

An investigation into the distance dependence of storativity in dual porosity aquifers

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Dissertation accepted in fulfilment of the requirements for the degree *Master of Science in Environmental Sciences with Hydrology and Geohydrology* at the North-West University

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Graduation May 2024

ACKNOWLEDGEMENTS

I would like to thank the following people who have helped me undertake this research:

My supervisor Prof. I Dennis for her feedback, support and patience.

The Centre for Water Sciences and Management, for approving my research and continued support.

For Dr. R Dennis for his feedback and guidance during the later stages of my research.

My fellow master's students for their feedback and fellowship.

For their contributions to data collection:

Jan Myburgh at AGES (Pty) Ltd

And finally, my parents, for their financial support, trust, advice and love.

ABSTRACT

Parameterisation of hydraulic systems becomes increasingly difficult in fractured aquifers. Storativity is one of the main parameters used to quantify the available water in aquifers. This dimensionless value is known to be spatially variable in fractured aquifers and is especially difficult to determine with analytical methods. Dual porosity, a feature in some fractured aquifers, involves two distinct types of porosity influencing flow. It can be identified through diagnostic plots comparing drawdown and derivative curves. In this study, approximately 60% of the drawdown data from fractured aquifers exhibited dual-porosity characteristics. The data were analysed using various analytical methods and a two-dimensional numerical model, RPTsolv. The study aimed to evaluate the accuracy of analytical methods compared to RPTsolv and establish correlations between storativity and the distance from the pumping borehole, enhancing our understanding of storativity's spatial variability. This process involved steps like data acquisition, diagnostic analysis, analytical methods, and the use of RPTsolv. The study revealed limitations of analytical methods in calculating storativity. Comparative analyses between analytical methods and RPTsolv suggested that all analytical methods face challenges in consistently determining storativity values at varying distances from the pumping borehole. In the Karoo Supergroup, storativity values derived analytically tended to increase or decrease with distance from the borehole depending on the method used, whereas most analytical methods tested, produced a decreasing trend with distance in the Cape Supergroup. RPTsolv, in contrast, demonstrated consistent storativity determination regardless of distance, maintaining the parameter within the same order of magnitude. These findings led to the development of an effective radius correlation methodology. Applying this methodology to the Cooper-Jacob analytical method enables more consistent storativity values, even without RPTsolv. The dissertation concludes that relying solely on analytical methods for analysing pumping test data in fractured or dual porosity aquifers is unreliable due to their inconsistency. While the numerical model RPTsolv provides more consistent results, its outdated nature of the software is a drawback. In scenarios, where a numerical model isn't feasible due to resource constraints, the developed method is recommended to achieve acceptable ranges of storativity and transmissivity values.

KEYWORDS

Pumping Test Analysis, Analytical Models, Dual Porosity Aquifers, Comparative Analysis, Hydraulic Parameters, Storativity, Cooper-Jacob

LIST OF ACRONYMS

2D	Two Dimensional
3D	Three Dimensional
IARF	Infinite Acting Radial Flow
IRF	Infinite Radial Flow
MODFLOW	Modular finite-difference flow model
REV	Representative Elementary Volume
RMSE	Root Mean Square Error
TMG	Table Mountain Group

NOMENCLATURE

K	Hydraulic conductivity (m/d)
R	Radius of influence of a borehole (m)
r	Borehole radius (m)
r_e	Effective radius (m)
S	Storativity
s	Drawdown (m)
S_f	Fracture Storativity
T	Transmissivity (m^2/d)
T_f	Fracture Transmissivity

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CHAPTER 1 INTRODUCTION

1.1 Introduction

Hydraulic parameters, such as storativity and transmissivity, play a crucial role in determining the ability of an aquifer to yield water (Kirlas & Katsifarakis, 2020). Accurately assessing these parameters are essential for efficiently exploiting the resources of an aquifer. Geohydrologists commonly use pumping tests (constant rate tests, recovery tests, and step discharge tests) to determine an aquifer's hydraulic parameters (Van Tonder *et al.*, 2002). The basic principle of a hydraulic test is to pump water at a controlled rate while measuring drawdown with time, which is then substituted into an appropriate curve fit model to calculate the hydraulic parameters (Kruseman & de Ridder, 1994). However, the evaluation of fractured aquifers poses a challenge due to their heterogeneity and spatial variability of hydraulic parameters, making it difficult to choose the correct model and associated curve fit (Van Tonder *et al.*, 2002:2).

Solutions for mathematical problems may be either analytical or numerical, according to Baalousha (2011:114). Analytical models require little data and can be used for a wide variety of simple problems, while numerical models are inherently more complex and can handle more complex problems than analytical models. However, the response of fractured aquifers to regional groundwater storage changes are too complex for a one-dimensional analytical solution (Osiensky *et al.*, 2000; Baumle, 2003; Neuman *et al.*, 2007; Maréchal *et al.*, 2003). Even though numerical models have been successful in analysing pumping and geohydrological data in modern times (Johnson *et al.*, 2001:582-592; Spiliotopoulos *et al.*, 2006:569-570), their complexity and input requirements sometimes act as a deterrent in terms of use. Despite their shortcomings, analytical models are still popular among geohydrologists due to their ease of use and efficiency with limited data.

Classical analytical analyses make use of radial flow models to simulate the aquifer's response to pumping. These models assume that an aquifer is ideal and homogeneous, meaning that parameters like storativity and transmissivity are constant throughout the aquifer. However, the reality of fractured aquifers is more complex since the flow to the borehole is not only governed by hydraulic gradients but also by the orientation, location, and aperture of individual fractures and the connectivity between these fractures to form a network (Cho *et al.*, n.d.:161-162). The extent of these fractures and the flow paths they form are unknown and spatially variable, making it challenging to account for them even in the most complex analytical models (Nyakeni, 2021:15). Despite these challenges, analytical models are still used by both practitioners and researchers

(Hsieh & Neuman, 1985:1655-1665; Yeh & Huang, 2009:1133-1147; Maréchal *et al.*, 2004) to aid in, or as the sole means of, analysing fractured heterogeneous aquifers.

Fractured aquifers have attracted attention due to their global occurrence and economic importance (Singhal & Gupta, 2010). For example, these basement aquifers underlie continents and extend from Southern China, North and South America, several areas in Europe, the majority of the United Kingdom, and the Southern and Northern parts of Africa. Arid and semi-arid regions are particularly affected since these communities rely on the use of groundwater for their basic needs, as surface water is limited (Dewandel *et al.*, 2011:474-487).

1.2 Problem Statement

The Cooper-Jacob (1946) and Theis (1935) analytical methods are widely used for hydraulic test analysis, but they are inadequate for accurate storativity values (Vivier & van Tonder, 1997; Kollet & Zlotnik, 2005; Baumle, 2003; Mawlood & Mustafa, 2016). Analytical methods based on the double-porosity concept, which describes fractured aquifers, have been more successful in modelling aquifer response but struggle to determine accurate storativity values (Kollet & Zlotnik, 2005:267-285; Burbey & Brandon, 2016:3289-3302). Previous literature highlights the distance dependency of storativity in pumping test analysis, although it is often overlooked (Gringarten, 1982; Verwey *et al.*, 1995; Neuman *et al.*, 2007).

Choosing the appropriate method or approach is complicated as different methods can result in a wide range of parameter values due to varying assumptions and requirements (Verhagen *et al.*, 2001:34-38; Baumle, 2003; Kollet & Zlotnik, 2005:267-285). Additionally, the duration of a pumping test is essential to derive accurate outer boundary conditions and formation behaviour. Insufficient test duration can lead to incorrect parameter values due to unexpected late-time no-flow boundaries in the borehole (Kruseman & de Ridder, 1994:43-44; van Tonder *et al.*, 2002:12; Allen, 1999:66). While the minimum duration is eight hours, a constant rate test should span several days based on aquifer type and conditions (Van Tonder *et al.*, 2002:12; Allen, 1999:66). However, the high costs associated with long-duration tests limit their practicality.

To address these challenges, alternative approaches have been explored. Complementary field data acquisition techniques such as tracer tests (Huang *et al.*, 2004), slug tests (Ji & Koh, 2015; Chen *et al.*, 2012), and geophysical methods (Soupios *et al.*, 2007; Day-Lewis *et al.*, 2017) offer valuable information on groundwater flow and underground structures. However, obtaining such data is often challenging for boreholes (Coptly *et al.*, 2011) and numerical models can provide a convenient solution.

Numerical models simulate complex systems by utilising conceptual models, and they require sufficient field data and initial conditions to simulate the specific aquifer (Bear *et al.*, 1992). Different numerical models, such as two-dimensional models like RPTsolv or three-dimensional models like MODFLOW, can be employed based on data availability and time constraints (Botha *et al.*, 1998; Van Tonder *et al.*, 2002). Numerical models have proven effective in recent years due to advancements in computational power, enabling simulation of complex fractured aquifers. However, data requirements for full numerical models are specific to the location and rarely available.

1.3 Research Aim and Objectives

The primary aim of the dissertation is to investigate the distance dependency of storativity through comparative analyses between various analytical models and RPTsolv (Botha *et al.*, 1998), as well as studying the results to quantify correlations and dependencies.

The objectives of the study include:

- The analysis of constant rate pumping tests of pumping and observational boreholes using a selection of analytical methods, including the Cooper-Jacob (1946) radial flow model; the Moench (1984) double-porosity model; and the Gringarten & Ramey (1974) single fracture flow model.
- RPTsolv, a two-dimensional model developed for South African aquifers, will be used to compare the analytical methods and determine consistent storativities. In this respect the RPTsolv model will serve as the benchmark for the study.
- The study of the correlation between the storativities produced by the modified Cooper-Jacob analytical model and the numerical model RPTsolv in order to produce an improved method of obtaining the parameter in South Africa.

1.4 Chapter Layout

The dissertation is divided into seven chapters, each of which will be given a brief description below.

Chapter 1 serves as an introduction to the research. It establishes the general background of the study, highlighting the significance and relevance of the topic being investigated. Moreover, the chapter clearly articulates the aim and objectives of the research, setting the foundation for the subsequent chapters.

In Chapter 2, a comprehensive overview of the study area is presented, providing relevant background information and describing its unique properties. Additionally, this chapter acknowledges the challenges faced by previous authors in quantifying hydraulic parameters within this area, emphasising the importance of addressing these difficulties in the current research.

Chapter 3 is dedicated to an extensive literature review. It encompasses a broad range of information related to fractured aquifers and available fracture flow models. The chapter delves into the concepts underlying each model, providing equations as well as discussing their advantages and disadvantages. Furthermore, it explores various analytical methods used to study flow regimes, drawing insights from case studies conducted by other researchers.

Chapter 4 outlines the methodology employed in the research. It details the step-by-step procedures followed to obtain the results. This chapter ensures transparency by providing a clear and comprehensive explanation of the methods utilised, enabling other researchers to duplicate the study if desired.

In Chapter 5, the obtained results are presented and thoroughly discussed. The discussions are centred around the performance of the methods employed and the valuable insights gained from graphical representations and error analysis. This chapter also highlights the findings, which incorporates the effective radius into a modified Cooper-Jacob model, enhancing the determination of storativity.

The penultimate chapter contains the general conclusion based on the research objectives. It summarises the key findings and their implications, ensuring that they align with the initial aims stated in Chapter 1. Additionally, this chapter provides recommendations for future studies, suggesting areas that could benefit from further exploration.

The final chapter includes the list of references cited throughout the dissertation, ensuring proper acknowledgment of the sources consulted. The appendices, attached at the end of the dissertation, contain supplementary material such as additional data, detailed calculations, or supporting documentation that is relevant to the research but not essential for the main body of the dissertation.

CHAPTER 2 STUDY AREA

The study was conducted in South Africa, covering the two main hard rock Supergroups. The predominant subsurface geology consists of fractured systems that in the past noted dual-porosity characteristics (Usher *et al.*, 2008:15). The data of 35 borehole pumping tests (single and multiple well) were mainly obtained from the six regions highlighted in Figure 2-1.

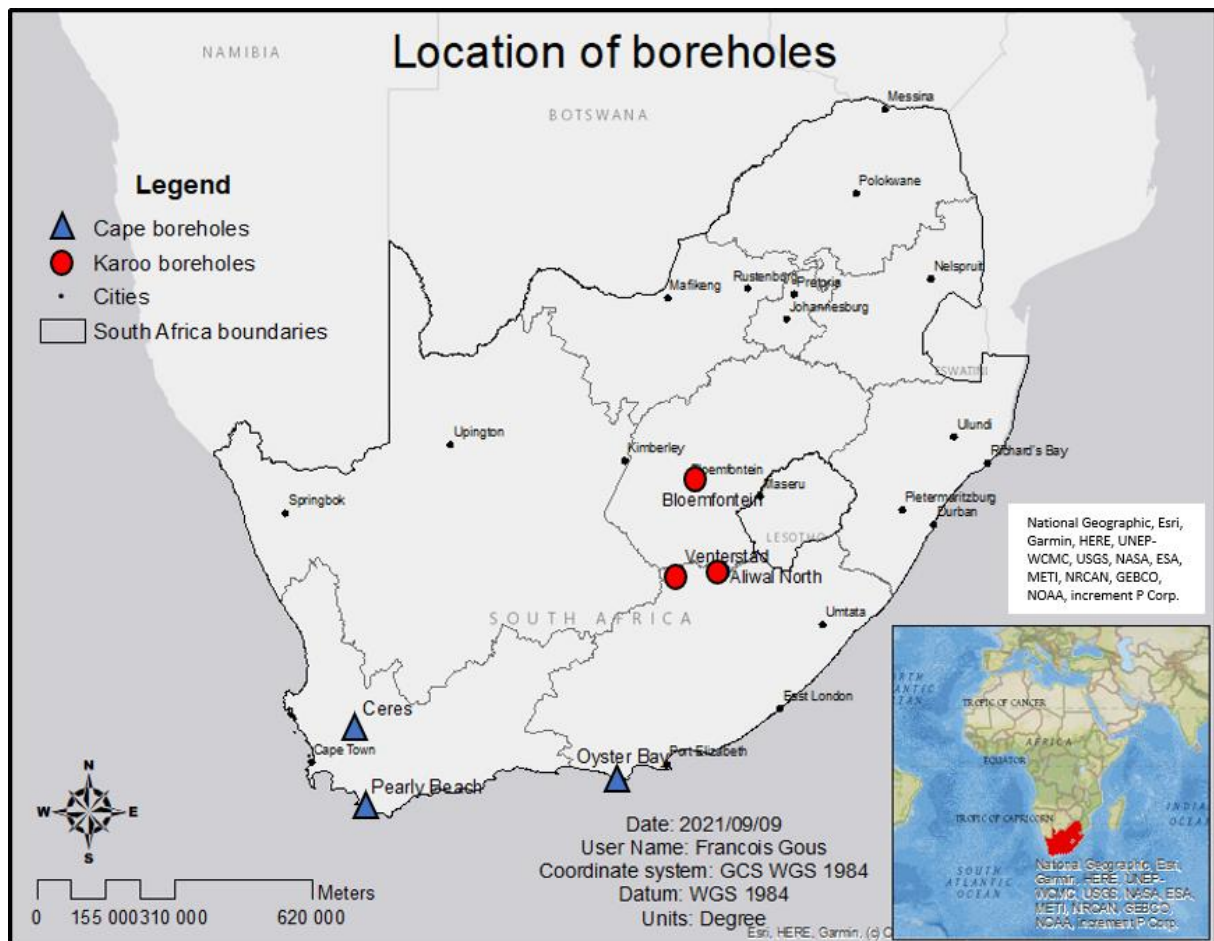


Figure 2-1: Location of boreholes where pump test data was recovered.

According to the groundwater dictionary (Fourie *et al.*, n.d.), a fractured aquifer is defined as a formation that contains sufficient fissures, fractures, cracks, joints, and faults to govern water flow that can yield economic quantities of water. Fractured aquifers are important because of the tendency of the secondary porosity (fractures) to transmit groundwater at high rates (Fourie *et al.*, n.d.). These systems have naturally been researched due to the need for groundwater development for communities underlain by the aquifers. This chapter present maps of the research sites located in South Africa.

The map presented in Figure 2-2 shows the distribution of the research sites and the lithology they are located on.

The research sites are situated in the following fractured aquifers:

1. The Karoo aquifers, which are the most widespread of the three, occurring in the Main Karoo and Kalahari basins across Southern Africa.
2. The Table Mountain Group aquifers (TMG aquifers) lie at the Southern end of the country and are only located in this region.

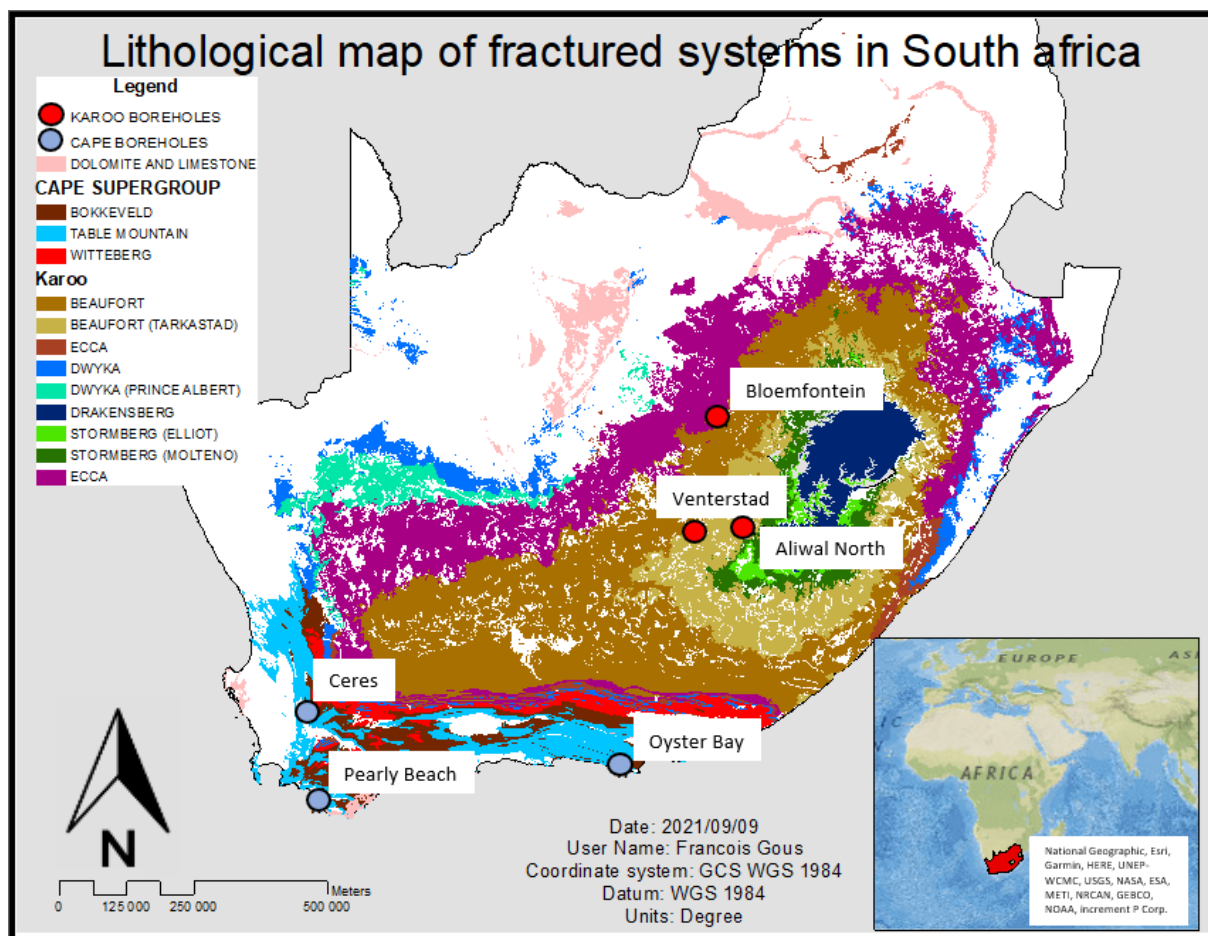


Figure 2-2: Map of the distribution of fractured aquifer systems in South Africa.

2.1 Karoo Supergroup

This Supergroup was deposited in basins created by the formation and breakup of Pangea with the deposition sequence ranging from the late Carboniferous to the middle of the Jurassic period, about 100 million years ago (Selden & Nudds, 2012). The Karoo Supergroup is an important aquifer due to its size and location (see Figure 2-3). Apart from occurring in the Main Karoo Basin

of South Africa, the Supergroup extends into the Kalahari Basin and subsidiary basins of Namibia, Botswana, Zimbabwe, and Mozambique. This Supergroup underlies many towns and rural areas of central South Africa where clean surface water is not always available year-round. Effective analysis of the aquifer is vital for the availability of clean water to consumers.

According to Mbiko (2016:8-22) and Svensen *et al.* (2015:349-362), the major flow characteristics of the Karoo Supergroup are the low permeability of the matrix and the intrusion of Dolerite sills and dikes. Mbiko (2016:8-22) further states that most boreholes in this region have low immediate yields (<1 l/s). Therefore, it is known from previous studies that the Karoo Aquifers are unreliable sources of groundwater (Botha & Cloot, 2004). However, Botha *et al.* (2004) noted large volumes of water being pumped daily from mines and buildings in and on the respective formation. This contradicts the previous statement and is not something one expects from aquifers with limited yields (Woodford & Chevallier, 2002).

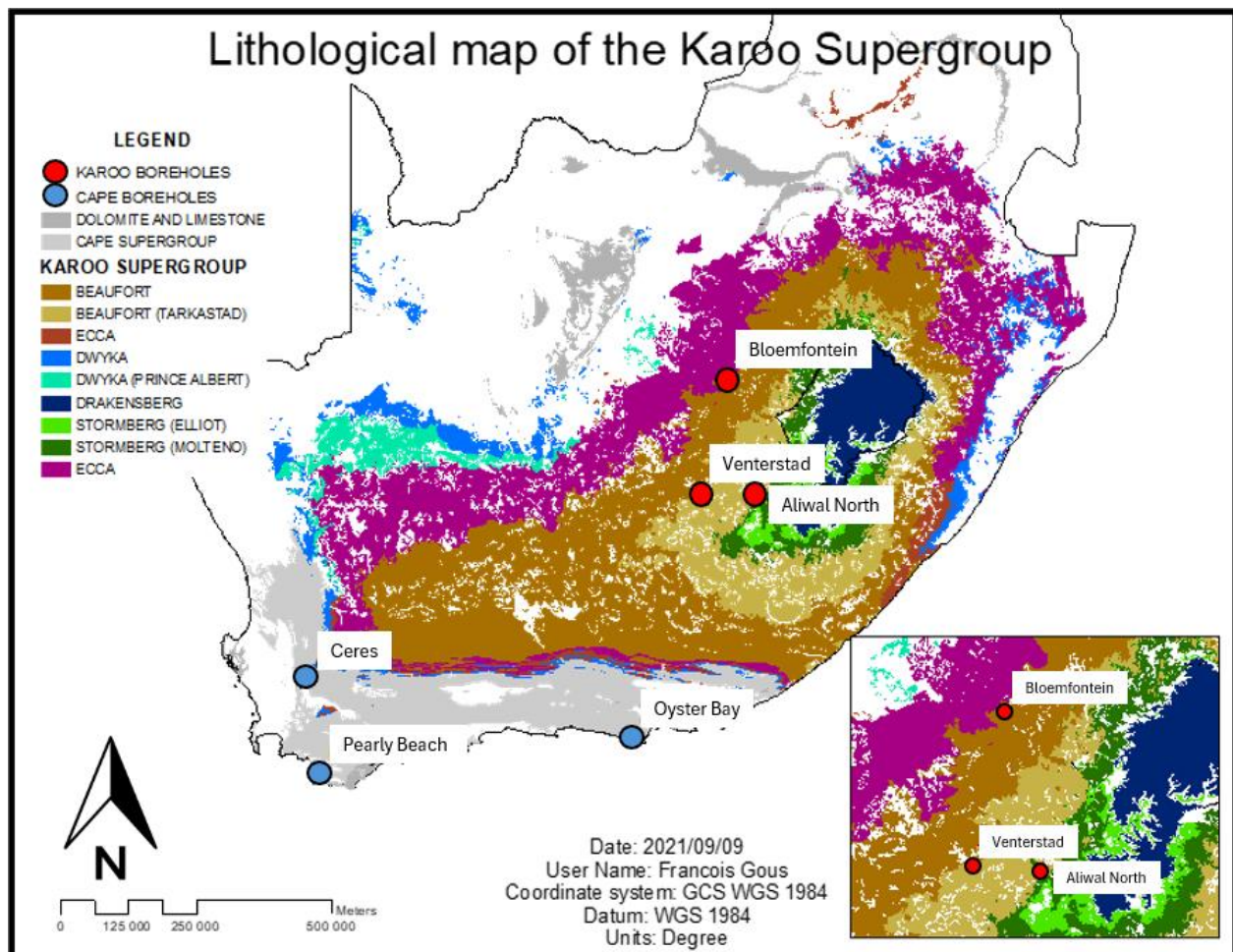


Figure 2-3: Map of the distribution of fractured aquifer systems in South Africa.

2.2 Cape Supergroup

The Cape Supergroup was deposited during the periods of the early Ordovician to early Carboniferous, up to 500 million years ago (de Beer, 2002). Ancient depositional environments included shallow marine and non-marine (fluvial) conditions. Consisting of quartz arenite and other hard sediments with a thickness of about 4000m (Duah, 2010). The Table Mountain Group (henceforth TMG) forms part of the lowest Cape Supergroup stratigraphy and extends from the Western Cape to the Eastern Cape, covering up to 248,400 km² (Jia, 2007). Refer to Figure 2-4 for the layout of the TMG aquifer.

The TMG aquifer has been disregarded for a long time because of the impermeable nature of the matrix and thus limited storativity of the fractures (Du Toit, 1954). The geology of TMG consists mostly of medium to coarse grain-sized quartz arenites. The quartz arenites are extremely resistant to weathering and susceptible to fracturing. Not only did folding and/or faulting of the group produce a fractured system but it also developed a secondary porosity. Secondary porosity allows the aquifer to permit flow through fractures and joints (Sun, 2014). These two factors enhanced both groundwater quality and exploitation potential (de Beer, 2002). This is why the Cape Supergroup exhibits potential to produce significant groundwater if utilised effectively.

This study offers a great opportunity to evaluate the accuracy of the different methods to calculate storativities from non-artesian TMG aquifers.

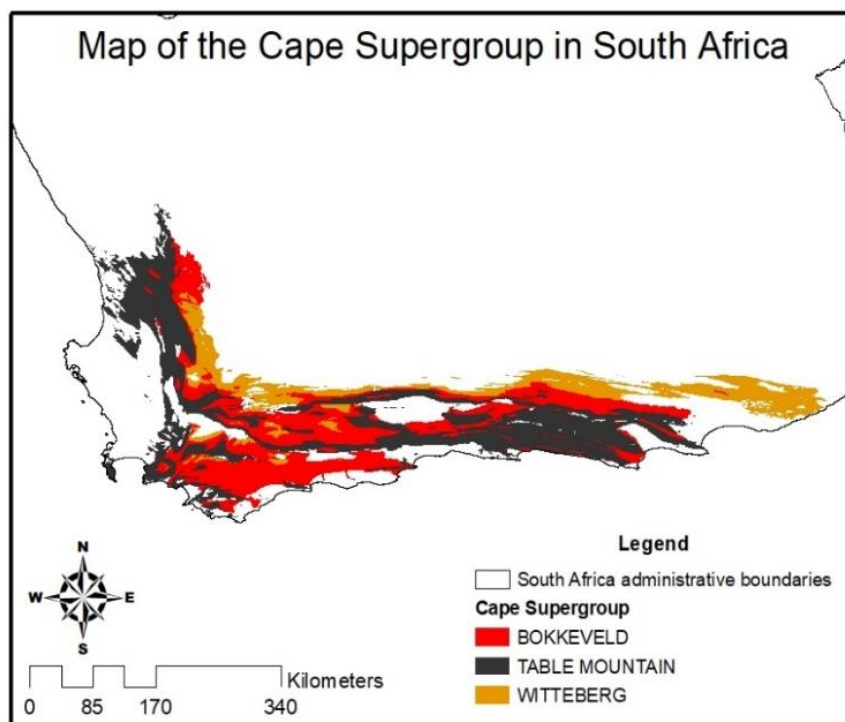


Figure 2-4: Map of the Cape Supergroup in South Africa.

CHAPTER 3 LITERATURE REVIEW

3.1 Introduction

The literature review will cover the theory behind pumping test analysis, which is the most used method to determine an aquifer's potential to transmit and produce water. The distance dependency of storativity in these aquifers is a critical issue that leads to incorrect assessments regarding groundwater potential (Bredenkamp *et al.*, 1995:160). The review will include discussions on the types of flow, concepts of fracture flow, and analytical methods for pump test analysis, as well as case studies.

3.2 The distance dependency of Storativity

The governing equations for groundwater flow typically involve two main parameters: transmissivity and storativity. Transmissivity, describes the ease with which water flows through an aquifer (as defined by Schwartz and Zhang, 2003). Storativity, or the storage coefficient, is a dimensionless parameter that represents the volume of water released from storage per unit surface area per unit decline in the head. These two parameters are determined by pumping tests or hydraulic tests. While transmissivity can generally be calculated numerically and analytically, storativity is subject to distance-dependent variations when calculated analytically (Bredenkamp *et al.*, 1995:160; Sanchez-Vila *et al.*, 1999; Akpen *et al.*, 2019:233-241; Meier *et al.*, 1998; Rivera & Geol, 2014).

The distance dependency of storativity has been previously observed and studied by Bredenkamp *et al.* (1995) and Botha *et al.* (1998). These studies determined storativity to be a function of the initial water level, the distance from the pumping borehole, and the discharge rate.

This phenomenon can be explained both practically and theoretically. An important term that will be used in the discussions is the representative elementary volume (REV). This term can be defined as a property of fractured medium, where given enough volume, it behaves homogenous. According to Long *et al.*, (1982) the volume can be determined where hydraulic parameters, which will normally vary with distance, first becomes constant as illustrated in Figure 3-1.

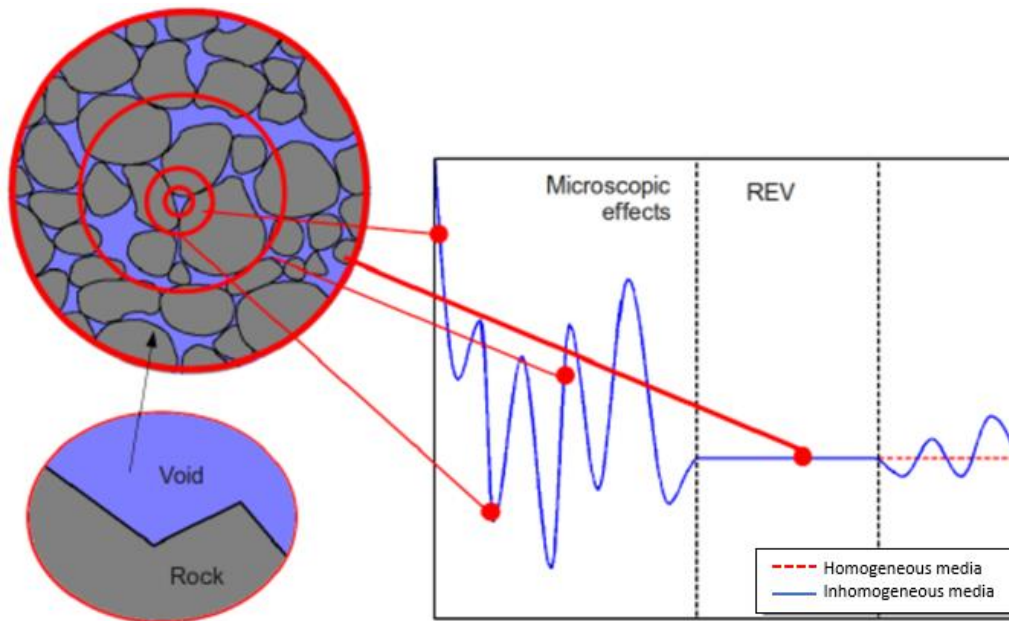


Figure 3-1: Simplified representation of the representative elementary volume (REV) of an aquifer (Lie & Mallison, 2015).

Practically, it is possible to explain how some parameters might change with distance. The following explanation was given by Woodford & Chevallier (2002). Close to the pumping borehole, large fractures with a high conductivity are sourced first, causing these fractures to lose hydraulic pressure rapidly. A high local pressure difference between the large and small fracture reservoirs now exists, therefore causing the small fractures to source water to the large fractures at a high rate, hence storativity is large. At some distance away from the pumping borehole, the pressure gradient between the two fracture types is relatively small. Minor fractures source water to the large fracture at a slow rate, therefore storativity is small.

Theoretically, researchers have pointed out the inability of analytical models to determine parameters that are distance dependent. While the equations of flow governing analytical models can deal with polygonal inhomogeneities, they are inherently unable to take a varying field of properties into account (Strack & Barnes, 2009). Therefore, there is no real theoretical justification for the results of analytical methods, particularly for the analysis of fractured aquifers. However, despite this limitation, analytical models still have advantages and should not be discarded in hydraulic analysis, but rather improved to better address these limitations.

3.3 Effective Radius

Effective radius (r_e) is used in pumping tests analysis to understand the influence of the pumping borehole on the surrounding aquifer. It is defined by Kruseman & de Ridder (1994:200) as the distance from the pumping borehole where the theoretical drawdown is equal to the measured

drawdown near the well screen. This should not be confused with the radius of the well (r) or the radius of influence (R). If drawdown is measured at the pumping borehole, the radial distance to the observation well is equal to the effective radius (Meier *et al.* 1998:1011). R_e is used as an input parameter in some of the most used equations to determine drawdown in an aquifer. Some of these equations include the radial flow methods of Theis (1935) and Cooper and Jacob (1946) and the straight-line fracture flow method from Warren and Root (1963).

3.4 Analytical models and implementing diagnostic tools

3.4.1 Analytical models

Groundwater flow models are important tools in the study of groundwater flow behaviour, and their foundation lies in mathematical equations that govern water flow through the earth. Darcy's law, which was introduced by Henry Darcy in 1856, was the first equation that described groundwater flow through sand columns. This law is similar to other physical laws such as Fourier's and Ohm's laws, which describe thermal and electrical conductance, respectively (Anderson, 2007). Darcy's law became the foundation of all groundwater flow calculations that followed.

