# A decision support system, using data envelopment analysis, to evaluate the efficiency of schools in the North West Province of South Africa

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# ABSTRACT

The South African school system is governed by the South African School Act 84 of 1996. Despite sufficient laws and regulations and what seems sufficient to be a proper school management system, there is a major and ongoing problem in terms of the efficiency and effectiveness of schools. This is evidenced by a large number of studies and research reports that confirm the general lack of appropriate school management and monitoring techniques.

To address the problem of a proper assessment technique that can be used to evaluate schools, this study proposes the development of a decision support system (DSS) based on mathematical programming techniques to assist decision makers in the field of education with school efficiency assessments. The mathematical models implemented are based on the well-known data envelopment analysis (DEA) techniques. A standard DEA model (based on input and output variables) was implemented as well as a class ranking model that utilises only output variables and that enables the implementation of Pareto optimal principles.

The DSS and the implemented models were applied to real world data pertaining to 54 secondary schools in the North-West province of South Africa. The study has shown that the adapted output model (which utilises the Pareto optimal principle) delivers more reliable and more useful results than the traditional DEA models. A significant strength of the proposed output only model is the construction of intermediate goals that will enable an inefficient school to progress step by step over a period of time. This is in contrast with the traditional DEA models that set targets to become immediately as effective as the top rated schools – something that is impractical as schools often do not have the resources for such a huge improvement over a short to medium term.

***Keywords:*** School efficiency; data envelopment analysis; Pareto optimal; mathematical programming; intermediate goals.

# OPSOMMING

Die Suid-Afrikaanse skole stelsel word gereguleer deur die Suid-Afrikaanse skole wet 84 van 1996. Ten spyte van voldoende wetgewing en ŉ oënskynlike behoorlike skoolbestuurstelsel, is daar steeds ŉ konstante probleem met betrekking tot die doeltreffendheid en effektiwiteit van skole. Hierdie probleem word gerugsteun deur ŉ groot aantal studies en navorsingsverslae wat bevestig dat daar ŉ algemene tekort in toepaslike skoolbestuurstelsels en moniteringstelsels bestaan.

Om die probleem van ŉ behoorlike assessseringstegniek vir die evaluering van skole aan te spreek, stel hierdie studie die onwikkeling van ŉ besluitsteunstelsel (BSS) gebaseer op wiskundige programmering voor. So ŉ stelsel kan dan besluitnemers in die opvoedkunde help om die effektiwiteit van skole te evalueer. Die wiskundige modelle wat geïmplementeer is, is gebaseer op die bekende data-omhullingsontleding (DOO) tegnieke. ŉ Standaard DOO model (gebaseer op invoer- en afvoerveranderlikes) is geïmplementeer asook ŉ klas rangordemodel wat gebruik maak van slegs afvoerveranderlikes wat ook die implementering van Pareto optimale beginsels moontlik maak.

Die BSS en die geïmplementeerde modelle is toegepas op werklike data van 54 sekondêre skole in die Noordwes provinsie van Suid-Afrika. Die studie het aangetoon dat die aangepasde afvoermodel (wat van die Pareto optimale beginsel gebruik maak) meer betroubare en meer bruikbare resultate lewer as die tradisionele DOO modelle. ŉ Betekenisvolle sterkpunt van die voorgestelde afvoermodel is die vermoë om intervlakdoelwitte te bereken wat ŉ oneffektiewe skool in staat stel om stapsgewys te verbeter oor ŉ tydperk. Dit is in teenstelling met die tradisionele DOO modelle wat doelwitte stel om dadelik net so doeltreffend soos die mees effektiewe skole te wees – dit is onprakties omdat skole dikwels nie genoegsame bronne het vir so ŉ groot verbetering oor ŉ kort tot medium termyn nie.

***Sleutelwoorde:*** Skool effektiwiteit; data-omhullingsontleding; Pareto optimaal; wiskundige programmering; intervlakdoelwitte.

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# 1 INTRODUCTION AND PROBLEM STATEMENT

## 1.1 Introduction

South Africa is a complex young democracy and struggles with a number of national problems which may be linked (in certain cases) to the legacy of the previous dispensation. The country has nine provinces and eleven official languages. One of the areas where serious problems are regularly reported is the South African education system and specifically the South African school system.

The South African school system is governed by the South African Schools Act (South Africa, 2011) and each of the nine provinces responsible for making sure that schools are managed effectively. This is done through a Department of Education in each province, which is responsible to a national Minister of Education. At school level there are School Governing Bodies (SGB) which consist of the School Principal and elected representation of teachers, parents, co-opted members, non-teaching staff and learners (in secondary schools). The SGB is responsible for the day to day operation of the school and despite this apparently good organisational structure many schools do not operate efficiently.

The problem of efficient and effective school management in South Africa seems to be a major and consistent problem. Many research studies and reports have been produced on this issue. Taylor (2011) has shown, for example, that there is a significant difference in numeracy skills of grade 4 learners that attend previously disadvantaged schools (called Homeland schools) and those that attend historically white schools. This may be an example of how schools were, and still are, impacted by the previous government of South Africa. Another example that can be linked to the unique circumstances of South Africa is the study by Shepherd (2011). This study indicates the differences in literacy skills of grade 5 learners in African language schools as opposed to the English and Afrikaans schools. In a certain sense, this study confirms the lack of appropriate school management techniques in a country with eleven official languages. The problem of school efficiency is succinctly described by Taylor (2011) as

*"School functionality or efficiency remains something of a 'black box': resources flow into the box and differential outcomes emerge, yet little is known or can be proven about what occurs within the box to determine the outcome"*

Other additional examples in the literature that confirm the problem of efficiency in the South African school system include studies by Mji and Makgato (2006), Holborn (2013) and Schwab and Sala i Martin, (2013).

Mji and Makgato (2006) focus on the poor mathematics and physical science results in schools while Holborn (2013) emphasizes the large amount of financial resources (6.4% of the GNP) that are spent on education with unsatisfactory results. Holborn also mentions that South Africa is rated only 140$^{th}$ out of 144 countries in terms of overall educational performance. The two subjects, mathematics and science, are rated even worse as 143$^{rd}$ out of 144 countries. Another alarming statistic quoted by Holborn is that 1.2 million children enrolled in grade one in 2001 of whom only 44% reached grade twelve. Based on these few examples it seems permissible to state that there is a need for efficiency measures that can be used to evaluate schools in South Africa.

The purpose of this chapter is to introduce the research study. A problem statement is presented and subsequently the research objectives are formulated. The research methodology is then outlined. The chapter concludes with a description of the layout of the study.

## 1.2 Problem Statement

As explained in the introduction, existing literature, research reports and general news bulletins strongly suggest that there is a need for proper efficiency measures in the South African school system. Current efforts, to measure the quality of South African schools, comprise mainly of research comparing different types of schools – this type of research, however, only leads to the conclusion that South African learners receive schooling of inferior quality than those of poorer countries (Taylor, 2008). Other examples of studies to measure the quality (efficiency) of schools in South Africa can be found in Christie *et al*. (2007) and Jonas (2005).

To address the problem of measuring and comparing the effectiveness of different schools, this study proposes the development of a decision support system (DSS) that is based on mathematical programming techniques to assist decision makers in the field of education with school efficiency assessments.

## 1.3 Research objectives

The primary objective of this study is to develop and formulate appropriate mathematical models that can be implemented in a DSS and that can be used to evaluate the efficiency of schools.

In order to achieve the primary objective certain secondary objectives have to be addressed. The secondary objectives include:

- Gain a good understanding of the school system in the North-West province of South Africa, as well as related studies to measure effectiveness of educational institutions.
- Provide an overview of linear programming models with specific reference to the models and techniques implemented in this study.
- Development of a DSS which implements appropriate models that can be used to assess the performance of schools.
- Apply and validate the proposed models using real data from the Department of Education in the North-West province of South Africa.

## 1.4 Research methodology

Oates (2006) defines a paradigm as "a set of shared assumptions or ways of thinking about some aspect of the world". Different research paradigms (a set of beliefs or assumptions that guide the perspective of a researcher) exist and the purpose of this section is to highlight three of these paradigms. It does not form part of the scope of this study to present a detailed overview of research paradigms and the discussion is therfore limited only to definitions of the three paradigms. Detailed discusions can be found in Oates (2006). The three paradigms presented in this section are the *positivistic paradigm*, *interpretivistic paradigm* and *critical social theory*. The discussion is then concluded in section 1.4.4 with an explanation of the research method used in this research study. The brief discussion are mainly based on the work by Oates (2006).

### 1.4.1 Positivism

The dictionary of philosophy (Mauther, 2005) refers to positivism as something that is used to designate a world view which is conceived of as being in tune with modern science. It rejects superstition, religion and metaphysics and claims that all knowledge is ultimately based on sense-experience. Genuine enquiries are therefore concerned with the description and explanation of empirical facts.

According to Oates (2006) positivism has certain characteristics which can be summarised as:

- The world exists independently from our mind.
- Research is focused on generalisations where indisputable facts, patterns, or universal laws are proven.
- The construction of models to explore the world by observing, analysing and measuring activities.
- Researchers are seen to be objective observers.
- The use of statistical and mathematical techniques and proofs for quantitative data analysis.
- The empirical testing of hypotheses and theories.

Although widely accepted as a research paradigm, Oates (2006) also presented criticism of positivism. These criticisms include aspects such as reductionism (may be impossible to break down complex systems in small researchable parts); repetition (repeating a study may not be possible); generalisation (often not desired); and the fact that everybody has their own world view and interprets the world differently.

### 1.4.2 Interpretivism

Oates (2006) describes interpretive research as an approach that "is concerned with understanding the social context of an information system: the social processes by which it is developed and construed by people and through which it influences, and is influenced by, it social setting". Interpretivism stands in contrast to positivism and research done in this paradigm normally focus on the formulation of a theory rather than to prove a hypothesis. The characteristics of interpretivism are summarised by Oates as follows.

- There exists more than one version of the truth.

- Reality can only be accessed and conveyed through communication, understanding and meaning.
- Researchers have to be self-reflective.
- People are studied in their natural social setting and the focus is on understanding individuals and their perspectives.
- Qualitative data analysis is preferred.
- There may be multiple interpretations to a research question.

As with positivism Oates also presents a number of criticisms to interpretivism. The most notable argument against the interpretivism paradigm is that it is non-scientific and ignores scientific verification procedures.

### 1.4.3 Critical social theory

Critical social theory is less known than the other two paradigms and Oates states that "it is concerned with identifying power relationships, conflicts and contradictions, and empowering people to eliminate them as sources of alienation and domination". The nature of critical social research is therefore to bring about change within a social environment and then observe and analyse any impact the changes may have. To achieve this, any technique (i.e. observations, case studies, action research) is used to collect data which may be either quantitative or qualitative in nature.

### 1.4.4 Research method used in this study

This study is concerned with the formulation of mathematical models and techniques that can be implemented in a DSS. The study is therefore positivistic in nature and follows a design and create strategy for the creation of an artefact (the DSS) in order to address a real world problem i.e. the measuring of efficiency of different decision making units such as schools.

## 1.5 Dissertation overview

This section presents the layout of the rest of the dissertation and briefly explains the purpose of each chapter.

Chapter 2 provides a summarised introduction to the school system in the North-West province of South Africa. The chapter also highlights previous studies on measuring efficiency in educational institutions.

The purpose of Chapter 3 is to present an overview and general understanding of mathematical programming techniques. The focus is on data envelopment analysis models and techniques for class ranking using these types of models.

In Chapter 4 the DSS is described. This is done by presenting a general description of a DSS followed by an outline of the mathematical techniques implemented in the DSS. An illustrative example is also presented.

The focus of Chapter 5 is on the application of the DSS (and the associated mathematical models) to assess the efficiency of schools in the North-West province of South Africa. The chapter concludes with a detailed discussion of the results.

The final chapter, Chapter 6, concludes the study by demonstrating how the goals set forth for the study were achieved. Limitations of the study as well as considerations for future research are also presented.

## 1.6 Conclusion

Chapter 1 served as an introduction to the research study. A problem statement was formulated after which the research objectives were detailed. The methodology was explained and the chapter was concluded by a brief explanation of the structure of the study.

# 2 BACKGROUND AND LITERATURE STUDY

## 2.1 Introduction

As explained in Chapter 1 the primary objective of the study is to formulate and implement a mathematical model that can be used to evaluate the efficiency of schools in the North-West province of South Africa. Prior to the development of a mathematical model, it is important to provide sufficient background to the schooling system in South Africa in general and in the North-West province in particular. The objective of this chapter is therefore to give a brief overview of teaching and teaching practices in South Africa. The chapter starts with a brief descriptive statistical overview of schools in South Africa and in the North-West province. This is followed by a short summary on existing methods used to evaluate school efficiency. The chapter then concludes with a literature review of mathematical models normally employed in school efficiency assessments. Specific attention is given to data development analysis (DEA) models that have been implemented or proposed in the literature.

## 2.2 Overview of teaching in South Africa

This section gives a brief introduction to the national school system in South Africa as well as a short descriptive statistical overview of schools in the North-West province.

### 2.2.1 The South African national school system

South Africa is a country with a new democracy that came into existence in 1994. There are nine provinces and eleven official languages in South Africa.

The South African government consists of a national government, provincial governments, and local governments. These governing bodies are interdependent and interrelated but also distinctive with their own legal and governing powers. The national government is responsible for the governing and the well-being of the whole country while provincial governments are responsible for specific provincial matters, such as roads, health services and school education. Local governments govern municipal regions within the different provinces.

As far as the educational system in South Africa is concerned, there is one National Department of Education. This National Department of Education consists of two different ministries - the one makes provision for basic education (primary and secondary schools) and the other one is concerned with higher education (tertiary institutions). The two departments are each headed by their own minister in the national government. On provincial level, there is a Provincial Department of Education with a Member of Executive Council (MEC) of Education from the Provincial Cabinet responsible for education and training. The main responsibility of the National Department of Education is governing and maintaining of policies, while the Provincial Department of Education are responsible for implementing the policies set by the National Departments of Education and Provincial Government. The complete educational system is regulated by the South African School Act 84 of 1996 (South Africa, 2011).

Every school is under the direct control of the Provincial Department of Education which reports to the National Department of Education. At school level, the public schools are governed by a school governing body which normally consists of the school headmaster, parents, educators, other staff, co-opted members, and learners (in secondary schools). The non-learner members are elected for a period not exceeding three years while learner members may not serve for a period exceeding one year.

The main function of the school governing body of a public school is the development and implementation of a school constitution, a mission statement and a code of conduct for the school. The school governing body is also responsible for the quality of education provided by the school while adhering to all the rules and regulations as stipulated in the School Act of South Africa.

The school system in South Africa consists of general education and training and ranges from grade 0 to grade 12. It is compulsory by law for all children to attend school from grade 1 (age 7 years) to grade 9 (age 15 years). Although grades 10 to grade 12 are not compulsory, the majority of pupils also attend these grades (South Africa, 2011).

According to the official 2013 school statistics (EMIS, 2013) there were 25 720 schools in the 9 provinces of South Africa. These schools were attended by 12 489 648 learners with 425 023 teachers teaching at the schools. Table 2.1 presents a summary of the school statistics for the nine provinces in South Africa.

| Province | Number of Learners | Number of Educators | Number of Schools | Number of Grade 12 Learners | Number who wrote National Senior Certificate (NSC) | % Pass Rate Grade 12 |
|---|---|---|---|---|---|---|
| Eastern Cape | 1 938 078 | 66 007 | 5 733 | 77 939 | 72 138 | 64.9 |
| Free State | 664 508 | 24 475 | 1 396 | 27 774 | 27 105 | 87.4 |
| Gauteng | 2 129 526 | 74 823 | 2 649 | 105 035 | 97 897 | 87.0 |
| KwaZulu-Natal | 2 866 570 | 96 057 | 6 156 | 157 300 | 112 403 | 77.4 |
| Limpopo | 1 714 832 | 57 108 | 4 067 | 86 650 | 82 483 | 71.8 |
| Mpumalanga | 1 052 807 | 34 936 | 1 885 | 52 321 | 50 053 | 77.6 |
| Northern Cape | 282 631 | 8 972 | 573 | 10 654 | 10 403 | 74.5 |
| North-West | 788 261 | 26 194 | 1 606 | 29 979 | 29 140 | 87.2 |
| Western Cape | 1 052 435 | 36 451 | 1 655 | 49 544 | 47 615 | 85.1 |
| South Africa | 12 489 648 | 425 023 | 25 720 | 597 196 | 562 112 | 78.2 |

Table 2.1: School statistics for South Africa Department of Basic Education, (EMIS, 2013).

### 2.2.2 Schools in the North-West province of South Africa

This study concentrates on the effectiveness of schools in the North-West province of South Africa and this section therefore provides a brief background to the education system in the North-West province.

The North-West province of South Africa is divided into four municipal districts. These districts are Bojanala Platinum (East), Dr Ruth Segomotsi Mompati/Bophirima (West), Ngaka Modiri Molema (Central) and Dr Kenneth Kaunda (South).

Table 2.2 lists some of the key towns in each municipality within the North-West province while Figure 2.1 graphically shows the location of the four district municipalities (Figure 2.1 continues on the next page).

| District Municipality | Key Towns |
|---|---|
| Bojanala Platinum (East) | Brits |
| | Bapong |
| | Rustenburg |
| | Phokeng |
| | Ledig |
| Dr Ruth Segomotsi Mopati/Bophirima (West) | Taung |
| | Vryburg |
| | Madibogo |
| | Ganyesa |
| | Morokweng |
| Ngaka Modiri Molema (Central) | Mafikeng |
| | Bodibe |
| | Lichtenburg |

| | |
|---|---|
| | Zeerust |
| | Swartruggens |
| Dr Kenneth Kaunda (South) | Potchefstroom |
| | Klerksdorp |
| | Wolmaransstad |
| | Leeudoringstad |

Table 2.2: Key towns in each district municipality (Schoolmedia, 2014).

Each one of the four municipal districts has its own department of education that is responsible for the management of schools in a municipal district. The total number of schools (primary and secondary) in all four districts is 1 606 with 788 261 learners and 26 194 teachers (EMIS, 2013).

Table 2.3 below presents school statistics in the North-West province of those schools that have more than 250 learners per school (Schoolmedia, 2014).

| District Municipality | Primary Schools | Primary Schools Learners | Secondary Schools | Secondary Schools Learners |
|---|---|---|---|---|
| Bojanala Platinum (East) | 205 | 129 427 | 90 | 61 513 |
| Dr Ruth Segomotsi Mopati/Bophirima (West) | 102 | 65 105 | 43 | 27 656 |
| Ngaka Modiri Molema (Central) | 182 | 108 194 | 76 | 47 414 |
| Dr Kenneth Kaunda (South) | 77 | 72 590 | 38 | 35 947 |

Table 2.3: School statistics for the North-West Province (Schoolmedia, 2014)

Figure 2.1 shows the North-West province with the four district municipality areas.

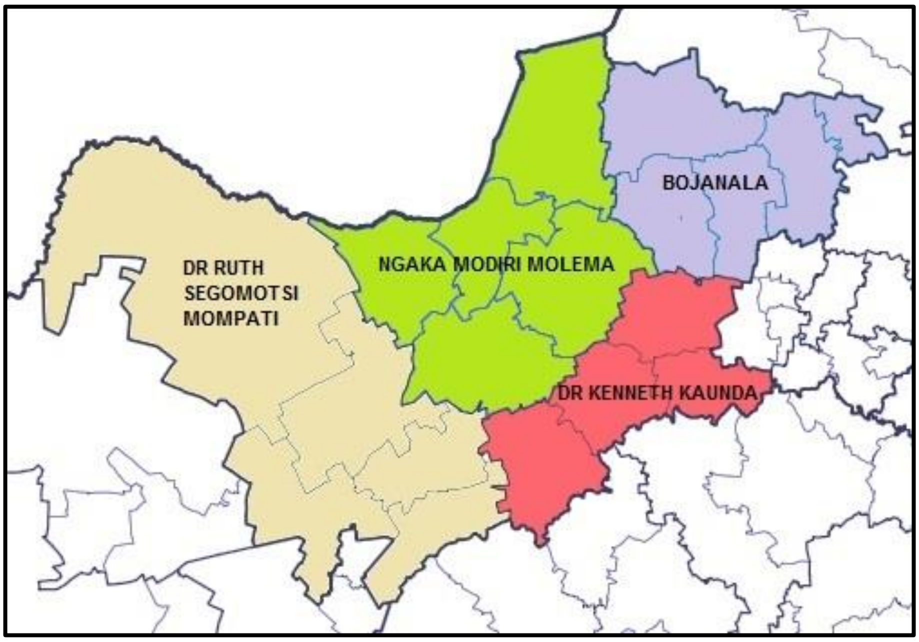



Figure 2.1: The district municipalities in the North-West province of South Africa

For reasons which are explained in Chapter 5 of this research project the study only considers data of secondary schools (grade 8 to 12) which offer mathematics and physical science at grade 12 level in the Dr Kenneth Kaunda district municipality. There were 54 secondary schools, with a total of 45 287 learners, that complied with the data requirements. It should be noted that the 54 secondary schools do not correspond with the 38 secondary schools indicated in Table 2.3 as Table 2.3 only present's schools with more than 250 learners. Table 2.4 presents some of the descriptive statistics for the 54 schools.

| | |
|---|---|
| Number of schools | 54 |
| Total number of learners | 45 287 |
| Number of grade 12 learners | 5 241 |
| Number of grade 9 learners | 12 006 |
| Total number of learners taking mathematics | 1999 |
| Total number of learners taking physics | 1783 |
| Total number of learners who passed grade 12 | 4 337 |

Table 2.4: Descriptive statistics for secondary schools in the Dr Kenneth Kaunda district municipality

### 2.2.3 Prior efforts to address efficiency in schools in South Africa and the North-West province of South Africa

During October 2013 a report entitled "South African education crisis: The quality of education in South Africa 1994-2011" was issued by the Centre for the development and Enterprise (Spaull, 2013). This report painted a rather poor picture of education in South Africa. It investigated the quality of education in South Africa since the transition to democracy and concludes that there is an ongoing crisis in the educational system and that the system is failing the majority of learners. The report also points out that South Africa has the worst education system of middle-income countries and also performs worse than many low income African countries. Based on comparative statistical data and other evidence, the overall conclusion of this report states that "the South African education system is grossly inefficient, severely underperforming and egregiously unfair".

It is clear from this report that research projects into the efficiency of schools in general, and in particular in South Africa, is a necessity that cannot be ignored. In this section an introductory literature review of such research projects is given. The remainder of this section presents details of some of the general research projects that were carried out in a South African context while section 2.4 highlights some other international studies on educational effectiveness with a focus on the use of DEA models.

There are not many formal studies that are directly linked to the efficiency and effectiveness of schools and especially not many where schools are compared in terms of their effectiveness. In the context of this study an effective school is a school that performs close to optimality while efficiency refers to utilising the best available methods and resources to achieve effectiveness. Different angles to the problem are often used for investigative purposes and most studies rely on a qualitative approach e.g. case studies or make use of questionnaires to obtain information.

Two prominent studies are Taylor (2011), who studied the numeracy skill of grade 4 learners and Shepherd (2011), who evaluated the literacy skills of grade 5 learners. The conclusion of the former study was that there was a significant difference in numeracy skills among learners who attended previously disadvantaged schools and those who attended historically white schools. The latter study indicated the differences in literacy skills among learners who attended African language school and learners who attended English and Afrikaans schools.

Taylor (2009) argues that a prerequisite for effective school improvement interventions is the identification of key problems on three levels namely classroom, school and administrative level. The same author also performed a study which resulted in a number of recommendations that would be effective in increasing educational opportunities particularly for poor children (Taylor, 2008).

Van der Berg *et al.* (2011) performed an investigation into the reasons for the poor education quality in South Africa while Van der Berg and Burger (2003) also conducted research into the question of resource availability in schools for the poor. They concluded that there was no significant correlation between performance and resource allocation in the group of schools used in their study.

A few other approaches were also followed to try and address school efficiency. Examples of such studies include Mestry (2006) who investigated the effective use of school funds; Prew (2009) focused on community involvement in school development, particularly at South African township schools; Mashiya (2011) identified factors that inhibited the use of mother tongue as the language of learning and teaching – although this study did not aim to address efficiency, it can be accepted that language plays a definite role in school efficiency; Ngidi (2004) conducted research into educators' perceptions of the efficiencies of school governing bodies – another vital factor in the overall efficiency level of a school. Spaull (2013) performed a study to evaluate the efficiency of primary schools.

There are apparently not many mathematical models used in the South African context to evaluate school efficiency in South Africa. One such study, which was not entirely aimed at school efficiency, is the work of Gustafsson (2007). In this work, the author employed ordinary least squares and hierarchical linear models in an effort to understand school production in South Africa.

It is clear from the studies quoted that research are indeed being conducted into the schools and educational setup in South Africa, with the objective of improving especially the quality of education. However, very little are being done in terms of using mathematical models and also in terms of comparing schools. Such a comparison may reveal certain best practice principles which can be used by the less efficient schools.

In the next section some examples of studies where mathematical models (particularly DEA models) were used to evaluate the efficiency of educational institutions are presented.

## 2.3 The use of DEA and other mathematical models

A popular and frequently used mathematical approach to evaluate efficiency of different, but homogenous units is the implementation of a linear programming model called data envelopment analysis (DEA) (Charnes *et al.*, 1978). This type of model is also used in this study and a more comprehensive introduction to DEA models is given in Chapter 3.

Briefly, DEA can be described as a non-parametric model that is used to empirically measure the efficiency of different decision making units (e.g. schools) by converting multiple non-parametric inputs into multiple non-parametric outputs (Cooper *et al.*, 2011; Porcelli, 2009). In the context of education, a school or university is modelled as a multi-input decision making unit (DMU) which attempts to maximize their outputs for a given set of inputs.

It is interesting to note that the authors of the seminal paper on DEA (Charnes *et al.*, 1978) frequently applied the DEA approach in an effort to improve the quality of education. See for example the following papers with W Cooper (one of the initial DEA model developers) as co-author (Ahn *et al.*, 1989; Arcelus, 1997; Brockett *et al.*, 2005; Charnes *et al.*, 1981; Cooper and McAlister, 1999). Johnes (2015) performed a comprehensive survey of the diverse problems in education and the techniques which have typically been applied to these problem areas. One of the areas identified by Johnes (2015) in education is the problem of efficiency and performance measurement, which is mainly addressed through the application of DEA models. According to Johnes (2015), who provided an extensive list of educational studies using DEA and related non-parametric methods, education is one of the top five areas of DEA.

Performance evaluation using DEA models has been extensively discussed in the literature. In the remainder of this section a few examples are presented followed by a summary (Table 2.5) of other research projects on this topic.

Blackburn *et al.* (2014) applied a DEA model to estimate the efficiency of Australian primary and secondary schools. They focused strongly on nondiscretionary environmental variables and employed a conditional estimator that did not allow a school with a better environment to serve as a benchmark for a school with a worse environment.

Sarrico and Rosa (2009) used a set of 4 inputs and 2 outputs in a DEA model to measure and compare the performance of 51 Portuguese secondary schools. They concluded that there was a considerable variance in the performance of Portuguese secondary schools and that significant improvement in the school system was possible.

There are also a large number of studies where DEA models were used to assess the efficiency of universities. Two examples of such studies include Tóth (2009), who used a DEA model to compare the efficiency of higher education institutions in 19 countries in Europe, and Abbott and Doucouliagos (2003). The latter research project experimented with various measures of outputs and inputs in a DEA model applied to

Australian universities. They concluded that regardless of the output-input mix, Australian universities were generally operating on high levels of efficiency.

An interesting variant of the original DEA concept is the so called “onion peeling” principle (Barr *et al*., 2000) which is also referred to as “measuring of attractiveness” (Seiford and Zhu, 2003). This technique is of particular importance in this study and is explained and elaborated on in Chapter 3. In short, the technique, which is normally used for ranking purposes, is a DEA based model without input variables that stemmed from the concept of Pareto optimality to stratify decision making units into classes of different levels. Examples of how this technique was applied in education can be found in Kao (1994) and Kao and Lin (2008).

Kao (1994) performed a case study using this version of DEA to evaluate junior colleges of technology in Taiwan. The study, and the models used, showed that the results were the same as those obtained by normal government evaluations. However, government evaluations are subject to a lengthy and costly process in order to get results. The application of the Pareto optimal model was much easier, quicker and less expensive and, in addition, through a dual analysis, suggestions could be provided for improvement for colleges in lower categories.

This work was followed-up by Kao and Lin (2008) with a class ranking study of 34 management colleges in Taiwan. A similar “onion peeling” model ensured that Pareto optimality was employed and it was shown once again that the technique was capable of ranking the colleges under incomparable criteria and also to provide intermediate goals for a college to improve to the next categories in stages.

There are also other techniques (non-DEA techniques) listed in the literature that were used to assess the efficiency of educational institutions. Some examples include Haelermans (2011), who used a meta-regression analysis approach; Conroy and Arguea (2008), who implemented a frontier production function estimation technique; and Wang (2003), who employed an adaptive neural network technique.

