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Explore learners' understanding of mathematics concepts using concept maps

By

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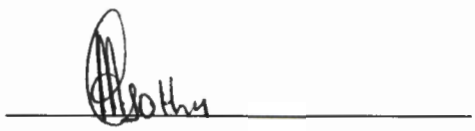
DECLARATION

I, **TUMISANG FREDRICK MOKGOTHU**, hereby declare that this dissertation for the Degree of Masters of Education (M.Ed) in the North West University hereby submitted, has not previously been submitted by me for any degree at this or any other University, that it is my work in design and execution and all the material contained in this report has been acknowledged.



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ABSTRACT

Understanding in mathematics learning has proved to be complex and challenging for many learners in rural schools. Understanding is so often defined as links and bonds, yet teachers do not teach bonds but are responsible for ensuring that understanding is occasioned. The study reports the usefulness of concept maps in exploring learners' understanding of mathematics concepts.

The aim of the study was to explore learners' understanding of mathematics and to determine causes that may lead to poor performance. The findings were significant in identifying lack of vocabulary for expressing mathematics concepts as one of the causes of poor understanding. Non-understanding was found to be at the initial stage of the continuum in which understanding exists. Understanding started to occur through constant re-organizations of internal schemes

Concept maps were qualitatively used to collect data. A multiple case study method involving Grade 10 learners in Dr Ruth Segomotsi Mompati District was used. Reflective interviews and observations were used as validation measures. The holistic approach of data presentation and analysis was adopted for use in the study.

Findings to elaborate on the understanding constructed by learners are expressed in a continuum consisting of the following stages: Non-understanding, incoherence and illogic; and coherence and logical reasoning. The study concludes that a learner understands a thing if he or she can do the following: identify its types, uses and components; and can provide relevant examples.

Concept mapping is recommended for use in classroom practices both as a tool of assessment and pedagogy. Multiple opportunities for doing concept mapping need to be created so as to allow learners to focus on omissions as that may activate their reflective thinking ability. The study was concluded by recommending that all mathematics teachers be trained in concept mapping for them to use it effectively as a tool of assessment for learning. Exposure of mathematics teachers in the use of concept maps as an assessment and teaching instrument may contribute towards better understanding.

TSHOBOKANYO

Go lemosegile fa go tlhaloganya mo dipalong go le matswakabele ebile go le thata mo barutwaneng ba dikolo tsa magae. Tlhaloganyo e tsewa jaaka kgokaganyo ya mafoko, fela barutabana ga ba rute kgokaganyo e mme ba ikarabela mo go e diragatseng. Thuto e, e bega fa mmapa wa mafoko o le botlhokwa mo go tlhotlhomisa se barutwana ba se tlhaloganyang mo thutong ya Dipalo.

Maikaelelo a thuto e, ke go tlhotlhomisa go tlhaloganya ga barutwana mo dipalong le go batlisisa mabaka a a tlhotlheletsang bokoa mo thutong. Diphitlhelelo e nnile katlego e e sedimositseng fa puo e le sebako sa tiragatso e e bokoa mo tlhaloganyong. Go fitlheletswe fa go akabala e le tiragatso ya seemo sa ntlha sa tlhaloganyo ebile tlhaloganyo e feto-fetoga mo diemong tse di farologaneng. Go tlhaloganya go dirafala dikakanyo di ntse di bopa-bopiwa mo bobokong jwa moakanyi.

Mmapa wa mafoko o dirisitswe jaaka mokgwa wa boleng mo go phutheng dintlha. Ditlhotlhomiso tsa bontsi tse di akaretsang Barutwana ba Kereiti ya lesome go tswa mo Kgaolo-tlhopong ya Dr Ruth Segomotsi Mompati di ne tsa dirisiwa. Barutwana ba rotloeditswe go golaganya mafoko ka maikaelelo a go lemoga gore ba a tlhaloganya jang. Dipotsolotso le go lebelela go dirisitswe jaaka mokgwa ya go netefatsa dintlha. Dintlha tse di phuthilweng di ne tsa seka-sekwa le go kwalwa ka mokgwa wa kakaretso.

Phitlhelelo e nnile gore tlhaloganyo ke kgokagano ya dikakanyo tse agilweng go akaretsa: kakabalo; kakanyo e e sa golaganang; le kakanyo e e rulaganeng. Thuto e fitlheletse fa tlhaloganyo ya barutwana e agilwe ka go: neela mefuta, mesola, diteng le dikao tse di maleba. Phitlhelelo e tsamaisana le systems theory; le mekgwa ya tlhaloganyo e e bidiwang instrumental le relational ka puo ya seeng.

Tiriso ya mmapa wa mafoko e tshwanetse go rotloediwa mo diphaposi-borutelong jaaka sediriswa sa tlhatlhobo le sa go ruta. Barutwana ba tshwanelwa ke go newa ditshono tse dintsi go netefatsa fa ba didrisa mafo a a tlogetsweng go aga dikakanyo tsa bone. Barutabana ba tshwanelwa go fiwa katiso ka ga go dirisa mmapa wa mafoko go ba kgontsa go o dirisa ka manontlhotlho. Fa barutabana ba dirisa mmapa wa mafoko mo tlhatlhobong le mo thutong, go tlhaloganya dipalo go tla tokafala thata.

ACRONYMS

APO – Area Project Office

HoD- Head of Department

ISC- Institutional Support Coordinator

MAA – The Mathematics Association of America

NCS – National Curriculum Statement

NCTM – National Council of Teachers of Mathematics

NWURC- North West University Research Committee

DoE – Department of Education

LO – Learning Outcome

LoLT – Language of Teaching and Learning

HSRC- Human Sciences Research Council

Timss – Third international Mathematics and science studies

OBE- Outcomes- Based Education

USA – United States of America

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CHAPTER 1
ORIENTATION

1.1 INTRODUCTION

Understanding has different meanings to people depending on the context in which it is used. Garegae (2001:232) defines understanding as an agreement or a contract, an act of sympathy, comprehension, or a mental state. Star (2007:6) describes understanding as involving the following words: know, explain, procedure, rote, problem solving, network, bonds, links, web and schema. Kinley (2008:17) describes understanding as a process consisting of three stages: concrete, representational and abstraction. Simple and common as it may look, understanding in mathematics learning is seemingly very problematic and in some instances difficult to achieve.

According to Baralos (2009:3), concept maps can be used to evaluate how learners organize their knowledge and give an observable record of their understanding. For this reason, this study problematises concept mapping as a tool best suited for exploring the complexity of understanding in mathematics learning. Concept mapping is seemingly a rare occurrence since it is not prescribed as an assessment tool in many textbooks which is where mathematics authority seems to lie. Conventional assessment methods are seemingly less able to portray the lapses in understanding and that paralyses the teachers' effort in helping learners to develop adequate understanding.

Many studies remain descriptive on how understanding occurs without guiding teachers on how to improve the possibility for learning with understanding. Star (2007:30) describes understanding as the knowledge that is tightly connected, is web, is a network and is organized in schemas. The metaphorical description and abstraction of the concept understanding as a link in many studies is helpful but not sufficient because teachers do not teach links and connections. Concept mapping as a tool of pedagogy could help educators to teach the links that are necessary for making understanding possible.

The complex and recursive nature of understanding has led to the fragmentation of the concept and partial descriptions. Two frameworks of understanding exist: Skemp's (1979:2) Instrumental and relational framework and procedural and conceptual framework which was

propagated by Hiebert, Carpenter, Fennema, Fuson, Wearrne, Murray, Olivier and Human (2000:4). The work of Skemp represents a landmark in the identification and distinction of relational and instrumental understanding.

The National Curriculum Statement (NCS) Grade 10-12 (Department of Education, 2002:9) observes that mathematical problem solving enables learners to understand the world and make use of that understanding in their daily lives. The NCS documents that the study of mathematics contributes to personal development through deeper understanding and successful application of its knowledge and skills. The NCS through its assessment standards presents understanding as an output of mathematics to the world and not as an input in the making of mathematics.

Teachers continue to use phrases like 'are you following?', 'are we together?' and 'is this OK?' to check whether students understand what is being taught. But this normally elicits the 'yes' answer from learners including those who are not concentrating. These phrases are used to manipulate learners to agree that they know what they do not know which in turn creates an impression that the teacher is making progress in teaching. This also shows that the word understand may be used for unintended outcomes.

Hough, O'Rode, Terman and Weissglass (2007:25) indicate that students from the traditional classroom come away with the impression that doing mathematics is rote and mechanistic, that all mathematics authority lies in the textbook, memorization is the way to succeed mathematically. Skemp (1979:35) agrees that students learn mathematics from the perception that mathematics learning consists of memorizing a collection of rules without reason. Traditional teachers believe that mathematical reasoning and explanations are matters of intellectual courtesy which comes only after mastering procedures. Mathematics teaching in the rural schools of South Africa remains textbook orientated and rule based.

1.2 PROBLEM STATEMENT

Learners have a great difficulty in constructing adequate understanding in mathematics learning. Reports on learning performance which is a possible indicator of how well understanding has been constructed suggest the high prevalence of inadequate understanding

among mathematics learners. For instance, in 2003, the Third International Mathematics and Science Studies (TIMSS) tested grade 8 learners' mathematical proficiency and South Africa came last of the 50 participating countries. Maree, Aldous, Hatting, Swanepool and Van den Linde (2006:3) note that the failure rate in mathematics in South African schools remains unacceptably high. The poor performance in mathematics as indicated by TIMSS is an indicator that understanding as the goal of mathematics teaching and learning has never been adequately realized.

The Human Sciences Research Council (HSRC), in the Grade 6 Systemic Evaluation Study of 2006, reported that learner achievements in mathematics for the National and North West provincial departments were 27% and 24% respectively. The percentages of learners at each achievement level for Bophirima (Dr Segomotsi Ruth Mompati District) are indicated as follows: 96% - Not achieved, 3% - Partly achieved and 1%- achieved. Learner participation and access to information were relatively low compared to those of Language of Learning and Teaching (LoLT) and the Natural Sciences.

Understanding supports improved performance for all learners at all levels. The statistics given above suggest that learners in Dr Segomotsi Ruth Mompati District have a problem of constructing mathematical understanding. The causes of this problem need to be identified in order to resolve it amicably lest it continue to cast gloom over the future aspirations of many learners and subsequently the economy of the country. The string of poor performance shows that the interactions and experiences that could lead to mathematical understanding need to be investigated and enhanced with special reference to the district identified above.

1.2 AIM

1.3.1 Primary aim

The purpose of this exploratory study was to investigate how learners construct their understanding in mathematics learning and to determine factors in the learning context that led to poor understanding in mathematics learning and suggest possible ways of resolving them.

1.3.2 Emergent subsidiary aims

The study intended to recognize the importance of learning mathematics with understanding and exploring ways of promoting the understanding of mathematics during teaching and determining ways of enhancing interaction and experiences that contribute to the promotion of mathematical understanding. The study intended to further explore the efficacy of concept mapping in identifying the lapses in learners' mathematical understanding.

1.4 RESEARCH QUESTIONS

- How do individual learners construct their understanding of mathematics concepts?
- What are the causes of the poor understanding in mathematics learning?

1.5 SIGNIFICANCE OF THE STUDY

Knowledge about understanding the subject could shed light on what successful learning might look like and help to guide the development of the curriculum, teaching, and assessment that can move students towards proficiency and understanding in mathematics learning. The study can promote the use of concept mapping as an effective tool of assessment and pedagogy. The study hopes to put an end to the annual ritual of hue and cry which often follow the publication of mathematics results for Grade 12 learners in the Dr Ruth Segomotsi Mompati District.

Understanding can significantly improve learner confidence and address the problem of mathematics avoidance as is the case with many high school learners. The study aims to test the validity of a few theories and a number of generalizations made in previous studies relating to the subject. The study can be seen as an ambitious effort to discover the problems that exist in the construct of understanding and to develop the theoretical perspective about the concept understanding.

1.6 LIMITATIONS

The instrument and interviews were administered in English which is the Language of Learning and Teaching (LoLT) to both teachers and learners involved in the study. All participants were of African origin whose home language is Setswana. The three mathematics educators concerned failed to participate in the drawing of concept maps sighting timelessness as the cause of their failure. The study involved grade 10 learners only. The researcher was also constrained by lack of finances for travelling to distant schools.

1.7 DELIMITATIONS

. This study has both geographical delimitations and conceptual delimitations.

1.7.1 Geographical Delimitations

The research was limited to three secondary schools in Taledi Central Cluster at Dr Ruth Segomotsi Mompati District. Taledi Central Cluster is situated approximately 42 kilometres south of Vryburg. It involved learners and teachers in villages of the North West Province.

1.7.2 Conceptual Delimitations

For the purpose of this study, the concept understanding was not defined as a contract, an agreement or sympathy. Skemp's (1979) relational understanding and Hiebert *et al.'s* (2000) conceptual understanding were taken to be similar and therefore were used interchangeably in the study.

1.8 RESEARCH ASSUMPTIONS

The following statements were set as research assumptions for the study:

- The teachers participating in the project were competent to teach mathematics. That is, they had the teaching skills, knowledge and understanding of, and good disposition towards the mathematics learning area.

- All students participating in the project were capable of understanding mathematical concepts and procedures that they were being taught. Understanding meant that learners were able to create links or connections between mathematical ideas.
- The teaching context was capable of generating multiple understandings of a given concept and that problem solving in mathematics was a reflection of the connections made or understanding constructed.

1.9 ETHICAL CONSIDERATIONS

Leedy and Ormrod (2001:107) present the following as pillars that ensure that ethical considerations are in place: Voluntary participation, no harm to respondents, anonymity and confidentiality, professionalism and no cheating of colleagues. The following paragraphs reveal how these considerations were realized.

1.9.1 Informed consent

Clarke-Cater (2004:14) notes that informed consent is one of the important ethical considerations to be put in place whenever human beings are involved in a research investigation. North West Department of Education (NWDoE), gatekeepers, teachers and learners were informed in detail about the nature of the investigation, the specific roles they were expected to play and the need for their cooperation before the information gathering phase.

Clark-Cater (2004:14) recommended that in some instances consent should be obtained from parents or someone acting in *loco parentis.* The researcher obtained consent from the parents of all learners who volunteered to take part. Personal details of the researcher were disclosed to all participants and the importance of their involvement in the research project was clarified to create trusting relation between the researcher and the researched.

1.9.2 Voluntary Participation

Many studies present voluntary participation as a fundamental ethical rule which allows respondents to volunteer their participation in the research project. For example, Leedy and Ormrod (2001:107) contends that participation in any study should be strictly voluntary. According to Clark-Cater (2004:15), undue pressure to take part should not be applied, and unnecessary inducements should not be offered because they may stultify the purpose of the study as learners would be participating for rewards.

The first visit was used to appropriately inform both teachers and learners about the project and to ask them to volunteer as respondents. No respondent was forced to take part in the research project. Participants, who withdrew before the completion of the investigation, were replaced to meet the specified sample. This was done in line with Clark-Carter's (2004:14) view that potential participants had the right to refuse, they should be approached politely and their rejection should be accepted gracefully.

1.9.3 Confidentiality and anonymity

Clarke-Cater (2004:15) recommended that all participants be assured of confidentiality. The researcher assured all participants that the information provided by them would be treated as confidential and their anonymity was also guaranteed. Differing Pseudonyms were attached to all participants during the first phase of the study. Real names of participants were not used in the research report to cater for the condition of anonymity. Actual names that were written on concept map sheets were deleted in an effort to satisfy the condition of anonymity. The researcher ensured that the collected information was only accessed by the people who were directly involved in the research project.

1.9.4 No harm to respondents

According to Bless and Higson-Smith (2000:100), social scientists should not subject respondents to harmful or uncomfortable situations. The investigator did not expose the

respondents to situations where their privacy was infringed upon. Sensitive questions were not asked and sensitive statements were not entertained at anytime during the investigation. All respondents were treated with respect and dignity. The researcher did not reveal damaging information about the researched in his report.

1.9.5 Professionalism and no cheating of colleagues

No cheating of colleagues was treated as the investigator's responsibility to the scientific community. Findings that were linked to the investigation were reported in a transparent and honest manner. As part of professionalism, the investigator provided respondents with accurate information regarding him, the aim, significance and purpose of the study. Respondents were advised to be honest in their responses.

According to Leedy and Ormrod (2001:108), researchers must report their findings in a complete and honest fashion. Leedy and Ormrod (2001:108) note that the use of other people's words and ideas without full acknowledgement constitutes plagiarism and documentary theft. The researcher pledged to observe the dictates of this egalitarian rule. Acknowledgements and consents were sought in an effort to observe this principle.

1.1O DEFINITION OF TERMS (as used in the context)

Concept map

- A concept map is a graphical representation consisting of nodes (concepts) and labelled lines. Lines denote a relation between a pair of concepts (Huerta, Galan and Granell, 2009:2).

System

- A system is an entity which can maintain some organization in the face of change from within and without (Rapaport, in Letseka, 2000:174).
- A system is a set of objects or elements in interaction to achieve a specific goal (Ryan, in Letseka, 2000:174).

Systems Theory

Systems theory refers to the transdisciplinary study of the abstract organization of phenomenon, independent of their substance, type, or spatial, or temporal scale of existence. It investigates both the principles common to all complex entities, and the (usually mathematical) models which can be used to describe them (Letseka, 2000: 171).

Theory

Theory is an explanatory framework which orders and makes connections between the currently known observations and information. Theories are not fixed: they are always evolving (Donald, Lazarus and Lolwana, 1997:202).

Perspective

Perspective refers to a broad, encompassing view or way of examining and understanding something (Donald, Lazarus and Lolwana, 1997: 202).

Intelligence

Intelligence refers to the cluster of mental abilities which together are very useful (Skemp, 1979:32).

1.11 AN OUTLINE OF CHAPTERS

The study was structured as follows:

Chapter 1- outlines the background, problem statement, significance of study, the aim, research questions, research assumption and ethical considerations.

Chapter 2- provides a detailed review of literature on the subject understanding and concept mapping in mathematics learning.

Chapter 3- gives an explanation of the research methodology used, outlines data collection strategies, the context and how data is analyzed.

Chapter 4- presents the data collection process (i.e. interpretation of concept maps, interviews and observations), data presentation and analysis, and the researcher values.

Chapter 5- is a synthesis of the critical findings, recommendations, summary and conclusive remarks.

1.12 CONCLUSION

The first chapter elaborated on the following: the background, problem statement, the significance of study, the aim, research questions, research assumption, and ethical considerations of the proposed Study. The chapter reports on why the study was necessary and how it planned to unfold.

CHAPTER 2
REVIEW OF LITERATURE

2.1 INTRODUCTION

The pervasive nature of poor learning performance in mathematics as displayed by South African schools as outlined in the previous chapter is undoubtedly a consequence of lack in mathematical understanding. It is a known fact that understanding supports improved performance for all learners at all levels. Accordingly, there seems to be confluence of the fact that both mathematical understanding and concept mapping are centred on interconnections, networks and interrelation between concepts.

This review of literature is focused on describing what understanding is, and identifying the role that concept mapping can play in the portrayal of understanding. Studies suggest that concept mapping has the potential of making concept relations more transparent. Brinkmann (2009:8) describes concept maps as powerful tools for identifying students' knowledge structures, especially misconceptions and other conceptions. Other studies such as, Baralos (2009:3) and Hough *et al.*(2007:39) have also elaborated on concept maps as a tool for promoting understanding. This serves as the rationale for problematising the use of concept maps in this study. Different views about understanding are being acknowledged, critiqued and used to inform the researcher's understanding of the researched phenomenon.

2.2 WHAT IS UNDERSTANDING?

Skemp (1979:46) is of the view that understanding something means assimilating something into an appropriate schema. Van de Walle (2009:1) describes understanding as the measure of how well an idea is integrated with or connected to other existing ideas in the learner's cognitive framework. Understanding is clearly the reflection of an internal relationship which the learner has with the learning content. As Van de Walle (2001:28) puts it, understanding is a measure of the quality and quantity of connections that an idea has with existing ideas.

Studies suggest that understanding can be limited and extensive. Pirie and Kierin (1989:8) view understanding in mathematics as a complex levelled phenomenon which is recursive

and levels are self referenced and dissimilar. Manon (2005:4) argues that understanding is something that is always changing and ever growing. Tall (1978:51) agrees that understanding exists in time and changes with time.

Despite the differing views about understanding, understanding in mathematics learning should remain a goal of teaching and a consistent part of all students' everyday mathematical experiences. Mwakapenda (2004:28) views understanding as one of the important traits associated with the attainment of educational goals. According to Garegae (2001:232), the entire world construes the importance of mathematical understanding as crucial. This implies that all forms of understanding irrespective of the framework or theory into which they fall are crucial for effective mathematics learning.

Garegae (2001:233) showed how the concept understanding subscribes the three main theories of learning – behaviourism, cognitivism and constructivism. In articulating Inhelder and Piaget's notion of understanding, Mouseley (2005:553) documents that developing understanding involves increasing ability to hold several relationships in mind, permitting further abstraction and anticipation.

Krathwohl (2002:215) projects understanding as a step in the cognitive process. Viewing understanding this way tends to reduce the essence and value of understanding as an overall goal of learning. Krathwohl has, however, acknowledged the role of understanding as forming the basement for the cognitive process dimension. According to Krathwohl (2002:215), the following sub-categories quantify understanding: interpreting, exemplifying, classifying, summarizing, inferring, comparing and explaining.

Two theoretical frameworks on the subject understanding exist: relational understanding and instrumental understanding. A better quantity of description and definition of the concept understanding subscribe to the constructivist theory. For Cognitive theorists, understanding is about making sense, interconnections and creating links. Garegae (2001:233) argues that cognitivism emphases the internal mental structures of the learner but lends itself to abstract information processing. For individuals who affiliate the behaviourist theory, understanding may refer to the possession of rules and the ability to apply those rules correctly. The types of understanding discussed in this study are instrumental and relational understanding.

2.2.1 Instrumental Understanding

Proponents of instrumental understanding believe that students learn best through memorization and repeated practice. Manon (2005:3) describes instrumental understanding as knowing what to do or the rule and the ability to use it. Skemp (1979:32) points out that instrumental mathematics is usually easy to understand in it, rewards are more immediate and one can often get the answers quickly. This portrays mathematics as something which is out there waiting to be transferred by a skilful teacher into the empty head of a learner.

Instrumental understanding supports learning by rote. Rote learning portrays mathematics as the embodiment of regimented, top-down, knower-dependant knowledge. Tall (1979:51) views instrumental understanding as no more than an exercise of memory. According to Skemp (1979:32), memorization arouses no interest, it gives learners no pleasure, work is boring and difficult, and learners do it because they have to. Cohen (1993:791) describes rote learning as a bankrupt or near sighted approach to learning.

On the contrary, Davis and Simmt (2003:153), argue that in a complex system, appropriate actions (e.g. understanding) can only be conditioned by the external authorities and not imposed. This implies that teachers do not have control of how learners comprehend a given concept. In case of rote learning, teachers tend to have a centralized control over ideas that are being learned and this may serve as a limiting factor for learners attaining higher levels of understanding. Considering Davis and Simmt' categorization of systems, instrumental understanding could therefore be classified under simple systems.

Van de Walle (2001:28) views understanding as something that exists along a continuum. According to him (Van de Walle, 2009:2), instrumental understanding has fewer and weak connections and gradually changes to relational understanding which has connections with more knowledge. This implies that instrumental understanding forms the basis for the development of relational understanding and there is no division point between the two forms of understanding.

Manon (2005:1) believes that all persons who ever fully understood arithmetic have been obliged to learn it over and over again in their own way. Skemp (1979:32) notes that habit learning helps children to learn the words for number concepts: 1, 2, 3. Teachers who believe in instrumental understanding will have a tendency of subjecting learners to repetitive drills and memorizations. Devlin (2006:3) points out that all learners who came to master multiplication tables did so after being subjected to these kinds of drills and repetitions. These support Van de Walle (2001:8) continuum theory which indicates that new concepts are at first understood instrumentally and with improvements they gradually develop to form a relational domain.

Unlike many researchers, Devlin is one of the mathematicians who believe that to follow the rule of practice is a necessary condition for the development of understanding. Devlin (2006:2) believes that understanding comes later as an emergent consequence of a regular use and practice. According to Devlin (2006:3), one of the things that school-based mathematics should definitely produce is the ability to apply rule based symbolic process without understanding. He (Devlin, 2006:3), cited the progression involved in learning to play chess as an example of how regular practice leads to understanding.

Practitioners of instrumental understanding believe that understanding comes from listening carefully to what the teacher has to say. Hiebert *et al.* (2000:8) argue that traditional teachers feel responsible to tell students important mathematical information, demonstrate procedures and ask students to practise what they have seen until they become proficient. The majority of tasks that involve instrumental understanding are seemingly in the topical zone where only evaluative and interpretive listening may be required. One could conclusively say that the form of understanding applied by an individual depends upon the kind of teaching to which the learners are exposed.

The discussion on instrumental understanding suggests that relational understanding emerges from instrumental understanding. The difference between the two forms of understanding seems to be the quantity of connections made between the known

concepts. In the next section, relational understanding is discussed as a form of understanding which is secondary to instrumental understanding.

2.2.2 Relational understanding

Tall (1978:51) views relational understanding as a mental activity change that is occasioned through a search for coherence in ideas taken from the universe we live in. Proponents of relational understanding hold the view that students learn best through active inquiry and investigation. Orton and Frosher (1996:12) contend that mathematics cannot be delivered to children and that traditional methods do not necessarily lead to understanding. Relational understanding has its philosophical base in the constructivist theory.

Several studies suggest that understanding can generally be characterized as interconnected knowledge. In support of the connection view, Mwakapenda (2004:34), points out that to know something is not to know it as an entity having a life of its own, but it is to know it in relation to something else: its context. According to Sponsel (2003:3), learners will have a great difficulty in understanding if concepts are isolated facts. However relational learners are capable of thinking instrumentally which means that the two forms of understanding should not be seen as parallels but rather as complementary.

Relational understanding subscribes to what Davis and Simmt (2000:141) term self-organization. Davis and Simmt (2003:141) portray self-organizing events like understanding as being 'bottom-up', as emergent macro-behaviours that arise through localized rules and behaviours of agents and not through imposition of top-down instructions. Laying emphasis on the importance of a concept during classroom instruction may not necessarily result in understanding being occasioned.

