HEDGE FUND PERFORMANCE EVALUATION USING THE KALMAN FILTER

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Abstract

In the capital asset pricing model, portfolio market risk is recognised through β while α summarises asset selection skill. Traditional parameter estimation techniques assume time-invariance and use rolling-window, ordinary least squares regression methods. The Kalman filter estimates *dynamic* αs and βs where measurement noise covariance and state noise covariance are known – or may be calibrated – in a state-space framework. These time-varying parameters result in superior predictive accuracy of fund return forecasts against ordinary least square (and other) estimates, particularly during the financial crisis of 2008/9 and are used to demonstrate increasing correlation between hedge funds and the market.

1 Introduction

Prior to the credit crisis of 2008/9, hedge funds were characterised by high returns compared with conventional funds and indices, large capital inflows and low market correlation (Tran, 2006). Hedge funds also charged high fees from their opaque investment strategies (usually a percentage of profits and a steep – up to 20% – management fee), and operated largely outside the heft of market regulations (Poloner, 2010: 308). The substantial out-performance of hedge funds prior to the crisis offered investors substantial returns by enhancing their portfolio risk-return trade-off.

The financial crisis, however, altered the hedge fund universe considerably and perhaps permanently. Since the crisis began in 2008, hedge fund investment performance has substantially deteriorated: large portfolio losses have fed a spiral of diminishing investment and investor withdrawals (Ben-David, Franzoni &

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Moussawi, 2011). Diversification benefits of hedge funds have continuously deteriorated as the correlation between hedge fund return and conventional asset class returns have slowly, but inexorably, increased (Guesmi, Jebri, Jabri & Teulon, 2014). This dismal return profile (Figure 1) no longer justifies the high fees enjoyed by hedge funds prior to the crisis (Authers, 2014). Investors have abandoned (and continue to abandon) hedge funds in favour of cheaper, higher return investments elsewhere (e.g. Schlaikier, 2014 and Marois, 2014), often with lower risk profiles.

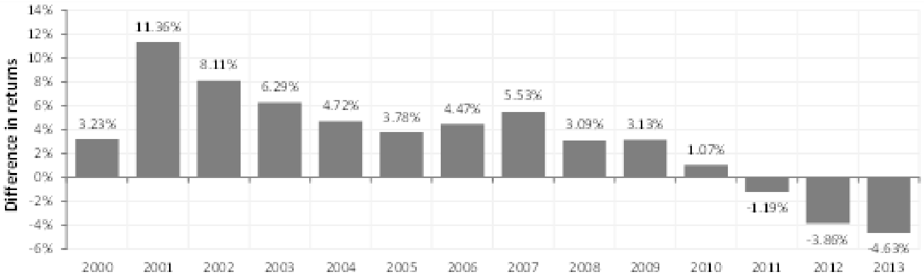

Figure 1: Difference between hedge fund returns and conventional asset class average returns from 2000 to 2013.

In the light of the waning popularity of hedge funds, financial performance measures have acquired renewed significance (Heuson, Hutchinson & Kumar, 2014 and Bussière, Hoerova & Klaus, 2014: 32). Hedge funds must now establish robust, consistent performance, with reasonable certainty, before yield-seeking investors participate in (traditionally) opaque investment strategies. Fortunately, several such measures exist, among them the Sharpe ratio (which measures excess fund returns relative to the risk free rate per unit of risk), the Information ratio (excess fund return relative to a benchmark per unit of tracking error, or relative risk), the Treynor ratio (excess fund return relative to the risk free rate per unit of systemic market risk, β) and several others. A detailed summary of hedge fund performance measures can be found in Aragon & Ferson (2006: 88) and more recently in Le Sourd (2009).

The capital asset market model (CAPM) – an equilibrium model – has profoundly influenced the way investors understand and react to the asset price/risk relationship (Sharpe, 1964: 428; Mossin, 1966: 779 and Fama & French, 2004: 26 – 28). The CAPM formulation asserts that fund excess returns over a benchmark (usually the market) include an abnormal portfolio return, α and a single risk measure of risk, β, which affects systematic differences in security returns. The CAPM coefficients (α and β) were originally assumed to be static – or relatively static – and most formulations still make this assumption (as evidenced by regression tests conducted to assess fund performance: these implicitly assume constant coefficients). Evidence indicates, however, that CAPM coefficients are dynamic, changing as market factors change and as portfolio components are altered by fund managers (e.g. Glabadanidis, 2009: 253–254). Measuring time-varying coefficients is non-trivial, yet it is important to establish whether they are static or behave dynamically (and, if dynamic, the question of whether this time-dependence is due to changing βs of the underlying assets or to the change of portfolio weights via the fund manager becomes important). The latter represents an active strategy justifying the high fees, as does the accurate, dynamic measurement of manager stock-selection skill (α) (Albrecht, 2005).

Various techniques have been developed to estimate time-varying coefficients such as time-weighted least-square estimators (including non-parametric weights) and GARCH-based estimators (Nieto, Orbe & Zarraga, 2014: 28 – 30). Bali & Engle (2010: 388) introduced and Engle (2014) tested a new method for formulating dynamic betas using conditional covariance matrices of both the exogenous and dependent variables. Called the Dynamic Conditional Beta (DCB), the approach was successfully applied to asset pricing and global systemic risk estimation (Engle, 2014).