As stated earlier, there are a vast number of studies that discuss efficiency evaluations of educational institutions. Apart from the few examples mentioned above, Table 2.5 provides a more comprehensive list of such studies. The list is not exhaustive and serves only as an additional resource of related studies. A further detailed source of studies and references on efficiency and performance measurement in education can be found in Johnes (2015).

| Author | Title | Techniques |
|---|---|---|
| De Witte and Rogge (2014) | Does ICT matter for effectiveness and efficiency in mathematics education? | Investigation into ICT infrastructure investments in educational institutions using Mahalanobis matching control groups. |
| Nazarko and Šaparauskas (2014) | Application of DEA method in efficiency evaluation of public higher education institutions. | CCR-CRS output-oriented DEA model used in a comparative efficiency study of 19 Polish universities of technology. One input variable, four output variables and two environmental variables were used. |
| Porter *et al.* (2014) | Blended learning in higher education: Institutional adoption and implementation. | Comparative study of 11 cases of institutional blended learning adoption. |
| Yalçin and Tavşancil (2014) | The comparison of Turkish students' PISA achievement levels by year via data envelopment analysis. | A DEA model was used to analyse data obtained from Turkish students for three different years. Five input variables and three output variables were used. |
| Agasisti (2013) | The efficiency of Italian secondary schools and the potential role of competition: a data envelopment analysis using OECD-PISA2006 data | Various DEA models including a DEA bootstrapping procedure and a Tobit regression model was used to calculate efficiency scores for a sample of Italian schools. Three input variables and one output variable were used. |
| Essid *et al*. (2013) | Small is not that beautiful after all: measuring the scale efficiency of Tunisian high schools using a DEA-bootstrap method. | A non-parametric statistical test procedure together with a smooth DEA-bootstrap method. Four input variables, two quasi-fixed input variables and four output variables were used. |
| de Figueiredo and Barrientos (2012) | A decision support methodology for increasing school efficiency in Bolivia's low-income communities. | DEA model and a correlation matrix were used to assess the efficiency of 439 Bolivian in-network schools among themselves and also against out-of-network schools. Three input variables and two output variables were used. |

| | | |
|---|---|---|
| Dutta (2012) | Evaluating the technical efficiency of elementary education in India: An application of DEA. | A DEA and regression model was used to assess the technical efficiency and efficiency differences in the elementary education system across the states of India. Four input variables and four output variables were used. |
| Haelermans and Blank (2012) | Is a schools' performance related to technical change? – A study on the relationship between innovations and secondary school productivity. | A nonparametric model using bootstrap based significance tests to study the statistical significance of variables. |
| Haelermans and De Witte (2012) | The role of innovations in secondary school performance – evidence from a conditional efficiency model. | The influence of educational innovations using a nonparametric conditional efficiency model. |
| Hirao (2012) | Efficiency of the top 50 business schools in the United States. | DEA model with two input variables and two output variables. |
| Klumpp (2012) | European universities efficiency benchmarking. | A comparative study of efficiency measures for a total of 370 comparable universities. One input variable and three output variables were used. |
| Worthington and Higgs (2011) | Economies of scale and scope in Australian higher education. | A cost function was employed to estimate the economies of scale and scope in Australian higher education. |
| Alexander *et al*. (2010) | A two-stage double-bootstrap data envelopment analysis of efficiency differences of New Zealand secondary schools | A DEA and regression analysis of the efficiency of 325 New Zealand secondary schools. Eleven inputs and three outputs were used in the DEA stage. Thirteen variables were used in the regression stage. |
| Kao and Hung (2008) | Efficiency analysis of university departments: An empirical study. | DEA model to evaluate 41 departments of similar characteristics categorized in 4 groups. Three input variables and three output variables were used. Cluster analysis was used to explain results. |
| Kao and Lin (2004) | Evaluation of the university libraries in Taiwan: total measure versus ratio measure. | DEA model to evaluate 24 university libraries. Two models were considered one taking the university size into account and the other one excluding it. One input variable and five output variables were used. |

| | | |
|---|---|---|
| Liu *et al*. (2004) | DEA approach for the current and the cross period efficiency for evaluating the vocational education. | A DEA model used to evaluate 38 technological institutes. A combination of eight input and output variables were used. |
| Abbott and Doucouliagos (2003) | Competition and efficiency: Overseas students and technical efficiency in Australian and New Zealand universities. | Stochastic Frontier Analysis (SFA). |
| Emrouznejad (2002) | The assessment of higher education institutions using dynamic DEA: A case study in UK universities. | A comparison of a dynamic DEA model (over time) and a static DEA model with other performance indicators. Two input variables and three output variables were used. |
| Kao and Liu (2000) | Data envelopment analysis with missing data: an application to university libraries in Taiwan. | A DEA model using membership functions to represent imprecise data. One input variable and five output variables were used. |
| Beasley (1990) | Comparing university departments. | DEA model used to evaluate chemistry and physics departments at universities. Three input variables and seven output variables were used. |
| Tyagi *et al*. (2009) | Efficiency analysis of schools using DEA: A case study of Uttar Pradesh state in India | DEA model to evaluate 348 elementary schools in India. Eight input variables and three output variables were used. |

Table 2.5: Studies on efficiency evaluations of educational institutions

## 2.4 Decision Support Systems in Education

One of the aims of this study is to develop a decision support system that implements DEA based models to evaluate school efficiency. Building the DSS and explaining DSS concepts are presented in Chapter 4. The purpose of this section is to present only a few background examples of other studies found in the literature and that is related to the use of DSS in education.

In section 2.4 the importance of models, such as DEA models, was pointed out in studies pertaining to efficiency and performance measurements in the education sector. The use of mathematical models and techniques may however be overwhelming in terms of complexity, ease of use and interpretation. To overcome this problem computerized systems (mainly DSS) are often developed to not only assist users but also to enhance the overall process of gathering, processing and interpretation of data and results. Following are a few examples of studies where specific educational systems were developed. A brief summary (Table 2.6) of additional literature resources on this topic is also provided.

Şuşnea (2013) argues that universities have become dependent on the collection, storage and processing of educational data. In order to make sense of the data and to improve decision making (which will maximise the performance of universities) an intelligent decision support system is proposed. The study describes a 3-component system; a data management system, a model management system (containing the analytic tools and models) and a user interface.

Dias and Diniz (2013) developed a fuzzy logic-based system that quantitatively estimates users' quality of interaction with a learning management system under blended learning. Users in this case refer to teachers/professors and learners. The quality of learning (effectiveness) is related to the quality of interaction which is enhanced through the fuzzy-logic model as it facilitates a better understanding of the relevant underlying aspects linked to a user's quality of interaction.

To achieve an acceptable level of administrative and operational efficiency, Miranda *et al.* (2012) proposed a web-based decision support system for course and classroom scheduling. The system implements an integer programming model that is capable of generating optimal schedules. Other functionalities include a direct interaction facility for instructors to gather and obtain spesific data.

Efficiency in educational institutions (especially higher education) is often dependent on the quality of student advising. To address this important determinant of efficiency Feghali *et al.* (2011) developed a web-based decision support tool to assist with accademic advising. The system enables users to make use of an already existing university information system and contributes to the relationship between an advisor and a student. Feghali *et al.* (2011) reported that a survey amongst students using this system showed a very high level of satisfaction amongst users. There exist also decision support systems that are not directly linked to the efficiency and effectiveness of an educational institution but they may have a significant impact on the institution and its performance. One such example is the web-based decision support system developed by Giannoulis and Ishizaka (2010) to rank British universities. Rankings of universities may have a sizable impact as it provides an indication of prestige which may directly influence the number and quality of students. These types of rankings can be done in various ways using different techniques of which DEA models are considered as one such technique. Giannoulis and Ishizaka (2010) refer to DEA as an possible option but implemented other multi-criteria decision methods in their decision support system.

This section is concluded with a brief summary of other studies involving decision support systems in education. The summary is presented in Table 2.6.

| **Author** | **Title of study** |
|---|---|
| Indrayani (2013) | Management of academic information system (AIS) at higher education in the city of Bandung |
| Chau and Phung (2012) | A knowledge-driven educational decision support system |
| Abu-Naser *et al.* (2011) | A prototype decision support system for optimizing the effectiveness of e-learning in educational institutions |
| Power *et al*. (2011) | Reflections on the past and future of decision support systems: perspective of eleven pioneers |
| Vohra and Das (2011) | Intelligent decision support systems for admission management in higher education institutes |
| Bresfelean and Ghisoiu (2010) | Higher education decision making and decision support systems |
| Zilli and Trunk-Širca (2009) | DSS for academic workload management |
| Bednarza and van der Schee (2006) | Europe and the United States: the implementation of geographic information systems in secondary education in two contexts |
| Vinnik and Scholl (2005) | Decision support system for managing educational capacity utilization in universities |
| Breiter and Light (2004) | Decision support systems in schools – From data collection to decision making |
| Deniz and Ersan (2002) | An academic decision-support system based on academic performance evaluation for student and program assessment |

Table 2.6: Studies on efficiency evaluations of educational institutions

## 2.5 Conclusion

The objective of Chapter 2 was to provide an introductory background to the study. This was achieved by presenting a brief overview of teaching and teaching statistics in South Africa and particularly in the North-West province of South Africa. A summarised literature review, covering the use of models (particularly DEA models) and decision support systems, was also presented. The next chapter gives an overview and background on DEA models which were employed in this research study.

# 3 MATHEMATICAL MODELS

## 3.1 Introduction

In this study a DEA methodology was used to develop a decision support system that can assist decision makers in evaluating school efficiency levels. The previous chapter presented a literature review of mathematical models, decision support systems and other concepts related to the efficiency of educational institutions. The aim of Chapter 3 is to provide an introductory theoretical background to the data envelopment analysis models that were employed in this study. The chapter starts with a summarized description of linear programming models in general, followed by a discussion on DEA models and related concepts. Class ranking, using a DEA approach, was of particular interest to this study and is presented in some detail.

## 3.2 Formulations of linear programming models

Data envelopment analysis techniques (which are explained in section 3.3) are based on linear programming models. This section therefore very briefly touches on the formulation of a linear programming model and its dual formulation, as an introductory background.

### 3.2.1 Linear programming model

It is widely accepted that general linear programming problems were first conceived by George B Dantzig around 1947 (Gass and Assad, 2005).

He was also responsible for developing the “simplex method” for solving linear programs.

Linear programming is concerned with the optimization (minimization and maximization) of a linear function while satisfying a set of linear equality and/or inequality constraints or restrictions (Bazaraa *et al.*, 2010). A given linear programming model can be represented in *standard* or *canonial* form. In standard form all constraints are expressed as equalities with all varialbles nonnegative. This is a prerequisite for the application of the simplex method. In canonical form all variables are nonnegative and the constraints are of the ≥ type (for minimization problems) or ≤

type (for maximization problems). Mathematically the standard form and the canonical form can be represented as follows (for a maximization problem):

Standard form: Maximize $$\sum_{j=1}^{n} c_j x_j \tag{3.1}$$

subject to $$\sum_{j=1}^{n} a_{ij} x_j = b_i \quad \text{for } i = 1 \cdots m, \tag{3.2}$$

$$x_j \geq 0 \quad \text{for } j = 1 \cdots n. \tag{3.3}$$

Canonical form: Maximize $$\sum_{j=1}^{n} c_j x_j \tag{3.4}$$

subject to $$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad \text{for } i = 1 \cdots m, \tag{3.5}$$

$$x_j \geq 0 \quad \text{for } j = 1 \cdots n, \tag{3.6}$$

where $c_j$ $(j = 1 \cdots n)$ are called cost coefficients and $a_{ij}$ and $b_i$ $(i = 1 \cdots m)$ and $(j = 1 \cdots n)$ are given constant values. $x_j$ $(j = 1 \cdots n)$ represent unknown decision variables.

Linear programming problems are also often stated in a more convenient form using matrix notation. Consider the standard form formulation given above. Denote the row vector $(c_1 \cdots c_n)$ by $\boldsymbol{c}$ and consider the column vectors $\boldsymbol{x}$ and $\boldsymbol{b}$ and the $m\ x\ n$ matrix $\boldsymbol{A}$, then

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} \qquad \boldsymbol{A} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}. \tag{3.7}$$

The model can then be formulated as

$$\begin{aligned} &\text{Maximize} \quad \boldsymbol{cx} \\ &\text{subject to} \quad \boldsymbol{Ax} = \boldsymbol{b} \ \text{ and} \\ &\qquad\qquad \boldsymbol{x} \geq \boldsymbol{0}. \end{aligned} \tag{3.8}$$

Linear programming problems in the canonical form can be converted to standard form through the implementation of slack, surplus and/or artificial variables (see for example

Bazaraa *et al*. (2010) and Render *et al*. (2012)). Detailed discussion on assumptions (i.e. proportionality, additivity, divisibility, and determinism); special cases (i.e. infeasibility, unbounded, redundancy and multiple solutions); and solution methods (i.e. the simplex method) can be found in Render *et al*. (2012) and Bazaraa *et al*. (2010).

### 3.2.2 The dual to a linear programming model

Every linear programming model has a corresponding linear programming problem associated with it. The original linear programming problem is termed the *primal* and the corresponding problem the *dual*. The dual model contains economic information and provides certain insights into the solution of a primal model. It may also be easier to solve in terms of less computation time.

The dual model can be derived from the primal model as follows.

Consider the following primal linear programming problem.

Find a column vector $\boldsymbol{X} = (x_1 \cdots\ x_n)$ which *minimises* the linear function

$$\begin{aligned} & f(\boldsymbol{X}) = \boldsymbol{cX} \\ \text{subject to} \quad & \boldsymbol{Ay} = \boldsymbol{b}, \\ & \boldsymbol{X} \geq \boldsymbol{0}. \end{aligned} \tag{3.9}$$

The dual linear programming model is then derived as follows.
Find a row vector $\boldsymbol{W} = (w_1 \cdots w_m)$ which *maximises* the linear function

$$\begin{aligned} & g(\boldsymbol{W}) \ = \ \boldsymbol{Wb} \\ \text{subject to} \quad & \boldsymbol{WA} \ \leq \ \boldsymbol{c} \end{aligned} \tag{3.10}$$

with the variables $w_i$ unrestricted in sign.

Associated with these model formulations is the important Duality Theorem which ensures that optimal solutions for the primal and the dual are equivalent. The theorem states, (Cooper *et al*., 2007)

- In a primal-dual pair of linear programs, if either the primal or the dual problem has an optimal solution, then the other does also, and the two optimal objective values are equal.
- If either the primal or the dual problem has an unbounded solution, then the other has no feasible solution.

– If either problem has no solution, then the other problem either has no solution or its solution is unbounded.

A complete proof of the Duality Theorem can be found in Cooper *et al.* (2007).

In the context of this study, the dual formulation is an important model representation. The study made use of a dual formulation of a spesific DEA model in order to rank and evaluate different schools. This is again explained in section 3.4 and subsequent chapters.

Section 3.2 provided only brief introductory comments on linear programming models and the associated dual formulations of such models. Comprehensive discussions and mathematical explanations can be found in Render *et al.* (2012) and Bazaraa *et al.* (2010).

## 3.3 The origin and concept of DEA

A large amount of research has been conducted in the area of efficiency measures and the measuring of relative productivity (relative efficiency). The two most widely used quantitative techniques in this area are stochastic frontier analysis (SFA) and data envelopment analysis (DEA) (Herrero and Salmeron, 2005). This section gives a brief overview of DEA which is, apart from model formulations, a non-technical discussion. The discussion is also of an introductory nature as the volume of research and literature is enormous and beyond the scope of this study. Emrouznejad *et al.* (2008) list, for example, more than 4000 research articles dealing with data envelopment analysis. This research involves more than 2500 authors and a large number of applications.

Section 3.3 proceeds as follows: a short summary on the history of DEA is presented, followed by an explanation of the DEA concepts. The basic DEA models are then covered and the section concludes with possible pitfalls in the application of DEA models.

### 3.3.1 The history of DEA

DEA has its roots in the work of Farrell (1957). Farrell argued that, although a number of attempts were made to solve the problem of measuring productive efficiency, they generally failed to combine the measurement of multiple inputs and outputs into a satisfactory overall measure of efficiency. Furthermore, Farrell was also concerned with the construction of index numbers, especially "indices of efficiency". These inadequacies of an overall measure of efficiency and separate indices lead to Farrell's work on the measurement of productive efficiency which was inspired by the activity analysis approach advocated by Koopmans (1951).

The initial DEA model was based on research by Eduardo Rhodes who evaluated an educational program for disadvantaged students in the USA. It was this challenge of estimating the relative "technically efficiency" of schools involving multiple inputs and outputs that resulted in the formulation of the so-called CCR (Charnes, Cooper and Rhodes) DEA model (Charnes *et al*., 1994). According to Charnes *et al.* (1994) the CCR model implemented a mathematical programming technique that generalized the work of Farrell into a multiple-output/multiple-input case by constructing a single "virtual" output to a single "virtual" input relative-efficiency measure. This model was applicable only to technologies characterised by constant returns to scale.

The original CCR model was extended by Banker *et al.* (1984) to accommodate variable returns to scale and the model became known as the BCC model, named after the three authors. Since then, many new developments and extensions to the DEA models have been recorded. Charnes *et al*. (1994) reported that more than 400 articles, books and dissertations were published between 1978 and 1992.

Section 3.3.3 presents the basic model formulations for the well-known CCR and BCC models. Extensions and other properties to these models are not discussed further. Such extensions and properties include isotonicity, non-concavity, economics of scale, piecewise linearity, discretionary and nondiscretionary inputs, value judgements, categorical variables and also ordinal relationships. Comprehensive discussions and mathematical details of these extensions can be found in Cooper *et al*. (2007); Thanassoulis, (2001); Fried *et al.* (1993); Seiford and Thrall (1990).

### 3.3.2 DEA – the concept

The DEA concept was developed by Charnes, Cooper and Rhodes (1978), following the work of Farrel (1957). Units or organisations under study are termed decision making units (DMUs) and may range from schools, hospitals, government departments or any other homogeneous set of units that perform similar operations and can be sensibly compared.

DEA is a non-parametric linear programming technique and is designed to construct specific benchmarks for evaluating the performance of individual DMUs. It can be seen as an extension of ratio analysis that also provides information regarding input and output targets to technically inefficient DMUs. This means that optimal weights are provided on DMUs that are used as references to benchmark each DMU being evaluated to obtain an objectively identified efficient peer group, called a reference set (Cooper, 2014; Johnes and Yu, 2008).

To illustrate the concept and how DEA works, an example from Thanassoulis (2001) is presented. Some of the work from Thanassoulis (2001) are therefore quoted without referencing it again.

Consider the case where a number of DMUs, using a single input to secure a single output, are assessed. The DMUs are plotted in Figure 3.1.

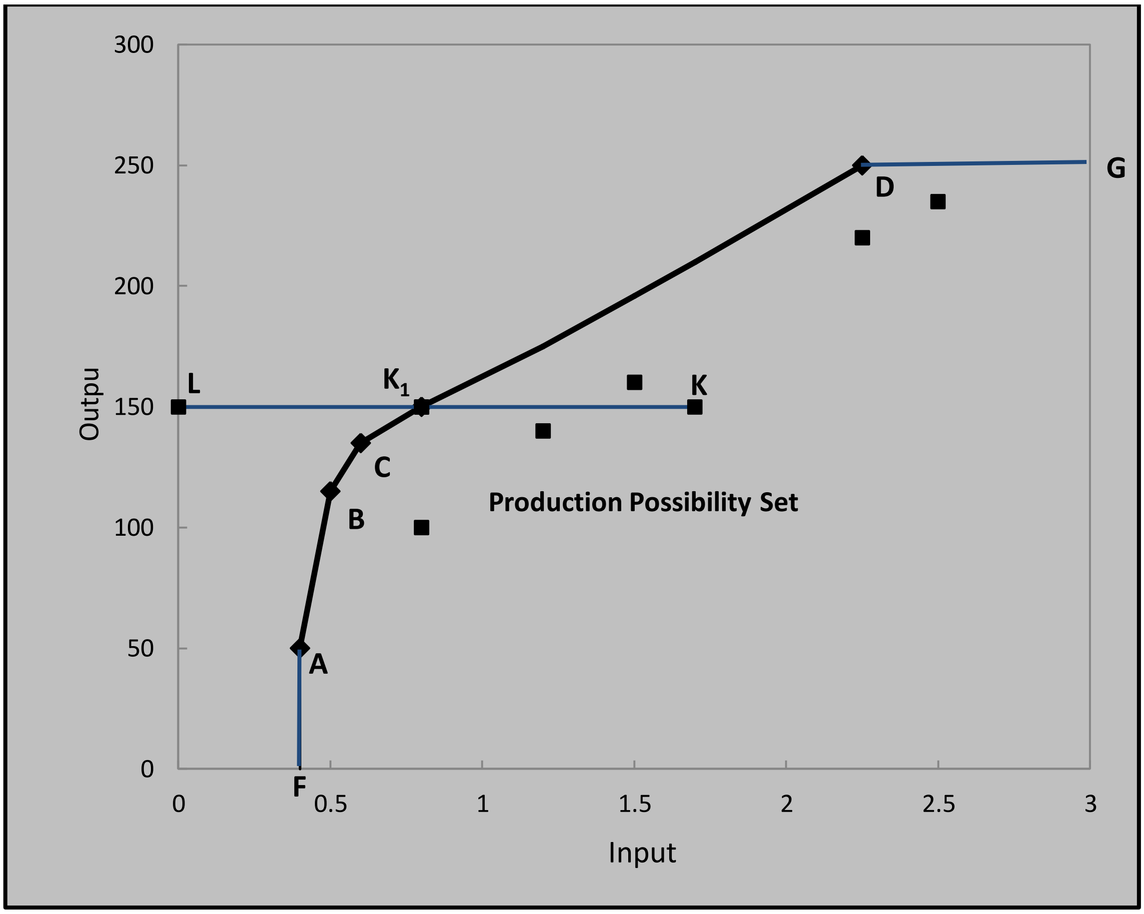



Figure 3.1: An illustration of an assessment by data envelopment analysis (Thanassoulis, 2001).

To construct the production possibility set (indicated in Figure 3.1), three assumptions are made

- Interpolation between feasible input-output correspondences leads to new input-output correspondences which are feasible in principle;
- Inefficient production is possible; and
- The production possibility set is the smallest set meeting the foregoing assumptions and containing all input-output correspondences observed at the units being assessed.

Based on these assumptions the linear segments AB, BC etc. are feasible in principle. The extensions DG and AF also contain input-output correspondences which are feasible in principle. Similarly, all input-output correspondences to the right and below the piece-wise linear boundary FABCDG are also feasible in principle. The smallest production possibility set satisfying the three assumptions and containing all units being assessed is the space to the right and below the boundary FABCDG, and including the boundary.

Once the production possibility set has been identified, the DMUs can be assessed. Suppose that the efficiency of $DMU_k$ needs to be estimated. The frontier FABCDG is used as a reference and, assuming an input oriented approach, a horizontal line is drawn from K to find the minimum output level that corresponds to a minimum input level. This is at point $K_1$ in Figure 3.1. Hence, the efficiency is $LK_1/LK$, the fraction to which $DMU_k$ could in principle lower its input level.

This assessment process yields much more information than a mere assessment of the efficiency of a DMU. Thanassoulis (2001) explains this as follows.

The target input for $DMU_k$ in Figure 3.1 was identified to be at $K_1$. This is based on the interpolation of the performance of DMUs C and D. The identification of DMUs such as C and D is important as they are on the boundary of the production possibility set and operate relatively efficient – i.e. there are no other DMUs or combinations of DMUs which dominate them in the sense of yielding more output for a given input or use less input for a given output. DMUs C and D can now be used as “role models” of which can be used by the inefficient $DMU_k$ to improve its performance.

In this example a single input and a single output were used to illustrate graphically how the concept of DEA works. The method works in the same way when multiple inputs and multiple outputs are involved. The only difference is that linear programming models are used to construct the production possibility set involved and to measure the distance of any unit within that space from the efficient boundary. Formulations of such models are presented in section 3.3.3.

It is important to note that there is a difference between DEA (a non-parametric technique) and other parametric approaches such as regression analysis. According to Charnes *et al.* (1994) the focus of DEA is on the individual observations as represented by the *n* optimizations (one for each observation) required in a DEA analysis, as opposed to the focus on the averages and estimation of parameters that are associated with single-optimization statistical approaches. A parametric approach requires the imposition of a specific functional form (e.g. a regression equation) which in turn requires specific assumptions (e.g. the distribution of error terms). DEA, as a non-parametric technique, does not require any assumption about the functional form and simply calculates a maximal performance measure for each DMU relative to all other DMUs. The sole requirement is that each DMU lies on or below the efficiency frontier. Each DMU that is not on the frontier is then scaled against a convex combination of the DMUs on the frontier facet closest to it.

The DEA methodology has opened up new avenues of addressing performance issues and has enabled decision makers to address various other issues which may have an effect on productivity. Thanassoulis (2001) lists the following examples of how DEA can be used:

- It can be used to decompose efficiency into components attributable to different layers of management or agents involved in the operations of DMUs being assessed;
- The impact of policy initiatives on productivity can be assessed through the use of DEA; and
- It can be used to measure the change over time in the productivity of the industry as distinct from that of the DMUs operating within it.

According to Charnes *et al.* (1994) the DEA approach affords new ways of organizing and analysing data and can result in new managerial and theoretical insights. It is therefore important to note the following characteristics of DEA calculations:

1. Focus on individual observations in contrast to population averages;

2. Produce a single aggregate measure for each DMU in terms of its utilization of input factors (independent variables) to produce desired outputs (dependent variables);
3. Can simultaneously utilize multiple outputs and multiple inputs with each being stated in different units of measurements;
4. Can adjust for exogenous variables;
5. Can incorporate categorical (dummy) variables;
6. Are value free and do not require specification or knowledge of a priori weight or prices for the inputs or outputs;
7. Place no restriction on the functional form of the production relationship;
8. Can accommodate judgment when desired;
9. Produce specific estimates for desired changes in input and/or outputs for projecting DMUs below the efficient frontier onto the efficient frontier;
10. Are Pareto optimal;
11. Focus on revealed best-practice frontiers rather than on central tendency properties of frontiers; and
12. Satisfy strict equity criteria in the relative evaluation of each DMU.

The purpose of section 3.3.2 was to explain the DEA concept briefly and in a non-technical manner. Mathematical model formulations of the basic DEA models are presented in the next section.

### 3.3.3 The basic DEA models

Since the original DEA model formulation by Charnes *et al*. (1978), many new models and/or extensions have been developed by a large number of researchers. In this section a summary of only the two initial basic DEA models is presented. These two models and their interpretive possibilities are (Charnes *et al.*, 1994):

The ***CCR ratio model*** (Charnes, Cooper, and Rhodes, 1978) which:

i. yields an objective evaluation of overall efficiency; and
ii. identifies the sources and estimates the amount of the thus-identified inefficiencies.

The ***BCC ratio model*** (Banker, Charnes, and Cooper, 1984) which distinguishes between technical and scale inefficiencies by:

i. estimating pure technical efficiency at the given scale of operation; and
ii. identifying whether increasing, decreasing or constant returns to scale possibilities are present for further exploitation.

Charnes *et al*. (1994) state that the selection of a basic DEA model for analysis should be made after careful consideration as the choice of model will determine the implicit returns-to-scale properties. A constant return to scale implies that output produced by a DMU is directly equal to the input consumed by the DMU. This means that if $(x, y)$ is a feasible input-output combination, then so is $(\alpha x, \alpha y)$ with $\alpha$ a non-zero positive constant. The CCR model is based on the assumption of constant returns to scale activities.

In contrast to the constant returns to scale assumption, the BCC model has its frontiers spanned by the convex hull of existing DMUs. These frontiers have piecewise linear and concave characteristics which leads to variable returns to scale. This may include increasing returns to scale and/or decreasing returns to scale.

Generally, there are also three orientations available to choose from. If the goal is to minimise inputs consumed while maintaining the same level of outputs the model is an input orientated model. Conversely, an output orientated model maximises output while maintaining a given input level. Lastly, a non-oriented model deals with input excesses and output deficiencies simultaneously. Figure 3.2 shows the classification by returns to scale and orientation (Charnes *et al*., 1994).

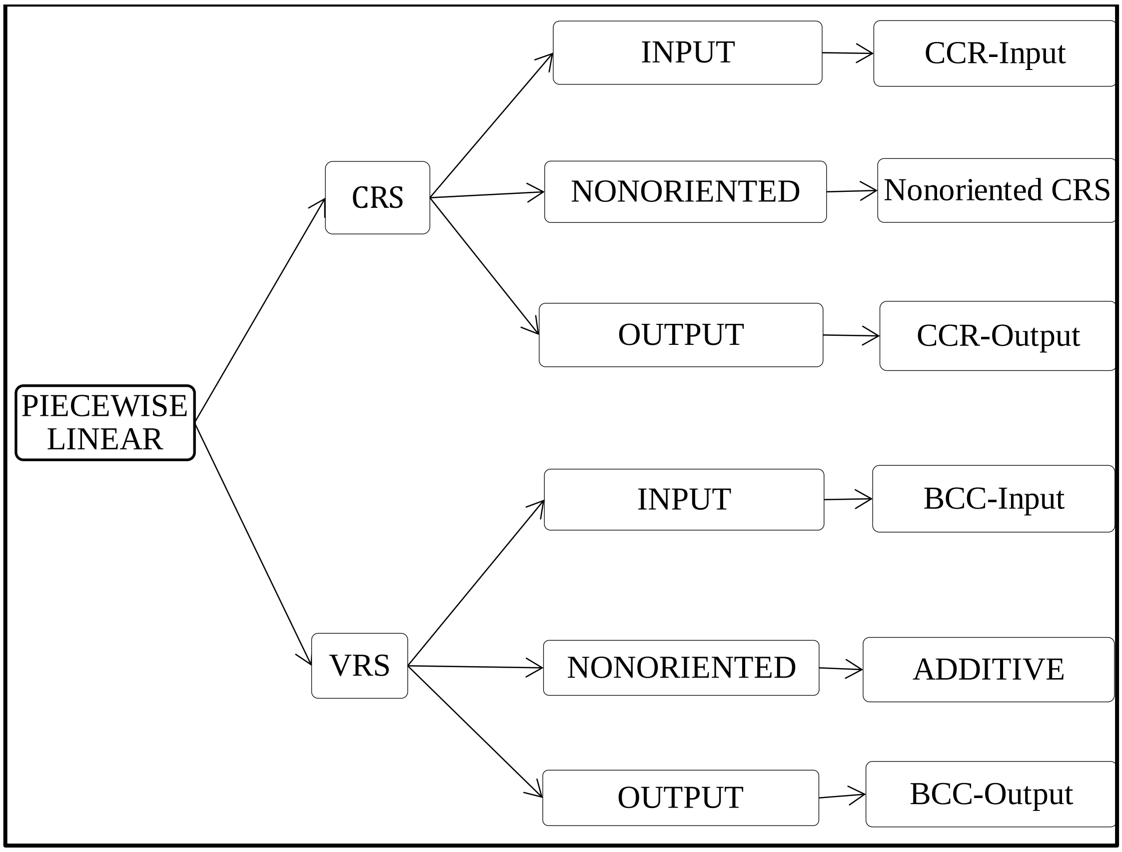



Figure 3.2: Classification by return to scale and orientation, (Charnes *et al*., 1994).

The next two sub-sections present the model formulations of the basic CCR and BCC models. The presentation and discussion is based on the work of Cook and Seiford (2009).

### 3.3.3.1 The CCR model

Assume that a set of $n$ DMUs need to be evaluated. Each $DMU_j$ ($j = 1,\ldots,n$) uses $m$ inputs $x_{ij}$ ($i = 1,\cdots,m$) and generates $s$ outputs $y_{rj}$ ($r = 1,\cdots,s$). If the prices (or multipliers) $\bar{u}_r$ and $\bar{v}_i$, associated with output $r$ and inputs $i$, respectively are known, then conventional benefit/cost theory suggests that the efficiency $\bar{e}_j$ of $DMU_j$ can be expressed as the ratio of weighted outputs to weighted inputs. Mathematically it can be denoted by:

$$\sum_{r=1}^{s} \bar{u}_r y_{rj} \Big/ \sum_{i=1}^{m} \bar{v}_i x_{ij} \qquad (3.11)$$

For the case of unknown prices (multipliers) Charnes *et al*. (1978) developed the CCR model to derive appropriate multipliers. If $DMU_0$ is considered, the CCR model for measuring the technical efficiency of $DMU_0$ is given by the solution to the following fractional programming problem:

$$e_0 = max \quad \sum_{r=1}^{s} u_r y_{r0} \bigg/ \sum_{i=1}^{m} v_i x_{i0} \tag{3.12}$$

$$\text{subject to} \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad \text{for all } j, \tag{3.13}$$

$$u_r, v_i \geq \varepsilon \quad \text{for all } i, r$$

where $\varepsilon$ is a non-Archimedean value designed to enforce strict positivity on the variables. This formulation is the original CCR model and provides for constant return to scale.

