A model for mathematics teachers to promote ESL acquisition through questioning strategies

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It all starts here ™

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ABSTRACT

Questions have been used in centres of learning and teaching all over the globe since time immemorial for student-teacher interaction, student learning and assessment. In most of South Africa's multilingual classrooms, these questions are phrased in English, a medium of instruction and also a second or third language for most of the students. However, the types of questions, their roles, questioning techniques and teacher strategies have not been widely explored, especially in mathematics classrooms in as far as the development of English Second Language (ESL) on the part of the students is concerned. The purpose of this study is therefore to explore this with the ultimate purpose of enabling grade 10 mathematics teachers to promote learners' understanding of mathematical discourse and ESL development through the types of questions used, questioning techniques and teacher strategies. The study also focuses on the functions of questions, questioning techniques and strategies that teachers can apply during lessons for learners to comprehend the lesson, to process and interact using language (ESL), to produce language in the form of output, and to receive feedback on their utterances. The research followed a qualitative approach within an interpretivist paradigm. The qualitative research design and multiple case study approach allowed the participants to give meaning to the construct by sharing their own experiences in mathematics classrooms. Ultimately the results from the data analysed and the literature reviewed, were used to design a hands-on tool to promote questioning and language acquisition in mathematics classrooms.

Key words:

Mathematical discourse, ESL acquisition, mathematical proficiency, questions, questioning techniques, teacher strategies, comprehensible input, language processing and interaction, output, and feedback.

OPSOMMING

Vrae word al sedert onheuglike tye by sentrums van onderrig en leer regoor die wêreld gebruik om leerder-onderwyser interaksie te fasiliteer en as basis virleerders se leer en assessering. In die meeste van die veeltalige klaskamers in Suid-Afrika word hierdie vrae in Engels gestel, aangesien dit die taal van onderrig is. Dit is egter ook vir die meeste van die leerders 'n tweede of derde taal. Die soorte vrae, hulle rolle, vraagstellingstegnieke en -strategieë is egter nog nie wyd ondersoek nie, veral nie wanneer dit kom by wiskundeklaskamers en rondom die ontwikkeling van Engels as Tweede Taal (ETT) by leerders nie. Die doel van hierdie studie is daarom om hierdie aspek te ondersoek, met die uiteindelike doelstelling om Graad 10 wiskunde-onderwysers te bemagtig om leerders se begrip van die wiskundediskoers te verbeter en om Engels as tweede taal te ontwikkel deur die gebruik van verskillende tipes vrae, vraagstellingstegnieke en -strategieë. Die studie fokus verder op die funksies van vrae en die verskillende vraagstellingstegnieke en -strategieë wat onderwysers kan gebruik gedurende hulle lesse om leerders instaat te stel om die les te verstaan, om te prosesseer en om interaksie te hê deur die gebruik van taal (ETT), om taal te produseer in die vorm van 'n uitset, en om terugvoer te ontvang. Die navorsing het die kwalitatiewe benadering gevolg met 'n interpretatiewe paradigma. Die kwalitatiewe navorsingsontwerp en veelvuldige gevalle studie benadering het aan die deelnemers die geleentheid gebied om betekenis tot die konstruk toe te voeg deur hulle eie ervaringe in wiskundeklaskamers te deel. Uiteindelik is die data gebruik om 'n praktiese instrument te ontwerp wat vraagstelling en taalverwerwing in die wiskunde klaskamers kan bevorder.

Sleutelwoorde:

Wiskundige geletterdheid, EAT-verwerwing, wiskundige bevoegdheid, vraagstellingstegnieke, onderwyserstratigieë, verstaanbare inset, taalprosessering en interaksie, uitset, terugvoer

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CHAPTER 1: INTRODUCTION

1.1 Introduction

The purpose of this chapter is to present the research problem and to contextualise it briefly by providing relevant background information. The chapter continues to identify the research questions and aims of the study and to outline the research methodology. It finally indicates the contribution that the study makes to the larger body of knowledge on this topic.

1.2 General problem statement

Mathematics is one of the subjects in which senior certificate students have been performing poorly over the past years in South Africa. This poor performance was exposed in the report issued to the South African Broadcasting Corporation (SABC) by the Concerned Maths Educators (CME) after the 2008 mathematics results of the senior certificate examination had been released. It states that the "the 2008 mathematics results do not reflect real improvement in mathematics education in South Africa" (CME, 2009:1).The main concern is the lack of improvement in mathematics education in South Africa. Their concern was that even though a total of 63 038 learners in 2008 had passed the subject and scored above 50%, these learners cannot be regarded as "adequately prepared to cope with mathematics related courses" at tertiary levels.

Similarly, the lack of improvement in mathematics education is evident from the increase in the percentage of candidates who scored 30% and above, but less than 40% in the years 2011, 2012, 2013, and 2014 (Motshekga, 2015:109). Also, the decline in the candidates' performance over the years has adversely affected the attitude of learners towards the subject, resulting in a decrease in the number of candidates who sat for the 2011, 2012, 2013, and 2014 examinations from 224 635, 225 874, 241 509, and 225 458 respectively (Motshekga, 2015:109).

This poor performance of learners in mathematics was also confirmed by the findings of the Third International Mathematics Science Study *(TIMSS 2011)*, which revealed that Grade 9 learners in South Africa, and also in Botswana and Honduras, performed very poorly in mathematics and science subjects. In fact, their national scores were among the bottom six countries out of a total of 42 countries that participated in the TIMMS 2011 study (IEA, 2011:4). It is also shocking to realise that even the average scores of the best performing schools in South Africa (Quintile 5 and Independent schools), were below the centre-point of 500, according to the report. The scores were 473.5 and 479 for mathematics and science respectively, and therefore far below the international benchmark of 550 for both mathematics and science (IEA, 2011:11*)*.

The study further revealed factors such as learners not speaking the medium of instruction at home, few parents having Grade 12 qualifications, and few or no books at home, etc. (IEA, 2011:7). The issue of the medium of instruction as a factor was also pointed out in the mathematics examiner's report on the candidates' performance in the November 2009 Mathematics Paper 1. Out of a total of 13 questions set, sub-sections of 11 questions "were poorly answered", according to the examiner. For example, learners found it difficult to answer questions with words such as *rate of change* and *interpret.* Also, according to the examiner, questions 5 to 9 were "the worst questions answered, … in fact, at 2 specific centres, 31 out of 70 learners scored no marks" for these questions NSC (2009:5). These sections contained words such as, *coordinates* and *axis of symmetry,* which made it difficult for learners to solve sub-sections of this question as they did not understand the meanings of such mathematical terms.

The majority of the examiner's comments on learners' problems with the language of instruction are found in the report on the Mathematics Paper 2. Comments made by the examiner indicated that candidates did not understand the mathematical concepts such as, *mutually exclusive events*, *mutually inclusive events*, and *independent events*. The report also showed that the candidates did not understand even the meaning of imperatives that are frequently used in mathematics classrooms, such as, *estimate*, *show that*, and *prove that* (Motshekga, 2015: 121).

Similarly, in a study analysing learners' errors in the scripts for the Grade 12 Geometry Paper written in 2008, the results show that in question 3.2.2, 75% of the learners could not "tell the difference in meaning of words such as *rotation*, *reflection*, and *translation*, and also between *rigid* and *non-rigid transformation*, and as a result, only 25% of the learners got this question correct" (Luneta, 2015:5).

The above examples illustrate that the learners do not understand the language that is used to phrase questions in mathematics examination question papers or the mathematical discourse used in these questions. One problem could be that the types of questions that improve learners' comprehension of mathematical problems are not used by mathematics teachers in their classrooms. The possible reason for that, according to Bellido *et al*. (2005:1), could be that such questions are not found in most mathematics textbooks.

Bellido *et al*.' s view was confirmed after an analysis of the types of questions used in Grades 10-12 prescribed mathematics textbooks, which shows that most of the questions used are imperatives, such as, *Expand*, *Factorise*, *Simplify*, etc. (Laridon *et al*., 2008:131). The imperatives or commands used give learners instructions on what to do with regard to the questions given without creating an opportunity for learners to acquire English, the medium of instruction. Probing imperatives like *explain how you got the answer* are used to a limited extent. For example, in the Grade 11 prescribed mathematics textbook, the imperative *explain your answer*, appears once in Chapters 1 to 7 (Laridon *et al*., 2006:165). These are the questions that 'foster deeper knowledge and access deeper understanding, ... questions that ask students to justify, clarify or extend their thinking strongly aligned with the ways of working as a mathematician' (Zevenbergen & Niesche, 2008).

Since learning begins with questions (Chuska,1995:7), it will be difficult for learners to perform well in mathematics when they do not understand the mathematical discourse used in the questions that are phrased in English. Further evidence of the significance of questions, especially in mathematics classrooms, is provided by Sutton and Krueger (2002), who assert that mathematics teachers who are highly rated by students ask a variety of questions.

1.3 Background

Research has shown that questions are an important teaching technique in a teaching and learning environment. Siposova (2007:34) lists the following functions that are fulfilled by teachers' questions:

- They give students the impetus and opportunity to produce language comfortably without having to risk initiating language themselves.
- They can serve to initiate a chain reaction of student interaction among themselves.
- They provide immediate feedback about student comprehension and opportunities to find out what they think by hearing what they say.

Brualdi (1998) and Rosenshine *et al*. (1996) also agree that teachers ask questions for a variety of reasons, such as, getting students' attention, enabling them to express their point of view, hearing different views from their peers, and evaluating learning. These functions emphasise the importance of teacher questions in facilitating and sustaining effective student participation, especially in mathematics classrooms where English is the language of learning and teaching (LOLT), but also a second or third language. This is the case in most high schools in South Africa, including the schools in the present study, as confirmed in the statement that “English is spoken by less than ten percent of the population” (Howie, 2003:1).

The problem of English as a medium of instruction is not limited to South Africa. In other countries where mathematics is taught using English as the medium of instruction (EMoI), learners experience problems understanding the content as well as the questions asked by the teachers. A study by Khisty and Chval (2002:156) conducted on Latino students in Illinois, USA, found that students in mathematics classrooms had to first acquire the language of instruction, English, and then the language of mathematics, which is totally different from the language of conversation. For example, in the expression $y = f(x)$, where f stands for the word “function”, a word which is totally different from the normal everyday meaning of the word ‘function’. Since mathematics is taught through the medium of English, a second and third language for most of the learners, this creates an additional burden for learners who have to battle with understanding the language of instruction and the complex mathematical discourse, before comprehending mathematical concepts, generalisations and thought processes.

In a study conducted by Abedi and Lord (2010: 221) in Los Angeles, California in the USA, the researchers gave 1174 grade 8 English Language Learners (ELL) a test with questions based on the unit on *Word problems* in the Algebra section to see how they could alleviate the additional burden indicated of first learning to understand the language of instruction. The learners had to choose between the two versions of the same test. One version had the original questions on *word problems*, the other had linguistic modifications like, among others, passive verb forms changed to active, shortening long nominals, removing relative clauses, complex question phrases changed into simple ones, etc. The majority of the learners, 83,1%, chose the linguistics modified version and during interviews those learners stated that the modified version was “easier to comprehend” (Abedi & Lord, 2010:221/222).

Failure to do well in mathematics due to the medium of instruction as shown in the studies mentioned above, is also applicable to learners in South Africa. This failure, according to Fleisch (2008), can be attributed to the “straight-for-English policies and early exit from mother-tongue” in primary schools, where the majority of the students have English as a second or third language. In fact, Fleisch (2008:98) asserts that “less than one South African child in ten speaks English as their first language” and that is a very small number indeed, as confirmed in Howie (2003:1). The important role of questions cannot be overlooked in helping learners acquire basic interpersonal cognitive skills (BICS) and cognitive academic language proficiency (CALP), which includes reading and writing skills as well as the understanding of subject-specific vocabulary (Cummins 1980). It is for this reason that the focus of the present study is on the effective use of different types of questions in teaching mathematics in high schools.

In order for mathematics teachers to assist their learners in their quest to promote the comprehension of mathematical discourse through the use of effective questions, code switching to the learners' mother tongue can be used on a limited scale, and not throughout the lessons, because English is used to set the question papers and the learners in turn are expected to respond to such questions using the medium of instruction as is the case in the grade 12 National Senior Certificate question papers and memoranda.

It is the responsibility of mathematics teachers to provide an environment in the classroom that enables students to understand the language of instruction as well as mathematical discourse to address the problems resulting from the fact that English is the medium of instruction. Such an environment would enable learners to have some control over their learning of the subject and to improve their mastery of the language of instruction.

1.4 Research questions

The research problems this study seeks to address can be conceptualised at three levels, namely the theoretical-methodological, descriptive and applicational levels.

The research problem that covers the theoretical-methodological aspect of the study can be formulated as follows:

1. What are the Second Language Acquisition theories and mathematics learning theories that underpin the effective questioning techniques to promote ESL acquisition?

At the descriptive level, the research problem can be formulated in terms of the following research questions:

2. What are the characteristics of the most frequently-used question types in grade 10 mathematics classrooms?

(a) How do they promote learners' understanding of mathematical discourse?

(b) How do they promote learners' understanding of mathematical discourse and ESL development?

(c) What are the functions of these questions in grade 10 mathematics classrooms?

(d) What are the questioning techniques used in grade 10 mathematics classrooms?

(e) What are the teacher strategies used in grade 10 mathematics classrooms?

At the applicational level, the research problem can be formulated in terms of the following research question:

3. What are the characteristics of a hands-on tool that could support grade 10 mathematics teachers in developing their questioning skills that promote English second language acquisition?

1.5 Aims of the study

The aims of this study can also be conceptualised at three levels, namely the theoretical-methodological, descriptive and applicational levels.

At the theoretical level, the aim of this study is to:

1. develop a theoretical model to illustrate the role of questions in the acquisition of mathematical discourse and ESL development.

The theoretical model discussed in Chapter 4 was developed after a literature survey on the topic. The literature survey is presented in Chapters 2, 3 and 4.

At the descriptive level, the study aims to:

2. describe the types of questions, questioning techniques and teacher strategies used in grade 10 mathematics classrooms.

Chapter 5 discusses the research methodology applied to investigate and describe the questioning types, questioning techniques and teacher strategies. Furthermore, Chapters 6 and 7 discuss the results and interpretation of the instrumental case studies and the collective study to answer this question.

At the applicational level, the findings and results derived from the focus on the research problems articulated, provided the researcher with guidelines on how to design a hands-on tool for mathematics teachers to use in their classrooms to promote ESL acquisition through questions, questioning techniques, and teacher strategies.

At the applicational level, the study, therefore aims to:

3. empower mathematics teachers with a hands-on tool that will guide them in the use of questions, questioning techniques and teacher strategies to promote learners' understanding of mathematical discourse and ESL development.

1.6 Research methodology

This section provides a brief introductory description of the research methodology of this study.

Research design

The study is qualitative in nature. Creswell's (2007:37) definition of qualitative research below contains the characteristics of a good qualitative study that are relevant to this study. He sees qualitative research as:

> *"the study of research problems inquiring into the meaning individuals or groups ascribe to a social problem, ... the collection of data in a natural setting and data analysis that establishes patterns or themes".*

Research has proven that most of the questions used in mathematics classrooms are closed and not open-ended questions that promote discussion and subsequently second language acquisition on the part of the learners (Brualdi, 1998; Sutton & Kreuger, 2002; Yeo & Zhu, 2009; Sadker, 2003; Zevenbergen & Niesche, 2009). The researcher could have gathered information on the types of questions used in these classrooms from mathematics tests and examination question papers only. However, the researcher was much more interested in the stories behind the types of questions used by the teachers in Grade 10 mathematics classrooms. In an effort to conduct a detailed analysis of the data collected in a natural setting in 4 Grade 10 mathematics classrooms, data were collected from the 4 teachers' lesson plans, lesson observations, interviews, and field notes to discover and use knowledge that "is constructed through communication and interaction", as is the case with all qualitative research studies. This knowledge is, according to Vanderstoep and Johnston (2009:166), "constructed and created by the people" and in this case the 4 teachers who prepared daily lesson plans using questions, teach in these classrooms using questions, and who were in a position to elaborate during interviews on the choice of questions they used to promote learners' understanding of mathematical discourse and ESL development. This information can finally provide themes or patterns that emerge from the analysis.

Methodology

The researcher has used the qualitative case study method which requires using multiple sources of data collection instruments such as interviews, observations, audio-visuals, documents and field notes. Since the study is qualitative in nature, it is a case study of each of the 4 grade 10 mathematics teachers' experiences of using the types of questions, questioning techniques and teacher strategies in their lessons. They were studied using lesson observations followed by individual interviews, field notes, and later in a focus group interview of all the four teachers.

Conceptual framework

The study draws on the interpretivist perspective that social life is a distinctly human product (Nieuwenhuis, 2007:59). The underlying assumption is that by observing people in their 'social contexts' "there is a greater opportunity to understand the perceptions they have of their own activities" (Hussey & Hussey, cited in Nieuwenhuis, 2007:59). In the case of this study the natural setting is the Grade 10 mathematics classrooms where mathematics teaching and learning had been taking place since the beginning of the 2012 academic year. This was the case with each of the four participants as each of them elaborated on 'the perceptions of their activities' with regard to the question types, questioning techniques and teacher strategies they used during the individual and focus group interviews.

Participant selection

Stratified purposeful sampling (Nieuwenhuis, 2010:79) was utilised for this study. Since the researcher was interested in the types of questions used by mathematics teachers in grade 10 classrooms, she used a sample of four teachers teaching mathematics to ESL learners at four high schools in a rural village in the North West province in South Africa. The sampling method used was selected based on its relevance to the research questions, the language of teaching and learning, and the curriculum offered at the four schools.

Researcher's role

The qualitative researcher is considered as an instrument of data collection (Denzin & Lincoln, 2003). This means that data are mediated through this human instrument (Simon (2011). To fulfil this role, the reader needs to know about this human instrument who is expected to describe relevant aspects of self, including any biases and assumptions, any expectations, any experiences to qualify his/her ability to conduct the research (Greenback, 2003). Furthermore, qualitative researchers should also explain if their role is emic – an insider, who is a full participant in activity, programme or phenomenon, or if the role is more etic – from an outside view, more of an objective viewer (Simon, 2011). In this qualitative research, the researcher is "the key instrument" (Creswell, 2007:38), as the researcher was responsible for data collection with regard to documents that are to be examined, observations and interviews to be conducted, and also in the development of the protocol, in other words, the instrument used to gather all the information in the form of questionnaires for the participants' interviews.

Data collection strategies

Data in the form of lesson plans, transcribed lesson observations, transcribed teachers' responses to the individual and focus group interviews, as well as, field notes from a diary, were collected in the 4 grade 10 mathematics classrooms. The study was collected specifically in grade 10 classes as this is the preparatory class for learners who will be writing the final matriculation or senior certificate that enables them to get admission into tertiary institutions, such as universities and further education training colleges, and also into the work place.

Data analysis

The data collected were analysed manually and the researcher identified patterns or themes in the data that could guide her to develop a model to empower mathematics teachers with a hands-on tool with questions, questioning techniques and teacher strategies to promote learners' understanding of mathematical discourse and ESL development. Data on the transcribed lesson observations and interviews were further analysed using the ATLAS.ti software to corroborate the findings of the data analysed manually. The software, as it analysed data, also captured the frequency of questions, questioning techniques and teacher strategies used in the transcribed lesson observations and interviews, something that is very exhausting when captured manually.

Validity and qualitative reliability or trustworthiness

For the sake of triangulation and to maintain construct validity of the results as required in a qualitative research study, following Creswell (2007:204) and Gast's (2010:12) suggestions, the researcher made use of multiple sources of data collection methods.

According to Friese (2012:146), the ATLAS.ti software used to analyse data, adds much to data analysis in terms of trustworthiness, credibility, transparency and dependability - the quality criteria by which good qualitative research is recognised.

1.7 Contribution of the study

After an extensive literature search, no study could be found that provides a theoretical model and a hands-on tool for mathematics teachers to use questions, questioning techniques and teacher strategies to assist learners with comprehension; language processing and interaction; opportunities to produce output; and to receive feedback. The argument put forward here is that in mathematics lessons, the types of questions used by teachers should promote learners' understanding of mathematical discourse and ESL development.

1.8 Limitations of the study

The generalisability of the study is limited as four schools and four teachers participated in the study, and as a result, this study cannot be generalised to the entire population of all the Grade 10 mathematics teachers in South Africa.

1.9 Chapter division

Chapter 1 provides an introduction of the research study in terms of its background, and subsequently positions the problem statement through a preliminary literature review. The research question and the associated aims and objectives are discussed in detail. The purpose of this study is to explore the characteristics of the most frequently-used question types in grade 10 mathematics classrooms, to identify the question type patterns, questioning techniques and teacher strategies that promote English Second Language Acquisition (SLA) on the part of the learners and to ultimately develop a model for promoting English SLA through questioning techniques.

Chapter 2 addresses mathematics as a language and the chapter elaborates on this concept by explaining what mathematics is, its different types of languages, as well as the challenges and solutions for mathematics teachers in as far as the teaching of the language of mathematics is concerned.

Chapter 3 examines learning language and mathematics and discusses the conditions for and the theories on the Teaching of English as a Second Language (TESL) and Mathematics Teaching (MT), as well as theories on English SLA and mathematics learning (ML). The roles of input, language processing and interaction, output, and feedback in teaching and learning in as far as English SLA and mathematics are concerned, are also discussed.

Chapter 4 on questioning discusses the types of questions and their functions, questioning techniques, and teacher strategies used in mathematics classrooms.

Chapter 5 on the qualitative research design explains the reasons why the researcher chose an interpretive qualitative case study to conduct the research. The steps of the detailed methods followed for data collection and analysis procedures relating to the research questions and its anticipated problems are discussed. Issues that are covered in this chapter also include the relevance of the research design for the study. In addition, the limitations of case studies and ethical aspects relating to this study are identified, described, while applicable administrative procedures for the research are also described.

Chapter 6 presents and discusses the results and findings of the research and sub-research questions of the four cases A, B, C and D. A summary of the findings is also provided.

Chapter 7 presents and discusses the results and findings of the research and sub-research questions of data on the transcribed lesson observations and interviews analysed using the ATLAS.ti software, and of the collective case study. A summary of the interpretation of the findings as it relates to the literature reviewed is provided.

Chapter 8 describes the hands-on-tool developed and offers suggestions on how it should be implemented by mathematics teachers. Lastly, the limitations, recommendations, suggestions for further research and a conclusion are provided.

CHAPTER TWO: THE LANGUAGE OF MATHEMATICS

2.1 Introduction

Many researchers have described mathematics as a language (Setati, 2002; Eisty, 1992). This chapter elaborates on this understanding of mathematics by explaining what it is and discussing the different types of languages that make up mathematics and the challenges and solutions for mathematics teachers in as far as the teaching of the language of mathematics is concerned. For learners to be able to understand mathematical language, they have to be mathematically proficient so as to be able to communicate their ideas mathematically.

2.2 What is mathematics?

Setati (2002:9) describes mathematics as a language as it uses notations, symbols, terminology, conventions, models and expressions to process and communicate information. Learners therefore need to be developed in the use of mathematical discourse, the language used in mathematics classrooms.

Similarly, Esty (1992:5) defines mathematics as a language, because, like other languages, it has its own grammar, syntax, vocabulary, word-order, synonyms, negations, conventions, idioms, abbreviations, and sentence and paragraph structure. Therefore, for English Second Language (ESL) and bilingual learners to understand mathematics, teachers should make mathematics intelligible and comprehensible for learners and assist them to be proficient in the different types of languages used in mathematics classrooms (Allen, 1988:4).

2.2.1 Types of mathematics language

According to Halliday (1978) cited in Molefe (2006:77), the language of mathematics is called the ‘register of mathematics,’ and it refers to the terms and grammatical structures that express mathematical purposes. In the same manner, Gaoshubelwe (2011:23) defines ‘mathematical register’ as the meanings belonging to the language specifically used in mathematics. According to Kersaint *et al*., (2009: 46), mathematical register refers to a subset of language composed of meaning appropriate to the communication of mathematical ideas, and it includes, vocabulary, syntax (sentence structure), semantic properties (truth conditions), and discourse (text) features.

2.2.1.1 Content language

The language that is specifically used in mathematics classrooms, classified as mathematical discourse, includes the following aspects, summarised in Figure 2-1 below.

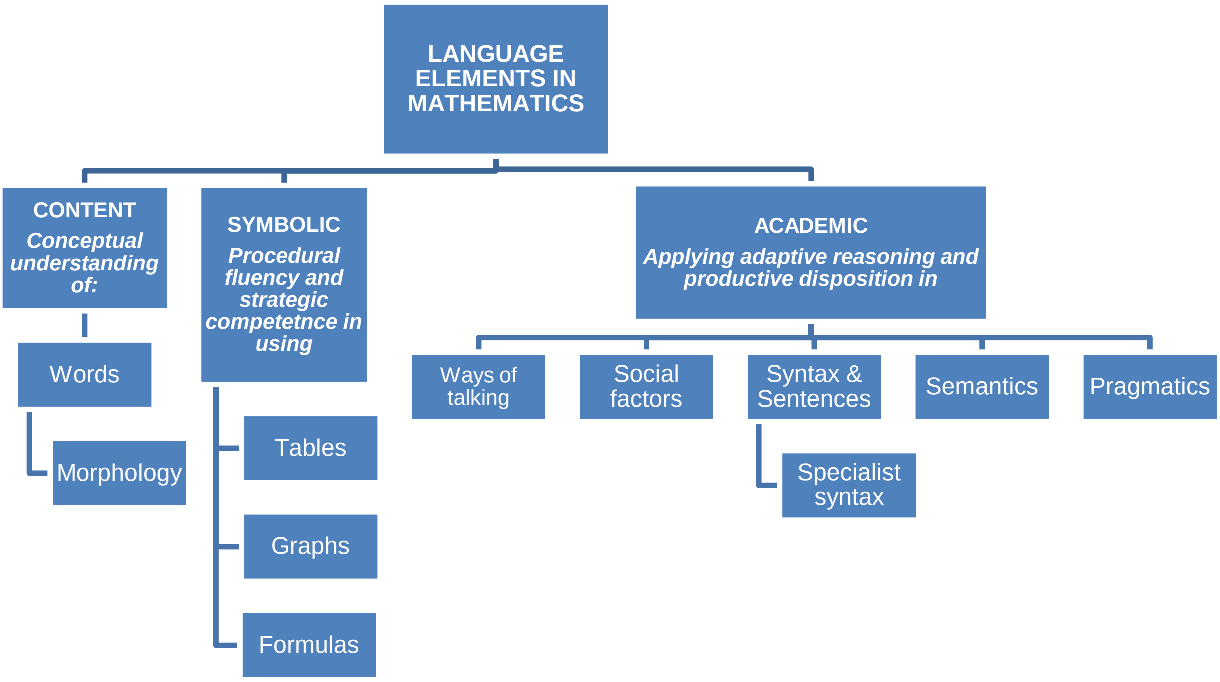

Figure 2-1: Types of language in mathematics

Adapted from Thompson and Rubenstein (2000:569); Barwell (2008:2) and Kenny (2005:3).

Academic language or technical vocabulary, according to Thompson and Rubenstein (2000:569), includes the following:

Words

- Some words are shared between mathematics and everyday English, but they have distinct meanings, e.g. *number*: prime, power, factor;

- Some mathematics words are shared with English and have comparable meanings, e.g. *number*: divide, equivalent, even, difference;

- Some mathematical terms are found only in mathematics classrooms, for example, *geometry*: quadrilateral, parallelogram, isosceles, hypotenuse;

- Modifiers may change mathematical meanings in important ways, e.g. *number* value, or *absolute* value, prime or *relatively* prime;

- Some mathematical phrases must be learned and understood in their entirety, e.g. *geometry*: if-then, if-and-only-if;

- Some words shared with science have different technical meanings in two disciplines, e.g. *number*: *divide*, *density*;

- Some mathematical words sound like every day English words, e.g. number: *sum* or some;

- Some mathematical words are related, but learners confuse their distinct meanings, e.g. number: *factor* and *multiple*, *hundreds* and *hundredths*, *numerator* and *denominator*;

- Technology may use language in special ways, e.g. algebra: *LOG* (for base-10 logarithms, not any logarithm, *scale*;

- A single word may translate into Spanish or any another language, in two different ways, e.g. *round* (redondear), as in "round off", or *round* (redondo), as in "circular"; and in Setswana, the number *0* (lefela) as in "zero" and it is also called (lee) meaning "an egg", as the number zero is similar in shape to an egg.

- Shorthand or abbreviations are often used in place of the complete word or phrase, even if learners must pronounce the entire word when verbalising the shorthand, e.g. *sin* for *sine*, *cos* for *cosine*, and *tan* for *tangent.*

Morphology

- Morphology or word structure is used for some of the words in mathematical discourse to make mathematical language come to life, making terms meaningful and revealing connections with relevant ideas. This reduces the number of things learners should learn, for example, the prefix *co-* means *together*, and therefore complementary means 'to make full', hence complementary angles are those angles that when added together, add up to 90° (Gaoshubelwe, 2011:29). Other examples with prefixes and suffixes are a *pentagon* from the Greek word *pente* and *gonia*, meaning 'five' and 'angle' respectively, and it means a five-sided polygon (a flat shape with straight sides); bisect with *bi-* meaning 'two', and 'sect' meaning cut, and bisect means to cut into two equal parts.

2.2.1.2 Symbolic language

Mathematical discourse includes the use of mathematical symbols, which range from numerals to more specialised notation, and these are confusing to the learners due to the reasons given below:

- Different representations are used to describe the same process, e.g. 2.2, 2 x 2, 2(2), 2^2 for multiplication;

- The symbols look alike, for example, the square-root sign $\sqrt{9}$ and the division $^{2}\sqrt{10}$ (Kenny, 2005: 3).

Diagrams and graphs

- Graphic representations may also be confusing to the learners, for example, bar graphs versus line graphs, because “they are not consistently read in the same direction” (Kenny, 2005:3).

2.2.1.3 Academic language

Ways of talking

Mathematical discourse, according to Barwell (2008:3), includes specialised ways of talking, including written and spoken forms of mathematical explanation, proof or definition, as well as text types like word problems, writing a solution, giving an explanation, e.g. *we multiply* (using plural forms).

Social factors

Mathematical language includes the particular ways that teachers and the learners talk in mathematics classes that are not specifically mathematical, but are associated with mathematics, for example, instructions such as *simplify*, *complete the following*. Teachers often use *we* to refer to ‘people who do mathematics’, e.g. *we use x to represent the unknown* (Barwell, 2008:2).

Syntax

Syntax is a part of linguistics that deals with the arrangement of words into phrases and phrases into sentences (Gashubelwe, 2011:12). Similarly in mathematics, syntax awareness reflects sensitivity for grammatical relationships between words, phrases and sentences. When the learners are aware of the syntax, it helps them translate word problems into symbolic form. For example, when looking at ‘*a is 7 less than b*’, the symbolic translation of the word problem is: $a = b - 7$, or $b - a = 7$, or $a + 7 = b$.

Sentences

Currently in mathematics teaching, statements and questions are often written in the passive voice (for example, *twelve (is) divided by three*), and there is no one-to-one correspondence between mathematical symbols and the words they represent. For example, in the word sum: *Ten times a number is five times the number*; learners must understand how key words relate to each other, that *a number* and *the number* refer to the same quantity (Dale & Cuevas, 1992), cited in (Molefe, 2006:79).

Specialist syntax

Mathematical discourse also includes specialist syntax, particularly in relation to the expression of logical relationships, for example, the use of *and*, or, *a*, *if, if and only if,* and *then* to define mathematical relationships (Barwell, 2011:2), as in the example, *if a = b, and b = c, then a = c.*

Semantics

Semantics refers to the process of making meaning from language (Kersaint, *et al*., 2009:49). For example, a student may have difficulty in understanding the statement: *8 times a number is 30 more than 6 times the number,* because s/he might not understand the rules of definite articles which suggest that *a number* in that statement is *the number* since it is no longer new information, but old information for the reader, hence *the number*. Some of the examples that are confusing to the learners are as follows: *the square of the number 4 indicated by* $4^2 = 16$, and *the square root of a number 4 indicated as* $\sqrt{4} = 2$.

Pragmatics

According to Rowland (2002), pragmatic meaning is how speakers convey affective messages to do with social relations, attitudes and beliefs. It gives them a way to associate or distance themselves from the propositions they articulate, to fulfil the interactional function of language. For example, after a lesson on a difficult section in mathematics, the teacher can say to the learners, "*Let us try to solve the following problems on the board*". By using the words '*Let us try'*, the teacher includes him/herself in the solution of the problems on the board, making the learners aware of the fact that s/he will be available to assist them; and putting them at ease if they experience some difficulties in getting some of the answers incorrect for the problems given.

According to NRC (2001:116), for learners to be able to learn mathematics successfully, the goal towards which mathematics learning should be aimed at is mathematical proficiency.

Mathematical proficiency has five components which are intertwined, and all are necessary for learners to learn mathematics successfully.

2.2.2 Components or strands of mathematical proficiency

Learners who have opportunities to develop all the components or strands of mathematical proficiency explained below, are more likely to become truly competent and mathematically proficient (NRC, 2009).

Conceptual understanding

Conceptual understanding involves a learner's comprehension of mathematical concepts, operations and relations. Such understanding results in learners having less to learn because they can see the deeper similarities between superficially unrelated situations, for example, *6 + 7 is just one more than 6 + 6* (NRC, 2009:120).

Procedural fluency

Procedural fluency refers to skills in carrying out procedures flexibly, accurately, efficiently and appropriately. It is intertwined with conceptual understanding because when learners learn with understanding, they can modify or adapt procedures to make them easier to use, for example, when they are required to add 598 and 647, they would recognise that 598 is only 2 less than 600, so they might add 600 and 647 minus 2 to get the answer (NRC, 2009:124).

Strategic competence

Strategic competence is the ability to formulate, represent, and solve mathematical problems. For learners to become proficient problem solvers, they learn how to form mental representations of problems, detect mathematical relationships, devise novel solution methods, for example, *86 – 59* can be solved by practically collecting 86 sticks and removing 59 to get the correct answer 27 (NRC, 2009:126).

Adaptive reasoning

Adaptive reasoning refers to a learner's capacity for logical thought, reflection, explanation and justification. One manifestation of adaptive reasoning is the ability to justify one's work. Learners have to be able to justify and explain ideas in order to make their reasoning clear, hone their reasoning skills, and to improve their conceptual understanding (NRC, 2009:130). In short, learners should be able to explain their thought process, in other words, how they arrived at their correct and incorrect answers so as to discourage guesswork or copying.

Productive disposition

Productive disposition is the learner's habitual inclination to see mathematics as sensible, useful, worthwhile, coupled with a belief in diligence in one's own efficacy. When learners see themselves as capable of learning mathematics and using it to solve problems, they become able to develop and further their procedural fluency or their adaptive reasoning abilities. For example, learners can assist their parents to calculate the number and price of tiles required to cover the floor of their kitchen or any room in their homes using the formula for calculating the area of a square or rectangle. Learners' disposition toward mathematics is a major factor in determining their educational success (NRC, 2009:131).

2.3 Some challenges teachers encounter with the language of mathematics in multilingual classrooms

According to Barwell (2008), teaching mathematics in multilingual classrooms is challenging and very complex because learners in these classrooms bring with them a wide range of languages, proficiencies, experiences, and expectations. These difficulties are discussed below.

2.3.1 Differences between English and learners' home language

The difficulties identified for English language learners (ELL) and English second language (ESL) learners, found in other countries, are also experienced by our learners in mathematics classrooms in South Africa. In fact the difficulties are many since English is different from the other ten official languages in South Africa, in terms of its spelling, pronunciation, syntax, semantics, and origin. For example, in Setswana, one of the 11 official languages in South Africa, the numbers 1 to 10 which pupils memorise in their first language (with the assistance of their parents and older siblings) before beginning pre-school, have no relationship whatsoever in meaning with their English counterparts. Since fingers of the two hands are used practically for counting these numbers, numbers 6 to 9 in Setswana, Sepedi, Sesotho, and other Nguni languages, are simply explanations of what happens when one continues counting from the left hand to the right hand, starting with the index finger as shown in the table below.

Table 2-1: Meanings of numbers 6 to 9 in learners' home language in South Africa

	English	Setswana	Sepedi	Ndebele	Pictures
Number	6	tshela/tshelela	Tshela	sitfupha	
Meaning	six	cross (to the right hand)	jump (to the right hand)	index	
Number	7	supa	supa	isikhombisa	
Meaning	seven	point	point	the one that points	
Number	8	robedi	seswai	yisishagalombili	
Meaning	eight	bend two (fingers)	ticking or drawing finger	leave out two (fingers)	
Number	9	robonngwe	senyane	yisishagalolunnye	
Meaning	nine	bend one (finger)	bend the little (finger)	leave out one (finger)	

As table 2-1 above indicates, the same alternative meaning of the numbers 6 and 7 is also found in Sesotho, Sepedi and Setswana languages. Similarly, the meaning of numbers 7 to 9 is found in Setswana, Sotho, Sepedi, Ndebele, Zulu, Swazi. As a result, the meanings ascribed to the numbers 6 to 9 make it is easier for learners speaking these languages to learn these numbers as they relate the numbers to the actions they perform when counting using their fingers. The last column shows the pictures of the meanings of these numbers.

Similarly, Kazima (in Barwell *et al*., 2007:116) reports that the findings of her research on Malawian learners' familiarity with probability terminology show that learners' understanding of words such as *certain*, *likely*, *unlikely*, and *impossible* could not be aligned with accepted mathematical meanings in English because learners bring a wide range of meanings to these words.

On the other hand, Clarkson's paper on multilingualism in mathematics classrooms claims that learners who are proficient in two languages are likely to be more successful in mathematics than learners who are highly proficient in just one language. This finding, according to Barwell *et al*., (2007:116), is in line with Cummins' (1989) threshold hypothesis. In explaining this finding, Clarkson suggests that switching between languages has a metacognitive function as it gives learners access to additional or alternative meanings and relationships when solving mathematics problems.

From the above examples, one can say that right from the beginning in the lower grade classes, ESL learners are faced with a variety of language related problems in mathematics classrooms. They have lessons and questions based on vocabulary, terminology, and other types of languages in mathematical discourse. As such, they end up memorising words that do not relate to the meanings they bring from their first languages. However, teachers should not dismiss learners' first language as not useful, they should consider it and know how to apply it for learners to understand mathematical language.

2.3.2 Finding a balance between focusing on mathematics and language when teaching ESL learners

When ESL learners take part in class discussions in mathematics classrooms, they make language-related mistakes in addition to those that relate to their understanding of mathematical language. Teachers are then confronted with the dilemma as to whether they should intervene or not intervene by correcting linguistic structures. However, Barwell (2008:3) cautions teachers to intervene only if the intervention will not shift learners' attention away from the mathematics they are grappling with.

Similarly, Adler (cited in Gaoshubelwe, 2011:27) argues that when language is used to clarify mathematics, it is invisible, but when attention is also paid to the use of the correct terminology and phrases, the meaning of words and the correct syntax becomes visible. She therefore cautions that when language is too visible, the learner can lose track of the mathematical argument while concentrating on language features, which may impede understanding.

2.3.3 Lack of content-specific pedagogical preparation to work with ELLs

In a survey of 5300 teachers in California, many teachers identified several challenges they experienced in teaching mathematics to ELLs, and these are summarised below.

- *Difficulty in communicating with learners and parents.* The teachers were unable to engage and discuss the learners' progress with their parents as they could not rely on parents to assist with homework.

- *Insufficient time to teach both subject matter and language.* The learners who enrolled during the latter years at high school did not have sufficient time to meet graduation requirements.

- *Variable academic levels among ELLs in their classrooms.* In any given classroom, ELLs and also ESL learners, with various levels of English language proficiency, cultural experience, and subject matter knowledge are found.

- *Lack of resources* (e.g. *ELL-friendly textbooks or assessments*). Teachers had to rely on the same materials and assessments that they used with their English speaking students and deemed these materials inappropriate for ELLs. These teachers cited a lack of appropriate "tools and materials" and lack of adequate support from educational policies.

- *Insufficient in-service training.* Forty-three percent of the teachers whose classes were composed of 50 percent or more of ELLs reported they had received no more than one in-service professional development session that addressed instruction of ELLs during the five years prior to completing the survey. In addition, those who had participated in professional development programmes reported that they found them inadequate.

- *Insufficient support.* Teachers expressed the need for additional support. Secondary teachers, in particular, wanted more opportunities for teacher collaboration, better materials, and more paraprofessional help.

- *Low percentage of in-*service *time that could be devoted to instruction of ELLS* (Kersaint *et al*., 2009: 58-59).

The next section discusses ways in which teachers can address some of these challenges to assist learners to understand mathematics.

2.4 Addressing challenges encountered in teaching the language of mathematics

According to Barwell (2008), research tells us that many ESL learners quickly develop a basic level of "conversational" English, but it takes several years to develop more specialised "academic" English in mathematics classrooms. This is also confirmed by Cummins (1999), who states that conversational English, referred to as Basic Interpersonal Communication Skills (BICS) in SLA, takes 2 years to be developed by ESL and ELL learners, but it takes 5 to 10 years to develop academic language, referred to as Cognitive Academic Language Proficiency (CALP). To develop learners' mathematical register or academic language, teachers themselves should actively use mathematical language during teaching. To develop learners' CALP, teachers are advised to do the following:

2.4.1 Be aware of learners' linguistic needs related to mathematics

When teachers are aware of learners' linguistic needs related to mathematics, they can come up with strategies to address these needs so that successful mathematics learning can take place. For example, in a study conducted by Charoula Stathopoulou and Fragiskos Kalabasis to explore some of the mathematical experiences and worldviews of Roma high school learners in Greece to compare them with their Greek peers, the findings showed that Roma learners were proficient at oral calculation methods at an early age, but were less familiar with the use of written mathematics (Barwell *et al*., 2007). That experience led to a conflict when these learners, a semi-nomadic minority within Greece, participated in formal (Greek) schooling (Barwell *et al*., 2007:117). When teachers are aware of their learners' linguistic demands, they can organise their lessons in such a way that such needs are attended to and fully addressed (Tipps *et al*. cited in Suliman, 2014: 35).

2.4.2 Highlight and discuss aspects of mathematical English with learners

The mathematics teacher should discuss mistakes in mathematical tasks with non-proficient learners to determine whether errors of reasoning or calculation may be caused by a lack in learners' understanding of the mathematical language involved in the task at hand. Sometimes it is necessary to go back to informal language to link the concept to the formal terminology (Gaoshubelwe, 2011:27).

2.4.3 Promote ESL learners' mathematical meaning-making by problem solving, problem posing, and opportunities for discussion

In a research conducted in the US by Khisty on two different English-Spanish bilingual, second grade classrooms, the findings showed that in the classroom where opportunities for discussion were provided for learners to negotiate mathematics with discussion, learners were able to explain their ideas and draw on previous experiences to make sense of new situations, and that led to learners making mathematical meaning for themselves by interacting with both the teacher and other learners (Barwell, 2008:3). Mathematics teachers are therefore advised to design activities that allow learners to bring their experiences and interests to mathematics. For example, in word problems teachers can use the aspect of money as a bridge to understanding mathematics, a common strategy used in many indigenous classrooms (Warren & Young, 2007:779).

In addition, the ability of learners to express themselves in the language of mathematics is a key aspect of learning for conceptual development. Mathematical classrooms that are highly interactive with frequent discussions and collaborative problem solving and inquiring activities, are more likely to encourage mathematical language development of the learners. Interactive discussions enable teachers to model and support the use of precise language and mathematical terms, but also provide opportunities to draw on everyday language (Gaoshubelwe, 2011:30).

2.4.4 Reformulate learners' incorrect word use

Mathematics teachers can teach mathematical language invisibly by modelling the correct terminology and syntax. The teacher can reformulate learners' incorrect words, for example, "corner to corner line" by the correct mathematical term *diagonal,* which is a line segment from one vertex of, for example, a rectangle to the opposite vertex. The teacher should ensure that the learners understand the mathematical term and how to explain concepts and relationships in the correct mathematical language (Gaoshubelwe, 2011:28).

2.4.5 Decode the terminology

The mathematics teacher can decode the terminology by exploring the origin and the historical background of words to help learners gain a firm grasp of the concept itself. When the learners understand the historical roots of certain words or if the word structure is analysed, it often helps them to understand the concept. For example, the origin of the word *isosceles* is Greek and 'isosceles' means 'with equal sides', being made up of "iso" and "skelos", which means 'equal' and 'legs' respectively. In other words, an isosceles triangle has two equal sides (Gaoshubelwe, 2011:28). In addition, teachers should also make use of oral strategies such as, explaining homophones that are confusing to the learners, such as, *pi/pie*, *plane/plain*, *rows/rose*, *sine/sign*, and also model the language and vocabulary they expect learners to use (Biro *et al*, 2005:3).

2.4.6 Maintain an open classroom climate

There are different types of classrooms, like the traditional classrooms where the teacher has a direct teaching approach, and open classrooms, where learners participate freely in classroom activities. The learners in an open classroom are free to communicate among themselves and to ask the teacher for assistance, and the teacher engages through discussion. The classroom climate affects not only how much is learned, but how long learning lasts, and how much future learning there is likely to be. The classroom climate impacts on the development of the learners' mathematical language (Gaoshubelwe, 2011: 30).

Teachers should not only teach learners to understand mathematical language, they should also teach them to be mathematically proficient and thus be able to communicate their ideas mathematically during classroom discussions.

2.5 How to achieve mathematical proficiency

Mathematics is a language that is replete with signs and symbols. In Vygotsky's (cited in Cottrill, 2003:3) view, this makes it the responsibility of the teacher to convey to the learners the "relationship that exists between the signs and the meaning of the signs". Below are the perspectives and conditions for learners to achieve mathematical proficiency to be able to communicate their ideas mathematically and thus develop mathematical proficiency.

Moschkovich (2002) proposes the following three perspectives for bilingual and ESL learners to 'communicate mathematically', both orally and in writing, and to participate in mathematical practices, such as, explaining solution processes, describing conjectures, proving conclusions, and presenting arguments (Moskovich, 2002:190).

2.5.1 Acquiring vocabulary

For ESL learners to communicate mathematically, they should acquire vocabulary, usually referred to as mathematical discourse. Acquiring vocabulary is emphasised in learning mathematics as it is the "central issue that second language learners are grappling with when learning mathematics" (2002:192). Learners can only communicate mathematically if they have acquired the vocabulary, which comprises the different types of languages discussed. However, in today's mathematics classrooms, acquiring vocabulary is not the only emphasis as it was the case in classrooms in which traditional approaches were practised, so learners have to construct meanings of the vocabulary acquired.

2.5.2 Constructing meanings

The second perspective describes mathematics learning as constructing multiple meanings for words rather than acquiring a list of words. Learning mathematics, therefore, involves a shift from everyday terms to more mathematical and precise meanings, referred to as 'mathematical register' (Moschkovich, 2002:194). To construct meaning, everyday meanings and learners' home language can also be used by the learners as resources to communicate mathematically for them to become proficient in using mathematical formulations.

For example, in lessons based on the graphs plotted along the *y*- and *x-axes*, the graph of $y = 0$ can be understood as the ground level for learners to construct meaning of the x-axes, and thus be in a position to explain why the graph of $y = 0.2x$ is less or more steep than the graph of $y = x$. Therefore, according to Moschkovich (2002:196), “instruction in mathematical communication needs to consider not only the obstacles that bilingual learners face, but also the resources these learners use to communicate mathematically”.

2.5.3 Participating in discourse

From this perspective, learning to communicate mathematically involves more than learning vocabulary or understanding meanings in different registers, and according to Moschkovich (2002:197), it is seen as “using social, linguistic, and material resources to participate in mathematical practices”. In other words, learners can use gestures or drawings, for example, to illustrate a rectangle. Objects like the teacher’s table or duster, can be measured and used to calculate the perimeter of a rectangle, and descriptions of a pattern can be used, for example, to explain steps followed to find solutions to problems based on *Solving for x* in equations, using phrases like, *what you do on the left-hand side should also be done on the right-hand side of the equation*.

2.6 Conclusion

Mathematics educators are cautioned to pay more attention to language learning because firstly, language learning is often an expected outcome of mathematics education, and secondly, there is evidence that language learning and mathematics learning are intimately related (Barwell, 2010:112). In fact, the academic language involved in mathematics has been referred to as a third language for ELL/ESL learners since research has shown that native English-speaking learners learning academic language faced many of the same challenges as those of learners learning ESL, and as a result, they should be paired during group work activities (Biro *et al*., 2005:3). Chapter three therefore discusses the relationship between mathematics and ESL in as far as their teaching and learning are concerned.

CHAPTER THREE: LEARNING LANGUAGE AND MATHEMATICS

3.1 Introduction

Learning a language is not a separate process that has no impact on mathematics learning (Barwell, 2010:112). In other words, the learning of mathematics in multilingual classrooms depends to a large extent on the acquisition of English as a second language. This interdependency on the learning of English as a second language and mathematics has resulted in some researchers defining mathematics as a language (Garrison & Mora, 1999:46). This chapter as a result discusses the conditions for and the theories on the teaching of English as a second language (TESL) and mathematics (MT), as well as those of English SLA and mathematics learning (ML). The roles of input, language processing and interaction, output and feedback in teaching and learning in as far as ESL, English SLA and mathematics are concerned, are discussed. The theories on TESL and mathematics teaching (MT), and English SLA and mathematics learning (ML) are visually represented based on a synthesis.

3.2 The role of language in mathematics teaching and learning

According to Barwell (2008:2), mathematics is about relationships between numbers, categories, formulae, geometric forms, symbols, variables, etc., and these relationships, which are abstract in nature, require language to express them. Therefore, mathematics does require language. In fact, it depends on language to be understood by learners as it is shown in the different types of languages discussed in chapter two.

According to Thompson and Rubenstein (2000:568), language plays at least three crucial roles in our classrooms, namely:

- We teach through the medium of language. It is our major means of communication;
- Learners build understanding as they process ideas through language; and
- We diagnose and assess learners' understanding by listening to their oral communication

  and by reading their mathematical writings.