Each of the above technique has drawbacks. Rolling-window ordinary least squares techniques require no parameterisation, but require window lengths to be pre-selected resulting in unstable β estimates (Lin, Chen & Boot, 1992: 522–523) and although GARCH(1,1) models describe volatility clustering and other important features of returns such as excess kurtosis, standard GARCH models do not capture important volatility properties (Prysyazhnyuk & Kirdyaeva, 2010). Engle's (2014) DCB approach does not observe the vector of conditional means nor conditional covariances directly, so models are required for each of these, requiring yet more assumptions.

The Kalman filter, originally developed to assist with aeronautics, may provide a solution. A principal advantage of the Kalman filter is that it can be applied in real time, i.e. for any observed value of the time series the forecast for the next observation can be calculated, making the method highly practical and important in the financial field (see, e.g., Jain, Yongvanich & Zhou, 2011). The filter uses only historical information, but it reacts rapidly to changing conditions. These expedient properties confirm the usefulness and applicability of the Kalman filter to the changing nature of hedge fund portfolios and the importance of it to detect crises or large market changes. Indeed, the Kalman filter is suited to take into consideration the multiple investment style variations of actively managed hedge funds (Swinkels & Sluis, 2006: 538 – 539), but its use has been rather limited in the literature due to the limited size of hedge fund databases. In addition, due to the complexity of understanding and implementing the Kalman filter, it has not been widely used over the traditional regression analysis in most of the statistical inference problems.

The remainder of this article proceeds as follows: Section 2 reviews the literature regarding the CAPM, its successes, failures and limitations and explores the validity of static CAPM coefficients in a dynamic investment environment. Work detailing the application of the Kalman filter to fund returns with a view to extracting the unobserved, time-varying CAPM coefficients (α and β) is also examined in this section. The data used for the study are characterised and defended in Section 3, along with a description of the mechanics of the Kalman filter assembly, construction and implementation. The results obtained are presented and interpreted in Section 4 as well as consequences of the results for hedge fund managers and investors. Section 5 concludes.

2. Literature review

In the risk premium model of the CAPM, excess return on a security (or portfolio) is calculated as a combination of 'abnormal return' generated by the skill of the fund manager (either through timing or asset selection) and the product of the market risk premium and systematic risk, i.e.:

$$\left(r_P - r_f\right) = \alpha + \beta\left(r_M - r_f\right) + \epsilon \tag{1}$$

where r_P is the security (or portfolio) return, r_f is the risk free rate, α is the abnormal rate of return on the security/portfolio, r_M is the market return, β is a measure of systemic risk given by

$$\frac{\sigma_P}{\sigma_M} \cdot \rho_{PM} \tag{2}$$

where σ_P is the portfolio volatility, ρ_{PM} is the correlation between the portfolio and market returns and σ_M is the market volatility. The noise term, ϵ, is assumed i.i.d. and $\sim N(0, \sigma_\epsilon^2)$. A detailed review of the principal concepts underlying the CAPM, its historical development and applications may be found in Fama & French (2004: 28 – 29) and Perold (2004: 17 – 18).

Forecasting accurate αs and βs is important for market participants: accurate predictions facilitate investment decisions and help evaluate fund manager performance. Corporate financial managers employ α and β forecasts to assist with capital structure decisions as well as appraise investment decisions (Choudhry & Hao, 2009 and Celik, 2013: 441 – 442). Celik (2013: 20 – 21) argued that the assumption of static α and β values could lead to erroneous assessment of fund manager performance. Although early empirical tests found the CAPM to be robust and reliable (Black, Jensen & Scholes, 1973; Fama and Macbeth, 1973: 619 – 620; He & Ng, 1994: 622 – 623 and Pettengill, Sundaram & Mathur, 1995: 108 – 111), later studies questioned the non-stationarity of β and the risk premium (Fama and French, 1992: 431 – 433; Davis, 1994: 1539 – 1541), the inadequacy of the market portfolio proxy (He & Ng, 1994: 601 – 603) and joint hypothesis test problems associated with unobservable expected returns (Burnie & Gunay, 1993: 22 – 23 and Pettengill *et al.*, 1995: 108 – 109).

The descriptive accuracy of constant (as opposed to time-varying) βs was also questioned by Chan & Chen (1988: 312 – 313) and later by Longstaff (1989: 875 – 876), Ferson & Harvey (1991: 400 – 403) and then Fama & French (1992: 27 – 28). Since the capital (and hence risk) structure of all companies change with the macroeconomic environment, Jagannathan & Wang (1993) asserted that the constant β assumption was unreasonable and that a more appropriate evaluation would be to examine the relationship between returns and time-varying βs. Jagannathan & McGrattan (1995: 9 – 11) blamed the focus on *constant* β values on the fact that the CAPM model had been originally developed to explain differences in risks across capital assets. Later work found considerable improvement in the description and accuracy of portfolio return behaviour if the constant β requirement was relaxed (Groenewold & Fraser, 1999: 538 – 539; Black & Fraser, 2000: 1020 – 1022; Fraser, Hamelink, Hoesli & Macgregor, 2000 and Prysyazhnyuk & Kirdyaeva, 2010).

