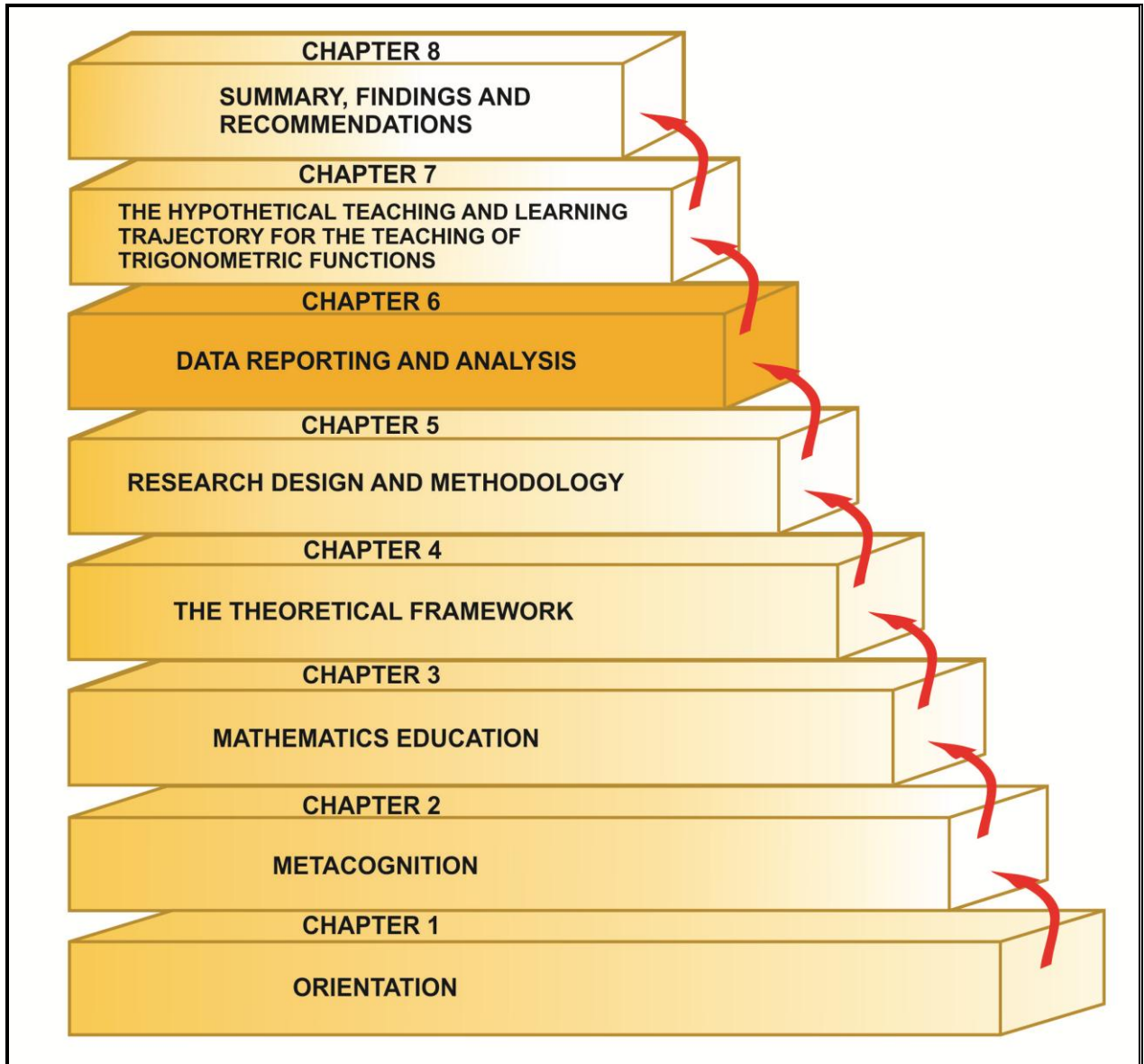
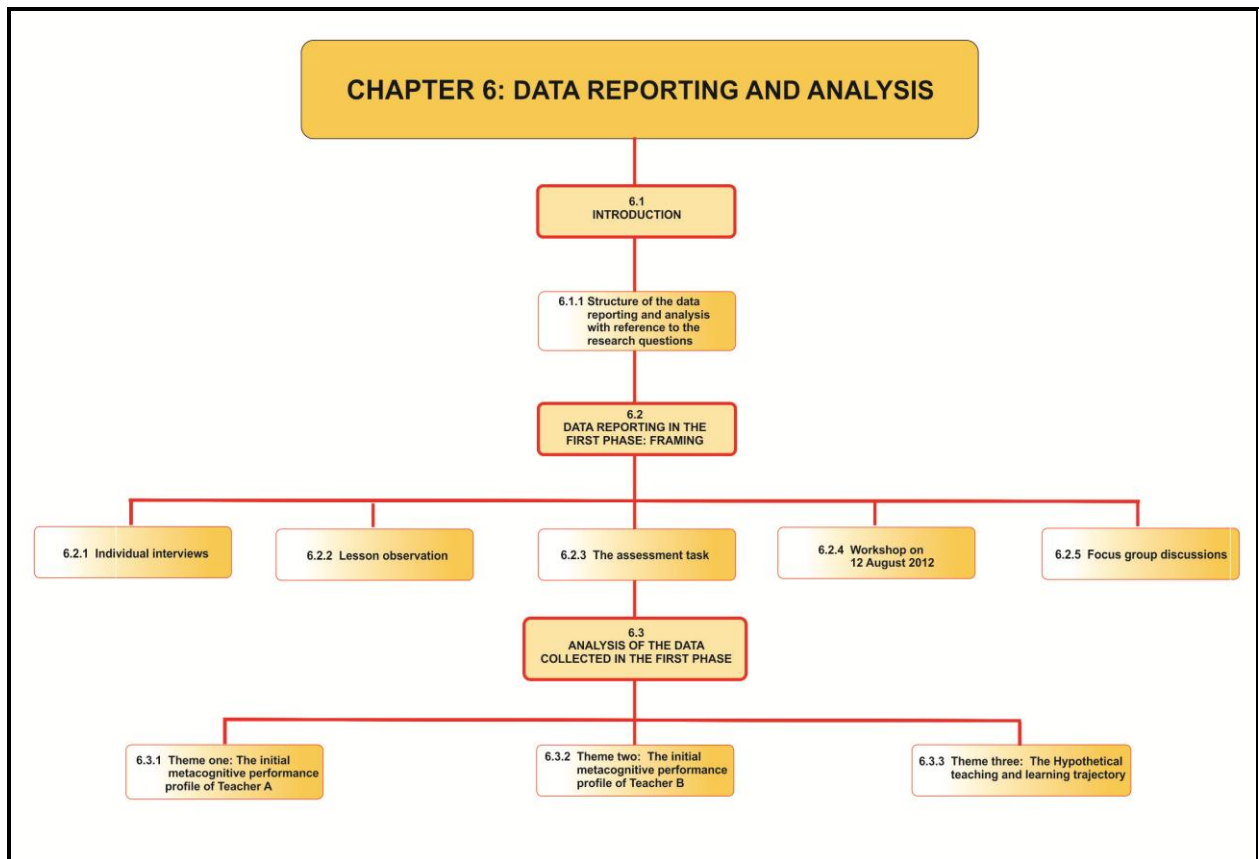


## CHAPTER 6: DATA REPORTING AND ANALYSIS

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## 6.1 INTRODUCTION

Chapter Five described the methods that were selected to empirically investigate the research propositions as well as the motivation for the decisions that were made with respect to the research design employed in the study. This chapter reports on the outcomes of the data-gathering in this design-based research study.

*Sometimes design-based research studies are seen to be reminiscent of the apocryphal story of the drunk looking for his lost keys under the streetlight where he can see, rather than in the dark alley where they were dropped (Dede, 2005, p. 5).*

In order to ensure that this mythical story is not true in the data analysis process embarked upon in this study, the data collected were analysed in relation to the overarching research question posed in this thesis:

*What are the characteristics of an in-service arrangement that facilitates the implementation of lesson activities focusing on the metacognitive skills and mathematical language of Mathematics teachers for the teaching of trigonometric functions in township schools in the Dr Kenneth Kaunda district?*

The research question implicates that Mathematics teachers do indeed use metacognitive skills and mathematical language in the teaching and learning of trigonometric functions in township schools. The notion of metacognitive instruction differentiates between teaching *with* metacognition and teaching *for* metacognition (Hartman, 2002). This notion is explored in the following three sub-questions:

- 1 How do Mathematics teachers apply metacognitive skills and mathematical language in the teaching of trigonometric functions?
- 2 Which challenges do Mathematics teachers face in the teaching of trigonometric functions?
- 3 How can the teaching of trigonometric functions be improved when focussing on the metacognitive skills and mathematical language used by Mathematics teachers?

All the questions were qualitative in nature and therefore individual interviews, an assessment task, lesson observations and focus group discussions were employed to address the ways in which the metacognitive skills and the mathematical language are used by the Mathematics teachers in the teaching of trigonometric functions in township schools. The participants were teachers, grade 10 learners and lecturers. The aforementioned data collection instruments were used in three distinct phases in line with design-based research (Mor, 2010) which was detailed in the proceeding chapter and are visually represented here in Figure 6.1. As the study consisted of three different phases, it was necessary to report and analyse the data in each of the phases in line with what Plomp (2010, p. 31) advises: “Each cycle has to take the findings of the previous ones into account”.

