Dynamic Modelling of the Natural Convection Water Cooling Principle

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by

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Executive summary

The aim of this project is to investigate the operating characteristics of the Reactor Cavity Cooling System (RCCS) of the Pebble Bed Modular Reactor (PBMR) demonstration power plant during the active and passive operation conditions. Fortunately, the RCCS experience no circulation anomalies during active operation conditions, due to the pumps circulating the water. However, in the event of the passive operating conditions, the pumps are not functioning, and the system is dependant on natural convection to circulate the water within the system. Some circulation anomalies develop during the passive operation conditions and the system tends to overheat.

During the passive operation conditions of the RCCS, the heat generated by the Reactor Pressure Vessel (RPV) is transferred to the standpipes of the system and natural convection is set into action. As the temperature in the system rises, the natural convection speeds up. Unfortunately, when the temperature surpasses a certain temperature the system tend to stop circulating properly and the water in the system start to boil, thus resulting in the system overheating.

The study aims to gain a better understanding of the dynamics of the RCCS by using dynamic modelling methods. A previous study on the RCCS has been done and a Flownex® simulation model of the system does exist. The simulation provides a representation of the hydraulic behavioural characteristics of the system during passive operating conditions. By developing a dynamic electrical circuit model of the existing RCCS model, it will be possible to study the natural convection phenomenon in another energy domain. The hydraulic simulation performed by means of Flownex will serve as reference for the results obtained from modelling the system by means of the developed equivalent electrical model. By manipulating the analogue circuit components analytically and by utilising fundamental circuit laws which includes Kirchoff's Law and Ohm's Law, state-space equations will be derived.

The research can be used to refine the design of the RCCS, thus ensuring circulation without anomalies, during passive operation conditions. This will enhance the safety factor of the PBMR nuclear plant dramatically. The state-space equations can be utilised to determine the poles of the system. The pole positions convey valuable information regarding the stability of the system elements.

There is still a lot of suspicion concerning the safety aspects of nuclear energy. The PBMR technology addresses and eliminates most of these suspicions and with the energy crisis the world is currently experiencing, nuclear energy is steadily becoming a more viable and much needed resource option.

Opsomming

Die doel van die projek is om die operasionele kondisies van die "Reactor Cavity Cooling System" (RCCS) van die Pebble Bed Modular Reactor (PBMR) demonstrasie werke te ondersoek gedurende die aktiewe en passiewe operationele kondisies. Gedurende normale operasionele kondisies ondervind die RCCS geen sirkulasie onreëlmatighede as gevolg van die hidroliese pompe wat die water deur die pypstelsel laat sirkuleer nie. Maar in die geval van die passiewe operasionele kondisies, funsioneer die pompe nie meer nie, en die stelsel is aangewese op die natuurlike konveksie verskynsel om die water te sirkuleer deur die pypstelsel. Gedurende die operasionele kondisies ervaar die pypstelsel dan sirkulasie onreëlmatighede, wat dan lei tot oorverhitting van die stelsel en gevolglik, gebarste pype.

Tydens passiewe operasionele kondisies van die RCCS, begin die water sirkuleer as gevolg van die hitte wat vanaf die reaktor druk silinder na die pypstelsel oorgedra word. Soos die temperatuur in die pypstelsel styg, vloei die water vinniger as gevolg van die verhoogde natuurlike konveksie verskynsel. Sodra die temperatuur in die pypstelsel 'n sekere punt oorskry, neig die stelsel om die voldoende sirkulasie te staak en die water in die stelsel begin kook, wat dan lei tot die stelsel wat oorvehit.

Die studie se doel is dus om 'n beter begrip van die dinamiese elemente van die RCCS te verkry deur gebruik te maak van dinamiese modellerings metodes. 'n Vorige studie is alreeds op die RCCS gedoen en 'n Flownex sagetware model van die pypstelsel bestaan, wat die dinamiese gedrag van die stelsel kan modelleer. Deur 'n dinamiese ekwivalente elektriese stroombaan model van die RCCS pypstelsel op te stel, kan die natuurlike konveksie verskynsel in 'n ander energie domein ondersoek word. Die hidroliese simulasie wat in Flownex gedoen is kan gebruik word as verwysings raamwerk vir die resultate wat verky word uit die modellering van die ekwivalenete elektirese stroombaan. Deur middel van analitiese manipulasie van die ekwivalente stroombaan komponente en deur gebruik te maak van die fundamentele elektriese wette soos dié van Kiirschoff en Ohm, kan die toestandsveranderlike vergelykings onttrek word.

Die studie se resultate kan dan gebruik word om die fisiese ontwerp van die RCCS te verfyn, en dus so te verseker dat die sirkulasie onreëlmatighede gedurende die passiewe operasionele kondisies tot die minimum beperk kan word. Dit sal die veiligheidsfaktor van die PBMR aansienlik verbeter.

Daar is nog steeds onsekerheid ten opsigte van die veiligheidsfaktore van kern energie. Die PBMR tegnologie adresseer en elimineer meeste van die onsekerhede en met die energie krisis voor die deur, kan kern energie 'n meer ekonomiese en betroubare bron van energie wees.

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I firstly would like to thank my Creator for granting me this opportunity and capability to further my knowledge and improve my capabilities.

Then, critical to my success, my wife, Marlize v.d Westhuizen whom I want to thank for her unconditional love and support during the good times and her supporting advice during the bad times. This stood as foundation for my successes in life.

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List of abbreviations and symbols

Abbreviations

$RCCS$	Reactor Cavity Cooling System
$PBMR$	Pebble Bed Modular Reactor
RPV	Reactor Pressure Vessel
DC	Direct Current

Symbols

m	The mass of the substance (kg)
g_C	The gravitational force $(9.81\ m/s^2)$
F	Force exerted (N)
A	The cross-sectional area of pipe (m^3)
d_i	The inside diameter (m^2)
$l_{segment}$	The length of the pipe increment (m)
f	The friction coefficient of the substance
L	The length of segment under consideration (m).
$\sum K$	The inside roughness of a pipe
D	The hydraulic diameter (m^2)
$\dot{m}$	The mass flow rate experienced in a pipe (kg/s).
A_{ff}	The free flow area (m^3)
ρ	The density of the substance density which is a function of the temperature (kg/m^3)
μ	The dynamic viscosity $(kg/m-s)$
Δp_{ol}	The total pressure drop due to frictional and other losses per segment length and per unit volume in (kPA)
$\rho \forall g$	The force working in on the pipe segment due to the gravitational force on the substance (N)
$\forall = A(\Delta z)$	The volume of the substance in that specific pipe segment (m^3/kg)
$\Delta z = z_{in} - z_{out}$	The height difference between the entrance and exit areas of the pipe segment (m)
$\bar{V}$	The velocity of the substance through the pipe segment (m/s)

PS	The pipe segment which is the control volume
SS	The control surface applicable to the control volume or pipe segment
f_1, f_2, f_3	The flow component in each bond
e_1, e_2, e_3	The effort components in each bond
w_i	The vector of integral causal storage field output variable
x_i	The vector of integral causal storage field input variable
z_i	The capacitor element matrix
r	The vector of dissipator field output variable
s	The vector of dissipator field input variable
D_R	The resistor element matrix
v_i	The source input
u_o	The source output
g_o, g_i	The output and input vector for the junction structure
A , B	The matrixes for the State-Space Equation
p_{0i} , p_{0e}	The pressure at the inlet and pressure at the outlet respectively
$p_{oe} = \rho g_C h_{oe}$, $p_{oi} = \rho g_C h_{oi}$	The pressure values at the exit and inlet of the pipe, w.r.t. height (kPA)
β	The volumetric coefficient of the thermal expansion of the substance
$\mathbf{x}$	State vector $\left[x_1 \cdots x_n\right]^T$
$\dot{X} = \frac{\partial x}{\partial t}$	Derivative of the state vector $\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{bmatrix}$
$\mathbf{u}$	Input vector $\left[u_1 \cdots u_n\right]^T$
t	The time in seconds
$\lvert e(t) \rvert$	The error between the simulations per time step
T_{oe} , T_{oi}	The outlet and inlet temperatures, respectively in (^oC).
T_O	The total temperature where $T_o = h\ (entalhpy) / c_p\ (heat\ capacity\ factor\ of\ water)\ (^oC)$
M_p	The highest value of the overshoot
fv	The final steady state value
P.O.	The calculated percentage overshoot
n	The number of dominant poles.
h_A	The integrated through-variable of the A-type element
v_A	The across-variable of the A-type element

C	The generalised capacitance of the A-type storage element
f_T	The through-variable of the T-type element
x_T	the integrated across-variables of the T-type element
L_i	The generalised inductance
f_D	the through-variable of the D-type element
v_D	the integrated across-variables of the D-type element
i	Generalized electrical current
R	Generalized electrical resistance

Chapter 1

1. Natural convection

1.1 BACKGROUND

The Reactor Cavity Cooling System (RCCS) is a heat removal pipe network system developed to remove excessive heat, radiated from the Reactor Pressure Vessel (RPV) wall of the Pebble Bed Modular Reactor. The pipe network encloses the RPV of the Pebble Bed Modular Reactor (PBMR) as illustrated in figure 1.1. The aim of the RCCS is to ensure that the concrete structure, encasing the PBMR, does not exceed an external temperature of 65°C. Under normal operating conditions pumps assist the water flow through the pipes, and passive operating conditions represent an event where the electrical power to the pumps is down. The aim is to develop an equivalent electrical analogy of the RCCS and to perform dynamic analyses on this equivalent developed electrical circuit. Figure 1.1 below illustrates the positioning of the various components within the reactor [2].

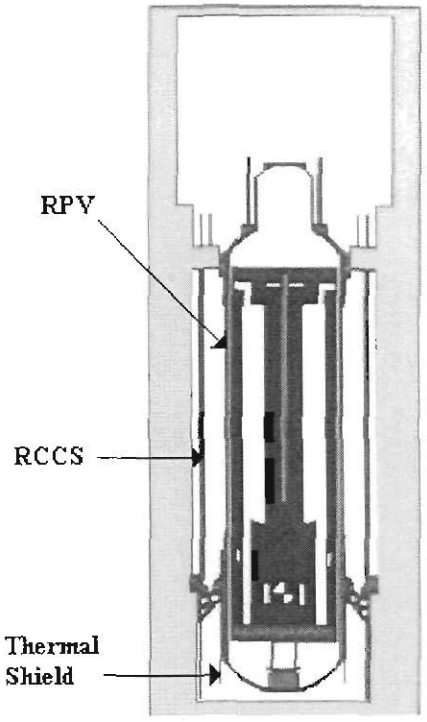


Figure 1.1: Schematic illustration of the positioning of the RCCS w.r.t. RPV [2]

To understand the significance of the RCCS to the PBMR, the following section discusses the PBMR research development and the role it will play in the race to cleaner and more sustainable energy. Thereafter the RCCS is discussed in detail with regards to its operating functionality during passive operating conditions.

1.2 THE PBMR

Due to the hazards and concerns normally associated with nuclear energy the technological development in the field of nuclear energy stagnated in the past few decades. What was thought to be a promising technology during the 1960's, is frowned upon today, despite the fact that nuclear power currently provide 17% of the world's energy needs. Accidents such as the incident at Three Mile Island (TMI) and Chernobyl resulted in circumstances where many preferred to rather give up the technology than to research nuclear technology further, to prevent mistakes like this from happening again.

Global warming and the fact that the world's fossil resources maybe on its way to depletion, forced the world to reconsider the benefits of nuclear power. Currently, the PBMR is being developed with the aim for commercial use by an international conglomerate that constitutes South African based Eskom, U.S. based Exelon Corporation and British Nuclear Fuels Limited [4].

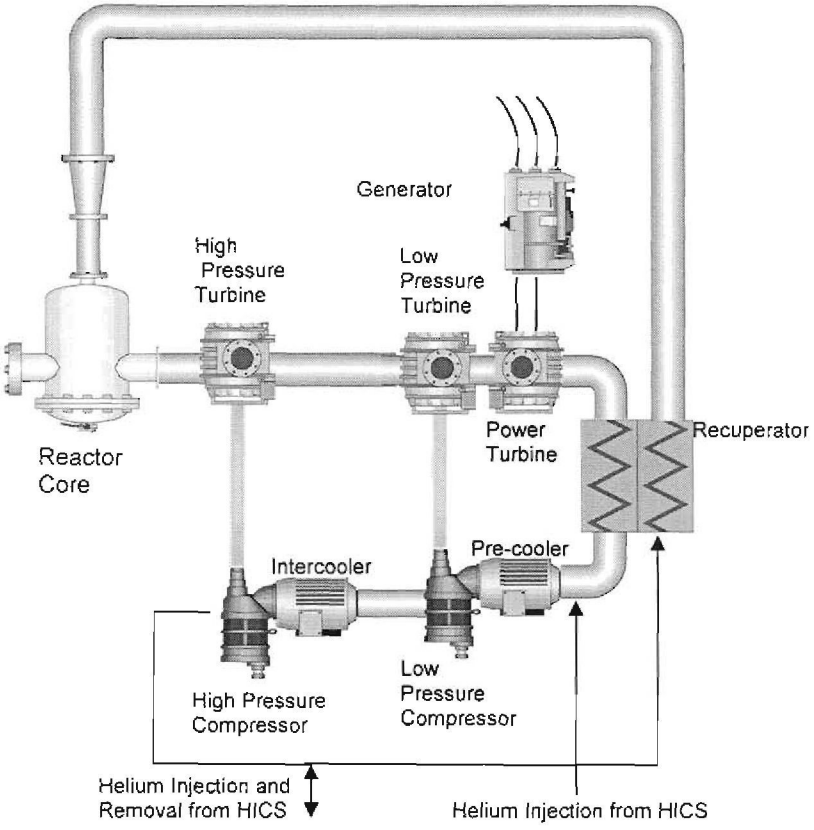


Figure 1.2: Simplistic layout of the PBMR [4]

The main objective of research in this area is to produce a nuclear plant that produces very little radioactive waste, that has a very little or no radiation hazard at the plant site boundaries and a plant that can be erected in the shortest possible time. Smaller reactor plants allow for better manufacturability and better safety manageability features.

The PMBR produces approximately 110 MW of power and can operate as a standalone unit or as part of a cluster unit, consisting out of two to ten units. Figure 1.2 displays the schematic diagram of the PBMR and figure 1.3 illustrates a cross section of the PBMR [4].

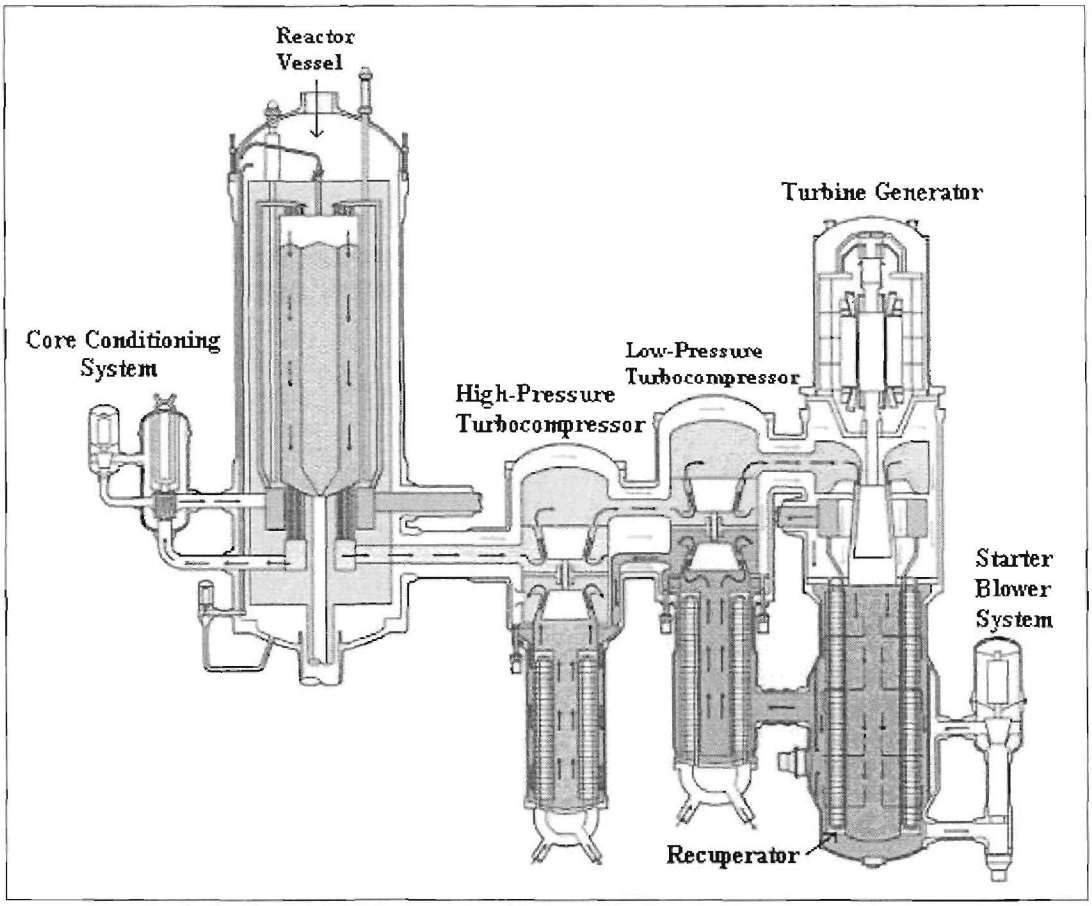


Figure 1.3: Cross Section of the PBMR [4]

The system uses a closed loop helium gas cycle to transfer the heat from the nuclear fusion elements to the power turbine this prevents any nuclear contamination to the environment. At the working temperature of the PBMR, the reactor vessel requires continuous cooling [4]. Through the use of a network of pipes which encloses the RPV of the PBMR, the RPV is cooled down. These pipe networks are known as the RCCS. The main purpose of the RCCS is to protect the building which houses the PBMR and to assist in dissipating radiated heat from the RPV. The following section discusses the RCCS in detail and also highlights the collapse of the natural convection phenomenon under certain conditions in the RCCS during passive operating conditions [2].

1.3 THE REACTOR CAVITY COOLING SYSTEM (RCCS)

The RCCS system consist out of a network of water circulating pipes that, by use of forced convection, remove the heat radiated by the reactor pressure vessel. The RCCS contains eighteen sub-systems, each consisting out of a feeder tank, cold feeding pipe from the heat exchanger pumps, inlet and outlet manifold, orifice and four standpipes as illustrated below in figure 1.4 [2].

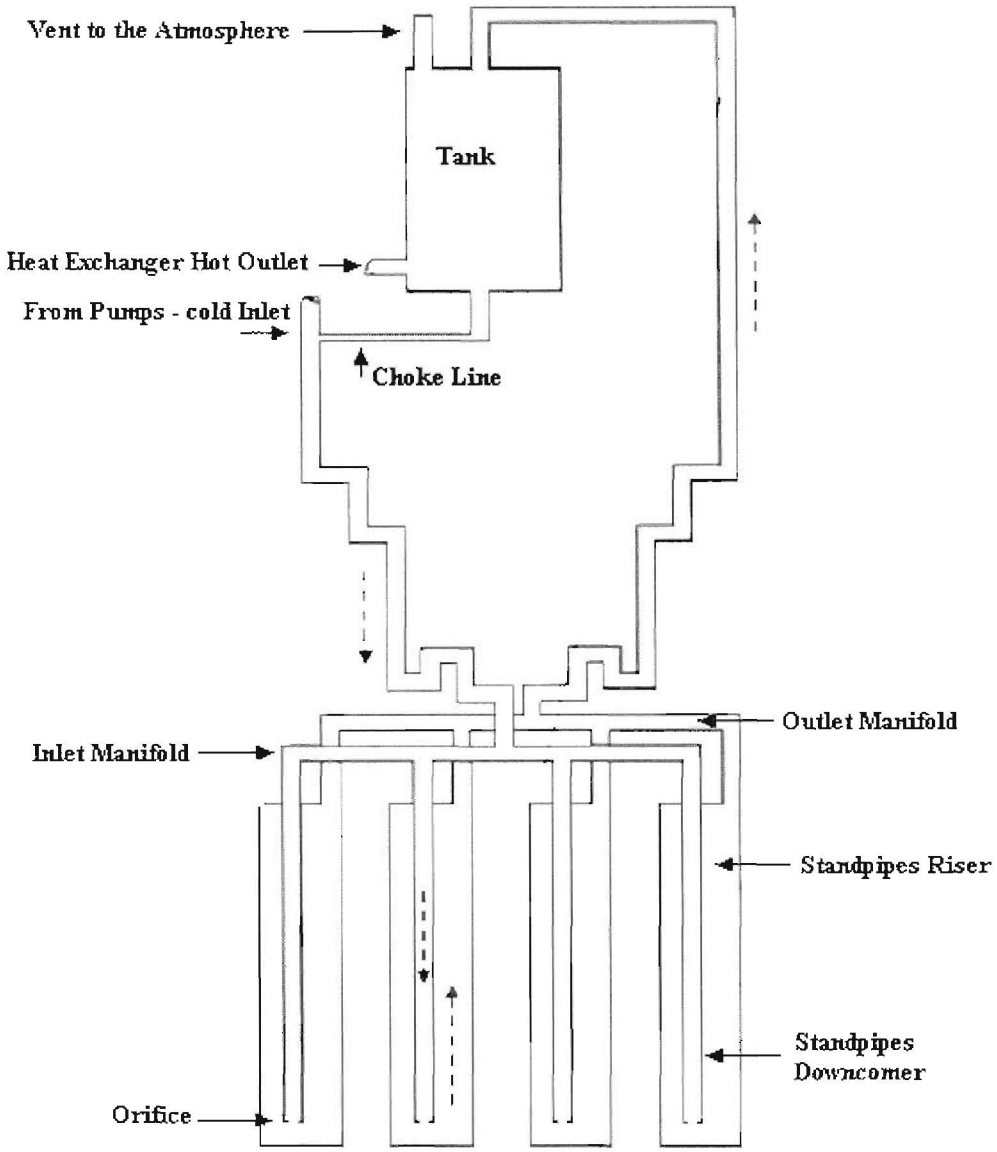


Figure 1.4: RCCS sub-system [2]

The flow through the pipes is forced through the pipe networks by means of pumps. The flow convention under normal conditions is illustrated in figure 1.5. The heat is then absorbed by the circulating water, which in turn disperses the heat through the external heat exchanger, situated in the sea. The sea serves as natural cooling agent for the heat exchangers. Under normal operating conditions the radiated heat is transferred from the RPV to the four stand pipes of the pipe network as illustrated in figure 1.6 [2].

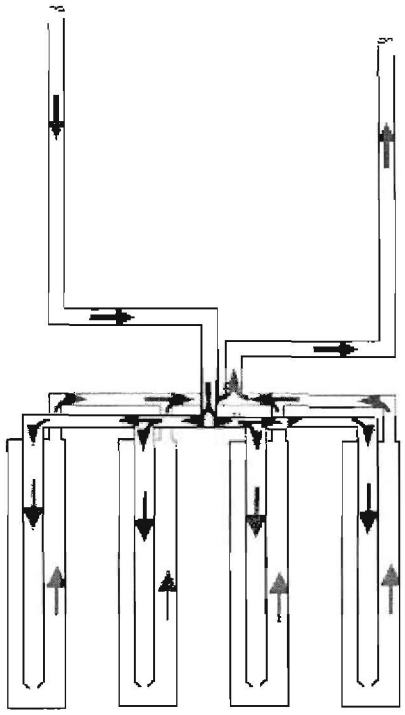


Figure 1.5: Flow convention under normal operating conditions [2]

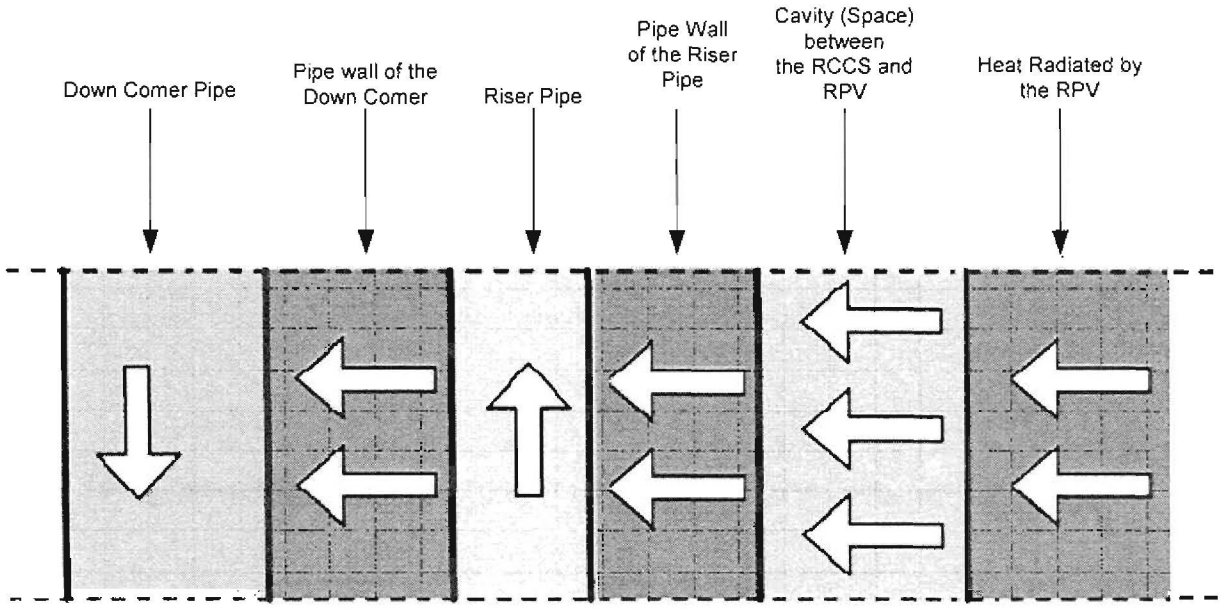


Figure 1.6: Illustration of the heat transfer path from the RPV to the water in the RCCS [2]

The heat is radiated from inside the RPV due to the nuclear reaction within the vessel. The heat is then transferred from the RPV to the cavity between the standpipes of the RCCS and the RPV. It is assumed that no fluid flow takes place inside the cavity that can contribute to the transfer of the radiated heat [2].

Heat is conducted through the outer wall of the riser pipe into the water inside the riser pipe. The heat is dissipated by means of the forced convection from the down comer pipe to the riser pipe and outwards to the tank and heat exchangers. The riser pipe absorbs the biggest portion of the radiated heat and has the biggest diameter. Only a small amount of heat is transferred to the wall of the down comer standpipe. The riser pipes are situated in such a way, that it shields the concrete structure that houses the RPV. Figure 1.7 illustrates this [2].

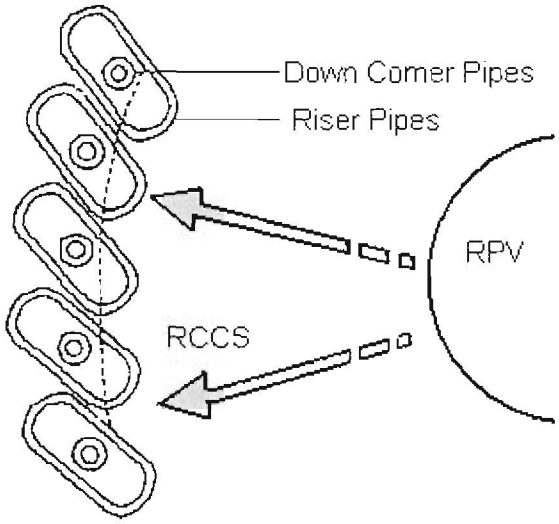


Figure 1.7: RCCS Standpipe Alignment [2].

1.4 PROBLEM STATEMENT

Under normal operating conditions the water within the RCCS is forced to flow through the system by means of the feeder pumps. During these conditions the radiated heat from the RPV is successfully dissipated by the system. Because of the high risk functionality the RCCS has within the PBMR, the design engineers took the lingering energy crisis into consideration, and designed the RCCS with certain contingencies. The purpose of the contingencies is to ensure that the water still circulates through the system in the event where the power supply to the feeder pumps falls away. This condition is known as the passive operating condition. According to the designers the RCCS was designed in such a way that the water would still circulate through the system during the passive operating conditions, due to natural convection. However, after performing a Flownex® flow analysis on the RCCS it was found that the natural convection flow collapses at a certain temperature. According to the software analysis the water stops circulating properly within the system, resulting in no heat being dissipated. The software simulation results show that the temperature of the water rises exponentially due to the collapse of the flow within the RCCS. The water inside the system starts to boil, resulting in the system overheating [3].

This is very dangerous to Nuclear Reactors because if the heat is not dissipated from the RPV, it will lead to a nuclear plant melt down. The aim of this study is to model the fundamental dynamics of the RCCS by means of an electrical circuit model. The resulting dynamic behaviour due to the natural convection phenomenon will be the main focus. To achieve this, it is required to understand the natural convection phenomenon and understand how the phenomenon can be expressed by means of drawing an analogue between the hydraulic domain and the electrical domain. The study will focus on the modelling of the fluid elements by means of electrical components and will use the RCCS as case study.

1.5 ISSUES TO BE ADDRESSED AND METHODOLOGY

1.5.1 Developing the reduced equivalent electrical circuit

It is possible to derive electrical equivalents for flow resistance, momentum and mass storage by using lumped elements such as resistors, inductors and capacitors. By incorporating these principals, an equivalent RLC electronic circuit model can be developed. The circuit gives structural information which together with fundamental laws such as Kirchhoff's law, can be used to construct mathematical models in state-space form.

To ensure analogous representation of the mechanical system, the electrical and analytical model must incorporate effects such as the resistance of the pipes, the temperature factor and the layout of the system. It is also important to simplify the system as far as possible without overlooking the fundamental characteristics of the system. During this exercise the limitations and assumptions of the intended model is also identified. The developed analogous electrical circuit model will be transformed into a state-space model by means of the network analysis method, for simulation purposes. The results of the state-space model simulation, by means of MATLAB®, will be compared with the performed Flownex® simulation.

1.5.2 The modelling approach

The first step in modelling the RCCS is to develop a detailed model specification, addressing all the relevant components of the system. System parameters which influence the heat transfer, the flow convection and the flow elements will be specified and used as a point of departure for the system modelling. The first step will be to analyse and model a single U-tube of the RCCS, which represent the pipe segment between the inlet manifold and the outlet manifold, and from here the model is extended to the final double U-tube system. The principle applied during the modelling phase is to keep the model as simple and representative as possible.

1.5.3 The Model analysis process

After completing the equivalent model, the developed model should be analysed to ensure the validity of the derived model. The network analysis method is used to develop an equivalent network graph network, to analyse the developed graph and to derive the state-space equations from the developed network graph. The network analysis method is discussed in detail in chapter 4.

1.5.4 Model evaluation

After developing the equivalent electrical model and deriving the state-space equations the model needs to be validated. The state-space equations will be solved and the results will be compared with the Flownex simulation of the reduced RCCS pipe network. Flownex is used as the benchmark because of the intensive validation processes it went trough. Conclusions regarding the value of the state-space model will be stated.

1.6 OVERVIEW OF DISSERTATION

In chapter 2 the dissertation discusses the natural convection phenomenon, to provide comprehension of the fundamental forces behind the manifestation. It aims to provide a understanding of the natural convection phenomenon by providing a mathematical model derived from the fundamentals of hydraulic engineering concepts.

Chapter 3 discusses the bond analysis method of analysing a system, which can either be a mechanical, hydraulic or electrical system. The chapter also illustrate the method to be followed when extracting the state-space model and provides an example of a simplified electrical circuit being analysed.

Chapter 4 discusses the network analysis method of analysing electrical and hydraulic systems. It explains in depth the hydraulic and electrical analogues between the energy domains, and illustrates how to generate a representative network graph for the equivalent electrical circuit. It also illustrates how to generate the state-space model for the developed network graph. At the end of chapter 4 the study provides the rational why the network graph was more appropriate for this study's purposes.

The focus of chapter 5 is on the development of the analogous electrical circuit, the simulations performed in Flownex®, for the hydraulic system, and the simulations performed in MATLAB® on the developed equivalent electrical circuit and the discussion of the simulation results.

Chapter 6 of the dissertation provides the concluding remarks and suggestions for future work.

1.7 CONCLUSION

The aim of this project is to develop an equivalent electrical circuit model of the RCCS pipe system and to extract a state-space model from a network graph representing the electrical circuit. The derived state-space model will then be used in a MATLAB® simulation for the RCCS system. The validity of the state-space model will be verified by means of comparing the MATLAB® and Flownex® simulations. All these issues will be considered in detail in the following chapters.

Chapter 2

2. Natural convection

2.1. BACKGROUND

In recent years a lot of attention has been given to the phenomenon known as natural convection. This is due to the fact, that the thermal performance of fluids and gasses within engineering applications, such as boilers, nuclear systems, energy storage and conservation, chemical, food and metallurgical industries are all affected by this phenomenon. For the purpose of this discussion a U-tube is used as model. The discussion looks at the forces within the U-tube under steady state conditions. The rationale behind this, is to simplify the discussion and to achieve the aim of illustrating which forces are the major role players in causing the natural convection phenomenon. It is the aim of this chapter, to just illustrate to the reader that the phenomena known as natural convection does exist and which hydraulic components need to be taken into account, during the development of the equivalent electrical circuit. The simulation of the U-tube in the forth coming modeling chapter, takes more non-linear conditions into consideration. When heat is added to a liquid or gas, the particles within the substance expands, resulting in a change of density. In the presence of gravity, the change in density within the substance results in the change of body forces. This phenomenon can be illustrated by means of a U-tube as illustrated by figure 2.1.

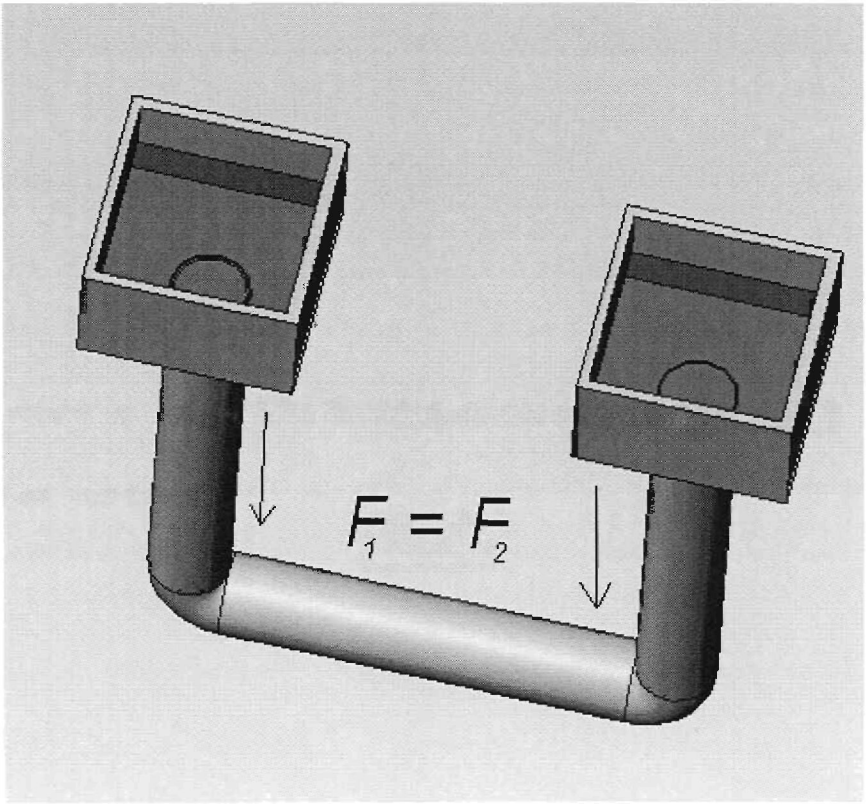


Figure 2.1: U-Tube with water tanks on both sides

The U-Tube as illustrated in figure 2.1 is in a steady state. No flow is experienced, due to the weight of water in both water tanks at the top of the U-tube. If one looks at a pipe segment of the tube on each side as illustrated in figure 2.2, the density of the substance under normal condition is the same for both sides. This means that the mass of the substance in each pipe segment is equal. The gravitational force is also equal in both sides and this ensures that the same downward force is experienced in both sides, therefore no flow is experienced.

