

Chapter 3

The geometric pulsar models

The γ -ray *emission* models discussed in the previous chapter, which are based on the emission processes described in Section 2.5.2, serve as the bases for the *geometric* γ -ray models that will be employed throughout the rest of this study: the geometric outer gap (OG) and two-pole caustic (TPC) models. The geometric models are not concerned with the underlying processes responsible for γ -ray emission, but rather with which configuration of the magnetospheric acceleration gaps (with regard to their locations and extents) best fits observations. Consequently they do not predict the spectral properties or observed intensity of the γ -ray emission, but only the shape of the profile. An empirical geometric radio model is employed alongside these γ -ray models.

The first section in this chapter describes the magnetic field assumed by all three geometric models. The geometric models themselves are discussed in Section 3.2 (for the γ -ray models) and Section 3.3 (for the radio model). Section 3.4 describes how the emission from these geometric models is affected by the rotation of the pulsar system, and how these rotational effects lead to strong peaks in the γ -ray emission. Section 3.5 introduces the tools with which the rest of this study will be conducted.

3.1 The magnetic field

The magnetic field structure is a fundamental assumption of the geometric models used in this study. For a non-rotating dipole, the magnetic field, expressed in spherical coordinates, is

$$\mathbf{B}_{\text{st}} = \frac{1}{r^3} [3(\boldsymbol{\mu} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \boldsymbol{\mu}],$$

where $\boldsymbol{\mu}$ is the pulsar's magnetic moment (Arendt et al., 1998; Cheng et al., 2000).

This structure has proven to be quite successful in describing low-altitude radio emission (e.g. Lyne & Manchester, 1988), but is not sufficient when the regions from which emission originates are close to the light cylinder where the effects of rotation become more important, as is the case in the γ -ray models (Dyks & Harding, 2004).

3.1.1 The retarded dipole

Deutsch (1955) derived the equations describing the magnetic field in the vicinity of an idealised, perfectly conducting, and sharply bounded sphere rotating rigidly in a vacuum, including the case

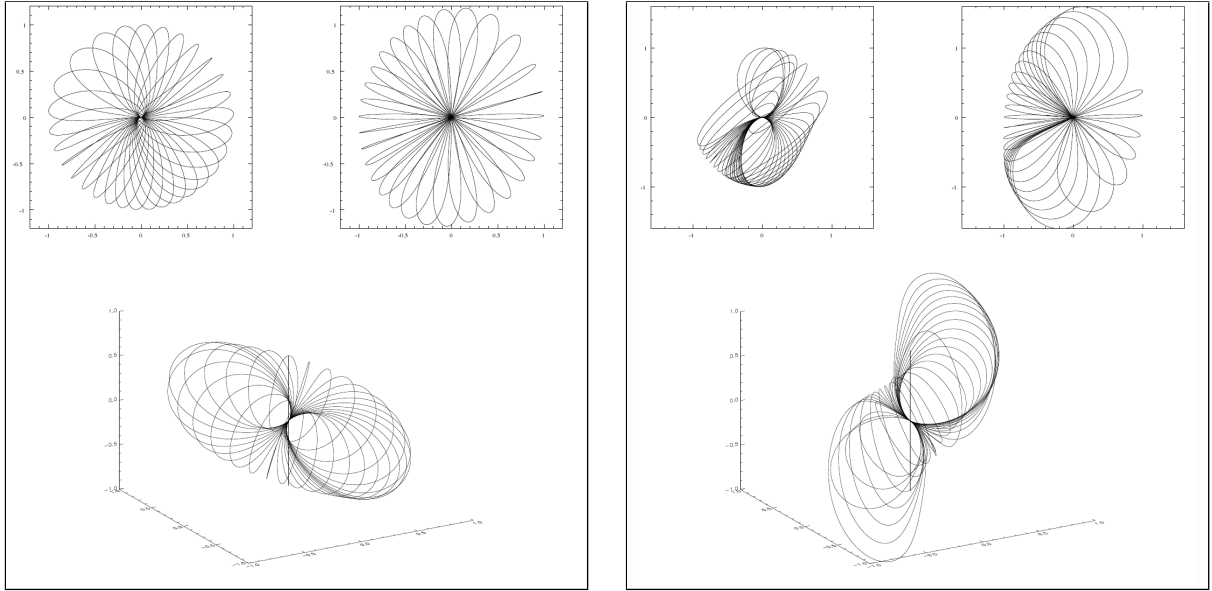


Figure 3.1: The last closed field lines for a retarded dipole inclined at $\alpha = 30^\circ$ (left), and 70° (right) with respect to the rotation axis. For each set of panels the top-left panel shows a projection looking down the rotation axis (rotation is counterclockwise), the top-right panel shows a projection looking down the magnetic axis μ , and the bottom panel shows a three-dimensional view, with the rotation axis indicated by a vertical line. The stellar radius is too small to be shown in these figures. From Arendt et al. (1998).