While Darcy's law itself doesn't calculate drawdown directly, the principles founded by Darcy have led to the creation of several analytical models and solutions for analysing groundwater drawdown in boreholes. Gringarten (1982) as cited in Woodford & Chevallier (2002) summarised methods for interpreting flow tests in geotechnical, groundwater, and petroleum literature, resulting in three main types of approaches for analysing fissure formations. These methods are still used in practice today and include the following:

1. The equivalent homogeneous approach, which uses a less complex model that only considers the dominant trend of drawdown behaviour.
2. The double-porosity medium approach. Matrix blocks and fissures are assumed to be homogeneously distributed throughout the aquifer.
3. The deterministic approach. Detailed and extensive data on individual fractures and structures are needed - mainly used for small-scale analysis.

Analytical models are a cost-effective and efficient way to evaluate and quantify hydraulic properties by analysing pump test data. These models were extensively researched in the late 1900s before the advent of numerical models. Analytical models simplified three-dimensional mathematical concepts, which were difficult to calculate at the time, by reducing them to one

dimension, making it possible to model drawdown analytically. This paved the way for new and more complex analytical models that can simulate flows and drawdowns associated with almost any type of aquifer (Kruseman & de Ridder, 1994). The following section explains the limitations of analytical models and compares it to numerical models.

3.4.1.1 Limitations of analytical models

Analytical groundwater flow models are widely used due to their ease of use. However, according to Strack and Barnes (2009), this ease of use may lead to less critical testing of the models and the incorrect parameterisation of certain aquifers. Analytical models suffer from non-uniqueness and uncertainty, which limit their predictability.

Non-uniqueness in analytical models relates to the fact that more than one combination of model inputs will yield results that match field data (Anderson *et al.*, 2015:12). Van Tonder *et al.* (2002:8) note that the results from the most commonly used analytical methods for dual porosity and fracture flow, namely the Moench (1984) method and the Gringarten method, are non-unique due to the number of parameters that need to be fitted. The non-uniqueness of analytical methods restricts their use as the sole means of analysing fractured aquifers.

The results obtained by analysing pump test data in fractured aquifers with analytical models does have a level of uncertainty. Uncertainty can arise from several factors. Firstly, models that simulate results over long periods may be affected by unknown factors outside the representative elementary volume (REV) of the model. If the pumping test duration is not sufficient and radial-acting flow was not reached, the estimated aquifer parameters are relatively uncertain (van Tonder *et al.* 2002:2). As mentioned by Anderson *et al.* (2015), future conditions will increase the uncertainty of the results. Secondly, any groundwater model is based on mathematical equations that rely on certain assumptions. These assumptions involve aquifer geometry, flow direction, and subsurface geology heterogeneities (Kumar, 2004:1-5). Therefore, selecting a model that performs under respective objectives, available resources, and data availability is crucial in minimising uncertainty (Bear *et al.*, 1992). If the assumptions made by the analytical method is not met by the aquifer being analysed, there will be an inherent uncertainty in the results obtained by said method.

Analytical models are therefore great tools in a geohydrologist's arsenal to determine the characteristics of an aquifer but should not be used as sole means of analyses as they are prone to limitations and uncertainties. If sufficient time and data are available, analytical models should be coupled with numerical models to produce the best results.

3.4.1.2 Analytical models versus numerical models

According to Botha (1988:1655-1658), there are two classes of flow models encountered in science: 1) Models governed by mathematical principles and phenomenon-dependent parameters (mostly analytical models) and 2) models that run on extensive and refined mathematical techniques to produce valid parameters (mostly numerical models). Table 3-1 presents a comparative summary of analytical and numerical models.

Table 3-1: A comparative summary between analytic and numeric models.

Item	Analytical	Numerical
Assumptions	High. Analytical methods are limited by the number of underlying assumptions (van Tonder <i>et al.</i> 2002:91) which may oversimplify the complexity of the aquifer being studied.	Low to medium. Less assumptions with regards to simplifying the natural world to fit a conceptual model (El-Rawy <i>et al.</i> , 2022) and the user can choose what assumptions to make (Johnson <i>et al.</i> , 2001).
Time and resources	Low. Data required is less compared to numerical models and requires less expertise (van Tonder <i>et al.</i> 2002:2 and Cook, 2003:92).	High. Numerical models take longer to calibrate and requires significantly more resources to function (van Tonder <i>et al.</i> 2002:2).
Complex geometry	No. Analytical models cannot account for a varying field of properties, but deal with polygonal inhomogeneities (Strack & Barnes, 2009). Classical and some of the most used methods are unable to account for vertical seepage or horizontal flows (Woodford & Chevallier, 2002).	Yes. An adequate numerical model can simulate complex geologies (Ramli, 2009:26-27)

Numerical models are used when a location needs to be assessed with a high certainty of accuracy, consequently complex numerical models are often rendered impractical because of the excessive data needed for a complete analysis. Numerical models are used when the project

objectives demand it and if adequate resources are allocated. Therefore, if these resources are unavailable (limited resources and time), analytical models are used instead.

3.4.2 Diagnostic plots

The use of analytical methods and diagnostic plots are well documented and widely used for groundwater analysis, as noted by Allen (1999), Baumle (2003), and Rivera & Geol (2014). Diagnostic plots, which are scatter plots of drawdown versus time, are typically plotted on a log-log or semi-log scale (Renard *et al.*, 2009). The purpose of these plots is to present aquifer characteristics in a graphical format that investigators can use for analysis. Renard *et al.* (2009) identified four primary advantages to analysing diagnostic plots:

- It offers a unified method to interpret pump test data.
- The derivative of the drawdown curve is highly sensitive to slight variations in shape. This can be used to detect anomalous behaviour that may not be apparent through other analyses.
- It promotes the selection of a conceptual model. Diagnostic plots can help select the most appropriate model for the specific aquifer and assist in calibrating the model to the field data.
- Derivatives can be used to rapidly estimate model parameters.

One of the first synthesis on diagnostic plots from different aquifer systems and boundary conditions and how to interpret them was produced by Kruseman and de Ridder (1994). Later, Renard *et al.* (2009) plotted the most typical diagnostic plots and included the derivative curves of each. The 10 most typical diagnostic plots are presented below and in Figure 3-2:

- a) Theis model for porous confined aquifers;
- b) double-porosity aquifers;
- c) infinite linear no-flow boundary;
- d) infinite linear constant head boundary;
- e) leaky aquifer;
- f) wellbore storage and skin effect;
- g) infinite conductivity vertical fracture;
- h) general radial flow and non-integer flow dimension smaller than 2;
- i) general radial flow model with non-integer flow dimension larger than 2;

- j) The combined effect of infinite linear constant head boundary and the effect of wellbore storage.

The importance of the derivative of the plotted curve is evident from the dotted line in the diagram presented in Figure 3-2, and it is usually included. The derivative is more sensitive to changes in the drawdown curve and can provide qualitative information on the borehole characteristics when used correctly. According to Van Bosch (2000), derivative plots offer an easier way to identify certain flow regimes, as characteristic straight-line slopes remain straight. Derivative plots also amplify slight variations in drawdown and make it easier to identify fractures and hydraulic boundaries. However, variations in terms of noise in the drawdown data will be enhanced by plotting the derivative, so it is recommended to smooth out the noise first. Overall, the use of derivative plots is an effective tool for analysing aquifers and identifying important features that may not be apparent in the original data.

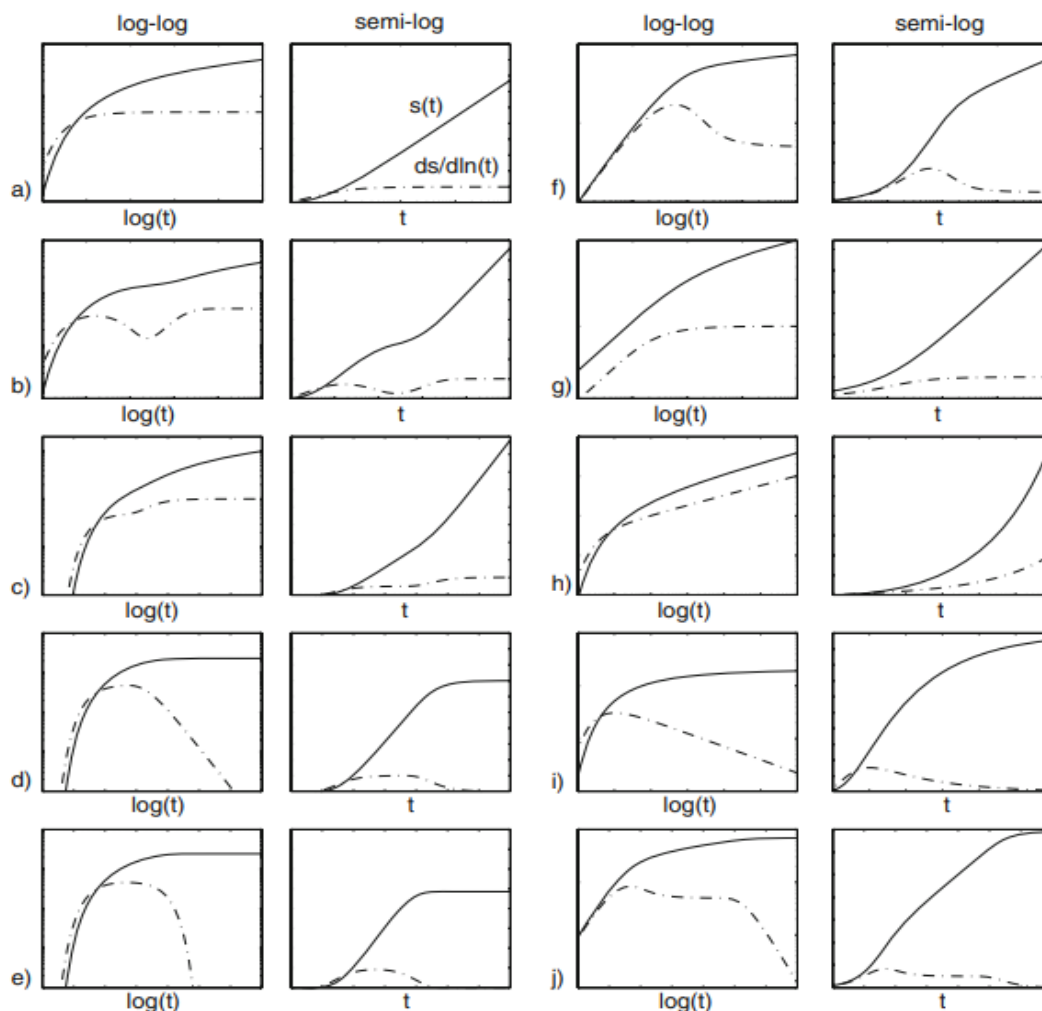


Figure 3-2: Most typical diagnostic plots in hydrogeology (after Renard *et al.* 2009:3).

Figure 3-3 was created by Van Tonder *et al.* (2002:19) and shows the information that a derivative plot may present on flow regimes and hydraulic boundaries. The increase in drawdown rate

observed at early-time was likely because of the combined effects of wellbore storage and limited water supply near the well. Once the hydraulic gradient reaches more flow paths, a clear decrease in drawdown rate is visible. This stage, more visible on the derivative, is called the dual-porosity dip and signals a change in the overall flow regime from inner well effects and flow to a more centralised fracture flow. Bi-linear flow is visible thereafter, with the derivative showing a good fracture network in a bi-linear flow regime. Late drawdown shows a relatively straight line in the log-log plot with the derivative indicating constant head or radial flow conditions.

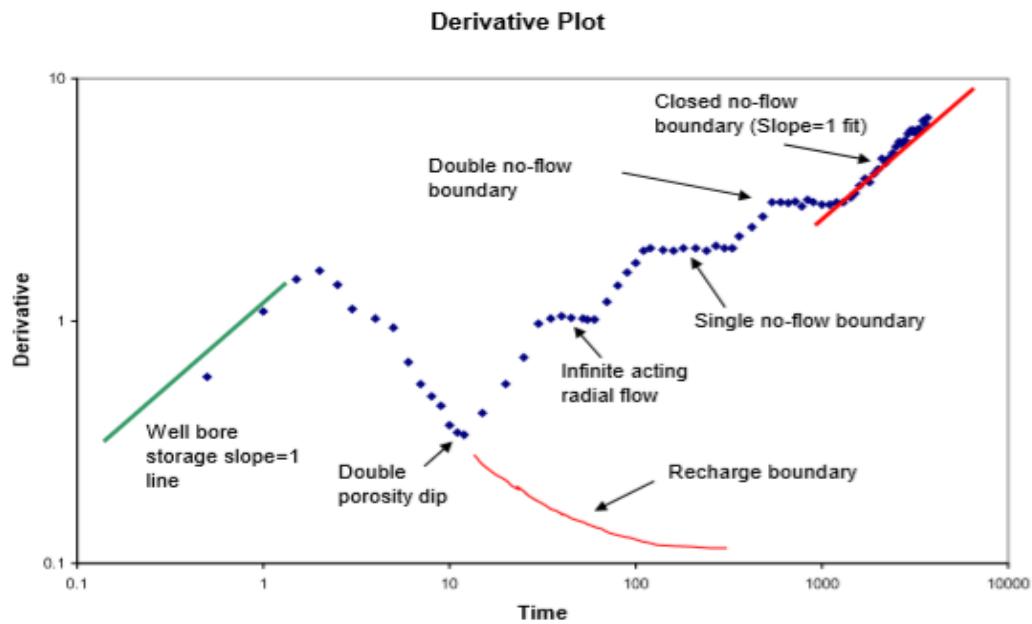


Figure 3-3: A derivative plot of drawdown data with diagnostic properties (After Van Tonder *et al.* 2002:19).

3.5 Types of flow in fractured aquifers

Four different flow types can be identified when examining the flow of water from the aquifer to the pumped borehole (excluding after flow/wellbore storage and boundary effects) as shown in Figure 3-4. These flow regimes include early-time linear flow, bi-linear flow, transitional flow and radial flow and will be discussed in the sections that follow.

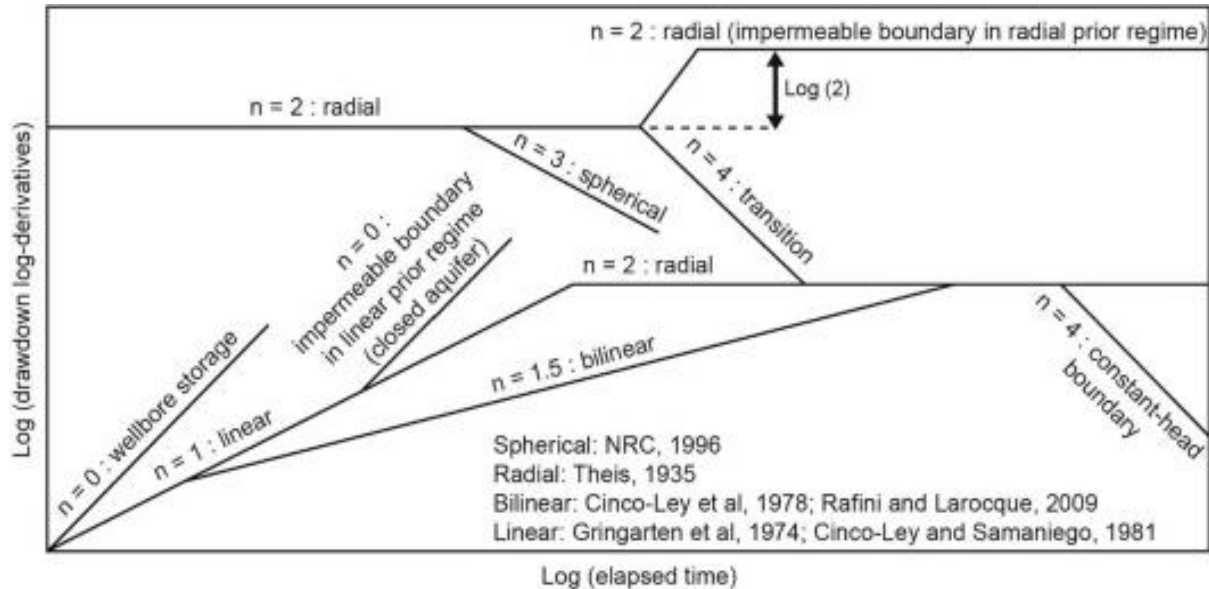


Figure 3-4: a Log(drawdown log-derivatives) versus Log(time) graph showing the flow regimes with their associated flow dimensions, n (from Ferroud *et al.*, 2019).

3.5.1 Linear flow

Linear fracture flow is a flow regime that occurs in hydraulically fractured boreholes where the fracture has finite conductivity within a low-conductivity matrix (Glover *et al.*, 2021). This flow regime is also observed in natural geological features, such as sub-vertical fractures, faults, and dikes (Bardenhagen, 2002). Linear flow occurs at an intermediate time and represents the flow of water from the formation to the fracture and then to the wellbore (Liu *et al.*, 2014).

The linear response of an aquifer to pumping can be observed in a time-drawdown plot, typically after the initial wellbore storage effects and before late-time radial flow effects (Cooper *et al.*, 1967). In a log-log plot of drawdown versus time, the slope of the linear flow regime follows a half-slope line, which is close to 1 (Bredehoeft *et al.*, 1982).

Representations of linear formation flow and linear fracture flow are presented in Figure 3-5.

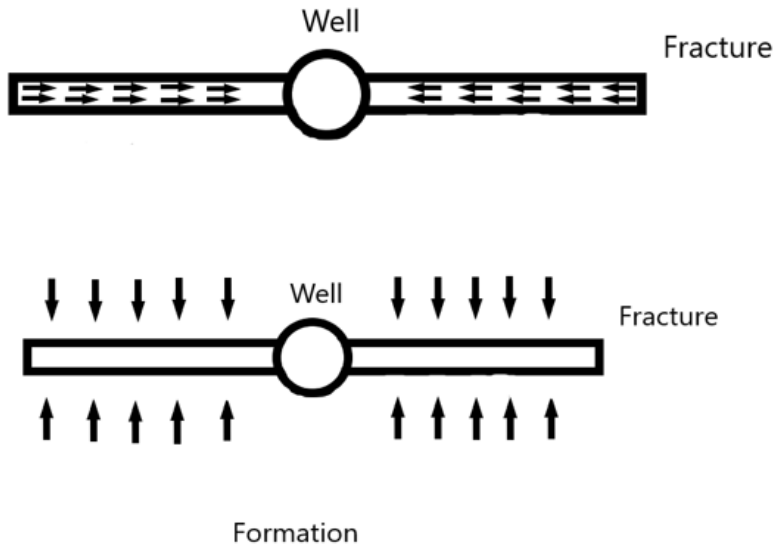


Figure 3-5: Linear fracture flow (top) and linear fracture flow (bottom) after Nashawi & Malallah (2007:303-320).

Figure 3-6 shows a practical example where linear fracture and linear formation flow is indicated.

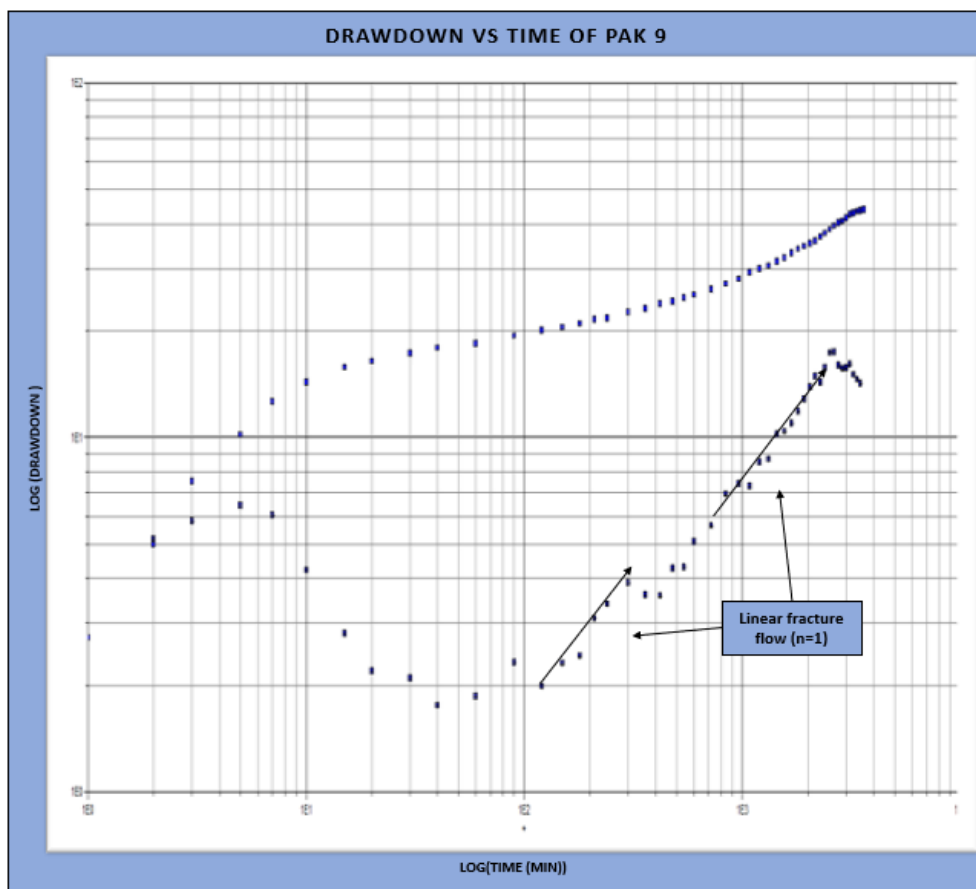


Figure 3-6: Linear flow can be identified as a slope with $n=1$ on a log-log ($\log(s)$ vs $\log(t)$) plot.

3.5.2 Bi-linear flow

In intermediate to late drawdown stages, when the piezometric head has lowered, water is sourced from both the fracture and matrix (see Figure 3-7). This flow regime is characterised by two types of linear flow; from the matrix to fracture and from fracture to the borehole, known as bi-linear flow (Cinco-Ley & Samaniego, 1977). The gradient of bi-linear flow follows a quarter-slope line on a graph, which means that for one log cycle of drawdown, four log cycles of time are required. The amount of water flowing in this regime depends on the pressure gradient between the fracture and matrix. Near the pumping borehole, the pressure gradient is steep, leading to the matrix supplying sufficient water to the fractures. Further from the borehole, the pressure gradient decreases, resulting in less water being sourced from the fractures (Van Bosch, 2000). The homogeneity assumption in analytical models does not account for this phenomenon, which is one of the primary reasons for distance-dependency of storativity.

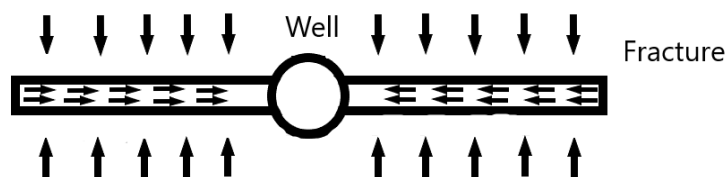


Figure 3-7: Bi-linear flow (Nashawi & Malallah, 2007:303-320).

3.5.3 Radial flow

Radial flow is defined as a type of groundwater flow that occurs when the cone of depression formed by pumping is nearly circular as seen in Figure 3-8, which is typically found in homogeneous aquifers (Bardenhagen, 2002:7). Although it is not commonly found in fractured aquifers, it can occur during late-time pumping. The onset of radial flow in fractured aquifers indicates that it behaves as a homogeneous medium, and the cone of depression extends beyond the REV. When an observation point is located outside the REV, only radial flow behaviour is observed (Van Bosch, 2000). However, Kruseman and de Ridder (1994:263) noted that achieving pseudo-radial flow in fractured aquifers may require uneconomically long pumping times, making it difficult to observe in borehole analyses. The gradient of the line representing radial flow remains constant and appears as a horizontal line on a diagnostic plot. Figure 3-9 gives a practical example of where radial flow was observed.

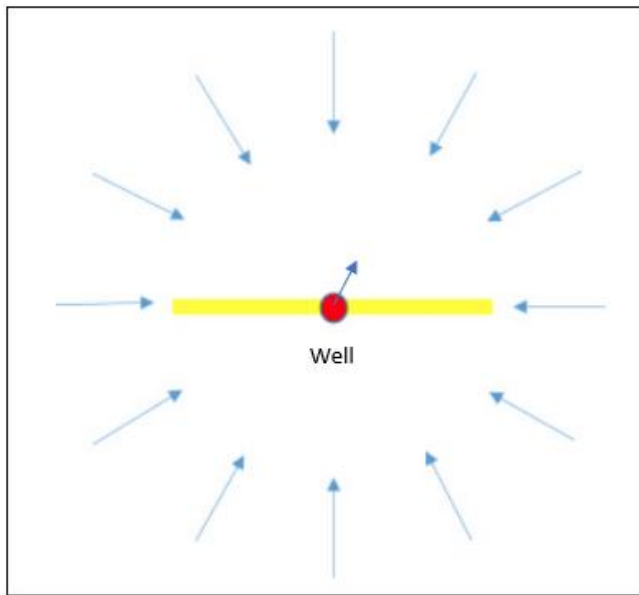


Figure 3-8: A two-dimensional representation of a circular cone of depression (radial flow) after Zhuang *et al.*, (2020).

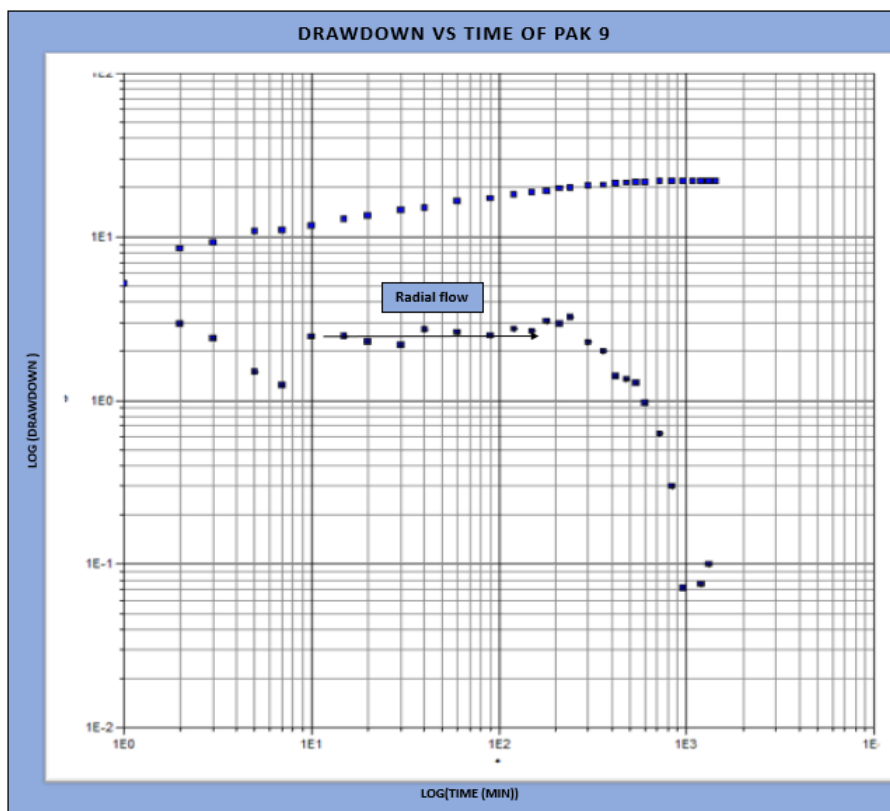


Figure 3-9: Radial flow on a log(s) vs log(t) plot.

3.5.4 Spherical flow

According to Ferroud *et al.* (2019), spherical flow occurs when the portion of the borehole receiving water is shorter than the aquifer thickness. This may be the case in the following scenarios:

- Partial penetrating well within a thick fractured aquifer.
- During a packer test.
- When pumping a partially penetrating fracture.

Spherical flow can be identified on a log-log plot of drawdown versus time by a period of decreasing drawdown and with a slope dimension of $n = 3$. The rate of decrease is lower than with constant-head boundaries but still represents a slowdown of the drawdown rate. See Figure 3-10 for a practical representation on a drawdown plot.

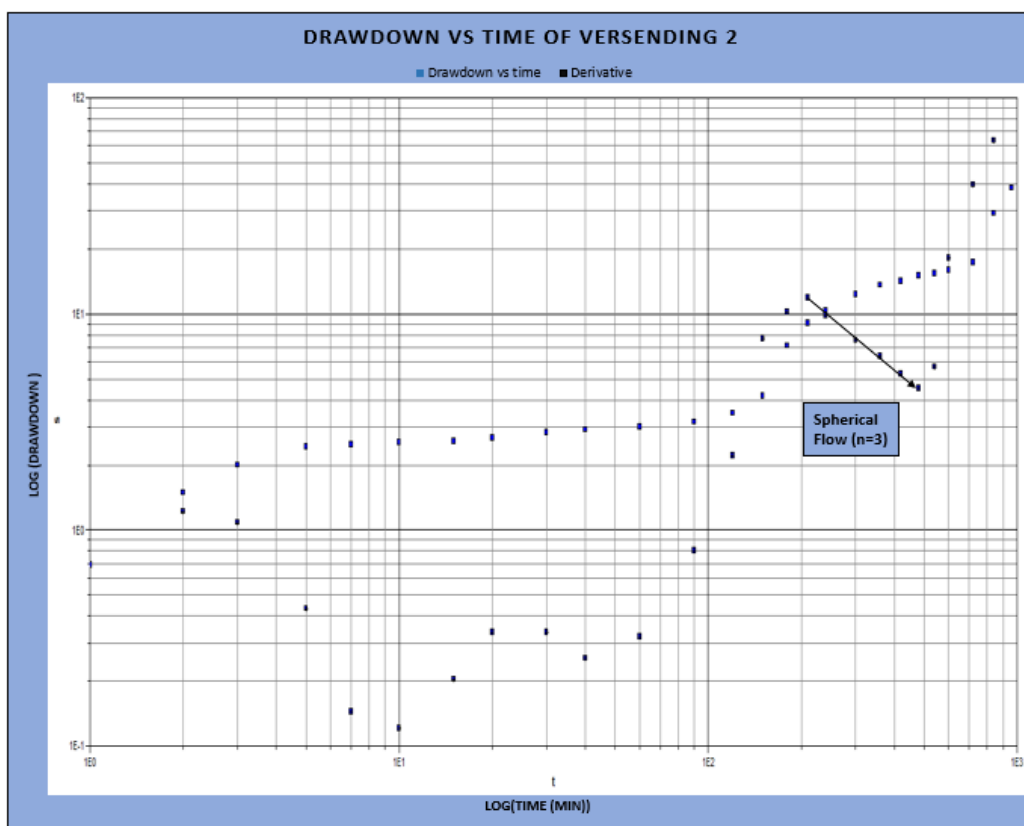


Figure 3-10: Spherical flow is characterised with an $n=3$ line on a log-log ($\log(s)$ vs $\log(t)$) plot

3.6 Concepts for fractured flow

The assumptions of the radial flow conceptual model are widely recognised. According to this model, drawdown creates a cone of depression that spreads out from the well in all directions.

Methods based on the radial flow concept, such as Theis (1935) and Cooper-Jacob (1946), are only appropriate for analysis if radial flow is present. However, in fractured bedrock aquifers, preferential flow paths are more likely to be dominated by spatially distributed fractures with varying geometries. As a result, double-porosity flow models assume that secondary (fracture) porosities contribute to overall flow at different stages. This means that the aquifer may consist of discretely oriented fractures, where a single fracture intersects the pumping borehole. This is known as the single fracture flow concept (Kruseman & de Ridder, 1994).

3.6.1 The dual-porosity concept

The dual-porosity concept was first introduced by Barenblatt *et al.* (1960) to modify classical seepage theory to account for both fracture and matrix pressures. This concept assumes a uniform distribution of homogeneous matrix and conductive fractures in space. Warren and Root (1963) developed a pseudo-steady state block-to-fracture flow solution by analysing pressure build-up data from quartzite aquifers using the dual-porosity concept as depicted in Figure 3-11. Later, Kazemi (1969) revised Warren and Root's one-dimensional analytical method for transient flow in sandstone aquifers with bedding planes. The results of Kazemi's tests showed that dual-porosity aquifers exhibit two parallel straight lines when plotting drawdown versus time on semi-log axes. These straight lines represent the two distinct regions of the reservoir, flow by fracture and flow by matrix.

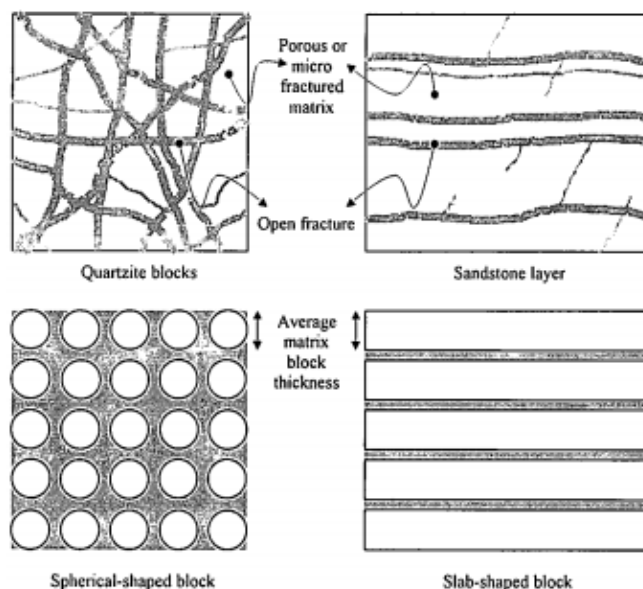


Figure 3-11: Warren and Root (1963) and Kazemi's (1969) representation of quartzite and sandstone aquifers based on the double-porosity concept from Bardenhagen (2002).

Moench (1984), on the other hand, introduced fractured skin to the concept. Moench (1984) as described by Woodford & Chevallier (2002:190) further stated that by introducing fractured skin,

pseudo-steady-state conditions may still apply. The proposed type-curve approach by Moench proved to be much more complex than the Warren and Root (1963) or Kazemi (1969) methods due to the inclusion of wellbore storage effects and borehole skin. Moench described the drawdown using a REV. The REV of any given system varies and may not be found if the scale of the investigation is too small. Figure 3-12 shows how Moench interpreted REV.