2.3 THE ROLE OF UNDERSTANDING IN LEARNING MATHEMATICS

The key question is not whether understanding is important or not, it is whether enough is being done to ensure that mathematics is learned with understanding. The need for implementing teaching for understanding in mathematics is undoubtedly a requisite in improving learner performance. Studies indicate that understanding facilitates retention and transfer. Manon (2005:22) acknowledges that without understanding, it is easy for students to forget procedures and discard them.

Manon (2005: 35) contends that understanding supports improved performance for students at all levels. Kilpatrick, Swafford and Findell (2001:119) argue that understanding enables learners to gain confidence which then provides the base from which they can move to another level of understanding. These views indicate that understanding is self-generative and somehow elastic. Manon (2005:9) argues that understanding breeds confidence and engagement whilst the lack thereof results in disillusionment and disengagement. Understanding enables learners to realize and correct their mistakes.

Van de Walle (2001:30) outlines the following as benefits of relational understanding: Intrinsically rewarding, enhancement of memory, reduction of things to be remembered, improved problem solving abilities, self-generating, improvement of attitudes and belief , and facility in learning new concepts and procedures. Understanding can lead to self-efficacy and self directed learning which contribute to improved performance. On the other hand, Orton and Frobisher (1996:18) acknowledge that children who have difficulty in understanding mathematics tend to generate the disliking attitude and rejection of mathematics.

As a learning principle, the National Council of Teachers of Mathematics [NCTM] (2000:20) contends that mathematics must be learned with understanding and students should actively build new knowledge from experience and prior knowledge. The 'must' emphasizes the need for understanding in mathematics learning and lays emphasis on the fact that mathematics can be learned with understanding. Studies suggest that understanding is an important goal of learning mathematics. Mwakapenda (2004:28) also agrees that students with conceptual understanding have knowledge and understanding of why a mathematical idea is important and the context in which the ideas are useful. Given the fact that understanding is self-

generative, the basic understanding that learners may have is used to generate an improved understanding. According to Stylianides and Stylianides (2007:107), learning principles embrace the fact that understanding supports autonomous learning.

2.4 TEACHING FOR UNDERSTANDING

There is a general expectation that teaching will ensure that understanding is occasioned. Orton and Frobisher (1996:18) acknowledge Richard's comment that learners do not become active by accident but by design. Teaching is supposed to ensure that learners are actively generating the intended mathematical connections (understanding). According to Davis and Simmt (2003:149), mathematics teachers are expected to occasion the emergence of a complex collective whose interactions and products are mathematical.

According to Van de Walle (2001:36), instruction should be structured in a way that includes students actively constructing their own knowledge with understanding whilst the relationship among concepts, skills and problem solving is emphasized. Skemp (1979:44) believes that exposition by the teacher is important for helping children's effort to understand for themselves. This implies that the teacher should ensure that instruction is well balanced to equitably cater for conceptual understanding and procedural knowledge. The content being taught should always fall within the learners' zone of proximal development.

According to Teppo (1991:210), Van Heile's theory propagates a teaching sequence which is constructed from direct instruction to independent understanding. Teppo (1991:210), also believes that the sequence provides three forms of insight which are: visual, descriptive and theoretical. This implies that learners have an understanding of what they are doing, why they are doing it and when to do it. Mousley (2004:2) articulates the 'scaffolding of learning' as a strategy that teachers could use to enhance learner understanding. Mousley (2004:1) also notes that scaffolding involves two interwoven processes, namely: active guidance and modelling by the teacher, and active construction of knowledge by the learner. The teaching action is a scaffold around the learning activity to bring about the active construction of ideas (understanding).

Miller and Koesling (2009:68) have proposed a multiple-reads strategy as a solution to the problem of understanding in mathematics. Van de Walle (2001:36) contends that the teacher should engage students by posing good problems and creating a classroom atmosphere of exploration and sense making. Emanuelsson (2001:229) acknowledges that skillful posing of questions by teachers during teaching makes students' understanding visible in the classroom. Selinger (1994:187) suggests that tests can be set in such a way that instrumental understanding cannot be useful for questions and relational understanding is required for all questions.

2.5 TOOLS AND MATERIALS FOR UNDERSTANDING

Van de Walle (2001:26) views elements of our physical surroundings as the material that we act on in building understanding. Our thought processes help us to create links between the ideas that we already possess. Skemp (1979:90) describes symbols as mental objects, about which and with which we think. Manipulative and concrete materials or teaching aids are helpful devices commonly used by teachers to develop pupils' understanding of concepts, procedures and other aspects of mathematics.

According to Mwakapenda (2004:34), lack of expertise in expressing mathematics and inadequate understanding in mathematical language and mathematics itself are likely to constrain student's ability to describe the links between concepts. Manon (2005:34) notes that language, material and symbols are mathematical tools which play an important role in learning mathematics with understanding. To promote mathematical understanding through teaching, Mousley (2004:5) proposes three strategies- the use of physical representations of a concept, building conceptual links and language based activities.

According to Uger (1994:8), it takes time to engage students in the performance of understanding. Uger (1994: 8) argues that giving students more time invites them to get involved in the kind of thinking that will foster the building of their own understanding. Experience has shown that most mathematics problems could not be solved in the first attempt which is why it takes time to understand mathematics. Hough *et. al* (2007:25) concurs that the doing of mathematics is a creative, sense making activity that entails

interpretations, hypothesizing, effort, and exploration. Understanding is simply generating internal connections among the ideas being learned.

The steps and bits done over a long period of time enables learners to gain the experience that is necessary and useful for solving other related problems. Sponsel (2003:1) observes that providing students with the formula $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ does not instantly create understanding. Unfortunately, most mathematics teachers are having massive workloads, crowded classrooms and insufficient support which deprive them of the opportunity to engage learners in activities that promote deeper levels of understanding.

The concept of understanding is often appealed to because policy-makers and curriculum writers want to capture everything that they find impossible to describe in more specific terms. Understanding is often used in the National Curriculum Statement as part of the generic assessment standards. Dean (1982:47) also points out that the understanding of something is always if any remark refers just to understanding. "This is expressed more generally by saying that mathematics education is a process which includes teaching pupils to full understanding of some further mathematical concept, symbol and experience" (Dean, 1982:47).

Kilpatrick *et al.* (2001:116), introduced the dimension of conceptual understanding as a critical component to mathematical proficiency which they view as a necessity for learning mathematics successfully. Kilpatrick *et al.* (2001: 116) define conceptual understanding as the comprehension of mathematical concepts; operations and relations. Mwakapenda (2004:28) advocates that the teaching and learning of mathematics are supposed to enable learners to develop deep conceptual understanding in order to make sense of the mathematics being learned.

Kilpatrick et *al.* (2001:118), note that students with conceptual understanding know more than isolated facts and methods and they advocate that conceptual understanding helps students to avoid many critical errors in solving problems. The literature suggests that procedures learned without understanding tend to be error prone, are easily forgotten and do not transfer easily to novel problem types. In support of these views, Skemp (1979:35) says that rules are only useful for the limited range of problems.

The National Academies Press (2002:19) described understanding as requiring that an individual should integrate a complex science structure of many types of knowledge, including the ideas of science, relationship between ideas, reasons for this relationship, ways to use the ideas to explain and predict other natural phenomena, and the way to apply them to many events. These tend to summarise what understanding is about. The different forms of understanding are clearly complementary and useful in the development of mathematics learning.

The next section explores the usefulness of concept maps to reveal how learner understanding is organized. The section describes concept mapping, acknowledges its uses and weakness and provides a background use of concept mapping as a research tool. Also covered in the next section is the relationship between concept mapping and conceptual understanding.

2.6 CONCEPT MAPS

Studies suggest that the aim of designing a concept map is to represent mathematical concepts spatially. Huerta *et al.* (2009:2) define a concept map as a graphical representation consisting of nodes (concepts) and labelled lines. According to them (Huerta *et al.*, 2009:2), lines denote a relation between a pair of concepts. Freeman and Urbaczewski (2009:41) view a concept map as an alternative method for measuring the knowledge of students using mental models. The rules, uses and linkage types are acknowledged as general features of a concept map. According to Brinkmann (2009:1), a concept map is a pedagogical tool for mathematics education.

2.6.1 General features of a concept map

As observed in the definitions of concept map given above, a concept has three basic elements, namely: concepts, line and linking words or phrases. Mwakapenda (2003:193) agrees that linking words on the lines indicate how pairs of concepts are related. Caldwell, Al-Rubaee, Lipkin, Caldwell and Campese (2006:1) observe that every concept map produced had 3 sequenced elements, namely: *concept- connector – concept.* According to them (Caldwell *et al.*, 2006:2), any sequence of concept-

connector-concept should stand alone as a correct and a mathematically meaningful sentence.

According to Huerta *et al.* (2009:2), the combination of two nodes (concepts) and a labelled line is called a proposition and they (Huerta *et al.*, 2009:2), describe a proposition as a basic unit of meaning in a concept map. A concept map may have several propositions, links and cross links and numerous hierarchy levels. Concept maps tend to differ in degrees of complexity depending on the nature of the links made and or quantity of propositions created.

2.6.2 Application of concept maps

Studies show that concept mapping is useful, flexible, rarely used but has a variety of applications. Brinkmann (2009:2) showed that concept mapping was introduced by Novak as a research tool showing in a graphical way the concept related concepts to a given topic. Mwakapenda (2003:193) indicates that the flexibility of concept mapping makes it a useful tool for investigating a wide range of aspects associated with student learning in mathematics.

Brinkman (2009:8) concedes that concept maps have many uses which include: a research tool in educational research, an assessment tool, a learning guide for students and as pedagogical tool for mathematics education. Despite the expansive nature of uses and the usefulness of concept maps, the use of this important tool both locally and globally remains a rare occurrence in educational settings. According to Caldwell *et al.* (2006:1), concept mapping has been very useful in a variety of disciplines, but has had a relatively little use in mathematics education in the United States of America (USA).

According to Brinkmann (2009:7), concept maps are more helpful for normally higher achievers than for lower achievers. The usage of concept mapping as a tool for assessment in the South African context is relatively minimal. According to Mwakapeda (2004:34) concept mapping is emerging as a reflective tool in mathematics learning in South Africa. Mind mapping seems to be a commonly used tool of

assessment in the Natural Sciences where it is used for the design of a water cycle, carbon cycle, etc.

The minimal usage of concept mapping in mathematics across all grade levels in South African classroom has resulted in both teachers and learners being unaccustomed to the use of this important tool. Mathematics Assessment guideline and the NSC policy documents in all phases of schooling are silent about the use of concept mapping as an assessment tool.

2.6.3 Advantages of concept mapping

Concept mapping seem to have a potential of making knowledge content more conceptually transparent. Hough *et al.* (2007:39), for example, agrees that concept mapping validates the mathematical knowledge that each participant holds. This implies that concept mapping can reveal details about the nature of understanding possessed by the learner on a particular concept. In concurring with this view, Freeman and Urbaczewski (2009:51) point out that concept maps may contain concepts and relationships that indicate a deeper understanding of materials.

Hough *et al.* (2004:34) argue that concept maps are in themselves valuable learning activities in which participants reflect on their own understanding. Brinkmann (2009:1) notes that concept maps may be efficient tools for building structures in mathematics. Freeman and Urbaczewski (2009:43) also indicate that concept maps support the notion of hierarchy. Brinkmann (2009:7) contends that concept maps help learners to organize information on the topic.

According to Brinkmann (2009:8), the following are the advantages of using concept maps:

- Concept maps facilitate meaningful learning, they aid organizing and understanding of new subject matter.
- Concept maps may be used for revision of a topic.
- Concept maps can be used as a design of instructional material

- Concept maps may serve as memory aid. i.e. easy commitment to one's memory and makes recall faster.

2.6.4 Disadvantages of concept mapping

As a disadvantage, Brinkmann (2009:80) acknowledges that concept maps may have a confusing effect. According to him (Brinkmann, 2009: 80), concept maps do not have open structures to allow for extended elaboration and expansion. Concept mapping is clearly an act of understanding. The literature on understanding suggests that acts of understanding require long periods of time and multiple opportunities to be performed. Concept mapping is a time consuming exercise but it remains a fruitful activity.

Freeman and Urbaczewski (2009:51) indicate that analyzing and interpreting concept maps is much more time consuming and intense than traditional assessment techniques. Teachers who know these challenges and the type of learners that our schools have may be dissuaded from effectively using concept mapping as a tool of assessment. Concept mapping may not be user-friendly in large classroom because of the length of time that it requires.

2.6.5 Concept maps versus conceptual understanding of school mathematics

Concept mapping supports the development of conceptual understanding. Kilpatrick *et al.* (2001:118) define conceptual understanding as an integrated functional grasp of mathematics ideas. Brinkmann (2009:1) notes that mathematical knowledge has a character of a network as mathematical objects. Conceptual understanding and concept mapping are critical requisites for mathematics learning.

2.7 THEORETICAL BACKGROUND

The concept "understanding" is firmly grounded in the following distinctive theories: Learning theory and Systems theory. Mwamwenda (1989:356) defines a theory as an explanation of observations. Donald *et al.* (1997:202) note that theories were not fixed but they were always evolving.

2.7.1 Learning theories

The three known perspectives of educational studies are Positivism, Interpretivism and the Critical theory. Garegae (2001:233) has pervasively shown that behaviourism, constructivism and critical pedagogy are respectively encapsulated in these perspectives. The concept of understanding is grounded in part to each of the three educational perspectives. It is associated with many theories of learning including the following: Social learning, Habit learning (in the behaviourist perspective) and intelligence learning in the Construtivist perspective.

According to Mwamwenda (1989:139), social learning is observational or imitation learning. Mwamwenda (1989: 140) has also acknowledged the following factors as being useful in the occurrence of social learning: attention, memory, motor skills, reinforcement and identification. These factors support Devlin's rule symbolic practice which leads to instrumental understanding. According to Garegae (2001:233), in behavariourism understanding is inferred as a product of learning measured by the ability to rehearse, retrieve, transform and code the information learnt.

Garegae (2001:233) also points out that in the constructivist paradigm, learning is viewed as an active process in which learners are engaged in constructing new concepts based upon current and past own knowledge. Jacobs, Vakalisa and Gawe (2004:46) also argue that what learners learn is as important as how they learn it. Jacobs *et al.* (2004:46) imply that constructivism lays more emphasis on the process of creating new information than on the content being learned. They (Jacobs *et al.,* 2004:47) note that constructivism agrees with ideas propagated in the critical inquiry theory. These two theories support relational understanding.

2. 7.1.1 Habit learning theory

Instrumental understanding is occasioned through habit learning. Skemp (1979:33) describes habit learning as learning by rote where behaviour is shaped by the environment and learning is preceded by action. This implies that drill and practice bring about the intended outcome which is the required understanding. Habit learning

and social learning theories are grounded in the behaviourist perspective. Understanding in this regard has no effect on subsequent learning. This implies that any form of understanding occasioned through habit learning is not self-generative and does not lead to any advanced form of understanding.

2.7.1.2 Intelligence learning theory

Relational understanding is grounded in the intelligence learning theory. According to Skemp (1979:34), intelligence learning theory is when learning precedes action and is used to shape our behaviour. The implies that learners will apply what they have learned. He (Skemp, 1979:35), views mathematics as a highly regular subject which requires a high proportion of intelligence learning. Skemp (1979:44) again points out that in intelligence learning the attainment of understanding is self-rewarding and can be seen in the learners' faces.

2.7.2 Systems theory

Systems theory consists of two theories known as chaos theory in simple system and complexity theory in the complex system. According to Letseka (2000:169), complexity theory is about flux, historical change, formations and dissolution of pattern and system behaviour.

2.7.2.1 Chaos theory

Letseka (2000:169) notes that complexity theory arises from chaos theory. Letseka (2000:170) conclusively notes that parts of a chaos system do not operate in harmony but somehow are involved in a dynamic interaction which gives rise to a new pattern while resulting in the dissolution of the old one. The concept "understanding" is a continuum that changes from the instrumental domain which is grounded in the Chaos (Simple) system to the rational domain which is grounded in the complex system.

2.7.2.2 Complex theory

The emergence of relational understanding from instrumental understanding is a resemblance of the rising of a complex theory from the Chaos theory. Letseka implies that instrumental understanding tends to die off as the quantity of interconnection increases resulting in a larger understanding called relational. Davis and Simmt (2003:139) have introduced the *Disorganised complex system* as median range between chaos and complex systems. The evolution of Understanding passes through the three systemic phases.

2.8 CONCLUSION

In the investigation, the relevant literature study suggests that understanding is a long continuum consisting of instrumental and relational characters. Appropriate teaching actions are requisites in ensuring that the understanding of mathematical content is occasioned. Understanding breeds engagement and confidence which are requisite for improving the learning performance. In the North West Province teaching seems to be centred on the promotion of procedural knowledge. Lack of conceptual knowledge, as explained in the review, is seemingly the cause of failure in mathematics learning.

The review problematises concept mapping as a vehicle best suited for identifying gaps in learner understanding. The theoretical background acknowledges the relation of the concepts understanding to the learning and systems theories. The next chapter explores the dynamics of the concept understanding using a qualitative research design and provides a presentation of data based on the research questions from the previous chapter.

CHAPTER 3
RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

Chapter 2 has problematised the concept mapping as a tool best suited for identifying the type of understanding constructed and the lapses that may be found in it. Chapter 3 gives an explanation of the research design and methodology used in responding to the following research questions: (1) How do individual learners construct their understanding of mathematics concepts? and (2) What are the causes of poor understanding in mathematics learning?. It outlines data collection strategies used. As part of the instrumentation, the chapter reveals the research content that was used in the study. It also prescribes validity issues considered in the next chapter.

3.2 RESEARCH DESIGN

Bless and Higson-Smith (2000:63) define a research design as a specification of the most adequate operations to be performed in order to test a specific hypothesis under given conditions. According to Schurink (2009:1), all research designs which focus on understanding and interpreting the life personal experiences and of inter-subjective realities of individuals using subjective experimentally captured data can be classified as qualitative research. A qualitative research design was used to collect responses based on the following research questions: (1) How do individual learners construct their understanding of mathematics concepts? and (2) What are the causes of poor understanding in mathematics learning?

Creswell (2007:73) defines a case study as a qualitative research approach in which an investigator explores a case or cases over time, through detailed, in-depth data collection involving multiple sources of information. A multiple case study method was selected to check how the subjects in each case have constructed their understanding of the given sets of concepts and to check if causes of poor understanding were similar. The researcher hopes to establish some general views about the construction of learner understanding and causes of poor understanding.

3.3 PARTICIPANTS IN THE STUDY

A group of twelve Grade 10 Mathematics learners (Subjects) were selected from three schools in Taledi 1(a cluster in Dr Ruth Segomotsi Mompati District) through convenience sampling methods. The selected schools were identified on the basis of proximity and from the researcher's reporting office to meet the constraint of time and finances. The three schools are classified as cases in this study. Each school had one mathematics teacher for the grade 10 learners. The educator concerned was requested to participate in the process of identifying participants (Snowball sampling). Pseudonyms used for Case study 1 participants were: Didi, Pinky and Tebogo. The pseudonyms for Case study 2 participants were: Thato, Olebogeng, Agnes, Joe, Katlego and Pule, and those for case study 3 were: Tshepang, Bontle and Thapelo.

3.4 INSTRUMENTATION

Qualitative research instruments were used in the process of exploring learners' understanding of mathematics learning. Concept maps, interviews and observations were used for collecting qualitative data from the participants. The three instruments were selected with the hope that the data collected would converge to give a solid response to the following research questions: (1) How do individual learners construct their understanding of mathematics concepts? and (2) What are the causes of poor understanding in mathematics learning?

3.4.1 Concept maps

Each learner was given two sets of concepts from algebra and geometry respectively. Each set had 15 content concepts which are critical in achieving Learning Outcome 1 and 3 respectively. Learners were asked to independently design two distinctive concept maps using the given sets of concepts. Learners were encouraged to draw the maximum possible links between the concepts within each setting. Sample concept maps (see Appendix D and E) from the Natural Sciences and Mathematics were used to demonstrate to learners what a concept map was so that they could know what the researcher expected from them. Aspects of understanding and concept mapping are not

unique to mathematics learning. They are generic and trans-disciplinary. The Natural science concept map was preferred because of its simplicity. The mathematics sample map was used as a consolidator to ensure that subjects develop the skills necessary for designing concept map since they were never exposed to concept mapping before the project. The following concept sets were given to learners.

Concept lists

Set one

Number systems	fraction	rational numbers
Real number	integer/s	ratio
Decimals	percentages	natural numbers
Whole number	proportion	exponent
Inverse	denominator	identity

Set two

Angle	line	acute
Obtuse	right angle	area
Prism	volume	equilateral
Triangle	Polygon	isosceles
Degree/s	sides	length

The concepts chosen in the first concept list represent the content embedded in learning outcome 1: Numbers, operations and relations (LO 1). The content subjects in LO 1 are interrelated. It helps learners to symbolically represent a given concept in different ways. Fraction can be represented as ratio, a percentage and can be written in exponential form with a negative power. Learners' ability to link the concepts was seen as a reflection of the quantity and quality of understanding they have in Learning Outcome 1.

The concept set 2 represents the content embedded in LO 3: Space, Shape and measurement. Learners were expected to create links between these concepts as a reflection of the knowledge and understanding they have in LO 3. Learners were expected to reveal more about the concepts. That is, the explanations, interdependence

and their deterministic ability on the central concept. The researcher expected that learners would have a starting concept or a central concept when drawing a concept map.

3.4.2 Interviews

The following open-ended questions were asked and follow-up probes were made to get greater detail of information of how knowledge was organized. The open-ended questions are derived the from research questions which are: (1) How do individual learners construct their understanding of mathematics concepts? and (2) What are the causes of poor understanding in mathematics learning? In a follow-up interview, TM = Interviewer and interviewees were represented by their pseudonyms.

- How did you start drawing the map?
- Why did you leave some of the concepts?
- Is there any concept that you know a lot about, and why?
- Is there any concept that you think you don't know much about?

Audiotapes were used to record data during the interview process. Learners were asked to explain the links that were created in the concept map. Learners were encouraged to speak more on the concepts which they knew a lot about. All interviewees were asked the above questions in a systematic and consistent way. Questions were adjusted depending on the need. Berg (2007:95) maintains that questions asked in semi-standardized interview can reflect awareness that individuals understand the world in varying ways. Berg (2007:131) contends that interviewers must be cordial and appreciative. For this reason each interviewee was thanked for his or her participation in the interview process.

Bless and Higson-Smith (2000:155) define an interview as a data collection technique based on a series of questions relating to the research topic to be answered by the research participants. The researcher has used a semi standardized type of interview as an information gathering tool. Berg (2007:93) describes semi-standardized interviews

as being semi structured, allowing reordering of questions and adjusting of language to give clarity. (*See* Appendix I for the learners' interview transcript)

3.4.3 Observation

The observations were recorded as field notes and analyzed for content. The researcher employed a simple observation method in the current study. Bless and Higson-Smith (2000:104) describe a simple observation method as the recording of events as observed by the outsider. The researcher employed active listening during interviews. The use of mathematical language was observed in the process.

3.5 VALIDITY ISSUES

According to Bless and Higson-Smith (2000:157), validity is the degree to which a study actually measures what it purports to measure. Concept mapping as an instrument was appropriately used for revealing details about how learners had constructed their understanding of mathematics learning. Observations were used to describe the behaviour of participants towards the study. Observations have led to a revelation that all participants were never exposed to concept mapping before their engagement in the project. Interviews were useful in identifying causes of poor understanding.

3.6 CONCLUSION

This chapter provides the rationale for using qualitative research design in finding out how learners' understanding of mathematical concepts is organized. The methodology selected is multiple case study because participants came from different schools. The instruments used for data collection were: simple observations, concept maps and interviews. The instruments were declared valid because the data collected appropriately addressed the research questions of this study. The next chapter provides a qualitative report on observations, interviews and concept maps. A detailed analysis of data collected is presented.

CHAPTER 4
DATA PRESENTATION AND ANALYSIS

4.1 INTRODUCTION

The 'Design and Method' chapter provided an overview of participants in the study, the rationale for using the instruments identified and confirmed their measurement as valid. This chapter presents the data collection process, data presentation and analysis, and the researcher values. The raw data collected include concept maps and interview transcripts (see appendices: H and I).

4.2 DATA COLLECTION PROCESS

Data from the twelve participants was categorically collected from the three research sites identified in the previous chapter. All the twelve participants were very cooperative and eager to stay behind after school to see the project through. Their background performance and gender orientation were not considered in the process of data collection. Data collection was done in the afternoons as a sign of conformity to the principle of protected teaching time as authorized by the North West Department of education. Data was collected through the use of concept maps, semi-structured interviews and observations.

Participants in each school were observed over an average total duration period of 10 hours. The observation process totaled 4 visits made to each of the participating schools. The lengths of the visits ranged from 30 minutes to 3 hours in every instance. The shortest time was 30 minutes in the first visit and the remaining three visits were averaged three hours each. Observation visits in all participating schools occurred over a period of two months.

In the first set of visits the researcher introduced himself and explained that the study was intended to learn more about how they understood mathematical concepts. Further that their real names would not be used and false names would be created and used in reporting their views. When learners were told that they were expected to draw concept maps, it seemed they were puzzled in all instances. Consent forms were given out for parents to complete and

the appointments for next series of visits were then secured in the respective cases. The researcher promised to collect the consents during the second visit.

Case study 1

The first case study included the following participants: Didi, Pinky and Tebogo (Not their real names). During the second visit, the researcher made a demonstration lesson on concept mapping using the sample concept map of the Natural Sciences and Mathematics repectively (see Appendix D and E). Learners participated in the process and made statements like "oho...this is a mind map". The researcher explained the difference between a mind map and a concept map.

When the researcher asked if they had understood what was presented, the learners were very slow in responding to these questions. Their mathematics teacher indicated that they were perhaps afraid of being told that they were wrong. Participants clearly lacked confidence. Consent forms were collected and participants were given a sheet of paper containing concepts prescribed for use in drawing the first concept map. Learners were each given sheet of paper on which they were expected to draw their respective concept maps. Pinky: "Oh, nna ke ne ke nagana gore re ya go dira maths mos". Researcher: "What do you mean? Pinky: "But there are no numbers. I thought we were going to do calculations". To Pinky, mathematics was about working with numerical symbols and not concepts.