The important role of language in mathematics learning is succinctly captured below:

Language is the cement that allows us to build upon prior knowledge learning.

If language is weak, so too is the ability to learn. (Harrison, 2014:12).

The above quotation implies that the role of language in mathematics determines learners' success or failure in as far as learning mathematics is concerned. It is, therefore, not surprising that language has also been considered as one of the contributing factors that hinder mathematics teaching and learning for ESL and bilingual learners in some of the factors discussed.

3.3 Some factors that hinder mathematics teaching and learning

There are many factors, inside and outside mathematics classrooms, which affect learners' performance in mathematics. The 2011 TIMSS (Trends in International Mathematics and Science Study) and PIRLS (Progress in International Reading Literacy Study) (Mullis *et al*., 2011) revealed factors, such as, learners not speaking the medium of instruction at home, among other things. The results show that only 26% of learners always speak English at home. These factors relate to English, the medium of instruction (MoI), and also the language of learning and teaching (LoLT).

3.3.1 English as a Medium of Instruction (MoI)

The problems regarding our learners' failure to do well in mathematics, according to Fleisch (2008), can be attributed to the "straight-for-English policies and early exit from mother-tongue" in primary schools in South Africa, where the majority of learners actually speak English as a second or third language. In fact, Fleisch (2008:98) asserts that less than one South African child in ten speaks English as their first language, and that is a very small number indeed, as confirmed in Howie (2003). In addition, the policies for changing the medium of instruction not only affect learners, but also most of the teachers who also speak English as their second or third language, and, as a result, experience language-related obstacles that hinder learners' performance in mathematics[1].

In studies conducted in Malaysia (Yahaya, *et al*., 2009), the United States of America (Echevaria, 2010; Khisty & Chval, 2002), and in South Africa (NSC, 2009), after the policy to change the medium of instruction from their native languages to English was implemented, the results showed that teachers and learners had a number of problems that affected learners' performance in mathematics. Some of the teachers and learners' findings are summarised.

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[1] It should be noted that English as a second language (SL) in South Africa, is not necessarily a second language, as it might be a third, fourth or fifth language – that is why in our curriculum it is referred to as an additional language (AL).

3.3.1.1 Using L1 to teach mathematics

The results of the different studies showed that teachers resorted to teaching mathematics in a mixture of both mother tongue and English (Yahaya *et al*., 2009:142). The use of mother tongue, according to Hong (cited in Yahaya *et al*., 2009), will not only affect the learners' language development, but also discourages the learners from using the target language. The teachers also maintained that learners' low proficiency in English was the main cause for using their mother tongue in mathematics classrooms.

3.3.1.2 ESL teachers not fluent in English

The study in Malaysia found that as a result of using Malay due to their being not fluent in English, the teachers encountered problems when explaining concepts to learners as they lacked the necessary skills to teach English. This was confirmed by 85.2% of the respondents, and 81.8% admitted using Bahasa Melayu, one of the Malaysian indigenous languages that is used as a first language (L1), to give explanations when faced with a breakdown in communication when using English. Similarly, in South Africa, English as a MoI affects most of the teachers who do not have English as their home or first language (Setati *et al*., 2008:14).

3.3.1.3 Materials provided are for proficient English speakers

In an attempt to empower teachers, the Ministry of Malaysia provided them with self-learning materials, such as, multimedia courseware and grammar books. However, the respondents interviewed claimed that the materials were more suitable for teachers and learners who are proficient in English as they had trouble following the content presented because of language difficulties (Yahaya *et al*., 2009: 144). Similarly in South Africa, according to Howie (2003:14), 50% of the population live in rural areas, where there are limited resources and facilities, and a large percentage of under-qualified teachers experience problems with materials that are aimed at proficient English speakers.

3.3.1.4 Vocabulary teaching in mathematics classrooms

In a study that observed 23 ethnically diverse classrooms in the USA, researchers found that in the core academic subject areas, teachers only spent 1.4 % of instructional time on developing vocabulary knowledge (Scott & Noel, cited in Taylor, 2009:7). The reasons for this could be similar to obstacles mentioned by the respondents in the Malaysian study, who indicated that they were unable to use self-learning materials such as multimedia courseware and grammar books provided by the Ministry due to a lack of time. As a result, learners are not sufficiently taught mathematical discourse, which, if taught, would assist learners to understand lessons in mathematics classrooms.

3.3.1.5 Traditional teaching approaches

Other difficulties ESL learners experience relate to the traditional teaching approaches used by teachers, such as teacher-centred learning in mathematics classrooms, which result in learners losing confidence in the subject, as pointed out by Thijse (cited in Jagals & van der Walt, 2013:9). In such traditional settings, learners sit passively in class, listening to teacher-talk, and that does not encourage engagement in the learning of the subject (Taylor, 2009:7). English being their second, third, fourth or additional language (in the S A context), makes it difficult and almost impossible for them to come up with strategies and activities that would make learners enjoy the subject.

3.3.1.6 English as the Language of Learning and Teaching (LoLT)

The problem of learners' poor performance in learning mathematics is related to English, the language of learning and teaching in most of our South African schools. This problem is not limited to South Africa's ESL learners only. In the United States of America (USA), where mathematics is also taught using English as the LoLT, learners experienced problems understanding the content as well as the questions asked by the teachers. In a study by Khisty and Chval (2002:156) conducted on Latino learners in Illinois, researchers found that learners in mathematics classrooms had to first acquire the language of instruction, English, and then the language of mathematics, which is different from the language of conversation. For example, in the expression $y = f(x)$, where f stands for the word "function", the word is different from the normal everyday meaning of the word 'function', and that creates an additional burden for the learners who have to first battle with understanding the language of instruction used in questions as well as the complex mathematical discourse, before comprehending mathematical concepts, generalisations and thought processes.

3.3.1.7 Questions used in examination papers

The issue of English, the MoI and LoLT, as a factor for learners' poor performance in mathematics, was also pointed out in the Mathematics Examiner's report on the candidates' performance in the November 2009 Mathematics examination Paper 1 consisting mostly of questions based on algebra, financial mathematics, equations and functions. Further comments on learners' problems with the language of instruction are found mostly with regard to Mathematics examination Paper 2 which consists mostly of questions based on geometry and trigonometry (NSC, 2009).

Similarly, the same problem of questions used in examination papers was pointed out in a study described in chapter 2 which identified the problem of LoLT for learners' poor performance in mathematics as a result of the types of questions used (Abedi & Lord, 2010:222).

The difficulties indicated above are experienced by all ELL and ESL learners when they are faced with test and examination questions phrased in English. The difficulties are a result of the language of mathematics being "limited largely to school" (Thompson & Rubenstein,2000:568), unlike the common English words that learners can hear, read on a daily basis from newspapers, television programmes, and other media sources respectively. This is also confirmed in Biro *et al*. (2005:1) where it is stated that learners do not pick up academic English subconsciously by talking to their friends, because it is not used in casual conversations.

The examples given in the examiners' reports include some of the types of questions learners did not understand due to the language and the vocabulary used in mathematics examination question papers. The problem could be that teachers do not use the types of questions that improve learners' comprehension of mathematical problems in their classrooms. The imperatives or commands used give learners instructions on what to do with regard to the problems given, without creating opportunities for learners to use and acquire English in the process. Probing imperatives like *Explain how you got the answer* are used to a limited extent. For example, in the Grade 11 classroom mathematics prescribed book, the imperative *Explain your answer*, appears once in chapters one to seven (Laridon *et al*., 2006:165).

3.3.1.8 Teachers' low expectations of ESL learners

According to Lee (cited in Taylor, 2009:7), mathematics teachers are not motivated to know their ESL learners, their culture, or their families, and as a result, "learners' poor performance is not only accepted, but expected". Biro *et al*, (2005:18), advise mathematics teachers to respect ELL and English SLA learners' customs by incorporating their languages and cultures within the classrooms, and communicate a message that says individual identities are valued. For example, learners may come from a culture where it is inappropriate for learners to express their opinion supported by evidence of thinking as expected in mathematics classrooms. Also, it may be inappropriate for some ESL learners to refer to the teacher using the second personal pronoun '*You'* but instead say *'the teacher',* as it is the case with learners speaking Setswana as a first language, thus confusing the teacher and other learners who do not understand Setswana and other cultures in the process. Therefore, teachers should adjust their teaching strategies to respect learners' customs.

The problems discussed with regard to the teaching and learning of mathematics provide a glimpse of what mathematics teaching and learning is, hence the next section elaborates on the parallels between teaching mathematics (MT) and ESL (TESL); and mathematics learning (ML) and English SLA.

3.4 Teaching and learning in both mathematics and ESL classrooms

Since mathematics has been defined as a second language, Garrison and Mora (1999) in their study on Latino learners state that the learning of mathematics is similar to that of learning a second language. As a result, they recommend the use of Krashen's formula (*i + 1*) on comprehensible input in the teaching and learning of mathematics. Krashen's input hypothesis on second language acquisition claims that "an important condition for second language acquisition to occur is that the acquirer understands (via hearing and reading) input language that contains structure 'a bit beyond' his or her current level of competence (Brown, 2007:295). This formula is used in the theories on and principles for English SLA and mathematics learning with regard to the roles of input, together with teacher strategies applied, as explained in the next sections. The formula (*i + 1*) is recommended because it provides comprehensible input for English language learners and the teacher strategies used to build mathematical concepts that are parallel in their potential for maximising learning, as they both work on the principle of teaching the unknown from the known.

3.4.1 Parallels in teaching mathematics and ESL (TESL)

In the 1960s, mathematics education in most parts of the world and in the Netherlands was dominated by a mechanistic teaching approach (Van den Heuwel-Panhuizen & Drijvers, 2014:521). This means that learners sat passively in mathematics classrooms while teachers demonstrated how problems are solved. Also, teachers asked closed questions that were followed up by learners' answers and teacher's feedback, engaging learners in the Initiate- Response - Evaluate (IRE) discourse in mathematics classrooms.

Similarly, the mechanistic teaching approach with regard to the audio-lingual method, emphasising the spoken language, became popular in the middle of the 20th century. It involved a systematic presentation of the structures of the second language, moving from simple to more complex, in the form of drills that learners had to repeat. It was influenced by a belief that the fluent use of a language was essentially a set of "habits" that could be developed with much practice. Much of this practice involved hours spent in the language laboratory repeating oral drills (Yule, 2010:190).)

Later on, Hans Freudenthal (1905-1990), a mathematician born in Germany, became interested in mathematics education and propagated a method of teaching mathematics that is relevant for learners. His method included carrying out thought experiments to investigate how learners can be offered opportunities for guided re-invention of mathematics, and in this way, contributed to the development of the Realistic Mathematics Education (RME) theory (Van den Heuwel-Panhuizen & Drijvers, 2014:522), which is similar to Task-based Instruction (TBI).

Realistic Mathematics Education (RME)

The main characteristics of RME are problematisation, construction, and reflection. The teacher is the activator in the process of problematisation and the tutor in the process of construction, taking learners' informal strategies as a starting point for the interactional development of mathematical concepts and insights (Van Eerde *et al*., 2008: 33).

Similarly, in recent years contemporary language teaching has moved away from dogmatic practices of 'right' or 'wrong', becoming much more eclectic in its attitudes, and more willing to recognise the potential merits of a wide variety of methods and approaches. As a result, the interest in the contribution of the learners in the teaching/learning dichotomy was resurrected, accommodating the learning strategies that learners employ in the process of language learning (Griffiths & Parr, 2001:248).

In Task-Based Instruction, according to Powers (2008:73), teachers prepare lessons that are constructed according to the language required to perform specific tasks. This means that learners will learn language structures through induction as they focus on task completion and meaning. Their interaction during the tasks facilitates transfer of information they have previously learned and incorporates it with new information they receive as they perform the task.

RME involves six core principles for teaching mathematics (Van Den Heuvel-Panhuizen & Drijvers, 2014:523), and these are compared with those applied to the teaching of the English as a second language.

The Activity principle

The activity principle emphasises that learners should be treated as "active participants in the learning process since mathematics is best learned by doing mathematics". This is strongly reflected in Freudenthal's interpretation of mathematics as a 'human activity' (Van Den Heuvel-Panhuizen & Drijvers, 2014:523). Learners should not be passive listeners but active participants in mathematics classrooms, and this can be achieved if learners are taught learning strategies such as metacognition to think about their learning process so as to make input comprehensible.

Similarly, TBI is based on Krashen's language acquisition hypothesis. Krahnke (1987) cited in Powers (2008:73) explains that the theory asserts the ability to use language is gained through exposure to and participation in using it, thus discouraging learners from being passive but active participants in the learning situation. Krahnke (1987) goes on to explain that TBI develops 'communicative competence' including linguistic, sociolinguistic, discourse and strategic competence. Thus, processing the information used during specific tasks through understandable input provides students with linguistic and sociolinguistic competence (Powers, 2008: 73).

The Reality principle

The reality principle can be recognised in RME in two ways. Firstly, it expresses the importance that is attached to the goal of mathematics education, including learners' ability to apply mathematics in solving 'real-life' problems. Secondly, it stresses the point that mathematics education should start from problem situations that are meaningful to learners and that offer them opportunities to attach meaning to the mathematical constructs they develop when solving problems (Van Den Heuvel-Panhuizen & Drijvers, 2014:523). In other words, learners should not be provided with abstract concepts that they do not understand, but with rich concepts that can be mathematised and put learners on the track of informal context-related solution strategies as a first step in the learning process.

Likewise, in task-based instruction (TBI), as pointed out in Ellis (2006:102), a 'focus on form' approach is valid as long as it includes an opportunity for learners to practise behaviour in communicative tasks, thus providing learners with opportunities also to apply mathematics in solving 'real-life' problems. The grammar taught emphasises not just form, but also the meanings and uses of different grammatical structures. As Krahnke (1987) cited in Powers (2008:73) points out, connecting tasks to real-life situations contextualises language in a meaningful way and provides large amounts of input and feedback to assist learners in the learning process.

The Level principle

The level principle underlines that learning mathematics means that learners pass various levels of understanding: from informal context-related solutions, through creating various levels of shortcuts and schematisations, to acquiring insight into how concepts and strategies are related. Particularly for teaching operating with numbers, this level principle is reflected in the didactical method of ‘progressive schematisation’ where transparent whole number methods of calculation gradually evolve into digit-based algorithms (Van Den Heuvel-Panhuizen & Drijvers, 2014:523).

Similarly, Long’s interaction hypothesis explains in detail how input is made comprehensible, therefore picking up where Krashen left off. Michel Long (1996, 1985) in his Interaction hypothesis posits that comprehensible input is the result of modified interaction (Brown, 2007:305). As a result, he accommodates various types of interactions, such as clarification requests, paraphrases, comprehension checks, etc., for learners to interact and process the language.

The Intertwinement principle

The intertwinement principle means mathematical content domains such as number, geometry, measurement, and data handling are not considered as isolated curriculum chapters, but as heavily integrated. Learners are offered rich problems in which they can use various mathematical tools and knowledge. This principle also applies within domains. For example, within the domain of angles, triangles, sines and cosines, quadrilaterals are taught in close connection with each other. In other words, different sections of mathematics should not be taught in isolation, but as a unit showing relationships between one another so as to make sense to the learners (Van Den Heuvel-Panhuizen & Drijvers, 2014:523).

Similarly, content-based instruction (CBI) is defined by Brinton *et al.* (1989) as ‘the concurrent study of language and subject matter, with the form and sequence of language presentation dictated by content material’. In other words, content and language are not taught in isolation. CBI can take place at all educational levels, and it refers to total immersion (approximately 90% of school time in the second language) or it can refer to content-based themes in language classes (Cenoz, 2014:19).

The Interactivity principle

The interactivity principle signifies that learning mathematics is not only an individual activity, but also a social activity. It encourages teachers to make full use of group work and whole-class discussions to provide learners with opportunities to share ideas and strategies on how they solve mathematical problems, and in this way, produce output. As learners share their strategies, they evoke reflection, which enables them to reach a higher level of understanding (Van Den Heuvel-Panhuizen & Drijvers, 2014:523).

Similarly, proponents of TBI (Nuna, 1989; Willis, 1996; Skehan, 1998; Ellis, 2003) agree that communicative activities used during pair- and group-work are appropriate vehicles; and that language learning activities should directly reflect what learners 'potentially or actually need to do with the target language' (Swan, 2005:377). Also, the role of the teacher in TBI classrooms is to supply task-related vocabulary where necessary, offering recasts, or acting as interlocutors, casting the teacher's role as a manager and facilitator of communicative activity rather than an important source of new language (Swan, 2005:391).

The Guidance principle

The guidance principle refers to Freudenthal's idea of "guided re-invention" of mathematics. It implies that in RME teachers should have a proactive role in learners' learning and that educational programmes should contain scenarios that have the potential to work as a lever to reach shifts in learners' understanding. To realise this, the teaching and the programmes should be based on coherent long-term teaching-learning trajectories (Van Den Heuvel-Panhuizen & Drijvers, 2014:523). According to Gravemeijer (2009:114), the principle means that learners should be provided with the opportunity to experience a process similar to the process by which a given piece of mathematics was invented. For example, a question like *Find the sum of 49 + 58*, should not require learners to crack their heads with adding the two numbers, but they should simply think of the internalised method of factorising common factors by applying the Distributive Law taught in class, and thus group like and unlike terms by expanding *49* into *40 + 9* and *58* into *50 + 8,* and finding the sum of like terms *40 + 50 = 90*, and *that of units 9 + 8 = 17* to get the answer *107*. In doing so, the learners would be outsourcing guidance or scaffolding by remembering what their teachers taught them before and also what they learned from mathematics textbooks. The role of the teacher in this case is to provide learners with the skills that are required to perform the tasks in the example given before giving them this exercise, thus providing them with scaffolding.

Similarly in ESL classrooms, Vygotsky's theory on Zone of Proximal Development (ZPD) stresses the fact that learners acquire language in the social world, and as a result, individuals learn best when working together with others during joint collaboration, and it is through such collaborative endeavours with more skilled persons that learners learn and internalise new concepts, psychological tools, and skills (Shabani *et al*., 2010:237). Likewise, Ellis (2006:102) believes that corrective feedback is important for learning grammar; and that it is best conducted using a mixture of implicit and explicit feedback types that are both input-based and output-based. To provide scaffolding, teachers can provide learners with the vocabulary that is required to perform a particular task, e.g. a visit to a doctor, by using a picture of a human skeleton with labels for learners to be able to use the vocabulary learnt to explain to the doctor what their problem is. Also fill-in-gap conversations can be used for learners to practise as they simulate the task.

RME furthermore led to new approaches to assessment in mathematics education process (Van Den Heuvel-Panhuizen & Drijvers, 2014: 524). Learners, for example, do not necessarily have to follow the same method of solving problems taught in the classroom, but are free to share the variety of methods they used to solve a particular problem. This provides them with opportunities to think and communicate mathematically during the whole assessment process. They apply the skills of writing and speaking during the process of learning the subject, therefore applying the skills that were neglected in the traditional, mechanistic way of learning mathematics as explained in the integrated and open-ended approach of language and mathematics teaching and learning.

Similarly, assessment in TBI involves evaluating the degree to which language learners can use their second language (L 2) to accomplish given tasks. A well-designed task and implemented assessment can also provide teachers and language learners with a detailed account of task performance that can inform future task-based instruction and L 2 development (Weaver, 2012: 287). It involved summative assessment, using tests for learners' competence in L 2; and formative assessment, using tasks to provide feedback to learners and teachers (Weaver, 2012:288).

The progression in the teaching approaches used in mathematics and ESL classrooms is illustrated in figure 3-1 below.

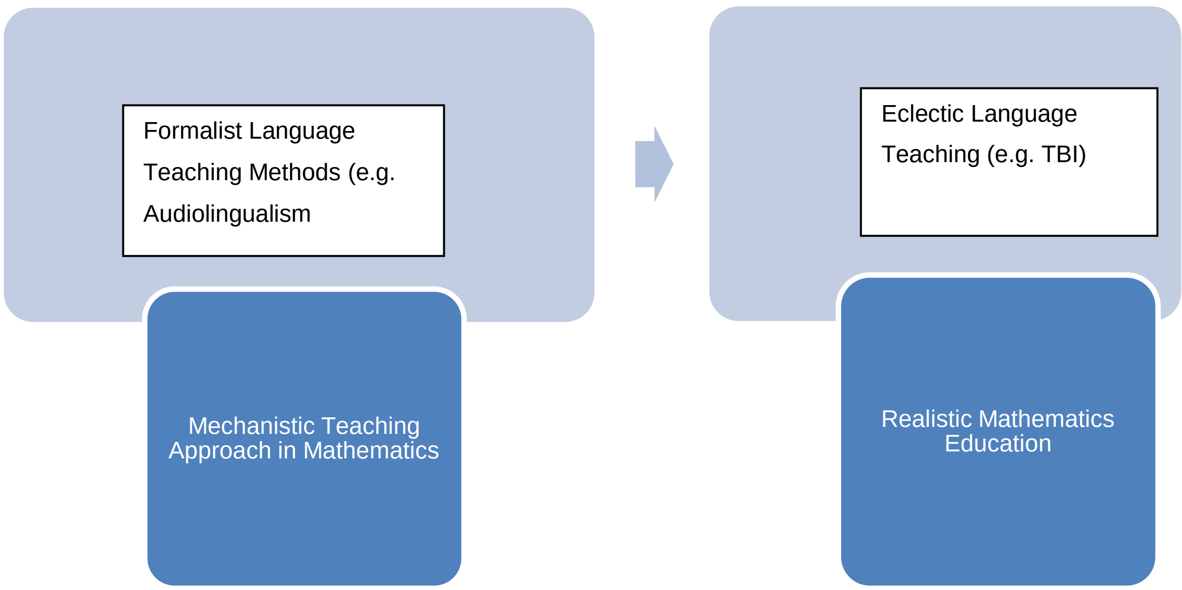


Figure 3-1: The order of the teaching approaches used in mathematics and ESL classrooms.

Mathematics and ESL teaching will be successful to assist teachers in achieving the intended outcomes discussed if the resources used in the classrooms relate to what is assessed as explained below.

Developing an integrated approach of language and mathematics teaching

Van Eerde and Hajer (2005: 1217), in their quest to promote interaction and to support teachers and learners in multi-ethnic mathematics classrooms at a Dutch secondary school, recommended instructional materials that can mediate integrated processes of learning language and mathematics, such as the Wisbaak, to make contexts in mathematics assignments accessible; to stimulate interaction as well as language production in mathematics classes; and to provide opportunities for teachers to give feedback on language (form) and mathematics (content). The designed lessons and tasks (which include lessons on *Promoting oral participation*, *Writing assignments*, *Vocabulary tests*) used by the researchers also enabled the researchers and the teachers to diagnose learners' learning difficulties and find starting point to stimulate their learning processes (Van Eerde & Hajer, 2005:1223).

Similarly, proponents of the task-based instruction (TBI) agreed on the following principles (Swan, 2005:377):

- Instructed language learning should primarily involve 'natural' or 'naturalistic' language use, based on activities concerned with meaning rather than language.
- Instruction should favour learner-centredness rather than teacher-control.
- Opportunities for learners to 'focus on form' should be provided to draw learners' attention to linguistic elements as they arise.
- Communicative tasks should be used in TBI classrooms.
- More formal pre- and post-task language study may be useful to boost 'noticing' of formal features.

3.4.2 Parallels in English SLA and mathematics language learning

Sociocultural theory, according to Van Eerde *et al*. (2008:34), had a major influence on language acquisition and consequently on (second) language learning. In the context of this study this is applied to mathematics language learning as shown in the theories and conditions discussed in the next section.

Theories and conditions

According to Krashen's Input hypothesis, If an acquirer is at stage or level *i,* the input he or she understands should contain (*i + 1)*' (Brown, 2007:295). It goes without saying that learners get bored if they are taught what they already know, i.e. input at (*i + 0*).Therefore, teachers should challenge learners by providing them with input that is above their level of understanding, but that input should not be too easy for them, at level (*i + 0*) or far above their level of understanding at level (*i + 2*), otherwise it could discourage or overwhelm them and interfere with their language development process and their understanding of what is taught.

Similarly, the first condition for mathematics language development emphasises comprehensible input as indicated below.

- Firstly, the availability of rich, challenging comprehensive oral written language input is crucial (Van Eerde *et al.,* 2008: 34).

Input

Input, according to Gass and Mackey (2006:5), refers to "language that is available to the learner through any medium (listening, reading or gestural in the case of sign language)". Similarly Ellis, (1986: 294) defines input as the language (for both English as a second language and mathematical language) that learners are exposed to. He further explains that it is possible for the input provided by the teachers and interlocutors to be comprehensible (i.e. input that learners can understand) or incomprehensible (i.e. input that they cannot understand); and when it is incomprehensible, it becomes "the impetus for learners to recognise the inadequacy of their own rule system" (Gass *et al.*, 1998:301). Krashen's input hypothesis and the reality principle on RME elaborate on the role of input in as far as English SLA and mathematics learning are concerned.

The role of input

From the definitions on input, one can outline the roles of input in language acquisition and mathematics learning as follows:

- Firstly, input plays a very important role in as far as language learning and acquisition is concerned. It provides the data that the learner must use to determine the rules of the target language. In the same way, the researchers of Universal Grammar view input as a "trigger that interacts with an innate system and/or the native language to promote learning" (Gass & Mackey, 2006: 5). Therefore, input forms the positive evidence that learners use as they construct their second language and mathematics grammars. This role of input has therefore resulted in many researchers describing the type of input learners receive in ESL classrooms as *foreigner talk* (Ferguson, 1971), the language addressed to non-native speakers (Gass & Mackey, 2006:5).

- Secondly, input, in the form of grammar rules (for both English and Mathematics), information from mathematics text books, and knowledge from the language teachers and interlocutors, and also from the learners, provides the stepping stone for any form of learning to take place, so it is up to the teachers and interlocutors to decide what they should do with all the input that they have to make it comprehensible for the learners to learn and acquire the languages, i.e. ESL and mathematics.

- Lastly, Seliger (1983) in Brown (2007:298) explains how the role of input gives credit to learners for successful acquisition to take place. The findings in the study showed that learners referred to as High Input Generators [HIGs] maintained high levels of interaction in the second language, both in the classroom and outside, and progressed at a faster rate than learners who interacted little, referred to as Low Input Generators [LIGs]. This is also supported by Cummins (1986: 27) when he states that "a pattern of classroom interaction which promotes instructional dependence ... liberates learners to become active generators of their knowledge." Teachers, at the beginning of the lesson, write down the vocabulary and symbols on the board. They discuss the definitions (in mother tongue if necessary) and representations. The learners then have a reference to the meaning of the words/terminology as well as how to use it.

Even though Krashen in his input hypothesis does not credit the role of learners in as far as input is concerned, when he states that "comprehensible input is the only causative variable in SLA" (Brown, 2007:297), many researchers with an increasing interest in social constructivist analyses of language acquisition, focus on the characteristics of successful language learners. They have come up with the following learning strategies that successful learners apply with regard to input to acquire language, including mathematics language, by making it comprehensible, and thus crediting learners' role with regard to input.

Learning strategies

The learning strategies, according to Brown, (2007:134-135), include:

- Meta-cognitive strategies

  Metacognitive is a term used in information-processing theory to indicate an 'executive function', and it includes strategies that involve planning for learning, thinking about the learning process as it takes place, monitoring one's production, and evaluating learning after an activity has been completed, and this evaluation includes self-monitoring, self-evaluation, advance organisers and delayed production (Wahl, 1999). For example, the learner could draw up a weekly study plan for each section of the chapters taught in the previous week to evaluate their understanding of the chapters learnt. If the learners are not satisfied with the progress made, they could make an appointment with the teacher to explain the section that is not clearly comprehended or use the internet or peers for assistance to reach the relevant goals.

- Cognitive

  Cognitive strategies are more limited to specific learning tasks and involve more direct manipulation of the learning material itself, and these include repetition, resourcing, translation, grouping, note-taking, deduction and others. For example, after calculating the values of *x* in a given exercise on *Solving for the values of x*, the learners could substitute the values of x that they got as the answer in the given equation to check if the answer is equal to zero without waiting for the teacher to check if the answer is correct or not.

- Socio-affective

  Socio-affective strategies have to do with social-mediating activity and interacting with others, for example, cooperation and asking questions for clarification. These also relate to output. Learning can be constrained by learners' or teachers' belief systems about and attitudes towards mathematics and the nature of mathematics, and how it should be learned. These inform learners' decisions to avoid or embrace challenges; and these may influence the learners or teachers attributing failure or success to cognitive (in)abilities rather than to effort.

Language processing and interaction

Even if input is understood, according to Ellis (1986:159), it may not be processed by the learner's internal mechanisms. That is what Krashen means when he stated that comprehensible input is not a sufficient condition for SLA. It is only when input becomes intake that SLA takes place. Input is the L2 data that the learner hears; intake is that portion of the L2 which is assimilated and fed into the inter-language system. Intake, according to Brown (2007: 297), is the subset of all input that actually gets assigned to our long-term memory store.

The second condition for English SLA, puts emphasis on opportunities for language processing and interaction to be provided for learners to use language, and therefore acquire English and mathematics language in the process, as stated in Long's Interaction Hypothesis.

Long's Interaction Hypothesis

Michael Long (1985, 1996) posits that comprehensible input is the result of modified interaction (Brown, 2007:305).

The role of interaction

The important role of interaction is revealed in the study conducted by Wong-Fillmore (1983) on Hispanic learners in ESL classrooms, which showed that "learners learned more English in classrooms that provided opportunities for reciprocal interaction with teachers and peers" (Cummins, 1986:29). This reciprocal interaction can be achieved, according to Gass and Mackey (2006:7), when L2 learners are presented with incomprehensible input that they do not understand, as that will force them to 'negotiate meaning' by using confirmation checks, clarification requests, and comprehension checks, in order to change it into comprehensible input, thus making it the result of modified interaction (Brown, 2007:305).

- Negotiation is defined as the process during which, "in an effort to communicate, learners and competent speakers provide and interpret signals of their own and their interlocutor's perceived comprehension, thus provoking adjustments to linguistic form, conversational structure, message content, or all three, until an acceptable level of understanding is achieved" (Gass & Mackey, 2006: 4). The combination of input and interaction, using forms of negotiation, according to (Long, 1996; Gass, Mackey, &Pica 1998; Gass, 2003; Swain & Lapkin, 1995), makes the two (input and interaction) major players in the process of acquisition (Brown, 2007:305). This is also the case when English in ESL and mathematics classrooms is used constantly for conversing, learning new ideas, concepts and vocabulary, thinking creatively, and problem solving, as it gives learners an opportunity to develop their language skills (Cummins, 1986:31). Learners can develop their language skills by reflecting on what they know already to understand what is taught. Teachers should also apply a variety of interactions discussed (in mathematics and ESLA classrooms), to assist learners to process input and interact using language.

Long in his Interaction Hypothesis posits that comprehensible input is the result of modified interaction (Brown, 2007:305). Similarly, Ellis (2006:100) states that input-based feedback models the correct form for the learner (e.g. by means of a recast); and output-based feedback elicits production of the correct form from the learner (e.g. by means of a clarification request). This section as a result discusses the different types of interactions, referred to as modifications or negotiations, applied by teachers and interlocutors to make input comprehensible to the learners in ESL and mathematics classrooms.

1. Confirmation checks

A confirmation check is defined by Long (1983) as “any expression ... following an utterance by the interlocutor which are designed to elicit confirmation that the utterance has been correctly heard or understood by the speaker” (Gass & Mackey, 2006: 7). It can be used for learners to receive comprehensible input. For example, when a learner leaves blank spaces in their utterances for the interlocutor to fill in to provide the correct answers, this is followed by feedback on their production, which points out the differences in the gaps between their language production and the target language. Therefore, interaction in this case through this confirmation check, makes L2 learners identify their problem areas in as far as the target language is concerned, and this enables them to recognise that there is an error and what the correct form should be.

2. Clarification requests or paraphrases

A clarification request as defined by Long (1983:137) is “any expression ... designed to elicit clarification of the interlocutor’s preceding utterances” (Gass & Mackey, 2006:8). It can be applied to incomprehensible input by saying an incorrect utterance in a rising intonation, thus changing it into a *Yes* or *No* question, for the learner to reflect on the answer provided and come up with the correct utterance. For example, if the learner says *denominator*, instead of *numerator*, the teacher could say, *denominator* with a rising intonation, and that could result in the learner reflecting on the wrong answer and saying the correct answer, *numerator.*

3. Comprehension checks

A comprehension check as defined by Long (1983:136) is an attempt “to anticipate and prevent a breakdown in communication” (Gass & Mackey, 2006:8). It can be used also in the form of a *Yes* or *No* question by the teacher to check if the learner understands the meaning of one of the utterances spoken for communication to continue, for example, *Given the right-angled triangle ABC, with* $<A = 90^{0}$, *and AB = 3 cm, and BC= 4 cm, use Pythagoras’ theorem to find the length of the hypotenus*. *Do you know Pythagoras theorem*? In response to the learner’s negative answer, the teacher will draw a right-angled triangle and show learners what s/he means by the word ‘*hypotenuse’,* thus assisting them on how to calculate the values of the hypotenuse of any right-angled triangle. This exercise would enable the learners to find the length of the hypotenuse in the problem initially given.

4. Recasts

Another form of negotiation for the learners' feedback is recasts, defined by Long (1996:434) as "utterances that rephrase a child's utterance by changing one or more sentence components (subject, verb, or object) while still referring to its central meaning" (Gass & Mackey, 2006:8). Recasts involve the teacher's reformulation of all or part of a learner's utterance minus the error (2006:9). In response to the learners' incorrect answer, the teacher may repeat the correct answer for the learner to identify the error that s/he has committed so as not to make the same error in the future. For example, in mathematics classrooms, in response to the learners' answer that the expression "*a^2 – 2ab is a difference of two squares"*, the teacher could reformulate the correct answer and say, "*What can we say about a in the first and the second term?",* and that question would enable the learner to say*, "a is the common factor in the expression a^2 – 2ab",* thus correcting the wrong answer initially provided.

The important issue to keep in mind is that learners do not speak or hear the specific terminology outside the classrooms, so it becomes necessary to actively involve learners in speaking or writing in mathematics classrooms. Learners can explain to other learners next to them, or they can write the explanations in words. This writing can be a reflection on what they learnt, and on what they still do not understand.

From the examples of negotiations and modifications given, one could say that these negotiations or modifications alert the learners to the mistakes they have made in their utterances, thus providing them with opportunity to focus their attention on language and the correct mathematical concepts; to search for more input in their future utterances; and to be more aware of their hypothesis about language and mathematics (Gass & Mackey, 2006:12).

Also, modifications in interactions, according to Long (cited in Meneves, 2013:405), are consistently found in successful SLA, therefore, they should be applied in English SLA as well as in mathematics classrooms. When these are applied by the teachers and interlocutors, they provide learners with opportunities to process their utterances and responses mentally before they can produce them, and also help them to reflect on their learning process, thus enhancing their acquisition and learning. This is confirmed by Cummins (1986:28), when he states that "pedagogical approaches that empower learners encourage them to assume greater control over setting their own learning goals and to collaborate actively with each other in achieving these goals". As learners respond to the modifications and interactions discussed, they ultimately produce output.

Output

Reading and listening are not enough for learners to learn the language, therefore teachers should provide learners with vast opportunities to try out and produce language using pair- and group-work activities. The three major functions of output in SLA, according to Swain (cited in Brown, 2007:298-299), emphasise the role of output in language production.

- **Firstly, learners become self-informed through their output:**

This function claims that learners, while attempting to produce the target language, may notice their erroneous attempts to convey meaning. This prompts them to recognise their linguistic shortcomings; thus becoming self-informed about their output. Output helps the learners to "try out" one's language: to test various hypothesis that are forming; and speech and writing can offer a means for the learner to reflect (productively) on language and mathematical language itself in interaction with peers.

Furthermore, Swain (cited in Gass & Mackey, 2006:14) also suggests that output provides an opportunity for learners to test hypotheses about the target language, ESL and Mathematics, and modify them where necessary. Also, for modified output to be useful, most interaction researchers suggest that it is necessary for learners to notice the relationships between their initially erroneous forms, the feedback they receive, and their output, since it is possible for learners' perceptions to differ according to the type of feedback they receive and the focus (Gass & Mackey, 2006:12).

Similarly, the second condition for mathematical language development emphasises language production.

- **Secondly, there should be ample opportunities for language production; (Van Eerde *et al.*, 2008: 34).**

In other words, the condition for output in mathematics learning is the promotion of active participation of pupils, giving them the opportunity to construct and verbalise their mathematical solutions, promoting classroom discussions, and asking for clarifications and justifications (Van Eerde *et al*., 2008:34).

Output is defined as the process of producing language (speaking and writing) (Brown, 2007: 293). Similarly Gass and Mackey (2006:13) define it as the language that learners produce. Krashen has been criticised by other researchers for disregarding the function of learners' output in SLA when he says that "output is too scarce to make any important impact on language development" (Brown, 2007:298). The roles of learners in as far as output is concerned are indicated below.

The role of output

Bot (cited in Brown, 2007:298), argues that "output serves an important role in second language acquisition … because it generates highly specific input the cognitive system needs to build up a coherent set of knowledge". As a result, interaction research, according to Gass and Mackey (2006:13), focuses on output that has been modified, and therefore modified output promotes learning since it stimulates learners to reflect on their original language.

Learners use a variety of communication strategies to request for assistance, to modify the output produced with the feedback they receive from their interlocutors, and thereby produce modified output. These include avoidance, compensatory, as well as socio-affective strategies that learners use as potentially conscious plans for solving what, to an individual, presents itself as a problem in reaching a particular communicative goal (Brown, 2007:137).

Communication strategies

Communication strategies include avoidance, compensatory, and socio-affective strategies discussed under learning strategies.

(a) Avoidance strategies:

Avoidance strategies include message abandonment, leaving a message unfinished because of language difficulties; and topic avoidance, avoiding topic areas or concepts that pose language difficulties (Brown, 2007: 138). Learners use these strategies by changing the topic or pretending not to understand it because it is too difficult for them to express.

(b) Compensatory strategies

Compensatory strategies are used for compensation for missing knowledge, and these include code-switching, circumlocution, appeal for help, and non-linguistic signals like miming, among others, (2007:138). For example, a learner can use the mnemonic *co-si*, short form of the famous soccer club in South Africa, Kaiser Chiefs, to assist him to remember the formula for the *cotangent* = *cosine/sine*, and invert it to remember the formula for the *tangent.*

(c) Socio-affective strategies

These have been discussed under communication strategies discussed above.

A combination of the learning and communicative strategies, known as strategies-based instruction (SBI) is discussed below.

Strategies-based instruction (SBI)

Strategies-based instruction is divided into direct and indirect strategies.

Direct strategies

Direct strategies include the following (Brown, 2007:141):

Memory strategies

Memory strategies include creating mental linkages, applying images and sounds, reviewing well, and employing action. For example, in order for learners to remember how they learn mathematics, they can simply look at the shape of their desks and use that mentally to remember the formulae for calculating the area of rectangles, squares, parallelograms, etc.

Cognitive strategies

Cognitive strategies include practising, receiving and sending messages, analysing and reasoning, and creating structure for input and output. For example, in solving for the value of x in equations, the learner could think of an equation as two sheets of paper that are equal in shape and size, and in order to make sure that the two sheets remain equal at all times, the learners should remember to cut off the same size and shape on each sheet, so as to remember what is always stressed in solving for equations, that is, *what you do on the left-hand side of the equation, should also be done on the right-hand side* to get the correct answer.

Compensation strategies

Compensation strategies include guessing intelligently and overcoming limitations in speaking and writing, as shown in the example under communication strategies.

Indirect strategies

Indirect strategies include the following (Brown, 2007:142):

Metacognitive strategies

Metacognitive strategies include centering your learning, arranging and planning your learning, and evaluating your learning; or self-questioning.

Affective strategies

Affective strategies include lowering your anxiety, encouraging yourself, and taking your emotional temperature. For example, learners could save their pocket money, and use it to buy a Streetwise Two KFC every time they get a good mark in monthly mathematics tests.

Social strategies

Social strategies include asking questions, cooperating with others, and empathising with others.

Learning strategies relate to input, whereas communication strategies relate to output (Brown, 2007:132). Therefore, it is up to language teachers to understand their learners' learning and communication strategies to apply those that have been identified by researchers as successful for ESL acquisition in mathematics classrooms.

For English SLA and mathematics learning to take place successfully, learners need support in the form of feedback from their teachers, interlocutors, adults, and peers, so as to perfect acquisition and the learning process.

The important role of output has resulted in many researchers claiming that output provides the forum for receiving feedback (Gass & Mackey, 2006:14). In other words, when learners produce L1 utterances, they rely on the interlocutors' feedback to ascertain as to whether they are on the right track in as far as language acquisition and mathematics learning are concerned.

Vygotsky's theory on the Zone of Proximal Development (ZPD) explains the steps or stages outlined in figure 3-2 to be followed for learners to be able to do tasks on their own when provided with support or scaffolding. ZPD is defined as the "difference between the child's capacity to solve problems on his own, and his capacity to solve them with assistance ... and it includes all the functions and activities that a child or a learner can perform only with the assistance of someone else" (Schütz, 2014:1-2).

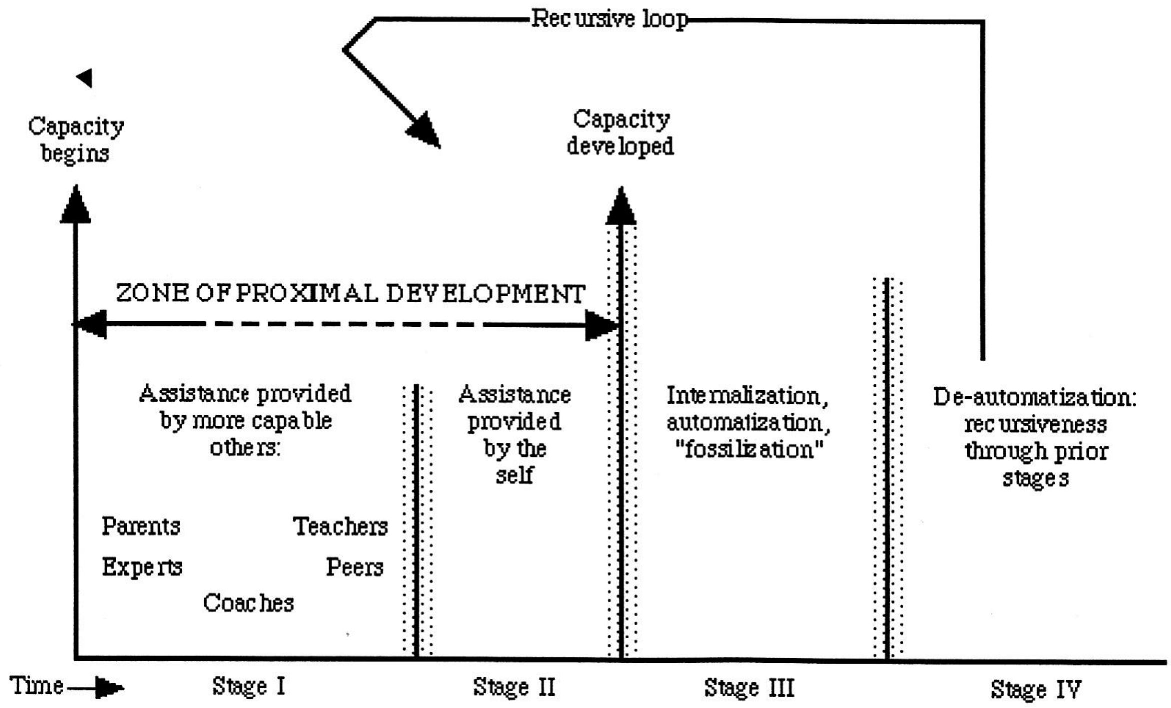


Figure 3-2: Model of four stages in the zone of proximal development (Gallimore & Tharp, 1990:185) Source: (Siyepu, 2013:5)

In explaining the learning of mathematics, Siyepu (2013:3) describes ZPD as the difference between what a learner can do without help and what a learner can do with help. Vygotsky believed that when a learner is at the ZPD for a particular task, providing the appropriate assistance will give the learner advancement to achieve the task (Galloway, cited in Siyepu, 2013:6). Siyepu (2013:5-6) describes the four stages in figure 3-2 as follows.

Stage I: The first stage demonstrates how learners develop an understanding of language that is appropriate to their study and the basics of the topic under study by relying on others such as instructors to perform the task.

Stage II: In the second stage learners use prior knowledge to carry out the task without any guidance. The ZPD occurs between the first and the second stages. Learners practise alone, which implies that they perform certain activities without assistance. However, they are not at a stage of perfect proficiency and require some assistance sometimes.

Stage III: In the third stage performance is developed, is happening without thinking and knowledge is fixed and it cannot be forgotten. This means that at this stage learners reach the stage of independence. At this stage a learner does not need help from an adult, nor to practise more exercises to reinforce the already existing knowledge (Gallimore & Tharp, cited in Siyepu, 2013:6).

Stage IV: In the fourth stage learners are at the de-automatisation of performance that leads to the process of repeating a function, each time applying it to the results of the previous stage through the ZPD.

In other words, ZPD theory allows the adult to be the "tool holder" as they hold tools in the form of relevant content from mathematics text books, questions, modifications, which gives them control over the concept for the child to be able to internalise external language. This means that adults, teachers and interlocutors should find the learners' ZPD, determine what the learners already know well, and then work from there until they reach the zone where they can successfully master new concepts without assistance.

Similarly, the third condition for mathematics language learning emphasises the need for learners to receive feedback.

- **Thirdly, language learners need feedback on their utterances (Van Eerde *et al*., 2008:34)**

Feedback

For learners' utterances and mathematics language to be perfected, learners need support or what is termed 'scaffolding' in the form of feedback from the teachers, interlocutors, peers, and adults, for them to be in a position to reflect on and correct the mistakes made and to perfect the acquisition and learning process. Scaffolding is understood as the assistance learners get from others (teachers, relatives, class-mates) and it enables them to perform learning tasks (Meneves, 2013:406).

Feedback from the teacher on pupils' contributions should not be immediate, but delayed to promote contributions from different pupils and horizontal interaction between pupils (Van Eerde *et al*., 2008:36).

This third condition shows that Vygotsky's theory on ZPD is also applicable in mathematics classrooms. The theory, as a result, puts emphasis on the role of feedback. Teachers and parents are therefore advised to offer learners this assistance and support, *scaffolding*, for successful learning and language development to take place in both ESL and mathematics classrooms, using the following strategies.

Strategies for scaffolding

In a study conducted in which a teacher was encouraged to employ the following seven strategies in a multilingual classroom, the results showed that the strategies used promoted pupils' language development (Smit & Van Eerde, 2013: 30).

Table 3-1: Strategies for scaffolding language Source (Smit & Van Eerde, 2013:24)

1	Reformulate pupils' utterances (spoken or written) into more academic language	[In response to the *graph goes higher and higher up*:] *Yes, the graph does rise steeply*.`
2	Ask pupils to be more precise in spoken language or to improve their spoken language	*What do you mean by 'it'*?
3	Repeat correct pupil utterances	*Yes, the graph does descend slowly.*
4	Refer to features of the text type (interpretative description of a line graph)	*Into how many segments can we split the graph?*
5	Use gestures or drawings to support verbal reasoning	E.g., gesturing a horizontal axis when discussing this concept
6	Remind pupils (by gesturing or verbally) to use a designed scaffold (i.e. word list or writing plan) as a supporting material	*Look, the word you are looking for is written down here.*
7	Ask pupils how written text can be produced or improved	*How can we rewrite this in more mathematical language?*

Similarly, Biro *et al*, (2005:5) encourages mathematics teachers to help ELLs to develop and practise academic language for learning mathematics using scaffolding strategies such as, having learners restate other learners' comments, using graphic organisers or gestures, correct errors and providing positive feedback, providing handouts to help learners structure and guide their work, among others.

A constructivist approach to teaching and learning (summarised below) should be applied for assessment in English SLA and mathematics language learning classrooms to take place in such a way that learners are able to reach the intended outcomes discussed.

3.4.3 Using a constructivist / open-ended approach to teaching and learning

A constructivist approach to teaching and learning, according to Mahlobo (2009:38), is characterised by the use of open-ended tasks and/or questions, and it encompasses the following:

The learners

(a) take the initiative in solving mathematical problems and do not depend on the teacher;

(b) determine their own approach when solving problems;

(c) express their own ideas more frequently when solving mathematical problems;

(d) modify other learners' ideas; and

(e) can stimulate the exploration of concepts and ideas and facilitate creative and critical thinking processes.

3.5 A visual representation of a theory on mathematics teaching (MT) and TESL, Mathematics learning (ML) and English SLA

The visual representation of a theory on TESL and mathematics teaching and English SLA and mathematics learning in figure 3-3 below, is a synthesis of the theories and conditions based on TESL and mathematics teaching (MT) and English SLA and mathematics learning (ML); together with the strategies used, which, when combined, provide learners with comprehensible input, opportunities for language processing and interaction, output, and feedback on their utterances.

