

An energy-based representation of an axial-flow compressor system

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PREFACE

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"I press toward the mark for the prize of the high calling of God in Christ Jesus."

Philippians 3:14

ABSTRACT

Large-scale industrial systems play a great role in our modern industry. One example of such a system is the Brayton cycle. The Brayton cycle consists of a compressor, combustor and turbine. Each interconnecting component can be modelled independently based on either the state variables or the energy of the system. Modelling of industrial systems for Control, Condition Monitoring and Fault Detection and Diagnosis (FDD) is general in engineering. The modelling of the nonlinearities pose difficulties in the analysis of such a system when multi-domain effects are considered in the model. Over the last decade, energy based-techniques delivered promising results in terms of the optimisation of nonlinear systems in the field of Control, condition monitoring and FDD.

Recent research has focused on viewing components in systems as energy-shaping devices. A compressor system demonstrates nonlinearities, commonly found in practical systems, so such a system can be considered as an ideal system for an energy-based representation. The most common axial-flow compressor model found in the literature is Moore and Greitzer's dimensional model derived in 1986. The dimensional model from Greitzer was used to obtain the state space model of the compressor system for this study. A set of equations were included in the model to simulate the dynamics of a leak in the plenum volume. The model can introduce one of three fault conditions. The first is a leak in the plenum volume. The second is rotating stall and thirdly surge, both types of unstable operations.

The axial-flow compressor model's response is validated by comparing it to the response of a Flownex[®] model. Flownex[®] is a computational CFD software package that is capable of modelling complex thermohydraulic systems. All of the components found in Flownex[®] are validated with experimental data. The energy analysis is done from the state variables of the system by deriving an energy-balance for each component in the system. The energy-balance for each component is derived from the first law of thermodynamics to verify the energy analysis. A visual energy representation is given for both stable and unstable conditions for each component. A sensitivity analysis based on the energy of each component is done that relates the energy parameter to a change of an input parameter. A distinct result is that the steady state energy characteristic can be mapped for each component. The power-energy plane for a fault condition can be superimposed on the power-energy plane of the normal operation when the operating point is modified. The energy and power response can be used to implement an FDD system. The sensitivity analysis indicates that the energy and power could be used for Controller design, Condition monitoring and FDD.

OPSOMMING

Industriële stelsels speel 'n belangrike rol in ons moderne industrie. Een voorbeeld van so 'n stelsel is die Brayton-siklus. Die Brayton-siklus bestaan uit 'n kompressor, verbrander en turbine. Elke tussenverbindende komponent kan onafhanklik gemodelleer word, gebaseer op die toestandveranderlikes of die energie van die komponente. Die modellering van industriële stelsels vir beheer, kondisiemonitering en foutopsporing en -diagnose is algemeen in die veld van ingenieurswese. So 'n stelsel is meer kompleks as gevolg van die nie-lineêre gedrag van die komponente in die stelsel wanneer multi-domein effekte in die model in ag geneem word. Oor die afgelope dekade het energie-gebaseerde tegnieke belowende resultate gelewer vir die optimalisering nie-lineêre stelsels vir beheer, kondisiemonitering en foutopsporingstegnieke.

Meer onlangse navorsing fokus op die komponente in 'n stelsel as energievormingstoestelle. 'n Kompressorstelsel is 'n voorbeeld van 'n stelsel wat die nie-lineariteit van komponente goed demonstreer, dus word so 'n stelsel beskou as 'n ideale stelsel vir die visualisering van energie. Die mees algemene aksiaal-vloei kompressorstelselmodel in die literatuur is die model van Moore en Greitzer wat afgelei is in 1986. In dié studie is die model van Greitzer gebruik om die toestand-veranderlike model van 'n aksiaal-vloei kompressorstelsel af te lei. 'n Stel vergelykings is in die model ingesluit om die dinamika van 'n lek in die stoorvolume te modelleer. Die model kan nog twee onstabiele modelleer waar die stelsel in 'n roterende glip of in 'n limietsiklus ingaan.

Die response van die analitiese model is gevalideer deur dit te vergelyk met die response van 'n Flownex[®]-model. Flownex[®] is 'n CFD sagtewarepakket wat komplekse termo-hidrolise stelsels kan simuleer. Al die komponente beskikbaar in Flownex[®] is gevalideer. Die toestand veranderlikes van die stelsel is gebruik om 'n energieanalise op die stelsel te doen. 'n Energiebalans is afgelei vir elke komponent wat gebaseer is op die eerste wet van termodinamika. 'n Visuele voorstelling van die energie is gedoen om beide die stabiele en onstabiele toestande van elke komponent voor te stel. 'n Sensitiwiteitsanalise wat gebaseer is op die energie in elke komponent is gedoen. Die sensitiwiteitsanalise poog om die sensitiwiteit van die energie respons te bepaal vir geselekteerde insetparameters. 'n Duidelike resultaat is dat die bestendige-toestand energie kenmerkend gekarakteriseer kan word vir elke komponent. Die response van die energie word visueel voorgestel deur drywing-energie vlakke wat die karakteristieke van die bestendige toestand en oorgangsresponse bevat. 'n Foutopsporingstelsel is geïmplementeer wat gebaseer is op die drywing-energie grafieke van die stelsel. Die sensitiwiteitsanalise dui aan dat die energie en drywing van die stelsel kan gebruik word vir beheerderontwerp, kondisiemonitering en foutopsporingstelsels.

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NOMENCLATURE

Abbreviation	Description
CCV	Close Coupled Valve
FDD	Fault Detection and Diagnosis
IAE	Integral of the Absolute error
IDA-PBC	Interconnection and Damping Assignment Passivity-Based Control
IEA	International Energy Agency
ISE	Integral of the Squared Error
ITAE	Integral of the Time multiplied by the Absolute error
ITSE	Integral of the Time multiplied by the Squared error
MG	Moore-Greitzer
PBC	Passivity-Based Control

Subscripts	Description
0	Initial time step
<i>amb</i>	Ambient
<i>c</i>	Compressor
<i>CV</i>	Control volume
<i>e</i>	Exit
<i>gen</i>	Generalised
<i>h</i>	Hole
<i>i</i>	Inlet
<i>in</i>	Into a node
<i>net</i>	Net, defined as the difference between the in and out
<i>o</i>	Outlet
<i>out</i>	Out of a node
<i>p</i>	Plenum
<i>ref</i>	Reference
<i>sf</i>	Scaling factor
<i>sh</i>	Shaft
<i>t</i>	Throttle

Latin lowercase	Unit	Description
a	-	Reciprocal time lag of the compressor passage
a_s	m/s	Speed of sound
c_p	kJ/(kg·K)	Specific heat at constant pressure
f_s	W/K	Entropy flow
h	kJ/kg	Enthalpy
k	-	Polytropic exponent
l_c	-	Non-dimensional length of the compressor
l_e	-	Non-dimensional length of the exit duct
l_i	-	Non-dimensional length of inlet duct
$\dot{m}$	kg/s	Mass flow rate
$\dot{m}_c$	kg/s	Mass flow rate through the compressor
$\dot{m}_h$	kg/s	Mass flow rate through the leak
$\dot{m}_t$	kg/s	Mass flow rate through the throttle
p	Pa	Pressure
p	N·s	Momentum
p_{amb}	Pa	Ambient pressure
Δp_c	Pa	Pressure difference across the compressor
$\Delta p_{c,ss}$	Pa	Compressor steady state pressure rise
Δp_h	Pa	Pressure difference across the leak
p_i	Pa	Pressure at the inlet of the compressor system
p_o	Pa	Pressure at the outlet of the compressor system
p_p	Pa	Pressure in the plenum volume
Δp_t	Pa	Pressure difference across the throttle
q	C	Charge
t	s	Time
u	J	Internal energy
$\mathbf{u}$	-	Input vector of the state space model
u_t	-	Throttle gain parameter

Latin lowercase continued

v	m/s	Velocity
x	m	Displacement
$\mathbf{x}$	-	State vector of the state space model
$\mathbf{y}$	-	Output vector of the state space model

Latin uppercase	Unit	Description
A	A	Current
$\mathbf{A}$	-	A matrix of the state space model
A_c	m ²	Flow area of the inlet duct
A_f	S	Flow admittance
A_{sf}	-	Flow admittance scaling factor
A_h	m ²	The area of the leak
A_o	m ²	Area of throttle opening
A_t	m ²	Flow area of the outlet duct
B	-	Greitzer B parameter
$\mathbf{B}$	-	B matrix of the state space model
$\mathbf{C}$	-	C matrix of the state space model
C_0	-	Shut-off value of dimensional characteristic
C_x	m/s	Absolute velocity
D	m	Diameter
$\mathbf{D}$	-	D matrix of the state space model
E	J	Energy
F	N	Force
H	-	Semi-height of cubic axisymmetric characteristic
J	-	Squared amplitude of rotating stall
L	m	Length
L_c	m	Length of the inlet duct, compressor and exit duct
L_t	m	Length of the outlet duct
N	r/min	Rotor speed

Latin uppercase continued

N_{stall}	-	Number of rotor revolutions required to develop a stall cell
P	J/s	Power
Q	m ³ /s	Volumetric flow rate
$\dot{Q}$	J/s	Heat flow
R	m	Rotor mean radius
S	J/K	Entropy
T	K	Temperature
ΔT_0	K	Temperature difference
U	m/s	Rotor speed at mean radius
U_t	m/s	Rotor speed at tip radius
V	V	Potential difference
V	m ³	Volume
V_p	m ³	Plenum volume
W	-	Semi-width of cubic characteristic
$\dot{W}_{sh}$	J/s	Shaft work

Greek lowercase	Unit	Description
γ	-	CCV gain/Specific heat ratio
ψ	-	Axisymmetric pressure-rise coefficient
ψ_{c0}	-	Shut-off value of axisymmetric characteristic
ϕ	-	Local axial flow coefficient
ϕ	V·s	Flux linkage
ρ_{amb}	kg/m ³	Ambient density
ρ	kg/m ³	Density
ρ_p	kg/m ³	Density of the working fluid in the plenum volume
τ	s	Compressor flow field time constant
ς	-	Constant in MG model
ζ	-	Loading temperature

Greek uppercase	Unit	Description
Φ	-	Non-dimensional mass flow coefficient
Φ_T	-	Non-dimensional throttle mas flow coefficient
Ψ	-	Non-dimensional pressure rise coefficient
Ψ_s	-	Non-dimensional in-stall characteristic
Ψ_T	-	Non-dimensional throttle characteristic
Γ	(N·s)/m ²	Pressure momentum

1 CHAPTER INTRODUCTION

Chapter 1 offers a brief background on compressor systems, energy analysis and energy visualisation. The latter part of the chapter presents a discussion of the problem statement, research scope and objectives, and methodology. The chapter concludes with the article contributions and a dissertation outline.

1.1 Background

The sustainability of modern society is dependent on the production and services delivered by large-scale industrial systems. The efficiency of industrial systems is a major concern due to the large energy demand of the industry. The International Energy Agency (IEA) estimated the total consumption of resources by industry in 2012 as follows: Coal consumption by industry at 80.0%, oil consumption by industry at 8.5%, natural gas consumption by industry at 36.5% and electricity consumption by industry at 42.3%.

A large percentage of the resources is consumed by industrial processes. This requires that systems should operate efficiently to minimise cost, increase reliability and to ensure the sustainability of the production process. The theoretic limitation of these systems with reference to their efficiency is that the systems are analysed and controlled based on linear techniques, whereas the operation of the systems in practice is nonlinear. Optimization based of the states was done on these systems from a direct approach in terms of observed data on a simplified model of the system and mostly for a single operating point [1]. An example of a common system found in industry is the Brayton cycle. This system includes a compressor, combustor and turbine. Each of these sub-systems contains multi-domain, nonlinearities noticeable in their practical operation [2]. The Brayton cycle for a gas turbine is shown in Figure 1.

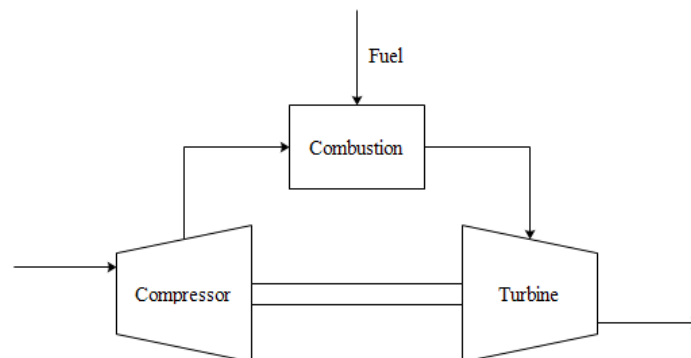


Figure 1: Brayton cycle diagram for a gas turbine [3]

Compressors play a crucial role in industry by performing mechanical work on the working fluid in the system and increasing the pressure of the working fluid. Two common types of compressors found in the industry include positive displacement and dynamic compressors. A dynamic compressor is a type of continuous-flow compressor, and unlike a positive displacement compressor, it delivers a fixed volumetric flow. Axial-flow and centrifugal compressors are two types of dynamic compressors. In axial-flow compressors the flow leaves the compressor in the axial direction and in the case of a centrifugal compressor, the air leaves in a perpendicular direction with respect to the shaft [4]. A schematic representation of an axial-flow compressor is shown in Figure 2.

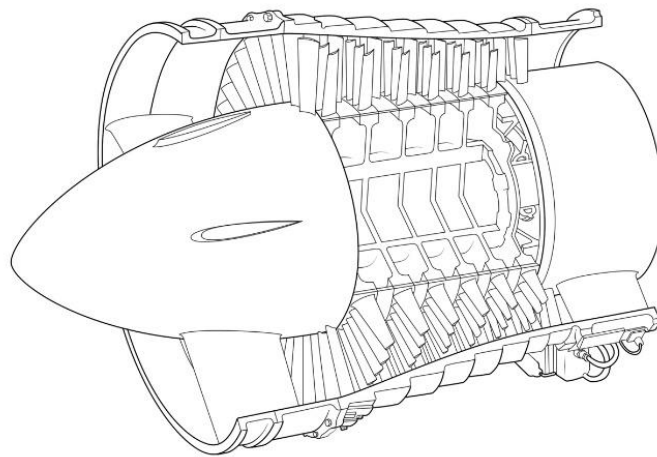


Figure 2: Schematic representation of an axial-flow compressor

Greitzer developed a nonlinear one-dimensional lumped parameter model of an axial-flow compressor in 1976. The model was mainly developed to predict if a compressor would enter rotating stall or surge when operated near the stall line [6,7].

The energy distribution in an axial-flow compressor system can be visually presented to portray the nonlinear dynamics of the system [7]. The energy response of such a system is useful to analyse and identify deficiencies in the system.

Energy is a fundamental concept in study fields such as physics, chemistry and engineering. Most of these disciplines can associate with the concept of energy. It can serve as a *lingua franca* between different research fields. In engineering, energy is regarded as a multi-domain concept that connects different domains. A complex system can be modelled by viewing its interconnecting subsystems from an energy perspective [8].

Most recent studies such as that of Ortega et al. focuses on viewing components in systems as energy shaping devices [9]. These studies are based on techniques such as the Lagrangian and Hamiltonian equations of motion where the key concept is energy storage [10].

Control strategies based on energy such as the controlled Lagrangian and Passivity-based control (PBC), which includes energy-balancing PBC [11] and Interconnection and Damping Assignment PBC (IDA-PBC) [12], have attracted the interest of specialists in the field of control. In the field of fault detection and diagnosis, energy-balance-based approaches showed great potential for uniquely detecting and isolating faults for model-based FDD systems [1].

1.2 Problem statement

Most control strategies are based on the concept of controlling a system by using its state variables. The limitation is that the control system is complex when controlling a nonlinear model. In the field of FDD, most recent studies focus on the energy in the system to generate residuals for the detection of and diagnosis of faults. The intention of the study is to derive a fundamental energy representation based on an analytic model of an axial-flow compressor system. The sensitivity of the energy representation should determine if the energy response could be used for controller design, condition monitoring and FDD.

1.3 Research scope

An analytic model is required for the modelling of an axial-flow compressor system. The response of the analytic model should be validated to ensure that the correct response is portrayed for the steady and transient operation of the compressor system.

The steady state compressor characteristic validation is limited to the three-stage axial-flow compressor system investigated by Gill in [13]. The compressor system investigated by Gravdahl shall be validated for the steady state and transient response.

The energy analysis shall be based on this single stage axial-flow compressor system that is capable of modelling unstable conditions such as rotating stall and surge. The single stage axial-flow compressor system model is restricted to a simulation environment.

1.4 Research objectives

The following objectives were identified for the study:

1.4.1 Derivation of an analytic model for an axial-flow compressor system

A nonlinear one-dimensional lumped parameter model is required to model the compressor system. A model can be used from literature and extended to be able to model all the required conditions. The model will consist of a set of first-order differential equations that can model the systems dynamics for the steady state and transient responses.

1.4.2 Construction of the axial-flow compressor system in Flownex®

A system CFD software package, namely Flownex®, shall be used to construct a model of the compressor system. The system CFD model is required to simulate the compressor system for both steady state and transient dynamics. The complexity of the model should match the complexity of the analytic model to ensure that the response is correct.

1.4.3 Verification and validation of the analytic model

The verification of the analytic model requires a comparison of the response to that of another model found in literature. All types of operating conditions that are investigated should be verified. The validation of the analytic system should be done for both the steady state response and the transient response of the analytic model.

1.4.4 Performing an energy analysis and visualising the energy of the system

An energy analysis is required to visually depict the system's response from an energy perspective. The visualisation should aim to show how the energy is shaped in the compressor system under different operating conditions.

1.4.5 Sensitivity evaluation

The sensitivity analysis is required to relate inputs of the system to the energy visualisation of the system. The analysis should determine how sensitive the energy response is to a change in input parameters.

1.5 Methodology

1.5.1 Derivation of an analytic model for an axial-flow compressor system

A nonlinear lumped parameter analytic model shall be derived for an axial-flow compressor system. This model shall be presented in a state-space format so that an energy analysis can be conducted on the state variables of the model. An analytic model is essential to analyse the store, flow and shaping of energy between interconnected components. Leak dynamics shall be included in the model to investigate the system's response for a fault condition. The analytic model shall be able to model instabilities such as rotating stall and surge that occur in compressor systems.

1.5.2 Construction of the axial-flow compressor system in Flownex®

A system CFD model of an axial-flow compressor system has to be constructed in Flownex®. The Flownex® model shall be used to validate the response of the analytic model. Effects such as compressibility and moment of inertia are not required.

1.5.3 Verification and validation of the analytic model

The verification of the analytical model includes the comparison of the response of the model with the response of other models found in literature.

The validation of the analytic model is subdivided into two parts. The first is the validation of the axial-flow compressor's steady state characteristic. Practical data from Gill [13] shall be used to validate the fitment of the steady state characteristic to the data points.

The second part is to validate the transient response of the analytic model. This response shall be validated by a comparison of the response to the response of the Flownex® model. Only by comparing the transient response, by producing the same operating point shift in both the analytic and Flownex® model, can the response of the analytic model be validated with that of the Flownex® models response.

1.5.4 Performing an energy analysis and visualising the energy of the system

An energy analysis shall be done on the compressor system using the state variables of the analytic model. This analysis shall include an evaluation of the steady state and transient energy of system.

The steady state energy shall be presented in vector form for different operating points and the transient energy response shall be visually presented on power-energy planes for the different components of the compressor system.

Different test cases shall be investigated through an energy analysis. These test cases includes the following:

- The normal operation of the compressor system
- The operation of the compressor system when a leak is introduced into the system
- The energy response of the compressor system during rotating stall
- The energy response of the compressor system during surge

1.5.5 Sensitivity evaluation

A sensitivity analysis shall be done on the energy analysis. This sensitivity analysis shall consist of a parametric sweep of selected inputs and parameters of the analytic model. The energy response shall be monitored for the sweep and shall be visually presented.

1.6 Article contributions

Two articles were written based on the study. The first article, entitled “Nonlinear state space modelling of an axial-flow compressor system for energy visualisation”, was presented at the 2015 South African Universities Power Engineering Conference (SAUPEC) and is given in Appendix C.

The second article, entitled “Energy-based visualisation of an axial-flow compressor system for the purposes of Fault Detection and Diagnosis” was submitted for the 2016 symposium on Dynamics and Control of Process Systems, including Biosystems (DYCPOS-CAB) and is given in Appendix C. At the time of the submission of the dissertation the article was still under review.

1.7 Dissertation outline

Chapter 2 contains a literature survey that discusses the most recent research on compressors, compressor maps, FDD techniques and the visualisation of compressor systems. The second part of this chapter discusses literature on compressors, FDD techniques and performance index calculations. The chapter concludes with a critical literature review. In Chapter 3, model is derived for an axial-flow compressor system. The model derived includes the capability of modelling a leak in the compressor system, as well as the capability to model instabilities such as rotating stall and surge.

In the latter part of this chapter the state space model of the system is presented. Chapter 4 contains the implementation of the Simulink® model, Flownex® model, verification and validation of the analytic model that was derived in Chapter 3. The verification and validation of the steady state and transient response is portrayed in the final part of this chapter. The first part of Chapter 5 introduces an energy analysis. The energy analysis includes four test cases where the power and energy in the system are visually presented. The chapter is concluded with a sensitivity evaluation of the power-energy planes for variations of the model parameters. The dissertation is concluded in Chapter 6 with a discussion of the findings in the study.

2^{CHAPTER}

LITERATURE STUDY

This chapter consists of three main sections. The first part includes a literature survey on the research of compressors, visualisations of compressor systems and energy-based FDD. The second part discusses literature on compressors, compressor maps, compressor models that include instabilities and faults, and FDD. The latter part of the chapter consists of a critical evaluation of the literature.

2.1 Introduction

Large-scale industrial systems form a great part of the industry. The reliability, performance and safety of such systems are considered important factors in their design and maintenance. Faults are likely to occur in these systems due to the complexity and precision of their design. The efficiency of a system is influenced by the occurrence of faults and faults can eventually result in the failure of a system.

2.2 Literature survey

Most research on compressors focus on the design of controllers for the active avoidance of rotating stall and surge. This includes the work of Gravdahl and Egeland in their recent publication on the modelling and control of surge and rotating stall in compressors [14]. The most common application of controller design for rotating stall and surge avoidance are found in the aviation industry.

A recent study was done by Benedikt and Hermann in [15] that focused on the development of a robust surge avoidance controller for a low pressure compressor of a turbo jet engine. Their contribution was to propose an additional feedback controller to compare a given reference value for the surge margin for a certain set of engine parameters. In their paper they also propose to implement H_∞ -control and a μ -synthesis approach to further improve the performance and fuel consumption of the engine.

Another recent study on fluid dynamic instabilities in axial compressor systems is the investigation of Ananth and Kushari in [16]. In this investigation the geometry, such as the effective length of the compressor, annulus area and plenum volume, are varied. The effects of how these parameters influence the system during rotating stall and surge are then monitored.

Leading research on compressor maps found in literature was done on the mapping of all four quadrants of a compressor characteristic by operating the compressor under special conditions so that the compressor does not enter an unstable condition. This includes the investigation of the negative rotation of a compressor and the reverse flow. Gill et al. investigated the high energy dissipation rates when a compressor is operated on a four quadrant compressor map in [17]. The investigation found that radially oriented vortices is formed between the blades in a blade row when the compressor is operated at low positive or negative flow rates. Gill et al. experimentally determined the four-quadrant compressor map for a three-stage low speed axial-flow compressor in [18] and [19].

One of the most renowned models of a compressor system is the non-linear, one-dimensional lumped parameter model of Greitzer. Numerous investigations was done on the model derived by Greitzer in the two part series [5] and [6]. The types of studies is mostly focused on the design of active avoidance controllers for rotating surge and stall. These investigations include the work of Meuleman [20], du Toit [3], Badmus [7] and Liaw et al, [21].

Another commonly found model in literature is the model derived by Moore and Greitzer in the two part series [22] and [23]. This model is described by three third order partial nonlinear differential equations. A Galerkin procedure is used to average the angular dependencies and the set of equations becomes first order in time. The complete model is presented in a non-dimensional form. Many other studies such as those of Gravdahl [24], Gu et al. [25] and Krstic et al. [26] are based on the model of Moore and Greitzer.

The visualisation of compressor systems includes a three dimensional visualisation by using a computational fluid dynamic (CFD) model. One example of such a modelling technique was done by Im et al. in [27]. Their investigation focused on using a sliding boundary condition to simulate a 3D multistage axial flow compressor.

Another visual approach was attempted by Gourdian et al. in [28], where a numerical simulation was done on an axial compressor system with non-asymmetric casing treatment. Two cases were investigated; one without the treatment of the casing and the other with a casing treatment configuration. In the axial velocity flow field's visualisation, reversed flow regions developed that lead to rotating stall for the untreated system. In the treated case the operating range of the system was extended. Another study of Gourdian and Leboeuf [29] was the unsteady simulation of an axial flow compressor stage with passive control strategies. Their focus was to increase the compressor performance by a passive control strategy that focused on the tip region of the blades.

Many fault detection methodologies can be found in literature, but the most common procedure is the model-based fault detection methodology. Isermann is one of the most renowned names in the field of fault detection and diagnosis. Isermann gave an introduction of fault detection and diagnosis based on methods such as parameter estimation, state estimation and parity equations in [30].

FDD systems can be applied to many types of systems, including compressor systems. Condition monitoring and fault detection was done by Zhang et al. in [31] on a compressor system based on collected vibration and other on-site data from a Bentley Nevada data acquisition system. Signal processing techniques such as a time-domain analysis, frequency-domain analysis, trend analysis and orbit analysis were used to monitor the system's operating condition and to detect faults in the system.

Another recent approach in FDD was that of Ahmed et al. where a fault detection strategy was applied to a reciprocating compressor using a model from principal component analysis (PCA) and vibrations. Five different faults were detected that included valve leakage, inter-cooler leakage, suction valve leakage, loose belt combined with inter-cooler leakage and loose drive belt combined with suction valve leakage [32].

In the study of García-Matos et al. in [33], a hybrid model-based fault detection approach was conducted on an axial-flow compressor of a combined-cycle power plant. The contribution of the study was to use the best of a physical model and a multilayer perceptron (MLP) model to detect faults in the compressor system. Leading research in the field of FDD systems examined the application of energy-based FDD to systems. In the thesis of Chen [1] he applied an energy-based FDD methodology to an RLC circuit, a pendulum and a robot manipulator benchmark. Another energy-based FDD methodology followed by Arogeti et al. is the energy-based mode tracking of hybrid systems for FDD [1].

2.3 Compressors

A compressor is a mechanical device that is used to increase the pressure of a working fluid or gas by means of compression. The inlet and outlet pressure of a compressor can vary from a deep vacuum to a high positive pressure, with the outlet pressure higher than the inlet pressure. Compressors can generally be classified into two distinct categories namely dynamic and positive displacement compressors [4]. The classification of compressors is shown in Figure 3.

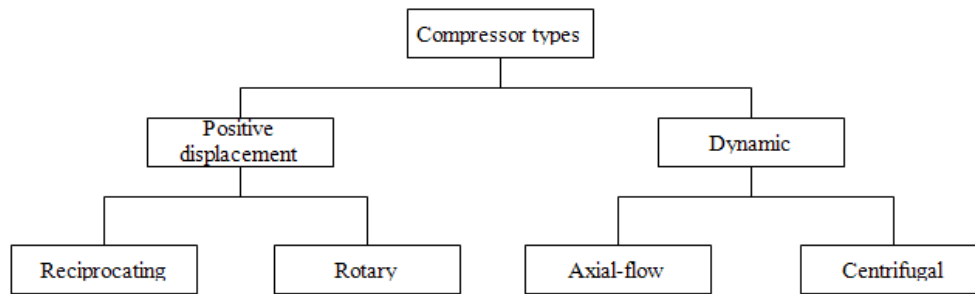


Figure 3: Classified compressor types based on operating principles [4]

Dynamic compressors deliver a continuous flow and is frequently used in the aviation industry, chemical plants and petroleum refineries. A dynamic compressor can be identified by a rotating impeller that adds velocity to a fluid and a stator blade row that decreases the velocity, subsequently increasing the pressure of the fluid [4]. The advantages and disadvantages of selecting a dynamic compressor for an application is listed in Table 1. Two types of dynamic compressors are found in the industry, namely axial-flow and centrifugal compressors.

Table 1: Advantages and disadvantages of dynamic compressors [4]

Type	Advantages	Disadvantages
Axial-flow	• High capacity for given size • High efficiency • Low maintenance	• Low compression ratios per stage
Centrifugal	• Wide operating range • High reliability	• Instability at reduced flow

Positive displacement compressors deliver a fixed volumetric flow of air at a high pressure that is not continuous. All positive displacement compressors operate by confining a certain inlet volume of gas by reducing the confined volume and expelling the fluid or gas through a discharge pipe. Positive displacement compressors are further characterised as either reciprocating or rotary.

Table 2: Advantages and disadvantages of positive displacement compressors [4]

Type	Advantages	Disadvantages
Reciprocating	• Wide pressure ratios • High efficiency	• Heavy foundation required • High maintenance
Rotary	• Wide application • High efficiency	• Expensive initial cost • Unsuitable for corrosive gases

All four types of compressors are used for different applications. Some applications have different requirements that are used to do a trade-off between different compressor types. Requirements such as weight, pressure ration, cost, maintenance and operating capabilities are considered in such a trade-off. The four compressors are presented in Figure 4 based on their capability to achieve a pressure ratio at a certain flow rate.

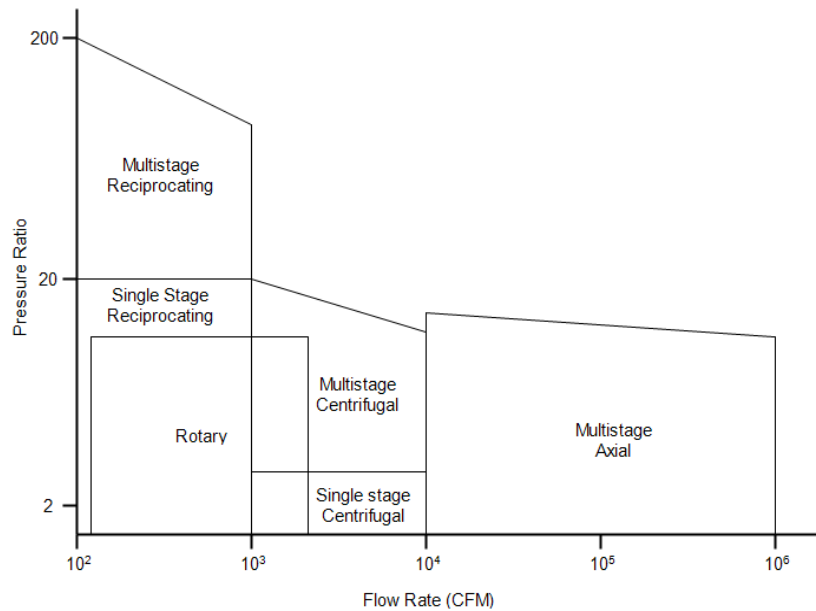


Figure 4: Compressor selection based on compressor capabilities [34]

2.3.1 Axial-flow compressor operation

Axial-flow compressors are mainly used in applications where a high intake volume of flow is required. An axial-flow compressor stage consists of rotating rotor blades that accelerate the working fluid, thus increasing the kinetic energy, and stationary stator blades that diffuse the fluid to increase the pressure. The rotor and stator blades are attached to a rotating drum where usually several rows of decreasing-height blades are used to achieve a desired pressure ratio [24]. One additional row of blades is usually found at the inlet of a compressor's intake to ensure that the fluid enters the first stage of rotor blades at a desired angle.

The operation of a compressor can generally be described by a compressor map or commonly referred to as the compressor characteristic. The compressor characteristic relates the pressure rise across the compressor to the mass flow rate through the compressor to depict its steady state performance.

A compressor map is determined experimentally and converted to non-dimensional form by using the parameters of the compressor and working fluid, such as the rotor speed, flow area and density. The data from the map can either be used in the form of a look-up table or function that is fitted through the data points. Moore and Greitzer propose the use of a cubic asymmetric function that relates the non-dimensional flow coefficient to the non-dimensional pressure rise coefficient [22]. This characteristic is given by

$$\psi_c(\phi) = \psi_{c0} + H \left[1 + \frac{3}{2} \left(\frac{\phi}{W} - 1 \right) - \frac{1}{2} \left(\frac{\phi}{W} - 1 \right)^3 \right] \quad (2.1)$$

with ψ_{c0} , H and W the parameters that define the characteristic. The characteristic is schematically shown in Figure 5.