It took learners a longer time and a bit of persuasion to produce the concept maps attached in appendices: 1 to 3. On the third visit, learners were asked to construct the concept map using the second set of concepts. The last visit was used for interviews based on the two maps they had drawn. The participants were thanked for their role in the study. The interview transcripts are attached as Appendix I.

Case study 2

The second case was made up of the following participants: Thato, Olebogeng, Agnes, Joe, katlego and Pule (Not their real names). On the second site visit, the researcher

started by collecting the consent forms. Concept mapping was then demonstrated by using the natural science sample map and mathematics sample maps. The collective then developed two other concept maps using the same set of concepts (*see* Appendix D & E). The researcher then gave out a sheet of paper containing the concepts prescribed for use in the construction of the first concept map. "But this is not mathematics," said Thato. "What is your view of mathematics?" asked the researcher. "Mathematics has numbers and patterns," Thato answered.

The researcher directed the participants to start drawing their concept maps on a given sheet of paper. The participants did not seem to struggle much with concept mapping after the presentation. Participants were persuaded to put as much information as they possibly could. The initial submissions were not neat, they were all asked to transcribe their work into a clean sheet and present both papers to the teacher for further submission to the researcher.

The third site visit, participants had a background experience of concept mapping. The researcher gave out a concept list prescribed for the construction of the second concept map. Clean sheets were given to all participants for them to construct their respective maps on. After assessing the concept maps for neatness, a few were returned. The affected learners were persuaded to transcribe the work onto a clean sheet of paper. The last visit was used for conducting reflective interviews on the concept maps that were drawn.

Case study 3

The subjects of research in the third case were: Tshepang, Bontle and Thapelo (Not their real names). The researcher started by collecting consent forms from the participants. The participants had no clue of what concept mapping was about. They indicated that they had not done it before. The researcher presented a demonstration lesson on concept mapping using the natural science and mathematics sample (see Appendix D and E). After a short presentation, the researcher checked with participants if they understood what they were expected to do. They all said they had understood.

The researcher gave out the concept list to be used in the construction of the first concept map. Learner were each given a sheet of paper and ordered to start constructing their concept maps. The concept maps submitted in the initial stages had exactly the same connectors as those in the sample map. The propositions were not standing as solid sentences. For example, "number system gives rational number". The researcher emphasized the use of appropriate connector to ensure that propositions read as sentences and ordered participants to rework their maps. Participants were advised to use as many concepts as they could.

On the third visit, the participants were familiar with concept mapping so the researcher gave out the second concept list and a clean sheet for use in the construction of the second concept map. During the last visit participants were interviewed to check how they had constructed their two maps. Participants were thanked for their participation in the project.

4.3 DATA PRESENTATION AND ANALYSIS

The 'Holistic approach' for presenting and analyzing research data was adopted for use in the current study. Burton, Brundrett and Jones (2008:144) advocate that data presentation and analysis are combined in the holistic approach. They (Burton *et al.*, 2008:144) note that data analysis should take into account: the research question, data relating to past research findings and the theoretical perspective from the literature.

4.3.1 Concept Mapping

A total of 24 concept maps were constructed (*see* Appendix H). Twelve concept maps were constructed from the first concept list and the other 12 from the second concept list. This section presents the preliminary interpretation of the raw data collected on concept mapping.

Case study 1

Concept map 1(*see* page 68)

In Didi's first concept map, the central concept (root) was real numbers and six connectors were radiated around the central concept. Connectors created were based on composition, definitions and examples. Four propositions were appropriately constructed, for example: "Integer includes positive and negative numbers". Two propositions were not labelled and 7 were inappropriately constructed.

Concept map 2 (*see* page 69)

Didi used the concept prism as a central theme in constructing her second concept map. Propositions were formed on the basis of components, type and uses. The propositions formed were radiated around the central concept. There were 3 inappropriate propositions, 4 non-propositions and 6 appropriately constructed propositions. The absence of connectors shows that Didi did not have the mathematical language for expressing the links that she identified.

Concept map 3 (*see* page 70)

Pinky used number systems as central theme. Connectors created were based on components and examples. Nine propositions were appropriately constructed; E.g. Zero and positive numbers are whole numbers. Two propositions were inappropriate; e.g. "rational numbers are integers" and one was not labelled. Concepts organization was hierarchical. Pinky shows a good understanding of natural numbers and whole numbers by showing that positive integers are natural numbers and that positive integers and zero form whole numbers.

Concept map 4 (*see* page 71)

Pinky used the concept angle as central concept. Her propositions were constructed on the bases of components and meanings. This is shown by the frequent use of "include"

as connector. Pinky had 7 inappropriate propositions, 1 non- proposition and 6 appropriately constructed propositions. The concept organization was hierarchical.

Concept map 5 (*see* page 72)

Tebogo used numbers system as a central theme. Concept connectors were meanings, components and examples. Five propositions were inappropriately created. Tebogo shows that a positive sign is a natural number and that zero and a positive sign combine to form whole numbers. This represents lack of language in expressing mathematics. Seven propositions were correctly constructed.

Concept map 6 (*see* page 73)

Tebogo used the concept angle as central concept. Her propositions were constructed on the bases of components and meanings. This is shown by the frequent use of "include" as connector. Tebogo had 9 inappropriate propositions and 6 appropriately constructed propositions. The concept organization was hierarchical.

Case study 2

Concept map 7 (*see* page 74)

Thato used number system as a central concept. Her propositions were all descriptive in character. Propositions made were based on components, types, uses and example. Seven propositions were inappropriately constructed. Nine propositions were appropriately constructed. The concept organization was a network. The concept identified the type of number systems and continued to show how natural numbers evolved through a series of other number systems to form real numbers.

Concept map 8 (*see* page 75)

Thato used angles as a central concept. Thato's propositions were based on uses and types. Ten propositions were not appropriately constructed, for example: "Area is used

to measure the capacity, an angle and to calculate side". This shows lack of skill in expressing mathematics. It implies that Thato has a lot of misconceptions about many concepts. Three propositions were appropriately constructed. The concept organization was a web.

Concept map 9 (*see* page 76)

Olebogeng used number systems as a central concept. Propositions made were based on components, examples and types. There were 15 appropriately constructed propositions. Four of the propositions were inappropriately constructed, for example: "fractions including numerator". This shows lack of skill in expressing mathematics. The concept organization was hierarchical. Olebogeng has a grasp of the content he was using.

Concept map 10 (*see* page 77)

Olebogeng used the concept line as a central concept. Propositions were formed on the basis of uses, components and types. Nine propositions were not appropriately constructed. Eighteen propositions were appropriate, for example: lines form an angle. The concept organization was a web.

Concept map 11 (*see* page 78)

Agnes used number system as a central concept. Propositions were expressed in terms of: definitions, types and examples. Propositions written were descriptive. There were 11 appropriately verbalized propositions. There were 4 non-propositions and 6 inappropriately constructed propositions, for example: "Number systems that can be used come from the type of numbers decimal". The concept organization was a network.

Concept map 12 (*see* page 79)

Agnes used the angle as a central concept. Propositions were expressed in terms of: uses and types. Propositions were descriptive. Agnes had 4 inappropriate propositions, 6 appropriate propositions and a non-proposition. The concept map organization was a network.

Concept map 13 (*see* page 80)

Joe used number systems as a central concept. Proposition were made on the basis of what they have or include, how they can be written and examples were used to reflect his understanding. Concepts were organized to form a hierarchy. Fifteen propositions were appropriately constructed. Five propositions were inappropriate. The inappropriate propositions show lack language in expressing mathematics. For example: "Exponents have powers and bases". A power has a base and an exponent.

Concept map 14 (*see* page 81)

Joe used lines as a central concept. Propositions were constructed on the basis of components, types, measuring units and definitions. There were ten appropriate propositions and five inappropriate propositions. The structure was a networked hierarchy. Joe intelligently used sketches to display his understanding of angle and angle types.

Concept map 15 (*see* page 82)

Katlego used number system as a central concept. Propositions were constructed on the basis of their composition structure, examples and types. All the sixteen propositions were appropriately constructed. The organization of concepts was a hierarchy. Examples were well used to reflect the adequacy of knowledge about the concepts.

Concept map 16 (*see* page 83)

Katlego used angle as a central concept. Propositions were constructed on the basis of their components and types. Nine propositions were appropriately constructed and 7 were inappropriately constructed. The concept map represents a networked hierarchy. There were no examples used.

Concept map 17 (*see* page 84)

Pule used number system as central concept. Propositions were constructed on the basis of their components, alternative writing and meanings. Seven propositions were inappropriate and 2 were appropriately constructed. The concept organization is a hierarchy. This shows a serious lack of skill in expressing mathematics. The examples given were correct. This implies that Pule has conceptual knowledge.

Concept 18 (*see* page 85)

Pule used the concept prism as a central concept. Propositions were constructed on the basis of meanings, components, types and uses. Ten propositions were not appropriately constructed. Three propositions were appropriate. The concepts radiated from the central concept. There were no examples used.

Case study 3

Concept map 19 (*see* page 86)

Tshepang used number system as a central concept. Propositions were constructed on the basis of components, types, alternative forms of writing and examples. The use of words like: has, included, involve and e.g. show that propositions were created on the basis of their component structures. Seventeen propositions were not appropriately constructed. Five propositions were appropriate. Examples were given but did not inform the constructed propositions. The concepts are radiated around rational numbers.

Concept map 20 (*see* page 87)

Tshepang used polygon as a central concept. Proposition were constructed on the basis of components, types, examples and uses. Nine propositions were inappropriately constructed. Eight propositions were appropriately constructed. The concept map organization is a web. Example of formulae and mathematical symbols representing the concepts were provided. Emerging from the map, one could conclude that she has a background of procedural knowledge.

Concept map 21 (*see* page 88)

Bontle used number systems as a central concept. Propositions were constructed on the basis of their components, example and uses. Bontle used examples to show her background knowledge of the concepts. Seven propositions were appropriately constructed. Six propositions were inappropriately constructed. The concept map drawn represents a web.

Concept map 22 (*see* page 89)

Bontle used polygon as a central concept. Her propositions were based on components, examples and uses. Six propositions were not appropriately constructed. Six propositions were appropriate. The map drawn represents a web. Some critical links were overlooked, for example the between angle and triangle.

Concept map 23 (*see* page 90)

Thapelo used number system as a central concept. Propositions were constructed on the basis of components and uses. Fifteen appropriate propositions were formed. Six propositions were inappropriate. The use of "can be" shows that the learner did not have the appropriate language to express mathematics. Examples were used to make propositions formed more meaningful. The concept map was organized in the form of a web.

Concept map 24 (*see* page 91)

Thapelo used line as her central concept. The propositions constructed included meanings, components and types. Thirteen propositions were not appropriately verbalized. Ten propositions were appropriate. The map drawn represents a web. The use of "can copy" shows lack of language or vocabulary in expressing mathematics.

4.3.2 Interviews

Twenty-four semi-structured interviews with participants were conducted. The learners were informed and shown the devise used to record the interviews and they were all comfortable with the process. I taped and transcribed data on interviews conducted in the three respective research sites. The interview transcripts (see Appendix I) represent the learners' actual words and nothing was done to correct impressions and grammar.
Didi's first interview

Didi said that she used real numbers as a central concept because it was a more inclusive concept. From the transcript it emerged that she knew a lot about exponents because of the frequency of exposition about the concept but could not explicate more on the concept despite the researcher's constant probing.

Didi's second interview

In the second interview Didi said she used prism as a starting concept because it had a lot of concepts on it. Didi implied that the concept prism was an inclusive word. Didi indicated that she knew more about angles because she learned about them in grade 9 and 10. She gave examples but could not provide accurate information about their measurements.

Pinky's first interview

Pinky indicated that she started her concept map with number systems because there were more things to say about the concept. She said that she knew a lot about fraction

because she could write their symbols and use them in problem solving. Pinky could not pronounce the word proportion appropriately. She claimed she had no idea about it.

Pinky's second interview

Pinky started her second concept map with prism because it had more things that could be used for other things. She said that she knew a lot about volumes but she could not provide the correct formulae for calculating volume. When probed to give more details she said that she just knew how to calculate it. What pinky seems to be having is procedural knowledge that was acquired by rote.

Tebogo's first interview

Tebogo started with the number systems because without the number systems anything else would not be there. Tebogo meant that the concept number system was broad and inclusive of all other concepts. The other concepts were left out because he did not have any understanding about them. Understanding in this regard will imply knowledge. Tebogo used examples as a mode showing his understanding of fractions. The concepts that were not known to Tebogo were blamed on the failure of classroom instruction to cover them.

Tebogo's second interview

Tebogo started this concept map with angle because it is essential for the formation of angle types and triangles. Tebogo explained how much he knew about triangle and declared that triangle could be found in a prism. Tebogo made a mistake of indicating that a triangle has a volume. Tebogo indicated that he was well informed on prisms.

Case study 2

Thato's first interview

Thato explained how she used number systems as a central concept and that she did not leave any concept out. Thato explained that she knew a lot about number systems and she categorically continued to outline its types through the use of examples. Thato could not confidently express her understanding of real numbers but the information she produced on the subject was accurate. This shows that confidence is not a reliable predictor of understanding.

Thato's second interview

Thato indicated that she used angle as a central concept because it had relations with all the concepts prescribed. She gave an overview of the concept map by naming other concept that relate to the central concept. She claimed that she knew a lot about areas but her explanation on the subject was rather inaccurate. She indicated that she did not know much about polygons but was able to give some accurate information on the subject.

Olebogeng's first interview

Olebogeng explained that he started her concept map with number systems and he logically explained how other systems fit into it. Olebogeng explained that he had no knowledge of percentage and inverse. This represents lack of vocabulary in expressing mathematics. Olebogeng revealed his procedural knowledge on exponents.

Olebogeng's second interview

Olebogeng started his concept map with lines and showed how line contributed to the formation of angle and triangles but gave inaccurate information on area and volume. Olebogeng explained the formula for calculating volume as the basis for his knowledge on the subject.

Agnes' first interview

Agnes explained that she started her concept map with number systems and proceeded to give a lengthy description of its component systems. Her lengthy description did not include real numbers. She indicated that she knew a lot about natural numbers and a little about identities. The identities she knew about were trigonometric identities.

Agnes' second interview

Agnes explained that she started her concept map with angles. Agnes failed to make sensible links between angle and area, and between area and volume. She said that polygons had two sides and that prisms were units for measuring angles. The utterances made in this interview showed that Agnes had a lot of misconceptions about the content of learning out 3. Her explanation of area showed that understanding was problematic.

Joe's first interview

Joe gave a logical explanation of the components of number systems which were his central concept. Joe left out the concept inverse because he did not have any conceptual grasp on the concept. He said that he knew a lot about natural numbers because they were part of the daily spoken language. Joe did not know much about inverses and identities.

Joe's second interview

Joe explained that he started the concept map with angle and indicated that angle had lines. Joe inaccurately pointed out that the types of angles are triangles and further mentioned that acute, obtuse and right-angles were types of triangle. This is a complete misrepresentation of fact which amounts to lack of adequate understanding of all related concepts.

Katlego's first interview

Katlego made a logical explanation by indicating how smaller number system fit into each other but could not complete the sequence. The explanation descended from number systems to whole numbers. In the process she failed to show that a rational number system was a much bigger set than integers and she made no mention of natural numbers. Katlego indicated that she did not know anything about identities. Katlego showed that she understood fraction by naming fraction types, giving examples and indicating that decimals and ratios were alternative forms of writing fractions.

Katlego's second interview

Katlego explained the concept angle and that it could be found in a triangle. She identified types of: angles, triangles, and polygons. She mentioned the properties of a triangular prism as being: area, volume, length and breath. Katlego said that she knew a lot about angles. She said that she did not know much about polygons.

Pule's first interview

Pule said that he started drawing his concept map with the word number systems because it involved all other concepts. He said that number systems form the basis of mathematics. He said that whole numbers were dividing to form exponents and that exponents had powers and bases. He also said that a fraction had a base and power. Pule confused fraction with powers. He did not have an adequate understanding of number systems.

Pule's second interview

Pule said that he started his concept map with polygon because it was inclusive of other concepts. Pule made incorrect statements about angle, triangles, ratios, area and volume. He said that triangles could be used to calculate area using length time breath. This showed that Pule did not have the necessary language skill to communicate mathematical ideas.

Case study 3

Tshepang's first interview

Tshepang's explanation of how she started her concept map was illogical. For example she said: "A number system is a type of a rational number and a rational number include real number, natural number, whole number and integers". She did not seem to know what she was talking about. The explanation given about fractions was adequate. She indicated that she did not know anything about inverses.

Tshepang's second interview

Tshepang gave a descriptive view of what the components of a polygon are. She could not explain that such components were all found in a triangular prism. She could not provide good explanation of other concepts except that she knew a lot about polygons.

Bontle's first interview

Bontle gave an illogical explanation of a number system. She did not seem to know how the components of a number system fit into each other. For example she said that whole numbers include integers; natural numbers involve rational numbers; and rational numbers include real numbers. This shows a very poor understanding of mathematical ideas.

Bontle's second interview

Bontle said something about this and the next about the other. There was no coherence of ideas presented. For example, she said: "Prisms have length and area and triangle". She proceeded to say: "examples of triangle are equilateral and isosceles". There is no logic and coherence to complement the adequacy of her understanding. Bontle said that she was coming across the concept isosceles for the first time and yet she mentioned that it was an example of a triangle. This showed that Bontle's understanding of mathematical concepts was poorly organized.

Thapelo's first interview

Thapelo said that she used number system as a central concept because it included everything there was in the concept list. She explained each system in terms of its component structure but made no mention of real numbers. She explained that she knew a lot about fractions and proceeded to mention types of fractions. She did not know a lot about proportions

Thapelo's second interview

Thapelo started by explaining how line contributed to the formation of different angles. She incorrectly stated that: "obtuse can copy a prism." Other statements were mathematically correct. For example, she said that polygons have many sides. She indicated that she did not know much about isosceles.

4.3.3 Coding and analysis

From the preliminary interpretations and initial inferences made in the data collection process and the data presentation the following issues: appropriateness and organization emerged. From the first issue the following themes emerged: appropriate and inappropriate. I then categorized appropriate data into four categories, namely: Coherent, not coherent, logical and illogical. Inappropriate data was mainly due to confusion and incorrect use of language. Inappropriate data is generally quantified as non-understanding

According to Huera *et al.* (2009:2) a proposition is a basic unit of meaning in a concept map. The concept map is a reflection of how an individual understands a set of given concepts. Combining these views leads to the perception that a proposition is the basic unit of understanding. Non- propositions were not consider in the coding of quantities. These propositions are characterized by instances of confusion and incorrect use of language (non-understanding), incoherent and illogically arranged propositions (one-on-one propositions lying far apart), and coherent and logical propositions (regarded as a chunks. Logical implies that the concepts are arranged in a proper sequence.

According to Tall (2009:4) non-understanding is more than just the absence of understanding. This clearly implies that non-understanding is not absence of understanding. For example, Didi indicated that fractions include rational numbers.

Case study 1

The data table in figure 4.3.1 represents the quantity of propositions according to the type of appropriateness, organization and the related sub-categories. The data presented in figure 4.3.1 represent a quantity propositions retrieved form concept maps drawn by Didi, Pinky and Tebogo. The subsequent discussion is based on the coded data and responses from the interview transcripts (*see* Appendix I)

Figure 4.3.1 (Themes for case study 1)

Name	Concept map	Quantity of units	Instrumental		Relational	organization
			Non-understanding	Appropriate		
			Confusion and Incorrect language use	Incoherent & illogical	Chunk	
Didi	1	13	7	4	2	Network
	2	9	3	6		Network
Pinky	3	11	2		9	Hierarchy
	4	13	7	6		Hierarchy
Tebogo	5	14	5		9	Hierarchy
	6	15	9	6		Hierarchy
Total		75	33	22	20	

The percentage analysis the given below is the sum of propositions (units) in a given category divided by the total quantity of units times 100%. The above table shows that 73,3% of the total propositions formed constitute instrumental understanding. Propositions that are categorized as non-understanding amount to 44%. Incoherent and illogical propositions make up 29,3% of the case study. The remaining 26.7% of propositions formed reflect relational understanding. These propositions were appropriate, coherent and logical.

Case study 2

The data table in figure 4.3.1 represents the quantity of propositions according to the type of appropriateness, organization and the related sub-categories. The data presented in figure 4.3.2 represent a quantity propositions that were constructed by the following subjects: Thato, Olebogeng, Agnes, Joe, Katlego and Pule.

Figure 4.3.2 (Themes for case study 2)

	Concept map	Quantity of units	Instrumental		Relational	Organization
			Non-understanding	Appropriate		
			Confusion & Incorrect language	Incoherent & illogical	Coherent & logical (Chunk)	
Thato	7	16	7		9	Network
	8	13	10	3		Network
Olebogeng	9	15			15	Hierarchy
	10	20	9		11	Hierarchy
Agnes	11	21	10	11		Web
	12	10	4	6		Web
Joe	13	20	5		15	Hierarchy
	14	15	5		10	Network
Katlego	15	16			16	Hierarchy
	16	16	7		9	Hierarchy
Pule	17	9	7	2		Network
	18	13	10	3		Radiated
Total		184	74	25	85	

The above table shows that 53,8% of the total propositions formed constitute instrumental understanding. Propositions that are categorized as non-understanding amount to 40%. Incoherent and illogical propositions make up 13,8% of the case study. The remaining 46,2% of propositions formed reflect relational understanding. Propositions that are quantified as relational were found to be appropriate, coherent and logical.

Case study 3

The data table in figure 4.3.1 represents the quantity of propositions according to the type of appropriateness, organization and the related sub-categories. The data presented in figure 4.3.1 represent a quantity propositions drawn by the following subjects: Tshepang, Bontle and Thapelo.

Figure 4.3.3 (Themes for case study 3)

	Concept map	Quantity of units	Instrumental		Relational	Organization
			Non- understanding	Appropriate		
			Confusion & Incorrect language	Incoherence & illogical	Coherence & logical (Chunk)	
Tshepang	19	18	10	8		Network
	20	17	9	8		Web
Bontle	21	13	6	7		Web
	22	12	6	6		Web
Thapelo	23	21	6		15	Web
	24	23	13	10		Web
Total		104	50	39	15	

The above table shows that 86% of the total propositions formed constitute instrumental understanding. Propositions that are categorized as non-understanding amount to 48%. Incoherent and illogical propositions make up 38% of the case study. The remaining 14% of propositions formed reflect relational understanding. These propositions were appropriate, coherent and logical. Case study 2 and 3 were analyzed using the method presented in figure 4.3.1. The following table reflects a summary of findings in the three case studies.

Figure 4.3.4 (Summary of findings)

	Understanding		Themes		
Case studies	Instrumental	Relational	Non-understanding	Incoherence & illogical	Coherence & logical
1	73,3%	26,7	44%	29,3	26,7
2	53,8	46,2	40%	13,8%	46,2
3	86%	14%	38%	38%	14%
Aggregate %	71%	29%	40,7%	30,3%	29%

Figure 4.3.4 shows that learners' understanding in mathematics learning is predominantly instrumental in all cases that were studied. Non- understanding which is characterized by confusion and lack of language in expressing is the initial stage of mathematical understanding. Many learners have constructed their understanding in this section of instrumental understanding. Incoherence and illogical stage emerges from non-understanding. Statistics show that a greater quantity of understanding constructed by learners is illogical and incoherent. Relational understanding which is categorized by coherence and logic has the least quantity of codes implying that fewer learners construct their understanding of mathematics learning relationally.

4.4 THE RESEARCHER VALUES

During this research process, I kept on reflecting on the ethical considerations, assumptions and the research questions that were identified in chapter 1. I looked for biases and prejudices and assessed their possible effect on the research process. I noted that the manner in which the three groups of participants were introduced to concept mapping were not the same. Perhaps it was influenced by some participants' level of curiosity and the researcher's expectations on the project.

Learners who provided lengthy explanations on how they started drawing their maps were not asked why they started with that specific central concept. I made a blanket assumption that the reason is captured in the lengthy response. I note that the sample used to demonstrate concept mapping might have had an effect on how most of the concept maps were designed.

In managing this bias, I influenced learners to show some creativity and originality in the design.

4.5 CONCLUSION

This chapter gave an in-depth analysis on observation, concept mapping and interviews. From the analysis process the following issues emerged: appropriateness, logic and coherence. The chapter found that a learner understands a thing if he or she can do the following: identify its types, uses and components, and provide relevant examples. The frequency of instrumental was found to be 71% whilst the frequency of relational understanding is 29%. Confusion, lack of language skills for expressing mathematics and the inability to think logically were key factors affecting the quality of one's understanding. Learners' understanding was found to be constructed in a continuum consisting of: non-understanding, incoherent and illogical clustering, and coherent and logical reasoning. Chapter 5 presents a synthesis of all findings and makes some recommendations for further study.

CHAPTER 5
FINDINGS AND RECOMENDATIONS

5.1 INTRODUCTION

The previous chapter presented an in-depth analysis of the collected data and critical findings that emerged from the data interpretation process. The purpose of this chapter is to synthesize the critical findings that emerged from the previous chapter. In fulfilling the purpose of the chapter, the following topics are discussed: Findings, recommendations, and summary and conclusions.

