The transition from secondary to tertiary mathematics: exploring means to assist students and lecturers

CG Benadé

Thesis submitted in fulfilment of the requirements for the degree

Philosophiae Doctor in Natural Sciences Education

at the North-West University (Potchefstroom Campus)

Supervisor:

Dr S Froneman

Assistant supervisors:

Prof HD Nieuwoudt

Dr M Hitge

May 2013

Potchefstroom

It all starts here

NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOM CAMPUS

ACKNOWLEDGEMENTS

Words cannot truly express my feelings, but my sincere thanks to the following people:

- Dr Sonica Froneman, my supervisor, for always being willing to help and for her expert advice and hours of reading, “editing” and support.
- Prof Hercules Nieuwoudt and Dr Mariette Hitge, my assistant supervisors, for their much needed support and expert advice.
- My husband Gerrie, my children (Sanet, Johan, Marlie, Sanmari, Zias) and my friends (Surika and Paul, Anita and Handjies, Linda) for their faith in my abilities and their daily encouragement, love and support.
- The students and the lecturers who were part of this study.
- Prof Christien Strydom for the opportunity to study and all the encouragement and support.
- My colleagues Mariana, Annami and Soekie for your friendship and support. A special thank you to Hannatjie for extra reading, valuable advice and much needed support.
- Dr Suria Ellis for the processing of the statistical data.
- Christien Terblanche for the language editing.
- Susan van Biljon for the technical aspects and the page lay-out.
- Anriette Pretorius for checking the technical correctness of the bibliography.
- The examiners for dedicating time and expertise to the development of new researchers in the field.
- Above all, my Heavenly Father, for giving me the strength, insight and persistence to complete this study.

CUM LAUDE LANGUAGE SERVICES

BA (Pol Sc), BA Hons (Eng), MA (Eng), TEFL

22 Strydom Street
BailliePark
2531

Tel 082 821 3083
cmeterblanche@hotmail.com

DECLARATION OF LANGUAGE EDITING

I, Christina Maria Etrecia Terblanche, id nr 771105 0031 082, hereby declare that I have edited the thesis of Ms CG Benadé, entitled **The transition from secondary to tertiary mathematics: exploring means to assist students and lecturers**, without viewing the final product.

Regards,

CME TERBLANCHE
Cum Laude Language Practitioners (CC)

ABSTRACT

Early in 2009 it became apparent from articles in the newspapers that first year mathematics students were not performing as well as the students of previous years. There was great concern regarding the insufficient transition from secondary to tertiary mathematics, as well as the preparedness of first year students for university studies. This research focuses on the different factors that are potential causes of the underachievement of first year mathematics students.

Students' and lecturers' beliefs are shaped by their experiences, the impact of continuous perceptions from the world around them, the present dominant paradigm, as well as the beliefs of their teachers. The different views of the nature of school mathematics show how a worldview has an effect on these views and the implications of this on the teaching of mathematics in secondary, as well as tertiary institutions. The paradigm shift from the modern era to the post-modern era caused an awareness of and interest in the construction of meaningful mathematical understanding. The gap between first year students' and lecturers' beliefs regarding the nature of mathematics and how mathematics is learned became apparent.

The changes in the thoughts about the structure of mathematics were investigated and a better understanding of the processes through which mathematical understanding develops emerged. This brought insight into the gap between the reasoning abilities of incoming students from secondary schools and the reasoning needed to succeed in university mathematics.

The theoretical study of the global theories of Piaget and Van Hiele gave insight into conceptual development through different stages and that a person should be on an appropriate conceptual level to make sense of what they learn. If not, then rote learning is likely to occur. The local theory of Tall implies that to facilitate understanding of a concept in mathematics, one should go through three worlds of mathematics: the embodied world, symbolic world and the formal world. The embodied view helps someone to give deep meaning to a

concept, otherwise one can be trapped in the symbolic world and not be able to move on to the formal world of mathematical thinking.

The theoretical investigations led to an empirical study in three phases. Phase 1 was an investigation into the views of mathematics held by the students and the lecturers. In phase 2 an investigation was done to establish the students' preferences on how they learn mathematics and how mathematics should be taught, using the Index of Learning Styles (ILS) questionnaire of Felder and Silverman. The results were compared with the way lecturers want their students to learn and how they themselves prefer to teach. Phase 3 included a classification of the questions in the first mathematics test written at tertiary level and subsequent analysis of the answers of students to obtain information on the type of reasoning required from students at tertiary level, as well as the reasoning abilities of the students.