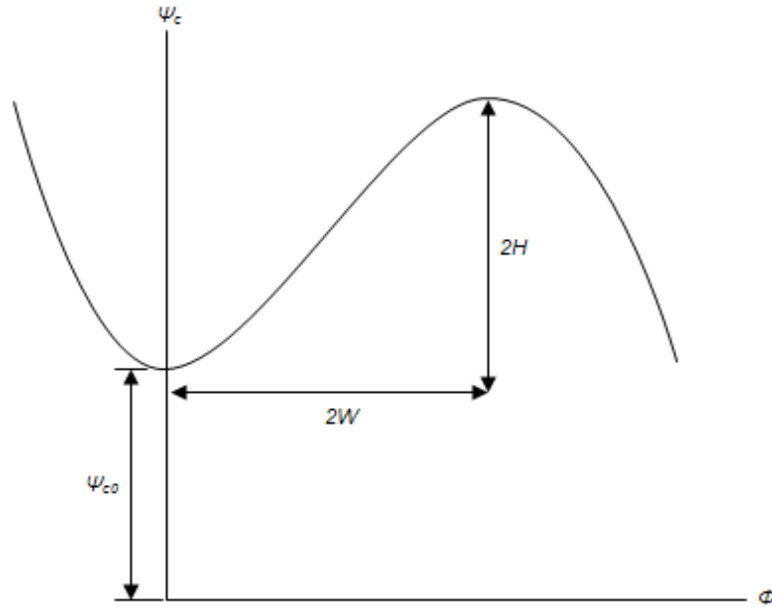


Figure 5: Steady state compressor characteristic [22]

2.3.2 Axial-flow compressor models

Many models are available in literature that can be used to model an axial-flow compressor system. These models vary from one-dimensional lumped parameter models to three dimensional distributed parameter models. The discussion is limited to one- and two-dimensional models, since the field of compressor modelling is so wide and there are too many models to discuss.

The trend of models followed is based on the model that Greitzer developed that can model rotating stall and surge in a compressor system. The models developed from the original model of Greitzer are shown in Table 3.

Table 3: Model development of compressors [35]

Model author	States	Compressor model
Greitzer [7, 8]	Φ, Ψ	Axial-flow
Hansen et al. [36]	Φ, Ψ	Centrifugal
Moore and Greitzer [22], [23]	Φ, Ψ, J	Axial-flow
Fink et.al [37]	Φ, Ψ, B	Centrifugal
Gravdahl and Egeland [38]	Φ, Ψ, B	Axial-flow
Gravdahl and Egeland [39]	Φ, Ψ, J, B	Axial-flow

Three models used for axial-flow compressor systems are discussed in the following section. The first is the one-dimensional lumped parameter model of Greitzer. The second is the first-order nonlinear model derived by Moore and Greitzer. The third model discussed is the model derived by Gravdahl that is based on the model of Moore and Greitzer.

2.3.2.1 Dimensional model derived by Greitzer

Greitzer developed a nonlinear model for an axial-flow compressor system in a two-part series. The first part focuses on the development of the model to predict the transient response of a compressor system due to perturbation from the steady state operation [5]. This study identified a non-dimensional B-parameter that determines if the compressor system is prone to enter rotating stall or surge. The second part of the study does an experimental investigation into rotating stall and surge in a three stage axial-flow compressor system [6]. The experimental results were compared to a theoretical model and showed that the model provided an adequate description of the motion of the system's operation during the induced transients. The model of Greitzer is used in this study and is further discussed in Chapter 3.

2.3.2.2 Non-dimensional model derived by Moore and Greitzer

The model derived by Moore and Greitzer [22] is a third order nonlinear model that consists of a set of three partial differential equations.

The angular dependencies of the partial differential equations are averaged by a Galerkin procedure and the three partial differential equations become first order in time. A schematic diagram of an axial-flow compressor used by Moore and Greitzer is shown in Figure 6.

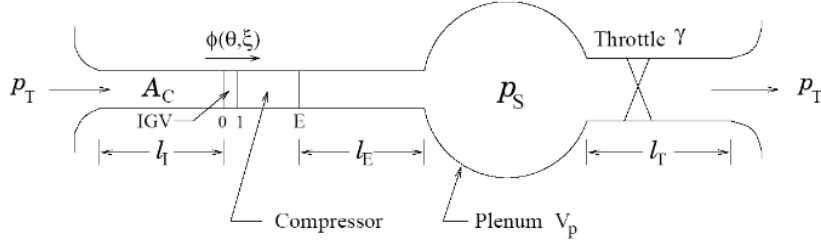


Figure 6: Compressor system diagram used by Moore and Greitzer [22]

All the distances used are non-dimensionalised by the mean compressor radius R . The time is denoted by the relation

$$\xi = \frac{Ut}{R}, \quad (2.2)$$

with U the rotor speed at mean radius and t the time. The pressure differences across the inlet duct, plenum and outlet duct are derived, where the compressor characteristic is included in the equation of the inlet duct and the throttle characteristic is included in the equation of the outlet duct. The overall pressure balance of the compressor system is derived with the transient dynamics included in the equations. The final set of governing equations is given by

$$\frac{d\Psi}{d\xi} = \frac{W/H}{4B^2} \left[\frac{\Phi}{W} - \frac{1}{W} F_T(\Psi) \right] \frac{H}{l_c}, \quad (2.3)$$

$$\frac{d\Phi}{d\xi} = \left[-\frac{\Psi - \psi_{c0}}{H} + 1 + \frac{3}{2} \left(\frac{\Phi}{W} - 1 \right) \left(1 - \frac{J}{2} \right) - \frac{1}{2} \left(\frac{\Phi}{W} - 1 \right)^3 \right] \frac{H}{l_c}, \quad (2.4)$$

$$\frac{dJ}{d\xi} = J \left[1 - \left(\frac{\Phi}{W} - 1 \right)^2 - \frac{1}{4} J \right] \frac{3aH}{(1+ma)W}. \quad (2.5)$$

This set of equations describes the circumferentially averaged flow coefficient Φ , total-to-total static pressure rise Ψ and the squared amplitude of angular variation J . This model is capable

of tracing the evolution of disturbances in an axial-flow compressor system to predict if the system is prone to enter pure rotating stall, pure surge or a combination of both.

2.3.2.3 Non-dimensional model derived by Gravdahl

The model employed by Gravdahl in [24] is based on the model developed by Moore and Greitzer in [22] and [23]. The model is extended by Gravdahl in [35] to include a closed coupled valve (CCV) located between the compressor and the plenum volume to be used for the purposes of control. The model derived is in non-dimensional form and is given and discussed below. A schematic of the compressor system used is shown in Figure 7.

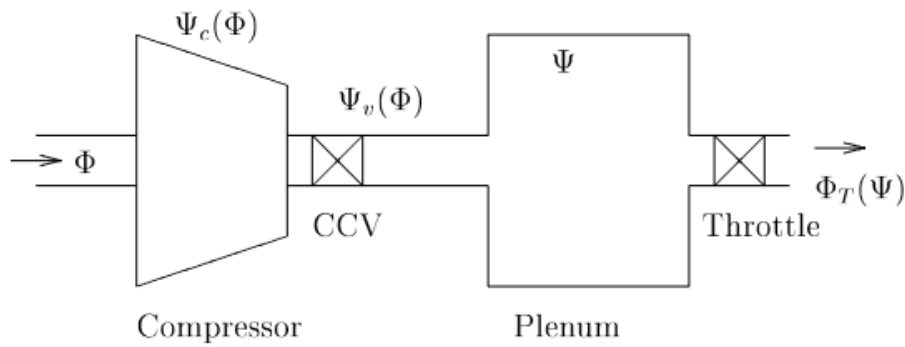


Figure 7: Compressor system including a CCV used by Gravdahl [24]

The three first order differential equations for the system arises from a Galerkin approximation based on the local momentum balance, annulus-averaged momentum balance and the mass balance of the plenum. The model that includes the CCV is given by

$$\dot{\Psi} = \frac{W/H}{4B^2} \left(\frac{\Phi}{W} - \frac{1}{W} \Phi_T(\Psi) \right) \frac{H}{l_c}, \quad (2.6)$$

$$\dot{\Phi} = \frac{H}{l_c} \left(-\frac{\Psi - \psi_{c0}}{H} - \frac{1}{2} \left(\frac{\Phi}{W} - 1 \right)^3 + 1 + \frac{3}{2} \left(\frac{\Phi}{W} - 1 \right) \left(1 - \frac{J}{2} \right) - \frac{1}{\gamma^2} \left(\frac{W^2 J}{2H} + \frac{\Phi}{H} \right) \right), \quad (2.7)$$

$$\dot{J} = J \left(1 - \left(\frac{\Phi}{W} - 1 \right)^2 - \frac{J}{4} - \frac{1}{\gamma^2} \frac{4W\Phi}{3H} \right) \zeta, \quad (2.8)$$

with Ψ the non-dimensional plenum pressure, Φ the annulus averaged mass flow coefficient, J the squared amplitude of the rotating stall.

The notation $\dot{\Phi}$ is defined as $\dot{\Phi} = \frac{d\Phi}{d\xi}$ with the time variable ξ defined by

$$\xi \triangleq \frac{Ut}{R}. \quad (2.9)$$

The non-dimensional throttle mass flow is given by the characteristic

$$\Phi_T(\Psi) = \gamma_T \sqrt{\Psi}, \quad (2.10)$$

with γ_T the throttle gain. The non-dimensional effective flow-passage length l_c of the compressor and ducts is defined by

$$l_c \triangleq l_I + \frac{1}{a} + l_E, \quad (2.11)$$

with l_I the non-dimensional length of the inlet duct, l_E the non-dimensional length of the exit duct and a the reciprocal time lag of the compressor passage. Greitzer's B-parameter is defined by

$$B \triangleq \frac{U}{2a_s} \sqrt{\frac{V_p}{A_c L_c}} \quad (2.12)$$

with U , the compressor tangential speed at mean diameter, a_s the speed of sound, V_p the plenum volume, A_c the flow area and L_c the lengths of the ducts and compressor. A non-dimensional compressor characteristic is assumed from equation (2.1) and is encompassed in the set of equations given in (2.6) to (2.8).

2.3.3 Compressor instabilities

Compressor systems are subjected to two distinct aerodynamic instabilities, namely rotating stall and surge [40]. These instabilities are prone to occur in the compression systems during operation and can lead to catastrophic failures of the system [41]. Extensive studies of models to effectively predict the probability that an instability may occur is researched in [44, 5, 6, 22, 23].

Many control techniques are presented in literature for the avoidance of rotating stall and surge for compressor systems. Greitzers' study in [43] investigated the stabilising effects that a downstream nozzle exerts on a compressor system and found it useful for controller design.

Badmus et al. proposed a control scheme for the simultaneous avoidance of rotating stall and surge across the full range of operation a compressor system in [7]. Wang et al. found in [44] that nonlinear stabilising techniques are necessary for controller design to avoid unstable operation.

A nonlinear robust stabilising feedback law is proposed in [45] to handle the control of compressor instabilities. A 3D numerical study was conducted where the stall inception process is investigated for a compressor with a smooth wall [46]. The second investigation in [46] investigated a compressor with casing treatment configuration. The result of this study revealed that the stable operating range of the compressor was extended by 6%. Greitzer proposed a B-parameter that is calculated by considering the compressor geometry and operating parameters to determine if a compressor is prone to enter rotating stall or surge. Greitzer investigated the transient response of a compressor system for different values of this B-parameter, varying from 0.45 to 5. Results showed that a compressor is prone to enter rotating stall for small values of the B-parameter and that large values of the B-parameter would result in a compressor entering surge. For the investigated compressor system, values up to 0.6 resulted in rotating stall when the system was operated to the left of the maxima and resulted in surge for values larger than 0.7.

2.3.3.1 Rotating stall

Rotating stall is an instability that occurs when there is a disturbance in the circumferential flow pattern. This can occur in either axial or centrifugal compressors. Stall is caused by one or more stall cells of reduced- or stall flow propagation around the compressor's annulus. The result of rotating stall is a reduction in the pressure rise of the compressor [24]. A compressor will enter rotating stall if it is operated left to the maximum of the compressor characteristic with a low B-parameter. The compressor will operate on the in-stall characteristics shown in Figure 8 where the in-stall characteristic is defined by

$$\Psi_s(\phi) = \Psi_{c0} + H \left[1 - \frac{3}{2} \left(\frac{\phi}{W} - 1 \right) + \frac{5}{2} \left(\frac{\phi}{W} - 1 \right)^3 \right]. \quad (2.13)$$

Two types of rotating stall can be distinguished, namely full-span and part-span. Full-span occurs when the complete height of the annulus is stalled and is most likely to occur in compressor systems with a large hub to tip ratio.

Part-span occurs when only a region of the blade passage is stalled. When the blades move in and out of the stalled flow regions, vibrations are induced in the blades [47]. If the frequency of

the vibrations matches the frequency with which the blades pass through the stall regions, the vibrations are resonated and can cause a failure in the compressor.

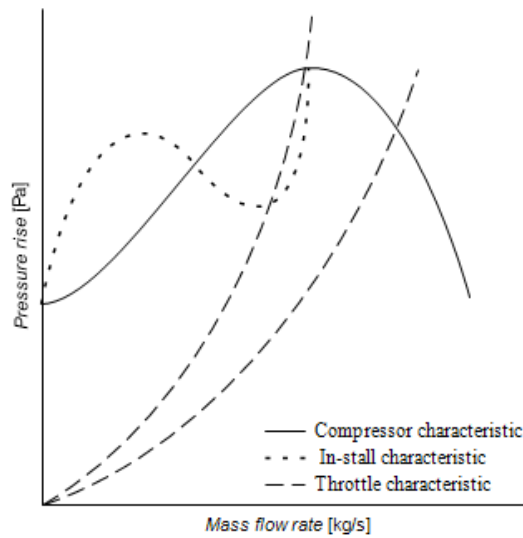


Figure 8: Compressor response during rotating stall [24]

2.3.3.2 Surge

Surge is a type of aerodynamic flow instability that can occur in compression systems such as gas turbines. Surge is an asymmetrical oscillation of the flow through and pressure rise of the compressor and is identified by a limit cycle on the compressor characteristic [24]. Surge occurs when the compressor system is operated to the left of the maximum on the compressor characteristic with a large value of the B-parameter. Figure 9 portrays a limit cycle that the compressor enters during surge.

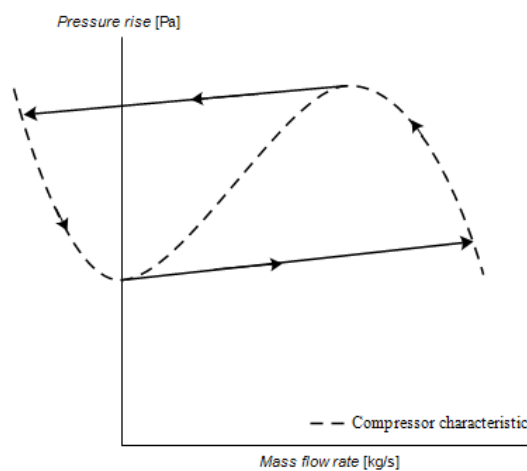


Figure 9: Compressor response during surge [48]

Surge can cause vibrations in other components in the system such as the ducts, due to the reversal of the mass flow through the system. Two types of surge are distinguished, namely classical and deep surge. A combination of rotating stall and surge is known as modified surge.

2.3.4 Faults and fault modelling of compressors

A fault model is a mathematical representation of a condition or a fault and how the fault influences the system. The term fault means that there is a deviation of the normal response of a component and it does not necessary mean that the component will cease to operate. When a component fails to operate it is called a failure. The goal is to detect a fault as soon as possible, before the component fails. A better fault model implies that less significant faults can be detected to prevent damage to a component in a system [49].

Faults can occur in any system, including compressor systems. Faults may result in the complete shutdown of production and can lead to significant economic loss [31]. Breese et al. [50] mentions examples of faults that can occur in compressor systems, namely erosion, fouling, tip clearance faults, leaks, bent blades, blade coats and blades missing. The method to detect and analyse faults in industrial systems are called Fault Detection and Isolation/Diagnosis.

2.4 Fault detection and diagnosis

Numerous FDD methodologies have been proposed over the last decade. Each methodology uniquely satisfies a certain requirement for the implementation of a FDD system.

A short classification of the methodologies follows that include, hardware redundancy, signal processing, and model-based fault detection.

2.4.1 Hardware redundancy FDD

The essential idea of this methodology is to use redundant hardware to reconstruct a system. If a fault occurs in the system, the outputs of the system will differ compared to the redundant system. This is an accurate and reliable technique to detect faults in a system. The drawback of this methodology is that it is limited to a number of system components and the use of redundant hardware results in a high cost of the application [51]. This methodology is mostly applied in systems where high safety and reliability are required.

2.4.2 Signal processing FDD

The fundamental concept of this methodology is to extract information from system signals to identify faults in the system. Process signals can carry information of the symptoms of faults. These indicators can either be time domain or frequency domain functions [51]. This approach is limited for the steady state of a system due to the fact that the dynamics of the system are not taken into account in the signal-process-based fault diagnosis [52].

2.4.3 Model-based FDD

The idea of a model-based fault detection scheme is to replace hardware redundancy with a mathematical model of the system. The process can then be reconstructed by the model and the response of the actual process can be compared to the mathematical model's response [1]. The system's model operates parallel to the actual system and the difference in the measured process variables is used to detect deficiencies in the actual system. The difference between the actual measured variables and that of the system model is called residuals.

Residuals are usually time-varying signals that are used as fault detectors. They are designed to be zero if an exact model is available and used for the system, or in the case of a realistic application it would be relatively small. In case of a fault, a residual deviates significantly from zero. The process of creating the residual signal is called residual generation. The post-processing evaluation of residuals that extract the information of the possible fault occurred is called residual evaluation. Three types of residuals are defined, namely fixed direction residuals, structured residuals and structured hypothesis tests [49]. Structured residuals are designed as a residual vector where the vector responds only in one direction depending on the fault that occurred [53].

These types of residuals are not extensively used in literature due to the design of a residual vector with desired properties. Structured residuals are implemented so that each residual is sensitive to a subset of faults [54]. These residuals are widely used in theoretical and practical systems for linear and nonlinear systems. A structured hypothesis test is proposed where the isolation method is formally defined and any fault modes can be used [55].

Model-based fault detection was done by Albas et al. in [56], where the investigation focused on detecting air gap faults. Specific research of compressor systems focused on the fault detection and diagnosis of an axial-flow compressor for a combined-cycle power plant as was done in [57]. The following section discusses different residual generation approaches.

2.4.3.1 Observer-based residual generation

In observer-based approaches, the estimation of the process output is done by an observer. The observer monitors the measured states of the system [1]. A basic observer-based residual topology is shown in Figure 10. The accuracy of the detection of faults is determined by the complexity of the model.

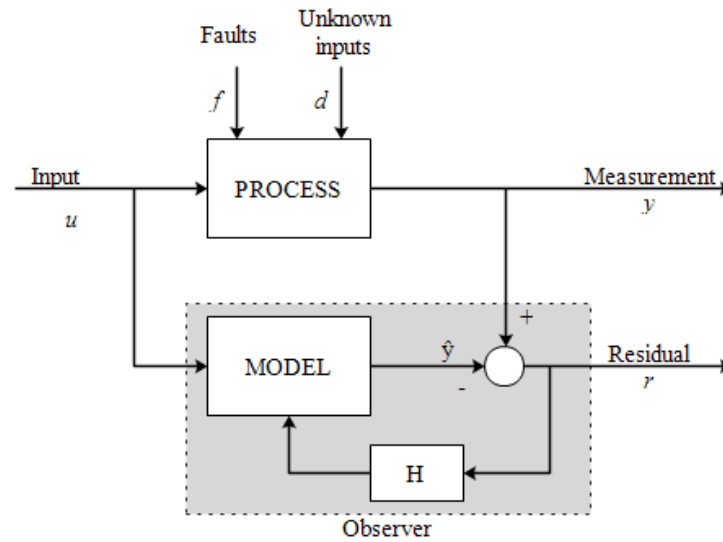


Figure 10: Basic configuration of an observer-based residual generator [58]

Two requirements are specified for an observer-based system. The first requirement is that the system model is robust for model uncertainties, disturbances and noise in the system. The second requirement is that the model is sensitive to faults.

The drawback of this type of approach is that there is a trade-off between the robustness and the sensitivity, thus increasing the sensitivity to faults may decrease the robustness due to disturbances.

2.4.3.2 Parity space-based residual generation

Parity space approach is structurally equivalent to that of an observer-based approach. Only the design procedure is different when compared to that of an observer-based approach. In the parity space approach, a parity relation is derived from the system model. The approach constantly checks the consistency of the parity relation based on a parity vector [51].

2.4.3.3 Parameter identification method

The parameter identification method operates based on the method of an on-line parameter estimation. The fault detection process is executed by comparing the estimated parameters with the nominal process parameters. This is useful due to the fact that the comparison holds the size of the deviation between the estimated and nominal parameters. The disadvantage is that a large excitation is needed to estimate the parameter correctly and this is not always possible in the system [1].

2.4.3.4 Energy-based fault detection

The application of energy-based fault detection is based on the same principles as conventional fault detection and diagnosis techniques. An energy-balance based approach is mostly used where an input-output model is impossible to obtain. The concept of an energy balance-based approach is applied as follows:

If a system is operated at a steady state operating point, the energy conservation principles can be used to establish an energy-balance of the system, which is then considered as an energy model. The main concept of an energy-balance based approach is based on the concept of a passive system where the energy stored cannot exceed the energy supplied to the system [51]. This is mathematically given by

$$V(x(\tau)) - V(x(0)) \leq \int_0^\tau y^T u dt, \quad (2.14)$$

where the functions V and $V(0)$ are non-negative functions that represent the store of energy in the system, $y^T u$ the supply rate that represents the external supply of energy to the system and x the state variables of the system. Equation (2.14) always holds in a fault-free case. In the case where a fault has occurred in a system the equation will be non-zero [51]. This can then be used to detect faults in a system by monitoring (2.14).

2.5 Energy-based modelling

The total energy of a component can be calculated by the sum of the energy of each domain. These domains include the hydraulic, thermodynamic, mechanic and electrical domain. The energy in each domain is calculated by using the equivalent elements of the domain.

The four equivalent elements are the flow f , effort e , generalised momentum p and generalised displacement q . The variables e and p are related by $\dot{p} = e$ and the variables q and f are related by $\dot{q} = f$ [8]. The four equivalent elements are defined for each domain in Table 4.

Table 4: Equivalent elements for different domains [8]

	Flow	Effort	Generalised momentum	Generalised Displacement
Electric	Current I	Voltage V	Flux linkage ϕ	Charge q
Fluid	Volumetric flow rate Q	Pressure p	Pressure momentum Γ	Volume V
Mechanic	Velocity v	Force F	Momentum p	Displacement x
Thermodynamic	Entropy flow f_s	Temperature T	-	Entropy S

Every domain has a pair of power variables that correspond to an effort and flow variable. This is given by

$$P(t) = e(t) f(t), \quad (2.15)$$

with P , the power, e an effort variable and f a flow variable. The energy is calculated by the time integral of the power as follows:

$$E(t) = \int_{t_0}^t e(\tau) f(\tau) d\tau + E(t_0), \quad (2.16)$$

with $E(t)$, the energy and $E(t_0)$ the initial energy at $t = t_0$.

2.5.1 Component diagram

From the concept of a thermodynamic system, a system boundary condition is defined for a selected analysis in Figure 11. Three types of forces can act on the control volume to transfer energy to and from the control volume.

Energy can be transferred to and from the control volume through shaft work $\dot{W}_{sh}$. Usually a compressor uses shaft work to transfer energy to a control volume, whereas a turbine extracts energy from the control volume. For an open system, energy can be transferred into or out of a control volume through mass transfer. This is indicated by the mass flow rate $\dot{m}$, in or out of the system. The third transfer method is through heat transfer $\dot{Q}$.

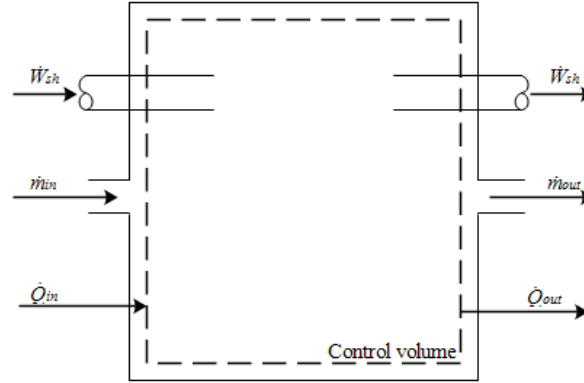


Figure 11: Forces acting on an open thermodynamic system adapted from [59]

2.5.2 Power equations

The power in and out of the control volume is derived by applying the conservation of mass law to the control volume [59]. This law is defined by

$$\sum_{in} \dot{m}(t) - \sum_{out} \dot{m}(t) = \frac{dM(t)}{dt}, \quad (2.17)$$

with $\dot{m}$, the mass flow rate and M the mass in the control volume. In steady state operation the change in mass is given by $\frac{dM}{dt} = 0$. Equation (2.15) is used to calculate the power. This calculation does not include the internal energy, kinetic energy and the potential energy of the fluid. The power through mass transfer is calculated from a flow and effort variable and is defined by

$$P(t) = Q(t)p(t), \quad (2.18)$$

with Q , the volumetric flow rate and p the pressure.

The volumetric flow rate is defined by

$$Q(t) = \frac{\dot{m}(t)}{\rho(t)}, \quad (2.19)$$

with ρ , the density of the fluid. When applying the first law of thermodynamics to the system, the power is defined by

$$P_{in}(t) - P_{out}(t) + \dot{W}_{net}(t) + \dot{Q}_{net}(t) = \frac{dE_{CV}(t)}{dt}, \quad (2.20)$$

with P_{in} the power flowing into the control volume, P_{out} the flowing out of the control volume, $\dot{W}_{net}$ the net shaft work, $\dot{Q}_{net}$ the net heat transfer. If shaft work is done on the fluid in the control volume, $\dot{W}_{net}$ is positive and if work is extracted from the fluid in the control volume, $\dot{W}_{net}$ is negative. The same applies to $\dot{Q}_{net}$ where a positive value represents a heat flow into the control volume, and negative value represents a heat flow out of the control volume. The left side of (2.20) represents the net power of the control volume and the net power is defined by

$$P_{net}(t) = P_{in}(t) - P_{out}(t). \quad (2.21)$$

In steady state operation, $P_{net} = 0$, $\dot{W}_{net} = 0$, $\dot{Q}_{net} = 0$. This means that a change in stored energy can only occur when an imbalance occurs in one or more of the net powers P_{net} , $\dot{W}_{net}$ or $\dot{Q}_{net}$.

2.5.3 Energy equations

The energy stored in a control volume is calculated by taking the time integral of the net power in equation (2.21). Energy can only be stored when an imbalance in the net power is established, thus a change in the stored energy ΔE_{CV} can only occur when $P_{net} \neq 0$, $\dot{W}_{net} \neq 0$ or $\dot{Q}_{net} \neq 0$. The energy stored in a control volume is given by

$$E_{CV} = \int_{t_0}^t (P_{net} + \dot{W}_{net} + \dot{Q}_{net}) dt + E_{CV}(t_0), \quad (2.22)$$

with $E_{CV}(t_0)$, the initial energy stored in the control volume at $t = t_0$.

2.5.4 Energy balance equations

From the first law of thermodynamics, the energy balance can be written for a control volume in terms of the state variables [59]. The energy balance of the control volume is given by

$$\frac{dE_{CV}}{dt} = \dot{Q}_{net} - \dot{W}_{net} + \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + gz_{in} \right) - \dot{m}_{out} \left(h_{out} + \frac{V_{out}^2}{2} + gz_{out} \right), \quad (2.23)$$

with h , the enthalpy, $\frac{V^2}{2}$ the kinetic energy and gz the potential energy. The enthalpy is given by

$$h = u + \frac{p}{\rho}, \quad (2.24)$$

with u , the internal energy, p the pressure and ρ the density. The internal energy of the working fluid is defined by

$$u = c_v(T - T_0), \quad (2.25)$$

with C_v , the specific heat at constant volume, T the temperature and T_0 the reference temperature.

2.6 Performance indices

The performance index is calculated from measurements to evaluate a system's performance. A quantitative measure is necessary to either improve the design of a system or to design a control system. A performance index is a number that is always positive or zero, and the best system defined that minimises this index is the optimal system. Four performance indices can be defined [60]. The integral of the square of the error is defined by

$$ISE = \int_0^T e^2(t) dt. \quad (2.26)$$

The upper limit of the integral T is a finite time chosen so the integral reaches a steady state value or chosen as the settling time T_s . The squared error is mathematically convenient for the analytic and computational purposes.

The integral of the absolute magnitude of the error is defined by

$$IAE = \int_0^T |e(t)| dt . \quad (2.27)$$

This index is useful for computer simulation studies. To reduce the contribution of the large initial error, the ITAE is derived from the IAE. The integral of time multiplied by the absolute error is defined by

$$ITAE = \int_0^T t |e(t)| dt . \quad (2.28)$$

The ITAE provides the best selectivity of the performance indices. The integral of time multiplied by the square error is given by

$$ITSE = \int_0^T t e^2(t) dt . \quad (2.29)$$

This index is also regarded as a good measurement as the integral is also selective to where the penalty is done on the response.

2.7 Critical literature review

In the first part of the chapter, literature on compressors, how compressors operate and compressor models are discussed. Literature indicates that an axial-flow compressor system is the best option for this study. This decision was based on the nonlinear dynamic complexity of a system that can be used to portray the energy in the system. The decision between an axial-flow and centrifugal compressor was based merely on the continuous flow ability of the axial flow compressor, the availability of literature of the compressor type and the data available from an experimental setup.

Based on this literature a decision was made on what type of model to use and how complex the model had to be. The focus of determining the modelling approach is based on how the energy analysis should be done on the system. Although a CFD model could be constructed for the compressor system, it should be pointed out that the computational power required to control such a model is not implementable in this study. Other higher order models give a more accurate description of the system. However, this is redundant for the use of an energy analysis of a system.

The selection of the model used from literature required that the model can be extended to model a fault condition such as a leak in the system. Another requirement for the selection of the model is that it has to be written in the form of a state space model. Using these requirements, a one-dimensional nonlinear lumped parameter model seems to be the best option to use based on literature such as that derived by Greitzer in [5]. The model derived by Moore and Greitzer in [22] is considered a sufficient model in terms of complexity, but it poses difficulty with extending the model to include leak dynamics in the system.

Another model that also satisfies the requirements is the model from Gravdahl in [24], although this model is derived from the model of Moore and Greitzer and two disadvantages disqualify it for use in the study. The first is that it incorporates additional components such as a CCV that is redundant for the analysis of this study. The second is that this model focuses on the avoidance of instabilities, rather than to model a complete compressor system. The limitation of most control schemes for the avoidance of compressor instabilities is that the nonlinear compressor system model is linearised at a specific operating point, as was done by Badmus et al. in [7]. This would decrease the complexity and accuracy of the analytic model of the compressor and it was decided that a nonlinear approach should be followed.

To determine the correlation between the responses of the analytic model to the responses of the model in Flownex[®] in the validation of the study, it was decided that a performance index should be used. This is advantageous due to the fact that the responses are analysed in both the steady and transient responses. By using a performance index the penalty can be increased or decreased based on whether the emphasis is on the steady or transient response. The IAE performance index was chosen for use, since it is commonly used in simulation studies and can easily be calculated for signals.

Considerable literature is available to calculate the energy from the state variables of a system. It is convenient to calculate the power and energy of the system from the first law of thermodynamics. The power and energy can be calculated for individual components as discussed in the literature by employing a first order differential equation for the power and using the time integral of the power to calculate the energy. The alternative method to follow is to calculate the power and energy from Bernoulli's principles. The limitation of this is that Bernoulli's law is stated in two parts. One for the steady state energy calculation and Bernoulli's extended equations for the transient energy calculation. It is more convenient to employ one differential equation to a component in the system than two additional calculations for the steady and transient response.

The field of FDD is widely defined and numerous techniques are available. Hardware redundancy is eliminated as a possible FDD method, since it was decided that a model-based FDD approach would be sufficient to calculate residuals for the system. A signal processing-based approach would require an in-depth study of signal processing techniques to identify faults in a system. This result was not formally applied in literature to the detection of faults from an energy perspective, so it is uncertain if it would be sufficient in this study. The FDD system that was selected was a combination between a qualitative model-based approach and an energy-balance-based approach.

3 CHAPTER

STATE SPACE MODEL

Chapter 3 contains the derivation of a non-linear state space model for an axial-flow compressor system. The dynamics of a leak in the system is derived and included in the model. The first order differential equations and the auxiliary equations, presented by Greitzer, are given, as well as the derivation of the leak dynamics. The state space model is derived from these equations. The state space model of the compressor system is simulated in Simulink®.

