iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Developing a graphical tool for students to understand air resistance and free fall: when heavier objects *do* fall faster

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2017 Phys. Educ. 52 034002

(http://iopscience.iop.org/0031-9120/52/3/034002)

View the table of contents for this issue, or go to the journal homepage for more

Download details:

IP Address: 143.160.10.83
This content was downloaded on 04/08/2017 at 08:16

Please note that terms and conditions apply.

You may also be interested in:

Motions of Celestial Bodies: Orbits of satellites and trajectories of missiles
E Butikov

From Newton to Einstein: Newtonian mechanics
F T Baker

Can an egg-dropping race enhance students' conceptual understanding of airresistance?
Yeung Chung Lee and Ping Wai Kwok

Angry Birds realized: water balloon launcher for teaching projectile motion with drag
Boyd F Edwards, David D Sam, Michael A Christiansen et al.

Are inertial forces ever of significance in cricket, golf and other sports?
Garry Robinson and Ian Robinson

Students' analogical reasoning in novel situations: theory-like misconceptions or p-prims?
Nikolaos Fotou and Ian Abrahams

Why did the apple fall? A new model to explain Einstein's gravity
Warren Stannard, David Blair, Marjan Zadnik et al.

Terminal velocity of skydivers
D C Agrawal

The equivalence principle comes to school—falling objects and other middle school investigations
Ann-Marie Pendrill, Peter Ekström, Lena Hansson et al.

PAPER

Phys. Educ. **52** (2017) 034002 (9pp) iopscience.org/ped

Developing a graphical tool for students to understand air resistance and free fall: when heavier objects *do* fall faster

Annalize Ferreira[1], **Albertus S Seyffert**[2] **and Miriam Lemmer**[1]

[1] School for Physical and Chemical Sciences, North-West University, Potchefstroom Campus, Private Bag X6001, Potchefstroom 2520, South Africa
[2] Centre for Space Research, North-West University, Potchefstroom Campus, Private Bag X6001, Potchefstroom 2520, South Africa

E-mail: albertus.seyffert@gmail.com

Abstract

Many students find it difficult to apply certain physics concepts to their daily lives. This is especially true when they perceive a principle taught in physics class as being in conflict with their experience. An important instance of this occurs when students are instructed to ignore the effect of air resistance when solving kinematics problems. To a student, this assumption disconnects from their everyday experience. Mathematically, ignoring the effect of air resistance is crucial, however, since it renders such problems tractable. However, this step is rarely, if ever, provided with justification in undergraduate texts, leading students to believe that what they are taught does not apply to their everyday experience. Taking the additional step of clarifying when it is reasonable to ignore air resistance makes students' reconciliation of their everyday experiences with the physics principle of free fall more likely. In this paper we develop a graphical tool intended to make this step as simple and effective as possible. We do this by summarising the results of a set of numerical simulations of various balls falling under the effects of both gravity and air resistance by means of a carefully chosen graph: a plot of an object's cross-sectional surface are versus its mass. We further use our numerical results to evaluate how these two variables relate to the effects of air resistance for balls dropped from varying heights.

1. Introduction

A number of studies have shown that students often have pre-instructional conceptions and ideas about a number of physical phenomena that are not in harmony with what they are taught in physics courses [1, 2]. These existing beliefs are often resistant to change, and influence the understanding students develop during their science tuition [3]. For example, when students are first taught about inertial motion, their experience that objects stop moving when one stops pushing them, conflicts with what they are taught (Newton's laws of motion). This conflict sometimes causes students to, for instance, subtly modify Newton's first law within their understanding to state that 'An object stays at rest unless a force acts on it,' omitting the offending concept that an object in motion will stay in motion unless a force acts on it. This subtle modification, while more easily reconciled with their experience, effectively negates their tuition since they still do not understand Newton's first law.