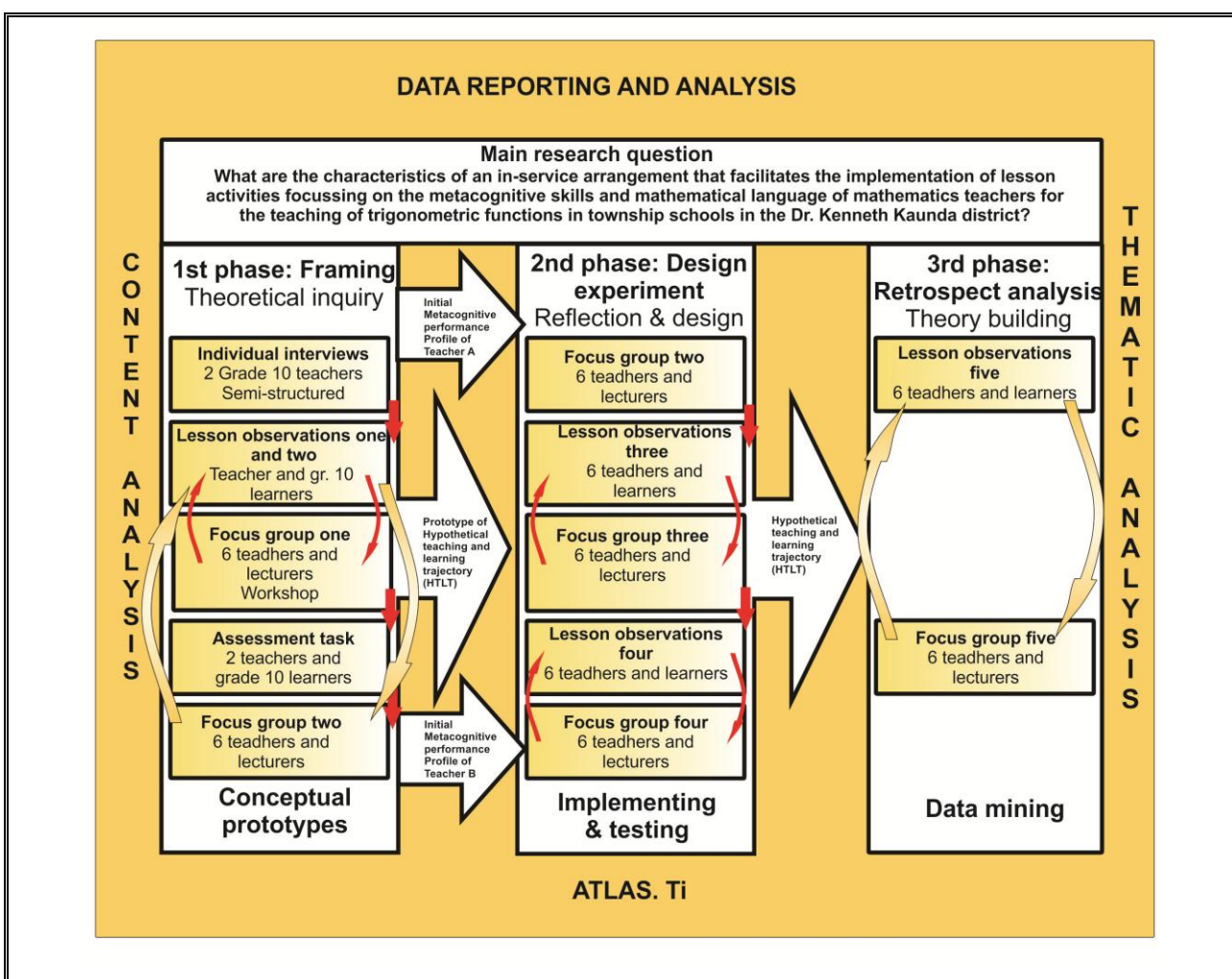


Figure.6.1: Data reporting and analysis in the three phases within this study

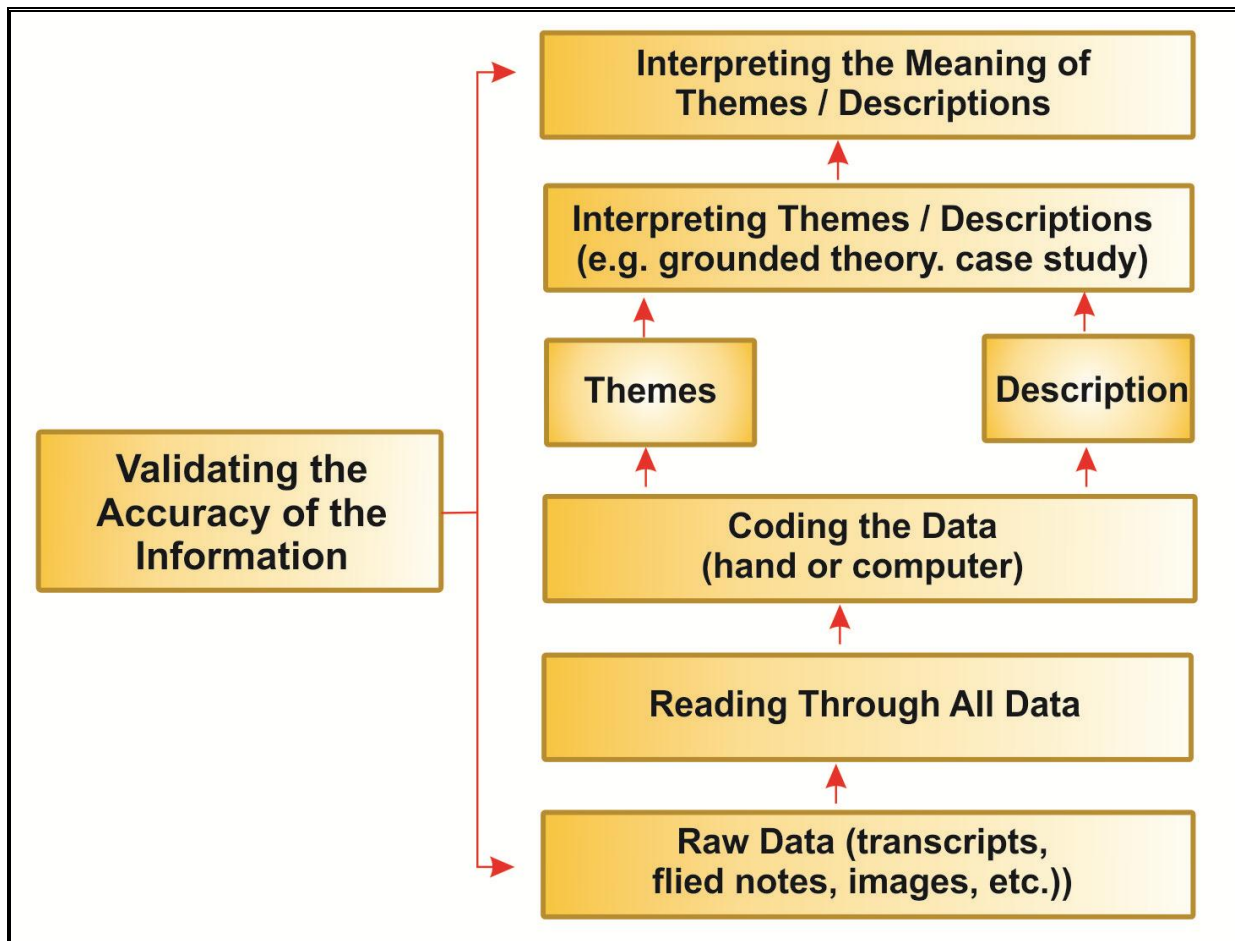
### 6.1.1 Structure of the data reporting and analysis with reference to the research questions

Table 6.1 provides an outline of the research questions with an indication of how and where data are reported and analysed in order to address each research question in this chapter.

**Table 6.1: References to sections that address the sub-questions in this chapter**

| SUB-QUESTIONS                                                                                                                                                | INDIVIDUAL INTERVIEWS            | TRIGONOMETRY ASSESSMENT TASK | LESSON OBSERVATIONS              | FOCUS GROUP DISCUSSIONS          |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------|------------------------------|----------------------------------|----------------------------------|
| 1. How do Mathematics teachers apply metacognitive skills and mathematical language in the teaching of trigonometric functions?                              | (§6.2.1)<br>(§6.3.1)<br>(§6.3.2) | (§6.2.3)                     | (§6.2.2)<br>(§6.3.1)<br>(§6.3.2) | (§6.2.5)<br>(§6.3.1)<br>(§6.3.2) |
| 2. Which challenges do Mathematics teachers face in the teaching of trigonometric functions?                                                                 | (§6.2.1)<br>(§6.3.3)             | (§6.2.3)                     | (§6.2.2)<br>(§6.3.3.1)           | (§6.2.5)<br>(§6.3.3.1)           |
| 3. How can the teaching of trigonometric functions be improved focussing on the metacognitive skills and mathematical language used by Mathematics teachers? |                                  | (§6.3.3)                     |                                  |                                  |

The data gathered were analysed following the general steps for the analysis of qualitative studies as outlined by Creswell (2009) and are indicated in Figure 6.2.



Source: Creswell, 2009, p. 185

Figure.6.2: Data Analysis in Qualitative Research

Although the outline in Figure 6.2 indicates a linear hierarchical approach, the actual data analysis in this study happened much more interactively with the three phases and five cycles more interrelated in an iterative way as shown in Figure. 6.3

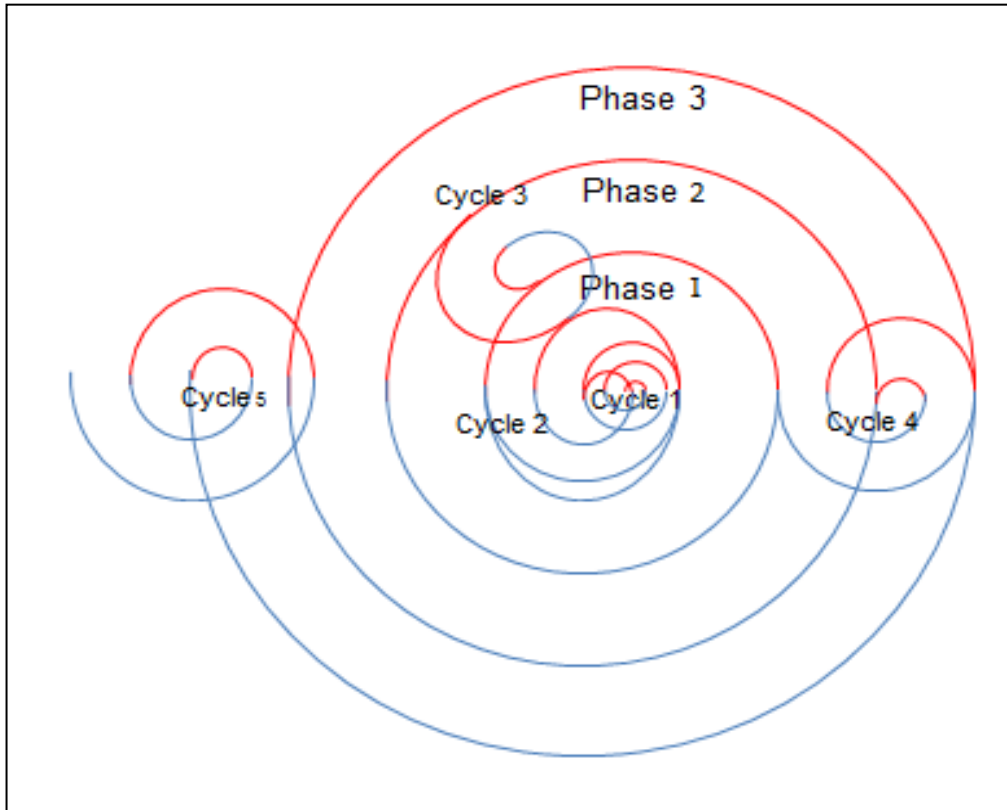


**Figure 6.3: Data analysis within the phases and cycles**

The data analysis required by the specific design approach suggested different phases and data-analysis was prescribed by the need for the analysis in each of the phases as analysis of one phase informed the planning of consecutive phases. Data was reported using a table format in order for readers to get an instantaneous view of what transpired during the interviews, lesson observations, assessment tasks as well as the focus group discussions. All the documents used were gathered into the hermeneutic unit of ATLAS.Ti and therefore form the primary documents. Quotations were used to support reporting and these notations, for example notation (P1:3) should be understood as follows: The first letter and number refer to the relevant primary document (P1 for the first primary document), the number which follows represents the number of that particular response. Therefore (P1:3) means primary document one, response number three. Figure 6.4 shows a visual presentation of all the themes that emerged from the data analysis of this inquiry.

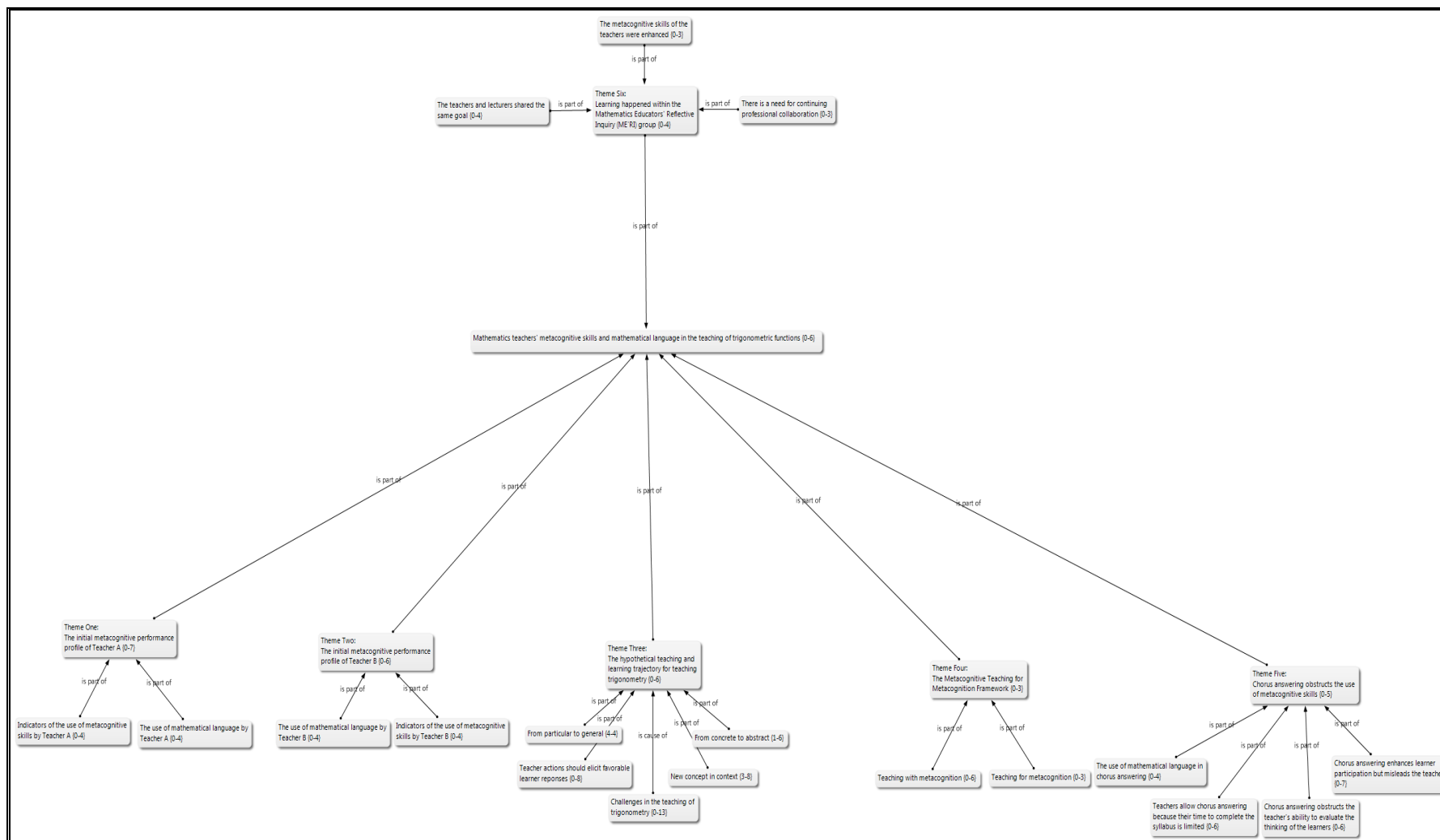


**Figure 6.4: Mathematics teachers' metacognitive skills and mathematical language in the teaching of trigonometric functions**

## **6.2 DATA REPORTING IN THE FIRST PHASE: FRAMING**

This first cycle can be seen as a continuation of the theoretical enquiry and the identification of the theoretical prototype that was done in Chapters Two and Three, especially the teaching of trigonometry and more specifically trigonometric functions. The collection and analysis of the empirical data in this framing phase informed firstly the metacognitive performance profiles of the two grade 10 teachers, Teacher A and Teacher B. Secondly the data collection and analysis of this first phase informed the design of the hypothetical teaching and learning trajectory of which the prototype was started in Chapter Three (§.3.9.6).