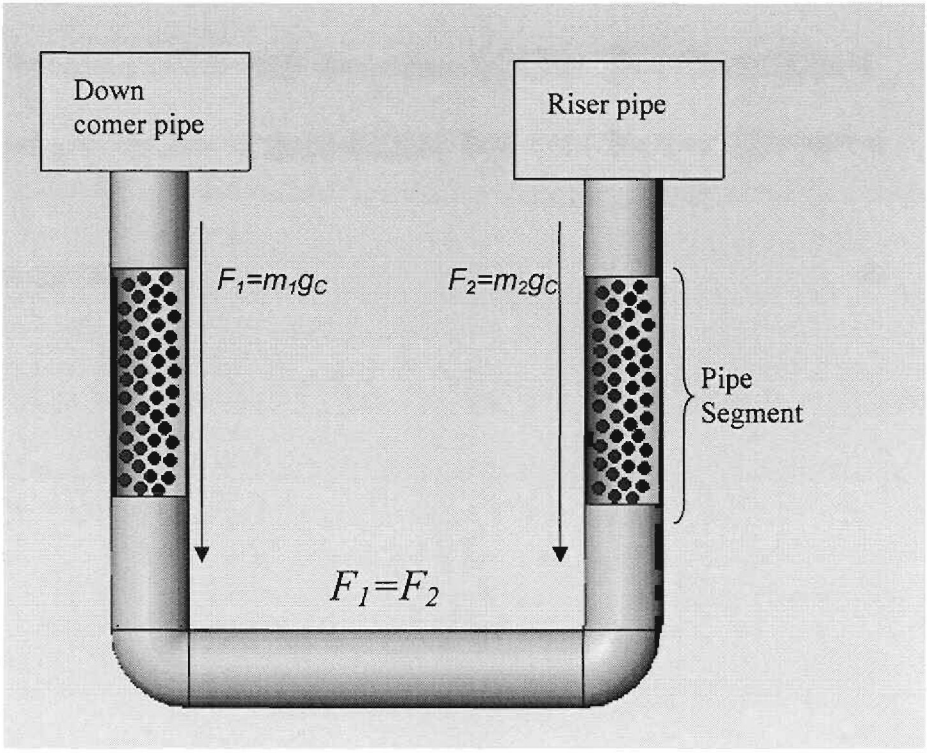


Figure 2.2: Illustration of the density of the substance in a segment of the pipe on both sides.

The mass of the substance is represented by m and the gravitational force is represented by g_C, which is the gravitational constant with a value of 9.81 m/s^2 . By assuming properties such as pipe resistance and viscosity to be constant, the downward force in each pipe can simply be written as [1]:

$$F_{segment} = m_{water} \times g_C \tag{2.1}$$

The mass of the pipe segment is dependant on the density of the substance and the density is dependant in turn, on the temperature of the substance. Higher temperatures will cause the density of the substance to be smaller. But in steady state, density of the substance is equal, therefore it is possible to state that the force in each segment in the riser (2) and the down comer (1) is the equal [1,5,8]:

$$F_1 = F_2 \tag{2.2}$$

It is possible to determine mathematically the total force experienced by the pipe segment in the down comer and the riser pipe. The following properties and formulas all play a major role in the phenomenon called natural convection.

2.1.1. The cross-sectional area of pipe

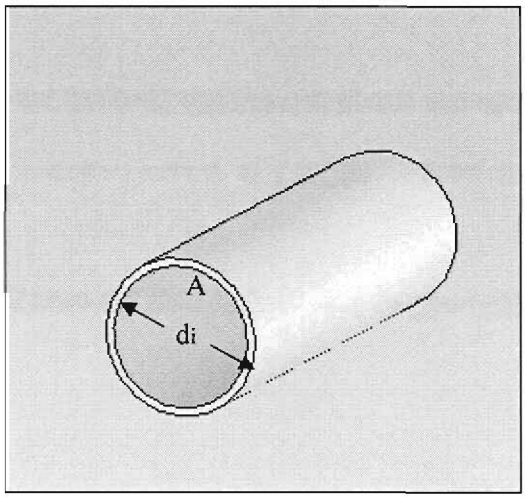


Figure 2.3: Cross-sectional cut of a pipe segment

The cross-sectional cut of a pipe segment is illustrated in figure 2.3. By using the dimensions of the pipe segment it is possible to determine the cross-sectional area and the mass of the substance within pipe segment. The cross-sectional area can be calculated as follows [15]:

$$A = \frac{\pi}{4} d_i^{\,2} \tag{2.3}$$

where, A represent the cross-sectional area of pipe and d_i the inside diameter.

2.1.2. Volume

The volume of the substance with in the Pipe segment can be calculated as follows:

$$\forall = A \cdot l_{segment} \tag{2.4}$$

where A represent the radius of the pipe and $l_{segment}$ the length of the pipe segment illustrated in figure 2.3. From this equation the volume in cubic meter can then be derived. The product of volume and density results in the obtaining of the mass, $m = \forall \rho$.

2.1.3. Pressure loss due to friction

To calculate the pressure loss due to friction within the pipe segment, It is calculated as follows [1,5,8]:

$$\Delta p_{ol} = (\frac{fL}{D} + \sum K) \frac{|\dot{m}|\dot{m}}{2\rho A_{ff}^{\ 2}} \tag{2.5}$$

where f represent the friction coefficient of the substance, L the length of segment under consideration, $\sum K$ the inside roughness of the pipe, D the hydraulic diameter, $\dot{m}$ the mass flow rate experienced in the pipe, A_{ff} the free flow area and ρ the density of the substance density which is a function of the temperature. The roughness of the pipe is usually read from the pipe characteristics table, received when the pipe is purchased.

2.1.4. Viscosity

The viscosity of a fluid, is a major role player in the analysis of the behaviour of a substance and also in the analysis of the motion of a substance, near solid boundaries. Viscosity is the resistance a substance exhibits to flow and is a measure of the adhesive/cohesive, or frictional property of the substance. The resistance is caused by intermolecular friction, exerted when layers of substances attempt to slide by one another. If a substance flows over a surface, the molecules against the solid boundaries (the ones clinging to the walls) have no movement [16].

As one progress further away from the solid boundary surface, the speed of the molecules increase. This difference in speed is the friction in the substance. It is the friction of molecules being dragged past each other. Thus, viscosity determines the amount of energy absorbed by the flow of the substance. The constant μ is known as the *coefficient of viscosity* or the dynamic viscosity and kinematic viscosity is derived by [16]:

$$v = \mu / \rho \tag{2.6}$$

Where μ represent the dynamic viscosity and ρ again the density of the substance under consideration.

2.2. MATHEMATICAL BACKGROUND

As illustrated in figure 2.4 the segment in the down comer has basically four forces working in on it. The first force is the product of the mass of the substance in the segment and the gravitational force (mg). This force is always present.

Then at the top part of the pipe segment, due to the water in the top tank, the segment experiences a pressure force (p_2A) per pipe area. The opposite pressure force (p_1A) is experienced by the segment in an opposite direction from the bottom, due to the rest of the substance that is already in the pipe. The forth force experienced by the segment, is the resistance force generated by the pipe per segment length ($\Delta p_{ol}A$) [14].

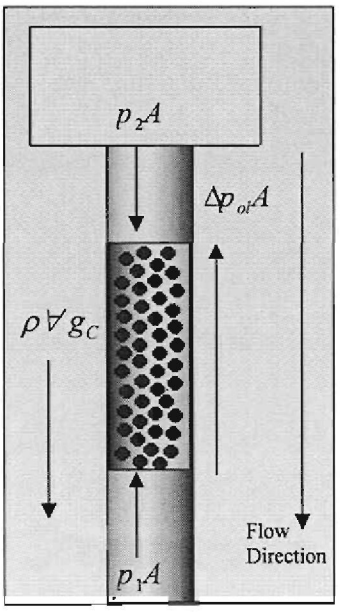


Figure 2.4: Forces working in on the segment in the down comer

Figure 2.4 illustrates the forces in the pipe segment, where A represents the cross-sectional area of the pipe in m^2, and Δp_{ol} represents the total pressure drop, due to frictional and other losses per segment length and per unit volume in kPA. The term $\rho \forall g_C$ represents the force working in on the pipe segment, due to the gravitational force on the substance, with $\forall = A(\Delta z)$ and $\Delta z = z_{in} - z_{out}$, which represents the height difference between the entrance and exit areas of the pipe segment as illustrated by figure 2.4. The pressure drop due to these losses can be calculated by using the following equation:

$$\Delta p_{ol} = (\frac{fL}{D} + \sum K) \frac{|\dot{m}|\dot{m}}{2\rho A_{fg}^{\ 2}} \tag{2.7}$$

To illustrate the concept, some assumptions are made to simplify the discussion. Assume that the flow direction, is from left to right in the U-tube under normal forced flow conditions. Also assume that the system is in steady-state, meaning that the mass of the pipe segment in the down comer is equal to the mass of the pipe segment in the riser segment. It is important to understand, that the mass of the substance stays the same for both sides of the U-tube, to enable us to derive an equation that illustrates the balance of forces within the system. By using the Reynolds transport equation the conservation of the mass in the pipe segment can be illustrated mathematically as follows [1,5,8]:

$$\frac{\partial}{\partial t}\iiint_{PS}\rho d\forall = -\oiint_{SS}(\rho\bar{V}dA) \tag{2.8}$$

where $\bar{V}$ represent the velocity of the substance through the pipe segment, ρ the density of the substance and $\forall$ the volume of the substance in that specific pipe segment. The subscripts identify which areas the integral needs to be performed upon. PS refers to the pipe segment which is the applicable control volume and SS refers to the applicable control surface of the pipe segment as illustrated in figure 2.4.

When taking the assumptions into account it is easy to see that the mass of the substance remains constant over time. This means that the rate of change, over time, over the pipe segment equals zero, which result in the pipe segment integral term also being zero. Therefore (2.8) can be written as [1,5,8]:

$$\iiint_{PS}\rho d\forall = 0 \rightarrow (time\ rate\ of\ change\ is\ zero)$$

$$\therefore \oiint_{SS}(\rho\bar{V}dA) = 0 \tag{2.9}$$

Thus, with reference to figure 2.4, this means that the total mass flow rate into the pipe segment, $\dot{m}_{in}$, equals the total mass flow rate out of the pipe segment, $\dot{m}_{out}$. Thus by assuming that flow are in steady-state and that the inlet and outlet flows are one-dimensional, it is possible to rewrite (2.9) as [1,5,8]:

$$\oiint_{SS}(\rho\bar{V}dA) = -\iint_{A_1}(\rho V\cdot dA) + \iint_{A_2}(\rho V\cdot dA) \tag{2.10}$$

A_1 and A_2 are respectively the entrance and exit areas of the segment surface of the segment illustrated in figure 2.4. Because of the assumption that the substance is incompressible, ρ and V becomes constants at each section of the pipe segment, and we can rewrite (2.10) as follows [1,5,8]:

$$0 = -\iint_{A_1}(\rho V dA) + \iint_{A_2}(\rho V dA)$$
$$\therefore \quad \rho V \iint_{A_1}(dA) = \rho V \iint_{A_2}(dA) \rightarrow \rho \text{ and } V \text{ is constants} \tag{2.11}$$

by integrating the resulting (2.11), it is possible to write:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \quad \rightarrow (\dot{m}_{out} = \dot{m}_{in}) \tag{2.12}$$

and with the assumption made that ρ is constant for each section it is possible to derive (2.12) to:

$$V_1 A_1 = V_2 A_2$$
$$\therefore \quad Q_1 = Q_2 \tag{2.13}$$

where Q_1 and Q_2 are respectively the volume flow for the entrance and the exit areas. With the above derivation explained mathematically, it is easy to see that the conservation of mass within the U-tube is in a steady-state condition.

The next step in understanding the natural convection phenomenon is to illustrate the behaviouristic characteristics of the forces present in the U-tube. It is possible to write the summation equation of the forces in the pipe segment, by using Newton's second law of momentum, as follows [1,5,8]:

$$\sum F_{pipe\ segment} = \underbrace{\frac{\partial}{\partial t}\iiint_{PS}\bar{V}\rho d\forall}_{\text{Control Volume}} + \underbrace{\oiint_{SS}\bar{V}\rho\bar{V}dA}_{\text{Control Surface}} \tag{2.14}$$

thus:

$$\sum F_{pipe\ segment} - \oiint_{SS}\bar{V}\rho\bar{V}dA = \frac{\partial}{\partial t}\iiint_{PS}\bar{V}\rho d\forall \tag{2.15}$$

where $\frac{\partial}{\partial t}\iiint_{PS} V\rho d\forall$ represents the rate of change of the momentum inside the pipe segment (control volume). By integrating this term it is possible to write[18]:

$$\frac{\partial}{\partial t}\iiint_{PS}\bar{V}\rho d\forall = \rho AL\frac{\partial \bar{V}}{\partial t} \tag{2.16}$$

The term $\oiint_{SS}\bar{V}\rho\bar{V}dA$ represents the forces p_1A_1 and p_2A_2, thus:

$$\oiint_{SS}\bar{V}\rho\bar{V}dA = p_1A - p_2A \tag{2.17}$$

and with reference to (2.1) it is also possible to rewrite the summation of the forces in the pipe segment, $\sum F_{pipe\ segment}$, as follows:

$$\sum F_{pipe\ segment} = \Delta p_{ol}A + \rho\forall g_C \tag{2.18}$$

where $\Delta p_{ol}A$ represents the surface force acting in on the pipe segment and $\rho\forall g_C$ represents the body force in the pipe segment. Therefore it is possible to finally rewrite the conservation of momentum as follows for the pipe segment in figure 2.4 [1,5,18]:

$$\rho AL\frac{\partial \bar{V}}{\partial t} = A(p_2 - p_1) + \Delta p_{ol}A + A\rho z\Delta g \tag{2.19}$$

thus to get the rate of change of the mass flow rate, (2.19) can be rewritten as:

$$\rho A\frac{\partial \bar{V}}{\partial t} = \frac{A}{L}(p_2 - p_1) + \Delta p_{ol}\frac{A}{L} + \frac{A}{L}\rho z\Delta g_C \tag{2.20}$$

where $\rho A\frac{\partial \bar{V}}{\partial t} = \rho A\bar{V}(\frac{\partial}{\partial t}) = \frac{\partial \dot{m}}{\partial t}$. It is therefore possible to write the rate of change of mass flow rate as:

$$\frac{\partial \dot{m}}{\partial t} = \frac{A}{L}(p_2 - p_1) + \Delta p_{ol}\frac{A}{L} + \frac{A}{L}\rho z\Delta g_C \tag{2.21}$$

Because the rate of change of momentum equals zero, the pressure drop over the down comer can be expressed as follows, with reference to (2.21):

$$0 = \frac{A}{L}(p_2 - p_1) + \Delta p_{ol}\frac{A}{L} + \frac{A}{L}\rho z \Delta g_C$$

$$\therefore \quad (p_2 - p_1) = \Delta p_{ol} + \rho z \Delta g_C$$

$$\therefore \quad \Delta p_{downcomer} = \Delta p_{ol} + \rho z \Delta g_C \tag{2.22}$$

The same mathematical procedure of calculating the forces present in the down comer pipe segment can be applied to calculate the forces experienced by the riser pipe.

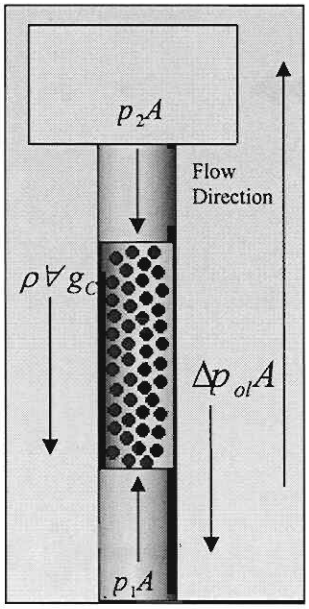


Figure 2.5: Forces working in on the segment in the riser pipe

The flow direction of the substance in the riser pipe is in the opposite direction compared to the flow of the substance in the down comer pipe. The pressure drop over the riser pipe can then be written as:

$$\therefore \quad \Delta p_{riser} = \Delta p_{ol} - \rho z \Delta g_C \tag{2.23}$$

In steady-state the pressure drop over the riser and down comer pipe are equal. This can be contributed to components such as the density in each pipe segment, the mass of the substance in each pipe segment, and the physical characteristics of each pipe segment, which are equal. Thus:

$$\therefore \quad \Delta p_{riser} \cong \Delta p_{downcomer} \tag{2.24}$$

therefore it is possible to rewrite (2.24) in terms of the forces:

$$A\Delta p_{ol} + A\rho z\Delta g_C = A\Delta p_{ol} - A\rho z\Delta g_C \tag{2.25}$$

this results in:

$$\left(\rho_{down\_comer}\forall_{down\_comer}g_C\right) = \left(-\rho_{riser\_pipe}\forall_{riser\_pipe}g_C\right) \tag{2.26}$$

This illustrates that the force present in the down comer, has an exact opposite force existing in the riser pipe side of the U-tube. The force in the down comer is positive, because the force is in the same direction as the flow of the substance under normal operational conditions. The force in the rise pipe is negative, because it exists in the opposite direction of the substance flow under normal conditions.

It is easy to see from equation (2.26) that in the event were heat is added to either side, the density will change resulting in the imbalance of the forces within the U-tube, which in turn will result in a flow towards the side with the smaller density coefficient. Natural convection occurs due to this imbalance between the two sides. One of the sides is heated and this results in a change of density of the substance. Figure 2.6 illustrates this concept.

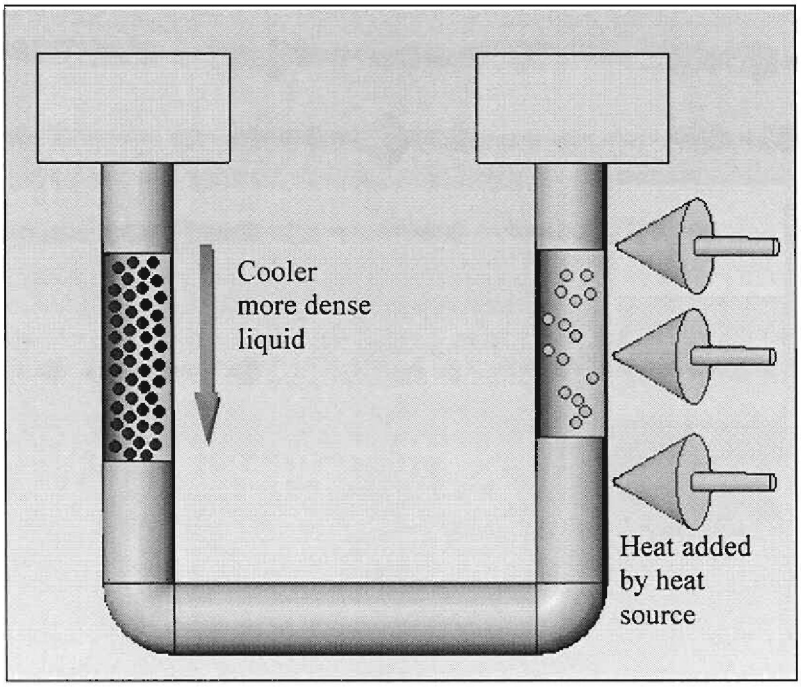


Figure 2.6: Substance less dense in up comer due to the added heat

The substance in the tube which receives no or very little heat has a greater density which means it has a greater mass value. This bigger mass value results in the tube having a larger pressure at the bottom of the tube and therefore natural flow of the substance will occur from left to right.

2.3. CONCLUSION

The concept of natural convection was discussed in depth, because under passive operating conditions, it is the fundamental phenomenon experienced in the RCCS. It is important to understand this phenomenon to ensure that the relevant components are taken into account when developing an equivalent model of the system. It is not the intention of this study to discuss transient conditions that results into more complex phenomena. This project aims to understand the phenomenon of natural convection in the RCCS, under predefined conditions. The following chapters discuss the analyses method intended to be used, to analyse the system and generate the state-space model.

Chapter 3

3. Bond graph analysis method

3.1. INTRODUCTION

Today various analysis methods are available to the industry to analyse a system. Each one of these analysis methods has pros and cons and these aspects should be taken into account when deciding which analysis method to use. For the purpose of this study, two dynamic analysis methods were investigated. The two methods are the bond graph and network graph analysis methods. The following chapters, chapters 3 & 4, discuss these two analysis methods in detail.

The fundamentals involved in modelling a system, is to disband the system into smaller components and therefore producing a simplified model that can be used to predict the system's behaviour. The model should be an approximation of the reality, therefore modelling takes a great deal of consideration. An unnecessary complex model may contain parameters that are physically impossible to analyse or solve. An over simplified model is again not the equivalent representation of the reality [21].

Graph based modelling enables the user to construct an equivalent diagram for an electrical, mechanical, thermal, hydraulic system, by simply identifying the components within the system and there interaction with each other [21].

The bond graph analysis modelling method is a graphical approach to system modelling. The fundamental feature of this method is the use of single line arrows that represent the interactions of the energy within a system. With the single line arrows it's possible to graphically represent the interaction between the system and the system's components [21].

The energy bond is the fundamental component of the bond graph. The energy bond is used to connect the energy ports of the components in the system. The bond graph method is a heuristic manner of investigating the energetic interconnections within a system prior to performing a detailed analysis [21].

3.2. BASIC BOND GRAPH COMPONENTS

Usually, when systems are modelled, it is done in terms of components that stores energy, components that dissipate energy. [21]. Components that stores energy normally represent components like sources and components dissipating energy are usually resistive of nature.

These components are usually either connected in series or in parallel, and occasionally some transformation or gyrating elements are also present in the system. It is therefore of essential importance to accurately define the bond conventions for basic one-port elements [21].

3.2.1. Energy Sources

For every system that exists, either a mechanical or electrical system, there must be an energy source within the system. The energy source causes the particles or electrons within the system to flow through the system. There are two basic sources within an electrical system namely an electrical current source and an electrical voltage source. For a mechanical system, the system sources consist out of a force source and a velocity source. In a fluid system, the system sources comprises out of a fluid pressure source and a flow rate source. The same is valid for thermal systems where the system sources are temperature and heat flow sources. These sources in which ever system they are, can general be categorised as either a flow source or an effort source within the bond graph. Table 3.1 categorise these pre-mentioned system sources with regards to effort and flow sources within the bond graph [21].

Table 3.1: Categorisation of mechanical, fluid and electrical sources as effort and flow sources within the bond graph [21]

Effort Source		Flow Source
SE e	Generalized component electrical	SF f
SE v		SF i
Voltage Source		Current Source
SE P	Fluids	SF Q
Pressure Source		Fluid flow source
SE v	Mechanical translation	SF F
Velocity Source		Force source
SE ω	Mechanical rotation	SF τ
Angular Velocity Source		Torque source
SE T	Thermal	SF Q
Temperature Source		Heat flow source

3.2.2. Energy storage components

There are basically two energy storage components in the bond graph arsenal, namely the flow and effort storage components. The flow storage components store energy, in terms of the time integral of the flow variable applied to the port of the flow storage component and the effort store component is described by the time integral of the flow variable and the material properties of the component [21]:

Table 3.2: Flow storage relations [21]

	Dynamic Relation	Linear constitutive relation
Electrical component [capacitor q=charge]	$q = \int i dt$	$v = \frac{1}{C} q$
Fluid components [fluid reservoir, V=fluid volume]	$V = \int Q dt$	$P = \frac{1}{C_f} V$

The relations as illustrated in table 3.2 are of an unpretentious nature. The network relation does not alter in the event of the occurrence of non-network relations. The effort storage component stores energy when the time integral of the effort variable is applied at its port. The equivalent flow is then given by the time integral of the effort variable and also the properties of the material of the device. As previously indicated, the relations are of a simple nature and therefore network relations do not vary during non-network relations [21]:

Table 3.3: Effort storage relations [21]

	Dynamic Relation	Linear constitutive relation
Generalized Components	$e_a = \int e dt$	$f = \frac{1}{L} e_a$
Electrical component [inductor, λ=flux linkages]	$\lambda = \int v dt$	$i = \frac{1}{L} \lambda$
Fluid components [fluid inertia, Γ=pressure momentum]	$\Gamma = \int P dt$	$Q = \frac{1}{L_f} \Gamma$

3.2.3. Energy Dissipators

There is universally only one type of dissipator of energy and that is a resistance of some sort. If it is a resistor in an electrical system or mechanical dashpots, they all function in the same way. They all dissipate energy when used in a system. The convention for the dissipator in various energy domains within the bond graph models are illustrated by table 3.4 [21]:

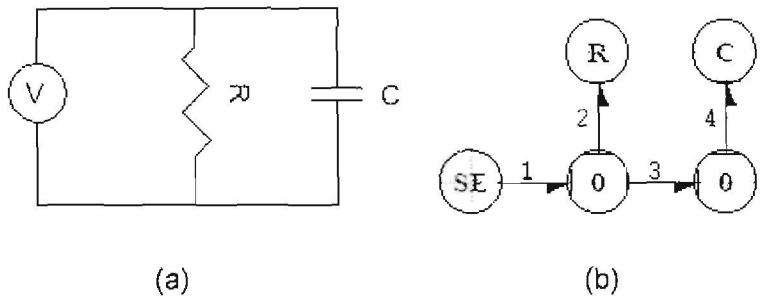
Table 3.4: Dissipator conventions [21]

	Linear constitutive relation
Generalized Components	$e = Rf$
Electrical component [resistor]	$v = Ri$
Fluid components [Fluid dissipator]	$P = R_f Q$
Mechanical components	$F = bV$
	$\tau = b\omega$
Thermal components	$T = RQ$

3.2.4. Inter component connections within the bond graph

The main difference between the network and bond graph modelling method, is the manner in which the components are interconnected to each other. Bond graphs are port orientated and to enable interconnection between two of more components, one must introduce explicit bond graph components. The network modelling method is terminal orientated and components are interconnected by simply joining terminals. There are basically two explicit ways of interconnecting the components within the bond graph, namely by an effort junction and a flow junction [21].

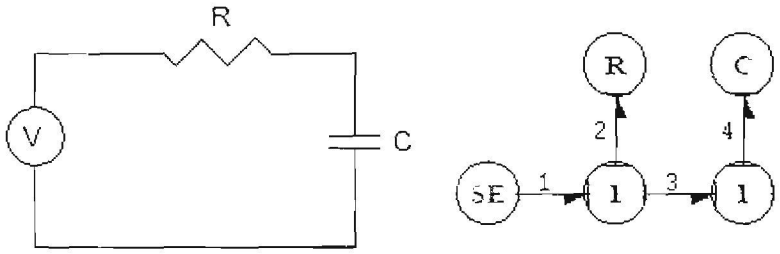
The flow junction represents a parallel connection within the system. The flow junction is connected to the circuit as follows:



(a) (b)

Figure 3.1: Illustration of (a) electrical circuit and (b) its equivalent bond graph [5]

The effort junction represents the connection of components connected in series. The flow junction is connected to the circuit as follows:



(a) (b)

Figure 3.2: Illustration of an effort junction connection (a) Circuit (b) Equivalent bond graph [5]

3.2.5. Causality

Between each component of the bond graph, an interconnection exists which indicates the direction of the energy flowing through the system. The interconnection passes certain causality characteristics which indicates wether the component is an output of a bond or if the component is the input for the next bond. The direction of the arrow indicates the direction of the power flow and the line indicates if the line segment serves as an input or output carrier [21]. For example:

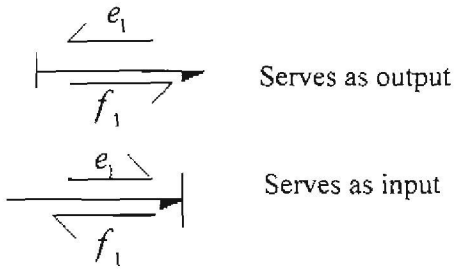


Figure 3.3: Indication of causality concerning inputs and outputs [21]

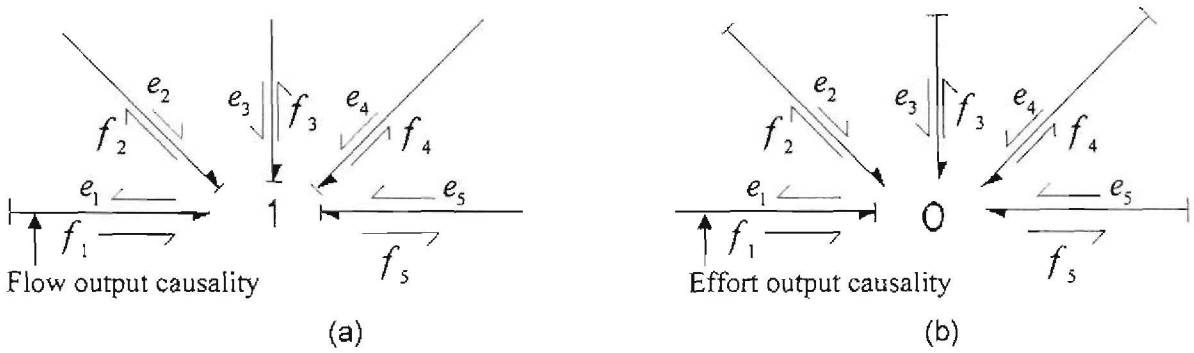


Figure 3.4: Causality convention of an effort source and flow source [21]

As indicated by figure 3.4, causality indicates what relation the specific bond has with its surrounding junctions. As indicated in figure 3.4 (a), the bond specified by the arrow is an output from a flow junction to the effort junction. The rest of the three bonds serve as inputs from effort junctions. The flow junction in figure 3.4 (b) has the same configuration but the causality is indicated differently. The bond indicated by the arrow point, serves as output from an effort junction to the flow junction. The rest of the bonds serve as input to the flow junction from other flow junctions [21].

3.3. BOND GRAPH STATE-SPACE MODELLING

The simplest way to model a system is to work systematically through the system starting at the source, continuing through the circuit to the last component connected. To illustrate the modelling technique using bond graphs, take the following fluid network into account. The network consists out of a pump, pipes and two water tanks.

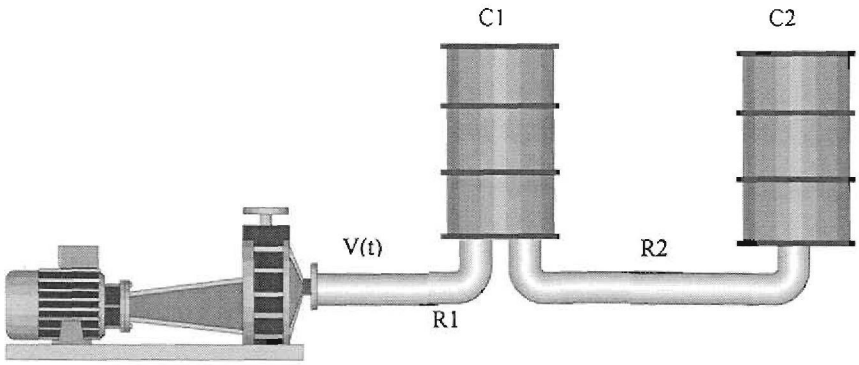


Figure 3.5: Simple fluid network

It is possible to derive the equivalent electrical circuit, due to the fact that the pressure produced by the pump is a direct analogy to a voltage source, the pipes to resistors and the tanks to electrical capacitance. The equivalent electrical circuit is as follows:

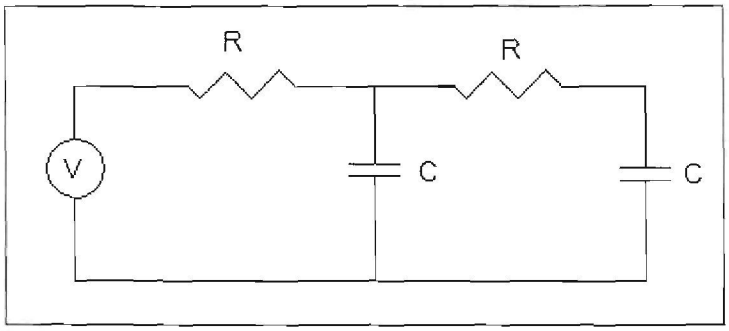


Figure 3.6: Electric circuit to be analysed by using the bond graph method

Figure 3.7 represents the equivalent bond graph of the electrical circuit as illustrated in figure 3.6. The SE (source effort) symbol represents the voltage source. The resistors are in series and the capacitors are in parallel. The "1" represent the series connections and "0" represent the parallel connections. The arrows between the connections within figure 3.7 indicate the direction of the energy flowing within the connections [5].

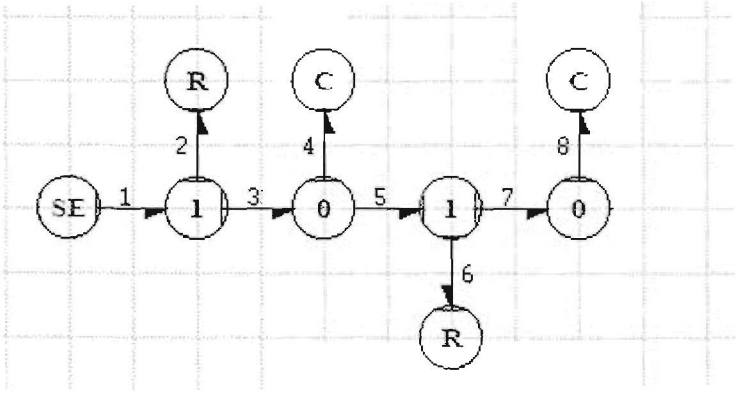


Figure 3.7: Equivalent bond graph for electrical circuit in figure 4.4

The first step in the state-space extraction algorithm is to construct the junction structures. This is done in the form of a matrix. To illustrate the process the first two components are taken into account, namely the voltage source (effort source) and the first resistor. The configuration is illustrated in figure 3.8.

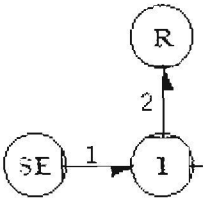


Figure 3.8: First two components of the equivalent bond graph

The effort variable of the bond flows in the direction of the causality sign. The flow variable flows in the opposite direction of the effort. By considering the causality of each bond, the directions of the effort and flow components can be determined as illustrated below.

