	Control	21.4% (9)	11.9% (5)	64.3% (27)	2.4% (1)
Total	Experimental	7.1% (6)	25.0% (21)	64.3% (54)	3.6% (3)
	Control	20.3% (17)	15.5% (13)	60.7% (51)	3.6% (3)

The results in the table above revealed that the learners in the experimental group produced low CR, compared to the control group with a percentage of 7.1% and 21.4%, respectively. Learners appeared to lack the conceptual knowledge of financial mathematics calculating the amount of money accrued using the simple interest formula $A = P(1 \pm in)$ and $A = P(1 \pm i)^n$. Kilpatrick et al. (2001:117) postulated that the learners should have conceptual knowledge to know the methods of solving mathematical problems. In addition, the data revealed a high percentage for IR, with 61.9% and 66.7%, and 57.1% and 64.3% for the experimental and control groups respectively, which showed little understanding of connecting and using methods of calculating the two question items. Again, the learners used the correct formulas to calculate the amount of money in the two question items, but they had incorrectly substituted the value of P, i and n . The learners also revealed a lack of procedures in finding the amount of money that could be paid monthly using the simple interest formula, and also calculating the amount of investment money using the compound interest formula.

5.2.4 LEARNER RESPONSES TO QUESTION 4 ITEMS

Table 5.4 indicates the data generated from Q4 question items, which assessed the learners on exponential expression and equations. The question items were coded as EE_{ex} for exponential expression, while the exponential equations were coded as EE_{eq1} and EE_{eq2}. The results reveal the high percentage of mathematical incorrect responses (IR) in the experimental group as compared to the control group in the three question items.

Table 5.4: Percentages (and absolute numbers) of Correct Responses (CR), Incomplete Responses (IR), Incorrect Responses (IR) and Blank Responses (BR) on Q4 question items for the experimental and the control groups.

Items	Group	Pre-test					
		CR		InR	IR		NR
EE _{ex}	Experimental	2.4%	(1)	7.1% (3)	69.1% (29)	21.4%	(9)
	Control	28.6%	(12)	2.4% (1)	52.4% (22)	16.7%	(7)
EEeq ₁	Experimental	23.8%	(10)	16.7% (7)	47.6% (20)	11.9%	(5)
	Control	61.9%	(26)	9.5% (4)	23.3% (10)	4.8%	(2)
EEeq ₂	Experimental	21.4%	(9)	14.3% (6)	50.0% (21)	14.3%	(6)
	Control	31.0%	(13)	11.9% (5)	50.0% (21)	7.1%	(3)
Total	Experimental	15.9%	(20)	12.7% (16)	55.6% (70)	15.9%	(20)
Total	Control	40.5%	(51)	7.9% (10)	41.9% (53)	9.5%	(12)

The results in Table 5.4 illustrate that the experimental group's CR, which ranged from 2.4% to 23.8%, was lower as compared to the control group's range from 28.6% to 61.9%, showing that learners had a little knowledge of procedures in simplifying EE_{ex}, and solving EE_{eq1} EE_{eq2}. Only 2.4% which is only one of the learners who produced CR and demonstrated a comprehension of exponential expression of the question item EE_{ex} for the experimental group. Despite the fact that Watson and Sullivan (2008) reported that the learners ought to comprehend relevant concepts, operations and relations in order to solve mathematical problems such as determining the EE_{ex}, EE_{eq1} and EE_{eq2}, EE_{ex}, using exponential laws.

5.2.5 LEARNER RESPONSES TO QUESTION 5 ITEMS

Table 5.5 below depicts the data generated from Question 5 items' learners' responses, revealing that a higher percentage was found in IR's in interpreting hyperbolic and exponential graphs in both the experimental and the control groups. The interpretation of the two graphs' question items were coded as HPEF₁, HPEF₂, HPEF₃, HPF, and HPF in table 5.5 below, were drawn from the two graphs given to the learners to interpret in order to respond to the question items.

Table 5.5: Percentages (and absolute numbers) of Correct Responses (CR), Incorrect Responses (IR), Incomplete Responses (InR) and Blank Responses (BR) on Q5 question items for the experimental and the control groups.

Items	Group	Pre-test			
		CR	InR	IR	BR
HPEF ₁	Experimental	19.1% (8)	33.3% (14)	7.1% (3)	40.5% (17)
	Control	38.1% (16)	2.4% (1)	52.4% (22)	7.1% (3)
HPEF ₂	Experimental	0.0% (0)	11.9% (5)	71.4% (30)	16.7% (7)
	Control	23.8% (10)	7.1% (3)	61.9% (26)	7.1% (3)
HPEF ₃	Experimental	0.0% (0)	14.3% (6)	66.7% (28)	19.1% (8)
	Control	19.1% (8)	4.8% (2)	66.7% (28)	9.5% (4)
HPF	Experimental	4.8% (2)	19.1% (8)	47.6% (20)	28.6% (12)
	Control	28.6% (12)	9.5% (4)	47.6% (20)	14.3% (6)
HPF	Experimental	0.0% (0)	21.4% (9)	40.5% (17)	38.1% (16)
	Control	14.3% (6)	14.3% (6)	54.8% (23)	16.7% (7)
Total	Experimental	4.9% (10)	20.0% (42)	46.7% (98)	28.6% (60)
	Comparison	24.9% (52)	7.6% (16)	56.7% (119)	10.9% (23)

The data above shows that the learners lacked devising strategies to recognise, formulate and solve problems, as suggested by Turner (2010:), in Q5 question items. Furthermore, the CR's percentage for the experimental group ranged between 0.0% to 19.1%, as compared to the control group's range between 14.3% and 38.1% in Q5 question items. The results also showed a high percentage of InR of question item HPEF₂'s 71.4% and IR's 66.7% for question item HPEF₂ for the experimental group, where the learners did not know when and how to use the procedures appropriately (Kilpatrick et al., 2001:117; Watson & Sulliman, 2008:115). The learners struggled to determine the value of x by using the graphs and finding the axis of symmetry of the new graph $h(x) = f(x)$ when $(x < 0)$ during the interpretation of the hyperbolic and exponential graphs.

The BR's percentage was found to be 73.8% for the experimental group, which was higher as compared to other categories control group for question item HPEF₁. The BR's results show that the learners in the experimental group lacked conceptual knowledge, in which this results are inconsistent with Kilpatrick et al.'s (2001:117). who reported that for learners to be successful problem-solvers, they should be able to comprehend mathematical concepts, operations and relations.

5.2.6 LEARNER RESPONSES TO QUESTION 6 ITEMS

Table 5.6 below depicts the data collected from Q6 question items, which expected learners to interpret the parabola graph. The question items were coded as PF₁, PF₂, PF₃, PF₄, PF₅,

PF₆, PF₇, PF₈, and PF₉, which reflected all the questions drawn from the parabola graph. The results show that all the learners in the experimental group struggled to solve the questions correctly.

Table 5.6: Percentages (and absolute numbers) of Correct Responses (CR), Incomplete Responses (InR) Incorrect Responses (IR) and Blank Responses (NR) on Q6 items for the experimental and the control groups.

Items	Group	Pre-test			
		CR	InR	IR	BR
PF ₁	Experimental	0.0% (0)	2.4% (1)	19.1% (8)	78.6% (33)
	Control	4.8% (2)	0.0 (0)	78.6% (33)	16.7% (7)
PF ₂	Experimental	0.0% (0)	0.0% (0)	11.9% (5)	88.1% (37)
	Control	16.7% (7)	0.0% (0)	54.8% (23)	28.6% (12)
PF ₃	Experimental	0.0% (0)	0.0% (0)	28.6% (12)	71.4% (30)
	Control	31.0% (13)	0.0% (0)	40.5% (17)	28.6% (12)
PF ₄	Experimental	0.0% (0)	0.0% (0)	26.2% (11)	73.8% (31)
	Control	28.6% (12)	0.0% (0)	40.5% (17)	31.0% (13)
PF ₅	Experimental	0.0% (0)	0.0% (0)	23.8% (10)	76.2% (32)
	Control	19.1% (8)	0.0 (0)	61.9% (26)	19.1% (8)
PF ₆	Experimental	0.0% (0)	0.0 (0)	23.8% (10)	76.2% (32)
	Control	64.3% (27)	0.0% (0)	19.1% (8)	16.7% (7)
PF ₇	Experimental	0.0% (0)	0.0% (0)	23.8% (10)	76.2% (32)
	Control	45.2% (19)	0.0% (0)	35.7% (15)	19.1% (8)
PF ₈	Experimental	0.0% (0)	0.0% (0)	11.9% (5)	88.1% (37)
	Control	14.3% (6)	0.0% (0)	54.8% (23)	31.0% (13)
PF ₉	Experimental	0.0% (0)	0.0% (0)	7.1% (3)	92.9% (39)
	Control	16.7% (7)	0.0% (0)	54.8% (23)	28.6% (12)
Total Experimental		0.0% (0)	0.3% (1)	19.6% (74)	80.2%
Control		26.7% (101)	0.0% (0)	49.0%	24.4

The results in Table 5.6 show that the experimental group obtained 0.0% for CR, suggested that learners were unable to formulate mathematical problems, to represent those problems, and to effectively solve them (Kilpatrick et al., 2001; Watson & Sulliman, 2008; Turner, 2010:59). The control group obtained between 4.8% and 64.3% for the CR category, which showed that some of the learners were able to interpret the graph with the given information to respond to the question items. The BR's of the learners' percentage was high, with 92.9% for question item PF₉ in the experimental group, and which did not suggest only a lack of conceptual understanding, but also a lack of strategic competence. The IR and InR's percentages for both the experimental and the control groups revealed that the learners lacked procedural understanding, as they did not know when and how to use the procedures. For

example, the samples below are taken from two different learners who answered PF4 in determining the new equation of f , showing the equation to be $f(x) = 2x^2 - 4x - 6$.

Learner's response: Sample 1

1. $F(x) = 2x^2 - 4x - 6$
 y -intercept put $x=0$
 $\Rightarrow 2(0)^2 - 4(0) - 6$
 $= -6$
 $\Rightarrow (0; -6)$
 x intercept put $y=0$
 $0 =$
 $2x^2 - 4x - 6 = 0$
 $(3; 0)$

Learners' response: Sample 2

6.4 $f(x) = 2x^2 - 4x - 6$
 $f(x) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $f(x) = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)}$
 $\frac{4 \pm \sqrt{16 + 48}}{4}$
 $\frac{4 - 8}{4}$ or $\frac{4 + 8}{4}$
 $f(x) = -1$ or $f(x) = 3$

The learners' samples above show that they did not know the correct procedures to determine whether the equation f given the turning point $(1, -8)$ and one x -intercept as -1 . In sample 1, the learner calculated the x - and y -intercepts, which illustrated that the procedure used here was inappropriate to finding the equation of f . In other words, the learner did not answer the question. On the other hand, in Sample 2, the values of x was calculated using the quadratic formula: $f(x) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ [instead of $y = a(x - p)^2 + q$], which demonstrated a lack of knowledge of the procedures in determining the equation of f . The learners' responses generally on Q6 revealed that lack knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately and efficiently (Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:115; Turner, 2010:59).

5.3 PRE-INTERVENTION SEMI-STRUCTURED TEACHER INTERVIEWS

As previously noted in Chapter 4, semi-structured interviews were conducted in the two schools immediately after the pre-test (see appendices J). The rationale for conducting a semi-structured interview with the teachers was to gain a deeper understanding of the teachers' perceptions about the learners' academic achievements in the Algebra test in the two schools. Bandura's (1986:459) theoretical resources that underpinned this study were used to analyse interview data, where *attention*, *motivation*, *retention*, and *production* were adopted as analytic framework.

In this section, the researcher discusses the results obtained from the semi-structured interviews conducted before the twinning process. The two teachers were asked the same questions in the same order, so as to understand the challenges the teachers encounter in the teaching and learning of mathematics. The pseudonyms Teacher A and Teacher B, were used

to indicate the two participating teachers in the experimental and control schools, respectively. The interview questions were presented in English. However, the interviewees were free to use the language of their choice and preferred to freely express themselves.

5.3.1 ANALYSIS OF THE SEMI-STRUCTURED INTERVIEWS WITH THE TEACHERS BEFORE THE TWINNING STRATEGY

The two teachers were asked the following questions in the general categories written in italics:

Challenges in mathematics: What do you think can be a challenge for the poor performance of the learners in mathematics and why?

Classroom interaction: How do you interact with your learners during the teaching and learning of mathematics?

Teaching and learning materials: Which teaching materials do you use to teach mathematics?

Language usage in teaching mathematics: Which language do you prefer to use in teaching mathematics?

Classroom assessment: What type of assessment do you use in the mathematics classroom?

Learner support in mathematics: What kind of support do you give the learners with difficulties?

Classroom culture: How do you establish a culture of teaching and learning in your mathematics classroom?

A few extracts of the twinned teachers' interviews are used as examples are presented below. Transcripts are not edited.

Transcript 1: What do you think can be a challenge for the poor performance of the learners in mathematics and why?

The first interview question needed to probe teachers' perceptions about the challenges faced by the learners in solving mathematical problems and the possible reasons that may have caused the perceived challenges in the participating classrooms.

Extract 4.1 below illustrates Teacher A's responses to the interview question of Transcript 1.

Extract 4.1

Teacher A: *The learners come to Grade 11 without the background of lower grades mathematics. They cannot solve even simple mathematical problems such as simple quadratic equations, which they should have learned in Grade 9. Some of them do not show any seriousness in the classroom like not writing homework. We also experience a problem of shortage of textbooks, four to five learners share one textbook, which also contributed towards learners not having enough time to practice given exercises. The learners don't return textbooks after passing a particular grade which causes shortage of the textbooks. We don't get support from parents, as most of them are not educated and cannot support their children to do their homework. Maybe the environment also plays a role in learner performance as most of the parents are not working.*

Teacher A's perceptions that "*the learners come to Grade 11 without the background of lower grades mathematics*" suggest the learners' lack of the knowledge of mathematics. This is evidenced when Teacher A indicated that his learners, "*...cannot solve simple quadratic equations,*" which support the pre-test results, where only a few learners were able to answer simple quadratic equations. Teacher A's response about the learners' poor background in mathematics was also found in the pre-test, as the question items used were from the Grade 10 syllabus, and most of the learners could not factorise simple quadratic equations (see Appendix H of question item 1.1). This poor background in mathematics could have been caused by the lack of learner commitment, given that the learners "*do not show any seriousness*". Furthermore, the learners' attitude and lack of motivation towards mathematics are viewed by researchers as factors that affect learner academic achievement in general (see for example Maat & Zacharia, 2010:272; Manoah, Indoshi & Othuon, 2011:965).

The statement "*four to five learners share one textbook*" signifies the unavailability of learning and teaching support materials in the school. The shortage of textbooks seemed to have contributed towards the learners' poor academic achievement in Algebra, as the learners could not practice exercises and refer to mathematics examples when they experience challenges. This statement of the shortage of textbooks supports what Mbugua (2011:114) reported that the shortage of textbooks affects the learners' mastery of mathematical concepts

through practising exercises. In other words, teaching and learning is effective when the schools have enough resources in the mathematics classes (Yara & Otieno, 2010:126).

Extract 4.2

Teacher B: *Most of our learners in the school are performing better in mathematics, but there are few who are not serious and are mostly not writing homework. I always keep them in the laboratory, where I stay with them to write homework together, and they drop my results [sic]. I do this because I know that their parents are unable to support [them] in homework, as [they] are not educated, and many of them are unemployed because of lack of education. Actually, all our learners remain at school after school to study under our supervision, and use ICT resources such as DVDs to reinforce understanding of the content taught. Some of the learners do not have [a] background of mathematics, especially those who are admitted from other schools. Some cannot even solve simple problems that you cannot [sic] expect them to struggle with.*

Teacher B in Extract 4.2 above indicated that most of the learners in his school are perceived to be performing well in mathematics. The teacher used “most” to represent a high number of learners who performed well in mathematics. From this extract, it appears that most of the learners committed themselves in mathematics, which is associated with motivation (Adeyinka et al., 2013:37), and the commitment the learners made in the mathematics developed their competence in the subject. Hence, Teacher B had the similar challenge, as in the low-performing school of learners who are not committed to their schoolwork. The category “not serious” signified the motivation and attention (Bandura, 1986:) of the learners in mathematics.

Teacher B took some initiative with learners who did not commit themselves in mathematics, by giving them support. Teacher B said, “*I always keep them in the laboratory after school*”, referring to the computer laboratory where he was able to engage with the learners for remedial support. Teacher B showed motivation to teach his learners, which can be associated with the learners’ performance in mathematics (Adeyinka, Asabi, & Adedotum, 2013:37) Shulman (2010:9) and Calman (2010:2) argued that well-performing schools created a supportive environment for learners, which is what Teacher B appeared to have been doing in his Grade 11 class.

The response by Teacher B shows that he used ICT resources to further support his learners to understand the concepts, which maybe were seen challenging to the learners by Teacher B. The rationale for infusing ICT tools in the teaching and learning of mathematics was to support the learners who experienced difficulties in solving mathematical problems. The use of ICT resources was encouraged in the teaching and learning of mathematics (Ndlovu, Wessels & de Villiers, 2011:4). Ndlovu et al. (2011:4) furthermore indicated that the use of ICT resources provided both the mathematics teachers, and the learners with the opportunities to teach and learn.

Transcript 2: What kind of support do you give learners who experience difficulties in Algebra?

Extract 4.3

Teacher A: *Learners remain at school after school for them to study as they don't work at home. We make sure that they do their homework here at school. What surprises me is that they don't come to seek help from me when they do their homework. They are not paying attention even when you teach.*

Learners in low-performing schools were supported by their teacher(s), because they attended study periods. “We” as a personal pronoun was used to signify all the teachers supporting the learners in the school in all the subjects, including mathematics, so as to ensure that learners do their homework at school. The issue of the commitment of the learners also emerged in this transcript, as was the case in Extract 4.1 “*what surprises me is that they don't come to seek help from me when they do their homework*” suggested the learners’ lack of motivation in their schoolwork. In Extract 4.4 below, Teacher B indicated a passion with his learners’ performance in mathematics.

Extract 4.4

Teacher B: *Learners leave the school at 16h30 every day and on Monday, and [on] Wednesday I play DVDs for them to watch and see how problems are solved. The computer software package programme is usually accessible in the afternoon and sometimes during the lesson I play DVDs, especially when learners did not understand the concept. The slow learners are always kept in my laboratory to see if they do write their homework.*

Teacher B, in the extract above, indicated that the learners in the school attend the study periods to get support. The first person singular ‘I’ was used to emphasise that teacher B had supported his learners by playing DVDs to reinforce the content taught that was not understood. The text in Extract 4.4 suggests that Teacher B’s Grade 11 Mathematics lessons integrated ICT tools so as to make teaching and learning fun for his learners and for those with special needs (DBE, 2011:11; Ndlovu, Wessels & De Villiers, 2011:4). The ‘*slow learners*’ used in the last sentence appeared to symbolise those learners in the control group who were kept in the computer laboratory to do their homework, as they were seen to be grappling with mathematics. The teacher also showed commitment in his learners’ school work, because he was eager to see all his learners do their work. The words “*to see if they do their homework*” reveal the attention and motivation displayed by the teacher in respect of the learners’ work. Transcript 3 below presents the results of teachers A and B, indicating how they interact with their learners.

Transcript 3: How do you interact with your learners during the teaching and learning of mathematics?

Extract 4.5 below describes how teacher A interacts with his learners during the teaching and learning of mathematics.

Extract 4.5

Teacher A: *I always use the question and answer method and choose learners who raised their hands up. Those who don’t raise their hands up show me that they don’t know the answer. These learners just keep quiet when I ask them questions.*

The text in Extract 4.5 above shows teacher A’s assertion that, “*I always use the question and answer method*” suggested a strategy used to engage his learners during teaching and learning. The teacher used ‘I’ in the sentence to symbolise himself using the method to ensure that the learners are involved in the lesson. These results support what was found during the pre-intervention classroom observations when Teacher A only involved those learners who raised their hands during teaching. The approach used by the teacher did not allow the learners to think for themselves or to interact with each other (Boaler, 2008:167; Ingram, 2008:282), in order to share their ideas during the teaching and learning of Algebra. Hence the method used to interact with the learners did not involve all the learners, but only those who could respond to the questions, or the fast learners. The teacher appeared to assumed that learners “*...those who don’t raise their hands up show me that they don’t know the answer*”

of the concepts taught. As a result, the teacher decided to engage only those learners who raised their hands because “...*these learners just keep quiet when I ask them questions*”.

Extract 4.6 below describes teacher B’s responses to how he interacts with his learners.

Extract 4.6

Teacher B: *Normally, I ask them questions to involve them in the discussions. Learners are chosen randomly to give the answer and I want them to give reasons why they think the answer is correct. If I am not happy about the answer, I probe, to understand his or her thinking about the concept. I also encourage small group discussion for learners to share ideas amongst themselves.*

Teacher B revealed that, “*normally, I ask them questions to involve them in the discussions*” which indicates that discussions and talking seem to be at the centre of teacher-learner interaction in this classroom. The strategy used by Teacher B to interact with the learners differed from that of Teacher A. The results seem to suggest that Teacher B used explicit instruction in his teaching of mathematics that encouraged interaction between the teacher and the learners (Steedly, Dragoo, Arafeh, & Luke, 2008:4). Teacher A interacted with the learners, who raised their hands, while Teacher B chose the learners randomly to give answers and also to justify why they thought their answers were correct.

Teacher B emphasised ‘I’ when he spoke to signify himself asking questions, so as to interact with his learners during his teaching and learning, namely “*if I am not happy, I probe, I want them to give reasons and I also encourage to give reasons for their answers*”, and suggested diagnosing the problems the learners have during the teaching and learning of mathematics, as for Bukova- Güzel (2010:1875).

Similar to reports by Shulman (1986:14), teacher B used questioning techniques as a strategy to make sense of the learners’ thinking about the concept taught, noting “*I want them to give reasons why they think the answer is correct*”. Learner-to-learner interaction was encouraged by teacher B, as he said, “*I encourage small group discussion for learners to share ideas*” to assist each other when discussing mathematical ideas. Zacharia, Solfitri, Daud and Abidin (2013:98) argued that the learners can work in a group to complete their mathematical tasks in order to support each other with ideas. Small-group discussions were further found to have a positive impact on the learners’ performance in Mathematics (Naomi & Githua, 2012:177; Zakaria et al., 2013:98; Zakaria & Iksan, 2007:37).

Transcript 4 describes the teachers' views about the teaching materials used to teach mathematics.

Transcripts 4: Which teaching materials do you use to teach mathematics?

Extract 4.7 below describes the illustration of Teacher A on the types of teaching materials he uses to teach Grade 11 Mathematics.

Extract 4.7

Teacher A : *I use a textbook and study guide to teach mathematical concepts, because we don't have enough resources in our school. I just said that four to five learners share one textbook.*

Researcher: Do you cope without enough text books?

Teacher A : *It is difficult, because the learners do not have the materials to use in their studies.*

Teacher A described how he struggled in a classroom that lacks teaching and learning resources. He indicated that, *"I just said that four to five learners share one textbook"* suggesting a lack of support materials (LTSM), which appeared to contribute towards the poor learner performance (Mbugua, 2011:113) in the pre-test results collected from the experimental group. Teacher A said, *"we don't have enough resources in our school"*,

which supported the statement in the Extract 4.1 transcript supporting the literature about the lack of textbooks in most schools (Oguntuase et al., 2013:4; Mbugua, 2011:113; Yara & Uganda, 2010:126). Teacher A also indicated that, *"the learners don't have materials to use for studying"* indicating the difficult situation the learners' face in learning mathematics and doing exercises for the mastery of the concepts in mathematics.

Extract 4.8 below describes Teacher B's teaching and learning resources in his school.

Extract 4.8

Researcher: Does your school have teaching and learning resources?

Teacher B : *In our school we have DVDs that demonstrates mathematical topics presented by experts in mathematics. I used software in the afternoon as that is the only time it is accessible. My learners share textbooks as they are few. Both of them also supply problems learners to solve before they show them*

how those problems can be solved. I use textbook also to teach our learners in the school to prepare teaching materials such as worksheets and handouts for every learner to have study material because of shortage of textbooks. Bana ba a ba buse ditextbook mafelelong a ngwaga ene ntho ye e re direla bothata ba dibuka [learners don't return the text books when they exit their grade at the end of the year, and it causes shortage of textbooks].

Researcher: How do you use them?

Teacher B : *I use hard copy materials during the periods and even software packages are used during the period, but mostly in the afternoon, because we don't have enough time. I give them either worksheets or handouts every day for practice.*

The text in extract 4.8 describes the types of teaching and learning materials the well-performing school had, and how they are used in the mathematics class. Teacher B indicated that the teaching and learning materials are mostly used to give the learners more problems to solve, as well as to reinforce the understanding of the concepts being taught. He incorporated ICT tools, namely “*in our school we have DVDs that demonstrate mathematical topics presented by experts in mathematics*” to teach mathematics to the learners. The shortage of textbooks was also highlighted by Teacher B, which is similar to those observed by teacher A. However, Teacher B prepared learning materials to supplement the shortage of textbooks, where the teacher “*prepare[s] teaching materials such as worksheets and handouts*” in order to enhance teaching and learning. Transcript 5 describes the types of classroom assessment teachers A and B prefer to use when teaching mathematics.

Transcript 6:

What type of assessment do you use in Mathematics, and how?

Extract 4.11 describes Teacher A's types of assessment used and how it is used in the mathematics classroom.

Extract 4.9

Teacher A: *As we are given assessment programme to follow such as projects, investigations, assignments, research, standardised tests and examinations. I prepare learners by giving them two classwork and one homework [assignment], or vice-versa and the other activities indicated above. I use*

textbook and study guide to give learners homework and classwork.

R: Why do you think classroom assessment is important in mathematics teaching?

Teacher A: *I am able to see how far learners understood the concept I have taught. I will also understand how the slow learners can be assisted.*

R: How do you assist the slow learners?

Teacher A: *I give them extra work so that they can practice, like giving them exercises to work on during teaching.*

Extract 4.9 above shows that Teacher A followed an assessment programme to assess his learners as prescribed by the Department of Basic Education (2011). He referred to the assessment programme as “*project, investigations, assignments, research, standardised tests and examinations*” seemed to refer to continuous assessment. Teacher A believes that, “*Giving them two classwork and one homework [assignment]*”, suggesting that the learners would be adequately prepared for summative assessment. In other words, the teacher wanted to be certain about his learners’ mathematical understanding through assessment “*prepare learners*”, suggesting that he diagnosed the learners’ understanding of the mathematical concepts taught, as Hattie (2009:260) and Popham (2008:13) have claimed.

Teacher A in Extract 4.9 further indicated that he assessed the learners for a particular reason. The teacher’s reasons were, “*I am able to see how far learners understood the concept I have taught*”, which appeared to suggest that he is able to monitor the academic progress of his learners (Van de Walle et al., 2013:78). Teacher A being able to monitor the progress of the learners seemed to have given him an opportunity to identify those learners who struggle with the concepts being taught, and how those learners can be supported. For example, “*I give them extra work so that they can practice*”, which seemed to demonstrate that the teacher tests the learners in order to know their performance in mathematics.

Extract 4.10

Teacher B: *Normally I follow the assessment programme given by the department. But I also give classwork, homework, worksheets and handouts to assess their progress in mathematics. I mostly prefer to give learners homework, as I mostly want to discuss problems with the learners during teaching.*

R: Why do you think assessment is important in mathematics teaching?

Teacher B: *I understand the level of understanding of the learners in mathematics. I also identify those who struggle to solve mathematical problems and assist them. I normally keep them in my laboratory in the afternoon and give them some work to do and assist them. These learners are also encouraged to give reasons for what they do so as to understand the type of mistakes they make in solving mathematical problems.*

Teacher B's response in Extract 4.11 is similar to Teacher A's about following the assessment programme provided by the Department of Basic Education. The two teachers also used classwork and homework to assess their learners, but teacher B gave learners some more activities: "*but I also give classwork, homework, worksheets and handouts to assess their progress in mathematics*" as exercises. Teacher B preferred to give the learners homework, as it appeared to play a major role for discussion purposes. The learners write the activity at home, and the teacher and learners are able to discuss the solutions in the classroom. The teacher gave the learners homework in order get feedback on what the learners understand (Bjorklund Boistrup, 2010a:144).

Both the two teachers' rationale for assessing the learners was to monitor their progress during the learning process and to identify the learners with learning barriers in order to provide extra support (Van de Walle et al., 2013:79). For example, Teacher B indicated that, "*I understand the level of understanding of the learners in mathematics*" and "*I am able to see how far learners understood the concept I have taught,*" suggesting the reason for assessing the learners in mathematics and understanding the learners' knowledge of mathematics. The two teachers used 'I' to show that the monitoring of the learners' progress was done by them personally in their respective classes. Teacher B gave his learners remedial support: "*I normally keep them in my laboratory in the afternoon and give them some work to*

do and assist them” in order to identify and understand the types of errors made by the learners.

Transcript 7 describes how teachers A and B established a culture of teaching and learning in mathematics classrooms in the two schools.

Transcript 7: How do you establish a classroom culture of teaching and learning in your mathematics classroom?

Extract 4.11

Teacher A: *I prefer to have enough resources when teaching and learning. I also want my learners to respect each other's opinions, so that each one of them is free to participate. My problem in the school are textbooks; I cannot mention any other resource, because this is the main one.*

Extract 4.12

Teacher B: *I prefer my learners to understand the importance of other learners in the classroom, because respect is the most important thing in learning. If a child feel[s] threatened, that particular child will not listen in the classroom and the child will fail at the end. I want them to respect each other even when some had made mistakes, they don't have to laugh. I want every child to enjoy my lessons so that they can all pass. We use the language of our choice to avoid learners' embarrassment during teaching, as English is not their mother tongue. Most of the learners mock each other if some lack vocabulary of English, and made me [sic] to allow them to use the language of their choice during discussion.*

Teachers A and B encouraged their learners to respect one another during classroom interaction, either between the teacher and the learner or the learner and the learner. The response of the two teachers shows that they were able to create an orderly and supportive classroom environment (Calman, 2010:2; Shulman, 2010:9). The issue of encouraging respect could have been influenced by the low participation of the low-achieving learners being afraid of being mocked by the other learners. Teacher A expected his learners, “...*to respect each other's opinions*”, and teacher B's belief that “...*respect is the most important thing in learning*” suggest respect as key to teaching and learning, as well as to promoting

learner-participation in mathematics teaching. Zacharia, Solfitri, Daud and Abidin (2013:36) contend that learner interaction and communication with others should take place harmoniously.

