

Expectations of the competencies of pre-service mathematics teachers

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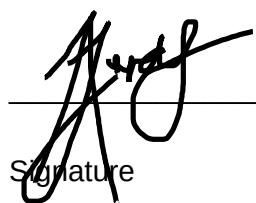
DEDICATION

I dedicate this study to my father, mother and brother; Christoffel van Aardt, Christine van Aardt, and Johann van Aardt, for always encouraging me to pursue my dreams.

DECLARATION

I, Judine van Aardt, solemnly declare that this work is original and the result of my own labour. It has never, on any previous occasion, been presented in part or whole to any institution or board for the award of any degree.

I further declare that all information used and quoted has been duly acknowledged by complete reference.



Signature

15 December 2020

Date

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ABSTRACT

The purpose of this concurrent triangulation mixed-method research was to determine to what extent the teacher-training curriculum for mathematics education at the North-West University (NWU), Vanderbijlpark Campus, prepares a heterogeneous group of pre-service teachers (n=19) to meet the competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge foregrounded in the Curriculum and Assessment Policy Statement (CAPS) for teaching mathematics.

Quantitative data were collected from the student participants using researcher-constructed tests, to determine subject content knowledge (factual, conceptual, and procedural). Qualitative data were collected through open-ended questions at the end of each section in the test, that focused on a specific subject content area, according to which participants had to reflect in writing about the use of relevant pedagogical strategies to teach the specific content on which the questions focused. The tests also contained open questions where participants reflected in writing about their metacognitive strategies to plan, monitor and evaluate learning, and the teacher-training curriculum and its qualities. Two closed five-point semantic differential scale questions also formed part of the tests, where participants reflected on their preparedness to teach mathematics subject content knowledge, and the variety of pedagogical strategies they have available to teach mathematics. The mathematics lecturers (n=4) were also given a questionnaire with open questions where they indicated their qualifications and had to reflect in writing about the qualities of the teacher-training curriculum.

In summary, the findings of the research revealed that the pre-service mathematics teachers who took part in the study appear to lack both subject content and pedagogical content knowledge to teach effectively, thus not fully meeting the competency expectations required of mathematics teachers. Subsequently, curriculum-related recommendations are made to strengthen, adapt or enrich the current mathematics teacher-training curriculum used at the NWU.

Key words/phrases: mathematics subject content knowledge, mathematics pedagogical content knowledge, common content knowledge, specialised content knowledge, factual, conceptual, procedural and metacognitive knowledge dimensions, cognitive knowledge dimensions.

TABLE OF CONTENTS

DEDICATION.....	I
DECLARATION	II
ACKNOWLEDGEMENTS	III
ABSTRACT	IV
CHAPTER 1: ORIENTATION AND PROBLEM STATEMENT.....	1
1.1 INTRODUCTION AND MOTIVATION OF THE STUDY	1
1.2 PROBLEM STATEMENT	5
1.3 RESEARCH QUESTIONS.....	6
1.4 AIM AND OBJECTIVES OF THE STUDY.....	7
1.4.1 Aim	7
1.4.2 Objectives.....	7
1.5 THEORETICAL AND CONCEPTUAL FRAMEWORKS.....	9
1.5.1 Theoretical framework	9
1.5.2 Conceptual framework.....	10
1.6 RESEARCH METHODOLOGY	13
1.6.1 Literature review.....	13
1.6.2 Empirical study	13
1.7 CHAPTER SUMMARY	18
CHAPTER 2: COMPETENCY EXPECTATIONS OF PRE-SERVICE TEACHERS: MATHEMATICAL SUBJECT CONTENT KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE	19
2.1 INTRODUCTION	19

2.2	EPISTEMOLOGICAL BELIEFS AND THE TEACHING OF MATHEMATICS	21
2.2.1	Introduction.....	21
2.2.2	The importance of epistemological beliefs and teaching mathematics	21
2.2.3	Epistemological beliefs about the nature and acquisition of knowledge	21
2.2.4	The significance of epistemological beliefs for the teaching and learning of mathematics	23
2.2.5	Epistemological belief systems for teaching and learning mathematics	24
2.3	A THEORETICAL FRAMEWORK FOR CONCEPTUALISING COMPETENCY EXPECTATIONS FOR TEACHING SUBJECT CONTENT KNOWLEDGE IN MATHEMATICS	26
2.3.1	Competency expectations concerning subject content knowledge for teaching mathematics: the South African framework and Bloom's revised taxonomy	27
2.3.2	The conceptualisation of mathematics subject content knowledge according to Bloom's revised taxonomy	32
2.3.3	Integrating subject content knowledge dimensions and cognitive process dimensions during the teaching of mathematics	42
2.3.4	Competency expectations concerning the qualities of teachers' mathematical subject content knowledge.....	45
2.3.5	Mathematical subject content knowledge in the CAPS and the teacher-training curriculum	47
2.4	THEORETICAL FRAMEWORK FOR CONCEPTUALISING COMPETENCY EXPECTATIONS ABOUT PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING MATHEMATICS	51
2.4.1	Direct teaching.....	54
2.4.2	Indirect teaching	55

2.4.3	Independent teaching	57
2.4.4	Interactive teaching	59
2.4.5	Learning-centred teaching	61
2.4.6	Pedagogical content knowledge in the CAPS and mathematics teacher- training curriculum	62
2.4.7	Competency expectations for pre-service mathematics teachers: the researcher's view	63
2.5	CHAPTER SUMMARY	65
CHAPTER 3: EMPIRICAL RESEARCH DESIGN		68
3.1	INTRODUCTION	68
3.2	RESEARCH PARADIGM	69
3.3	EMPIRICAL RESEARCH	70
3.3.1	Aim	70
3.3.2	Objectives.....	70
3.4	RESEARCH DESIGN.....	70
3.4.1	Research strategy	72
3.5	DATA COLLECTION METHODS.....	72
3.5.1	Quantitative study: a self-constructed test and questionnaire with closed and open test items	72
3.5.2	Qualitative study: a document study and written reflections	82
3.6	QUALITY CRITERIA: RELIABILITY AND VALIDITY OF THE QUANTITATIVE STUDY	84
3.6.1	Reliability	84
3.6.2	Validity of the research design.....	84

3.6.3	Validity of the data collection instruments	85
3.7	QUALITY CRITERIA: TRUSTWORTHINESS OF THE QUALITATIVE STUDY	87
3.7.1	Credibility.....	87
3.7.2	Dependability.....	87
3.7.3	Transferability.....	87
3.7.4	Confirmability.....	88
3.8	THE ROLE OF THE RESEARCHER IN QUALITATIVE RESEARCH	88
3.8.1	Assumptions.....	88
3.8.2	Personal connection to the research site	88
3.9	RESEARCH PARTICIPANTS	89
3.9.1	Participants for the quantitative study	89
3.9.2	Participants for the qualitative study	90
3.10	DATA ANALYSIS	90
3.10.1	Quantitative test and closed questionnaire items: descriptive statistics.....	90
3.10.2	Qualitative data analysis: written reflections to open questions.....	91
3.10.3	Qualitative document analysis	91
3.11	TRIANGULATION OF DATA	92
3.12	ETHICAL CONSIDERATIONS.....	93
3.12.1	Ethical issues in the research problem	93
3.12.2	Ethical issues in the purpose and questions	93
3.12.3	Ethical issues in data collection	93
3.12.4	Ethical issues in data analysis and interpretation.....	94

3.12.5	Ethical issues in writing and disseminating the research.....	94
3.13	CHAPTER SUMMARY	94
	CHAPTER 4: DATA ANALYSIS AND INTERPRETATION.....	97
4.1	INTRODUCTION	97
4.2	BIOGRAPHIC INFORMATION OF RESEARCH PARTICIPANTS	98
4.3	QUANTITATIVE DATA ANALYSIS AND INTERPRETATION: OVERALL TEST RESULTS OF PARTICIPANTS.....	100
4.3.1	Descriptive data analysis	100
4.3.2	Data analysis guidelines	105
4.3.3	Analysis and interpretation of data: knowledge and cognitive process dimensions – FET Phase.....	108
4.3.4	Analysis and interpretation of knowledge and cognitive process dimensions – Senior Phase	138
4.3.5	Analysis and interpretation of knowledge and cognitive process dimensions – Intermediate Phase.....	153
4.4	QUALITATIVE DATA ANALYSIS AND INTERPRETATION: STUDENT PARTICIPANTS	172
4.4.1	Student perceptions: the metacognitive knowledge dimension	172
4.4.2	Student perceptions: preparedness to teach and availability of teaching strategies.....	178
4.4.3	Student perceptions: strengths of teacher training	193
4.4.4	Student perceptions: weaknesses of teacher training	196
4.4.5	Student perceptions: changes to improve teacher training.....	200
4.4.6	Student perceptions: reasons for being a good teacher	203
4.4.7	Student perceptions: reasons for not being a good teacher	208

4.4.8	Student perceptions: the easiest and most difficult test questions.....	211
4.4.9	Student perceptions: teaching strategies used to teach subject content areas	217
4.5	QUALITATIVE DATA ANALYSIS AND INTERPRETATION: LECTURERS..	224
4.5.1	Biographic information: lecturers.....	224
4.5.2	Lecturer perceptions: strengths and weaknesses of the mathematics teacher-training curriculum	225
4.5.3	Lecturer perceptions: limitations of the mathematics teacher-training curriculum for teaching students	227
4.5.4	Lecturer perceptions: adequacy of content and pedagogical knowledge in the teacher-training curriculum	229
4.6	TRIANGULATING QUANTITATIVE AND QUALITATIVE DATA	231
4.7	CHAPTER SUMMARY	233
CHAPTER 5: SUMMARY FINDINGS AND RECOMMENDATIONS		236
5.1	INTRODUCTION	236
5.2	AN OVERVIEW OF THE STUDY	236
5.3	FINDINGS FROM THE LITERATURE REVIEW.....	238
5.3.1	Findings concerning subject content knowledge	238
5.3.2	Findings concerning subject content knowledge and cognitive process dimensions	239
5.3.3	Findings concerning pedagogical content knowledge	239
5.3.4	Findings concerning the relationship between the different knowledge dimensions	240
5.3.5	Findings concerning the relationship between subject content knowledge, pedagogical knowledge, and epistemological beliefs	240

5.3.6	Findings concerning the competency expectations for mathematics teachers in the CAPS and teacher-training curriculum.....	241
5.3.7	Findings concerning the attributes of teachers' mathematical subject content knowledge.....	241
5.4	FINDINGS FROM THE EMPIRICAL STUDY	242
5.4.1	Findings concerning biographic information	242
5.4.2	Findings concerning competency expectations for subject content knowledge of the pre-service teachers	243
5.4.3	Findings concerning competency expectations for metacognitive knowledge of the pre-service teachers	252
5.4.4	Findings in relation to the competency expectations concerning pedagogical content knowledge of the pre-service teachers	253
5.4.5	Additional findings	256
5.4.6	Contradictory findings	256
5.5	FINDINGS CONCERNING THE AIM AND OBJECTIVES OF THE STUDY ...	257
5.6	RECOMMENDATIONS TO STRENGTHEN THE TEACHER-TRAINING CURRICULUM	259
5.6.1	Recommendations concerning lecturers (Finding 4) (cf. 5.4.4)	259
5.6.2	Recommendations concerning subject content knowledge (factual, conceptual, and procedural) of pre-service teachers (Findings 1, 4, 5, 6, 7, 8) (cf. 5.4.2)	260
5.6.3	Recommendations concerning the development of metacognitive knowledge (Findings 1, 2) (cf. 5.4.3)	262
5.6.4	Recommendations to foster realistic perceptions about competency among mathematics pre-service teachers (Findings 2, 3) (cf. 5.4.2)	263
5.6.5	Recommendations to addressing specific problems related to subject content areas (Finding 10) (cf. 5.4.2)	263

5.6.6	Recommendations for developing cognitive process dimensions in conjunction with subject content knowledge (Finding 11) (cf. 5.4.2).....	264
5.6.7	Recommendations concerning pedagogical content knowledge (Findings, 1, 2) (cf. 5.4.4)	264
5.6.8	Recommendations to address limitations in the teacher-training curriculum: subject content knowledge and cognitive process dimensions (Findings 9, 11, cf. 5.4.2; Finding 3, cf. 5.4.3), and pedagogical content knowledge (Finding 3, cf. 5.4.4).....	265
5.6.9	Recommendations to build on the strengths observed in the teacher-training curriculum (Finding 1, 2, 3) (cf. 5.4.5).....	267
5.6.10	Recommendations for creating awareness of the importance of framing teaching within a variety of epistemological beliefs (Finding 4) (cf. 5.4.5).	267
5.7	LIMITATIONS OF THE STUDY	269
5.7.1	Limitations: aim and objectives, research design, data collection, sample	269
5.7.2	Possible limitations: collection and analysis of test data	270
5.8	SUGGESTIONS FOR FURTHER RESEARCH	272
5.9	CONTRIBUTION OF THE STUDY	273
5.9.1	Contribution to theory	273
5.9.2	Contribution to practice	274
5.10	CONCLUSION	274
	REFERENCES.....	276

LIST OF TABLES

Table 2.1: Epistemological beliefs systems in mathematics.....	25
Table 2.2: The cognitive process dimensions of Bloom's revised taxonomy	29
Table 2.3: The knowledge dimensions of Bloom's revised taxonomy	30
Table 2.4: Questions to prompt the development of metacognitive self-regulation.....	38
Table 2.5: Cognitive levels and skills: CAPS curriculum	42
Table 2.6: Analysis of knowledge dimensions and cognitive processes.....	44
Table 2.7: Assessment of mathematics subject content and weighting (according to the highest grade in the Phase) according to CAPS and the teacher-training curriculum: Intermediate Phase (Department of Basic Education, 2011c, p. 11).....	48
Table 2.8: Assessment of mathematics subject content and weighting (according to the highest grade in the Phase) according to CAPS and the teacher-training curriculum: FET and Senior Phase (Department of Basic Education, 2011a, p.10; 2011b, p. 11)	50
Table 2.9: Direct teaching strategies	55
Table 2.10: Indirect teaching strategies	56
Table 2.11: Independent teaching strategies	57
Table 2.12: Interactive/cooperative teaching strategies	59
Table 3.1: Assessment of mathematics subject content and weighting.....	75
Table 4.1: Biographic information of the research participants.....	98
Table 4.2: Test results: FET Phase	100
Table 4.3: Test results: Senior Phase.....	102
Table 4.4: Test averages: FET and Senior Phase participants	102
Table 4.5: Test results: Intermediate Phase	104

Table 4.6: Linking verbs in the context of the CAPS curriculum to Bloom’s revised taxonomy.....	107
Table 4.7: Data analysis: FET Phase Question 1 – differential calculus	108
Table 4.9: Data analysis: FET Phase Question 2 – algebra and equations.....	111
Table 4.11: Data analysis: FET Phase Question 3 – numbers, patterns, sequences, and series.....	115
Table 4.13: Data analysis: FET Phase Question 4 – finance, growth, and decay	120
Table 4.15: Data analysis: FET Phase Question 5 – functions	123
Table 4.17: Data analysis: FET Question 6 - Probability.....	130
Table 4.19: Data analysis: FET Question 7 - Statistics	133
Table 4.21: Data analysis: Senior Phase Question 1 – patterns, functions, and algebra	139
Table 4.23: Data analysis: Senior Question 2 – numbers, operations and relationships	141
Table 4.25: Data analysis: Senior Phase Question 3 – space and shape (geometry)	144
Table 4.27: Data analysis: Senior Phase Question 4 – measurement	146
Table 4.29: Data analysis: Senior Phase Question 5 – data handling.....	149
Table 4.31: Data analysis: Intermediate Phase Question 1 – Numbers, operations, and relationships	154
Table 4.33: Data analysis: Intermediate Phase Question 2 – patterns, functions, and algebra	160
Table 4.35: Data analysis: Intermediate Phase Question 3 – measurement.....	163
Table 4.37: Data analysis: Intermediate Phase Question 4 – space and shape.....	165
Table 4.39: Data analysis: Intermediate Phase Question 5 – data handling	168
Table 4.44: Participant perceptions: preparedness to teach content areas.....	178
Table 4.45: Perceptions: availability of teaching strategies to teach subject content areas....	185

Table 4.64: Summary: the easiest and most difficult questions.....	217
Table 4.68: Biographic information: lecturers.....	224
Table 4.73: Triangulation of data.....	232

LIST OF FIGURES

Figure 1.1: Competency expectations concerning subject content and pedagogical content knowledge (Hill et al., 2008, p. 377).....	9
Figure 2.1: Growth of epistemological beliefs	24
Figure 2.2: Competency expectations for teaching mathematics in South Africa (Department of Basic Education, 2018, p. 14)	27
Figure 2.3: The differences between the cognitive dimensions in Bloom's taxonomy and Bloom's revised taxonomy (Wilson, 2016, p. 4)	29
Figure 2.4: Pedagogical subject content knowledge (Killen, 2015, p. 31)	52
Figure 2.5: Conceptualisation of competency expectations concerning subject content knowledge and pedagogical content knowledge in teaching mathematics	64
Figure 3.1: Triangulation (Creswell, 2014, p. 283)	71
Figure 3.2: Concurrent triangulation mixed method research.....	71

LIST OF GRAPHS

Graph 4.1: Comparison: FET and Senior Phase test results	103
Graph 4.2: Question 1 – differential calculus	110
Graph 4.3: Question 2 – algebra and equations	114
Graph 4.4: Question 3 – numbers, patterns, sequences, and series	118
Graph 4.5: Question 4 – finance, growth, and decay	122
Graph 4.6: Question 5: Functions	129
Graph 4.7: Question 6: Probability	132
Graph 4.8: Question 7: Statistics	136
Graph 4.9: Question 1: patterns, functions, and algebra	140
Graph 4.10: Question 2: Numbers, operations, and relationships	143
Graph 4.11: Question 3: space and shape (geometry)	145
Graph 4.12: Graph 4.12: Question 4: measurement	148
Graph 4.13: Question 5: Data handling	150
Graph 4.14: Question 1 – numbers, operations, and relationships	158
Graph 4.15: Question 2 – patterns, functions, and algebra	162
Graph 4.16: Question 3 – measurement	164
Graph 4.17: Question 4: Space and shape	167
Graph 4.18: Question 5 – data handling	169
Graph 4.19: Preparedness to teach – FET Phase	179
Graph 4.20: Preparedness to teach – Senior Phase	181
Graph 4.21: Preparedness to teach – Intermediate Phase	183

Graph 4.22: Availability of teaching strategies – FET Phase	187
Graph 4.23: Availability of teaching strategies – Senior Phase	189
Graph 4.24: Availability of teaching strategies – Intermediate Phase.....	191

CHAPTER 1: ORIENTATION AND PROBLEM STATEMENT

In Chapter 1, the researcher identifies and motivates the research problem on which the study focused, delineates the focus of the research, and clarifies the research methodology employed to collect data to solve the research problem. The chapter unfolds according to the following structure.

1.1 INTRODUCTION AND MOTIVATION OF THE STUDY

1.2 PROBLEM STATEMENT

1.3 RESEARCH QUESTIONS

1.4 AIM AND OBJECTIVES OF THE STUDY

1.5 THEORETICAL AND CONCEPTUAL FRAMEWORKS

1.6 RESEARCH METHODOLOGY

1.7 CHAPTER SUMMARY

1.1 INTRODUCTION AND MOTIVATION OF THE STUDY

Grade 12 mathematics results in South Africa are disturbing (Avalos, Tellez, & Navarro, 2010; Mij & Makgato, 2006; Volmink, cited by BusinessTech, 2020), and mathematics consistently appears to be the lowest performing subject with a percentage pass of between 30% and 35% (Shay, 2019). Research links this problem to among others, language barriers (Setati & Barwell, 2008), as teachers who are not English first-language speakers (Bohlmann & Pretorius, 2008, Setati & Barwell, 2008), find it difficult to understand and explain mathematical terminology (Siyepu, 2013). Also, teachers lack factual subject content knowledge (Bansilal, Mkhwanazi, & Brijlall, 2014; Jojo, 2019; Loyiso & Olubenga, 2019; Malaty, 2004; Shepherd, 2013; Venkat & Spaul, 2015), as well as a fundamental understanding of mathematical concepts (May, 2017; Venkat & Spaul, 2015), procedural knowledge (May, 2017), and solid pedagogical content knowledge (how to teach mathematics effectively) (Luneta, 2018; Sepeng & Webb, 2012).

In South Africa, teachers in KwaZulu-Natal were tested on the Grade 12 subject content knowledge that they possess. The disturbing results showed that teachers have limited subject content knowledge, scoring 57% on average and a low of 29% on problem-based questions (Bansilal et al., 2014). A teacher cannot teach what a teacher does not know or understand (Cazzola, 2008; Plotz, Froneman, & Nieuwoudt, 2012).

The mathematics subject content knowledge that learners need to acquire comprises factual, conceptual, procedural, and metacognitive knowledge (Department of Basic Education, 2018; Long & Wendt, 2019; May, 2017) (cf. 2.3.2.1 – 2.3.2.4). In order to teach learners effectively,

teachers need to possess common subject content knowledge, specialised subject content knowledge, and horizon subject content knowledge (Hill, Ball, & Schilling, 2008) (cf. 1.5.1, 2.3.2.4). Common subject content knowledge refers to basic knowledge regarding the content of the mathematics curriculum for each grade. Specialised subject content knowledge comprises the depth and breadth of knowledge (Hill et al., 2008), and horizon subject content knowledge concerns an awareness of how mathematics topics that might not be dealt with as part of the curriculum relate to one another (Ball & Bass, 2009, p. 5; Ball, Thames, & Phelps, 2008, p. 403) (cf. 2.3.4.2).

International and national studies identified some gaps concerning the competency expectations for mathematics teachers regarding their subject content knowledge and pedagogical content knowledge. Plotz et al. (2012) assert that during tertiary education, students who wish to become mathematicians, study very high levels of mathematics which include several modules of calculus, algebra, and applied mathematics up to third-year university level. Mathematics teachers need to understand the depth and breadth of each mathematical subject content area that they teach to their learners (Kanyongo & Brown, 2013; Plotz et al., 2012). The depth of knowledge is also associated with knowledge progression within grades across teaching phases (Department of Education, 2003), and includes the following thinking levels, namely: memorisation, understanding, problem-solving, and reasoning (Holmes, 2012). Depth concerns insight into different methods that would result in obtaining a correct answer or solving a problem (Wineburg, 1997) (cf. 2.3.4.3). However, it seems as if mathematics pre-service teachers only have compressed knowledge that lacks depth and breadth (Kanyongo & Brown, 2013; Plotz et al., 2012). It appears that school content is not stressed in the subject content modules at university (Reddy, Nair, & Reddy, 2014; Taylor, 2018; Zazkis & Leikin, 2010), and that student teachers mostly manage mathematics content at Intermediate Phase level (Bowie & Reed, 2016). According to the research of May (2017), the majority of Grade 6 teachers have mathematics content levels below Grade 6.

Kanyongo and Brown (2013) and Plotz et al. (2012) add that teachers lack certain types of content knowledge, especially in the field of statistics. When South Africa implemented the CAPS for Gr. 10, Gr. 11, and Gr. 12 learners, the mathematics teachers, for example, did not have enough content knowledge regarding the section on statistical content that was added to the mathematical syllabus (Adler, 2005; Blignaut, Luus, Latief, Lombard, & Kotze, 2013). Research highlighted problems with teachers' understanding of the subject content in calculus (Reddy et al., 2014; Tsanwani & Sinthumule, 2015) and geometry (Reddy et al., 2014). The research of Tsanwani and Sinthumule (2015) reported a mean score of 37% in a calculus pre-test, which increased to 61% in the post-test after teachers completed a professional development programme in calculus.

As mathematics teachers at school appear to be unfamiliar with some of the content topics, Blignaut et al. (2013) and Siyepu (2013) argue that students enter university unprepared, which automatically increases their risk of failure. Jacobs and Pretorius (2015) therefore argue that the baseline knowledge of mathematics of first year students could also be viewed as disturbing, and research conducted by Reddy et al. (2014) reveal that only 3 % of a group of 203 first year students studying in the field of engineering, optometry, food technology and biotechnology at universities in South Africa obtained more than 50 % for the 2014 mathematics matric examination. Similarly, research conducted by Bowie and Reed (2016) report that third year pre-service teachers only obtained a 27% average for the 2014 mathematics matric examination. In this regard, Mahlobo (2015) also points out that there is a problematic transition between mathematics at secondary and tertiary level. According to Mahlobo (2015), students entering tertiary level cannot solve problems unless they are given formulae, and are mainly interested in solving routine problems and are not interested in gaining insight into how formulae are derived. The aforementioned is compounded by the fact that pre-service teachers do not perform well at the tertiary level, but still, they graduate and become teachers (Blignaut et al., 2013; Siyepu, 2013). Given the identified lack of subject content knowledge among mathematics teachers (Plotz et al., 2012), teacher-training institutions cannot afford to advance teachers with poor content knowledge (Blignaut et al., 2013), as this will exacerbate the problem of poor mathematics performance at the school level.

Apart from limited subject content knowledge, teachers' pedagogical content knowledge and teaching skills (Ikeda, 2013; Luneta, 2018; Ng, 2013, Sepeng & Webb, 2012; Spangenberg, 2018; Zhao et al., 2014) that among others, refer to understanding how learners learn, and employing teaching strategies to enable learners to acquire subject content effectively (Blömeke, Suhl, & Kaiser, 2011; Hill et al., 2008; Kanyongo & Brown, 2013; Sibuyi, 2012) (cf. 2.4), also seem questionable. In general, South African mathematics teachers tend to teach the same way that they were taught; by making use of brief explanations on the board without any interactive or critical thinking classroom activities (Plotz et al., 2012), or by using a transmission-mode of teaching (De Beer & Kriek, 2018). Luneta (2013) extends the argument by stating that teachers provide limited opportunities for learners to conceptualise and analyse problems that require reasoning, interpretation, reflection, and evaluation (Ang, 2010; Kang & Noh, 2012).

The choice of teaching strategies appears to be influenced by factors including gender, age, experience, and the content area to be taught. Umugiraneza, Bansilal, and North (2017) contend that at the national level, teachers tend to feel more comfortable with using a single teaching strategy when teaching statistical topics. Besides, female teachers are more likely to use only one teaching approach in class, whereas males are more likely to combine teaching approaches.

Teachers younger than 40 years of age, and who had less than ten years of teaching experience, seem to use more than one teaching strategy.

Research reports on the unpreparedness of mathematics teachers to employ reform-oriented, inquiry-based teaching strategies that could encourage learner engagement in the mathematics classroom (Brodie & Molefe, 2010; Kilpatrick, Swafford, & Findell, 2001), and support the development of conceptual understanding in mathematics, such as modelling (Durandt & Jacobs, 2014; Ikeda, 2013; Ng, 2013), problem-based teaching (Cazzola, 2008; Luneta, 2018), discussion (Kunene & Sepeng, 2018; Sepeng & Webb, 2012; Venkat, 2013), and cooperative learning (Spangenberg, 2018). Learner engagement could bridge the gap between theory and practice (Boyd, 2011). Moreover, it could be argued that learner engagement could make mathematics more understandable for learners because they can see how mathematics is applied. However, for mathematics teachers to implement reform-oriented, inquiry-based teaching strategies, they must first experience and understand the application of these strategies (Cazzola, 2008).

Ball, Bass, Sleep, and Thames (2007), Blömeke et al. (2011), Hill et al. (2008), and Pimentel, Palhares, and Vale (2007) conclude that a good mathematics teacher should possess solid subject content knowledge and pedagogical content knowledge. Research points to a strong link between solid subject content knowledge and effective pedagogical content knowledge (Adler & Davis, 2006; Brijlall & Isaac, 2011; Fung et al., 2017; Kanyongo & Brown, 2013; Marshall & Connelly, 2012; Stylianides & Stylianides, 2010) for learner achievement (Fuentes, Bloom, & Peace, 2014; Kanyongo, & Brown, 2013). A combination of low-level subject content knowledge and low pedagogical efficacy can be associated with poor achievement (Blömeke et al., 2011). Low levels of subject content knowledge, but moderate pedagogical efficacy could still produce higher achievement (Fung et al., 2017). The research of Fung et al. (2017), and Rowland, Martyn, Barber, and Heal (2001) postulate that subject content knowledge only partially contributes to a teacher's efficacy. Higher achievement in mathematics is strongly related to teachers' pedagogical efficacy within the classroom (Farooq, Chaudhry, Shafiq, & Berhanu, 2011; Fung et al., 2017; Mij & Makgato, 2006). Therefore, Plotz et al. (2012) argue that teachers need more pedagogical knowledge than content knowledge to be effective at teaching mathematics. Brijlall and Isaac (2011) report that teachers with a lower subject content knowledge tend to implement teaching strategies that involve smaller classroom-based group activities, quizzes, and tests. A higher level of subject content knowledge on the other hand, ensures facilitation during teaching rather than coaching learners how to solve a problem (Brijlall & Isaac, 2011).

Along with the decline in the pass rate of mathematics, many learners do not choose mathematics as a subject (Adler, 2005) and for the past five years significantly less than 50% of the matric final examination writers wrote mathematics as a subject (Shay, 2019), which in turn leads to fewer students continuing tertiary studies in the field of mathematics. Mij and Makgato (2006) comment

that students who do continue with mathematics at the tertiary level are not likely to choose teaching as a career. Thus, the students choosing teaching as a career might not necessarily be the top-performing learners at school, which sequentially could influence the quality of graduate mathematics teachers.

Based on the aforementioned evidence, the quality of initial mathematics teacher-training seems to be far from sound (Avalos et al., 2010; Durandt & Jacobs, 2014; Feza, 2014; Malaty, 2004; Maseko, 2015; May, 2017; Mij & Makgato 2006; Sorto, 2010; Taylor, 2014, 2018; Teza, 2014; Wendt, Kasper, & Trendtel, 2017). Consequently, Lam (2017), Pournara, Hodgen, Adler, and Pillay (2015), and Sorto (2010) argue that teachers do not acquire a solid understanding of the content that they have to teach to learners (Pournara et al., 2015; Sorto, 2010), lack specialised subject content knowledge (Avalos et al., 2010), and the effect of their training is annulled when they start teaching (Durandt & Jacobs, 2014). It therefore seems reasonable to argue that mathematics teachers are not meeting the expectations for competency to teach mathematics in terms of subject content knowledge and pedagogical content knowledge, which could directly be linked to the quality of teacher-training. The next section, elucidates the problem statement for the study.

1.2 PROBLEM STATEMENT

Studies concerning the effectiveness of initial mathematics teacher-training programmes are limited (May, 2017). An important and relevant study conducted by Taylor (2014) that focused on a broad overview of the subject content and pedagogical content offered to pre-service teachers at five universities in South Africa reported that the universities do not fully meet the challenge to prepare teachers that would be able to uplift the low quality school system concerning mathematics.

From the preceding line of inquiry (cf. 1.1), and the study of Taylor (2014), the researcher identified the following problem: teacher-training seemingly does not enable pre-service teachers to meet the competency expectations concerning mathematical subject content knowledge and pedagogical knowledge which affect their ability to teach effectively (Adedoyin, 2011; Avalos et al., 2010; Malaty, 2004; Mij & Makgato 2006; Taylor, 2018), and subsequently learners' achievement in mathematics is hampered (Farooq et al., 2011; Fung et al., 2017; Plotz et al., 2012). This study continues and extends the aforementioned line of inquiry by exploring the interaction between teacher-training, and the acquisition of subject content knowledge and pedagogical content knowledge among pre-service mathematics teachers at a university in South Africa.

To the best knowledge of the researcher, no studies have yet been done in the South African context to determine to what extent pre-service mathematics teacher-training enables prospective teachers to meet competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge as foregrounded in the Curriculum and Assessment Policy Statement (CAPS) – the policy document guiding the teaching of mathematics in South Africa. For this reason, the study compared the competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge addressed in the current mathematics teacher-training curriculum used at the North-West University, Vanderbijlpark Campus, with what is expected in the CAPS. The study identified three areas according to which competency expectations among pre-service teachers could be viewed:

- Competency expectations related to equipping learners at school with factual, conceptual, procedural, and metacognitive knowledge.
- Competency expectations concerning the possession of common, specialised, and horizon mathematics subject knowledge as a teacher.
- Competency expectations in applying a variety of pedagogical approaches during teaching.

In this study, the focus was on competency expectations related to equipping learners with factual, conceptual, procedural, and metacognitive knowledge, as well as competency expectations in applying a variety of pedagogical approaches during teaching.

1.3 RESEARCH QUESTIONS

The following primary research question drove the research:

To what extent does the teacher-training curriculum for the Intermediate (Gr. 4-6), Senior (Gr.7-9) and Further Education and Training (FET) (Gr. 10-12) Phases at the North-West University (NWU), Vanderbijlpark Campus, prepare mathematics teachers to meet the competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge foregrounded in the CAPS for teaching mathematics?

Within the primary question, the following secondary questions unfolded.

- What mathematical subject content knowledge should form part of a teacher-training curriculum in mathematics?
- What pedagogical content knowledge should form part of a teacher-training curriculum in mathematics?

- To what extent does the teacher-training curriculum at the NWU, Vanderbijlpark Campus prepare pre-service mathematics teachers to meet competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge?
- How qualified and experienced are lecturers who train pre-service mathematics teachers at the NWU, Vanderbijlpark Campus to prepare pre-service mathematics teachers to meet the competency expectations concerning subject content and pedagogical content knowledge?
- How do pre-service mathematics teachers and mathematics lecturers involved in teacher training at the NWU, Vanderbijlpark Campus perceive the quality of the mathematics teacher-training curriculum?
- Which curriculum-related recommendations could be made to strengthen, adapt or enrich the current mathematics teacher-training curriculum used at the NWU, Vanderbijlpark Campus to ensure that pre-service teachers meet the competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge?

The researcher wishes to mention that although the research focused on one of the campuses of the NWU, the teacher-training curriculum for mathematics, also applies to one of the other NWU campuses; the Potchefstroom Campus. The recommendations emanating from the research could therefore be considered by both campuses.

1.4 AIM AND OBJECTIVES OF THE STUDY

From the research questions, the following aim and objectives were formulated for the study.

1.4.1 Aim

The aim of the study was to determine to what extent the teacher-training curriculum for the Intermediate (Gr. 4-6), Senior (Gr. 7-9) and the FET (Gr. 10-12) phases at the NWU, Vanderbijlpark Campus, prepares mathematics teachers to meet the competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge foregrounded in the CAPS for teaching mathematics.

1.4.2 Objectives

The aim was operationalised in the following objectives:

- To identify what mathematical subject content knowledge should form part of a teacher-training curriculum. This objective was achieved with a literature review and by qualitatively comparing the subject content knowledge foregrounded in the CAPS with the subject content knowledge specified in the teacher-training curriculum.

- To establish what pedagogical content knowledge should form part of a teacher-training curriculum in mathematics. This objective was achieved with a literature review and by qualitatively comparing the pedagogical content knowledge foregrounded in the CAPS with the pedagogical content knowledge specified in the teacher-training curriculum.
- To evaluate to what extent the teacher-training curriculum at the NWU, Vanderbijlpark Campus, prepares teachers to meet competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge. This objective was achieved as follows:
 - Quantitatively by using test data to establish levels of subject content knowledge, preparedness to teach, and the adequacy of subject content and pedagogical content knowledge.
 - Qualitatively by using open questions in the test that asked the pre-service teachers to reflect on their metacognitive skills and the pedagogical strategies they have to teach mathematics.
 - Qualitatively by comparing the CAPS and teacher-training curricula to establish convergence about competency expectations.
- To determine how qualified and experienced lecturers who train pre-service mathematics teachers at the NWU, Vanderbijlpark Campus are, to prepare pre-service teachers to meet the competency expectations concerning subject content and pedagogical content knowledge. This objective was achieved by obtaining lecturers' responses to open items in a questionnaire.
- To explore how pre-service mathematics teachers and mathematics lecturers involved in teacher training at the NWU, Vanderbijlpark Campus perceive the quality of the mathematics teacher-training curriculum. This objective was achieved by exploring qualitative written reflection data obtained from lecturers and pre-service teachers in response to open items in a questionnaire (lecturers) and open items as part of the student tests (pre-service teachers).
- To make curriculum-related recommendations that could strengthen, adapt, or enrich the current pre-service mathematics teacher-training curriculum used at the NWU, Vanderbijlpark Campus, to ensure that teachers meet the competency expectations concerning mathematical subject content and pedagogical content knowledge. This objective was achieved by making recommendations based on the findings obtained with the quantitative and qualitative data collection.

The following section elucidates the conceptual and theoretical frameworks applied in the study.

1.5 THEORETICAL AND CONCEPTUAL FRAMEWORKS

The broad relationships between the variables as revealed in the literature, and the assumptions of the researcher that play a role in a study, make up the theoretical framework of the study (Imenda, 2014; Kachchhap & Mishika, 2015; Regoniel, 2010). A conceptual framework sheds light on the variables that were explored in a study and their interrelatedness (Regoniel, 2010).

1.5.1 Theoretical framework

The theoretical framework of Hill et al. (2008) guided the researcher's understanding of the variables that constitute the competency expectations about the qualities of a teacher's subject content knowledge in mathematics and pedagogical content knowledge for teaching mathematics. The framework of Hill et al. (2008) is presented in Figure 1.1

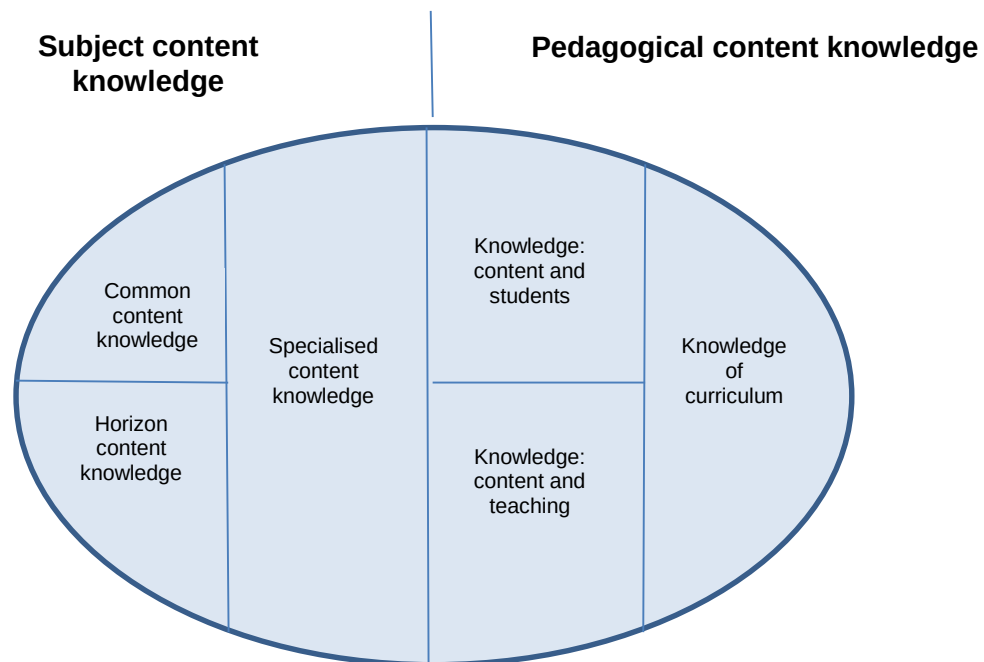


Figure 1.1: Competency expectations concerning subject content and pedagogical content knowledge (Hill et al., 2008, p. 377)

According to Figure 1.1, meeting expectations for competency to teach subject content knowledge to learners, requires from teachers to ensure that their mathematical subject content knowledge comprises common subject content knowledge, specialised subject content knowledge and horizon subject content knowledge (cf. 1.1, 2.3.4). Solid subject content knowledge will enable teachers to equip learners with factual, conceptual, procedural, and

metacognitive knowledge that they have to acquire in their respective grades (Department of Basic Education, 2018; Krathwohl, 2002) (cf. 2.3.2.1 – 2.3.2.4).

In conjunction to the framework of Hill et al. (2008), the revised taxonomy of Bloom (Anderson et al., 2001) provided the theoretical framework for understanding and qualifying the competency expectations concerning the variables that constitute subject content knowledge that learners need to acquire, namely: factual knowledge, conceptual knowledge, procedural knowledge, and metacognitive knowledge (cf. 2.3). In conjunction with the four subject knowledge dimensions, the following cognitive process dimensions should be acquired, namely: remembering, understanding, analysing, applying, evaluating, and creating.

The competency expectations concerning pedagogical content knowledge involve knowledge of subject content and how students learn, knowledge of subject content, and how to best teach the subject content, as well as subject knowledge and the curriculum that involves planning instruction to achieve and assess learning outcomes (cf. 2.4). The researcher explains the revised taxonomy of Bloom, and clarifies its application in the study in Chapter 2 (cf. 2.3.1).

1.5.2 Conceptual framework

The conceptual framework indicates how the researcher investigated the research problem in an integrated manner by delineating from the theoretical framework the concepts on which the research focused. In the study, the relationship between the concepts implied that meeting competency expectations to become effective at mathematics teaching implies solid subject content knowledge and solid pedagogical content knowledge.

The expectations for competency about subject content knowledge are viewed two folded, namely:

- subject content knowledge learners need to achieve (factual, conceptual, procedural, and metacognitive) (cf. 2.3.2.1 – 2.3.2.4), and
- subject content knowledge teachers need to possess (common content, specialised content, and horizon content knowledge) (cf. 2.3.4). Although the latter is important, it was not part of the aim and objectives of the study.

To enable learners to master subject content knowledge effectively, teachers need to possess subject content knowledge in conjunction with knowledge about learners and their learning, knowledge about teaching approaches, and knowledge about planning to teach and assessing the achievement of outcomes (pedagogical content knowledge). In this study, the teaching approaches to teach subject content knowledge in mathematic

s were foregrounded, and considered direct teaching approaches, indirect teaching approaches, independent teaching approaches, and interactive teaching approaches (cf. 2.4.1 – 2.4.4).

The researcher clarifies how the concepts and phrases central to the study were understood in the section below.

1.5.2.1 Concepts and phrases central to the study

- ***Teacher-training curriculum and CAPS***

The focus was on the pre-service mathematics teachers-training curriculum presented at the NWU, Vanderbijlpark Campus, which comprised the following subject content modules (MATE 111/121; MATE 211/221; MATE 311/321 (Senior and FET mathematics); MALA 211/221; MALA 311/321 (Intermediate Phase mathematics) and pedagogical content modules presented to all students (MATD 211; MATD 321; MATD 411, 421, MATD 413 (only Intermediate Phase students) (cf. 2.4.6). To assess the quality of the subject content and pedagogical content knowledge in the teacher-training curriculum, the CAPS for mathematics that specifies what should be taught at the school-level in the Intermediate, Senior and FET phases were used as guiding documents (cf. 2.3.5). The researcher presents a synthesis of the comparison between the two curricula in Chapter 2, sections 2.3.5 (subject content knowledge) and 2.4.6 (pedagogical content knowledge).

