

Modeling of the Polytopic Index and Temperature Anisotropy in the Solar Wind

Ephrem Tesfaye Desta^{1,2} , R. D. Strauss^{1,3} , and N. E. Engelbrecht¹ 

¹ Center for Space Research, North-West University, Potchefstroom, 2522, South Africa; ephysics.tesfaye@gmail.com

² Department of Physics, Addis Ababa University, Addis Ababa 1176, Ethiopia

³ National Institute for Theoretical and Computational Sciences (NITheCS), South Africa

Received 2024 February 2; revised 2024 March 26; accepted 2024 March 26; published 2024 May 2

Abstract

The polytopic index is a fundamental physical parameter related to the thermodynamic processes present in space and astrophysical plasmas. This paper investigates the theoretical relationship between the polytopic index and the temperature anisotropy for flow parameters relevant to space plasmas. The derivation is based on the Chew–Goldberger–Low double-adiabatic equations of state and the finite Larmor radius correction. On the basis of this, we present the polytopic index relation, taking into account the temperature anisotropy, flow speed, and magnetic field of the plasma. This relation was further analyzed for the limit of the quasi-parallel and quasi-transversal cases. The quasi-transversal limit gives a polytopic index as a function of the anisotropic temperature $\gamma = 1 + 2[2T_{\perp} - T_{\parallel}]/[2T_{\perp} + T_{\parallel}]$. Using this result, we analyze the polytopic index for the bulk proton parameters derived from Ulysses spacecraft data spanning the interval from 1992 January 1 to 2009 June 30, and we find an average polytopic index of $\gamma \sim 1.43$. This value is close to that of recently published results. However, unlike previous statistical studies, this research computes the polytopic index without relying on power-law fitting, and its variation is now associated with the anisotropic temperature.

Unified Astronomy Thesaurus concepts: Polytopes (1281); Heliosphere (711); Space plasmas (1544); Theoretical techniques (2093); Solar wind (1534); Solar energetic particles (1491)

1. Introduction

In the heliosphere, solar wind protons exhibit a dynamic flow influenced by both large-scale cooling and turbulent heating. This turbulence originates from several sources: solar fluctuations occurring on a grand scale, the excitation of plasma waves by newly introduced interstellar pickup ions, stream-shear effects in the solar wind, and large-scale mixing, to name a few (see, e.g., Zhou & Matthaeus 1990; Roberts et al. 1992; Zank et al. 1996; Isenberg 2005; Zank et al. 2017; Oughton & Engelbrecht 2021, and references therein). These turbulent phenomena have a significant impact on the polytopic behavior of solar wind plasmas.

The polytopic index is a fundamental physical parameter for investigating the thermodynamic processes present in space and astrophysical plasma, and the formulation of the theoretical relation between the polytopic index and the anisotropic temperature is currently of considerable recent interest in space plasma (see, e.g., Pang et al. 2022; Livadiotis & McComas 2023; Ghag et al. 2024), as the polytopic index has broad applications in the field of space and astrophysical science (see Horedt 2004). For the quasi-static process, Ledoux (1958) describes the relationship between density, temperature, and thermal pressure. Consequently, a polytopic expansion or compression is defined as a process in which pressure, temperature, and density have a $p \propto n^{\gamma}$ or $T \propto n^{\gamma-1}$ relation, which implies that

$$p = cn^{\gamma}, \quad (1)$$

where c is a streamline constant, $S = c/K_B$ is the entropy, and K_B is the Boltzmann constant. If the polytopic index γ is

greater than 1, the temperature will decrease as the gas expands and increase as the gas compresses. However, in rare cases, space plasmas have negative correlations between their density and temperature (that is, $\gamma < 1$). Such conditions can be found in, for example, the outer heliosphere (Livadiotis et al. 2012; Livadiotis 2017; Livadiotis & Nicolaou 2021; Elliott et al. 2019), inner heliosheath (Livadiotis 2016, 2017), planetary magnetospheres (e.g., Schopke et al. 1981), central plasma sheet (e.g., Pang et al. 2015), bow shock (e.g., Pang et al. 2020), magnetosheath and boundary layer (e.g., Nicolaou et al. 2014), and Kronian magnetosphere (e.g., Dialynas et al. 2018).

The polytopic index also has the advantage of relating the variation of temperature and number density with the radial distance R from the Sun (see Feldman et al. 1998; Skoug et al. 2000, and references therein). The value of the polytopic index is characteristic of the type of process that plasma experiences. For example, when an interplanetary coronal mass ejection (ICME) expands outward from the Sun, it undergoes multiple expansion and compression processes that include isochoric $\gamma = \pm \infty$, explosive $\gamma = -1$, isothermal $\gamma = 1$, and adiabatic $\gamma = 5/3$ expansions (Livadiotis & McComas 2012; Livadiotis 2016). Several authors studied polytopic indices in different astrophysical plasmas such as coronal mass ejections (CMEs; Liu et al. 2006; Mishra & Wang 2018), coronal plasma (Prasad et al. 2018), and planetary magnetospheres (e.g., Dialynas et al. 2018) that are characterized as having subadiabatic ($1 < \gamma < 5/3$) or superadiabatic values ($5/3 < \gamma < +\infty$).

Detailed theoretical considerations on the relationship between the polytopic index and the anisotropic temperature were presented by Pudovkin et al. (1997, 2000). However, this theoretical result was not confirmed by direct experimental data. Following this, Pudovkin et al. (1999) and Meister et al. (2011) investigated the characteristic plasma motion parameter using solar wind data from the Interplanetary Monitoring



Original content from this work may be used under the terms of the [Creative Commons Attribution 4.0 licence](https://creativecommons.org/licenses/by/4.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

Platform-8 spacecraft. Furthermore, using Active Magnetospheric Particle Tracer Explorer satellite data as well as the double-adiabatic Chew–Goldberger–Low (CGL) and anisotropic temperature, Meister et al. (2011) studied this parameter in the magnetosheath and confirmed that the effective polytropic index ranges between 1.4 and 1.9 in the bow shock and may be less than unity in the magnetopause.

In the turbulent solar wind, anisotropy is produced because spectral energy transfer is more rapid in wavevectors that are nearly perpendicular to the large-scale magnetic field than it is in nearly parallel wavevectors (see Oughton & Matthaeus 2020). In quasi-two-dimensional turbulence, the dominant excitations are in (Fourier) wavevectors that are transverse to both the directions of the mean magnetic field and the magnetic fluctuation. From this perspective, the solar wind fluctuations contain a subpopulation with wavevectors nearly transverse to both the mean magnetic field and the fluctuations about the mean (e.g., Matthaeus et al. 1990). In particular, fast streams are more dominated by fluctuations with wavevectors quasi-parallel to the local magnetic field, while slow streams, which appear to be more fully evolved turbulence, are more dominated by quasi-perpendicular fluctuation wavevectors (Dasso et al. 2005; Horbury et al. 2005; Gary & Smith 2009). Subsequent correlation analysis (Marsch & Richter 1987) indicated that the polytropic equation of state can be applicable beyond 0.3–1 au for wave damping or dissipation of anisotropic solar wind turbulence. The investigation of the theoretical relationship between the polytropic index and the temperature anisotropy has been conducted by Pudovkin et al. (1997, 1999) and more recently by Livadiotis & Nicolaou (2021).

Several authors investigated the value of the polytropic index in space plasmas with spacecraft data, using a scatter plot of temperature or pressure versus densities to fit a power law (Kartalev et al. 2006; Nicolaou et al. 2014; Livadiotis 2018). The connection of the polytropic index with kappa distributions has been systematically shown for the first time by Livadiotis (2019). Additionally, valuable theoretical insights can be obtained from previous studies (Nicolaou et al. 2019; Livadiotis & Nicolaou 2021; Nicolaou et al. 2023). Using partial temperature tensor measurements and solar wind proton observations near 1 au, Nicolaou et al. (2021) investigated the correlations between the temperature tensor elements and the scalar temperature and deduced that the use of the perpendicular temperature leads to more accurate calculations. Recently, using a correlation analysis between small-scale variations in plasma density and temperature in selected subintervals and Ulysses data over a wide range of heliocentric distances and latitudes during the solar minimum period, spanning from 1992 January 1 to 1998 December 31, Nicolaou et al. (2023) derived an average polytropic index of 1.4.