Teacher B furthermore indicated that if the learners are not aware of the outcomes of not observing mutual respect, especially the achieving learners, they may ultimately become a threat to the under-achieving learners in the form of intimidation. Cohen (1992:10) argued that the teachers should give the learners the opportunity to accept, support and respect each other, so as to communicate and resolve their differences constructively. Teachers A and B expected their classes to be free and fair in terms of participation of learners, in which every learner has to be free to participate in class. No learner should intimidate another by laughing at him or her when wrong answers are given. Shulman (2010:9) contends that schools should act as a hub to support the learners in teaching, instilling in learners' self-esteem and confidence, ensuring their safety and well-being. Schools that lack order, discipline and social control present difficulties for the teachers to attain a high level of learner attention and interaction within the classroom Shulman (2010:9).

5.4 PRE-INTERVENTION OBSERVATIONS

Four classroom observations were held with the experimental group before the intervention process, with the purpose of investigating the pedagogical approaches used by Teacher A in teaching Grade 11 Algebra. The study made use of the classroom observation schedule adapted from Sepeng (2010) to collect the data generated (see Appendix H) through lesson observations, before the intervention was implemented. The following information was gathered, namely the type of interaction during classroom practice, the type of resources, the teachers' teaching methods and style, and the classroom observations.

The theoretical resources such as scaffolding and the zone of proximal development were used to analyse the description of Teacher A's lesson observations. These pre-intervention observations in the two schools intended to respond to the objective, namely to establish the pedagogical approaches used by Teacher A to teach Grade 11 Algebra.

5.4.1 THE EXPERIMENTAL GROUP

The classroom observations were held in one month, namely one lesson per week, and revealed the following themes that respond to the objective of understanding the pedagogical practices in the classroom.

5.4.1.1 TEACHING METHODS AND STYLES

A ‘telling method’ was mostly used as a fundamental approach in teaching and learning Grade 11 Algebra. The teaching methods used by Teacher A did not follow scaffolding as a strategy to present the lessons (Vygotsky, 1978:86). The chalkboard and chalk method dominated for the most part, as the learners were seen sitting passively receiving information from the teacher. The teacher-centred approach was followed in all the lessons presented by the teacher in the experimental group, as the learners were the passive recipients of information imparted by the teacher. For example, in Lesson 2, the teacher would introduce his lessons by saying, *“today we are going to discuss...”* and he wrote the topic on the chalkboard, such as, *“...solving quadratic equations by completing a square”* by writing an example on the board. One of the examples used was $x^2 + 7x + 4 = 0$, and he explained step by step until getting the answer. He did not give himself enough opportunity to observe and listen carefully to the learners’ ideas and alternative conceptions, meaning that the knowledge was not built on the learners’ prior knowledge (Van der Stuyf, 2002:5). The teaching and learning of algebra in the experimental group was guided and influenced by the quest for completing the syllabus on time because *“...we have to finish all topics to be covered in the first quarter according to the pace setter”* suggested their topics have to be completed according to the pace setter adapted from DBE. The teacher was under pressure to complete the syllabus as stipulated by the DBE, for them to cover the syllabus of the term each time. He used to say, *“sometimes we have to come during weekends to cover the topics lacking behind”*. He always referred to a pace setter to the learners about the syllabus when teaching algebra to complete the stipulated topics on time.

5.4.1.2 CLASSROOM INTERACTIONS

The classroom interactions followed the form of the teacher’s questioning style to involve his learners. The role of teacher A during the interaction when using questioning method did not support the learners’ development, and also did not provide support structures to learning get develop learning as Vygotsky (1978:86) argued. The teacher posed questions and the learners raised their hands up in responding to the questions during the lessons. The teacher would say, *“okay, what are the factors of the equation $x^2 - x - 20 = 0$? tell us, Tebatso”* [pseudonym], which supports what Teacher A indicated in the semi-structured interviews as support the statement that indicates he interacted with the achieving learners who were used to raise their hands. The classroom atmosphere did not encourage any form of the learners’ discussion where they could have shared mathematical ideas amongst themselves to reach a common understanding. For example, the teacher would say, *“whenever I give an activity to*

discuss in pairs, you don't show any interest and some don't do anything." The teacher here gave the learners an equation to solve in pairs, but most of the learners just talked, and did not do the work that was supposed to be done. This was seen when the learners were supposed to give feedback after the discussion. Some were observed to be disengaged with the lesson. The teacher mostly led this type of discussion by looking for a specific answer. This supports what the teacher said in the interviews when he noted, *"they don't pay attention even when you teach"*, which showed that the learners were not motivated (Bandura, 1986:459) to learn the concepts taught. Classroom interaction through questioning was not able to motivate the learners' interests in relation to the tasks discussed in the classroom.

5.4.1.3 TEACHING AND LEARNING RESOURCES

Tachie and Chirese (2013:67) argued that teaching and learning ought to be supported by resources. Teacher A mostly used the textbook, the study guide, chalk and chalkboard when presenting his lessons. The results revealed that the learners in the experimental group shared one mathematics book in a group of four or five, as was also indicated in the semi-structured interviews with Teacher A. Information and communication technology (ICT) is encouraged in the teaching and learning of mathematics (Ndlovu, Wessels, & De Villiers, 2011:4), which was inconsistent with the findings in the control group. The school had an overhead projector and some broken computers to be used by the teacher, which meant that the ICT tools could not be used during teaching and learning. Teaching resources were scarce in the school, which led the teacher to use available resources, such as the textbook and the study guide. The teacher used the study guide and the textbook to select examples and questions for activities during classroom discussions. Mbugua (2011:114) has suggested that the textbook is a powerful tool in the teaching and learning of mathematics, which was a challenge to the experimental group' learners and could have had a negative impact towards the learners' performance (Oguntuase et al., 2013:4; Mbugua, 2011:114; Yara & Uganda, 2010:126) in Algebra. For example, the teacher used to copy exercises on the chalkboard, or ask one of the learners to copy them if the period was over. This appeared to have been a challenge to the learners as they did not have any references when they did their homework, and also could not study, due to the lack of textbooks.

5.4.1.4 CLASSROOM ASSESSMENT

Teacher A's classroom assessment was done in the language of teaching and learning, as Sepeng and Webb (2012:61) and Sepeng (2014:756) indicated, namely that assessment in teaching and learning was done in English. In this classroom, the approach of administering activities and then providing the learners with solutions did not form part of the classroom

assessment, which may be used to enhance the quality teaching and learning (Van de Walle, Karp, & Bay-Williams, 2013:79), as Teacher A just gave learners activities and the following day he gave the solutions to the learners. The results showed that the learners were not given an opportunity to discuss or give their solutions before the teacher could give the final answer. The learners also did not show any motivation or interest in relation to homework or classwork (Bransford, Brown, & Cocking, 2000:155). More than 90% of the questions of the exercises of homework and class activities were taken from the textbook, and some of the exercises were taken from the study guide, to give to learners in the experimental group. The teacher used to write those questions on the chalkboard due to the scarcity of textbooks as indicated earlier. The learners were allowed to discuss questions of the homework and class activities in class after the lesson presentations, as most of the activities given to them were supposed to be completed at home in order to assist the underachieving learners.

5.5 TWINNING: THE INTERVENTION

The twinning strategy was implemented in Phase 2 of this study. Twinning was then introduced to the two teachers in order to share their classroom practices in teaching Grade 11 Algebra. This strategy intended to achieve the following objectives, namely:

- to investigate the key elements of a successful twinning model;
- to explore the possible barriers for implantation;
- to understand the benefits and the disadvantages of twinning; and
- to develop a framework for the successful implementation of the twinning model.

5.5.1 CLASSROOM OBSERVATIONS DURING THE TWINNING PROCESS

Classroom observations were done in this study in order to be able to answer the following research questions: 1) what pedagogical practices do the two teachers use to teach Grade 11 algebra during the twinning process; and 2) what are the benefits of using twinning as a strategy in teaching Grade 11 mathematics? Twelve classroom observations were done in the experimental group during the twinning process. The twinned teachers planned their lessons together in respect of the topics selected to be taught during the twinning process. As noted earlier, twinning followed Bandura's (1986:459) observational learning, where Teacher A was the participant-observer and Teacher B was teaching the lessons. The data that were gathered from the classroom observations were used to achieve the objectives of this study.

5.5.1.1 LESSON PREPARATIONS

The participating teachers prepared ten lessons together on the topics they identified. The lessons were broken up into chunks and were planned the day beforehand, on topics such as

“quadratic equations (word problems), number patterns, exponents, financial mathematics, hyperbolic functions, exponential functions, exponential functions and parabolic functions”. The twinning process took place twice a week. Teacher B travelled from his school to the experimental group to present the lessons. The lessons prepared by the two teachers and presented by Teacher B seemed to have been effective when compared to the lessons presented by Teacher A in the experimental group. For example, one of the learners said, *“sir, ke be ke sa kwisisi gore ge re nyaka general term ya second common difference re e hwetsa jwang [sir, I did not understand how to determine the general term of a sequence of the second common difference], but now I know.”* Calman (2010:3) indicated that the good-performing schools were able to create an orderly and supportive environment. The sharing, planning, and acting of the twinned teachers during the intervention seemed to have provided the learners in the experimental group with the necessary support.

5.5.1.2 TEACHER-LEARNER INTERACTION

The classroom interaction followed the method of Teacher B, namely asking questions, followed by the learners’ responses. Teacher B made use of scaffolding (Vygotsky, 1978:86) by first asking questions in order to know how to provide the necessary support to the learners. In the first week of the intervention process, the learners seemed to be puzzled, as they were not actively involved, and only a few learners responded to the teacher’s questions. The results of classroom observations indicated that the questions were asked mostly to be able to understand the learners’ prior knowledge before introducing the lesson. In the lessons presented by Teacher B, it seemed that he asked the questions based on what the learners had learnt before, in order to arouse the learners’ interest in the new lesson. Moreover, Teacher B asked the learners questions, so as to enable him to understand their thinking, by probing further questions about the concepts being taught. For example, Teacher B in Lesson 4 asked, *“what type of a sequence is this one, 7; 11; 15;...? What is the difference between the consecutive terms? What can you say about the difference between the terms?”* He further asked the learners to *“...find the general term of the sequence”*, whereby he wanted to understand if the learners were able to understand linear sequences, and how to determine their general term before he could introduce the sequence of the second difference. Teacher B wanted to ascertain whether the learners might be able to determine the common difference between the consecutive terms and also if they could determine the general terms of the sequence.

Teacher B involved all the learners in his teaching. He chose the learners randomly to answer his questions. Sometimes the learners responded all together. During the lessons the learners

asked questions for clarity and consolidation of the mathematical concepts taught. One of the learners asked, *“is there any other method of finding the general term except this one?”* He was referring to the method used by the teacher to determine the general term of the second common difference of the number patterns.

As Bandura (1986:459) indicated, the model should demonstrate their behaviour in the presence of the observer, Teacher A appeared to have paid attention to Teacher B’s teaching practices during classroom interaction, as he sometimes intervened when Teacher B asked the learners questions. For example, Teacher A in Lesson 7 asked the learners if they *“...understand how to determine the number of terms and the n^{th} terms of a sequence”*, which showed that he was observing how Teacher B interacted with the learners. The learners appeared not to have knowledge of procedures when solving problems in number patterns, such as determining the number of terms and the n^{th} of the given sequence. Teacher A’s attention was seen by his participation in Lesson 6, when Teacher B presented the number patterns of the second difference, saying *“remember that you were shown yesterday how to determine the value of a , b and c of the sequence that you must first find the difference between the consecutive terms.”* Teacher B was asking the learners to determine the equation of the quadratic sequence.

5.5.1.3 COOPERATIVE LEARNING IN A TWINNING CONTEXT

Cooperative learning in Teacher B’s lessons made use of learner interaction and communication in a small group (Zacharia, Solfitri, Daud & Abidin, 2013:98). Teacher B used a small number of learners with different abilities (heterogeneous groups) grouped together (David & Rodger, 2001:100). He did not follow the group discussion method of learners sitting together, but he preferred to use mixed abilities to guide the grouping of his learners. For example, Teacher B said, *“I want to mix you because if you sit as friends, you will not work”*, and was assisted by Teacher A to mix them, as he knew his learners’ abilities in Mathematics. The classroom observations’ results indicated that the learners were given different learning activities that were planned by both Teachers A and B. The interaction of the learners and communication were encouraged by Teacher B for learners not to undermine each other’s thoughts. For example, *“you need to respect each other’s’ ideas because you are all learning”*. Teacher B suggested that all the learners should accommodate one another’s opinion during group discussions, and that they also pay attention to each other’s ideas. The twinned teachers agreed upon a clear set of specific learning objectives, a clear and complete set of task or instructions for the learners’ discussions and the post-group reflections (or debriefing) on group-work.

The results show that the learners in the experimental group shared their knowledge when solving algebra problems during the group discussions. The learners helped each other, challenged each other's ideas, and encouraged each other for the mutual benefit of all. Unlike in the first four days of the intervention process, the learners were not motivated because they seemed not to be attentive to each other's opinions when they engaged in group discussions. Teacher A commented, *"these learners now show commitment in group discussions, they used to play and now showing seriousness,"* indicating that his learners were improving in mathematics and that the group discussions appeared to have been effective during the intervention process. This cooperative teaching strategy encouraged the learners in small groups to get to know and to respect each other's ideas. They accepted each other's thoughts during small group discussions and supported each other when they did not understand the problems.

5.5.1.4 PROBLEM-SOLVING IN A TWINNING CONTEXT

During the course of the lessons, Teacher B gave the learners problems to solve, either in groups, or individually. The twinned teachers mostly prepared problems for the learners to solve that did not have immediate solutions or algorithms to provide the answers (Schoenfeld, 1992:382). Most of the learners appeared to struggle to find the immediate algorithms to solve the given problems, such as in Question 1.4 (see Appendix H). They constantly requested assistance from their teachers. Vygotsky's (1979:30) notion of scaffolding was used to provide support for the learners in solving the word problems. For example, Teachers A and B guided them by asking questions in their respective groups, such as, *"what is Tumi's and his father's ages?"* They furthermore asked, *"what does the product of their ages mean to you?"* to assist the learners in understanding the given scenario. The learners were encouraged to analyse and interpret problems before attempting to solve them. Teacher B said, *"you have to analyse the problem before you can start solving it."* He suggested that if they merely started by looking for the algorithms to solve the problems, they may struggle to find solutions. This ignorance of analysing the problem may contribute to their poor performance.

5.5.1.5 TEACHING AND LEARNING RESOURCES IN A TWINNING CONTEXT

As Tachie and Chirese (2013:67) mentioned, teaching and learning resources are essential in the mathematics class. Teacher B designed worksheets and handouts for the learners to use in the experimental group to use, as the school lacked textbooks. Teacher B mostly preferred to introduce a lesson, and during the lesson, would give the learners handouts for the discussion

of the problems on the worksheets or the handouts. The teacher did, however, seem to prefer to make use of the worksheets rather than the handouts, as handouts were prepared for learners to use as notes. The worksheets were used to engage the learners in doing the exercises on the concepts that they were taught.

The use of information and communication technology (ICT) has been encouraged in the teaching and learning of mathematics (Ndlovu, Wessels, & De Villiers, 2011:4). Teacher B preferred to incorporate DVDs to make his lessons effective. For example, DVD lessons were used in five lessons, especially in financial mathematics and functions. The teacher asked the learners to pay attention to the DVD lessons; and the learners asked the teacher to pause the DVD in order to gain clarity on the lesson taught. For instance, one of the learners in Lesson 9 asked the teacher to pause the DVD in order to receive clarification on the parabolic function in $f(x) = a(x - x_1)(x - x_2)$, which appeared to have confused the learners as they compared it with $f(x) = ax^2 + bx + c$. The learners found the first parabolic function more difficult to understand than the second. The lessons that were integrated with the DVDs seemed to have been fun, interesting and more effective, as the learners' participation was high when compared to the lessons which were not integrated with the ICT tools.

5.5.1.6 ASSESSMENT DURING THE INTERVENTION

Classwork and homework were used as tools to monitor the progress of learners during assessment. Teacher B preferred to give learners an activity after each lesson during the intervention in order to monitor the learners' progress and the level of understanding of the concepts taught (Van de Walle, Karp & Bay-Williams, 2013:81). He gave the learners homework, such as in Lesson 4, 6, 7, 9, and 10 and classwork in Lesson 1, 2, 3 and 5. Mbugua (2011:114) indicated that the textbook plays an important role in teaching and assessment. Teacher B used the textbook to design worksheets and handouts to give to the learners as assessment in order to master the algebra concepts. For example, he designed worksheets and handouts containing questions for the learners to answer, either during the lesson as a group activity or as classwork, such as in Lessons 1, 2, 3, and 5. The two designed instruments assisted the learners in the experimental group, as they did not have textbooks for class and home activities.

5.5.2 TEACHER A SEMI-STRUCTURED INTERVIEW

The semi-structured interviews with Teacher A (see Appendix K) were conducted immediately after the intervention classroom observations. The transcriptions below are

examples from the semi-structured interview conducted with teacher A of the experimental group, to establish his perceptions about the benefits of the intervention, if any.

Transcript 1: Do you think twinning is implemented according to your plans and why?

Extract 4.13

Teacher A: *Yes, we implemented twinning according to our plans, as we did combined lesson preparations, shared resources such computer software that played on the laptop, and I observed how the model teacher taught algebra. We also agreed on the days that twinning can take place during the periods. We shared our timetables and found that he is free on Tuesdays and Wednesdays. I checked my timetable also, and found that my Mathematics periods for the two days suit[ed] our plans. The problem is the funding, because if the schools don't assist us with petrol money, the twinning cannot go far.*

As twinning is a joint commitment of two schools (Berliner, 1990:5), the text in the extract above describes how the twinning was planned and implemented according to the twinned teachers' plan prior to the classroom observations. The twinned teachers intended to share the resources, lesson preparations and teaching practices. Teacher A used 'we' in the first sentence, suggesting his own self and the identity of Teacher B as those responsible to plan and implement the twinning process. These criteria of delegating duties amongst the twinned teachers also revealed that the implementation of twinning was carried out according to the plan. This implementation of twinning came to light when Teacher A said, "*we implemented twinning according to our plans as we did combined lesson preparations, shared resources*" and "*we shared our timetables*", so as to indicate how twinning was implemented by the twinned teachers.

Transcript 2: Does the model teacher possess the appropriate skills and knowledge he shares with you and why?

Extract 4.14

Teacher A: *Yes, the model teacher shows an experience of teaching mathematics. He improved learners' participation in the classroom. I like his questioning style as all learners are encouraged to listen as he just*

chooses anyone to give the answer. I also gained knowledge on how to introduce the topics, as he is able to link the prior knowledge and new topic to be taught, such as quadratic functions; he wanted to understand if learners were able to solve quadratic equations.

My learners were very weak and [I] found that after this twinning the learners started to change and participate. Those who gave me problems in writing activities started to write and had shown me that this learners are motivated. I really appreciate this twinning, and wish that it is sustained as we all learn, the teachers and learners. He introduced small groups discussions and my learners participated well. These learners did not play, as they did with me in their group discussions. The support he gave with their resources also played a role in motivating our learners to treat mathematics serious[ly].

Tracey and Morrow (2012:112) indicated that a model is someone who ought to be an expert who performs an action, which is consistent with what Teacher A said about Teacher B's skills, and the knowledge of mathematics he possessed. In addition, the skills and knowledge of Teacher B were supported when he said, *"yes, the model teacher shows an experience of teaching Mathematics"*, indicating that Teacher B possessed the expertise and experience that could change Teacher A's teaching practices in this subject. Teacher A indicated that the teaching approaches used by Teacher B seemed to have been effective, improving learner participation in small group discussions during twinning, as compared to learners' participation prior to the intervention. Teacher B's questioning techniques engaged most of the learners during the intervention than before the intervention. Teacher A said, *"I like his questioning style, as all learners are encouraged to listen, as he just chooses anyone to give the answer"*, suggesting the involvement of the learners in the lesson. The questioning techniques used by Teacher B were found to be engaging to most of the learners during the classroom observations.

The topics taught during the intervention have been well-taught, and were well-introduced by linking the new topic and the topic learners learned in teaching Algebra, as Teacher A indicated in the extract. For example, *"he is able to link the prior knowledge and new topic to be taught"*, showed that the prior knowledge was prioritised in teaching concepts by teacher B. The examples that were cited were quadratic functions, where Teacher B wanted to know if the learners were familiar with working with quadratic equations. Furthermore, teacher B

was seen testing prior knowledge in most of his lessons, for example, *“can you please find the formula for this sequence 3, 5, 7,...?”* The teacher wanted to teach the sequences of the second common difference by firstly understanding if the learners were able to determine the formula of the first common difference of the sequence. His intention was for the learners to identify the difference between the formula of the first difference and the second difference. Also, learners ought to know how to determine the common differences of the first and second formulas.

Most of the learners appeared to experience problems when solving algebraic problems before the intervention, as the pre-test results showed. The intervention seemed to have assisted in improving the learners' performance in algebra in the experimental group. For example, Teacher A mentioned, *“my learners were very weak and found that after this twinning, the learners started to change and participate.”* These learners appeared to have become motivated by the teaching strategies used by Teacher B. The teaching strategies in this school during pre-intervention classroom observations indicated the passiveness of the learners in not being able to respond to the questions asked by Teacher A, which support the interpretation of learners' weaknesses in mathematics. Only two or three learners were able to interact with the teacher, and the others remained silent or passive. Unlike during the twinning process, the observation *“the learners started to change and participate”* indicated improvement in the classroom interaction. Even those learners who did not take their work seriously, as noted in the comment *“those who gave me problems in writing activities started to write and had shown me that these learners are motivated,”* indicated that the learners understood the concepts taught during the intervention.

The results of Teacher A's interview indicated cooperative learning, which encouraged the interaction of the learners amongst themselves and communication with others (Zacharia, Solfitri, Daud & Abidin, 2013:98). The sharing of knowledge amongst the learners was encouraged by Teacher B during the intervention, which promoted social learning. Teacher A said, *“he introduced small group discussions and my learners participated well”*, suggesting the belief that learning is effective when the learners share ideas and work together in a group to solve mathematical tasks. Teacher A used the second person pronoun *‘he’* to identify Teacher B, who introduced working small group discussion approach in the experimental group. Teacher A's assertion that *“...these learners did not play as they did with me in their group discussions”* seemed to indicate a lack of mathematical knowledge prior to the intervention. Such perception about the small group discussions was consistent with the data collected from the classroom observations, which indicated that the group-discussion

approach appeared to have been ineffective during the pre-intervention observations. *‘These learners’* in the experimental group interacted in their groups by helping and challenging one another, providing feedback, encouraging one another and striving for mutual benefits in learning.

The group discussions encouraged most of the learners to be engaged in the lesson presented by Teacher B, as most of them participated voluntarily even when giving solutions on the board during the class discussion. Extract 4.14 of Transcript 3 below describes the skills and knowledge Teacher B had, and the reasons why he was regarded to have those skills and knowledge in mathematics teaching.

Transcript 3: Do you think the model teacher’s skills and knowledge can be used in the future teaching of mathematics and why?

Extract 4.15

Teacher A: *He used another approach, he sometimes gives learners who understood the concepts problems to solve and attend those who don’t understand, explaining each steps [sic]. He gives them enough time and found that it was good to treat them like that to avoid the boredom of the learners who understand the concept. Most of the learners tend to understand the concepts as he gives himself enough time to explain to them.*

The text in Extract 4.15 indicates that the different approach used by Teacher B had addressed most of the learners with difficulties in learning Algebra. The results showed that Teacher B was able to support learners with difficulties to solve problem (Vygotsky, 1978:86) . Teacher A used *‘he’*, referring to Teacher B, who supported learners who struggled to solve problems alone, and were able to solve them after the support. The statement above is confirmed when saying, “...*he sometimes gives learners who understood the concepts problems to solve and attend[ed] those who don’t understand explaining each steps [sic]*”, suggesting that the fast learners were given complex problems to solve in order for Teacher B to support those learners who struggled to understand the concepts taught followed procedural knowledge, step-by-step, in solving problems. This approach transpired again during the intervention classroom observations, where Teacher B grouped the learners according to their abilities and gave those learners who appeared to understand the concepts complex problems to solve in their groups, and attended those who had difficulties in learning the concepts taught. This type of support showed that Teacher B assisted most underachieving learners during the lesson. Vygotsky (1978:86) has said that a teacher should

provide support to learners who struggle to solve problems independently, needing the support of a more capable peer.

This type of group discussion was seen as important for both the achieving and underachieving learners, as the achieving learners did not feel left out when the teacher supported the slow learners. Teacher A's text in Extract 4.15 indicated that the group discussions in the experimental group appeared to be effective as "*most of the learners tend to understand the concepts*", which showed an improvement in the learning of algebra, as compared to the pre-intervention observations. Zacharia and Iksan (2007:36) argued that learners' small group discussions are effective, as learning occurs when the learners share ideas and work in groups to execute mathematical tasks. Extract 2.16 in Transcript 4 describes what Teacher A learned during the twinning process.

Transcript 4: What did you gain in this twinning process and why?

Extract 4.16

Teacher A: *I have learned many things, how to approach topics and engaging with the learners. The content knowledge of the model teacher also played a role. I had content knowledge of mathematics, but was impressed [with] the way this teacher presented the knowledge of mathematics in my mathematics class. Sometimes you will think that you have knowledge of mathematics, but found that there are other aspects of the content that need to be learned, like teaching number patterns and functions. These two concepts really motivated me and my learners did well in class activities and home activities. Learners did much better in the two tests we gave together, and this had made me to be confident [sic] that my learners are improving in mathematics. My learners also appreciated the collaboration of model teacher and me [sic]. I really enjoyed the collaboration during this twinning and will use the teaching approaches I gained from the model teacher mixing with the ones I used to apply.*

Extract 4.16 described how Teacher A rated Teacher B in respect of content and pedagogical content knowledge (Shulman, 1986:14) in the teaching of Grade 11 Algebra. Teacher A indicated that he experienced problems in teaching some of the algebra content "*...sometimes you will think that you have knowledge of mathematics, but found [sic] that there are other aspects of the content that need to be learned*", suggesting that he learned the skills and

knowledge of teaching those concepts during twinning in the experimental group. He used 'you', referring to his own identity, and positioned himself in a sense that the concepts seemed to challenge him before the intervention, and therefore could not influence learners' academic performances.

Hill (2010:515) has indicated that mathematics teachers ought to have content knowledge of the subject, as this is related to classroom teaching, and also as it influences learners' academic achievement. The concepts cited, namely '*number patterns and functions*' that were difficult to learn and to teach before the intervention, were addressed by Teacher B. For example, Teacher B gave the learners worksheets in small groups with problems involving functions to discuss in small groups, in order to determine the x -intercepts, y -intercepts, axis of symmetry, turning points, and the plotting of the graphs using the function $f(x) = x^2 - 2x - 3$. Teacher B explained to the learners how to find the parameters before they could solve problems.

Extract 4.17 of transcript 6 below describes the challenges that may need attention in planning the twinning process.

Transcription 5: Are there any challenges that need attention in order to develop this process?

Extract 4.17

Teacher A: *The challenge is time and money to run this twinning. If maybe we can get financial support, things will be much better because it is costly for the model teacher to travel from his school to our school twice a week.*

R: Who should provide a financial support?

Teacher A: *The schools in twinning, the department or NGOs.*

Twinning has its own challenges, such as the cost of travelling from one school to the other (De Young & Howley, 1990:65); the time involved in planning meetings and discussions (Nachtigal, 1990:9); as well as additional work responsibilities for teachers, and work-related stress (Rees 2003:24). The text in the extract above describes the issue of time and money as the most challenging factors in the planning and implementing an effective twinning. The '*time*' in the text refers to the time-tabling of twinned schools to be drafted and the days to be used to implement twinning, and the '*money*' refers to the funding that is needed either for travelling from one school to another, in order to implement twinning. The twinned teachers suggested that if the Department of Basic Education (DBE) and non-governmental

organisations were able to support the twinning project with funding, this project would prove effective. Teacher A used the word *'things'* to signify the logistics of this, such as travelling expenses, and other materials, such as ICT tools, worksheets, and handouts to be prepared for learners that may be required in the twinning process.

Transcripts 6: What are the most important benefits of twinning?

Extract 4.18

Teacher A: *In our twinning, we pay attention to sharing best teaching practices and resources, combined lesson preparations, team teaching, collaboration and transfer of knowledge. One of the factors can be change in learners' performance in mathematics.*

R: What do you mean by sharing best teaching practices?

Teacher A: *The teaching approaches the other teacher can use, sharing LTSM, plan lessons together, co-teach especially when learners discuss during twinning.*

Rees (2003:24) mentioned that the teachers ought to benefit in the twinning process. Bandura (1986:459) indicated that the observer pays attention to the modelled behaviour in order to gain information such as skills, knowledge and strategies that can be modelled by the expert teacher. The text in the extract indicates that the expert teacher ought to model his teaching approaches, while the other teacher observes him to learn the skills and expertise during twinning. The schools that embark on twinning should share “resources”, suggesting the learner-teacher support materials (LTSM), according to the DBE (2011:11). “Combined lesson preparations” were suggested, which supported what the researcher observed as the twinned teachers shared their lessons, which was found to be effective.

5.6 POST-INTERVENTION CLASSROOM OBSERVATIONS

Post-intervention classroom observations were held in order to reflect on the intervention strategy, namely whether Teacher A paid attention to the expertise and skills (Bandura, 1986) of Teacher B. Four classroom observations were held with Teacher A in the experimental group after the intervention.

5.6.1 TEACHER-LEARNER INTERACTION

The data obtained from a lesson conducted by Teacher A revealed that he was able to transfer the knowledge gained during the intervention (Bandura, 1986:459). The introduction of the lesson followed question-and-answer method to reveal what the learners' knew, in order to understand how to introduce new knowledge of algebra.