3.1 Introduction

Many compressor models can be found in literature. Most of these models are derived from the three basic laws of thermodynamics. The differentiating factor among these models is the assumptions made prior to deriving the model. More assumptions result in a less complex model. This study is focused on an axial-flow compressor system. The basic model derived in 1967 by Greitzer is used as a starting point. The conceptual system is given together with the assumptions that were made prior to the derivation. Additional to the model of Greitzer, dynamic equations were added to model a leak in the plenum volume. The complete set of equations is then written in state space format.

3.2 Modelling approach

First, a conceptual diagram of an axial flow compressor system is presented in Figure 12. The assumptions made are discussed prior to the derivation process. The derivation of the model is then discussed for the compressor, plenum and throttle components. The steady state characteristic of the compressor is firstly discussed. In this discussion the characteristic is given in dimensional and non-dimensional form.

The governing equations are derived from the mass and momentum balances for the inlet duct. These governing equations incorporate the steady state characteristic of the compressor component. The governing equation of the plenum is derived from the conservation of mass. This governing equation excludes the leak dynamics of the system. The steady state characteristic of the throttle is given and discussed in the subsequent section. The governing equation of the outlet duct is then derived based on the conservation of mass. This governing equation of the outlet duct includes the steady state throttle characteristic.

The characteristic of the leak is given and the governing equation of the plenum is re-validated to include the leak effects in the equation. The state space representation is given in the final part of the chapter.

3.2.1 Conceptual system

The diagram portraying the compressor system used in the derivation of the analytic model is shown in Figure 12. The system consists of an inlet duct, connected to the inlet of a plenum volume, and an outlet duct, connected to the outlet of the plenum volume. The compressor component is located in the inlet duct and the throttle component is located in the outlet duct.

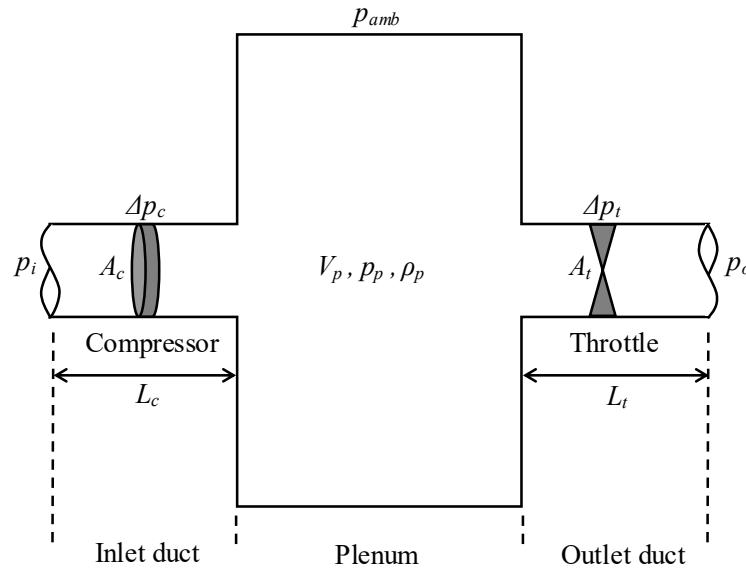


Figure 12: Axial-flow compressor system diagram

3.2.2 Modelling assumptions

The first assumption concerning the system is that the flow through the ducts is regarded as one-dimensional and incompressible. Due to this assumption, the following is expected concerning the ducts in the system:

- Equal axial velocity of the fluid in the ducts
- Flow in the ducts are regarded as incompressible with a density being equal to ambient density
- Kinetic energy is associated with the motion of the fluid in the inlet and outlet ducts
- The compressor is modelled as an infinitely thin actuator disk and the fluid in the compressor is assumed incompressible

An assumption with regard to the plenum volume is that the fluid velocity in plenum is negligible and that the gas in the plenum is assumed compressible. This implies the following:

- The static pressure is uniform throughout the plenum
- Potential energy is associated with the compression of the gas in the plenum

Temperature effects in the compressor system are disregarded, since it is assumed that the temperature throughout the system stays constant.

3.3 Modelling equations

The modelling equations are derived for the compressor system in the next part of the chapter. These modelling equations include the first order differential equations that are derived from the conservation of mass and momentum for the inlet duct, plenum and throttle duct. The leak dynamics are derived in the subsequent part and are incorporated into the set of first order differential equations in the section thereafter.

3.3.1 Inlet duct modelling equations

The non-dimensional compressor characteristic that depicts the steady state operation of the compressor is given. The dimensional characteristic is then derived by using two relations for the mass flow rate and the pressure rise. The governing equations for the inlet duct are then derived from the conservation laws of mass and momentum.

3.3.1.1 Non-dimensional steady state compressor characteristic

Moore and Greitzer propose a cubic characteristic equation in [23] that is a good estimate for the steady state pressure rise across the compressor. This equation is presented in non-dimensional form and is given by

$$\varphi_c(\phi) = \varphi_{c0} + H \left[1 + \frac{3}{2} \left(\frac{\phi}{W} - 1 \right) - \frac{1}{2} \left(\frac{\phi}{W} - 1 \right)^3 \right], \quad (3.1)$$

with φ_c the axisymmetric pressure rise coefficient, φ_{c0} the shut-off value of the axisymmetric characteristic, ϕ the local axial flow coefficient, H the semi-height of the cubic asymmetric characteristic and W the semi-width of the cubic asymmetric characteristic.

3.3.1.2 Dimensional steady state compressor characteristic

Equation (3.1) is converted from non-dimensional to dimensional form by means of the following two relations in (3.2) and (3.3). The relation between the non-dimensional axial-flow coefficient and the mass flow rate is described by

$$\phi = \frac{\dot{m}_c}{\rho U A_c}, \quad (3.2)$$

with $\dot{m}_c$, the mass flow rate through the inlet duct, ρ the density, U the rotor speed at mean radius and A_c the flow area of the inlet duct. The second relation is between the non-dimensional pressure rise coefficient and the steady state pressure rise across the compressor, given by

$$\varphi = \frac{\Delta p}{\rho U^2}, \quad (3.3)$$

with Δp the pressure difference across the compressor. U is calculated by the rotor speed N and the rotor mean radius R as

$$U = \frac{2\pi R N}{60}. \quad (3.4)$$

The dimensional steady state compressor characteristic can be derived from (3.1) by using the two relations in (3.2) and (3.3). The dimensional steady state compressor characteristic is given by

$$\Delta p_{c,ss}(\dot{m}_c, N) = \frac{C_0 \rho (NR\pi)^2}{900} - H \left(\frac{15 \dot{m}_c^3(t)}{\pi R N \rho^2 (A_c W)^3} - \frac{3 \dot{m}_c^2(t)}{2 \rho (A_c W)^2} \right). \quad (3.5)$$

3.3.1.3 Governing equations of the inlet duct

The differential equation for the conservation of momentum for the inlet duct is given by

$$\frac{d\dot{m}_c(t)}{dt} = \frac{A_c}{L_c} (\Delta p_c(t) - \Delta p_{in}(t)), \quad (3.6)$$

with L_c , the length of the inlet duct and Δp_c the pressure rise across the compressor.

The pressure difference across the inlet duct Δp_{in} is defined by

$$\Delta p_{in}(t) = p_p(t) - p_i, \quad (3.7)$$

with p_p , the plenum pressure and p_i the inlet pressure of the compressor system.

The compressor system is assumed to not operate as a quasi-steady system. The result is that the transient response of the compressor deviates from the steady state characteristic. Due to this, a differential equation is presented to model the lag effect of the compressor and is given by

$$\frac{d\Delta p_c(t)}{dt} = \frac{1}{\tau} (\Delta p_{c,ss}(\dot{m}_c, N) - \Delta p_c(t)), \quad (3.8)$$

with $\Delta p_{c,ss}$, the steady state compressor pressure rise, specified in (3.5). The compressor flow field time constant τ is defined by

$$\tau = \frac{2\pi R N_{stall}}{U}, \quad (3.9)$$

with N_{stall} , the number of rotor revolutions required to develop a stall cell.

The compressor is modelled as an actuator disk, therefore energy is added to the system by the compressor in the form of a pressure rise of the working fluid. A compressors steady state pressure rise is described by a compressor map relating the pressure rise across the compressor to the mass flow rate through the compressor.

3.3.2 Governing equations of the plenum

The differential equation derived from the conservation of mass for the plenum is given in (3.10). This equation represents the change in density of the plenum with respect to time.

$$\frac{d\rho_p(t)}{dt} = \frac{1}{V_p} (\dot{m}_c(t) - \dot{m}_t(t)). \quad (3.10)$$

In equation (3.10), ρ_p is the density of the working fluid in the plenum, V_p the plenum volume and $\dot{m}_t$ the mass flow rate through the outlet duct. The process in the plenum is polytropic.

This allows the change in density to be related to the change in pressure by

$$\frac{d\rho_p(t)}{dt} = \frac{\rho_p(t)}{p_p(t)k} \frac{dp_p(t)}{dt}, \quad (3.11)$$

where k is the polytropic exponent. k is assumed to be equal to the value of the specific heat ratio γ [6]. The differential equation for the plenum can be derived by substituting (3.11) into (3.10) and is given by

$$\frac{dp_p(t)}{dt} = \frac{p_p(t)\gamma}{\rho_p(t)V_p} (\dot{m}_c(t) - \dot{m}_t(t)). \quad (3.12)$$

3.3.3 Outlet duct modelling equations

The throttle in the compressor system is modelled as a nonlinear resistance with a quadratic characteristic. The function that describes the pressure drop across the throttle with relation to the mass flow rate through the throttle is given by

$$\Delta p_t(\dot{m}_t) = \frac{\dot{m}_t^2(t)}{2u_t\rho A_t^2}, \quad (3.13)$$

with u_t , the throttle gain. The value of u_t is proportional to the throttle opening, where a value of 0 relates to a closed throttle and a value of 1 relates to an open throttle.

3.3.3.1 Governing equations of the outlet duct

The conservation of momentum equation for the outlet duct is given by

$$\frac{d\dot{m}_t(t)}{dt} = \frac{A_t}{L_t} (\Delta p_{out}(t) - \Delta p_t(\dot{m}_t(t))), \quad (3.14)$$

with A_t , the flow area of the outlet duct and L_t the length of the outlet duct. The pressure drop across the throttle Δp_t is specified in (3.13). The pressure drop across the outlet duct Δp_{out} is defined by

$$\Delta p_{out}(t) = p_p(t) - p_{out}, \quad (3.15)$$

with p_{out} , the pressure downstream of the compressor system.

3.4 Dynamic modelling of a leak

One fault that can occur in this system is a leak in the plenum. The effects of the leak are related to the pressure in the volume and should therefore be dynamically implemented. An assumption is that the brim formed by the leak is infinitely thin.

3.4.1 Derivation of the equations for a leak in the plenum

The characteristic of the leak can then be given by a quadratic equation that relates the pressure drop across the leak to the mass flow rate through the leak by

$$\Delta p_h(\dot{m}_h) = \frac{\dot{m}_h^2(t)}{2\rho A_h^2}, \quad (3.16)$$

with Δp_h , the pressure drop across the leak, $\dot{m}_h$ the mass flow rate through the leak and A_h the effective leak area. The dynamics of the leak can be described by the conservation of mass equation for the leak and is given by

$$\frac{d\dot{m}_h(t)}{dt} = \frac{A_h}{L_h} (\Delta p_{amb}(t) - \Delta p_h(\dot{m}_h(t))), \quad (3.17)$$

with L_h , the edge thickness of leak. The pressure difference Δp_{amb} is defined by

$$\Delta p_{amb}(t) = p_p(t) - p_{amb}, \quad (3.18)$$

with p_{amb} , the ambient pressure. By substituting (3.16) into (3.17), the conservation of mass equation can be written as

$$\frac{L_h}{A_h} \frac{d\dot{m}_h(t)}{dt} = p_p(t) - p_{amb} - \frac{\dot{m}_h^2(t)}{2\rho A_h^2}. \quad (3.19)$$

The edge thickness is chosen to be 0, due to the assumption that $L_h \rightarrow 0$. Equation (3.19) is then given by

$$p_p(t) - p_{amb} - \frac{\dot{m}_h^2(t)}{2\rho A_h^2} \approx 0. \quad (3.20)$$

Equation (3.20) can be rewritten in terms of the mass flow rate through the leak and be given by

$$\dot{m}_h(p_p(t)) = \sqrt{2\rho A_h^2(p_p(t) - p_{amb})}. \quad (3.21)$$

Since the leak is implemented in the plenum of the system and affects the conservation of mass, equations (3.11) and (3.12) can be rewritten with the leak effects included and given by

$$\frac{d\rho_p(t)}{dt} = \frac{1}{V_p}(\dot{m}_c(t) - \dot{m}_t(t) - \dot{m}_h(p_p)), \quad (3.22)$$

for the change in density and

$$\frac{dp_p(t)}{dt} = \frac{p_p(t)\gamma}{\rho_p(t)V_p}(\dot{m}_c(t) - \dot{m}_t(t) - \dot{m}_h(p_p)), \quad (3.23)$$

for the change in the plenum pressure. Substituting the equation depicting the mass flow rate through the leak (3.21) into (3.22) and (3.23), the density of the plenum is given by rewriting the conservation of mass equations as

$$\frac{d\rho_p(t)}{dt} = \frac{1}{V_p}(\dot{m}_c(t) - \dot{m}_t(t) - \sqrt{2\rho A_h^2(p_p(t) - p_{amb})}), \quad (3.24)$$

and the pressure in the plenum is given by

$$\frac{dp_p(t)}{dt} = \frac{p_p(t)\gamma}{\rho_p(t)V_p}(\dot{m}_c(t) - \dot{m}_t(t) - \sqrt{2\rho A_h^2(p_p(t) - p_{amb})}). \quad (3.25)$$

3.5 State space model

The equations derived above can now be written in the following state space form [60]:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}\mathbf{u}. \quad (3.26)$$

Notice that the matrix $\mathbf{A}$ is dependent on the states that indicate the model is nonlinear. Let n indicate the number of states and m the number of inputs. Then $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$, $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times m}$.

The output equation of the state space model is defined by

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}. \quad (3.27)$$

Let $\mathbf{y} \in \mathbb{R}^p$ with p the number of outputs. Then $\mathbf{C} \in \mathbb{R}^{p \times n}$ and $\mathbf{D} \in \mathbb{R}^{p \times m}$.

The set of first order differential equations of the compressor system consisting of (3.12), (3.14), (3.24), (3.25) and (3.8) is used to construct the nonlinear state space model of the system. The state variables of the system is chosen as

$$\mathbf{x}(t) = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \dot{m}_c(t) \\ \dot{m}_t(t) \\ \rho_p(t) \\ p_p(t) \\ \Delta p_c(t) \end{bmatrix}, \quad (3.28)$$

with $\mathbf{x} \in \mathbb{R}^5$ and the input vector $\mathbf{u} \in \mathbb{R}^3$ as

$$\mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = \begin{bmatrix} p_{in}(t) \\ p_{out}(t) \\ N(t) \end{bmatrix}. \quad (3.29)$$

The $\mathbf{A}$ and $\mathbf{B}$ matrices are constructed using the set of differential equations and the state vector in (3.28). Some of the state variables are present in the $\mathbf{A}$ matrix, resulting in a nonlinear model.

The $\mathbf{A}$ matrix is then given by

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & -\frac{A_c}{L_c} & \frac{A_c}{L_c} \\ 0 & \frac{x_2(t)}{2u_t \rho A_t L_t} & 0 & \frac{A_t}{L_t} & 0 \\ \frac{1}{V_p} & -\frac{1}{V_p} & 0 & \frac{1}{V_p \sqrt{x_4(t)}} \sqrt{2\rho A_h^2 \left(1 - \frac{p_{amb}}{x_4(t)}\right)} & 0 \\ \frac{\gamma x_4(t)}{x_3(t) V_p} & -\frac{\gamma x_4(t)}{x_3(t) V_p} & 0 & -\frac{\gamma}{x_3(t) V_p} \sqrt{2\rho A_h^2 (x_4(t) - p_{amb})} & 0 \\ -\frac{H}{\tau} \left(\frac{15x_1^2(t)}{\pi R N \rho^2 (A_c W)^3} - \frac{3x_1(t)}{2\rho (A_c W)^2} \right) & 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix}. \quad (3.30)$$

and the input matrix **B** is given by

$$\mathbf{B} = \begin{bmatrix} \frac{A_c}{L_c} & 0 & 0 \\ 0 & -\frac{A_t}{L_t} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{C_0 \rho N (\pi R)^2}{900 \tau} \end{bmatrix}. \quad (3.31)$$

3.6 Conclusion

A nonlinear state space model of an axial-flow compressor system that is capable of simulating rotating stall, surge and a leak in the plenum is derived. It was found that five state variables is necessary to model this system. The state space format is useful for the purpose of FDD and control. Five state variables are selected in the state space model that shall be verified and validated in the next chapter. The energy analysis in Chapter 5 shall be based on the response of the five state variables of the compressor system.

4 CHAPTER

SIMULATION RESULTS

The simulation results of the model discussed in the previous chapter are presented. The initial conditions and parameters of the system are given. The steady state and transient response of the system are shown and validated. The steady state compressor characteristic is validated with practical results from a compressor system. The transient response is validated with Flownex[®], a computational fluid dynamic software package.

4.1 Introduction

The nonlinear lumped parameter model of the compressor system that was derived in Chapter 3 is simulated. The analytic model is implemented in Simulink[®], where a variable step solver is used to solve the set of first order differential equations. The simulation parameters for the system and the fluid properties are given in Table 5. A system CFD model is implemented in Flownex[®]. This model is used in the latter part of the chapter to validate the response of the analytic model.

The verification and the validation of the analytic model is given and discussed for the steady state and transients of the compressor system.

4.2 Analytical model implementation in Simulink[®]

The analytic model is implemented in Simulink[®], a graphical programming environment that is capable of simulating, modelling and analysing multi-domain systems. The implementation of the analytic model in Simulink[®] can be seen in Appendix A.1. The system parameters and the fluid properties used during the simulation are given in Table 5. These parameters were obtained from the compressor system used by Gravdahl [24].

Table 5: Parameters of the compressor system investigated by Gravdahl

Parameter	Value	Parameter	Value	Parameter	Value
L_c	3 m	p_{amb}	100 kPa	A_c	0.01 m ²
L_t	0.8 m	p_{in}	100 kPa	A_t	0.01 m ²
R	0.1 m	p_{out}	100 kPa	V_p	1.5 m ³
ρ_{amb}	1.15 kg/m ³	γ	1.401	N_{stall}	2

The steady state compressor characteristic is generated by using the parameters portrayed in Figure 5. The characteristic of the compressor is included in the state space model and the parameters can be substituted into the model. The initial conditions are determined for the compressor system when the system is operated at an initial operating point with a rotor speed of 4583 r/min and a throttle opening of 100%, that is $u_t = 1$. The initial conditions of the state variables are given by

$$\mathbf{x}_0 = \begin{bmatrix} x_1(t_0) \\ x_2(t_0) \\ x_3(t_0) \\ x_4(t_0) \\ x_5(t_0) \end{bmatrix} = \begin{bmatrix} 0.4165 \\ 0.4165 \\ 1.144 \\ 100754 \\ 745 \end{bmatrix}. \quad (4.1)$$

The values for the input vector is given by

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 100000 \\ 100000 \\ 4583 \end{bmatrix}. \quad (4.2)$$

The values given in (4.1) are used throughout the simulation as the initial steady state operating point unless otherwise indicated. The values of the input vector given in (4.2) are constant for the simulation.

4.3 Analytic model implementation in Flownex®

Flownex® is a systems CFD software package capable of simulating complex thermal-fluid systems. It was developed by M-Tech Industrial for the purposes of simulating large-scale industrial systems. Each component in the library of Flownex® is validated with practically measured data. The compressor system implemented in Flownex® is shown in Figure 13.

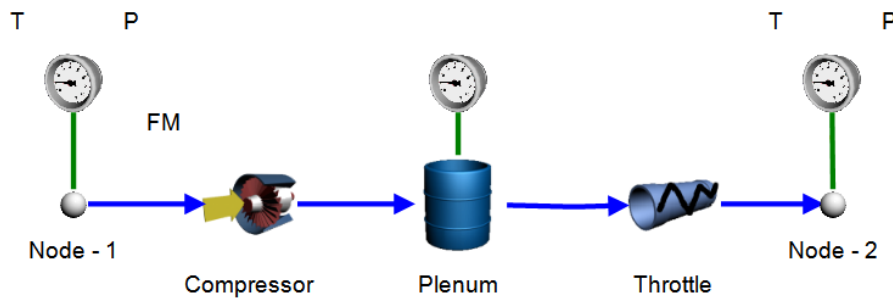


Figure 13: Flownex® implementation of the axial-flow compressor system

The setup of the Flownex[®] model for the axial-flow compressor system can be seen in Appendix A.2. The setup of the components is discussed as well as additional values that are calculated for the implementation.

4.4 Verification of the analytic model

The analytic compressor system model is simulated and the results are presented for the steady state characteristic and the transients between two operating points.

4.4.1 Steady state simulation

The compressor's steady state characteristic is shown in Figure 14. The third order polynomial equation is used to relate the mass flow rate through the compressor to the pressure rise across the compressor for a fixed rotor speed of 4853 r/min. The characteristic has a maximum at a mass flow rate of 0.275 kg/s.

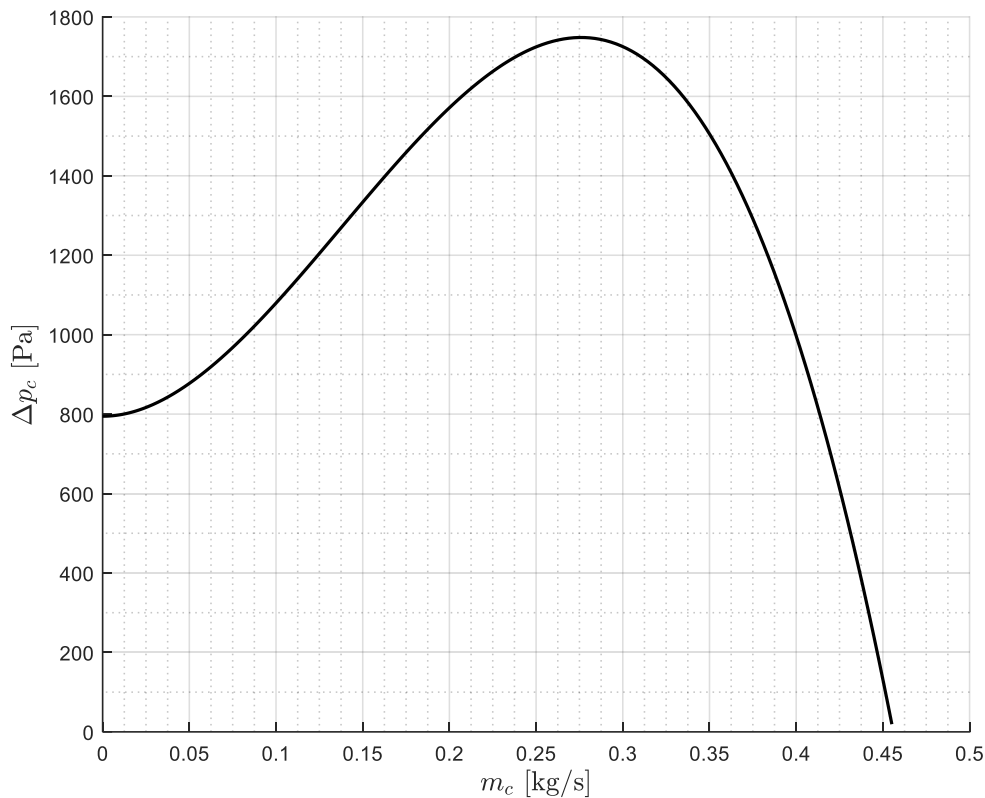


Figure 14: Steady state response of the compressor

The unstable operating region is located to the left of the maximum of the compressor characteristic. If the compressor is operated in this region, it is prone to enter either rotating stall or surge, depending on the value of the B-parameter that is determined by the rotor speed.

The stable operating region is located to the right of the maximum of the compressor characteristic. This region is the preferred operation range where the compressor is operated.

4.4.2 Transient response simulation

The transient simulation is done by operating the compressor system at an initial operating point at a constant rotor speed. The throttle opening is changed from the initial opening of 100% to an opening of 30%.

The response of the compressor follows a trajectory to a different steady state operating point on the compressor characteristic. The throttle opening is then changed back to an opening of 100%. The response follows a different trajectory to the initial operating point.

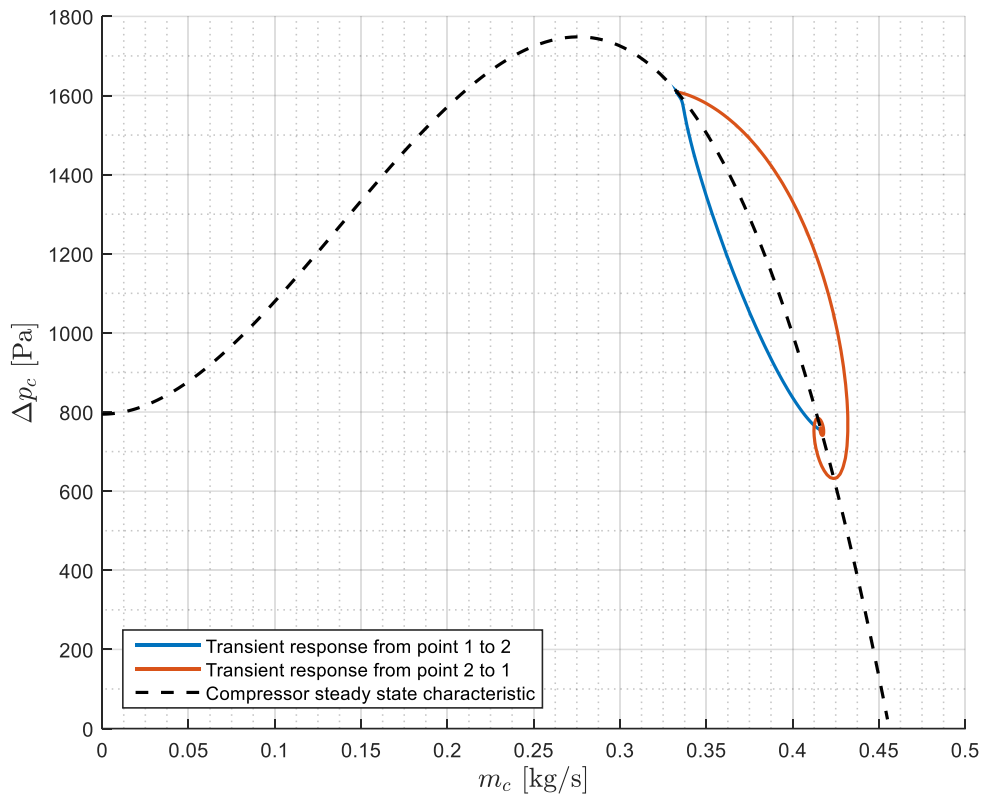


Figure 15: Compressor response superimposed on the steady state characteristic

The response in Figure 15 is due to the nonlinearities in the compressor and throttle components. The response of the compressor indicates that the system is not operated at quasi-steady (pseudo-steady) due to the time lag that is modelled in the compressor.

The five state variables of the system is shown for a step input in of the throttle opening in Figure 16. The responses of the state variables shown are the natural responses of the compressor system.

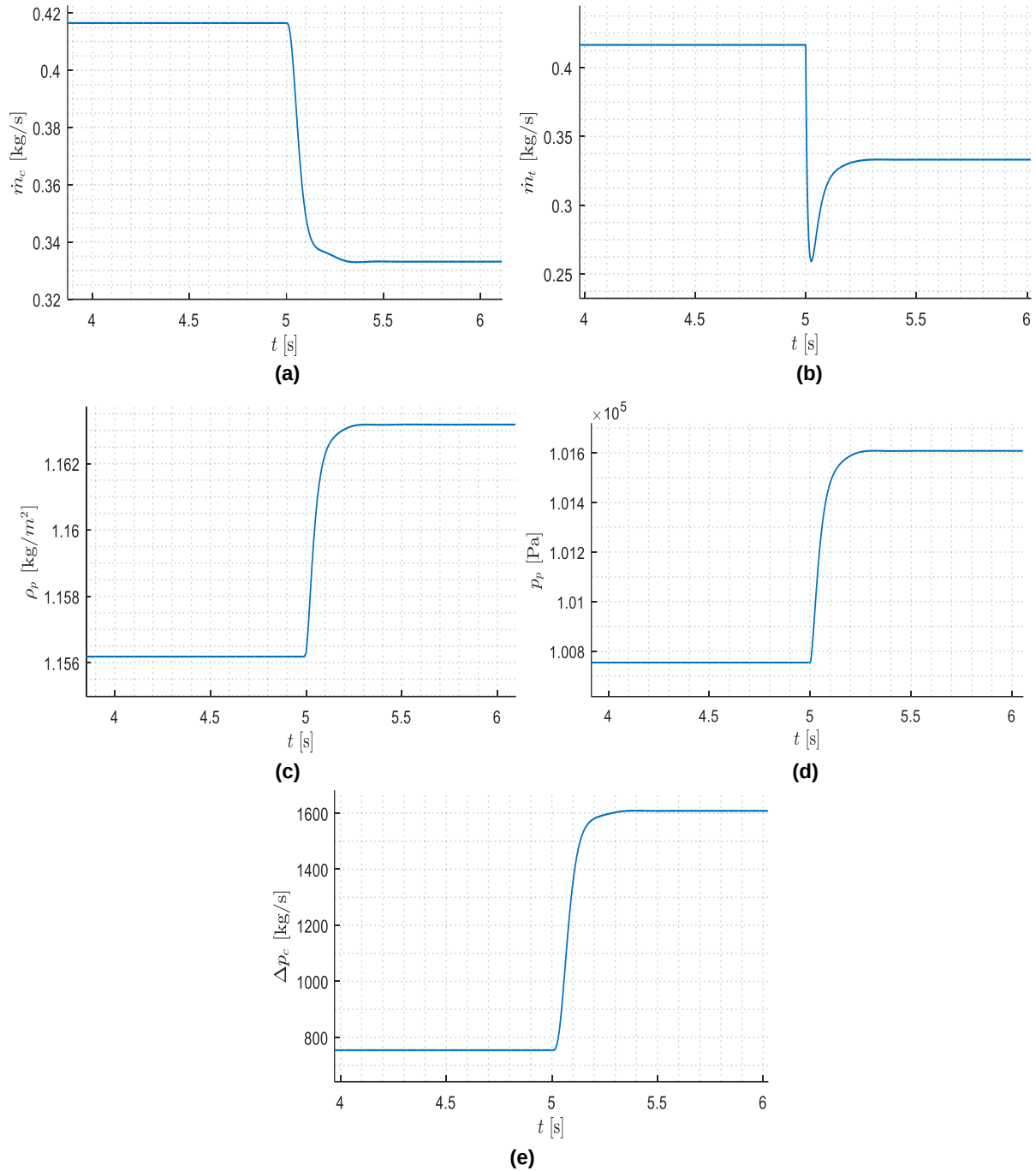


Figure 16: Transient response due to a step input of the throttle opening for the (a) Compressor mass flow rate (b) Throttle mass flow rate (c) Plenum density (d) Plenum pressure (e) Compressor pressure rise

The transient response of the compressor system is shown in Figure 17 for an inverted step of the throttle opening. The response for an inverted step to a less efficient operating point is different than the response of the system during a normal step to a higher efficiency point.