where the magnetic axis is inclined with respect to the rotation axis. Shitov (1983) first considered the effect of rotational sweepback on magnetic field structure in the context of pulsar observations, and this effect has been studied in some detail by, e.g., Arendt et al. (1998), Cheng et al. (2000), and Dyks & Harding (2004). The magnetic field structure, when taking the effects of rotation into account, is given by the equation (Cheng et al., 2000)

$$\mathbf{B}_{\text{ret}} = \hat{\mathbf{r}} \hat{\mathbf{r}} \cdot \left[3 \frac{\boldsymbol{\mu}(t)}{r^3} + 3 \frac{\dot{\boldsymbol{\mu}}(t)}{cr^2} + \frac{\ddot{\boldsymbol{\mu}}(t)}{c^2 r} \right] - \left[\frac{\boldsymbol{\mu}(t)}{r^3} + \frac{\dot{\boldsymbol{\mu}}(t)}{cr^2} + \frac{\ddot{\boldsymbol{\mu}}(t)}{c^2 r} \right], \quad (3.1)$$

where $\boldsymbol{\mu}(t)$ is the time-dependent magnetic moment of the dipole, inclined at an angle α with respect to the angular velocity vector $\boldsymbol{\Omega}$ (directed along the $\hat{\mathbf{z}}$ axis). This magnetic moment and its first and second order derivatives can be expressed as

$$\boldsymbol{\mu}(t) = \mu (\sin \alpha \cos \Omega t \hat{\mathbf{x}} + \sin \alpha \sin \Omega t \hat{\mathbf{y}} + \cos \alpha \hat{\mathbf{z}}), \quad (3.2)$$

$$\dot{\boldsymbol{\mu}}(t) = \Omega \mu \sin \alpha (-\sin \Omega t \hat{\mathbf{x}} + \cos \Omega t \hat{\mathbf{y}}), \text{ and} \quad (3.3)$$

$$\ddot{\boldsymbol{\mu}}(t) = -\Omega^2 \mu \sin \alpha (\cos \Omega t \hat{\mathbf{x}} + \sin \Omega t \hat{\mathbf{y}}). \quad (3.4)$$

These time derivatives approach zero as Ω approaches zero. This means that in the static limit $\mathbf{B}_{\text{ret}}$ reduces to $\mathbf{B}_{\text{st}}$. From the above equations the spherical components of $\mathbf{B}_{\text{ret}}$ are as follows (Dyks & Harding, 2004):

$$\begin{aligned}
\mathbf{B}_{\text{ret},r} &= \frac{2\mu}{r^3} [\cos \alpha \cos \theta + \sin \alpha \sin \theta (r_n \sin \lambda + \cos \lambda)] \\
\mathbf{B}_{\text{ret},\phi} &= -\frac{\mu}{r^3} \sin \alpha [(r_n^2 - 1) \sin \lambda + r_n \cos \lambda] \\
\mathbf{B}_{\text{ret},\theta} &= \frac{\mu}{r^3} \{ \cos \alpha \sin \theta + \sin \alpha \cos \theta [-r_n \sin \lambda + (r_n^2 - 1) \cos \lambda] \},
\end{aligned}$$

where $r_n = r/R_{\text{LC}}$, R_{LC} is the light cylinder radius, and $\lambda = r_n + \phi - \Omega t$. These equations again reduce to those for the components of $\mathbf{B}_{\text{st}}$, in the limit where $r_n \rightarrow 0$.

Figure 3.1 shows two examples of the magnetic field described above. The set of panels on the left are for a magnetic field inclined at $\alpha = 30^\circ$ while those on the right are for one inclined at $\alpha = 70^\circ$ (Arendt et al., 1998).