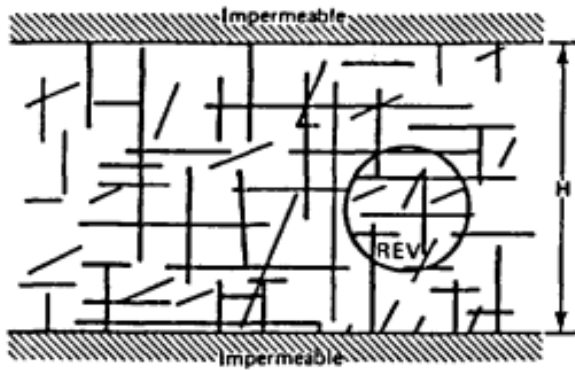


Figure 3-12: Schematic diagram of the representative elementary volume of double-porosity aquifers (by Moench, 1984).

According to the dual-porosity concept proposed by Moench (1984:832), fractured aquifers can be simplified into two interacting continuums. One continuum is of low permeability and comprises primary porosity or matrix, while the other continuum is of high permeability and consists of secondary porosity or fractures.

Figure 3-13 provides a visual representation of the concept. Thus, the flow process of fractured aquifers involves two distinct media, hence the name dual-porosity flow. Fracture flow is dominant early on as high permeable waterways source the water. Later, flow from the matrix system, characterised by low permeability, recharges the higher permeability zone and dominates flow at later times (Rivera & Geol, 2014:1). As a result, early-time drawdown data from a pumping test will reflect the borehole sourcing water from the wellbore storage and fracture(s), whereas a longer pumping test, which duration is determined by the REV, will reveal the true behaviour of the aquifer. Behaviours such as late-time fracture flow, boundary conditions, and eventually radial flow will only be visible on the drawdown graph at late-time.

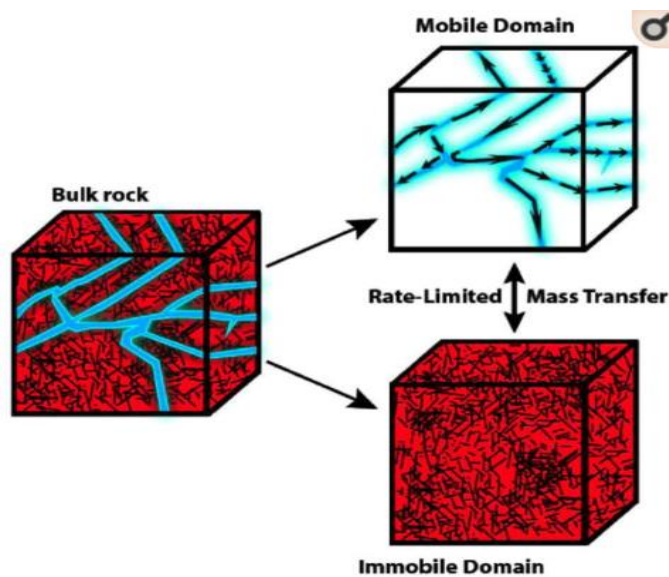


Figure 3-13: Visual representation of dual-porosity aquifers. From Day-Lewis *et al.* (2017:709-720).

To diagnose dual-porosity behaviour, one must utilise type curves and drawdown derivative plots. Because of the development of diagnostic plots by Gringarten (1987) for a wide range of aquifer types, it is used to characterise groundwater flow.

3.6.2 Single vertical fracture concept

This concept is based on a single vertical fracture intersecting a well in an otherwise uniform and homogeneous aquifer. Analytical models were developed by Gringarten and Witherspoon (1972) and later by Gringarten-Ramey-Raghavan (1974) for the petroleum industry. Kroes *et al.* (2008:263-273) explains that the analytical solution assumes a single plane, vertical fracture of limited spatial extent intersecting the borehole mid-length, and the aquifer is bounded above and below by impermeable layers as shown in Figure 3-14. The Gringarten and Witherspoon (1972) method and other analytical solutions derived from this concept are useful for short-term analysis of field data but become less applicable when the flow to the well reaches infinite acting radial flow. At that point, classical methods of analysis will yield the same results.

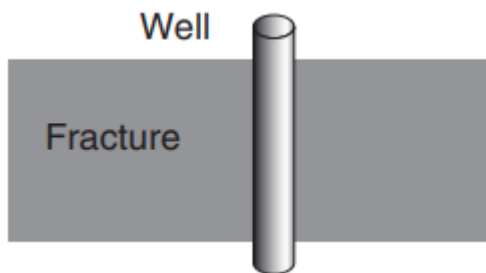


Figure 3-14: Conceptual sketches of a well intersecting a single vertical fracture (Kuchuk & Biryukov, 2013).

3.7.1.1 Cooper-Jacob (1946) method

The standard Cooper-Jacob method (1946) is a straight-line approximation derived from the Theis (1935) method. Cooper and Jacob found that for small values of u (thus large values of time and small radius) the Theis well function $W(u)$ could be approximated by only using the first two terms. Studies of the dimensionless value of u have since stated that u must be $u \leq 0.05$ by Driscoll (1986) or $u \leq 0.01$ by Kruseman and de Ridder (1994) for the Cooper-Jacob to function. While the Theis (1935) method is only valid for infinite radial flow (IRF), the Cooper-Jacob method can be applied after IRF conditions. Wellbore storage must be neglected due to the solution assuming a line source for the pumped well (Duffield, 2007).

Transmissivity and storativity can be determined by fitting the Cooper-Jacob straight-line approximation to drawdown or recovery data. Assumptions based on the Theis (1935) equation are true for the Cooper-Jacob (1946) method. Consequently, the conceptualisation, assumptions, and data requirements of the Cooper-Jacob (1946) method are similar to that of Theis (1935). The majority of the assumptions are listed below as given by Duffield (2007):

- Aquifer is homogeneous and has an infinite areal extend
- Aquifer is unconfined
- Flow is unsteady
- Storage in the well can be neglected
- Drawdown is small compared to the overall saturated thickness if the aquifer

As previously mentioned, radial flow occurs when the slope of the drawdown curve shows a constant derivative. The method of constraining a radial flow model to radial flow conditions is necessary to decrease errors in the results. By using this method, the storativity can be calculated by fitting the line on the appropriate late-time data, when the dominant effect of the fracture flow on drawdown has ceased.

Advantages

- The Cooper-Jacob method is a straight-line approximation, meaning the data can be constrained to data adequate for analyses.
- It produces consistent transmissivity when constrained to radial flow.
- It is faster and less subjective than the Theis method (Allen, 1999).

Disadvantages

- It is based on the Theis equation, and thus includes the same assumptions.
- Results show diverse storativity values.

- It is limited to radial flow conditions.

3.7.1.2 Flow patterns of radial flow models

Radial flow plots are useful tools to identify several behaviours (Duffield, 2007):

1. Inner-boundary effects (wellbore storage) at very early-time, are recognised by a unit slope on a log-log radial flow plot.
2. Infinite acting radial flow at late-time, are exhibiting a straight-line in a semi-log plot.
3. Boundary conditions can be interpreted as late-late time data on a log-log plot.

3.7.1.3 Case studies

The study conducted by Vivier and van Tonder (1997) provided evidence of the applicability of the Cooper-Jacob (1946) method to analyse fractured aquifers. They used the method to estimate aquifer parameters for both the matrix and the fractures by fitting the Cooper-Jacob straight-line approximation to the corresponding segment of the drawdown data. This practice of segmenting the data to obtain more accurate results is not unique to the Cooper-Jacob (1946) method; it is also applicable for other straight-line analytical methods.

It is important to note that fitting the straight-line approximation to the appropriate early-time or late-time data that correspond to the fractures or matrix, respectively, is crucial in obtaining reliable results. This is because the Cooper-Jacob (1946) and other straight-line methods assume certain flow conditions and may not produce accurate results if these conditions are not met. Therefore, it is recommended to use this common practice of segmenting the data when applying straight-line analytical methods like the Cooper-Jacob (1946) and Warren and Root (1963) method.

Neuman *et al.* (2007:1-15) conducted a study on the accuracy of the Cooper-Jacob (1946) and Neuman methods in analysing time-drawdown data from hard rock aquifers. According to the authors, using the Theis (1935) method leads to significant errors. The study compared the results obtained from the Cooper-Jacob method with a method developed by Neuman *et al.* (2004). The Cooper-Jacob method underestimated the transmissivity by 30% compared to the Neuman *et al.* (2004) method, and by 40% compared to the transmissivity calculated by measuring hydraulic conductivity with three hundred and twenty (320) flowmeters. Despite being less accurate than the Neuman *et al.* (2004) method when compared to the flowmeter readings, the Cooper-Jacob (1946) method still provided consistent transmissivity estimates.

Akpen *et al.* (2019:233-241) conducted a similar study using the Cooper-Jacob (1946) straight-line method to estimate aquifer parameters in areas of Benue State, Nigeria. The authors noted that the geology of the region comprises both complex basement rocks and sedimentary rocks. The results obtained from the Cooper-Jacob (1946) method were acceptable in terms of transmissivity as it corresponded with previous studies. Storativity ranged from 0.01 to 1 and 0.01 to 2.4 in certain areas. It must be noted that storativity can never exceed a dimensionless factor of 1 because 1 represents just water. Values exceeded 1 since the authors used the radius of the borehole and not the effective radius. Consequently, the Cooper-Jacob (1946) method is recommended for producing parameter estimations for modelling, but it is not recommended for the sole analysis of fractured aquifers.

The aim of Meier *et al.*'s (1998:1011) report was to demonstrate that the Cooper-Jacob (1946) method performs very well in calculating effective transmissivities in heterogeneous media when using late-time data. Similar to Vivier and van Tonder's (1997) method, Meier *et al.* (1998) utilised straight-line analytical models to fit late-time data, which resulted in consistent transmissivity values. However, for single-hole and cross-hole tests, the estimates for storativity varied greatly and the results are shown in Figure 3-16.

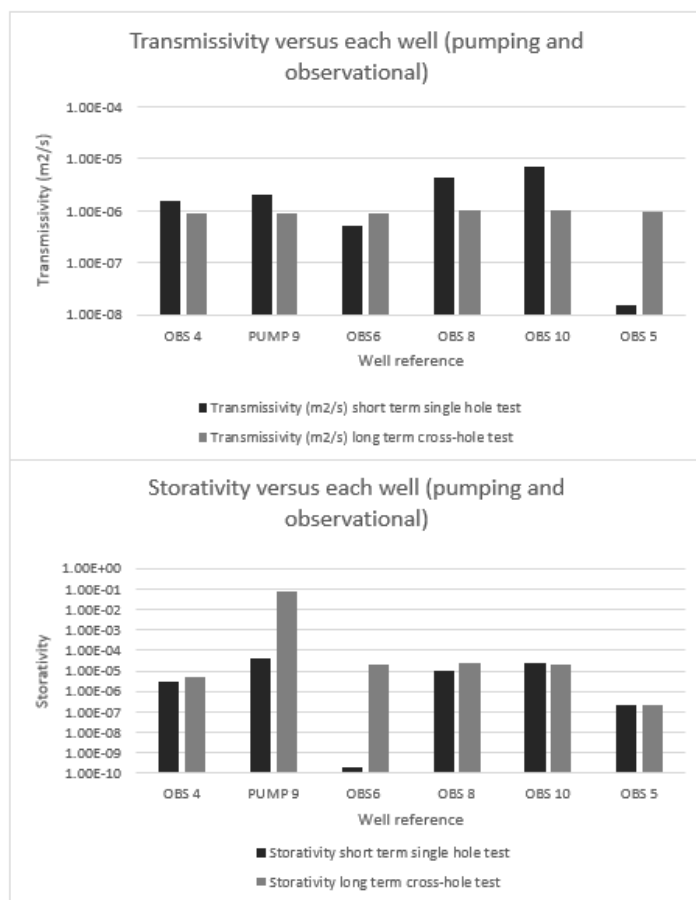


Figure 3-16: Results obtained using the Cooper-Jacob (1946) method in fractured aquifers (by Meier *et al.*, 1998).

Osiensky *et al.* (2000:30-50) observed that the estimation of transmissivity and storativity values from independent drawdown plots often lead to uncertainty in values. According to the author, the variability of these parameters is due to the analysis of drawdown data independently and the short duration of pumping tests. The report further suggested that consistent transmissivity values could be obtained by using long-duration pumping tests and family drawdown curves via the Cooper-Jacob (1946) method, which is consistent with the findings of Rivera and Geol (2014) and Vivier and van Tonder (1997). All these studies emphasised the importance of constraining the models to late-time data to obtain more consistent parameters.

However, the Osiensky *et al.* (2000:50-51) study differed from others in that drawdown curves were analysed as integral members of a family of curves that include late-time drawdown. MODFLOW was used in the study to simulate two-dimensional, transient flow to a pumping well. Rather than analysing individual drawdown plots of pumping and observation wells separately, they were plotted on the same graph, allowing for the estimation of tighter ranges of average transmissivity and storativity values representing the entire aquifer. The report concluded that the estimates of storativity were affected by the vertical and horizontal positions of the plotted drawdown curves. These variations represented spatial distortions in the cone of depression resulting from variations in transmissivity, even when the actual storativity is constant throughout the aquifer.

Although originally derived for homogeneous aquifers, the Cooper-Jacob (1946) method can also calculate late-time transmissivity in heterogeneous aquifers. However, its accuracy in estimating storativity varies. This finding is consistent with the results of several previous studies, including those by Sanchez-Vila *et al.* (1999), Akpen *et al.* (2019:233-241), Meier *et al.* (1998), and Rivera and Geol (2014).

There are two main reasons to include classical analytical methods in a study of flow in fractured aquifers: (1) these methods, especially the Cooper-Jacob (1946) method, are still commonly used in practice (Allen, 1999; Neuman *et al.*, 2007:1-15; Akpen *et al.*, 2019:233-241; Meier *et al.*, 1998:1011; Osiensky *et al.*, 2000:30-50; Alexander, Berg & Illman, 2011) (2) fractured aquifers can sometimes behave like homogeneous aquifers at late time (Meier *et al.*, 1998:1011).

3.7.2 Double-porosity flow models

The double flow porosity concept is seen as the soundest method when simulating fractured aquifers analytically. The assumptions of the method most closely resemble reality of the study areas in this dissertation.

3.7.2.1 Moench method

Moench (1984) derived an analytical solution for unsteady-state flow in an isotropic fractured aquifer. Moench assumed a double-porosity model that accounted for fractured skin and slab or spherical matrix blocks (Duffield, 2007). The author further extended his method to include partially penetrating wells in 1988 and delayed response in 1997.

Moench (1984:831) used Figure 3-12 to illustrate what a typical REV of dual-porosity aquifers looks like in theory. The REV represents the whole aquifer and its properties. Thus, adding more volume will not change the hydraulic parameters of the aquifer. The Moench (1984) method assumes the block and fissure continue to be homogeneous and isotropic concerning their hydraulic properties. As confirmed by Wei *et al.* (1998), this is only true if the cone of depression is larger than the REV, otherwise individual fractures will not be negligible. Furthermore, the aquifer has an infinite areal extent and uniform thickness with unsteady-state flow.

The Moench (1997) method's fundamental assumptions are that the aquifer has (1) infinite areal extent, (2) uniform thickness with (3) unsteady-state flow. The aquifer type is furthermore assumed to be (4) confined with double-porosity.

Data requirements (Duffield, 2007)

- Pumping and observation well locations
- Pumping rates
- Observation well measurements
- Aquifer thickness
- Thickness of slab blocks (or diameter of spherical blocks)

Advantages

- It takes wellbore storage, well losses, and partial penetration into account - the most advanced double-porosity flow method.
- It determines the hydraulic conductivity and specific storage of both fracture and matrix.
- It is applicable for multiple well tests.

Disadvantages

- The cone of depression must be larger than the matrix and fracture REV, thus a long-duration pumping test is required. If this is not the case, individual fractures will not be negligible (Wei *et al.*, 1998).

3.7.2.2 Warren and Root method

This solution is a pseudo-steady-state double-porosity flow model. As previously stated, Barenblatt *et al.* (1960) altered the seepage theory to contain both fracture and matrix pressures and not just effective pressure. The author thus discovered the dual-porosity concept, which was later modified into an analytical model by Warren and Root. Warren and Root (1963) created an analytical solution to study pressure build-up performance tests in aquifers containing both primary and secondary porosities. This was possible by adding two parameters to the governing equation that characterised flow in fractured aquifers. These parameters are:

σ - the ratio of fracture storage to bulk storage and

λ - the inter-porosity flow factor controlling the flow from the matrix to fracture.

It assumes that the primary porosity contributes to the total pore space and the secondary porosity contributes mainly to fluid transport. Therefore, this analytical model was created for pseudo-steady-state flow conditions. Since then, the Warren and Root method has become the industry standard for calculating drawdown in fractured aquifers (Mohammadi *et al.*, 2012; He *et al.*, 2017). It is also important to note that Warren and Root (1963) simplified the actual reservoir system by presenting it as an idealised system with an orthogonal set of cubic matrix blocks with intersecting fractures of identical sizes as shown in Figure 3-17.

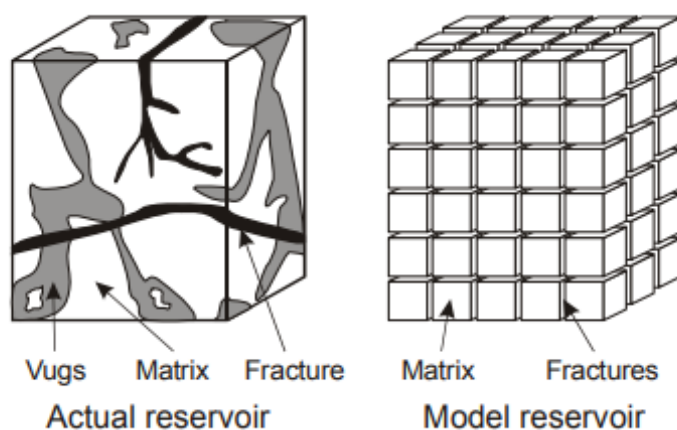


Figure 3-17: Idealisation and simplification of the heterogeneous porous medium by Warren and Root (1963:246).

The assumption by Warren and Root (1963) is oversimplified if the contrast in permeability between the matrix and the fractures is not significant (Lu & Qu, 2017).

Warren and Root (1963) stated that the model assumes (1) the material containing the primary porosity is homogeneous and isotropic; (2) the secondary porosity is contained within an orthogonal stem of uniform continuous fractures; (3) flow can occur between the fracture and the block, but there is no flow between any two blocks; (4) fluid flow is single phase.

In addition, Warren and Root (1963:252) warned that the build-up curve of double-porosity aquifers is like that of stratified reservoirs and that more information on the geometry is needed for an unambiguous interpretation. The authors also stated that the final stages of a pressure build-up test should be used with extreme caution.

The following advantages and disadvantages were documented to familiarise oneself with the potential pitfalls of this concept, for example, if wellbore storage persists in early-time.

Advantages

- Straight-line method. This method can be constrained to data adequate for analyses.
- Accurate parameter determination in true double-porosity aquifers.
- It produces storage coefficients of both the matrix and fracture systems.

Disadvantages

- It is not as complex as other analytical methods, assuming double-porosity.
- It is only applicable in type 2 NFR; thus aquifers where the fracture network provides the reservoir permeability.
- This method will yield incorrect results for storage coefficients if wellbore storage is not negligible.
- It is only used for analyses of pumping boreholes

3.7.2.3 Flow patterns of dual-porosity aquifers

The following discussion will be based on Figure 3-18 and Figure 3-19. Different stages of groundwater flow can be recognised and discussed when pumping a borehole in dual-porosity aquifers.

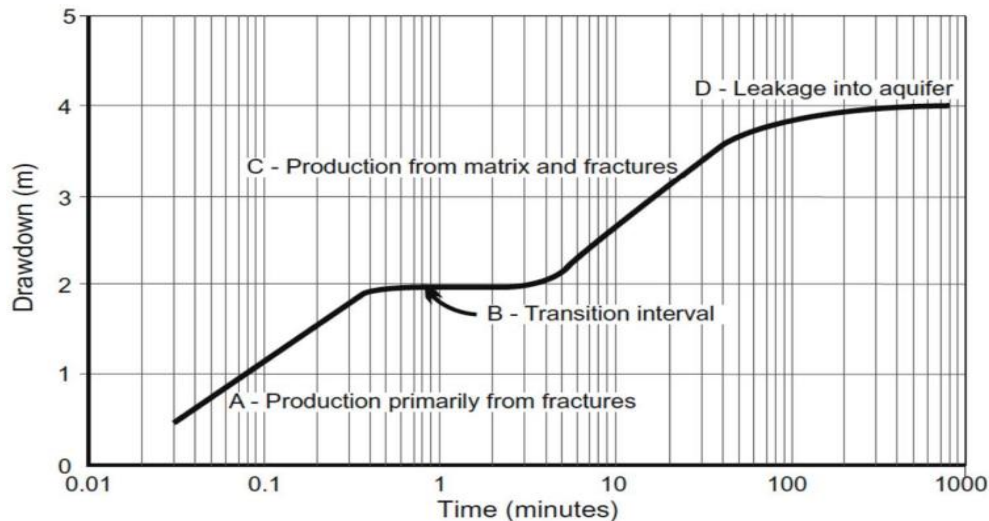


Figure 3-18: A hypothetical drawdown curve indicating the flow regime of a fractured aquifer.

3.7.2.3.1. Wellbore storage

When pumping begins, a steep drawdown slope is expected and is known as the wellbore storage effect. As previously mentioned, the water present inside the borehole will be extracted first and can be identified by a slope of 1 on a log-log graph as seen in Figure 3-19. Wellbore storage only affects the drawdown for a short period because of the limited volume the wellbore possesses.

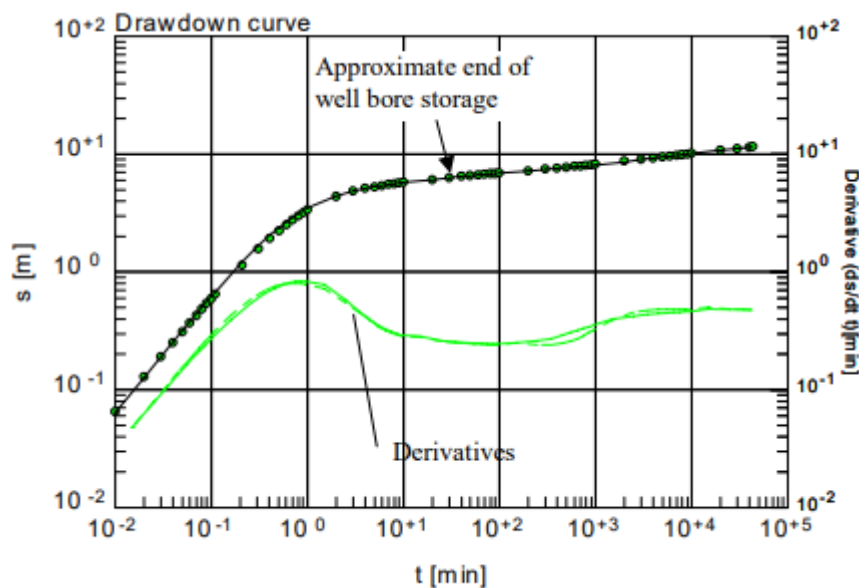


Figure 3-19: Wellbore storage effects on a pumping well during early-time drawdown (from van Tonder *et al.* (2002:32)).

3.7.2.3.2 Production primarily from fractures

Pumping the borehole further will increase the hydraulic gradient between the borehole and the surrounding area. Therefore, fractures nearest to the borehole will be pumped first due to the high permeability of the fractures relative to the matrix. A decrease in drawdown rate compared to the wellbore storage stage is expected. Since the storativity of the fractures is very low, this stage only lasts a limited time.

3.7.2.3.3 Transition period

Streltsova-Adams (1978) described this period as a re-equilibration of pressure differentials between matrix and fracture. In this situation, the drawdown will be constant during the transition period from fracture flow to flow from fracture and matrix. This transition yields a “U” shape in the derivative drawdown plot of a fractured or double-porosity aquifer.

3.7.2.3.4 Production from fracture and matrix

The matrix system recharges the depleted fractures until the discharge reaches a rate at which the two systems are said to be in equilibrium. The matrix is said to store most of the water while the fractured system transports the water. This behaviour, where a fractured aquifer reaches

radial flow, will only happen at late-time drawdown and if no recharge or outer boundaries are present.

3.7.2.3.5 Late-time boundary effects

A long-duration pumping test can yield information about no-flow and recharge boundaries at late-time. If the cone of depression reaches such a boundary, the drawdown will either increase or decrease depending on the type of boundary present. A vertical dyke, for example, represents a no-flow boundary and will cause the drawdown to increase at late-time. A river or stream represents a recharge boundary that will produce a decreased drawdown at late-time (see D in Figure 3-2). These boundary effects are seldom considered when determining sustainable yield due to insufficiently long pumping tests and the inability of analytical models to account for late-time drawdown fluctuations.

3.7.2.4 Case studies

3.7.2.4.1 Moench method

Kollet and Zlotnik (2005:267-285) investigated the origin of highly variable and unrealistic parameter estimates. The analytical models of Moench (1995) and Neuman (1974) were applied to data obtained at a site in east-central Nebraska, USA. Previous research on the area shows that the aquifer is heterogeneous and unconfined. The application of Moench (1995) and Neuman (1974) with the correction for aquifer return flow, has estimated highly variable storativity (S) and hydraulic conductivity (K) values. The Moench (1995) method was more accurate than the Neuman (1974) method, with estimates of S and K showing a reduced root-mean-square error between measured and calculated data. A global analysis was also performed and yielded similar results with the Moench (1995) and the Neumann method. A global analysis is where all drawdown versus time plots are plotted on one graph and analysed by producing a best fit to all the plots. The authors recommended the use of global analysis with applied juxtaposition over individual curve analysis, as this has yielded more consistent results. A 3D numerical model of high K-inclusion was constructed by using the MODFLOW code that, according to the author, successfully modelled the aquifer in terms of specific yield and hydraulic conductivity.

The methods of Moench (1984), Theis (1935), and Barker (1988) were implemented to analyse hydraulic tests in crystalline rocks. The investigation, by Burbey and Brandon, (2016:3289-3302), was proposed to quantify the properties and characteristics of an isolated fracture. Because the Moench (1984) method produced parameters for both the fracture and the matrix, it could identify the relation between the fracture and matrix storativity and transmissivity. The report concluded

that the Moench method provided the best estimate for specific storage, as the results were similar to previous estimates.

Furthermore, Verhagen *et al.* (2001:34-38) studied the response of a fractured multi-layered aquifer in the Orapa/Letlhakane area in Botswana. A case study where the methods of Theis (1935), Walton, and Moench were used to analyse the drawdown versus time data from the Orapa/Letlhakane area. The study concluded that storativity declines the further an observation borehole is situated from a pumped borehole when using classical methods of analyses. Interpretations according to the Moench method (block system) yielded “consistent” results for storativity in that the storativity produced the same decline as the distance between the pumping and observation borehole increased. The results obtained for storativity versus distance were hand-fitted and can cause unforeseen errors. Nonetheless, the graph in Figure 3-20 visualises the similarity in response between storativity and distance to the borehole and can hence derive a more representative storativity for the aquifer.

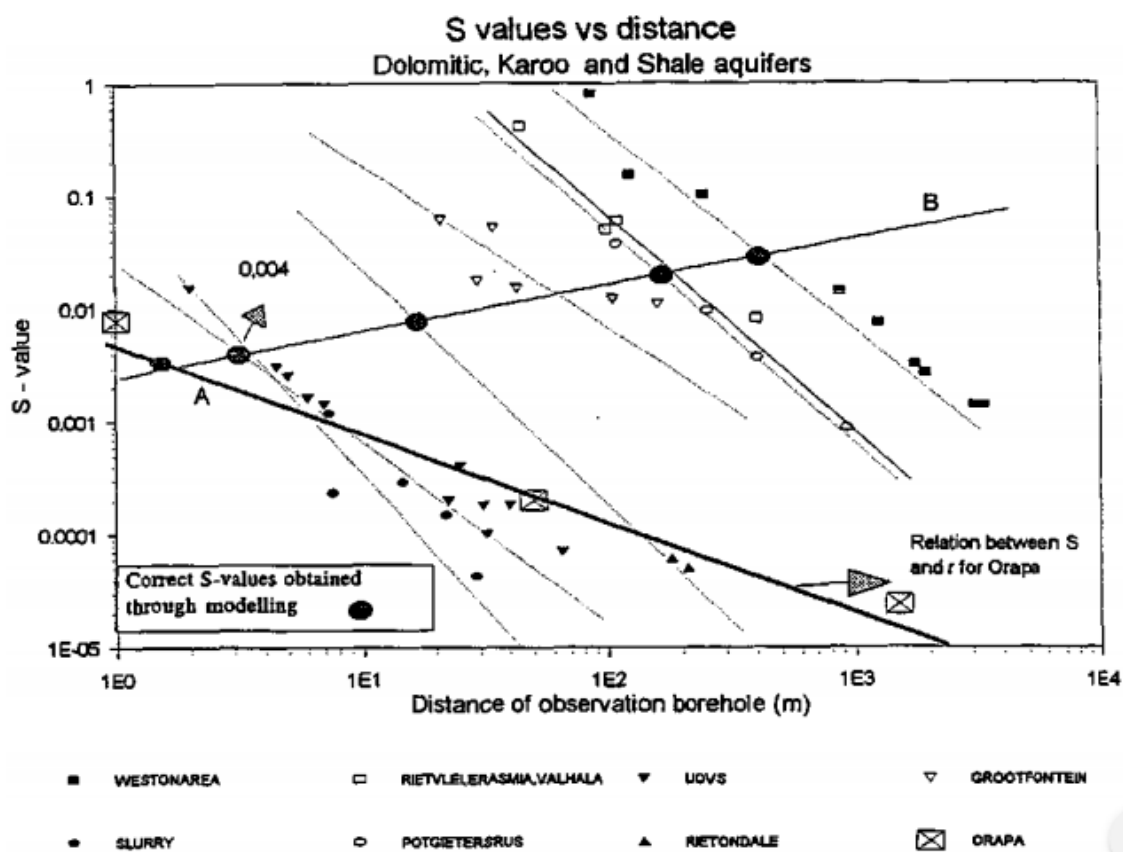


Figure 3-20: Storativity values derived from the Moench method (Block system) from Verhagen *et al.* (2001).

Verhagen *et al.* (2001) discussed the declining storativity phenomenon with the help of the renowned Professor Neuman from Arizona. Professor Neuman was presented with the South

African results concerning the spatial variability of storativity and volunteered to give his explanation of the phenomenon:

“Close to the pumping well, pressure in the large fractures declines rapidly relative to its rate of decline in the small fractures. The latter therefore releases a relatively large amount of water into the large conductive fractures due to a sizeable local pressure gradient between the small and large fractured reservoirs. Hence S is large. Far from the pumping well, the pressure gradient between the two reservoirs is relatively small. Therefore, water release from the small to the large fractures occurs very slowly. Most of the initial measurable drawdown (in the large fractures) is associated with water release from storage in these same fractures. Therefore, S is small.” Professor Neuman further stated that *“With time, local pressure differentials between the reservoirs stabilise and flow everywhere within a given radius approaches a steady radial pattern. Therefore, I would expect S everywhere within this radius to approach a uniform value representing both reservoirs.”*

This statement is consistent with the results produced from the case study by Verhagen *et al.* (2001) in which the storativity reduces as the distance between the pumping borehole and the observation borehole increases.

3.7.2.4.2 Warren and Root method

Four complex analytical techniques were used to derive the hydraulic properties of fractured hard rock in South India. The authors (Maréchal *et al.*, 2003:1-10) noted that South India is dominated by fractured hard rock and general techniques could not simulate the conditions, thus producing inaccurate parameters. Methods utilised to analyse pumping tests carried out in the same study area were: The Neuman method for anisotropy of permeability at observation wells, the Gringarten method for horizontal fracture intersecting the pumping well, the dual-porosity model of Warren and Root (1963) and the Barker (1988) theory for connectivity and fractional dimension flow. A short conclusion stated that these methods were successfully applied to the case studies and produced parameters within acceptable ranges. Maréchal *et al.* (2003:1-10) did not use extensive pumping data and only two observation wells were available. Not all four methods were used on each well. Instead, the diagnostics of the drawdown plots were used to choose the most appropriate method. This highlights the importance of decision-making regarding what analytical model to use after studying the drawdown curves when compared to diagnostic curves.

The two straight-line methods of Warren and Root (1963) and Kazemi *et al.* (1969) were used to analyse pumping test data of bedrock aquifers in Colombia. In this regard, Allen (1999) noted that

it was very difficult to determine if dual-porosity models were adequate for some aquifers. The results obtained were higher than the results achieved from fracture flow methods. This may be because of the aquifer not exhibiting true dual-porosity behaviour or not long enough pumping times. Consequently, Allen (1999) recommended a detailed site investigation with multiple observation wells to form a strong conclusion.

Furthermore, the Warren and Root method was tested on heterogeneous media in California, USA. Wu (2002) checked the accuracy of the analytical method by using a numerical method and concluded that the Warren and Root method was accurate most of the time. Additionally, the author also concluded that the Warren and Root method will yield accurate estimates when the double-porosity concept is valid (type 2 NFR). Thus, $k_f \gg k_m$ and $\phi_m \gg \phi_f$, meaning that the hydraulic conductivity of the fractures present represents the whole aquifer, and the porosity of the matrix represents the aquifer's porosity. This statement by Wu (2002:5-6) agrees with Egeya *et al.* (2018:30-49) who stated that the Warren and Root method is only applicable for a type 2 reservoir. If this assumption is valid, the Warren and Root method will prove to be accurate for medium to late-time analyses.

The Warren and Root method, though not the most complex, was immensely popular due to being the first real method assuming the dual-porosity concept. With a straight-line application and assumptions based on fractured aquifers, the Warren and Root method is both easy to analyse data with and it yields proper parameters.

3.7.3 Single fracture flow models

The concept of single fracture flow will be discussed further as researchers have incorporated the concept into analytical methods to analyse pumping tests.

3.7.3.1 Gringarten and Ramey (1974)

This solution assumes unsteady-state flow in fractured aquifers with a single horizontal plane fracture intersecting the control borehole (Gringarten and Ramey, 1974:413-426). The analytical solution was first developed for hydraulic fracturing, but its importance in hydrology has since been recognised. The method can determine hydraulic conductivity (K_f), the hydraulic conductivity anisotropy ratio (K_z/K_f), specific storage (S_s), and fracture radius (R_f).

The model assumptions are as follows: (1) the aquifer has an infinite areal extent and (2) uniform thickness. (3) the potentiometric surface before pumping is horizontal (h_0), (4) the fractured aquifer is represented as a single plane horizontal fracture, (5) the radius of the observation well is very small, and (6) water release from storage happens instantaneously with a decline in head.

The solution can apply to variable pumping rates and multiple observation wells if the required data are available.