- ***Mathematical subject content knowledge: Learners***

The mathematical subject content knowledge learners need to acquire involve the following dimensions.

Factual knowledge

The basic facts in a specific discipline that students should know to enable them to solve problems in the discipline:

- knowledge of terminology; and
- knowledge of specific details and elements (Anderson et al., 2001) (cf. 2.3.2.1).

Conceptual knowledge

Understanding the interrelationships among basic facts in a specific discipline:

- knowledge of classifications and categories;
- knowledge of principles and generalisations; and
- knowledge of theories, models and structures (Anderson et al., 2001) (cf. 2.3.2.2).

Procedural knowledge

The application of the information in a specific discipline. How to do something; methods of inquiry; and criteria for using skills, algorithms, techniques, and methods:

- Knowledge of subject-specific skills and algorithms;
- Knowledge of subject-specific techniques and methods; and
- Knowledge of criteria for determining when to use appropriate procedures (Anderson et al., 2001) (cf. 2.3.2.3).

Metacognitive knowledge

Having knowledge and an awareness of one's thinking:

- Strategic knowledge;
- Knowledge about cognitive tasks, including appropriate contextual and conditional knowledge; and
- Self-knowledge (Anderson et al., 2001) (cf. 2.3.2.4)

The application of several cognitive process dimensions is always embedded in subject content knowledge. These skills involve lower-order skills such as remember, understand, and analyse, as well as higher-order skills such as apply, evaluate, and create (Anderson et al., 2001; Krathwohl, 2002) (cf. 2.3.3). Addendums E and F provide an analysis of the subject content knowledge dimensions and cognitive process dimensions that learners need to acquire according to the CAPS and teacher-training curriculum, respectively.

• *Mathematical subject content knowledge: Teachers*

In the context of the study, expectations for competency about the qualities of teachers' mathematical subject content knowledge were conceptualised as three dimensions, namely: common content knowledge (basic knowledge), specialised content knowledge (knowledge that reflects depth and breadth) (Pimentel et al., 2007, p. 1), and horizon subject content knowledge that emphasises insight into the interrelatedness of mathematics topics (Hill et al., 2008) (cf. 2.3.4). Although the research did not explicitly assess these knowledge dimensions during the research, the researcher was able to make preliminary deductions about them, based on the responses of the student participants to the test questions that focused on mathematics subject content knowledge.

• *Pedagogical content knowledge*

In this study, pedagogical content knowledge was seen as the "how" in the transference of knowledge (Goos, 2013). Pedagogical content knowledge forms the bridge between mathematical subject content knowledge and the understanding thereof (Brodie, 2001; Park &

Oliver, 2008; Plotz et al., 2012). Section 2.4 clarifies how pedagogical content knowledge was conceptualised in the study.

The next section presents an overview of the research methodology employed in the study.

1.6 RESEARCH METHODOLOGY

The research comprised two phases, namely, a literature review and an empirical study.

1.6.1 Literature review

Various sources of information (i.e. books, journal articles, and the internet) were consulted to identify the themes that guided the execution of the study. The key concepts and phrases that were explored during the literature review are teacher-training, teacher-training curriculum, the CAPS for mathematics, mathematical subject content knowledge, knowledge of mathematics teachers, and pedagogical content knowledge of mathematics teachers.

1.6.2 Empirical study

The researcher aimed to answer the research questions by collecting relevant quantitative and qualitative data (Kothari, 2004) utilising a variety of data collection instruments (cf. 1.7.2.4).

1.6.2.1 Research paradigm

Achieving the objectives of the study required the collection of both quantitative and qualitative data. The researcher therefore employed mixed-method research framed within a pragmatic research paradigm (Taylor, Kermode, & Roberts, 2007; Teddlie & Tashakkori, 2009). A description of, and a motivation for the choice of the research paradigm, which acted as a lens through which the researcher viewed the research process, are presented in Chapter 3 (cf. 3.2).

1.6.2.2 Research design

A research design is a plan for collecting and analysing data about a certain phenomenon. A concurrent triangulation mixed methods design (Creswell, 2014) was used in the study, which is motivated and explained in Chapter 3 (cf. 3.4.1). The study included data collection with both quantitative and qualitative data collection methods to confirm, cross-validate, or corroborate findings within the study.

1.6.2.3 Strategies of inquiry

The strategy of inquiry outlines the different ways in which the data were collected.

As part of the quantitative study, the researcher used a non-experimental, descriptive research strategy to collect data from the pre-service teachers and their lecturers by using researcher-constructed tests with open and closed items for the pre-service teachers, and survey research that employed a researcher-constructed questionnaire with open items for the lecturers (McMillan & Schumacher, 2014).

The qualitative research component involved critical educational research as a strategy of inquiry (Merriam, 2009) as the researcher envisaged to critically evaluate the context that shapes the competencies of pre-service mathematics teachers by using written reflections about the teacher-training curriculum. In addition, as part of the qualitative study, a document study was employed. In the study, the CAPS and the teacher-training curriculum were critically analysed to establish evidence of the extent to which competency expectations about subject content knowledge and pedagogical knowledge in the CAPS, are supported by the teacher-training curriculum.

The use of the quantitative and qualitative strategies of inquiry are clarified and motivated in Chapter 3 (cf. 3.4.1).

1.6.2.4 Data collection methods

Quantitative data were collected through the researcher-constructed tests, as the researcher focused on collecting data about knowledge levels and opinions (closed questions), as well as establishing understanding (open questions that formed part of the test) (Wilson & McLean, cited by Cohen, Manion, & Morrison, 2000).

Three self-constructed tests with open and closed items were administered to establish pre-service teachers' levels of subject content and pedagogical knowledge, across the FET, Senior and Intermediate teaching phases respectively. Closed items probed the participants' perceptions/opinions concerning their preparedness to teach, and the availability of teaching strategies to teach. Open items were added to the participants' tests, to explore the pre-service teachers' strategies to apply metacognition and the pedagogical strategies they have available to teach subject content. Also, the open questions explored the participants' perceptions about the strengths and weaknesses of the teacher-training curriculum; the questions which they found the easiest and most difficult, changes to improve the teacher-training curriculum, and possible reasons why they would view themselves as good or bad teachers.

As part of the qualitative study, a document study (Guion, 2002; Nieuwenhuis, 2007a; Strydom & Delport, 2002, 2011) was done to explore whether the competency expectations concerning mathematical subject content knowledge and pedagogical knowledge in the CAPS are addressed in the current mathematics teacher-training curriculum used at the NWU, Vanderbijlpark Campus (cf. 2.3.5, 2.4.6).

The lecturers received a questionnaire with open items to understand their training, qualifications, and experience in teaching/lecturing mathematics. Also, the lecturers' perceptions about the strengths and weaknesses, and limitations of the teacher-training curriculum as well as the adequacy of subject content knowledge and pedagogical content knowledge in the teacher-training curriculum, were explored using open items in a questionnaire.

The quantitative and qualitative data collected were triangulated and integrated to obtain a comprehensive understanding of the research problem (cf. 4.6). The quantitative and qualitative data-collection procedures and their application, are explained in detail in Chapter 3 (cf. 3.5).

1.6.2.5 Population and sampling

For the quantitative study, a purposive and convenient sample was chosen (Palys, 2008), which comprised a heterogeneous group of fourth-year pre-service teachers with mathematics as a major subject at the NWU, Vanderbijlpark Campus (n = 21). Nineteen participants were willing to take part in the study and comprised the following groups: FET and Senior Phase (n = 7), and Intermediate Phase (n = 12). Four of the mathematics lecturers on the Vanderbijlpark Campus (n = 4), who were all white, female lecturers from different age groups, with different qualifications and experience, took part in the study.

All the pre-service teachers who completed the tests took part in the completion of the qualitative written narratives. A purposeful selection of documents was sampled for the qualitative part of this study. The CAPS and the teacher-training curriculum concerning the following teaching phases, namely the Intermediate Phase (Gr. 4-6), the Senior Phase (Gr. 7-9), and the FET Phase (Gr. 10-12) for study years 1 to 4, were the focus of the qualitative document analyses.

A detailed description and motivation of the sampling process used in the study follow in Chapter 3 (cf. 3.9).

As the researcher works in China, the recruitment procedure, obtaining informed consent, and the data collection, took place with the assistance of an independent researcher as explained in the informed consent forms (cf. Addendums K1 and K2).

1.6.2.6 Quality criteria

The researcher upheld criteria for reliability and validity for the quantitative study and ensured that the qualitative component of the study complied with criteria for trustworthiness.

Reliability and validity: Quantitative study

A small-scale and informal dynamic pilot study was conducted before data collection (Brace, 2008) with three mathematics lecturers who were identified for this purpose, and who on a rotation basis evaluated the researcher's constructed tests.

The following criteria identified by McMillan and Schumacher (2014) and Leedy and Ormrod (2005) were taken into consideration to ensure the validity of the quantitative research design, namely: internal validity, external validity, construct validity and statistical conclusion validity.

The validity of the data collection instruments was guaranteed by adhering to criteria for face validity, content validity, construct validity, and criterion validity (Leedy & Ormrod, 2014).

Trustworthiness: Qualitative study

To guarantee the trustworthiness of this study the researcher adhered to the criteria identified by Babbie and Mouton (2002), and Lincoln and Guba (1985), which included credibility, dependability, transferability, and conformability.

How the researcher adhered to criteria for reliability and validity as well as trustworthiness in the study, is clarified in Chapter 3 (cf. 3.6, 3.7).

1.6.2.7 The role of the researcher in qualitative research

According to Creswell (2014), a qualitative researcher has to consider the following ethical and personal issues that could compromise the collection of trustworthy data, namely: historical, social and cultural experiences, status, race, gender, assumptions, personal connection to the site and sensitive ethical issues. The researcher only regarded assumptions and personal connection to the site as issues that could have compromised the collection of data. The researcher explains in Chapter 3 (cf. 3.8), what she did to avoid the above-mentioned aspects compromising data collection.

1.6.2.8 Data analysis

Quantitative data were analysed utilising descriptive statistics with frequencies, percentages, and means (Leedy & Ormrod, 2014) for the various responses to the tests and closed questionnaire items. The responses of all questions were analysed in terms of factual, conceptual, and procedural knowledge, that involved the application of cognitive process actions the participants had to perform in answering the questions (Tables 4.8, 4.10, 4.12, 4.14, 4.16, 4.18, 4.20 (FET); 4.22, 4.24, 4.26, 4.28, 4.30 (Senior); 4.32, 4.34, 4.36, 4.38, 4.40 (Intermediate)).

The various curricula were analysed qualitatively by using a-priori codes (Nieuwenhuis, 2007b) as identified in the literature. The following aspects were utilised for the objective and structured analysis of the various curricula (cf. Addendums E, F), to explore evidence concerning competency expectations related to the following:

- Desired learning outcomes related to subject content would be the successful attainment of knowledge in the factual, conceptual, procedural, and metacognitive dimensions, as well as mastering the cognitive process dimensions' required of learners at school level in conjunction with subject content knowledge (cf. Addendums E, F).
- The time allocated to teaching different content topics as an indication of the weighting of the content in the curriculum (cf. 2.3.5).
- The teaching approaches (methods and strategies) suggested achieving learning outcomes (cf. 2.4.5).

An inductive analysis procedure (Leedy & Ormrod, 2005) was used to analyse the responses to open-ended questions posed to the pre-service teachers in the test and the lecturers in the lecturer questionnaire. Text data were examined and re-occurring themes extracted to make meaning of the participants' reflections (Creswell, 2014; Strydom & Delport, 2011).

An extensive clarification of the quantitative and qualitative data analysis procedures follows in Chapter 3 (cf. 3.10).

1.6.2.9 Ethical considerations

The researcher adhered to several ethical considerations during the execution of the study, as identified by Creswell (2009). These considerations included the following: ethical issues in the research problem, ethical issues in the purpose and questions, ethical issues in data collection, ethical issues in data analysis and interpretation, and ethical issues in writing and disseminating the research (Creswell (2009).

The researcher explains in Chapter 3 (cf. 3.11) how she dealt with the ethical considerations in practical terms, and the informed consent form for participants is attached in Addendums K1 and K2.

1.6.2.10 Chapter layout

The study unfolded according to the following structure:

Chapter 1: Introduction and motivation of the study

Chapter 2: Literature study: Competency expectations for pre-service teachers: Mathematical subject content knowledge and pedagogical content knowledge

Chapter 3: Empirical research design

Chapter 4: Data analysis and interpretation

Chapter 5: Summary, findings, and recommendations

1.7 CHAPTER SUMMARY

Chapter 1 provided an outline for achieving the aim of the study, namely to determine whether the teacher-training curriculum at the NWU, Vanderbijlpark Campus prepares pre-service mathematics teachers with mathematical subject content knowledge and pedagogical content knowledge to meet the competency expectations outlined in the CAPS for teaching mathematics (cf. 1.4). To achieve the aim of the study, concurrent triangulation mixed-method research was employed, and data collected from fourth-year pre-service mathematics teachers and their lecturers utilising researcher-constructed tests and questionnaires with closed and open items, as well as a document study.

Quantitative data were analysed with descriptive statistics, and a content analysis was employed to understand the qualitative data. Subsequently, the researcher aimed to suggest recommendations to strengthen, adapt, or enrich the current mathematics teacher-training curriculum used at the NWU (cf. 1.4 -1.6).

The theoretical framework of Hill et al. (2008) guided the researcher's conceptualisation of the competency expectations concerning teachers' subject content knowledge in mathematics and pedagogical content knowledge for teaching mathematics (cf. 1.5.1). The revised taxonomy of Bloom (Krathwohl, 2002) provided the theoretical framework for understanding and qualifying the competency expectations concerning the subject content knowledge that learners have to acquire in mathematics, namely factual, conceptual, procedural and metacognitive knowledge dimensions that have to be developed in conjunction with cognitive process dimensions (cf. 1.5.1).

In Chapter 2, the competency expectations for pre-service mathematics concerning subject content and pedagogical content knowledge are explored.

CHAPTER 2: COMPETENCY EXPECTATIONS OF PRE-SERVICE TEACHERS: MATHEMATICAL SUBJECT CONTENT KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE

2.1 INTRODUCTION

This chapter aims to provide a comprehensive overview of the expectations about the competencies of pre-service mathematics teachers regarding subject content knowledge and pedagogical content knowledge. These expectations, which were identified in Chapter 1, involve the following:

- Teachers need to equip learners with factual, conceptual, procedural, and metacognitive knowledge (cf. 2.3).
- Teachers need to possess common, specialised, and horizon mathematics subject knowledge as a teacher (cf. 2.3). This aspect was not explicitly investigated in the study.
- Teachers need to apply a variety of pedagogical approaches during teaching (cf. 2.4).

In summary, the chapter addresses the following issues:

2.2 EPISTEMOLOGICAL BELIEFS AND THE TEACHING OF MATHEMATICS

2.2.1 Introduction

2.2.2 The importance of epistemological beliefs and teaching mathematics

2.2.3 Epistemological beliefs about the nature and acquisition of knowledge

2.2.4 The significance of epistemological beliefs for the teaching and learning of mathematics

2.2.5 Epistemological belief systems for teaching and learning mathematics

2.3 A THEORETICAL FRAMEWORK FOR CONCEPTUALISING COMPETENCY EXPECTATIONS FOR TEACHING SUBJECT CONTENT KNOWLEDGE IN MATHEMATICS

2.3.1 Competency expectations concerning subject content knowledge for teaching mathematics: the South African framework and Bloom's revised taxonomy

2.3.2 The conceptualisation of subject content knowledge according to Bloom's revised taxonomy

2.3.2.1 Factual knowledge and epistemological beliefs

2.3.2.2 Conceptual knowledge and epistemological beliefs

2.3.2.3 Procedural knowledge and epistemological beliefs

2.3.2.4 Metacognitive knowledge and epistemological beliefs

2.3.2.5 The relationship between the different subject content knowledge dimensions in mathematics

2.3.3 Integrating subject content knowledge dimensions and cognitive process dimensions during the teaching of mathematics

2.3.4 Competency expectations concerning the qualities of teachers' mathematical subject content knowledge

2.3.4.1 Common subject content knowledge

2.3.4.2 Specialised subject content knowledge

2.3.4.3 Horizon subject content knowledge

2.3.5 Mathematical subject content knowledge in the CAPS and the teacher training curriculum

2.4 A THEORETICAL FRAMEWORK OR CONCEPTUALISING COMPETENCY EXPECTATIONS ABOUT PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING MATHEMATICS

2.4.1 Direct teaching

2.4.2 Indirect teaching

2.4.3 Independent teaching

2.4.4 Interactive teaching

2.4.5 Learning-centred teaching

2.4.6 Pedagogical content knowledge in the CAPS and mathematics teacher training curriculum

2.4.7 Competency expectations for pre-service mathematics teachers: the researcher's view

The next section examines the role of epistemological beliefs in mathematics. Although the epistemological beliefs of the participants were not explicitly investigated in the study, the researcher used the information extracted from the literature review to make preliminary conclusions about the extent to which the absence or presence of an epistemological belief might

have contributed to meeting expectations for being competent regarding subject content and pedagogical content knowledge.

The researcher could not find any explicit evidence that specifies epistemological beliefs as a competency expectation for teachers who wish to teach mathematics. However, from the literature it is clear that epistemological beliefs influence how subject content is viewed and taught (Depaepe, De Corte, & Verschaffel, 2016; Schoenfeld, 1992; Soulios & Psillos, 2016). How teachers view the subject mathematics will influence how they will understand and teach the subject (Plotz et al., 2012), as well as the type of subject content knowledge they will favour, namely factual knowledge, conceptual understanding, or procedural application (Kanyongo & Brown, 2013, p. 108). This chapter, therefore, firstly presents an overview of the importance of developing an epistemological belief(s) and the role that epistemological beliefs play in understanding the nature of mathematics subject content knowledge and which pedagogical strategies to employ to enable learners to acquire subject content knowledge.

2.2 EPISTEMOLOGICAL BELIEFS AND THE TEACHING OF MATHEMATICS

2.2.1 Introduction

Epistemological beliefs are seen as the knowledge of how an individual distinguishes what is truthful (Sing & Khine, 2008), what could be regarded as facts and opinions (Green & Hood, 2013), and how it is best acquired (Schraw & Olafson, 2003). Brownlee et al. (2016), and Lodewyk (2015) suggest that an epistemological belief is inseparable from the way one teaches. The following section pays attention to different epistemological beliefs that could play a role in the teaching of mathematics.

2.2.2 The importance of epistemological beliefs and teaching mathematics

Epistemological beliefs are developed by different aspects that influence the psychological framework of an individual, for example, socio-economic background, culture, geographical location, gender, education, etcetera (Brownlee & Berthelsen, 2008; Brownlee et al., 2016).

In the following sections, the researcher explores three main general epistemological belief systems, namely: realist, relativist, and contextualist (Schraw & Olafson, 2003).

2.2.3 Epistemological beliefs about the nature and acquisition of knowledge

2.2.3.1 The realist epistemological belief system

Realism supports a dualistic approach to knowledge, where knowledge is viewed as either true or false, which means that only one truth exists (Brownlee et al., 2016; Green & Hood, 2013;

Päuler-Kuppinger & Jucks, 2017; Sing & Khine, 2008), and postulates that truth is the knowledge that can be acquired by observation or the study of facts (Cohen, 1999). Within education, a realist curriculum focuses on the systematic study of the subject content, teaching focuses on the direct transmission of content by teachers to learners (cf. 2.4.1), and repetition as a method of study to remember the prescribed subject content by passive learners (Brownlee et al., 2016; Cohen, 1999; Green & Hood, 2013; Schraw & Olafson, 2003). According to Brownlee et al. (2016), and Green and Hood (2013), teachers believe there is one true way of answering questions or solving problems.

A realist pedagogy is therefore underpinned by behaviourist learning theory that is based on three assumptions, namely: learning involves a *change in behaviour* that is shaped by the *environment* through *conditioning* (Arends, 2009). The realist epistemology differs from the relativist and the contextualist epistemologies which are more considerate of the 21st-century thinking skills and dispositions that learners and pre-service teachers need to develop.

2.2.3.2 The relativist epistemological belief system

Relativism is grounded in differentiation between cultures, languages, gender, attitudes, norms, and values (Baghrarian, 2004; Greiffenhagen & Sharrock, 2006). Moreover, relativism claims that there are multiple truths, and that the value of the truth depends on the perspective of a person (Baghrarian, 2004; Brownlee & Berthelsen, 2008; Green & Hood, 2013; Schraw & Olafson, 2003; Stoel et al., 2017), that might change as a person acquires more knowledge over time (Borgerding, Deniz, & Anderson, 2017). In a classroom context, learners may have different perspectives on what is right or wrong, making learning very complex (Alexander & Winne, 2012). Green and Hood (2013) refer to the relativist epistemology as a multiplistic view on knowledge; acknowledging there is more than one way to solve a problem, although it is still believed that there is only one correct answer (Alexander & Winne, 2012).

Greiffenhagen and Sharrock (2006, p. 4-5) mention that different cultures could have differences in counting principles or variation in numbers:

French call 80 “quatre-vingt” (which could be ‘represented’ as 4×20) while the English call it “eighty” (8×10); while the English group three digits together (thousands) the Chinese group four digits together (ten thousand, wan in Chinese), hence “fifty thousand” ($50 \times 1,000$) in English but “wu wan” ($5 \times 10,000$) in Chinese; and the Oksapmin have devised a body counting system up to twenty-seven, rather than the Western finger-counting up to ten.

A relativist focuses on individual project-based learning, and learners are encouraged to construct their understanding of the content independently (cf. 2.4.3) (Ertmer & Newby, 2013; Schraw & Olafson, 2003). Teaching is framed within the cognitivist learning theory (Wallace, Ross, Davies,

& Anderson, 2007), viewing the learner as an active, organised processor of information, and the teacher merely a facilitator who provides feedback when necessary (Lodewyk, 2011).

2.2.3.3 The contextualist epistemological belief system

According to contextualist epistemology, knowledge changes as the need for change arises due to a specific situation, therefore, knowledge can differ depending on the situation (Aypay, 2010; DeRose, 1992, 1999, 2009; McKenna, 2015; Pynn, 2016; Schraw & Olafson, 2003). A contextualist curriculum is mainly learner-centred and focuses on interactive, problem-based or inquiry-based learning (cf. 2.4.3, 2.4.4). Learning is viewed from a constructivist perspective, thus regarding learners as actively involved in the independent construction of knowledge (Kim, 2005), or socially engaged in understanding information (Maxim, 2010; Powell & Kalina, 2011; Schraw & Olafson, 2003) by collaborating with peers and solving problems (Bada, 2015; Lodewyk, 2011). Within the curriculum, there are set skills and knowledge that need to be mastered, but the knowledge and skills are taught to be used in real-life situations (Schraw & Olafson, 2003).

In summary, the key differences between the three epistemological worldviews concern the following. Realists believe in one truth and absolute knowledge that is transmitted. Relativists believe that there are many different truths as the knowers construct understanding. Contextualists believe that truth depends on the setting and there can be one truth within a specific setting, but not one truth above all.

The significance of the aforementioned general epistemological beliefs for teaching and learning mathematics is motivated in the following section.

2.2.4 The significance of epistemological beliefs for the teaching and learning of mathematics

Epistemological beliefs, of pre-service teachers need to develop from realistic to contextualistic to relativistic (Green & Hood, 2013; Sing & Khine, 2008), as depicted in Figure 2.1.

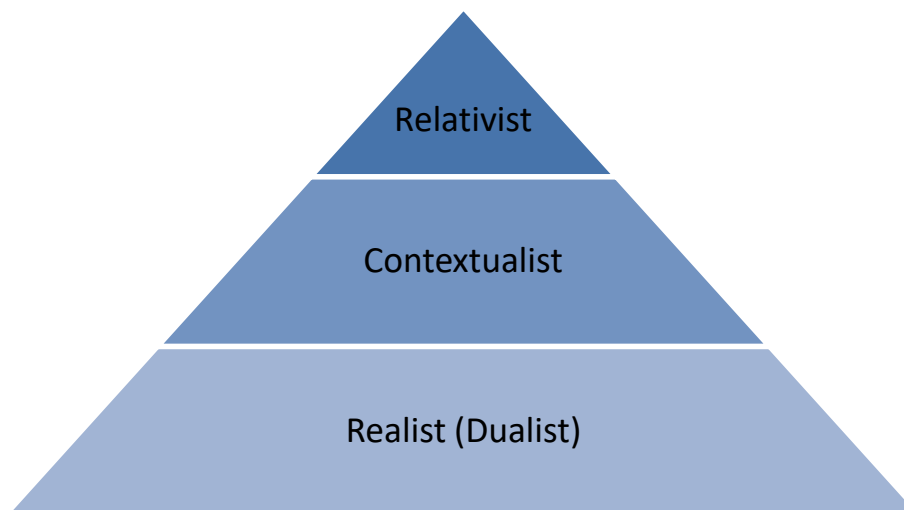


Figure 2.1: Growth of epistemological beliefs

According to Figure 2.1, the focus of the classroom needs to shift from teacher to learner (realist), to a teacher that needs to assume the role of a facilitator (contextualist) (Green & Hood, 2013), and ultimately to a classroom where the focus is on indirect or independent learning (relativist) (Green & Hood, 2013; Sing & Khine, 2008) (cf. 2.4).

Although most learners prefer realist belief system teaching and learning should challenge and enhance these realist conceptions about teaching and learning (Green & Hood, 2013). Enhanced teaching and learning that creates a critical thinking environment underpinned by contextualist and relativist beliefs will enable learners to link mathematical concepts for deeper understanding, thus doing away with rote learning.

The subsequent section aims to make the previously discussed general epistemological beliefs about the nature and transmission of knowledge specific to the mathematics classroom.

2.2.5 Epistemological belief systems for teaching and learning mathematics

It is imperative that pre-service mathematics teachers acquire knowledge about epistemological beliefs for teaching mathematics to enable them to understand the nature of the subject content knowledge and how to best teach it. This will avoid that teachers merely teach the way they were taught (Depaepe et al., 2016; Schoenfeld, 1992).

Ernest (1989) and Blömeke et al. (2008) identified the following mathematics teacher epistemologies, which are summarised in Table 2.1. The researcher links the mathematics teacher epistemologies with the previous general epistemological beliefs.

Table 2.1: Epistemological beliefs systems in mathematics

	Instrumentalist and scheme epistemology	Platonist and formalist epistemology	Problem solver and process epistemology	Application epistemology
View of mathematics	Accumulation of unrelated facts, skills, and rules.	A formal, static, exact body of logical, carefully structured, and interrelated knowledge should be discovered rather than learned.	Focus on the mathematics process rather than the product. It is seen as a dynamic and a relative human invention. Mathematics is a science conceived by problem-solving	Mathematics is primarily perceived as relevant to society and life.
Belief orientation	Absolutist (realist)	Absolutist (realist), relativist	Fallibilist (relativist, contextualist)	Fallibilist (relativist)

From Table 2.1, it can be concluded that an instrumentalist (Ernest, 1989) and a scheme perspective (Blömeke et al., 2008) both share the belief that mathematics consists of rules, formulae, and skills. The instrumentalist and the scheme perspective are both underpinned by absolutist or realist epistemological beliefs (Depaepe et al., 2016) (cf. 2.2.3.1).

The platonist (Ernest, 1989) and the formalist epistemologies (Blömeke et al., 2008) both share the belief that mathematics is structured, logical, and formal. Both the platonist and the formalist epistemologies are underpinned by absolutist or realist beliefs (in Depaepe et al., 2016) (cf. 2.2.3.1). Besides, they also emphasise the element of knowledge discovery rather than merely acquiring knowledge through transmission (relativist epistemology) (cf. 2.2.3.2). The problem solver (Ernest, 1989) and the process (Blömeke et al., 2008) epistemologies both share the belief that mathematics has a process nature and is conceived by problem-solving (Depaepe et al., 2016); thus underpinned by fallibilist or relativist beliefs (Depaepe et al., 2016) (cf. 2.2.3.2). The application epistemology (Blömeke et al., 2008), foregrounds the importance of mathematics for living.

Baker, Czarnocha, Dias, Doyle, and Kennis (2012), assert that teachers are trained to teach mathematics using direct instruction (cf. 2.4.1) framed within a realist epistemological belief, which, according to Baker et al. (2012) is to promote factual and procedural proficiency rather than conceptual understanding. Depending on direct teaching methods only can indicate that a teacher has an absolutist or realist view of mathematics, meaning that the teacher sees mathematics as fixed where math truths are transmitted and not discovered or invented (Depaepe et al., 2016). Although there is always a place for teaching mathematics using direct, realist

teaching practices (Kilpatrick et al., 2001), mathematics teaching can therefore not only be approached from a realist stance. In this regard, Brownlee and Berthelsen (2008) postulate that most teacher training programmes are based on a cookie-cutter epistemological belief, or a realist belief (Brownlee & Berthelsen, 2008). Learners are seen as passive within their learning and knowledge is seen as only facts to be transmitted (Sing & Khine, 2008).

Holliday (2006) claims that the importance of epistemological beliefs in pre-service teachers should be enhanced through implicit and/or explicit instruction (Kistner et al., 2010; Soulios & Psillos, 2016). It would be ideal if teachers' beliefs could be altered to a fallibilist or relativist view of mathematics where mathematics knowledge is seen as fallible and open to revision (Depaepe et al., 2016). Depaepe et al. (2016) conclude that teachers with a higher level of mathematical subject content knowledge (cf. 2.3.4) will be more likely to have a fallibilist (relativist) view of mathematics. Having a fallibilist view of mathematics will also mean that problem-solving is no longer seen as a skill to be taught, but as an art to be mastered (Schoenfeld, 1992).

Education reform in mathematics advocates for the improvement of critical thinking inside classrooms (Brodie & Molefe, 2010; Kilpatrick et al., 2001; Luneta, 2013), emphasising the importance of a fallibilist approach to the teaching of mathematics. To reform how mathematics is learned and taught, the focus must be on seeking solutions, exploring patterns, and formulating conjectures (Schoenfeld, 1992). If mathematical teaching focusses on these three areas, the general idea of mathematics as a rigid or absolute will change to become dynamic and evolving (Schoenfeld, 1992), and the learning process of learners will be based on their understanding and application of knowledge (Brownlee & Berthelsen, 2008).

The next section explores the expectations for being competent concerning the dimensions of subject content knowledge that teachers need to develop among learners in the context of teaching mathematics.

2.3 A THEORETICAL FRAMEWORK FOR CONCEPTUALISING COMPETENCY EXPECTATIONS FOR TEACHING SUBJECT CONTENT KNOWLEDGE IN MATHEMATICS

In the context of the study, the revised Bloom's taxonomy of Anderson et al. (2001) guided the conceptualisation of competency expectations concerning mathematical subject content knowledge to be taught to learners. Using the revised taxonomy of Bloom for conceptualising knowledge in the context of teaching mathematics, aligns well with the framework for teaching mathematics in South Africa (Department of Basic Education, 2018) as explained in the next section.

2.3.1 Competency expectations concerning subject content knowledge for teaching mathematics: the South African framework and Bloom's revised taxonomy

The framework presented in Figure 2.2, does not replace the CAPS, but visually summarises pathways for teachers to meet competency expectations during the teaching of mathematics in South Africa.

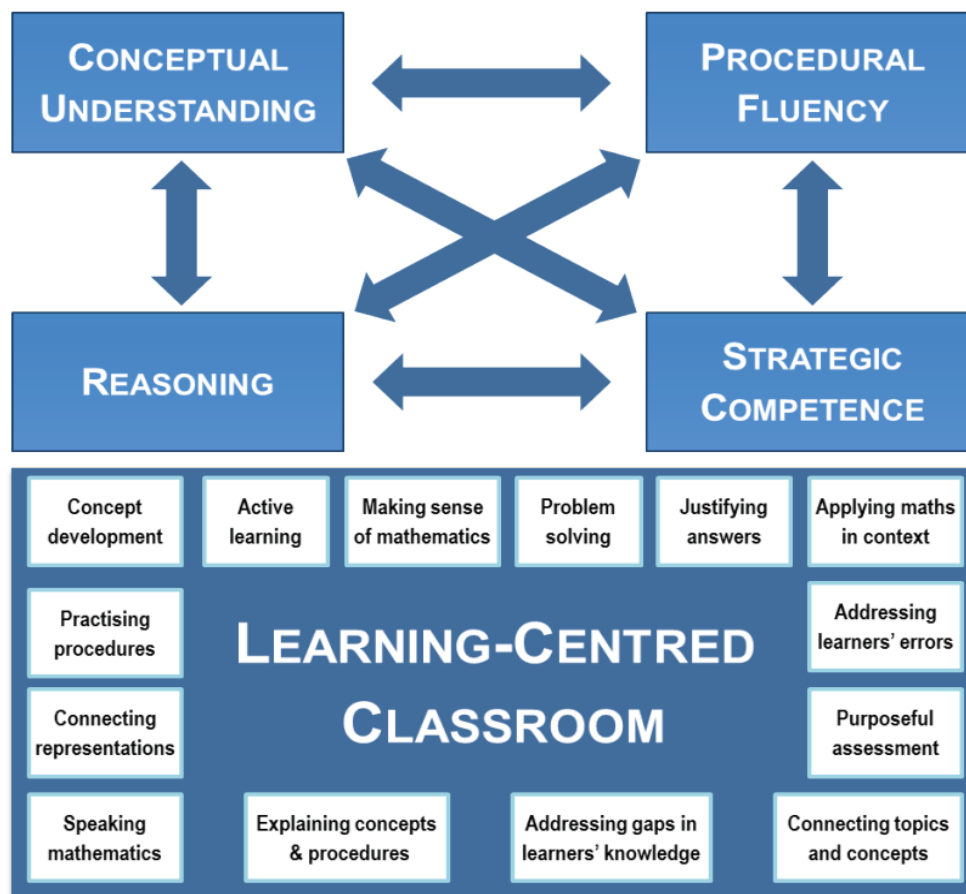


Figure 2.2: Competency expectations for teaching mathematics in South Africa (Department of Basic Education, 2018, p. 14)

According to Figure 2.2, one could argue that the competency expectations that teachers need to achieve during the teaching of mathematics involve the following:

- Conceptual understanding: a comprehension of mathematical concepts, operations and relations.
- Procedural fluency: carrying out procedures accurately and flexibly.
- Strategic competence: formulating and identifying appropriate strategies to solve problems.

- The development of reasoning skills: logical thinking, reflection about work, explanation, and justification of working ways.
- Promoting a learning-centred classroom: learning characterised by teacher and learner engagement (Department of Basic Education, 2018, p. 8). (This part of the framework will be dealt with in the section that focuses on pedagogical content knowledge) (cf. 2.4).

Linked to the aforementioned, the CAPS focuses on *knowledge*, *application*, and *problem-solving* that need to stand central to teaching mathematics (Department of Basic Education, 2011a, b, c). Although the framework in Figure 2.2 and the CAPS present a clear understanding of the competency expectations that teachers should achieve when teaching mathematics, some important knowledge dimensions, according to the researcher, appeared to be implied in the CAPS and the framework of the Department of Basic Education, such as factual and metacognitive knowledge. In particular, the conceptual knowledge dimension seems to be implied in the CAPS that only mention *knowledge*, *application*, and *problem-solving*.

The researcher believes that the factual, conceptual and metacognitive knowledge dimensions are crucial knowledge dimensions, which need to be more explicitly acknowledged, as in the revised taxonomy of Bloom (Krathwohl, 2002). A close examination of the framework presented in Figure 2.2 and the CAPS reveals meaningful links with the revised taxonomy of Bloom, which was subsequently identified as a theoretical framework for the present study. The revised taxonomy of Bloom and its aligned application to the framework of the Department of Basic Education (2011 a, b, c; 2018), is explained in the following sections.

In comparison to the original taxonomy of Bloom, the revised version of Bloom's taxonomy (Anderson et al., 2001, Krathwohl & Anderson, 2010), reflects a changed view on thinking and learning and comprises two dimensions, namely a cognitive process dimension reflecting verbs and not nouns (original taxonomy) (cf. Figure 2.3), as well as a knowledge dimension, which was not part of the original taxonomy (Krathwohl, 2002).

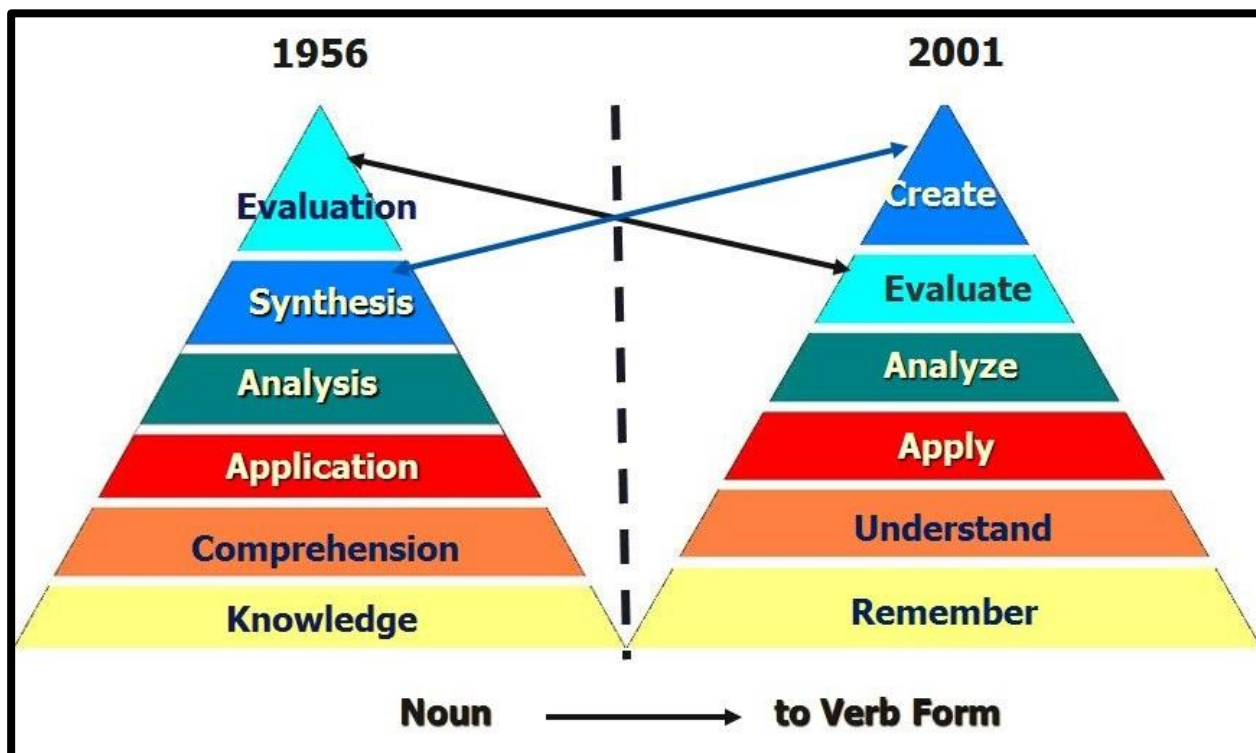


Figure 2.3: The differences between the cognitive dimensions in Bloom's taxonomy and Bloom's revised taxonomy (Wilson, 2013, p. 4)

Anderson et al. (2001, p. 214) provide the following structure (cf. Table 2.2) to indicate the increase in the difficulty level for the various cognitive process dimensions (cf. Figure 2.3) of the revised taxonomy (cf. Table 2.2). According to the revised taxonomy, remembering is viewed at the lowest and creating at the highest level of difficulty.

Table 2.2: The cognitive process dimensions of Bloom's revised taxonomy

Remember: Retrieving relevant knowledge from long-term memory.

- Recognising
- Recalling

Understand: Determining the meaning of teaching messages. Including oral and graphic communication.

- Interpreting
- Exemplifying
- Classifying
- Summarising
- Inferring
- Comparing
- Explaining

Apply: Carrying out or using a procedure in a given situation.

- Executing
- Implementing

Analyse: Breaking material into constituent parts and detecting how the parts relate to one another and an overall structure or purpose.

- Differentiating
- Organising
- Attributing

Evaluate: Making a judgment based on criteria and standards.

- Checking
- Critiquing

Create: Putting elements together to form a novel, coherent whole, or make an original product.

- Generating
- Planning
- Producing

(Krathwohl, 2002, p. 215)

Table 2.2 indicates that similar to the older version of the taxonomy, the six levels in the revised taxonomy can still be classified from simple to complex (Krathwohl, 2002). Synthesis in the original taxonomy was replaced by *creating*, while *knowledge* was replaced by *remembering*, and *comprehension* replaced by *understanding*. Similar to the older version of Bloom's taxonomy, the challenge with the revised taxonomy is again in its hierarchical six-level classification, which makes it difficult to shy away from the effect of packaging thinking accumulatively from less complex to more complex. The strict hierarchy has however been reduced by allowing overlapping between cognitive levels, for example: *understand* that replaced *comprehend* in the old taxonomy, has a much broader scope with more cognitively complex processes (Krathwohl, 2002, p. 215). The cognitive process involved in *differentiating* could for example be regarded as more complex than the cognitive process involved in *executing* that is located on the level of *applying*. However, when judging the six levels on a scale of complexity, Krathwohl (2002, p. 215) argues that one would still be able to classify them from simple to complex.

Anderson et al. (2001, p. 215) suggest the following structure (cf. Table 2.3) to clarify the new knowledge dimensions of Bloom's revised taxonomy.

Table 2.3: The knowledge dimensions of Bloom's revised taxonomy

Factual knowledge: The basic elements that learners must know to be acquainted with a discipline or solve problems in it.

- Knowledge of terminology.
- Knowledge of specific details and elements.

Conceptual knowledge: The interrelationships among the basic elements within a larger structure that enable them to function together.

- Knowledge of classifications and categories.
- Knowledge of principles and generalisations.
- Knowledge of theories, models, and structures.

Procedural knowledge: How to do something; methods of inquiry; and criteria for using skills, algorithms, techniques, and methods.

- Knowledge of subject-specific skills and algorithms.

• Knowledge of subject-specific techniques and methods • Knowledge of criteria for determining when to use appropriate procedures. <i>Metacognitive knowledge:</i> Knowledge of cognition in general as well as awareness and knowledge of one's cognition. • Strategic knowledge. • Knowledge about cognitive tasks, including appropriate contextual and conditional knowledge. • Self-knowledge.

(Krathwohl, 2002, p. 214)

Each of the four main subject content knowledge dimensions as indicated in Table 2.3, is clarified below in the context of the teaching framework for mathematics in South Africa in order to clarify the suitability of the revised taxonomy of Bloom as framework for the present study.

The researcher found existing frameworks that classify mathematics knowledge in terms of cognitive difficulty. The following frameworks have reference:

- Skemp (1976): the framework focuses on relational understanding (deep content understanding) and instrumental understanding (rote memorisation of facts and processes).
- Shulman (1986): the framework of Shulman (1986) emphasises the development of substantive knowledge (understand and explain key concepts, facts, and principles), and syntactical knowledge (understand structures underlying mathematical concepts).
- Hiebert and Carpenter (1992): the framework focuses on conceptual understanding (understand relationships among concepts and schemes), and procedural understanding (knowledge of rules, algorithms, and procedures).
- Porter (2002): the framework of Porter (2002) distinguishes between memorising, performing procedures, communicating understanding, solving non-routine problems, generalising and proving.