3.1.2 The shape of the PC

The retarded dipole field described in the previous section, which takes the effects of rotation into account, is significantly different from the static dipole field when considering the shape of the last open field lines. The differences between these two fields may be summarised by considering the shape of the PC in both instances. The PC is the region on the star's surface bounded by the footpoints of the last open field lines.

It is useful to introduce the so-called open volume coordinates r_{ovc} and l_{ovc} , as described by Dyks et al. (2004a). These coordinates label the footpoints of the magnetic field lines on the PC surface in a very specific way. The coordinate $r_{\text{ovc}} \equiv 1 \pm d_{\text{ovc}}$ labels the set of footpoints located at a normalised distance d_{ovc} from the PC boundary. The radius of the static dipole PC is used to normalise d_{ovc} , and $r_{\text{ovc}} = 1$ for the footpoints of the last open field lines. The coordinate l_{ovc} denotes the distance along the ring of constant r_{ovc} at which a footpoint is located, measured from the fiducial plane containing both the rotation and magnetic axes.

Figure 3.2 shows the shape of the PC for various inclination angles (α). In the static dipole approximation (the white dots in Figure 3.2), the PC is a circle with radius $r_{\text{PC}} = R_{\text{NS}} (R_{\text{NS}}/R_{\text{LC}})^{1/2}$ for an aligned rotator (e.g., Sturrock, 1971). As α increases, the PC's shape becomes more elliptical, compressed in the latitudinal direction by up to $\sim 60\%$ for an orthogonal dipole (Roberts & Sturrock, 1972; Biggs, 1990). In the retarded dipole regime (the black dots in Figure 3.2) the shape of the PC is markedly different from what it is in the static approximation. This is because the shape of the PC is sensitive to the shape of the field lines at the light cylinder (Dyks & Harding, 2004), and these are strongly distorted for a nonaligned rotator. These high-altitude distortions give rise to a PC shape which is neither circular nor simply elongated (Arendt et al., 1998). Instead, two distinct points can be identified where the PC's boundary gradually becomes indented. The sharper of the two, labelled N_1 in the panes with $\alpha > 30^\circ$ in Figure 3.2, is called the notch. The other point, labelled N_2 in the panes with $\alpha > 45^\circ$, is less pronounced and does not have a very large effect. In Section 4.1 it will be shown that the distorted portion of the PC boundary between these two points can be associated with a prominent emission feature.