Other econometric methods have been employed to estimate time-varying βs (Brooks, Faff & McKenzie, 1998: 17 – 19): two well-known approaches are GARCH models (various types are discussed in Choudhry & Hao, 2009: 441 – 443) and the Kalman filter (e.g. Black, Fraser & Power, 1992: 1018 and Well, 1994: 79 – 81). The former construct the conditional β series using conditional variance information while the latter uses an initial set of priors to estimate the β series recursively, thereby generating a series of conditional αs and βs. Das & Ghoshal (2010: 1928 – 1929) found that estimating dynamic βs in the CAPM using traditional (auto-regressive) methods, yielded suboptimal results while the Kalman filter was able to estimate an optimal dynamic β even where measurement noise and state noise covariances were unknown, but themselves dynamically determined.

Albrecht (2005) concluded that models which assumed time-varying αs and βs provided superior return forecasts to those that assumed static CAPM coefficients (such as OLS regression models). Albrecht (2005) applied dynamic exposure results – derived from the Kalman model – to fund returns and found that dynamic CAPM coefficients could be partially explained by the active adjustment of portfolio-weights, confirming that value generated by fund managers arises not only from asset selection skills, but also from dynamic portfolio management.

Jain *et al.* (2011) assessed the skills-based component (α) of US fund returns from data spanning 18 years (1993 – 2011). Jain *et al.* (2011) regressed the fund returns against S&P 500 index returns using ordinary least squares regression (OLS), but obtained spurious results. Factor sensitivities were found to vary over time and the regression analysis averaged α and β over the data set. A rolling regression technique was then attempted, in which α and β were calculated over a fixed window of 36 months (this being found to optimally balance variance and bias) and then rolled forward by one month to obtain the next month's α and β. A time-weighted regression, in which decreasing weights are assigned to observations the longer ago they occurred in the past, was also attempted. Although the last two techniques are common choices for estimating average α and β, Jain *et al.* (2011) found them to be inadequate at capturing the dynamic nature of the CAPM coefficients, in particular when funds' strategic investment horizons were smaller than the window size.

Jain *et al.* (2011) then employed a Kalman filter to establish the dynamics of hedge fund exposures. Although the filter requires a substantial amount of data (roughly an order of magnitude more than standard OLS regressions) this limitation was partially ameliorated by imposing more structure to the model, e.g. by assuming no correlation between α and β. Both α and β retain their unique variances, but the covariance between α and β (off-diagonal elements in the process covariance matrix, $\boldsymbol{Q}$ – see (4)) is set to 0. Despite noisy data, Jain *et al.* (2011) were able to extract reasonable values of the hidden variables: fund market risk exposure (α) and market sensitivity (β).

Although the Kalman filter has been shown to be better than most models at capturing exposure dynamics, this flexibility is sometimes disadvantageous. If the model specification describing the underlying process is inaccurate, or if too few return data are used to generate forecasts, the filter fits the excess noise rather than the core signal (Punales, 2011). While obsolete data renders regression model results *and* Kalman filter model results inaccurate if substantial shifts in hedge fund risk profiles occur, the dynamic quality of the state space model allows it to adapt more quickly (Punales, 2011). Assumptions used in the formulation of the Kalman filter are thus more robust than the assumption of constant exposure employed in regression analysis (Tsay, 2010).

The inherent flexibility of the Kalman model is required to capture dynamic fund exposure behaviour and, as a result, the technique employed to estimate exposures may vary over time and across hedge funds. So, while OLS multivariate regression may be suitable for a hedge fund characterised by slowly-varying exposures, the Kalman filter proves the superior approach for hedge funds during volatile periods (Roll & Ross, 1994: 113 – 114 and Faff, Hillier & Hillier, 2000: 528).

This article explores the conditional relationship between hedge fund returns and non-stochastic time-varying βs using a Kalman filter (time series) analysis technique.

3. Data and methodology

3.1 Data

The data employed were monthly returns of global hedge fund indices representing various investment styles as well as a broad, 'hedge fund index' (HFI) for the entire industry. These data were procured from the Hedge Fund Research (HFR) Global Hedge Fund database spanning 18 years, i.e. from January 1997 to December 2014 inclusive (HFR, 2014).

A summary of the annualised descriptive statistics for all funds is provided in Figures 2(a) through (d) as well as the Morgan Stanley Capital International (MSCI) world index and the broad HFI for comparison. Annualised returns are determined using $(1+\overline{r}_m)^{12}-1$, where $\overline{r}_m$ is the average monthly return over the full period, annual volatility is $\sigma_m \cdot \sqrt{12}$, where σ_m is the monthly standard deviation of all monthly returns, annual skew is $s_m/\sqrt{12}$, where s_m is the skew of all monthly returns, and annual excess kurtosis is $(k_m + 3 \cdot 11)/12 - 3$, where k_m is the kurtosis of all monthly returns for the relevant index. These are standard scaling techniques for the first four moments of return distributions.

Hedge fund return data are generally negatively-skewed and leptokurtic.

