An exploration of reflection and mathematics confidence during problem solving in senior phase Mathematics

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For the LORD shall be thy confidence and shall keep thy foot from being taken

(Proverbs 3:26)

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'I think I could, if I only know how to begin', said Alice, 'I almost think I can remember feeling a little bit different.' 'What do you mean by that?' said the Caterpillar sternly. 'Explain yourself.'

(C.S. Lewis, *Alice in Wonderland*)

Summary

Empowerment through proficiency in mathematics could better not only one's life, but also one's chances in study and work. The current study is an exploration of what reflection and mathematics confidence entail during mathematics problem solving. Reflections on experiences with mathematics create awareness of the individual's level of confidence in the social, psychological and intellectual domains. Personal, strategic and task knowledge enhances meaning and promotes the understanding of mathematics tasks during problem solving. The level of mathematics confidence can be described as either fearful or fearless when solving mathematics problems. Reflecting on achievement, with or without fear, is regarded as vital for higher-order reasoning by means of metacognitive processes, moderates mathematics confidence and fosters achievement. Although research in metacognition is increasing, literature involving mathematics confidence and reflection is scarce.

The current study explores this link between reflection and mathematics confidence by focusing on metacognitive reflective skills. A mixed-method design consisting of positivist and interpretivist paradigms is employed. Merging of the quantitative and qualitative findings indicates that metacognitive strategies include reflecting on task, personal and strategic awareness. Regulating understanding, planning, monitoring and evaluating during problem solving occurs in accordance with these active internal processes. Mathematics confidence during problem-solving emerges from experiences relating to a variety of contexts involving mathematics. The findings confirm the dimensionality of mathematics confidence and present sources of participants' mathematics confidence and metacognitive skills as reflected upon.

The schools in the sample represent single-gender (all-boys and all-girls) and co-ed schools and findings should not be generalised to all schools. Reflection on metacognitive knowledge and regulation deepens the awareness of the level of confidence and promotes, to some extent, a knowing of knowledge. The study therefore evaluates the role reflection and mathematics confidence play during problem solving in senior phase mathematics.

Keywords

Mathematics

Mathematics anxiety

Mathematics confidence

Metacognition

Mixed-method approach

Monitoring

Problem solving

Reflection

Senior phase

Video recording

Regulation

Strategies

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Chapter 1

Orientation

1.1 International surveys comparing mathematics achievement in South Africa with that in other countries

The Third Trends in International Mathematics and Science Study (TIMSS), conducted in 1995 and 1999, which samples schools, classes and learners to gain scientifically based information regarding Mathematics performance at school level (Howie, 2001:3), made a startling discovery: Learners in South African schools do not achieve the same level of results, compared to schools in other participating countries. The Trends in Mathematics and Science Study (1999) placed South Africa in the last place in a study conducted in 46 countries (TIMSS, 2003:3). South Africa compare its similar Grade 8 and Grade 9 Mathematics and Physical Science results, with the results of countries like Jordan, Scotland, Indonesia, Morocco, Saudi Arabia and Botswana (TIMSS, 2003). Countries like Korea, Japan, Switzerland, Belgium, Russia and the United States of America obtained the highest scores (TIMSS, 2003:10). In 2002, a study by the Southern African Consortium for Monitoring Educational Quality (SACMEQ) was conducted in 15 East African and Southern African countries (Moloi & Strauss, 2005:1). This study also concluded that learners in Grades 6 to 9 performed poorly in subjects like Physical Science and Mathematics. The TIMSS (2003) and TIMSS (2007) confirmed that South African learners performed poorly when doing Mathematics tasks and solving problems that involved geometry and word sums (Van der Walt & Maree, 2007:224; TIMSS, 2007:2-4). In 2011, 40 countries participated in both PIRLS-2011 and TIMSS-2011 studies, which served as an evaluation between Language, Mathematics and Physical Science on a global scale (TIMSS, 2011). Focussing on both content and cognitive domains (Gonzales et al., 2009:5) the TIMSS-2011 report assesses content such as number, algebra, geometry, data and chance together with knowing, applying and reasoning skills.

1.1.1 National standards and policies in South African schools

The National Curriculum Statement (Department of Education, 2002) declares that social transformation is imperative in South Africa and aims to ensure equal education opportunities for all by removing all artificial barriers to learning (DoE, 2003:2-4). The promotion of higher-order skills and knowledge, including human rights and indigenous systems of knowledge, is considered vital to development (DoE, 2003:3). The transformation of skills and knowledge, as embraced by all disciplines within the structure of the national curriculum, sets the stage for classroom practice.

1.1.2 Mathematics performance of South African learners

The media alarmingly predicts that 40% of South African Grade 12 learners will face unemployment in the future (Rademeyer, 2009; Maree, Olivier & Swanepoel, 2004:58). Grade 12 learners who underachieve in subjects like Mathematics and Physical Sciences are less likely to be admitted to university or to secure a full-time occupation. The critical shortage of top achievers in Mathematics who can gain university entrance not only hinders the country's economic growth, but also means that the demand for skilled persons in South Africa is not met (Maree, Olivier & Swanepoel, 2004:54). According to the Centre of Development and Entrepreneurship (CDE), no less than fifty thousand Grade 12 learners need to pass Mathematics to provide the skills required in South Africa every year (Rapport, 2008:18). According to the Action Plan for 2014 aimed at the Realisation of Schooling 2025, only one in every eight youths will pass Grade 12 with full university exemption – a very sobering statistic (DoE, 2010:13). This means that only one in every eight learners may become a doctor, a teacher or an accountant. With a shortage in these professions South Africans are bound to experience poor health, below standard education and financial instability.

1.2 Problem statement and rationale

First, a discussion follows regarding the rationale of the study.

1.2.1 Rationale

For the past year I have been involved at a Dinaledi school in the North-West Province, acting as a cluster leader for the subjects Mathematics and Mathematical Literacy. Different schools in the region have been divided into clusters in order to provide support

for Physical Science and Mathematics teaching and learning[1]. The Mathematics outreach programme organises quarterly meetings as part of professional development to discuss the participating schools' achievements and to conduct workshops with the focus on mathematics education. Lesson plans, methodological approaches and mathematics problem solving are merely a few examples of topics that are of importance during these meetings. Concern is often expressed regarding learners' mathematics achievement and the number of learners that enrol for Mathematics in the Further Education and Training phase. Due to my experiences in the programme, I have developed a conscious yearning to live a meaningful life and a desire to aid others. This has led me to explore the possibility of reflection and mathematics confidence and their role in mathematics problem solving.

1.2.2 Purpose of the research

The central purpose of this research is to explore the role mathematics anxiety[2] and reflection[3], as a metacognitive skill, play during mathematics problem solving. Literature on research that investigates the relationship between mathematics confidence and mathematics problem solving concludes that low confidence results in poorer performance (Maree, Prinsloo & Claassen, 1997b; Sherman & Wither, 2003:138; Ashcraft & Kirk, 2001). Little literature is available on research that explores mathematics confidence and its relation to reflection during problem-solving situations (Jain & Dawson, 2009).

The current study attempts to bridge this shortcoming in literature by focusing on individual differences in mathematics confidence and the implementation of reflection (as a facet of metacognition). The researcher examines a possible relationship between metacognitive ability, reflection and mathematics confidence during problem solving in senior phase Mathematics.

With this in mind, the purpose encompasees the following aims of the proposed study to better understand and explore each the following:

[1] Use of the terms 'education' and 'teaching' is often misguiding. For the purposes of this study, teaching implies a guided experience towards development and growth by means of education or learning.
[2] Recent literature expresses mathematics anxiety as a low (negative) motivational construct. After collaborating with my study leader, I decided to adopt a more positive stance and rather to refer to mathematics confidence.
[3] In this study the term 'reflection' is also adopted throughout (a facet of metacognition).

- A possible correlation between mathematics confidence and reflection on metacognitive knowledge during mathematics problem solving.
- The relationship between mathematics confidence and reflection on metacognitive regulation during mathematics problem solving.
- The reflective strategies that learners in the senior phase implement, if any.
- The signs of mathematics confidence, if any.

The study also attempts to detect a possible relationship between the models of reflection and mathematics confidence.

1.2.3 Research question

The primary question in this study is: What is the role of reflection and mathematics confidence during problem solving in senior phase Mathematics?

1.2.3.1 Secondary research questions

The study comprises the following secondary research questions:

- Question 1: Is there a correlation between mathematics confidence and reflection on metacognitive knowledge during problem solving?
- Question 2: Is there a correlation between mathematics confidence and reflection on metacognitive regulation during problem solving?
- Question 3: Which reflection strategies or skills do learners in senior phase mathematics implement, if any?
- Question 4: What does the mathematics confidence experienced by learners entail?

1.3 Definition and overview of keywords

In order to correctly interpret the title of this study, it is necessary to explain the following concepts:

1.3.1 Senior phase

The senior phase covers five learning outcomes of school mathematics for learners in Grades 7 to 9, as stipulated in DoE (2005:6). These learners are usually between the ages of 12 and 16.

1.3.2 Mathematics

The following table summarises various definitions of mathematics:

Table 1.1 Definitions of mathematics

Definition	Author
A helpful and significant discipline to study	Dungan & Thurlow (s.a.)
Mathematics can be considered a science	MSEB (1989:31)
An art of patterns and regularity	Van de Walle (2004:13)

Due to its dynamic and disciplined nature (Nieuwoudt, 2006:17), Mathematics is for the purpose of this study viewed as defined by the Department of Education (2003:9):

> *It is a distinctly human activity practised by all cultures. Mathematics is based on observing patterns; with rigorous logical thinking, this leads to theories of abstract relations.*
>
> *Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change.*

1.3.3 Mathematics problem solving

> *Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives* (DoE, 2003:9).

The following views of mathematical problem solving indicate how problem solving as a part of everyday experience is connected with mathematical phenomena.

Table 1.2 Mathematics problem solving definitions

Definition	Author
Transferring ideas learnt in one context to various and new situations	Van de Walle (2004:26)
Engagement in a task with an unknown solution	Cangelosi (2003:156)
Transference of skills and knowledge to unfamiliar contexts physically and socially	DoE (2003:1, 5); DoE (2010:8)
Requirement of thinking towards achieving a goal	Davidson & Sternberg (1998)

For the purpose of this study, mathematical problem solving will be viewed as the use of strategies and methods to effectively meet a specified outcome or goal accomplished by transferring skills and knowledge from one situation to another.

1.3.4 Mathematics anxiety and mathematics confidence

Test stress, low self-confidence, fear of failing, *and negative attitude towards learning mathematics* (Bassant, 1995:327) can be defined as low mathematics confidence or, in more colloquial terms, as math anxiety.

Table 1.3 Definitions of mathematics anxiety

Definition	Author
Fear or tension associated with mathematics tasks	Legg & Locker (2009:471)
Feelings of nervousness experienced when facing mathematical problems	Sheffield & Hunt (2007:19)
An avoidance of mathematics and an interference with conceptual and memory processes; a threat towards performance and participation in mathematics	Newstead (1999:54)
A combination of mathematics test anxiety and numerical anxiety in everyday life, with a correlation between performance in mathematics and gender	Neser (2009:59)

Mathematics confidence is viewed in this study as a psychological factor that influences performance and learning in Mathematics and it is symptomatically described as low (feelings of loss, failure and nervousness) or high (positive and motivated attitude) mathematics confidence (Maree, Prinsloo & Claassen, 1997a:7).

1.3.5 Metacognition

Metacognition is the process of monitoring, planning, controlling and evaluating (Flavell, 1976). Definitions of metacognition by various authors are compared in Table 1.4:

Table 1.4 Definitions for metacognition

Definition	Key components	Author
Actively applying cognitive skills to solve and understand problems	Cognitive skills, applied Problems	Gavalek & Raphael (1985)
Self-directing, goal-orientated information seeking, incorporating the application of strategies to optimise performance	Awareness of skills and their uses	Ertmer & Newby (1996:1)
During information processing in the cognitive transaction, the active monitoring, regulation and coordination of processes that are aimed towards achieving a goal	Information processing Active monitoring, regulation, coordination A goal	Flavell (1979:232)

The key concepts of these definitions can be combined to view metacognition as follows:

A cognitive series of processing that allows a learner to use previous knowledge and experiences in an organised manner by selecting, seeking and applying skills and strategies. One such overarching skill identified in this study is reflection.

1.3.6 Reflection

A distinction can be made between Reflection-on-action and Reflection-in-action. Reflection-on-action is the active process of linking past experiences with prior knowledge and skills as part of discovering meaning in successive experiences (Dewey, 1933:76). Reflection-in-action is defined by Bormotova (2010:13) as the growth of consciousness and management of learning when information changes while a learner is reflecting on past experiences (Ertmer & Newby, 1996:14). Reflection is defined as follows by the authors as indicated in Table 1.5:

Table 1.5 Definitions for reflection

Definition of reflection	Author
Reflection supplies the link between higher-order knowledge and regulation; a powerful link between thought and action	Ertmer & Newby (1996:14)
Reflection leads to changes in the future processing of knowledge and learning	Simons et al. (2006:294)
An active process of exploration and discovering	Boud, Keogh & Walker (1985:7)
Coming to some positive or negative conclusion	Bormotova (2010:14)

During this study, reflection will be viewed as a self-regulated process during which the learner recalls skills and experiences, and faces them with selective and thoughtful consideration.

1.4 Conceptual framework

The following sections introduce the conceptual framework of the study

1.4.1 Introduction

Mathematics has its origins in the necessity for societal, technological and cultural growth or leisure (Ebrahim, 2010:1). The advancement of concepts and theories over time to meet the needs of various cultures has left its imprint on nature, architecture, medicine, telecommunications and information technology. Mathematics has overcome centuries of problems and continues to solve everyday problems. In the development, progress and influences of mathematics, there are connections between the cognitive, connotative and affective psychological domains. The increasing demand to process and apply information in the South African society (which is characterised by increasing unemployment and immense demands on schools), remains something to recover from and to clarify before these cognitive and metacognitive challenges can be met (Maree & Crafford, 2005: 84). From a socio-constructivist perspective, developing, adapting and evolving more complex systems should be the aim and goal of mathematics education (Lesh & Sriraman, 2005). English (2007:123-125) furthermore lays down powerful ideas for developing Mathematics towards the 21st century.

Some of these ideas include the following:

- A social constructivist view of problem solving, planning, monitoring and communication
- Effective and creative reasoning skills
- Analysing and transforming complex data sets
- Applying and understanding school Mathematics
- Explaining, manipulating and forecasting complex systems through critical thinking and decision making

These ideas all form part of transformative schooling (DoE, 2010:25) and serve as a basis for problem solving.

1.4.2 Problem solving in Mathematics

Problem solving in Mathematics has featured in the policy documents of numerous organisations, both internationally (TIMSS, 2003; SACMEQ, 2009; PIRLS, 2009; Moloi & Strauss, 2005; NCTM, 1989) and nationally (DoE, 2010; DoE, 2010:3). Problem solving is emphasised as a method of inquiry and resolution (Fortunato, Hecht, Tittle & Alvarez, 1991:38). Generally two types of mathematical problems exist: routine problems and non-routine problems. The use and application of non-routine problems, unseen mathematical processes and principles are part of the scope of Mathematics education in South Africa (DoE, 2003:10).

A distinction can be made between a problem-solving approach, a problem-centred approach, and a problem-based approach to teaching. Maree and Crafford (2005:85) claims that an adequate study orientation in Mathematics (based on a problem-solving approach) is a significant factor in determining success in learning and teaching in schools. Factors relevant to a learner's study orientation include mathematics anxiety, attitude towards learning, study habits, problem-solving behaviour and study milieu.

Cangelosi (2003:98) expects learners first to identify the problem, articulate it and discuss the Mathematics before sufficient problem solving can occur as also claimed by Piaget (Piaget, 1979). Strengths and weaknesses may also be identified as this could be meaningful over time (Reynolds, 2006:17). Reflecting on thoughts and own capabilities

to solve problems can be seen as an important and cautious consideration of knowledge and skill in the problem solving process. Problems are solved mainly in three steps as claimed by Polya (1973) and Schoenfeld (1994) namely: planning, solving the problem and the control, evaluation or reflection on the solution.

Malmivuori (2006) illustrates two strategies applied within problem-solving processes: conceptually based strategies with a deeper comprehension of problems (metacognitive), and a demand-based strategy that is designed when processing steps with a given order (cognitive). This effect of reduced intrinsic cognitive load will lead to lowered performance in problem solving and therefore also in Mathematics.

1.4.2.1 Possible influences on mathematics performance in South Africa

A clear distinction must be made between mathematics performance factors in developed and developing countries (Howie, 2005:125). Howie (2005:123) explored data from the TIMSS-R South African study which revealed a relationship between contextual factors and performance in Mathematics. School level factors seem to be far less influential (Howie, 2005: 124, Reynolds, 2006:79) than classroom factors. According to Maree et al. (2005:85), South African learners perform inadequately due to a number of traditional approaches towards Mathematics teaching and learning. Maree (1997b:95) also classifies problems in study orientation as cognitive factors, external factors, internal and intra-psychological factors, and factors of a contextual nature in the subject content.

One psychological factor measured in the Study Orientation in Mathematics questionnaire (SOM) by Maree, Prinsloo and Claassen (1997b) is the level of mathematics confidence of Grade 7 to 12 learners in a South African context. Sherman and Wither (2003:138) also documented a case where a psychological factor causes an impairment of mathematics achievement, while a distillation of a study done by Wither (1998) established that low mathematics confidence causes underachievement in Mathematics. Due to insufficient evidence it could not be proved that underachievement results in low mathematics confidence. The present study did nevertheless indicate that a third factor, metacognition, could possibly cause both low mathematics confidence and underachievement in Mathematics (Sherman & Wither, 2003:149).

Ashcraft and Kirk (2001) associated mathematics confidence and underachievement with working memory capacity as the third factor, while Adams and Holcomb (1986) identified mathematics efficiency as the third factor. Hadfield and Maddux (1988) described a cognitive style independent from a cognitive field. The focus of this study will however be on mathematics confidence as the third (as well as a psychological) factor.

1.4.3 Development of mathematics confidence domains

Low mathematics confidence may be defined as an irrational fear of Mathematics. Panic, helplessness and mental paralysis are symptomatic when affected individuals are required to solve mathematics problems (Whitacre, 1998:12). Panic and low self-esteem during problem solving also relates to low mathematics confidence (Sekao, 2004:17). According to Strawderman (2010:1), Mathematics is probably the only subject that causes this adverse reaction as it is a clear and concentrated example of intelligent learning. Newstead (1999:2) states a number of possible causes for low mathematics confidence (anxiety), including teaching and learning environment issues, the characteristics of Mathematics, past experiences with Mathematics and previous failure. Both Newstead (1999:4) and McLeod (1993) argue that the root of mathematics confidence lies in classroom experiences (see also Newstead, 1999:7; McLeod, 1993; Tobias, 1978; Stodolsky, 1985).

Three domains exist within the context of an individual's mathematics confidence, namely social, intellectual and psychological. Strawderman (2010:1) mentions a natural overlap between boundaries where the social, intellectual and psychological domains coincide. Figure 1.1 illustrates these three mathematics confidence domains, and a brief discussion of their respective spheres of influence follows afterwards.

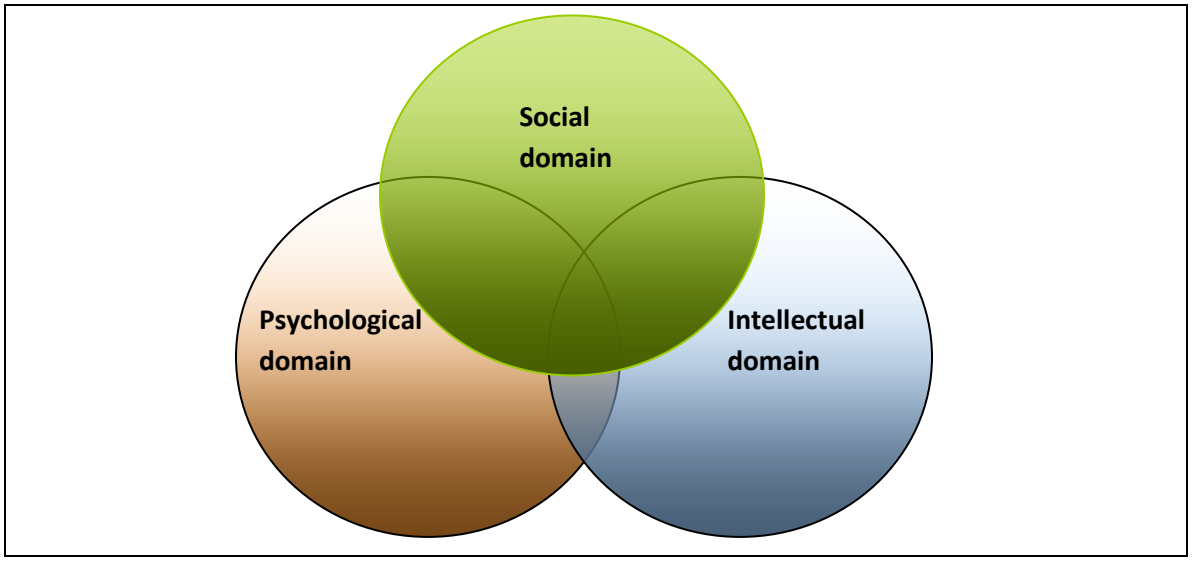


Figure 1.1 Classification and overlapping of the mathematics confidence domains

Source: Strawderman (2010:2); Bergh & Theron (2009:86)

- The **social domain** illustrates external factors outside the individual's control such as family, friends and teachers (Strawderman, 2010:2; Bergh & Theron, 2009: 86). The psychological field linked with this domain is the behaviour of individuals, and ranges between involving oneself in mathematics activities or classes and complete avoidance of such situations.
- The **intellectual domain** entails cognitive influences. This includes skills and knowledge of problem-solving procedures and strategies. Since personal performance is measured in this domain, it is associated with the field of personal achievement and related perceptions (Strawderman, 2010:3). Fluctuating between success and failure, the individual evaluates the acquirement and use of mathematics skills and concepts.
- The **psychological domain** extends further because of affective factors. Emotional history, familiar experiences and stimulus reactions are associated with the individual's feelings of confidence, anxiety or discomfort and pleasurable experiences.

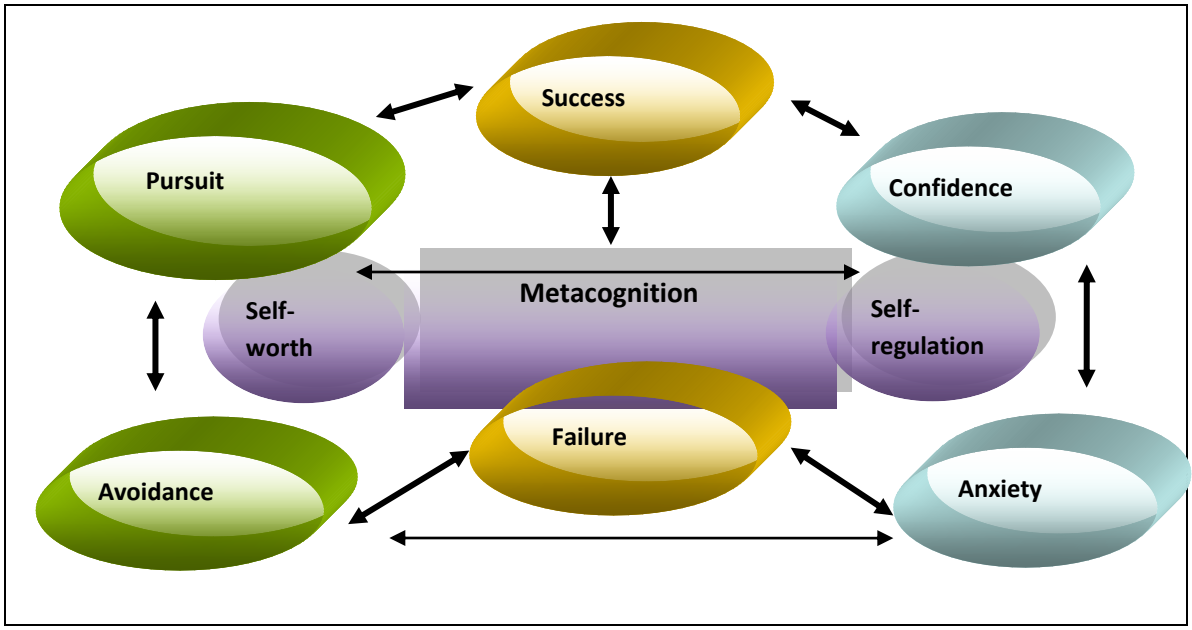


Figure 1.2 Overlap between the fields and domains of mathematics confidence

Source: Adapted from Strawderman (2010:2), Bergh & Theron (2009:86); Hadfield & Maddux (1988)

Figure 1.2 illustrates a mathematics confidence model connecting these domains, as adapted from Strawderman (2010:2). The three domains are now clearly connected and encircling another domain. Understanding or the lack of understanding is an element of higher-order cognitive development. It would appear that the third factor hypothesised by Sherman and Wither (2003:149) could exist here, interconnected with the three domains of mathematics confidence.

The phenomenon of learners reflecting on their own achievement with fear, low self-motivation, distraction and some mental disorganisation, will be regarded as an aspect of their higher-order reasoning and reflective questioning. Higher-order reasoning and reflection are elements of metacognition.

1.4.4 Metacognition

The higher-order cognitive domain (metacognition) is associated with problem solving in Mathematics (Jacobs, 2010:1; Van der Walt & Maree, 2007:224) and related to understanding Mathematics as part of the problem-solving process (Van der Walt et al., 2006:7). This conscious control over learning and problem-solving strategies is crucial in

Mathematics (Livingston, 1997:2). Ridley, Schutz, Glanz, Weinstein and Grabinger (1996) have shown that metacognition has two main components, encircling all its elements, nature and its meaning. The two components include knowledge about one's own cognition and the regulation of cognition.

In order to self-regulate cognitive activities, the learner must first be aware of his/her own cognition and must continuously reflect on decisions, strategies and their outcomes. Looking at the ideas set forward by English (2007) for Mathematics in the 21st century (English, 2007:123-125), it seems that all the ideas form part of the elements and nature of the metacognitive activity 'reflection'.

The construction of individual meaning improves when learners think about what they learn. Brookfield (1995) explains that reflective practices are essential for problem solving, learning and teaching. It seems as though these reflective practices all include the subcomponents of metacognition. The latter are self-regulatory, which means that individuals have to engage in self-reflection **(**Ridley et al., 1996).

Figure 1.3 below illustrates the components and subcomponents of metacognition.

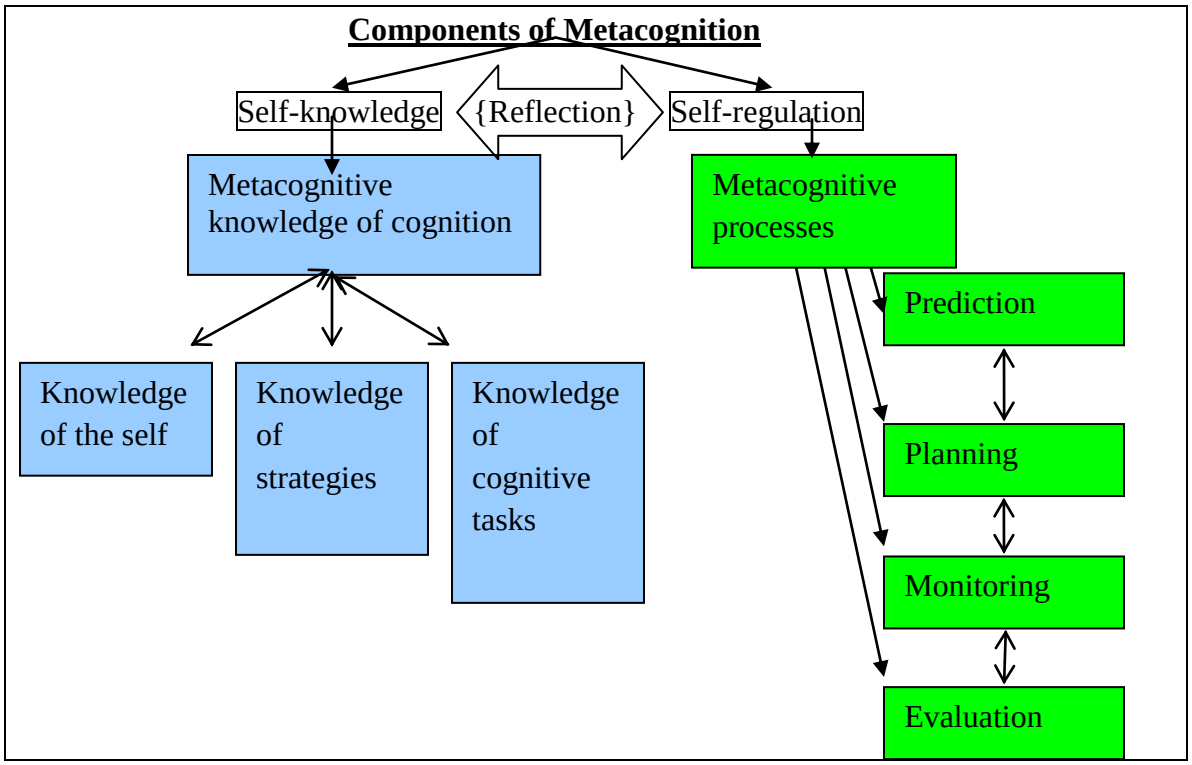


Figure 1.3 Components of metacognition

Source: Adapted from Ridley et al. (1996)

Metacognition can be divided into two basic components, each with its own subcomponents. These subcomponents can further be classified into four classes: metacognitive knowledge, metacognitive experiences, goals or tasks, and actions or strategies (Panaoura, Philippou & Christou, s.a.; Flavell, 1971; Rheeder, Rexhepi-Johansson & Wykes, 2010:49-50). Reflection constantly takes place between the subcomponents.

1.4.4.1 Reflection

Reflection, as a metaphor for various cognitive processes, is described by Sjuts (1999:40) as comparison and scrutinising, thinking, examining, specific direction, finding differences, detachment and delving deeper into cognition. Reflection could therefore be understood as an exploration of the self, one's own reasoning capacity. Kaune (2006:350) quotes Dubinsky (1991) when stating *that we somehow move into another dimension when we reflect on what we have done.*

Reflection as a high-level cognitive thinking process can help to solve mathematics problems and to understand Mathematics (Kaune, 2006:351). Differentiating between the *on* and *in* components of reflection, Kaune (2006) provide scaffolds by referring to "reflection-in-action" and "reflection-on-action" as well as a "reflective social discourse". Figure 1.4 illustrates the progress of the metacognitive ability (reflection) as a strategic approach to problem solving, adapted from NCREL (1995:3). The three phases of reflection can be described as reflection-before-action, reflection-on-action and reflection-after-action.

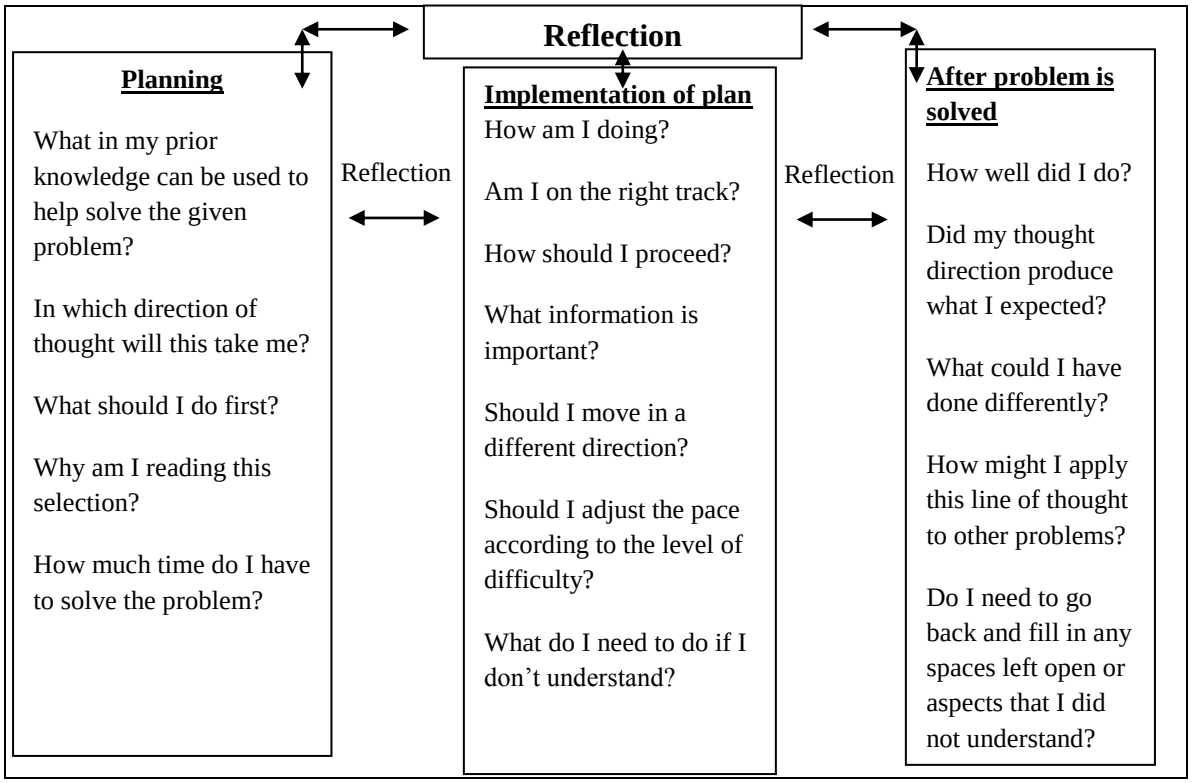


Figure 1.4 **Three phases of reflective thinking and metacognitive prompting during problem solving**

Source: *Adapted from NCREL (1995:3)*

The following cycle indicates the connection between prior knowledge and the acquirement of new knowledge during reflection activities:

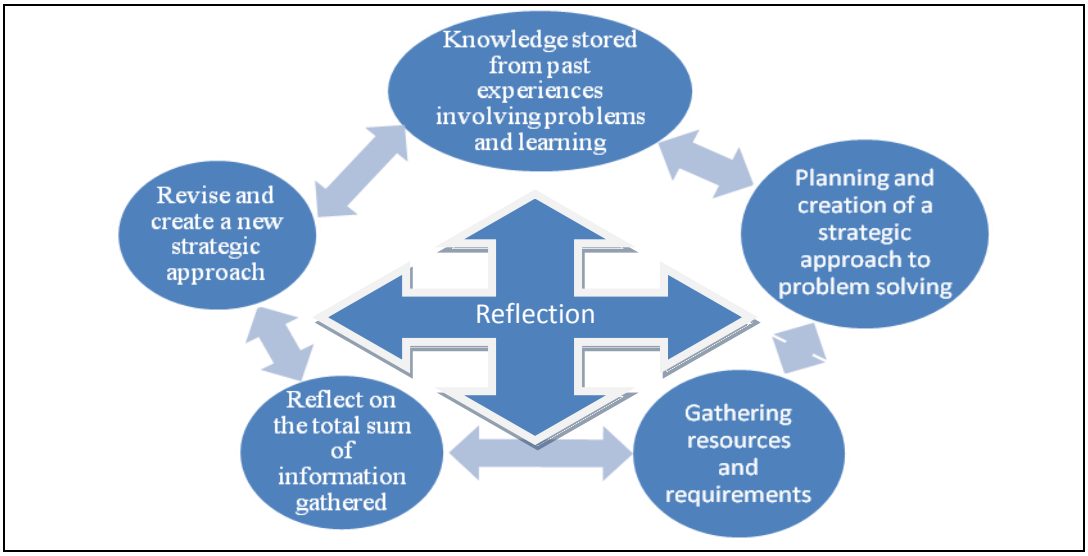


Figure 1.5 **Reflection cycle**

Source: *Adapted from Ertmer and Newby (1996:17)*

1.4.4.2 Synthesising performance in mathematics problem solving, mathematics confidence and reflection

A pre-post-test experimental study done among university students in Japan (Saito & Miwa, 2008) revealed that reflective activities supported by instructional design improved performance significantly. Lin and Lehman (1999) point out that learners' reflection on problem solving can support their achievement in problem solving. Evidence also shows that there is a link between mathematics confidence and metacognition within the verbal domain (Legg & Locker, 2009:474). Because it contributes to the reflective ability of learners, mathematics confidence could play a crucial role in problem solving during metacognitive processes (Goos, Brown & Makar, 2008:509). As obvious as this might be, the problem-solving process could also cause low mathematics confidence and it is hypothesised that this may, in turn, hinder reflective abilities. Figure 1.6 summarises the connection between mathematics confidence and reflection during problem solving.

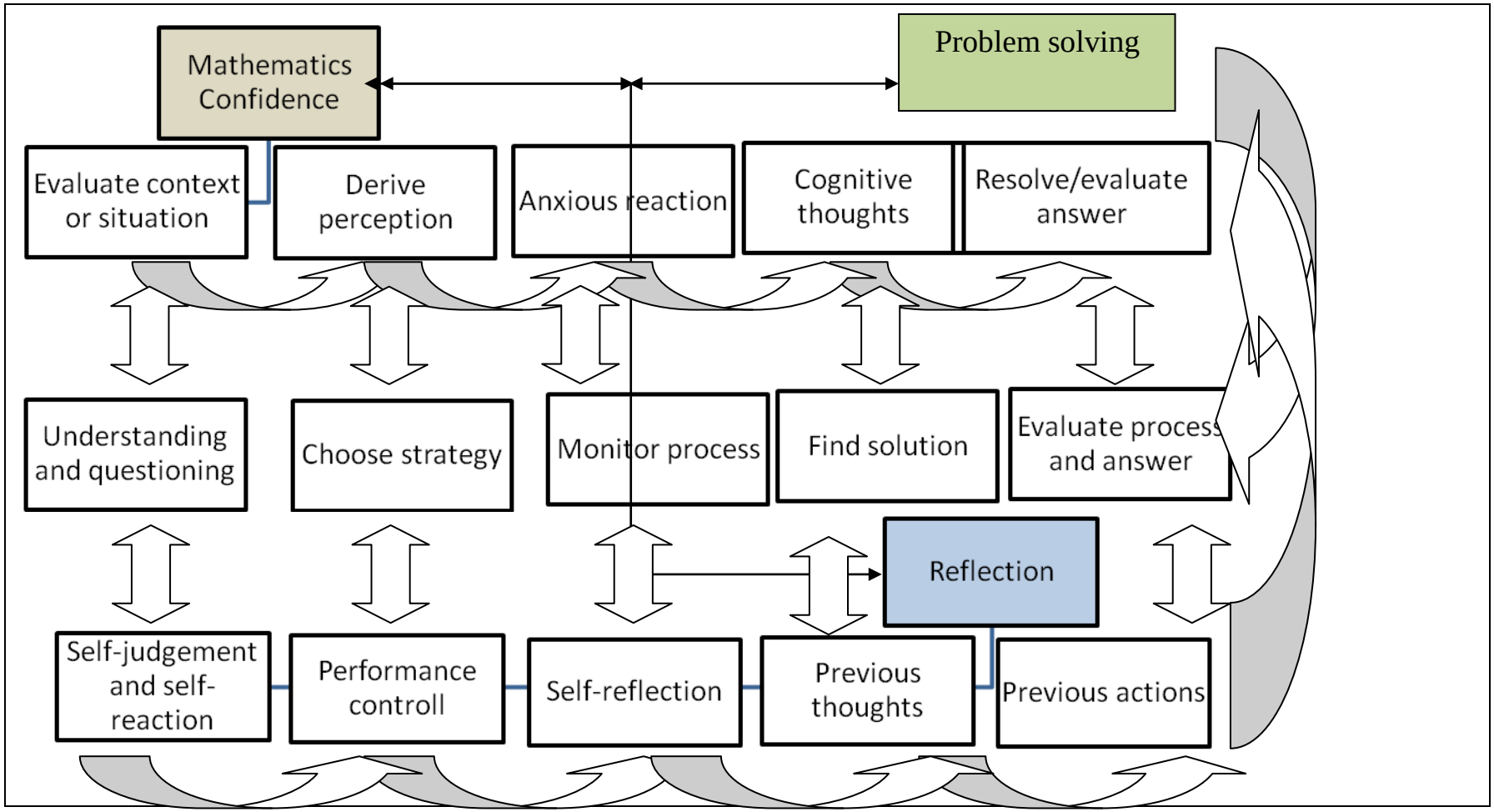


Figure 1.6 Components of mathematics confidence and reflection during the problem-solving process

Source: Adapted from Reynolds, 2006; Maree, Prinsloo & Claassen, 1997; Van der Walt, 2006

1.5 Research design

The following sections elucidate the approaches and methods for the mixed-method research design:

1.5.1 The paradigm complexity: aspiring to use a mixed-method approach

The integration of a mixed-method research design that contains both quantitative and qualitative approaches remains a problem as noted by Flick (quoted by Denzin, 2010:1). Triangulation (combining different research methods) is seen as a *lesser strategy for validating results* (Flick, 2002:227). Silverman refers to the perception that triangulation is bound by ground rules (Denzin, 2010:1). It seems as though a paradigm war exists along the trail of mixed-method designs in education research (Denzin, 2010:1; Morell & Tan, 2009:242-243; Creswell, 2009:102). With regard to the fields where mixed methods are most often adopted (psychology and education), Creswell (2010:103) suggests *new thinking about research designs*. In essence, the primary approach in this design should stipulate the combination of quantitative and qualitative approaches, rather than to rely on a single approach.

In support of the research question (problem) in this study, one approach therefore did not dominate another. Instead, the researcher weighed the quantitative (post-positivist) and qualitative (social constructivist) views (paradigms) in order to present reliable, fair and equal-valued representations (Creswell, 2010:104). Denzin (2010:2-4) calls for a paradigm expansion. A new paradigm dialogue anticipating a post-paradigm moment together with *transgressive* methodologies and a *colouring of epistemologies* should be initiated (Denzin, 2010:2). Instead of obtaining data directly from participants in a solitaire study, a mixed-research synthesis (Barrosso, Sandelowski & Voiles, 2006:29) was conducted. The data used was the findings of primary qualitative and quantitative studies in empirical research. The researcher utilised an exploratory convergent design and alternate between the quantitative and qualitative data as illustrated in Figure 1.7. Note how the quantitative study was integrated with the qualitative study.

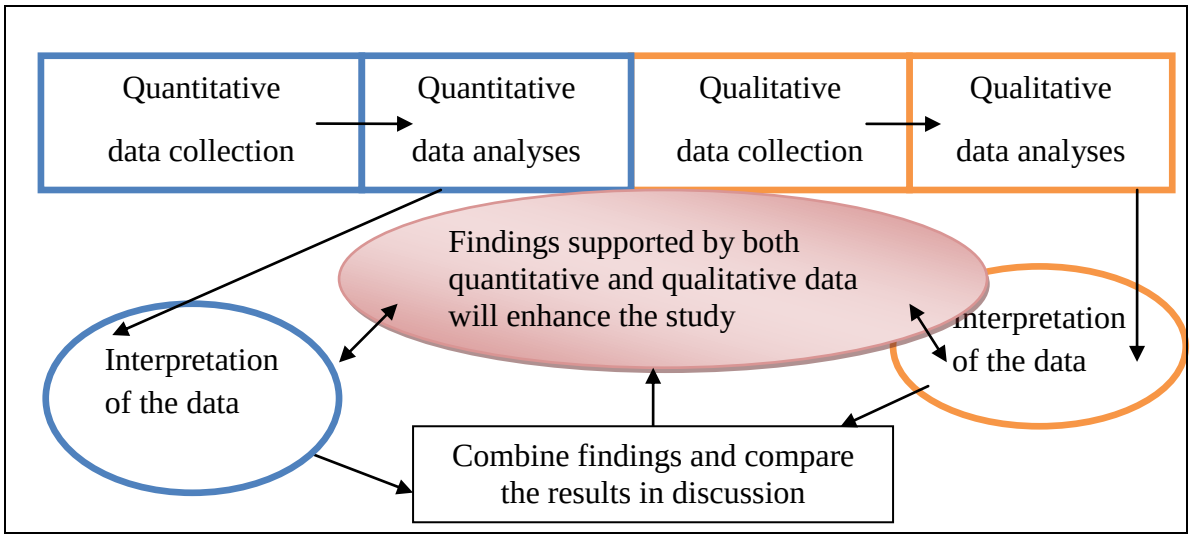


Figure 1.7 Exploratory convergent design

Source: Adapted from Creswell (2011:221)

The research proceeded as portrayed in Figure 1.7, which demonstrates that the study followed an exploratory convergent design (Creswell, 2011).

Table 1.6 Mixed-method data collection plan

Planned progression	Activity and technique administered
Quantitative data collecting	Administering the SOM (Study Orientation in Mathematics) and the RPSQ (Reflection and Problem-Solving Questionnaire).
Qualitative data collecting	Video recording of individual interview sessions.
Capturing of quantitative data on computer system for future analysis	Data captured according to the instructions of the Statistics Consultation Services of the North-West University. A statistics consultant will also examine the data by conducting a statistical analysis.
Observing, noting and interpreting qualitative data	Credibility, transferability, dependability and conformability will be advised and revised.
Making use of triangulation	"Deepened, complex, thoroughly partial understanding of the topic", merging of quantitative and qualitative data.

Source: Maree et al., (2010)

1.5.2 Validating the use of a mixed-method research approach

According to Morel and Tan (2009:242), mixed-method research designs can be implemented to support various constructs of validity aimed at strengthening the argument and providing a lens to understand the research question and measuring instrument (Morel & Tan, 2009:244). A problem mentioned by Morel and Tan (2009:242-243) in mixed-method research concerns the concept of validity, which must be clarified to merge quantitative and qualitative approaches. During the evaluation of mixed-method approaches (Taylor & Dionne, 2000:413-425; Morel & Tan, 2009:245-255), multiple data collection methods allow a more *complete* understanding of processes and performance of the concepts and variables in this study.

Methods to be implemented are chosen based on their complementary strengths and weaknesses, as this ensures equal opportunities for interpreting findings (Creswell, 2011:12). Data is collected independently and analysed subsequent to integration, explanation and interpretation (Morell & Tan, 2009:246).

1.5.3 Research premises: paradigmatic assumptions and perspectives

According to Cohen et al. (2001), knowledge can be viewed from either a positivist or interpretivist approach. In the current study, the researcher used a lens of epistemological assumptions, which gave rise to the use of scientific methods (quantitative study). Together with this empirical study, the researcher also adopted a more subjective participatory (qualitative) role by examining the research practice (as characterised by Cohen in Maree et al., 2010:38) and taking on the role of participant observer.

Opposing the interpretivist approach, the study also upheld a natural scientific approach which is the norm in human behavioural research (Welman, Kruger & Mitchell, 2009:19-25). The paradigmatic assumptions for this study therefore included the following:

- The continuing need for adequate performance in Mathematics
- Progress in education research from a social constructivist perspective
- Low mathematics confidence that hampers learners' mathematics performance

1.5.4 Delineated mode of inquiry

The mixed-method mode of enquiry ensures internal validity by visibly describing the combination of reflection and mathematics confidence during problem-solving activities (Maree et al., 2010:37). Zimmerman (2000) agrees with Maree, Prinsloo and Claassen (1997) as well as with Reynolds (2006: 34) in that the social environment can influence learners' self-regulative processes. Learners in the learning environment (classroom) standardise the context and content of learning due to their self-evaluation, social feedback and motivational factors (Zimmerman, 2000:619; Reynolds, 2006:36). Learners' self-worth can identify with their mathematics knowledge and strategies during problem-solving (Schunk, 2000:106; Reynolds, 2006:18).

1.6 Research methodology

The first aspect within the extent of the research methodology involves a brief overview of the relevant concepts in literature study.

1.6.1 Literature study

A library and Internet study was conducted to retrieve and collect suitable literature related to the following key aspects of the study:

> *Mathematics, mathematics confidence, mathematics anxiety, metacognition, reflection, problem solving, cognitive psychology, learning environment, self-regulative learning, information processing, metacognitive activities* and *monitoring.*

The first steps in the research involved data collection, which formed part of the quantitative phase of the study.

1.6.2 Quantitative phase of the study

During the quantitative phase of the study, an *ex post facto* design was adopted. In the quantitative phase, a correlation study, each individual was assessed according to the variables 'reflection', 'problem solving' and 'mathematics confidence'.