In solar wind plasmas, at sufficiently small spatial scales comparable to the gyroradius ($L \simeq r_L$) of the particles, the contributions of nongyrotropic pressure become significant. Therefore, it is necessary to include first-order finite Larmor radius (FLR) corrections (p^{FLR}) in the fluid description of gyrotropic plasmas. The FLR correction is ignored for electrons, due to the smallness of Ω_e^{-1} compared to Ω_i^{-1} , and the pressure tensor $\vec{p}$ can be presented for the first order in Ω_i^{-1} as a form.

In this paper, we investigate the theoretical relation between the polytropic index, temperature anisotropy, magnetic field, and solar wind flow speed. We accomplish this through a theoretical analysis, considering the role of the FLR correction in the context of the double-adiabatic CGL theory. Using the derivation of the Liouville theorem, Schulz & Eviarar (1973)

showed the validity of the double-adiabatic CGL equation of state for solar wind problems. Recently, Hunana & Zank (2017) showed the importance of CGL for the first-order description of plasmas with anisotropic temperatures, such as the solar wind. The results obtained are then compared with data from the Ulysses spacecraft and aligned with recent studies for validation.

This paper is organized as follows: In Section 2, we analyze the relationship between the polytropic index and temperature anisotropy, with details provided in Appendices A and B. In that section, the polytropic index will be formulated for the case of quasi-transversal and quasi-parallel propagation. Section 3 presents the data set and compares it with previous theoretical and observational studies using Ulysses spacecraft data. Using this result, we investigate the polytropic index during Ulysses fast latitude scans and for full solar cycle periods. Finally, we conclude in Section 4.

2. Polytropic Index and Temperature Anisotropy

The relation in Equation (1) is valid for the adiabatic indices of isotropic plasmas. However, for the case of anisotropic plasmas, because of the independent variation of the parallel and perpendicular temperatures, it is impossible to describe them using a single equation of the type of Equation (1).

To solve this problem, one needs to find a single equation for the polytropic index that may be valid for anisotropic plasmas. We approach the problem by considering the time evolution of the pressure and density in the solar wind, and hence from the time derivative of Equation (1), the polytropic index can be related to these quantities as

$$\gamma = \frac{(1/p)(dp/dt)}{(1/\rho)(d\rho/dt)}, \quad (2)$$

which is identical to Equation (11) in Pudovkin et al. (1999) and Equation (2) in Pudovkin et al. (1997). For an anisotropic system, the total pressure $p = (2p_\perp + p_\parallel)/3$ can be used in Equation (2), and it has the form

$$\gamma = \frac{\left(\frac{1}{p_\perp} \frac{dp_\perp}{dt} + \frac{1}{p_\parallel} \frac{dp_\parallel}{dt} \frac{p_\parallel}{2p_\perp}\right)}{\left(\frac{1}{\rho} \frac{d\rho}{dt}\right) \left(1 + \frac{p_\parallel}{2p_\perp}\right)}. \quad (3)$$

To evaluate the polytropic index using Equation (3), we need to determine the values of $\frac{1}{\rho} \frac{d\rho}{dt}$, $\frac{1}{p_\perp} \frac{dp_\perp}{dt}$, and $\frac{1}{p_\parallel} \frac{dp_\parallel}{dt}$, which can be determined from the continuity equation, magnetic frozen-in condition, double-adiabatic equations of states, and applying the CGL theory with an FLR correction analysis, as presented in Appendix B. Equation (A5) of Appendix A can be further expressed as

$$\gamma = 1 + \left[\frac{1 + \frac{p_\parallel}{p_\perp} \Lambda}{(1 + \Lambda) \left(1 + \frac{p_\parallel}{2p_\perp}\right)} \right], \quad (4)$$

where $\Lambda = \frac{\nabla_\parallel \mathbf{v}_\parallel}{\nabla_\perp \cdot \mathbf{v}_\perp}$ is the plasma flow parameter and describes the characteristic motion of plasma.

The model presented in Equation (4) coincides with that of Equation (18) introduced by Livadiotis & Nicolaou (2021) for $\Lambda = p_\parallel/2p_\perp$.

Using the relation $p_{\parallel}/p_{\perp} = T_{\parallel}/T_{\perp}$ and for the wavevector $(k_{\perp}, k_{\parallel}) = k (\sin \theta_{BV}, \cos \theta_{BV})$, for arbitrary propagation $k_{\parallel}/k_{\perp} = \cot \theta_{BV}$ in Equation (B18) of Appendices A and B, one can easily obtain the effective polytropic index as a function of the anisotropic temperature. This is given by

$$\gamma = 1 + \left[\frac{1 - \frac{T_{\parallel}}{2T_{\perp}} \left[1 + 2 \cot^2 \theta_{BV} \left(2 \frac{T_{\parallel}}{T_{\perp}} - 1 \right) \right]}{\left(1 - \frac{1}{2} \left[1 + 2 \cot^2 \theta_{BV} \left(2 \frac{T_{\parallel}}{T_{\perp}} - 1 \right) \right] \right) \left(1 + \frac{T_{\parallel}}{2T_{\perp}} \right)} \right], \quad (5)$$

where θ_{BV} is the flow angle between the magnetic field and plasma flow speed, such that $\theta_{BV} = \cos^{-1}(\frac{\mathbf{B} \cdot \mathbf{V}}{|\mathbf{B}| |\mathbf{V}|})$ in the wavevector plane. Then, we can express $\cot \theta_{BV}$ in terms of flow speed and magnetic field as

$$\cot \theta_{BV} = \frac{\mathbf{B} \cdot \mathbf{V}}{\sqrt{|\mathbf{B}|^2 |\mathbf{V}|^2 - (\mathbf{B} \cdot \mathbf{V})^2}}. \quad (6)$$

Therefore, the polytropic index in Equation (5) depends on the magnetic field, the flow speed, and the anisotropic temperature (pressure). Limits, depending on the wave propagation direction, can then be deduced, as described below.

2.1. For Quasi-parallel Propagation ($k_{\parallel}/k_{\perp} \gg 1$)

For this limit in Equation (B18), applying L'Hôpital's rule, the polytropic index becomes

$$\gamma = 1 + \frac{T_{\parallel}/T_{\perp}}{1 + T_{\parallel}/2T_{\perp}}. \quad (7)$$

2.2. For Quasi-perpendicular Propagation ($k_{\parallel}/k_{\perp} \ll 1$)

For this limit in Equation (B18), the polytropic index is

$$\gamma = 1 + 2 \frac{1 - T_{\parallel}/2T_{\perp}}{1 + T_{\parallel}/2T_{\perp}}. \quad (8)$$

For the case of an isotropic plasma, where $T_{\perp} = T_{\parallel}$, Equations (7) and (8) maintain adiabaticity where the polytropic index $\gamma = 5/3$. In the case of quasi-parallel propagation, the polytropic index has a value greater than one ($\gamma > 1$).

3. Discussion

Wavevector anisotropy is generally observed. That is, for a given magnetic fluctuation, the energy is stronger at quasi-perpendicular propagation ($k_{\perp} \gg k_{\parallel}$) than it is at quasi-parallel propagation ($k_{\perp} \ll k_{\parallel}$) (Matthaeus et al. 1990). Following Nicolaou et al. (2023) and Livadiotis (2016) for the purposes of comparison, we consider the polytropic index relation for the case of quasi-transversal propagation $k_{\perp} \gg k_{\parallel}$ limit.