The theory of fractional programming (Charnes and Cooper, 1962) suggests that the fractional programming model can be converted to a linear programming model by implementing the following variable substitutions:

$$\mu_r = t u_r \text{ and } \nu_i = t v_i \tag{3.14}$$

where

$$\text{t} = \left( \sum_{i=1}^{m} v_i x_{i0} \right)^{-1} . \tag{3.15}$$

The linear programming formulation is then given by:

$$\text{e}_0 = \max \quad \sum_{r=1}^{s} \mu_r y_{r0} \tag{3.16}$$

$$\text{subject to} \quad \sum_{i=1}^{m} \nu_i x_{i0} = 1, \tag{3.17}$$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} \leq 0 \quad \text{for all j}, \tag{3.18}$$

$$\mu_r, \quad \nu_i \geq \varepsilon \quad \text{for all } i, j, r.$$

The dual formulation of this linear programming model is as follows:

$$\text{Min } \theta_0 - \varepsilon \left( \sum_{r=1}^{s} s_r^+ - \sum_{i=1}^{m} s_i^- \right) \tag{3.19}$$

$$\text{subject to } \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta_0 x_{i0} \quad i = 1 \cdots m, \tag{3.20}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r0} \quad r = 1 \cdots s, \tag{3.21}$$

$$\lambda_j, s_i^-, s_r^+ \geq 0 \quad \text{for all } i, j, r,$$

and $\theta_0$ is unconstrained.

The production possibility set T can mathematically be expressed as:

$$T = \left\{ (X, Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j, \quad Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \quad Y_j, \lambda_j \geq 0 \right\} \tag{3.22}$$

To perform the DEA evaluation would require the solution of *n* linear programming problems – one for every DMU in the set of DMUs.

A detailed description of each mathematical expression (3.11 – 3.22) can be found in Charnes *et al*. (1978) and Charnes and Cooper (1962).

Figure 3.3 provides a graphical representation of the CCR DEA model and shows 7 DMUs in a single input and single output case. The solid line represents the efficient frontier. Only $DMU_2$ is efficient and is in the reference set for all the other DMUs. The efficiency of $DMU_3$ can now be measured as the ratio $A/B = 4.2/6 = 0.7$ or $70\%$. For $DMU_3$ to be efficient (move onto the efficient frontier), its inputs should be reduced by $30\%$ while maintaining the same level of output. It should be noted that a DMU identified as efficient in the CCR input-oriented model, will also be efficient in the output-oriented CCR model.

| DMU | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| X | 2 | 3 | 6 | 9 | 5 | 4 | 10 |
| Y | 2 | 5 | 7 | 8 | 3 | 1 | 7 |

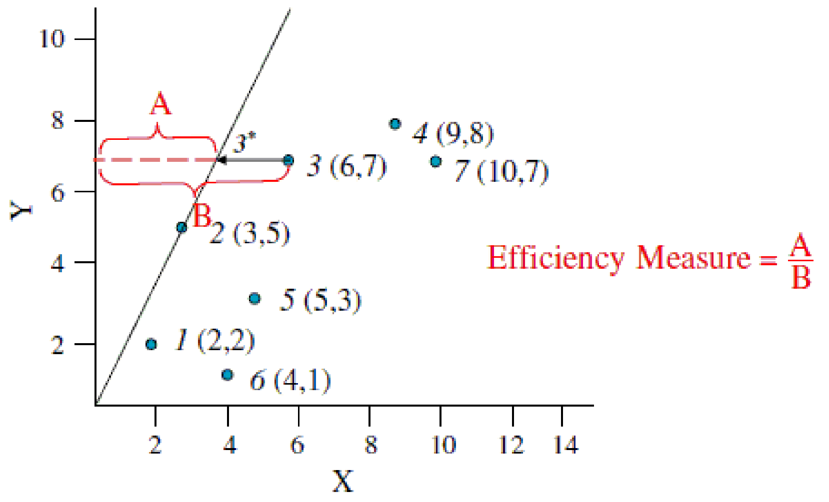



Figure 3.3: Classification by returns to scale and orientation (Cook and Seiford, 2009).

3.3.3.2 The BCC model

One of the prominent extensions to the original CCR model is the extended model (BCC model) from Banker *et al*. (1984) to provide for variable returns to scale.

The BCC model differs from the CCR model by way of an additional variable. The ratio model is formulated as follows (for the same DMU set as in section 3.3.3.1):

$$e_0^* = max \quad \left[\sum_{r=1}^{s} u_r y_{r0} - u_0\right] \Big/ \sum_{i=1}^{m} v_i\, x_{i0} \tag{3.23}$$

$$\text{subject to} \quad \sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j = 1 \cdots n, \tag{3.24}$$

$$u_r\,,\; v_i \geq \varepsilon \quad \text{for all } i, r$$

and $u_0$ unrestricted in sign.

As with the CCR model, the BCC ratio model can be converted to the following linear programming equivalent:

$$e_0^* = \max \quad \sum_{r=1}^{s} \mu_r\, y_{r0} - \mu_0 \tag{3.25}$$

$$\text{subject to} \quad \sum_{i=1}^{m} \nu_i\, x_{i0} = 1 \tag{3.26}$$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \mu_0 - \sum_{i=1}^{m} \nu_i\, x_{ij} \leq 0 \quad j = 1 \cdots n, \tag{3.27}$$

$$\mu_r, \nu_i \geq \varepsilon \quad \text{for all } i, r$$

and $\mu_0$ unrestricted.

The dual formulation is given by the following:

$$Min\, \theta_0 - \varepsilon \left( \sum_{r=1}^{s} s_r^+ - \sum_{i=1}^{m} s_i^- \right) \tag{3.28}$$

$$\text{subject to} \quad \sum_{j=1}^{n} \lambda_j\, x_{ij} + s_i^- = \theta_0\, x_{i0} \qquad i = 1 \cdots m, \tag{3.29}$$

$$\sum_{j=1}^{n} \lambda_j y_{r0} - s_r^+ = y_{r0} \qquad r = 1 \cdots s, \tag{3.30}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \tag{3.31}$$

$$\lambda_j\,,\, s_i^-\,,\, s_r^+ \geq 0 \qquad \text{for all } i, j, r$$

and $\theta_0$ unrestricted.

A detailed description of each mathematical expression (3.23 – 3.31) can be found in Banker *et al*. (1984).

It should be noted that the dual formulation of the BCC model differs from the dual formulation of the CCR model in that it has the additional convexity constraint (3.31) on the $\lambda_j$.

The BCC model is graphically depicted in Figure 3.4 using the same example as for the CCR model in section 3.3.3.1. The dotted line represents the original efficiency frontier for the CCR model while the solid line, connecting DMUs 1, 2, 3 and 4, represents the BCC efficiency frontier.

The variable returns to scale in the BCC model can be seen from Figure 3.4. The section on the frontier from $DMU_1$ up to $DMU_2$ (but not including $DMU_2$) represents increasing returns to scale. $DMU_2$ is experiencing constant returns to scale while all points to the right of $DMU_2$ (from $DMU_2$to $DMU_3$ and from $DMU_3$ to $DMU_4$) make up the decreasing returns to scale. It is also clear that any DMU identified as efficient in the CCR model, will also be efficient in the BCC model – but the opposite is not always true.

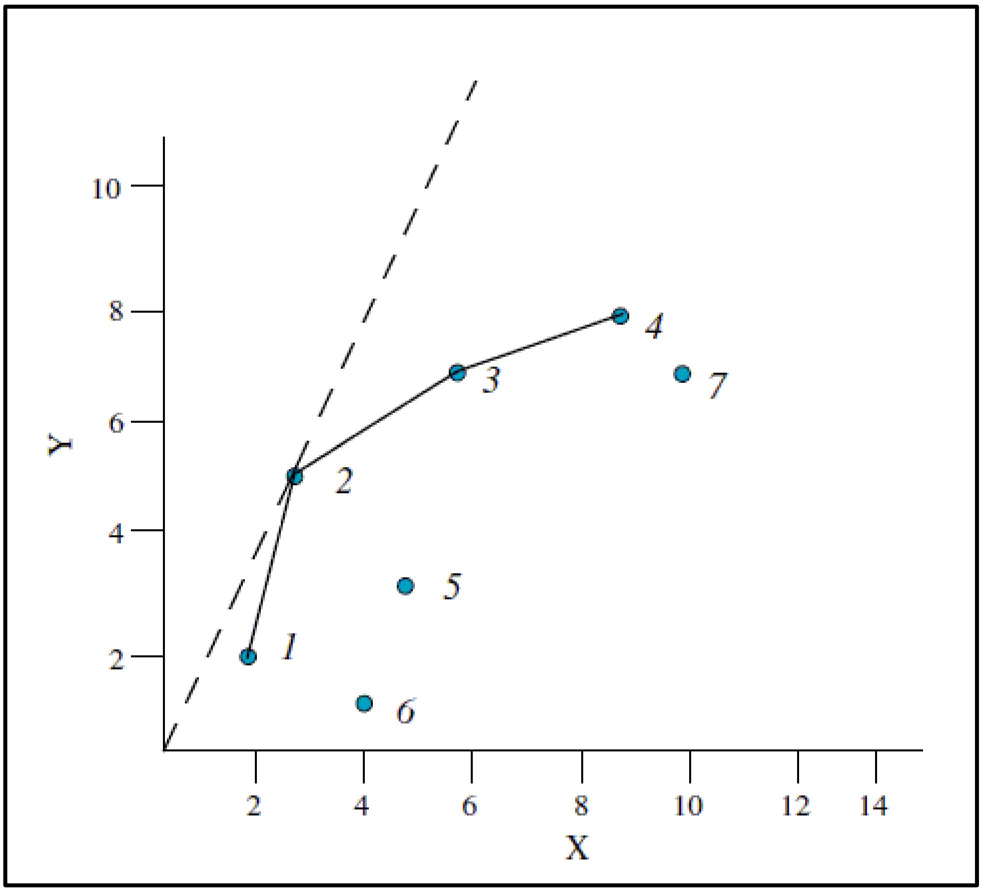



Figure 3.4: The variable return to scale Frontier (Cook and Seiford, 2009).

The BCC and CCR models are the two most well-known and basic DEA models. Several extensions and new models have already been developed and mathematical details and comprehensive overviews of other existing DEA models can be found in Cooper *et al*. (2007), Thanassoulis (2001) and Cook and Seiford (2009).

### 3.3.4 Pitfalls in DEA

Cooper (2014) warns that a considerable effort is required from an analyst to understand DEA models and the context in which a DEA model is applied. The results of a DEA application depend on the composition and the understanding of the model itself. Dyson *et al*. (2001) listed a number of issues that may influence the outcome of a DEA model while DEA pitfalls, specifically related to the financial services sector, are described by Brown (2006). For completeness purposes this section gives a summarized overview of the DEA pitfalls identified by Dyson *et al*. (2001).

The following 5 areas that may present difficulties in a DEA modelling process were identified (Dyson *et al*., 2001):

i. Homogeneity assumptions

A number of assumptions are made in the DEA modelling process. These include the assumption that DMUs undertake similar activities and produce comparable products; that a similar range of resources is available to all DMUs; and, that DMUs are operating in similar environments. The pitfalls, derived directly from these assumptions, are the comparison of non-homogeneous DMUs; a non-homogeneous environment; and economics of scale (too small or too large DMUs).

ii. The input/output set

With regard to the input/output set, four key assumptions are normally made:

- it covers the full range of resources used;
- captures all activity levels and performance measures;
- the set of factors are common to all DMUs; and
- environmental variation has been assessed and taken into consideration.

These four input/output assumptions may lead to difficulties such as the number of inputs and outputs to be used (inappropriate number may reduce the level of discrimination); correlation factors (omission of a highly correlation variable may lead to significant changes in efficiency); and mixing indices (associated with performance measures); and volume measures (associated with activity levels).

iii. Factor measurement

The general assumptions with regard to factor measurement include that inputs and outputs should conform to ratio scales (have a meaningful zero and equal intervals should have equal values) and that inputs and outputs are also isotonic (increased input reduces efficiency and increased output increases efficiency). Four pitfalls are related to these factor measurement assumptions. They are the mixing of volume measurements with percentages and other normalized data; the incorporation of qualitative data into an analysis; the occurrence of undesirable inputs and outputs (e.g. undesirable output such as the emission of a pollutant); and exogenous and constrained factors over which DMUs have no control (e.g. environmental, geographical or legislative factors).

iv. Weights

Certain limiting technical assumptions exist pertaining to the selection and manipulation of weights in the assessment of DMUs. Firstly, the linearity assumption may become problematic when, for some output the value of an additional output may begin to diminish. Secondly, the permission of a zero or mathematical infinitesimal (usually denoted by $\varepsilon$) weight attached to a factor means that the factor is effectively omitted from the assessment – inclusion of factors implies that factors are important and should be taken into account. Thirdly, freely chosen input and output weights may violate some agreed relative value (e.g. the weight attached to a *very* important factor should be higher than the weight attached to an important factor). Finally, input and output weights may sometimes be linked – given a high weight to one factor and a low weight to a linked factor may contradict desired outcomes or expectations.

v. Weight restrictions

The incorporation of weight restrictions has the potential to introduce numerous pitfalls. Examples of such pitfalls include the following. The justification of weight restrictions (explanation of why particular weight restrictions were used); the non-transferability of weight restrictions (weight restrictions may be different in different DEA models); interpretation of results (interpretation of results with weight restrictions is not the same as interpretation of results without weight restrictions); the absolute versus relative efficiency (relative efficiencies may be underestimated with weight restrictions); and, redundant weight restrictions (weight restrictions may have no effect on relative efficiencies).

It should be noted that Dyson *et al*. (2001) also offered solutions to the identified pitfalls. These solutions, termed protocols, generally suggest techniques to be used to overcome or lessen the impact of the pitfalls. A discussion of the protocols does not form part of this presentation and details can be found in Dyson *et al*. (2001).

## 3.4 Class ranking

This section presents a method of class ranking that stemmed from the concept of Pareto optimality to stratify DMUs (schools in this study) into classes of different levels. The method is based on a technique called “peeling the DEA onion” and was suggested by Barr *et al*. (2000). The technique entails the separation of DMUs into a series of nested efficient-frontier layers and provides a new efficiency metric and explanation for those DMUs that are seen as inefficient.

### 3.4.1 Background to the ranking procedure

In many applications it makes sense that every DMU should focus on its strong areas to develop a distinct characteristic – this principle may be called context-dependent data envelopment analysis (Seiford and Zhu, 2003). Accepting this principle implies that the conventional way of aggregating scores for certain variables (e.g. input and/or output variables) to form a composite index for ranking is inappropriate. A DMU is superior to another DMU only if it performs better in all aspects – this is the concept of dominance and such a DMU is Pareto optimal (Tomkins and Green, 1988).

The concept of dominance and Pareto efficiency in the context of input and output efficiency for a number of decision making units can be explained as follows (Thanassoulis, 2001).

Consider a homogeneous set of decision making units that consume one or more inputs to secure one or more outputs, then,

For an *output oriented* model: A decision making unit is Pareto efficient if it is not possible to raise anyone of its output levels without lowering at least another one of its output levels and/or without increasing at least one of its input levels. Mathematically this can be expressed as follows.

Let $y_{rj}$ $(r = 1, \cdots, s)$ be the output levels secured by decision making $j$ and $x_{ij}$ the levels of inputs $(i = 1, \cdots, m)$ it uses. Decision making unit $j_0$ is Pareto efficient if there exist no observed or feasible in principle decision making unit $j$ with $j \neq j_0$ such that $y_{r'j} > y_{r'j_0}$ for some $r'$ and $y_{rj} \geq y_{rj_0}$ for all $r \neq r'$ while $x_{ij} \leq x_{ij_0}$ for all $i$.

For an *input orientated* model: A decision making unit is Pareto efficient if it is not possible to lower anyone of its inputs levels without increasing at least another one of its input levels and/or without lowering at least one of its output levels. The mathematically representation is as follows.

Let $y_{rj}$ $(r = 1, \cdots, s)$ be the output levels secured by decision making $j$ and $x_{ij}$ the levels of inputs $(i = 1, \cdots, m)$ it uses. Decision making unit $j_0$ is Pareto efficient if there exist no observed or feasible in principle decision making unit $j$ with $j \neq j_0$ such that $x_{i'j} < x_{i'j_0}$ for some $i'$ and $x_{ij} \leq x_{ij_0}$ for all $i \neq i'$ while $y_{rj} \geq y_{rj_0}$ for all $r$.

Pareto efficiency and optimality can also be defined in general (not in a DEA context). Basic explanations and definitions for the general case can be found in Niu *et al.* (2013) and Ehrgott (2012).

To illustrate the concept of dominance and Pareto optimality the following example, adapted from Kao and Lin (2008) may be considered.

Suppose that six DMUs, labelled A, B, C, D, E and F are being evaluated using two criteria (output variables). The sample DMUs and their associated outputs are as follows

| DMU | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Output 1 | 1 | 4 | 5 | 3 | 4 | 3.1 |
| Output 2 | 5 | 4 | 1 | 4 | 2 | 3.1 |

The DMUs and their associated output values are depicted in Figure 3.5 on the next page.

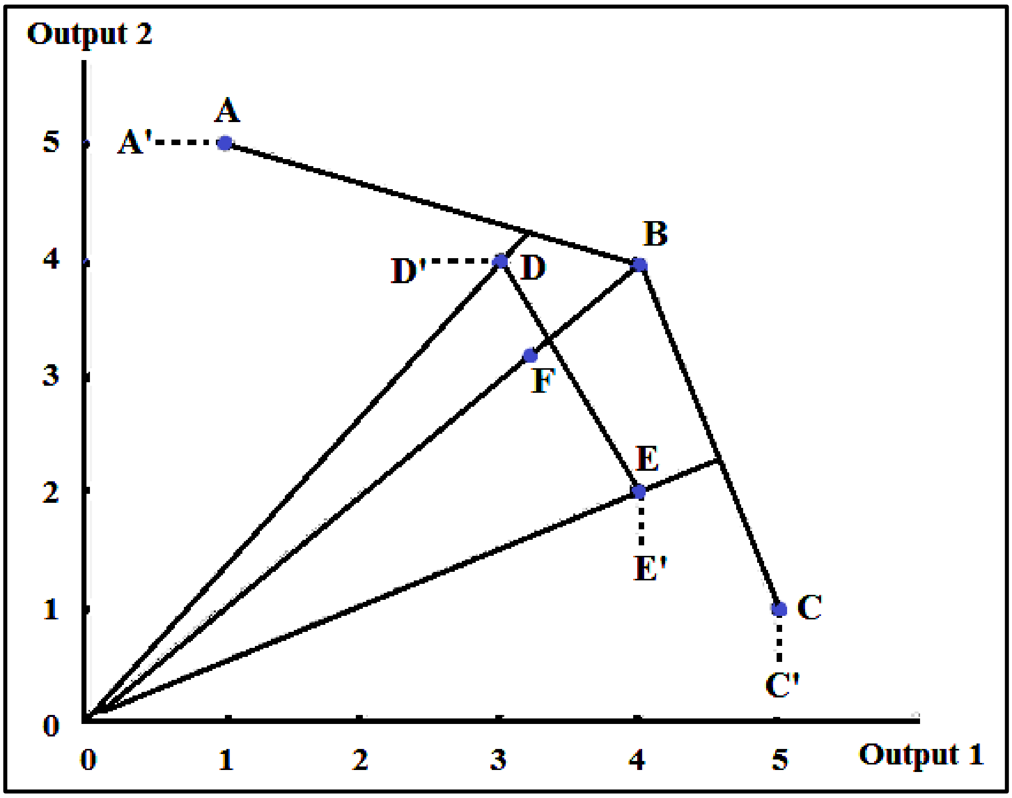



Figure 3.5: Dominance relationship of six DMUs.

If the importance of the two criteria is irrelevant, $DMU_B$ would be the best performing DMU with a total score (output) of 8. $DMU_D$ would be second best (total score of 7) with $DMU_F$ the third best (total score of 6.2). DMUs A, C and E would be tied in fourth place, each with a total score of 6. It is clear that assigning different weights to the two criteria would result in a different ranking.

Consider the case where the two criteria are incomparable – this is equivalent to seeking for dominating DMUs. Dominating DMUs will now belong to a class of the same level while the dominated DMUs will belong to a class of a lower level. In Figure 3.5 the DMUs A, B and C are the dominating DMUs and are also Pareto optimal. These DMUs are therefore in the same class on a higher level. DMUs D, E and F are dominated by $DMU_B$ and would therefore belong to a class at a lower level. Kao and Lin (2008) stated that the dominance relationship can be identified graphically by connecting the DMUs on the northeast frontier with all the DMUs inside the frontier. This can be seen in Figure 3.5 where the frontier is composed of the piece linear line segment A'ABCC'. The Pareto optimal DMUs are those on the frontier (A, B and C) while those inside the frontier (D, E and F) are the dominated DMUs. This process results in a set of DMUs that can be divided into different levels of efficient frontiers – in Figure 3.5 are three different frontiers i.e. A'ABCC', D'DEE' and F. Removing the original efficient frontier (A'ABCC') the remaining inefficient or dominated DMUs will form a new second-level efficient frontier (D'DEE'). If this new second-level frontier is removed, a third-level efficient frontier is formed (in Figure 3.5 a frontier consisting only of $DMU_F$) and so on until no DMUs are left. Each of these different frontiers provides a class ranking as well as a means for context evaluation of the DMUs.

### 3.4.2 Model formulation for class ranking

To generate non-dominated sets of DMUs, i.e. the nested efficient frontiers, the following context-depended DEA model may be considered (Seiford and Zhu, 2003). Suppose $DMU_j$ $(j = 1 \cdots n)$ produces a vector of outputs $\boldsymbol{y}_j = (y_{1j} \cdots y_{sj})$ using a vector of inputs $\boldsymbol{x}_j = (x_{1j} \cdots x_{mj})$.

Let $\boldsymbol{J}^l = \{DMU_j;\ j = 1 \cdots n\}$ be the set of all $n$ DMUs. Define $\boldsymbol{J}^{l+1} = \boldsymbol{J}^l - \boldsymbol{E}^l$ where $\boldsymbol{E}^l = \{DMU_k \in \boldsymbol{J}^l \mid \phi^*(l,k) = 1\}$, and $\phi^*(l,k)$ is the optimal value to the following linear programming model

$$\phi^*(l,k) = \max_{\lambda,\phi(l,k)} \phi(l,k) \tag{3.32}$$

$$\text{subject to} \quad \sum_{j \in F(\mathrm{J}^l)}^{n} \lambda_j y_j \geq \phi(l,k) y_k, \tag{3.33}$$

$$\sum_{j \in F(\mathrm{J}^l)}^{n} \lambda_j x_j \leq x_k, \tag{3.34}$$

$$\lambda_j \geq 0, \tag{3.35}$$

$$j \in F(\boldsymbol{J}^l) \tag{3.36}$$

where $(\boldsymbol{x}_k, \boldsymbol{y}_k)$ represents the input and output vector of $DMU_k$ and $j \in F(\boldsymbol{J}^l)$ means $DMU_j \in \boldsymbol{J}^l$, i.e., $F(\cdot)$ represents the correspondence from a DMU set to the corresponding subscript index set.

When $l = 1$ the formulated model becomes the original output-oriented CCR model and DMUs in the set $\boldsymbol{E}^l$ define the first-level efficient frontier. That is the set of Pareto optimal DMUs (non-dominated) that will be ranked into the first class. When $l = 2$, the model solution will give the second-level efficient frontier after the exclusion of the first-level Pareto optimal DMUs. This process is then repeated until all different levels of efficient frontiers have been identified. For every $l, \boldsymbol{E}^l$ will contain the Pareto optimal set of DMUs (class ranking) for the $l^{th}$ level efficient frontier.

Algorithm 3.1 presents the algorithm for class ranking of a group of DMUs based on the nested efficient frontier procedure discussed.

**Algorithm 3.1** (Seiford and Zhu, 2003)

*Step 1:* Set $l = 1$. Evaluate the entire set of DMUs, $\boldsymbol{J}^l$, by the model (3.32) to (3.36) to obtain the first-level efficient DMUs, set $\boldsymbol{E}^l$ (the first-level efficient frontier).

*Step 2:* Exclude the efficient DMUs from future DEA runs. $\boldsymbol{J}^{l+1} = \boldsymbol{J}^l - \boldsymbol{E}^l$. (If $\boldsymbol{J}^{l+1} = \emptyset$ then stop.)

*Step 3:* Evaluate the new subset of "inefficient" DMUs, $\boldsymbol{J}^{l+1}$, by model (3.32) to (3.36) to obtain a new set of efficient DMUs $\boldsymbol{E}^{l+1}$ (the new efficient frontier).

Step 4: Let $l = l + 1$. Go to *step 2.*

Stopping rule: $\boldsymbol{J}^{l+1} = \emptyset$, the algorithm stops.

With reference to Figure 3.5, the procedure would yield a three class stratification with A, B and C in the first class, D and E in the second and F in the third. The first class is higher than the second, of which is higher than the third and DMUs of the same class are of the same rank. Comparing these rankings with the initial equal-weight rankings, two differences can be noted. First, A and C who had the lowest total score (6 each) are now classified into the first class. Second, F, which has the third highest score, has now been classified in the lowest class. The reason for this is that the equal-weighted method favours those DMUs that perform equally well in both criteria while the dominance method favours those performing extraordinarily well in specific criteria (Kao and Lin, 2008).

### 3.4.3 Alternative formulation

This study uses a slightly different model to generate non-dominated sets of DMUs. The formulation is based on a DEA model where only outputs are considered and was suggested by (Kao and Lin, 2008). In this section only the formulation will be presented. The reasons for using this model are explained in Chapters 4 and 5 where the DSS and school application are presented.

Kao and Lin (2008) performed a class ranking exercise on Management Colleges in Taiwan. The same algorithms presented in section 3.4.2 was used but the model used to generate the Pareto optimal DMUs was based on a simple DEA model without inputs, i.e. only outputs are considered. This model and algorithm were also used by other researchers (Takeda and Satoh, 2000; Barr *et al*., 2000; Seiford and Zhu, 2003).

The DEA model without inputs is formulated as follows.

$$\text{Maximise } E_k = \sum_{j=1}^{m} y_{ij}\, w_j \tag{3.37}$$

$$\text{subject to } \sum_{j=1}^{m} y_{ij}\, w_j \leq 1 \qquad \text{for } i = 1 \cdots n, \tag{3.38}$$

$$w_j \geq \varepsilon \qquad \text{for } j = 1 \cdots m, \tag{3.39}$$

$$\varepsilon > 0 \tag{3.40}$$

where ε is a small positive number used to restrict a DMU from ignoring unfavourable criteria.

In this model, each DMU would select the most advantageous weights to calculate the composite index $E_k$. If $E_k = 1$ then $DMU_k$ is Pareto optimal, otherwise it is Pareto non-optimal.

In order to obtain more information regarding target values for a specific DMU it is more appropriate to make use of the dual formulation of the DEA model without inputs.

The basic solutions of the primal model (3.37) to (3.40) and the newly derived dual model would be the same as explained in section 3.2.2.

This formulation of the dual is as follows

$$\text{Minimize } E_k = \sum_{i=1}^{n} \lambda_i - \varepsilon \sum_{j=1}^{m} s_j \tag{3.41}$$

$$\text{subject to } \sum_{i=1}^{n} y_{ij}\, \lambda_i - s_j = y_{kj} \qquad \text{for } j = 1 \cdots m, \tag{3.42}$$

$$\lambda_i \geq 0 \qquad \text{for } i = 1 \cdots n, \tag{3.43}$$

$$s_j > 0 \qquad \text{for } j = 1 \cdots m. \tag{3.44}$$

The optimal value $E_k$, obtained from this model is exactly the same as from the primal model formulation. However, the targets for a specific DMU can be calculated using the output from the dual formulation. Let $\theta = \sum_{i=1}^{n} \lambda_i$ , the target for the $k^{th}$ DMU is then given by

$$(y_{kj} + s_j)/\theta \quad \text{for} \quad j = 1 \cdots m. \tag{3.45}$$

The multi-stage class ranking model will therefore provide intermediate goals for each DMU (from one class to the next) – thus is in contrast to the conventional one-stage DEA model that only provides an ultimate goal of getting directly to the initial efficient frontier (the top class). For a DMU belonging to class $f$, the $f-1$ targets in the previous $f-1$ stages can be calculated. Each of these targets will then show the improvement needed by a DMU in each category to become a member of the corresponding class.

In the context of this study, the application of the explained models provides a ranking of schools under incomplete criteria. Furthermore it also provides intermediate goals for a school to become efficient in stages via the DEA-type model. Further motivation and explanation of the claims is provided in Chapters 4 and 5.

## 3.5 Conclusion

Chapter 3 serve as an introductory overview of DEA models. The overview covered the formulation of a general linear programming model and the associated dual model. This was followed by a discussion on DEA models which included a short background on the origins and history of DEA, the DEA concept and model formulations of the basic DEA models. The final section of the chapter dealt with class ranking using DEA related models. Chapter 4 is devoted to the development of the decision support system and the implementation of the models and algorithms.

# 4 DECISION SUPPORT SYSTEMS

## 4.1 Introduction

The development of a decision support system (DSS) that implements some of the mathematical models presented in the previous chapter, forms an integral part of this study. The objective of Chapter 4 is therefore to present an overview of DSSs and how these concepts were implemented in the development of a novel DSS to evaluate school efficiency.

The chapter consists of three main sections. First an introductory overview of concepts and methodologies of DSSs is presented. This is followed by a description of the high level features of the newly developed DSS. Finally, an illustrative example of an application of the DSS is presented. This example, which is based on the ranking and the evaluation of universities, serves two purposes - in the first instance it validates the models and the techniques used as being correct, and secondly, it demonstrates the usefulness of results produced by the DSS.

## 4.2 Concepts of Decision Support Systems

The study of DSSs is a vast field that involves a large number of sub-fields and a comprehensive description of the entire DSS discipline would be impossible in one subsection of one chapter. This section therefore only provides a brief and introductory overview of some of the concepts pertaining to a DSS. The discussion is mainly based on the authoritative work by Turban *et al*. (2011) and some of the work is quoted without referencing the authors again.