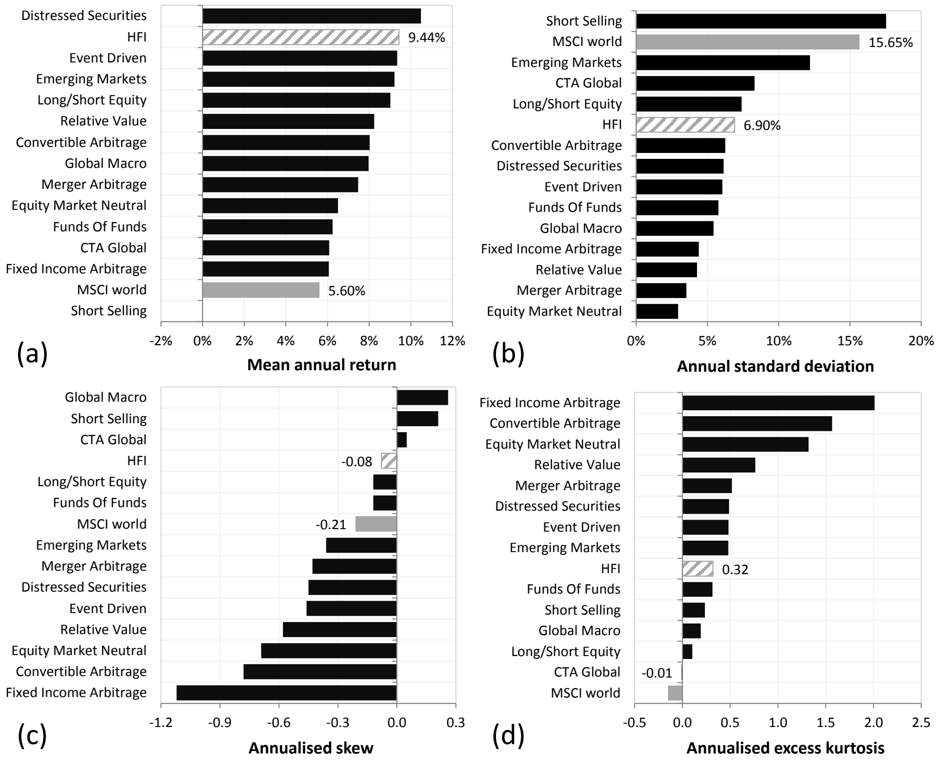

Figure 2: Hedge fund return statistics for various investment types

Figure 3(a) shows the monthly HFI returns (left axis) for the period under investigation as well as the cumulative return profile (right axis). MSCI world index returns are included for comparison on the same timescale. Monthly volatilities were calculated using an exponentially weighted moving average (EWMA) approach with λ (the weighting constant – see Hendricks, 1996: 49 – 52) calibrated as $\lambda = 0.950$ (calibrated using representative global market data as at December 2014). Figure 1(b) shows the monthly volatility profile of both indices over the same time period.

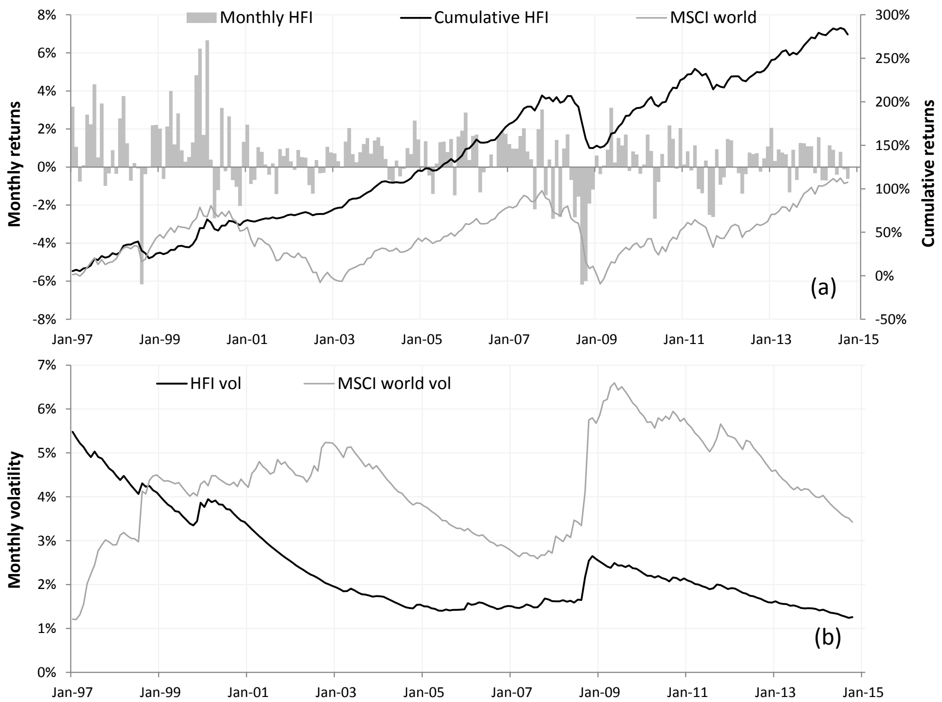

Figure 3: (a) Monthly and cumulative HFI returns and monthly MSCI world returns for comparison and (b) EWMA volatilities of HFI and MSCI.