Data requirements (Duffield, 2007):

- Measurements and locations of pumping and observation boreholes
- Pumping rates
- Saturated thickness
- Radius and depth of horizontal fractures

Advantages

- It is used to analyse pumping data from short-duration pumping tests.
- It is applicable for multiple well tests.
- Parameters calculated represent the aquifer system.

Disadvantages

- If the effects of the horizontal fracture are misinterpreted as wellbore storage, unrealistic high storativity will be obtained (Bardenhagen, 2002).
- It is only applicable when a piezometer is used at the observation borehole.

3.7.3.2 Gringarten and Witherspoon (1972)

The response of the aquifer to pumping will differ from the Theis solution if a well intersects a single vertical fracture (Kruseman & de Ridder, 1994). Various solutions have been proposed, but most have produced erroneous results. The conceptualisation of the fracture as a vertical plane (no storage capability) of short length and infinite hydraulic conductivity was a major step forward. As a result, aquifers could now be modelled as homogeneous, anisotropic, and porous media with single plane fractures (Kruseman & de Ridder, 1994). The Gringarten and Witherspoon (1972) method is based on this single vertical fracture flow concept. This method presents an analytical solution for unsteady-state flow to a fully penetrating well intersecting a single uniform-flux vertical fracture. Figure 3-21 illustrates the concept of the method. It also assumes the aquifer to be confined, anisotropic, and infinite.

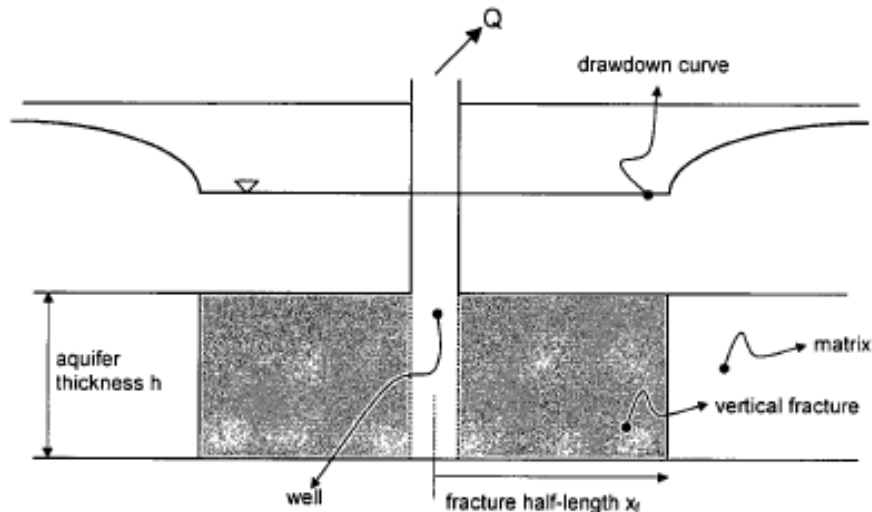


Figure 3-21: Gringarten and Witherspoon (1972) method of a well intersecting a uniform-flux vertical fracture (from Bardenhagen, 2002).

The output parameters of the Gringarten and Witherspoon (1972) method are the hydraulic conductivity in the x-direction (K_x), specific storage, the hydraulic conductivity anisotropy ratio (K_y/K_x), and the length of the fracture.

Data requirements (Duffield, 2007)

- Measurements and locations of pumping and observation boreholes
- Pumping rates
- Saturated thickness
- Length of vertical fracture

Advantages

- Only short-duration pumping data are required.
- Applicable for multiple well tests.

Disadvantages

- Do not take wellbore storage into account.
- Are only used for analyses of observation boreholes.

3.7.3.3 Flow patterns of single fracture flow models

3.7.3.3.1 Flow patterns of single horizontal fractured aquifers

The flow of water to a borehole intersecting a single horizontal fracture was studied by Gringarten and Ramey (1974). Four flow stages can be used for diagnosis:

1. Fracture storage flow where the duration depends on the thickness of the fracture.
2. Linear flow from the fracture to the well at intermediate time with a 0.5 slope as shown in Figure 3-22.
3. Transient flow.
4. Radial flow at late-time starting at $t_d = 5$. Radial flow is characterised by a straight horizontal line on a log-log plot.

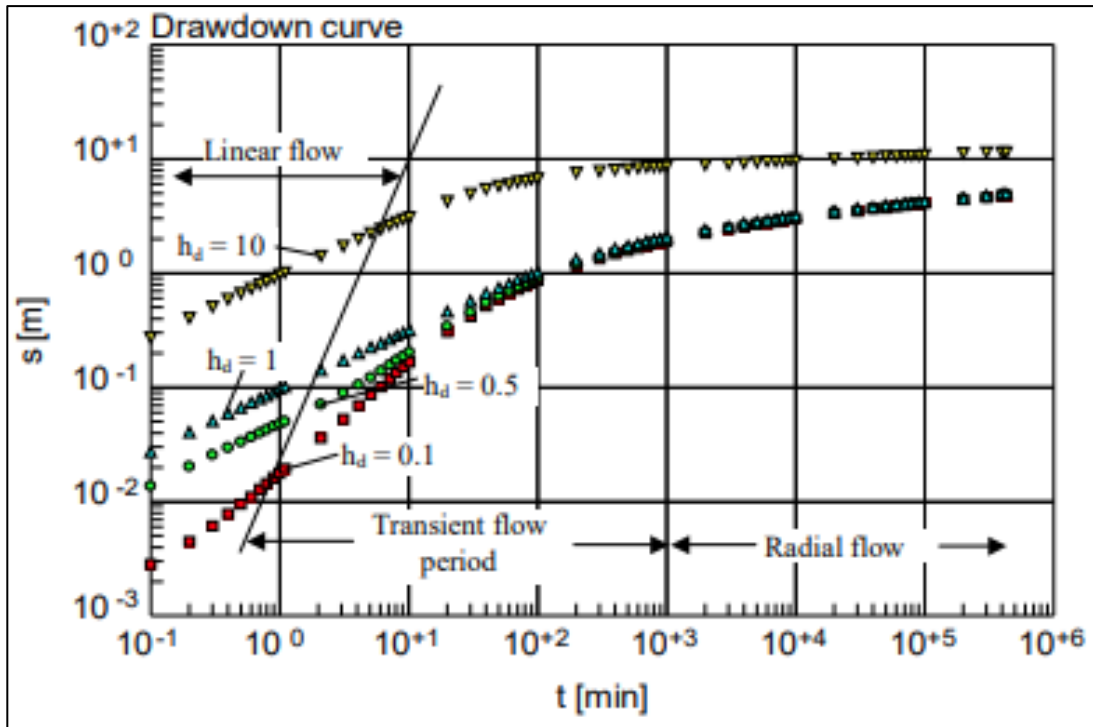


Figure 3-22: The flow periods of a borehole intersecting a penny shaped bedding plane fracture (after van Tonder *et al.*, 2002).

3.7.3.3.2 Flow patterns of single vertical fracture models

The stages of flow and flow regimes of a well intersecting a single vertical fracture were studied by Gringarten *et al.* (1974) and Cinco-Ley and Samaniego (1981). The flow can be subdivided into four different regimes:

1. Linear flow from fracture to the borehole at a very early-time. Linear flow exhibits a 0.5 slope on a log-log plot.
2. Bi-linear flow at early-time after linear flow has ceased. Bi-linear flow typically shows a slope of 0.25 on a log-log plot.
3. Linear flow from the formation at an intermediate time, which shows a slope of 0.5 on a log-log plot.

4. Radial flow from the formation at late-time. A straight horizontal line on a log-log plot characterises radial flow.

3.7.3.4 Case studies

A report by Baumle (2003) investigated the geohydraulic characteristics of fractured flow regimes. The author covered a wide variety of methods commonly used in fractured aquifers. Information on well hydrograph analysis, hydraulic test analysis, groundwater tracing techniques, hydrochemical and isotope studies, and groundwater modelling are all included in the report. Baumle (2003) obtained time-drawdown data from pumping tests conducted for 72 hours. Theis (1933) and Cooper-Jacob (1946) were used, as well as Moench (1982), Gringarten *et al.* (1974), and Gringarten and Ramey (1974). The author noted that sometimes the analysis was ambiguous and more than one method yielded similar results. These methods were not compared as only certain methods have been used based on diagnostic plot analyses. Baumle (2003) concluded that, in general, a large regional variance in parameters (transmissivity and storativity) was observed. This may be due to the wellbore storage that was present in early-time drawdown. Only one of the analytical methods chosen assumes wellbore storage and that is the Moench (1984) double-porosity method.

Similar to previous comparative studies, Allen (1999) assessed different methodologies for analysing hydraulic test data. The author studied fractured bedrock aquifers in British Columbia through analyses with analytical methods that account for three different concepts (radial flow, double-porosity flow, and single fracture flow concepts). Hydraulic test data for 28 water supply wells were analysed. The single vertical fracture models of Gringarten *et al.* (1975) and Gringarten and Witherspoon (1972) were used for analysing pumping and observation well data, respectively. The results show that single fracture flow models were appropriate for several of the hydraulic tests and that good estimates of T and S were obtained. Parameters showed similarities when compared with the parameters calculated with Theis (1935) at late-time radial flow. Since single fracture flow methods are used at early to intermediate time, this similarity is important. Long-duration pumping tests to reach radial flow are expensive and rarely done. Thus, a method that yields similar results from shorter pumping tests will prove to be economic.

A detailed pumping test analysis was conducted by Gernand and Heidtmann (2005) to characterise a fractured bedrock aquifer. Pumping included 24 wells and a duration of 21 days on a low-yielding fractured gneiss aquifer. The time-drawdown response revealed linear flow in early times and IARF at line times. Investigators also noted that all observation wells responded

in accordance with radial flow models. This suggests that certain bedrock aquifers may behave similarly when a large area is investigated.

The major advantage of single fracture flow models is the ability to analyse short-term pumping data. The method provided good estimates of transmissivity and storativity in most cases. This method will therefore calculate early-time storativity values.

3.8 Numerical model (RPTsolv)

According to Botha *et al.* (1998:133-147), the use of analytical methods and type curves can only produce rough estimates of parameters in the field, especially in fractured aquifers. A study was conducted on the flow characteristics of the Karoo aquifer by Botha *et al.* (1998). Botha (1998: viii) stated that the flow in the Karoo aquifers is mainly vertical and that horizontal two-dimensional numerical models are inadequate to characterise said aquifers. Consequently, the development of a new vertical two-dimensional flow model based on an existing program GCON (by Botha *et al.*, 1992) for horizontal flow was created, called RPTsolv.

RPTsolv uses inverse modelling to reach a solution. This type of modelling includes matching observation data by specifying and adjusting parameters until the produced curve fits the observation data. Botha *et al.* (1998) noted that the inverse problem has an infinite number of solutions, consequently there is no guarantee that the parameters calculated represent the aquifer. The user must not take the parameters at face value but compare the value with known parameters to see how they correlate. RPTsolv utilises the Liebenberg-Marquardt algorithm (LM), a robust non-linear least square algorithm. The algorithm is used as a curve fitting method for inverse modelling. Furthermore, the LM algorithm is fast to converge and can handle models with multiple parameters that are not precisely known (Marquardt, 1963:431-441). Multiple disadvantages of the non-linear algorithm are that it cannot be used for flat functions and it can take longer to converge when the model has over ten parameters.

By altering the horizontal flow equation to represent vertical flow together with a robust non-linear least square algorithm, Botha *et al.* (1998) developed the numerical model, RPTsolv. The model has since been successful in determining parameters of Karoo aquifers (Verwey *et al.*, 1995; Hartnady & Hay, 2000; Nel, 2001:42-44; Gomo, 2009) more accurately than analytical models.

Botha *et al.* (1998) noted a few problems. Firstly, the model is not free of the distance dependency of storativity but just more consistently accurate than other models. The presentation of the perturbed borehole in the program needs attention. The authors stated that the presentation of the perturbed borehole as a line source (Neumann boundary) causes the distance dependency

of storativity. It was therefore recommended that the Neumann boundary be replaced by the Dirichlet approximation of Huang (1973) in future versions of the program. Users may experience two difficulties when using RPTsolv. The first is that the program uses the three-dimensional hydraulic conductivity and specific storativity, therefore transmissivity and storativity before they are multiplied with the aquifer depth. This is not a real problem, as the depth of the aquifer had to be known as one of the geometric parameters for RPTsolv. As a result, transmissivity and storativity can be derived from hydraulic conductivity and specific storage if needed. The second difficulty the program imposes on its users is its inability to handle arbitrarily spaced observation boreholes. Observation boreholes must lie in a straight horizontal line or must be projected that way.

A case study was done by Botha *et al.* (1998) in which Theis and RPTsolv were used to calculate storativity. This was done to see if RPTsolv is free of the distance dependency behaviour. The results of the analysis of a constant rate test with Theis and RPTsolv are given in Table 3-2:

Table 3-2: Comparison of the hydraulic parameters observed during a constant rate test with the RPTsolv and Theis methods. From Botha *et al.* (1998:151).

Borehole	Distance from PBH (m)	RPTsolv		Theis	
		T (m ² /s)	S	T (m ² /s)	S
UO15	14.234	3.13E-04	4.60E-03	2.03E-04	4.22E-04
UO7	26.117	2.66E-04	3.60E-03	2.11E-04	1.51E-04
UO9	35.72	2.32E-04	1.90E-03	2.02E-04	7.83E-05
UO11	43.91	1.97E-04	3.00E-03	1.74E-04	1.19E-04
UO13	52.98	2.32E-04	1.10E-03	1.86E-04	6.54E-05

As can be seen in the results above, the Theis-fitted values were not acceptable, but the numerical model calculated storativity more accurately than the analytical method. RPTsolv has since been cited to accurately determine storativities in fractured aquifers. Nel (2001:42-44) used RPTsolv to analyse the results of a constant rate test in the Karoo aquifers. The estimated storativities were obtained and compared between Theis (1935), Cooper-Jacob (1946), FC-method, and RPTsolv. Furthermore, the numerical model produced the most consistent storativity values, while the analytical methods struggled to achieve the same consistencies. Likewise, Hartnady and Hay (2000) analysed pumping test results using RPTsolv and the FC-method. Based on the results, a recommended storativity of 10^{-3} and $10^{-5} - 10^{-3}$ was given for deeper confined aquifers and shallow aquifers, respectively. The results were in accordance with a previous study in the same area.

3.9 Summary

The parameters transmissivity and storativity are critical for understanding groundwater flow and drawdown, with storativity being particularly challenging to determine due to its variation with distance from the pumping borehole. The distance dependency cannot be accounted for by current analytical methods used in practice (Bredenkamp *et al.*, 1995:160; Sanchez-Vila *et al.*, 1999; Akpen *et al.*, 2019:233-241; Meier *et al.*, 1998; Rivera & Geol, 2014). Despite their widespread use, analytical models have limitations such as non-uniqueness and uncertainties, and they are less suited for accounting for horizontal and vertical flows in fractures (Strack & Barnes, 2009; Anderson *et al.*, 2015; Woodford & Chevallier, 2002). Therefore, when resources allow, they should be supplemented with numerical models for enhanced accuracy. Diagnostic plots remain a valuable tool in groundwater analysis, aiding in model selection and parameter estimation, by providing a visual representation of aquifer behaviour against time on a logarithmic scale (Renard *et al.*, 2009; Van Tonder *et al.*, 2002:19).

In order to study the distance dependency of storativity when calculating analytically, a 2-dimensional numerical model called RPTsolv was used in the research. RPTsolv was specifically created for South African aquifers and produces more accurate and reliable parameter values than analytical methods (Verwey *et al.*, 1995; Hartnady & Hay, 2000; Nel, 2001:42-44; Gomo, 2009). See Appendix B which contains a summary of the flow regimes of each method compiled during the study.

CHAPTER 4 METHODOLOGY

4.1 Introduction

This chapter explains how the dissertation aims to accomplish the research objectives. To investigate the distance dependency of storativity, the researcher used both analytical and numerical methods, drawing upon the relevant literature as a foundation for the approach. Analytical models based on the three distinct concepts that describe fractured and dual-porosity flow were considered. Additionally, this chapter explains how the numerical model known as RPTsolv is used in these analyses and how it is incorporated into an Excel method. This combination of analytical and numerical tools allows for a comprehensive examination of the variation of storativity with distance.

4.2 Data acquisition and preparation

The data were primarily obtained from geohydrological consultants and the Centre for Water Sciences and at the North-West University. The data consist of constant rate tests, as well as some data containing information of step-drawdown tests and slug-test analyses. The necessary data are identified and organised for the study, which specifically focuses on constant rate pump test data paired with observation boreholes. The study involves thirteen (13) pumping tests with observation wells and ten (10) single-well tests. Please refer to Appendix D for all pumping test drawdown graphs. Data are gathered from four key areas in South Africa, as indicated in the description of the study areas, with two locations each in the Karoo Supergroup and Cape Supergroup. The analysis focuses on drawdown data from boreholes with associated observational data, allowing for the study of the distance dependency of storativity. Meanwhile, these single borehole tests are used for comparative analysis purposes.

4.3 Diagnostic process

All time-series data and the information contained in the data files were extracted and catalogued into a database for easy visualisation. An example dataset is presented in Figure 4-1.

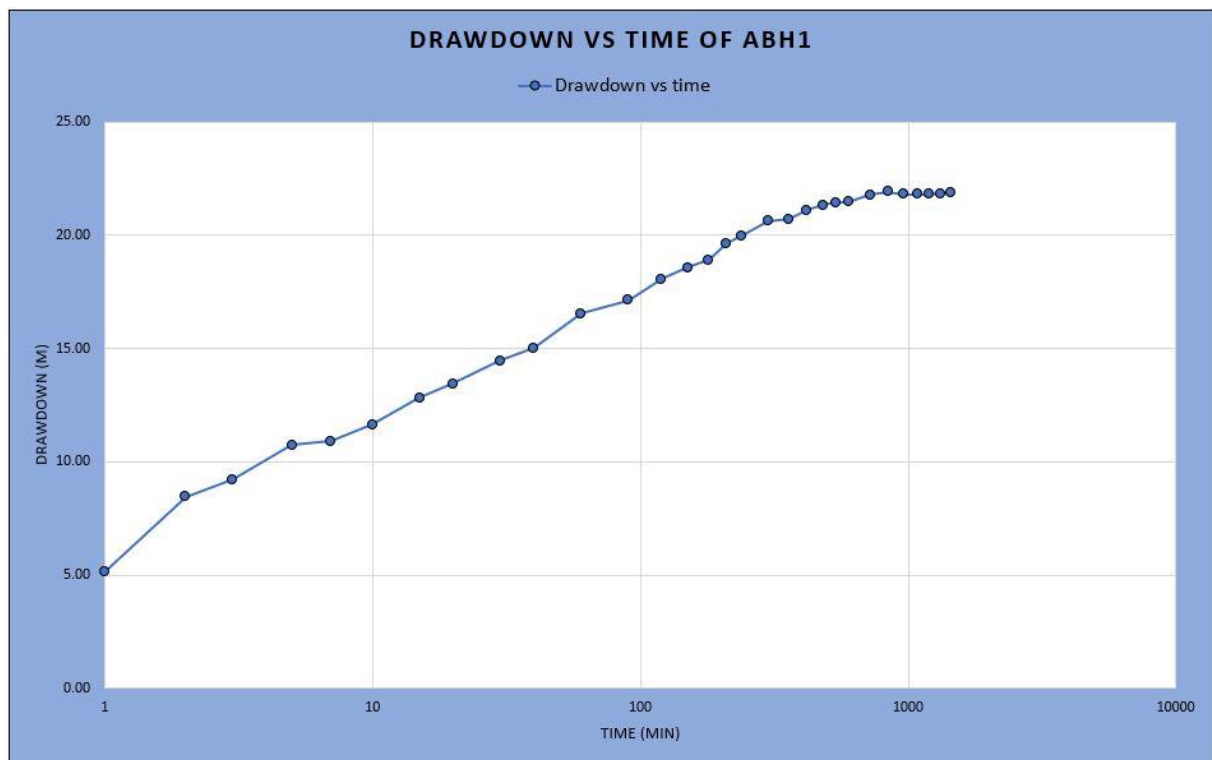


Figure 4-1: Linear-log plot of drawdown versus time is a standard way of visualising data obtained from the pumping test.

Some observation boreholes may have noisy or unusable data due to their distance from the pumping borehole. By applying post processing on then data to filter out noise, a usable sub dataset is obtained. The result of this aforementioned process is illustrated in Figure 4-2, showing the before and after picture of a selected dataset subject to noise.

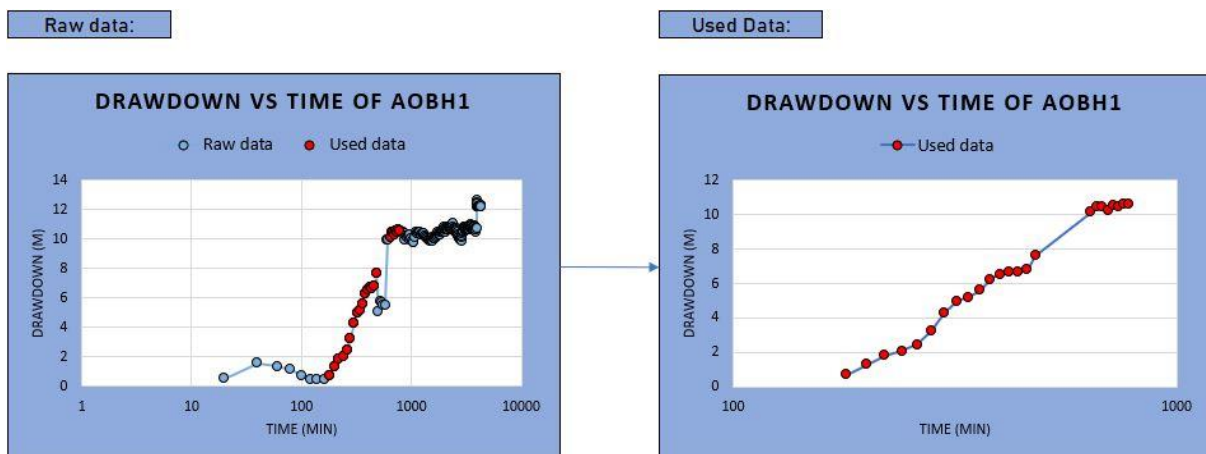


Figure 4-2: Two linear-log plots of drawdown versus time showing the raw data (left) to the data used for analyses (right).

These drawdown plots are used for data visualisation before the diagnostic process. The diagnostic procedures on the log-log graphs aim to identify specific flow regimes, while the

derivative analysis focuses on pinpointing fracture positions and boundaries, as minor variations in drawdown are amplified. Log-log graphs of drawdown versus time allows for the identification of flow regimes as shown in the example presented in Figure 4-3. A summary of the flow regimes with associated temporal scale are presented in Table 4-1. This step yields valuable information on key aquifer characteristics, such as whether wellbore storage persists during the early time or if radial flow is reached in the late time phase.

Log-log diagnostics: For identifying regime flows

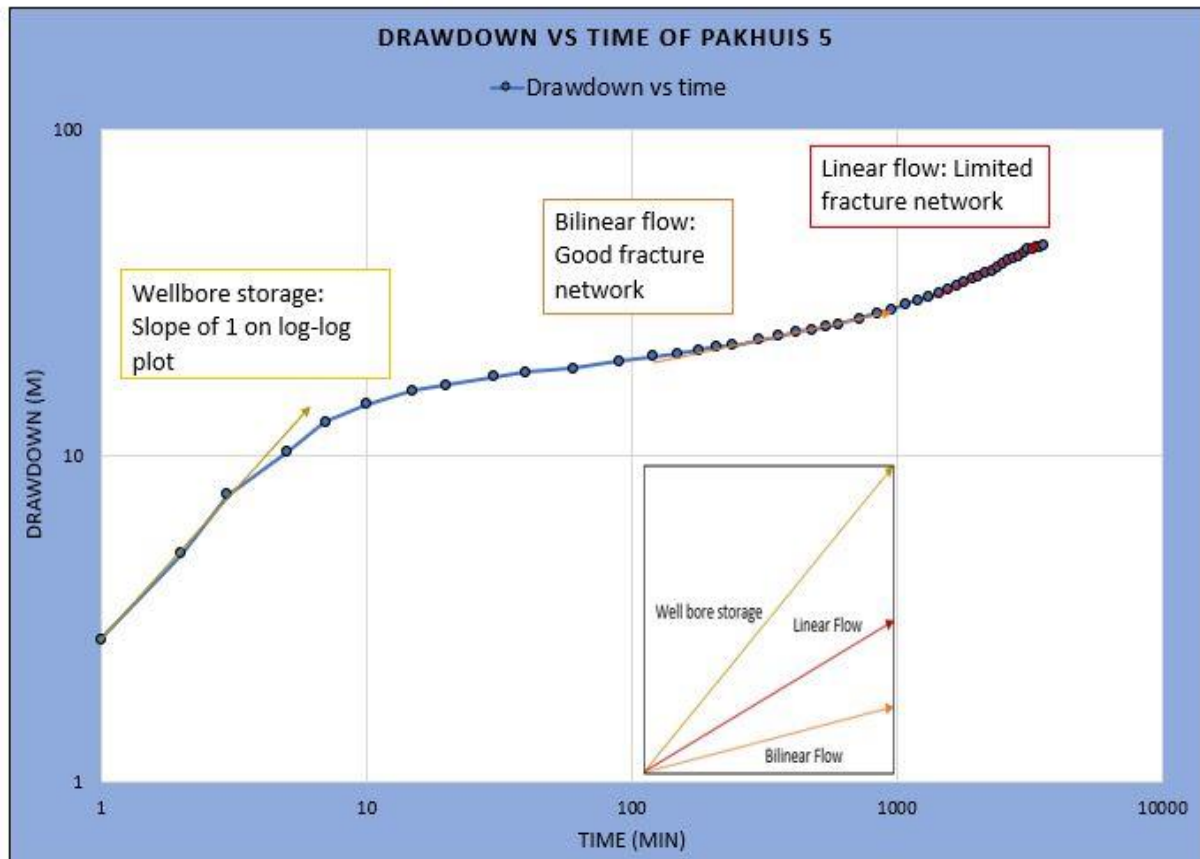


Figure 4-3: Log(s)-Log(t) graph with diagnostics aimed at identifying the flow regimes.

Table 4-1: This table notes the flow regimes of different time periods pertaining to different flow concepts and methods.

Flow type	Flow regimes in diagnostic plots			Methods
	Early time	Intermediate time	Late time	
Radial flow	Inner-boundary effects (wellbore storage). Slope of 1 on a log-log plot.	Transition period to IARF.	Radial flow from formation. Shows a horizontal line on a log-log plot.	Theis (1935) Cooper-Jacob (1946)
Double porosity	Radial flow from fracture network to borehole. Straight-line on semi-log plot (s vs. log(t)).	Transition period. The derivative curve is characterised by a U-shaped double-porosity dip.	Radial flow from the matrix and fracture. Straight-line on the semi-log plot (s vs. log(t)).	Warren and Root (1963) Moench (1984)
Single horizontal fracture	Inner-boundary effects (wellbore storage). Slope of 1 on the log-log plot.	Linear flow from the fracture to the well. Slope of 0.5 on a log-log plot.	Pseudo-radial flow. A horizontal line on a log-log plot.	Gringarten & Ramey (1974)
Single vertical fracture	Linear fracture flow with a slope of 0.5 on a log-log plot. Linear flow is followed by bilinear flow with a slope of 0.25 on a log-log plot.	Linear flow from the formation. Slope of 0.5 on a log-log plot.	Pseudo-radial flow. Horizontal line on a log-log plot.	Gringarten and Witherspoon (1973)

As previously discussed, derivative analyses are essential as it enhance the visibility of subtle changes in the drawdown plot that might be challenging to discern using a standard log-log drawdown plot. This analytical technique serves a dual purpose, providing a deeper

characterisation of water flow within the aquifer and serving as a valuable tool for identifying fracture positions and boundaries.

The insights obtained from a derivative plot have proven invaluable in discerning various elements as shown in the example presented in Figure 4-4. It is within the derivative plot that the identification of the dual-porosity dip is most prominent, a pivotal step in this dissertation.

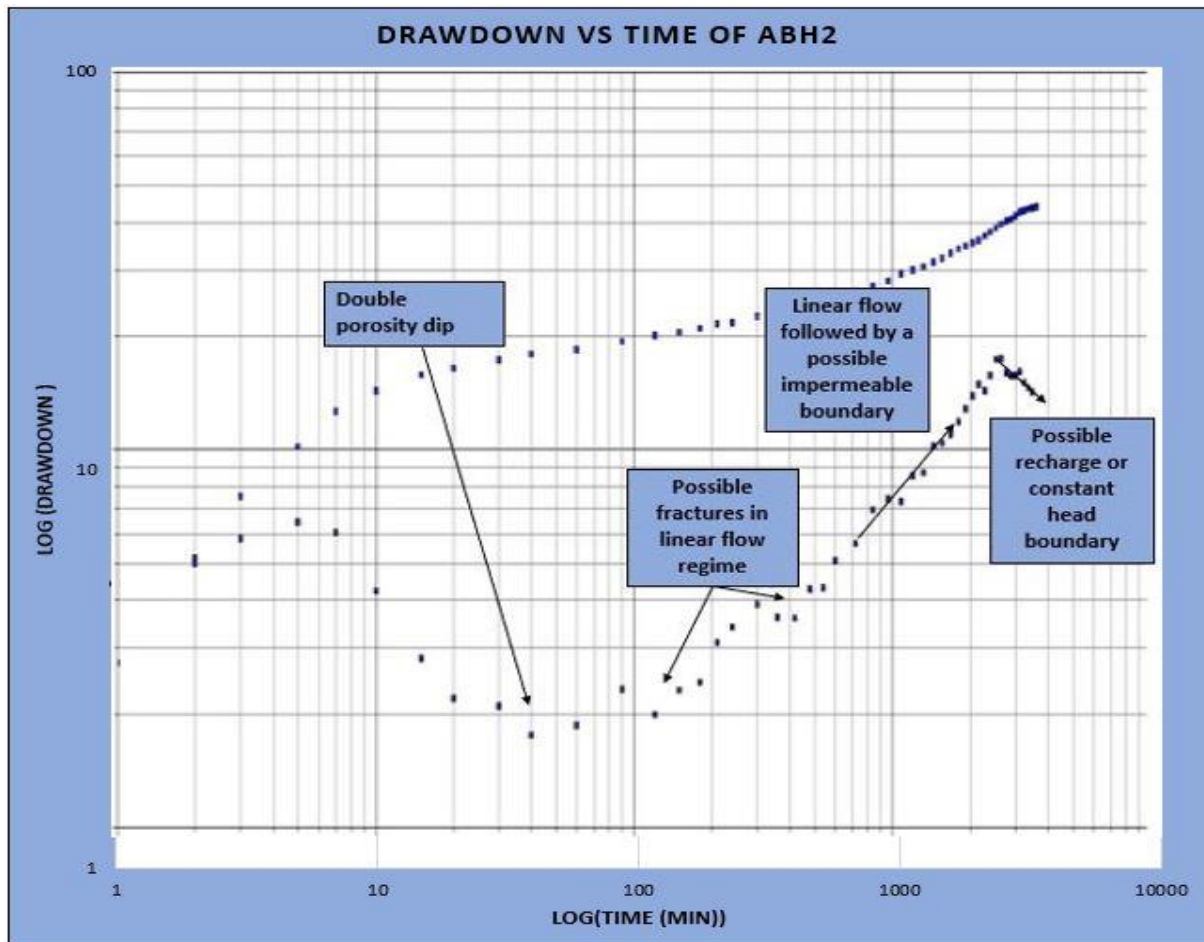


Figure 4-4: Log-Log graph of a drawdown plot with the derivative for derivative analysis.

The final step in the diagnostic process involves consolidating all the gathered information from the preceding steps into a structured table format. Table 4-2 displays the flow regime of the log-log plot during four distinct time periods, highlighting occurrences of well bore storage, dual-porosity dip, and boundaries encountered throughout the pump test duration. By considering all these aspects, a recommended analytical model for analysing the specific pump test is proposed. This diagnostic process serves to enhance the utilisation of analytical models in two crucial ways:

1. Justifying Model Selection - It allows for the justification of selecting a particular model over an alternative option.

2. Aiding in Fitting Procedure - The process assists with the fitting procedure by identifying relevant slopes and time periods, streamlining the model fitting process.

Table 4-2: The results of the diagnostic process.

	n (flow dimension)	Regime flow	Time (min)
Very early time	4	Transition	10-50
Early time	1	Linear	100-1000
Medium time	0	Impermeable boundary	1000-2500
Late time	4	Constant head	2500-3500
Well bore storage		Yes	
Dual porosity dip		Yes	
No flow or recharge boundary		Recharge	
Suggested model		Dual porosity (Moench) or Cooper-Jacob	

4.4 Analysis Methods

As discussed in the literature review, both AquiferTest and Aqtesolv are software packages designed for the interpretation and analysis of pump test data, utilising a diverse array of analytical techniques. By utilising the tools and functions provided by the software, pump test data can be effectively and efficiently analysed, with users following the outlined steps in the user manual. It is essential to acknowledge that while the process may seem straightforward, it will not always be that simple in practice. A limitation of analytical methods is the absence of a definitive correct answer. Therefore, the user's expertise and experience will be crucial in ensuring that the results obtained are both appropriate and realistic.

AquiferTest is used for analyses with the Warren and Root double porosity method and the Moench (1984) fracture flow method. Aqtesolv is used to analyse the data with the straight-line linear flow methods of Gringarten and Ramey (1974) for pumping borehole and Gringarten and Witherspoon (1972) for observational boreholes. The results are documented in a database where storativity was used in comparative analyses to further characterise each method's adequacy in dual porous fractured aquifers.

4.4.1 Cooper-Jacob (1946) method

The Cooper-Jacob (1946) method is chosen because it is a straight-line approach, and its parameters can be easily calculated and documented. The straight-line approach makes it easier to obtain an adequate fit to the data plot and gives the user the ability to fit the data to a suitable tangent line (see Appendix C). The input required is drawdown-time series data, the discharge rate (l/s), and the radius from the borehole (0.08m for pumping borehole). The data are visualised

by a semi-log plot. An early-time and a late-time tangent line are fitted to the data. The following steps are then followed to make required calculations:

Step 1 : Transmissivity is calculated using the solution from Cooper and Jacob, (1946):

$$T = \frac{2.3Q}{4\pi\Delta s} \quad (1)$$

Where,

- T = Transmissivity (m²/d)
- Q = Discharge rate (m³/d)
- Δs = Drawdown (m) over one log cycle
- t = Time (days)

Note that both s and t represent a point on the tangent line.