### 4.2.1 Definition of a DSS

The concept of a DSS is very broad and a number of researchers have tried to give an all-inclusive definition for a DSS without success (Gachet, 2001). Definitions range from short descriptions, such as “a computer based system that aids the process of decision making” (Finlay, 1994), to long explanations based on different viewpoints. Turban *et al*. (2011), for example describe the concept of a DSS as a conceptual methodology that may fit into different views such as

- DSS may be used as an umbrella term that describes any computerised system that supports decision-making.
- The term DSS may be used to refer to a specific application or to the process to build a customised application for structured or semi-structured problems.
- DSS may also refer to a specific DSS architecture which includes data as the first component of a DSS architecture. This is then followed by a model component, a knowledge component, a user component and finally an interface component.
- There are also different types of DSSs and the term may therefore refer to a specific type of DSS e.g. a model-oriented DSS or a data-oriented DSS.

For the purpose of this study, the simple definition given by Finlay (1994) at the beginning of this section is accepted.

### 4.2.2 Characteristics and Capabilities of a DSS

Owing to the fact that there is no consensus on an all-inclusive definition, Turban *et al*. (2011) proposed a set of fourteen key characteristics and capabilities that can be used to describe a DSS. Figure 4.1 shows a graphical representation of these characteristics.

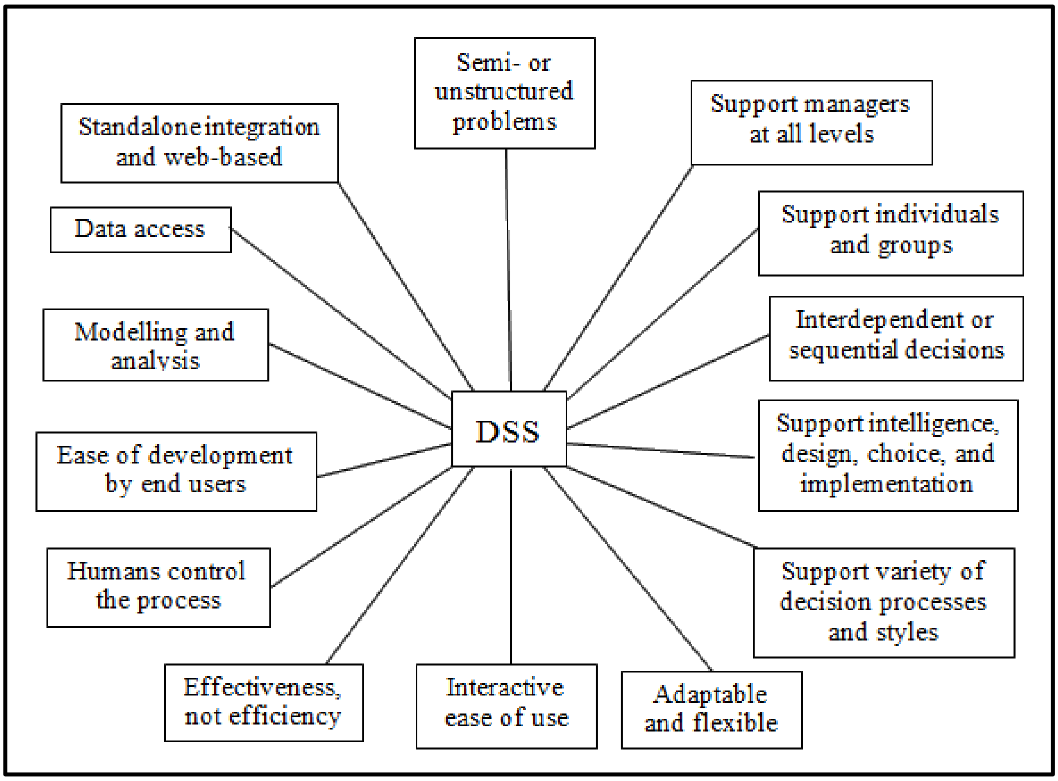



Figure 4.1: Key characteristics and capabilities of a DSS, (Turban *et al.*, 2011).

The fourteen DSS characteristics and capabilities will allow decision makers to improve their decision making and produce better and more consistent decisions. They can be further described as follows.

*Semi structured or unstructured problems*

These types of problems are normally solved with difficulty if only standard systems and methods are used. A DSS brings together human judgment and computerised information and in this way enables decision makers to make more informed decisions.

*Support managers at all levels*

A DSS will support all levels of management, ranging from top executives to line managers.

*Support individuals and groups*

Well-designed DSSs are able to support individual as well as group work. This implies that individual decision making is supported as well as groups of decision makers who may work also independently.

*Interdependent or sequential decisions*

Some decisions have to be made repeatedly or several times while some decisions constitute a once off action. All these types of decisions are normally supported by a DSS.

*Support intelligence design, choice and implementation*

Decision making is often seen as a process that consists of an intelligence-, design-, choice-, and implementation step. All these different steps are supported by a DSS.

*Support variety of decision processes and styles*

Different managers/decision makers have different styles and processes that they may follow to make a decision. A DSS will support all these different styles and processes.

*Adaptable and flexible*

Decision makers are regularly confronted with change which may require reaction or adaptation. A good DSS will therefore be flexible so that it can be readily adapted or modified to solve other (but similar) problems.

*Interactive, ease of use*

To increase the effectiveness of DSSs, the system should have attributes such as user-friendliness, graphical capabilities, natural language interactive human-machine interface etc.

*Effectiveness and efficiency*

DSSs place the emphasis on effectiveness of decision making i.e. accuracy, timeliness, quality etc. The efficiency (cost of making the decision) may in some cases be less important. The implementation of a DSS often means that decision making takes longer, but the decisions are better.

*Humans control the process*

The goal of a DSS is to support a decision maker and not to replace him/her. This means that the decision maker has complete control over all the steps in the decision making process.

*Ease of development by end user*

End users are able to develop and modify simple systems themselves while larger systems may be developed with the help of information specialists.

*Modelling and analysis*

The majority of DSSs utilize models to analyse decision making situations. By changing parameters, input, strategies or configurations, different experiments and analyses can be performed.

*Data access*

The use of a DSS normally enables a user to have access to a variety of data sources, formats and types.

*Stand-alone, integration and web-based*

DSSs are not limited to a stand-alone tool, used by a single user in one location. It may also be distributed throughout an organisation and integrated with other applications and/or DSSs. To distribute a DSS externally it may be a web-based DSS.

### 4.2.3 Classification of a DSS

The lack of a universal DSS definition and the existence of a set of general characteristics indicate that every DSS cannot fit neatly into one category. Turban *et al*. (2011) argue that the design, operation and implementation of a DSS depend on the *type* of DSS. Five general categories of DSSs currently exist (Turban *et al*., 2011) and are highlighted briefly:

*Communications-driven and group DSS*

This category refers to all DSSs that support any kind of group work. A DSS in this category will make use of computer, collaboration and communication technologies to fulfil its supporting role. Knowledge management systems that are developed around communities may also fall in this category.

*Data-driven DSS*

As the name indicates, these types of DSSs are primarily involved with data, data processing and presenting information to a decision maker. Databases and Data Warehouses play a major role in these types of DSS structures.

*Document-driven DSS*

The main objective of a document-driven DSS is to provide support for decision making using documents in various forms such as oral, written and multimedia. This type of DSS is text based with minimal use of any mathematical models.

*Knowledge-driven DSS*

This category includes all artificial intelligence-based DSSs. Technologies such as artificial neural networks and expert systems (or other rule-based systems) are also included.

*Model-driven DSS*

The model-driven DSS category refers to a DSS that is primarily developed around one or more mathematical model such as an optimization or simulation model. Important activities here include model formulation, model maintenance, model management and what-if analysis.

Apart from the 5 categories listed above, there may also be hybrids that combine two or more categories. These are called *compound DSSs*. An example may be a data-driven DSS that feeds into a large-scale optimization model (model-driven DSS).

### 4.2.4 Basic components of a DSS

This final subsection of DSSs will present a high level view of the components of a DSS. According to Turban *et al.* (2011) a DSS application consists of four sub systems. They are

- A data management subsystem which includes a database that contains the relevant data. This subsystem is managed by a Database Management System (DBMS) and may be interconnected with various other data sources.
- A model management subsystem that includes financial, statistical, management science and other qualitative models to provide the required analytical capabilities.
- A user interface subsystem that allows for interaction between the decision maker and the computer.
- A knowledge-based management subsystem that can support any of the other subsystems or can act as an independent component. It provides intelligence and can be interconnected to other repositories.

These four subsystems are then connected to other systems, technologies, data etc., as shown in Figure 4.2.

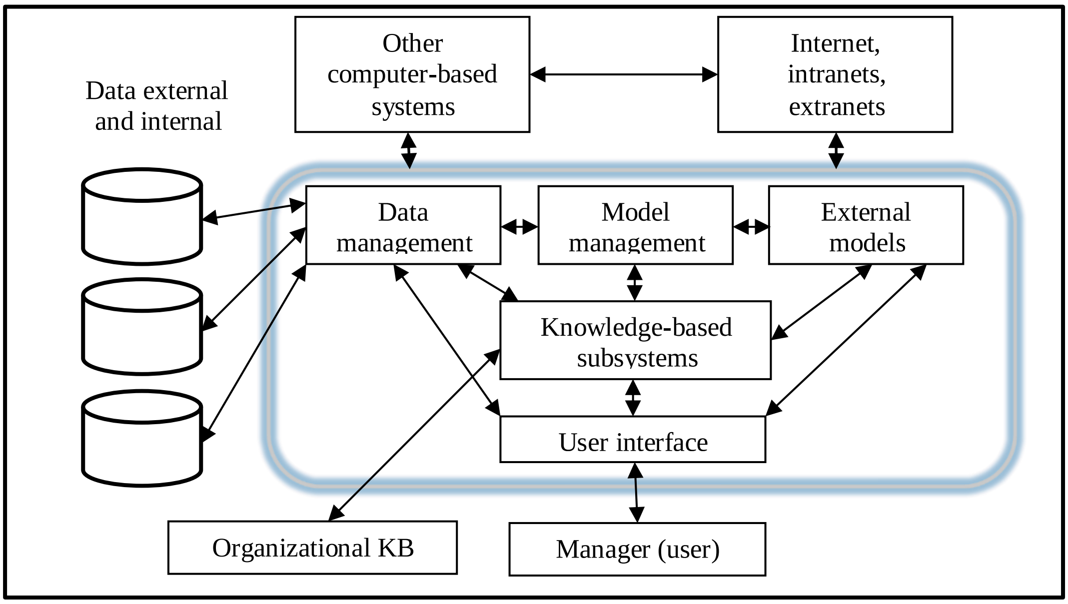



Figure 4.2: Schematic view of a DSS, (Turban *et al*., 2011).

Section 4.2 has presented some basic background concepts of DSSs which is important to this study as a DSS will be developed for the evaluation of schools. The next section deals with this new DSS.

## 4.3 DSS development

A DSS to evaluate school efficiency and to rank schools accordingly was developed as one of the objectives of this study. The previous section presented a brief introductory overview of DSS concepts and in this section a description of the newly developed DSS is given. The first sub-section presents a high level overview of how the DSS was constructed and the modules implemented. Next, a description of the various mathematical models implemented is given.

### 4.3.1 Basic structure of the newly developed DSS

The DSS that was developed is essentially a model-driven DSS as it implements mathematical models and techniques to perform the required activities. The system consists of four basic components; a module where input data is received, a database where the data and relevant information regarding the mathematical techniques are stored, an optimization model, and, an output module. Figure 4.3 is a schematic representation of the basic structure of the DSS

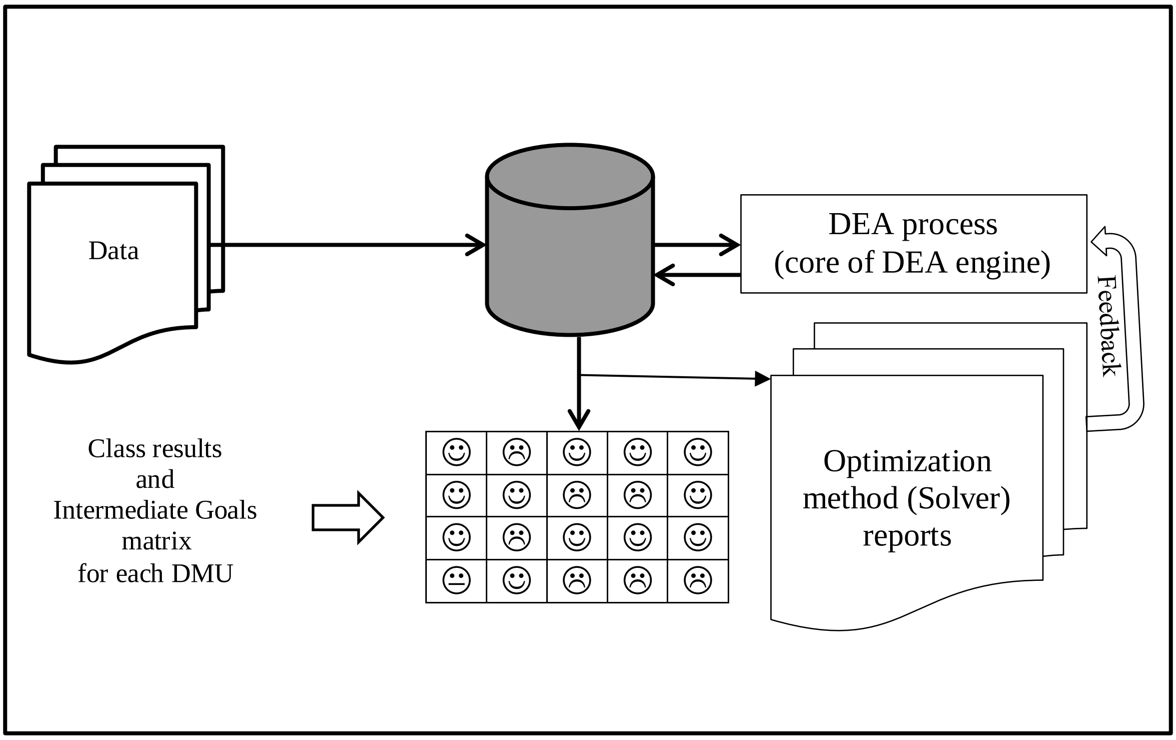



Figure 4.3: Basic structure of the DSS

*The data input module*

The DSS primarily utilises DEA models (described in Chapter 3) and to perform the efficiency calculations and class rankings, the input data should be in a format typically required by DEA models. The required data therefore consist of a list of decision making units (DMUs) – schools in this study – each with a number of pre-defined input and output variables. For example

| | Input 1 | Input 2 | $\cdots$ | Input m | Output 1 | Output 2 | $\cdots$ | Output s |
|---|---|---|---|---|---|---|---|---|
| DMU 1 | $i_{11}$ | $i_{12}$ | $\cdots$ | $i_{1m}$ | $o_{11}$ | $o_{12}$ | $\cdots$ | $o_{1s}$ |
| DMU 2 | $i_{21}$ | $i_{22}$ | $\cdots$ | $i_{2m}$ | $o_{21}$ | $o_{22}$ | $\cdots$ | $o_{2s}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| DMU n | $i_{n1}$ | $i_{n2}$ | $\cdots$ | $i_{nm}$ | $o_{n1}$ | $o_{n2}$ | $\cdots$ | $o_{ns}$ |

where $i_{kl}$ indicate input $l$ for DMU $k$ and $o_{kl}$ indicates output $l$ for DMU $k$. The input is received via a Microsoft Excel file with a predetermined structure and layout. Pre-programmed VB macros are then utilised to link with the next module i.e. the database.

*The database module*

The database module serves as a repository where the input data is stored and retrieved for processing. It also contains information regarding the models and processes to be executed. The DEA models can be implemented with both input and output variables or with output variables only. The class rankings are also performed using the dual formulation of a DEA model and depending on which model or technique will be used, data and information are retrieved from the database.

The database is implemented in an Excel file which makes the integration with the optimization module easy.

*The optimization module*

The optimization module sets up the required DEA models by formulating the models, integrating the input and/or output variables and then performs the evaluation by solving the model(s). The module utilizes the Microsoft Excel solver together with a number of pre-programmed macros. Section 4.3.2 provides more detail on the mathematical models and class ranking techniques implemented in the optimization module.

*The output module*

This module presents the results obtained from the optimization module. Depending on the task, the model may present basic DEA results from an input/output model or results from an output only model. The module also presents class ranking results in a user friendly manner with clear recommendations on how a school should go about, to move from one class to the next.

The complete DSS was created by developing a desktop application that integrates with Microsoft Excel Solver and that was programmed in Excel 2010 Visual Basic using Microsoft Visual Basic for Application 7.0. System requirements for the developed DSS are listed in Appendix A. Examples of code for core functions in the DSS are presented in Appendix D.

### 4.3.2 Mathematical models and techniques implemented

The optimization module, mentioned in the previous section, is where the formulation of the mathematical models and techniques takes place. The models implemented in this study are all DEA-related models of which the background theory was presented in Chapter 3. The aim of this section is therefore to present the models implemented in the newly developed DSS.

The DSS performs mainly two tasks depending on the need of the user. These tasks are

- Perform an ordinary input/output DEA analysis to evaluate school efficiency – i.e. apply the CCR-model described in Chapter 3 in section 3.3.3.1.
- Perform a class ranking of schools in order to advise how a school should progress from an existing class of schools to a higher class of schools. This task is based on a DEA model that utilizes only output variables which was described in section 3.4.2 of Chapter 3.

The techniques and models implemented for these two activities are briefly highlighted below.

4.3.2.1 Ordinary input/output DEA model (CCR-model)

To perform a basic efficiency analysis of a set of schools the system implements the standard CCR-model that was described in section 3.3.3.1. For completeness sake the model formulation is repeated below.

$$e_0 = max \quad \sum_{r=1}^{s} u_r y_{r0} \Big/ \sum_{i=1}^{m} v_i x_{i0} \tag{4.1}$$

$$\text{subject to} \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad \text{for all } j, \tag{4.2}$$

$$u_r , v_i \geq \varepsilon \quad \text{for all } i, r \tag{4.3}$$

where $\varepsilon$ is a non-Archimedean value designed to enforce strict positivity on the variables. This formulation is the original CCR model and provides for constant return to scale.

The DSS requires a data set consisting of a number of schools (DMUs) together with the associated pre-identified input and output variables. The output procedure then includes a list of efficient and inefficient DMUs. Reference sets for the inefficient schools then enable an analysis to advise how the inefficient schools may become as efficient as their peers.

Although the CCR-model is used in this evaluation, it can be easily changed to other variations (e.g. the BCC model), depending on the situation.

4.3.2.2 Class ranking

In many instances when evaluating schools, it seems to be more appropriate to use a model that focus on output variables and exclude possible input variables. One of the reasons for this is that the distribution of resources to schools, particularly in South Africa, is not equal. Other reasons for this are referred to again in Chapter 5.

The DSS in this study implements a class ranking technique that makes use of an “output only” DEA model. The objective is to stratify schools into different classes which can then be used to set targets for a school in a specific class to move to the next class. Moving from one class to a next higher (better) class is more appropriate in the context of schools in South Africa than the normal DEA approach where targets are set to move immediately to the final efficiency frontier. The technique employed here (and discussed in Chapter 3, section 3.4) entails the separation of schools into a series of nested efficient frontier layers – an analysis of these, will then enable a school to move to a better efficient frontier (or class).

The technique requires from a user to provide a set of *n* schools to be evaluated under *m* criteria (the output variables). A simple DEA model (with output variables only) is then formulated as follows

$$\text{Maximise } E_k = \sum_{j=1}^{m} y_{ij}\, w_j \tag{4.4}$$

$$\text{subject to } \sum_{j=1}^{m} y_{ij}\, w_j \le 1 \qquad \text{for } i = 1 \cdots n, \tag{4.5}$$

$$w_j \ge \varepsilon \qquad \text{for } j = 1 \cdots m, \tag{4.6}$$

$$\varepsilon > 0, \tag{4.7}$$

where $\varepsilon$ is a small positive number.

$E_k$ is the composite index for school *k*. If $E_k = 1$, then school *k* is Pareto optimal, otherwise it is Pareto non-optimal.

To perform the class ranking, algorithm 3.1 (see section 3.4.2) is implemented. This can be summarized as follows.

1. Define the set of schools: N = {$\text{School}_1$, $\text{School}_2$, $\cdots$ $\text{School}_n$} and set a counter $g = 1$.
2. Apply the output only model, (4.4) to (4.7) to calculate the composite index $E_k$ for every k ∈ N.
3. Schools with $E_k = 1$ are categorized into a set $C_g$ and deleted from the set N.
4. If $N = \emptyset$ stop, else let $g = g + 1$ and return to step 2.

The number of times this procedure is repeated (value of $g$) indicates the number of classes the schools have been stratified in. The schools in the same class are of the same rank and schools in $C_g$ is of higher rank than those in $C_f$ (with $f < g$).

The output of this procedure gives a class ranking of schools in the following form,

| School | Output 1 | Output 2 | ⋯ | Output m | Class |
|---|---|---|---|---|---|
| School $C_{11}$ | O$_1$(School $C_{11}$) | O$_2$(School $C_{11}$) | | O$_m$(School $C_{11}$) | $C_1$ |
| School $C_{12}$ | | | | | $C_1$ |
| ⋮ | ⋮ | ⋮ | ⋱ | ⋮ | ⋮ |
| School $C_{1r_1}$ | O$_1$(School $C_{1r_1}$) | O$_2$(School $C_{1r_1}$) | | O$_2$(School $C_{1r_1}$) | $C_1$ |
| School $C_{21}$ | O$_1$(School $C_{21}$) | O$_2$(School $C_{21}$) | | O$_m$(School $C_{21}$) | $C_2$ |
| School $C_{22}$ | | | | | $C_2$ |
| ⋮ | ⋮ | ⋮ | ⋱ | ⋮ | ⋮ |
| School $C_{2r_2}$ | O$_1$(School $C_{2r_2}$) | O$_2$(School $C_{2r_2}$) | | O$_m$(School $C_{2r_2}$) | $C_2$ |
| ⋮ | ⋮ | ⋮ | | ⋮ | ⋮ |
| School $C_{g1}$ | O$_1$(School $C_{g1}$) | O$_2$(School $C_{g1}$) | | O$_m$(School $C_{g1}$) | $C_g$ |
| School $C_{g2}$ | | | | | $C_g$ |
| ⋮ | ⋮ | ⋮ | ⋱ | ⋮ | ⋮ |
| School $C_{gr_g}$ | O$_1$(School $C_{gr_g}$) | O$_2$(School $C_{gr_g}$) | | O$_m$(School $C_{gr_g}$) | $C_g$ |

where School $C_{ij}$ indicates the $j^{th}$ school in class $i$, $O_t$(School $C_{ij}$) the $t^{th}$ output of the $j^{th}$ school in class $i$ and $C_i$ the $i^{th}$ class.

To extend the class making into an analysis that can provide intermediate targets for a school to move from class $g$ to class $g-1$, it is more appropriate to implement the dual formulation of the "output only" model. The reason for this is that the dual formulation provides more information regarding the coordinates of the target for the $k^{th}$ school. These models were discussed in Chapter 3 and the dual formulation is repeated here for completeness sake.

$$\text{Minimize } E_k = \sum_{i=1}^{n} \lambda_i - \varepsilon \sum_{j=1}^{m} s_j \tag{4.8}$$

$$\text{subject to } \sum_{i=1}^{n} y_{ij}\,\lambda_i - s_j = y_{kj} \quad \text{for } j = 1 \cdots m, \tag{4.9}$$

$$\lambda_i \geq 0 \quad \text{for } i = 1 \cdots n, \tag{4.10}$$

$$s_j > 0 \qquad \text{for } j = 1 \cdots m, \qquad (4.11)$$

$$\varepsilon > 0. \qquad (4.12)$$

Let $\theta = \sum_{i=1}^{n} \lambda_i$, the targets for the $k^{th}$ school is then given by

$$(y_{kj} + s_j)/\theta \qquad \text{for } j = 1 \cdots m. \qquad (4.13)$$

The class ranking can now be performed in a similar fashion as for the primal model described earlier. During each iteration the intermediate targets are calculated. This process is performed for every school and an intermediate target "schedule" to move from one class to the next can be constructed. Table 4.1 shows the format of an intermediate target schedule for a hypothetical inefficient school *k*.

| Class | Output 1 | Output 2 | $\cdots$ | Output m |
|---|---|---|---|---|
| Class 1 (Highest) | $O_{11}$ | $O_{21}$ | $\cdots$ | $O_{m1}$ |
| Class 2 | $O_{12}$ | $O_{22}$ | $\cdots$ | $O_{m2}$ |
| Class 3 | $O_{13}$ | $O_{23}$ | $\cdots$ | $O_{m3}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| Class $g$-1 | $O_{1g-1}$ | $O_{2g-1}$ | $\cdots$ | $O_{mg-1}$ |
| Class $g$ (Current) | $O_{1g}$ | $O_{2g}$ | $\cdots$ | $O_{mg}$ |

Table 4.1: Output for school *k*

where $O_{ij}$ indicates output *i* in class *j*.

With reference to Table 4.1: School *k* is currently in class $g$. To improve to the next higher class ($g - 1$) the school needs to improve *all* those outputs where the outputs in class g is less than (worse) the outputs in class $g - 1$. For example, if $O_{2g} < O_{2g-1}$, then school *k* needs to improve output 2 which will move the school to the next class $g - 1$. By analysing all the outputs at every class level a roadmap can be devised to move from class to class until class 1 has been reached. By comparing the targets in the different classes to the original class rankings it is also possible to determine which schools, school *k* should treat as a benchmark.

Section 4.3.2 has highlighted the mathematical models and techniques that are implemented in the DSS's optimization module. The background theory and more detailed discussions of the models and techniques were presented in Chapter 3. The next section presents an illustrative example which further explains and clarifies the models implemented in the DSS.

## 4.4 Illustrative example

The third part of this chapter presents an illustrative example of the mathematical models and techniques (discussed in the previous section) that where implemented in the DSS. The objective is to show that the DSS performs according to expectations and that results produced are valid. The next chapter then focuses on the school application for which the system was developed.

The application selected for the illustrative example is concerned with the ranking and evaluation of universities. Reasons for this choice include the following

- Universities form part of the education sector which is the same for schools.
- The ranking of universities is a contemporary and important problem. It is seen by many as proxy indicators of reputation and performance (Ramakrishna, 2013) and may influence the financial matters of a university considerably. There exist a number of reports that frequently report on university rankings (Rauhvargers, 2011) while it is also an active research field – examples of research projects pertaining to this subject can be found in Daraio *et al.* (2015), Millot (2015) and Sidorenko and Gorbatova (2015).
- Suitable data related to universities was readily available (the data set used is described in section 4.4.1).
- Results obtained from the university example application can easily be related to the school's application.

The illustrative example focuses on the models that utilize outputs only. The normal input/output DEA model (CCR-model) does not form part of the example. The output only model forms the focus of the DSS and the study. Inputs for universities (and schools) are often not the same while outputs may indicate a set of incomparable criteria. The objective is therefore to rank universities (and schools) under criteria (which may be incomparable) while excluding unfair distribution of resources (inputs).

### 4.4.1 The dataset

The data chosen for the example application was obtained from a company called QS-Stars (Quacquarelli-Symonds, 2013). QS-Stars is well known and perform regular university ranking exercises. Further details on QS-Stars and its activities can be found at www.qs.com. The assessment system used by QS-Stars is based on a pre-defined set of criteria where each criterion can earn a number of stars depending on the assessment. From this an overall rating (number of stars) is then determined for each university which constitutes the ranking.

The initial dataset used in this illustrative example consist of 106 universities world-wide that where related according to nine criteria (facilities, internationalization, teaching, employability, research, innovation, engagement, specialist criteria and access). Not all records in this initial dataset were usable (due to missing data values) and the dataset was reduced to a 50 university dataset with 6 criteria to ensure a complete and usable dataset. The six criteria used include facilities, internationalization, teaching, employability, research and innovation. Table 4.2 below shows an extraction of the data. The names of the universities have been replaced by numbers. The complete dataset can be found in Appendix B.

| University | Facilities | Internationalization | Teaching | Employability | Research | Innovation | Overall rating |
|---|---|---|---|---|---|---|---|
| 1 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 2 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 3 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 45 | 4 | 2 | 3 | 3 | 2 | 1 | 2 |
| 46 | 5 | 4 | 1 | 3 | 1 | 1 | 2 |
| 47 | 3 | 1 | 1 | 4 | 2 | 3 | 2 |
| 48 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 49 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 50 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |

Table 4.2: Extraction of dataset used in illustrative example

The data in Table 4.2 refers to the ratings (number of stars) for the different universities for each criterion. The ratings ranges from 1 star (worst performance) to 5 stars (best performance). For example, university 45 received a rating of 4 stars for the criterion facilities, 2 stars for internationalization and so forth. The last column "Overall rating" indicates the rating that was finally given to the university based on the ratings of the criteria. In total there were 19 universities who received a top overall rating of 5 stars.

## 4.4.2 Application and results

The dual formulation of the output only model presented in section 4.3.2.2 as (4.8) to (4.13) was solved using the university dataset. At the same time the DSS calculated the intermediate targets for all those universities not ranked in the top class. The intermediate targets were calculated using the formula 4.13 presented in section 4.3.2.2. Two outputs (results) were then produced, first a class ranking of the universities and secondly intermediate targets. Each of these two outputs (results) are discussed below.

*Class Ranking*

The results of the class ranking are presented in Table 4.3. From this table the following can be observed. There are nine different classes in which the 50 universities were categorised. The first class (see the last column in in Table 4.3) has nine universities each having a total of 30 stars and with the same rating in every criterion. The second class has two universities with a total of 29 stars each and is clearly dominated by every university in class 1. The third class has seven universities. It should be noted that universities 20 and 26 in class 3 have less stars (in total) than the other universities in class 3. This is due to the Pareto principle and nested efficiency frontiers explained in section 3.4.1 in Chapter 3. Consider, for example, university 26 with a total of 25 stars. It can be seen that no other university in class 3 or in any other lower class (e.g. class 4) completely dominate (in all criteria) university 26. University 26 therefore belongs to class 3. The same is true for all other universities in a specific class with a total number of stars less than the expected number. It is therefore also possible that universities with the same number of total stars be stratified into different classes, e.g. university 23 (class 4) and university 24 (class 5) both have 26 stars in total.