In support of Holmes (2012), the researcher believes that compared to the revised taxonomy of Bloom, the mentioned frameworks do not provide sufficient depth to the different knowledge levels highlighted in the revised taxonomy of Bloom. Moreover, the metacognitive knowledge dimension is not included in any of the mentioned frameworks.

The researcher is aware of the criticism against the use of the revised taxonomy of Bloom in mathematics (Long, Dunne, & De Kock, 2014; May, 2017) due to among others, the difficulty to interpret the cognitive process dimensions in the context of mathematics. As an alternative, the aforementioned authors suggest frameworks more applicable to mathematics. Long et al. (2014) propose that the knowledge dimensions of the revised taxonomy of Bloom be combined with

cognitive process dimensions proposed by Usiskin (2012), namely *skills-algorithm*, *property-proof*, *use-application*, and *representation*. As an alternative, May (2017) suggests the combination of the knowledge dimensions of the revised taxonomy of Bloom with the cognitive dimensions identified by Lithner (2008), namely *imitative* reasoning (memorised reasoning and algorithmic reasoning), and *creative mathematically founded reasoning* (local creative reasoning and global creative reasoning).

Although the frameworks suggested by Long et al. (2014) and May (2017) are applicable to mathematics, the researcher was of the opinion to work within the CAPS framework that is grounded in Bloom's taxonomy, as mathematics teaching needs to adhere to this framework. In order to address the concern about the meaningful operationalisation of the cognitive process dimensions for mathematics in the revised taxonomy of Bloom, the researcher extended the verbs that form part of the cognitive process dimensions in the revised taxonomy to more specifically reflect a mathematics context. For this purpose, the researcher examined the outcomes formulated in the CAPS that need to be achieved in each of the subject content areas (cf. Addendums E1 – E3) to extract a diverse set of verbs that would qualify the different cognitive process dimensions in the revised taxonomy of Bloom more accurately for mathematics (cf. Table 4.6).

2.3.2 The conceptualisation of mathematics subject content knowledge according to Bloom's revised taxonomy

In the subsequent sections, the researcher clarifies how the different dimensions (types) of knowledge in mathematics could be understood and highlights the importance of a variety of epistemological beliefs (cf. 2.2.2) that are required to enable learners to acquire the different dimensions of knowledge.

2.3.2.1 Factual knowledge and epistemological beliefs

Factual knowledge refers to absolute truth and a 100% certainty (Dretske, 1981) and can be seen as the subject-specific theory, terminology, algorithms and rules which aid learners in inferencing about findings (Ball et al., 2008; Kahan, Cooper, & Bethea, 2003; Löwe & Müller, 2010).

Although the framework in Figure 2.2, does not explicitly indicate factual knowledge as a knowledge dimension, the learning outcomes in the CAPS have a strong factual focus (cf. Addendums E1 - E3). Besides, the researcher views factual knowledge as fundamental to conceptual understanding, procedural fluency, and reasoning in mathematics. Learners need to know the terminology and details of each topic before they can attempt the application of knowledge. Basic theory relating to a preceding grade's content knowledge is a very fundamental

part of succeeding in mathematics. Without knowing the theory of a preceding grade, the application difficulty of new content will increase.

The acquisition of factual knowledge in mathematics aligns well with the realist (cf. 2.2.3.1), or the absolutist, instrumentalist, scheme, and formalist epistemological beliefs (cf. Table 21.), that focus on the importance of the acquisition of facts. It would therefore be important that pre-service teachers are made aware of and understand the importance of the mentioned epistemological beliefs for teaching factual knowledge.

2.3.2.2 Conceptual knowledge and epistemological beliefs

Similar to the framework in Figure 2.2, the revised taxonomy of Bloom emphasises the importance of conceptual understanding that enables one to compare, relate, and infer (Department of Basic Education, 2018).

In the context of mathematics, conceptual knowledge could be viewed as puzzle pieces. If fit together, the pieces form a whole, which connects all mathematical procedures and different ideas to form meaning (Eisenhart et al., 1993; Haapasalo, 2003; Lauritzen, 2012; Rittle-Johnson & Schneider, 2015; Star & Stylianides, 2013; Tossavainen, 2007). Conceptual knowledge therefore relates to mathematical theory, which is the foundation of the subject, as well as to basic terminology and the larger structure of mathematical concepts, such as proofs, laws, and principles (Anderson et al., 2001, Lauritzen, 2012; Rittle-Johnson, Schneider, & Star, 2015). Conceptual knowledge refers to knowing why a specific procedure/concept is necessary and understanding how the procedure/concept is applied to solve problems (Crooks & Alibali, 2014; Department of Basic Education, 2018; Lauritzen, 2012; Rittle-Johnson et al., 2015), rather than getting the applications right (Lauritzen, 2012; Rittle-Johnson et al., 2015; Stols, 2013). Conceptual knowledge is therefore important to derive a deeper understanding of procedural knowledge.

A correct solution therefore relates to good procedural knowledge, but not necessarily to conceptual understanding (Lauritzen, 2012; Stols, 2013). Knowing the different methods to solve problems, is considered conceptual knowledge, where knowing how to apply each procedure the right way is considered procedural knowledge (Crooks & Alibali, 2014; Ginsburg, 1997; Rittle-Johnson et al., 2015; Star & Stylianides, 2013). Conceptual knowledge provides the learners with flexible problem-solving skills and a way to reflect on their answers, and to check if the answer is reasonably correct (Crooks & Alibali, 2014, Lauritzen, 2012).

With the analysis of the CAPS curriculum, the researcher, in support of Lauritzen (2012) and Stols (2013), did not find evidence that the conceptual knowledge dimension is singled out as an individual knowledge dimension, but rather integrated with procedural knowledge. As a result, the

researcher feels that teachers might not focus on the details of different concepts, but rather on the application thereof. Without a deeper understanding of why a concept is used mathematics becomes nothing more than rote learning of procedural knowledge (Lauritzen, 2012; Rittle-Johnson et al., 2015). Stols (2013) asserts that the depth to which concepts are explained to learners appears to be left to the discretion of the teacher.

Conceptual knowledge in mathematics is important so that learners can also understand the relationships between applications and therefore link different subject areas to mathematics (Rittle-Johnson et al., 2015). An example of a subject that can be linked to mathematical functions is economics – more specifically economic curves (Lauritzen, 2012). A better understanding of various representations of functions (equation, graph, table) can help learners to interpret functions in economics better (Lauritzen, 2012).

Teachers need efficient knowledge – both pedagogical and subject content knowledge to be effective at developing conceptual knowledge (Long et al., 2014; Stols, 2013). To teach learners conceptual knowledge, a multiplistic approach (cf. Table 2.1) is suitable as it values more than one truth. For conceptual knowledge, one needs to understand that many different methods can lead to the same solution if applied correctly, but one also needs to know that some methods will be easier to apply and solve the problems (or less time consuming) than others.

2.3.2.3 Procedural knowledge and epistemological beliefs

Aligned to the framework in Figure 2.2, which argues for procedural fluency, the revised taxonomy of Bloom proposes a procedural knowledge dimension too.

In the context of mathematics, procedural knowledge refers to being skilled in the mathematical computation, and knowledge of procedures for identifying mathematical components, rules, algorithms, definitions, procedures, and routine steps to solve problems (Eisenhart et al., 1993; Haapasalo, 2003; Rittle-Johnson et al., 2015; Star & Stylianides, 2013; Tossavainen, 2007). In other words, procedural knowledge can be seen as the knowledge of identifying a problem, and knowing how to correctly solve it (Crook & Alibali, 2014; Department of Basic Education, 2018; Eisenhart et al., 1993; Rittle-Johnson, et al., 2015; Star & Stylianides, 2013). Learners need to have solid factual and conceptual knowledge before procedural knowledge can be developed (Rittle-Johnson et al., 2015; Schoenfeld, 1992), as procedures are based on mathematical principles (Rittle-Johnson et al., 2015). Procedural tasks need to include familiar and unfamiliar problems where learners will require either recognition that a known procedure is applicable or make small adaptations to a known procedure to accommodate the unfamiliar problem (Canobi, Reeve, & Pattison, 1998; LeFevre et al., 2006; Renkl, Stark, Gruber, & Mandl, 1998; Rittle-Johnson, 2006; Rittle-Johnson & Schneider, 2015; Schneider & Stern, 2010).

Building on the aforementioned, Hiebert (1986) and Star and Stylianides (2013) view procedural knowledge as knowledge of the format and the syntax of the symbol representation system. Lauritzen (2012) defined procedural knowledge as the knowledge of forms. Knowledge of forms relates to the recognition of, for example, the correct form of a function, or to the application of step-by-step algorithms and knowing that one step determines the next (Eisenhart et al., 1993; Lauritzen, 2012; Rittle-Johnson et al., 2015; Star & Stylianides, 2013). Teaching procedural knowledge to learners include exemplified step-by-step presentation of rules and algorithms along with strategies for remembering them (Eisenhart et al., 1993, p. 9).

According to Eisenhart et al. (1993), Porter (1989) and Stols (2013), emphasis is put on procedural knowledge in most schools where learners are expected to know the application of rules and algorithms and the teaching of conceptual knowledge seems to be underemphasised, and found to be difficult. Teachers also seem to feel that their procedural knowledge of mathematics and mathematics teaching is much stronger than their conceptual knowledge (Eisenhart et al., 1993; Stols, 2013).

The personal epistemological stance of teachers teaching for procedural knowledge needs to be either dualistic or multiplistic (cf. Table 2.1) (Crooks & Alibali, 2014). Although there may be many different methods to solve the same problem (Crooks & Alibali, 2014), each method has only one correct answer (Crooks & Alibali, 2014). By following a dualistic approach, the learners will know why a specific procedure is used (conceptual knowledge) and know that each procedure will only have one correct answer (procedural knowledge) (Lauritzen, 2012). Many learners get an incorrect answer due to not checking the procedure they followed (Lauritzen, 2012). By teaching learners to reflect on their calculations, learners will develop some sense of metacognition (Lauritzen, 2012) (cf. 2.3.2.4).

Although the framework in Figure 2.2 emphasises conceptual understanding as a pathway in teaching mathematics, the CAPS does not seem to foreground deep conceptual understanding (cf. Table 2.5), which could lead to a superficial understanding of the content and the acquisition of steps via rote learning to solve problems without having the underlying conceptual knowledge. To cultivate a deeper understanding in learners concerning the different possible procedures that can be used in solving problems, a teacher can draw similarities between different concepts when explaining the procedures to solve problems (Lauritzen, 2012). Indicating cross conceptual similarities to learners could show learners how different concepts could be applied to the same problem (Stols, 2013), thus broadening and deepening learners' perspectives on mathematical problem-solving.

The following section pays attention to the metacognitive knowledge dimension.

2.3.2.4 Metacognitive knowledge and epistemological beliefs

The framework presented in Figure 2.2, mentions the importance of strategic competence and reasoning in mathematics. According to the Department of Basic Education (2018), strategic competence refers to learners who are capable of deciding what strategies to employ, or able to devise their own strategies to solve problems. Learners must not always depend on fixed and prescribed strategies. Also, reasoning involves reflection on work and explaining and justifying working ways. Problem-solving forms the basis of mathematics content thus requiring metacognitive learning strategies that will transfer the responsibility of learning to the learners by developing the learners' self-regulating skills (Flavell, 1979; Schoenfeld, 2008; Umaru, 2010). Therefore, although strategic competence and reasoning expect a learner to become metacognitively involved in learning, the emphasis on 'metacognition' as the main area of competence to be developed seems vague.

Metacognition is said to be a carefully planned, intentional, purposeful, and future-oriented process of thinking (Al-Kayyali & Al-Haq, 2018; Flavell, 1971). It involves self-awareness and self-regulatory action which enables one to identify or evaluate one's knowledge or gaps within one's knowledge base; thus, thinking about one's thinking (Schneider & Artelt, 2010). According to Crescenzi (2016), Flavell (1979), and Roebbers (2017), metacognition can be divided into three major parts, namely: metacognitive knowledge, metacognitive experiences, and metacognitive skills.

- **Metacognitive knowledge**

- Declarative knowledge: declarative knowledge involves the knowledge one has of one's abilities to process information. Flavell (1985, p. 15) speaks about "person knowledge"; namely how much students understand about their memories and how they learn (Sperling, Howard, & Staley, 2004, p. 118).
- Procedural knowledge: procedural knowledge is "task knowledge" (Flavell, 1985, p. 15), and comprises the knowledge that one should possess to solve problems successfully.
- Conditional knowledge: conditional knowledge is also referred to as "strategy knowledge" (Flavell, 1985, p. 15). Strategy knowledge means having the sense to know when to use specific learning strategies to solve problems successfully (Paris & Paris, 2010, p. 1240).

- **Metacognitive experiences** involve self-knowledge about how one feels (anxiety, fear, excitement, etcetera) when involved with problem-solving (Flavell, 1979; Schneider & Artelt, 2010).

- **Metacognitive skills** are required to understand problems and how to communicate the solutions to the problems verbally (Flavell, 1979; Schneider & Artelt, 2010); thus, communication ones' understanding.

The main focus of metacognition is self-regulation, to eliminate common mistakes or select the best procedures to solve problems successfully (Flavell; 1979; Schneider & Artelt; 2010), which was the focus of this study. Self-regulation is an active and constructive process where learners apply metacognitive strategies to plan and set goals for their learning, as well as monitor, regulate, control, and evaluate their thinking, motivation, behaviour, and contextual factors to ensure that learning outcomes are successfully achieved (Basso & Abrahão, 2018; Eze, 2007; Darshanand & Zimmerman, 2011; Umaru, 2010; Yeager, Mische-Lawson, & Cook, 2014; Zimmerman & Schunk, 2011). In this study, the researcher was interested to explore how the participants self-regulate their working on mathematics tasks, thus, how they plan, monitor and evaluate the completion of mathematics tasks, as posed in the tests that were used for data collection.

Reflection plays a role during the planning, monitoring, and evaluation (Paris & Paris, 2010; Zimmerman & Schunk, 2011) of learning or the completion of tasks. Through reflection, one concludes from experiences to effect a positive change in future processes to complete a task successfully (Paris & Paris, 2010; Yeager et al., 2014). The metacognitive strategies involved in planning, monitoring, and evaluation, was the focus of the study.

- **Planning**

Planning involves setting clear goals, understanding task demands, selecting appropriate strategies to complete the task (e.g. how to paraphrase or summarise critical information), rereading instructions and the task itself, and identifying unknown information and obstacles that might hamper the successful completion of a task (Bannert & Reimann, 2012; Cazan, 2012; Hartwig & Dunlosky, 2012; Zumbunn, Tadlock, & Roberts, 2011). Proper planning ensures successful task completion and a final product of quality.

- **Monitoring**

Monitoring follows the planning stage, during which a learner makes sure that he is making progress towards attaining the set goals. Monitoring includes strategies to evaluate whether work is on track and correct to achieve goals and to establish if adjustments need to be made to employ other strategies to achieve goals (Finn & Metcalfe, 2013; Schunk, 2005). Monitoring could be done by using a checklist specifying the success criteria for a task (Hattie & Donoghue, 2016). Learners might have to seek help from peers, parents, and teachers during the monitoring process.

- **Evaluation**

Finally, during the evaluation phase, the learner evaluates how successful he was in attaining goals (Paris & Paris, 2010; Zimmerman & Schunk, 2011; Yeager et al., 2014). Learners need to determine if they should modify their strategies for completing similar tasks in the future (Pintrich, 2000). During evaluation learners may find that their poor achievement might be related to a lack of motivation or that the study environment presented obstacles that influenced success (cf. Table 2.4). Acknowledging the obstacles that influence academic performance negatively, could bring about changes in the learning behaviour of the learners. They might put in more effort to enhance their motivation and study environments in the future (Schunk, 2005).

Learners can be taught metacognitive strategies that help them become skilled, independent, confident, and self-regulated learners, who perform better academically (Alexander, Fabricius, Fleming, Zwahr, & Brown, 2003; Kron-Sperl, Schneider, & Hasselhorn, 2008; Schneider & Artelt, 2010; Umaru, 2010). Solid conceptual knowledge will aid learners when evaluating the reliability of their answers or procedures, which again brings to light the important role of conceptual knowledge for mathematical reasoning (Crooks & Alibali, 2014).

If a teacher creates an environment where questioning is harboured and where learners can express their meanings freely (cf. relativist, contextualist, and fallibilist epistemological beliefs – Table 2.1) the learners will more likely be able to develop critical thinking and hence metacognitive skills (Department of Basic Education, 2018). The CAPS is not explicit in guiding teachers on how to develop metacognitive self-regulation in mathematics (Du Plooy & Long, 2014). Table 2.4 below provides examples of questions that teachers can use to prompt the development of metacognitive self-regulation.

Table 2.4: Questions to prompt the development of metacognitive self-regulation

Cognitive	Motivational	Behavioural/environmental
PLANNING		
What is the goal of the task?	How do I feel about this kind of task?	When and where do I study/ work best?
What strategies would be most effective for this task?	Do I like this kind of work?	Do I have sufficient time to study/do this task?
What do I know about this task?	Does the task demand a lot of concentration and effort	What kind of study conditions will I need to complete this task?
What useful skills do I have?		
MONITORING		
Are the strategies I have chosen to work with on this task, the correct strategies?	How am I feeling as I work on the task?	Am I giving myself the time I need?
	What is my level of confidence?	

Do I understand what I am doing? Am I making progress towards achieving the goal?	Is the task interesting, and holding my attention?	Is the environment in which I am working supportive, or do I need to find another place? Do I need to add outside resources to help me complete the task?
EVALUATION		
How well did my approach work with this task? What did I do when strategies did not work? Did I achieve the goal? What new goals do I have now after completing the task? How could I improve the approach that I used to complete the task?	How much effort was required to do this task? Did I remain motivated throughout the completion of the task? How am I feeling about the outcome of the task?	How well did I organise my study environment? What could I have changed? Did I choose a good time and place to study? What obstacles did I encounter during the completion of the task, and how did I deal with the obstacles?

(Adapted from Ertmer & Newby, 1996, p. 20)

When learners are prompted to think and talk aloud about the way they approach learning or task completion, they can eliminate obstacles that could obstruct successful task completion. Teachers must make time for metacognitive reflection and guide learners' efforts to enable them to realise the advantages.

Learners need to add depth and breadth to their subject content knowledge (cf. 2.3.4) before they can apply metacognition in mathematics. For learners to plan their process of solving a problem they first need to have subject content knowledge regarding the specific problem. The learners also need to know different approaches to solve a problem as part of the planning phase of metacognition. For example, according to the Department of Basic Education (2018), when learners are given two fractions with different denominators and then asked which one is bigger, or if the total of the two fractions will be less than $\frac{1}{2}$, the learners will have to understand that the fractions are not just a part of a whole but also represented by numbers on a number line. The learners will have to use the knowledge that they have of fractions being part of a whole to reason that addition is smaller, bigger or equal to $\frac{1}{2}$. Different ways of reasoning could be correct, and therefore learners need to communicate their answers in groups to develop their communication skills and reasoning.

The various knowledge dimensions should not be viewed in isolation; as strong relationships exist between the various knowledge dimensions in mathematics. The following sections highlight the relationship between the various knowledge dimensions in mathematics

2.3.2.5 The relationship between the subject content knowledge dimensions in mathematics

Without solid factual knowledge a learner will not be able to make sense of the concepts (Kahan et al., 2003; Löwe & Muller, 2010). Conceptual knowledge develops as knowledge increases and conceptual knowledge becomes more enriched as the learner becomes more of an expert in the subject (Rittle-Johnson & Schneider, 2015).

Procedural knowledge often calls for the automatic and unconscious application of steps, whereas conceptual knowledge requires conscious thinking about rules, algorithms, or procedures (Crooks & Alibali, 2014; Haapasalo, 2003; Long et al., 2014). Conceptual knowledge connects the different mathematical principles to form understanding that can be applied in the form of procedures (Crooks & Alibali, 2014; Long et al., 2014; Rittle-Johnson & Alibali, 1999).

According to Crooks and Alibali (2014), Haapasalo (2003), Lauritzen (2012), Rittle-Johnson and Alibali (1999), Rittle-Johnson et al. (2015), and Star and Stylianides (2013) the close relationship between conceptual and procedural knowledge can be summarised as follows:

- Conceptual knowledge is the knowledge of the various theoretical methods (procedures), and procedural knowledge is the application thereof to solve problems.
- Procedural knowledge involves the rote memorisation of knowledge application to solve problems without analysing a problem critically. Conceptual knowledge on the other hand requires an in-depth understanding of mathematics and creatively and critically analysing a problem and its solution.
- Conceptual and procedural knowledge do not develop independently Bidirectional relationships exist from conceptual to procedural knowledge and from procedural to conceptual knowledge (Rittle-Johnson et al., 2015). Either procedural or conceptual knowledge may develop first, but one type does not as a rule develop before the other. One type might be better developed at a particular point in time, but this does not mean that the other type is completely absent (Rittle-Johnson, Siegler, & Alibali, 2001). Concerning the counting principles, however, it can be said that learners first learn the procedure of counting before the concept is taught (Rittle-Johnson & Alibali, 1999). The learners thus acquire abstract conceptual knowledge from the counting experience. Unfortunately, following the correct procedure does not necessarily lead to a better understanding of the underlying concepts (Rittle-Johnson & Alibali, 1999).

The researcher is of the opinion that when teachers are aware of the importance of conceptual understanding, they will not teach procedural skills before concept mastering has taken place. Expertise in mathematics involves both conceptual and procedural knowledge, and if learners

acquire procedures without understanding, their knowledge may be limited to meaningless routines (Department of Basic Education, 2018). An increase in conceptual knowledge promotes the generation of one's procedures to solve problems. Conceptual knowledge, therefore, has a strong role to play in developing procedural knowledge.

Problem-solving stands central to mathematics education, yet many learners have difficulty in solving problems (Schneider & Artelt, 2010; Verschaffel, 1999). The interaction between conceptual and procedural knowledge is what brings forth problem-solving (Du Plooy & Long, 2014). It has been proven that if conceptual knowledge is developed alongside procedural knowledge, then learners become better problem solvers (Du Plooy & Long, 2014). To be an effective problem-solver, learners need a multidimensional understanding of mathematical procedures, which hinges on conceptual knowledge (Du Plooy & Long, 2014).

Although metacognition is the key when it comes to checking a final answer (Schneider & Artelt; 2010; Verschaffel; 1999), conceptual knowledge is needed to know what to look for or how to determine the reliability of an answer (Brownell in Crooks & Alibali, 2014). The learners need to reflect and evaluate their thinking processes to understand the solution they give to problems (Schneider & Artelt; 2010; Verschaffel; 1999).

Through the ages, mathematics has been taught to learners by having the learners implement formulas of different difficulty levels, thus focusing on procedural knowledge (Crooks & Alibali, 2014). Besides, learners tend to question the relativity of mathematics within real-life situations and very few learners ever truly understand the influence that mathematics has on their everyday lives (Schneider & Artelt, 2010). Schoenfeld (1987) was one of the first people to highlight the importance of metacognition within the field of mathematics education. Schoenfeld (1987) argued that learners should understand the use of mathematics in everyday life to find mathematics meaningful (Schneider & Artelt, 2010). By re-creating mathematics as a meaningful tool for living and understanding the world around us, learners will think of mathematics as an integral part of life, which in turn helps them make connections between mathematical concepts and their everyday lives, and enhances the chances that learners retain useful information (Schneider & Artelt, 2010; Schoenfeld, 1987).

The different knowledge dimensions should always be developed in conjunction with cognitive processes (Krathwohl, 2002). When teaching subject content knowledge, it is important to also develop cognitive processes. The subsequent section explains the integration of knowledge and cognitive processes during the teaching of mathematics.

2.3.3 Integrating subject content knowledge dimensions and cognitive process dimensions during the teaching of mathematics

The researcher believes that the reasoning component presented in Figure 2.2, latches on to the cognitive process dimensions that form part of Bloom's revised taxonomy. According to the Department of Basic Education (2018) reasoning also implies that learners need to acquire skills that they do not possess naturally, such as higher-order thinking skills. Each of the knowledge dimensions (factual, conceptual, procedural, and metacognitive) in the revised taxonomy of Bloom, involves the application of several cognitive process dimensions comprising easier lower-order thinking skills (remember, understand, analyse), and higher-order thinking skills (apply, evaluate, create) (Krathwohl, 2002).

The CAPS specifies four cognitive process levels that should be achieved, namely: *knowledge*, *routine procedures*, *complex procedures*, and *problem-solving* (Department of Basic Education, 2011a, b, c). Once again, if compared to the revised taxonomy of Bloom the focus tends to be on the application of conceptual knowledge, thus placing value on procedures and not the understanding thereof (Stols, 2013), and no mention is made of metacognitive knowledge.

According to the CAPS, the four cognitive levels, and the percentage of time learners need to be involved in tasks at the various cognitive levels are set out in Table 2.5.

Table 2.5: Cognitive levels and skills: CAPS curriculum

FET Phase		Senior and Intermediate Phase	
Knowledge	20%	Knowledge	25%
Routine procedures	35%	Routine procedures	45%
Complex procedures	30%	Complex procedures	20%
Problem-solving	15%	Problem-solving	10%

At the FET phase the cognitive levels focus on the application of the following skills:

- Knowledge: recalling of facts; identification of correct formula, and the appropriate use of mathematical vocabulary.
- Routine procedures: estimation and appropriate rounding of numbers; proofs of prescribed theorems and derivation of formulae (no changing of a subject); identification and direct use of correct formula; performing well-known procedures, simple applications, and calculations involving a few steps; the derivation from given information, and identification and use of correct formula (after changing the subject).

- Complex procedures: complex calculation and higher-order reasoning; following no obvious route to a solution; solving problems based on a real-world context; making significant connections between different representations; and conceptual understanding.
- Problem-solving: working on non-routine problems; solving problems where higher-order reasoning and processes are involved; and breaking problems down into their constituent parts (Department of Basic Education, 2011a, p. 53).

At Senior and Intermediate Phase level, the knowledge, routine procedures, and problem-solving levels comprise the demonstration of similar skills, namely:

- Knowledge: estimation and rounding of numbers, recall, identification, and direct use of correct formula; the use of mathematical facts, and the appropriate use of mathematical vocabulary.
- Routine procedures: performing well-known procedures, simple applications, and calculations that might involve many steps, the derivation from given information, and identification and use of correct formula (after changing the subject).
- Problem-solving: similar to the FET Phase, problem-solving involves solving non-routine problems (which are not necessarily difficult); higher-order understanding and processes; and breaking problems down into their constituent parts (Department of Basic Education, 2011b, p.157; Department of Basic Education, 2011c, p. 296).

At the Intermediate Phase level dealing with complex procedures refers to the application of the following skills:

- Complex calculations and/or higher-order reasoning.
- Investigations to describe rules and relationships (often not the obvious route to derive a solution).
- Solving problems not based on real-world context and making connections between different representations.
- Conceptual understanding (Department of Basic Education, 2011c, p. 296).

Dealing with complex procedures in the Senior Phase involves the following:

- Solving problems involving complex calculations and/or higher reasoning.
- Investigating elementary axioms to generalise them into proofs for straight-line geometry, congruence, and similarity.
- Dealing with problem-solving that does not have an obvious route to the solution.

- Solving problems that are not necessarily based on real-world contexts.
- Making significant connections between different representations.
- Conceptual understanding (Department of Basic Education, 2011b, p. 157).

In the study, the knowledge dimensions were applied as presented in the revised taxonomy of Bloom, but the cognitive process dimensions in the revised taxonomy were extended by identifying additional verbs (cf. Table 4.6) for each of the six cognitive process levels to meaningfully characterise the mathematics context. The knowledge and cognitive process dimensions guided the analysis of the participants' test answers. The researcher presents an example from the FET test (Question 1) to explain how the answers to the test questions were analysed in terms of factual, conceptual and procedural knowledge (cf. Memorandum for Question 1, Addendum C1, p.7).

A table was drawn to reflect the analysis of the participants' responses to the test questions. Table 2.6 serves as an example of how the assessment of the test responses was dealt with.

Table 2.6: Analysis of knowledge dimensions and cognitive processes

Knowledge dimension	Cognitive process dimension					
	1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
A Factual knowledge	x 1 2	x 1 6 7				
B Conceptual knowledge	x	x	x 1 5 8	x		
C Procedural knowledge			x			

According to Table 2.6, an X indicated the ideal responses for the question, thus the cognitive process dimensions that had to be applied successfully in order to answer the question. The numbers of the participants who were successful with the application of a cognitive process dimension, were placed next to the X. The numbers of participants who were unsuccessful with the application of the cognitive process dimensions, were omitted from the table. On completion of the assessment, the researcher could make tentative conclusions about knowledge and cognitive process dimension that seem to be in need of attention.

Prospective teachers do not only need to meet competency expectations concerning the teaching of subject content knowledge to learners, but also need to meet competency expectations concerning their personal mathematical subject content knowledge. Teachers need to take

cognisance of the qualities of their mathematical knowledge to ensure that learners acquire depth and breadth in their mathematical knowledge.

2.3.4 Competency expectations concerning the qualities of teachers' mathematical subject content knowledge

Although expectations of the qualities of teachers' mathematical subject content knowledge are not explicitly communicated in the CAPS and the teacher-training curriculum, Ball et al. (2009) and Hill et al. (2008) argue that teachers need to possess common mathematical subject content knowledge, specialised mathematical subject content knowledge, and horizon subject content knowledge in order to be effective at teaching mathematics.

2.3.4.1 Common subject content knowledge (basic knowledge)

Common subject content knowledge can be seen as the original knowledge of mathematics that can be used in any other profession. It is the subject-specific knowledge needed to solve mathematics problems (Ball et al., 2008). Common subject content knowledge also provides the vocabulary needed to discuss or transfer mathematical knowledge (Holmes, 2012).

2.3.4.2 Specialised subject content knowledge (knowledge that reflects depth and breadth)

Specialised subject content knowledge of mathematics refers to knowledge for specific teaching tasks (e.g. how to provide mathematical explanations for common procedures or methods) (Ball, Hill, & Bass, 2005; Hill et al., 2008). Shulman (in Lam, 2017, p. 9) states that "the teacher need not only understand that something is so, the teacher must further understand why it is so". Knowledge has long been seen as the passing on of facts (Green & Hood, 2013), which is no longer true. Specialised subject content knowledge also has a level of depth (saturated knowledge within a specific content topic) and breadth (knowledge of a variety of content topics) (cf. 2.3.2) (Green & Hood, 2013; Wineburg, 1997).

Ball et al. (2008, p. 400) add that specialised subject content knowledge refers to knowledge and skills unique to the teaching of mathematics. The depth of knowledge is intertwined with progression, where depth indicates the level of understanding and progression indicates the difficulty level of the question that needs understanding (Department of Education, 2003). The depth of knowledge consists of three levels: (i) memorisation, (ii) understanding, and (iii) problem-solving and reasoning (Holmes, 2012), and also refers to having insight into different methods to answer questions or solve problems (Wineburg, 1997).

Teachers with more depth and breadth to their subject content knowledge of mathematics will focus on the underlying concepts of each mathematics topic within the curriculum, whereas

teachers with less depth and breadth to their content knowledge will focus more on the mathematical algorithms of each mathematical topic (Kanyongo & Brown, 2013, p. 108). When a teacher focuses not only on the algorithms of each mathematical topic within the curriculum, but also on the underlying mathematical concepts of each topic, the learners will have a broader understanding of how mathematical concepts fit together.

Specialised subject content knowledge involves the unpacking of concepts and problems (Fuentes et al., 2014), and enables a teacher to nurture conceptual understanding in learners, rather than just factual and procedural understanding (Fuentes et al., 2014). The effectiveness of a teacher reflects in a learner's level of in-depth conceptual understanding of mathematics and not just the understanding of mathematical procedures (Bowie et al., 2014; Fuentes et al., 2014). To be an effective mathematics teacher a teacher must have a comprehensive understanding of all mathematical concepts and their interrelatedness to guide learners in forming connections and conceptual understanding (Fuentes et al., 2014; Goos, 2013).

If a teacher has no knowledge or limited knowledge of mathematical concepts, it cannot be expected of the teacher to transfer the knowledge to the learners meaningfully (Plotz et al., 2012). The more comprehensive the knowledge of mathematics teacher is, the easier it would be for the teacher to relate concepts and thus to transfer knowledge to the learners in a more comprehensible form (Plotz et al., 2012).

2.3.4.3 Horizon subject content knowledge

Horizon knowledge refers to an awareness of the interrelatedness between mathematical topics, which might not be contained in the curriculum (Ball & Bass, 2009, p. 5; Ball et al., 2008, p. 403). Horizon knowledge is regarded as useful to learners, as it presents the larger significance and navigation of the content area that is only revealed in part in the curriculum (Ball & Bass, 2009). It is therefore important that teachers' knowledge expands beyond the school curriculum. Zazkis and Leikin (2010, p. 263-264) define horizon knowledge as advanced knowledge that will be acquired in mathematics courses to be taken as part of a degree from a university or college. The advanced knowledge will enable teachers to make meaningful connections between the advanced mathematics they study in a university and the school mathematics they will teach. For the example of $((x + y)^2 = x^2 + 2xy + y^2)$, algebra teachers need to understand how previous topics like the order of operations, exponents, and the distributive property relate to this problem. Algebra teachers would need to understand how this problem relates to factoring polynomials and working with rational expressions.

The greatness of a mathematics teacher lies within the amount and understanding of their mathematical subject content knowledge (Shulman, 1998). Studies have confirmed the relationship between a teacher's mathematical subject content knowledge and their ability to be

effective teachers (Ball et al., 2008; Lam, 2017; Ma, 1999; Schmidt, 1999; Shulman, 1998; Tsanwani & Sinthumule, 2015).

In the next section, the researcher provides insight into the mathematical subject content knowledge to be acquired by learners according to the CAPS, and the extent to which the teacher-training curriculum that was analysed in the study, aligns with the CAPS guidelines.

2.3.5 Mathematical subject content knowledge in the CAPS and the teacher-training curriculum

To establish to what extent the teacher-training curriculum addresses the outcomes in the CAPS that teachers need to achieve during mathematics teaching at the school level, the researcher firstly, analysed the CAPS in terms of the outcomes to be achieved for the various knowledge dimensions identified by Krathwohl (2002) (cf. 2.3.2), and secondly, analysed the teacher-training curriculum to establish how well the teacher-training curriculum is aligned to the outcomes of the CAPS. Evidence of the curriculum analyses is attached in Addendum E (CAPS analysis) and Addendum F (teacher training curriculum analysis).

The pre-service mathematics teachers' curriculum included the following modules, for which the various study guides that focused on mathematics subject content, were consulted:

- MATE 111/121 – first year: Senior and FET Phases, first and second-semester modules;
- MATE 211/221 – second year: Senior and FET Phases, first and second-semester modules;
- MATE 311/321 – third year: Senior and FET Phases, first and second-semester modules;
- MALA 211/221 – second year: Intermediate Phase, first and second-semester modules;
- MALA 311/321 – third year: Intermediate Phase, first and second-semester modules;
- MATD 211 – second year, pedagogical content knowledge, first semester (all phases);
- MATD 321 – third year, pedagogical content knowledge, second semester (all phases);
- MATD 411/421 – fourth year, pedagogical content knowledge, first and second-semester modules (all phases); and
- MATD 413 – fourth year, pedagogical content knowledge, first semester (only Intermediate Phase students).

Succinctly, the researcher's analysis of the mathematics subject content in the CAPS revealed the following:

- The CAPS indicates the progressive development of knowledge across the grades within a particular phase.
- Although the content specified in the CAPS does not clearly distinguish between factual, conceptual, and procedural knowledge, an analysis of the various outcomes to be achieved does reveal whether the acquisition and/or application of facts, concepts, or procedures are implied (see the researcher's analysis in Addendum E).
- The researcher could not find evidence in the CAPS that guides teachers towards dealing with the development of metacognitive strategies (cf. 2.3.2.4) during mathematics teaching.
- The formulation of the CAPS outcomes reveals various cognitive processes that learners need to perform. However, the importance of ensuring that cognitive processes are developed in conjunction with subject content knowledge is not explicitly communicated to teachers. Mention is however made of the following cognitive levels that need to guide learner assessment across all teaching phases: *knowledge, routine procedures, complex procedures, and problem-solving* (Department of Basic Education, 2011a, b, c) (cf. 2.3.3). As part of the researcher's analysis, the outcomes in the CAPS were scrutinised and linked to the various knowledge and cognitive process dimensions identified by Krathwohl (2002) (cf. Chapter 4, Table 4.6).

Table 2.7 provides a comparative overview of the mathematics subject content knowledge and weighting according to CAPS and the teacher-training curriculum for the Intermediate Phase. The CAPS weighting guided the researcher in setting the Intermediate Phase test to ensure balanced coverage of all the content areas. The study emphasised the content related to the highest grade in a particular phase in each of the tests.

Table 2.7: Assessment of mathematics subject content and weighting (according to the highest grade in the Phase) according to CAPS and the teacher-training curriculum: Intermediate Phase (Department of Basic Education, 2011c, p. 11)

CAPS Intermediate phase				Teacher-training curriculum	
Content area	Mark allocation in test	% Weighting	Weighting (Grade 6)	Modules: MALA 211/221, 311/321	Weighting credits
Numbers, operations and relationships	20	51.3	50%	MALA 211	16

Patterns, functions and algebra	5	12.8	10%	MALA 221 & MALA 311	16 8
Space and shape	5	12.8	15%	MALA 321	8
Measurement	3	7.7	15%	MALA 321	8
Data handling	6	15.4	10%	MALA 311	8
Total	39		100%		

According to Table 2.7, the content areas in the CAPS curriculum Grade 6 are covered across three years of teacher training in four, 16-credit modules (160 notional hours each). The Intermediate Phase students only start their subject content training in the second year of study. The weighting of the subject content according to the CAPS guided the researcher in setting the test paper to ensure balanced coverage of all the content areas.

The Senior Phase students do not follow a phase-specific curriculum during their training but attend lectures with the FET students. According to the researcher, this arrangement could pose a limitation to the students who will specifically teach in the Senior Phase, who do not receive training specifically geared at the Senior Phase content areas (numbers, operations and relationships, patterns, functions and algebra, space and shape, measurement, and data handling). However, if Senior Phase students are employed in the FET Phase, FET training could qualify them better when they are faced with this challenge.

Table 2.8 provides a comparative overview of the mathematics subject content and weighting according to CAPS and the teacher-training curriculum for the FET Phase. Again, the weighting guided the researcher in setting the FET test to ensure balanced coverage of all the content areas. The study emphasised the content related to the highest grade in the FET Phase, Grade 12.

It is clear from Table 2.8 that the teacher-training curriculum covers the content areas required of FET students in six equally-weighted modules of 16 credits (160 notional hours) each.

Table 2.8: Assessment of mathematics subject content and weighting (according to the highest grade in the Phase) according to CAPS and the teacher-training curriculum: FET and Senior Phase (Department of Basic Education, 2011a, p.10; 2011b, p. 11)

Senior and FET phase CAPS				Teacher-training curriculum	
Content area	Mark allocation in test	% Weighting	Weighting (Grade 12) Mark allocation	MATE 111/121, 211/221, 311/321	Weighting credits
Number patterns, sequences, series	6	10,5	25 ± 3	Study section in MATE 221	5,3
Functions	12	21,1	35 ± 3	MATE 111	16
Algebra and equations	8	14	25 ± 3	MATE 221 & 321	5,3 16
Finance, growth, and decay	9	15,8	15 ± 3	Study section in MATE 221	5,4
Differential calculus	10	17,5	35 ± 3	MATE 311	16
Probability	6	10,5	15 ± 3	MATE 121 (50%)	8
Statistics	6	10,5	20 ± 3	MATE 121 (50%)	8
Euclidean and spherical geometry	n/a	n/a	50 ± 3	MATE 211	16
Analytical geometry	n/a	n/a	n/a	n/a	n/a
Trigonometry	n/a	n/a	n/a	n/a	n/a
Total	57		150		

The teacher-training curriculum for the FET Phase does not include content on the following topics that form part of the CAPS: analytical geometry, and trigonometry. Six equally-weighted modules of 16 credits each (160 notional hours) proved coverage of the remaining content areas specified in the CAPS.

The content area 'space and shape' in the Senior Phase is not adequately covered in the module Euclidean and spherical geometry as there is no focus on Euclidean and spherical geometry in the topics dealt with in MATE 211. No questions on geometry and trigonometry were included in the FET test written by the research participants, as complete coverage of the content area geometry did not form part of the teacher-training curriculum. The module on Euclidean and spherical geometry that formed part of the curriculum also focuses on content that differs from

school content. The Senior Phase and Intermediate Phase tests included a question about space and shape (geometry).

The analysis of the teacher-training curriculum did not reveal evidence of how metacognitive self-reflection is developed. Also, although the outcomes to be achieved with the different modules are phrased in cognitive terms, the explicit development of cognitive process dimensions in conjunction with subject content knowledge is not disclosed in the study guides of the various modules that were also scrutinised (cf. Addendum F).

Apart from solid subject content knowledge, a good mathematics teacher also needs to possess sound pedagogical content knowledge, which will be explored in the section below.

2.4 THEORETICAL FRAMEWORK FOR CONCEPTUALISING COMPETENCY EXPECTATIONS ABOUT PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING MATHEMATICS

Hill et al. (2008) divide pedagogical content knowledge into three components, namely:

- Knowledge of content and students is focused on how learners learn particular content, determines what makes concepts difficult or easy to learn, and identifies what preconceived ideas learners from different age groups and backgrounds might have about the content (Shulman, 1998). Killen (2015) talks about understanding the processes underlying how people learn, and knowledge about how to learn particular types of content.
- Knowledge of content and teaching is concerned about the development of subject content knowledge through teaching; thus, considering different methods and strategies to enable learners to effectively acquire knowledge and to guide learning appropriately (Killen, 2015; Sibuyi, 2012).
- Knowledge of content and curriculum presents a detailed plan for instruction that comprises the outcomes to be achieved, what learners need to be taught to meet the outcomes, and how what is taught needs to be assessed.

According to Figure 2.4, the overlapping of three forms of knowledge (A, B, C) contributes to producing pedagogical content knowledge.