Step 2 : Fracture aperture is calculated using the equation given by Witherspoon (2000) and Faybishenko *et al.* (2000):

$$D = \sqrt{\frac{\left(\frac{T}{d}\right)}{\left(\frac{\rho g}{12\mu}\right)}} \left(\frac{T}{d}\right) \quad (2)$$

Where,

- D = Fracture aperture (m)
- T = Transmissivity (m²/d) from Equation (1)
- d = Aquifer thickness (m)
- ρ = Fluid density (kg/m³)
- g = Gravitational acceleration (m/s²)
- μ = Fluid viscosity (MPa/s)

The term $\frac{\rho g}{12\mu}$ is known due to these being constants of water and gravity.

Step 3 : Theoretical fracture storativity is calculated from the following equation (Kruseman & de Ridder, 1994:23):

$$S_f = pgD(\alpha + n\beta) \quad (3)$$

Where,

- S_f = Fracture Storativity
- ρ = Fluid density (kg/m³)
- g = Gravitational acceleration (m/s²)
- D = Fracture aperture (m) from Equation (2)
- α = Compressibility of the fracture (5x10⁻⁹ m²/N)
- β = Compressibility of water (4.74x10⁻¹⁰ m²/N).

Fracture storativity can therefore only be calculated after fracture thickness is known.

Step 4 : Transmissivity of the fracture is calculated using the solution from Cooper and Jacob, (1946):

$$T_f = \frac{2.3Q}{4\pi\Delta s} \quad (4)$$

Where,

- T_f = Transmissivity (m²/d)
- Q = Discharge rate (m³/d)
- Δs = Drawdown (m) over one log cycle in early time
- t = Time (days)

Step 5 : Calculate the effective radius making use of the Theis (1935) equation:

$$s(t) = \frac{Q}{4\pi T_f} \times W(u) \quad (5)$$

$$u = \frac{r_e^2 S_f}{4T_f t} \quad (6)$$

Where,

- r_e = Effective radius of the borehole (m)
- s = Drawdown (m)
- Q = Discharge rate (m³/d)
- T_f = Transmissivity of the fracture (m²/d) from Equation (4).

$W(u)$	=	Theis wellfield function
S_f	=	Storativity of the fracture from Equation (3)
t	=	Time (days)

Once the all parameters are calculated using Step 1 through to Step 5, storativity can then be calculated making us of the Cooper-Jacob equation as expressed in Equation 7.

$$S = \frac{2.25 T t}{r_e^2 10^{\left(\frac{4\pi T s}{2.3Q}\right)}} \quad (7)$$

Storativity is dimensionless and dependent on the tangent line used for fitting. In principle, the early time tangent line will yield storativity for fracture flow and late time tangent line for matrix and fracture flow. The flow diagram presented in Figure 4-5, depicts the method described in the steps above, simplifying it through a visual representation of the process.

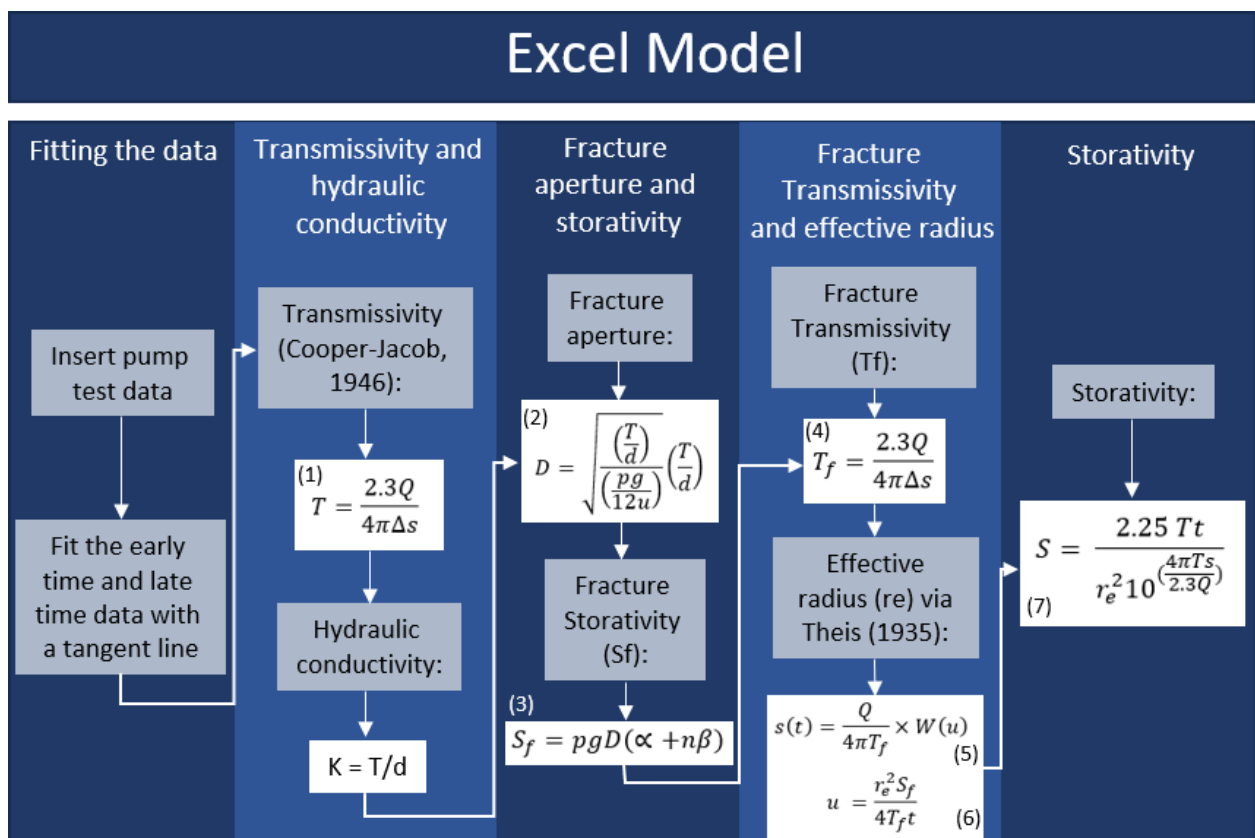


Figure 4-5: Flow diagram of the process used in the Cooper-Jacob (1946) method.

4.4.2 RPTsolv Model

As discussed in the literature review, RPTsolv was created for dual porosity aquifers. The governing equation incorporates horizontal flow and is two-dimensional. Once data from a pump test are correctly imported into the model, a semi-log plot showing the time versus drawdown data is produced by the model. An example dataset imported into the model is shown in Figure 4-6.

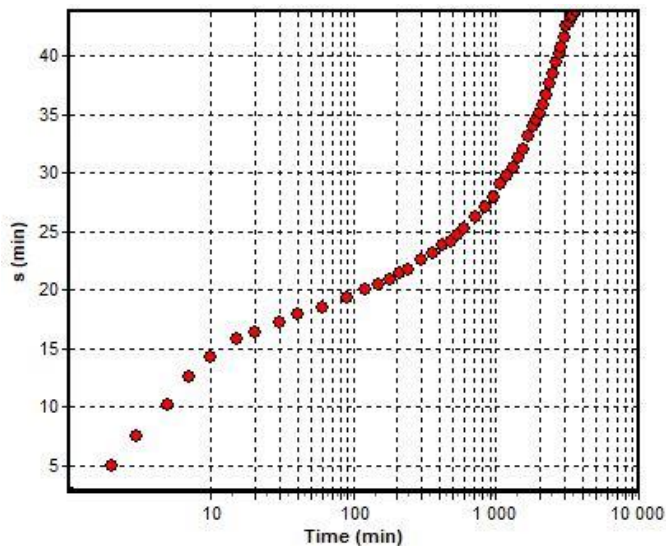


Figure 4-6: RPTsolv drawdown graph before analysis.

Before the analyses can start, the user must first create the mesh and specify the parameters of the aquifer. The 2D mesh tab is used to create two-dimensional finite element mesh elements for the model domain. The 2D mesh is a fundamental component in the model and is used to represent the spatial domain where groundwater flow is being simulated. For the mesh to be constructed correctly, the default parameters presented in Table 4-3 must be updated in the Mesh subtab of RPTsolv.

Table 4-3: Mesh construction in RPTsolv with default values.

Mesh construction	
	Default
Thickness of the aquifer (m)	40
The ratio of the fracture (0-1)	0.2
Distance to the opposite boundary (m)	1024
Observation distance (m)	1
Observation position	Fracture

The default values presented are based on a study conducted by Botha *et al.* (1998) regarding the Karoo aquifers in South Africa. These default values can be applied if a parameter is unknown,

such as the ratio of the fracture. The remaining parameters will be calculated using the methods outlined below:

1. *Thickness of the aquifer* - If the actual aquifer thickness for the borehole is not known. This calculation is based on the depth of the borehole and water level information.
2. *Ratio of fracture* – Used as a fitting parameter.
3. *Distance to the opposite boundary* – This parameter is calculated using the distance to boundary method. The equation: $r = \sqrt{\frac{T_f t_{bound}}{S_f}}$, where r is the distance (m), T_f and S_f are fracture transmissivity and fracture storativity and t_{bound} is the time where the fitted line leaves the drawdown plot.
4. *Observation distance* – Use 0.08m for the pumping borehole and for observation boreholes, it is the distance from the observation borehole to the pumping borehole.

A list of parameter selection options are presented in Table 4-4. Each parameter can be changed by choosing which of the options best match the aquifer's diagnostics.

Table 4-4: Parameter specifications in RPTsolv.

Parameter	Possible options
Pump position	Fracture and matrix Fracture Matrix
Opposite boundary	Constant head No-flow
Water level observations	Drawdown Meter above sea level

RPTsolv further makes provision to specify the maximum and minimum values for each parameter (storativity and transmissivity), along with an option to disregard several initial drawdown points. Initially, large ranges of maximum and minimum values are specified to ensure that RPTsolv has the freedom to obtain a good fit. When the model converges, a report will be presented which shows the user essential information, including the error estimation and the number of model iterations.

4.4.3 Effective radius correlation method

After the completion of the aforementioned analyses, two sets of data for exist for each borehole, one for the Cooper-Jacob (1946) results and one for the RPTsolv results. These datasets are then compared to one another with respect to transmissivity and storativity, where the RTPsolv values are assumed to represent the baseline or observed values and the Cooper-Jacob values are seen as the predicted values. The error between observed and predicted values are expressed making use of the following error metrics:

- Root mean square error (RMSE) – Quantifies the difference between predicted and actual observed or true values. The formulation of this metric is shown in Equation 8.

$$RMSE = \sqrt{\frac{\sum(p_i - o_i)^2}{n}} \quad (8)$$

Where,

- $RMSE$ = Root Mean Square Error
- p_i = Predicted value at point i
- o_i = Observed value at point i
- n = Number of reference points

- Percentage difference – Measures the difference between two related values in percentage as presented in Equation 9.

$$\%Difference = \left| \frac{S1 - S2}{\frac{S1 + S2}{2}} \right| * 100 \quad (9)$$

Where,

- $S1$ = RPTsolv Storativity
- $S2$ = Cooper-Jacob Storativity

- Pearson correlation coefficient (R) - Determines the degree to which two variables are linearly related. The Pearson's correlation is calculated as the covariance divided by the product of the standard deviations of observed data and simulated data. The results may range from -1 (negative linear relationship) to 1 (positive linear relationship), and 0 indicates no relationship present between the predicted data and the observed data. The calculation of the Pearson's correlation coefficient is presented in Equation 10.

$$R = \frac{\sum(O_i - \bar{O})(P_i - \bar{P})}{\sqrt{\sum(O_i - \bar{O})^2 \sum(P_i - \bar{P})^2}}$$

Where,

- | | | |
|-----------|---|------------------------------|
| P_i | = | Root Mean Square Error |
| O_i | = | Predicted value at point i |
| $\bar{O}$ | = | Observed value at point i |

(10)

The effective radius approach to determine storativity has two main advantages and is therefore considered as a fitting parameter for two main reasons:

1. By using the effective radius, the problem of calculates storativities with values of greater than one is resolved (the theoretical maximum for storativity is one, which implies 100% void space).
2. The transmissivity calculation is not dependent on the effective radius.

The steps to produce the final results from the analysis of pump test data using the original Cooper-Jacob (1946) method to produce predicted values and RPTsolv to generate observed data is presented below.

1. Pump test data are analysed using the RPTsolv numerical model and the Cooper-Jacob (1946) method, and the results are documented. Two sets of storativity values are now available.
2. This data are re-analysed using the Cooper-Jacob (1946) analytical model, applying the effective radius as a fitting parameter. This approach involves adjusting the effective radius until the predicted storativity of the Cooper-Jacob (1946) aligns with the observed storativity generated by the RPTsolv model.
3. The original effective radius and the adjusted effective radius used in the fitting are also documented.
4. The regression line equation produced by the comparison between the two sets of effective radii are incorporated in the Cooper-Jacob (1946) analytical model. The analytical method with the effective radii correlations incorporated be called the modified Cooper-Jacob method for the purpose of this study.
5. The effective radius will no longer be calculated the traditional way, but will be dependent on the correlation equation obtained between the original effective radius and the adjusted effective radius.

- Finally, the pump test data are analysed using the modified Cooper-Jacob model incorporating the correlation equation to generate the corrected set of storativities.

Figure 4-7 is a visual representation steps outlined above for the proposed methodology.

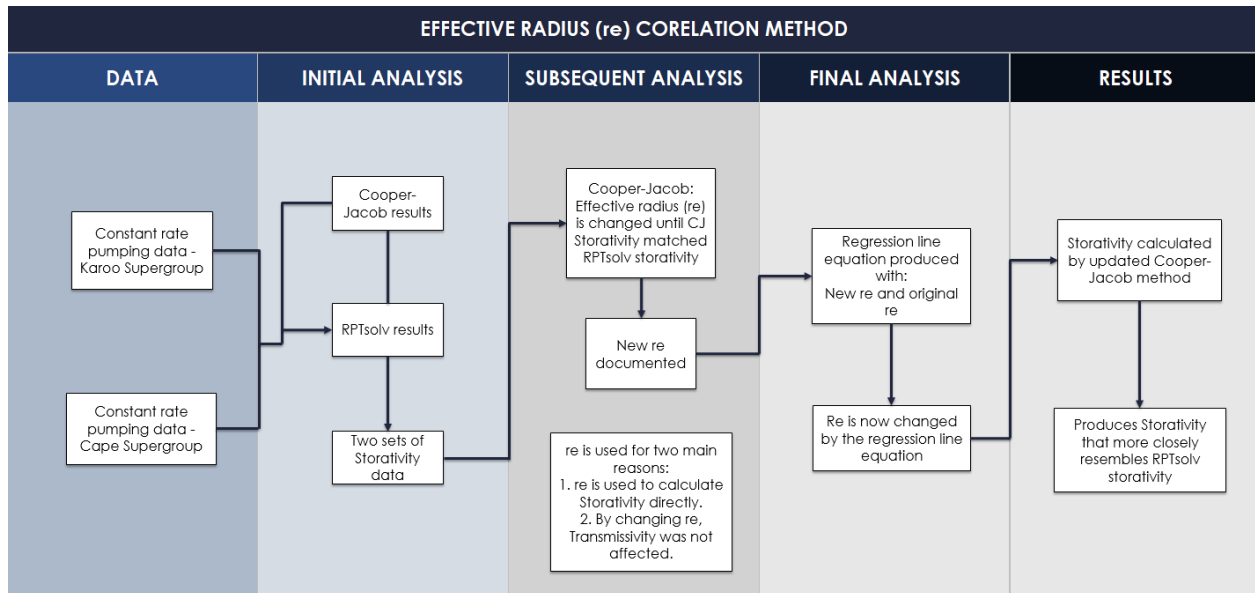


Figure 4-7: Effective radius correlation method methodology.

CHAPTER 5 RESULTS AND DISCUSSION

Two separate methods were followed to achieve the aim of the dissertation: 1) Comparative analysis between analytical methods and RPTsolv in order to study their validity in South African aquifers and 2) an effective radius correlation modelled into the Cooper-Jacob (1946) analytical method to enhance the model's storativity calculations.

5.1 Comparative analysis

This chapter presents and discusses the results of the various analytical methods used in this study. First the data are compared by calculating the averages of transmissivity and storativity. RPTsolv was used as the control value in these comparisons since literature and past studies have shown that this method produces acceptable values, more so than the analytical models. Further comparisons in terms percentage difference for storativity and transmissivity are also used. The data are then plotted on a graph to visualise the results and comparisons before plotting storativity to distance. The discussions are mainly based on storativity although the overall results are also taken into account.

5.1.1 Karoo Supergroup

Time series data from three (3) multi well tests and ten (10) single well tests were collected from the Karoo Supergroup in South Africa. The majority of tests (9 out of 13) were diagnosed to have dual porosity flow. All of the multi-well tests exhibit dual porosity characteristics.

5.1.1.1 Comparisons

Table 5-1 shows the average storativity and transmissivity of all the pumping tests combined. One outlier was removed per set if the data point was considered an anomaly with respect to the other data points. The outliers removed were most likely caused due to the pumping test not being sufficiently long enough to reach radial flow or human error in fitting the data with the analytical method.

Table 5-1: Averages of the results obtained for the Karoo Supergroup.

	Average S	Average T
S RPTsolv	7.58E-05	55.40
Cooper-Jacob (1946) (excel)	5.29E-04	33.07
Dual Porosity (Aquifertest)	2.44E-03	62.16
Moench (Aqtesolv)	3.89E-03	72.03
Gringarten & Ramey (Aqtesolv)	5.60E-02	115.69

For ease of comparison between the different methods, Table 5-2 was created:

Table 5-2: Showing the comparative results by taking the percentage difference of the storativity (left) and the percentage difference of the transmissivity (right).

Storativity	%difference	Transmissivity	%difference
RPTsolv and Cooper-Jacob	149.87	RPTsolv and Cooper-Jacob	50.48
RPTsolv and Dual Porosity	187.95	RPTsolv and Dual Porosity	11.50
RPTsolv and Moench	192.35	RPTsolv and Moench	26.10
RPTsolv and Gringarten & Ramey	199.46	RPTsolv and Gringarten & Ramey	70.48

Transmissivity and storativity were compared by taking the percentage difference between RPTsolv and each of the analytical methods.

Table 5-2 also reveals several key observations. Notably, when comparing RPTsolv to each analytical method, higher percentage differences for storativity compared to transmissivity were calculated. The Cooper-Jacob (1946) method specifically yielded the closest storativity values, with a percentage difference of 149.87%, while the Gringarten & Ramey (1974) method produced the largest deviation at 199.46%. In contrast, transmissivity exhibited a much narrower range of differences, with the Dual Porosity method showing only an 11.5% variation from RPTsolv.

These results underscore a significant discrepancy in the calculation of storativity as opposed to transmissivity, suggesting a potential limitation in the analytical methods' ability to accurately estimate storativity values that closely align with RPTsolv. To provide a visual representation of these findings, Figure 5-1 displays the difference in storativity between each analytical method and RPTsolv. Therefore, the closer the method was to RPTsolv the lower the value will be.

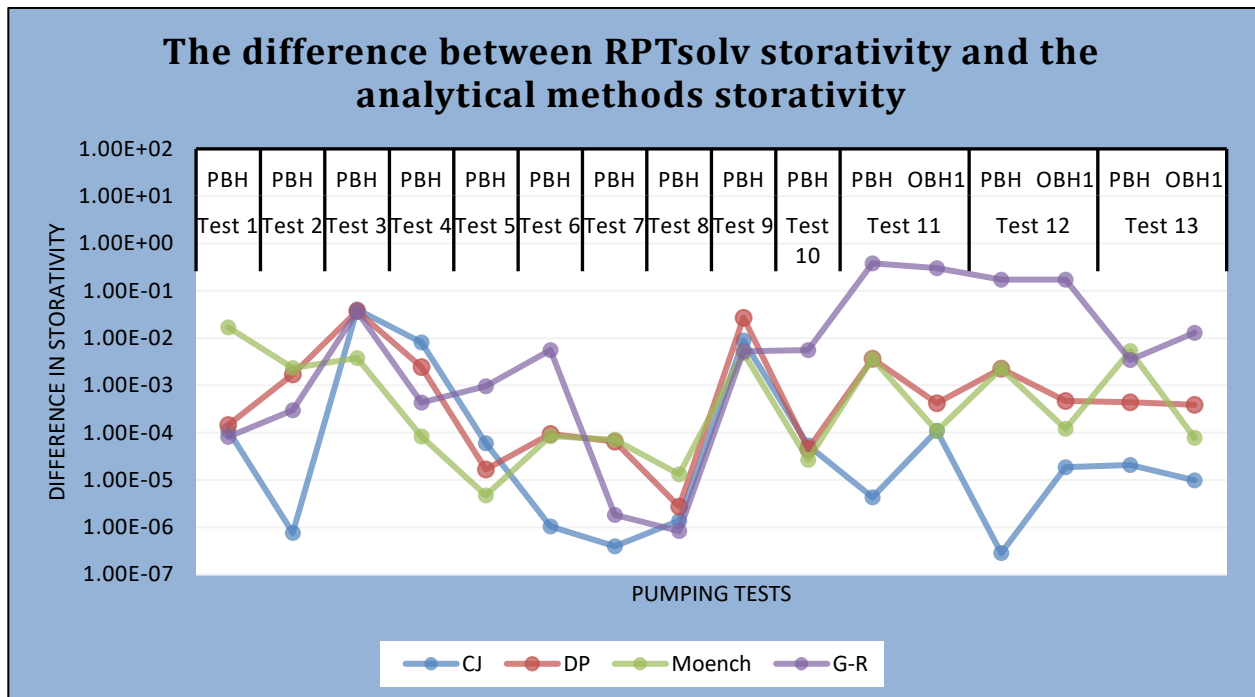


Figure 5-1: The differences in storativity between RPTsolv and the analytical methods for the Karoo Supergroup.

The Cooper-Jacob (1946) method consistently demonstrates the smallest discrepancies when compared to other analytical methods. Although it may not consistently yield the closest storativity values to RPTsolv in every instance, it exhibits greater overall consistency. In contrast, the Gringarten and Ramey (1974) method faces challenges in replicating storativity values from RPTsolv.

For the most part, the Dual Porosity and Moench (1984) analytical methods closely follow each other's curves. While they may not achieve storativity values as close to RPTsolv as the Cooper-Jacob (1946) method, they represent an improvement over the Gringarten and Ramey (1974) method. It is important to note that this is a general trend. In some tests, the other three analytical methods produce storativity values closer to RPTsolv than the Cooper-Jacob (1946) method. However, their results tend to be more sporadic and are less consistent in achieving low discrepancies, as was observed with the Cooper-Jacob (1946) method.

5.1.1.2 Distance dependency of storativity

Figure 5-2 shows the distance to the pumping borehole versus storativity in order to visualise the dependency. Only the results of the multi-well pumping tests were used in generating Figure 5-2.

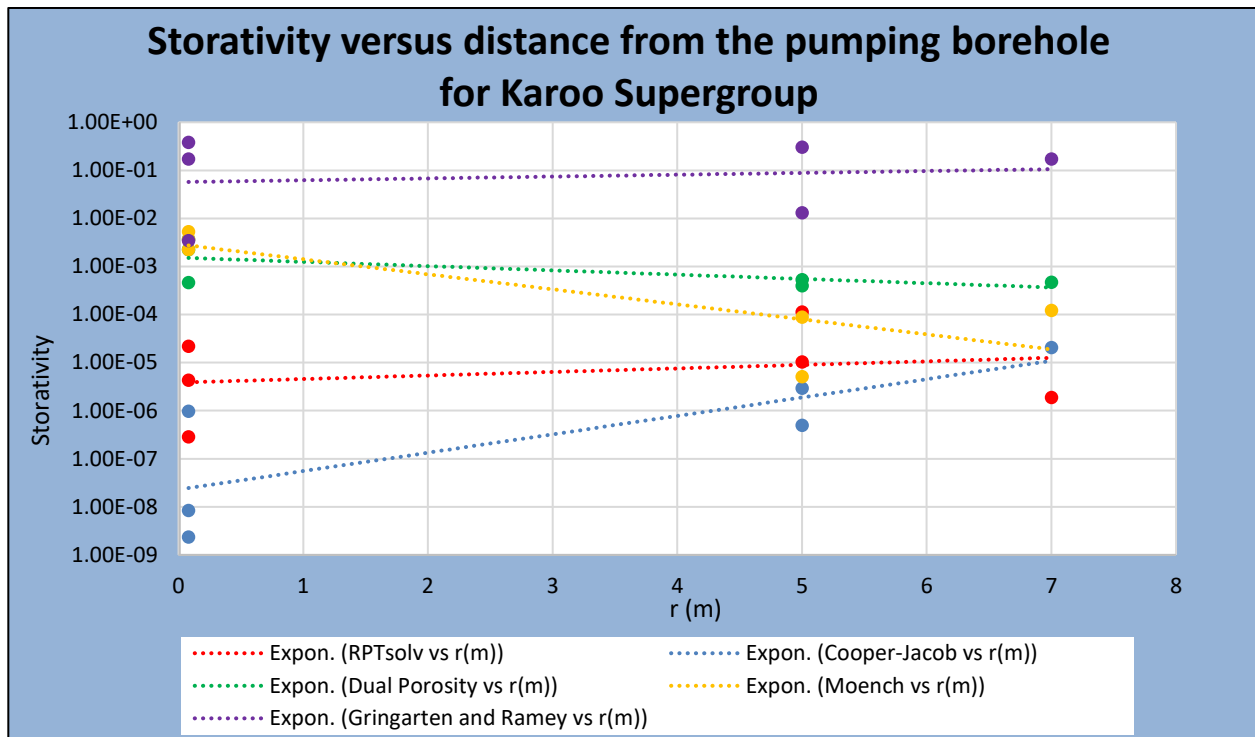


Figure 5-2: Storativity versus distance from the pumping borehole for the Karoo Supergroup.

The data represents three multiple well pumping tests. Two pumping tests had observational wells 5m away from the pumping well with one pumping test having an observational well 7m away. A color was used to represent each method's results when the pumping tests was analysed. The red trendline shows the change of storativity with distance when calculated with RPTsolv. Two of the tests produced constant storativities with the trendline remaining horizontal for the most part. One test produced storativity with an upwards trend indicating that for this pumping test the storativity increased with distance. The Gringarten and Ramey (1974) method also produces constant storativity with distance but calculates it up to 4 orders of magnitudes higher than RPTsolv. If storativity is determined with the Cooper-Jacob (1946) method, the trend suggests that storativity will increase with distance by 2 or 3 orders of magnitude over a distance of just 7m for 2 of the 3 pumping tests. The dual porosity method and Moench method tended to produce storativities that decreased with distance.

Storativity shows a clear distance dependency with all methods used and the overall trend would suggest that the parameter increases with distance from the pumping borehole. RPTsolv determined storativity with the least amount of variance, further highlighting the numerical model's superiority over the analytical methods.

5.1.2 Cape Supergroup

Data from 10 multi-well hydraulic tests were collected of which 7 were diagnosed to exhibit dual porous flow. As mentioned previously, a maximum of one outlier was removed per set if the data point was considered an anomaly when compared to the other data points. The outlier is most likely due to human error, or the pump test data analysed did not reach radial flow.

5.1.2.1 Comparisons

Table 5-3 below shows the average storativity and transmissivity of all the pumping tests combined.

Table 5-3: Averages of the results obtained for the Cape Supergroup.

	Average S	Average T
S RPTsolv	4.99E-05	81.19
Cooper-Jacob (excel)	1.59E-05	101.78
Dual Porosity (Aquifertest)	1.63E-03	26.81
Moench (Aqtesolv)	1.12E-02	67.61
Gringarten & Ramey (Aqtesolv)	5.95E-02	435.53

Table 5-4 gives the results of further analysis of the values above by taking the percentage difference when compared to RPTsolv.

Table 5-4: Showing the comparative results by taking the percentage difference of the storativity (left) and the percentage difference of the transmissivity (right).

Storativity	%Difference	Transmissivity	%Difference
RPTsolv and Cooper-Jacob	103.59	RPTsolv and Cooper-Jacob	22.52
RPTsolv and Dual Porosity	188.09	RPTsolv and Dual Porosity	100.71
RPTsolv and Moench	198.22	RPTsolv and Moench	18.25
RPTsolv and Gringarten & Ramey	199.66	RPTsolv and Gringarten & Ramey	137.15

In general, the percentage difference between storativity and RPTsolv is notably higher compared to the percentage difference in transmissivity. The majority of the findings align closely with those observed in the Karoo Supergroup. The Cooper-Jacob (1946) method specifically produced the storativity values closest to RPTsolv, with a percentage difference of 103.59%, while the Gringarten & Ramey (1974) method exhibited the largest deviation at 199.66%. In contrast, transmissivity values displayed a much closer proximity to RPTsolv, with the Cooper-Jacob (1946) and Moench (1984) methods yielding averages within 23% of RPTsolv.

These results closely mirror those found in the Karoo Supergroup analysis. It is evident that storativity calculations consistently result in higher percentage differences compared to certain

transmissivity differences. While the analytical methods demonstrate the capability to accurately estimate transmissivity values in certain instances, they struggle to achieve the same level of accuracy in storativity calculations.

To visually represent these findings, Figure 5-3 illustrates the differences in storativity between each method and RPTsolv. In this context, lower values on the graph indicate a closer alignment between the method and RPTsolv.

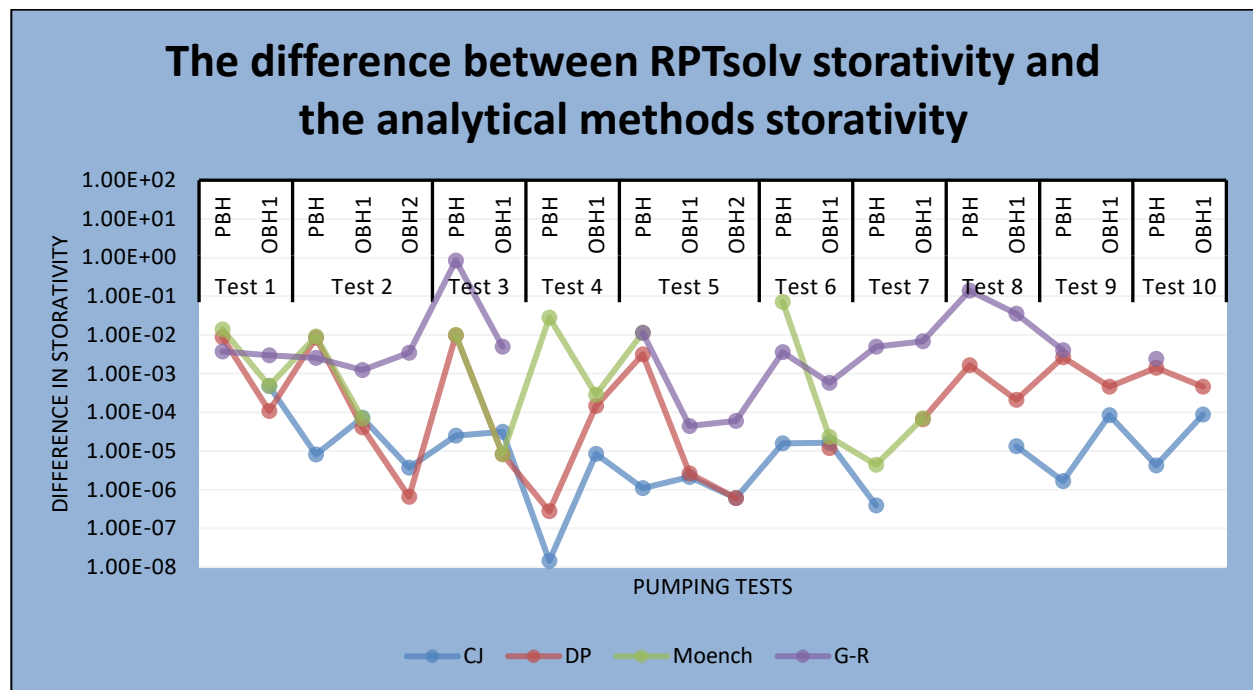


Figure 5-3: The differences in storativity between RPTsolv and the analytical methods for the Cape Supergroup.

Similar to the findings in the Karoo Supergroup analysis, the Cooper-Jacob (1946) method consistently records the smallest discrepancies when compared to other analytical methods. In contrast, the Gringarten and Ramey (1974) method consistently produces the highest differences in most cases.

It is noteworthy that the Gringarten and Ramey (1974) method encounters challenges in producing storativity and transmissivity results that closely resemble RPTsolv in both study areas. On the other hand, the Cooper-Jacob (1946) method consistently produces values that are closest to RPTsolv.

5.1.2.2 Distance dependency of storativity

Figure 5-4 plotted the distance to the pumping borehole versus storativity in order to visualise the dependency. The graph below had to be created differently due to the amount of data making the

graph area clustered and hard to read when plotting it the same as Figure 5-2. First the distance to the pumping well is showed in Logarithmic scale in order to see storativity values at small distances. Second, not all the data points were plotted but rather the average of each method per distance increment from the pumping well. This created a clear understandable graph which still showed the overall distance dependency of each method.