5.2 FINDINGS

5.2.1 How do individual learners construct their understanding of mathematics concepts?

- A learner understands a thing if he or she can do the following: identify its types, uses and components; and can provide relevant examples.
- The understanding constructed by learners was a continuum consisting of the following stages: Non-understanding (confusion and incorrect use of language), incoherence and illogical clustering, coherent and logical abstractions (Chunking). The initial stage of continuum is termed non-understanding because agents of understanding are mixed up. Non-understanding is not the total absence of understanding but represents a situation where things that are not related were linked directly. For example, indicating that fractions were components of a set of natural numbers. During interviews Bontle remarked that whole numbers included integers. (*see* Bontle's first interview in section 4.3.2)
- Learner understanding occasioned was twofold: i.e. it was 71% instrumental and 29% relational. (*see* Figure 4.3.4) Non-understanding and incoherence illogical cluster are recursively organized to form instrumental understanding. Coherent and logical abstraction emerges from incoherence and illogical

cluster through re-organization of components which is effected through further learning. (*see* the sequence of theme in figure 4.3.1)

- The organization of learner understanding is such that it flows from a more inclusive concept to the least inclusive concept. Learners were mostly starting the concept maps with the concepts which were seen to be big. (*see* Pinky' first interview in Section 4.3.2)

5.2.2 What are the causes of poor understanding in mathematics learning?

- Learners lack the language skills for expressing the mathematical relation.(*see* concept map 23 in section 4.3.1) It also includes inappropriate positioning of concepts at the end of a connector. The propositions are not appropriately verbalized. For example, number system is a type of rational number. This suggests that the learner concerned is confused. Confusion is a major cause of poor understanding.(*see* Tshepang's first interview in section 4.3.2)

5.3 RECOMMENDATIONS

5.3.1 How do individual learners construct their understanding of mathematics concepts?

- The content embedded in any mathematical theme should include types, uses and components, and examples to initiate instrumental understanding.
- Pedagogical practice should be context based, organized logically and coherently to include the integratedness of concepts to ensure that relational understanding is occasioned.

5.3.2 What are the causes of poor understanding in mathematics learning?

- If assessment had included concept mapping, educators would have discovered the deficiencies in the learners' inability to comprehend mathematical concepts. Subsequent teaching could have been used to redress the lapses identified. For this reason, concept mapping is recommended for

use in classroom practices both as a tool of assessment and pedagogy. All mathematics teachers should be trained on concept mapping for them to use it effectively as a tool of assessment.

- The omissions in concept mapping were due to inadequate exposure to mathematical concepts embedded in a specified learning outcome. Given the integrated nature of learning and that comprehensively learning was done in isolated bits or pieces, it is worth recommending that teachers should always have the integratedness of concepts whenever they are teaching.
- Deliberate effort to reach out to all teachers to increase their use of language in mathematics in mathematics teaching is recommended. Mathematics teachers should be encouraged to prescribe language based activities for learners. Poorly developed language skill aggravates the misconceptions of the questions asked on externalized question papers and these result in underperformance. The effort to improve learners' language competence could aid the understanding of mathematical concepts, improve mathematics learning performance and put an end to the hue and cry that follow the release of the end of year results.

5.4 SUMMARY AND CONCLUSION

There are several conclusions that can be drawn from the study. First, it supports the instrumental and relational framework. Secondly, it supports the systems theory by showing how instrumental understanding emerges into a relational form of understanding. Third, a learner understands a thing if he or she can do the following: identify its types, uses and components; and provides relevant example. Fourth, that concept mapping both as tools of assessment and pedagogy is worthwhile. Fifth, the study has achieved its purpose and the limitations did not have an adverse effect on the findings. Future directions must be geared towards the development of teaching activities that could aid the attainment of the understanding that is both coherent and logical.

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APPENDICES

APPENDIX A



NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
MAFIKENG CAMPUS

07 JULY 2009

TO: The APO Manger

Taledi APO

Dear Sir

REQUEST FOR PERMISSION TO CONDUCT A RESEARCH STUDY

This letter serves as a request to conduct a research titled: "Utility of concept maps as a tool to assess learners' understanding of mathematics concepts" under the supervision of Prof Thapelo Mamiala. The research will involve Mathematics learners as the main participants. The purpose of the research is to explore how learners construct their understanding in mathematics learning

I am a part time Masters Degree student in Mathematics/Science Education at North West University (Mafikeng Campus), South Africa. Mathematics is integrated in all learning areas and it is needed in almost all careers. The researcher hopes to help both teachers and learners in their quest to improve mathematics instruction, mathematics learning and the learning attainment in mathematics.

Confidentiality will be exercised throughout the data collection process and the collected information will only be used for the purpose of the research study. No reference will be made to the schools or the teachers and learners that will participate in the study, pseudonyms will be used.

Thank you in advance for your co-operation

Yours faithfully

.....

Tumisang Mokgothu

Faculty of Education, School of Under Graduate Studies
NorthWestUniversity
Private Bag X2046, Mmabatho,2735
Tel: 018 389 2032/2081, Fax: 018 389 2342 www.nwu.ac.za

APPENDIX B

education
Lefapha la Thuto
Onderwys Departement
Department of Education
NORTH WEST PROVINCE gvaltyn@nwpg.gov.za

17 van Niekerk Street
Vryburg 8600
Tel: +2753 927 0405
Fax: +2753 927 5407
Email:

DR RUTH SEGOMOTSI MOMPATI DISTRICT

OFFICE OF THE APO MANAGER
TALEDI AREA PROJECT OFFICE

Re: Approval in respect of a request to conduct research

This letter serves to indicate that approval is hereby granted to Mr. T.F. Mokgothu to proceed with research in respect of the study about the 'Utility of concept maps as a tool to assess learners' understanding of mathematics concepts'.

The onus lies with the researcher to negotiate appropriate and relevant time schedules with the schools and all gatekeepers concerned. The Department of Education whishes him well in this endeavor. We look forward to derive benefit from his effort.

Kind regards

GP Valtyn

APO Manager

APPENDIX C

07 JULY 2009

TO: The Parent



NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
MAFIKENG CAMPUS

CONSENT FOR PARENTS

I ..... (Full names of parent/guardian), parent/guardian of ..... (Full names of learner) agree (s) that my child may participate in the research. I understand that the research is undertaken to provide a better understanding on how learners may become more interested in mathematics and not to compile marks that may contribute towards my child's CASS. I was also informed that the Researcher will not use real names in the final research report.

....

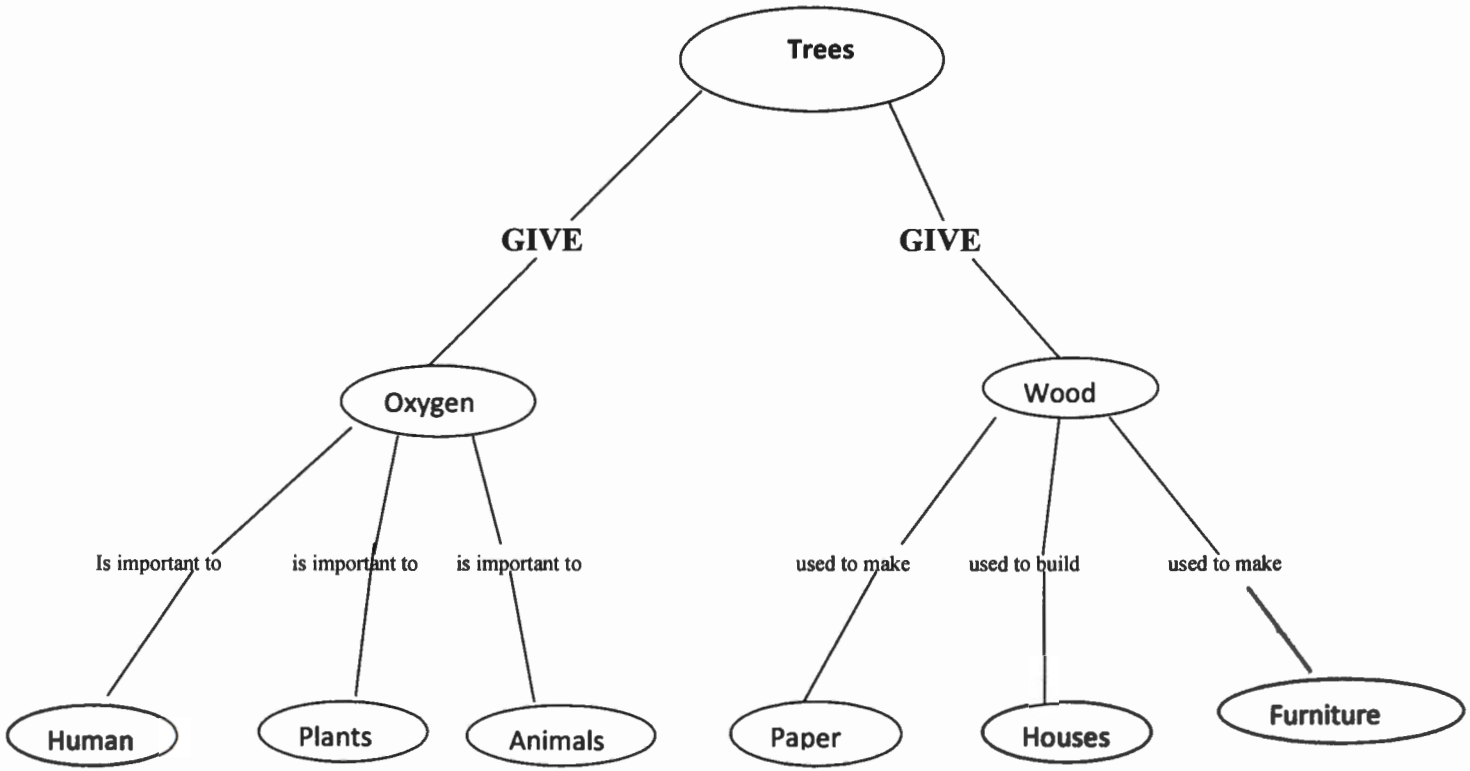
Signature of the Parent

Date signed

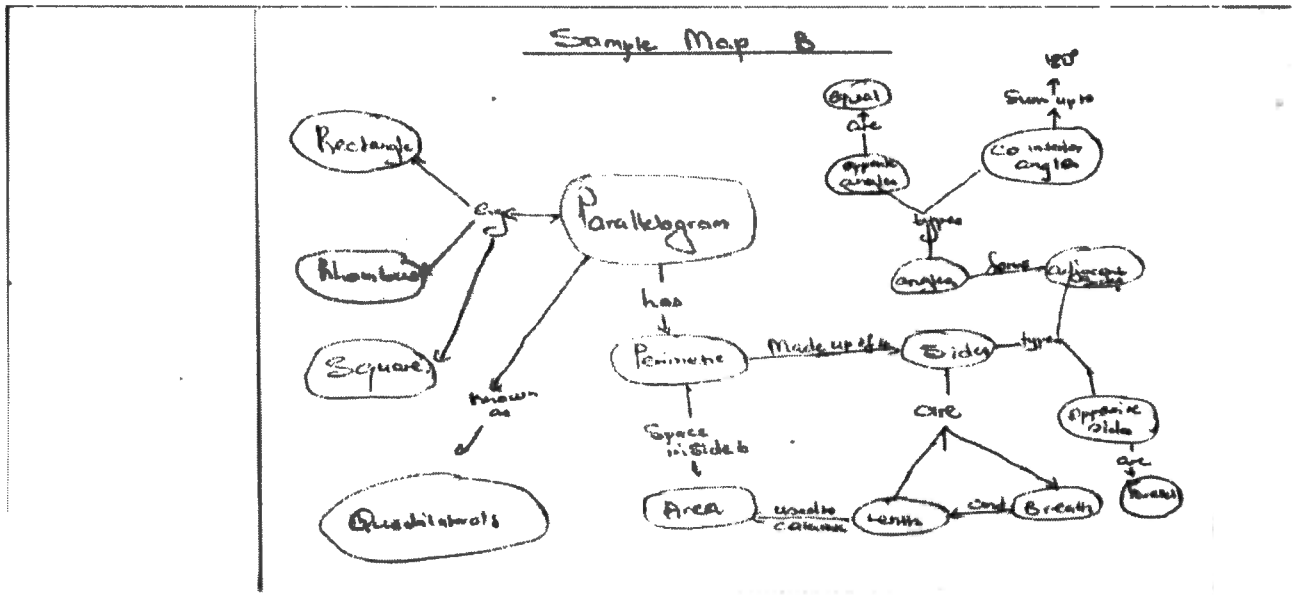
Faculty of Education, School of Under Graduate Studies
North West University
Private Bag X2046, Mmabatho, 2735
Tel: 018 389 2032/2081, Fax: 018 389 2342 • www.nwu.ac.za

APPENDIX D

SAMPLE CONCEPT MAP: NATURAL SCIENCES

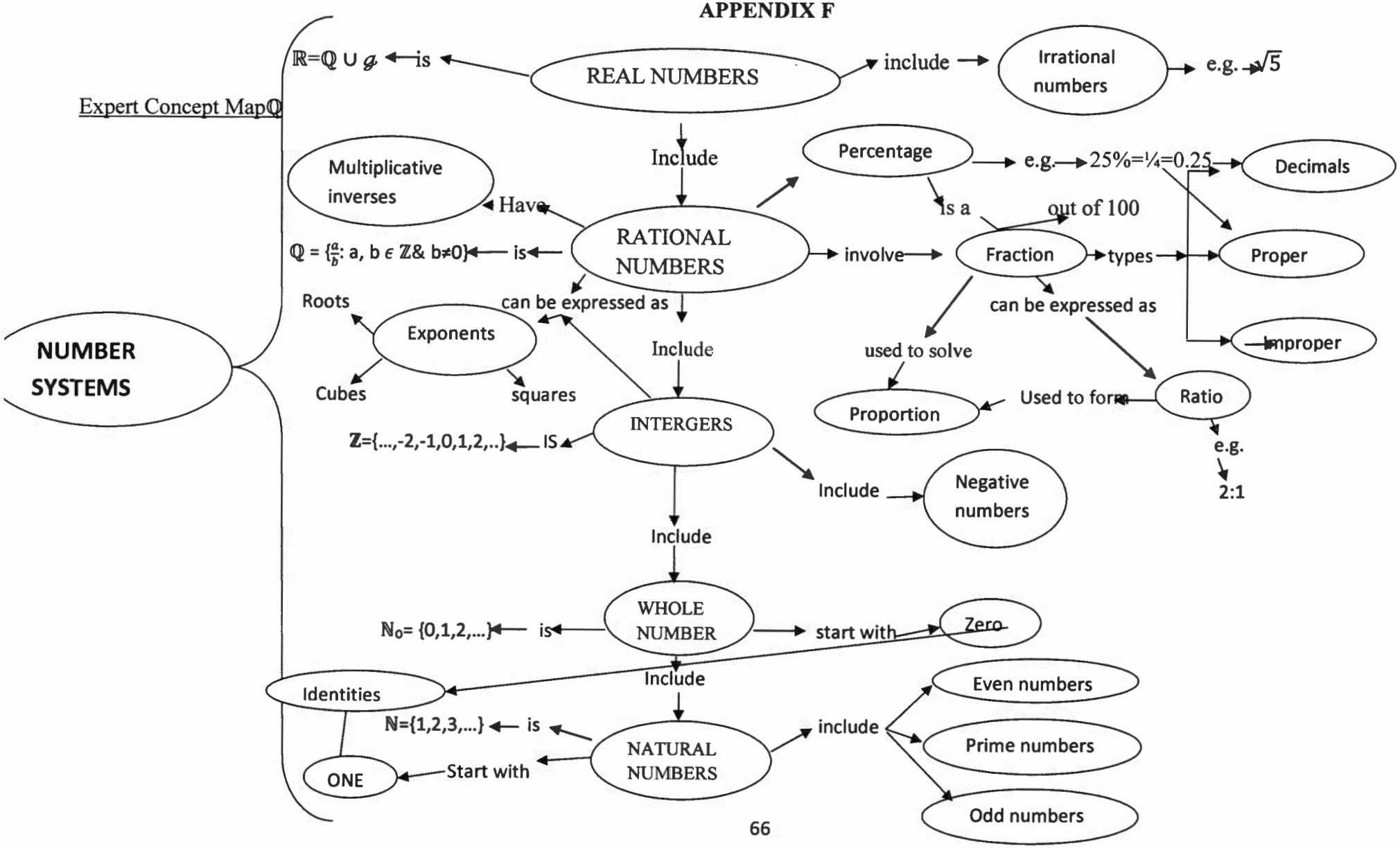


APPENDIX E

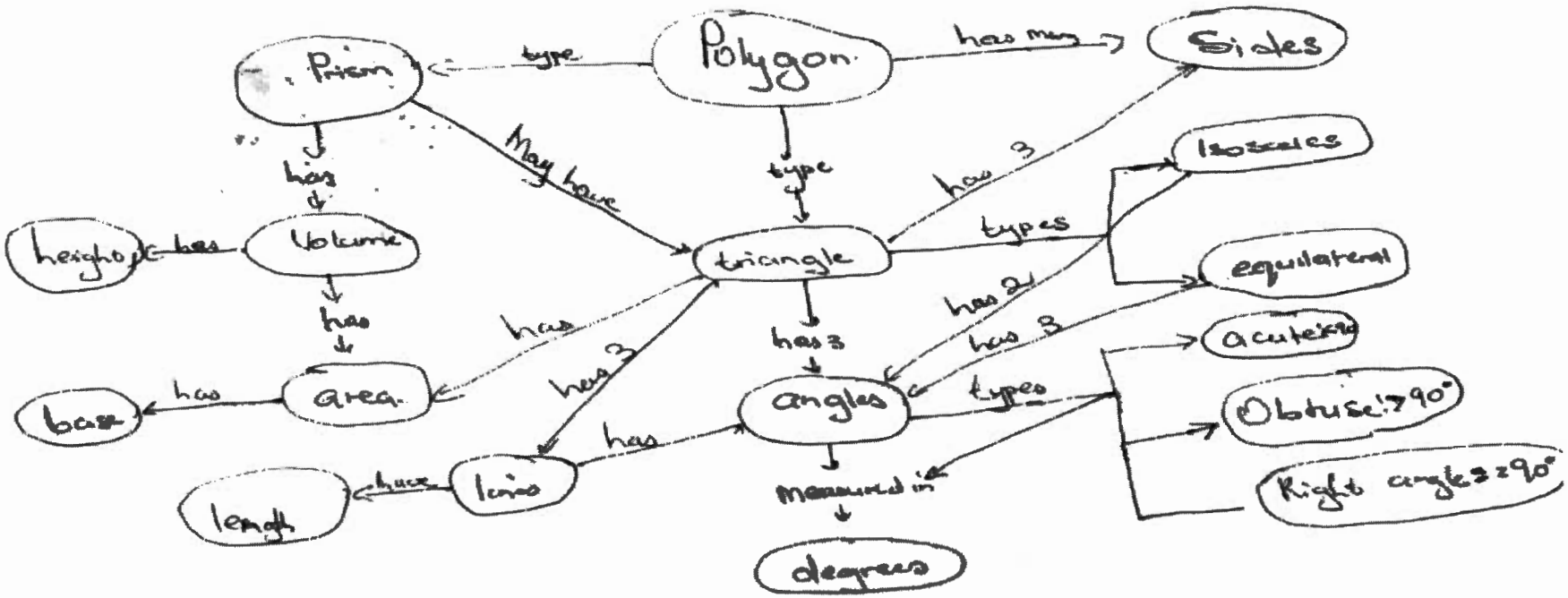

Sample Map B
Rectangle
Rhombus
Square
Parallelogram
has
Perimeter
Made up of
Sides
type
are
Space inside
Area
used to calculate
Length
and
Breath
Opposite sides
are
Parallel
Equal
are
Opposite angles
Co interior angles
Sum up to
180°
types
angles
forms
Quadrilaterals
known as

APPENDIX F

Expert Concept Map



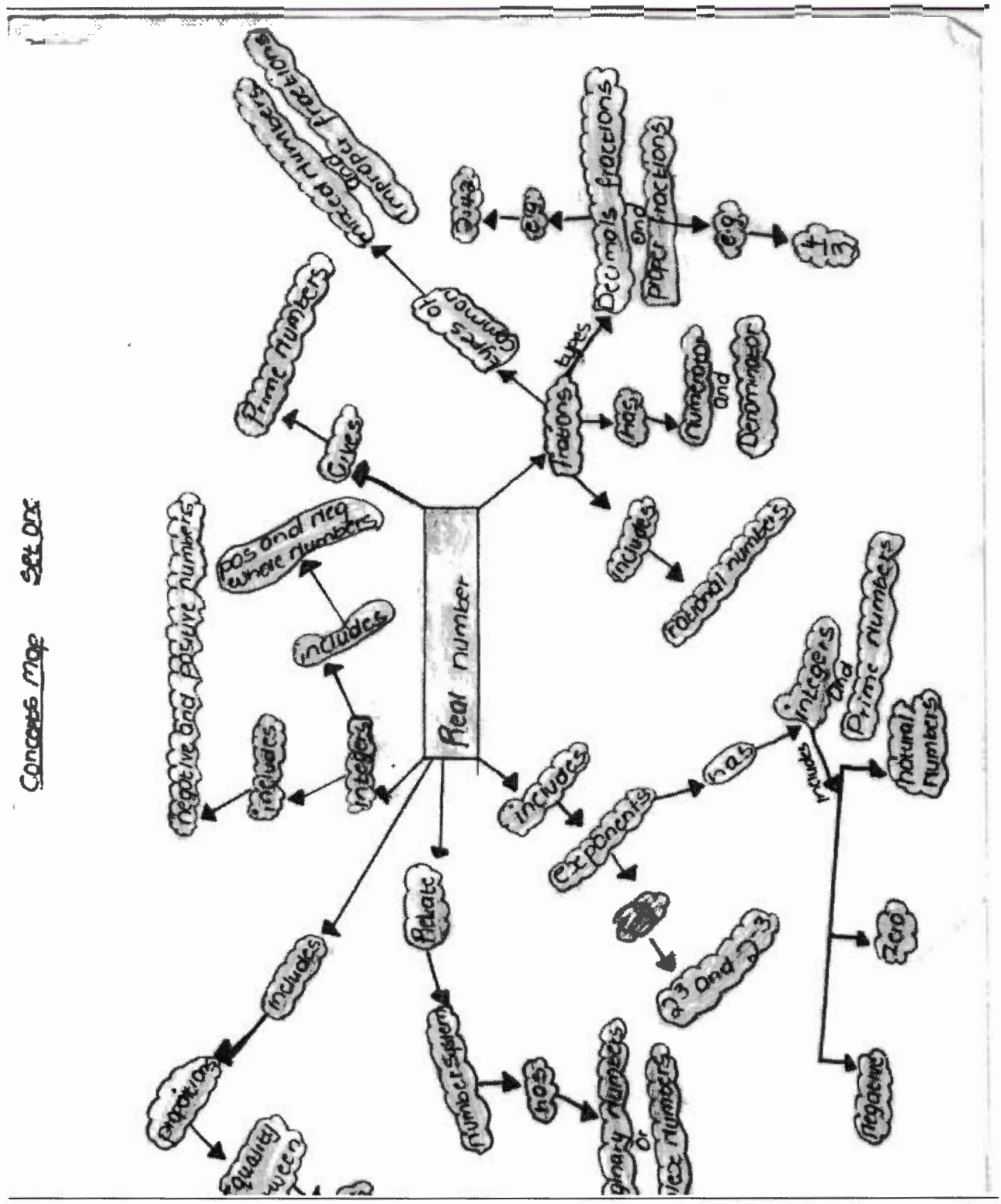
APPENDIX G


Polygon
Prism
type
has many
Sides
May have
type
has
Volume
has
height
has
area
has
base
has
triangle
has 3
Isosceles
types
equilateral
has 2
has 3
has 3
has
has 3
angles
types
acute
Obtuse
Right angle
has
lines
have
length
measured in
degrees

APPENDIX H (Learners' concept maps)

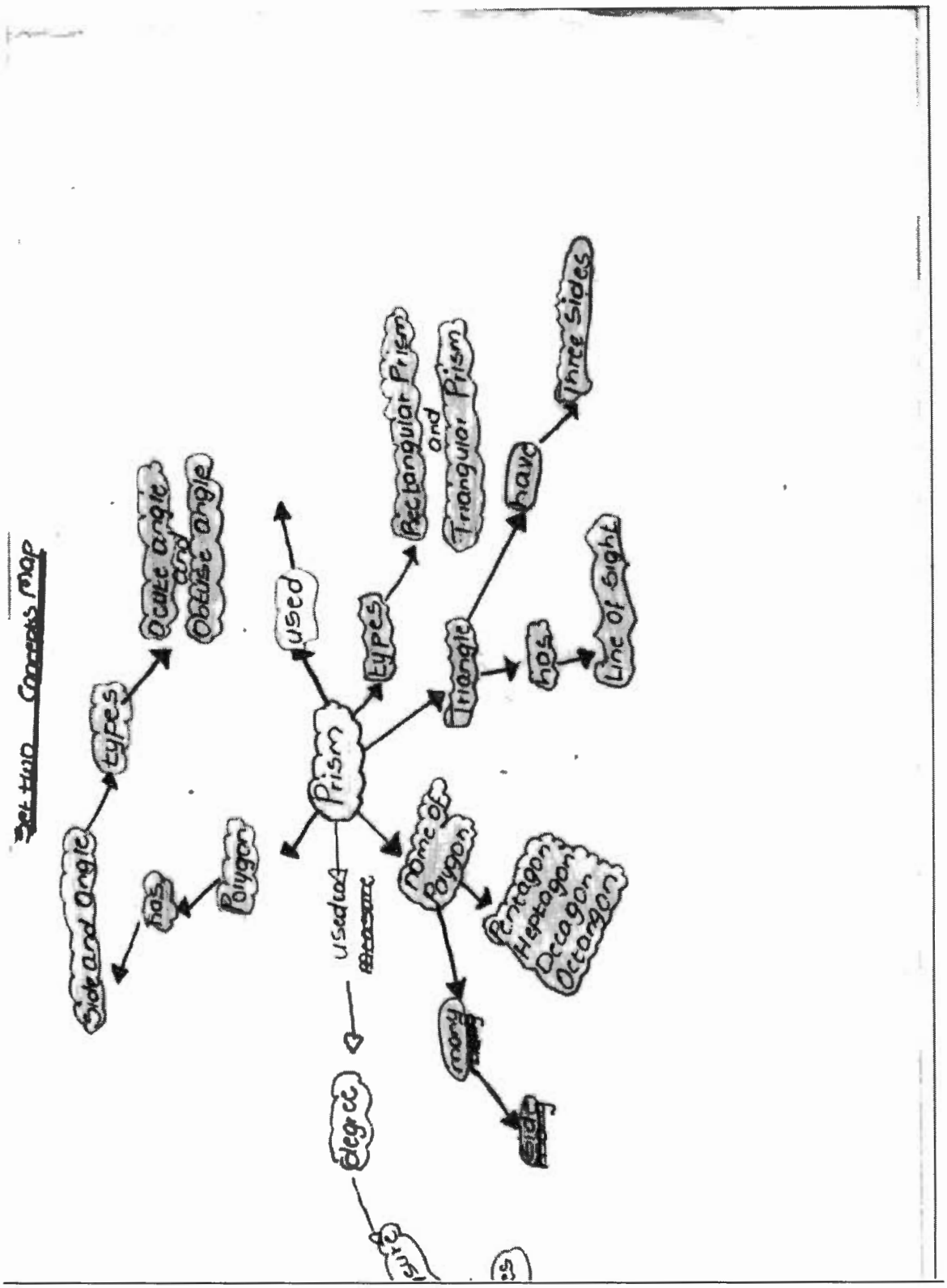
CONCEPT MAP 1

Didi's first concept map (on Learning Outcome 1)



