Chapter 3

Lumped parameter thermal modelling

This chapter explains the derivation of a thermal model for a PMSM by combining a lumped parameter (LP) model and an analytical distributed model. The whole machine can be modelled using LPs, which is the focus of this chapter.

3.1 Modelling approach

All electric machines are limited by thermal thresholds, making thermal modelling an important part of the machine's design. High speed machines are physically smaller than low speed machines for the same power rating. This results in a smaller heat removal area in high speed machines. When a PMSM is used as a high speed machine, care must be taken of the PM temperature. Rising temperature causes a reduction in a PM's residual flux density and coercivity. This will negatively influence the PMSM's operation. If the PM is heated above its Curie temperature, demagnetization will occur. In a thermal model of a PMSM, the most important part is thus the PM and it should be modelled as accurately as possible.

In reality, all the components of a PMSM are three dimensional. A thermal model should include only the dimensions in which significant heat flow occurs since 3-D models are difficult to derive and solution times are long. The PM in the TWINS machine has an axial length of 60 mm and its ID and OD are 46 mm and 60 mm, respectively. The left of Figure 3.1 shows a scale drawing of the PM and shielding cylinder, in the rz - plane. Since the PM will be rotating at high speed, the convection heat flow in the z - direction will be significant, even if the surface area is much smaller than in the r - direction. This effect can only be included when using a 2-D model.

The thermal distribution can be determined using a distributed or a lumped model. Numerical models are widely used and employed in commercial simulation packages. The absence of distributed models in electric machine thermal modelling literature warrants investigation and is one of the contributions of this thesis. The convection boundary conditions should be modelled since the PM is located on the rotor. The distributed model of the PM will be discussed in

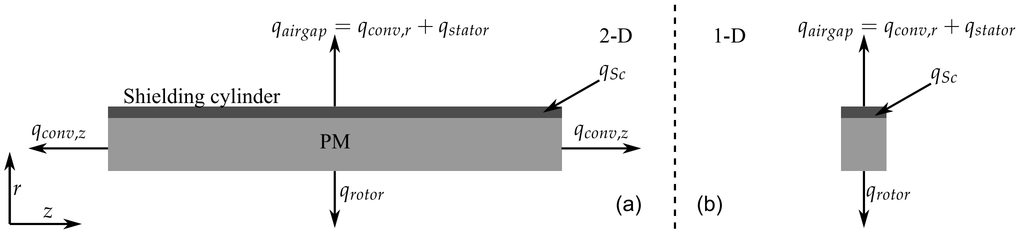

Figure 3.1: PM thermal energy system in (a) 2-D and (b) 1-D

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Modelling the whole electric machine in 2-D using a distributed model is difficult due to the complex geometry. The non-critical components, from a thermal point of view, should be modelled in 1-D to decrease model complexity. A thermal model of the entire machine is necessary since the temperatures in the critical components are influenced by the overall thermal distribution. The PM is modelled using a 2-D distributed model and the remainder of the machine using 1-D LPs. The 1-D LP components can be interconnected to model heat flow in the axial and radial direction.

3.1.1 Combination of the LP and distributed models

Figure 3.1 illustrates the difference between the 2-D (left) and 1-D (right) modelling approaches in the PM. The shielding cylinder loss (q_{Sc}) are due to eddy currents and will be discussed in section 5.5. On three of the sides, convection heat flow ($q_{conv,z}$, $q_{conv,r}$) removes some of the heat from the PM. A portion of the heat can flow into the rotor (q_{rotor}) and a portion to the stator (q_{stator}). These two heat transfer rates are influenced by the machine's overall thermal distribution. The convection heat fluxes are dependent on the machine's inner air temperature and convection coefficient, which in turn depends on the rotational speed of the rotor. The 1-D model does not take into account the convection heat flow in the z - direction, thus the PM temperature determined using this model will be an overestemation.

