

Mathematical modelling of blood spatter with optimization and other numerical methods

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Summary

The current methods used by forensic experts to analyse blood spatter neglects the influence of gravitation and drag on the trajectory of the droplet. This research attempts to suggest a more accurate method to determine the trajectory of a blood droplet using multi-target tracking. The multi-target tracking problem can be rewritten as a linear programming problem and solved by means of optimization and numerical methods.

A literature survey is presented on relevant articles on blood spatter analysis and multi-target tracking. In contrast to a more advanced approach that assumes a background in probability, mathematical modelling and forensic science, this dissertation aims to give a comprehensive mathematical exposition of particle tracking. The tracking of multi-targets, through multi-target tracking, is investigated. The dynamic programming methods to solve the multi-target tracking are coded in the MATLAB programming language. Results are obtained for different scenarios and option inputs. Research strategies include studying documents, articles, journal entries and books.

Key terms

1. Bloodstain analysis
2. Fluid mechanics
3. Multi-target tracking
4. Linear programming
5. Dynamic programming
6. *K*-shortest path algorithms

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List of symbols

Fluid mechanics

a	acceleration
D	fall distance
d	drag coefficient
D_0	diameter of stain
D_1	diameter of droplet
F	force magnitude
Fr	Froude number
f	external force
F_d	drag force
F_g	gravitation force
g	gravitation
h	vertical height
L	length of ellipse
m	mass
p	pressure
r	radius of droplet
Re	Reynolds number
T	kinetic energy
V	volume
v	velocity
W	width of ellipse
We	Weber number
γ	surface tension
μ	viscosity
ρ	density

Graph Theory

e	edge
v	vertex
E	Set of edges
G	graph
P	path matrix
V	Set of vertices
W	weight matrix

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