This is an illustration of the exceptional strength of the ranking method to produce a ranking that is Pareto optimal and not simply a ranking based on a number of stars or even an ordinary input/output DMU efficiency ranking. It should be kept in mind that, according to the explanations in Chapter 3, universities in the same class is better than those in lower classes. This result in an improvement of the rating system used by QS Stars where 19 universities have an overall rating of 5 stars while the Pareto optimal analysis indicates that only 9 universities can be regarded as the set of top universities.

| | Facilities | International-ization | Teaching | Employability | Research | Innovation | Total Stars | Class |
|---|---|---|---|---|---|---|---|---|
| University 1 | 5 | 5 | 5 | 5 | 5 | 5 | 30 | 1 |
| University 2 | 5 | 5 | 5 | 5 | 5 | 5 | 30 | 1 |
| University 3 | 5 | 5 | 5 | 5 | 5 | 5 | 30 | 1 |
| University 4 | 5 | 5 | 5 | 5 | 5 | 5 | 30 | 1 |
| University 5 | 5 | 5 | 5 | 5 | 5 | 5 | 30 | 1 |
| University 6 | 5 | 5 | 5 | 5 | 5 | 5 | 30 | 1 |
| University 7 | 5 | 5 | 5 | 5 | 5 | 5 | 30 | 1 |
| University 8 | 5 | 5 | 5 | 5 | 5 | 5 | 30 | 1 |
| University 9 | 5 | 5 | 5 | 5 | 5 | 5 | 30 | 1 |
| University 10 | 5 | 5 | 5 | 5 | 4 | 5 | 29 | 2 |
| University 11 | 5 | 5 | 5 | 4 | 5 | 5 | 29 | 2 |
| University 12 | 5 | 5 | 4 | 5 | 4 | 5 | 28 | 3 |
| University 13 | 5 | 5 | 5 | 5 | 3 | 5 | 28 | 3 |
| University 14 | 5 | 5 | 5 | 4 | 5 | 4 | 28 | 3 |
| University 15 | 5 | 5 | 5 | 5 | 3 | 5 | 28 | 3 |
| University 16 | 5 | 5 | 5 | 5 | 3 | 5 | 28 | 3 |
| University 20 | 5 | 5 | 5 | 2 | 5 | 5 | 27 | 3 |
| University 26 | 4 | 4 | 4 | 3 | 5 | 5 | 25 | 3 |
| University 17 | 5 | 5 | 4 | 5 | 3 | 5 | 27 | 4 |
| University 18 | 5 | 5 | 4 | 4 | 5 | 4 | 27 | 4 |
| University 19 | 5 | 5 | 5 | 4 | 3 | 5 | 27 | 4 |
| University 21 | 5 | 5 | 5 | 5 | 3 | 3 | 26 | 4 |
| University 22 | 5 | 3 | 5 | 5 | 3 | 5 | 26 | 4 |
| University 23 | 5 | 5 | 4 | 2 | 5 | 5 | 26 | 4 |
| University 24 | 5 | 5 | 5 | 3 | 3 | 5 | 26 | 5 |
| University 25 | 5 | 5 | 4 | 5 | 3 | 3 | 25 | 5 |
| University 27 | 5 | 5 | 3 | 5 | 2 | 5 | 25 | 5 |
| University 30 | 5 | 4 | 5 | 5 | 3 | 2 | 24 | 5 |
| University 33 | 5 | 3 | 5 | 4 | 1 | 5 | 23 | 5 |
| University 35 | 5 | 5 | 5 | 4 | 1 | 1 | 21 | 5 |
| University 28 | 5 | 5 | 4 | 3 | 3 | 5 | 25 | 6 |
| University 31 | 4 | 5 | 5 | 3 | 3 | 3 | 23 | 6 |
| University 32 | 5 | 4 | 3 | 5 | 3 | 3 | 23 | 6 |
| University 34 | | 4 | 5 | 3 | 1 | 5 | 23 | 6 |
| University 37 | 5 | 3 | 2 | 4 | 1 | 5 | 20 | 6 |
| University 29 | 5 | 5 | 4 | 2 | 3 | 5 | 24 | 7 |
| University 36 | 4 | 5 | 1 | 3 | 3 | 4 | 20 | 7 |
| University 38 | 4 | 5 | 5 | 3 | 1 | 1 | 19 | 7 |
| University 39 | 4 | 3 | 2 | 3 | 2 | 5 | 19 | 7 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| University 40 | 5 | 4 | 4 | 3 | 2 | 1 | 19 | 7 |
| University 47 | 3 | 1 | 1 | 4 | 2 | 3 | 14 | 7 |
| University 41 | 5 | 4 | 3 | 2 | 1 | 3 | 18 | 8 |
| University 42 | 4 | 5 | 2 | 2 | 1 | 4 | 18 | 8 |
| University 43 | 4 | 5 | 2 | 2 | 1 | 4 | 18 | 8 |
| University 44 | 4 | 5 | 1 | 3 | 2 | 1 | 16 | 8 |
| University 45 | 4 | 2 | 3 | 3 | 2 | 1 | 15 | 8 |
| University 46 | 5 | 4 | 1 | 3 | 1 | 1 | 15 | 8 |
| University 48 | 2 | 2 | 1 | 1 | 1 | 1 | 8 | 9 |
| University 49 | 2 | 2 | 1 | 1 | 1 | 1 | 8 | 9 |
| University 50 | 2 | 2 | 1 | 1 | 1 | 1 | 8 | 9 |

Table 4.3: Class ranking of the 50 universities

Figure 4.4 shows a graphical display of the number of universities in each class.

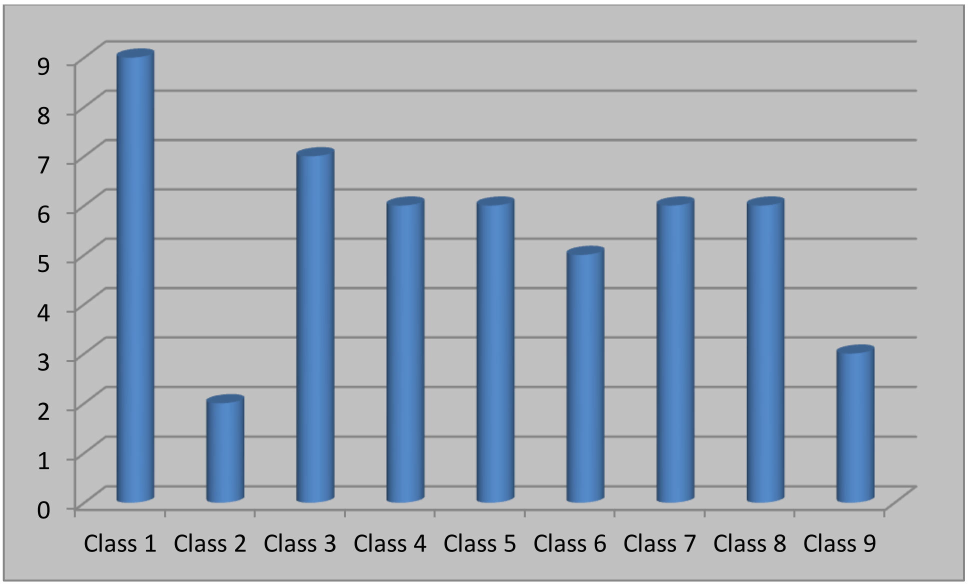



Figure 4.4: Number of universities in each class

*Intermediate targets*

One of the advantages of the DSS is that it is also capable of providing intermediate targets for those universities that are not ranked in the top class. The intermediate targets which should be seen as short term goals, provide a roadmap for a university to move from one class to the next (from one efficiency frontier to the next). This is of particular interest as it is, in practice not possible to move from a lower class straight to the top. Moving from any lower class straight to the top class (the final efficiency frontier) is what ordinary DEA models are used for.

The DSS provide intermediate targets for each university and for illustrative purposes the targets for university 46 in class 8 is presented below in Table 4.4.

| University 46 | | | | | | |
|---|---|---|---|---|---|---|
| Class | Facilities | Internat. | Teaching | Employability | Research | Innovation |
| 1 | 5 | 5 | 5 | 5 | 5 | 5 |
| 2 | 5 | 4 | 5 | 3 | 1 | 5 |
| 3 | 5 | 5 | 4 | 5 | 4 | 5 |
| 4 | 5 | 5 | 4 | 5 | 3 | 5 |
| 5 | 5 | 5 | 5 | 3 | 3 | 5 |
| 6 | 5 | 5 | 4 | 3 | 3 | 5 |
| 7 | 5 | 4 | 4 | 3 | 2 | 1 |
| 8 (Current) | 5 | 4 | 1 | 3 | 1 | 1 |

Table 4.4: Intermediate targets for university 46

From the intermediate targets in Table 4.4 the following can be observed.

University 46 is currently ranked in class 8. It may be impossible for university 46 to improve over a short period of time to be ranked in class 1. For example, university 46 may be situated in a developing country with poor infrastructure and minimal resources. It therefore makes sense for university 46 to rather try and improve its ranking from class 8 (the current ranking) to class 7 and then proceed step by step from one class to the next higher class. The intermediate roadmap in Table 4.4 indicates that for university 46 to be ranked in class 7 it should improve its teaching activities considerably to reach a 4-star rating (the current rating is 1 star). It should also improve its research activities from a 1-star rating to a 2 star rating. The other criteria are already on the same level as for class 7 and needs no further improvements. Furthermore, the target values also indicate which university, or universities, university 46 should treat as benchmarks. The target values in class 7 correspond to the ratings of university 40 in class 7 (see Table 4.3). This means that university 40 should be treated as the benchmark for university 46 for moving from class 8 to class 7. Once university 46 is in class 7, the next benchmark university to reach class 6 would be university 28 (targets for class 6 is the same as the ratings for university 28 in Table 4.3). The process can now be repeated over a period of time until the top class is reached.

It should be noted that in some instances the target (to improve to a higher class) for a specific criterion may be lower as the current performance. See for example criteria employability in Table 4.4. In class 3 the performance rating is 5 while in class 2 the performance is 3. This simply means that if university 46 is at class 3 and it wants to move to class 2 then it already performs better than expected in employability and further improvement in this area is not necessary. Ratings like this are due to the nature of the mathematical model that is solved repeatedly and is in line with results of other similar studies – see for example the class ranking of Taiwan colleges (Kao, 1994).

It should furthermore also be noted that the intermediate target for a university such as in Table 4.4 provides a roadmap at a specific point in time. It is advisable that whenever a university moves from one class to the next higher class the models are solved again and a new roadmap be produced. The reason for this is that when a university has improved in some areas (and moved to a higher class) the ratings have changed which may cause the intermediate values to change as well. Despite this, the intermediate targets at any point still provide a good and reliable picture of what is expected of a university. Improvement in universities (and schools) are in any case not something that occur overnight and the intermediate targets provide a useful plan with shorter term goals that can guide universities to make the right decisions on where to focus resources.

In addition to the intermediate target table as presented in Table 4.4, the DSS also provide a more user friendly visualization of the intermediate targets. The well-known traffic light colour scheme was implemented using pictures of a face. A green smiling face would indicate that current performance is satisfactory and no additional work is needed. A yellow straight face indicates that some improvement is necessary (e.g. a 1 star improvement) while a red sad face indicates that a considerable improvement (2 or more stars) are required. Table 4.5 below shows the user friendly output formed for user 46.

| University 46 | | | | | | |
|---|---|---|---|---|---|---|
| Class | Facilities | Internat. | Teaching | Employability | Research | Innovation |
| 1 | ☺ | ☺ | ☺ | ☺ | ☺ | ☺ |
| 2 | ☺ | 😐 | ☺ | ☹ | ☹ | ☺ |
| 3 | ☺ | ☺ | 😐 | ☺ | ☺ | ☺ |
| 4 | ☺ | ☺ | ☺ | ☺ | 😐 | ☺ |
| 5 | ☺ | ☺ | ☺ | ☹ | ☺ | ☺ |
| 6 | ☺ | ☺ | 😐 | ☺ | ☺ | ☺ |
| 7 | ☺ | 😐 | ☺ | ☺ | 😐 | ☹ |
| 8 (Current) | ☺ | ☺ | ☹ | ☺ | 😐 | ☺ |

Table 4.5: Intermediate targets for university 46

This concludes the illustrative example. The example has indicated the theory and models implemented in the DSS deliver sound results that are logic and applicable. It has shown that the models, and the DSS, can rank universities under incomparable criteria and it can provide intermediate goals for a university to become efficient in stages.

## 4.5 Conclusion

The aim of Chapter 4 was to present an overview of DSSs in general and to describe the DSS developed in this study. The chapter comprises of three main sections. Firstly, the general concepts of a DSS were introduced followed by an overview of the newly developed DSS. Finally, an illustrative example was presented to validate the models and techniques implemented. The next chapter describes the school application.

# 5 DEA MODEL APPLICATION FOR SCHOOLING SYSTEM

## 5.1 Introduction

The aim of this study was to develop a DSS which implements a mathematical model that can be used to evaluate the efficiency of schools. The objective was to implement and apply the DSS and associate models specifically to schools in the North-West province of South Africa. Following the background theory of the mathematical models in Chapter 3 and the DSS description in Chapter 4, this chapter focuses on the school application.

The goal of the chapter is to describe the school efficiency assessment using the DSS. The chapter starts with a synopsis of the dataset used in this study. This is followed by a discussion of the application which includes both an input/output and an output only model. A discussion of the findings and recommendations concludes the chapter.

## 5.2 The school data set

The data used in the analysis was limited to schools in one of the four main municipality districts of the North-West province (see Chapter 2 for a description of the different municipality districts). Reasons for this limitation include

- Easy access to the Department of Education for this specific district.
- The number of schools in this district was sufficient to perform an efficiency assessment.
- Data for this district was readily available.
- Lack of, or incomplete data for the other districts mean that they would be unsuitable for the study unless a huge amount of cost and effort is invested to obtain proper data.
- Permission to include schools from other districts could not be obtained.

To ensure a homogeneous dataset it was decided to use only secondary schools (grade 8-12) in the analysis – primary schools were therefore excluded. Furthermore, only schools offering mathematics and physical science were included in the dataset. This decision was based on the importance of mathematics and physical science as subjects and a specific need to include these two subjects in the efficiency analysis. An initial data analysis was performed to ensure the dataset is complete and accurate – records with missing data values were removed and, where appropriate, data transformations were done. This resulted in a usable dataset of 54 secondary schools from which input and output variables were selected.

It should be noted that the selection of input and output variables was completely dependent on the availability of data. In the literature one may find different input and output variables which may enhance the results of an efficiency analysis. See for example the paper by Lovell *et al*. (1994) that was published in Charnes *et al*. 1994. In this study the authors made use of variables such as physical facilities and extra curricular activities. This type of data was not available in the North-West province and hence the remark that the variable selection was dependent on the availability of data.

The final input and output variables selected were as follows.

*Input variables*

- Number of educators – this refers to the number of teachers in a school.
- Number of subjects – the number of subjects is the number of different subjects offered by a school.
- Learner/educator ratio – this ratio indicates the number of learners per teacher in a school.
- Qualification – the qualification variable refers to the number of teachers with an honours or higher degree at a school.
- Total income per learner – this represents school fees and a departmental subsidy per learner.

*Output variables*

- Grade 12 pass rate – the grade 12 pass rate expressed as a percentage for each school.
- Grade 12 mathematics average – expressed as a percentage, this represents the average mark obtained in mathematics (Grade 12) for each school.
- Grade 12 physical science average – expressed as a percentage, this represents the average mark obtained in physical science (Grade 12) for each school.

- School pass rate – this variable represents the overall pass rate in all grades for a school and is expressed as a percentage.
- Grade 9 pass rate – represents the pass rate (percentage) for grade 9 learners for each school. The grade 9 pass rate was deemed to be important as grade 9 is a national examination (same as for grade 12).

Table 5.1 presents the complete dataset.

| | Outputs | | | | | Inputs | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate | Number of educators | Number of subjects | Learner/Educator ratio | Qualifications | Total income per learner |
| School 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 | 34.0 | 15.0 | 15.1 | 20.0 | 14475.0 |
| School 2 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 | 9.0 | 14.0 | 9.8 | 5.9 | 7585.0 |
| School 3 | 100.0 | 61.8 | 61.1 | 98.6 | 100.0 | 38.0 | 19.0 | 19.0 | 21.1 | 9736.2 |
| School 4 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 | 49.0 | 21.0 | 19.7 | 17.9 | 8569.9 |
| School 5 | 98.0 | 58.7 | 47.9 | 96.8 | 93.0 | 24.0 | 19.0 | 18.3 | 13.7 | 75.0 |
| School 6 | 100.0 | 56.2 | 55.4 | 96.8 | 97.0 | 25.0 | 21.0 | 19.9 | 30.8 | 8075.0 |
| School 7 | 97.0 | 51.1 | 59.5 | 96.0 | 94.0 | 57.0 | 22.0 | 19.9 | 18.3 | 7025.0 |
| School 8 | 98.0 | 54.0 | 60.6 | 95.0 | 95.0 | 46.0 | 20.0 | 19.1 | 15.1 | 8905.8 |
| School 9 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 | 14.0 | 16.0 | 13.6 | 18.2 | 5575.0 |
| School 10 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 | 38.0 | 16.0 | 21.0 | 13.2 | 7995.0 |
| School 11 | 95.0 | 39.9 | 46.7 | 91.2 | 88.0 | 14.0 | 14.0 | 30.0 | 18.5 | 4975.0 |
| School 12 | 86.0 | 39.8 | 40.3 | 90.4 | 96.0 | 46.0 | 13.0 | 20.7 | 6.3 | 4675.0 |
| School 13 | 94.0 | 40.9 | 41.4 | 88.8 | 92.0 | 26.0 | 15.0 | 21.2 | 23.4 | 8575.0 |
| School 14 | 92.0 | 46.7 | 46.9 | 82.8 | 64.0 | 49.0 | 17.0 | 21.1 | 10.6 | 7775.0 |
| School 15 | 88.0 | 31.4 | 48.4 | 81.2 | 81.0 | 33.0 | 17.0 | 23.5 | 20.8 | 75.0 |
| School 16 | 92.0 | 38.5 | 46.4 | 80.6 | 64.0 | 39.0 | 15.0 | 29.4 | 16.7 | 75.0 |
| School 17 | 87.0 | 34.6 | 43.5 | 79.6 | 76.0 | 33.0 | 16.0 | 22.5 | 13.7 | 75.0 |
| School 18 | 85.0 | 30.1 | 34.0 | 78.8 | 100.0 | 34.0 | 13.0 | 30.7 | 12.1 | 75.0 |
| School 19 | 96.0 | 40.3 | 45.9 | 73.4 | 54.0 | 26.0 | 13.0 | 32.7 | 9.5 | 150.0 |
| School 20 | 90.0 | 40.4 | 38.9 | 72.0 | 46.0 | 17.0 | 14.0 | 28.2 | 0.0 | 150.0 |
| School 21 | 87.0 | 34.5 | 46.5 | 70.8 | 75.0 | 50.0 | 16.0 | 26.4 | 13.4 | 75.0 |
| School 22 | 95.0 | 28.6 | 34.4 | 70.2 | 25.0 | 28.0 | 14.0 | 27.4 | 4.8 | 75.0 |
| School 23 | 57.0 | 21.0 | 25.9 | 69.4 | 68.0 | 26.0 | 14.0 | 18.9 | 14.9 | 75.0 |
| School 24 | 63.0 | 46.2 | 46.6 | 68.6 | 85.0 | 29.0 | 13.0 | 33.9 | 9.1 | 75.0 |
| School 25 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 | 30.0 | 12.0 | 36.6 | 15.8 | 125.0 |
| School 26 | 72.0 | 33.8 | 32.9 | 68.0 | 59.0 | 32.0 | 13.0 | 37.2 | 4.3 | 150.0 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| School 27 | 63.0 | 28.9 | 39.8 | 67.2 | 67.0 | 40.0 | 15.0 | 28.7 | 8.1 | 575.0 |
| School 28 | 87.0 | 28.7 | 37.4 | 66.4 | 61.0 | 36.0 | 16.0 | 26.3 | 14.8 | 75.0 |
| School 29 | 82.0 | 25.0 | 29.9 | 66.4 | 79.0 | 14.0 | 10.0 | 23.6 | 13.6 | 75.0 |
| School 30 | 72.0 | 25.9 | 19.2 | 65.8 | 69.0 | 34.0 | 20.0 | 35.4 | 16.4 | 125.0 |
| School 31 | 90.0 | 47.8 | 42.4 | 64.0 | 31.0 | 30.0 | 14.0 | 34.8 | 10.5 | 150.0 |
| School 32 | 88.0 | 35.3 | 26.2 | 63.0 | 54.0 | 31.0 | 16.0 | 38.2 | 9.7 | 75.0 |
| School 33 | 62.0 | 28.9 | 30.6 | 62.2 | 63.0 | 36.0 | 16.0 | 32.8 | 9.7 | 125.0 |
| School 34 | 86.0 | 20.9 | 35.1 | 61.2 | 30.0 | 34.0 | 13.0 | 32.6 | 8.6 | 125.0 |
| School 35 | 87.0 | 33.6 | 36.0 | 60.6 | 41.0 | 16.0 | 14.0 | 31.7 | 0.0 | 75.0 |
| School 36 | 63.0 | 17.4 | 24.5 | 60.4 | 54.0 | 31.0 | 13.0 | 19.2 | 16.1 | 425.0 |
| School 37 | 84.0 | 27.6 | 31.9 | 60.2 | 48.0 | 42.0 | 13.0 | 28.1 | 5.7 | 125.0 |
| School 38 | 74.0 | 44.2 | 36.5 | 59.4 | 57.0 | 44.0 | 17.0 | 32.2 | 8.7 | 75.0 |
| School 39 | 76.0 | 34.8 | 41.6 | 58.6 | 57.0 | 67.0 | 19.0 | 25.8 | 12.1 | 725.0 |
| School 40 | 76.0 | 29.1 | 34.4 | 58.6 | 29.0 | 40.0 | 15.0 | 28.3 | 12.5 | 75.0 |
| School 41 | 70.0 | 32.0 | 38.0 | 58.4 | 19.0 | 19.0 | 12.0 | 30.9 | 17.2 | 175.0 |
| School 42 | 69.0 | 27.5 | 31.7 | 56.6 | 41.0 | 52.0 | 16.0 | 25.7 | 3.8 | 175.0 |
| School 43 | 89.0 | 31.0 | 34.6 | 56.2 | 36.0 | 20.0 | 13.0 | 39.2 | 25.0 | 125.0 |
| School 44 | 80.0 | 29.1 | 31.8 | 54.8 | 37.0 | 36.0 | 15.0 | 31.1 | 12.8 | 75.0 |
| School 45 | 52.0 | 24.5 | 27.2 | 54.8 | 50.0 | 24.0 | 15.0 | 30.0 | 13.8 | 75.0 |
| School 46 | 73.0 | 23.6 | 28.1 | 54.6 | 61.0 | 9.0 | 11.0 | 49.3 | 11.1 | 75.0 |
| School 47 | 66.0 | 25.6 | 37.8 | 53.2 | 39.0 | 40.0 | 14.0 | 32.9 | 13.5 | 150.0 |
| School 48 | 72.0 | 23.5 | 25.6 | 53.0 | 26.0 | 34.0 | 16.0 | 34.6 | 28.8 | 150.0 |
| School 49 | 94.0 | 31.1 | 34.1 | 47.4 | 34.0 | 28.0 | 14.0 | 25.9 | 7.5 | 125.0 |
| School 50 | 74.0 | 29.7 | 34.5 | 47.4 | 37.0 | 35.0 | 16.0 | 29.6 | 9.1 | 75.0 |
| School 51 | 70.0 | 25.6 | 29.8 | 44.6 | 2.0 | 25.0 | 17.0 | 30.1 | 7.4 | 150.0 |
| School 52 | 86.0 | 20.4 | 30.3 | 42.6 | 2.0 | 33.0 | 16.0 | 31.4 | 21.3 | 75.0 |
| School 53 | 74.0 | 23.4 | 29.0 | 34.6 | 2.0 | 40.0 | 13.0 | 21.9 | 4.0 | 75.0 |
| School 54 | 46.0 | 21.2 | 14.1 | 30.4 | 3.0 | 19.0 | 15.0 | 32.4 | 10.6 | 75.0 |

Table 5.1: School dataset

## 5.3 The school application

The school dataset described in the previous section was used to perform two different efficiency analysis. First the well-known CCR-model (see Chapter 3, section 3.3.3.1) which utilizes both input and output variables was implemented and secondly, a school ranking model utilizing only output variables was implemented. In this section the results of these two techniques are presented as well as a comparison of the results.

### 5.3.1 The CCR-model

To perform the input/output efficiency analysis the DSS implements the CCR-model that was formulated and explained in Chapter 3 section 3.3.3.1. The 5 input variables and 5 output variables defined in section 5.2 were used for the set of 54 schools. The results are presented in Table 5.2 below.

| School | Efficiency rating | Schools in the reference set | | | | | |
|---|---|---|---|---|---|---|---|
| School 1 | 1.000 | | | | | | |
| School 2 | 1.000 | | | | | | |
| School 3 | 0.956 | 1, | 5, | 9, | 24, | 25 | |
| School 4 | 0.926 | 2, | 8, | 9, | 10, | 20 | |
| School 5 | 1.000 | | | | | | |
| School 6 | 0.830 | 1, | 2, | 5, | 9, | 24, | 25 |
| School 7 | 0.928 | 2, | 5, | 8, | 9 | | |
| School 8 | 1.000 | | | | | | |
| School 9 | 1.000 | | | | | | |
| School 10 | 1.000 | | | | | | |
| School 11 | 1.000 | | | | | | |
| School 12 | 1.000 | | | | | | |
| School 13 | 0.865 | 1, | 2, | 29 | | | |
| School 14 | 0.880 | 1, | 2, | 10, | 20 | | |
| School 15 | 1.000 | | | | | | |
| School 16 | 1.000 | | | | | | |
| School 17 | 0.975 | 5, | 15, | 16, | 24, | 25, | 29 |
| School 18 | 1.000 | | | | | | |
| School 19 | 1.000 | | | | | | |
| School 20 | 1.000 | | | | | | |
| School 21 | 0.995 | 5, | 16, | 24, | 35 | | |
| School 22 | 1.000 | | | | | | |
| School 23 | 0.897 | 2, | 5, | 29 | | | |
| School 24 | 1.000 | | | | | | |
| School 25 | 1.000 | | | | | | |
| School 26 | 0.953 | 12, | 18, | 20, | 24, | 29 | |
| School 27 | 0.863 | 5, | 9, | 12, | 20, | 24 | |
| School 28 | 0.918 | 5, | 16, | 22, | 29 | | |
| School 29 | 1.000 | | | | | | |
| School 30 | 0.578 | 5, | 18, | 20, | 29 | | |
| School 31 | 1.000 | | | | | | |
| School 32 | 0.921 | 5, | 22, | 29 | | | |
| School 33 | 0.693 | 5, | 12, | 18, | 20, | 24, | 29 |
| School 34 | 0.908 | 19, | 20, | 29 | | | |
| School 35 | 1.000 | | | | | | |
| School 36 | 0.803 | 2, | 5, | 29 | | | |
| School 37 | 0.924 | 20, | 22, | 29 | | | |
| School 38 | 0.896 | 5, | 24, | 35 | | | |
| School 39 | 0.789 | 5, | 9, | 20, | 24 | | |
| School 40 | 0.812 | 5, | 16, | 22 | | | |
| School 41 | 0.997 | 11, | 19, | 29, | 46 | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| School 42 | 0.760 | 2, | 5, | 20 | | |
| School 43 | 0.867 | 2, | 20, | 25, | 29 | |
| School 44 | 0.842 | 5, | 16, | 22, | 29 | |
| School 45 | 0.662 | 5, | 16, | 18, | 24, | 29 |
| School 46 | 1.000 | | | | | |
| School 47 | 0.780 | 5, | 9, | 24, | 25 | |
| School 48 | 0.579 | 2, | 5, | 29 | | |
| School 49 | 0.998 | 2, | 5, | 22, | 29 | |
| School 50 | 0.804 | 5, | 16, | 24, | 35 | |
| School 51 | 0.646 | 2, | 5, | 20, | 22, | 29 |
| School 52 | 0.894 | 5, | 22 | | | |
| School 53 | 0.948 | 5, | 20, | 22, | 35 | |
| School 54 | 0.509 | 5, | 29, | 35 | | |

Table 5.2: Results of the CCR-model with 5 input and 5 output variables

With reference to Table 5.2. The first column shows the 54 schools while the middle column indicates the efficiency of each school as calculated by solving the CCR-model. An efficiency of 1.00 indicates a 100% efficiency, while efficiencies less then 1.00 imply that the school should improve its performance. For example, school 1 is 100% efficeint while school 36 is only 80% efficient. The last column shows the reference set for the inefficient schools. The CCR-model compares inefficient schools with a data reference set of other selected schools (the reference set) all of which have an efficiency value of one. The efficiency reference set for an inefficient school is determined by examining the opportunity cost (part of the output of the mathematical model) of the inefficient school. An opportunity cost value greater then zero would indicate that a school (associated with the opportunity cost) belongs to the inefficient school's reference set. Furthermore, this value indicates also how much the objective function (i.e. the efficiency) would be improved if inputs were reduced by one unit. This is a typical sensitivity analysis activity that is usually performed with linear programming models. A comprehensive explanation of this type of target calculations can be found in Ragsdale (2010) and Metzger (1994).

It can be seen from the results in Table 5.2 that 20 schools (37%) were evaluated as efficient given the set of input and output variables. The remainder of the schools are inefficient and through their reference sets spesific targets can be calculated to improve their performance to 100% efficiency. This was not done for the inefficient schools as the goal of this study (and the DSS) is to focus on the ranking of schools and a stepwise improvement plan rather than making recommendations to become immediately 100%

efficient. Such a recommendation to become immediately 100% efficient would be inappropriate (and useless) as schools simply do not have resources to become immediately as efficient as the schools in their reference sets. The argument is elaborated on in the discussion section (section 5.4).

Figure 5.1 shows the geographical location of the 54 schools in the spesific municipality district. The green labels indicate the 100% efficient schools according to the input/output CCR-model.