3.2 Kalman filter specification

The Kalman filter (Kalman, 1960: 38 – 39) is a Bayesian updating scheme that maximises the likelihood of correctly estimating unknown parameter values (Koch, 2006). The filter addresses the general problem of attempting to estimate the state $[x \in \Re^n]$ of a discrete, time-controlled process governed by the linear stochastic difference equation:

$$x_t = \boldsymbol{F}x_{t-1} + \boldsymbol{B}u_{t-1} + w_{t-1}$$

with a measurement $[z \in \Re^n]$:

$$z_t = \boldsymbol{H}x_t + v_t.$$

The random variables w and v represent process white noise and measurement white noise respectively. These are assumed to be independent of each other (i.e. 0 correlation between them) with normal probability distributions:

$w(\cdot)\sim N(0,\boldsymbol{Q})$
$v(\cdot)\sim N(0,\boldsymbol{R})$.

In practice, the process noise covariance $\boldsymbol{Q}$ and measurement noise covariance $\boldsymbol{R}$ matrices (here variance matrices since $\rho = 0$) might change with each time step, however here they are assumed to be constant (Koch, 2006). These values were obtained by maximum likelihood methods.

The 2×1 (in this case) state transition matrix $\boldsymbol{F}$ links the state at the previous time step $t-1$ to the current state at step t, assuming no driving function or process noise. The 2×2 control matrix $\boldsymbol{B}$ relates the optional control input $\boldsymbol{u} \in \Re^{l}$ to the state x. The 2×1 matrix $\boldsymbol{H}$ in the measurement relates the state to the measurement z_k. In practice $\boldsymbol{F}$ and $\boldsymbol{H}$ might change with each time step, but here they are both assumed to be constant.

The mechanical process to be followed is:

PREDICT

Project state 1 time step ahead	$\hat{\boldsymbol{x}}_{t\|t-1} = \boldsymbol{F}_t\hat{\boldsymbol{x}}_{t-1\|t-1} + \boldsymbol{B}_t\boldsymbol{u}_t$	(3)
Project error covariance 1 step ahead	$\boldsymbol{P}_{t\|t-1} = \boldsymbol{F}_t\boldsymbol{P}_{t-1\|t-1}\boldsymbol{F}_t^T + \boldsymbol{Q}_t$	(4)

UPDATE

Compute Kalman gain	$\boldsymbol{K}_t = \boldsymbol{P}_{t\|t-1}\boldsymbol{H}_t^T\left(\boldsymbol{H}_t\boldsymbol{P}_{t\|t-1}\boldsymbol{H}_t^T + \boldsymbol{R}_t\right)^{-1}$	(5)
Update estimate with measurement $\boldsymbol{y_t}$	$\hat{\boldsymbol{x}}_{t\|t} = \hat{\boldsymbol{x}}_{t\|t-1} + \boldsymbol{K}_t\left(\boldsymbol{y}_t - \boldsymbol{H}_t\hat{x}_{t\|t-1}\right)$	(6)
Update error covariance	$\boldsymbol{P}_{t\|t} = (\boldsymbol{I} - \boldsymbol{K}_t\boldsymbol{H}_t)\boldsymbol{P}_{t\|t-1}$	(7)

where $\hat{\boldsymbol{x}}$ is the estimated state, $\boldsymbol{F}$ is the state transition matrix (i.e., transition between states), $\boldsymbol{u}$ represents the control variables, $\boldsymbol{B}$ is the control matrix (i.e., mapping control to state variables), $\boldsymbol{P}$ is the state variance matrix (i.e., error of estimation), $\boldsymbol{Q}$ is the process variance matrix (i.e., error due to process), $\boldsymbol{y}$ represents the measurement variables, $\boldsymbol{H}$ is the measurement matrix (i.e., mapping measurements onto the state), $\boldsymbol{K}$ is the Kalman gain and $\boldsymbol{R}$ is the measurement variance matrix (i.e., error from measurements).

Subscripts represent:

$t\|t$:	the current time period
$t-1\|t-1$:	the previous time period, and
$t\|t-1$:	intermediate steps.

3.3 CAPM equations

The observation equation is the CAPM model, given by:

$$r_P(t) = \alpha(t) + \beta(t) \cdot r_M(t) + \epsilon(t) \qquad \epsilon(t) \sim N(0, \sigma_\epsilon^2) \tag{8}$$

where r_P is the security (or portfolio) excess return (c.f. (1)), α is the abnormal rate of return on the security/portfolio, β is the systematic risk as defined earlier in (2), r_M is the excess market return (1) and ϵ is a noise term, assumed i.i.d. and $\sim N(0, \sigma_\epsilon^2)$.

The form of the transition equation depends on the form of stochastic process that the time-varying αs and βs are assumed to follow, so the transition equation may use an autoregressive, mean-reverting – AR(1) – model or a random walk process. It has been established that the random walk model provides the most robust characterisation of time-varying βs, while AR(1) forms of transition equation experience convergence problems (symptomatic of transition equation misspecification) for some return series (Faff *et al.*, 2000: 538).