1.6.2.1 Pilot study for the RPSQ

The Reflection and Problem-Solving Questionnaire (RPSQ), a measuring instrument adapted by the researcher, had to be tried out before it could be administered to the actual sample (Welman, Kruger & Mitchell, 2009:148). A pilot study was undertaken to determine whether the quantitative questions and statements were well within the grasp of the sampling population, taking into account their socio-economic and cultural background. This added validity to the aim and focus of this study and reviewed learners' conceptual and literal understanding of the questions in the measuring instrument. The measuring instrument was reviewed by critical readers, my study promoter, teachers, learners, colleagues and experts at other universities before the pilot study was executed. During the pilot study, the following aspects of the questionnaire were evaluated:

- Ambiguous instructions, time allocated, significance of the different variables measured
- Identification of unclear statements or questions
- Any noticeable non-verbal behaviour

After the pilot study had been completed, the questionnaire was modified to correct possible flaws in the measurement procedure.

1.6.2.2 Sampling of respondents in the quantitative phase of the study

This study made use of non-probability sampling for reasons of convenience and economy as described by Welman, Kruger and Mitchell (2005:56). Schools in the North-West Province were divided into different clusters. These clusters consisted of schools that varied in respect of their medium of instruction (Afrikaans and English), as well as socio-economic and cultural context. Although they were not participants or respondents, the teachers in these schools came from various ethnic backgrounds. The learners who took part in this study were also from various ethnic backgrounds, and no group or part of the sample was favoured. The eventual research sample included 609 learners in Grades 8 and 9 from within a single cluster of schools (high schools).

1.6.2.3 Research instruments used

First, the Study Orientation in Mathematics (SOM) questionnaire developed by Maree et al. (1997b) for Grade 7 to 12 learners was administered to participants in the study seeing that it is standardised for South African learners. This questionnaire aims to assess the level of mathematics confidence in the cognitive and affective domains. Secondly, learners had to solve a non-routine mathematics problem. This task was administered to assess their mathematics achievement and was followed by their completion of a Reflection and Problem-Solving Questionnaire (RPSQ) for senior phase Mathematics. Learners had to assess their ability to apply metacognitive skills, reflection and awareness of reflective practices (refer to the reflection cycle in Figure 1.4) during problem solving. The RPSQ was subsequently adapted by the researcher by applying statements and questions from Schraw and Dennison (2001:1-3), Lucangeli and Cornoldi (1997:121-139), as well as Fortunato et al. (1991:38).

1.6.2.4 Quality assurance and verification: the quantitative phase of the study

First, a pilot study was conducted to check whether the statistical analysis of the quantitative data (obtained by means of the measuring instruments) validated the results of the study. The quantitative phase of the study was evaluated through analysis by the Statistical Consultation Services of the North-West University. This was done to ensure that the data collected and measured would be sufficient for answering the proposed research question and secondary research questions.

1.6.2.5 Construct validity of the RPSQ

In order to ensure that the instrument measured exactly what it was supposed to measure, the researcher examined the construct validity of the scores obtained. Since any given construct can also measure irrelevant constructs, more than one measure of the same construct was used (Welman, Kruger & Mitchell, 2009:142). This prevented respondents from faking their responses and prevented inconsistent answers.

1.6.2.6 Reliability of the RPSQ

The validity of the RPSQ would be enhanced and its reliability be assured when the findings obtained by means of it could be generalised, in other words, if it would reveal the same results whenever the instrument was administered, no matter who was administering it. The instrument was tested and retested to determine whether it yielded the same results. Cronbach alpha values were also determined.

1.6.2.7 Statistical procedures for analysing the quantitative data

The five-point scale SOM (Maree, 1997a) and the four-point scale RPSQ, as adapted by the researcher, were both implemented (Schraw & Dennison, 2001:1-3; Lucangeli & Cornoldi, 1997:121-139; Fortunato et al., 1991:38) and the data obtained was analysed quantitatively. The statistical procedures employed included the following:

- Exploratory and confirmatory factor analysis, as well as Cronbach alpha values of the data, to determine validity and reliability of the constructs in the measuring instruments. Descriptive statistics was also performed on the construct scores. These analyses included averages, medians, percentages and standard deviations. Inferential statistics (Chi-square analysis), variance analysis and Spearman rank correlations (rho) were also performed.

- Spearman rank correlations were used to compare constructs and investigate the relationship between mathematics confidence, reflection and achievement in problem solving.

The following diagram demonstrates the data analysis plan that was adopted for the quantitative phase of the mixed-method sequential research design:

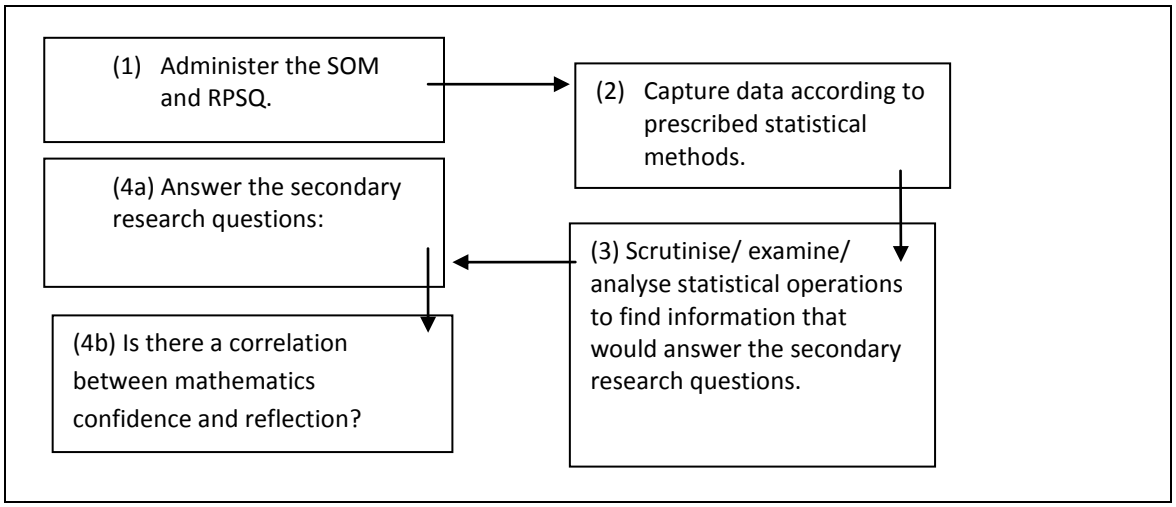


Figure 1.8 Quantitative data analysis plan

Source: Derived from Joubert and Creswell (in Maree et al., 2010:40)

Once the participants had been identified, the study progressed towards the qualitative phase of the research design.

1.6.3 Qualitative phase of the study

The research also included a qualitative element.

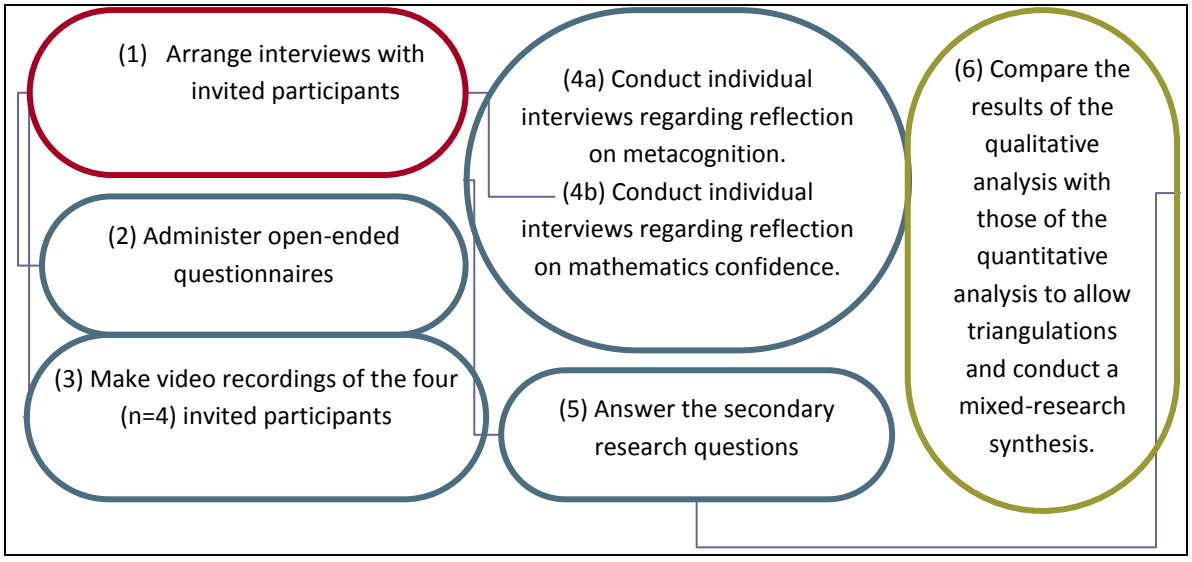


Figure 1.9 Qualitative data analysis plan

Source: Derived from Joubert and Creswell (in Maree et al., 2010:40)

1.6.3.1 Sampling of participants in the qualitative phase

A purposive sample of four (n = 4) participants was invited to take part in the qualitative study. The participants represented learners in Grades 8 and 9 who took Mathematics as one of their school subjects. Through purposive, convenience sampling (Maree et al., 2010:79) four participants were selected, which included two learners with high achievement in Mathematics and two learners with an average achievement. These participants constituted a desired group with regard to mathematics achievement, age, gender and race.

1.6.3.2 Role of the researcher

The researcher developed a collaborative partnership with the participants (Maree et al., 2010:41). This allowed him to delve deeper and explore the phenomena related to the variables mathematics confidence and reflection during problem-solving activities (refer to Figure 1.6). The following functions identified by Joubert (in Maree et al., 2010:41) describe the role that the researcher played in the current research:

- Administering open-ended questionnaires
- Preparing and conducting interviews (video recordings, flash cards, notes and qualitative questioning)
- Analysing a verbatim description of the data
- Interpretation and triangulation of the data

1.6.3.3 Data-collection procedures

Open-ended questionnaires, interviews (with all four participants), metacognitive statements (action cards) associated with awareness, evaluation and regulation, video recordings and focus groups served as resources in the current research design. Figure 1.8 reveals a mixed-method data collection plan as derived from Joubert (in Maree et al., 2010:35-36), while Figure 1.9 illustrates the data collection and analysis plan adopted as part of the research design.

1.6.3.4 Validity and trustworthiness of the qualitative data

In order to ensure that the results of the study would be repeatable even on different occasions or with various assessment techniques (Maree et al., 2010:46), the researcher facilitated quality assurance and the verification of data. This is not the case in the social-constructivist approach, since qualitative data could not yield the same results if the study is repeated (Maree et al., 2010:48). The reason for this 'inconsistency' is clarified by McMillan and Schumacher (in Maree et al., 2010:46-48) who reiterate that human nature is never a static entity. The information gathered from the interviews and video recordings were analysed and sorted to capture data about the examined variables. The collection of information and conclusions was validated by the study leader, the participants (during the pilot study) and specialists. Reports on the first interview session were also compared to reports on the second interview session to determine whether the statements supported the nature of the participants' metacognitive reflection. They did indeed validate the findings. The use of multiple statements in the action cards were also used to improve validity (Wilson, 2010:5).

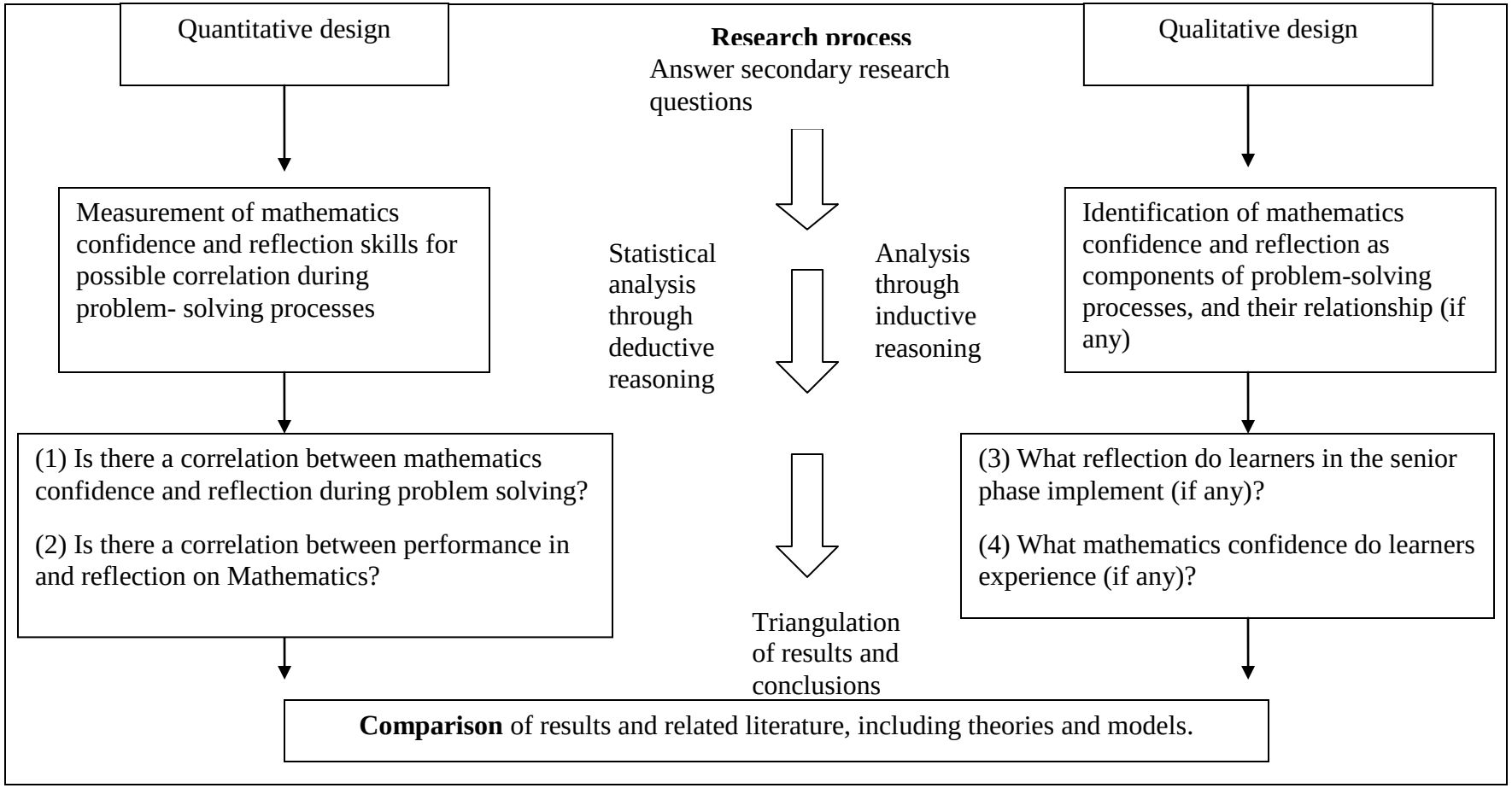


Figure 1.10 Overview of analysis plan for quantitative and qualitative designs – a mixed-method exploratory convergent design

Source: Derived from Joubert and Creswell (in Maree et al., 2010:40)

1.7 Ethical considerations

The learners were invited to take part in the study after gaining approval from their parents, guardians, school principals, the provincial Department of Education and the Ethical Committee of the North-West University. Such permission and consent were obtained before the learners were asked to participate in interviews and complete questionnaires.

1.8 Preliminary structure of the study

The dissertation consists of six chapters and the study has the following chapter layout:

Table 1.7 Layout of chapters

Chapter	Heading	Contents
Chapter 1	Orientation	Introduction, orientation, purpose and methods used in the proposed study.
Chapter 2	Literature review	A literature review covers the epistemological theories in the scholarly field of metacognitive reflection and mathematics confidence. This chapter clarifies the development of theories involving metacognition, reflection and mathematics confidence. A description of different models and theories was amalgamated, forming a theoretical framework from which conclusions were drawn for possible recommendation for further studies.
Chapter 3	The mixed-method research design	The research design, research methodologies and data analysis methods used during the quantitative and qualitative phases of the study.
Chapter 4	Analysis of the quantitative data	Presentation of the results obtained in the quantitative phase of the study.
Chapter 5	Analysis of the qualitative data	Presentation of the data gathered by means of the qualitative procedures.
Chapter 6	Summary, discussions and recommendations	Summary of the study, comments and discussions, conclusions and possible recommendations for future research.

Chapter 2

Literature review

2. Introduction to the literature review

This chapter reviews the literature related to mathematics confidence, reflection and problem solving. It begins with an introduction to mathematics and is followed by educational changes and concerns in South Africa. It examines the metacognitive activity 'reflection' and its various facets along with affective issues in Mathematics. Then, differentiating between past and current research, the focus is moved to the way in which mathematics confidence and reflective thinking relate to learners' level of achievement and performance in Mathematics problem-solving processes. According to Thijsse (2002:34) Mathematics is an *emotionally* charged subject, evoke feelings of dislike, fear and failure. Mathematics involves cognitive and affective factors that form part of the epistemological assumptions regarding mathematical learning (Thijsse, 2002:7) and these will be discussed in the following paragraph. The chapter concludes with a discussion that illustrates the relationship between reflection and mathematics confidence during problem-solving processes, thus outlining the conceptual framework of the study.

2.1 Mathematics, its nature and structure

Mathematics can be seen as a combination of calculation skills and reasoning (Hannula, Maijala & Pehkonen, 2004:17) and successful performance will therefore depend on an individual's mathematical understanding. According to Chamoso, Cáceres and Azcárate (2012:154), Mathematics is a process, fixed to a certain person, a topic, an environment or an idea. It is the collaborative study of quantity, structure, space and change, which allows us to look for and establish numerical and geometrical patterns. This *distinctly human activity* enables us to understand and represent the physical and social world across all cultures (DoE, 2011:6).

Mathematics emerged as an absolute necessity, not only for societal, technological and cultural growth, but also for leisure (Ebrahim, 2010:1). This compulsion led to the advancement of concepts and theories to meet the needs of various cultures across time. With its imprint in nature, architecture, medicine, telecommunications and information technology, the use of

Mathematics has overcome centuries of problems and continues to fulfil the needs of problem solvers to unravel everyday problems. Although Mathematics has changed over time, its progress and influence show interwoven connections between the cognitive (Maree, 1997), conative (Van der Walt, 2008; McCutcheon, 2008) and affective (Hannula, Maijala & Pehkonen, 2004) psychological domains. In the South African society – a society characterised by increasing unemployment and immense demands on schools – the increasing demand for processing and applying information still awaits resolution, recovery and substance from these cognitive and metacognitive challenges (Maree & Crafford, 2005:84). From a socio-constructivist perspective, developing, adapting and evolving more complex systems should be the aim and goal of mathematics education (Lesh & Sriraman, 2005). Epistemological assumptions are discussed in the next paragraph.

2.1.1 Epistemological assumptions regarding mathematics learning

From a constructivist perspective, learning can be viewed as the active process within and influenced by the learner (Yager, 1991:53). Mathematical learning is therefore an interactive consequence of the encountered information and how the learner processes it, based on perceived notions and existing personal knowledge (Yager, 1991:53). According to DoE (2003:3), competence in mathematics education is aimed at integrating practical, foundational and reflective skills.

While altering the paradigms in learning, mathematics education was *turned upside down* with the shift being towards instructing, administering and applying metacognitive activity-based learning in schools as claimed by Yager (1991:53) and Leaf (2005:12-18). The change and reform in education and education paradigms are illustrated in Figure 2.1.

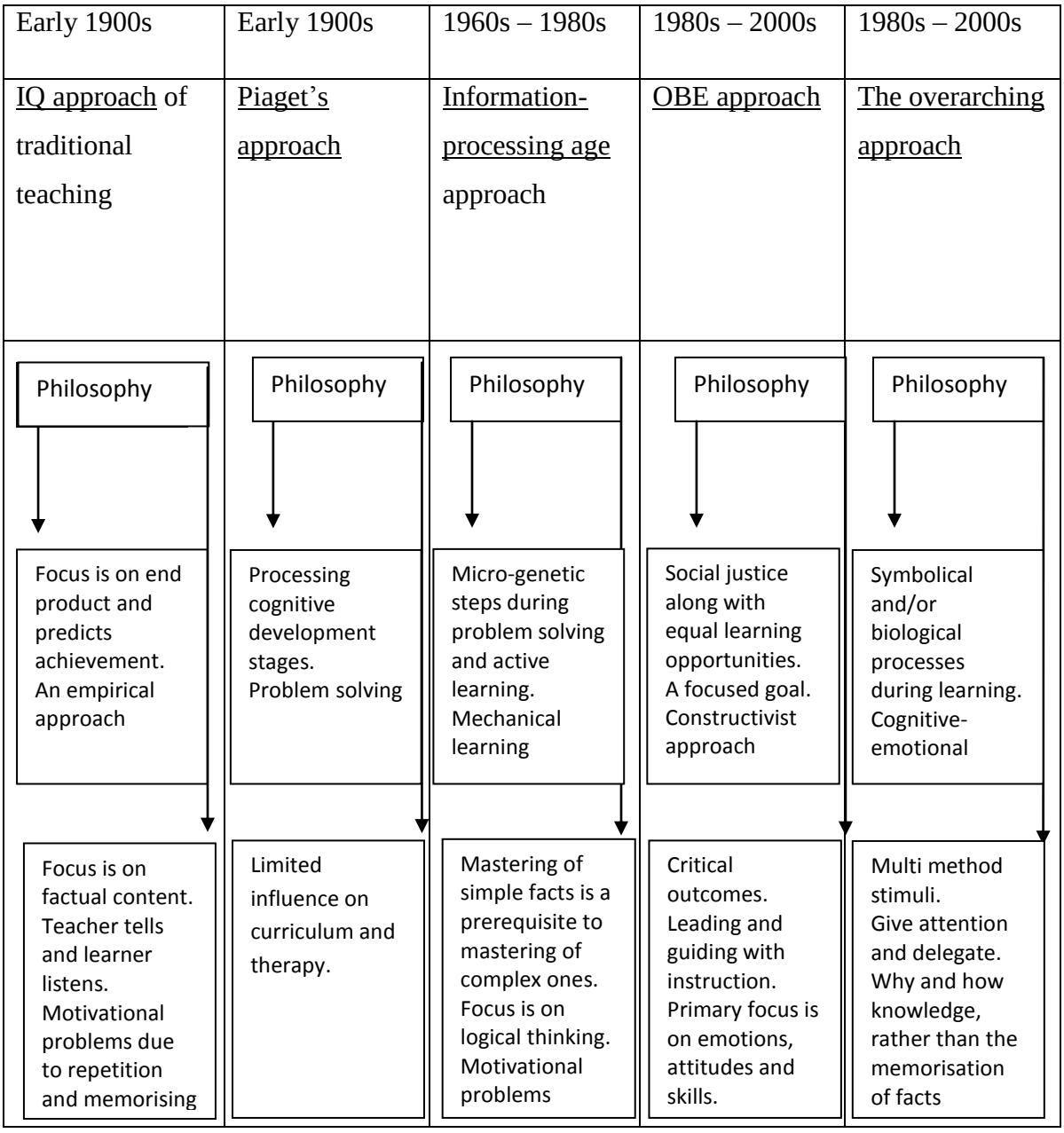


Figure 2.1 The paradigm shift in learning theories over the past century

Source: Adapted from Leaf (2005:12-18)

In Figure 2.1, Leaf (2005:4) states that the intelligence quotient (IQ) is one of the greatest paradigm dilemmas. This approach was designed in the early twentieth century by F. Galton and labelled too many learners as either *slow* or *clever*. IQ tests did assess logical, mathematical and language preference and dominance in learners, but left little or no room for other ways of thinking or mental aptitude (Leaf, 2005:5). The IQ approach was ousted by Piaget's approach,

named after its founder, Jean Piaget. Focusing on cognitive development, Piaget suggested timed stages or learning phases in a child's cognitive development as a prerequisite to the learning process. He claimed that if a stage was overseen, learning would not take place. A third paradigm, based on the Information-processing age, divided problem solving into three phases: input, coded storing and output. Designed in an era when technological advances and computers entered schools and the school curriculum, information processing was seen as comparing the learner with a microchip. Thus, retrieving and storing data and information was seen as a method where practising and *learning* was considered the focus of learning. This learning took place in a hierarchical order, and one phase had to be mastered before the learner could proceed with a more difficult task. The fourth learning theory, Outcomes-Based Education (OBE), was implemented after the 1994 national democratic elections in South Africa. After 1997, the local school system underwent drastic change away from the so-called *apartheid* era. In terms of the Revised National Curriculum Statement (DoE, 2003) the curriculum was now based on development of the learners' full potential in a democratic South Africa. The creation of lifelong learners has been the focus of this paradigm.

According to Dewey (1933) constructivism, as a paradigm, is primarily cognitive in nature and is based on learning and knowing as a result of doing mathematics. Furthermore, the transformative process of learning creates new knowledge structures, ideas and thoughts that ultimatthe proficiency of a comunity.

The unsuccessful transformation of mathematics education in South Africa reiterated the need for a theory to challenge some of the shortcomings of the paradigms mentioned so far. The overarching approach proposed by Leaf (2005:12) focused on learning dynamics – in other words, *what* makes learning possible. This paradigm utilised emotions, experiences, backgrounds and cultural aspects to facilitate learning and problem solving (Leaf, 2005:12-15). The above-mentioned aspects were also known to associate with performance in mathematics problem solving (Maree, Prinsloo & Claassen, 1997a; Leaf, 2005:12-15).

2.1.2 Factors associated with performance in Mathematics

Large-scale international studies that focus on school mathematics compare countries in terms of learners' cognitive performance over time (TIMSS, 2003; PISA, 2003). A clear distinction must

be made between mathematics performance factors in developed and developing countries (Howie, 2005:125). Howie (2005:123) explored data from the TIMSS-R South African study, which revealed a relationship between contextual factors and performance in Mathematics and showed that school level factors were far less influential (Howie, 2005:124, Reynolds, 2006:79). According to Maree et al. (2004:85), South African learners performed inadequately due to the authorities' adherence to a number of traditional approaches towards mathematics teaching and learning. Maree (1997b:95) also classified problems in study orientation as cognitive factors, external factors, internal and intra-psychological factors, and facilitation of subject content.

One psychological factor in the Study Orientation in Mathematics (SOM) questionnaire of Maree, Prinsloo and Claassen (1997b) measures the level of mathematics confidence of Grade 7 to 12 learners in a South African context. Sherman and Wither (2003:138) documented a case where a psychological factor, mathematics anxiety, causes an impairment of mathematics achievement. A pre-distillation of a study done by Wither (1998) concluded that low mathematics confidence causes underachievement in Grade 8 learners' mathematics. Due to insufficient evidence, it could not be proved that underachievement results in low mathematics confidence. The study nevertheless indicated that a possible third factor (metacognition) could cause both low mathematics confidence and underachievement in mathematics (Sherman & Wither, 2003:149).

Academic underachievement[4] on the one hand and performance in Mathematics on the other hand are the result of a number of variables, as identified by Maree, Prinsloo and Claassen (1997), as well as Lesh and Zawojewski (2007). These variables include beliefs and attitudes demonstrated by the learner, as well as environmental factors and factors that emerge during the process of instruction.

2.1.2.1 Associated factors demonstrated by the learner

Affective issues revolve around an individual's environment within different systems and how that individual matures in and interacts with the systems (Beilock, 2008:339). Depending on the learner's position in these systems, he/she has a positive or negative attitude towards

[4] Note that underachievement refers to a negative result of mathematics problem-solving confidence. Some factors associated with underachievement will be attended to in the discussion. These factors are in contrast to those that may influence performance. Thus performance is seen as the opposite of underachievement.

Mathematics (Maree, Prinsloo & Claassen, 1997a). Beliefs about one's own capabilities and the perception that success cannot be linked to effort and hard work are seen as affective factors in problem solving (Dossel, 1993:6; Thijsse, 2002:18). Distrust of one's own intuition, not knowing how to correct mistakes and a lack of personal effort are regarded as factors that facilitate mathematics anxiety (Thijsse, 2002:36).

2.1.2.2 Associated environmental factors

Timed testing environments such as oral exams or test situations where answers must be given quickly and/or verbally are seen as environmental factors that cause underachievement in mathematics. Public contexts where the learner has to express mathematical thought in front of an audience or peers may also be seen as an environmental factor that limits performance. A systematic evaluation report (DoE, 2006:80) expresses the importance of knowing what complex dynamics the Department of Education has to face when it comes to environmental factors that are accountable for underachievement in mathematics. Environmental factors associated with learner achievement include social economic status, school safety, education resources available at home and at school, assessment practices and teacher knowledge (DoE, 2006:81-82).

2.1.2.3 Associated factors during the process of instruction

Knowledge about study methods, the implementation of different strategies and domain specific knowledge are seen as factors that influence performance in Mathematics. It seems as though performance is measured according to the learner's ability to apply algorithms dictated by a figure of higher authority such as a parent or teacher (Thijsse, 2002:35). Thijsse (2002:19) agrees with Dossel (1993:6) and Maree (1997) that the teacher's attention to the right or wrong dichotomy stresses the fact that mathematics education can also be associated with performance. A brief discussion on mathematics problem solving follows below.

2.2 Mathematics problem solving

A mathematics problem can be defined as a mathematically based task that indicates realistic contexts in which the learner creates a model for solving the problem under various circumstances (Chalmers, 2009:3). Making decisions within these contexts is only one of the elementary concepts of human behaviour. In a technology-based information age, computation,

conceptualisation and communication are basic challenges South Africans have to face (Maree, Prinsloo & Claassen, 1997; Lesh & Zawojewski, 2007). These changing challenges have led to different mathematical thinking and approaches to problem solving (English, Lesh & Fennewald, 2008:1). Developing problem-solving abilities for academic success should stretch beyond school level. According to Kleitman and Stankov (2003:2), managing uncertainty in one's understanding is essential in mathematical problem solving and research into mathematical problem solving has become increasingly more complex than in previous years (Lester & Kehle, 2003:510). Problem solving therefore continues to feature in the policy documents of various organisations, both nationally (DoE, 2010:3) and internationally (TIMSS, 2003; SACMEQ, 2009; PIRLS, 2009; Moloi & Strauss, 2005; NCTM, 1989). According to Lesh and Zawojewski (2007:764) *… the pendulum of curriculum change again swings back towards an emphasis on problem solving…*Therefore, Realistic Mathematics Education (RME) envisions mathematics as a combination of learning and problem solving within formal, informal and other social contexts (Lesh & Zawojewski, 2007).

Problem solving is emphasised as a method that involves inquiry and decision making (Fortunato et al., 1991:38). In general, two types of mathematical problems exist: routine problems and non-routine problems. The use and application of non-routine problems, unseen mathematical processes and principles are part of the scope of mathematics education in South Africa (DoE, 2003:10). Through reflection, keeping track *of* and *on* the process of information seeking and decision making, mathematics problem solving is linked to the content and context of the problem situation (Lesh & Zawojewski, 2007:764). It seems as though mathematics concept development and the development of problem- solving abilities should be part of mathematics education; furthermore, beliefs, feelings or other affective factors should be taken into account. In the next paragraph a discussion follows regarding research done on mathematics problem-solving behaviour.

2.2.1 Previous research on mathematics problem-solving behaviour

Numerous studies on mathematics problem-solving behaviour share complexities as far as research on individual differences within mathematical aptitude is concerned. Studies on learners' performance in Mathematics and their mathematics behaviour vary in their approaches to perform, and research has been conducted on mathematics problem solving since the 1930s

(Dewey, 1933; Piaget, 1970; Flavell; 1976; Schoenfeld, 1992; Lester & Kehle, 2003; Lesh & Zawojewski, 2007:764). Good problem solvers were generally compared to poor problem solvers (Lester & Kehle, 2003:507) while Schoenfeld (1992) suggested that the former not only know more mathematics, but also know mathematics *differently* (Lesh & Zawojewski, 2007:767). English, Lesh and Fennewald (2008:1) voiced their concern about the decline in problem-solving research and attributed it to a number of *cyclic* trends in education policies, limited research on conceptual development and the changing nature of mathematical problem solving. Though the nature and development of mathematics problems are widely researched (Lesh & Zawojewski, 2007:768) – especially with the focus on how learners see and approach Mathematics and mathematical problems – Polya-style (heuristic) problems involve strategies such as picture drawing, working backwards, looking for a similar problem or identifying necessary information (English, Lesh & Fennewald, 2008:1; Lesh & Zawojewski, 2007:768). Confirming the use of these strategies, Zimmerman (2000:8-10) and Wilson (2001) describe dimensions for academic self-regulation by involving conceptually based questioning by means of a technique called prompting. Examples of these prompts are questions starting with why, how, what, when and where, in order to provide so-called scaffolding (guidance) for information processing and decision making. These prompts are also known as metacognitive strategies that aim to solve problems as a form of reflective thinking in the course of the problem-solving process.

2.2.2 Aspects of the working memory and its connection to mathematics problem-solving behaviour

Problem solving is initially seen as an approach to achieving a goal (Goldstein, 2008:404) and involve search and adapt strategies. The working memory is essential for storing information about mathematical problems and problem-solving processes, such as knowledge and strategies (Sheffield & Hunt, 2007:2). Cognitive effects like anxiety disrupt processing in the working memory system and hence underachievement will follow (Ashcraft, Hopko & Gute, 1998:343; Ashcraft, 2002:1). Intrusive thoughts such as worrying overburden the system, interfere with the system's speed and accuracy (Van der Walt, 2008:86) and reduce confidence. The working memory consists of three components: the psychological articulatory loop, visual-spatial sketchpad and a central executive (Ashcraft, Hopko & Gute, 1998:344).

The learner, either an expert or a novice problem solver, is researched on his/her ideas, strategies, representations or habits in mathematical contexts (Ertmer & Newby, 1996). Expert learners are found to be well-organised individuals who have integrated networks of knowledge so as to succeed in mathematics problem situations (Lesh & Zawojewski, 2007:767; Zimmerman, 1994). Clearly, learners' problem-solving personalities affect their achievement. According to Thijsse (2002:33), learners who trust their intuition and perceive intuition as insight into a rational mind, rather than emotional and irrational feelings, are more confident. Attributes such as anxiety and confidence are included in reflective processes either cognitively or metacognitively, and these processes will be discussed in the next paragraph.

2.3 Cognitive and metacognitive factors

Although cognitive and metacognitive processes are compared in the available literature, Lesh and Zawojewksi (2007:778) agree with Livingston (1997:2) that mathematics concepts (cognitive) and higher-order thinking (metacognitive) should be studied correspondingly and interactively. Identifying individual trends, behaviour patterns and feelings could relate to mathematics problem-solving success (Lesh & Zawojewksi, 2007:778). According to Livingston (1997:2), definitions about knowledge and the regulation of knowledge (commonly known to define metacognition) concern cognition and metacognition. Livingston (1997:2) claims that a clear distinction can be made when referring to Flavell's (1979) notable quote: *thinking about thinking*. English, Lesh and Fennewald (2008:1) define metacognition as processes operating the lower-order knowledge or abilities. Cognitive strategies are known to help with achieving a goal (e.g. understanding fractions), whereas metacognitive strategies are focused more on ensuring that the goal can be reached (e.g. deciding what available strategies to select for different kinds of fractions tasks) (Livingston, 1997:2). Overlapping of cognitive and metacognitive strategies is possible as in the example of questioning, which can be seen as a cognitive or metacognitive strategy, depending on the purpose of the question. A question could be asked to obtain knowledge (cognitive) or to monitor what is already known (metacognitive). Comparative discussions on cognition and metacognition will follow.

2.3.1 Cognitive processes during mathematics problem solving

Newstead (1999:25) describes an individual's cognitive levels as being either convergent (knowledge of information) or divergent (explaining, justifying and reasoning with knowledge). According to McCutcheon (2008:507-508), the cognitive domain thinks about awareness of own knowledge, as well as the strengths and weaknesses of understanding mathematical concepts. Cognition refers to the processes involved with knowing, understanding, storing and retrieving. Some cognitive factors like academic self-concept, attitudes towards Mathematics in general, confidence in learning Mathematics, locus of control and problem-solving abilities describe the thinking process by means of associating, reasoning and evaluating own knowledge (Maree, 1997; McCutcheon, 2008:507). It appears that thinking processes in the cognitive domain make use of metacognitive strategies, which are discussed in the next paragraph.

2.3.2 Metacognition

Knowledge can be seen as metacognitive knowledge if it utilises planning as part of a goal-driven approach. Before, during and after the cognitive task, metacognition acts upon cognition and regulates cognition's product, performance (Jacobs, 2010:1). Different literature regarding metacognition confirms difficulty defining this higher (meta-) approach to cognition and cognitive tasks (Bormotova, 2010). After over thirty years of investigating and exploring this realm of the mind, some researchers still quote Flavell (1979) by stating that metacognition is simply *thinking about thinking*[5] (Flavell, 1979; Livingston, 1997:1; Bormotova, 2010; Legg & Locker, 2009:471; Lai, 2011:33). It seems that metacognition can be viewed as an individual's own facilitation of knowledge towards and about oneself. Metaphorically speaking, Jacobs (2010:2) states that metacognition allows us to change gears in our thinking. Figure 2.2 illustrates the connection between the components of metacognition.

[5] Although this quote may be regarded as outdated, the complex nature of metacognition cannot be labelled in a more describing yet simplistically skilful manner.

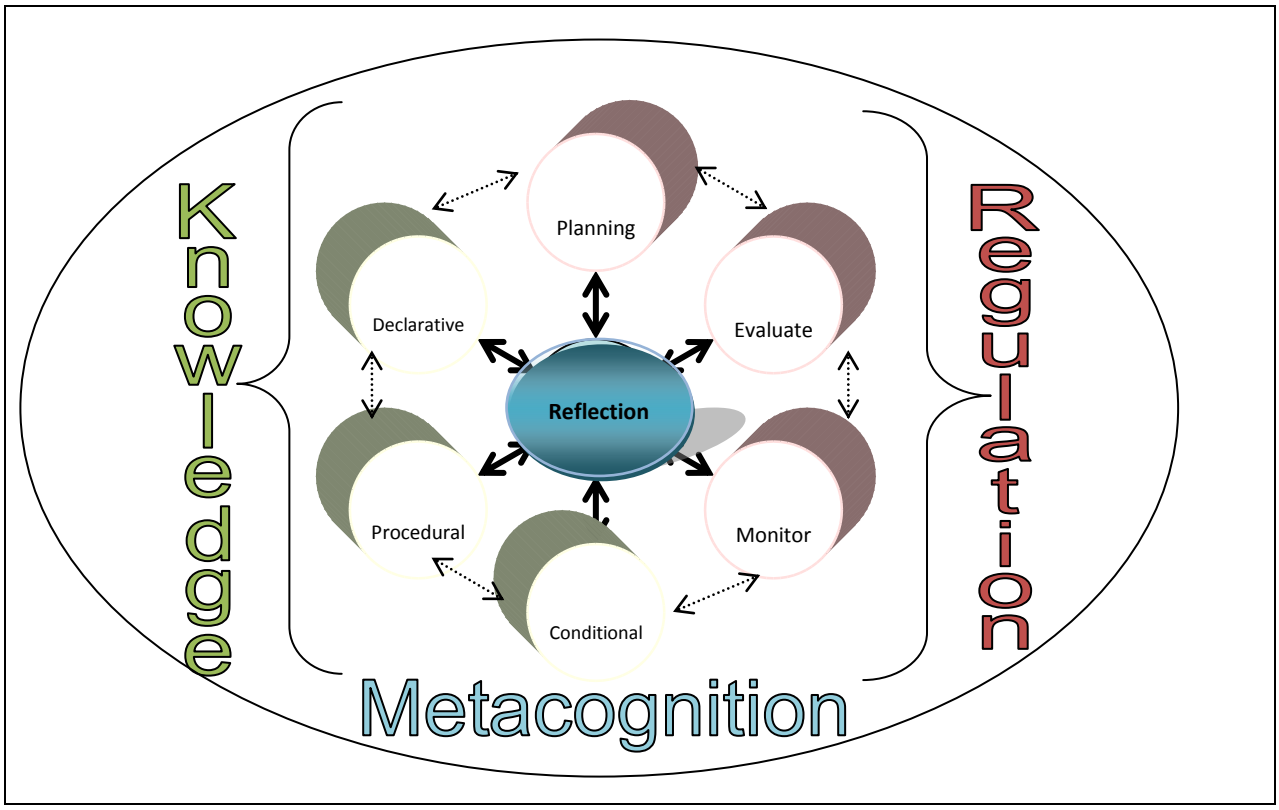


Figure 2.2 **Components of metacognition**

Source: *Adapted from Jacobs (2010); Van der Walt (2006); Ertmer and Newby (1996)*

To illustrate the overflowing connection between the components of metacognition, Figure 2.2 is now discussed in more detail. Metacognition is illustrated in the figure as consisting of two dimensions with integrating roles about the sub-dimensions. Rheeder, Rexhepi-Johannson and Wykes (2009:49) as well as Schraw and Moshman (1995) refer to knowledge about cognition and the regulation of cognition as the two dimensions of metacognition. Both dimensions are now discussed along with sub-dimensions.

2.3.2.1 Knowledge about cognition: declarative, procedural and conditional

Knowledge about one's own cognition includes knowledge about the functionality or management of the mind and insight. There are three sub-dimensions of metacognitive knowledge: declarative, procedural and conditional (Schraw, Grippen & Hartley, 2006). Declarative knowledge indicates knowledge about the factors that can influence one's learning, for example knowing that there is not enough light in a room to study sufficiently. Procedural

knowledge refers to strategies on how to perform tasks. Skills are used automatically and the strategies are selected efficiently. Conditional knowledge includes knowing when and why certain strategies are used. Selecting appropriate strategies will also enable the learner to think about previous experiences with similar problems and different approaches can be considered to solve one problem.

2.3.2.2 Self-regulation: monitoring, planning and evaluation

The process of regulation refers to controlling cognition or managing the thinking process (Legg & Locker, 2009:473). This can be done by merging the processes of monitoring, planning and evaluation. Planning procedures involve predictions and expectations that are known to occur before a certain problem-solving behaviour begins (Legg & Locker, 2009:473). Monitoring, a sub-dimension of cognitive regulation, refers to the awareness and understanding of personal achievement in mathematics problem solving. The awareness involves constantly checking if the correct understanding is in place for what the problem asks. Correcting any mistakes made so far and paraphrasing the questions are some of the strategies employed as part of the regulatory process. Evaluation of the completed task refers to *making sure* the question is answered and the problem is solved, and appraising the performance of the cognitive components. However, it appears that metacognition does not consist of reflection as a component or as a sub-dimension; instead, it seems as though reflection is at the heart, or foundation of all metacognitive components. It seems that metacognition as a skill relies on the reflective skills of the individual, thus completing the three stages: before, during and after the task reflection. Therefore, in this study, reflection will be seen as parallel to metacognition. Reflection in a sense kindles metacognition.

Although metacognition is generally seen as a second or higher form of thinking, cognition and metacognition must co-exist (Smith & Beran, 2008:679). The Polya-style heuristics on problem-solving strategies is noted by Lesh and Zawojewski (2007:368) as an *after-the-fact-of-past-activities* process. This review process (that occurs between interpreting the problem and selecting appropriate strategies that may or may not have worked in the past) is linked with experiences (negative or positive) that provide a framework for reflective thinking. It shows that reflection is the connection between metacognition and cognition.

2.3.3 Reflection as a facet of metacognition

Reflection as defined by Glahn, Specht and Koper (2009:95) is an active reasoning process that confirms experiences in problem solving and involves social interaction. Reflection can be seen as a transformational process through which our experiences affect our objective way of thinking (Garcia, Sanchez & Escudero, 2009:1). According to Van der Walt (2008:79), all metacognitive knowledge is actively and collectively unified by reflection. It appears that reflection is a key component in the regulation of metacognitive knowledge and self-regulation. There are three stages of reflection – before, during and after action in the problem-solving process (see Figure 2.2). After these three stages, the individual (as a reflective practitioner) may experience moments where synchronous reflection facilitates the so- called *aha* moment, which is a moment of acquired insight (Schon, 2010:3). Schon (2010:3) refers to metacognitive reflection and argues that a distinction can be made between cognitive reflection, metacognitive reflection and self-reflection.

2.3.3.1 Development of reflective thinking

Thinking about mathematics problems and reflecting on them is essential for interpreting the given problem. Provided details about what is needed are reflected upon in order to solve the problem (Lesh & Zawojewski, 2007:368). Schoenfeld (1992) mentions an *examining of special cases* for selecting appropriate strategies from a hierarchical description, but Lesh and Zawojewski (2007:369) argue that this will involve a too long (prescriptive process) or too short conventional list of prescribed strategies. Lesh and Zawojewski (2007:770) rather suggest a *descriptive process* to reflect on and develop sample experiences. The process should be focused on various facets of individual persona and differences, such as prior knowledge and experiences, which differ from one individual to the next.

2.3.3.2 Expansion models for reflective practice

According to Pletzer et al. (1997), applying reflective practice is a powerful and effective way of learning. Three models for reflective practice feature in the next discussion. They include the reflective cycle of Gibbs (1988), Ertmer and Newby (1996), John's (200) model for structural reflection and the framework for reflective practice proposed by Rolfe, Freshwater and Jasper (2001). The first model to be discussed is that of Gibbs (1988).

- **Gibbs's (1988) model for reflection**

Gibbs' model is mostly applied during reflective writing. This model for reflection is adopted during problem-solving situations to assess first and second cognitive levels.

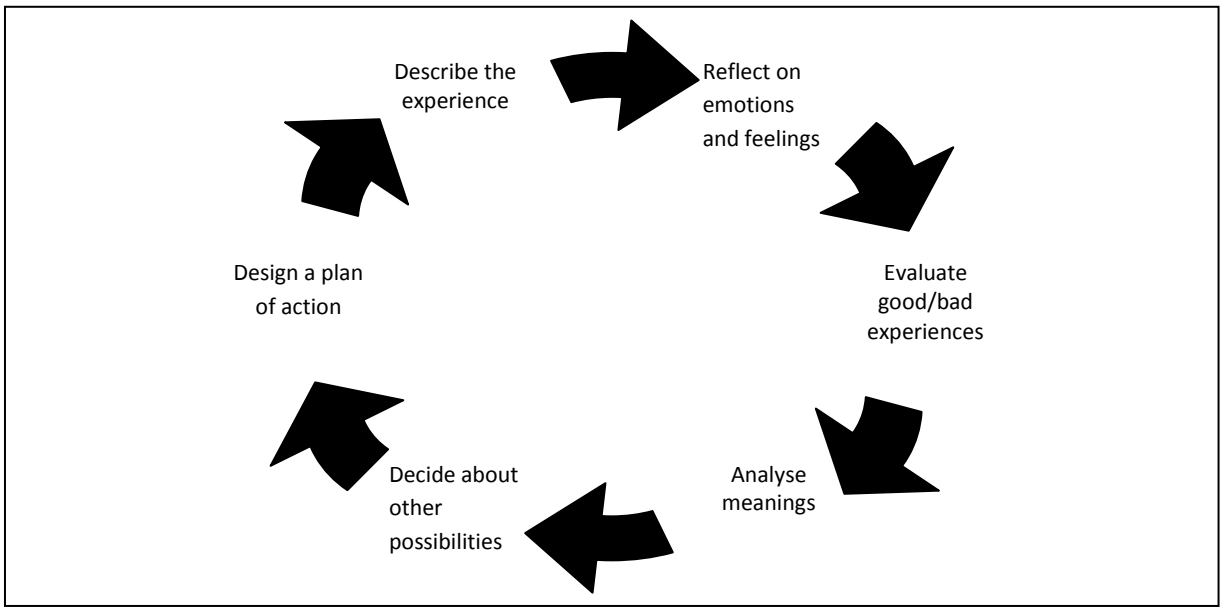


Figure 2.3 Gibbs's model for reflection

Source: Adapted from Gibbs (1988), Ertmer and Newby (1996)

In a particular situation, such as in Figure 2.3, the learner has to solve a mathematical problem. The situation is accompanied by feelings and emotions that will be remembered and reflected upon. A conscious cognitive decision must be made to determine whether the experience caused a positive (good) or negative (bad) emotion or feeling. By analysing the sense of the experience, other options are considered to reflect upon (Gibbs, 1988; Ertmer & Newby, 1996).