3.1. Comparison with Previous Studies

Using the relationship between the effective dimensionality and polytropic index presented by Livadiotis (2015a, 2015b), we consider the temperature ratio $\alpha = T_{\perp}/T_{\parallel}$, and rewrite Equation (8) in terms of an effective dimensionality (d_{eff}) as

$$\gamma = 1 + 2d_{\text{eff}}^{-1}, \quad (9)$$

where

$$d_{\text{eff}} = \frac{2\alpha + 1}{2\alpha - 1}. \quad (10)$$

The relationship between the effective dimensionality and the temperature anisotropy can satisfy several conditions:

$$\begin{aligned} \alpha = \infty &\iff d_{\text{eff}} = 1 \\ \alpha = 3/2 &\iff d_{\text{eff}} = 2 \\ \alpha = 1 &\iff d_{\text{eff}} = 3 \\ \alpha = 0 &\iff d_{\text{eff}} = -1. \end{aligned} \quad (11)$$

Livadiotis (2016) studied the general thermodynamic process of the solar wind using analytical derivations of state, Bernoulli's integral, and the polytropic spectrum analysis diagram given in Figure 1 of Livadiotis (2016). For a similar analysis of different thermodynamic processes in the solar wind, we rewrite the result in Equation (8) as a function of the parallel and transverse mean energy using $T_{\parallel}/2T_{\perp} = \varepsilon_{\parallel}/\varepsilon_{\perp}$. One can then write

$$\gamma = 1 + 2 \left(\frac{1 - \mathcal{E}_r}{1 + \mathcal{E}_r} \right), \quad (12)$$

where $\mathcal{E}_r$ denotes the coefficient of relative energy, which is the ratio of the mean parallel to transversal energy $\mathcal{E}_r = \varepsilon_{\parallel}/\varepsilon_{\perp}$. For $\mathcal{E}_r = 1$, the value of the polytropic index becomes $\gamma = 1$, and thus the solar wind undergoes isothermal expansion or compression. If $\mathcal{E}_r = 1/2$, the polytropic index $\gamma = 5/3$ (isotropic and also adiabatic expansion or compression). If $\mathcal{E}_r = 3$, the value of the polytropic index $\gamma = 0$ (isobaric process). If the total mean thermal energy is set to zero, $\varepsilon = \varepsilon_{\parallel} + \varepsilon_{\perp} = 0$, for the limit of loss or gain of complete energy transfer between the mean parallel and transverse energy, then $\mathcal{E}_r = -1$. For this limit, the polytropic index has infinite ∞ values, and the solar wind undergoes an isochoric process.

For the limit of larger parallel mean energy $\varepsilon_{\parallel} \gg \varepsilon_{\perp}$, the polytropic index $\gamma = -1$. Thus, the negative polytropic index characterizes the explosive behavior of the plasma, and the system undergoes a thermal explosion thermodynamic process. Regarding the limit of transverse mean energy, which is much greater than the parallel one $\varepsilon_{\perp} \gg \varepsilon_{\parallel}$, the polytropic index becomes $\gamma = 3$. In this case, the system undergoes a super-adiabatic expansion. Therefore, for the polytropic index in the limit of the quasi-transversal case, Equation (8) describes the thermodynamical process in the polytropic spectrum diagram given in Figure 1 of Livadiotis (2016).

3.2. Comparison with Ulysses Measurements

The European Space Agency (ESA) launched the Ulysses mission in 1990 October. After a close encounter with Jupiter in 1992 February, it entered a solar orbit nearly perpendicular to the ecliptic plane. This made Ulysses the first spacecraft ever to enter the polar regions. Ulysses completed three distinct orbits around the Sun. The first orbit, which spanned from 1992 to 1998, coincided with a period of minimum solar activity during the 11 yr solar cycle. The second orbit, from 1998 to 2004, coincided with the peak of solar cycle 23. The final orbit, from 2004 to the end of the mission in 2009. For more information, see, e.g., Wenzel et al. (1992) and Wenzel (1995).

In the timescale range from 10^{-2} to 10^1 s, the solar wind is characterized by the proton gyroperiod Ω_p , which has a further

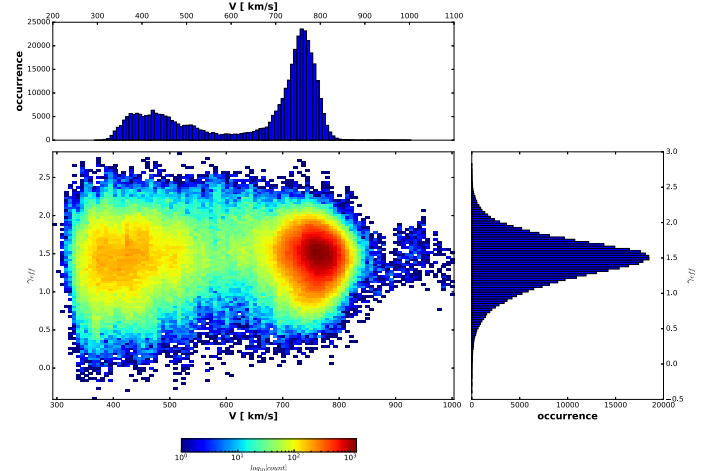
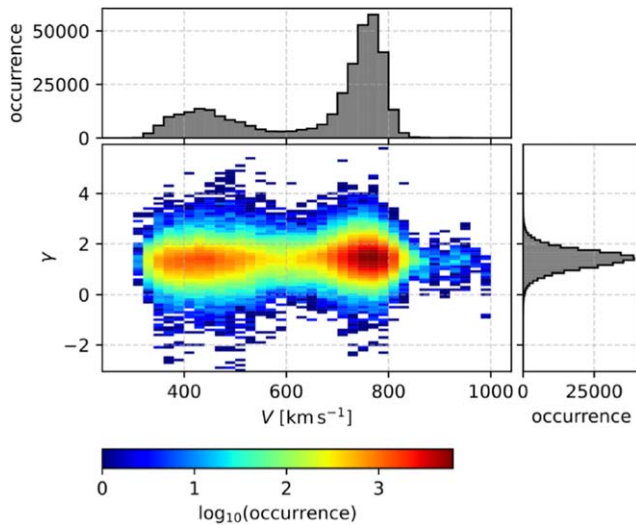


Figure 1. A 2D histogram of γ as a function of plasma flow speed. Presented here for comparison are the findings reported by Nicolaou et al. (2023) in the left panel and the results obtained in this investigation in the right panel.

contribution for FLR, and length scales such as the proton inertial length scale (d_p) and the proton gyration radius scale (ρ_p) range from $\sim 5 \times 10^3$ to 10^5 m (see, e.g., Verscharen et al. 2019). Having these characteristics of solar wind plasma, for our analysis, we used proton bulk parameter data from NASA’s SPDF-Coordinated Data Analysis Web, for Ulysses spacecraft SWOOPS observations of the distribution function of protons-moments⁴ spanning from 1992 January 1 up to 2009 June 30 (for more information, see McComas et al. 1999), which includes first-orbit, second-orbit, and third-orbit observations. To compare our result with the recently conducted polytropic index analysis by Nicolaou et al. (2023), we also used Ulysses spacecraft data spanning the interval from 1992 January 1 to 1998 December 31. For Solar Cycle analysis, we also used sunspot data from the American Association of Variable Star Observers.⁵

Using Equation (8), we calculate the polytropic index from observations and generate 2D histograms of the polytropic index and the flow speed as shown in Figure 1. We used a high sampling rate for the analysis of the result depicted in the right panel of Figure 1. In contrast, the left panel depicts results from Figure 5 of Nicolaou et al. (2023), who used data with a resolution of 4 min and performed a correlation analysis between plasma density and temperature. Both figures share similar features in the 1D and 2D histogram distributions. The first peak appears in both results to be centered at $V \sim 430$ km s⁻¹, observed at low latitude ($|\theta| < 30^\circ$), and the second centered close to 770 km s⁻¹, observed at high latitude ($|\theta| > 30^\circ$); these correspond to slow and fast solar wind regions, respectively. Our result shown in the right panel of Figure 1 is the average value of $\gamma \sim 1.44$, and the result in the left panel indicates that the value of γ has one peak at ~ 1.40 . Our analysis results in values of $\gamma < 2.5$, while in Nicolaou et al. (2023) the range of values is nearly < 4.0 . However, the spread of γ reported by Nicolaou et al. (2023) is much larger than our result.

Figure 2 shows the mean value of the polytropic index and its standard error for each radial and latitude (θ) bins, respectively. Compared to the result of Nicolaou et al. (2023), our results have similar trends in the value of γ , except for $\theta = -60^\circ$, and have the same values at $R = 3.5$ au and $R = 4.0$ au.