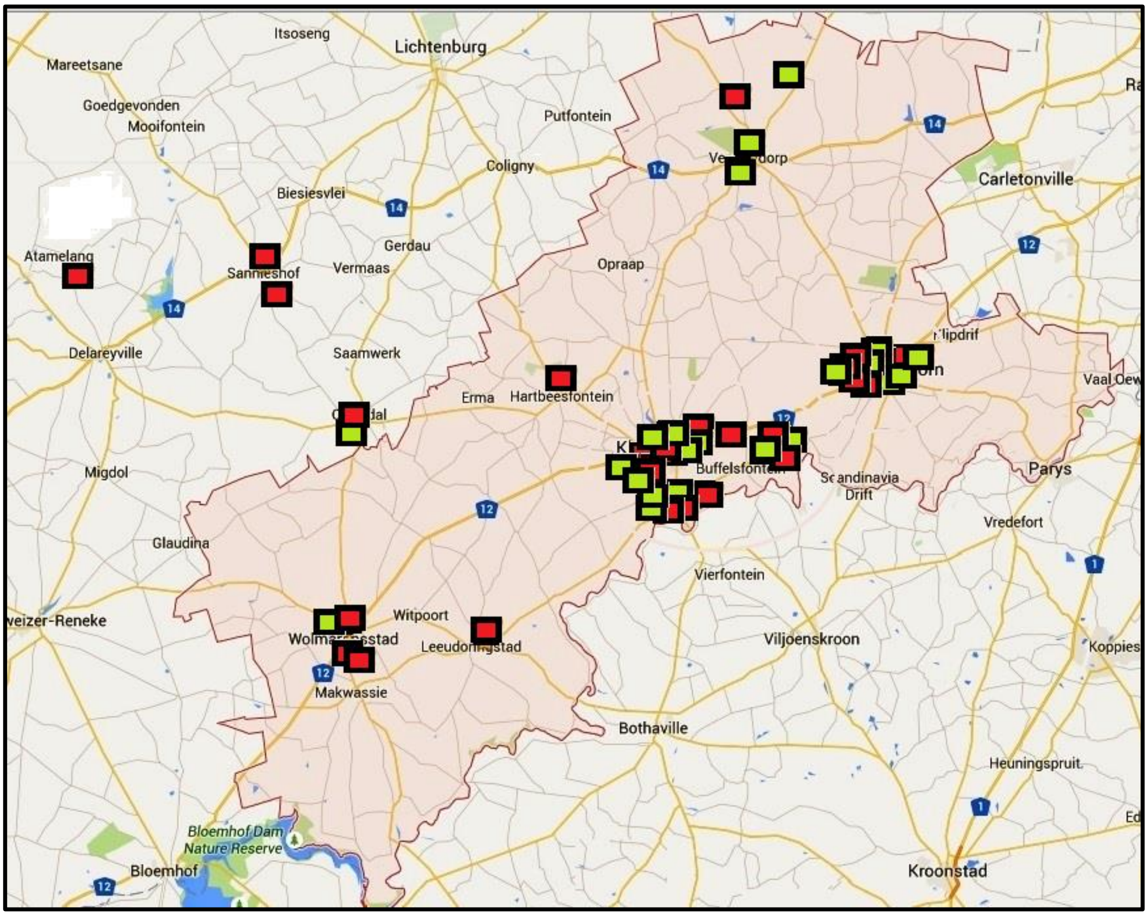



Figure 5.1: Geographical location of efficient schools according to the CCR-model

The map in Figure 5.1, as well as the data shows no spesific trend or clustering of efficient or inefficient schools. Efficient schools occur in various setups such as urban areas, rural areas and townships. Other than providing a list of efficient schools, the results seem to have no additional value. One reason for this is the set of input variables that was used. As explained in the previous section there may be other (better) input variables that was inaccessable for this study. Further comments on this are also presented in the discussion (section 5.4).

It is of course possible to solve the CCR-model also for different combinations of input and output variables e.g. using just one input with 5 outputs. Another 5 combinations (varying the number of input variables) were also solved by the DSS. The results revealed nothing extraordinary and in fact seems to be unreliable because of the differences in data values for spesific input variables. For the purpose of this section the presentation of the results of the 5 input and 5 output model therefore suffices.

### 5.3.2 Class ranking using an output only model

Due to spesific reasons (e.g. a presumed unfair distribution of resources (inputs) to schools) it was decided to focus on class ranking of schools utilising an output only model. Spesific reasons for this are futher elaborated on in section 5.4.

To perform the class ranking of schools and at the same time produce a more realistic improvement plan for inefficient schools, the dual formulation of the output only model (3.41) to (3.44) presented in section 3.4.3 was solved. Intermediate targets for inefficient schools were simultaneously calculated using the formula (3.45) explained in section 3.4.3.

The input for this exercise was the dataset of schools presented in section 5.2 but excluding the 5 input variables – i.e. a model utilising only the 5 output variables.

Two types of output were produced; first, a class ranking of the 54 schools, and secondly intermediate targets for all the schools not ranked in the top class. These two types of output are described in the following paragraphs.

***Class Ranking***

Table 5.3 on the next page presents the results of the class ranking performed by the application of algorithm 3.1 in solving the mathematical model (see section 3.4.2).

| School | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate | Class |
|---|---|---|---|---|---|---|
| School 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 | 1 |
| School 3 | 100.0 | 61.8 | 61.2 | 98.6 | 100.0 | 1 |
| School 2 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 | 2 |
| School 4 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 | 2 |
| School 6 | 100.0 | 56.2 | 55.4 | 96.8 | 97.0 | 2 |
| School 8 | 98.0 | 54.0 | 60.6 | 95.0 | 95.0 | 2 |
| School 5 | 98.0 | 58.7 | 47.9 | 96.8 | 93.0 | 3 |
| School 7 | 97.0 | 51.1 | 59.5 | 96.0 | 94.0 | 3 |
| School 10 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 | 3 |
| School 12 | 86.0 | 39.8 | 40.3 | 90.4 | 96.0 | 3 |
| School 18 | 85.0 | 30.1 | 34.0 | 78.8 | 100.0 | 3 |
| School 9 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 | 4 |
| School 13 | 94.0 | 40.9 | 41.4 | 88.8 | 92.0 | 4 |
| School 11 | 95.0 | 39.9 | 46.7 | 91.2 | 88.0 | 5 |
| School 14 | 92.0 | 46.7 | 46.9 | 82.8 | 64.0 | 5 |
| School 15 | 88.0 | 31.4 | 48.4 | 81.2 | 81.0 | 5 |
| School 19 | 96.0 | 40.3 | 45.9 | 73.4 | 54.0 | 5 |
| School 24 | 63.0 | 46.2 | 46.6 | 68.6 | 85.0 | 5 |
| School 25 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 | 5 |
| School 31 | 90.0 | 47.8 | 42.4 | 64.0 | 31.0 | 5 |
| School 16 | 92.0 | 38.5 | 46.4 | 80.6 | 64.0 | 6 |
| School 17 | 87.0 | 34.6 | 43.5 | 79.6 | 76.0 | 6 |
| School 20 | 90.0 | 40.4 | 38.9 | 72.0 | 46.0 | 6 |
| School 21 | 87.0 | 34.5 | 46.5 | 70.8 | 75.0 | 6 |
| School 22 | 95.0 | 28.6 | 34.4 | 70.2 | 25.0 | 6 |
| School 29 | 82.0 | 25.0 | 29.9 | 66.4 | 79.0 | 6 |
| School 38 | 74.0 | 44.2 | 36.5 | 59.4 | 57.0 | 6 |
| School 23 | 57.0 | 21.0 | 25.9 | 69.4 | 68.0 | 7 |
| School 26 | 72.0 | 33.8 | 32.9 | 68.0 | 59.0 | 7 |
| School 27 | 63.0 | 28.9 | 39.8 | 67.2 | 67.0 | 7 |
| School 28 | 87.0 | 28.7 | 37.4 | 66.4 | 61.0 | 7 |
| School 30 | 72.0 | 25.9 | 19.2 | 65.8 | 69.0 | 7 |
| School 32 | 88.0 | 35.3 | 26.2 | 63.0 | 54.0 | 7 |
| School 35 | 87.0 | 33.6 | 36.0 | 60.6 | 41.0 | 7 |
| School 39 | 76.0 | 34.8 | 41.6 | 58.6 | 57.0 | 7 |
| School 49 | 94.0 | 31.1 | 34.1 | 47.4 | 34.0 | 7 |
| School 33 | 62.0 | 28.9 | 30.6 | 62.2 | 63.0 | 8 |
| School 34 | 86.0 | 21.0 | 35.1 | 61.2 | 30.0 | 8 |
| School 37 | 84.0 | 27.6 | 31.9 | 60.2 | 48.0 | 8 |
| School 41 | 70.0 | 32.0 | 38.0 | 58.4 | 19.0 | 8 |
| School 43 | 89.0 | 31.0 | 34.6 | 56.2 | 36.0 | 8 |
| School 46 | 73.0 | 23.6 | 28.1 | 54.6 | 61.0 | 8 |

| | | | | | | |
|---|---|---|---|---|---|---|
| School 47 | 66.0 | 25.6 | 37.8 | 53.2 | 39.0 | 8 |
| School 36 | 63.0 | 17.4 | 24.5 | 60.4 | 54.0 | 9 |
| School 40 | 76.0 | 29.1 | 34.4 | 58.6 | 29.0 | 9 |
| School 42 | 69.0 | 27.5 | 31.7 | 56.6 | 41.0 | 9 |
| School 44 | 80.0 | 29.1 | 31.8 | 54.8 | 37.0 | 9 |
| School 45 | 52.0 | 24.5 | 27.2 | 54.8 | 50.0 | 9 |
| School 50 | 74.0 | 29.7 | 34.5 | 47.4 | 37.0 | 9 |
| School 52 | 86.0 | 20.4 | 30.3 | 42.6 | 2.0 | 9 |
| School 48 | 72.0 | 23.5 | 25.6 | 53.0 | 26.0 | 10 |
| School 51 | 70.0 | 25.6 | 29.8 | 44.6 | 2.0 | 10 |
| School 53 | 74.0 | 23.4 | 29.0 | 34.6 | 2.0 | 10 |
| School 54 | 46.0 | 21.2 | 14.1 | 30.4 | 3.0 | 11 |

Table 5.3: Class ranking of the 54 schools

From Table 5.3 the following can be observed. The 54 schools were categorised into 11 different classes. The top class (class 1 – see last column in Table 5.3) consist of only two schools. Class 2 has 4 schools and so forth. The Pareto optimality is also clear from the results – e.g. a school in class 1 clearly dominates any other school in a lower class. It should therfore be noted that schools in a higher class perform better than those in lower classes and that they are not comparable (as in the input/output model presented in section 5.3.1).

The actual school names are not provided for confidentiality and sensitivity reasons. However, a closer look at the results revealed the following interesting facts

- All schools categorised in classes 1 to 4 are mainly urban area schools (traditionally the more advantaged schools).
- All schools categorised in classes 7 to 11 are rural, farm and township schools (traditionally the more disadvantaged schools).
- Schools categorised in classes 5 and 6 represent a mix of all schools (traditionally advantaged and disadvantaged schools).

It can thus be concluded that the model has produced logical and intuitively correct results by making these clear distictions that is consistent with expectations.

Figure 5.2 shows a bar chart indicating the number of schools per class while Figure 5.3 presents a map of schools. Classes 1 to 4 are indicated in green, class 5 and 6 in yellow and classes 7 to 11 in red.

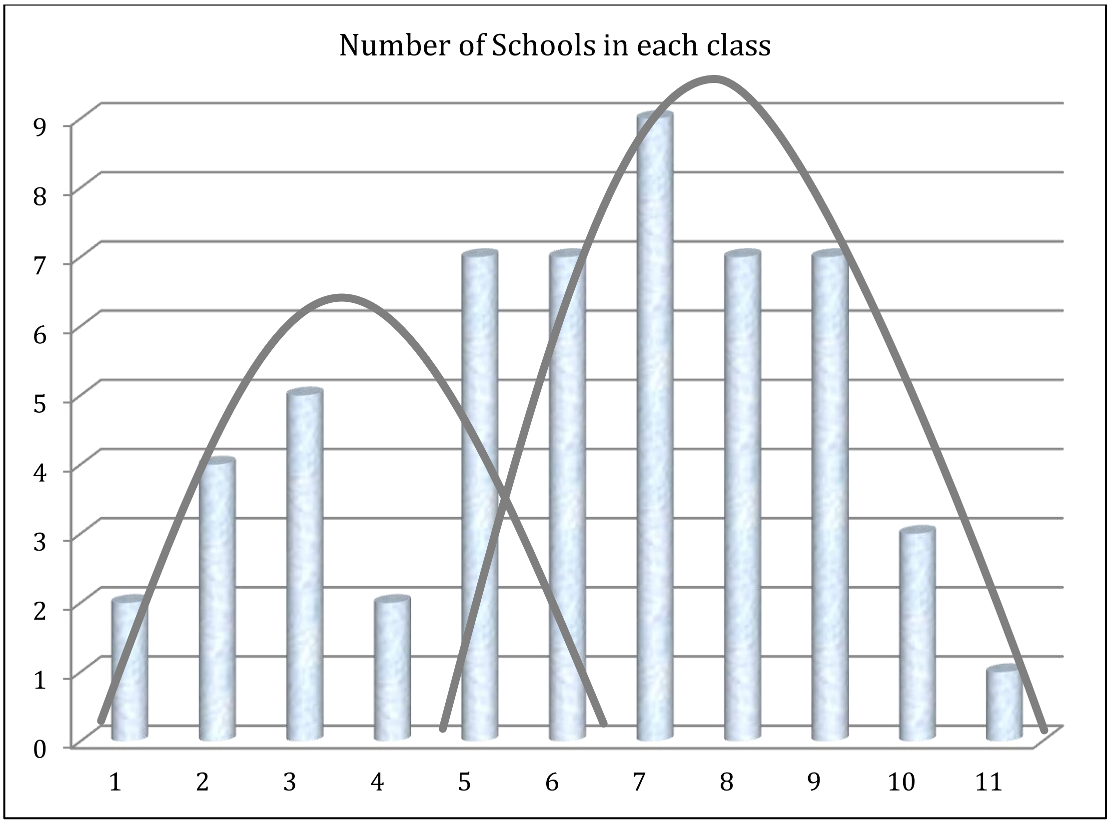



Figure 5.2: The number of schools per class

For clarity purposes two regions (Potchefstroom and Klerksdorp-Orkney-Stilfontein) have been extracted from Figure 5.3 and are presented as Figures 5.4 and 5.5 on the next page. Figures 5.4 and 5.5 visually confirm the earlier statement that schools categorised in higher classes are located in urban areas while those schools in lower classes are from the traditionally disadvantaged areas.

A discussion of these class ranking results and the results of the CCR-model (section 5.3.1) are presented in section 5.4.

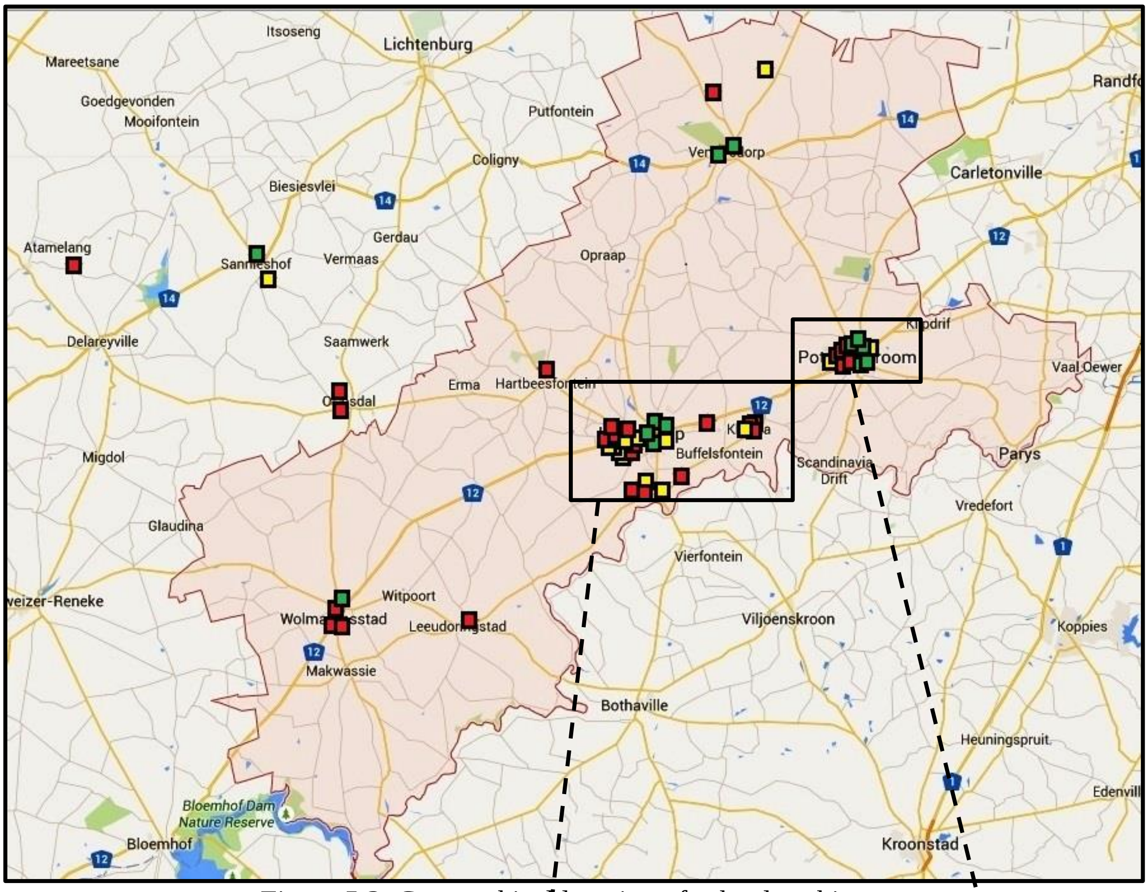
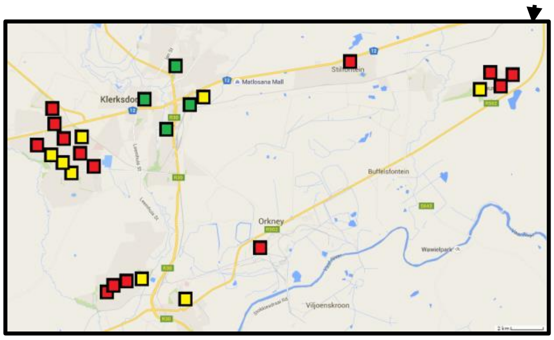



Figure 5.3: Geographical location of school rankings



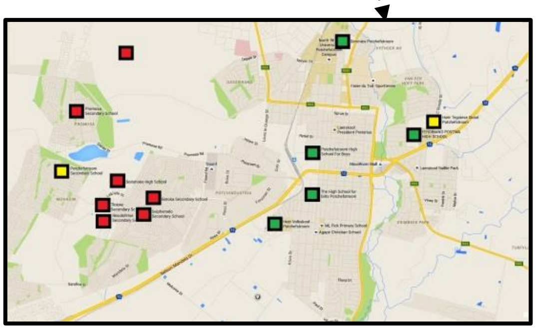


Figure 5.4: Klerksdorp-Orkney-Stilfontein area and Figure 5.5: Potchefstroom area

***Intermediate targets***

Using the dual formulation of an output only model, the DSS is able to calculate and provide intermediate targets for those schools that are not ranked in the top class. These intermediate targets indicate improvements necessary to move from the school's current class to the next higher class. As explained in the illustrative example in Chapter 4 (section 4.4.2) this is of particular interest as a school does not have resources to move from a low class immediately to the top class – moving from any low efficiency to the top performers is what the CCR-model suggest. Utilising the nested efficiency frontier principle (class ranking) therefore enables the DSS to provide a roadmap to improve in a more realistic way step by step.

Intermediate targets for all 52 schools not ranked in class 1 were produced and is presented in Appendix C. For explanatory purposes the intermediate targets for school 40 are presented and discussed here. Table 5.4 presents the intermediate targets for school 40.

| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
|---|---|---|---|---|---|
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 45.8 | 48.2 | 74.0 | 60.5 |
| 5 | 96.0 | 40.3 | 45.9 | 74.0 | 55.1 |
| 6 | 92.9 | 35.6 | 42.8 | 77.5 | 52.4 |
| 7 | 82.7 | 31.7 | 37.4 | 63.8 | 55.8 |
| 8 | 77.1 | 29.5 | 34.9 | 59.4 | 33.9 |
| Current (9) | 76.0 | 29.1 | 34.4 | 58.6 | 29.0 |

Table 5.4: Intermediate goal matrix for school 40

From the intermediate targets presented in Table 5.4 the following can be observed. School 40 is currently ranked in class 9. As explained earlier, schools in classes 7 to 11 are rural or township schools and school 40 is indeed a school located in a township. It is nearly impossible for any school in a township in the North-West province to improve in a short period of time from its current inefficient position to the top class efficient performers (as the CCR-model suggests). Resources for such a drastic improvement are simply not available in poorer communities (schools). To illustrate this, consider the grade 9 pass rate for school 40 in Table 5.4. The current pass rate for school 40 is 29%. To improve to class 1, the grade 9 pass rate has to improve immediately from 29% to 100% which is very unlikely. It is much more realistic for school 40 to try and improve its grade 9 pass rate from the current 29% to the next level which is only 33.9%. It is more logical to suggest an improvement plan that can enable a school to proceed from one class to the next in a step by step manner. The intermediate targets or roadmap in Table 5.4 provides the necessary framework for such a step by step improvement. To improve from the current class 9 to class 8 the 5 outputs should be increased only marginally; for example, grade 12 pass rate from 76% to 77.1%. Once the outputs for a class 8 ranking have been achieved the next step would be to improve the outputs to the next level (level 7) as indicated in Table 5.4. This process of a step by step improvement is now repeated over time until school 40 has reached either class 1 or a class ranking where it performs satisfactory. The intermediate targets, which can be seen as a roadmap for improvement over time, therefore provide a more realistic framework in which a school can become as efficient as the other schools.

As explained in section 4.4.2 (the illustrative example), some points need to be noted.

- If the current level of an output is greater than the level of output (for the same criteria) in the next higher class, it means that the school already performs efficient in that specific criteria and improvement are only necessary in the other criteria. For example, in Table 5.4 the grade 9 pass rate in class 7 is higher (55.8%) than the pass rate in class 6 (52.4%). This means to move from class 7 to class 6, the school has already reached a satisfactory performance in terms of the grade 9 pass rate – improvements are only necessary in the other 4 output criteria.
- For smaller datasets and especially in cases where the range of data values are small (e.g. from 1 to 5 stars in the university example), the target values will also indicate which school, or schools, should be treated as a benchmark for the school under consideration. For example, in Table 5.4, if school 40 was at class level 4 and it wants to improve to class 3 it may use school 10 as a benchmark as the target values in class 3 (see Table 5.4) correspond to the data of school 10 (see Table 5.3). In the example of school 40 it is however not always true. To move from the current class 9 to class 8 the target values in class 8 do not correspond to any one particular school's data in Table 5.3. This is due to the fact that more than one school (a combination of schools) may be treated as a benchmark – a similar idea as in the CCR-model where reference sets with more than one school may exist. This situation is not seen as a problem as it only means (in practise) that a school will work towards certain targets (given by the intermediate target Table 5.4) without referencing to another specific school.

Benchmarking schools are used for target setting and if the targets are already available there is not such a great need to know which schools were in the reference or benchmarking set.

- The roadmap (Table 5.4) provides targets at a specific point in time and it is advisable to solve the models again as schools progress from one class to the next. Moving from one class to the next (especially if a number of schools improved over time) may imply a significant change in data which may result in new target values.

The DSS also provides a more user friendly visualization of the intermediate targets. A green smiling face indicates satisfactory performance, a yellow face indicates that certain improvements are necessary and a red sad face indicates that a considerable improvement is required. Table 5.5 shows the user-friendly roadmap for school 40 while Table 5.6 presents another example of a roadmap for the inefficient school 54. It can be seen that school 54 needs a considerable amount of improvements. The intermediate targets for all schools (not ranked in class 1 and 2) are presented in Appendix C.

| Class step | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
|---|---|---|---|---|---|
| 2 to 1 | ☺ | ☺ | 😐 | 😐 | 😐 |
| 3 to 2 | ☺ | ☹ | 😐 | 😐 | ☹ |
| 4 to 3 | 😐 | 😐 | ☹ | ☹ | ☹ |
| 5 to 4 | ☺ | ☹ | 😐 | ☺ | ☹ |
| 6 to 5 | 😐 | 😐 | 😐 | ☺ | 😐 |
| 7 to 6 | ☹ | 😐 | ☹ | ☹ | ☺ |
| 8 to 7 | ☹ | 😐 | 😐 | 😐 | ☹ |
| 9 to 8 | 😐 | 😐 | 😐 | 😐 | 😐 |

Table 5.5: Intermediate goal matrix for school 40

| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
|---|---|---|---|---|---|
| 1 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 |
| 2 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |
| 5 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |
| 6 | 88.6 | 40.8 | 38.7 | 70.9 | 47.0 |
| 7 | 88.0 | 35.3 | 26.2 | 63.0 | 54.0 |
| 8 | 70.0 | 32.0 | 38.0 | 58.4 | 19.0 |
| 9 | 74.0 | 29.7 | 34.5 | 47.4 | 37.0 |
| 10 | 70.1 | 25.4 | 29.5 | 45.2 | 3.6 |
| Current (11) | 46.0 | 21.2 | 14.1 | 30.4 | 3.0 |

Table 5.6: Intermediate goal matrix for school 54

In the next section a discussion and recommendation pertaining to the CCR-model and the class ranking approach are given.

## 5.4 Discussion and Recommendations

The previous section presented the results of the models implemented in the DSS. This included results from the CCR-model (utilising both input and output variables) and the class ranking model (using only output variables). The purpose of this section is to provide a more formal discussion of the results as it pertains to schools and to indicate the differences between the two techniques.

**CCR-model**

The CCR-model, which is based on both input and output variables, has been applied successfully in many cases and in many different application areas. The model may also be well suited to assess the efficiency of schools (or educational institutions) where schools are operating in a homogenous environment. In the context of the schooling system in South Africa (and probably in all developing countries) it seems however that the CCR-model did not perform very well. To motivate this, the following points (based on the results of this study) may be considered.

- There is a high probability that important input variables will always be excluded when using a model that utilises input variables. The general reason for this is that the required data is simply not available or it would be too difficult to obtain the data. For example, in this study it was clear that the number of "additional facilities" available at a school is of great importance and should be treated as a crucial input to deliver satisfactory outputs. Examples of such additional facilities may include school libraries, computer centres, music centres, sports facilities etc. These types of data were not available for this study and the only way to obtain it was to either visit each school to get the data or to request every school individually to provide the data. This was impractical and the "additional facility" input variable had to be excluded from the model.
- Generally, the input data used in models such as the CCR-model is dependent on data provided by a school. There is no real control over the quality of the data and it may be questionable. This is in contrast with outputs from a school such as pass rates which are normally checked and published widely. The problem of questionable input data provided by schools was also applicable to this study.
- South Africa is a country with a history of separate development. This means that resources were never distributed equally to all schools in the country. As part of the legacy of the old political system, schools still do not have access to equal resources. There are many so-called poor schools that simply cannot afford the same type of resources that more wealthy schools can afford. If schools do not have the same resources (inputs) it would be unfair to include resources in any model to assess schools. A traditional input/output model therefore seems to be inappropriate in this case.
- The CCR-model compares all schools relatively to each other and ignores the concept of class ranking where only schools in a specific class are compared to each other. The result of this is that a school may be evaluated (by the CCR-model) as 100% efficient while, according to the Pareto optimal principle, it may be ranked only in class 5 (for example). See for example school 11 in Table 5.2 with an efficiency rating of 100% that was classified as a class 5 school in Table 5.3. It seems therefore that in the context of a school assessment it is not appropriate to compare all schools with each other but rather analyse different classes (groups) of schools.

- Closely related to the above point is the fact that results are not consistent between the two techniques. Schools that are rated as 100% efficient by the CCR-model are not in the top classes when class ranking (using the Pareto optimal principle) is performed. For example, schools 9 and 11 are both rated as 100% efficient by the CCR-model (see Table 5.2). However, applying the Pareto optimal principle in class ranking, school 9 is classified as a class 4 school and school 11 as a class 5 school (see Table 5.3). The opposite is also true. School 3 is classified as a class 1 school (Table 5.3) while the CCR-model rated school 3 as only 95% efficient (Table 5.2).
- By producing reference sets for inefficient schools, the CCR-model is able to provide targets for inefficient schools to become as efficient as their efficient peers. These targets are aimed at improving a school immediately from its inefficient level to the same level as the top efficient schools. This target setting may work well in other applications but in a school application (especially in South Africa) the targets are senseless and of no use. The reason for this is that a school simply does not have resources to become 100% efficient within a short period of time. A recommendation from the CCR-model to improve a certain output will almost always be useless as a school do not have sufficient resources for such large and significant improvements. For example, school 54 has an efficiency rating of only 51% (see Table 5.2). This school (which is probably situated in a rural disadvantage area) will not have the necessary resources to become 100% efficient over a short, or even medium, term. Target setting from the CCR-model does not make sense in the context of South African schools. It makes more sense to provide an improvement plan that that can be followed over time and which will enable a gradual improvement. The class ranking and output only model makes provision for such an improvement plan and is discussed next.

**Class ranking and output only model**

Class ranking in this study was based on an output only model that implements a Pareto optimal principle for the different schools. This enables the comparison of schools in the same class and also provides a suggested improvement plan for schools in lower classes to gradually move from one class to the next higher class. The results, and the nature of the results, seem to be much more reliable and appropriate for a school application. To motivate this, the following can be considered.

- Schools in South Africa are heavily judged by their outputs – i.e. pass rates. The input (resources) to produce satisfactory pass rates are however not equally distributed or available. It makes therefore sense to exclude the uneven distribution of inputs and to focus only on outputs and to classify schools accordingly. The use of the Pareto optimal principle ensures that schools with the same output are ranked within the same class where they can be compared to only the other members (schools) of that specific class.
- Output variable data for schools (e.g. pass rates) are more readily available than the data for input variable. The data is also much more reliable. This implies that the output only model provides a better quality solution.
- Class ranking makes it easier to identify trends or clusters of interest. In the South African schools application, the model results clearly indicates (and confirms) that the top performing schools are clustered into urban areas while the weaker schools are located in previously disadvantaged areas.
- As mentioned earlier, the output only model and class ranking techniques also provides an improvement plan where a school in a lower class can improve to the next class and then progress gradually from one class to the next until it has reached class 1 – as opposed to the CCR-model that provides a target to move immediately to a 100% efficiency level. For example, school 40 is currently ranked as a class 9 school (see Table 5.4). It would be more appropriate (and much easier) for school 40 to try and improve to class 8 instead of becoming immediately 100% efficient.

Based on the results of this study and the discussion above it seems permissible to recommend that class rankings and the output only model should be seriously considered when schools are evaluated for efficiency. Schools in South Africa are unique and the efforts to help and improve the schooling system should be dynamic and aimed at the different requirements of different schools. The poor schools in the North-West province, seems to be focussed on survival while the wealthier schools are focussed on performance with a lot less concerns about resources. These differences make a class ranking and a gradual improvement plan the ideal tool to assist with addressing efficiency problems in schools.