While a variety of alternative conceptions exist, including the example above, those relating to gravity and the Galilean principle of free fall are of particular interest. A good understanding of these two concepts is critical in founding students' knowledge regarding a host of other physics concepts, including, but not limited to, Newton's Laws, the laws of conservation of energy and momentum, and projectile motion. In this paper we focus on one such alternative conception, specifically relating to free-fall motion, that 'heavier objects fall faster'. This conception is, of course, in conflict with the Galilean principle of free fall, and hinders students in their attempts to solve problems where a physics conception of free fall is crucial.

In the majority of undergraduate physics textbooks, the physics of falling objects is primarily discussed in terms of the Galilean principle of free fall, ignoring the effects of air resistance. While these texts do touch on the effects of air resistance, they typically limit their discussion thereof to a brief explanation of the concept of terminal speed [4, 5]. A predictable consequence of this focus on pure free fall in undergraduate tuition is that the majority of mechanics problems students are asked to solve are accompanied by statements like '...ignoring air resistance.' Physics problems accompanied by such statements, however, rarely serve their intended purpose of aiding the student in developing an accurate conception of Newtonian mechanics. Sanborn *et al* [6] reported that accurate conceptions of Newtonian mechanics only develop within physics problems that match students' terrestrial experience. Since air resistance plays a significant role in many phenomena students encounter in their daily lives, statements like '...ignoring air resistance' serve only to disconnect what they are being taught from their everyday experience.

These qualifying statements are not unwarranted, though, since they greatly simplify the mathematics involved in introductory mechanics problems by eliminating the variables associated with the effects of air resistance [7, 8]. For many learners, however, this seemingly unjustified simplification calls into question the applicability of what they are taught. As a result, students doubt the integrity of physics as a coherent system of concepts to accurately describe and predict real-world phenomena [6].

This paper aims to reconcile students' everyday experience of falling objects with the Galilean principle of free fall. To do this we investigate the circumstances under which air resistance can justifiably be ignored. Additionally, this paper aims to develop a graphical tool which intuitively, and simply, encodes this newly-established understanding. This tool should allow educators to sufficiently motivate statements like '...ignoring air resistance' without resorting to in-depth mathematical arguments, partially re-establishing students' confidence in the applicability of what they are taught in undergraduate physics courses.

There have been a number of previous explorations, within the context of physics education, of the effect of air resistance on falling bodies. Grant [9], for example, investigates the effect air resistance has on a variety of school sports involving projectile motion using a simple experiment. Kincanon [10] uses skydiving as an example for teaching about resistive forces. Using basic physical principles, a skydiver's speed as function of time is remarkably well reproduced. Agrawal [11] investigates the relation between a skydiver's mass and terminal velocity in the horizontal spread eagle position. Theilmann *et al* [12] and Greening [13] both investigate supersonic free fall, specifically discussing Felix Baumgartner's record-breaking jump, also known as the *Red Bull Stratos* space diving project. While both of these papers focused on the relationship between air density and air resistance, and the resultant effect on terminal velocity, Theilmann *et al* also modelled the free fall itself.

2. Graphs as effective conveyors of information

Graphs are powerful tools that can visually communicate complicated concepts in an approachable manner [14]. Their utility as conveyors of information has led some researchers to argue that correct understanding of a concept will more likely be developed if it is first introduced using graphical representations of its core elements, instead of mathematical representations (e.g. Jackson *et al* [15]).

When deciding how to best convey the important aspects of the physics described in this paper, it proved useful to draw inspiration from a similar situation: evaluating how significant relativistic gravity is for a given astrophysical system. In that case the magnitude of the effect of relativistic corrections is characterised by the ratio of two numbers associated with the system or object in question. In some textbooks, these graphs are used to very good effect to convey this characterisation simply to the student. As an example, Hartle [16, p 5] uses a graph of the characteristic mass versus the characteristic radius of various astrophysical (and everyday) objects to show whether relativistic gravity is necessary when describing them, or whether classical Newtonian gravity will suffice. An important addition made to this graph is that of a diagonal line whose slope corresponds to specific value for the ratio between an object's characteristic mass and radius. This diagonal line allows the reader to instantly, and intuitive, evaluate how significant relativistic gravity is for a given system, since relativistic gravity is more important for objects that lie closer this line.