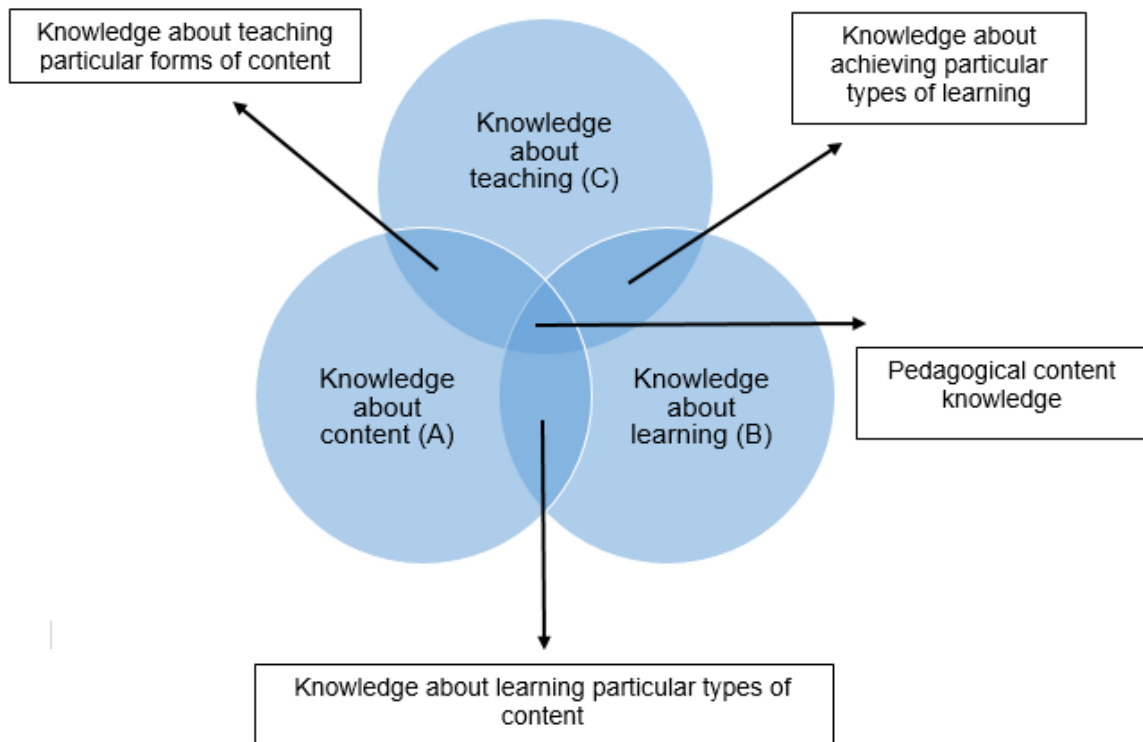


Figure 2.4: Pedagogical subject content knowledge (Killen, 2015, p. 31)

The overlapping of knowledge implies the following:

- The overlap of A and B: teachers require a thorough understanding of subject content and learning theories to understand how certain content should be learned.
- The overlap of A and C: teachers need to understand how to teach certain types of content.
- The overlap of B and C: teachers need to understand how to enable learning from different perspectives, for example, a constructivist perspective. Thus, comprehensive knowledge of different learning theories and the teaching approaches they underpin, namely: direct, indirect, independent, and interactive approaches (cf. 2.4).
- The interaction of all the sections in the centre, namely the pedagogical content knowledge refers to understanding how to teach to enhance certain ways of learning, i.e. surface or deep learning. Surface learning involves studying unrelated pieces of information and memorising information without understanding to avoid failure. Deep learning involves self-regulated learning, where learners are actively and metacognitively involved in learning and develop meaning and understanding of the information acquired and consolidated during surface learning. Learners are constructively involved in planning, monitoring, and evaluating their learning (Pintrich, 2000).

From the aforementioned, one could conclude that the effectiveness of teaching and learning depends on the interrelationship between a teacher, the learner, and the content (Killen, 2015), as depicted in Figure 2.4.

The interrelationship: Teacher and content

The effectiveness of a mathematics teacher relies on the amount and understanding of their own mathematical subject content knowledge (Shulman, 1986; Ünsal, Jakobson, Molander, & Wickman, 2018). Teaching for conceptual understanding requires that teachers have a deep understanding of mathematical concepts themselves (Fuentes et al., 2014), thus having common content knowledge, specialised content knowledge, and horizon content knowledge (cf. 2.3.4). Studies have confirmed the relationship between a teachers' mathematical subject content knowledge and their ability to be effective teachers (Ball et al., 2008; Tsanwani & Sinthumule, 2015).

The interrelationship: Teacher and learner

Although the content and pedagogical content knowledge of a teacher greatly contributes towards effectiveness in the profession, the general personality traits, attitudes, and dispositions of the teacher could also affect effectiveness and influence learners' attitudes towards learning (Stronge, 2018). Examples of these traits, attitudes, and dispositions are: being knowledgeable, enthusiastic, confident, committed, passionate, willing to share and collaborate, resourceful, patient and persistent, and reflective about their teaching practice (Killen, 2010; Stronge, 2002).

The interrelationship: learner and content

For effective learning to take place a learner needs to be ready for the content that will be taught (Killen, 2010). In mathematics, the content of a new year builds on prior knowledge acquired during the previous school years, and if a learner did not acquire prior knowledge the learner will not be able to effectively learn the new content (Killen, 2010). This will cause a negative learning experience and a lot of frustration for the learner.

In this study, the researcher placed focus on the knowledge of content and teaching, thus how teachers present and transfer subject content knowledge to the learners (Livy & Vale, 2011; Shulman, 1998). Pedagogical and subject content knowledge relates in the sense that pedagogical content knowledge is the teacher's knowledge of when to intervene, and what suggestions to make that might help a learner solve a problem while leaving the solution in the learner's hands (Schoenfeld, 1992).

The four major teaching approaches: direct, indirect, independent, and interactive (Killen, 2015) all play a very important role in the teaching and learning of mathematics. The researcher argues that no teacher can make use of just one pedagogical approach to teach, given the diversity of knowledge dimensions that learners have to acquire (cf. 2.2.4, 2.3.2.1-2.3.2.4). The following sections elaborate on the four major teaching approaches concerning the teaching of mathematics.

The researcher acknowledges that there are many mathematics-specific strategies, specifically reform-oriented strategies such as modelling, solving problems, discussion, and inquiry-based learning, available to teachers to teach the individual content areas (Kunene & Sepeng, 2018; Luneta, 2013; Sepeng & Webb, 2012; Spangenberg, 2018; Ng, 2013; Staples, 2007; Van de Walle, Karp, & Bay-Williams, 2010; Venkat, 2013). However, as the teacher-training curriculum did not specifically equip the participants with mathematics-specific strategies (as revealed through the analysis of the teacher-training curriculum), and the participants were mainly exposed to generic teaching and learning approaches as part of their Professional Studies modules, the study, therefore, focuses on the generic approaches to teaching and learning.

2.4.1 Direct teaching

Direct teaching emphasises the transfer of knowledge from the teacher to the learners, and no group work or experiments are involved (Killen, 2010, 2015; Markusic, 2012). Direct teaching is a powerful teaching style when the learners need to learn specific principles, facts, or rules, as the teaching style is structured and easy to follow (Markusic, 2012; Rüttmann & Kipper, 2011). The direct teaching style calls for an introduction, stating the outcomes of the lesson, and after that the teaching itself (Markusic, 2012). After the lesson, the teacher will provide the learners with a summary and assessment. According to Burden and Byrd (2010), the teacher is in full control of the subject content and skills to be learned. Direct teaching is very structured, and learners may be hindered in thinking creatively about the learning process (Markusic, 2012).

Direct teaching follows a realist epistemological belief system (cf. 2.2.2.1) since the focus is on mastering the standard curriculum. Also, subject content knowledge within the curriculum is very important and the same goes for teachers using the direct method of teaching.

Table 2.9 lists some teaching strategies that could be used during direct teaching.

Table 2.9: Direct teaching strategies

Strategies	Application
Lectures	The teacher presents information, learners listen and take down notes.
Presentations/Explanations	The teacher disseminates information, explains difficult concepts/terminology.
Demonstration	The teacher presents a visual lesson.
Questioning	The teacher guides the development of cognitive processes using questions (Green & Murris, 2014).
Practice and drill	The teacher revises material; learners repeat information until the information is fixed in their minds.
Guided practice	Learners practice knowledge and skills in class under the guidance of the teacher.
Homework	Learners practice knowledge and skills independently at home.

(Adapted from Burden & Byrd, 2010)

As Kilpatrick et al. (2001), the researcher believes that direct teaching is a necessary teaching approach, but it should not be the norm. Direct teaching is good for the explanation of new principles and rules of a topic, for example, the exponential laws. Learners will be listening to explanations to remember, understand, and finally, apply the factual knowledge related to the exponential rules. Direct teaching might lose the depth of a topic and reduce it to just a theoretical approach that focuses on breadth. According to the researcher, this happens in most mathematics classrooms. However, if direct teaching could be combined with indirect teaching (cf. 2.4.2), learners would get the opportunity to investigate principles by building on prior knowledge. By building on what the learners already know increases the chance of a learner succeeding at the task, and learners will feel more confident when solving a problem (Campbell & Hackett, 1986; Kapur, 2016).

2.4.2 Indirect teaching

Indirect teaching aligns with cognitive and constructivist learning theories where learners are actively involved in and more in control of their learning (Killen, 2015). Indirect teaching stimulates learners' curiosity and encourages learners to discover information independently and to apply acquired information (Killen, 2015; Kramer, 2006). Teachers with more sophisticated epistemological beliefs make use of indirect teaching (Lodewyk, 2011). Teachers using the indirect approach to teaching tend more towards the relativist (cf. 2.2.3.2), and contextualist epistemological beliefs (cf. 2.2.3.3, 2.2.5), as learners construct knowledge from their understanding and background (Lodewyk, 2011).

Teachers can help learners understand mathematics concepts better by making use of indirect teaching, where learners observe the application of concepts from an inquiry-based perspective (Rüütman & Kipper, 2011). Learners can be encouraged to recognise the concept needed to answer a question or solve a problem instead of just teaching the learners the methods for answering the question or solving a problem (Rüütman & Kipper, 2011). To achieve this, the teacher will have to provide the learners with guided questions that build on their prior knowledge (Rüütman & Kipper, 2011). The learners will then answer the questions in sequence and the questions will guide them in exploring a new concept (Rüütman & Kipper, 2011). The learners will thus discover the application of a concept before the concept is formally introduced to them (Rüütman & Kipper, 2011). Felder (2006) states that indirect teaching emphasises concept learning through inquiry-based and problem-based strategies (Rüütman & Kipper, 2011).

Examples of indirect teaching strategies include the application of the following strategies listed in Table 2.10.

Table 2.10: Indirect teaching strategies

Strategies	Application
Class activities	Use activities that relate to the real lives of the learners.
Concept maps/Thinking Maps	Learners create visual displays of information
Harvesting	Learners reflect after activity in class on “what” they learned, and what to learn next.
Case studies	Learners are given real fictitious situations and answer questions to demonstrate their understanding of facts, principles, and concepts.
Simulations	Learners are given problem-based learning situations which they have to solve, make decisions, and face the consequences of their decisions.
Field trips	Learners undertake practical experiments at authentic sites outside the classroom but are related to the curriculum.
Investigations and experiments	Learners are requested to develop their investigations about a topic/theme.
Interviews	Learners gather information on various topics/themes.
Learners' self-evaluation	Teachers engage learners in critical self-evaluation of their work and thereby promote self-regulated learning.

(Adapted from Kramer, 2006; Rüütman & Kipper, 2011)

In the context of mathematics, an example of a topic that could be dealt with by applying indirect teaching is the introduction to a new function, such as the parabola. Learners will already know how to plot the straight-line graph by using the table method to find coordinate points (x and y). A guided worksheet can be created where the learners have to use the table method to find coordinate points to the given function (not knowing it is the parabola yet). The worksheet can guide the learners in finding the intercepts and instruct the learners to plot the coordinate points

from the table on a Cartesian plane, along with the intercepts. With the help of the guided worksheet, the learners will sketch the parabola. By introducing the parabola using prior knowledge, the learners would feel less intimidated by the new concept as they are applying skills (prior knowledge) that they have already mastered, to acquire new skills (sketching a parabola).

2.4.3 Independent teaching

According to Learning (2002), independent teaching allows learners the opportunity to think for themselves. Teachers are there to provide guidance, but the teacher must decide when guidance is needed. Independent teaching focuses on developing learners' self-regulation, self-reliance, self-awareness, self-improvement, and initiative so that a deeper understanding of the content can be cultivated (Learning, 2002; Philpott, 2009). Also, independent teaching guides learners to make connections between real-life and subject content (Learning, 2002). During independent teaching, the teacher observes the learners and identifies problems the learners may experience (Burden & Byrd, 2010), but the learners plan and execute learning activities without close management by the teacher (Borich, 2007). The independent teaching approach links well with a relativistic epistemological belief system since the teacher will allow the learners to find multiple truths for themselves to develop their critical and creative thinking skills (Sing & Khine, 2008).

Table 2.11 lists some teaching strategies that teachers could apply to nurture independent learning.

Table 2.11: Independent teaching strategies

Strategies	Method
Learning centres/stations	Certain areas in the class that is created for learners to use resource material to find information to complete tasks.
Contracts	A structure that provides the teacher and learner to agree on tasks to allow learners to work on their own.
Video/movie clips	Learners provided with URL/hyperlinks to websites with curriculum-related content; learners must provide answers/explanations to given questions.
Blogs	Teachers create blogs to place questions/problems that would elicit discussions among learners. Learners work independently to solve problems.
Homework	Learners complete homework independently outside the classroom.
Research projects	Learners perform projects (investigation or experiment) outside the classroom and submit a report (example: scientific method).
Interviews	Learners conduct interviews and present results in class.
Assignments	Learners answer question-related subject content case studies and experiments, etcetera.
Equipment-assisted learning	Learners use computers, mobile phones, etcetera, for learning.

Strategies	Method
Independent worksheets	Learners work through investigative/experimental worksheets independently.

(Adapted from Burden & Byrd, 2010)

When the learners have been introduced to the angle of elevation and angle of depression within the topic of trigonometry an example of independent teaching could be a real-life assignment where learners build a hypsometer. Learners can then be given a worksheet with the explicit teaching of how to use the hypsometer to find the height of buildings around the school grounds. Specific buildings and objects around the school can be identified and learners can be instructed to find the height by using the hypsometer they have built. This links the angle of elevation to real-life applications and might spark the curiosity and interest of the learners.

However, confidence is important when it comes to solving mathematics problems since it opens the door for curiosity (Borowske, 2005). Curious learners will want to understand the depth of a topic, especially when an investigative method is implemented in class (Borowske, 2005). The researcher thinks that independent teaching can only be implemented if learners are already confident in their mathematics capabilities. Independent teaching does not have to be a daily pedagogical approach to mathematics, but rather linked to specific content suitable to deal with separately. Independent teaching also creates the opportunity for learners to acquire a deeper understanding of concepts as it requires knowledge across topics and insight (metacognition).

In the context of independent teaching and learning, problem-solving stands central as it bridges the gap between theory and practice (Boyd, 2011). Problem-solving in mathematics has been the same for decades – teachers explain the materials, show an example problem on the blackboard and learners solve problems on their own (Schoenfeld, 1992). This however is not problem-solving, because problem-solving should be based on investigative activities that require innovation (Schoenfeld, 1992). Problem-solving involves inquiry and encourages learners to become independent in discovering information and understanding and solving problems (Dostál, 2015; Schoenfeld, 1992; Snyder & Snyder, 2008). Besides, Anthony and Walshaw (2007), and Hmelo-Silver, Duncan, and Chinn (2007) assert that problem-based learning promotes the development of self-directed learning skills, enables learners to collaboratively solve problems, and reflect on their experiences, as well as enhance the understanding of different concepts and their interrelatedness (Organisation for Economic Co-operation and Development (OECD), 2007; Siyepu, 2013).

There are different kinds of activities that could be used for problem-solving and they include the following.

Experiments: Investigations linked to formal curriculum-based problems involving hypotheses.

Case studies: Real-life events that focus on problems that need to be solved.

Role-play and simulations: Teacher-constructed real-life problem situations from which learners should learn certain things.

Project work: Research or investigations are conducted to collect information that is presented in a structured and coherent manner (Killen, 2015; South African Institute for Distance Education (SAIDE), 2013).

Problem-solving is educationally useful to nurture critical thinking because learners learn to apply cognitive and metacognitive skills and strategies, such as interpretation, analysis, evaluation, making inferences, explanation, and metacognitive self-regulation. It also engages learners actively in learning, and solutions to problems can lead to a deeper understanding of the concepts about the procedures (Crooks & Alibali, 2014; Killen, 2015).

2.4.4 Interactive teaching

Interactive teaching relies on two-way communication (cooperation and interaction) between the teacher and the learners and between learners (Burden & Byrd, 2010). One way of how interactive/participative teaching can be introduced into classrooms is using cooperative learning, which is a structured and systematic way of teaching in which learners work together in small groups to ensure that everybody in the groups learns and understands the same content (Burden & Byrd, 2010). Cooperative activities involve solving problems, answering questions, and communicating the outcome of the activities (Grosser, 2014). The more learners practise to communicate their mathematics knowledge, the more critical they will think about the solutions, and the more confident mathematicians they will become (Ziemba, 2007).

Table 2.12 summarises some teaching strategies that promote interactive/cooperative learning.

Table 2.12: Interactive/cooperative teaching strategies

Strategies	Application
Think-pair-share	Teachers pair learners up and then present them with a problem. The learners would have to think about the problem individually first, then share their thoughts with their partner, and afterward with the class. This method will get learners involved with their thinking process and most importantly it will help them communicate their understanding.
Round-robin	Place learners in groups and they share their answers to questions with others verbally.
Pass the pointer	Place a complex/intricate/detailed image on a screen (for example, in Geometry). Ask learners to identify key features with aid of a pointer. Ask questions about items learners do not understand.

Strategies	Application
Ask the winner	Learners solve the problem individually. The teacher reveals the answer and asks learners who had the answer correct to raise their hands. All learners who answered incorrectly need to pair with someone with a raised hand to rectify their answer.
Incident process	The teacher divides the class into small groups. Each group receives a real-life problem and the learners are asked to develop a workable solution.
Brainstorming	Learners can use various brainstorming techniques to help them make sense of the content and communicate their understanding to other learners, for example, structured and unstructured, reverse or negative thinking, nominal group relationships, and online interaction, such as chats, forums, and email.
Question and answer	The teacher hands out paper or cards to the learners. Each learner is asked to write down a question. The cards are taken back in and mixed up. The teacher then picks a random card from the mix and the class answers the question together.
Relay questions	The class is divided into groups of four learners each. Each group member receives a number from 1 to 4 and a four-step question-answering procedure numbered from 1 to 4. The teacher places a timer on the board and gives the class a certain amount of time to answer the questions. Learners are responsible for answering the parts of the question corresponding with their respective numbers in chronological order. The learners have to start with the first before passing it on the next group member, in numerical order. The next group member needs to check the step of the previous group member and do the step of the question related to his/her number. Again, the process will be repeated until all the steps are completed. At the end of the activity, each group member would have completed all four steps. The answers can either be taken in and graded or the first group to finish with the correct answer can be awarded a prize.

(Adapted from Burden & Byrd, 2010; Concordia University of Portland, 2018; Grosser, 2014).

Interactive teaching is a way to involve learners in the process of understanding. The learners get to share opinions and different strategies for solving problems. Mathematics is a subject where problems can be solved by making use of various strategies, but teachers seldom introduce a variety of strategies to the learners. This can be due to a lack of time in the teaching schedule or lack of pedagogical content knowledge (how to employ interactive teaching) in mathematics. By introducing interactive learning learners develop critical thinking, reasoning skills, and refine their thought process (Florea & Hurjui, 2015; Panitz, 1999).

The researcher argues that interactive teaching could work well with the topic of statistics, where learners have to make inferences about answers, such as interpreting the slope of a regression line, the correlation coefficient, or the coefficient of determination. The learners can work in groups and be given a problem to solve where they have to create a scatter plot of data values, indicate the regression line, and interpret their findings by making use of the correlation coefficient, coefficient of determination, and/or slope of the regression line. Each group will then get the opportunity in class to explain their answers to the rest of the class. A class discussion can follow after all the groups had the opportunity to share their answers.

With all this said, to change teaching strategies is not an easy task (Wang, Lin, & Spalding, 2008). Most teachers teach the way they were taught, namely utilising direct teaching (Ho, Watkins & Kelly, 2001; Sing & Khine, 2008; Wang et al., 2008). In addition, teacher-training also seems to add to the unpreparedness of mathematics teachers to apply strategies such as problem-solving, modelling, discussion, and cooperative learning (Ikeda, 2013; Luneta, 2013; Ng, 2013; Spangenberg, 2018). Luneta (2013) adds that teachers mostly require learners to perform specific procedures such as those in most textbooks, that direct learners to a specific algorithmic process. Limited opportunities to conceptualise and analyse problems, to reason and interpret, and exposure to challenging questions are provided to learners (Ang, 2010; Kang & Noh, 2012). Ng (2013) continues by stating that teachers tend to perceive mathematics as being formula-based involving linear track solutions.

For a consistent long-term change in teaching strategy continued personal development courses that stretch across several months would be most effective for teachers, since changing a teaching strategy is no small matter (Wang et al., 2008). Practising teachers would need consistent help and guidance to gradually and continuously move towards more sophisticated teaching strategies (Wang et al., 2008).

The following section elucidates the view of the Department of Basic Education (2018) to apply teaching strategies that would nurture a learning-centred teaching approach.

2.4.5 Learning-centred teaching

The emphasis in a learning-centred classroom is placed on a teacher who designs learning experiences to help learners to master mathematics by employing any strategies the teacher deems to be most suitable for a specific lesson. Teachers need to control learning pathways, but interactions between teachers and learners need to stand central, and learners need to be exposed to opportunities to express their ideas, ask questions, and to discuss with others their ways of thinking about solving problems. It is reasonable therefore to argue that the characteristics of the learning-centred classroom are supported by relativist, fallibilist and problem-solving epistemologies (cf. Table 2.1). Besides, the researcher believes that indirect (cf. 2.4.2), independent (cf. 2.4.3), and interactive (cf. 2.4.4) approaches to teaching would benefit a learning-centred classroom.

The Department of Basic Education (2018) provides several pointers to guide teachers in creating a learning-centred pedagogy thus implying the integration of direct, indirect, independent, and interactive approaches (cf. 2.4.1 – 2.4.4) to teaching in the mathematics classroom, namely to: focus on concept development; provide active learning; emphasise problem-solving; expect of

learners to justify answers; apply mathematics in context; connect topics and concepts; explain concepts and procedures; connect representations; and practise procedures.

The next section reports on the comparison of the competency expectations regarding pedagogical content knowledge between the CAPS and the mathematics teacher-training curriculum.

2.4.6 Pedagogical content knowledge in the CAPS and mathematics teacher-training curriculum

The researcher's analysis of the CAPS did not yield any evidence of pedagogical content knowledge that provides teachers with suggestions about teaching strategies or tools according to which they could achieve subject content outcomes. Mention is made of teaching guidelines, but these guidelines provide clarity on the allocation of teaching time and the sequencing and pacing of work and do not provide guidance in terms of teaching approaches towards achieving the set outcomes. Besides, clarification is given to teachers about the concepts and skills learners need to acquire.

The researcher's analysis of the CAPS points to three of the four important components that a standard curriculum should comprise of, namely: aims and objectives (outcomes), content, and evaluation (assessment) according to leaders in the field of curriculum design (Krüger & Müller, 1988; Tyler, 1949). An important fourth component that would assist teachers in their lesson planning and help them choose an appropriate vehicle for delivering content, namely 'method', does not form part of the CAPS. Guidance about teaching methods and strategies is important for pre-service teacher-training, as teaching methods and strategies form the core of a curriculum and put the goals and use of the content into action to achieve the curriculum outcomes (Adedoyin, 2011; Alfayed, Latif, & Raihan, 2014; Avalos et al., 2010)

The teacher-training curriculum focuses on equipping all students with pedagogical content knowledge in their second, third, and fourth years of study (cf. Addendum F), namely MATD 411 (fourth year, first semester); MATD 421 (second semester, fourth year), MATD 321 (third year, second semester), and MATD 211 (second year, first semester). One module, MATD 413 (first semester, fourth year) is specifically presented for the Intermediate Phase students, in addition to the other aforementioned modules. An analysis of the module outcomes and content revealed some gaps in the preparation of mathematics teachers who need to possess well-rounded pedagogical content knowledge in addition to subject content knowledge (Adedoyin, 2011; Avalos et al., 2010) (cf. 1.2).

The module outcomes to be achieved emphasise knowledge of the curriculum, lesson planning and presentation, and how to include technology as part of the lesson presentation. Only in one

of the modules, MATD 321 (Study unit 2), evidence of the use of specific teaching strategies during teaching could be found. In this regard, large group presentations, discussions, cooperative small groups, and independent work were addressed (cf. Addendum F). The MATD 321 module (Study unit 2) presented a section on the importance of personal learning theory for teaching and learning mathematics. The researcher finds the inclusion of this topic encouraging, as the use of a learning theory directly implies the choice and application of teaching methods and strategies aligned to one's theory for teaching and learning mathematics. Unfortunately, the information in the study guide appeared to be scanty with no information suggesting learning theories or teaching methods and strategies appropriate for the mathematics classroom. Students are referred to a textbook in this study unit, which possibly provided some guidance on choosing teaching methods and strategies aligned to a specific theory for learning and teaching mathematics.

It seems reasonable to conclude that the students have to rely on the information about teaching and learning theories, and teaching methods and strategies that they receive during their Professional Studies modules. Unfortunately, the Professional Studies modules have a generic outlook and the ideal would be to receive guidance about methods and strategies that are meaningful for the mathematics classroom.

To conclude the chapter, the researcher applies the preceding literature review to construct her view on the conceptualisation of competency expectations concerning subject content and pedagogical content knowledge for pre-service mathematics teacher-training.

2.4.7 Competency expectations for pre-service mathematics teachers: the researcher's view

Given the preceding discussions about the different epistemological beliefs that guide mathematics teaching (cf. 2.2.2), the competency expectations about dimensions of knowledge and cognitive processes that need to be developed among learners during the teaching of mathematics (2.3.2), the competency expectations about the qualities of mathematical subject content knowledge teachers need to possess (cf. 2.3.4), and competency expectations about the pedagogical content knowledge that is needed to facilitate understanding in the mathematics classroom (cf. 2.4), the researcher presents Figure 2.5 to illustrate her view on the interrelated nature of the components specifying the competency expectations for teachers who become involved in the teaching of mathematics.

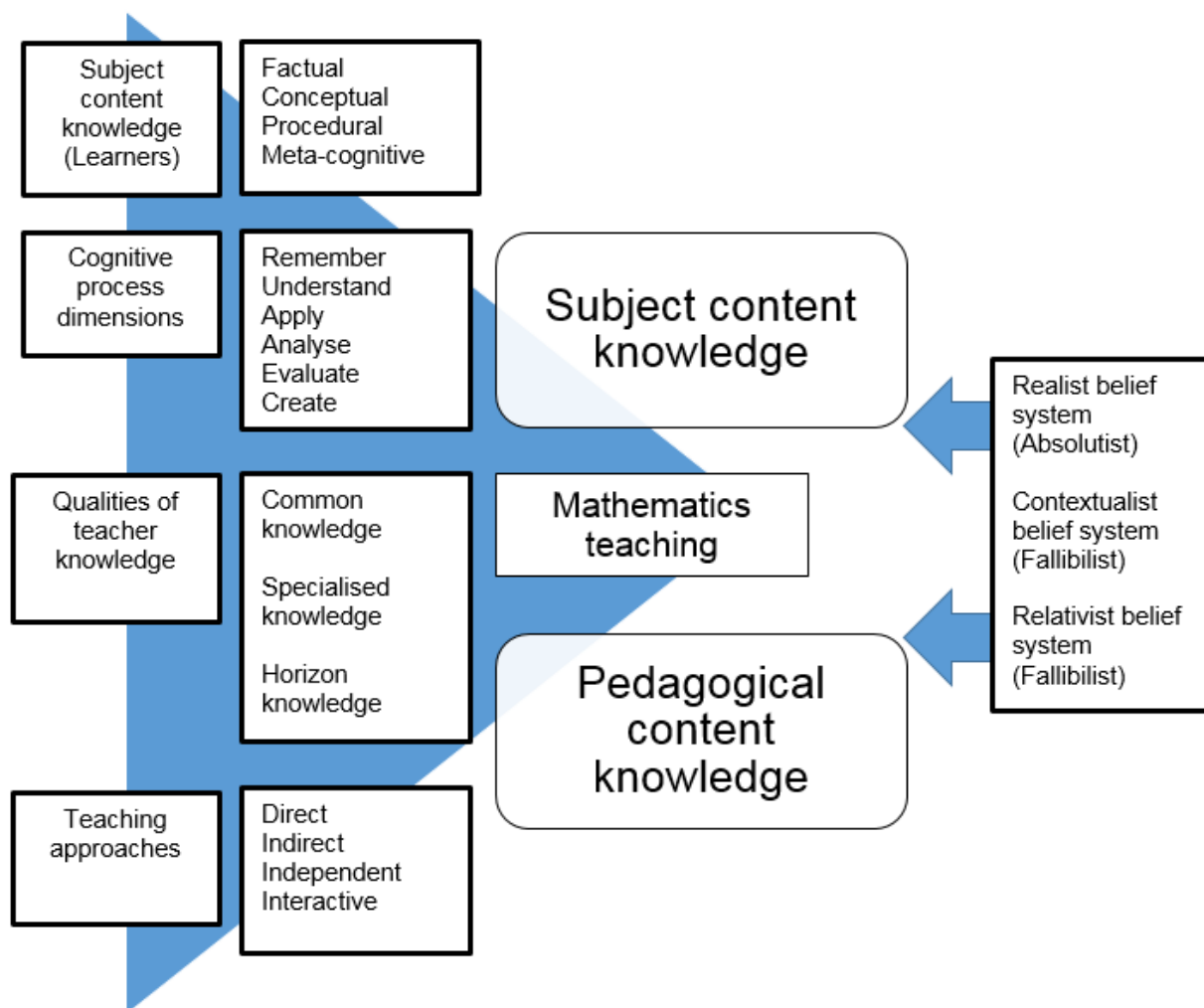


Figure 2.5: Conceptualisation of competency expectations concerning subject content knowledge and pedagogical content knowledge in teaching mathematics

Figure 2.5 indicates that the epistemological beliefs of a teacher (how a teacher perceives the nature of knowledge and how it is best acquired) influence how learners will acquire subject knowledge. Pedagogy (the method of transferring knowledge) is the key to enable learners to understand the subject content knowledge and the teacher's epistemological beliefs influence pedagogy greatly. A teacher might be epistemologically orientated to teach directly, indirectly, independently, or interactively. However, the researcher argues that the nature of the mathematics subject content that includes factual knowledge, conceptual understanding, and procedural competence will expect a teacher to balance and vary their epistemological beliefs to ensure that the learning outcomes in each of the subject content knowledge dimensions are achieved effectively.

Learners need to acquire factual, conceptual, procedural, and metacognitive knowledge. To effectively equip learners with knowledge, teachers need to possess common subject content knowledge, which is subject-specific knowledge needed to solve mathematics problems (Ma,

1999). Also, teachers need specialised knowledge that will enable them to teach for conceptual understanding in learners, rather than just factual and procedural understanding (Fuentes et al., 2014). Teaching mathematics would also greatly benefit if teachers acquire horizon knowledge – an awareness of the interrelatedness between mathematical topics, which might not be contained in the curriculum (Ball & Bass, 2009, p. 5; Ball et al., 2008, p. 403). Learners have to acquire different cognitive processes, such as remember, understand, apply, analyse, evaluate, and create, in conjunction with the subject content knowledge (Anderson et al., 2001).

Many generic pedagogical strategies are implemented during teaching and learning of mathematics, for example, advance organisers, concept mapping, and group activity strategies (Idowu, 2002; Okebukola, 1994; Umaru, 2010). However, the mentioned strategies often appear to be inadequate for teaching mathematics as problem-solving forms the basis of mathematics content thus requiring metacognitive learning strategies that will transfer the responsibility of learning to the learners by developing in the learners' self-regulating skills (Flavell, 1985; Schoenfeld, 2008; Umaru, 2010). Problem-solving can however only be developed if the learners already have a solid conceptual understanding and if they know how to transition the concepts to procedures (Du Plooy & Long, 2014).

A good and balanced combination of subject content knowledge and pedagogical content knowledge is what makes a good teacher, but sometimes gaps in one or both of these fields may be identified. Teachers need to think about their epistemological beliefs and alter them to ensure that the best pedagogical approach(es) is/are selected to teach specific subject content (Bromme, Pieschl, & Stahl, 2010; Hofer & Sinatra, 2010; Soulios & Psillos, 2016).

2.5 CHAPTER SUMMARY

This chapter clarified the competency expectations for mathematics teachers by consulting the CAPS (Department of Basic Education, 2011a, b, c) and the teacher-training curriculum of the NWU, Vanderbijlpark Campus. In sum, competency expectations involve the following:

- Teachers need to equip learners with factual, conceptual, procedural, and metacognitive knowledge that involve the application of various cognitive process dimensions (cf. 2.3.2, 2.3.3).
- Teachers need to possess common, specialised, and horizon mathematics subject knowledge (cf. 2.3.4).
- Teachers need to applying a variety of pedagogical approaches during teaching (cf. 2.4).

The gist entailed in each of the sections is summarised below.

Although not explicitly cited as a competency expectation, an epistemological belief stands central to the view a teacher upholds concerning the nature of subject content knowledge and how learners may acquire the knowledge best. Three general and relevant epistemological beliefs aligned to specific epistemological beliefs in the context of mathematics were explored in this chapter, namely the realist perspective (cf. 2.2.3.1), the relativist perspective (cf. 2.2.3.2), and the contextualist perspective (cf. 2.2.3.3) (Brownlee et al., 2016; Green & Hood, 2013; Pynn, 2016; Stoel et al., 2017). Specific epistemological beliefs related to mathematics teaching were aligned to the general epistemological beliefs, namely the absolutist or realist, and the fallibilist or relativist and contextualist epistemological beliefs (Depaepe et al., 2016) (cf. 2.2.4).

This chapter motivated the choice to use Bloom's revised taxonomy (Anderson et al., 2001) as the framework that guided the conceptualisation of subject content knowledge in the context of the study (cf. 2.3.1). Aligned to the framework of the Department of Basic Education (2018), and CAPS (Department of Basic Education, 2011a, b, c), the researcher defines the dimensions of mathematical subject content knowledge as: factual, conceptual, procedural, and metacognitive (cf. 2.3.2.1 – 2.3.2.4). Infused in the dimensions of knowledge are various cognitive processes, namely: remember, understand, apply, analyse, evaluate, and create (Anderson et al., 2001) (cf. 2.3.3).

Teachers need to possess solid subject content knowledge, which refers to possessing common subject content knowledge (cf. 2.3.4) that comprises mathematical knowledge applied across professional fields, specialised subject content knowledge that refers to depth and breadth of the common content knowledge, and horizon subject content knowledge (cf. 2.3.4.1 – 2.3.4.3) (Hill et al., 2008).

Pedagogical content knowledge was classified into three components, namely: knowledge of content and students, thus understanding how learners learn, and how to best learn particular types of content (Hill et al., 2008; Killen 2015), knowledge of content and teaching which refers to considering different methods and strategies to enable learners to learn mathematics effectively (Hill et al., 2008; Killen, 2015; Sibuyi, 2012), and knowledge of content and curriculum which involves the planning for instruction, considering the outcomes to be achieved, and assessing what has been taught what learners need to be taught to meet the outcomes, and how what is taught needs to be assessed (Hill et al., 2008) (cf. 2.4).

In the study, the researcher paid attention to the application of different approaches to teaching that foregrounded four main approaches, namely: direct teaching, indirect teaching, independent teaching, and interactive teaching (cf. 2.4). Direct teaching refers to the transfer of knowledge from a teacher to a learner (cf. 2.4.1). Indirect teaching encourages learners to discover information on their own and to apply acquired information (cf. 2.4.2). Independent teaching

involves learners becoming more independent and self-regulated during learning and allow them to develop their skills by investigating concepts in real-life situations (cf. 2.4.3). Interactive teaching involves learners actively in social, group-based learning activities (cf. 2.4.4). Indirect, independent and interactive teaching supports a learning-centred approach to the teaching of mathematics (Department of Basic Education, 2018) (cf. 2.4.5).

The chapter also presented an overview of the comparison of the subject content knowledge (cf. 2.3.5) and pedagogical content knowledge (cf. 2.4.6) competency expectations of mathematics teachers evident in the CAPS and teacher-training curriculum of the university where the research was conducted.

Although epistemological beliefs (cf. 2.2) and teachers' content knowledge (cf. 2.3.4) were not specified as expectations in the curricula that were analysed, the researcher finds their contribution to mathematics teaching essential and made tentative conclusions about both aspects in the findings that emanated from the study. In Figure 2.5, the researcher presented her view of the interrelatedness of the categories of competency expectations for pre-service mathematics teachers.

In the following chapter, Chapter 3, the research methodology used in this study is discussed in detail.

CHAPTER 3: EMPIRICAL RESEARCH DESIGN

3.1 INTRODUCTION

Chapter 2 highlighted the competency expectations concerning subject content and pedagogical content knowledge for pre-service mathematics teachers. This chapter seeks to clarify and motivate the methodological intent of the study and explains how the research was conducted empirically to conclude whether pre-service mathematics teachers meet the competency expectations concerning subject content and pedagogical content knowledge. The following layout guides the sequencing of information in the chapter.

3.2 RESEARCH PARADIGM

3.3 EMPIRICAL RESEARCH

3.3.1 Aim

3.3.2 Objectives

3.4 RESEARCH DESIGN

3.5 DATA COLLECTION METHODS

3.5.1 Quantitative study: a self-constructed test and questionnaire with closed and open items

3.5.2 Qualitative study: a document study and written reflections

3.6 QUALITY CRITERIA: RELIABILITY AND VALIDITY OF THE QUANTITATIVE STUDY

3.6.1 Reliability

3.6.2 Validity of the research design

3.6.3 Validity of the data collection instruments

3.7 QUALITY CRITERIA: TRUSTWORTHINESS OF THE QUALITATIVE STUDY

3.8 THE ROLE OF THE RESEARCHER IN QUALITATIVE RESEARCH

3.8.1 Assumptions

3.8.2 Personal connection to the research site

3.9 RESEARCH PARTICIPANTS

3.9.1 Participants for the quantitative study

3.9.2 Participants for the qualitative study

3.10 DATA ANALYSIS

3.10.1 Quantitative test and closed questionnaire items: descriptive statistics

3.10.2 Qualitative data analysis: written reflections to open questions

3.10.3 Qualitative document analysis

3.11 TRIANGULATION OF DATA

3.12 ETHICAL CONSIDERATIONS

3.12.1 Ethical issues in the research questions

3.12.2 Ethical issues in the purpose and questions

3.12.3 Ethical issues in data collection

3.12.4 Ethical issues in data analysis and interpretation

3.12.5 Ethical issues in writing and disseminating research

3.13 CHAPTER SUMMARY

3.2 RESEARCH PARADIGM

According to Taylor et al. (2007), a paradigm is stance or a point of view (Teddlie & Tashakkori, 2009), or, according to the researcher, a lens through which a researcher views the research world.

To determine a research paradigm, the researcher must answer four questions (Andrew, Pederson, & McEvoy, 2011; Brisolara, Seigart, & SenGupta, 2014; Creswell, 2014; Hall, 2012; Taylor & Medina, 2013). The first of these questions being: What role will the participants play in the research? In this study, the participants played a non-interactive role, since they only completed a test and/or questionnaire, and written reflections, and were not involved in making shared meaning with the researcher by talking to the researcher.

The second question concerns the position of the researcher. During this study, the researcher played an external role and internal role. Firstly, the researcher was not involved in meaning-making with the participants. The researcher expected the participants to respond to predetermined questions in the tests and questionnaires. However, the researcher also fulfilled an internal and subjective role, as she was subjectively involved in the construction of meaning during the qualitative data analysis of the curriculum documents, and also constructed meaning from the written reflections of the pre-service teacher participants and the lecturer participants.

The third question asks what the best data collection method for a study would be. In this study, quantitative and qualitative data were regarded as important to gather rich data to answer the research questions. The tests containing closed and open questions were used to collect quantitative and qualitative data. Qualitative data were also collected through document analyses, and written student and lecturer reflections. Both data collection methods were used to ensure that the optimum amount of data was gathered so that the best possible conclusions could be drawn regarding the competency expectations of the pre-service mathematics teachers at the NWU, Vanderbijlpark Campus.

The last question focuses on the purpose/aim of the study. This study aimed to make conclusions about whether pre-service mathematics teachers meet the competency expectations concerning subject content knowledge and pedagogical content knowledge. In-depth information about meeting the competency expectations had to be gathered through quantitative and qualitative data to ensure that reliable conclusions could be drawn.

Observing the answers to the four questions, convinced the researcher that concurrent triangulation mixed methods research with pragmatism as the research paradigm would be apposite to gather substantial data. This study was therefore framed within a pragmatic paradigm.

Pragmatism drives mixed-method research designs that focus on a problem and understanding the possible solutions to a problem from different angles (Andrew et al., 2011).

3.3 EMPIRICAL RESEARCH

3.3.1 Aim

The aim of the study was to determine to what extent the teacher-training curriculum for the Intermediate (Gr. 4-6), Senior (Gr. 7-9) and the FET (Gr. 10-12) Phases at the NWU, Vanderbijlpark Campus, prepare mathematics teachers to meet the competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge foregrounded in the CAPS for teaching mathematics.

3.3.2 Objectives

The aim was operationalised in six objectives (cf. 1.4.2).

The next section explains and motivates the research design that was employed in the context of the research.

3.4 RESEARCH DESIGN

To conduct the research a concurrent triangulation mixed methods design was used. A mixed-method research design makes use of different sources of data collection to determine an outcome (Guion, 2002). The study included both quantitative and qualitative data collection methods to derive the findings. This study supported the view of Creswell (2014) who explains that triangulation can involve gathering data from different participants and making use of different types of data collection instruments (Creswell, 2014) as explained in Figure 3.1. Data were collected from pre-service mathematics teachers and lecturers, and a variety of data collection methods were used, namely: (i) tests for pre-service teachers (with closed and open items), (ii) a questionnaire for lecturers (with open items), and (iii) a document study.

Data from different individuals	Example: Pre-service teachers and lecturers
Different types of data	Example: Qualitative and quantitative data
Different Data collection methods	Example: Tests (with open and closed items, questionnaires (with open and closed items , document study))

Figure 3.1: Triangulation (Creswell, 2014, p. 283)

A concurrent triangulation mixed-method design collects quantitative and qualitative data concurrently, and compares, and integrates the data as indicated in Figure 3.2.