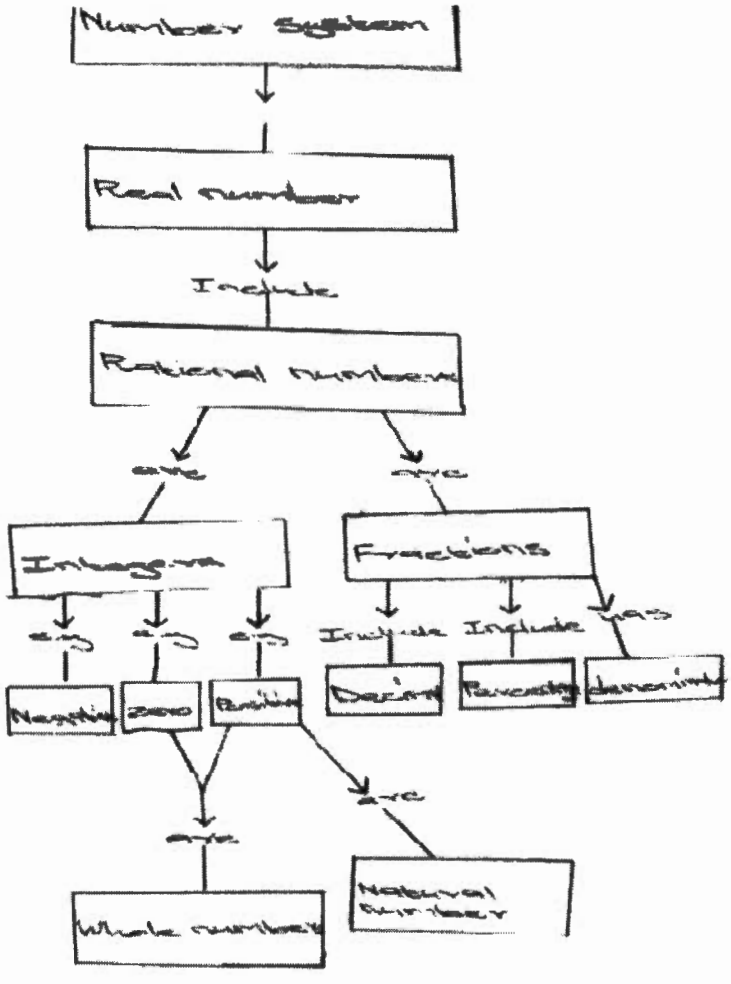
CONCEPT MAP 2

Didi' s second concept map (on Learning Outcome 3)



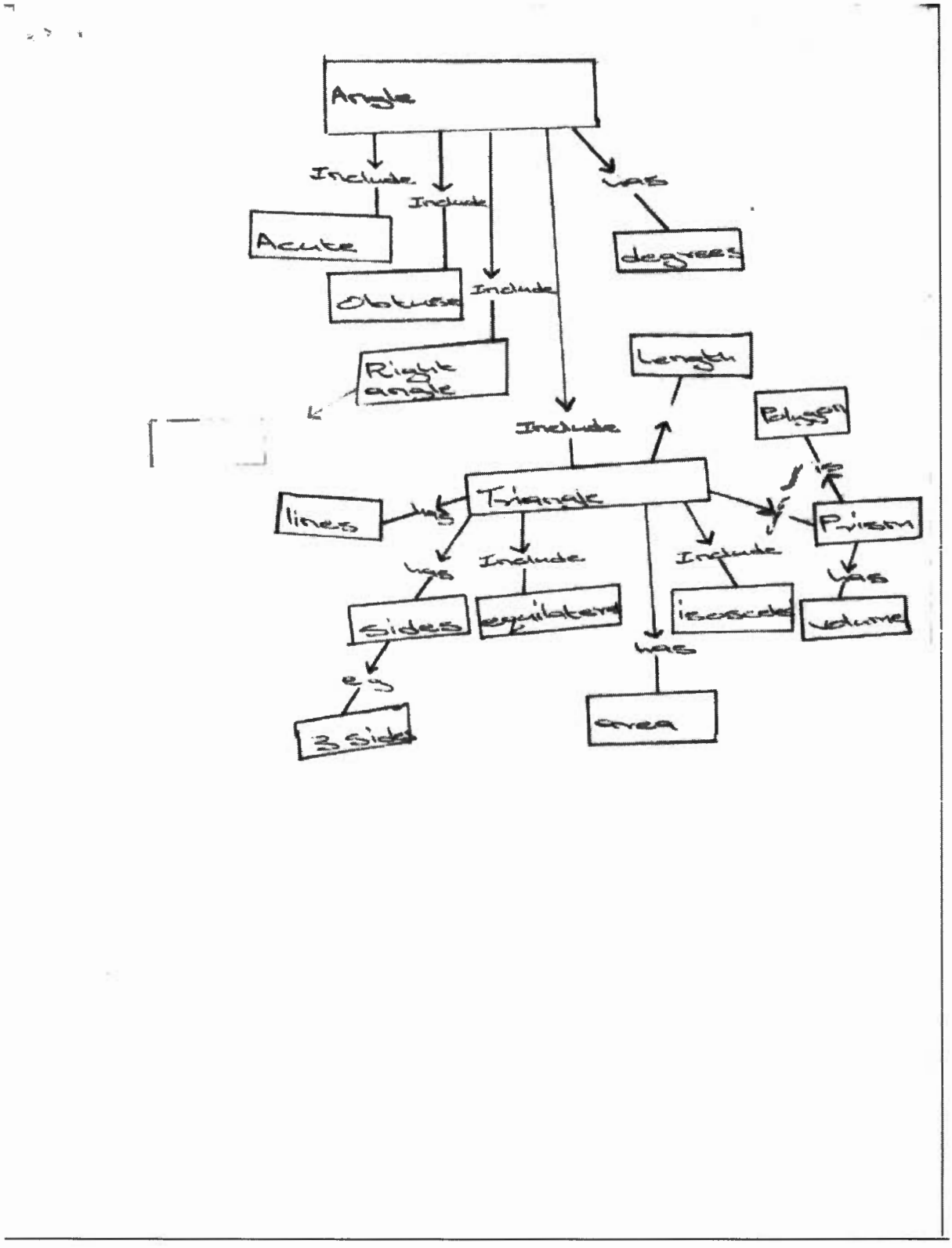
CONCEPT MAP 3

Pinky's first concept map (on Learning Outcome 1)



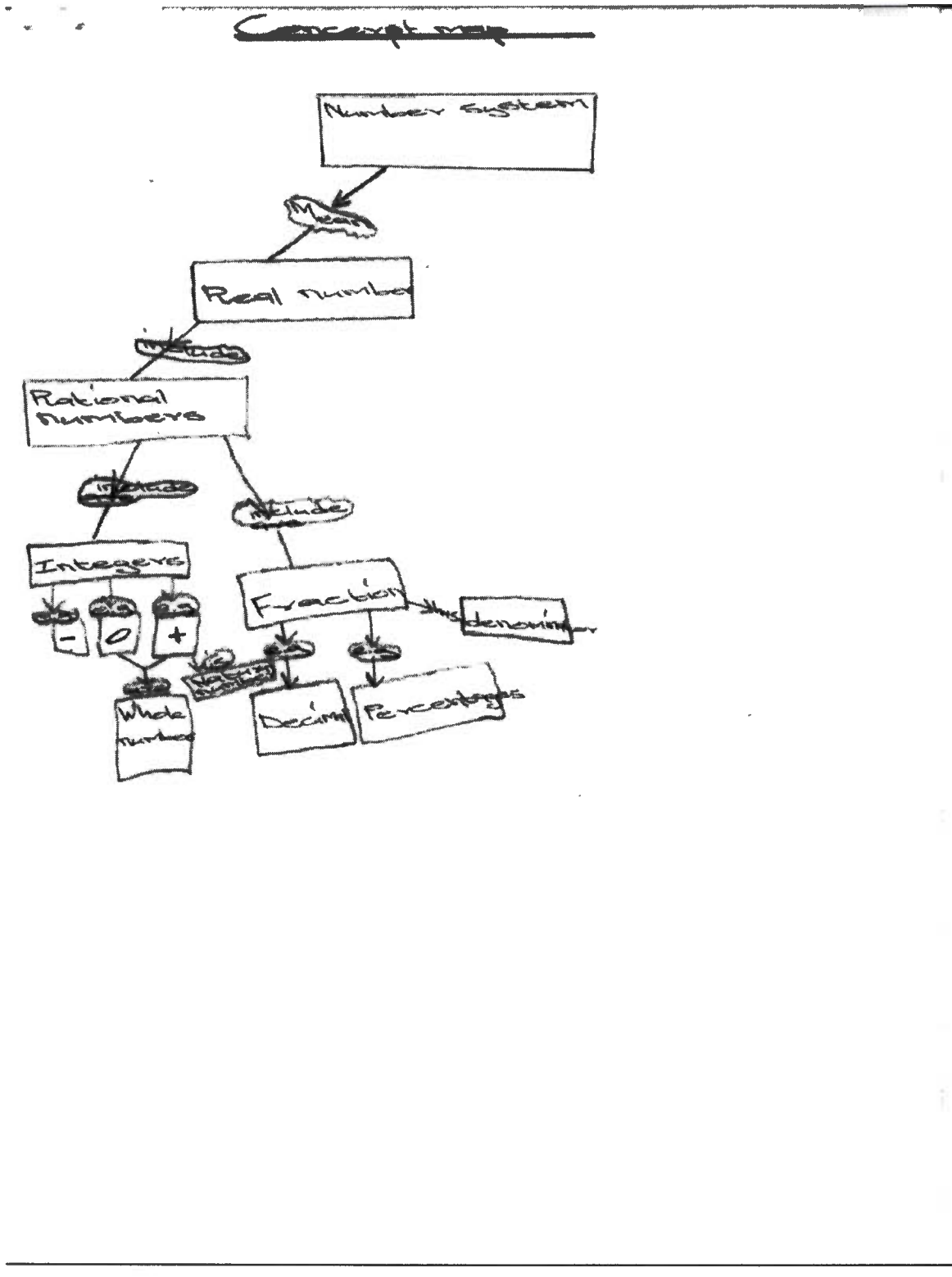
CONCEPT MAP 4

Pinky's second concept map (on Learning Outcome 2)



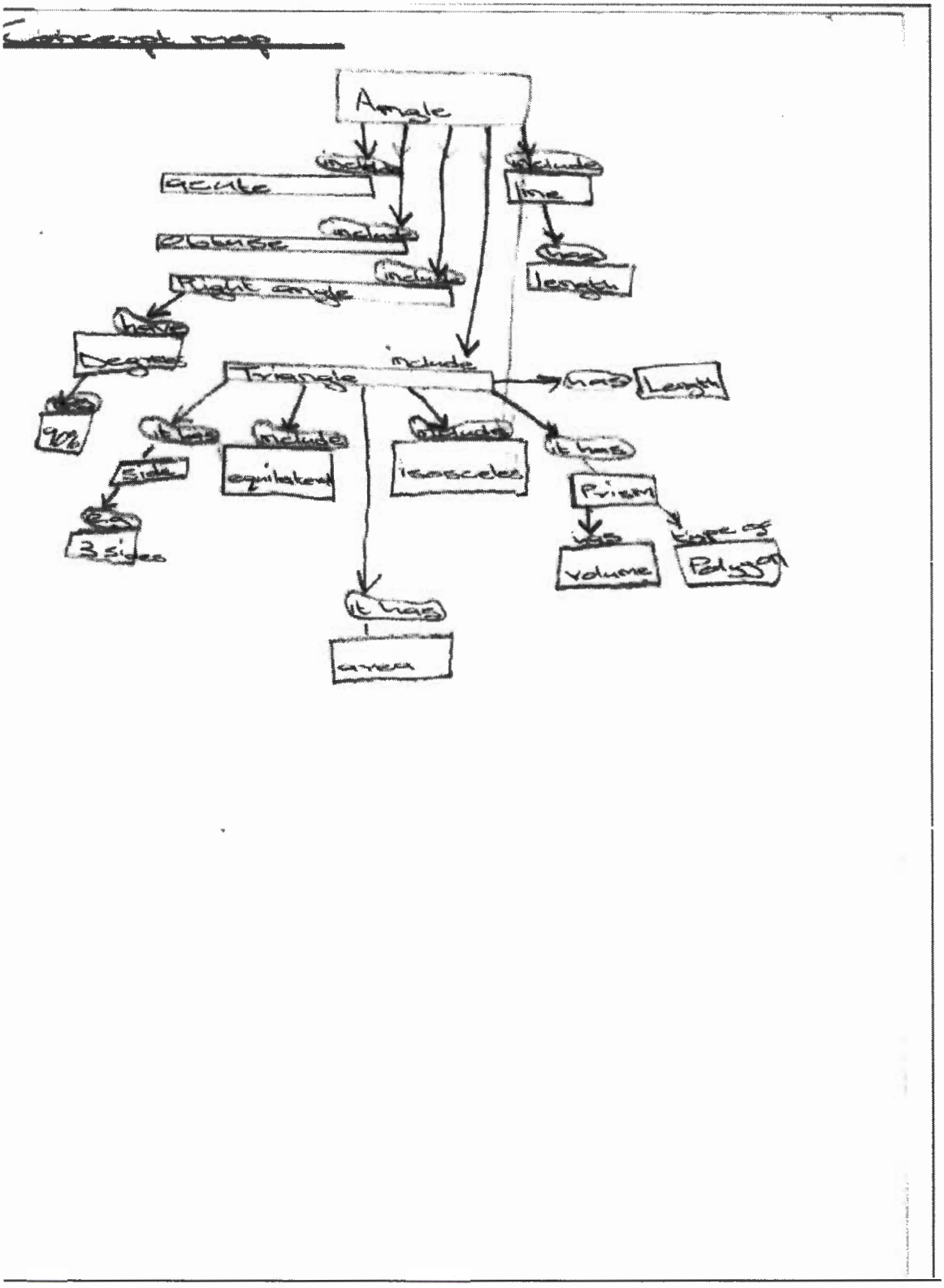
CONCEPT MAP 5

Tebogo's first concept map (on Learning Outcome 1)



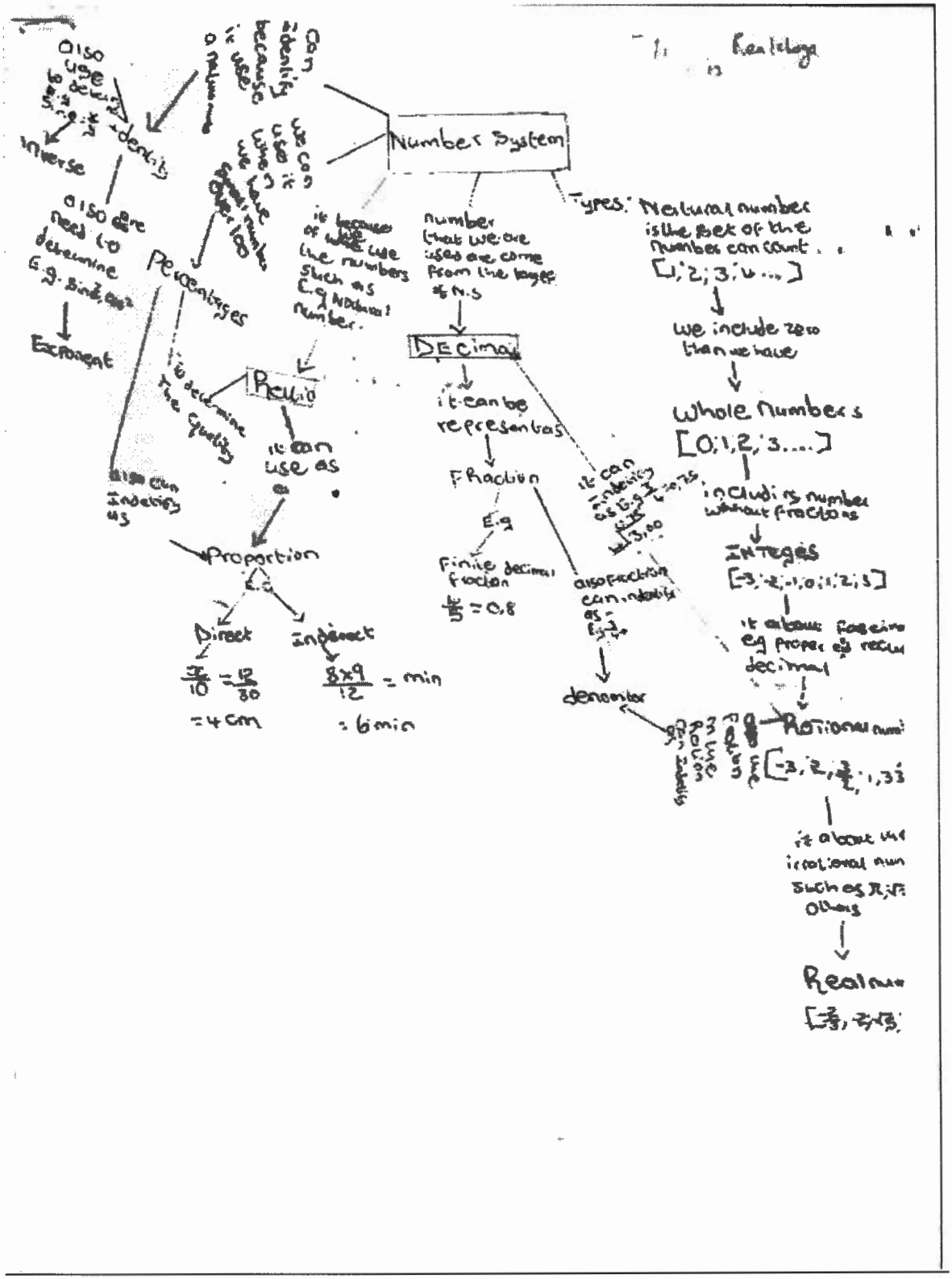
CONCEPT MAP 6

Tebogo's second concept map (on Learning Outcome 3)



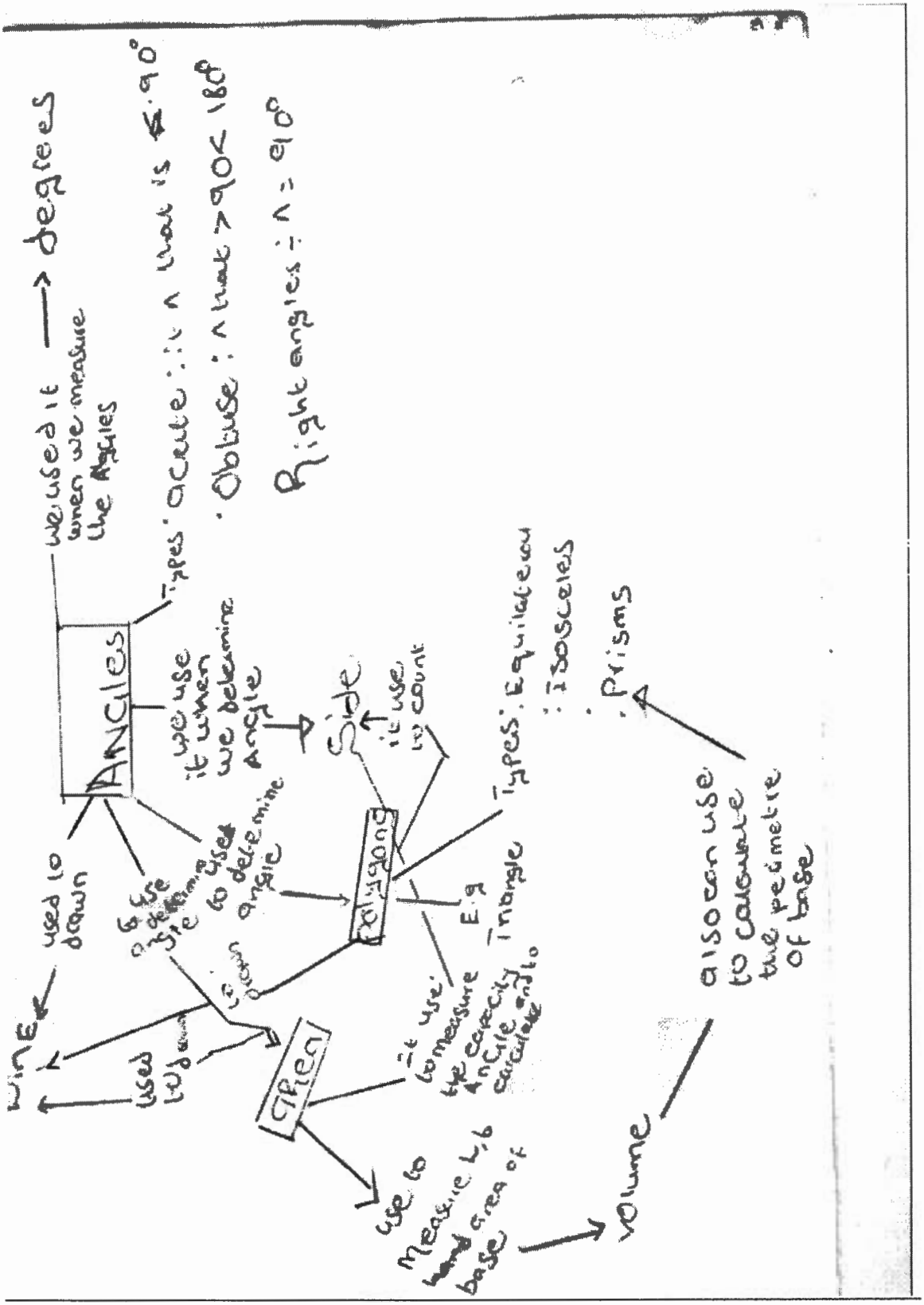
CONCEPT MAP 7

Thato's first concept map (on Learning Outcome 1)



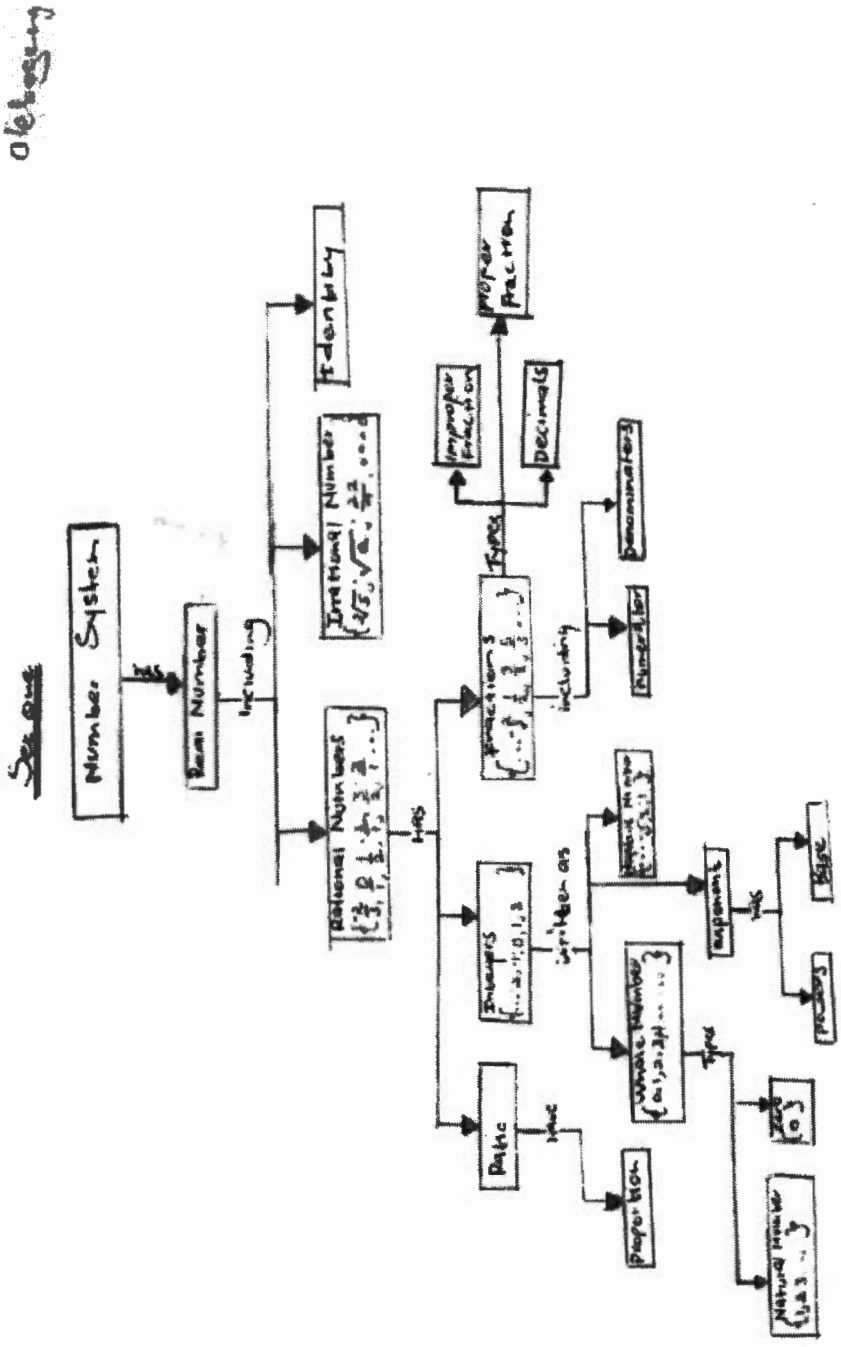
CONCEPT MAP 8

Thato's second concept map (on Learning Outcome 3)