The combination of the LP and distributed models is illustrated in Figure 3.2. The transient heat flow of the overall machine can be modelled using the LP model where q_{Sc} is an input. The heat flow in the radial direction into the rotor (q_{rotor}) is determined from the LP model and used as an input for the distributed model. The distributed model is used to determine the convection heat flow on the sides of the PM ($q_{conv,z}$), as well as the thermal distribution of the PM in 2-D. The heat flowing in the z - direction will reduce the heat flowing in the r - direction. To account for this reduction, the loss inside the shielding cylinder can be reduced by $q_{conv,z}$. The new shielding cylinder loss; $q^*_{Sc} = q_{Sc} - q_{conv,z}$ is used as the input to the LP model, which is solved again. The temperature distribution in the PM is known from the distributed model and the temperatures in the rest of the machine, from the LP model.

The heat removed from the rotor through convection raises air temperature in the end winding region. Through convection, the heat flows from the end winding air to the stator housing and then to the ambient air. Due to the high rotational speed, as well as forced air cooling in the end winding area, complex airflow is expected. The end winding air will not be modelled during this study since it is assumed that it does not have a significant effect on the thermal distribution in the machine.

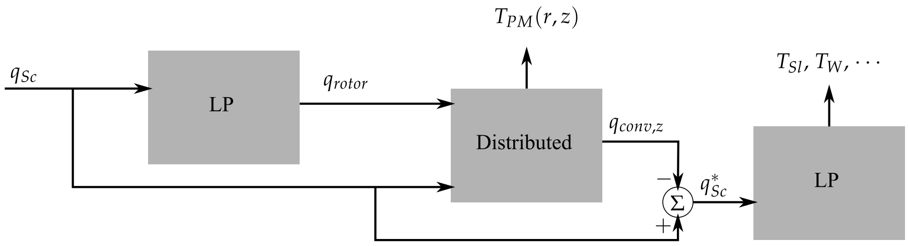

Figure 3.2: LP model and distributed model combination

3.2 Background

Most of the analytical thermal modelling done on electric machines in recent years are based on the work by Mellor *et al.* [4]. The article does not show how the resistance equations are derived but rather applies the equations to an induction machine. Even though this approach to lumped parameters is widely used, there are still some misinterpretations of the technique. The root of these misinterpretations can be illustrated by modelling a cylinder with internal heat generation ($\dot{q}$) in 1-D, as shown in Figure 3.3. The stator windings in an electric machine is a common example of a cylinder with internal heat generation. The thermal resistance (R_{cyl}) of a cylinder can be calculated using:

$$R_{cyl} = \frac{\ln(r_{out}/r_{in})}{2\pi Lk} \tag{3.1}$$

where r_{in} and r_{out} are the inner and outer radii, L is the axial length and k is the thermal conductivity of the cylinder. The error lies in assuming the distributed loss source can be lumped into a single heat flux source [3]. The error can clearly be seen in the temperature distribution

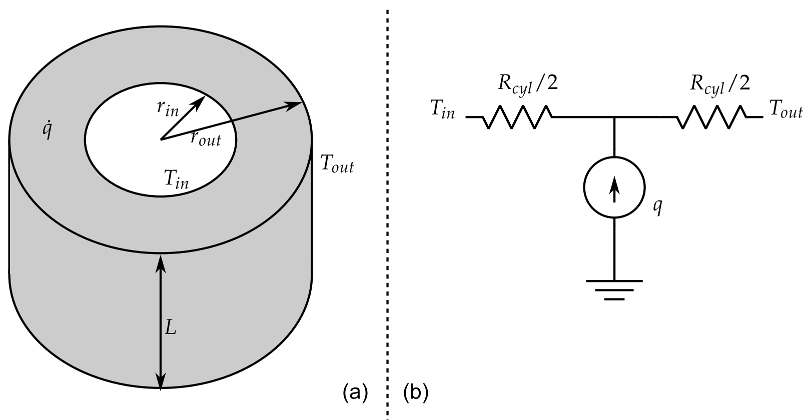

Figure 3.3: Erroneous LP model. (a) Cylinder with internal heat generation, (b) erroneous LP model.

of the cylinder, shown in Figure 3.4. The solid line shows the temperature calculated using the exact solution of the diffusion equation and the dashed line shows the temperature when using the erroneous LP circuit. It should be noted that the heat flux flowing through the boundaries of the cylinder will be correct since the heat source in the LP ($q = \dot{q} \times \pi(r_{out}^2 - r_{in}^2)L$) results in the same total loss inside the cylinder. The results shown in Figure 3.4 are for $r_{in} = 40$ mm, $r_{out} = 80$ mm, $T_{in} = 293.15$ K, $T_{out} = 298.15$ K, $k = 1$ [W/(m.K)], $L = 60$ mm and $q = 20$ W.