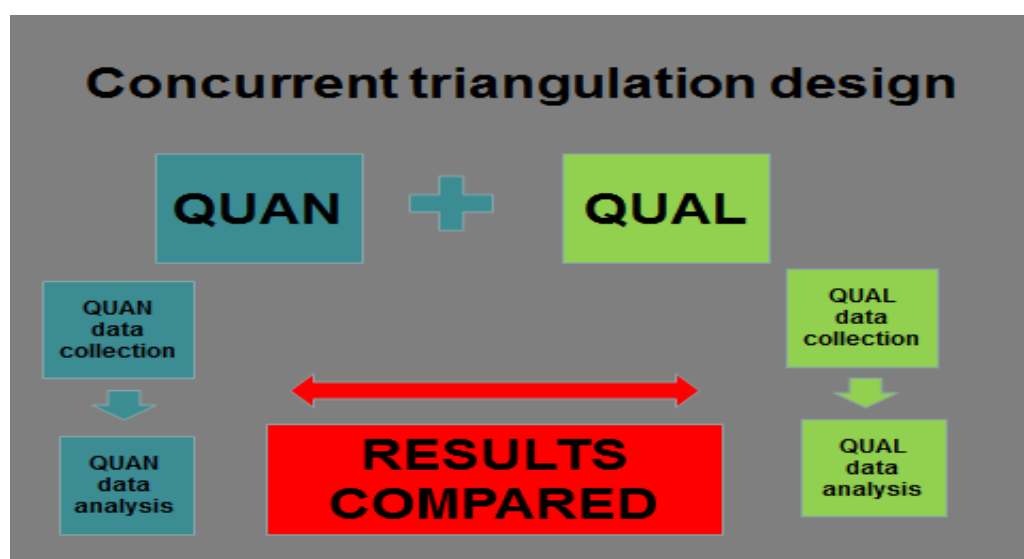


Figure 3.2: Concurrent triangulation mixed method research

According to Figure 3.2, both sets of data (quantitative and qualitative) play an equally important role in the study to answer the research questions. The two sets of data confirmed, cross-validated, or corroborated findings within the study (Ivankova, Creswell, & Plano Clark, 2007).

3.4.1 Research strategy

A research strategy is a step-by-step plan the researcher uses to direct the research to gather the needed information within a certain period (Dinnen, 2014).

3.4.1.1 Quantitative study

As part of the quantitative study, the researcher used a non-experimental, descriptive, survey research strategy (Delpont & Roestenburg, 2011; Lodico, Spaulding, & Voegtle, 2010; Maree & Pietersen, 2016b). According to McMillan and Schumacher (2014) and Maree and Pietersen (2016b), a non-experimental research strategy can be seen as descriptive, since it does not change or manipulate anything. Since no interference or intervention took place during the study, it can be seen as only being descriptive. The quantitative or descriptive part of the study made use of numbers to summarise the outcome of the responses to test questions and closed semantic differential scale items included in the test (Maree & Pietersen, 2016a; McMillan & Schumacher, 2014).

3.4.1.2 Qualitative study: critical educational research

A qualitative study entails the study of how peoples' worldviews can respectively create different perceptions of reality (Hancock, Ockleford, & Windridge, 2009). Qualitative studies focus on experiences or data that cannot be recorded numerically (Hancock et al., 2009).

As part of the qualitative research, critical research was employed as a strategy. The goal of critical research is to critique, challenge, transform and empower (Merriam, 2009). In the context of this study, the researcher envisaged querying the context that shapes the competencies of pre-service mathematics teachers. This was done by analysing the teacher-training curriculum of pre-service mathematics teachers critically in comparison to what is expected in the CAPS for teaching mathematics at the school-level. Moreover, the open items in the tests and the lecturer questionnaire probed for information about the quality of the teacher-training curriculum from the student participants' and lecturers' perspectives and requested suggestions for changes or improvement of the teacher-training curriculum.

3.5 DATA COLLECTION METHODS

3.5.1 Quantitative study: a self-constructed test and questionnaire with closed and open test items

Maree and Pietersen (2016a), Lodico et al. (2010), and Wilson and McLean (cited by Cohen et al., 2000), mention that questionnaires and tests are widely used as a useful instrument for collecting opinions and establishing understanding. Three self-constructed tests (FET Phase,

Senior Phase, and Intermediate Phase) with closed and open questions for student participants and a self-constructed questionnaire for lecturers comprising open questions were used.

3.5.1.1 Pre-service teacher tests

Open and closed test items to establish subject content and pedagogical content knowledge

The main focus of the tests was to determine if participants meet the competency expectations for teaching factual, conceptual, and procedural knowledge to learners. The tests focused on the subject content areas for each of the teaching phases as specified by the CAPS (cf. Table 3.1).

The test also comprised closed-scale items (Maree & Pietersen, 2016a) to determine the pre-service teachers' biographic information, and semantic differential scale items to establish preparedness to teach the various mathematics subject content areas and the availability of teaching strategies. The preparedness to teach the subject content areas and the availability of teaching strategies was assessed on a five-point semantic differential scale (Maree & Pietersen, 2016a). The following scales applied:

- Preparedness to teach content areas: 1 = well prepared; 5 = not well prepared.
- Availability of teaching strategies: 1 = possessing a wide variety of strategies; 5 = possessing limited strategies.

Semantic scales involve opposites at the two extremes of a scale, usually a 7 to 11-point scale, on which participants chooses a point that best qualifies their feelings about an issue (Maree & Pietersen, 2016a). The researcher decided to use a five-point scale to avoid too many response options that might complicate the data analysis and interpretation

Open items: written reflections

This section of the test aimed to determine if the pre-service mathematics teachers know how to apply different teaching strategies in a mathematics classroom. To establish the pedagogical knowledge of the participants they were provided with certain problematic mathematical test scenarios based on content in the school curriculum relevant to their teaching phases. They had to explain and motivate their choice of a specific pedagogical strategy they regard as the most suitable for teaching the content related to the given scenario.

The test also comprised a section where pre-service teachers had to provide written reflections concerning their perceptions of their metacognitive skills, the strengths, and weaknesses of the mathematics teacher-training curriculum, suggestions for improving the teacher-training curriculum, questions in the test they found the easiest and the most difficult, and the type of teachers they perceive themselves to be.

The researcher took into account the following aspects during the design of the test items that focused on subject content knowledge.

3.5.1.2 Constructing a test

According to Cohen, Manion and Morrison (2007), Lodico et al. (2010), and Norris and Ennis (1989) a researcher should consider the following during the construction of a test:

- The purpose of the test. What outcomes must be achieved?
- The type of test for example, diagnostic, achievement, etcetera.
- The test items must relate to the specific outcomes of the programme or curriculum tested.
- What is tested? The content that will be included in the test.
- Reliability and validity of the test.
- Who will administer the test? To ensure reliability the researcher cannot administer the test so other people must administer the test.

The researcher elaborates in the subsequent sections, how the mentioned aspects were handled in the context of the research.

Identifying the purpose of the test

The purpose of the test must be clear, and the following should be considered as possible purposes, before constructing the test.

In this study, the researcher envisaged testing whether students have effectively acquired mathematics subject content knowledge and pedagogical knowledge to teach mathematics subject content. The following aspects were considered:

Judging students versus judging training programmes: if students want to be appraised, the test must be more reliable and valid for each student in comparison to when the aim is to appraise a programme. In the study, the researcher focused on judging the teacher-training programme (curriculum).

Aspect-specific versus comprehensive testing: the researcher wanted to identify students' strengths and weaknesses regarding the specific subject content areas in the FET, Senior, and Intermediate Phases, the mathematics teacher training curriculum and limitations concerning the application of teaching strategies to teach mathematics.

Norm-referenced testing versus criterion-referenced testing: although test scores were comparatively presented for the research participants to provide an overview of the test results for the group of participants, the aim was to establish what each student has learned and could do (Cohen et al., 2007). It was important for the researcher to identify gaps in the subject content

knowledge of the students, and based on the findings, highlight possible trends about strengths and weaknesses in the mathematics teacher-training curriculum.

Making a table of specifications

After a test purpose was identified, the specific aspects on which the test would focus were considered, and the weighting of the different test aspects clarified (Norris & Ennis, 1989). The weighting of each content area included in the tests was aligned to previous national standardised tests used by the Department of Basic Education, namely the Annual National Assessment (ANA) used for systemic evaluation of education performance, and the Gr. 12 final examination. The tests were set to take no more than 60 minutes and the mark allocation, as far as possible, more or less corresponded with the weighting according to the national standardised assessments.

The only exception was the exclusion of geometry and trigonometry in the FET test. The exclusion of geometry was due to the exclusion of geometry in the FET curriculum of the pre-service mathematics teachers at the NWU (cf. 2.3.5).

Table 3.1 indicates how the subject content knowledge was assessed in the context of the research. Three different tests were used, one for each of the pre-service teacher-training phases, namely: the Intermediate Phase (Gr. 4–6), the Senior Phase (Gr. 7–9) and the FET Phase (Gr. 10–12). It was expected of the pre-service teachers, irrespective of the Grade that they taught within a phase, to possess mathematical content knowledge relevant to the entire teaching phase, as their training prepared them to teach across an entire phase. The tests were however set to focus on the content in the highest Grade in each phase, namely: Grade 6, Grade 9, and Grade 12. Copies of the tests are attached (Addendums G1 – G3).

Table 3.1: Assessment of mathematics subject content and weighting

Intermediate phase		
Content area	Mark allocation in test	CAPS % Weighting
Numbers, operations, and relations	19	51,3
Patterns, functions, and algebra	5	12,8
Space and shape	5	12,8
Measurement	3	7,7
Data handling	6	15,4
Total	38	

Senior phase		
Content area	Points in test	CAPS % Weighting
Numbers, operations, and relations	11	28,9
Patterns, functions, and algebra	8	21,1
Space and shape	11	28,9
Measurement	4	10,5
Data handling	4	10,5
Total	38	

FET phase		
Content area	Mark allocation in test	CAPS % Weighting
Number patterns, sequences, series	6	10,5
Functions	12	21,1
Algebra	8	14
Finance, growth and decay	9	15,8
Differential calculus	10	17,5
Probability	6	10,5
Statistics	6	10,5
Total	57	

Selecting the contents of the test

According to Cohen et al. (2007), and Lodico et al. (2010), the test items must be clear and culturally fair. The questions that were used were selected from standardised examinations found on the website of the Department of Education, South Africa, and could, therefore, be regarded as fair and clear to understand. Also, the tests were examined by three different lecturers at the NWU (as part of the pilot study), to ensure applicability and clarity. The questions were adapted to the lecturers' suggestions.

Cohen et al. (2007) specifies that attention should be given to the presentation, operation, and responses to a test. The test questions for this study were open-ended mathematical problems that had to be solved and written down step-by-step.

Considering the form of the test

According to Cohen et al (2007), and Lodico et al. (2010), the form of the test must be considered since there are many forms outside pen-and-paper tests. According to Cohen (2007, p. 426) a test "can be written, oral, practical, interactive, computer-based, dramatic, diagrammatic, pictorial,

photographic, involve the use of audio and video material, presentational and role-play, simulation". The form of the test for this study was a pen-and-paper test. The researcher argued that a written test would be most suitable in mathematics to give participants the opportunity to reflect on information before responding and to present their responses in writing. Written responses would also enable the researcher to assess the responses of participants more accurately.

While setting the test the researcher took into account the reliability, validity, difficulty of questions, marking and grading of answers, and the time needed to answer each question (Cohen et al., 2007). As the test questions were taken from previous standardised examinations (ANA and Grade 12 examinations) the mark allocation was already provided, and questions could be considered reliable. To ensure reliability, validity, and difficulty the tests were however moderated as part of the pilot study by three NWU lecturers (cf. 3.6.3).

Writing the test items

According to Cohen et al. (2007), space left after each question, where the student needs to answer indicates the possible length of the response required. Space in the form of blocks or lines were provided after each question, but participants were however not restricted, and could extend the space if they wished.

Considering the layout of the test

Guided by Cohen et al. (2007), the researcher took the following into account when the layout of the test was planned:

- Together with the co-supervisor the length of the test, number of items and the type of instructions to guide test completion were decided.
- In order not to overload participants with instructions on the first page, instructions were spread out for each section of the test.
- The point allocation for each question was specified.
- Similar content was grouped together.

Considering the timing of the test

The time frame for completing the test was 60 minutes. Participants were given extra time if they needed in order to avoid that a time limit influences the quality of their answers. The Intermediate Phase participants only completed the Intermediate Phase test, but the FET Phase participants were requested to complete both the FET and the Senior Phase paper. The FET participants might teach either FET or Senior Phase mathematics, although the focus of their study is the FET

phase. Therefore, the FET participants took a minimum of 120 minutes to complete both test papers. To reduce and avoid stress and fatigue, the participants received refreshments between tests. The open items in the test were completed simultaneously with the closed-test items.

Planning the scoring of the test

Points should be awarded to each test question according to the difficulty of the question and the completion of the answers (Cohen et al., 2007). This means more difficult questions will count more points and that points will be awarded for specific steps shown in the solution (Cohen et al., 2007). In this study, study marks were allocated mainly according to the mark allocation as evident on the memoranda of the standardised tests from which the questions were taken. The assessment of the different knowledge dimensions was guided by the following principles identified in the literature review (cf. 2.3.2.5).

- Factual knowledge and related cognitive process dimensions
- Conceptual knowledge and related cognitive process dimensions
- Procedural knowledge and related cognitive process dimensions

Marks were allocated for specific steps applied in solving the mathematical problems and for correct final answers. The memoranda used to score the tests are available in Addendums C1 – C3. The researcher had several consultation sessions with the co-supervisor to ensure the correctness of the memoranda. After the marking of the test scripts, the co-supervisor assessed the researcher's marking for accuracy and correctness. Several guidelines (cf. 4.3.2) were applied in marking the tests, as the researcher had to make some exceptions due to the intensive nature of the data collected.

Ethical issues in preparing for tests

To adhere to ethical conduct in administering a test that assesses subject content knowledge, it is important that a teacher does not teach for the test. By teaching for the test, learners are prepared for the test questions, which might lower the reliability of the test instrument. The researcher did not teach for the test, and participants were not prepared in advance to write the tests. The researcher provided an information sheet (cf. Addendum L) with relevant mathematical factual information from previous study years that participants might have forgotten that could assist them in answering the questions.

The next section provides information about the researcher-constructed lecturer questionnaire that was used for data collection.

3.5.1.3 Lecturer questionnaire

Open questions: training and experience

The qualifications and experience of the lecturers who teach pre-service mathematics teachers from their first to the fourth year of study, were also established. Four of the lecturers were requested to participate as all of them contributed to the subject content and pedagogical knowledge across the students' four years of study. The researcher's co-supervisor is also a lecturer, but due to conflict of interest, she did not take part. The researcher used the data to tentatively conclude if the lectures' qualifications and experience could have had any influence on the observations made about the competencies concerning subject content and pedagogical content knowledge of the pre-service mathematics teachers.

Open questions: qualities of the teacher-training curriculum

The lecturers were also asked to reflect on the present strengths and weaknesses of the teacher-training curriculum, its limitations posed to teaching, and adequacy in terms of subject content and pedagogical knowledge to prepare pre-service teachers adequately. A copy of the lecturer questionnaire can be viewed in Addendum D.

In addition to the guidelines for constructing tests specifically (cf. 3.5.1.1.3), the researcher also additionally applied some of the guidelines developed by Cohen et al. (2007) and Maree and Pietersen (2007, 2008a) to guide the construction of the tests and the questionnaire. These guidelines included appearance, instructions, completion time, criteria for formulating questions, and wording of questions.

3.5.1.4 Developing test and questionnaire items: technical aspects

The appearance of the tests and questionnaire

The researcher ensured that the tests and questionnaire were user friendly. The typing was done neatly, and the font was easy to read and not too small. The layout was professional with clear instructions and the purpose of participation was provided.

Instructions

The instructions on the tests and the questionnaire were simple, clear, and concise to make understanding as easy as possible (Maree & Pietersen, 2016a).

Completion time of the tests and questionnaire

The lecturers received their questionnaires via e-mail and were asked to complete the questionnaires on their own time. The lecturers were asked to e-mail the completed questionnaire back to the researcher or her supervisors (for time completion of tests, cf. 3.5.1.1).

Types of questionnaire questions

The questionnaire started by asking for biographical information from the lecturers. Thereafter the lecturers were asked to share their perceptions in writing by reflecting on the following:

- The strengths and the weaknesses of the mathematics teacher-training curriculum at the NWU, Vanderbijlpark Campus.
- The limitations to teaching that the teacher-training curriculum at the NWU, Vanderbijlpark Campus present.
- Whether the teacher-training curriculum at the NWU, Vanderbijlpark Campus prepares the pre-service teachers to acquire adequate subject content and pedagogical content knowledge.

Types of test questions

The student tests comprised the following sections:

- Section A: sections to obtain biographic information and information about the preparedness of the participants to teach mathematics, and the availability of teaching strategies to teach mathematics.
- Section B: questions on subject content knowledge and written reflections about pedagogical content knowledge for each of the teaching phases.
- Section C: questions about metacognitive knowledge, strengths and weaknesses of the teacher-training curriculum, suggested improvements to the teacher-training curriculum, reasons for being regarded as good or bad teachers, and examples of teaching strategies to teach the various subject content areas.

Criteria considered for formulating questions (wording)

Suitable language must be used when constructing test and questionnaire items so that misunderstandings are avoided and meaningful data gathered (Maree & Pietersen 2016a). For this study, the test questions were taken from previously standardised national examinations (ANAs and Gr. 12 examinations), for which language had already been approved. To ensure that the language was clear and as simplified as possible the texts were edited by the NWU University

language department. The tests were also moderated by three NWU lecturers, as part of the pilot study to ensure the quality and clarity of the questions.

The researcher ensured the correct wording of the questions in the tests and the questionnaire so that the items were understood and were meaningful. The following guidelines as suggested by Cohen et al. (2007, p. 334) and Maree and Pietersen (2016a, p. 179-180) were applied:

- Clear, unambiguous language was used.
- Questions and statements were formulated clearly and to the point.
- No double-barrelled and vague questions or statements were included.
- No sensitive or offensive questions, and statements were included.

Scoring the tests

- The researcher met with the co-supervisor to discuss the marking of the tests according to the memoranda. The researcher compiled her own guidelines in terms of identifying evidence for factual, conceptual and procedural knowledge in the answers of the participants, which is reflected on the memoranda, and aligns with Table 4.6.
- On completion of the marking, the assessment of the researcher was verified by the co-supervisor, and suggestions were made by the co-supervisor to make changes to the assessment, after which the marks were re-calculated. The verification of the marking by the co-supervisor appears electronically on the test scripts.

The researcher acknowledges that the use of open test and questionnaire items for data collection has advantages and disadvantages.

3.5.1.5 Advantages and disadvantages of open test and questionnaire items

The researcher considered the following advantages of open test and questionnaire items, as indicated by Norris and Ennis (1989), as important in the context of this study.

- Open questions allow for clarification of answers and reducing superficial responses.
- Participants can justify responses.
- Responses can be assessed more generously, however, not overly, when responses are interpreted.
- Strengths and weaknesses within individual responses can be identified.
- Making inferences on the presence or absence of aspects concerning the focus of the study.

The researcher acknowledges the following disadvantages of open test and questionnaire items in the research, as pointed out by Cohen et al. (2007).

- It is difficult to use open-ended questions for comparison purposes.
- Participants might overlook the instructions because they might be concentrating too much on writing in their own words than reading what the question expects them to do.
- Participants might not be equally capable of transferring their thoughts into writing. The researcher observed this in the reflections of the participants. Answers were often not clear and information provided scanty.
- Open responses are more difficult to score and assess.

3.5.2 Qualitative study: a document study and written reflections

3.5.2.1 A document study

As part of the qualitative study, a document study was used. This was done to explore to what extent the mathematical content knowledge and pedagogical knowledge addressed in the current mathematics teacher-training curriculum used at the NWU, Vanderbijlpark Campus, align with what is expected in the CAPS for mathematics.

A document study involves a process where different documents are analysed and interpreted to gather data, which is then recorded as part of a study (Guion, 2006). Nieuwenhuis (2016). Strydom and Delport (2002) indicate that a document study involves the study of personal documents, official documents, mass media, and archival material. In this study, the official school curriculum and teacher-training curriculum were analysed to collect data concerning the competency expectations of teachers.

The teacher-training curriculum used at the NWU, Vanderbijlpark Campus was analysed (all four years of study for each of the teaching phases). The teacher-training curriculum used at the Mafikeng Campus of the NWU was not included in the study because the mathematics pre-service teacher-training curriculum on this campus differs from the curriculum used on the Vanderbijlpark and Potchefstroom Campuses. At the Vanderbilpark and Potchefstroom campuses, all the modules for pre-service mathematics teachers are aligned and are taught in the Faculty of Education, towards obtaining a BEd-degree. At the Mafikeng Campus, the pre-service mathematics teachers' mathematics modules are taught in the Faculty of Agriculture, Science and Technology, towards obtaining a BSc-degree.

Permission to analyse the pre-service mathematics teacher-training curriculum and collect data from the NWU students and lecturers was granted by the NWU Research Gate Keeper Committee (NWU-RSGC) (cf. Addendum B).

3.5.2.2 Written reflections

The written reflections adopted a naturalist approach to gauge the research participants' experiences with the teacher-training curriculum by making use of the participants' true words (Etherington, 2013). Participants had to look back at events, question these and consider various alternative ways of building on strengths and weaknesses (Bold, 2012). The aim was to involve the participants in the research as active partners in the development of the recommendations presented in Chapter 5.

The focus was placed on reflecting for action (Schön, 1984), to think of steps that need to be taken to enhance the teacher-training curriculum. The researcher took a stance towards a problem and gradually elaborated it through the experiences of the student and lecturer participants. Reflection is an intentional, dynamic process that allows improvement in one's actions, abilities, and knowledge by learning from experiences to identify and find solutions to weaknesses (Castleberry et al., 2016; Kember, McKay, Sinclair, & Wong, 2008). The written reflections of the student participants appear in Addendum J, and the lecturers' reflections in Addendum D.

The researcher acknowledges that the use of written reflections for data collection had advantages and limitations.

Advantages of written reflections

- Written reflections provided the researcher access into the pre-service teachers' private views and provided rich data (Méndez, 2013).
- The written reflections are easily accessible to readers (McAlpine, 2016).
- The act of reflecting could contribute to pre-service teachers' lives by making them think about what is important during teacher-training (Méndez, 2013).
- Written reflections might make it easier for participants to share the truth since they have anonymity and do not have to talk directly to a person (Aarikka-Stenroos, 2010).

Limitations of written reflections

- It might be difficult to keep the anonymity of participants as they shared locations or relationships in their answers (McAplene, 2016). Anonymity was ensured by indicating to participants to submit their reflections without names.
- The written reflections captured only a limited number of experiences during the participants' training and the researcher should thus be mindful that not all experiences were taken into account (McAplene, 2016).

3.6 QUALITY CRITERIA: RELIABILITY AND VALIDITY OF THE QUANTITATIVE STUDY

3.6.1 Reliability

A pilot study was done to ensure the reliability of the study. A pilot study is a small-scale version of the study to check the study's effectiveness in providing answers to the research questions and to establish to what extent the research questions will be answered (Creswell, 2014). A pilot study forms a vital part of the research, since it increases the success of the actual study (Creswell, 2014; Strydom, 2011c).

A small scale, informal dynamic pilot study was conducted before data collection (Brace, 2008). Three mathematics lecturers were identified for this purpose, who on a rotation basis evaluated the researcher's constructed tests. After receiving the changes suggested by Lecturer 1, the researcher made the changes, after which the adapted test was circulated to Lecturer 2. Similarly, the changes suggested by Lecturer 2 were made, and the adapted test circulated to Lecturer 3. After changes suggested by Lecturer 3 were incorporated, the test was administered as part of the actual study. The questions in the tests were also verified with the co-supervisor who has many years of experience in teaching and lecturing mathematics to students at school and at the university level, to determine whether they measured the constructs in question.

3.6.2 Validity of the research design

The following criteria identified by McMillan and Schumacher (2006, p. 134-142), and Leedy and Ormrod (2005, 2014) were taken into consideration.

3.6.2.1 Internal validity

Leedy and Ormrod (2005) explain that internal validity is the extent to which the design and the data it yields allow the researcher to draw accurate conclusions about cause-and-effect and other relationships within data. As it was not possible to control the influence of biographical variables, such as gender, home language, ethnic group, and teaching phase on the outcomes of the research due to the small sample size, internal validity could not be guaranteed in the study.

3.6.2.2 External validity

External validity, as defined by Leedy and Ormrod (2014), can be seen as the generalisation of the research to other contexts. Since the researcher made use of a small, convenient sample, the findings of the study cannot be generalised and external validity thus compromised. The purpose of the study was not to generalise findings.

3.6.2.3 Construct validity

By making use of tests, questionnaires and a document study to conduct research, the researcher believes that construct validity was adhered to, since the use of a test to understand competency concerning subject content knowledge, and a test and questionnaire with open and closed questions to measure perceptions and views of participants about pedagogical knowledge and the qualities of the teacher-training curriculum can be regarded as suitable data collection instruments (Leedy & Ormrod, 2005). Moreover, a document study could be regarded as suitable to explore the nature and quality of the competency expectations concerning mathematical subject content and pedagogical content knowledge among pre-service teachers. The findings obtained from the tests and questionnaire data as well as the document study were combined to answer the research questions to support construct validity.

3.6.2.4 Statistical conclusion validity

Suitable statistical analyses were done to analyse the data that the researcher collected. The researcher and her supervisors were responsible for capturing and analysing the data and constructing the graphs that represented the data. As only basic descriptive data were used, a statistical consultation service was not employed. The researcher believes that for the study that did not focus on a comparison of data, a descriptive data analysis could be regarded as valid and appropriate, and is therefore convinced that statistical conclusion validity was guaranteed in the study.

3.6.3 Validity of the data collection instruments

3.6.3.1 Face validity

Face validity was guaranteed as the tests measured what it was supposed to measure (Leedy & Ormrod, 2014), namely: if pre-service mathematics teacher meet the competency expectations to teach mathematics subject content knowledge in their different teaching phases. The other closed and open questions in the test also supported the measurement of aspects concerning the pre-service teachers' availability of pedagogical strategies to teach mathematics, and qualities of the teacher-training curriculum that were of relevance to the study. Similarly, the open questions in the lecturer questionnaire also contributed to gaining information about the qualities of the teacher-training curriculum and were used to support the data obtained with the student tests and written reflections.

During the document analyses, the pre-service mathematics curriculum was analysed according to the competency expectations as prescribed by the CAPS concerning the following aspects:

- Evidence of factual, conceptual, procedural, and metacognitive knowledge (Department of Basic Education, 2011a, b, c; 2018; Krathwohl, 2002) (cf. 2.3.5)
- The time or weighting allocated to teaching different content topics (cf. Table 3.1).
- The teaching approaches (methods and strategies) suggested for achieving learning outcomes (cf. 2.4.5).

3.6.3.2 Content validity

Content validity implies that the tests and the questionnaire gather all the information needed by the researcher to complete the study (Maree & Pietersen, 2008b). The tests contained questions about both mathematical subject content and pedagogical content knowledge of pre-service mathematical teachers. The themes/topics of the mathematical subject content knowledge questions were weighted in the construction of the tests according to the weight distribution within the school curriculum. As a result, certain topics within the tests were more prominent (cf. Table 3.1).

The pedagogical knowledge questions included mathematical problem scenarios used in the tests where pre-service teachers were requested to write their responses to explain the strategies they would use to teach the subject content in the specific scenario.

3.6.3.3 Construct validity

Construct validity was guaranteed by giving the test and questionnaire items, as well as the criteria that were used for the document analysis, to experts in the field of mathematics teaching and teacher-training to establish if the test and questionnaire items and the criteria for the document analysis provided a true reflection of what the researcher envisaged to measure.

3.6.3.4 Criterion validity

Criterion validity relates to the correlation of the measurement instruments' results with the results of other related measurement instruments (Chin & Lee, 2008; Pietersen & Maree, 2016). As far as the researcher is aware, there are no similar tests that examine the subject content and pedagogical content knowledge of pre-service mathematics teachers according to the different knowledge dimensions against which the researcher-developed tests could be compared. Thus, criterion validity could not be guaranteed.

3.7 QUALITY CRITERIA: TRUSTWORTHINESS OF THE QUALITATIVE STUDY

To ensure the quality of the qualitative part of the study the research must be trustworthy to ensure that the study is unbiased (Babbie & Mouton, 2002). To guarantee the trustworthiness of this study the researcher adhered to the criteria identified by Babbie and Mouton (2002), and Lincoln and Guba (1985). These included credibility, dependability, transferability, and conformability.

3.7.1 Credibility

The researcher aimed to achieve credibility by adhering to the following procedures.

- **Prolonged engagement:** the researcher stayed in the field until she was sure that data saturation occurred. Follow-up questions were, for example, posed to the lecturers to obtain more clarity on their perceptions in the written reflections.
- **Peer debriefing:** The researcher made use of a knowledgeable colleague in the field to verify her interpretations and initial findings. The researcher's supervisors also provided on-going oversight of the data collection and findings.

3.7.2 Dependability

For research to be dependable, it is important as a qualitative researcher to provide evidence that findings would be similar if the research were to be conducted with the same participants in the same context (Babbie & Mouton, 2002). To enhance the dependability of the study, the researcher adhered to the following criteria as set out by Lincoln and Guba (1985).

- The researcher's supervisors did an inquiry audit. The inquiry audit involved an examination of the document analyses, as well as the interpretations of the researcher to verify accuracy, credibility, and acceptability.
- The checking sessions with the researcher's supervisor and co-supervisor ensured that the researcher's interpretations were true reflections of what was contained in the curriculum documents of the university.

3.7.3 Transferability

Lincoln and Guba (quoted by Babbie & Mouton, 2002) assert that a qualitative study cannot be regarded as credible unless it is also transferable. Transferability refers to the extent to which the findings can be applied in other contexts or with other participants. Although it was not the purpose of the study to generalise findings, the researcher guaranteed transferability according to the following criteria identified by Babbie and Mouton (2002).

- **Thick description:** the findings contained a thick description of what transpired from all the analyses of the curriculum documents, the tests, and written reflections. The study also

provided a thorough description of the participants' contexts and biographical characteristics to make duplication of the study in similar contexts possible.

3.7.4 Confirmability

Research findings should not be based on the biases of a researcher (Babbie & Mouton, 2002), and a researcher should leave evidence to enable somebody to trace the interpretations and findings made (Lincoln & Guba, 1985). The audit trail of the document analyses and written reflections, which the researcher included, will assist a reader to determine if the conclusions, interpretations, and findings formulated are supported by the data obtained.

3.8 THE ROLE OF THE RESEARCHER IN QUALITATIVE RESEARCH

According to Creswell (2014), a qualitative researcher has to consider the following range of ethical and personal issues that could compromise the collection of trustworthy data, namely: historical, social, and cultural experiences, status, race, gender, assumptions, personal connection to the site, and sensitive ethical issues. As the qualitative part of the research did not involve working directly with people to obtain their opinions or experiences, the researcher only regards assumptions and personal connection to the site as issues that could compromise the collection of data.

3.8.1 Assumptions

Assuming certain positions regarding the research before collecting and analysing the data is easy. This could happen due to arguments in the literature and reports in the media insinuating that mathematics teachers have a poor academic background and that it is their fault that South Africa has such a poor mathematics pass rate (cf. 1.1). The researcher, therefore, had to be open-minded and form her conclusions regarding the outcome of the study despite what the media reported. It was the researcher's duty to report only what transpired from the data.

3.8.2 Personal connection to the research site

The researcher is a post-graduate student at the NWU, Vanderbijlpark Campus, known to many of the students and lecturers, and a mathematics teacher as well, which poses a threat to conflict of interest. The researcher made use of objective criteria when she conducted the document analysis to avoid that the analysis was biased or that the researcher's assumptions clouded the analyses of the curricula. Also, an independent researcher was responsible for obtaining informed consent to avoid that participants were coerced in taking part in the research and assisted in the administering of the tests to the participants. The researcher had no direct involvement with the research participants.

3.9 RESEARCH PARTICIPANTS

3.9.1 Participants for the quantitative study

The population of the research was all pre-service mathematics teachers and mathematics lectures involved in teacher-training at universities in South Africa. As it was not possible to conduct research with all the pre-service teachers in South Africa, a purposive and convenient study population was identified.

3.9.1.1 Study population

The study population for this study comprised all the pre-service teachers (year 1 to 4) with mathematics as a major subject at the NWU, Vanderbijlpark Campus (N = 21) and their lecturers (N = 4).

3.9.1.2 Sample

A sample can be defined as a smaller part of the population used for a research study (Creswell, 2014; Maree & Pietersen, 2016c; Strydom, 2011b). As it was important for the researcher to obtain the perceptions of pre-service teachers who were the closest to completing their studies and should have mastered mathematical subject content and pedagogical content knowledge across the four years of study, the fourth-year students were purposively approached to take part in the study. Purposive sampling can be defined as selective sampling to address a specific purpose (Maree & Pietersen, 2016c; Palys, 2008).

The NWU, Vanderbijlpark Campus was chosen conveniently for the following reasons. Firstly, the researcher qualified as a mathematics teacher at the NWU and therefore had good contact and working relationships with many of the lecturers to gain support for their and their students' involvement in the study. Convenient sampling is classified as non-probability sampling and in the context of the study, it implied that the research participants were easily accessible, available, and willing to take part in the study (Creswell, 2014; Strydom, 2011b). Secondly, at the time of the commencement of the research the NWU was in the process of developing and implementing a new curriculum for mathematics teacher-training across its campuses. The researcher argued that her study could provide valuable information to the lecturers involved in the development of the new mathematics curriculum for future adaptations.

Given the training model followed at the Mafikeng Campus where students with mathematics as a major subject are lectured in the Faculty of Science, the study population only comprised the students on the Vanderbijlpark Campus. Unfortunately, the Potchefstroom Campus did not want to participate in the study.

Due to the small number of students that enrol to become mathematics teachers, all final-year pre-service mathematics teachers were approached to take part in the study. Student numbers were confirmed closer to the study, and the final number of fourth-year pre-service teachers with mathematics as a major subject involved 21 participants ($N = 21$). Of the 21, 19 participants, seven FET participants ($n = 7$) and 12 ($n = 12$) Intermediate Phase participants were willing to take part in the study. The seven FET participants also constituted the sample for the Senior Phase, as FET and Senior Phase participants were exposed to the same training. The sampled participants were a heterogeneous group in terms of biographical variables such as gender, language, ethnic group, age, and phase of training. The lecturers on the Vaal Triangle Campus ($n = 4$) were all white, female lecturers with different qualifications and teaching experience.

3.9.2 Participants for the qualitative study

All the pre-service teachers who completed the tests were also involved in the completion of the qualitative, written reflections that formed part of the test. The four lecturers were requested to complete the open questions in the lecturer questionnaire.

3.10 DATA ANALYSIS

The data were analysed in two separate ways, as data were collected quantitatively and qualitatively. Descriptive statistical procedures were used for the analysis of the test responses and the closed items in the test. A content analysis was employed for the document study and the open items in the test, and the written reflections.

3.10.1 Quantitative test and closed questionnaire items: descriptive statistics

The data obtained from the various responses to the test and closed questionnaire items were analysed using descriptive statistics, namely: frequencies, means, and percentages. Descriptive statistics provided the researcher with a summary of the data (Leedy & Ormrod, 2015; Maree & Pietersen, 2008a; Pietersen & Maree, 2016).

Due to the large volumes of data obtained, and in order to adhere to guidelines concerning the length of the study, only syntheses of what transpired from the data analyses are included in Chapter 4. A comprehensive and detailed account of all descriptive analyses for the individual participants is presented in Addendums I1 – I3, and Addendums M1 – M3.

The results obtained by the participants for each of the questions concerning the various content areas in the tests were summarised in tables and graphical form. Also, the average percentages obtained by each of the participants for the entire test were reported (cf. Tables 4.2 – 4.4). The overall and collective performances of the participants in a teaching phase were presented in graphical form to synthesise data and identify trends concerning strengths and weaknesses for

the various subject content knowledge dimensions and cognitive process dimensions on which the research focused. The verbatim written reflections of the participants are presented in tabular form and appended in Addendum J (pre-service teachers) and Addendum D (lecturers).

3.10.2 Qualitative data analysis: written reflections to open questions

The open-ended questions posed to the pre-service teachers in the test and the questions posed to the lecturers in the questionnaire were content analysed to explore patterns in the data that could be clustered together. Attention was also paid to diverse opinions. A content analysis is “a detailed and systematic examination of the contents of a particular body of material for the purpose of identifying patterns, themes, or biases” (Leedy & Ormrod, 2005, p. 142.) There is no fixed process to analyse and present reflection data (Bold, 2012). The data analysis drew on patterns that emanated from the data (Bold, 2012). The researcher carried out the following procedures to analyse the written reflection data as described by Creswell (2009).

Step 1: Several close readings of the data to obtain a sense of its overall meaning and usefulness.

Step 2: Analysis of data by highlighting those parts in the participants’ reflections that contributed to answering the question asked. The researcher looked for similarities and expected information (reflected in the literature and known) as well as novel or surprising information.

Step 3: Grouping similar data by using different colours to extract prominent patterns.

Step 4: Constructing a composite that summarised the major findings that emanated from the reflections.

3.10.3 Qualitative document analysis

A content analysis systematically summarises message content and is a process typically performed on all forms of human communication, such as books, brochures, written documents, transcripts, news reports, and visual media. It was therefore regarded as a suitable way to analyse and interpret the curriculum (Leedy & Ormrod, 2005; Nieuwenhuis, 2016).

A comparative analysis of the pre-service teacher-training curriculum for mathematics for each teaching phase (Intermediate Phase (Gr. 4–6), the Senior Phase (Gr. 7–9) and the FET Phase (Gr. 10–12) and the corresponding teaching phases in the CAPS, was done in respect of the mathematical subject content knowledge and pedagogical content knowledge according to a priori codes (Nieuwenhuis, 2016). The following procedures were utilised for the objective and structured analysis of the curriculum:

- Verbs were identified in the description of outcomes in the CAPS and the teacher-training curriculum to classify cognitive process in relation to factual, conceptual, procedural, and metacognitive knowledge (cf. Table 4.6).
- The time allocated to teaching different content topics in relation to the weighting of the content in the CAPS and the teacher-training curriculum.
- Identifying guidance offered in applying teaching approaches and strategies suitable to achieve learning outcomes.

Based on the suggestions of Creswell (in Leedy & Ormrod, 2005, 2014) and Nieuwenhuis (2016), the researcher included the following steps as part of the document analysis:

- Step 1: The researcher identified the specific body of material to be studied.
- Step 2: The researcher defined the characteristics or qualities to be examined in precise, concrete terms concerning cognitive processes related to factual, conceptual, procedural, and metacognitive knowledge, as well as suggested teaching methods and strategies.
- Step 3: The researcher broke down the content into small, manageable segments and analysed them separately.
- Step 4: The researcher carefully examined the material for instances of each characteristic or quality defined in Step 2.
- Step 5: A description of similarities (and/or differences) that the data reflected was compiled (CAPS versus the teacher-training curriculum) (cf. 2.3.5; 2.4.5)

3.11 TRIANGULATION OF DATA

The collected data were integrated to enable the researcher to deduce research findings for each of the three teaching phases (cf. 4.6).

The final section in the chapter clarifies how the researcher complied with ethical principles during the research.

3.12 ETHICAL CONSIDERATIONS

The researcher adhered to ethical principles concerning the following aspects: the research problem, the purpose of the research and the research questions, the data collection procedure, data analysis and interpretation, and writing up and disseminating the research. In the following sections, the researcher explains in practical terms how she adhered to ethical principles.

3.12.1 Ethical issues in the research problem

As identified by Creswell (2009), and Strydom (2011a), it is important that the researcher identifies problems to study that will benefit the research participants. Most learners fail mathematics seemingly due to a lack of knowledge, which they need to gain from their teachers, who in turn do not possess adequate content or pedagogical knowledge (Kanyongo & Brown, 2013). This study will not only benefit the pre-service mathematics teachers who participate, but also the learners, whom they will teach one day. When the findings of the study are communicated to the participants, they might realise that their mathematical content knowledge is not up to standard or that they do not possess sufficient pedagogical knowledge. As a result, the participants could sharpen up knowledge to become better mathematics teachers. The learners who in turn will be taught by the participants could then also benefit from instruction that would probably be more effective. The participating university could also benefit from the study if the recommended guidelines could be followed to enhance the present teacher-training curriculum. In this sense, this study focuses on an important issue.

3.12.2 Ethical issues in the purpose and questions

It is of vital importance to the researcher to state the purpose of the study clearly so that all the participants will understand what the study is about (Creswell, 2009; Strydom, 2011a). The independent researcher explained the purpose of the study to all participants before the study commenced and clarified any questions that the participants might have. Participants were well-informed about what their involvement in the study would entail.

3.12.3 Ethical issues in data collection

An informed consent form (cf. Addendums K1, K2) was handed to each participant by the independent researcher to sign before the participants engaged in the research (Creswell, 2009). This form acknowledged that the participants' rights would be protected during data collection. In line with Creswell (2009), and Strydom (2011a), the researcher's consent form provided information regarding the following: identification of the researcher; participation that is voluntary; identification of how the participants were selected; identification of the purpose of

the study; the risks and benefits of participation; identification of the level and type of participant involvement; a guarantee of anonymity and confidentiality to the participant; assurance that the participant can withdraw at any time; and the researcher's details should the participants have any queries.

3.12.4 Ethical issues in data analysis and interpretation

When the researcher analyses and interprets data, some issues may emerge that call for good ethical decisions (Creswell, 2009). The researcher applied the following criteria identified by Creswell (2009):

- the anonymity of the participants was protected at all times by using a coding process to identify participants; and
- the test and questionnaire data were only available to the researcher and her supervisors to ensure confidentiality.

3.12.5 Ethical issues in writing and disseminating the research

This study did not use language or words that were biased against participants in terms of gender, age, campus, ethnicity, language, disability, or teaching phase (Creswell, 2009). Thus, in no circumstances did the researcher allow herself to form conclusions based on these pre-assumptions to conduct this study. The researcher also guarded against falsifying or inventing findings to meet the researcher's needs (Creswell, 2009). Therefore, the findings of this study were only based on the data obtained by the empirical study.

3.13 CHAPTER SUMMARY

The research methodology applied to this study was explained and motivated in this chapter.

As quantitative and qualitative data were collected in the context of the study, a pragmatic research paradigm (cf. 3.2) was used. The aim of the study was to determine to what extent the teacher-training curriculum at the NWU, Vanderbijlpark Campus prepare mathematics teachers to comply with competence expectations concerning mathematical subject content knowledge and pedagogical content knowledge as specified in the CAPS for teaching mathematics. Subsequently, the study aimed to make recommendations to strengthen, adapt, or enrich the current mathematics teacher-training curriculum used at the NWU, Vanderbijlpark campus.