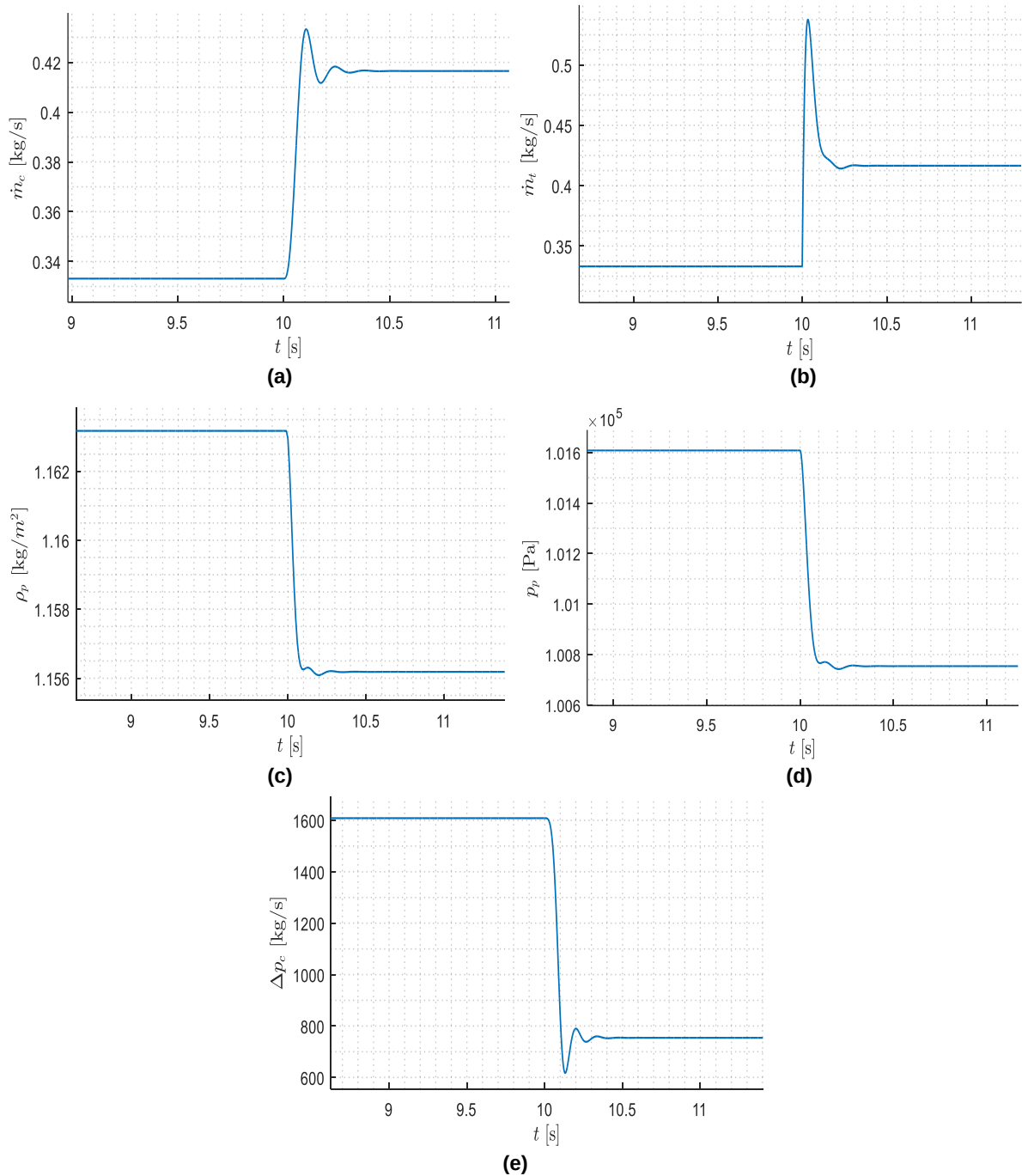


Figure 17: Transient response due to an inverse step of the throttle opening for the (a) Compressor mass flow rate (b) Throttle mass flow rate (c) Plenum density (d) Plenum pressure (e) Compressor pressure rise

4.5 Validation of the analytic model

The validation of the analytic model is done with two systems: a comparison of the analytic and Flownex® models with experimental data from the compressor investigated by Gill [13] and a comparison of the analytic and Flownex® models for the compressor system investigated by Gravdahl [24]. The validation with the compressor system investigated by Gill aims to validate the steady state characteristics used by the analytic and Flownex® models. The second validation aims to validate the steady state and transient response of the analytic model with the response of the Flownex® model. In each case a performance index is calculated to determine the correlation between the response signals of the different systems.

The methodology followed for the steady state characteristic validation is shown in Figure 18. It should be noted that the validation includes a comparison of the responses of the systems and a performance index to determine the correlation between the response signals.

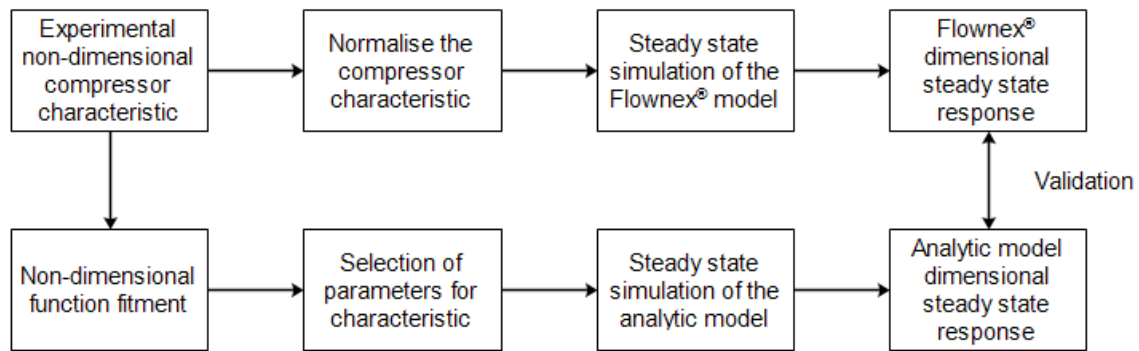


Figure 18: Methodology followed for the steady state validation

The methodology followed for the validation of the transient response is shown in Figure 19. The validation includes the comparison of the state variables of the systems and a performance index for each state variable.

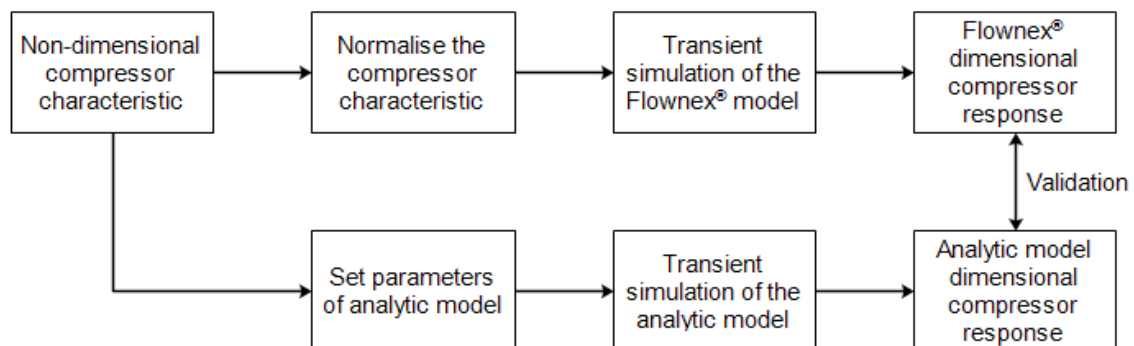


Figure 19: Methodology followed for the transient response validation

4.5.1 Validation with the compressor model investigated by Gill

The compressor system investigated by Gill [13] is a three-stage axial flow compressor system. The system is designed to operate at rotor speed of 3000 r/min with a mass flow rate of 2.7 kg/s that corresponds to a flow coefficient of 0.503 and a nominal total to total pressure ratio of 1.022. The compressor's hub and casing diameters are 0.3 m and 0.24 m respectively. The rotor consists of 43 blades and 41 stator blades. The geometric parameters are calculated from the specification given for the system. These geometric parameters used in the analytic and Flownex® models are given in Table 6.

Table 6: Parameters of the compressor investigated by Gill

Parameter	Value	Parameter	Value
D_h	0.3 m	A_h	0.070686 m ²
D_t	0.42 m	A_t	0.138544 m ²
R_h	0.15 m	A_c	0.067854 m ²
R_t	0.21 m	U_t	65.97344 m/s
R_m	0.105 m	U_m	32.98672 m/s

4.5.1.1 Steady-state compressor characteristic validation

The parameters for the compressor characteristic function used in the analytic model are determined through trial and error. This is done so that the best fitment of the cubic function fits the experimental data. These parameters were determined and are given in Table 7.

Table 7: Compressor characteristic parameters determined for the compressor investigated by Gill

Parameter	Value
C_0	0.5
H	0.43
W	0.19

The non-dimensional and the dimensional steady state characteristic of the experimental results, analytic model and Flownex® model is shown in Figure 20. The experimental response is used as the reference response and the IAE for the analytic and Flownex® models responses are calculated and given in Figure 20.

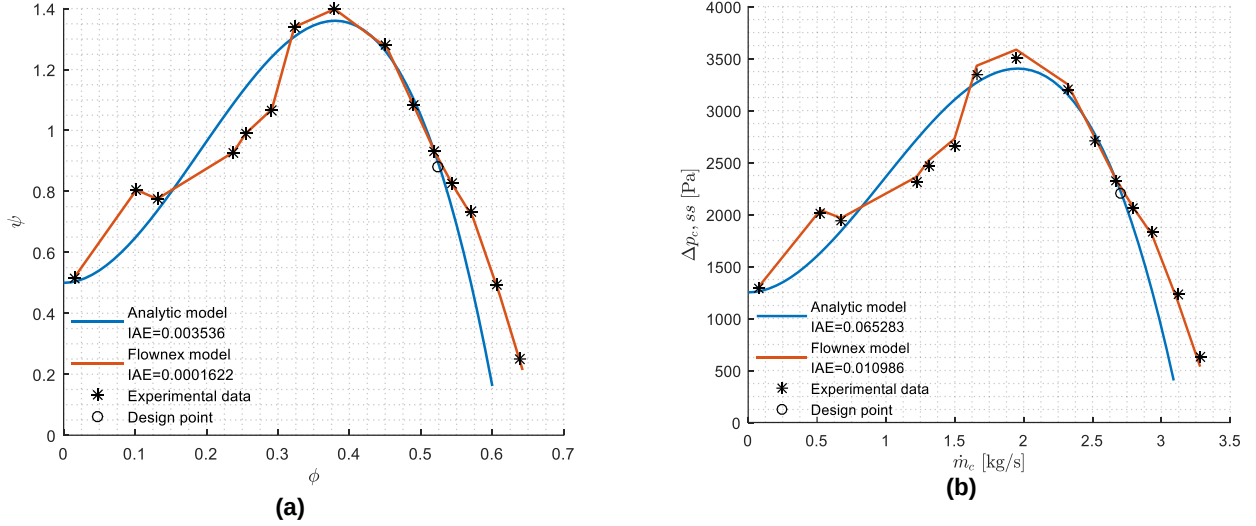


Figure 20: Validation of the compressor system investigated by Gill in [13] for (a) The steady-state non-dimensional compressor characteristic (b) The steady-state non-dimensional compressor characteristic

The pressure response of the Flownex[®] model is scaled by a factor of 2, since the relations of converting the non-dimensional response to the dimensional response of the pressure differs for the Flownex[®] model and the relation used by Gill. The relations used by Gill to convert the non-dimensional compressor characteristic to the dimensional compressor characteristic are given by

$$\phi = \frac{\dot{m}_c}{\rho U A_c}, \quad (4.3)$$

$$\varphi = \frac{\Delta p}{\frac{1}{2} \rho U^2}. \quad (4.4)$$

The relations used by Flownex[®] are given and discussed in Appendix A.

4.5.2 Validation with the compressor system model investigated by Gravdahl

The validation of the analytic model compared to the model investigated by Gravdahl is done for the steady state compressor characteristic and the transient response. The methodology followed for validating the steady state compressor characteristic is depicted in Figure 18. The methodology followed to validate the transient response is shown in Figure 19.

The non-dimensional compressor characteristic is normalised and introduced to the compressor component in Flownex[®] and the parameters of the system are set for all components. The parameters of the analytic model are set and correspond to the compressor characteristic.

The simulation of the analytic and Flownex[®] model is done and the responses of the two models are compared.

4.5.2.1 Steady-state compressor characteristic validation

The parameters used for the steady state compressor characteristic are given in Table 8. These parameters are used from the model investigated by Gravdahl [24].

Table 8: Compressor characteristic parameters determined for the compressor system investigated by Gravdahl

Parameter	Value
C_0	0.3
H	0.18
W	0.25

The non-dimensional and dimensional steady state characteristics for the compressor system are shown in Figure 21. There is a good correlation between the two responses of the steady state characteristics, as indicated by the performance index.

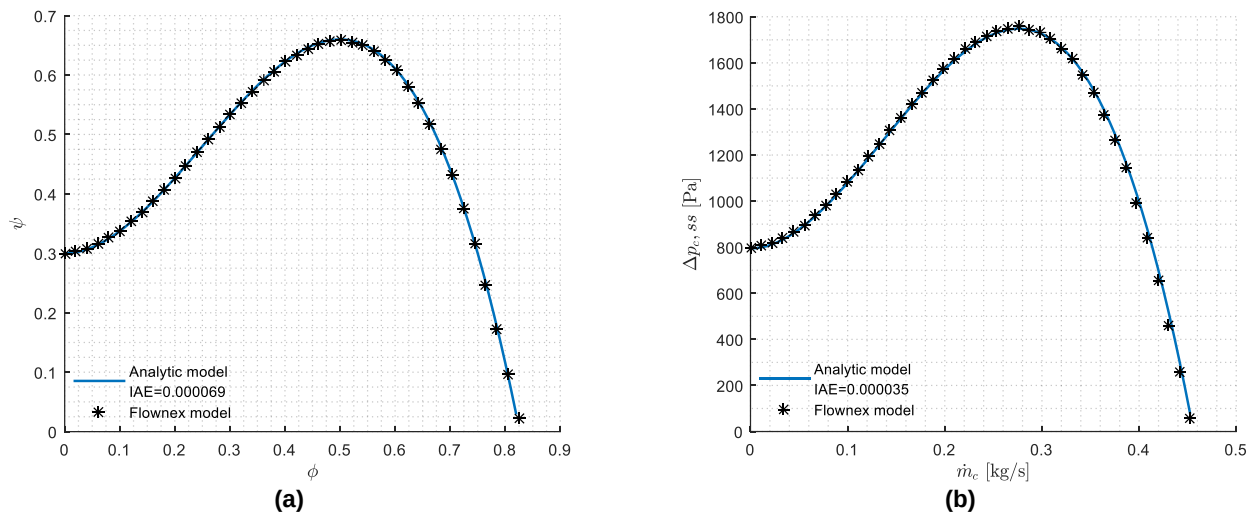


Figure 21: Validation of the compressor system investigated by Gravdahl in [24] for (a) The steady-state non-dimensional compressor characteristic (b) The steady-state non-dimensional compressor characteristic

4.5.2.2 Transient response validation of the compressor system

The response of the five state variables for the transitions between the operating points are shown in Figure 22. The response indicates a transition from one operating point to another by changing the size of the opening in the throttle.

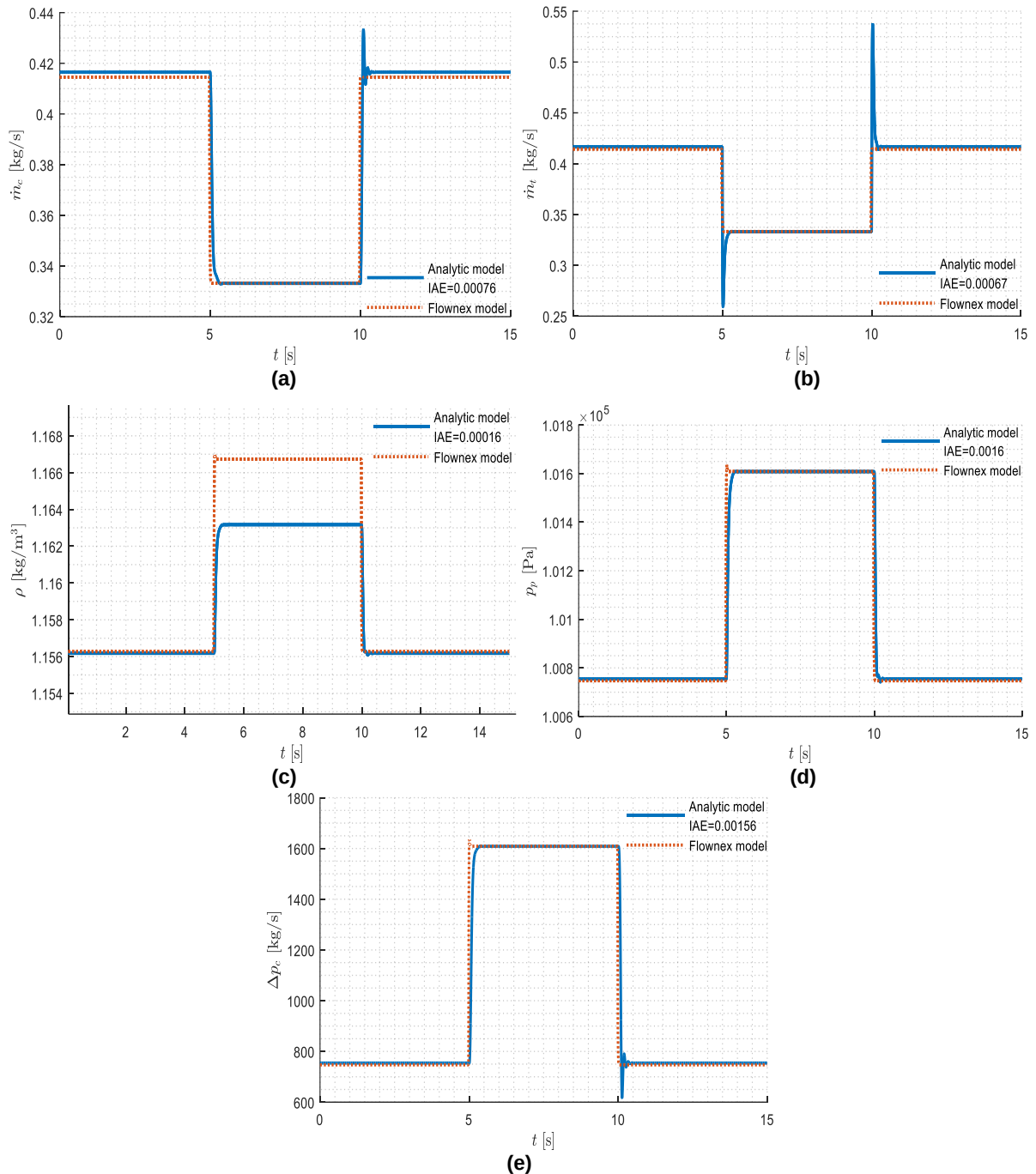


Figure 22: Transient response of the (a) Compressor mass flow rate (b) Throttle mass flow rate (c) Plenum density (d) Plenum pressure (e) Compressor pressure rise

The IAE is calculated for each state using the Flownex[®] response as the reference signal. The IAE indices show good correlation between the signals of all 5 state variables. In the response of the compressor mass flow rate in Figure 22 (a) and compressor pressure rise in Figure 22 (e) at $t=10$ s, a large overshoot in the responses are observed compared to the responses of the Flownex[®] model.

This is due to the fact that the analytic model includes a lag parameter for the compressor component. This causes the analytic model to not operate quasi-steady, whereas the Flownex[®] operates quasi-steady. This response of the analytic model is more likely to be observed in a physical system when a large transition between operating points are made.

In the throttle mass flow rate response, the transients are observed to have a large overshoot. Again this is due to the sudden change in throttle opening where the differential equations used to model the system are sensitive for this type of transient where a large transition is made between operating points.

4.6 Conclusion

In the validation of the steady state characteristic it is noted that the analytic model's response is accurate compared to the response of the experimental system and the Flownex[®] models. The transient validation indicated a satisfying response of the analytic model. Some differences can be found in the response of the analytic model compared to the Flownex[®] model. These differences in the transient response are caused by the lag parameter introduced in the analytic model and the sudden change of the throttle opening.

5 CHAPTER

ENERGY VISUALISATION

This chapter discusses an energy analysis by applying the first law of thermodynamics to the compressor system. Node diagrams are used to represent the compressor system for different types of operations. The power and energy of the system are visually presented in power-energy graphs, showing how energy is shaped by components in the system. The visualisation are depicted in four test cases: Normal operation; A fault operation where a leak is introduced in the plenum; An unstable operation where the compressor enters rotating stall; An unstable operation where the compressor enters surge.

5.1 Introduction

Energy is a well-known concept among scientists and engineers. The basic operation of systems can be described by considering the energy that is shaped by components in the system. The energy of interconnected components can be individually calculated or as a whole for the complete system. The energy analysis is done by applying the power and energy equations discussed in Chapter 4 on the compressor system.

5.2 Test case methodology for the energy visualisation

The energy visualisation of the compressor system is done in four test cases. The compressor system is represented by a node diagram for the purposes of the energy analysis. This diagram portrays the compressor system when the system is operated in the stable operating region or when the compressor enters rotating stall or surge. The direction of the arrows shows the energy flow in and out of each component.

The system is divided into three nodes: The compressor node C that represents the compressor component in the inlet duct; the second node P that represents the plenum volume; and the third node T represents the throttle component in the outlet duct. Power due to shaft work P_B enters the compressor node and power is dissipated by the throttle P_E .

5.2.1 Test case 1 methodology

The node diagram of the compressor system used in test case 1 is portrayed in Figure 23. The aim of test case 1 is to set a benchmark test for the normal operation of the compressor system at two operating points and the transients between the two operating points.

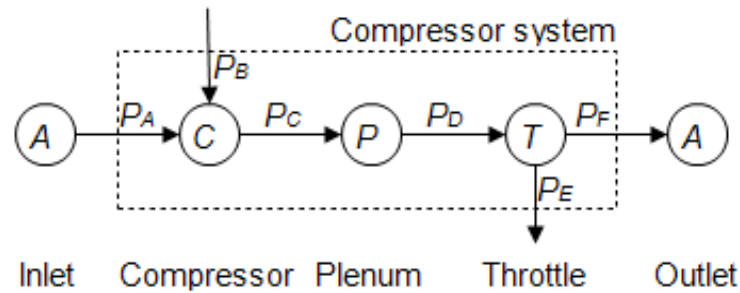


Figure 23: Compressor system node diagram used in test case 1 and 3

The first test case investigates the compressor's energy response for a steady and transient operation. The operating point is changed for a fixed time period and then back to the initial operating point. The methodology followed for test case 1 is shown in Figure 24.

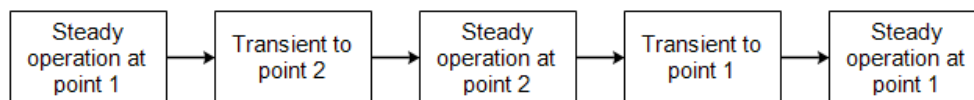


Figure 24: Methodology followed for test case 1

5.2.2 Test case 2 methodology

The node diagram portrayed in Figure 23 is extended for use in test case 2. In the first part of test case 2, a compressor system is operated as in test case 1, followed by an operation with a leak introduced in the plenum.

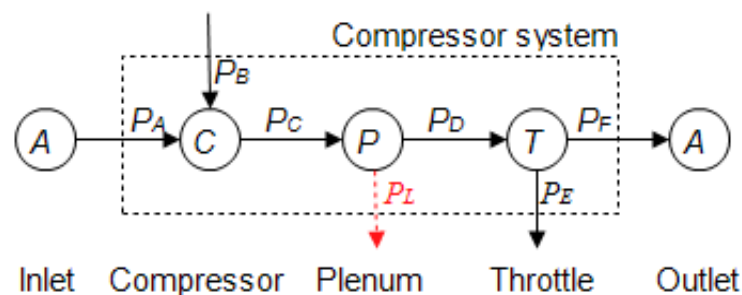


Figure 25: Compressor system node diagram with a leak in the plenum used in test case 2

In the second part of the test case, two identical systems is operated parallel to each other. One system is operated under normal conditions and the other system is operated with a leak in the plenum. The energy response of the two systems is used to generate residuals for the purposes of a FDD system. The methodology followed for test case 2 is shown in Figure 26 below.

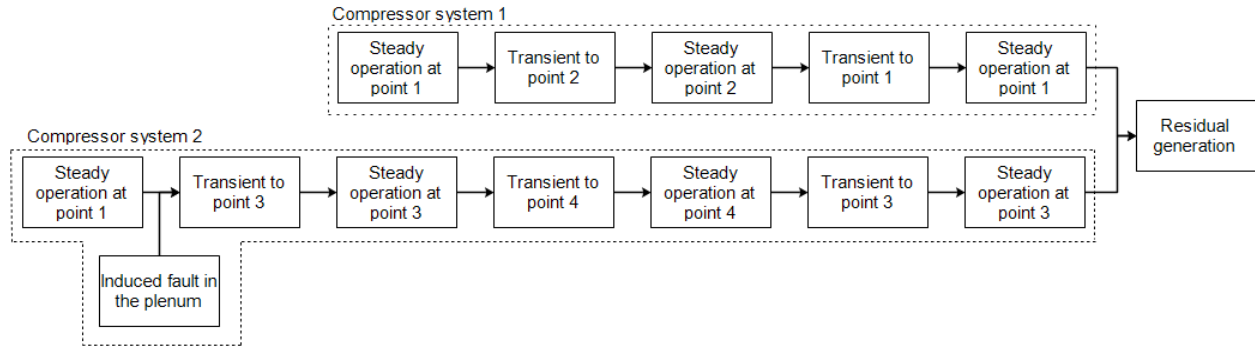


Figure 26: Methodology followed for test case 2

5.2.3 Test case 3 methodology

The third test case focuses on visualising the energy and power in the plenum during rotating stall. The compressor is operated in the stable region near the maximum of the compressor characteristic and is transitioned to the unstable region so the system enters rotating stall. The node diagram portrayed in Figure 23 is used for test case 3. The methodology followed for this test case is shown in Figure 27.

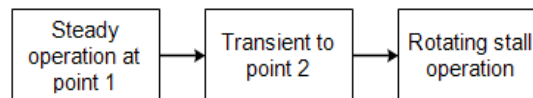


Figure 27: Methodology followed for test case 3

5.2.4 Test case 4 methodology

The node diagram portrayed in Figure 23 is modified for test case 4. This is done since oscillations in the mass flow are present, which results in a change of direction of the energy flow. The node diagram that portrays the compressor system during surge is shown in Figure 28. The fourth test case investigates the compressor's power-energy response when the compressor enters an unstable operating condition, namely surge. This is done by operating the compressor at an operating point near the maximum of the characteristic at a higher rotor speed where a B-Parameter of 1.5 is achieved.

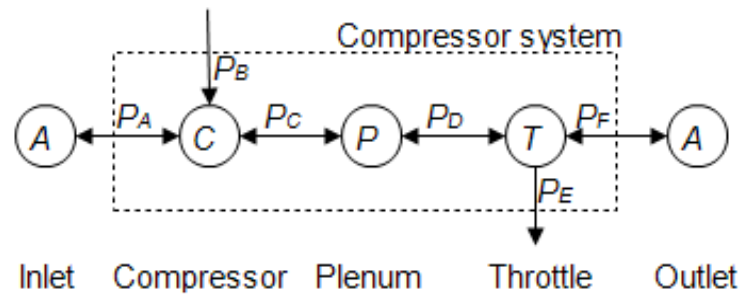


Figure 28: Compressor system node diagram used in test case 4

The throttle opening is changed so that the compressor is operated in the unstable region and enters surge. The methodology followed in test case 4 is shown in Figure 29 below.

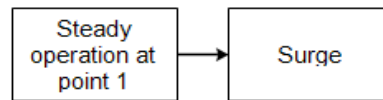


Figure 29: Methodology followed for test case 4

5.3 Energy visualisation

The investigation of the four test cases are done in the following part. Each test case is described and the parameters used for the specific test case are given. The power-graphs for each test case are given and the residuals generated in test case 2 are discussed.

5.3.1 Test case 1: Normal operation

In the first test case, the compressor system is operated with a throttle opening of 100 % at operating point 1. The operating point is changed to operating point 2 by changing the throttle opening to 30 %. A trajectory is followed and the system settles at a steady operating point 2.

The system is transitioned back to the initial operating point and a different trajectory is followed. The power-energy planes for the compressor, plenum and throttle node are portrayed in Figure 30, Figure 31 and Figure 32. The parameters used for test case 1 are shown in Table 9.

Table 9: Parameters used for test case 1

Time	Response	Throttle gain	Throttle opening	Rotor speed	B-parameter
$0 \leq t < 3$	Steady operation at point 1	$u_t = 1$	100 %	4583 r/min	0.5
$3 \leq t < 3.5$	Transient from point 1 to point 2	$u_t = 0.3$	30 %		
$3.5 \leq t < 6$	Steady operation at point 2	$u_t = 0.3$	30 %		
$6 \leq t < 6.5$	Transient from point 2 to point 1	$u_t = 1$	100 %		
$6.5 \leq t \leq 9$	Steady operation at point 2	$u_t = 1$	100 %		

The power-energy plane of the compressor node is portrayed in Figure 30. The two operating points and the transitions between them are depicted. The trajectory from 1 to 2 is negative due to the fact that the power entering the node decreases whilst the rate of power leaving the node increases. The result of this transient is a decrease in the stored energy in the compressor node. The trajectory from 2 to 1 is positive and results in an increase in the stored energy of the compressor node.

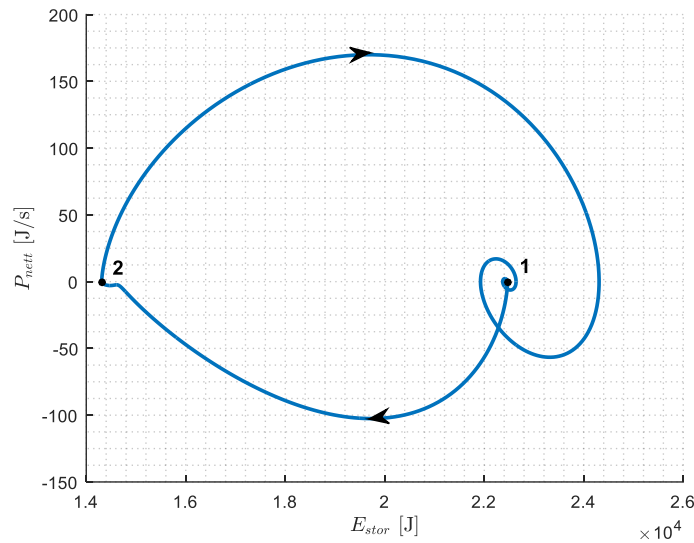


Figure 30: Power-energy graph for the normal operation of the compressor node

The power-energy diagram that depicts the response of the plenum in steady operation and during the transients is shown in Figure 31. The transition from 1 to 2 follows a positive trajectory that results in an increase in the stored energy in the plenum.

The exchange of energy is large compared to the compressor and throttle node due to the large storage capacity of the plenum. The transition back to operating point 1 results in a negative net power trajectory as expected where the stored energy in the plenum decreases.

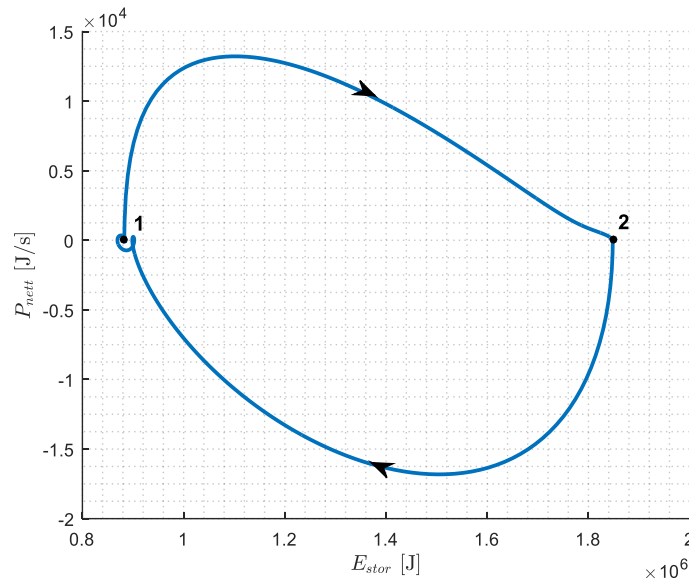


Figure 31: Power-energy graph for the normal operation of the plenum node

The throttle node's power-energy plane is depicted in Figure 32. The amount of energy that is stored in the outlet duct is much less compared to the stored energy in the inlet duct and plenum. The throttle node's trajectories that are followed is clearly differentiable from the compressor and plenum node's trajectories. The instantaneous change in the throttle opening causes the energy response to follow this trajectory.