- **John's (2009) model for structural and guided reflection**

This model provides a framework for analysing and critically reflecting on a general problem or experience. The 2009 John's model provides scaffolding or guidance for more complex problems encountered on cognitive levels three and four.

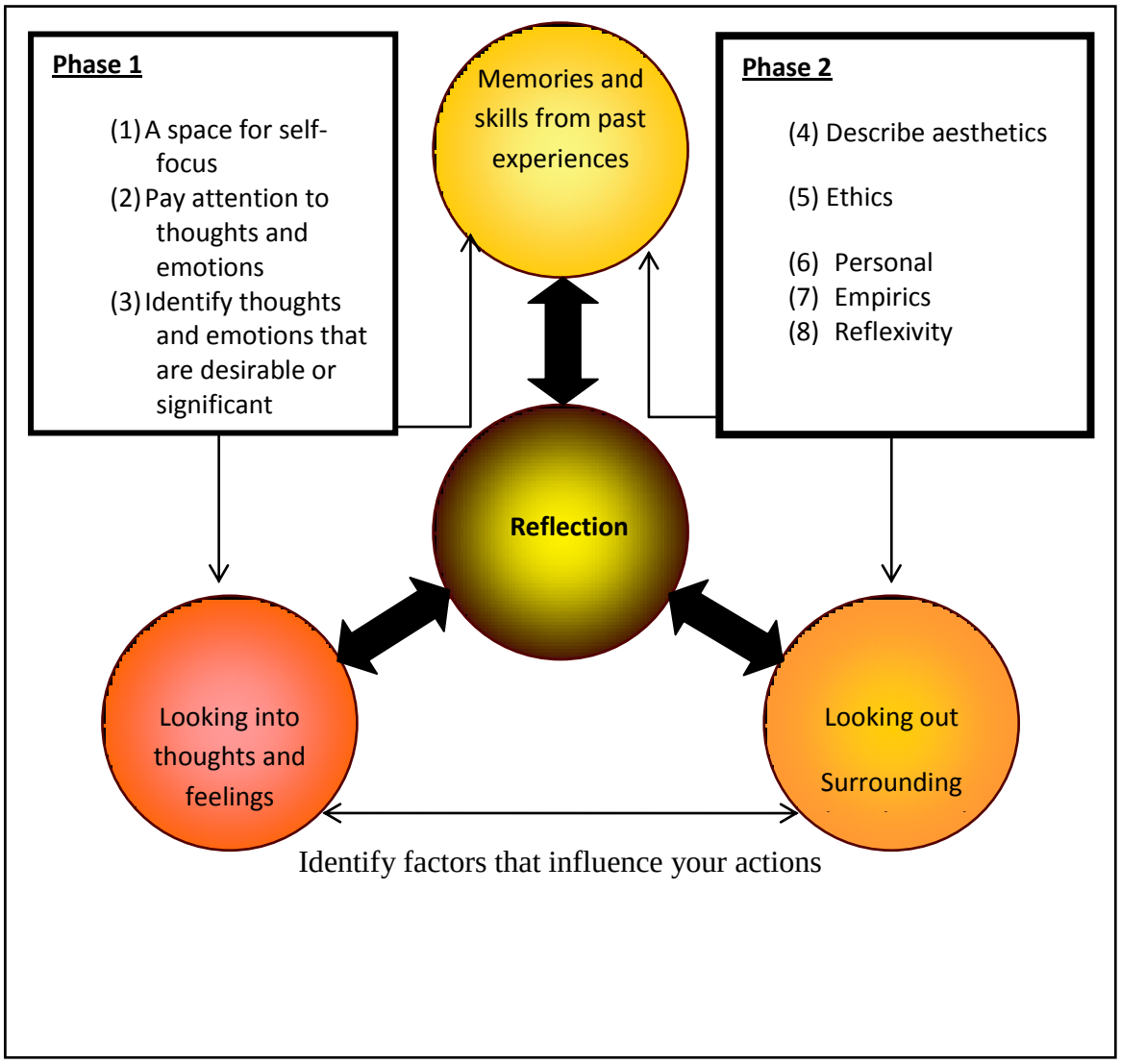


Figure 2.4 John's model for reflective practice

Source: *Adapted from John (2000)*

John's (2009) model (Figure 2.4) is divided into two phases. Phase 1 refers to the recall of past memories and skills from previous experiences, where the learner identifies goals and achievements by reflecting on his/her past. A proposed way to do this is by using a video recording of a situation where the learner solves a problem (Wilson, 2001). It is in this phase that learners take note of their emotions and what strategies are used or not. Phase 2, on the other hand, describes the feelings, emotions and surrounding thoughts that accompany their memories. A deeper clarification is given when learners have to motivate why certain steps were left out or why some strategies were used and others not. They have to explain how they feel and what

makes them feel that way. In the end the learners reflect on the inputs and outputs of factors that could have affected their emotions or thoughts in any way.

The third model that is proposed is the one by Rolfe et al. (2001), known as a framework for reflexive practice.

- **Rolfe et al.'s model for reflexive practice**

According to Rolfe et al. (2001) the questions 'what?', 'so what?' or 'now what?' can stimulate reflective thinking. The use of this model is simply descriptive of the cognitive levels and can be seen as a combination of Gibbs' (1988) and John's (2009) model. The learner first reflects on a mathematics problem in order to describe it. In the second phase, the learner constructs a personal theory and knowledge about the problem in order to learn from it. Lastly, the learner reflects on the problem and considers different approaches or strategies to understand or make sense of the problem situation.

Table 2.1 demonstrates the model of Rolfe et al. (2001) and of Gibbs (1988) and John (2009) as adapted by the researcher. It shows the movement of thoughts, actions and emotions between the different stages of reflection (before, during and after) in problem solving.

Table 2.1 Integration of reflective stages and the models for reflective practice

Stage 1 **Reflection before action**	**Stage 2** **Reflection during action**	**Stage 3** **Reflection after action**
Descriptive level of reflection (planning and description phase)	**Theory and knowledge building of reflection (decision-making phase)**	**Action-orientated level (reflecting on implemented strategy/action)**
Identify the level of difficulty of the problem and possible reasons for feeling, or not feeling, "stuck", "bad" or unable to go to the next step. Pay attention to thoughts and emotions and identify them.	Describe negative attitude towards mathematics problems, if any.	Observe and notice expectations of self and others like parents, teachers or peers.
Evaluate the positive and negative experiences.	Analyse and understand the problem and plan the next step, approach or strategy.	Perform the planned action.
Create an awareness of ethics, beliefs, personal traits and motivations.	Recall strategies that worked in the past.	Reflect on the solution, reactions and attitudes.

Source: Adapted from John (2009), Gibbs (1988) and Rolfe et al. (2001)

In Table 2.1 the first stage illustrates reflection before action, when planning and description take place as metacognitive tasks. The problem's level of difficulty is measured and accompanying feelings or negative thoughts are identified. These positive and negative experiences are evaluated and the individual will reconcile them with applicable beliefs, attitudes and traits that

enhance, or reduce, motivation. In the second stage, reflection occurs as part of the problem-solving process. This *in action* decision-making phase describes negative and/or positive attitudes while the problem is being analysed and understood. Reflection-in-action resembles patterns in everyday conversation, with self-directed speech. The plan or strategy is implemented while past experiences with strategies, similar to the chosen one, may be reflected upon. In the third stage, reflection occurs after the problem has been solved. Observations and expectations are only some of the individual factors constructed from current beliefs and attitudes. Evaluating and reflecting on the performed plan or strategies chosen (or not chosen) creates new or previously unthought-of plans of action. Eventually, reactions to the problem-solving process will venture new attitudes and reflections, depending on personal beliefs.

2.3.3.3 Concluding remarks on the reflection process

While some research claims that *seeing* and *doing* Mathematics is useful in the interpretation of and decision making about problem-solving processes (Lesh & Zawojewski, 2007), a more affective approach would involve *feelings* in general or the feelings about Mathematics in particular (Sheffield & Hunt, 2007), in other words, affective factors. High levels of negative metacognitive beliefs are associated with various affective factors or a lack of confidence (Rheeder, Rexhepi-Johansson & Wykes, 2009:50). Some of these affective factors will be discussed next.

2.4 Affective factors relating to performance in Mathematics

Rapidly changing states of feelings, moderately stable tendencies, internal representations and deeply valued preferences are all categories of affect in Mathematics (Schlogmann, 2003:1). Reactions to Mathematics are influenced by emotional components of affect. Some of these components include negative reactions to Mathematics, such as stress, nervousness, negative attitude, unconstructive study orientation, worry[6], and a lack of confidence (Wigfield & Meece, 1988; Maree, Prinsloo & Claassen, 1997). Meta-worry refers to worry about worry (Wells & Cartwright, 2004: 386). Learners' self-concept is strongly connected to their self-belief and their success in solving mathematics problems is conceptualised as important (Hannula, Maijala &

[6] Wells and Cartwright (2004:386) distinguish between three types of worry: social worry, health worry and meta-worry. Since our focus is on metacognition in this study, reference will only be made to meta-worry.

Pehkonen, 2004:17). A study done by Ma and Kishor (1997) confirms that belief, as a form of affect on mathematics achievement, correlates weakly with mathematics achievement among children from Grade 2 to 8. In their study with learners in Grades 7 to 12, Hannula, Maijala and Pehkonen (2004) however concluded that there was a strong correlation between their beliefs and achievement in Mathematics. Beliefs are related to self-reflective factors and involve feelings. According to Lesh and Zawojewski (2007:775), the self-regulatory process is critically affected by beliefs, attitudes, confidence and other affective factors.

2.4.1 Beliefs as an affective factor in mathematics performance

Belief, in a mathematics learner, forms part of constructivism as a comcomitant understanding of affect and its nature. The constructed knowledge and belief is a perspective formed about the individual and his/her interaction(s) with the environment. This can be distinctive of an individual's understanding of his/her own feelings, and as personal concepts formed, when engaging in mathematical problem solving (Hannula, Maijala & Pehkonen, 2004:3). Beliefs play an important role in attitudes and emotions due to their cognitive nature and, according to Goldin (2000:5), learners attribute a kind of *truth* to their beliefs. This is because they are shaped by a series of background experiences involving perception, thinking and actions (Furinghetti & Pehkonen, 2000:8) developed over a long period of time (Mcleod, 1992:578-579). Beliefs about mathematics can be seen as a mathematics world view (Schlogmann, 2003:2) and can be divided into four major categories (Hannula, Maijala & Pehkonen, 2004:17): beliefs on Mathematics (e.g. there can only be one correct answer), beliefs about oneself as a mathematics learner or problem solver (e.g. Mathematics is not for everyone), beliefs on teaching Mathematics (e.g. Mathematics taught in schools has little or nothing to do with the real world) and beliefs on learning Mathematics (e.g. Mathematics is solitary and must be done in isolation) (Hannula, Maijala & Pehkonen, 2004:17). Faulty beliefs about problem solving allow fewer and fewer learners to take Mathematics courses or to pass Grade 12 with the necessary requirements for university entrance. Beliefs are known not only to work against change or act as a consequence of change, but also to have a predicting nature (Furinghetti & Pehkonen, 2000:8). Affective issues such as beliefs generally form part of the cognitive domain, anxiety (Wigfield & Meece, 1988), which will be dealt with in the next paragraph.

2.4.2 Anxiety

Anxiety, an aspect of neuroticism, is often linked with personality traits such as conscientiousness and agreeableness (Morony, 2010:2). This negative emotion manifests in faulty beliefs that cause anxious thoughts and feelings about mathematics problem solving (Ashcraft, Hopko & Gute, 1998:344; Thijsse, 2002:17). A distinction can be made between the different types of anxieties as experienced by learners across all age groups. Some of these anxieties include general anxiety, test or evaluation anxiety, problem-solving anxiety and mathematics anxiety. The widespread phenomenon, mathematics anxiety[7], threatens performance of learners in Mathematics and interferes with conceptual thinking, memory processing and reasoning (Newstead, 1999:2).

2.4.2.1 Mathematics anxiety

The pioneers of mathematics anxiety research, Richardson and Suinn (1972), defined mathematics anxiety in terms of its effect on performance in mathematics problem solving as

> *... feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations*

This anxious and avoidance behaviour towards Mathematics has far-reaching consequences as is stressed by a number of researchers (Maree, Prinsloo & Claassen, 1997; Newstead, 1999; Sheffield & Hunt, 2007; Morony, 2009). Described as a chain reaction, mathematics anxiety consists of stressors, perceptions of threat, emotional responses, cognitive assessments and dealing with these reactions. A number of researchers expand the concept of mathematics anxiety to include facilitative and debilitative anxiety (Newstead, 1999:2). It appears that Ashcraft, Hopko and Gute (1998:343) sees mathematics anxiety in the same locale as the working memory system. Both areas consist of psychological, cognitive and behavioural components. Although they agree on the same components, Eysenck and Calvo (1992) state that it is not the experience of worry that diverts attention or interrupts the working memory process,

[7] Mathematics anxiety is commonly expressed in the literature as math anxiety or math phobia. A negative attitude towards doing Mathematics refers to a negative motivational construct. In this study mathematics anxiety or low mathematics confidence is seen as a motivational factor as illustrated in the mathematics anxiety and confidence scale in 2.5.3.9.

but rather ineffective efforts to divert attention away from worrying and to focus on the task at hand.

2.4.2.2 Symptoms for identifying mathematics anxiety

Mathematics anxiety is symptomatically described as low (feelings of loss, failure and nervousness) or high (positive and motivated attitude) confidence in doing Mathematics (Maree, Prinsloo & Claassen, 1997a:7). Dossel (1993:6) and Thijsse (2002:18) state that these negative feelings are associated with a lack of control and that uncertainty and helplessness are experienced when facing *danger*. Avoidance, as well as the inability to think rationally and perform adequately, causes anxiety and negative self-beliefs (Thijsse, 2002:17). Anxious children show signs of nervousness when a teacher comes near. They will stop writing and cover their work with their arm, hand or book, in an approach to hide their work (Maree, Prinsloo & Claassen, 1997; Newstead, 1999; Thijsse, 2002:16). Panicking, anxious behaviour and worry manifest in the form of nail biting, crossing out correct answers, habitual excuse from the classroom and difficulty to verbally express oneself (Maree, Prinsloo & Claassen, 1997a). Maree (1997:7) states that mathematics anxiety could be expressed as an antonym for self-confidence. Rapid heartbeat, fear and low confidence are associated with mathematics anxiety symptoms.

From the above it is clear that mathematics anxiety includes many features. The causes of mathematics anxiety are described in the next paragraph.

2.4.2.3 Some causes of mathematics anxiety

Both Newstead (1999:4) and McLeod (1993) argue that the root of mathematics anxiety lies in early classroom experiences as documented by Newstead (1999:7); McLeod (1993); Tobias (1978); Stodolsky (1985:126); Bormotova (2010) and Strawderman (2010). Newstead (1999) investigated affective differences between traditional and alternative approaches to teaching mathematics. Teaching methodologies like the *explain-practise and memorise* paradigm are seen as a major cause of the mathematics anxiety *syndrome* (Thijsse, 2002:19). The findings from research done by Mcleod (1993), Maree, Prinsloo and Claassen (1997), Thijsse (2002), Bormotova (2010) and Stawderman (2010) indicate that learners exposed to traditional teaching methods had higher levels of mathematics anxiety than those who were taught in alternative classrooms. Traditional approaches included drill practice, word problems and rote-memorised

operations. Thijsse (2002) has a similar opinion as Tobias (1987:129), who claims that *word problems are at the heart of mathematics anxiety* and that the degree of accuracy in number manipulation is identified as a source of anxiety. With its many facets, mathematics anxiety contains physical, cognitive and psycho-behavioural components. The teacher's own mathematics anxiety could cause learners to experience anxious reactions towards the subject (Thijsee, 2002:20-22). For some learners, merely doing mathematics in front of peers or in public is a cause of mathematics anxiety, stipulating a social domain.

The following paragraphs give an overview of the possible reduction and treatment of mathematics anxiety to serve as an aid towards building confidence in respect of Mathematics.

2.4.2.4 Reducing mathematics anxiety and building mathematics confidence

In the presence of a supporting teacher, *in a transmission-type* classroom, mathematics anxiety can be reduced. As Thijsse (2002:25) puts it, *math anxiety often begins with the teacher.*

Reducing anxiety builds self-confidence and sets the basis for a favourable attitude towards doing Mathematics. Schlogmann (2003:7) explains that individuals have certain avoidance behaviour strategies, which cause them to replace their understanding of Mathematics with avoiding Mathematics in everyday life. However, it is not possible to avoid Mathematics in the Mathematics classroom, where learners are forced to learn and do Mathematics. The surrounding context or environment will still evoke certain affective reactions and the pressure of passing or failing the exam is unavoidable. In their search for a strategy to cope with anxiety, learners approach Mathematics with a belief system. They either believe that they can do Mathematics and understand it, or they believe that they cannot do Mathematics or that Mathematics is simply not for them. A constant reminder about writing tests or examaminations creates a link between the fear and the negative emotions within information processing systems and causes test or evaluation anxiety (Schlogmann, 2003:8). Possible approaches towards reducing mathematics anxiety and building confidence manifest in three forms: teaching, psychological and cognitive approaches.

- **Some teaching approaches as adapted from Thijsse (2002) and Maree (1997)**

  a Provide learners with success experiences

b Demonstrate strategies

c Be a role model with confidence in subject knowledge

d Vary teaching strategies

e Provide opportunities for discussions

f Allow learners to evaluate their own work

g Have a positive attitude towards errors and use them as a way to learn from mistakes

h Be aware of learners who experience anxiety and identify the causes thereof

- **Some psychological approaches as adapted from Thijsse (2002) and Maree (1997)**

a Propose relaxation techniques

b Change attitude in respect of one's own abilities and beliefs

c Introduce anxiety management training

d Consult a psychologist

e Verbalise fears and frustrations

f *Anchoring* (to block mental tasks)

g Write in a journal (about experiences – either positive or negative)

h Introduce bibliotherapy (reading about other people's problems with Mathematics)

i Initiate behaviour modification (classical conditioning: reward behaviour)

- **Some cognitive approaches as adapted from Thijsse (2002) and Maree (1997)**

a Become involved in group activities

b Apply the work in real-life situations

c Stress motivation[8]

d Develop a personal belief in one self

It is essential that learners take control of their mathematics performance. This will encourage them to take control of other aspects of their lives as well. According to Ashcraft (2002:2) all learners suffer from some degree of mathematics anxiety that relates to gender, teaching approaches, ethnic background, age, attitude and previous experiences. It appears that we all have some degree of apprehension about Mathematics, and Stuart (2000: 331) states that *anxiety is nothing but a lack of confidence.*

With this in mind, mathematics confidence will be discussed next.

2.4.3 Mathematics confidence

Mathematics confidence, a psychological factor, influences performance and learning in Mathematics either positively or negatively (Maree, Prinsloo & Claassen, 1997a:7). Learners who lack confidence with regard to Mathematics experience anxiety with the subject based on a number of causes that vary from personal and environmental to instructional. Ashcraft (2002:2) agrees with Dodd (1999) that a lack of confidence is the anxious learner's greatest barrier to successful mathematics problem solving.

2.4.3.1 Relations to mathematics confidence and anxiety

In the mid-eighties, a study by Fergusson (1986:149) found that mathematics confidence is multidimensional. Numerical anxiety, test anxiety and abstraction anxiety are found to be strongly correlated to achievement in problem solving and are noted factors of the mathematics confidence construct.

2.4.3.2 Gender comparisons in respect of mathematics confidence

Performance may suffer due to a stereotyped threat (Quest, Linn & Hyde, 2010:103). Achievement in mathematics problem solving and comparison between gender *not proven* (Hannula, Maijala & Pehkonen, 2004:17; Bohlin, 1994; Hannula & Malmivuori, 1997; Pehkonen, 1997). A study comparing the mathematics confidence of Grade 5, 6, 7 and 8 learners

[8] Keep in mind that anxiety can be employed as a form of motivation.

(Hannula, Maijala & Pehkonen, 2004:20) shows that Grade 8 learners' confidence is the lowest of the four grade groups. It seems that the older the learners get, the further their mathematics confidence declines. Hannula, Maijala and Pehkonen (2004:21) nevertheless found that girls in Grade 8 had higher confidence levels than boys in Grade 8. Achievement and confidence differences between genders are considered to be determined by learners' spatial ability, interest in Mathematics and Mathematics courses and biological issues (Mcleod, 1988:139). Societal stratification based on gender shows fewer opportunities for girls. According to Quest, Linn and Hyde (2010:136), girls experience less positive attitudes and more negative affect. A more detailed description of these differences related to confidence follows next.

- **Mathematics confidence in boys**

According to Hannula, Maijala and Pehkonen (2004:17), younger boys had more confidence and higher achievement in mathematics problem solving than girls, although older girls showed more confidence and achieved better than older boys.

- **Mathematics confidence in girls**

Beilock (2008:339) mentions that female learners assume that *everyone knows girls cannot do Mathematics.* Such stress-loaded situations have highly negative consequences and stand in sharp contrast to the motivation required for optimal performance (Beilock & Carr, 2001:702-703). Data from studies done by McLeod (1988:139) suggests that girls do not achieve as well as boys on problem solving.

2.4.3.3 Age difference and its relation to mathematics confidence

Research done by Newstead (1999:66) found that children between nine and eleven years old experienced a considerable amount of anxiety about social and public aspects in the presence of teachers, parents or peers.

2.4.3.4 Relationship between gender and cultural differences with regard to mathematics confidence

Bernstein, Reily and Cote-Bonanno (2001) did a study on college students and found that Caucasian females had higher mathematics confidence than Caucasian males, while Asian and

Native American male students had lower mathematics confidence than their counterpart females. It seems that, besides age and gender, mathematics confidence also differed with regard to culture and its relation to mathematics confidence. Evidence supports the fact that different cultural groups attribute performance in Mathematics to different attitudes and beliefs (Stevenson, 1987; Tijsse, 2002:34). For example, Americans believe that a person's mathematics ability is hereditary (inborn) while Asians believe that success in Mathematics is the result of hard work (Aschraft, 2002:1). According to Aschraft (2002) cultural differences may also be responsible for lower confidence in respect of Mathematics, as African American, Hispanic, Asian and Native American males and females had low mathematics confidence. He found no significant difference between college students' confidence, compared with Bernstein, Reily and Cote-Bonanno's study in 2001. Aschraft (2002:2) argued that this absence of a significant difference was the result of similar mathematics backgrounds, which should also be taken into consideration.

The variation between low and high levels of mathematics confidence is described in more detail in the next paragraph where mathematics anxiety and confidence are illustrated by means of a scale.

2.4.3.5 Mathematics anxiety and confidence scale

According to Hopko (1998:344) the distinction between high- and low-anxious learners is not confined to harbouring worrisome thoughts, but to their effectively paying attention to those thoughts. High anxiety is associated with easy distraction and lower attention control. Experience with Mathematics comprises positive (good grades, teacher and parent praise, overcoming difficulty in Mathematics) and/or negative (failure, criticism and possible future difficulties) memories and feelings (Ashcraft, Hopko & Gute, 1998:345). Figure 2.5 illustrates a comparison between mathematics anxiety and mathematics confidence.

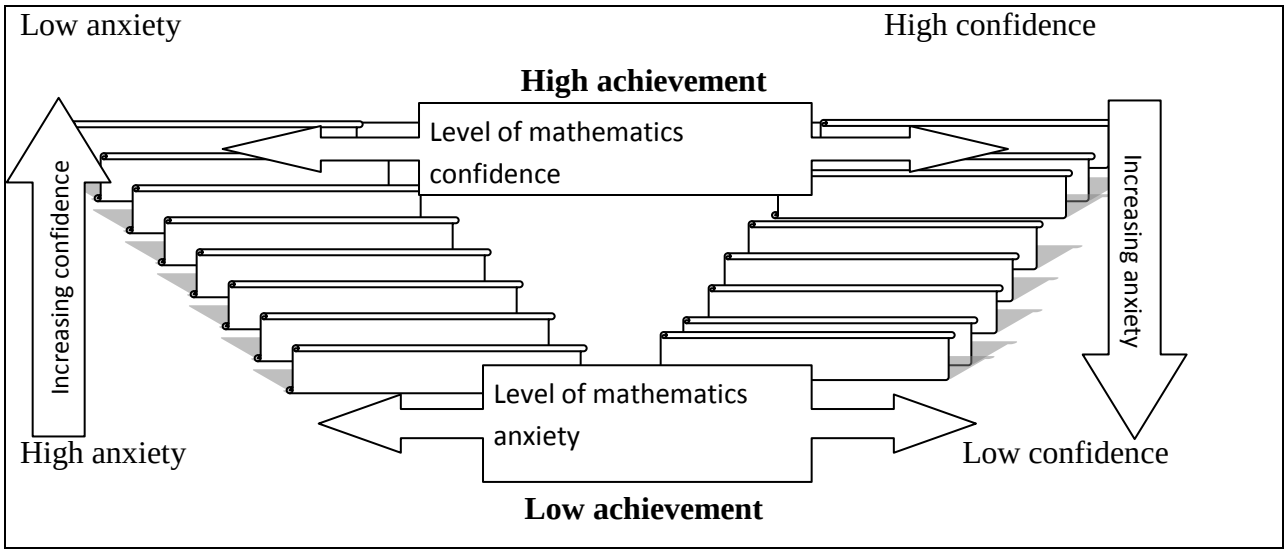


Figure 2.5 The staircase phenomenon

Source: Adapted from Stawderman (2010) and Maree (1997)

Sheffield and Hunt (2007:2) distinguish between *math* anxious[9] and non-*math*[10] anxious learners[11]. Figure 2.5 illustrates an individual's mathematics confidence on two sets of staircases going up or down different levels. Underlying the steps on the bottom right is a low level of confidence[12] where such a learner will have a low achievement in Mathematics (bottom left) and will also experience a high level of mathematics anxiety. When the level of anxiety is reduced, due to constructive approaches, the learner's confidence will increase (going up the staircase on the left) and move towards a level of high confidence. On this level, the learner will have a high achievement in mathematical problem solving. As soon as the approach becomes less constructive (going down the staircase on the right), mathematics anxiety will increase and the level of confidence will drop.

However, not all anxiety has a destructive effect on high achievement. By comparing anxiety with a problem's level of cognitive difficulty, Ashcraft and Kirk (2001) claim that mathematics

[9] Math anxiety, a colloquial term for mathematics anxiety, is a term commonly found in literature. For the purpose of this study, in a South African context, math anxiety is rather referred to as mathematics anxiety, which is academically more acceptable.

[10] Literature referring to mathematics anxiety generally views this affect as a negative factor in performance. In this study, the focus is rather on the level of confidence, seen as a form of low of anxiety.

[11] This sentence could also read as: 'Sheffield and Hunt (2006:2) distinguish between learners who have confidence in doing mathematics and those who do not.'

[12] I will start at the low confidence level to illustrate how the section on reducing anxiety and building confidence can relate to Figure 2.5. One may wish to start at the level of high confidence instead, and the figure could be interpreted either way.

anxiety will have an unwanted effect on achievement if the level of difficulty of the problem increases. Increasing the variety of procedures in the problem raises anxiety and lowers the learner's analysing ability and synthesis that relates to the cognitive domain (Vermeulen, 2002; Sheffield & Hunt, 2007). Equilibrium is found once the learner's creative, analytic and adaptability skills balance on the mathematics problem and cause anxiety, frustration and boredom to be cancelled out. Schunk, Pintrich and Meece (2008) state that the complexity of the problem should be motivating and challenging without making too low or too high demands on the working memory. If these requirements are not met, the learner's confidence will decrease and anxiety will increase, resulting in low achievement. Figure 2.6 illustrates this equilibrium phase when the problem-solving circumstances are effective.

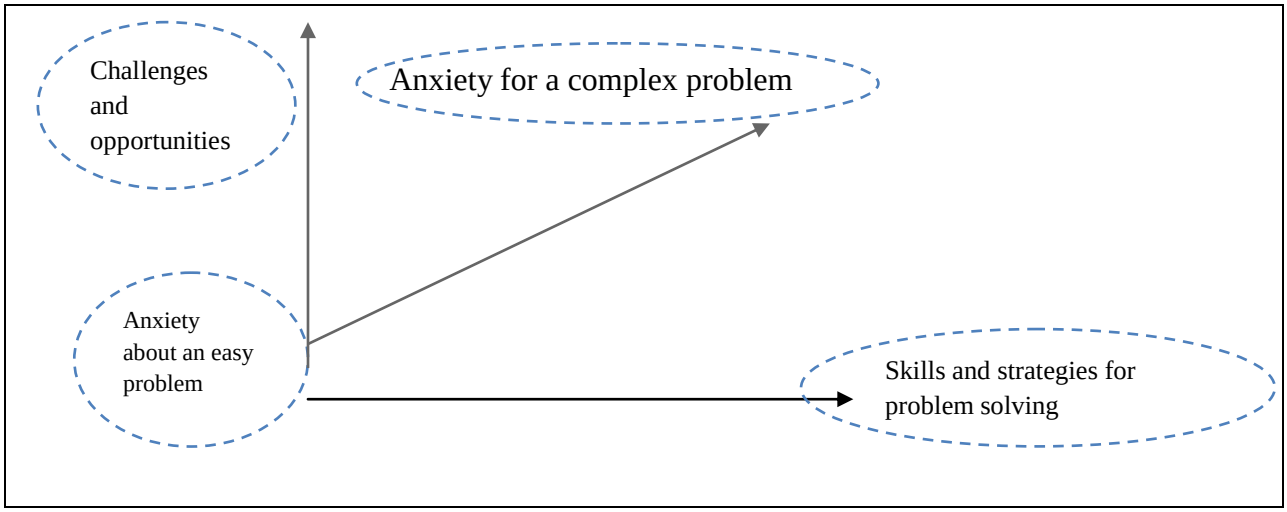


Figure 2.6 Problem solving and anxiety equilibrium

Source: Schunk, Pintrich and Meece (2008:256)

Figure 2.6 illustrates the outcomes when the mathematics problem makes a too complex cognitive demand on the learner. The anxiety level will increase to such an extent that confidence will drop and more opportunities and challenges are needed. If the problem is too easy, the skills and current knowledge will find the problem boring and, as a result, there will be little anxiety, too much confidence and less motivation for future problems or challenges that are more difficult. Research indicates that anxiety can in some cases lower the achievement during problem solving, while it can also serve as a basis of motivation and thus allow opportunities for higher achievement (Maree, 2005). Figure 2.7 illustrates this occurrence.

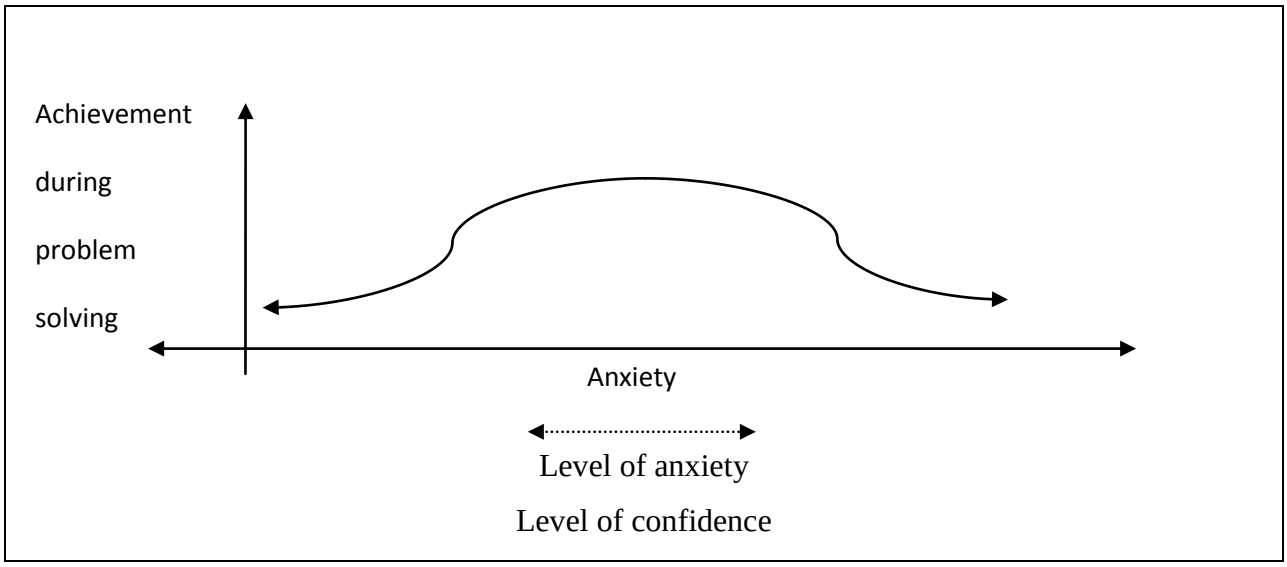


Figure 2.7 Possible relationship between achievement and anxiety

Source: Adapted from Maree (2002)

Figure 2.7 illustrates that anxiety possibly affect achievement. Therefore it appears as if the level of high confidence may possibly result in high achievement (Maree, 2002). In this case, anxiety may serve as a motivational facet of affect.

2.5 Mathematics confidence domains

From the literature (Bormotova, 2010; Strawderman, 2010; Newstead, 1999; Maree, 2005; Schunk, Pintich & Meece, 2008) it is evident that mathematics confidence is a multi-dimensional construct. In view of all the psychological, affective and cognitive issues involved, Strawderman[13] proposes a model that unifies the dimensionality of mathematics confidence.

2.5.1 Modelling mathematics confidence

Mathematics experiences are described as successful or failed attempts accompanied by feelings of high or low confidence. There are three components within the context of an individual's mathematics confidence, namely social, intellectual and psychological domains (Strawderman, 2010:1).

[13] Strawderman's model for mathematics anxiety was adapted for this study to include the antonym 'mathematics confidence'.

2.5.2 The three domains of mathematics confidence

The following discussion focuses on Strawderman (2010)'s three mathematics confidence domains**.**

2.5.2.1 The social domain

Bergh and Theron (2009:86) explain the social domain in terms of Bronfenbrenner's ecological systems model. The environment includes four levels on which human influences develop. The first is a micro system consisting of persons and organisations that have the most frequent contact with the individual. Teachers, family members and peers take responsibility for the individual's reflection on and restructuring of these immediate sources. A second level is the meso system consisting of interaction with groups such as schools or churches. The exo system involves aspects outside immediate contacts such as hospitals, social groups, chat rooms and clubs. The fourth system involves the macro system and includes habits, laws, socio-economic and political influences. Values and beliefs are often regulated in this domain.

2.5.2.2 The psychological domain

In this domain, the behaviour of the individual ranges between involving oneself in and avoiding situations like mathematics activities or classes. It consists of affective factors such as emotional history, feelings and awareness of likes and dislikes. The psychological domain extends further because of affective factors. Emotional history, familiar experiences and stimulus reactions are associated with the individual's feelings of confidence, anxiety or discomfort and pleasurable experiences. Bergh and Theron (2009:156) made a clear distinction between emotionally motivated and de-motivated individuals. Reinforcement, from the psychological behaviourist view, has a positive or negative motivational effect on individuals and the outcomes of their actions. Rewarding the *correct* action, approach or behaviour and punishing the *incorrect* has complete diverse consequences. Positive motivation involves setting goals and helping individuals to achieve those goals, which leads to their handling of more tasks that are complex. However, punishing could lead to negative motivation and result in undesirable effects such as hostile behaviour and avoidance of tasks related to the punishment.

2.5.2.3 The intellectual domain

The intellectual domain entails cognitive influences and is frequently judged as the individual's ability. This includes skills and knowledge of the problem-solving procedures, approaches and strategies. Personal performance is measured in this domain and it is associated with the field of personal achievement and the perception thereof (Strawderman, 2010:3). Fluctuating between success and failure, the individual evaluates the acquirement and use of Mathematics skills and concepts. Guilford's structural model of intellect (Bergh & Theron, 2009:149) reviews the intellectual domain as a combination of various aspects that involve visual, auditory, symbolic, semantic and behavioural constructs. The operations related to these aspects appear to be mainly cognitive (memory, divergent production, convergent production) and metacognitive (evaluation and monitoring) in nature. The product of these operations is found to influence group settings, relationships, systems, transformation of knowledge and implications for problem solving.

2.5.2.4 Synthesising the three mathematics confidence domains

The three domains have individual characteristics as well as a natural overflow between their components, due to cause and effect of one component on another. In the social domain, family members, friends (peers) and society influence the values, beliefs and views the learner might develop. The result is a motivated or de-motivated individual and positive or negative emotions. Engaging with mathematics tasks or avoiding them in the intellectual domain either allows the individual to succeed or fail in their use of strategies or approaches.

Combining the components of the three domains of Strawderman (2010) leads to an affective product in mathematics problem solving, namely mathematics confidence. The overflow and connection of the three domains are illustrated in Figure 2.8.

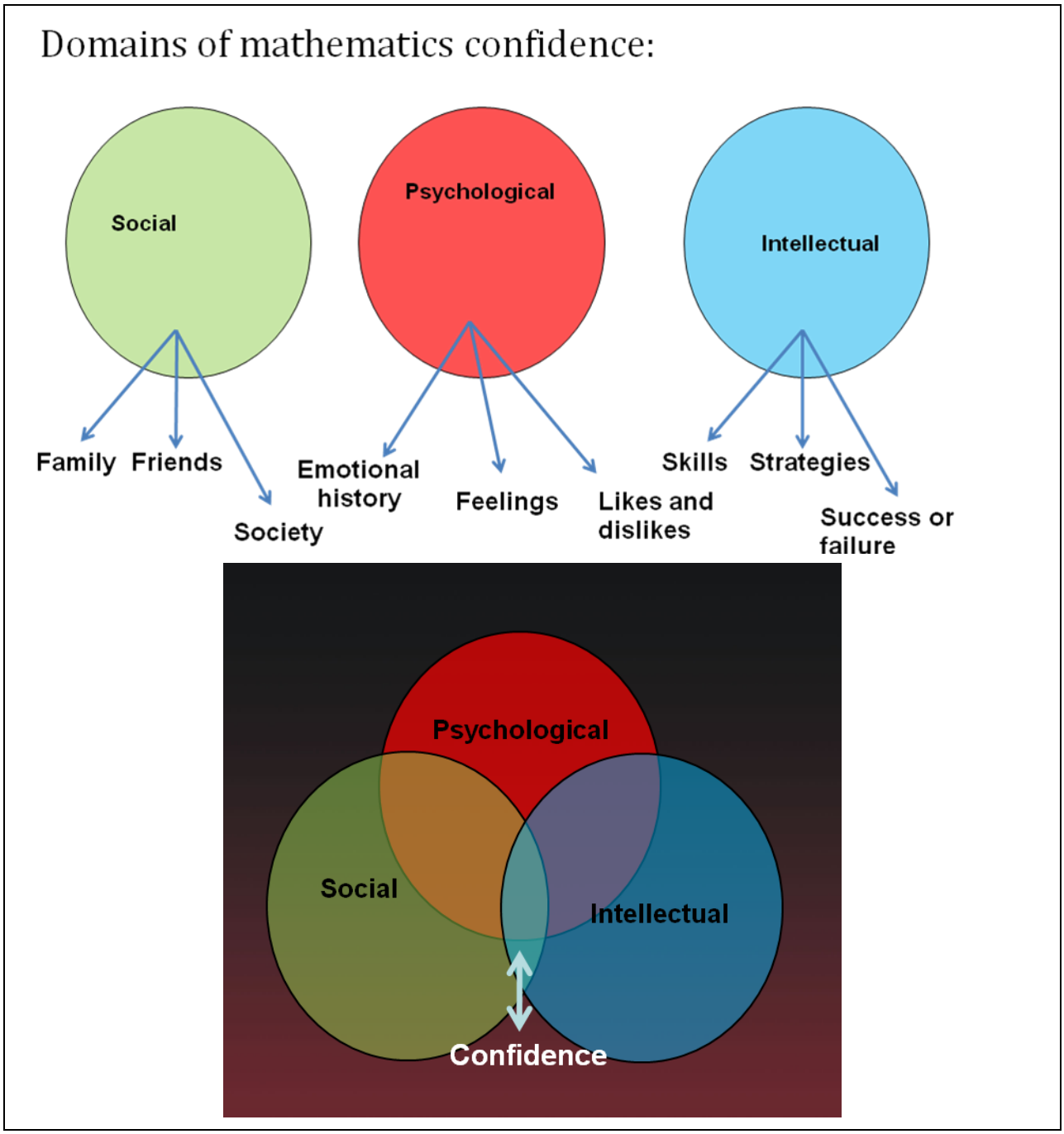


Figure 2.8 Domains of mathematics confidence

Source: Adapted from Strawderman (2010) and Bergh and Theron (2009)

The individual problem solver is a carrier of many unique and diverse facets. In the next paragraph a discussion follows regarding reflection on some of the aspects regarding problem solving.

2.6 Social, affective and reflective aspects of problem solving

The social aspect to consider regarding affect involves group work during problem solving.

2.6.1 Group work

Research has shown that learners working successfully in groups will achieve higher cognitive levels of thinking than if they worked alone. Chalmers (2009:1) argues that although problem solving in group settings is necessary, it is not always sufficient. Mathematical problem solving in a group setting allows learners to access a wide range of strategies and solutions. Thinking strategies will provide scaffolding for future reference when learners engage in problem-solving tasks again. During group work settings, the teacher visits learners in their groups after a mathematics problem was given to solve. This may show the teacher's interest in the group's performance but this action might seem insulting to some students (Brookfield, 2005:1). On the other hand, when a teacher joins such a group, some learners may try to impress the teacher by talking louder, getting more active on the use of strategies or discussions regarding the task. It is necessary to model reflective behaviour in group settings since the group as a whole consists of learners that are vulnerable to affective and metacognitive end products. Applying metacognition in group settings is also an essential element of mathematical problem solving within the group (Chalmers, 2009:1).

2.6.1.1 Group metacognition and reflection

Within group settings, a storming stage (where conflicts emerge) and an adjourning stage (where the group reflects on the problem once the solution has been found) may develop (Chalmers, 2009:2). Metacognition involves the development of cognitive strategies and thinking in a group setting. This development requires planning, monitoring and the evaluation of the group's behaviour towards the task. Studies by Xiaodong (2001) and Chalmers (2009) found that learners would only engage in metacognitive thinking when they were told to do so. Johnson, Johnson and Johnson-Holubec (1993), together with Chalmers (2009), suggest four elements that must be included in learning activities:

- Face-to-face interaction
- Developing social skills

- Individual liability
- Group processing

Group members must think about the problem and consider how they are going to solve it by implementing these four elements. The learners need to monitor – at a metacognitive level – first their own, and then the group's performance (Goos et al., 2002; Hisz, 2004). The requirement for implementing metacognition in group settings also stipulates that the metacognitive skill of 'reflection' is required (Chalmers, 2009:6). Implementing metacognition can be accomplished by using scaffolding questioning or metacognitive prompting. According to Chalmers (2009:5), scaffolding questions are divided into specific question cycles such as planning, monitoring and evaluation – in other words, reflection. Reflection can be done by asking questions such as: What does the question ask? How is it solved? Is the question answered meaningfully?

2.7 Conceptualising the conceptual/theoretical framework

The social domain illustrates external factors outside the individual's control (Strawderman, 2010:2; Bergh & Theron, 2009: 86). The psychological field concerns the behaviour of individuals and includes their feelings, emotions, likes and dislikes, whereas the intellectual domain entails cognitive influences. The latter include skills and knowledge of the problem-solving procedures and strategies. Personal performance is measured in this domain and is associated with the field of personal achievement and perceptions thereof (Strawderman, 2010:3). Fluctuating between success and failure, the individual evaluates the acquisition and use of mathematics skills and concepts.

The psychological domain extends further because of affective factors. Emotional history, familiar experiences and stimulus reactions are associated with the individual's feelings of confidence, anxiety or discomfort and pleasurable experiences. The following conceptual framework portrays the literature and incorporates the concepts for the current research design. The three domains associated with mathematics confidence namely: social, psychological and intellectual have, to some, extent integrated overlaps. Strawderman (2010) asserts that these overlaps explain mathematics confidence. Learners reflect on their level of mathematics confidence and on the metacognitive components of knowledge and regulation. This internal process is illustrated in Figure 2.9 and serves as the conceptual framework of this study.

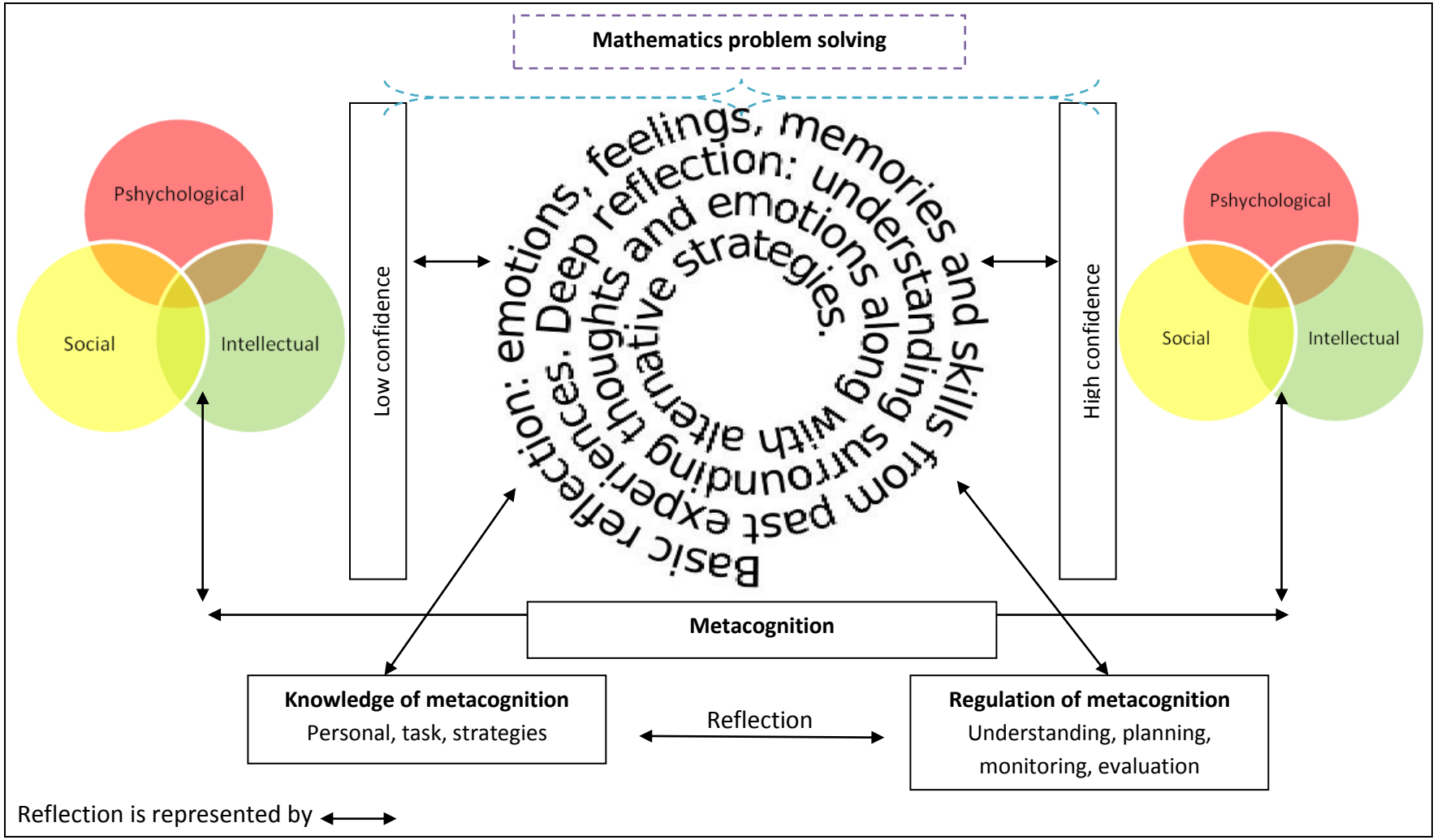


Figure 2.9 Conceptual framework for reflection on metacognition and mathematics confidence

Source: Adapted from Strawderman (2010); Johns (2009); Jasper (2003); Gibbs (1988); Bergh and Theron (2009)

2.7.1 Summary and conclusion of the literature review

In this chapter it was indicated that, inadequacy (which is described by Kogelman (1981:32), Newstead (1999:6) and Thijsse (2002:20) as a product of pressure to perform during timed testing, speed drills and flash cards) causes increased anxiety. Negative thoughts and worries disrupt the individual's working memory system and hamper the system's resources (Ashcraft; Hopko & Gute, 1998:344). Causes of mathematics anxiety may be related to events occurring inside classrooms and may be triggered by a lack of support or understanding. This may in return generate a lack of confidence (Aschraft, 2002:5). If mathematics anxiety is not serving as a motivational factor, it results in decreased achievement. Teaching, psychological and cognitive approaches can be implemented to alleviate the problem of low mathematics confidence. Metacognitive dimensions, cognitive knowledge and cognitive regulatory processes may ease this problem and build learners' mathematics confidence, provided they are employed as motivational constructs. It appears that metacognitive success is determined by the individual's reflective skills. By enhancing the learner's understanding of reflection, he/she will better understand self-regulatory processes and, as a result, implement meaningful strategies to solve mathematics problems.