This graph for relativistic gravity serves as the inspiration for the graphs we develop in this paper. As will be shown in section 3.2, the significance of air resistance similarly depends on the ratio between two characteristic parameters of an object: its cross-sectional surface area (A_c; perpendicular to its motion) and its mass. Our approach is therefore to distil the physics of air resistance into a similar graph, which can serve as a tool with which students and educators can evaluate the significance of air resistance for a given system, and decide whether or not it is justified to ignore air resistance.

Before considering the development of this set of graphs we first investigate the physics of falling objects in more detail. While this investigation is crucial for developing the graphs, understanding and using them does not require an understanding of the physics described here.

3. The physics of falling objects

3.1. Free-fall motion

Objects falling only under the influence of gravity are said to be in free fall. In the one-dimensional case, their motion can be described by a simple linear differential equation (from Newton's Second Law):

$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = g, \tag{1}$$

where $(\mathrm{d}^2 s/\mathrm{d}t^2)$ is the object's acceleration, and s, t, and g have their usual meaning. Solving this equation is well within the reach of most first-year undergraduate students. Doing so requires only simple integration, and yields the well-known expression

$$s(t) = ut + \frac{1}{2}gt^2, \tag{2}$$

where u is the object's initial speed.

3.2. Drag-effected motion

The real-world projectiles students typically have experience of, however, also experience a drag force F_d due to air resistance [5]. To successfully describe the motion of objects for which this force is significant, then, we need to solve a modified version of equation (1):

$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = g - \frac{F_\mathrm{d}}{m}, \tag{3}$$

where m is the object's mass.

For an object moving with speed $v = (\mathrm{d}s/\mathrm{d}t)$ through air with density ρ, the drag force acting on it can be written as [5]:

$$F_\mathrm{d} = \frac{1}{2}C_\mathrm{d}\rho A_\mathrm{c} v^2, \tag{4}$$

where A_c is the object's cross-sectional area perpendicular to its direction of motion, and C_d is the drag coefficient associated with that object. This force arises as a result of turbulence effects as air flows around an object. It is therefore only valid for objects that are large enough, and are moving fast enough, for these turbulence effects to become important. For most everyday objects more than ~3 cm across, equation (4) is a very good approximation of the drag force. The value of C_d is determined by the shape and smoothness of an object. In this discussion it is only important to know that, in general, smooth objects that have the same shape, have the same C_d, regardless of their relative size (as long as they are not too small). For the sake of simplicity, we will assume that all the objects considered here have the same value for C_d. Equation (3) therefore becomes:

$$\frac{d^2s}{dt^2} = g - \frac{1}{2}C_d\rho\left(\frac{A_c}{m}\right)v^2. \tag{5}$$

This differential equation is nonlinear in s (due to the v^2), and is therefore much more difficult to solve than its free-fall counterpart. The mathematical tools needed to solve such equations generally lie outside the scope of typical undergraduate courses, which inhibits the degree to which the effects of drag can be investigated. Fortunately, useful physical understanding regarding the effects of drag can already be extracted from equation (5) without solving it.

First, we can qualitatively understand how an object experiencing drag will fall, and how its motion will differ from free-fall motion. Initially, for an object dropped from rest, its acceleration is g, just as in the free-fall case. As the object accelerates, and v increases, the magnitude of the net acceleration it experiences decreases. Compared to the free-fall case, therefore, the object will fall less rapidly. Additionally, if the object is allowed to fall long enough, the net acceleration will become zero. The speed for which the net acceleration is zero, is the terminal speed v_t. Conversely, an object initially moving at a speed greater than v_t will decelerate (since the second term would be larger than g) until its speed is v_t.