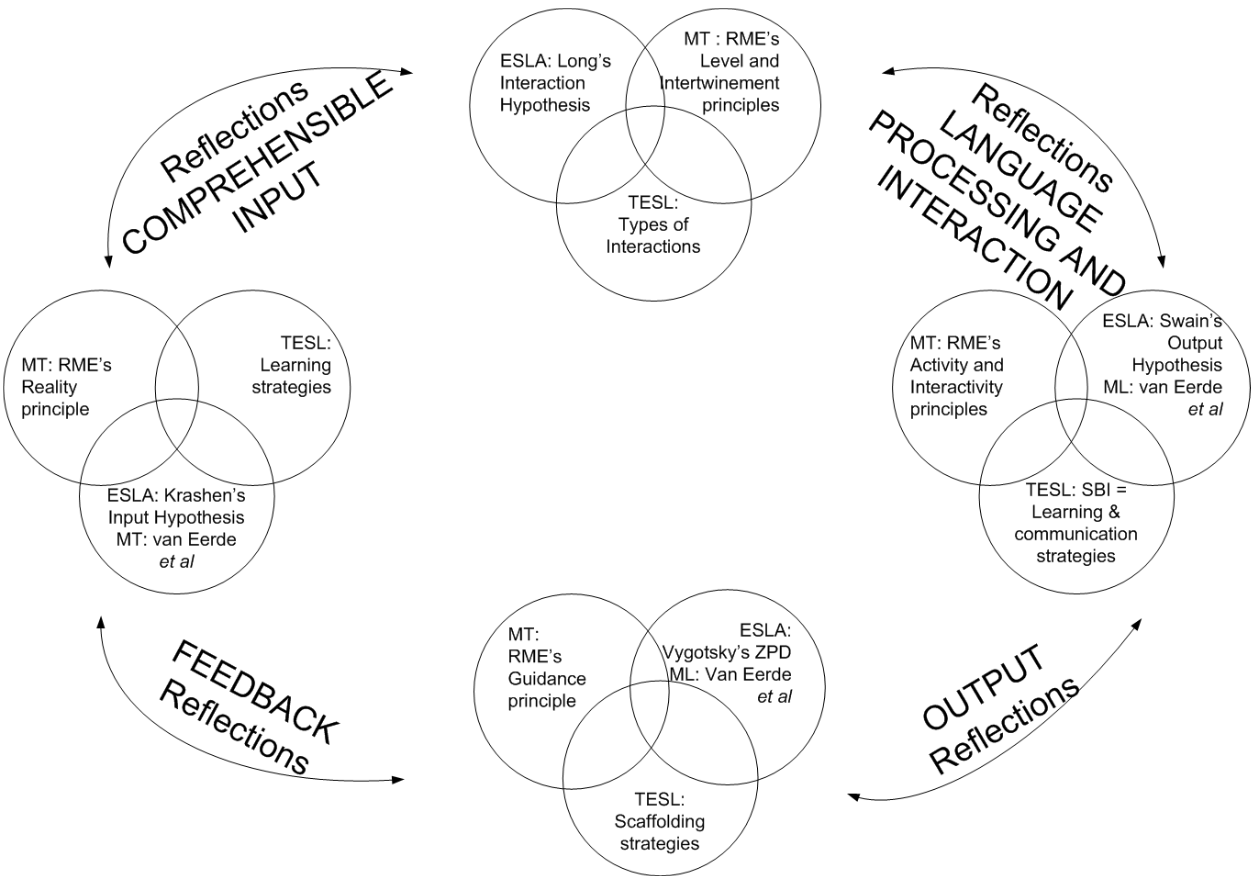


Figure 3-3: A visual representation of a theory on TESL and mathematics teaching, and English SLA and mathematics learning

3.5.1 Comprehensible input

Krashen's Input Hypothesis for ESLA and the first condition for mathematics learning (ML) respectively, (Krashen, 1981a; van Eerde *et al*., 2008), together with RME's Reality principle on mathematics teaching (MT) (Van den Heuwel-Panhuizen & Drijvers, 2014), all emphasise that learners should be provided with comprehensible input to be able to understand what is taught. Learning strategies used in ESL classrooms that assist learners to understand language to make it comprehensible are also included (Brown, 2007:134). These should also be used in mathematics classrooms.

3.5.2 Language processing and interaction

Long's Interaction Hypothesis on ESLA, together with RME's Level and Intertwinement principles on mathematics teaching (MT), (Long, 1985, 1996; Cummins, 1986; Van den Heuwel-Panhuizen & Drijvers, 2014) stress the fact that learners are required to process input and interact using the language provided in ESL and mathematics classrooms. The types of interactions that assist learners to interact and process the language are also included in the visual representation (Brown, 2007:305). Even though the interactions are used in ESL classrooms, they can also be used in mathematics classrooms for learners to process and interact using language.

3.5.3 Output

Swain's Output Hypothesis and the second condition for mathematics learning (ML), (Swain, 1995, 2005; van Eerde *et al*., 2008), together with RME's Active and Interactivity principles (MT), (Van den Heuwel-Panhuizen & Drijvers, 2014); all emphasise the fact that learners should be provided with vast opportunities to try out language in order to produce output . A combination of learning and communication strategies referred to as Strategies-Based Instruction (SBI) that assist learners to try out and produce language in ESL classrooms (Brown, 2007:134-135, 137-138, 141-142), can also be used in mathematics classrooms.

3.5.4 Feedback

Vygotsky's theory on ZPD for ESLA and the third condition for mathematics learning (ML) (Vygotsky, 1978; Van Eerde *et al*., 2008) together with the RME's Guidance principle, (Van den Heuwel-Panhuizen & Drijvers, 2014); all emphasise the fact that learners should be provided with feedback on their utterances in the form of 'scaffolding' or support to enable them to correct and improve on their utterances. Strategies for scaffolding used in ESL and mathematics classrooms, that provide learners with feedback are also part of the visual representation (Smit & Van Eerde, 2013:24).

In addition, the reflection processes described in the visual representation are cyclical in nature as the teachers provide input throughout the learning situation when they also reflect on the input provided to learners and on the output produced. Similarly, the learners reflect on what is taught and also on what they bring into the learning environment by applying meta-cognitive processes to speedy up the process of producing language and acquiring it in the long run. Hence the arrows on both sides show reflections throughout the learning process as teachers and learners reflect on the input provided and output received by using meta-cognitive knowledge (e.g. declarative, procedural and conditional; and/or person, task and strategy variables) as well as the self-regulated processes (e.g. planning, monitoring, evaluation) (Van der Walt, s.a.). These processes are illustrated in figure 3-4, and explained thereafter.

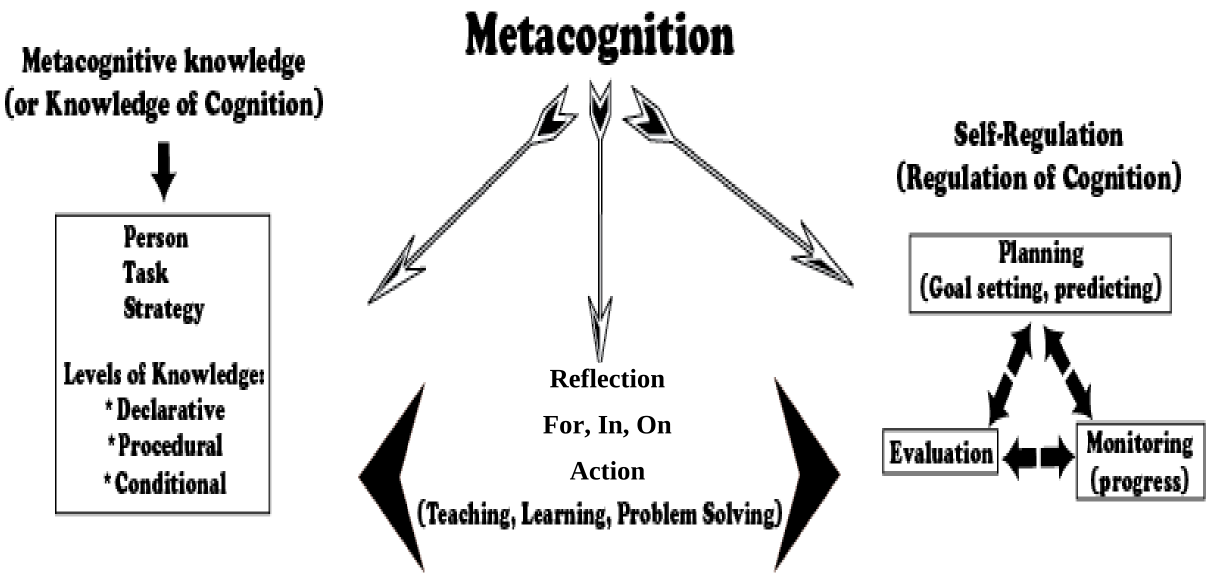


Figure 3-4: A conceptual model of metacognition Source: van der Walt (s.a.)

A conceptual model of metacognition

Metacognition is a complex phenomenon (Senduvur *et al*., 2011), involving three distinct but interrelated domains of metacognitive behaviours (Ertmer & Newby, 1996), and these are, metacognitive knowledge, self-regulation, and reflection in various contexts, including teaching and learning.

Metacognitive knowledge concerns knowledge and beliefs about cognition, and can be associated with person variables (how we learn/teach), task variables (nature of particular tasks and cognitive processing of the demands of the tasks), and strategy variables (cognitive and metacognitive teaching/learning strategies) for a particular task (Ertmer & Newby, 1996; Flavell, 1979).

Managing one's own thought processes is called self-regulation. Self-regulation is the active part of planning for the particular task, monitoring progress and evaluating the outcomes of the task (Bransford *et al*., 2000; Dunlosky & Metcalfe, 2009). It includes those plans a person makes before the task, the adjustments made during the execution of the task, as well as the revision of the solution at the end of the task (Paris & Winograd, 1990).

All the metacognitive knowledge and self-reflection are purposefully held together throughout by the phenomenon "reflection" (Van der Walt *et al*., 2008). The tendency to think about a task and to use metacognitive knowledge and self-regulation is called reflection (Cornoldi, 2009). Reflection on metacognitive knowledge precedes (planning), accompanies (monitoring), and is followed by (evaluation of cognitive strategies) (Livingston, 1997).

Reflection/reflective thinking transforms the knowledge acquired during execution of a task and after completion of the task, into knowledge that is available for the next task (Ertmer & Newby, 1996). In other words, reflection can promote the awareness of teachers and learners about teaching and learning mathematics and the related language.

3.6 Conclusion

Learning a subject like mathematics through a second or additional language involves socialisation within both mathematics and language. A number of discursive practices, such as repetition, stressing key words, elaborating on the meaning of words and statements, echoing by learners, completion and direct requests contribute to the learners' socialisation within mathematics and English (Barwell, 2010:115). The discursive practices are used to assist learners to comprehend lessons, to process and interact with the language, to produce output, and to receive feedback in both ESL and mathematics classrooms. Discursive practices in this study involve the use of questions, questioning techniques, and teacher strategies used in mathematics classrooms, and these are discussed in the next chapter.

CHAPTER FOUR: QUESTIONING

4.1 Introduction

The theories on ESLA, and the conditions for mathematics learning (ML), and the RME's principles on mathematics teaching (MT), together with strategies used in ESL and mathematics classrooms, discussed in Chapter 3, are linked to the themes that emerged from the literature reviewed, namely comprehensible input, language processing and interaction, output, and feedback. These are illustrated in the visual representation, showing the movement in the form of questions, questioning techniques and strategies. Chapter 4 discusses the types of questions and their functions, questioning techniques, and teacher strategies used in mathematics classrooms. Tables are provided in each case to introduce and summarise each section. Finally, a theoretical model synthesising all the links is illustrated and discussed.

4.2 The role of questions in mathematics classrooms

The number of questions most teachers ask is rated at 400 a day and 7000 a year, and that, according to Hastings (2003:1), accounts for a third of all teaching time, confirming Manouchehri and Lapp's (2003:563) assertion that teachers' speech consists mainly of questions. Therefore, questioning is very important and it is a critical skill for teachers as it promotes a lifelike, motivating, and challenging climate in the classroom because it engages both the teachers' and learners' minds and hearts (Hess & Pollard,1995:3).

Furthermore, according to Manouchehri and Lapp (2003:564), questioning is a sophisticated skill that makes teaching exciting since it provides the teacher with opportunities to experiment with ideas, learn about his/her learners, and to discover their true potential. In short, questioning is the most frequently used form of communication in the classroom and a crucial feature fostering development of second language abilities, as asserted by Shan Wen (2004:1) and Brock (1986:47). Therefore, it has to be taught in mathematics classrooms.

Long and Sato (cited in Shomoossi, 2004:97) suggest that output in the form of learners' responses depends on the types of questions used by the teachers in mathematics classrooms.

4.3 Types of questions used in mathematics classrooms

According to Borich (2004:258), any oral statement or gesture that is intended to evoke a learner response is considered to be a question. Questions are classified into procedural questions, recall or closed questions, and process or open questions (Hargie, cited in Tuan &Nhu, 2010:32). Table 1 below summarises the types of questions from the literature reviewed and these will be discussed in detail in the sections that follow.

Table 4-1: Types of questions used in mathematics classrooms

QUESTIONS	FUNCTIONS	EXAMPLES
Procedural	To give out instructions; provide learners with alternative options.	*Do class-work on p. 120;* *Yes* or *No, e.g. Is sine 60° = Cos 60°?* *True* or *False, e.g. Tan 45°=1.*
Closed	To retrieve knowledge	*What*? *Fill in*, *Where*?, *When*? *Either ... or ...Which*? What is the square root of 16?
Open-ended	Require learners to process and produce language	*What*? *Why*? *What if*? *How*? *Explain...*, e.g. The lengths of the sides of a triangle are 3, 4, and 5. Explain why the triangle is a right-angled triangle or not.

4.3.1 Procedural questions

Procedural questions are part of the language used by the teacher to give out instructions for learners to cooperate in class, and these do not require learners to produce language. Furthermore, procedural questions are sometimes referred to as alternative questions that include *Yes* or *No* and *True* or *False* questions.

Yes or No questions

Yes or *No* questions, according to Manouchehri and Lapp (2003:562), are used after a lesson has been taught and before the teacher assigns more exercises for class work practice based on the lesson on any section of the subject content. They are also used for control purposes. Feedback on these questions can be probed by using *why* questions for learners to explain why their responses are *Yes* or *No*.

True or False questions

Like *Yes* or *No* questions, *true* or *false* questions promote limited interaction in the classroom and do not promote learners' understanding of mathematical discourse and language development as they require learners to listen or read and think about the statements spoken or written before they can say that they are *true* or *false*. As it is the case with *Yes*/*No* questions, feedback on these questions can also be used to encourage learners to provide explanations for their choice of answers, thereby engaging them in a mathematical discourse and English SLA in the process as the learner explains the reasons for the statement to be *true* or *false*.

Studies on procedural questions

Terrell (2009:7) in her study aimed at stimulating learners' interest and curiosity, used *True* or *False* questions in her Calculus classes on the Intermediate Value Theorem, and her findings showed that these questions, followed by commands like, "*Be ready to offer a proof or counter example*", motivated her learners to perform better than they did in previous years. Furthermore, she stated that learners were also asking her questions, and in that way they were communicating with the teacher, and thus improving their English SLA.

Lomen (2009) also used *True or False* questions specific to each section of the Calculus and used learners' feedback to these questions to guide the discussion for each day's lesson.

Procedural questions require short answers (Donnelly, 2009:5). This is the reason why Shomoosi (2004:101) cautions teachers that these questions on their own reduce classroom interaction, so teachers should follow them up with *Why* questions and other probing questions to promote open-ended discussion that will promote learners' understanding of mathematical discourse and English SLA in the process. Researchers suggest that even these questions can be changed into opportunities for interaction through the teacher's involvement by facilitating the classroom situation. To support this statement, Sadker (2003:2) states that using probing questions on feedback from these questions can be as effective as using open-ended questions. In other words, for effective lessons that engage both the teacher and learners in the learning process, teachers should use a combination of procedural, closed as well as open-ended questions.

4.3.2 Closed questions

Closed or recall questions are used to retrieve the knowledge learners have learnt and do not require them to apply high-cognitive capacity. They are classified as closed questions as they require learners to present their knowledge in a few words or phrases.

Zevenbergen and Niesche (2008:6) describe closed questions as low-level or lower-order questions used for memory, rote and simple recall, and they do not produce high levels of intellectual quality. Shan Wen (2004:7) further describes them as non-creative questions as they fail to stimulate creative thinking on the part of the learners. According to Shahrill (2013), these types of questions demand learners to recite what they have learnt. An example of a low-level or lower-order question is as follows:

What is the formula to find the area of a rectangle?

(Shahrill, 2013:226).

Closed questions are referred to as closed in terms of their function, and according to Shan Wen (2004:7), these include display, closed-referential and convergent questions, all which involve lower-level thinking processes and invite only one answer. Display questions require learners to display the knowledge and information they have already learnt and put emphasis on memorisation and recognition (Shan Wen, 2004:3). An example of a display question is as follows:

Write down the factors of all the numbers from 1 to 20.

(Laridon *et al.,* 2008:5).

Furthermore, a closed-referential question is a question to which the teacher does not know the answer but to which there is either only one or a very limited set of possible answers (Shan Wen, 2004: 4). It is also described as a question that requests unknown information or opinions in which the questioner is interested. An example of a closed-referential question is:

What did you read about?

(Brulhart, 1986: 32).

Closed questions require learners to recall, to recognise or to organise material in a predictable way, they are concerned with the right answers or very limited possible answers (Shan Wen, 2004:6). All these questions constitute closed questions that are described by Donnelly (2009:5) as *thin* questions that require short answers and seek a particular answer known by the teacher. An example of a convergent question is as follows:

Which is smaller, $^{5}/_{16}$ *or* $^{3}/_{8}$*?*

(Ebert *et al*., 2006:2).

Studies on closed questions

Borich (2004:317) found that 80% of all questions asked in mathematics classrooms were closed questions that he describes as *direct* questions since they limit learners' responses to the questions asked. Similar studies conducted in 1912, 1935 and 1970 described in Hastings (2003:3), indicated that 60% of teachers' questions required learners to recall information. Similarly, reviews of research in the United States of America, United Kingdom, Germany, Australia and Iran on the types of questions used in mathematics classrooms have also shown that most of the questions asked by the teachers were closed (Long & Sato, 1983; Brualdi, 1998; Sutton & Kreuger, 2002; Sadker, 2003; Zevenbergen & Niesche, 2008; Shomoossi, 2004).

The above statements are evidence of the use of mostly closed questions in mathematics classrooms. Yeo and Zhu (2009:6) in their study aimed at investigating the extent of higher-order thinking in mathematics classrooms in Singapore, found that learners repeatedly regurgitate and replicate the knowledge they are taught, and that high-order thinking has not been encouraged in Singapore mathematics classrooms as a result of the use of these questions.

Closed questions have been used traditionally to determine what has been learnt, and as a result, establish and validate learners' perceptions about what is important to know to succeed in mathematics classes (Manouchehri & Lapp, 2003:565). Closed questions are used to measure learners' mastery of basic skills, but they are not adequate for determining what learners can do beyond solving problems.

Hastings (2003:3) cautions teachers that closed questions might intimidate learners who are not sure of the 'correct factual answer' and might make those who do not know the answer feel like failures. As a result, communication is shut down in the classroom due to 'peer fear' and this will consequently hamper learners' language development.

Chorus responses

In a study conducted on the roles of unison (chorus) responses in the teaching and learning of mathematics at two under-resourced township schools in South Africa, findings showed that unison or chorus responses was a major teaching strategy in those schools (Watson, 2002:35). Further analysis of chorus responses elicited by closed questions in an algebra lesson in which learners were required to multiply a binomial with a binomial to make it a trinomial, learners were able to do the exercises given on the lesson (Watson, 2002:39). An example of the chorus responses is shown below.

Teacher: *the first multiplied by the first gives the ...?*

Pupils: *first term*

Teacher: *and the first multiplied by the second gives the*... ?

Some pupils: *second term*

Teacher: *and the second multiplied by the first gives the ... ?*

Some pupils: *third term*

Teacher: *and the last by the last gives you the...?*

Pupils: *last term*

Responses by 'some pupils', according to the researcher who was able to look at learners' exercises based on the lesson, resulted in her deducing that even those pupils who did not respond with '*second and third term*' knew the correct answers (Watson, 2002:40).

Open-ended questions

Open-ended or process questions, on the other hand, require learners to go through more complex mental processes to be able to give their opinions or to justify and evaluate any given statement. They are classified as open questions because they provide learners with more chances of interaction at advanced levels, and therefore encourage them to participate and produce language (Tuan & Nhu, 2010:33).

Open-ended questions are described by Maley (2009) as questions that have a variety of answers that are open for discussion and negotiation. Shan Wen (2004:7), as well as Tuan and Nhu (2010:34), describe open-ended questions as creative questions because they involve learners in higher-level thinking processes and require them to think critically and creatively as they call for interpretation, opinion, evaluation, inquiry, making inferences, and synthesising.

Open-ended questions are responses on several levels of Bloom's taxonomy of learning categories, which has been reorganised into the Cunningham model. In this model, open-ended questions are named *evaluative* or higher level questions in reaction to which learners are required to elaborate more on their responses when the teacher probes further to find out why they gave such answers (Shahrill, 2013:226-227). These questions are open to a range of possible responses from learners and do not have one particular answer known by the teacher as is the case with closed questions. They elicit slightly longer and more learner utterances (Brock, 1986; Long, 1983; Shomoossi, 2004: 97).

Sadker (2003:2) describes open-ended questions as high-order for more demanding and exacting thinking as they demand thinking and talking on the part of the learners. Unless learners are requested to explain their thinking by using open-ended questions like *why*, which give learners an opportunity to communicate their reasoning process, a teacher may not know which concepts the learners understand (Manouchehri & Lapp, 2003:564). Examples of open-ended questions are *why*, *how* and *what if*, categorised as *analysis* questions in Kelly (2012). These types of questions give learners an opportunity to communicate their reasoning process, therefore providing the teacher with a better understanding of learners' knowledge (Manouchehri & Lapp, 2003:564). An example of an open-ended question is as follows:

How could you simplify this equation: 9x + 27y = 153?

(Ebert II *et al*., 2006:2)

Open-ended questions are further described as questions that require the reader to provide the answer by explaining something as they ask learners to think deeper about what they are doing and why what they are doing works or not (Bellido *et al*., 2009). During that process, language development is promoted as they use language to elaborate on the steps they followed to arrive at a particular answer. An example of a divergent question in number relationships is as follows:

I'm thinking of a number. The sum of its digits is divisible by 2.

The number is a multiple of 11. It is greater than 4 x 5.

It is a multiple of 3. It is less than 7 x 8 + 23. What is the number?

Is more than one answer possible?

(Pelfrey, 2000:38).

Furthermore, open-ended prompts and questions can be used for a conversational approach in the classrooms to achieve the following intended outcomes:

- To prompt more thinking, for example:
    - *You are on to something important. Keep going*.
    - *You are on the right track. Tell us more*.
    - *There is no right answer, so what would be your best answer*?
    - *What did you notice about* …?
- To fortify or justify a response, for example:
    - *That's a good probable answer*. .. *How did you get to that answer*?
    - *Why is what you said so important*?

  - *What is your opinion (impression) of … and Why*?

- To see other points of view, for example:

  - *That's a great start. Keep thinking and I'll get back to you*.

  - *If you were in that person's shoes*, *what would you have done*?

  - *Would you have done (or said) it like that*? *Why or why not*?

- To consider consequences, for example:

  - *Should she have* …?

  - *What if he had not done that*?

  - *Some people think that …is [wrong, right, and so on*.] *What do you think*? *Why*?

  - *How can we apply this to real life*?

(Taylor, 2009:10).

Studies on open-ended questions

Open-ended questions were used by Johnson (2000:1) in her study of mathematics learners in which she found that gifted learners differed from their classmates with regard to the pace at which they learnt, the depth of their thinking, and the interests that they held. As a result, she suggested that gifted learners should be taught differently by using, among other things, multiple resources and more high-level questions in justification and discussion of their problems, such as, *why*, and *what if*. In addition, their teachers should choose textbooks that provide more enriched opportunities to create assessments that allow for differences in understanding, creativity and accomplishments; they should also ask learners to explain their reasoning both orally and in writing.

In her study aimed at developing discourse in mathematics classrooms, Sherin (2002:229) found that when the research participant (the mathematics teacher she called David), used three questions like 1) *What do people think about this idea?*, its response followed up with 2) *Why*?, and the open-ended response followed up with 3) *What do other folks think about that*?, they generated open-ended discussion in the classroom.

The type of discourse presented by these kinds of questions, according to Sherin (2002:218), was totally different from the traditional classroom discourse in which discussion followed a pattern of IRE – 1) Initiation by the teacher, 2) Reply from the learner, followed by 3) an evaluative comment from the teacher. A review of these questions to generate more open-ended discourse in mathematics classrooms resulted in their being rephrased as follows: 1) *What do you think*? 2) *Why*? Or *Can you explain* that? 3) *What do other people think*?

Further analysis of the learners' responses to these questions revealed, among other things, that ideas were not only generated, but were also preliminarily elaborated and evaluated by other learners in the classroom. David used the three questions at the beginning of a discussion to "draw out kids' ideas" and to give the learners a sense of ownership over the discourse (Sherin, 2002:219). Once several ideas had been generated, the teacher moved on to the next step where the ideas were compared and evaluated by both the teacher and learners, followed by the next step called *filtering*, a term used to "emphasise that any new content raised by the teacher was based on a narrowing of ideas raised already by the learners" (Sherin, 2002: 220). During that step, the teacher refocused the content of discussion on areas that s/he felt were mathematically significant and productive for enhancing learners' understanding of lessons in mathematics.

David also used 'focus questions' that directed learners' attention to the key elements of a particular solution strategy (Sherin, 2002:220). Findings and recommendations on using these three questions suggested that the teacher was able to orchestrate discussions that were based on learner ideas, and were productive and worthwhile mathematically. Sherin (2002: 228-229) therefore emphasises that teachers should view discourse as a viable tool for learners' learning of mathematical concepts.

The importance of questions in the classrooms was confirmed in studies conducted in ESL, EFL and mathematics classrooms. Brock's (1986:55) study on the effect of open-referential questions in ESL classroom discourse revealed that learners' responses to open-referential questions were on average more than twice as long and more than twice as syntactically complex as their responses to display questions, and that the use of these types of questions increased the amount of learner output (1986:56). Findings similar to these were noted in Shomoossi (2004:102) and Sherin (2002: 228-229) in EFL and Mathematics classrooms respectively, leading to their conclusion that open-referential questions were important tools in language production in the classroom.

Researchers have also shown that only a small number of open-ended questions are used during teaching lessons. In a study of secondary school lessons conducted in 1989 in the USA, findings described in Hastings (2003) showed that only 4% of questions used in the

US primary classes were of a higher-order nature and in 1999 Ted Wragg, replicating the same research study in primary schools, found that 8% were higher-order questions. A report of 37 research projects by US educationist Kathleen Cotton in 1988 based on the questions used across the US concluded that, by increasing the number of higher-order questions by 50%, learners' attitude and performance were improved significantly (Hastings, 2003: 3). The report implied that something could be done to address the low percentage of open-ended questions used in mathematics classrooms.

Chorus responses

An analysis of chorus responses to open-ended questions in Watson (2002)'s study in a geometry lesson, showed that the learners' chorus response, '*vertically opposite angles are equal*' did not assist learners, when doing the exercise given in class, to use the statement to deduce the equality of, and hence ascribe values to angles in the diagram (Watson, 2002:40).

Furthermore, from the examples on the chorus responses, one could deduce that closed questions elicited many chorus responses, and open-ended questions elicited few chorus responses (Watson, 2002: 39).

The studies discussed above encourage teachers to be aware of the fact that good questioning is an excellent aid to teaching that they should utilise to the fullest extent. Questions should be major determinants of teaching and learning outcomes. The time invested by teachers to plan effective questions could be worthwhile if they focus on essential learning; help learners add to their knowledge and transfer it to other subjects; motivate learners to take a real interest in the subject; and help learners apply essential learning to real problems, issues and decisions (Chuska, 1995:7).

Summary

In summary, teachers still use more closed questions, though the use of such questions hampers learners' understanding of mathematical discourse and ESL development. This should be discouraged as research has proven that "effective questioning skills have been linked with learners' achievement in mathematics" (Shahrill, 2013:230). Therefore, teachers are advised to ask higher-order questions that challenge learners and allow them to explain and elaborate on their responses in mathematics classrooms, thus promoting interaction and English SLA in the process. For the purposes of this study, the distinction is therefore made between closed and open-ended questions as research has shown that teachers use many closed and few open-ended questions in mathematics classrooms, and these types of questions, which are also used in mathematics classrooms, are the focus of this research study.

4.4 The functions of questions used in mathematics classrooms

Teachers use closed and open-ended questions throughout the lessons for a number of reasons and for a variety of purposes as questions play a major role in learners' learning and language development. The following functions of questions listed in Table 4-2 have been identified from the literature reviewed and will be discussed in more detail in the next section.

Table 4-2: Functions of questions used in mathematics classrooms

FUNCTIONS	PURPOSE	EXAMPLES
Diagnostic	Diagnose, check if learners understood what has been taught	*Is that clear*?
Managerial	Discipline and control purposes	*What do you think about that Peter*? A question addressed to Peter who was making noise at the back.
Constructive	Elicit prior knowledge	*What is the length of the hypotenuse in the $\triangle ABC$ with $<A = 90^{\circ}$?*
Corrective	Structure and redirect wrong answers	*What can you add to your solution to make it clearer to the reader*?
Cognitive	Foster learners' critical thinking skills	*What do you notice about the sides of this triangle PQR with 3, 4 and 5*?
Language acquisition	Use mathematical language to communicate mathematical ideas	*Explain how you arrived at that answer*.
Evaluative	Evaluate the lesson	*What did you like best in today's Geometry lesson*?
Affective	Apply mathematics to real-life situations	Learners taking photographs that resemble rectangles outside the classroom

Source: Own compilation from the literature reviewed (Borich, 2004; Capacity Building Series # 21, 2011; Manouchehri & Lapp, 2003; Orbit, 2012; Pelfrey, 2000)

4.4.1 The Diagnostic function

According to Borich (2004:259), closed and open-ended questions are asked during the course of the lessons to diagnose, check, and verify whether or not learners are following what is being taught. This function of questions is also captured in Ng'ambi and Brown (2009:317), who stated that embodied in questions is implicit knowledge about levels of learners' current understanding. Long and Sato (1983: 272) refer to the types of questions that perform this function as '*referential questions'* since they provide contextual information about situations, events, and action.

Traditionally questions were used to evaluate what learners know, but according to Chin (2006:1319), the purpose of questions in inquiry-oriented lessons is to elicit what learners think, to encourage them to elaborate on their previous answers and ideas, and to help them construct conceptual knowledge.

Examples of these questions are as follows:

- *What information are you/we going to use to solve this problem*?
- *What other problem have you/we solved that is similar to this one*?

(Capacity Building Series # 21, 2011:7)

4.4.2 The Managerial function

For learning to take place without any disruptions, teachers have to maintain order and discipline in their classrooms by asking closed or lower-order questions for managerial purposes to provide an atmosphere that is conducive to learning. As indicated in Hastings (2003:3), a teacher can fire questions like bullets at learners who are making noise at the back of the classroom just to make them aware of the fact that their whispered conversation is a barrier to learning. According to Borich (2004:259), such questions are also used by teachers to capture learners' interest and attention, especially at the beginning of a lesson and after the learners have been distracted by something else. They control learners' learning as they focus learners' attention on specific features of the concepts that they explore in class (Manouchehri & Lapp, 2003:563).

For example, the question below was used to bring a pupil's attention back to the task in hand:

What do you think about that Peter?

Or

Do you agree?

(Orbit, 2012:2)

Managerial questions are important tools for managing the classroom as they help to draw individuals into the lesson and to keep them interested and alert (Hastings, 2003:2). They also send a clear message that pupils are expected to be active participants in the learning process, and that they should always concentrate.

4.4.3 The Constructive function

Before teachers continue with a lesson that is based on a chapter or section that was started the day before, they normally ask questions to elicit prior knowledge and link it to new knowledge. Martino and Maher (1999:56-57) encourage teachers to ask such questions as these questions stimulate learners to grow in their mathematical discourse. For example, to encourage learners to explain in detail how they finally arrived at a particular answer and to justify the given answer, the following question in number relationships was used:

(a) *In any collection of seven natural numbers, show that there must be two whose sum or difference is divisible by 10*.

(b) *Find six numbers for which the conclusion of part (a) is false*.

(Pelfrey, 2000:38).

As learners explain their thoughts, they develop their mathematical discourse as they follow the given instructions.

4.4.4 The Cognitive function

Open-ended questions can be used to foster learners' critical thinking skills for them to fully use their cognitive and thinking operations in conceptualisation. To achieve this, Tang (2003:23) recommends the framework illustrated below on Questioning for Understanding: Empowering learner thinking (Qu:Est).

Core questions for Individual Thinking Operations in Conceptualising (as illustrated by Tang, 2003:23)

Table 4-3: Core questions for Individual Thinking Operations in Conceptualising (Tang, 2003:23)

Operations	Core questions
Observing	*What do you notice about ...?*
Recalling	*What do you remember about ...?*
Comparing	*What similarities are there between ... and ...?*
Contrasting	*What differences are there between ... and ...?*

Operations	Core questions
Grouping	*In what way do these items go together*?
Labelling	*What can we call ...*?
Classifying	*How can we classify ...*?

Tang's framework can be used after lessons in Geometry, such as the *Mid-point theorem, squares, rectangles*, which involve figures, and it is highly recommended to nourish learning in mathematics (Tang, 2003: 26). In addition, these types of questions are also used to encourage high-level thought processes. The exercise below on *Consecutive Sums* enables learners to engage in mathematical thinking and to develop their higher-order thinking skills:

Can all numbers be presented in this way?

For example: 9 = 3 + 4 + 5, 11 = 5 + 6,

12 = 3 + 4 + 5, 20 = 2 + 3 + 4 + 5 + 6.

(Orbit, 2012:3).

Another exercise written below, on *Exploring properties of rectangles: Perimeter and area*, also encourages learners to engage in mathematical thinking by using questions for this function:

Do two rectangles that have the same area also have the same perimeter?

(Orbit, 2012:5).

According to Tuan and Nhu (2010:32), the aim of such pedagogical questions is to motivate, sustain and direct the thought-processes of a pupil.

4.4.5 The Corrective function

Questions are also used to structure and redirect wrong answers (Sherin, 2002: 220). As the learners' ideas were generated with the three questions indicated below for the constructive function, David also used "focus questions" that directed learners' attention to the key elements of a particular solution strategy.

1. *What do you think*?
2. *Why*? or *Can you explain that*? and
3. *What do other people think*?

(Sherin, 2002: 229).

Examples of such questions are as follows,

1. *What does this part represent in your solution*?
2. *What can you add to your solution to make it clearer for the reader*?

(Capacity Building Series #21, 2011:7).

4.4.6 The Language acquisition function

The role of ESL as the language of instruction in mathematics classrooms as pointed out in Setati (2002:9) is that learners should be able to use mathematical language to communicate mathematical ideas, concepts, generalisations and thought processes. According to Wachira *et al*., (2013:2) a key element in discourse is the need to use mathematics language and articulate mathematical concepts in order to learn both the language and the concepts.

To address the role of ESL as the language of instruction, the authors in the section titled "Did You Know?" in the Grade 10 Classroom Mathematics prescribed book, provide learners with mathematical discourse to be learnt at the beginning of a section or unit. For example, before a chapter on Trigonometric functions, one full page is devoted to information and diagrams on the relevance of a chord and how Trigonometry began with chords. Mathematical discourse used in this section, such as *angles*, *circle*, *radius*, *sine*, *chords*, *perpendicular*, etc., are mentioned, explained, and illustrated in the form of diagrams (Laridon *et al*., 2008: 286).

4.4.7 The Evaluative function

Open-ended questions can also be used to evaluate the lesson. Due to time constraints, it is not always possible for mathematics teachers to address all the learners' responses to the questions asked during the course of the lessons, and that means some of the learners may leave the classroom with some of their concerns not addressed. Others may find it difficult to ask and respond to questions because they are not proficient in the language of instruction (Yushau & Bokhari, 2005:7).To take care of such concerns, Lomen (2009:2) in his study of his learners' feedback in a mathematics classroom, advised teachers to use the following two open-ended statements during the last two minutes of every mathematics lesson for learners to evaluate the lessons and for the teacher to make sure that such concerns are addressed before the beginning of the next day's lesson.

- *One thing you understood clearly after today's lesson is ...*
- *One thing you wish you could have a better understanding of after today's class is* ...

Similarly, Samuel (2002:2) requires his learners to answer the following questions in their diaries from the first day of instruction:

What were the goals/objectives of today's lesson in Geometry?

Why did I learn this?

What strategies can I use to accomplish today's goals?

What did I like best about today's Geometry class?; and

What was most frustration about today's class?

As learners read and respond to these questions, they learn to communicate freely in the privacy of writing in their journals or sheets of papers provided, using the medium of instruction, thus promoting their understanding of mathematical discourse and developing the language of instruction in the process. Their responses, which constitute learners' comprehended output, enable them to produce their own messages in the target language, proving that output is an important factor in successful SLA (Brock, 1986: 55). Their responses, which are submitted to the teacher, provide the teacher with feedback on the lesson as learners are provided with an opportunity to express in writing their concerns, which would be addressed by the teacher on the next day. The whole process promotes their understanding of mathematical discourse and ESLA development.

4.4.8 The Affective function

For learners to enjoy mathematics and to apply mathematics to real-life situations, Samuel (2002:4) gave learners a quarterly project that required all the learners to take photographs of objects from nature and the world around them outside the classroom and to link them with geometrical items used in the classroom. One of the learners commented that the technique helped him to learn a lot more about Geometry.

Summary

In summary, Shahrill (2013:225) advises teachers not to rely on questions found in mathematics textbooks, but to use questions with a variety of functions in mathematics classrooms for learners to be given an opportunity to describe the solution to the problems given, irrespective of whether the response is correct or not, and to direct the responses to the teacher and classmates to provoke class discussion. During discussions, as learners listen to different views expressed, "learners will gain from this experience of learning".

4.5 Questioning techniques used in mathematics classrooms

Using Brown's (2007:132) definition, questioning techniques can be defined as specific 'attacks' that teachers employ with the questions they pose to solve problems. In her summary on the importance of questioning, Philpott (2009:72) states that questioning is a vital skill that allows learners to explore and articulate their own understanding and to challenge others' thinking as it can introduce, develop and consolidate areas of study or raise and crush learners' self-esteem.

Similarly, Molefe (2006:79) states that questions inform the direction of the discourse and the learners' reflection on their learning and formative assessment, and therefore the questioning techniques a teacher employs are important tools. As a result, teachers should be taught the questioning techniques and they should use them with caution to lead learners to be more actively involved in the lessons, and in that way promote learners' understanding of mathematical discourse and ESL development. The following questioning techniques in Table 4-4 have been identified from the literature reviewed and discussed in more detail in the section that follows.

Table 4-4: Questioning techniques used in mathematics classrooms

QUESTIONING TECHNIQUES USED IN MATHEMATICSS CLASSROOMS		
TYPES	**FUNCTION**	**EXAMPLES**
Modifications	Clarifying questions for learners to be able to understand concepts	Cues, translations, clarification requests, explanations, repetitions, paraphrasing, expansions
• Repetition and rephrasing	For learners to hear the question for the second time and for it to be clarified to the learners	*What do we mean by a difference of 2 squares*? (2 times) What is a difference?
• Paraphrasing the question	To provide content and linguistic scaffolding	*What is the length of the hypotenuse*? *What is the length of BC*?
• Redirecting the question	Asking another learner to respond to the question for more learner participation	*John, what is your opinion to Mary's answer*?
Probing	To investigate the learner's incorrect answer	*Could you explain why you divided by half*?
Moving from closed to open-ended questions	To get a qualitative answer	*How many sides does a quad have?, the closed question is changed to be open-ended: What do you notice about this figure*?
Wait-time	Giving learners 3 - 4 seconds to think about the question asked to facilitate higher-cognitive thinking.	T: *What is the value of x in the expression* x^2 +

QUESTIONING TECHNIQUES USED IN MATHEMATICSS CLASSROOMS		
TYPES	**FUNCTION**	**EXAMPLES**
		$5x + 6$? I will give you 2 minutes to give me the answer.
Revoicing	Affirm learners' responses and make them available to all especially to those who did not know the correct answer	L1: *$a^2 - b^2$ is a difference of two squares*. T: *Yes, $a^2 - b^2$ is a difference of two squares*.

Source: Own compilation from literature reviewed (Bellido *et al*., 2009; Borich, 2004; Cashin, 1995; Chin, 2006; Kazemi & Stipek, 2001; Moyer & Milewicz, 2002; Philpott, 2009; Swain, 2002; Tuan & Nhu, 2010)

4.5.1 Modifications

Questioning techniques like modifications are suggested by Swain (2002:620-621) and these include clarification requests, statements of non-understanding, requests for reformulation, explanations, expansions, repetitions, paraphrasing or elaboration. The questioning techniques are in line with Long's (1996) Interaction Hypothesis which claims that comprehensible input is the result of modified interaction. The hypothesis makes sense, especially with regard to the questions asked by the teachers during the lessons. When realising that learners do not understand the questions asked, teachers resort to modifications like verbal and non-verbal cues, code-switching and translations to make questions 'comprehensible' to the learners. All these are modifications that ultimately assist learners to try and respond to the questions asked. Some of them are discussed below.

Repeating and rephrasing

Repetition and rephrasing of the teachers' questions, according to Cashin (1995), are questioning techniques used to ensure that the entire class hears the question for the second time and to check the learners' understanding of the question. They also give other learners time to think about the question and possible answers to it.

Redirecting

The teacher can also redirect the question by asking another learner to respond to it or the whole class in general to comment or elaborate on it. This procedure not only encourages more learner participation, but it also implies that peers are a resource for learning (Cashin, 1995).

Paraphrasing

The teacher can also paraphrase the learner's answer to allow those with weak language abilities and who may have difficulties in verbalising their thoughts the opportunity to co-construct a response with their teacher and peers. In doing so, the teacher provides not only conceptual, but also linguistic scaffolding (Chin, 2006:1336). This is also supported by Swain's (2005) Output Hypothesis when she states that learners can improve the accuracy of output if they receive feedback from teachers. That is why she encourages teachers to offer adequate input, to manage and push the learners to produce the target language by giving more opportunities and much more practice time to learners during the process of language learning (Tuan & Nhu, 2010:42).

Probing

Probing is another questioning technique defined as the use of different types of questions to invite answers or further investigate the child's incorrect answer (Moyer & Milewicz, 2002:301). Its function is to communicate to the child that the answer is still open for discussion and to assess what the child is thinking (2002:306). Probing and follow-up questions used in a study to examine the questioning techniques of 48 pre-service teachers, resulted in their learners not only providing answers to the questions, but also providing stimulating, relevant discussion about the child's thinking. During that process, learners' interaction using language took place when the learners' responses to the probing questions were analysed (Moyer & Milewicz, 2002:294).

Similarly, in a study that involved four teachers teaching the addition of fractions to Grades 4 and 5 mathematics classes in three schools, the researchers' analysis of video-taped learner-teacher interactions revealed that high press exchanges for conceptual thinking was produced using probing questions such as,

Could you explain why you divided it in half?

What does that mean if there are eight halves?

Could you tell us why you chose eights?; etc.

(Kazemi & Stipek, 2001:65).

The probing questions, according to the researchers, resulted in learners going beyond the explanations or summaries of the steps they had taken to arrive at the correct answer. This resulted in explanations consisting of mathematical arguments that were characterised by the following socio-mathematical norms:

- the general ways in which learners participated in the classrooms that are specific to mathematical activities, such as an explanation consisting of a mathematical argument, not simply a procedural description;
- mathematical thinking involving understanding relations among multiple strategies;
- errors providing opportunities to reconceptualise a problem, explore contradictions in solutions, and pursue alternative strategies; and
- collaborative work involving individual accountability and reaching a consensus through mathematical argumentation

(Kazemi & Stipek, 2001:60).

Low press exchanges, on the other hand, in which the above-listed probing questions were not used, resulted in the teachers, and not the learners, ending up explaining how the problems were solved, subsequently resulting in limited conceptual discourse (Kazemi & Stipek, 2001:70).

Furthermore, to promote higher-order thinking skills, the teacher could modify a basic mathematics question by asking a learner for a qualitative answer in which the learner demonstrates application, analysis or synthesis. For example,

1. The temperature of the Sun's core is estimated to be 20 000 000 ^{0}C.

a) If a star is 50 times hotter than the Sun, what is its temperature?

b) If a planet is 100 000 times colder than the Sun, what is its temperature?

(Laridon *et al*., 2008:154).

Learners' feedback can be used as a starting point after it has been affirmed as correct by the teacher to raise new questions to take learners' thinking forward. This can include the use of probing questions like *explain why you said Yes or No*, and in that way, extend what the learners were really thinking (Chin, 2006:1336).

4.5.2 Moving from closed to open-ended questions

Changing closed questions into open-ended questions is another questioning technique teachers can use in mathematics classrooms. For example, instead of asking a learner to point at a figure that represents $^2/_3$, a teacher could show all the figures that represent $^2/_3$ in slightly different contexts and request learners to identify and explain why they say the figures represent $^{2}/_{3}$. That, according to Bellido *et al*. (2009:6), will help learners realise that fractions have different meanings in different contexts and in that way, conceptual questions will not encourage learners to memorise, but they will provoke learning and not just confirm what has already been learned.

Again closed questions can be changed into open-ended questions when teachers ask for a qualitative answer by modifying a rote memory or basic arithmetic question and asking for further explanation. For instance, on the rules governing formulae of triangles, instead of being satisfied with the learner's answer to the question, *How many sides does a quadrilateral have?*- a closed question -the teacher could ask the learners the question, *What do you notice about these figures*?- an open-ended question (Capacity Building Series, 2011:2).

Furthermore, the teacher can elicit alternative ideas and approaches to closed questions (Charles, 2012:1). For example, a closed question like *Do you agree that* $2x^4$ *is the answer*?, can be rephrased as follows: *Does anyone have the same answer* $2x^4$ *but a different way to explain how they got it*? In that way the learner would be processing input into intake for second language acquisition to take place, since "input alone is not sufficient for acquisition" (Gass, cited in Tuan & Nhu, 2010:43).

By changing closed into open-ended questions, teachers promote learners' understanding of mathematical discourse and ESL development. Bellido *et al*., (2009: 2) discourage teachers from focusing only on the correct answer. Teachers are encouraged to check how the learner got the answer right, using open-ended questions. They used an example of $^{16}/_{64}$ = ¼, to illustrate that it is possible for a learner to get a correct answer by using a wrong method. The answer is correct even though the learner had used the wrong method of deleting the number *6* that appears in the numerator *16* as well as in the denominator *64.* To avoid such mistakes, learners should be given a chance to explain the rationale for their correct answers. As they explain, they will be using language in the form of sentences, clauses and also mathematical discourse, proving that they do understand the problem given.

4.5.3 Wait time

Wait time is defined as the duration of pauses separating utterances during verbal interaction (Tobin, 1987:69). Wait time facilitates higher cognitive level learning by providing teachers and learners with additional time to think. Philpott (2009:67) suggests wait time of about '1-30 seconds', between the question and learners' responses, depending on the level of difficulty of the question for all the learners to be given a chance to think about the question asked. This enables learners to think deeply about the answer and about the language they have to use to make their answers logical and comprehensible to both the teacher and other learners.

Borich (2004:277) and Philpott (2009:67) concur on the advantages of wait time, stating that it enables learners to give long answers to questions. Learners' responses were more confident and there were fewer unanswered questions. To encourage teachers to make use of wait-time, Shahrill (2013:228) states that, "the time spent in waiting is worth the wait and is made up in the responses teachers gain by making wait time an important element in questioning."

4.5.4 Revoicing

Revoicing, according to Adler and Setati (cited in Gaoshubelwe, 2011:36), is the technique where the teacher listens to the learners' mathematical talk and repeats what the learner has said in a well-constructed sentence in order to lead them towards the correct and formal mathematics discourse. So, when the learners respond to the teachers' questions during the course of the lessons, the teacher can respond to the learners' feedback in a variety of ways. For example, the teacher can 'revoice' the learner's response, such as, $a^2 - b^2$ *is a difference of two squares*, the teacher will say $a^2 - b^2$ *is a difference of two squares*, to affirm it and to make the idea available to all in the class, thus making it common knowledge, especially in the case of those who did not know the answer.

Summary

In summary, to challenge the current state of affairs with regard to teachers using mostly closed questions in their classrooms, Philpott (2009:66) advises that questioning is a skill that can be learned and successfully executed through the use of simple techniques, some of them discussed in this section, and through deeper understanding of lesson objectives and subject progression. Reflecting on the advantages of using skilful questioning skills, Chuska (1996:7) and Shomoossi (2004:98) state that questioning can also lead to more interaction among learners and more learner-initiated questions - output that improves language learning. In addition, learners' feedback on the questions and the questioning skills used by the teacher provide valuable information for the teacher's immediate attention. This relates to the difficulty level of the topic addressed, the ability level of the learners, the curriculum time available and the teacher's preferred style of teaching (Chin, 2006:1338).

4.6 Teacher strategies used in mathematics classrooms

Questioning techniques refer to what the teacher does with respect to the questions used and learners' correct and incorrect responses. Teacher strategies, on the other hand, are specific methods of approaching a problem or a task, modes of operation for achieving a particular end, and planned designs for controlling and manipulating certain information (Brown, 2007:119). Therefore, teacher strategies can be defined as teacher's methods of approaching a problem that arises during the course of teaching and assessing the lesson to make it fully comprehensible for the learners. Shahrill (2013:226) advises teachers when asking open-ended questions to "generate an essence of curiosity and wanting to know" and that can be achieved by making use of a variety of strategies discussed in the next section.

Table 4-5 summarises teacher strategies from the literature reviewed. These will be discussed in detail in the section.