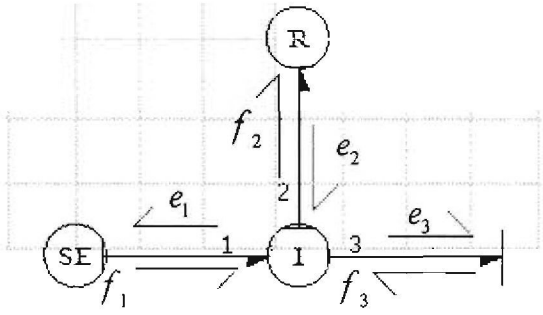


Figure 3.9: Illustrates the flow and effort component in each connection between the components and node

The junction structure can now be constructed in matrix form. Every junction consists out of internal and external bonds. The interconnections indicated by number 3, illustrated in figure 3.10 is known as the internal bond and the interconnection indicated by number 2, as illustrated in figure 3.10, is known as the external bound. The internal bond connects two junctions with each other and the external bond connects the element with a junction. The external bonds within the system are indicated by the vector g and the vector h represents the internal bonds [21]. This convention is illustrated in figure 3.10.

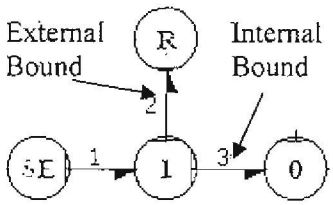


Figure 3.10: Illustrates the internal and external bound convention

The generalised equations which represent the configuration illustrated in figure 4.9 are:

$$\begin{bmatrix} g_{out} \\ h_{out} \end{bmatrix} = \begin{bmatrix} J_{11} J_{12} \\ J_{21} J_{22} \end{bmatrix} \begin{bmatrix} g_{in} \\ h_{in} \end{bmatrix} \tag{3.1}$$

thus taking this into account, it is easy to see that the equations representing the illustrated configuration in figure 3.6 can be derived as:

$$\begin{bmatrix} e_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{3.2}$$

In the above equations f_1, f_2, and f_3 represent the flow component in each bond and e_1, e_2, and e_3 represent the effort components in each bond. The sign convention of each branch is determined by using Kirchoff's current law regarding currents flowing into and out the junction. Thus, for each junction/node, a junction structure/matrix can be derived. It is therefore easy to see that the matrices derived below represent the junctions illustrated in figure 3.7 [21].

3.4. STEPS FOR STATE-SPACE EXTRACTION

3.4.1. Step 1 – Generate the representing matrix for every junction

3.4.1.1. Effort Junction 1

This effort junction is the first junction in the system. The voltage source is connected to it and the first resistor within the system. The source and the resistor are connected in series therefore the junction has a ***1*** notation. The representing matrix for effort junction 1 is:

$$\begin{bmatrix} f_1 \\ e_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ f_2 \\ e_3 \end{bmatrix} \tag{3.3}$$

3.4.1.2. Flow Junction 2

This flow junction represents the capacitor which is connected in parallel with the rest of the circuit. The capacitor is connected in parallel therefore the junction has a ***0*** notation. The representing matrix for flow junction 2 is:

$$\begin{bmatrix} f_4 \\ e_3 \\ e_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_4 \\ f_3 \\ f_5 \end{bmatrix} \tag{3.4}$$

3.4.1.3. Effort junction 3

This effort junction represents the second resistor within the system. The resistor is connected in series with the rest of the circuit therefore the junction has a ***1*** notation. The representing matrix for effort junction 3 is:

$$\begin{bmatrix} e_6 \\ f_5 \\ f_7 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_6 \\ e_5 \\ e_7 \end{bmatrix} \tag{3.5}$$

3.4.1.4. Flow junction 4

This flow junction represents the second capacitor which is connected in parallel with the rest of the circuit. The capacitor is connected in parallel therefore the junction has a ***0*** notation. The representing matrix for flow junction 4 is:

$$\begin{bmatrix} f_8 \\ e_7 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} e_8 \\ f_7 \end{bmatrix} \tag{3.6}$$

3.4.2. Step 2 – Combine the generated matrices in one general matrix

The next step in utilising the bond graph method to analyse the electrical circuit illustrated in figure 3.6 is to combine all the equations into one combined matrix, as illustrated below:

$$\begin{bmatrix} A\,l\,l - g_{out} \\ A\,l\,l - h_{out} \end{bmatrix} = \begin{bmatrix} J_{11} J_{12} \\ J_{21} J_{22} \end{bmatrix} \begin{bmatrix} A\,l\,l - g_{in} \\ A\,l\,l - h_{in} \end{bmatrix} \tag{3.7}$$

As expressed in equation (3.7) all the g_{out} vectors are grouped and all the h_{out} vectors are grouped to form the following equations:

$$\begin{bmatrix} G_{out} \\ H_{out} \end{bmatrix} = \begin{bmatrix} J_{11} J_{12} \\ J_{21} J_{22} \end{bmatrix} \begin{bmatrix} G_{in} \\ H_{in} \end{bmatrix} \tag{3.8}$$

thus utilising (3.8) it is possible to group the junction matrix into one combined matrix:

$$J = \left[\begin{array}{c} f_1 \\ e_2 \\ f_4 \\ e_6 \\ f_8 \\ -- \\ f_3 \\ e_3 \\ e_5 \\ f_5 \\ f_7 \\ e_7 \end{array}\right] = \left[\begin{array}{ccccc|ccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \multicolumn{5}{c|}{- -} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \left[\begin{array}{c} e_1 \\ f_2 \\ e_4 \\ f_6 \\ e_8 \\ -- \\ e_3 \\ f_3 \\ f_5 \\ e_5 \\ e_7 \\ f_7 \end{array}\right] \tag{3.9}$$

3.4.3. Step 3 – Generate the final junction matrix

From the above combine matrix the next step is to extract the J_{11}, J_{12}, J_{21} and J_{22} matrices. These matrices are used in calculating the A and B matrices of the state-space equation. Therefore it is easy to see, with reference to (3.8), that the required matrixes can be written as follows:

$$J_{11} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.10}$$

$$J_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.11}$$

$$J_{21} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.12}$$

$$J_{22} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.13}$$

3.4.4. Step 4 – Generate the permutation matrix

Step four in the process of developing an equivalent representative model for the electrical circuit in figure 3.6, is to generate the permutation matrix. When using the bond graph modelling method it is required to re-order h_{in} in the form of h_{out}. The reason for this is that both h_{out} and h_{in} relates to the internal junction structure bonds. The permutation matrix fulfils this role and is generated by taking only the internal bonds into account. To be able to do this, the relation between h_{out} and h_{in} must be evaluated [21]. The convection is expressed as follows:

$$[h_{out}] = [P][h_{in}] \tag{3.14}$$

and by utilising equation (3.14) it is possible to derive the permutation matrix as follows:

$$\begin{bmatrix} f_3 \\ e_3 \\ e_5 \\ f_5 \\ f_7 \\ e_7 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e_3 \\ f_3 \\ f_5 \\ e_5 \\ e_7 \\ f_7 \end{bmatrix} \tag{3.15}$$

therefore the P-matrix is:

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{3.16}$$

After developing the J-matrixes and the P-matrix the final junction matrix can be calculated by means of the following equation, namely:

$$J = J_{11} - J_{12} \cdot P \cdot ((J_{22} \cdot P - I)^{-1} \cdot J_{21}) \tag{3.17}$$

and by utilising equation (3.17), the J-matrix can be expressed as follows :

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{3.18}$$

where I is:

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.19}$$

3.4.5. Step 5 – Generate matrixes for Resistors and capacitors within the system

Step five in the process is to develop the matrixes for the capacitors and the resistors [21]. For the capacitors the following equation is used:

$$w_i = z_i x_i \tag{3.20}$$

where w_i represent the integral vector of the causal storage field output variable, x_i the integral vector of the causal storage field input variable and z_i represent the capacitor element matrix. Only the bonds where capacitors are connected are evaluated [21]. The capacitors can only be connected by effort junctions, thus the following matrix can be extracted from the effort junctions:

$$\begin{bmatrix} e_4 \\ e_8 \end{bmatrix} = \begin{bmatrix} \frac{1}{C_4} & 0 \\ 0 & \frac{1}{C_8} \end{bmatrix} \begin{bmatrix} q_4 \\ q_8 \end{bmatrix} \tag{3.21}$$

meaning that:

$$z_i = \begin{bmatrix} \frac{1}{C_4} & 0 \\ 0 & \frac{1}{C_8} \end{bmatrix} \quad ; \quad w_i = \begin{bmatrix} e_4 \\ e_8 \end{bmatrix} \text{ and } x_i = \begin{bmatrix} q_4 \\ q_8 \end{bmatrix} \tag{3.22}$$

where

$$q = \int f(t)\, dt$$

The same can be done for the dissipators within the system. For the resistors the following equation is used:

$$r = D_R s \tag{3.23}$$

where r represents the vector of the dissipator field output variable, s the vector of the dissipator field input variable and D_R represents the resistor element matrix as illustrated in equation (3.25). The resistor element can be used in tow different ways within the bond graph. In the event where the causality exists on the side of the resistor, the effort components flow towards the resistor and the flow component flows towards the bond. The resistor is seen as being connected in parallel with the rest of the circuit. However when the causality exists on the side of the bond, the effort component flows towards the bond, away from the resistor and the flow component towards the resistor, then the resistor shall be seen as being connected as in series [21]. The junction matrix for the resistor elements can be expressed as follows for the bond graph illustrated in figure 3.7:

$$\begin{bmatrix} f_2 \\ f_6 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_2} & 0 \\ 0 & \frac{1}{R_6} \end{bmatrix} \begin{bmatrix} e_2 \\ e_6 \end{bmatrix} \tag{3.24}$$

where:

$$r = \begin{bmatrix} f_2 \\ f_6 \end{bmatrix} \quad ; \quad D_R = \begin{bmatrix} \frac{1}{R_2} & 0 \\ 0 & \frac{1}{R_6} \end{bmatrix} \text{ and } \quad s = \begin{bmatrix} e_2 \\ e_6 \end{bmatrix} \tag{3.25}$$

According to the bond graphs the following matrices are declared to obtain the junction structure constitutive relation with regards to the input and output of the systems. The vectors g_o and g_i represent the output and input vector for the junction structure, respectively and can be expressed as follows:

$$g_o = \begin{bmatrix} \dot{x}_i \\ v_i \\ s \end{bmatrix} \quad \text{and} \quad g_i = \begin{bmatrix} w_i \\ u_o \\ r \end{bmatrix} \tag{3.26}$$

where v_i is the source input and u_o is the source output. The junction structure constitutive matrix contains block entries as illustrated in the following form:

$$g_o = J\ g_i \tag{3.27}$$

thus:

$$\begin{bmatrix} \dot{x}_i \\ v_i \\ s \end{bmatrix} = \begin{bmatrix} \mathrm{J_{x\,w}} & \mathrm{J_{x\,u}} & \mathrm{J_{x\,r}} \\ \mathrm{J_{v\,w}} & \mathrm{J_{v\,u}} & \mathrm{J_{v\,r}} \\ \mathrm{J_{s\,w}} & \mathrm{J_{s\,u}} & \mathrm{J_{s\,r}} \end{bmatrix} \begin{bmatrix} w_i \\ u_o \\ r \end{bmatrix} \tag{3.28}$$

3.4.6. Step 6 – Determining the A and B matrix of the state-space equation

The sixth step is to determine the A and B matrixes for the state-space equation [21]. This is done by mathematically manipulating the junction structure constitutive matrix defined in equation line 3.28. By deleting the second row the following matrix can be derived:

$$\begin{bmatrix} \dot{x}_i \\ s \end{bmatrix} = \begin{bmatrix} \mathrm{J_{x\,w}} & \mathrm{J_{x\,u}} & \mathrm{J_{x\,r}} \\ \mathrm{J_{s\,w}} & \mathrm{J_{s\,u}} & \mathrm{J_{s\,r}} \end{bmatrix} \begin{bmatrix} w_i \\ u \\ r \end{bmatrix} \tag{3.29}$$

and by considering the store and dissipator constitutive relations from equations (3.21) and (3.24), respectively, the following matrix can be derived:

$$\begin{bmatrix} \dot{x}_i \\ s \end{bmatrix} = \begin{bmatrix} \mathrm{J_{x\,w}} & \mathrm{Z_i}\ \mathrm{J_{x\,u}} & \mathrm{J_{x\,r}} & \mathrm{D} \\ \mathrm{J_{s\,w}} & \mathrm{Z_i}\ \mathrm{J_{s\,u}} & \mathrm{J_{s\,r}} & \mathrm{D} \end{bmatrix} \begin{bmatrix} x_i \\ u \\ s \end{bmatrix} \tag{3.30}$$

and by eliminating the vector s from the state-space equation the state-space equation is derived in the following manner:

$$\dot{x} = A\ x_i + B\ u \tag{3.31}$$

where:

$$A = J_{x\,w} Z_i + (J_{x\,r} D(I - J_{s\,r} D)^{-1} J_{s\,w} Z_i) \tag{3.32}$$

$$B = J_{x\,u} + (J_{x\,r} D(I - J_{s\,r} D)^{-1} J_{s\,u}) \tag{3.33}$$

By utilising the above technique, the state-space model for the circuit in figure 3.6 can be expressed as follows:

$$A = \begin{bmatrix} -\dfrac{1}{R_2 C_4} - \dfrac{1}{R_6 C_4} & ; & \dfrac{1}{\mathrm{R_6 C_8}} \\ \dfrac{1}{\mathrm{R_6 C_4}} & ; & -\dfrac{1}{\mathrm{R_6 C_8}} \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{1}{R_2} \\ 0 \end{bmatrix} \tag{3.34}$$

3.5. CONCLUSION

The bond graph method for extracting the state-space models is a systematic way of modelling. This helps the system designer to eliminate unnecessary mistakes, such as the possibly of violating the laws of nature. As with all analysis methods, there are advantages and disadvantages for using the specific method. This method also has disadvantages in terms of the size of the matrices generated by the modelling technique. The disadvantage of using this method is that the junction structure constitutive matrix (3.16), permutation matrix (3.14) and junction matrix (3.7) expand with two rows and two columns for every node/junction added to the bond graph. This result in matrices which grows exponentially with every junction added. It is therefore very difficult to generate a detailed bond graph which represents systems that consist out of many components. The modelling technique also lends itself to errors due to the wrong identification of the causalities between the involved components. A modelling technique that is able to cope with systems that have a large number of components is needed. Network modelling techniques may be more convenient in this respect. The network graph analysis method is discussed in detail in the next chapter.

Chapter 4

4. Network analysis method

4.1. INTRODUCTION

After gaining a good understanding of the bond graph analysis method and the network graph analysis method, it was decided to utilise the network graph analysis method rather than the bond graph method. This chapter discuss the network analysis method in detail.

The network graph analysis method is a graphical analysis technique which is widely used for the representation and formulation of dynamic system models. As the name suggests, the representative network graph of a system illustrates the relationships of the system elements and the interconnections between the elements by means of line segments [21].

The chapter discusses the methods of drawing the system network, the method of generating the algebraic statements of the interconnectivity constraints and the formulation of the models. It also discusses the similarities of one-port elements in both the electrical and fluid energy domains. By defining analogies between elements and the variables in the various energy domains it is possible to derive a systematic modelling technique using these generalised elements and variables. To be able to do this we need to discuss the analogy between the elements and variables in the various energy domains.

4.2. THE NETWORK GRAPH STRUCTURAL PROPERTIES

4.2.1. Network representation and tree definition

The first step when developing the network graph is to develop the system graph. The system graph contains all the network graph "nodes" and the all the "branches" which refers to the connections between the nodes. These so called "nodes" represent the connection points between the elements in the physical system and the "branches" represent the elements within the physical system. In each branch the graph contains an across-variable and through-variable as illustrated in figure 4.1.

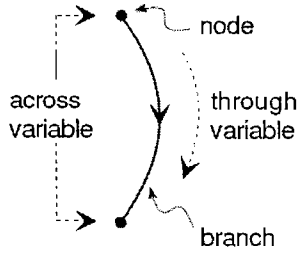


Figure 4.1: Line segment of a one-port element [21].

Figure 4.2 illustrates the convention of the system graph for a simple electrical circuit and its equivalent network graph. Notice that the branches represent the elements and the nodes represent the connection points between the elements [21].

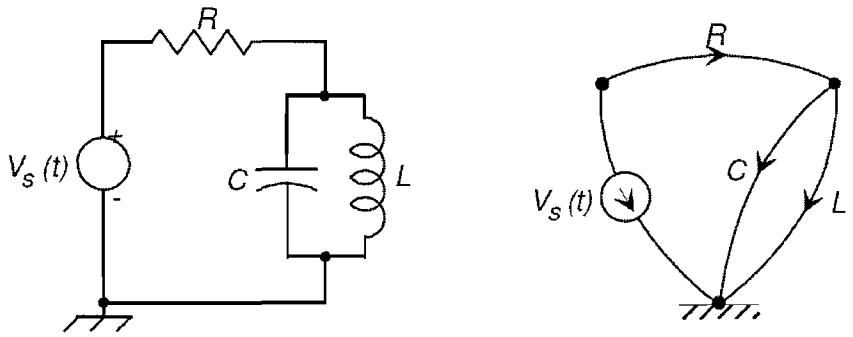


Figure 4.2: Illustration of the system graph of a simple electrical circuit and its equivalent network graph.

The second step is to generate the tree from the network graph. A tree is a connected graph without any loops, as illustrated in figure 4.4. The tree plays a mayor role in the extraction of the state-space algorithm. The tree contains no links and only contains the branches not responsible for creating any loops. The branches responsible for creating the loops are known as the links. Figure 4.3 illustrates a network graph were the links are represented by dotted lines [21].

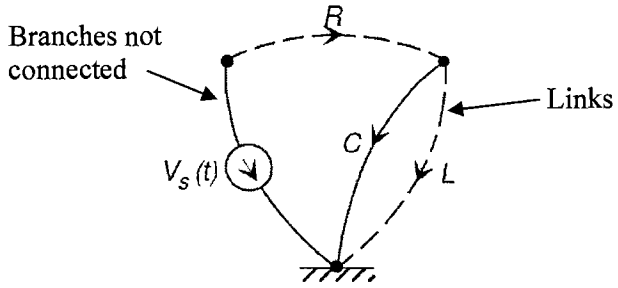


Figure 4.3: Dotted lines in network graph are known as links [21].

By removing the links the tree for the system illustrated in figure 4.1 can be constructed as illustrated in figure 4.4:

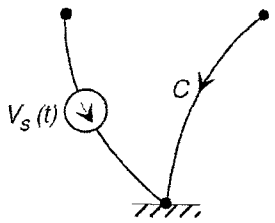


Figure 4.4: The graph tree of the electrical circuit illustrated in figure 4.1[21]

As can be seen from figure 4.4, the tree can consist out of a number of branches. The number of branches and links are depicted by the number of nodes in the system. Assume the number of branches is represented by T_B, the number of links by T_L and the total number of branches by B_T. The number of branches and the number of links can now be calculated in the following mathematical manner, namely $T_B = N - 1$ and $T_L = B_T - N - 1$, where N represent the number of nodes in the system. The tree is used to derive (a) the primary and secondary variables of the system, (b) the system order n, (c) the state variables and (d) the set of independent compatibility and continuity equations [21]. The following sections discuss the various structural properties of the network graph system in detail.

4.2.2. Through- and across-variables

This section discusses the components known as the across- and through-variables, which represent the various analogous components in the system. Figure 4.5 (a) represents a mechanical spring with external forces being exerted on it, and figure 4.5 (b) illustrates an electrical inductor connected to a power source. The forces ($F(t)$) applied to the spring over time as illustrated in figure 4.5 (a) compresses the spring, meaning that the force is applied "through" the spring. This variable present in the spring due to the forces, is known as the through-variable of the spring. The same convention can be illustrated by utilising an electrical inductor as illustrated in figure 4.5 (b). The current flowing "through" the inductor is analogous to the forces being conveyed into the spring and also is known as the through-variable for the inductor.

A branch has two terminals that represent the inflow and outflow of energy. The line segment therefore contains a physical quantity which can be measured across the terminals. This is known as the across-variable of the line segment. For example, in figure 4.5 (a) the spring contains two variables namely the velocity and the force exerted on the spring. As already mentioned the force is the through-variable meaning that the velocity is the across-variable in the system, because it must be measured at the point of maximum compression and at the point of maximum release [21].

Measuring across the two terminals of the spring is analogues to measuring the potential difference across the terminals of the inductor [21]. Figure 4.6 provides a more practical methodology of the concept and figure 4.7 illustrates schematically the concept of the through- and across-variable [21].

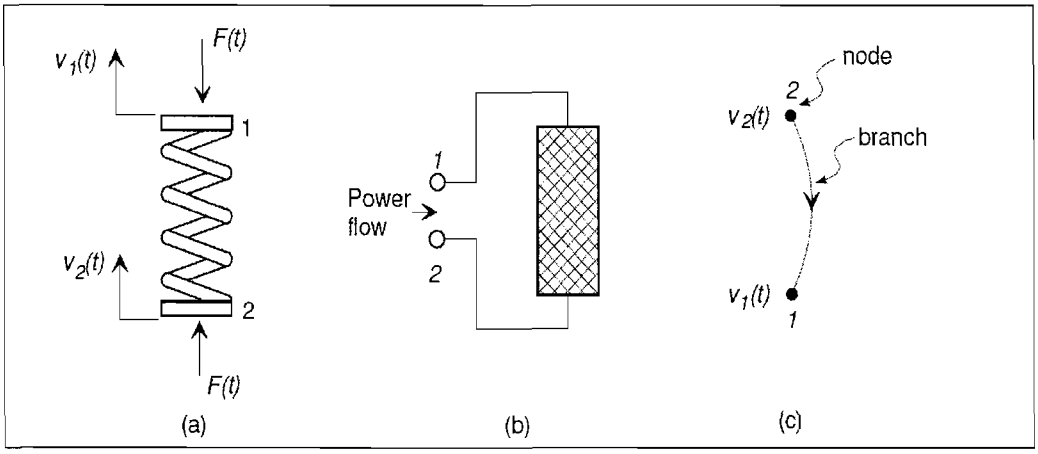
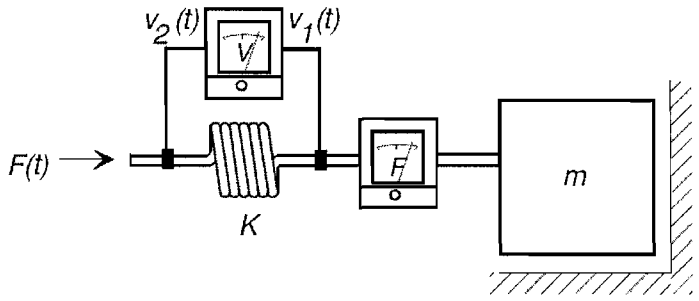
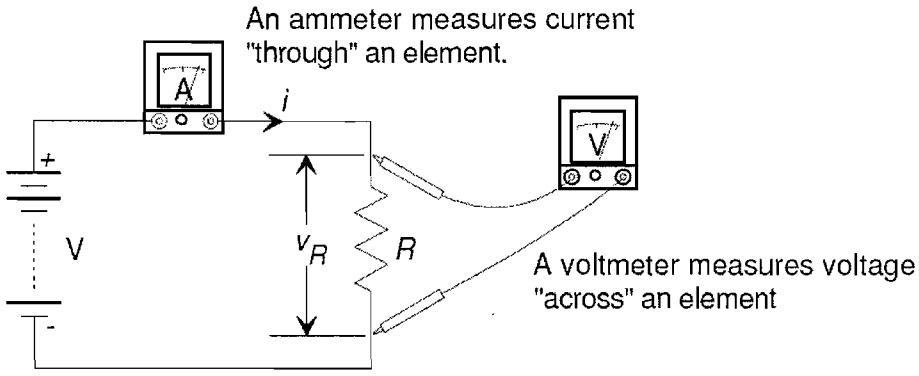


Figure 4.5: Illustration of (a) Mechanical spring, (b) Electrical Inductor, and (c) equivalent one-port network graph line segment [21].



(a)



(b)

Figure 4.6: Schematic illustration of (a) Mechanical System and (b) Electrical System concerning the through- and across-variables [21].



Figure 4.7: Illustration of a (a) through-variable and a (b) across-variable [21]

The study uses only two of the various analogies that exist in the engineering field namely, the electrical system analogy and the hydraulic analogy. As illustrated in figure 4.7 (b) the across-variable in an electrical system is the voltage drop across an element and the through-variable is the current flowing through the element. With hydraulic systems the across-variable is the pressure difference over an element and the through-variable is the flow rate through the element. Table 4.1 provides definition convention regarding the across- and through variables in each energy domain.

4.2.3. General Representation of the One-port Element

To be able to model one-port elements in various energy domains it is necessary to define a general set of elements and equations for these elements. For the across- and through-variables the following generalised notation is used in the network graph method [21]:

Generalised across-variable: v
Generalised integrated across-variable:

$$x = \int_0^t v dt + h(0) \tag{4.1}$$

Generalised through-variable: f
Generalised integrated through-variable:

$$h = \int_0^t f dt + h(0) \tag{4.2}$$

The power passing through the element can be represented by the following equation:

$$P = fv \tag{4.3}$$

and the work done by the element over a specific period of time can be determined by the following equation:

$$W = \int P dt \tag{4.4}$$

There are three fundamental elements in every energy domain namely, a storage element, a dissipative element and a source element. These elements are characterized as a certain "type" of element within the network graph system. The following section discusses these types of elements within the system when the network graph method is used to analyse the system [21].

4.2.4. A-type storage elements

Some of the storage elements are a function of the across-variable. For example, a capacitor is a function of its across-variable. This type of element is classified in the network graph method as the A-type storage element [21]. The energy within the A-type element can be written as:

$$E = \frac{1}{2} C v_A^2 \tag{4.5}$$

For the purpose of this study only the network relationship of the elements is discussed. The network equation for the A-type storage elements can be written as:

$$h_A = C v_A \tag{4.6}$$

where h_A represent the integrated through-variable and v_A is the across-variable. C, is known as the generalised capacitance of the A-type storage element. By differencing (4.6) the generalised A-type element equations can be written as:

$$f_A = C \frac{dv}{dt} \tag{4.7}$$

The lumped elements included in the A-type category includes translational mass, rotational inertia, electrical capacitance, fluid capacitance and thermal capacitance [21]. For the purpose of the study only the electrical capacitance and the fluid capacitance is considered. Some elements which forms part of the A-type elements group are illustrated in figure 4.8 and the relationships are depicted in table 4.2.

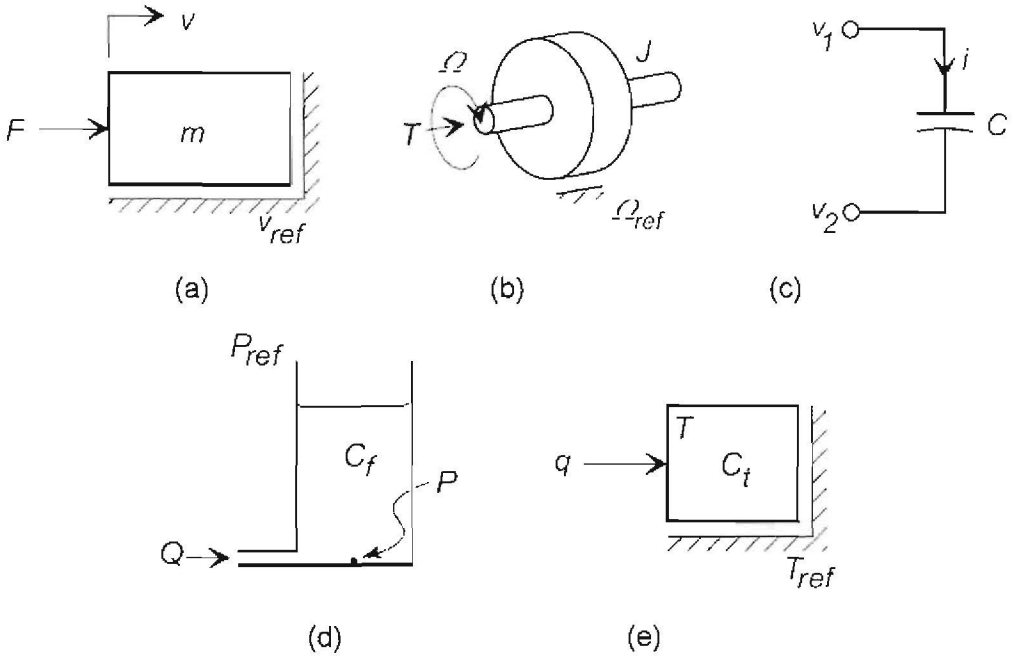


Figure 4.8: Schematic illustration of the groupe of A-type elements which inlcude (a) the translational mass, (b) the rotational inertia, (c) the electrical capcitor, (d) the fluid capacitance and (e) the thermal capacitance [21].

This study only addresses the electrical equivalent component for the hydraulic elements. The relationships between these two energy domains are depicted in table 4.1 and table 4.2.

Table 4.1: Definitions concerning the across-variables and through-variables within an electrical and fluid system [21].

System	**Across-variable**	**Through-variable**
Electrical	Voltage drop (v)	Current (i)
Fluid	Pressure Difference (p)	Volume flow rate (Q)
System	**Integarted across-variable**	**Integrated through-variable**
Electrical	Flux Linkage (λ)	Charge
Fluid	Pressure Difference Moment (Γ)	Volume (V)

Table 4.2: Summery of the relationships of the generalised A-type elemnets and the A-type elements in an electrical and fluid system [21].

System	**Constitutive equation**	**Elemental equation**	**Energy**
Generalised A-type Element	$h_A = Cv_A$	$f_A = C\frac{dv_A}{dt}$	$E = \frac{1}{2}Cv_A^2$
Electrical	$q = Cv_A$	$i = C\frac{dv_A}{dt}$	$E = \frac{1}{2}Cv_A^2$

System	Constitutive equation	Elemental equation	Energy
Fluid	$V = C_f P$	$Q = C_f \frac{dP}{dt}$	$E = \frac{1}{2} C_f P^2$

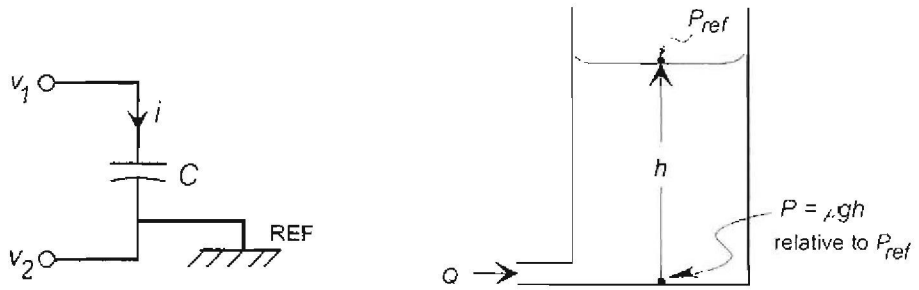


Figure 4.9: Illustration of typical reference connections to a reference node w.r.t. A-type elements (a) electrical capacitance, and (b) fluid capacitance [21].

It is evident from the type of elements included in the A-type category that these elements are capable of either absorbing power or supplying it. In the event where the power is greater than zero at the reference node, the power is flowing into the element, meaning that the element is both absorbing and storing the power. When the power measured at the reference node is smaller than zero, it indicates that the element is supplying power.

4.2.5. T-Type Storage Elements

Elements in which energy is stored can be expressed as a function of the through-variable and are known as the T-type elements. The generalised notation for these elements is inductance [21]. The T-type element's constitutive equation is written as follows:

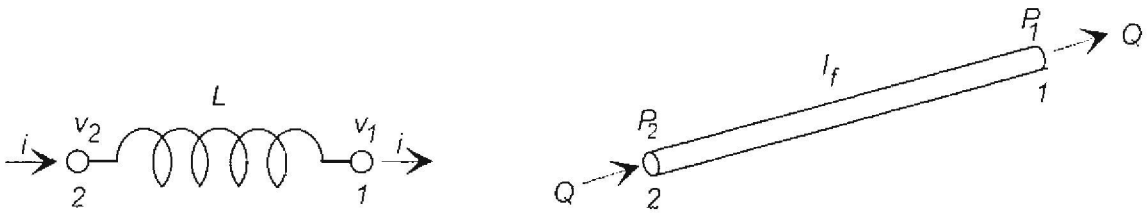
$$x_T = F(f_T) \tag{4.8}$$

where x_T represent the integrated across-variables of the system, and f_T the through-variable of the element. This means that the across-variable of the T-type element is a monotonic function of the through-variable. Equation (4.8) can be written in a simpler fashion, namely:

$$x_T = L_i f_T \tag{4.9}$$

with L_i as the generalised inductance. The differentiation of the simpler constitutive equation (4.9) gives the generalized elemental equation:

$$v_T = L_i \frac{df}{dt} \tag{4.10}$$



(a) (b)

Figure 4.10: T-Type elements in the (a) electrical domain (Inductor) and (b) fluid domain (Fluid Inertance) [21]

Table 4.3: The table provide a summary of the T-type elemental relationships [21]

System	Constitutive equation	Elemental equation	Energy
Generalised D-type Element	$x_T = L_i f_T$	$v_T = L_i \frac{df}{dt}$	$E = \frac{1}{2} L_i f_T^2$
Electrical	$\lambda = L_i i$	$v_T = L_i \frac{di}{dt}$	$E = \frac{1}{2} L_i i^2$
Fluid	$\Gamma = I_f Q$	$P = I_f \frac{dQ}{dt}$	$E = \frac{1}{2} I_f Q^2$

4.2.6. D-Type Dissipation Elements

The D-type elements represent elements that dissipated energy. The elements do not store energy and only transform energy into heat. The D-type element is defined by an algebraic relationship between the across- and through-variables and the constitutive equation of the D-type element is expressed in the following form [21]:

$$v_D = F(f_D) \tag{4.11}$$

where v_D represent the integrated across-variables and f_D the through-variable of the system. As with the T-type element is the D-type elements across-variable also a monotonic function of their through-variable. The simpler constitutive equation with R as the generalised resistance component can be expressed as follows:

$$v_D = R f_D \tag{4.12}$$

or

$$f_D = \frac{1}{R} v_D \tag{4.13}$$

Table 4.4: The table provide a summary of the T-type elemental relationships [21]

Element	Constitutive equation	Energy
Generalised T-type Element	$f_D = \frac{1}{R} v_D \quad v_D = R f_D$	$P = \frac{1}{R} v_D^2 = R f_D^2$
Electrical	$i = \frac{1}{R} v_D \quad v_D = Ri$	$P = \frac{1}{R} v_D^2 = Ri^2$
Fluid	$Q = \frac{1}{R_f} P \quad P = R_f Q$	$P = \frac{1}{R_f} p^2 = Q^2 R_f$

As previously discussed, the across-variable of the above-mentioned types of element must be defined with respect to a constant reference value. For instance, suppose a mass element exists. The velocity on the mass element is then defined with respect to the constant velocity inertial reference structure. The branches representing these type elements must therefore have one node connected to the reference point. In the network graph each branch contains an arrow that indicates the sign convention of the through- and across-variables. The direction of the arrow is drawn as follows:

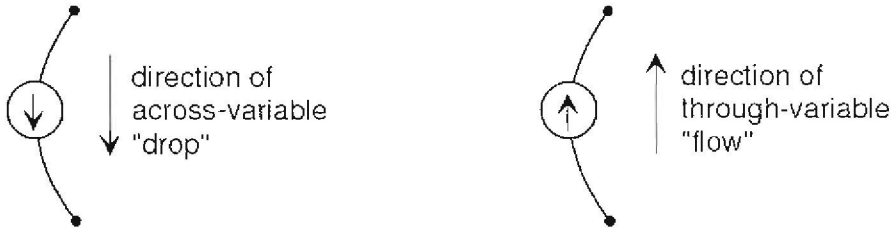
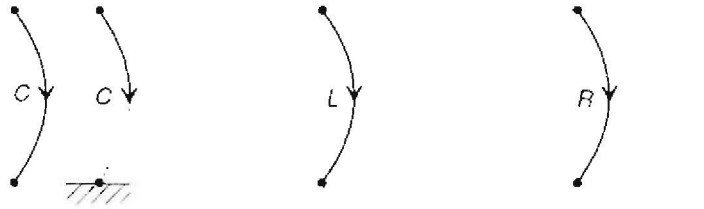


Figure 4.11: The direction of the arrow in the (a) across-variable and in the (b) through-variables [21]

As illustrated in figure 4.7 the across-variable v, is indicated as negative. This is in the direction of the presumed voltage drop. The through-variable f, is defined as positive in figure 4.7. When the source and the through-variables have the same power flow direction it means that the power is flowing into the element and therefore it is positive. The direction of the arrows in the elements is a convention to indicate the direction of the flow of current or fluid flow through the system. It simplifies the task of generating the representative system graph.