### **6.2.1 Individual interviews**

This part of the data reporting focuses on only the two grade 10 teachers, although the entire study involved the analysis of the experiences of all six teachers. The intent with interviewing only the two core teachers is to enable a more in-depth analysis of how each of these teachers was learning throughout the study and to sketch the metacognitive performance profile of each of these two teachers. Size constraints make it impossible to show this level of detail with all the teachers. Specific questions that were asked to participants focused on (a) Attitude towards Mathematics; (b) Motivation to keep teaching Mathematics; (c) Challenges in teaching Mathematics and in particular the teaching of Trigonometry; (d) Preferences in teaching particular strands in Mathematics: Trigonometry, Algebra and Geometry; (e) Own learning of Mathematics at school level and also tertiary training as Mathematics teacher.

The individual interviews (See primary documents one and two in the Hermeneutic Unit) with each of the two participating teachers yielded the following domino effect.

**Table 6.2: Table reporting on interviews with Teachers A and B**

| Categories                          | Teacher A (P1)<br>Red school                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Teacher B (P2)<br>Blue School                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|-------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Attitude towards Mathematics        | <p>Teacher A likes Mathematics because it helps in solving problems. His liking for Mathematics already started in Primary school. In Secondary school he used to do extra problems and then asked the teacher whether they were correct.</p> <p><i>Yes I like Mathematics because it helps in thinking, problem solving and uh . when there more especially in problem solving when you have problems, people with maths they can easily come up with solutions. (P1:3)</i></p> | <p>Teacher B, the only female teacher, likes Mathematics very much because of the challenging nature thereof and the fact that Mathematics deals with numbers. Her liking for Mathematics came from working with numbers and figures from a very early age.</p> <p><i>Uhh, because it's a subject that is really challenging and uh and also its a subject that's dealing with numbers If you are also inclined with this Mathematics it does not need you to teach like other learning areas, you just want the numbers and you know it become so interesting...especially when it comes to the Geometry part. (P2:5)</i></p> |
| Motivation for teaching Mathematics | <p>Teacher A is motivated by his desire to improve the situation of learners who are failing Mathematics.</p> <p><i>You know for me to teach Maths is one of my major subjects I mean consider the fact that learners are failing and then I only wanted to say I know I am good in Maths then maybe I can make an improvement or influence in teaching Mathematics. (P1:11)</i></p>                                                                                             | <p>Her love for numbers and self-confessed “inclination with numbers” keeps her motivated to go to the Mathematics classroom.</p> <p><i>What motivates me its the issue that I've already spoken about that its all dealing with the figures and the numbers...if you're so inclined with it you get so much interested in continue with the learners. (P2:7)</i></p>                                                                                                                                                                                                                                                          |

| Categories                                               | Teacher A (P1)<br>Red school                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Teacher B (P2)<br>Blue School                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|----------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Experiences of own Mathematics learning at school level: | <p>Teacher A initially experienced Mathematics during his early grade (Primary School) very positively.</p> <p><i>Oh when and where Uh It started in Primary...cause my class teacher my math teacher in primary she was very good in Mathematics I enjoyed Mathematics more than any other learning area in Primary that's why I continued with Mathematics...then I think ya. It started in an early stage and then I enjoyed actually working with problems. (P1:9)</i></p> <p>However, later in high school, Teacher A had negative experiences of how he was taught Mathematics at school level. His high school Mathematics teacher used to ask them to leave Mathematics if they do not understand the work.</p> <p>This situation resulted in teacher A, together with some of his then classmates, forming their own study groups with particular “strong” learners to help others in the different subjects. Teacher A was the “strong” learner for Mathematics and used to help other learners with Mathematics, although he admits that he himself struggled with word problems at</p> | <p>Teacher B experienced Mathematics at school level as difficult. She recalled not having a Mathematics teacher in grade 12 and subsequently had to rely on some bright grade 11 learner to teach her and her classmates Mathematics in grade 12.</p> <p><i>At the school it was so difficult. Uhh...I'm going to talk specifically about my high school life...You know there when I was doing grade 10, uhh there was a certain boy that he was my classmate..he was so brilliant...that boy...you know what happened he was demoted to grade 11...he was demoted to grade 11... When we were in grade 12 we didn't have a teacher by then....So what we did we requested the principal to go and call that boy to come and teach us... actually Maths and Science in grade 12 we were taught by teachers who were not qualified who only had grade 12. (P2:17)</i></p> |

| Categories                                    | Teacher A (P1)<br>Red school                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Teacher B (P2)<br>Blue School                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|-----------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                               | <p>school.</p> <p><i>I realise when I was in Matric I started to experience some difficulties but because on my own ehh...with my classmates by then we having a study group and in our study groups there was some other guy who was helping us with Physical Science and he was good in Physical Science. I was helping with Mathematics because our teacher by then when in class he was just like when you ask him a question he would say: No, if you don't understand this question you rather leave . (P1:11)</i></p> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| Own learning of Mathematics at tertiary level | <p>Teacher A went to the local university and achieved a degree in Mathematics education. He experienced Mathematics learning at tertiary level much better than the Mathematics learning at school.</p> <p>Teacher A admitted that he overcame his struggle with word problems at university and since then was able to understand more clearly how to tackle word problems.</p> <p><i>Uhm...my training was ...was excellent at university. I started to master some more. (P1:33)</i></p>                                 | <p>Teacher B received several certificates and diplomas in Mathematics and Science education and went to the local university and achieved a Bed Hons degree in Mathematics education. She found her tertiary training better than the learning of Mathematics at school level.</p> <p><i>Tertiary training...there it was bit, compared to high school it was a bit better, but the lecturer also was also seem to be a bit struggling with Mathematics. (P2:17)</i></p> |

| Categories                                                                     | Teacher A (P1)<br>Red school                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Teacher B (P2)<br>Blue School                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|--------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Challenges in Mathematics teaching and in particular, in trigonometry teaching | <p>The negative attitude of learners towards Mathematics.</p> <p><i>The challenges in Mathematics teaching are when the learners got a negative attitude towards Maths and then sometimes you might blame the learners some time are the educators because I've realized that. (P1:13)</i></p> <p>The negative attitude towards Mathematics of some Mathematics educators.</p> <p>Lack of content knowledge of some Mathematics educators.</p> <p><i>The teachers when they don't understand a certain part they don't take some extra mile to try to explain to the learners you see? (P:15)</i></p> <p>The pride of some Mathematics teachers not to ask help in cases where they do not understand the content.</p> <p><i>I don't know whether maybe its pride or what .we cannot do it. (P1:17)</i></p> <p>More definitions in Trigonometry.</p> | <p>Poor learner performance in Mathematics.</p> <p>Learners do not practise Mathematics at home.</p> <p><i>Uhh The challenges are the learners are not really performing good in Mathematics...that is the only challenge in Mathematics because it looks like they don't practice because in Mathematics if you don't practise you wont make it....if the teacher taught something learner must go home and make sure on the particular aspect the teacher has taught he does maybe two.. three examples and also practice to ensure that what the teacher has taught in class it become much easier for them. (P2:11)</i></p> |

| Categories                                  | Teacher A (P1)<br>Red school                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Teacher B (P2)<br>Blue School                                                                                                                                                                                                                          |
|---------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                             | <p>Learners are failing to see the big picture.</p> <p><i>For me...I think because of uhh you know you have some more definitions in Trigonometry. So if you don't understand those definitions of Trigonometry the basics actually of Trigonometry, then you cannot continue with other difficult questions because if you miss the basics you won't be able to continue. For me trigonometry it means you must have a broader picture of what you are doing so that you can be able to apply that's why and then you don't have that thing. (P1:25)</i></p> |                                                                                                                                                                                                                                                        |
| Preference of areas to teach in Mathematics | <p>Most difficult to teach for Teacher A is Geometry, followed by Trigonometry and then lastly Algebra.</p> <p><i>I can say...Geometry then Trigonometry and the last one it would be Algebra. (P1:29)</i></p>                                                                                                                                                                                                                                                                                                                                                | <p>For teacher B Trigonometry is the most difficult to teach, followed by Algebra. Teacher B found Geometry to be the easiest to teach.</p> <p><i>Most difficult to I think it will be Trigonometry and then Algebra and then Geometry. (P:13)</i></p> |

## Discussion

From Table 6.2 it appears as if teachers A and B's attitudes towards Mathematics are based on a love for numbers which drives their motivation to keep teaching Mathematics and can be characterised as positive. A common occurrence in the experiences of the two teachers with Mathematics learning at school level, is the phenomenon of being taught by other learners to compensate for the lack of competent Mathematics teachers which confirms statements made by Howie and Plomp (2002), Macrae *et al.* (1994), Van der Flier *et al.* (2003) and Fiske and Ladd (2004) of inadequate training during the previous political dispensation (§3.2.1). The data from the table regarding the attitude and motivation inform the beliefs of the two teachers and are critical for affective self-regulation which, according to Hartman (2002a), entails the self-regulation of values, expectations, beliefs and attitudes necessary for effective metacognitive functioning (Table 2.2, Hartman and Steinburg's model). Furthermore, data from the above table informed the development of the metacognitive performance profiles of each of the two teachers.

### 6.2.2 Lesson observations

Five lesson observations in total were done in this study, but for this first phase in the study only the first two lesson observations (see Primary documents three and four in the Hermeneutic Unit) were considered for reporting and analysis because they were offered by Teachers A and B, and in this study were used to further inform the metacognitive performance profiles of the two grade 10 teachers.

#### 6.2.2.1 Metacognitive skills used by Teacher A and Teacher B in the lessons

The following table (Table 6.3) reports on what transpired in these first two lessons with respect to how Teachers A and B used their metacognitive skills during the lessons.

**Table 6.3: Indicators of metacognition from lessons one and two towards the metacognitive performance profiles of Teachers A and B**

| Metacognition<br>(Artzt & Armour-Thomas, 2002) | Components and indicators thereof<br>(Artzt & Armour-Thomas, 2002)                                                                                                             | Teacher A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Teacher B                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Overarching                                    | Knowledge: Learners                                                                                                                                                            | Facilitation by Teacher A indicated only a general knowledge of the learners in relation to the content                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Teachers B's facilitation revealed some knowledge of learners in relation to the content.                                                                                                                                                                                                                                                                                                                                                                             |
|                                                | Knowledge: Content<br>Indicators: Mastering and use of:<br>Declarative knowledge;<br>Procedural knowledge;<br>Conditional knowledge.<br>Linkages to other areas in Mathematics | <p>Facilitation showed mastering of declarative knowledge of content without real linkages to other relevant areas in Mathematics.</p> <p>Teacher A had the following misconceptions:<br/> <i>Now let me ask you a question: If I want to calculate angle C then it would be what? To get angle C...We don't want tan C we want angle C. How are we going to get it? (P3:110)</i><br/> <i>We divide both sides by tan. Its correct ne. (P3:112)</i><br/> <i>We have said what people? Remember if you round to two decimal places you look for the third number behind the comma, ne now the third number we have said if it is greater than five then you add one, if it is less than five you write it as it is ne. in other word this number is greater than five so then you add one on this one then it would be 18,06 degrees ne. Is that correct? (P3:132)</i></p> | <p>Facilitation showed mastering of declarative and procedural knowledge with some linkages to other areas of Mathematics, like Geometry:</p> <p><i>and that one is equal to that and again this angle is equal to that. That is the first one when we talk about similar triangle. The second one is that: The corresponding sides of our triangles must also be in proportion. I remember we have spoken about this when you were in grade 9 ager? (P4:016)</i></p> |

| Metacognition<br>(Artzt & Armour-Thomas, 2002) | Components and indicators thereof<br>(Artzt & Armour-Thomas, 2002)                                                  | Teacher A                                                                                                                                                                                                                                                                                               | Teacher B                                                                                                                                                                                                                                                                                                                                                                                                 |
|------------------------------------------------|---------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                | Knowledge: Pedagogy<br><br>Indicators: Teaching style, strategies and approach                                      | Teacher-centred.<br><br>Only talk and chalk<br><br>Teacher provided all the information                                                                                                                                                                                                                 | Teacher-centred<br><br>Talk-and chalk approach<br><br>Teacher provided all the information                                                                                                                                                                                                                                                                                                                |
|                                                | Beliefs: Learner role                                                                                               | From the facilitation by Teacher A it is evident that Teacher A believes that the role of the learner is to pay attention and listen to the teacher.                                                                                                                                                    | Facilitation of teacher B indicated the belief that learner has a passive role of listening to the teacher.                                                                                                                                                                                                                                                                                               |
|                                                | Beliefs: Teacher role                                                                                               | It is evident that Teacher A believes that the role of the teacher is to explain the work to the learners.                                                                                                                                                                                              | Facilitation by teacher B shows belief that the teacher has to explain the work to the learners.                                                                                                                                                                                                                                                                                                          |
|                                                | Goals                                                                                                               | It appears from the facilitation as if the goal of Teacher A in this lesson was for learners to understand trigonometric functions and their application to solving angles and sides in a triangle.                                                                                                     | Facilitation by Teacher B displays that the goal with this lesson was understanding of the ratios of trigonometric functions.                                                                                                                                                                                                                                                                             |
| Pre-Active                                     | Planning the Learning Experience<br><br>Does the lesson plan show:<br><br>Objectives<br><br>Structure<br><br>Phases | Teacher A's lesson plan (see Addendum E18) consisted only of worked out examples that were done in the class. No outcome for the lesson was written down.<br><br>Some structure is showing indicating example 1 and 2 to be done.<br><br>Teacher A did not indicate the different phases of the lesson. | Teacher B's lesson plan (see Addendum E18) indicated some planning, but no written outcome for the lesson. However, there were good indicators that this lesson was "coached" before the time:<br><br><i>What is the opposite? This? No I'm not yet there. Why is that? (P4:052)</i><br><br><i>No I did not even speak about the adjacent side. I said the side opposite theta here.. Yes..? (P4:057)</i> |