Table 4-5: Strategies used in mathematics classrooms

STRATEGIES	PURPOSE	EXAMPLES
No-hands strategy	No hands are raised for everyone to think about the question asked and respond when requested to do so.	T: *Yes Mary, give us the answer*.
Basketball method	Teacher becomes part of the whole class activity.	T:*In the example 5a + 6b + 7c, we have how many terms*? L1: *3* T: *What is the first term*? L 2: *5a* T: *What is the second term*? L 3: *6b* T: *What is the third term*? L 4: *7c* T: *Very good*
Linking concepts	Linking target concepts to its related mathematical category	Drawing figures of rectangles to introduce a lesson on squares
Visual, graphic and aural representations	To represent learners' mathematical thinking	Teacher draws line AB parallel to line CD

STRATEGIES	PURPOSE	EXAMPLES
Varying learners' responses	Incorporate the four basic skills into the teaching and learning of mathematics	T: *Write an essay on Pythagoras explaining how he came up with his theorem*.
Pre-teaching vocabulary	For learners to learn mathematical discourse	At the beginning of a new unit, learners are requested to make a list of new words and find their meanings
Adding extra data that make sense	Simplify mathematics and relate it to real-life experiences	Using the examples of cows and sheep for learners to understand the mathematical concept that $5a + 6b = 5a + 6b$
Process approach	Make learners aware of problem solving techniques they should apply when solving problems	Realising that 53 + 64 can be simplified as (50 + 3) + (60 + 4) and like terms 50 + 60 and 3 + 4. added to find the answer.
Motivation	Create a conducive environment for teaching and learning	T: *We must all try, but we should not fail to try. This is how we learn*.
Collaborative practice	Employ pair- and group-work activities	T: *Question 1 is for group 1 and questions 2 and 3 should be done in pairs*.
Building Strategic Techniques	Providing an atmosphere that is conducive to SLA and mathematics learning	For cooperative learning in the classrooms

Source: Own compilation from the literature reviewed (Boaler, 2008; Brown, 2007; Philpott, 2009; Samuel, 2002; Turner et al., 2002;)

4.6.1 The No-hands strategy

During the course of the lessons, teachers may find themselves getting answers from the same small group of learners. To encourage all the learners to participate and engage in the lessons, Rowie (1974, cited in Philpott, 2009:67), recommends the *'no-hands' strategy,* which means exactly what it says. Learners are not allowed to raise their hands to respond to questions asked. They are expected to think deeply about the answer during the wait-time period and should be in a position to provide a response, and if they do not know the answer, they should state that they do not know it. As they think about the answer, they also think about the language to be used for their answers to be understood by both the teacher and their peers, thus developing language in the process. The strategy also forces each learner to think about the answer and give a response rather than embarrass her/himself by admitting to the whole class that s/he does not know it.

4.6.2 Basketball versus Ping-Pong

Philpott (2009:69) recommends the Basketball method over the Ping-Pong method of questioning during classroom lessons. The Ping-Pong method describes a situation where the teacher throws questions back and forth to the learners using broadcast questions. This results in learners getting bored and the teacher getting exhausted. The Basketball method on the other hand describes a situation in which the teacher is part of a whole class activity in which all learners are engaged and responsive, thus encouraging development of learners' understanding of mathematical discourse and the language of instruction.

4.6.3 Linking concepts

Other strategies for creating mathematical discourse involve the teacher asking questions by linking the target concept to its related mathematical category. For example, for learners to understand the meaning of 'quadrilateral', other figures within this category (unilateral, trilateral, squares, etc.) are also mentioned as part of the question. Related to this strategy is the technique where teachers repeat the target concept in the question in the feedback and in written format, subsequently practising a second language development strategy - repetition - classified under cognitive strategies used in ESL classrooms (Philpott, 2009:70).

4.6.4 Visual, graphic and aural representations

Garrison and Mora (1999:45) recommend the use of graphics and writing as they are necessary tools for all learners to be able to represent their mathematical thinking. For example, $3 \times 2 = 6$ can be illustrated graphically and in verbal or written form with three circles written in a row or column two times, making a total of 6, to explain the concept of multiplication, which means *how many times we do something.* This enables learners to visualise abstract ideas and relationships and to develop a fuller understanding of the underlying concepts.

For aural representations, Samuel (2002:3) incorporates the writing skills in mathematics classrooms by requesting learners to write poems that could be used to increase vocabulary skills in Geometry. Learners were required to write a poem that rhymes and contains four definitions and two properties from Geometry. In that way, learners were able to learn Geometry, thus enhancing their mathematical discourse and their ESL development as they recited to the whole class what they wrote and learnt.

In short, Carlan (2007:7) states that comprehensible input in the form of visuals, demonstrations, and hands-on experience can be used to improve learners' understanding of mathematical discourse.

4.6.5 Varying learner responses

Adair and Houston (1996:3) in their study to incorporate writing skills in mathematics classrooms, encourage activities such as narrative writing, impromptu writing prompts, writing word problems, journal writing, etc. Their findings revealed that the writing activities provided learners with the opportunity to reflect on the work of the day or week, thus acting as a revision aid to question themselves about their own thought processes and to take responsibility for their own learning. Such open-ended writing activities enable learners to explain and understand how they learn mathematics. This means that the learners employ meta-cognitive strategies and take conscious control of learning and analysing the effectiveness of learning strategies; and thus gain confidence and become more independent as learners (Wahl, 1999). This strategy, according to Bond (2007:20), "provides more opportunities for interaction, thus, reducing management problems".

4.6.6 Pre-teaching vocabulary

In an attempt to assist English as a Second Language (ESL) Fairfax high school learners who avoided taking Geometry because of its difficult vocabulary and instead chose Algebra as a major in order to graduate, Samuel (2002:1-2) came up with the following strategies that incorporate all four basic skills into the teaching and learning of mathematical discourse and ESL development to assist them to learn the vocabulary used in the subject.

- At the beginning of a new unit, each learner had to create a list of new words that were peculiar to that unit to increase their vocabulary in mathematics;
- During the lessons, learners were required to give the definition of the words they had listed, consequently enhancing their mathematical discourse and ESL development in the process.

4.6.7 Adding extra data that makes sense to the learners

By adding extra data that make sense to the learner, the teacher provides learners with opportunities to engage in practices that are represented and required in everyday life (Boaler, 2008:118).This simplifies mathematics by closing the gap that sometimes exists between mathematics learnt at school and its application in real life situations. This was confirmed in Boaler's (1997) findings which showed that learners who were taught mathematics in schools using the ‘traditional textbook approach’ were outperformed by those who were taught in schools that used ‘open-ended’ projects at all times (Boaler, 2008:113).

4.6.8 Process approach

To make use of the process approach to problem solving questioning strategy, teachers are advised to make learners aware of problem-solving techniques that they should apply while asking themselves questions. Questions like ‘*what*’, ‘*how*’ and ‘*where*’ should be asked during each step as teachers assist learners while they attempt to solve problems given in exercises as class-work or homework: understanding a problem by getting familiar with every aspect of the problem; devising a plan by finding the relation between the condition and the unknown; carrying out the plan you made in the previous step; looking back to check the answer in many ways (Biryukov, 2004:2).

4.6.9 Motivation

There is a myth that mathematics is a difficult subject (NRC, 2001:142-144), so when learners do not perform well in the subject, they believe that it is a fact and that makes them feel discouraged and less motivated. A study was conducted by Turner *et al*., (2002) to examine aspects of a learning environment, like for example a teacher’s instructional or classroom discourse’s effects on learners’ attitude towards mathematics. In their findings, learners reported that they were highly motivated and did well in the subject in classrooms perceived as emphasising learning, understanding, effort and enjoyment, in other words in classrooms in which teachers provided instructional and motivational support for learning. Data collected revealed that teachers conveyed mastery messages to their learners, messages that emphasised that being unsure; learning from mistakes; and asking questions were natural and necessary parts of learning (Turner *et al*., 2002:103).

The above strategy is in line with Krashen’s Affective Filter Hypothesis, which claims that acquisition would occur in environments where anxiety was low and defensiveness was absent, in other words in contexts where the ‘affective filter’ is low (Brown, 2007:295). If the learners are anxious about what they learn, they will find it difficult to understand, let alone improve their language development in the process. In such classrooms where teachers provided instructional support or what the researchers refer to as ‘scaffolding’, learners used less avoidance strategies like self-handicapping, avoidance of help seeking, and a preference to avoid novel approaches to engaging in academic discourse (Turner *et al*., 2002:102-103), as they enjoyed the subject, and as a result, performed well in the end. Conversely, instructional discourse practices that emphasised cognitive aspects such as ‘final answers’ lacked the teacher’s sense of caring and humour and negatively affected learners’ motivation, resulting in the learners’ poor performance in the subject.

4.6.10 Collaborative practice

To enable language development and production to occur, Carlan (2007:13) encourages teachers to employ pair- and group- work strategies as they promote language practice in listening, speaking, reading and writing activities. Many researchers assert that practice is the most beneficial when carried out in collaboration with small groups or peers, rather than with the teacher or in a whole-class setting. Group work allows learner-learner interaction and peer interaction to take place, which enables learners to practise language in a more relaxed atmosphere, free from the teacher’s scrutiny; to provide language models; and to interact with one another (Tuan & Nhu, 2010: 36).

Similar research conducted by Doughty (1985) found that teacher-learner interaction generated less input from learners than learner-learner interaction (Tuan & Nhu, 2010:41). Furthermore, second language learners need comprehensible input, they need to be in situations that provide maximum personal involvement in the communication, and they need opportunities to use the target language (2010:43). Therefore, it can be concluded that collaborative practice during pair- and group- work activities does facilitate language development (Ellis, 2003; Mercer, 2004; Light & Glachan, 1985; Harmer, 2001) as discussed in (Tuan & Nhu, 2010:36).

4.6.11 Building strategic techniques

Some of the strategies discussed in this section are also suggested for ESL teachers in what is called ‘building strategic techniques’, to provide an atmosphere in the classroom that is conducive to ESL development on the part of the learners (Brown, 2007:146). The strategies were compiled from the results of a Strategy Inventory for Language Learning (SILL) questionnaire that was tested and distributed to language learners to complete in many countries around the globe and even translated into several languages (Brown, 2007:143). Some of these teacher strategies can also be used in mathematics classrooms and these include, among others, promoting cooperative learning by encouraging learners to work as a team in group activities and not compete with one another; getting learners to make their mistakes work for them, letting the learners explain how they arrived at the answer; and with peer-correction, identifying the mistake in the process; promoting tolerance for ambiguity by encouraging learners to ask questions if they do not understand; getting learners set their own goals by encouraging them to go beyond the classroom goals by giving them credit for extra work done.

These strategies, if applied in mathematics classrooms, would empower mathematics teachers to organise their lessons in such a way that learners see a relationship between mathematics and real-life situations. Mathematics teachers can in this way develop an atmosphere that is conducive to learning and teaching as they will be demystifying mathematics and making it enjoyable as a subject. Researchers Khisty and Chval (2002:167) in their attempt to promote that kind of environment, recommended that teachers should make speaking mathematically a ‘critical part’ of learning the subject since the teacher is the only one who engineers the learning environment. This can be achieved if, and only if, learners can be stretched mentally through sensitive teacher-led, but not teacher-dominated discourse (Chin, 2006:1343).

Summary

In summary, despite some teachers’ complaints like, for example, not having enough time to get through the syllabus, Jones and Tanner (2002:269), with regard to using the suggested questioning techniques and strategies to promote learners’ understanding of mathematical discourse and English SLA development during lessons, the researchers indicated that there were also positive spin-offs that were captured during interviews with regard to teachers who changed their old traditional methods of teaching to accommodate teacher-led discourse during lessons. Some of them stated that, “the culture in my classroom is now one where children feel at ease to develop their own methods, have the confidence to participate in the discussion, and view mathematics as fun and achievable” (Jones & Tanner, 2002:269). Consequently they were able to do away with a myth that mathematics is a difficult subject for a chosen few, a myth that we still have to get rid of in our South African schools by discouraging teachers from clutching their old teaching styles and encouraging them to collaborate with ESL teachers to embrace and apply the suggested questioning techniques and teacher strategies that are applicable in both ESL and the mathematics classrooms as shown in the next section.

4.7 The theoretical model

The theoretical model

Chapter 2 elaborates on the teaching and learning of mathematics which has been described as a language, and Chapter 3 elaborates on the teaching and acquisition of ESL. As a result, the theoretical model in Figure 4.1, shows the similarities between the theories of the teaching of mathematics and ESL, and the learning of mathematics and ESL acquisition. From the literature reviewed, the themes that emerged, namely, comprehensible input, language processing and interaction, output, and feedback, are in response to the first research question 1:

> **What are the Second Language Acquisition theories and mathematics learning theories that underpin the effective questioning techniques to promote ESL acquisition**?

The themes are shown in the theoretical model and discussed thereafter.

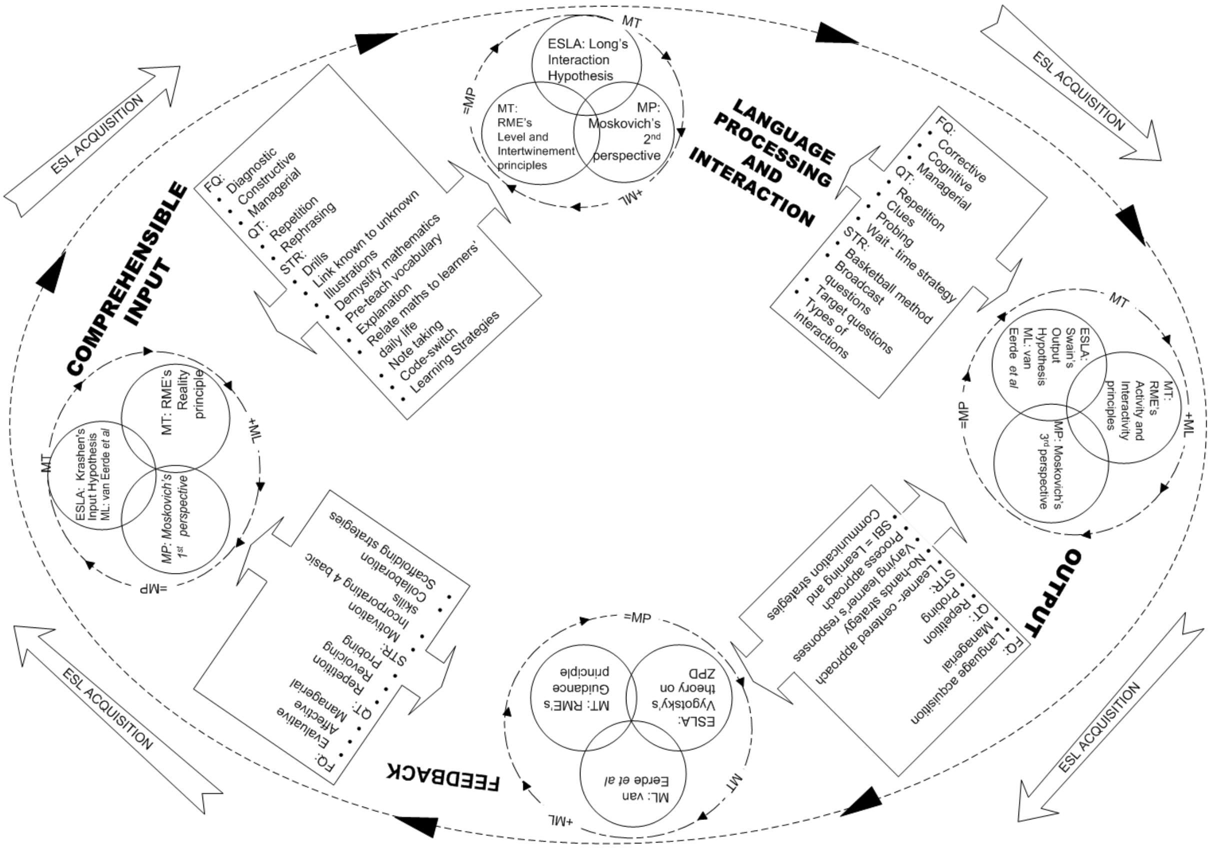


Figure 4-1: The theoretical model

4.7.1 Comprehensible input

According to Krashen's Input hypothesis in ESLA (Krashen, 1981, 1984), as well as van Eerde *et al* (2008)'s 1st condition for mathematics learning, learners in ESL and maths classrooms should be provided with comprehensible input at level (*i + 1*), i.e. input that is challenging to the learners, and not input that is very easy at level (*i + 0*), or difficult at level (*i + 2*). Furthermore, for learners to be able to understand what is taught in mathematics classrooms and achieve mathematical proficiency, according to Moskovich (2002)'s 1st perspective, provisions should be made for learners to *acquire vocabulary* referred to as ‘*mathematical discourse’*, and also learners should be given real-life problems that are meaningful to them, according to the RME’s Reality principle (Van den Heuwel-Panhuizen & Drijvers, 2014), for them to understand what is taught in mathematics classrooms.

The types of questions indicated, together with the questioning techniques and strategies, when used in these classrooms, will also provide learners with comprehensible input. In other words, for input to be made comprehensible, teachers in ESL and mathematics classrooms should use questions with diagnostic, managerial, and constructive functions that assist teachers in identifying learners’ problems and to address them in remedial lessons. Questions for the managerial function are used to maintain control and discipline for lessons to run smoothly, and to address any disruptions. Also, as soon as teachers realise that learners do not understand the questions asked, they can apply questioning techniques such as modifications, repetition and rephrasing, to make them comprehensible. Again, when teachers realise that learners do not comprehend the questions and lessons, strategies such as pre-teaching vocabulary, using visuals, graphs, and aural representations, linking the known to the unknown, and other learning strategies, can be used for learners to comprehend the questions and lessons.

4.7.2 Language processing and interaction

For learners to be able to process and interact using language, according to Longman’s Interaction Hypothesis (1996, 1985), comprehensible input has to be modified using the different types of interactions. Similarly, Moskovich (2002)’s 2nd principle emphasises *constructing meaning*, implying that everyday meanings and learners’ home language can be used for mathematical formulations and concepts for learners to acquire mathematical proficiency. Furthermore, RME’s Level and the Intertwinement principles (Van den Heuwel-Panhuizen & Drijvers, 2014), underline that learning mathematics means that learners should be taught in such a way that they see the concepts taught as inter-related, and not isolated from each other.

Furthermore, to engage learners in language processing and interaction in ESL and mathematics classrooms, questions which have the managerial, cognitive and corrective functions, can be used during the lessons for them to interact and process language. Also, questioning techniques like repetition, providing clues, wait-time, and probing incorrect responses, provide learners with opportunities to interact and process language as they explain steps followed in arriving at their incorrect responses, and thus identify their mistakes in the process. In addition, when teachers get responses from the same learners, strategies such as the no–hands, basketball method, broadcast and target questions, and other types of interactions, when applied, will provide all the learners with opportunities to think about the questions asked and use language to process and interact with one another.

4.7.3 Output

For learners to produce output, according to Swain's Output Hypothesis in ESLA**,** and van Eerde *et al* (2008)'s 2nd condition for mathematics learning**,** learners should be provided with opportunities to produce output. Also, to achieve mathematical proficiency, according to Moskovich (2002)'s 3rd perspective, learners should be given opportunities to *participate in discourse.* Similarly the RME's Activity and Interactivity principles (Van den Heuwel-Panhuizen & Drijvers, 2014), emphasise that learners should be treated as active participants in mathematics classrooms, and therefore group-work activities should be used in these classrooms for learners to be provided with opportunities to produce output. Therefore, questions for the listed functions, questioning techniques and strategies, should be used by the teachers in these classrooms to provide learners with opportunities to produce utterances in the form of output.

In an effort to engage learners in producing language, questions with the language production and managerial functions, can be used for learners to use language in their responses. Also, learners should be encouraged to produce output by repeating and probing their correct responses for them to explain how they arrived at their correct responses, and thus discourage copying and guessing. Furthermore, teacher strategies such as, no-hands, learner-centred approach, varying learners' responses, and Strategy Based Instruction, a combination of learning and communication strategies, when applied in these classrooms, will provide learners with opportunities to practise and produce language.

4.7.4 Feedback

For learners to do well in acquiring both ESL and mathematical discourse**,** they should be provided with feedback on their utterances. This is emphasised in Vygotsky's ZPD theory on feedback and also in van Eerde *et al (*2008)'s 3rd condition for mathematics learning. This is also stated in the RME's Guidance principle (Van den Heuwel-Panhuizen & Drijvers, 2014), where teachers are encouraged to provide scaffolding or support in the form of feedback on learners' utterances**.**

Teachers in these classrooms therefore should use the listed questions for the functions indicated, questioning techniques, and strategies, to provide learners with feedback, for them to be able to improve on their language acquisition in both ESL and mathematics classrooms. In other words, for learners to be provided with feedback on their utterances, questions with managerial, evaluative, and affective functions should be used in as far as their oral utterances and written assignments are concerned. Also, questioning techniques such as repetition, probing, revoicing, will assist learners to know as to whether their utterances are correct, and if not, they will learn from the teacher's feedback how to rephrase them correctly. In addition, teacher strategies such as, motivation, incorporating the 4 basic skills, collaboration with ESL teachers to assist them with a variety of methods to teach mathematics, and scaffolding strategies, when applied in these classrooms, will provide learners with feedback on their utterances, to improve the target language, i.e. mathematics.

It should be noted that the managerial function of questions appears under all the processes for the 4 themes, because for all these processes to run smoothly without any disruptions, teachers have to maintain control and discipline throughout the lessons by using this function of questions.

In addition, the inner concentric circle shows that English SLA is taking place throughout the processes, while the outer concentric circle shows that mathematics teaching, together with mathematics learning, produces mathematical proficiency in all the four aspects discussed. Also, the arrows which are cyclical in nature, with 'Reflections', on it, show movement created by questions, questioning techniques and teacher strategies, thus moving learners from input to language processing and interaction, from language processing and interaction to output, from output to feedback, and vice versa.

Summary

Since current research on second language acquisition and mathematics learning show that learners also need to actively use comprehensible input to process language through interactions; to produce new linguistic elements in meaningful contexts; and to receive feedback (Krashen, 1981, 1984; van Eerde *et al*., 2008; Moskovich, 2002; RME's principles, (Van den Heuwel-Panhuizen & Drijvers, 2014; Long, 1996, 1985; Swain, 2005; Vygotsky, 1978), the theoretical model illustrated assisted the researcher to design a hands-on tool discussed in chapter 8 to assist mathematics teachers to plan lessons that would provide opportunities for "classroom interaction to form a critical resource for learning" (Van Eerde *et al*., 2008:33).

4.8 Conclusion

Chapter 4 discussed the literature review regarding the types of questions and their functions, together with questioning techniques and teacher strategies used in mathematics classrooms. These are related to input, language processing and interaction, output, and feedback. Finally, a theoretical model based on the theories, principles, and perspectives as well as conditions for mathematics teaching, mathematics learning, mathematical proficiency, and English SLA, is illustrated and discussed. The functions of questions, questioning techniques, and teacher strategies, are also included in the model. Chapter 5 discusses the qualitative research design and the case study method carried out during the data collection and analysis processes. The ethical considerations applied during the research process are also discussed.

CHAPTER FIVE: RESEARCH METHODOLOGY

5.1 Introduction

Chapter 4 on questioning explained how the theoretical model synthesised the theories and conditions for mathematics and ESL teaching, mathematics learning and English SLA, and mathematical proficiency, together with questioning techniques and teacher strategies. Chapter 5 discusses how the research was conducted using the interpretivist paradigm, a qualitative research approach with a case study method. The reasons for choosing this method are also explained. Tables on the participating schools and the participants are provided and discussed. The data collection and analysis procedures, as well as the limitations of qualitative research and how they were addressed, together with ethical considerations, are discussed. The chapter also explains how triangulation, validity, and qualitative reliability and trustworthiness were maintained throughout the research process.

5.2 Research paradigm

In choosing qualitative research, researchers make certain assumptions about the nature of reality (ontology), how they know what they know (epistemology), the role of values in the research (axiology), and the language of the research (rhetoric) (Creswell, 2007:16). These assumptions shape their research by bringing to the inquiry paradigms or world views. A paradigm is "a basic set of beliefs that guides action" (Creswell, 2007:19). It informs qualitative researchers to identify how the set of their beliefs or worldviews shape the direction of the research to be chosen. The researcher chose the interpretive paradigm to utilise the qualitative method, which supports the view that there are many truths and multiple realities. The research also utilises a triangulation approach to explore the types of questions used in grade 10 mathematics classrooms (Michel, 2008: 41).

5.2.1 Interpretivist paradigm

The researcher, as a former mathematics teacher at high school, chose the interpretivist paradigm to examine the role of questions in mathematics classrooms, and to find out whether questions, among other aspects, also have an impact on learners' poor mathematics performance reported annually when the grade 12 results are made public (DBE, 2014).The purpose of the research was to interview the participants (mathematics teachers) to recognise the value and depth of their individual responses to the open-ended questions asked. As a result of the data collection procedures followed, the results were descriptive and explanatory in nature after they had been analysed.

The study further draws on the principle that the sample drawn should be representatives who are able to provide expertise from different points of view (Michel, 2008:41-42). It also draws on the perspective that social life is a distinctly human product (Nieuwenhuis, 2007:59). The underlying assumption is that by placing teachers in their natural 'social contexts' provides a better opportunity to understand the perceptions they have of their own activities (Hussey & Hussey, cited in Nieuwenhuis, 2007:59). In the case of this study the natural setting was the grade 10 mathematics classrooms where mathematics teaching and learning had been taking place since the beginning of the 2012 academic year. Also, the researcher focussed on the multi-perspective stories of individuals (four mathematics teachers), who told their stories about the types of questions, questioning techniques, and teacher strategies they used in their classrooms.

Interpretivist qualitative researchers use case studies, ethnographic studies, phenomenographic studies (Weber: 2004), methods that include focus groups, observations, interviews, documents and field notes in research diaries. The researcher's goal, as a result, is to make sense or interpret the meanings others have about the world, hence qualitative research is often called "interpretive "research (Creswell, 2007:21)

Furthermore, interpretive projects can represent underrepresented or marginalised groups (Creswell, 2007: 24). These include mathematics teachers in rural schools who are expected to produce good results at the end of each academic year just like those in advantaged urban schools, even though they live in rural communities and disadvantaged schools that have very limited resources.

5.3 Research approach

5.3.1 Qualitative approach

The researcher, in the case of this study, chose the qualitative approach as it is subjective and relies on the personal experiences of the researcher as a former mathematics teacher at high school. The purpose was to investigate the roles of questions, questioning techniques, and teacher strategies used by mathematics teachers to promote learners' understanding of mathematical discourse and mathematical language in the classrooms. Since the study is qualitative in approach, words in the form of lesson plans, field notes, transcribed lesson observations, as well as open-ended questions during interviews, were used to collect open-ended data with regard to information relevant to the research questions based on the types of questions, questioning techniques, and strategies used in grade 10 mathematics classrooms, with the primary intent of developing themes from the data (Creswell, 2003:18).

Also, the researcher's goal in using the qualitative approach was to comprehend how the participants 'constructed meanings explained the events of their world' as they encounter them in their daily natural setting by conducting interviews (Creswell, 2007:24). The underlying assumption is that by placing teachers in their natural 'social contexts' in grade 10 mathematics classrooms where teaching had been taking place since the beginning of the 2012 academic year, "there is a greater opportunity to understand the perceptions they have of their own activities" (Hussey & Hussey, cited in Nieuwenhuis, 2007:59).

In addition, the qualitative approach was chosen because of the richness and depths of exploration and the descriptions it yields (Nieuwenhuis, 2007:60). This was the case with data collected from each of the four grade 10 mathematics teachers' week-long daily lesson plans, lesson observations, interviews, field notes, followed by a focus group interview. After data were analysed manually, and also after data on the transcribed lesson observations and interviews was analysed using the ATLAS.ti software, it yielded results on the rich descriptions from each of the four cases.

Qualitative research answers the questions *what* or *how,* and in the case of this study *what* types of questions, questioning techniques, and teacher strategies, were used in mathematics classrooms; and *how* all these promoted learners' understanding of mathematical discourse and English SLA, were answered. The data collected were first analysed manually and thereafter using the ATLAS.ti software.

Crabtree and Miller (1999 cited in Harling, 2002:6), argue that the ultimate test of qualitative studies is that the work carries sufficient conviction to enable someone else to have the same experience as the original observer and to appreciate the truth of the account. In other words, good methods are important, but what really matters is good thinking. The next section therefore discusses the method used to investigate during the study, and how it relates to the phenomenon being investigated.

5.4 Case study

Qualitative research methods, according to Harling (2002:5), have developed to serve the view that a phenomenon, particularly when humans are involved, includes complex interactions and is seldom simply caused. To understand the event, all aspects of the situation have to be considered and this inclusiveness tends to mean that each situation is unique. The result is that qualitative researchers consider many variables in a case or a few cases. They probe deeply into a situation, describing the full range of influences associated with the phenomenon. They see benefit in understanding a particular phenomenon and hope that some of the understanding developed will transfer to other phenomena.

5.4.1 Definition of a case study

A case study is a holistic inquiry that investigates a contemporary phenomenon within its natural setting (Harling, 2002). The terms used in the definition are explained as follows:

- The **phenomenon** in the case of this study was the four participants teaching mathematics in grade 10 classrooms.

- The **natural setting** is the grade 10 mathematics classrooms at the four schools. The **phenomenon and the setting** bounded the system; that is, there are limits on what is considered relevant or workable, such as English and not Setswana as the MoI. The boundaries are set in terms of time, place, events, and processes, for example, conducting research during the period from 08h00 till 12h00, the period on the school time-table allocated for mathematics lessons in most of the classes observed. **Holistic inquiry** involves collection of in-depth and detailed data that are rich in content and also involves multiple sources of information in the form of transcribed lesson observations, field notes, individual and focus group interviews. The multiple sources of information provide the wide array of information needed to provide an in-depth picture (Harling, 2002: 2).

5.4.1.1 The collective case study

The collective case study, also known as the multiple case study approach, was followed to provide a general understanding using a number of instrumental case studies that either occur on the same site or come from multiple sites. The four case studies used came from multiple sites of four grade 10 mathematics classrooms at the four different schools. When collective case studies are used, a typical format is to provide a detailed description of each case and then present the themes within the case (within case analysis) followed by thematic analysis across cases (cross-case analysis). In the final interpretative phase, the researcher reports the lessons learnt from the analysis. When using multiple cases, the question of how many arises. Again the researcher has to provide a rationale for the specific number of cases used (Harling 2002).

The collective case study was therefore chosen as it allows for as many variables to be recorded as possible to have meanings that are varied and multiple, and also to rely as much as possible on the information provided by the situation. The types of questions to get the varied meanings from data collected are open-ended for the researcher to get an opportunity to listen carefully to what the participants say and do in their life setting, and from their interactions, a theory was developed.

The following conditions are relevant for the application of the case study method.

(i) Setting

In the case of this study, data were collected in natural settings, namely grade 10 mathematics classrooms, bringing the experiences closer to the researcher (in the form of the types of questions, questioning techniques, and teacher strategies) of the four teachers who participated in the study. The recordings even captured the noises of the teacher-learners' interaction or lack thereof as found in real-life classroom situations. The particulars of the four schools where data were collected are captured in Table 5-1

Table 5-1: Particulars of the four schools

INSTITUTION	SCHOOL A	SCHOOL B	SCHOOL C	SCHOOL D
Location	Village	Village	Village	Village
APO region	Zeerust	Zeerust	Zeerust	Zeerust
School category	Sub-system P	Sub-system P	Sub-system P	Sub-system P
Province	North West	North West	North West	North West
Number of learners in 2012	660	763	452	721
No of members in management team	5	5	2	3
No of HoDs	3	2	4	3
No of members in SGB	12	14	15	17
No of teachers	18	28	23	28
Medium of instruction	English	English	English	English
No of Grade 10 learners	160	233	203	108

The study was conducted at 4 grade 10 mathematics classrooms at the 4 high schools falling under the Zeerust APO region, situated in the Ngaka Modiri Molema district in the Ramotshere Moiloa Municipality of the North West Province in South Africa. Zeerust is a small town that lies in the Marico valley, approximately 240 kilometres north-west of Johannesburg. It lies on the N4, the main national road linking South Africa and Botswana.

Only one grade 10 mathematics lesson was observed at each of the four grade 10 mathematics classrooms at the 4 high schools in 2012 since each class was taught by each of the four participants. In all the four schools, mathematics is taught using English as the medium of instruction, a second language to both the teachers and the learners.

(ii) Aims

Case studies open the possibility of giving a voice to the powerless and voiceless (Nieuwenhuis, 2007:75). As the researcher writes the report on the findings of this study, the multi-faceted stories of the four teachers, together with their own voices are presented. The 4 teachers teach in rural areas, and as a result, they are greatly disadvantaged as they work in schools that have no basic resources such as decent ablution facilities, let alone fully-functioning libraries. Case studies also facilitate the construction of detailed, in-depth understanding as they help the researchers and the readers to understand complex inter-relationships in terms of the focus on the significance of the idiosyncrasies brought about as a result of multiple case studies (Hodkinson & Hodkinson, 2001:5).

(iii) Methods

The outcome of a case study is a description and interpretation of, in this instance, the four cases. The analysis section subsequently reports on the description of a theme based on each one of the four cases A, B, C, and D, as well as cross-cased themes based on all the four cases captured during focus group interviews. This makes the researcher's voice much more apparent (Vanderstoep & Johnston, 2009:210).

5.4.2 Participants

Case study involves qualitative research that focuses on the uniqueness of individuals. The participants selected were unique in as far as their age, educational qualifications, gender, and teaching experience are concerned. The sample included the four grade 10 mathematics teachers (N=4) who teach mathematics using English as a medium of instruction to ESL learners speaking Setswana, one of South Africa's official languages. Table 2 below provides information on the profile of each of the 4 participants.

Table 5-2: Particulars of the four cases A, B, C, and D

PARTICIPANTS	CASE A	CASE B	CASE C	CASE D
Teacher's academic qualifications	B.Ed (Hons)	Dip. In Educ.	B. Tech (Man.)	Grade 12
Teacher's professional qualifications	None	Diploma	UDES	ACE
Qualifications in mathematics	ACE	Diploma	ACE	ACE
Teaching experience in mathematics in years	17	6	17	9
Age in years	41	29	47	35
Gender	Female	Male	Female	Female
Teacher's Home Language	Setswana	Shona	Setswana	Setswana
No of G.10 maths classes teaching	1	5	2	3
No of G.10 mathematics learners in class for lesson observations	46	41	42	23
Number of boys	29	11	38	15
Number of girls	17	30	29	8
Range in learners' ages	15 to 22	14 to 17	14 to 17	16 to 20
Learners' Home Language	Setswana	Setswana	Setswana	Setswana
2011 Grade 12 Maths results	100%	63%	93.40%	69%
Prescribed book for Grade 10 Mathematics	Classroom Mathematics	Classroom Mathematics	Classroom Mathematics	Classroom Mathematics

The participants' ages range from 29 to 47 in years, and their teaching experience from 6 to 17 years. Even though Case A is more qualified than the rest of the participants with a B. Ed., a postgraduate degree in Education, she does not have a diploma for bridging the gaps in school teachers with current methods of teaching, known as University Diploma in Education Secondary (UDES). Cases A and C have many years of experience in teaching mathematics in Grades 10 to 12 classes, hence the good maths results they obtained in Grade 12 classes in the year 2011, (the results for the 2011 grade 12 learners are the pass rates for Case A, Case B, Case C, only) as Case D did not teach grade 12 learners. All the participants are females and speak Setswana as their home language, excepting Case B, a male who speaks Shona, a native language spoken in Zimbabwe, Mozambique and Botswana. All the four cases use the same prescribed book for grade 10 mathematics.

5.4.2.1 Criteria for case selection and their number

According to Perry (1998:793), there are no rules for sample size in a qualitative study. The number of cases suggested, it is cautioned, should be a mere guideline for the researchers, as long as the validity, meaningfulness and insights generated from qualitative inquiry have more to do with the information-richness of the cases selected and the observational/analytical capabilities of the researcher (Perry, 1998). In the case of this study, a sample of four teachers, namely Cases A, B, C and D, was selected for the researcher to observe how the participants used questions and their individual questioning techniques and teacher strategies in their natural setting to promote learners' understanding of mathematical discourse as well as English SLA development.

5.4.2.2 Multiple cases regarded as multiple experiments

Yin (2012:10) advises that multiple cases, also known as a collective case study, in the form of the study of each of the four participants' lesson plans, lesson observations, interviews, and field notes with regard to this study, be regarded as 'multiple experiments' and not as 'multiple respondents' in a survey. Yin (2012:10) further goes on to say that good case studies benefit from multiple sources of evidence. The relevant data collected thus came from multiple and not singular sources of evidence, namely lesson plan documents, field notes, transcribed lesson observations and face-to-face interviews, of each of the four cases, and also from the combined input captured during the focus group interview. The process therefore enabled the researcher to identify features that appear common to the four cases' data and also those that are significantly different.

5.4.2.3 Replication logic followed and not sampling logic

Literal replication logic was followed as four cases were chosen to predict similar results, as pointed out in (Yin, cited in Shakir, 2002:195). Replication logic was used to achieve the analytic generalisation, the generalisation of a particular set of results to some broader theory (Yin, cited in Shakir, 2002:192).

5.4.2.4 Stratification

Stratified random sampling was used to select the research participants. According to Vanderstoep and Johnston (2009), stratified random sampling involves selecting research participants based on their membership in a particular sub-group or stratum (Vanderstoep & Johnston, 2009:32). In the case of this study, the participants were selected based on their membership to mathematics teachers, using English as a medium of instruction, to teach mathematics to ESL learners speaking Setswana as their L1. In addition, purposive sampling was used as the researcher chose specific people within a population to use for a particular study. The aim was to concentrate on people with particular characteristics who will be able to assist the researcher with relevant research. Since the researcher was interested in the types of questions used by mathematics teachers in grade 10 classrooms, she used a sample of four grade 10 teachers teaching mathematics at the four high schools. As a result, the purposive sampling method used was selected based on its relevance to the research question, the language of learning and teaching (LoLT), and the curriculum offered at the four high schools.

5.5 Data collection

Data collection involves the data collection procedures and methods discussed in the next section.

5.5.1 Data collection procedures

The execution of this current study complies with the approach recommended by Easterby-Smith *et al*., (1991:5-10) in terms of the research project as follows:

Preliminary investigations

Preliminary investigations were conducted prior to the empirical data gathering phase by doing a literature survey and document analysis of the schools' curriculum, policy documents and studies related to the use of English as the medium of instruction in the teaching and learning of mathematics in grade 10 classrooms as well as ESL learners who speak Setswana as their first language in sub-system P schools. Also, the four case studies were selected after taking into account both the focus of the research study and the level of commitment from the participants.

Data collection stage

The data collection stage included the gathering of qualitative data through a series of transcribed lesson observations, individual and focus group interviews, and the gathering of documents in the form of lesson plans and field notes, to achieve what is referred to as data triangulation, as recommended in Creswell (2007:204) and Gast (2010:12). Data on the transcribed lesson observations and interviews were also collected using the ATLAS.ti software's Textbank in the form of Primary Documents, before it was assigned to its Hermeneutic Unit (HU) or project file.

5.5.2 Data collection methods

According to Gillham (cited in Easterby-Smith *et al*., 1991: 5-16), case study is an overarching method and within it there are different sub-methods like interviews, observations and document and record analysis. Data were collected from all the four cases in the following order as illustrated in Figure 5-1.

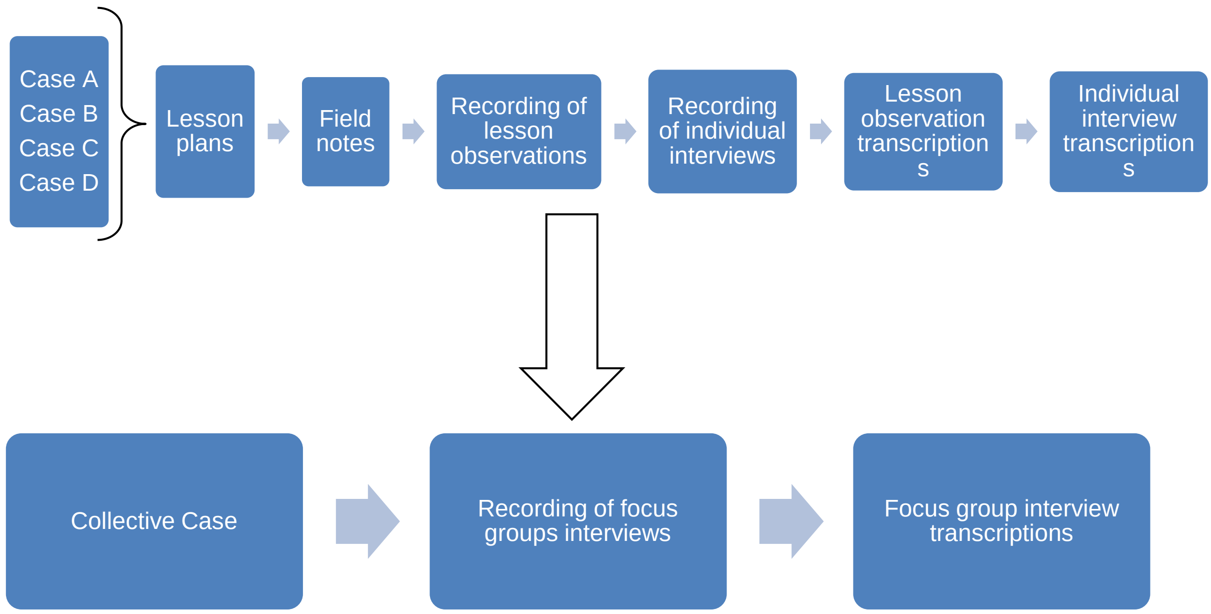


Figure 5-1: Data collection methods used

In response to the first research question on the types of questions, questioning techniques and teacher strategies used in grade 10 mathematics classrooms, the following data collection methods captured in Figure 5-1 are discussed.

5.5.2.1 Lesson plans

Documents as a data gathering technique serve to corroborate the evidence from other sources (Nieuwenhuis, 2007:83). Documents in the case of this study were in the form of lesson plans for each of the four grade 10 mathematics lessons to be observed in each of the four classrooms at the four schools for a period of a week at each school. These were used to enable the researcher to underline and note down the question types that were going to be used during the lesson observations in each classroom (see Appendix A). As the researcher went through the lesson plans and underlined the questions written, she was able to capture the types of questions (if any), that were going to be used during the lesson observations. The present situation of work was determined through the analysis of the participants' lesson plans, as they gave the researcher a general picture of what was going to take place in each of the four grade 10 mathematics classrooms during lesson observations.

5.5.2.2 Field notes

To maintain quality assurance, Mouton (2008:107) recommends that researchers should also use a diary to capture decisions and actions in field notes during fieldwork. Vanderstoep and Johnston (2009:239) also recommend field notes during the process of observation as they could be less intrusive than the recording equipment. They also provided the researcher with the opportunity for reflection, interpretation and analysis of what transpires during the process of the lesson observations.

The diary enabled the researcher to note the types of questions written on the chalkboard as class work, together with the behaviour of the participants, specifically their non-verbal behaviour, which is part of modifications or questioning techniques and teacher strategies used to make questions comprehensible to the learners. These enabled the researcher to respond to the sub-questions and the second research question, to note down the functions of the questions used, how they were phrased, to produce closed and open-ended responses, thus developing learners' mathematical discourse and ESL development in the process.

5.5.2.3 Lesson observations

Lesson observations as a data collection method have become popular in classroom research as implied by Seliger and Shohamy (1989:165). Furthermore, observation as a data gathering tool in qualitative research, according to Nieuwenhuis (2007:84), enabled the researcher to gain a deeper insight and understanding of the phenomenon being observed. This was the case with the researcher during lesson observations as she not only observed what was written on the board and the lessons taught, but also gained a deeper insight into whether the learners understood or did not understand the lesson through observing teacher-learner interactions.

Also, the focus of the researcher during lesson observations was to listen to and write down the types of questions used in each of the four grade 10 mathematics classrooms and to capture that in writing and recording. This put the researcher in a position to analyse the different types of questions used, like closed questions that promoted learners' understanding of mathematical discourse, and open-ended questions that promoted learners' understanding of mathematical discourse and ESL development. This process is referred to as structured observation as the researcher had identified predetermined categories of behaviour, for example, the types of questions, closed and open-ended used during the lesson observations (Nieuwenhuis, 2007:85).

The data collection tool for lesson observations (see Appendix B) enabled the researcher to respond to the research questions and sub-questions in terms of the types of questions used in the classrooms, the functions of these questions; how the participants phrased such questions to promote learners' understanding of mathematical discourse and English SLA development.

In response to why the participants used the types of questions, questioning techniques, and teacher strategies captured during lesson observations, the following data collection instruments were used.

5.5.2.4 Individual and focus group interviews

The following general considerations as suggested in Easterby-Smith *et al*. (1991:5-19) explain how the interviews should be conducted. Those relevant to this study are summarised below.

Purpose of the interviews

A brief explanation of the purpose and format of the interview is given in the introductory paragraph of the interview protocol (see Appendix C for the interview protocol).

Individual interviews

According to Seliger and Shohamy (1989:166), the purpose of an interview is to obtain information by actually talking to the subject to probe for information and obtain often unforeseen data. This is the reason why after the lesson observations, individual face-to-face interviews were used to probe the participants' reasons for the choice of questions, questioning techniques and teacher strategies used during the lesson observations. The interviews were used after each lesson observation on the advice of Nieuwenhuis (2007:87), to corroborate data emerging from other sources such as the lesson plans and lesson observations. Responses to the interview questions were also recorded, transcribed and stored in computer files where they were retrieved for analysis purposes.

At the end of each week, the schedule for the lesson observations and individual interviews was captured on a form (see Appendix D), bearing the school stamp and completed and signed by each participant, the researcher and the school principal. The form indicated the dates and periods of the weekly scheduled lesson observations and interviews conducted at each of the four schools

Focus group interviews

The purpose of a focus group interview, according to Vanderstoep and Johnston (2009:235), is to bring together a group of six to ten people who, under the guidance of a moderator, engage in a group question-and-answer discussion. Therefore, focus group interviews were used by the researcher to enable the teachers to be productive in widening the range of responses and for the researchers to hear about the responses that might have been forgotten by individual teachers during face-to-face interviews and, in so doing, unexpected comments and new perspectives added value to the study (Nieuwenhuis, 2007:90).

After all the data of the different cases had been analysed, a meeting with the research participants was convened for a focus group interview to be conducted, so as to corroborate the findings of the data collected. Therefore, multiple methods of data collection were engaged and that, according to Nieuwenhuis (2007:80), would lead to trustworthiness.

Length of interviews

The semi-structured interviews' duration varied from 15 minutes to 30 minutes, according to the setting and the purpose of the interview. The interview sessions were slightly affected by unforeseen circumstances prevailing at each particular school, for instance impromptu staff and South African Democratic Teachers' Union (SADTU) meetings, as well as changes in the schools' draft timetables, especially at schools A and B, where the research was conducted at the beginning of the academic year 2012. The focus group interview was conducted during a lunch hour period.

Size of the group interview

Individual interviews were conducted on a one-on-one basis for an in-depth discussion of the reasons for the types of questions, functions of the questions, the questioning techniques and strategies used during the lesson observations. After data on each of the participants had been analysed, a focus group interview followed with three instead of four respondents to review the data captured and its analysis as they responded to the primary and secondary research questions. (One of the participants was unable to attend the focus group interview on the date scheduled due to unforeseen family issues).

Mixture of locations

The individual interviews were conducted at each of the four grade 10 mathematics teacher's office, while the focus group interview was conducted at a central location at the Case A's office, which was convenient for both the researcher and the participants.

Language issues

English was used as the language of the interview as it is the medium of instruction for mathematics at the four schools, and also the lingua franca of the four teachers and the researcher as one of the teachers did not speak Setswana, but Shona as his first language.

Use of digital recorder and transcribing software

It was appropriate to use a digital voice recorder to capture the lesson observations and interviews verbatim. These were transcribed using the Home Dragon Naturally Speaking programme, a speech-recognition software famous for its ability to turn 'talk' into 'text' as it transcribes data with accuracy on to a computer or laptop. Data collected were stored in files and retrieved for analysis purposes during the analysis stage. For data to be transcribed with accuracy, the technician installed the software in my laptop, and thereafter, guided me through the dictation exercises. As I dictated to the Dragon microphone at my natural pace, a small Dragon icon indicated that the software was processing my speech, and transcribed what was said on the laptop screen. I had to go through the dictations for an hour for the software to capture and recognise my voice using its '*Recognition mode'*. Data was captured and transcribed with 100% accuracy, because when the software misrecognised some words dictated, I could use the correction menu for the command '*Correct that'* or '*delete that'*. By using the 'Tool menu' I could dictate to the microphone to respond to commands for '*selecting*' and '*editing the text*', '*moving the cursor to the left or right*', '*adding lines*, *paragraphs and spaces'*, and also for capturing special characters, such as punctuation marks, numbers, dates, times, units, prices, and others.

Tracking of interview data

More specific data was gathered through the in-depth exploration of the lesson observations, and following them up with individual and focus group interviews.

The interview protocol has sub-headings indicating when and where the interviews took place, information on the participants who took part in the interviews was also captured (See Appendix C).

The role of the researcher

In the case of this study, data on the lesson plans, field notes, lesson observations and interviews were conducted in a natural setting, where the researcher assumed the role of the observer as a participant as she was not immersed in the day-to-day lives of the participants (Creswell, 2007:68-69). The researcher chose that role to be detached and less prone to bias, even though that role did not allow the intimacy and depth of observation that a participant as observer can achieve as pointed out by Vanderstoep and Johnston (2009:91). Also, the researcher as a former mathematics teacher at high school, and a lecturer in ESL classrooms, thus qualifies to conduct this research.

5.6 Data analysis procedures

Data collected in qualitative studies produce large volumes of information that can be overwhelming to the researchers, and as a result, researchers are advised to read the transcripts in their entirety several times, to immerse themselves in the details, write memos in the margins of field notes or transcripts in the initial process of exploring a database (Creswell, 2007:150). It was easy for the researcher to follow these procedures as she single-handedly analysed manually data on the lesson plans, field notes, transcribed lesson observations and interviews.