According to Legg and Locker (2009:471), little research has been done in the field of examining metacognition and affective factors during the process of problem solving. The variables in this study are identified as confidence and reflection during the process of mathematical problem solving. The research design and methodology that were used to examine these variables are described in Chapter 3.

Chapter 3

The mixed-method research design

3.1 Introduction

In Chapter 2, literature dealing with reflection on mathematics and mathematics confidence during problem solving was reviewed. The mixed-method research approach that was implemented in the current study is outlined in Chapter 3. The nature of the instruments administered, the strategies involved and the data-collecting and analysis procedures are described in this chapter.

3.2 Assumptions made by the researcher

The aim of this study was to promote a better understanding of, to clarify, and to explore the characteristics of learners' metacognitive skills with special reference to reflection and mathematics confidence during mathematics problem-solving processes. To this end, the researcher implemented a mixed-method exploratory design based on the positivist as well as social-constructivist/interpretivist approaches.

There is a general paucity of literature on mathematics confidence, the awareness of the metacognitive strategy and reflection on mathematics (Efklides, 2011:6). Since little is known about the effect of mathematics confidence and reflection on senior phase Mathematics (Andrews, 2011:99), the aim of this study was to provide insight into affective (mathematics confidence) and metacognitive aspects (reflection) of mathematics problem-solving situations.

In the last decade, research on affect often linked learners' experience with Mathematics to current beliefs and motivational factors (McLeod, 1998; Newstead, 1999). In order to investigate this link, the theoretical stance of the research design and the philosophical nature of the researcher are described in the next paragraph.

3.3 Theoretical assumptions

In this study, according to the features noted by Cohen et al. (2001), knowledge was viewed from a positivist (quantitative) as well as a social-constructivist (interpretivist/qualitative) perspective.

The researcher used a lens of epistemological assumptions that gave rise to the use of scientific methods (quantitative part of the study). Together with this empirical study, the researcher also adopted a more subjective participatory role (qualitative). He examined the research practice as characterised by Cohen (in Maree et al., 2010:38) and took on the role of a participant observer in the interviews.

Supporting the interpretivist approach, the study also included a natural scientific approach, which is the norm in human behavioural research (Welman, Kruger & Mitchell, 2009:19-25). The paradigmatic assumptions for this study therefore included:

- The continuing need for adequate performance in mathematics
- Progress in educational research from a social constructivist perspective
- Mathematics confidence that curtails learners' mathematics performance

It is important to understand the stance of this study from the researcher's perspective, before interpreting the theoretical and paradigmatic views. For this reason, a short description of the researcher's philosophic aspirations follows.

3.3.1 Philosophic aspirations of the researcher

The researcher sees man[14] in the light of Bronowski's (2011:21) description that regards man as having a *set of unique gifts*. Being the *sculpture* and the *artist* and not merely the landscape, the learner as the artist of his/her own knowledge defines him-/herself as being unique as far as his/her own creative talents, knowledge, skills and beliefs are concerned. The researcher agrees with Bronowski (2011:19-21) that every individual, in body and in mind, is the explorer of *nature*[15] and a reflection of experiences and explanations bounded by that nature. The learner reflects on a multitude of factors varying between imagination, creativity,

[14] Reference is made to *man*, not as a restriction to gender, but to rather view the whole of mankind. In other words: the learner or the individual.
[15] Some synonyms for *nature* include personality, environments, life and character (Eksteen, 1997:1140).

reasoning and emotional subtlety, allowing change and development. One such development that has transformed over time is the field of mathematics.

According to Hawking (2005:11-12), no intellectual endeavour has more significantly changed our worldview as did Mathematics. Mathematics, as a *tool* and *language* for science (Hawking, 2005:12), is defined by Bronowski (2011:119) as a ladder for mystical and rational thought. Being able to reflect on previous and current experiences within Mathematics allows the individual to develop a cyclic connection between emotions and knowledge about oneself (Gibbs, 1988). In the realm of awareness of one's own emotions and thinking processes, the researcher aspires to examine and understand any possible relationship between the levels of mathematics confidence and the success in mathematics problem solving that is associated with reflection.

3.4 Paradigmatic perspective

Scientific enquiries consider scientific knowledge relevant and valuable when it describes and explains phenomena. The phenomena in this study were learners' self-confidence in doing Mathematics and their reflection on mathematics problem-solving processes. According to Cecez-Kecmanovic (2011:5), the research approach or paradigm serves as a theoretical framework. This paradigm, a consensus, differentiates between various scientific communities. In the quantitative part of this study, an interpretivist paradigm view was adopted as it seemed to fit the description of the typology given by Cecez-Kecmanovic (2011:5) of research paradigms in social sciences. As mentioned in paragraph 3.2, the current study involved two research approaches.

3.4.1 The positivist (quantitative) paradigm

Within the paradigm of the positivist views (quantitative), only observable and factual phenomena were emphasised over the theoretical (found in the literature) or metaphysical philosophies. In this study, the quantitative part constituted the positivist paradigm, as the emphasis was on numerical measurements, statistical analyses and graphical interpretations. The empirical usage of the data obtained by means of the questionnaires, along with the method of quantitative analysis, limited the inquiry to be an open scientific exploration of the possible relationships between the examined variables. These measuresments indicate that a level of anxiety and the components of metacognition exist in the sample. In order to understand these phenomena in more depth, a qualitative study was undertaken from an

interpretevist paradigm. The interpretivist or qualitative study was conducted parallel to the positivist research design. The results and findings were triangulated in order to answer the researcher question(s).

3.4.2 The interpretivist (qualitative) paradigm

In the qualitative part of the study, the researcher as interpretivist aimed to understand the social reality[16] as a phenomenon, and interpret the reasons for participants' actions and thoughts. This interpretivist approach gave meaning to individuals' experience without being affected by the researcher's judgement from any ethical or normative point of view (Cecez-Kecmanovic, 2011:6). The interpretation and explanations of the qualitative data were of a social-constructivist nature. Figure 3.1 illustrates the paradigms as implemented in the theoretical framework for this study.

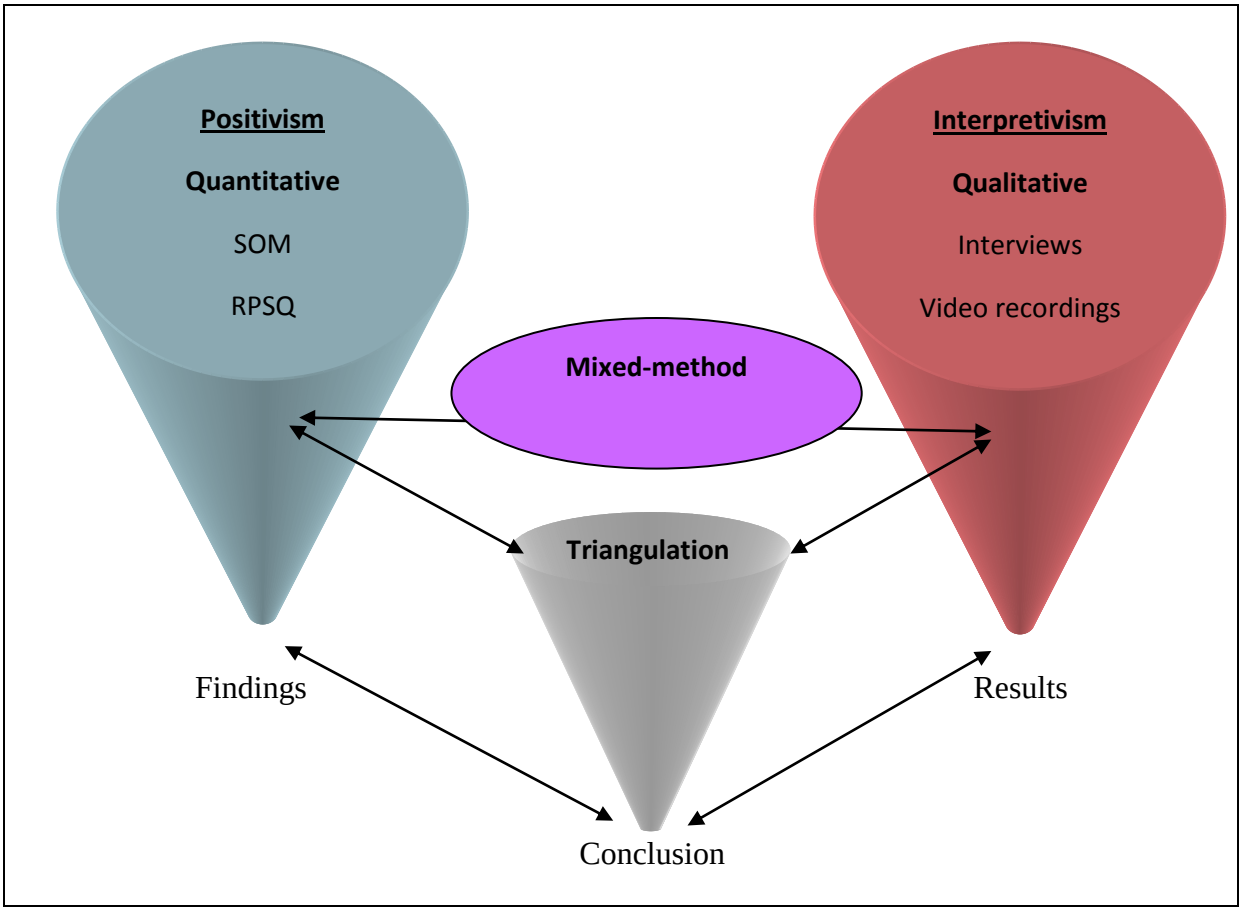


Figure 3.1 Overview of the positivist and interpretivist paradigms

Source: Adapted from Cecez-Kecmanovic (2011:7)

[16] For the purposes of this study, the social reality was considered learners' self-confidence in respect of Mathematics and their usage of reflection on mathematical processes.

In Figure 3.1 the funnel effect is clearly visible in the process of triangulation (discussed in paragraph 3.10). The positivist (quantitative) and the interpretive (qualitative) paradigms were applied in the study at different times and for different purposes as indicated by the exploratory nature of a convergent mixed-method research design (Creswell & Plano Clark, 2011:221). Firstly, the positivist view was employed to collect and analyse the quantitative data, while the interpretivist view was employed to collect and analyse the qualitative information. The results and findings of the two paradigms were supportive of this mixed-method design and triangulated the results and findings, and in a sense, merged the data. A description of the research methodology applied in the study follows next.

3.5 Research methods used

The researcher used an exploratory convergent design consisting of two analogous (parallel) studies that included quantitative and qualitative parts. The researcher alternated between the quantitative and qualitative data collection and analysis procedures. The exploratory convergent mixed-method design made use of non-experimental (quantitative and qualitative) approaches that ranged from the administration of questionnaires, solving a mathematics problem, video recordings, field observations, to individual interviews. Note how the quantitative study was integrated within the qualitative study, as illustrated in Figure 3.2.

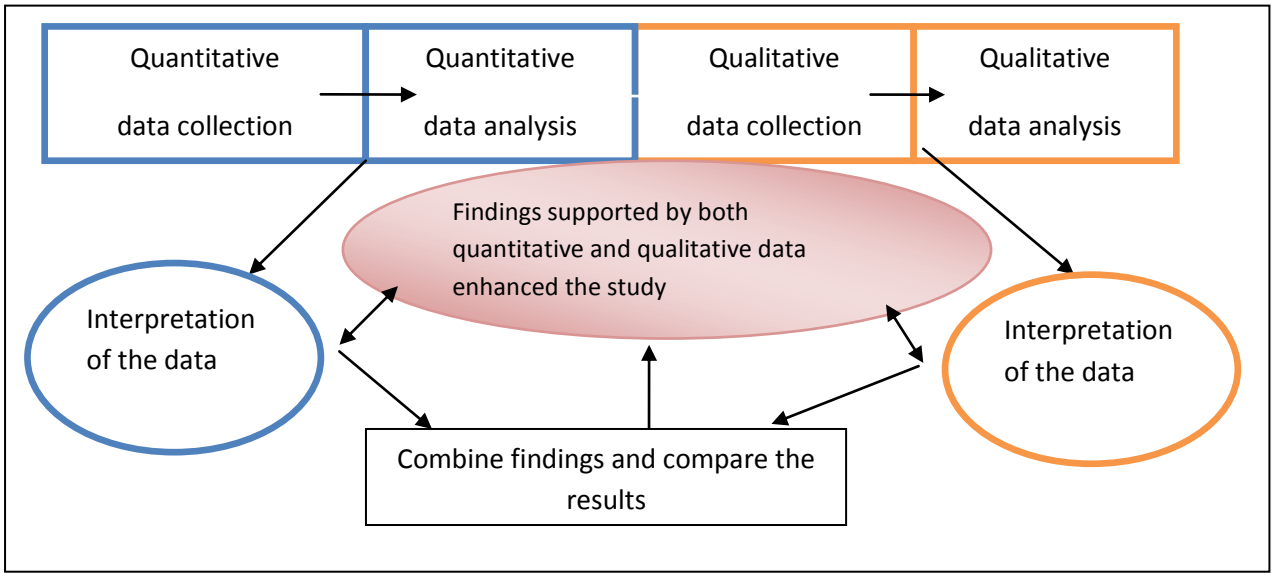


Figure 3.2 Exploratory convergent mixed-method research design

Source: Adapted from Creswell (2009:209)

The quantitative as well as the qualitative data was captured and organised for statistical analysis. When both analyses were completed, the quantitative results were triangulated with

the qualitative findings (based on video recordings, verbatim transcribed notes) to interpret and answer the research questions. This is followed by a summary of the results and concluding remarks in the discussion chapter.

3.6 Sampling and participants

This study made use of convenience sampling for the quantitative part of the study. Sampling strategies were considered based on economy and convenience (as described by Welman, Kruger and Mitchell (2009:56)), but eventually participants were selected for the qualitative part of the study by means of purposeful sampling.

The population and participants in this study included senior phase learners in three secondary schools in one of the educational clusters in the North-West Province. The medium of instruction in these schools was English and learners came from different socio-economic and cultural backgrounds. Both the participants and their teachers also came from various ethnic backgrounds. (The latter did, however, not form part of this study.) The sample included 609 Grade 8 and 9 learners, from a single cluster of secondary schools.

Arrangements were made with the principals[17] of the participating schools for learners to complete the questionnaires in the classrooms. This took place during the June 2011 exams, on days that the Grade 8 and Grade 9 learners did not write any exams but instead had a study, read and learn session.

3.6.1 School diversity in the sample

According to Welman, Kruger and Mitchell (2009:73), the research environment should show optimal similarity to real-life situations and therefore all data collection occurred inside the natural education environment, the classroom. The following schools were included in the study:

- School A: A secondary school for boys educated through medium of English First Language, who enjoy good socio-economic conditions and come from diverse ethnic backgrounds.

[17] This was done after approval was received from the Department of Education and the Ethics Committee of the North-West University. The parents also signed the informed consent form (letter to parents) referred to in paragraph 1.6.

- School B: A secondary school for boys and girls educated through medium of English First Language, who have a poor socio-economic background and show diverse ethnicity. This school is close to a township.
- School C: A secondary school for girls educated through medium of English First Language, who enjoy a good socio-economic background and show diverse ethnicity.

Note that this sample was not purposely selected for a retrospective design. However, School B was situated in a rural area, and the mother tongue of most of the learners in all three of the schools was Setswana.

Next follows a description of the sampling of respondents for the quantitative component of the study.

3.6.2 Quantitative sampling of respondents

Conceptually, each respondent was assessed according to two independent variables, namely reflection on Mathematics and mathematics confidence, and the relationship between these variables. The groups of units of analysis (learners) within the sample (n = 609) were divided according to the distinctions between schools as identified in Table 3.1.

Table 3.1 Summary of the quantitative sampling of learners

School	Number of Grade 8 learners	Number of Grade 9 learners	Medium of instruction	Gender depiction
A	122	129	English	All boys
B	72	86	English	Boys and girls
C	94	106	English	All girls
Total	288	321		

The sample (n = 609) included both boys and girls from the three selected schools in Grades 8 and 9 and was therefore representative of the population. The sample also displayed variance (heterogeneity) since it contained an all-girls school, all-boys school and a co-ed (mixed gender) school (see Table 3.1).

3.6.3 Qualitative sampling of participants

A purposive sample of four (n = 4) participants was invited to take part in the qualitative study. The participants represented learners in Grade 8 and 9 taking Mathematics as a compulsory subject. The sample (Maree et al., 2010:79) of participants included two learners with high achievement and two learners with an average achievement in Mathematics. According to their teachers, the purposively selected participants could express themselves verbally and would not be shy to share information about themselves. The participants included three girls and one boy. They were invited to the interviews to answer the secondary research questions regarding reflective behaviour and mathematics confidence.

3.7 Data collection instruments

The data was collected in analogous parts. The quantitative design was implemented when the researcher administered two quantitative instruments – the SOM and RPSQ. The qualitative design was implemented by conducting two interview sessions with the participants individually. The next paragraph describes the quantitative instruments that were employed.

3.7.1 Quantitative instruments

Two instruments, namely the SOM (Maree, Prinsloo & Claassen, 1997b) and the RPSQ (adapted from Lucangeli & Cornoldi, 1997; Schraw & Dennison, 2001:1-3; Maree, Prinsloo & Claassen, 1997; Fortunato et al., 1991:38; Wilson, 2001) were administered for purposes of quantitative data collection. A brief description of each of the instruments follows:

3.7.1.1 The Study Orientation in Mathematics (SOM) questionnaire

The SOM, developed by Maree, Prinsloo and Claassen (1997b) for Grade 7 to 12 learners, was administered seeing that it is standardised for South African learners. This questionnaire was used to assess the level of mathematics confidence in the cognitive and affective domains. The items in the SOM (Maree et al., 1997b) are based on a five-point Likert-type scale and learners are required to think about and rate their current or past experiences within the Mathematics context. The SOM includes 76 statements for Grades 7 to 9 and respondents had to choose a rating (rarely, sometimes, frequently, generally or almost always) by selecting one (1) to five (5) on a Likert-type scale. The following are some examples of the statements in the SOM (Maree, Prinsloo & Claassen, 1997):

- *When answering questions in tests or exams I tend to get nervous*
- *I get nervous in the Mathematics classroom*
- *I lose marks for correcting answers that were correct in the first place (cross out and do over)*

Table 3.2 illustrates the reliability of the SOM for the various grade, gender and language groups in the senior phase as stipulated in Maree, Prinsloo and Claassen (1997:25-26).

Table 3.2 Reliability of the SOM

Field of the SOM	Grade 8	Grade 9	Boys	Girls	English	Afrikaans	Other African languages
Mathematics confidence	0.71	0.73	0.70	0.74	0.74	0.83	0.69

3.7.1.2 The Reflection and Problem-solving Questionnaire (RPSQ)

An instrument was needed to measure reflection (the metacognitive handling) of learners in the sample taking senior phase Mathematics. A collection was made of the statements, theories and ideas of metacognitive skills, reflection on Mathematics and mathematics confidence made by the following researchers: Lucangeli and Cornoldi (1997); Schraw and Dennison (2001:1-3); Maree, Prinsloo and Claassen (1997); Fortunato et al. (1991:38) and Wilson (2001). A pilot study was undertaken to refine the Reflection and Problem-Solving Questionnaire for senior phase Mathematics, and this pilot study is next discussed briefly.

3.7.1.3 The pilot study for refining the RPSQ

A pilot study was conducted to determine whether the statistical analysis of the quantitative data (obtained from the measuring instruments) in fact validates the results of the study. The Statistical Consultation Services at the North-West University, which ensured that the data collected was sufficient for answering the proposed research question and the secondary research questions, supported the quantitative analysis.

A first draft of the RPSQ was *tested out* before administering it to the respondents in the sample. This was done by purposively selecting 43 learners taking Mathematics in the senior phase (Grade 9). These learners varied as far as background, gender, age, home language and

culture were concerned and they did not take part in the main study. The learners acted as respondents by completing the questionnaire and answering some questions afterwards. Some of these initial questions read as follows:

- *Did you understand what you have read?*
- *Is the font big enough for you to read clearly?*
- *Are there any questions you did not understand?*
- *Was the problem difficult or easy?*

Afterwards, the responses were considered and, in consultation with my promoter and statistician, I decided that the ratings in the Likert-type scale should be reduced from five options to four. The various statistical approaches were discussed with the statistics consultant. After gaining enough information from the sample respondents, the analysis procedures were planned and the questionnaire was language edited (past to present tense). The mathematics problem was adapted to make it easier for the respondent to picture/imagine the problem in real life. Eventually, the statistics consultant, language editor, some colleagues in education, critical readers and study promoter agreed that the instrument piloted was ready to be administered for gathering data for the main study.

3.7.1.4 The RPSQ as adjusted following the pilot study

The final instrument consisted of three sections. In the first section (1.1) respondents had to solve a Mathematics problem that was adapted by the researcher using examples from Fortunato et al. (1991:38). The problem was assessed and given a mark out of five. This mark then represented the problem-solving achievement of the respondents. Since this study only focused on reflection and mathematics confidence during problem solving, the achievement was not analysed. Instead, the Mathematics word problem provided the respondents with something to reflect on.

After the problem was solved, learners had to respond to statements in the second section (1.2) taken from Schraw and Dennison (2001:1-3) and Fortunato et al. (1991:38). The statements were rated according to a four-point Likert-type scale, similar to the one used in the SOM. The Likert-type scale assessed the respondents' application, awareness, understanding and use of their ability to reflect on mathematics experiences and applications.

This assessment was done on a scale that allowed them to choose between the following four ratings: *almost never*, *sometimes, usually* and *almost always.*

In the third section (1.3) respondents wrote what they did in each step of the solution to the Mathematics problem and why they did it. This had to be done in complete sentences, to give the researcher an idea of which respondents could put their thoughts and actions into words. This instruction provided them with a resource (their own words) that measured their ability to write down their reflections (like journal keeping). The written reflections were not analysed as they served as a means or a tool to engage in a reflective activity. They did, however, allow the researcher to measure the learners' reflective behaviour.

Table 3.3 summarises the reliability of the RPSQ for the variables as measured by it.

Table 3.3 Reliability of the RPSQ

Variable	Cronbach *alpha*
Monitoring task	0.71*
Planning task	0.61*
Monitoring task, strategies and person	0.81*
Planning task, strategies and person	0.62*

(* According to Cotterall and Murray (2009:35), coefficients of 0.5 or higher indicate reliability.)

3.7.2 Qualitative instruments

Participants generally have difficulty expressing their metacognitive thoughts (Wilson, 2001). Difficulties associated with assessing internal processes like metacognition and other cognitive and unobservable thoughts constitute a problem for assessing the reliability of the study (Panaoura & Philippou, 2005:262). According to Wilson (2001:5), these difficulties can be overcome if the internal processes are *externalised.* In the quantitative study, the solving of the word sum and the writing down (wording) of reflective thoughts formed part of and contributed to this externalisation. Thoughts can be externalised by using statement cards (a qualitative approach) as suggested by Wilson (2001:5), as well as playing back video recordings as part of the interview process to allow and help the participant to recall what happened. With this in mind, the qualitative instruments in this study included metacognitive statement cards and two video-recorded interview sessions.

3.7.2.1 The metacognitive statement cards

The idea of the statement cards was taken from Wilson (2001). These cards provided the participants with the necessary language to express their thoughts and their thinking about their own thinking. According to Wilson (2001:5) the cards eliminate the '*I don't know'* responses during the interviews and serve as validation of the verbal reports. Researchers like Randhawa (1994), Leaf (2006) and Davis (2006:285) also referred to such metacognitive statements as cues or metacognitive prompts.

Each of the 15 metacognitive statement cards represented one aspect of the functions of metacognitive processes: prediction, awareness, monitoring, regulation, evaluation and cognitive handling (Van der Walt, 2006:85). Some examples of the wording on the statements cards are as follows:

- *I thought about something that I had done in the past that was helpful* (Knowledge of strategies)
- *I thought about a different way to solve the problem* (Regulation of strategies)
- *I looked back at each step before I continued* (Monitoring)
- *I drew a diagram to better understand the question* (Knowledge of task, understanding)

The metacognitive statement cards were only employed during the first interview session. A discussion of all the instruments used follows below.

3.7.2.2 I**nstruments used during the interviews**

During the first interview, the following instruments were employed:

- Metacognitive statement cards
- A Mathematics problem
- A video camera for video recordings
- Two labelled boxes for the statement cards
- Some blank cards

- Video recording playback
- Semi-structured open-ended questions
- Verbatim transcriptions

In the second interview, the researcher asked questions about the participant's experiences with and in Mathematics. The qualitative open-ended questions led to in-depth conversations about the participants' early memories of and experiences with Mathematics.

3.7.3 The role of the researcher

Conducting an in-depth literature review, knowing what information to obtain from the participant and identifying participants suitable for providing information were all functions added to the role of the researcher.

The researcher developed a collaborative partnership with the participants and respondents (Maree et al., 2010:41). This allowed him to delve deeper and to explore the phenomena and consider the variables mathematics confidence and reflection on Mathematics during problem-solving activities (refer to Figure 1.6 and the conceptual framework in Figure 2.9). The following functions suggested by Joubert (in Maree et al., 2010:41) formed part of and clarified the role of the researcher:

- Scheduling appointments for the interviews with participants
- Developing open-ended questions
- Selecting an appropriate mathematics problem
- Preparing and conducting interviews (video recordings, statement cards, notes and qualitative questioning)
- Making verbatim transcriptions
- Analysing data
- Assessing the solution to the Mathematics problem
- Interpreting and triangulating the data

It should be kept in mind that the researcher had no control over the participants' gender, age, race, physical appearance, or background – all of which might have affected the study in some way.

3.8 Data collection procedures

The data was captured, organised for statistical analysis and processed (quantitative component of the study). Afterwards, participants were invited to take part in the individual interviews (qualitative), which were intended to better understand and explore the research question(s).

3.8.1 Procedures for quantitative data collection

With reference to Figure 3.2 in the convergent mixed-method exploratory research design, the first phase of the study commenced with quantitative questioning. After permission had been granted by the appropriate authorities and arrangements had been made with the participating schools, the SOM (Maree, Prinsloo & Claassen, 1997b) and the RPSQ (as adapted from Lucangeli & Cornoldi, 1997; Schraw & Dennison, 2001:1-3; Maree, Prinsloo & Claassen, 1997; Fortunato et al., 1991:38; Wilson, 2001) were administered. The three participating schools scheduled appropriate time slots in the June 2011 exam timetable to ensure that the learners do not miss any schooling when they completed the questionnaires. The instruments were administered in the following order: first the SOM, followed by the RPSQ. Details about the administering of the instruments follow in the next paragraph.

3.8.1.1 Administering the SOM

The Study Orientation in Mathematics (SOM) by Maree et al. (1997b) for Grade 7 to 12 learners was administered as it is standardised for South African learners. The instrument was used to assess the level of Mathematics confidence in the cognitive and affective domains. Although the SOM measured the respondents' level of Study Orientation in Mathematics, only the domain of Mathematics anxiety was regarded of significance for this study. Approximately 30 minutes was allowed (Maree et al., 1997) for completion of this questionnaire. Thereafter the respondents had a short break (about 15 minutes) and then completed the second questionnaire, the RPSQ.

3.8.1.2 Administering the RPSQ

Next, a mathematics problem was administered to assess respondents' mathematics achievement, followed by their completion of the Reflection and Problem-Solving Questionnaire (RPSQ) for senior phase Mathematics. Learners assessed their ability to apply metacognitive skills, reflection and awareness of reflective practices during their mathematics problem solving. The instrument took about 40 minutes to complete and measured respondents' reflective skills and achievement when solving a Mathematics problem. The data was subsequently organised, arranged and prepared for the data analysis.

3.8.2 Procedures for qualitative data collection

The qualitative research was seen as an approach rather than a design or a fixed set of techniques (Welman, Kruger & Mitchell, 2009:188). These approaches comprised various interpretive procedures that were employed in this study. Open-ended questions, coding, transcription of notes and video recordings were ways of collecting data of the natural phenomena – in this case, reflection on Mathematics and mathematics confidence during mathematics problem-solving processes (Welman, Kruger & Mitchell, 2009:188).

Open-ended questions in semi-structured interviews, metacognitive statement cards associated with metacognitive functions, together with video recordings, served as resources in this research approach. A description of the procedures employed in the individual interviews follows next.

3.8.2.1 Individual interviews

Before the interviews were held, ethical clearance was considered (see paragraph 3.11 on ethical issues). After the necessary permission had been obtained from the parents, the Department of Education (Addendum B), school principals and the North-West University's ethics committee (Addendum E), preparation started for the interviews. Each individual interview was conducted in two sessions.

- **First session**

In the first session participants had to read the metacognitive statement cards (see paragraph 3.7.2.1) in order to make sure they understood the statements and meanings on the cards. Whenever they were unsure about any of the statements, the researcher explained the meaning to clarify any uncertainties. Afterwards the participant had to solve a Mathematics

problem based on the area of circles and his/her attempt to solve the problem was video recorded. Once the problem was solved, the participant had to select those metacognitive statement cards that he/she had used and put them in one box (labelled 'used'), while the ones not used were put in the second box (labelled 'not used'). Blank cards and a pen were also provided in case the participant wanted to add to the metacognitive statements.

The participant then had to arrange the cards in the order that he/she remembered using them, starting from the beginning of the problem-solving process (i.e. the moment the Mathematics problem was given). The video recording was then played back via the computer and data projector. The researcher played, paused, rewound and stopped throughout the video playback and asked the participant semi-structured open-ended questions about what he/she had been doing or thinking at a particular moment. The participants were asked questions regarding what they had done, what they had been thinking and why they did what they had done. The participants were allowed to change their selection and order of the metacognitive statement cards at any time. The interview was then verbatim transcribed for further analysis.

- **Second session**

In the second interview session, participants were asked semi-structured questions about their experiences with and in Mathematics, currently and in the past. Three types of qualitative questions were asked: main questions, probes or exploring questions, and follow-up questions (Bormotova, 2010:69). The qualitative open-ended questions asked during the second session were adapted from some questions by Swanson (2006). Examples of these questions included the following:

(a) *Did the teacher(s) you had for Mathematics have any effect on your fondness or dislike of Mathematics? How so? (Please give as much detail as possible.)*

(b) *In your opinion, what makes Mathematics difficult or easy to understand?*

(c) *What do you think being confident means?*

Semi-structured interviews were conducted for the following specific purposes as identified by Welman, Kruger and Mitchell (2009:197): to identify the important variables so to formulate breakthrough questions about the variables and to generate a hypothesis for further investigations. In the unstructured interviews the researcher mentioned a theme and a topic of discussion, and posed a question while the discussion contained open-ended questions and answers. Note that although the unstructured interviews did not need to follow an interview

schedule, the researcher nevertheless prepared questions that served as a framework for the interview. Fontana and Frey (1999) state that a wealth of information can be gathered from conducting unstructured interviews. The researcher observed each participant during the interviews while asking open-ended questions. He intentionally did not ask questions of a sensitive nature so as to keep the participant from feeling nervous, embarrassed or uncomfortable. In this case, the researcher acknowledged the participant's rights and considered possible ethical issues.

A discussion on the analysis procedures follows in the next paragraph.

3.9 Analysis procedures: quantitative and qualitative parts of the study

A description of the analysis procedures of the quantitative and qualitative data was necessary to gain insight into how the data had to be arranged and prepared.

3.9.1 Analysis of the quantitative data

The five-point scale SOM (Maree, 1997a) and the four-point scale RPSQ, as adapted by the researcher (Schraw & Dennison, 2001:1-3, Wilson, 2001, Lucangeli & Cornoldi, 1997:121-139; Fortunato et al., 1991:38) were implemented. Data was analysed quantitatively. The statistical procedures that were applied included the following:

3.9.1.1 Exploratory factor analysis of the SOM and RPSQ

Exploratory factor analyses were performed to examine the internal reliability of the measures (Newsom, 2005:2). Any theoretical constructs or factors present in the items were investigated to determine if they were correlated or orthogonal. The Cronbach alpha values of the data were calculated to determine the reliability of the constructs in the measuring instruments.

First, an exploratory factor analysis was performed on the SOM. The instrument contains 76 statements but only the 14 statements measuring respondents' confidence in respect of Mathematics was considered. The subsequent factor analysis of the RPSQ revealed a pattern matrix that was later reduced from 12 to 4 factors.

3.9.1.2 Descriptive statistics

The following descriptive statistics were calculated on the construct scores:

- Averages ($\bar{x}$) of the responses in the three schools were determined for each factor measured by the SOM and RPSQ.
- Percentages of data regarding language, gender and grade were descriptive of the respondents' biographical data.
- Standard deviation and effect sizes determined any practical significance between the groups (schools)

3.9.1.3 Inferential statistics

Interferential statistics included the following analyses:

- Chi-square analysis
- Analysis of variance (ANOVA)

Constructs were compared to investigate the relationship between mathematics confidence, reflection on Mathematics and achievement in problem solving by using Spearman rank correlations.

3.9.2 Analysis of the qualitative data

The qualitative data was first organised, then analysed. Three phases of analysis procedures were involved, namely content analysis, verbatim transcriptions and coding.

3.9.2.1 Content analysis

Content analysis was applied to analyse and code the notes of observations, verbatim transcriptions, responses to open-ended questions and video recordings obtained from the individual interviews (Maree et al., 2009:101). Analysis was done during and after both interview sessions. According to Welman, Kruger and Mitchell (2009:221), *frequencies and sequencing of particular words, phrases or concepts* described the qualitative data as quantitative. The content was also described and summarised using network display analysis.

3.9.2.2 Verbatim transcriptions of the qualitative data

Data obtained from the video recordings were transcribed verbatim and included non-verbal cues. Laughter, gestures, pausing to think, silence, frowning or any other form of non-verbal

behaviour or reaction are examples of cues mentioned in the literature (Fortunato et al., 1991). The transcribed data was then coded for further analysis.

3.9.2.3 Coding the qualitative data

Coding started by means of descriptive words or themes as found in the literature. Although the interviews were semi-structured, the researcher made use of a priori coding (first interview session) as well as inductive coding (second interview session) (Maree et al., 2009:107). The verbatim transcribed notes were coded by reading through every sentence and labelling them according to meaningful analytical themes or a priori codes. Patterns were identified and participants' reaction and responses were compared and summarised in terms of these themes. Three coding procedures were used.

- **A priori codes in the first interview session**

Table 3.4 shows a priori codes that were used to analyse the transcriptions of the first session of the individual interviews. This table stands parallel with Table 5.2 regarding the identified codes.

Table 3.4 A priori codes for the qualitative data from the first interview

Theme	Code	Category
K Metacognitive Knowledge	KP	Reflection on knowledge of the self (person)
	KT	Reflection of knowledge of the task
	KS	Reflection on knowledge of strategies
R Metacognitive regulation	RU	Reflection on regulation of understanding
	RP	Reflection on regulation of planning
	RM	Reflection on regulation of monitoring
	RE	Reflection on regulation of evaluation

Source: Adapted from Yeap (1998:2), Wilson (2001), Strawderman (2010) and Bormotova (2010)

The codes in Table 3.4 were identified prior to the interview as they included themes relevant to the research topic. The following steps were taken to analyse the first interview sessions (Bormotova, 2010:83):

a) Firstly, the interviews were transcribed verbatim, entered into the computer as a Word document and saved using pseudonyms for the participants (Learners A, B, C and D).

b) Secondly, a priori codes were identified and categorised in the transcriptions. The researcher read and reread the transcriptions and labelled related sentences or words according to the a priori codes. The particular sentences, words or phrases were then cross tabulated according to the codes in Table 3.4. These labelled responses were subsequently compared to determine if there was a pattern or structure in participants' problem-solving orientation, heuristic problem-solving behaviour, functioning in the cognitive domain, affect and/or reflective attributes.

c) Codes represented in the transcriptions (in terms of responses and phrases) were summarised and then tabled in four comparing columns. The columns included the identified codes for each participant. Patterns emerging from the data were compared between participants' overall application of metacognitive strategies. Care was taken to identify which reflection strategies or skills the participants implemented, and at what stage of the problem-solving process the implementing took place.

d) Similar patterns for the different themes were joined together and comparisons were made between all participants and relevant themes.

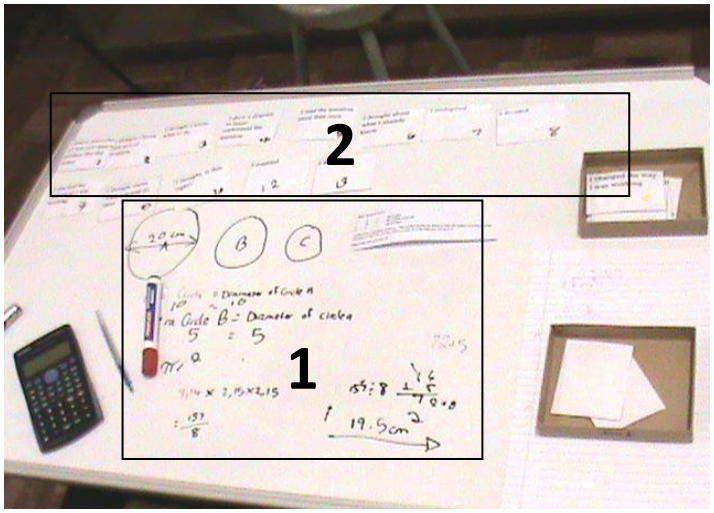
e) The patterns found were summarised and discussed.

f) Finally, the findings were reported and interpreted in terms of the theories that emerged from the review of the literature (Chapter 2) and compared with similar studies done in the past by other researchers.

The participants also had to make a selection from the metacognitive statement cards. After the Mathematics problem was solved, these cards were placed in the order they had been used and then numbered. A discussion of the codes that were used to analyse the order of the statement cards follows in the next paragraph.

- ***A priori codes for analysing the order of the metacognitive statement cards***

An example of the arrangement of the cards and the solved Mathematics problem is given in Picture 3.1.



Picture 3.1: Attempted solution of the Mathematics problem with statement cards

In this example, and throughout all first session interviews, the participants solved the Mathematics problem (labelled 1 in the photo) and thereafter made a selection of the metacognitive statement cards (see paragraph 3.8.2.1). The cards were then arranged and numbered after being placed in sequence (labelled 2 in the photo). Because the statement cards provided the participant with the necessary vocabulary or language (see paragraph 3.7.2.1), the cards were used to determine what metacognitive functions (knowledge or regulation) or cognitive handling was implemented during which specific phase of reflection (before, during or after action). This validated the statements made while watching the video recording.

A priori codes (Table 3.5) were also used to analyse the selection and order of the metacognitive statement cards. Note that metacognitive regulation was seen as regulatory awareness, regulatory monitoring and regulatory evaluation (Legg, 2009:17).

Table 3.5 A priori codes for analysing the order of selected metacognitive statement cards

Metacognitive components	Description of statement	No.[18]
Knowledge of the person /self (KP)	Awareness of own cognitive processes	KP1
Knowledge of the task (KT)	Identifying the type of Mathematics problem	KT1
	Knowledge about the problem	KT2
	Familiarity with the problem	KT3
Knowledge of the skills and/or strategies (KS)	Routine/non-routine strategies identified	KS1
	Past experiences with similar problems	KS2
	Alternative approaches	KS3
Regulation of understanding (RU)	Interpreting the problem	RU1
	Rereading or drawing, modelling the problem	RU2
Regulation of planning (RP)	Altering the approach	RP1
	Planning or predicting approaches	RP2
Regulation of monitoring (RM)	Looking back	RM1
	Checking current developments	RM2
	Controlling successes	RM3
Regulation of evaluation (RE)	Determining the correctness of the answer	RE1

The following steps were employed to analyse the order of the cards:

a) Firstly, a photo (see Picture 3.1) of the order in which the cards was taken.
Each card represented a metacognitive function or cognitive handling (see paragraph 3.7.2.1).

b) Secondly, the video recording was played back to identify the metacognitive statement(s) that were used during particular phases of reflection (before, during or after). The phases of reflection were determined by visible (observable) regulatory actions such as starting the next step, getting an answer on the calculator or going back to reread the question.

c) The order of the cards was then used to categorise the themes identified in Table 3.4 in the order in which the functions (understanding, planning, monitoring or evaluation) had been used.

d) The information found was compared to determine whether there was a pattern.

e) The findings of the first interview session were lastly summarised, interpreted and discussed.

The second interview session also needed to be coded and analysed.

[18] This number represents the category, code and relevant statement and should be read in accordance with Table 5.12.

- **Coding the second interview session**

In the second interview, inductive coding was used to analyse the data obtained. Categories were determined in the theoretical framework, including for the social, psychological and intellectual domains. The study involved an interpretive natural approach (Bormotova, 2010:73) towards the understanding of the research topic. The interview was verbatim transcribed, after which the following analysis procedures were followed:

a) The information was organised and arranged according to Welman, Kruger and Mitchell (2009:216-217), as well as Maree et al. (2009:106).

b) The data was read, reread and compared with reflective and observation notes.

c) Network displays (like mind mapping) were created to link questions with the participant's answers, thus creating an overview of the participant's profile.

d) Because participants had to reflect on past experiences, the researcher also used a matrix diagram to list events (Welman, Kruger & Mitchell, 2009:220-221).

e) Each part of the network and matrix was divided into meaningful themes relating to the research topic.

f) These themes were categorised (broken down into sentences relating to a particular theme) and coded (labelled with key words or phrases represented by abbreviations or codes).

g) Codes were compared to determine possible patterns between themes and participants' responses.

h) Interpretation, discussion and summarising were done before the process of triangulation was proceeded to.

3.10 Triangulation: merging the quantitative and qualitative data

Triangulation involved the combination of the reports based on the quantitative and the qualitative analyses. This enhanced the reliability and trustworthiness of the study in both stages of the design. The researcher also applied the process of triangulation to ensure quality verification. As defined by Bormotova (2010:110), triangulation entails the use of multiple methods, data collection strategies and/or data sources, and the sequential combination of the results for comparison purposes.

The results of both the quantitative and qualitative data were merged. Results from the quantitative study were employed to better understand the results from the qualitative study and vice versa. Figure 3.4 illustrates this process of triangulation as employed in the study.

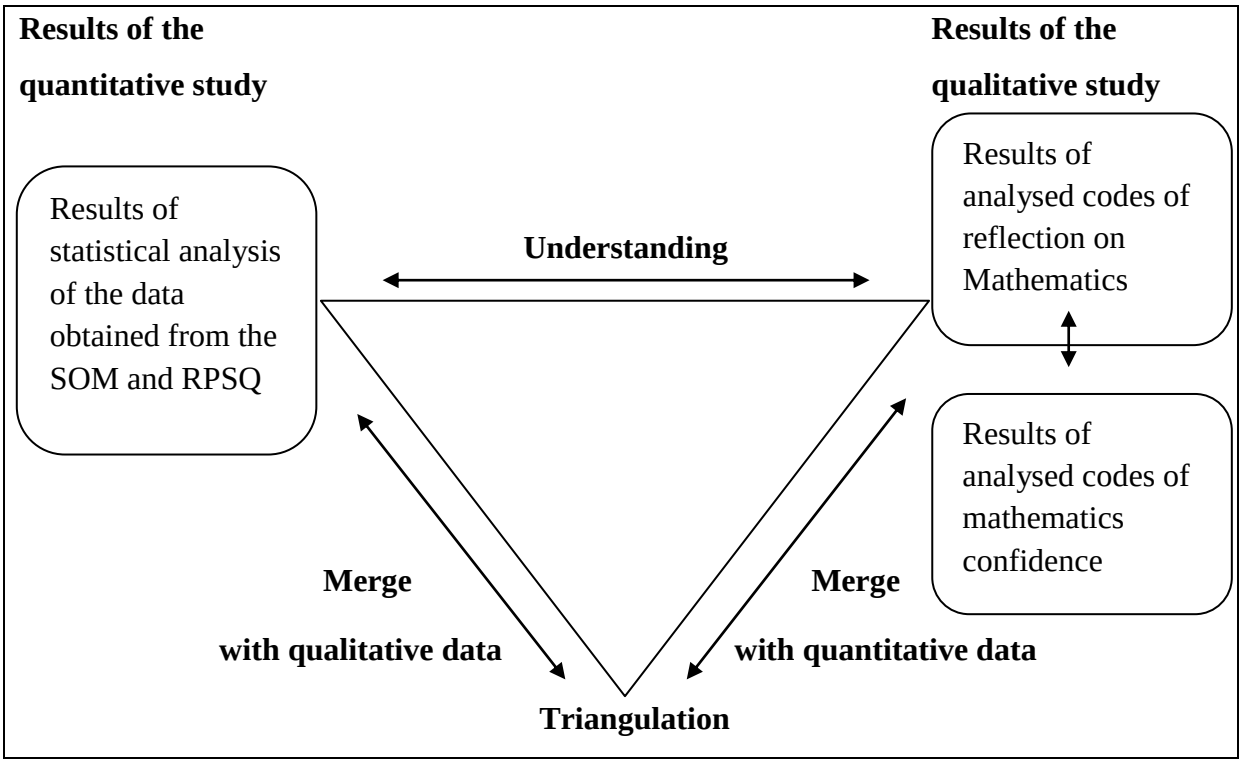


Figure 3.3 The process of triangulation

Source: Adapted from Creswell (2003:196) and Creswell (2011:221)

Figure 3.4 illustrates that the results of the two stages of the study were merged, and in a sense this implies a triangulation of the conclusions. Conclusions were made firstly about the quantitative results and secondly about the qualitative findings. The process of triangulating the quantitative and qualitative data is discussed in the paragraph below.

3.10.1 The role of quantitative data in triangulation

First, the results obtained from the instruments (SOM and RPSQ) were arranged and compared. Statistical analysis was done to determine reliability, as well as to compare and identify significant findings in the analysis. Followed by a discussion, these steps served as the first phase of triangulation. An understanding of the results was important for clarifying observed behaviour in the data obtained from the qualitative phase.

3.10.2 The role of qualitative data in triangulation

Secondly, the data obtained from the qualitative part of the study was organised into two main themes: first, reflection on metacognitive knowledge and regulation during the problem-solving processes (in the first interview), and second, mathematics confidence (in the second interview). Qualitative analysis of the individual interviews and personal observations was used to build a coherent set of coded categories. The findings obtained from the analysis of these coded categories were seen as the second phase of triangulation.

Triangulation enhanced the trustworthiness and reliability of the findings in this study through an in-depth understanding – the third phase – of the phenomena mathematics confidence and reflection on metacognitive components.

3.10.3 Summary of the research design

The research design included quantitative and qualitative approaches. In this study, learners were assessed in terms of their awareness and use of reflection on Mathematics as a metacognitive skill and the role that mathematics confidence played during Mathematics problem solving. This was done by sampling 609 respondents in three selected schools, with each respondent completing the SOM (Maree, 1997a) and RPSQ (Schraw & Dennison, 2001:1-3, Wilson, 2001, Lucangeli & Cornoldi, 1997:121-139; Fortunato et al., 1991:38). Four participants were subsequently invited to two individual interview sessions. Quantitative and qualitative analyses were made and the results were triangulated.

Figure 3.5 summarises the research design as discussed in Chapter 3, according to the different parts of the study.