Second, we see that the magnitude of the drag term in equation (5) is directly proportional to the ratio (A_c/m). Therefore, if two objects have different values for (A_c/m), they will fall at different rates. In the case where both objects have the same A_c, this boils down to something that closely resembles the alternative conception students report: the more massive object, having a lower value for (A_c/m), will fall more rapidly.

This is, of course, in stark contrast with the free-fall case, where all objects fall at the same rate, regardless of their mass. Fortunately, this attribute of free-fall motion is not totally lost in the drag-effected case, and its analogue can be stated as: **all objects that have the same value for ($\mathbf{A_c/m}$) fall at the same rate**. When trying to determine which model for the motion of a falling object is appropriate, i.e. under which circumstances air resistance can be ignored, this attribute of drag-effected motion proves to be quite useful.

3.3. When to resort to the drag-effected model

The two models described above are related in a very specific way: free-fall motion is a limiting case of drag-effected motion in the limit where $(F_d/m) \rightarrow 0$ (see equation (3)). Therefore, the free-fall model is perfectly valid to use in some cases, while the drag-effected model is more appropriate in others. There is, however, no clear-cut boundary between these two categories. This is problematic since students encounter both kinds of motion in their everyday lives, leading to confusion when confronted with statements like '... ignoring air resistance.' Such statements therefore give students the impression that their everyday experience should be ignored when solving physics problems, and that what they learn in their physics courses does not apply in their daily life.

A good first step in addressing this issue is to enable the students to recognise which phenomena or systems fall into which category. Since we are trying to say something about things that they have observed, it is best to do this using quantities they can easily recognise from their experience. To this end, the boundary we define in this paper between the two regimes (free-fall and drag-effected) is based on the distance an object falls. Specifically, for an object dropped from a height h, we say that the drag-effected model is more appropriate if the object in question falls less than some fraction k of h in the time it would take a free-falling object to fall the entire height. Symbolically this becomes:

$$s_d(t_h) < kh, \tag{6}$$

where $s_d(t)$ is the object's displacement as function of time for the drag-effected model, and t_h is the time it would take a free-falling object to fall a distance h. The specific choice for the value of k is less important, and a range of values could feasibly make sense. In our numerical method we chose a convenient value of $k = 0.95$, implying that a drag-effected object falls less than 95% of the height, to illustrate what we found.

This boundary definition connects directly to the reported alternative conception (that 'heavier objects fall faster'), since two objects falling at different rates will fall different distances in the same time. It also allows the construction of a very simple, and intuitive experiment to demonstrate how the two regimes differ (see section 4). Most importantly, we provide a tool with which the student can quantify the significance of air resistance in various everyday phenomena without solving equation (5). This is especially true within the context of an experiment, since students essentially obtain the solution to equation (5) empirically and compare it to the familiar free-fall solution (equation (2)).

This boundary definition is not complete, however, since we have not yet related $s_\mathrm{d}(t)$ to parameters characterising different objects. As stated in section 3.2, the rate at which an object falls is inversely proportional to the ratio (A_c/m). Since the object's displacement is merely the integral of its rate of fall, this implies that

$$s_\mathrm{d}(t_h) \sim \frac{1}{(A_\mathrm{c}/m)}. \tag{7}$$

Since the deviation between the two cases, for a given height, depends on $s_\mathrm{d}(t_h)$, and $s_\mathrm{d}(t_h)$ decreases as (A_c/m) increases, there has to be some value $\sigma_k(h)$ for (A_c/m) where $s_\mathrm{d}(t_h) = kh$ (i.e. where equation (6) becomes an equality). Once this value is known for a specified height (and choice for k), we can easily know whether an object will display drag-effected motion or free-fall motion by comparing its (A_c/m) to it. Stated mathematically, the drag-effected model is appropriate if

$$\frac{A_\mathrm{c}}{m} > \sigma_k(h). \tag{8}$$

In section 5 we present a graph that demonstrates how this condition translates to a useful tool with which to evaluate the significance of air resistance for a given system. We also give an empirical form for $\sigma_k(h)$ based on our numerical solution of equation (5).