The random walk model (RWM) – employed in this article – assumes that both α and β evolve according to a random walk, i.e. current market exposure is a normally-distributed random variable taking as mean the exposure of the last period. The uncorrelated system noises are also normally distributed.

The state variables $x(t) \in \Re^2$ are the time-varying coefficients:

$$x(t) = \begin{bmatrix} \alpha(t) \\ \beta(t) \end{bmatrix}$$

at each time t.

Both state variables are assumed to follow the random walk model. The state equation is:

$$\begin{bmatrix}\alpha(t+1)\\ \beta(t+1)\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \cdot \begin{bmatrix}\alpha(t)\\ \beta(t)\end{bmatrix} + \begin{bmatrix}\gamma\\ \delta\end{bmatrix} \quad (9)$$

where

$$\begin{bmatrix}\gamma\\ \delta\end{bmatrix} \sim N\left(\begin{pmatrix}0\\ 0\end{pmatrix}, \begin{bmatrix}\sigma_\gamma^2 & 0\\ 0 & \sigma_\delta^2\end{bmatrix}\right)$$

and the measurement equation is:

$$r_P(t) = \begin{bmatrix}1 & r_M(t)\end{bmatrix} \begin{bmatrix}\alpha(t)\\ \beta(t)\end{bmatrix} + \epsilon(t) \quad (10)$$

which is merely the matrix representation of the CAPM, i.e. (9).

It is important to note that the data used are not returns of hedge funds, but rather returns of hedge fund *indices*. Using indices rather than individual hedge funds drafts away from specific performance evaluation: calculating α for an index only makes sense if that index is investable, otherwise it is not a measure of performance, but more an indication of return related to the index characteristics.

3.4 Procedure

Hedge fund returns were grouped by geography (US and Canada, Europe and Asia) as this facilitated the use of a single risk free rate time series and a single market return index for each region). Monthly, annualised hedge fund *excess* returns were run through the Kalman model as specified above and the resulting α and β extracted.

For comparison and for later assessment, rolling 36 month OLS regression αs and βs (using excess market returns and excess fund returns – i.e. the annualised risk free rate was subtracted from both returns time series) were also calculated and plotted on the same time scale as those derived from the Kalman filter results.

4. Results and discussion

4.1 Diversification and hedge funds

Traditionally, hedge funds have been considered good sources of diversification due to their low correlation with conventional asset classes (e.g. Brown, Gregoriou

& Pascalau, 2011 and Teo, 2013: 5). Early hedge funds assembled such hedging strategies by being long in certain securities that were expected to outperform the market in upturns and short in unfavourable securities to provide protection in falling markets, thereby eliminating most systematic investment risk (Chan, Getmansky, Haas & Lo, 2006: 72). However, since the turn of the century hedge fund returns have shown *increasing* correlation with the market, leading to concerns (Amenc & Goltz, 2007) and criticisms (Markowitz, 2014) about their effectiveness as an effective *hedging* asset class.

Figure 4 shows the EWMA correlation ($\lambda = 0.950$) between two representative hedge fund indices and the HFI with the market index from Jan 97 to Dec 14. Although relatively high levels of correlation persist prior to the 2007-8 credit crisis for some fund indices, *during* the crisis correlations all decrease dramatically – indeed, fall to zero within eight months and then become negative. This may be an indication of hedge fund agility; the speed at which these funds can enter and exit markets and hedge positions effectively. However, high correlations prior to the crisis still signify minimal diversification when it may be most required. The shaded regions in Figure 4 indicate the 1998 Long Term Capital Management hedge fund collapse and the 2008/9 credit crisis, both periods of heightened correlation with the market and both preceding considerable market perturbations. Also, since the start of 2010, correlations have increased and continue (December 2014) to do so.

Note that diversification observations using only correlation coefficients, especially when the returns are from hedge fund *indices* (not from funds themselves), is not necessarily representative of hedge fund performance.

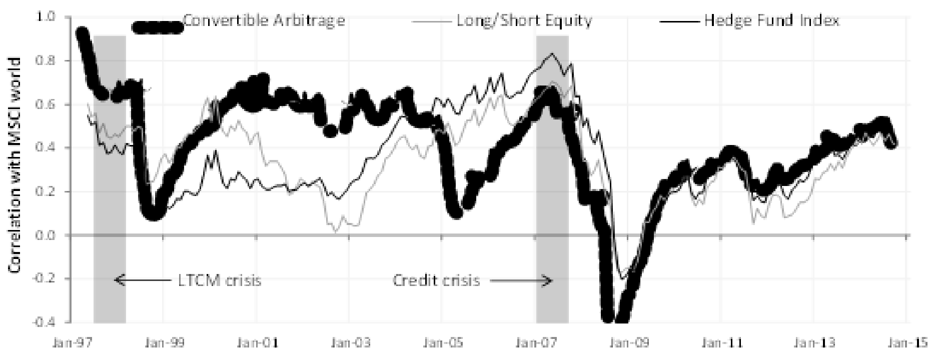

Figure 4: Correlation of selected hedge fund strategies with MSCI world

4.2 Comparison of regression and Kalman α and $\boldsymbol{\beta}$

The CAPM asserts that portfolio returns in excess of the risk free rate are the combination of abnormal return generated by manager skill, denoted by α, and the interaction between excess return on the market and systemic risk factor β. Figure 5 shows the evolution of the α for the Convertible Arbitrage Fund index over the period under investigation, as measured by traditional 36-month linear unweighted regression and the Kalman filter. Index excess returns are shown on the secondary axis. Similarities between the α values obtained from the two approaches are evident (e.g. between 2003 and 2005), but the regression α is slower to react to changes in the market compared with the Kalman approach. This is to be expected: regression weighs each regression component equally, while Kalman is far more adaptive – filtering out extraneous signal noise and capturing contemporaneous α rather than an OLS-averaged α spread over 36 months. To our knowledge the superiority of the Kalman filter's results (over the regression results) has not yet been demonstrated in the literature.