## 5.5 Contributions of this study

This study has made a general contribution to model development for measuring the efficiency of schools in the North-West province of South Africa. The results and the improvement plans for individual schools that are generated by the proposed techniques may lead to an improved school management system which will hopefully have an impact on the educational sector as a whole. This means that the study has ultimately contributed to education in South Africa. More specific contributions can be summarised as follows.

- The study has shown that a class ranking approach combined with intermediate improvement plans may be more appropriate in the context of schools then the traditional DEA models.
- A DSS was developed that automate the school evaluation process and that communicate results in a user friendly manner.
- The DSS provides a centralised method to evaluate and manage schools in different geographically locations.
- Clusters of schools that may need help and specific areas of improvements where identified for secondary schools in the North-West province.

## 5.6 Conclusion

In this chapter the application of the newly developed DSS to the schools in the North-West province was described. The dataset used in the application was explained followed by a discussion of the results of both the input/output and output only models. Findings and recommendations were presented and the chapter was concluded by highlighting the contributions of the study.

# 6 SUMMARY AND CONCLUSIONS

## 6.1 Introduction

The purpose of Chapter 6 is to present the final comments and concluding remarks for the study. The original research objectives of the study are restated along with a brief description of how these objectives were achieved. Problems experienced during the study are highlighted and the chapter then concludes with an outline of new possible research opportunities.

## 6.2 Research objectives

In Chapter 1 the primary objectives of the study were formulated as the development and implementation of appropriate mathematical models in a DSS that can be used to evaluate the efficiency of schools. In order to achieve these primary objective four secondary objectives were defined. These objectives were

- Gain a good understanding of the school system in the North-West province of South Africa, as well as related studies to measure effectiveness of educational institutions.
- Provide an overview of linear programming models with specific reference to the models and techniques implemented in this study.
- Development of a DSS which implements appropriate models that can be used to assess the performance of schools.
- Apply and validate the proposed models using real data from the Department of Education in the North-West province of South Africa.

The remainder of this section presents a summary of how the objectives were achieved.

*Gain a good understanding of the school system in the North-West province of South Africa, as well as related studies to measure effectiveness of educational institutions.*

This first objective was addressed by providing an introductory overview of schools and teaching practices in South Africa and the North-West province (Chapter 2, section 2.2). First the South African national school system was briefly introduced (Chapter 2, section 2.2.1) followed by a description of the education system in the North-West province (Chapter 2, section 2.2.2). To ensure a sufficient background, related studies in the literature were presented. Prior efforts to address efficiency in schools in South Africa were presented (Chapter 2, section 2.3) as well as the use of DEA and other mathematical models in efficiency studies (Chapter 2, section 2.4). Finally, a summarised overview of decision support systems in education was presented (Chapter 2, section 2.5).

*Provide an overview of linear programming models with specific reference to the models and techniques implemented in this study*

This study utilises data envelopment models and certain adaptations of such models. A theoretical background of these types of models and techniques was presented in Chapter 3. First, a summarised description of general linear programming models was presented (Chapter 3, section 3.2) followed by a discussion on DEA models and related concepts (Chapter 3, section 3.3). Finally, an overview of a class ranking technique that stemmed from the concept of Pareto optimality and used to stratify schools into different levels of classes was presented (Chapter 3, section 3.4).

*Development of a DSS which implements appropriate models that can be used to assess the performance of schools*

The development of a DSS that implements appropriate mathematical models and techniques formed an integral part of this study. This objective was achieved by presenting an overview of concepts and methodologies of decision support systems in general (Chapter 4, section 4.2). The high level features of the newly developed DSS was described (Chapter 4, section 4.3) followed by a comprehensive illustrative example to validate the models and to demonstrate the usefulness of results obtained (Chapter 4, section 4.4).

*Apply and validate the proposed models using real data from the Department of Education in the North-West province of South Africa*

The final secondary objective was achieved by applying the newly developed DSS to a real world school data set. A synopsis of the dataset used was given (Chapter 5, section 5.2) followed by a comprehensive discussion of the application of two types of models, i.e. an input/output model and a class ranking model based on an output only model (Chapter 5, section 5.3). A final discussion and recommendations were presented to conclude the school application (Chapter 5, section 5.4).

To summarise, all the objectives set forth in Chapter 1 were successfully addressed and achieved. The study has made some significant contributions (see Chapter 5, section 5.4) and results indicated that the proposed class ranking method and the associated Pareto optimal principles proved to be more suitable than the normal DEA models used in school's efficiency studies.

## 6.3 Problems experienced

A limiting factor in this study was the lack of suitable data (especially for certain input variables of the input/output models). Due to this, certain variables (which may have had a significant effect) had to be excluded from the study. Efforts to obtain these data from the North-West Provincial Education Department were unsuccessful.

The ideal situation (which was not feasible due to a pre-determined study period) would be to implement and use the DSS over a longer period of time to determine the effect of the DSS results in practise.

## 6.4 Further research opportunities

A number of research opportunities have emerged from this study. Examples of such opportunities may include:

- The inclusion of a more comprehensive set of variables.
- The feasibility of different model formulations e.g. an input only model as opposed to an output only model.
- The inclusion of primary schools and the application of the DSS on a national level.
- The effect of the implementation of the intermediate progress plans on future model formulations.

## 6.5 Conclusion

Chapter 6 was the final chapter of this study. The research objectives formulated in Chapter 1 and how they were achieved was presented followed by some of the problems experienced during the study. The chapter concluded with a brief mention of further possible research opportunities.

# References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# APPENDICES

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# APPENDIX A SYSTEM SPECIFICATION

The DSS developed in this study has no additional specific system requirement. However, certain basic requirements have to be met in order to run the DSS. The requirements include both hardware and software requirements and are presented below. Memory does have the greatest impact – therefore more memory will give better performance.

<u>Software requirements</u>

Operating system: Windows 7 or higher.

Application: Microsoft office Excel 2010 and visual studio ultimate 2010.

Add-In: Microsoft Excel Solver Add-In

<u>Hardware requirements</u>

CPU: Intel Core i3.2.0 GHz or Higher and 32/64-bit OS

Memory: 4 GB RAM (The more the better)

Hard drive: 20 MB available on hard drive.

# APPENDIX B ORIGINAL DATA SET FROM QS STARS (2013)

Appendix B presents the complete university dataset used in the illustrative example in chapter 4, section 4.4. An extraction of this dataset was also presented in Table 4.2 in section 4.4.1.

| University | Facilities | Internationalization | Teaching | Employability | Research | Innovation | Overall rating |
|---|---|---|---|---|---|---|---|
| 1 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 2 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 3 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 4 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 7 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 8 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 9 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 10 | 5 | 5 | 5 | 5 | 4 | 5 | 5 |
| 11 | 5 | 5 | 5 | 4 | 5 | 5 | 5 |
| 12 | 5 | 5 | 4 | 5 | 4 | 5 | 5 |
| 13 | 5 | 5 | 5 | 5 | 3 | 5 | 5 |
| 14 | 5 | 5 | 5 | 4 | 5 | 4 | 5 |
| 15 | 5 | 5 | 5 | 5 | 3 | 5 | 5 |
| 16 | 5 | 5 | 5 | 5 | 3 | 5 | 4 |
| 17 | 5 | 5 | 4 | 5 | 3 | 5 | 5 |
| 18 | 5 | 5 | 4 | 4 | 5 | 4 | 5 |
| 19 | 5 | 5 | 5 | 4 | 3 | 5 | 5 |
| 20 | 5 | 5 | 5 | 2 | 5 | 5 | 4 |
| 21 | 5 | 5 | 5 | 5 | 3 | 3 | 5 |
| 22 | 5 | 3 | 5 | 5 | 3 | 5 | 4 |
| 23 | 5 | 5 | 4 | 2 | 5 | 5 | 4 |
| 24 | 5 | 5 | 5 | 3 | 3 | 5 | 4 |
| 25 | 5 | 5 | 4 | 5 | 3 | 3 | 4 |
| 26 | 4 | 4 | 4 | 3 | 5 | 5 | 4 |
| 27 | 5 | 5 | 3 | 5 | 2 | 5 | 4 |
| 28 | 5 | 5 | 4 | 3 | 3 | 5 | 3 |
| 29 | 5 | 5 | 4 | 2 | 3 | 5 | 4 |
| 30 | 5 | 4 | 5 | 5 | 3 | 2 | 3 |
| 31 | 4 | 5 | 5 | 3 | 3 | 3 | 4 |
| 32 | 5 | 4 | 3 | 5 | 3 | 3 | 3 |
| 33 | 5 | 3 | 5 | 4 | 1 | 5 | 3 |
| 34 | 5 | 4 | 5 | 3 | 1 | 5 | 3 |
| 35 | 5 | 5 | 5 | 4 | 1 | 1 | 4 |
| 36 | 4 | 5 | 1 | 3 | 3 | 4 | 4 |
| 37 | 5 | 3 | 2 | 4 | 1 | 5 | 2 |
| 38 | 4 | 5 | 5 | 3 | 1 | 1 | 3 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 39 | 4 | 3 | 2 | 3 | 2 | 5 | 3 |
| 40 | 5 | 4 | 4 | 3 | 2 | 1 | 3 |
| 41 | 5 | 4 | 3 | 2 | 1 | 3 | 2 |
| 42 | 4 | 5 | 2 | 2 | 1 | 4 | 2 |
| 43 | 4 | 5 | 2 | 2 | 1 | 4 | 2 |
| 44 | 4 | 5 | 1 | 3 | 2 | 1 | 2 |
| 45 | 4 | 2 | 3 | 3 | 2 | 1 | 2 |
| 46 | 5 | 4 | 1 | 3 | 1 | 1 | 2 |
| 47 | 3 | 1 | 1 | 4 | 2 | 3 | 2 |
| 48 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 49 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 50 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |

Full dataset used in illustrative example (section 4.4.1)

# APPENDIX C SCHOOLS INTERMEDIATE GOAL MATRIXES

The DSS provides an intermediate goal matrix for each school that was not ranked in class 1 or class 2. This intermediate goal matrix can then be used to progress from a current class to the next higher class. These results were detailed in chapter 5 and intermediate goal matrixes were presented for school 40 (see Tables 5.4 and 5.5) and school 54 (see Table 5.6). This appendix presents the efficiency improvement plans for all the other schools that were not in class 1 or class 2 and which need to improve.

## Improvement plans for schools currently ranked in class 3

| School 5 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 59.9 | 58.2 | 98.8 | 100.0 |
| 2 | 100.0 | 59.1 | 59.3 | 97.4 | 96.0 |
| 3 | 98.0 | 58.7 | 47.9 | 96.8 | 93.0 |

| School 7 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 61.7 | 61.1 | 98.6 | 100.0 |
| 2 | 99.3 | 57.2 | 59.8 | 96.5 | 95.6 |
| 3 | 97.0 | 51.1 | 59.5 | 96.0 | 94.0 |

| School 12 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 95.9 | 44.7 | 42.0 | 94.2 | 100.0 |
| 2 | 100.0 | 49.0 | 41.7 | 99.1 | 99.2 |
| 3 | 86.0 | 39.8 | 40.3 | 90.4 | 96.0 |

| School 18 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 85.0 | 30.1 | 34.0 | 78.8 | 100.0 |
| 2 | 85.0 | 30.1 | 34.0 | 78.8 | 100.0 |
| 3 | 85.0 | 30.1 | 34.0 | 78.8 | 100.0 |

# Improvement plans for schools currently ranked in class 4

| School 9 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 52.2 | 52.7 | 98.3 | 96.8 |
| 2 | 100.0 | 55.3 | 52.6 | 98.2 | 97.2 |
| 3 | 98.0 | 55.7 | 51.6 | 96.3 | 92.8 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |

| School 13 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 49.4 | 44.1 | 98.7 | 97.9 |
| 2 | 100.0 | 49.5 | 44.1 | 98.4 | 97.9 |
| 3 | 96.4 | 50.0 | 58.2 | 95.1 | 94.3 |
| 4 | 94.0 | 40.9 | 41.4 | 88.8 | 92.0 |

# Improvement plans for schools currently ranked in class 5

| School 11 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 48.6 | 49.2 | 96.1 | 92.6 |
| 2 | 100.0 | 48.6 | 49.2 | 96.1 | 92.6 |
| 3 | 98.8 | 55.1 | 50.4 | 95.8 | 91.5 |
| 4 | 95.3 | 46.3 | 47.2 | 92.3 | 88.2 |
| 5 | 95.0 | 39.9 | 46.7 | 91.2 | 88.0 |

| School 14 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 50.7 | 51.0 | 97.6 | 95.4 |
| 2 | 100.0 | 54.4 | 51.0 | 98.4 | 97.5 |
| 3 | 99.7 | 50.6 | 53.7 | 94.6 | 89.6 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 92.0 | 46.7 | 46.9 | 82.8 | 64.0 |

| School 15 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 56.7 | 55.0 | 97.9 | 96.8 |
| 2 | 100.0 | 56.7 | 55.0 | 97.9 | 96.8 |
| 3 | 98.9 | 51.7 | 54.4 | 95.2 | 91.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 88.0 | 31.4 | 48.4 | 81.2 | 81.0 |

| School 19 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 51.5 | 47.8 | 100.0 | 100.0 |
| 2 | 100.0 | 52.6 | 47.8 | 98.7 | 98.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 40.3 | 45.9 | 73.4 | 54.0 |
| 5 | 96.0 | 40.3 | 45.9 | 73.4 | 54.0 |

| School 24 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 96.5 | 54.5 | 54.9 | 94.0 | 100.0 |
| 2 | 100.0 | 55.2 | 53.4 | 97.1 | 97.3 |
| 3 | 95.8 | 51.3 | 51.8 | 94.1 | 94.4 |
| 4 | 95.5 | 47.5 | 48.5 | 93.1 | 87.4 |
| 5 | 63.0 | 46.2 | 46.6 | 68.6 | 85.0 |

| School 25 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 61.8 | 61.2 | 98.6 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |
| 5 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |

| School 31 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 53.2 | 56.7 | 95.6 | 92.4 |
| 2 | 100.0 | 53.2 | 56.5 | 95.5 | 91.8 |
| 3 | 99.3 | 52.8 | 52.1 | 95.2 | 90.5 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 90.0 | 47.8 | 42.4 | 64.0 | 31.0 |

# Improvement plans for schools currently ranked in class 6

| School 16 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 54.1 | 50.4 | 98.4 | 97.6 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 45.1 | 48.4 | 84.1 | 70.5 |
| 5 | 93.8 | 39.6 | 47.3 | 82.2 | 76.1 |
| 6 | 92.0 | 38.5 | 46.4 | 80.6 | 64.0 |

| School 17 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 56.2 | 55.4 | 96.8 | 97.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 49.1 | 50.4 | 92.7 | 83.9 |
| 5 | 94.2 | 39.4 | 47.0 | 86.7 | 82.3 |
| 6 | 87.0 | 34.6 | 43.5 | 79.6 | 76.0 |

| School 20 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 56.2 | 55.4 | 96.8 | 97.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 43.1 | 47.2 | 76.8 | 60.6 |
| 5 | 95.6 | 43.0 | 47.2 | 76.5 | 65.8 |
| 6 | 90.0 | 40.4 | 38.9 | 72.0 | 46.0 |

| School 21 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 89.6 | 35.6 | 47.9 | 81.9 | 77.2 |
| 6 | 87.0 | 34.5 | 46.5 | 70.8 | 75.0 |

| School 22 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 45.3 | 47.8 | 70.9 | 56.7 |
| 5 | 96.0 | 40.3 | 45.9 | 73.4 | 54.0 |
| 6 | 95.0 | 28.6 | 34.4 | 70.2 | 25.0 |

| School 29 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 |
| 2 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 |
| 3 | 97.3 | 53.2 | 56.3 | 96.2 | 93.7 |
| 4 | 94.4 | 42.4 | 43.1 | 89.8 | 90.9 |
| 5 | 95.0 | 39.9 | 46.7 | 91.2 | 88.0 |
| 6 | 82.0 | 25.0 | 29.9 | 66.4 | 79.0 |

| School 38 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 59.7 | 57.9 | 98.8 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 98.0 | 58.5 | 48.3 | 96.8 | 93.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 78.5 | 46.9 | 45.0 | 69.3 | 60.5 |
| 6 | 74.0 | 44.2 | 36.5 | 59.4 | 57.0 |

# Improvement plans for schools currently ranked in class 7

| School 23 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 46.6 | 37.2 | 100.0 | 100.0 |
| 2 | 100.0 | 46.6 | 37.2 | 99.9 | 99.9 |
| 3 | 96.9 | 51.0 | 59.4 | 96.0 | 94.0 |
| 4 | 94.9 | 44.6 | 45.4 | 91.2 | 89.4 |
| 5 | 95.0 | 39.9 | 46.7 | 91.2 | 88.0 |
| 6 | 86.4 | 33.4 | 41.8 | 77.9 | 76.4 |
| 7 | 57.0 | 21.0 | 25.9 | 69.4 | 68.0 |

| School 26 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 50.5 | 45.7 | 100.0 | 100.0 |
| 2 | 100.0 | 51.4 | 45.7 | 99.0 | 98.4 |
| 3 | 99.9 | 49.7 | 54.2 | 94.3 | 89.2 |
| 4 | 96.0 | 47.8 | 49.8 | 90.7 | 80.3 |
| 5 | 93.5 | 43.2 | 46.8 | 87.1 | 76.2 |
| 6 | 87.5 | 38.2 | 43.8 | 76.9 | 66.7 |
| 7 | 72.0 | 33.8 | 32.9 | 68.0 | 59.0 |

| School 27 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 56.8 | 59.3 | 100.0 | 100.0 |
| 2 | 100.0 | 57.7 | 57.4 | 97.1 | 96.5 |
| 3 | 95.1 | 49.0 | 56.1 | 94.7 | 94.4 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 86.4 | 34.8 | 47.9 | 80.9 | 80.7 |
| 6 | 87.0 | 34.6 | 44.9 | 75.7 | 75.4 |
| 7 | 63.0 | 28.9 | 39.8 | 67.2 | 67.0 |

| School 28 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 Math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 56.2 | 55.4 | 96.8 | 97.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 46.8 | 48.8 | 79.4 | 67.3 |
| 5 | 95.6 | 40.1 | 46.2 | 80.2 | 67.0 |
| 6 | 91.8 | 38.4 | 46.3 | 80.6 | 64.4 |
| 7 | 87.0 | 28.7 | 37.4 | 66.4 | 61.0 |

| School 30 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 61.8 | 61.2 | 98.6 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 97.5 | 55.2 | 53.3 | 96.4 | 93.5 |
| 4 | 94.5 | 43.0 | 43.7 | 90.2 | 90.5 |
| 5 | 95.0 | 39.9 | 46.7 | 91.2 | 88.0 |
| 6 | 84.8 | 30.3 | 37.5 | 73.7 | 77.3 |
| 7 | 72.0 | 25.9 | 19.2 | 65.8 | 69.0 |

| School 32 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 |
| 2 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 41.7 | 46.6 | 76.6 | 58.9 |
| 5 | 95.9 | 40.2 | 46.0 | 75.9 | 58.8 |
| 6 | 92.4 | 37.1 | 44.7 | 79.1 | 58.5 |
| 7 | 88.0 | 35.3 | 26.2 | 63.0 | 54.0 |

| School 35 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 48.5 | 41.4 | 100.0 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 96.0 | 40.3 | 45.9 | 73.4 | 54.0 |
| 6 | 92.8 | 35.8 | 43.1 | 77.8 | 53.5 |
| 7 | 87.0 | 33.6 | 36.0 | 60.6 | 41.0 |

| School 39 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.7 | 54.7 | 99.0 | 100.0 |
| 2 | 100.0 | 49.4 | 54.7 | 94.2 | 89.1 |
| 3 | 100.0 | 49.3 | 54.7 | 94.2 | 89.1 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 90.8 | 40.0 | 47.7 | 72.9 | 65.5 |
| 6 | 91.6 | 38.6 | 46.1 | 80.1 | 63.8 |
| 7 | 76.0 | 34.8 | 41.6 | 58.6 | 57.0 |

| School 49 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 61.8 | 61.1 | 98.6 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |
| 5 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |
| 6 | 94.2 | 31.2 | 37.5 | 72.9 | 35.2 |
| 7 | 94.0 | 31.1 | 34.1 | 47.4 | 34.0 |

# Improvement plans for schools currently ranked in class 8

| School 33 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 99.1 | 50.9 | 48.5 | 98.7 | 100.0 |
| 2 | 100.0 | 52.1 | 47.7 | 98.1 | 98.2 |
| 3 | 93.3 | 47.0 | 52.9 | 93.6 | 94.8 |
| 4 | 94.5 | 43.1 | 43.9 | 90.3 | 90.4 |
| 5 | 92.9 | 40.3 | 46.7 | 89.7 | 87.8 |
| 6 | 86.8 | 34.7 | 43.3 | 79.2 | 75.7 |
| 7 | 64.9 | 30.0 | 38.3 | 67.4 | 65.3 |
| 8 | 62.0 | 28.9 | 30.6 | 62.2 | 63.0 |

| School 34 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 47.0 | 40.8 | 98.7 | 97.5 |
| 2 | 100.0 | 47.0 | 40.8 | 98.7 | 97.5 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 44.8 | 47.5 | 68.3 | 53.4 |
| 5 | 96.0 | 44.5 | 47.3 | 68.3 | 53.1 |
| 6 | 94.0 | 31.8 | 38.3 | 73.6 | 37.8 |
| 7 | 88.3 | 29.2 | 36.8 | 62.8 | 55.9 |
| 8 | 86.0 | 21.0 | 35.1 | 61.2 | 30.0 |

| School 37 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 46.9 | 37.9 | 100.0 | 100.0 |
| 2 | 100.0 | 47.0 | 37.9 | 99.8 | 99.8 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 40.5 | 46.0 | 74.0 | 54.9 |
| 5 | 96.0 | 40.3 | 45.9 | 73.8 | 54.8 |
| 6 | 92.8 | 35.7 | 43.0 | 77.7 | 53.1 |
| 7 | 88.2 | 29.1 | 36.9 | 63.2 | 56.4 |
| 8 | 84.0 | 27.6 | 31.9 | 60.2 | 48.0 |

| School 41 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 61.8 | 61.2 | 98.6 | 100.0 |
| 2 | 100.0 | 56.3 | 54.3 | 98.0 | 96.9 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 93.0 | 40.2 | 47.7 | 73.4 | 63.3 |
| 6 | 91.3 | 38.7 | 46.0 | 79.8 | 63.7 |
| 7 | 73.6 | 33.7 | 40.0 | 61.4 | 58.9 |
| 8 | 70.0 | 32.0 | 38.0 | 58.4 | 19.0 |

| School 43 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 56.2 | 55.4 | 96.8 | 97.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |
| 6 | 93.8 | 32.7 | 39.3 | 74.5 | 41.0 |
| 7 | 90.0 | 31.3 | 35.0 | 56.8 | 44.3 |
| 8 | 89.0 | 31.0 | 34.6 | 56.2 | 36.0 |

| School 46 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 61.8 | 61.2 | 98.6 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 47.8 | 49.8 | 90.6 | 80.2 |
| 5 | 95.2 | 40.0 | 46.5 | 86.8 | 79.6 |
| 6 | 88.0 | 35.4 | 44.0 | 79.8 | 73.6 |
| 7 | 77.9 | 27.6 | 30.0 | 66.2 | 65.1 |
| 8 | 73.0 | 23.6 | 28.1 | 54.6 | 61.0 |

| School 47 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 61.8 | 61.1 | 98.6 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 98.8 | 50.0 | 56.6 | 94.9 | 91.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 88.8 | 32.7 | 48.3 | 79.9 | 78.3 |
| 6 | 87.0 | 34.5 | 46.5 | 70.8 | 75.0 |
| 7 | 76.0 | 34.8 | 41.6 | 58.6 | 57.0 |
| 8 | 66.0 | 25.6 | 37.8 | 53.2 | 39.0 |

# Improvement plans for schools currently ranked in class 9

| School 36 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 51.5 | 56.1 | 95.9 | 92.2 |
| 2 | 100.0 | 54.4 | 57.1 | 95.9 | 92.7 |
| 3 | 99.3 | 52.7 | 52.1 | 95.2 | 90.5 |
| 4 | 96.0 | 48.4 | 50.1 | 92.0 | 82.4 |
| 5 | 95.0 | 39.9 | 46.7 | 91.1 | 87.8 |
| 6 | 88.9 | 36.0 | 44.5 | 80.0 | 71.5 |
| 7 | 71.0 | 30.9 | 32.2 | 68.1 | 60.9 |
| 8 | 64.8 | 28.0 | 31.1 | 62.1 | 59.2 |
| 9 | 63.0 | 17.4 | 24.5 | 60.4 | 54.0 |

| School 40 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 45.8 | 48.2 | 74.0 | 60.5 |
| 5 | 96.0 | 40.3 | 45.9 | 74.0 | 55.1 |
| 6 | 92.9 | 35.6 | 42.8 | 77.5 | 52.4 |
| 7 | 82.7 | 31.7 | 37.4 | 63.8 | 55.8 |
| 8 | 77.1 | 29.5 | 34.9 | 59.4 | 33.9 |
| 9 | 76.0 | 29.1 | 34.4 | 58.6 | 29.0 |

| School 42 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 42.6 | 47.1 | 78.7 | 62.1 |
| 5 | 95.7 | 40.2 | 46.1 | 78.5 | 63.8 |
| 6 | 92.5 | 36.9 | 44.4 | 78.9 | 57.7 |
| 7 | 79.7 | 31.8 | 36.6 | 65.3 | 59.1 |
| 8 | 73.1 | 29.1 | 33.6 | 59.9 | 43.4 |
| 9 | 69.0 | 27.5 | 31.7 | 56.6 | 41.0 |

| School 44 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 57.0 | 59.7 | 100.0 | 100.0 |
| 2 | 100.0 | 56.2 | 55.4 | 96.8 | 97.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |
| 6 | 93.4 | 34.0 | 40.9 | 75.9 | 46.3 |
| 7 | 88.2 | 32.1 | 35.0 | 60.4 | 46.6 |
| 8 | 83.9 | 30.5 | 34.2 | 57.4 | 38.8 |
| 9 | 80.0 | 29.1 | 31.8 | 54.8 | 37.0 |

| School 45 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 52.3 | 49.6 | 100.0 | 100.0 |
| 2 | 100.0 | 53.2 | 48.9 | 98.6 | 97.9 |
| 3 | 98.0 | 58.7 | 48.0 | 96.8 | 93.0 |
| 4 | 96.0 | 49.3 | 50.5 | 94.3 | 86.1 |
| 5 | 94.7 | 40.5 | 46.7 | 90.5 | 85.9 |
| 6 | 88.2 | 35.7 | 44.2 | 79.7 | 72.7 |
| 7 | 67.2 | 30.4 | 33.7 | 68.0 | 62.6 |
| 8 | 63.3 | 28.5 | 30.8 | 62.1 | 61.2 |
| 9 | 52.0 | 24.5 | 27.2 | 54.8 | 50.0 |

| School 50 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 61.8 | 61.2 | 98.6 | 100.0 |
| 2 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 96.0 | 40.3 | 45.9 | 73.4 | 54.0 |
| 6 | 92.4 | 37.1 | 44.7 | 79.1 | 58.5 |
| 7 | 83.0 | 33.3 | 38.7 | 54.4 | 48.2 |
| 8 | 75.2 | 30.2 | 35.0 | 57.8 | 37.6 |
| 9 | 74.0 | 29.7 | 34.5 | 47.4 | 37.0 |

| School 52 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12<br>pass rate | Grade 12<br>math average | Grade 12<br>science average | School<br>pass rate | Grade 9<br>pass rate |
| 1 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 |
| 2 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 96.0 | 40.3 | 45.9 | 73.4 | 54.0 |
| 6 | 95.0 | 28.6 | 34.4 | 70.2 | 25.0 |
| 7 | 94.0 | 31.1 | 34.1 | 47.4 | 34.0 |
| 8 | 89.0 | 31.0 | 34.6 | 56.2 | 36.0 |
| 9 | 86.0 | 20.4 | 30.3 | 42.6 | 2.0 |

# Improvement plans for schools currently ranked in class 10

| School 48 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 61.8 | 61.2 | 98.6 | 100.0 |
| 2 | 100.0 | 59.1 | 59.4 | 97.4 | 96.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 45.2 | 47.8 | 70.7 | 56.3 |
| 5 | 96.0 | 42.5 | 46.7 | 70.7 | 53.5 |
| 6 | 94.3 | 30.9 | 37.2 | 72.6 | 34.1 |
| 7 | 87.7 | 29.0 | 37.1 | 64.5 | 58.4 |
| 8 | 82.2 | 26.8 | 32.2 | 60.5 | 47.0 |
| 9 | 77.6 | 29.1 | 33.4 | 57.1 | 32.1 |
| 10 | 72.0 | 23.5 | 25.6 | 53.0 | 26.0 |

| School 51 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 49.1 | 42.6 | 100.0 | 100.0 |
| 2 | 100.0 | 47.3 | 42.6 | 98.1 | 96.4 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 96.0 | 40.3 | 45.9 | 73.4 | 54.0 |
| 6 | 93.3 | 34.1 | 41.1 | 76.0 | 46.7 |
| 7 | 87.0 | 31.8 | 37.1 | 55.4 | 46.7 |
| 8 | 83.5 | 31.3 | 35.6 | 56.8 | 31.1 |
| 9 | 78.0 | 28.5 | 33.2 | 56.2 | 29.8 |
| 10 | 70.0 | 25.6 | 29.8 | 44.6 | 2.0 |

| School 53 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 61.8 | 61.2 | 98.6 | 100.0 |
| 2 | 100.0 | 47.8 | 39.2 | 99.7 | 99.6 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 49.4 | 50.6 | 94.4 | 86.0 |
| 5 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |
| 6 | 94.4 | 30.7 | 37.0 | 72.4 | 33.4 |
| 7 | 90.9 | 30.0 | 35.6 | 55.9 | 46.1 |
| 8 | 88.5 | 31.0 | 34.7 | 56.3 | 35.6 |
| 9 | 81.3 | 25.7 | 31.9 | 51.0 | 21.4 |
| 10 | 74.0 | 23.4 | 29.0 | 34.6 | 2.0 |

# Improvement plans for schools currently ranked in class 11

| School 54 | | | | | |
|---|---|---|---|---|---|
| Class level | Grade 12 pass rate | Grade 12 math average | Grade 12 science average | School pass rate | Grade 9 pass rate |
| 1 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 |
| 2 | 100.0 | 46.3 | 36.7 | 100.0 | 100.0 |
| 3 | 100.0 | 49.2 | 54.6 | 94.2 | 89.0 |
| 4 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |
| 5 | 96.0 | 44.7 | 47.4 | 68.0 | 53.0 |
| 6 | 88.6 | 40.8 | 38.7 | 70.9 | 47.0 |
| 7 | 88.0 | 35.3 | 26.2 | 63.0 | 54.0 |
| 8 | 70.0 | 32.0 | 38.0 | 58.4 | 19.0 |
| 9 | 74.0 | 29.7 | 34.5 | 47.4 | 37.0 |
| 10 | 70.1 | 25.4 | 29.5 | 45.2 | 3.6 |
| 11 | 46.0 | 21.2 | 14.1 | 30.4 | 3.0 |

# APPENDIX D EXAMPLES OF CODE FOR CORE FUNCTIONS IN THE DSS

Chapter 4 detailed the DSS developed in this study. The purpose of this appendix is to present a few selected examples of code for the core functions of the DSS. The complete system is not included with the dissertation as it needs human interaction to run. The complete system is available from the author.