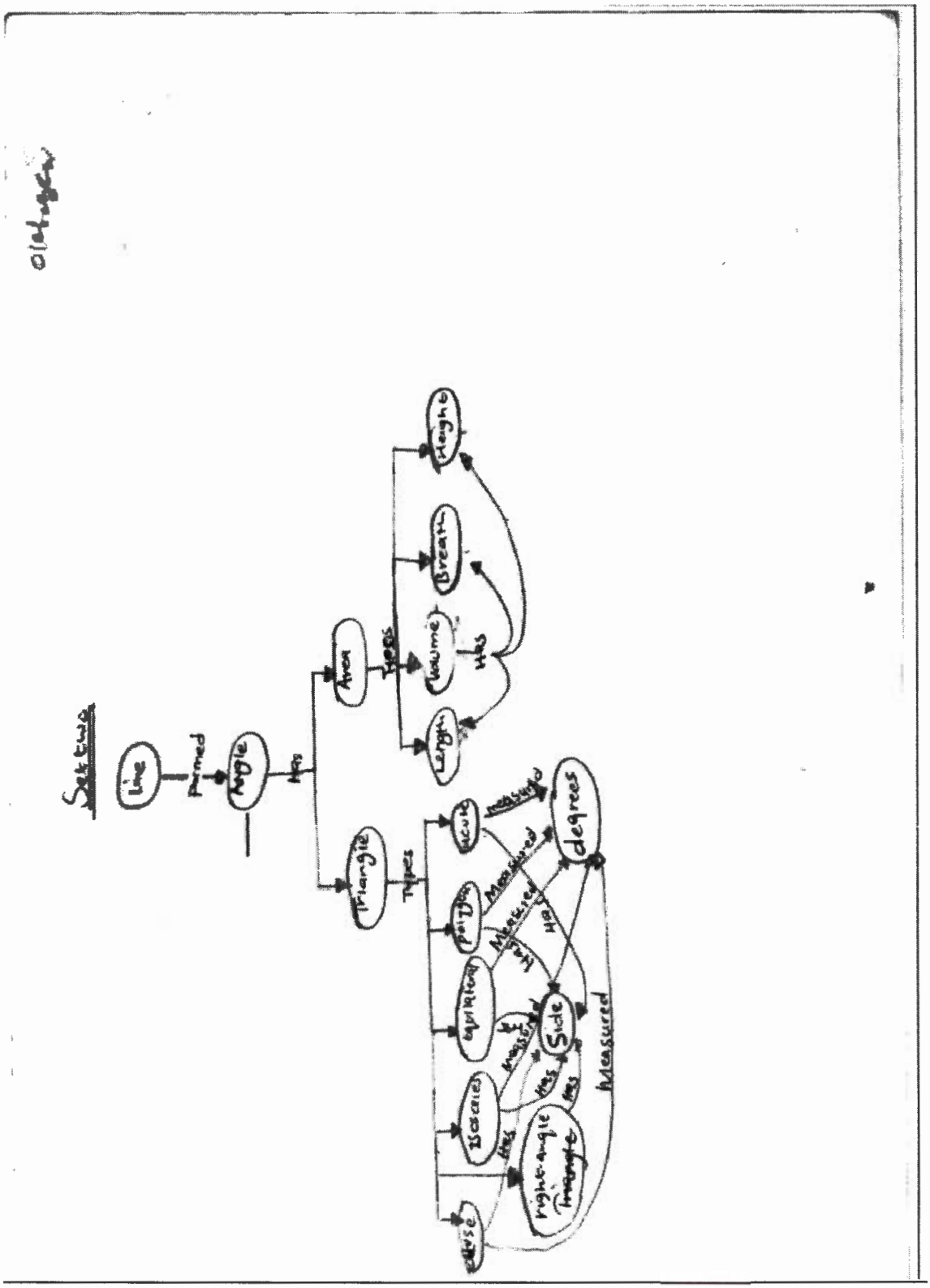
CONCEPT MAP 9

Olebogeng's first concept map (on Learning Outcome 1)



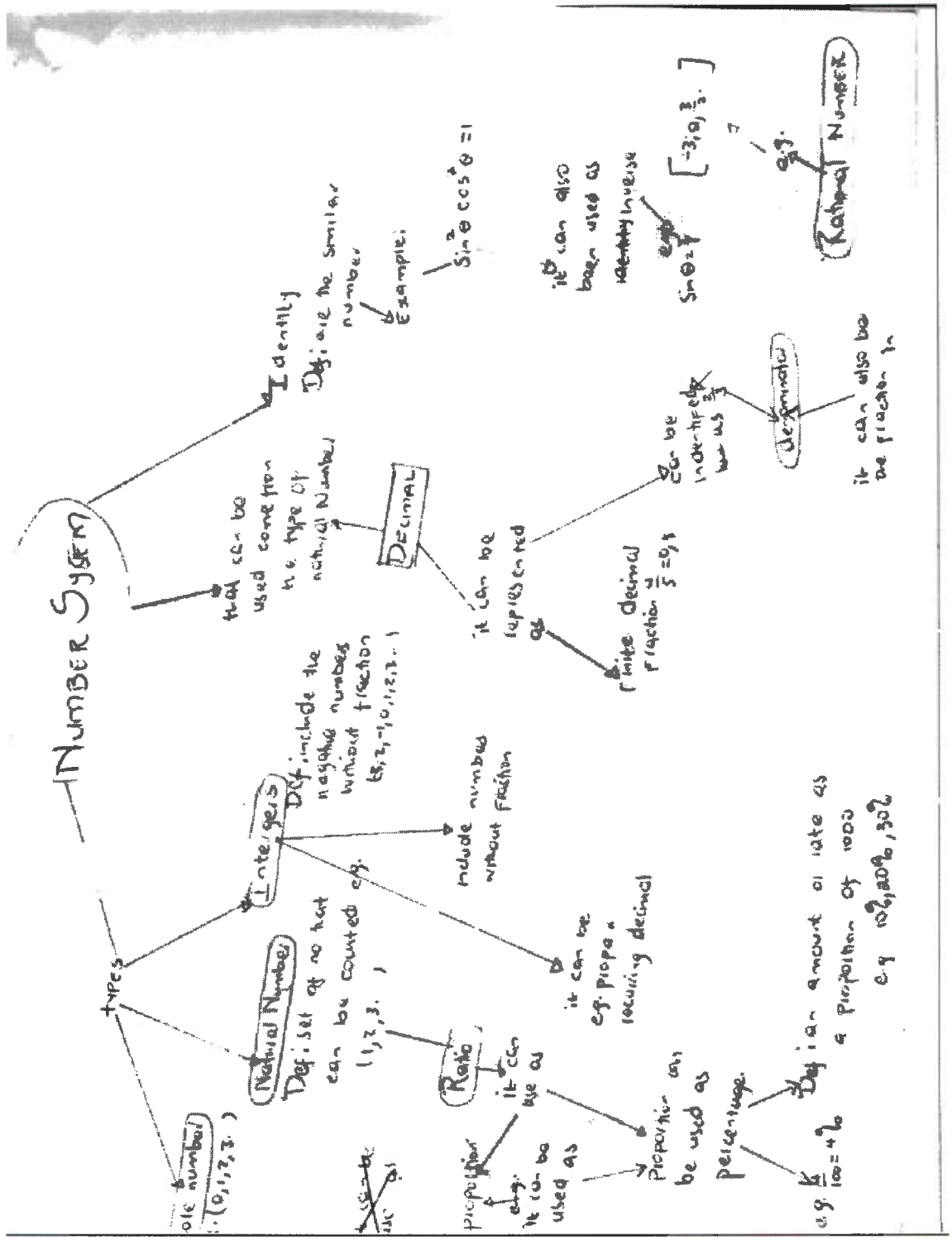
CONCEPT MAP 10

Olebogeng's second concept map (on Learning Outcome 3)



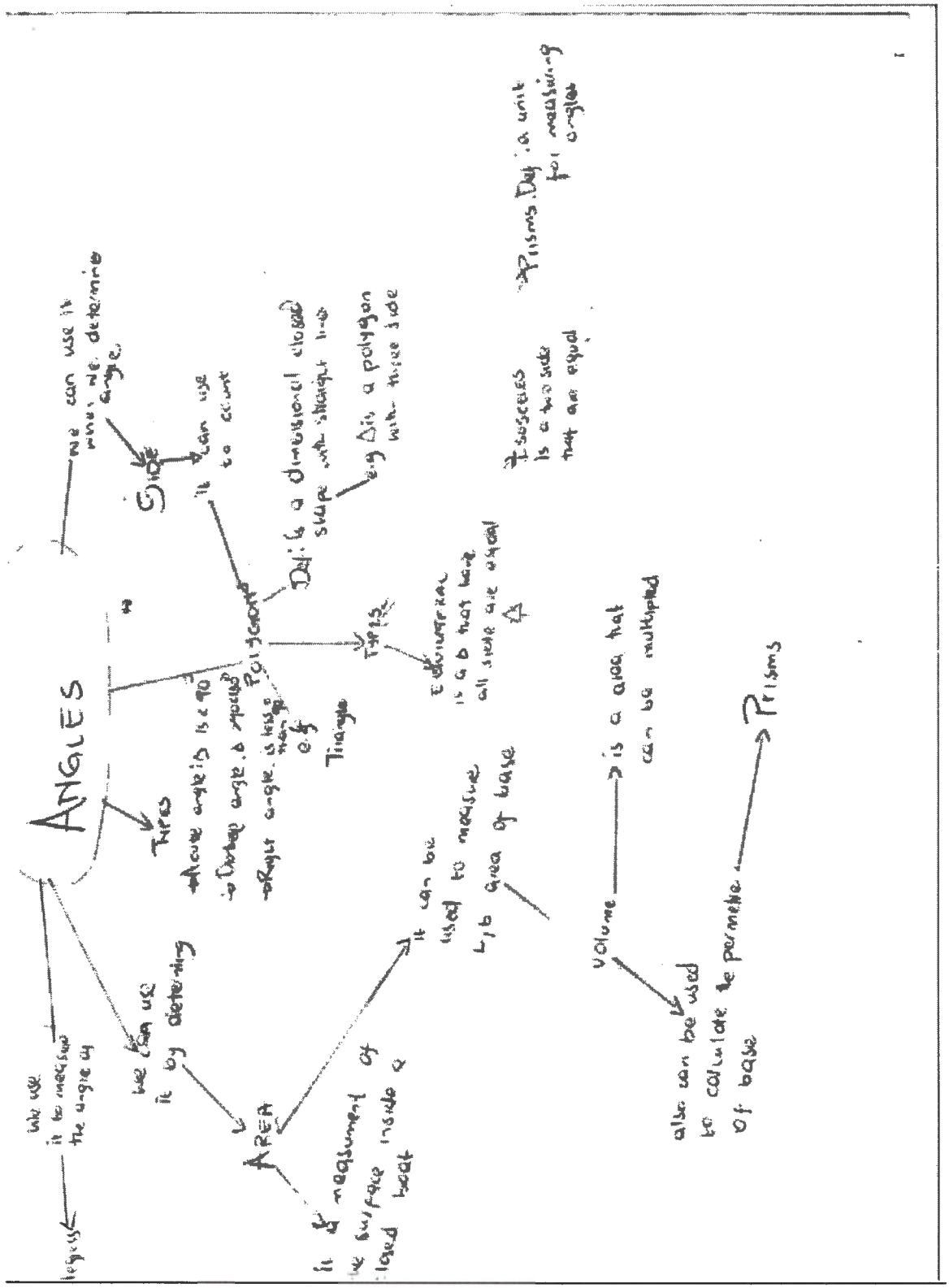
CONCEPT MAP 11

Agnes's first concept map (on Learning Outcome 1)



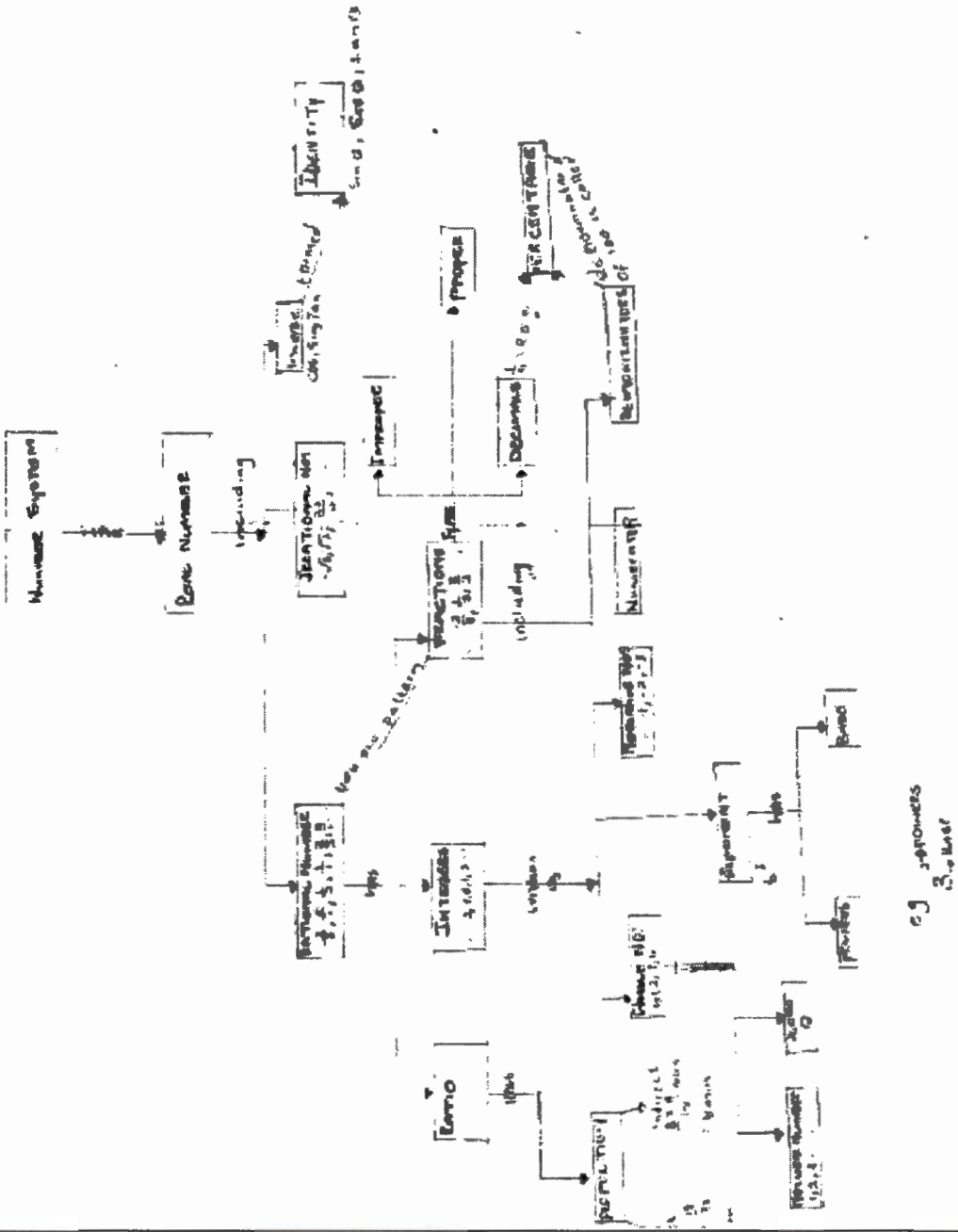
CONCEPT MAP 12

Agnes's second concept map (on Learning Outcome 3)



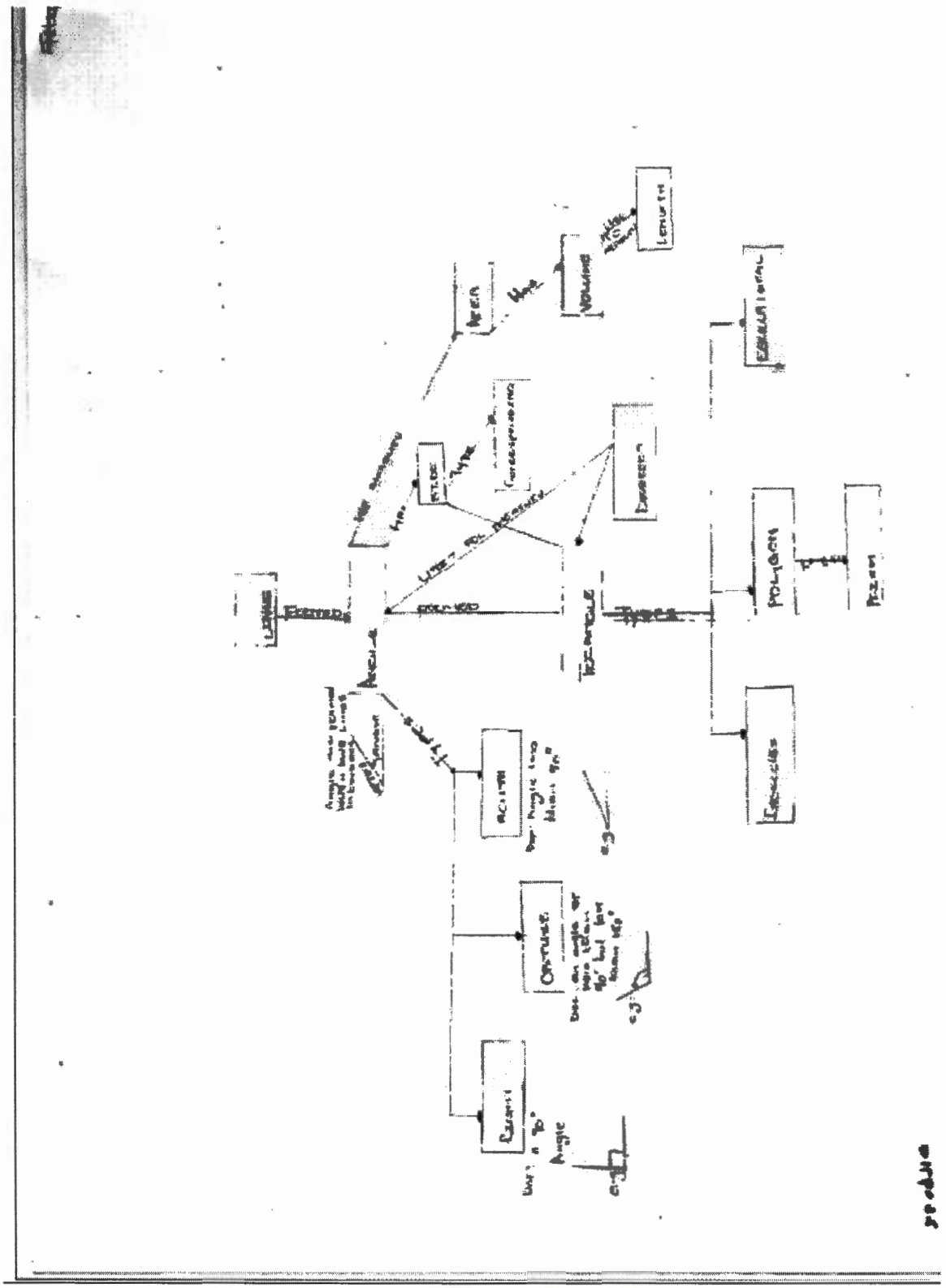
CONCEPT MAP 13

Joe's first concept map (on Learning Outcome 1)



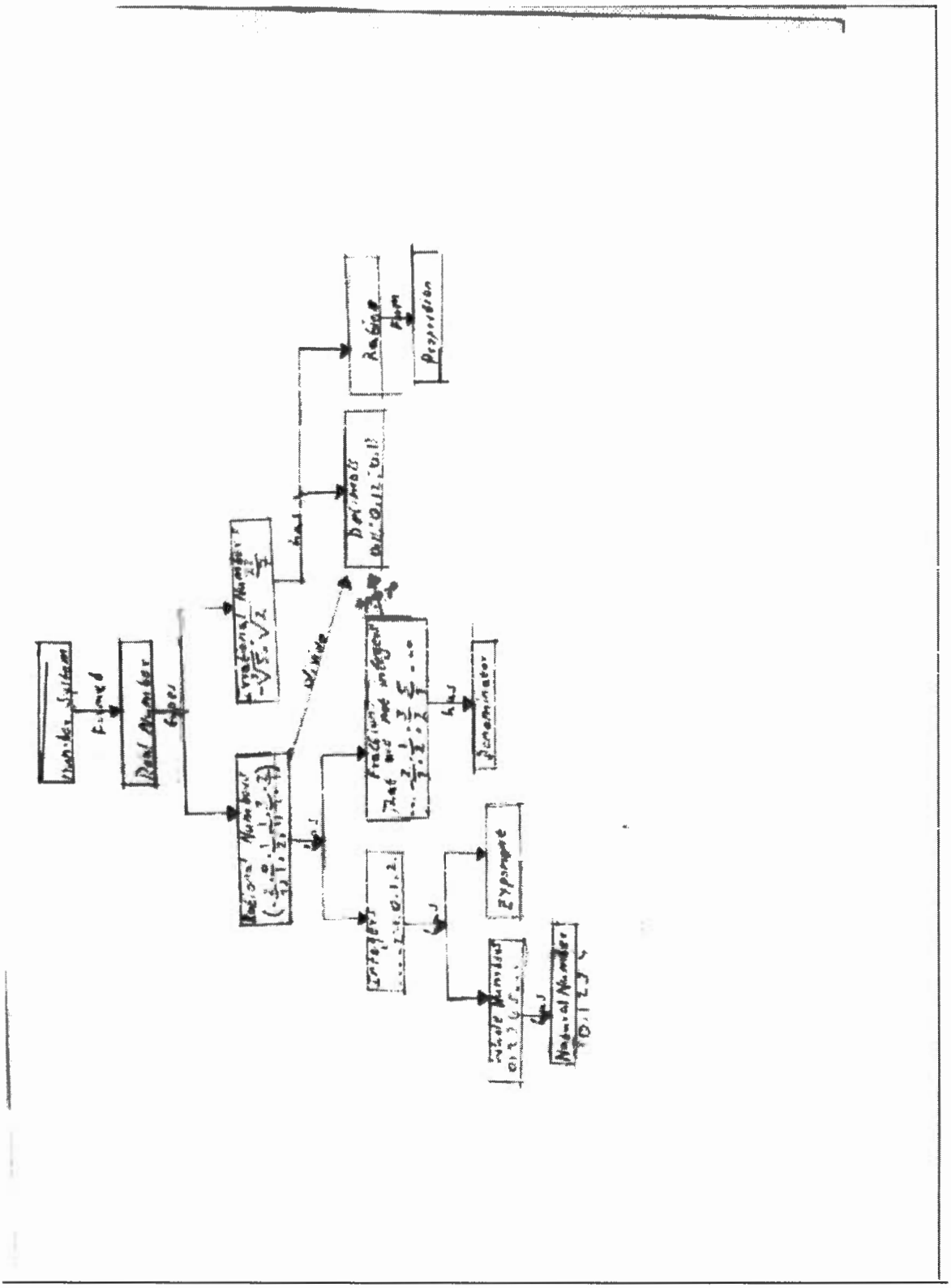
CONCEPT MAP 14

Joe's second concept map (on Learning Outcomc 3)



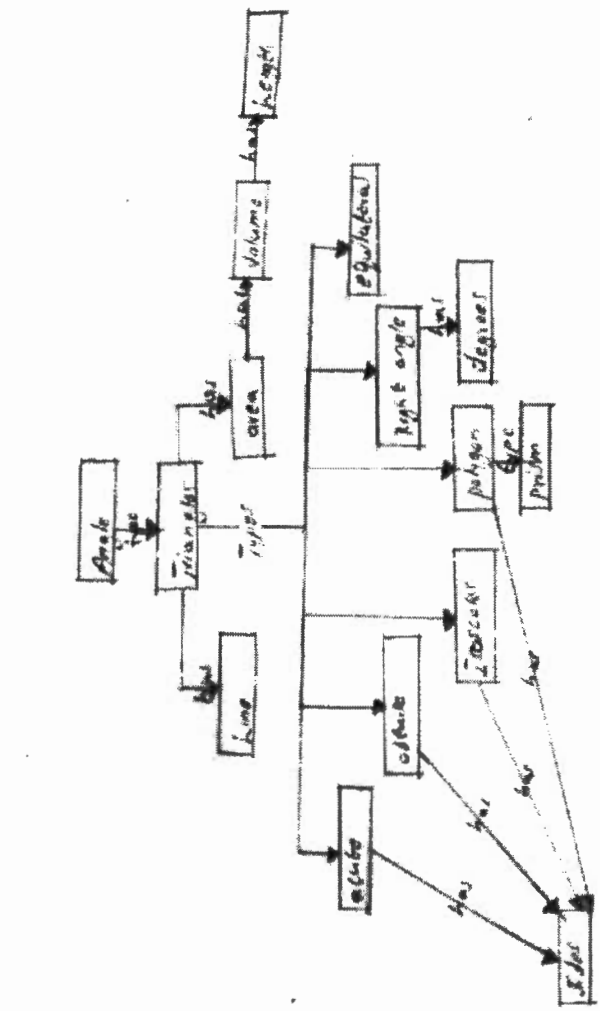
CONCEPT MAP 15

Katlego's first concept map (on Learning Outcome 1)