4. The numerical method

Our initial attempts to produce useful approximate solutions to equation (5) by means of proportional reasoning, specifically attempting to estimate the time an object would take to fall a given distance, proved unsuccessful. Our attempts especially failed to enable a satisfactory evaluation of whether air resistance was important for a given situation. Eventually it became evident that our initial choice to characterise the motion's deviation from free-fall motion by a deviation from the free-fall predicted fall time, was a bad one. This was due to the fact that for a given fall time deviation, the displacement deviation depends on the speed of the object at the end of its motion. For a light object, like a feather, which typically moves slowly at impact, a deviation of 0.5 s is small from a height of 2 m, while for a heavy object, like a bowling ball, which typically moves much more quickly at impact, a deviation of 0.5 s is large.

Our final choice, to characterise an object's motion rather by the deviation in its displacement, proved more effective. To obtain this result, equation (5) was numerically integrated by means of a *finite difference method*. First, the integration was transformed into a summation by discretizing the time t. The problem then reduces to an iterative calculation, with each iteration corresponding to an incrementing of t by a finite time difference Δt. At each of these steps, the instantaneous acceleration a, and instantaneous speed v, are calculated, and the latter is used to estimate the distance the object falls during the interval Δt. For the ith iteration, these quantities are calculated, in order, as follows:

$$\begin{aligned} a_i &= g - \frac{F_{\mathrm{d},i-1}}{m}, \\ v_i &= v_{i-1} + a_i \Delta t, \\ s_i &= s_{i-1} + v_i \Delta t. \end{aligned} \tag{9}$$

where the subscripts i and $i-1$ refer to the values for the subscripted variable for the current and previous iteration, respectively, and $F_{\mathrm{d},i-1}$ is the drag force defined in equation (4) for v_{i-1}. For the first iteration the initial conditions for a ball falling from rest are used as the '$i-1$' values. These initial conditions are $v_0 = 0$, $s_0 = 0$, and (in the interest of consistency) $a_0 = g$.

Upon completing this integration for a given ball and height, we are left with a numerical solution for $s_\mathrm{d}(t)$. This solution can then be compared to the prediction for free fall, and the condition defined in equation (6) can be checked for that motion (telling us whether or not the motion is significantly drag-effected). Doing this for a variety of balls, each having a different value for (A_c/m), dropped from a range of heights,

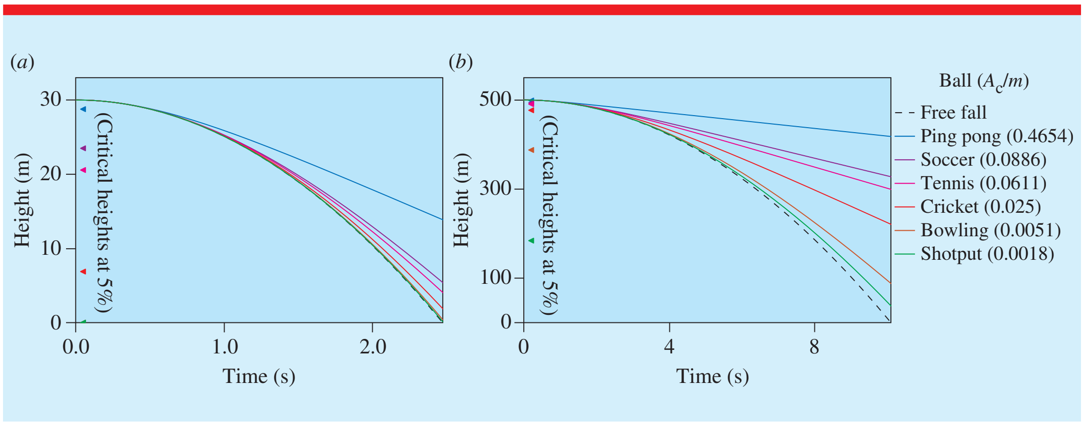

Figure 1. Simulated trajectories for a variety of balls dropped under the influence of air resistance from two different heights: 30 m and 500 m. The dashed line indicates the trajectory predicted for free fall. For each ball, the height at which its displacement deviates by 5% from free fall is indicated with a ◁. (a) $h = 30$ m, (b) $h = 500$ m.

then allows us to estimate the function $\sigma_k(h)$ we defined in equation (8).