| Metacognition<br>(Artzt & Armour-Thomas, 2002) | Components and indicators thereof<br>(Artzt & Armour-Thomas, 2002)                                                                                                                                                                        | Teacher A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Teacher B                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Interactive                                    | <p>Monitoring</p> <p>Indicators:</p> <p>Identifying what learners already know</p> <p>Monitoring of learners' comprehension of work as lesson progresses</p> <p>Observing</p> <p>Listening to learners</p> <p>Eliciting participation</p> | <p>Teacher A did not attempt to identify what learners already knew.</p> <p>Very few instances of monitoring of the learners' comprehension of the work could be observed from the lesson presented by Teacher A.</p> <p><i>Do we all know what Navigation mean?</i> (P3:006)</p> <p>It must be noted that Teacher A did not wait for an answer and continued to explain what navigation means.</p> <p><i>Are we correct?</i> (P3: 224)</p> <p>Teacher A did not really observe the learners. He was busy writing on the board while talking to the learners with his back towards them for most of the time.</p> <p>Teacher A listened to the learners who for most of the time chorus-answered his questions.</p> <p>To elicit participation, Teacher A asked questions most of the time to the whole class on which the learners answered in a chorus. He therefore did elicit participation from the learners, but mostly whole class participation and not individual participation.</p> | <p>Teacher B referred to the prior knowledge (grade nine work) of her learners.</p> <p>One way of monitoring by teacher B is visible when she wanted to know from the learners whether they "are together" with her. However, she did not really seem to be interested in the answer; she seems to have expected the learners to say yes on this question.</p> <p><i>We cannot apply it here because there we don't have any 90 degrees. Are we together?</i> (P4:038)</p> <p>Never do the learners respond with a "no" to this question.</p> <p>More observation of the learners happened in this lesson than in lesson one.</p> <p>More listening to the learners when compared to Teacher A.</p> <p>More individual participation was elicited from learners by Teacher B than by Teacher A.</p> |

| Metacognition<br>(Artzt & Armour-Thomas, 2002) | Components and indicators thereof<br>(Artzt & Armour-Thomas, 2002)                                                                                                                  | Teacher A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Teacher B                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                | <p>Regulating</p> <p>Indicators: Adapting instruction based on monitoring</p> <p>Excluding planned examples based on monitoring</p> <p>Adding more examples based on monitoring</p> | <p>As in the case with his monitoring skills, Teacher A did not display many instances of regulating his thinking during the lesson facilitation. There were, however, episodes when Teacher A was checking whether the answers are correctly approximated.</p> <p><i>I've said read ..71,94261263. Now if you are not told to round off to one decimal number or two decimal number we say the standard to round of ... you must always round off to two decimal numbers. So it would be 71,94. Now let me ask you the question: Why we said our final answer would be 71,94 metres or actually this one is an angle so it is 71.94 degrees. Why we say it will be 71,94 not 95 or 93? (P3:080)</i></p> <p>Judging from the lesson plan which shows the two examples Teacher A planned to do, no examples were excluded or added than those already planned for the lesson</p> | <p>Teacher B also displayed only a few episodes in which regulation of thinking could be observed. She usually explained the work in such a manner that learners had to complete the sentence. She was also, most of the time, asking questions that needed declarative knowledge.</p> <p><i>We need a...what type of a triangle here? (P4:040)</i></p> <p>Teacher B one altered her instruction by using a different method of making learners understand the "opposite side: Teacher B used A4 paper covering the two sides so that only the opposite side was visible, making sure learners really saw the opposite side. She called one learner to the board to help holding papers to the diagram, closing the other two sides to only open up the opposite side, successfully regulating thinking.</p> <p><i>I want to demonstrate something here so that we don't forget what do we mean when we talk about opposite. Can you see what is happening here? (P4:065).</i></p> <p>She also added more examples to show the different ratios</p> |

| Metacognition<br>(Artzt & Armour-Thomas, 2002) | Components and indicators thereof<br>(Artzt & Armour-Thomas, 2002)                      | Teacher A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Teacher B                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|------------------------------------------------|-----------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Post active                                    | Assessing<br><br>Indicators: Finding out whether goal was accomplished                  | <p>Teacher A gave homework only at the end of the lesson to evaluate the learning experience. The bell had already rung when he gave the homework. He did not manage to go through the homework with the learners.</p> <p><i>So we have used the theorem of Pythagoras we got 4.67. We have used the trig functions we got 4.67 metres. So I want you to do exercise 5.7 exercise 5.6 on page 116. number 1 number 2 a and b as well as number 3. Number 1, number 2 as well as number 3...that will be your homework.ne. (P3:226)</i></p> | <p>Teacher B also gave homework towards the end of the lesson. It must be noted, however, that she managed to go through the homework with her learners:</p> <p><i>Now I want us to go to page...exercise 5.2 on page 105. I want us to look at the triangles there. We are given three triangles which are said to be similar...Similar meaning corresponding sides I mean corresponding angles are equal Ager? and then again the ratio of the corresponding sides will also be the same. I want you to quickly fill...uhh..to answer those questions in your book...exercise 5.2 from number a to b. Now before you answer the questions let me quickly explain here...we are given triangle... three triangles the one is triangle ABC the other one is triangle PQR the other one is triangle XYZ. (P4:113)</i></p> |
|                                                | Revising<br><br>Indicators: Revisiting previous work done in lesson or previous lessons | <p>Teacher A went back to the definitions of the trigonometric ratios quite a few times.</p> <p><i>Ok, lets go back to the trigonometry definitions. (Teacher moves back to the trigonometry ratios he wrote earlier on the board.) What is tan theta? (Answering his own question) Tan theta is opposite over adjacent. Sine theta is opposite over hypotenuse and cos theta is adjacent over hypotenuse (learners</i></p>                                                                                                                | <p>Teacher B revised the previous year's work during her lesson:</p> <p><i>Now If you have the right-angled triangle you must also be able to apply the Pythagoras theorem (Teacher writes on board while learners are saying this together with teacher (choir answering). Remember we had done the investigation last year in grade 9 where we were proving that the square on the longer side is equal to...? (P4:022)</i></p>                                                                                                                                                                                                                                                                                                                                                                                        |

| Metacognition<br>(Artzt & Armour-Thomas, 2002) | Components and indicators thereof<br>(Artzt & Armour-Thomas, 2002) | Teacher A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Teacher B |
|------------------------------------------------|--------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|
|                                                |                                                                    | <p><i>join in reciting the ratios). Now have a look at the diagram there. The hypotenuse we have said is an angle which is opposite to the 90 degrees. Now we don't know the hypotenuse in this case, ne. (P3:044)</i></p> <p>He also revisited the rule of rounding off more than twice during the lesson.</p> <p><i>3,58...Ok now we have said if the 3rd number is greater than five but this one it equals five so we add one but if it was less than 5 it then we would write it as it is ne in other words we have 3.58. Now 3.58 it would be this side. ne 3,58 ne. (P:197)</i></p> |           |

## Discussion

From Table 6.3 it is evident that Teacher A was still very much using the chalk and talk approach. As far as metacognitive teaching is concerned, very little evidence could be found hereof. Teacher A's planning of the lesson consisted of only worked out examples of the trigonometric functions. Although this was supposed to be the introductory lesson on trigonometric functions it was clearly not the first time the learners heard about trigonometric functions. They definitely knew about the different trigonometric ratios judging from their answers. Instances of teaching with metacognition appeared only sporadically. Teaching for metacognition, in that Teacher A gave opportunities for thinking about thinking, was never observed within this lesson. No mention of the unit circle was made in this lesson. Furthermore it must be noted that the facilitation done by Teacher B was still very teacher-centred. It was confusing that Teacher B introduced Trigonometry as "new" concept but learners seemed already to know the different trigonometric ratios. It was very obvious that the terms adjacent, hypotenuse and opposite had been explained before, because on two occasions learners used the new terms even before teacher came to the new terms. However, Teacher B connected the new concept (trigonometry) with prior knowledge of Pythagoras' theorem and used the word hypotenuse here as well. Similar to the first lesson, there was also no reference to the unit circle in this lesson.

### 6.2.2.2 Use of mathematical language in the lessons

Thinking requires language and therefore it was imperative that this inquiry also analysed how Teachers A and B used mathematical language to make visible their thinking. The following table shows the use of mathematical language by Teachers A and B and attempts to further crystallize the performance profile of these teachers focusing on mathematical language usage in the two lessons.

**Table 6.4: Mathematical language usage in lessons one and two**

| Conceptualizing | Language elements in Mathematics (San Diego County Office of Education, 2007)                                           | Teacher A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Teacher B                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|-----------------|-------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                 | Content language (such as technical language, for example fraction, equation, degree and exponent),                     | Teacher A made use of content language to define the concept "Trigonometry". He made use of content language about 57 times.                                                                                                                                                                                                                                                                                                                                                                                                     | Teacher B made use of content language about 43 times                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|                 | Symbolic language (for example numbers, tables, graphs and formulas)                                                    | Teacher A made use of symbolic language about 20 times                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | Teacher B made use of symbolic language about 10 times                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|                 | Academic language (such as language used in the instruction of Mathematics, for example simplify, evaluate and convert) | The word "calculate" was also used by Teacher A to define the concept Trigonometry.<br>Teacher A made use of academic language about 28 times                                                                                                                                                                                                                                                                                                                                                                                    | Teacher B only used academic language 6 times. This might be because the lesson presented by teacher B was mostly about identifying the three sides, opposite, adjacent and hypotenuse in the triangle. Only towards the end, Teacher B introduced the trigonometric functions, Sine theta, Cos theta and Tan theta.                                                                                                                                                                                                    |
| Misconceptions  | Teacher                                                                                                                 | Teacher A's use of the word "dimensions" when referring to the three sides and the three angles in the triangle was incorrect. The word "dimensions" in Mathematics usually refer to a measurement of length in one direction and therefore it can never refer to the angles in the triangle.<br><br><i>Now today we are going to start with our chapter which is the three dimensions. (P:006)</i><br><br>Teacher A corrected himself later on during the lesson:<br><br><i>Now we say we use trigonometry to calculate the</i> | Teacher B introduced trigonometric functions as only applicable in a right-angled triangles. This is a misconception as trigonometric functions can be used in any triangle:<br><br><i>Right what happen here. we now have here our trigonometric functions. Right we can only be able to apply the trig functions only if our triangle is a right-angled triangle (learners say along with teacher(choir answering)). We cannot apply it here because there we don't have any 90 degrees. Are we together? (P:038)</i> |

| Conceptualizing | Language elements in Mathematics (San Diego County Office of Education, 2007) | Teacher A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Teacher B                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|-----------------|-------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                 |                                                                               | <i>three things nê...the angles as well as the sides, nê.</i> (P:007)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|                 | Learners                                                                      | <p>The learners had misconceptions relating to approximating answers which were read from their calculators.</p> <p>They had the misconception that they needed to round off each reading from their calculator and not only at the final answer.</p> <p>Secondly, rounding off to two decimals seemed to have the connotation for learners that in all cases the digit right after the comma should change and then they just had to make sure that there would be two digits after the comma judging from the next teaching episode (P3:118-129):</p> <p>Teacher looked around for learners who raised their hands, indicating that they had calculated the answer on their calculator then asks one particular girl learner to answer.</p> <p>Learner: 18,05738736</p> <p>Teacher: (Wrote the answer on board) 18,05738736. So when you round that to two decimal places it would be what?</p> <p>Learner: 18.1</p> <p>Teacher: Is it correct?</p> <p>Learners: (choir answering) No (while one learner clearly say) yes</p> <p>Teacher: Let's hear. When you round 18,057 to two decimal places, two decimal places... it would be what. (Teacher ask another learner) You...</p> <p>Learner: 18,10</p> | <p>It appears as if learners were confused about the angles and the sides, judging from the next teaching episode (P4:055 – P4:065):</p> <p>Teacher: Which side here is opposite angle theta. Thembane? Which side here is opposite angle theta?</p> <p>Learner: The adjacent side.</p> <p>Teacher: Why is that? No, I did not even speak about the adjacent side. I said the side opposite theta here. Yes..?</p> <p>Learner: Hypotenuse</p> <p>Teacher: Hypotenuse? Opposite...opposite..no..</p> <p>Learner: Angle</p> <p>Teacher: Angle? Is that angle? (16:00) Is this angle?</p> <p>Learner: Side...</p> <p>Teacher: side a ..Look at that... Can you see what is happening here? Can you see?</p> <p>Learners: (choir answering) Yes</p> |

| Conceptualizing | Language elements in Mathematics (San Diego County Office of Education, 2007) | Teacher A                                                                                                                                                                  | Teacher B |
|-----------------|-------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|
|                 |                                                                               | <p>Teacher: <i>18.10?</i></p> <p>Learners: (choir answering) <i>No</i></p> <p>Teacher: <i>You don't know how to round off...</i>( asking another learner) S... (P:128)</p> |           |

| Conceptualizing | Language elements in Mathematics (San Diego County Office of Education, 2007) | Teacher A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Teacher B                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|-----------------|-------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Code switching  | Teacher                                                                       | <p>Only once did Teacher A make use of code switching (aker), but only as a way of finding out if learners understood him.</p> <p><i>So now I said for us to calculate the trigonometry to calculate the unknown side or to calculate the unknown angle.ne In this case we've been calculating the unknown side aker we have calculate the unknown side I mean the unknown angles which is angle C and angle A.. Now lets see an example where we calculate the unknown side. (P:150)</i></p> | <p>Although Teacher B made use of code switching ("bagaitsu" and "aker") several times, it was just done as a habit of approaching the learners and hearing whether they understood and were following her.</p> <p><i>Right. Let's come back to this right-angled triangle. Now.. In a right-angled triangle remember we also have ...we saying in order for you to be able to apply trigonometry you need to have the right-angled triangle. Are we together bagaitsu? (P4:020)</i></p> <p>Teacher B also made use of the word "tsa":</p> <p><i>I mean everything that we do remember all these theorems tsa Mathematics they are link to one another you cannot separate them. (P:046)</i></p> |
|                 | Learners                                                                      | The learners listened most of the time to Teacher A and did not really communicate with each other.                                                                                                                                                                                                                                                                                                                                                                                           | Teacher B addressed the learners on their names. However, learners did not use code switching when responding to her questions, even when the teacher addressed them using "bagaitsu"                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| Mnemonic        | Teacher                                                                       | Teacher A did not use the mnemonic SOHCAHTOA or any other mnemonic for that matter.                                                                                                                                                                                                                                                                                                                                                                                                           | Teacher B used the mnemonic SOHCAHTOA to help learners remember the trigonometric ratios.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