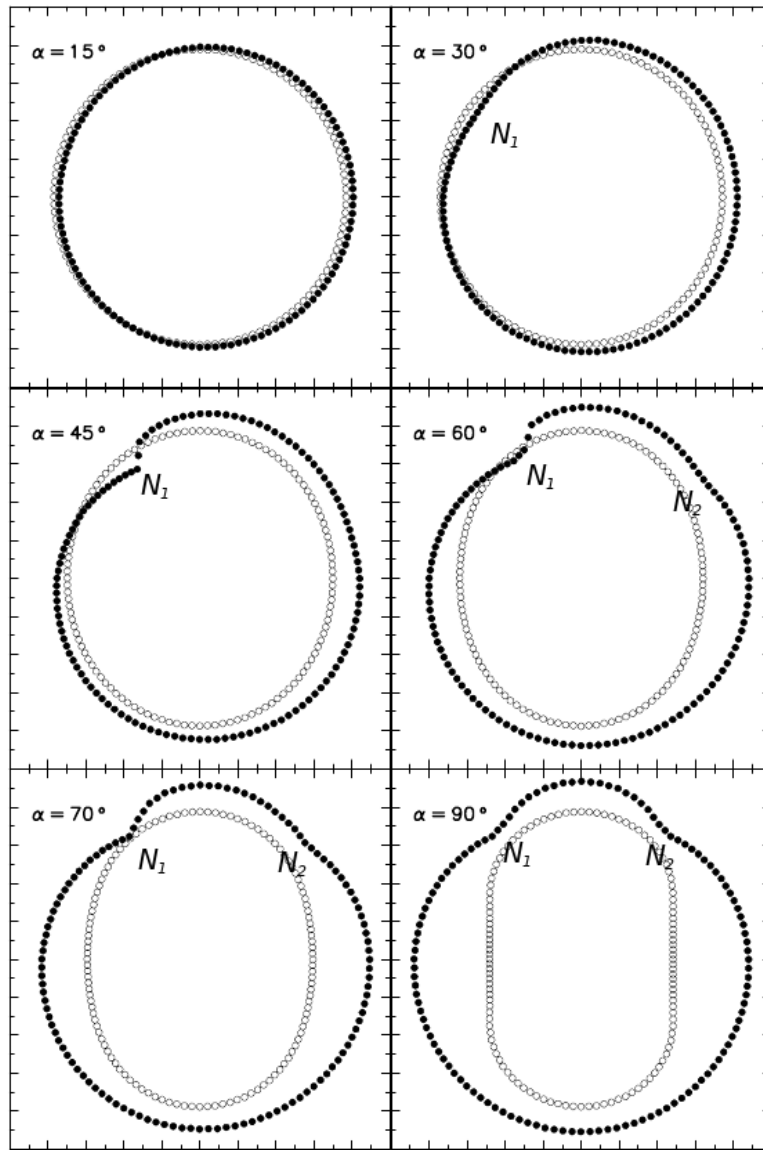


Figure 3.2: PC shapes for various inclination angles in both the static and rotating magnetosphere approximations. The white dots indicate the shape associated with the static case, while the black dots indicate the shape associated with the rotating case. The angular velocity vector Ω points toward the left in these figures (Cheng et al., 2000). The notch, indicated as N_1 , starts appearing at $\alpha = 30^\circ$ and is present throughout. Second, less pronounced, notch-like structure starts appearing at $\alpha = 60^\circ$.

3.2 The geometric γ -ray models

Figure 3.3 shows the location and extent of the magnetospheric emission regions assumed by the geometric TPC and OG models. In both models the gaps are located along the last open field line, and their associated emission regions are assumed to have constant emissivity in the corotating frame. It is further assumed that emission occurs tangentially to the local magnetic field direction due to the strong beaming associated with CR. The widths of the emission regions are taken as $0.05r_{\text{PC}}$ in Chapter 6, and an investigation into the effect of this choice is performed in Section 4.3.3. In the TPC model the emission regions are co-located with the acceleration gaps. The emission regions thus extend from $r_{\text{ovc}} = 0.95$ to $r_{\text{ovc}} = 1.00$, and from the stellar surface up to the light cylinder, as indicated by the dashed lines in Figure 3.3. In the OG model the emission regions

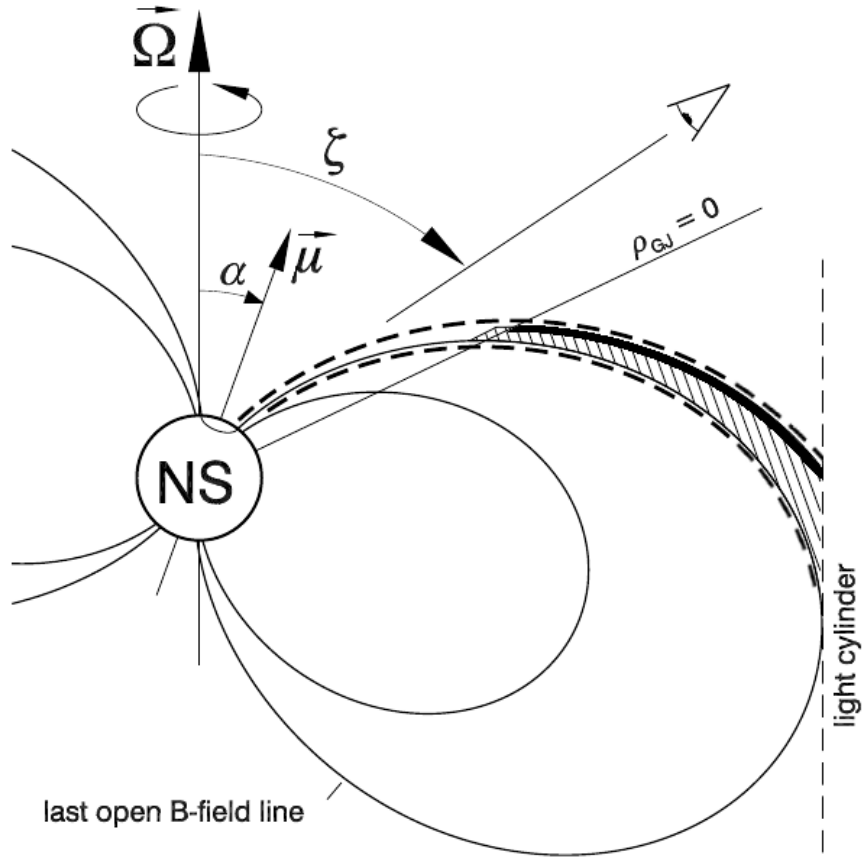


Figure 3.3: Illustration of the geometric TPC and OG models. The radiation region (between the thick dashed lines) is confined to the surface of the last closed field lines. It extends from the stellar surface all the way to the light cylinder. The shaded area shows the location of the outer gap acceleration region (for comparison), which extends from the null-charge surface to the light cylinder (Dyks & Rudak, 2003).