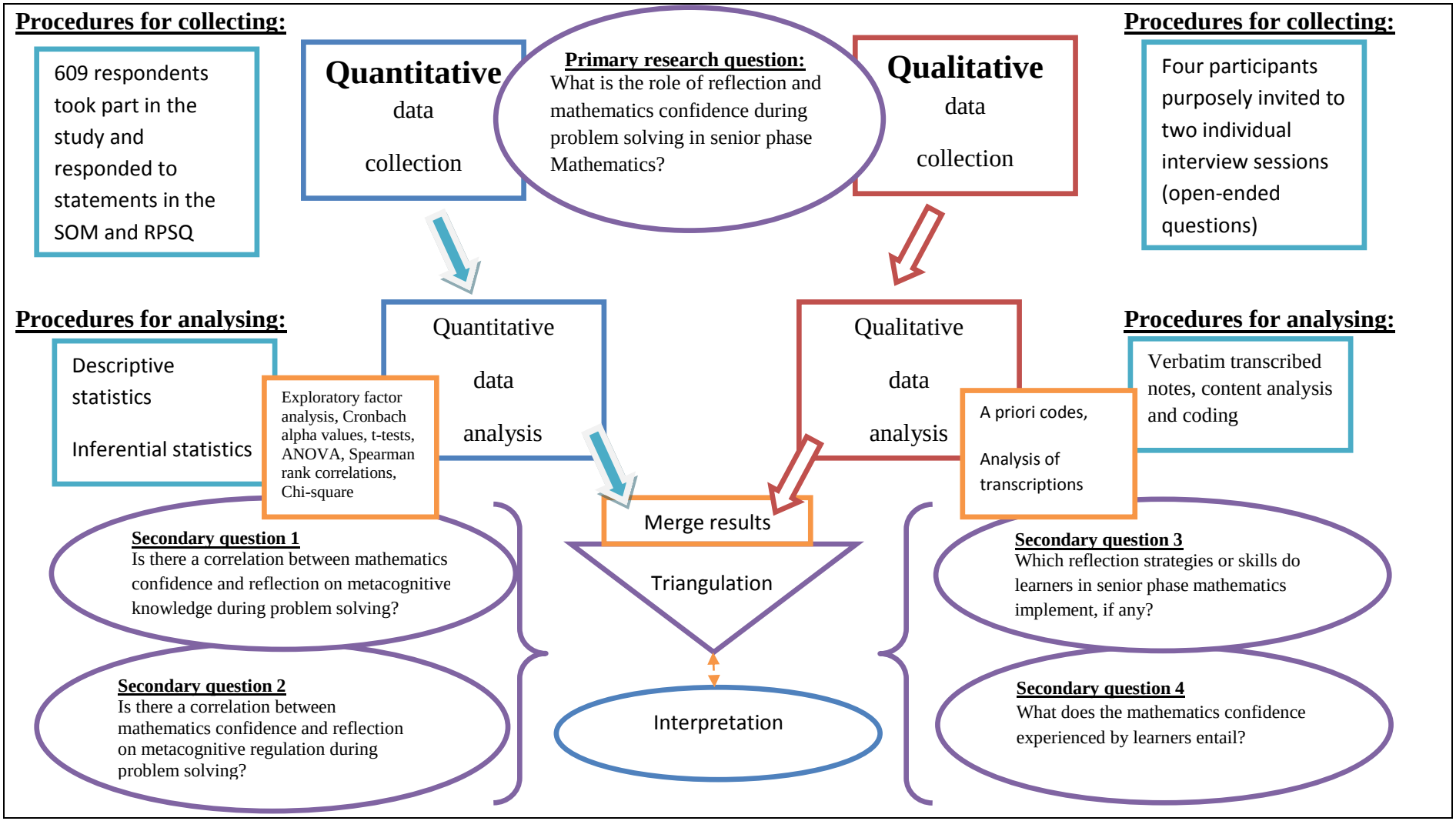


Figure 3.4 Summary of the exploratory convergent parallel mixed-method design

Source: Adapted from Creswell (2011:118)

3.11 Ethical considerations

According to Leedy and Ormrod (2005:101), ethical implications that are at issue involve protection from harm, informed consent, privacy and honesty.

Permission was obtained from the North-West Province's education authorities in Potchefstroom, as well as from the school principals who agreed to allow learners in their schools to participate. Written permission was sought and obtained from the parents of the learners who participated in the interviews. Learners' parents, teachers, school principals, and district offices signed an informed consent form that allowed participants to engage in the research. Respondents had to write their names on the answer sheets so that the researcher could identify the respondents when arranging and organising the data, yet no names were referred to in the results or summary of this research. The following ethical issues (see Table 3.6) derived from Creswell (2009: 89); Van der Walt (2006:133), Maree et al. (2010:42) and Welman, Kruger and Mitchell (2009:201-202) were also considered:

Table 3.6 Ethical issues considered

Ethical issues	**Considered approaches**
Anonymity	Learners' right to privacy was protected by using pseudonyms – thus keeping them anonymous at all times
Confidentiality	All information regarding respondents, participants and schools was kept confidential
Biased usage of language	Language or words in the study were not biased against any person or organisation
Emotional /physical risk	No respondent or participant was at risk of any physical or emotional harm
Rights of participants	Respondents and participants were not disempowered or marginalised and were free to withdraw from the study at any time
Generalisation of results	Findings and discussions were generalised to the sample population with great care
Research instruments	Research instruments suitable to answer the research problem were used

Some limitations of the current research will now be discussed.

3.12 Limitations and research problems

Due to the limited number of respondents and participants, the results have been restricted to the specific described sample and great care will be taken not to generalise findings outside the sample. Researchers may vary their paradigmatic perspectives and therefore the study can also be viewed from another angle.

The schools in the sample are individual representatives of single gender and co-ed schools and the results should therefore not be generalised to all-girls, all-boys or co-ed schools. The instrument that was adapted by the researcher for the quantitative study (RPSQ) had been standardised for American learners. Although the researcher adapted the RPSQ to measure the reflective abilities of South African learners, the instrument may yield different results across different provinces, districts, cities, towns or schools.

In Chapter 4, the results of the quantitative study will be presented and this excludes analysis of other affective factors such as attitudes, views and beliefs. In the SOM's analysis, not all items were analysed as only mathematics confidence was under consideration. In the analysis of the RPSQ results, Item 1.3 of the questionnaire required the respondents to write down their reflections. Although this allowed them to confirm their responses on the Likert scale (Bormotova, 2010) these written reflections were not analysed in this study.

Chapter 4

Analysis of the quantitative data

4.1 Introduction

Chapter 3 described the mixed-method research methodology as it was used to explore reflection and mathematics confidence during mathematical problem solving. The statistical procedures that were employed to analyse the data obtained from the two administered questionnaires are presented in this chapter, thus elucidating the quantitative part of this study.

The presentation on the data collected in the quantitative part of the study was done with the conceptual framework of Figure 4.1 in mind. At the end the two studies (quantitative and qualitative) were merged and through triangulation the research questions were answered.

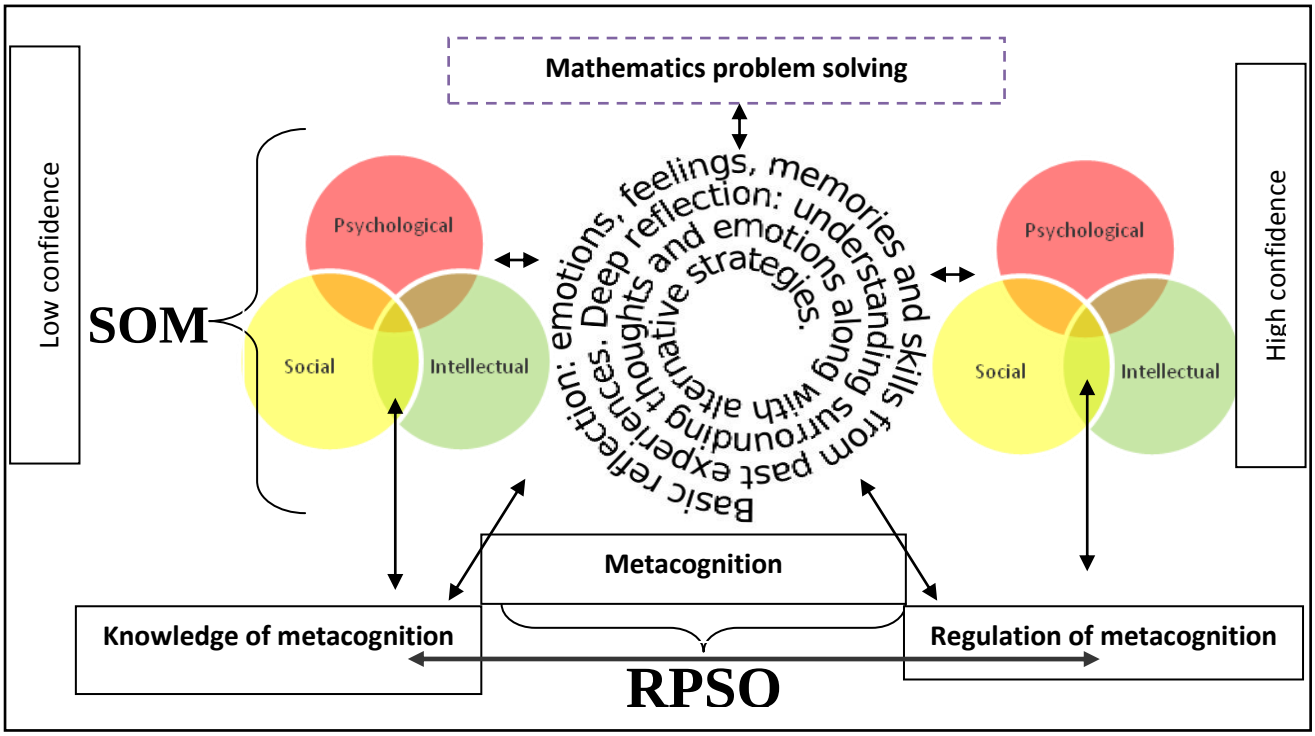


Figure 4.1 Theoretical framework for mathematics confidence and metacognitive reflection

Source: Adapted from Strawderman (2010); John (2000); Ertmer and Newby (1996)

This figure serves a different purpose than Firgure 2.9 as it indicates what constructs (mathematics confidence and metacognition) are being measured in the quantitative part of the study. The quantitative results were obtained by administering two questionnaires (SOM and RPSQ) to Grade 8 and 9 learners in three conveniently sampled schools. The SOM

measured respondents' mathematics confidence, while the RPSQ measured their reflections during mathematics problem solving.

4.1.1 Research focus during analysis

The Study Orientation in Mathematics (SOM) questionnaire by Maree, Prinsloo and Claassen (1997) measures seven fields of the cognitive, external, internal and intrapsychological orientation in Mathematics of respondents in Grades 7 to 12. The SOM consists of 76 items measuring attitude towards study, mathematics confidence, study habits, problem-solving behaviour, study milieu, information processing and study orientation. For the purposes of this study, and to answer the research questions, the focus was on mathematics confidence (14 items). The other fields were excluded from the analysis (Maree et al., 1997). The Reflection and Problem-Solving Questionnaire (RPSQ- see Table 4.8) for senior phase Mathematics consists of 44 items, a Mathematics word problem and an instruction to respondents to write down what they do in each step when solving the word problem, as well as why they do what they do.

4.1.2 Research questions

The primary research question to answer in this study was the following:

What is the role of reflection and mathematics confidence during problem solving in senior phase Mathematics?

Table 4.1 Overview of the quantitative data analysis

Analogous part in the study	Secondary research questions	Secondary research question(s)	Variables	Data collection method	Data analysis method
Quantitative data analysis	Secondary research question 1	Is there a correlation between reflection on metacognitive knowledge and mathematics confidence?	Metacognitive knowledge components and mathematics confidence	Administering the SOM and RPSQ	Factor analysis, reliabilities, Spearman rank correlations
	Secondary research question 2	Is there a correlation between reflection on metacognitive regulation and mathematics confidence?	Metacognitive regulative components and mathematics confidence		

An outline of the statistical analysis employed in this study, along with the relevant paragraphs, is presented in Figure 4.2.

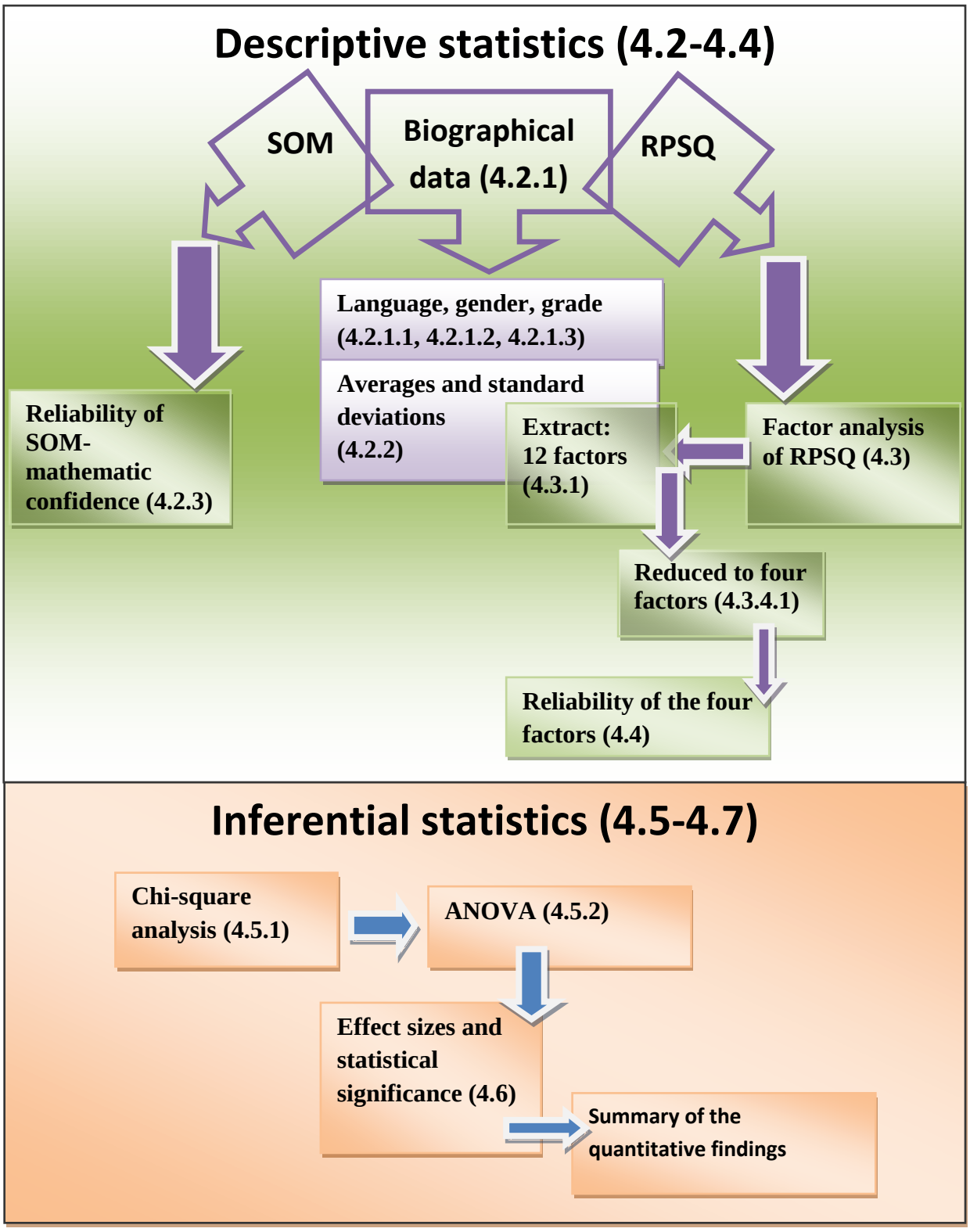


Figure 4.2 Order of statistical analyses and presentation

4.2 Descriptive statistics

The first part of the descriptive statistics refers to the biographical data of the respondents.

4.2.1 Biographical data of the respondents

The three schools that took part in this study are referred to as School A, B and C. Of these secondary schools, School A represented an all-boys school, School B a co-ed school for boys and girls, and School C an all-girls school (see paragraph 3.6.1). The biographical data of the respondents in the participating schools reveals frequencies of respondents' home language, gender and grades. We first consider home language.

4.2.1.1 Frequencies of respondents' home language

With such diverse backgrounds and ethnicity, nine groups of languages were identified as summarised in Table 4.2.

Home language	Afrikaans	English	Setswana	isiXhosa	N.Sotho	S.Sotho	isiZulu	Tshivenda	Other	**Total**
Total	12	84	283	26	18	37	28	4	117	**609**

Table 4.2 Frequencies of respondents' home language

Figure 4.3 contains a 3-D pie chart that shows the individual percentages of respondents' home language.

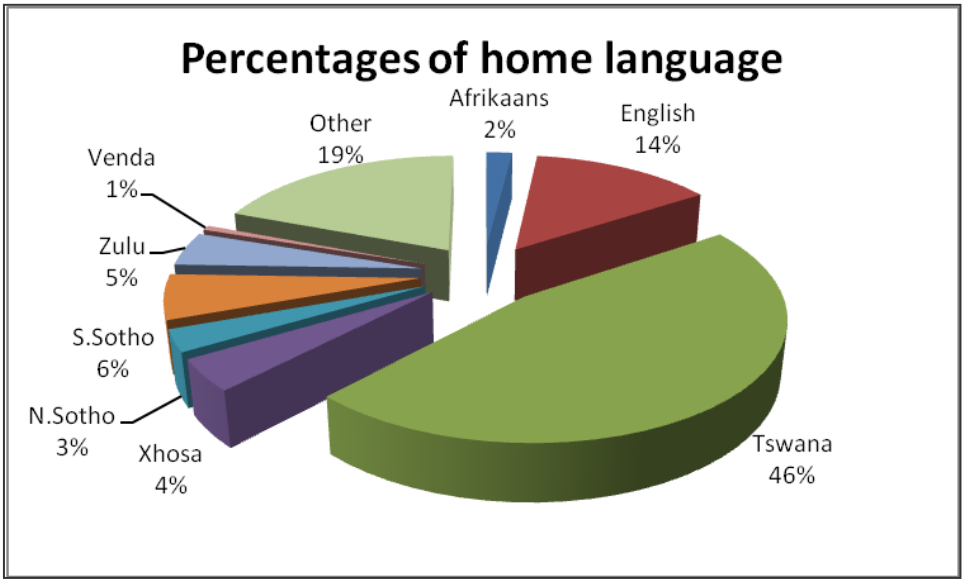


Figure 4.3 Pie chart representing home languages

From Table 4.2 and Figure 4.2, it is clear that more Setswana-speaking[19] respondents than English or 'Other language' speakers took part in this study.

4.2.1.2 Frequencies of respondents' gender

Two-way tables with the number of respondents according to gender were constructed.

Table 4.3 Frequencies of respondents according to gender

School	Female	Male	Total
School A	0	251	251
School B	110	48	158
School C	200	0	200
Total	310	299	609

Table 4.3 shows that approximately the same number of male and female respondents was included in this sample. School B, a co-ed school, had noticeably more girls than boys, whereas School A only had boys and School C only girls. The number of respondents from each school therefore varied with regard to gender.

4.2.1.3 Frequencies of respondents per grade

Figure 4.4 shows the comparisons of the grade totals across schools.

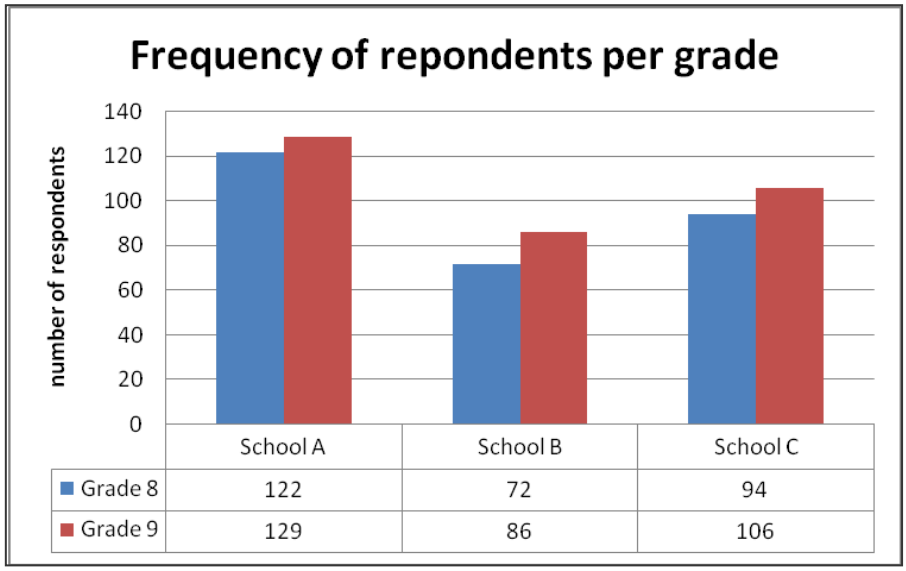


Figure 4.4 Frequencies of respondents per grade

[19] Note that the language of instruction in the participating schools was English.

The number of respondents per grade differed across the three schools. Although comparisons within the schools did not vary to a great extent, it is noticeable that there were more Grade 9 than Grade 8 respondents in each school.

4.2.2 Averages and standard deviations

Calculated mean scores ($\bar{x}$) and standard deviations (s) of the respondents' (n) responses on the Likert-type scales of the SOM (5-point) and RPSQ (4-point) are summarised in the tables that follow. The SOM measured responses in respect of the mathematics confidence items, while the RPSQ measured aspects of the monitoring and planning tasks, persons and strategies.

4.2.2.1 Synthesising the mean and standard deviation scores

From the scores obtained in Table 4.4 it seems that the three schools had almost equal average monitoring scores. High mathematics confidence scores were almost equal among Schools A ($\bar{x}$ = 55.51) and C ($\bar{x}$ = 55.14), together with equal achievement in mathematics problem solving(School A: $\bar{x}$ = 2.68; School C: $\bar{x}$ = 2.60). Problem-solving behaviour scored the highest in School B's ($\bar{x}$ = 57.80) responses, despite the fact that this school had the lowest mathematics confidence ($\bar{x}$ = 51.57), achievement ($\bar{x}$ = 1.45) and planning scores (1.38). School C had the lowest scores on problem solving behaviour and equal monitoring and planning scores compared to School A.

These observations explain the relationships between the three partaking schools in terms of the variables measured in addition to contribute to answering the secondary research questions: Question 1 and Question 2 (see section 1.2.3.1).

Table 4.4 Mean scores and standard deviations

	Mean scores and standard deviations (SOM)								
Instrument	**5-point Likert scale**	**School A n = 251**		**School B n = 158**		**School C n = 200**		**Total n = 609**	
Study Orientation in Mathematics	**Variable**	$\bar{x}$	(s)	$\bar{x}$	(s)	$\bar{x}$	(s)	$\bar{x}$	(s)
	Mathematics confidence	55.51	7.48	51.57	7.92	55.14	9.80	53.54	8.51
	Problem solving behaviour	56.77	9.91	57.80	9.51	53.83	9.06	56.07	9.66
Instrument	**4-point Likert scale**	**Mean scores and standard deviations (RPSQ)**							
Reflection and Problem Solving Questionnaire	Achievement	2.68	1.66	1.45	1.00	2.60	1.55	2.34	1.57
	Monitoring Task	2.92	0.50	2.94	0.46	2.95	0.54	2.93	0.50
	Planning Task	2.89	0.42	1.38	0.50	2.85	0.47	2.87	0.46
	Monitoring Task, Person, Strategies	2.96	0.48	2.88	0.46	2.85	0.53	2.88	0.50
	Planning Task, Person, Strategies	2.74	0.40	2.68	0.40	2.85	0.36	2.76	0.40

4.2.3 Reliability of the SOM

Cronbach alpha coefficients measured the internal consistency of the SOM. Table 4.5 represents the reliabilities for the factors in the SOM in terms of Cronbach α.

Table 4.5 Reliability of the SOM

Factors	**Cronbach alpha (α)**
Mathematics confidence	0.73*
Problem-solving behaviour	0.84*

* According to Cotterall and Murray (2009:35) coefficients of 0.5 or higher indicate reliability. Table 4.5 shows that the SOM measured respondents' mathematics confidence and problem-solving behaviour reliably. The RPSQ was an adapted product of various theories, statements and questions taken from the literature. An exploratory factor analysis was done in order to determine the factors of the RPSQ, as well as their reliabilities and possible correlations.

4.3 Exploratory factor analysis of the RPSQ

Exploratory factor analysis, according to Winter and Dodou (2012:695), is one of the most widely employed statistical analysis procedures in psychology research. In the current study, this statistical method was used to determine whether a series of factors existed in the data and whether these factors could be interpreted in a theoretical sense (Hooper, 2012:2). When conducted on the items of the RPSQ, co-variance between the items identified factor structures, which sufficiently illuminated inter-correlations between the constructs. The factor analysis was done in four stages that involved the calculation of Eigen values, a matrix of associations, principle axis factoring and a rotated solution through Oblimin rotation with Kaiser normalisation (Darlington, 2004).

4.3.1 Extraction of factors

Exploratory factor analysis was employed to refine the number of factors and determine their nature (Hooper, 2012). The factors in the RPSQ were extracted by firstly determining the statistical significance of the correlation between its items. The following account describes this procedure.

4.3.2 The Bartlett's test and the Kaiser-Meyer-Olkin test

Two tests were completed before factor analysis could be done – the Kaiser-Meyer-Olkin measure of the sampling adequacy and Bartlett's test for sphericity.

The Bartlett's test for sphericity determined that there was a correlation of statistical significance ($p < 0.001$) between the items of the RPSQ. In this study, the test measured whether correlations between the items were big enough for meaningful factor analysis. The Kaiser-Meyer-Olkin test measured 0.882, indicating a compact structure between the factors of the RPSQ (Field, 2005). The Kaiser-Meyer-Olkin measure of sampling adequacy extracted 12 factors with Eigen values greater than 1.0. These 12 factors of the RPSQ are represented along with their initial Eigen values and percentage of variance in Table 4.6.

Table 4.6 Initial Eigen values and percentage of variance for the 12 RPSQ factors

Factor	Initial Eigen value	Percentage of variance
1	8.592	19.528
2	2.793	6.347
3	1.959	4.453
4	1.646	3.742
5	1.531	3.479
6	1.324	3.008
7	1.235	2.807
8	1.185	2.694
9	1.140	2.591
10	1.092	2.481
11	1.051	2.388
12	1.013	2.302
		Total variance: 55.821

Table 4.6 indicates that the 12 factors explained 55.821% of the total variance. In order to determine the number of factors to extract from the 44-item RPSQ and minimise the 12 factors obtained from the factor analysis, Kaiser's criterium was used.

4.3.3 Principle axis factoring with Oblimin rotation

Principle axis factor analysis was used to extract the common factors in the RPSQ. The Oblimin rotation method was selected (Brown, 2009) to obtain simple structured and interpretable factors. Each row of the factor loading was normalised. Oblimin rotation with Kaiser normalisation converged in 28 iterations. Through principle axis factoring and Oblimin rotation with Kaiser normalisation, a pattern matrix of the 12 factors was obtained. Table 4.7 illustrates this pattern matrix along with the extracted factors' communalities.

Table 4.7 Pattern matrix for the RPSQ items with item communalities

Items	1	2	3	4	5	6	7	8	9	10	11	12	Communalities
32	.546							.214					0.577
31	.436					.282							0.501
33	.389												0.402
2	-.330	.229				.280					.250		0.551
25	.298								.229				0.383
8		-.656							.256				0.614
40		-.635											0.366
7		-.634											0.517
10		-.472										.319	0.504
9		-.462											0.381
28		-.387			.215			.263					0.423
37			.890										0.720
21			.751										0.578
43			.612				.261						0.538
36				.539									0.393
35	.268			.550									0.468
12				.444									0.286
3				.438									0.336
20				.346									0.318
39				.314	.215					.220			0.393
19				.340									0.270
18				.233									0.289
13				.222									0.251
38					.680								0.403
29	.294				.314								0.437
27						.527							0.381
30				.229		.459							0.368
24						.427							0.356
23						.425							0.356
1						.296				.283			0.437
15							.38						0.203
44							.333						0.182
14							.263	-.219				.211	0.439
34							.219	.543					0.568
42									.582				0.430
26									.422				0.422
41							.290		.321				0.271
16									.34				0.312
6										-.490			0.179
17											.561		0.338
11											.380		0.146
4		.221		.218						-.223		.349	0.372
22						.230						.332	0.425
5												.272	0.339

Extraction method: Principle axis factoring.

Rotation method: Oblimin rotation with Kaiser normalisation, converged in 28 iterations.

4.3.4 The 12-factor pattern matrix

According to SPSS-Inc (2005), recalculation of the common variance or individual factors is not possible after rotation. Further factor analysis was needed to reduce the number of factors in this 12-factor pattern matrix. This was completed by combining the items according to their internal correlations, consistencies and theoretical relationships (as compared among all items in the instrument).

4.3.4.1 Factor analysis of the 12-factor pattern matrix

Internal correlations between the items were calculated. Item correlations were subsequently compared and grouped (clustered) according to the items' relevance to one another in terms of the correlation in the pattern matrix and the theoretical construct of the factors. Clusters were formed to reduce the number of factors in the pattern matrix. Table 4.8 summarises the four factors as constructs with related items. The highlighted colours indicate which factors were grouped (clustered) according to their respective internal correlations[20] and theoretical descriptions.

[20] Refer to Table 4.8 and Addendum F to see the highlighted internal correlations of the items and corresponding factors.

Table 4.8 Reduction table: Organisation, description and identification of factors in the 12-factor pattern matrix

Factor number of the pattern matrix	Clusters of factors with internal item correlations	Item number on the RPSQ	Short theoretical description of the items	Summary of items' description	Identified factors
1	1	32 31; 33 25	Check calculations Look back at calculations and check work again Continuously check written work	Reflecting on what was done and whether it makes sense. It involves rereading, checking work that was done, looking back at the question, and understanding the problem	**Factor 1** Monitoring of tasks
2	5	38 29	Predict what will be done next Review steps all the time		
3	8	34	Check if answers make sense		
4	9	42 26 41 16	Carefully read instructions Redo steps in case something goes wrong Write down important information Focus on important information	Reflecting on how the problem was solved with the focus on written or mental planning, backup plan or approach, looking for important or relevant information and determining what strategies to utilise/employ	**Factor 2** Planning of tasks
5	10	6	Identify unnecessary information		
6	12	4 22 5	Familiarise yourself with the problem Think about all the steps Identify information needed		
7	4	20 12 3 19 18 36 35 13 39	Use own examples to compare Consider different strategies Put the problem in own words Think of several approaches Use self-questioning before the task Try alternative approaches Think about different options Use strategies that worked before Select specific methods/strategies	Reflecting on own understanding and views, making sure about strategies selected and approaches carried out, being aware and having knowledge of cognitive processes	**Factor 3** Monitoring of task, person and strategies
8	6	27 30 24 23	Question your own understanding Check understanding Reconsider what was written Look back at the problem		
9	2	2 25 8 40 7 10 9 28	Understand the problem Check work systematically Be confident about the correctness Cannot decide what to do, confused Continue with self-questioning ability Identify prior successes Be sure of capability to do the sums Exercise self-belief and control of knowledge	Reflecting on the utilisation and management of tasks and personal resources, including but not limited to self-questioning	**Factor 4** Planning of task, person and strategies
10	3	37 21 43	Draw picture Understand through pictures Do modelling/ model building, drawing		
11	7	15 44 14	Be aware of your own strengths Identify familiar problems Identify important information		
12	11	17 11	Focus on time to finish the task Have self-awareness regarding skills		

4.3.4.2 The four factors

After compiling the items into clusters with each cluster measuring a different construct, four factors were extracted and identified as follows:

- Monitoring of tasks (MT)
- Planning tasks (PT)
- Monitoring tasks, strategies and person (MTSP)
- Planning tasks, strategies and person (PTSP)

The four factors are described below with the relevant items arranged according to their internal correlations obtained from the factor analysis.

4.3.4.3 Factor 1: Monitoring of tasks (MT)

From the pattern matrix, factors 1, 5 and 8 had closely related correlations. By combining factors 1, 5 and 8 according to their related descriptions (theoretically), the measuring of respondents' monitoring of tasks (MT) was identified as Factor 1. Items 32, 31, 33, 25, 38, 29 and 34 involved monitoring skills such as rereading the question, checking calculations to see if they are correct, looking back at your work and looking back at the problem to see if the answer makes sense. These 7 items measures respondents' reflective skills on metacognitive regulation.

4.3.4.4 Factor 2: Planning of tasks (PT)

Table 4.8 indicates that factors 9, 10 and 12 constitute the second cluster. Factor 2 is identified as respondents' planning of tasks (PT). Items 42, 26, 41, 16, 6, 4, 22 and 5 included reflection on planning skills, such as seeking relevant information and reading instructions carefully and selectively. In other words, planning is a regulative skill that is needed during problem solving.

4.3.4.5 Factor 3: Monitoring tasks, person and strategies (MTSP)

When clustering factors 4 and 6 according to internal item correlations, the third factor is identified as monitoring tasks, person and strategies (MTSP). Items 20, 12, 3, 19, 18, 36, 35, 13, 39, 27, 30, 24 and 23 were concerned with self-talk, self-evaluation and the success and availability of knowledge and strategies. This factor resembles the individual's reflection on how he/she copes with the problem-solving task and specifies metacognitive knowledge.

4.3.4.6 Factor 4: Planning tasks, person and strategies (PTSP)

Once factors 2, 3, 7 and 11 were clustered, Factor 4 was identified as the planning of task, person and strategies (PTSP). Items 2, 25, 8, 40, 7, 10, 9, 28, 37, 21, 43, 15, 44, 14, 17 and 11 displayed reflection on the planning done before, during and after the problem-solving process. Identifying metacognitive knowledge regarding personal strengths and weaknesses, gathering important information and comparing strategies are examples of the components relating to this factor.

4.4 Reliability of the RPSQ

The reliability of the four factors of the RPSQ was measured once the factor analysis was completed. All four factors had Cronbach alpha coefficients of 0.6 or higher, indicating reliable internal consistency between the items of the four factors. Table 4.9 provides a summary of the four factors of the RPSQ's reliability coefficients.

Table 4.9 Reliability coefficients of the four factors of the RPSQ

Factors	Cronbach alpha (α)
Monitoring task	0.71*
Planning task	0.61*
Monitoring task, strategies and person	0.81*
Planning task, strategies and person	0.62*

* According to Cotterall and Murray (2009:35) coefficients of 0.5 or higher indicate reliability.

Table 4.9 indicates that monitoring of tasks, strategies and person (self) measured a high reliability of 0.81. Monitoring of tasks had a moderately high reliability, compared to the reliability of the planning of tasks, person and strategies. Monitoring of tasks had a high reliability of 0.71. For the aim of this study, the Cronbach alpha values of Table 4.5 and Table 4.9 indicate that there was a sufficient number of reliable items and factors in both the SOM and RPSQ.

4.5 Inferential statistics

Inferential statistics is concerned with the inferences made about the population, based on the indicators obtained from an analysis of the sample (Welman, Kruger & Mitchell, 2009:236). This field of statistics relies a great deal on the probability theory (Maree et al., 2010:198) and as such, statistics are calculated from the sample data.

4.5.1 Chi-square analysis

Chi-square (x^2) analysis was done to compare observed ratios with expected ratios. The chi-square statistic for language, gender and grade is presented in Table 4.10.

Table 4.10 Chi-square of language, gender and grade

Statistic	Language	Gender	Grade
Pearson Chi-square	266.3421	482.6075	0.3686305
M-L Chi-square	293.8744	655.7697	3687583
Phi	0.6613194	**0.8902016**	**0.0246029**
Contingency coefficient	0.5516086	0.6649114	0.0245955
Cramer's V	0.4676234	**0.8902016**	**0.0246029**

4.5.2 Analysis of variance (ANOVA)

With three schools participating, the mean scores had to be compared to determine any statistical differences between the schools (see paragraph 3.6.1 regarding the different schools in the sample). Analysis of variance was determined to explore statistical differences between the schools as discussed in section 4.6.

The analysis of variance (ANOVA) tests for significant differences between the three groups by comparing the mean scores of each. After the completion of the ANOVA (Table 4.11), the post hoc test was used to determine any significant differences between the groups in the analysis of variance.

Table 4.11 Analysis of variance (ANOVA)

Variables	SS Effect	dF Effect	MS Effect	SS Error	DF Error	MS Error	F	p
Achievement	168.7630	2	84.38152	1327.230	606	2.190149	38.52775	0.000000
Confidence	1125.582	2	562.7908	42917.52	606	70.82098	7.946667	0.000392
Problem-solving behaviour	1597.627	2	798.8136	55100.34	606	90.92465	8.785447	0.000173
Monitoring task	0.087	2	0.0434	154.09	606	0.25428	0.170639	0.843167
Planning task	0.234	2	0.1171	127.49	606	0.21038	0.556629	0.573431
Monitoring task, person and strategies	2.991	2	1.4957	144.68	606	0.23874	6.264824	0.002028
Planning task, person and strategies	3.018	2	1.5090	91.90	606	0.15165	9.950960	0.000056

4.6 Effect sizes and statistical significance

Calculating the effect sizes between schools in addition to the statistical significance indicates any practical significance of the ρ-values. Effect sizes were determined for the difference between two mean scores regarding the comparison of schools and for the relationship between variables. This was done because the effect size is independent of the sample size and shows the relationship between the variables to answer the secondary research questions in section 1.2.3.1 (Ellis & Steyn, 2003:51). According to Maree et al. (2010:211), effect sizes serve a purpose with or without statistical significance. Table 4.12 summarises the effect sizes to determine the practical significance of the variables in this study.

Table 4.12 Effect sizes between schools as compared to the different variables

Variables	Mathematics confidence	Problem-solving behaviour	Monitoring task	Planning task	Monitoring task, person and strategies	Planning task, person and strategies
School A with B	0.24	0.10	0.04	0.06	0.34	0.15
School A with C	0.17	0.30	0.06	0.09	0.21	0.28
School B with C	0.36	0.42	0.02	0.02	0.11	0.43

Table 4.12 indicates the effect sizes between the schools and the variables of this study ranging between 0.02 and 0.43 (Table 4.12). According to Ellis and Steyn (2003:52), a small effect is indicated by values in the region of 0.2, while a medium effect is indicated by 0.5 and a large effect by 0.8. When comparing School A with School B, small effect sizes were identified, except for the variable *monitoring task, planning and strategies* which has a higher, although below medium¸ magnitude of 0.34 (Table 4.12). When comparing School A with School C, the

effect sizes were also small, ranging from 0.06 to 0.30 (Table 4.13) with the highest effect 0.30 (Table 4.13) for *problem-solving behaviour*. School B and C's highest score was a medium effect of 0.42 and 0.43 (Table 4.13) for *problem-solving behaviour* and *planning tasks, person and strategies* respectively. School C's performance in *planning* (TSP) was visibly better than that of School B, with a medium effect of 0.43 (Table 4.12).

4.6.1 Scores on mathematics confidence

With reference to Table 4.4 and Table 4.12 the average score obtained for respondents' *mathematics confidence* was 53.54 (Table 4.4), whereas the standard deviation was 8.51 (Table 4.4). School A and C correspondingly obtained approximately equal mean scores (Table 4.4) of 55.51 and 55.14 respectively, with a small effect of 0.17 (Table 4.12) that is indicative of equal levels in *mathematics confidence*. School B had the lowest mean score of 51.57 with a standard deviation of 7.92 (Table 4.4) when compared to the other two schools, indicating that the respondents from School B had lower *mathematics confidence* (varying moderately between effect sizes of 0.24 and 0.36) as compared to those of Schools A and C. The non-similar standard deviations of School A (7.48), School B (7.92) and School C (9.80) show that the responses of School A (an all-boys school) in respect of *mathematics confidence* was higher (55.51) than the responses of School B. The *mathematics confidence* mean responses of Schools A and B can be regarded as similar, with less related standard deviations and a small effect of 0.24 (Table 4.12).

4.6.2 Scores on problem-solving behaviour

School A's *problem-solving behaviour* is almost equal to the total mean of 56.07 (Table 4.4). School B's *problem-solving behaviour* is the highest at 57.80. This score for *problem-solving behaviour* is close to that of School A's score (56.77 - Table 4.4), with a small effect size of 0.10 (Table 4.12). School C has the lowest average for *problem-solving behaviour* at 53.83 and shows less problem-solving skills than Schools B and A with moderate effect sizes of 0.30 and 0.42 respectively (Table 4.12). The standard deviations of the three schools are almost alike, ranging between scores of 9.06 to 9.91 (Table 4.4).

4.6.3 Scores on achievement

Having the highest score for *problem-solving behaviour*, it is surprising that School B has an achievement mean score of 1.45 (see Table 4.4), the lowest in comparison to Schools A and C.

School A has the highest achievement score (2.68), which is almost equal to that of School C (2.60). The standard deviation (1.00) for School B is also the lowest, while those of Schools A (1.66) and C (1.55) are closer together.

4.6.4 Scores on monitoring tasks

The three schools' mean scores on *monitoring tasks* are considered equal. Scores around 2.93 also have a smaller standard deviation, which ranges between 0.54 and 0.46 (Table 4.4). However, on a small scale (2.94), School B has a higher average for *monitoring tasks* than School A with a very small effect of 0.04 (Table 4.12). This is noticeable due to the range of these schools' mean scores on *achievement* and *mathematics confidence*. School C has the highest (on a small scale) score of 2.95. It is comparable to the *problem-solving behaviour* of Schools C and B, with a small effect of 0.02.

4.6.5 Scores on planning tasks

School B's mean score for *planning tasks* is the lowest (1.38) compared to the almost equal scores of Schools A (2.89) and C (2.85) in this regard. This corresponds to the differences regarding School B's *mathematics confidence* and *achievement*. With standard deviations varying between 0.42 (School A) and 0.50 (School B), School A has the least standard variance compared to the total score of 0.46. School A and C's equal scores, and the small effect of 0.09 correspond to their *achievement* and *mathematics confidence* mean scores.

4.6.6 Scores on monitoring (task, person, strategies)

With closely related scores totalling an average of 2.88, School B (2.88) and C (2.85) share the closest mean scores (with a small effect of 0.21 – Table 4.12). School A (0.48) and B (0.46), on the other hand, have the closest standard deviations and a moderate effect of 0.34. The closely related mean scores, for *monitoring* (TSP) of 0.48 and 0.46 are also comparable with the *monitoring task* scores of schools A and B.

4.6.7 Scores on planning (task, person, strategies)

The mean scores for *planning the task, person and strategies* differ for the three schools. School A's score (2.74) is closest to the total average score of 2.76 (Table 4.4). School B has the lowest

score of 2.68, which compares with similar mean scores for *mathematics confidence*, *achievement* and *planning tasks*. School C has the highest score of 2.95 for *monitoring tasks* and the lowest score for *problem-solving behaviour*. The standard deviations for the three schools are close to 0.40. School A differs with a small effect of 0.15 (Table 4.12) compared to School B, while A also differs slightly from C with an effect size of 0.28. A moderate effect size of 0.43 is found when comparing School B with School C.

4.7 Spearman rank correlations of all variables in the study

Spearman rank correlations were determined between respondents' *mathematics confidence*, *problem-solving behaviour* and reflection on the *monitoring of tasks, planning of tasks, monitoring of task, person and strategy*, and *planning of task, strategy and person*. These correlations are represented in Table 4.13 and serve to answer the secondary research question (section 1.2.3.1).

Table 4.13 Spearman rank correlations for the variables in the SOM and RPSQ

Spearman rank correlation coefficients (Spearman's rho)						
Variables	**Confidence**	**Problem solving**	**Monitoring Task**	**Planning Task**	**Monitoring TSP**	**Planning TSP**
Confidence	1.000					
Problem-solving behaviour	**0.126****	1.000				
Monitoring task	**0.228****	**0.394****	1.000			
Planning task	**0.187****	**0.447****	**0.583****	1.000		
Monitoring TSP	0.058	**0.462****	**0.587****	**0.579****	1.000	
PlanningTSP	0.069	**0.361****	**0.472****	**0.446****	**0.471****	1.000

****** Correlation is significant at the 0.01 level (2-tailed).

Spearman rank correlations were calculated to determine any possible statistical significance ($p < 0.05$) and practical significance ($r > 0.5$) between the correlations of the variables of both the SOM and the RPSQ. It appears from Table 4.13 that there are statistically significant

correlations between the variables in this study; however, lower correlations exist between *mathematics confidence* and the other variables.

4.7.1 Correlations with mathematics confidence

From Table 4.13 it can be observed that there is a low correlation between *mathematics confidence* and *problem-solving behaviour* (0.126). *Mathematics confidence* correlates the highest with reflection on the *monitoring of tasks* (0.228). The second highest correlation of *mathematics confidence* is with reflection on *planning of tasks* (0.187). These positive correlations are statistically significant at the 0.01 level. *Monitoring* (TSP) shows the lowest correlation with *mathematics confidence* (0.058), along with reflection on *planning* (TSP), the second lowest at 0.069 (Table 4.13). These lowest correlations are not statistically significant. *Mathematics confidence* appears to be moderately correlated to the reflection on *problem-solving behaviour, monitoring and planning of tasks* of the respondents in this study.

4.7.2 Correlations with problem-solving behaviour

Problem-solving behaviour correlates moderately with *mathematics confidence*, although the *monitoring of tasks* correlates at a statistically significant Cronbach alpha of 0.394. This correlation is very similar to the correlation with *planning* (TSP) at 0.361. The correlations with *monitoring* (TSP) at 0.462 and *planning of tasks* at 0.447 are the highest positive correlations with *problem-solving behaviour*.

4.7.3 Correlations with reflection on the monitoring of tasks

The factor '*monitoring of tasks*' correlates moderately with *mathematics confidence* and *problem-solving behaviour*. The correlation with internal factors of the RPSQ such as *planning of tasks* (0.583), *monitoring* (TSP) (0.587) and *planning* (TSP) (0.472) is high as alpha values are close to 0.5. It appears from Table 4.13 that among all factors in this study, reflection on the *monitoring of tasks, person and strategies* shows the highest correlation that is statistically significant ($p = 0.84$).

4.7.4 Correlations with reflection on the planning of tasks

The *planning of tasks* correlates highly with *monitoring tasks* and moderately with *problem-solving behaviour* and *mathematics confidence*. The correlation with *monitoring* (TSP) is high enough to establish a relationship at $\alpha = 0.579$. *Monitoring* (TSP) correlates moderately with *planning* (TSP) at $\alpha = 0.446$ (see Table 4.13).

4.7.5 Correlations with reflection on the monitoring of tasks, person and strategies

The reflection on *monitoring* (TSP) correlates moderately with all the above factors at a significant level, including the factor of reflection on *planning* (TSP) with $\alpha = 0.471$. A low correlation (0.069 according to Table 4.13) with *mathematics confidence* was noticed.

4.7.6 Correlations with reflection on the planning of tasks, person and strategies

Planning (TSP) correlates significantly with all factors in the study as shown above, except with *mathematics confidence*.

The results of this quantitative study indicate that there is a relationship between the variables mentioned in Table 4.13. This correlation is either statistically significantly moderate or high; or non-statistically significantly low.

4.8 Summary of the quantitative results

Monitoring and *planning*, both regulatory components, are diverse facets of metacognition linked to *mathematics confidence*. *Monitoring* (TSP) differs from *monitoring tasks* only in the sense that correlations with *mathematics confidence* were higher when reference was made to tasks alone. Perhaps this difference is explained by the SOM measuring *mathematics confidence* of only the psychological domain. It seems to exclude the social (person) and intellectual (strategies) domains. This distinction might result from a hesitation to integrate personal knowledge with task knowledge. Personal qualities and reflection on strategies correlated low with *mathematics confidence*, pointing out the absence of deep reflection when planning tasks relate to personal experiences. However, *monitoring* (TSP) and *planning* (TSP) correlated moderately with *mathematics confidence* and with *problem-solving behaviour*. The results of this study agree with findings by Efklides (2006:7) that metacognitive experiences produce a level of

mathematics confidence where progress, feelings and goal settings are monitored. Reflection associates feelings and emotions with metacognitive experiences.

The factors and their correlations will be discussed further in Chapter 6. The next chapter (Chapter 5) is devoted to the findings of the qualitative study.

Chapter 5

Analysis of the qualitative data

5.1 Introduction

Chapter 3 described the data collection procedures and analysis plan that were implemented in order to examine and understand the variables proposed in Chapter 1 and described in Chapter 2. The mixed-method exploratory convergent research design was evaluated in two analogous (parallel) studies consisting of both quantitative and qualitative parts. Chapter 5 presents the qualitative results that were obtained.