5.6.1 Data analysis methods

The following data analysis methods used for each of the 4 participants are captured in Figure 5-2

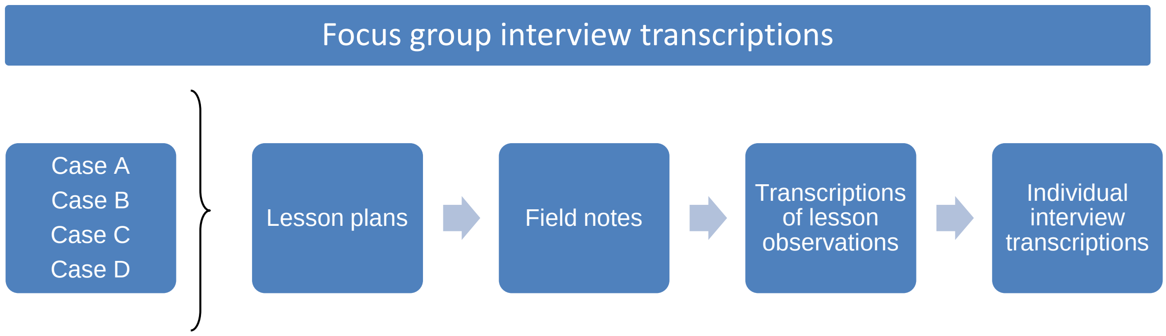


Figure 5-2: Data analysis methods used manually

Figure 5-2 shows the data analysis methods followed with regard to the data collected from each of the 4 participants, starting with Case A, then Case B, Case C, and Case D. Presenting an analysis of findings and recommendations for action was achieved through the manual data analysis process of the lesson plans, transcribed lesson observations and interviews, field notes, and lastly the feedback provided during focus group interviews. It was also achieved by revisiting analysed data several times, and finally by downloading data on the transcribed lesson observations and interviews in the Hermeneutic Unit (HU) file of the ATLAS.ti software, analysing it, and finally presenting it in figures in its Network view. It should be noted that the researcher was the only person involved in the manual data analysis and also when using the ATLAS.ti software. Finally, the analysis involved member checks before the production of the final research report on findings and recommendations. The methods used are discussed.

5.6.1.1 Manual data analysis procedures

Table 8 (Creswell, 2007:149) lists Miles and Huberman's (in Creswell, 2007:149) analytic strategies to analyse data, and these were followed in the manual data analysis methods discussed below.

Sketching ideas by writing margin notes in field notes

The desk and chair provided for the researcher in each of the 4 grade 10 mathematics classrooms and in their offices during lesson observations and interviews respectively, enabled her to sit down and be able to note non-verbal questioning techniques, such as 'writing on the board', used by the teachers in the diary during the lesson observations.

Taking notes by writing reflective passages in notes

Two columns were drawn in the diary to write down and enter the types of questions that produced closed and open-ended responses during lesson observations. The classroom setting, traditional or learner-centred was also sketched in the diary.

Summarising

The researcher's reflections were drafted in summary sheet in a diary as field notes at each visit to each of the four schools on a daily basis.

Identifying codes

Codes were used solely by the researcher to identify data collected from each of the four cases. Codes identified in the literature review included closed questions coded CL, questioning techniques coded as QT, and strategies as STR. For example, data on Case A's first lesson observation at school A was coded as Case A: L O1: 53-56, and data on Case C's fifth lesson observation as (Case C L O 5: 10-15), the colon indicating the number of the lines from which the transcribed lesson observations have been taken. Also data collected on the second interview based on Case B's lesson observation was coded as (I Case B 2: 33 39), the colon showing the number of lines from which the transcribed data on lesson observations have been taken.

Reducing themes to codes by noting patterns and themes

Symbols like = were used to indicate similar types of questioning techniques, and teacher strategies used to promote learners' comprehension. These were listed under Comp = comprehension to differentiate them from those which promote output listed under Output.

Counting frequency of codes

With regard to the total number of questions used in each of the 4 grade 10 mathematics classrooms and in all the classrooms, tables were drawn to show the total number of the types of questions used by each of the 4 participants, resulting in tables capturing the types of questions used by each of the 4 cases.

Relating categories by factoring, noting relations among variables, building a logical chain

With regard to each participant's questions, the questions that produced closed and open-ended responses on the part of the learners, and that promoted learners' understanding of mathematical discourse only as well as mathematical discourse and ESL development respectively, were identified and noted. Furthermore, procedures on how to phrase such questions to elicit learners' responses that promoted their understanding of mathematical discourse and ESL development were also noted.

Displaying the data by making contrasts and comparisons

Information on the closed questions that promoted learners' understanding of mathematical discourse and also on open-ended ones that promoted learners' understanding of mathematical discourse as well as ESL development, captured during lesson observations, was displayed in tables and figures from the Network file for comparison and contrast.

Report writing

Elements of the report were produced after the analysis of the results was completed and the production of the report continued until final submission towards the end of the project, both in the form of a report back to the 4 cases and the formal submission of the report for academic assessment.

Furthermore, the data analysis procedures discussed above are in line with what was recommended by several authors, including Hussey and Hussey (1997), Patton (2002) and Yin (1994), who list procedures such as organisation of data, its categorisation, its interpretation of single instances, identification of patterns, and its synthesis and generalisations.

5.6.1.2 ATLAS.ti data analysis procedures

The procedures followed in analysing data using the ATLAS.ti software are illustrated in Figure 5-3

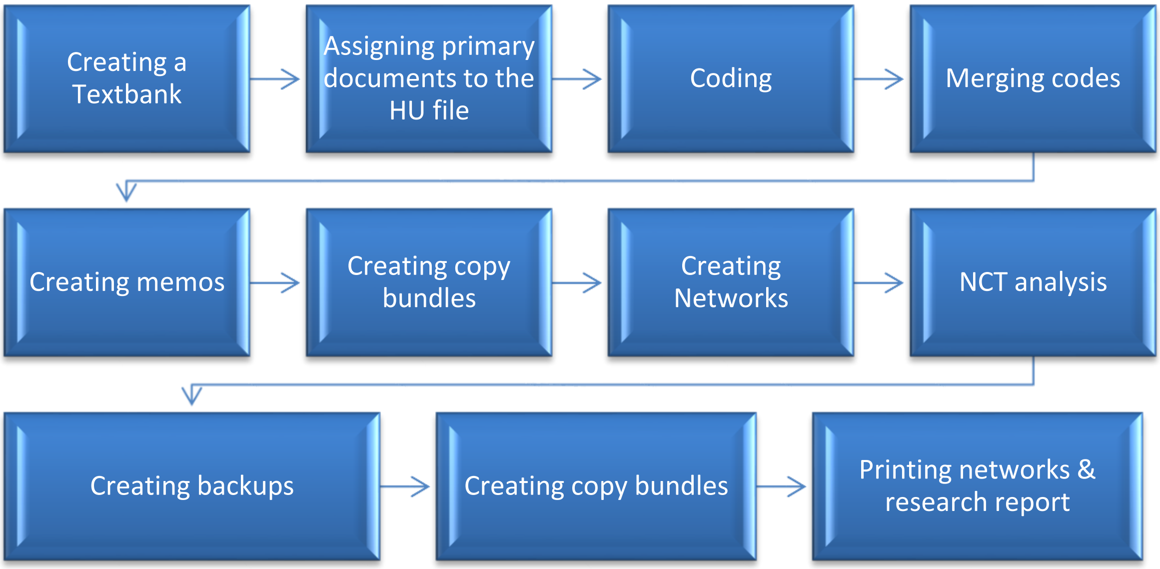


Figure 5-3: Data analysis methods procedures using the ATLAS.ti software

Figure 5-3 on the data analysis procedures using the ATLAS.ti software is briefly discussed.

Creating Textbank

Thirty-seven transcribed documents in the form of 18 lesson plans, 18 interviews and one focus group interview were edited and saved in a Textbank before they could be assigned to the Hermeneutic Unit (HU) or project file, these were thereafter referred to as Primary Documents;

Assigning Primary documents to Hermeneutic Unit (HU) File

Segments of each of the Primary Documents for each of the 4 participants' transcribed lesson observations and interviews, were assigned to a HU File, where they are saved for retrieval and analysis purposes.

Counting frequency of codes

The ATLAS.ti was able to produce the total number of procedural, closed questions and open-ended questions used, as well as the responses elicited in figures 1 and 2 respectively from the Network files in Addendum A.

Coding

Data analysis procedures in response to each of the research questions were highlighted and coded using the coding techniques described below.

Open-coding

Open-coding was achieved by highlighting the word or sentence/paragraph and right-clicking on the selected text to choose and enter the code by typing a code name, for example ***What*** questions. Open-coding simply means to create a new code (Friese, 2012:64);

Code in Vivo

Code in Vivo simply means highlighting the text and using the selected or highlighted text as the code name. This was used at the beginning of the coding process as it is easy and quick to create (Friese, 2012:73);

Selecting codes from the list

When the highlighted text is more or less similar to the code name that has already been entered, the code name was selected from the previous code on the list (Havenga, 2013:17). For example, if the text previously coded was the *What* question, the following selected texts falling under *What* questions, would be entered under that code name since it is already in the code list.

Last used code

If the selected text was more or less similar to the previous code used, the last code name was applied (Havenga, 2013:17). For example, if the selected text, a *what* question, coded as a *Closed* question, is followed by another selected text of a *what* question, then the previous code *Closed* question was used to code the text.

Merging codes

After the initial coding explained above has been completed, codes with more or less similar names or meanings were merged to clean up the code list to reduce the number of codes and to push codes from a descriptive to a conceptual, more abstract level (Friese, 2012:106). A total of more than 200 codes initially created were reduced to a code list of 20 categories all coloured in different colours for distinction purposes, for example, green could be used for closed questions (CLOSED QUESTIONS), comprising of *what*, *which*, *where*, *who*, *when*, *either* ...*or* questions, in response to the first research question on the types of questions used in grade 10 mathematics classrooms. Further merging of codes were done with regard to the functions of questions, questioning techniques, and teacher strategies for comprehension, language processing and interaction, output, and feedback as %FQ_FUNCTIONS OF QUESTIONS, $QUESTIONING TECHNIQUES, and &RQ3STRATEGIES respectively.

Creating memos

Writing memos, according to Friese (2012:234-235), is an essential step in qualitative data analysis as they represent analytic work in progress that will be used as building blocks in writing up a research report. Using the Memo manager function, free stand alone memos, linked memos, theory or literature memos and research question memos were created and linked to quotations. The Memo function, using its Query tool, enabled the researcher to review and export the results on each quotation and the printer symbol respectively. The data analytic process was also documented in the Research Diary where it would be retrieved during presentations at meetings, seminars or conferences. The Output was also created to be used as a building block for the research report. The Theory or Literature Memo was used to capture literature review relevant to the findings of the study.

Creating copy bundles

A copy bundle file was created after each work session as a backup and saved in an external drive, and also in the memory stick in case the computer crashes and the file gets lost.

Creating networks

Networks are graphical representations of a semantic type created by linking codes to codes and quotations to quotations through specific meanings. The Network tool allowed the researcher to explore data visually, and that could be used throughout the analysis process as a tool to integrate all the findings (Friese, 2012:191). For example, to show the relationships that exist between the codes, the hyperlinks structures in the Network editor enabled the researcher to specify and illustrate that relationship using options such as, '*part of'*, '*contradicts*', '*causes*', and others.

Using the Network function, the results were presented visually by showing the Primary Document Manager, (See Figures 1 to 5 in the Network file of Addendum A), explaining major categories, showing the coding schemes used, illustrating the relationships between the codes, and using research question memos to explain how conclusions were derived (Friese, 2012:220).

The analysis stages described started once data collection activities had commenced (in order to give further direction to the latter part of the empirical data collection stage), continuing through the remainder of the field work, leading up to the design of the model and the writing of the report. As a result, the suggested data analysis procedures were followed, conclusions were drawn that might have implications beyond the specific case that had been studied, as stated in Leedy and Ormrod (cited in Easterby-Smith *et al*., 1991:5-23).

Summary

As stated in Hodkinson and Hodkinson (2001:6), one of the strengths in using case study research is its ability to show the process involved in causal relationships. Most conventional studies of causal relationships are based on statistical correlation. The depth and complexity of a case study data can illuminate the ways in which such correlated factors influence each other.

5.7 Maintaining triangulation, validity, qualitative reliability and trustworthiness

To maintain triangulation, validity, qualitative reliability and trustworthiness, the following procedures were applied:

5.7.1 Triangulation

Triangulation as an approach is used to increase the quality and validity of the qualitative research method to avoid bias on the part of the researcher. Therefore, it helps to overcome both these potential sources of bias even if bias is not totally eliminated, as cautioned by (Easterby-Smith *et al*., 1998:5-18). Triangulation can be classified as follows.

5.7.1.1 Data triangulation

For the sake of triangulation and to maintain construct validity of the results as required in a qualitative research study, following the advice in Creswell (2007: 204) and Gast (2010:12), the researcher made use of multiple sources of data collection methods in the form of the lesson plans, transcribed lesson observations, face-to-face and focus group interviews, as well as field notes captured in the diary. Easterby-Smith *et al*. (1998:5-18) refer to this method of data collection as data triangulation. Seliger and Shohamy (1989:122-123) concur that this process facilitates ‘validation and triangulation’, and also increases the reliability of the conclusions reached.

5.7.1.2 Methodological triangulation

Methodological triangulation is achieved when the researchers employ both the qualitative and quantitative methods, to bridge the gap that exists between the two (Vanderstoep & Johnston, 2009: 179). For example, using more than one methodology to address the same question, such as a quantitative survey combined with qualitative interviews. Methodological triangulation in this study was achieved through the use of two methods for data analysis, namely the manual data analysis and also the ATLAS.ti software in addressing all the five sub-questions of the second research question. According to Friese (2012:146), the memo manager function of the ATLAS.ti software used to analyse data, adds a lot to data analysis in terms of qualitative reliability, credibility, transparency and dependability - the quality criteria by which good qualitative research is recognised.

Validity

Nieuwenhuis (2007:77) states that validity in the study is maintained in the analysis of lesson observations and the participants' interview responses since the 'thick' detail of data from the participants' lesson observations usually fulfils the key criterion of validity far better than data obtained from other methods. Furthermore, to ascertain and maintain the validity of the results, the researcher took the write-up of the results back to the participants for review to provide the participants with more of a voice and perspective in the framing of conclusions and interpretations (Vanderstoep & Johnston, 2009:193).

5.7.2 Qualitative reliability and trustworthiness

To maintain reliability in as far as the data collection instruments are concerned, as suggested by Creswell (2007:209), the researcher made use of the digital voice recorder that has functions to record in verbatim data on lesson observations, teachers' responses to individual face-to-face and focus group interviews, as well, as the Home Dragon Naturally Speaking, a speech recognition software to transcribe data onto the laptop. Also, to maintain qualitative reliability with regard to data collection, the first 5 minutes of the lesson observations were not recorded to eliminate the observer effect on the participants and the learners.

Furthermore, for the sake of validity and qualitative reliability of the data analysis during the analytic phase, the researcher went back to the mathematics teachers whose lessons were observed and thereafter interviewed them, and scheduled the 28th March 2012 for a focus group interview to cross-check whether her analysis and understanding correlate with the conclusions of the research subjects before the results were made public (Mahlomaholo, 2009:10) (see Appendix E, an attendance register for the focus group interviews). Vanderstoep and Johnston (2009:192) refer to this three-phased type of inquiry as reflexive validity.

Furthermore, the researcher as the 'research instrument' was able to collect 'rich data' from the analysis of lesson observations, interviews and documents in the form of teachers' lesson plans and field notes. She was also able to collect data in the form of transcribed lesson observations and interviews in a Textbank, and all these procedures, according to Nieuwenhuis (2007:80), would lead to trustworthiness.

5.7.2.1 Qualitative reliability, credibility, transparency and dependability

Both the manual data analysis and the ATLAS.ti software were used a lot to data analysis in terms of qualitative reliability, credibility, transparency and dependability. The memo output captures the research questions, the results based on the research questions, the literature reviews relating to the findings, and also the process followed by the researcher in compiling the report. Using the memo, output can be called up in data segments in the form of coded data. For example, the reasons why the participants used more closed than open-ended questions, all these reasons can be highlighted by clicking on the code ###RQ1CLQ_Reasons for closed questions, for the audience to see during presentations at seminars and conferences, thus adding a lot to data analysis.

Inter-rater reliability

Inter-rater reliability is often used for behavioural observations. A measure has high inter-rater reliability if two people who are observing behaviour agree on the nature of the behaviour (Vanderstoep & Johnston, 2009:65). Even though the researcher was the only one observing the research participants during lesson observations and analysing data, the low inter-rater reliability was improved by recording the lesson observations and transcribing them using a speech recognition software for the researcher to have ample opportunities to focus on the behaviour of each of the four cases and that of the learners as a whole, and note that in the diary as data in the form of field notes. As a result, she was able to note learners' behaviour when they did not understand a lesson and raised the issue during the individual interviews with the particular participant.

5.8 Limitations of the case study research and how they were handled

As outlined in Hodkinson and Hodkinson (2001:8-11), the limitations relevant to this study are briefly discussed.

5.8.1 There is too much data for analysis

All case study researchers are aware of the fact that they are overwhelmed by the amount of data to be analysed, for example the researcher had 18 instances of field notes, lesson plans, transcribed lesson observations, interviews, and 1 focus group interview, resulting in a total of 73 documents for data analysis. That could inevitably result in the production of the reports in which quotations are drawn from a small number of participants and subsequent results are not published. This was avoided by the researcher revisiting the data several times to analyse and write about such issues that were not reported initially. Furthermore, the researcher was able to use the ATLAS.ti software to analyse the entire data corpus of 37 transcribed lesson observations and interviews in a period of two weeks, to take care of that limitation. That process enabled the researcher to compare the results and findings with those of data analysed manually.

5.8.2 Case study research is very expensive if attempted on a large scale

Case study data collection is time-consuming, (it took the researcher 16 days in February 2012 just to collect data from the four schools), and even more time-consuming to analyse it. Fortunately, the researcher was on study leave during that period and could focus only on her research. Case study research is also very expensive; the researcher had to apply for funding from the North West University's Research committee to be able to visit the four schools within a period of four weeks to pay for travelling, accommodation expenses, and also for editing, binding and printing the thesis. Without any funding, the research would not have been completed.

5.8.3 The complexity examined is difficult to represent

Even though case studies are successful in revealing some of the complexities of social or educational situations, there is often a problem of representation as it is often difficult to present a realistic picture of that complexity in writing. To take care of that limitation, Chapters 6 and 7 on the results and findings show a with-in case analysis of the results of each of the four participants, followed by an analysis of the transcribed lesson observations using the ATLAS.ti software, and followed by a cross-case analysis of data from all the 4 participants.

5.8.4 Some aspects of case study work do not lend themselves to numerical representation

Some aspects of case study work can be fairly easily presented in numerical form, but much of it cannot, in fact it is even more problematic. As a result, most of the reports contain phrases like, "most/many of our sample..." This lack of precision is regarded as a serious weakness by many researchers, thus doing away with the many strengths of case study research discussed earlier. Triangulation of the data collection instruments took care of that limitation as some of the findings were illustrated in tables and figures using numerical representations, and phrases indicating a lack of precision were not at all used in the data analysis section. Also, the Network editor of the ATLAS.ti software, in the figures 1 to 5 represented, indicated the number of responses captured inside the brackets as shown in the example on Procedural questions (##RQ1TYPES PROCEDURAL QUESTIONS {181~1}.

5.8.5 Case studies are not generalisable in the conventional sense

By definition, case studies can make no claims to be typical. We cannot generalise that the types of questions used in the 4 grade 10 mathematics classrooms where data were collected are similar or different from those in other grade 10 mathematics classrooms around the schools in South Africa. Furthermore, because the sample is small and idiosyncratic, and because the data are predominantly non-numerical, there is no way we can say with certainty that the data are representative of some larger population. For many researchers, this renders any case study findings of little value and unreliable. However, certain trends and themes which are found in the cross-case analysis, for example, from the findings, the trend of open-ended questions eliciting individual open-ended responses and not chorus ones is repeated in each of the 4 case studies, and in the cross-case analysis, thus making the case study generalisable.

5.8.6 Case study reports are strongest when researcher expertise and intuition are maximised, but this raises doubts about "objectivity"

In case study research, researcher expertise, knowledge and intuition are vital parts of the case study approach. Case study researchers choose what questions to ask, how to ask them, what to observe, and what to report. They also decide on how to present individual stories, what data to include and focus on, and what to exclude, and in that way they are constantly making judgements about the significance of the data. As a result, reports on case study research contain construction of data around issues that researchers judge to be important. So, no matter how hard researchers can work, it means the research cannot be objective. The researcher revisited the data collected manually more than once during the analysis stage and incorporated as many views as possible when checking data with the participants afterwards, and also confirmed the findings through the literature reviewed.

5.8.7 Findings of case study research are easy to dismiss by those who do not like the messages that they contain

If the case study research presents findings that are not popular, especially in the view of policy makers, they are likely to be dismissed, using statements like, "*the sample was too small, it's not like that everywhere*, and *the researchers were biased*". This limitation was taken care of in form of the results on the trends and themes that were found in the cross-case analysis which seemed to be convincing.

5.8.8 Case study research findings cannot answer a large number of relevant and appropriate research questions

According to Hodkinson and Hodkinson (2001:10), case studies are neither ubiquitous nor a universal panacea as there are many important research questions that cannot be answered using this method. To address that limitation, the researcher focussed only on the three research questions and five sub-questions addressed and outlined in the study.

5.8.9 Theory can be transposed beyond the original sites of study

Where case studies generate new thinking, that thinking has a validity that does not entirely depend upon the cases from which it is drawn. For instance, in the case of this study, the open-ended questions used produced learners' output during classroom interactions. That in itself made the researcher believe that the results of this study are slightly and possibly generalisable to most of the population, especially in South Africa where English is taught as a second, third or an additional language, and is also used as a medium of instruction in mathematics classrooms. Therefore, the theory is much more than a story of the 4 grade 10 mathematics teachers at the 4 schools as it is transposed beyond the 4 grade 10 mathematics classroom walls by also confirming that open-ended questions produced output in mathematics classrooms as pointed out in Hastings (2003) and Brock (1986).

5.9 Ethical considerations

According to Creswell (2007:141), qualitative researchers are faced with many ethical issues that surface during the data collection process in the field and in the analysis and dissemination of reports. Ethical issues include, among others, informed consent procedures; covert activities; confidentiality towards participants, sponsors, and colleagues; benefits of research to participants; other risks; and participants' requests that go beyond social norms.

Similarly, Hatch (2002:68) lists a number of questions relating to these ethical issues that the researcher should honestly respond to in order to abide by the guidelines for ethical practices. Some of the ethical issues relevant to this study are discussed below.

5.9.1 Granting permission to conduct research

Before the researcher could visit and conduct research at the 4 schools, she was granted permission by the Department of Education, North West province, (See Appendix J). Thereafter the ethical clearance was obtained from the North-West University's Ethical Committee, Faculty of Educational Sciences, granting her permission to conduct research at the 4 schools after making sure that conditions that satisfied ethical issues were taken care of (see Appendix F), minutes of the meeting captured in document number NWU-00100-11-S2, dated the 24th November 2011.

5.9.2 Openness with interviewees

The researcher was open with the participants as it is required before one could conduct research at the 4 schools by following the procedures summarised below.

5.9.2.1 Debriefing

To follow this procedure, the researcher made an appointment with each of the 4 school principals in order to request for permission to conduct research at each of the 4 schools' grade 10 mathematics classrooms. On the agreed day, the researcher hand-delivered letters addressed to the School Principal and the School Governing Body (SGB) of each of the 4 schools, requesting for permission to conduct research at the schools (see Appendix G). The letter outlined the procedures on how the research would be conducted. Each of the principals thereafter, on the same day, summoned the grade 10 mathematics teacher and explained the purpose of my visit. The teacher was also given the letter (see Appendix H), requesting his/her permission for research to be conducted in the grade 10 mathematics classroom for a period of one week. The letter also outlined the data collection procedures to be followed.

5.9.2.2 Informed consent

The participants were also given informed consent forms (see Appendix I) that outlined the data collection procedures to be followed during a period of one week at each of the 4 schools. The participants were informed that they were required to sign the consent form on Friday preceding the week in which the research would be conducted at their respective schools.

5.9.2.3 Opportunity to withdraw

The informed consent form that outlined the data collection procedures to be followed during a period of one week also stated that the participants were free to withdraw from participating in the research project, even though that would not be encouraged.

5.9.2.4 Offering incentives

Although participation in research is voluntary, according to Vanderstoep and Johnston (2009:14-15), the participants should be compensated for their time and effort. However, incentives are not always needed. This was the case in this project. However, as the focus group interviews were conducted during lunch time, the participants were offered a light lunch. This was also offered during another meeting with the participants to cross-check whether the analysis and understanding correlate with the conclusions of the participants before the results could be made public.

5.9.2.5 Using deception

The participants were also informed in the consent form that they signed during the research week that there were no known risks and/or discomforts associated with the study and they were provided with an information sheet on the topic of the research.

5.9.2.6 Plagiarism

Research ethics prohibit an investigator from presenting the ideas or data of others as his or her own (Vanderstoep & Johnston, 2009:18). The sources of ideas presented throughout the chapters from the literature reviewed have been acknowledged, and the data analysed were personally collected and analysed by the researcher. Also, before the thesis was submitted to the examination office, the supervisor had to subject the thesis through Turn-it-in, a software employed by the North West University to detect any form of plagiarism, and produce a report confirming that plagiarism was not detected. This was submitted together with the bound thesis to the examination officer.

5.9.3 Research ethics

According to the Belmont Report, researchers must be concerned with the following three ethical issues (Vanderstoep & Johnston, 2009:12).

5.9.3.1 Respect for persons

Even though two of the participants, cases A and C, were former mathematics students of the researcher at high school, they were also treated with respect and as autonomous in their respective grade 10 mathematics classrooms. Each of the participants were formally addressed as *Mam,* (short form of Madam) with regard to the female participants, and *Mister,* for the male participant during visits at the respective schools. All the participants were addressed as *ladies and a gentleman* during the focus group interview.

5.9.3.2 Beneficence

The researcher, as a former mathematics teacher at one of the participating schools, School C, thought of the welfare of the participants, and did not in any way expose them to any form of risk. In fact the researcher maximised the benefits of the participants as a former mathematics teacher by sharing with them some of the methods of teaching mathematics that she applied and considered as successful.

5.9.3.3 Justice

There was fairness in the distribution of benefits for all the participants and no possible risks. After the focus group interviews were conducted, the participants, when asked to comment on how they felt about the research conducted in their respective mathematics classrooms, expressed appreciation for how they benefitted from the questioning techniques and strategies we discussed, such as not using Setswana to teach mathematics, but group-work activities to engage learner-learner interaction and produce mathematical discourse in mathematics classrooms.

5.9.4 Appropriate treatment of confidential information

The letters of the alphabet, A, B, C, and D, were used to identify the teachers and their schools to protect their identity.

Summary

From what has been discussed in this chapter, one cannot help it but agree with Harling (2002), when he states that qualitative research has all the problems its detractors claim. Yet Creswell (cited in Harling,2002:6), encourages qualitative researchers to see “qualitative research as sharing good company with the most rigorous quantitative research, arguing that it should not be viewed as an easy substitute for quantitative study”.

5.10 Conclusion

Chapter 5 explained the reasons why the researcher chose the interpretivist paradigm and the qualitative research method that led to the investigation of a case study of 4 teachers teaching mathematics in grade 10 classrooms to ESL learners. Furthermore, the data collection and analysis methods and procedures followed, their qualitative reliability and its limitations were also discussed. Ethical procedures followed and considerations taken were explained. Chapter 5 presents the synthesis of the results and findings.

CHAPTER SIX: ANALYSIS OF DATA

6.1 Introduction

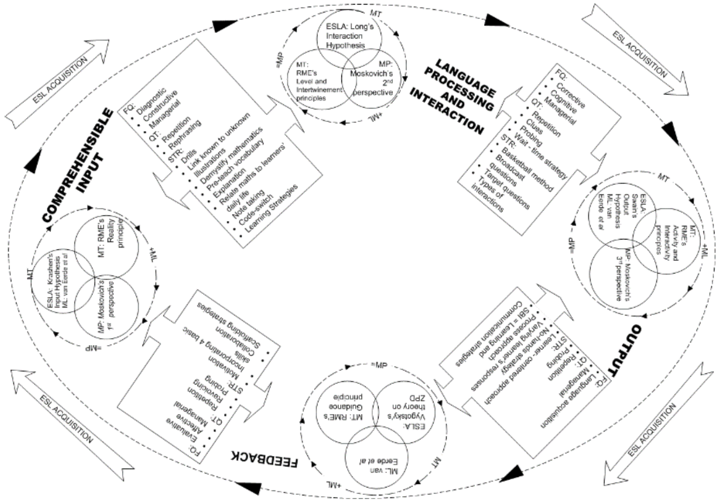
Chapter 6 discusses the results of each individual case study. It provides the results based on the patterns of data for each research question, based on each of the participants' lesson plans, field notes, transcribed lesson observations, individual interviews. The results, presented in tables and figures, are discussed, analysed, and a within-case summary is provided for each case. The results of the text-based data analysed using the ATLAS.ti software and the collective case study are illustrated in figures and summarised in Chapter 7. A summary of the findings based on all the results with regard to all the research questions is also provided in Chapter 7. These results are related to the review of the study in the conclusion of Chapter 7.

The results and findings of the four cases (A, B, C, and D) are reported first according to the following sub-headings for the sub-research questions:

- The most frequently-used question types in grade 10 mathematics classrooms;
- The questions used to promote learners' understanding of mathematical discourse;
- The questions used to promote mathematical discourse and ESL development;
- The functions of the questions used in grade 10 mathematics classrooms;
- Questioning techniques used in grade 10 mathematics classrooms;
- Teacher strategies used in grade 10 mathematics classrooms.

Table 6-1: Sections where the results and findings are captured in Chapter 6 with regard to the four case studies list the research and sub-research questions for this study with regard to each of the four cases to avoid unnecessary repetition and for the reader to refer to the relevant section in as far as the results and findings are concerned.

Table 6-1: Sections where the results and findings are captured in Chapter 6 with regard to the four case studies

1	What are the theories that underpin the effective questioning techniques and strategies to promote ESL acquisition				Chapters 2 - 4
2	At the descriptive level, the research problem can be formulated in terms of the following research questions:	Case A	Case B	Case C	Case D
	What are the characteristics of the most frequently-used question types in grade 10 mathematics classrooms?	6.1.1.1	6.2.1.1	6.3.1.1	6.4.1.1
	2.1 How do the questions used promote learners' understanding of mathematical discourse?	6.1.1.2	6.2.1.2	6.3.1.2	6.4.1.2
	2.2 How do the questions used promote mathematical discourse and ESL development?	6.1.1.3	6.2.1.3	6.3.1.3	6.4.1.3
	2.3 What are the functions of the questions used in Grade 10 mathematics classrooms?	6.1.1.4	6.2.1.4	6.3.1.4	6.4.1.4
	2.4 What are the questioning techniques used in Grade 10 mathematics classrooms?	6.1.1.5	6.2.1.5	6.3.1.5	6.4.1.5
	2.5 What are the teacher strategies used in Grade 10 mathematics classrooms?	6.1.1.6	6.2.1.6	6.3.1.6	6.4.1.6

6.2 Case study A

6.2.1 Results and interpretation for Case A

6.2.1.1 The characteristics of the most frequently-used question types in Grade 10 mathematics classroom

Table 6-2 Imperatives used in Case A's lesson plans

LESSON PLANS FOR	QUESTION TYPES USED
Day 1	*Multiply* a binomial with a trinomial *Collect* like terms *Remove* brackets first *find* the product of $(x^2 - 2x - 3)(x - 5)$ (L P Case A: 1)[2]
Day 2	First, *take out* the common factor Second, *take out* the common variables Then, *take* out the common factor, *Factorise* a) $15x^2 - 5x^2y + 5xy$ (L P Case A: 2)
Day 3	(No lesson plan provided)
Day 4	*Factorise* $25x^2 - 64y^2$ (L P Case A: 4)

From the lesson plans provided for the four lesson observations, Table 6-2 shows that Case A used 9 imperatives, for example, *Multiply* a binomial with a trinomial, (L P Case A: Day 1); *Factorise* (L P Case A: Day 4), and no lesson plan was provided for day 3. So, only imperatives were used in the lesson plans.

Case A used imperatives only as shown in Table 6-2, and as a result, no questions were used in the lesson plans provided. Since the lesson plans provided were photocopies of exercises from the prescribed text book, they have imperatives only as confirmed in an analysis of the types of questions used in grades 10 -12 prescribed mathematics textbooks, which shows that most of the questions used are imperatives, such as, *Expand*, *Factorise*, *Simplify*, etc. (Laridon *et al*., 2008: 131). Also, as a result of using photocopies of the exercises from the textbooks, there was no official document specifically used for capturing daily lesson preparations.

[2] (L P Case A:1) the code starting with L P stands for transcripts from each the cases followed by the day of the lesson plan

Figure 6-1 captured the types of questions used by Case A during lesson observations.

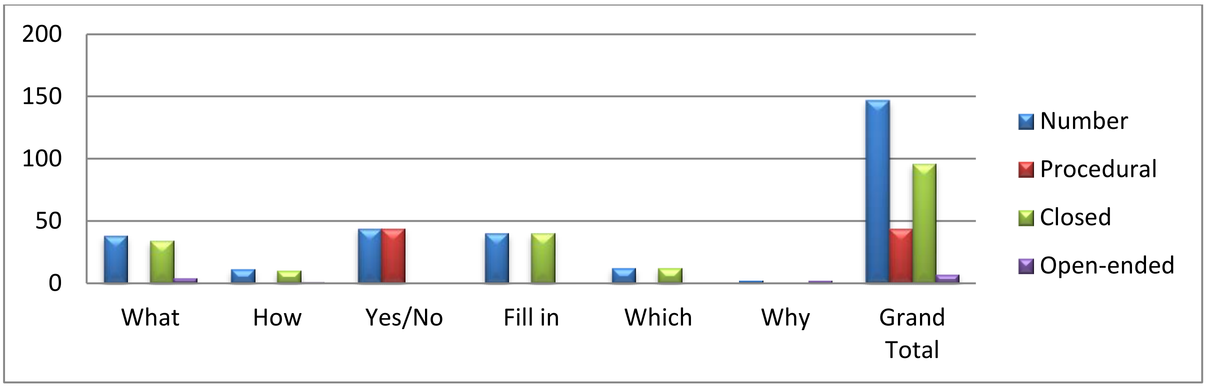


Figure 6-1: Types of questions used by Case A during lesson observations

From Figure 6-1, it seems that 80 closed questions, 44 procedural and 6 open-ended questions were used. Therefore, the number of procedural questions, and that of closed ones, is more than that of open-ended ones.

The results and findings on Case A, show that during lesson observations, the most frequently-used question types were closed. In response to why she used many closed questions during her lessons, Case A gave the following reason during the individual interview,

> *Mathematics teachers were directed by the content of mathematics to use closed questions most of the time.* (I Case A 4: 49-50)[3].

Case A's comments are similar to findings on reviews of research in the United States of America, United Kingdom, Germany, Australia and Iran on the types of questions used in mathematics classrooms, which have also shown that most of the questions asked by the teachers were closed (Long & Sato, 1983; Brualdi, 1998; Sutton & Kreuger, 2002; Sadker, 2003; Zevenbergen & Niesche, 2008; Shomoossi, 2004).

6.2.1.2 Questions used to promote learners' understanding of mathematical discourse

From the analysis of data collected on the questions that promote learners' understanding of mathematical discourse, the following observations captured in Table 6-3 are illustrated and summarised.

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[3] Code starting with (I Case A 4: 49-50) stands for the transcripts for the number of the individual interview of each of the 4 cases followed by the line number(s) from which the transcribed interview has been taken.)

Table 6-3: Questions used by Case A to promote learners' understanding of mathematical discourse

INPUT	EXAMPLES	OUTPUT	FUNCTION
What-question	Case A: *What is it*? (pointing at the expression)	Chorus: *A binomial.*	State or mention what is pointed out on the board
Fill-in question	Case A: *4x multiplied by4, it is.* ..?	Chorus: *16x* (Case A L O 1: 22 - 23)[4]	Provide responses using the multiplication operation
Which-question	Case A: *Now we take the last one, which is -1 x 4, which is* ...?	Chorus: *-4* (Case A L O 1: 32 – 33)	Fill in the blank spaces left open in the question asked

Table 6-3 shows that closed questions such as *What, Fill in*, and *Which*, produced closed responses in the form of one word answers or phrases. Closed responses provided learners with limited opportunities to promote mathematical discourse, as most of the responses were in chorus form, thus making it difficult for Case A to ascertain as to whether all the learners' mathematical discourse has been promoted.

Table 6-3 further shows that closed questions promote learners' understanding of mathematical discourse when phrased in such a way that requires learners to perform the following functions:

- to state or mention what is pointed out on the board;
- to provide responses using the multiplication operations; and
- to fill in the blank spaces left open in the question asked.

Table 6-3 also shows that closed questions, *What*, *Fill-in*, and *Which*, elicited closed responses that promote learners' understanding of mathematical discourse. However, most of the learners' closed responses were in chorus form, so it was difficult to ascertain as to whether or not all the learners' understanding of mathematical discourse was promoted. This could be attributed to the fact that case A did not *call learners by their names* to respond individually to the questions asked, a strategy used successfully by Cases C and D, to

[4] The code e.g. (Case A L O 1: 22 – 23) stands for each of the 4 cases' lesson observation number, followed by the lines from which the transcribed lesson observations have been taken.

ascertain that all the learners (who responded individually to the question asked), had their understanding of mathematical discourse promoted.

6.2.1.3 Questions used to promote mathematical discourse and ESL development

The results on questions used to promote mathematical discourse and ESL development, captured in Table 6-4, show input provided in the form of the types of questions and examples used, followed by learners' responses and the functions of these questions in as far as the learners are concerned.

Table 6-4: Questions used by Case A to promote learners' understanding of mathematical discourse and ESL development.

INPUT	EXAMPLES	OUTPUT	FUNCTION
What-question	Case A: *What did you mean when you say that something is common*?	L 1: *It is the same* (Case A L O 3: 12 - 14	Define mathematical terms
What-question	Case A: *When we say "difference" in mathematics, what are we referring to*?	L 2: *The difference is the answer we get after subtracting* (Case A L O 4: 10 - 11)	Define mathematical terms
Why-question	Case A: *We also have $x^2 + y^2$, Is this a difference of two squares*? Case A: *No, Why*? Why do we *say that this one is not a difference of two squares? Why are we saying that*?	Chorus: *No* L 3: *Because it has a positive sign in between*. (Case A L O 4: 34 38).	Probing *Yes* or *No* responses with *Why*-questions
How-question	Case A: *You have got 2a +4b + 6c. How many terms do we have*?	L 1: *We have three terms.* (Case A L O 3: 17-19).	Say, using a full sentence the number of the sum added on the board.

Table 6-4 shows that open-ended questions such as, *What*, *Why*, and *How*, promoted mathematical discourse and ESL development when they are phrased in such a way that required learners to perform the following functions:

- to define mathematical terms;
- to respond to the *Why*-question used to probe learners' *Yes* or *No* responses; and
- to respond, using a full sentence, giving the sum of the numbers added.

Table 6-4 shows that Case A used open-ended questions, *What*, *How,* and *Why*, which elicited open-ended responses, even though on a small scale, to promote learners' mathematical discourse and ESL development. As a result of the small number of open-ended questions used (6), learners were not sufficiently provided with opportunities to use and produce language throughout the lesson observations. In response to why the participant used few open-ended questions that promote mathematical discourse and ESL development, Case A responded as follows,

> *"I think it is the role of the language teacher because myself as a content teacher, I don't have much time for teaching these learners the language, the syllabus is too much, and I've got to teach the syllabus of the lower classes due to the barrier of the language"* (I Case A 1: 107-109).

Case A's excuse is similar to the one expressed in the results of a survey of 5300 ELLs mathematics teachers who stated that they did not have time to teach both the mathematics and language (Kersaint *et al*., 2009:58). However, such excuses should be discouraged since, according to Philpott (2009:66), questioning is a skill that can be learned and successfully executed through the use of simple techniques.

6.2.1.4 Functions of questions used

Table 6-5 shows a summary of the input and examples on the functions of questions used during lesson observations and interviews with Case A.

Table 6-5: Functions of questions used by Case A

FUNCTIONS	INPUT AND EXAMPLES
Diagnostic - to diagnose learners' problems	Case A: *The 'what' question at the beginning of a lesson is used to test prior knowledge*. (I Case A 1: 42)
Constructive - to test prior knowledge	Case A: *The 'what' question is used to revise work done in the previous grade 9.* (I Case A 1: 49)
Language acquisition - For learners to use and produce language	Case A: *When we say the difference in mathematics, what are we referring to*? L 2: *The difference is the answer we get after subtracting*. (Case A L O 4: 10 - 11).
Evaluative - to evaluate the lessons and receive feedback	Case A: *In the expression 2a + 4b + 6c, 2 is common because we have 2 in the first number, we have 2 in the second number, and we have 2 in the third number. That means our common number here is 2*. (repeated twice) *Are we fine*? Chorus: *Yes, Mam*. (Case A L O 3: 40 - 44).

Table 6-5 shows that the *What*-question can be used for the diagnostic function (I Case A 1:42), to identify learners' problems; for the constructive function (I Case A 1: 49), to test learners on what has been taught; and for the language acquisition function (Case A L 0 4: 10-11), for learners to use and produce language and output. Furthermore, it should be noted that the procedural question producing a "*Yes*" response in chorus form was used for the evaluative function to check if learners understood what was taught.

The different functions of questions were used on a limited scale. However, the functions of questions that assist learners with language processing and language interaction, which include corrective, cognitive, managerial, and affective feedback, were not used at all. This could be attributed to the fact that Case A did not prepare any questions in her lesson plans for use during lesson observations, and also because questions for these functions are not found in mathematics textbooks (Bellido *et al*., 2005).

Furthermore, most of the questions used for the evaluative function were procedural questions, for example "*Are you fine*?", and these elicited *Yes* responses in chorus form only. The chorus responses did not provide the teacher and learners with feedback as to whether the lessons had been understood or not. Case A did not use the *thumbs up and down*, used for *Yes* or *No questions*, to make it possible for her to obtain feedback from all the learners by viewing their responses simultaneously and privately (Kersaint *et al*., 2009:89).

Furthermore, there were no questions used to stimulate the cognitive function of learners to encourage higher-level thought processes, and also for the affective functions of questions for learners to research on topics they enjoy in the subject. This could be attributed to the fact that the school had no library where learners could research topics that they enjoy, as pointed out by the participant during the interview (I Case A 4:110).

6.2.1.5 Questioning techniques used

The results on questioning techniques used in Case A's classroom are captured in Table 6-6 below.

Table 6-6: Questioning techniques used by Case A

QUESTIONING TECHNIQUES	**EXAMPLES**	**PURPOSE**
Repetition	*And the next step what do we do? What do we do?* (Case A L O 2: 78).	Comprehension and Clarification

QUESTIONING TECHNIQUES	**EXAMPLES**	**PURPOSE**
Rephrasing	Case A: Is 4 divisible by 2? *(No response)* Case A: Can 2 get into 4 without leaving a remainder? *Chorus: Yes* (Case A L O 3: 30 - 33)	Language processing
Clues	Case A: 3, 6 and 12, so which one is the smallest? Chorus: 3 (Case A L O 3: 69 - 71)	Language processing
Learner reads the question	Case A: for learners to get a different meaning because listening is not the same as reading... Not explaining, just reading, (I Case A: 68-69).	Comprehension
Ignoring learner's incorrect response	Case A did not respond to the learner's incorrect answer, and during the interview, she provided the following reason for that: Case A: *I do not want to hurt the learner, ... So by going to the next learner, the learner will know that his/her answer is wrong ... In most cases it is with the signs, If we are supposed to get 7x and he says -7x, then I go, so next time he will remember that the signs are very important*. (I Case A 2: 40 – 44)	Language processing
Wait-time	Case A: For the learners to think about the *the question for about 10 seconds.* (I Case A 4: 85-87) and (I Case A 2:40 -41)	Language processing and interactions

The results captured in Table 6-6 show that the questioning techniques were used for the following outcomes:

- Comprehension:
  Repetition and rephrasing of questions were used for learners to comprehend the questions asked;

- Language processing and interaction:
  Clues in the form of *Either ...or...* questions were used for learners to process and

interact using language; together with *wait-time* for learners to think and process the questions asked before responding; and

- Feedback
  Ignoring learners' incorrect response was used to provide learners with feedback.

The questioning techniques of repetition and rephrasing were used frequently for learners to comprehend the questions asked; and clues were used for language processing and interaction. The questioning techniques for providing learners with opportunities to use language and produce output, and also for learners to receive feedback, were not used. The questioning technique used, *ignoring learner's incorrect response*, after it had been used, left the learner concerned with disappointment on her face as a result of her response not being acknowledged and her not knowing if it was correct or not. This did not provide her and other learners with any feedback. The participant explained this questioning technique as follows,

> *I do not want to hurt the learner, the emotions of the learner, so by going to the next learner, the learner will know that his or her answer is wrong. The learner will also know that it means the answer is not correct. (I Case A 2: 40-43).*

Due to the participant's lack of professional qualifications, despite her many years of experience, she was insensitive in her use of this questioning technique. As advised in Kersaint *et al.* (2009:119), classroom environments should encourage the development of respect and mutual appreciation of all cultures, backgrounds, and experiences.

The questioning technique, *learner reads the question* for comprehension, was not used during lesson observations. It was mentioned during the individual interviews with Case A, but it was not possible for the researcher to see how its purpose was achieved. Reading skills can be incorporated into mathematics learning since mathematics is described as a language, and therefore the SQ3R method used in ESL classrooms can be used for learners to S-survey the lesson by looking at the headings, bold text, titles, to get the sense of the lesson; generate Q-questions; R-read the lesson and answer the question; R-recite what they learned; and R-review by summarising information (Louw *et al*., 2013:12).

Questioning techniques used to enable learners to process and interact using language included the *wait-time* strategy.

Also, no questioning techniques for output were used as Case A used mostly those for comprehension, especially repetition, which also allow learners to hear, say, and use mathematics vocabulary if they engage in mathematics communication (Kersaint *et al*., 2009:84).

6.2.1.6 Teacher strategies used

Table 6-7 summarises the teacher strategies captured from Case A's field notes, lesson observations and interviews, showing the strategies used with examples, followed by the purposes of using such strategies.

Table 6-7: Strategies used by Case A

STRATEGIES	EXAMPLES	PURPOSE
Teacher-centred setting	A traditional seating arrangement is maintained throughout the lesson observations (F N Case A:1)[5]	Learners sit passively without interacting with one another The teacher talks and does work on the board.
Revision	Case A: *Firstly remove the brackets and then multiply with the first term. Multiply each term of the binomial with each of the trinomial. Collect the like terms, ...* (Case A L O 1:39 – 44).	Learners should comprehend the steps learnt when they factorise the exercises given as class-work.
Doing many examples on the board	Examples 1, 2, 3 and 4 are done before giving learners class work (Case A L O 3:17; 64; 115; 152).	Enabling learners to comprehend the steps to be followed when they do the class work to be given.
Note-taking	Case A: *So you can take down the notes on the board*. (Case A L O 3: 186 – 187).	Enables comprehension of the exercise given by referring to the notes if experiencing problems.
Question and answer method	In the example: *2a + 2b + 3c*, Case A: *then we go to the variables, our variables are a, b and c. Which one is the same as the other*? *Is a the same as b*? Chorus: *No* Case A: *Is b the same as c?* Chorus: *No* Case A: *No, that means there is no common factor here*. (Case AL O 3: 47-52).	Teacher-learner interactions
Broad-cast questions and target questions	Case A uses broadcast and target questions, *so as to cater for a variety of learners in my class* (I Case A 3: 51-53). (Case A L O 3: 47 – 52) above is an example of a Broadcast question.	Language processing and interaction in the class
Code-switching	Case A: *Learners are struggling with the medium of instruction. Learners understand me better than when I use English*. (I Case A 1: 75, 92-93).	Comprehension of the lesson and mathematical concepts.

[5] The code e.g. (F N Case A: 1) stands for the Field Note of each of the 4 cases followed by the number of the day on which it was captured.

STRATEGIES	EXAMPLES	PURPOSE
Demystifying mathematics	Case A: *I think I should relate mathematics to their everyday life. In that way, that will be able to make kids relate to mathematics* (I Case A 1: 117-134).	Comprehension of the mathematical concepts
Being fully prepared	Case A: *For learners to love the subject*. (I Case A 3: 66-67, and 69).	Comprehension of the lesson
No-hands strategy	Case A: *It can affect the discipline in the classroom*. (I Case A 4: 97-100).	Language use and output

The results captured in Table 6-6 show that a teacher-centred approach to learning was used throughout the lesson observations as pointed out in the field notes (F N Case A 1), resulting in the teacher talking most of the time while learners listened and sat passively.

Furthermore, Table 6-6 shows that the teacher strategies were used for the following outcomes:

- Comprehension:
  The questioning techniques used to assist learners in comprehending the lesson included *revision*, *doing many examples* on the board, *code-switching*, *note-taking*, *demystifying mathematics*, and *being fully prepared*;

- Language processing and interaction:
  Strategies used to enable learners to process and interact using language included the *question and answer* method or *broadcast* and the *target* questions;

- Feedback:
  With Case A, *doing corrections on the board,* provided learners with the opportunity to receive feedback on whether their answers were correct or not.

- Output:
  No strategies were used for this function.

Table 6-6 shows that teacher strategies used were mostly for lesson comprehension and very few were used for language processing and interaction. Case A did not use any teacher strategies that provided learners with opportunities to produce output as she maintained a teacher-centred approach throughout her lessons. This is the state of affairs in many mathematics classrooms as confirmed in Kersaint *et al*. (2009:53), who states that “in many mathematics classrooms, instructional practices are teacher-centred and place learners in a passive role as recipients of information”.

Case A did not embrace the “*No hands*” strategy, giving the reason below:

> *I do not agree with the no-hands strategy. In most cases this gives learners an opportunity to disrespect teachers, e.g. a learner may sometimes not answer even if s/he knows the answer to questions, just to be a bully, and this can affect the discipline of the class* (I Case A 4: 97-100).

Case A dismissed the use of the *No-hands* strategy which the researcher suggested to encourage learners to produce language.