Some researchers proposed additional heat sources to rectify the overestimated temperature [77, 124–127]. Only half of the losses are applied in the middle of the resistances and the rest of the losses are applied to the sides of the resistances. Mejuto *et al.* proposed discretization of the winding to decrease the error [79]. Through the derivation of the LP cylindrical component developed by Mellor *et al.*, it will be shown that the internal heat generation is indeed applied correctly. This is done by adding a negative resistance to the LP model. The derivation of a general cylindrical component is discussed next.

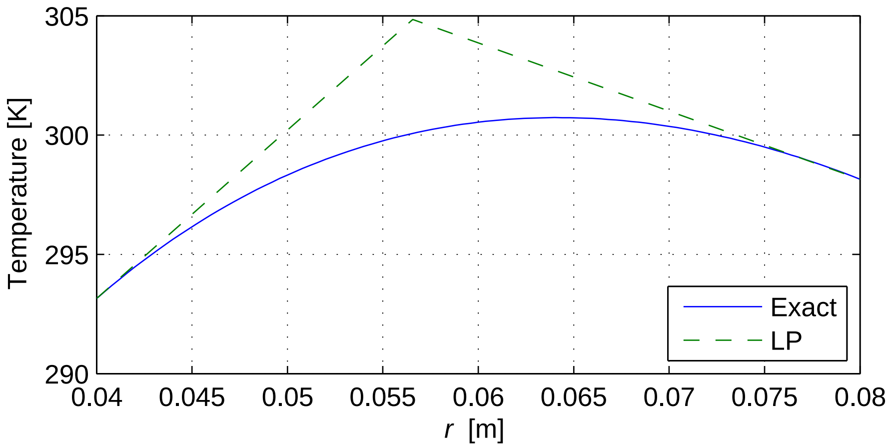

Figure 3.4: Temperatures of a cylinder with internal heat generation

3.3 Derivation of the general cylindrical component

This section derives the general cylindrical component as proposed by Mellor *et al.* The derivation rests on the application of superposition where the temperature distribution for two scenarios are determined and the results added to determine the final distribution. The diffusion equation is solved for: 1) zero internal heat generation and 2) zero surface temperature with internal heat generation. The generalised cylindrical component is shown in Figure 3.5. The temperatures T_1, T_2, T_3 and T_4 are the radial outside, radial inside, axial left and axial right temperatures, respectively. The cylinder has an inner radius of r_2, an outer radius of r_1 and an axial length L. The mean temperature of the cylinder is T_m. The thermal resistances related to the axial and radial directions, have subscripts a and r, respectively.

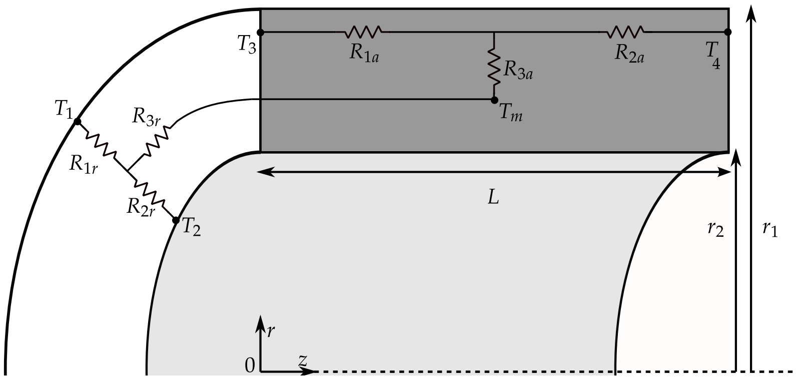

Figure 3.5: Generalised cylindrical component

3.3.1 Zero internal generation

In this scenario the internal generation is zero and the resulting thermal distribution will be only due to the boundary temperatures.