are located slightly inward of the acceleration gaps, and are chosen to extend from $r_{\text{ovc}} = 0.90$ to $r_{\text{ovc}} = 0.95$ and from the null-charge surface up to the light cylinder. The locations of the OG emission regions correspond to the location of the boundary layer as discussed in Section 2.5.5, and are indicated by the thick black band in Figure 3.3.

While the slot gap (SG) model discussed in Section 2.5 serves as a possible physical basis for the existence of the emission regions in the TPC model, the SG model does not represent the origin of the TPC model. The TPC model was proposed as a purely geometric model by Dyks & Rudak (2003) in an attempt to better account for some of the properties of the observed pulsar light curves. The model can be thought of as an extension of the geometric OG model, and has the advantage of producing two-peak profiles for the majority of geometric parameters (α and ζ , see Section 4.1), in contrast to the PC and OG models. Furthermore, both the PC and OG models have a preferred range of geometric parameters over which two-peak profiles are produced, with the PC model preferring nearly aligned rotators ($\alpha \sim 0^\circ$) and the OG model highly oblique rotators (Section 4.1.3). In the PC case this preference is due to the high duty cycle of most observed γ -ray pulsars.

3.3 The radio model

The empirical model used for the radio emission in this study is basically the model put forward by Story et al. (2007), but ignoring the core component of the radio emission beam. The model is based on the assumption that radio emission at a given frequency originates from a single-altitude annular region above the PC. The intensity of the emission originating from this region is modelled to have an offset Gaussian profile, with the part of the profile where the emission intensity is 0.1% of the peak emission lying on the last open magnetic field line. This region's height above the PC is

$$r_{\text{KG}} = 40\nu^{-0.26}\dot{P}^{0.07}P^{0.3}, \quad (3.5)$$

where r_{KG} is in units of R_{NS} , ν is the frequency of the emission in GHz, and $\dot{P}$ is the period derivative in units of $10^{-15} \text{ s.s}^{-1}$ (Kijak & Gil, 1998).

An important aspect of this empirical relation is the so-called radius-to-frequency mapping of the radio emission. This means that the higher the frequency of the radio emission, the lower the altitude at which it is emitted. This mapping is a consequence of the radiative losses the emitting particles suffer as they move along the magnetic field lines. Coupled with the dipole shape of the magnetic field, this implies that the opening angle of the emission cone will decrease as one goes to higher frequencies (for a specific pulsar). This is reflected in the expression for the opening angle of the beam, which is

$$\rho_{\text{cone}} = 1.24^\circ \sqrt{\frac{r_{\text{KG}}}{P}}. \quad (3.6)$$

A complete discussion of the effects of this model's parameters can be found in Section 4.4.

3.4 Pulsar rotation and the formation of caustics

A key difference between the geometric models for the radio and γ -ray emission is that in the former, emission originates from a single altitude, while in the latter the emission region extends across a range of altitudes. This becomes important when considering what effect the travel time of an emitted photon has on the predicted LC.

Let the starting point of the rotation (where $\phi = 0$) be when $\boldsymbol{\mu}$ is directed toward the observer. Now consider two photons emitted simultaneously (in the observer's reference frame) in the direction of the observer at $\phi = 0$, one at the surface of the NS, and one at some point closer to the observer. Due to the constant speed of light, the photon closest to the observer will reach the observer first, and will thus be observed to arrive earlier in phase than the photon emitted at the surface. This means that the closer the emission point of a photon is to the observer, the earlier in phase it will be observed. Conversely, this means that for a pair of photons emitted at different distances from the observer to arrive at the observer simultaneously, the farther one must be emitted earlier than the closer one. A simple analogy is to think of the two photons as two athletes racing toward a finish line. The starting points for the two athletes aren't the same distance from the finish line. For them to tie, the athlete farther from the finish line must be given a head start, and the closer athlete only allowed to start when his competitor passes his starting point (assuming, of course,