6.3 Case study B

6.3.1 Results and interpretations for Case B

6.3.1.1 The most frequently-used question types

Table 6-8 Imperatives used in Case B’s lesson plan

LESSON PLANS FOR	QUESTIONTYPES USED
Day 1	***Draw*** the graph of $y = 3x + 1$ ***Find*** the *x- intercept*; ***Find*** the *y- intercept*; ***Find*** the *x- and y- intercepts of* a) $-y= x^2 - 16$ b) $y= {}^1/_2x^2 + 2$ (L P Case B: 1)

It should be noted that the lesson plans provided by Case B did not have questions, but imperatives only. The lesson plan submitted and reflected in table 6-8 was for day 1 only as the participant did not provide any lessons for the remaining 3 days, giving the reason that he was correcting the homework exercises given for the rest of the week. He had no formal document on which to prepare his lesson plans. Case B used four imperatives in his lesson plan.

Even though Case B has a diploma in academic and professional mathematics, the research findings of the NCTM, (1989, cited in Kersaint *et al*., 2009:74) have shown concern with regard to the programmes of pre- and in-service mathematics teachers for them to effectively teach mathematics to ELLs. The problem could be the same with regard to the programmes for mathematics teachers teaching ESL learners in South Africa.

Figure 6-2 below captured the types of questions Case B used during lesson observation.

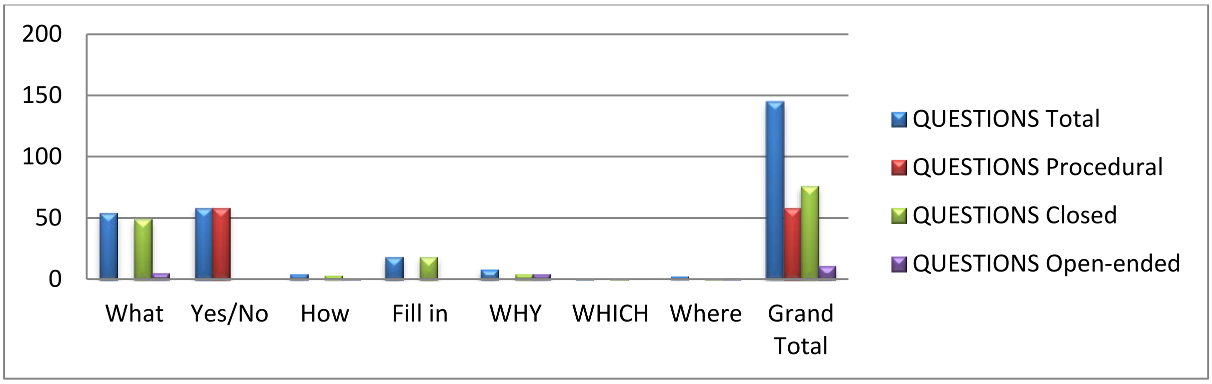


Figure 6-2: Types of questions used by Case B during the lesson observations

Figure 6-2 reveals that 58 questions were procedural, 65 closed, and 10 open-ended.

Figure 6-2 shows that during lesson observations, the number of procedural and closed questions is more than that of open-ended ones. It shows that the most frequently-used question types are closed. In response to why the participant used more closed than open-ended questions, his responses were as follows:

> *Like I said yesterday, most of the questions that teachers give students involve calculations. In questions that involve inherent calculations, it becomes difficult for the learner to say something* (I Case B 2: 26-28).

As with Case A, Case B also gives excuses for not using open-ended questions in his mathematics classrooms. The provision of excuses for lack of expected and necessary skills indicates that teachers lack support. As advised in Kersaint *et al*. (2009:59), in-service training time should be devoted to instruction of ELLs and ESL learners.

6.3.1.2 Questions used to promote learners' understanding of mathematical discourse

Table 6-9 presents the questions used by Case B to promote learners' understanding of mathematical discourse.

Table 6-9: Questions used by Case B to promote learners' understanding of mathematical discourse

INPUT	EXAMPLES	OUTPUT	FUNCTION
What question	Case B writes $y = x^2 - 25$ on the board Case B: *What are the first things That we will be looking for?*	Chorus: *y- intercept*. (Case B L O 1: 12-15).	Retrieve prior knowledge

INPUT	EXAMPLES	OUTPUT	FUNCTION
Fill-in question	Case B: *Yes, and x times -2 will give us...?*	Chorus: *-2x* (Case B L O 2: 97-98).	*Apply the multiplication operation.*
Either ... Or ... question	Case B: *What is the gradient of M N? Is it 3 or -3*? Case B: *Some are getting 3 and others -3. Someone is wrong here*. Case B does the calculation on the board and the answer is 3.	Chorus: *3, -3* (Case B L O 4: 54-55).	Direct learners to the correct answer, using options to choose answers from.

Table 6-9 shows that the following types of questions were used to promote learners' understanding of mathematical discourse: the *What*, the *Fill-in*, and the *Either ... or ...* questions. All the questions elicited one word or phrase responses.

Table 6-9 also shows that closed questions promoted learners' understanding of mathematical discourse when phrased in a way that required learners to perform the following functions:

- The *What*-question was used to retrieve prior knowledge when stating the steps followed in solving a particular problem.
- Case B engaged learners in the multiplication operations using the *fill-in* questions.
- Case B used the *Either ... or ...* question to provide learners with options from which the correct answer can be chosen.

The results and findings from Table 6-9 also show that closed questions can be used to promote learners' understanding of mathematical discourse when they are phrased in a way that required learners to mention or state the steps to be followed when solving problems; to apply the multiplication operations; and to choose the correct answer from the options provided.

However, most of the learners' responses are also in chorus form and due to the many closed questions used, it is difficult for the teacher to make sure that all the learners' understanding of mathematical discourse has been promoted. Case B did not use the *thumbs up and down* strategy to get feedback from all the learners with regard to the *Yes* or *No* responses. This could be due to inadequate pre- and in-service programmes that are found wanting, as reported by mathematics teachers in a survey conducted in California (Kersaint *et al*., 2009:59).

6.3.1.3 Questions used to promote mathematical discourse and ESL development.

The results for questions used to promote mathematical discourse and ESL development, captured in Table 6-10, show input in the form of questions used and in the form of examples, followed by learners' responses and the functions of these questions in as far as the learners are concerned.

Table 6-10: Questions used by Case B to promote mathematical discourse and ESL development

INPUT	EXAMPLES	OUTPUT	FUNCTION
What-question	Case B: *We are looking for the gradient and PR, and if they are equal, it means what*...? Case B: *They are collinear, and if they are not equal, it means what ...?*	L 6: *Points are collinear*. L 7: *They are not collinear*. (Case B L O 3: 58-61).	Define mathematical terms
How-question	Case B: *And that gives us 2 over 2, and we are having a gradient of 1 over 1. How do we know that the points are collinear?* *When we say that P, Q, and R are collinear points, what are we talking about?*	L 5: *They must have equal gradients* (Case B L O 3: 25-28).	Explain the reason for the mathematical terms used.
Why-question	Case B: *Why is the graph turning at -16*?	L 4: *Because the value of y is - 16*. (Case B L O 2: 51-53).	Explain the reason for the question asked

Table 6-10 shows that open-ended questions like *What*, *How* and *Why* elicited responses in the form of sentences and clauses, consequently enhancing learners' mathematical discourse and ESL development. It also shows that responses to open-ended questions are not in chorus form, confirming that individual learners' understanding of mathematical discourse and ESL development are promoted.

Furthermore, Table 6-10 shows that open-ended questions promoted mathematical discourse and ESL development when they are phrased in a way that required learners to perform the following functions:

- to define mathematical terms, using the *What*-question;
- to explain the meaning of mathematical terms, using the *Why*-question; and
- to explain the reasons for the question asked, using the *How*-question.

However, open-ended questions were used on a small scale to provide the intended outcomes, even though Case B supported the use of such questions as pointed out below,

Yes, learners should be asked questions that demand thinking and talking.

(I Case B 4:59).

The possible reason for his positive response with regard to the use of questions for this function could be the absence of such questions in mathematics textbooks as pointed out in (Bellido *et a*l., 2005). Case B also submitted one lesson plan for four days, and that could be attributed to the fact that he is overloaded, teaching five grade 10 mathematics classes (see Table 5-2 on the particulars of the participants). As a result he does not have time to prepare and write five lesson plans for each of his 5 grade 10 mathematics classes per day.

6.3.1.4 The functions of questions used

Table 6-11 shows the summary of the functions of the questions used during lesson observations and interviews with Case B.

Table 6-11: Functions of the questions used in Case B's Grade 10 mathematics classroom

FUNCTIONS	INPUT AND EXAMPLES
Corrective - to direct learners towards the correct answer	Case B: *What is the gradient of MN? Is it 3? or is it -3? Some are getting 3 and others are getting -3, someone is wrong here.* (Case B L O 4: 69-72).
Constructive - to test prior knowledge	Case B: *The reason why I used the 'what?' question is to test the previous knowledge, so most of the questions I was asking them, I was expecting them to know their answers. We were doing graphs where in most of the questions they are expected to solve for x to get*

FUNCTIONS	INPUT AND EXAMPLES
	the x -intercept because I have already taught them *how to solve for x.* (I Case B 1:27-30).
Cognitive - to encourage higher-level thought processes	Case B: *The gradient of A B= 2, and the gradient of B C is = -1/2*. *Prove that A B is perpendicular* *to B C.* (Case B L O 4: 9-10).
Managerial - for control and discipline	Case B addressed learners who made noise by saying "*Are you fine?*", and they stopped making noise and focussed on what the teacher was teaching in class. (I Case B 1: 71-80).
Language acquisition - to communicate mathematical ideas	Case B: *Why is the graph turning at -16*? L 4: *Because the value of y is -16*. (Case B L O 2: 51-53).
Evaluative - to evaluate the lesson and receive feedback	Case B: *When I ask them, "Do you understand*?" *I want them to respond positively as to whether they have understood the concept that I've been teaching them or not*. (I Case B 1: 48-49).

Table 6-11 showed that the function of questions for comprehension was used. The constructive function was used to test prior knowledge; and the managerial function to maintain discipline and control during lessons. For language processing and interaction, the corrective function was used to direct learners towards the correct answer from the options given. The imperative *Prove* was used for the cognitive function for learners to apply high cognitive skills. For output, *Why* questions were used for the language acquisition function for learners to use and produce language. For feedback, the evaluative function for the participant to check whether learners understand what has been taught (I Case B 1: 48 - 49).

It should be noted that Case B used questions for the language acquisition on a small scale. This could be attributed to the fact that he did not speak the learners' home language, and as a result communication was stunted due the vast differences between Case B's L1 (Shona) and that of the learners (Setswana). This was conspicuous even though Case B continued with the lessons despite realising that the learners did not understand what was taught as pointed out during the interview (I Case B 3: 16-30). As a result, Case B was unable to use learners' L 1 to allow learners to understand and process information, for their concept development and language acquisition (Kersaint *et al*., 2009:136).

The diagnostic function, to diagnose learners' problems was used interchangeably with the evaluative function of questions, even though they did not achieve the intended outcomes of diagnosing learners' problems and verifying that learners were provided with the correct feedback. The affective function of questions, for learners to show that they enjoy the subject, was not used at all despite the school having a fully-functioning library as pointed out during the interviews (I Case B 3:79).

It should also be noted that Case B used many procedural questions for the evaluative function, and these elicited *Yes* responses only in the chorus form, making it difficult for the teacher and the researcher to ascertain as to whether all the learners understood what had been taught, despite what the participant said as pointed out above in (I Case B 1:48-49). The strategy, *thumbs up and down*, for *Yes* or *No questions,* if used, could assist Case B to obtain feedback from all the learners by viewing their responses simultaneously and privately (Kersaint *et al*., 2009:89).

6.3.1.5 Questioning techniques used

Table 6-12 shows the results of the questioning techniques used during lesson observations

Table 6-12: Questioning techniques used in Case B's Grade 10 mathematics classroom

QUESTIONING TECHNIQUES	EXAMPLES	PURPOSE
Repetition	Case B: *Okay, now we have got all the coordinates that will help us to Draw the graph of $y = x^2 – 16$, so let us look at the next equation. We have got $y = {}^1/_{2x} + 2$,now we have to look for y- and -x intercepts. Looking for our x- intercept, it means that y = 0, then it means that ${}^1/_{2x} + 2 = 0$, ${}^1/_{2x} = -2$, so what do we do next*? L 4: *We multiply by 2* Case B: *We multiply by 2 to get the value of x*. (Case B L O 2: 31 - 38).	Provide learners with feedback that the answer is correct.

QUESTIONING TECHNIQUES	EXAMPLES	PURPOSE
Probing	Case B: *Why is the graph of* $Y = x^2 - 16$ *turning at -16*? L 4: *Because the value of y is -16*. (Case B L O 2: 51 - 53). Case B: *By probing learners will also know that the answer is not correct*. (I Case B 2: 40 - 42).	For learners to produce output
Rephrasing	Case B: (He draws perpendicular lines on the board) *What do they do*? (No response) Case B: *When they meet they create what*? Chorus: *90 degrees* Case B: *Very good*. (Case B L O 4: 31 -35).	Language processing
Wait-time	Case B: For the learners to think about *analysing the question before they start answering it*. (I Case B 3: 52-53).	Learners have to think about the question and the language to be used

Table 6-12 shows that the following questioning techniques were used for the following purposes:

- Comprehension:
  The question was rephrased to simplify it and to make it comprehensible to the learners;

- Feedback:
  Repetition of learners' correct answers provided learners with feedback that the answer is correct;

- Output:
  Probing learners to explain the reason for the question asked;

- Language processing and interactions:
  No questioning techniques aimed at this outcome were used.

Case B used questioning techniques for comprehension, output and feedback. It should be noted that questioning techniques for language processing and interaction were not used at all, despite the participant's support for the *wait-time* strategy as indicated below.

> *So, when you ask a question, learners have to think about analysing the question before they start to answer*. (I Case B 3: 53-54).

Similarly, as stated by Case B, the *wait-time* strategy provides learners in mathematics classrooms with opportunities to process language and interact. This allows ESL learners to process information, reflect on presented information, and grapple with mathematics ideas (Kersaint *et al*., 2009:86).

6.3.1.6 Teacher strategies used

Table 6-13 summarises the teacher strategies used with examples, followed by the purposes of using such strategies.

Table 6-13: Teacher strategies used in Case B's Grade 10 mathematics classroom

TEACHER STRATEGIES	**EXAMPLES**	**PURPOSES**
Teacher-learning setting	The traditional seating arrangements maintained throughout the lesson, resulting in a teacher-centred learning environment (F N Case B: 1: 2).	Teacher-dominant environment without learner participation
Revision	Case B revised the previous lesson before a new topic was introduced to enable the learners to link the known with the unknown, the new topic. (F N Case B: 3).	Linking known to the unknown for lesson comprehension
Using illustrations	Case B: *They (perpendicular lines) are like these*. (Case B draws perpendicular lines on the board). *The perpendicular lines*. (Case B L O 4: 27-28).	Illustrations are used to help learners comprehend what had been taught, i.e. mathematical concepts
Checking learners' class work and homework	Case B: (Case B checks the learners' individual graphs and explains to them the reasons why they are facing in the opposite direction). (Case B L O 2: 76-77).	Provides learners with feedback
Doing corrections	Case B: *Okay, I hope you are ticking what is correct, and if your answer is wrong, You should do the corrections*. (Case B L O 3: 78-80)	Provides learners with feedback so that they can avoid making the same mistakes in the future
Doing many examples on the board	Case B: *To illustrate what I am teaching, to make them understand*. (I Case B 2: 35-37).	Examples help learners to comprehend the work that will be given thereafter

TEACHER STRATEGIES	EXAMPLES	PURPOSES
Using drills	Case B used drills, *For the learners to understand new concepts and use them in class.* (I Case B 2: 46-47).	Drills are used to teach learners formulae that they should know by heart
Being fully prepared	Case B: *I go to class fully prepared and I should also say positive things about mathematics.* (I Case B 2: 69-70).	The teacher prepares so that learners can understand fully
No-hands strategy	Case B: *It has its positive and negative aspects*. .... *I don't think it should be an everyday strategy.* (I Case B 4: 24-36).	Learners have to use and produce language output
Demystifying mathematics	Case B: *By teaching them how it (*Mathematics*) relates to their everyday life e.g. Trigonometry is related to constructions*. (I Case B 4: 69-73).	The teacher attempts to simplify mathematics and make it comprehensible
Incorporate the writing skills	Case B: *to expand learners' creativity and level of understanding*. (I Case B 4: 78-86).	The teacher tries get feedback from learners
Broadcast and	Case B uses Broadcast *for all the learners*	These questions are aimed at comprehension, e.g. (The teacher writes the following on the chalkboard): *1.* $y = x^2 - 25$. Case B: *Now, with this one we are talking about a quadratic graph. What are first things that we will be looking for here*? Chorus: *y – intercept* (Case B L O 1: 11-15).
Target questions	*As well as target questions for the gifted learners*. (I Case B 1: 108).	These questions are aimed at language processing and interactions, e.g. Case B: *She says that the value of x is -16. What is another reason for the graph to be turning at -16*?(*There was no response). The graph turns at -16 because it is where it cuts the Y-axis*. (Case B L O 2: 54-57)
Wait-time	Case B: *For the learners to think about analysing the question before they start to answer it.* (I Case B 3: 52-53).	Learners are given time to think about the question and the language to be used

Table 6-13 shows that for learners to achieve the necessary outcomes, the following teacher strategies were used:

- Comprehension:
  The teacher strategies included *revision*, *illustrations*, *checking* if the learners had done the home work given, *doing many examples* before giving them class work, *drills* for learners to understand formulae and concepts, *being fully prepared*, and *demystifying* mathematics.

- Language processing and interaction:
  Teacher strategies used included *broad cast* questions for all the learners, *target* questions for gifted learners; and *wait time* for learners to think about the question and the responses.

- Feedback:
  Case B did *corrections* on the board for work given as class work and homework;

- Output:
  No teacher strategies were used to achieve language output.

Learners were not provided with opportunities to produce language in the form of output as Case B maintained a teacher-centred approach throughout the lesson observations (F N Case B 1: 2). This was also the case with Case A. who spent most of the time talking while learners sat listening passively. As a result of the teacher-centred approach maintained, Case B was unable to provide learners with opportunities to take risks in language production on a small scale, like for example with a statement like, "*Tell your partner what you think the answer is and why*" (Kersaint *et al*., 2009:87).

Furthermore, Case B did not embrace the *no-hands* strategy, a strategy for learners to produce language, stating that,

> *Yes, it has some positives and negatives to me. (I Case B 4: 24-36).*

Cases A and B did not use questioning techniques and teacher strategies for output. This is attributed to the fact that mathematics textbooks are written for all the learners, irrespective of whether they are proficient in English or not, as is the case with ESL learners. This was also attested by mathematics teachers teaching ELLs in California who mentioned 'a *lack of ELL-friendly textbooks or assessments',* as one of their challenges (Kersaint *et al*., 2009: 58).

6.4 Case study C

6.4.1 Results and interpretations for Case C

6.4.1.1 The most frequently-used question types

Table 6-14: Imperatives used in Case C's lesson plans

LESSONS PLAN FOR	QUESTION TYPES
Day 1	*Factorise* $(x^2 + 2x + 1$ *Find* the y - intercept *Find* the x - and y - intercepts of a) $-y = x^2 - 16$ b) $y = {}^1/_2x + 2$
Day 2	(No lesson plan provided)
Day 3	*Factorise* (a - 4)/ 2(4 - a)
Day 4	*Simplify*: $\frac{x^2 - 4}{4} \div \frac{2x^2 + 8}{x^2} \div \underline{4x^2 - 2x^2}$
Day 5	*Simplify*
	a) ${}^{5a}/_{b} - {}^{3a}/_{2b} \div {}^{a}/_{3b}$ b) $x + y/x \ \ y^2 - x^2/x^2$

The results on Case C' lesson plans show that six imperatives were used in the lesson plans over a period of five days, and no questions were used. As it was the case with the other participants, Case C did not write questions in her lesson plans. She submitted photocopies of exercises from the prescribed textbook, which had imperatives only.

As it was the case with the other two participants, Case C also did not have an official document used for capturing lesson plans. This could be due to the fact that teaching, and not questioning, is emphasised in mathematics classrooms, despite that the fact that learning begins with questions (Chuska, 1995:7).

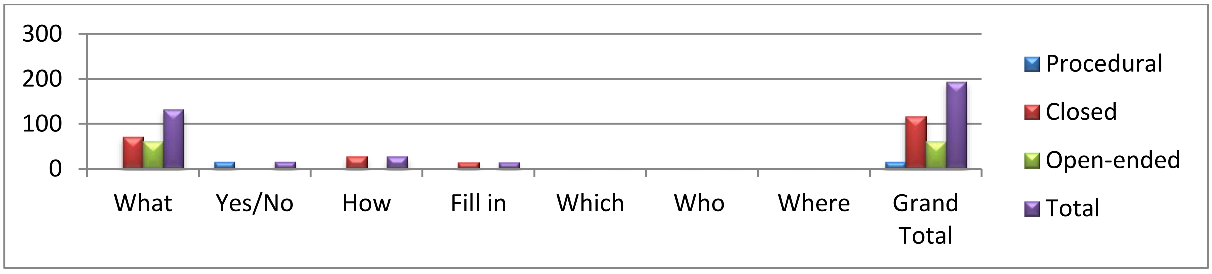


Figure 6-3: Types of questions used during Case C's lesson observations

Figure 6-3 shows that Case C used 105 closed questions, 10 procedural and16 open-ended questions.

Case C also used more closed questions than open-ended ones. The findings are similar to those in Borich (2004:317) which found that 80% of all questions asked in mathematics classrooms were closed questions. He describes them specifically as *direct* questions since they limit the learners' responses to the questions asked.

6.4.1.2 Questions used to promote learners' understanding of mathematical discourse

Table 6-15: Questions used by Case C to promote learners' understanding of mathematical discourse

QUESTIONS	INPUT	OUTPUT	FUNCTIONS
What-question	Case C writes on the board $\underline{21p^3 - 9qp^2}$ X $\underline{4qp + 4q^2}$ 6^2 - $14p^2q$ $7p$ Case C: *What is the common factor*? (Case C L O 4: 21-22).	L 2: p^2	Mentions or states what is happening on the board.
How-question	Case C: *4 into 4, how many times*?	Chorus: *1* (Case C L O 5: 45 -46).	Requests for the answer after applying the division operation.
Fill in question	Case C: *Yes, a number can also be a common factor*. *What about 2x + 6*? *Yes, Seun*? Case C: *Yes, 2 is the common factor, then we have* *2(x + ...)*?	Seun: *2 is the common factor* . Chorus: *3* (Case C L O 3: 20-24).	Completes the blank space left open in the question asked

The following questions were used to promote learners' understanding of mathematical discourse:

- The *What*-question was used to ask learners to mention or state what was happening on the board;
- The *How*-question was used to ask learners to apply the division operation;
- The *Fill-in* question was used to ask learners to complete the blank spaces left open.

It should be noted that the *What*-question elicited a one-word individual response. However, other questions elicited chorus responses that made it difficult for the researcher and the participant to conclude that all the learners' understanding of mathematical discourse was promoted. Similarly, as a result of the use of closed questions, Yeo and Zhu (2009:6) in their findings in the research aimed at investigating the extent of higher-order thinking in mathematics classrooms in Singapore, showed that learners repeatedly regurgitated and replicated the knowledge they had been taught and that higher-order thinking is not encouraged in Singapore mathematics classrooms.

6.4.1.3 Questions used to promote mathematical discourse and ESL development

Table 6-16: Questions used by Case C to promote mathematical discourse and ESL development

INPUT	EXAMPLES	OUTPUT	FUNCTIONS
What-question	Case C: *Look at* $(ab - a^2)$ *What can you say about it*?	L 2: *Our common factor is a.* they see happening on the board. (Case C L O 2: 18-19).	Learners provide opinions on what

Table 6.16 shows the following:

- Open-ended questions elicited individual responses in the form of simple sentences, and these promoted mathematical discourse and ESL development.

However, the number of open-ended questions to promote mathematical discourse and ESL development was very small.

In response to the low number of open-ended questions used to promote mathematical discourse and ESL development, Case C responded as follows:

> *We believe as mathematics teachers that we do not have to know more about the English. We do not talk too much in mathematics – we do too much we do not believe in acquiring more English even in our language as we speak you can hear that this person does not know English, in fact we are not interested in knowing English.*(I Case C 3:79-83).

The excuses provided by Case C are similar to those provided by Cases A and B, as all of them exonerate themselves from the responsibility of teaching language in mathematics classrooms. The excuses should be discouraged as research has proven that “effective questioning skills have been linked with learners’ achievement in mathematics” (Shahrill, 2013:230).

6.4.1.4 Functions of the questions used

Table 6-17: Functions of questions used in Case C's lesson observations

FUNCTIONS	INPUT AND EXAMPLES
Diagnostic - to diagnose learners' problems	Case C used *Yes* or *No* questions to check if learners understood what has been taught, e.g. Case C: *Are we alright*? Chorus: *Yes, Mam* (Case C L O 1: 74 - 75).
Constructive - to test prior knowledge and recall information	Case C: *+3 into 6 how many times*? Chorus: *2 times* (Case C L O 1: 91 - 92).
Cognitive - to engage learners in the thinking process	Case C: *You develop learners' thinking when you use What-questions*. (I Case C 1: 29 - 30)
Language acquisition - for learners to produce the language	Case C: *If you look at* $ab - a^2$, *what can you say*? L 1: *Difference of two squares* Case C: *Difference of two squares*? *Is it the difference of two squares*? Chorus: *No*
Corrective - to direct learners towards the correct answer	Case C: *Look at* $(ab - a^2)$ L 2: *Our common factor is a* Case C: *Our common factor is a. It means in the brackets we are left with what*? Chorus: *(b - a)* (Case C L O 2:14 - 21).

Table 6.17 shows how the teacher reached the different outcomes by using questions with their respective functions.

- Comprehension:
  The diagnostic function of questions was used to diagnose learners' problems; and the constructive function, to test prior knowledge;

- Language processing and interaction:
  The cognitive function was used for learners to use higher levels of thinking; and the corrective function was used to direct learners to the correct answers;

- Output:
  The language acquisition function was used for learners to use and produce language as output.

- Feedback:
  No questions were used for the evaluative and affective functions.

Case C used the functions of questions for comprehension, language processing and interaction, and output. As a result, Case C's learners were very lively and energetic as she used questions with the functions of stimulating language processing and interaction as pointed out below.

> *I apply the Socratic method, i.e. the question and answer method and in that way they get where I want them.* (I Case C 2:30-31).

However, no questions with the function of asking for feedback were used. This could be attributed to the fact that mathematics teachers lack effective questioning techniques, just like mathematics teachers teaching ELLs due to lack of adequate support from educational policies (Kersaint *et al*., 2009:50).

6.4.1.5 Questioning techniques used

Table 6-18: Questioning techniques used in Case C's Grade 10 mathematics

QUESTIONING TECHNIQUES	EXAMPLES	PURPOSE
Repetition	Case C: (ab - $^{a2}/_{b2}$ - a^2 + $^{a2}/_{b2}$ + ab is written on the board Case C: *If you look at ab - a^2, what can you say? Raise up your hands*. L 1: *Difference of two squares*. Case C: *Difference of two squares? Is it a difference of two squares*? Chorus: *No* (Case C L O 2: 13 - 17).	To provide learners with feedback that the answer is not correct.

QUESTIONING TECHNIQUES	EXAMPLES	PURPOSE
Probing learner's incorrect response	Case C writes $^3/_2 \times ^1/_4$ on the board Case C: *What is the LCD of 2 and 4*? *Yes, Tlago*? Tlago: *6* Case C: *Is 4 divisible by 6 Tlago*? *What do you think, Tlago*?	For the learner to think about the answer she gave and thus process it.
Rephrasing	*Come on Tlago, give us a number which is divisible by 2 and 4. Is 2 divisible by 4*? Tlago: *Yes* Case C: *Is 4 divisible by 4*? Tlago: *Yes* Case C: *So what is the LCD of 2 and 4* Tlago: *4* (Case C L O 5: 25 39).	To simplify the question and make it comprehensible
Wait-time	Case C: *Yes, normally I ask the question and the learner who is bright will raise up his/her hand immediately and I will wait for those who are average, five or six students and select one of them to answer the question* (I Case 2: 41- 43).	Probes learners to think and process questions and answers

Table 6.18 shows that the following outcomes were achieved by using the following questioning techniques:

- Feedback:
  The questioning technique, *repetition* was used for learners' incorrect responses;

- Language processing and interaction:
  The questioning technique, *probing* was used for learners' incorrect responses, and *wait-time,* for learners to think about the questions and process them;

- Comprehension:

  The questioning technique, *rephrasing* the questions previously asked was used to simplify them; and

- Output:

  No questioning techniques were used for learners to use language and produce output.

Case C used repetition for comprehension during her lessons, giving the reason below,

> *That is why I repeated it several times so that they can hear while they are looking at it and see that this is straight forward subtraction.* (I Case C 4: 27-29).

Similarly, repetition is encouraged in mathematics classrooms as it allows learners to hear, say, and use mathematics vocabulary if they are to engage in mathematics communication. In fact, according to Murray (cited in Kersaint *et al*., 2009:84), "learners need to use a new word at least 30 times to own it".

However, no questioning techniques were used that allowed learners to produce output, despite the large number of *Yes* or *No* responses that could be probed with *Why*-questions to provide learners with opportunities to provide reasons for such responses, and thus produce language.

6.4.1.6 Teacher strategies used

Table 6-19: Teacher strategies used in Case C's Grade 10 mathematics classroom

STRATEGIES	EXAMPLES	PURPOSE
Learner-centred setting	Case C: *During group work activities, extroverts lead the discussions and introverts learn from their peers. The advantage of group work is that ...* (I Case C 1: 88 – 90)	Language use and output
Doing many examples on the board	Case C does examples 1, 2 and 3 on the board before class work is written on the board. (Case C L O 2: 13; 39 and 51).	Examples enable learners to comprehend the lesson and mathematical concepts

STRATEGIES	EXAMPLES	PURPOSE
Inviting learners to do exercises on the board	Case C: (*The strategy) helps the learners to develop confidence in the subject and a positive attitude towards the subject.* (I Case C 1: 71-81).	Motivates learners
Revision	Case C revised rules on addition and subtraction of like and unlike terms, *for the learners not to make a common mistake of adding the exponents when adding and subtracting like terms.* (I Case C 4: 34 – 37).	Revision enables learners to comprehend the lesson
Linking known to the unknown	After the learners have failed to give the answer to the question, *what is* $4x^3 - 2x^3$ Case C decided to ask the following questions Case C: *What is 4 – 2?* Chorus: *2* Case C: *What is* $4x - 2x$? Chorus: $2x$ Case C: *so, what is* $4x^3 - 2x^3$ Chorus: $2x^3$ (Case C L O 4: 98 – 103).	Comprehension
Praising learners	Case C writes on the board $^3/_2$ *x ¼.* Case C: *What do we do in this case, baby*? L 1: *We find the L C D of the denominator*. Case C: *Excellent! We find the LCD of the denominator*. (Case C L O 5: 19 – 22).	Motivates and boosts learners' confidence
No hands strategy	Case C uses that when, *only one learner is the only one whose hand is raised,* (I Case C 2: 41 – 45).	Encourages learners to use and produce language
Calling learners by their name	Case C, calling learners by their names *helped them to participate in class since they could not hide from me; they easily open up to me and thus share with me their problems at home as they regard me as someone who cares about them.*	Helps learners to be involved and to participate in class and enjoy the lesson.

STRATEGIES	EXAMPLES	PURPOSE
	(I Case C 4: 78 – 83).	
Code-switching	Case C: *their eyes light up showing that they have heard something that is very important than when it was put in English* (I Case C 1: 102 – 104).	Helps learners to comprehend the mathematical concepts
Demystifying mathematics	Case C: *for the unknown, variables like a, b, c, x, etc., are used to challenge them to ultimately find the known, i.e. the values of a, b, c, x, etc.* (I Case C 3: 61 – 69).	Simplifies mathematics and make it comprehensible
Question & answer method	Case C: *to get many responses from the learners, and also to promote learners' understanding of mathematical discourse*. (I Case C 4: 45 – 46)	Learners' interactions
Touching learner on the shoulder	Case C: *Learners are assured that they are loved, the teacher has their welfare heart that is why they are being followed up like that,* (I Case C 5: 48 – 53).	Motivates passive learners
Broadcast and target questions	Case C use the questions , *to identify slow learners and to help them and average learners to catch up as they also attempt to respond to target questions.* (I Case C 2: 21 – 24).	Encourages lesson interactions

Table 6-19 shows that the following outcomes were achieved by using the respective types of teacher strategies:

- Comprehension:

  Teacher strategies used included *doing many examples* on the board, *revising work done* in previous lessons and grades, *linking known to unknown*, *code-switching* to the learners' L 1; and *demystifying* mathematics.

- Language processing and interaction:

  Teacher strategies used included the *question and answer method* or *broad cast* questions, and *target* questions; and

- Output:

  Teacher strategies used included *learner-centred setting* during the lesson observations, *group work activities*, *no hands* strateg*y*, *touching learners on the shoulder*, *calling them by their names*; and

- Feedback:
  The questioning strategy of *praising learners* was used.

Case C used teacher strategies for comprehension; language processing and interaction and output as a result of using the learner-centred approach in her lesson observations from Monday to Thursday. She was able to use group-work activities that provided learners with opportunities for problem solving while interacting with their peers. She also provided opportunities for her learners to do the corrections for the exercises given on the board, while other learners asked questions and commented on what was written on the board. As part of interaction, learners learnt and used words related to the context of the problem to communicate their ideas and respond to ideas presented by others (Kersaint *et al*., 2009: 113).

One could see that learners in that classroom enjoyed the lessons and the variety of strategies applied, like *touching passive learners' shoulder*, *calling learners by their names*, which she defended as follows,

> *It is very very important, that one is very very important because if you do not know the child's name, s/he will always hide and will think that this one does not want to know my name so I'm not going to answer any question even though they know the answer.* (I Case C 4:67-69).

Case C also motivated those learners who were discouraged and had a negative attitude towards the subject. She explains this as follows:

> *Most of the learners come from the middle schools and come with the perception that mathematics is a difficult subject...they say we hear about mathematics as a difficult subject but I tell them that there is nothing difficult about mathematics, that mathematics is user-friendly. Working very hard is all that we need from them, that is what I tell them.* (I Case C 2:76-77; 84-87).

Furthermore, it should be noted that Case C is also very passionate about mathematics and she goes all out for her learners as shown below in collaborating with other content subject teachers to address the problem of ESL learners' poor performance in mathematics.

> *We also take our problems to the English department especially with financial mathematics and to the Business studies and Economics teachers and there the learners present as a class now talking about business studies and not mathematics and this will lead them to that. I try by all means to integrate with English teachers, give them essay topics on financial mathematics for them to talk about their own finances at home*

and then in the L O (Life Orientation) also to make a budget and in that way we are addressing the problem. There in the L O, they are also recorded and this is how this language problem is addressed.

(I Case C 1:54-61).

In defending the use of code-switching in mathematics classrooms, Case C responded as follows:

But when doing the equations, you can hear that they (learners) do the equation in English when they say 'solving for x' and they will say 'factorisation' since there is no word in Setswana for that, so you can see that the content is done in English but the expressions, 'Do you understand'?, are done in Setswana. (I Case C 5: 102-105).

Similarly, code-switching to learners' L1 is encouraged as it helps learners to understand that their L1 and culture are valued, and that the learning of mathematics is more important than the language in which mathematics is learned (Kersaint *et al*., 2009:137).

However, the questioning techniques used for feedback were very limited. This is attributed to the fact that Case C did not incorporate the writing skills, a valuable strategy for feedback, into the teaching and learning of mathematics. Writing is important because, "as learners write, they can indicate what they know about a concept; explain their methods; justify their reasoning; and reflect on their learning experiences" (Kersaint *et al*., 2009:107).

6.5 Case study D

6.5.1 Results and interpretations for Case D

6.5.1.1 The most frequently-used question types

Table 6-20: Imperatives used in the lesson plans by Case D

LESSON PLANS	QUESTION TYPES
Day 1	***Do*** corrections ***Factorise*** {3a(p - 3q) - 2b}
Day 2	***Factorise*** ac - ad + bc - bd
Day 3	***Look*** for the common terms ***Open*** the brackets ***Take out*** the common factor ***Write*** the common factor firstly ***Open*** the bracket for terms outside the brackets (Instructions for exercises on the grouping of terms given)

LESSON PLANS	QUESTION TYPES
Day 4	***Factorise*** $6p^3 - 4x^3 + 3p^2x - 8px^2$
Day 5	***Factorise*** $(a^2 - y^2)$

In a period of five days, Case D used 10 imperatives excepting questions. Case D, just like the other 3 cases, used photocopies of exercises from the prescribed textbook, which also use imperatives and not questions. This accounts for the absence of questions in her lesson plans.

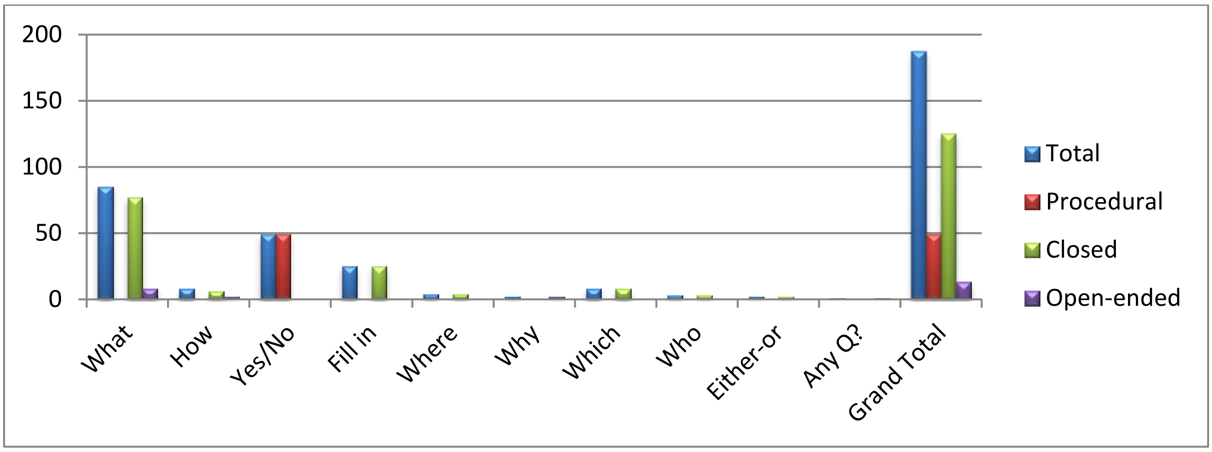


Figure 6-4: Types of questions used by Case D during lesson observations

Figure 6-4 reveals that 125 of the questions were closed, 49 procedural, and 13 open-ended. The most frequently-used question types are closed questions.

Figure 6-4 shows that Case D also used few open-ended questions than closed ones.

Similarly, in a study of secondary school lessons conducted in 1989 in the USA, findings reported by Hastings (2003) showed that only 4% of primary questions used in the US classes were of a higher-order nature; and in 1999 Ted Wragg replicated the same research study in primary schools and found that 8% were higher-order questions.

6.5.1.2 Questions used to promote learners' understanding of mathematical discourse

Table 6-21: Questions used by Case D to promote learners' understanding of mathematical discourse

INPUT	EXAMPLES	OUTPUT	FUNCTION
Which	Case D: *We have* $a^2 + 3a + ab + 3b$ *So which terms can we group now? Yes Mooki*? Case D: a^2 *and ab, very good and continue Mooki*? Case D: *+ 3a + 3b, very good Mooki.* *Do you see people?*	. Mooki: a^2 *and ab* Mooki: *+ 3a + 3b* Chorus: *Yes Mam*. (Case D L O 3: Lines 140 - 146).	State or mention what learners see on the board.

Case D promoted learners' understanding of mathematical discourse by inviting learners to perform the following function:

- State or mention what they see on the board.

It should be noted that closed questions that elicit individual responses promoted learners' understanding of mathematical discourse. However, those eliciting chorus responses made it difficult for the participant to see if the mathematical discourse has been promoted.

6.5.1.3 Questions used to promote mathematical discourse and ESL development

Table 6-22: Questions used by Case D to enhance mathematical discourse and ESL development

INPUT	EXAMPLES	OUTPUT	FUNCTIONS
What-question	Case D: *What do we do now*? (No response) Case D: *What do we do*?	Makgetha: *We take out the negative sign.* (Case D L O 2: 130 - 134).	Explain the steps to be followed when solving a particular problem.
How-question	E.g.2: *-3x + 15y* Case D: *How do we factorise this according to the examples we have done?* *Yes, Malepa*?	Malepa: *We are going to find the highest common multiple.* (Case D L O 2: 10 - 14).	Explain the steps to be followed when solving a particular problem.

INPUT	EXAMPLES	OUTPUT	FUNCTIONS
Why-question	Case D: *So I'm going to say -3 into x* *and* she writes *-3(x ?). Is it going to be a plus or a minus sign*? Case D: *Minus, Why Minus? Yes, Sefuno*?	Chorus: *Minus* Sefuno: *Because a minus time a minus is equal to a positive sign*. (Case D L O 1: 49 - 53).	*Probing responses to* procedural questions with a *Why*-question.
Any question	Case D: *So the answer is (9c + 7(d - e)(9c -7(d - e), Any questions*? Case D: *There is nothing wrong in that as long as the signs are opposite*.	Mabeleng: *What if I start with the negative sign in the first bracket and a positive sign in the second bracket*? (Case D L O 5: 128 - 133).	Providing learners with *opportunities to ask questions on the lesson* taught.

An analysis of the open-ended questions shows that they promoted mathematical discourse and ESL development when they were phrased in a way that required learners to perform the following functions:

- Explain the steps to be followed when solving a particular problem, using the *what*-and the *How*-questions;
- Explain the reasons for their *Yes* or *No* responses, using the *Why*-question;
- Provide learners with opportunities to ask questions on the lesson taught, using *Any question,* even though it was used once within a period of five days.

The table also shows that open-ended questions always elicited open-ended responses. The findings during this observation are similar to those found in the other cases' classes in as far as the outcomes of open-ended questions in mathematics classrooms are concerned. This is the reason why Shan Wen (2004:7) and Tuan and Nhu (2010:34), describe open-ended questions as creative questions. They involve learners in higher-level thinking processes and require them to think critically and creatively as they call for interpretation, opinion, evaluation, inquiring, making inferences, and synthesising.

Functions of questions used

Table 6-23: Functions of questions used in Case D's Grade 10 mathematics classroom

<table>
<tr><th>FUNCTIONS</th><th>QUESTIONS AND EXAMPLES</th></tr>
<tr><td>Evaluative- to check if learners understand</td><td>Case D: Keitumetse. Are you with us?
Keitumetse: Yes Mam.
(Case D L O 3: 79 - 80)</td></tr>
<tr><td>Constructive - to test prior knowledge
and</td><td rowspan="2">(Case D writes on the board -3x + 15y)
Case D: According to the examples we have done, what are we supposed to do? How are we going to factorise these according to the examples?
Yes, Malepa?
Malepa: We are going to find the highest common multiple
Case D: He is saying we are going to find the highest common multiple, is that correct?
Chorus: No, Mam.
Case D: No Malepa, Yes Sefuno?
Sefuno: The highest common factor.
Case D: The highest common factor.
(Case D L O 1: 9 - 21).</td></tr>
<tr><td>Language acquisition -
to encourage learners to use and produce language</td></tr>
<tr><td>Evaluative -for learners
to evaluate the lesson and to receive the feedback</td><td>Case D writes (9c + 7(d - e) (9c - 7(d - e)
Case D: Any questions?
Mabeleng: What if I start with a negative sign in the first bracket and a positive in the second bracket?
Case D: Mabeleng says what if you start with a negative sign in the first bracket and a positive sign in the second bracket.
There is nothing wrong in that as long as the signs are opposite.
(Case D L O 5: 128 - 133).</td></tr>
</table>

An analysis of Table 6-23 shows that the following outcomes were achieved by using the different functions of questions:

- Comprehension:
  The constructive function was used to test prior knowledge and the evaluative function, to check if learners understood what had been taught.

- Language processing and interaction:
  No function of questions were used; and

- Output:
  Case D used "*Any questions*" for the language acquisition function of questions.

- Feedback:
  The evaluative function of questions was not used.

Case D used many questions for the *diagnostic* and *constructive* functions for learners' comprehension and "Any questions?" for the language acquisition function. However, no questions for the function of language processing, interactions and feedback were used. This could be attributed to the fact that the prescribed mathematics textbooks used by the participants did not have questions for these functions as shown in the findings on the types of questions used (Laridon *et al*., 2008: 131).

6.5.1.4 Questioning techniques used

Table 6-24: Questioning techniques used in Case D's Grade 10 mathematics classroom

QUESTIONING TECHNIQUES	EXAMPLES	PURPOSE
Repetition	Case D writes (a^3 - a^3b) on the board Case D: *What will our common factor be*? *What is it going to be*? L 5: a^3 (Case D L O 1: 64, 79, 81).	Gives learners a chance to hear it for the second time and comprehend it
Probing	Case D writes a^3 - a^3b on the board Case D: *Do we have a in the first term*? Chorus: *Yes* Case D*: Is that the only a we have in the first term*? Chorus: *No* Case D: *So we have the second a in term one and two and the third a in term one and*	Makes sure that learners understand what is explained

QUESTIONING TECHNIQUES	EXAMPLES	PURPOSE
	term two. Look at term two, it has b. Do *we have b in term one?* Chorus: *No* (Case D L O 1: 72 - 77).	
Rephrasing	Case D writes (*$-3x + 15y$*) on the board Case D: *What are we supposed to do here*? *How do we factorise this according* *to the examples*? *Yes, Malepa*? (Case D L O 1: 11- 13).	Simplifies the question and makes it comprehensible to the learners.
Clues	Case D writes $2a^2(x - y) + b(-x + y)$ on the board Case D: *Which bracket are we going to swop*? (No response) Case D: *Which one*? *The first or the second* *bracket*? *Yes, Mooki*? Mooki: *The second bracket*. (Case D L O 2: 64 - 69).	Leads the learners to the correct answer and helps them comprehend it

Table 6-24 showed that the following outcomes were achieved when the different questioning techniques were used:

- Comprehension:
  Questioning techniques used included *repetition* of questions asked and *rephrasing* the question asked to simplify it for better comprehension;

- Language processing and interaction:
  Questioning techniques used included *probing* learners' closed responses with procedural questions; and providing learners with *clues* by using two or three options from which the correct answer can be chosen;

- Output:
  No questioning techniques for this outcome were used; and

- Feedback:

  No questioning techniques used for this outcome.

Table 6-24 shows that most of the questioning techniques used were for comprehension and language processing and interactions, and none were used for output and feedback. Case D, just like Cases A, B and C, did not use questioning techniques for learners to use and produce language on a large scale. The inability of the participants to use effective questioning techniques confirms the findings of a study where mathematics teachers who had participated in professional development programmes reported that they found them inadequate (Kersaint *et al*., 2009: 59).

6.5.1.5 Teacher strategies used

Table 6-25: Teacher strategies used by Case D

STRATEGIES	EXAMPLES	PURPOSE
Traditional setting	A traditional seating arrangement was maintained throughout the lesson observations. (F N Case D: 1).	Absence of learner Interaction and language production
Calling learners by their names	Case D: *to show them that they are* very important, ... and that encourages them to do the right things. ... It keeps them alert and makes them feel special, and as a result, they work hard in order not to disappoint me. (I Case D 3: 32 – 49).	Invites them to produce output during the lessons.
Demystifying mathematics	In the example: *3a(p –q) -2b(p – q)* Case D: *Inside the brackets we have Kefilwe* *and Tom, and outside the brackets, we have* *Mother and Father. This our Father is 3a* *and what is our Mother*?, *Akanyang*? Akanyang: *-2b* Case D: - *2b. Very good*. (Case D L O 2: 13 – 17).	Provides better comprehension of the lesson and mathematical concepts.
Praising learners	Not only did Case D confirm the answer by repeating it in the lesson above, but she also praised the learner.	Acknowledge that the learners did well and encourages them to have a positive attitude towards the subject - Feedback
Revision	a^2 +a(3 + b) +3b T D: *After a, you could say between a and the*	

STRATEGIES	EXAMPLES	PURPOSE
	brackets, okay we have the multiplication	
	sign and an addition sign, so now according to the BODMAS Rule, which sign must we deal with first? *Yes, Sedumedi*? Sedumedi: *The multiplication sign*. (Case D L O 3:107 – 110).	
Question & answer method	Case D: *The word 'difference' means subtraction isn't it*? Chorus: *Yes* Case D: *And for addition we use*? Chorus: *Sum* Case D: *And for division we use*? Chorus: *Quotient*. Case D: *And for multiplication we use*? Chorus: *Product*. Case D: *Product, very good*! (Case D L O 5: 26 – 35).	Language processing and interactions
Learners do corrections on the board	Case D gives learners a chance to do class work on the board to check whether they understand what I was teaching. (I Case D 3: 21- 26).	Feedback in written form
Group and pair work activities	Case D: *Group work and pair work activities I used them to engage learners in a discussion*. (I Case D 2: 90 – 96).	Learner-interaction and output
Broadcast questions	Case D embraces using broadcast questions, *To help the learners recall what they have learnt, e.g. The BODMAS rule*. (I Case D 3: 55 – 61).	Language processing and interactions
Target questions	T D also uses target questions, *for*	Language output

STRATEGIES	EXAMPLES	PURPOSE
	bright or gifted learners. (I Case D 3: 62 – 77).	and processing
No hands strategy	Case D also embraces the *No-Hands* strategy only if all the learners did not respond to the question, *I moved from row to row*, *asking learners to give her the answer*. (I Case D 2: 106 -117).	Language output and processing
Incorporating the writing skills	Case D: *learners are encouraged to use journals to capture the steps followed to solve a problem,.. for reference purposes*. (I Case D 5: 62 – 67).	Provides feedback
Code-switching	Even though Case D is sometimes forced to code-switch to the learners' first language during lessons, she promises to discourage learners from using their mother tongue as she has realised *it is a barrier in learners' performance in mathematics.* (I Case D 5: 85 – 105).	Helps with lesson comprehension and mathematical concepts.