Using the result of Equation (8) and the data set observed by the Ulysses spacecraft, we calculate γ for each θ bin that corresponds to a width of 20° , as shown in Figure 3. Our results in Figure 3 show distribution patterns similar to those shown in Figure 9 of Nicolaou et al. (2023), especially for bins with equal solar wind speed intervals.

3.3. Polytropic Index during Polar Passes and as a Function of Solar Cycle

We analyze the proton polytropic index for three consecutive Ulysses orbits as shown in Figure 4; for the first orbit during solar minimum ($\gamma = 1.44$), for the second orbit during solar maximum ($\gamma = 1.40$), and for the third orbit during solar minimum ($\gamma = 1.45$); these results suggest that there are slight variations in the polytropic index that correspond to solar activity. For the case of the full spacecraft orbit, we calculate that the average value of γ is ~ 1.43 .

Figure 5 depicts the trajectory of the Ulysses spacecraft during its first orbit mission. This mission executed south-north pole passes, often known as fast latitude scans. The scans took place when the spacecraft was over 70° heliographic latitude in either the northern or southern hemisphere. The first polar pass (over the southern solar pole) commenced on June 26 and ended on 1994 November 5. The second polar pass (over the northern solar pole) commenced 1 yr later on June 19 and ended on 1995 September 29 (see, e.g., Wenzel et al. 1990, for more detail as to these orbits).

The red shaded region in Figure 5 indicates the first polar pass where Ulysses made its rapid pole-to-pole passage; the band of solar wind variability in the various parameters when the spacecraft is at ~ 1.4 au, having swept nearly 43° in latitude, from -22° to $+21^\circ$. In our analysis on 2001 April 20–June 10 at $R \sim 1.3$ au, in this latitude interval, we obtain a value of the polytropic index ~ 1.3 . Using observations from three different spacecraft at 1 au and using a power-law fitting,

⁴ <https://cdaweb.gsfc.nasa.gov/index.html>

⁵ https://lasp.colorado.edu/lisird/data/american_relative_sunspot_number_daily

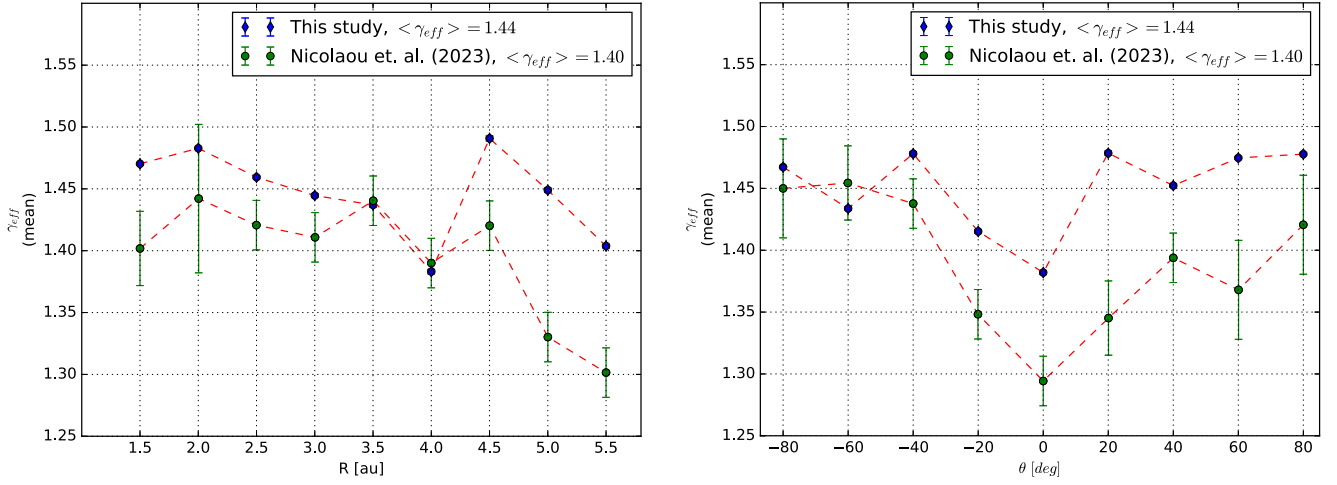


Figure 2. Mean value of γ as a function of the heliographic distance (the left panel) and as a function of the heliocentric latitude (the right panel). The green data points are the results of Nicolaou et al. (2023), and the blue symbols are our results.

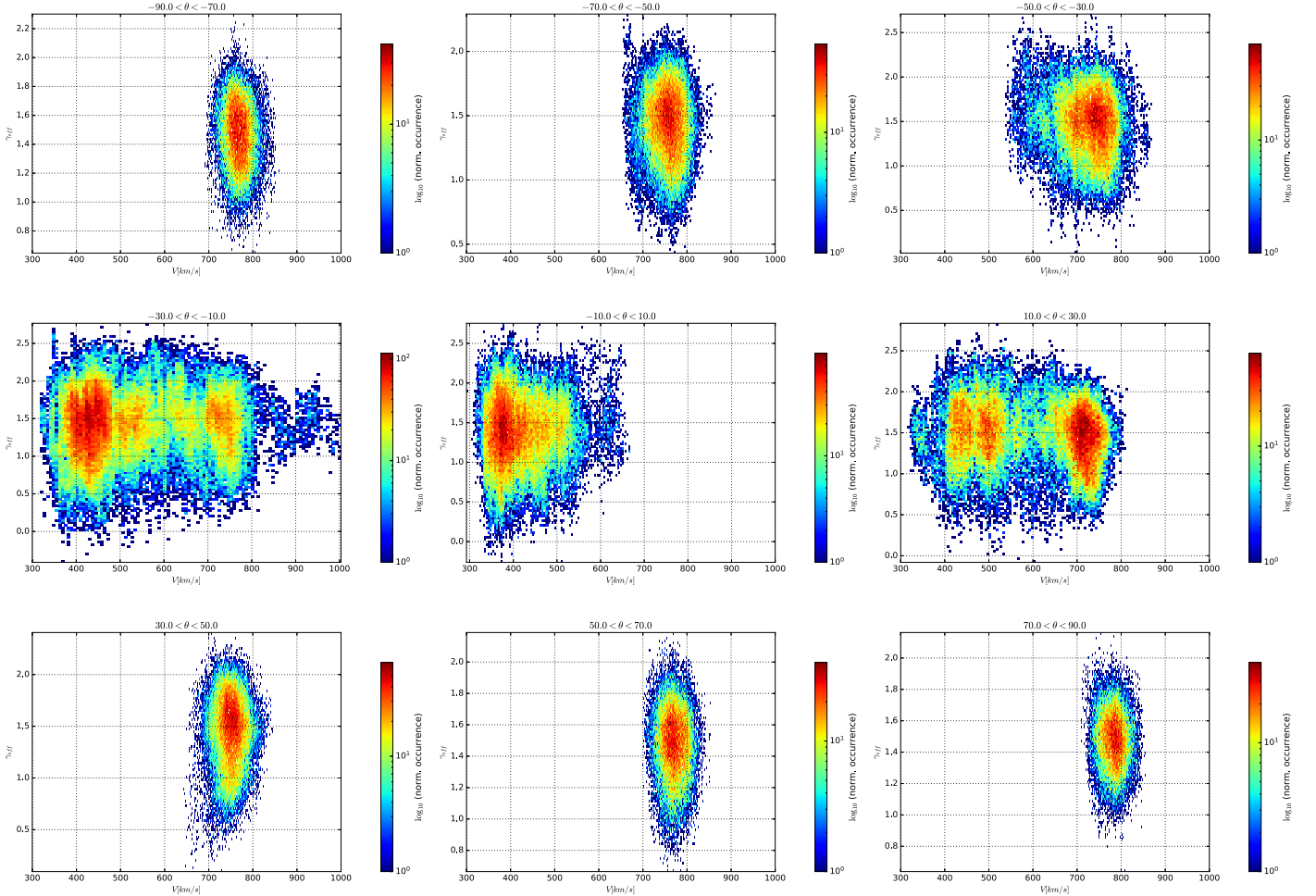


Figure 3. Two-dimensional histogram of polytropic index (γ) vs. solar wind speed (V) at different heliographic latitudes (HGL) from -90° to 90° for nine bins, each with a width of 20° , for the first orbit of Ulysses during the solar minimum period.