The rationale for scaffolding is to provide structures to move the learners from one level to another, through the support of the teacher (Vygotsky, 1978:89). 'Scaffolding' is a teaching strategy that ought to facilitate the learners' ability to build on prior knowledge, and internalise information through the support of the teacher (Van der Stuyf, 2002:5). The teacher randomly asked the learners questions, and further probed them so as to gain an understanding of what the learners knew, unlike what he did before the intervention, by asking questions to gain the correct answers. For example, Teacher A, in the fourth lesson, asked the learners, "*what is the nature of the roots when $\Delta = 0$?*" He wanted the learners to tell him that the roots were real and equal. Teacher A followed the narrative or teacher-centred approach, and the learners in the experimental group appeared to have been puzzled during the lessons before the intervention. For example, Teacher A explained to learners how to find critical values "*you have to get the left and right hand values of the value factorised $x = 2$ and $x = 3$ before determining the inequalities*", as learners appeared not to have understood what to do in order to solve for x of quadratic inequalities.

Teacher A's teaching approaches were mostly learner-centred, as the learners were able to construct knowledge through the guidance of the teacher (Olson & Platt, 2001:175). These researchers argued that in the learner-centred approach, the teacher becomes the facilitator of knowledge, rather than the dominant content expert. The approaches used during the post-intervention classroom observations were different from those Teacher A used before the intervention, which marginalised learners' involvement. Teacher A had acquired the appropriate skills and gained the relevant knowledge of teaching from Teacher B. He was eager to understand the learners' ways of thinking. He diagnosed the learners' errors and the sources of those errors and misconceptions underlying those errors knowledge of various ways of representing specific topics (Shulman, 1986:14).

5.6.2 LEARNER-LEARNER INTERACTION

The learners in the experimental group were given the opportunity to share their ideas in the groups during the post-intervention classroom observations. Teacher A appeared to have seen this type of interaction as effective, by giving the learners problems to solve so as to assist

them to provide an atmosphere of achievement (Naomi & Githua, 2012:178). This type of interaction appeared to be effective, when compared to the one used before the intervention, where most of the learners were passive, and merely received the information transmitted by the teacher.

Teacher A encouraged his learners “...to share ideas and respect each one another”, when introducing small group discussions of algebraic concepts. The teacher also: made a clear set of learning outcomes; a complete set of task instructions; gave the learners sufficient time during the discussions; and time for debriefing (Naomi & Githua, 2012:178). The learners were also active in solving problems in groups, as they were exposed to this approach by Teacher B during the interventions. The learners exchanged ideas, and challenged one another on solving problems, and asked questions when experiencing problems. The teacher walked around to assist the learners during the group discussions. He appeared to have learned this approach from Teacher B’s lesson presentations.

5.6.3 TEACHING AND LEARNING RESOURCES

Teacher A adapted Teacher B’s method of using handouts and worksheets in teaching his Grade 11 class. Yara and Uganda (2010:126) indicated that the availability of resources enhance the effectiveness of the schools, where Teacher A refrained from using only the textbook and the study guide to teach his learners. The teacher always gave his learners handouts and worksheets to use when discussing problems in class. The DVDs were used three times when teaching the concepts, which showed that the teacher was motivated by what Teacher B did during the intervention. The teacher used DVDs for almost twenty-five minutes. While the learners watched, he sometimes paused for the learners to solve some of the problems, before the solutions were given by the DVD presenter. The teacher would say, for example, “okay, let’s see if you can solve this problem $\frac{2}{x+3} \leq \frac{1}{x-3}, x \neq \pm 3$.” Teacher A seemed to have realised the importance of incorporating ICT as an opportunity to improve classroom teaching (Ndlovu, Wessels & De Villiers, 2011:4).

5.6.4 CLASSROOM ASSESSMENT

The teacher gave the learners homework and classwork to be used as formative assessment to diagnose the learners’ understanding of the concepts. The classroom assessment used in Teacher A’s class was the monitoring of the learners’ progress in algebra. Van de Walle, Karp, and Bay-Williams (2013:78) supported that assessment ought to enable the teachers’ to monitor a learners’ progress. This was unlike before the intervention, when the learners were not doing activities (classwork and homework) given by Teacher A, and where monitoring

could not take place. Formative assessment ought to inform the teacher about the future action in classroom learning (Williams, 2010:10). Teacher A gave the learners feedback after each activity in order that the learners may understand the mistakes made in solving problems.

5.7 POST-TEST RESULTS: QUANTITATIVE DATA

The descriptive statistics generated from the post-test data are discussed below in the light of the research objective, in an effort to make sense of twinning as a strategy in Grade 11 Algebra. The data generated from the post-test followed Didis and Erbas' (2014:1141) categories of the learners' responses for analysis as used in the pre-test data analysis: correct responses (CR); incomplete responses (InC); incorrect responses (IR); and blank responses (BL); as for the pre-test.

5.7.1 LEARNER RESPONSES TO QUESTION 1 ITEMS

Table 5.7 below depicts the data generated from the learners' post-test results on Q1 question items in quadratic equations, quadratic inequality, simultaneous equations, and one-word problems. The overall percentage of Q1 question items for both the experimental and the control groups were 33.6% and 56.2%, respectively. The results showed an improvement in the learners' responses in CR's experimental group as compared to the Q1 question items in the pre-test.

Table 5.7: Percentages (and absolute numbers) of Correct Responses (CR), Incorrect Responses (IR) and Blank Responses (NR) on Q3 items for the experimental and the control groups.

Items	Group	Post-test			
		CR	InR	IR	NR
QE ₁	Experimental	33.3% (14)	14.3% (6)	31.0% (13)	19.1% (8)
	Control	100% (42)	0.0% (0)	0.0% (0)	0.0% (0)
QE ₂	Experimental	45.2% (19)	11.9% (5)	26.2% (11)	16.7% (7)
	Control	90.5% (38)	0.0% (0)	9.5% (4)	0.0% (0)
QI	Experimental	21.4% (9)	21.4% (9)	31.0% (13)	26.2% (11)
	Control	85.7% (36)	0.0% (0)	14.3% (6)	0.0% (0)
SE	Experimental	54.8% (23)	16.7% (7)	19.1% (8)	9.5% (4)
	Control	92.9% (39)	0.0% (0)	7.1% (3)	0.0% (0)
WP	Experimental	28.6% (12)	11.9% (5)	31.0% (13)	28.6% (12)
	Control	61.9% (26)	14.3% (6)	21.4% (9)	2.4% (1)
Total	Experimental	36.7% (77)	15.2% (32)	27.7% (58)	20.0% (42)
	Control	87.8% (181)	2.9% (6)	10.5% (22)	0.5 (1)

The results in Table 5.7 indicate that Q1 items' percentage ranged from 21.4% to 54.8% and 61.9% to 100% for CR in the experimental and the control groups, respectively. Though the experimental group showed less of an improvement, namely 33.4% as compared to the control group with 38.1% in the CR category, the CR category was satisfactory but the learners performed better in the Q1 items. The high percentage in IR for the experimental group was produced in and QE₁, QI and PW, with 31.0% by experimental group, the IR category suggesting a lack of knowledge of correct procedures (Kilpatrick et al., 2001:116). The experimental and control groups' percentage change in the post-test for InR, IR, and BR respectively, suggested the improvement of the learners in using procedures to solve the given equations, inequality and word-problems in comparison to the pre-test results. The overall results showed an improvement in the experimental group's learner achievement in Q1 which suggests that the connection of the concepts and the methods to solve the question items was made by the learners (Watson & Sulliman, 2008:110).

5.7.2 LEARNER RESPONSES TO QUESTION 2 ITEMS

Table 5.8 below indicates the detailed results on the number patterns (NP₁ – NP₄) question items, which required from the learners to determine the next term of a sequence, the general term and the number of terms in the sequence. The overall results revealed 39.1% and 52.6% respectively, in Q2 items, for the experimental and the control groups.

Table 5.8: Percentages (and absolute numbers) of Correct Responses (CR), Incorrect Responses (IR) and Blank Responses (NR) on Question 3 items for the experimental and the control groups.

Items	Group	Post-test			
		CR	InR	IR	NR
NP ₁	Experimental	97.6% (41)	0.0% (0)	0.0% (0)	2.4% (1)
	Control	85.7% (36)	0.0% (0)	14.3% (6)	0.0% (0)
NP ₂	Experimental	45.2% (19)	26.2% (11)	28.6% (12)	0.0% (0)
	Control	47.6% (20)	14.3% (6)	35.7% (15)	2.4% (1)
NP ₃	Experimental	28.6% (12)	23.8% (10)	14.3% (6)	33.3% (14)
	Control	31.0% (13)	21.4% (9)	45.2% (19)	2.4% (1)
NP ₄	Experimental	7.1% (3)	7.1% (3)	21.4% (9)	64.3% (27)
	Control	35.7% (15)	28.6% (12)	31.0% (13)	4.8% (2)
Total	Experimental	44.6% (75)	14.3% (24)	16.1% (27)	25.0% (42)
	Control	50.0% (84)	16.1% (27)	31.6%	2.4% (4)

The results in the table above indicate that the CR showed learners' percentage ranged from 7.1% to 97.6% and from 31.% to 85.7%, for the experimental and control groups respectively, which showed an improvement in the comprehension of the number patterns. Kilpatrick et al. (2001:117) postulated that most learners taught conceptual knowledge are

able to connect concepts in solving mathematical problems. A high percentage for NP₄ was found in BR, with 64.3% for the experimental group, which showed that the learners lacked the knowledge of determining the number of terms in the sequence, as they failed to attempt to answer the question item. The results for InR showed that NP₂'s percentage was 26.2%, where the learners showed little knowledge of the correct procedures to determine the general term of the sequence.

5.7.3 LEARNER RESPONSES TO QUESTION 3 ITEMS

Table 5.9 below depicts the data generated from Question 3 items, which provide detailed information on financial mathematics (FM₁ and FM₂). The overall results for Q3 are 45.3% and 58.9% for the experimental and the control groups, respectively. The results in the table below revealed better correct mathematical responses (CR) for the experimental group as compared to InR, IR, and BR.

Table 5.9: Percentages (and absolute numbers) of Correct Responses (CR), Incorrect Responses (IR) and Blank Responses (NR) on Question 3 items for the experimental and the control groups.

Items	Group	Post-test			
		CR	InR	IR	NR
FM ₁	Experimental	19.1% (8)	42.9% (18)	31.0% (13)	2.4% (1)
	Control	50.0% (21)	11.9% (5)	31.0% (13)	7.1% (3)
FM ₂	Experimental	21.4% (9)	45.2% (19)	33.3% (14)	4.8% (2)
	Control	52.4% (22)	7.1% (3)	28.6% (12)	11.9% (5)
Total	Experimental	20.1% (17)	44.1% (14)	32.2% (27)	3.6% (3)
	Control	51.2% (43)	9.5% (8)	29.8% (25)	9.5% (8)

The results in Table 5.9 reveal the CR's percentage range scores from 19.1% to 20.1% and from 50.0% to 52.4% for the experimental and control groups, in calculating the monthly premiums and investments using a simple interest formula and compound interest formula for FM₁ and FM₂ question items. The percentage scores showed that the learners still needed conceptual knowledge (Kilpatrick et al., 2001:117) when calculating the monthly instalment and investment for the two question items.

The IR's 31.0% and 33.3% were produced which showed the learners misinterpreted the statements given to them by means of which to calculate the monthly instalments using the simple interest formula and the total amount of investment of the money saved using the compound interest formula. The IR's results showed that the learners had little knowledge and skills in respect of financial mathematics, but could not apply them appropriately, flexibly, accurately and efficiently (Kilpatrick et al., 2001:117). Kilpatrick et al. (2001:117)

and the National Council of the Teachers of Mathematics (NCTM, 2014:9) have argued that procedural fluency builds on conceptual understanding, strategic reasoning and problem-solving. The percentages found in IR's revealed learners' lack of the conceptual knowledge as the procedures used to calculate the monthly instalment and investment were not all correct.

5.7.4 LEARNER RESPONSES TO QUESTION 4 ITEMS

Table 5.10 below indicates the results generated from Q4 question items, which provide detailed information on the exponential expressions and equations for the experimental and the control groups. The overall results for Q4 were found to be 47.8% and 48.0% for the experimental and the control groups, respectively. The results showed a bettered mathematical response of the learners in simplifying exponential expressions and equations.

Table 5.10: Percentages (and absolute numbers) of Correct Responses (CR), Incorrect Responses (IR) and Blank Responses (BR) on Question 3 items for the experimental and the control groups.

Items	Group	Post-test			
		CR	InR	IR	NR
EE _{ex}	Experimental	38.1% (16)	11.9% (5)	21.4% (9)	28.6% (12)
	Control	40.5% (17)	16.7% (7)	26.2% (11)	16.7% (7)
EE _{eq}	Experimental	47.6% (20)	4.8% (2)	21.4% (9)	23.8% (10)
	Control	35.7% (15)	11.9% (5)	31.0% (13)	21.2% (9)
EE _{eq}	Experimental	40.5% (17)	14.3% (6)	21.4% (9)	23.8% (10)
	Control	38.1% (16)	7.1% (3)	28.6% (12)	26.2% (11)
Total	Experimental	42.1% (53)	10.3% (13)	21.4 (18)	25.4% (32)
	Control	38.1% (48)	11.9% (15)	28.6% (36)	21.4% (26)

Table 5.10 above indicates the results for the experimental and the control groups, namely that the CR's percentages ranged from 38.1% to 47.6% and 35.7% to 40.5%, respectively, which suggested that both the experimental and the control groups' learners were able to connect the exponential laws in simplifying exponential expressions (EE_{ex}) and exponential equations (EE_{eq1} and EE_{eq2}). The experimental group produced the highest percentage of 14.3% for InR, which showed the learners lacked the relevant procedures in solving the value(s) of x in EE_{eq2}. The learners did not realise that the equation expected them to use the common factor method to find the values of x for EE_{eq2}. The IR and BR's highest percentages were 21.4% and 28.6 respectively, for the experimental group, which showed an improvement, though the learners lacked the conceptual knowledge of both the exponential expressions and the exponential equations in comparison to the pre-test results.

5.7.5 LEARNER RESPONSES TO QUESTION 5 ITEMS

The table below depicts the data generated from Question 5 items, which required the learners to interpret the hyperbola and exponential graphs. The overall results for Q5 were 50.0%, and 44.5% for the experimental and the control groups, respectively.

Table 5.11: Percentages (and absolute numbers) of Correct Responses (CR), Incorrect Responses (IR) and Blank Responses (BR) on Question 3 items for the experimental and the control groups.

Items	Group	Post-test			
		CR	InR	IR	NR
HPEF ₁	Experimental	73.8% (31)	0.0% (0)	14.3% (6)	11.9% (5)
	Control	59.5% (25)	9.5% (4)	28.6% (12)	2.4% (1)
HPEF ₂	Experimental	50.0% (21)	0.0% (0)	28.6% (12)	21.4% (9)
	Control	54.8% (23)	0.0 (0)	38.1% (16)	7.1% (3)
HPEF ₃	Experimental	33.3% (14)	0.0% (0)	42.9% (18)	23.8% (10)
	Control	33.3 (14)	0.0% (0)	45.2% (19)	21.4% (9)
HPF ₄	Experimental	42.9% (18)	0.0% (0)	38.1% (16)	19.1% (8)
	Control	31.0% (13)	0.0% (0)	50.0% (21)	19.1% (8)
HPF ₅	Experimental	28.6% (12)	0.0 (0)	33.3% (14)	38.1% (16)
	Control	33.3% (14)	0.0 (0)	42.9% (18)	23.8% (10)
Total	Experimental	45.7% (96)	0.0% (0)	31.4% (66)	22.9% (48)
	Control	42.4% (89)	1.9% (4)	41.0% (86)	14.8% (31)

The results for Q5 question items showed the CR's percentage ranged from 28.6% to 73.8% and 31.0% to 59.5% for the experimental and the control groups, respectively, which indicated the learners' knowledge of interpreting the hyperbola and exponential graphs in order to calculate the unknown values using the given information. The learners should be able to connect the related concepts and methods in the appropriate ways in order to solve mathematical problems (Kilpatrick et al., 2001:118). The results revealed that IR and BR yielded 42.9% and 38.1% for TI₃ and TI₅, which is consistent with the knowledge of variety of procedures to be used to solve mathematical problems effectively National Research Council (NRC, 2001:12).

5.7.6 LEARNER RESPONSES TO QUESTION 6 ITEMS

Table 5.12 depicts the data generated from Question 6 items on parabola graph drawn with the turning point and one x-intercept given to the learners to interpret. The overall results were 11.8% and 39.6% for the experimental and the control groups, respectively. The results revealed that the experimental group showed more IR and BR in the question items.

Table 5.12: Percentages (and absolute numbers) of Correct Responses (CR), Incorrect Responses (IR) and Blank Responses (BR) on Question 3 item for the experimental and the control groups

Items	Group	Post-test			
		CR	InR	IR	NR
PF ₁	Experimental	11.9% (5)	0.0% (0)	19.1% (8)	66.7% (28)
	Control	26.1% (11)	0.0% (0)	47.6% (20)	26.2% (11)
PF ₂	Experimental	7.1% (3)	0.0% (0)	19.1% (8)	73.8% (31)
	Control	26.1% (11)	0.0% (0)	52.4% (22)	21.4% (9)
PF ₃	Experimental	4.8% (2)	0.0% (0)	16.7% (7)	78.6% (33)
	Control	21.4% (9)	0.0% (0)	52.4% (22)	26.2% (11)
PF ₄	Experimental	19.1% (8)	0.0% (0)	19.1% (8)	61.9% (26)
	Control	35.7% (15)	0.0% (0)	47.6% (20)	16.7% (7)
PF ₅	Experimental	26.2% (11)	14.3% (6)	14.3% (6)	45.2% (19)
	Control	50.0% (21)	16.7% (7)	19.1% (8)	14.3% (6)
PF ₆	Experimental	23.8% (10)	0.0% (0)	14.3% (6)	61.9% (26)
	Control	52.4% (22)	0.0% (0)	28.6% (12)	19.1% (8)
PF ₇	Experimental	14.3% (6)	0.0% (0)	40.5% (17)	45.2% (19)
	Control	35.7% (15)	0.0% (0)	52.3% (22)	11.9% (5)
PF ₈	Experimental	31.0% (13)	0.0% (0)	14.3% (6)	54.8% (23)
	Control	33.3% (14)	0.0% (0)	52.4% (22)	14.3% (6)
PF ₉	Experimental	31.0% (13)	0.0% (0)	9.5% (4)	59.5% (25)
	Control	38.1% (16)	0.0% (0)	50.0% (21)	11.9% (5)
Total	Experimental	18.8% (71)	1.6% (6)	18.5% (70)	60.8% (230)
	Control	35.4% (134)	1.9% (7)	44.7% (169)	18.0% (68)

The results in Table 5.12 indicate the CR's percentages ranged from 7.1% to 31.0% and 21.4% to 52.4% for the experimental and the control groups in interpreting the graph with given points. The CR's highest percentage of 31.0% was found in TI₈ and TI₉ for the experimental group, with the learners showing an understanding of the translation of the graph determining the minimum and maximum values of the new graph. The IR and BR's percentages found were 78.6% and 40.5% respectively for the experimental group, suggesting a lack of interpretation of the graph in order to respond to the given questions. The learners appeared to lack the conceptual knowledge to connect the related concepts and the methods (Kilpatrick et al., 2001) to use the information given in the graph to respond to the question items.

5.8 STATISTICAL ANALYSIS

The descriptive statistics generated from the pre-test and post-test data are discussed below in the light of the research objectives of the study. The statistical data are analysed using the Wilcoxon-Rank Sum test for statistics to test for statistical significance. All the variables are

not normally distributed (all *p-values* are less than 0.05). The use of a parametric test is warranted due to this normality distribution. The Rank Sum Test is used to compare the two study groups. Furthermore, the interpretation is performed at a 95% confidence limit. Two permutations were used to analyse the statistical data, the difference between and within the two study groups.

5.8.1 ANALYSES OF PRE- AND POST-TEST RESULTS

The analyses of the pre- and post-test results are discussed separately as per question, Q1 to Q6, by comparing the Rank Sum scores, testing the statistical significance between the two study groups.

Table 5.13: Analysis of the pre- and post-test results for Q1: The Wilcoxon Rank-Sum (Mann-Whitney) Test

School	Pre-test			Post-test		
	Obs	Rank-sum	Expected	obs	Rank-sum	Expected
			d			d
Experimental	43	1069.5	1849	43	1389	1849
Control	42	2585.5	1806	42	2266	1806
Combined	85	3655	3655	85	3655	3655
		<i>p-value = 0.0000</i>			<i>p-value=0.0001</i>	

The analysis of the results for Q1 revealed that the two study groups performed significantly differently in Q1 in the pre-test ($p - value = 0.0000$), which is less than 0.05 at the 95% confidence limit. The control group performed better in comparison to the experimental group, and recorded significantly higher scores in Q1, which suggested learners in the experimental group grappled with quadratic equations, quadratic inequalities, simultaneous equations and word-problems.

The post-test analysis of the results showed that the two study groups indicated a significant difference, namely $p - value = 0.0001$, also still less than 0.05, with the experimental group performing poorly in comparison to the control group a 95% confidence limit. Therefore, the control group still recorded significantly higher scores than the experimental group after the intervention.

The experimental group, however, recorded low scores in the post-test, but showed a better score improvement in comparison to the pre-test results. Therefore, the twinning effect had improved the experimental group's scores, as the learners performed better in the post-test in comparison to in the pre-test.

Table 5.14: Analysis of the pre-and post-test results for Q2: The Wilcoxon Rank-Sum (Mann-Whitney) Test

School	Pre-test			Post-test		
	obs	Rank-sum	Expected	obs	Rank-sum	Expected
Experimental	43	1862	1849	43	1608.5	1849
Control	42	1793	1806	42	2046.5	1806
Combined	85	3655	3655	85	3655	3655
			$p\text{-value} = 0.9078$			
						$p\text{-value}=0.0333$

The data before the intervention indicated that the analysis of the results in the pre-test showed no statistical significant difference between the two study groups for Q2, namely ($p - value = 0.9078$), greater than 0.05 at a 91% confidence limit. Even though the experimental group recorded a higher score than the control group, the difference between the groups was too small in Q2 to yield the significant difference.

The analysis of the results after the intervention yielded a significant difference between the two groups for Q2 in the post-test ($p - value = 0.0333$), less than 0.05 at a 95% confidence limit.

The control group recorded a significantly higher score when compared to the experimental group in the post-test. Therefore, twinning did not have any effect, as the experimental group's performance dropped in the post-test, when compared to the pre-test.

Table 5.15: Analysis of the pre-and post-test results for Q3: The Wilcoxon Rank-Sum (Mann-Whitney) Test

School	Pre-test			Post-test		
	obs	Rank-sum	Expected	obs	Rank-sum	Expected
Experimental	43	1720	1849	43	1657.5	1849
Control	42	1935	1806	42	1997.5	1806
Combined	85	3655	3655	85	3655	3655
			$p\text{-value} = 0.2467$			
						$p\text{-value}=0.0889$

The results before the intervention for Q3 in the pre-test indicated no significant difference between the two study groups ($p - value = 0.2467$), greater than $p - value$ 0.05 suggesting that the two groups displayed similar results. The results showed that the control group recorded higher scores when compared to the experimental group, but the difference of the scores was not significant for Q3. The results suggested that the learners struggled to

calculate the amount of money to be paid using the simple and compound interest formulas in questions items.

The data after the intervention in the post-test indicated no statistical significant difference between the two study groups ($p - value = 0.0889$), greater than $p - value 0.05$ at a 89% confidence limit. The experimental group recorded a declined score in the post-test results, which suggested the intervention did not have any effect.

Table 5.16: Analysis of the pre-and post-test results for Q4: The Wilcoxon Rank-Sum (Mann-Whitney) test

School	Pre-test			Post-test		
	obs	Rank-sum	Expected	obs	Rank-sum	Expected
Experimental	43	1627.5	1849	43	1849	1849
Control	42	2027.5	1806	42	1808	1806
Combined	85	3655	3655	85	3655	3655
		$p-value = 0.0486$			$p-value = 1.0000$	

The data in Table 5.16 above depicts that the Q4 pre-test's analysis of the results indicated a significant difference between the two study groups ($p - value = 0.0486$), which is less than $p - value 0.05$ at a 95% confidence limit. The control group recorded a significantly higher score compared to the experimental group, which suggests a poor performance in Q4 in the experimental group before the intervention in the pre-test, when simplifying the exponential expressions and solving the exponential equations.

The results in Table 5.16 furthermore reveal the post-test results for Q4, which yielded no significant difference between the two study groups ($p - value = 1.0000$), greater than 0.05. Though the results did not indicate the significant difference between the study groups, the experimental group showed an improvement in their post-test scores, when compared to the pre-test scores. The results suggest that the intervention had a positive impact in the post-test in the experimental group, as they showed an improvement in simplifying exponential expressions and exponential equations.

Table 5.17: Analysis of the pre- and post-test results for Q5: The Wilcoxon Rank-Sum (Mann-Whitney) Test

School	Pre-test			Post-test		
	obs	Rank-sum	Expected	obs	Rank-sum	Expected
Experimental	43	1386.5	1849	43	1950.5	1849
Control	42	2268.5	1806	42	1619.5	1806
Combined	85	3655	3655	85	3655	3655
		<i>p-value= 0.0000</i>			<i>p-value=0.1350</i>	

The analysis of the pre-test results indicated a statistical significant difference between the two study groups ($p - value = 0.0000$), less than $p - value 0.05$ at a 95% confidence limit. The control group recorded a significantly higher score when compared to the experimental group, suggesting a poor performance in Q₅, before the intervention in interpreting the exponential and hyperbolic functions.

The post-test analysis of these results for Q₅ revealed no significant difference between the two groups ($p - value = 0.1350$), greater than $p - value 0.05$. The experimental group recorded a significantly higher score when compared the control group in the post-test after the intervention.

Furthermore, the experimental group showed an improvement in the post-test after the intervention, which suggested that the intervention had a positive impact on the learners' performance in interpreting the exponential and hyperbolic functions.

Table 5.18: Analysis of the pre- and post-test results for Q6: The Wilcoxon Rank-Sum (Mann-Whitney) test

School	Pre-test			Post-test		
	obs	Rank-sum	Expected	obs	Rank-sum	Expected
Experimental	43	1053.5	1849	43	1172.5	1849
Control	42	2601.5	1806	42	2397.5	1806
Combined	85	3655	3655	85	3655	3655
		<i>p-value= 0.0000</i>			<i>p-value=0.0000</i>	

The data in Table 5.18 depicts the analysis of the Q₆ pre-test results, indicating significant different scores between the two study groups ($p - value = 0.0000$), less than $p - value 0.05$ at a 95% confidence limit. The results showed the control group recorded a significantly higher score than the experimental group before the intervention. Thus, the

experimental group performed poorly in the pre-test, compared to the control group in the pre-test, when interpreting parabolic functions.

The analysis of the post-test results in Table 5.18 revealed a significant difference between the two study groups ($p - value = 0.0000$), less than $p - value 0.05$ at a 95% confidence limit. The control group still recorded a significantly high score when compared to the experimental group, which suggests that the experimental group performed poorly in the post-test, after the intervention. Though the experimental group performed poorly in Q6 after the intervention, the group showed an improvement in the post-test, when compared to the pre-test results in Q6. The results after the intervention in the post-test suggested that twinning had a positive impact in improving the learners' performance in Grade 11 Algebra.

5.8.2 SUMMARY OF THE PRE- AND POST-TEST RESULTS

The pre-test and post-test results are summarised in Graph 5.7 and are compared as per question below. The different mean scores of the pre-test and post-test are compared between and within groups.

Table 5.19: Summary of pre- and post-test results based on the rank sum scores between the experimental and the control groups: The Wilcoxon Rank-Sum (Mann-Whitney) Test

<u>Question</u>	<u>Experimental</u>			<u>Control</u>		
	<u>Pre-test</u>	<u>Post-test</u>	<u>p-value</u>	<u>Pre-test</u>	<u>Post-test</u>	<u>p-value</u>
Q1	<u>1273.5</u>	<u>2467.5</u>	<u>0.0000</u>	<u>1590</u>	<u>1980</u>	<u>0.0805</u>
Q2	<u>1768.5</u>	<u>1972.5</u>	<u>0.3730</u>	<u>1480</u>	<u>2090</u>	<u>0.0060</u>
Q3	<u>1552</u>	<u>2189</u>	<u>0.0050</u>	<u>1443</u>	<u>2127</u>	<u>0.0019</u>
Q4	<u>1491.5</u>	<u>2249.5</u>	<u>0.0009</u>	<u>1572</u>	<u>1998</u>	<u>0.0553</u>
Q5	<u>1099</u>	<u>2556</u>	<u>0.0000</u>	<u>1295.5</u>	<u>2274.5</u>	<u>0.0000</u>
Q6	<u>1376</u>	<u>2279</u>	<u>0.0000</u>	<u>1572.5</u>	<u>1997.5</u>	<u>0.0550</u>
Total rank	<u>8560.5</u>	<u>13713.5</u>	<u>0.0010</u>	<u>8953</u>	<u>12467</u>	<u>0.0460</u>

The results in Table 5.19 show the analysis of the two study groups, revealing different rank-sum score between the pre-test and post-test for Q1. The experimental group's results showed a significantly different score between the pre-test and post-test ($p - value = 0.0000$), less than 0.05 at a 95% confidence limit. On the other hand, the control group indicated no significantly different scores for Q1 between the pre-test and the post-test ($p - value = 0.0805$) greater than the $p - value 0.05$, which suggests no significant improvement when compared to the experimental group, which improved significantly in the post-test. The

results of the experimental group for Q1 showed that the learners improved significantly in solving quadratic equations, quadratic inequities, simultaneous equation and word-problems.

The data in Table 5.19 indicates the analysis of the results for Q2, which revealed no statistical significant difference for the experimental group ($p - value = 0.3730$), greater than the $p - value$ 0.05, which suggested no significant improvement. While the control group indicated a significant difference ($p - value = 0.0060$), less than $p - value$ 0.05 at a 95% confidence limit. Therefore, the control group indicated a statistical significant improvement in the post-test, when compared to the experimental group, which showed no significant improvement in the post-test. In other words, the intervention in the experimental group did not significantly alter the way in which the general terms and the number of terms of the sequence in number patterns were determined.

The analysis of the results in the table above for the experimental group between the pre-test and the post-test for Q3 revealed a statistically significant difference ($p - value = 0.0050$) below the $p - value$ 0.05 at a 95% confidence limit. On the other hand, the analysis of the control group's results also indicated a statistical significant difference between the pre-test and post-test ($p - value = 0.0019$) below the $p - value$ 0.05 at a 95% confidence limit. The results showed a significant improvement in the two study groups in the post-test, which suggested that the learners understood how to calculate the amount of money to be paid monthly, and the investment, using simple and compound interest formulas. Though the two study groups improved significantly, the experimental group improved more than the control group, which suggests that the intervention had a positive impact in improving the learners' performance in Q3, in calculating the amount of money using simple and compound interests.