(a) A-type elements (b) T-type elements (c) D-type element

Figure 4.12: Various types of elements within the Network Graph system [21]

4.2.7. Causality

Elemental equations define the primitive elements in the system. These elemental equations are related via the through-variables and the across-variables present in the equations. These relations represent the actual constraints between the across-variable and the through-variables within the system. The analyst must ensure that the elemental elements are an equivalent representation of the real system. The two variables cannot be specified simultaneously and must be specified independently form each other. One of the variables must be specified by the system or external input and the other variable must be specified by the elemental equation. Thus, causality represent the convention where the direction of the energy flow, flowing through the branch or element has been determined [21].

In the energy storage element the relation between the through-variables and the across-variables is expressed as a differential or integral relationship, which in turn defines the element as having a derivative or integral causality [21]. For example, suppose an element *m* existed which represent the mass of a system. If the mass was driven by velocity *v (t)* the elemental relationship of the element is usually written as:

$$F = m\frac{dv}{dt} \tag{4.14}$$

The above equation determines the force required to drive the mass at a predefined velocity, *v(t)*. To determine the through-variable *F(t)*, the equation requires the differentiation of the velocity *v*, and is then said to have a derivative causality. On the other side of the coin, suppose the element was driven by a predefined force *F(t)*, the resulting velocity of the mass can be determined with the following elemental equation [21]:

$$\frac{dv}{dt} = \frac{1}{m}F \qquad or \qquad v(t) = \frac{1}{m}\int_0^t F dt + v(0) \tag{4.15}$$

This elemental equation is known as the integral causality form of the elemental equations. Dissipative elements like electrical resistors, always operates in the algebraic causality because the through– and across-variables are related by algebraic equations. The causality concept becomes important when the analyst develops the system models of the interconnected elements. The causality of the system is determined by the system structure of the interconnected system elements.

4.2.8. The Interconnection Laws of the Network Graphs

The fundamental purpose of the network graph method is to enable the user to generate a graph that represents the structure of a system model. This section describes the general interconnection laws relating to the across-variables and the through variables.

4.2.8.1. Compatibility Laws

The compatibility law represents a set of constraints on across-variables in the graph that may be related to physical laws that govern the interconnection of lumped elements. Compatibility equations are required to be developed for every closed loop within the developed network graph. Consider figure 4.13 which illustrates a simplified electrical system and its equivalent network graph.

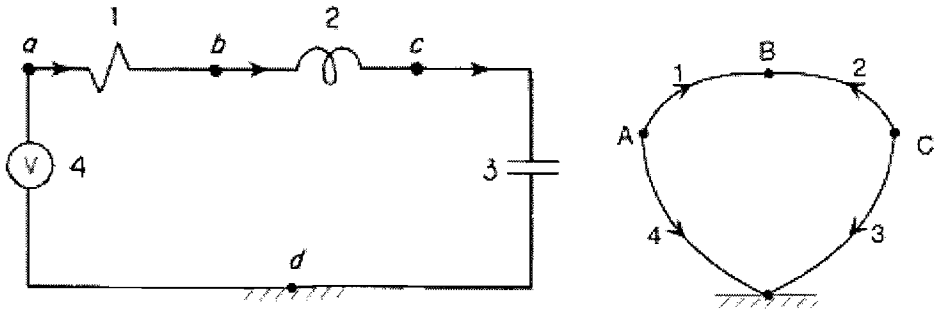
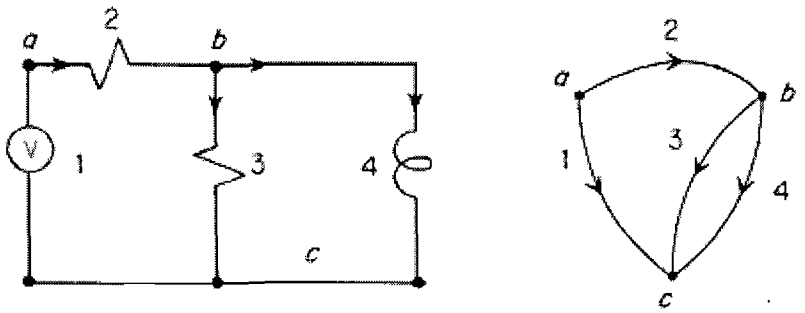


Figure 4.13: Simplified schematic of a series electrical circuit with its equivalent network graph



(b)

Figure 4.14: Simplified schematics of parallel electrical circuit system with its equivalent network graph.

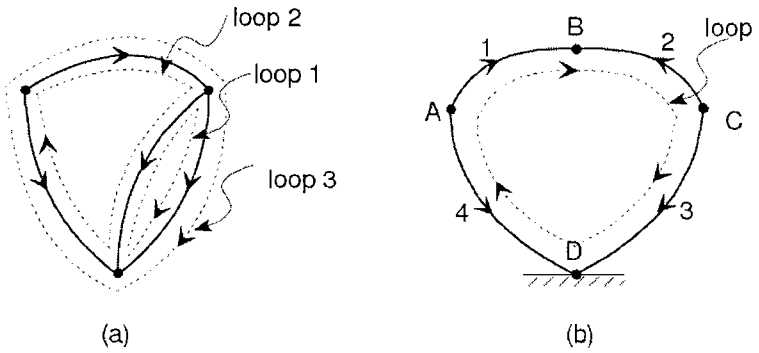


Figure 4.15: Possible loop configurations within the network graphs [21]

As illustrated in figure 4.15 (a), three closed loops can be identified within the network graph of the parallel network and in figure 4.15 (b) only one loop can be identified within the series network. The sum of the across-variables on the branches within a closed loop equals zero [21]. The representative equation can be written in the following form:

$$\sum_{i=1}^{N} v_i = 0 \tag{4.16}$$

for *N* elements within the closed loop.

It is now possible to develop a compatibility equation for any closed loop within the network graph, and this includes the inner and outer loops as illustrated in figure 4.15. The branches indicates the direction of the across-variable, thus the term concerning the branches is assigned in accordance with the direction of the across-variable's arrow. If the arrow on the branch points in the same direction as the arrow of the source, the term is positive. If It is in the opposite direction, the sign of the term is negative [21]. For example consider figure 4.15 (b), the compatibility equation can be written in the following form:

$$\sum_{i=1}^{4} v_i = v_1 - v_2 + v_3 - v_4 \tag{4.17}$$

With this compatibility equation the system can be analysed further. The across-variable drop is the difference across the two nodes of the branch, for example; $v_1 = v_A - v_B$. We can apply this rule to all terms/components in the compatibility equation [21]. The compatibility equation can be written as follows:

$$\sum_{i=1}^{4} v_i = (v_A - v_B) - (v_C - v_B) + (v_C - v_D) - (v_A - v_D) = 0 \tag{4.18}$$

The compatibility law refers to physical laws within each energy domain. In an electrical system the compatibility law is identical to Kirchoff's voltage law. The law states that the summation of voltage drops around a closed loop should be equal to zero. For a fluid system the compatibility law represent the law of energy conservation where the pressure is a scalar potential which must sum to zero around any closed path in a fluid system.

4.2.8.2. Continuity

As discussed in the previous section, the compatibility law specifies the constraints on the across-variable within the system. The constraints on the through-variable are specified by the continuity laws. This law relates physically to the law that governs the interconnection between the elements [21]. The continuity law can be written in the following form:

$$\sum_{i=1}^{N} f_i = 0 \qquad \text{for any } N \text{ Branches} \tag{4.19}$$

As illustrated by the above equation, the sum of the through-variables flowing into the closed loop must be equal to zero. Figure 4.16 illustrates the convention:

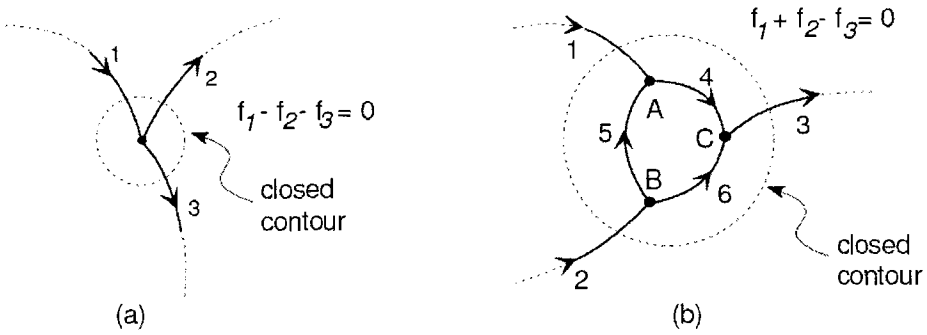


Figure 4.16: Illustration of the continuity laws within a closed loop [21]

As illustrate in figure 4.16 the continuity law represent the current law where the through-variables moving towards the closed contour is positive and the through-variable moving away from the closed contour is negative. The continuity law also dictates the fact that the sum of the through-variables moving towards and from the drawn closed loop must equal zero. Consider figure 4.16 (b) to illustrate the concept:

$$\begin{aligned} f_1 - f_4 + f_5 &= 0 \quad \text{at node A} \\ f_2 - f_5 - f_6 &= 0 \quad \text{at node B} \\ -f_3 - f_4 + f_6 &= 0 \quad \text{at node C} \end{aligned} \tag{4.20}$$

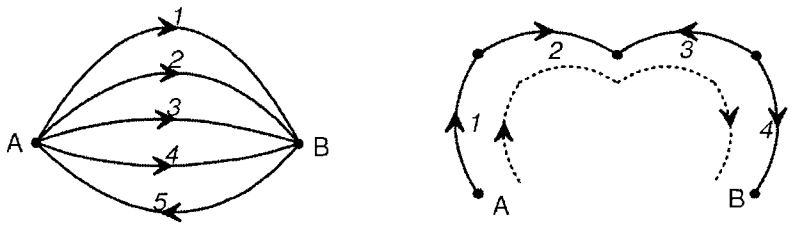
For the contour enclosing all three nodes, the sum of the through variables is:

$$f_1 + f_2 - f_3 = (f_4 - f_5) + (f_5 + f_6) - (f_4 + f_6) = 0 \tag{4.21}$$

This convention is valid for any node where the accumulation of the through-variable is possible. If the principle was not valid at that node it would suggest that the sum of the through-variable at that node is not zero, meaning that the node either store or dissipate energy, acting summarily to a primitive elements [21]. The contnuity law refers to physical laws within each energy domain. In an electrical system the contnuity law is identical to Kirchoff's current law. The law states that the summation of currents flowing into a node should be equal to zero. For a fluid system the continuity law refers to the requirement where the sum of the volume flow rate into a junction must be zero.

4.2.8.3. Series and Parallel Connections of Elements

This section discuss the possibility for elements to be connected in either series of parallel. Figure 4.17 illustrates this two possible connections of elements within a network graph.



(a) Parallel connection (b) Series connection

Figure 4.17: Schematic Illustration of (a) a parallel connection and (b) a series connection [21]

In figure 4.17 (a) five branches are connected in parallel with each other. This means that the branches all have the same starting node and ending node. By considering the direction of the energy flow (arrows) within each branch it is possible to write the compatibility equations in the following form:

$$v_1 = v_2 = v_3 = v_4 = -v_5 \tag{4.22}$$

Similarly, it is possible to write the contnuity equations for node B:

$$f_1 + f_2 + f_3 + f_4 - f_5 = 0 \tag{4.23}$$

The through-variable within parallel connected elements is divided amongst the elements between the two shared nodes. The element also share a common across-varaible because they are connected in parallel. In the series connected configuration the continuity condition equation can be written as follows:

$$f_1 = f_2 = -f_3 = f_4 \tag{4.24}$$

As illustarted by the above equation the arrows dictates the sign convention of each element in the equation. The element f_1 dictates the direction of the general energy within the system as illustrated in figure 4.17 (b). The energy within branch f_3 flows in the opposite direction therfore the sign covention for the element is negative [21].

In the event where the elements, are connected in serie and forms part of a loop, the sum of each individual potential drops over each branch, must be equal to the total value of the across-varaible between node A and B. That means:

$$v_{AB} = v_1 + v_2 - v_3 + v_4 \tag{4.25}$$

Unlike the parallel connected elements where the through-varaible is divided between the parallel connected elements, the series connected elements share a common through-variable [21].

4.2.9. Sign Convention of the Elements within a Network Graph

To illustrate the sign covention of elements within a network graph, consider the following simple electrical circuit where the circuit contains a DC voltage source and a resistor. Figure 4.18 illutrates the electrical ciruit and also the equivalent network graph with the arrows, indicating the flow direction of the energy within each branch [21].

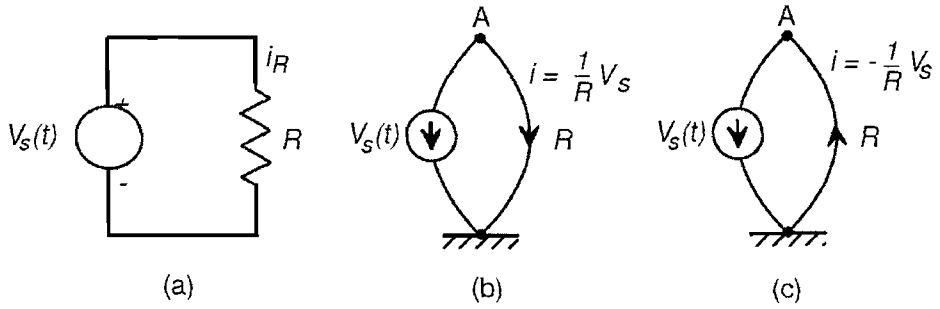


Figure 4.18: Schematic illustration of a simple electrical circuit and its equivalent network graphs indicating various sign convention possibilities [21].

As illustrated in figure 4.18 (a) the positive pole of the voltage source is north of the element and the negative pole is south, meaning that the energy flow is upwards. The system has only two nodes, namely, node A as the positive pole of the voltage source and node B as the reference node, abritrarly chosen as the negative pole of the voltage source. The arrow direction of the source is in the direction of the voltage drops, which is pressumed to be positive with the positive energy flowing from the negative pole to the positive pole. Therefore as illustrated in figure 4.18 (b), the across-variable points downwards.

The sign convention for the D-type element (resistor) is randomly assigned and there exist two possible sign configurations, namely in a positive direction as illustrated in figure 4.18 (b) and a negative direction, as illustrated in figure 4.18 (c). In figure 4.18 (b) the direction is assumed in the positive direction of the general energy flow within the system [21]. The compatibility equation for the graph can be written as:

$$\begin{aligned} -V_s &= v_R \\ -V_s + v_R &= 0 \end{aligned} \tag{4.26}$$

with the elemental equation for the D-type element as $v_R = Ri_R$, which give expression to the current following through the energy dissapating element. The current flowing through the resistor can then be expressed as:

$$i_R = \frac{V_s}{R} \tag{4.27}$$

When the through-variable is positive it means that the power flowing through the element is positive. The power is then flowing into the resistor, thus the result is a physical equivalent of the conditions illustrated in figure 4.18 (b).

In figure 4.18 (c) the flow of energy through the resistor is assumed to be in the oppiste direction of the general energy flow within the system.The compatibility equation for the graph can be written as:

$$\begin{aligned} V_s &= -v_R \\ V_s + v_R &= 0 \end{aligned} \tag{4.28}$$

The current flowing through the resistor can then be expressed as:

$$i_R = -\frac{V_s}{R} \tag{4.29}$$

When the through-variable is negative it means that the power flowing through the element is negative. This means that the power is flowing out of the resitor, thus the result is a physical equivalent of the conditions illustrated in figure 4.18 (c). When assuming the direction of the arrows, it is good pratice to direct all arrows concerning passive elements away from the source and towards the reference node within the system. The source should be defined is such a manner that it is comapatible with the convention of the source within the physical system.

4.3. NETWORK GRAPH GENERATING METHOD

The elements within a physical system are represented by interconnecting one-port branch elements within the network graph. The development of this equivalent system graph requires various modelling judgements, engineering conclusion and decisiveness from the analyst's side. When developing the equivalent model for a physical system it is required that person performing the modelling has an acute understanding of the physics involved in the energy domain being modelled. The general procedure for developing the equivalent network graph for a physical system can be summarised as follows [22]:

1. Define the system boundaries. These boundaries must include all the elements to be included as system inputs and outputs, relating to the applicable energy domain.
2. Generate a schematic representation of the physical system. Also assume a sign convention for the elements within the physical system.
3. Determine the characteristics with regards to each element in the physical system. Identify the appropriate system source elements, the energy storage elements and the dissipation elements.
4. Identify the across-variables and draw the set of nodes.
5. Insert each lumped elements into the graph.
6. Set the sign convention for the passive elements in accordance with the system source element. This should be consistent with the physical system. Draw the arrows.

Step 1 to 3 of this procedure is perhaps the most difficult part of generating an equivalent network graph for a physical system. This is because the analyst requires detail knowledge of the energy domain involved in the analysis. The analyst must draw the correct system relating assumptions and must make the correct system engineering judgements [22].

4.3.1. Hydraulic and electrical network graph examples

To illustrate the convention of the network graph method, the following example will clarify the concept. The most uncomplicated example is a conventional electrical system. Figure 4.19 illustrates the network graph of a typical electrical circuit.

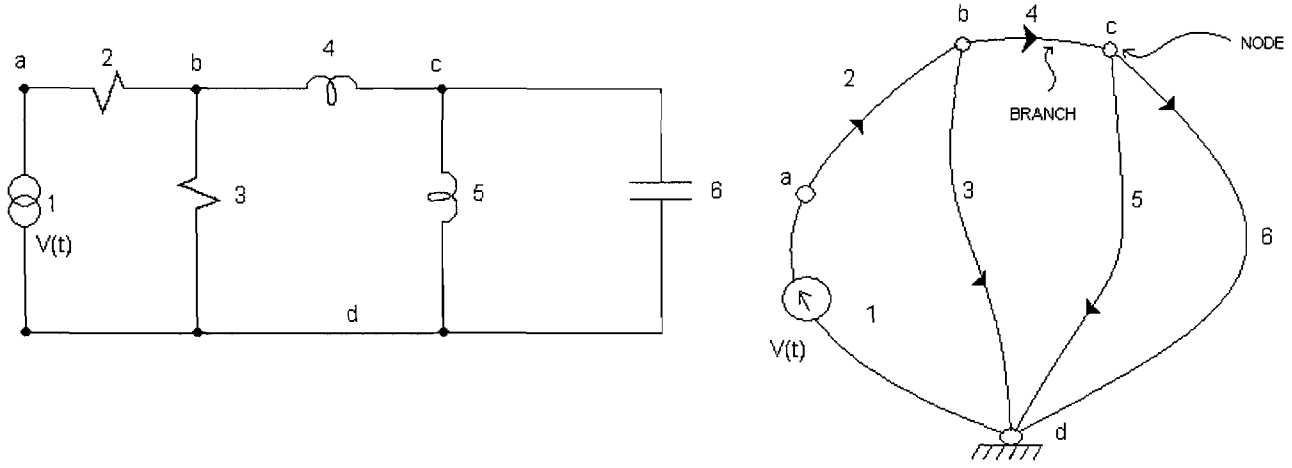


Figure 4.19: Illustrates the network graph of a typical electrical circuit [21].

Figure 4.19 illustrates a network graph that represents an electrical network which consists out of a single current source and with five one-port elements. In this case there are four nodes, which represent each distinctive point where a distinctive voltage or current can be measured. Node *d* is the reference node of the system. In the electrical systems the reference node usually represents ground or the zero-voltage point and in hydraulic systems the reference node represents the reference pressure from which all the other pressures in the system are measured.

Each port consists out of a branch between two appropriate nodes. With each branch a through-variable and across-variable is associated, where the through-variable is assumed to pass through the branch and where the across-variable represent the difference in potential value between the two nodes.

Figure 4.20 illustrates the equivalent network graph of a fluid system. In figure 4.20 the pressure due to the pump in the system is illustrated in the graph as the through-variable. The pipes are illustrated in the same manner as the graph would illustrate resistance. The pipe's resistance are the dissipative elements therefore they are D-type elements. Node *d* is the reference node which refers to the reference pressure used in the system. The components are as follows:

1. Pressure due to Pumps
2. Tank
3. Resistor
4. Inductor
5. Tank
6. Valve

Long pipes tend to have turbulent flow and therefore component 4 is indicated as an inductor, because the inductor is a direct analogy to the effect of turbulent flow.

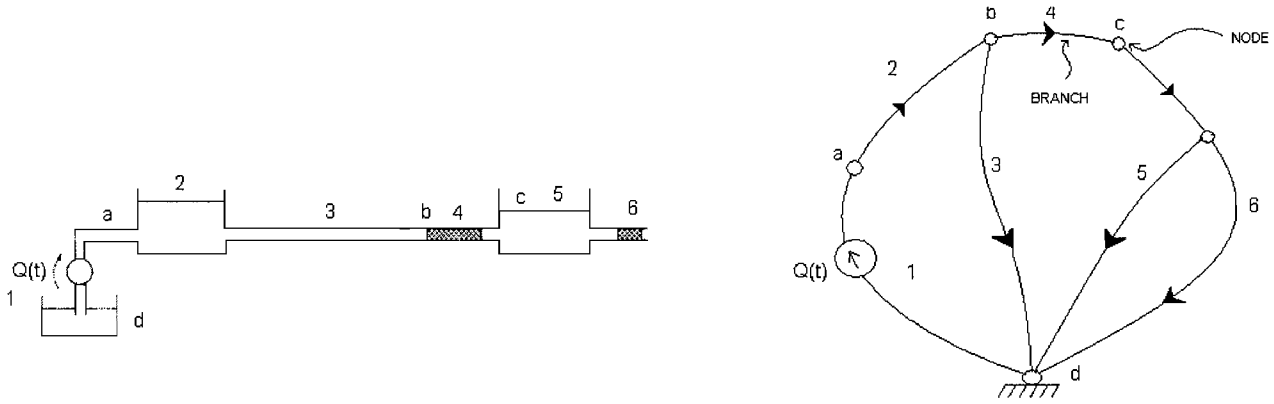


Figure 4.20: Fluid System Element [21]

4.4. FORMALISING THE STATE-SPACE MODEL

The network method has a number of well developed ways of deriving the state-space model of the system being analysed. For most of these methods the most efficient way of obtaining the state-space model for the system is to utilise the direct approach by combining the interconnective constraints, the constitutive relations and the dynamic relations of the system.

The fundamental concept of the network method is that in the system the flow store components and the effort store components form a set of state variables. The constitutive relation relates the accumulation variables to the terminal variables and therefore it became custom to use the terminal efforts of the flow stores and the terminal flows of the effort stores as the state variable is this specific analysis method [23].

By adopting this convention it is quite clear that certain redundancies can easily be seen. To explain this more clearly, suppose a loop exists which contains only flow store components and within this loop an effort source exists. The effort compatibility constraint around the loop will then be a network constraint for the terminal effort of the flow store components [23].

The choice of state variables and the possibility of redundancy dictate the state-space model. The formulation of the state variables must be in such a manner that the tree contains the following components, namely[23]:

- All the effort sources
- The maximum number of flow stores
- All the dissipators
- The minimum number of effort stores

The edge flow and effort vectors must be partitioned in terms of the tree and co-tree [23]:

$$e = \begin{bmatrix} e_t \\ e_c \end{bmatrix} \text{ and } f = \begin{bmatrix} f_t \\ f_c \end{bmatrix} \tag{4.30}$$

and the branch vectors [23] can be written as:

$$e_t = \begin{bmatrix} e_1 \\ e_{Ct} \\ e_{Rt} \\ e_{Lt} \end{bmatrix} \text{ and } f_t = \begin{bmatrix} e_1 \\ e_{Ct} \\ e_{Rt} \\ e_{Lt} \end{bmatrix} \tag{4.31}$$

The co-tree is represented by the links removed from the tree graph as discussed in Section 4.2. The co-tree vectors can be derived as follows [23]:

$$e_t = \begin{bmatrix} e_{Cc} \\ e_{Rc} \\ e_{Lc} \\ e_2 \end{bmatrix} \text{ and } f_t = \begin{bmatrix} e_{Cc} \\ e_{Rc} \\ e_{Lc} \\ e_2 \end{bmatrix} \tag{4.32}$$

The next step is to identify the cutset and loopset relations for the tree. The loopset consist out of a set of branches and nodes configured in such a way that each node in the loopset has a degree of two. This means that each node has two branches attached to it. Note that the nodes of the representative network graph of a physical system always have a degree of two. In the case of the example used in figure 4.14, the loopset is as follows:

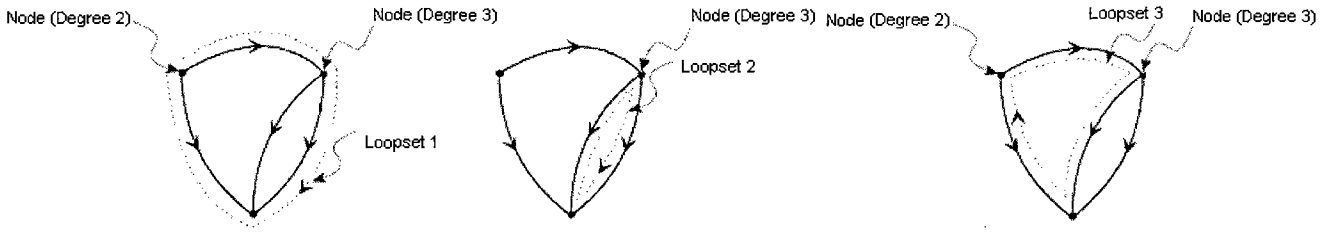


Figure 4.21: Illustrate the loopset convention [21]

The cutset represents the set of branches of the network graph, which when cut, results in the network graph being divided in two connected graphs. The relation between the two line segments with regards to the current flow is then identified. The cutset and loopset relation for the tree can then be expressed by utilising the following equation [23]:

$$e_c = H'e_t \ (cutset), \ f_t = -Hf_c(loopset) \tag{4.33}$$

where H is in the form of [23]:

$$H = \begin{bmatrix} H_{1c} & H_{1R} & H_{1L} & H_{12} \\ H_{CC} & H_{CR} & H_{CL} & H_{C2} \\ H_{RC} & H_{RR} & H_{RL} & H_{R2} \\ H_{LC} & H_{LR} & H_{LL} & H_{L2} \end{bmatrix} \tag{4.34}$$

where the matrices H_{RC}, H_{LC} and $H_{LR} = 0$. In addition to the interconnectivity relations that exist, the following component relations matrices also need to be identified [23]:

$$
\begin{aligned}
\begin{bmatrix} f_{Ct} \\ f_{Cc} \end{bmatrix} &= \begin{bmatrix} \mathrm{C_t}\ 0 \\ 0\ C_c \end{bmatrix} \frac{d}{dt} \begin{bmatrix} e_{Ct} \\ e_{Cc} \end{bmatrix} \\
\begin{bmatrix} e_{Lc} \\ e_{Lt} \end{bmatrix} &= \begin{bmatrix} L_c\ 0 \\ 0\ L_t \end{bmatrix} \frac{d}{dt} \begin{bmatrix} f_{Lc} \\ f_{Lt} \end{bmatrix} \\
\begin{bmatrix} e_{Rc} \\ e_{Rt} \end{bmatrix} &= \begin{bmatrix} R_c\ 0 \\ 0\ \mathrm{R}_t \end{bmatrix} \frac{d}{dt} \begin{bmatrix} f_{Rc} \\ f_{Rt} \end{bmatrix} \\
f_{Rc} &= R_c^{-1} e_{Rc} \\
f_{Rt} &= R_t^{-1} e_{Rt}
\end{aligned}
\tag{4.35}
$$

where $\mathrm{C_t}$ represent the capacitors in the tree, $\mathrm{C_C}$ signify the capacitor is the co-tree, $\mathrm{L_t}$ symbolize the inductors in the tree, and $\mathrm{L_C}$ stands for the inductors in existing in the co-tree. $\mathrm{R_t}$ represent the resistors in the tree and $\mathrm{R_C}$ indicates the resistors in the co-tree. If no redundant flow store components exists, $\mathrm{C_C} = 0$ and if no redundant effort store components exists in the loop, $\mathrm{L_t} = 0$. By combining (4.30) and (4.31) the resulting equations are [23]:

$$
\begin{bmatrix} C\ 0 \\ 0\ L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} e_{Ct} \\ f_{Lc} \end{bmatrix} = \begin{bmatrix} \mathrm{A_{11}}\ \mathrm{A_{12}} \\ \mathrm{A_{21}}\ \mathrm{A_{22}} \end{bmatrix} \begin{bmatrix} e_{Ct} \\ f_{Lc} \end{bmatrix} + \begin{bmatrix} \mathrm{B_{11}}\ \mathrm{B_{12}} \\ \mathrm{B_{21}}\ \mathrm{B_{22}} \end{bmatrix} \begin{bmatrix} e_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} \mathrm{D_{11}}\ 0 \\ 0\ \mathrm{D_{22}} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} e_1 \\ f_2 \end{bmatrix} \tag{4.36}
$$

where

$$
\begin{aligned}
C &= C_t + H_{CC} C_c H'_{CC} \\
L &= L_c - H'_{LL} L_t H_{LL} \\
A_{11} &= -H_{CR} (R_c + H'_{RR} R_t H_{CR})^{-1} H'_{CR} \\
A_{22} &= -H'_{RL} (R_t^{-1} + H_{RR} R_c^{-1} H'_{RR})^{-1} H_{RL} \\
A_{12} &= -H_{CL} + H_{CR} (R_c + H'_{RR} R_t H_{RR})^{-1} H_{1R} \\
B_{11} &= -H_{CR} (Rc + H'_{RR} R_t H_{RR})^{-1} H_{1R} \\
B_{22} &= -H'_{RL} (R_t^{-1} + H'_{RR} R_c^{-1} H'_{RR})^{-1} H_{RJ} \\
B_{12} &= -H_{C2} + H_{CR} (R_c + H'_{RR} R_t H_{RR})^{-1} H'_{RR} R_t H_{R2} \\
B_{21} &= H'_{1L} - H'_{RL} (R'_t + H_{RR} R_c^{-1} H'_{RR})^{-1} H_{RR} R_c^{-1} \\
D_{11} &= -H_{CC} C_c H'_{1C} \\
D_{22} &= -H'_{LL} L_t H_{L2}
\end{aligned}
\tag{4.37}
$$

In the following chapter, this method will be implemented. The state-space equation, derived as prescribed by the above discussion, will be used in the MATLAB® simulation and the results will be compared with the Flownex® simulation.

4.5. CONCLUSION

After evaluating the bond and network graph analysis method, it was decided to use the network graph method to analyse the RCCS system. Both of the methods are excellent methods to use, however the network graph method was found to have a more direct representation of the actual system. The bond graph's look is not a direct representation of the actual system and the user may have problems identifying the correct causality for each connection. The equation matrix illustrated in equation (3.9), for the bond graph method, grows exponentially. When an element is added within a close bond graph loop, the matrix developed from that matrix grows with two columns and two rows. Compared to the network graph, when an element is added the matrix only grow with one column and one row. It is for this reason that the decision was made to use the network graph analysis method. The rest of the study utilise this method to analyse the developed equivalent electrical circuit and to generate the intended state-space equation.

Chapter 5

5. Modelling of the double U-tube system

5.1. INTRODUCTION

In the previous chapters two different analysis methods have been discussed. After considering the two methods, it appeared that the network method would be more appropriate for the study's requirements. However, before any analysis work can commence on the system, it is necessary to understand the relationship between the electrical and hydraulic components. This chapter discusses the relation between the electrical and hydraulic energy domains, the development of the analogous electrical circuit and the simulation of these systems. The aim is to illustrate the steps required for the development of the equivalent electrical circuit for the reduced hydraulic system.

Almost all engineering systems are dynamic, meaning that there is no lasting steady-state condition for real systems. Even though the designing of a new system usually entails making assumptions and taking steady-state aspects into consideration, a dynamic analysis approach must be applied to the designed system to determine the effects of unforeseen transient peak loads. Stability analysis performed on a system also determines the capability of the system to withstand quick changes concerning inputs and the ability of the system to operate without violent oscillations when disturbed. System analysis enables the analyst to predict these problems before a system is physically built. In this case the Reactor Cavity Cooling System (RCCS) is such a system where a dynamic analysis was performed by using the computational fluid dynamics (CFD) program, named Flownex®.

5.2. THE REDUCED HYDRAULIC MODEL OF THE RCCS

A Typical dynamic system can be defined as a system which constitutes of a collection of matter, parts or components included inside a specific boundary [8]. The boundary of a dynamic system is the conceptual separation from its outside environment. The interaction by the system with its environment affects the behaviour of the system. Factors from outside the system's boundaries are the inputs into the system and these inputs leads to the resulting outputs of the system [8]. An example of such a dynamic system is the RCCS discussed in chapter 1. The system has momentum storage, mass storage and resistive components. The only effect from the environment outside the boundary of the RCCS, is the heating effect from the reactor pressure vessel (RPV).

To ensure proper understanding of a system we need to define the inputs and outputs which relates to the cause-and-effect relationship in the real physical system.

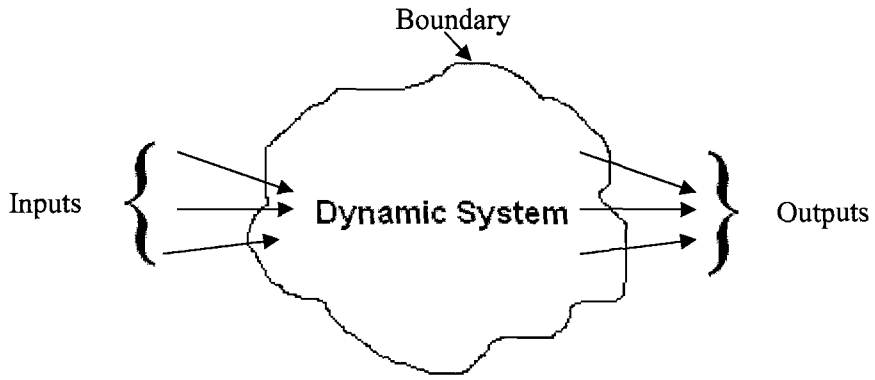


Figure 5.1: Illustration of a Dynamic System concerning the effect of the inputs w.r.t. outputs [33]

As discussed in Chapter 1, the RCCS contains a number of sub-systems which consists out of the pipe system illustrated in figure 5.2. From a modelling perspective, it will be sufficient to model only two of the four standpipes. Two standpipes are the minimum needed to describe the fundamental dynamics of natural convection. The configuration as indicated by the drawn block, in figure 5.2, will not form part of the analysis.