Ghag et al. (2024) determined the polytropic index to be ~ 1.3 , and suggested that the polytropic index has a coherent nature within the $[-20^\circ, +20^\circ]$ longitude separation between observations.

As shown in the bottom panel of Figure 5, the first orbit began when the sunspot number was in the declining phase,

implying solar minimum conditions, while the second orbit (second red band) coincided with the solar maximum period. The results in Figure 6 are calculated using data taken during the rapid pole-to-pole latitude scan of the Ulysses spacecraft, as indicated by the red shading in Figure 5. The data are binned into 20° intervals in the HGL during the solar minimum and

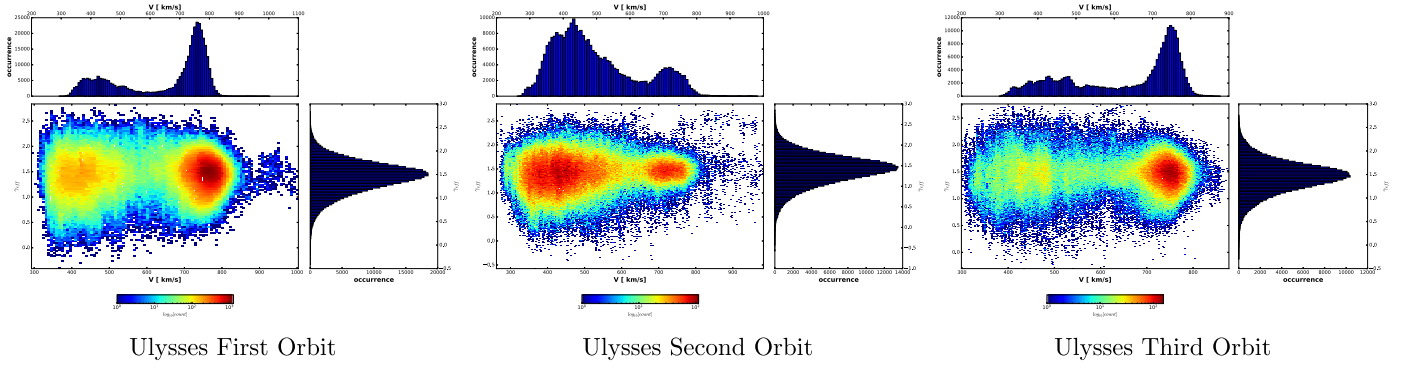


Figure 4. Two-dimensional histogram of γ as function of plasma flow speed, as well as the occurrence (top left) 1D histogram of the proton speed. Panels present data from the first, second, and third solar orbits of Ulysses, respectively.

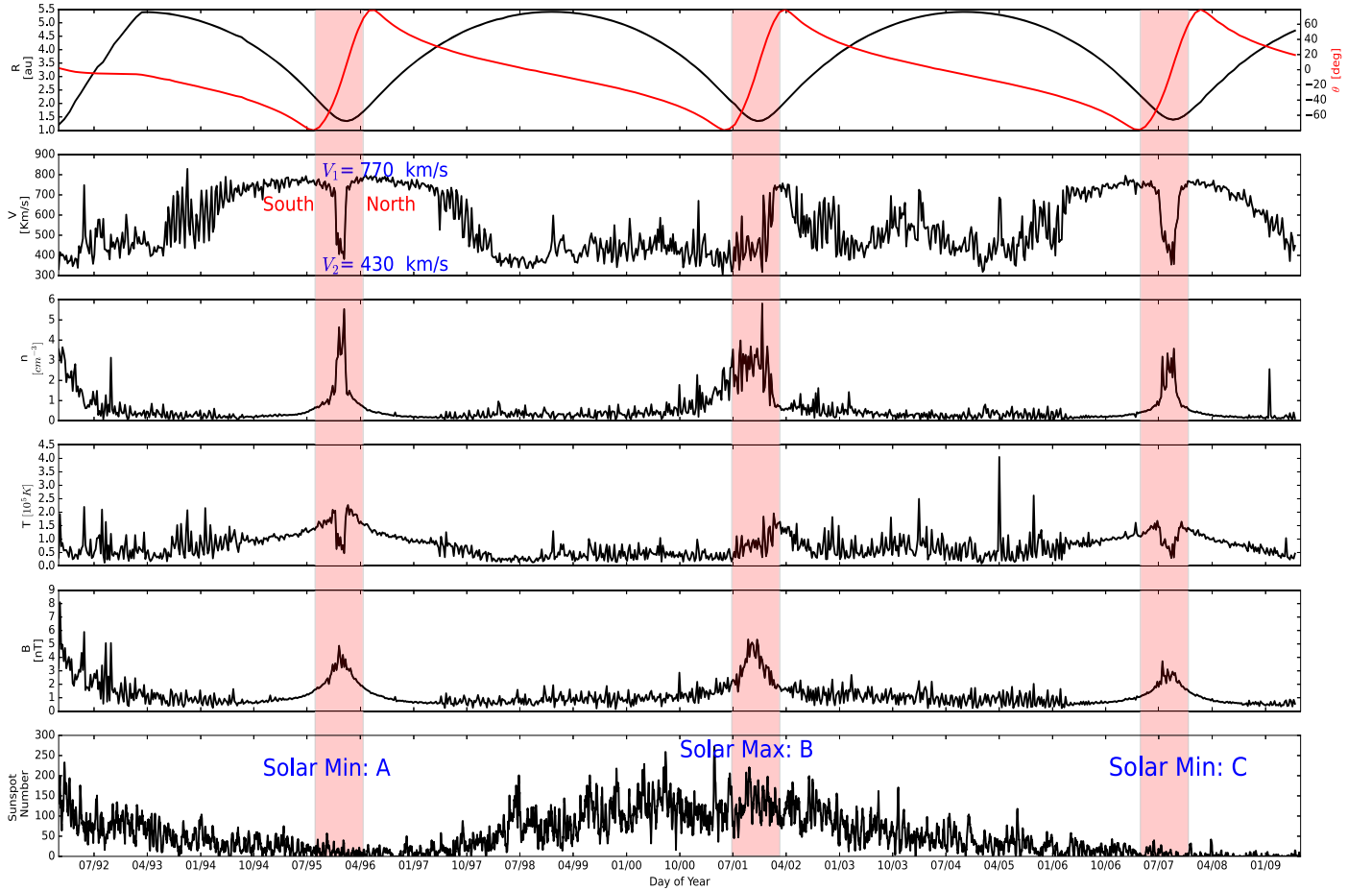


Figure 5. The trajectory and plasma parameters from the Ulysses spacecraft, encompassing both solar maximum and minimum periods. The bottom panel shows the sunspot number.

maximum periods. In our analysis, we find a minimum polytropic index of approximately 1.34 during this solar maximum period, as shown in panel 2 of Figure 6.

4. Conclusion

We investigate the polytropic index as a function of the temperature anisotropy theoretically using the FLR correction and double-adiabatic polytropic equations. We obtain an expression for the polytropic index γ as a function of anisotropic temperature, magnetic field, and flow speed.

Following Nicolaou et al. (2023), for our analysis, we used Ulysses spacecraft data from 1992 January 1 to 2009 June 30 with a high sampling rate. We consider the polytropic index relation for the quasi-transversal propagation $k_{\perp} \gg k_{\parallel}$ case, and calculate the value of γ as a function of speed for different data bins, such as the heliocentric distance (R) and the heliographic latitude (θ).