In practice the estimation of $\sigma_k(h)$ was done by dropping a set of test balls numerically (as described above) by means of a Python implementation of the numerical calculations detailed here. Python is a high-level programming language designed to be both easily human-readable and powerful. These balls all had the same radius (and therefore equal A_c), but their masses were chosen such that the effect of drag decreased linearly as the balls' masses increased. The solutions for $s_d(t)$ obtained for this set of balls were then used to estimate the value of (A_c/m) where $s_d(t_h) = kh$ for a set of heights (1 m, 10 m, 100 m, and 1 km). From these calculations the results in section 5 were produced.

5. Results

Using the numerical method detailed in section 4, two sets of graphs and an empirical solution to $\sigma_{0.95}(h)$ were produced. Figure 1 shows the trajectories of a variety of balls as predicted by the drag-effected model. These trajectories initially closely resemble those predicted by the free-fall model (the dashed line). As the balls fall, their trajectories increasingly deviate from free fall, with the degree of deviation determined by the (A_c/m) ratio associated with each ball. Figure 1 shows that higher values for (A_c/m) correspond to more severe deviations. In figure 1(b) we also see that the balls' trajectories tend toward straight lines as they fall. This corresponds to the balls reaching their terminal speed, and the slope of their trajectory once it's straight is its terminal speed. Lastly, by comparing figure 1(a) to (b) we see that the deviation monatonically increases for any ball as time passes, just at varying rates.

In figure 2 a variety of balls are plotted according to the A_c and m values, along with a set of straight lines corresponding to different values of $\sigma_{0.95}(h)$ (for the sake of clarity this graph is plotted at two different zoom levels). For any such line, corresponding to a specified drop height, the drag-effected model is appropriate for balls that lie above it, and the free-fall model for balls that lie below it (in accordance with equation (6)). As the drop height is increased, the slope of the corresponding line decreases, and an increasing number of balls become drag-effected. At a drop height of 400 m all the balls in our sample are drag-effected.

For balls that have similar sizes (comparable A_c; e.g. the Tennis and Cricket balls), we see that lighter balls are more greatly effected by air resistance. This behaviour corresponds well with what the reported alternative conception predicts: that heavier objects fall faster.

Table 1 gives the empirical solution obtained for $\sigma_{0.95}(h)$.

6. Discussion

Figure 2 shows the conditions shows the conditions under which the effect of air resistance cannot be ignored. As discussed earlier, the critical ratios $(\sigma_k(h))$ for the different heights are related to the