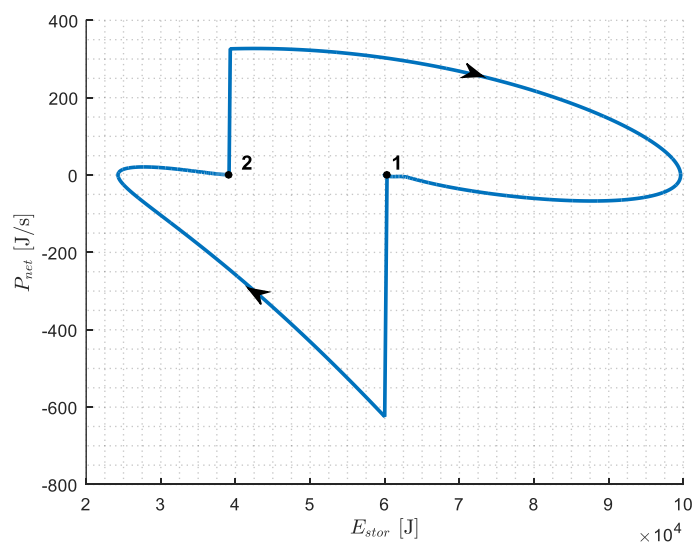


Figure 32: Power-energy graph for the normal operation of the throttle node

Features can be extracted for a healthy response between the two operating points from the three power-energy graphs of the compressor, plenum and throttle nodes. These Features include the length of the trajectories between the operating points, areas enclosed by the trajectories, centre of mass of the enclosed trajectory and the distances between the steady operating points. These characteristics can be paired with the corresponding operating points and be used to detect if there is a deviation in the system's operation.

5.3.2 Test case 2: Leak implementation

The second test case employs a leak in the plenum of the compressor system observed in Figure 33, Figure 34 and Figure 35. The compressor system is operated at point 1 and undergoes a transient to point 2 and back. A leak is employed in the plenum at $t = 9$ and undergoes a transient to 3. The system still operates with a throttle opening of 100 % at point 3. The throttle opening is changed to an opening of 30 % and back. The change in throttle opening results in transients from 3 to 4 and back. The parameter changes during the simulation is shown in Table 10.

Table 10: Parameters used for test case 2

Time	Response	Throttle gain	Throttle opening	Leak area	Rotor speed	B-parameter
$0 \leq t < 3$	Steady operation at point 1	$u_t = 1$	100 %	-	4583 r/min	0.5
$3 \leq t < 3.5$	Transient from point 1 to point 2	$u_t = 0.3$	30 %			
$3.5 \leq t < 6$	Steady operation at point 2	$u_t = 0.3$	30 %			
$6 \leq t < 6.5$	Transient from point 2 to point 1	$u_t = 1$	100 %			
$6.5 \leq t < 9$	Steady operation at point 1	$u_t = 1$	100 %			
$9 \leq t < 9.5$	Transient from point 1 to point 3	$u_t = 1$	100 %	0.0008 m ²		
$9.5 \leq t < 12$	Steady operation at point 3	$u_t = 1$	100 %			
$12 \leq t < 12.5$	Transient from point 3 to point 4	$u_t = 0.3$	30 %			
$12.5 \leq t < 15$	Steady operation at point 4	$u_t = 0.3$	30 %			
$15 \leq t < 15.5$	Transient from point 4 to point 3	$u_t = 1$	30%			
$15.5 \leq t < 18$	Steady operation at point 3	$u_t = 1$	100 %			

Figure 33 depicts the normal and fault transients followed by the compressor node when a leak is introduced in the plenum. The compressor node stores more energy during the steady operations at 3 and 4 compared to the energy stored at the normal operating points 1 and 2. It is noted that a different trajectory is followed when a leak is employed.

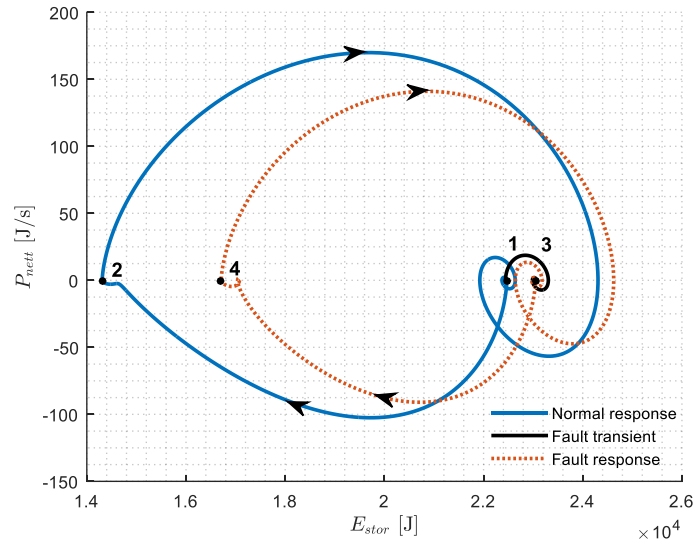


Figure 33: Power-energy graph of the compressor node for a leak in the plenum

The effects of the leak on the power-energy plane for the plenum node can be observed in Figure 34. The plenum stores less energy compared to the energy stored during normal operation in the plenum. In the power-energy plane, it can be noted that the enclosed trajectory shrinks and moves to the left.

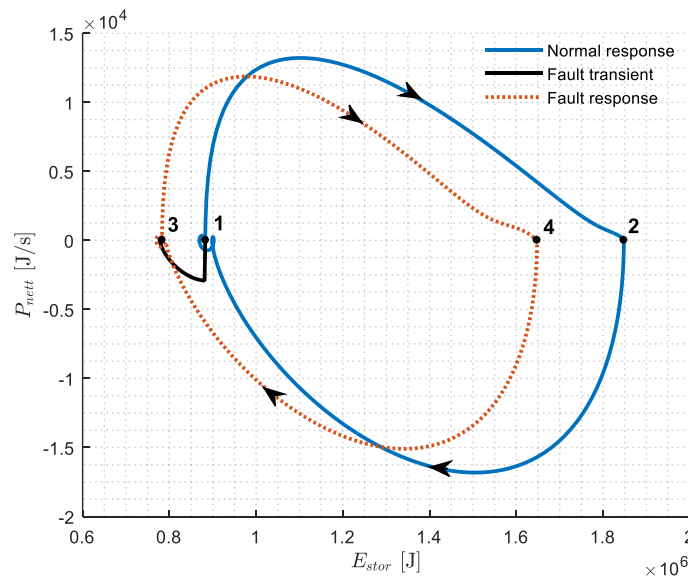


Figure 34: Power-energy graph for the plenum node for a leak in the plenum

The power-energy graph for the throttle node is depicted in Figure 35. The response is as expected for the component downstream of where the leak occurred. The throttle node stores less energy in the same manner that the plenum stores less energy due to the loss of energy through the leak. The trajectory differs from the one followed under normal operating conditions.

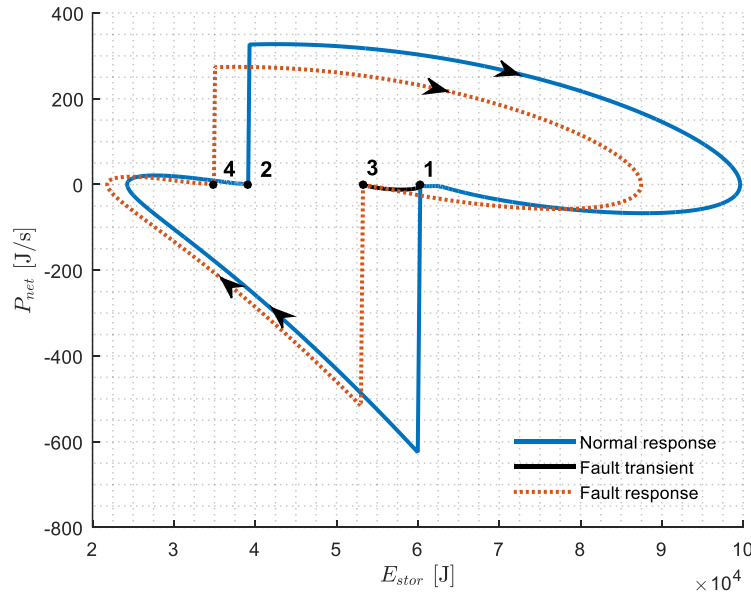


Figure 35: Power-energy graph of the throttle node for a leak in the plenum

From these power-energy planes, residuals are calculated for the steady state operation and transients of the system. These residuals hold certain information in respect of the difference between the normal and fault response. The investigation focuses on two specific residuals, namely residuals calculated from the power and residuals calculated from the energy in each node. These two types of residuals are compared to the residuals calculated from the state variables of the system.

5.3.2.1 State variable residuals

Two systems are operated parallel to each other. The one system is operated under normal conditions as investigated in test case 1. The second system operates under a fault condition, where a leak is employed in the plenum at $t=1.5$. The operating parameters for both systems during the simulation are shown in Table 11 below.

Table 11: Parameters used for the parallel operation of two systems for FDD

Time	Throttle gain	Throttle opening	System 1		System 2	
			Leak area	Response	Leak area	Response
$0 \leq t < 1.5$	$u_t = 1$	100 %	-	Steady operation at point 1	-	Steady operation at point 1
$1.5 \leq t < 2$	$u_t = 1$	100 %		Steady operation at point 1	0.0008 m ²	Transient from point 1 to point 3
$2 \leq t < 3$	$u_t = 1$	100 %		Steady operation at point 1		Steady operation at point 3
$3 \leq t < 3.5$	$u_t = 0.3$	30 %		Transient from point 1 to point 2		Transient from point 3 to point 4
$3.5 \leq t < 6$	$u_t = 0.3$	30 %		Steady operation at point 2		Steady operation at point 4
$6 \leq t < 6.5$	$u_t = 1$	100 %		Transient from point 2 to point 1		Transient from point 4 to point 3
$6.5 \leq t \leq 9$	$u_t = 1$	100 %		Steady operation at point 1		Steady operation at point 3

The state vector of system 1 is defined by equation (3.28) and given by

$$\mathbf{x} = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5]^T. \quad (5.1)$$

The state vector of system 2, which operates under a fault condition, is defined by

$$\mathbf{x}_f = [x_{1,f} \quad x_{2,f} \quad x_{3,f} \quad x_{4,f} \quad x_{5,f}]^T, \quad (5.2)$$

with the subscript f denoting a fault condition in the system. The steady state residual vector of the state variables for a single operating point is defined by

$$\mathbf{r}_{i,x} = \mathbf{x}_i - \mathbf{x}_{i,f}, \quad (5.3)$$

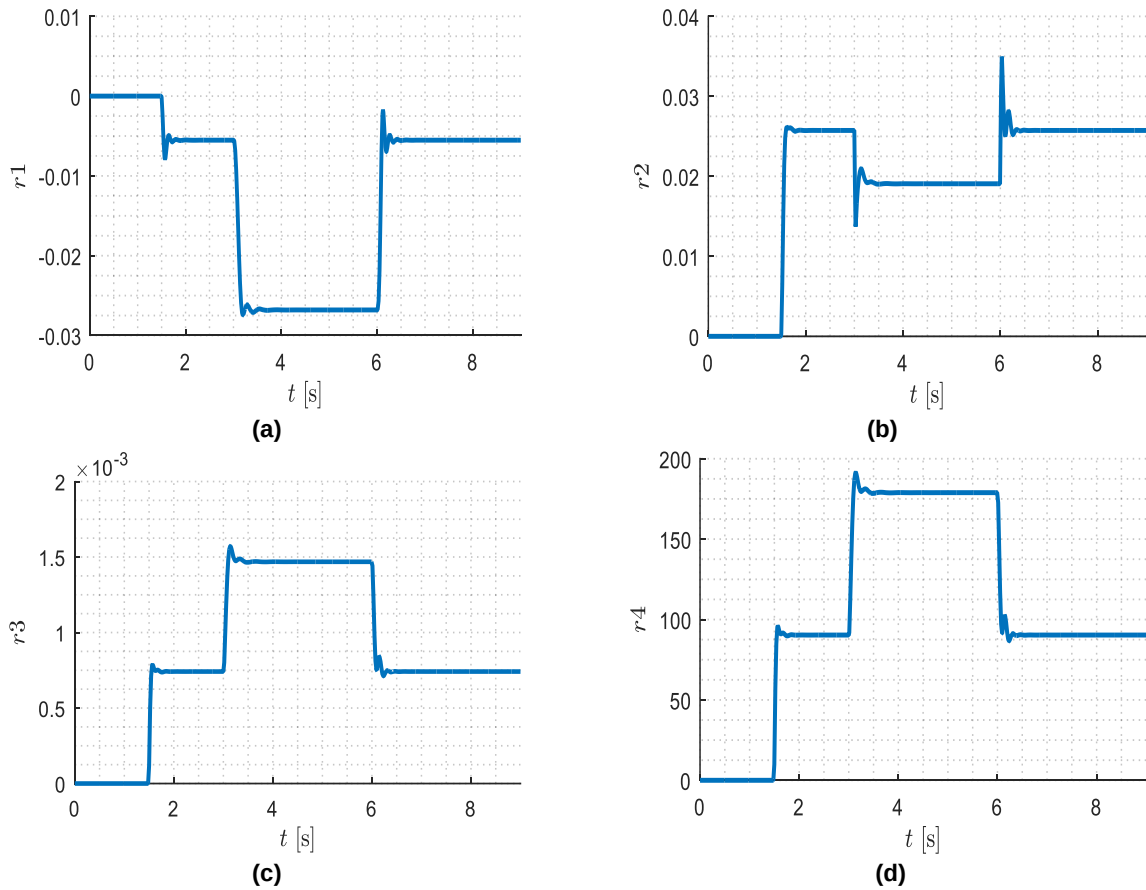
with the subscript x denoting the state variable vector and the subscript i denoting the operating point. The state residual vectors for operating points 1 and 2 are calculated from equation (5.3) and given by

$$\mathbf{r}_{1,x} = \begin{bmatrix} -0.0055 \\ 0.0257 \\ 0.001 \\ 91 \\ 91 \end{bmatrix} \text{ and } \mathbf{r}_{2,x} = \begin{bmatrix} -0.3599 \\ 0.019 \\ -0.998 \\ 179 \\ 179 \end{bmatrix}. \quad (5.4)$$

The dynamic residual vector that is time dependent is defined by

$$\mathbf{r}_x(t) = \mathbf{x}(t) - \mathbf{x}_f(t). \quad (5.5)$$

Both systems 1 and 2 are operated at operating point 1. At $t = 1.5$, a fault is introduced into one of the systems. At $t = 3$, a shift to operating point 2 is made and at $t = 6$ a shift back to operating point 1 is made. The residuals for the five state variables are portrayed in Figure 36.



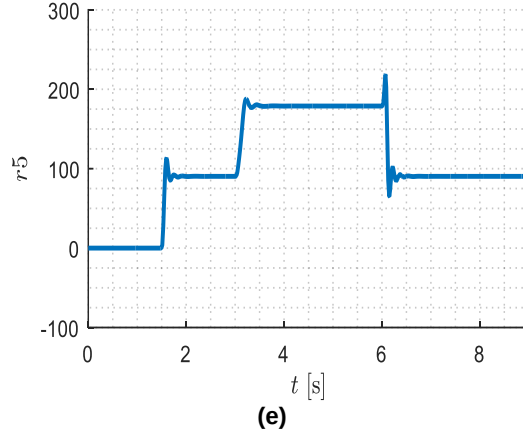


Figure 36: Dynamic residuals calculated from the state variables for the (a) Compressor mass flow rate (b) Throttle mass flow rate (c) Plenum density (d) Plenum pressure (e) Compressor pressure rise

A change in residual values can be observed in all 5 residuals when the leak is introduced into the system. Residual 3, 4 and 5 increase when operating at point 2, and residual 1, 2 and 3 decrease in value at this operating point. This response of the residuals is observed due to the nonlinear effects of the leak in the system.

5.3.2.2 Power residuals

To calculate the residuals for the steady state power, the power vector is defined by

$$\mathbf{P}_i = [P_A \ P_B \ P_C \ P_D \ P_E \ P_F]^T, \quad (5.6)$$

where the subscript i indicates the operating point and P_A to P_F indicate the power at different sections. The power dissipated through the leak is defined by P_L and shown in the node diagram in Figure 25. The two power vectors are given by

$$\mathbf{P}_1 = \begin{bmatrix} 36491.12 \\ 273.17 \\ 36491.12 \\ 36491.12 \\ 273.17 \\ 36491.12 \end{bmatrix} \text{ and } \mathbf{P}_2 = \begin{bmatrix} 29434.67 \\ 465.95 \\ 29434.67 \\ 29434.67 \\ 465.95 \\ 29434.67 \end{bmatrix}, \quad (5.7)$$

for normal operation at point 1 and 2 with $P_L = 0$.

The vectors are calculated for point 1 and 2 under fault conditions and are given by

$$\mathbf{P}_{1,f} = \begin{bmatrix} 36941.80 \\ 243.64 \\ 36941.80 \\ 34205.37 \\ 226.56 \\ 34205.37 \end{bmatrix} \text{ and } \mathbf{P}_{2,f} = \begin{bmatrix} 31746.60 \\ 447.44 \\ 31746.60 \\ 27700.66 \\ 390.41 \\ 27700.66 \end{bmatrix}, \quad (5.8)$$

with the subscript f denoting the vector under a fault condition and $P_L = 2736.43$ for operating point 1 and $P_L = 4045.94$ for point 2. The power residual vector is defined by the difference between the normal and fault power vector and is given by

$$\mathbf{r}_{i,p} = \mathbf{P}_i - \mathbf{P}_{i,f}, \quad (5.9)$$

where the subscript p denotes the power vector. The power residual vectors for points 1 and 2 are given by

$$\mathbf{r}_{1,p} = \begin{bmatrix} -450.68 \\ 29.53 \\ -450.64 \\ 2285.75 \\ 46.61 \\ 2285.75 \end{bmatrix} \text{ and } \mathbf{r}_{2,p} = \begin{bmatrix} 2311.91 \\ 18.15 \\ 2311.91 \\ 1734.01 \\ 75.54 \\ 1734.01 \end{bmatrix}. \quad (5.10)$$

These residual vectors indicate the difference between the power for normal operation and when a leak is introduced into the system. For point 1 and 2 the vectors are noticeably different due to the nonlinear dynamics that the leak exerts on the system. This is even more noticeable when the dynamic residuals are considered power. The dynamic power residual is defined by

$$\mathbf{r}(t) = \mathbf{P}(t) - \mathbf{P}_f(t). \quad (5.11)$$

Figure 37 portrays the dynamic power residuals for the compressor system. Residuals 1 and 2 are the residuals for the power entering the compressor node. The residual of the power leaving the compressor node and entering the plenum node is represented by residual 3. Residual 4 is

the residual calculated for the power leaving the plenum node and power that enters the throttle node. Residual 5 and 6 are the residuals for the power leaving the throttle node.

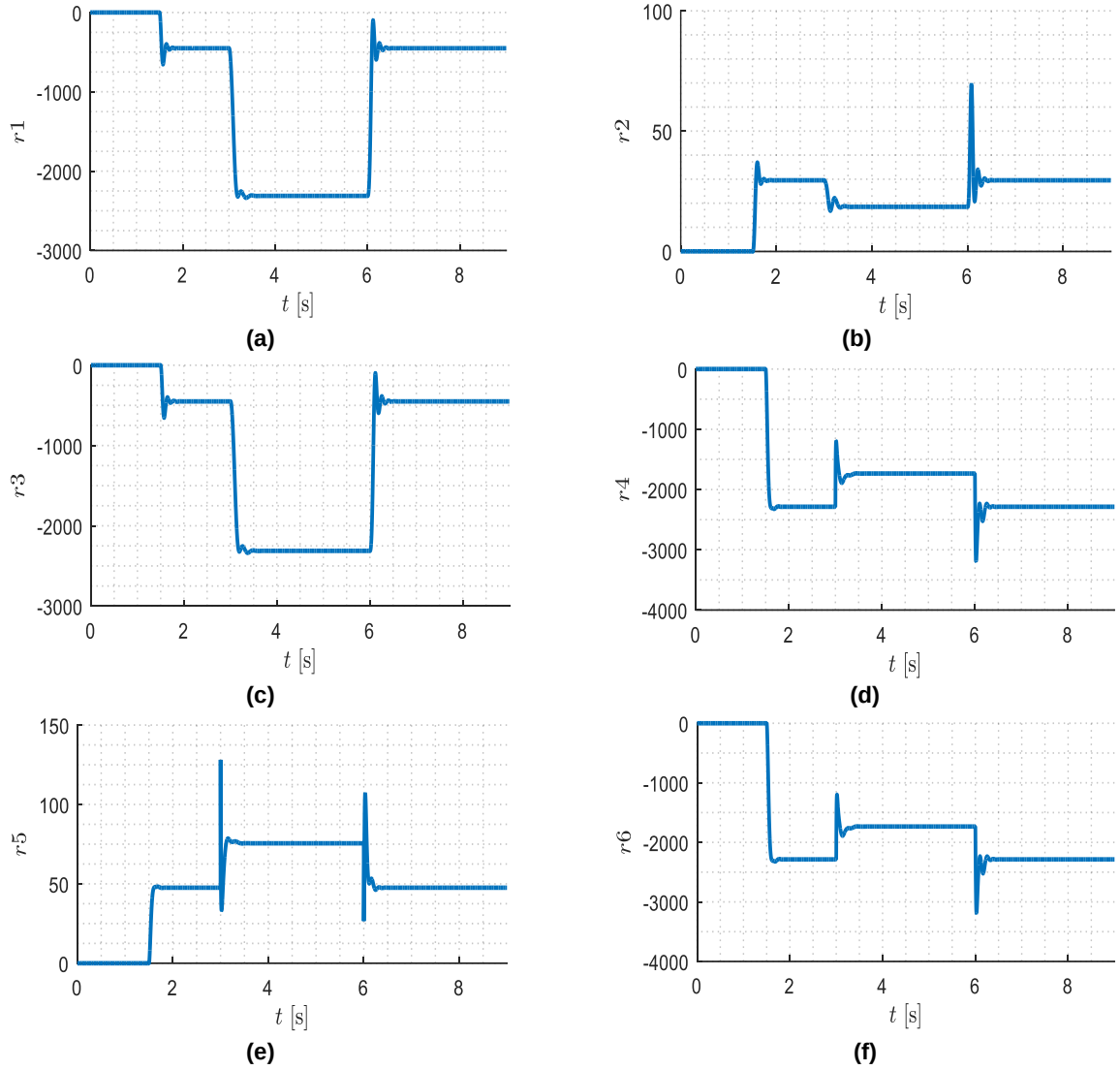


Figure 37: Dynamic residuals calculated from the power vector for the power in (a) P_A (b) P_B (c) P_C (d) P_D (e) P_E (f) P_F

At the time instance where $t = 1.5$ s, a leak is implemented in system 2. The system undergoes a transient to a new operating point. The fault can be identified by the non-zero residuals between $1.5 \leq t \leq 9$. When the transition is made at $t = 3$ s to operating point 2, it can be observed that residuals 1, 3 and 5 are much more sensitive to operating point 2, whereas residuals 2, 4 and 6 are more sensitive to operating point 1.

5.3.2.3 Energy residuals

Residuals are calculated for the stored energy in each component of the system. The stored energy in the components are defined in vector form by

$$\mathbf{E}_i = \begin{bmatrix} E_C \\ E_P \\ E_T \end{bmatrix}, \quad (5.12)$$

where the subscript i denotes the operating point and subscripts C , P and T denote the compressor, plenum and throttle nodes respectively. From equation (2.22) the energy stored in each component is calculated for operating point 1 and 2 and given by

$$\mathbf{E}_1 = \begin{bmatrix} 22468.77 \\ 882690.57 \\ 5994.93 \end{bmatrix} \text{ and } \mathbf{E}_2 = \begin{bmatrix} 14309.80 \\ 1848086.13 \\ 4390.77 \end{bmatrix}. \quad (5.13)$$

The energy vectors for both operating points under the fault condition are calculated and given by

$$\mathbf{E}_{1,f} = \begin{bmatrix} 23047.10 \\ 782891.04 \\ 5444.68 \end{bmatrix} \text{ and } \mathbf{E}_{2,f} = \begin{bmatrix} 16709.39 \\ 1648046.43 \\ 4035.23 \end{bmatrix}. \quad (5.14)$$

Equation (5.13) and (5.14) is used to calculate the steady state residual vector for an operating point as

$$\mathbf{r}_{i,E} = \mathbf{E}_i - \mathbf{E}_{i,f}, \quad (5.15)$$

with the subscript e denoting the energy vector. The calculated steady state residual vectors for operating point 1 and 2 are given by

$$\mathbf{r}_{1,E} = \begin{bmatrix} -578.33 \\ 99799.53 \\ 550.25 \end{bmatrix} \text{ and } \mathbf{r}_{2,E} = \begin{bmatrix} -2399.59 \\ 200039.70 \\ 355.54 \end{bmatrix}. \quad (5.16)$$

It can be observed from the residuals that the stored energy increases in the component upstream from where the fault occurred, while the stored energy decreases in the plenum and throttle.

The response of the components downstream is different from the response of the components upstream of where the fault occurred. The dynamic energy residual vector is defined by

$$\mathbf{r}(t) = \mathbf{E}(t) - \mathbf{E}_f(t). \quad (5.17)$$

The dynamic energy residuals for the three nodes are portrayed in Figure 38. Residual 1 depicts the difference in energy stored in the compressor node, residual 2 the difference in energy stored in the plenum and residual 3 the difference in energy stored in the throttle node, all in relation to the energy stored in the node under normal operation.

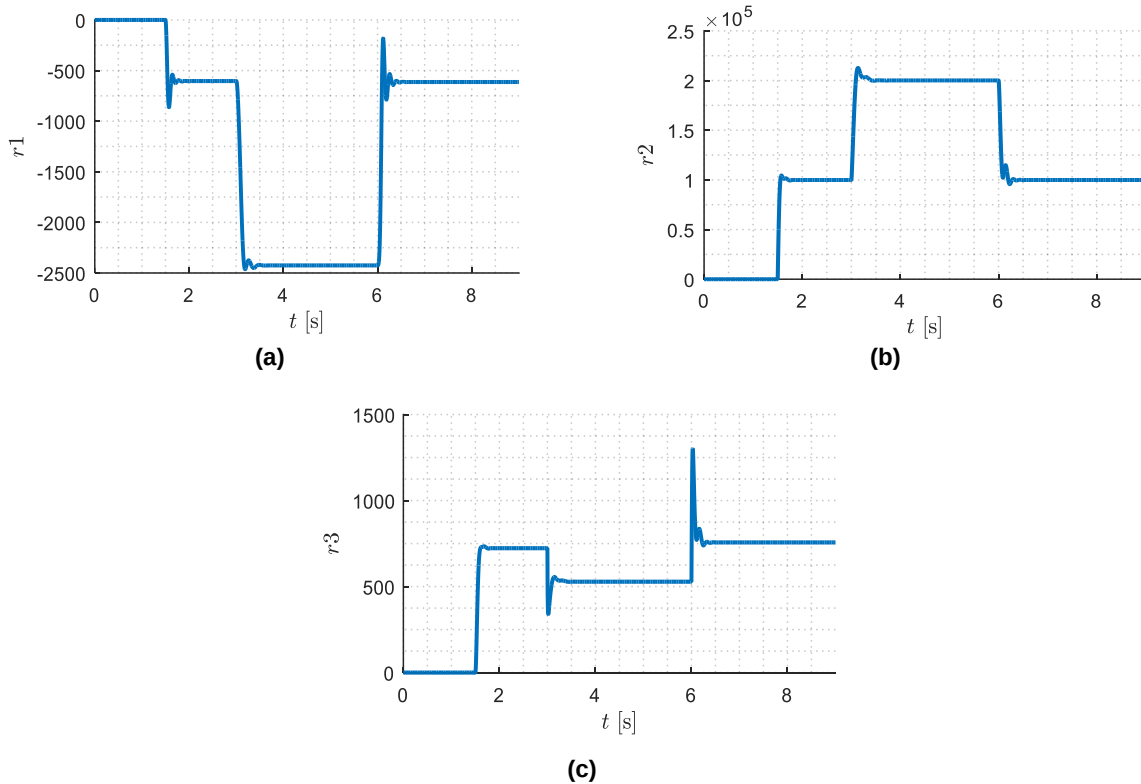


Figure 38: Dynamic residuals calculated from the energy vector for the (a) Stored energy in the compressor node E_c (b) Stored energy in the plenum node E_P (c) Stored energy in the throttle node E_T

It can be observed that residuals 1 and 3 are less sensitive compared to residual 2 when the fault is introduced at $t = 1.5$. Again it can be noted that residuals 1 and 2 are more sensitive at operating point 2, that is at $3 \leq t < 6$, whereas residual 3 is more sensitive at operating point 1 where $1.5 \leq t < 3$ and $6 \leq t < 9$. This is due to the nonlinearities in the system. The energy response is more sensitive at different operating points.

5.3.3 Test cases 3: Rotating stall

In this test case, the compressor system is initially operated at an operating point near the maximum of the compressor characteristic, namely operating point 1, where the throttle opening is set to 20 %. The throttle opening is changed to an opening of 16 % at $t = 1$ s. The parameters used during the test case is shown in Table 4 below.

Table 12: Parameters used for test case 3

Time	Response	Throttle gain	Throttle opening	Rotor speed	B-parameter
$0 \leq t < 1$	Steady operation at point 1	$u_t = 0.2$	20 %	4583 r/min	0.5
$1 \leq t < 9$	Transient from point 1 to point 2	$u_t = 0.16$	16 %		
$9 \leq t$	Steady operation at point 2	$u_t = 0.16$	16 %		

This results in a case where the compressor's operation shifts to the left of the maxima on the compressor characteristic, and the compressor enters rotating stall. The power-energy plane of the plenum is portrayed in Figure 39 for the transition to point 2. The time response of the plenum is portrayed in Figure 40.

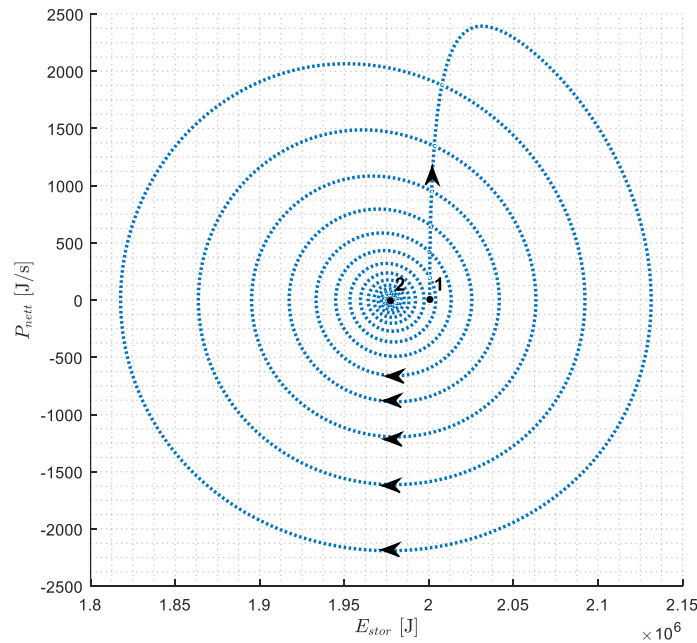


Figure 39: Power-energy plane for the plenum during rotating stall

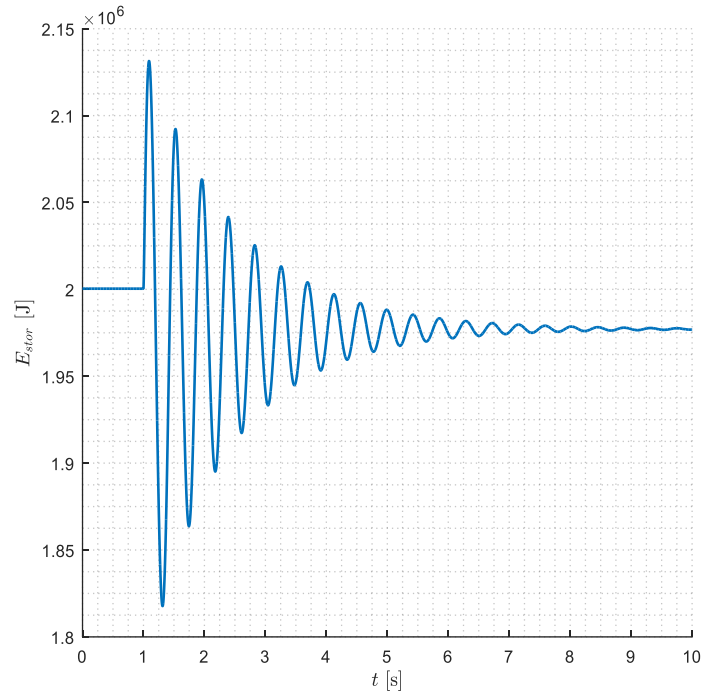


Figure 40: Stored energy in the plenum during rotating stall

A damped oscillation is observed from the instance when the throttle opening is changed. The plenum energy settles at a new steady state with a lower value than the initial operating point. Usually the stored energy would increase due to a smaller opening of the throttle, but due to rotating stall, the stored energy decreases.