In every bin of R and angle θ , we compute the average value of γ and the associated standard error. In the case of slow solar wind anisotropic plasma, we identified the maximum value at a speed of 430 km s^{-1} , with the peak centered within the heliographic latitude range $\theta < 30^\circ$, and for fast-flow solar

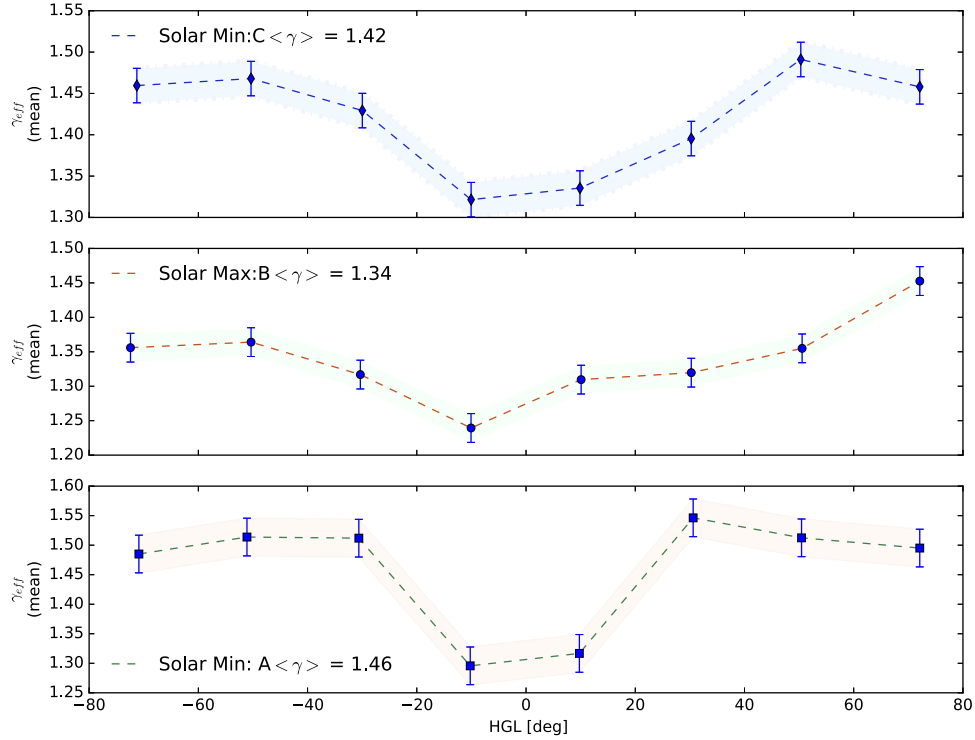


Figure 6. Mean value of γ as a function of the heliocentric latitude during Ulysses pole-to-pole scans.

wind anisotropic plasma, we determine the peak value centered at 770 km s^{-1} in the heliographic region $|\theta| > 30^\circ$. These results agree with the results obtained by Nicolaou et al. (2023).

In our analysis, the polytropic index exhibited is 1.44 during periods of solar minimum, 1.40 for the second orbit during solar maximum, and 1.45 during solar minimum, as shown in Figure 4. The corresponding temperature anisotropy ratios, $T_\perp/T_\parallel$, for these values are 0.78, 0.75, and 0.79, respectively. These values are similar to those found in the study conducted by Lentz et al. (2021) using data from the Mars Atmosphere and Volatile Evolution spacecraft to examine instabilities caused by proton temperature anisotropy at Mars. Their research focused on the southern hemisphere during the fall, winter, spring, and summer seasons. The predominant temperature anisotropy values observed were approximately 0.80–0.85, 0.70–0.75, and 0.60–0.65 respectively.

Using the polytropic results of this study, we calculate the mean value of the polytropic index for the first full cycle of Ulysses and obtain $\gamma = 1.44$. This is close to the result of Nicolaou et al. (2023), which is 1.40. However, this is for the first Ulysses orbit during the solar minimum period. In our first radial bin ($1.25 < R < 1.75$) for the inner heliosphere, the average γ is 1.47. This result is close to the polytropic index value determined by Totten et al. (1995), using the temperature and density fitting of the Helios data from 0.3 au to 1 au, and obtaining a value of ~ 1.46 .

The polytropic index was investigated by several authors and was found to be independent of plasma velocity and solar activity (Livadiotis 2018; Nicolaou et al. 2023), but in our analysis as given by Equation (5), the value of the polytropic index depends on the magnetic field, flow speed, and anisotropic temperature (pressure), which may also exhibit a solar cycle dependence.

Acknowledgments

We acknowledge the NASA/GSFC’s Space Physics Data Facility’s CDAWeb (<https://cdaweb.gsfc.nasa.gov/index.html>) for Ulysses spacecraft data. The Sunspot data presented in this document rely on data produced by the American Association of Variable Star Observers (AAVSO; <https://www.aavso.org/>) and are available at <https://www.aavso.org/solar>. These data were accessed via the LASP Interactive Solar Irradiance Datacenter (LISIRD; <https://lasp.colorado.edu/lisird/>). The authors are grateful to the anonymous reviewer for providing constructive comments and helpful suggestions.

Appendix A Plasma Flow Parameters

To compute the plasma parameters in Equation (3), consider a system of particles with number density n , mass density $\rho = mn$, and anisotropic temperature and thermal pressure $p_{\perp,\parallel} = nk_B T_{\perp,\parallel}$ that flows with bulk velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$. From the continuity equation

$$\frac{1}{\rho} \frac{d\rho}{dt} = -\nabla \cdot \mathbf{v}, \quad (\text{A1})$$

using the frozen-in condition $\frac{d}{dt}(\mathbf{B}/\rho) = (\frac{\mathbf{B}}{\rho} \cdot \nabla)\mathbf{v}$,

$$\begin{aligned} \frac{1}{B} \frac{dB}{dt} &= \mathbf{b} \cdot (\mathbf{b} \cdot \nabla)\mathbf{v} + \frac{1}{\rho} \frac{d\rho}{dt} \\ &= -\nabla_\perp \cdot \mathbf{v}_\perp, \end{aligned} \quad (\text{A2})$$

and following Chew et al. (1956), the double-adiabatic equations of state have the form

$$\begin{aligned} \frac{1}{p_{\perp}} \frac{dp_{\perp}}{dt} - \frac{1}{\rho} \frac{d\rho}{dt} - \frac{1}{B} \frac{dB}{dt} &= 0, \\ \frac{1}{p_{\parallel}} \frac{dp_{\parallel}}{dt} - \frac{3}{\rho} \frac{d\rho}{dt} + \frac{2}{B} \frac{dB}{dt} &= 0. \end{aligned} \quad (\text{A3})$$

Using Equations (A1) and (A2) in Equation (A3), the rate of pressure anisotropy change $\frac{dp_{\perp}}{dt}$ and $\frac{dp_{\parallel}}{dt}$ can be related with plasma flow parameters as follows:

$$\begin{aligned} \frac{dp_{\perp}}{dt} &= -p_{\perp}(\nabla \cdot \mathbf{v} + 2\nabla_{\perp} \cdot \mathbf{v}_{\perp}), \\ \frac{dp_{\parallel}}{dt} &= -p_{\parallel}(\nabla \cdot \mathbf{v} + 2\nabla_{\parallel} \cdot \mathbf{v}_{\parallel}), \end{aligned} \quad (\text{A4})$$

where $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$ is the total derivative. Plugging Equations (A1) and (A4) into Equation (3), the polytropic index can be expressed as follows:

$$\gamma = 1 + \left[\frac{1 + \frac{p_{\parallel}}{p_{\perp}} \frac{\nabla_{\parallel} \cdot \mathbf{v}_{\parallel}}{\nabla_{\perp} \cdot \mathbf{v}_{\perp}}}{\left(1 + \frac{\nabla_{\parallel} \cdot \mathbf{v}_{\parallel}}{\nabla_{\perp} \cdot \mathbf{v}_{\perp}}\right) \left(1 + \frac{p_{\parallel}}{2p_{\perp}}\right)} \right], \quad (\text{A5})$$

which is similar to Equation (6) in Meister et al. (2011). The problem here is to determine $\frac{\nabla_{\parallel} \cdot \mathbf{v}_{\parallel}}{\nabla_{\perp} \cdot \mathbf{v}_{\perp}}$.