The empirical study assisted in understanding the problematic transition from secondary to tertiary mathematics with regard to the nature of mathematics, the beliefs on teaching and learning of mathematics, as well as the reasoning skills that the students possess when entering university.

Key words for indexing:

"teaching and learning mathematics", "gap between secondary and tertiary mathematics", "transition from secondary to tertiary mathematics".

OPSOMMING

Vroeg in 2009 het dit uit koerantartikels duidelik geword dat eerstejaar wiskundestudente nie presteer soos die studente van vorige jare nie. Daar was groot kommer oor die ontoereikende oorgang van sekondêre na tersiêre wiskunde en die onvoorbereidheid van eerstejaarstudente vir universiteitstudies. Die navorsing fokus op die verskillende faktore wat potensiële oorsake kan wees vir die onderprestasie van eerstejaar wiskundestudente.

Studente en dosente se sienings word gevorm deur hulle ervaringe, die impak van voortdurende gewaarwordinge uit die wêreld rondom hulle, die huidige dominante paradigma en die oortuigings van leerders se onderwysers. Die verskillende sienings van die aard van skoolwiskunde wys hoe ‘n wêreldbeskouing ‘n effek het op ‘n student se sienings en wat die implikasies daarvan is vir wiskunde-onderrig by sekondêre sowel as tersiêre instellings. Die paradigmaskuif vanuit die moderne era na die post-moderne era bring meer bewustheid en belangstelling in die konstruksie van betekenisvolle verstaan van wiskunde. Die gaping in die sienings van eerstejaarstudente en dosente oor die aard van wiskunde en hoe wiskunde geleer word het duidelik geword.

Die veranderinge in denke oor die struktuur van wiskunde is ondersoek en daaruit het ‘n beter begrip na vore gekom van die prosesse waardeur wiskundebegrip ontwikkel. Dit bring insig in die gaping tussen die redenasievermoë van studente wat inkom uit sekondêre skole en die redenasievermoë wat nodig is om sukses te behaal met universiteitswiskunde.

Die teoretiese studie van die globale teorieë van Piaget en Van Hiele belig konsepsuele ontwikkeling deur die verskillende stadiums en die feit dat ‘n persoon op ‘n sekere toepaslike konsepsuele vlak moet wees om sin te maak uit wat hy leer. Indien dit nie die geval is nie, sal gewoonteleer heel moontlik plaasvind. Die lokale teorie van Tall impliseer dat om begrip van ‘n konsep in wiskunde te fasiliteer moet die persoon deur drie wiskundewêrelde gaan: die “beliggaamde wêreld” (*embodied world*), die simboliese wêreld en die formele wêreld. Die beliggaamde wêreld help ‘n persoon om diep betekenis te gee aan

‘n konsep. As dit nie gebeur nie kan die persoon vasgevang wees in die simboliese wêreld sonder die vermoë om die wêreld van wiskundige denke in te gaan.

Die teoretiese ondersoek het gelei na ‘n empiriese studie in drie fases. Fase 1 was ‘n ondersoek na die sienings wat studente en dosente het oor wiskunde. Fase 2 het ondersoek ingestel na die studente se voorkeure met betrekking tot hoe hulle wiskunde leer en hoe wiskunde onderrig moet word deur die gebruik van die *Index of Learning Styles* (ILS) (Indeks van Leerstyle) vraelys van Felder en Silverman. Die resultate is vergelyk met die manier waarop dosente wil hê hulle studente moet leer en hoe hulle verkies om onderrig te gee. Fase 3 sluit ‘n klassifikasie in van die vrae in die eerste wiskundetoets wat op tersiêre vlak geskryf is, gevolg deur ‘n analise van die antwoorde van studente om inligting te verkry oor die tipe redenasievermoë wat studente op tersiêre vlak nodig het en die vermoëns waaroor hulle reeds beskik.