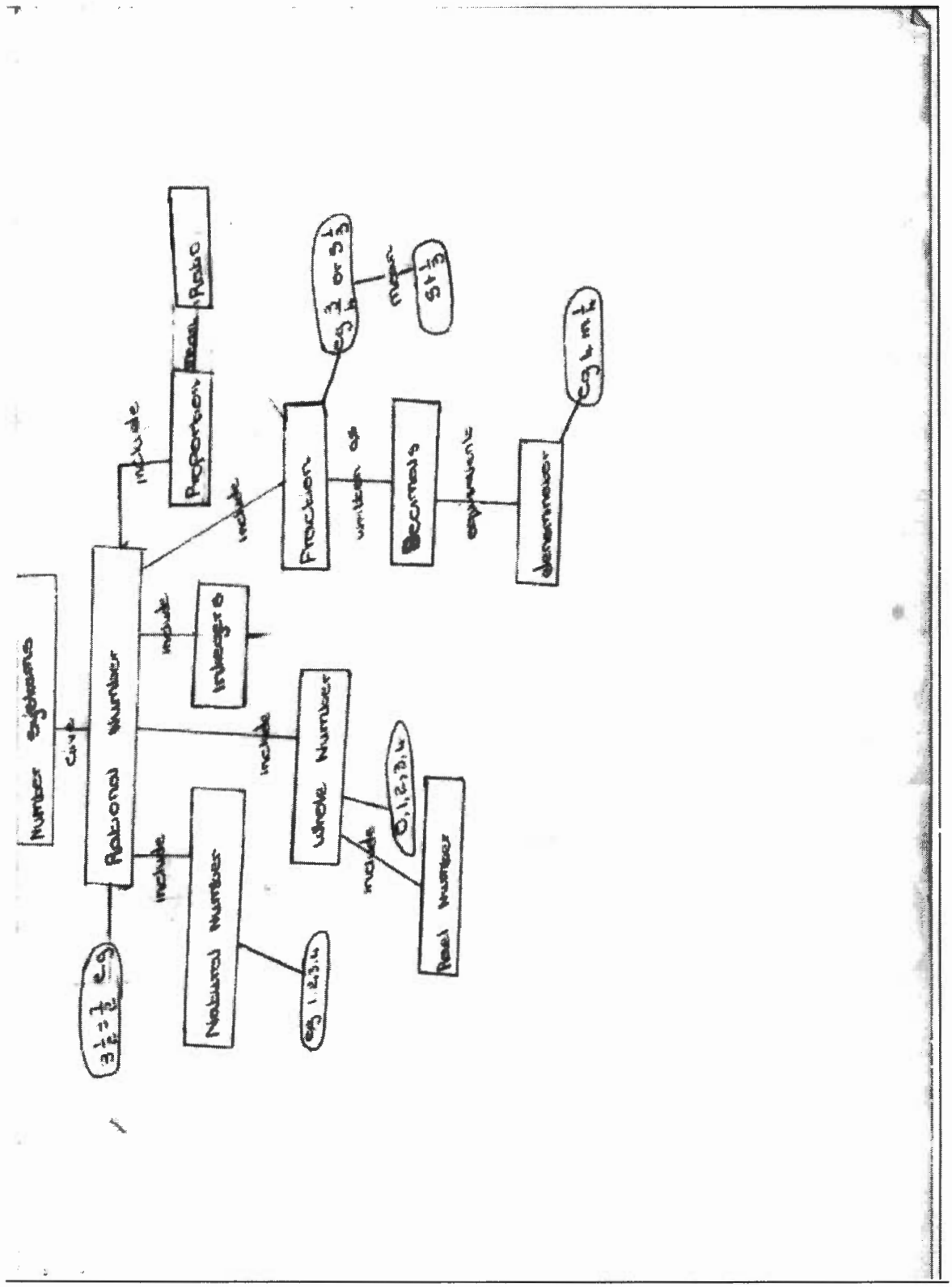
CONCEPT MAP 16

Katlego's second concept map (on Learning Outcome 3)



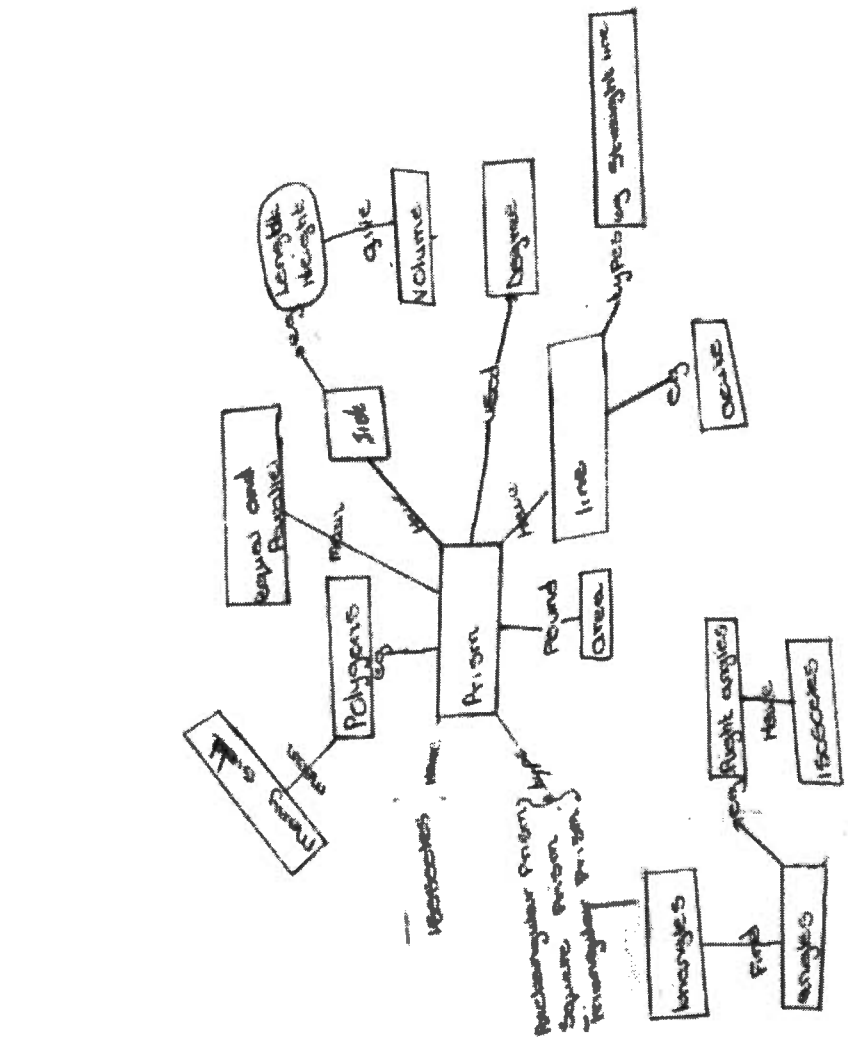
CONCEPT MAP 17

Pule's first concept map (on Learning Outcome 1)



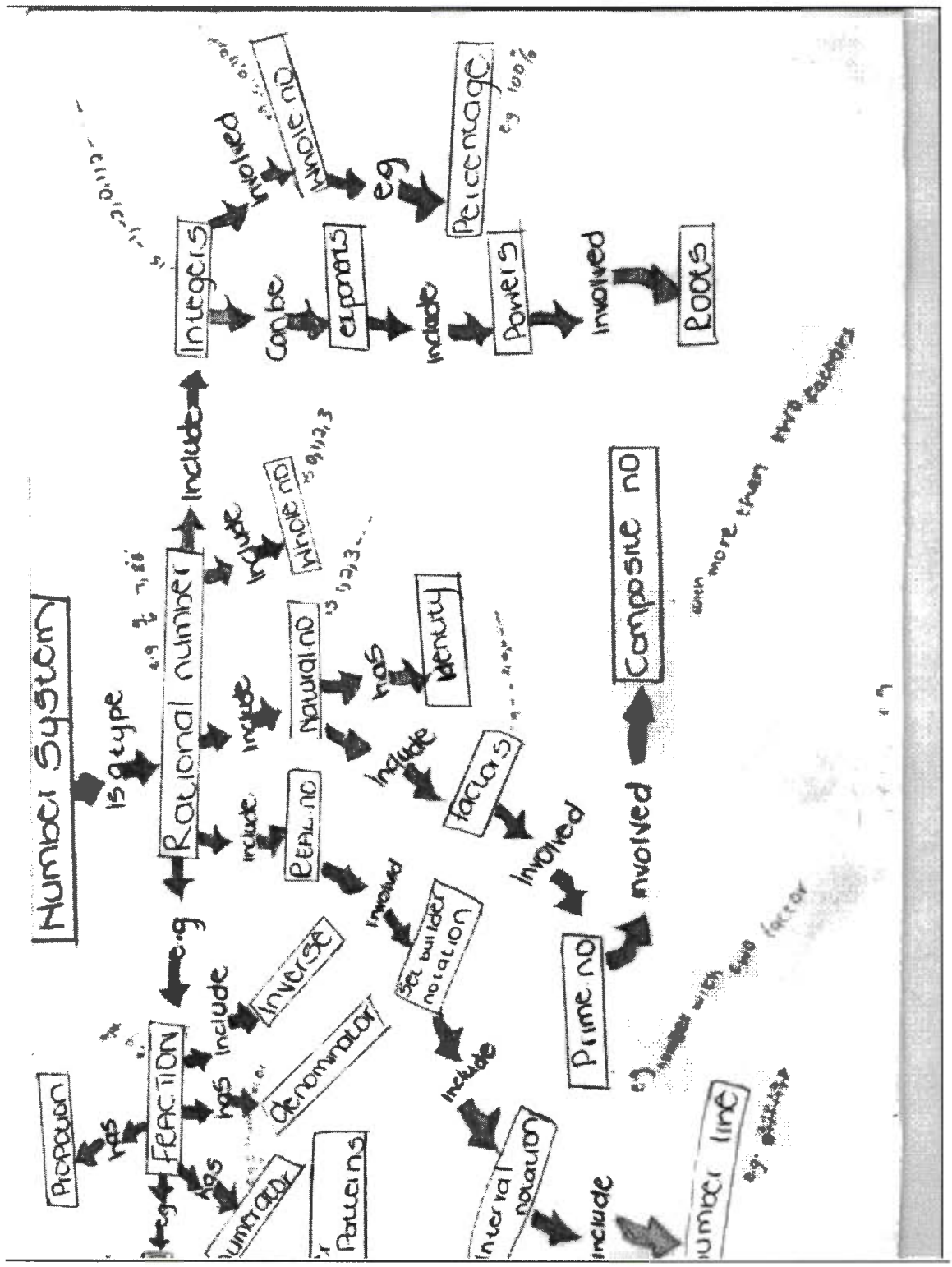
CONCEPT MAP 18

Pule's second concept map (on Learning Outcome 3)



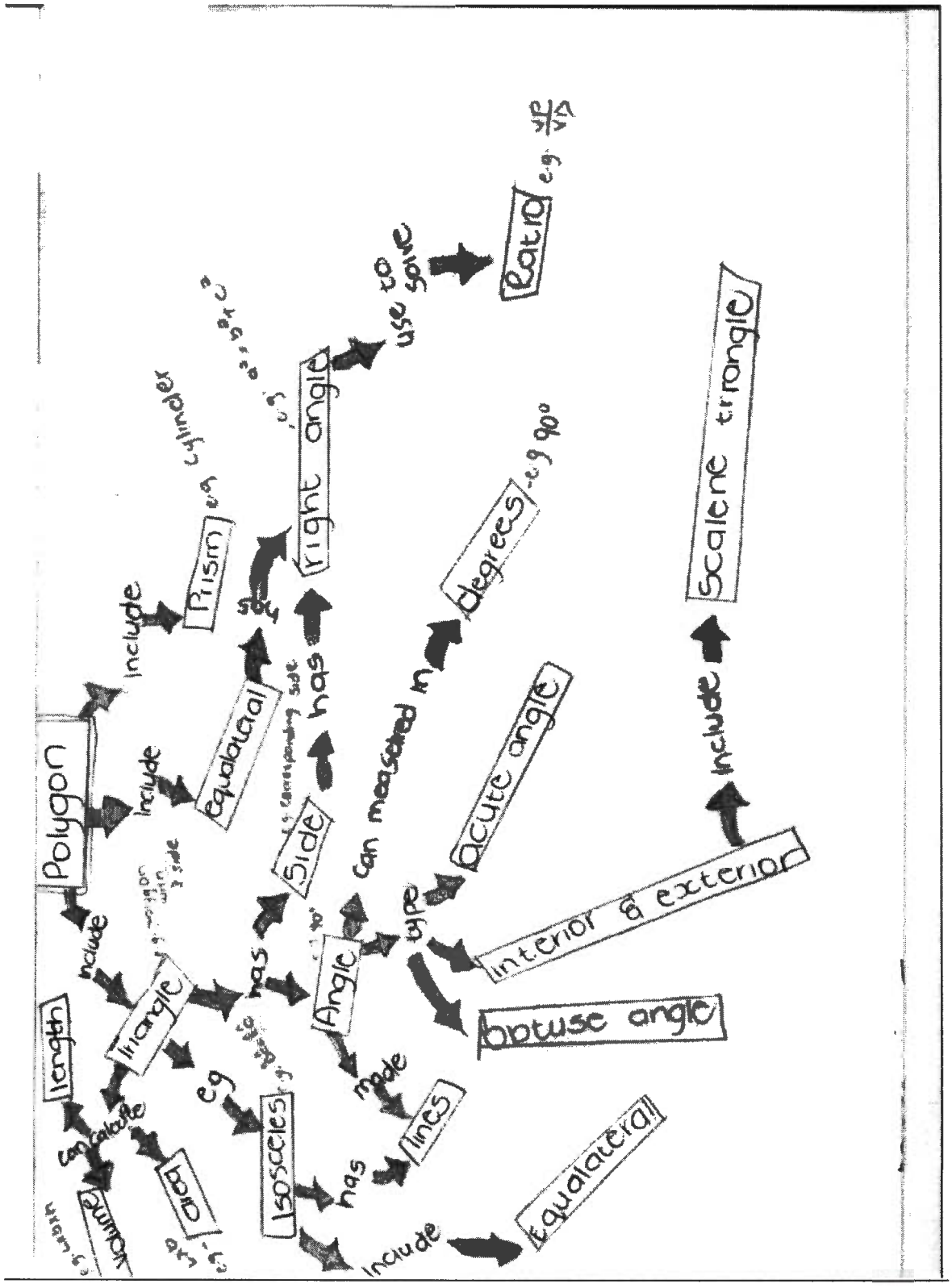
CONCEPT MAP 19

Thepang's first comcept map (on Learning Outcome 1)



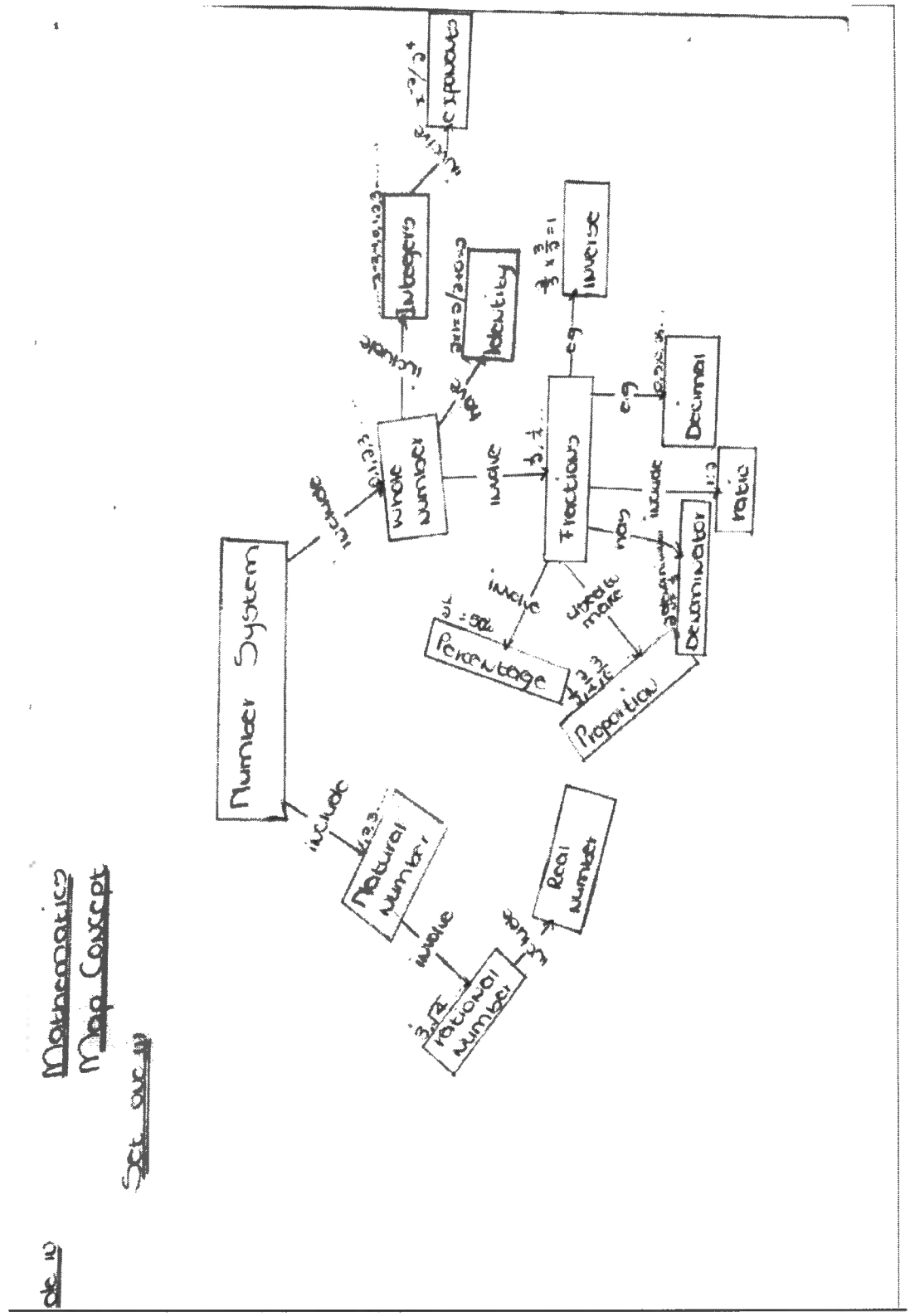
CONCEPT MAP 20

Tshepang's second concept map (on Learning Outcome 3)



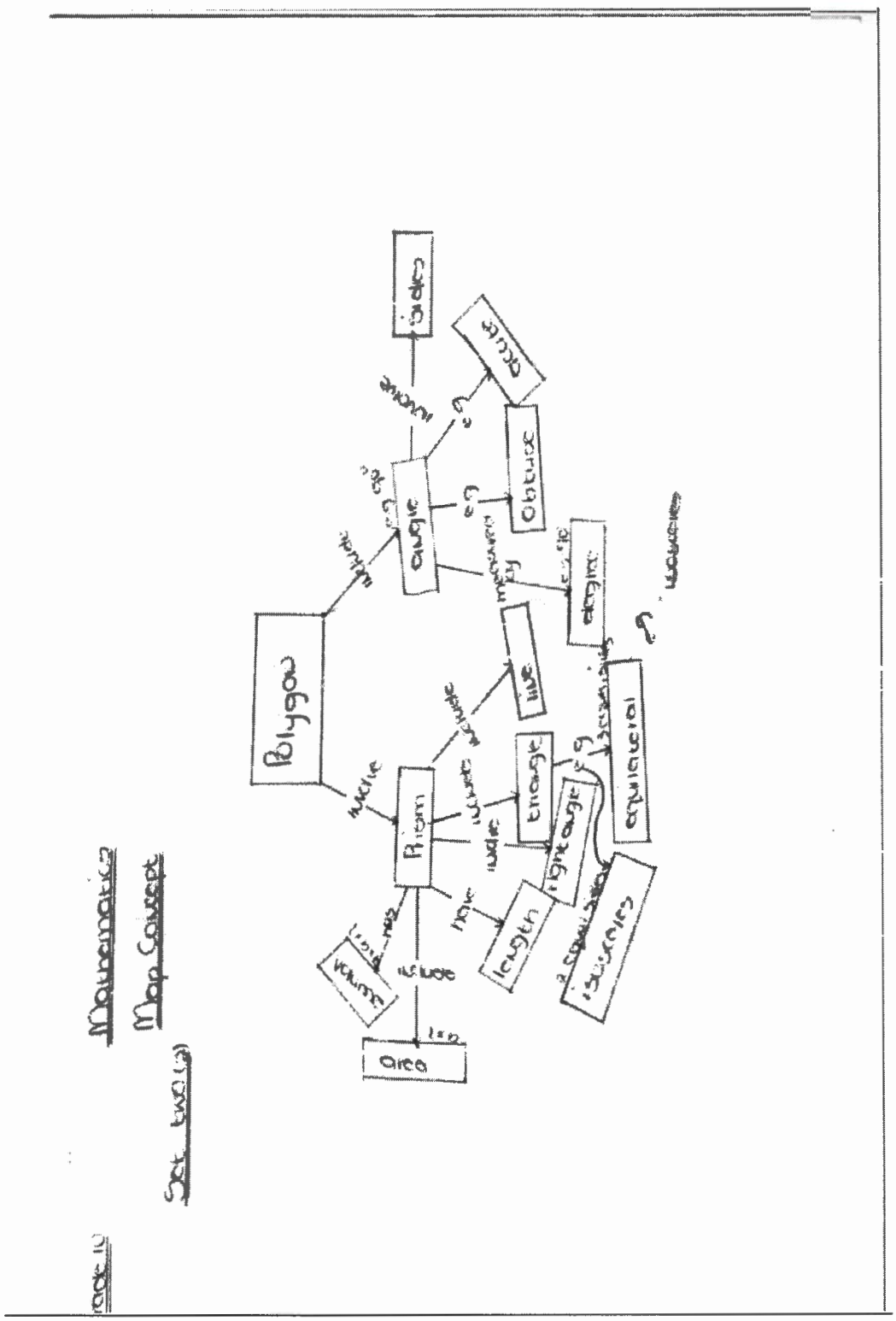
CONCEPT MAP 21

Bontle's first concept map (on Learning Outcome 1)



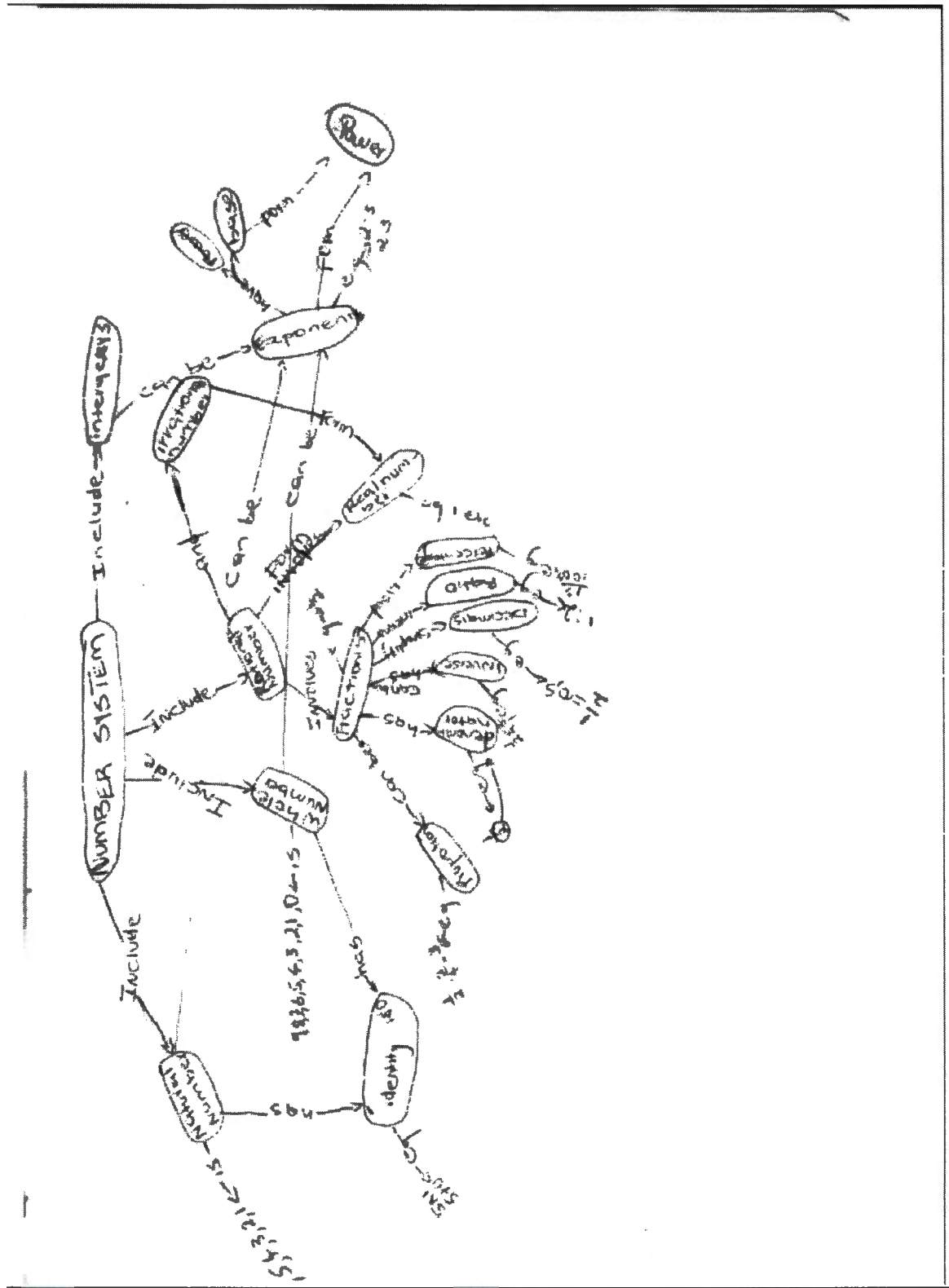
CONCEPT MAP 22

Bontle's second concept map (on Learning Outcome 3)



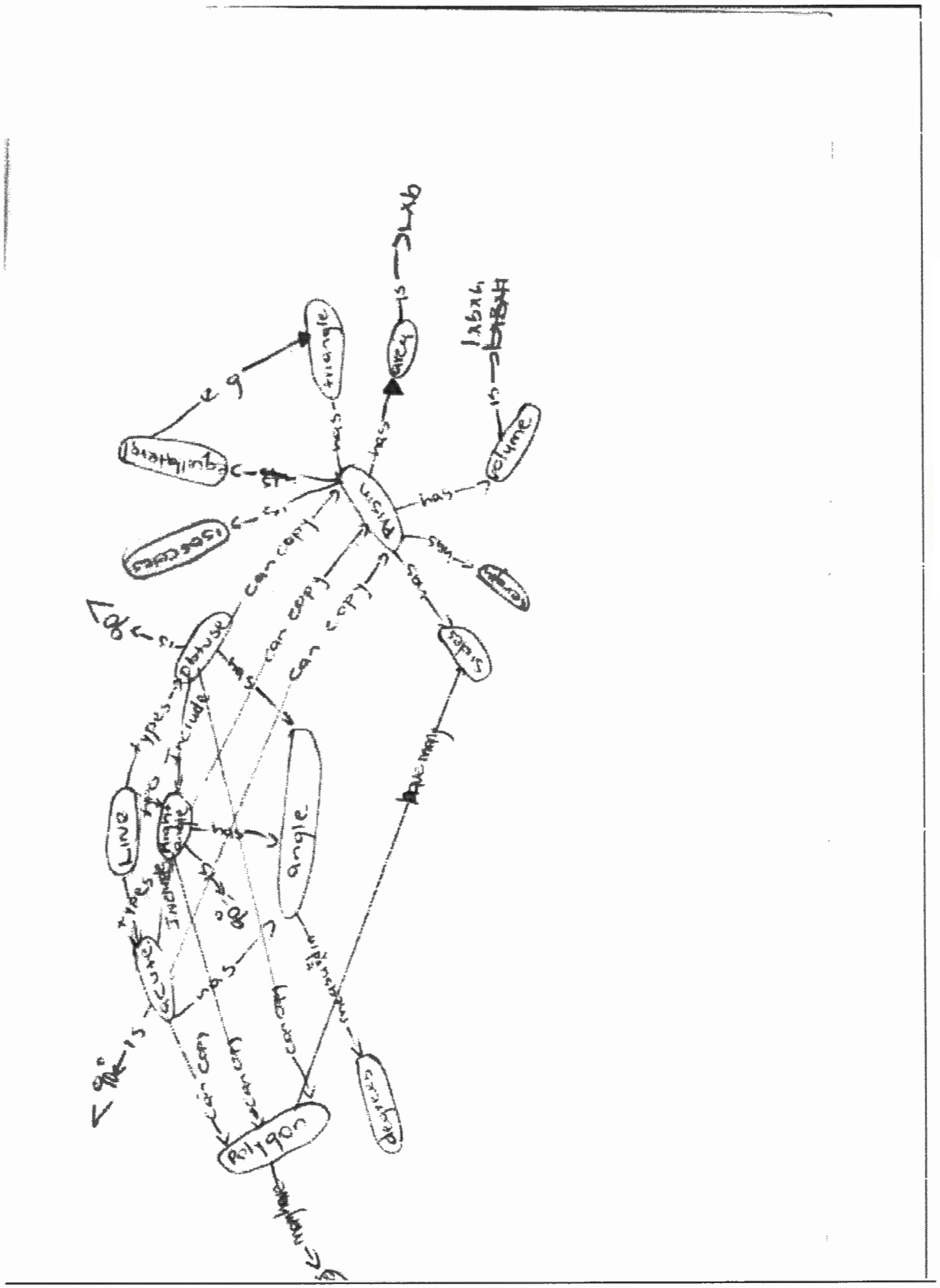
CONCEPT MAP 23

Thapelo's first concept map (on Learning Outcome 1)



CONCEPT MAP 24

Thapelo's second concept map (on Learning Outcome 3)


Line
Angle
Polygon
Circle
Sphere
Cube
Cylinder
Cone
Prism
Pyramid
Triangle
Square
Rectangle
Pentagon
Hexagon
Heptagon
Octagon
Nonagon
Decagon
Dodecagon
Tetradecagon
Hexadecagon
Octadecagon
Icosagon
Triacontagon
Tetracontagon
Pentacontagon
Hexacontagon
Heptacontagon
Octacontagon
Enneacontagon
Hectogon
Chiliagon
Myriagon
Megagon
Apeirogon
has
can copy
can be
can be part of
can be made up of
can be made of
can be found in
can be found on
can be found

APPENDIX I (Interview transcripts)

Didi's first interview

TM: Didi, how did you start drawing your concept map?