In the z - direction

The diffusion equation reduces to (3.2), where it is assumed that the conductivity k_z is constant.

$$\frac{d}{dz}\left(k_z \frac{dT}{dz}\right) = 0 \tag{3.2}$$

Integrating twice gives:

$$\begin{aligned} \int \frac{d}{dz}\left(\frac{dT}{dz}\right) dz &= \int 0 dz \\ \int \frac{dT}{dz} dz &= \int C_1 dz \\ T(z) &= C_1 z + C_2. \end{aligned} \tag{3.3}$$

The constants are determined using the boundary conditions. Assuming the temperatures are T_3 and T_4 where z is zero and L, respectively, the solution is:

$$T(z) = \left(\frac{T_4 - T_3}{L}\right) z + T_3. \tag{3.4}$$

Applying Fourier's law gives the heat flux q_z'',

$$\begin{aligned} q_z'' &= -k_z A_z \frac{dT}{dz} \\ &= -k_z \left(\pi r_1^2 - \pi r_2^2\right) \left(\frac{T_4 - T_3}{L}\right) \\ &= \frac{k_z \pi \left(r_1^2 - r_2^2\right)}{L} \left[T_3 - T_4\right], \end{aligned} \tag{3.5}$$

where A_z is the surface area in the z direction. The lumped parameter approach draws analogies between electrical current and heat flux, potential difference and temperature difference as well as electrical resistance and thermal resistance. The thermal resistance with no internal heat generation is:

$$R_z = \frac{L}{\pi k_z \left(r_1^2 - r_2^2\right)}. \tag{3.6}$$

Dividing the total resistance into two then gives the first two resistances in the axial direction (R_{1a}, R_{2a}) for the cylindrical component:

$$R_{1a} = R_{2a} = \frac{L}{2\pi k_z \left(r_1^2 - r_2^2\right)}. \tag{3.7}$$

In the r - direction

The diffusion equation in this direction reduces to:

$$\frac{1}{r}\frac{d}{dr}\left(k_r r \frac{dT}{dr}\right) = 0. \tag{3.8}$$

Following the same method as done for the z - direction, the equivalent resistance is

$$R_r = \frac{\ln\left(r_1/r_2\right)}{2k_r \pi L}. \tag{3.9}$$

This is then divided where the mean temperature would be if no internal generation is present, to give the first two radial resistances:

$$R_{1r} = \frac{1}{4\pi k_r L}\left[1 - \frac{2r_2^2 \ln\left(r_1/r_2\right)}{\left(r_1^2 - r_2^2\right)}\right] \tag{3.10a}$$

$$R_{2r} = \frac{1}{4\pi k_r L}\left[\frac{2r_1^2 \ln\left(r_1/r_2\right)}{\left(r_1^2 - r_2^2\right)} - 1\right]. \tag{3.10b}$$

3.3.2 Zero surface temperature, internal generation

In this scenario, the surface temperatures are assumed to be zero and the internal heat generation is applied.

In the z - direction

In this case the heat generated ($\dot{q}$) is also part of the diffusion equation:

$$\frac{d^2T}{dz^2} + \frac{\dot{q}}{k_z} = 0 \tag{3.11}$$

Again, integrating twice gives:

$$T(z) = -\frac{\dot{q}}{2k_z}z^2 + C_1 z + C_2. \tag{3.12}$$

The boundary conditions require zero temperature at the surfaces where $z = 0, L$; giving the general solution in this case as:

$$T(z) = -\frac{\dot{q}}{2k_z}z^2 + \frac{\dot{q}L}{2k_z}z. \tag{3.13}$$

The average temperature $\bar{T}$ can then be calculated:

$$\begin{aligned}
\bar{T} &= \frac{1}{L}\int_0^L T(z)dz \\
&= \frac{1}{L}\int_0^L \left[-\frac{\dot{q}}{2k_z}z^2 + \frac{\dot{q}L}{2k_z}z\right] dz \\
&= \frac{1}{L}\left[-\frac{\dot{q}}{6k_z}z^3 + \frac{\dot{q}L}{4k_z}z^2\right]_0^L \\
&= \frac{\dot{q}L^2}{12k_z}.
\end{aligned} \tag{3.14}$$