The data in Table 5.19 for Q4 of the experimental group revealed a significant difference ($p - value = 0.0009$) below the $p - value$ 0.05 at a 95% confidence limit between the pre-test and the post-test. In contrast, the analysis of the control group's results between the pre-test and post-test indicated no significant difference ($p - value = 0.0553$) above the $p - value$ 0.05. Therefore, the results for Q4 showed a significant improvement in the post-test for the experimental group, suggesting that the intervention had a positive impact on the learners learning the exponential expressions and equations.

The data for Q5 that were analysed for the two study groups revealed a statistical significant difference between the pre-tests and post-tests. Both had the same p-value ($p - value = 0.0000$) below the $p - value$ 0.05 at a 95% confidence limit. The results of the two study groups indicated a significant improvement in the post-test. Though the two study groups

showed a significant improvement in the post-test, the experimental group improved more than the control group, suggesting the intervention had an effect in improving the learners' interpretation of the exponential and hyperbolic functions.

The analysis of the results for Q6 revealed a statistically significant difference between the pre-test and post-test of the experimental group ($p - value = 0.0000$) below the $p - value$ 0.05 at a 95% confidence limit. On the other hand, the control group's results indicated no statistically significant difference between the pre-test and post-test scores ($p - value = 0.0550$) above the $p - value$ 0.05, suggesting a low improvement in the post-test. The analysis of the results revealed a significant improvement in the post-test of the experimental group when compared to the control group. The results suggest that the intervention showed a positive impact on the experimental group, namely that the learners' interpretation of parabolic functions improved in the post-test.

In summary, the analysis of the results in the pre-test and post-test in the two study groups, revealed statistical significant different scores for the experimental group ($p - value = 0.0010$) below the $p - value$ 0.05 at a 95% confidence limit. On the other hand, the control group also showed a statistical significant difference in the pre-test and post-test ($p - value = 0.0010$) below the $p - value$ 0.05 at a 95% confidence limit. The results revealed that the two study groups improved significantly in the post-test, suggesting that the learners improved generally in solving Grade 11 Algebra. Although the two study groups both improved significantly, the experimental group yielded a greater improvement in the learners' performance in Grade 11 Algebra, when compared to the control group. The results suggest that the intervention had a positive impact on Grade 11 Algebra in the experimental group during twinning. Table 5.20 below depicts the total percentage (improvement) between the pre-test and post-test using the Didis and Erbas' (2015:1142) correct response (CR), incomplete response (InR), incomplete response (IR), and blank response (BR).

Table 5.20: Total percentages of CR, InR, IR, and BR on six question items of the experimental and the control groups.

	Pre-test				Post-test			
	CR	InR	IR	BR	CR	InR	IR	BR
Q1 Experimental	5.3%	36.6%	37.2%	22.7%	36.7%	15.2%	27.7%	20.0%
Control	59.2%	15.2%	19.2%	12.4%	87.8%	2.9%	10.5%	0.5%
Q2 Experimental	28.6%	6.6%	25.0%	39.9%	44.6%	14.3%	16.1%	2.4%
Control	41.7%	13.1%	20.3%	25.0%	50.0%	16.1%	31.6%	2.4%
Q3 Experimental	7.1%	25.0%	64.3%	3.6%	20.1%	44.1%	32.2%	3.6%
Control	20.3%	15.5%	60.7%	3.6%	51.2%	9.5%	29.8%	9.5%
Q4 Experimental	15.9%	12.7%	55.6%	15.9%	42.1%	10.3%	21.4%	25.4%
Control	40.5%	7.9%	41.9%	9.5%	38.1%	11.9%	28.6%	21.4%
Q5 Experimental	4.9%	20.0%	46.7%	28.6%	45.7%	0.0%	31.4%	22.9%
Control	24.9%	7.6%	56.7%	10.9%	42.4%	1.9%	41.0%	41.0%
Q6 Experimental	0.0%	0.3%	19.6%	80.2%	18.8%	1.6%	18.5%	60.8%
Control	26.7%	0.0%	49.0%	24.4%	35.4%	1.9%	44.7%	18.0%

Table 5.20 indicates the improvement in the results of the experimental and the control groups, when comparing the pre-test and the post-test. The results in the table show the improvement in the two study groups in the post-test when compared to the pre-test, with the control group still performing better when compared to the experimental group. Although the table above showed an improvement in the results for the two study groups, the focus of the study was on the experimental group.

The experimental group performed poorly before the intervention, when compared to the post-test after the intervention. The performance of the experimental group ranged between 0.0% and 28.6% before the intervention. A high percentage was found in the InR, IR, and BR categories with 36.6%, 64.3%, and 80.2% respectively, suggesting that the learners mostly struggled with solving quadratic equations, calculating the monthly instalment in financial mathematics, and also interpreting parabolic functions.

The experimental group performed better in the post-test when compared to the pre-test after the intervention. The performance of the learners in the experimental group in the post-test ranged between 18.8% for Question 6 items and 45.7% for Question 5 items (see Appendix H) after the intervention. The improved percentages indicate that the learners improved in respect of the knowledge of procedures, that is, of when and how to apply the procedures (Kilpatrick et al., 2001:117; Watson & Sulliman, 2008:115). The InR, IR, and BR categories of the experimental group were seen to have been better in percentages after the intervention when compared to the pre-tests. The percentages of the three categories were below 36.6%,

which the lowest percentage in InR before the intervention, except in the BR category with 60.8 percent.

Therefore, the results in Table 5.20 reveal that the intervention improved the learners' in the experimental group's performance in the CR category. Number patterns (Q2), exponential expressions and exponential equations (Q4), and the interpretations of exponential and hyperbolic functions (Q5) showed the highest percentages after the intervention of 44.6%, 42.1%, and 45.7% respectively, suggested that the intervention had a positive impact than the Q1, Q3, and Q6 question items. However, Q1 and Q5 showed a greater improvement after the intervention, namely 31.4% and 40.8%, respectively, when compared to the other question items. The percentage difference of the question items was calculated by finding the difference between the percentages in the pre-test and post-test. The improvement in the performance suggests that the learners were able to calculate the amount of money using the simple and compound interests. Also, the learners demonstrated an understanding of interpreting the exponential and hyperbolic functions with given points after the intervention. Though the learners did not perform well in Q1 if compared to the other question items, they showed an improvement of 31.4% when compared to Q3 and Q6, with 13.0% and 18.8%, respectively. The CR's categories showed that the intervention had improved the performance of the learners, as the results of all the question items showed the positive difference between the pre-test and post-test.

5.9 CONCLUSION

This chapter presented the results of this study, which indicated that the experimental group performed poorly when compared to the control group in the pre-test with the Wilcoxon-Rank Sum Test ($p\text{-value} < 0.05$), which revealed significantly difference scores. However, the experimental group's rank-sum score ($\chi=2639$) showed a significant improvement in comparison to the control group ($\chi=2134$) after the intervention ($p = 0.0010$), suggesting the positive impact of the intervention in the experimental group in improving the learners' learning of Grade 11 Algebra. The rank-sum score percentage was used to compare the improvement of the two study groups in the post-test.

The analysis of the qualitative results after the intervention showed that the sharing of teaching expertise, experience and resources by teacher B in the experimental group improved teacher A's teaching practice in Grade 11 Algebra. The learners' inclination to learn was found to have improved through their participation and performance.

The next chapter presents a discussion of both the qualitative and quantitative results framed by both the literature and theories adopted for the purposes of this study.

CHAPTER 6

DISCUSSION OF THE FINDINGS

6.1 INTRODUCTION

This chapter discusses the findings generated from the qualitative and quantitative data presented in Chapter 5. The analyses of the findings of the classroom observations done before, during and after the twinning with the experimental group and the semi-structured interviews conducted with both the two participating teachers are presented. The results of the quantitative data analysis of the pre- and post-tests are discussed within the contexts of categories noted earlier, in terms of the analytic framework adopted for the purposes of this study. In addition, the findings are interpreted within the context of both the theories underpinned and the literature used for this study, in order to respond to the research questions.

6.2 QUALITATIVE FINDINGS

In this section, the researcher discusses the qualitative data generated from the classroom observations and the semi-structured interviews conducted with the participating teachers. As noted earlier, the classroom observations intended to ascertain whether the teacher in the experimental group benefitted from the teaching strategies and resources shared during the intervention. Furthermore, the interviews were used to understand the teacher's views about the key elements in the implementation of twinning and the possible barriers of the twinning process.

6.2.1 CLASSROOM OBSERVATIONS

The findings from the classroom observations (before, during and after the intervention) revealed the following points:

Pre-intervention findings

The attention that was given to the under-achieving learners during the questioning did not create an academic space for the learners to learn new concepts, because the teacher interacted only with those who raised their hands (Vygotsky, 1978:86). The classroom interaction was mainly focused on the achieving learners and the under-achieving learners were ignored before the intervention.

The cooperative learning technique did not encourage the learners to interact and communicate their mathematical ideas in the small groups (Zacharia, Solfitri, Daud & Abidin, 2013:98). In fact, the small-group discussions were characterised by the learners sitting in pairs at the same desks. It was not effective, due to the low level of participation.

The lack of teaching and learning materials was found to derail the progress of teaching (Oguntuase et al., 2013:4; Mbugua, 2011:114; Yara & Uganda, 2010:126). The learners did not have access to the amount of information in algebra because of the shortage of resources that disabled them to practice mathematical exercises.

Mid-intervention findings

Although the low-achievers were marginalised, the learners seemed to have been involved during the interventions (Zacharia, Solfitri, Daud & Abidin, 2013:98). The learners appeared to have been motivated through the interaction (Bandura, 1986:459). This finding seems to suggest that the intervention encouraged learner-centeredness during the intervention.

Post-intervention findings

The attention paid by the teacher in the experimental group to the teaching practices by the teacher from the control group, motivated to apply them after the interventions in his classroom teaching (Bandura, 1986:459). The results suggest that teacher B's attention on the skills and knowledge during the intervention process seemed to have developed his motivation to teach algebra and other mathematical concepts and hence his efficacy appeared to have increased. After the intervention the teacher's lessons focused on small-group discussions and questioning that needed to understand learners' thinking through probing and prepared materials for the learners such as handouts and worksheets. This finding from the classroom observations seemed to suggest that the intervention changed the teacher A's own teaching practices by exposure to the new practices gained from the other teacher through twinning. There could be some factors that improved teacher A's practices except twinning such as his mathematics teaching experience in the school and the reflection of his own practices.

The data that were collected during classroom observations before, during and after the intervention were intended to respond to the following research objectives:

- to make sense of how the two teachers teach Grade 11 Algebra;
- to investigate the key elements of successful twinning;
- to explore the possible barriers in the implementation of twinning; and

- to understand the benefits and disadvantages within a context of twinning.

The data on the classroom observations collected before, during, and after the intervention are discussed below, with reference to the themes that emerged.

6.2.2 TEACHING METHODS

The teacher A's teaching methods during the intervention did not demonstrate his broader knowledge, multiple teaching methods and a deep knowledge of mathematics (Livy & Vale, 2011:40). The teaching method mostly used was teacher-centred, as he depended on a single method of teaching to reinforce the learners' understanding. The lack of broader knowledge of the teacher was inconsistent with the relationship of the content knowledge with effective teaching, which could have contributed towards the poor performance in algebra (Stylianides and Stylianides, 2006:206). Furthermore, the teaching and learning was not able to reach most of the learners' intra-psychological and intra-psychological planes (Vygotsky, 1978:86). Most of the learners were unable to participate in class when asked questions, and could also not engage in classroom discussions.

Sepeng (2014:756) indicated that the effective teaching of school mathematics requires from the teachers to have a good command of subject content knowledge. The ineffective teaching of algebra before the twinning process showed that teacher A lacked a good command of the subject content knowledge, by mostly using the telling method. The teacher could not apply multiple methods in teaching Algebra, as the teacher had focused on the telling method to explain mathematical problems to the learners. The classroom interaction before the intervention did not motivate learners, and also did not provide the direction for the learners to learn Algebra.

Moreover, classroom observations were conducted so as to understand whether the teacher A's teaching methods considered the learners' ways of thinking and their ability to diagnose the learners' errors (Shulman, 1986:4). The findings from the classroom observations showed that the teacher did not focus on what learners' know, and that this caused him not to be able to diagnose learners' mistakes when learning algebra. The reason for observing the subject matter knowledge was to understand how the teacher knew, how *much* he knew, and what he ought to know (Leavit, 2008:19).

The teacher A's teaching style before the intervention followed procedural understanding for his learners to learn Algebra without the foundation of conceptual understanding. This teaching strategy, following procedural understanding, appeared to have an impact on learners being independent in their problem-solving (Hartman, 2002:26). The learners were

encouraged to memorise and practice procedures to solve algebra problems without justifying those procedures (The National Research Council, 2012). The teacher would say, “*Do you understand*” and the whole class would just say “yes” without the teacher collecting evidence of their understanding of the concepts. The teacher involved learners by giving them problems to solve in pairs, but most of the learners were passive and the teacher ultimately provided them with solutions to the given problems.

The teacher from the control group used the telling method, whole-class discussions and small group discussions, which were effective in comparison to the teacher in the experimental group’s teaching approaches. His lessons appeared to have been effective as they aroused learners’ curiosity, most of the learners started to participate and also prompted learners’ prior knowledge to form new knowledge during classroom teaching (Van der Stuyf, 2002). The learners ought to be actively involved and not regarded as passive recipients (Van der Stuyf, 2002). The effective teaching during the intervention revealed that teacher B had a good command of algebra, as he used methods that met the needs of the learners.

The teacher B’s teaching style also considered the learners’ learning styles, what the learners know and what they do not know, and applied this during twinning. This teaching style of learner-centeredness showed that prior knowledge was a priority to the teacher, in order to determine how to support the learners (Vygotsky, 1978). Research showed that teachers with good pedagogical content knowledge understand learners’ way of thinking, their ability to diagnose learners’ errors and the sources of those errors, and can use various methods to teach mathematics (Shulman, 1986:4; Tirosh, Even & Robinson, 1998:55). The teaching approaches used by the teacher in the experimental group focused mainly on what learners know and the types of mistakes to provide the necessary support for the learners to learn new knowledge (Raymond, 2000:176).

Teacher B gave the learners the opportunity to explain their steps and justify why the answer of a particular problem was correct or incorrect. These learners in the experimental group were able to solve problems independently after the support they received from the teacher (Chang, Chen, & Sung, 2002:19). The learners gave their solutions on the chalkboard to demonstrate their abilities to solve algebra problems justifying the facts, procedures and methods used (Kilpatrick et al., 2001:116).

6.2.3 CLASSROOM INTERACTION

In classroom interaction during the intervention, teacher B promoted the participation of all the learners, and ensured a better achievement for each learner in the experimental group (Stoll & Fink, 1996:63). The role of this teacher from the control group was to support the learners to develop in learning Algebra, and also to provide support structures by means of which to promote the learners' learning (Vygotsky, 1978:88). The teacher's role during classroom interaction was consistent with that of the good-performing schools. The classroom interaction did not merely create an orderly learning environment, but also fostered a supportive learning environment (Shulman, 2010).

The classroom interaction took the form of a questioning style in the experimental group during the intervention to ascertain what learners know or don't know in order to know the type of support that can be given to the learners (Vygotsky, 1978:88). The teacher who came from the control group asked questions to assess learners' understanding of algebra, so that those learners could gain the necessary support. In contrast, the teacher's interaction with the learners did not promote progress and a supportive learning environment before the intervention. The teacher engaged those learners who raised their hands to get the correct answer, where the classroom environment did not promote learner participation (Shulman, 2010:9). However, both the twinned teachers were able to keep order in the classroom during the lessons.

Teacher B asked questions to involve all the learners, so as to ascertain what the learners knew in order to provide the necessary support during the intervention (Shulman, 1986:14; Tirosh, Even & Robinson, 1998:55). The teacher from the control group had chosen learners randomly to give answers, and probed when the answer was wrong and also when the answer was correct. The teacher's intention was to understand learners' thinking about the concepts taught. The teacher also required his learners to justify their answers in order to understand the underlying misconceptions the learners possessed.

He further probed when learners gave the correct answers, so as to understand their reasoning beyond those answers. Different methods when teaching algebra were used by the teacher (Livy & Vale, 2011:23). The effective interaction between the teacher and learners in the experimental group during the intervention appeared to have developed learners' participation in algebra. The teacher and learners' interaction was improved during the intervention compared with before the intervention.

Before the intervention, the teacher in the experimental group asked questions to get the correct answers, by choosing learners who raised their hands. When they gave the incorrect answers he just corrected them without asking the learners to justify their responses. The teacher's questioning style did not allow him to understand learners' thinking about algebra in order to know how to support them.

As noted earlier, Shulman (1986:55), Tirosh, Even and Robinson (1998:14) indicated that teachers ought to teach the subject, understand the learners' ways of thinking, diagnose the learners' errors and their sources, and display knowledge of various alternative ways of presenting the topics. Teacher A ought to interact with all the learners to understand their responses, by probing in order to understand what learners thought about the concepts. Again, the teacher might be able to diagnose the mistakes learners make in algebra in order to address them.

6.2.4 TEACHING AND LEARNING RESOURCES

The twinned teachers seemed to have similar resources, but differed in the way in which they used them in the classroom. The resources, when effectively used, are seen as essential in mathematics, according to Tachie and Chirese (2013:67). Mbugua (2011:114) mentioned that teaching and learning resources in schools, such as textbooks, the chalk board, and mathematical three-dimensional figures, are regarded as material resources. The availability of teaching and learning resources enhance the effectiveness of the schools and have a positive impact on the learners' academic achievement (Mbugua, 2011:114). Although the schools may have teaching and learning resources, if they are not effectively used or not used at all, the schools cannot be effective and the learners' performance in mathematics may be poor.

The experimental and control groups lacked textbooks, which are powerful tools in the teaching and learning of mathematics (Mbugua, 2011:114; Yara & Uganda, 2010:126). Local research has found that most schools have a shortage of mathematics textbooks (Oguntuase et al., 2013:4; Mbugua, 2011:114; Yara & Uganda, 2010:126). The shortage of mathematics textbooks in the control group did not derail its progress, as the teacher developed worksheets and handouts for his learners. The teacher in the experimental group relied on the textbook and the study guide, which denied learners access to exercises in the textbook. Four to five learners shared one mathematics textbook.

However, the teacher in the experimental group had confidently improved his learners' learning by using the worksheets and handouts, and by observing the teacher from the control

group (Hoy, 2000:480). The finding suggests that the teacher retained the idea of using worksheets and handouts when observing the teacher from the control group during the intervention. The use of worksheets and handouts seemed to have encouraged the learners' participation in algebra, as most were able to practice many algebra problems, both during and even after the lessons.

The experimental and control groups had ICT tools, such as DVDs and laptops, to be used by both twinned teachers. The DVDs were played on the laptop, and viewed using a data projector. During the intervention, the teacher from the control group integrated ICT tools in algebra in the experimental group to reinforce the lesson (Ndlovu, Wessels & De Villiers, 2011:14). The use of DVDs by the teacher made lessons fun, interesting and more effective (Hamdane, Khaldi, & Bauzihab, 2013). The DVDs were used to promote the teaching and learning of algebra during the intervention (Ndlovu, Wessels & De Villiers, 2011:14).

Although the experimental group had DVDs and a laptop, the teacher did not incorporate them in classroom teaching before the intervention (Kilicman, Hassan & Husain-Said, 2010:13). The teacher preferred to use the textbook and the study guide to present his lessons, and did not integrate his teaching with ICT tools (Hamdane et al., 2013:321). These reasons for only using the textbook and study guide appeared to have played a role in the poor performance of learners in the experimental group (Ndlovu et al., 2011:14). Research revealed that the use of ICT in mathematics may complicate the life of teachers (e.g. Robert & Rogalski).

However, teacher A incorporated DVDs and his laptop so as to complement his normal teaching utilising the textbook and study guide after the intervention. The literature supports the argument that ICT needs to be integrated in the process of teaching and learning (La Velle & Nichol, 2000:101, Lever-Duffy, McDonald, & Mizell, 2003:15). Hamdane et al. (2013:321) postulated that ICT is seen as an important tool to be used in mathematics. It makes learning fun, interesting and more effective than just using textbooks, worksheets and handouts. These teaching materials should complement each other to make learning effective.

6.2.5 CLASSROOM ASSESSMENT

Classwork and homework were used as instruments by the twinned teachers to enhance the learners' learning. It also assisted the teachers to make informed decisions about the classroom instruction during the intervention (Sattler, 2008:5; Van de Walle, Karp & Bay-Williams, 2013:80). The reason for giving the learners the activities was for the twinned teachers to collect data about the learners' knowledge and skills in algebra (The National

Council of the Teachers of Mathematics, 1995:575). Again, the twinned teachers aimed to measure the learners' level of understanding of algebra, and to monitor the progress of the learners (Van de Walle et al., 2013:80). These two instruments used to assess learners appeared to have served the purpose of classroom assessment, enhanced the learning through the given exercises, and helped the two teachers to make informed decisions about the learners' progress.

The learners in the experimental group were given feedback of the activities by the twinned teachers, so as to identify their errors and about the misconceptions learners possessed in the learning process (Hattie, 2009:260). Teacher B gave the learners the opportunity to present their solutions on the chalkboard during the intervention, while the teacher in the experimental group gave the learners solutions to the tasks written the previous day, before the intervention. The teacher seemed to have understood the importance of feedback, as it caused him to uncover the learners' errors and misconceptions underlying those concepts taught during the intervention. The teachers' feedback sessions seemed to be more learner-centred, (as informed the teachers about learners' problem solving abilities in Grade 11 Algebra (Van de Walle et al., 2013:80). Rather, the teacher in the experimental group seemed not to have understood his learners' difficulties in learning the concepts taught, as he mostly gave the solutions himself. The teacher did not use the feedback session to understand challenges learners encountered when solving the given problems. It was difficult for the teacher in the experimental group to be informed about his learners' problem-solving abilities and growth towards understanding algebra. The teacher mostly gave solutions and explained them on the chalkboard.

Furthermore, before the intervention, the teacher could not monitor the progress of his learners, due to the learners' passiveness in the teaching and learning during the feedback session (Van de Walle et al., 2013:80). However, the teacher from the control group was able to monitor his learners' progress during the intervention, as these learners were active during the learning process, and participated during the feedback sessions. The learners were observed voluntarily giving solutions, and interrogated each other during the feedback, for learners to justify their solutions. If teachers are able to monitor the learners' progress during assessment, they will be able to get an ongoing feedback towards learning objectives (NCTM, 1995:575; Van de Walle et al., 2013:80). Teacher B and the learners seemed to be informed by the assessment about learners' problem-solving abilities towards understanding mathematical concepts, practices, and procedural fluency during the intervention. Teachers

and learners corrected tasks together, so as to clarify the difficulties learners encountered when writing the activities.

6.2.6 LEARNERS' PROBLEM-SOLVING ABILITIES

Problem-solving is a process where the learners encounters a problem which does not have an immediate solution or algorithm to the correct answer (Kantowski, 1977:162; Lester, 1980:286; McLeod, 1988:19; Schoenfeld, 1992:342). The learners were given problems to solve in small groups during classroom practice during the intervention, in order for the learners to share ideas and work together by communicating with others (Zacharia, Solfitri, Daud & Abidin, 2013:98). Teacher B made use of cooperative learning in problem-solving so as to understand the abilities of the learners. This method appeared to have been effective, as the learners helped one another, challenged each other's conclusions, provided feedback amongst themselves, encouraged one another, and strove for mutual benefit (Cohen, 1992:10). The teacher in the experimental group did not test the problem-solving abilities of the learners. He did not make use of cooperative learning. Therefore, the learners did not have the chance to share their ideas in groups (Zacharia & Iksan (2007:36). The teacher in the experimental group mostly wrote the example on the chalkboard and led the whole class-discussion in solving the problems.

As cooperative learning was encouraged in the experimental group by the teacher from the control group, a small number of learners were grouped together. David and Rodger (2001:182) indicated that learners with different abilities ought to be grouped together to perform the tasks in order to understand the subject matter. Naomi and Githua (2012:177) mentioned that during cooperative learning, each member of a group ought to have the responsibility to assist the others to learn, which provides an atmosphere of achievement. Moreover, this cooperative approach encourages the interaction of the learners and communication with others in the group in harmony (Zacharia, Solfitri, Daud & Abidin, 2013:98). The teacher did not consider the abilities of the learners when forming small groups, but grouped them according to their seats during the intervention. Furthermore, the learners were not assigned responsibilities in their groups. They merely solved problems together, sharing ideas, so as to understand the concept taught. Nevertheless, the small groups appeared to have been effective, as most of the learners in those groups were actively participating. The learners in the experimental group did not have group discussions before the intervention; they just discussed problems in pairs. The discussions were, however, not effective, as only a few learners could give answers to the given problems.

Much research has been conducted on cooperative learning, which has a positive impact on learner performance in mathematics (Naomi & Githua, 2012:177; Slavin, 1985:38; Zakaria et al., 2013:98; Zakaria & Iksan, 2007:36). Shimazoe and Aldrich (2010:53) concur that this strategy promotes the deep learning of the material and helps the learners to achieve better grades. This strategy seemed to have had positive results in the experimental group after the intervention. Teacher B mostly used it for learners to share mathematical ideas to assist one another for learning (David & Rodger, 2001:182). The teacher from the control group appeared to have motivated the teacher in the experimental group to make use of cooperative learning, in order to promote the learners' sharing of ideas and working together in small groups after the intervention (David & Rodger, 2001:182). The learners in the experimental group were able to engage each other in problem-solving, with the assistance of their peers in their respective groups (Effandi, 2005:63).

6.3 SEMI-STRUCTURED INTERVIEWS WITH TEACHERS

The interviews with the teachers were semi-structured, as noted earlier in the methodology in Chapter Four. It enabled the interviewer to rephrase the questions he asked the teachers and also probed for clarity during the process. The purpose of the semi-structured interviews was to find out why the experimental and control groups performed differently in mathematics and the importance of twinning of the two teachers before the intervention. The interviews were conducted with the two teachers before the interventions, and with the teacher in the experimental group during and after the interventions.

The interview sessions after the interventions aimed to measure the effect of twinning in the experimental group. The intention of conducting the interviews was to respond to the following objectives of the study:

- to investigate the key elements of a successful twinning model;
- to explore possible barriers of the implementation of twinning;
- to understand the benefits and disadvantages within the context of twinning; and
- to develop a framework for the successful implementation of the twinning model.

Below are the general categories that emerged from the semi-structured interviews which referred to the above.

6.3.1 GRADE 11 LEARNERS' PERFORMANCES IN MATHEMATICS

The results revealed that the experimental group performed poorly in comparison to the control group, before the intervention. The poor performance in mathematics was characterised by various factors, as previously mentioned, such as poverty (Taylor, 2008:4); the learners' attitude towards mathematics (Maat & Zakaria, 2010:272; Manoah, Indoshi & Othuon, 2011:965; Mohamed & Waheed, 2011:277; Mohd, Mahmood & Ismail, 2011:49; Tahar, Ismail & Adnan, 2010:476; Tezer & Karasel, 2010:5808); and the teachers' motivation (Adeyinka, Asabi & Adedotum, 2013:37, Makgato & Mji, 2006:253). The data indicated other factors rather than the ones identified by the authors above, that contributed to the poor performance of the learners in mathematics, namely a lack of mathematical content knowledge, and commitment of the learners and learner-teacher support materials (LTSM). The control group consisted of a small number of learners, who performed poorly in mathematics when compared to the experimental group. The teacher in the experimental group indicated that his learners lacked the courage to do mathematics, due to a poor background in mathematics.

6.3.2 THE LEARNERS' KNOWLEDGE OF MATHEMATICS

The lack of the learners' mathematical knowledge in the experimental indicated the learners' lack of comprehension of the mathematical concepts, operations and of relations (Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:125). These learners' lack of conceptual knowledge was indicated during the classroom observations before the twinning exercise, where the learners were merely passive when listening to the teacher teaching Algebra. The finding shows that the learners were not fully engaged in the lessons (Van der Stuyf, 2002:5). Bransford, Brown and Cocking (1999:55), and Carpenter and Lehrer (1999:25) argued that learners who are taught how to develop conceptual understanding can organise knowledge into a coherent whole, and connect new and old ideas. This was not in line with what learners in the experimental group displayed before the twinning, as revealed by their lack of conceptual knowledge of algebra.

Although the experimental group lacked mathematical knowledge before the twinning exercise, their improvement was obvious during the twinning process, when the twinned teachers worked together through observational learning (Bandura, 1986:459). The teachers' teaching approaches during the twinning exercise mediated between the cognitive, behavioural and environmental influences (Bandura, 1986:459; Groenendijk, Jansen,

Rijlaarsdam, & Van den Bergh, 201:21). This observational learning appeared to have assisted the learners in developing conceptual knowledge, which mostly works hand-in-hand with the procedural knowledge the learners gained during the twinning procedure. The Council of Chief State School Officers (2010:29) and The National Council of the Teachers of Mathematics (2000:8, 2014:9) argued that procedural fluency builds on a foundation of conceptual understanding, strategic reasoning and problem-solving. The participation of the learners during and after the intervention showed an improvement in the experimental group, which implied that this intervention was effective in the learning of algebra.

6.3.3 THE LEARNERS' COMMITMENT

The learners in the experimental group appeared not to have been committed to their school-work, which led to their poor performance in mathematics. This was possibly due to the attitude they had towards mathematics, and also to ineffective methods (Miheso, 2012:81; Opolot-Okurot, 2005:168). These learners were not seen doing classroom activities and homework. Lupton, Noden, Brady and West (2013:5) mentioned that the home learning environment can impede the learners' learning. The home learning environment incorporates different aspects, such as attitude towards learning, educational resources at home, actual support with learning from the parents, and the parents' warmth and affection, all of which is normally known as the learners' emotional responses to learning and behaviour (Branson & Zuze, 2014:70; Hartas, 2011:72). In contrast, this home learning environment was dissimilar to the control group as these two groups shared the same geographical location as the experimental group. The home learning environment was not seen as a contributing factor for learners not committing themselves in their school-work.