Die empiriese studie het bygedra tot ‘n begrip van die problematiese oorgang van sekondêre na tersiêre wiskunde met betrekking tot die aard van wiskunde, die oortuigings oor onderrig en leer van wiskunde en die redenasievermoë wat studente het wanneer hulle die universiteit betree.

Sleutelwoorde vir indeksering:

“onderrig en leer van wiskunde”, “gaping tussen sekondêre en tersiêre wiskunde”, “oorgang vanaf sekondêre na tersiêre wiskunde”.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS ..... **i**

CUM LAUDE LANGUAGE SERVICES ..... **ii**

ABSTRACT ..... **iii**

OPSOMMING ..... **v**

TABLE OF CONTENTS ..... **vii**

LIST OF TABLES ..... **xii**

LIST OF FIGURES ..... **xiv**

CHAPTER 1

BACKGROUND AND OVERVIEW OF THE STUDY **1**

1.1 PROBLEM STATEMENT AND MOTIVATION FOR THE STUDY 1

1.2 REVIEW OF LITERATURE 2

1.3 RESEARCH AIMS 5

1.4 LITERATURE STUDY 5

1.5 EMPIRICAL STUDY 6

1.5.1 Research design 6

1.5.2 Study population and sample 7

1.5.3 Measuring instruments 7

1.5.4 Statistical techniques 7

1.6 CHAPTER OUTLINE 8

1.7 VALUE OF THE RESEARCH 9

CHAPTER 2

THE GAP BETWEEN SECONDARY AND TERTIARY LEVEL CONCERNING BELIEFS ON THE NATURE AND LEARNING OF MATHEMATICS **10**

2.1 INTRODUCTION 10

2.2 THE INFLUENCE OF WORLDVIEWS AND PARADIGM SHIFTS ON BELIEFS 10

2.3 THE INFLUENCE OF THE CURRENT PARADIGM SHIFT ON EDUCATION IN GENERAL 11

2.4 THE INFLUENCE OF THE CURRENT PARADIGM SHIFT ON MATHEMATICS EDUCATION 13

2.5 CURRENT VIEWS OF SCHOOL MATHEMATICS 14

2.5.1 Platonist (static-formalist) view 14

2.5.2 Dynamic, problem-solving (constructivist) view 16

2.5.3 Instrumentalist view 17

2.6 VIEWS OF MATHEMATICS IMPLICIT AND EXPLICIT TO THE PRESENT SCHOOL CURRICULUM (NCS) 18

2.7 THE GAP BETWEEN BELIEFS OF MATHEMATICS AT SECONDARY AND TERTIARY LEVEL 22

CHAPTER 3

THE GAP BETWEEN SECONDARY AND TERTIARY LEVEL CONCERNING BELIEFS ON THE STRUCTURE OF MATHEMATICS 25

3.1 INTRODUCTION 25

3.2 VIEWS ON THE STRUCTURE OF MATHEMATICS 26

3.3 MATHEMATICAL UNDERSTANDING 27

3.4 PROCEDURAL AND CONCEPTUAL KNOWLEDGE 29

3.5 THE COGNITIVE PROCESSES THROUGH WHICH MATHEMATICAL UNDERSTANDING DEVELOPS 32

3.5.1 Representations 32

3.5.2 Connections 36

3.5.3 Reasoning 36

3.6 THE GAP DUE TO DIFFERENT PERCEPTIONS OF THE STRUCTURE OF MATHEMATICS 45

CHAPTER 4

THE GAP IN THE TRANSITION FROM ELEMENTARY TO ADVANCED THINKING 48

4.1 INTRODUCTION 48

4.2 JEAN PIAGET'S COGNITIVE DEVELOPMENT THEORY 49

4.3 THE VAN HIELE LEVELS OF THE DEVELOPMENT OF GEOMETRIC THOUGHT 52

4.3.1 The Van Hiele levels of reasoning 53

4.3.2 Stages of instruction to reach a next level 54

4.3.3 Reduction of the five levels 55

4.4 TALL'S THREE WORLDS OF MATHEMATICAL THINKING 55

4.5 INTEGRATING PIAGET, VAN HIELE AND THE THREE WORLDS OF MATHEMATICS OF TALL 58

4.6 THE SHIFT FROM ELEMENTARY TO ADVANCED MATHEMATICAL THINKING 60

4.7 CONCLUSION 64

CHAPTER 5

EMPIRICAL STUDY 65

5.1 INTRODUCTION 65

5.2 THE RESEARCH METHODOLOGY 66

5.2.1 Research design 66

5.2.2 Sampling 66

5.2.3 The collection of the data 67

5.3 THE BELIEFS QUESTIONNAIRE (ANNEXURE A) 68

5.3.1 Construction of the beliefs questionnaire 68

5.3.2 The reliability and validity of the beliefs questionnaire 70

5.3.3 Analysis of the beliefs questionnaire 72

5.4 THE INDEX OF LEARNING STYLES (ILS) QUESTIONNAIRE (ANNEXURE B AND C) 81

5.4.1 Background 81