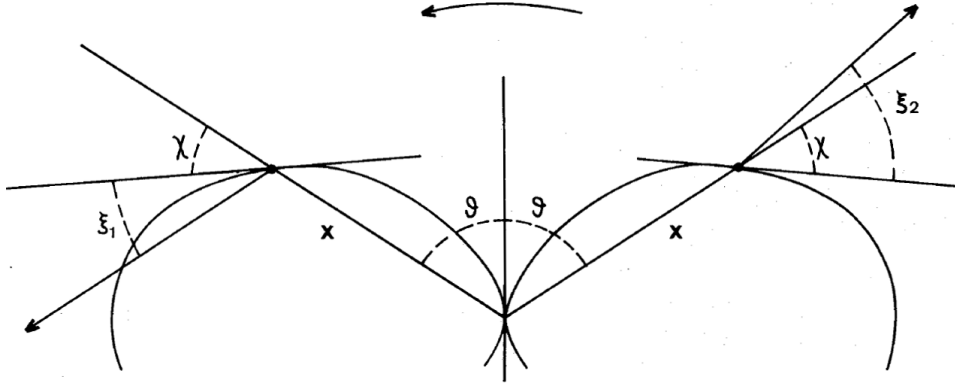


Figure 3.4: Emission field lines geometry for an orthogonal rotator. The curved arrow indicates the direction of rotation (Morini, 1983).

that the athletes instantaneously run at their top speeds, and that those top speeds are identical). Thus two photons emitted at different times and different locations in the magnetic field can be bunched in phase as seen by the observer. Mathematically this effect is summarised (for a single photon, and to first order in r/R_{LC}) as a deviation from its original phase of

$$\Delta\phi_{\text{tof}} \simeq -\frac{r}{R_{LC}}, \quad (3.7)$$

due to photon travel time (time-of-flight delays) (Dyks & Harding, 2004). The shift to an earlier phase is indicated by the minus sign.

Now consider how this effect interacts with the effect that the rotating magnetic field has on the emission directions of the photons. If the effect of photon travel time is neglected, the phase at which a particular photon would be observed is entirely determined by the geometry of the magnetic field. Figure 3.4 shows an orthogonal rotator with a simple dipole magnetic field structure rotating counter-clockwise. It also shows two points on the last open field lines, located symmetrically with respect to the magnetic axis. The point on the left is located on the leading field line, while the point on the right is located on the trailing field line. At both points the line tangent to the magnetic field is also indicated, making an angle χ with respect to the local radial direction. Consider first the trailing part of the magnetic field, and an observer located at $\theta = 0$ at $\phi = 0$. As the pulsar rotates, the altitude at which the tangent line is parallel to the observer direction, increases. This means that observer-directed emission from lower altitudes will be emitted earlier than observer-directed emission emitted at higher altitudes. The differences in their time-of-flight, however, will cause photons emitted along this trailing field line to be bunched together in phase as these delays have the opposite effect on the photon arrival times. Thus the difference in the travel times of the two photons cancels out some portion of the difference in phase between the times they were emitted due to the magnetic field geometry. For the leading field line the opposite effect is seen, i.e., that the emission emitted along it is more spread out in phase. This is due to the fact that the altitude at which observer-directed emission occurs decreases as the pulsar rotates. In the case of the leading field line, then, the difference in the travel time (and hence observed phase) of the two photons adds to the difference in emission phase of the photons.

Returning to the analogy of the two athletes, how the two effects interact can be easily visualised. This time not only two athletes are competing in the race, but an entire field of athletes. As the

leading and trailing field line regions are symmetric, a single configuration (corresponding to one half of Figure 3.4) for the athletes' starting positions will suffice, and only their starting order will be changed. In both cases the athletes' distances from the finish line represent the distances of emitted photons from the observer. Thus, the athlete starting at the starting line (farthest from the finish line) corresponds to a photon emitted at $r = 0$, while the athlete corresponding to the two points shown in Figure 3.4 would be positioned about halfway between the starting line and the athlete closest to the finish line.