5.3.4 Test case 4: Surge

In this test case, the compressor is operated with a rotor speed of 16500 r/min. This results in a Greitzer B-parameter of 1.5 that indicates that the compressor is prone to enter surge if operated to the left of the maxima on the compressor characteristic. The throttle is initially set to an opening of 20 %. The throttle opening is changed to 16 % and the compressor enters surge. The parameters used during the test case are shown in Table 13.

Table 13: Parameters used for test case 4

Time	Operating point	Throttle gain	Throttle opening	Rotor speed	B-parameter
$0 \leq t < 1$	Steady operation at point 1	$u_t = 0.2$	20 %	16500 r/min	1.5
$1 \leq t$	Response oscillates	$u_t = 0.16$	16 %		

Operating point 1 is indicated on the power-energy plane of the plenum node. When the compressor system is transitioned by decreasing the throttle opening, the compressor system enters a limit cycle as observed in Figure 41. The time response of the plenum energy is depicted in Figure 42.

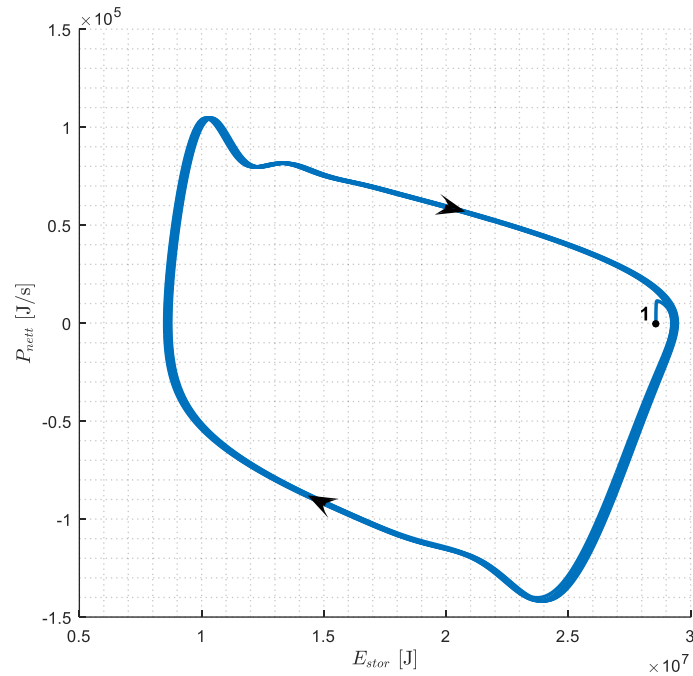


Figure 41: Power-energy plane of the plenum during surge

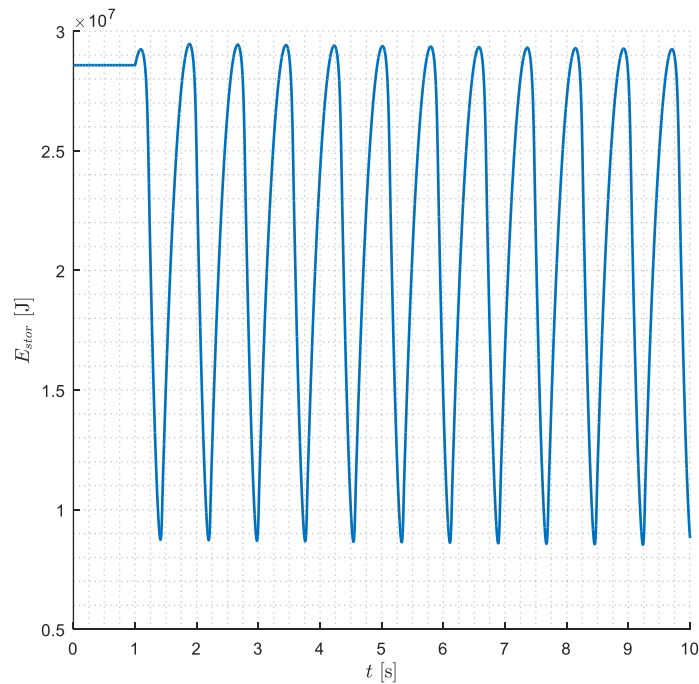


Figure 42: Stored energy in the plenum during surge

During $0 \leq t < 1$ the compressor system is operated in steady state. At $t = 1$ the plenum pressure starts to oscillate at a frequency of 1.275 Hz. This is an unwanted response due to fluctuations in the pressure and mass flow rate. When the compressor is in surge, a large amount of energy is exchanged to and from the plenum volume. This energy is not transferred through the throttle as desired, but also to and from the compressor. This could cause the compressor to break down when the energy is exerted in the opposite direction than the intended design.

5.4 Evaluation of the sensitivity of the power-energy plane

To evaluate the sensitivity of the power-energy planes of the three components a parametric sweep is done. This is done by varying a selected parameter and taking the system through a transition from one operating point to another and back. Three parameters were selected to evaluate the sensitivity of the power-energy graphs. These parameters are the leak size, rotor speed and plenum volume. These parameters were selected since they portray the most common variations found in a compressor system.

A benchmark test is used to determine the response of the power-energy planes under normal operating conditions. The parameters used for the benchmark test is given in Table 14. The system is transitioned from an operating point with a throttle opening of 100 % to an operating point with a throttle opening of 30 % and back to the initial operating point.

Table 14: Parameters used for the benchmark response

Parameter	Value
Rotor speed	4583 r/min
Plenum volume	1.5 m ³
Leak size	0 m ²

The power-energy planes' sensitivities are evaluated for three different types of parametric variations as shown and discussed in the next section. In each variation, the parameter is set to five different values with constant increments.

5.4.1 Evaluation of the sensitivity of the power-energy plane by varying the leak size

This evaluation is done by varying the leak size through the plenum volume in the compressor system. A leak with a size of 0.0002 m² is firstly implemented in the plenum. The system is taken through a transition from one operating point to another and back. This is done for five different leak sizes shown in Table 15.

Table 15: Parameters used for the sensitivity evaluation of a leak

Parameter	Value
Rotor speed	4583 r/min
Plenum volume	1.5 m ³
Leak sizes	0.0002 m ²
	0.0004 m ²
	0.0006 m ²
	0.0008 m ²
	0.001 m ²

The power-energy planes of the compressor, plenum and throttle nodes are shown in Figure 43.

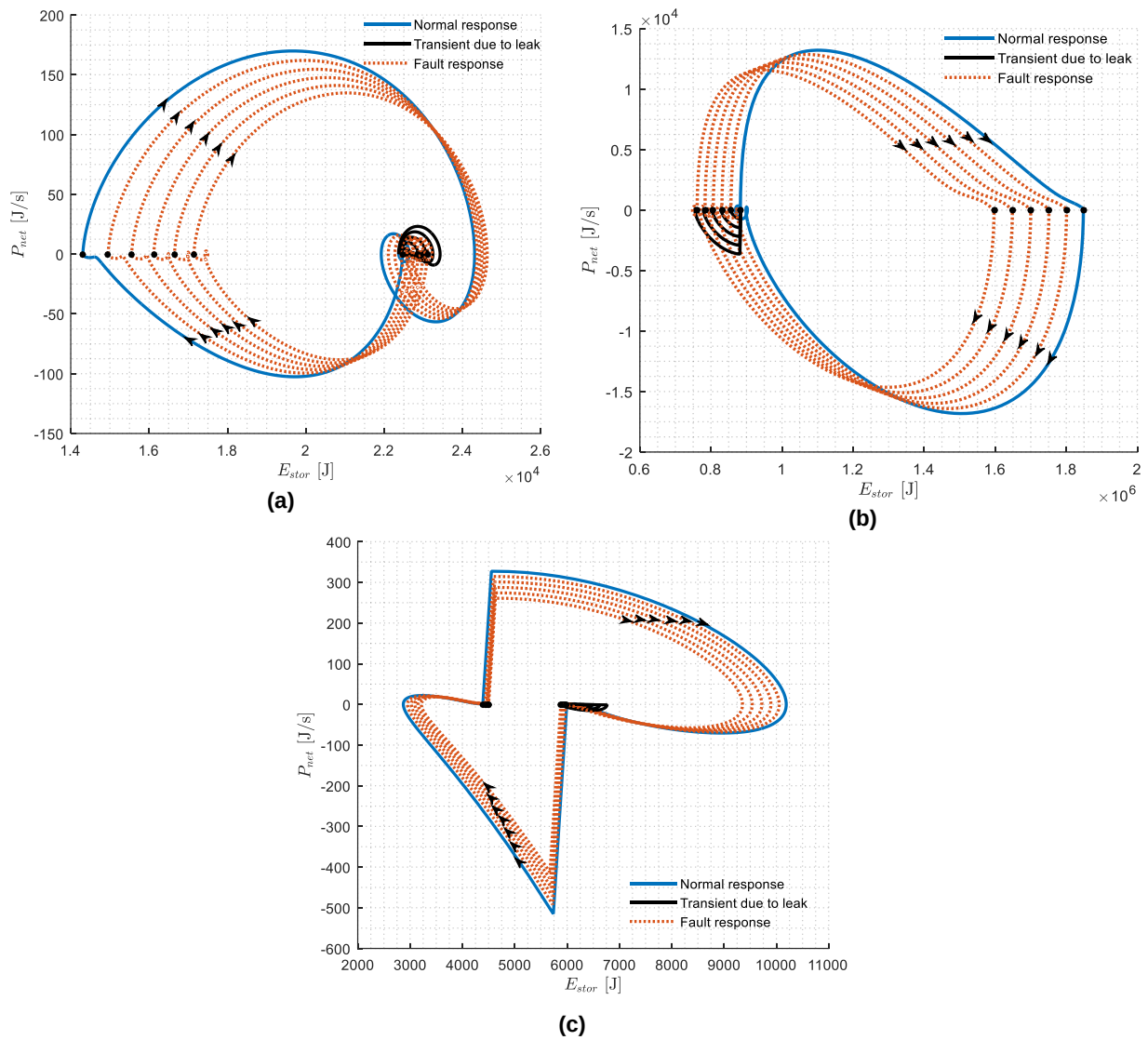


Figure 43: Fault sensitivity of power-energy planes for the (a) Compressor node (b) Plenum node (c) Throttle node

When the leak size is increased, it is noted that the enclosed trajectory moves either to the left or the right. In all three nodes the enclosed trajectory shrinks resulting in a shorter length of the trajectory. The result of this is that the compressor's operation moves to the right of the compressor characteristic, thus the compressor operates at a lower efficiency as indicated in Table 18 in Appendix B. All the nodes in the system reach steady state in a shorter time and trajectory length than when operated normally, but with a decrease in stored energy in the plenum. The normalised trajectory lengths are shown in Table 18, Table 19 and Table 20 in Appendix B.

5.4.2 Evaluation of the sensitivity of the power-energy plane by varying the rotor speed

The evaluation of the sensitivity of the power-energy planes due to an increase in the rotor speed is discussed in the next section. The rotor speeds are selected with constant increments of 100 r/min, thus ranging from 4683 r/min to 5083 r/min and given in Table 16.

Table 16: Parameters used for the sensitivity evaluation for a change in rotor speed

Parameter	Value
Rotor speed	4683 r/min
	4783 r/min
	4883 r/min
	4983 r/min
	5083 r/min
Plenum volume	1.5 m ³
Leak size	0 m ²

The simulation is done by transitioning the system from a one operating point with a throttle opening of 100 % to an operating point with a throttle opening of 30 % open and back for each rotor value. Figure 44 portrays the power-energy planes for the compressor, plenum and throttle nodes.

It is noted that the enclosed trajectory increases in size on the power-energy plane of the compressor node, plenum and throttle node. The result of this is that the power-energy response of the nodes has a longer settling time. This confirms the statement from Greitzer that at a higher B-parameter value, which is determined by the geometry and the rotor speed, the compressor is more prone to enter surge.

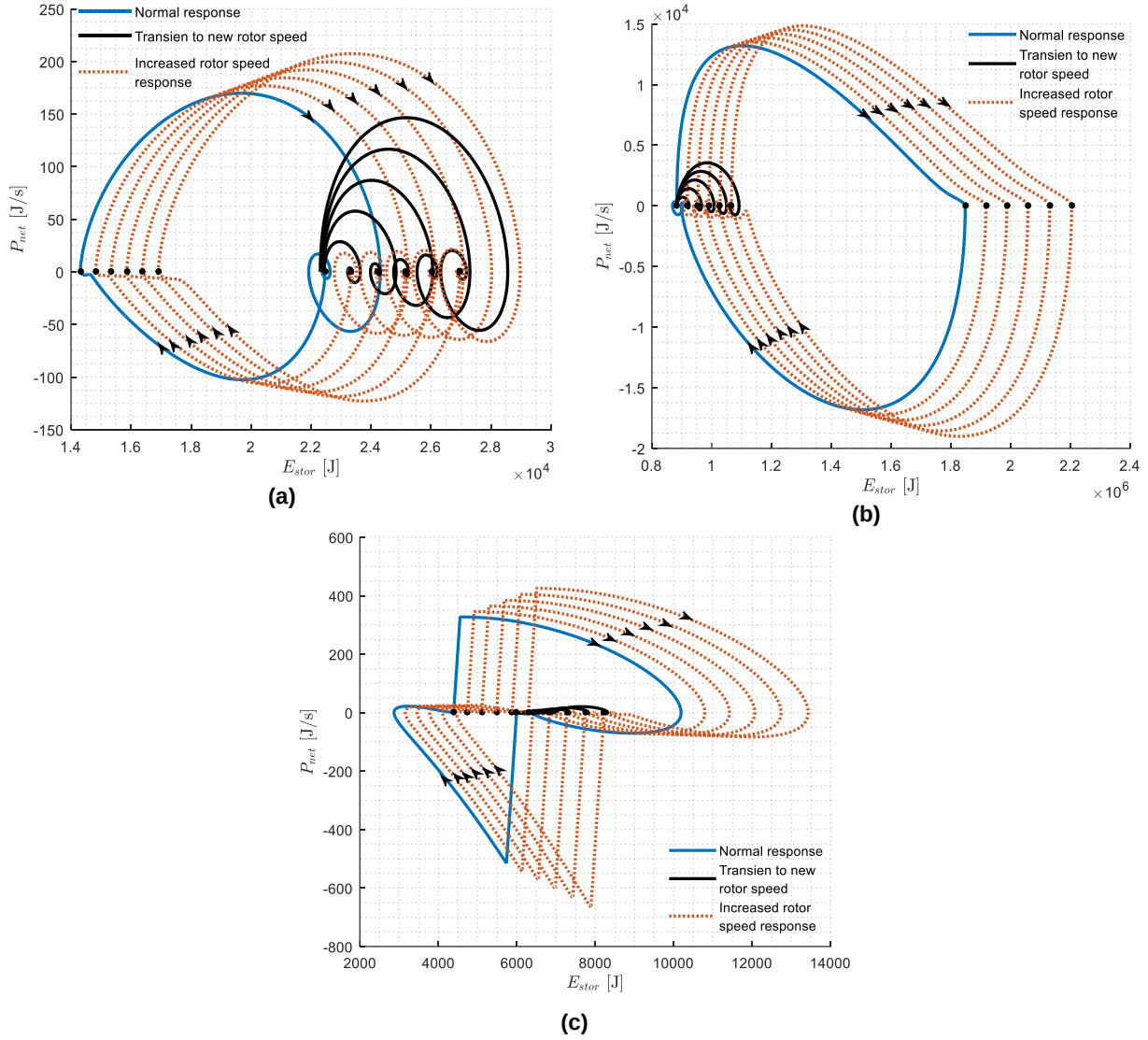


Figure 44: Rotor sensitivity of power-energy planes for the (a) Compressor node (b) Plenum node (c) Throttle node

The power-energy response of all three node moves to the right, thus more energy is stored in all of the nodes as expected at a higher rotor speed. The pressure ratio for the compressor is shown to increase as the rotor speed increases as shown in Table 21 in Appendix B. Other features such as the normalised trajectory lengths and normalised enclosed trajectory lengths are calculated for the three nodes and shown to increase in Table 21, Table 22 and Table 23 in Appendix B.

5.4.3 Evaluation of the sensitivity of the power-energy plane for different plenum sizes

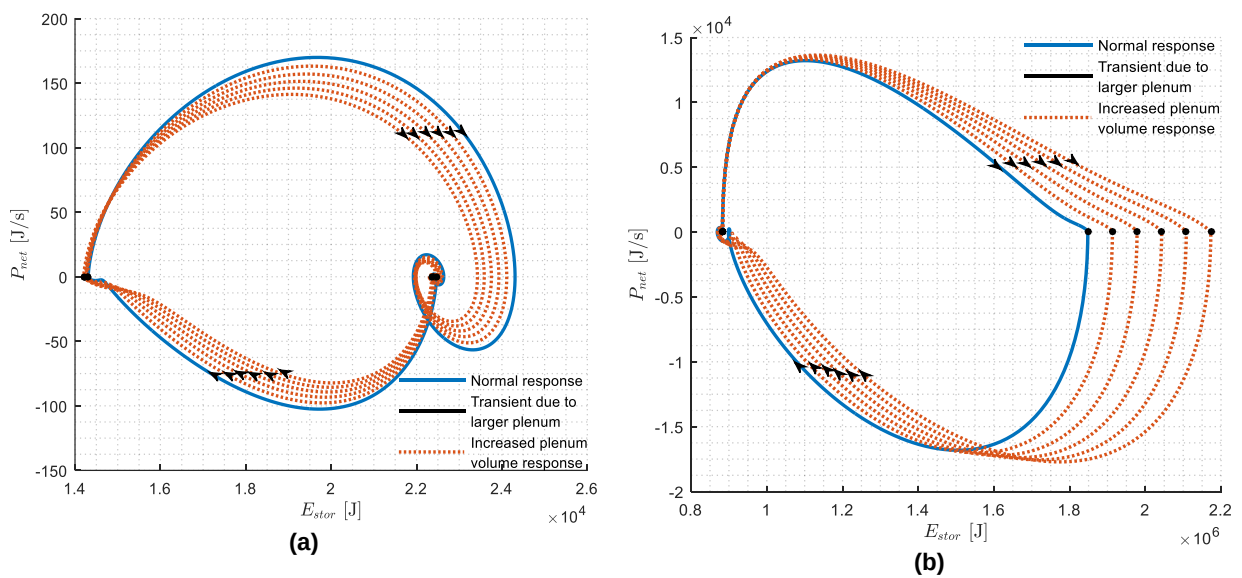
The sensitivity evaluation of the three power-energy planes for different plenum sized are investigated in the next section. The plenum sizes vary from 1.6 m^2 to 2 m^2 as shown in Table 17. Transitions with the throttle opening from 100 % to 30% and back are made for each plenum size.

Table 17: Parameters used for the sensitivity evaluation for a change in plenum size

Parameter	Value
Rotor speed	4583 r/min
Plenum volume	1.6 m ³
	1.7 m ³
	1.8 m ³
	1.9 m ³
	2 m ³
Leak size	0 m ²

Figure 45 (a) to (c) portrays the power-energy plane for the compressor, plenum and throttle nodes respectively. It is noticed that as the normalised trajectory length between the operating points of the compressor nodes decreases as the plenum volume increases. This leads to the result that a larger plenum forces the compressor to follow a more stable trajectory. The stored energy in the plenum increases as expected for a larger plenum size. The power energy-plane of the throttle node shifts to the right, thus storing more energy than in the baseline test. The pressure ratio of the compressor, as well as the normalised trajectory lengths for a variation in the plenum size are given in Table 24, Table 25 and Table 26 in Appendix B.

From the sensitivity of the power-energy graph it is evident that a unique response can be achieved for a change in parameter value. It is proven that the components shape the power and energy uniquely in different components and valuable information to optimise a system can be extracted from the power-energy planes.



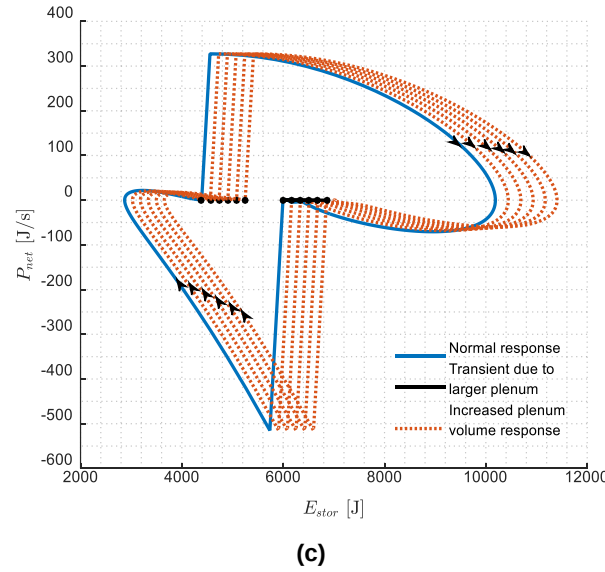


Figure 45: Plenum volume sensitivity of power-energy planes for the (a) Compressor node (b) Plenum node (c) Throttle node

5.5 Conclusion

The investigation of the compressor system proved to be supplemental to the analysis of the state variables. It is evident that a power-energy profile can be created for a system that transitions between operating points as was done in Test case 1.

In Test case 2, the observation was made that a different trajectory is followed on the power-energy graph of the system when a fault is introduced into the system. This lead to the investigation of generating residuals for identifying and isolating faults in the system. When the power and energy residuals are compared to the residuals generated from the state variable, the power and energy residuals were more sensitive to faults than the residuals of the state variables.

Test case 3 and 4 depict the compressor entering rotating stall and surge. The instabilities of these two cases in the power-energy plane and in the time response can be seen. These responses are more sensitive in the energy domain than in the hydraulic domain and can be used for controller design to avoid the unstable operation of a system.

It was determined that the sensitivity of the power-energy planes of the system are more sensitive than the state variables of the system for different input parameters. This confirm the intention to use the power, energy and features of the power-energy planes to design a control system or use it or the design of a FDD system.

6 CHAPTER CONCLUSION

In this chapter the energy visualisation of an axial-flow compressor system is discussed in terms of its attributes that make it useful for the purposes of control and FDD. Recommendations are made for future work in energy visualisation.

6.1 Introduction

This study is divided into four sections. These sections include the derivation of the analytic model, the validation of the analytic model with experimental data and Flownex®, an energy analysis and visualisation of the energy and lastly a sensitivity analysis of the energy response. In this chapter, the conclusions for each of the sections are discussed. The conclusion is that the energy representation can be used for the purposes of control and FDD. Further research is needed to design a complete control strategy or FDD system based on the features of the energy visualisation of the system.

6.2 Conclusions

An analytic model was derived for an axial-flow compressor system. This model was based on the model derived by Greitzer in 1978. The one-dimensional non-linear lumped parameter model served as a sufficient model to be used for an energy analysis. This model includes the capability of modelling unstable conditions such as rotating stall and surge. The model could also be extended to include the leak dynamics of a hole in the plenum volume.

One point that should be noted is that the state space representation portrayed a nonlinear model. This is so since there are state variables present in the A matrix. These type of models are not commonly found in literature due to the fact that most models are linearised.

The validation of the state space model was done for two systems. The first was the steady state validation of the three stage axial-flow compressor system investigated by Gill [13]. This validation of the steady state characteristic showed a good correlation between the fitted function used by the analytic model and the steady state response of the practical system of Gill. The validation of the second system with the geometrical dimensions used from Greitzer and Gravdahl was done for the steady state and transient response of the compressor system. The compared steady state response of the analytic and Flownex® model showed a good correlation for an operating point where a low pressure increase is established over the compressor component.

This is due to the fact that Flownex[®] models the fluid throughout the system as a compressible gas, whereas the analytic model models the working fluid as a compressible gas only in the plenum volume. A difference was noted in the transient response of the two models. This was due to the fact that the compressor lag effect is not modelled in the Flownex[®] model. However, it is included in the analytic model. Although there are differences in the response of the analytic and Flownex[®] model, the performance index that was calculated stated that there was a good correlation between the responses of the two models.

During the simulation of the state space model it became evident that a backwards Euler implementation was not sufficient to solve the set of first order differential equations for time increments larger than 0,0001 s. This was noticed by a discrepancy in the stored energy of the throttle. The solution to this problem was the implementation of the state space model in Simulink[®]. Here a variable step solver was used to solve the problem encountered by the backward Euler method.

The energy analysis was done on the system by modelling the interconnected components of the system independently. The power and energy flowing into and out of each node was calculated based on the first law of thermodynamics. In the four test cases investigated, the following was noted.

In test case 1 the compressor system was mapped in terms of a power-energy plane graph. This graph portrayed both the steady state and transient response of the system by using the stored energy and the net power in a selected components. In this test case the healthy energy response was documented for the compressor system when the system was transitioned between two selected operating points.

In the second test case the energy response of the different components changed when a leak was introduced in the system. This revealed that different features, such as the area enclosed by the trajectory, the length of the trajectory and distance between steady operating points, could be extracted from these power-energy planes. These features could be used for the detection and unique isolation of faults in a system by generating residuals from these features.

Test case 3 and 4 investigated the energy response of the compressor system when the system operated in the unstable region. The same oscillation as in the state variables were noted in the energy response. These test cases also revealed that the power and energy responses are more sensitive during the operation in the unstable region, compared to the state variables of the system.

The energy response portrayed the nonlinearities found in the compressor and throttle components. These nonlinearities are noticed in the residuals generated in test case 2 of Chapter 5.

The sensitivity of the power-energy response was analysed by a parametric sweep of a fault parameter, input parameter and a geometric parameter of the compressor system. It was found that the trajectory of the power-energy planes uniquely changed for each parametric sweep. The systems efficiency decreased for a larger leak in the plenum. In addition, the pressure ratios changed for each parameter and operating point where the system was operated.

The study made two contributions. The first is the implementation of a dynamic leak in the plenum of the compressor system. The equations were derived to model a leak in the plenum volume by specifying the size of the leak. The second contribution relates to an energy representation. The concept of a power-energy plane is derived from the concept of representing the response of a system on a phase portrait graph. The net energy was used in this representation, since features of the steady state and transient responses can be extracted from this representation.

6.3 Recommendations

It is recommended that a lumped parameter model be used when an energy analysis is conducted on a system. Demonstrating the usefulness of the energy visualisation, a controller could be designed to control the system based on its energy. Further investigation could include the comparison of the controller to conventional control systems. The complexity of the model could be increased by taking temperature changes into account for further investigations. This will lead to a more accurate model that could be used for an FDD system. Further investigation is needed to extract all the possible features from the energy visualisation and to generate residuals for these features. The analysis of the exergy of the system could further help with the optimisation of the systems energy. Concepts such as an entropy-enthalpy graph could be investigated for such a system. Lastly it is recommended that a variable stiff solver should be used to solve the first order differential equations of the system.

6.4 Closure

The objectives of deriving a nonlinear analytic model for a compressor system that includes fault conditions and instabilities were achieved. The model was extended to include a leak in the system and was successfully validated with experimental data and with the system CFD software package, Flownex®. The energy in the system was visually presented and the usefulness of the representation were proved by the residual generated and sensitivity analysis of the system.

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A APPENDIX

SIMULIK® AND FLOWNEX® IMPLEMENTATION

A.1 Simulink® model

The state space model derived in Chapter 3 was implemented in Simulink®. The implementation of this model is shown in Figure 46. An ode23 Rosenbrock stiff solver was used to solve the set of first order differential equations. The solver was selected so a variable-step approach could be followed.

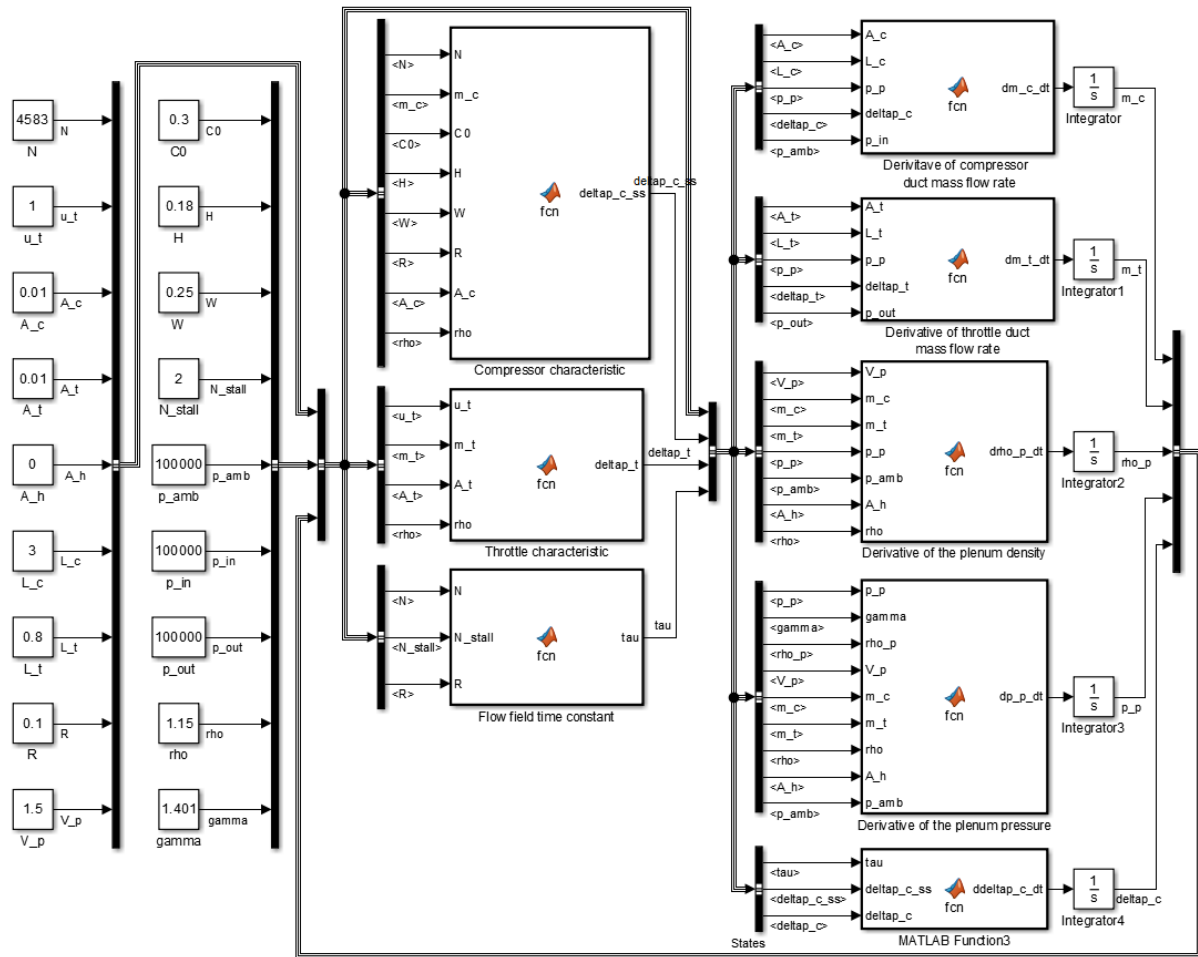


Figure 46: Implementation of the analytic model in Simulink®

A.2 Flownex® model setup

Three components were used for the implementation of the compressor system in Flownex®: a compressor stage component, a reservoir and nonlinear flow resistance. The implementation of the compressor stage and flow resistance is discussed for the compressor system used in Chapter 3.

A.2.1 Compressor stage component

The compressor component requires a generalised compressor chart that relates the flow coefficient ϕ to the pressure rise coefficient ψ . The flow and pressure rise coefficients are generalised by the two relations given by

$$\phi_{gen} = \frac{\phi}{\phi_{ref}} \text{ and } \psi_{gen} = \frac{\psi}{\psi_{ref}}, \quad (\text{A.18})$$

with ϕ_{nor} the generalised flow coefficient, ψ_{nor} the generalised pressure rise coefficient, ϕ_{ref} the reference flow coefficient and ψ_{ref} the reference pressure rise coefficient. The mass flow rate is calculated from the flow coefficients by

$$\frac{c_p \Delta T_0}{N^2 D^2} = f \left(\frac{\dot{m}}{\rho_{01} N D^3} \right), \quad (\text{A.19})$$

with c_p the specific heat at constant pressure, U the local blade speed, D the impeller diameter, N the rotor speed, $\dot{m}$ the mass flow rate through the compressor and ρ_{01} the density. The temperature difference ΔT_0 of a compressor is defined by

$$\Delta T_0 = T_{0e} - T_{0i}, \quad (\text{A.20})$$

with T_{0e} the temperature at the exit of the compressor and T_{0i} the temperature at the inlet of the compressor. From the continuity, $\dot{m} \propto \rho_{01} C_x D^2$, the loading temperature ζ or the flow coefficient ϕ is defined by

$$\zeta = \frac{c_p \Delta T_0}{U^2} = f \left(\frac{C_{x1}}{U} \right) = f(\phi), \quad (\text{A.21})$$

with C_{x1} the inlet absolute velocity in the axial direction. The pressure rise coefficient is calculated from the inlet and outlet pressure by

$$\psi = c_p T_{01} \left[\left(\frac{p_{03}}{p_{01}} \right)^{\frac{(\gamma-1)}{\gamma}} - 1 \right] / U^2, \quad (\text{A.22})$$

with T_{01} the temperature at the inlet, p_{01} the pressure at the inlet, p_{03} the pressure rise across the compressor and γ the ratio of specific heats. The pressure rise across the compressor, as a function of the pressure rise coefficient, is given by

$$p_{03} = p_{01} \left(\frac{\psi U^2}{c_p T_{01}} + 1 \right)^{\frac{\gamma}{\gamma-1}}. \quad (\text{A.23})$$

Equations (A.19) and (A.23) are used to calculate the mass flow rate and pressure rise across the compressor from the non-dimensional flow and pressure rise coefficients.