Appendix B CGL-FLR Analysis

To calculate the polytropic index using Equation (4), one can obtain the values of $p_{\parallel}$ and $p_{\perp}$ from the measurements taken by the Ulysses spacecraft. However, the value of the plasma flow parameter still needs to be determined. Therefore, to assist in observation, it is necessary to have a theoretical estimation of this characteristic of the plasma motion parameter (Λ). For the purpose of our polytropic analysis, we consider a two-dimensional plasma flow $\mathbf{v}(v_x, 0, v_z)$ to determine Λ .

Following Prajapati et al. (2022), the linearized forms of the momentum transfer equation including the FLR correction within the frame of the double-adiabatic theory of Chew, Goldberger, and Low (Chew et al. 1956) are given as

$$\rho \frac{\partial \delta \mathbf{v}}{\partial t} = -\nabla \cdot (\delta \mathbf{p}) + \frac{1}{\mu_0} [(\nabla \times \delta \mathbf{b}) \times \mathbf{b}], \quad (\text{B1})$$

$$\frac{\partial \delta \rho}{\partial t} = -\rho(\nabla \cdot \delta \mathbf{v}), \quad (\text{B2})$$

$$\frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \times (\delta \mathbf{v} \times \mathbf{B}), \quad (\text{B3})$$

$$\frac{\delta p_{\perp}}{p_{0\perp}} = \frac{\delta \rho}{\rho} + \frac{\delta B}{B_0}, \quad (\text{B4})$$

and

$$\frac{\delta p_{\parallel}}{p_{0\parallel}} = 3 \frac{\delta \rho}{\rho} - 2 \frac{\delta B}{B_0}, \quad (\text{B5})$$

where $\delta \mathbf{v}$ (v_x, v_y, v_z), $\delta \mathbf{B}$ (B_x, B_y, B_z), ρ , and μ_0 are perturbations in the velocity, magnetic field, density, and magnetic permeability in free space, respectively. The variables $\delta p_{\parallel}$ and $\delta p_{\perp}$

denote perturbations in the pressure along and perpendicular to the direction of the magnetic field. The magnetic field is considered to be $\mathbf{B}(0, 0, B)$ in the frame of reference.

The pressure tensor ($\mathbf{p}$) with FLR correction can be linearized as

$$\begin{aligned} \overleftrightarrow{\mathbf{p}} &= \delta p_{\perp} \overleftrightarrow{\mathbf{I}} + (\delta p_{\parallel} - \delta p_{\perp}) \mathbf{b} \mathbf{b} + (p_{0\parallel} - p_{0\perp}) \mathbf{b} \delta \mathbf{b} \\ &\quad + (p_{0\parallel} - p_{0\perp}) \delta \mathbf{b} \mathbf{b} + \delta p^{\text{FLR}}. \end{aligned} \quad (\text{B6})$$

The vector $\delta \mathbf{b}$ can be determined from equation

$$\delta \mathbf{B} = \delta B \mathbf{b} + B \delta \mathbf{b}, \quad (\text{B7})$$

where $\delta \mathbf{b}$ the unit vector along the magnetic field. Taking into account the first-order terms with respect to the disturbances, we find

$$\delta b = \frac{\mathbf{B}_0 \cdot \delta \mathbf{B}}{B_0} = \frac{\delta B_z}{B_0} = \delta b_z, \quad \delta b_x = \frac{\delta B_x}{B_0}, \quad \text{and} \quad \delta b_y = \frac{\delta B_y}{B_0}. \quad (\text{B8})$$

Following Yajima (1966), the perturbed FLR tensor ($\delta \mathbf{p}_F$) is given by

$$\begin{aligned} (\nabla \cdot \delta \mathbf{p}_F)_x &= \frac{1}{4\Omega_i} [p_{\perp} k_{\perp}^2 + 2k_{\parallel}^2 (2p_{\parallel} - p_{\perp})] \delta v_y, \\ (\nabla \cdot \delta \mathbf{p}_F)_y &= -\frac{1}{4\Omega_i} [(4p_{\parallel} k_{\parallel}^2 + p_{\perp} (k_{\perp}^2 - 2k_{\parallel}^2)) \delta v_x \\ &\quad + 2p_{\perp} k_{\perp} k_{\parallel} \delta v_z], \\ (\nabla \cdot \delta \mathbf{p}_F)_z &= -\frac{1}{2\Omega_i} k_{\perp} k_{\parallel} (p_{\perp} - 2p_{\parallel}) \delta v_y. \end{aligned} \quad (\text{B9})$$

Substituting Equations (B2)–(B9) into Equation (B1), one obtains the following:

$$\begin{aligned} \rho_0 \frac{\partial}{\partial t} \delta v_x &= -\left((p_{0\parallel} - p_{0\perp}) \frac{\partial}{\partial z} \delta b_x + \frac{\partial \delta p_{\perp}}{\partial x} \right) - \nabla \cdot \delta \mathbf{p}_x^{\text{FLR}} \\ &\quad - \frac{B_0}{\mu_0} \left(\frac{\partial}{\partial x} \delta B_z - \frac{\partial}{\partial z} \delta B_x \right), \end{aligned} \quad (\text{B10})$$

$$\begin{aligned} \rho_0 \frac{\partial}{\partial t} \delta v_y &= -\left((p_{0\parallel} - p_{0\perp}) \frac{\partial}{\partial z} \delta b_y + \frac{\partial \delta p_{\parallel}}{\partial y} \right) - \nabla \cdot \delta \mathbf{p}_y^{\text{FLR}} \\ &\quad - \frac{B_0}{\mu_0} \left(\frac{\partial}{\partial y} \delta B_z - \frac{\partial}{\partial z} \delta B_y \right), \end{aligned} \quad (\text{B11})$$

and

$$\begin{aligned} \rho_0 \frac{\partial}{\partial t} \delta v_z &= -\left((p_{0\parallel} - p_{0\perp}) \left(\frac{\partial}{\partial x} \delta b_x + \frac{\partial}{\partial y} \delta b_y \right) + \frac{\partial \delta p_{\parallel}}{\partial z} \right) \\ &\quad - \nabla \cdot \delta \mathbf{p}_z^{\text{FLR}}. \end{aligned} \quad (\text{B12})$$

From Equation (A5), the polytropic index depends on the plasma flow parameters Λ and the anisotropy pressure (temperature). To evaluate the polytropic index and illustrate this dependence, we constrain the two-dimensional plasma motion $\delta \mathbf{v}(\delta v_x, 0, \delta v_z)$ in a system with wavevector $\mathbf{k}$ lying in the x - z plane. In this case, the plasma flow parameter becomes $\Lambda = \frac{k_{\parallel} \delta v_z}{k_{\perp} \delta v_x}$.

Using normal mode analysis, assuming perturbations of the form of $\delta f \propto f_0 \exp[i(k_x x + k_z z - \omega t)]$, where f_0 is the amplitude of the perturbed quantity and ω is the perturbation frequency. This simply gives the space derivatives $\partial/\partial x = ik_x$ and $\partial/\partial z = ik_z$ and the time derivative $\partial/\partial t = -i\omega$.

The above perturbations, Equations (B10)–(B12), have the form

$$[\omega^2 - k^2 V_A^2 + k_{\parallel}^2 C_{s\parallel}^2 - k_{\perp}^2 C_{s\perp}^2] \delta v_x - k_{\perp} k_{\parallel} C_{s\perp}^2 \delta v_z = 0, \quad (\text{B13})$$

$$\begin{aligned} & -\frac{\omega}{4\Omega_i} [k_{\perp}^2 C_{s\perp}^2 + 2k_{\parallel}^2 (2C_{s\parallel}^2 - C_{s\perp}^2)] \delta v_x \\ & -\frac{\omega}{2\Omega_i} C_{s\perp}^2 k_{\perp} k_{\parallel} \delta v_z = 0, \end{aligned} \quad (\text{B14})$$

and

$$-k_{\perp} k_{\parallel} C_{s\perp}^2 \delta v_x + (\omega^2 - 3k_{\parallel}^2 C_{s\parallel}^2) \delta v_z = 0, \quad (\text{B15})$$

where $C_{s\parallel,\perp} = (p_{\parallel,\perp}/\rho)^{1/2}$, are the thermal speeds parallel and perpendicular to the direction of the magnetic field and $V_A = B/(\mu_0 \rho)^{1/2}$ is the Alfvén speed.