Quantitative and qualitative data were collected as part of concurrent triangulation mixed methods research (cf. 3.4). A concurrent triangulation mixed method design collects quantitative

and qualitative data concurrently, compares, and integrates the data. The data were collected using various data collection methods, namely: researcher-constructed tests, a researcher-constructed questionnaire, and a document study (cf. 3.5). Tests with open-ended questions that focused on subject content knowledge according to the different teaching phases (Intermediate, Senior and FET) were administered to participating pre-service teachers (cf. 3.5.1). The qualitative data collection included open-ended questions at the end of each section in the test that focused on a specific subject content area to establish the teaching strategies and metacognitive strategies that the pre-service teachers possess. By using written reflections, the participants had to explain the use of relevant pedagogical strategies to teach the specific content on which the questions focused (cf. 3.5.2). The tests also contained some open questions (cf. 3.5.2), where student participants were requested to reflect in writing about the teacher-training curriculum and its qualities to enable them to become effective teachers. Two closed five-point semantic differential scale questions also formed part of the tests, where participants had to indicate their preparedness to teach mathematics subject content knowledge, and the variety of pedagogical strategies they have available to teach mathematics.

The mathematics lectures were also given a questionnaire with open questions where they were asked about their qualifications and to reflect in writing about the qualities of the teacher-training curriculum (cf. 3.5.2).

Lastly, the qualitative data collection also included a comparative document analyses of the NWU teacher-training curriculum against the CAPS to establish congruence about the competency expectations for mathematics teachers (cf. 3.5.2.1).

To ensure the reliability of the questionnaire, a pilot study was done which confirmed that the tests complied with face, content, and construct validity. Also, criteria for internal, external, construct, and statistical conclusion validity were complied with, to confirm the validity of the quantitative research instruments (cf. 3.6.1, 3.6.2). To support the trustworthiness of the qualitative data, the research complied with criteria for credibility, transferability, dependability, and confirmability (cf. 3.7).

The data collection was not compromised by the role of the researcher. To ensure this the researcher paid attention to her personal assumptions and the personal connection she had to the research site (cf. 3.8).

Non-probability, convenient sampling was used to select participants for the study. The participating pre-service mathematics teachers were recruited from the NWU, Vanderbijlpark Campus (cf. 3.9.1). All the final year pre-service mathematics teachers at the NWU,

Vanderbijlpark Campus (N =21) were invited to take part in the study. Nineteen of the participants were willing to be included in the sample (n = 19).

Ethical considerations were firmly adhered to during the research by identifying a meaningful research problem, clarifying the purpose of the research clearly to participants, and complying with ethical principles during data collection, analysis, and interpretation of data, and the writing and dissemination of the research (cf. 3.11).

The next chapter, Chapter 4, focuses on the analysis and interpretation of the data.

CHAPTER 4: DATA ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

This chapter aims to provide a comprehensive overview of the preparedness of the pre-service mathematics teachers who took part in the study concerning the expectations concerning subject content knowledge and pedagogical content knowledge based on the analyses of the data collected.

The chapter addresses the following issues:

4.2 BIOGRAPHIC INFORMATION OF THE RESEARCH PARTICIPANTS

4.3 QUANTITATIVE DATA ANALYSIS AND INTERPRETATION: OVERALL TEST RESULTS

4.3.1 Descriptive data analysis

4.3.1.1 Further Education and Training Phase (FET)

4.3.1.2 Senior Phase

4.3.1.3 Intermediate Phase

4.3.2 Data analysis guidelines

4.3.3. Analysis and interpretation of data: knowledge and cognitive process dimensions – FET Phase

4.3.4 Analysis and interpretation of data: knowledge and cognitive process dimensions – Senior Phase

4.3.5 Analysis and interpretation of data: knowledge and cognitive process dimensions – Intermediate Phase

4.4 QUALITATIVE DATA ANALYSIS AND INTERPRETATION: STUDENT PARTICIPANTS

4.4.1 Student perceptions: the metacognitive knowledge dimension

4.4.2 Student perceptions: preparedness to teach and availability of teaching strategies

4.4.3 Student perceptions: strengths of teacher training

4.4.4 Student perceptions: weaknesses of teacher training

4.4.5 Student perceptions: changes to improve teacher training

4.4.6 Student perceptions: reasons for being a good teacher

4.4.7 Student perceptions: reasons for not being a good teacher

4.4.8 Student perceptions: the easiest and most difficult test questions

4.4.9 Student perceptions: teaching strategies used to teach subject content areas

4.5 QUALITATIVE DATA ANALYSIS AND INTERPRETATION: LECTURERS

4.6 TRIANGULATING QUANTITATIVE AND QUALITATIVE DATA

4.7 CHAPTER SUMMARY

To make sense of the data analysis presented in this chapter, the following should be observed:

Due to large volumes of data generated and restrictions in terms of page numbers that the final dissertation should comprise of, this chapter only presents syntheses of what transpired from the data analyses. The comprehensive data analyses for each of the research participants that informed the syntheses are attached in Addendums I (1-3) and (Quantitative data) and Addendum J (Qualitative data), and elucidate the syntheses presented in the chapter. Please note that some tables do not follow in chronological order, as many of the tables were moved to the Addendum to reduce the length of the study. The discussion in the chapter however references the tables in their original chronological order, and refers the reader to the tables attached in the Addendums.

The first section of the chapter presents the biographic information of the student participants.

4.2 BIOGRAPHIC INFORMATION OF RESEARCH PARTICIPANTS

Table 4.1 summarises the biographic information of the research participants.

Table 4.1: Biographic information of the research participants

Age	20-22	23-25	25-30	Older	Other
FET	3	3	1		
Senior	3	3	1		
Intermediate	8	3		1	
Gender	Male		Female		
FET	4		3		
Senior	4		3		
Intermediate	3		9		
Involved in extra mathematics classes at school	No	Daily	Weekly	Monthly	
FET	2	1	3	1	
Senior	2	1	3	1	
Intermediate	10		1		1
Name of school attended					
FET					
Participant 1	Sizme Senior Secondary School (Township)				
Participant 2	Kgoro Ye Thuto Secondary School (Township)				
Participant 3	Tokelo Secondary School (Township)				
Participant 4	Sasolburg High School (ex Model C)				
Participant 5	Rutesetjhaba Secondary School (Township)				
Participant 6	Hoërskool Dr Malan (ex Model C)				
Participant 7	E.P.P Mhunga Secondary School (Township)				

	Senior	
Participant 1	Hoërskool Dr Malan	
Participant 2	Rutesetjhaba Secondary School	
Participant 3	Sasolburg High School	
Participant 4	Sizme Senior Secondary School (Township)	
Participant 5	Tokelo Secondary School (Township)	
Participant 6	Kgoro Ye Thuto Secondary School.	
Participant 7	E.P.P Mhunga Secondary School	
	Intermediate	
Participant 1	Rutasetjhaba Secondary School	
Participant 2	Fundulwazi Secondary School	
Participant 3	----	
Participant 4	Graceland Education Center	
Participant 5	Boksburg High School (ex Model C)	
Participant 6	Meyerton High School (ex Model C)	
Participant 7	Vaalpark Articon (ex Model C)	
Participant 8	Hoërskool Overvaal (ex Model C)	
Participant 9	Hoërskool Dr Malan (ex Model C)	
Participant 10	Sasolburg High School	
Participant 11	Hoërskool Drie Riviere	
Participant 12	-----	

*'Other' in the table refers to participants who are already in a full-time teaching position.

According to Table 4.1, the majority of the participants across the three phases fall in the age group 20–22. An equal number of male and female participants comprised the FET and Senior Phases in the study, and mainly female participants were representative of the Intermediate Phase. Most of the FET and Senior Phase participants, namely three, were involved in weekly extra mathematics classes at school, whereas the majority of the Intermediate Phase participants, namely ten, were not involved in any extra mathematics classes at school. The participants completed their matric at 14 different public schools, of which six were township schools. No private schools were involved in the study. Participants also attended seven of the so-called "model C" schools, a term which is still commonly used to describe former whites-only government schools.

The following section presents the analysis and interpretation for the quantitative test data.

4.3 QUANTITATIVE DATA ANALYSIS AND INTERPRETATION: OVERALL TEST RESULTS OF PARTICIPANTS

4.3.1 Descriptive data analysis

The sections below summarise the descriptive data for the test results obtained by the participants in the different phases.

4.3.1.1 Further Education and Training Phase (FET Phase)

Table 4.2 summarises the results obtained by the FET participants. The table reflects the total achieved by each participant for each of the test questions, as well as the average percentage obtained by each participant for the test in total.

Table 4.2: Test results: FET Phase

Participants	Question							Total	%
	1	2	3	4	5	6	7		
1	0	4	2	3	7	0	0	16	28,07
2	0	4	2	8	7	0	0	21	36,84
3	0	3	2	0	12	0	3	20	35,08
4	0	2	2	0	7	0	0	11	19,29
5	0	4	2	0	7	0	4	17	29,84
6	0	2	1	0	4	0	3	10	17,54
7	0	0	1	2	6	0	1	10	17,54
Question total	10	8	6	9	12	6	6	57	100,00
Test average %									26,31
Question average %	0	33,92	28,57	20,63	59,52	0	26,19		

*Key:

Question 1: Differential calculus

Question 2: Algebra and equations

Question 3: Number patterns, sequences, and series

Question 4: Finance, growth, and decay

Question 5: Functions

Question 6: Probability

Question 7: Statistics

Table 4.2 displays troublesome results. None of the participants scored above 50% for the FET test. The average for the FET test was 26,31%. When comparing the test averages to the scores of the participants it can be seen that only participants 2, 3, and 5 obtained scores above the test average.

Participants 2 and 3 scored the highest marks with an average of 36,84% and 35,08% respectively, and Participants 6 and 7 obtained the lowest test averages, with 17,54% each. If

the percentage score is seen as the amount of subject content knowledge that the participants possess, it can be assumed that the participants with the highest averages only knew approximately 35-37% of the FET subject content covered in the test. The researcher is concerned that if the highest-scoring pre-service teachers in this group know less than 50% of the FET content, the effectiveness of teaching in the FET might be compromised. The participants obtained the best scores in Question 5 (functions), with an average of 59,52 % for the question. It seems like the most difficult questions were Question 1 (differential calculus) and Question 6 (probability). None of the participants could score any points for Questions 1 and 6.

The following section summarises the descriptive test results for the Senior Phase participants.

4.3.1.2 Senior Phase

The FET and Senior Phase participants wrote both the FET and Senior Phase tests. Based on the experience of the researcher, teachers qualifying in either one of the two phases are often expected to teach both phases. The participants were given the FET phase test after finishing the Senior phase test (the first to finish the Senior Phase test was the first to receive the FET Phase test), therefore due to ensuring anonymity and testing circumstances, the FET and Senior Phase numbering of the participants did not necessarily correspond. Table 4.3 reflects the results obtained by the participants in the Senior Phase. The numbering of the participants that appear in brackets indicate the original numbering on their Senior Phase tests. The numbering without brackets was added to the table to correspond with the numbering of the FET Phase participants. The numbers in brackets guided the analysis and interpretation of the Senior Phase test data in section 4.3.4.

Table 4.3: Test results: Senior Phase

Participants	Question					Total	%
	1	2	3	4	5		
FET 1 (4) Senior	8	11	5	3	3	30	78,94
FET 2 (6) Senior	8	11	11	3	3	36	94,73
FET 3 (5) Senior	8	10	4	3	1	26	68,42
FET 4 (3) Senior	7	11	11	3	3	35	92,10
FET 5 (2) Senior	5	9	8	3	1,5	26,5	69,73
FET 6 (1) Senior	4	11	5	0	2,5	22,5	59,21
FET 7 (7) Senior	8	11	3	3	2,5	27,5	72,36
Question total	8	11	11	4	4	38	100,00
Test average %							76,49
Question average %	85,71	96,10	61,03	64,28	58,92		

*Key:

Question 1: Patterns, functions, and algebra

Question 2: Numbers, operations, and relationships

Question 3: Space and shape (geometry)

Question 4: Measurement

Question 5: Data handling

The clarification of the data in Table 4.3 is based on the numbers of the participants within brackets. Compared to the FET data in Table 4.2, the data in Table 4.3 reveal an encouraging picture. All the participants managed to score above 50% average for the test. The average for the Senior Phase test was 76,49%, and three of the participants scored above the test average. Participant 6 had the highest score for the test, namely 94,73%. Good results were also obtained by Participant 3 (92,10 %) and Participant 4, who scored 78,94%. Participant 1 had the lowest score of 59.21% for the test. The lowest scores were obtained for Question 5 (data handling), as the average was only 58,92%. The highest scores were obtained for Question 2 (numbers, operations, and relationships), leading to an average of 96,10%. The results for patterns, functions, and algebra were also encouraging, namely an average of 85,71%.

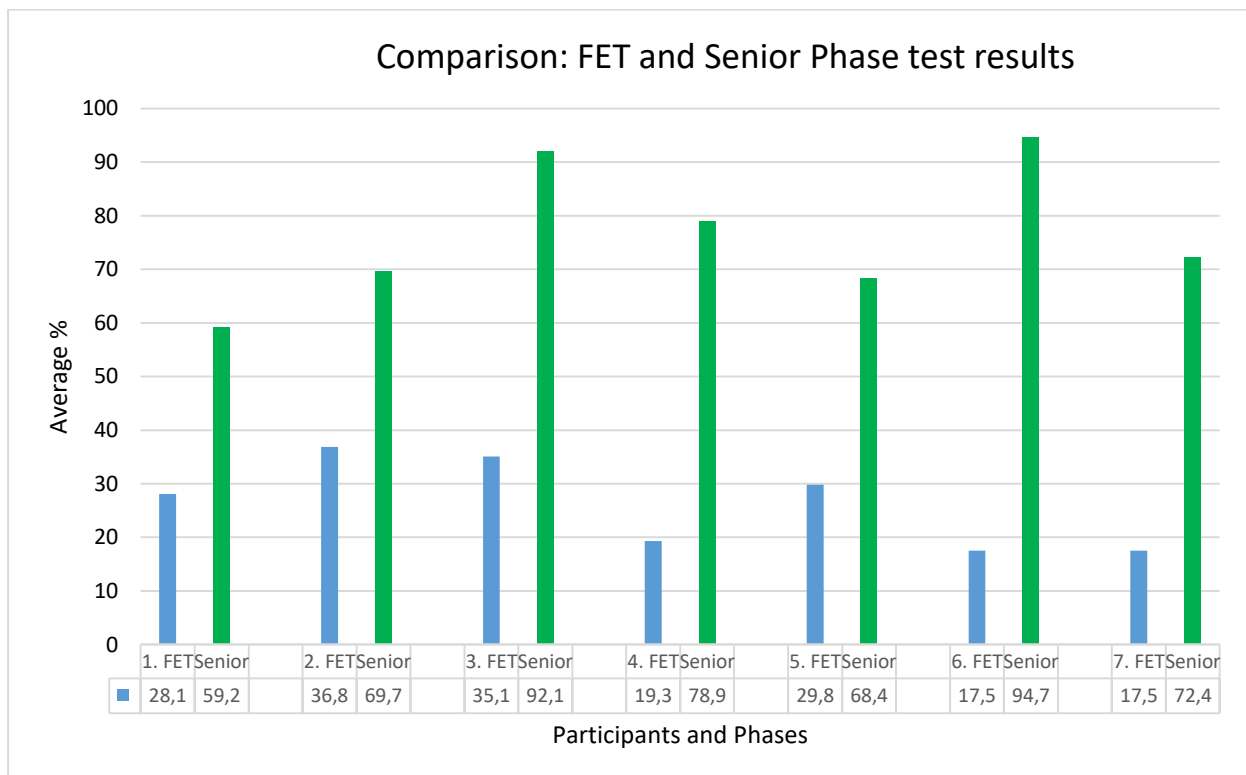
Table 4.4 reflects a comparison of the test averages obtained by the participants who wrote both the FET (cf. Table 4.2) and Senior Phase tests (cf. Table 4.3 (numbering within brackets)).

Table 4.4: Test averages: FET and Senior Phase participants

Participants	Phase	$\bar{x}$
Participant 1	FET	28,07
	Senior	59,21
Participant 2	FET	36,84
	Senior	69,73
Participant 3	FET	35,08

	Senior	92,10
Participant 4	FET	19,29
	Senior	78,94
Participant 5	FET	29,84
	Senior	68,42
Participant 6	FET	17,54
	Senior	94,73
Participant 7	FET	17,54
	Senior	72,36

Graph 4.1 graphically displays the results in Table 4.4.



Graph 4.1: Comparison: FET and Senior Phase test results

(Values in Table 4.4 were converted to the nearest integers on the graph)

From Graph 4.1 it can be seen that the participants all scored much higher marks in the Senior Phase test than in the FET Phase test. Participant 6 had the highest average in the Senior Phase test (94,7%), but had a low average (17,5%) in the FET phase test. The second-highest score for the Senior Phase test was Participant 3 (92,1%), who again scored a mere 35,1% average in the FET Phase test. Participant 4 obtained the third-highest score, namely 78,94%.

Participant 1 scored the lowest average in the Senior Phase test (59,2%), and only 28,1% in the FET Phase test. Participant 2 scored 36,8 % in the FET test, and obtained 69,7% in the Senior Phase test. Participant 7 who together with Participant 6 scored the lowest averages in the FET Phase, namely 17,5%, obtained averages of 72,4% (Participant 7) and 94,7% (Participant 6) respectively for the Senior Phase test. A similar tendency was observed for Participant 5, who obtained a low score for the FET test, namely 29,8% and 68,4% respectively for the Senior Phase test.

The next section pays attention to the descriptive test results obtained by the participants in the Intermediate Phase.

4.3.1.3 Intermediate Phase

Table 4.5 provides an overview of the test data for the participants who wrote the Intermediate Phase test.

Table 4.5: Test results: Intermediate Phase

Participants	Question					Total	%
	1	2	3	4	5		
1	17	3	1	1	6	28	71,79
2	10	5	1	4	6	26	66,66
3	12	5	1	0	6	24	61,53
4	10	3	1	1	6	21	53,84
5	4	5	3	4	6	22	56,41
6	3	5	2	2	5	17	43,58
7	8	5	1	2	6	22	56,41
8	12	3	2	1	6	24	61,53
9	17	5	3	3	6	34	87,17
10	15	5	3	2	6	31	79,48
11	11	0	0	0	0	11	28,20
12	0	1	1	3	6	11	28,20
Question total	20	5	3	5	6	39	100,00
Test average %							57,90%
Question average %	49,58	75	52,77	38,33	90,27		

*Key:

Question 1: Numbers, operations, and relationships

Question 2: Patterns, function, and algebra

Question 3: Measurement

Question 4: Space and shape (geometry)

Question 5: Data handling

The data in Table 4.5 reveal that the test average for the Intermediate Phase participants was 57,90%. Six of the 12 participants scored below the test average, and two participants scored above 80% for the test. Participants 9 and 10 obtained the highest scores, namely 87,17% and 79,48%, respectively. Participants 11 and 12 also had the same scores, namely 28,20%, which

were the lowest scores obtained for the test. Participant 11 only scored points in Question 1 (numbers, operations, and relationships), whereas Participant 12 scored no marks for Question 1. The participants had the lowest average for Question 4 (space and shape), with an average of 38,33%, and the highest average for Question 5 (data handling), with an average of 90,27%.

In the following section, the researcher presents guidelines that directed the analysis of the participants' test data.

4.3.2 Data analysis guidelines

The researcher wishes to state the following upfront to clarify the data analysis procedure that was followed for each of the phases:

- The data analysis focuses on the main problems or inadequacies observed among the participants concerning answering each of the questions. Detailed information regarding the participants' responses to questions, can be found in the copies of the test scripts that are provided in Addendums G1 – 3.
- Each question was analysed concerning the researcher's observation of the factual, conceptual, and procedural knowledge dimensions and related cognitive process dimensions that had to be in place for a participant to answer a question successfully. For each question the cognitive process dimensions for each of the knowledge dimensions are specified in table form. These tables (cf. Addendums I1 – I3) should be consulted to understand the syntheses provided in the chapter.
- To be successful with the application of any cognitive process dimension as part of conceptual and procedural knowledge, a solid factual knowledge base that requires remembering and understanding was regarded as a prerequisite.
- Although the participants might have provided a correct answer to some of the easy questions, the researcher's assessment of their answers often indicated a lack of conceptual knowledge if the participants could not provide evidence in their answers of solid conceptual understanding.
- All participants received an information sheet to use during the completion of the test. As the researcher anticipated that not all the participants would have been able to remember all of the formulas and important content dealt with across their years of study, the information sheet was provided as a way to avoid participants being disadvantaged for not remembering important information to answer the questions.
- The cognitive process dimensions, according to the revised taxonomy of Bloom, were linked to relevant and meaningful cognitive actions in the context of mathematics as presented in the CAPS (cf. Addendums E1 - 3). The cognitive actions are depicted by

verbs, which were collaboratively identified by the researcher and verified by the co-supervisor. These cognitive actions had to be executed by the participants in order to be regarded as successful (cf. Table 4.6).

- Table 4.6 classifies the verbs used in the context of the mathematics learning outcomes to be achieved in the various mathematics content areas (as specified by the CAPS) across all the phases, linked to the cognitive process dimensions according to the revised taxonomy of Bloom. The researcher acknowledges that people might perceive the identification of verbs linked to the revised taxonomy levels of Bloom different from the researcher. However, in the context of the study, the identified verbs were found to be the most suitable and identified in consultation with the co-supervisor who is a lecturer in Mathematics. The verbs were therefore applied uniformly to analyse the responses of all the participants.
- The researcher continued marking a participant's answer to the end, even if a mistake was made prior to the final answer, thus not complying to the normal procedure for marking mathematics. It was important to continue marking as the researcher wanted to obtain information about all the knowledge dimensions involved in answering a specific question.
- Reference made to the 'majority' or 'most' of the participants in the data analysis refers to more than half of the number of participants.
- The main purpose with the tests was to get an initial impression of the solidity of subject content knowledge that the research participants possess, and not to provide an in-depth analysis of the mistakes they made. It was however necessary to take cognizance of the mistakes made in order to derive initial conclusions about the absence or presence of solid subject content knowledge.

Table 4.6: Linking verbs in the context of the CAPS curriculum to Bloom's revised taxonomy

Knowledge Dimension		Cognitive Process Dimension					
		Remember	Understand	Apply	Analyse	Evaluate	Create
	Factual	List Read Recall Recite Name	Summarise Compose and decompose numbers Locate points Understand	Classify Choose Follow simple routine procedures Apply (an algorithm) Represent words in diagrams	Order Characterise Retrieve information from a graph or table Identify or locate specific information in graphs, charts or tables	Rank	Combine Brainstorm
	Conceptual	Describe Define	Interpret Discuss Give examples Evaluate an expression Specify and explain relationships Identify main ideas Understand	Experiment Instruct Establish Determine Select a procedure Solve a routine problem by applying multiple concepts Retrieve information from a graph to solve a problem with multiple steps Substitution Apply Manipulate	Select appropriate displayed data Organise and interpret simple data or variables Extend patterns Identify relevant or irrelevant information Order	Assess Prove	Plan Formulate Propose Generate conjectures or hypotheses based on observation or prior knowledge Derive
	Procedural	Tabulate Reproduce Write	Predict Report Explain Illustrate	Calculate Compute Produce Solve Show Demonstrate Use concepts to solve non-routine problems Apply concepts to problems Manipulate Determine Identify Count Find Substitute	Differentiate Compare information Analyse and draw a conclusion Organise or interpret data Select	Describe, compare and contrast solution method Verify the reasonableness of results Justify conclusions made Substitute Find Interpret Conclude	Develop a complex model for a given situation Compose Manipulate Integrate Adapt Formulate

The next section provides the data analysis and interpretation of the knowledge and cognitive process dimensions concerning the FET phase.

4.3.3 Analysis and interpretation of data: knowledge and cognitive process dimensions – FET Phase

For each of the questions in each of the teaching phases a table provides information about the ideal assessment (x) and the individual assessments of each of the participants for the development of the applicable cognitive process dimensions in the context of subject content knowledge. The participants' responses compared to the ideal assessment are indicated by presenting the number of the successful participants next to the ideal responses. Next to the X, the numbers of the participants who were successful with a cognitive process dimension linked to a knowledge dimension are indicated.

It was important for the researcher to obtain an overall view of which of the knowledge dimensions and cognitive process dimensions appeared to require attention. For each of the questions in each of the phases the data obtained are presented graphically to display problem areas pertaining to knowledge and cognitive process dimensions for each of the subject content areas among the pre-service teachers who took part in the study in the content area data handling.

Table 4.7 reflects the assessment for Question 1.

Table 4.7: Data analysis: FET Phase Question 1 – differential calculus

	Knowledge dimension	Cognitive process dimension					
		1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Q1	Factual	x 1 2 3 4 5			x		
	Conceptual		x 3 5	x 4	x 4	x	x
	Procedural			x			

4.3.3.1 Data analysis: Question 1 – differential calculus

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.8 (see Addendum I1, p. 116).

Differential calculus is a topic that requires knowledge application from various content areas within mathematics, in this case, mainly measurement. The factual knowledge related to content areas of preceding grades is required to answer this question. If the participants could not link the content areas space, algebra, and calculus, they would not be successful in answering the question. As part of conceptual knowledge, the participants could have used different methods to obtain the same solution, if the methods are applied correctly. Also, knowing multiple concepts in mathematics is important to understand the relationships between concept applications, as well as to link different content areas within mathematics (Lauritzen, 2012; Rittle-Johnson et al., 2015) (cf. 2.3.2.2).

Question 1 tests the participant's depth and breadth of knowledge, since it requires not only knowledge from one, but various content areas/topics (algebra, Euclidean geometry, measurement, analytical geometry, trigonometry, and differential calculus). By not indicating a clear plan for solving the problem (even with an incorrect answer), it could be argued that the participants lack conceptual knowledge. A "plan" in Question 1 was associated with some indication of understanding by writing down steps or drawing a sketch to show reasoning or planning of a solution, but they were not explicitly required to do this.

In summary, important observations that emanate from the analysis concerning differential calculus (Question 1, cf. Table 4.8), are as follows:

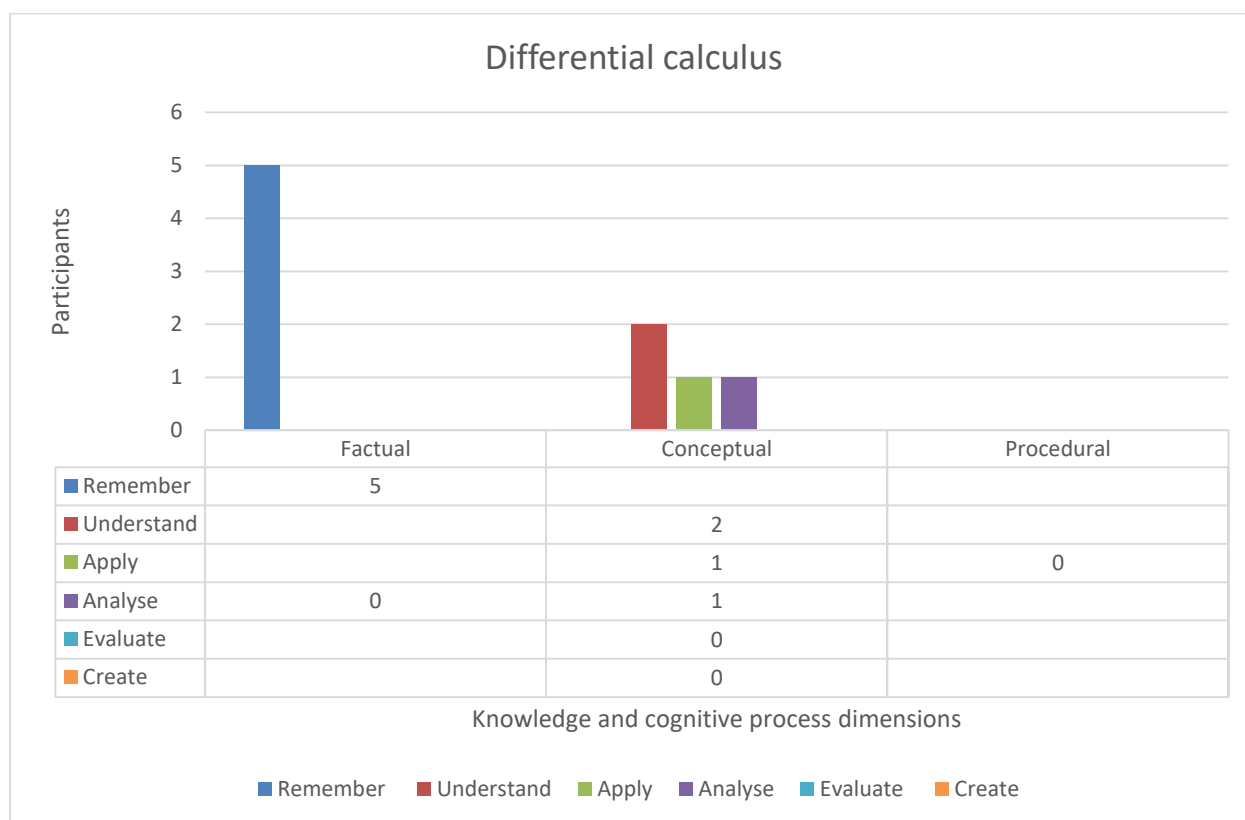
- According to Table 4.7, participants 1, 2, 3, 4, and 5 were able to remember information as part of the factual knowledge dimension. Furthermore, participants 3 and 5 were successful at understanding the conceptual knowledge to answer question 1. Participant 4 appeared to be the only participant who was able to be successful at the application and analysis of the conceptual knowledge, and not one of the participants was successful at applying procedural knowledge. Two participants, participants 6 and 7, were not successful with any of the cognitive process dimensions.
- None of the participants appeared to be effective at creating and evaluating knowledge as part of the conceptual knowledge dimension. It could be a result of not remembering sufficient factual and conceptual knowledge, or a lack of understanding how to connect the concepts from different content areas in the FET mathematics curriculum as indicated in the CAPS (algebra, Euclidean geometry and measurement, analytical geometry, trigonometry, and differential calculus).
- Only two participants (participants 3 and 5) could remember factual knowledge dimension, as well as understand information as part of the conceptual knowledge dimension.
- Application as part of the conceptual knowledge dimension and analysis that formed part of the factual and conceptual knowledge dimensions appeared to be the most troublesome

cognitive processes, as only participant 4 could apply and analyse in the conceptual knowledge dimension. None of the participants could analyse the factual knowledge.

- None of the participants were successful with the application of the information in the procedural knowledge dimension.
- The majority of the participants (participants 1, 2, 3, 4, and 5) were successful at remembering factual knowledge. They could remember a formula relating to the factual knowledge of the question, but seemingly remembering this information was not adequate to support their being successful in the other knowledge dimensions.

Not one of the participants managed to score any marks for Question 1 (cf. Table 4.2), although some of them were capable of understanding (participants 3 and 5) and applying and analysing information (only participant 4) in the conceptual knowledge dimension. It appears as if their unsuccessfulness with the cognitive process dimensions related to apply and analyse (all participants, except participant 4) and in particular to evaluate, and create (all the participants) across all the knowledge dimensions, contributed to the incorrect answer.

Graph 4.2 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 1 (differential calculus).



Graph 4.2: Question 1 – differential calculus

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required in a particular knowledge dimension

The researcher observes the following trends from the graph:

- The cognitive process dimensions related to understand, apply, analyse, evaluate, and create across all the knowledge dimensions (cf. Table 4.7), could be viewed as problematic.
- Noteworthy is that the participants seem to be the most successful with remembering factual knowledge.
- Both the conceptual and procedural knowledge dimensions, according to the graph, seem to have been troublesome. The higher the cognitive level within the conceptual knowledge dimension (analyse, evaluate, create), the more demanding it became for the participants to answer the question.

The following section pays attention to the interpretation of Question 2, algebra and equations.

4.3.3.2 Data analysis: Question 2 – algebra and equations

Table 4.9 reflects the assessment for Question 2.

Table 4.9: Data analysis: FET Phase Question 2 – algebra and equations

	Knowledge dimension	Cognitive process dimension					
		1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Q2	A Factual	2.1	x 2 3 6 7		x 2 3 6 7	x 2 3 6 7	
		2.2	x 1 2 3 4 5 6			x 1 2 3 4 5	
	B Conceptual	2.1		x	x		
		2.2		x 1 2 3 5	x 1 2 3 4 5 6	x 1 2 5	
	C Procedural	2.1			x		
		2.2			x 1 2 5		

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.10 (see Addendum I1, p. 119).

Algebra and equations relate to the number system (including real, rational and irrational numbers, etcetera), rounding, estimation, and the application of all mathematical operations to find roots, products, simplifications, and factors when solving equations. Quadratic inequalities, polynomials, simultaneous equations, and word problems also form part of this section.

Question 2.1

As indicated in Table 4.9, participants 2, 3, 6, and 7 could remember factual knowledge, and also appeared to be effective at applying and analysing factual knowledge. None of the participants seemed to be effective at understanding and applying conceptual knowledge, and applying procedural knowledge (cf. Table 4.10).

In summary, important observations that emanate from the analysis (cf. Addendum I1) concerning algebra and equations for Question 2.1, are the following:

- Although four participants (participants 2, 3, 6, and 7) appeared to have factual knowledge in place and could remember, apply, and analyse, they could not arrive at the correct answer. In the context of the question, the factual knowledge referred to having factual knowledge of the exponential rules, but not necessarily knowing which rules to apply to get the correct solution. Exponential rules were used as part of the solution, but as the rules were used incorrectly it did not help the participants to be successful in answering the question correctly. A possible reason for this can be that students might not be given the opportunity to deeper understand factual knowledge during their training but are only taught to give answers for straightforward questions. Students might know that the square of a number results in a number greater than, or equal to zero, but when it is part of an equation they cannot apply the factual knowledge.
- The key to answering the question correctly was recalling that squared numbers (or in this case parenthesis) are equal to zero or a positive number. Participants 2, 3, 6, and 7 all failed at recalling this. The participants could only recall the order of calculations, which means the answer will be incorrect as the main principle that needed recalling was missing for all four of these participants. All four participants who could remember factual knowledge, were able to apply and analyse the factual knowledge (order of calculations). Often learners are exposed to rote learning, and no deeper understanding is encouraged.
- All the cognitive process dimensions related to the conceptual and procedural knowledge dimensions appear to be problematic (cf. Table 4.10), as none of the participants were successful.

Question 2.2

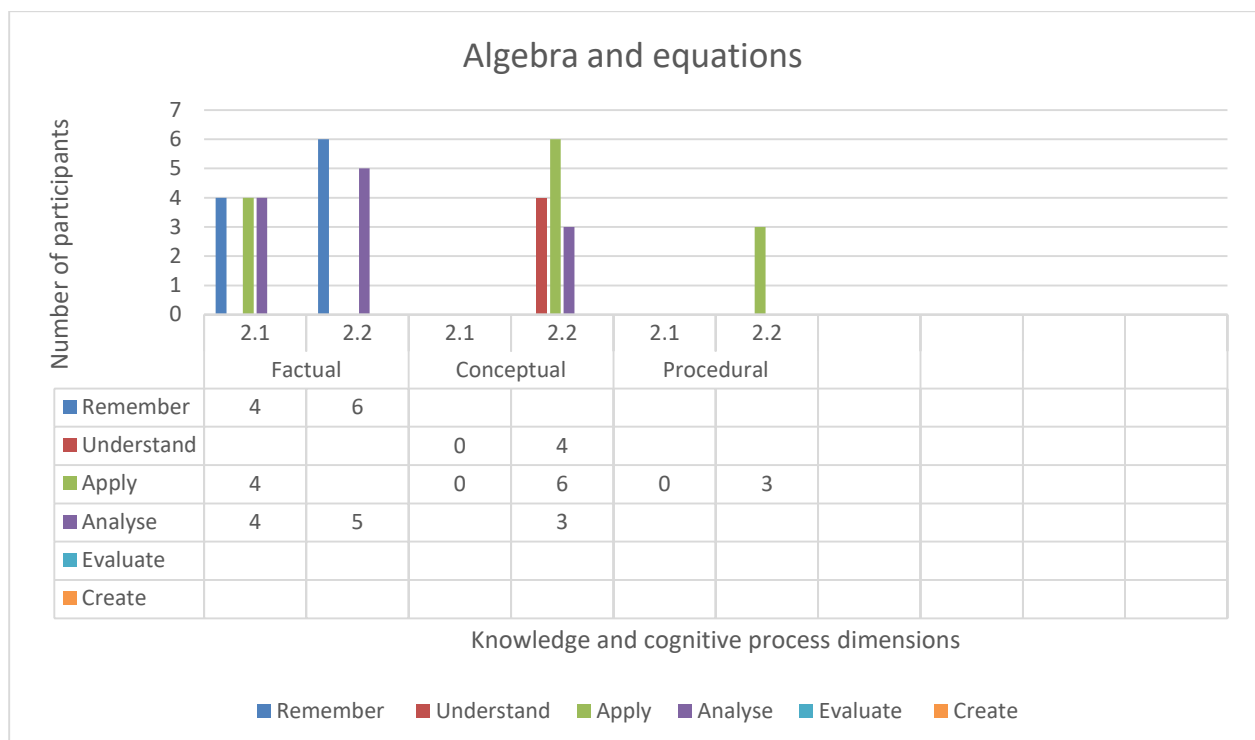
According to Table 4.9, all participants, except participant 7 seemed able to remember factual knowledge, and only participants 1, 2, 3, 4, and 5 could analyse factual knowledge. Participants 1, 2, 3, and 5 were successful at understanding the conceptual knowledge to answer question 2. Furthermore, participants 1, 2, and 5 appeared to be the only participants able to analyse factual and conceptual knowledge. Participants 1, 2, 3, 4, 5, and 6 who were successful at remembering factual knowledge, were also successful at applying conceptual knowledge. Participant 7 was the only participant who was unsuccessful in all the knowledge dimensions, and participants 1, 2, and 5 were successful in all the knowledge dimensions.

In summary, important observations that emanate from the analysis (cf. Addendum I1) concerning algebra and equations for Question 2.2, are the following:

- Only participants 1, 2, 3, 4, and 5 displayed skill in analysing factual knowledge and only participants 1, 2, and 5 could analyse in the conceptual knowledge dimension. The majority of the participants could execute the lower-order cognitive processes, such as remembering in the factual knowledge dimension (participants 1, 2, 3, 4, 5, 6), understanding in the conceptual knowledge dimension (participant 1, 2, 3, 5), and apply as part of the conceptual knowledge dimension (participants 1, 2, 3, 4, 5, 6).
- Except for one participant (participant 7), all the others were successful at remembering factual knowledge and applying conceptual knowledge. Only participants 1, 2, and 5 were successful with the application of procedural knowledge.

Not one of the participants managed to score more than 4 out of 8 for Question 2. Participants 1, 2, and 5 scored 4 out of 8. It appears that the success with the cognitive process dimensions related to analysing conceptual knowledge, and applying procedural knowledge (cf. Table 4.10), contributed to their partially correct answers.

Graph 4.3 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 2.



Graph 4.3: Question 2 – algebra and equations

*If no values are displayed in some of the blocks in the data table, it implies that a particular cognitive process dimension was not required in a specific knowledge dimension.

The researcher observes the following trends from Graph 4.3:

- Most of the participants seem to be successful in remembering factual knowledge (Questions 2.1, 2.2).
- Understanding of conceptual knowledge appears to be in need of attention, as only four of the seven participants were successful with understanding conceptual knowledge (Question 2.2), and none of the participants displayed conceptual knowledge for Question 2.1.
- The application of knowledge was quite successful in the conceptual knowledge dimensions for Question 2.2 (six of the seven participants).
- Application and analysis were executed successfully in the factual knowledge dimension (Question 2.1) by four of the seven participants. However, in the conceptual and procedural knowledge dimension, none of the participants was successful with the application of knowledge. Application of information in the conceptual knowledge dimension in the context of Question 2.2 appeared to be more successful. The reason participants might have found Question 2.1 more difficult in the conceptual and procedural

knowledge dimensions might be due to the question having two variables in one equation. The participants might not have been used to questions with multiple variables.

- Analysis of factual knowledge for both questions seemed to be more successful than what it was for analysing conceptual knowledge for Question 2.2. This might be because factual knowledge for Question 2 is based on recalling and identifying of rules or principles, whereas conceptual knowledge is about selecting the correct rule to answer the question, analysing and understanding the question, and applying the rules or principles, thus involving cognitive processes at a higher-order cognitive level.
- Overall, when comparing the responses for the two questions, the participants displayed greater success with the factual, conceptual, and procedural knowledge dimensions related to Question 2.2.

The following section pays attention to the interpretation of Question 3, numbers, patterns, sequences, and series.

4.3.3.3 Data analysis: Question 3 – numbers, patterns, sequences, and series

Table 4.11 reflects the assessment for Question 3.

Table 4.11: Data analysis: FET Phase Question 3 – numbers, patterns, sequences, and series

Q3	Knowledge dimension	Cognitive process dimension						
		1 Remember		2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
	A Factual	3.1	x 1 2 3 4 5 6 7		x 1 2	x 1 2 3 4 5 6 7		
		3.2	x 3 5 6		x			x
	B Conceptual	3.1		x 1 2 3 4 5 6 7	x 1 2 3 4 5 6 7	x 1 2 3 4 5 6 7		
		3.2			X 3 5	x 3 5		x
	C Procedural	3.1			x 1 2 3 4 5 6 7			
		3.2			x			

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, is provided in Table 4.12 (see Addendum I1, p. 123).

The content area, numbers patterns, sequences, and series in FET mathematics comprises the following: identifying a general term and constant difference for a number pattern, it refers to quadratic patterns, geometric and arithmetic sequences as well as series, sigma notation, derivation, and application of the formulae for the sum of arithmetic and geometric series. In the topic of number patterns, sequences, and series it is important to identify the type of question, whether it is a sequence or series, and whether the sequence or series is either arithmetic or geometric. It is important to identify and recall the general formula as an indication of factual knowledge. Correctly identifying the sequence or series type would provide evidence of conceptual understanding. Written out steps are argued to be a sign of conceptual understanding since the problems in this topic contain multiple variables that need to be taken into consideration during a calculation and identifying the general term will make the thought process evident in the solution.

Participants should have had the necessary conceptual understanding to identify the general term to find the next value in the sequence. If the participant does not follow the steps, there is no evidence of any subject content knowledge and it only means the participant identified the pattern, which is not FET level understanding, but rather General Education and Training level (Grade 9) of understanding.

Question 3.1

According to Table 4.11, all the participants were successful at remembering and analysing factual knowledge, while only participants 1 and 2 could apply factual knowledge. The reason for the majority of the participants not being able to apply factual knowledge could be because the participants did not make use of the general formula to answer the question. It is assumed that the participants identified the common ratio if the answer was correct, and even though the participants did not make use of the formula, they appeared to be able to analyse the factual knowledge. It is important to recognise the sequence type and not just the number pattern to be successful. The application of factual knowledge relates to the use of the geometric sequence formula to indicate that the participant could indeed apply the sequence formula and that the participant did not just identify the pattern. All the participants were able to understand, apply, and analyse conceptual knowledge, and they were all successful with the application of procedural knowledge.