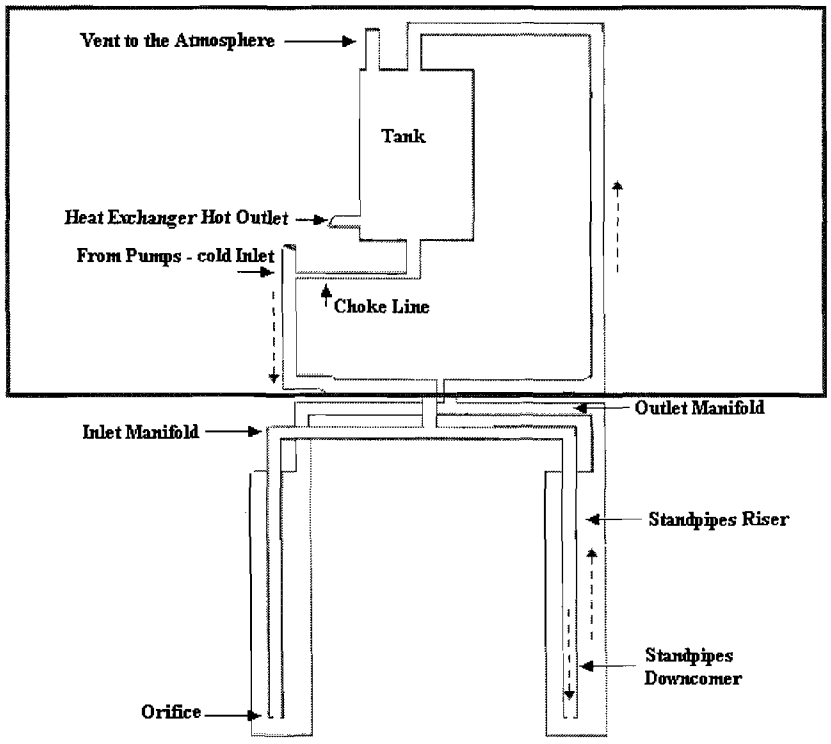


Figure 5.2: Reduced model of one of the RCCS pipe systems [2]

For the reduced hydraulic model of the RCCS, which is represented by the double U-tube, the only input is the heat from the RPV and the output is the natural convection generated by the density differences caused by the input. In the modelling process some assumptions will be made regarding the system section confined in the red block, as illustrated in figure 5.2. The thermal information which includes the heat transfer coefficient will be obtained from Flownex® and the derived electrical model will only characterize the hydraulic characteristics of the double U-tube system.

The analysis process will involve the formulation of a conceptual model containing all the basic hydraulic building elements which essentially represents the physical system. It is therefore of critical importance to identify these basic elements properly, to ensure that the model will behave practically in the same manner as the real system. The best model is the simplest one which still yields the required information [8]. It is for this reason that the model of the sub-system was simplified. The concept of natural convection stays true to the model and with the reduced RCCS system the same flow effects ought to be observed.

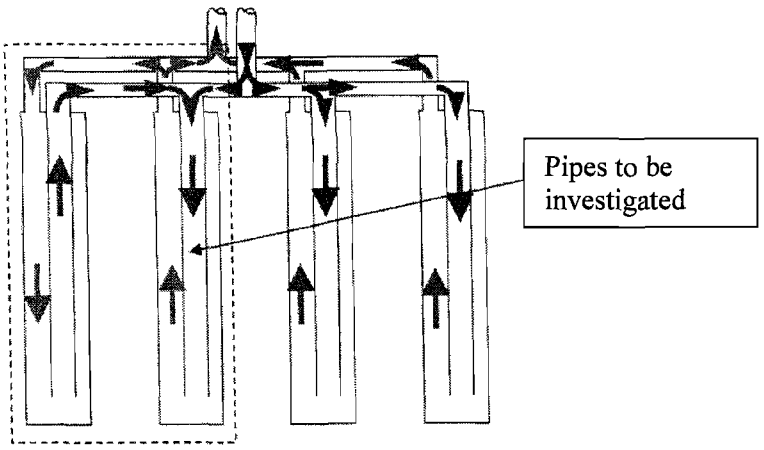


Figure 5.3: Double U-tubes under investigation [2]

5.2.1. The Flownex® model

Flownex® is a systems-computational fluid dynamics (SCFD) program developed by M-Tech Industrial (Pty) Ltd. The software package has been put through meticulous validation and verification tests, and is currently used by the PBMR as one of their prominent fluid simulation software packages. This bestows good credibility to the software. The SCFD or network approach, on which Flownex® is based, makes use of a collection of one-dimensional elements connecting to nodes in an unstructured manner as shown in Figure 5.4 [18].

The elements are symbolized by circles while the nodes are symbolized by squares. The elements can be any type of thermal-fluid component which include but is not limited to pipes, pumps, fans compressors or heat exchanger elements. The nodes can either just be a connection between two elements with no physical significance or it can have a volume, height, pressure or temperature in order to represent a reservoir, tank, manifold or any element equivalent to a fluid capacitance [18].

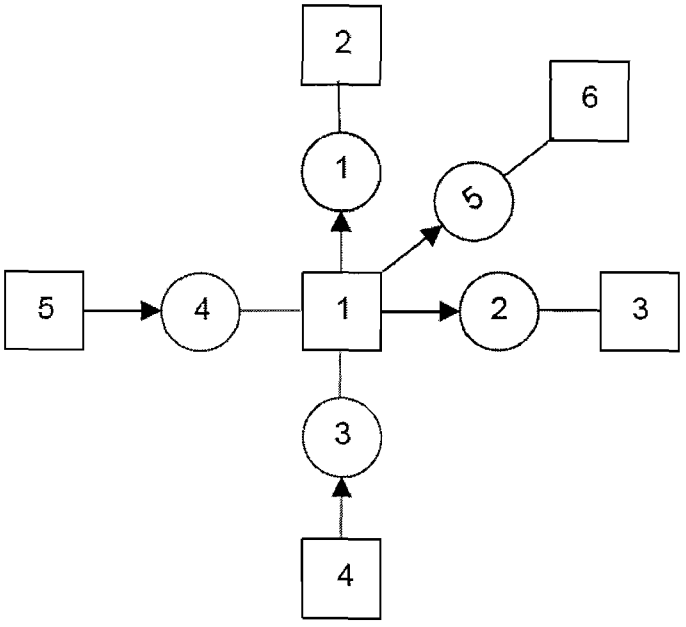


Figure 5.4: Typical configuration for a SCFD approach [18]

The CFD method of solving a dynamic fluid system entails the solving of the differential equations representing the conservation of mass, the conservation of momentum and the conservation of energy equations. Flownex® assumes that the properties of the fluids in a node are represented by a single average value. The conservation of mass and energy is node specific and that of momentum is element specific which serves as connections between the nodes [18].

Flownex® also uses certain conservation laws which are fundamental in the results calculated by the SCFD software package. By utilising the SCFD method the conservation of momentum is derived in terms of the conservation of the total pressure, and that of energy is derived in terms of the conservation of the total enthalpy. The following simplified conservation equations for fluids are applicable and used by Flownex® [18]:

Table 5.1: Fundamental conservation laws for incompressible fluids during steady state [18]

Conservation law	Equation
Mass	$\dot{m} = \dot{m}_e = \dot{m}_i$
Momentum	$p_{0e} - p_{0i} + p_{height} = -\Delta p_{0L}$

where $\dot{m}$ represent the mass flow rate in kg/s, $\dot{m}_{0i}$ and m_{0e} refers to the mass flow rate in and out of the system respectively and, p_{0i} and p_{0e} indicates the pressure at the inlet and the outlet respectively.

Fundamentally the natural convection phenomenon is caused by the condition where the buoyancy force of the fluid overcomes the friction forces inside the pipe. Thus, the mass flow rate through the U-tubes is a function of the buoyancy and frictional forces. The friction force can be represented as already illustrated in chapter 2, by the equation [18]:

$$A\Delta p_{ol} = A\left[\left(\frac{fL}{D} + \sum K\right)\frac{|\dot{m}|m}{2\rho A_{ff}{}^2}\right] \tag{5.1}$$

and the buoyancy force can be represented by the equation [9]:

$$Buoyancy\ Force = A\left(\left(\overline{\rho}\beta\Delta T\right)g\Delta z\right) \tag{5.2}$$

where Δz represents the height difference between the entrance and the exit areas, ΔT reefers to the temperature difference between the inlet and outlet areas and β is known as the volumetric coefficient of the thermal expansion of the substance ($\beta = \frac{1}{\rho}\frac{\partial\rho}{\partial T}\Big|_P$) [9,5,6]. The condition where the buoyancy force exceeds the frictional force causes the substance to flow in the direction of the buoyancy force. Even though it is not part of this study to calculate these parameters by hand, it is quite possible to do so. This can be achieved by calculating the energy balance of the system and the result can be used to calculate the corresponding densities and friction factors.

For the purpose of this study, only the conservation of momentum and the conservation of mass laws is considered. The conservation law of energy which involves the adding of energy to the system by means of heat, will automatically be taken into account by the Flownex® simulation.

The resulting substance densities in the pipes, due to the added heat energy, will be extracted from the Flownex® simulation results and used in the MATLAB® simulation of the electrical equivalent circuit. The change in density of the substance is a non-linear function which involves the conservation of the total enthalpy within the system. This does not form part of this study and will also be taken from Flownex®.

The abovementioned section discussed the reduced hydraulic system of the RCCS and a brief description of the inner workings of Flownex® was provided. The following section explains the development of the representative models of the single and double U-tube system in Flownex® and the development of the analogues electrical circuit.

5.2.1.1. Representing Flownex® model of the reduced hydraulic system

When simulating a system it implies that the characteristics of the system are known to the person performing the simulation. The reason why the simulation is performed is to predict the system's functionality and performance. For this study the result of the simulation serves to confirm the existence of natural convection and to verify the system's reaction when it occurs.

The best modus operandi to follow is to start with the simulation of the single U-tube pipe system, and after evaluating the simulation results extend the simulation to the parallel connected U-tube system, called the double U-tube. Figure 5.5 illustrates the configuration as it was set up in Flownex® for the single U-tube pipe system.

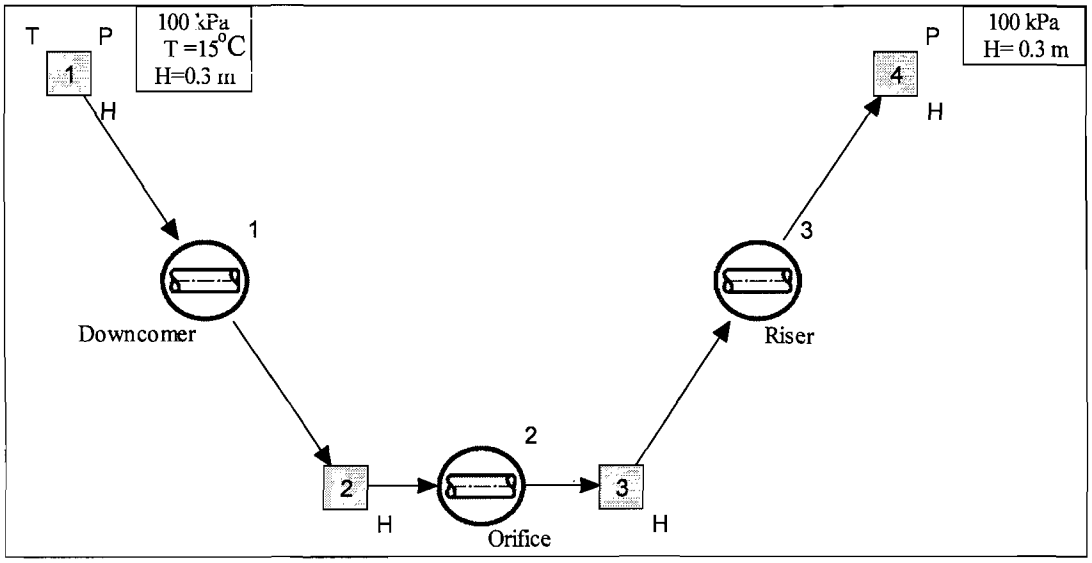


Figure 5.5: Flownex® configuration of the single U-tube pipe system

At node 1 the temperature, height and initial pressure was specified. At node 2 and 3 the height was specified as zero to provide an indication to the program that some elevation exists with regard to pipe element 1. At Node 4 the height and outlet pressure was specified.

This was done again to emphasise the elevation of pipe element 3 and to provide some reference outlet pressure. Pipe element 2 represents the orifice between the down comer and riser pipe. For each pipe segment the length and diameter was specified. It was also necessary to define the required number of increments Flownex® needed to divide the pipe segment into. Flownex® then performs discretisation analysis on each pipe segment during the simulation.

Discretisation analysis involves the process where the pipe segment is divided in small increments. Each increment is then separately evaluated by Flownex®. The purpose of this is that, due to the small size of the increment compared to the total length of the pipe, the increment can actually be evaluated under steady-state conditions. The impact non-linear factors have on the increment is then so insignificant it can be ignored. This simplifies the calculation required to determine the mass flow rate, pressure differences and density changes of the substance due to the added heat over the length of the pipe segment.

The result of each increment is then consolidated into the final result for the total length of the pipe. The higher the discretisation value for the pipe, the more accurate the result will be. There is however a down side to this, because as the discretisation value increases so does the time it takes to complete the simulation.

Figure 5.6 illustrates the double U-tube configuration used in Flownex® to perform the simulation of the reduced hydraulic system. It is clearly evident from figure 5.6 that the double U-tube configuration is two single U-tubes connected in parallel.

It was previously mentioned that assumptions were going to be made for the section of the system indicated by the red block in figure 5.2. Node 10 and 9 represent the inlet and outlet manifolds and for node 10 the temperature, pressure and height were assumed and for node 9, the pressure and height was depicted.

At node 2, 3, 6 and 7, which represent the orifices, the height was specified as zero to provide an indication to the program that some elevation exists with regards to pipe element 1, 3, 4 and 6. Pipe element 2 and 5 represent the orifices between the down comer, represented by pipe elements 1 and 4, and the riser pipes, represented by pipe elements 3 and 6, respectively.. The configuration in Flownex® is shown in figure 5.6:

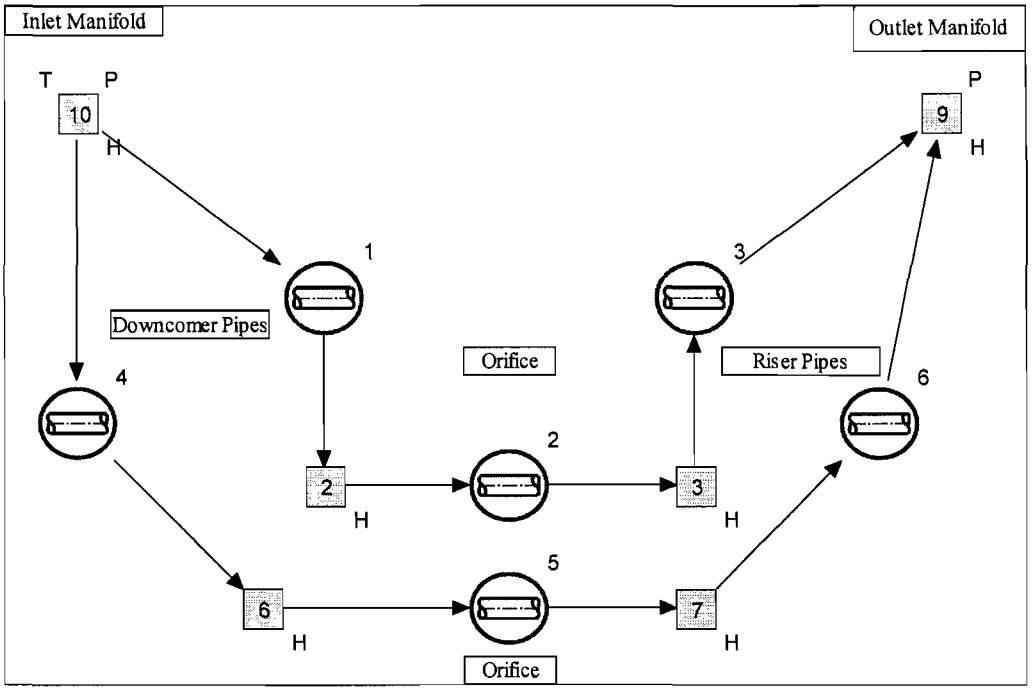


Figure 5.6: Flownex® model of the two U-tubes connected in parallel via manifolds

It is of critical importance to understand the natural convection phenomenon to ensure that all the fundamental aspects are identified for developing the equivalent electrical circuit. The rationale behind it is to develop an equivalent electrical circuit where by the same phenomenon can be observed with the current flowing through the circuit. The following section discusses the development of the equivalent electrical circuit for the single and double u-tube pipe systems, which represent the reduced hydraulic system.

5.3. DEVELOPMENT OF THE ANALOGUES ELECTRICAL SYSTEM

The aim is to develop the analogues electrical circuit and therefore verify the behaviour of the reduced hydraulic system of the RCCS when exposed to certain predefined parameters. The behaviour of electrical elements within an electrical circuit is very similar to the behaviour experienced within a system containing hydraulic elements. Before the equivalent electrical circuit can be developed, a thorough understanding of what role the voltage and current plays in a system, is required.

In an electrical circuit, voltage always coexists with current. As with a hydraulic system where work, energy and power are fundamental factors of the system, so are voltage and current fundamental factors within electrical systems. The voltage or potential difference between two nodes can be defined as the work required transporting a charged unit from one point to another.

Figure 5.7 is an illustration of a charged unit moving from one point to another due to potential difference that exists between the two points [21].

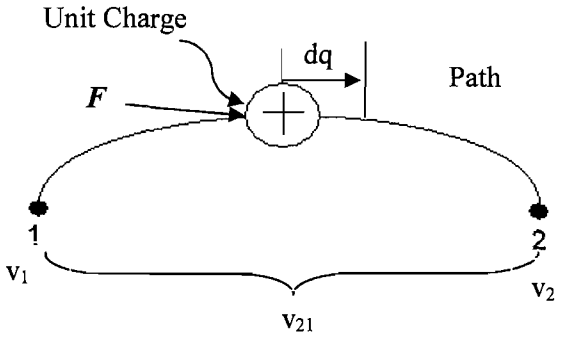


Figure 5.7: Unit Charge moving from one point to another due to potential difference between the points [21].

From the above figure it can be seen that $v_2 - v_1 = v_{21}$, meaning that v_{21} is positive when $v_2 > v_1$ and negative when $v_1 > v_2$. The amount of work done when moving the unit charge dq from point 1 to 2 can be written as [21]:

$$v_{21} = \frac{dW_{21}}{dq},\ v = \frac{J}{coul} \tag{5.3}$$

The above equation plays a fundamental part in linking the equivalent electrical circuit with the hydraulic system being modelled. The current can be defined as the rate of flow of the charged unit across a cross-sectional area of the wire between two points. Figure 5.8 illustrates the convention concerning the current flow through the wire [21].

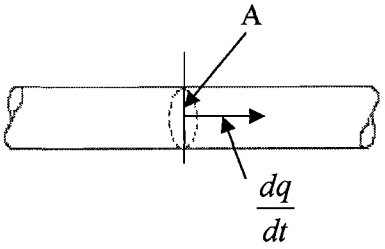


Figure 5.8: Convention concerning the flow of current through a wire [21]

Taking figure 5.8 into account, the current can be written as follows when the charge dq crosses the area A in the time [21]:

$$i = \frac{dq}{dt},\ Amp = \frac{coul}{\sec} \tag{5.4}$$

The current illustrated in figure 5.8 contains two components namely direction and magnitude. The arrow in figure 5.8 illustrates the direction of the flow of the current. If the flow of the current is in the direction of the unit charge, the current is positive.

In the event where the current flow in the opposite direction, the current will be negative. As with any system the most important part of the modelling process is to take the physics of the system into account. The ***principle of conservation of electric charge*** is one such aspect required to be taken into account when modelling any electrical circuit. What it means is, simply put, the charge entering point 1 must equal the charge leaving point 2. This conservation law for electrical systems is identical to the ***conservation law of mass*** within a hydraulic system. Electrical systems transmit current as hydraulic systems transmit force [18 & 21].

5.3.1. Analogue between hydraulic and electrical elements

The previous sections of this study have discussed a number of basic principles and fundamentals in the modelling of hydraulic and electrical systems. It is quite evident that the hydraulic and electrical elements display a number of similarities between certain elements. It is evident that hydraulic force can be substituted by electrical current, mass storage can be substituted by capacitance and hydraulic pressure can be substituted by voltage [21].

All relates back to the through- and across-variable concept. Force and current are variables that move through elements. Voltage and pressure are thought of as existing across two points. In the following sections the types of fluid elements will be discussed identifying the direct electrical analogous corresponding to the specific mechanical element [21].

5.3.1.1. Fluid pressure and flow rate

Fluid flow is the consequence of pressure being exerted on one side of a pipe. Pressure is thought of as the work done in passing a unit volume of fluid across an area upon which pressure exerts a force. As voltage and current coexist, pressure and the mass flow coexist as well. It is thus quite evident that pressure is directly analogous to electrical potential and the mass flow is directly analogous to the current. The mass flow rate is the variable which indicates the quantity of fluid passing through a pipe during a certain time span. It is also quite evident from this statement that mass flow rate is a through-variable in the same way as electrical current. To illustrate the analogous relation between pressure and electrical potential consider the following piece of element illustrated in figure 5.9. In the hydraulic domain this element represents a piece of pipe and in the electrical domain this element represents a piece of electrical wire.

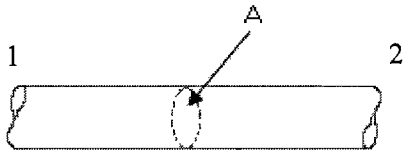


Figure 5.9: Element representing a piece of pipe or a piece of wire. [21].

The pressure and electrical potential across the element is the result of the difference of the pressure or potential difference between point 1 and 2. The pressure difference between the two points can be written as $\Delta P = P_2 - P_1$ and in the same way the potential difference between the two points can be written as $\Delta v = v_2 - v_1$. The direct analogous relation is therefore quite evident [21].

5.3.1.2. Fluid capacitance

Any type of element where fluid can be stored can be defined as a fluid capacitor. This may include but is not limited to tanks, reservoirs and even pipes. It is common sense to see that the fluid capacitance is directly analogous to electrical capacitance. Thus the capacitance of fluid with relation to electrical capacitance can be written as follows [34]:

$$C = \frac{\rho A L}{B} \tag{5.5}$$

where A represents the area of the fluid capacitor, ρ the fluid density, B the bulk modulus and L reefers to the length of the fluid capacitor. The bulk modulus represents the substance's resistance to uniform compression and it can be defined as the pressure required to cause a given relative volume to decrease in substance. The decrease in volume causes an increase in density of the substance. The bulk modulus of a substance can be read from a standard hydraulic table, containing the bulk modulus of certain substances at certain temperatures.

5.3.1.3. Fluid resistance

As with the fluid capacitance the resistance present in the pipe due to the roughness of the pipe and the viscosity of the fluid can be expressed as follows as an electrical resistance value [34]:

$$R = \frac{\left(\frac{fl}{D} + k\right)|\dot{m}||\dot{m}|}{2A^2 \rho g_C} \tag{5.6}$$

where D represents the hydraulic diameter of the pipe, l the length of the pipe, A the cross-sectional area of the pipe, k the secondary losses due to elbows, t-pieces etc, and f refers to the friction coefficient of the pipe. The rest of the parameters are already known.

5.3.1.4. Fluid inertia

Fluid inertia is the resistance to change by a substance within the pipe. Inertia is analogous to electrical inductance. Fluid inertia is thought of as the change in momentum of the fluid flow within the pipe. When the fluid considered is incompressible the flow rate in every section of the pipe is the same. The velocity of the fluid particles is uniform across any cross-section of the pipe. However in the case of the orifice, these assumptions are not valid anymore and therefore the orifice is represented by a resistor which represents the friction losses and an inductor representing the inertia that can be caused by turbulent flow. Fluid inertia can be represented as follows [34]:

$$I = \frac{L}{A} \tag{5.7}$$

where L represents the length of pipe, and A represents the cross-sectional area of pipe.

5.3.2. The equivalent electrical model

The first step in developing the equivalent electrical circuit is to identify the system specific component in the hydraulic system. The assumption is made that the top part of the RCCS, which includes the tank and the pipe extending from the tank to the manifolds, will be represented by a predefined pressure component. The pressure component will then be represented by a voltage source. The voltage source represents the initial pressure value transferred by the inlet and outlet manifolds from the top part of the RCCS to the double U-tube pipe system.

The friction in the pipe segment can be represented by a resistor and the inertia due to turbulence in the pipe can be represented by an inductor. The pressure that exists within the pipe due to the elevation of the substance in the pipe is represented by another voltage source. This source is calculated by utilising the following equation $\rho g_C h$. After analysing the circuit the following system components have been identified:

1. Inlet Pressure – The pressure is represented by a voltage source.
2. Outlet Manifold Pressure - The pressure is signified by a voltage source.
3. Pressure within tubes due to gravitational force and height – represented by a voltage source.
4. Friction factor – The friction factor is characterized by an electrical resistor.
5. Inertia – The inertia of the substance within the pipe is symbolized by an inductor.
6. Control volume, like a tank or reservoir – this is represented by an electrical capacitor.

The resistor which represents the friction within the pipe and the inductor, which in turn signifies the inertia in the pipe, is connected in series. The capacitance is connected in parallel and the voltage source representing the pressure due to the elevation of the pipe, is connected in series with these components. The voltage source V_{i1}, in figure 5.10, represents the reference pressure from the manifold due to the top pipe system and is also connected in series.

The voltage source V_{h1}, in figure 5.10, symbolize the pressure due to the elevation and gravitational force working in on the substance within the U-tube. It is therefore easy to see that the equivalent electrical circuit for the down comer pipe within the hydraulic system can be developed as illustrated in figure 5.10. The configuration illustrated in figure 5.10, can either be used to model the equivalent electrical circuit for the down comer or the riser pipe, respectively.

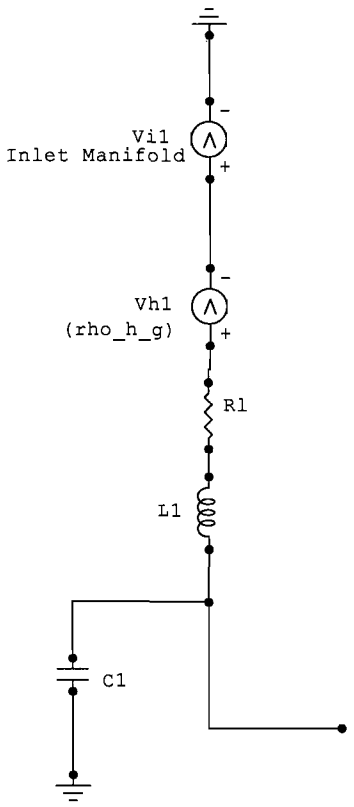


Figure 5.10: Equivalent electrical circuit for the down comer pipe in the U-tube pipe

By connecting two circuits with these characteristics as illustrated in figure 5.10, it is possible to generate the analogous electrical circuit for a single U-tube. Figure 5.11 illustrates this convention. The resistor and inductor connecting the two sides represent the orifice present in the hydraulic system. The inductor models the momentum due to the movement of the fluid. It is important to notice that both electrical circuits make use of the reference voltage sources, V_{i1} and V_{01}, which serves as the inlet and outlet manifold pressure respectively. All the other voltage sources in each circuit represent the pressure present in the tubes due to gravitational forces, $P = \rho gh$.

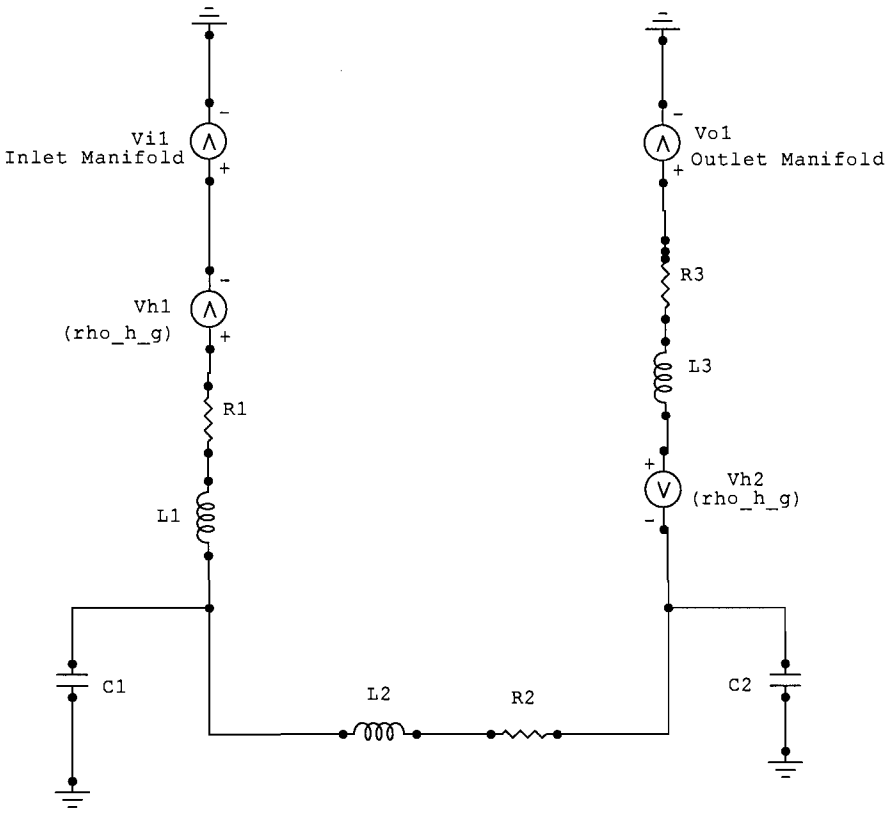


Figure 5.11: Electrical equivalent circuit for the single U-tube pipe system

By connecting two of the circuits as illustrated in figure 5.11 in parallel, it is possible to construct the double U-tube analogous electrical circuit for the reduced RCCS system. Figure 5.12 illustrates this convention:

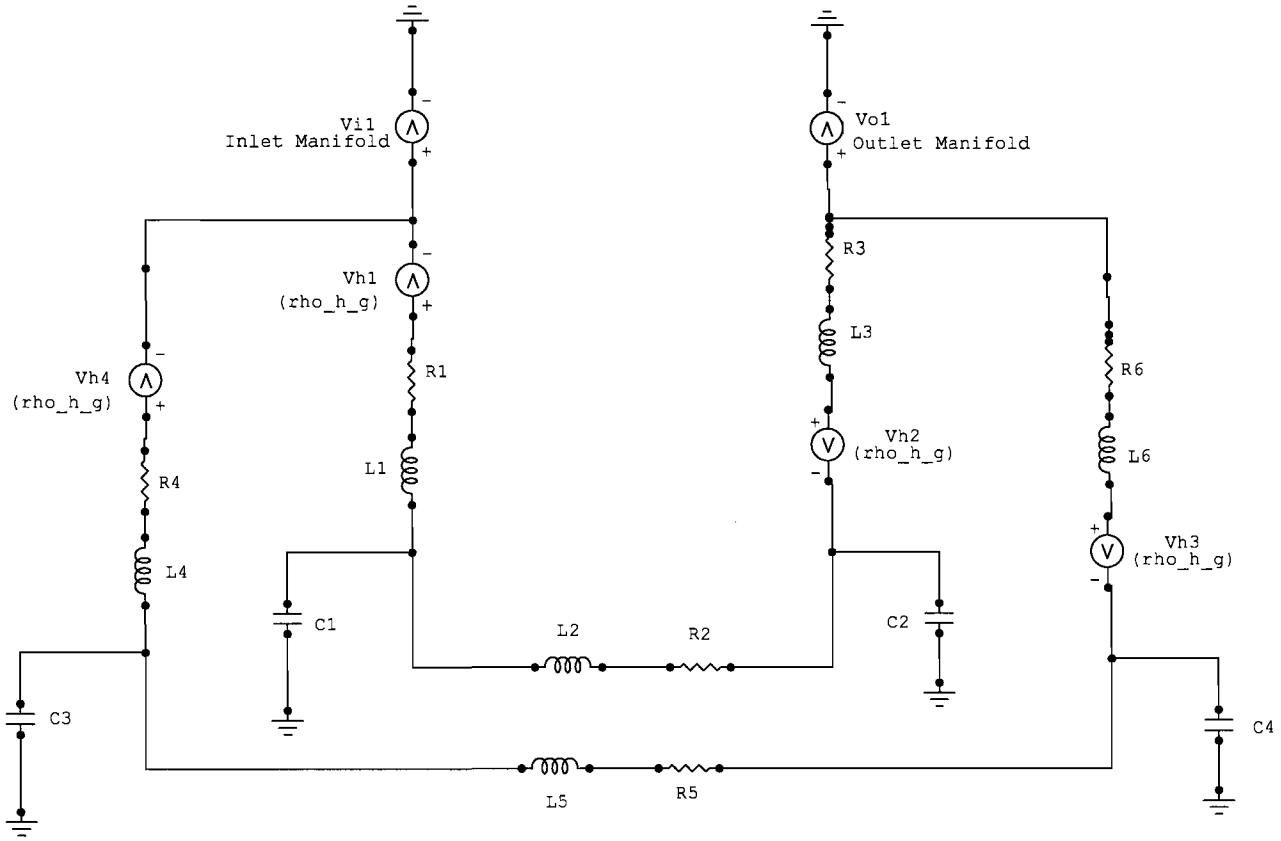


Figure 5.12: Electrical equivalent circuit for the double U-tube system

Figure 5.12 represent the analogous electrical circuit for the reduced hydraulic system of the RCCS as illustrated in figure 5.3. Each resistor, inductor and voltage source series connected component combination represent a pipe in the reduced RCCS hydraulic system. The next step in the analysis process is to determine the state-space equations of the analogous electrical circuit illustrated in figure 5.12.