## Code for data capturing and filtering

```
using System;
using System.Collections.Generic;
using System.ComponentModel;
using System.Data;
using System.Drawing;
using System.Linq;
using System.Text;
using System.Windows.Forms;
using Excel = Microsoft.Office.Interop.Excel;
using System.Runtime.InteropServices;
using Office = Microsoft.Office.Core;
using System.IO;

namespace WindowsFormsApplication1
{
    public partial class Form1 : Form
    {
//      private int schoolIdx;
        private ClassSchool[] school = new ClassSchool[256];

        public Form1()
        {
            InitializeComponent();
            school = new ClassSchool[256];
        }

        private void button1_Click(object sender, EventArgs e)
        {
            OpenExcelFile();
        }

        public void newSchool(int schoolIdx)
        {
            school[schoolIdx] = new ClassSchool();
        }

        private void OpenExcelFile()
        {

// Unfiltered data files

string workbookPath = "C:/MSc-projek/School.xlsx";

            Excel.Application objApp = new Excel.ApplicationClass();
```

```
            Excel.Workbook objBooks =
objApp.Workbooks.Open(workbookPath, 0, false, 5, "", "", false,
Excel.XlPlatform.xlWindows, "", true, false, 0, true, false, false);
            objApp.Visible = true;
            Excel.Sheets excelSheets = objBooks.Worksheets;
            Excel.Worksheet excelWorksheet;
            excelWorksheet = (Excel.Worksheet)excelSheets.get_Item(1);

            Excel.Worksheet excelWorksheet2;
            excelWorksheet2 =
(Excel.Worksheet)excelSheets.get_Item(2);

            Excel.Worksheet excelWorksheet4;
            excelWorksheet4 =
(Excel.Worksheet)excelSheets.get_Item(4);

            Excel.Worksheet excelWorksheet7;
            excelWorksheet7 =
(Excel.Worksheet)excelSheets.get_Item(7);

            Excel.Worksheet excelWorksheet10;
            excelWorksheet10 =
(Excel.Worksheet)excelSheets.get_Item(10);

            Excel.Worksheet excelWorksheet11;
            excelWorksheet11 =
(Excel.Worksheet)excelSheets.get_Item(11);

            Excel.Worksheet excelWorksheet13;
            excelWorksheet13 =
(Excel.Worksheet)excelSheets.get_Item(13);

            Excel.Worksheet excelWorksheet14;
            excelWorksheet14 =
(Excel.Worksheet)excelSheets.get_Item(14);

            Excel.Worksheet excelWorksheet18;
            excelWorksheet18 =
(Excel.Worksheet)excelSheets.get_Item(18);

            string[] s9 = new string[15];
            string[] s10 = new string[15];
            int[] i9 = new int[15];
            int[] i10 = new int[15];

            String s0, s1, s2, s3, s4, s5, s6, sNo, s7;
            int i1, i2, i7 = 0;
            double d1, d2 = 0;
            excelWorksheet.Cells[1, 1]  =
((Microsoft.Office.Interop.Excel.Range)excelWorksheet2.Cells[1,
1]).Value2.ToString();
            excelWorksheet.Cells[1, 2]  =
((Microsoft.Office.Interop.Excel.Range)excelWorksheet2.Cells[1,
2]).Value2.ToString();
            excelWorksheet.Cells[1, 3]  = " Input 1. Number of
Educators  (Col 3)";
            excelWorksheet.Cells[1, 4]  = " Input 2. Number of
Subjects  (Col 4)";
            excelWorksheet.Cells[1, 5]  = "          % Edu. Present
Col 5";
```

```
            excelWorksheet.Cells[1, 6]  = "          % Learners
Present Col 6";
            excelWorksheet.Cells[1, 7]  = " Input 3.  Learner/Educator
Ratio (Col 7)";
            excelWorksheet.Cells[1, 8]  = " Input 4.  Hons.and higher
% (Col 8)";
            excelWorksheet.Cells[1, 9]  = "           Library (No of
books etc. (Col 9))";
            excelWorksheet.Cells[1, 10] = " Input 5.  Total cost per
learner (Col 10)";
            excelWorksheet.Cells[1, 11] = "           Cost per learner
(Col 11)";
            excelWorksheet.Cells[1, 12] = "           Subsidized cost
per learner (Col 12)";
            excelWorksheet.Cells[1, 13] = " Output 2. Grade 12 Pass
rate (Col 13)";
            excelWorksheet.Cells[1, 14] = " Output 4. Grade 12 Math
Average % (Col 14)";
            excelWorksheet.Cells[1, 15] = " Output 5. Grade 12 PHSC
Average % (Col 15)";
            excelWorksheet.Cells[1, 16] = " Output 1. School pass rate
(Col 16)";
            excelWorksheet.Cells[1, 17] = " Output 3. Grade 9 Pass
rate (Col 17)";
            excelWorksheet.Cells[1, 18] = " Output 6. Grade 9 Math
Pass rate % (Col 18)";
            excelWorksheet.Cells[1, 19] = " Output 7. Grade 12 Math
Lit. Average % (Col 19)";
            excelWorksheet.Cells[1, 20] = " Output 8. Grade 12 Math
Lit. Average % (Col 19)";

            excelWorksheet.Cells[1, 23] = "23. Honours, Masters and
Doctors";
            excelWorksheet.Cells[1, 24] = "24. All certificates";
            excelWorksheet.Cells[1, 25] = "25. Grade 9 Math pupils ?
(> Col 18)";
            excelWorksheet.Cells[1, 25] = "Grade 9 Math no passed ? (>
Col 18)";

            excelWorksheet.Name = "Page1";

            int schoolIdx;
//
            for (int i = 2; i < 257; i++)
            {
                schoolIdx = i - 2;
                school[schoolIdx] = new ClassSchool();

                //  excelWorksheet.Cells[i, 1] =
((Microsoft.Office.Interop.Excel.Range)excelWorksheet2.Cells[i,
1]).Value2.ToString();
                //  excelWorksheet.Cells[i, 2] =
((Microsoft.Office.Interop.Excel.Range)excelWorksheet2.Cells[i,
2]).Value2.ToString();

                s0 = ((Excel.Range)excelWorksheet2.Cells[i,
1]).Value2.ToString();
                s1 = ((Excel.Range)excelWorksheet2.Cells[i,
2]).Value2.ToString();
```

```
                s2 = ((Excel.Range)excelWorksheet2.Cells[i,
21]).Value2.ToString();
                s3 = ((Excel.Range)excelWorksheet2.Cells[i,
22]).Value2.ToString();

                s6 = ((Excel.Range)excelWorksheet2.Cells[i,
20]).Value2.ToString();

//sNo = ((Excel.Range)excelWorksheet7.Cells[i, 20]).Value2.ToString();

                school[schoolIdx].setSchoolId(s0);
                school[schoolIdx].setSchoolName(s1);

                i1 = 0;
                int.TryParse(s2, out i1);
                i2 = 0;
                int.TryParse(s3, out i2);
                school[schoolIdx].setNoOfPersonnel(i1, i2);
                excelWorksheet.Cells[i, 1] =
school[schoolIdx].getSchoolId();
                excelWorksheet.Cells[i, 2] =
school[schoolIdx].getSchoolName();
                excelWorksheet.Cells[i, 3] =
school[schoolIdx].getNoOfPersonnel();

                d2 = i1 + i2;
                i1 = 0;
                int.TryParse(s6, out i1);
                d1 = i1;

                d1 = d1 / d2;
                excelWorksheet.Cells[i, 7] = "" + d1;

                for (int k = 0; k < 15; k++)
                {
                    if (((Excel.Range)excelWorksheet2.Cells[i, k +
5]).Value2 != null)
                    {
                        s9[k] = ((Excel.Range)excelWorksheet2.Cells[i,
k + 5]).Value2.ToString();
//MessageBox.Show("" + ((Excel.Range)excelWorksheet2.Cells[i, k +
6]).Value2.ToString());
                    }
                    else s9[k] = "0";

                    int.TryParse(s9[k], out i9[k]);
                    school[schoolIdx].setNoInGrade(i9[k], k);
                }
            }
// 20140213
            // Gr9 Math Sheet13 col 18 (?), 19 (HL), 20 (FAL)
            int iGr9MathE, iGr9MathF;
            string sGr9MathE = "", sGr9MathF = "";
            for (int j = 3; j < 1510; j++)
            {
             if (((Excel.Range)excelWorksheet13.Cells[j, 1]).Value2 !=
null)
                {
                    sNo = ((Excel.Range)excelWorksheet13.Cells[j,
1]).Value2.ToString();
                    schoolIdx = 0;
```

```
                    for (int i = 2; i < 257; i++)
                    {
                        schoolIdx = i - 2;
                        if (school[schoolIdx].getSchoolId() == sNo)
                        {
                            sGr9MathE =
((Excel.Range)excelWorksheet13.Cells[j, 18]).Value2.ToString();
                            int.TryParse(sGr9MathE, out iGr9MathE);
                            school[schoolIdx].setGr9MathE(iGr9MathE);

                            sGr9MathF =
((Excel.Range)excelWorksheet13.Cells[j, 20]).Value2.ToString();
                            int.TryParse(sGr9MathF, out iGr9MathF);
                            school[schoolIdx].setGr9MathF(iGr9MathF);
                        }
                    }
                }
            }
//=================================================================
//================  Skool slaag syfer ?? ========================

            int iGrEnterPos, iGrWrotePos, iGrNotPromotePos,
iGrPromotePos, iGrEnter, iGrWrote, iGrNotPromote, iGrPromote;
            string sGrEnter = "", sGrWrote = "", sGrNotPromote = "",
sGrPromote = "";
            for (int j = 3; j < 1281; j++)
            {
                if (((Excel.Range)excelWorksheet14.Cells[j, 1]).Value2
!= null)
                {
                    sNo = ((Excel.Range)excelWorksheet14.Cells[j,
1]).Value2.ToString();
// excelWorksheet4.Cells[j, 5] = "" + sCentre;

                    schoolIdx = 0;
                    for (int i = 2; i < 257; i++)
                    {
                        schoolIdx = i - 2;
                        if (school[schoolIdx].getSchoolId() == sNo)
                        {
 //excelWorksheet4.Cells[j, 6] = "2 " + chool[schoolIdx].getSchoolId()
+ "=" + "3 " + sNo;
                            for (int iGr = 0; iGr < 12; iGr++)
                            {
                                 iGrEnterPos = iGr * 4 + 2;
                                 iGrWrotePos = iGr * 4 + 3;
                                 iGrNotPromotePos = iGr * 4 + 4;
                                 iGrPromotePos = iGr * 4 + 5;

                                 if
                                 (((Excel.Range)excelWorksheet14.Cells
                                 [j, iGrEnterPos]).Value2 != null)
                                  {
                                    sGrEnter =
                              ((Excel.Range)excelWorksheet14.Cells[j,
                              iGrEnterPos]).Value2.ToString();
//excelWorksheet4.Cells[j, 8] = "1 " + sGrEnter;

                              int.TryParse(sGrEnter, out iGrEnter);
//excelWorksheet4.Cells[j, 12] = "2 " + iGrEnter;
```

```
school[schoolIdx].setGrE(iGrEnter, iGr);
//excelWorksheet4.Cells[i, iGrEnterPos] =
school[schoolIdx].getGrE(iGr);
                                                  }
if (((Excel.Range)excelWorksheet14.Cells[j, iGrWrotePos]).Value2 !=
null)
                         {
sGrWrote = ((Excel.Range)excelWorksheet14.Cells[j,
            iGrWrotePos]).Value2.ToString();
int.TryParse(sGrWrote, out iGrWrote);
school[schoolIdx].setGrW(iGrWrote, iGr);
                          }
if (((Excel.Range)excelWorksheet14.Cells[j, iGrNotPromotePos]).Value2
!= null)
                          {
sGrNotPromote = ((Excel.Range)excelWorksheet14.Cells[j,
iGrNotPromotePos]).Value2.ToString();

int.TryParse(sGrNotPromote, out iGrNotPromote);

school[schoolIdx].setGrNotP(iGrNotPromote, iGr);
 //
excelWorksheet4.Cells[i, iGrNotPromotePos] =
school[schoolIdx].getGrNotP(iGr);

                           }
if (((Excel.Range)excelWorksheet14.Cells[j, iGrPromotePos]).Value2 !=
null)
                           {
sGrPromote = ((Excel.Range)excelWorksheet14.Cells[j,
iGrPromotePos]).Value2.ToString();
int.TryParse(sGrPromote, out iGrPromote);
school[schoolIdx].setGrP(iGrPromote, iGr);
                           }
                             }
                         }
                     }
                 }
             }
//============================  Kwalifikasies Hons, MSc en DSc

                       for (int j = 3; j < 9596; j++)
                         {
s7  = ((Excel.Range)excelWorksheet7.Cells[j, 4]).Value2.ToString();
sNo = ((Excel.Range)excelWorksheet7.Cells[j, 1]).Value2.ToString();

                            i7 = 0;
                            int.TryParse(s7, out i7);
//                  MessageBox.Show("i " + s7);
                            for (int i = 2; i < 257; i++)
                            {
                               schoolIdx = i - 2;
                           if (school[schoolIdx].getSchoolId() == sNo)
                                {
                                    switch (i7)
                                    {
                                        case 19: case 20: case 21:
                                        {

                                    school[schoolIdx].setNoOfQual(0);
```

```
                                            break;
                                        }
                                    default:
                                        {

                                school[schoolIdx].setNoOfQual(1);
                                            break;
                                        }
                                    }
                                }
                            }
                        }

                        for (int i = 2; i < 257; i++)
                        {
                            schoolIdx = i - 2;
                            excelWorksheet.Cells[i, 8] = 100 *
school[schoolIdx].getQualRatio();

                            excelWorksheet.Cells[i, 18] =
school[schoolIdx].getGr9MathPerc();
                            excelWorksheet.Cells[i, 25] =
school[schoolIdx].getGr9MathE();
                            excelWorksheet.Cells[i, 26] =
school[schoolIdx].getGr9MathF();

                            excelWorksheet.Cells[i, 23] =
school[schoolIdx].getNoOfQual(0);
                            excelWorksheet.Cells[i, 24] =
school[schoolIdx].getNoOfQual(1) + school[schoolIdx].getNoOfQual(0);
                        }
//excelWorksheet.Cells[1, 11] = "11. Cost per learner";

            string sIdx;
            int ii = 0, jj = 0, kk = 0;
            for (int j = 3; j < 369; j++)
            {
                ii = 0;
                sNo = ((Excel.Range)excelWorksheet10.Cells[j,
1]).Value2.ToString();
                sIdx = ((Excel.Range)excelWorksheet10.Cells[j,
3]).Value2.ToString();
                int.TryParse(sIdx, out ii);
                ii++;
                s10[ii] = ((Excel.Range)excelWorksheet10.Cells[j,
4]).Value2.ToString();
                int.TryParse(s10[ii], out i10[ii]);

                for (int i = 2; i < 257; i++)
                {
                    schoolIdx = i - 2;
                    if (school[schoolIdx].getSchoolId() == sNo)
                    {
                     school[schoolIdx].setCostPerGrade(i10[ii], ii);
                    }
                }
            }
            for (int i = 2; i < 257; i++)
            {
                schoolIdx = i - 2;
                school[schoolIdx].setLearnerCost();
```

```
                excelWorksheet.Cells[i, 11] =
school[schoolIdx].getLearnerCost();
            }
            for (int j = 3; j < 1615; j++)
            {
                sNo = ((Excel.Range)excelWorksheet10.Cells[j,
6]).Value2.ToString();
                sIdx = ((Excel.Range)excelWorksheet10.Cells[j,
16]).Value2.ToString();
                int.TryParse(sIdx, out ii);

                for (int i = 2; i < 257; i++)
                {
                    schoolIdx = i - 2;
                    if (school[schoolIdx].getSchoolId() == sNo)
                    {
                        school[schoolIdx].setQuintile(ii);
                        excelWorksheet.Cells[i, 12] =
school[schoolIdx].getQuintileCost();
                    }
                }
            }
// excelWorksheet.Cells[1, 12] = "12. Subsidized cost per learner";

            for (int i = 2; i < 257; i++)
            {
                schoolIdx = i - 2;
// excelWorksheet.Cells[i, 10] = school[schoolIdx].getQuintileCost();
excelWorksheet.Cells[i, 10] = school[schoolIdx].getTotalFees();
            }
//=========== Skool laagste en hoogste standerd
excelWorksheet.Cells[1, 21] = "21. Schools lowest grade";
excelWorksheet.Cells[1, 22] = "22. Schools higest grade";

            string sLgrade, sHgrade;
            int iL, iH;
            for (int j = 3; j < 304; j++)
            {
sNo = ((Excel.Range)excelWorksheet18.Cells[j, 1]).Value2.ToString();
                for (int i = 2; i < 257; i++)
                {
                    schoolIdx = i - 2;
                    if (school[schoolIdx].getSchoolId() == sNo)
                    {
                        sLgrade =
((Excel.Range)excelWorksheet18.Cells[j, 35]).Value2.ToString();
                        int.TryParse(sLgrade, out iL);
                        sHgrade =
((Excel.Range)excelWorksheet18.Cells[j, 36]).Value2.ToString();
                        int.TryParse(sHgrade, out iH);

                        school[schoolIdx].setLowestGrade(iL);
                        school[schoolIdx].setHighestGrade(iH);
                        excelWorksheet.Cells[i, 21] =
school[schoolIdx].getLowestGrade();
                        excelWorksheet.Cells[i, 22] =
school[schoolIdx].getHighestGrade();
                    }
                }
            }
// ===== Graad 12 slaag syfer
```

```
                string sEnter, sWrote, sPass, sCentre;
//                excelWorksheet.Cells[1, 13] = "13. Grade 12 Pass rate";
                for (int j = 2; j < 380; j++)
                {
                  if (((Excel.Range)excelWorksheet11.Cells[j, 12]).Value2
                  != null)
                    {
                        sNo = ((Excel.Range)excelWorksheet11.Cells[j,
                        12]).Value2.ToString();
                        sCentre = ((Excel.Range)excelWorksheet11.Cells[j,
                        6]).Value2.ToString();
//MessageBox.Show(sNo);
                        sEnter = ((Excel.Range)excelWorksheet11.Cells[j,
                        8]).Value2.ToString();
                        int.TryParse(sEnter, out kk);
                        sWrote = ((Excel.Range)excelWorksheet11.Cells[j,
                        9]).Value2.ToString();
                        int.TryParse(sWrote, out ii);
//MessageBox.Show(ii);

                        sPass = ((Excel.Range)excelWorksheet11.Cells[j,
                        10]).Value2.ToString();
                        int.TryParse(sPass, out jj);
//MessageBox.Show(jj);
                         for (int i = 2; i < 257; i++)
                         {
                              schoolIdx = i - 2;
                              if (school[schoolIdx].getSchoolId() == sNo)
                              {
                                  school[schoolIdx].setCentreNo(sCentre);

                                  school[schoolIdx].setWroteExam(ii);
                                  school[schoolIdx].setGrW(ii, 12);

                                  school[schoolIdx].setAchieveExam(jj);
                                  school[schoolIdx].setGrP(jj, 12);

                                  school[schoolIdx].setGrE(kk, 12);

                                  excelWorksheet.Cells[i, 13] =
                                  school[schoolIdx].getGr12PassPerc();
                                  // 2014/02/11
                                  excelWorksheet.Cells[i, 16] =
                                  school[schoolIdx].getHighSchoolPassPerc();

                                  excelWorksheet.Cells[i, 17] =
                                  school[schoolIdx].getGr9PassPerc();
// school[schoolIdx].setGrE(iGrEnter, 12);
// sGrWrote = ((Excel.Range)excelWorksheet14.Cells[j,
iGrWrotePos]).Value2.ToString();
// int.TryParse(sGrWrote, out iGrWrote);
// school[schoolIdx].setGrW(iGrWrote, iGr);
// sGrPromote = ((Excel.Range)excelWorksheet14.Cells[j,
iGrPromotePos]).Value2.ToString();
// int.TryParse(sGrPromote, out iGrPromote);
// school[schoolIdx].setGrP(iGrPromote, iGr);
                              }
                         }
                    }
                    else sNo = "";
                }
```

```
//excelWorksheet.Cells[1, 14] = "14. Math Pass Rate";
//excelWorksheet.Cells[1, 4] = " 4. Number of subjects";
//excelWorksheet.Cells[1, 15] = "15. PHSC Pass Rate";
            string sSubject = "", sMathAve = "", sSciAve = "",
            sMathLitAve = "", sLifeAve = "";
            double dMathAverage, dSciAverage, dMathLitAverage,
            dLifeAverage;
            for (int j = 2; j < 4785; j++)
            {
              if (((Excel.Range)excelWorksheet13.Cells[j, 37]).Value2
              != null)
                {
                  sCentre = ((Excel.Range)excelWorksheet13.Cells[j,
                  37]).Value2.ToString();
                   sSubject = ((Excel.Range)excelWorksheet13.Cells[j,
                   33]).Value2.ToString();

                    for (int i = 2; i < 257; i++)
                    {
                     schoolIdx = i - 2;
                     if (school[schoolIdx].getCentreNo() == sCentre)
                        {
                            school[schoolIdx].setNoOfSubjects();
                            if (sSubject == "MATH")
                            {
                              excelWorksheet13.Cells[j, 38] =
                              school[schoolIdx].getSchoolId();
                              sMathAve =
                              ((Excel.Range)excelWorksheet13.Cells[j,
                              36]).Value2.ToString();
                              double.TryParse(sMathAve, out
                              dMathAverage);
//school[schoolIdx].setMathPassRate(dMathAverage);
school[schoolIdx].setMathAvv(dMathAverage);
//excelWorksheet.Cells[i, 14] = school[schoolIdx].getMathPassRate();
excelWorksheet.Cells[i, 14] = school[schoolIdx].getMathAvv();

                            }
                            if (sSubject == "PHSC")
                          {
                          excelWorksheet13.Cells[j, 39] =
                          school[schoolIdx].getSchoolId();
                          sSciAve =
                          ((Excel.Range)excelWorksheet13.Cells[j,
                          36]).Value2.ToString();
                          double.TryParse(sSciAve, out dSciAverage);
                          school[schoolIdx].setSciAvv(dSciAverage);
                          excelWorksheet.Cells[i, 15] =
                          school[schoolIdx].getSciAvv();
                          }
                          if (sSubject == "MLIT")
                          {
                          excelWorksheet13.Cells[j, 39] =
                          school[schoolIdx].getSchoolId();
                          sMathLitAve =
                          ((Excel.Range)excelWorksheet13.Cells[j,
                          36]).Value2.ToString();
                          double.TryParse(sMathLitAve, out
                          dMathLitAverage);
```

```
                                school[schoolIdx].setMathLitAvv(dMathLitAver
                                age);
                                excelWorksheet.Cells[i, 19] =
                                school[schoolIdx].getMathLitAvv();
                                }
                                  if (sSubject == "LIFE")
                                {
                                excelWorksheet13.Cells[j, 39] =
                                school[schoolIdx].getSchoolId();
                                sLifeAve =
                                ((Excel.Range)excelWorksheet13.Cells[j,
                                36]).Value2.ToString();
                                double.TryParse(sLifeAve, out dLifeAverage);

                                school[schoolIdx].setLifeAvv(dLifeAverage);
                                excelWorksheet.Cells[i, 20] =
                                school[schoolIdx].getLiveAvv();
                                }
                                excelWorksheet.Cells[i, 4] =
                                school[schoolIdx].getNoOfSubjects();
                              }
                          }

                      }
//else excelWorksheet13.Cells[j, 31] = excelWorksheet13.Cells[j - 1,
31];
                 }
            excelWorksheet4.Cells[1, 1] = "Test";
            for (int i = 2; i < 257; i++)
          {
                schoolIdx = i - 2;
                excelWorksheet4.Cells[i, 1] =
                school[schoolIdx].getSchoolId();
                excelWorksheet4.Cells[i, 2] =
                school[schoolIdx].getSchoolName();
                excelWorksheet4.Cells[i, 3] =
                school[schoolIdx].getNoOfPersonnel();
                excelWorksheet4.Cells[i, 4] =
                school[schoolIdx].getNoOfSubjects();
              for (int iGr = 0; iGr < 13; iGr++)
              {
                  iGrEnterPos = iGr * 4 + 5;
                  iGrWrotePos = iGr * 4 + 6;
                  iGrNotPromotePos = iGr * 4 + 7;
                  iGrPromotePos = iGr * 4 + 8;
                  excelWorksheet4.Cells[i, iGrEnterPos] =
                  school[schoolIdx].getGrE(iGr);
                  excelWorksheet4.Cells[i, iGrWrotePos] =
                  school[schoolIdx].getGrW(iGr);
                  excelWorksheet4.Cells[i, iGrNotPromotePos] =
                  school[schoolIdx].getGrNotP(iGr);
                  excelWorksheet4.Cells[i, iGrPromotePos] =
                  school[schoolIdx].getGrP(iGr);
              }
          }
      }
}
```

# Code to solve the CCR Model for each school

```
Sub M1S54V5W5EffS1()
'
' M1S54V5W5EffS1 Macro
'
For Row = 16 To 69
    For Col = 3 To 7
        Cells(15, Col) = Cells(Row, Col)
        Cells(70, Col + 5) = Cells(Row, Col + 5) * (-1)
    Next Col

 SolverOK SetCell:="$P$15", MaxMinVal:=1, Valueof:=0, ByChange:="$C$83:$L$83", Engine:=2

    SolverLPOptions MaxTime:=100, Iterations:=100, Precision:=0.000001, _
    StepThru:=False, Scaling:=False, AssumeNonNeg:=True
    SolverSolve True
    SolverFinish KeepFinal:=1

    If (Cells(Row, 15) < "1") Then Cells(Row, 15).Interior.Color = 65535
    If (Cells(Row, 15) >= "1") Then Cells(Row, 15).Interior.Color = 5296274
    Cells(Row, 15) = Cells(15, 16)
    Cells(Row, 18) = Cells(15, 16)

    Cells(7, 12) = Row

  Next Row
End Sub
```

# Class ranking:

## example code to generate one specific class group of schools

```
Sub M1OOV5C59_Button1_Click()
' M1OOV5C59 Macro
iPos = 0
iEnd = 69
For Row = 16 To iEnd

   For Col = 3 To 7
      Cells(15, Col) = Cells(Row, Col)
   Next Col

   SolverOk SetCell:="$K$15", MaxMinVal:=1, ValueOf:=0, ByChange:="$C$77:$G$77", Engine:=2
   SolverLPOptions MaxTime:=100, Iterations:=100, Precision:=0.000001, _
   StepThru:=False, Scaling:=False, AssumeNonNeg:=True
   SolverSolve True
   SolverFinish KeepFinal:=1

   Cells(Row, 10) = Cells(Row, 11)
   Cells(Row, 13) = Cells(15, 11)

   If (Cells(Row, 10) = "1") Then
         Cells(Row, 10).Interior.Color = 5296274
         Cells(Row, 13).Interior.Color = 5296274
         Cells(Row, 13) = Cells(Row, 13)
         jPos = 71 - (iEnd - iPos)
         Sheets("Class").Cells(jPos, 1).Value = Sheets("M1OOV5C59").Cells(Row, 1).Value
         Sheets("Class").Cells(jPos, 2).Value = Sheets("M1OOV5C59").Cells(Row, 2).Value
         Sheets("Class").Cells(jPos, 3).Value = Sheets("M1OOV5C59").Cells(Row, 13).Value
         iPos = iPos + 1
   Else
         Cells(Row, 10).Interior.Color = 65535
         Cells(Row, 13).Interior.Color = 65535
   End If

   Cells(7, 12) = iEnd - Row
  Next Row
End Sub
```

# Improvement (intermediate) plan for inefficient schools:

## Example code using the dual formulation model to construct an intermediate step for each school.

```
 Sub M2OOV59C10_Button1_Click()
'    Sheets.Add.Name = "Matrix"
    Sheets("Matrix").Select
    Rows("1:100").Select
    Selection.NumberFormat = "0.0"
    Selection.HorizontalAlignment = xlCenter
For Col = 2 To 55
    Range(Cells(2, (5 * (Col - 1) - 4)), Cells(2, (5 * (Col - 1)))).Merge
    Range(Cells(3, (5 * (Col - 1) - 4)), Cells(3, (5 * (Col - 1)))).Merge
    Sheets("Matrix").Cells(2, 5 * (Col - 1) - 4).Value = Sheets("M2OOV59C10").Cells(12, Col).Value
    Sheets("Matrix").Cells(3, 5 * (Col - 1) - 4).Value = Sheets("M2OOV59C10").Cells(13, Col).Value
Next Col
For Col = 2 To 55
 Sheets("M2OOV59C10").Select
    For Row = 16 To 20
       Cells (Row, 62) = Cells(Row, Col)
    Next Row
    JCol = 63
    SolverOk SetCell:="$BL$15", MaxMinVal:=2, ValueOf:=0, ByChange:="$B$28:$BH$28", Engine:=2
    SolverLPOptions MaxTime:=100, Iterations:=1000, Precision:=0.0000001, _
    StepThru:=False, Scaling:=False, AssumeNonNeg:=True
    SolverSolve True
    SolverFinish KeepFinal:=1
    For Idx = 1 To 5
          Iswift = (Idx + 5 * (Col - 2))
          Sheets("Matrix").Cells(4, Iswift).Value = Sheets("M2OOV59C10").Cells(Idx + 15, JCol).Value
          If Col Mod 2 = 0 Then
             Sheets("Matrix").Cells(4, Iswift).Interior.Color = vbWhite
          Else
             Sheets("Matrix").Cells(4, Iswift).Interior.Color = rgbLightGray
          End If
    Next Idx
Next Col
End Sub
```