One example of this slow reaction time for regression α can be seen in Figure 5 over the period Jan 2009 to Jan 2011. Regression α in early 2006 embraces poor performance data measured over the crisis since it employs three years of equally weighted monthly data. The index's market outperformance after the end of the crisis improves α more slowly over this period than that observed for the Kalman α as the poor performance exits the regression data one month at a time. The Kalman approach is more responsive to improved (or deteriorating) performance – at times the differences between the two techniques are more than double. These observations could prove influential in fund manager compensation.

Figure 5: Comparison of Kalman filter and regression α estimates for the convertible arbitrage strategy with fund returns on the same timescale for comparison

This trait is also observed for the β coefficient, as illustrated in Figure 6 for the equity long-short hedge fund style. In this example, βs calculated for the HFI using the Kalman filter lead the regression values, responding more quickly to market changes. A clear example is represented in the shaded period, when the Kalman β doubled (from 0.125 to 0.250) before the regression βs showed any signs of rallying. This slow responsiveness of regression βs could prove detrimental to fund managers using CAPM to estimate this important market statistic.

Figure 6: Comparison of Kalman filter and regression β estimates for the equity long-short investment style

The superiority of the Kalman α and β measurements over the regression estimates of these parameters may be demonstrated by the forecast excess index values obtained using both methods. Using (1), forecast excess index returns may be calculated using α and β obtained from the Kalman approach and the regression methodology and a comparison may be drawn between forecast and measured values. The results for the long-short index are shown in Figure 7. Higher R^2 values for the Kalman forecast fit were observed for all indices.

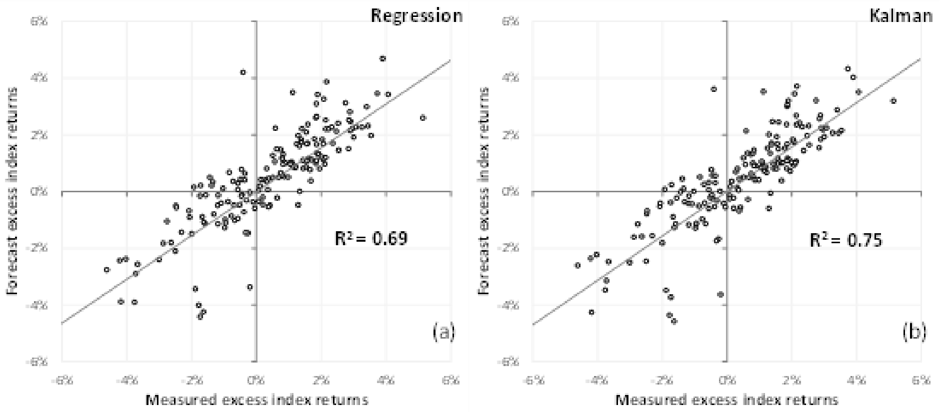

Figure 7: Comparison of excess index return forecasts using (a) rolling regression and (b) the Kalman filter

4.3 Constituents of $\boldsymbol{\beta}$

The constituents of β were defined in (2) as:

$$\beta = \left(\frac{\sigma_P}{\sigma_M}\right) \cdot \rho_{PM}$$

Two factors may be identified which influence the systemic risk coefficient, namely the ratio of index to market return volatilities, and the correlation between index and market returns.

Figure 8(a) shows the Kalman β for the equity market neutral index while Figures 8(b) and (c) illustrate how the components of (2) evolve over time.