During the intervention, the learners indicated having been motivated to do mathematics through the support of the teacher from the control group (Shulman, 2010:9; Calman, 2010:4). Most of the learners showed improvement in participation, and all of them submitted their work every day. During the twinning process, the teacher supported the learners by encouraging them to practice mathematical problems and do their school-work. Shulman (2010:9) further argued that effective schools act as hubs to support the learners, instilling their self-esteem and confidence, ensuring their safety and well-being. The learners in the experimental group appeared to have gained self-confidence in doing mathematics during the twinning process. The learners in the experimental group seemed to have been motivated. Studies have indicated that motivation goes hand-in-glove with learners performance (e.g. Schiefele, Krapp & Winteler, 1992:184; Keeley, Zayak, and Correia, 2008:8). The motivational process is an internal condition that are influenced by the model's expertise that

can activate the observer, provide energy, and direct behaviour (Endler, Rey & Butz, 2012:1124).

6.3.4 LEARNER SUPPORT IN MATHEMATICS

The teacher from the control group supported the learners to promote progress for each learner in learning mathematics (Stoll & Fink, 1996:63). Moreover, the teacher was able to create an orderly and supportive environment for his learners (Calman, 2010:4). The interview findings support the classroom observation that the teacher from the control group supported his learners by using DVDs in the afternoons, and by giving the competent learners more complex problems to solve, in order to give him time to clarify and explain concepts to the low-achieving learners. In contrast, the support given to the learners in the experimental group was minimal, as the teacher reinforced learning by solving many examples before the intervention. The finding from the teacher support suggests that the teacher was unable to cater for all the learners, including those learners with difficulties (Adams & Carnine, 2003:404). Consequently, the learners in the experimental group received the same treatment as the ones in the control group during the intervention. The teacher gave the same support to the learners that he used in his school to the low-achieving learners, developing positive attitude towards learning mathematics (Kilpatrick et al., 2001:120).

The data showed that the teacher in the experimental group also paid attention to the strategies of the teacher from the control group during the intervention in supporting the low-achieving learners to become independent, self-regulating and problem-solvers (Van der Stuyf, 2002:6). The teacher paid attention to the teaching strategies during the observational learning, with the teacher from the control group and applied those strategies after the intervention to improve algebra topics (Bandura, 1986:459). The strategies gained by the teacher during the twinning process were found to be effective, because he was able to transform knowledge into easily remembered structures to produce better outcomes in algebra. The expertise of the teacher demonstrated during the twinning process did not overburden the teacher in the experimental group. That is the reason why he was able to apply expertise gained from the teacher from control group in teaching algebra, as well as other concepts of mathematics, after the intervention. If the expertise modelled by the teacher from the control group was at a higher rate or level of complexity, which might overburden the teacher in the experimental group's cognitive skills, the observational learning would have been fragmentary (Bandura, 1986:459).

6.3.5 TEACHERS' CONTENT AND PEDAGOGICAL CONTENT KNOWLEDGE

The data from the teachers' interviews show that the teacher in the experimental group seemed to have gained content knowledge of algebra during the intervention, as he demonstrated a good command of the algebra topics after the intervention (Hill, 2010:515). The teacher appeared to have pedagogical content knowledge, but the knowledge did not meet learners' needs before the intervention; however, after the intervention he indicated that this had changed (Livy & Vale, 2011:27). Observational learning during the intervention improved the quality of teaching in the experimental group (Hill, 2007:12). Furthermore, the teacher appeared to have contradicting ideas about his content knowledge as he thought that he did have the knowledge. He was, however, surprised by the pedagogical content knowledge that the teacher from the control group displayed during the process. Leavitt (2008:15) argued that content knowledge shows an understanding of what the teacher knows, how much he or she knows, and what that teacher ought to know. Thus, the teacher in the experimental group acknowledged the deeper mathematical content knowledge the teacher from the control group had, and he confirmed to have gained knowledge of mathematics during the intervention through observational learning.

The teacher in the experimental group further acknowledged the pedagogical content knowledge (CK and PCK) of mathematics that the teacher from the control group displayed during the intervention (Shulman, 1986:8). Apparently the teacher learned how to approach topics that were difficult for his learners and also how to engage the learners in robust discussions in Algebra (Shulman, 2007:14).

PCK is viewed as knowledge of the subject that needed to be taught, and general PCK (knowledge of teaching strategies, classroom management strategies and assessment strategies) and content knowledge (knowing about the background of learners and knowing the organisational culture of the school) of the teacher (Zeidler, Walker, Ackett & Simmons, 2002:344; Banks, Leach and Moon, 2005:333). Furthermore, the teacher from the control group showed this knowledge during the classroom observations, as he always wanted to understand learners' ways of thinking and diagnosed learners' errors and misconceptions, by probing questions during twinning. This data concurred with the description of PCK by Shulman (1986:8); Tirosh, Even, & Robinson (1998:55) that described PCK as the type of knowledge required by teachers to teach the subject and knowledge of the various alternative ways of presenting specific topics. The teacher in the experimental group seemed to have paid attention to the teachers' teaching methods and strategies during the intervention, and was seen to be motivated by the CK and the PCK of the teacher after the intervention.

6.3.6 TEACHERS' EFFICACY

The results on teacher-efficacy revealed that the teacher in the experimental group gained confidence in promoting the learning of algebra (Hoy, 2000:480). Hoy (2000:480) defines *teacher-efficacy* as the teacher's confidence in his/her ability to promote the learners' learning in the classroom. The teacher in the experimental group seemed to have gained confidence in teaching Grade 11 algebra and other mathematics topics by using the knowledge and skills gained during the intervention. The teacher showed his confidence when he said, "*I am now confident to teach subjects like functions and number patterns*", suggesting that these topics were challenging for him to present before the intervention had taken place.

A number of researchers assert that teacher-efficacy is related to the learners' success (Tschannen-Moran, Woolfolk & Hoy, 1998:783) and to the classroom environment (Raudenbush, Rowan & Cheong, 1992:153). Teaching efficacy may results in a better teacher, one who can influence the learner achievement in a positive way, and also improve learner participation (Ball, 2005:180). The teacher in the experimental group appeared to gained confidence in teaching other concepts in algebra such as integration of function and number patterns. The learners also gained confidence in doing mathematics, as their participation was improved gradually during the intervention and remained effective even during the post-intervention observations.

6.3.7 THE IMPLEMENTATION OF TWINNING

The two teachers in the study agreed about how twinning was to be implemented. Lock (2011:13) argued that the implementation of twinning should be guided by the clustering together of schools to advance their common purpose, such as the learners' academic improvements, through working together with a peer. According to the participating teachers, they implemented twinning during two days of a week for a period of six months.

The results after the twinning exercise revealed that the sharing of teaching and learning resources, such as DVDs, worksheets and handouts, as a result of twinning two teachers appeared to have been effective due to the fact that learners' performances were impacted positively. In addition, and consistent with Berliner's (1990:5) definition of twinning, the sharing of resources made it possible for the learners to participate during the lessons, and in the process increased their chances of academic success.

The implementation of the twinning process took the form of a continuous interactive learning process, which made it possible for the learners' cognitive and behavioural aspects

of the learning of Grade 11 Algebra to be effective (Bandura, 1986:459; Groenendijk, Jansen, Rijlaarsdam & Van den Bergh, 2011:2). In other words, the twinning process facilitated what Galef (2003:137) refers to as the transmission of information from one person to another, where both participating teachers benefited from one another.

Therefore, these findings seem to suggest that the *twinning process* ought to be implemented in an environment where the participants understand the key elements of successful implementation, such as the proper and effective planning together, the sharing and use of learner-teacher support materials, as well as the creation of an intelligent space for continuous interaction, where the participants share their knowledge, expertise and good practice.

6.4 THE QUANTITATIVE RESULTS

The quantitative results indicate the rank-sums score between and within the marks of the experimental and control group in the pre-test and post-test results. The statistical significance difference using the Wilcoxon-Rank Sum test's p-value is discussed below. The t-test and the mean scores were not used, as the data was not normally distributed. The improvement rank-sum scores within and between the experimental and control groups are interpreted as before and after the twinning process.

6.4.1 THE PRE- AND POST-TESTS

The main purpose of administering the pre-test and post-test in the two groups was to measure the nett improvements/nett decline of the learners' performances in Algebra before and after twinning. In particular, the pre-test and post-test aimed at measuring the nett-effect of twinning in the learners' performances. School twinning is a joint commitment of two schools sharing resources for the sake of mutual benefits, and in particular, to promote better school results (Berliner, 1990:6). Lock (2011:13) argued that school twinning advances school improvement, that is, working together for peer support with an external colleague.

The statistical software package that was used to analyse the collected data is Stata V13. The Mann-Whitney *U* test (also called the Mann-Whitney-Wilcoxon (MWW), Wilcoxon rank-sum test, or Wilcoxon-Mann-Whitney test) is defined as a non-parametric test of the null hypothesis, where two samples come from the same population against an alternative hypothesis, especially that a particular population tends to have larger values than the other. Therefore, we used the Wilcoxon rank-sum test to assess the nett effect of twinning, and the interpretation of the results was performed at $\alpha = 0.05$ error rate. Thus, the results were

declared significant if $p < 0.05$. The analysis of learner performances per item before and after twinning is given below.

6.4.1.1 QUESTION 1 ITEM RESULTS BEFORE AND AFTER TWINNING

Kilpatrick et al. (2001:117) and Watson and Sullivan, 2008, refer to *procedural fluency* as the knowledge of procedures, of when and how to use them appropriately, and the skill in performing them flexibly, accurately, and efficiently. Learners should acquire procedural knowledge that ought to be supported by conceptual understanding in mathematics (The National Governors' Association Centre for Best Practices and The Council of Chief State School Officers, 2010:14; NCTM, 2014:9).

The results for Q1 question items indicate that the control group performed significantly differently to the experimental group in the pre-test ($z = 4.051$; $p < 0.001$). In other words, the learners in the control group (rank-sum = 2585.5) over those in the experimental group (rank-sum = 1069.5) were consistently scoring higher marks (see Appendix O). The results revealed that the learners in the experimental group appeared to lack the knowledge of solving quadratic equations, quadratic inequalities and simultaneous equations, as well as word-problems, before twinning.

Therefore, the learners in the experimental group appeared to have lacked the skills in performing procedures flexibly, accurately and efficiently (Kilpatrick et al., 2001:117). The results showed that the learners in the experimental group lacked conceptual understanding and strategic reasoning in solving those problems. The learners needed to have a deeper and more flexible knowledge of a variety of procedures, along with the ability to make critical judgements about which procedures are appropriate for use in a particular situation (The National Research Council, 2012:13; The Star, 2005:8).

The same conclusion (as in the pre-testing) was reached in the post-testing, where, for the experimental group, the control group performed significantly better ($z = -4.052$; $p < 0.001$). In other words, the learners in the control group ($rank - sum = 2266$) than in the experimental group ($rank - sum = 1389$) were consistently scoring higher marks (see Appendix O). However, the performance of the experimental group had an improved rank-sum score of 319.50. The results suggested that the procedural knowledge of finding the factors, solving the inequalities, and solving word-problems of the experimental group improved, as most of the learners knew when and how to use the procedures appropriately, and had the skills of performing those procedures flexibly, accurately and efficiently (Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:125). The experimental group

demonstrated knowledge of procedures when solving algebra problems (The National Research Council, 2012:9; The Star, 2005:5). The NCTM (2014:9) argued that those learners who used the correct procedures in solving problems displayed a foundation of conceptual understanding and reasons why they used those procedures to solve the given problems.

6.4.1.2 QUESTION 2 ITEM RESULTS BEFORE AND AFTER TWINNING

The results of Q2 question items revealed that the experimental group performed significantly differently from the control group on Q2 pre-test ($z = 0,116$; $p < 0.9078$). The experimental group recorded ($rank - sum = 1862$) significantly higher scores than the control group ($rank - sum = 1793$) on Q2. The learners in the experimental group indicated a conceptual understanding in determining the number of terms and the general terms on Q2 of the given sequence (see Appendix O; Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:125). Kilpatrick et al. (2001:117) and Watson and Sullivan (2008:125) indicated that *procedural knowledge* is the knowledge of procedures, of how and when to use these procedures and the skills of performing them flexibly, accurately and efficiently. Most of the learners in the experimental group knew when and how to use a particular procedure to determine the number of terms and the general terms of the linear and quadratic sequences.

The post-test results after twinning reached a different conclusion in the post-test. The control group performed significantly differently from the experimental group ($z = -2,128$; $p = 0.0333$) at a 95% confidence limit. The results showed that the learners in the control group ($rank - sum = 2046.5$) than the experimental group ($rank - sum = 1608,5$) scored higher marks (see Appendix O). Furthermore, the learners in the experimental group showed a nett decline of rank-sum score by 253, which suggested that twinning did have an impact in Q2 (see Appendix O). The results in Q2 are not the same with those who were taught how to develop conceptual knowledge, can organise knowledge into a coherent whole, and connect new and old ideas to determine the number of terms general terms (Bransford, Brown & Cocking, 1999:155; and Carpenter & Lehrer, 1999:25). Skemp (1976:26) argued that it is not enough for the learners to understand how to perform mathematical tasks, but they also have to know why a particular task is performed.

6.4.1.3 QUESTION 3 ITEM RESULTS BEFORE AND AFTER TWINNING

The pre-test results for Question 3 items indicate the control group performed significantly differently to the experimental group ($z = -1.58$; $p = 0.2427$). The finding shows that the learners in the control group consistently scored higher marks ($rank - sum = 1935$) than those in the experimental group ($rank - sum = 1720$) before the intervention. The learners

appeared to lack the knowledge of procedures to be followed in Q3 (see Appendix O) in calculating the monthly instalment, using the simple interest formula $A = P(1 \pm in)$ and the investment money, using the compound interest formula $A = P(1 \pm i)^n$ (Kilpatrick et al., 2001). As The National Governors' Association Centre for Best Practices and The Council of Chief State School Officers (2010:10) and The National Council of the Teachers of Mathematics (2014:8) supported the procedural fluency has to be built on a foundation of conceptual understanding, strategic reasoning and problem-solving. The learners' knowledge of procedures appeared not to have been built on conceptual knowledge. Most of the learners did not pay attention to the 10% of the costs in Question Item 3.4 (see Appendix H), and merely calculated the total amount instead of the monthly instalment.

The same conclusion as in the pre-test was reached in response to the post-test results after twinning. The control group performed significantly differently to the experimental group ($z = -1.701; p = 0.0889$). In other words, the learners in the control group ($rank - sum = 1997.5$) compared to those in the experimental group ($rank - sum = 1657.5$) consistently scored higher marks. Though the experimental group received treatment during the intervention, their nett rank-sum score declined by 63 in the post-test, when compared to the pre-test rank-sum scores. The learners in the experimental group appeared to have struggled to understand the procedures in calculating the amount of money, using the simple interest formula, and the investment money, using the compound interest formula during the intervention (Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:125).

6.4.1.4 QUESTION 4 ITEM RESULTS BEFORE AND AFTER TWINNING

The pre-test results for the Q4 (see Appendix O) test items before twinning showed that the experimental group performed significantly differently to the control group ($z = -1.972; p = 0.0486$) at a 95% confidence limit. Therefore, the learners in the control group ($rank - sum = 2027.5$) than the experimental group ($rank - sum = 1627.5$) scored consistently higher marks (see Appendix O). The learners in the experimental group had difficulty in simplifying the exponential expression and exponential equations. Therefore, the learners could not apply the exponential law $\frac{a^m}{a^n} = a^{m-n}$ to simplify the exponential expression $\frac{(y^{-3}) \times \sqrt[3]{x^8}}{(xy^{-2n+3})}$ for Question Item 4.1 (see Appendix H), suggesting that the learners lacked the knowledge of mathematical relations (Kilpatrick et al., 2001). The results revealed that learners lacked the skills of solving these kinds of problems in the experimental group before the intervention (Schoenfeld, 1992:394; Kantowski, 1977:162; Lester, 1980:286). McLeod (1988:19) and Blum and Niss (1991:38) argued that problem-solving is a process

where the learners encounter a problem which does not have an immediate solution or an algorithm to the direct answer, such as simplifying the exponential expression and exponential equations in the context of this study. The experimental group lacked the understanding of how and when to apply the exponential laws to simplify the exponential expression to solve exponential equations (Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:125).

A different conclusion (as compared to the pre-test) was reached in the post-test results after twinning, where the experimental group performed significantly better ($z = 0.000$; $p = 1.0000$). This results means that the learners in the experimental group (rank – sum score = 1849), when compared to the control group (rank – sum score = 1806), scored higher marks. The learners in the experimental group appeared to have acquired knowledge of mathematics with understanding during the intervention and also were provided the basis for generating new knowledge, and were able to solve new and unfamiliar problems (Bransford et al., 1999). Therefore, the experimental group's rank-sum score between the pre- and post-test improved by 221.5, which suggests that the improvement was due to the intervention. Kilpatrick et al. (2001:117) argued that the learners with a conceptual understanding avoided critical errors in solving mathematical problems. The experimental group in the post-test appeared to have improved in simplifying exponential expressions and solving exponential equations, as they gained knowledge of how to use procedures to solve Question 4 items (Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:125).

6.4.1.5 QUESTION 5 ITEM RESULTS BEFORE AND AFTER TWINNING

The results before twinning of Q5 (see Appendix O), indicate that the learners in the control group performed significantly differently to the experimental group in the pre-test ($z = -4.677$; $p = 0.0000$) at a 95% confidence limit. The results show that the learners in the control group (rank – sum = 2268.5) than the experimental group (rank – sum = 1386.5) consistently scored higher marks (see Appendix O). The learners in the experimental group appeared to have lacked procedural fluency, along with the problem-solving skills of the interpretation of exponential and hyperbolic graphs in Q5 (Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:125). Problem-solving is recognised as an important skill, involving a range of processes that include analysing, interpreting, and reasoning, predicting, evaluating and reflecting (Anderson, 2009:3). Most of the learners did not answer Q5, which showed that they could not analyse and interpret the two graphs in order for them to find the appropriate procedures to solve their problems. Also, they did not know the correct

procedures (Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:125) to calculate the values q , k , a , and b for Question Item 5.1, using the given graphs.

In conclusion, the results indicate that the learners in the experimental group performed significantly differently those to the control group ($z = -4.677$; $p = 0.0000$) at a 95% confidence limit. In other words, learners in the experimental group (rank – sum = 1950.5) than in the control group (rank – sum = 1619.5), scored higher marks (see Appendix O). Moreover, the learners in the experimental group improved by the rank-sum score of 564, which could have been impacted by the implementation of the intervention. The learners in the experimental group showed that they had developed knowledge of analysing and interpreting the exponential and hyperbolic functions on Question 5 items (Anderson, 2009:3). Also, the learners understood when they were taught during the intervention, how to develop conceptual understanding of the two functions, in order to respond to the question items (Bransford, Brown, & Cocking, 1999:155).

6.4.1.6 QUESTION 6 ITEM RESULTS BEFORE AND AFTER TWINNING

The pre-test results before the twinning indicate the learners in the control group performed significantly differently to the experimental group ($z = -7.739$; $p = 0.0000$), at a 95% confidence limit. The findings show that the learners in the control group (rank – sum score = 2601.5) than in the experimental group (rank – sum = 1053.5) consistently scored higher marks (see Appendix O). Research found that once a learner had memorised and practiced procedures that are difficult for them to understand, that this can lead to a lesser motivation to understand the meaning and reasoning behind those procedures. The learners in the experimental group appeared to have memorised the procedures of solving problems related to parabolic functions, which caused them to use incorrect procedures when solving the difficult problems in Q6 (see Appendix O). Furthermore, the learners in the experimental group lacked the foundation of conceptual knowledge, as they did not know the appropriate procedures to be used by means of which to answer parabolic function problems in the post-test (Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:125).

The same conclusion as in the pre-test results was reached in the post-test after twinning. The learners in the control group performed significantly differently to those in the experimental group ($z = -5.543$; $p = 0.0000$) at a 95% confidence limit. In other words, the control group (rank – sum = 2397.5) consistently scored higher marks than the experimental group (rank – sum = 1172.5). However, the performance of the learners in the experimental group showed an improvement of 119 in the rank-sum scores, between the pre-test and the

post-test. Therefore, some of the learners in the experimental group appeared to have acquired knowledge of how to analyse and interpret the parabolic functions with given points on the graph (Anderson, 2009:3). The learners also appeared to have gained knowledge of interpreting and using the procedures to respond to the parabolic functions question items during the intervention (NRC, 2002:14; Watson & Sullivan, 2008:115).

6.5 SUMMARY OF THE PRE- AND POST-TEST RESULTS

The summary of the pre-test and post-test results made use of the rank-sum total percentage between the experimental group and the control group. Furthermore, the summary of results is discussed in the light of the performance between the pre-test and post-test in the experimental group. The table below presents the summary of the results between the experimental and control groups in the pre-test and post-test.

Table 6.1: Summary of the pre-test results

School	Observations	Rank sum	z-score	p-value
Experimental	43	1195.5	-5.750	0.0000
Control	42	2459.5		
Combined	85	3655		

As Table 6.1 above highlighted, the results the learners in the control group performed significantly differently to those in the experimental group in the pre-test ($z = -5.750$; $p = 0.0000$) at a 95% confidence limit. In other words, the learners in the control group (rank – sum = 2459.5) consistently scored higher marks than those in the experimental group (rank – sum = 1195.5) (see Appendix O). The learners in the experimental group appeared to not have possessed deeper and more flexible knowledge of the procedures by means of which to solve algebra problems before the intervention (NRC, 2014:12). Furthermore, the findings showed that the learners’ procedural understanding was based on conceptual knowledge and strategic reasoning in solving algebra problems (Kilpatrick et al., 2001:116; NRC, 2012:12; Watson & Sullivan, 2008:115). The table below gives a summary of the post-test results in the experimental and control groups.

Table 6.2: Summary of the post-test results

School	Observations	Rank sum	z-score	p-value
Experimental	43	1437.5	-3.619	0.0003
Control	42	2217.5		
Combined	85	3655		

The same conclusion (as in the pre-test) was reached in the post-test after twinning; the learners in the control group performed significantly differently to those in the experimental group ($z = -3.619$; $p = 0.0003$) at a 95% confidence limit. The finding shows that learners in the control group ($rank - sum = 2217.5$) consistently scored higher marks than those in the experimental group ($rank - sum = 1437.5$). However, the learners in the experimental group improved their the rank-sum score by 242, which is the difference between the pre-test and the post-test results. The learners in the experimental group appeared to have gained a deeper knowledge of the procedures needed to solve algebraic problems, as most of them were aware of the procedures required to solve these problems (The National Research Council, 2012:12; Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:125).

6.6 OVERVIEW OF QUALITATIVE AND QUANTITATIVE RESULTS

As noted earlier, twinning is seen as a joint commitment of two schools sharing resources, experiences and expertise (Berliner, 1990:5; Oxfam, 2007:6; Rees, 2003:24; Rees & Woodward, 1998:28), for peer support with an external colleague (Galton & Hargreaves, 1996:179; Hargreaves, 1996:25; Lock, 2011:13), and to improve the quality of education (Hill, 2007:12; Smith, 2009:14). Twinning also benefits both the teacher and the learner (Lock, 2011:13; Nachtigal, 1992:8; Rees, 2003:24; Rees & Woodward, 1998:28). Furthermore, mathematical knowledge is transferred more readily, and the teacher becomes more efficient in using the resource (Hargreaves, 2010:13; Rees, 2003:24); the one teacher acts as a specialist or a model (Galton & Hargreaves, 1995:179; Bandura, 1986:459); while the teachers work in a social environment during twinning (Galton & Hargreaves, 1996:179; Lock, 2011:13; Nachtigal, 1992:8; Rees, 2003:24; Rees & Woodward, 1998:28), and school twinning discourages the spirit of competition and control amongst schools (Nachtigal, 1992:8).

6.6.1 TEACHERS' EXPERIENCES AND EXPERTISE

The results of this study showed that the twinned teachers from the experimental and control groups worked together to share teaching practices and resources in Grade 11 Algebra. As noted earlier, twinning took place in an experimental group, where the teacher from the control group acted as a model, with the other was the observer. The study was underpinned by Bandura's (1986:459) observational learning, where, for the purpose of this study, one teacher acted as expert, and the other observed the expert teaching Grade 11 Algebra.

'Observational learning' refers to continuous interactive learning which mediates between cognitive, behavioural and environmental influences (Bandura, 1986:459). Observational learning, in this study, took the form of interaction between the twinned teachers on behavioural, environmental and cognitive influences (Groenendijk, Jansen, Rijlaarsdam, & Van den Bergh, 2011:2). Furthermore, the twinned teachers worked collaboratively during the intervention to enjoy learning in a social environment, through observational learning (Galton & Hargreaves, 1995:179; Lock, 2011:13; Nachtigal, 1992:8; Rees & Woodward, 1998:28; Rees, 2003:24). The results revealed that during observational learning in the twinning process, the teacher's teaching practices appeared to have been effective, and provided experiences that are able to help the learners connect procedures with underlying concepts (NCTM, 2014:9). Furthermore, the teacher from the control group provided the learners in the experimental group with an opportunity to practice strategies through problem-solving, by justifying the procedures used (NCTM, 2014:9)

6.6.2 THE SHARING OF THE TEACHING AND LEARNING RESOURCES

As noted earlier, school twinning is seen as a joint commitment of two or more schools sharing resources (Berliner, 1990:5; Oxfam, 2007:6; Rees, 2003:24; Rees & Woodward, 1998:28), in order to provide peer support with an external colleague (Galton & Hargreaves, 1996:179; Hargreaves, 1996:179; Lock, 2011:13) and to improve the quality of education (Hill, 2007:12; Smith, 2009:14). The results revealed that the teacher from the control group brought DVDs, worksheets and handouts he prepared to share with the teacher and his learners in the experimental group (Yara and Uganda, 2010:126). Indeed these teaching and learning resources were used to complement the scarcity of textbooks in the two study groups, as the control group also lacked mathematics textbooks. Furthermore, professional knowledge is transferred more readily when teaching resources are well used and the teacher in the experimental group became more efficient in using them (Hargreaves, 2010:13; Rees, 2003:24). However, the teacher seemed to have DVDs in his school, but could not use them in classroom teaching, before the intervention (Hamdane et al., 2013:321). Eventually, in the

post-intervention, the teacher started to use those DVDs during and after twinning, so as to reinforce mathematical knowledge.

The use of teaching and learning resources in mathematics has been extensively researched in respect of mathematics learners' performance (Oguntuase et al., 2013:4; Yadar, 2007:9; UNESCO, 2008:9; Mbugua, 2011:114; Yara & Uganda, 2010:126; Ruthven & Hennesy, 2003:38; Sun & Pyzdrowski, 2009:38). The results during the twinning process revealed that the learners' performances in the experimental group improved as the learners participated by responding to questions asked on the worksheets and the handouts.

Moreover, learning started to be fun when the teacher used DVDs to teach algebra topics during twinning (Hamdane, Khaldi and Bauzihab, 2013:321). The post-test results also support the fact that the learners in the experimental group improved in algebra, as the rank-sum percentages improved from 1195.5 to 1437.5.

6.6.3 MOTIVATION IN A TWINNING CONTEXT

School twinning is beneficial to both the teachers and learners (Lock, 2011:13; Nachtigal, 1992:8; Rees, 2003:24; Rees & Woodward, 1998:28). In the experimental group, the benefits of twinning appeared to be motivation and learner performance. The motivational process is an internal condition that can activate, provide energy and direct the behaviour (Endler, Rey & Butz, 2012:1124). The teacher asserted that he would implement some of the strategies learned during twinning, such as using DVDs, developing worksheets and handouts for his learners, questioning techniques to understanding the learners' thinking by probing, and using effective classroom discussions.

Several studies correlate motivation with the learners' learning performance (Schiefele, Krapp & Winteler, 1992:184; Song and Keller, 2001:8; Keeley, Zayak, & Correia, 2008:8). Andrews (2011:60) asserts recognition to be a positive strategy, which can produce improved teacher motivation and respect. The results revealed that the teacher in the experimental group was motivated by twinning as a strategy to change his own teaching practices and wished it could be sustained so as to further share experiences with the other twinned teacher. The motivation of the teacher correlated with the learners' performance. The teacher in the experimental group indicated that learners who grappled with mathematics had improved in terms of performance and participation (Song & Keller, 2001:8). As a result, the teacher in the experimental group seemed to have had challenges in teaching algebra and other mathematical concepts before the intervention. The intervention appeared to have motivated him to confidently teach algebra.

6.6.4 TEACHER SELF-EFFICACY

The teacher in the experimental group did not seem to have much confidence in some of algebra topics before the intervention. This finding, of a lack of confidence in teaching certain algebra topics, is inconsistent with the kind of teacher-efficacy that promotes learners' learning (Hoy, 2000:480). This teacher's efficacy appeared to have improved during the twinning process, as the teacher confirmed the skills and knowledge gained during the intervention developed his confidence in teaching topics like functions and number patterns after the intervention (Stripling, Ricketts, Roberts, & Harlin, 2008:126). The learners' participation also seemed to have improved his confidence. The motivation of the learners was also improved by the teacher's change in teaching practices (Wolters & Daugherty, 2007:182). The beliefs the teachers harboured in relation to their own effectiveness are known as *teacher-efficacy*, and underlie their instructional decisions, which ultimately shape the learners' educational experiences, and in turn, improve the learners' academic performance (Woodcock, 2011:84). The results from the interviews after the twinning process indicated that teacher A gained confidence that his learners would perform better in the post-test, and also in the mid-year and end-of-year examinations.

6.6.5 IMPROVEMENT OF THE LEARNERS' PERFORMANCE

This study sought to explore the effectiveness of twinning two mathematics teachers teaching Grade 11 Algebra, in order to improve the learners' performance. In this regard, it appeared that twinning the two teachers teaching Grade 11 Algebra, had a positive impact on the learners' ability in solving algebra problems in the experimental group (Berliner, 1990:5; Hill, 2007:12). As noted earlier, the pre-test results showed that the learners in the experimental group experienced difficulties in solving algebra problems. As a result, the experimental group appeared to have lacked conceptual and procedural knowledge, as well as problem-solving skills, in dealing with algebra concepts (Schoenfeld, 1992:340). Though the Grade 11-learners were tested on Grade 10's content, they performed poorly in all the questions in the pre-test. All the learners in the experimental group were unable to answer Question 6, involving the interpretation of the parabola graph, and only two of the learners out of the whole class answered Question 5, which involved the interpretation of hyperbola and exponential graphs.