Didi: I start with real number.

TM: Why did you start with real number?

Didi: Because Real numbers have more concepts on it.

TM: Is there any concept that you know a lot about?

Didi: Yes, is exponent.

TM: Tell us more about exponents.

Didi: An exponent is a concept that I understand a lot more than the other concepts on the map I was taught.

TM: Can you give us an example of an exponent:

Didi: Two to the exponent of three, four to the exponent of five, two to the exponent of four.

TM: Why do you know so much about exponents, what is the reason?

Didi: Because exponent I understand it more than the other exponent, because I was taught it in Grade 9 and in Grade 10 again that is why I understand it better than other concept.

TM: From all the concepts that were given to you, is there any concept that you do not know a lot about?

Didi: Yes, Sir. Proportions

TM: You don't know a lot about proportions. What's the problem?

Didi: Because it is the first time I come across the word proportions.

TM: Is there any other concept that you do not know a lot about, except proportion?

Didi: Yes Sir, is Fraction.

TM: What do you think the problem is with fractions?

Didi: Because I don't understand more than the other concept.

Didi's Second Interview

TM: How did you start drawing your map?

Didi: I start with prism.

TM: Why did you start with prism and not any other concept?

Didi: Because prism has a lot of concept on it that's why I started with it.

TM: Did you leave other concepts out?

Didi: Yes Sir, I leave Isosceles.

TM: Can you tell us why?

Didi: Because I don't understand it.

TM: Is there any concept from all these concepts that you know a lot about and why?

Didi: Angles, because I learned it in Grade 9 and Grade 10.

TM: Give us examples of angle that you know about.

Didi: Obtuse angle, acute angle and right angle.

TM: Tell us more about those types of angle.

Didi: Obtuse angle .....

TM: Can you tell us about a right angle.

Didi: Right angle has 90°.

TM: What about acute angle?

Didi: 60° and 90°

TM: Is there any concept that you do not know a lot about and why?

Didi: Yes sir, is a volume and area, because I don't teach it a lot.

TM: Sorry

Didi : Because I don't understand it better.

TM: Have you left any of the concepts out?

Didi: Yes sir, Isosceles and quadrilateral.

TM: Why did you leave them out?

Didi: Because I don't understand it better.

Pinky's first interview

TM: How did you start drawing your map?

Pinky: I started with a number system.

TM: and why did you start with the concept number system?

Pinky: Because number systems have more things that I can say about it.

TM: and why did you start with the concept number system?

Pinky: Yes, fractions I know about it more because I know how to put a fraction and how to use a fraction in a concept.

TM: Is there any concept that you do not know a lot about and why?

Pinky: I don't know a lot about profectile. I don't know what proportions talk about; I don't have an idea about proportions.

Pinky's Second interview

TM: How did you start drawing your second concept map?

Pinky: My second concept I started with Prism.

TM: why did you start with Prism?

Pinky: Because Prism has more things that I can say about it and have things that we can use it at other things.

TM: Have you included all the fifteen concepts?

Pinky: No, Sir

TM: Why have you left some of the concepts out?

Pinky: Because I don't know how to use it in this map.

TM: Is there any concept that you know a lot about?

Pinky: Yes, Sir

TM: Which one is that?

Pinky: Volume

TM: Tell us more about volume

Pinky: The thing that I know about volume, I know how to calculate because I use a length times height to get a volume.

TM: What else do you know about volume?

Pinky: I just know how to calculate it.

TM: Is there any concept that you think you do not know much about?

Pinky: Yes, Sir Acate.

TM: what?

Pinky: Aih ..cate.

TM: What is the problem? Is it the first time that you see this one?

Pinky: Yes, Sir

TM: is there any other concept that you think you do not know a lot about?

Pinky: Yes, Sir. Is Obtacles

TM: Obtuse you mean. Which other concept?

Pinky: others I think I know something about it but I don't have more ideas about it.

Tebogo's first interview

TM: Tebogo, can you tell us about your first concept map?

Tebogo: I started with the word 'number system' because without number system there would have been no real numbers and so on.

TM: What do you mean by that?

Tebogo: I mean number system is the word that is big more than other words.

TM: Have you left other concepts or other words out?

Tebogo: Yes.

TM: Which one and why?

Tebogo: I left ratios, exponents and . I left them because I didn't understand more about them.

TM: Is there any concept that you know a lot about?

Tebogo: Yes sir. It is rational numbers. Rational numbers have inverse which are positive numbers and negative numbers and it also have fractions which are decimals and percentages. Example of a fraction is one over two. Example of a decimal fraction is zero comma five.

TM: Is there any concept that you think you do not know a lot about?

Tebogo: I do not know a lot about inverse identity and proportion because they didn't teach me about them.

Tebogo second interview

TM: Tebogo, how did you start drawing your second concept map?

Tebogo: I started with the word angles and give example of angle which is acute, obtuse, right angle and triangle.

TM: Why did you start with angle?

Tebogo: I started with it because without angle there would have not been acute, obtuse, right angle and triangle.

TM: Have you left any concept out?

Tebogo: No.

TM: Is there a specific concept that you know a lot about?

Tebogo: I know a lot about triangle because we have two types of triangles in set two which equilateral and isosceles. And triangle has sides which are three and it also has lines and can be prism and has volume.

TM: Why is it that you know so much about triangles?

Tebogo: I started doing about triangles since from grade eight.

TM: Is there any concept or any word that you think you do not know a lot about?

Tebogo: Ee, Prism

TM: Can you tell us why?

Tebogo: Because I don't have a lot of information about prisms.

Thato's first interview

TM: Thato, can you explain how you started drawing your concept map?

Thato: I started my concept map by choosing the number system then I started to share the example of it, and the types of number system and the way it relate with other numbers.

TM: What else did you include in your concept map?

Thato: I include the decimal number to show the relation with the number system, also rational number, percentage and identity. I also elaborate the types of number systems, its examples.

TM: Did you leave any concept out?

Thato: No, I didn't leave any concept out.

TM: Is there any concept that you know a lot about?

Thato: Yes. If there is a concept that I know a lot about is like types of number system.

TM: Can you tell us more about number system.

Thato: In types of number system we have natural number. Natural number is the type of number that you can count. For example: 1; 2; 3; 4 and etc. We move to the whole

number and in the whole number we will include zero and also 1; 2; 3, and also we have an integer. Intergers have negative and positive numbers. Rational numbers, in rational numbers we have the fractions, the recurring number, decimal numbers and also natural numbers we have it in natural numbers.

TM: Is there any concept that you think you do not know a lot about?

Thato: The real numbers.

TM: Can you tell us about real numbers?

Thato: I have a little bit of information about real numbers because it has so many numbers because it includes the rational, irrational number, natural numbers and also whole numbers are included in the real numbers.

Thato's second interview

TM: Thato, we would like you to explain the second concept map?

Thato: In my second concept map I choose angle because everything that is in set two is related to angles.

TM: How did you link the remaining concepts to angle?

Thato: I started with the type of angle. Types of angle are acute, obtuse and right angle. I use a degree after because when you measure angle you must have a degree by a protractor; and also the sides, and polygons and area the way it relate with angles.

TM: Is there any concept that you know a lot about?

Thato: The concept that I know a lot about is areas. Area is when we compare the capacity and in area we must have the volume, the prism. In volume we measure the area the base and also the height. In prism we also use it to calculate the perimeter of the base

TM: Is there any concept that you think you do not know a lot about?

Thato: The concept that I do not know a lot about is polygons.

TM: Why do you know so little about polygons?

Thato: Because in polygons in polygons we deal with two dimensional shapes and also three dimensional shapes. For example, in prism we deal about the three dimensional shape. Equilateral is about the size of the shape. And in equilateral we... I don't know.

Olebogeng's first interview

TM: How did you go about doing your first concept map?

Olebogeng: My first concept is Number system. Number system have the real number, real number include irrational number, rational number and identity. Rational number has ratio, integers and fraction. Ratio has proportion. Integers can be written as exponent and, whole number and negative numbers. Types of real numbers are natural number and zero. Exponent has power and base. Fractions include numerator and denominator. Type of fraction: proper fraction, decimal and improper fraction.

TM: Did you leave any of the words or concepts out?

Olebogeng: Yes, sir. Percentages and inverses.

TM: Why did you leave those two out?

Olebogeng: Because I don't know them.

TM: Is there any concept that you think you know a lot about?

Olebogeng: Yes, sir. It is exponent.

TM: Can you tell us about exponents?

Olebogeng: For example, when you multiply the same base you add the exponents. When you divide the same base you subtract the exponents.

TM: Is there any concept that you think you do not know a lot about?

Olebogeng: no response

Olebogeng's second interview

TM: How did you start your second concept map?

Olebogeng: I started with a line. Lines are forming angles. Angle has triangles and area. Types of angles are obtuse, isosceles, equilateral, polygon and acute. All the types of triangles are measured with degree. Types of area are length, volume, breath and height. Volume has breadth height and length. All the triangles have sides. Right angles are also type triangle.

TM: Did you leave any concept out.

Olebogeng: No

TM: Is there any concept that you know a lot about?

Olebogeng: Volume. When you want volume you measure with height, breath and length. For example the formula of the volume equals to length, breath and height when you multiply them it give volume.

TM: Is there any concept that you think you do not know much about?

Olebogeng: I know a lot.

Agnes' s first interview

TM: How did you go about doing your first concept map?

Agnes: I started with a number system. I have mentioned the types of number system and their uses. First type of number system is the whole numbers. In the whole numbers I mentioned the examples of whole numbers which is zero, one, two, three and etc. The next type is the natural number. Natural numbers are the set numbers that can be counted. For example: one, two, three, four and etc. My last type of number system is integers. Integers include all the negative numbers without fractions like negative three, negative two, negative one and etc. We can use natural numbers as a decimal and we can represent it as a finite decimal fraction. For example $\frac{4}{8}$ will be 0,8 and can be identified as $\frac{2}{3}$. Two will be our numerator and three will be our denominator. We can connect the number system with an identity. The definition of identity is the similar numbers. An example of an identity is $sin^2\theta + cos^2\theta = 1$. It can also be used as inverse. For example, $\sin\theta = \frac{x}{r}$. A rational numbers can be identified from this. A denominator can be used in a fraction. A ratio can be used as proportion and as a percentage. For example $\frac{4}{10}$ is equal to 4%. The definition of a percentage is an amount of rate as a fraction of 100. An example of that is going to be 10%, 8%, etc.

TM: Is there any concept that you know a lot about?

Agnes: A natural number. Natural numbers are a set of numbers that can be counted. For example: one, two, three, until the end of numbers.

TM: Is there any concept that you think you do not know much about?

Agnes: identity, I know a little bit about it.

Agnes's Second interview

TM: How did you go about doing your Second concept map?

Agnes: I started with angles. On the angles, I started by mentioning the types of angles. The types of angles are acute angle, obtuse angle and the right angle. We can use the angle to measure the degrees. We can also use it by determining the area. The area is a measurement of the surface inside the closed board. An area can be used to measure length, breath and the area of a base. A volume is an area that can be multiplied. We can also use a volume to calculate the perimeter of a base. We have polygon on an angle. The definition of a polygon is a dimension closed shape on the straight line. For example, a polygon has a two side. The types of polygons are equilateral, isosceles and prisms. Prisms are a unit for measuring angles. Isosceles has two sides and two angles that are equal. Equilateral has all sides equal. Angle you can use it when you want calculate an angle like side.

TM: Is there any concept that you think you know a lot about?

Agnes: It is area. Area is like when you measure a volume you can measure it by the length and breadth of a base. I know about a type of angle, a right angle which is less than ninety degrees.

TM: Is there any concept that you think you do not know much about?

Agnes: Yes, sir. Isosceles. I know that it has equal side but I don't know a lot about it.

Joe's first interview

TM: How did you go about doing your first concept map?

Joe: I started with number systems which are formed by real numbers. I then gave the types of real numbers which are rational and irrationals. When we talk about irrationals it involves integers and fractions. When we divide fractions they give us decimal numbers. Rational number has decimals and ratio. Ratios are formed by proportions. The examples of integers are negative two, negative one and zero and one, two, etc. The whole numbers are zero, one two, etc.

TM: Did you leave any concept out?

Joe: Yes, sir. It is inverse.

TM: Why did you leave it out?

Joe: I don't know anything about inverse.

TM: Is there any concept that you know a lot about?

Joe: It is natural numbers which are used every day when we talk.

TM: Is there any concept that you think you do not know much about?

Joe: That is inverse and identity.

Joe's second interview

TM: How did you go about doing your second concept map?

Joe: I started my second concept map with angles which has lines and then I go to the types of angles which are triangles. Triangles are formed by area, volume and length. Then I look for the types of triangle which are acute angle, obtuse angles, isosceles, polygons, right angle and equilateral angles. All angles have sides. Types of polygons are prism. Prisms are formed by degrees.

TM: Why did you start with angle?

Joe: Because angles have more information than the others.

TM: Is there any concept that you know a lot about?

Joe: Yes, Sir. I know a lot about polygons. I know the difference between the types of polygons. The difference between the types of polygons is the side. Some polygons have three sides others have four. I know that it is prism, eh.. I forgot others, but I know a lot about polygons.

TM: Is there any concept that you think you do not know much about?

Joe: No, Sir.

Katlego's first interview

TM: How did you start drawing your first concept map?

Katlego: I started with the number system. The number system has the real numbers. The real numbers *di na le* (they have) the integers, the whole number, the rational number. In the real numbers we have the irrational numbers and the rational numbers. Rational numbers can be written as a percentage. We have the fractions. Types of fractions are mixed fraction, proper fraction, and improper fraction. In the fraction we have the denominator and the numerator. Fraction can be written as a ratio, just like $\frac{3}{4}$ can be

written as a ratio. In a fraction we also have a proportion. In a proportion we can have 3 over 4 and we say 3 divide by 3, it gives you 12 and you say 4 divided by 4 *e go fa* (it gives you) 16. I have also the exponent. In the exponent we have the power and the base.

TM: Did you leave any concept out?

Katlego: Yes, Sir. I left identity.

TM: Why did you leave Identity out?

Katlego: I don't know anything about identity.

TM: Is there any concept that you know a lot about?

Katlego: Yes, Sir. It is fractions. I know the types of fractions: mixed fraction like $3\frac{1}{2}$, proper fraction can be $\frac{6}{10}$ and the improper fraction can be $\frac{9}{5}$. I know that fractions can be written as numerator and denominator. It can be written as ratio or the decimal and it can be written as a proportion.

TM: Is there any concept that you think you do not know much about?

Katlego: *Ke di* identity. (They are identities)

TM: Why do you know so little about identities?

Katlego: I only know that we have $\tan\beta$ *and* $\sin\beta$

Katlego's second interview

TM: How did you go about doing your second concept map?

Katlego: I started with the angles. I mentioned the types of angles like acute angle. Acute angle is an angle that is less than 90°. A right angle is a 90° angle. The obtuse angle is an angle greater than 90°. I also started with the triangles. In the triangle we can have the angles, the sides and the lines. The types of line are parallel line, line segments, perpendicular line and bisecting line. The type of side is the corresponding side. The examples of the triangle are Isosceles, Equilateral. Isosceles is the triangle that has two equal lines. Equilateral is when the three angle are equal and the three line are equal. I also have the polygon. The example of the polygon is the prism. An example of the prism is the triangular prism. You can measure an area on the triangle by measuring the volume including the length and breath.

TM: Is there any concept that you know a lot about?

Katlego: I know about angles. I know the types of angle and the definitions. There are types of angle that I have not mentioned which are reflection angle and Straight angle. I know that angle can be measured I n degrees.

TM: Is there any concept that you think you do not know much about?

Katlego: I don't know much about the polygon. I only know the triangular prisms and that is the only thing I know about the polygon.

Pule's first interview

TM: How did you start drawing your First concept map?

Pule: I started drawing my concept map with a concept that can involve all these concepts that concept is a number system. A number system is a basic of mathematics and involves all of these including real numbers, including natural numbers. Natural numbers, for example are number 1 up to number 9. Natural numbers include integers and integers make a whole number. Whole numbers divide to exponent. An exponent has a base and a power. A natural number has an identity. Integers divide by fraction. The fraction is a decimal fraction. The fraction has a base and a power.

TM: Is there any concept that you know a lot about?

Pule: I know a lot about rational numbers. Decimal numbers are the number that when you multiply give you recurring answers. Rational numbers are made up of real numbers. An example of rational numbers can be 3 divided by 4, it give you the answer that recurs.

TM: Is there any concept that you think you do not know much about?

Pule: I don't know much about proportions. What I know about proportion is that they are triangles that are equal to each other. For example, when you have triangle ABC and say that triangle ABC is equal to AB.

Pule's second interview

TM: How did you start drawing your second concept map?

Pule: I started drawing my concept map with a concept that can involve all these concepts. That concept is a polygon. Polygon is a figure that has three or more sides including prism and prism has a line. Then I included a triangle and equilateral. A triangle has a right angle and then has an angle. There are two types of angle which are acute angle and obtuse angle. An angle is measured in degrees. For instance 90°. A Right angle is used to make a ratio. A triangle can calculate the area using the formula length times breadth. And can calculate the volume using the formula: length times breadth times height.

TM: Is there any concept that you know a lot about?

Pule: I know a lot about a triangle because a triangle is a type of polygon with three sides equal. For instance a triangle has an angle and a triangle has the lines joined together to be popular and to be success. Triangle can calculate an area and has an angle. A triangle has an example which is an isosceles triangle.

TM: Is there any concept that you think you do not know much about?

Pule: It is prism. An example of a prism is a cylinder. I don't know much about a prism.

Tshepang's first interview

TM: How did you start drawing your first concept map?

Tshepang: I started drawing my concept map by using number system. A number system is a type of a rational number and a rational number include real number, natural number, whole number and integers. Integers can involve whole numbers. Whole number is an example of percentage. Integers can be exponents. Exponents include powers and powers involve roots. Natural numbers have identity, include fraction. Natural numbers involve prime numbers. Prime numbers involve composite number. A Whole number involve self builder notation. Self builder notation has interval notation and interval notation include number line. A fraction has a proportion. Example of fraction is a decimal number. Example of decimal number is zero comma seventy-five. Decimal numbers include number patterns. A fraction has denominator and a numerator. A fraction include inverse.

TM: Is there any concept that you know a lot about?

Tshepang: A concept that I know a lot about is a fraction. A fraction is a number between zero and one. A fraction can be expressed in a common fraction for example and in decimal fraction as zero comma seventy-five. On the $\frac{3}{4}$, 3 is the numerator and 4 is a denominator. A fraction can be changed to its equivalent fraction by multiplying the denominator and the numerator by the same number. For example $\frac{3}{4} \times \frac{2}{2}$ is going to be $\frac{6}{8}$.

TM: Is there any concept that you think you do not know much about?

Tshepang: I don't know a lot about an inverse. In fact, I don't know anything about an inverse.

Tshepang's second interview

TM: How did you start drawing your second concept map?

Tshepang: I started my second map with polygon. Polygons include triangle, equilateral, and prism. A triangle has sides and angles. An angle can be measured in degrees. Angles have the following types: acute, interior and exterior, and an obtuse angle. An angle has line. The examples of a triangle are isosceles and equilateral. In a triangle we can calculate area and length. Right angle is used to solve ratio. Exterior and interior include scalene triangle.

TM: Is there any concept that you know a lot about?

Tshepang: I know a lot about polygon because I have investigating about polygon.

TM: Is there any concept that you think you do not know much about?

Tshepang: It is equilateral.

Bontle's first interview

TM: How did you start drawing your first concept map?

Bontle: I started drawing my concept map with number systems. Numbers systems include whole numbers and natural numbers. Whole numbers include integers. Integers involve exponents. Whole numbers have identity. Whole numbers involve fractions. Examples of fractions are inverse and decimals. Fractions include ratio. Fractions have denominators. Fractions are used to make proportions and fractions involve

percentage. Natural numbers involve rational numbers and rational numbers include real numbers.

TM: Is there any concept that you know a lot about?

Bontle: Its fractions. A fraction is a number between zero and two. It can be expressed in a common fraction form or a decimal fraction form. We can write the number 1 as a fraction and we multiply denominators together and numerators together. In a common fraction $\frac{1}{2}$, 1 is numerator and 2 is a denominator. It can be changed to equivalent fraction by multiplying the numerator and denominator by the same number.

TM: Is there any concept that you think you do not know much about?

Bontle: It is proportion. It is the first time I hear the word proportion.

Bontle's second interview

TM: How did you start drawing your second concept map?

Bontle: I started with polygon. Polygon include angle and involve prism. Prism includes lines, triangle and right angle. Prism has length and area and volume. Examples of triangle are equilateral and isosceles. Angles have side. Examples of angles are acute angle and obtuse angle. An angle is measured by degree.

TM: Is there any concept that you know a lot about?

Bontle: I know a lot about triangle. A triangle is a polygon with three sides. The angles of a triangle add up to 180°. an exterior angle of a triangle is equal to the interior opposite side.

TM: Is there any concept that you think you do not know much about?

Bontle: Isosceles. I am coming across the word isosceles for the first time.

Thapelo's first interview

TM: How did you start drawing your First concept map?

Thapelo: I started drawing my concept map starting with number system because I think that number system is a basic of mathematics. Numbers systems include all numbers that I have used. For an example, Number systems include natural numbers which is number 1 up to number 9. Number systems include integers. Integers can be an

exponent. Exponent and base can form a power. Natural number has identity which is one. Number systems again include whole numbers. Whole numbers have identities which are one and zero. Number systems include a rational number and rational numbers and irrational numbers can from a real numbers. Rational numbers involve fractions. Fraction can be a proportion. Fraction has a denominator. Fraction can be an inverse. We simplify a fraction to be a decimal fraction. Decimal fractions include ratios and are used to make percentages.

TM: Is there any concept that you know a lot about?

Thapelo: I know a lot about fractions because they are simple. I know many types of fractions which are improper fraction, proper fractions and decimal fractions.

TM: Is there any concept that you think you do not know much about?

Thapelo: A proportion. I know that proportion have a denominator and numerator only.

Thapelo's second interview

TM: How did you start drawing your second concept map?

Thapelo: I have started drawing mu concept map using a line because I think that lines makes acute angle which is less than 90°. It makes obtuse angle which is greater than 90°. I think that without lines we can't see an acute angle and a right angle. Acute angle and obtuse angle include a right angle. A right angle is added up to 90°. obtuse can copy a prism. We have prism triangle. Prism has an area, triangle, lengths and sides. Polygons have many sides. An angle is measured in degrees.

TM: Did you leave any concept out.

Thapelo: I left isosceles and equilateral because I don't know a lot about them.

TM: Is there any concept that you know a lot about?

Thapelo: I know a lot about triangles. I know that a triangle is made with straight horizontal line and a vertical line to make it 90°.

TM: Is there any concept that you think you do not know much about?

Thapelo: I do not know a lot about isosceles.