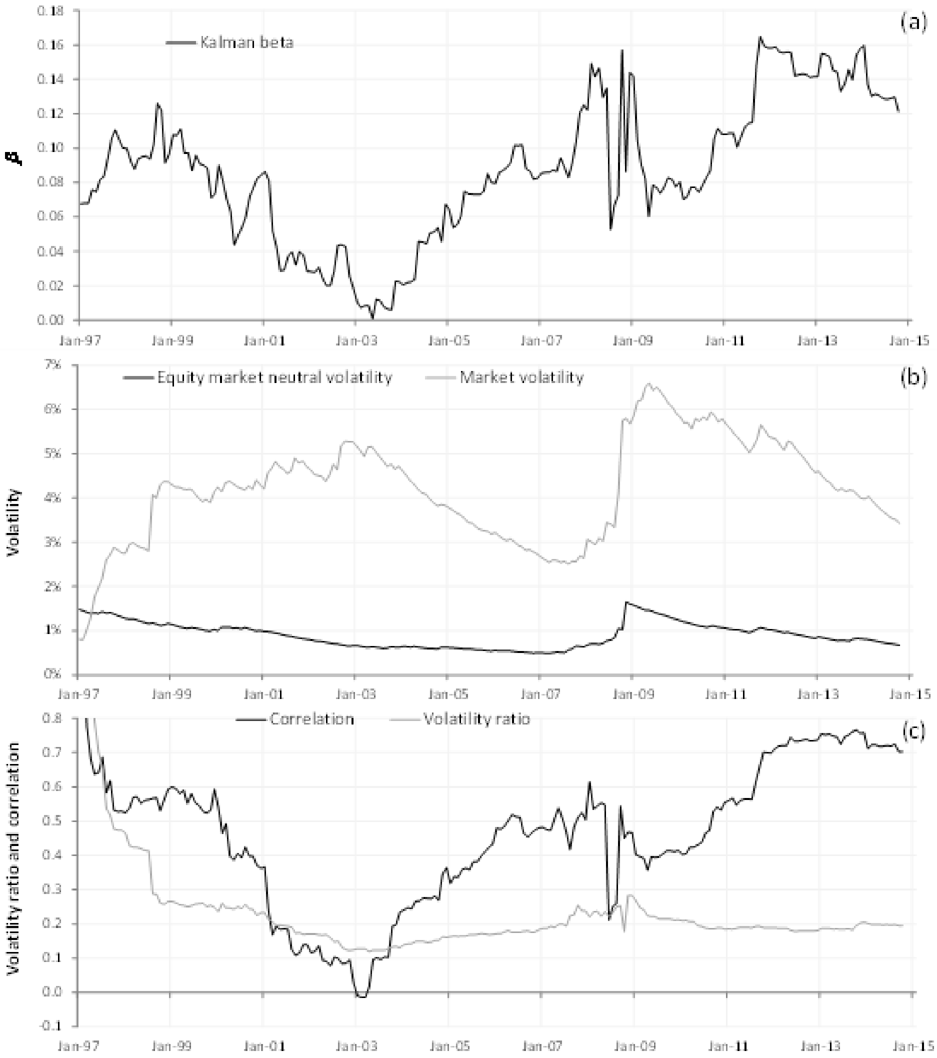

Figure 8: (a) Kalman β, (b) market and fund EWMA volatility and (c) volatility ratio and correlation between fund and market for the equity market neutral fund strategy

The interaction between these variables demonstrates how the correlation component dominates the β estimation. Most hedge fund index volatilities were *lower* than that observed for the market in absolute terms for the period under investigation: Figure 8(b) is a representative example. However, the volatilities (of index and market) generally move in unison, i.e. they are correlated to some extent, so the ratio of $\left(\frac{\sigma_P}{\sigma_M}\right)$ does not deviate far from its mean (0.20 in this case – see Figure 8(c)). The only remaining component of β is the correlation between index and market returns; Figure 8(c) illustrates a close resemblance between the principal features of index β (Figure 8(a)) and the correlation between index and market returns (Figure 8(c)). Only when the correlation is flat, i.e. unchanging, can the volatility ratio influence the shape of β.

5. Conclusions and suggestion for future work

Since the turn of the century, broad hedge fund performance has deteriorated. Measured αs have diminished steadily since 2000 and correlations between fund returns and the market have remained stubbornly high (as revealed by high βs), decreasing diversification and violating the very principal underlying 'hedge' funds. Arbitrage opportunities that were previously exploited to great effect have shrunk and lacklustre performance compared with simply buy-and-hold 'strategies' have resulted in considerable asset outflows as well as calls for enhanced regulatory pressure and reduced management fees. Because manager compensation for hedge funds is almost entirely linked to execution, accurate and timely measurement of fund performance has become critically important to hedge fund management, investors and regulators.

Traditional CAPM αs and βs are usually used to gauge fund performance, but the standard regression methodology obscures their measurement due to the rapidly-changing modern market. The Kalman filter provides an elegant solution. Suitably calibrated, the filter tracks the underlying processes more accurately and more timeously, as evidenced by the higher R^2 values for forecast excess fund (or index) returns versus measured excess returns using a Kalman filter rather than the regression methodology. By reacting more quickly to market changes and filtering out noisy market signals, the Kalman estimates of CAPM coefficients have been demonstrated to be superior.

Research demonstrates that it is more difficult to consistently exhibit good market timing skills than it is to consistently exhibit good stock selection skills (Jain, *et al*., 2011). This would suggest that traditional hedge fund screening measures might be improved by focusing on stock selection α as opposed to market timing α. Future work will attempt to differentiate between timing α and stock selection α using the

Kalman filter. The filter is ideally suited to this type of research as it reacts quickly to changing market conditions and can thus identify and distinguish between these contributory factors and thereby ensure more efficient allocation of manager rewards and performance allocation.

Further potential work could be to develop a Kalman filter-based model for asset prices that could be used in conjunction with a control theory oriented state space model for portfolio optimisation. CAPM with time-varying coefficients estimated using the Kalman filter could be connected to a formulation of a stochastic receding horizon control framework. Together, these applications could provide a methodology for estimating an asset pricing model and performing portfolio optimisation.

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