The total heat generation $q_z = \dot{q}A_zL$, mean temperature and resistances R_{1a}, R_{2a} can then be used to calculate the third axial resistance:

$$\begin{aligned}
R_{3a} &= \frac{\bar{T}}{\dot{q}A_zL} - \frac{R_{1a}R_{2a}}{R_{1a}+R_{2a}} \\
&= \frac{\dot{q}L^2}{12k_z}\frac{1}{\dot{q}\pi\left(r_1^2 - r_2^2\right)L} - \left[\frac{R_{1a}}{2}\right] \\
&= \frac{1}{12}\frac{L}{k_z\pi\left(r_1^2 - r_2^2\right)} - \frac{L}{4\pi k_z\left(r_1^2 - r_2^2\right)} \\
&= -\frac{L}{6\pi k_z\left(r_1^2 - r_2^2\right)}.
\end{aligned} \tag{3.15}$$

If the distrubuted heat generated inside the cylinder was injected between R_{1a} and R_{2a}, the calculated center temperature would be incorrect, as was shown in the previous section. This final resistance ensures that the mean temperature of the cylinder is correct.

In the r - direction

The diffusion equation is:

$$\frac{1}{r}\frac{d}{dr}\left(k_r r\frac{dT}{dr}\right) + \dot{q} = 0 \tag{3.16}$$

Applying the same method as for the z - direction, the general solution is:

$$T(r) = -\frac{\dot{q}r^2}{4k_r} + \frac{\dot{q}r_1^2}{4k_r} - \left[\frac{\dot{q}\left(r_2^2 - r_1^2\right)}{4k_r\ln(r_1/r_2)}\right]\ln r + \left[\frac{\dot{q}\left(r_2^2 - r_1^2\right)}{4k_r\ln(r_1/r_2)}\right]\ln r_1 \tag{3.17}$$

which agrees with results given by Incropera *et al.* [3]. The final resistance in this direction is:

$$R_{3r} = \frac{-1}{8\pi\left(r_1^2 - r_2^2\right)k_rL}\left[r_1^2 + r_2^2 - \frac{4r_1^2r_2^2\ln\left(r_1/r_2\right)}{\left(r_1^2 - r_2^2\right)}\right]. \tag{3.18}$$

3.3.3 Thermal capacitance

The thermal energy stored inside a component can be modelled using a thermal capacitance. The thermal capacitance C of the general cylinder is given by:

$$\begin{aligned} C &= cm \\ &= \rho c \pi ({r_1}^2 - {r_2}^2) L \end{aligned} \tag{3.19}$$

where m is the mass of the component, ρ is the density and c is the specific heat capacity. The thermal capacitance must be included in a thermal model if transient behaviour is investigated.

3.3.4 LP assumptions

Before the LP model can be applied, the assumptions of this method must be discussed.

Independent heat flow Heat flow in the axial and radial directions do not influence each other. Heat flux can thus not change direction, e.g. flow from a radial boundary to an axial boundary directly without raising the mean temperature.

Single mean temperature The heat flow in both directions are defined by a single mean temperature.

No circumferential heat flow It is assumed that there are no temperature differences in the circumferential direction.

Uniform thermal capacitance If the material is isotropic, the specific heat capacity and density will be constant, resulting in a uniform thermal capacitance.

Uniform heat generation The heat generation must be uniform throughout the component. This is generally true for stator windings and induction motor rotor bars, where Joulean heat generation occurs due to current flow. In the case of eddy current loss in the PM of a PMSM due to switching harmonics, this is NOT true. The penetration depth of the magnetic field causing the eddy currents can cause the currents to only flow in part of the PM. Eddy current loss in the PM is discussed in detail in section 5.5.