## Synthesis

Although Teacher A started the lesson with defining trigonometry as concept; he referred once to the trigonometric ratios as the definition of trigonometry. From Table 6.4 it appears as if Teacher A mostly used correct mathematical language, but failed to give opportunities to his learners to use mathematical language. Teacher A also linked this new area (for the learners) in Mathematics to the use in real life, which is commendable as learners need always to understand what the functionality of Mathematics to real life is. This is said to enhance the understanding of the work.

When Teacher A started his lesson on the trigonometric functions, he introduced the trigonometric functions as Tan, Sin and Cos. But these concepts do not refer to trigonometric functions if they are not used in relation to an angle, whether this angle be known or unknown.

*Trig functions, we have Tan..we have Sine (learners say this together with teacher(choir answering), we also have Cos (learners say this together with teacher (choir answering). (P3:007)*

A few seconds later Teacher A referred to the functions in relation to an angle theta, but this time around for Teacher A it constituted the definitions of the trigonometric functions:

*Now this three functions ...in definition we say Tan is Tan of an angle let's say Tan of angle theta, Sine of and Cos of theta. (P:007)*

But he then carried on with writing the concepts **without** referring to them in relation to the angle theta:

*So now...Tan in definition we say Tan is...(P:007).*

Later on in the lesson Teacher A was trying to solve the following equation:

$$\tan \hat{C} = 0,326027397$$

*Now let me ask you a question: If I want to calculate angle C then it would be what? To get angle C...We don't want tan C we want angle C. How are we going to get it? (P:110)*

And then Teacher A divided by tan only on both sides of the equal to sign and got to

$$\hat{C} = \frac{1}{\tan(0,326027397)}:$$

*Now in other words we are going to remain with angle C that side. Now remember here if we divide by tan, it is going to give you what? (answering his own question without waiting for the learners): One over tan ne. (P:116)*

And by using exponent rules, Teacher A got to  $\tan^{-1}$ :

*Let's say tan theta... And if it is one over tan when we write it in negative exponents then we are going to... in exponential form then it's going to give you tan negative one...because of that one it will be negative one, ne.(P3:118)*

So Teacher A then got to  $\hat{C} = \tan^{-1} 0,326027397$

Which Teacher A and his learners correctly calculated on their calculators as  $\hat{C} = 18,05738736$  and then rounded it off to 18, 06°.

Teacher A's thinking, which he made visible, using the board and mathematical language for his learners of how to get to  $\hat{C} = \tan^{-1} 0,326027397$  is mathematically not sound as we cannot divide only by tan to get the angle alone. Tan cannot be separated from the angle as tan only make sense as a trig function when it is used in relation to the angle, in this case the angle C. The understanding that  $\tan \hat{C} = 0,326027397$ , which means that the opposite side of angle C was divided by the adjacent side of angle C in the triangle, was not mastered by Teacher A.

Similar to how Teacher A started the lesson, Teacher B also started this second lesson with a break-down of the concept "Trigonometry", but this time around there was no linkage to real life or the use of Trigonometry in real life by Teacher B at the start of the lesson. Linking these concepts to real life only happened towards the end of the lesson. The teacher went ahead and explained the term "Trigonometry" fairly well. In this second case, the teacher linked the Pythagoras theorem with the new term (trigonometry) mostly because of the hypotenuse term which also appears in trigonometric functions. Mathematical language was used by the teachers fairly well.

Code switching happened in this class with the use of “aker” and “bagaitsu” which were used continuously throughout the lesson. Individual learners had a little bit more opportunity to use mathematical language in this second lesson compared to the first lesson. Bagaitsu means “my good people” and is usually used to reflect a person’s familiarity, but at the same time also respect for the people who are addressed in this way. No mathematical terms and seemingly difficult terms were translated to the mother tongue of the learners. Aker has the same meaning as the English phrase “Do you agree” and therefore in the lesson, learners were responding to this word all the time by saying “Yes’. The question to be asked is whether learners always understood what they agreed to? Did they really think about this, or is it just a way of response to which they have grown used to? The word “tsa” that was used by Teacher B in her lesson, has the same meaning as “from” which has a connecting function in the context it is used. The teacher was connecting the theorems with Mathematics. She also explained in this same context that the theorems are linked. It is evident that Teacher B wanted her learners to clearly understand that the theorems belong to Mathematics and are interlinked. It is interesting that in this paragraph she used all three the Setswana words: “Aker”, “Bagaitsu” and “tsa”.

The next section aims to crystallize what happened in each classroom with the lessons.

### **Lesson one at the Red School**

**Presenter: Teacher A**

**Physical set-up:** Lesson was presented in a classroom with area approximately 80 square meters big. Learners were sitting in seven single rows of six learners. The total learners were 40. The table was situated in front of the room with a board ruler, board triangle, textbook and an A4 paper on top of it. Five posters were pasted neatly in a row next to each other at the back of the class on the wallboard with one poster hanging loosely onto the wall. These posters were Science posters and not Mathematics posters. In front of the class, next to the board there were another five posters pasted directly onto the wall. Three big windows on one pair of opposite sides of the room allowed enough light into the room, making the classroom a spacious setting used for the teaching and learning of Mathematics.

**Physical appearance:** Learners appeared very orderly and neat in their red school uniforms. Learners seemed not to be bothered by the video camera.

**Teacher movement:** The teacher used only the space between the board and the table as well as at the sides of the table. The teacher stopped each time at the end of the table and then returned back to the area behind the table. The teacher tended to speak to the learners while doing the sums on the board. Diagrams were drawn neatly on the board with the help of only the board ruler. Not once did the teacher move around the tables of the learners.

**Learner involvement:** Learners were sitting passively, yet listening attentively to the teacher's explanations, many of them with their hands folded on the table in front of them. Only some learners had calculators with which they calculated the angles and sides when the instruction came from the teacher to use them. Only three learners had pens in their hands and appeared to make notes while the teacher explained. The textbooks were closed, lying on the table, and remained in this way during the lesson until the teacher gave homework at the end of the lesson. This was the only time learners consulted their textbooks, just to mark the exercises which they had to do for homework.

**Instructional practice:** Teacher gave the lesson in the traditional way of talk and chalk. There was very little interaction between teacher and learners, except for the occasional questions to the class on which learners responded by chorus-answering. Although the teacher addressed particular learners a few times, he mostly addressed the class in general. A lot of the time the teacher talked with his back to the class and paid little attention to what learners were doing behind his back. The blackboard was used most of the time and the teacher did almost all the talking. This approach provides no opportunities for learners to engage in mathematical thinking and does not help to promote learners' development of problem solving. It is evident that a set of heuristics is needed to guide the teacher in helping his/her learners to engage in an inquiring approach and develop mathematical relations in their activities.

**Interaction between learners and teacher:** The teacher occasionally asked the learners to calculate the angle on their calculators. Otherwise he asked questions which were answered in a chorus by the learners.

**Interaction between learners:** No opportunities to interact with peers were given in this lesson. Interaction between learners happened very informally, almost sneakily.

**Questioning:** Questions asked by teacher were mostly the ones that needed completion of the sentence, which learners did indeed complete in a choir. Teacher A was also asking confirmation questions which were usually confirmed with the word “yes” by the learners. Occasionally the teacher was asking: Is this correct? Sometimes the teacher asked a “why question” on which learners did not respond. In these cases the teacher waited for an answer but when no-one replied, went back to the trigonometric ratios. He also asked them to raise their hands if they knew the answer, but then learners always went back to chorus answering. Once the teacher asked a specific learner to respond, but the learner also did not give a correct answer.

## **Lesson two at the Blue School**

**Presenter: Teacher B**

**Physical set-up:** This second lesson was presented in a fairly new science laboratory with long desks and about six learners sitting next to one another at one such a long desk. Learners were 32 in total. Two science posters hung on the wall in front of the class. The long teacher’s desk was situated in front of the class.

**Physical appearance:** Learners appeared very orderly and neat in their blue school uniforms and did not seem to be bothered by the video camera. Learners were listening attentively to the teacher and appeared interested in the lesson.

**Teacher movement:** Teacher B remained behind the table and for the most of the time explained the work on the board. Not once did the teacher leave the space in front to move between the learners.

**Learner involvement:** Learners sat passively at table and listened attentively. They occasionally answered the teacher when she required an answer from them.

**Instructional practice:** Instructional practice was characterized by the talk and chalk approach, just like in the first lesson. Teacher also used a diagram to refer to relevant sides.

**Interaction between learners and teacher:** One learner was asked to display his thinking on the board, which clearly shows some indication of metacognitive skills usage. Because the teacher used the names of the learner, pinpointing every time who should answer the question, much less chorus answering occurred.

**Interaction between learners:** Much similar to the first lesson, no opportunities to interact with peers were given in this lesson.

**Questioning:** Although there were still some chorus answering, teacher gave learners more time to think because she asked a question to a specific learner and then waited patiently for the answer. No why questions indicate again that learners did not get the opportunity to find out for themselves why or to reflect on what they knew and did not know. Wrong answers were responded to with a loud “no” by the teacher, instead of “Why do you say that?”

Next the performance in the trigonometric task was analysed. This analysis also ensured gauging of the adequacy of their knowledge of trigonometric functions. Simultaneously with the analysis of the assessment task done by the teachers, the analysis of the assessment task done by the learners was completed. In both cases the task provided a good indicator of their (teachers and learners) interpretation of mathematical language that was used in the task. All of the above measurement tools were used in establishing the profiles of the participating teachers.

### **6.2.3 The assessment task**

Teachers A and B completed the assessment task (Addendum D3) during the first workshop, in which a trigonometry sum had to be done with some questions on his/her thinking while completing the task. The sum required the drawing of a diagram. The important feature of this trigonometry assessment task is that the drawing gave away the thinking of the person who made the drawing.

#### **6.2.3.1 Metacognitive skills used by Teacher A and B in the assessment task**

It was imperative to also focus on how the teachers used their metacognitive skills when they had to solve a mathematical problem themselves. The following table (Table 6.5) reports on how Teacher A and B used their metacognitive skills in the assessment task.