5.3.3. Determining the state-space equations

To determine the state-space model for the equivalent electrical circuit, illustrated in figure 5.12, the analyst is required to develop the equivalent network graph for this circuit. The derivation of the state equation is straight forward from this point forward. The fundamental concept, dictating the process of generating the equivalent network graph successfully, is to identify the power flow within the branches and also the connections between the elements within the system, correctly [22]. The network graph for the electrical circuit illustrated in figure 5.12 is as follows:

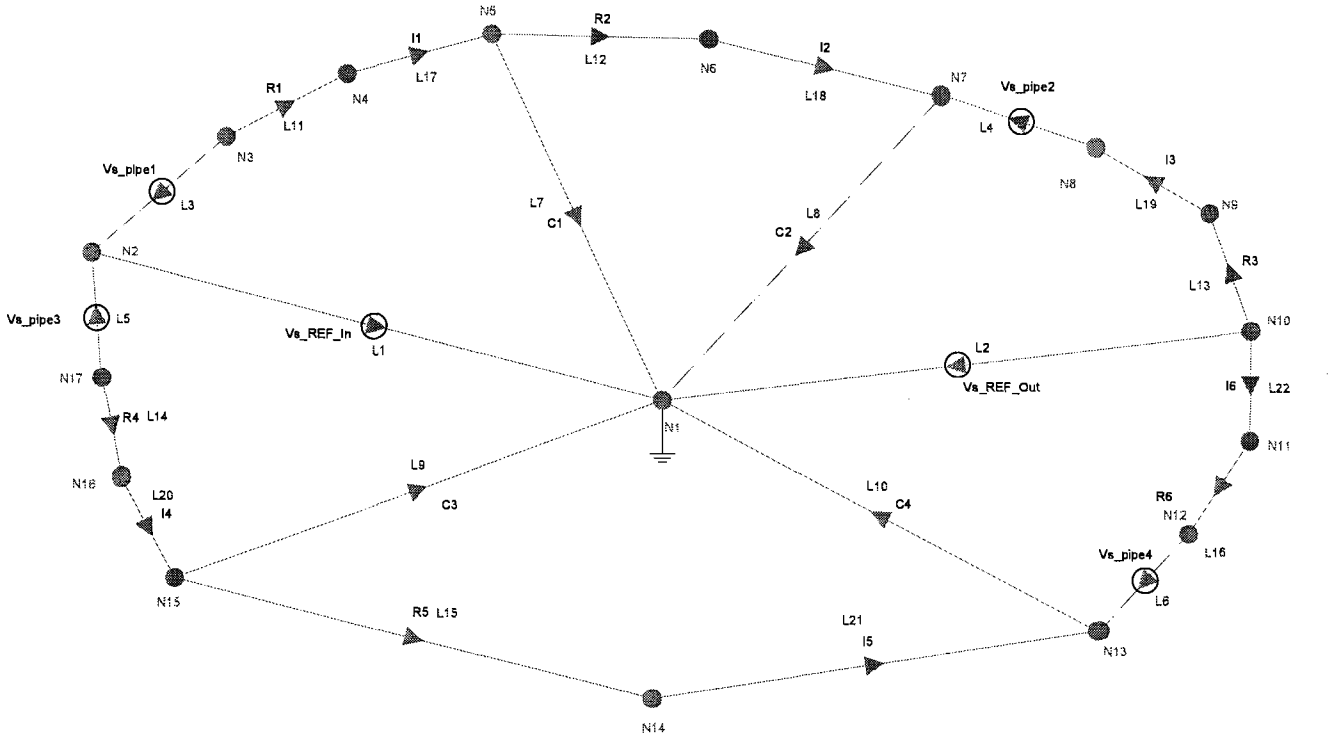


Figure 5.13: Network graph of the U-tube pipe system.

Node 1 ($N1$) of the network graph illustrated in figure 5.13 represents the ground or reference node of the system. The network graph consists out of six loops with twenty two line segments. Each line segment represents an element in the electrical circuit. Each arrow in each line segment indicates the flow direction of the current through the electrical component and this is straight forward.

With elements like the resistor and the inductor the arrow points in the direction of the current flow. It is a little trickier with storage elements, like capacitors, and sources. As illustrated in figure 5.14, with a voltage source the pass through variable is in the opposite direction than the direction of the potential difference.

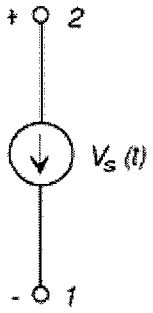


Figure 5.14: Illustration of the direction indication of the through variable in a voltage source [21]

Storage elements like capacitors also demonstrate the same convention as the sources, meaning that the current flows in the opposite direction of the potential difference over the capacitor elements as illustrated by figure 5.13. The next step is normalising the network graph. The normalised graph is as follows:

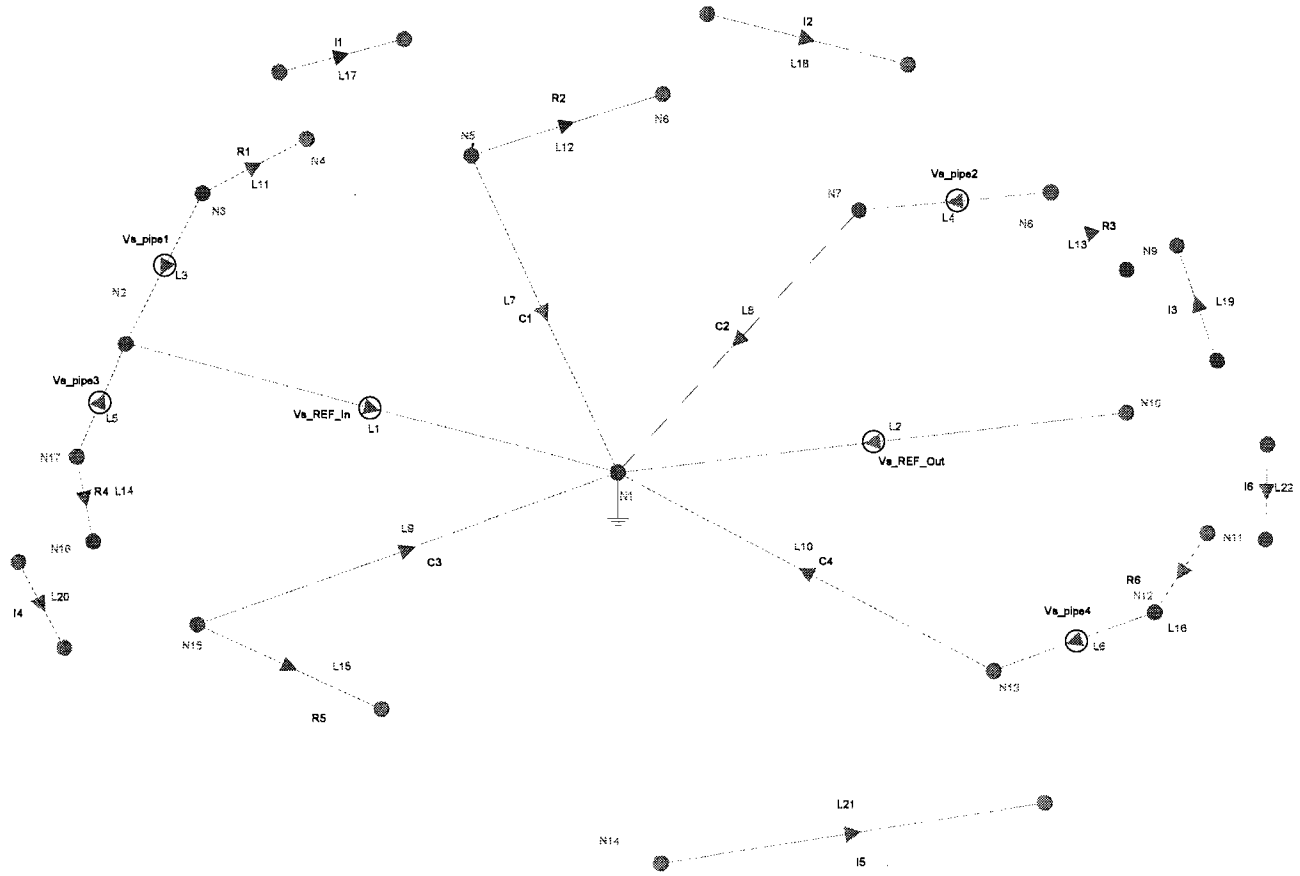


Figure 5.15: Normalised network graph consisting out of the co-trees and the tree.

The normalisation of the graph as illustrated in figure 5.15 is achieved by breaking all the existing loops in the network graph. It is quite clear from figure 5.15 that the A-type storage and D-type dissipative elements are still contained in the tree. The T-type storage elements which are the inductors are contained in the co-tree [23].

After generating the normalised network graph as illustrated in figure 5.15, the matrix which provides indication of the positioning of each element within the system, needs to be generated. If a branch x is connected to a node y and the through-variable, points away from the node y, the element is indicated by a positive "1" in the normalised matrix [23]. If a branch x is connected to a node y and the through-variable, points towards the node y, the element is indicated by a negative "-1" in the normalised matrix. If the branch x is not connected to the node y, the element is indicated by a "0" in the normalised matrix [23]. Thus, the normalised matrix for the normalised graph is as follows:

$$A = \begin{array}{c} \\ N1 \\ N2 \\ N3 \\ N4 \\ N5 \\ N6 \\ N7 \\ N8 \\ N9 \\ N10 \\ N11 \\ N12 \\ N13 \\ N14 \\ N15 \\ N16 \\ N17 \end{array} \begin{array}{c} \begin{array}{cccccccccccccccccccccc} L1 & L2 & L3 & L4 & L5 & L6 & L7 & L8 & L9 & L10 & L11 & L12 & L13 & L14 & L15 & L16 & L17 & L18 & L19 & L20 & L21 & L22 \end{array} \\ \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad (5.8)$$

The matrix above contains redundant information and can be reduced by removing the entries corresponding to the reference node. This matrix is also known as the reduced "normalised matrix" (Nodes-1). The reduced normalised matrix for the network graph can be expressed as follows:

$$A = \begin{array}{c} \\ N1 \\ N2 \\ N3 \\ N4 \\ N5 \\ N6 \\ N7 \\ N8 \\ N9 \\ N10 \\ N11 \\ N12 \\ N13 \\ N14 \\ N15 \\ N16 \end{array} \begin{bmatrix} L1 & L2 & L3 & L4 & L5 & L6 & L7 & L8 & L9 & L10 & L11 & L12 & L13 & L14 & L15 & L16 & L17 & L18 & L19 & L20 & L21 & L22 \\ 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.9)$$

After generating the reduced normalised matrix, the following step is to extract the matrix representing the "tree" and the "co-tree". Nodes 1 to 16 represent the "tree" and nodes 17 to 22 represent the co-tree of the network graph [23]. The "tree" matrix is as follows:

$$Tree = \begin{array}{c} \\ N1 \\ N2 \\ N3 \\ N4 \\ N5 \\ N6 \\ N7 \\ N8 \\ N9 \\ N10 \\ N11 \\ N12 \\ N13 \\ N14 \\ N15 \\ N16 \end{array} \begin{bmatrix} L1 & L2 & L3 & L4 & L5 & L6 & L7 & L8 & L9 & L10 & L11 & L12 & L13 & L14 & L15 & L16 \\ 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (5.10)$$

and the "co-tree" matrix includes all the links and is as follows [23]:

$$CoTree = \begin{array}{c} \\ N1 \\ N2 \\ N3 \\ N4 \\ N5 \\ N6 \\ N7 \\ N8 \\ N8 \\ N10 \\ N11 \\ N12 \\ N13 \\ N14 \\ N15 \\ N16 \end{array} \begin{array}{c} \begin{bmatrix} L17 & L18 & L19 & L20 & L21 & L22 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \tag{5.11}$$

The next step in the process of determining the state-space model is to develop the fundamental cutset matrix Qf. The fundamental cutset matrix is developed in terms of the currents (i.e., the through-variables) within the system and is equivalent to Kirchoff's current equations. The Qf matrix's rows correspond to the (N-1) fundamental cutsets, and the column is generated with reference to the network graph branches [23]. The Qf matrix is as follows:

$$Qf = \begin{array}{c} \\ N1 \\ N2 \\ N3 \\ N4 \\ N5 \\ N6 \\ N7 \\ N8 \\ N9 \\ N10 \\ N11 \\ N12 \\ N13 \\ N14 \\ N15 \\ N16 \end{array} \begin{array}{c} \begin{bmatrix} L1 & L2 & L3 & L4 & L5 & L6 & L7 & L8 & L9 & L10 & L11 & L12 & L13 & L14 & L15 & L16 & L17 & L18 & L19 & L20 & L21 & L22 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \tag{5.12}$$

From the fundamental cutset matrix it is possible to develop the continuity matrix H. This matrix is developed in terms of the currents flowing through the elements and is equivalent to the current equations of Kirchoff [23]. The rest of the Qf matrix consists out of the identity matrix I. The continuity matrix, H is as follows:

$$H = \begin{array}{c} \\ N1 \\ N2 \\ N3 \\ N4 \\ N5 \\ N6 \\ N7 \\ N8 \\ N9 \\ N10 \\ N11 \\ N12 \\ N13 \\ N14 \\ N15 \\ N16 \end{array} \begin{array}{c} \begin{bmatrix} L1 & L2 & L3 & L4 & L5 & L6 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array} \tag{5.13}$$

By utilising the equations defined in equations 4.35, 4.36 and 4.37, the state-space model can be derived quite easily. With the help of MATLAB® the state-space model was derived and the model can be expressed as follows:

$$A = \begin{bmatrix} 0, & 0, & 0, & 0, & 1/C1, & -1/C1, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 1/C2, & -1/C2, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1/C3, & -1/C3, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1/C4, & 1/C4 \\ -1/L1, & 0, & 0, & 0, & -1/L1*R1, & 0, & 0, & 0, & 0, & 0 \\ 1/L2, & -1/L2, & 0, & 0, & 0, & -1/L2*R2, & 0, & 0, & 0, & 0 \\ 0, & 1/L3, & 0, & 0, & 0, & 0, & -1/L3*R3, & 0, & 0, & 0 \\ 0, & 0, & -1/L4, & 0, & 0, & 0, & 0, & -1/L4*R4, & 0, & 0 \\ 0, & 0, & 1/L5, & -1/L5, & 0, & 0, & 0, & 0, & -1/L5*R5, & 0 \\ 0, & 0, & 0, & -1/L6, & 0, & 0, & 0, & 0, & 0, & -1/L6*R6 \end{bmatrix} \tag{5.14}$$

$$B=\begin{bmatrix} 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 1/L1, & 0, & 1/L1, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 1/L3, & 0, & 1/L3, & 0, & 0, & 0 \\ 1/L4, & 0, & 0, & 0, & 1/L4, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 1/L6, & 0, & 0, & 0, & 1/L6, & 0 \end{bmatrix} \quad (5.15)$$

Thus the state-space equation can be written as follows:

$$\dot{\mathrm{X}}=\mathrm{A}\mathbf{x}+\mathrm{B}\mathbf{u} \quad (5.16)$$

where $\mathbf{x}$ represents the state vector $\left[x_1 \cdots x_n\right]^T$, $\dot{X}=\frac{\partial x}{\partial t}$ the derivative of the state vector $\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{bmatrix}$ and

$\mathbf{u}$ refers to the input vector $\left[u_1 \cdots u_n\right]^T$ [26]. Where the state vector component

x_1 represents the initial inlet manifold pressure for U-Tube #1,

x_2 shows the initial inlet manifold pressure for U-Tube #2,

x_3 signifies the initial pressure for the outlet manifold for U-tube #2,

x_4 represents the initial pressure for the outlet manifold for U-tube #2,

x_5 indicates the initial mass flow rate in the down comer pipe in U-tube #1,

x_6 represents the initial mass flow rate in the orifice pipe in U-tube #1,

x_7 embodies the initial mass flow rate in the riser pipe in U-tube #1,

x_8 represents the initial mass flow rate in the down comer pipe in U-tube #2,

x_9 stands for the initial mass flow rate in the orifice pipe in U-tube #2,

x_{10} shows the initial mass flow rate in the riser pipe in U-tube #2 and where the input vector components u_1 symbolizes the input value for the inlet manifold pressure,

u_2 represents input value for the outlet manifold pressure,

u_3 signifies input value for the pressure in the down comer pipe for U-tube #1,

u_4 indicates input value for the pressure in the riser pipe for U-tube #1,

u_5 represents input value for the pressure in the down comer pipe for U-tube #2, and

u_6 represents the input value for the pressure in the riser pipe for U-tube #2.

The only way to make sure that the above derived state-space model is correct is to validate the model. This is done by utilising the model in a simulation and then comparing the results to the simulation results of Flownex® simulation. The following section discusses the simulation of the hydraulic system by means of Flownex® and the simulation of the developed analogues electrical circuit by means of MATLAB®.

5.4. THE FLOWNEX® AND MATLAB® SIMULATIONS

5.4.1. Non-linear aspects in the MATLAB® simulation

To ensure that the simulation is representative of the real system, some non-linear aspects are required to be included in the MATLAB® simulation. In chapter 2, the natural convection phenomenon was discussed in steady-state for the purpose of understanding the fundamental forces responsible for the phenomenon. However, when simulating the real system, the assumptions made in chapter 2 are no longer valid. The state-space model derived is dynamic. This means that:

$$\frac{\partial}{\partial t}\iiint_{PS}\rho d\forall \neq 0 \tag{5.17}$$

resulting in the fact that $\frac{\partial \dot{m}}{\partial t} \neq 0$ either. Thus with reference to equation (2.23) the mass flow rate of the system can be written as:

$$\frac{\partial \dot{m}}{\partial t} = \frac{A}{L}(p_2 - p_1) + \Delta p_{ol}\frac{A}{L} + \frac{A}{L}\rho z \Delta g_C \tag{5.18}$$

where with reference to equation (5.6) and (5.7), the equation for the mass flow rate can be rewritten as:

$$\frac{\partial \dot{m}}{\partial t} = \frac{1}{I}(p_2 - p_1) + R\frac{1}{I} + \frac{1}{I}\rho z \Delta g_C \tag{5.19}$$

where $\frac{A}{L} = \frac{1}{I}$, $\Delta p_{ol} = R = \frac{\left(\frac{fl}{D} + k\right)|\dot{m}|\dot{m}}{2A^2\rho g_C}$ and the source voltage for the pipe increment is represented by $\rho z \Delta g_C$. Even though the values used for the capacitance is linear, due to the fact that the steady state results from the Flownex® simulation are used to calculate the capacitance, the equation (5.5) used in the MATLAB® simulation is still very dynamic in nature. As mentioned previously the density of the substance is a function of the temperature and the change in pressure is a function of the change in density. The coefficient of the volume expansion due to the temperature difference can be written as follows:

$$\begin{aligned}\beta &= \frac{1}{\forall}\left(\frac{\partial \forall}{\partial t}\right) \\ &= -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t}\right)_P\end{aligned} \tag{5.20}$$

and in terms of finite changes the above equation can be written as:

$$\begin{aligned}\beta &\approx \frac{\Delta \forall / \forall}{\Delta \mathrm{T}} = -\frac{\Delta \rho / \rho}{\Delta T} \\ &= \frac{\rho_1 - \rho_2}{\bar{\rho}(T_1 - T_2)} \\ \therefore\ \rho_1 - \rho_2 &= \bar{\rho}(T_1 - T_2)\beta\end{aligned} \tag{5.21}$$

The result is the buoyancy component which is proportional to the density [9]. This result can be used to expand the mass flow rate equation even more to increase the level of non-linearity present in the MATLAB® simulation. The mass flow equation can be written in the following manner:

$$\frac{\partial \dot{m}}{\partial t} = \frac{1}{I}(p_2 - p_1) + R\frac{1}{I} + \frac{1}{I}\bar{\rho}(T_1 - T_2)\beta z \Delta g_C \tag{5.22}$$

As can be seen from this final equation, the buoyancy force is proportional to the density differences, which in turn is proportional to the temperature differences at a constant pressure [18]. The above equation is used in the MATLAB® simulation to determine the mass flow rate.

The programs in Appendix 8.1, 8.2, 8.3 and 8.4 were developed in MATLAB® to provide a tool to evaluate and validate the developed electrical circuit illustrated in figure 5.11. The program depicted in Appendix 8.1 and 8.3 derives the state-space model by utilising the equations described in chapter 4 for the single and double U-tube systems, respectively. The program described in Appendix 8.2 and 8.4, uses the derived state-space model and solves the model for the conditions dictated in table 5.1 for the single and double U-tube systems, respectively. The methodology used in the MATLAB® simulation is:

1. Identify the system parameters,
2. Calculate the resistor, capacitor and inductor values for the system,
3. Utilise the state-space model with the calculated value in the MATLAB® solver.

The following sections describe the simulations performed in Flownex® and MATLAB®. The section also compares and discusses the results obtained from the simulations.

5.4.2. Natural convection simulations for the single U-tube pipe system

In this section the simulation of the single U-tube and double U-tube pipe systems are discussed. The simulation results from Flownex® are then compared to the simulations performed for the electrical circuit by means of MATLAB®. The boundary parameters for the first simulation are purely experimental as dictated by table 5.2.

The specified values serve as the initial values of the system and through utilising the transient simulation function of Flownex®; the system determines what the flow rate ought to be. The flow rate is calculated proportional to the temperature changes within the system. The outlet temperature will be affected by the added heat and if not specified, Flownex® will calculate the real-time outlet temperature. The mass flow rate was calculated by Flownex® by using the initial values which were specified as follows:

Table 5.2: Experimental System Parameters used for the simulations performed in Flownex®

Parameter	Initial Value
Inlet Temperature(T_{inlet})	15 ^{o}C
Inlet Pressure (P_{inlet})	100 kPA
Diameter of orifice	0.01m
Height of orifice	0 m
Length of orifice	0.001m
Gravitational constant (g_C)	9.81m/s^2
Height of the inlet manifold node	1 m
Height of the outlet manifold node	1 m
Diameter of down comer pipe	0.1 m
Length of down comer	1 m
Length of riser pipe	1 m
Diameter of riser pipe	0.1 m
Outlet pressure	100 kPA

Flownex® was used to calculate the required changes in mass flow and pressure. The heat transfer rate was also calculated by Flownex® by means of indicating a constant heat transfer from the RPV source. It was found when in steady-state, where no heat is added to the system, no mass flow was experienced. The next step was to add heat to the system, by specifying a constant heat transfer value in Flownex®, for pipe element 3. This is achieved by setting the heat transfer coefficient in Flownex® to 1 kW for pipe element 3.

After the simulation it was quite evident that natural convection did take place in the U-tube pipe system. Figure 5.16 confirms that a temperature difference was experienced in pipe element 3. For the specific configuration the results obtained are as follows:

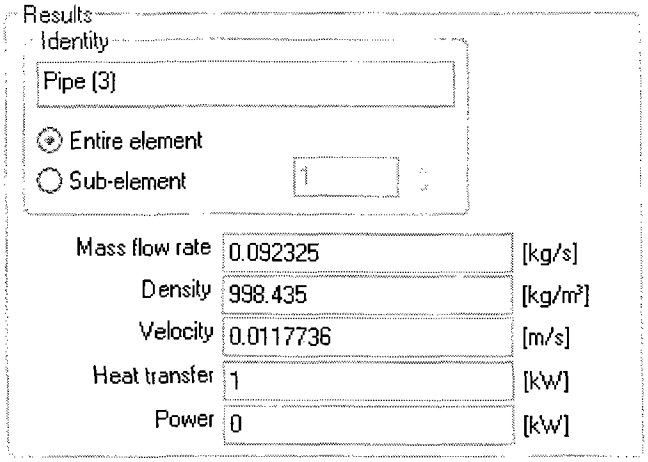


Figure 5.16: Result of the single U-tube simulation performed in Flownex®

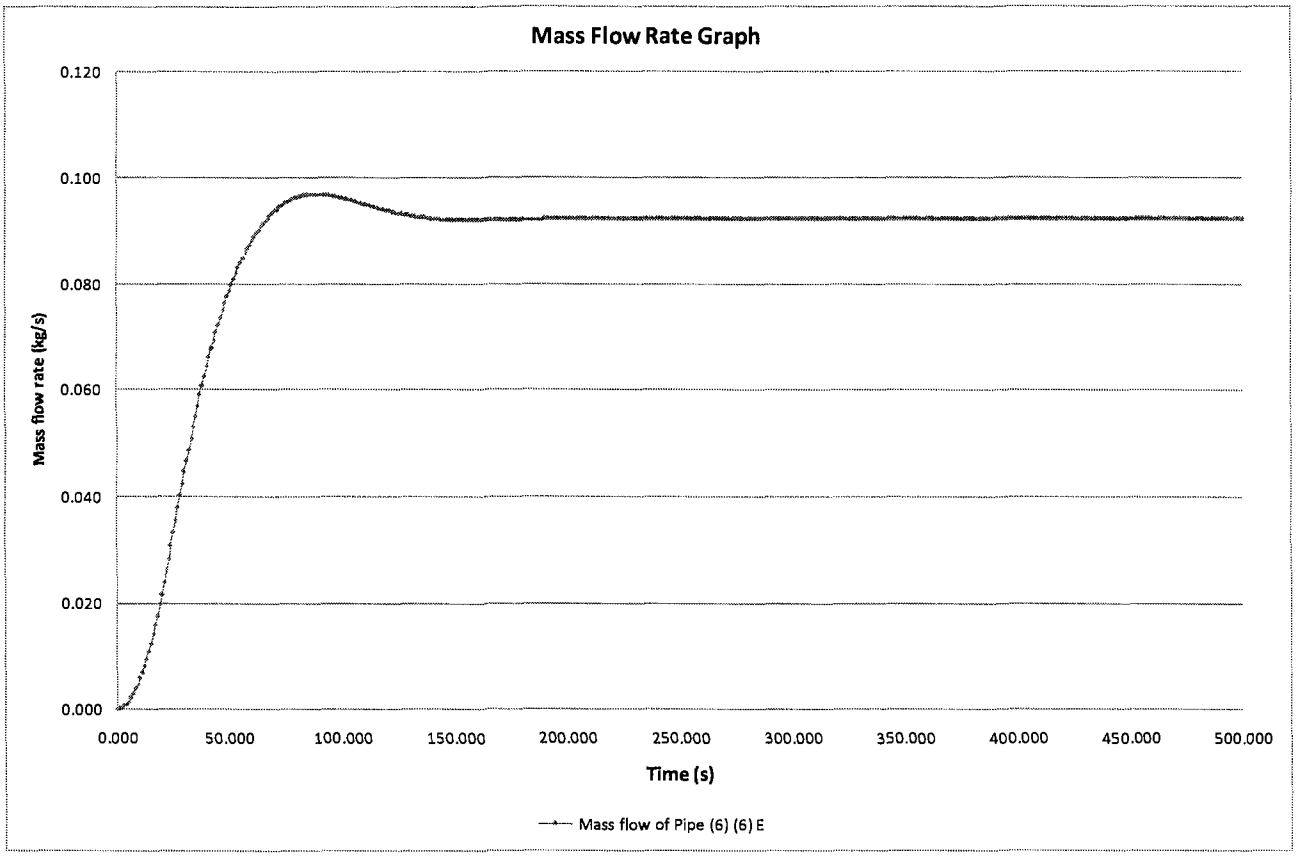


Figure 5.17: Mass flow rate during the simulation in Flownex® of the single U-tube pipe system

Even though the mass flow rate experienced is very small, the simulation confirms that by adding heat to one of the U-tube pipes, natural convection does occur. The result from MATLAB® is as follows:

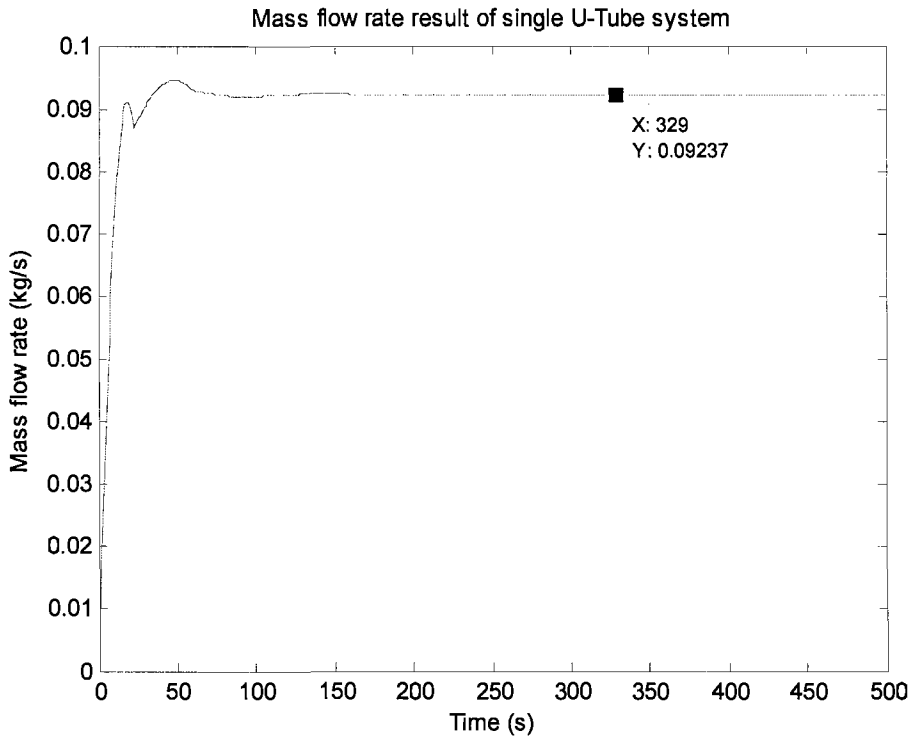


Figure 5.18: Results from the MATLAB® simulation of electrical circuit

In the following graph the results from Flownex® is super imposed onto the MATLAB® results. From figure 5.19 it is quite clear that the transient response time of the two simulations differs somewhat.

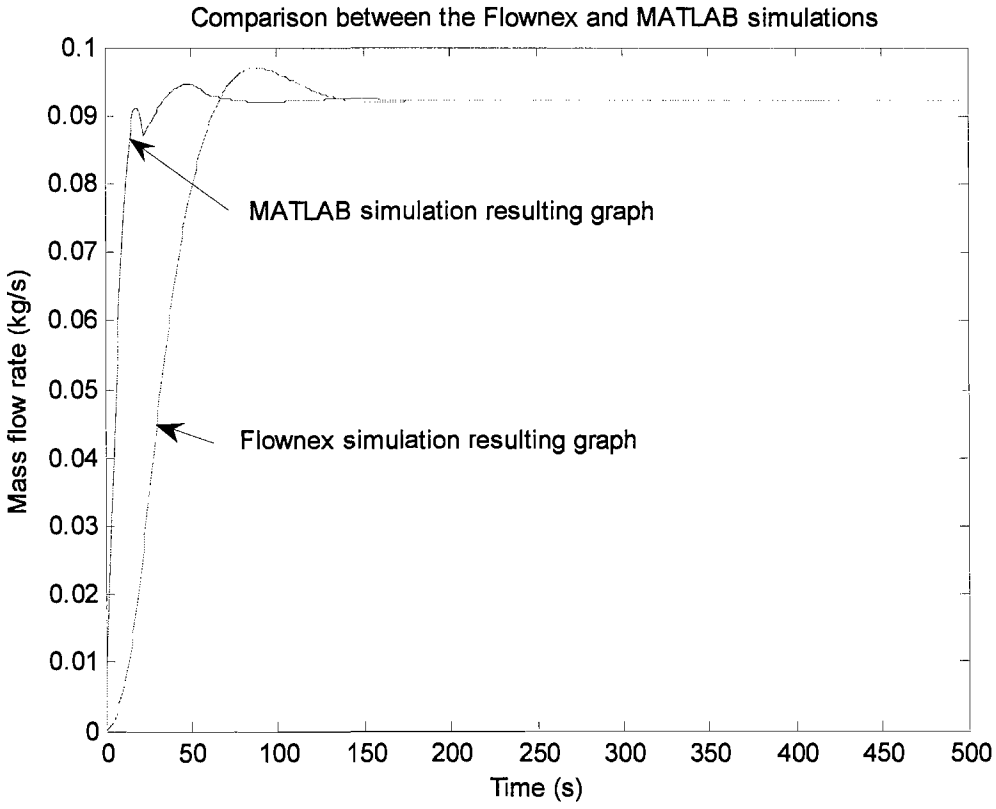


Figure 5.19: Comparison between the Flow and MATLAB simulations

The Flownex® simulation was performed and from there the resulting density was used to simulate the state-space model. As can be seen in figure 5.18 the MATLAB® simulation and the Flownex® simulation compares extremely well. Notice that the rise time of the two simulations differs, this may be due to the fact that the Flownex® has the time history of the densities. The density change is nonlinear and this may be the cause of the slower rise time of Flownex® transient.

The second scenario investigation involved the conditions where the flow collapses or turns around inside the pipe system. This condition is caused when the down comer pipe experience a higher heat transfer coefficient than the riser pipe. The result is also natural convection but in the opposite direction as illustrated in figure 5.20. When the added heat was moved from the riser pipe to the down comer pipe in the simulation, the reversed effect was seen.

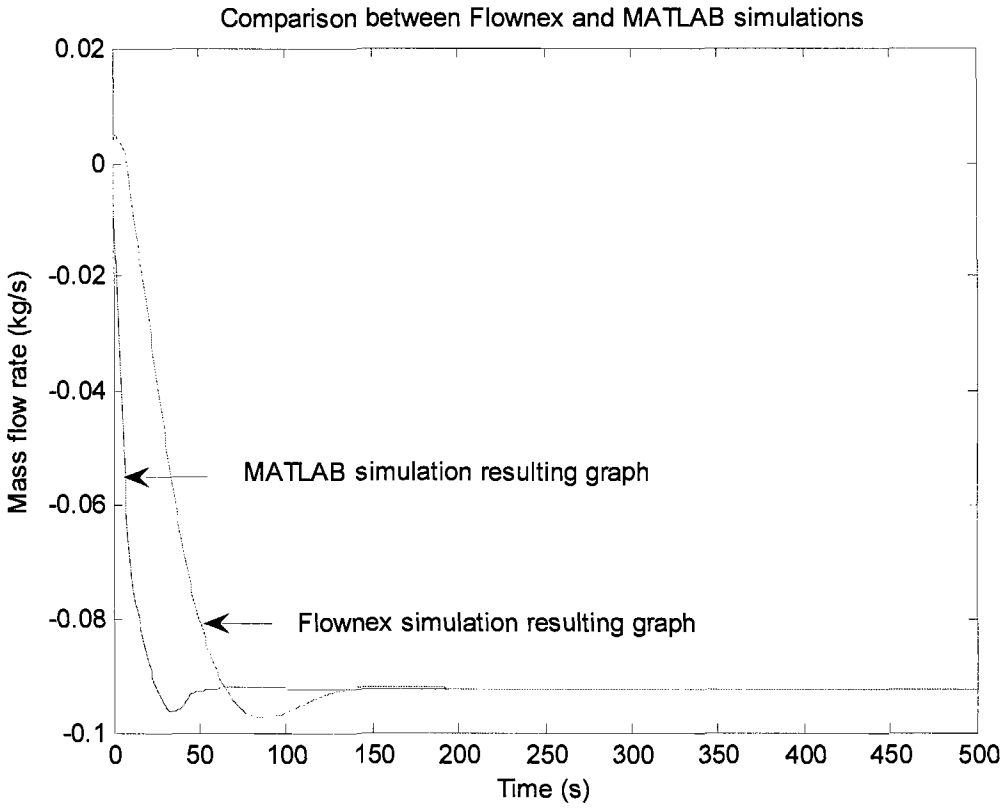


Figure 5.20: Resulting mass flow graph of reversed flow simulations

Good correlation was found between the results of the MATLAB® and Flownex® simulations and the results can be summarised as follows:

Table 5.3: Summary of the simulation results

Parameter	Simulated Result		
	Flownex® Simulation	MATLAB® simulation	Deviation %
Mass flow Rate	0.09236836kg/s	0.092368kg/s	0.001%
Pressure in riser pipe element	104.895kPA	109.8 kPA	0.045%

To quantify the performance of the MATLAB® simulation, and thus the performance of the developed electrical circuit, it is required to identify a mathematical method capable of doing this. The performance indices known as IAE criterion was used to calculate the error margin between the Flownex® and MATLAB® simulations [26]. The IAE equation can be written as follows:

$$IAE = \int_0^t |e(t)| \partial t \tag{5.23}$$

where t represents the time in seconds and $|e(t)|$ indicates the error between the simulations per time step [26]. The following figure illustrates the error between the two simulations per time step. It is quite evident from figure 5.21 that the transient response time between the two simulations do not compare very well. From a time stamp of, 200 seconds, the error between the two simulations becomes imitable. This error during the transient response time can be attributed to the complexity of the Flownex solver. However, the simulation's steady-state final value compares excellent.