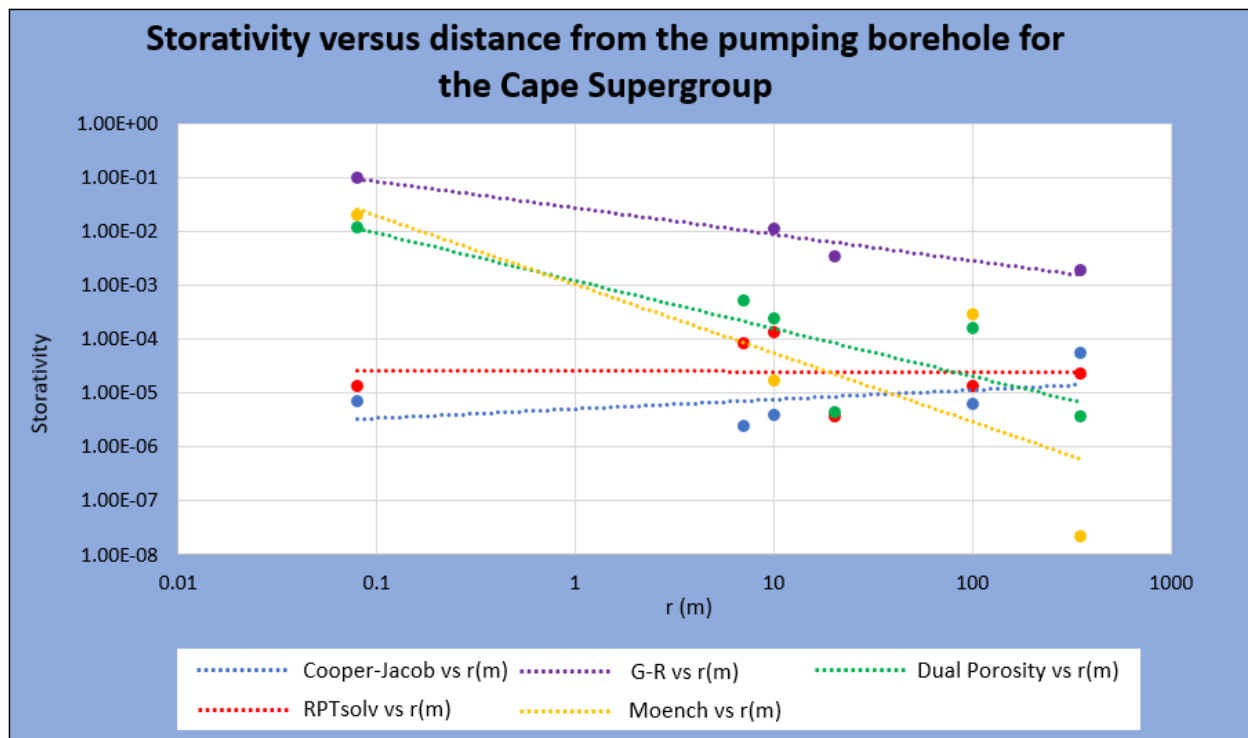


Figure 5-4: Storativity versus distance from the pumping borehole for the Cape Supergroup.

From Figure 5-4 it is clear that the red line (RPTsolv versus distance from pumping borehole) produces a very straight horizontal line, suggesting that most of the results are within the same order and magnitude and are not changing due to the distance from the pumping borehole. The Cooper-Jacob (1946) method produces a trendline at 1 order of magnitude lower than RPTsolv and suggests that storativity increases slightly with distance. The Dual Porosity method, Moench (1984) method, and Gringarten-Ramey (1974) method produce trendlines that suggests that storativity is reducing with distance from the pumping borehole. Note that the distance from the pumping borehole significantly increases compared to the Karoo Supergroup. This is also visible in the data which shows higher variances due to the increased distance.

This graph indicates that all the analytical methods yield storativity that differs with distance and only the numerical model seems to be free from this distance dependency.

5.2 Effective radius correlation method

The results produced to obtain the effective radius correlation equations are discussed in the section that follows. Note that a power trendline was used in all the graphs discussed. The power trendline recorded the best fit for almost all the data. The Karoo and Cape Supergroup pumping test data was combined in order to have sufficient data for this method.

5.2.1 Analyses with original Cooper-Jacob (1946) model (before fitting)

The drawdown versus time pump test data was analysed with both the Cooper-Jacob (1946)-model and the RPTsolv numerical model. As mentioned in the methodology, the two datasets were compared and analysed based on three error metrics: Root mean square error (RMSE), % difference, and Pearson correlation coefficient (R). The correlation of the transmissivity was first tested and the results are presented in Figure 5-5 with the associated error metrics summarised in Table 5-5.

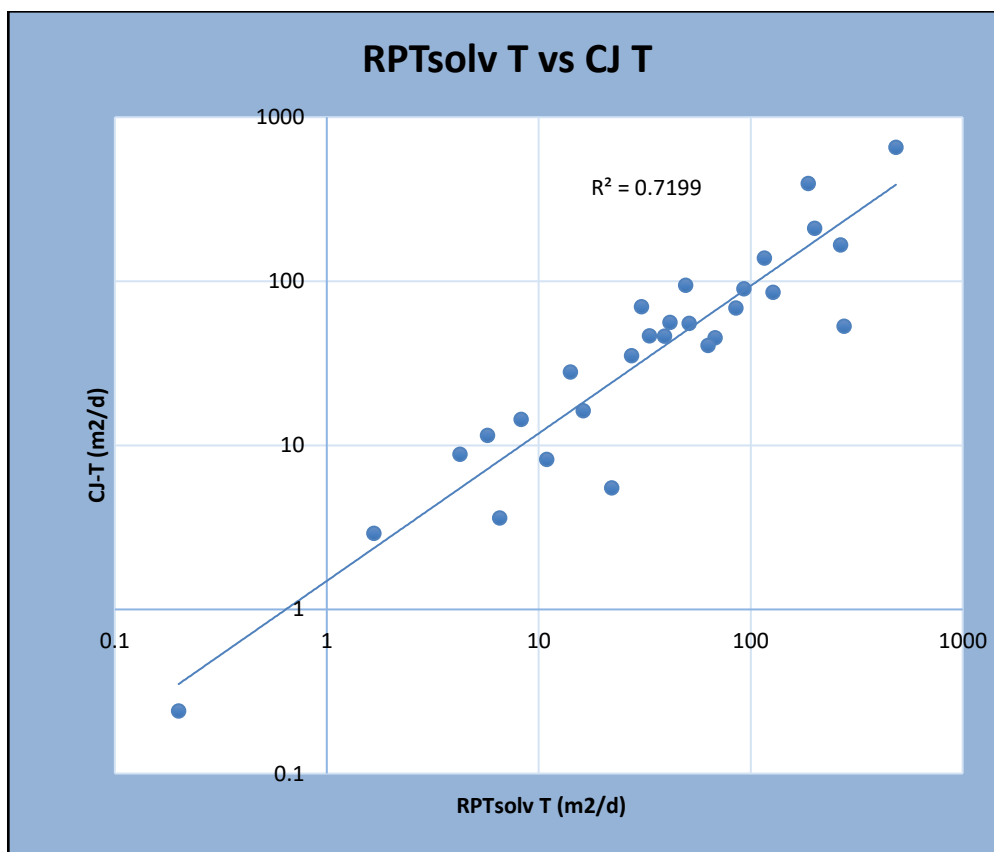


Figure 5-5: Correlation of Cooper-Jacob (1946) T and RPTsolv T.

Table 5-5: Error metrics between Cooper-Jacob (1946) T and RPTsolv T

Error metric	Value
RMSE	70.68
R	0.85
% difference	44.29

The transmissivities produced by the two different models show a good power regression correlation with a factor of $R = 0.85$. This suggests a strong, positive association between the two variables. Both these methods calculated transmissivity in the same ranges for low and high transmissivity scenarios. In other words, both models remain in the same zone of transmissivity, even when storativity calculations differ. This is due to transmissivity calculations not being dependent on the storativity.

Figure 5-6 shows the correlation between storativity values produced by these two methods and Table 5-6 summarises the results of the error metrics.

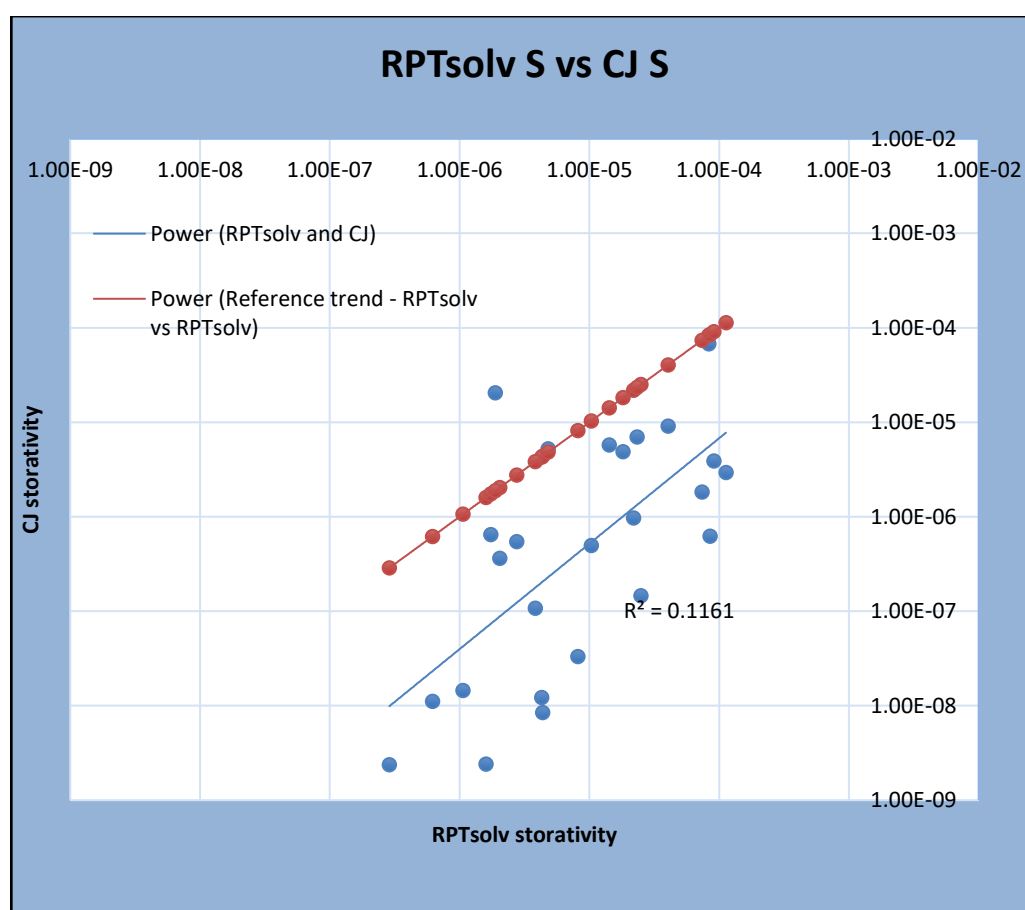


Figure 5-6: Correlation between the original Cooper-Jacob (1946) S and RPTsolv S

Table 5-6: Error metrics between Cooper-Jacob (1946) S and RPTsolv S

Error metric	Value
RMSE	3.76E-05
R	0.34
% difference	156.75

The storativity values produced by RPTsolv and Cooper-Jacob (1946) indicate a weak, positive correlation between the two models. As seen in the comparative analysis, the Cooper-Jacob (1946) calculates storativity at 1 or 2 orders of magnitude lower than RPTsolv. This highlights the incapability of the analytical model to produce storativity values in the same ranges as RPTsolv.

5.2.2 Effective radius correlations

Figure 5-7 shows the correlation between the effective radii in the original Cooper-Jacob (1946) method to the effective radius after changing the storativity values of the Cooper-Jacob (1946) method to match the RPTsolv storativity.

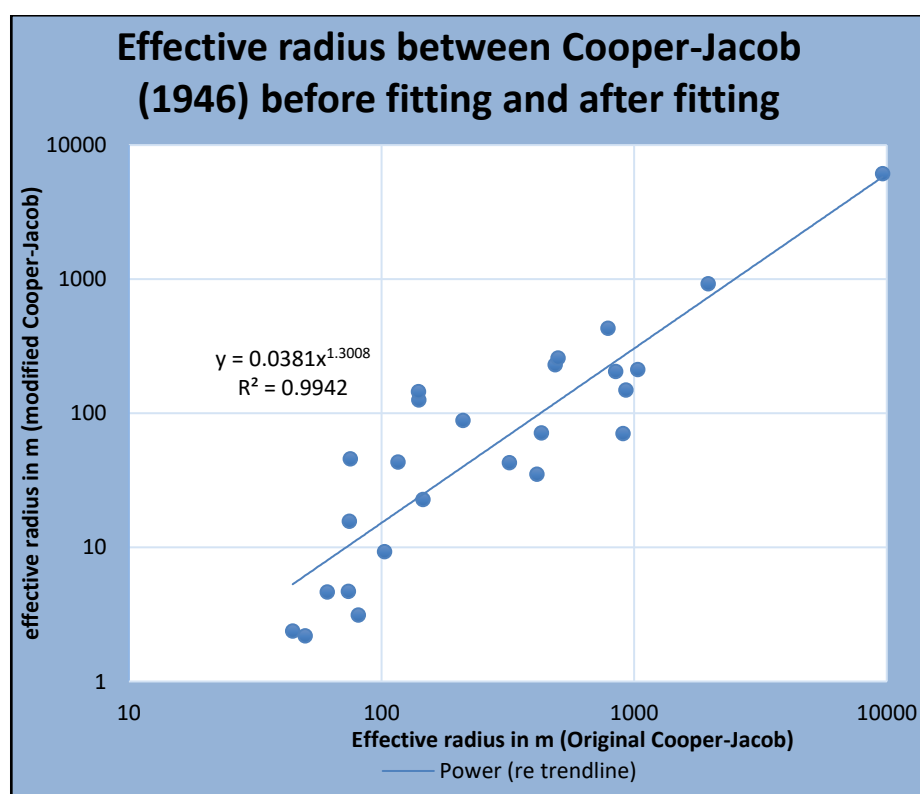


Figure 5-7: Original Cooper-Jacob (1946) method effective radius is plotted against the effective radius of the modified Cooper-Jacob method

A power regression trend of $R = 0.997$ indicates a very strong, positive correlation between the two sets of effective radii. The high correlation does not mean that all data correlates well. There are some effective radius points that shows orders of magnitudes difference. Nonetheless, the high correlation is a good sign that the methodology will work. This is called the “mean” correlation because both sets of data (pumping and observation) were used to produce the correlation equation.

The correlation plots of the pumping and observation datasets are presented in Figure 5-8 and Figure 5-9 respectively.

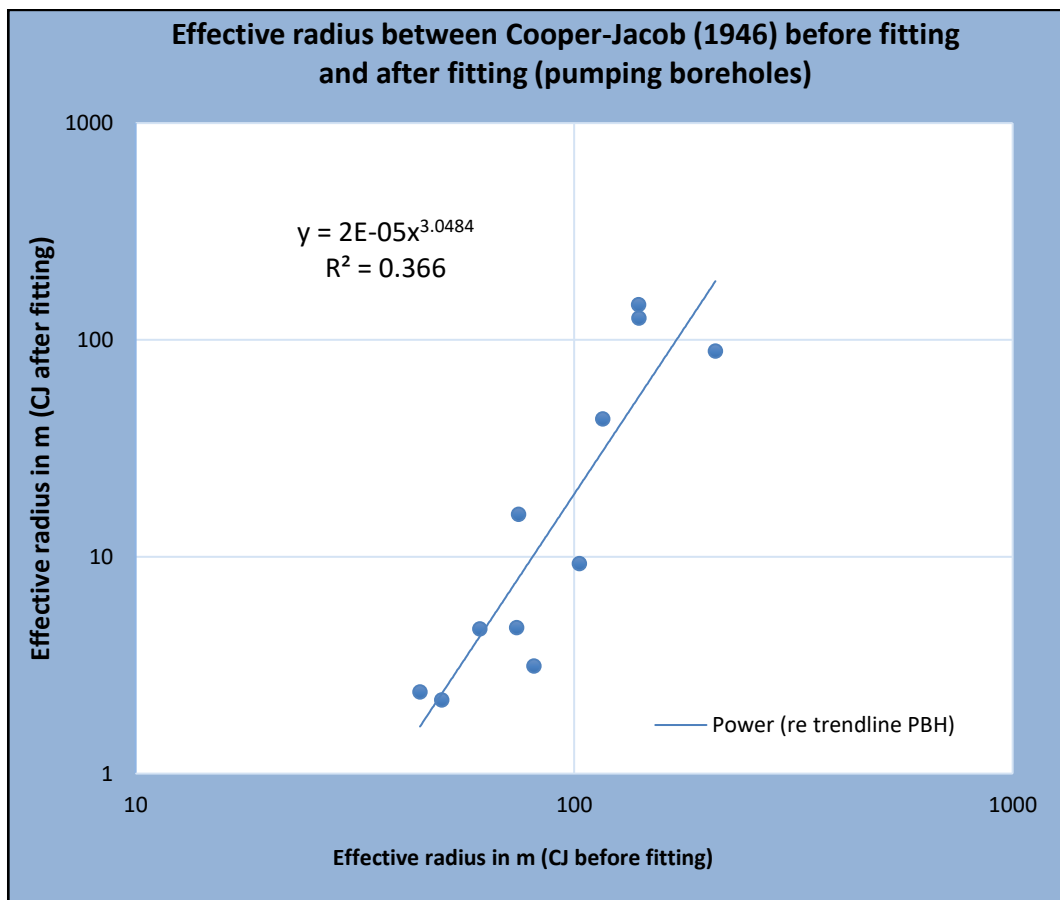


Figure 5-8: Effective radius between Cooper-Jacob before and after the fitting process are plotted against one another, only showing pumping borehole data.

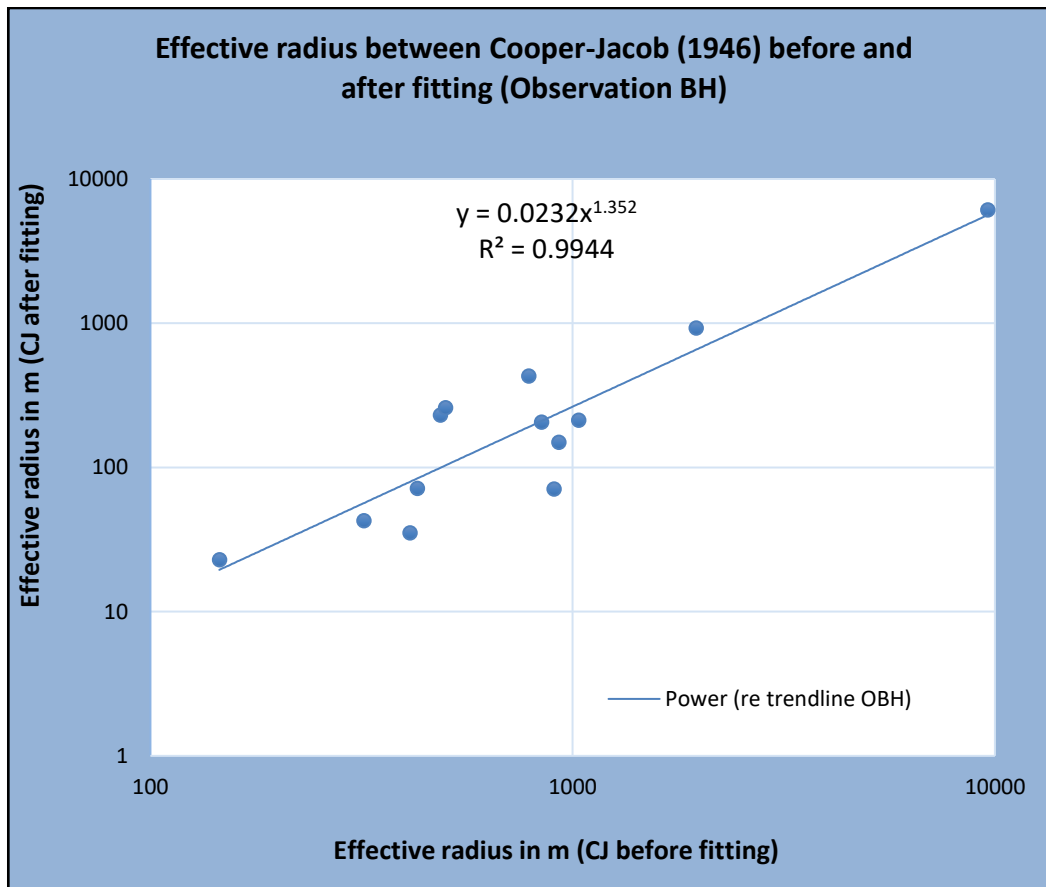


Figure 5-9: Effective radius between Cooper-Jacob before and after the fitting processes are plotted against one another, only showing observation borehole data.

The power regression of the pumping borehole indicates a moderate positive correlation between the two effective radii with an $R = 0.6$. The same cannot be said for the observation borehole regression with an $R = 0.997$. The equations produced are called the “separate” correlation equations because the two sets of data (pumping and observation) were used separately to produce two correlation equations.

5.2.3 Analyses using the correlations.

When analysing the results, the R value only shows the goodness of fit of the data to the regression line and is not an overall indicator of the method’s potential. In the sections that follow, the red line indicates the trend of RPTsolv storativity versus itself (reference line) and is used for fast visual analysis of the accuracy of the results. If the blue trendline closely matches the red trendline, it indicates that the results are generally within the same order of magnitude for both sets of data.

5.2.3.1 Mean correlation equation

The following results and subsequent discussion are based on the analysis of the pump test data with the correlations noted in section 5.2.2 incorporated into the Cooper-Jacob (1946) model. This correlation equation will change the effective radius of said model in order to determine storativity values closer to RPTsolv storativity. The first discussion is based on the analysis with the mean effective radius correlation equation. The storativity correlation, by analysing the pumping data, is plotted in Figure 5-10 below followed by the Table 5-7 showing the results from the error metrics.

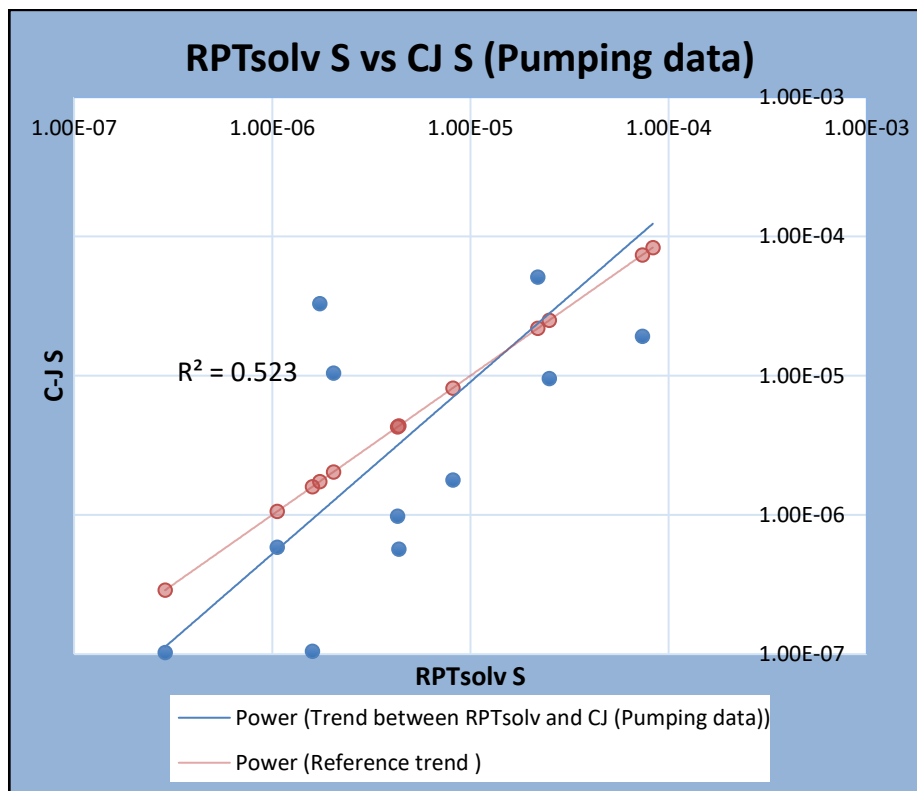


Figure 5-10: RPTsolv S is plotted against the modified Cooper-Jacob S for the pumping borehole data (mean correlation equation).

Table 5-7: Error metrics between the modified Cooper-Jacob S and RPTsolv S for the pumping data results (mean correlation equation).

Error metric	Value
RMSE	6.64E-04
R	0.72
% difference	126.97

The power regression of the pumping boreholes indicates a moderate, positive correlation between RPTsolv storativity and modified Cooper-Jacob storativity, with an R value of 0.723. The

trend line is a bit steeper than the reference trend line indicating that although the R value shows a moderate correlation, the two methods still struggles to produce results within the same order of magnitude. The storativity correlation, by analysing the observational data, is plotted in Figure 5-11 below followed by the Table 5-8 showing the results from the error metrics.

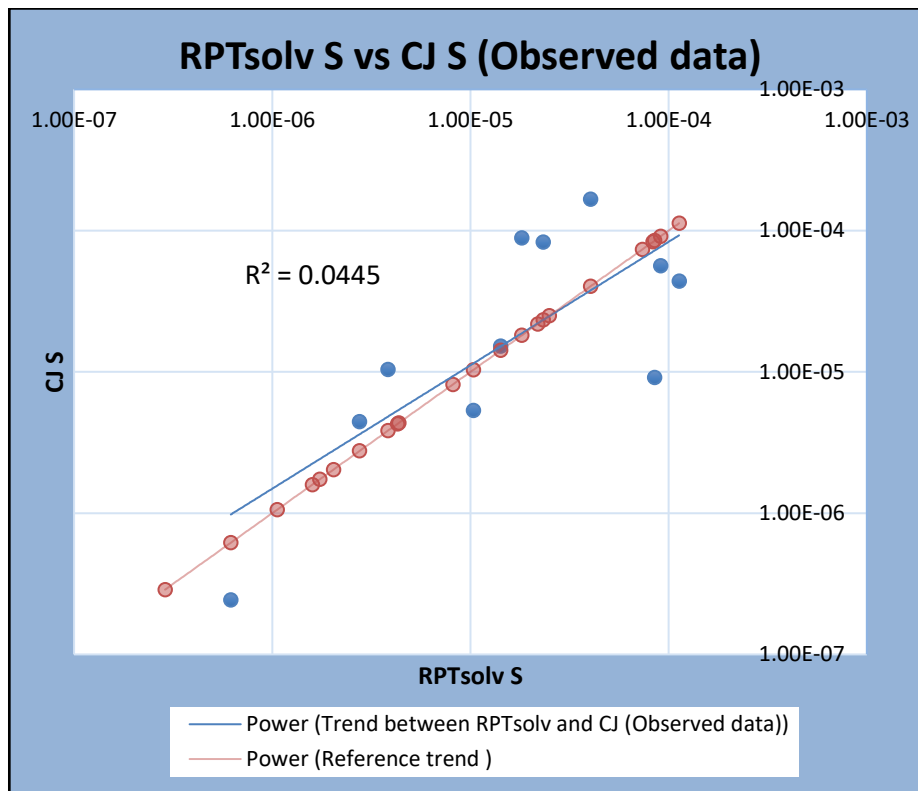


Figure 5-11: RPTsolv S is plotted against the modified Cooper-Jacob S for the observational borehole data (mean correlation equation).

Table 5-8: Error metrics between the modified Cooper-Jacob S and RPTsolv S for the observational data results (mean correlation equation).

Error metric	Value
RMSE	5.74E-05
R	0.21
% difference	87.22

The R value of the observation boreholes is less impressive with a value of 0.21 which indicates a weak positive relation. The overall power regression trendline still closely follows that of the reference trend. The individual data points suggest that the modified Cooper-Jacob method now stays within 1 order of magnitude from the RPTsolv method most of the time. This is an improvement from the original Cooper-Jacob (1946) method which calculated storativity between

1 and 2 orders of magnitude too low. The % difference shows a decrease from the pumping data results with 87.22%.

Figure 5-12 visualises all data with the mean effective correlation in place and Table 5-9 presents the error metrics.

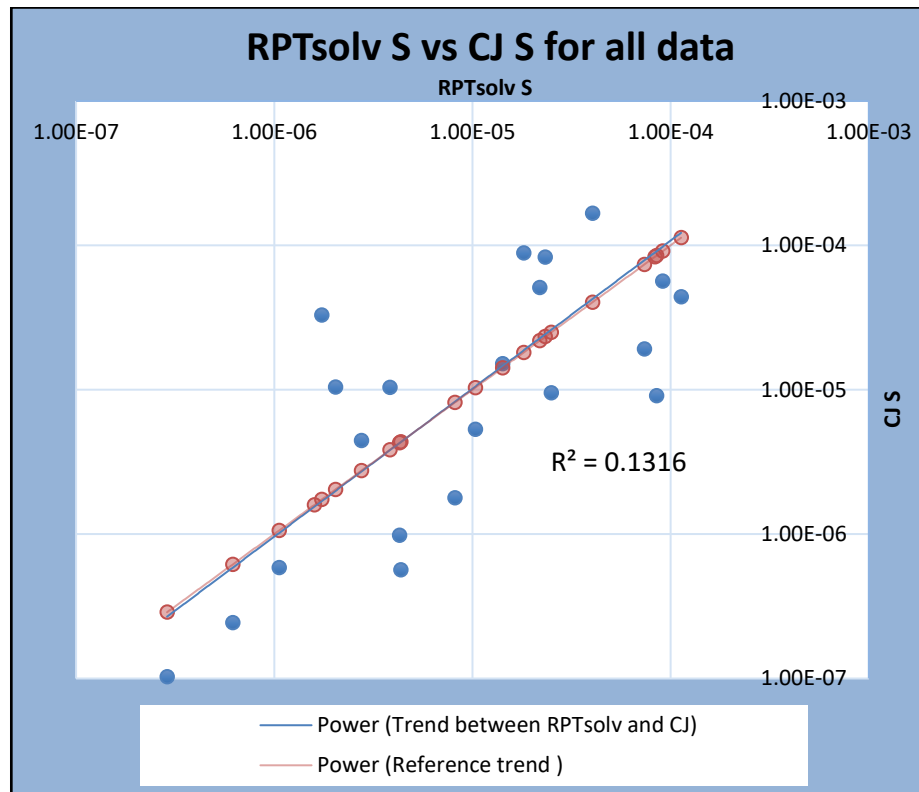


Figure 5-12: Storativity produced by RPTsolv and the modified Cooper-Jacob method, for pumping data and observed data, are plotted against one another.

Table 5-9: Error metrics between the modified Cooper-Jacob S and RPTsolv S (mean correlation equation).

Error metric	Value
RMSE	4.92E-04
R	0.36
% difference	107.96

An R value of 0.36 indicates a weak, positive correlation between the two variables. Instead of calculating storativities lower than RPTsolv, the modified Cooper-Jacob method now tries to mimic the results as can be seen by the individual data points. The overall trend line suggests that, on average, the modified Cooper-Jacob method yields S within the same order of magnitude to RPTsolv S.

5.2.3.2 Separate correlation equation

Figure 5-13 show the correlation between RPTsolv storativity and modified Cooper-Jacob storativity if the effective radius correlation equation produced by the pumping data (Figure 5-10) is used in the modified Cooper-Jacob model to analyse the pumping boreholes. This is also true for the observational data, the effective correlation equation produced by the observational data (Figure 5-11) is used to analyse the observation boreholes. Table 5-10 presents the error metrics.

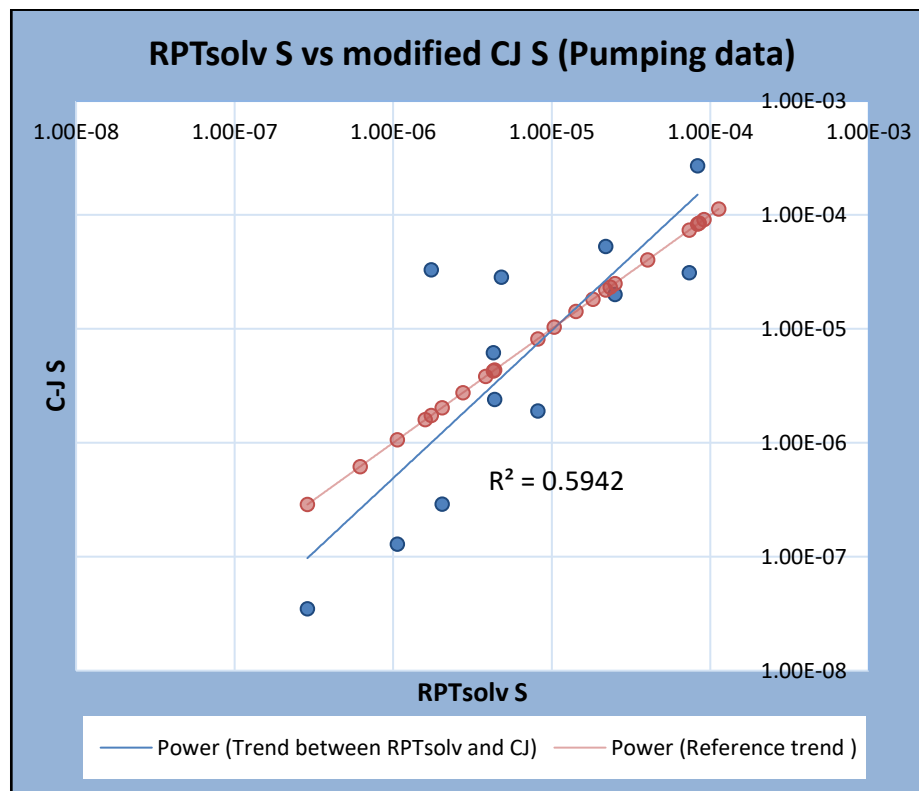


Figure 5-13: Modified Cooper-Jacob storativity versus RPTsolv storativity produced by analysing pumping borehole data and the separate effective radius correlation equation.

Table 5-10: Error metrics between the modified Cooper-Jacob S and RPTsolv S for the pumping data results (separate correlation equation).

Error metric	Value
RMSE	1.07E-05
R	0.77
% difference	102.02

With a R value of 0.77, the individual data points show a good positive correlation which closely resembles the RPTsolv trendline. A RMSE of 1.07×10^{-5} is lower compared to the mean

correlation method which shows that on average the difference between RPTsolv S and Cooper-Jacob S is smaller.

The results obtained from using the separate correlation equation is presented in Figure 5-14 below followed by the error metrics which is presented in Table 5-11.

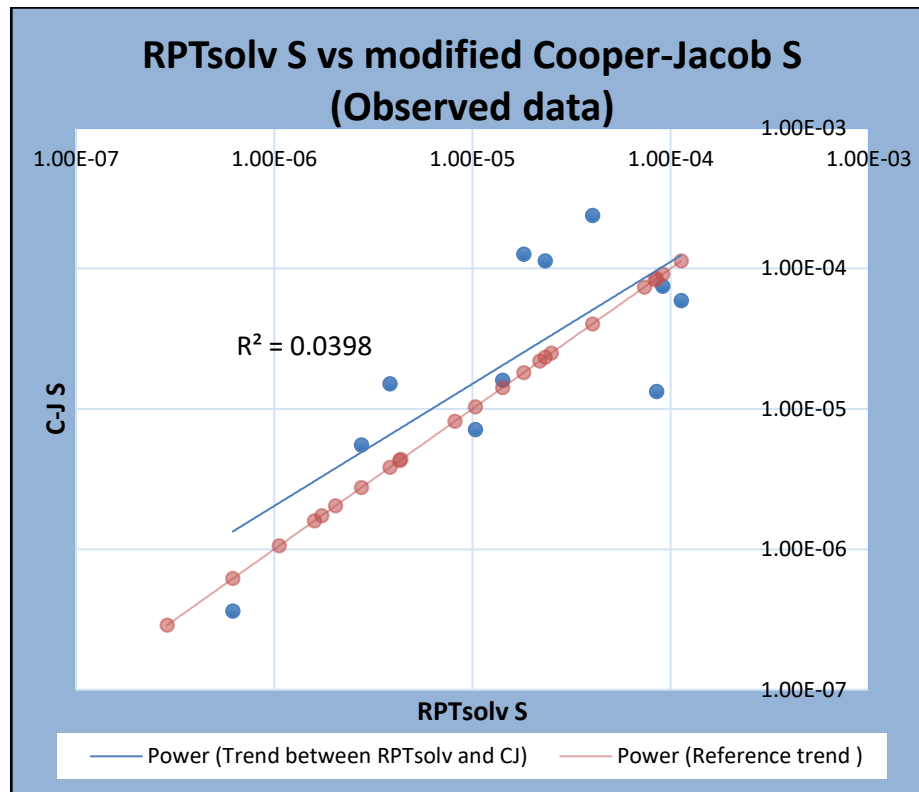


Figure 5-14: Modified Cooper-Jacob S versus RPTsolv S produced by analysing the observational data with the separate effective radius correlation equation.