**Table 6.5: Metacognitive skills usage by Teacher A and B in the assessment task**

| Components of Metacognition | Assessment task Teacher A (P7)                                                                                                                                                                                                                                                                                                                                                                          | Assessment task Teacher B (P8)                                                                                                                                                                                                                                                                                                                                                                                        |
|-----------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Prediction                  | Teacher A predicted that he would be able to do the question correctly, but ended up doing it totally wrong.                                                                                                                                                                                                                                                                                            | Teacher B was absolutely sure that she would do the question correctly, but as was the case with Teacher A, did the task wrong.                                                                                                                                                                                                                                                                                       |
| Planning                    | It seems that Teacher A had difficulty in planning his thinking. He claimed that he first drew the diagram before he read the information well. He then highlighted important information but the question does not show any markings on the form which he was completing (see primary document 7 in the hermeneutic document in ATLAS.Ti). Lastly he extracted the information necessary for the task. | According to Teacher B, the planning of solving the task was as follows: Firstly she read the assignment well, then she extracted the information necessary for the task. She claimed that she next highlighted important concepts and terminology, but nothing was highlighted in the question of the task given to her (See primary document 8 in the hermeneutic unit in ATLAS.Ti). Lastly she drew the diagram.   |
| Awareness                   | Teacher A seemed not to be aware of his thinking as he answered negatively on questions whether he tried to remember previous experiences that he could link with the task he needed to do, although he admitted to thinking about what he already knows.                                                                                                                                               | Teacher B asserted that she thought about what she already knew, but failed to try to remember if she had ever done a problem like that before, though she admitted that she did think about something she had done that had been helpful. This is contradictory because she did not try to remember if she had done a problem like that before and at the same time she was thinking about something she did before. |
| Monitoring                  | In describing the strategies he was using to solve the task, Teacher A explained that he read the information first then he analysed it. Next he repeated the whole process, making sure that he understood what was asked. He then drew the diagram before looking for information to solve the task. He then answered the question.                                                                   | In her attempt to describe the strategies she used to solve the task, Teacher B said that she first read the question, and then she analysed it. Interestingly she “identified” next the most important and guiding words (again there was no trace of this step in her task as the information was never “marked” or highlighted). She then drew the diagram and solved the task.                                    |
| Regulation                  | Teacher A claimed that he did make a plan, and thought about what he would do next, while admitting to not changing the way he had worked in any stage of solving the task.                                                                                                                                                                                                                             | In terms of regulating her thinking, Teacher B claimed that she did make a plan, and thought about she would do next, but did not change the way she was working at any stage while doing the task.                                                                                                                                                                                                                   |
| Evaluation                  | Teacher A evaluated his attempt on answering the task correctly, although it was done incorrectly.                                                                                                                                                                                                                                                                                                      | Teacher B was absolutely sure that she did the sum correctly, although it was wrongly done.                                                                                                                                                                                                                                                                                                                           |

## Discussion

From the table (Table 6.5) it is evident that Teacher A was contradicting himself in the ordering of the planning steps: He marked “drawing a diagram” as his first step while when asked to rethink his strategy; he claimed that he did read the question repeatedly before he drew the diagram. Teacher A’s difficulty in reflecting upon his own problem solving raises some serious concerns and widens the gap between the expected level of Teacher A’s problem solving and the low level of his metacognitive performance (Kozulin, 2005) which is also indicative of a low self-directed learning tendency. Teacher A did not complete the question of why learners can’t solve such tasks (See Primary document 7 in the Hermeneutic Unit). Teacher B made a diagram indicating only one building. The drawing shows an “extended” ladder to cater for the two different angles (30 and 40 degrees) indicated in the sum (See Primary document 8 in the Hermeneutic Unit). She applied the correct trigonometric ratio for the first part of the sum and calculated the answer correctly, although she approximated this answer pre-maturely as she was only supposed to approximate the final answer.

### 6.2.3.2 Mathematical language used by Teacher A and B in the assessment task

**Mathematical language used by Teacher A:** In terms of mathematical language usage, the diagram drawn by Teacher A illustrates wrong interpretation of the information in the task. This may bear evidence of not understanding the mathematical language, but could also indicate a lack of mathematical content knowledge.

**Mathematical language by Teacher B:** As was stated before, one diagram instead of two diagrams was drawn, when the question explicitly stated that one building was on one side of the street and another building was on the other side of the street. This incorrect drawing indicates wrong interpretation which might be due to a lack of understanding of the mathematical language in the phrasing of the question or might be due to a lack of content knowledge.

## Synthesis

Both Teachers A and B predicted that they would do the task correctly, and were even absolutely sure that they would do the task correctly, but failed to get it correct which concurs with what Martinez (2006) asserts that the illusion often exists amongst individuals that they know (meta-memory) and understand (meta-comprehension) a piece of work while this is not actually the case. The ability to say beforehand what one can do is to have control over self-

knowledge which is critical for metacognition. The most recent thoughts about metacognition acknowledge metacognition as one of the most relevant predictors of accomplishing complex problems (Roebers, et al., 2012; Van der Stel and Veenman, 2010). Furthermore, the observations made above, confirms findings by Kozulin (2005) that the teachers' previous educational experiences, both in school and also during tertiary training, failed to prepare some of them for cognitive problem solving. Solving the problem incorrectly has far-reaching implications for the quality of teaching in this particular school if one keeps in mind that teachers are expected to be principally skilled in analysing and explaining the problem solving process of their learners (Kozulin, 2005). Kozulin's study (2005) employed quantitative methods of data collection and analysis. This inquiry employed qualitative methods and it also indicated that cognitive tasks, which are usually intended for the learners, pose a certain difficulty for their teachers who are actually expected to teach these tasks in the classroom (Kozulin, 2005).

#### **6.2.4 Workshop on 12 August 2012**

This one and only workshop in the study was held to ensure that all participants were on the same page as far as metacognition, mathematical language and trigonometric functions, the key concepts in the study, were concerned. As teachers are not usually knowledgeable about research, the adapted lesson study and design-based research approach as the methodology used in this study were also discussed. It was also during this workshop that the outcomes of the lesson were identified while the study lesson was designed.

In the next section lesson one and two were analysed in order to inform the prototype of the hypothetical teaching and learning trajectory (HTLT) in addition to the initial prototype (§ 3.9.6) from the literature review.

### **6.3 ANALYSIS OF THE DATA COLLECTED IN THE FIRST PHASE**

Analysing the data gathered in this first phase gives effect mainly to the first two sub-questions *viz*:

- (i) How do Mathematics teachers apply metacognitive skills and mathematical language in the teaching of trigonometric functions?
- (ii) Which challenges do Mathematics teachers face in the teaching of trigonometric functions?

As design-based research envisages that the analysis of one cycle informs the design of consecutive cycles, the data gathered during this first phase are subsequently analysed. As mentioned earlier, the aim of the data collection in this first frame was twofold: Firstly it served to inform mainly the development of an metacognitive performance profile for each of the two grade 10 teachers, Teacher A and Teacher B. Secondly the analysis of the first phase served to inform the design of the hypothetical teaching and learning trajectory. Table 6.7 and Figures 6.5 and 6.6 show the main codes, sub-categories, categories used and themes which crystallized from the analysis of the data in this first phase, using ATLAS.Ti.

**Table 6.7: Data analysis in the first phase**

| Codes                                                                                                                                              | Sub-categories                                                                                | Categories                                                                    | Themes                                                                                                      |
|----------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|
| Declarative knowledge, procedural knowledge, conditional knowledge, knowledge of self, knowledge of learners, knowledge of others                  | Metacognitive knowledge                                                                       | Category one: Indicators of metacognitive performance for Teacher A           | Theme one: The metacognitive performance profile of Teacher A                                               |
| Task, strategy, regulating, monitoring, reflection-on-action, reflection-in-action meta-memory, meta-comprehension, evaluation, assessing          | Metacognitive skills                                                                          | Category two: Indicators of metacognitive performance for Teacher B           |                                                                                                             |
| Cognitive awareness, revisiting, revising, critical thinking, beliefs, prediction,                                                                 | Metacognitive experiences                                                                     |                                                                               |                                                                                                             |
| Clarifying concepts, symbolical language, academic language, content language, mnemonic, defining trigonometry as concept                          | Conceptualizing                                                                               | Category three: The use of mathematical language by Teacher A                 | Theme two: The metacognitive performance profile of Teacher B                                               |
| Teacher mistake, learner mistake, incorrect solution.                                                                                              | Misconceptions                                                                                | Category four: The use of mathematical language by Teacher B                  |                                                                                                             |
| Code switching                                                                                                                                     | Code switching                                                                                |                                                                               |                                                                                                             |
| Definition, concrete, abstract, language, challenge,                                                                                               | <i>From concrete to abstract.</i>                                                             | Category five: Challenges in the teaching-learning of trigonometric functions | Theme three: The hypothetical teaching and learning trajectory for the teaching of trigonometric functions. |
| Particular, general, practice,                                                                                                                     | From particular to general                                                                    |                                                                               |                                                                                                             |
| Content language, Defying trigonometry as concept, academic language, symbolic language, mnemonic, explaining in another way, mathematical jargon. | <i>Context of any new concept before technicalities, intricacies and mathematical jargon.</i> |                                                                               |                                                                                                             |
| Individual response, chorus answering, desire to know, critical thinking, interaction, incorrect solution, reading of from board.                  | <i>Favourable reactions from the students.</i>                                                |                                                                               |                                                                                                             |