Table 6-25 shows that the different teacher strategies used to achieve the following outcomes:

- Comprehension:
  Teacher strategies used included *demystifying mathematics*, *revision of work done* in previous lessons; and *code-switching* to the learners' L 1 when they did not understand mathematical concepts;

- Language processing and interactions:
  Teacher strategies used included *question and answer method* or *broadcast* questions and t*arget* questions;

- Output:
  The teacher strategies used included *inviting learners to do corrections on* class and homework given on the board; *calling learners by their names* to motivate them to produce output, and make them feel special (I Case D 3:32-49), and

- Feedback:

  Teacher strategies used included *praising learners* for their correct responses.

Most of the teacher strategies used were for comprehension, language processing and interaction as a result of the teacher maintaining a teacher-centred approach. For example, Case D also code-switched extensively during the lessons, and her learners expected her to do so during the lesson observations. Very few strategies for output and feedback were used as a result.

Even though Case D identified learners who had a negative attitude towards mathematics, she blamed them as pointed out below,

> *They take it to be difficult, they are lazy. When they get home, they just close their books. It seems like parents do not encourage them to practise on their own, to read, that is why learners come to school without having done their homework.* (I Case D 1: 77-79).

It seems that Case D had difficulty communicating with learners and parents as was the case with teachers teaching mathematics to ELLs in a survey of 5300 teachers in California. The teachers were unable to engage and discuss the learners' progress with their parents as they could not rely on parents to assist with homework (Kersaint *et al*., 2009: 58). Teachers are advised to involve parents as resources to help the children learn mathematics by providing them with opportunities to even supervise their children's completion of the assignments (2009:138).

Case D demystified mathematics by relating it to learners' everyday life as advised in Kersaint *et al*., (2009:113). The strategy assists teachers to explain the meaning of words, especially in the *Word problems* section. However, the strategy can still be used in other mathematics lessons as it is the case in *Factorisation* as shown in the example provided (Case D L O 2: 13 – 15).

Case D also praised learners for the correct answers provided to provide them with feedback that their answers are correct (Case D L O 2:16–17).

6.6 Conclusion

In conclusion, the findings on each of the four cases show that in mathematics classrooms, closed questions can be phrased in such a way that they promote learners' understanding of mathematical discourse (Cases A, B, C, and D); and open-ended questions can be used to promote mathematical discourse and ESL development (Cases A, B, C, and D). Furthermore, the results show that there are questions with specific functions, questioning techniques, and teacher strategies that can be used to assist learners with comprehension, language processing and interaction, opportunities to produce output, and feedback (Cases A, B, C, and D). Chapter 7 discusses the results and findings as derived from data analysed using the ATLAS.ti software and the collective study. A summary of all the findings and their relation to the literature reviewed is also discussed in Chapter 7.

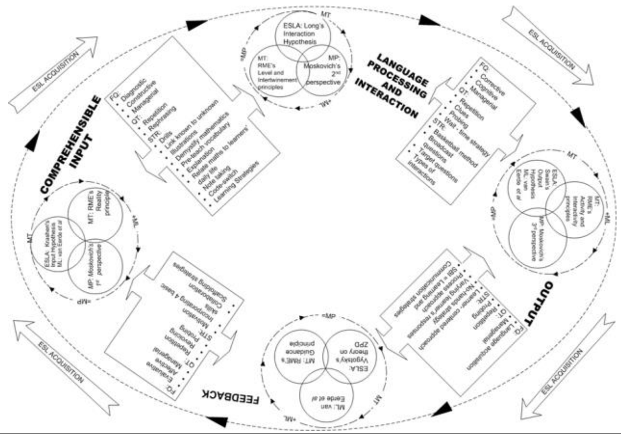
CHAPTER SEVEN: FURTHER ANALYSIS OF DATA USING THE ATLAS.ti SOFTWARE AND THE INTERPRETATION OF THE COLLECTIVE CASE STUDY

7.1 Introduction

Chapter 6 contains the results and interpretations of each of the 4 case studies. Chapter 7 subsequently discusses the results and interpretation of the data on the transcribed lesson observations and interviews analysed using the ATLAS.ti software, and the collective case study. A summary of the findings relating to the results on the data analysed using the ATLAS.ti, the collective case study, the literature reviewed, and the theoretical model developed is discussed as these results relate to the research questions.

Table 7-1 below lists the results of the research and sub-research questions. It presents the data analysed using the ATLAS.ti software and the collective case study. The table also lists the relevant section in the thesis to avoid unnecessary repetition.

Table 7-1: Sections where the results of the case studies and the statistical analyses are captured in Chapter 6

	Research questions	Data analysed using the ATLAS.ti software	Collective case study	Chapters
1	What are the theories that underpin the effective questioning techniques and strategies to promote language acquisition?			2,3, and 4
2	At the descriptive level, the research problem can be formulated in terms of the following research question: What are the characteristics of the most frequently-used question types in Grade 10 mathematics classrooms?	7.1.1	7.2.1.	5, 6, and 7
	2.1 How do the questions used promote learners' understanding of mathematical discourse?	7.1.2	7.2..2	
	2.2 How do the questions used promote learners' understanding of mathematical discourse and ESL development?	7.1.3	7.2.3	
	2.3 What are the functions of the questions used in grade 10 mathematics classrooms?	7.1.4	7.2.4	
	2.4 What are the questioning techniques used in grade 10 mathematics classrooms?	7.1.5	7.2.5	
	2.5 What are the teacher strategies used in grade 10 mathematics classrooms?	7.1.6	7.2.6	
3	What are the characteristics of a hands-on tool that could support grade 10 mathematics teachers in developing questioning skills to promote language acquisition?	The PTQS tool		8

7.2 Data analysed using the ATLAS.ti software

This section answers the second research and sub-research questions based on the findings of the data analysed using the ATLAS.ti software. The discussion is followed by the interpretation of the data based on the findings.

7.2.1 Answering the second research question

This section answers the second research question of the study, namely:

QUESTION 2. What are the characteristics of the most frequently-used question types in Grade 10 mathematics classrooms?

In response to the second research question, Figure 1 on the characteristics of the most frequently-used questions in the 4 grade 10 mathematics classrooms (see Addendum A on the Network File), shows that the participants used 181 procedural questions, 317 closed, and 97 open-ended ones. The findings also show that the participants used more procedural and closed questions than open-ended ones during the lesson observations. The results are similar to those of the 4 case studies (see sections 6.1.1.1, 6.2.1.1, 6.3.1.1, 6.4.1.1). The results are also similar to the findings on the research conducted in mathematics classrooms in the United States of America, United Kingdom, Germany, Australia and Iran, which have also shown that teachers mostly ask closed questions (Long & Sato, 1983; Brualdi, 1998; Sutton & Kreuger, 2002; Sadker, 2003; Zevenbergen & Niesche, 2008; Shomoossi, 2004).

The large number of closed questions used by the participants could be attributed to the fact that the participants' lesson plans had imperatives only, as they prepared no open-ended questions as part of their lesson plans to be used during the lesson observations. The reasons for the use of many procedural and closed questions could be that the participants were not aware of the fact that questions should be made part and parcel of the lesson preparations for the success of every lesson, as "they are major determinants of teaching and learning outcomes" (Chuska, 1995:7).

Figure 1 also shows that the participants provided reasons such as the following for using many closed questions in their classrooms, and these are similar to the reasons evident from the 4 cases.

Case A: *I don't know but I can say that we are used to that and we are directed* *by the content of the subject* (P 31: 049-050)[6].

One participant commented on the ‘content of mathematics’,

> *Like I said yesterday, most of the questions that teachers give students involve calculations. In questions that involve inherent calculations, it becomes difficult for the learner to say something* (P24: 26-28).

The ‘content of mathematics’ therefore refers to the contents found in the prescribed textbooks, which contains exercises such as *Solve for x and y*, and as a result require them to ask closed questions such as *What*, *Yes* or *No*, *Which*, *Either*... *or*, which provide answers for the values of *x* and *y*. This is not surprising because in a research conducted on the role of textbooks in South Africa, the results showed that textbooks do play an important role in curriculum development in South Africa, and also worldwide (Martins, 2013:187). The results further showed that between 61% and 100 % of the curriculum is based on the textbook contents or structure.

7.2.2 Answering the first sub-research question of the second research question

This section answers the following first sub-research question of the second research question.

Question 2.1	**How do questions used in grade 10 mathematics classrooms promote learners’ understanding of mathematical discourse?**

In response to the first sub-research question of the second research question, the results captured in Figure 2 (see Network file from Addendum A) on the types of responses elicited by the types of questions used, show that 171 procedural responses, 290 closed, and 70 open-ended responses were elicited by the types of questions used. It should be noted that procedural responses were mostly in *Yes* chorus form, and closed responses were also in chorus form as shown in the examples below from the instrumental cases.

Example 1 on procedural responses:

> Case C: *If you look at ab – a*^{2}*, what can you say?*
> L 1: *Difference of 2 squares*?
> Case C: *Is it a difference of 2 squares*?
> Chorus*: No*

[6] The code e.g. (P 31: 049-050) stands for number of the participant captured in the ATLAS.ti data analysis, followed by the lines from which the transcripts have been taken.

(P 7:14-17)

Example 2 on closed responses:

Case B writes $y = x^2 - 25$ on the board.
Case B: *What are the first things that we will be looking for*?
Chorus: *y- intercept*
(P 3: 12-15).

The results show that most of the procedural and closed responses were in chorus form. The results are similar to those captured in the 4 case studies (see sections 6.1.1.2, 6.2.1.2, 6.3.1.2, 6.4.1.2). Similarly, the findings in Singapore mathematics classrooms showed that chorus responses were elicited as "learners repeatedly regurgitate and replicate the knowledge they are taught and that high-order thinking has not been encouraged in these classrooms as a result of the use of closed questions" (Yeo & Zhu, 2009:6).

The findings are also echoed in the definitions of closed questions that show that they do not promote learners' mathematical discourse. Closed questions are described as questions that require learners to 'recall, to recognise or to organise material in a predictable way, they are concerned with the right answers or very limited possible answers" (Shan Wen, 2004:6).

Figure 2 on the responses elicited from the Network file shows that closed questions elicited many chorus responses. The findings are similar to those shown in Watson's (2002) study on the roles of unison or chorus responses in mathematics classrooms (Watson, 2002: 39). She also indicated that, as a result of the chorus responses elicited, it was impossible for the teachers to ascertain as to whether all the learners understood the lesson as filling in the gaps left open in the teacher's closed question, is "more to do with rhythm and participation than active, intelligent, informed choice" (Watson, 2002:39).

As a result of mathematics teachers using mostly closed questions that do not promote mathematical discourse, learners' performance in our South African schools is affected. An analysis of the grade 12 learners' geometry examination paper scripts written in 2008 showed that 75% of the learners in response to question 3.2.2, could not tell the differences in meaning of words such as *rotation*, *reflection*, and *translation*, and also between *rigid* and *non-rigid*, and only 25% got this question correct (Luneta, 2015:5).

The use of many closed questions that do not promote learners' mathematical discourse in mathematics classrooms could be attributed to the fact that the participants do not know that mathematics, just like other content subjects, has its special discourse or register. These subject-specific discourses are referred to as academic language or Cognitive Academic Language Proficiency (CALP), which is acquired only in mathematics classrooms, in other words from the teachers and mathematics textbooks, and it takes five to ten years to develop. This is different from the English known as conversational English or Basic Interpersonal Communication Skills (BICS) in SLA, which can be acquired inside and outside mathematics classrooms, and which takes only two years to be developed by ESL and ELL learners (Cummins, 1999). In-service training workshops for mathematics teachers should therefore include programmes for teaching mathematical discourse, the language used in mathematics classrooms, so that the teachers could acquire it and assist learners to acquire it and improve learners' performance in the subject.

7.2.3 Answering the second sub-question of the second research question

This section answers the following second sub-research question:

Question 2.2: How do questions used promote learners 'mathematical discourse as well as ESL development?

The results in Figure 2 further show that the open-ended responses were few compared to closed and procedural ones as a result of few open-ended questions used to promote mathematical discourse and ESL development in Figure 1. The results are also similar to those captured from the case studies (see sections 6.1.1.3, 6.2.1.3, 6.3.1.3, and 6.4.1.3). Hastings (2003) reveals in his 1989 report on mathematics classrooms in the USA that only 4% of questions used in the US primary classes were of a higher-order and Borich (2004:260) shows that teachers used 20% of open-ended questions in mathematics classrooms.

Open-ended responses provide learners with opportunities to use language and produce output as shown in the example below from one of the instrumental cases,

T writes on the board P(3;5), Q(2;1), and R(-3;2)
Case B: *What are we proving here*?
L 2:*To find their gradients*
Case B: *To find their gradients and prove what*?
L 6: *To prove that they are collinear*.
(P10: 44 -47)

Open-ended questions elicit open-ended responses that provide learners with opportunities to produce language, as shown in the above examples. This is also confirmed by Manouchehri and Lapp (2003:564), when they state that the questions give learners an opportunity "to communicate their reasoning process, thus providing the teacher with a better understanding of learners' knowledge".

Figure 1 from the Network file also captured the participants' reasons for using few open-ended questions. In response to why the participants used few open-ended questions, the participants provided the same reasons provided by other mathematics teachers, such as "not having enough time to complete the syllabi" (Jones & Tanner, 2002:269), and also that "open-ended questions take more time of the teachers" (Moyer & Milewicz, 2002:296). The reasons provided by mathematics teachers are simply acknowledgements on their part that they are not conversant with ESL methods of teaching mathematical discourse and ESL development, thus resulting in teacher-centred approaches found in many mathematics classrooms in which learners sit passively while the teacher talks most of the time (Kersaint *et al*., 2009:53).

However, this was not the approach used by one of the case studies, Case C, who provided learners in each group with vast opportunities to do exercises given on the board, asking other learners questions to explain all the steps followed to arrive at the correct answer (P 1:11-47). The exercises given were solved on the board by learners, using language and interacting with other learners.

Some of the reasons for not using open-ended questions to promote mathematical discourse and ESL development, showed that mathematics teachers have exonerated themselves from the role of teaching learners the language used in mathematics classrooms as shown below:

> *I think it is the role of the language teacher because myself as a content teacher, I don't have much time for teaching these learners the language, the syllabus is too much, and I've got to teach the syllabus of the lower classes due to the barrier of the language* (P 19: 107-109).

The reasons are simply acknowledgements on the part of the participants that they are not conversant with ESL methods of teaching mathematics, and are not aware of the cognitive and corrective functions of questions for learners to process and interact using language. There is also the acquisition function for learners to produce language and output. In an effort to address this problem, they need collaboration with ESL teachers as expressed by mathematics teachers teaching ELLs (Kersaint *et al*., 2009:59).

Figure 2 on the responses elicited from the Network file showed that few open-ended responses were elicited by open-ended questions. The results are similar to those in Watson's 2002 study which also revealed that some learners had difficulty using the open-ended chorus responses in the exercise given as "they might only have a vague idea about what they were saying" (Watson, 2002:39).

In summary, teachers are advised to take time to plan effective questions for their lessons as "the time invested to do so could be worthwhile if they focus on essential learning; help learners add to their knowledge and transfer it to other subjects; motivate learners to take real interest in the subject; and help learners apply essential learning to real problems, issues and decisions" (Chuska, 1995:7).

7.2.4 Answering the third sub-question for the second research question

This section answers the following third sub-question of the second research question.

Question 2.3:	**What is the function of the questions used in Grade 10 mathematics classrooms?**

Figure 3 on the functions of questions used in mathematics classrooms show that the participants used many questions for comprehension, 237 for the constructive function, to retrieve prior knowledge, and 61 for diagnostic functions. Most of these closed questions used are classified under the diagnostic function of questions, to diagnose learners' problems; and the constructive function, to test prior knowledge. The functions of these questions are used for comprehension in the theoretical model, in other words, the focus of the participants' lessons is to check whether learners experience problems with the taught content, but then they do not address those problems adequately.

Also, the results showed that 78 questions for the evaluative functions were used to check feedback, and that 73 questions for the language acquisition functions were used to engage learners to produce language. The results are similar to that of the instrumental cases, which show many questions for comprehension, and few for language processing and interaction, and output (see sections 6.1.1.4, 6.2.1.4, 6.3.1.4, and 6.4.1.4). This could be attributed to the absence of a variety of the functions of questions in mathematics prescribed textbooks (Laridon *et al*., 2008), and as a result, they are not available for use and not known by mathematics teachers.

Figure 3 further shows that many questions in the form of procedural questions were used for the evaluative function, even though the responses elicited were in *Yes* chorus form, and as a result, not convincing that the learners understood the lessons. However, by probing the *Yes* responses with *Why* questions, the participants would have prompted the learners to process language and interact using it, to assist learners to use "mathematical language to communicate mathematical ideas, concepts, generalizations and thought processes" (Setati, 2002:9). However, the participants were not aware of the function of these questions as they are not found in most mathematics textbooks (Bellido *et al*., 2005:1).

To evaluate teachers' lesson, Lomen's (2009) two statements have been included in the hands-on-tool discussed in chapter 8, to prompt learners to provide written feedback on a daily basis so that teachers are aware of their problem areas, and prepare remedial lessons on these for the following day before proceeding with that day's lesson.

Lack of resources has been pointed out (see P22: 85-93) as one of the reasons for the absence of the affective function of questions that require learners to research topics that they enjoy in mathematics, and present them as projects in written and oral form, thus incorporating the four basic skills into the teaching and learning of mathematics.

Despite the lack of resources and other problems experienced by mathematics teachers, questions with a variety of functions should be used in mathematics classrooms as research has pointed out that mathematics teachers who are highly rated by their students ask a variety of questions (Sutton & Krueger, 2002).

7.2.5 Answering the fourth sub-question for the second research question

This section answers the following fourth sub-question for the second research question of the study:

Question 2.4: What are the questioning techniques used in grade 10 mathematics classrooms?

The results in Figure 4 from the Network file on the questioning techniques used by the participants show that the questioning technique, *repetition* was used 125 times for questions asked, for learners to understand the question; for learners' correct and incorrect responses, and to provide learners with feedback as to whether the answers are correct or not correct. The results are similar to those from the instrumental cases (see sections 6.1.1.5, 6.2.1.5, 6.3.1.5, and 6.4.1.5).

The results also show that *repetition* and *rephrasing* were used for learners to comprehend the questions. This is also confirmed as asserted in Cashin (1995:1), "to ensure that the entire class hears the question for the second time and to check the learners understanding of the question" respectively.

However, the questioning technique, *learner reads the question*, is mostly used for learners to read the questions given as class and homework, for teachers to write them on the board to be solved. However, Case A supports its use for comprehension during the interviews as shown below:

Case A: *for learners to get a different meaning because listening*

is not the same as reading ... not explaining, just reading.

(P 23: 68-69).

One could rather say it is used for managerial purposes, for learners to focus on the question that is being read out aloud by one of them, for the teacher to write it on the board, and therefore to draw learners' attention to focus on the board and prepare themselves to think about the steps to be followed to solve it.

Questioning techniques for language processing and interaction include *repetition, clues, and probing* learner's incorrect responses. In the example below, Case A provides clues for the learners to identify the correct answer from the options given.

Case A: *3, 6 and 12, so which one is the smallest*?

Chorus: *3*

(P 9:70 -71).

As a result of many responses elicited in chorus form, questioning techniques such as probing were used on a small scale. Probing, when used for learners' chorus responses, provide learners with opportunities to use language and produce output. Probing is encouraged in mathematics classrooms as it is mostly used, "to invite or further investigate the child's incorrect answer" (Moyer & Milewicz, 2002:301). As a result, it is used to discourage learners from copying answers.

Furthermore, probing should be used to discourage learners from doing guess-work. Bellido *et al.* (2009:2) relates the case in a mathematics classroom where the learner simplified $^{16}/_{64,}$ and gave the correct answer as $^{1}/_{4}$even though the wrong method of deleting the 6 that appears in both the numerator and the denominator was used to get the correct answer. The questioning techniques for language processing and interaction and output are captured in the theoretical model for teachers to follow up learners' correct and incorrect responses, "to enable the learners to go beyond the explanation or summaries of the steps they had taken in arriving at the correct answer and resulted in explanations consisting of mathematical arguments" (Kazemi & Stipek, 2001:64).

7.2.6 Answering the fifth sub-question for the second research question

This section answers the following fifth sub-question for the second research question:

Question 2.5: What are the teacher strategies used in grade 10 mathematics classrooms?

The results in Figure 5 in the Network file show that most of the teacher strategies used were for comprehension: 79 for *writing on the board*, 169 for *explanations*, and 9 for *code-switching*. Similarly, the results from the instrumental case show that most of the teacher strategies used were for comprehension, and very few for language processing and interaction, output, and feedback (see sections 6.1.1.6, 6.2.1.6, 6.3.16 and 6.4.1.6). This could be attributed to the fact that the teacher-centred approach in most mathematics classrooms limits the variety of teacher strategies used by the participants. In an effort to address these problems, teacher strategies that are recommended to assist ESL learners with language processing and interaction, producing output and feedback should include, the *question and answer method*, *no-hands* strategy, *group-work* activities, for learners to enjoy the lessons as they are "stretched out mentally through sensitive teacher-led, but not teacher-dominated discourse" (Chin, 2006:1343).

As a result of using the learner-centred approach, Case C used a variety of teacher strategies and came up with unique ones summarised below.

Calling learners by their names

Learners were called upon by their names to respond to questions asked, and Case C used the strategy to "make them feel important and special" (P30: 32-35). When learners realised that teachers knew their names, they were motivated to work very hard and not disappoint them as pointed out by Cases C and D. This is in line with what Turner (2002:103) says, "learners did well in the subject in classrooms perceived as emphasising learning, effort and enjoyment", so motivation in the form of scaffolding is therefore necessary to promote learners' feedback. The strategy is in line with Krashen's Affective Filter Hypothesis, which claims that "acquisition would occur in environments where anxiety is low and defensiveness is absent, in contexts where the 'affective filter' is low" (Brown, 2007:295).

Touching a learner's shoulder

Case C touched one of the learners on the shoulder as she was struggling to respond to the question asked, to relax her so that she does not feel stressed, and she eventually provided a correct answer. During the interviews, Case C indicated that she used the strategy 'for passive learners who normally do not participate in class' (see P33:47-57).

Motivating learners

Case C also motivated learners to perform well in the subject by *going the extra mile*, for example by organising afternoon, Saturday and holiday classes to accommodate struggling learners to assist them to perform better in the subject (see P35:79-88). Some of the strategies suggested by the participants to motivate learners included *saying positive things about mathematics*, *organising maths week events* that show a bright future in careers for learners with mathematics as a subject (see P29:40-53), and *being taught ESL methods of teaching* since mathematics is defined as a language.

Incorporating the writing skills

Due to lack of resources such as fully-functioning libraries and computer rooms, it was not possible for some of the participants to incorporate the four basic skills into the teaching and learning of mathematics. However, this can be achieved by creating a collaborative relationship between mathematics and English teachers as suggested by the participants (See P35:41-53). This is also indicated in Samuel (2002:3), who incorporated the writing skills in mathematics classrooms by requesting students to write poems that could be used to increase vocabulary skills in Geometry. Similarly, Adair and Houston (1996:3) in their study incorporated writing skills into the teaching of mathematics by encouraging activities such as narrative writing, impromptu writing prompts, writing word problems and journal writing.

7.3 Collective case study

This section summarises the results of the collective case study and those captured from the focus group interviews. During the focus group interviews, the participants expressed disbelief when each one of them went through the transcribed lesson observations and interviews, and realised that the transcriptions were captured in verbatim. They all agreed that what was written was a true reflection of what transpired during the lesson observations and interviews. The results are summarised briefly in the sections below.

7.3.1 Answering the second research question

This section answers the second question of the study.

QUESTION 2: What are the characteristics of the most frequently-used question types in Grade 10 mathematics classrooms?

The most frequently-used questions were closed questions, and very few open-ended questions were used. The results are similar to those captured in the case studies and also in Figure 1 in the Network file (see sections 6.1.1.1, 6.2.1.1, 6.3.1.1, 6.4.1.1 and 7.1.1).

The findings are attributed to the fact that no questions are found in the participants' lesson plans (see L P A, L P B, L P C, and L P D). Another reason could be that open-ended questions are not found in the grade 10 prescribed mathematics books (Bellido *et al*., 2009; Laridon *et al*., 2008). Therefore, in-service mathematics workshops should include programmes on questioning skills since research has proven that effective questioning skills have been linked to learners' achievement in mathematics classrooms (Shahrill, 2013: 230).

7.3.2 Answering the first sub-question of the second research question

This section answers the first sub-question of the second research question in the study:

Question 2.1	**How do the questions used promote learners' understanding of mathematical discourse?**

The results show that not all the closed questions elicit responses that promote learners' understanding of mathematical discourse (see sections 6.1.1.2, 6.2.1.2, 6.3.1.2, 6.4.1.2 and 7.1.2).

The case study findings showed that closed questions can promote learners' mathematical discourse when learners are called upon by their names. The strategy is captured in the theoretical model to be included in the hands-on-tool developed in Chapter 8 to empower mathematics teachers with such skills, as encouraged by Philpott (2009:66), stating that questioning is a skill that can be learned and successfully executed through the use of simple techniques.

7.3.3 Answering the second sub-question of the second research question

This section answers the following second sub-question of the second research question:

Question 2.2:	**How do questions used promote learners' mathematical discourse and ESL development?**

The results for this research question show that very few open-ended questions that promote mathematical discourse and ESL acquisition were used in the case studies and in Figure 1 of the Network file (see sections 6.1.1.3, 6.2.1.3, 6.3.1.3, 6.4.1.3 and 7.1.3). Therefore, due to the large number of procedural and closed responses, open-ended responses can be increased by probing procedural and closed responses with *why* questions to change them into open-ended questions, since the findings of the case studies showed that open-ended questions elicited individual open-ended responses that promote mathematical discourse and ESL development because they elicit slightly longer and more learner utterances (Brock, 1986; Long, 1983; Shomoossi, 2004: 97).

Also, closed questions can be changed into open-ended questions and produce responses that promote mathematical discourse and ESL development when teachers "ask for a qualitative answer by modifying a rote memory or basic arithmetic question and asking for further explanation" (Capacity Building Series, 2011:2).

The findings also showed that open-ended questions elicit open-ended responses that promote mathematical discourse and ESL acquisition, and therefore they should be used in large numbers to achieve the intended outcomes of this study.

7.3.4 Answering the third sub-question for the second research question

This section answers the following third sub-question of the second research question:

Question 2.3:	**What is the function of the questions used in Grade 10 mathematics classrooms?**

The findings show that the diagnostic and constructive function of comprehension were mostly used by the participants, and very few questions for the functions of language processing and interaction, and output, were used (See sections 6.1.1.4, 6.2.1.4, 6.3.1.4, 6.4.1.4 and 7.1.4). Also there were many evaluative functions of questions for feedback even though they elicited *Yes* responses that did not convince the participants that the lessons were understood. The theoretical model, however, includes Lomen's (2009:2) two statements that teachers can use towards the end of the lesson for learners to evaluate his/her lesson in written form on a daily basis, for them to prepare remedial lessons before the beginning of the next day's lesson.

7.3.5 Answering the fourth sub-question for the second research question

This section answers the following fourth sub-question for the second research question.

Question 2.4:	**What are the questioning techniques used in Grade 10 mathematics classrooms?**

The findings from the collective study show that repetition as a questioning technique was used mostly by the participants, for the questions asked, and for learners' incorrect and correct responses (see sections 6.1.1.5, 6.2.1.5, 6.3.1.5, 6.4.1.5 and 7.1.5). However, the technique did not encourage learners to produce language, it simply acknowledged learners' responses and confirmed them as correct or incorrect. The questioning technique, *probing* was used on a small scale. However, if used for the many closed and procedural responses elicited, it would have provided learners with many opportunities to produce output as it is pointed out that "probing questions like *explain why you said Yes or No*, extend what the learners were really thinking" (Chin, 2006:1336).

7.3.6 Answering the fifth sub-question for the second research question

This section answers the following fifth sub-question for the second research question of the study

Question 2.5:	**What are the teacher strategies used in Grade 10 mathematics classrooms?**

The results for the third sub-question of the second research question showed that the teacher strategies for comprehension, such as *writing on the board*, *explanation*, and others, were used mostly, and very few were used for language processing and interaction, output, and feedback (see sections 6.1.1.6, 6.2.1.6, 6.3.1.6, 6.4.1.6 and 7.1.6). Therefore, strategies for language processing and interaction, output, and feedback were very few. These are captured in the theoretical model to be used in the hands-on tool. Examples of these include the *group-work* activities, which allow "learner and peer interaction as they enable learners to practise language in a more relaxed atmosphere, free from the teacher's scrutiny" (Tuan & Nhu, 2010:36). The absence of the teacher strategies for these outcomes are attributed to the teacher-centred approach used in many mathematics classrooms. However, the tool developed can be used in mathematics classrooms to change out-dated methods of teaching mathematics, which is classified as a third language.

7.4 Summary of the findings

In summary, the results of the study have shown the following:

7.4.1 The most frequently-used questions types

The findings showed that only imperatives were used in the participants' lesson plans and that there were no official documents for lesson plan preparations at the 4 schools. Furthermore, the results showed that the participants used many closed and procedural questions, and very few open-ended questions during the lesson observations.

In an attempt to address the problems above, the researcher has developed a hands-on tool, described in Chapter 8, to assist mathematics teachers to prepare daily lesson plans with a variety of questions that promote learners' mathematical discourse and ESL acquisition.

7.4.2 How questions used promoted learners' understanding of mathematical discourse

The findings show that not all closed questions promoted learners' mathematical discourse, but that closed questions can be changed into open-ended questions to promote learners' mathematical discourse and ESL development.

Chapter 2 on the types of questions used in mathematics classrooms discusses ways in which closed questions can be changed into open-ended questions. The hands-on tool also describes ways in which closed questions can be phrased in such a way that promote learners' understanding of mathematical discourse. Examples of questions to be used are also provided in the tool. Furthermore, teachers are encouraged to call learners by their names to avoid eliciting chorus responses that do not promote mathematical discourse.

7.4.3 How the questions used promoted learners' mathematical discourse and ESL development

The results showed that very few open-ended questions were used to promote mathematical discourse and ESL development, and also that open-ended questions always elicit individual open-ended responses that enhance learners' mathematical discourse and ESL development and acquisition.

In an effort to address the small number of open-ended questions used by the participants, Chapter 2 discusses ways in which closed questions can be changed into open-ended ones; and some of the questioning techniques and teacher strategies used are illustrated in the theoretical model and in the hands-on-tool with examples to guide the teachers. The tool has guidelines in the form of examples for mathematics teachers to refer to on how to phrase questions in such a way that they elicit open-ended responses only.

Furthermore, by probing procedural and closed responses with *Why* questions, open-ended responses will be elicited.

7.4.4 Functions of questions used in mathematics classrooms

The results showed that many questions were asked for comprehensible input, but very few for language processing, output, and feedback. They also showed that evaluative questions for feedback were mostly in *Yes* responses that did not convince the participants that the lessons had been comprehended.

This shortfall with regard to the small number of questions used for the intended outcomes, was addressed in Chapter 2, which discusses the variety of functions of questions used in mathematics classrooms. This should make teachers aware of the functions of these questions so that they can use them, for instance for learners to acquire mathematical discourse and ESL acquisition. Examples in each case are provided to guide the teachers. Furthermore, the functions of these questions are captured in the tool to empower mathematics teachers to use them in their classrooms, together with Lomens' (2009) two statements to be used by learners to evaluate teachers' lessons on a daily basis.

7.4.5 Questioning techniques used in mathematics classrooms

The results also showed that the questioning technique, *repetition* was mostly used for comprehension, language processing and interaction, output, and very few questioning techniques were used for processing language and interaction, output, and feedback.

Chapter 2 on the questioning techniques elaborates on a variety of questioning techniques that can be used in mathematics classrooms for learners to process and interact using language, by producing output, and feedback. These are captured in the theoretical model and the tool to empower mathematics teachers.

7.4.6 Teacher strategies used in mathematics classrooms

Similarly, the results showed that participants mostly used teacher strategies for comprehension, which included *writing on the board* and *explanations* and very few for language processing and interaction, output, and feedback, as a result of the participants maintaining a teacher-centred approach in their mathematics classrooms.

However, this kind of setup can be changed in these classrooms as one of the participants did. As a result of maintaining a learner-centred approach in her classroom, she used teacher strategies that have been incorporated into the tool. Also Chapter 2 elaborates on the teacher strategies used in mathematics classrooms to empower the teachers.

In summary, there are ways in which open-ended questions can be phrased to elicit open-ended responses. The importance of using such questions, questioning techniques and teacher strategies, for lesson comprehension, language processing and interaction, output and feedback, is discussed in the study, and also captured in the theoretical model and the hands-on tool developed. The importance of using all these is captured in what Vygotsky (cited in Chaiklin, 2003:12) proposed for teachers after giving a problem to a child:

> *We show a child how such a problem must be solved and watch to see if he can do the problem by imitating the demonstration. Or we begin to solve the problem and ask the child to finish it. Or we propose that the child solve the problem that is beyond his mental age by cooperating with another, more developed child or, finally, we explain to the child the principle of solving the problem, asking leading questions, analysing the problem for him, etc.*

7.5 Conclusion

In conclusion, the results of the study show that in mathematics classrooms, there are closed questions that can be used to elicit individual responses which promote learners' understanding of mathematical discourse (Cases A, B, C, and D). There are also open-ended questions that always elicit open-ended responses to promote learners' mathematical discourse and ESL development (Cases A, B, C, and D). Furthermore, there are questions with specific functions, questioning techniques, and teacher strategies that can be used by mathematics teachers to assist learners with comprehension, language processing and interaction, provide learners with opportunities to produce output, and feedback (Cases A, B, C, and D). The results of the study form the foundation for the hands-on tool discussed in the next chapter, to empower mathematics teachers with the types of questions that elicit responses that promote learners' understanding of mathematical discourse and ESL development to address the problem of ESL identified as a barrier in learners' performance in the subject, and thus achieve the specified intended outcomes. Suggestions on how to use the tool are also included.

CHAPTER 8: THE HANDS-ON-TOOL FOR MATHEMATICS TEACHERS TO PROMOTE ESL ACQUISITION THROUGH QUESTIONING TECHNIQUES

8.1 Introduction

Chapters 2 to 4 attempted to answer research question 1; and Chapters 5 to 7 research question 2. Chapter 8 attempts to answer research question 3.

Question 3: What are the characteristics of a hands-on tool that could support grade 10 mathematics teachers in developing the correct questioning skills that promote English second language acquisition?

The hands-on tool entitled, "**A Planning Tool to promote Questioning Skills**" or PTQS upon which the theoretical model for effective questions, questioning techniques and teacher strategies is based and developed, is discussed. The tool was developed with the aim of addressing questioning skills problems identified in mathematics classrooms, to provide mathematics teachers with questions, questioning techniques, and teacher strategies, together with examples from data collected from the literature reviewed and the participants' data, for them to apply these in mathematics classrooms. The tool is illustrated and each section is discussed in the chapter. Suggestions on how the PTQS should be implemented by grade 10 mathematics teachers in their classrooms, limitations of the study, and recommendations for future research, are also discussed.

8.2 Findings considered when developing the PTQS tool

Findings from the results of the data from the four case studies, the collective case study, and data analysed using the ATLAS.ti software were considered when developing the tool. These are discussed below.

No standard lesson plan

The lesson plans provided by the participants showed that no standard lesson plan was used by the 4 cases (6.1.1.1, 6.2.1.1, 6.3.1.1, and 6.3.1.1), and that resulted in some of them photocopying and pasting exercises from the textbooks and using that as daily lesson plans. The tool would probably encourage mathematics teachers to prepare daily lesson plans, and be in a position to use the questions, questioning techniques, and teacher strategies, to address the problem of ESL, identified as a barrier in learners' performance in mathematics (Abedi & Lord, 2010; CME, 2009; Fleisch, 2008; Howie, 2003; Lunetta, 2015; Yahaya *et al*., 2009). All these should be captured in the lesson plans for the teachers to engage learners and to use them for comprehensible input; language processing and interaction; output; and feedback; and for the learners to love mathematics, and dismiss the myth that it is a subject for a chosen few.

Types of questions used

Findings on the types of questions used in grade 10 mathematics classrooms, showed that the 4 participants used many procedural questions for learners to evaluate their lessons (6.1.1.1, 6.2.1.1, 6.3.1.1, 6.4.1.1, 7.1.1, 7.2.1, and 7.3.1). Learners' responses to these questions were mostly in *Yes* chorus form, and as a result the feedback received was not convincing as to whether or not the learners understood what was taught throughout the lessons. In the hands-on-tool, the questions for the evaluative and affective functions have been included, to provide learners with feedback and to incorporate all the four basic skills into the teaching and learning of mathematics.

The results further showed that the participants used closed questions that also elicited chorus responses (6.1.1.2, 6.2.1.2, 6.3.1.2, 6.4.1.2, 7.1.2, 7.2.2, and 7.3.2), which made it difficult for the participants to ascertain as to whether all the learners' understanding of mathematical discourse was promoted. To address this problem, teachers are advised to make use of name tags to be able to call learners by their names to respond individually to the closed questions asked, and thus ascertain that all the learners' understanding of mathematical discourse is promoted. Furthermore, research also suggest the use of the *thumbs up and down* method to be used for teachers to be aware of those learners who understand and those who do not understand the lesson, and thus prepare remedial lessons for them.

The results also showed that the participants used few open-ended questions which promoted learners' understanding of mathematical discourse and ESL understanding (6.1.1.3, 6.2.1.3, 6.3.1.3, 6.4.1.3, 7.1.3, 7.2.3, and 7.3.3). To address this shortfall, a balance of closed, procedural, and open-ended questions, with examples, has been captured in the tool, with examples, to promote learners' understanding of mathematical discourse and ESL development. This would probably result in all the questions used, eliciting responses that engage learners in language processing and interaction; output; and feedback.

Functions of the questions used

The results based on the functions of the questions used show that there are many questions used for the diagnostic, constructive, and evaluative functions (6.1.1.4, 6.2.1.4, 6.3.1.4, 6.5.1.4, 7.1.4, 7.2.4, and 7.3.4). By balancing the different functions of questions included in the tool, teachers would increase the number of questions for the corrective, cognitive and the language acquisition functions, as well as for the evaluative and affective functions, to provide them and the learners with feedback.

Questioning techniques used

The results on the questioning techniques show that the questioning techniques for language processing and interaction, output, and feedback are very few, especially with regard to probing (6.1.1.5, 6.2.1.5, 6.3.1.5, 6.4.1.5, 7.1.5, 7.2.5, and 7.3.5). Probing learners' responses with procedural questions, such as *Why* questions, and also probing learners with imperatives such as*, Explain how you arrived at that answer*, would address this shortfall, as learners would process and interact using language, produce language when they explain the steps followed in arriving at the incorrect and correct responses. This would likely encourage learners to work very hard as the use of such imperatives would discourage guess work and copying from other learners.

A questioning techniques such as *ignoring learners' incorrect responses,* (6.1.1.5) which confused the learner with regard to the feedback received, is not included in the tool. Philpott (2009: 72) cautions teachers to be careful with questioning techniques as they can be used to raise and crush learners' self-esteem.

In addition, questioning techniques from the literature reviewed such as, *revoicing/recasting/paraphrasing* discussed in chapter three are included in the tool as they provide learners with the correct answer and the grammatical structure of responses that are elicited by open-ended questions.

Teacher strategies used

The results from the field notes showed that in three out of four classrooms visited, a traditional setting, a teacher-centred approach was maintained throughout the lessons (6.1.1.6, 6.2.1.6, 6.4.1.6, 7.1.6, 7.2.6, and 7.3.6), and that resulted in the participants spending most of the time explaining, doing exercises, and writing on the board, while learners sat passively during the course of the lessons. By changing this traditional set-up into a learner-centred approach, as it was the case in Case C's classroom (6.3.1.6), learners would not sit passively during the lessons. They were engaged in group- and pair-work activities, referred to as collaborative practice, and thus processed and interacted using language; and produced output in the process, thus acquiring mathematical language, and performing well as research has proven that learners in traditional classrooms have been outperformed by those in the learner-centred ones (Boaler, 2008: 113).

Furthermore, the following teacher strategies that are unique to the 4 cases have been included in the PTQS tool.

- *Praising learners for correct responses*

Learners were praised after providing correct answers with expressions such as '*very good*, *excellent'*, etc. to provide them with feedback, and also to encourage them to do well in the subject as pointed out by the participants.

- *Calling learners by their names*

Some of the participants, Cases C and D, called learners by their names to respond to the questions asked, to encourage learners to work very hard so as not to disappoint the teachers who took it upon themselves to know their names.

- *Touching struggling learners on the shoulder*

During one of Case C's lesson observations, one of the learners provided an incorrect answer to the question asked. The participant went behind her, touched her shoulder as she probed her responses, using the question-and-answer method, until she finally came up with a correct response. Case C indicated that she used this strategy for passive learners to say something at least and take part in classroom discussions.

- *Giving learners time to take down notes*

One of the participants, before giving learners class work, wrote down notes on the board on the steps they should follow when multiplying trinomials with binomials, (See Case A L O 1: 39 – 43), and thereafter gave learners three minutes to take down the notes that they could refer to when doing the exercises given.

- *Motivating learners*

Case C motivated learners to do well in the subject by going extra miles in arranging morning, afternoon, and Saturday classes for learners to do well in the subject. She, together with her colleagues, further went above the call of duty by visiting learners from poor families, especially on Mandela Day, and provided them with gifts to encourage them to attend classes regularly. They also arranged maths week events during which students dramatised successful careers that are available for students who have chosen mathematics as one of their subjects, to encourage others to take it as one of their subjects at grade 10 level.

- *Speaking positively about mathematics*

There is a myth that mathematics is a difficult subject, and the only way to do away with this myth is for mathematics teachers to speak positively about the subject for learners to be encouraged to take it as one of their subjects at grade 10 level, as pointed out by the participants.

In addition, strategies captured from the literature reviewed in chapter four, such as the *no-hands strategy, was* included in the tool even though some of the participants did not fully embrace it (6.1.1.6, and (6.2.1.6), so as to encourage all the learners to take part in classroom discussions, and thus address the problem of ESL and mathematical discourse in these classrooms. Also included *in the tool are meta-cognitive strategies*, *SBI*, and *Scaffolding* from chapter three.

8.3 From a theoretical model to a PTQS tool

The PTQS tool was designed after synthesising the theories on SLA and mathematics learning, principles for mathematics and ESL teaching, and perspectives for mathematical proficiency conditions. Also, findings from the literature reviewed, as well as data analysed based on the research questions of the study, have also been used. The tool is illustrated and briefly discussed below.

8.3.1 Discussion of the tool

The tool has four pages arranged in flaps namely, the cover page flap with the title, "*A planning tool to promote questioning skills*", the first flap on "*Preparing learners for new content*", the second flap on "*Processing new content*", the third flap on "*Practising new content*", and the fourth flap on "*Feedback*". The tool is illustrated below, and each of the flaps is summarised in this section

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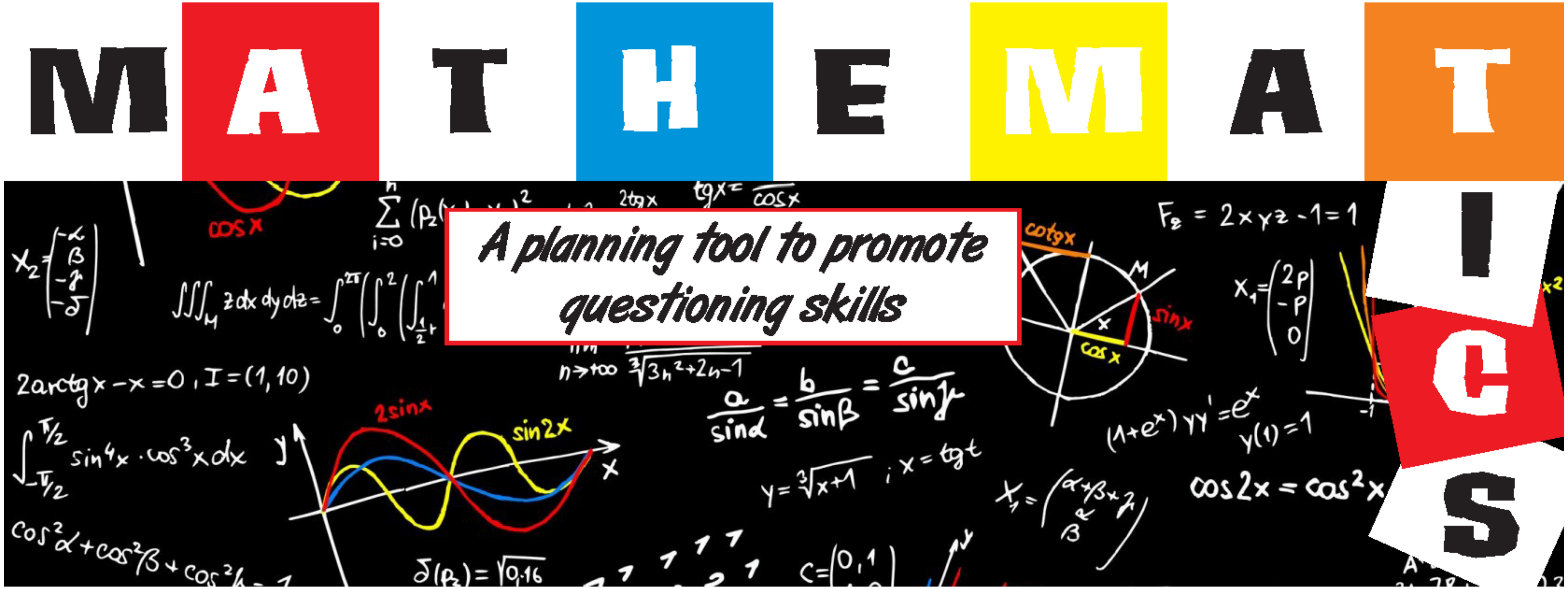
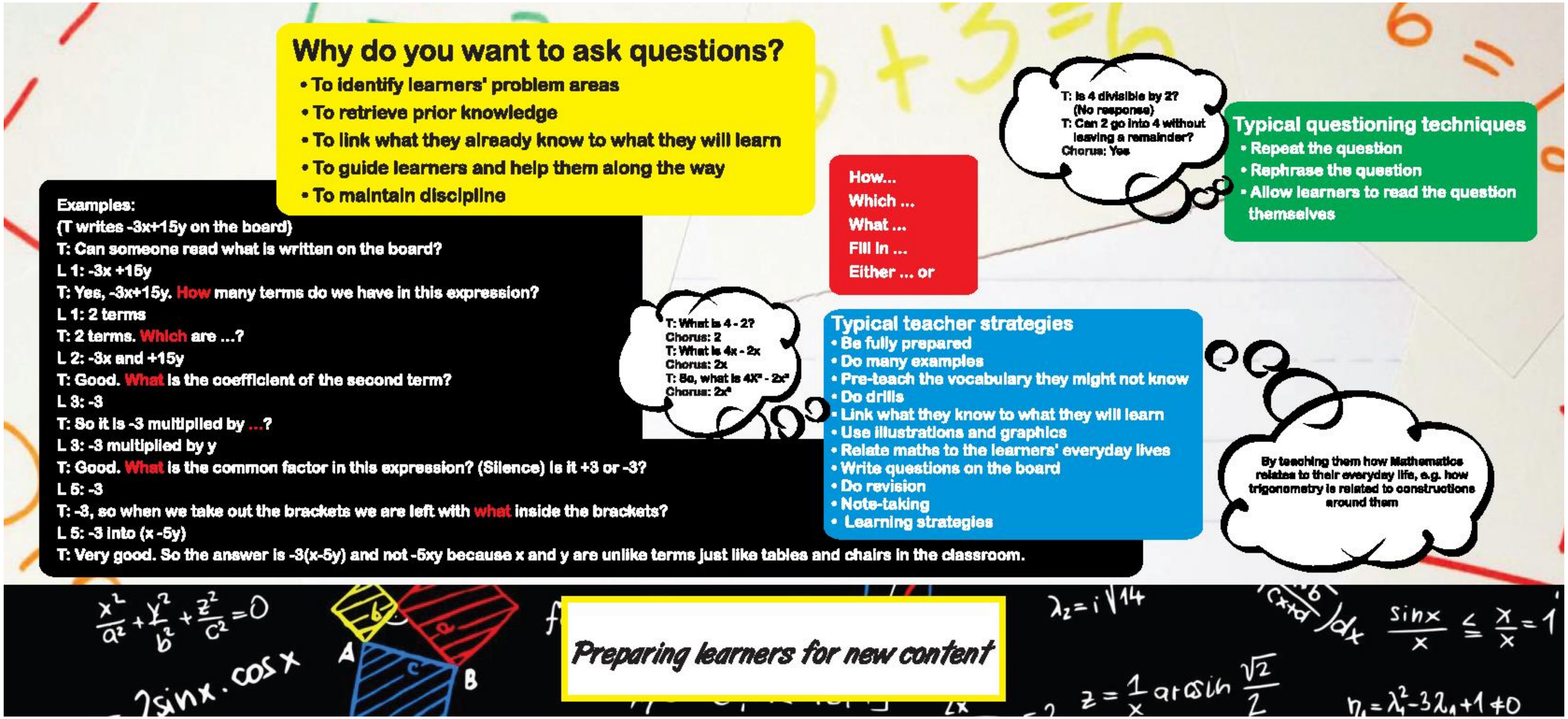
8.3.1.1 The cover and title page: A Planning Tool to promote Questioning Skills



The cover page above is beautifully decorated with mathematical equations against a black background. The title "*A planning tool to promote questioning skills*" appears below the word MATHEMATICS written in capital letters and in black and white font colours on highlighted white, red, blue, yellow and peach colours, to make it appealing to the users.