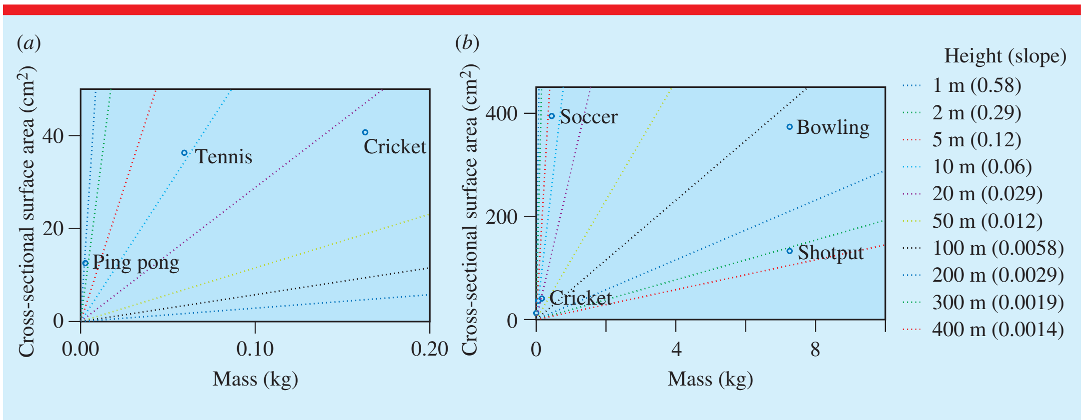

Figure 2. Graphs of A_c versus m for a variety of balls. The dotted lines each correspond to a drop height, and indicate the boundary between free-fall and drag-effected motion for that height. For such a line, if a ball lies above it, that ball's motion is best described by the drag-effected model when dropped from that height. This means that their displacement will deviate by more than 5% from free fall when dropped from the corresponding height. Balls that lie below a given line, are well described by free fall when dropped from the corresponding height.

slopes of the lines. The position on the graph of a given ball relative to one of these lines determines whether that ball is best described by the free fall or drag-effected model when dropped from the corresponding height. If the ball lies above the line, the effect of air resistance is large enough to warrant the use of the drag-effected model. To understand this relation, consider the most massive ball in the study, the shotput. This ball lies above the 400 m line, but below the 200 m line. The (A_c/m) ratio of 0.0018 for the shotput is smaller than the critical ratio of 0.0029 for the 200 m line, but larger than the critical ratio of 0.0014 for the 500 m line. This means that the shotput's displacement will deviate by $<5\%$ form free fall when dropped from 200 m, but by $>5\%$ when dropped from 400 m. Consequently, for the latter case the drag-effected model is more appropriate (for our chosen tolerance)

This behaviour can be summarised as follows: (i) the lower the (A_c/m) ratio is, the less significant the effect of air resistance on the object, and (ii) if the (A_c/m) of a ball is more than the critical ratio for a certain height (i.e. if the ball lies below the corresponding line) the effect of air resistance can be ignored.

Table 1. The obtained values for $\sigma_k(h)$ at various drop heights for $k = 0.95$. The value of $\sigma_{0.95}$ at a given height is the maximum allowed value for the ratio (A_c/m) a ball may have to still be considered as in free fall under equation (6). Air resistance is important, at a given height, for any object with $(A_c/m) > \sigma_{0.95}(h)$.

Height h (m)	$\sigma_{0.95}(h)$ (m^2 kg^{-1})
1	0.58
2	0.29
5	0.12
10	0.06
20	0.029
50	0.012
100	0.0058
200	0.0029
300	0.0019
400	0.0014

6.1. Revisiting students' alternative conception

Examples used to introduce and assess the concept of free fall typically describe the motion of two objects (usually equally sized balls with different masses) being dropped simultaneously from the same height (see for example Item 1 of the Force Concept Inventory presented in Hestenes *et al* [17]). If a student who believes that heavier objects fall faster is asked to predict which ball will hit the ground first in such an example, ignoring air resistance, they generally answer that the heavier of the two balls will hit the ground first.

The qualification of equal sizes for the balls, i.e. equal A_c, is never emphasized, and students consequently do not consider it to have an influence on the motion. The cross-sectional area plays an equally important role in the determination of the motion, however. Discussions regarding its impact are generally limited to specific examples, e.g. a parachutist experiencing an increase in drag when he opens his parachute. These examples usually intend to illustrate the concept of terminal speed, and neglect to point out that both mass and cross-sectional area are important when considering the magnitude of the experienced drag force. For example, in comparing the motion of a leaf and a nut falling from the same tree and the nut landing first, the difference in motion is more often explained in terms of the bigger mass of the nut than in terms of the bigger cross-sectional area of the leaf.

Having little or no knowledge of drag, students do not realize that a heavier object is less restricted by drag while a bigger object is more restricted by drag, and that the real falling rate (along with the drag coefficient) is determined by the ratio of these two quantities. As a consequence, students tend to focus on the influence mass has on the motion, concluding that heavier objects fall faster.