3.4 Natural convection resistance

Section 2.4.1 introduced natural convection and showed that the convection coefficient (h) has a non-linear relation to the temperature difference between the surface and ambient fluid. The equations needed to determine the convection resistance (R_{conv}) are:

$$\mathrm{Ra} = \frac{g\beta(T_s - T_\infty)L^3\mathrm{Pr}}{\nu\alpha} \tag{3.20a}$$

$$\mathrm{Nu} = 0.59\mathrm{Ra}^{1/4} \tag{3.20b}$$

$$h = \frac{\mathrm{Nu}k}{L} \tag{3.20c}$$

$$R_{conv} = \frac{1}{hA} \tag{3.20d}$$

where Ra, Nu, Pr are the Rayleigh, Nusselt and Prandtl dimensionless numbers, respectively. The length of the convection wall is L and its area A. The fluid properties: β is the volume expansivity, ν is the kinematic viscosity, α is the thermal diffusivity, k is the conduction coefficient and g is the gravitational acceleration. To accurately model the convection heat flow, a temperature dependent resistor should be used. The thermal resistance of a component is the amount of heat flux flowing through it when a temperature difference is applied over it. A variable current source can thus be used to model a variable resistor. A Simulink® implementation of this concept is shown in Figure 3.6. The temperature difference (voltage difference) over the resistor is used to determine the Ra number. The Ra number is used to determine the Nu number, which is used to determine h. From h, A and the temperature difference, the heat flux flowing through the resistor is determined.

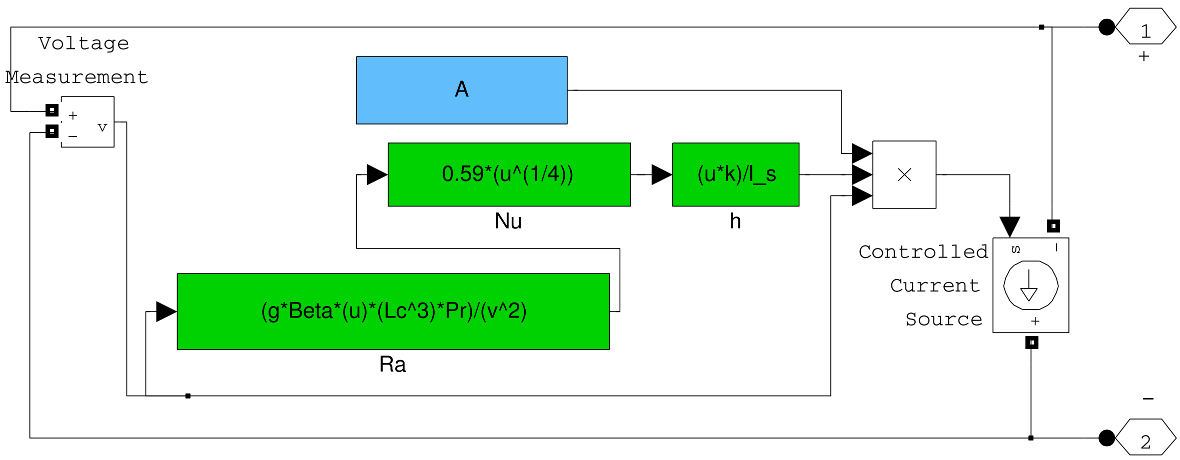

Figure 3.6: Variable natural convection resistor implementation

3.5 Implementation on the TWINS machine

This section discusses the implementation of Mellor's LP technique on the TWINS machine.

The LP circuit model of the TWINS is shown in Figure 3.7. The stator housing, stator laminations, winding, end winding, shielding cylinder, permanent magnet and rotor laminations are all modelled using the T-network proposed by Mellor. It is assumed that the rotor eddy current loss due to high frequency magnetic fields are contained in the shielding cylinder, according to its purpose. Loss sources are present in the winding, shielding cylinder, end winding and stator laminations. It is assumed that the thermal storage of the coil former is not significant due to its size and the material used.