The analysis of the data collected in the qualitative part of the study was done with the following conceptual framework (see Chapter 2) in mind.

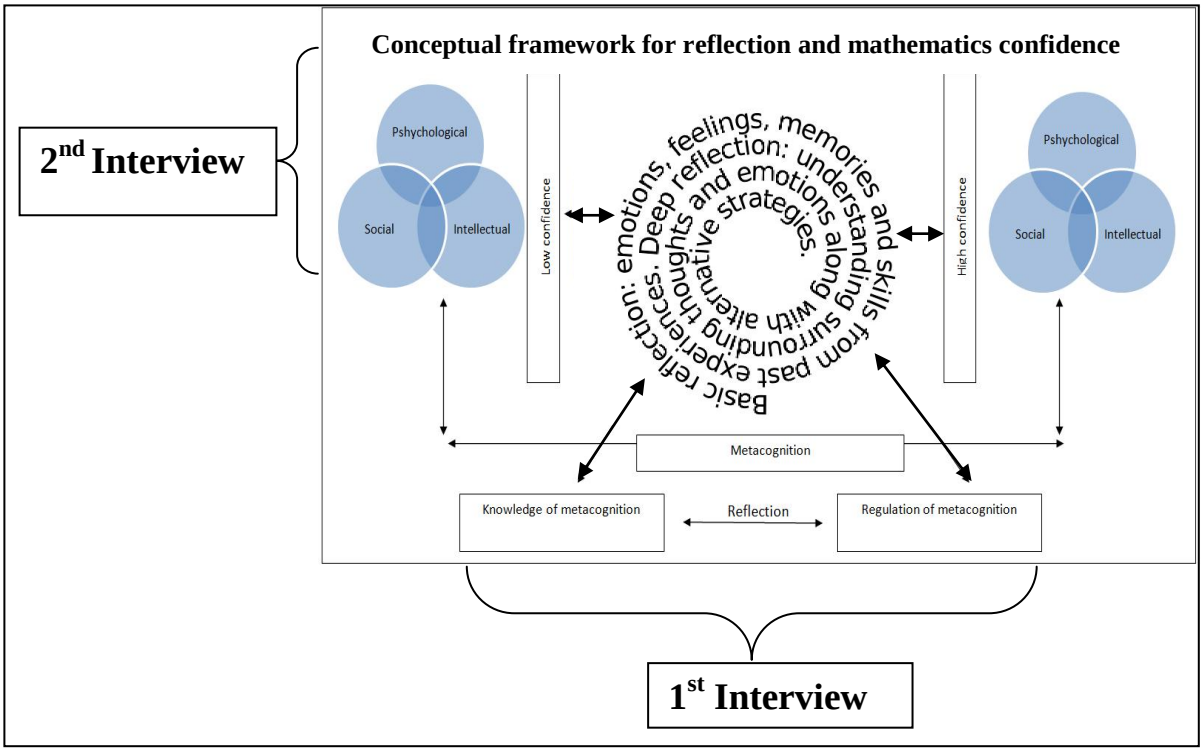


Figure 5.1 Conceptual framework for mathematics confidence and metacognitive reflection

Source: Adapted from Strawderman (2010:3); John (2000); Ertmer and Newby (1996)

The qualitative results were obtained by interviewing four participants in two separate individual sessions. The interview sessions focused on reflection as a metacognitive skill (first interview session) and experiences of mathematics confidence (second interview session) during mathematical problem solving. Conceptually, metacognition was analysed according to the participants' metacognitive knowledge and metacognitive regulation. Mathematics confidence was analysed in terms of three domains: social, psychological and intellectual. The results are presented here in three stages, each relating to the concepts as described in the literature review. Once the analysed results have been presented, the data is summarised.

5.1.1 Research focus during analysis

The primary research question to answer was:

What is the role of reflection and mathematics confidence during problem solving in senior phase Mathematics?

To answer this, two secondary research questions were identified and explored.

Table 5.1 Overview of the qualitative data analysis

Analogous part of the study	Secondary research questions	Research question(s)	Variables	Data collection method	Data analysis method
Qualitative data analysis	**Secondary research question 3**	Which reflection strategies or skills do learners in senior phase mathematics implement, if any?	Reflection on metacognitive knowledge and regulation	Individual interviews Video recordings Selection and arrangement of metacognitive statement cards	A priori codes Content analysis and coding
	Secondary research question 4	What does the mathematics confidence experienced by learners entail?	Mathematics confidence	Individual interviews	Content analysis and coding

In order to answer the research question, and to explore the role of reflection and mathematics confidence, two individual interview sessions were conducted with each of the four participants. After analysing the transcripts of the interviews, two themes emerged, namely reflection on

metacognition and mathematics confidence. The themes were further categorised under the following sections:

- Reflection on metacognitive knowledge
- Reflection on metacognitive regulation
- Reflection on mathematics confidence in the social domain
- Reflection on mathematics confidence in the intellectual domain.

It is important for analysis and triangulation purposes that the research design stays within the margins of the conceptual framework (Trafford & Leshem, 2010:96). In the current study, the researcher found concepts and themes in the literature relating to mathematics confidence and reflection. However, these concepts and themes, such as beliefs, values and attitudes, were not part of the research topic or conceptual framework and were excluded from analysis. The analyses only pertained to those concepts and variables that were measured and searched for in the described methodology. The methodology aimed only to answer the research question(s) and did not linger on analysing information that was not relevant to the research topic. Some researchers might include concepts of affect for mathematics in the framework. One such example is found in the work of Hudlicka (2005:3) which involves the components of habitual and compulsive reflection on planning and monitoring. Again, in this particular study, the focus was only on mathematics confidence as a component of affect and reflection during mathematics problem solving. A discussion on how the results are presented follows next.

5.1.2 Preliminary discussion on the presentation of the results

To ensure that the methodology would support the process of answering the research question(s), the design in Figure 5.2 was implemented based on the conceptual framework.

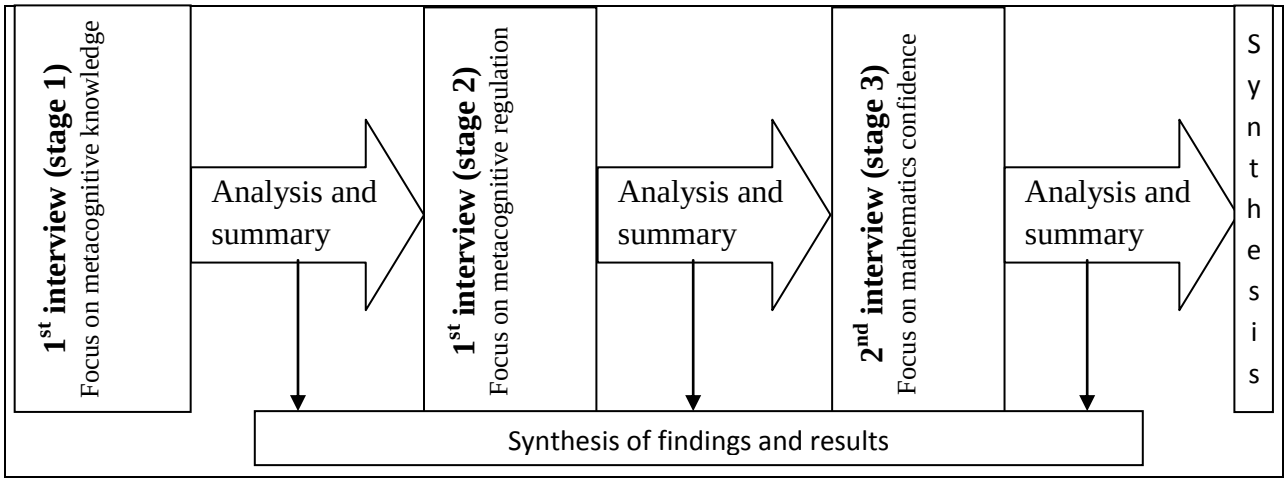


Figure 5.2 Three stages of the qualitative interview and analysis processess

The results of the study were presented according to the structure illustrated in Figure 5.2. First, the data obtained from the first interview session (interview in stage 1) was organised, arranged and analysed. The results for each of the different categories, formulating the a priori codes, were presented and discussed separately. The findings were summarised and compared after having been cited in the conceptual framework, and they constituted a model for analysis.

Second, the data obtained from the selection of the metacognitive statement cards was arranged in table format. Participants' selection of the cards was compared and discussed with reference to the relevant transcriptions. Each participant's results were compared with those of other participants. The reason for this was to validate the metacognitive processes during which participants reflected on their knowledge and their regulation. Stages 1 and 2 in Figure 5.2 appear similar but they explore different aspects of metacognition. This corresponds with Wilson (2001) who mentions the difficulties in metacognition assessment. The statement cards (stage 2) serve as validation of the participants' responses.

Third, the data from the second interview session was arranged according to the three domains (categories) of mathematics confidence: social, psychological and intellectual. The transcriptions of each interview were collectively presented and then summarised in terms of these three categories. Afterwards, each category's summary was compared and sited in the conceptual framework. A discussion of the synthesis of findings follows. It involves the role that reflection on metacognitive knowledge and regulation, as well as mathematics confidence plays during problem solving.

5.1.3 Identified categories for the stages in first and second interviews

The first interview session explored reflection on metacognition while the second interview focused on mathematics confidence. For each interview session, a priori codes and categories were identified. The categories also determined the order in which the data was analysed and arranged for presentation and discussion. Only a collective overview of the categories is shown in this paragraph. For a more detailed outline of the categories, see paragraph 3.9.2.3.

5.1.3.1 Themes, codes and categories for the interviews

Analysis started immediately after each interview. The transcriptions were read and re-read. Each participant's responses were grouped according to the identified a priori codes in Table 5.2.

Table 5.2 Identified a priori codes and categories

Themes	Categories	Codes	Description
Reflection on metacognitive knowledge			
K Metacognitive knowledge	Reflection on knowledge of the self (person)	KP	Awareness of one's own abilities and skills
	Reflection of knowledge of the task	KT	Identifying procedures and information relevant to the task
	Reflection on knowledge of strategies	KS	Strategies employed to solve the problem
Reflection on metacognitive regulation			
R Metacognitive regulation	Reflection on regulation of understanding	RU	Knowing what to do or how to do it
	Reflection on regulation of planning	RP	Signs of arranging and organising throughout the process
	Reflection on regulation of monitoring	RM	Looking back at what was done up to a certain point
	Reflection on regulation of evaluation	RE	Making sense of the solution and comparing answers
Mathematics confidence in the psychological domain			
C Mathematics confidence in the psychological domain	Confidence in the psychological domain regarding feelings	CPSF	Conveying emotions or feelings
	Confidence in the psychological domain regarding enjoyment	CPSE	Identifying likes and dislikes
	Confidence in the psychological domain regarding comfort	CPSC	Realising what makes you feel comfortable and what not
Mathematics confidence in the social domain			
C Mathematics confidence in the social domain	Confidence in the social domain regarding family and friends	CSOD	Influences as a result of what family and friends said/did
	Confidence in the social domain regarding persons such as teachers and society	CSOP	Influences as a result of what teachers or other persons said/did
Mathematics confidence in the intellectual domain			
C Mathematics confidence in the intellectual domain	Confidence in the social domain regarding success in mathematics	CINS	Achievement in Mathematics prior to the interview and how successful problems were solved. This resembles confidence
	Confidence in the social domain regarding failure in mathematics	CINF	Failures and non-successes that might showcase anxiety like symptoms
	Confidence in the social domain regarding pursuit in mathematics	CINP	The effort and ongoing trial and error that the participant shows during the problem-solving process

5.1.4 The four participants who were interviewed

Learner A, a fifteen year old girl, has a seemingly quiet nature. She had an average of 70% for Mathematics at the end of Grade 8 and took 9 minutes and 20 seconds to solve the given word problem.

Learner B was fifteen years old and from an all-girls school. She smiles a lot and seemed to enjoy the discussion and questions in the interviews. She averaged 80% for Mathematics at the end of Grade 7 and solved the word problem in 16 minutes and 58 seconds.

Learner C was fourteen years old. She seemed more interested in the study than were the other participants. She asked questions about why the study was done and whether she was allowed to ask questions during the interview. This learner had obtained an average of 76% at the end of Grade 7 and solved the word problem in the smallest amount of time, only 4 minutes.

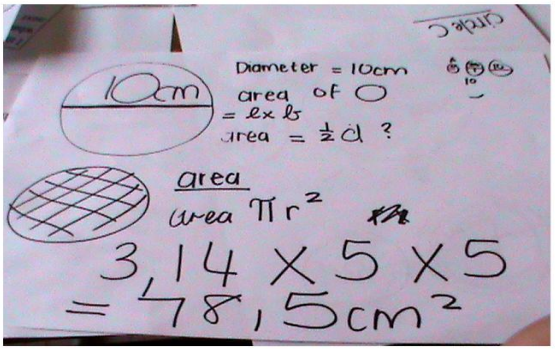
Learner D was a thirteen-year-old boy, the youngest, and from an all-boys school. His teachers considered him a top student, achieving 96% at the end of Grade 8. He took 9 minutes and 30 seconds to solve the problem, almost equal to Learner A.

The four participants were purposively invited after consulting with the Grade 8 and 9 Mathematics teachers of the participating schools. The researcher asked the teachers to identify learners who would be able to solve a word problem, would not be shy in front of a camera and could express themselves verbally.

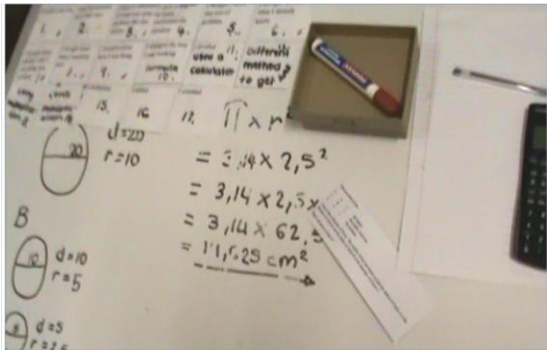
Table 5.3 Biographical information of the four participants

Participant	Time taken to solve the problem	Was the solution correct?	Gender	Age	Was the area of a circle done as part of the curriculum during 2011?	Grade in the year 2011	Achievement in Mathematics at the end of the previous year (2010)
Learner A	00:09:20	No	Female	15	No	9	70%
Learner B	00:06:58	Yes	Female	15	Yes	8	80%
Learner C	00:04:00	Yes	Female	14	Yes	8	76%
Learner D	00:09:30	No	Male	13	Yes	9	96%

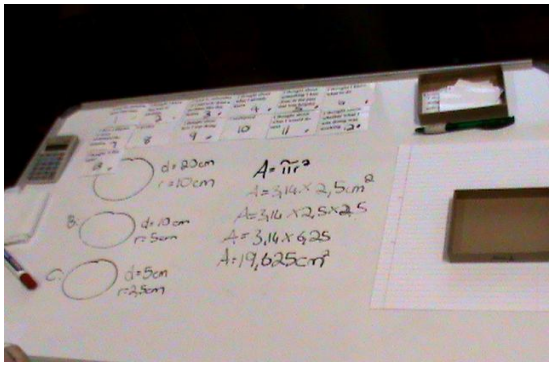
The following pictures illustrate the participants' final solutions.



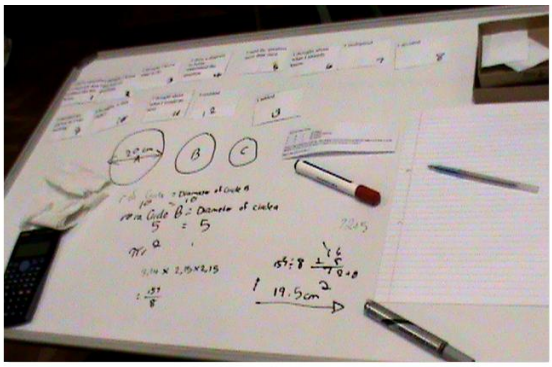
Picture 5.1: Learner A's solution



Picture 5.2: Learner B's solution



Picture 5.3: Learner C's solution



Picture 5.4: Learner D's solution

The first interview involved two stages of data collection, namely the interview itself (stage 1) and the selection and order of the metacognitive statement cards (stage 2). The findings gained from the first interview are presented first.

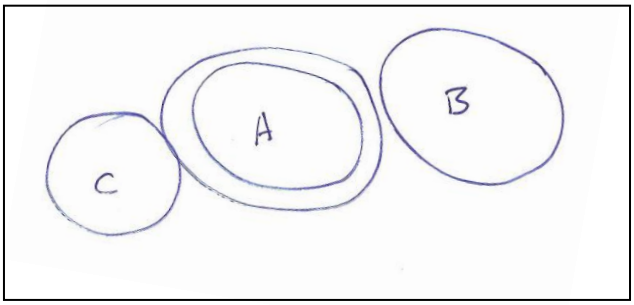
5.2 Stage 1: Analysis of the video-recorded first interview session(s)

Two themes were identified as role-playing factors in the first interview. In the first stage, only an account of participants' reflection on metacognitive knowledge and regulation is given. Note that these accounts are given according to each of the categories in Table 5.2 (KT; KP; KS; RU; RP; RM; RE) and for each of the four participants individually. The interconnecting and overlapping themes, categories and findings are synthesised and presented in this chapter and discussed in more detail in Chapter 6. The analysis of the participants' responses and actions is presented individually and consecutively, starting with Learner A and ending with Learner D.

The account of participants' problem-solving practice during the first interviews will now follow.

5.2.1 Account of Learner A's problem-solving practice

Learner A solved the Mathematics problem in nine minutes and twenty seconds. After drawing three circles, she labelled them in the centres as A, B and C. She circled the middle circle, labelled A, again and reread the question before drawing a line through A, as if picturing a diameter. She reread the question concentrating more and reading longer, while looking up at her written work and drawings.



Picture 5.5: Learner A's three circles

Learner A wrote down the given value of the diameter of 10 cm, before taking on the steps. Her first challenge was when:

> *I couldn't remember the formula* [1A:4][21]

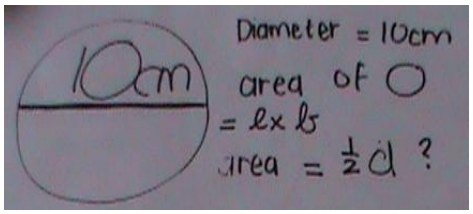
Trying three different approaches, she reflected on her knowledge and practices from experiences. The three approaches included (1) trying half of the diameter, (2) then drawing a big circle with a shaded area, and (3) eventually writing the word *area* on a piece of paper and underlining it. After just a few seconds, she remembered the formula and wrote it down.

> *I think that's why I underlined the word area…I remembered the formula*[22] *for the area* [1A:34-36]

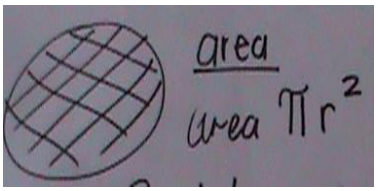
Almost immediately after getting the answer (Picture 5.8), the learner made another attempt using the formula known to calculate the circumference of a circle[23].

In Picture 5.9, after calculating the area, she crossed out the sum and wrote her final answer for the area of circle C as *78,5 cm squared*.

[21] The reference indicated by [1A:4] should be read as follows: the first interview [1] with Learner A [A], line four [4].
[22] Note that the formula in pictures 5.6 and 5.7 contains the word *area* and not "*A"* as expected. Also when referring to Picture 7.7 it is noted that there is no equal sign in the formula.
[23] Note that circumference is one-dimensional and in Picture 5.9 the solution to the circumference is written as 62,8 cm squared, clearly the unit of area, a two-dimensional aspect.

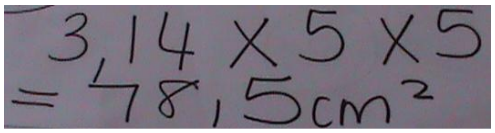


Picture 5.6: Big circle (A) with diameter and attempted formulae

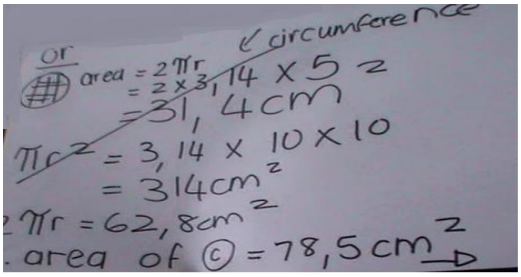


Picture 5.7: Shaded circle with correct formula

She then substituted the values of the radius[24] in the formula and used a calculator to calculate the area as 78,5 cm squared.



Picture 5.8: Substitution and answer



Picture 5.9: Other approaches as part of evaluation.

The account of the problem-solving process in the next paragraph pertains to the categories of reflection on metacognitive knowledge and regulation.

5.2.1.1 Reflection on the knowledge of the person (KP)

Reflecting on her own knowledge, Learner A stated that she could not remember the formula to calculate the area of a circle. After remembering the formula, the learner looked at her work again and did not look satisfied about what she had written down. She claimed that she was not sure.

When asked why she used a calculator, she said that: *I didn't know what, I couldn't calculate it in my head.* [1A:86]

The learner claimed to check the calculator twice: *I always just check on my calculator twice, because it sometimes gives me wrong answers.* [1A:107-108]

She mentions that

> *If I can't remember something, then I do it, even if it is wrong, I just try it and I do other things as well just to maybe think okay, this one's right.* [1A:122-123]

The learner summarised her 'person' knowledge as experienced in this problem when she stated:

[24] The use of the radius as *5cm* is incorrect as that is the diameter of circle C (or the radius of circle B) and not the radius of circle C. However, she does know the formula and how to distinguish between diameter and radius as illustrated in Picture 4.6. In this case, it was used to calculate the diameter of 10cm.

I don't know, I wasn't sure about the whole thing. I wasn't sure. Yes, I just wasn't sure if I was doing it right. I couldn't remember. [1A:179-180]

5.2.1.2 Reflection on knowledge of the task (KT)

The learner watched herself solve the problem in the video-recorded session from the start through to the end. While watching herself draw the three circles, she said:

Now I'm writing the diameter… [1A:8] *I just said the area of, and then I drew the circles and then I said length times breadth. I didn't think it was that but I just did that to, I don't know.* [1A:2] *I couldn't remember the formula*. [1A:4]

She used knowledge learned in the previous year to get the radius if the diameter of a circle was given, but she was confused with this concept as she stated:

I just got things I remembered from last year, to remember the formula. [1A:14] *I figured out that half of the diameter is not the formula and I think I remembered what the formula was.* [1A:22 & 32]

She realised her confusion and lack of understanding when she said:

I couldn't remember what was the difference between circumference and area, and then I remembered that area has a squared at the end. [1A:38-39]

She demonstrated self-questioning by trying to remember the formula and confirmed her surety about the formula when saying:

I think I was trying to work out how to put the formula, is it the radius plus radius or is it radius times radius? [1A:61-62] *Here I wrote radius times radius and I knew it was radius times radius and not radius plus radius*. [1A:79-80] *Then I remembered having a, I actually remembered having a radius of ten last year and I knew you just move the comma.* [1A:159]

5.2.1.3 Reflection on knowledge of strategies (KS)

Drawing circles was used as a strategy to better understand the question. Learner A drew three circles and every time, when she wondered about something, she drew another circle. She attempted different strategies to get or remember the formula.

So I tried half of the diameter and put a question mark there. I then drew a circle and thought area is like shaded area. The whole circle and then I remembered what the formula was. [1A:8-10]*… and I just drew a picture for myself.* [1A:47]

I wasn't sure if that was the formula for area, so I tried to make other formulas because I didn't know what the formula was for the area and so I just made it as well. [1A:112] *And do other things as well…* [1A:123]

5.2.1.4 Reflection on metacognitive regulation of understanding (RU)

Trying to solve the problem, the learner made sure that she understood what was asked and she regulated her understanding by drawing three circles. She used the method of drawing pictures and comparing the formulas in order to understand:

> *I made another circle just to show that the area is like the shaded area, the whole circle.* [1A:19] *Half the diameter is not the formula;* [1A:22] *I tried other formulas of a circle, like I tried circumference … I just drew a picture for myself.* [1A:50]

5.2.1.5 Reflection on metacognitive regulation of planning (RP)

Regulative planning took the form of picture drawing and rereading the problem. It appears that Learner A's planning related to her understanding. She started planning immediately after getting the word problem. She prepared her workspace by moving the calculator, pen and problem into position. After this, her planning involved analysing information (reading the question) and what was done with that information so far (looking at the three circles drawn). For example, after drawing the three circles (see Picture 5.5) the participant filled in the given information as she read the problem.

> *I made another circle just to show that the area is like shaded, the whole circle.* [1A:22] *I remembered the shaded area … and I just drew a picture for myself.* [1A:47]

At some stage just after reading the word problem, Learner A also imagined herself drawing diameters or other circles. This observation was made while replaying the video recording. She would hold the pen, as though writing, and pretend to write or draw. Doing so her eyes would look up and down, browsing her written and sketched work. At one stage, she "circled" circle A as if to show to herself the circumference of the circle. It appears as though planning also took place when going from one strategy to another and indicates a form of imaginative planning. The learner imagined and predicted a specific situation *seeing* what her outcome would be.

5.2.1.6 Reflection on metacognitive regulation of monitoring (RM)

Learner A showed clear signs of monitoring behaviour. This could be noticed by observing her hand-eye movements as well as her verbal report. The researcher made observation notes by asking himself what the participant was doing or looking at. The following verbal and non-verbal account was given regarding Learner A's reflection on monitoring. Notice the

accompanying physical indicators that correspond with both the verbal and non-verbal accounts in Table 5.4 where the self-reflective and research observations are compared.

Table 5.4 Verbal and non-verbal account of Learner A's reflection on monitoring (Self-reflective and research observations)

Verbal account – after solving the problem (participant's self-reflective comments)	Indicators while reflecting – after action
…and then I put a question mark [1A:9]	She kept her hands on her lap and only explained with them underneath the desk.
…looking at the work, reading the question again [1A:31; 1A:44; 1A:67]	Fidgeted with her fingers. Explained with her hand above the desk, not fidgeting any more but pointing confidently at her work. She held her left forearm.
I was checking if I'm right because I wrote it down [1A:82]	Getting closer to the answer, the participant was more relaxed, explaining with her hands and pointing with her finger at what she did.
…and that's when I remembered I knew exactly what the formula was [1A:50]	Smiled, and held her right forearm
…I think I was trying to work out how to put the formula [1A:61]	Scratched her nose with her right hand
Non-verbal account during the problem-solving process (researcher's observations)	**Indicators while monitoring – during action**
As she read and reread the problem, she stopped and wrote down or drew her own version of what she had read.	Her lips started twitching She bit her lower lip
She paused looking at her work and then away from her work, staring in the air with her lips slightly moving.	Restarted the solution on a different page, not looking at the first page of the solution until the end, when evaluating
Her eyes browsed through her written and sketched work just before she wrote the next step or completed filling in the information of the next circle.	Rubbed her neck while biting lower lip
She pointed with a pen towards her written and sketched work while reading the word problem.	Frowned
After every sketch or step, she paused and scanned her page from top to bottom.	Rubbed her nose
After every answer, she compared the answers and then tried another way to solve the problem.	Scratched her chin, lower lip and arm

5.2.1.7 Reflection on metacognitive regulation of evaluation (RE)

Learner A was the only participant who showed apt evaluation skills. By using three different methods, she managed to arrive at closing stages when comparing various answers.

> *And then I realised that it was wrong because it couldn't fit to, the amount was too small, three one comma four, the amount was too small to be the area. And then I tried another formula.* [1A:144-146] *… because maybe, I thought, I was mixed up* [1A:149]
>
> *Yes, I'm looking at it, then I thought that the second formula is too small and the third formula is way too big. So then, I thought, it must be the first one that I did.* [1A:167-168]

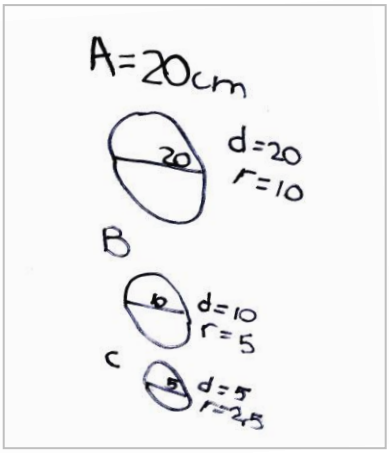
The learner compared her answers and evaluated their meaning to check if they made sense. Although her answer was wrong, she did manage to evaluate her answer using different approaches.

Table 5.5 Summary of the account of Learner A's problem-solving practice

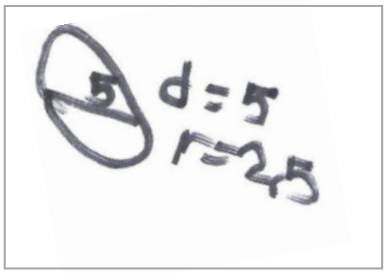
Categories	Description
Reflection on metacognitive knowledge	
Reflection on knowledge of the self (person) (KP)	The learner realised that she could not remember the formula and it felt confused and uncertain.
Reflection of knowledge of the task (KT)	She was unsure about why she was doing what she was doing and lacked knowledge of the formula., After trial and error she got it right. She did however lack to grasp the meaning of circumference and area in applications. She remembered, from experience, what to do and tried using that.
Reflection on knowledge of strategies (KS)	Knowledge of strategies to enhance understanding by drawings, writing in words and comparing approaches. Approaches proved an existing background in similar contexts.
Reflection on metacognitive regulation	
Reflection on regulation of understanding (RU)	The learner showed an understanding of what the problem was about and what it expected from her. However, she lacked a thorough understanding of concepts like circumference, diameter, radius and area.
Reflection on regulation of planning (RP)	Learner A's planning is related to her understanding of the first principles in Euclidean geometry. Planning involved organising her stationary, calculator, reading, re-reading and drawing pictures related to the information in the problem.
Reflection on regulation of monitoring (RM)	She monitored throughout the process, excessively. She showed signs of low and high confidence and varied between uncertainty and certainty.
Reflection on regulation of evaluation (RE)	Through monitoring, this learner evaluated her work. She also tried other strategies, comparing their solutions and judging their accuracy.

5.2.2 Account of Learner B's problem-solving practice

Learner B solved the problem in sixteen minutes and fifty-eight seconds. Unlike Learner A, this participant first read the whole problem three times before drawing the circles. She seemed to have a calm and relaxed approach, doing what she does slowly and double-checking her work regularly. She first drew circle A and then labelled it. Then she drew circles B and C and labelled them. After drawing each circle, she read the question again and wrote the information given next to each of the three circles as illustrated in Picture 5.10 and Picture 5.11.



Picture 5.10: Learner B's three circles

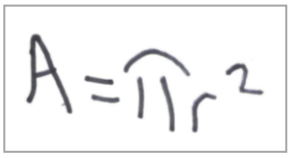


Picture 5.11: small circle C with diameter and radius

When the video recording was played back, the researcher asked at this stage what the learner was doing, and she commented:

> *I was putting like into a picture format, yes, like individual, like a, b, c.* [1B:8]

She moved closer to the question on the desk. She read it, pointing at some parts of the question and looking up at her drawings, then back to the question. She moved the calculator slightly out of the way and began writing *A* =. *S*he paused, holding the pen against her lips, while looking at circle C and then finished the first step with the following formula and a substituted value for pi.



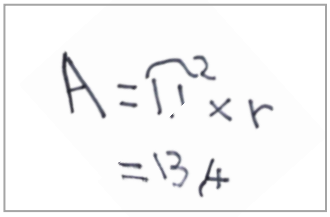
Picture 5.12: Formula for area of a circle

When the researcher asked the learner why she was pausing, she said:

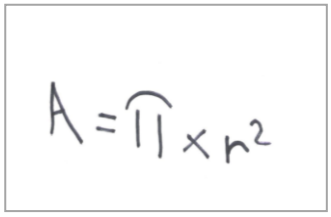
> *I wanted to write my formula so I went back to check what are we doing, what area, perimeter or volume, so then I paused to double check; okay what is the area, so I specifically went to look for, if its area.*[1B:73]

She paused, sat back in her chair and held the pen against her lips while looking at her work. She then erased the square and *r* in the formula; wrote times r; hesitated (paused) and placed the square at pi as shown in Picture 5.13. At this stage, the learner commented: *I was very uncertain there.* [1B:138].

She paused again, pen between the lips, and after some time corrected the formula as seen in Picture 5.14.

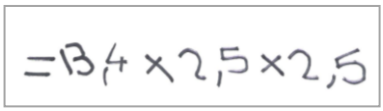


Picture 5.13: Formula for area after it was changed

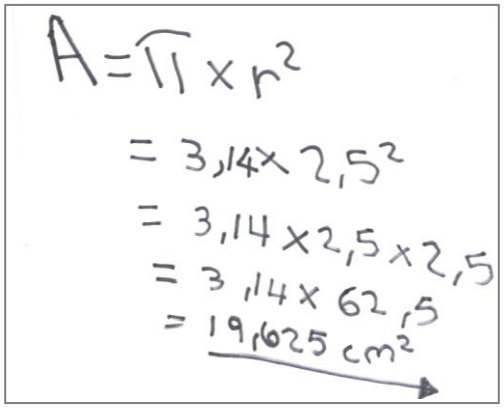


Picture 5.14: Correct formula for the area of the circle

After substituting the value for pi and the radius into the second step and using a calculator to get the answer, the learner evaluated her answer. She recalculated using other strategies and realised that pi was wrongly substituted. She corrected this and wrote the final answer as shown in Pictures 5.15 and 5.16.



Picture 5.15: Second step completed



Picture 5.16: Solution to the problem

The following account reveals how and on what the participant reflected during the playback of the video recording.

5.2.2.1 Reflection on the knowledge of the person (KP)

While reflecting on her problem-solving process, the learner expressed her opinion regarding her hesitant pauses throughout the sum. She claims:

> *I'm trying to figure out if it's right because I'm uncertain.* [1B:124]

The learner used different approaches, like multiplying twenty-five with twenty-five and putting in the commas at the end instead of writing two comma five times two comma five.

> *...I took the comma away so that it would be easier for me* [1B:201]

The learner made use of a non-programmable scientific calculator and could not understand why she got a fraction for an answer. The researcher did however mention to the learner that

she could ask questions. The learner showed clear evidence that she knew and predicted what the outcome should be and she identified information, concepts or skills that might help her.

> *…and that gave me a fraction sign so I was confused, I was like what?* [1B:146]*…I kept looking for a decimal.* [1B:148]*…I did long multiplication…it would be much easier* [1B:1507]

When the learner noticed that she had paused frequently, she confirmed her awareness and the knowledge she had of herself as well as of her shortcomings during problem solving by saying:

> *I think I'm used to doing that, I will wait and count before I write it.* [1B:233]. *To be honest, most things are forgotten, I'm very forgetful.* [1B:243]. *I think I was mixed up* [1B:249]. *I don't know how to check my answer.* [1B:257]

5.2.2.2 Reflection on knowledge of the task (KT)

She stated that her first approach was not only reading the word sum: *I'm reading,* but also, *I'm analysing* [1B:2]. *I went back to read, so sort of looking for information* [1B:43]

She demonstrated having knowledge of this kind of task when she said:

> *The circle has a point in the middle, so if the diameter is twenty, its* [radius] *obviously is going to be half of twenty, which is ten.* [1B:53; 1B:77]

A lack of specific, detailed knowledge regarding the formula and the value of pi seemed to be a cause of uncertainty for this learner:

> *Sir I was uncertain. I was thinking, wait, is the r supposed to come before or am I not supposed to put the multiple sign, or am I supposed to put it, so I wrote and then thought, wait, is this right?* [1B:116; 1B:81; 1B:87; 1B:63]

One major concern in this participant's first take on the solution involved confusion of the value for pi. *I think I was thinking if the pi is right* [1B:81]

Learner B's effort to get the correct answer (without the fraction) gave insight into how she performed in unfamiliar situations, for example, how she dealt with a difficult topic such as fractions. She would calmly specify what she knew of the task by writing or drawing it.

5.2.2.3 Reflection on knowledge of strategies (KS)

Reading and analysing are strategies to better understand the problem and context.

> *I'm reading. I'm analysing.* [1B:2]. *I was like putting into a picture format.* [1B:8]

Deriving radius from the diameter and referring to the midpoint of the circle were tactics this learner employed but did not show. She mentioned that she did this: *But in my head.* [1B:57] She wrote down the matching information and the answers of her mental calculations next to each of the circles.

> *I'm just writing the extra points down so that I understand…I write the radius so that I understand what I'm doing.* [1B:67]

Claiming to have knowledge of relevant contexts of circles, such as *area, perimeter* or *volume* [1B:73], this learner got confused with the value of pi and afterwards, when realising this, changed this. She also made changes to the formula as portrayed in Picture 5.11 and Picture 5.12. It appears that these changes contributed to her understanding of the question and her knowledge of the task. She remembered the formula later when saying:

> *…I recognised it after I changed the formula*. [1B:108]*…I think when I changed, when I added the multiplication signs.* [1B:128]

Although she had a scientific calculator, she did not have sufficient knowledge of the strategies needed to use the calculator. This was evident when the answer she obtained from using it was unexpected.

> *I'm thinking because maybe there was like another way to do this*. *I'm not fond of scientific calculators so I just did everything.* [1B:152]

She explained how she typed the values on the calculator and looked at her answer:

> *…and then it gave me a fraction sign so I was confused. I was like; what?* [1B:146]

One strategy to help in the process of calculating involved long multiplication. The learner did this because

> *I … used plan B … to do long multiplication*. [1B:185] *… I thought it would be much easier.* [1B:149] *… I first took out the comma and wrote it as a whole number.* [1B:221]

In such a situation of confusion, the learner went back to her initial approach and sought help in the question:

> *I'm reading because I was mixed up.* [1B:249] *… I went back to read and I understand it now.* [1B:249]

In the end, when asked if she thought her answer was right or wrong, the learner mentioned that her lack of knowledge might have been the cause of her uncertainty:

I'm a bit uncertain [1B:155]*... Because I don't know how to check my answer* [1B:257]

By reflecting on her work, she explained what she was doing. Most of the time it sounded as if she was quoting her self-talk during the problem-solving process:

I think I was thinking…If I had to do it a different way… [1B:81]

5.2.2.4 Reflection on metacognitive regulation of understanding (RU)

Learner B regulated her understanding by reading and analysing the problem. Her first approach to understanding was: …*I think I just predicted.* [1B:4] She drew three circles and labelled them with the necessary information. She took some time between remembering the formula and getting confused about the value of pi. Eventually the learner achieved despite these difficulties by applying her knowledge and strategies to correct the problem. Understanding (or a lack of it) was noticeable from the beginning of the problem solving. Her approaches in Picture 5.8 and 5.9 explain her understanding. However, in Picture 5.10, a mistaken value for pi was substituted. The learner found it difficult to work with decimals and this, in turn, tapered her understanding of the value of pi.

The learner understood, or tried to understand, some parts of the problem-solving task. By employing other metacognitive strategies such as monitoring, she pinpointed or clarified her lack of understanding:

> *I wanted to check, where does the ten come from?* [1B:49]*... and went back to reading.* [1B:65]… *I wanted to write the formula so I went back to check what we are doing.* [1B:73]… *I still don't understand so I kept on looking back to figure it out.* [1B:166]

She expressed her own understanding of what to do by interpreting and analysing the question. She even put it in her own words: *I then rephrased…* [1B:108]

5.2.2.5 Reflection on metacognitive regulation of planning (RP)

Learner B's regulative planning started immediately when she first received the problem. She read some parts more and some longer. She arranged the writing space, cleared the desk and started drawing. Labelling and completing information for the three circles confirmed reading, rereading and analysing as important starting points:

...I'm reading, I'm analysing. [1B:2]… *I just predicted.* [1B:4]

Rereading the questions and searching for information were trademarks of this learner's regulation of planning. Picture drawing agreed with her take on understanding. The drawings themselves showed a big circle A and a smaller circle B with a smallest circle C. This was a clear suggestion that she was planning to use three different-sized circles and these served as an aid to understanding.

> *I was putting like into a picture format…* [1B:8]*... I read that piece then went back to the circle …* [1B:43]*... so sort of looking for information…* [1B:43]

Planning also occurred and confirmed its link with understanding when, during self-talk, the learner mentioned:

> *I was thinking, wait, is the r supposed to come before, or am I not supposed to put the multiplication sign, or am I supposed to put it – so I wrote and then I thought…*[1B:116]

Knowing, and even identifying, another strategy called *plan B* [1B:185] confirms that this learner had a planned approach to the problem. She looked at her work from top to bottom and bottom to top while pausing from rereading. She looked away from her work from time to time and carried on after a few seconds, seeming to acquire a moment of insight. Regulative planning was an important element in this learner's problem-solving practice.

5.2.2.6 Reflection on metacognitive regulation of monitoring (RM)

Learner B provided evidence of monitoring. Observations are based on her overall body language such as repeatedly looking at the question, placing her pen between her lips, smiling, laughing, pointing with her finger or pen at parts of the question. While watching the video she realised something regarding her choice of metacognitive statements:

> *…I am not sure; I think I put some of the questions, statements, in the wrong box.* [1B:171]

This, on the task reflection serves as monitoring behaviour. Throughout the problem-solving practice, she monitored her work:

> *…checking* [1B:10]*... I paused … I was checking if I was on the right track.* [1B:19; 1B:21]

Checking her own work to control what she was going to do next, explains how she monitored throughout the process: *If it looked like it fitted … I would have probably used plan B…* [1B:185]

She looked at her work in small parts, focusing on one at a time. This physical and non-physical behaviour repeated throughout the process. The following two tables compare Learner B's self-reflective comments and observations. Notice the verbal and non-verbal indicators while monitoring. Overall, it appears as if self-talk occurred along with monitoring.

Table 5.6 Verbal and non-verbal account of Learner B's reflection on monitoring (self-reflective)

Verbal account – after solving the problem (participant's self-reflective comments)	**Indicators while reflecting – after action**
I was checking [1B:10; 1B:37; 1B:69]	Reread the word problem and looked at the circles drawn and the written work, pointing with finger/pen
I paused [1B:15&17]	Pointed with the pen
I think I looked at that because it looked similar[1B:106]	Looked up at the video playback and down at her written work correspondingly
I check everything, if everything makes sense [1B:245]	Explained with her hands
If it looks like it fitted with the whole sum [1B:185]	Explained with her hands as if comparing the answers on a scale
Non-verbal account during the problem-solving process (researcher's observations)	**Indicators while monitoring – during action**
Appeared to read some parts more and longer	Hands on her lap and her lips slightly moving as she was reading
Learner read, stopped, read again	When pausing she pointed with a pen or finger towards some parts of the word sum or pictures.
Learner seemed uncertain about a question asked during the interview	Looked down at her written work
After reading, before writing	Looked away from her work for a few seconds and suddenly carried on writing

5.2.2.7 Reflection on metacognitive regulation of evaluation (RE)

Learner B evaluated her work but not with as much variety in methods as did Learner A. Self-talk guided her monitoring and evaluation actions. *I was thinking, is this right?* [1B:81] At one stage when replaying the video recording, the learner said softly: *That's where I made my mistake.* [1B:82] It appears that there were two options after reflecting on evaluation. The participant realised what was done is incorrect: *I have to go back…* [1B:67]*…is this right or am I doing something wrong?* [1B:124] She agreed with the approach so far: *Then I thought it was right and I carried on* [1B:81]. After using the calculator to find out the answer, she evaluated her answer by comparing it with what her expectations were: *…and that gave me a fraction sign … I was like – what?* [1B:146]

To conclude the account of Learner B's problem-solving practice, she was asked if she thought her answer was right. Her response to this correlated her metacognitive knowledge with her regulation:

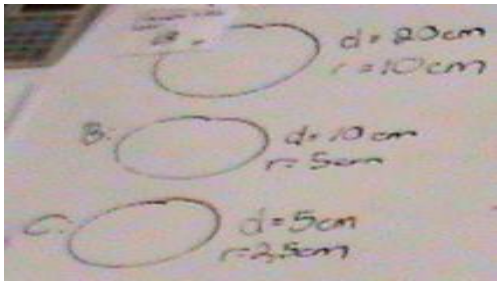
After doing all the calculations, I went back… I'm a bit uncertain … because I don't know how to check my answer. Yes, I don't know how to check my answer. [1B:249-257]

Table 5.7 Summary of the account of Learner B's problem-solving practice

Categories	Description
Reflection on metacognitive knowledge	
Reflection on knowledge of the self (person) (KP)	The learner was uncertain about her approach. She knew how to make it easier. She claimed to be very forgetful.
Reflection of knowledge of the task (KT)	Fractions confused her. She remembered components of the task from a previous experience. Unsure of the value for pi.
Reflection on knowledge of strategies (KS)	She took the comma away. She did long multiplication.
Reflection on metacognitive regulation	
Reflection on regulation of understanding (RU)	The learner regulated her understanding by reading and analysing. She drew circles. Difficulties included pi and what the formula is.
Reflection on regulation of planning (RP)	Read some parts of the problem more. Organised her desk, stationary. Systematic drawing. Planning order of variables. *Plan B.*
Reflection on regulation of monitoring (RM)	She monitored her work in small consecutive parts. Kept on reading every time after she wrote something. Included physical behaviour. Self-talk.
Reflection on regulation of evaluation (RE)	After reflecting, she realised what was incorrect or agreed with the approach. Compared answer with expectations (plan/predict).

5.2.3 Account of learner C's problem-solving practice

Learner C solved the problem effortlessly, and mechanically, in 4 minutes. She read the question only partly, focusing on some particular aspects at a given moment. The steps were taken without doubting the formula, units or substituted values – unlike Learner A or Learner B. After reading the question, she drew three circles of similar sizes. Without showing lines to picture radius or diameter, she wrote down the given and mentally calculated information next to the matching circles.



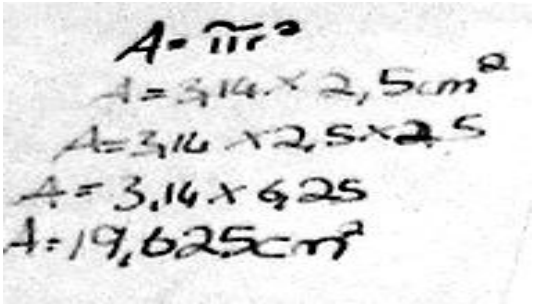
Picture 5.17: Learner C's three circles

The learner calculated or deduced the information for circle B and C mentally, not showing any written work for her conclusions about the diameter or radii. She read information every time before she wrote something, thus breaking the question up into smaller manageable parts. While referring to the video recording afterwards, the learner had difficulty describing what she had done. She kept her answers short and sounded uncertain. The dialogue between the participant and researcher explains this:

Researcher: *Why did you read the question again?* [1C:106]

Learner C: *Sir I was looking at this (pointing with her finger towards her solution)* [1C:107]

When asked questions involving *what* she was doing and *why* she did something, her response had a futile motive. She commented on a particular step: *I was writing and then I closed the pen and then I wanted to work out the sum.* [1C:119-120]



Picture 5.18: Learner C's final solution

Learner C could mostly not reflect on what she had done nor could she provide clear reasons for doing what she had done. She monitored her work less often than the other participants did, and only reread the question. She also did not evaluate her answer. When finished with the interview, the researcher mentioned that she had finished her sum in four minutes, and she asked nervously: *Is that bad sir?* [1C:149] The researcher said that it was *not bad at all* and thanked her for participating in the interview. The account of Learner C's reflective practice will now follow.

5.2.3.1 Reflection on the knowledge of the person (KP)

For this participant it appears that feeling comfortable was important. She confirmed this and showed her awareness in this regard as she moved her question and calculator out of the way. She said: *It was uncomfortable, the way I was sitting, I wanted to move it closer* [1C:4]

This learner did not read the whole question through from beginning to end. Instead, she broke it up and later explained why she did it: *… I read that part like three times…* [1C:12]

> *It's better for me to understand what I'm doing. If I read it, like the whole question, then I don't understand what I'm doing.* [1C:21-22]

She mentioned the reason why she labelled the circles: *It was bothering me…* [1C:75] Realising a lack of knowledge and strategy, she said: *I wasn't sure if I should write the square or not.* [1C:136]

5.2.3.2 Reflection on the knowledge of the task (KT)

Learner C read the question, seeking information to enhance her understanding. She only read the question partly at each moment, working with smaller quantities at a time, relevant to what information she was seeking. She wrote the formula without hesitation and showed knowledge of this task. This learner was confident at first. Doing what she did effortlessly, however, when reflecting on her work, she said after she had watched herself write the answer:

> *I wasn't sure, I was like I couldn't remember because we did circles a long time ago*. [1C:135]

5.2.3.3 Reflection on the knowledge of strategies (KS)

Drawing the three circles and reading the question in smaller parts were seen as strategies that Learner C employed to better understand the question.