A.2.1.1 Compressor characteristics for the compressor investigated by Gill

The non-dimensional compressor characteristic of the compressor system investigated by Gill in [13] is generalised with the relations given in (A.18). The generalised non-dimensional compressor characteristic for this compressor implemented in Flownex[®] is shown in Figure 47.

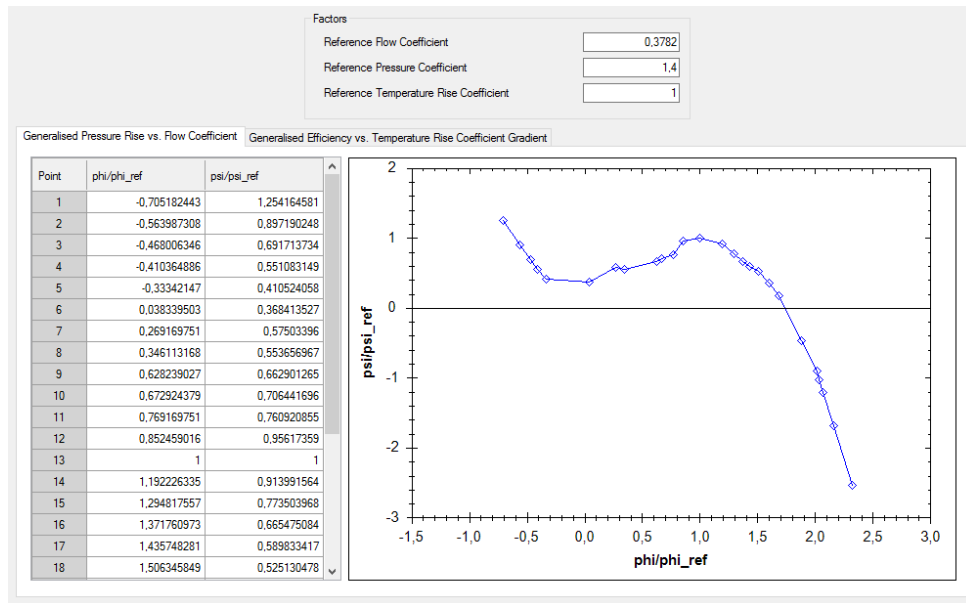


Figure 47: Implementation of the generalised compressor characteristic for the system investigated by Gill in Flownex[®]

A.2.1.1 Compressor characteristics for the compressor investigated by Gravdahl

The non-dimensional compressor characteristic of the compressor system investigated by Gill in [13] is generalised with the relations given in (A.18). The generalised non-dimensional compressor characteristic for this compressor implemented in Flownex® is shown in Figure 47.

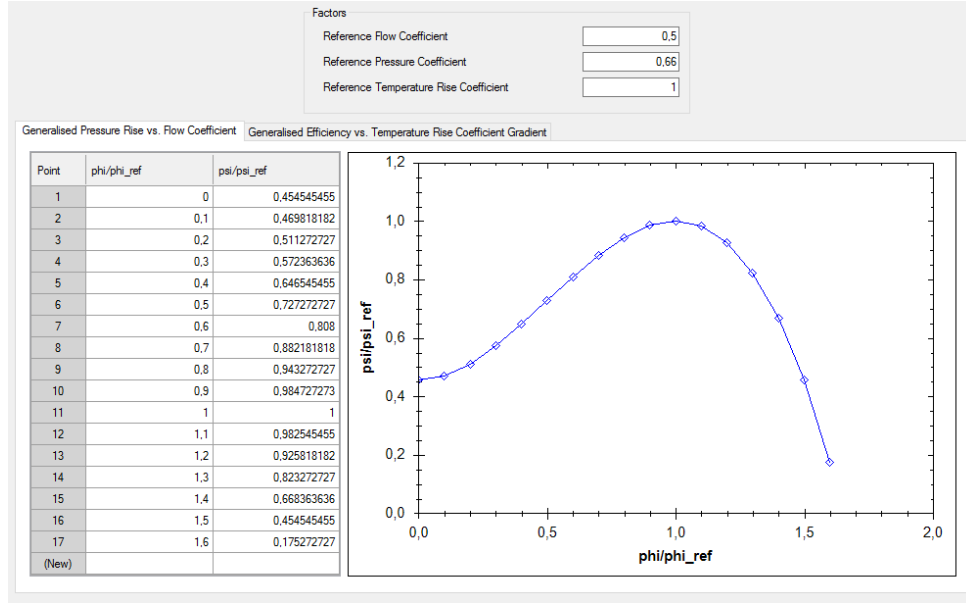


Figure 48: Implementation of the generalised compressor characteristic for the system investigated by Gravdahl in Flownex®

A.2.2 Throttle stage component

The throttle is implemented in Flownex® as a nonlinear flow resistance with a quadratic behaviour. The pressure drop across the flow resistance is defined in Flownex® by

$$\Delta p_0 = \frac{|\dot{m}| \dot{m}}{A_f A_{sf} A_o} + pg \Delta z, \quad (\text{A.24})$$

with $\dot{m}$ the mass flow rate through the throttle, A_f the flow admittance, A_{sf} the admittance scaling factor and A_o the opening size. The term that includes the admittances and the opening size is given by

$$\frac{1}{A_f A_{sf} A_o} = \left(\frac{fL}{d} \right) \frac{1}{2\rho A^2}, \quad (\text{A.25})$$

with f the friction factor, L the length of the pipe, D the inside diameter of the pipe, ρ the fluid density and A the flow area. By setting the pressure drop across the throttle of the analytic model equal to that of the Flownex[®] model and disregarding the pressure drop due to height, the pressure drop is given by

$$\frac{\dot{m}_t^2}{2u_t \rho A^2} = \frac{|\dot{m}| \dot{m}}{A_f A_{sf} A_o} . \quad (\text{A.26})$$

For this equality, where $\dot{m}_t = \dot{m}$, $u_t = A_o$ and $A_{sf} = 1$, the admittance can be calculated by

$$A_f = 2\rho A_t^2 . \quad (\text{A.27})$$

This admittance value is used as an input to the throttle component in Flownex[®].

B APPENDIX

FEATURES OF THE POWER-ENERGY PLANES

B.1 Normalised trajectory length

All the lengths in this section are non-dimensional lengths. The trajectories is normalised by the max value of the power and energy in the power-energy planes. The normalised energy of a trajectory is defined by

$$E_{nor} = \frac{E}{E_{\max}}, \quad (B.1)$$

with E the energy and $E_{\max}$ the maximum value of the energy. The normalised power for a trajectory is defined by

$$P_{nor} = \frac{P}{P_{\max}}, \quad (B.2)$$

with P the energy and $P_{\max}$ the maximum value of the energy. The lengths of the trajectories are calculated by using the normalised values and shown in the next section.

B.2 Power-energy plane features for a variation of the leak size

Table 18: Features of the power-energy plane of the compressor node for a variation of the leak size

Leak size [m ²]	Top line length [-]	Bottom line length [-]	Trajectory length [-]	Pressure ratio at p1 [-]	Pressure ratio at p2 [-]	Efficiency at p1 [%]	Efficiency at p2 [%]
0	3.105	1.302	4.408	1,0075	1,0161	100,00	100,00
0,0002	2.952	1.274	4.227	1,0073	1,0157	96,81	97,36
0,0004	2.813	1.251	4.065	1,0071	1,0152	93,75	94,60
0,0006	2.685	1.230	3.916	1,0069	1,0148	90,83	91,75
0,0008	2.564	1.211	3.776	1,0066	1,0143	88,02	88,87
0,001	2.452	1.191	3.644	1,0064	1,0138	85,34	86,00

Table 19: Features of the power-energy plane of the plenum node for a variation of the leak size

Leak size [m ²]	Top line length [-]	Bottom line length [-]	Trajectory length [-]
0	2.130	2.893	5.023
0,0002	2.073	2.791	4.865
0,0004	2.017	2.695	4.713
0,0006	1.962	2.603	4.566
0,0008	1.909	2.515	4.425
0,001	1.857	2.430	4.288

Table 20: Features of the power-energy plane of the throttle node for a variation of the leak size

Leak size [m ²]	Top line length [-]	Bottom line length [-]	Trajectory length [-]
0	2.901	3.391	6.292
0,0002	2.776	3.245	6.022
0,0004	2.665	3.106	5.772
0,0006	2.552	2.974	5.527
0,0008	2.439	2.848	5.287
0,001	2.326	2.729	5.056

B.3 Power-energy plane features for a variation of the rotor speed

Table 21: Features of the power-energy plane of the compressor node for a variation of the rotor speed

Rotor speed [r/min]	Top line length [-]	Bottom line length [-]	Trajectory length [-]	Pressure ratio at p1 [-]	Pressure ratio at p2 [-]
4583	2.549	1.071	3.620	1,0075	1,0161
4683	2.651	1.110	3.761	1,0078	1,0167
4783	2.758	1.151	3.909	1,0081	1,0173
4883	2.867	1.193	4.061	1,0085	1,0179
4983	2.978	1.236	4.215	1,0088	1,0185
5083	3.092	1.281	4.373	1,0091	1,0191

Table 22: Features of the power-energy plane of the plenum node for a variation of the rotor speed

Rotor speed [r/min]	Top line length [-]	Bottom line length [-]	Trajectory length [-]
4583	1.884	2.563	4.448
4683	1.931	2.615	4.547
4783	1.979	2.669	4.649
4883	2.028	2.723	4.751
4983	2.076	2.777	4.854
5083	2.125	2.832	4.958

Table 23: Features of the power-energy plane of the throttle node for a variation of the rotor speed

Rotor speed [r/min]	Top line length [-]	Bottom line length [-]	Trajectory length [-]
4583	2.608	2.226	4.834
4683	2.749	2.323	5.072
4783	2.894	2.437	5.332
4883	3.044	2.554	5.599
4983	3.198	2.675	5.874
5083	3.357	2.799	6.156

B.4 Power-energy plane features for different volumes of the plenum

Table 24: Features of the power-energy plane of the compressor node for different plenum sizes

Plenum volume [m ³]	Top line length [-]	Bottom line length [-]	Trajectory length [-]	Pressure ratio at p1 [-]	Pressure ratio at p2 [-]
1,5	3.108	1.304	4.412	1,0075425	1,0160844
1,6	2.926	1.242	4.168	1,0075425	1,0160838
1,7	2.767	1.190	3.958	1,0075425	1,0160836
1,8	2.624	1.144	3.769	1,0075425	1,0160832
1,9	2.496	1.103	3.599	1,0075425	1,0160827
2	2.379	1.065	3.444	1,0075425	1,0160821

Table 25: Features of the power-energy plane of the plenum node for different plenum sizes

Plenum volume [m ³]	Top line length [-]	Bottom line length [-]	Trajectory length [-]
1,5	2.040	2.780	4.820
1,6	2.062	2.760	4.823
1,7	2.085	2.750	4.836
1,8	2.108	2.750	4.858
1,9	2.130	2.759	4.890
2	2.152	2.777	4.929

Table 26: Features of the power-energy plane of the throttle node for different plenum sizes

Plenum volume [m ³]	Top line length [-]	Bottom line length [-]	Trajectory length [-]
1,5	2.832	3.373	6.205
1,6	2.805	3.369	6.174
1,7	2.796	3.366	6.163
1,8	2.790	3.364	6.154
1,9	2.784	3.362	6.147
2	2.780	3.360	6.140

C APPENDIX ARTICLE CONTRIBUTIONS

C.1 Article contributions

The first article was submitted to the 2015 South African Universities Power Engineering Conference (SAUPEC). This article focused on the modelling of an axial-flow compressor system, a steady state energy analysis and the visualisation of the steady state power and energy.

The second article was submitted for the 2016 symposium on Dynamics and Control of Process Systems, including Biosystems (DYCPOS-CAB). At the time of the submission of the dissertation the article was still under review. This article focused on the transient power-energy visualisation of an axial-flow compressor system for the purposes of FDD.

NONLINEAR STATE SPACE MODELLING OF AN AXIAL-FLOW COMPRESSOR SYSTEM FOR ENERGY VISUALISATION

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Abstract: Energy is associated with all engineering disciplines including electrical, mechanical and chemical engineering, and therefore a multi-domain concept. An energy visualisation for large-scale industrial systems can portray the system's steady state response from an energy perspective. If a controller can be based on the energy in a system, then any discipline can form a reasonable understanding of how to optimise a system from an energy perspective. This paper aims to derive a nonlinear model for an axial-flow compressor system similar to Greitzer (1986) in state space format. From the state variables the steady state energy flow rate in the system is calculated and visually presented. Evaluating the representation, a clear concept of the energy flow of the system is formed to determine the energy conversion at two different operating points.

Keywords: Axial-flow compressor, state space, nonlinear, energy, visualisation

1. INTRODUCTION

A model of a large-scale industrial system is most often derived from the conservation of mass, momentum and energy equations. Such a set of equations can either be linear or nonlinear depending on the complexity of the system. These equations are an accurate depiction of the system, but in most cases these models have been linearised at a certain operating point. The limitation of a linear model is that dynamics over a large operating range cannot be represented accurately.

Compressors form part of many of the systems found in industry. One of the most common systems that include a compressor is the Brayton-cycle. This system consists of a compressor, combustor and a turbine. Compressors can be classified into two types namely: positive displacement and dynamic compressors. Dynamic compressors are a type of continuous-flow compressors and unlike positive displacement compressors, they deliver a fixed volumetric flow [1]. Dynamic compressors can then be categorised into axial-flow compressors and centrifugal compressors. In axial-flow compressors the flow leaves the compressor in the axial direction and in centrifugal case, the air leaves in a perpendicular direction with respect to the shaft [2].

Greitzer developed a nonlinear one dimensional lumped-parameter model for an axial flow compressor in [3] to predict the transient response of a compressor system and to determine if compressor instability, rotating stall or surge will be encountered by operating the compressor near the stall line. In [4] and [5] a model was presented for the analysis of stall effects of a multi-stage axial compressor.

Considering literature focusing on the analysis of multi-domain nonlinear systems, passivity-based approaches came out as the most prevalent. The underlying idea

behind passivity-based techniques is the consideration of complex systems as energy manipulation devices. Lyapunov theory stemming from stability analysis also plays a vital role. Recent literature also considers complex systems in port-Hamiltonian framework which combines a bond-graph approach with theoretical mechanics. These approaches solely rely on energy principles [6].

In a multi-domain environment where thermal, hydraulic and the electric domains are present, a controller can be designed to control a nonlinear model from first principles of energy and to visually represent the energy and energy flow in the system.

An energy visualisation can be in the form of an energy signature for the system at a specified operation point. From these operating points it can be determined if the system is operating at maximum efficiency. Condition monitoring and fault detection and isolation (FDI) can be done on the system by using the energy visualisation.

The contribution of this paper is to describe the nonlinear, one-dimensional, lumped-parameter compressor model in state space format and evaluate the energy with Bernoulli's equation for the flow and kinetic energy of the system from the state variables. The paper also focuses on visually presenting the energy flow of the system.

The paper starts in section 2 with the derivation of the model for an axial-flow compressor system. In section 3 the state space model for the system is derived. Section 4 shows the simulation of the state space equations and the response of the state variables for different operating points. An energy visualisation is depicted in section 5 for the compressor system for different operating points and the paper concludes with future work and a conclusion of the findings in this paper.

2. MODEL DERIVATION

From the conservation of the mass and momentum equations, the model consisting of the compressor, plenum and throttle for the compressor system is derived. Unlike more recent compressor models the equations are preserved in dimensional form and the derivation of the states is kept with respect to time. The compressor system for which the equations are derived is depicted in figure 1.

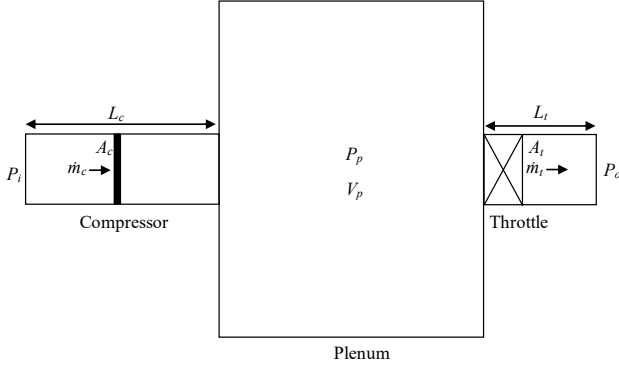


Figure 1 : Diagram of compressor system

A quadratic equation is used to describe the operation of the throttle and a cubic equation is used for the compressor's steady state response. This yields a nonlinear model for the compressor system. A result of the derivation is a one-dimensional, lumped-parameter model describing the response of the compressor system.

2.1 Mass and momentum equations

The first order state equations for the compressor's mass flow, throttle mass flow, plenum pressure and compressor's pressure rise are given by

$$\frac{d\dot{m}_c}{dt} = \frac{A_c}{L_c} (\Delta p_c - \Delta p), \quad (1)$$

$$\frac{d\dot{m}_t}{dt} = \frac{A_t}{L_t} (\Delta p - \Delta p_t), \quad (2)$$

$$\frac{dp_p}{dt} = \frac{a^2}{V_p} (\dot{m}_c - \dot{m}_t), \quad (3)$$

$$\frac{d\Delta p_c}{dt} = \frac{1}{\tau} (\Delta p_{c,ss} - \Delta p_c), \quad (4)$$

where A_c [m²] and A_t [m²] denote the active compressor and throttle area, L_c [m] and L_t [m] the length of the compressor and throttle duct, Δp [Pa] the pressure difference from the plenum to atmosphere, Δp_t the pressure difference across the throttle, $\Delta p_{c,ss}$ the steady state compressor pressure rise, a [m/s] the speed of sound and V_p [m³] the plenum volume. The compressor's time lag τ is given by

$$\tau = \frac{2\pi R N_{Stall}}{U}, \quad (5)$$

with R [m] the mean compressor radius, N_{Stall} the number of revolutions required to develop a stall cell and U [m/s] the mean rotor velocity.

2.2 Compressor characteristic

The compressor is modelled as an actuator disk, therefore energy is added to the system by the compressor in the form of a pressure rise in the working fluid. Equation (6) assumes a cubic equation to describe the compressor's steady state pressure rise for a given mass flow rate and rotor speed as

$$\Delta p_{c,ss} = \frac{C_0 (NR\pi)^2 \rho}{900} - H \left(\frac{15m_c^3}{\pi R N (\rho A_c W)^2} - \frac{3m_c^2}{2\rho (A_c W)^2} \right), \quad (6)$$

with C_0 the shut off value, H and W the compressor semi-height and width and N the rotor speed.

2.3 Throttle characteristic

A quadratic equation is used to describe the throttle characteristic for a given mass flow rate and throttle position so that the pressure drop over the throttle is given by

$$\Delta p_t = \frac{u_t \dot{m}_t^2}{2\rho A_t^2}, \quad (7)$$

with u_t the throttle position and ρ [kg/m³] the density.

3. STATE SPACE MODEL

The states variables of the compressor system are chosen as follows:

$$\begin{aligned} x_1(t) &= \dot{m}_c(t), \\ x_2(t) &= \dot{m}_t(t), \\ x_3(t) &= p_p(t), \\ x_4(t) &= \Delta p_c(t). \end{aligned} \quad (8)$$

with $\dot{m}_c$ [kg/s] the compressor mass flow rate, $\dot{m}_t$ [kg/s] the throttle mass flow rate and p_p [Pa] the plenum pressure of the system. The state differential equation [7] is then given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \quad (9)$$

where the $\mathbf{A}$ and $\mathbf{B}$ matrices are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & -\frac{A_c}{L_c} & \frac{A_c}{L_c} \\ 0 & -\frac{x_2(t)}{2\rho L_t A_t} & \frac{A_c}{L_t} & 0 \\ \frac{a^2}{V_p} & -\frac{a^2}{V_p} & 0 & 0 \\ \frac{-15Hx_1^2(t)}{\tau\rho^2\pi R A^3 W^3} + \frac{3Hx_1(t)}{2\tau\rho A^2 W^2} & 0 & 0 & \frac{1}{\tau} \end{bmatrix}, \quad (10)$$

$$\mathbf{B} = \begin{bmatrix} \frac{A_c}{L_c} & 0 & 0 \\ 0 & -\frac{A_c}{L_t} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{C_0\pi^2\rho R^2}{900\tau} \end{bmatrix}. \quad (11)$$

It can be seen that the A-matrix contains some of the state variables and therefore the state space model can be considered as nonlinear. The output equation that relates the inputs of the system with the outputs of the system is given by

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \quad (12)$$

4. SIMULATION

4.1 Initial values and parameters

The initial values for the states are given in (13) with the corresponding units of the state variables as discussed in (8).

$$\mathbf{x}_0 = [0.3 \quad 0.3 \quad 101700 \quad 1700]^T \quad (13)$$

The parameters used to describe the system are given in table 1. These parameters remain fixed for the duration of the simulation.

Table 1 : Simulation values

Parameter	Value	Parameter	Value
a	340 [m/s]	N_{stall}	2
A_c	0.1 [m ²]	p_i	100 [kPa]
A_t	0.1 [m ²]	p_o	100 [kPa]
C_0	0.3	ρ	1.15 [kg/m ³]
L_c	3 [m]	R	0.1 [m]
L_t	0.8 [m]	V_p	1.5 [m ³]
N	4591 [rpm]		

4.2 System response

The compressor system is started from a steady state operating point with a throttle position u_t of 4. Instantaneously the throttle position u_t is changed to 1 and

a new steady state operating point is reached within a time window of 0.3 s.

In figure 2 the compressor pressure rise is superimposed on the compressor and throttle characteristics. The intersection of these two characteristics determines the steady operating point of the compressor system.

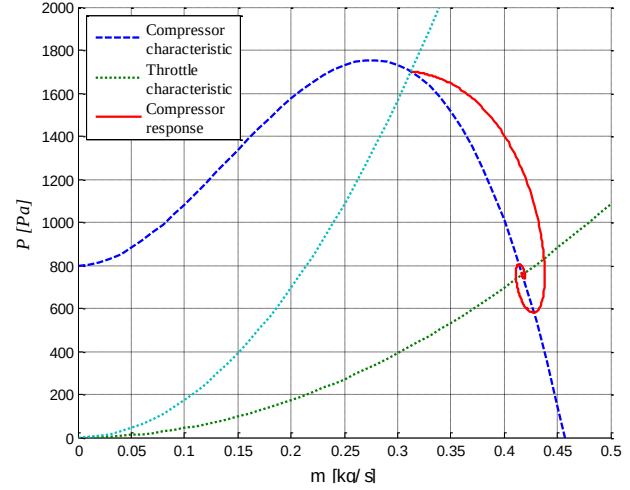


Figure 2 : Compressor pressure rise response superimposed on system characteristic

The response of the four states of the system with respect to time is shown in figure 3. It can be seen that all of the states settle to a new value when steady state is reached for the new throttle position.

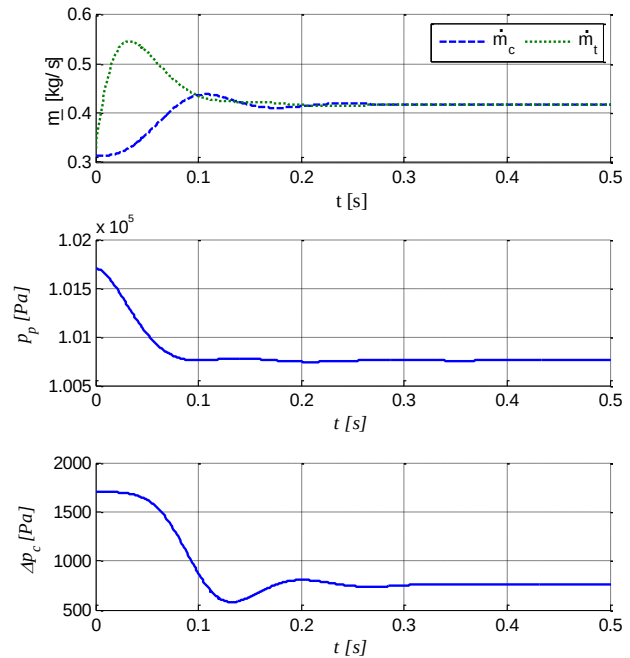


Figure 3 : Transient response of state variables

4.3 Model validation

Flownex® is a systems CFD software package capable of simulating thermo-fluid systems. Flownex® incorporates a variety of components and the components are validated.

The values for different steady state operating points for both the derived model and a Flownex® model are shown in figure 4. Only an operating range where the compressor is stable is shown in the figure.

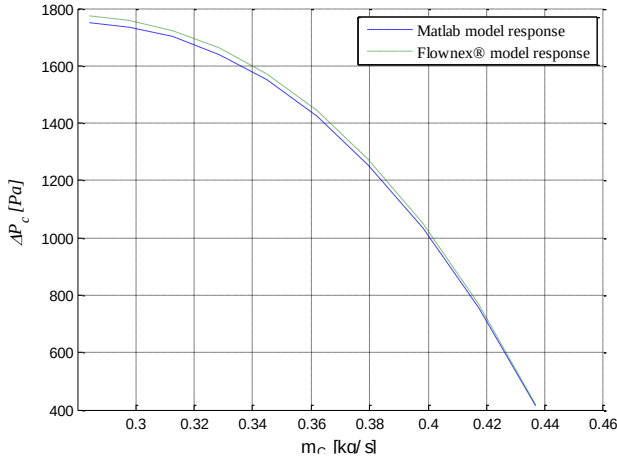


Figure 4 : Steady state response comparison of derived model and Flownex®

The percentage error for all operating points is calculated and shown in figure 5. The error is due to the fact that Flownex® takes temperature rise across the system into account, where the derived model neglects the temperature rise.

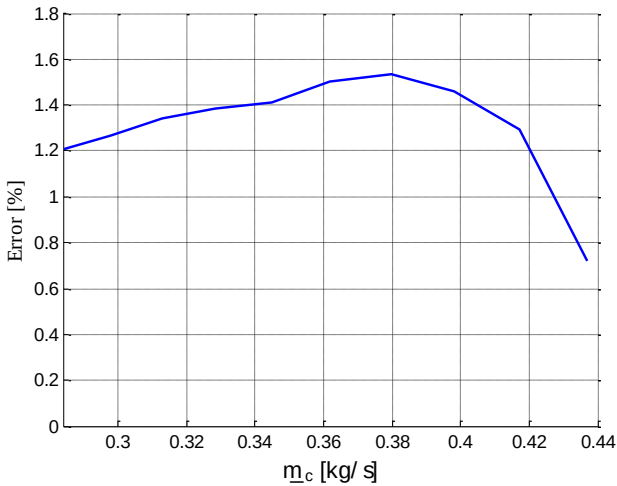


Figure 5 : Difference of the derived model and Flownex® model as a percentage error

5. ENERGY VISUALISATION

Energy is a fundamental concept in systems and can be used to describe multi-domain systems. Therefore energy can be seen to connect all domains and can depict a systems state using energy as the unifying domain [8].

5.1 Bernoulli equation

The Bernoulli equation is concerned with the conservation of kinetic, potential and flow energy to describe the mechanical energy of a particle in a streamline [9]. Bernoulli's equation is an approximate relation between the pressures, velocity and elevation. Bernoulli's equation is derived by using the conservation of linear momentum principle and can be evaluated at any point in a streamline where pressure, velocity, density and elevation are known for steady incompressible flow, and is given by

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{const}, \quad (14)$$

with P [Pa] the pressure, ρ [kg/m³] the density, V [m/s] the velocity, g [m/s²] the gravitational acceleration and z [m] the height of a particle from a reference point. The first term P/ρ is recognized as the flow energy, $V^2/2$ as the kinetic energy and gz as the potential energy. The equation that relates to the pressure to the energy flow rate can be calculated from the Bernoulli equation as

$$E_f = \dot{m} \left(\frac{P}{\rho} \right), \quad (15)$$

to obtain the flow energy flow rate or a type of potential energy flow rate due to pressure E_f [J/s]. The energy flow rate that relates to the velocity can be calculated by

$$E_k = \dot{m} \left(\frac{V^2}{2} \right), \quad (16)$$

to obtain the kinetic energy flow rate E_k [J/s]. The potential energy flow rate can also be calculated by multiplying the potential energy by the mass flow rate.

5.3 Node selection

The selection of nodes in the compressor system is done such that each node can represent both the pressure and mass flow rate of the corresponding component at a node. The selection of nodes must also enable the evaluation of the flow energy rate at each node. The node diagram of the system is depicted in Figure 6.

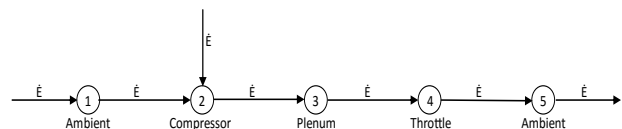


Figure 6 : Compressor system node diagram

Nodes 1 and 5 represent ambient conditions and nodes 2, 3 and 4 represent the compressor, plenum and throttle respectively.

Energy enters the node 2 from node 1 as well as external energy that is converted by the compressor to flow energy. Because the model does not account for losses in the system, the energy that enters node 3 will flow through node 4 and exit at node 5. In the flow of energy through the nodes the energy is only converted to and from flow and kinetic energy.

5.3 Steady state energy flow rate

In this section the energy flow rate for the flow and kinetic energy of the compressor system at the nodes is graphically portrayed through bar graphs. The potential energy is disregarded because, it is assumed that the height change is negligible small. In steady state it is assumed that the mass flow rate is steady and the density at the different nodes is fixed, therefor the energy flow rate for a certain operating point of the system can be evaluated. A visualisation cannot be analysed with the standard Bernoulli equation when the compressor enters rotating stall or surge, since the flow is unsteady under these conditions and Bernoulli's equation does not hold in the case of unsteady flow.

The results are obtained for a pressure reference of 100kPa. The compression system is initially set to a throttle position that restricts the mass flow of the system and results in a steady operating point. Figure 6 depicts the total energy flow rate entering and exiting each node.

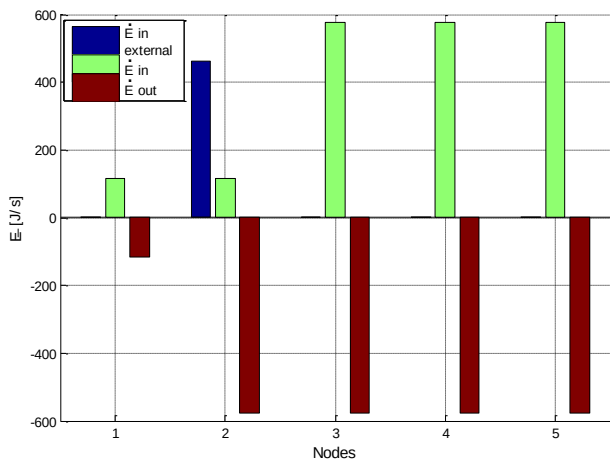


Figure 6: Total energy flow rate with a throttle setting of 4

The total energy flow rate for each node is discretised into the flow and kinetic energy components. This can be seen in figure 7 and how the energy is converted to and from flow and kinetic energy at nodes 2 and 4.

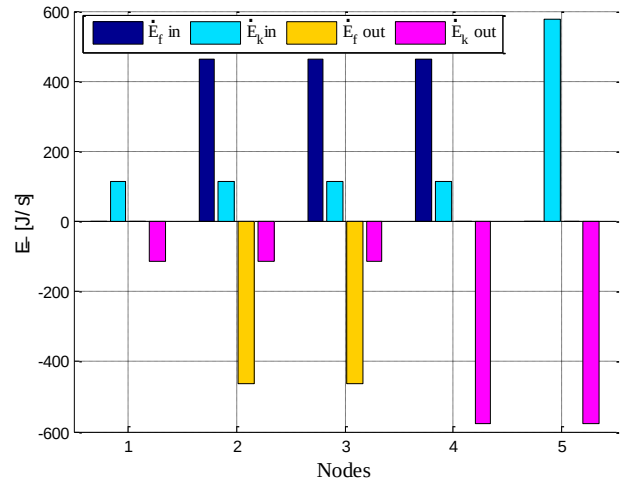


Figure 7 : Energy flow rate with a throttle setting of 4 (discretised)

When the throttle position is changed, meaning that the throttle restriction of the flow changes, a new steady state operating point is reached within 0.3 seconds. This leads to a change in state variables, thus changing the energy flow rate.