Four interface resistances are included in the model to take into account the manufacturing uncertainties. On the rotor it is between the rotor laminations and permanent magnet (R_{rRI}) since this interface is filled with glue. All the other interfaces are established through shrink fits and thus have a good thermal contact. Interface resistances in the axial (R_{aEI}) and radial (R_{rEI}) directions are placed between the end winding and stator housing. The stator laminations' loose fit in the stator housing also necessitates an interface resistance (R_{rSI}).

The convection heat transfer due to natural convection ($R_{nat,conv}$) and that due to the forced cooling system ($R_{for,conv}$) are also shown.

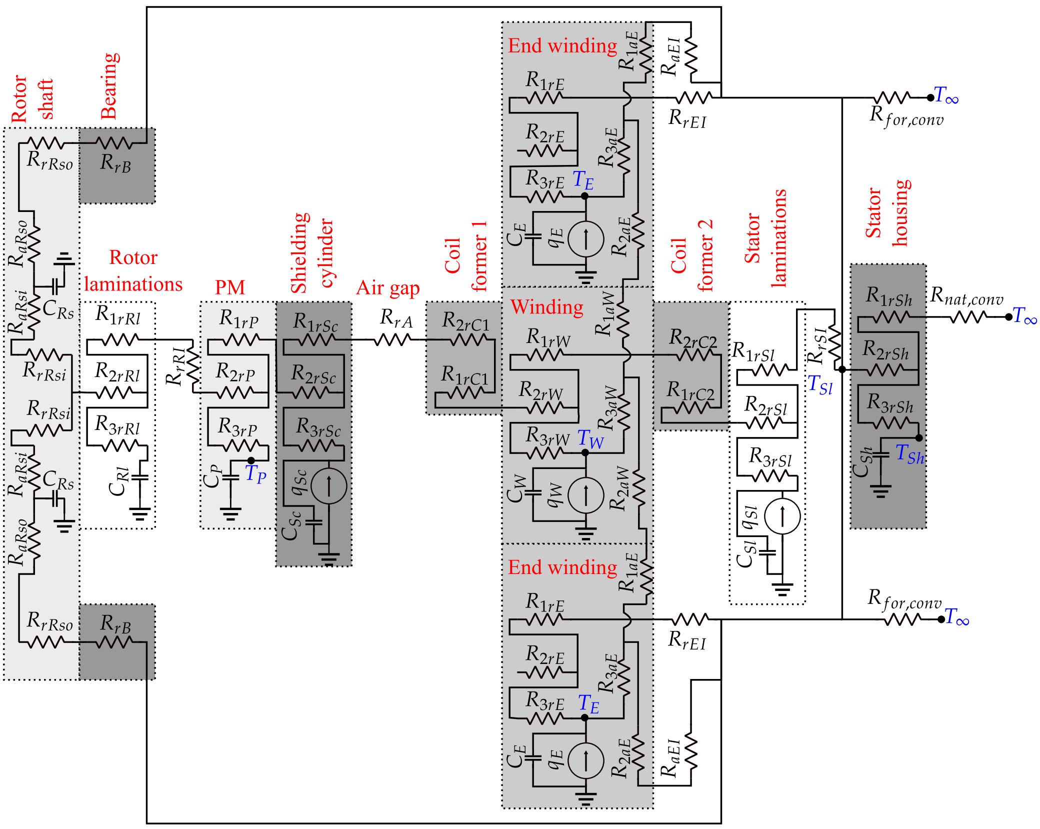

Figure 3.7: LP thermal model implementation on TWINS machine

3.6 Conclusion

This chapter begins with discussing the thermal modelling approach of a high speed PMSM. The PM is a vital component of a PMSM, but it is thermally sensitive. Since the heat flows into and out of the PM are dependent on the machine's overall temperature distribution, a thermal model for the whole machine is needed. LPs can be used to derive a model of the entire machine. This chapter derived the general cylindrical component that forms the basis for the LP model. In conclusion, the LP model for a high speed PMSM, based on the TWINS machine, is given.

In the next chapter, a 2-D analytical distributed model for the thermal distribution inside the PM is derived. The convection heat flow in the z - direction can then be determined and used in the LP model to include the 2-D effects found in the PM.