If we neglect the effect of FLR, Equations (B13)–(B15) are the same as the results of Meister et al. (2011) that are reduced for two-dimensional flow. From Equation (B14), one can easily compute the plasma flow parameters $k_{\parallel} \delta v_z / k_{\perp} \delta v_x$ as

$$\Lambda = \frac{1}{2} \left[2 \frac{k_{\parallel}^2}{k_{\perp}^2} \left(1 - 2 \frac{C_{s\parallel}^2}{C_{s\perp}^2} \right) - 1 \right]. \quad (\text{B16})$$

Equation (B16) can be expressed as function of anisotropy,

$$\Lambda = (k_{\parallel}^2 / k_{\perp}^2) (1 - 2\alpha^{-1}) - 1/2, \quad (\text{B17})$$

where $\alpha = T_{\perp}/T_{\parallel}$.

In Equation (B16), since the flow parameters Λ depend on the flow speed, the polytropic index depends on the pressure (temperature) anisotropy and the flow speed of the plasmas. On substituting Equation (B16) into Equation (4), the polytropic index has the following form:

$$\gamma = 1 + \left[\frac{1 - \frac{p_{\parallel}}{2p_{\perp}} \left[1 + 2 \frac{k_{\parallel}^2}{k_{\perp}^2} \left(2 \frac{p_{\parallel}}{p_{\perp}} - 1 \right) \right]}{\left(1 - \frac{1}{2} \left[1 + 2 \frac{k_{\parallel}^2}{k_{\perp}^2} \left(2 \frac{p_{\parallel}}{p_{\perp}} - 1 \right) \right] \right) \left(1 + \frac{p_{\parallel}}{2p_{\perp}} \right)} \right]. \quad (\text{B18})$$

ORCID iDs

Ephrem Tesfaye Desta  <https://orcid.org/0000-0001-5502-3064>

R. D. Strauss  <https://orcid.org/0000-0002-0205-0808>

N. E. Engelbrecht  <https://orcid.org/0000-0003-3659-7956>

References

Chew, G. F., Goldberger, M. L., & Low, F. E. 1956, *RSPSA*, **236**, 117
Dasso, S., Milano, L. J., Matthaeus, W. H., & Smith, C. W. 2005, *ApJL*, **635**, L181

Dialynas, K., Roussos, E., Regoli, L., et al. 2018, *JGRA*, **123**, 8066
Elliott, H. A., McComas, D. J., Zirmstein, E. J., et al. 2019, *ApJ*, **885**, 156
Feldman, W. C., Barraclough, B. L., Gosling, J. T., et al. 1998, *JGR*, **103**, 14547
Gary, S. P., & Smith, C. W. 2009, *JGRA*, **114**, A12105
Ghag, K., Pathare, P., Raghav, A., et al. 2024, *AdSpR*, **73**, 1064
Horbury, T. S., Forman, M. A., & Oughton, S. 2005, *PPCF*, **47**, B703
Horedt, G. P. 2004, *Polytropes—Applications in Astrophysics and Related Fields*, 306 (Dordrecht: Kluwer)
Hunana, P., & Zank, G. P. 2017, *ApJ*, **839**, 13
Isenberg, P. A. 2005, *ApJ*, **623**, 502
Kartalev, M., Dryer, M., Grigorov, K., & Stoimenova, E. 2006, *JGRA*, **111**, A10107
Ledoux, P. 1958, *C&T*, **74**, 379
Lentz, C. L., Chasapis, A., Qudsi, R. A., et al. 2021, *JGRA*, **126**, e29438
Liu, Y., Richardson, J. D., Belcher, J. W., Kasper, J. C., & Elliott, H. A. 2006, *JGRA*, **111**, A01102
Livadiotis, G. 2015a, *JGRA*, **120**, 1607
Livadiotis, G. 2015b, *ApJ*, **809**, 111
Livadiotis, G. 2016, *ApJS*, **223**, 13
Livadiotis, G. 2017, *Kappa Distribution: Theory and Applications in Plasmas* (Amsterdam: Elsevier)
Livadiotis, G. 2018, *JGRA*, **123**, 1050
Livadiotis, G. 2019, *ApJ*, **874**, 10
Livadiotis, G., & Nicolaou, G. 2021, *ApJ*, **909**, 127
Livadiotis, G., & McComas, D. J. 2012, *ApJ*, **749**, 11
Livadiotis, G., & McComas, D. J. 2023, *ApJ*, **956**, 88
Livadiotis, G., McComas, D. J., Randol, B. M., et al. 2012, *ApJ*, **751**, 64
Marsch, E., & Richter, A. K. 1987, *AnGeo*, **5**, 71
Matthaeus, W. H., Goldstein, M. L., & Roberts, D. A. 1990, *JGR*, **95**, 20673
McComas, D. J., Funsten, H. O., Gosling, J. T., & Pryor, W. R. 1999, *GeoRL*, **26**, 2701
Meister, C. V., Maurer, C., & Hoffmann, D. H. H. 2011, *CoPP*, **51**, 639
Mishra, W., & Wang, Y. 2018, *ApJ*, **865**, 50
Nicolaou, G., Livadiotis, G., & Desai, M. I. 2021, *Appl. Sci.*, **11**, 4019
Nicolaou, G., Livadiotis, G., & McComas, D. J. 2023, *ApJ*, **948**, 22
Nicolaou, G., Livadiotis, G., & Moussas, X. 2014, *SoPh*, **289**, 1371
Nicolaou, G., Livadiotis, G., & Wicks, R. T. 2019, *Entrp*, **21**, 997
Oughton, S., & Engelbrecht, N. E. 2021, *NewA*, **83**, 101507
Oughton, S., & Matthaeus, W. H. 2020, *ApJ*, **897**, 37
Pang, X., Cao, J., Liu, W., et al. 2015, *JGRA*, **120**, 4736
Pang, X., Geng, X., Wang, S., et al. 2022, *ApJ*, **940**, 120
Pang, X., Wang, F., Geng, X., et al. 2020, *ApJ*, **904**, 167
Prajapati, R. P., Desta, E. T., Fok, M.-C., & Eritro, T. H. 2022, *MNRAS*, **515**, 1444
Prasad, S. K., Raes, J., Van Doorselaere, T., Magyar, N., & Jess, D. 2018, *ApJ*, **868**, 149
Pudovkin, M. I., Lubchich, A. A., Zaitseva, S. A., Meister, C.-V., & Besser, B. P. 2000, *PhPl*, **7**, 4683
Pudovkin, M. I., Meister, C.-V., Besser, B. P., & Biernat, H. K. 1997, *JGR*, **102**, 27145
Pudovkin, M. I., Meister, C. V., & Besser, B. P. 1999, *AN*, **320**, 87
Roberts, D. A., Goldstein, M. L., Matthaeus, W. H., & Ghosh, S. 1992, *JGR*, **97**, 17115
Schulz, M., & Eviatar, A. 1973, *JGR*, **78**, 3948
Skopke, N., Paschmann, G., Haerendel, G., et al. 1981, *JGR*, **86**, 2099
Skoug, R. M., Feldman, W. C., Gosling, J. T., et al. 2000, *JGR*, **105**, 27269
Totten, T. L., Freeman, J. W., & Arya, S. 1995, *JGR*, **100**, 13
Verscharen, D., Klein, K. G., & Maruca, B. A. 2019, *LRSP*, **16**, 5
Wenzel, K. P. 1995, *NuPhS*, **39**, 59
Wenzel, K. P., Marsden, G. R., Page, E. D., & Smith, J. E. 1990, *ICRC* (Adelaide), **7**, 252
Wenzel, K. P., Marsden, R. G., Page, D. E., & Smith, E. J. 1992, *A&AS*, **92**, 207
Yajima, N. 1966, *PTEP*, **36**, 1
Zank, G. P., Adhikari, L., Hunana, P., et al. 2017, *ApJ*, **835**, 147
Zank, G. P., Matthaeus, W. H., & Smith, C. W. 1996, *JGR*, **101**, 17093
Zhou, Y., & Matthaeus, W. H. 1990, *JGR*, **95**, 10291