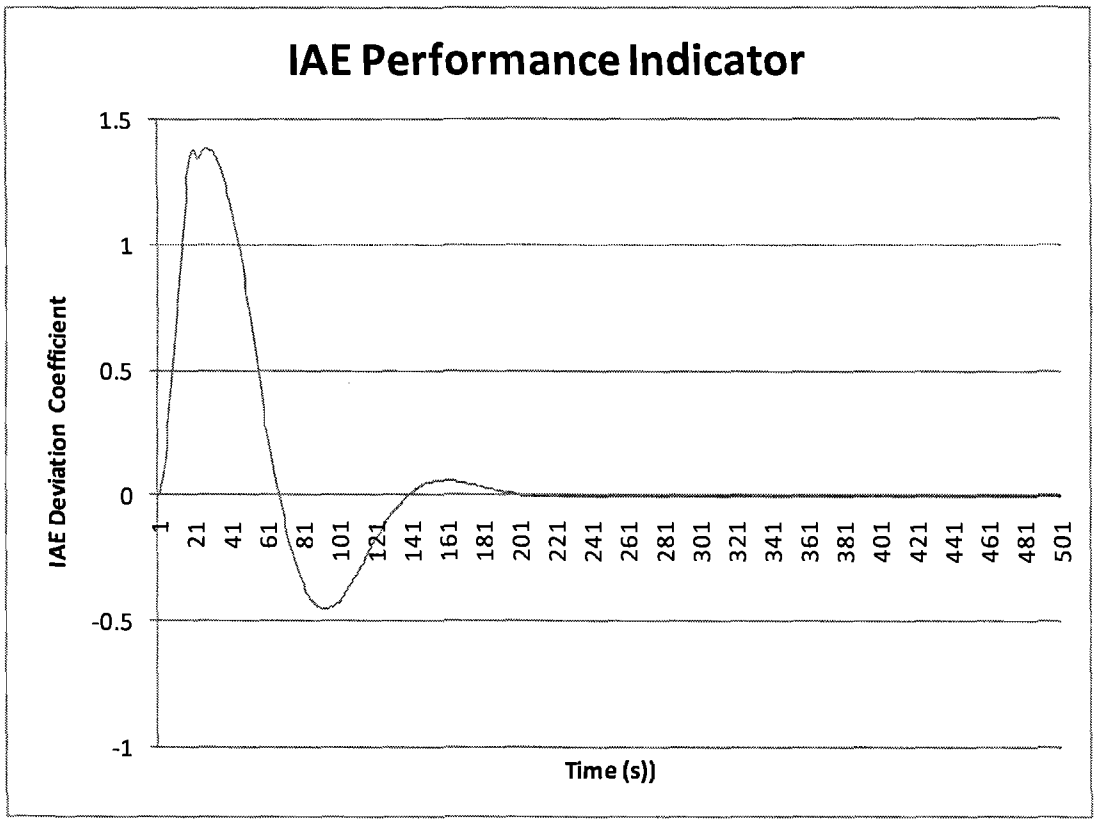


Figure 5.21: IAE performance graph for the single U- tube simulations

From the results from Flownex® and MATLAB® it is quite evident from figure 5.20, that when the system experience a situation where the down comer experience a higher heat transfer coefficient compared to the riser pipe, the fluid starts to flow in the opposite direction. It is also quite clear that natural convection does occur and the pipe segment that receives the added heat dictates the direction of the flow.

The above comparison between the simulations also validates the validity of the developed equivalent electrical circuit. Therefore it is quite possible to develop an equivalent electrical circuit for the mechanical and hydraulic domain by utilising the techniques described in this study. It also confirms that the results of Flownex® under the predefined conditions are correct. To provide more real system results the following section discusses the same type of simulation that was performed on the reduced hydraulic system of the RCCS.

5.4.3. Natural convection simulations for the double U-tube pipe system

From the discussion in section 5.4.1, it is easy to see that natural convection did occur in the single U-tube pipe system under the predefined conditions stipulated in table 5.2. This was confirmed by the results obtained by the MATLAB® simulation of the equivalent electrical circuit. In this section the double U-tube pipe system is simulated with predefined physical parameters and boundaries representative of the physical RCCS system.

The parameters as depicted in table 5.4 includes all the physical characteristics of the system and also the assumed initial values of the system. The results of the simulations are then compared. As already mentioned the top structure of the RCCS will be represented by the predefined inlet pressure and temperature and outlet pressure in the outlet manifold. The temperature difference due to the added heat to the system will first be calculated by Flownex® and afterwards used to perform the simulation in MATLAB®. The physical parameters used are as follows:

Table 5.4: Physical parameters of the reduced double U-tube system

Parameter	Initial Value
Inlet pressure	100 kPA
Inlet temperature	15 °C
Height of orifice	0 m
Length of orifice	0.01 m
Diameter of the orifice	0.03 m
Gravitational constant (g_C)	9.81m/s^2
Height of the inlet manifold node	23.45 m
Height of the outlet manifold node	23.45 m
Diameter of down comer pipe	0.217m
Length of down comer	23.45 m
Length of riser pipe	23.45 m
Diameter of riser pipe	0.217 m
Outlet pressure	100 kPA

During normal operation where the system experiences forced flow by means of the pump system, the system experiences no collapse in internal flow. This is largely due to the fact that all water is forced to move through the system. However, this situation is totally different during the passive condition, where the power failed and the pumps cannot be used. The buoyancy within the fluid and the heat transferred to the pipe system causes the water to circulate by means of natural convection. The scenario investigated involves the conditions where only the riser pipe of each U-tube was exposed to the heat radiated from the RPV. The heat transfer coefficient used for each riser pipe is the same due to the assumption that the RPV radiates heat evenly through the cavity between the RPV and the RCCS. As evident from figures 5.22 and 5.23, the mass flow rate in both U-tubes are equal.

The same mass flow rate means that the two U-tubes experience the same pressure within the U-tube and therefore no opportunity exist for inter circulation between the U-tubes. This only happens when one of the U-tubes experiences a smaller pressure drop than the other U-tube, and the substance tend to flow back into this U-tube. The flow rate for the simulation is as follows:

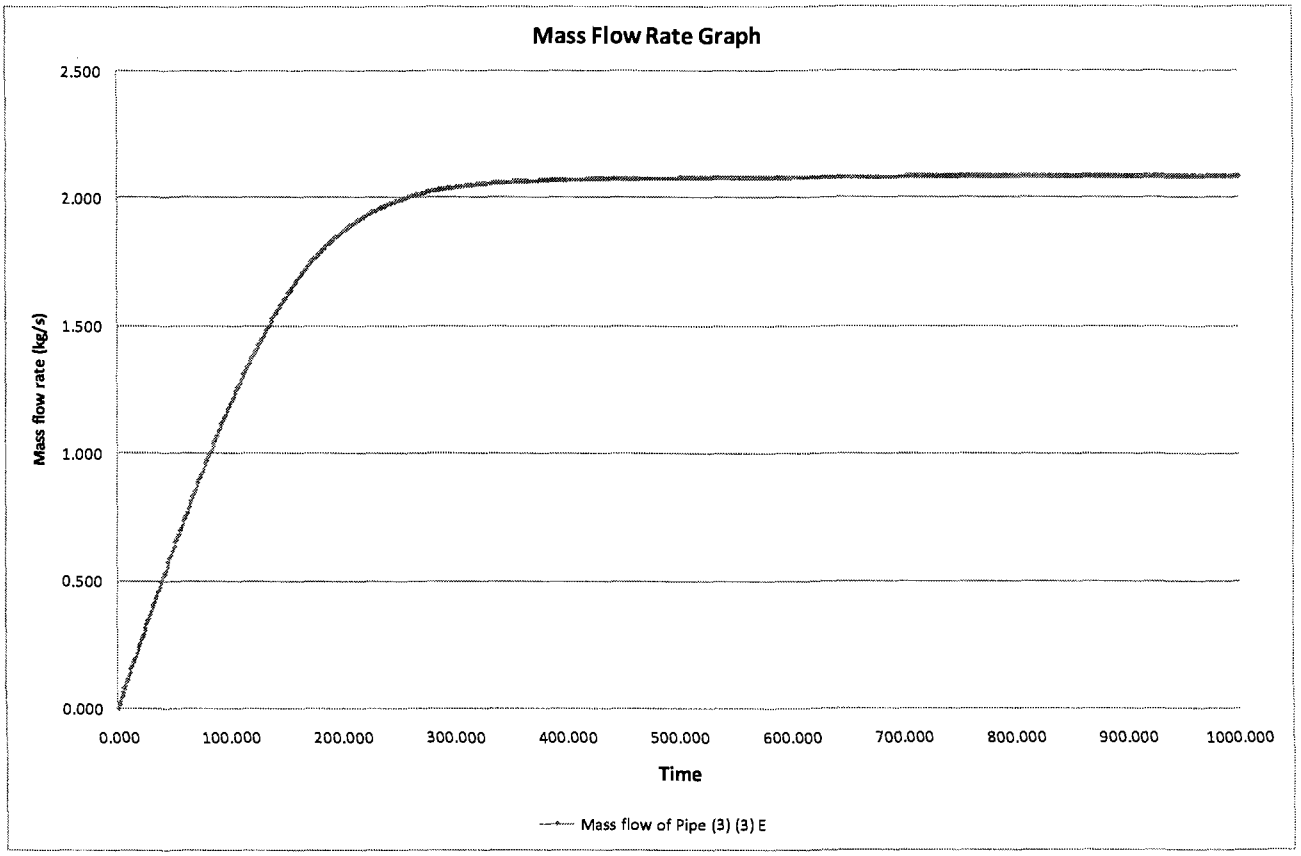


Figure 5.22: Results of the simulation performed in Flownex® using the physical parameters of the reduced hydraulic system

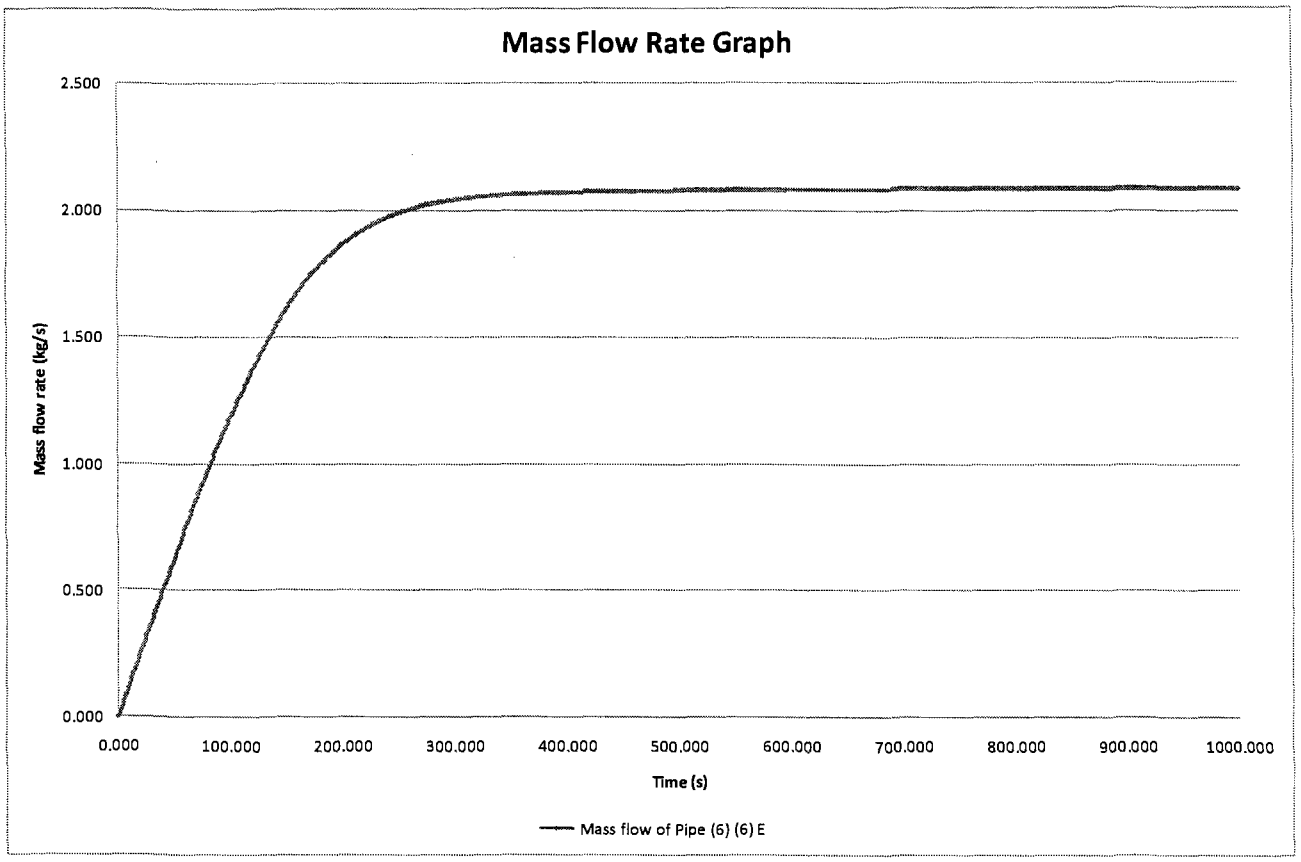
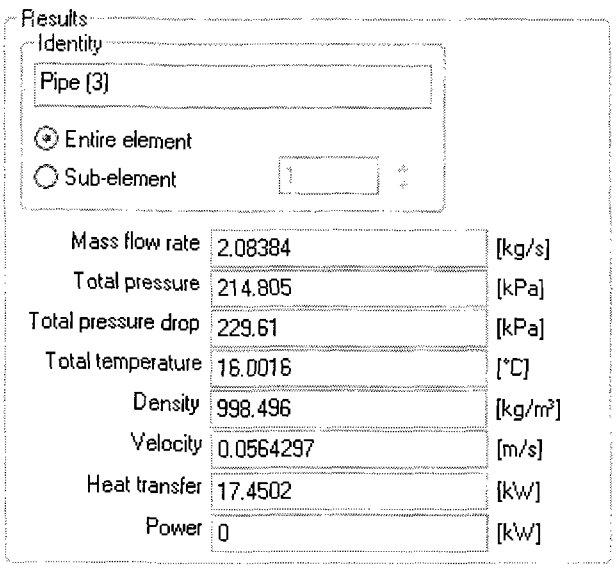
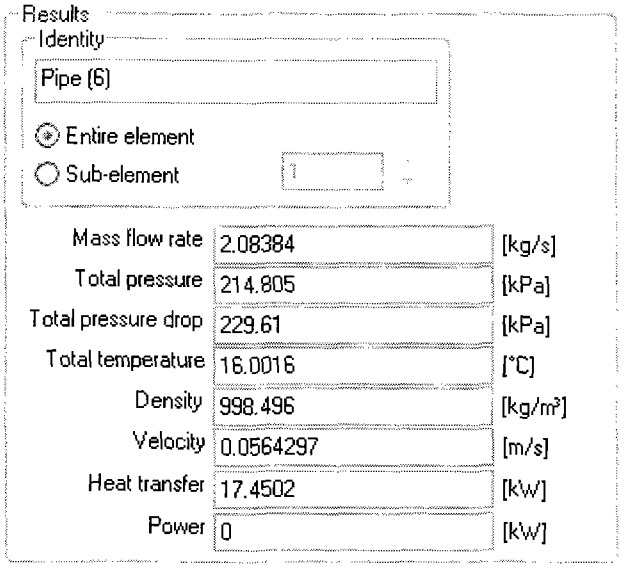


Figure 5.23: Results of the simulation performed in Flownex® using the physical parameters of the reduced hydraulic system

Summary of the results concerning the pressure, mass flow rate in each riser pipe and temperature for the simulation performed in Flownex® is indicated below.



(a)



(b)

Figure 5.24: Summary of the results of the simulation performed in Flownex® for (a) pipe element # 3 and (b) pipe element #6.

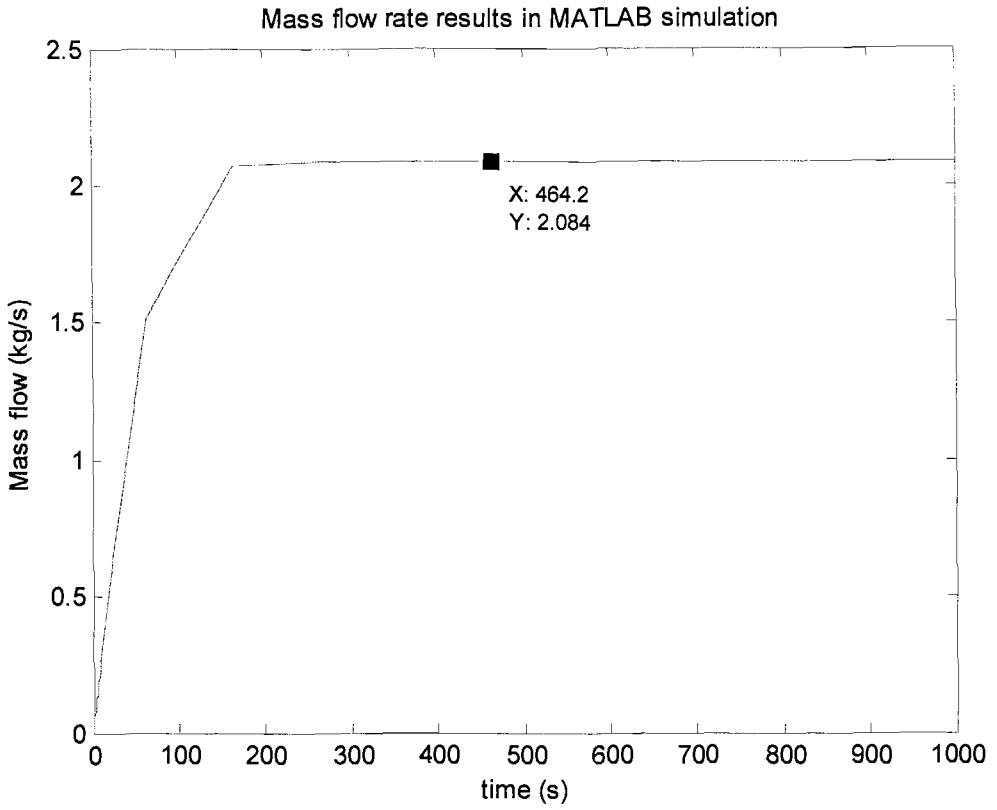


Figure 5.25: Results of the simulation performed in MATLAB® using the physical parameters of the reduced hydraulic system

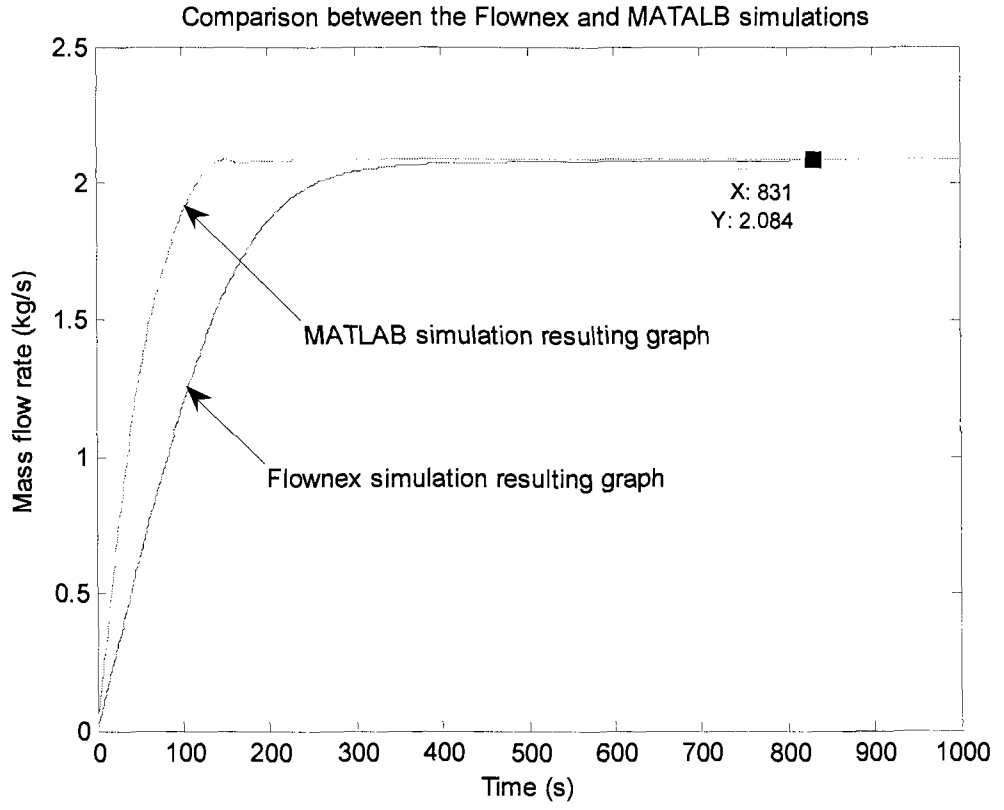


Figure 5.26: Comparison w.r.t. the mass flow graph of Flownex® and MATLAB® simulation

In this simulation however, the thermal effect on the pressure was not neglectable due to the long lengths of the pipes. It was found that it was required to include the pressure relating components which are affected by the heat transfer coefficient. The pressure component due to the temperature difference had to be added to the initial condition values of the MATLAB® simulation to ensure a correct pressure drop result.

It was therefore found that it is impossible to ignore the pressure components relating to the substance temperature when working with long pipes. The pressure component due to the temperature difference was calculated using the following equations [5, 6]:

$$p_0 = (T_{oe} - T_{oi}) \frac{1}{T_o} \frac{(\dot{m}^2)}{2\rho A^2} \tag{5.24}$$

where T_{oe} and $,T_{oi}$, represent the outlet and inlet temperatures, respectively in ^oC, A indicates the area of the pipe in m^2, ρ indicates the density of the substance in kg/m^3, $\dot{m}$ signifies the mass flow rate in kg/s, and T_O represents the total temperature where $T_O = h\ (entalhpy)/c_p\ (heat\ capacity\ factor\ of\ water)$. The results for the pressure within each tube are as follows for pipe element 3 and 6 respectively:

Table 5.5: Summary of the pressure drop results of the simulations

Parameter	Simulated Results		
	Flownex® Simulation	MATLAB® simulation	Deviation %
Pressure drop in pipe element #3	229.61 kPA	229.7 kPA	0.0391%
Pressure drop in pipe element #6	229.61 kPA	229.7 kPA	0.0391%

It is quite evident from the results between the MATLAB® and Flownex® simulations that the derived state-space model is a very good representative model of the physical system. The system can however not be evaluated only by the steady-state results and should therefore be compared with relating to the settling time, the rise time and the percentage overshoots. These performance indices were also calculated and the error margin between the Flownex® and MATLAB® simulations are as follows:

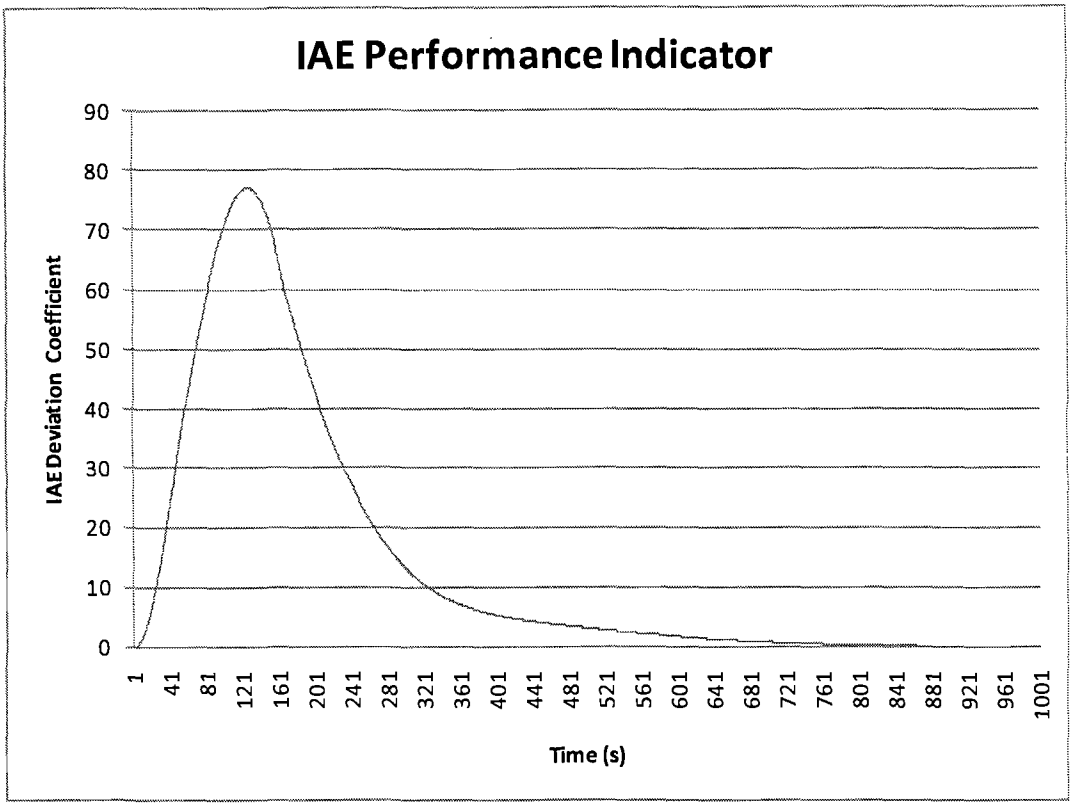


Figure 5.27: IAE performance graph for the double U-tube simulations

Yet again it is evident from the figure above that the transient response time differs, but after a certain period of time the same steady-state values are achieved. The results illustrated that correlation between the MATLAB® and Flownex® simulations is very good and the representative electrical model is a good analogy of the physical system. The results can be summarized as follows:

Table 5.6: Summary of the simulation results

Parameter	Simulated Result		
	Flownex® Simulation	MATLAB® simulation	Deviation %
Mass flow Rate	2.086836 kg/s	2.0868 kg/s	0.003%
Pressure drop in pipe element #3	229.61 kPA	229.7 kPA	0.0391%
Pressure drop in pipe element #6	229.61 kPA	229.7 kPA	0.0391%

The above simulations in Flownex® and MATLAB® illustrate the natural convection principal. Each time the U-tubes were exposed to the heat transfer coefficient from the simulated heat source, corresponding mass flow rates were observed. It was confirmed that the mass flow rate is a function of the heat transfer coefficient. It is also quite evident from the resulting mass flow rate and pressure drop illustrated in figures 5.24-31, that the results from the MATLAB® and Flownex® simulations compares extremely well with each other.

5.5. REMARKS REGARDING THE SIMULATION RESULTS

5.5.1. Performance comparison of the simulations

The good comparison of the final result between the simulations serves as substantiation that the developed analogues electrical circuit for the reduced hydraulic is truly representative. There is however still some deviations between the simulation results of the Flownex® and MATLAB® simulations. The first deviation is the rise time difference between the simulations. The rise time of the mass flow rate during the Flownex® simulations tend to be longer than the rise time of the resulting mass flow rate graph of the MATLAB® simulation.

How swift the system reacts to a step input introduced to the system, is measured by the rise and peak times. With over damped systems, as in the case of this simulation, the peak time of the system is not defined and the 10-90% rise time assessment ratio is used. To clarify this more, it means that the system can be evaluated by evaluating the time the system takes from 10% to 90% of the steady-state final value. The settling and rise time results can be summarized as follows:

Table 5.7: Summarization of the Flownex® and MATLAB® simulations performance

Parameter	**Simulated Result**		
	Flownex® Simulation (seconds)	**MATLAB® simulation (seconds)**	**Deviation %**
Time to get from 10% to 90% of final value.	180.5 s	121.5 s	48.56%
Settling time to get to within 2-5% of the final Mass flow rate value (2.00 kg/s).	259 s	157.5 s	64.45%
Settling time to get to within 1% of the final Mass flow rate value (2.064kg/s).	360 s	174 s	107%

Though the settling times between the two simulations differ up to 107%, the comparisons between the final results of the simulations are still good. The deviation in settling time between the two simulations is a further substantiation of the fact that Flownex® takes more non-linear effects into account during the simulation.

The deviation on performance relating to the transient response can be attributed to the fact that the initial pressure, capacitor, inductor and resistor values used in the MATLAB® simulation are dependent on the final steady-state density results of the Flownex® simulations. The steady-state results generated by the Flownex® simulation for the densities in the U-tube are used as input values for the MATLAB® simulation. Flownex® also considers more non-linear effects with regards to the thermal effect the heat transfer coefficient has on the system. The MATLAB® simulation also experienced a percentage overshoot which can be calculated by the following equation [26]:

$$\text{P.O} = \frac{M_p - fv}{fv} \text{ x } 100\% \tag{5.25}$$

where M_p represents the highest value of the overshoot and fv shows the final steady-state value. The resulting over shoot of the MATLAB® simulation can then be calculated as follows:

$$\begin{aligned} \text{P.O.} &= \frac{2.086 - 2.084}{2.084} \text{ x } 100\% \\ &= 0.096\% \end{aligned} \tag{5.26}$$

It is quite evident that the percentage overshoot of the system is omissible small. From the discussion above it is quite evident that the Flownex® simulation has a higher damping ratio than the MATLAB® simulation. This can be directly related to the complexity of the solver in Flownex®, taking more non-linear effects into account. It was found that flow-pressure coefficients in hydraulic control systems, directly affects the damping ratio of the system [31], and therefore it can be assumed with certainty that the cause of the deviations in MATLAB® is due to the fact that the steady-state density results from the Flownex® simulation is used as input parameters for the MATLAB® simulation. It thus makes sense to investigate the dominant poles which are responsible for the system performance of the MATLAB® simulation. The following section discusses this topic in detail.

5.6. CONCLUSION

The results verify that the network method is a credible modelling method. The correct definition of the equivalent electrical elements also contributed to the good results. The state-space model can now be used by systems and control engineers for system analysis and control purposes. It also sets a platform for future studies on the RCCS system. The following chapter summarise the findings of the study and provide the concluding remarks.

Chapter 6

6. Conclusions and recommendations

6.1. INTRODUCTION

M-Tech Industrial (Pty) Ltd performed a thorough software analysis on the RCCS by means of Flownex®. This was done to ensure that the RCCS will be able to dissipate sufficient heat from the RPV during active and passive operating conditions.

The analysis confirmed that under normal operating conditions, the RCCS will be able to dissipate the required heat. However, it was found that the flow collapsed during passive operating conditions, when the water reached a certain temperature. The request from M-Tech Industrial (Pty) Ltd and PBMR was to model the RCCS by means of an electrical network that included only the dominant dynamics. Such a simplified system would facilitate a better understanding of the natural convection process.

The problem was basically broken up into five sub-problems. First, the dissertation addresses the natural convection phenomenon, to provide a comprehension understanding of the fundamental forces behind the manifestation. Secondly, an analysis method was identified to analyse the system in another energy domain, namely the electrical domain. Thirdly, a representative electrical circuit was developed for the hydraulic system. Fourthly the developed system was analysed using the identified analyses method and lastly the analysed system was simulated by the use of MATLAB® and the results was compared to the results of the Flownex® simulations of the same system. In this chapter a conclusion of the results of the study is provided and possible future studies that can be done are also identified.

6.2. CONCLUDING REMARKS

It was decided to investigate the natural convection phenomenon, to gain a proper understanding of the force responsible for the phenomenon. By doing this, all the important components were taken into account for the development of an analogous electrical circuit. By identifying the fundamental aspects responsible for the phenomenon, the system could be reduced, resulting in a good representation and simplified system. This made it possible to reduce the RCCS to a double U-tube system. Initial values were given to the inlet and outlet manifolds of the double U-tube and this represented the effect the top pipe configuration may have had on the reduced system.

The equivalent electrical circuit was developed by understanding the correlation of each of the hydraulic components with regards to the electrical components. It was found that fundamental mathematical characteristics of various components in the hydraulic domain showed good resemblance to the fundamental mathematical characteristics of the corresponding components in the electrical domain.

Two methods of analysis were investigated. The methods were the bond and network graph analysis. It was found that the network analysis method was more appropriate and was used for the analysis. The reduced hydraulic system was then analysed using the network analysis method and the state-space model describing the system was derived.

MATLAB® was used to develop a program to simulate the behaviour of the system by using the state-space model. The simulation methodology followed was that the single U-tube system was first simulated in Flownex®. The resulting pressure and densities from the Flownex® simulation was used as part of the inputs for the state-space model simulation. It was found that the results of the Flownex® simulation of the reduced single U-tube system and the MATLAB® simulation of the reduced single U-tube system compared very well. The simulation was then extended to the double U-tube system and the same modus operandi was followed concerning the resulting pressures and densities. These results also compared very well.

There was however still some deviations between the results, but this can be contributed to the fact that Flownex® takes much more non-linear aspects into account. Even though some non-linear aspects for the resistor values of the system were taken into account, the capacitor and inductor values were based on linear values. The results of the simulations are as follows.

Table 6.1: Summary of the Flownex® and MATLAB® simulations performed on the reduced hydraulic system of the RCCS.

Parameter	**Simulated Result**		
	Flownex® Simulation	**MATLAB® simulation**	**Deviation %**
Mass flow Rate	2.084kg/s	2.084 kg/s	0.00%
Pressure in pipe element #3	229.61 kPA	229.7 kPA	0.0391%
Pressure in pipe element #6	229.61 kPA	229.7 kPA	0.0391%

The results served as validation for the developed equivalent electrical circuit. The dominant elements identified by the correlating dominant poles also compared well with the real system.

It was however found that the transient response time simulations done in Flownex® and MATLAB® did differ somewhat. The reason for this is that due to the fact that most of the aspects simulated were based on linear models, taken from the Flownex® simulation and used in the MATLAB® simulation.

6.3. FUTURE WORK

Even though a good comparison between the final simulation results of the hydraulic domain and the electrical domain was obtained, the simulation in MATLAB® needs to be extended in terms of the non-linear affects taken into account. The capacitor and inductor values need to be represented by a non-linear mathematical equation or simulation that takes the thermal effect on the thermal capacitance in the system into consideration, and simulate the effect in conjunction with a dynamic table which contains changing density values.