For the leading field line, the athlete closest to the finish line starts the race first, followed by the second closest athlete, and so forth. The closest athlete thus had both an initial lead over his competitors, and a head start, and the difference between his arrival time and those of his competitors will be greater than the difference in their starting times. For the trailing edge the opposite effect is seen. The athlete farthest from the finish line starts first and has time to make up some of the lead that the athletes ahead of him have at the start of the race. In other words, the difference in arrival times would, in this case, be less than the difference in starting times. The more closely matched the differences in the starting positions of the athletes and the distances they are able to travel in the extra time they have are (their relative head-starts), the more tightly bunched the field of runners will arrive at the finish line. Thus, the rotation of the pulsar turns the trailing last open field line into a caustic surface, the effect of which can be seen as a bunching of large portions of the emission originating from along it in phase, or as peaks in the observed LCs.

This is, however, not the complete picture, although it gives an excellent understanding of how caustics form. The element neglected here is the effect the corotation of the emitting particles has on their emission directions in the observer's frame. While it is true that in the corotating frame charged particles basically move along the magnetic field lines, their direction of motion in the observer's reference frame has an added component due to their corotation velocity. This means that their direction of motion will be aberrated from what it is in the corotating frame by an amount proportional to their corotation velocity. At lower altitudes this effect isn't very significant, as the corotation velocity isn't close to the speed of light, but at higher altitudes this effect becomes quite important. Mathematically the shift in phase due to the effect of aberration (or an instantaneous Lorentz transformation), to first order in r/R_{LC} , is (Dyks & Harding, 2004)

$$\Delta\phi_{\text{ab}} \simeq -\frac{r}{R_{\text{LC}}}. \quad (3.8)$$

To summarise then, the phase at which a photon is observed is determined by the geometry of the magnetic field (tangentially to which the emitting particles are moving in the corotating frame), the magnitude by which the emission direction of the photon is aberrated (due to the corotation of the particles in the pulsar magnetosphere), and by the distance the particle is from the observer, compared to that of the star itself. For the leading field line, these three effects interact in such a way as to smear out the emission over a larger portion of phase than would have been the case had aberration and time-of-flight not been considered, while for the trailing field line these effects interact to bunch the emission originating from it in phase.