As mentioned earlier, the results of the post-test indicated the rank-sum score difference between and within the experimental and control groups. The post-test results showed a statistical significant difference ($z = -3.619$; $p - value 0.0003$) of the rank-sum total percentage between the two groups, with the experimental group still performing poorly in

comparison with the control group. However, the experimental group improved significantly in the post-test, after the intervention.

6.7 ANSWERING THE RESEARCH QUESTIONS

This study intended to answer the following primary research question, as indicated in Chapter 1:

What is the impact on teacher practice of twinning two teachers as a strategy for improving the learners' academic achievement in teaching Grade 11 Algebra?

The secondary research questions are used in responding to the primary research question emanating from the objectives of this study. The secondary research questions, as indicated, are answered in the next sections.

6.7.1 PEDAGOGICAL APPROACHES

The data obtained from the classroom observations revealed that the teacher in the experimental group approached Grade 11 algebra differently, as compared to the teacher of the control group. His teaching approaches were characterised by teacher-centeredness, where the learners assumed the role of passive recipients of the information. The learners appeared to not have been supported by the teacher before the intervention (Vygotsky, 1978:86). Furthermore, the teacher in the experimental group did not consider testing the learners' prior knowledge, in order to know what type of support he should provide (Van der Stuyf, 2002:6). Learning in the experimental group before the intervention did not focus on supporting and developing the learners for them to become independent, self-regulating problem-solvers (Hartman, 2002:26).

The teacher from the control group preferred to use prior knowledge of learners to understand what learners know before introducing the new (Van der Stuyf, 2002:6). The teacher from the control group appeared to have been interested in understanding what the learners know and what they have to be taught in order to be able to support them (Van der Stuyf, 2002:6). Probing during the introduction of the lessons was used mostly to understand the learners' thinking about the concepts taught previously, as it guided the teacher from the control group to know where to start teaching new concepts (Margerum-Leys & Marx, 2004:426; Shulman, 1986a:3). The learners in the experimental group actively participated in small group and whole class discussions, and positively responded to the questions asked by the teacher in the introduction and during course of the lessons.

The teacher in the experimental group was present, observing how the teacher from the control group introduced the lessons, by engaging the learners in respect of what they had learnt previously (Bandura, 1986:459; Berliner, 1990:5; Rees, 2003:24). The learners in the experimental group demonstrated what they had learnt before, by responding to the questions asked by the teacher, and the teacher further probed to understand the thinking of the learners by justifying their mathematical knowledge (Margerum-Leys & Marx, 2004:426; Shulman, 1986a:3). The teacher from the control group provided support to those learners who struggled to deal with the concepts taught previously, so that learning could take place effectively, unlike his teaching before the intervention (Vygotsky, 1978:86).

During the intervention, the teacher from the control group used a combination of teacher-centred and learner-centred approaches to provide the learners with direction to achieve their goals (Bransford, Brown & Cocking, 2000:155). In cases where the learners did not understand the concepts, he used the telling method in order to create space for learners to understand the concept taught or to guide them to internalise the new information (Van der Stuyf, 2002:6). The teacher in the experimental group observed the teacher from the control group teaching Grade 11 Algebra during the intervention, in order to acquire the skills and knowledge displayed during the process (Bandura, 1986:459).

The teacher from the control group during the intervention also took the form of cooperative learning, where the learners worked in small groups, so as to interact and communicate with one another (Zacharia, Solfitri, Daud & Abidin, 2013:98). Learners in the experimental group solved mathematical problems in small groups sharing ideas and assisting one another (Naomi & Guthua, 2013). The teacher supported learners where they experienced challenges when solving the given problems in small groups, in order to develop their knowledge and thinking (Vygotsky, 1978:89). Van der Stuyf (2002:6) has argued that scaffolding is not permanent when the teacher assists learners to solve during the learning of new materials; the teacher gave learners an opportunity to solve problems independently after giving them guidance. Furthermore, the learners were given the opportunity to interact amongst themselves by interrogating each other for clarity on the procedures used to solve problems (Kilpatrick et al., 2001:117; Watson & Sullivan, 2008:128). The participation of the learners in the experimental group during the intervention improved, in comparison with the ones before the intervention.

When it came to cooperative learning, the teacher in the experimental group was more ineffective at first than after the intervention. The teacher gave the learners activities to discuss in pairs before the intervention. Their participation was initially ineffective, as the

learners were observed to merely proceed to sit out the task passively, without sharing any ideas to complete the given tasks (Zacharia & Iksan, 2007:36). The learners were not fully supported in learning algebra, as the teacher just gave the solutions to them, not giving learners the chance to solve problems by themselves.

The learners support in the experimental group was demonstrated by the teacher from the control group, as he gave learners problems to solve independently, and guided them in how to solve them (Berliner, 1990:5; Oxfam, 2007:6). The teacher in the experimental group, post- intervention, was observed using cooperative learning, ensuring that each learner in the group was participating in order to share ideas and to assist each other, promoting learning (Naomi & Guthua, 2013:178). This type of teaching method appeared to have been influenced by the intervention when the teacher from the control group presented lessons in the experimental group.

The lessons taught during the intervention also took the form of a questioning technique, where the teacher interacted with the learners in the experimental group in order to understand the learners' thinking about the concepts taught (Vygotsky, 1978:89). The teacher from the control group modelled how to use this questioning technique so as to engage the learners in the teaching and learning of algebra (Bandura, 1986:459). The teacher asked the learners questions to try and understand how the learners thought, by means of prompting, when the learners gave wrong answers and also when they gave the correct answers to explain the procedures used to solve the problems (Shulman, 1986:8). Furthermore, the teacher requested the learners to demonstrate their understanding by writing the solutions on the chalkboard to diagnose the errors and misconceptions, in order plan the future lessons (Shulman, 1986:8; Tirosh, Even & Robinson, 1998:55). The teacher then provided support to the learners, by clarifying the misconceptions the learners had, asking further questions.

The teacher in the experimental group preferred to make use of textbooks and the study guide to prepare and present the lessons before the intervention. Mbugua (2011:114) has argued that the mathematics textbook plays an important role in guiding the sequence of teaching and also in providing the mastery of the concepts. The learners in the experimental group had a shortage of textbooks, and they did not have the opportunity to practice the exercises provided in the textbooks. This shortage of textbooks was not in line with the improvisation of the teachers when the resources that yield good results in mathematics are scarce (Mbugua, 2011:114; Oguntuase, Awe & Ajayi, 2013:4; Yara & Uganda, 2010:126). The shortage of textbooks in the experimental group seemed to have had a negative impact in learning Grade 11 Algebra before the intervention.

Then, during the intervention, the teacher from the control group brought worksheets and handouts to give to the learners and to complement the textbook. The teacher used these two teaching resources in every lesson. The learners appeared to have gained knowledge of algebra, as they had something to use for exercises, which is consistent to the learners' performance in mathematics (Mbugua, 2011:114). The teacher in the experimental group learned from the teacher from the control group how to develop the worksheets and handouts for his learners after the intervention (Tracey & Morrow, 2012:122). In other words, the modelling that was shown by the teacher from the control group was found to make learning effective during the intervention. Bandura (1989:222) argued that modelling is an important step to take in developing competencies during observational learning.

Furthermore, the teacher from the control group preferred to use DVDs to teach Grade 11 Algebra in order to promote the knowledge of the teacher in the experimental group (Mendonça & Justi, 2011:480; Souza & Justi, 2012:386). The DVDs were integrated in algebra during the intervention, as the use of ICT promotes learning in mathematics (Ndlovu, Wessels & De Villiers, 2011:4). In contrast, the teacher in the experimental group did not use the ICT tools in teaching algebra before the intervention. He may possibly have found it challenging to integrate technology in his teaching practices, e.g. operating computers (Robert & Rogalski, 2005:126). However, the teacher appeared to be motivated after observing the teacher from the control group integrating the ICT tools into his teaching practices (Endler, Rey & Butz, 2012:1124). Andrews (2011) has argued that motivation in the teaching fraternity produces quality teachers. In other words, the motivation of the teacher in the experimental group can also encourage the teachers who are demotivated.

6.7.2 REQUIRMENTS FOR SUCCESSFUL TWINNING

Twinning is described as a joint commitment of two schools sharing resources for the sake of mutual benefit, to promote better schools results as indicated earlier (Berliner, 1990:5). Oxfam (2007:6) has argued that twinning is a partnership between two schools, forming a joint commitment for the benefit of the learners' education. Again, twinning was seen as a programme that ought to bring two schools together for peer support, with an external colleague who acts as a specialist (Lock, 2011:5).

In this study, twinning followed observational learning, where the teacher from the control modelled the teaching practices for the observing teacher (Bandura, 1986:459; Lock, 2011:5). The teacher facilitated learning in the presence of the observer, observing how algebra topics were taught (Tracey & Morrow, 2012:124). The reason for the two teachers to work together was to transform the two teachers through twinning, but mainly for the teacher in the

experimental group to improve a means of facilitating the learning of Grade 11 Algebra (Hill, 2007:12). The two teachers planned how to implement twinning, such as the topics that they taught together, the time to work together, the resources that they shared, and the activities to be given to the learners during the twinning process.

The information collected by means of the semi-structured interviews and classroom observations indicated that the twinned teachers preferred observational learning as a key component of successful twinning (Bandura, 1986:459). Galef (2003:137) argued that observational learning facilitates the transmission from one person to another. Groenendijk, Jansen, Rijlaarsdam & Van den Bergh (2011:2) concur that in observational learning, the apprentice observes the expert demonstrating his or her behaviour. Therefore, twinning takes the form of the teacher in the experimental group observing the teacher from the control group teaching Grade 11 Algebra.

Observational learning was seen to be effective during the intervention, as the teacher from the control group modelled his behaviour, that is, teaching Grade 11 Algebra, in the presence of the teacher in the experimental group (Schunk & Hanson, 1985:72). The teacher in the experimental group gained or mastered skills and knowledge during the interaction with the teacher from the control group, during observational learning (Haston, 2007:27).

The twinned teachers shared their expertise, resources and knowledge of teaching algebra, such as quadratic equations and functions, number patterns and financial mathematics, which were difficult for the learners in the experimental group (Rees & Woodward, 1998:28). Furthermore, the twinned teachers shared classroom activities such as classwork and homework, where the teachers aimed to observe the learners' improvement in mathematics in the experimental group (Berliner, 1990:5). In essence, the sharing of the best teaching practices and the appropriate available resources constitute another key element of an effective twinning process.

6.7.3 POTENTIAL BARRIERS

Intervention in the form of twinning presented its own barriers in the context of this study. Other commitments that the twinned teachers had in their respective schools were a problem to fulfil the agreements of twinning. The finding suggests that school twinning presents additional responsibilities and work-related stress (Rees, 2003:24). The twinned teachers had commitments in their schools, except teaching, such as organising sports activities and participating in other activities. Similar to Nachtigal's (1990:9) study on twinning, the issue of time was also a major challenge for the two teachers. Money was another problem, as the

one teacher had to do some travelling to the experimental group using his own car, and the researcher had to supply funding for petrol. Streifel, Foldesy and Holman (1991:16) argued the cost of transportation to be one of the drawbacks in twinning, on the grounds of the teacher having to travel between the schools. In brief, twinning ought to consider these factors when two or more teachers work jointly.

6.7.4 BENEFITS OF TWINNING TO THE GRADE 11 ALGEBRA CLASSROOM

The impact of twinning in this study is measured against the learners' improvement in their performance when solving algebra problems, and the teacher's change in practices by the exposure to new practices gained during the twinning process.

6.7.4.1 CHANGE IN LEARNER PERFORMANCE

The analysis of data generated from the pre-test, the post-test, the classroom observations and the semi-structured interviews showed that the learners' performances in the post-test improved after the twinning process. This finding suggests that the intervention assisted in improving the learners' performance in Grade 11 Algebra, which supports one of the noted benefits of twinning, namely the improvement of the learners' learning (Oxfam, 2007:6). The pre-test and post-test results showed that the learners also improved in their conceptual understanding of algebra during the twinning process, as well as after the twinning, once the teacher implemented the skills and knowledge gained during the interaction. The learners knew when, how and why to use a particular procedure to solve algebraic problems during and after the intervention (Kilpatrick et al., 2001:117; Watson & Sulliman, 2008:110).

Furthermore, the classroom observations during and after the interventions showed the learners' participation to have been improved in the Algebra classroom. The learners participated effectively with the teacher from the control group in the small groups sharing mathematical ideas and working together to perform the tasks given to them (Zacharia & Iksan, 2007:36). The NCTM (2014:9) indicated that procedural understanding is built on the foundation of conceptual understanding, strategic reasoning and problem-solving. The results supported the three mathematical proficiencies, as the learners were able to explain the procedures used to solve problems through mathematical reasoning.

The statistical results showed that the rank-sum total percentage for the experimental group was performed poorly (rank – sum = 1195.5), when compared to the control group's rank-sum total percentage before the twinning in the pre-test ($\chi = 2459.5$). The results between the two groups revealed a statistical significant difference with ($z = -5.750$; $p - value < 0.0000$) at a 95% level of confidence in favour of the control group. The two study groups

showed a rank-sum difference in total percentage of 1264 in the pre-test results before the intervention. The findings show that the learners in the experimental group lacked the appropriate knowledge of procedures to solve algebra problems (Kilpatrick et al., 2001:).

The same conclusion (as in the pre-test) was reached in the post-test after twinning. The learners in the control group performed significantly differently than the learners in the experimental group ($z = -3.619$; $p - value < 0.0003$ at a 95%) confidence limit. The experimental group still performed poorly, with a rank-sum total percentage (rank – sum = 1437.5) as compared to the control group’s rank-sum total percentage (rank-sum= 2217.5). However, the experimental group showed an improvement from the pre-test to the post-test of the rank-sum’s total percentage of 242 after the intervention. The learners in the experimental group appeared to have gained conceptual and procedural knowledge of Grade 11 Algebra (Kilpatrick et al., 2001:118; NCTM, 2014:9).

6.7.4.2 CHANGE IN TEACHING PRACTICE

The analysis of the qualitative data generated from the semi-structured interviews and classroom observations revealed that the teacher in the experimental group changed his teaching practices by the exposure to the new practices gained during twinning (Lock, 2014:5; Rees, 2003:24). Other researchers agreed that teachers working collaboratively share their experiences, expertise and resources during the clustering or the twinning process (Lock, 2011:5; Rees, & Woodward, 1998:28).

The teacher in the experimental group preferred to use small group discussions implemented by the teacher from the control group during the twinning process, which had benefits for both the teacher and his learners (Naomi & Githua, 2012:178; Zacharia, Solfitri, Daud & Abidin, 2013:98).

Furthermore, the teacher gained knowledge of how to use questioning techniques with the learners to understand his learners’ thinking, by prompting questions for learners to clarify their the problem solving in Algebra (Shulman, 1986:459). Again, the teacher in the experimental group improved his questioning style by not looking for the correct answers before the intervention, but to test the learners’ prior knowledge before the beginning of the new lesson (Van der Stuyf, 2002:6). The teacher also considered the learners’ thinking abilities, by prompting them when asking questions.

The teacher in the experimental group started to develop worksheets and handouts after the intervention, for the benefit of his learners, as the experimental group had a shortage of

textbooks (Mbugua, 2011:114). The teacher also started to integrate learning with ICT tools after the intervention, which showed that he benefitted from the twinning. Lock (2011:5) asserts that twinning develops and supports schools with stimuli used for the improvement of a teacher's teaching practice, through working collaboratively with the teacher from the control group. Therefore, the teacher from the control group's teaching approaches appeared to have a positive impact in changing the teacher's teaching practices in the experimental group, by the exposure to the new practices gained during twinning.

6.7.4.3 DEVELOPING EFFECTIVE TWINNING MODELS

The results of twinning two teachers indicated that one teacher was the expert or model who was an external colleague (Lock, 2011:5). The other teacher was the observer, who paid attention to the behaviour of the model. Twinning in the context of this study was focused on two schools that performed differently in Grade 11, where the two teachers shared their expertise, experiences and resources to improve the other school (Berliner, 1990:5). The one school was a well-performing school, and the other was a poor-performing school. The status of the two schools differed in terms of their performance in mathematics.

Before two or more schools or teachers can embark on twinning, the participants of the twinning of schools or teachers ought to understand the needs of poor-performing school(s), such as their access to resources, and the type of support they need. The teachers who embark on twinning should have a joint plan for the sake of mutual benefit, so as to improve the results in the poor-performing school (Berliner, 1990:5). Twinning should encourage a spirit of collaboration between the teachers so as to regularly exchange ideas, expertise and new knowledge among the participating teachers (Rees, 2003:24). The learners also benefit from twinning, because the more the teacher improves his or her skills in mathematics, the more he or she will be able to apply them in teaching the learners.

The teacher(s) who will be observing ought not to just sit and observe the expert teacher, but ought also to participate, in order to demonstrate attention to the process. It is clear that they will be motivated by the sharing of good teaching practices and resources and be able to apply the new skills acquired from the expert teacher(s) during twinning. The learners in the twinned schools may be considered for twinning if possible, so as to share skills and knowledge of mathematics with one another as they learn.

6.8 DEFICIENCY IN BANDURA'S THEORY

The theoretical framework that has underpinned this study took the form of Bandura's observational learning incorporated in the socio-cognitive theory. Observational learning is a process where an observer observes the behaviour of the model for the same observer to practice what was observed during the process (Bandura, 1986:459). In the context of this study, the teacher in the experimental group observed the teacher from the control group teaching Grade 11 Algebra.

Bandura (1986:459) recommends that the observer should pay attention to the behaviour of the model during observational learning. The teacher in the experimental group paid attention to the skills and knowledge the teacher from the control group demonstrated during twinning, in order to retain them for future practice. The teacher appeared to have been paying attention to the skills and knowledge the teacher from the control group demonstrated during the intervention, by showing his participation in the lessons. The teacher in the experimental group, during the intervention process, intervened to show that he was paying attention when the teacher from the control group presented his lessons, by assisting the learners in group discussions and clarifying concepts.

The results of this study revealed that the teacher in the experimental group showed his commitment in observational learning, by supporting the learners during the classroom discussions. The participation of the teacher showed that he was motivated by the lesson presentations of the other teacher during the intervention, and that he implemented some strategies in teaching Grade 11 Algebra, and other mathematical concepts. Thus, motivation appears to be functional when the observer participates in observational learning, during the intervention, and hence improve the learners' performance in mathematics.

6.9 CHAPTER SUMMARY

The qualitative and quantitative data generated by this study were discussed in this chapter. The pre-test and post-test analyses as well as the semi-structured interview responses and classroom observations were synthesised with the theoretical perspectives and the literature review. The quantitative data indicates that the learners in the experimental group performed poorly in the pre-test, when compared to those in the control group. The semi-structured interviews with the teacher in the experimental group supported the challenges that contributed towards the poor performance in this group. The classroom observations revealed that the learners in the experimental group appeared to have a problem in learning algebra, as they remained passive during the course of the lessons, before the intervention. The learning

appeared to have been teacher-centred in the experimental group as the teacher mostly tried to explain the procedures of solving algebraic problems. However, the teacher of the control group used teacher-centredness when the learners seemed to have problems in understanding the concepts taught, only sometimes using the learner-centred approach to make learning effective (Black, 2006:4).

The data from the classroom observations showed an improvement in the learners' participation during the intervention. As such, the learners started to participate in the lessons presented during the twinning process, by asking questions for clarity and engaging in group discussions. Furthermore, the learners who were not used to doing classroom activities changed, and started to do activities such as homework, which showed that they understood what they were supposed to do in algebra. The learners also indicated their motivation during and after the intervention observations. The experimental group participated by giving the solutions to the homework and the classwork, and moreover, they challenged each other for the justification of the answers given on the chalkboard. During the intervention, the teacher introduced small group discussions in the experimental group, which were not used before the intervention, and which appeared to have been effective, as the learners participated and learned among themselves.

Again, the teacher from the control group was interested in understanding learners' thinking, by asking questions about the answers given by the learners during the twinning process. The lessons presented appeared to be fun for the learners in the experimental group when learning was integrated with the ICT tools. The teacher played DVDs for the reinforcement of concepts. The teacher from the control group would introduce the topic for about twenty minutes, and then start playing the DVDs, which showed that the learners were interested in learning by means of ICT tools by sometimes asking the teacher to pause, and then start a classroom discussion about the topic.

Statistical analysis revealed the statistical significant difference between the two study groups before the intervention, with the learners in the experimental group performing poorly when compared to the ones in the control group. The study showed an improvement in the rank-sum difference between the pre-test and post-test for the experimental group after the intervention. Although the results showed that the experimental group's learners performed poorly in comparison to the control group, there was significant improvement in the performance of the experimental group, and only a marginal improvement of the control group.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 INTRODUCTION

Twinning is a joint commitment of two schools in aid of peer support, with an external colleague (Lock, 2011:5) sharing resources for their mutual benefit, namely to promote better school results (Berliner, 1990:6). The teachers become more efficient in the use of resources (Hargreaves, 2010:13), they regularly exchange ideas, their expertise and new knowledge (Lock, 2011:5; Rees & Woodward, 1998:28; Rees, 2003:24). Furthermore, twinning improves the quality of teaching and learning (Hill, 2007:12; Lock, 2011:5; Smith, 2009:14).

For the purpose of this study, the impact of twinning two mathematics teachers teaching Grade 11 Algebra was explored for the sake of the teacher A's change in his/her practices, by the exposure to the new practices gained during the intervention. The study also investigated the way in which twinning benefitted the learners in poor-performing school in terms of academic performance. This study consisted of a case study of two schools which produced different results in Mathematics, that is, one school performing poorly, and the other one performed well in this subject.

The participating learners in the two schools displayed a different status in terms of their performance in mathematics; the one school was a well-performing school, labelled the control group, and the other one was a poor-performing school, labelled the experimental group. The two schools indeed performed differently, as supported by the evidence collected in the previous Grade 12 Mathematics examination results from 2010 to 2013. The pre-test was administered to the experimental and the control groups to measure the level of algebraic proficiency before the twinning process. A post-test was administered to the experimental group to measure the impact of twinning as strategy on the Grade 11 Algebra learner performance.

This final chapter reflects on the rationale for and the research design of the study in order to draw a conclusion on the benefits of twinning as a teaching strategy. The limitation of this study is also discussed, as well as the implication of the framework used in the study in order to modify it. Finally, recommendations for further research are suggested.

7.2 IMPORTANCE AND DESIGN OF THE STUDY

As mentioned in Chapter 3, twinning is seen as a process that brings two schools together for the teachers to share their experiences, expertise and resources (Ministry of Education and Sports Republic of Serbia Vocational Education and Training Reform Program, 2003; Rees, 2003:24). Twinning was introduced in the Capricorn District of Limpopo Province. It appeared to have had little effect, and marginalised the rural schools, like the experimental and control groups. In a study reported here, twinning was also used to combat school isolation in rural areas, where bonding and bridging were encouraged among the participating teachers (Collaborne & West-Burnham, 2008:28; Lock, 2011:13). West-Burnham (2009:28) describes *bridging* as a network, cluster or partnership. The rural schools, which were not involved in twinning, appeared to have been denied the opportunity to share their expertise, skills and new knowledge, and instead, were seen to be competing in Grade 12 results. Therefore, this study was designed to explore with teachers the impact of twinning as a teaching strategy in the experimental group, in relation to their teaching practices. Furthermore, the study also measured the impact of twinning at learner level, using the pre-test and post-test. Defining and describing the twinning process to the teachers before the implementation was a key to this study, for the twinned teachers to have an understanding of the intervention.

This study used a pre-test-intervention-post-test design to measure the impact of twinning in the experimental group at learner level. The pre-test and the post-test were also used to compare the results between the experimental and the control groups. A pre-test was administered to investigate the level of performance in Grade 11 Algebra of the experimental and control groups before the intervention. The pre-test revealed that the experimental group performed poorly when compared to the control group in Grade 11 Algebra. Semi-structured interviews that were conducted with the two participating teachers assisted in understanding the nature of the difficulties experienced in Grade 11 Algebra. Classroom observations were conducted in the experimental groups before, during, and after the twinning process. As noted earlier, a post-test was administered after the intervention. The twinning process took place over a period of six months in the experimental group.

7.3 MAIN FINDINGS

The pre-test results revealed that the experimental group performed poorly in comparison to the control group, which indicated a statistical significant difference between the two groups. The experimental group demonstrated low proficiency in algebra, which needed attention

such as that offered by this intervention. The data generated from the quantitative study showed a statistical significant difference between the two groups in the pre-test (the Wilcoxon-Rank Sum test). However, the results in the post-test after the twinning showed a statistically significant improvement on rank-sum total score of 2639.5 at learner level in the experimental group. The improvement suggested that twinning may have had a positive impact in the teacher A's changing his own practices, by the exposure to the new practices gained during twinning, and hence improved learner performance.

Quantitative and qualitative data gathered before the twinning suggested the learners' lack of mathematical knowledge due to their poor background in the subject (Kilpatrick et al., 2001:118; Watson & Sullivan, 2008:125). The teacher A's content and pedagogical content knowledge before the intervention appeared to have been ineffective (Stylianides & Stylianides, 2006:206). Hill (2010:515) has argued that effective teaching is bound to influence the learners' academic performance. The teacher's teaching approach before the intervention was mostly narrative, as he spent most of the time explaining concepts without involving the learners. Only a few learners were observed to be participating before the intervention, while other learners sat passively, listening to his explanations. The prior knowledge of learners was not taken into consideration to inform the teacher about what the learners knew, and what they were supposed to know, to guide the lessons (Van der Stuyf, 2002:6). Moreover, the teacher in the experimental group neither understood the learners' thinking when asking question, nor supported the learners experiencing difficulties. In fact, most of the learners did not make sense of algebra in responding to the pre-test question items. Almost all of them could not answer the interpretation of graphs, where all the question items were left blank.

The results of the post-test in the experimental group revealed an improvement on Q1 when solving quadratic equations, with the rank-sum total score of 2556, and on Q5 when interpreting the hyperbolic and exponential graphs. The rank-sum total score also improved, with 2639.5 in the post-test, as compared to the pre-test's rank-sum score total of 1101.5.

The rank-sum total difference score was found to be 1534, which appeared to be as a result of the twinning process, which was introduced in this group. During the intervention, the use of resources such as worksheets and handouts to complement the Grade 11 textbook in Algebra in the experimental group was another key factor in yielding positive improvements in the learners' academic performances (Mbugua, 2011:114). In addition to the worksheets and handouts, the integration of extra-learner support resources such as DVDs also appeared

seemed to be successful during the intervention. Such initiatives that may be regarded as strategies by the researcher seemed to suggest the benefits associated with the improvement in learning amongst participating learners in the study. It is therefore reasonable to suggest that the appropriate use of teaching and learning resources may be essential in promoting effective teaching. The use of resources by the teacher from the control group showed to have motivated and improved the teacher's efficacy (Bandura, 1986:459). The teacher A's motivation was observed when he used DVDs when teaching algebra in the post-intervention classroom observations.

Qualitative data during the intervention suggested that there were pockets of improvement on and shifts towards classroom interaction through mathematical small group discussion and talk in the classroom (Sepeng 2010:104, Sepeng & Webb 2012:61, Sepeng 2014:756). In actual fact, the small group discussion method used by the teacher in the experimental group seemed to encourage the learners to share ideas amongst themselves, and in the process, learner participation was evident in the classroom. In addition, the classroom interaction and learner participation appeared to be mostly influenced and triggered by the teacher's teaching approaches in the experimental group, used to stimulate the learners' thinking about the concepts taught.

The results of this study appear to suggest that the research has achieved its objectives, namely, investigating and reporting on how the teachers teach Grade 11 Algebra. The key elements of twinning and the possible barriers for the implementation of were identified, explored and recorded. The benefits of using twinning as a strategy were outlined in this study, and reported in order to develop a framework for a successful twinning model. Twinning was introduced in the experimental group, and implemented by the two participating teachers as a teaching strategy, which allowed the researcher to statistically measure its impact in relation to change in learners' academic performances in algebra.

Cooperative learning was a common feature throughout the twinning process, which seemed to suggest that the heterogeneous group needs to be maintained in order to encourage the knowledge sharing of learners in the small groups (David & Rodger, 2001:100).

In other words, the learners appeared to learn more when they are mixed in groups in terms of their learning abilities, so as to support each other in small groups when learning algebra. Cooperative learning techniques seemed to have been cemented by the use of whole-class discussions, with the learners voluntarily giving solutions to the problems on the chalkboard. In essence this finding of effective cooperative learning suggests that the learners should be

given an opportunity to engage with each other, and amongst themselves, in order to deal with alternate conceptions, and make sense of the types of errors emerging during discussions and problem-solving processes.

The key elements identified in and for the purposes of this study for successful twinning are: 1) a need for the twinned teachers to share their knowledge of the content and pedagogical content of Grade 11 Algebra; 2) the sharing of teaching and learning resources such as handouts, worksheets and DVDs; 3) sufficient time to implement twinning; and 4) money to travel between schools. However, time and money seemed to be the most significant challenges in the implementation of twinning. Other logistical issues such as the academic programmes and activities of the twinned schools may disrupt the twinning processes, minimising its benefits for the participants. As such, there needs to be a clear and well-thought-out plan to have a successful twinning programme that can benefit both the teachers and the learners in the rural areas such as the one used in this study.

7.4 THE RESEARCHER'S VOICE AND THEORETICAL FRAME

The study was underpinned by observational learning. The results of this study provided insight and understanding into how twinning can be implemented. Recommendations can be drawn about this strategy of observational learning. The analysis of the qualitative data suggests that the implementation of twinning benefitted the teacher in the experimental group in changing his teaching practices. The results show that the teacher in the experimental group gained content knowledge of Grade 11 Algebra and pedagogical content knowledge during the implementation of the intervention. The learners in the experimental group seemed to have a positive gain in twinning, and have improved their participation in the group discussions, responding to questions asked during twinning, and voluntarily giving solutions of the activities given to those learners during twinning. The quantitative analysis supports the positive gain of the learners in the twinning process, as the results showed the significant improvement in solving algebraic problems in the post-test.