> *I didn't read the whole question…I read it like this part, I read that part like three times and then I went to the next part and I read that like three times.* [1C:10-12]

The learner drew three circles in succession, looked for the information that was relevant, and filled that in. *I counted. I drew them …* [1C:38]. Labelling the circles was not important to this learner and later she mentioned it was bothering her. When asked why she had labelled them, she said: *I don't actually know*. [1C:74] She employed only one approach to solve the problem. Identifying the context of the question, choosing the correct formula and substituting the deduced values seemed effortless. She did the sum in parts.

> *I'm calculating two comma five times two comma five*. [1C:125]… *Now I'm working out what is three comma one four times six comma two five.* [1C:126]

5.2.3.4 Reflection on metacognitive regulation of understanding (RU)

Learner C's understanding seemed to be linked to her knowledge of strategies. Reading the word sum, focusing only on some parts at a time and drawing pictures constituted patterns that were important for her understanding:

> *I read it, like this part* [1C:10]*…it's better for me to understand* [1C:21] *… I'm drawing a* circle [1C:27] *… So that I can see the picture of what I must do.* [1C:29-30]

The learner drew three circles of the same size, with no symbols or depictions of what their radii and diameter entail. Her understanding of the word sum was algorithmic. Although she solved the problem correctly, it is not certain whether the learner understood what she did.

The necessary information to start with the sum, like radius of circle C, was available mentally and quickly. She did this unseen, non-routine problem, routinely. The fact that she had difficulty in describing what she did in each step and why, shows this. She reflected (in a basic sense) on what was done, but could not describe surrounding thoughts (deeper reflection) about why she did what she had done.

5.2.3.5 Reflection on metacognitive regulation of planning (RP)

Planning took the form of picture drawing and purposive reading. Learner C looked for commands by reading the word sum. This was linked with her understanding when she said:

> *I drew the picture so that I can see what I must do* [1C:30]

Neither counting the number of circles nor labelling the three circles was seen as a planned action, as the reason why she labelled the circles is inferred. *I don't actually know* [1C:74]. The learner claimed: *It was bothering me that there were no labels.* [1C:75, 81]

5.2.3.6 Reflection on metacognitive regulation of monitoring (RM)

Learner C monitored her work by rereading parts of the word sum that were pertinent at the moment. Just before she started writing, she read the second part of the question again. Before writing down each circle's radius and diameter, she went back to the question and reread that particular part. *I was reading what the diameter was* [1C:52]

She looked at her pictures, read the question, looked at her pictures again, and wrote the first step. From here onwards, she looked at her pictures and the word sum only once. She completed a second and third step, without looking back at previous written work. On the calculator, she typed in the third step once, moving to the fourth step and typed that in once as well. She did not monitor her answer by checking on the calculator again. After writing the final answer, she hesitated briefly before writing down the square for unit of the area of the circle.

Table 5.8 Verbal and non-verbal account of Learner C's reflection on monitoring (Self-reflective and researcher observations)

Verbal account – after solving the problem (participant's self-reflective comments)	Indicators while reflecting – after action
I read like this part [1C:10; 1C:29-30; 1C:91-92; 1C:52]	Pointed with her index finger at the question while looking down at her written work, reading in parts
I went back to the question [1C:60]	Looked at the question and her drawings
I was making sure what I was supposed to write [1C:95]	Looked at her question and written work; then for a slight moment away from her work as if looking at something in the air in front of her
I was looking at this because I was making sure what I was supposed to write here [1C:102-103]	Pointed with her right index finger at her second step
Non-verbal account during the problem-solving process (researcher's observations)	**Indicators while monitoring – during action**
Learner reads, stops reading, writes the values down and reads again. Appears to read some parts longer.	When pausing, she looked at the word sum with her left hand on her lap and her lips slightly moving. At some stages, she frowned.
Getting an answer on the calculator and writes it down	She first looked at her previous step and then wrote down the answer.
After writing the answer, hesitates and then writes the square at the unit	While hesitating, her mouth twitched and she looked away from her work

Rereading some parts of the word sum more often than others also served as monitoring. When Learner C was asked if she had learned anything from watching the recorded interview, she said:

> *I went back to the question to read the second part again, to check…* [1C:44] *I went back to the question a lot, like I didn't know I do that.* [1C:144]

5.2.3.7 Reflection on metacognitive regulation of evaluation (RE)

Although the participant monitored her work, she did not evaluate her answer at all. After writing down the final answer as shown in step 5 of Picture 5.14, she announced that she was finished. When asked whether she had gone back to the question again afterwards, she simply said: *No.* [1C:146]. She did however double-check the answer she had written down from the calculator in step 4 of Picture 5.18.

> *I was checking if it was a six, I was making sure what the rest of the number was so I didn't write it wrong.* [1C:130-131]

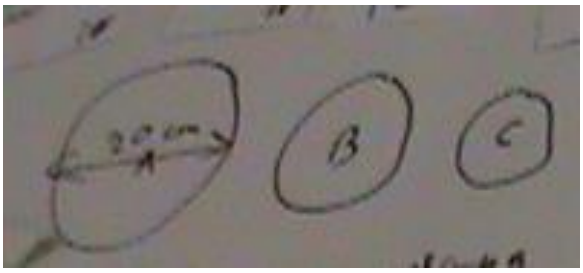
The only self-evaluated comment the participant made was when the researcher told her how long she had taken to solve the problem, and she asked nervously whether it was bad.

Table 5.9 Summary of the account of Learner C's problem-solving practice

Categories	Description
Reflection on metacognitive knowledge	
Reflection on knowledge of the self (person) (KP)	Identified that it was important to feel comfortable in the class. She knew what to do to understand..
Reflection of knowledge of the task (KT)	Read the question to seek information. The question was read in parts. She knew the formula and values needed to substitute. However she admitted that she wasn't sure because this work had been done a long time ago.
Reflection on knowledge of strategies (KS)	Drew picture and reread the question. Used one strategy to solve the problem, and did the sum in parts.
Reflection on metacognitive regulation	
Reflection on regulation of understanding (RU)	Regulated understanding by rereading selectively and drawing pictures. Did not show differences between pictures' sizes, all were the same size. Wrote no symbols or lines to show diameter or radii. Uncertain if this learner understood the task.
Reflection on regulation of planning (RP)	Planning was done mentally and searched for commands by rereading the question. Did not know why she did what she had done.
Reflection on regulation of monitoring (RM)	Rereading and checking were the most relevant steps. Went back to the question and pictures after she had written. Did not monitor from the third step onwards.
Reflection on regulation of evaluation (RE)	Did not evaluate her answer. Only recalculated the value given by the calculator and wrote that down.

5.2.4 Account of Learner D's problem-solving practice

This learner solved the problem in 9 minutes and 30 seconds. He started drawing the circles almost immediately after being given the word problem. Although all four participants drew the three circles and filled in information about each circle, Learner D only included information for circle A. He also did not make use of algorithms or so-called *steps;* instead he used a method of picture drawing, descriptive words and mental division.

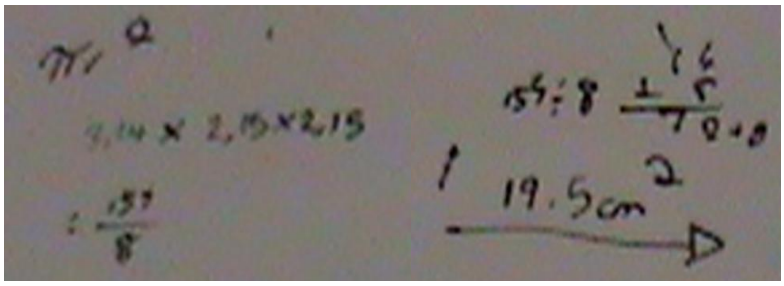


Picture 5.19: Learner D's three circles

Learner D drew the circles and only wrote information in circle A. Circles B and C were just drawn and labelled but no values were written inside or outside these circles.

> *I drew it, what they said it is, I drew it on. So if it's a big circle, I draw a big circle if it's a small circle I draw a small circle.* [1D:3]

After drawing the circles, the learner started calculating the area by first making sure he understood the given information, which was not written down. He wrote the formula for the area of a circle and continued to substitute the values for pi, as well as the predetermined mentally calculated value for the radius of circle C.



Picture 5.20: Steps from formula to answers

Learner D seemed surprised when he wrote down the answer as shown on the calculator's screen. This fraction led him to solve the problem, manually, by doing division. The following descriptions of Learner D's problem-solving practice elucidate his reflective skills during this process.

5.2.4.1 Reflection on the knowledge of the person (KP)

Learner D's knowledge of the person echoes his father's instruction of some time ago. Knowledge of the ability of the personal self is traced back to a reflection on something learned in the primary school. When reflecting, he said:

> *My dad always tells me that instead of using the calculator I must do it in my head* [1D:139-140] *The way I worked in primary school is…*[1D:146-148; 1D:153]

After reflecting on this experience and the doing of division *in my head*, the learner showed some awareness of his mind's functionality when he was less confident about the strategy selected:

> *I don't understand why I do it.* [1D:157] *I don't know.* [1D:174-175]

This learner could identify inconveniences in problem solving. These difficulties included knowledge of himself, choice and personal awareness and shortcomings:

> *I always take things that I know, like it's stuck in my head.* [1D:249-251] *I have a problem with making sure my adding and subtracting is right.* [1D:271]

5.2.4.2 Reflection on the knowledge of the task (KT)

Learner D did not write much detail at the sketches of the three circles or in the steps pertaining to this problem. Instead, he explained: *I decided to use my head.* [1D:140; 1D:144; 1D:205]

One problem that this learner identified was:

> *Normally on my calculator, when I divide 357 by 8 it gives me the decimal; the scientific calculator didn't do that, so I did it in my head instead.* [1D:134]

He used a division approach that he had learned in primary school and provided the following details with regard to his knowledge of this task:

> *I first said 8 can go into 15 once and then fractions you normally say how many times 8 can go into15 and then you write your answer.* [1D:172-173]

5.2.4.3 Reflection on the knowledge of strategies (KS)

Knowledge of the task(s) that this participant had to deal with was evident in his knowledge of strategies. One strategy that he utilised to understand the problem was similar to that used by the other participants involved:

> *I drew it. What it said it is, I drew it on. Every time it gives me a fact I put it on paper. That's why I drew all the arrows, to show the diameter… and radius of circle A.* [1D:3-4]

When asked how many times he had read the question, he replied:

> *Once. The first time and then the second time.* [1D:10] *I go back to the question to read the next fact.* [1D:17]

Another previously noted strategy was determining the radius of the next circle. Although the learner decided not to use the answer obtained from the calculator, he tried despite knowing that it might be wrong:

> *They didn't give you the fact, I had to make it up for myself.* [1D:38] *I would half, half the diameters.* [1D:32] *Not a different strategy, it just looks different*. [1D:238] *…you know you can't do this, you can't go into 157 once, so I knew it was wrong.* [1D:174] *I just wanted to see if I did that, if it would be right.* [1D:165]

Dividing, multiplying, correctly remembering the formula for the area of a circle, substituting the value for pi and calculating the radius of circle C were all strategies employed by Learner D to solve the problem.

5.2.4.4 Reflection on metacognitive regulation of understanding (RU)

The process of understanding this problem seemed very important to Learner D. Rereading the question was also part of his understanding, besides drawing the pictures:

> *I took the problem; I worked with it to make sense.* [1D:3] *Every time it gives me a fact, I put it on paper, so that's why I drew.* [1D:6]

On writing the information comparatively as illustrated in Picture 5.20, the participant said:

> *I wanted to write r again and then I realized that r can be anything. So I wrote r to make it more, to make more sense of it.* [1D:62-64]

5.2.4.5 Reflection on metacognitive regulation of planning (RP)

Learner D was the only participant who mentioned his plan of approach and he stated it with awareness and confidence in his abilities.

> *I can calculate the formula. I can use the formula to calculate the answer.* [1D:52] *I try to make sure that everything is actually right before I go on.* [1D:111]

He strategically compared the various circles' radii. On the skill of mentally calculating, the participant appeared to have had training in primary school. However, not everything was anticipated, as he at one stage mentioned**:**

> *If the calculator didn't give me what I wanted I use my head.* [1D:140] *I knew it was wrong. I was just doing it to, just, I don't know, I just did it.* [1D:175]

Another uncertain element of planning was when he drew the circles:

> *I was making a dot in the middle. I don't know why I did that, maybe just to make sure that I was right.* [1D:124]

5.2.4.6 Reflection on metacognitive regulation of monitoring (RM)

Monitoring was not observed as much, although he did comment on his thinking when reflecting on steps done so far:

> *I go back to see what I was doing.* [1D:11] *Pausing, I was making sure the facts are correct by reading the question again.* [1D:72-74] *If I do a sum, I see that the first sum fits onto the other sums.* [1D:110; 1D:157; 1D:272]

Learner D summarised his own monitoring throughout the process when saying:

> *I wasn't sure if the answer was right, so I was checking my remainder again to make sure ;so I got 77 and then I was checking basically the remainders, just to make sure it was right.* [1D:295-297]

Table 5.10 Verbal and non-verbal account of Learner D's reflection on monitoring

Verbal account – after solving the problem (participant's self-reflective comments)	**Indicators while reflecting – after action**
I go back to see what I was doing [1D:11]	Looked down at his work
I went back to the question [1D:17]	Pointed at the question
That's what I was trying to do [1D:43]	Looked unsure and looked down at written work
Non-verbal account during the problem-solving process (researcher's observations)	**Indicators while monitoring – during action**
Pointed with his pen towards the work	Frowned
Read, stopped and looked away from written work	Bit his lower lip
Paused after substituting values	Moved his feet around

5.2.4.7 Reflection on metacognitive regulation of evaluation (RE)

Throughout the problem-solving process, Leaner D monitored his work and also evaluated the answers between steps.

> *To see if they were right.* [1D:110] *I knew that was wrong once I remembered 8.* [1D:165]

The participant seemed to double-check his answer slightly at the end. After saying he was done, he looked down at his work again, saying:

> *I think I'm just making sure if everything is alright, because I don't want an answer to be wrong.* [1D:327] *I'm still making sure everything is right.* [1D:332]

Table 5.11 Summary of the account of Learner D's problem-solving practice

Categories	Description
Reflection on metacognitive knowledge	
Reflection on knowledge of the self (person) (KP)	Knowledge linked with parental involvement. Calculations were preferably done mentally. Knew how he worked in primary school. Knew how his mind worked. Did not know why he had done it. He identified problems with fractions, addition and subtraction checking.
Reflection of knowledge of the task (KT)	Calculations were done mentally. Utilised division methods learnt in primary school.
Reflection on knowledge of strategies (KS)	Drawing, rereading the question and using strategies to help in case aid was needed during uncertainty.
Reflection on metacognitive regulation	
Reflection on regulation of understanding (RU)	Making sense of the problem was important. Facts given were written down or drawn.
Reflection on regulation of planning (RP)	Identified what to utilise to solve the problem and compared various radii. If the calculator gave him an unanticipated answer, he would work it out using his head (a plan B approach).
Reflection on regulation of monitoring (RM)	Made a dot to check if something was right. Paused after every step. Reread the question to check if what was written down was actually correct. Saw if one sum fitted into another. Went over parts of the steps.
Reflection on regulation of evaluation (RE)	Only double-checked answer slightly at the end.

In the next paragraph, the findings of the four learners' selection of the metacognitive statement cards substantiate and validate the findings from paragraph 5.2.2.1 up to and including 5.2.2.4.

5.3 Learners' selection of metacognitive statement cards

This mixed-method interview technique (Wilson, 2001:1) provided a non-contradictory report of the participants' statements during the interviews. For further clarification of the employment and description of the metacognitive statement cards, refer to paragraph 3.7.2.1 concerning research methodology. The statements were divided and arranged according to the two metacognitive domains: knowledge and regulation. Each statement was coded, similar to the categories and codes of Table 5.2. Each participant's selection of the statements was recorded and compared with their verbal reports. This was done in order to establish the validity of the verbal reports in the first interview sessions.

5.3.1 Overview of the selected metacognitive statement cards for the four participants

Data obtained on the two domains of metacognition is presented in the summative table below (Table 5.12). Relevant categories and codes for the domains are bordering examples of

the statements and an indication is given of the participant(s) who selected, employed and reflected on the identified metacognitive components. Participants received the metacognitive statement cards after they had solved the Mathematics problem. They read the statements and had a chance to ask questions if there was any confusion relating to the statements' meaning.

Only one participant, Learner A, asked about the meaning of the statement: *I thought about whether what I was doing was working.* The researcher explained to clarify any doubts. The participants were then asked to select the statements that they had utilised or that were relevant to what they had thought, and to place them in a box labelled *used*. The ones they did not use were placed in a box labelled *not used*. Before watching the video recording of their own interview, participants arranged and numbered their selected statements in the order that they remembered applying them. Participants were free to change the order and selection of cards during and after the video playback. Table 5.12 illustrates that all learners reflected on both metacognitive knowledge and metacognitive regulation. However, not all four participants reflected on the same item at the same time. For this reason, a brief summary is given of participants' selection of the metacognitive statement cards.

Table 5.12 Participants' selection of metacognitive statement cards

Metacognitive knowledge: Categories and codes	**Examples of statements**	**No.[25]**	**Participants[26]**
Knowledge of the person /self (KP)	I thought I can't do it	KP1	A, B
Knowledge of the task (KT)	I thought I know this sort of problem	KT1	A, C, D
	I thought about what I already know	KT2	A, B, C, D
	I tried to remember if I had ever done a problem like this before	KT3	A, B, C, D
Knowledge of the skills and/or strategies (KS)	I thought I know what to do	KS1	A, C, D
	I thought about something I had done in the past that was helpful	KS2	A, C
	I thought about a different way to solve the problem	KS3	A,
Metacognitive regulation: Categories and codes	**Examples of statements**	**No.**	**Participants**
Regulation of understanding (RU)	I read the question more than once	RU1	A, B, C, D
	I drew a diagram to better understand the question	RU2	A, B, C,D
Regulation of planning (RP)	I changed the way I was working	RP1	A, B, D
	I thought about what I would do next	RP2	A, B, C
Regulation of monitoring (RM)	I thought about how I was doing	RM1	A, B, C
	I checked my answer as I was working	RM2	A, D
	I thought about whether what I was doing was working	RM3	A, B, C
Regulation of evaluation (RE)	I thought, is this right?	RE1	A, C, D

[25] This number represents the category, code and relevant statement and should be read in accordance with Table 3.5.
[26] The participants were given the pseudonyms of Learners A, B, C and D.

5.3.1.1 Selection and arrangement of metacognitive statement cards

As mentioned in paragraph 3.7.2.1, the use of metacognitive statement cards serves as validation for the findings obtained during the first interview. The summary in Table 5.12 validates the verbal reports. Two of the four participants, A and B, selected a negative self-reflective statement regarding their knowledge of themselves. The statement *I thought I can't do it* was the first card selected by Learner B and Learner A's third last choice. All four participants had knowledge of this task, and they reflected on it by thinking about what they knew about this problem and if they had done a similar problem like this before. Learner B did not select either of the cards *I thought I know this sort of problem* or *I thought I know what to do*. Nonetheless, three participants found the Mathematics problem or parts thereof familiar. Two learners claimed to have thought about something they had done in the past that was helpful. Only Learner A tried different strategies to solve the problem; however, learners B and D approached the problem differently by calculating the product with long division. All four participants regulated their understanding of the problem, read the question more than once and drew a picture.

Three participants reflected on the skills and strategies learned in the past, and changed their way of working. Learner A, in particular, employed three different approaches (see paragraph 5.2.1) to evaluate her answer. All four participants monitored their work, although Learner C did not check her answer and Learner D did not monitor his use of strategies. Three of the participants, Learners A, C and D, evaluated their work by asking themselves *is this right?* Table 5.12 summarises the four participants 'selection of statements regarding their reflection on metacognitive knowledge and regulation. The components of the foundation of the theoretical and conceptual framework is noticed in participants' selection of metacognitive statement cards.

5.4 Learners' reflection on their mathematics confidence experiences

In the first interview session, all participants experienced some level of confidence – whether high or low – in respect of problem solving in Mathematics. The four participants were interviewed for a second time. During this second individual interview, which followed the first interview as soon as possible, the focus was on what the participants' mathematics confidence entailed. Throughout the second interview, participants connected with their feelings regarding experiences in mathematics by reflecting on past and more recent instances. They identified and developed an understanding and awareness of their

mathematics confidence by reflecting on their confidence in the subject. Their confidence (an umbrella term for their psychological, social and intellectual domains) was related to their metacognitive knowledge and regulation.

According to Strawderman (2010:1-3), mathematics confidence comprises of three domains. Therefore, the three themes – social, psychological and intellectual – are presented individually for the various categories obtained from the analysis. Overall, after analysing the transcribed data, nine categories were formulated when the mathematics confidence experienced by learners was examined. The categories that were identified through content analysis of the transcriptions of the second interviews are summarised in Table 5.13.

Table 5.13 Themes, categories, codes and total number of responses for the second interview

Theme	Category	Codes	Participants	Total low[27]	Total high[28]
Social domain	1. Relations with family members and friends	CSOD	C, D	2	1
	2. Relations with teachers	CSOP	A, B, C	2	3
Psychological domain	3. Emotional or feelings (history)	CPSF	A, B, C, D	4	1
	4. Low or high confidence	CPSH	A, B, C, D	4	3
	5. Enjoyments (likes and dislikes)	CPSE	A, B, C, D	4	3
	6. Comfortableness	CPSC	A, B, C, D	2	2
Intellectual domain	7. Pursuits	CINP	A, B, D	1	2
	8. Skills and strategies	CINT	A, B, D	1	2
	9. Success and failure	CINS	A, B, C, D	4	4

All four participants experienced some level of confidence in each of the domains. It appears that most responses were of a psychological nature and involved emotions and feelings of like or dislike. Learner C reported the most low confidence statements, while Learner D stated the largest number of high confidence statements. All four participants reflected on their mathematics confidence in both successful and failed attempts.

[27] The number here indicates how many of the four participants had low mathematics confidence regarding a specific category.
[28] The number here indicates how many of the four participants had high mathematics confidence regarding a specific category.

5.4.1 Brief overview of the mathematics confidence codes

Figure 5.1 analyses the categories and codes in Table 5.2. Figure 5.3 on the other hand presents the components and codes of the dimensionality of mathematics confidence.

The three domains together vary in respect of the states and sources of confidence. Confidence in the social domain (CSO) motivates behaviour and originates from sources such as family members (CSOD), friends or peers (CSOD) and society or teacher influences (CSOP). The classroom, a natural education setting (Bormotova, 2010:75) allows the learner to be influenced by others, just as in real life. The academic environment must therefore be of such a nature that it allows learners to learn how to learn. Learners' attitude towards the subject can be the result of a positive or negative psychological aspect.

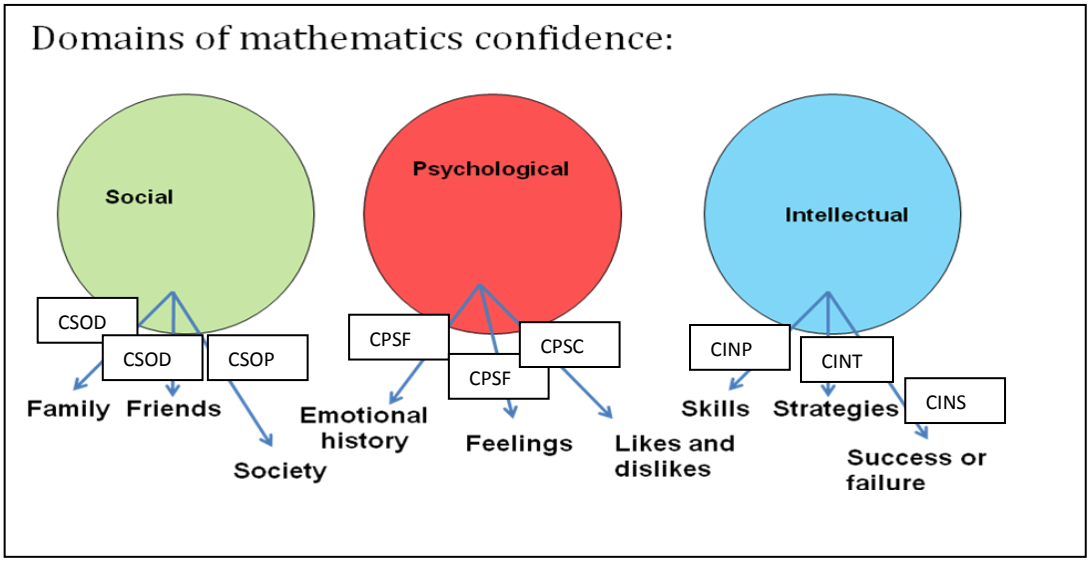


Figure 5.3 Domains and codes of mathematics confidence

Source: Strawderman (2010:2)

The psychological domain comprises emotional history and thus concerns participants' feelings, emotions (CPSF) and likes or dislikes (CPSC) with regard to Mathematics. Maree, Prinsloo and Claassen (1997a), together with Hannula, Maijala and Pekhonen (2004:18), suggest strong correlations between achievement and affect throughout Grades 7 to 12.

The intellectual domain comprises cognitive influences such as skills (CINP) and strategies (CINT). These influences, to a certain extent, determine the degree to which failures or successes (CINS) in mathematics problem solving exist. A study by Newstead (1999:1)

showed that learners who had been exposed to traditional teaching environments and practices experienced lower confidence in respect of Mathematics. These sources accumulate in the domains of mathematics confidence and result in the pursuit or avoidance of Mathematics.

Findings obtained on the identified categories in stage 3 of the study (see Table 5.2) follow in the accounts of interviews that were conducted with each of the four participants. The findings represent the categories in Table 5.13 and show accounts of low and high mathematics confidence traits.

5.4.2 Reflection on mathematics confidence in the social domain (CSO)

The following components comprise the social domain:

5.4.2.1 Reflections on relations with family members and friends (CSOD)

None of the participants responded with high confidence when they reflected on the contributions of their family members or friends in the social domain. Two participants mentioned a low mathematics confidence characteristic:

> *I didn't enjoy the work, even if my parents explained it to me.* [2D:56-57]
>
> *This year everybody tells us our marks will drop because it's high school* [2C:43]

One participant compared himself with his classmates:

> *I was like; I was one with the group. Most kids would say that I don't like it and it's boring. Other people are still doing it, but not me.* [2D:45-46] *They don't have the passion so they think, why should I do it?* [2D:47-48]

5.4.2.2 Reflections on relations with teachers (CSOP)

Regarding teachers' attributes, each participant mentioned something that benefitted their confidence.

> *Most of them taught me something that wasn't maths.* [2A:23] *If you like the teacher, you will like the subject. My math teacher was a real down-to-earth person that I can relate to.* [2A:23-25]

One participant claimed that a previous teacher used to play music in the class and made *maths* fun. In this particular class, learners were seated together and worked in groups. There were also some low mathematics contributions: *I liked the teacher, but not the way he did it.* [2A:18]

Some destructive, low mathematics confidence indicators were identified as the participants reflected on their teachers and society.

> *I didn't know the teacher or what to expect from her.* [2A:45-46]
>
> *I couldn't understand, she jumped around using different methods.* [2B:18]
>
> *You don't like the teacher, so you don't concentrate. We got this other maths teacher, he was a male. I think it was the teacher that made me dislike maths.* [2C:86-88]

The society of which teachers and learners are part also constitutes the environment, culture and educational surroundings of the learner. Three participants claimed that they had moved to a new home and school during their primary school years.

5.4.3 Reflection on mathematics confidence in the psychological domain (CPS)

Most of the responses involved traits of the psychological domain and included aspects of high and low mathematics confidence as discussed below.

5.4.3.1 Reflections on emotional history and feelings (CPSF)

Three of the four participants claimed that they found mathematics interesting and that they were not scared of anything. Learner D confidently said:

> *I just don't feel stressed like in other tests. Maths is just there, it comes natural. I don't feel stressed. I feel excited to see what questions to expect; anxious to see what's happening in the tests*. [2D:92-93]

Features of low mathematics confidence in the psychological domain were identified by each of the participants when they said:

> *I tend to get frustrated.* [2A:32]
>
> *I sometimes feel that I didn't practice at all.* [2B:84] *It doesn't feel good when you don't understand.* [2B:56]
>
> *I'm a bit worried.* [2C:75] *Sometimes I get lost during the sum and I don't know what to do.* [2C:45] *I get very scared.* [2C:45] *I feel lost.* [2C:35]
>
> *I was shaking before an unprepared test*. [2D:90]

5.4.3.2 Reflections on low or high mathematics confidence (CPSH)

Two participants, Learners A and B, gave their own opinions about what mathematics confidence meant to them:

It means trusting yourself with maths or believing in yourself concerning maths. [2A:34]

To know what you're doing and to do it right. To be able to excel. Not to let anyone discourage you or put you down. Have pride in your maths work. [2B:41]

When asked if they had ever been confident in Mathematics, three participants gave the following replies:

I actually laugh at myself when I think about how I struggled last term. [2A:37]

I was good since Grade 5, because in Grade 5 I got 98% [2B:39].

My math teacher always said there is a time when you start finding math easy. I think that was in Grade 5. [2D:15-17]

Learner C did not mention any high confidence remark; instead she said:

This was my lowest confident year ever. [2C:46]

The three other participants also experienced instances of low mathematics confidence.

In grade 5, I lacked confidence. [2A:44]

I'm not confident in my school work. You didn't have the confidence to ask the teacher. [2B:20]

Before Grade 5, I was not confident in maths. [2D:50]

5.4.3.3 Reflections on likes and dislikes (CPSE)

In this category, which was coded by responses consisting of likes and enjoyment, the statements made by Learners A, B and D completely differed. Learner A said that she liked *maths* and enjoyed being challenged. Learner B restricted this claim by saying that she only liked Maths if she understood it. Learner D enjoyed doing Mathematics, especially the most difficult sums. He referred back to his statement regarding being confident in Mathematics and said:

Enjoying maths in class and preferring Maths to other subjects. That's confidence. [2D:38-39]

Three of the participants enjoyed Maths; however, all four participants mentioned disliking Mathematics in some way or at some time.

I won't enjoy something if I don't understand how and why it has to be done. [2A:65]

I don't like to compete against someone. [2B:65]

It's not just one specific thing, I just don't like doing Maths. [2C:93]

Word problems are just something I don't like. Didn't like maths in Grade 4 because I didn't understand it. [2D:28]

5.4.3.4 Reflections on comfortableness (CPSC)

Being comfortable with learning and the learning environment seemed to be important to the participants. When asked whether they would change something in their classroom, three of them mentioned something they would not change with regard to the contents of Mathematics.

I wouldn't change anything. [2A:20]

Fractions are fun to do. [2D:8]

Most of the time but I understand most of the Maths. [2C:31]

Each participant mentioned something they would change regarding the surroundings or classroom environment.

The way the desks were organised [2A:20]

Sun shine, brighter colours. The area isn't really math like. [2B:28]

Just the way that we sit, it looks weird to sit in rows [2C:11]

The curtains [2D:23]

5.4.4 Reflection on mathematics confidence in the intellectual domain (CIN)

The first code (CINP) refers to the refletions on pursuits as an aspect of mathematics confidence

5.4.4.1 Reflections on pursuits (CINP)

Three of the participants reasoned to some extent about why they did not avoid Mathematics. Learners A, B and D reflected on this pursuit during the first interview.

I just keep trying over and over again. [2A:52]

I must compete. I think it will boost my marks and I'll become a better person. [2B:71]

If I can pinpoint them, I realise that I won't make this mistake again. …even if you get it wrong, you still try to get it right. [2D:36]

On the other hand, three of the participants indicated that they avoided the subject.

I get frustrated and just leave it alone. [2A:32]

I don't really care much; I just do it and get it over with [2B:22]

I feel tired and it makes me sleepy every time I go to maths class. My body just gets into sleep mode. When the bell rings, I'm all energetic again. [2C:49]

5.4.4.2 Reflections on skills and strategies (CINT)

Two learners who employed metacognitive strategies in the first interview (e.g. picture drawing and trying different approaches) reflected with confidence on their own skills and strategies in doing Mathematics:

When I know I can do it another way, that is easier [2A:69]

…if it's in pictures [2C:73]

Strategies are important, at least for learner C.

You have to know how to solve the problem… the teachers didn't explain [2C:68]

5.4.4.3 Reflections on success and failures (CINS)

Learner C reflected on her success and failure and stated:

You don't understand something, you have to learn to work out anything. [2C:69]

This branch of pursuit appears to increase or reduce mathematics confidence. Learner A claimed that she keeps on going until she gets it right. Learner D mentioned that it eventually becomes interesting. Learner C said: *I go through my test and wondering what I did wrong.* [2C:80], while Learner B said: *I wasn't getting the marks I expected to get. I didn't reach my goal.* [2B:51-52]

5.5 Summary of the four participants' mathematics confidence

The four participants reflected on experiences they had in Mathematics that reminded them of moments when they had low and high mathematics confidence. They also elaborated on what they thought (i.e. deeper reflection) was the cause for this. The following paragraphs represent the findings in the two extremities of low and high mathematics confidence. From the analysis, some findings in respect of reflection on metacognitive components also include data not assessed in that set. For example, although learners reflected on knowledge, symptoms of low or high mathematics confidence were evident. In the same way, learners

talked about their mathematics confidence and in turn reflected on components of metacognition.

5.5.1 Low and high confidence extremities

In this section a distinction is made between the participants' mathematics confidence. Participants mentioned, with some regard to metacognition, their reflections on the three domains of mathematics confidence: social, psychological and intellectual. Figure 5.4 sketches a summative connection between the extremities of their low and high mathematics confidence in each of the domains. It displays statements and gives examples of the sources of low and high mathematics confidence in each of the three domains.

5.5.2 Summary of the qualitative findings

The reflective process identifies knowledge that stems from metacognitive experiences. However, when a learner could not find a suitable strategy to impose, he/she chose between three alternatives. Alternative A was giving up and going on with another, different problem, and making a decision to revisit the previous one later. Alternative B involved giving up and not revisiting the problem, while Alternative C involved selecting the knowledge and strategy component that he/she was most confident with and performing it before carrying on to solve the problem. Participants reflected on person, tasks and strategies by looking for a self-regulative domain and identified this as reflective knowledge (Cox, 2005). Learners reflected on their confidence as well as resources such as planning, understanding and monitoring regarding social, psychological and intellectual qualities.

According to Sheffield and Hunt (2007), the role of affect in mathematics problem solving is determined by regulating the self-system, including knowledge of the self. Pauses during mathematics problem solving have underlying sources of metacognitive and affective features. These characteristics occasionally interrupt the problem-solving process and moderate achievement. Past influences coordinate beliefs and knowledge of the self's confidence during problem solving. These social, psychological and intellectual ranges contribute affectively to an individual's performance in Mathematics. The reflective, metacognitive and affective nature of the individual problem solver is complex and disciplined by constructs of knowledge and control. During reflection, conscious and subconscious thought revisit these constructs during problem solving. The self reflects on these constructs and selects an approach to implement for solving the problem. This

instinctive selection scaffolds problem-solving behaviour and promotes achievement in Mathematics. The demands and social beliefs regarding mathematics learning are considered vital for any country's development (Hannula; Maijala & Pehkonen, 2004:17) and collectively, mathematics confidence (through observing and reporting) model a self-reflective knowledge.

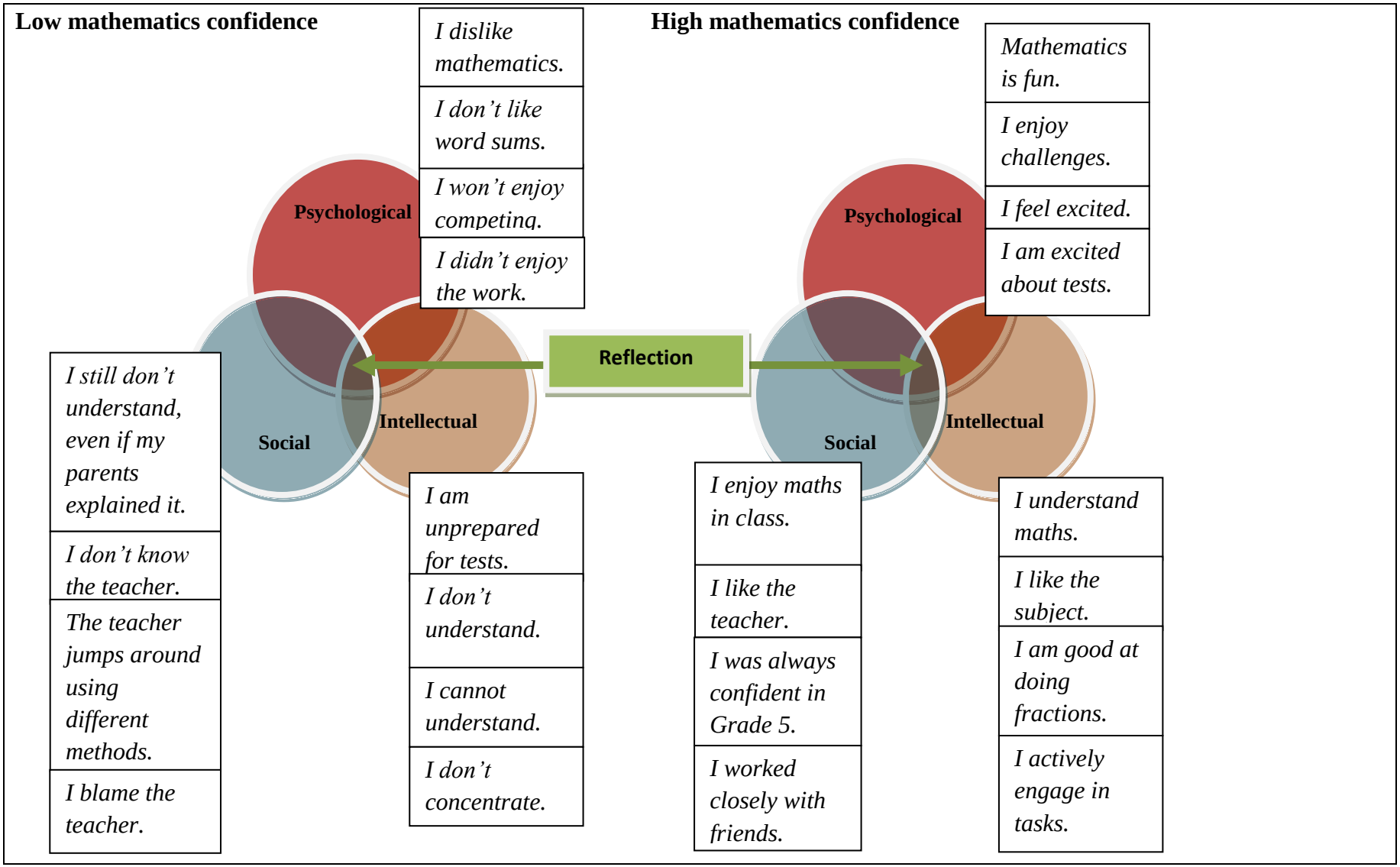


Figure 5.4 Extremities of components between low and high mathematics confidence domains

Chapter 6

Summary, discussions and recommendations

6.1 Summary of the previous chapters

In this mixed-method exploratory convergent design (Creswell, Plano & Clark, 2011:215) the independent sets of data (quantitative and qualitative) were collected, analysed and presented separately. Chapter 4 presents the empirical set of quantitative data and Chapter 5 the qualitative information. Chapter 6 discusses and merges the data sets by triangulating the findings to answer the secondary research questions. After contextualising the findings with the literature and theory, possible explanations, limitations and recommendations follow in an attempt to answer the primary research question.

In Chapter 1, the research paradigms and perspectives were described leading to the primary and secondary questions. The use of a mixed-method approach was validated by a short discussion on the quantitative and qualitative data collection and analysis procedures. The methods for collecting data were introduced by a description of the collecting instruments and analysis procedures. Keywords defined the terms and described the variable of this study in a short review of the literature.

Chapter 2 provided insight into the background of theories and past studies done on metacognition, reflection, mathematics anxiety and mathematics confidence. Problem solving played an important role in defining the need to examine reflection and confidence in respect of Mathematics. The literature review examined what metacognitive and reflective strategies were involved during mathematics problem solving. The idea of mathematics anxiety was introduced as low and high mathematics confidence.

In Chapter 3, the theoretical stances, research methodology and paradigmatic perspectives were discussed in broad terms. The researcher's philosophical aspirations and motive for conducting the study were described. The need for adequate instruments to measure reflection and mathematics confidence was also discussed. A description of the pilot study and the administration of the SOM and RPSQ instruments to respondents in the sample followed. The collection procedures of the quantitative part of the study were discussed, as well as qualitative methods for collecting information from the invited participants. The participants took part in two individual interview sessions. These sessions were video

recorded and examined participants' responses and behaviour regarding reflection on mathematics problem solving.

Chapter 4 presents the quantitative analysis of the data obtained from administering the SOM and the RPSQ. Statistical analysis included factor analysis to determine the four factors measured by the RPSQ. Cronbach alpha values were calculated to explore correlations between reflection and mathematics confidence. Factor analysis of the SOM indicated a relationship between the factors of the RPSQ (metacognition) and those of the SOM (mathematics confidence).

Chapter 5 presents the findings of the qualitative data analysis. The first interview explored metacognition, while the second interview focused on the aspect of mathematics confidence. However, in the second interview it became apparent that mathematics confidence was multi-dimensional and included confidence in the social, psychological and intellectual domains. The first interview revealed the integrative role that reflection plays during problem solving by associating metacognitive knowledge components with regulative components.

A discussion follows on the quantitative and qualitative findings through which the research questions are answered.

6.1.1 Contextualising the literature

A widespread, comprehensive literature search was conducted before, during and after collecting the data (started in 2010 and continued until 2012). This literature search included ideas and interlinked characteristics of the theoretical framework for this study. The literature studied was published between the years 1990 and 2012 and provided a theoretical background on relevant concepts researched over the past two decades. In order to contextualise the findings and results of this study, detailed comparisons and association with findings in the literature was done.

6.1.2 Structure of the discussion section

In the following sections, the quantitative and qualitative analyses attempt to answer the secondary research questions. In the discussions, the factors identified in the study (quantitative and qualitative) are compared to similar findings in the literature. Findings contradictory to the literature are also noted. The researcher evaluates the individual and merged results with regard to the research questions and summarises his findings. This is followed by explanations, limitations and recommendations that are suggested in the

researcher’s attempt to answer the secondary and primary research questions. Table 6.1 summarises the themes, codes, results and sources referred to in the discussion.

An outline of all the components of the quantitative and qualitative study is displayed in Table 6.1. This summary lists the themes, factors and categories, codes, relevant paragraphs and sources that constitute the results and findings of this study.

Table 6.1 Summary of the quantitative results and qualitative findings

Quantitative part					Qualitative part			
Literature agreeing with traits and results	**Analysis and presentation in Chapter 4**	**Codes**	**Factors**	**Themes**	**Categories**	**Codes**	**Analysis and presentation in Chapter 5**	**Literature agreeing with traits and findings**
Zimmerman, Moylan, Hudesman, Flugman & White (2011:11)	4.6.6 4.7.5	M/P TSP	Monitor/Plan Person	**K Metacognitive knowledge**	Reflection on knowledge of the self (person)	KP	5.2.1.1 5.2.2.1 5.2.3.1 5.2.4.1	Lai (2011) Flavell (1979) Kuhn & Dean (2004 Schraw et al. (2006).
Garrett, Mazzocco & Baker (2006:8) Muis (2008:193)	4.7.6 4.6.7		Monitor/Plan Task		Reflection of knowledge of the task	KT	5.2.1.2 5.2.2.2 5.2.3.2 5.2.4.2	Kuhn & Dean (2004)
Muis (2008:193) Efklides (2006)	4.6.7 4.7.3		Monitor/Plan Strategies		Reflection on knowledge of strategies	KS	5.2.1.3 5.2.2.3 5.2.3.3 5.2.4.3	Schraw et al. (2006)
Seitz (2000:26)	4.7.6 4.7.4 4.3.4.4	P	Planning task	**R Metacognitive regulation**	Reflection on regulation of understanding	RU	5.2.1.4 5.2.2.4 5.2.3.4 5.2.4.4	Cox (2005)
Muis (2008:193)	4.6.5				Reflection on regulation of planning	RP	5.2.1.5 5.2.2.5 5.2.3.5 5.2.4.5	Efklides (2006)
Muis (2008:193)	4.3.4.3 4.7.3	M	Monitoring task		Reflection on regulation of monitoring	RM	5.2.1.6 5.2.2.6 5.2.3.6 5.2.4.6	
	4.6.4				Reflection on regulation of evaluation	RE	5.2.1.7 5.2.2.7 5.2.3.7 5.2.4.7	Garrett, Mazzocco & Baker (2006:8)
Kleitman (2011)	4.6.1	MC	Mathematics confidence	**C Mathematics confidence**	Reflection on mathematics confidence in the social domain	CSO	5.4.2.1 5.4.2.2	Lai (2011) Flavell (1979) Kuhn & Dean (2004) Schraw et al. (2006).
Bormotova (2010)	4.7.2				Reflection on mathematics confidence in psychological domain	CPS	5.4.3.1 5.4.3.2 5.4.3.3 5.4.3.4	Kuhn and Dean (2004)
Brady & Bowd (2005:41)					Reflection on mathematics confidence in the intellectual domain	CIN	5.4.4.1 5.4.4.2 5.4.4.3	Schraw et al. (2006)

6.1.3 Merging the results of the study (triangulation)

After comparing the quantitative and qualitative discussions, the merged data is fused in a triangulated view based on the results. The individual and merged results are evaluated within the sections, thus answering the secondary research questions. Each secondary research question is answered by the merged data that was obtained within the three themes (see Table 6.1).

6.1.4 Answering the secondary research questions

The discussions of the secondary research questions represent a contextualisation of the results and findings. The literature and merged data stipulate the findings within the discussion and serve as answers to the research questions. Each theme is explored quantitatively and qualitatively by answering the secondary research questions jointly. Secondary research questions 1 and 3 are discussed by exploring the themes metacognitive knowledge and mathematics confidence, while reference is made to the social and psychological aspects of secondary research question 4. Secondary research questions 2 and (partly) 4 (intellectual aspects) discuss the themes of reflection on metacognitive regulation and mathematics confidence.

6.2 Discussion of the relation between the themes of metacognitive knowledge and mathematics confidence (K vs. C)

The results of this study point to a practical and statistically significant relationship between mathematics confidence and metacognitive knowledge.

6.2.1 Discussion of the quantitative results: secondary research question 1

Is there a correlation between mathematics confidence and reflection on metacognitive knowledge during problem solving?

To determine whether a correlation exist between confidence and reflection, descriptive and inferential statistics were calculated on the data obtained from the SOM and RPSQ instruments by focusing on mathematics confidence, monitoring (TSP) and planning (TSP). Reflecting on metacognitive knowledge correlated positively with factors such as mathematics confidence, problem-solving behaviour, as well as regulative components: monitoring and planning of tasks. This seems to mirror the suggestion by Muis (2008), namely that monitoring relates to performance and planning at a practical and statistically significant level.

6.2.1.1 Mathematics confidence (MC)

Mathematics confidence correlates with metacognitive knowledge components such as monitoring (TSP) and planning (TSP). With regard to reflection, the three schools differed in their responses to mathematics confidence questions. This was to the extent that the effect sizes varied between small and medium (0.17 to 0.36) (see Table 4.12), indicating that the responses regarding mathematics confidence of participants in a single gender school differ significantly from those in co-ed schools. In the all-boys and all-girls schools, the confidence averages were almost similar and averaged a little higher than in the co-ed school.

6.2.1.2 Metacognitive knowledge (MTSP & PTSP)

In the quantitative study, the findings concern metacognition through monitoring and planning tasks, strategies and personal qualities. These are similar to the findings of Zimmerman, Moylan, Hudesman, Flugman and White (2001). Muis (2008:189) calculated the same Cronbach alpha for the factor 'monitoring' (0.71) (see Table 4.8) as is the case in this study's factor analysis of the items in the RPSQ when he performed a factor analysis of the Motivated Strategies for Learning Questionnaire (MSLQ).