In summary, important observations that emanate from the analysis (cf. Addendum I1) concerning patterns and sequences (Question 3.1) are as follows:

- Participants 3, 4, 5, and 6 could not or did not deem it necessary to identify the geometric sequence formula explicitly as part of the solution despite getting the answer correct. This could be as a result of not being taught the importance of showing the main steps as proof of conceptual understanding even if a question seems easy. The lack of indicating the geometric sequence formula did not affect the answer to the question. The researcher feels that it is important for a pre-service teacher to know the importance of indicating concepts as part of a solution to teach that to the learners. According to the researcher, indicating concepts as part of a solution helps to prevent mistakes.
- It appears as if the factual, conceptual, and procedural knowledge dimensions for many of the participants are in place for Question 3.1. Application of knowledge in the factual knowledge dimension seems to be the most problematic aspect for the majority of the participants. All the participants got the correct final answer except for participant 7. Participant 7 was indicated as successful in the procedural knowledge dimension as the participant could satisfy one of the three requirements for procedural knowledge (cf. 5.7.2 – Limitations).

Question 3.2

As indicated in Table 4.11, participants 3, 5, and 6 were successful in remembering factual knowledge while only participants 3 and 5 could analyse and apply conceptual knowledge. None of the participants was successful in applying or creating factual knowledge, creating conceptual knowledge, and at applying procedural knowledge.

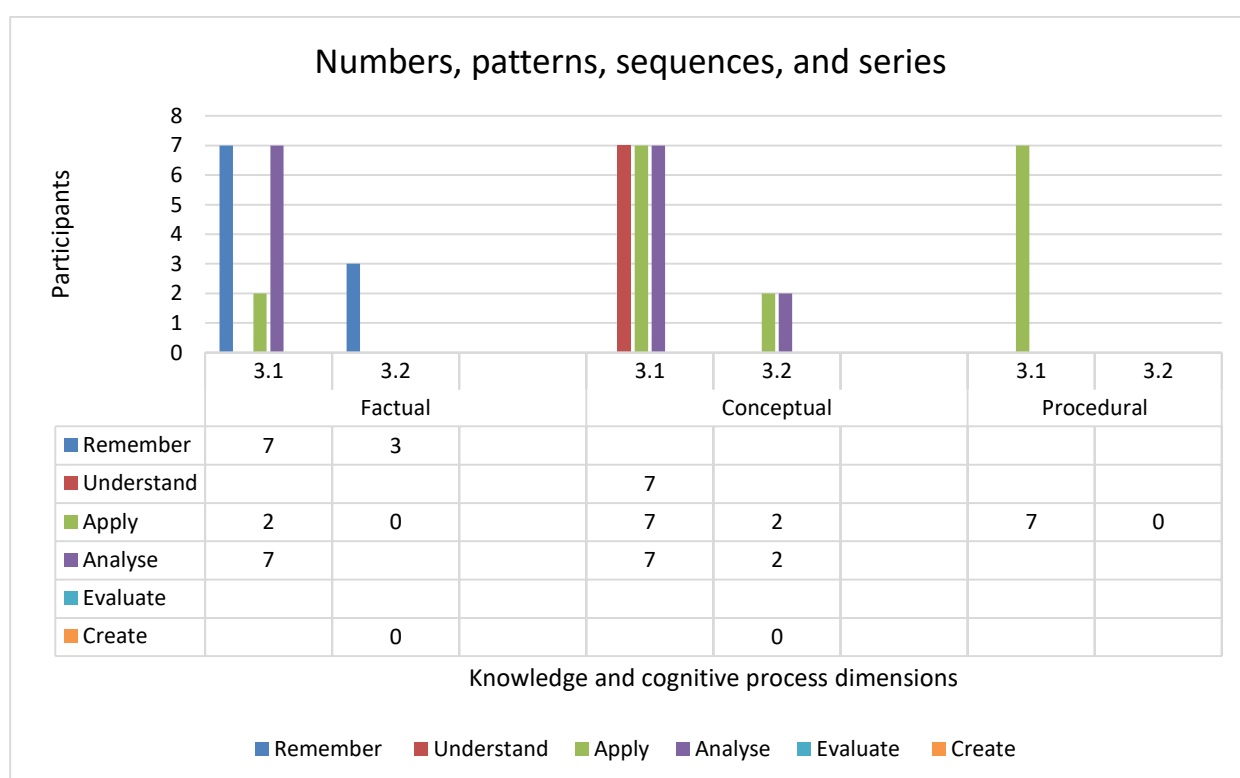
In summary, important observations that emanate from the analysis (cf. Addendum I1) concerning numbers, patterns, sequences, and series (Question 3.2) are the following:

- Participants 3 and 5 were the most successful in answering Question 3.2, in the factual and conceptual knowledge dimensions. The only knowledge missing from their solutions was that of the infinite series (factual and conceptual knowledge), which influenced their success in the procedural knowledge dimension.
- Although three participants (participants 3, 5, and 6), could remember factual knowledge, only participants 3 and 5 could apply (substitute the radii) and analyse (identify the relevant variables) conceptual knowledge.
- It appears as if the factual, conceptual, and procedural knowledge dimensions were problematic for the majority of the participants. Question 3.2 is a high-level question that requires knowledge of at least four mathematical topics (algebra, Euclidean geometry, measurement, and sequences and series). Question 3.2 is a series question, where 3.1 is a sequence question. Learners start practising sequences related to number patterns in

primary school, which means the knowledge paths could be well developed. In contrast, series is only started in the higher school grades. The knowledge networks for series might not be as developed as that of sequences. But, Question 3.2 is also not just a straight-forward application of knowledge question, it is phrased in the form of a word problem, which in itself makes it more difficult for learners (or in this case students) to comprehend.

For Question 3, five participants scored 2 out of 6 and two participants had a score of 1 out of 6 each. The problem seems to be with Question 3.2. There seems to be a lack of content knowledge regarding series notation and connecting the different topics within mathematics to answer Question 3.2. Question 3.1 seemed to be well answered although not all the steps were written out as the researcher would have preferred.

Graph 4.4 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 3.



Graph 4.4: Question 3 – numbers, patterns, sequences, and series

*If no values are displayed in some of the blocks in the data table, it implies that a particular cognitive dimension was not required as part of a specific knowledge dimension.

An overall view of which of the knowledge dimensions and cognitive process dimensions appeared to need attention in the content area, numbers patterns, sequences, and series, comprise the following:

- The participants seemed more successful with the cognitive process dimensions in all the knowledge dimensions for Question 3.1, except for application of factual knowledge (cf. Table 4.12). The reason could also be that participants were challenged to think, and possibly prefer straight-forward questions that are less demanding.
- Participants were not very successful with remembering and applying factual knowledge for Question 3.2. Also, problems were experienced with analysing, applying, and creating conceptual knowledge, as well as with the application of procedural knowledge for Question 3.2. Only three of the seven participants could remember factual knowledge for Question 3.2 (cf. Table 4.12).
- All the cognitive process dimensions on which Question 3.1 focused, namely remember, understand, analyse, application and creation could be regarded as being successful, except for the application of factual knowledge. The reason for the lack of success with application in the factual knowledge dimension seems to be due to only writing an answer and not using the general formula. As the participants are FET pre-service teachers, they have to demonstrate comprehension by writing down steps. A correct answer only would not provide solid evidence of a participant's success with the application of cognitive process dimensions in relation to the various knowledge dimensions. The cognitive process dimensions related to Question 3.2, namely remember, analyse, create, and apply, appeared to be troublesome for the majority of the participants.
- In particular, the researcher finds the following cognitive process dimensions troublesome: Application in the factual knowledge dimensions for Question 3.1 and 3.2, application in the conceptual and procedural dimensions for Question 3.2, create in both the factual and conceptual dimensions for Question 3.2, and analysis in the conceptual dimension for Question 3.2.
- Overall, the participants displayed better factual, conceptual, and procedural knowledge for Question 3.1 than for Question 3.2. The results for Question 3.1 were much better (6 out of 7 answered correctly) than that of Question 3.2 (none of the participants got the answer correct).

The following section pays attention to the interpretation of Question 4, Finance, growth, and decay.

4.3.3.4 Data analysis: Question 4 – finance, growth, and decay

Table 4.13 summarises the assessment for Question 4 (finance, growth, and decay).

Table 4.13: Data analysis: FET Phase Question 4 – finance, growth, and decay

	Knowledge dimension	Cognitive process dimension					
		1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Q4	A Factual	4.1	x 1 2 4 5 6 7		x 2	x 1 2 4 6 7	
		4.2	x 1 2 4 5 6		x 2 5		
	B Conceptual	4.1		x 1 2 4 7	x 2 5 6 7	x 1 2	x 5
		4.2		x 1 2 5 6		x 1 2 4 5 6	x 2 5
	C Procedural	4.1			x 2		x 1 2 4 5 6 7
		4.2			x		x 2

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.14 (see Addendum I1, p. 127).

To be successful at answering questions about finance, growth, and decay, students need to know that simple interest is calculated only based on the initial amount invested, which is called the principal amount. The students also need to know that compound interest is earned based on the principal amount and its accumulated interest. This means that interest is being earned on interest, hence the word compounded. The final amount consists of the sum of the principal amount and the amount of interest earned.

Question 4.1

In general, the most important observations that emanate from the analysis (cf. Addendum I1) concerning finance, growth, and decay for Question 4.1, are the following:

- The majority of the participants were successful with remembering factual information for Question 4.1 (cf. Table 4.13).
- The majority of the participants experienced problems with the application of the information in the factual and procedural knowledge dimensions, with the analysis of conceptual knowledge, and with creating as part of conceptual and procedural knowledge (cf. Table 4.13).
- Only participant 2 was successful with the application and creation of procedural knowledge.
- Participant 3 was not successful with the application of any cognitive process dimension.

Question 4.2

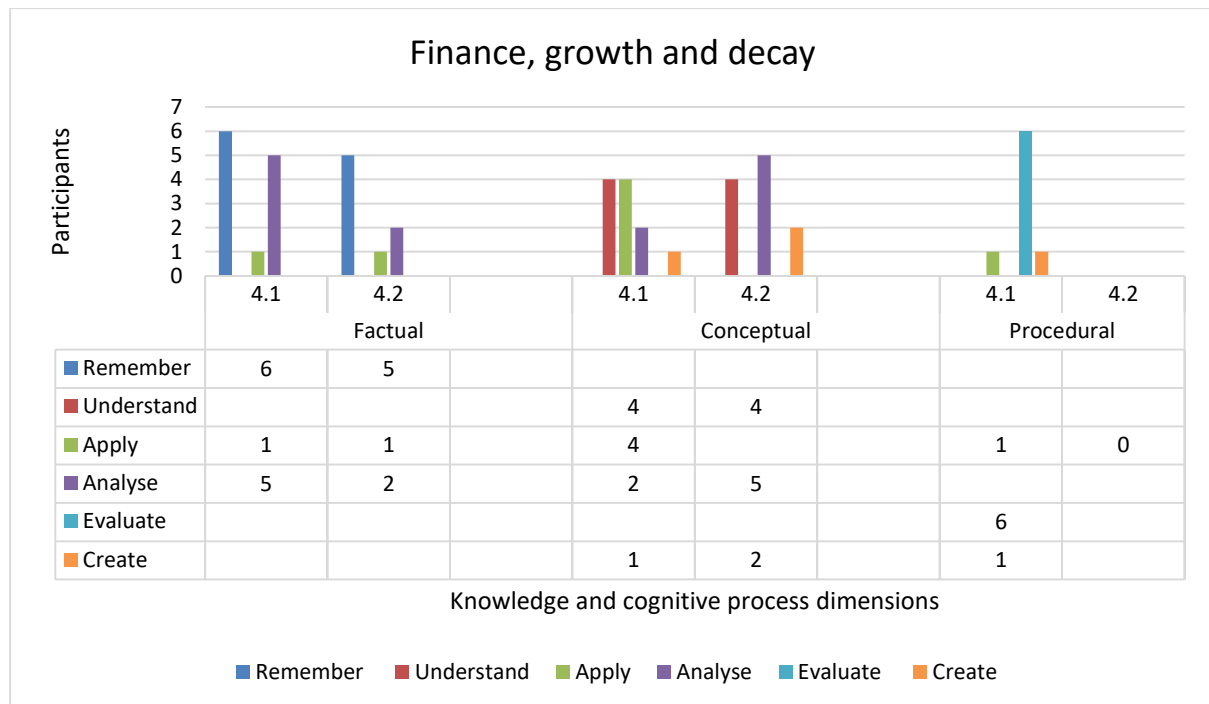
According to Table 4.13, participants 1, 2, 4, 5, and 6 were successful with remembering factual knowledge, and with analysing conceptual knowledge. None of the participants was successful with application in the procedural knowledge dimension. Participants 3 and 7 did not demonstrate success with any of the cognitive process dimensions across the knowledge dimensions.

In general, the most important observations that emanate from the analysis (cf. Addendum I1) for Question 4.2, are the following:

- None of the participants was successful in all the cognitive process and knowledge dimensions (cf. Table 4.13).
- Overall, the participants experienced problems with creating in the conceptual knowledge dimension, and with application and analysis in the factual dimension, creating in the conceptual knowledge dimension, as well as with application in the procedural knowledge dimension (cf. Table 4.13).

Only one participant (participant 2) scored 8 out of 9 for this question, while all other participants scored 3 and less out of 9.

Graph 4.5 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 4.



Graph 4.5: Question 4 – finance, growth, and decay

*If no values are displayed in some of the blocks in the data table, it implies that a particular cognitive dimension was not required in a specific knowledge dimension

Graph 4.5 enables the researcher to conclude the following:

- Overall, the factual and procedural knowledge for Question 4.1 seemed to be more solid than the factual and procedural knowledge displayed for Question 4.2. The participants might have had limited exposure to questions containing two graphs on one plane, which might have caused the lacking knowledge in Question 4.2. Participants might not possess enough skills and lack experience to integrate concepts from different topics in one question. This might lead to the assumption of not a deep enough understanding of content knowledge.
- The application of knowledge in the factual knowledge dimension for both questions appears to be difficult for the participants. Apply in the factual knowledge dimension relates to the application of formulas, rules, or principles. It seems like the participants had difficulty doing so, as it was expected from them to use a single graph to deduce many

facts in 4.1, and in 4.2 they had to substitute values correctly according to alphabetical characters.

- Greater success was achieved with evaluating in the procedural knowledge dimension for Question 4.1, than with creating in the conceptual and procedural knowledge dimensions for both Questions 4.1 and 4.2.
- Analysing in the factual knowledge dimension for Question 4.1, proved to be better than analysis in the factual knowledge dimension for Question 4.2. In Question 4.1 the analysis refers to the identification of the coordinate point representing the future and initial value of the car, some participants could identify the coordinate but did not correctly distinguish between the two values. The difficulty experienced with Question 4.2 might be due to not remembering the formula very well. For Question 4.2, the analysis included the identification of the interest as per annum and therefore they had to divide by 12. Analysis in the conceptual knowledge dimension however seemed to be more successful in the context of Question 4.2 than in the context of Question 4.1. The layout and structure of Question 4.1 might have been unfamiliar to the participants. In Question 4.1 the participants were given a graph and they needed to get information from a graph, instead of from words as in Question 4.2, which might have been more challenging.

The following section pays attention to the analysis and interpretation of data for Question 5, functions.

4.3.3.5 Data analysis: Question 5 – functions

Table 4.15 summarises the assessment for Question 4 (finance, growth, and decay).

Table 4.15: Data analysis: FET Phase Question 5 – functions

	Knowledge dimension	Cognitive process dimension						
		1 Remember		2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Q5	A Factual	5.1	x 1 2 3 4 5 6 7		x 1 2 3 4 5 6 7	x 1 2 3 4 5 6 7		
		5.2	x 2 3 4 5	x 1 3 5	x 3	x 1 2 3 4 5		
		5.3	x 1 2 3 4 5 6 7					x 1 2 3 4 5 7
	B Conceptual	5.1		x 1 2 3 4 5 6 7	x 1 2 3 4 5 7	x 1 2 3 4 5 6 7		x 1 2 3 4 5 6 7
		5.2		x 1 2 3 4 5	x 3			x 3
		5.3		x 1 2 3 4 5 6 7		x 1 2 3 4 5 7		
	C Procedural	5.1			x 1 2 3 4 5 6 7		x 1 2 3 4 5 6 7	
		5.2			x 3			x 3
		5.3				x 1 2 3 4 5 7	x 1 2 3 4 5 7	x 1 2 3 4 5 7

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.16 (see Addendum I1, p. 133).

To be successful at answering a question concerning functions students should know the characteristics of functions to distinguish between them, for example; linear, quadratic, hyperbolic, exponential, and inverse functions. Students should know that functions can have one-to-one relations or many-to-one relations. A function is considered a many-to-one relation if there are two or more values of the independent variable with a single value of the dependent variable. Students need to understand the dimension of a function as a set of x values and the range is a set of y values.

Question 5.1

The data in Table 4.15 suggest that success was observed for remembering, applying and analysing knowledge in the factual knowledge dimension by participants 1, 2, 3, 4, 5, 6, and 7. Participants 1, 2, 3, 4, 5, and 7 were successful in all the knowledge and cognitive process dimensions. Participant 6, was the only participant who was unsuccessful with application in the conceptual knowledge dimension. The success in the understanding of the conceptual knowledge dimension (identify intercepts and turning point) did not necessarily indicate an understanding of both the intercepts and turning point, as the intercepts and turning point could have been identified correctly but not necessarily applied correctly as part of procedural knowledge to answer the questions. The incorrect or lack of correct use of the intercepts and turning point in the procedural knowledge might indicate that the identification of the points does not ensure understanding of how to use them. All the participants were however successful with analysis and creating in the conceptual knowledge dimension, and with application and evaluation in the procedural knowledge dimension.

Understanding conceptual knowledge related to three separate actions as indicated in Table 4.16: (i) the identification of different functions on the Cartesian plane, (ii) their relationship to one another, and (iii) the identification of the y -intercept and coordinates of the turning point. If the participant correctly identified either one of the three actions mentioned before, the participant would be awarded an “x” (successful) in Table 4.15 for understanding in the conceptual knowledge dimension (see Limitations – 5.7.2). Therefore, although some participants made a mistake with the sign of the p -value, all participants could identify the y -intercept and would thus have been indicated as successful in Table 4.15.

Analysis of conceptual knowledge implied the correct use of the order of calculations (even if the formula was incorrect, the participant could still correctly use the order of calculations to be successful with analysis). Creating in the conceptual knowledge dimension refers to planning the calculation of the only remaining variable after the substitution, variable a . Some participants made a sign error during the substitution, but could still plan the calculation of the variable a , therefore the sign error might have influenced the answer (in most cases it did not influence the answer due to the squared parenthesis cancelling out the sign error), but not the planning. So, despite a sign error during substitution, creating in the conceptual knowledge dimension was dealt with successfully by all participants as they could simplify the equation to having just one variable, a .

Evaluation as part of procedural knowledge comprised the identification of the function as a hyperbola and therefore using the hyperbola formula. The application of procedural knowledge relates to three separate things, (i) showing the y -intercept as well as (ii) the turning point and (iii) calculating the answer. If any one of the afore mentioned applications was successful, the participant would have the “x” (successful) in the table under apply in the procedural knowledge dimension (see Limitations – 5.7.2). Noteworthy is the fact that all the participants were successful with the application (showing the coordinate points and/or calculating the answer correctly) and evaluation (identifying the correct function) of procedural knowledge despite the sign errors.

In sum, important observations that emanate from the analysis (cf. Addendum I1) concerning functions for Question 5.1, are the following:

- Although all the participants appeared to have factual knowledge in place to answer the question not all of them arrived at the correct answer or wrote down the correct steps. Some mistakes were made due to sign errors with the formula or when substituting, or not writing down the final equation in terms of x and y . According to the researcher, the mistake was not made due to a lack of factual knowledge, but in the researcher’s opinion possibly due to the way that the students were taught to write out the steps (or maybe not being taught how to write out the steps). If a student does not know how to write out steps correctly, the student might still get the correct answer, but the student might not necessarily be awarded all the marks allocated for the indication of conceptual understanding. The researcher thinks that since mathematics is very systematic, incomplete steps (a lack of indicating conceptual understanding) might influence the solution as a whole, especially if the question is a high-level question. In this case only one participant seemed to be unsuccessful in the conceptual knowledge dimension due to writing out steps, namely participant 6. The researcher acknowledges that due to time constraints, the participant might have left out steps in order to save time. Students are

sometimes encouraged to leave out routine steps that require mental calculations to save time. The steps that indicate conceptual understanding were specified on the standardised test memorandums that were used to construct and guide the assessment of the tests used for data collection in the study for which marks had to be awarded.

- It appears as if the solid factual knowledge only enabled some of the participants to be successful in the conceptual knowledge dimension. All the participants were successful at understanding the mathematical content, but that does not necessarily mean both the y -intercept and the turning point coordinates. Only participants 1, 2, 3, 4, 5, and 7 could apply their conceptual knowledge, but all participants were successful at analysing and creating knowledge in the conceptual knowledge dimension. To be successful with apply in the conceptual knowledge dimension, the participants needed to correctly substitute into the formula (if the variables were incorrectly identified, however indicated before substitution, the participants were still considered being successful with application). To be successful with analysis in the conceptual knowledge dimension the participants needed to make use of the order of calculations (even if the formula was incorrect, the participant could still correctly use the order of calculations to be successful with analysis). Create in the conceptual knowledge dimension refers to planning to change the subject within the formula. If the formula was incorrect, but the participant could still correctly change the subject to a , it indicated planning on the participant's part.
- It could be argued that the factual knowledge was seemingly in place among all the participants, and possibly enabled them to be more effective in the procedural dimension; specifically, concerning the application and evaluation of information. Factual knowledge involved recalling the formula and variables needed to answer the question along with the functions used. If the participant had factual knowledge in place, the participant should have been able to evaluate procedural knowledge. To evaluate procedural knowledge refers to interpreting the two functions individually, along with interpreting their variables concerning the formula. Identifying the correct functions and variables would enable a participant to calculate, thus being successful with application in the procedural knowledge dimension (the final answer). If the participant has the required factual knowledge, it is possible to answer the question correctly. Some participants made a mistake with the conceptual knowledge (turning point or formula) but still managed to apply the formula correctly for the substituted values (whether the values were correct or not).
- Application of factual knowledge appears to be a problematic cognitive process dimension. This might be due to either a sign error in the formula or a sign error after or during substitution.

- All the participants were effective with understanding, analysing, and creating conceptual knowledge. Understand refers to the identification of the needed variables, although some participants made a mistake with the sign of the p -value, all participants could identify the y -intercept. Analyse involves to the use of the order of calculations, which all participants executed correctly. Create comprises to the reordering of the equation after substitution to make the only remaining variable a , the subject of the equation, which all the participants could do correctly.
- All the participants were effective with the application and evaluation of procedural knowledge. All the participants seemed to have good factual and conceptual knowledge, which could have contributed to the correct application of the concepts to find the answer to a .

Question 5.2

According to the analysis of the data, participants 2, 3, 4, and 5 could remember factual information to answer the question, whereas participants 1, 3, and 5 were effective at understanding the factual information, and of the three participants, only participant 3 could apply factual information. Participants 1, 2, 3, 4, and 5 could also analyse the factual information required for answering this question. Participants 1, 2, 3, 4, and 5 seemed successful at understanding the conceptual knowledge on which the answering of the question was based, however, only participant 3 could apply and create information within the conceptual knowledge dimension. Only participant 3 was effective at applying and creating information in the procedural dimension.

Overall, important observations that emanate from the analysis (cf. Addendum I1) concerning functions for Question 5.2, are the following:

- The participants were ineffective at applying factual, conceptual, and procedural knowledge. Only one participant, participant 3, was successful with application in all the knowledge dimensions. Many of the participants could not apply the formula (factual knowledge), which affected the conceptual knowledge as no substitution could be done (or the incorrect substitution was done). The procedural application (calculations) cannot be done with an incorrect formula or substitution. A lack of solid factual knowledge possibly influenced the conceptual and procedural knowledge dimensions.
- Creating in the conceptual and procedural knowledge dimensions seemed troublesome for the participants. Only one participant, participant 3, could create in the conceptual and procedural knowledge dimensions. To be successful with creating knowledge in both the

conceptual and procedural knowledge dimensions, a participant had to correctly make use of simultaneous equations to answer the question.

- Remembering and understanding factual knowledge could also be regarded as troublesome for the majority of the participants. Some participants could not recall the horizontal asymptote as d , but not recall simultaneous equations in factual knowledge, which was the main problem that the participants faced. Not recalling simultaneous equations as a solution method, influenced the conceptual formulation and the procedural calculation, composition, and manipulation. A lack of factual knowledge again influenced the conceptual and procedural knowledge dimensions. Only one participant got the answer to Question 5.2 correct (participant 3), although participants 1, 2, 4 and 5 could identify the correct d -value.

Question 5.3

According to Table 4.15, all the participants displayed success with all the knowledge and cognitive process dimensions, except participant 6.

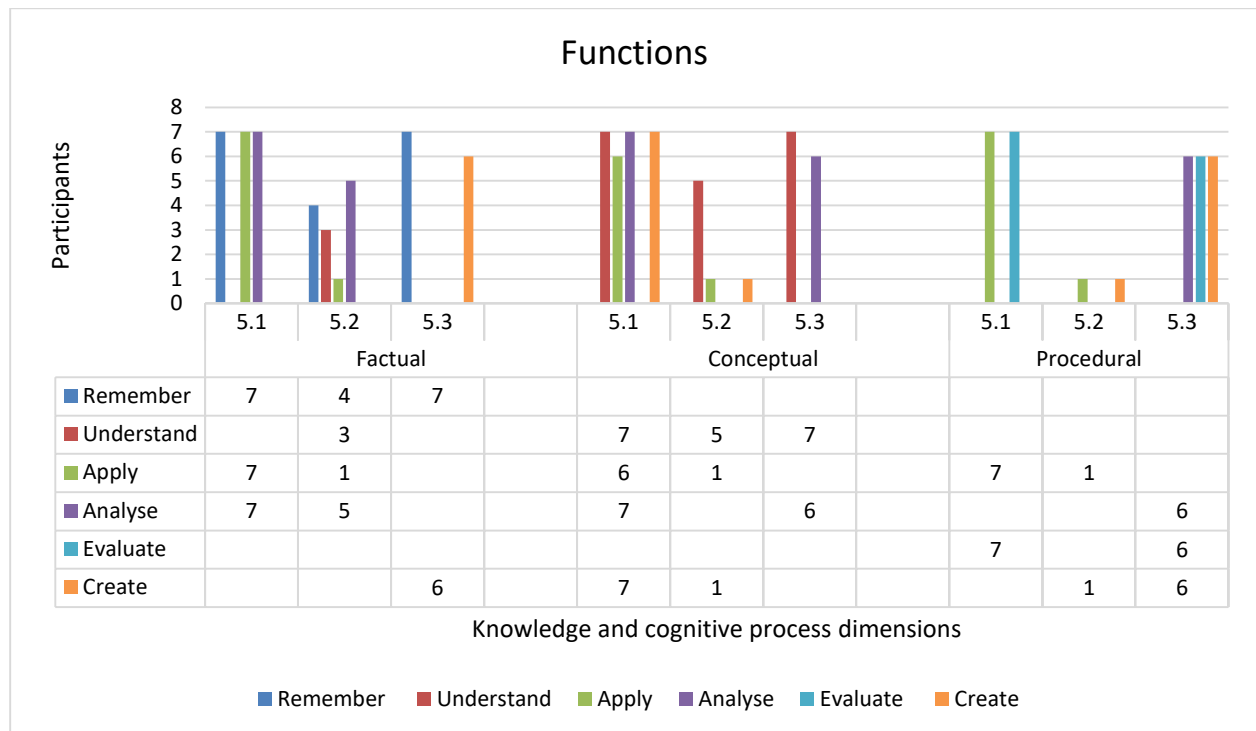
In short, important trends that emanate from the analysis (cf. Addendum I1) concerning functions for Question 5.3, are the following:

- All participants, except participant 6, were effective at applying all the cognitive process dimensions successfully concerning the factual, conceptual, and procedural knowledge dimensions (cf. Table 4.16, Addendum I1, p. 133).
- All the participants, except participant 6, could compare and interpret the data from the functions to compose an inequality for the dimension where $g(x) \geq f(x)$. Participant 6 seemed to lack some factual and conceptual knowledge which influenced the procedural knowledge, and thus the final answer. Participants 1, 2, 3, 4, 5, and 7 got the answer to Question 5.3 correct

In sum, only participant 3 managed to get full marks for Question 5. Participants 6 and 7 obtained the lowest scores, namely 4 and 6 out of 12, respectively. Participant 7 attempted a solution but made use of the wrong variables and possibly formula, whereas participant 6 did not even attempt a solution. The lack of factual knowledge influenced both conceptual and procedural knowledge for both participants 6 and 7. Despite the sign error made during the identification of the p -value after the calculation in Question 5.1, participant 7 still got both Questions 5.1 and 5.3 correct. The sign error in Question 5.1 related to the variable p , which was identified as a positive instead of a negative, but it did not influence the final answer of a . Therefore, the answer to 5.1 is still correct. The participant got Question 5.2 incorrect due to a lack of factual knowledge and conceptual

misconceptions. The participant could recall most of the general formulas for the functions in Question 5, but none of the variables or other details needed to answer Question 5.2. The participant could identify the function as a hyperbola, but the formula was not correctly substituted in as the variables were incorrect, and the participant could not identify the concept needed to solve the problem (simultaneous equations).

Graph 4.6 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 5.



Graph 4.6: Question 5: Functions

*If no values are displayed in some of the blocks in the data table, it implies that a particular cognitive process dimension was not required in the specific knowledge dimension.

According to Graph 4.6, the researcher concludes the following:

- Application in the factual, conceptual, and procedural knowledge dimension appears to be worrying, specifically for Question 5.2. Also, to create in the conceptual and procedural dimensions, in particular for Question 5.2, appears to be demanding for the participants. Furthermore, compared to Questions 5.1 and 5.3, the participants seemed not so successful with remembering in the factual knowledge dimension for Question 5.2. Understanding knowledge in the conceptual dimension was also more successful in the contexts of Questions 5.1 and 5.3. Question 5.2 seems to have been the most

troublesome, in terms of all the knowledge and cognitive process dimensions, in comparison to Questions 5.1 and 5.3.

The following section pays attention to the data analysis of Question 6, probability.

4.3.3.6 Data analysis: Question 6 - Probability

Table 4.17 summarises the assessment for Question 6 (probability).

Table 4.17: Data analysis: FET Question 6 - Probability

Q6	Knowledge dimension	Cognitive process dimension						
		1 Remember		2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
	A Factual	6.1	x 4 5 7					
		6.2	x 5					
	B Conceptual	6.1		x				x
		6.2				x 5		x
	C Procedural	6.1			x			
		6.2			x			

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.18 (see Addendum I1, p. 140).

To be successful at answering a probability question, a student needs to know the terminology of the topic very well as to be able to identify what is asked in the question. Secondly, it is very important to know all of the probability rules to calculate the answers. If the student knows the terminology, but not the rules, the student would not be able to solve the question.

Question 6.1

In general, the most important observations that emanate from the analysis (cf. Addendum I1) concerning probability, are the following:

- All the participants experienced problems with the cognitive process dimensions in the factual, conceptual, and procedural knowledge dimensions, and none of the participants was successful in answering the questions correctly. Only three of the seven participants (participants 4, 5, and 7) could remember some factual knowledge to answer the question (cf. 5.7.2 - Limitations). It could be argued that the participants might not have understood the question, especially as many did not attempt to answer the question (participants 1, 2, 3), and some only displayed partial knowledge in answering the question (participants 4, 5, 7).
- Similar to the participants who remembered factual knowledge (participants 4, 5, 7), those who did not remember the factual knowledge (participants 1, 2, 3, 6) were also not successful at understanding and creating in the conceptual knowledge dimension, and with applying procedural knowledge. This could be because “remember” in the factual knowledge dimension refers to three aspects, namely (i) factorials, (ii) permutations, and (ii) grouping, and the participants could not recall all three. Most of them could only recall factorials, which was not enough to answer the question correctly. In the case of participants 1, 2, 3, and 6 the absence of factual and conceptual knowledge could have contributed to the problems experienced with procedural knowledge in the question (Rittle-Johnson et al., 2015; Schoenfeld, 1992) (cf. 2.3.2.3). However, the factual knowledge remembered by participants 4, 5, and 7 did not enable them to be effective the conceptual and procedural knowledge dimensions, as they could not recall factual knowledge concerning factorials, permutations, and grouping to answer the question.

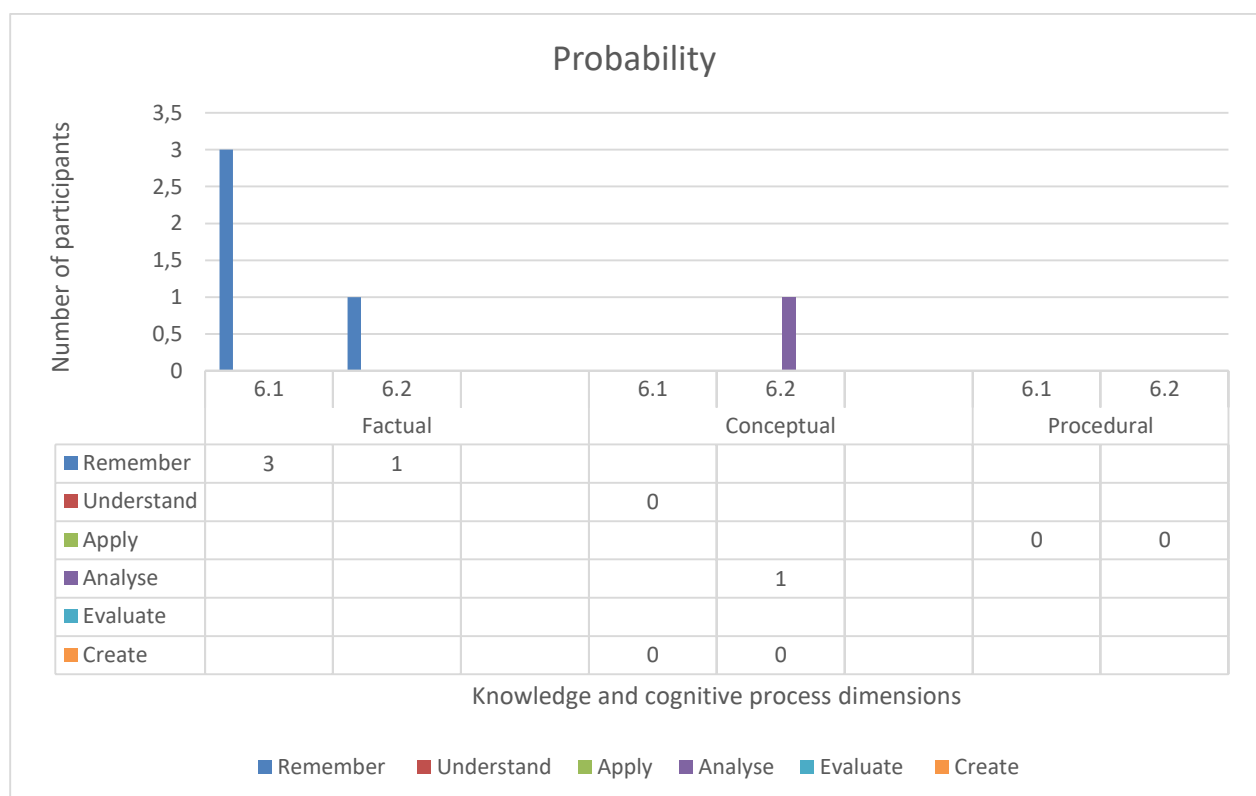
Question 6.2

No success was achieved by any of the participants in creating information in the conceptual, and applying the information in the procedural dimension, respectively. None of the participants was able to answer the question correctly.

To sum up, important trends that emanate from the analysis (cf. Addendum I1) concerning probability for Question 6 are the following:

- Overall, the participants experienced problems with the application of cognitive process dimensions in all the knowledge dimensions (cf. Table 4.18), except for participant 5 who could remember factual knowledge and analyse information as part of conceptual knowledge. Participant 5 was the only participant who attempted to answer the question, but could only recall the use of factorials and was therefore not successful with the application of procedural knowledge. Solid factual knowledge thus appears to be lacking among all the participants (cf. Table 4.18, Addendum I1).

Not one of the participants provided a correct answer for Question 6. Graph 4.7 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 6.



Graph 4.7: Question 6: Probability

*If no values are displayed in some of the blocks in the data table, it implies that a particular cognitive process dimension was not required in a specific knowledge dimension.

The comparison between Question 6.1 and 6.2 in the graph is disturbing. According to Graph 4.7, it could be concluded that the cognitive process dimensions related to remember, understand,

apply, analyse and create across all the knowledge dimensions for both Question 6.1 and 6.2, could be viewed as problematic. Only a few participants seem to be successful in remembering factual knowledge.

Although three of the participants (4, 5, and 7) seemed to have solid factual knowledge concerning Question 6.1, it did not benefit them in being successful in the conceptual and procedural knowledge dimensions. The participants could recall factorials, but Question 6.1 required the recollection of a lot more than just factorials, for example, also permutations, to correctly answer the question. None of the mentioned participants were successful in the procedural knowledge dimension for both questions, which the researcher argues to be as a result of poor conceptual understanding. In Table 4.2 it can be seen that all the participants scored zero for Question 6.

The following section pays attention to the data analysis of Question 7, statistics.

4.3.3.7 Data analysis: Question 7 – Statistics

Table 4.19 summarises the assessment for Question 7 (statistics).

Table 4.19: Data analysis: FET Question 7 - Statistics

	Knowledge dimension	Cognitive process dimension					
		1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Q7	A Factual	7.1	x 2 3 4 5 6 7			x 3 5 6	
		7.2	x				
	B Conceptual	7.1			x 3 5 6		
		7.2	x		x		
	C Procedural	7.1		x 5			
		7.2		x			

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.20 (see Addendum I1, p. 142).

Answering questions about statistics requires of a student to know all of the terminology that goes hand-in-hand with the topic. If the student knows the terminology, the student will be able to understand the question. For this test, the students needed knowledge of curves, plotting, and extracting information from the curves and plotting.

Question 7.1

In the memorandum for Question 7.1, the researcher refers to “the shape of the graph”, which in the context of this question means that a cumulative frequency curve looks like a 'stretched S' which is also called an ogive.

In short, important observations that emanate from the analysis (c. Addendum I1) concerning statistics for Question 7.1, are the following:

- Some of the participants who displayed that they could remember (participants 3, 5, and 6) and analyse (participant 6) factual information, were also successful with the analysis of information in the conceptual dimension. Participant 5 however, was the only participant among the three who was successful with the application of procedural knowledge. Remembering factual knowledge refers to multiple terms (class, class limits, cumulative frequency, the shape of a cumulative frequency curve, and coordinate plotting), but not all participants could recall all these terms. For example, participant 4 could only recall the plotting of coordinates, but nothing else. Therefore, participant 4 could not interpret the question in terms of the concepts needed to draw the correct graph. Participant 6 made a mistake with the grounding point even though the upper limits, cumulative frequency and smooth shape of the curve were correct.
- Participant 7 could identify the period and recalled everything except for the cumulative frequency and shape of the graph. Participant 7 could draw a graph but drew a histogram instead of a cumulative frequency graph. Participant 2 could only recall the coordinate plotting and nothing else.
- Participant 1 was unsuccessful in applying any of the cognitive process dimensions. Participant 1 did not attempt to answer the question and only drew the axis, which might be as a result of insufficient factual knowledge.

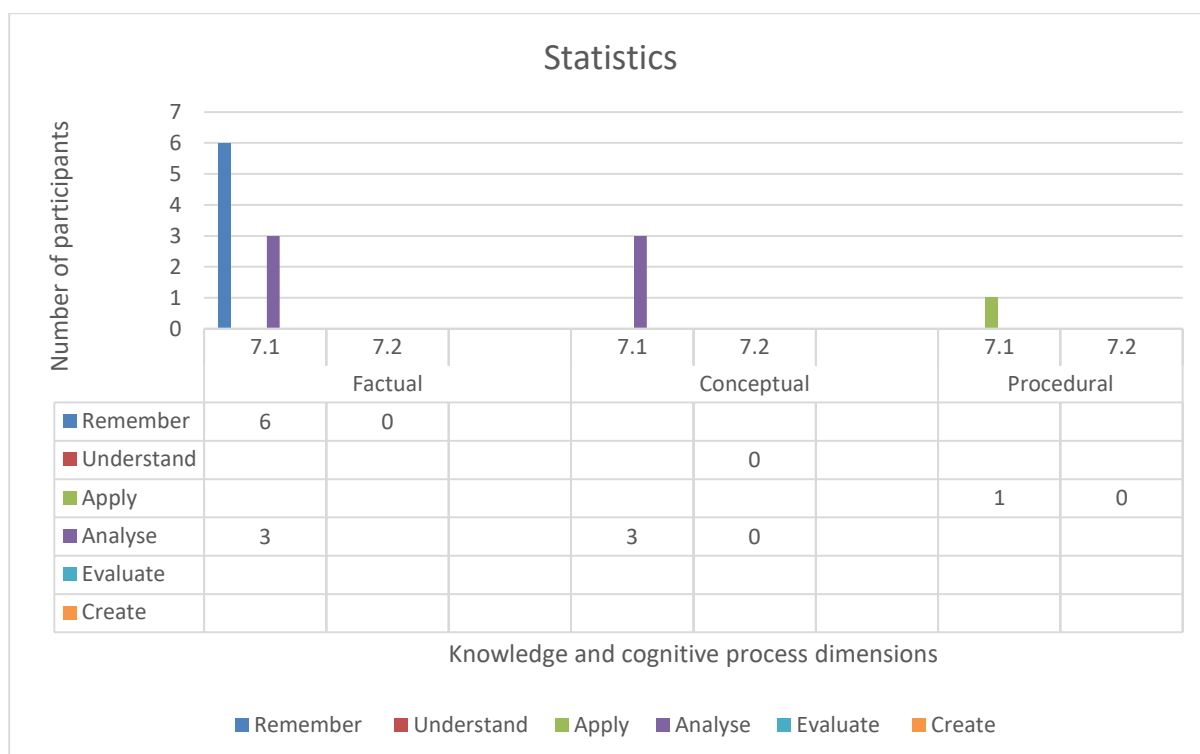
Question 7.2

Overall, factual, conceptual, and procedural knowledge appeared to be lacking among all the participants.

In sum, important observations that emanate from the analysis (cf. Addendum I1) concerning statistics for Question 7, are the following:

- Factual knowledge is indicated for some of the participants in Question 7.1 (participants 2, 3, 4, 5, 6, and 7). In the context of Question 7.1, factual knowledge should be viewed as broad and comprising many different terms (class, class limits, cumulative frequency, the shape of a cumulative frequency curve and coordinate plotting). The test analysis indicates that if a participant did not remember all the terms the participant could not correctly answer Question 7.1. In Question 7.2, none of the participants seemed to possess solid factual knowledge, which could have influenced performance in the conceptual and procedural knowledge dimensions negatively.
- The highest score out of 6 obtained for Question 7, was 4 out of 6 (participant 5), and the lowest score was 0 out of 6 (participants 1, 2, and 4). Question 7.1 assisted participants 3, 5 and 6 to obtain the better scores Question 7, namely 3, 4, and 3 out of 6, respectively.