8.3.1.2 Preparing learners for new content in mathematics classrooms: Planning questions for your introduction

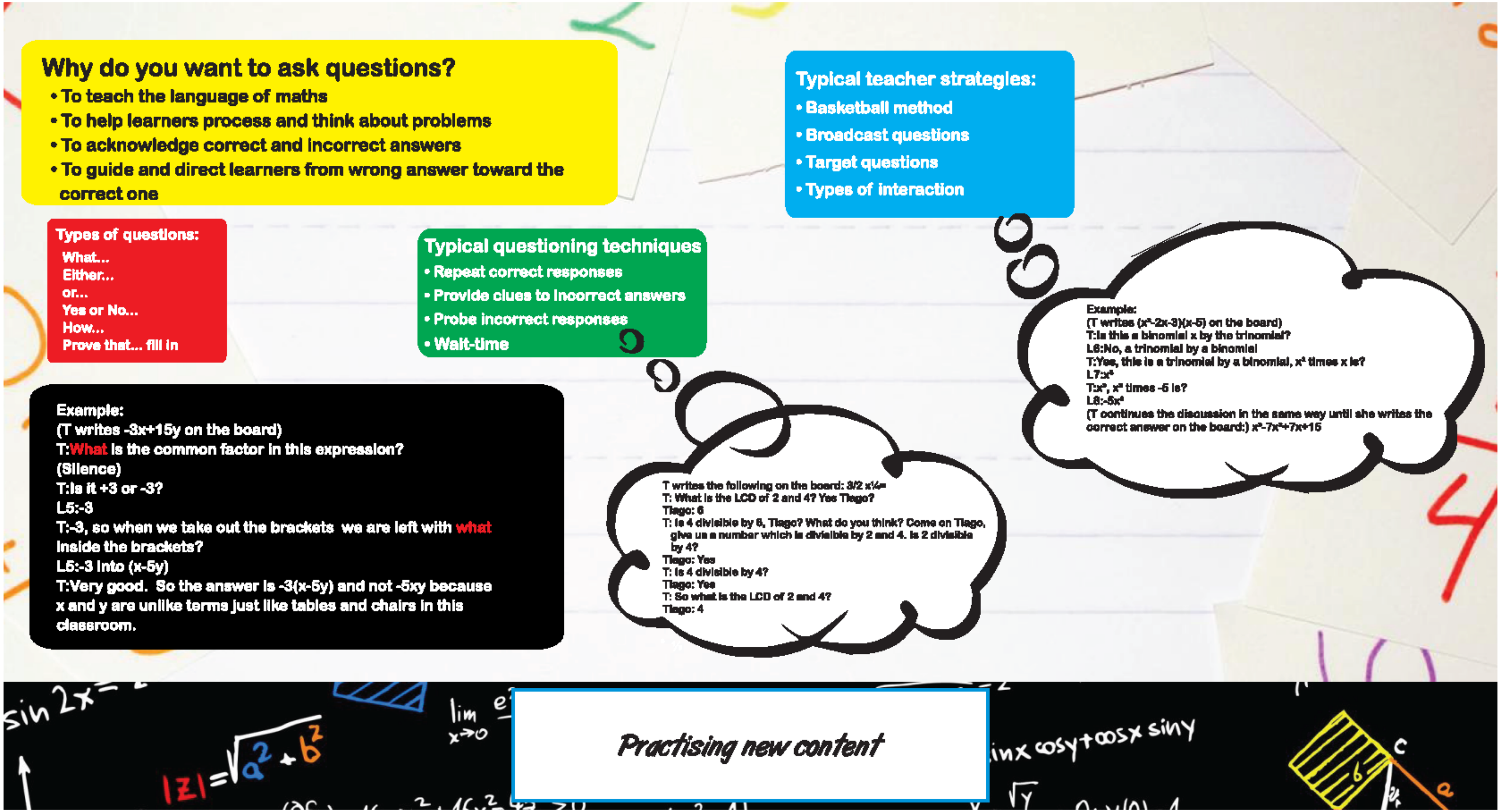


The first flap titled, ***Preparing learners for new content****: Planning questions for your introduction*, has a list of the types of closed questions, questioning techniques, and teacher strategies that mathematics teachers can use, for learners to comprehend the lesson and the questions asked, and thus achieve the following outcomes as shown in the illustration:

- Identifying learners' problem areas;
- Retrieving prior knowledge;
- Linking the known to unknown
- Maintaining discipline; and
- Guiding learners and helping them along the way.

Examples of the types of closed questions, questioning techniques, and teacher strategies, captured from the lesson observations and literature reviewed are shown in the illustration, to guide teachers when preparing lesson plans to make provision for learners to receive comprehensible input.

8.3.1.3 Language processing and interaction in mathematics classrooms: Practising new content

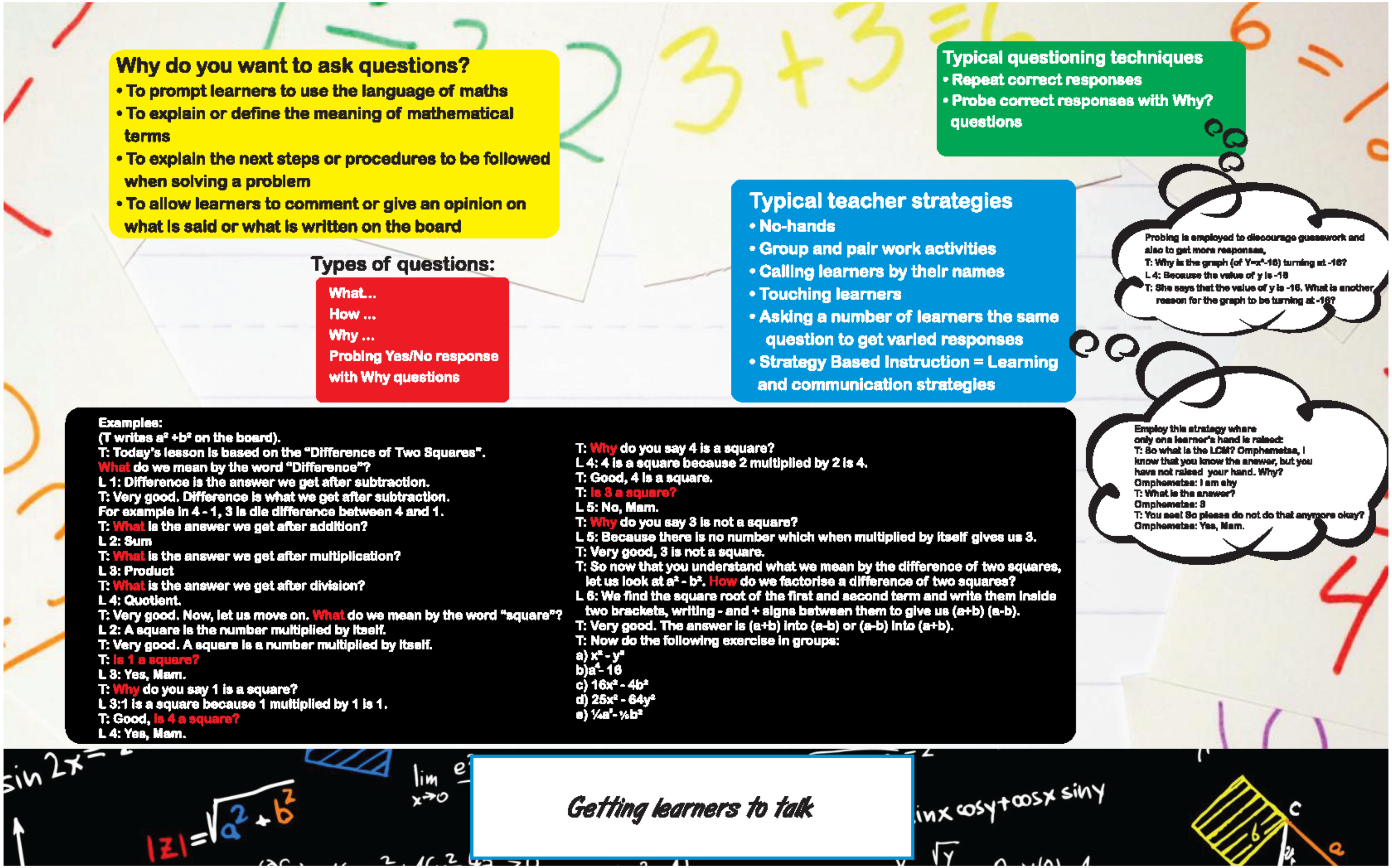


The second flap titled ***Practising new content***, has closed and open-ended questions, questioning techniques and teacher strategies that teachers can use to promote learners' understanding of mathematical discourse, and to achieve the following outcomes:

- Teaching mathematical discourse;
- Learners process and use new content;
- To interact using language – both mathematical (formal and informal) and academic language;
- To acknowledge learners' incorrect responses; and
- To direct and guide learners to correct responses.

Examples of the types of questions, questioning techniques, and teacher strategies captured from the lesson observations and literature reviewed, are given in each case as shown in the illustration, to guide the teachers to provide learners with opportunities to process and interact using language as they prepare lesson plans.

8.3.1.4 Output: Getting learners to talk in mathematics classrooms

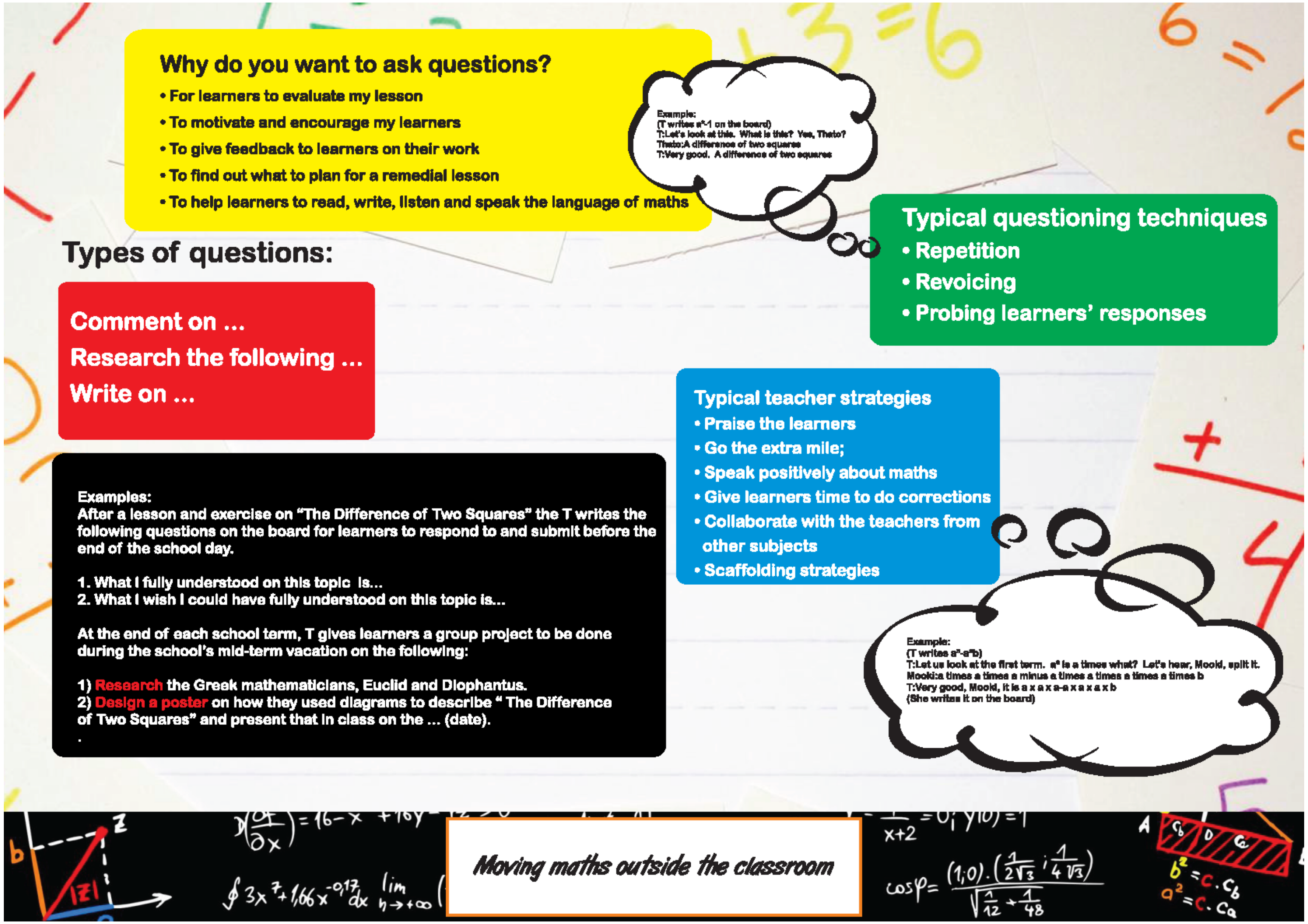


The third flap titled, ***Getting learners to talk***, presents the types of questions (procedural, closed, and open-ended), questioning techniques and teacher strategies that teachers can use to promote learners' understanding of mathematical discourse as well as ESL development, for the learners to be able to perform the following outcomes:

- Explain or define the meaning of mathematical terms;
- Explain the next steps or procedures to be followed when solving a particular mathematics problem;
- Provide reasons, comments or opinions on what is said or written on the board; and
- To prompt learners to use the language of mathematics and acquire ESL in the process.

Examples of the types of open-ended questions, imperatives, questioning techniques and teacher strategies are given in each case as shown in the illustration, to guide the teachers to provide learners with vast opportunities to produce output as they prepare lesson plans.

8.3.1.5 Feedback: Planning questions to use mathematics outside the classroom



The fourth flap titled, ***Moving maths outside the classroom***, has *Planning questions to use outside mathematics classrooms;* the types of questions, for example, open-ended questions and imperatives; questioning techniques and teacher strategies; that teachers can use to incorporate the four basic skills; *Listening*, *Reading*, *Speaking*, and *Writing*, into the teaching and learning of mathematics as they prepare lesson plans, for learners to be able to achieve the following outcomes:

- To evaluate teachers' lessons daily;
- Be motivated and encouraged to perform well in the subject;
- Receive feedback on their progress, and for teachers to prepare remedial lessons;
- Teachers to provide scaffolding and support for learners' understanding of mathematics; and
- To incorporate the 4 basic skills into the teaching and learning of mathematics.

Examples of the types of questions (open-ended questions and imperatives), questioning techniques, and teacher strategies are given in each case as shown in the illustration, to guide the teachers to provide learners with feedback on their utterances, and thus improve their language acquisition.

The five flaps are joined at the top end in such a way that the teacher is able to turn over each of the four flaps with ease while preparing daily lesson plans.

8.4 Suggestions for implementing the PTQS

It should be noted that the functions of questions used, questioning techniques, as well as teacher strategies, may not follow one another in the order outlined in the model. The order will be at the discretion of the teachers and will be determined by the situations that the teacher will be faced with at that particular moment during the lesson. For example, if the previous lesson ended with an unanswered difficult question, the teacher, when preparing a lesson plan for the next lesson, would use questioning techniques and teacher strategies that they could use to make that particular question comprehensible for learners to be able to respond to it and thus promote their understanding of mathematical discourse and ESL development in the process.

Also, if the teachers realized that learners were unable to answer a question because they had no clue regarding the rules that should have been taught at a previous grade, they would be forced to revise the rules for the learners to understand so as to move on with the topic for that particular day. Suggestions on how to achieve the intended outcomes are included in the tool in the form of examples taken from the data collected from the lesson observations at the 4 grade 10 mathematics classrooms.

8.5 Limitations

It should be noted that the tool was developed based on the results and findings from the data collected during lessons observations on “Factorisation”, and, as a result, it may not be relevant to other sections in the mathematics syllabus for grade 10 classes. Nevertheless, it can still be tested in other sections of the grade 10 mathematics syllabus to find out whether it does work or not. Also, not only the findings from the research study were incorporated into the tool, but also those from the literature reviewed, for example, the *wait-time* and *no hands* strategies which teachers were not aware of as they taught during the lesson observations. Therefore, findings that were found relevant and practical in terms of the tool were incorporated into its design.

Limitations with the regard to the tool are as follows:

- The PTQS has not yet been tested in grade 10 mathematics classrooms;
- The tool needs to be accompanied by a training manual, to guide the teachers, the manual will be made available after the tool has been tested in grade 10 mathematics classrooms.

8.6 Recommendations and suggestions for future research

The following recommendations are made:

The tool should be used on a daily basis to prepare lesson plans for the lessons to be taught, and for the teachers to understand the questions, questioning techniques, and teacher strategies recommended.

Due to lack of resources in some of the rural areas, the functions of some of the questions, and teacher strategies, were not used. The Department of Education should therefore address the problems experienced in schools categorized as P sub-system, where teachers do not even have decent basic ablution facilities. Twenty-one years into democracy, there is absolutely no excuse the Department can give for learners in rural areas to be attending classes under the trees, to say the least. This is unacceptable and a violation of children’s rights to education.

The tool, after it has been tested in grade 10 mathematics classrooms, should be used in other grades for mathematics learners, and also at teacher-training institutions, to produce teachers who will maintain a learner-centred approach in mathematics classrooms, and also for learners to love and enjoy mathematics, as it is the only subject in which one can score 100%.

The tool will also be recommended to authors of mathematics prescribed textbooks for them to avoid using imperatives only in the textbooks, but the suggested variety of questions, questioning techniques, and teacher strategies. This would assist mathematics teachers to make learners enjoy mathematics and to develop a positive attitude towards it so that they can finally consider it as a subject for everyone and not for a chosen few as some of them think.

Suggestions for further research can include the following:

- Testing the tool in other topics in the grade 10 mathematics syllabus;
- Testing the tool in other grades;
- A comparative study of classrooms that use the tool versus those in which it is not used; and the results and findings would also be recommended to develop the school curriculum for mathematics teaching and learning.
- Feedback on the results and findings from research to be conducted would be welcomed to improve on the tool.

The researcher, in designing the tool, hopes that the teachers, by using the tool and following its suggestions to the letter, would finally be able to achieve "the educative process" in which "the student is active, the teacher is active and the milieu which they have created is active" (Vygosky, cited in Davydov, 1995:17)   .

8.7 Conclusion

I was a former mathematics and English teacher at high school in the late 80's in Bophuthatswana, a former home-land in the North West Province in South Africa. Our grade 12 learners did well in mathematics during that period as a result of incorporating the ESL teaching methods into mathematics classrooms. However, over the last few decades, the results in mathematics in S A have dropped dismally, prompting the researcher to investigate what exactly is happening inside our mathematics classrooms for some of the learners to even drop mathematics at high school. As a current ESL lecturer at a university, I have incorporated the skills of teaching ESL into the hands-on tool to possibly change the sad status quo, because research has proven that mathematics is also a language that can be acquired in our classrooms just like ESL.

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ADDENDUM A: NETWORK FILE

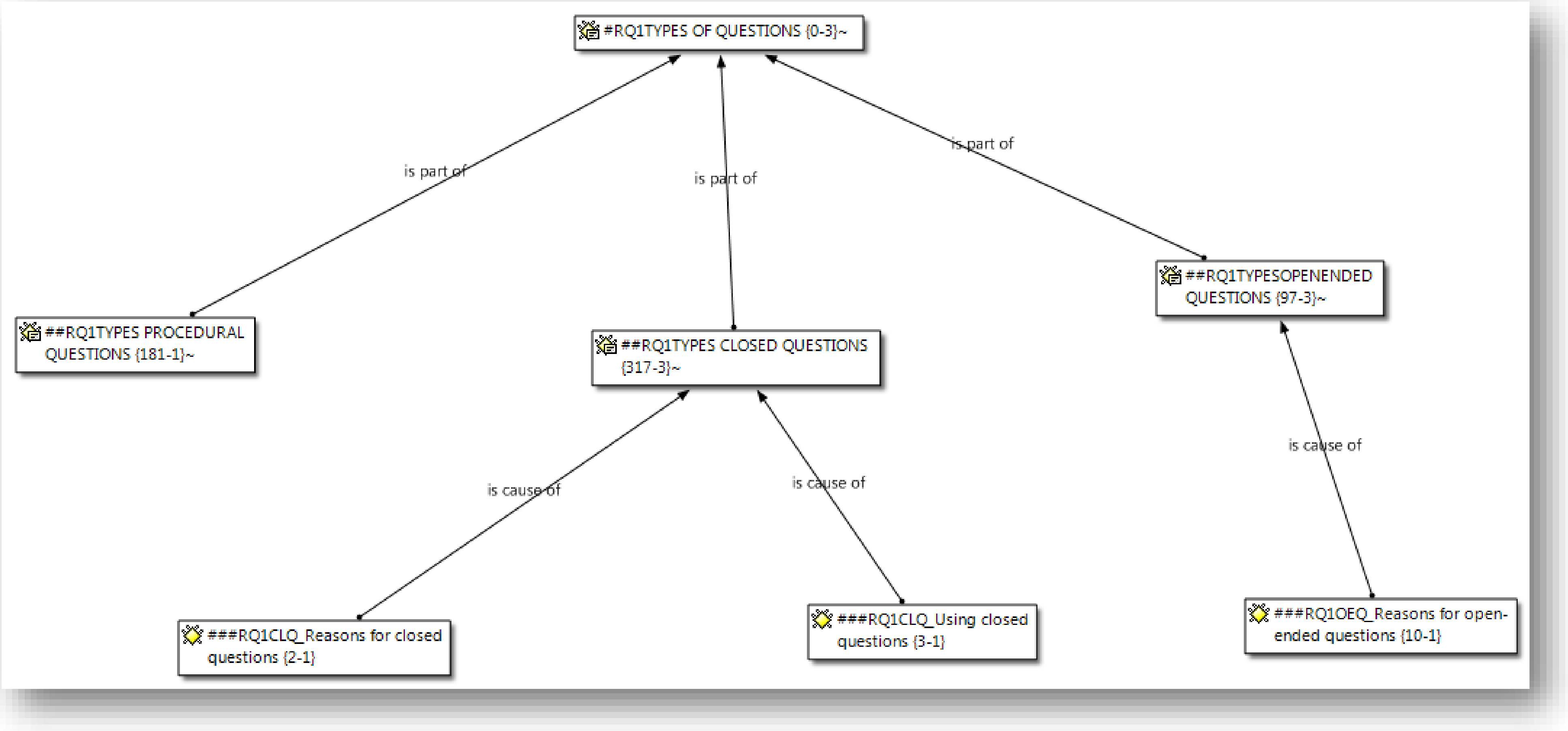

FIGURE 1: TYPES OF QUESTIONS USED IN GRADE 10 MATHEMATICS CLASSROOMS

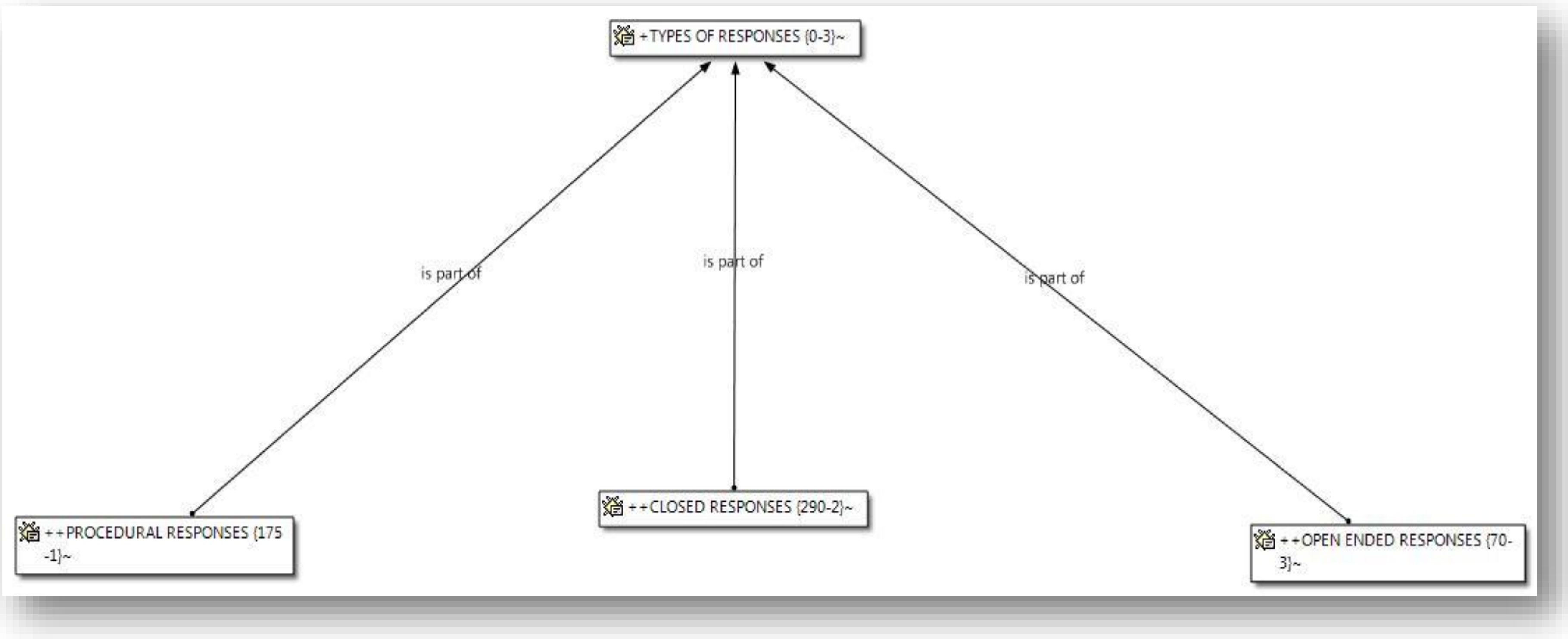

FIGURE 2: TYPES OF RESPONSES ELICITED BY QUESTIONS USED IN GRADE 10 MATHEMATICS CLASSROOMS

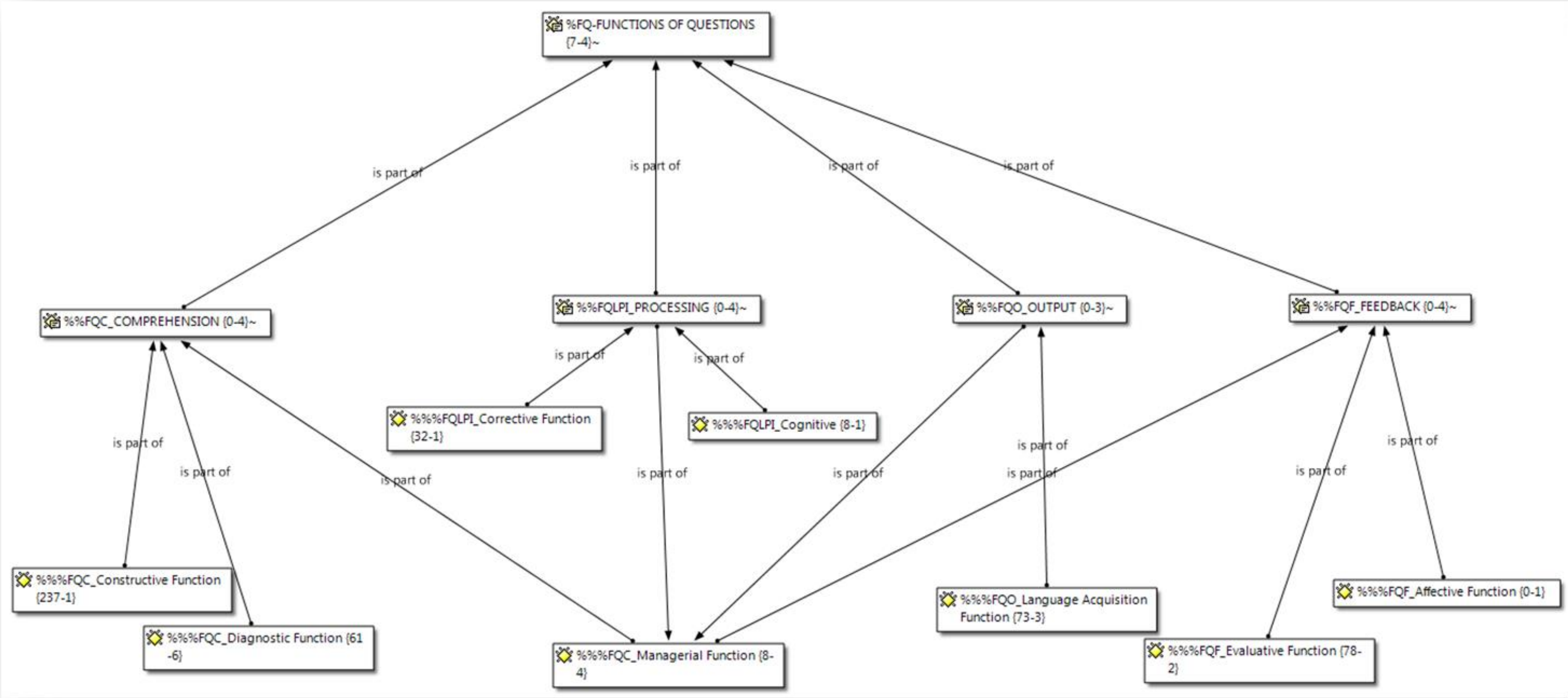

FIGURE 3: FUNCTIONS OF QUESTIONS USED IN GRADE 10 MATHEMATICS CLASSROOMS

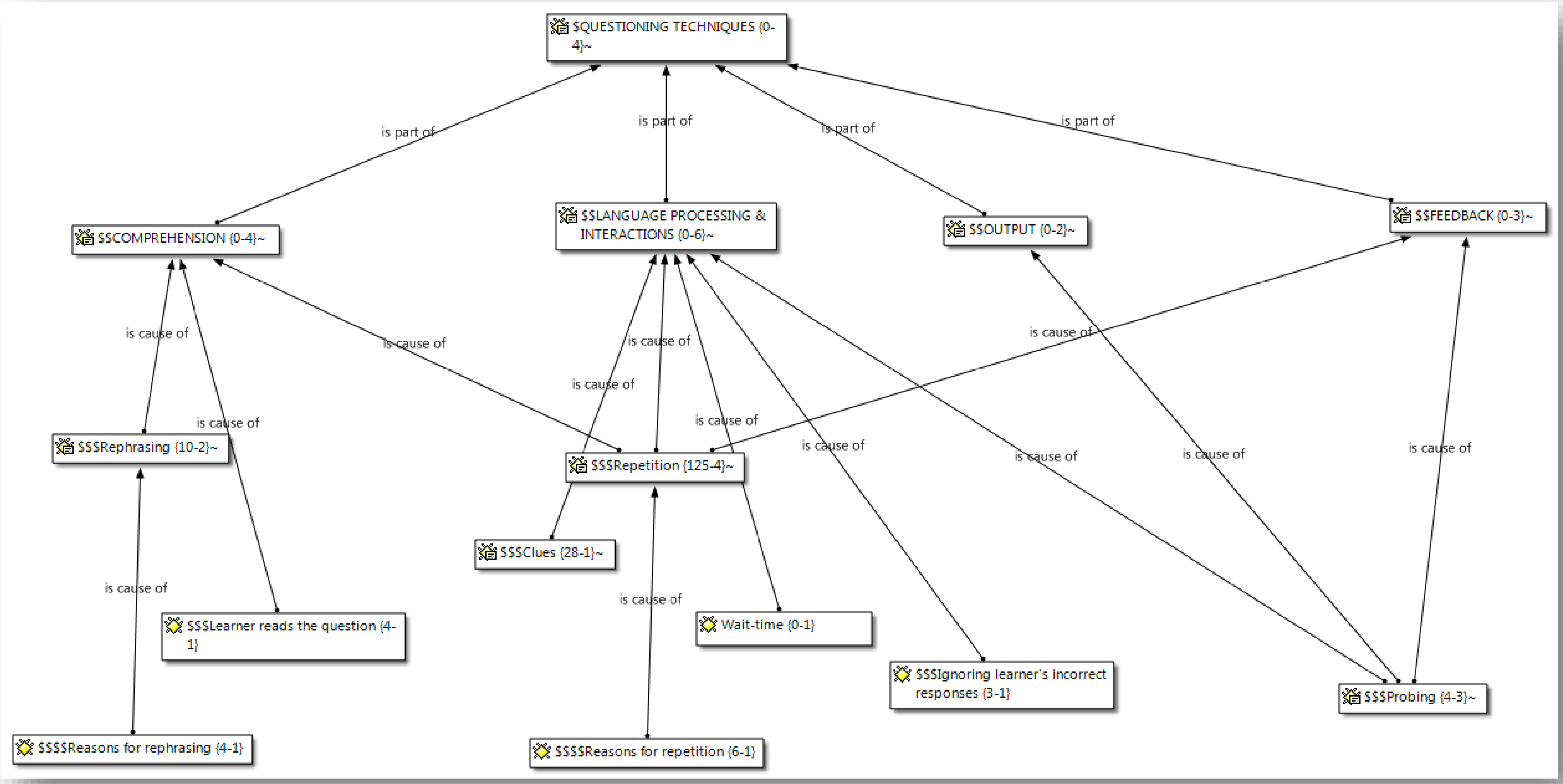

FIGURE 4: QUESTIONING TECHNIQUES USED IN GRADE 10 MATHEMATICS CLASSROOMS

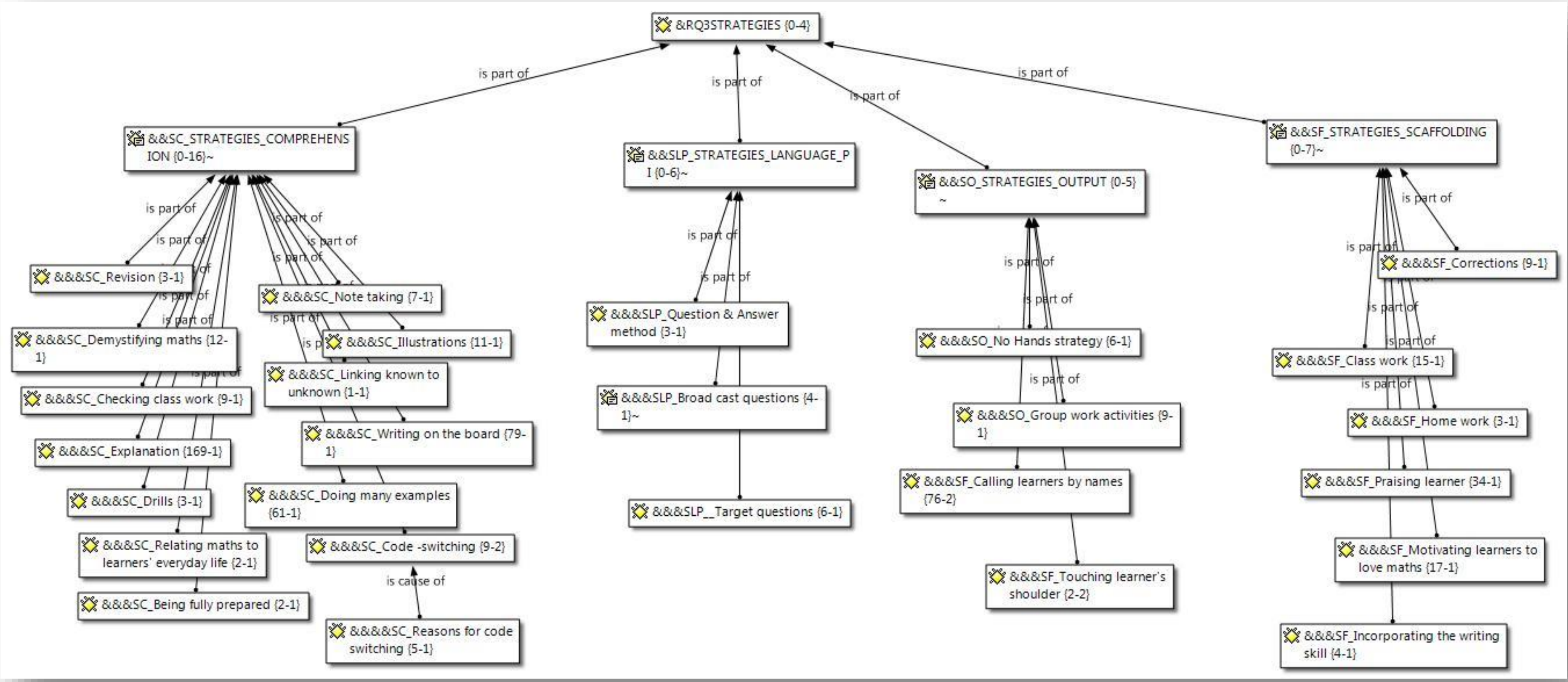

FIGURE 5: TEACHER STRATEGIES USED IN GRADE 10 MATHEMATICS CLASSROOMS

ADDENDUM B: APPENDICES

APPENDIX A : ON DATA COLLECTION TOOL FOR LESSON PLANS

	DATA COLLECTION TOOL FOR LESSON PLANS			
	AT THE SCHOOLS			
Lesson plan for	**Date**	**Question Types**	**Examples**	
Day 1				
Day 2				
Day 3				
Day 4				
Day 5				

APPENDIX B: Data collection tool for lesson observations

DATA COLLECTION TOOL FOR LESSON OBSERVATIONS				
AT THE SCHOOLS				
School:		**Class:**	**Teacher:**	**Date:**
Time:.............		**Lesson on:..**		**........................**
Questions	**Types**	**Learners' responses**		

APPENDIX C

INTERVIEW PROTOCOL FOR INDIVIDUAL GRADE 10 MATHEMATICS TEACHERS

School: ....

Date:.................................

Venue:..

Interviewer:............................... **Lesson:..**

Time: .... **Interviewee:.........................**

Position of interviewee:...

R: Good morning Sir/Madam, as previously agreed, I am here to get more information about the questions that you have used during your lesson today. Remember that the lesson will be recorded for me to be able to capture all your responses accurately. Are you ready to respond to the interview questions?

T: ....

R: Thank you for your time. Without wasting any time, let us start. During the course of the lesson for today, you used the .... question most of the time, in fact .... times? What were you trying to achieve by using that question .... times?

T: ....

....

....

R: You used the ....question at the beginning of your lesson. What is the role of that question?

T: ....

....

....

R: Your learners responded actively, using language and acquiring it, to the .... question. What could be the reason behind that?

T: ....

....

....

R: Your learners did not respond at all to the .... question (optional). What could have caused that?

T: ....

....

....

R: You modified the question ....by ....
What was the reason behind that?

T: ....

....

....

R: You modified the question................ by...

What were you trying to achieve by doing that?

T:...

....

....

R: You used the question .... once during your lesson. Do you think this question has a significant role to play in mathematics lessons?

T:...
....
....
When you prepared the lesson plan for your learners, did you think of including questions that enhance learners' mathematical discourse and ESL? Elaborate on your response.

T: ....

....

....

R: Do you think it is necessary for mathematics teachers to use questions that enhance learners' mathematical discourse and ESL development?

T: ....

Elaborate on your answer ....

....

....

R: You code-switched to Setswana to explain the question .... (optional). Why did you do that?

T: ....

....

....

R: Which questions would you recommend that can enhance learners' understanding of mathematical discourse and ESL development?

T:..
....
....

R: How does the ...... question(s) you have recommended above enhance(s) learners' ESL development? (this question will be repeated depending on the number of questions recommended by the teacher).

T:..

....

....

R: You employed the .... technique(s) / strategy(ies) during the lesson. What was the purpose? (This question will also be repeated depending on the number of techniques / strategies employed).

T:..

....

....

R: What other strategies and techniques do you recommend in your lessons to enhance learners' mathematical discourse and ESL development? Explain in details.

T:..

....

....

R: Is there anything you would like to comment about on the questions you have used in today's lesson?

T:..

....

....

R: Thank you very much for your time. If there is anything you would like to add or comment on regarding information on this interview and today's lesson, you are welcomed to let me know. Once more thank you very much for allowing me to observe your lesson today, to go through the questions in this interview and also for allowing me to record today's proceedings. Be assured that your responses will be kept confidential as promised in the letter requesting for your consent to participate in this research project. Have a wonderful day and wishing you and your learners a very successful academic year.

APPENDIX D

LESSON OBSERVATIONS AND INTERVIEWS CONDUCTED

AT SCHOOLS A, B, C and D COMPLETED

SCHOOL ....

FOR WEEK ENDING .... February 2012.

DAY	PERIOD	TIME	LESSON
Monday			
Tuesday			
Wednesday			
Thursday			
Friday			

Mathematics Teachers signs:...

Researcher signs:...

School Principal signs:...

School's stamp and date

APPENDIX E

ATTENDANCE REGISTER FOR THE FOCUS GROUP INTERVIEWS

The following research participants attended the focus group interviews organised by the researcher, M.M. Ledibane at Nelson Mandela School on the 28th March 2012 from 13.00 - 14.00 .

1. Teacher A: Signature:......Apology sent......
2. Teacher B: Signature ....
3. Teacher C: Signature..
4. Teacher D: Signature..........Mogoroi..............................
5. Researcher : M.M. Ledibane:...MLedibane...........

Principal signs:.......................................

School stamp

N.R. MANDELA COMMERCIAL SECONDARY SCHOOL
28/03/2012
P.O BOX 695
LERATO 2880

APPENDIX F

NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOM CAMPUS

Fakulteit Opvoedingswetenskappe
Faculty of Education Sciences

Verwysingsnr: / *Reference nr.*

FAKULTEIT OPVOEDINGSWETENSKAPPE / *FACULTY EDUCATION SCIENCES*

Notule

Vergadering

Fakulteit Opvoedingswetenskappe

Navorsingsetiekkomitee

Datum: ***Donderdag, 24 November 2011, 14:15***

Plek: ***Seminaarkamer 299E, C6***

ITEM		Bladsy/ Page
1.1.1 **Projekhoof**	Dr K Kaiser (Notule van 22 September 2011)	
Studente/Span	MM Ledibane	
Etieknommer	NWU-00100-11-S2	
Titel	A model for mathematics teachers to promote learners' ESL acquisition through questioning strategies	
Werksverdeling	Dr Tiaan Kirsten Prof Lukas Meyer Dr Betty Breed	
Besluit	**Magtiging / *Approved***	

APPENDIX G

P.O. Box 6354

Mmabatho

7th March 2011

The Principal and School Governing Body

School A, B, C and D

Dinokana

2868

Dear Sir,

REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT YOUR SCHOOL

I hereby request for permission to conduct research at the school under your administration for my PhD studies for a period of one week in February 2012.

Research has proved that language has always been a barrier in the performance of mathematics for learners taking English as Second Language (ESL). The focus of my research study is therefore on the types of questions used in grade 10 mathematics classes, to determine the extent to which they enhance mathematical discourse and ESL development through questioning techniques and teacher strategies.

In order to be able to conduct the research, I need permission to be able to do the following at your school so as to be in a position to validate the findings through triangulation:

- Observe and audio-tape grade 10 mathematics lessons during their scheduled periods on the school's composite time-table;
- Get copies of each of the lesson plans for the classes to be observed;
- Interview the teacher whose lessons will be observed during his/her free periods;
- Interview the teachers after data has been collected and analysed to verify some of the facts;
- Audio – tape the interviews using a digital voice recorder; and
- Note down what transpires during lesson observations in a diary.

The study will enable the researcher, with data to be collected from the lessons observed and interviews, to develop a model that will empower mathematics teachers to enhance mathematical discourse and ESL development through questioning techniques and teacher strategies. After the model has been developed, tested and approved, it will be shared with the teachers who took part in the research project.

I also promise that the anonymity of the schools and teachers will be protected by using letters of the alphabet and numbers to identify the schools and the teachers respectively. In addition teachers are allowed to withdraw from the research at any stage even though that would not be encouraged.

Thanking you in anticipation and hoping to hear from you in due course.

I remain,

Yours in education,

....

[illegible]

Email: [illegible]

PhD student at the North West University, Potchefstroom Campus

Student number: [illegible]

Supervisor: Dr K. [illegible]

APPENDIX H

English Department

NWU, Mafikeng Campus

Mmabatho

2735

18th November 2011

The Mathematics Teacher,

School A, B, C and D

Dinokana

2868

Dear Sir/Madam,

REQUEST FOR PERMISSION TO OBSERVE YOUR GRADE 10 MATHEMATICS CLASSROOM LESSONS

I humbly request for your permission to observe and audio-tape your mathematics lessons during your normal school teaching periods during February 2012 for a period of one week in order to collect data for my studies on the types of questions used in grade 10 mathematics classrooms.

Research has proved that language has always been a barrier in the performance of mathematics for learners taking English as Second Language (ESL). The focus of my research study is therefore on the types of questions used in grade 10 mathematics classes, to determine the extent to which they do (not) promote ESL acquisition through questioning techniques.

In order to be able to conduct the research, I need your permission to be able to do the following in your mathematics classes at your school so as to be in a position to validate the findings through triangulation:

- Observe and audio-tape grade 10 mathematics lessons during your scheduled periods on the school's composite time-table for the whole week;
- Get copies of each of the lesson plans for the lessons to be observed;
- Interview you after each lesson that I have observed during your free period;
- Interview you after data has been collected and analysed to verify some of the facts; and
- Audio – tape the lesson observations as well as the interviews using a digital voice recorder for 100 % accuracy.

The study will enable the researcher, with data to be collected from the lessons observations and interviews, to develop a model that will empower mathematics teachers to promote English Second Language acquisition through questioning techniques. After the tool or model has been developed, tested and approved, it will be shared with the teachers who took part in the research project.

I also promise that the anonymity of the schools and the teachers will be protected by using letters of the alphabet and numbers to identify the schools and the teachers respectively. In addition teachers are allowed to withdraw from the research at any stage even though that would not be encouraged.

I look forward to visiting your mathematics classes and learning a lot from you as an expert in the teaching of the subject.

Thanking you in anticipation and hoping to hear from you in due course.

I remain,

Yours in education,

....

Maureen Matlakala Ledibane

PhD student at the North West University, Potchefstroom Campus.

Student number: 20560966

Supervisor: Dr K. Kaiser.

APPENDIX I

A model for mathematics teachers to promote ESL acquisition through questioning techniques.

Dear Participant,

The following information is provided for you to decide whether you wish to participate in the present study. You should be aware that you are free to decide not to participate or to withdraw at any time without affecting your relationship with the researcher or the North West University.

The purpose of the study is to understand the types of questions used in grade 10 mathematics classrooms with the ultimate purpose of developing a model for mathematics teachers to enhance learners' understanding of mathematical discourse and ESL (English as Second Language) developmentthrough questioning techniquesand teacher strategies.

For the researcher to be able to achieve that, data will be collected at four points; namely in your mathematics classroom during lesson observations for a period of one week, from copies of the lesson plan for eachlesson to be observed, duringface-to-face interviewsafter each lesson observation, and duringa focus group interview of all the teachers whose lessons have been observed. Please note also that the lesson observations and responses to the interviews will be recorded using a digital voice recorder for 100 % accuracy. These will be transcribed and stored in computer files for dataanalysis purposes. Fieldnotes will also be used to capture the teachers' non-verbal behaviour during the lesson observations.

Do not hesitate to ask any questions about the study either before participating or during the time that you are participating. Before the study could be made public, findings on the study will be shared with you to check their accuracy. We would also be happy to share our findings with you after the research is completed.However your name will not be associated with the research findings in any way, and your identity as a participant will be known only to the researcher.

There are no known risks and/or discomforts associated with this study. The expected benefits associated with your participation are the information about the experiences in teaching mathematics in grade 10 classrooms and the opportunity to participate in a qualitative research study. If submitted for publication, a by-line will indicate the participation of all grade 10 mathematics teachers in public schools in the North West Province.

Please sign your consent with the full knowledge of the nature and purpose of the procedures. A copy of this consent form will be given to you to keep.

....

Signature of the participant Date

Phd Student

Student Number 20560966

<u>Dr K. Kaiser</u> <u>Prof. M van der Walt</u>

Supervisor Co-supervisor

Appendix I: Consent-to-participate form

Adapted from: Sample Human Subjects Consent-to-participate form (Creswell, 2007:124)

APPENDIX J

Lefapha la Thuto
Onderwys Departement
Department of Education
NORTH WEST PROVINCE

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Mmabatho 2735
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Telephone: 018-3883411 Fax: 018 388 4097
E-mail:

21 November 2011

To: Maureen Matlakala Ledibane: PhD student
Faculty of Education Sciences
University of North West: Potchefstroom Campus

From: Dr M A Seakamela
Acting Superintendent General

SUBJECT: REQUEST TO CONDUCT RESEARCH IN SECONDARY SCHOOLS IN ZEERUST AREA OFFICE: LANGUAGE AS A BARIER IN THE PERFORMANCE OF MATHEMATICS FOR LEARNERS TAKING ENGLISH AS SECOND LANGUAGE (ESL)

I acknowledged receipt of your request in respect of the above.

Please be informed that permission has been granted for you to conduct research at 5 secondary schools within the Zeerust Area office in the North West Department of Education. Approval is therefore granted under the following provisos:

- That consultation with the relevant School Principal is done
- That any publication of information pertaining to the Department should be done with the permission from the department
- That learning and teaching process is not compromised
- That service delivery is not compromised
- That the department be furnished with the outcomes of the research

Your contribution in improving the standard of Education is immensely appreciated

Regards

p.p. ....
Dr M.A. Seakamela
Acting Superintendent General

"Together doing more, better" Page 1

ADDENDUM C: DECLARATION OF LANGUAGE EDITING

cumlaude
languagepractitioners

Director: CME Terblanche - BA (Pol Sc), BA Hons (Eng), MA (Eng), TEFL

22 Strydom Street
Baillie Park, 2531

Tel 082 821 3083
cumlaudelanguage@gmail.com

DECLARATION OF LANGUAGE EDITING

I, Christina Maria Etrecia Terblanche, hereby declare that I edited the research study titled:

A MODEL FOR MATHEMATICS TEACHERS TO PROMOTE ESL ACQUISITION THROUGH QUESTIONING

for MM Ledibane for the purpose of submission as a postgraduate dissertation. Changes were suggested and implementation was left to the discretion of the author.

Regards,

CME Terblanche

Cum Laude Language Practitioners (CC)

SATI accr nr: 1001066

PEG registered