5.4.2 The four dimensions of learning styles 83

5.4.3 Scoring of the ILS questionnaire 84

5.4.4 The reliability of the ILS 86

5.4.5 Analysis of the ILS data 87

5.5 ANALYSIS OF THE FIRST MATHEMATICS TEST WRITTEN AT TERTIARY LEVEL (ANNEXURE D) 98

5.5.1 Introduction 98

5.5.2 Framework for classification of reasoning 98

5.5.3 The validity and reliability of the task classification 102

5.5.4 The classification of the questions in the test 103

5.5.5 Discussion of results 109

5.6 CONCLUSION 111

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS 113

6.1 INTRODUCTION 113

6.2 CONCLUSIONS 113

6.2.1 The gap regarding the beliefs of the nature of mathematics 113

6.2.2 The gap regarding the learning of mathematics 114

6.2.3 The gap regarding the development from elementary to advanced mathematical thinking 115

6.2.4 Final conclusions on the gap 116

6.3 RECOMMENDATIONS 118

6.3.1 Recommendations for lecturers 118

6.3.2 Recommendations for students 119

6.4 A FINAL NOTE 120

REFERENCES 121

ANNEXURE A: Beliefs questionnaire 135

ANNEXURE B: Index of Learning Styles Questionnaire (Lecturers) 138

ANNEXURE C: Index of Learning Style Questionnaire (Students) 145

ANSWER SHEET for the ILS Questionnaire 151

ANNEXURE D: Memo 152

LIST OF TABLES

Table 4.1 Local and global stages of cognitive development 60

Table 5.1: Questions linked to views of mathematics..... 69

Table 5.2 Student and lecturers percentages for the beliefs questionnaire 73

Table 5.3: Keywords about the nature of mathematics where students strongly agreed or disagreed ($d \geq 0.8$) 74

Table 5.4: Keywords about the nature of mathematics where lecturers strongly agreed or disagreed ($d \geq 0.8$)..... 77

Table 5.5: Statements with a statistical and practical significant difference between the students and the lecturers on beliefs 79

Table 5.6 Statistics for statements with $p < 0.05$ and $d \geq 0.5$ of the beliefs questionnaire..... 80

Table 5.7: An example of a scoresheet of the ILS questionnaire... 85

Table 5.8: Cronbach alpha coefficients of the dimensions of the ILS 87

Table 5.9: Questions in the visual/verbal dimension that showed statistically significant differences 88

Table 5.10: Summary of the mean percentages and d-values for the questions in the active/reflective dimension..... 90

Table 5.11: Summary of mean percentages and d-values for the questions in the sensing/intuitive dimension 92

Table 5.12: Summary of the mean percentages and d-values for the questions in the visual/verbal dimension..... 93

Table 5.13: Summary of mean percentages and d-values for the questions in the sequential/global dimension 96

Table 5.15: Average percentages for reasoning types 111

LIST OF FIGURES

Fig. 3.1 A representation of relational and instrumental understanding (Van de Walle, 2007:25) 28

Fig 3.2: Five different representations of mathematical ideas (Lesh *et al*., 1987:34)..... 33

Fig. 3.3 The five representations of a function (Van de Walle, 2007:280) 35

Fig 3.4 Reasoning types (Lithner, 2006:5) 42

Fig 4.1: A spectrum of performance in using mathematical procedures, processes and procepts (Gray *et al*., 1999). 58

Fig. 5.1: An example of a summary of a scoresheet with learning preferences 86

Fig 6.1: Schematic representation of the teaching and learning of mathematics with understanding..... 117