To ensure that the zeros and poles extracted for control purposes are a representative illustration of the behaviour relating to the flow within the system, the MATLAB® program needs to be adapted to simulate the positions of the zero and poles continuously during the simulation of the flow. With this it will be possible to monitor the positions of the zeros and poles and also identify the conditions where the flow collapses in the system.

6.4. CLOSING REMARKS

The goal of this study has been achieved by deriving a representative, but reduced electrical circuit model of the RCCS. A state-space model has also been derived from this circuit model that can be helpful for future investigations of RCCS dynamics. This study paves the way for further improvements to the state-space model regarding the complexity and possibly non-linear considerations.

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8. Appendix

8.1. MATLAB® Program that determine the state-space model for single u-tube pipe system

```
%Program: For Extracting the state space model of a network graph for
%the single u-tube Pipe system
%=========================================================================

% Normalised matrix derived from network graph (ground node included)

%    1  2  3  4  5  6  7  8  9  10  11  12 links
A = [-1 -1  0  0 -1 -1  0  0  0   0   0   0 ;%1 NODE
     1  0 -1  0  0  0  0  0  0   0   0   0 ;%2 NODE
     0  0  1  0  0  0  1  0  0   0   0   0 ;%3 NODE
     0  0  0  0  0  0 -1  0  0   1   0   0 ;%4 NODE
     0  0  0  0  1  0  0  1  0  -1   0   0 ;%5 NODE
     0  0  0  0  0  0  0 -1  0   0   1   0 ;%6 NODE
     0  0  0 -1  0  1  0  0  0   0  -1   0 ;%7 NODE
     0  0  0  1  0  0  0  0  1   0   0   0 ;%8 NODE
     0  0  0  0  0  0  0  0 -1   0   0   1 ;%9 NODE
     0  1  0  0  0  0  0  0  0   0   0  -1 ]%10 NODE

%STEP 1: ELIMINATE GROUND NODE
%================================
SIZE_A = size(A)
Nodes = SIZE_A(1,1)
Branches = SIZE_A(1,2)
A = A(2:Nodes,1:Branches)
%================================

%STEP 2: ORGANIZE A INTO CO-TREE AND TREE MATRICES (A =
%[A11(CO-TREE)|A12(TREE)])
%=========================================================================
A11 = A(:,10:12)  %Co-Tee matrix
A12 = A(:,1:9)    %Tree matrix
A = [A11,A12]
Qf = inv(A12)*A

%STEP 3: DERIVE H MATRIX FORM Qf MATRIX(Qf = [H I])
H = Qf(:,1:3)
H_ = H'
%=========================================================================
 %|eCc| = | H1C_ HCC_ HRC_ HLC_ | | e1  |
 %|eRc| = | H1R_ HCR_ HRR_ HLR_ | | eCt |
 %|eLc| = | H1L_ HCL_ HRL_ HLL_ | | eRt |
 %|e2 | = | H12_ HC2_ HR2_ HL2_ | | eLt |

%STEP 3: IDENTIFY THE RELATIONSHIPS

H1C_ = 0; H1C = H1C_';
HCC_ = 0; HCC = HCC_';
HRC_ = 0;
HLC_ = 0;

H1R_ = 0; H1R = H1R_';
HCR_ = 0; HCR = HCR_';
HRR_ = 0; HRR = HRR_';
```

```
HLR_ = 0; HLR = HLR_';

H1L_ = H_(1:3,1:4); H1L = H1L_';
HCL_ = H_(1:3,5:6); HCL = HCL_';
HRL_ = H_(1:3,7:9); HRL = HRL_';
HLL_ = 0; HLL = HLL_';

H12_ = 0; H12 = H12_';
HC2_ = 0; HC2 = HC2_';
HR2_ = 0; HR2 = HR2_';
HL2_ = 0; HL2 = HL2_';

%STEP 4: IDENTIFY THE RCL MATRICES
%=========================================================================
%Resitors
syms R1 R2 R3
Resistors_cotree = [0];
Resistors_tree   = [R1,R2,R3];
Rt = diag(Resistors_tree);
Rc = diag(Resistors_cotree);

%Capacitors
syms C1 C2
Capacitors_cotree = [0];
Capacitors_tree   = [C1,C2];
Ct = diag(Capacitors_tree);
Cc = diag(Capacitors_cotree);
C = Ct + HCC*Cc*HCC';

%Inductors
syms L1 L2 L3  %If there is no inductors make matrices 0
Inductors_cotree = [L1,L2,L3];
Inductors_tree   = [0];
Lc = diag(Inductors_cotree);
Lt = diag(Inductors_tree);
L = Lc + HLL'*Lt*HLL;

%STEP 4: CALCULTAE A & B MATRICES
%A-MATRIX
%=========================================
if(HCR == 0)
   A11 = [0];
else
   A11 = -HCR*inv(Rc + HRR'*Rt*HRR)*HCR';
end
%=========================================
if(HCR == 0)
            A12 = -HCL;
            A21 = -A12';
elseif(HRR' == 0)
            A12 = -HCL;
            A21 = -A12';
elseif(HRL == 0)
            A12 = -HCL;
            A21 = -A12';
else
             A12 = -HCL + HCR*inv(Rc + HRR'*Rt*HRR)*HRR'*Rt*HRL;
             A21 = -A12';
end
```

```
%=============================================
if(HRL == 0)
        A22 = 0;
elseif(HRR == 0)
        A22 = -HRL'*inv(inv(Rt) + 0)*HRL;
else
        A22 = -HRL'*inv(inv(Rt) + HRR*inv(Rc)*HRR')*HRL;
end
%=============================================

%B-MATRIX
%=============================================
if(HCR == 0)
        B11 = 0;
elseif(H1R' == 0)
        B11 = 0;
elseif(HRR == 0)
        B11 = -HCR*inv(Rc + 0)*H1R';
else
        B11 = -HCR*inv(Rc + HRR'*Rt*HRR)*H1R'
end
%=============================================
if(HCR == 0)
        B12 = -HC2;
elseif(HR2 == 0)
        B12 = -HC2;
elseif(HRR' == 0)
        B12 = -HC2;
else
        B12 = -HC2 + HCR*inv(Rc + HRR'*Rt*HRR)*HRR'*Rt*HR2;
end
%=============================================
if(HRL == 0)
        B21 = H1L';
elseif(H1R' == 0)
        B21 = H1L';
elseif(HRR == 0)
        B21 = H1L';
else
  B21 = H1L' - HRL'*inv(inv(Rt) + HRR*inv(Rc)*HRR')*HRR*inv(Rc)*H1R';
end
%=============================================
if(HRL == 0)
  B22 = 0;
elseif(HR2 == 0)
  B22 = 0;
elseif(HRR == 0)
  B22 = -HRL'*inv(inv(Rt) + 0)*HR2;
else
  B22 = -HRL'*inv(inv(Rt) + HRR*inv(Rc)*HRR')*HR2;
end

%=============================================
 %STEP 6:CL matrix
%=============================================
CL = [C,zeros([length(C) length(L)]);
    zeros([length(L) length(C)]),L]
%=============================================

%Generate final A and B matrices
%=============================================
 %States - these are the physical conditions the system is in
%=============================================
```

```
syms W1 W2 W3  %W =  massflows
syms P1 P2    %P = internal pressures
eCt = [P1;P2];
fLc = [W1;W2;W3];
%=======================================

%Sources - sources of energy
%=======================================
syms P01 P02 PH1 PH2  %Pressures
e1 = [P01;P02;PH1;PH2];
f2 = [0];
%=======================================
%Define size of the matrices A and B and fill them with Zeros were
%applicable
%=============================================================================
Siz_eCt = size(eCt);
Siz_fLc = size(fLc);

Siz_e1 = size(e1);
Siz_f2 = size(f2);

AA = [zeros([Siz_eCt(1,1) Siz_eCt(1,1)])  zeros([Siz_eCt(1,1) Siz_fLc(1,1)]);
    zeros([Siz_fLc(1,1) Siz_eCt(1,1)])  zeros([Siz_fLc(1,1) Siz_fLc(1,1)])]

BB = [zeros([Siz_eCt(1,1) Siz_e1(1,1)])  zeros([Siz_eCt(1,1) Siz_f2(1,1)]);
    zeros([Siz_fLc(1,1) Siz_e1(1,1)])  zeros([Siz_fLc(1,1) Siz_f2(1,1)])]
%=============================================================================
%Make sure all matrices correct size
%=============================================================================
%B-MATRICES
sizB11 = size(B11);
for(i=1:Siz_eCt(1,1)-sizB11(1,1))
   B11(i+sizB11(1,1),:) = 0;
end
   for(j=1:Siz_e1(1,1) - sizB11(1,2))
      B11(:,j+sizB11(1,2)) = 0;
   end


sizB12 = size(B12);
for(i=1:Siz_eCt(1,1)-sizB12(1,1))
   B12(i+sizB12(1,1),:) = 0;
end
   for(j=1:Siz_f2(1,1) - sizB12(1,2))
      B12(:,j+sizB12(1,2)) = 0;
   end

sizB21 = size(B21);
for(i=1:Siz_fLc(1,1)-sizB21(1,1))
   B21(i+sizB21(1,1),:) = 0;
end
   for(j=1:Siz_e1(1,1) - sizB21(1,2))
      B21(:,j+sizB21(1,2)) = 0;
   end


sizB22 = size(B22);
for(i=1:Siz_fLc(1,1)-sizB22(1,1))
   B22(i+sizB22(1,1),:) = 0;
end
   for(j=1:Siz_f2(1,1) - sizB22(1,2))
      B22(:,j+sizB22(1,2)) = 0;
```

```
   end

% FOR A MATRIX
%-----------------------------------------------------------
%A-MATRICES
sizA11 = size(A11);
for(i=1:Siz_eCt(1,1)-sizA11(1,1))
   A11(i+sizA11(1,1),:) = 0;
end
   for(j=1:Siz_eCt(1,1) - sizA11(1,2))
      A11(:,j+sizA11(1,2)) = 0;
   end
     sizA12 = size(A12);
for(i=1:Siz_eCt(1,1)-sizA12(1,1))
     A12(i+sizA12(1,1),:) = 0;
end
   for(j=1:Siz_fLc(1,1) - sizA12(1,2))
      A12(:,j+sizA12(1,2)) = 0;
   end
     sizA21 = size(A21);
for(i=1:Siz_fLc(1,1)-sizA21(1,1))
   A21(i+sizA21(1,1),:) = 0;
end
   for(j=1:Siz_eCt(1,1) - sizA21(1,2))
      A21(:,j+sizA21(1,2)) = 0;
   end
      sizA22 = size(A22);
for(i=1:Siz_fLc(1,1)-sizA22(1,1))
   A22(i+sizA22(1,1),:) = 0;
end
   for(j=1:Siz_fLc(1,1) - sizA22(1,2))
      A22(:,j+sizA22(1,2)) = 0;
   end

%=============================================================================

%Construct A and B matrices
%=============================================================================
AA = [A11 A12; A21 A22]

BB = [B11 B12; B21 B22]
%=============================================================================

if(C == 0)
   A21 = inv(L)*A21;
   A22 = inv(L)*A22;
   B21 = inv(L)*B21;
   B22 = inv(L)*B22;
   A = [A11 A12; A21 A22]
   B = [B11 B12; B21 B22]
elseif(L == 0)
   A11 = inv(C)*A11;
   A12 = inv(C)*A12;
   B11 = inv(C)*B11;
   B12 = inv(C)*B12;
   A = [A11 A12; A21 A22]
   B = [B11 B12; B21 B22]
else
   A = inv(CL)*AA
   B = inv(CL)*BB
end
```

END OF PROGRAM

8.2. MATLAB® Program that use state-space model to simulate the behavioural characteristics of single U-tube system

```
function StateSpaceNonLinSolve
%Program: Simulates the state space model derived from the network
%Methodology: The state space model is used as input in the MATLAB solver.
%The Resistor, capacitor and Inductor values are calculated and fed into
%the solver

clc
clear all
close all
%Initial Conditions of the system;
%[P1;   P2;   m1;m2;m3] - initial pressures in manifolds and mass flows in each pipe
X0_h = [100e3; 100e3; 0; 0;  0];

%Inputs
%Set up height differences between nodes, let the lowest level node be equal to h = 0 m
h1 = 1;
h2 = -1;
rho = 999.1;            %Density of water at 15 degree Celsius
g = 9.81;               %Gravitational acceleration constant

%inputs
     %P01     P02       Ph1       Ph2     f0
u_h =  [100e3; 100e3;  999.1*g*h1;  998.435*g*h2; 0]

%Timespan - time the simulate will run in seconds
tspan = [0,200];

%Simulation of the state space model performed - solver called here

options = odeset('RelTol',1);

%call the solver
[t,X] = ode23s(@State_Space_Function, tspan,X0_h,options,u_h);
disp(X)
plot(t,X(:,4));
xlabel('time')
ylabel('mass flow')
figure
plot(t,X(:,1));
xlabel('time')
ylabel('P1')
figure
plot(t,X(:,2));
xlabel('time')
ylabel('P2')

%Function organise the model in state space form - used as input at solver
function [dx_h]=State_Space_Function(t,x,u_h)

%Set up the component values
LL1 = 1;            %Length of the links  - down comer (m)
LL2 = 0.001;        %Length of the links  - orifice (m)
LL3 = 1;                    %Length of the links  - riser (m)
```

```
LCV1 = 0.10005;          %Length of the control volumes's (m)
LCV2 = 0.10005;          %LCV1=LCV2=1+1+0.001 = 2.001/2


d1 = 0.1;               %Diameter of pipe (m)
d2 = 0.01;              %Diameter of orifice (m)
Area1 = (pi*d1^2)/4;        %Crossectional area of pipe(m^2)
Area2 = (pi*d2^2)/4;        %Crossectional area of orifice(m^2)
rho = 999.1;            %Density of water at 15 degree Celsius
m_u = 0.001138;             %Viscosity of water at 15 degrees Celsius
g = 9.81;               %Gravitational acceleration constant
f = 0.007993;           %Fanning friction factor
k = 0.00011;            %head loss coefficient

%Values for the Resistor
R1 = ((f*LL1)/d1 + k)/(2*Area1^2*rho*g);
R2 = ((f*LL2)/d2 + k)/(2*Area2^2*rho*g);
R3 = ((f*LL3)/d1 + k)/(2*Area1^2*rho*g);

%Values for the Inductors
L1 = LL1/Area1;
L2 = LL2/Area2;
L3 = LL3/Area1;

Bulk = 200e6;                   %Bulk Modulus 200 Mpa

%Values for the Capcitors
C1 = (999.1*Area1*LCV1)/Bulk;
C2 = (998.435*Area1*LCV2)/Bulk;

%Derived Sate space model

A_h = [      0,      0,    1/C1,   -1/C1,      0;
             0,      0,       0,    1/C2,   -1/C2;
          -1/L1,      0, -1/L1*R1*abs(x(3,1)),      0,      0;
           1/L2,   -1/L2,       0, -1/L2*R2*abs(x(4,1)),      0;
             0,    1/L3,       0,       0, -1/L3*R3*abs(x(5,1))];


B_h = [    0,    0,    0,    0,    0;
           0,    0,    0,    0,    0;
        1/L1,    0,  1/L1,    0,    0;
           0,    0,    0,    0,    0;
           0, -1/L3,    0,  1/L3,    0];


  dx_h = A_h*x + B_h*u_h; %Hidroliese state space
  return
```

END OF PROGRAM

8.3. MATLAB® Program that determine the state-space model for the double U-tube pipe system

```
%Program: For Extracting the state space model for network graph for a double  %U-Tube system
%=========================================================================
 % Normalized matrix derived from network graph (ground node included)

clear
clc
% STEP 1
% Develop the linear graph matrix with the reference node included
%=========================================================================
%
%   1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17  18  19  20  21  22 links
A = [-1 -1  0  0  0  0 -1 -1 -1  -1   0   0   0   0   0   0   0   0   0   0   0   0;%1 NODE
     1  0 -1  0 -1  0  0  0  0   0   0   0   0   0   0   0   0   0   0   0   0   0;%2 NODE
     0  0  1  0  0  0  0  0  0   1   0   0   0   0   0   0   0   0   0   0   0   0;%3 NODE
     0  0  0  0  0  0  0  0  0   0  -1   0   0   0   0   1   0   0   0   0   0   0;%4 NODE
     0  0  0  0  0  0  1  0  0   0   1   0   0   0   0  -1   0   0   0   0   0   0;%5 NODE
     0  0  0  0  0  0  0  0  0   0   0  -1   0   0   0   0   1   0   0   0   0   0;%6 NODE
     0  0  0 -1  0  0  0  1  0   0   0   0   0   0   0   0  -1   0   0   0   0   0;%7 NODE
     0  0  0  0  0  0  0  0  0   0   0   0  -1   0   0   0   0   1   0   0   0   0;%8 NODE
     0  0  0  1  0  0  0  0  0   0   0   1   0   0   0   0   0   0   0   0   0   0;%9 NODE
     0  1  0 -1  0 -1  0  0  0   0   0   0   0   0   0   0   0   0   0   0   0   0;%10 NODE
     0  0  0  0  0  1  0  0  0   0   0   0   0   0   1   0   0   0   0   0   0   0;%11 NODE
     0  0  0  0  0  0  0  0  0   0   0   0   0   0  -1   0   0   0   0   0   0   1;%12 NODE
     0  0  0  0  0  0  0  0  0   1   0   0   0   0   0   0   0   0   0   0  -1  -1;%13 NODE
     0  0  0  0  0  0  0  0  0   0   0   0   0   0  -1   0   0   0   0   0   1   0;%14 NODE
     0  0  0  0  0  0  0  0  1   0   0   0   0   0   1   0   0   0   0  -1   0   0;%15 NODE
     0  0  0  0  0  0  0  0  0   0   0   0   0  -1   0   0   0   0   0   1   0   0;%16 NODE
     0  0  0  0  1  0  0  0  0   0   0   0   0   1   0   0   0   0   0   0   0   0]%17 NODE

%=================================================
%STEP 2
%%reduced normalised matrix (node-1)
%Eliminating the Ground Node
%=================================================
%================================
SIZE_A = size(A)
Nodes = SIZE_A(1,1)
Branches = SIZE_A(1,2)
A = A(2:Nodes,1:Branches)
%================================

%=====================================================================
%STEP 3
%Developing the Tree and Co-Tree matrices
%=====================================================================

A11 = A(:,17:22)   %Co-Tee matrix
A12 = A(:,1:16)    %Tree matrix
A = [A11,A12]
Qf = inv(A12)*A    %Fundamental Cutset Matrix (Qf = [H I])
H = Qf(:,1:6)
H_ = H'

%To devide the Unit matrix from the H matrix within the Qf matrix (Qf = [H:I])
%       | H1C H1R H1L H12 |
%       | HCC HCR HCL HC2 |
%H    = | HRC HRR HRL HR2 |
%       | HLC HLR HLL HL2 |
 %|eCc| = | H1C_ HCC_ HRC_ HLC_ | | e1  |
 %|eRc| = | H1R_ HCR_ HRR_ HLR_ | | eCt |
 %|eLc| = | H1L_ HCL_ HRL_ HLL_ | | eRt |
 %|e2 | = | H12_ HC2_ HR2_ HL2_ | | eLt |
```

```
%================================
% % STEP 4
% % Determine the Edge flow and Effort matrixes
% %================================
H1C_ = 0; H1C = H1C_';
HCC_ = 0; HCC = HCC_';
HRC_ = 0;
HLC_ = 0;

H1R_ = 0; H1R = H1R_';
HCR_ = 0; HCR = HCR_';
HRR_ = 0; HRR = HRR_';
HLR_ = 0; HLR = HLR_';

H1L_ = H_(1:6,1:6); H1L = H1L_';
HCL_ = H_(1:6,7:10); HCL = HCL_';
HRL_ = H_(1:6,11:16); HRL = HRL_';
HLL_ = 0; HLL = HLL_';

H12_ = 0; H12 = H12_';
HC2_ = 0; HC2 = HC2_';
HR2_ = 0; HR2 = HR2_';
HL2_ = 0; HL2 = HL2_';


%================================
% STEP 5
% Identify the Resistor Matrix
%================================
syms R1 R2 R3 R4 R5 R6
Resistors_cotree = [0];
Resistors_tree   = [R1,R2,R3,R4,R5,R6];
Rt = diag(Resistors_tree);
Rc = diag(Resistors_cotree);

%================================
% STEP 6
% Identify the Capacitor Matrix
%================================
syms C1 C2 C3 C4
Capacitors_cotree = [0];
Capacitors_tree   = [C1,C2,C3,C4];
Ct = diag(Capacitors_tree);
Cc = diag(Capacitors_cotree);
C = Ct + HCC*Cc*HCC';

%================================
% STEP 7
% Identify the inductor Matrix
%================================
syms L1 L2 L3 L4 L5 L6 %If there is no inductors make matrices 0
Inductors_cotree = [L1,L2,L3,L4,L5,L6];
Inductors_tree   = [0];
Lc = diag(Inductors_cotree);
Lt = diag(Inductors_tree);
L = Lc + HLL'*Lt*HLL;
```

```
%==============================
% STEP 8
%Identify the States and sources
%======================================
syms W1 W2 W3 W4 W5 W6  %W =  massflows
syms P1 P2 P3 P4        %P = internal pressures
eCt = [P1;P2;P3;P4];
fLc = [W1;W2;W3;W4;W5;W6];

syms P01 P02 PH1 PH2 PH3 PH4   %Pressures
e1 = [P01;P02;PH1;PH2;PH3;PH4];
f2 = [0];
%======================================

%==============================
% STEP 9
%Calculate A and B matrice entries
%==============================
%A-MATRIX
%=========================================
if(HCR == 0)
   A11 = [0];
else
   A11 = -HCR*inv(Rc + HRR'*Rt*HRR)*HCR';
end
%=========================================
if(HCR == 0)
            A12 = -HCL;
            A21 = -A12';
elseif(HRR' == 0)
            A12 = -HCL;
            A21 = -A12';
elseif(HRL == 0)
            A12 = -HCL;
            A21 = -A12';
else
             A12 = -HCL + HCR*inv(Rc + HRR'*Rt*HRR)*HRR'*Rt*HRL;
             A21 = -A12';
end
%=========================================
if(HRL == 0)
            A22 = 0;
elseif(HRR == 0)
            A22 = -HRL'*inv(inv(Rt) + 0)*HRL;
else
            A22 = -HRL'*inv(inv(Rt) + HRR*inv(Rc)*HRR')*HRL;
end
%=====================================================

%B-MATRIX
%=====================================================
if(HCR == 0)
               B11 = 0;
elseif(H1R' == 0)
               B11 = 0;
elseif(HRR == 0)
               B11 = -HCR*inv(Rc + 0)*H1R';
else
               B11 = -HCR*inv(Rc + HRR'*Rt*HRR)*H1R'
end
%=====================================================
if(HCR == 0)
               B12 = -HC2;
```

```
elseif(HR2 == 0)
            B12 = -HC2;
elseif(HRR' == 0)
            B12 = -HC2;
else
            B12 = -HC2 + HCR*inv(Rc + HRR'*Rt*HRR)*HRR'*Rt*HR2;
end
%=================================================
if(HRL == 0)
            B21 = H1L';
elseif(H1R' == 0)
            B21 = H1L';
elseif(HRR == 0)
            B21 = H1L';
else
   B21 = H1L' - HRL'*inv(inv(Rt) + HRR*inv(Rc)*HRR')*HRR*inv(Rc)*H1R';
end
%=================================================
if(HRL == 0)
   B22 = 0;
elseif(HR2 == 0)
   B22 = 0;
elseif(HRR == 0)
   B22 = -HRL'*inv(inv(Rt) + 0)*HR2;
else
   B22 = -HRL'*inv(inv(Rt) + HRR*inv(Rc)*HRR')*HR2;
end
%=================================================

%================================
% STEP 10
%Generate Capacitor and Inductor matrix
%================================
%=======================================
CL = [C,zeros([length(C) length(L)]);
    zeros([length(L) length(C)]),L]
%=======================================

%================================
% STEP 11
%Generate final A and B matrices
%=======================================

%Define size of the matrice A and B and fill them with Zeros
%=============================================================================
Siz_eCt = size(eCt);
Siz_fLc = size(fLc);

Siz_e1 = size(e1);
Siz_f2 = size(f2);

AA = [zeros([Siz_eCt(1,1) Siz_eCt(1,1)])  zeros([Siz_eCt(1,1) Siz_fLc(1,1)]);
    zeros([Siz_fLc(1,1) Siz_eCt(1,1)])  zeros([Siz_fLc(1,1) Siz_fLc(1,1)])]

BB = [zeros([Siz_eCt(1,1) Siz_e1(1,1)])  zeros([Siz_eCt(1,1) Siz_f2(1,1)]);
    zeros([Siz_fLc(1,1) Siz_e1(1,1)])  zeros([Siz_fLc(1,1) Siz_f2(1,1)])]
%=============================================================================
%Fill matrices with Zeros if they do not have the same size as what is desired
%=============================================================================
%B-MATRICES
sizB11 = size(B11);
for(i=1:Siz_eCt(1,1)-sizB11(1,1))
```

```
   B11(i+sizB11(1,1),:) = 0;
end
   for(j=1:Siz_e1(1,1) - sizB11(1,2))
      B11(:,j+sizB11(1,2)) = 0;
   end
      sizB12 = size(B12);
for(i=1:Siz_eCt(1,1)-sizB12(1,1))
   B12(i+sizB12(1,1),:) = 0;
end
   for(j=1:Siz_f2(1,1) - sizB12(1,2))
      B12(:,j+sizB12(1,2)) = 0;
   end
      sizB21 = size(B21);
for(i=1:Siz_fLc(1,1)-sizB21(1,1))
   B21(i+sizB21(1,1),:) = 0;
end
   for(j=1:Siz_e1(1,1) - sizB21(1,2))
      B21(:,j+sizB21(1,2)) = 0;
   end
      sizB22 = size(B22);
for(i=1:Siz_fLc(1,1)-sizB22(1,1))
   B22(i+sizB22(1,1),:) = 0;
end
   for(j=1:Siz_f2(1,1) - sizB22(1,2))
      B22(:,j+sizB22(1,2)) = 0;
   end

%-----------------------------------------------------------
%A-MATRICES
sizA11 = size(A11);
for(i=1:Siz_eCt(1,1)-sizA11(1,1))
   A11(i+sizA11(1,1),:) = 0;
end
   for(j=1:Siz_eCt(1,1) - sizA11(1,2))
      A11(:,j+sizA11(1,2)) = 0;
   end
      sizA12 = size(A12);
for(i=1:Siz_eCt(1,1)-sizA12(1,1))
   A12(i+sizA12(1,1),:) = 0;
end
   for(j=1:Siz_fLc(1,1) - sizA12(1,2))
      A12(:,j+sizA12(1,2)) = 0;
   end
      sizA21 = size(A21);
for(i=1:Siz_fLc(1,1)-sizA21(1,1))
   A21(i+sizA21(1,1),:) = 0;
end
   for(j=1:Siz_eCt(1,1) - sizA21(1,2))
      A21(:,j+sizA21(1,2)) = 0;
   end
     sizA22 = size(A22);
for(i=1:Siz_fLc(1,1)-sizA22(1,1))
   A22(i+sizA22(1,1),:) = 0;
end
   for(j=1:Siz_fLc(1,1) - sizA22(1,2))
      A22(:,j+sizA22(1,2)) = 0;
   end

%===========================================================================
%STEP 12
%Construct prelimary A and B matrices
%===========================================================================
AA = [A11 A12; A21 A22]
```

```
BB = [B11 B12; B21 B22]
%=========================================================================

%Generate final A and B matrices
if(C == 0)
   A21 = inv(L)*A21;
   A22 = inv(L)*A22;
   B21 = inv(L)*B21;
   B22 = inv(L)*B22;
   A = [A11 A12; A21 A22]
   B = [B11 B12; B21 B22]
elseif(L == 0)
   A11 = inv(C)*A11;
   A12 = inv(C)*A12;
   B11 = inv(C)*B11;
   B12 = inv(C)*B12;
   A = [A11 A12; A21 A22]
   B = [B11 B12; B21 B22]
else
   A = inv(CL)*AA
   B = inv(CL)*BB
end
```

END OF PROGRAM

8.4. MATLAB® Program that solves the state-space model for the double U-tube pipe system

```
function StateSpaceNonLinSolve
%Program: Simulates the state space model derived from the linear network
%Methodology: The state space model is used as input in the MATLAB solver.
%The Resistor, capacitor and Inductor values are calculated and fed into
%the solver

clc
clear all
close all


%initial condition Parameter identification;

X0_h = [100e3;100e3;100e3;100e3;0;0;0;0;0;0]; %[P1 P2 P3 P4;W1;W2;W3;W4;W5;W6]

T_oe = 16.0016;        % Temperature exiting - Info from Flownex
T_oi = 15.0016;        % Temperature entering - Info from Flownex
m_dot=2.08384;         % Resulting mass flow rate in pipe - Info from Flownex
rho1 = 998.66;         %Density of water at 15 degree Celsius
rho2 = 998.496;        % Resulting density in pipe - Info from Flownex
d=0.217;               % diameter in m
A=(pi/4)*d^2;          % Area
h_1=288.5 %[kJ/kg]     %entalphy for water at temperature 15 degree Celsius
h_2=290.5 %[kJ/kg]     %entalphy for water at temperature 17 degree Celsius
cp_1=4.184 %[kJ/kg-K]  %cp for water at temperature 15 degrees Celsius
cp_2=4.183 %[kJ/kg-K]  %cp for water at temperature 17 degrees Celsius
T1=h_1/cp_1;
T2=h_2/cp_2;
P_exstra1 = (T_oe - T_oi)*(1/T1)*(m_dot^2/(2*rho1*A^2)); %  thermal effect on pressure taken into account
P_exstra2 = (T_oe - T_oi)*(1/T2)*(m_dot^2/(2*rho2*A^2)); %  thermal effect on pressure taken into account
```

```
%Heights inputs
%The lowest level node be equal to h = 0 m
h1 = 23.45;
h2 = -23.45;

%Define the substance charateristics
rho = 998.66;              %Density of water at 15 degree Celsius
g = 9.81;                  %Gravitational acceleration
m_u = 0.001138;            %Viscosity of wate at 15 degrees Celsius

% Physical Condition inputs
    %P01 P02  Ph1           Ph2                Ph3
u_h =  [0;0;rho*g*h1+P_exstra1; rho2*g*h2+P_exstra2; rho*g*h1+P_exstra1;
Ph4              f0
rho2*g*h2+P_exstra2;0];

%timespan of simulation
tspan = [0,1000];

%simulate
options = odeset('RelTol',1);

[t,X] = ode23s(@State_Space_Function, tspan,X0_h,options,u_h);      %call the solver
disp(X(:,1:6))
plot(t,X(:,9));
xlabel('time')
ylabel('mass flow')
figure
plot(t,X(:,1));
xlabel('time')
ylabel('P1')
figure
plot(t,X(:,2));
xlabel('time')
ylabel('P2')


%The Function organise the state space model in a form required by the solver

function [dx_h]=State_Space_Function(t,x,u_h)

%Set up the component values again
LL1 = 23.45;           %Length of Down Comer -U-tube 1(m)
LL2 = 0.01;            %Length of Orifice -U-tube 1(m)
LL3 = 23.45;           %Length of Riser -U-tube 1(m)

LL4 = 23.45;           %Length of Down Comer -U-tube 2(m)
LL5 = 0.01;            %Length of Orifice -U-tube 2(m)
LL6 = 23.45;           %Length of Riser -U-tube 2(m)

LCV1 = 23.505;         %Length of the Control volumes (m)
LCV2 = 23.505;
LCV3 = 23.505;
LCV4 = 23.505;
d1 = 0.217;            %Diameter of pipe (m)
d2 = 0.03;             %Diameter of Orifice (m)
Area1 = (pi*d1^2)/4;       %Cross-sectional area of the pipe(m^2)
Area2 = (pi*d2^2)/4;       %Cross-sectional area of orifice(m^2)
rho = 998.66;          %Density of water at 15 degree Celsius
rho2 = 998.496         %Density after simulation in Flownex
g = 9.81;              %Gravitational acceleration
```

```
m_u = 0.001138;          %Viscosity of wate at 15 degrees Celsius
f = 0.1872;              %Friction factor
k = 0.00805;             %Head loss coefficient


%Nonlinear values of the resistor within the system
R1 = ((f*LL1)/d1 + k)/(2*Area1^2*rho*g);
R2 = ((f*LL2)/d2 + k)/(2*Area2^2*rho*g);
R3 = ((f*LL3)/d1 + k)/(2*Area1^2*rho2*g);
R4 = ((f*LL4)/d1 + k)/(2*Area1^2*rho*g);
R5 = ((f*LL5)/d2 + k)/(2*Area2^2*rho*g);
R6 = ((f*LL6)/d1 + k)/(2*Area1^2*rho2*g);



%Calculate the values of the inductors within the system
L1 = LL1/Area1;
L2 = LL2/Area2;
L3 = LL3/Area1;
L4 = LL4/Area1;
L5 = LL5/Area2;
L6 = LL6/Area1;

 Bulk = 200e6;                %Bulk Modulus 200 MPA

%Calculate the values of the capacitors within the system
C1 = (rho*Area1*LCV1)/Bulk;
C2 = (rho2*Area1*LCV2)/Bulk;
C3 = (rho*Area1*LCV3)/Bulk;
C4 = (rho2*Area1*LCV4)/Bulk;

%Calculated A and B matrices – calculated by program in Appendix 8.3


A_h = [  0, 0, 0, 0, 1/C1, -1/C1, 0, 0, 0, 0;
        0, 0, 0, 0, 0, 1/C2, -1/C2, 0, 0, 0;
        0, 0, 0, 0, 0, 0, 0, 1/C3, -1/C3, 0;
        0, 0, 0, 0, 0, 0, 0, 0, 1/C4, -1/C4;
        -1/L1, 0, 0, 0, -1/L1*R1*abs(x(5,1)), 0, 0, 0, 0, 0;
         1/L2, -1/L2, 0, 0, 0, -1/L2*R2*abs(x(6,1)), 0, 0, 0, 0;
         0, 1/L3, 0, 0, 0, 0, -1/L3*R3*abs(x(7,1)), 0, 0, 0;
         0, 0, -1/L4, 0, 0, 0, 0, -1/L4*R4*abs(x(8,1)), 0, 0;
         0, 0, 1/L5, -1/L5, 0, 0, 0, 0, -1/L5*R5*abs(x(9,1)), 0;
         0, 0, 0, 1/L6, 0, 0, 0, 0, 0, -1/L6*R6*abs(x(10,1))];


B_h = [   0,    0,    0,    0,    0,    0,    0;
          0,    0,    0,    0,    0,    0,    0;
          0,    0,    0,    0,    0,    0,    0;
          0,    0,    0,    0,    0,    0,    0;
          1/L1,  0,  1/L1,    0,    0,    0,    0;
          0,    0,    0,    0,    0,    0,    0;
          0, -1/L3,    0,  1/L3,    0,    0,    0;
          1/L4,  0,    0,    0,  1/L4,    0,    0;
          0,    0,    0,    0,    0,    0,    0;
          0,  -1/L6,    0,    0,    0,  1/L6,    0];


  dx_h = A_h*x + B_h*u_h; %Hydraulic state space

return
```

END OF PROGRAM