The experimental group appeared show a better improvement in the post-test, as compared to the pre-test, which showed the positive impact of twinning when compared to the pre-test results. However, for the teachers to implement twinning successfully through observational learning, they are obliged to consider the needs of the respective schools in which they teach such as their available resources and the topics that have to be shared during twinning. It is necessary for the teachers who embark on twinning to organise a meetings where they are able to share ideas on how to implement twinning strategy, such as choosing topics that are

difficult for the learners to solve, the resources they require and which are available, and also to agree on the date and time of the implementation. The school management team ought to be involved in twinning process for the smooth implementation of twinning. One of the teachers ought to be an expert in the subject, who will be teaching the selected concepts and the other teachers may be observing the teaching. The other teachers should also participate in the process in order for them to become motivated, and retain the knowledge that would be shared during twinning (Bandura, 1986:459). During the twinning process, the teachers embarking on this course of teaching should agree again on the resources that will be required to improve learning in mathematics in their classrooms. Lastly, twinning can be effective and sustainable when it has a legal framework for every teacher, ballasting a commitment of himself or herself to the process. Thus, any intervention that is not legally framed may not be sustainable, because the teachers and the school management team are not legally bound to support it.

The key element of this study was of a successful collaboration in which both teachers respected each other's thoughts and ideas. The results suggested that the joint planning for implementation of twinning, the regular exchange of ideas, resources, expertise and new knowledge, and lastly, the teacher acting as a specialist, appeared to have made twinning successful in this study (Lock, 2011:6). Moreover, the results showed that if twinning can involve the school management team for full support, the challenge of time and money to travel from one school to another would minimise the challenges of twinning. Again, the poorly-performing schools, which are mostly marginalised in remote areas, would benefit by the twinning process if they could be sustained and implemented, and those schools would not feel isolated, and competition amongst well-performing and poor-performing schools would be discouraged (Nachtigal, 1992:8). The results revealed that twinning can change teachers' teaching practices, as well as learners' performance in mathematics, if it is implemented according to plan.

The policy of South African school requires that Mathematics teachers integrate teaching with Integrated Computer and Technology (ICT) in the classrooms (Curriculum and Assessment Policy Statement, 2012). The results showed that the use of ICT in teaching can improve learner participation, and encourage them to engage in problem-solving, make learning fun and interesting.

Observational learning is guided by four interrelated processes during modelling phenomena, namely, attention, retention, motivation and production (Bandura, 1986:459). These four interrelated processes can be achieved only if the teacher who observes can participate during

twinning. It is not guaranteed that the teacher, who observes the specialist teacher or expert during twinning, is paying attention to the skills and knowledge the other teacher is modelling. The results of this study showed that if teacher A who observes has a role to play, such as intervening during teaching process when paying attention to the model teacher, is much more likely that he or she will apply the knowledge and skills of mathematics gained during twinning. Then, the teacher who observes can be motivated to participate in twinning, and hence, due to the causal link between teacher and learner motivation, learner academic performance can be improved.

The teacher who observes will gain confidence in teaching other mathematical concepts that would be challenging to the teacher and the learners, in order to promote learning in mathematics classroom. It is important to enhance teacher efficacy during twinning, as it can assist with encouragement for the teacher with mathematical concepts that are difficult to teach, and encouragement likewise for the learners to learn (Woodcock, 2011:84). The teacher who observes may gain both the personal teacher-efficacy and general teacher-efficacy, as he or she can control, manage and motivate learners in mathematics. Therefore, teachers who have high levels of teacher-efficacy tend to be motivated and persevere through challenges in teaching mathematics.

7.5 PROPOSED EFFECTIVE TWINNING

A proposed effective twinning strategy can be designed in schools based on the findings of this study, the literature and the theoretical framework underpinned the study. The literature indicated that school twinning is implemented for various reasons, such as bringing two schools together for the benefit of small schools or low-performing schools through the sharing of resources, experiences and expertise, either by principals, teachers and learners (Berliner, 1990:5; Nachtigal, 1992:8).

For the purpose of this study, twinning was focused in the poor-performing school, with the teacher from the well-performing school shared teaching practices and resources in teaching algebra. The Limpopo Department of Basic Education (DBE, 2011:18) proposed twinning projects in schools that are in the Dinaledi Project, which perform well in Mathematics and Science, and those that are seen to be underperforming.

For twinning to be effective, the DBE ought to propose a policy framework that can be put in place and be monitored and supported by the departmental officials for the smooth implementation of the twinning project. The policy framework that was proposed for twinning suggested that the Dinaledi schools ought to twin schools that do not perform well

in Mathematics and Science, to share best practice (DBE, 2011:18). However, the policy did not state how these schools ought to be twinned, such that the teachers and learners will be involved, and the other stakeholders that ought to participate, such as parents, in allowing learners to travel from one school to another. This type of teacher and learner involvement was the reason why the twinning project was proposed by the department in the Limpopo Province, and was neither well-implemented, and also not sustained.

Based on the findings of this study, twinning should involve all the stakeholders, such as the DBE, the School Management Teams (SMT), the teachers, the learners and the parents, to support the implementation. Monitoring and support of the DBE officials can be put in place to ensure there is enough funding travelling between schools and resources that can be used during twinning. Twinning should also involve the rural schools so as to limit the influx of rural learners in urban schools for a better education, as well as to capacitate the teachers in content and pedagogy. Effective twinning can also assist participating schools in retaining good mathematics and science teachers in the rural schools. Most of the schools based in rural areas, according to my teaching experience in secondary schools, can be found to compete when it comes to matric results. Those schools do not instead consider supporting each other in subjects that are difficult for learners, such as Mathematics and Science (Lock, 2011).

Twinning should focus on the needs of the schools, such as resources, and take into consideration topics that are challenging to learners and the teachers who would participate. The participating schools should hold meetings to agree on how to plan, identify the participants such as teachers and learners, and implement twinning. During the planning stage of the twinning process, the participating teachers ought to identify mathematical topics that are challenging to both the teachers and the learners, in order to improve performance.

Twinning should also focus on the types of resources, expertise and experiences that should be shared during twinning (Rees, 2003:24; Lock, 2011:8). Furthermore, expert teachers ought to be identified from any other schools to take the lead in the project and other teachers' participation will be to observe how the expert teachers teach the selected mathematical topics.

For effective twinning, it is proposed that observational learning be employed in the implementation of the project. In observational learning, one teacher should be a model or an expert teacher, and the other teacher(s) should observe and learn new and effective practices during the twinning process (Bandura, 1986:459). The rationale for using observational

learning is to encourage the teachers to learn within social contexts, while sharing teaching approaches employed by the expert teachers.

7.6 LIMITATIONS OF THE STUDY

Although the study seemed to be successful in the twinning of the two teachers drawn from the experimental and control groups, certain limitations in respect of the nature of the results ought to be acknowledged, since the study reported on a case of only two teachers out of hundreds of teachers in the Limpopo Province, where the findings of the study cannot be generalised to the entire population.

The study intended to focus on the two teachers' teaching practices with their learners to measure the impact of twinning on teacher A's learners' academic achievement. This study used a small sample as was an in-depth case study of these two teachers teaching Grade 11 Algebra.

7.7 SUGGESTIONS FOR FUTURE RESEARCH

The main findings of the study appear to suggest that the twinning of two teachers whose schools perform differently is effective when the expertise, knowledge and resources in the poorly-performing school are shared. Through the reflection of the intervention in this study, the benefits of the twinning process, and mathematics in general, ought to be explored by involving more than one grade in Mathematics. The nature and effects of the twinning of the school management teams between schools should be investigated in order to understand how teachers can be supported towards best teaching practices.

On the other hand, twinning can be implemented for the whole school for the purpose of keeping small schools open. Therefore, studies that would further probe and investigate the benefits of the twinning of clusters of schools are needed, so as to operate within the framework of all key elements of successful twinning that were identified in the study reported here. Other questions that may enhance an understanding of the influence of twinning on learners' academic performances are: what is the greatest influence and impact of twinning on the learners' academic performances? Is it mostly the results of twinning the school management (such as the principals) or the academic managers of units (such as the heads of academic departments)? Or perhaps twinning the entire schools (such as whole-school twinning) where the twinned schools would plan together and share resources?

7.8 CONCLUSION

This study explored the impact of twinning two mathematics teachers teaching Grade 11 algebra as a strategy on learner academic performances in a poorly-performing school. Twinning appeared to have positive gains for both the teachers' teaching practices of Grade 11 Algebra, and the learners' academic performances. Moreover, the importance of this study was to change the teacher's own practices by exposing him or her to effective teaching practices observed during the twinning process. The data generated from the study ought to be able to make a contribution to the teacher development of the Department of Basic Education in South Africa, regarding the way in which they should implement twinning in their districts and their schools. The benefits of twinning two mathematics teachers teaching Grade 11 Algebra, as indicated in this study, should encourage education stakeholders in South Africa to carefully implement this strategy widely in schools.

The results of this study should furthermore provide insight into the impact of twinning for school managers, mathematics teachers, and other teachers of various subjects, on how twinning can be implemented to promote the quality of teaching in their classrooms. The results should also provide the participating schools with the benefits of this strategy, in order to effect change in the learners' academic performance in general.

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Appendix A: Letter to the Head Office

The Head of Department
Department of Basic Education
Limpopo Province
Polokwane
0700
10 February 2015

Dear Sir/ Madam

RE: Request for permission to conduct a PhD research

This letter serves to request permission to conduct a PhD research study at two schools in Capricorn District. I am a PhD student at University of North West and completed the research proposal thus far. The title of the research is: **Twinning of two mathematics teachers teaching grade 11 algebra: A strategy for change in practice**. The purpose of the study is to explore the impact of twinning two teachers teaching grade 11 algebra topics using problem solving approach. The study aims to achieve the following:

1. to identify the root causes of the poor academic achievement in mathematics in the low-performing school in relation to lesson practices;
2. to establish why these challenges or factors exist;
3. to identify factors contributing towards better academic achievement in mathematics in the performing school in relation to lesson practices;
4. to understand the type of teacher professional development the two schools have;
5. to establish what is needed for a successful twinning and possible barriers; and
6. to develop framework for use of twinning.

The study follows a mixed method approach. Both quantitative and qualitative methods will be used to collect data. Mainly, a quasi-experimental (classroom -based) consisting of pre-test and post-test measure for learners, will involve the experimental and control groups. However, aspects of descriptive survey design, involving lesson observations, interviews with teachers, will be included to determine whether or not the qualitative results will account for the outcomes of the quasi-experimental study.

The ethical clearance certificate to conduct this research will be sought from the Research Ethics Committee (REC) in the Faculty of Education at University of North West (NWU). Prior to the commencement of the study, permission, consent and/or assent letters in which the purpose of the study is outlined will be given to prospective participants. Participation in this research is voluntary and the right to withdraw from the study at any time, without adverse consequences, is upheld.

Thanking you in anticipation

Yours truly



Mr Sello Makgakga
(PhD Student: Mathematics, Science and Technology Education, NWU)

Appendix B: Permission letter from the Head Office



LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF EDUCATION

Ref: 2/2/2 Eng. MC Makola P1D Tel No: 015 290 9448 E-mail: MakolaMC@edu.limpopo.gov.za

Makgaka S

UNISA
PO BOX 392
Pretoria
0003

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.
2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"TWINING AS A STRATEGY FOR CHANGE IN PRACTICE: A CASE STUDY OF TWO MATHS TEACHERS TEACHING GRADE 11 ALGEBRA"**
3. The following conditions should be considered:
 - 3.1 The research should not have any financial implications for Limpopo Department of Education.
 - 3.2 Arrangements should be made with the Circuit Office and the schools concerned.
 - 3.3 The conduct of research should not anyhow disrupt the academic programs at the schools.
 - 3.4 The research should not be conducted during the time of Examinations especially the fourth term.

Request for permission to Conduct Research: Makgaka S

CONFIDENTIAL

Cnr. 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X9489, POLOKWANE, 0700
Tel: 015 290 7600, Fax: 015 297 6920/4220/4494

The heartland of southern Africa - development is about people!

- 3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).
- 3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.
- 4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.
- 5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.



MUTHEIWANA NB
HEAD OF DEPARTMENT (ACTING)



DATE

Request for permission to Conduct Research: Makgaka S

CONFIDENTIAL

Appendix C: Informed consent form



NORTH-WEST UNIVERSITY
YUNIBESITHI YA BOKONE-BO PHIRIMA
NORDBES-UNIVERSITEIT
INSTITUTIONAL OFFICE

NORTH-WEST UNIVERSITY (MAFIKENG CAMPUS)
FACULTY OF EDUCATION AND TRAINING
SCHOOL OF TEACHER EDUCATION AND TRAINING

Consent Form for Grade 11 teachers

Twinning two teachers teaching the 11th grade algebra: A strategy for change in practice

I have been given information about "Twinning two teachers teaching the 11th grade algebra: A strategy for change in practice". I have discussed this research project with Mr Sello Makgaka, a student at North-West University. This is part of PhD degree supervised by Professor Percy Sepeng from the Faculty of Education at North-West University.

I understand that if I consent to participate in this project I will allow the researcher to observe me teaching the 11th grade algebra. The researcher will also be allowed to video-tape and audio-tape my lessons. I also consent to participate in a survey and an interview to be conducted by the researcher during and after the academic session has concluded. My contribution in this study is agreed to be treated confidentially and that there will be no personal identification in the data that I agree to allow to be used in the study. I understand that there are no potential risks or burdens associated with the study.

I have had an opportunity to ask Mr Sello Makgaka any questions I may have about the research and my participation. I understand that my participation in this research is voluntary and I am free to refuse to participate and I am free to withdraw from the research at any time. My refusal to participate or withdrawal of consent will not affect my relationship with the Faculty of Education at North-West University.

If I have any queries about the research, I can contact Mr Sello Makgaka (012 352 4268 or 072 475 9363 and/or Prof Percy Sepeng 018 389 2887). If I have any concerns or complaints regarding the way the research is or has been conducted, I can contact the supervisor, Prof Percy Sepeng at 018 389 2887.

By signing below I am indicating my consent to participate in the research. I understand that the data collected from my participation will be used primarily for a PhD thesis, and will also be used in summary form for journal publication, and I consent for it to be used in that manner.

Signed

Date

MABOJA M A

Name (please print)

29.01.15

Signed

Date

Name (please print)

29.01.15

Appendix D: Teacher questionnaires

TEACHER 1 SURVEY INSTRUMENT QUESTIONNAIRE

Dear Participant

This is a survey questionnaire for a PhD research project. It is used to analyse the factors that affect learners' performances in mathematics. From a perspective of a case study, the study focusses on the two schools that would be twinned at a later stage of the study. Please note that the information that would be gathered during the study will be treated confidentially and you are advised to withdraw at any stage of the study should you wish not to participate further. Thank you.

Demographic Information

1. Gender	Male	Female		
2. Highest academic qualifications	diploma	Bed/BSc	Med/MSc	Other
3. Subject majors				
4. Experience in teaching mathematics	≤ 5	6 - 10	11 - 15	16 and above
5. Highest level of mathematics	Diploma	Degree	Masters	Others (specify)
6. How many workshops have you attended in the last two years?				
7. How were those workshops	Effective	Ineffective		

1. What keeps a better standard of mathematics in your school? *Hard work, insight and the understanding of importance of mathematics in real life.*
- _____
- _____
- _____
- _____
- _____

2. Which topics in mathematics are most challenging to learners? Especially in algebra. Formula mathematics
3. Do you get any support from the SMT or other stakeholders? Yes/No. how and why? yes, through workshops and in-service training
4. If yes, what type of support is that one? _____
5. How would you characterise your working relationship with your SMT? Good.
6. What is your mathematics pass average in your school for 2013? _____
7. How do you view your learners in terms of attitude, attendance and motivation? Please explain in each of the items? learners attitude towards mathematics looks better attendance is good, and they look motivated.
8. How is your learners' socio-economic background in general? poor
9. How is the parents' educational background? poor and educated
10. How is parental support in your school? Good or bad, why? bad because they do lack orientation of their roles in education as parents.
11. Does your school have enough textbooks and other teaching/learning resources? Yes/ No. if no, how do your learners cope? No, they do share textbooks and some other learning materials
12. How many periods do you teach per week? 5
13. Do they tally with the departmentally allocated periods per week per teacher? Yes/ No. If no, why? yes

14. Are you satisfied with your Teaching post? Yes/ No. If no, why? yes
15. How do you motivate your learners to develop interest in mathematics? by orientating them on the importance and opportunities that comes with Mathematics
16. What approaches do you use to teach algebra topics and why? investigative, telling and communicative approaches
17. What is your approach to classroom management and learners' discipline? by developing a list of classroom rules that govern the behaviour of learners
18. What if your learners don't understand a concept? In other words, if a lesson is not working for all your learners, do you have a plan for remediation? How do you execute that plan? repeating a lesson, giving extra work on challenging concepts and change the approach of teaching method
19. Describe for me a lesson you taught that went well. Why did the lesson work so well? Number pattern because of prior knowledge
20. How often do you give learners tasks? during lesson, after daily, weekly, monthly, quarterly, complete of homework and classwork, test, assignment, etc
21. What types of tasks do you give your learners? Homework, classwork, test, and assignment, project.
22. What is the response of learners towards the given tasks? positive
23. Do they all write activities such as homeworks? If no, what their reasons? yes
24. How often do you give your learners feedback and how? during lesson, after completion of tasks

25. How do you interact with your learners during mathematics lessons? through
question and answer method
26. Do your learners participate voluntarily? If no, why? yes
27. What type of support do you give the slow learners? extra lesson,
outsourcing of teachers, team work
and

Thanks for your time

Appendix E: Classroom observation schedule

DEMOGRAPHIC DETAILS

1. Name of School: _____
2. Physical Address of School: _____
3. Postal Address of School: _____
4. Tel: _____ Fax: _____
5. Name of Principal: _____

Male	Female
------	--------
6. Name of Teacher: _____

Male	Female
------	--------
7. Grade Observed: _____
8. Number of Learners: _____

OBSERVING CLASSROOM PRACTICE

1. How does teaching and learning of Mathematics occur? (Please list e.g. *whole class*)
(i) _____ (ii) _____
(iii) _____ (iv) _____
2. How is the classroom arranged? (*Furniture*)

3. What methodology / approach is being used?

4. Which resources are used?

5. How does the teacher deal with correct or incorrect responses?

OBSERVING TEACHING AND LEARNING IN THE CLASSROOM

The PEER system underlies the lessons in a classroom situation. It might not be possible to incorporate all of them in a particular lesson but each lesson will contain some aspects of this system. Please tick(✓) your rating.

A	PRODUCTIVE SKILLS	Excellent	Good	Average	Needs More Attention	Not Applicable to the lesson
1.	Learners are able to do reading on the concept being taught.					
2.	Learners write notes on the concept taught.					
3.	Learners are able to solve problems given as exercises					
4.	Learners are able to relate and apply the concept in real life problems.					
5.	Learners are able to use their knowledge of and experience in the concept in formulating their own responses.					
6.	Learners are able to accomplish work given on the concept independently					
7.	Learners are able to define and describe learned terms encountered when dealing with the concept.					
8.	Learners are able to follow the steps in solving exercises based on the concept.					
9.	Learners competently use technology (calculators) in areas where it is required in the concept.					

10.	Learners are able to deal with problems in real and abstract context using the concept.				
11.	Learners' ways of making decisions in problem solving is enhanced.				
B	EVOCATIVE SKILLS		Excellent	Good	Average
	1.	Learners ask questions for clarification.			
	2.	Learners ask questions to consolidate their understanding of the concept			
	3.	Learners are puzzled by certain areas of the concept and hence very inquisitive.			
	4.	Learners are able to interpret new information on the concept.			
	5.	Learners ask critical questions to ensure that methods used are appropriate.			
	6.	Learners use their referencing skills to acquire better understanding of the concept.			
	C				
	EVALUATIVE SKILLS				
	1.	Learners are able to do self-assessment tasks in the concept learned.			
	2.	Learners are capable of evaluating their own work on the concept.			
	3.	Learners are able to evaluate procedures followed in problem solving in the concept.			
	4.	Learners are able to identify errors committed when dealing with the concept.			
	5.	Learners are able to discuss pros and cons in using specific methods to solve problems.			
	6.	Learners are able to identify incorrect ways of solving problems.			
	7.	Learners have alternative ways to solve problems based on the concept.			

Appendix F: Pre-test and post-test

Pre-test and post-test

Grade 11 Mathematics

Dear Learner

This is a diagnostic test which comprises seven questions with their sub-questions. Please read each question carefully before you answer. Show all your calculations and non-programmable calculator may be used. This paper will take you 1h30 to complete.

Question 1 [22]

1.1 Solve for x:

1.1.1 $5x^2 = 2x$ (3)

1.1.2 $\frac{2x-1}{x+2} = \frac{2x-3}{x+2}$ (4)

1.2 Given: $-8 \leq -3x + 2 < 2$, solve the given inequality for x. (4)

1.3 Solve for x and y:

$$2y - 6x = 4$$

$$y - 3x^2 = 6x - 4$$
 (5)

1.4 The length of a rectangle is twice the breadth. If the area is 128cm^2 , determine the length and the breadth. (4)

Question 2 [11]

2.1 A sequence is given as follows:

6; 8; 12; 18; 26; ...

2.1.1 What number will come next? (1)

2.1.2 Determine the formula for the n^{th} term of this sequence. (4)

2.2 Given: 5; -1; -7;; -283

2.2.1 Determine the expression for T_n , the general term of the sequence. (3)

2.2.2 Hence, determine the number of terms in the sequence. (3)

Question 3 [8]

- 3.1 Nakedi purchases a stove costing R4800-00, using a hire purchase agreement. She pays a deposit of 10% of the purchase, and pays a balance off at 15% per annum over 3 years. She is required to pay a monthly insurance premium of R15-50 as well. Calculate her monthly instalment. (4)
- 3.2 Raesetja invests R10 000 in a savings account on the 1st of January 2011, at 9% interest per annum compounded monthly. How much will her investment be worth on the 1st April 2013? (4)
-

Question 4 [13]

- 4.1 Simplify

4.1.1 $\frac{(y^{-3}) \times \sqrt[3]{x^8}}{(xy^{-2n+3})}$ (4)

- 4.3 Solve for x :

4.3.1 $16^x = \sqrt{8}$ (4)

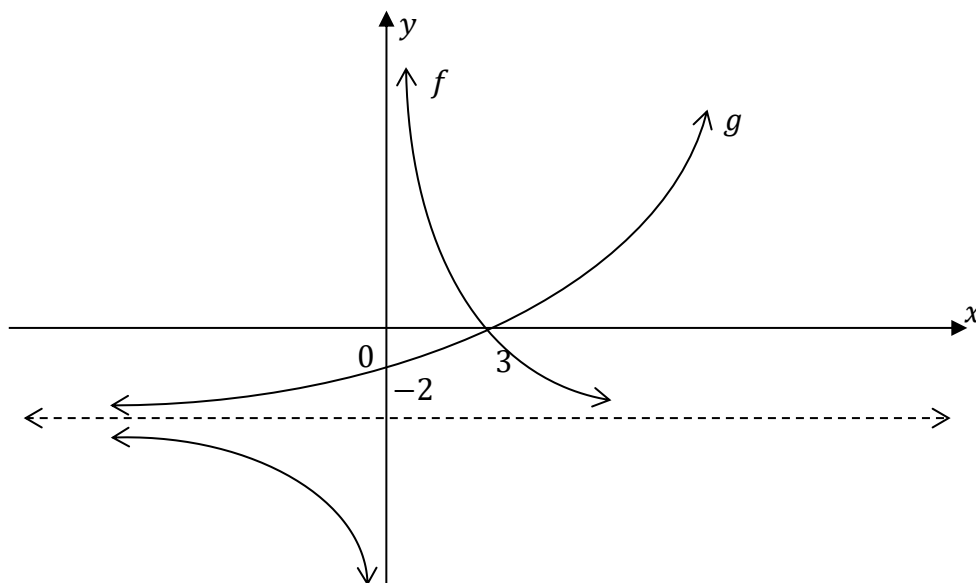
4.3.2 $3 \cdot 5^{2x-1} - 5^{2x} = -2$ (5)

Question 5 [10]

Shown below are graphs of:

$$f(x) = \frac{k}{x} + q \text{ and } g = a \cdot b^x + q$$

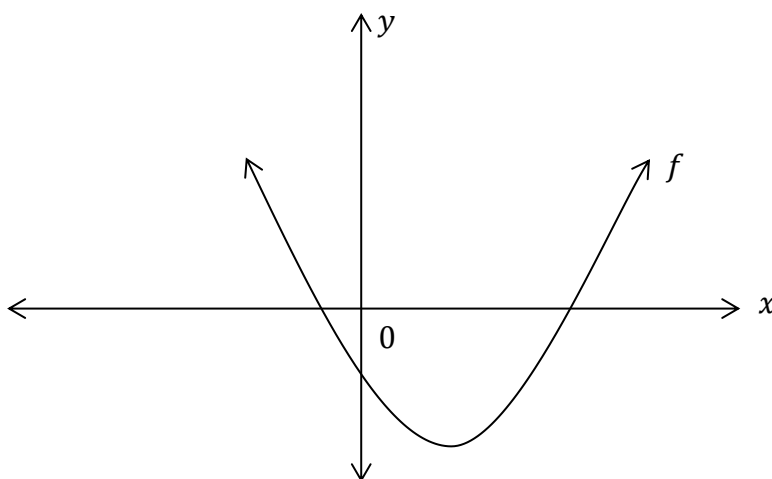
f and g have the same horizontal asymptote of $y = -4$.



- 5.1 Calculate the values of q , k , a and b . (6)
- 5.2 Use the graphs to solve for x if:
- 5.2.1 $f(x) - g(x) < 0$ (1)
- 5.2.2 $f(x) \cdot g(x) \geq 0$ (1)
- 5.3 Write down the range of f . (1)
- 5.4 State the axis of symmetry of h , if: $h(x) = f(x)(x < 0)$ (1)

Question 6 [16]

In the sketch below, the parabola, f , has a turning point of $(1, -8)$, and one x -intercept is -1 .



- 6.1 Write down the equation of the axis of symmetry of f . (2)
- 6.2 State why f is a function:
- 6.2.1 Using an appropriate line test (2)
- 6.2.2 Using the formal definition of a function in terms of x 's and y 's. (2)
- 6.3 Explain why the coordinates of the other x -intercept of f will be $(3; 0)$. (1)
- 6.4 Determine the equation of f , showing that it will be $f(x) = 2x^2 - 4x - 6$. (3)
- 6.5 For f , write down the:
- 6.5.1 domain (1)
- 6.5.2 range (1)
- 6.6.
- 6.6.1 If $h(x) = f(x) - 3$, what would be maximum value that h attains? (2)
- 6.6.2 At which value of x will h attains this minimum value? (2)

Thanks for your time

Appendix I: Pre-test and post-test memorandum

PRE-TEST AND POST TEST MEMORANDUM

QUESTION 1

$$\begin{aligned}1.1.1. \quad 5x^2 &= 2x \\ 5x^2 - 2x &= 0 \\ x(5x - 2) &= 0\end{aligned}$$

$$x = 0 \quad \text{or} \quad x = \frac{2}{5}$$

$$1.1.2. \quad \frac{2x-1}{x+2} = \frac{2x-3}{x+2}$$

$$(2x-1)(x+2) = (2x-3)(x+2)$$

$$2x^2 + 3x - 2 = 2x^2 + x - 6$$

$\therefore$ No solution

$$\begin{aligned}1.2.1. \quad -8 &\leq -3x + 2 < 2 \\ -8 - 2 &\leq -3x < 2 - 2\end{aligned}$$

$$\begin{aligned}-10 &\leq -3x < 0 \\ 0 &< x \leq \frac{10}{3}\end{aligned}$$

$$1.2.2. \quad \begin{array}{c} \leftarrow \quad \circ \quad \text{---} \quad \bullet \quad \rightarrow \\ \quad \quad 0 \quad \quad \quad \frac{10}{3} \end{array}$$

$$x \in (0; \frac{10}{3}]$$

$$\begin{aligned}1.3. \quad y &= 3x + 2 \\ y - 3x^2 &= 6x - 4 \quad \dots \textcircled{2}\end{aligned}$$

$$(3x + 2) - 3x^2 = 6x - 4$$

$$3x + 2 - 3x^2 = 6x - 4$$

$$-3x^2 - 3x + 6 = 0$$

$$-x^2 - x + 2 = 0$$

$$x^2 + x - 2 = 0$$

$$(x-1)(x+2) = 0$$

$$\therefore x = 1 \quad \text{or} \quad x = -2$$

$$\text{and } y = -4 \quad \text{or} \quad y = 5$$

$$1.4 \quad l \times b = 128$$

$$\therefore 2b \times b = 128$$

$$2b^2 = 128$$

$$b^2 = 64$$

$$b = \pm 8$$

$$\therefore b = 8 \text{ cm and } l = 2b = 16 \text{ cm.}$$

QUESTION 2

$$2.1.1. \quad 36$$

$$2.1.2. \quad 2a = 2$$

$$a = 1$$

$$3a + b = 2$$

$$3(1) + b = 2$$

$$b = -1$$

$$a + b + c = 6$$

$$1 + (-1) + c = 6$$

$$c = 6$$

$$T_n = n^2 - n + 6$$

$$2.2.1. \quad a = 5 \quad d = 6$$

$$T_n = a + (n-1)d$$

$$= 5 + (n-1)(6)$$

$$= 11 + 6n$$

$$2.2.2. \quad -283 = 5 + (n-1)(-6)$$

$$n = 49$$

Question 3

3.1 Deposit = R 480

Balance = R 4320

$$A = P(1 + i \cdot n)$$

$$= 4320 [1 + (0,5)(3)]$$

$$= R 6264$$

Monthly payment = R 189

3.2. $A = P(1 + i)^n$

$$= 10\,000 \left(1 + \frac{0,09}{12}\right)^{27}$$

$$= R 12\,235,35$$

Question 4

4.1. $\frac{(y^{-3}) \times \sqrt[3]{x^8}}{(xy^{-2n+3})}$

$$= \frac{y^{-3} \times x^{8/3}}{x y^{-2n+3}}$$

$$= y^{-3+2n-3} x^{8/3-1}$$

$$= y^{2n-6} x^{5/3}$$

4.2. $16^x = \sqrt{8}$

$$2^{4x} = 2^{3/2}$$

$$4x = 3/2$$

$$x = 3/8$$

4.3. $3 \cdot 5^{2x-1} - 5^{2x} = -2$

$$5^{2x} (3 \cdot \frac{1}{5} - 1) = -2$$

$$5^{2x} (\frac{3}{5} - 1) = -2$$

$$5^{2x} (-\frac{2}{5}) = -2$$

$$5^{2x} = 5$$

$$x = 1/2$$

QUESTION 5

5.1. $f: y = kx + q$

$g: y = a \cdot b^x + q$

$q = -4$

$f: y = kx - 4$

$(3, 0)$

$0 = k \cdot 3 - 4$

$k = 12$

$g: y = a \cdot b^x - 4$

$(0, 2)$

$-2 = a \cdot b^0 - 4$

$a = 2$

$\therefore y = 2 \cdot b^x - 4$

$(3, 0)$

$0 = 2 \cdot b^3 - 4$

$b^3 = 2$

$b = \sqrt[3]{2}$

$b = 1.26$

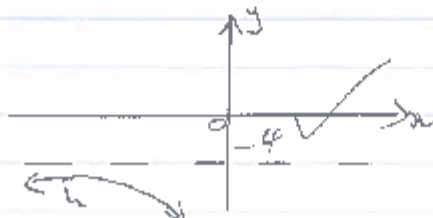
5.2. $y_f \neq y_g$

$\therefore x \in (-\infty, 0) \text{ or } 3$

5.3. $y \in \mathbb{R}, y \neq -4$

$y \in (-\infty, -4) \text{ or } (-4, \infty)$

5.4.