Table 5-11: Error metrics between the modified Cooper-Jacob S and RPTsolv S (mean correlation equation).

Error metric	Value
RMSE	9.42E-05
R	0.20
% difference	88.65

The observational borehole correlation produces a R value of 0.2 which suggests a weak positive relation. The difference between RPTsolv and Cooper-Jacob S is only 88.65% which indicates a reduction in the overall differences between the 2 methods.

The data indicate the same overall trend compared to using the mean correlation equation. The pumping borehole correlations are moderate to good and positive while the observational borehole correlations are weak and positive. Figure 5-13 and Figure 5-14 are now plotted together

in Figure 5-15. This plotting method produces an overall idea of the effectiveness of the separate correlation equation method. Table 5-12 gives the results from the error metrics.

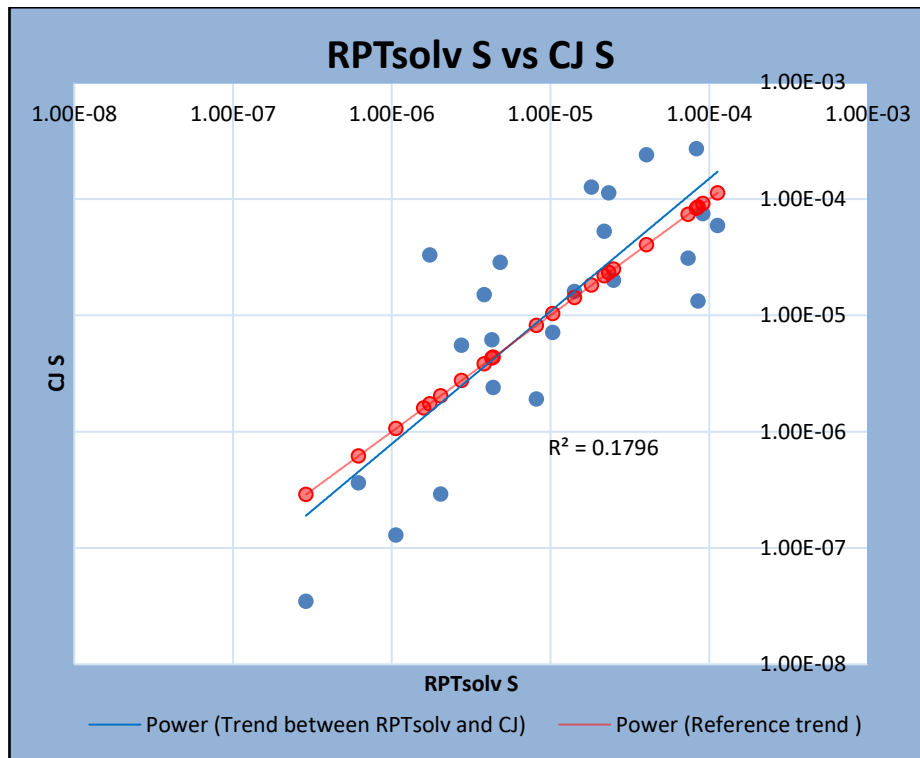


Figure 5-15: Modified Cooper-Jacob S versus RPTsolv S for both pumping and observation data, using the separate effective radius correlation equation.

Table 5-12: Error metrics between the modified Cooper-Jacob S and RPTsolv S (separate correlation equation).

Error metric	Value
RMSE	6.68E-05
R	0.42
% difference	101.60

The regression trendline from all these data plotted together is very close to the RPTsolv reference trend. This indicates that these two methods produce storativity calculated storativity in the same order of magnitude. A correlation coefficient of 0.42 shows a moderate to weak positive correlation.

5.2.3.3 Overall results

Table 5-13 contains a summary of these results and is used as a point of discussion:

Table 5-13: A summary of the storativity results against the three chosen error metrics, with yellow cells indicating the best option between ‘Before fit’, ‘After fit (mean)’, and ‘After fit (separate)’.

	Before fit	After fit (mean)	After fit (separate)
Correlation (R) - All	0.34	0.36	0.42
Correlation (R) - OBH	0.09	0.21	0.20
Correlation (R) - PBH	0.97	0.72	0.77
% difference	156.75	107.96	95.94
RMSE	3.76E-05	4.92E-04	6.68E-05

The above table shows the result in terms of these three chosen error metrics. These error metrics are compiled by comparing these results of the original and modified Cooper-Jacob method to that of RPTsolv. The cells highlighted in yellow indicate the method that produced the lowest error or deviates the least from RPTsolv results.

5.2.3.3.1 Correlation (R)

The Cooper-Jacob (1946) method produced a correlation coefficient of 0.09 to 0.97 which indicates that the correlation is either weak or strong depending on whether pumping or observational boreholes were used. It is worth noting that the R value of 0.97, when comparing Cooper-Jacob (1946) storativity to RPTsolv storativity, is much higher than anticipated. The graph shows that the trendline of the Cooper-Jacob (1946) storativity is 1 or more order of magnitude lower than that of RPTsolv. The modified Cooper-Jacob method, with the mean effective radius equation included, produced R values between 0.21 and 0.72. This shows that the Storativity calculated with this method indicates a moderate positive trend with the data points exhibiting some dispersion from the trendline. The best overall correlations were produced by the modified Cooper-Jacob method fitted with the separate effective radius equation. R values of between 0.20 and 0.77 show a moderate to strong positive correlation with less dispersion of data points along the trendline.

5.2.3.3.2 Percentage Difference

This error metric shows the difference between the original and modified Cooper-Jacob method and RPTsolv by using percentages. Here there is a clear decline in the percentage difference from the original Cooper-Jacob to the modified Cooper-Jacob. The percentages from the original method, the method with the mean effective radius equation, and the method with the separate effective radius equation, produced 156.75%, 107.96%, and 95.94% respectively. By using the

effective radius correlation equation, this study could reduce the error between the Cooper-Jacob method and RPTsolv by between 48.7% and 60.8%.

5.2.3.3.3 Root mean square error (RMSE)

The RMSE when the original and modified Cooper-Jacob S was compared to RPTsolv S, was calculated. Surprisingly, the original Cooper-Jacob (1946) method produced the smallest RMSE despite having the highest percentage difference. The RMSE of the two sets when the Cooper-Jacob was analysed with the Mean correlation noted the highest RMSE with $4.92\text{E}-04$ followed by the RMSE when the Cooper-Jacob storativity was analysed with the separated correlation equation of $6.68\text{E}-05$. The reason the original Cooper-Jacob method yielded the lowest RMSE might be because the RMSE takes into account the mean of the errors in the data and does not account for each individual point.

5.2.4 Analytical model versus the modified Cooper-Jacob method

Figure 5-16 was created to visualise the ratio between RPTsolv storativity and the storativity of all other analytical models, including the modified Cooper-Jacob model.

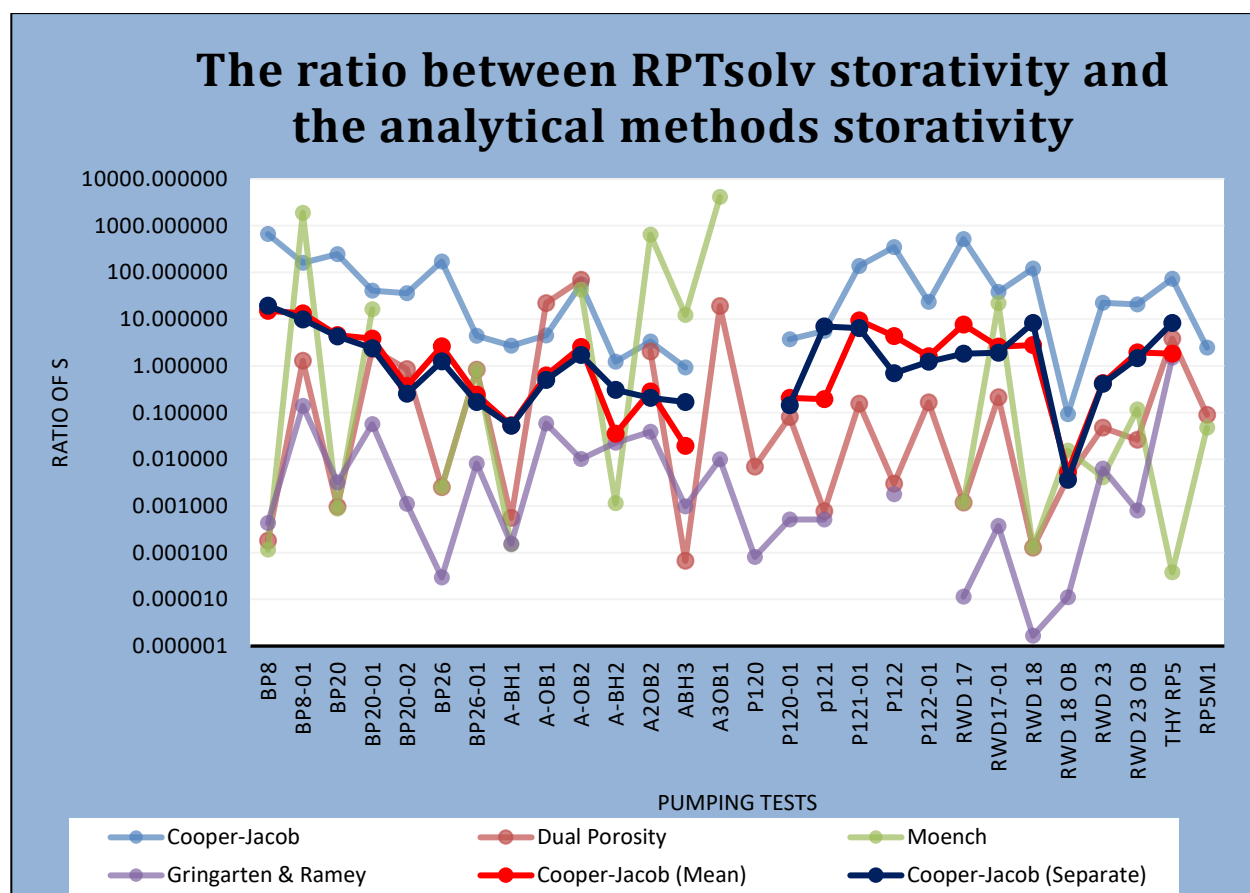


Figure 5-16: Ratio between RPTsolv S and the S of all analytical models.

This graph visualises the harsh truth when comparing storativities from RPTsolv to that of the analytical models. A ratio was taken by dividing the RPTsolv storativity with each storativity estimate of the analytical models. If the storativities were equal, the ratio would have been 1. If RPTsolv produced a storativity larger than the analytical method, the ratio would be lower. The original Cooper-Jacob (1946) model produced storativities larger than RPTsolv by 1 to 3 orders of magnitude. The models of Moench (1984), Gringarten & Ramey (1974) and the dual porosity model produced storativities consistently lower than that of RPTsolv by 1 to 5 orders of magnitude. The two modified Cooper-Jacob models produced the best ratios when compared to RPTsolv. The ratio line is consistently between 10 and -10, which indicates that the modified methods produce storativity within the same order of magnitude for most cases.

5.2.5 Modified Cooper-Jacob method validation

The modified Cooper-Jacob is calibrated to the drawdown data used in this study. The method has therefore not been analysed with data that it has not seen during the calibration stage. In order to validate the methodology, two random pumping tests was removed from the original data pool. The new data pool produced different effective radius correlation equations as per the methodology set out in Chapter 4. The correlation equations were then used in the Cooper-Jacob method and the two data sets removed in the beginning was analysed to see how accurate the Storativity was compared to RPTsolv Storativity for the same data.

The new effective radius correlations were noted by Table 5-14:

Table 5-14: New effective radius correlation equations used for the validation step.

Mean re correlation equation	$y = 0.0351 \times (re)^{1.3307}$
R value	0.91
re correlation equation for pumping data	$y = 0.00003 \times (re)^{2.96}$
R value	0.6
re correlation for the observational data	$y = 0.0176 \times (re)^{1.4105}$
R value	0.906

As is shown in the table, the correlations between the two sets of effective radii are still strong and positive after removing two pump test data sets. These equations were now modelled into the Cooper-Jacob method in order to analyse two removed data sets. See the results in Table 5-15 below:

Table 5-15: Results of the two new data sets analysed with the modified Cooper-Jacob method.

	1-PBH	1-OBH1	2-PHB	2-OBH
RPPTsolv S	1.06E-06	1.42E-05	1.59E-06	4.84E-04
Original CJ S	1.45E-08	5.76E-06	2.40E-09	2.99E-06
Mean re correlation S	5.06E-07	1.08E-05	1.07E-07	2.70E-05
Separate re correlation S	1.29E-07	9.98E-06	8.83E-08	3.62E-05

The results were plotted on Figure 5-17 below to visualise the data.

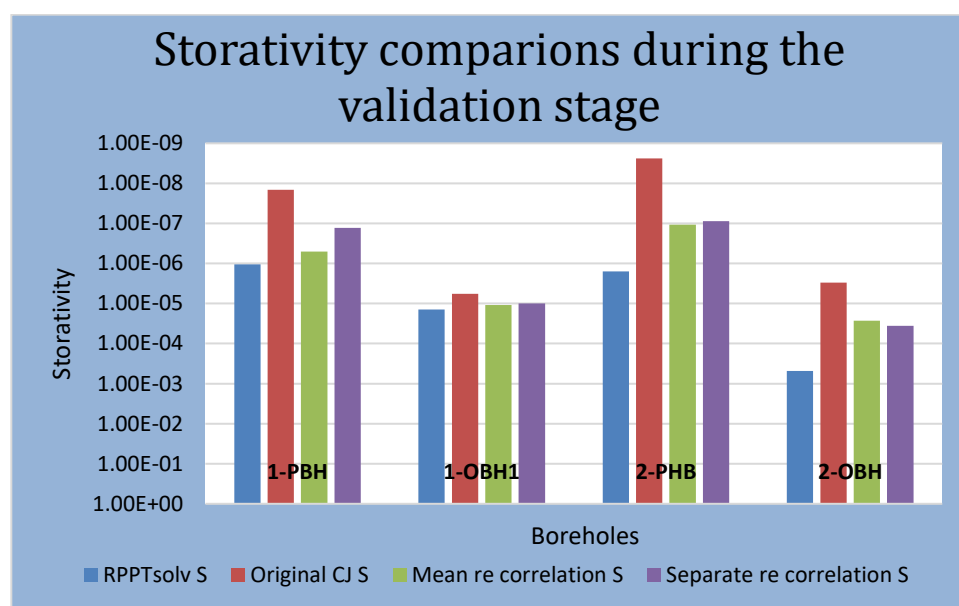


Figure 5-17: RPTsolv S and the S of all analytical models during the validation stage.

The modified Cooper-Jacob method calculated the Storativities closer to that of RPTsolv compared to the original Cooper-Jacob method. As stated previously, the Cooper-Jacob method tends to produce Storativities lower than RPTsolv by a few orders of magnitudes. The modified Cooper-Jacob method tries to mimic RPTsolv S by increasing the storativity of the Cooper-Jacob method.

CHAPTER 6 CONCLUSION AND RECOMMENDATIONS

This dissertation has aimed to identify the distance dependency of storativity in fractured dual porous aquifers when analysed with analytical methods.

This research has had two main objectives, to compare and evaluate analytical methods for the use in fractured, dual porosity aquifers; and to study the correlations between the storativity values produced by the analytical method of Cooper-Jacob (1946) and the numerical model of RPTsolv, in order to improve the results of the analytical method with regards to storativity.

6.1 Comparative analysis conclusion

The key observations revealed that the Cooper-Jacob (1946) method, on average, yielded results for storativity closer to RPTsolv when compared to other analytical methods, although it did not always provide the closest values in every individual test. Gringarten & Ramey's (1974) method reported the greatest deviations in storativity from RPTsolv and may point to the inability of straight-line single fracture flow models of predicting reliable storativities in dual porous aquifers.

The inability of analytical methods to calculate accurate and consistent storativity values was further studied by examining their dependence on distance. It became clear that these methods struggled to produce consistent storativities as the distance varied. In the Karoo Supergroup, both the Dual Porosity and Moench methods produced storativities that decreased with increasing distance, whereas the Gringarten & Ramey (1974) and Cooper-Jacob (1946) methods showed an increase. It is worth noting that the furthest monitoring wells were only 7m from the pumping well. Despite this proximity, the variations were quite noticeable, especially with the Cooper-Jacob method. In the Cape Supergroup, the Gringarten & Ramey (1974) method produced storativities that slightly decreased with distance from the pumping borehole. For both supergroups, the Gringarten & Ramey (1974) method produced storativities with acceptable variations, but the storativities calculated were orders of magnitude higher than those produced by other methods. Storativities calculated by the Dual Porosity and Moench methods decreased significantly with distance from the pumping borehole, while those calculated by the Cooper-Jacob method increased slightly with distance. The monitoring wells were much further away from the pumping borehole when compared to the Karoo Supergroup. This was noticeable due to the higher variations in Storativity for this Supergroup. In both Supergroups, RPTsolv produced consistent storativities across distances. This supports the suggestion that RPTsolv is free from the distance dependency issue that plagues other analytical methods.

These results clearly showed that the analytical methods used in the study were more reliable in estimating transmissivity than storativity. While the range of transmissivity values was narrower and more consistent across the methods, storativity calculations varied significantly. These findings suggest that current analytical methods can produce reliable transmissivity values but requires different approaches to improve storativity estimations.

6.2 Effective radius correlation method conclusion

For transmissivity, both models produced similar values, suggesting that the Cooper-Jacob (1946) method can estimate this parameter reliably. However, the storativity estimations initially revealed a wide range of values, with the Cooper-Jacob (1946) method producing values significantly lower than those from RPTsolv, indicating the limitations of the standard Cooper-Jacob (1946) method.

Upon modifying the Cooper-Jacob model's effective radius by using the method set out in chapter 4.6, there was a noticeable improvement in its storativity estimates. By changing the effective radius of the Cooper-Jacob (1946) method, it has enhanced the models results to more closely resemble that of RPTsolv. Although the match was not exact, it represented an improvement from the baseline model.

The summary table, Table 5-3, has clarified these improvements made by using the effective radius correlations. By integrating the effective radius correlations into the modified Cooper-Jacob method, the wide range of storativity estimates compared to RPTsolv were reduced considerably, as seen by the lower percentage differences and lower R values. The original Cooper-Jacob (1946) model though, did have the lowest RMSE when compared to RPTsolv. This could indicate that although RMSE is a valuable error metric, it must not be looked at as the sole mean of determining a successful model.

Furthermore, by comparing the ratio of the RPTsolv storativity when it is divided by the storativity of the analytical methods, it is clear that the modified Cooper-Jacob produces storativities much closer to RPTsolv than the other analytical models. The model was validated by taking out two data sets from the original drawdown data pool. The reduced data pool was then used to create new effective radius correlation equations which was modelled into the Cooper-Jacob method . The two sets that was removed was then analysed with the modified Cooper0Jacob and yielded Storativity results closer to RPTsolv S compared to the original Cooper-Jacob method.

In conclusion, the modified Cooper-Jacob model, with effective radius adjustments, has offered a more reliable means of estimating storativity compared to the original model. While the results

were not always accurate, they represent a step in the right direction towards improving the model's output reliability. With that being said, these results were obtained from certain areas in the Cape Supergroup and Karoo Supergroup of South Africa and must only be used in these areas. If the same modified Cooper-Jacob model was used elsewhere, it might produce erroneous results.

Based on these conclusions, practitioners should consider adapting the same methodology set out in this dissertation in more areas. This will ensure that the modified Cooper-Jacob method can be used in more areas and not just the study areas set out in this dissertation. This study was based on very limited pumping test data. To improve the correlation equations, the study should be replicated with more pump test data.

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APPENDIX A: ANALYTICAL MODELS USED IN THE STUDY

Cooper-Jacob (1946) for fractured aquifers:

Table A-1: The equations for the Cooper-Jacob method.

Cooper-Jacob for fractured aquifers: 1. $T = \frac{2.3Q}{4\pi\Delta s}$ 2. $Q = T\left(\frac{s_w m}{r}\right)$ 3. $\log S = \frac{2.25Tm}{r^2} - \frac{s_w m \pi T m}{2.30Qm}$		
Parameter	Parameter description	Dimension
T	Transmissivity	L ² /T
Q	Discharge rate of the borehole	L ³ /T
S_w	Drawdown measured	L
r	Distance from pumping borehole	L
S	Storativity	-

Moench (1984) used the Laplace transform solution for dimensionless drawdown. The equations are as follows:

Table A-2: The Laplace transform equation used by Moench (1984).

The Laplace transform: 1. $\hat{h}_{WD} = \frac{2 [K_0(x) + xS_w K_1(x)]}{p[pW_D[K_0(x) + xS_w K_1(x)] + xK_1(x)]}$ (Pumped well)	
2.	$\hat{h}_D = \frac{2K_0(r_D x)}{p[pW_D[K_0(x) + xS_w K_1(x)] + xK_1(x)]}$ with $x = \sqrt{p + qD}$ (Observation well)
3.	$\bar{q}D = \frac{y^2 m \tan h(m)}{1 + S_f m \tan h(m)}$ (Slab block)

4. $\bar{q}D = \frac{3y^2[m \cot h(m)-1]}{\{1+S_f[m \cot h(m)-1]\}}$ (Spherical blocks)		
5. $m = \frac{\sqrt{\sigma p}}{y}$	9. $S_f = \frac{2K' b_s}{k_s b'}$	
6. $y = \frac{2r_w}{b'} \sqrt{\frac{K'}{K}}$	10. $W_D = \frac{\pi r_c^2}{2\pi r_w^2 S_s b}$	
7. $\sigma = \frac{S_s'}{S_s}$	11. $t_D = \frac{Kt}{S_s r_w^2}$	
8. $r_D = \frac{r}{r_w}$	12. $h_D = \frac{4\pi K b}{Q} (h_0 - h)$	
Parameter	Parameter description	Dimension
h	Head in aquifer	[L]
h_0	Static head in the aquifer	[L]
$\hat{h}_{wD}$	Dimensionless drawdown	-
K	Fracture hydraulic conductivity	[L/T]
K'	Matrix hydraulic conductivity	[L/T]
K_i	Modified Bessel function of the second kind	-
K_s	Fracture skin hydraulic conductivity	[L/T]
p	Laplace transform variable	-
Q	Pumping rate	[L ³ /T]
r	Radial distance from the pumping well to the observation well. r_c = Casing radius	[L]

	r_w = Well radius	
S	Specific storage S_s = Fracture specific storage S_s' = Matrix specific storage	$[L^{-1}]$
S_f	Fracture skin factor	-
S_w	Wellbore skin factor	-
t	Elapsed time since pumping started	$[T]$

Warren & Root (1963):

Table A-3: The equation used by Warren and Root (1963) for the double-porosity flow model.

Warren & Root (1963):		
1. $-q_\alpha = S'_s \frac{dh'}{dt}$	(flow equation in the matrix)	
2. $q_\alpha = \alpha k'_h (h' - h)$	(flow rate between matrix and fractures)	
3. $\alpha = \frac{A}{lV}$	(geometry of the block)	
Dimensionless parameters:		
4. $\lambda = \frac{\alpha r^2 k'_h}{k_h}$	5. $\sigma = \frac{S'_s}{S_s}$	6. $r_D = \frac{r}{r_w}$
Parameter	Description	Dimension
q_α	Flow	$[L/T]$
S	Specific storage coefficient with S_s - Fractures S'_s - Matrix blocks	-

h	Water level height (hydraulic head) with h - Fractures h' - Matrix blocks	[L]
t	Time	[T]
k	Hydraulic conductivity with k_h - Fractures k'_h - Matrix blocks	[L/T]
r	Distance/radius with r - Distance from pumping well to the observation well r_w - Effective radius of the pumping well r_D - Dimensionless distance	[L] [L] -
α	Parameter for describing geometry (matrix)	[L ⁻²]

Gringarten-Ramey (1974):

Table A-4: Equations used for the Gringarten-Ramey (1974) model.

Gringarten-Ramey (1974):

$$1. \quad s = \frac{Q}{4\pi\sqrt{K_r K_z}} \frac{2}{H_D} \int_0^{t_D} P \cdot Z \frac{d\tau}{\tau}$$

$$2. \quad P = e^{-\frac{r_D^2}{4\tau}} \int_0^1 I_0\left(\frac{r_D v}{2\tau}\right) e^{-\frac{v^2}{4\tau}} v dv$$

$$3. \quad Z = 1 + 2 \sum_{n=1}^{\infty} e^{-\frac{n^2 \pi^2 \tau}{H_D^2}} \cos\left(n\pi \frac{z_f}{b}\right) \cos\left(n\pi \frac{z}{b}\right)$$

$$4. \quad H_D = \frac{b}{r_f} \sqrt{\frac{K_z}{K_r}}$$

5. $r_D = \frac{r}{r_f}$ 6. $t_D = \frac{K_r t}{S_s r_f^2}$		
Parameters	Description	Dimension
s	Drawdown	[L]
Q	Pumping rate	[L ³ /T]
K	Hydraulic conductivity where K_z – vertical hydraulic conductivity K_r- radial hydraulic conductivity	[L/T]
H_D	Dimensionless drawdown	-
τ and ν	Variables of integration	-
z	Distance in z-direction	[L]
b	Aquifer saturated thickness	[L]
r	Distance/radius with r - Distance from pumping well to observation well r_D- Dimensionless distance r_f- Radius of the horizontal fracture	[L] - [L]

I_0	Modified Bessel function of the first kind	-
S_s	Specific storage	[L ⁻¹]
t	Time elapsed since pumping started	[T]

The equations for Gringarten and Witherspoon (1972) are given below:

Table A-5: The equation for the single vertical fracture solution by Gringarten and Witherspoon (1972).

Gringarten and Witherspoon (1972): $1. s = \frac{Q}{4\pi\sqrt{T_x T_y}} \frac{\sqrt{\pi}}{2} \int_0^{t_D} \left[\operatorname{erf} \frac{1-X_D}{2\sqrt{\tau}} + \operatorname{erf} \frac{1+X_D}{2\sqrt{\tau}} \right] e^{\frac{Y_D^2 T_x}{4x T t}} \frac{d\tau}{\sqrt{\tau}}$ $2. t_D = \frac{T_x}{S X_f^2}$ $3. x_D = \frac{x}{X_f}$ $4. Y_D = \frac{y}{X_f}$ $5. S_D = \frac{4\pi\sqrt{T_x T_y}}{Q} s$		
Parameters	Description	Dimension
s	Drawdown	[L]
Q	Pumping rate	[L ³ /T]
T	Transmissivity where T _x – Transmissivity in x direction	[L ² /T]

	T_y – Transmissivity in y direction	
t	Time	[T]
S	Storativity	-
X_f	Fracture half-length in x-direction	[L]
x and y	Coordinate distances	[L]

APPENDIX B: TABULATED SUMMARY

Flow regimes

Table B-1: Summary of regime flow periods of the proposed analytical models.

Flow type	Flow regimes in diagnostic plots			Methods
	Early time	Intermediate time	Late time	
Radial flow	Inner-boundary effects (wellbore storage). Slope of 1 on a log-log plot.	Transition period to IARF.	Radial flow from formation. Shows a horizontal line on a log-log plot.	Theis (1935) Cooper-Jacob (1946)
Double porosity	Radial flow from fracture network to borehole. Straight-line on semi-log plot (s vs. log(t)).	Transition period. The derivative curve is characterised by a U-shaped double-porosity dip	Radial flow from the matrix and fracture. Straight-line on semi-log plot (s vs. log(t)).	Warren and Root (1963) Moench (1984)
Single horizontal fracture	Inner-boundary effects (wellbore storage). Slope of 1 on the log-log plot.	Linear flow from the fracture to the well. Slope of 0.5 on a log-log plot.	Pseudo-radial flow. A horizontal line on a log-log plot.	Gringarten & Ramey (1974)
Single vertical fracture	Linear fracture flow with a slope of 0.5 on a log-log plot. Linear flow is followed by bilinear flow with a slope of 0.25 on a log-log plot.	Linear flow from the formation. Slope of 0.5 on a log-log plot.	Pseudo-radial flow. Horizontal line on a log-log plot.	Gringarten and Witherspoon (1973)

APPENDIX C: CASE STUDY IN THE COOPER-JACOB EXCEL METHOD

The following discussions are based on the two figures below. It is known by now that some flow characteristics can be deduced by observing the drawdown plot and its derivative. The increase in drawdown rate at early-time were likely because of the combined effects of wellbore storage and limited water supply near the well. Once the hydraulic gradient reaches more flow paths, a clear decrease in drawdown rate is visible. This stage, more visible on the derivative, is called the dual-porosity dip and signals a change in the overall flow regime. Bilinear flow is visible thereafter, with the derivative showing a good fracture network in a bilinear flow regime. Late drawdown showing a relatively straight line in the log-log plot with the derivative indicating constant head or radial flow conditions.

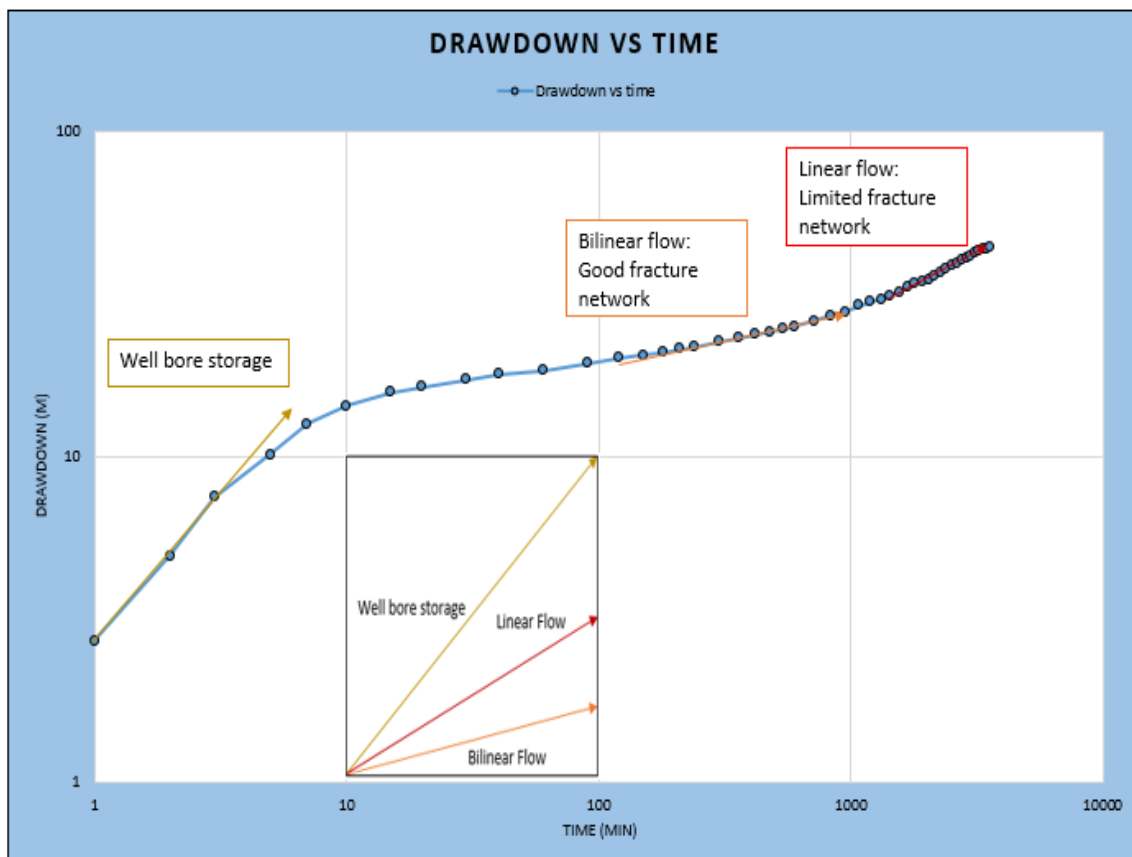


Figure C-1: Log-log drawdown versus time plot indicating different flow regimes.