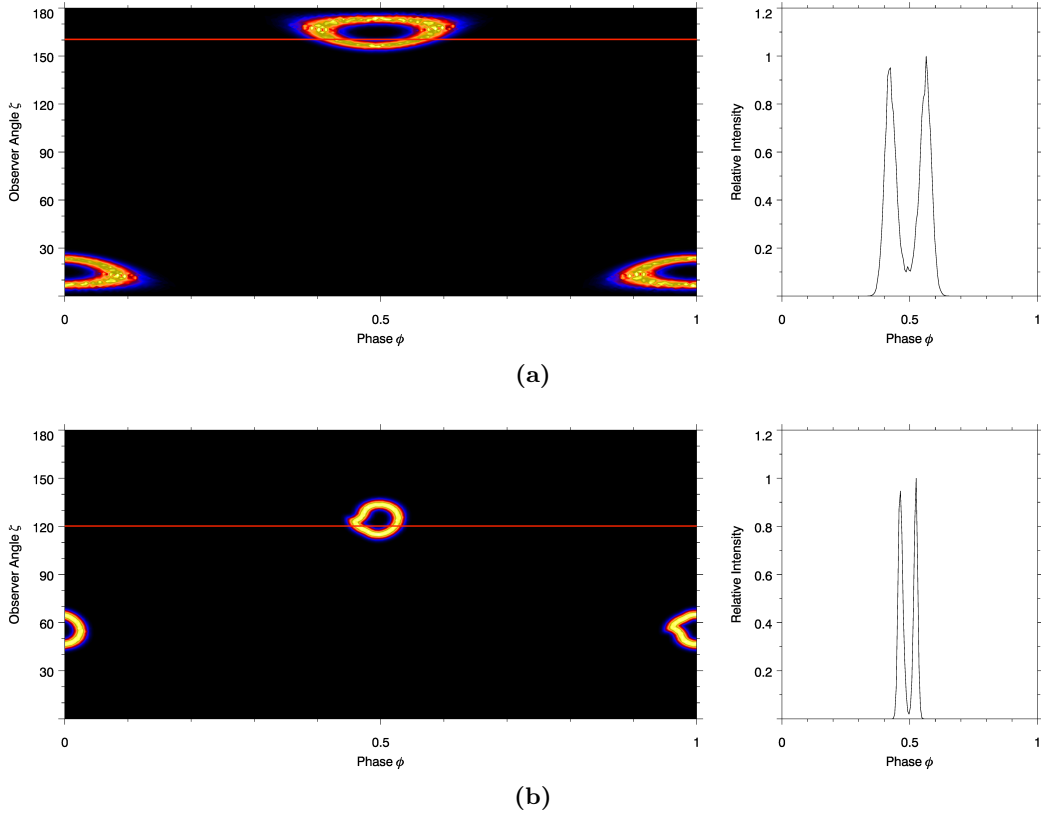


Figure 3.5: Constant ζ cuts through two sample radio phaseplots, with (a) $\alpha = 15^\circ$ and (b) $\alpha = 55^\circ$. The cuts are made at $\zeta = 160^\circ$ and $\zeta = 120^\circ$ for (a) and (b) respectively. In the interest of clarity, the cuts are made at $\zeta > 90^\circ$ as the peaks in the profiles then appear at the centre of the LC figure. The cuts illustrated here are equivalent to the ones at $180^\circ - \zeta$ under the symmetry discussed in the text.

3.5 Obtaining model predictions

With the geometric models now fully introduced it is useful to briefly discuss how predictions can be obtained from them. This discussion also serves as an introduction to the visual tools used throughout this study. A discussion of how these models are implemented is outside the scope of this work. See Dyks et al. (2004b) and Dyks & Harding (2004) for more details.

3.5.1 Phaseplots and LCs

Geometrically, a single LC corresponds to a single set of geometric parameters α and ζ . For a specific pulsar, however, with its fixed inclination angle, the shape of the LC observed is solely determined by the position of the observer. To most naturally represent the radiation a single simulated pulsar emits, a so-called phaseplot is drawn. Simply put, a phaseplot contains the entire set of LCs that can possibly be seen by an observer for a single simulated pulsar system of constant α . A phaseplot is an equirectangular projection of the emission intensity per solid angle of the pulsar's emission. Figure 3.5 shows example radio phaseplots for a pulsar with an inclination angle of $\alpha = 15^\circ$ and $\alpha = 55^\circ$ for panels (a) and (b) respectively, as well as predicted LCs produced through cuts at constant ζ . For the purposes of obtaining predictions for the shape of an LC and fitting it to that of an observed LC, it is only necessary to know what the relative intensity of the emission depicted in the LC is. Therefore, the LCs used in this study are normalised such that

the peak intensity is at unity. In Figure 3.5 (b) the emission geometry described in Section 3.3 is seen to result in two rings on the phaseplot, one centred on the point $[\phi = 0, \zeta = 55^\circ]$, and one centred at the point $[0.5, 130^\circ]$. These two points correspond to the locations of the two PCs on the phaseplots. In Figure 3.5 (a) the distortions due to the projection process can be clearly seen as the rings which would be associated with the annular emission regions are stretched in the ϕ direction. On the corresponding LC this results in a broadening of the predicted peaks.

An important property of the phaseplots is that they remain unchanged under the operation of vertical reflection, and shift in phase by 0.5. This symmetry is due to the symmetry between the two hemispheres of the pulsar assumed in the geometric models. The most significant implication of this symmetry is that the top half of the phaseplot can be obtained from the bottom half by mapping $[\phi, \zeta]$ onto $[\{\phi + 0.5\}, 180^\circ - \zeta]$, where $\{\phi + 0.5\}$ is the fractional part of the shifted phase. As $\phi = 0$ is chosen to correspond to the phase at which the magnetic axis is pointing toward the observer (Weltevrede et al., 2010), consideration of only the bottom half of the phaseplots will be sufficient when attempting to obtain LC fits.

3.5.2 The flux correction factor f_Ω

Another useful quantity which can be obtained by evaluating the produced phaseplots is the flux correction factor f_Ω , which plays an important role when estimating the true γ -ray all-sky luminosity L_γ of a pulsar based on the observed phase-averaged flux G_{obs} . Watters et al. (2009) estimated L_γ as

$$L_\gamma = 4\pi d^2 f_\Omega G_{\text{obs}}, \quad (3.9)$$

where d is the distance to the pulsar, and

$$f_\Omega = \frac{\iint F_\gamma(\alpha, \zeta, \phi) \sin(\zeta) d\zeta d\phi}{2 \int F_\gamma(\alpha, \zeta_E, \phi) d\phi}, \quad (3.10)$$

with F_γ being the photon flux per solid angle, and ζ_E the Earth line-of-sight.