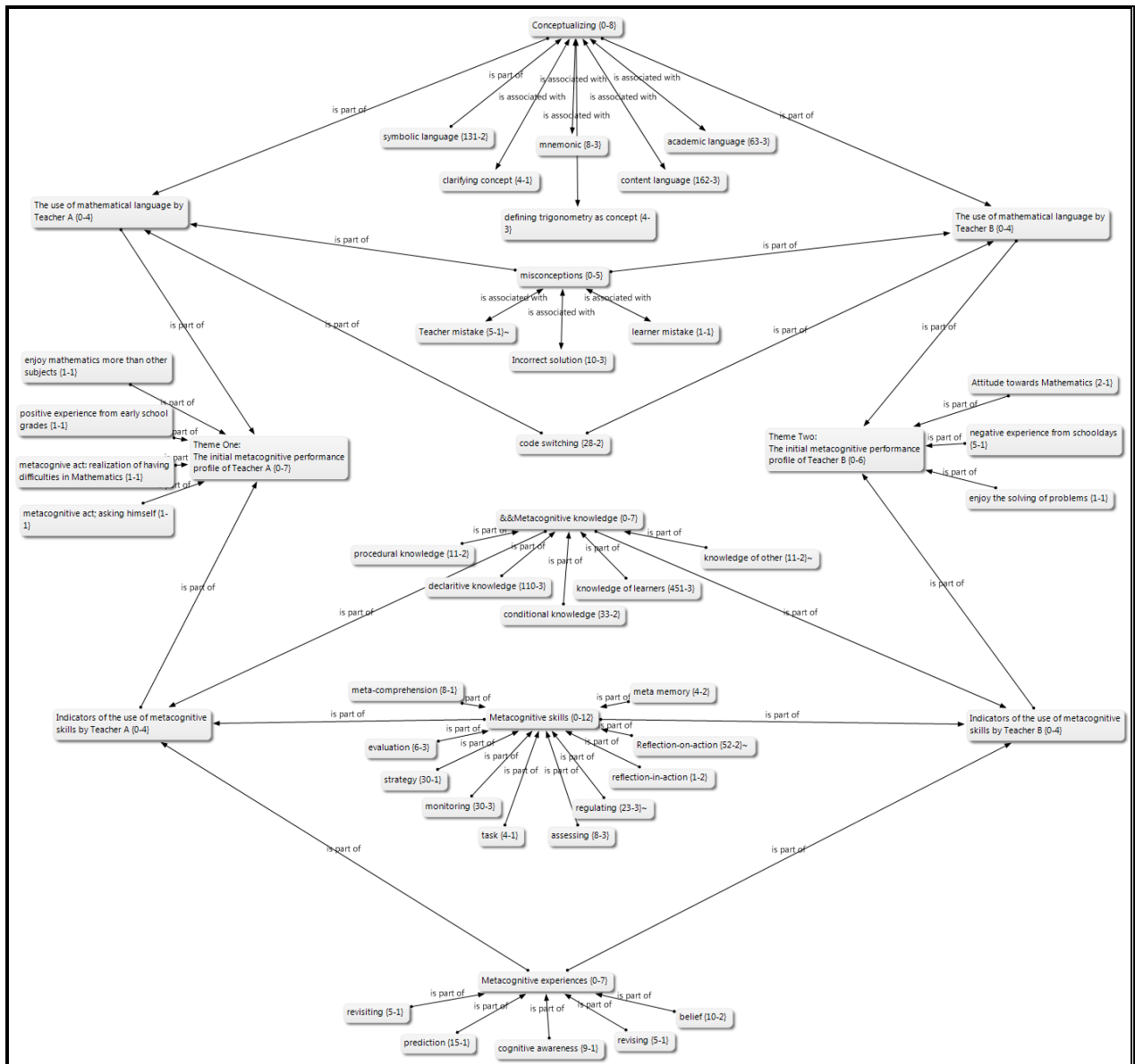


**Figure 6.5: The metacognitive performance profiles of Teacher A and B**

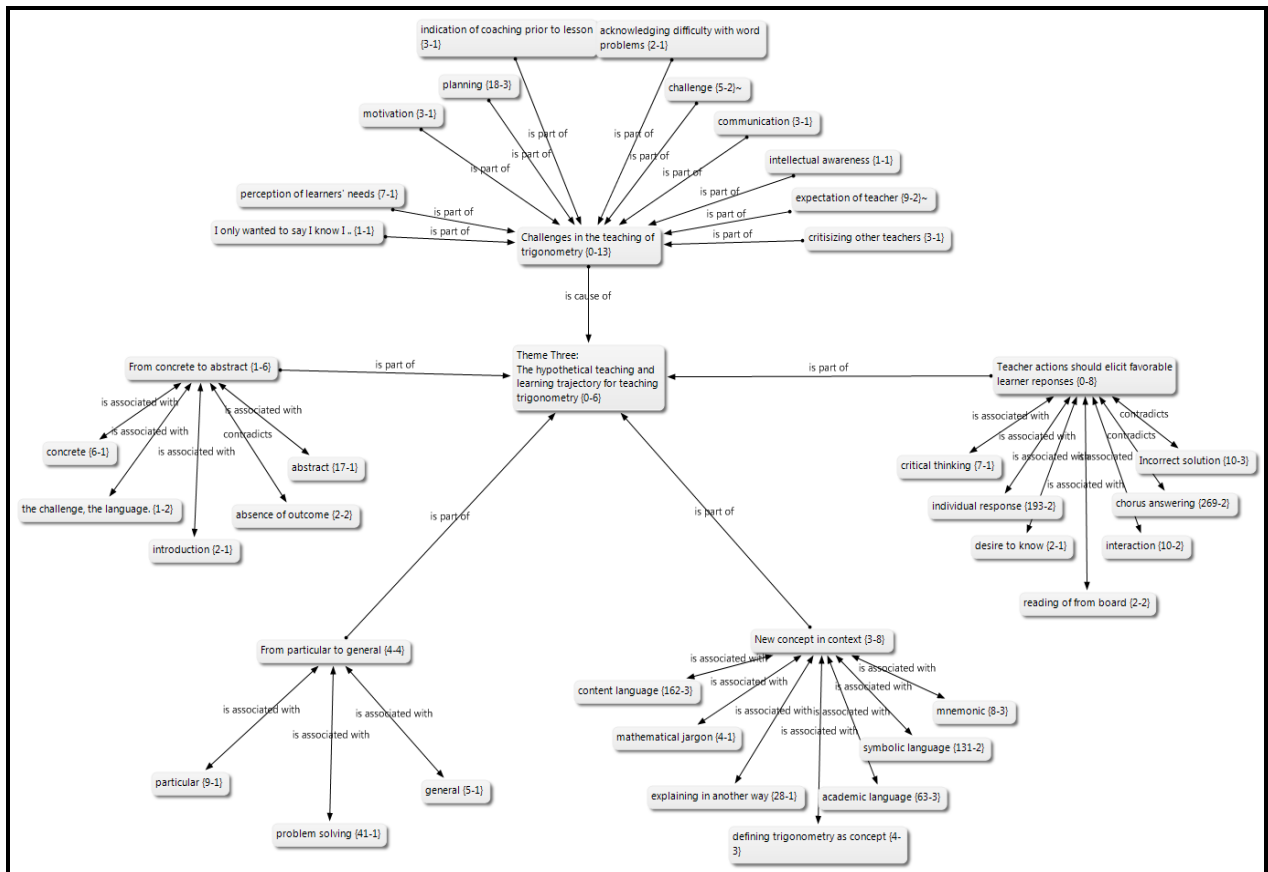


**Figure 6.6: The hypothetical Teaching and Learning Trajectory for Trigonometric functions**

The analysis of the first three data collection instruments, viz. the individual interviews, the lesson observations and the assessment tasks, informed the development and crystallization of a metacognitive performance profile for both Teacher A and Teacher B. The nature of the study required the establishment of the metacognitive performance profile of the participating teachers in order to gauge whether metacognitive skills and mathematical language could catalyze a shift for the better in the way the teachers have been teaching. Figure 6.4 show the network heuristic of how themes formed from the codes, into sub-categories, into categories.

The next section aims to provide a profile for each of the teachers using Table 2.4 (§2.7.2) in which the work of different scholars (Lin et al, 2005: 253, Shofield, 2012: 59, Wilson and Bay, 2010; 269) has been adapted to form a distinction between the metacognitive teacher and the non-metacognitive teacher.

## **6.2.5 Focus group discussions**

### **6.2.5.1 Cycle one of the adapted lesson study: First Focus group discussion at the Red School**

This first focus group discussion happened directly after the workshop that was held at the Red School and One of the lecturers from the Mathematics subject group at the university where the researcher is based, agreed to be part of this first focus group discussion. Only three of the six teachers attended due to other commitments: One teacher had to go to District Office while the other two attended workshops. It must be mentioned that although teachers were in general enthusiastic and positive about the study that the study was not their first priority. They had other commitments that were either work related or related to their families.

The researcher played a video recording of the lesson that was presented by Teacher A to his grade 10 learners. Only two questions were asked at this first focus group discussion (See Addendum D4):

1. What is your general impression of the lesson?
2. Comment on the way the trigonometric functions had been introduced within the lesson.

Three participating teachers as well as one of the lecturers, who were present at this first focus group discussion, watched the video. Although the video recording of this first lesson elicited much discussion, teachers were a bit reluctant to really let go as this was their first experience of reflecting on another teacher's lesson in the study. Teacher A, who actually was the teacher that presented the lesson, opened the discussion by interestingly stressing the importance of planning when a teacher is presenting a lesson. He elaborated on this by acknowledging that a teacher might sometimes go to a lesson thinking that learners have prior knowledge only to find out that they do not. The rest of the teachers then followed on this by sharing their impressions of this lesson. Although this was a reflection on lesson one, towards the end of the discussion some suggestions on the consecutive lesson were also verbalized. The general impression that can be reported for this first focus group discussion is that it went reasonably well, given the

sensitive nature in terms of the adapted lesson study and the inputs that were given, was neither judgmental nor evaluative.

#### **6.2.5.2 Cycle two of the adapted lesson study: Focus group discussion two at the University**

This second focus group discussion took place at the university and was attended by four teachers, two lecturers the study leader and the researcher. The study leader acted also as the moderator of this discussion. The other two teachers had other commitments. In this second focus group discussion the video recordings of the first two lessons were shown. This second focus group discussion took place at the university and was attended by four teachers, two lecturers the study leader and the researcher. The study leader acted also as the moderator of this discussion. The other two teachers had other commitments. In this second focus group discussion the video recordings of the first two lessons were shown. Thereafter the questions for the focus group discussion (see Addendum D4) were presented for discussion. These questions were set to achieve two aims: Firstly it should reflect on the past lesson/s and secondly it should inform the next lesson. Only the first three questions were used for analysis in this cycle which aimed to discuss the thinking, engagement and behavior of the learners in the previous two lessons.

1. Compare this lesson to the previous two lessons in terms of:
  - a. The behavior of the learners
  - b. The metacognitive skills used by the teacher.
2. Comment on the visibility of the thinking by the learners.
3. Comment on the mathematical language usage by
  - a. The learners and
  - b. The teacher

It was done in this way in order to identify some patterns of metacognitive behavior and mathematical language usage, as well as to establish what challenges could be observed in the teaching of trigonometric functions within the two lessons. In a way these focus group discussions can be seen as the analysis of the lessons which were presented by the teachers by the research team which consisted of the teachers themselves, the lecturers, the study leader

and the researcher. This second focus group discussion was characterized by a true sense of sharing expertise and valuable experiences. Table 6.6 shows what transpired in the first and second focus group discussion.

**Table 6.6: Focus group discussions one and two**

| Categories                                                                                    | Focus group one                                                                                                                                                                                                                                                                                                                                                                                     | Focus group two                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-----------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>From concrete to abstract.</i>                                                             | <p>Reflections by the teacher who presented the first lesson acknowledged the importance of planning:</p> <p><i>I realized something...it is always important when you go to a class, you must be prepared. (P11:16)</i></p>                                                                                                                                                                        | <p>Learners should have the opportunity to show what they know in the introduction:</p> <p><i>Ok, to describe these classes, would be difficult, unless we set up a lesson where, just after introduction, the learners have the opportunity, to show their learning, just with the introduction, to see how much they know about application, practical application, even before the teacher can explain more about Mathematics, if mean that will describe the kind of learners we want. (P5:041)</i></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| From particular to general                                                                    | <p>Teacher A, the presenter of the first lesson, also admitted to assuming that learners have the required prior knowledge when going to the classroom:</p> <p><i>My thinking is... sometimes when you present a lesson you might have that knowledge or thinking that the learners have that prior knowledge on what you going to treat only to find whereas it is not like that. (P11:16)</i></p> | <p>Teachers do not know about going from the particular to the general. They only considered the time issue:</p> <p><i>There is also the issue of time. We want to cover everything in a very short period of time and we want to finish. When the learner, we have three different learners there. The others are slow, the others are fast learners. You have such type of learners. And you would even find, learners aren't interested in the problem. So, but in the end you want to reach that learner also. So, the issue is first impressions last. So it seems to me that when we are working through the lesson, a lot of work is to be done, so that, at least in the end, even though it is not 100%, but almost all of them should have solved the problems. You don't want to miss time, you want to cover time. You also want to, that at the end of the lesson, everything that you brought to the learners, they should reach it. So quite a number things have to be planned beforehand." (P5:099)</i></p> |
| <i>Context of any new concept before technicalities, intricacies and mathematical jargon.</i> | <p>Teacher A jumped straight to the technicalities and mathematical jargon with the teaching of trigonometric functions:</p> <p><i>you see... because if you heard him when he started explaining and introducing the lesson that it's all about triangles we going to do trigonometric ratios and then the opposite and adjacent that on itself... (P11:31)</i></p>                                | <p>The teacher thinks the learners know nothing and as a result fails to address the context of a new concept before the mathematical jargon:</p> <p><i>There's one thing that I've seen in almost all the teachers. Normally when we get to class, whatever lesson that we are going to present to them, we think that the learners know nothing about it. We think it's new. So that's why when we get there we have to introduce a lesson to them, let's say trig, trigonometry,</i></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |

| Categories                                     | Focus group one                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Focus group two                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | <i>they have to think about it, come up with concepts. That will also help. The teacher will be greatly helped because we will know who understands more about the lesson or how to study the same problem. But normally when we start the lesson we would introduce the topic. The other thing that I think which is very important is to give lesson outcomes when we start, and point them. That at the end of the lesson we can check this and this and this. And then you can work towards that along with the learners. (P5:095)</i> |
| <i>Favourable reactions from the students.</i> | <p>Reactions from the learners were mostly “chorus-answering”. In this first lesson learners answered as a class about 65 times:</p> <p><i>And sort of in a group also they answered. (P5:016)</i></p> <p>Teacher “opened up” the minds of the learners:</p> <p><i>The beautiful thing that I also saw from the lesson... when he introduced the learners ...let's say maybe we have tangent theta, cosine theta and sine theta and we placed it in the triangle and he showed them that if the theta is positioned here, the opposite will be here, the adjacent will be there and the hypotenuse will be there and then lets now change it and place it there on top...that's the beautiful thing...on both...most of the teachers...they just concentrate on the first...maybe just explain it and then from there give an activity... He placed on the other side, positioned it on a different side and then it opens the learner's mind that's very interesting. (P11:53)</i></p> | <p>Chorus answering was less in the second lesson with only 36 times. However, the research team is of the opinion that chorus answering is not a favourable reaction form the learners and that it is misleading the teacher :</p> <p><i>That's why I say group answer is misleading...(P5:230)</i></p>                                                                                                                                                                                                                                   |

## Discussion

It appears from Table 6.6 as if the four principles that should guide the introductory teaching of trigonometric functions, were to a large extent not adhered to in the first two lessons.

One member of the research team was of the opinion that the teacher was much too active in the first lesson:

*What I saw...was...everything was explained very well ...but the teacher was talking too much...it was the teacher more the students were just a little bit...so in other words most of the lesson the teacher was working hard.....I thought maybe he could have given them chance to do...after explaining one part, give them a chance to do a little bit for themselves on their papers because they did not even have anything to write on everything was done on the chalkboard they never had anything that they have done on the chalkboard...that's what I saw.. (P5:19)*

### 6.3.1 Theme one:

#### **The metacognitive performance profile of Teacher A**

Teacher A can be described, in terms of the way he displayed metacognitive skills usage in the individual interview, lesson observation and assessment task as follows: Teacher A has a positive attitude towards Mathematics and is motivated by his desire to help learners pass Mathematics. Teacher A does not really plan his lesson using a lesson plan, and his planning to solve the assessment task was contradicting his actual problem solving. Teacher A tends to only set taken-for-granted goals that are aimed to induce almost the same activity from everybody and displays no real understanding of metacognition in the assessment task. Teacher A also tends to teach strategies using the talk and chalk approach which aims to control learners' behaviour and does not engage them in activities that encourage construction of own knowledge. Teacher A did not try to identify what learners know and do not know at the start of the lesson. He tended to tell learners what to do and assessed their understanding by giving them homework only at the end of the lesson. Teacher A has some serious misconceptions, but in most of the cases he corrected himself and was also talking to himself once. This shows some traces of metacognitive behaviour. Teacher A did not succeed in solving the assessment task correctly and could not detect absence of the important information very well because he failed to develop the habits of gathering more information so that he could determine what strategies and solutions to apply. In terms of Teacher A's mathematical language usage, he could articulate fairly well the material he

intended to teach using mathematical language, but should think more frequently about what he is saying.

### **6.3.2 Theme two:**

#### **The metacognitive performance profile of Teacher B**

Teacher B's love for numbers and her self-confessed inclination for numbers keeps her motivated to keep returning to the Mathematics classroom. She has negative experiences of how she was taught Mathematics at school. Teacher B tends to plan her lessons but failed to articulate the goals of her lesson. She, however, succeeded in linking the prior knowledge with the new knowledge. Teacher B does have a fairly good understanding of metacognition from the way she completed her assessment task, but failed to give the correct solution to the mathematical problem. Teacher B made use of the talk and chalk approach and learners did not get any opportunity to construct their own knowledge. Teacher B monitored or regulated the thinking of the learner to a reasonable extent by asking individual learners to respond, and also adjusted her teaching by using another way to show the learners how to identify the opposite side using paper on the board and asking one learner to help her with this. Teacher B made use of code switching and this encouraged more interaction from the learners. Only one misconception could be identified from the lesson presented by Teacher B.