Question 6

6.1. $(-5; 2)$ and $(-2; 0)$

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{-2 - 0}{-5 - (-2)} \\ &= \frac{-2}{-3} \\ &= \frac{2}{3}\end{aligned}$$

Sub in B (or A)

$$y = \frac{2}{3}(-2) + C$$

$$0 = -\frac{4}{3} + C$$

$$C = \frac{4}{3}$$

$$y = \frac{2}{3}x + \frac{4}{3}$$

$$6.2. \quad m_g = -\frac{3}{2} \quad g \perp f$$

$$g = -\frac{3}{2}x + \frac{4}{3}$$

$$x = 1$$

6.2.1. Vertical line cuts the graph once.

6.2.2. for each x value, there is only 1 y -value.

6.2.3. AD is a way between the x -intercepts
by explicit construction.

$$6.4. \quad y = a(x - x_1)(x - x_2)$$

$$y = a(x + 1)(x - 3)$$

$$-8 = a(1 + 1)(1 - 3)$$

$$-8 = a(2)(-2)$$

$$-4a = -8$$

$$a = 2$$

$$\therefore y = 2(x + 1)(x - 3)$$

$$= 2(x^2 - 2x - 3)$$

$$= 2x^2 - 4x - 6$$

6.5.1. $x \in \mathbb{R}$ ✓

6.5.2. $y \geq -8$ ✓

6.6.1.

6.6.1. -11 ✓✓

6.6.2. $x = 1$ ✓✓

Appendix H: Semi-structured interview before twinning

Phase 1: Pre-intervention semi-structured interviews

Q1: What do you think can be a challenge for poor performance of learners in mathematics and why?

Q2: What kind of support do you give learners with difficulties?

Q3: How do you interact with your learners during teaching and learning of mathematics?

Q4: Which teaching materials do you use to teach mathematics?

Q5: Which language do you prefer to use in teaching mathematics?

Q6: What type of assessment and how do you use in mathematics classroom?

Q7: How do you establish a culture of teaching and learning in your mathematics classroom?

Appendix I: Semi-structured interview during twinning

Phase 2: Interviews during twinning process

Q1: Do you think twinning is implemented according to your plans and why?

Q2: Does the model teacher possess the appropriate skills and knowledge he shares with you and why?

Q3: Do you think the model teacher's skills and knowledge can be used in future teaching of mathematics and why?

Q4: Do you think the model teacher meets the learners' needs in teaching and learning, and why?

Q5: How does the model teacher deal with the learners with difficulties?

Q6: Does the model teacher consider the cognitive level of learners in teaching and learning of algebra?

Q7: What do you gain in this twinning and why?

Q8: Are there any challenges (if any) that need attention in order to develop this process?

Q9: What are the most important factors of this twinning?

Appendix J: Semi-structured interview after twinning

Phase 2: Interviews after twinning process

Q1: What best practices have you shared with the model teacher in teaching and learning Grade 11 algebra?

Q2: How was the ease for classroom interaction between the model teacher and the learners?

Q3: What have learned through observing the model teacher teaching Grade 11 algebra?

Q4: How is learner participation as compared to the one prior twinning?

Q5: How can you compare combined lesson preparations and single planned lesson preparations?

Q6: How did teaching and learning resources assisted learners' learning algebra?

Q7: Do you think twinning can be sustained between the two schools and why?

Q8: According to you, was this twinning effective or not and why?

Q9: How can we make this twinning effective?

Appendix K: Sample of learner's script (Pre-Test)

Name Chene
 Grade 11
 Mathematics
 Test

28 January 2015

Question 1

1.1 Solve for x

1.1.1 $5x^2 = 2x$

~~$= \sqrt{5x^2} = \sqrt{2x}$
 $= 5x = \sqrt{2x}$~~

$= \sqrt{5x^2} = \sqrt{2x}$
 $= \frac{5x}{5} = \frac{\sqrt{2x}}{5}$
 $= \frac{5x}{5} = \frac{\sqrt{2x}}{5}$
 $= \therefore x = \frac{\sqrt{2x}}{5}$

1.1.2 $2x-1 = 2x-3$
 $x+2 \quad x+2$
 $= x+2(2x-1) = x+2(2x-3)$
 $= x+4x-2 = x+4x-6$
 $= x+4-x-4x = -6+2$
 $\frac{20}{0} = \frac{-4}{0}$
 $\therefore x = 0$

1.2 Given $-8 \leq -3x + 2 \leq 2$

~~$-8 \leq -3x + 2 \leq 2$~~

~~$-8 \leq -3x + 2$~~

~~$\frac{-8}{-3} \times 1 \leq \frac{-3x + 2}{-3} \times \frac{3}{-3} \leq \frac{2}{-3} \times \frac{3}{-3}$~~
 ~~$\frac{8}{3} \leq x \leq 0$~~



1.3 Solve for x & y

$2y - 6x = 4$ --- ①

$y - 3x^2 = 6x - 4$ --- ②

from ①

$y = 6x + 4 - 2$ --- ③

sub ③ into ②

$2(6x + 4 - 2) - 6x = 4$

$12x + 4 - 4 - 6x = 4$

$12x - 6x = 4 - 8 + 4$

$\frac{6x}{6} = \frac{0}{6}$

$x = 0$

sub $x = 0$ into ①

$2y - 6(0) = 4$

$2y = 4$

$y = 2$

$y = 2$



1.4 The length of a rectangle is twice the breadth.

$$\begin{aligned}\text{Area} &= l \times b \times h \\ &= 2 \times 20 \times h \\ &= 800 \text{ cm}^2 \\ h &= \frac{800 \text{ cm}^2}{40} \\ &= 20 \text{ cm}\end{aligned}$$

Question 2

2.1 A sequence is given as follows:

6, 8, 12, 18, 26, 36

2.1.1 36

$$\begin{aligned}2.1.2 T_n &= \frac{T_n - T_1}{T_3 - T_2} \\ &= \frac{18 - 6}{12 - 8} \\ &= \frac{12}{4} \\ T_n &= 3\end{aligned}$$

2.2 Given S, -1, -7, ..., -283

2.2.1 $T_n = 94, 33$

2.2.2 The number of terms in the sequence
= 12

Question 3

3.1 $P = R 4800 = 4800$

$A = ?$

$n = 3$

$i = 15\% \div 100 = 0,15$

~~$A = P(1+i)^n$
 $= 4800(0,15+1)^3$
 $= 7300,2$~~

~~$A = P(1+i)$
 $= 4800(3 \times 0,15+1)$
 $= 4800+1,45$
 $= 4801,45$~~

3.2 $P = R 10\,000$

$A = ?$

$i = 9\% \div 100 = 0,09$

~~$A = P(1+i)^n$
 $= 10\,000(0,09+1)^n$~~

~~$n = 10\,000 \times 1,09$
 $= 10900,09 \times 12$~~

~~$n = 1290013,08$~~

Question 4

4.1 Simplify

4.1.1 $\frac{(y^{-3})x\sqrt{x^8}}{(xy^{-2n+3})}$

$= \frac{y^{-3-2n+3}}{\sqrt{x^8}}$

$= \frac{y^{-2n}}{\sqrt{x^8}}$

$= (\sqrt{y^{-2n}})(\sqrt{x^8})^2$

$= (\sqrt{y^{-2n}})(x^4)^2$
 $= y^{-2n} \cdot x^8$

Appendix L: Sample of learner's script (Post-Test)

Barbara Mapule
Grade 11
Mathematics

27

Post test

1.1

1.1.1 $5x^2 = 2x$

$$5x^2 - 2x = 0$$

$$x(5x - 2) = 0$$

$$x = 0 \text{ or } 5x - 2 = 0$$

$$\frac{5}{5}x = \frac{2}{5}$$

$$x = \frac{2}{5}$$

3

1.1.2 $\frac{2x-1}{x+2} \times \frac{2x-3}{x+2}$

$$(2x-1)(x+2) = (2x-3)(x+2)$$

$$2x^2 + 4x - x - 2 = 2x^2 + 4x - 3x - 6$$

$$2x^2 + 3x - 2 = 2x^2 + x - 6$$

$$-2x^2 + 2x^2 + 3x - x = -2 + 6 = 0$$

$$2x + 4 = 0$$

$$\frac{2}{2}x = -4$$

$$x = -\frac{4}{2}$$

$$x = -2$$

4

1.2. $-8 \leq -3x + 2 < 2$

$$-8 - 2 \leq -3x < 2 - 2$$

$$-10 \leq -3x < 0$$

$$\frac{-10}{-3} \leq x < \frac{0}{-3}$$

$$\frac{10}{3} \leq x < 0$$

2

$$1.3 \quad 2y - 6x = 4 \quad \dots (1)$$

$$y - 3x^2 = 6x - 4 \quad \dots (2)$$

From (2)

$$y = 3x^2 + 6x - 4 \quad \dots (3)$$

Sub (3) into (1)

$$2(3x^2 + 6x - 4) - 6x = 4$$

$$6x^2 + 12x - 8 - 6x = 4$$

$$6x^2 + 6x - 8 = 4$$

$$6x^2 + 6x - 8 - 4 = 0$$

$$\frac{6x^2}{6} + \frac{6x}{6} - \frac{8}{6} - \frac{4}{6} = 0$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x+2=0 \quad | \quad x-1=0$$

$$x = -2 \quad \cdot \quad x = 1$$



Sub $x = -2$ into (3) / Sub $x = 1$ into (3)

$$y - 3(-2)^2 = 6(-2) - 4$$

$$y - 3(1)^2 = 6(1) - 4$$

$$y - 12 = -12 - 4$$

$$y - 3 = 6 - 4$$

$$y = -12 - 4 + 12$$

$$y = 6 - 4 + 3$$

$$y = -4$$

$$y = 5$$

$$(-2, -4)$$

$$(1, 5)$$

Question 2

2.1.1



$$\frac{2a}{2} = \frac{2}{2}$$

$$a =$$

$$2.1.1. \quad 3b$$

$$2.1.2 \quad \frac{3a}{3} = \frac{3}{2}$$

$$a = 1$$

2

$$T_n = x^2 + 3x + 3$$

$$3a = b$$

$$3(1) = b$$

$$b = 3$$

$$a + b + c = 0$$

$$(1)(3) = c$$

$$c = 3$$

$$2.1.3$$

$$2.2. \quad \begin{matrix} 5, -1, -7 \\ -6 \quad -6 \end{matrix}$$

$$2.2.1 \quad T_n = a + (n-1)d$$

$$T_n = 5 + (n-1)-6$$

$$T_n = -n + 6$$

$$T_n = -n + 11$$

6

$$2.2.2 \quad \text{linear pattern}$$

?

Question 3

3.1. $P = 4800$

$i = 15\%$

$n = 3$

$$4800 = (1 + 0,15 - 3)$$

$$) \frac{4800 \cdot 3 \cdot 15}{100}$$

~~82160~~

3.3 $P = (1 + n \cdot i)^n$

~~$10000 = (1 + 0,09)^3$~~

~~$10000 = 1,29$~~

~~$\therefore R = 10000,29$~~

$) = \frac{10000 \cdot 3 \cdot 9}{100}$

$\therefore 2700$

Question 4

4.1.1. $\frac{(y^{-3}) \times \sqrt[3]{x^6}}{xy^{-2n+3}}$

$$\frac{y^{-3} \times (x^6)^{\frac{1}{3}}}{xy^{-2n+3}}$$

$$\frac{y^{-3} \times x^2}{xy^{-2n+3}}$$

$$\frac{y^{-3+2n+3} \cdot x^{\frac{2}{1}-1}}{y^{2n} \cdot x^{\frac{2}{1}-1}}$$



Rakula Mapula
Grade 11^C
Mathematics

Question 4

4.3.

4.3.1 $16^x = \sqrt{8}$

$$(16^x)^2 = (\sqrt{8})^2$$

$$16^{2x} = 8$$

$$(2^4)^{2x} = 2^3$$

$$2^{4 \cdot 2x} = 2^3$$

$$\frac{4 \cdot 2x}{4} = \frac{3}{4}$$

$$x = \frac{3}{4}$$

4.1.2. $3 \cdot 5^{2x-1} - 5^{2x} = -2$

$$3 \cdot 5^{2x} \cdot 5^{-1} - 5^{2x} = -2$$

$$5^{2x} \left(\frac{3}{5} - 1 \right) = -2$$

$$5^{2x} \left(\frac{3-5}{5} \right) = -2$$

$$5^{2x} = -2 \cdot \frac{5}{-2} = \frac{10}{2}$$

$$5^{2x} = \frac{10}{2} = 5$$

$$\frac{2x}{2} = \frac{1}{2}$$

$$x = \frac{1}{2}$$

Question 5

5.4. $h(x) = 2$ ~~$h(x) = 2$~~

$h(x) = 2$ ~~$h(x) = 2$~~

Question 6

6.1. $y = -x$

6.2 Because it represent $y = -x$

6.3

6.3.1.

6.3.2

6.3 Because y-intercept is doesn't have co-ordinate

6.4. $f(x) = a(1+p)^x + b$

6.5.

6.5.1. $\forall x \in \mathbb{R}$

6.5.2 $y \in \mathbb{R}$

6.6

6.6.1. $h(x) = 3$

6.6.2. -3

Appendix L: Statistical Results

Chapter 1

The statistical software package used to analyse the data is Stata.

variable	N	min	max	p50	mean	sd
Q1	170	0	20	8	7.988235	5.895529
Q2	170	0	12	4	4.405882	2.907849
Q3	170	0	8	3	3.352941	2.479291
Q4	170	0	13	4	5.041176	3.952912
Q5	169	0	10	3	2.946746	2.778018
Q6	169	0	16	2	3.230769	3.670993
TotalPerce~e	170	3.75	91.25	31.25	33.66176	20.13021
Total	170	3	73	25	26.92941	16.10416

The above table presents the summary statistics for all variables.

Skewness/Kurtosis tests for Normality						
Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	adj chi2(2)	Prob>chi2	joint
Q1	170	0.2843	0.0000	40.69	0.0000	
Q2	170	0.0319	0.0085	10.12	0.0063	
Q3	170	0.4069	0.0000	21.68	0.0000	
Q4	170	0.1185	0.0000	48.02	0.0000	
Q5	169	0.0125	0.0001	17.91	0.0001	
Q6	169	0.0000	0.0497	23.61	0.0000	
TotalPerce~e	170	0.0046	0.1446	9.07	0.0107	
Total	170	0.0046	0.1446	9.07	0.0107	

All of the variables are not normally distributed (all **p-values** are less than 0.05)
The use of non-parametric tests is warranted.

The rank-sum test will be used to compare the two study groups. The interpretation
will be performed at 95% confidence limit.

Chapter 2

```
-> bys Test : ranksum Q1, by(School)
```

```
-----
--
-> Test = POST
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

School	obs	rank sum	expected
-----+	-----	-----	-----
Experimental	43	1389	1849
Control	42	2266	1806
-----+	-----	-----	-----
combined	85	3655	3655

unadjusted variance 12943.00

adjustment for ties -56.28

adjusted variance 12886.72

Ho: Q1(School==Experimental) = Q1(School==Control)

z = -4.052

Prob > |z| = 0.0001

The two schools performed significantly differently on Q1 in the post-test(p = 0.0001). Therefore, Control group than Experimental group recorded significantly higher scores on Q1.

Section 2.1

```
-> Test = Pre
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

School	obs	rank sum	expected
-----+	-----	-----	-----
Experimental	43	1069.5	1849
Control	42	2585.5	1806
-----+	-----	-----	-----
combined	85	3655	3655

unadjusted variance 12943.00

adjustment for ties -248.51

adjusted variance 12694.49

Ho: Q1(School==Experimental) = Q1(School==Control)

z = -6.918

Prob > |z| = 0.0000

```
-> bys Test : ranksum Q2, by( School)
```

The two schools performed significantly differently on Q1 in the pre-test(p = 0.0000). Therefore, Control than Experimental recorded significantly higher scores on Q1.

```
-----
-----
-> Test = POST
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

School	obs	rank sum	expected
-----+	-----	-----	-----
Experimental	43	1608.5	1849

```

Control |      42      2046.5      1806
-----+-----
combined |      85      3655      3655

unadjusted variance    12943.00
adjustment for ties    -175.79
-----
adjusted variance      12767.21

```

```

Ho: Q2(School==Experimental) = Q2(School==Control)
      z = -2.128

```

```

Prob > |z| = 0.0333

```

Seshigo performed significantly differently from Makgabo in the post-test for Q2 (p = 0.0333).

```

-----
-> Test = Pre

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

```

      School |      obs      rank sum      expected
-----+-----
Experimental |      43      1862      1849
Control      |      42      1793      1806
-----+-----
combined     |      85      3655      3655

unadjusted variance    12943.00
adjustment for ties    -339.70
-----
adjusted variance      12603.30

```

```

Ho: Q2(School== Experimental) = Q2(School==Control)
      z = 0.116
      Prob > |z| = 0.9078

```

```

-> bys Test : ranksum Q3, by( School)

```

```

-----
-> Test = POST

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

```

      School |      obs      rank sum      expected
-----+-----
Experimental |      43      1657.5      1849
Control      |      42      1997.5      1806
-----+-----
combined     |      85      3655      3655

unadjusted variance    12943.00
adjustment for ties    -274.95
-----
adjusted variance      12668.05

```

```

Ho: Q3(School== Experimental) = Q3(School== Control)
      z = -1.701
      Prob > |z| = 0.0889

```

```

-----
-> Test = Pre

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

```

      School |      obs      rank sum      expected
-----+-----
Experimental |      43      1720      1849
Control      |      42      1935      1806

```



```

-----+-----
combined |      85      3655      3655

unadjusted variance    12943.00
adjustment for ties    -542.69
-----
adjusted variance      12400.31

Ho: Q3(School== Experimental) = Q3(School== Control)
      z =  -1.158
      Prob > |z| =  0.2467

```

```
->  bys Test : ranksum Q4, by( School)
```

```
-----
-> Test = POST
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

School	obs	rank sum	expected
Makgabo	43	1849	1849
Control	42	1806	1806
combined	85	3655	3655

```

unadjusted variance    12943.00
adjustment for ties    -135.58
-----
adjusted variance      12807.42

```

```

Ho: Q4(School== Experimental) = Q4(School== Control)
      z =  0.000
      Prob > |z| =  1.0000

```

```
-----
-> Test = Pre
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

School	obs	rank sum	expected
Experimental	43	1627.5	1849
Control	42	2027.5	1806
combined	85	3655	3655

```

unadjusted variance    12943.00
adjustment for ties    -326.80
-----
adjusted variance      12616.20

```

```

Ho: Q4(School== Experimental) = Q4(School== Control)
      z =  -1.972
      Prob > |z| =  0.0486

```

```
->  bys Test : ranksum Q5, by( School)
```

```
-----
-> Test = POST
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

School	obs	rank sum	expected
Experimental	42	1950.5	1785
Control	42	1619.5	1785

```

combined |      84      3570      3570

unadjusted variance    12495.00
adjustment for ties    -237.33
-----
adjusted variance      12257.67

Ho: Q5(School== Experimental) = Q5(School== Control)
      z =      1.495
      Prob > |z| =    0.1350

```

```

-----
-> Test = Pre

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

School	obs	rank sum	expected
Experimental	43	1386.5	1849
Control	42	2268.5	1806
combined	85	3655	3655

```

unadjusted variance    12943.00
adjustment for ties    -3166.06
-----
adjusted variance      9776.94

```

```

Ho: Q5(School== Experimental) = Q5(School== Control)
      z =    -4.677
      Prob > |z| =    0.0000

```

```

-> bys Test : ranksum Q6, by( School)

```

```

-----
-> Test = POST

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

School	obs	rank sum	expected
Experimental	42	1172.5	1785
Control	42	2397.5	1785
combined	84	3570	3570

```

unadjusted variance    12495.00
adjustment for ties    -286.66
-----
adjusted variance      12208.34

```

```

Ho: Q6(School== Experimental) = Q6(School== Control)
      z =    -5.543
      Prob > |z| =    0.0000

```

```

-----
-> Test = Pre

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

School	obs	rank sum	expected
Experimental	43	1053.5	1849
Control	42	2601.5	1806
combined	85	3655	3655

```

unadjusted variance    12943.00

```

```

adjustment for ties      -2377.90
-----
adjusted variance        10565.10

Ho: Q6(School== Experimental) = Q6(School== Control)
      z =  -7.739
      Prob > |z| =   0.0000

->  bys Test : ranksum TotalPercentage, by( School)

-----
-> Test = POST

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

      School |      obs      rank sum      expected
-----+-----
Experimental |      43      1437.5      1849
Control      |      42      2217.5      1806
-----+-----
combined     |      85      3655      3655

unadjusted variance      12943.00
adjustment for ties      -16.57
-----
adjusted variance        12926.43

Ho: TotalP~e(School== Experimental) = TotalP~e(School== Control)
      z =  -3.619
      Prob > |z| =   0.0003

-----
-> Test = Pre

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

      School |      obs      rank sum      expected
-----+-----
Experimental |      43      1195.5      1849
Control      |      42      2459.5      1806
-----+-----
combined     |      85      3655      3655

unadjusted variance      12943.00
adjustment for ties      -26.43
-----
adjusted variance        12916.57

Ho: TotalP~e(School== Experimental) = TotalP~e(School== Control)
      z =  -5.750
      Prob > |z| =   0.0000

->  bys Test : ranksum Total, by( School)

-----
-> Test = POST

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

      School |      obs      rank sum      expected
-----+-----
Experimental |      43      1437.5      1849
Control      |      42      2217.5      1806
-----+-----
combined     |      85      3655      3655

unadjusted variance      12943.00
adjustment for ties      -16.57

```

```

-----
adjusted variance      12926.43

Ho: Total(School== Experimental) = Total(School== Control)
      z =  -3.619
      Prob > |z| =  0.0003

-----
-----
-> Test = Pre

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

      School |      obs      rank sum      expected
-----+-----
Experimental |      43      1195.5      1849
Control      |      42      2459.5      1806
-----+-----
combined     |      85      3655      3655

unadjusted variance    12943.00
adjustment for ties    -26.43
-----
adjusted variance      12916.57

Ho: Total(School== Experimental) = Total(School== Control)
      z =  -5.750
      Prob > |z| =  0.0000

. set more off

-> bys School : ranksum Q1, by( Test )

-----
-----
-> School = Experimental

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

      Test |      obs      rank sum      expected
-----+-----
POST      |      43      2467.5      1870.5
Pre        |      43      1273.5      1870.5
-----+-----
combined   |      86      3741      3741

unadjusted variance    13405.25
adjustment for ties    -375.49
-----
adjusted variance      13029.76

Ho: Q1(Test==POST) = Q1(Test==Pre)
      z =   5.230
      Prob > |z| =  0.0000

-----
-----
-> School = Control

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

      Test |      obs      rank sum      expected
-----+-----
POST      |      42      1980      1785
Pre        |      42      1590      1785
-----+-----
combined   |      84      3570      3570

unadjusted variance    12495.00
adjustment for ties    -46.81

```

```

-----
adjusted variance      12448.19

Ho: Q1(Test==POST) = Q1(Test==Pre)
      z =      1.748
      Prob > |z| =    0.0805

```

The two tests are not significantly different with respect to Q1 (p = 0.0805). In another words, the observed difference between the two tests is just marginal.

```

-> bys School : ranksum Q2, by( Test )

```

```

-----
-> School = Experimental

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
POST	43	1972.5	1870.5
Pre	43	1768.5	1870.5
combined	86	3741	3741

```

unadjusted variance      13405.25
adjustment for ties      -298.72
-----

```

```

adjusted variance        13106.53

```

```

Ho: Q2(Test==POST) = Q2(Test==Pre)
      z =      0.891
      Prob > |z| =    0.3730

```

```

-----
-> School = Control/

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
POST	42	2090	1785
Pre	42	1480	1785
combined	84	3570	3570

```

unadjusted variance      12495.00
adjustment for ties      -162.94
-----

```

```

adjusted variance        12332.06

```

```

Ho: Q2(Test==POST) = Q2(Test==Pre)
      z =      2.747
      Prob > |z| =    0.0060

```

```

-> bys School : ranksum Q3, by( Test )

```

```

-----
-> School = Experimental

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
POST	43	2189	1870.5
Pre	43	1552	1870.5
combined	86	3741	3741

```

unadjusted variance      13405.25

```

adjustment for ties -511.57

adjusted variance 12893.68

Ho: Q3(Test==POST) = Q3(Test==Pre)
 z = 2.805
 Prob > |z| = 0.0050

-> School = Control

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
-----+-----			
POST	42	2127	1785
Pre	42	1443	1785
-----+-----			
combined	84	3570	3570

unadjusted variance 12495.00
adjustment for ties -329.93

adjusted variance 12165.07

Ho: Q3(Test==POST) = Q3(Test==Pre)
 z = 3.101
 Prob > |z| = 0.0019

-> bys School : ranksum Q4, by(Test)

-> School = Experimental

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
-----+-----			
POST	43	2249.5	1870.5
Pre	43	1491.5	1870.5
-----+-----			
combined	86	3741	3741

unadjusted variance 13405.25
adjustment for ties -340.33

adjusted variance 13064.92

Ho: Q4(Test==POST) = Q4(Test==Pre)
 z = 3.316
 Prob > |z| = 0.0009

-> School = Control

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
-----+-----			
POST	42	1998	1785
Pre	42	1572	1785
-----+-----			
combined	84	3570	3570

unadjusted variance 12495.00
adjustment for ties -138.65

adjusted variance 12356.35

```
Ho: Q4(Test==POST) = Q4(Test==Pre)
      z = 1.916
      Prob > |z| = 0.0553
```

```
-> bys School : ranksum Q5, by( Test )
```

```
-----
-> School = Experimental
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
POST	42	2556	1806
Pre	43	1099	1849
combined	85	3655	3655

```
unadjusted variance 12943.00
adjustment for ties -1594.92
-----
adjusted variance 11348.08
```

```
Ho: Q5(Test==POST) = Q5(Test==Pre)
      z = 7.040
      Prob > |z| = 0.0000
```

```
-----
-> School = Control
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
POST	42	2274.5	1785
Pre	42	1295.5	1785
combined	84	3570	3570

```
unadjusted variance 12495.00
adjustment for ties -302.73
-----
adjusted variance 12192.27
```

```
Ho: Q5(Test==POST) = Q5(Test==Pre)
      z = 4.433
      Prob > |z| = 0.0000
```

```
-> bys School : ranksum Q6, by( Test )
```

```
-----
-> School = Experimental
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
POST	42	2279	1806
Pre	43	1376	1849
combined	85	3655	3655

```
unadjusted variance 12943.00
adjustment for ties -5275.85
-----
adjusted variance 7667.15
```

```

Ho: Q6(Test==POST) = Q6(Test==Pre)
      z =      5.402
      Prob > |z| =      0.0000

```

```

-----
-> School = Control

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
POST	42	1997.5	1785
Pre	42	1572.5	1785
combined	84	3570	3570

unadjusted variance 12495.00

adjustment for ties -229.10

adjusted variance 12265.90

```

Ho: Q6(Test==POST) = Q6(Test==Pre)
      z =      1.919
      Prob > |z| =      0.0550

```

```

-> bys School : ranksum TotalPercentage, by( Test )

```

```

-----
-> School = Experimental

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
POST	43	2639.5	1870.5
Pre	43	1101.5	1870.5
combined	86	3741	3741

unadjusted variance 13405.25

adjustment for ties -27.44

adjusted variance 13377.81

```

Ho: TotalP~e(Test==POST) = TotalP~e(Test==Pre)
      z =      6.649
      Prob > |z| =      0.0000

```

```

-----
-> School = Control

```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
POST	42	2134	1785
Pre	42	1436	1785
combined	84	3570	3570

unadjusted variance 12495.00

adjustment for ties -7.72

adjusted variance 12487.28

```

Ho: TotalP~e(Test==POST) = TotalP~e(Test==Pre)
      z =      3.123
      Prob > |z| =      0.0018

```



```
-> bys School : ranksum Total, by( Test )
```

```
-----  
-----  
-> School = Experimental
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
-----+-----			
POST	43	2639.5	1870.5
Pre	43	1101.5	1870.5
-----+-----			
combined	86	3741	3741

unadjusted variance 13405.25

adjustment for ties -27.44

adjusted variance 13377.81

Ho: Total(Test==POST) = Total(Test==Pre)

z = 6.649

Prob > |z| = 0.0000

```
-----  
-----  
-> School = Control
```

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

Test	obs	rank sum	expected
-----+-----			
POST	42	2134	1785
Pre	42	1436	1785
-----+-----			
combined	84	3570	3570

unadjusted variance 12495.00

adjustment for ties -7.72

adjusted variance 12487.28

Ho: Total(Test==POST) = Total(Test==Pre)

z = 3.123

Prob > |z| = 0.0018

Chapter 4

4.1 Introduction

L;rlgkrgrgB;t;gtlkgt<bvngttg
Rgkbbjrgklbrgbrgbtrgb
rkjvnbgb

Section 4.1.1

This section presents the statistical results...

	Q1	Q2	Q3	Q4	Q5	Q6	Total
School	Rank	Rank	Rank	Rank	Rank	Rank	Rank
Control	1126	4569					
Experimental	69	2354					
p-Value	0.001	0.046					

Table 4-1: Comparisons of schools - Post-test

Table 4-1 presents the p-values after comparing the performances of the two schools in the post-test setting. It can be seen that the performance favoured Seshigo....

--	--