6.2. Addressing their alternative conception

Figure 2 (either (a) or (b)) captures the essence of the constraint derived in equation (8), and provides an easy-to-use tool with which to determine whether air resistance is relevant for a specific circumstance. Included in an ungraduate physics text, such a graph would provide an invaluable reference against which to justify the qualifying phrase '... ignoring air resistance' throughout the text. With little more than a brief introduction to how this graph is to be used, a student can easily assess and understand why air resistance can be ignored for a given problem. Additionally, this graph serves to assure the student that the physics they are being taught is indeed applicable to their everyday experience.

7. Conclusion

The physics argument in this paper enriches the concept of free fall and provides a comprehensive explanatory framework that encompasses both idealized and observable situations. The study validates the use of the conditional statement '... ignoring air resistance' by providing convincing evidence that there are circumstances when, although present, the effect of air resistance can realistically be ignored. We also illuminate two factors that influence the effect of drag on falling balls, namely the falling height (h) and the cross-sectional area to mass ratio (A_c/m). The most valuable contribution of the study is the graph in figure 2, which captures this conceptual understanding in a very simple and concise way.

8. Implications for teaching

An advantage of the approach described above is making students aware of how one abstracts real-world phenomena into theoretical framework, and how simplifying assumptions like the one they generally encounter regarding motion through air, come about. In essence, it shows them when to disregards certain aspects of real-world problem without losing the desired applicability, and when not to. Students' real-world alternative conception that 'the heavier ball falls faster' is not discredited, but used for meaning-making by relating the practical and theoretical worlds. This, in turn, enhances students' view of physics as a collection of coherent concepts, applicable and valuable in their daily lives.

The study has another very important implication for the teaching of Galilean free fall. The graphical representations offer a way for both students and teachers to interpret and understand the conditions under which this principle is valid. The condition of free fall facilitates the learning of other physics topics, e.g. conservation of energy and projectile motion and also serves as an example of the application of conditional statements elsewhere in physics.

Acknowledgments

The authors gratefully acknowledge the invaluable input provided by C Venter and R Gunstone, without which this work would have been lessened.

Received 1 February 2017, in final form 6 March 2017
Accepted for publication 10 March 2017
https://doi.org/10.1088/1361-6552/aa65da

References

[1] Larkin D 2012 *Sci. Educ.* **96** 927–59
[2] von Aufschnaiter C and Rogge C 2015 *Encyclopedia Sci. Educ.* 209–18
[3] Duit R H and Treagust D F 2012 *Issues and Challenges in Science Education Research* (Berlin: Springer) pp 43–54
[4] Giordano N 2012 *College Physics* vol 1 (California: Brooks/Cole)
[5] Halliday D, Resnick R and Walker J 2011 *Fundamentals of Physics* 9th edn (New York: Wiley)

[6] Sanborn A N, Mansinghka V K and Griffiths T L 2013 *Psychol. Rev.* **120** 411

[7] Gunstone R F 1987 *Am. J. Phys.* **55** 691–6

[8] Redish E F 2004 arXiv:physics/0411149

[9] Grant A R 1990 *Phys. Educ.* **25** 288–92

[10] Kincanon E 1990 *Phys. Educ.* **25** 267

[11] Agrawal D 2000 *Phys. Educ.* **35** 281

[12] Theilmann F and Apolin M 2013 *Phys. Educ.* **48** 150

[13] Greening F 2013 *Phys. Educ.* **48** 139

[14] Mäntylä T and Hämäläinen A 2015 *Sci. Educ.* **24** 699–723

[15] Jackson J, Dukerich L and Hestenes D 2008 *Sci. Educator* **17** 10

[16] Hartle J B 2003 *Gravity: an Introduction to Einsteins General Relativity* (Reading, MA: Addison Wesley)

[17] Hestenes D, Wells M and Swackhamer G 1992 *Phys. Teach.* **30** 141–58