### **6.3.3 Theme three:**

#### **The Hypothetical teaching and learning trajectory**

According to Davis and Simmt (2003), teachers are advised to concentrate on the construction of mathematical ideas by sharing and discussing their ideas within a community in a collaborative engagement with co-teachers, Mathematics educators and mathematicians. This engagement was made possible in this inquiry through the focus group discussions in which discussions focused on the extent to which the four principles in the theoretical prototype were adhered to. Not only does the next section address each of these principles, but it also attempts to show how the mathematical language and metacognitive skills played out in the two lessons. The prototype for the hypothetical teaching and learning trajectory (Figure 6.7) is given here once again as springboard for the analysis of how the design principles (Quinlan, 2004:20) were addressed:

1. *Go from concrete to abstract. Avoid starting with definitions.*
2. *Go from particular to general.*

3. *Immerse students in the context of any new concept before explicating its technicalities and intricacies and mathematical jargon.*
4. *The lesson introducing a new concept should be one that results in favourable reactions from the students.*

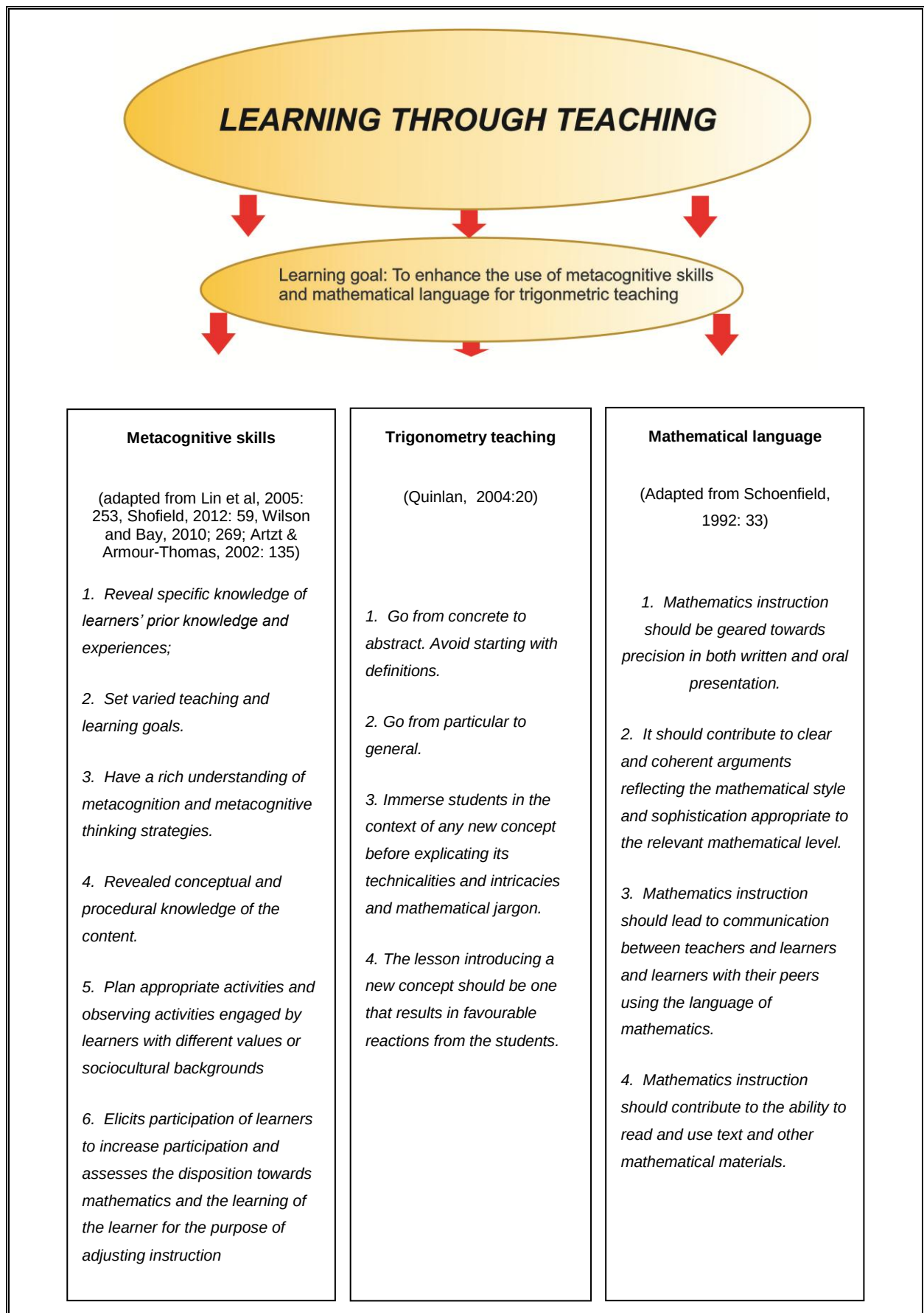


**Figure 6.7: Prototype for the hypothetical teaching and learning trajectory**

1. *Go from concrete to abstract. Avoid starting with definitions (Quinlan, 2004:20).* This did not happen at all during lesson one. In fact, Teacher A indeed started with the definitions. Teacher B in the second lesson also did not go from the concrete to the abstract, although she did attempt to connect the prior knowledge of the learners with the new work. Both Teachers A and B also failed to mention the outcomes. The general idea from the discussions in both the first and the second focus group discussions was that the absence of mentioning what the teacher aims with the lesson should also be part of this first principle. This first principle then adjusted to: ***“Go from the concrete to the abstract, while explicitly mentioning the outcome/s of the lesson. Avoid starting with definitions”*** after the first two cycles of lesson observations and focus group discussions.
2. *Go from particular to general (Quinlan, 2004:20).* Teacher A also did this in reverse: He went from general to particular. He was doing the general definitions before he went to particular cases of application of the trigonometric functions. Teacher B adhered to this second principle only to some extent: She started with particular examples of triangles and then went to the general definitions later on. This principle was not adjusted in any way.
3. *Immerse students in the context of any new concept before explicating its technicalities and intricacies and mathematical jargon (Quinlan, 2004:20).* New concepts were addressed with their technicalities and mathematical jargon without a real context. Except for some uses of trigonometry in real life which teacher A mentioned at the start of the lesson and Teacher B mentioned later on in her lesson, neither applied the problems to any real life context. Regarding this principle, the opinion of the research team was that it was very important and therefore it was not adjusted in any way.
4. *The lesson introducing a new concept should be one that results in favourable reactions from the students (Quinlan, 2004:20).* From observing the lesson, the learners did not appear very excited and motivated during the lesson which can be seen as non-favourable reactions from the learners. This principle elicited much discussion which centred on chorus answering as a not so favourable reaction from the learners. It was decided that the use of metacognitive skills could prevent chorus answering to a large extent. The question that also arises is to what extent chorus answering can be seen as group metacognition. This raises issues of the usefulness of chorus answering as indeed a favourable reaction from the learners. More about this phenomenon of chorus answering is provided in the next phase. However, this principle differs from the other three in that it focuses more on the learners and not explicitly on the teacher, therefore it was adjusted to: ***“The actions of the teacher***

*should elicit favourable reactions from the learners when a new concept is introduced".*

**Metacognitive skills usage:** The particular case of the first example indicated that Teacher A did not use metacognitive skills: When Teacher A asked for the trigonometric ratio and the learner answered "tan", he responded with a short "no" answer which actually misguided the learners as can be viewed from the next teaching episode in lesson one: (P3:162-P3:164)

Teacher: *Side LB then then there we have small letter h. Now which trig function can you use to calculate side LB. Have a look on your side LB? Which trig function can you use to calculate that side? In other words we have 40 degrees we have 90 degrees. Which one will we be able to use to calculate side h?*

Learner: *Tan?*

Teacher: *Tan...no...*

This is incorrect information...Tan theta was the correct answer as the unknown side could have been calculated with the trigonometric ratio, Tan theta, as Teacher A realized only later in the lesson because both the angle and the opposite side would have been known with only the adjacent side to be unknown. No metacognition was present for the pre-active phase of the lesson, as these answers should have been worked out already with all possible answers. This would have made Teacher A aware that Tan theta could also have been used.

Another example showing the absence of the use of metacognitive skills by Teacher A followed a few minutes later when he asked for a side instead of the trigonometric ratio that needed to be used to solve the problem in the following teaching episode: (P3:165-P3:170)

Teacher: *I said if you know an answer raise up your hand...Which side will be correct?*

Learner: *Cos theta*

Teacher: *Cos theta ok let's see...you say Cos theta. If you say cos then what? Cos of which angle?*

Learner: *adjacent over hypotenuse*

Teacher: *Which angle?*

Learner: *angle B*

As can be seen from the previous teaching episode (P3:165-P3:170), the learners also did not display the use of metacognitive skills either, because when the teacher asked for an angle the learner gave a ratio.

Teacher B on the other hand displayed to a reasonable extent the use of metacognitive skills. She altered her way of explaining when she realized that the learners do not understand what the opposite side means, during the following teaching episode: (P4:055-P4:066)

Teacher: *Which side here is opposite angle theta. Thembane? Which side here is opposite angle theta.*

Learner: *The adjacent side.*

Teacher: *Why is that? No I did not even speak about the adjacent side. I said the side opposite theta here.. Yes..?*

Learner: *Hypotenuse*

Teacher: *Hypotenuse? Opposite...opposite..no..*

Learner: *Angle*

Teacher: *Angle? Is that angle? (16:00) Is this angle?...*

Learner: *Side...*

Teacher: *side a ..Look at that... Can you see what is happening here? Can you see?*

Learners: (choir answering) Yes

Teacher: (Teacher is using A4 paper covering the two sides so that only the opposite side is visible making sure learners really see the opposite side)*The side which is... Can you see? The side Come here* (call one learner to the board to help holding papers to the diagram closing the other two sides to only open up the opposite side) *.I want to demonstrate something here so that we don't forget what do we mean when we talk about opposite. Can you see what is happening here?*

Learners: (choir answering) Yes...

**The use of mathematical language:** Both teacher A and B were using mostly declarative knowledge which is actually the content and academic language. Teacher A made use of a bit more of symbolic language also as he was solving problems on the board. Looking back to the mathematical language usage of Teacher A when he explained to his learners how to find the corresponding angle when we know the value of the trigonometric function, the following can be concluded: Actually the idea that was "lost" is that of the inverse of the tangent function, although the context within which the teacher and learners were working on the "reverse" process is important to take into consideration as well: With the inverse not yet been introduced at this stage, it was a situation of: "If we know the value of  $\tan A$ , what is the corresponding angle(s)?" This was the question that should have been posed by Teacher A to his learners. The correct use of mathematical language in this case is once again shown to be of critical importance to avoid misconceptions at this crucial stage when trigonometric functions are introduced in grade 10. It is crucial because this was the basis on which learners needed to build on for further calculations with trigonometric functions in grades 11 and 12 and even for post school mathematics education. It was evident that the concept of "Tan theta" was not fully understood by Teacher A. However, discussing it with Teacher A afterwards and also during the focus group discussion clarify this concept for Teacher A. It can be concluded that the correct mathematical language and the effective use of metacognitive skills within the group could help Teacher A to conceptualize the concept "Tan theta". In Teacher B's case, she could clarify her learners's understanding of "opposite" side by the correct mathematical language and also metacognitive skills. She then succeeded in conceptualizing the term "opposite" side for her learners.

**Rounding off:** The correct way of rounding of was excessively focused on by Teacher A. Teacher A was complimented in the focus group discussion by the group for doing that. However, working with nine decimal digits in itself indicates some misplaced use of mathematics/mathematical language, Any digit after rounding to two decimals does not really have any significance - but Teacher A tend to think that one has to use "all digits shown on the calculator or computer screen just because "it is mathematics" .In this case it was more critical to let learners understand the significance of the angle as reference angle to make sense of the trigonometric functions.

### **6.3.3.1 The challenges in teaching trigonometric functions**

The following challenges in the teaching of trigonometry could be derived from the analysis of the four data instruments (interviews, lesson observations, assessment tasks and the focus group discussions) which informed this first phase of the inquiry.

1. The negative attitude of learners towards mathematics.
2. The negative attitude of some mathematics educators towards mathematics.
3. Lack of content knowledge of some mathematics educators.
4. The pride of some mathematics teachers not to ask help in cases where they do not understand the content.
5. More definitions in Trigonometry.
6. Learners are failing to see the big picture.
7. Poor learner performance in mathematics.
8. Learners do not practise mathematics at home.

These challenges are also addressed in the next chapter in which the implementation and the analysis of the Hypothetical teaching and learning trajectory are reported and analyzed.

In summary of the analysis which informs the design principles, the following principles were used in consecutive phases (The added parts are bold to show the adjustments):

1. Go from the concrete to the abstract, **while explicitly mentioning the outcome/s of the lesson.** Avoid starting with definitions
2. Go from particular to general.
3. Immerse students in the context of any new concept before explicating its technicalities and intricacies and mathematical jargon.
4. **The actions of the teacher should elicit favourable reactions from the learners when a new concept is introduced** (Adjusted from Quinlan, 2004:20).

These adjusted principles were used in consecutive cycles of the adapted lesson study and subsequently in the remaining two phases of this design-based inquiry in order to address the research question. The next chapter focuses on the data reporting and analysis of the second phase.