6.2.1.3 Monitoring tasks, strategies and person (MTSP)

This form of self-evaluation regarding tasks, strategies and personal influences (TSP) correlated highly with self-efficacy in a similar study done by Zimmerman et al. (2011). These authors reported that students who are less confident in Mathematics will not check their answers as frequently as those with high mathematics confidence. From the results of this study, it seems that the monitoring of tasks, strategies and the person relates to problem-solving behaviour and planning of tasks. However, mathematics confidence does not appear to correlate with reflection on monitoring (TSP) – a low correlation of $\alpha = 0.058$ is indicated in Table 4.13. This indicates that there are possible variations of monitoring, co-dependent on mathematics confidence, especially with its link to strategies, tasks and personal or self-evaluation.

6.2.1.4 Planning tasks, strategies and person (PTSP)

According to Efklides (2006:4), planning involves the monitoring of goals and the level of confidence in achieving those goals. In the present study, respondents' planning of tasks, strategies and the person, appeared to vary between the three schools. A moderate effect size of 0.43 (see Table 4.12), when comparing the all-girls school and the co-ed school, indicates

that there is a practical and statistically significant difference in respect of planning (TSP) by the respondents of these schools. A possible explanation could involve the confidence of the learners of these schools (noted in paragraph 4.7.2) and their problem-solving behaviour. From the results of this study, planning (TSP) appears to correlate with monitoring of tasks to the extent where confidence is dismissible.

6.2.2 Discussion of the qualitative findings: secondary research question 3 (focusing on metacognitive knowledge)

What reflection strategies or skills do learners in senior phase Mathematics implement?

To answer this, a discussion follows on the reflective strategies that participants employed during problem solving as evidence of their awareness and use of metacognitive skills.

6.2.2.1 Reflection on metacognitive knowledge (K)

According to Legg and Locker (2009:471), memories and cognitive strategies complexly and diversely contribute to mathematical thinking and problem solving. The four participants' reflection on the components of their personal and task knowledge (Lai, 2010:7) is discussed in this paragraph with reference to the following three types of knowledge: person and task knowledge (Flavell, 1979) as declarative (Schraw et al., 2006); knowledge of strategies as procedural; and knowledge of why and when strategies are used as conditional knowledge.

6.2.2.2 Reflection on knowledge of the person (KP)

Participants in this study reflected on the knowledge they have of themselves – which corresponds with findings in a study conducted by Bormotova (2010). Participants mentioned that they were aware of their uncertainties, the strength of their memory and their preferences and styles with regard to the use of strategies and content knowledge. All of them mentioned that, at some stage, they have trouble remembering some strategy, formula or a value to substitute. As one learner said, *I am very forgetful*. Problems with memory of content matter are regarded as traits of anxiousness and depict the influence of affect in the working memory (Ashcraft & Kirk, 2001). One learner mentioned that she changed her strategy because the previous approach bothered her. This self-awareness aid serves as a guide for metacognitive thinking and could be seen avoidance behaviour, where learners rather focus on something other than the task. Knowledge of feelings, such as confusion and frustration, along with the awareness of likes and dislikes was evident in this study. The four participants' knowledge of and insight into their personal self showed that reflection allows them to alternate their

thinking between the problem (task) and the self (personal) (Legg & Locker, 2009:472). A self-reflective knowledge emerges by reflecting on the knowledge of the task and comparing (reflectively) that knowledge with the knowledge of the person. Usher and Pajares (2009:89) verified this self-reflection as a trait of self-comparative experiences before, during and after problem solving.

6.2.2.3 Reflection on knowledge of the task (KT)

Participants also reflected on their knowledge of the task before, during and after the problem-solving process (Jacobs, 2010:1). According to Bormotova (2010:36), this is a tacit, spontaneous subconscious and conscious reflection described in terms of their understanding and choice of strategies. Two of the participants could not immediately remember the formula for the area of a circle but, through a process of reflecting on experiences as well as trial and error, managed to remember it. Participants seemed to be challenged when it came to the content knowledge the task required. In each case participants experienced confusion regarding the formula and the value of pi, had trouble adding or subtracting fractions, and were confused about the area, volume or circumference of a circle. Identified problems in respect of understanding and remembering were overcome by recovering the information needed through reflecting on strategies used in the past. These strategies required participants to have knowledge of other tasks such as reading the question to gain information and picture drawing to get a new kind of knowledge of the task.

6.2.2.4 Reflection on knowledge of strategies (KS)

Each participant read the problem in parts and not from beginning to end every time, which agreed with the findings of Yeap (1998) and Yimer and Ellerton (2002). This effort to understand the problem, to identify necessary information and to decide what to do next, agrees with the findings of Schraw and Dennison (2001), as well as Fortunato et al. (1991). Participants furthered their reflection by using different kinds of strategies to help with their understanding and planning of the task. Strategies included the use of pictures, symbols and labels to make the context and content of the problem more familiar or meaningful. Cognitive load was reduced by familiarising the problem. Participants did this by taking away decimal commas (a primary school tactic), using long multiplication when unsure about the calculator's answer, and breaking the sum up into smaller manageable parts. According to Malmivuori (2006), this decrease of the cognitive load lowers performance in Mathematics. However in this study, the participants solved the problem correctly, possibly through their

awareness and reduction of cognitive load. Learners reflected by monitoring their actions and thoughts, and they gained confidence and constructed a picture or diagram to help them understand or make sense of it all. The participants reflected on what they know and do not know. During such reflection, they reviewed and projected their knowledge. They became aware of their knowledge, learning and understanding, as well as of the intuitive, cognitive and affective aspects of their problem solving. Whenever these factors are communicated, learners reflect on what was done, what is and what will be (Yancey, 1998:6 & Bormotova, 2010:33).

According to Legg and Locker (2009:473), this regulation checks the behaviour and performance before, during and after the task. It involves planning procedures, understanding what to do and how to do it, as well as monitoring and evaluating the answers.

6.2.3 Discussion of the qualitative results: secondary research question 4 (focusing on the social and psychological domain)

What does learners' mathematics confidence entail?

To answer this question, participants' responses to the interview questions are discussed in terms of two of the three domains of mathematics confidence. Even though learners reflect, there appears to be underlying factors that cause low mathematics confidence while reflecting. Some of these factors emerge from negative social or psychological aspects. The following discussion focuses on mathematics confidence in terms of some social (CSO) and psychological (CPS) aspects.

6.2.3.1 Discussion of mathematics confidence in the social domain (CSO)

Mathematics confidence in the social domain involves reflecting on school, personal and social affairs. According to Ernest (2002:2), the social realm influences personal growth in mathematics confidence. Some researchers (Ernest, 2002; Bormotova, 2010; Strawderman, 2010) have confirmed that social aspects are influenced by psychological traits. If positive, these traits strengthen the development and utilising of skills during mathematics problem solving. Stankov and Lee (2009) explain that social attributes include values and attitude that originate from society. Learners face the social context in a variety of contexts, as is discussed below.

- **Relations with peers**

Learners face and have to cope with social acceptance at school. According to MacFarland (2010:4) learners in the senior phase experience changes associated with puberty and have to make social and psychological adjustments in their everyday lives. The current study agrees. Influences that peers might have on an individual's mathematics confidence involve attitudes towards Mathematics in general and towards problem solving in particular, especially geometry and word sums.

- **Relations with parents**

Seeking independence could also influence learners' interaction with parents (MacFarland, 2010). Non-teaching parents who assist learners with homework and other school based mathematics tasks might find it difficult to explain or understand the mathematical content necessary for these tasks. Schoenfeld (2004) states that parents' might have trouble with this assistance becasue unlike teachers or trained parents, they do not have the relevant background and trained skills Encouraging parents boost learners' confidence.

- **Relations with teachers**

Learners' reflections on mathematics problems related to a large extent to primary school experiences. Reflections on feelings towards teachers that were due to current, high school (middle grade) experiences with mathematics problem solving, also occur in the findings of MacFarland (2010:9). Thus the entire academic environment must be of such nature that it allows learners to learn how to learn.

6.2.3.2 Discussion of mathematics confidence in the psychological domain (CPS)

Reflecting on experiences involving Mathematics in Grades 5 and 6 indicate that for the participants in this study, mathematics confidence seemed to start to change at around age ten or eleven, having its foundations in primary school years. This reflection of learners' emotional experiences with Mathematics agrees with the findings of Newstead (1999), and Sheffield and Hunt (2007:25). Fear of failing, shame, guilt and negative feelings towards the self and subject comprise affective and cognitive dimensions (Pantziara & Philippou, 2011). Nervousness and frustration were confirmed in participants' problem-solving reflections during the first interview session. Connections to likes, enjoyments and dislikes were evident and varied. Some sources for liking and disliking mathematics problem solving that were also identified by Strawderman (2010) included environmental factors, sums involving fractions

and word sums. Bormotova (2010:32) found that positive feelings and emotions enhance the learning process. These feelings keep the learner focused on the task and inspire new learning. In terms of positive experiences, Malmovouri (2006) found that a relationship between regulating skills and strategies resulted in experiences of enjoyment.

6.2.4 Merging the quantitative and qualitative discussions regarding reflection on metacognitive knowledge and mathematics confidence

A practical, statistically significant relationship between mathematics confidence and metacognitive practices confirms the dimensionality of mathematics confidence and metacognitive knowledge. From the interviews it was established that Learner D, a boy, had a high level of confidence, similar to Learner A, a girl. Confidence averages across single-gendered schools were almost similar and averaged higher than those of the co-ed school. Effect sizes varied between 0.17 and 0.36 (see Table 4.12), indicating that responses in respect of mathematics confidence differ significantly from those in co-ed schools. Variations of monitoring that are co-dependent on mathematics confidence are linked to strategies, tasks and personal or self-evaluative skills.

Uncertainties and preferences regarding strategies and content knowledge indicate problems with regard to memory of content matter. This trait – anxiousness – depicts the influence of affect in the working memory. While participants monitored their tasks frequently, Learner C did not monitor her work as frequently; nor did she evaluate her answer, suggesting that she had low mathematics confidence (Zimmerman et al., 2011). Problems with understanding and remembering were overcome by reflecting on information and available knowledge. Different strategies help with understanding and planning of the task. Monitoring actions and thoughts influences confidence in the social domain and involves reflecting on school, personal and social affairs.

Fear of failing, shame, guilt and negative feelings towards the self and subject comprise affective and cognitive dimensions, whereas positive feelings and emotions enhance the learning process. Although a very low correlation existed between planning (TSP) and monitoring (TSP) with regard to mathematics confidence (see Table 4.13), the researcher believes that the findings of the qualitative study support the explanation in the quantitative study (see paragraph 4.8). In other words, there exists a relationship between mathematics confidence and metacognitive knowledge, particularly a domain-specific relationship.

6.3 Discussion of the relation between the themes of metacognitive regulation and mathematics confidence (R vs. C)

Metacognitive regulation, such as reflection on the planning and monitoring of tasks, correlated moderately with mathematics confidence.

6.3.1 Discussion on the quantitative results: secondary research question 2

Is there a correlation between mathematics confidence and reflection on metacognitive regulation during problem solving?

Problem-solving behaviour had higher correlations with the planning and monitoring of tasks. The planning of tasks correlated with mathematics confidence and monitoring, suggesting that some level of reflection and confidence was established before solving the given problem. The relationship between monitoring and problem-solving behaviour varied across the three schools in this study. According to Muis (2008:15), planning is an individual action carried out before the multitude of actions that precede problem solving.

Knowledge about one's cognitive processes can be altered and controlled by using metacognitive regulation (Muis, 2008:15). When referring to the mean scores in Table 3, it seems that the respondents in the participating schools monitored their own problem-solving processes and progress in almost equal terms. The small effect sizes between schools A, B and C about their monitoring of tasks (ranging from 0.02 to 0.06) (see Table 4.12) confirmed this. A statistically significant correlation between the monitoring of tasks and mathematics confidence agrees with the findings of Efklides (2006). With a high internal correlation with monitoring (TSP), monitoring of tasks allows the learner to focus on task-orientated and heuristic reflections. This excludes monitoring personal and strategic features.

6.3.2 Discussion of the qualitative findings: secondary research question 3 (focusing on metacognitive regulation)

What reflection strategies or skills do learners in senior phase Mathematics implement?

6.3.2.1 Reflection on regulation of understanding (RU)

Hannula, Maijala and Pehkonen (2004:17) view understanding as a process, predetermined to the person, topic and setting. The understanding of participants in this study is described firstly as an understanding of the problem and its contextual content and, secondly, as using and manipulating strategies. In this study, it appears that participants' understanding was goal

orientated. To solve the problem, or to understand it, they eliminated the problems in their understanding by reflecting on similar understandings and experiences in similar contexts. This understanding agrees with their knowledge of the task and person. Whenever the learners were uncertain of something, they reflected on both successful and non-successful attempts and were in a way relieved of their confusion. By identifying a different, less difficult problem and solving it instead, they each travelled between different levels of confidence ranging from low confident to high confident. Learners reflected on the problem, on their explorations, on the approaches, the appropriateness, the entire solution process and their persistence. Approaches to understanding involved reading, re-reading, selective reading, picture drawing, adaptations to and labelling of drawings, analysing, rewriting the question and writing down the relevant facts or information. Participants' approaches were seen as problem-solving aids to better understand and explain the problem. The content problems that participants experienced, involved their understanding of concepts like radius, diameter, area, volume and circumference, as well as calculations involving decimals and common fractions. The problems experienced by the participants in this study were common to the findings of the TIMSS (2007) survey.

6.3.2.2 Reflection on regulation of planning (RP)

Regulative metacognitive skills include planning as part of the selection, monitoring and conscious control of how to approach a given problem (Bormotova, 2010:51). Bormotova (2010:214) suggests that planning should be utilised to re-channel worrisome and anxious thoughts. The participants in this study seemed to have planned their approach from the start by drawing, labelling, identifying and completing information for each circle. They adopted a systematic approach that could be divided into seven stages. The current study identified these stages of planning and summarised them in Figure 6.1.

Participants regulated their planning by reflecting on these seven stages of planning. Although not exactly, these stages correlate with the findings of Yimer and Ellerton (2002), as well as of Hudlicka (2005). Stages of problem solving were similar for all participants in this study and involved the following: read, draw, read again, look at pictures and complete information needed to solve the problem, in other words identify a goal (what must be done?). Look for information to use as knowledge and understanding. Select the appropriate strategic approach to solve the problem, get the answer, and check if the answer makes sense.

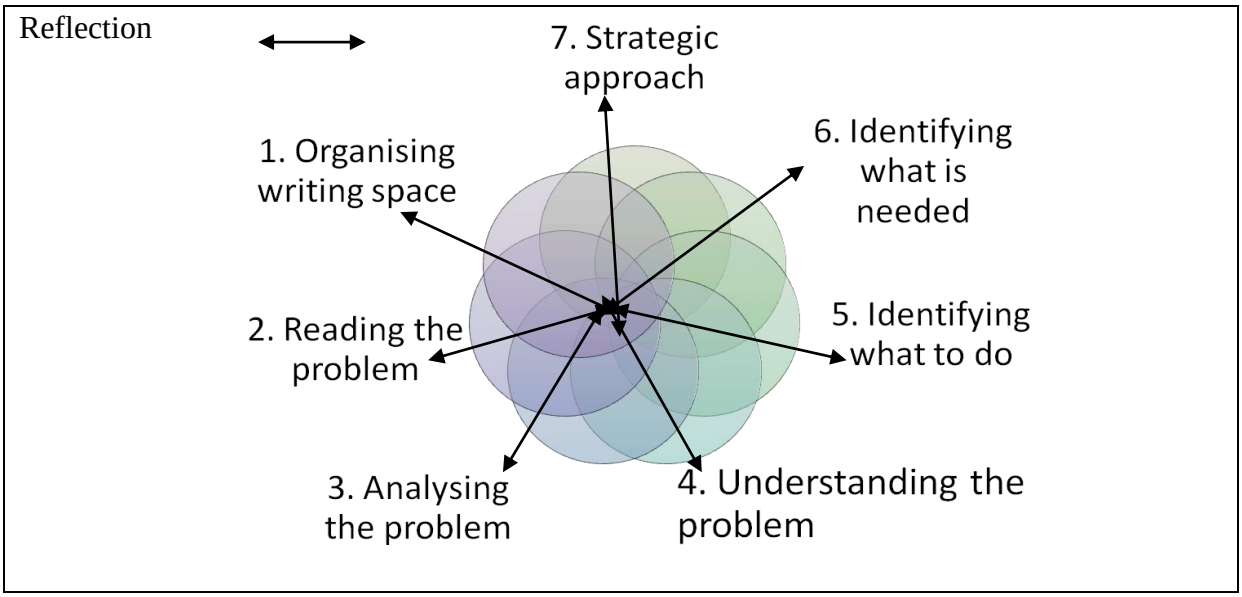


Figure 6.1 Seven stages of metacognitive planning

6.3.2.3 Reflection on regulation of monitoring (RM)

Monitoring was possibly the most frequent reflective strategy that participants employed. It also had the largest number of underlying changes and variances. Participants read the problem, and looked at the steps and at the pictures often. These facets of comprehension and understanding controlled problem solving by allowing the correction of errors and misconceptions, and by looking into the efficiency of the strategies chosen. Monitoring allowed adjustment in behaviour and strategies whenever this was considered necessary (Ridley et al., 1992; Malmivuori, 2006:19). Participants in the current study monitored their work by using symbols, and by looking at, for and with information. They imagined themselves completing at least parts of the sum, and reflected on their calculations. All of these monitoring actions had physical indicators assigned to them, indicating a strong link between monitoring and mathematics confidence.

- **Representative or symbolic monitoring**

Representative monitoring was seen as an aid by illustrating the process of monitoring through writing or drawing symbols or pictures. This involved participants' imagery in the form of writing or drawing, and their reflections on this. It was also an indication of their understanding of what they are doing or should be doing.

- **Informative or knowledge monitoring**

Informative monitoring was seen as participants' reflective actions by means of identifying and utilising information. Participants had difficulty monitoring their comprehension effectively, as was also the case in a study done by Wilson (2001). Tobias, Everson and Lin (2001) declare that this kind of monitoring is an essential metacognitive skill. It includes participants' skill to identify what they do and do not know. They monitor their knowledge and gain a new knowledge of themselves and of the task.

- **Imaginative modulation monitoring**

Imaginative modulation monitoring (reflective modelling) is both a verbal and non-verbal action. Participants monitored by looking at and sometimes away from their work. They then remembered and identified useful information, and imagined completing or going on to the next step or part thereof. They used that information before they actually wrote the step down. This type of monitoring is not widely identified in the literature. However, Legg and Locker (2009:473) describe monitoring as a comprehension and retention attempt without looking at the page. Schon's reflective practicum and Bormotova's (2010:24) statement about reflecting in action do not merely hold words. Reflective learners actually imagine the results of their thoughts.

- **Success monitoring**

By determining whether they have enough knowledge of content and strategies to solve the problem, participants reflect on what was done so far and what parts thereof are successful enough to be monitored and reflected on as informative knowledge. This leads to completing more complicated or successive parts of the problem.

- **Strategic monitoring**

Strategic monitoring consists of participants' reflection on their use of strategy and the sequence of these strategies. It appears that participants' monitoring contributes to their planning and understanding as regulative components. By monitoring their person knowledge, task requirements and available strategies, monitoring is also connected to their metacognitive knowledge. During monitoring, various symptoms of anxiety were noted. These symptoms indicate a link between monitoring and mathematics confidence.

6.3.2.4 Reflection on regulation of evaluation (RE)

The analysis of the employed strategies and compared solutions was evident of reflection as an evaluative skill. Participants evaluated their answers by using up to three different strategies. Throughout the process of evaluation, participants compared their solutions with that suggested by the calculator and recalculated their answer. This action resembles reflection on representative or symbolic monitoring.

One participant did not evaluate her answer. Although Brown (1987) describes this as an *autopilot state* and argues that expert learners evaluate their progress as an automatic reflection (in other words unaware and subconsciously), this particular learner said that she *did not evaluate* her answer. This contrast to the fact that some learners do not question themselves about strategies and have no or little idea of what they are doing or why, points to the fact that not all learners are aware of their metacognitive thinking. In fact, Hartman (2001) goes further by saying that these learners have knowledge and strategies, but are (metacognitively) not aware of them because of a lack of confidence.

6.3.2.5 Discussion on the qualitative findings: secondary research question 4 (focusing on the intellectual domain)

What does learners' mathematics confidence entail?

To answer this question, participants' responses to the interview questions are discussed in terms of the intellectual domain of mathematics confidence. Sources include intellectual aspects. According to their reflections, participants recalled low confidence connected to unsuccessful experiences with Mathematics, as well as moments where their confidence was high and performance successful. The following discussion focuses on mathematics confidence in terms of the intellectual (CIN) sources.

6.3.3 Merging the quantitative and qualitative results regarding reflection and metacognitive regulation and mathematics confidence

Understanding was goal orientated. To understand, problems had to be eliminated by reflecting on similar understandings and experiences in similar contexts. The facets of comprehension and understanding of mathematics controlled problem solving by regulating approaches. Strategic approaches were selective, and monitoring through symbols or pictures involved participants' imagery in the form of writing and the skill to identify what they know

and do not know. Sources included intellectual aspects such as low confidence connected to learners' unsuccessful experiences with Mathematics, as well as moments when their confidence was high and performance was successful. This internal correlation between the items of the RPSQ indicates that reflection serves as a means for metacognitive thinking that bonds the knowledge and regulation components.

6.4 Putting it all together: answering the primary research question

The primary research question that this study sought to answer, is:

What is the role of reflection and mathematics confidence during problem solving in senior phase Mathematics?

By answering this question partially in the secondary research questions, the researcher concluded that reflection as a metacognitive skill allows the individual problem solver to identify and use previous knowledge and experiences to alter his/her awareness of the task, strategies and the self. It appears that reflection and mathematics confidence are multi-dimensional constructs relating to the components of metacognition, namely reflection on metacognitive knowledge and regulation. These factors have diverse and joint origins of a social, psychological and intellectual nature. Mathematics confidence moderates regulatory skills, which is a metacognitive component, through the process of reflection.

In this study, Mathematics confidence and reflective skills appeared to be different across the gender-specific and co-ed schools, and the components of metacognitive knowledge correlated low with mathematics confidence while components of metacognitive regulation had a high correlation. Monitoring and planning, both regulatory components, are diverse facets of metacognition that are linked to mathematics confidence. Monitoring (TSP) differs from monitoring tasks only in the sense that correlations with mathematics confidence were higher when referred to tasks alone. Perhaps this difference is best explained by the SOM, which measured mathematics confidence of only the psychological domain and seemed to exclude the social (person) and intellectual (strategies) domains. This distinction might involve an avoidance of integrating personal knowledge with task knowledge. The mathematics confidence differences between knowledge and regulatory components indicate reflection on two levels: firstly, a basic level of awareness (task orientated) and secondly a deeper reflection in respect of reasons and sources for confidence (person and strategy orientated). The act of reflection seems to stimulate the level of confidence related to the

planning and monitoring of tasks. It seems that the acts of reflecting possibly manipulate and vary the knowledge and feelings associated with person, strategy and task characteristics during problem solving. Planning related to learners' confidence regarding their understanding of and approaches towards the solving of the mathematics problem (Molenaar, 2010:1731). Personal attributes and reflection on strategies correlated low with mathematics confidence, which indicated a possible absence of deep reflection when the planning of tasks pertained to personal experiences. However, monitoring (TSP) and planning (TSP) correlated least with mathematics confidence and with problem-solving behaviour, indicating intercorrelations between the components of metacognition as possible diverse domain-specific reflections. The researcher in this study agrees with Efklides (2006:7) that metacognitive experiences relate to a process where progress, feelings and goal settings are monitored and in turn produce a level of mathematics confidence.

6.4.1 Recommendations from a philosophic reflective stance

The learner, who develops and reflects on personal, strategic and task knowledge gained from experience, interprets meanings from these experiences according to his/her own cognitive structures. Several aspects of mathematics confidence and reflection require further investigation. These aspects include possible affective and metacognitive connections to mathematics problem solving. The following might serve as potential considerations for both researchers and teachers in mathematics education.

6.4.1.1 For researchers

Research on the variables in this study should not retire from delivering literature on affect and metacognitive thinking. Developed concepts in research methodology, social sciences and mathematics education, with regard to reflection, may possibly contribute to constructs in lesson study, educational paradigm epistemologies and research methodology outputs. Due to the number of respondents in this study speaking Setswana and other home languages, the questionnaires administered in the quantitative study might provide insight into respondents' verbal domain and identify possible language barriers in Mathematics. Cultural and socio-economic differences could perhaps be explored to determine possible relationships between environmental backgrounds and their relation to mathematics confidence and reflection. Domain- and component-specific research is suggested to explore and possibly better understand the dimensionality of metacognition and affect.

6.4.1.2 For teachers

It is the researcher's opinion that learners and teachers alike understand the role of reflection and mathematics confidence. The use of standardised tests can assist with the measuring of learners' confidence with regard to Mathematics and can identify and diagnose causes and resources to reduce the level of anxiety and increase confidence.

With regard to metacognitive, affective and reflective attributes, sources identified ought to be assessed and understood in order to stimulate the successful teaching and learning of Mathematics, which is recognised as a collective need.

6.4.2 General limitations of this study

Although grade, achievement and written reflections on mathematics confidence and metacognitive components could be done, these aspects were excluded from analysis in this study because of time constraints.

6.4.2.1 Limitations of the quantitative study

One limitation of using the SOM includes the absence of items that focus on components of the social, psychological and intellectual domains, which might reveal higher correlations with monitoring (TSP). Such items could well increase the validity of the findings in the qualitative study. The majority of respondents had Setswana as their first language. This might have had an effect on learners' understanding of some of the statements in the questionnaires. Possible correlations could exist between the factors of milieu and those of the RPSQ, since this involves the environmental factors 'social' and 'psychological'. Other factors measured by the SOM could be included in the factor analysis of the RPSQ, allowing possible research opportunities that would link affect, metacognition and study orientation in Mathematics.

6.4.2.2 Limitations of the qualitative study

The passage of time may have affected the quality of participants' reflections on their earlier Mathematics experiences as probed during the second interview. Similar reporting found in the literature could serve as a validation of the findings. Journal writing as a form of reflection was not considered as an approach as it was too time-consuming. Yet, the researcher believes that it could provide a broader insight into participants' reflections and the sources of their mathematics confidence.

6.5 Concluding remarks – a reflection

Social empowerment through mathematics could better one's life, chances in study and work. Self-reflective judgement creates a self-belief system where positive or negative emotions and feelings arise and affect the individual's current and future engagement with mathematics tasks. The availability and appropriateness of the task, instruction methods, knowledge and strategies determine the degree of success attained in problem solving. The level of confidence associated with reflection regulates the understanding and solving of unfamiliar mathematics problems. The results of this study indicate that confidence with regard to Mathematics has certain implications for the individual, and that the social, psychological and intellectual success of the community as a whole has its roots in diverse experiences with Mathematics. With its complex and profound interactions, reflection stems from and stirs both metacognitive and affective aspects. Reflection on metacognitive knowledge and regulation deepens the awareness of the level of confidence and fosters, to some extent, a knowing of knowing.

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ADDENDUM A

Letter to the Department of Education requesting permission to conduct research in selected schools

Miss S. Yssel
APO: Manager
Potchefstroom
Contact no: 018 297 4201
Fax: 086 514 3195
e-mail: syssel@nwpg.gov.za

Date: 13 March 2011

Dear Madam

Permission requested to conduct research in Mathematics in Potchefstroom schools

I am currently enrolled for the degree MEd specializing in Mathematics at the North-West University Potchefstroom. The purpose of my study is to examine the *role reflection and Mathematics confidence plays during problem-solving processes.* The number of learners that enrol at universities with high pass rates in subjects like Physical Science and Mathematics is regarded as a concern nationally and internationally. It is known that performance in Mathematics is a major concern in South Africa, as also stipulated in the TIMSS-survey (2003) and CAPS (2010).

This study aims to contribute in the design and preparation of Mathematics teaching and learning contexts.

To complete this study successfully, I hereby request your permission for the following:

(i) To distribute questionnaires among learners in grade 8 and 9 in Potchefstroom schools
(ii) To conduct interviews with some of these learners

Permission will also be requested from the Ethics Committee of the North-West University, school principals and parents involved. The learners and schools will be kept confidential and they will by no means be mentioned in the research. All respondents and participants will be kept anonymous.

I trust that my request will be taken into consideration

Kind regards,

(Mr) D. Jagals

ADDENDUM B

Letter from the Department of Education granting permission to conduct research in selected schools

education
Lefapha la Thuto
Onderwys Departement
Department of Education
NORTH WEST PROVINCE

Private Bag X 919
Potchefstroom
2520
Tel: 018-297 4201
Fax: 018-294 6094
E-mail: Syssel@nwpg.gov.za
Dpretorius@nwpg.gov.za

DR KENNETH KAUNDA DISTRICT

POTCHEFSTROOM AREA OFFICE

OFFICE OF THE AREA OFFICE MANAGER

TO: MR D. JAGALS

FROM: MS S.S. YSSEL
AREA MANAGER
POTCHEFSTROOM

DATE: 18 MARCH 2011

SUBJECT: PERMISSION TO CONDUCT RESEARCH IN MATHEMATICS AT POTCHEFSTROOM SCHOOLS

The above matter refers.

Permission is herewith granted to you to visit the schools identified in Potchefstroom Area under the following provisions:

- the activities you undertake at school should not tamper with the normal process of learning and teaching;
- you inform the principal of your identified school of your impending visit and activity;
- you provide my office with a report in respect of your visit;
- you obtain prior permission from this office before availing your findings for public or media consumption.

Wishing you well in your endeavour.

MS S.S. YSSEL
AREA MANAGER
POTCHEFSTROOM

AO1137/dp

ADDENDUM C

Letter to school principals inviting the schools to participate in the research

Private Bag X1217
Potchefstroom 2520
Contact: 079 338 7881
E-mail: jagalsdivan@yahoo.com

To: The Principal

Dear Sir/Madam

INVITING YOUR SCHOOL TO PARTICIPATE IN A RESEARCH PROJECT

I am currently enrolled for the degree MEd specializing in Mathematics at the North-West University Potchefstroom. The purpose of my study is to examine *the role reflection and Mathematics confidence plays during problem-solving processes.*

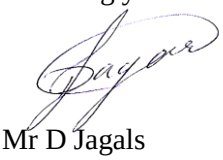
The Mathematics results and achievement of learners in the North-West Province is generally a matter of concern. The number of learners that enrol at universities with high pass rates in subjects like Physical Science and Mathematics is regarded as a concern nationally. It is known that performance in Mathematics is a major concern in South Africa, as also stipulated in the TIMMS-survey (2003) and CAPS (2010). Research has shown that a great deal of learners experience anxiety when doing Mathematics and this in turn influence their achievement and understanding in the subject. One factor that can help understand this cause is the learner's ability to reflect on experiences with Mathematics in the past.

The research aims to contribute in the design and preparation of Mathematics teaching and learning contexts. To complete this study successfully, I hereby request your permission for the following:

(i) To administer questionnaires among learners in grade 8 and 9 in your school
(ii) To conduct interviews with some of these learners

Permission will also be requested from the Ethics Committee of the North-West University, the Department of Education and parents of the school. The information regarding learners and the school will be kept confidential and they will by no means be mentioned in the research. All respondents and participants will be kept anonymous.
Feedback will be given by means of published results available to the school. Arrangement may also be made if you would wish for me to discuss my findings with parents of the school on a parent's day.

Thanking you kindly,



Mr D Jagals

ADDENDUM D

Letter of consent to parents of respondents and participants, inviting them to participate in the study

Dear Mr/Mrs **June 2011**

INVITING YOUR CHILD TO PARTICIPATE IN A RESEARCH PROJECT

I am currently busy with research focussing on the role Mathematics confidence plays on the use of reflective strategies during Mathematics problem solving.

The Mathematics results and achievement of learners in the North-West Province is generally a matter of concern. Research has shown that a great deal of learners experience anxiety when doing Mathematics and this in turn influence their achievement and understanding in the subject. One factor that can help understand this cause is the learner's ability to reflect on experiences with Mathematics in the past.

Learners who participate in this study will be requested to solve one Mathematics problem and complete two questionnaires. Some of the learners will be invited to an interview. The interview will consist of two sessions held on different days. During the interview, the learner will solve one Mathematics problem while being video recorded. This is to capture any physical behaviour (like frowning or making corrections). Afterwards the learner will receive action cards that describe, in words, what he or she did or thought when solving the problem. The video will be played back and paused to question the learner about certain behaviours and thoughts like (What were you thinking here? or Why did you change that step?). The learners are allowed to change their approach to the problem at any time even after they finished. The choices made and the reaction found during the interview will be analysed and interpreted to better understand behaviour during Mathematics problem solving.

Participation in the project is free willingly and the learner is allowed to withdraw from the study at any time.

Your permission as parent is therefore necessary to allow your child to participate in the study. Information gathered will be regarded as confidential and learners will be kept anonymous.

The research will be conducted during the month of June 2011. Only learners that are invited to the interviews will be notified and appointments will be made during this time for after school sessions. Sessions for the interviews will take about an hour.

Feedback will be provided by arrangement with the researcher.

Thanking you kindly,

Mr D Jagals
Cell: 079 338 7881
E-mail: jagalsdivan@yahoo.com

……

Permission

I have read the above and understand the nature of the research. I understand that by giving permission the researcher will conduct the study in a professional manner and will consider the child's human rights. I understand that I can contact the researcher or any queries regarding the study. Undersigning, I hereby give my child permission to participate in the above study.

Name and surname of learner:_____

Grade:_____ Contact number:_____

Parent's / Guardian's signature:_____Date:_____

ADDENDUM E

Ethics Committee: Approval

NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT

Private Bag X6001, Potchefstroom
South Africa 2520

Tel: (018) 299-4900
Faks: (018) 299-4910
Web: http://www.nwu.ac.za

Ethics Committee
Tel +27 18 299 4850
Fax +27 18 293 5329
Email Ethics@nwu.ac.za

ETHICS APPROVAL OF PROJECT

This is to certify that the next project was approved by the NWU Ethics Committee:

Project title : An exploration of reflection and Mathematics confidence during problem-solving in senior phase Mathematics
Student : D Jagals
Project leader: Dr M Van Der Walt
Ethics number: **NWU-00043-11-A2**

Status: S = Submission, R = Re-Submission, P = Provisional Authorisation, A = Authorisation

Expiry date: June 2016

The Ethics Committee would like to remain at your service as scientist and researcher, and wishes you well with your project. Please do not hesitate to contact the Ethics Committee for any further enquiries or requests for assistance.

The formal Ethics approval certificate will be sent to you as soon as possible.

Yours sincerely

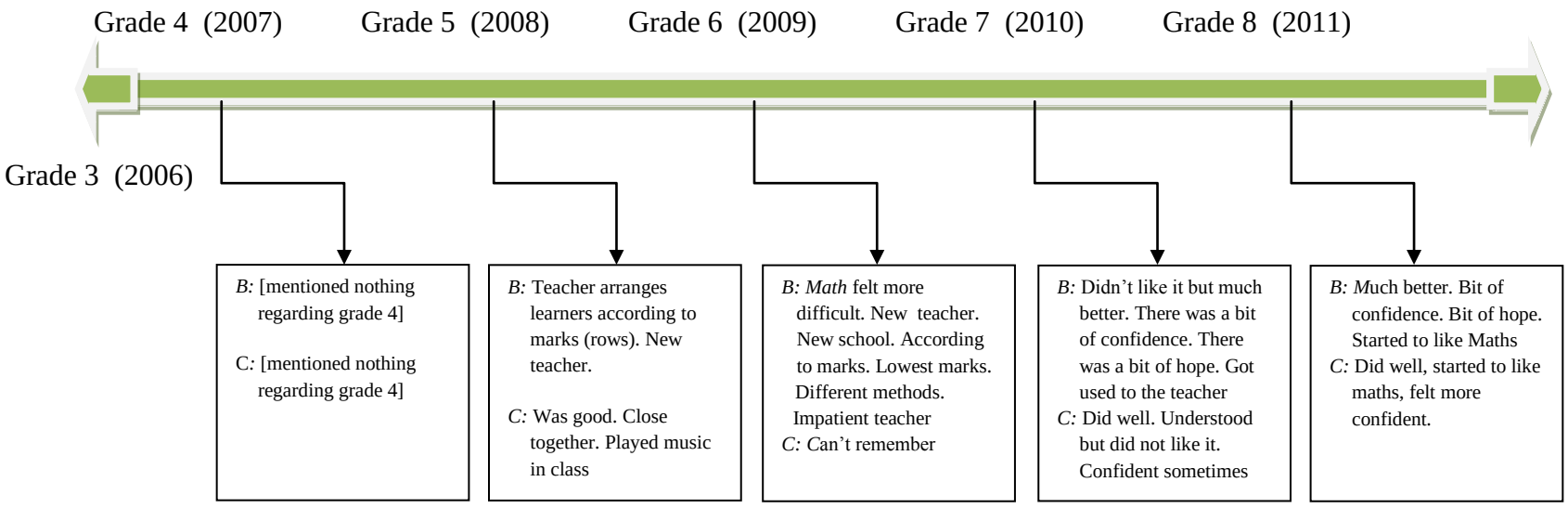
Me.Marietjie Halgryn
NWU Ethics Secretariate

ADDENDUM F

Summative tables and figures – collective overviews of chapters

Time line for Grade 8 participants

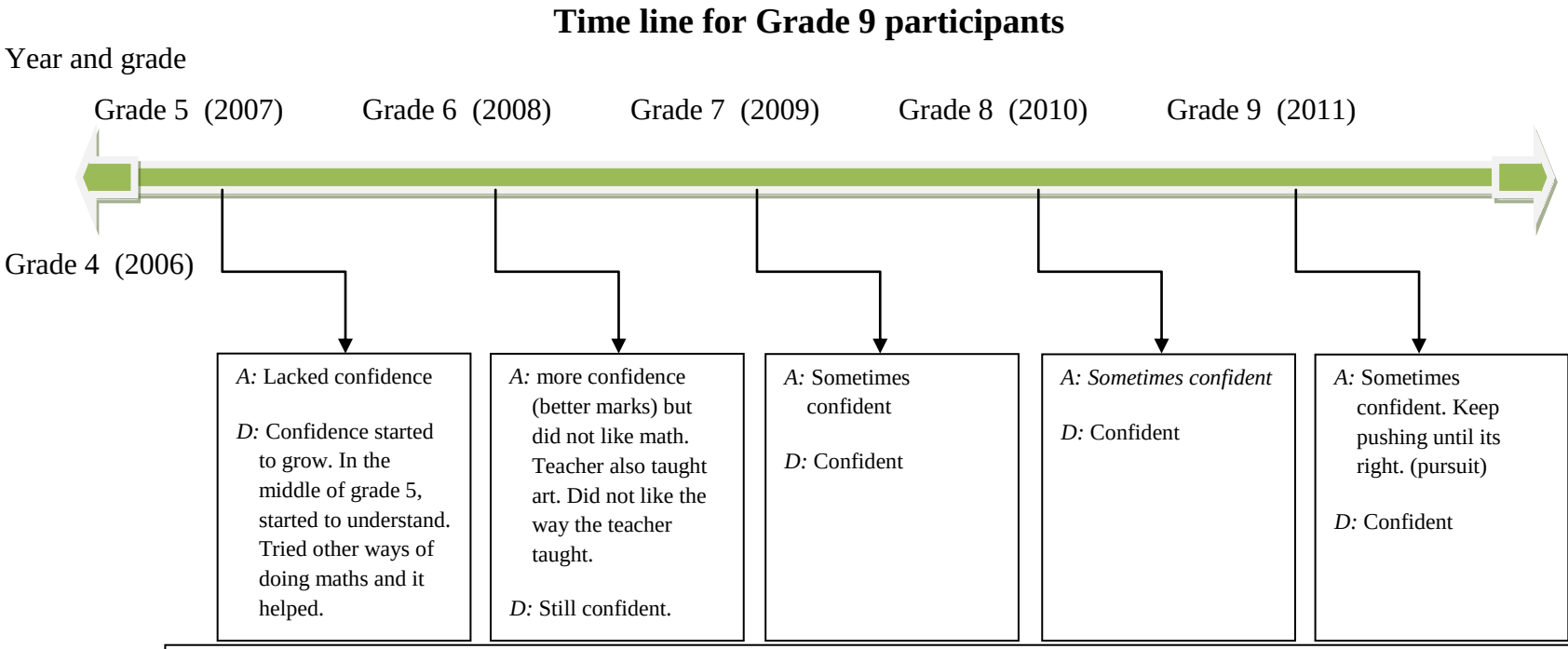
Year and grade



B: *In grade 5 it didn't feel as difficult as in grade 6…In grade 5 we had a new teacher who would put us according to our marks. Back or front…Grade 6, the worst year. Where I got the lowest marks ever. I went to a new school. My math teacher was my register class teacher. I couldn't understand. She jumped around using different methods…Didn't like it, but it was much better than in grade 6...realized I should focus more in class... Because I think I got used to the teacher.* – [the same as in grade 6]*…since the first exam* [not confident]*…Grade 7, there was a bit of confidence…In grade 7 there was a bit of hope. I didn't reach my goal. And in grade 8, it's the same as in grade 7. I got used to the teacher (same as grade 6)…if the teacher just had patience…*

C: *In grade 5 we had a female teacher. She just made maths fun. The class was small. We were all close together. She would play around with us. Sometimes when she gave us work to do she would play music. Nice music…I was good since grade 5. In grade 5 I got 98%, the lowest mark was 90%. In grade 7 I was still doing good. Now I do well also. I understood grade 7 but did not like it…I didn't really care much…I can't remember my grade 6 teacher… started to like maths…*

Time line for Grade 9 participants



A: *...in grade 5 I lacked confidence...grade 6 made up for grade 5...In grade 6 I did not like math...I found my own way of doing things...I had to go home and do everything all over again...I liked the teacher but not the way he did it...My grade 6 teacher [he] also taught art...I tend to get frustrated and just leave it alone ...were sometimes confident in grade 9...I just keep pushing until I get it right... In grade 6 I understood everything because I taught it to myself...in grade 4 [liked mathematics] if you like the teacher you will like the subject, my math teacher was a real down to earth person that I can relate to...*

D: *...it only started in grade 5, in grade 4 I hated math...There was a time I didn't like maths. I was like I was with the group. Most kids would say that I don't like it and it's boring. Other people are still doing it but not me. I think I understood it even though I did not like it. Before grade 5 I was not confident in maths, but in grade 5 and afterwards...tried new ways to better understand and it helped...I didn't enjoy the work, even if my parents explained it to me. I think I was that stage when I couldn't understand maths...Not the same as in grade 4 and 5. I can't remember much of my grade 4 teacher... [Confidence started] I was sitting in the second row from the front. I was asked a question and I got it right. She [my teacher] looked at me and said I "I can see you started to like maths"...In the middle of grade 5 I started to understand...my math teacher always said there is a time when you start finding math easy.*

Category(ies)	Learner A	Learner B	Learner C	Learner D
Reflection on knowledge of the self (person) (KP)	The learner realised that she could not remember the formula and it felt confused and uncertain.	The learner is uncertain whether she has the right approach. She knows how to make it easier. She claims to be very forgetful.	This learner is identifies that for her, personally, it is important to feel comfortable in the learning environment. She identifies what is better for her to do like understanding what she's doing. Some factors, like labelling the circles, are only done because they were *bothering* her.	This learner is identifies that for her, personally, it is important to feel comfortable in the learning environment. She identifies what is better for her to do like understanding what she's doing. Some factors, like labelling the circles, are only done because they were *bothering* her.
Reflection of knowledge of the task (KT)	She was unsure about why she was doing what she was doing and lacked knowledge of the formula, after trial and error getting it right, and lacked proper meaning of circumference and area in applications. She remembered, from experiences, what to do and tried using that.	The fraction sign confused her. She remembers components of the task from a previous experience "from when we were doing circles". Unsure of a value that should be substituted (pi).	Reading the question to seek information. The question was read in parts. Knows the formula and values needed to substitute. However she did comment that she wasn't sure because this work was done a long time ago.	Reading the question to seek information. The question was read in parts. Knows the formula and values needed to substitute. However she did comment that she wasn't sure because this work was done a long time ago.
Reflection on knowledge of strategies (KS)	Knowledge of strategies to enhance understanding by drawings, writing in words and comparing approaches. The learner' approaches, for the sake of understanding and strategy selection, proves an existing background in similar contexts.	She took the comma away. She did long multiplication.	Picture drawing and rereading the question. Only one strategy was used to solve the problem and no others to evaluate. The sum was done in parts.	Picture drawing and rereading the question. Only one strategy was used to solve the problem and no others to evaluate. The sum was done in parts.
Reflection on regulation of understanding (RU)	The learner shows understanding of what the problem was about and expected from her but she lacked thorough understanding of concepts like circumference, diameter, radius and area.	The learner regulated her understanding by reading and analysing. Drawing circles. Confused about the value of pi and what the formula is. Understanding sums with decimals was a problem.	Regulates understanding by rereading selectively, picture drawing. Does not show identities of pictures' sizes, all were the same size. No symbols or lines to illustrate diameter or radii. Uncertain if this learner understood the task meaningfully.	Regulates understanding by rereading selectively, picture drawing. Does not show identities of pictures' sizes, all were the same size. No symbols or lines to illustrate diameter or radii. Uncertain if this learner understood the task meaningfully.
Reflection on regulation of planning (RP)	Learner A's planning is related to her understanding of the first principles in Euclidean geometry. Planning involved organising her stationary, calculator, reading, re-reading and drawing pictures related to the information in the problem. She also made us of an imaginative planning.	At the start. Reading some parts of the problem more than others. Organising her desk, stationary. Picture drawing, systematically. Planning where to put the variables of the formula, in what order. The identifying and employing of a *plan B.*	Planning was done mentally, searching for commands by rereading the question. Doesn't know why she did what she did.	Planning was done mentally, searching for commands by rereading the question. Doesn't know why she did what she did.
Reflection on regulation of monitoring (RM)	Monitored throughout the process, in a way excessively. She showed signs of low confidence when monitoring work, which she was uncertain of, and high confidence when she was certain about her effort.	She monitored her work in small successive parts. Kept on reading every time after she wrote something. Includes physical behaviour. Self-talk.	Rereading and checking most relevant steps. She goes back to the question and pictures after she writes. She did not monitor after the third step onwards.	Rereading and checking most relevant steps. She goes back to the question and pictures after she writes. She did not monitor after the third step onwards.
Reflection on regulation of evaluation (RE)	This learner, through monitoring, evaluated her work by trying other strategies, comparing their solutions and judging their accuracy.	After reflecting, she realised what was incorrect or agrees with the approach. Comparing answer with expectations (plan/predict).	This learner did not evaluate her answer. She did however recalculate the value given by the calculator and wrote that down.	This learner did not evaluate her answer. She did however recalculate the value given by the calculator and wrote that down.

Table 4.8 Reduction table: Organisation, description and identification of factors in the 12-factor pattern matrix

Items	1	2	3	4	5	6	7	8	9	10	11	12	Communalities
32	.546							.214					0.577
31	.436					.282							0.501
33	.389												0.402
2	-.330	.229				.280					.250		0.551
25	.298								.229				0.383
8		-.656							.256				0.614
40		-.635											0.366
7		-.634											0.517
10		-.472										.319	0.504
9		-.462											0.381
28		-.387			.215			.263					0.423
37			.890										0.720
21			.751										0.578
43			.612				.261						0.538
36				.539									0.393
35	.268			.550									0.468
12				.444									0.286
3				.438									0.336
20				.346									0.318
39				.314	.215					.220			0.393
19				.340									0.270
18				.233									0.289
13				.222									0.251
38					.680								0.403
29	.294				.314								0.437
27						.527							0.381
30				.229		.459							0.368
24						.427							0.356
23						.425							0.356
1						.296				.283			0.437
15							.38						0.203
44							.333						0.182
14							.263	-.219				.211	0.439
34							.219	.543					0.568
42									.582				0.430
26									.422				0.422
41							.290		.321				0.271
16									.34				0.312
6										-.490			0.179
17											.561		0.338
11											.380		0.146
4		.221		.218						-.223		.349	0.372
						.230						.332	0.425
5												.272	0.339