The energy flow rate for the new throttle position is shown in figure 8.

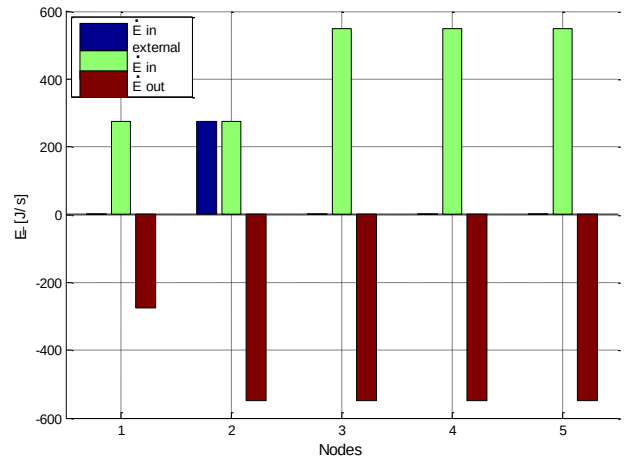


Figure 8: Total energy flow rate with a throttle setting of 1

Again the total energy flow for each node is discretised into the flow and kinetic energy. The new discretised energy flow rate for each node is shown in figure 9.

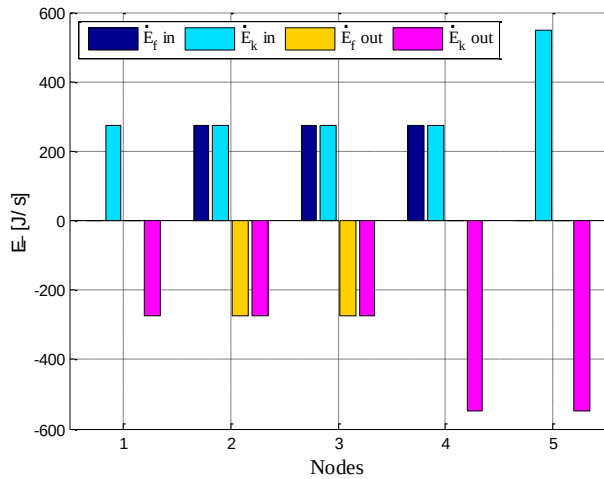


Figure 9: Energy flow rate with a throttle setting of 1 (discretised)

There can be seen that the energy flow rate due to the work done by the compressor decreases and the energy flow rate due to the mass flow rate entering the compressor increases when compared to the energy flow rate of the initial throttle position.

The energy balance is achieved through the summation of the energy flow rate in and out of each node. With this it can be determined if energy is lost at a node and the efficiency of a system can be determined.

6. FUTURE WORK

Future work includes adding a thermal domain to the compressor system and re-evaluating the representation at the different nodes for the flow energy rate. The future research also aims to evaluate the transient energy in each node with the extended Bernoulli equation and describe the stored energy in each node.

From this, the energy requirement to move from one operating point to another can be determined and suggest if the transition is energy efficient.

7. CONCLUSION

Energy visualisation seems relevant to use in research for alternative multi-domain system analysis and optimisation.

A nonlinear model for an axial-flow compressor system accurately depicts the response of the system and the flow and kinetic energy flow rates are determined with the Bernoulli equation from the state variables. From the steady state energy flow representation it is possible to

determine the amount of energy entering and exiting the system and how it changes at different operating points.

Another advantage seen in the visualisation is to distinguish between flow and kinetic energy flow rate and how energy is shaped from one sub-subsystem to another.

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Energy-based visualisation of an axial-flow compressor system for the purposes of Fault Detection and Diagnosis

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Abstract: As the complexity of industrial systems increases, the necessity also increases for more reliable, robust and accurate Fault Detection and Diagnosis (FDD) techniques. This article proposes a methodology for the design of an FDD system, based on an energy-visualisation of an axial-flow compressor system. This method investigates the steady state and transients of the system for residual generation. A nonlinear model is derived for this compressor system, capable of modelling compressor instabilities such as rotating stall and surge as well as leaks. A power-energy analysis is done using the state variables of the system for normal and fault conditions and are visually presented. Two fault conditions are analysed in terms of power and energy. The first test case introduces a leak in the system. The second focuses on the more complex situation when the system enters an unstable operation, namely rotating stall. The results indicate that the power and energy of the system may be useful for the purposes of FDD.

Keywords: Energy-based visualisation, Axial-flow compressor, Fault detection and diagnosis, Rotating stall, leak.

1. INTRODUCTION

As the complexity of large-scale industrial systems increases, issues such as safety, reliability, performance and cost become all the more important. Faults that occur in a system can influence all four of these factors. A properly designed FDD system can minimise the severity of the consequences of these faults.

Gertler (1998) categorised FDD systems as either model-free or model-based. Model-free methods are based on physical redundancy, where additional hardware is used to detect any deviation in the system's operation. Isermann (1997) stated that model-based methods implement analytical redundancy by using a model of the observed system to detect abnormalities. Model-based methods are mostly favoured over model-free methods since they do not impose penalties such as volume, mass and cost according to Bokor and Szabó (2009). Model-based methods such as observer-based, parity-space and parameter identification are well known methods in literature.

García-Matos et.al. (2013) conducted a model-based FDD on an axial-flow compressor of a Combined-Cycle Power Plant. Their methodology is based on a hybrid approach that entails a physical model and a multilayer perceptron (MLP) model for residual generation. This was done for general fault conditions that may occur in a compressor system. Typical faults, such as leaks, blockages and fouling can occur in a compressor systems according to Breese et al. (1992). Leaks are caused by corrosion and development of high-pressures in components that results in an undesired discharge of pressure from the system. Instability are also common in axial-flow

compressors. Gravdahl (1998) discussed two examples of such instabilities namely rotating stall and surge.

Faults cause abnormalities in the response of a system, which in turn result in undesired energy exchanges within the system. Advances in nonlinear energy-based FDD methods have made it possible to monitor this energy exchanges. Chen (2011) has applied this approach to the energy balances of a pendulum and robot manipulator in two test cases.

This paper aims at illustrating the potential of using an energy-based visualisation of an axial-flow compressor system for designing an FDD system. In order to illustrate this potential, a leak is introduced in the compressor system, and the effects on the energy response are analysed. A second test case is investigated, where a change in operating point causes the compressor to enter rotating stall.

In this paper, the state space model of an axial-flow compressor system is derived in Section 2. The model is extended to include the dynamic effects of a leak in the plenum. Section 3 contains the simulation and validation of the model. The validation of the model is done by comparing the model's response to the response of a Flownex[®] model. The two test cases are investigated in section 4 from an energy perspective, one case for a leak and the other for unstable operation. Section 5 concludes the paper with a discussion of the results and planned future work.

2. STATE SPACE MODEL

A number of axial-flow compressor models can be found in literature. These models range from simplistic 1-D models of Greitzer (1976) to complex 3-D models of Takata and Nagano (1972). The set of differential equations in Greitzer (1976) is used for the state space model in this paper. The model is capable of representing the dynamics of both rotating stall and surge. Temperature effects are however not included. The model is further expanded to include the dynamics of a leak in the plenum volume.

The system considered in the derivation of the state space model is shown in Fig. 1. Three main components are identified in the system. These components include the compressor, plenum and throttle. The compressor is located in the inlet duct, while the throttle is located in the outlet duct.

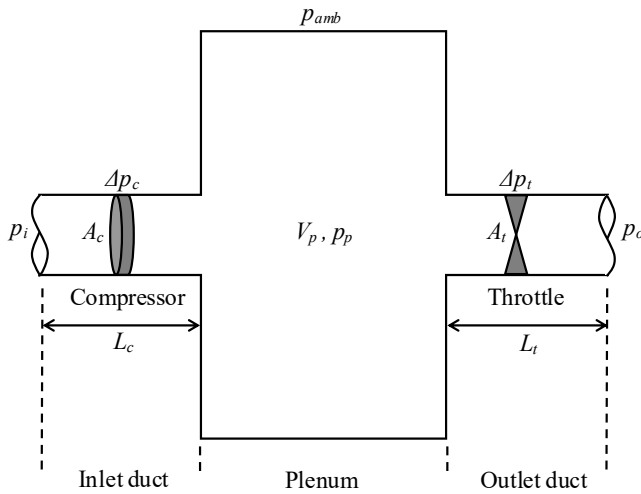


Fig. 1. Compressor system.

The set of first order differential equations is given in section 2.2. These equations govern the dynamics of the system and include the characteristics of the compressor and throttle components.

In section 2.3, a fault condition is implemented within the governing equation of the plenum. In the latter part of this section, the complete model, including the fault condition, is presented in state space format.

2.1 Model parameters

The model parameters and values for the compressor system considered in this paper are given in Table 1.

Table 28. Model parameters

Symbol	Parameter	Value	Unit
a	Speed of sound	340	m/s
U	Compressor mean rotor speed	48	
A_c	Compressor duct area	0.01	m ²
A_t	Throttle duct area	0.01	
L_c	Compressor duct length	3	m

Model parameters (continued)

Symbol	Parameter	Value	Unit
L_t	Throttle duct length	0.8	m
R	Rotor mean radius	0.1	
V_p	Plenum volume	1.5	m ³
ρ	Density	1.15	kg/m ³
C_o	Shut off value of compressor characteristic	0.3	-
H	Comp. characteristic semi-height	0.18	
W	Comp. characteristic semi-width	0.25	
N	Rotor speed	4583	r/min

2.2 Compressor system governing equations

The set of first-order differential equations that describe the dynamics of the compressor system is derived from the conservation of mass and momentum balances for the compressor system. The set of equations is given by

$$\frac{d\dot{m}_c}{dt} = \frac{A_c}{L_c} (\Delta p_c - \Delta p_i), \quad (1)$$

$$\frac{d\dot{m}_t}{dt} = \frac{A_t}{L_t} (\Delta p_o - \Delta p_{t,ss}), \quad (2)$$

$$\frac{dp_p}{dt} = \frac{a^2}{V_p} (\dot{m}_c - \dot{m}_t), \quad (3)$$

$$\frac{d\Delta p_c}{dt} = \frac{1}{\tau} (\Delta p_{c,ss} - \Delta p_c), \quad (4)$$

with p_p the plenum pressure, Δp_c the pressure rise across the compressor, Δp_i the pressure difference between the plenum and inlet and Δp_o the pressure difference between the plenum and outlet. The compressor time lag τ found in (4) is given by

$$\tau = \frac{2\pi N_{stall}}{U}, \quad (5)$$

where N_{stall} is the number of rotor revolutions required to develop a stall cell and is selected as 2, as in Greitzer (1976).

The characteristic that describes the pressure rise across the compressor is modelled by a third-order polynomial function. This function relates the mass flow rate through the compressor $\dot{m}_c$ [kg/s] to the steady state pressure rise across the compressor $\Delta p_{c,ss}$ [Pa], and is given by

$$\Delta p_{c,ss} = \frac{C_o \rho (NR\pi)^2}{900} - H \left(\frac{15 \dot{m}_c^3}{\pi R N (\rho A_c W)^2} - \frac{3 \dot{m}_c^2}{2 \rho (A_c W)^2} \right). \quad (6)$$

The throttle characteristic is defined by a second-order polynomial function. The function relates the mass flow rate through the throttle $\dot{m}_t$ [kg/s] to the pressure drop across the throttle Δp_t [Pa], and is given by

$$\Delta p_{t,ss} = \frac{\dot{m}_t^2}{2u_t \rho A_t^2}. \quad (7)$$

2.3 Fault dynamics

A leak in the plenum is one type of fault that may occur in a compressor system. The result of this fault is a decrease in the performance of the system. Certain effects may be observed

when the fault occurs. The effect of a leak may be included in the mass balance equation as follows:

$$\frac{dp_p}{dt} = \frac{a^2}{V_p} (\dot{m}_c - \dot{m}_t - \dot{m}_h), \quad (8)$$

with $\dot{m}_h$ [kg/s] the mass flow rate through the leak. Deriving the first-order differential equation for the mass flow rate through the leak results in

$$\frac{d\dot{m}_h}{dt} = \frac{A_h}{L_h} (\Delta p_h - \Delta p_{amb}), \quad (9)$$

with A_h [m²] the area of the outflow, L_h [m] the thickness of the brim formed, Δp_h [Pa] the pressure drop across the leak and Δp_{amb} [Pa] the pressure difference between the plenum and ambient pressure.

The mass flow rate through the leak is derived from (9). Two assumptions are made for this derivation. Firstly the thickness of the brim is negligibly small, that is $L_h=0$. Finally, the characteristic of the leak is considered a quadratic polynomial function as in (7), with a fixed opening. The mass flow rate is given by

$$\dot{m}_h = p_p \sqrt{2\rho A_h^2 \left(\frac{1}{p_p} - p_{amb} \right)}. \quad (10)$$

2.4 State space representation

The nonlinear state space model consists of the state differential equation and the output equation as follows:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}\mathbf{u}, \quad (11)$$

$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t). \quad (12)$$

The state variables of the compressor system are selected as in (13) and the input vector as in (14). The state space model allows the next step of analysing the system from an energy perspective.

$$\mathbf{x} = [\dot{m}_c \quad \dot{m}_t \quad p_p \quad \Delta p_c]^T = [x_1 \quad x_2 \quad x_3 \quad x_4]^T \quad (13)$$

$$\mathbf{u} = [p_i \quad p_o \quad N]^T = [u_1 \quad u_2 \quad u_3]^T \quad (14)$$

The $\mathbf{A}$ and $\mathbf{B}$ matrices of the compressor system are given by (15) and (16) respectively.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & -\frac{A_c}{L_c} & \frac{A_c}{L_c} \\ 0 & -\frac{x_2(t)}{2u_1\rho L_t A_t} & \frac{A_t}{L_t} & 0 \\ \frac{a^2}{V_p} & -\frac{a^2}{V_p} & \frac{a^2}{V_p} \sqrt{2\rho A_h^2 \left(\frac{1}{x_3(t)} - p_{amb} \right)} & 0 \\ -\frac{15Hx_1^2(t)}{7\rho^2\pi R N A_c^3 W^3} + \frac{3Hx_1(t)}{2\eta\rho A_c^3 W^2} & 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \quad (15)$$

$$\mathbf{B} = \begin{bmatrix} \frac{A_c}{L_c} & 0 & 0 \\ 0 & -\frac{A_t}{L_t} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{C_0\pi^2\rho R^2}{900\tau} \end{bmatrix} \quad (16)$$

3. MODEL SIMULATION

The state space model is validated with Flownex[®], a systems CFD software package capable of simulating complex thermal-fluid systems. Flownex[®] incorporates a large variety of system components, all of which are validated with measured data. Fig. 2 depicts the compressor system as implemented in Flownex[®].

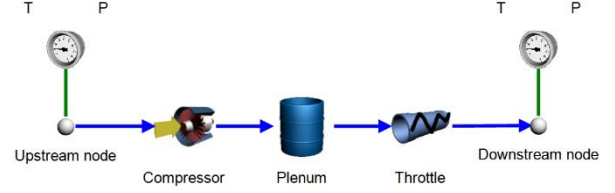


Fig. 2. A Flownex[®] model of the compressor system.

The data for the compressor characteristic are generated from (6), converted to non-dimensional form and introduced to the compressor component in Flownex[®].

3.1 Model simulation and validation

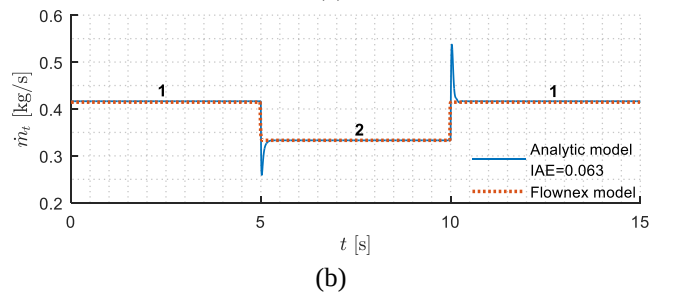
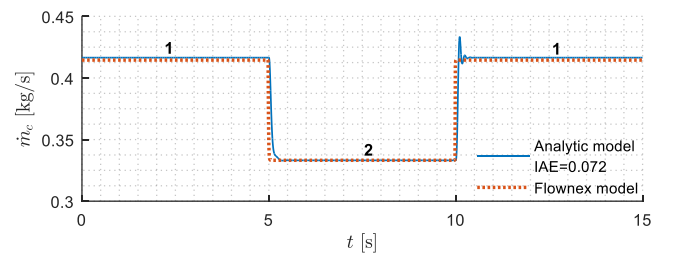
The ambient pressure is chosen as 100 kPa and the initial conditions of the state variables are given by

$$\mathbf{x}_0 = [0.4165 \quad 0.4165 \quad 100754 \quad 754]^T. \quad (17)$$

The inputs to the system are given by

$$\mathbf{u} = [100000 \quad 100000 \quad 4583]^T. \quad (18)$$

The analytic model is simulated and validated by comparing its response to the response of the Flownex[®] model. The compressor system is initially operated at a particular operating point 1 with a throttle opening of 100%. After 5 seconds, the throttle opening is instantly changed to 30% and the system settles at a new operating point 2. At 10 seconds the throttle opening is changed back to 100%, bringing the system back to the initial operating point. The comparison of the state variables is shown in Fig. 3 for the transition between the operating points.



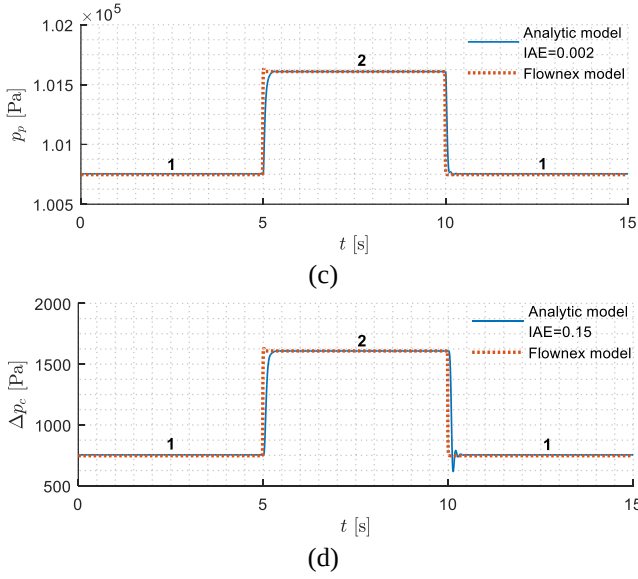


Fig. 3. (a) Compressor mass flow rate. (b) Throttle mass flow rate. (c) Plenum pressure. (d) Compressor pressure rise.

The discontinuities observed in the transients of the state variables are caused by the compressor time lag that is considered in the analytic model. The IAE performance index indicates the error between the response of the analytic and Flownex[®] models. The calculated IAE is shown within the legend of each state variable.

4. ENERGY ANALYSIS

The main contribution of this paper is the energy visualisation based on the energy analysis of the system energy.

Ortega et al. (2001) describe dynamical systems as energy transformation devices. This is advantageous when dividing complex nonlinear systems, such as compressor systems, into sub-systems. The energies of the sub-systems add together to represent the complete system's energy profile.

The compressor system is divided into three nodes for the energy analysis as portrayed in Fig. 4. These nodes are labelled as follows: the compressor node C, the plenum node P and throttle node T. The power flowing in and out of each node is denoted by the subscripts A to F, and the power loss due to a leak is denoted by the subscript L. The stored energy in each node is denoted by subscripts C, P and T.

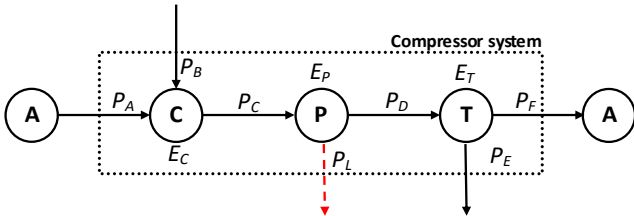


Fig. 4. System node diagram.

The power throughout the compressor system is calculated by

$$P_i(t) = \frac{\dot{m}(t)}{\rho} \times p(t) = Q(t) \times p(t), \quad (19)$$

with $\dot{m}_i$ [kg/s] the mass flow rate, ρ_i [kg/m³] the density, Q_i [m³/s] the volumetric flow rate and p_i [Pa] the pressure. Using the convention that all power values are positive, the net power of an element can be calculated by

$$P_{net}(t) = P_{in}(t) - P_{out}(t) - P_{dis}(t), \quad (20)$$

with P_{in} [J/s] the power in, P_{out} [J/s] the power out and P_{dis} [J/s] the power dissipated. The net power P_{net} calculated in (20) is equal to 0 for steady state conditions.

An imbalance of the power in, power out and power dissipated cause energy to be stored in components. The energy stored can be calculated by the time integral of the net power:

$$E_{stor}(t) = E_0(t_0) + \int_{t_0}^t P_{net}(t) dt, \quad (21)$$

where E_0 [J] is the stored energy at $t = t_0$.

4.1 Steady state energy response under normal conditions

The steady state power is calculated from (19) and the steady state energy is calculated from (21). The power matrix at operating point 1 is given by

$$\mathbf{P}_1 = \begin{bmatrix} P_A & P_B \\ P_C & P_D \\ P_E & P_F \\ P_L & 0 \end{bmatrix} = \begin{bmatrix} 36491.12 & 273.17 \\ 36491.12 & 36491.12 \\ 273.17 & 36491.12 \\ 0 & 0 \end{bmatrix} \quad (22)$$

and the energy vector is given by

$$\mathbf{E}_1 = \begin{bmatrix} E_C \\ E_P \\ E_T \end{bmatrix} = \begin{bmatrix} 22468.77 \\ 882690.57 \\ 5994.93 \end{bmatrix}. \quad (23)$$

When the system operates at operating point 2 the power matrix is given by

$$\mathbf{P}_2 = \begin{bmatrix} 29434.67 & 465.95 \\ 29434.67 & 29434.67 \\ 465.95 & 29434.67 \\ 0 & 0 \end{bmatrix} \quad (24)$$

and the energy vector is given by

$$\mathbf{E}_2 = \begin{bmatrix} 14309.80 \\ 1848086.13 \\ 4390.77 \end{bmatrix}. \quad (25)$$

These matrices indicate the ideal steady state operating points on the power-energy plane and can now be used for residual generation.

4.2 Power and energy residual generation for a leak

Power and energy residuals may be useful to identify faults that can occur in systems. These residuals are investigated by introducing a leak with an area of 0.0008 m² to the plenum of the compressor system. The power matrices for both operating point 1 and 2 are recalculated and given by

$$\mathbf{P}_{1,f} = \begin{bmatrix} 36941.80 & 243.64 \\ 36941.80 & 34205.37 \\ 226.56 & 34205.37 \\ 2736.43 & 0 \end{bmatrix}, \mathbf{P}_{2,f} = \begin{bmatrix} 31746.60 & 447.44 \\ 31746.60 & 27700.66 \\ 390.41 & 27700.66 \\ 4045.94 & 0 \end{bmatrix}. \quad (26)$$

The stored energy at both operating points is given by

$$\mathbf{E}_{1,f} = \begin{bmatrix} 23047.10 \\ 782891.04 \\ 5444.68 \end{bmatrix}, \mathbf{E}_{2,f} = \begin{bmatrix} 16709.39 \\ 1648046.43 \\ 4035.23 \end{bmatrix}. \quad (27)$$

A power residual matrix $\mathbf{r}_{i,p}$ is defined as the difference between the power matrix under normal conditions $\mathbf{P}_i$ and the power matrix under a fault condition $\mathbf{P}_{i,f}$. This matrix is defined as

$$\mathbf{r}_{i,p} = \mathbf{P}_i - \mathbf{P}_{i,f}. \quad (28)$$

The energy residual vector $\mathbf{r}_{i,e}$ is defined as the difference between the energy vector under normal conditions $\mathbf{E}_i$ and the energy vector under fault conditions $\mathbf{E}_{i,f}$ as

$$\mathbf{r}_{i,e} = \mathbf{E}_i - \mathbf{E}_{i,f}. \quad (29)$$

The calculated power and energy residual matrices for operating points 1 and 2 are

$$\mathbf{r}_{1,p} = \begin{bmatrix} -450.68 & 29.53 \\ -450.64 & 2285.75 \\ 46.61 & 2285.75 \\ -2736.43 & 0 \end{bmatrix} \text{ and } \mathbf{r}_{2,p} = \begin{bmatrix} 2311.91 & 18.15 \\ 2311.91 & 1734.01 \\ 75.54 & 1734.01 \\ -4045.94 & 0 \end{bmatrix}. \quad (30)$$

The calculated error vectors for operating point 1 and 2 are

$$\mathbf{r}_{1,e} = \begin{bmatrix} -578.33 \\ 99799.53 \\ 550.25 \end{bmatrix} \text{ and } \mathbf{r}_{2,e} = \begin{bmatrix} -2399.59 \\ 200039.70 \\ 355.54 \end{bmatrix}. \quad (31)$$

The residual matrices and vectors indicate that when a fault occurs in the system, the power flowing through the leak becomes an additional loss. The residuals indicate a shift in the steady operating points to less efficient steady state operating points. This is portrayed in the power-energy planes of Fig. 5-7 in section 4.3.

4.3 Transient energy response under normal and fault conditions

The transient behaviour of the compressor system in the power-energy plane can be observed when a transition between the two operating points occurs. This is shown for the compressor node in Fig. 5, plenum node in Fig. 6 and throttle node in Fig. 7.

A leak is introduced in the plenum with a selected area of 0.0008 m². The result of the leak is a transient from point 1 to 3. The same throttle adjustment is introduced with the leak present. The result is a transition from point 3 to 4 and back.

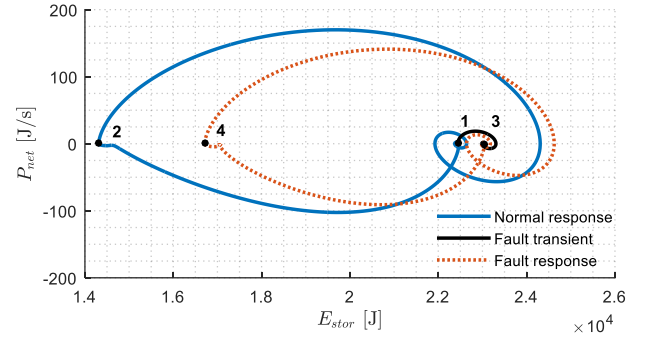


Fig. 5. Compressor net power vs. energy stored for a leak in the plenum.

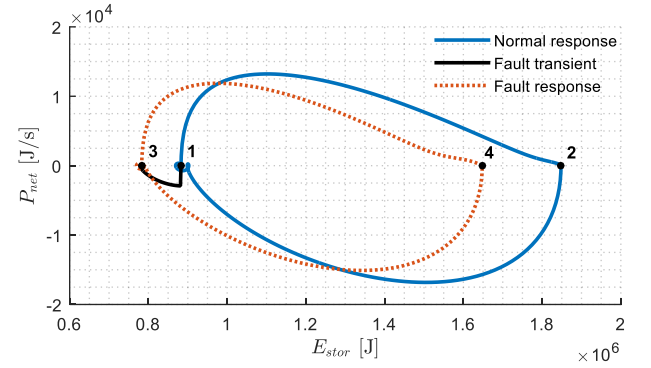


Fig. 6. Plenum net power vs. energy stored for a leak in the plenum.

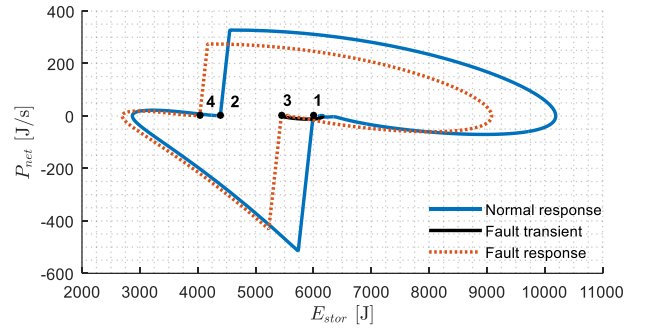


Fig. 7. Throttle net power vs. energy stored for a leak in the plenum.

In Fig. 5-7, the responses of the three components indicate changes in the trajectories and shifts of the power-energy characteristics after a fault has occurred. The compressor dissipates more power, resulting in a slightly smaller net power characteristic. This can be noted in a comparison of the maximum and minimum values of power-energy characteristics before and after the fault. Also, one can see a considerable increase in the stored energy in the compressor node after the fault has occurred. On the other hand, the leak introduced a loss of stored energy in the plenum and throttle node.

From these power-energy graphs, features such as path length, area and centre of mass may be extracted and used for residual generation.

4.4 Rotating stall

An axial-flow compressor is prone to become unstable when operated to the left of the maximum on its compressor characteristic as stated by Greitzer (1998). In such an event, the compressor enters a condition known as rotating stall. According to Gravdahl (1998), rotating stall occurs when the circumferential flow pattern is disrupted. This leads to a decrease in the pressure of the compressor and plenum.

For this test case shown in Fig. 8, the compressor is operated at operating point 1 with a throttle opening of 20%. This operating point occurs near the maximum of the compressor's characteristic. The operating point is changed to point 2 with a throttle opening of 16% that is located to the left of the maximum of the compressor characteristic. At operating point 2, the compressor pressure rise as well as the energy stored in the plenum decreases due to the occurrence of rotating stall. In Fig. 8(a), the power-energy cycle that is portrayed after the transition indicates considerable unwanted energy exchange. This is confirmed in Fig. 8(b) which shows the plenum energy for the shift in operating point.

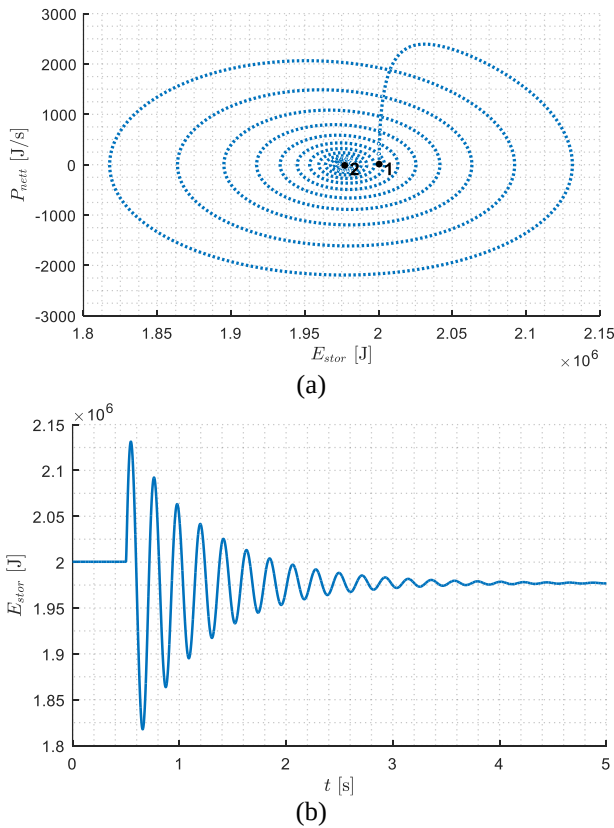


Fig. 8. (a) Plenum net power vs. energy stored in rotating stall. (b) Energy stored time response.

A damped response can be observed in the transient before the response converges to a steady operating point. Such characteristics are to be expected for an unstable operation such as rotating stall. Within the power-energy plane, this operation is also easily differentiated from a fault such as a leak, which does not necessarily cause instability. In fact, it is interesting to note that the introduction of a close-coupled valve was the solution proposed by Gravdahl (1998) for combatting instability of rotating stall.

5. CONCLUSION

Initial experiments have shown the potential of the power-energy plane for calculating residuals. The premise is that a predictable pattern may be observed for faults within these different system components, ultimately leading to the isolation of the fault to a certain component of the system. This is indeed the most easily observed for the positions of the steady state responses. However, generating and analysing a residual for the transient response may also give a clear indication of a fault. It is encouraging to notice that instability is also clearly differentiable from other faults using the energy-visualisation perspective. The authors aim to continue research of this perspective for fault detection and diagnosis. The research further proposes to investigate how the energy representation may be used for control and condition monitoring. There is also interest to see how a fundamental approach to a system's energy relates to modern energy-based control techniques.

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