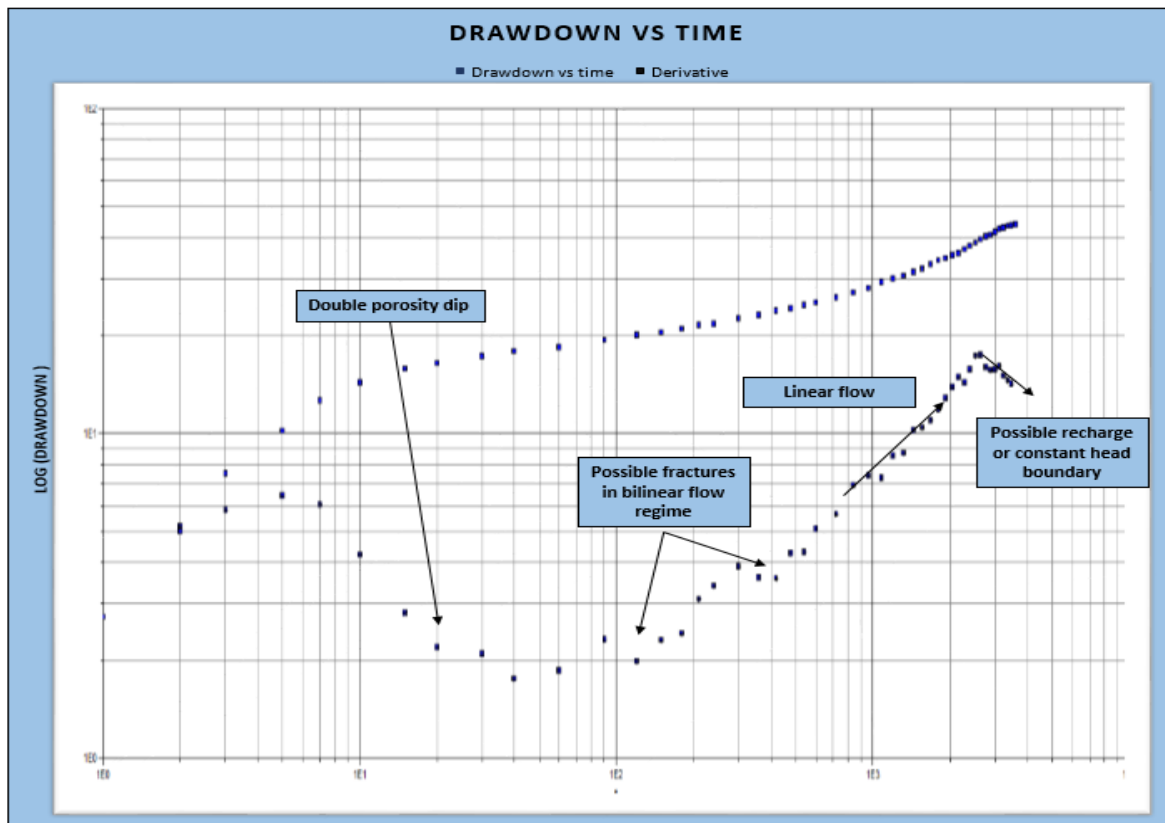


Figure C-2: Analysing the derivative of the drawdown plot for fractures and boundaries by using flow dimensions.

The early-time line was fitted on the bilinear flow regime period to ensure parameters represented fracture flow. The late-time line was fitted after the transition between bilinear and linear flow, in a period presumed to be matrix and fracture flow representative of the aquifer. Once the fitting procedure was complete, the results were documented in an excel sheet.

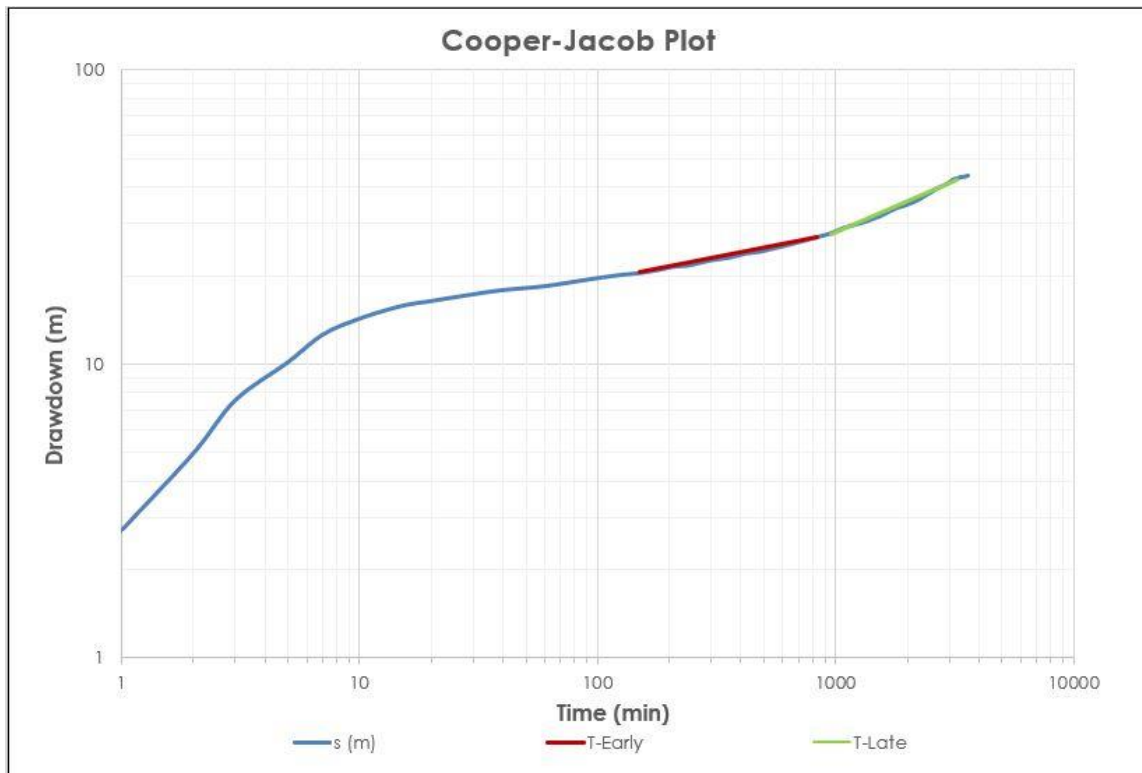
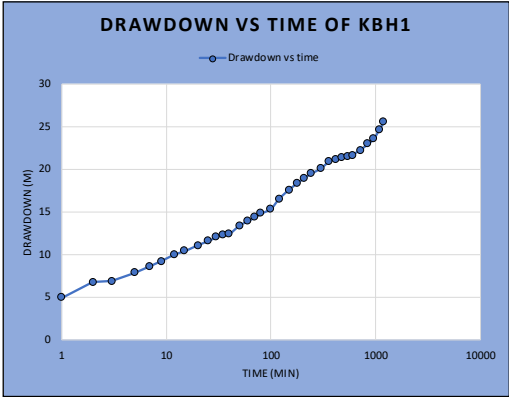


Figure C-3: Semi-Log graph of drawdown versus time showing lines fitted for the Cooper-Jacob method.

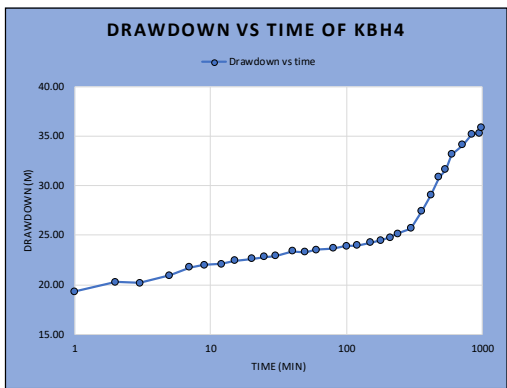
APPENDIX D: DATA

Karoo Supergroup:

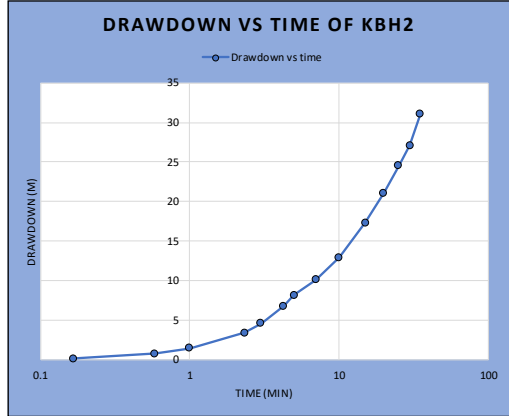
Borehole Name Borehole location rehole coordinates (if availab
KBH1 Aliwal North NA



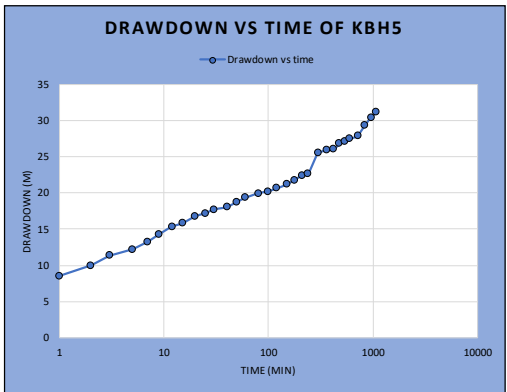
Borehole Name Borehole location Borehole coordinates
KBH4 Ventersdorp NA



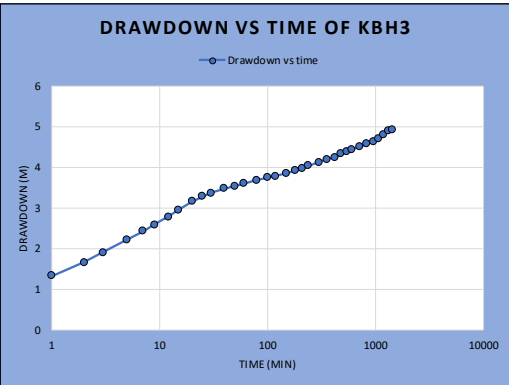
Borehole Name Borehole location Borehole coordinates
KBH2 Bloemfontein NA



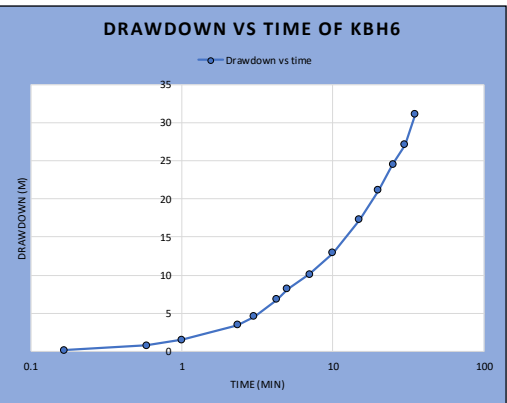
Borehole Name Borehole location Borehole coordinates
KBH5 Ventersdorp NA



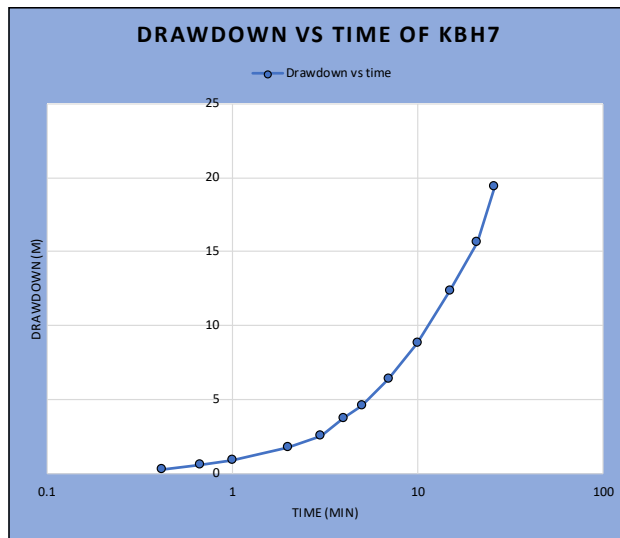
Borehole Name Borehole location Borehole coordinates
KBH3 Bloemfontein NA



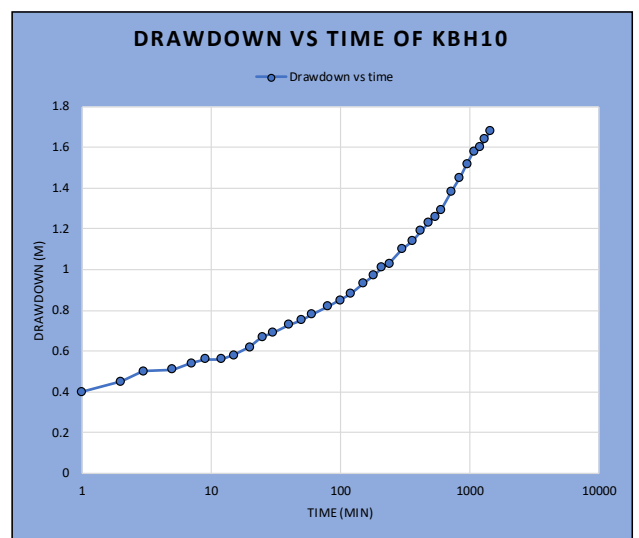
Borehole Name Borehole location Borehole coordinates
KBH6 Bloemfontein NA



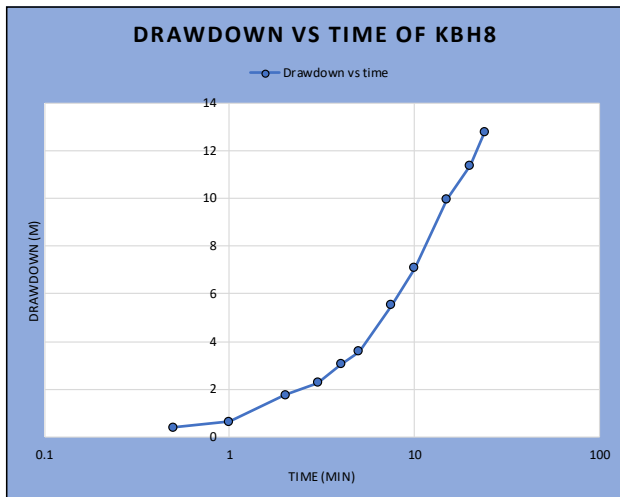
Borehole Name KBH7
Borehole location Bloemfontein
Borehole coordinates NA



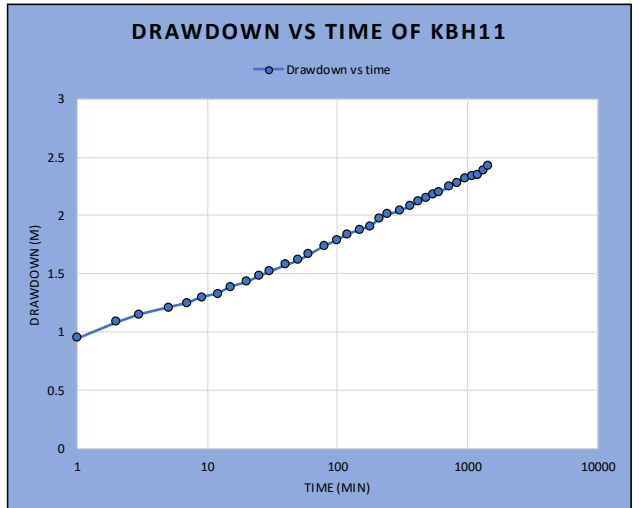
Borehole Name KBH10
Borehole location Venterstad
Borehole coordinates NA



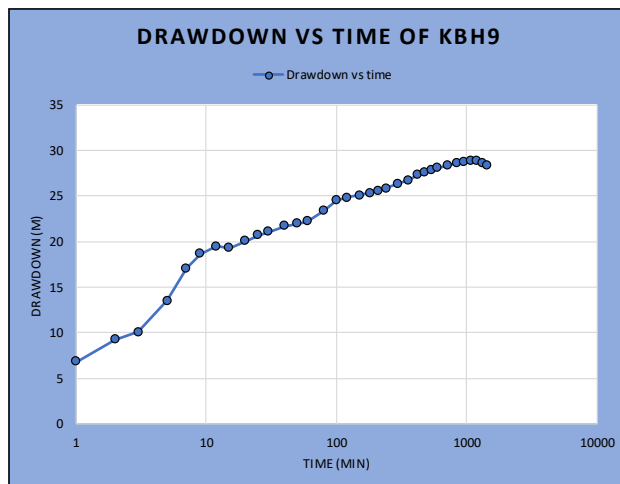
Borehole Name KBH8
Borehole location Bloemfontein
Borehole coordinates NA



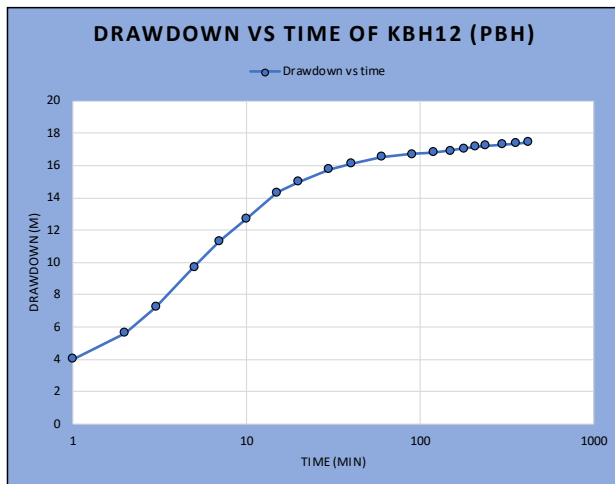
Borehole Name KBH11
Borehole location Venterstad
Borehole coordinates NA



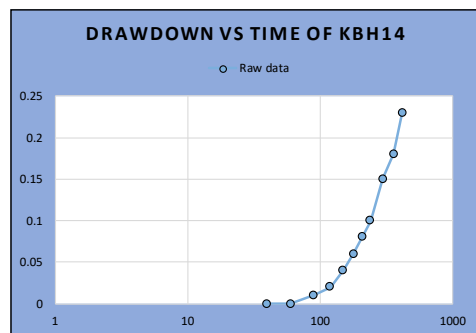
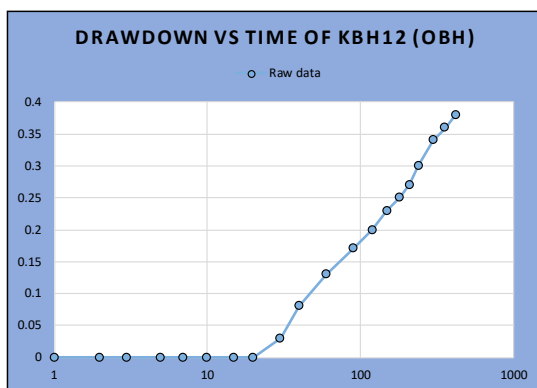
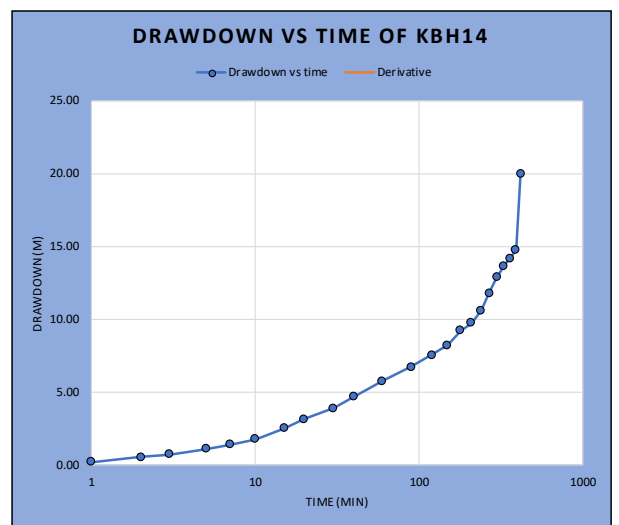
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Borehole location Venterstad
Borehole coordinates NA



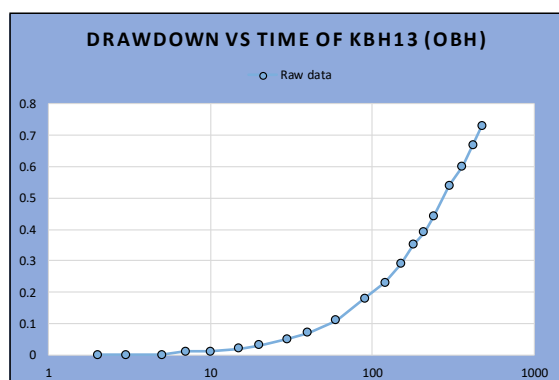
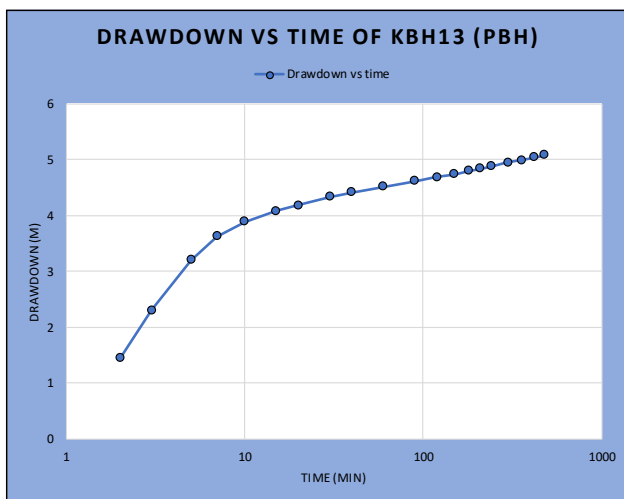
Borehole Name KBH12
Borehole location Bloemfontein
Borehole coordinates NA



Borehole Name KBH14
Borehole location Bloemfontein
Borehole coordinates NA

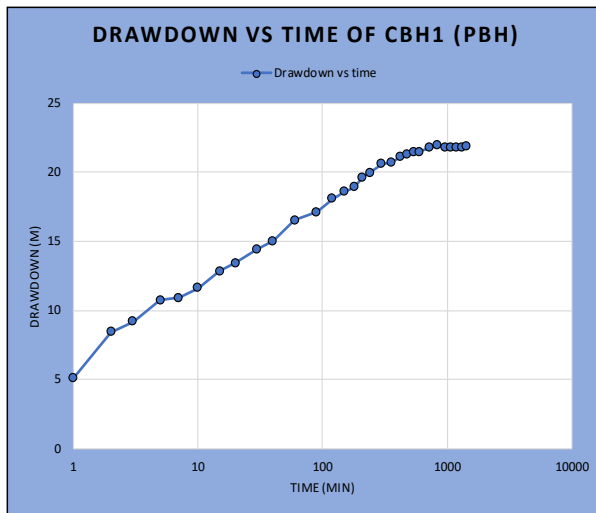


Borehole Name KBH13
Borehole location Bloemfontein
Borehole coordinates NA

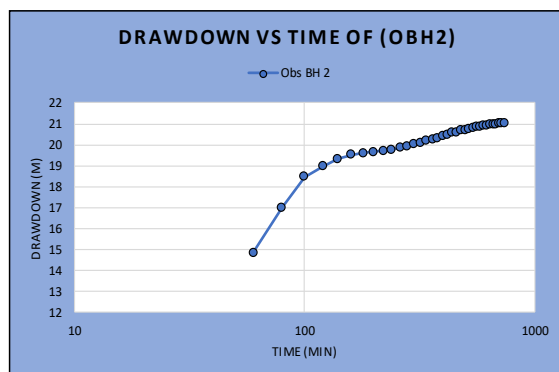
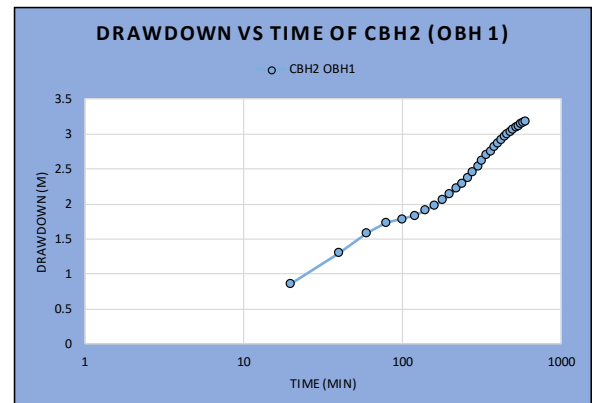
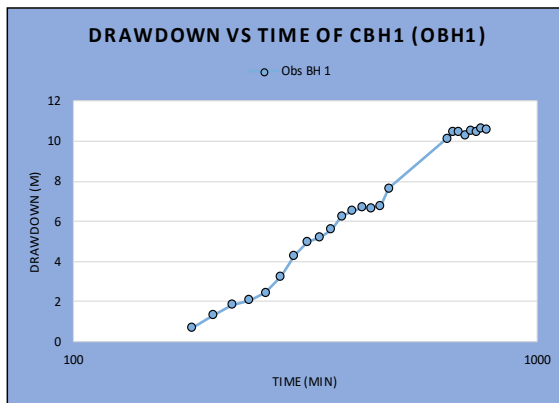
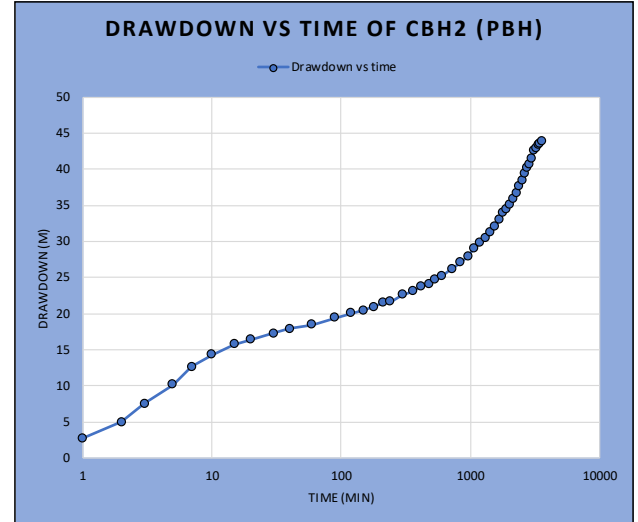


Cape Supergroup:

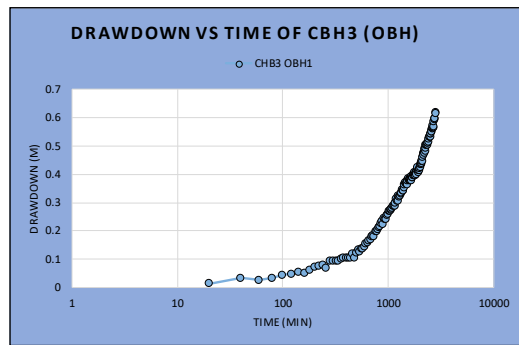
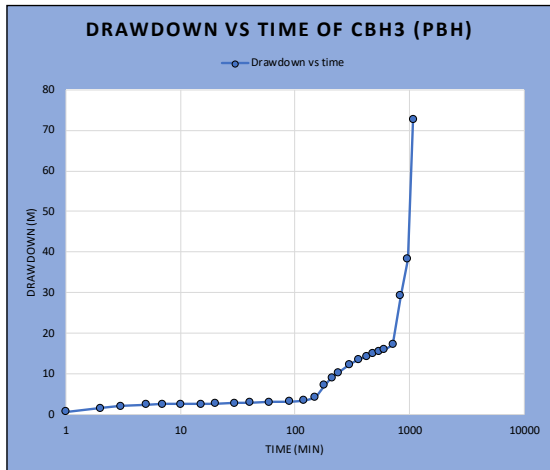
Borehole Name	Borehole location	Borehole coordinates
CBH1	Ceres	NA



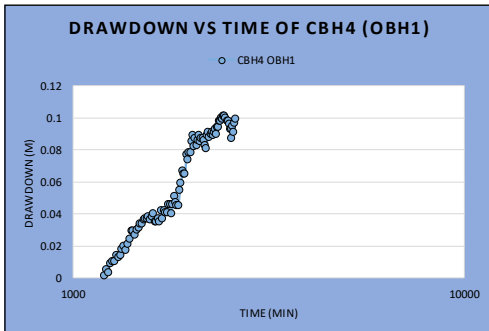
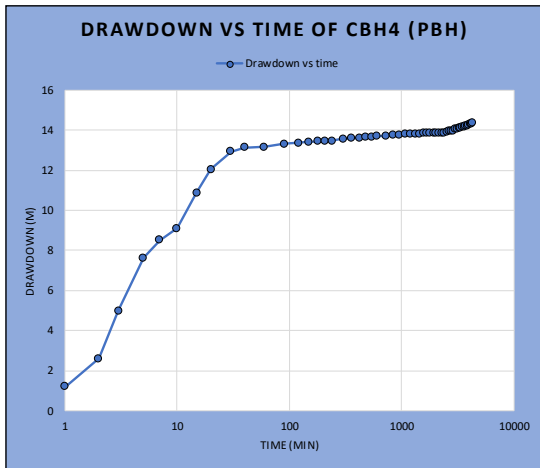
Borehole Name	Borehole location	Borehole coordinates
CBH2	Ceres	NA



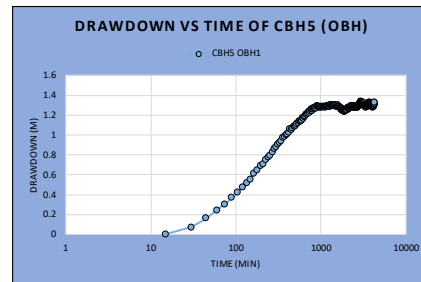
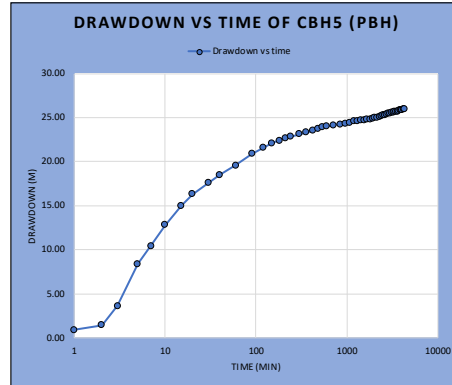
Borehole Name Borehole location Borehole coordinates
CBH3 Ceres NA



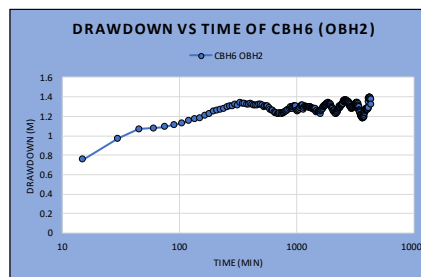
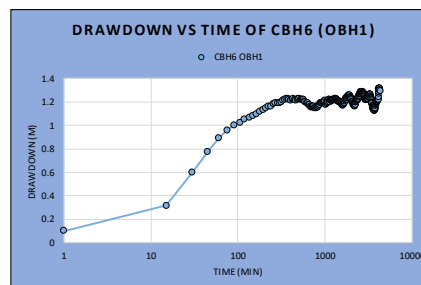
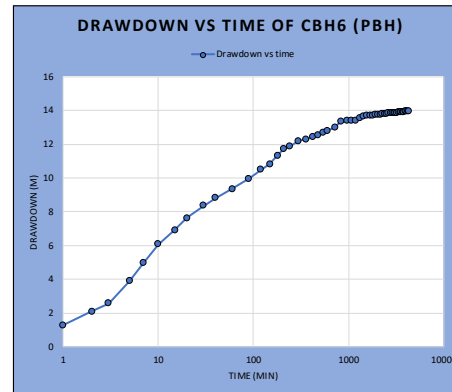
Borehole Name Borehole location Borehole coordinates
CBH4 Oyster Bay NA



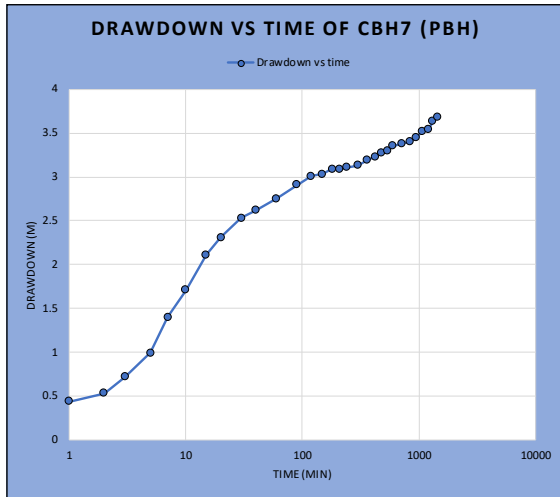
Borehole Name Borehole location Borehole coordinates
CBH5 Overberg, Pearly beach NA



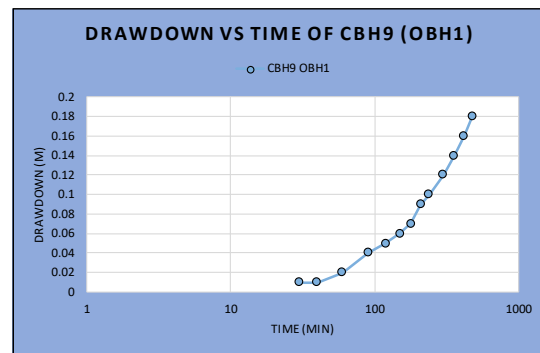
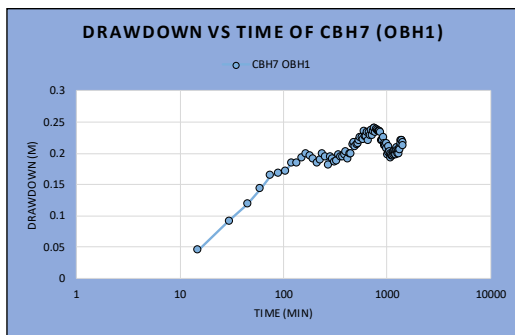
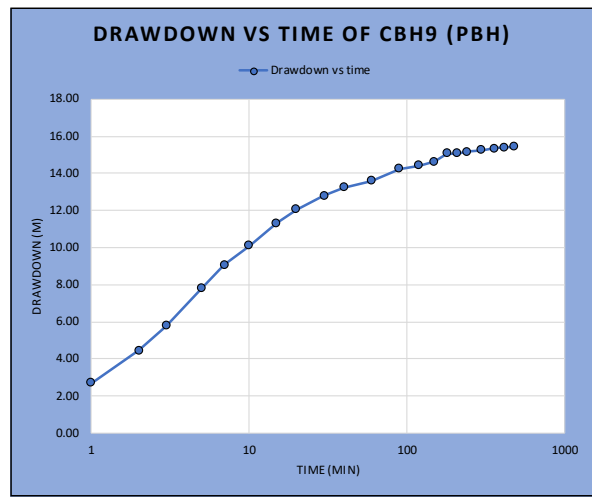
Borehole Name Borehole location Borehole coordinates
CBH6 Overberg, Pearly beach NA



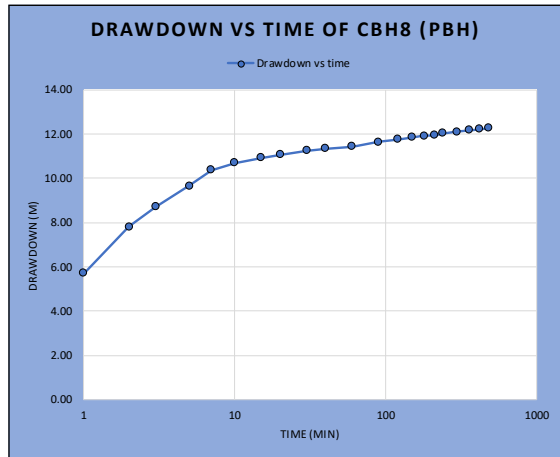
Borehole Name Borehole location Borehole coordinates
CBH7 Overberg, Pearly beach NA



Borehole Name Borehole location Borehole coordinates
CBH9 Western Cape NA



Borehole Name Borehole location Borehole coordinates
CBH8 Western Cape NA



Borehole Name Borehole location Borehole coordinates
CBH10 Western Cape NA