Graph 4.8 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 7.



Graph 4.8: Question 7: Statistics

The comparison of Questions 7.1 and 7.2 in the graph enables the researcher to derive the following conclusions.

- Graph 4.8 displays a gloomy picture for success concerning the content area statistics for both questions.
- Although not optimal, some participants seem to be relatively successful with remembering factual knowledge related to Question 7.1. However, the cognitive process dimensions in Question 7.1 related to the analysis of factual and conceptual knowledge, as well as the application of procedural knowledge could be viewed as problematic.
- Question 7 was the last question of the FET test and it might also have contributed to the poor responses or lack of responses, due to participants being tired. Participants might also have encountered time problems and could not finish the test properly.

4.3.3.8 A synthesis: Overall conclusions FET Phase

When comparing the average results obtained by the participants for the various questions (cf. Table 4.2), and considering the analyses for each of the questions (cf. Tables 4.7, 4.9, 4.11, 4.13, 4.15, 4.17, 4.19; Graphs 4.2 - 4.8) the following trends are observed:

- The participants obtained the highest average for Question 5 (functions), namely 59,52%.

- The second and third highest averages were obtained for Question 2 (algebra and equations) and Question 3 (number patterns, sequences, and series), with 33,92% and 28,57%, respectively.
- The fourth highest average was obtained for Question 7 (statistics), with 26,19%.
- In the fifth place, the participants obtained an average of 20,63% for Question 4 (finance, growth, and decay).
- The lowest averages were obtained for Question 1 (differential calculus), and Question 6 (probability), with 0%.

Combining the average results with the analyses of the participants' performance (cf. Addendum I1) concerning the knowledge and cognitive process dimensions for each of the questions, the following can be concluded:

- The highest average obtained for Question 5 (functions), could be linked to the fact that the factual knowledge and related cognitive process dimensions for specifically Questions 5.1 and 5.3, appeared to be relatively well developed. This contributed to success in the conceptual and procedural knowledge dimensions and benefitted the overall result, although the participants were not so successful in answering Question 5.2, and factual knowledge appeared to be limited.
- For Questions 2 (algebra and equations) and 7 (statistics), factual knowledge appeared to be strong for Question 2.2 and 7.1. The strong factual knowledge possibly benefitted the execution of cognitive process dimensions in the conceptual and procedural knowledge dimensions, in particular for Question 2.2. Strong factual knowledge did not seem to benefit application of knowledge in the procedural knowledge dimension for Question 7.2. Where the factual knowledge seemed to be less solid, or absent (Question 2.1 and 7.2), the participants seemed to find the execution of the cognitive process dimensions in the conceptual and procedural knowledge dimensions demanding.
- Remembering information in the context of Question 4 (finance, growth, and decay) seemed to be in place, which possibly assisted the participants to achieve some success in the conceptual knowledge dimension, but not in the procedural knowledge dimension.
- Similar to Questions 2 and 7, the researcher comes to a related conclusion for Questions 3 (number patterns, sequences, and series). The results that displayed the stronger factual knowledge base (Questions 3.1) also seemed to contribute to greater success in the conceptual and procedural knowledge dimensions. For Question 3.2, where factual knowledge appeared to be absent, the execution of cognitive process dimensions in the conceptual and procedural knowledge dimensions was unsuccessful.

- The lowest average obtained for Question 1 (differential calculus) and Question 6 (probability) could be linked to the absence of solid conceptual and procedural knowledge. Although the participants demonstrated that they could remember some factual knowledge, the factual knowledge did not benefit the conceptual or procedural knowledge dimensions. In the context of both questions, the cognitive process dimensions related to understanding, and the application, analysis, evaluation, and creation of conceptual and procedural knowledge seem demanding for the participants.

The information sheet handed to participants to aid the recall of knowledge they might have forgotten, was not used by all the participants. The researcher finds it surprising that not many of the participants used the information sheet to assist them in answering the questions concerning the different content areas:

- Differential calculus: used by one participant.
- Algebra and equations: used by four participants.
- Numbers, patterns, sequences, and series: used by four participants.
- Finance, growth, and decay: used by four participants.
- Functions: used by one participant.
- Probability: used by one participant.
- Statistics: used by none of the participants.

The following section presents a synthesis of the analysis and interpretation of the test data for the Senior Phase.

4.3.4 Analysis and interpretation of knowledge and cognitive process dimensions – Senior Phase

In the subsequent sections, each of the questions posed to the Senior Phase participants are analysed and interpreted.

4.3.4.1 Data analysis: Question 1 – patterns, functions, and algebra

Table 4.21 reflects the assessment for Question 1.

Table 4.21: Data analysis: Senior Phase Question 1 – patterns, functions, and algebra

Knowledge dimension		Cognitive process dimension					
		1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Factual	1.1	X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7			
	1.2.1	X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7	X 1 2 3 4 5 6 7		
	1.2.2	X 1 2 3 4 5 6 7		X 4 5 6 7			
	1.3	X 2 3 4 5 6 7		X 2 3 4 5 6 7	X 2 3 4 5 6 7		
Conceptual	1.1		X 2 3 4 5 6 7		X 1 2 3 4 5 6 7		
	1.2.1		X 1 2 3 4 5 7		X 1 2 3 4 5 6 7		
	1.2.2		X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7	X 4 5 6 7	
	1.3		X 2 3 4 5 6 7	X 2 3 4 5 6 7	X 2 3 4 5 6 7		
Procedural	1.1			X 3 4 5 6 7			
	1.2.1			X 1 2 3 4 5 6 7			
	1.2.2			X 4 5 6 7			
	1.3			X 2 3 4 5 6 7			

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.22 (see Addendum I2, p. 146).

Patterns, functions, and algebra is a topic that requires knowledge of relationships between numbers in given patterns, rules, formulae, and equations for which input and output values can be found and equations that can be solved. It also requires learners to make use of symbolic expressions, graphs, and tables. Lastly, it focuses on the identification and analysis of regularities and change in patterns, and relationships that enable learners to make predictions and solve problems.

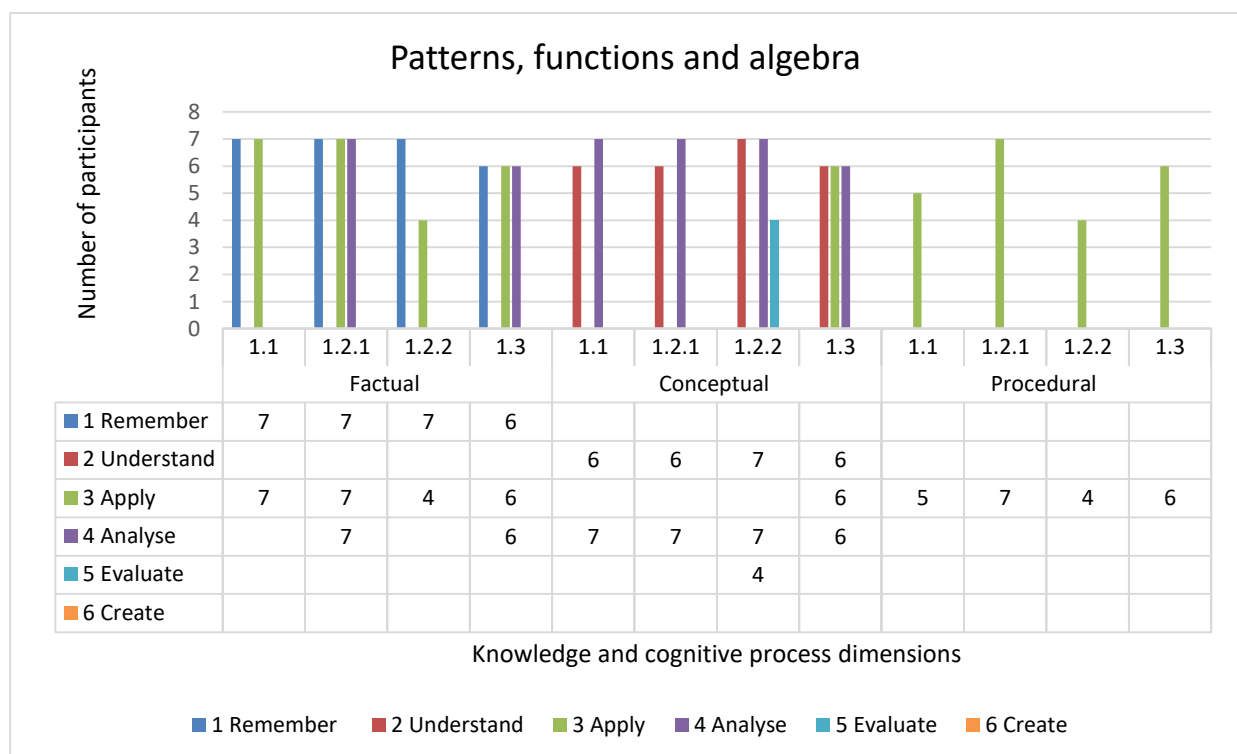
Question 1 tests the participants' mathematical skills as it requires the participants to apply rules, formulae, and operations to find solutions. The participants needed to plan the solutions, evaluate, and monitor the steps by checking calculations performed throughout the solution.

In summary, important observations that emanate from the analysis (cf. Addendum I2) concerning patterns, functions, and algebra, Question 1, are the following:

- All of the participants were successful with remembering factual knowledge, except participant 1 in Question 1.3. Participants 1, 2, and 3 seemed to have trouble with the application of factual knowledge in Question 1.2.2, and participant 1 with application and analysis of factual knowledge for Question 1.3.
- All the participants were successful at understanding conceptual knowledge except participant 1 in Question 1.1 and 1.3. Participants 1, 2, and 3 were unsuccessful at evaluating conceptual knowledge (Question 1.2.2). Participant 1 also experienced trouble with the application and analysis of conceptual knowledge for Question 1.3.
- Participant 1 seemed to be the only participant who experienced problems in the procedural knowledge dimension with the application of knowledge in Questions 1.1 and 1.3, whereas participants 1, 2, and 3 experienced difficulties with application in the procedural knowledge dimension for Question 1.2.2.

From the scores in Table 4.3 it can be seen that Question 1's average is the second-highest in the Senior Phase test, namely 85,71%. Only Participants 1, 2, and 3 lost points. The lowest score for Question 1 was 4 out of 8, scored by participant 1. Participants 4, 5, 6, and 7 all scored full marks for Question 1 (cf. Table 4.3).

Graph 4.9 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 1.



Graph 4.9: Question 1: patterns, functions, and algebra

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required in the specific knowledge dimension.

The researcher observes the following trends from Graph 4.9.

- Overall, the participants seemed to be successful with the cognitive processes dimensions across the knowledge dimensions, except for application in the factual and procedural knowledge dimensions for Question 1.2.2, and evaluation in the conceptual dimension for Question 1.2.2. Evaluate relates to assessing the expression to correctly find the two different factorising concepts needed to answer the question. Some participants could only identify the first concept and not the second concept. Apply in the procedural knowledge dimension was also considered unsuccessful if the answer was incomplete or incorrect.
- The unsuccessfulness with applying and evaluating in the conceptual knowledge dimension for Question 1.2.2 probably also influenced the lower success that was observed for apply in the procedural knowledge dimension.
- Noteworthy is that most participants were successful in the factual knowledge dimension and it can be argued that success in the factual knowledge dimension possibly contributed to successful execution of the cognitive process dimensions in the conceptual and procedural knowledge dimensions as a result. Progression of knowledge from factual to procedural seems to be true for this question.

The following section pays attention to the interpretation of Question 2, numbers, operations, and relationships.

4.3.4.2 Data analysis: Question 2 – Numbers, operations, and relationships

Table 4.23 reflects the assessment for Question 2.

Table 4.23: Data analysis: Senior Question 2 – numbers, operations and relationships

Knowledge dimension		Cognitive process dimension					
		1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Factual	2.1	X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7			
	2.2	X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7			
	2.3	X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7			X 1 2 3 4 5 6 7
	2.4	X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7			
Conceptual	2.1		X 1 2 3 4 5 6 7	X 1 2 3 4 5 6 7			

	2.2		X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7		
	2.3				X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7
	2.4		X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7		
Procedural	2.1			X 1 2 3 4 6 7			
	2.2			X 1 3 4 5 6 7			
	2.3			X 1 2 3 4 5 6 7		X 1 2 3 4 5 6 7	X 1 2 3 4 5 6 7
	2.4			X 1 2 3 4 5 6 7			

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.24 (see Addendum I2, p. 152).

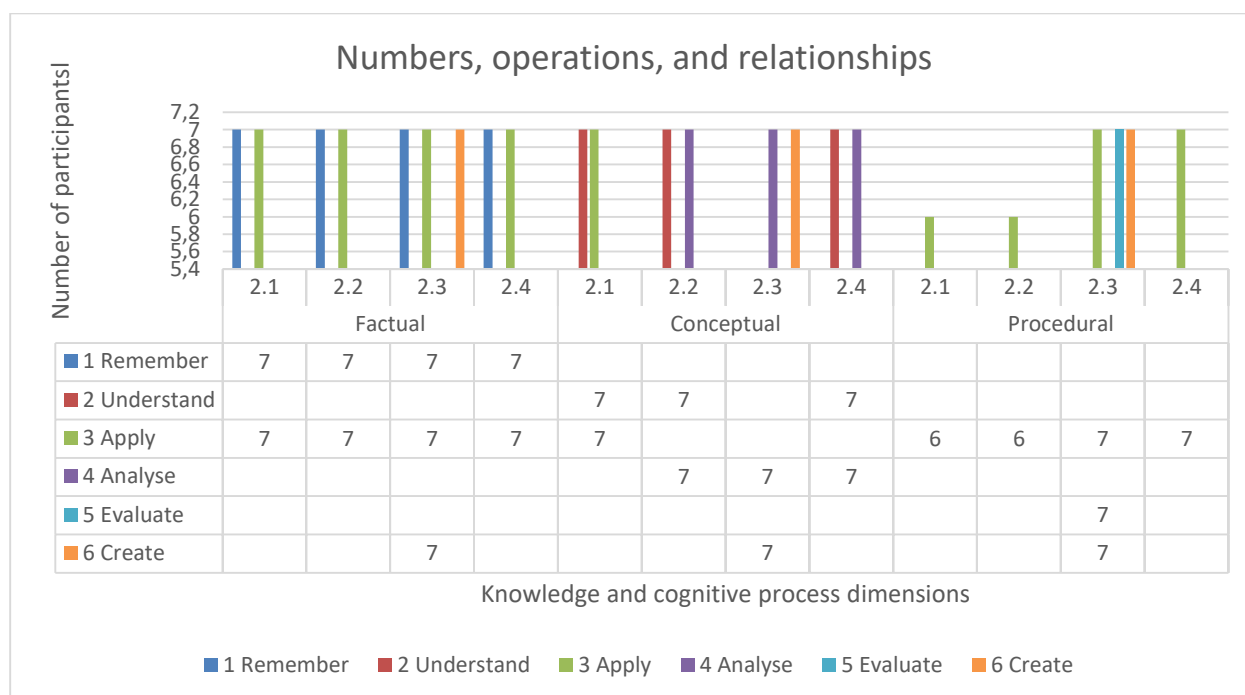
The subject content area numbers, operations, and relationships focuses on the meaning and relationship between different kinds of numbers, the relative size of different numbers, the representation of numbers in various ways, the effect of operating with numbers, and the ability to estimate and check solutions.

In summary, the following emanates from the analysis (cf. Addendum I2) concerning numbers, operations, and relations for Question 2:

- The participants all seem to have solid factual, conceptual, and procedural knowledge in the content area of numbers, operations, and relationships as they were successful in executing all the cognitive processes. Question 2 had the highest question average for the Senior Phase test, 96,10%. Only participants 2 and 5 lost points in Question 2. Participant 2 had the lowest score for Question 2 with 9 out of 11, and participant 5 scored 10 out of 11.

It was important for the researcher to obtain an overall view of which of the knowledge dimensions and cognitive process dimensions appeared to pose problems to the participants. The comparison in the graph enables the researcher to conclude where problem areas concerning knowledge and cognitive process dimensions seem to occur among the pre-service teachers who took part in the study in the content area numbers, operations and relationships.

Graph 4.10 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 2.



Graph 4.10: Question 2: Numbers, operations, and relationships

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required in the specific knowledge dimension

The researcher observes the following trends from Graph 4.10:

- Noteworthy is that all the participants seem to be successful across the knowledge and cognitive process dimensions.
- Question 2.3 involved the execution of more higher-order cognitive processes than the other questions, but the participants were successful in all the cognitive processes within the three knowledge dimensions.
- Participants 2 and 5 who did not get full marks for Question 2, were the only participants who experienced problems with application of information in the procedural knowledge dimension for Question 2.1 (participant 5) and Question 2.2 (participant 2) (cf. Table 2.23).

The following section pays attention to the interpretation of Question 3, space and shape (geometry)

4.3.4.3 Data analysis: Question 3 – space and shape (geometry)

Table 4.25 reflects the assessment for Question 3.

Table 4.25: Data analysis: Senior Phase Question 3 – space and shape (geometry)

Knowledge dimension		Cognitive process dimension					
		1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Factual	3.1	X 1 2 3 5 6 7		X 1 2 3 5 6 7	X 1 2 3 5 6 7		
	3.2	X 1 2 3 4 5 6			X 2 3 4 6		
Conceptual	3.1		X 1 2 3 5 6 7		X 1 2 3 5 6 7		
	3.2		X 2 3 4 6		X 2 3 4 6		
Procedural	3.1			X 1 2 3 5 6 7		X 1 2 3 5 6 7	
	3.2			X 2 3 4 6		X 2 3 4 6	

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.26 (see Addendum I2, p. 156).

The topic, space and shape, improves understanding and appreciation of the pattern, precision, achievement, and beauty in natural and cultural forms. It focuses on the properties, relationships, orientations, positions, and transformations of two-dimensional shapes and three-dimensional objects. The participants needed to identify the required principle to organise and interpret the questions (conceptual knowledge dimension). To be successful in the procedural knowledge dimension the participants needed to interpret and answer the questions according to the parallel line principle. Written out steps are also argued to be a sign of conceptual understanding since the problems in this topic contain multiple variables that need to be taken into consideration during a calculation.

In summary, important observations that emanate from the analysis (cf. Addendum I2) concerning space and shape, Question 3.1, are the following:

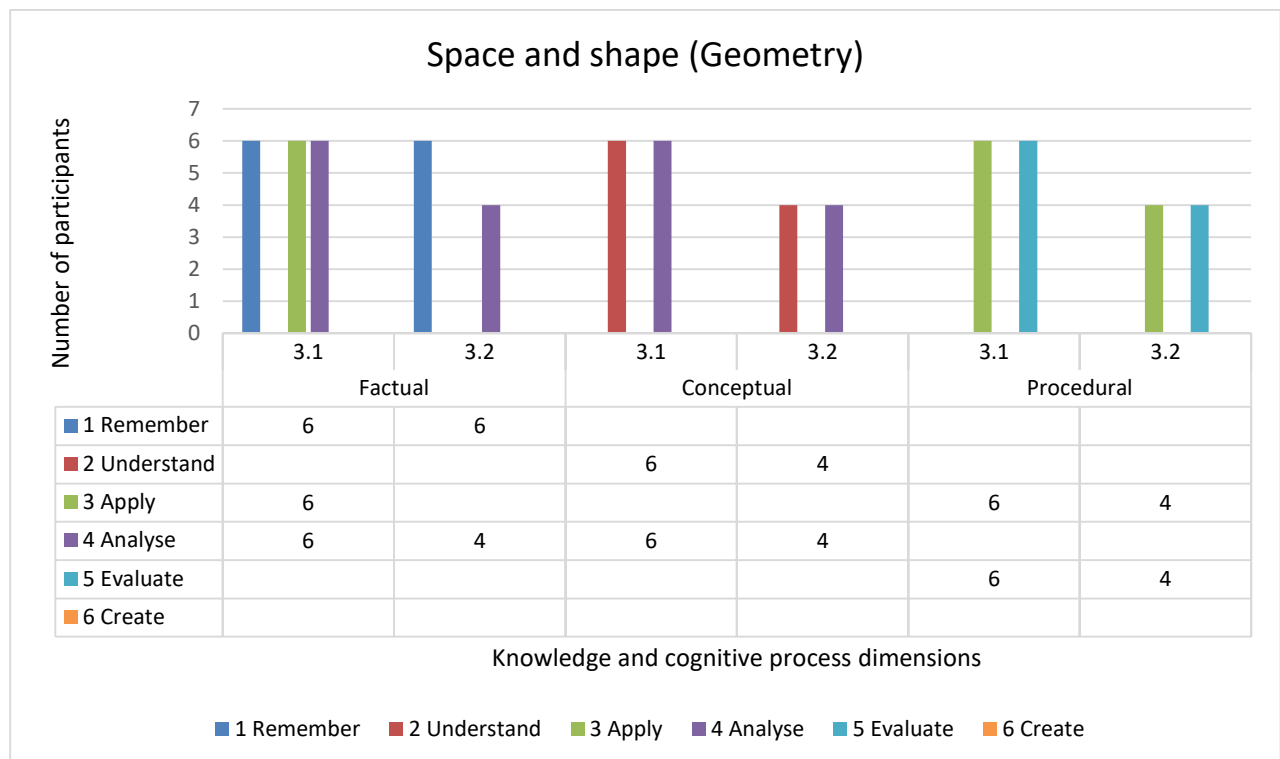
Important observations that emanate from the analysis (cf. Addendum I2) concerning space and shape, Question 3.2, are the following:

- The researcher argues that not being able to remember factual knowledge (participant 7), not being able to analyse information in the factual knowledge dimension (participants 1, 5, and 7), as well as poor conceptual understanding and inability to analyse information as part of conceptual knowledge (participants 1, 5, and 7), might have contributed to problems the participants experienced with application and evaluation of information in the procedural knowledge dimension.

- All the participants, except participant 4, were successful in dealing with all the cognitive process dimensions across all the knowledge dimensions. Their solid factual knowledge likely enabled the participants to be successful in both the conceptual and procedural knowledge dimensions. Participant 4 was not successful in the factual, conceptual, and procedural knowledge dimensions.
- Despite the researcher's comment regarding the procedural notation (the participants did not start the solution to the problem posed in the question by identifying which two triangles are being investigated for congruency), participants 2, 3, 4, and 6 correctly answered Question 3.2, which the researcher assumes to be because of their solid factual knowledge.

For question 3 participants obtained an average of 61,03%. Participants 3 and 6 both scored full marks for Question 3, and participants 5 and 7 had the lowest scores with 4 and 3 out of 11, respectively. Participants 3 and 6 both had the highest test averages too, and scored 92,10% and 94,73% respectively.

Graph 4.11 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 3.



Graph 4.11: Question 3: space and shape (geometry)

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required in the specific knowledge dimension

The researcher derives the following conclusions from Graph 4.11.

- The participants seemed to be successful with remembering factual knowledge for both Questions 3.1, and 3.2.
- Analysis of factual and conceptual knowledge for Question 3.2 appears to require attention, as only four of the seven participants were successful with the analysis of factual and conceptual knowledge. The understanding of conceptual knowledge for Question 3.2 also seemed to be troublesome.
- Question 3.1 was answered successfully by all, except for one participant, namely participant 4. Participant 4 made use of the incorrect parallel line principle and therefore got the answer to the question incorrect.
- In Question 3.2 the participants seemed less successful with carrying out the higher-order cognitive processes related to factual knowledge (analyse), conceptual knowledge (analyse), and procedural knowledge (evaluate). In addition, only four participants were successful with application in the procedural knowledge dimension for Question 3.2.
- From Graph 4.11 it seems like the participants were more successful in performing the cognitive process dimensions applicable to Question 3.1 than for Question 3.2.

The following section pays attention to the interpretation of Question 4, measurement.

4.3.4.4 Data analysis: Question 4 – measurement

Table 4.27 reflects the assessment for Question 4 (measurement).

Table 4.27: Data analysis: Senior Phase Question 4 – measurement

Knowledge dimension		Cognitive process dimension					
		1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Factual	4.1	X 2 3 4 5 6 7					X 2 3 4 5 6 7
	4.2	X 2 3 4 5 6 7		X 2 3 4 5 6 7			
Conceptual	4.1		X 2 3 4 5 6 7				X 2 3 4 5 6 7
	4.2			X 2 3 4 5 6 7	X 2 3 4 5 6 7		
Procedural	4.1						X 2 3 4 5 6 7
	4.2			X			

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.28 (see Addendum I2, p. 160).

To be successful at answering the questions related to measurement the participants need to know the area and perimeter of 2D shapes, surface area, and volume of 3D objects and the Pythagorean theorem to solve problems involving right-angled triangles. Participants also need to use formulae for measuring area, perimeter, surface area, and volume of geometric figures and solids, select and convert between appropriate units of measurement.

In general, the most important observations that emanate from the analysis (cf. Addendum I2) concerning Measurement for Question 4.1, are the following:

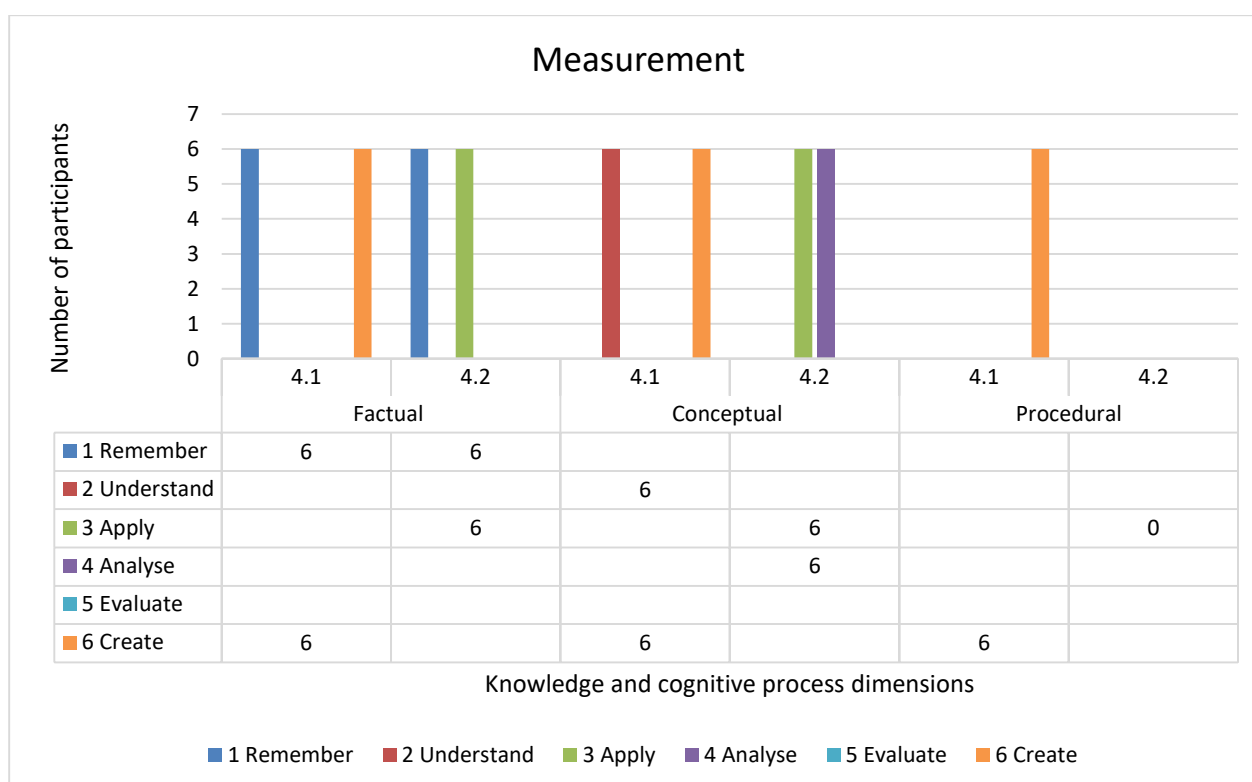
- Of all the participants, only participant 1 was unsuccessful with executing of all the cognitive process dimensions across all the knowledge dimensions.
- Participants 2, 3, 4, 5, 6, and 7 could recall the circle area formula to rewrite it as well as the meaning or use of a factor. The participants could also combine the area formulas of two circles to formulate a new integrated formula to answer the question, and were successful with carrying out all the cognitive processes across the knowledge dimensions.

The most important observations that emanate from the analysis (cf. Addendum I2) relating to the content area measurement for Question 4.2, are the following:

- The solid factual and conceptual knowledge displayed by Participants 2, 3, 4, 5, 6, and 7 seemingly did not guarantee their success in the procedural knowledge dimension.
- Not one of the participants was successful with application as part the procedural knowledge dimension.

From Table 4.3 it is evident that the question average for Question 4 is 64,28%, which is the third-lowest score for the Senior Phase test. Participant 1, who scored the lowest test average in the Senior Phase test (59,21%), also had the lowest score for Question 4 (0%). Participants 2, 3, 4, 5, 6, and 7 each scored 3 out of 4 for Question 4. None of the participants obtained full marks for the question.

Graph 4.12 below, visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 4.



Graph 4.12: Question 4: measurement

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required in a specific knowledge dimension.

The researcher observes the following trends from Graph 4.12:

- Graph 4.12 indicates that most of the cognitive processes linked to the knowledge dimensions were carried out successfully, except for the procedural knowledge dimension of question 4.2, where the application of knowledge seemed troublesome. The marks in Question 4.2 were lost due to a missing unit or not leaving the answer in terms of π , which influenced only the procedural knowledge dimension.
- According to Graph 4.12, there are six successful participants in the factual knowledge dimension for both Questions 4.1 and 4.2. The success is also evident in the conceptual knowledge dimensions, which could indicate that solid factual knowledge possibly resulted in solid conceptual knowledge, as well as procedural knowledge specifically for Question 4.1.

The following section pays attention to the interpretation of Question 5, data handling.

4.3.4.5 Data analysis: Question 5 – data handling

Table 4.29 reflects the assessment for Question 5.

Table 4.29: Data analysis: Senior Phase Question 5 – data handling

Knowledge dimension	Cognitive process dimension					
	1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Factual	X 1 2 3 4 5 6 7			X 1 2 3 4 6 7		
Conceptual		X 1 2 3 4 5 6 7		X 1 6		
Procedural			X 3 4 5 7			

To support the analysis of the data, a summary of the expectations that participants had to meet regarding knowledge dimensions and cognitive process dimensions in answering the question, follows below in Table 4.30 (see Addendum I2, p. 163).

To be successful at answering the questions related to data handling the participants needed to know how to organise and summarise data, draw and interpret graphs, as well as to interpret, summarize, analyse and report data and and make predictions about situations. Participants also needed to know how to pose questions for investigation. Participants needed knowledge of the probability of outcomes including both single and compound events and their relative frequency in simple experiments.

In general, the most important observations that emanate from the analysis (cf. Addendum I2) relevant to the content area measurement for Question 5, are the following:

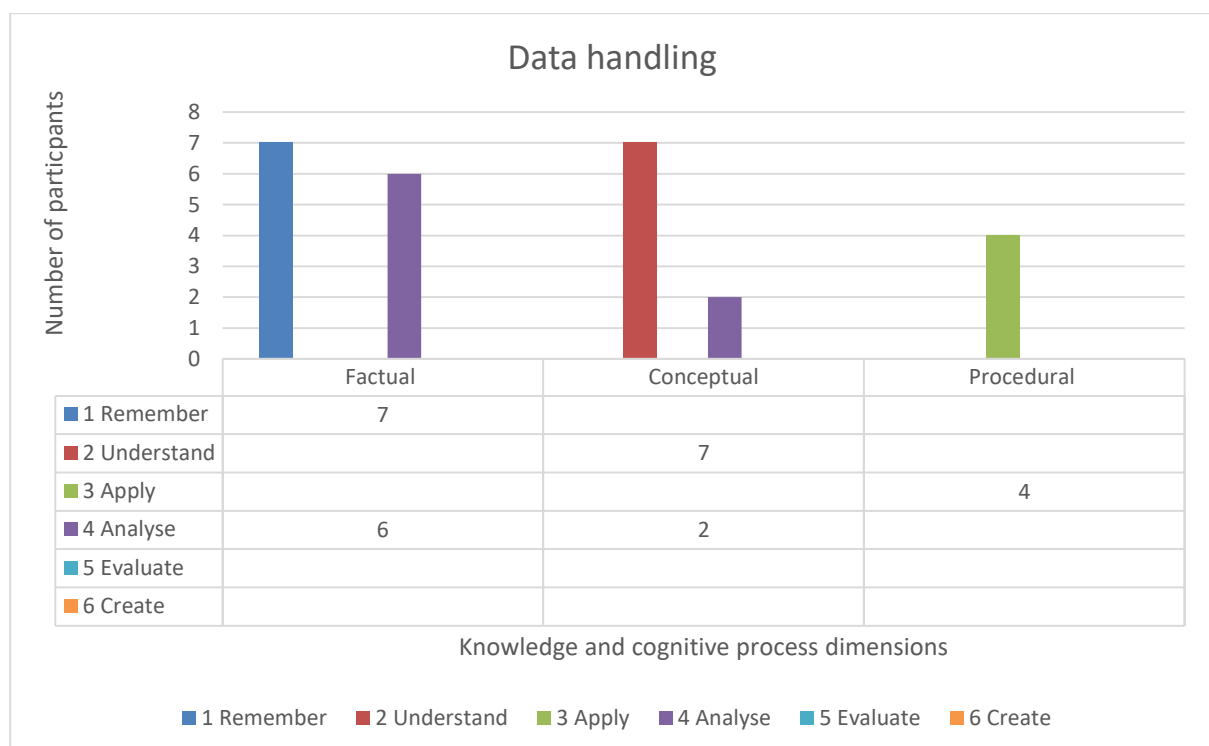
Table 4.29 reveals that all the participants were successful at remembering in the factual knowledge dimension. All the participants, except for participant 5, were also successful with the analysis of information in the factual knowledge dimension. All the participants were successful with understanding in the conceptual knowledge dimension, but only participants 1, and 6 could analyse knowledge in the conceptual dimension. Only participants 3, 4, 5, and 7 were successful at applying knowledge in the procedural knowledge dimension.

In the factual knowledge dimension, the concepts range, median, and mean had to be recalled according to the memorandum. If one of these concepts were recalled by the participant, they were assessed as successful with remember in the factual knowledge dimension (cf. 5.7.2 - Limitations). The data analysis of each participant (cf. Addendum I2) explicitly states which concepts could not be recalled by a participant, for example participants 1, 2, and 5 did not seem to recall range but could recall median and mean.

Analysis of factual knowledge relates to the questions about the mode and median. Problems with the analysis of conceptual knowledge might not only be as a result of insufficient factual knowledge. The researcher acknowledges that not enough evidence was provided in the tests to make a deduction, but suggests that it might be due to a lack of exposure during the participants' training to questions where explanations are required, as the participants did not seem to understand how to do so. Alternatively, the participants might not have read the question properly or just left it out on purpose. Only participant 5 struggled with analysis in the factual knowledge dimension, whereas participants 2, 3, 4, 5, and 7 experienced problems with analysis in the conceptual knowledge dimension. The lack of conceptual knowledge demonstrated by participants 2, 3, 4, 5, and 7, only seemed to influence the procedural knowledge of participant 2, as participants 3, 4, 5, and 7 could calculate the answers in order to be successful in the procedural knowledge dimension (Table 4.29). Many of the participants were unsuccessful with analysing conceptual knowledge due to the median being calculated incorrectly.

Question 5 has a question average of 58,92%. Participant 5 had the lowest score for the question, namely 1 out of 4. Participants 3, 4, and 6 had the highest scores for the question; namely 3 out of 4.

Graph 4.13 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 5.



Graph 4.13: Question 5: Data handling

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required in the specific knowledge dimension.

The researcher observes the following trends from Graph 4.13.

- The data in Graph 4.13 display that the participants seemed to be successful in the factual knowledge dimension, with only one participant (participant 5) who experienced analysis as troublesome.
- The participants could also be regarded as successful with understanding as part of conceptual knowledge, but analysis as part of conceptual knowledge seems problematic, as only 2 participants (participants 1 and 6) succeeded in analysing knowledge.
- Only four participants had success with the application of knowledge as part of the procedural knowledge dimension (participants 3, 4, 5, and 7).

As seen in Graph 4.13, the cognitive process of analysing (identifying the mode and the middle value - median) that formed part of the factual knowledge dimension was less successful, which possibly contributed to the unsuccessfulness with analysis as part of the conceptual knowledge dimension (organising values in chronological order to find the middle value – median). The procedural knowledge dimension involved calculating range and the median. Therefore, if the participant could not recall the range or median as part of the factual knowledge dimension, the participants would also not be able to do the calculations as part of the procedural knowledge dimension.

4.3.4.6 A synthesis: overall conclusions of the Senior Phase

When comparing the average results obtained by the participants for the various questions (cf. Table 4.3) and considering the analyses for each of the questions (cf. Tables 4.21, 4.23, 4.25, 4.27, 4.29; Graphs 4.9 - 4.13) the following trends are observed:

- Question 2 (numbers, operations, and relationships) had the highest average of 96,10%.
- Question 1 (patterns, functions, and algebra) had a question average of 85,71% and was thus the question that was answered second-best.
- For question 3 (space and shape) an average of 61,03% was obtained.
- Question 4 (measurement) the participants achieved an average of 64,28%.
- Question 5 (data handling) had the lowest average, namely 58,92%.

Combining the average results with the analyses of the participants' performance (cf. Addendum I2) concerning the knowledge and cognitive process dimensions for each of the questions, the following can be concluded:

- The highest average obtained for Question 2 (numbers, operations, and relationships), could be linked to the fact that the factual knowledge appeared to be solid as the carrying out of related cognitive process dimensions was effective. Although the participants did not all get the correct answer due to the lack of a unit, or not writing the answer in terms of pi, the factual knowledge possibly contributed to success in the conceptual and procedural knowledge dimensions, which benefitted the overall test result. It is however encouraging that only participants 2 and 5 did not supply the correct answers, for Question 2.2 and 2.1, respectively.
- In Question 1, where factual knowledge appeared strong, the participants were successful with executing cognitive processes in both the conceptual and procedural knowledge dimensions. However, the factual knowledge seemed to be less solid for Question 1.2.2, in particular for applying knowledge, it might have influenced the participants' success in the conceptual dimension, specifically in relation to the evaluation of knowledge, as well as in the procedural knowledge dimension with application of knowledge.
- The absence of conceptual knowledge about understanding and analysing information, especially for Question 3.2 possibly influenced the lower average obtained for Question 3 (61.03%). Although the participants demonstrated that they could remember some factual knowledge, the gaps in the factual knowledge (mostly related to the recollection of the geometric principles) seemed to hinder success with enacting understanding and analysis as part of conceptual knowledge, and with application and evaluation as part of procedural knowledge.
- In Question 4, the results displayed a strong factual knowledge base which possibly influenced the success observed in the conceptual and procedural knowledge dimensions. Participant 1 was the only participant who was unsuccessful with the carrying out of all cognitive processes across the knowledge dimensions.
- For Question 5 the participants also displayed strong factual knowledge. The lowest average obtained for Question 5 (data handling), namely 58,92%, which the researcher thinks might be linked to the problems experienced with the analysis of conceptual knowledge, and the application of procedural knowledge.

The information sheet handed to participants to aid the recall of knowledge they might have forgotten, was not used by all the participants. The researcher wishes to mention the following about how many of the seven participants used the information sheet to assist them in answering the questions concerning the different content areas:

- Patterns, functions, and algebra: used by one participant.

- Numbers, operations, and relationships: used by one participant.
- Space and shape: used by none of the participants.
- Measurement: used by none of the participants.
- Data handling: used by none of the participants.

In this section the researcher analyses and interprets the data obtained for each of the individual participants in the Intermediate Phase.

4.3.5 Analysis and interpretation of knowledge and cognitive process dimensions – Intermediate Phase

The following sections analyse and interpret the data obtained for the Intermediate Phase participants in relation to each of the questions on which the test focused.

4.3.5.1 Data analysis: Question 1 – Numbers, operations, and relationships

The table below, Table 4.31, reflects the assessment for Question 1.

Table 4.31: Data analysis: Intermediate Phase Question 1 – Numbers, operations, and relationships

Knowledge dimension		Cognitive process dimension					
		1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
A Factual	1.1	X 1 2 3 4 5 7 8 9 10 11					
	1.2	X 1 2 3 4 7 8 9 10 11			X 1 2 3 4 7 8 9 10 11		
	1.3	X 1 2 3 4 5 7 8 9 10 11					
	1.4	X 1 2 4 7 8 9 10 11					
	1.5	X 5 6 8 9 10 11					
	1.6	X 1 2 3 4 9 10 11					
	1.7	X 1 2 3 4 5 9 10					
	1.8	X 1 2 4 5 6 7 8 9 10 11					
B Conceptual	1.1		X 1 2 3 4 5 7 8 9 10 11		X 1 2 3 4 5 7 8 9 10		
	1.2				X 1 2 3 7 9 10 11		
	1.3		X 1 2 3 4 5 7 8 9 10 11	X 1 2 3 7 8 9 10			X 1 2 3 7 8 9 10 11
	1.4			X 1 7 8 9 10 11	X 1 7 8 9 10 11		
	1.5		X 5 6 8 9 10 11		X 5 6 8 9 10		
	1.6				X 1 2 3 4 9 10 11	X 1 2 3 4 9 10 11	
	1.7			X 1 2 9			X 1 2 3 4 9 10
	1.8		X 1 2 4 5 6 7 8 9 10 11		X 1 2 4 5 6 7 8 9 10 11		
C Procedural	1.1			X 1 2 3 4 5 7 8 9 10			
	1.2			X 1 2 3 9 11			
	1.3			X 1 3 7 8 9 10 11			
	1.4			X 1 8 9 10 11			
	1.5			X 6 8 9 10			
	1.6			X 1 3 4 9 11			
	1.7			X			
	1.8			X 1 2 4 6 7 8 9 10 11			

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows below in Table 4.32 (see Addendum I3, p. 165).

Numbers, operations, and relationships is a topic that requires knowledge of:

- whole numbers (mental calculations, number ranges for counting, ordering, comparing, etcetera, calculation techniques, number ranges for multipliers and factors, properties of whole numbers, problem-solving, common and decimal fractions, etcetera);
- patterns, functions, and algebra (numeric patterns, input and output values, equivalent forms, geometric patterns, equivalent forms, and number sentences, etcetera);
- space and shape (range of shapes, characteristics of shapes, angles, properties and characteristics of 3D objects, symmetry, and transformations, etcetera);
- measurement (length, measurement instruments, units, calculations and problem-solving involving length, mass, capacity/ volume, time, and temperature, etcetera); and
- data handling (collecting and organising data, representing data, analysing, interpreting and reporting data, and probability, etcetera).

Question 1 tests the participants' mathematics skills as it requires the participants to recall operations and terminology; understand, apply, analyse, and evaluate concepts; and apply procedures to calculate answers.

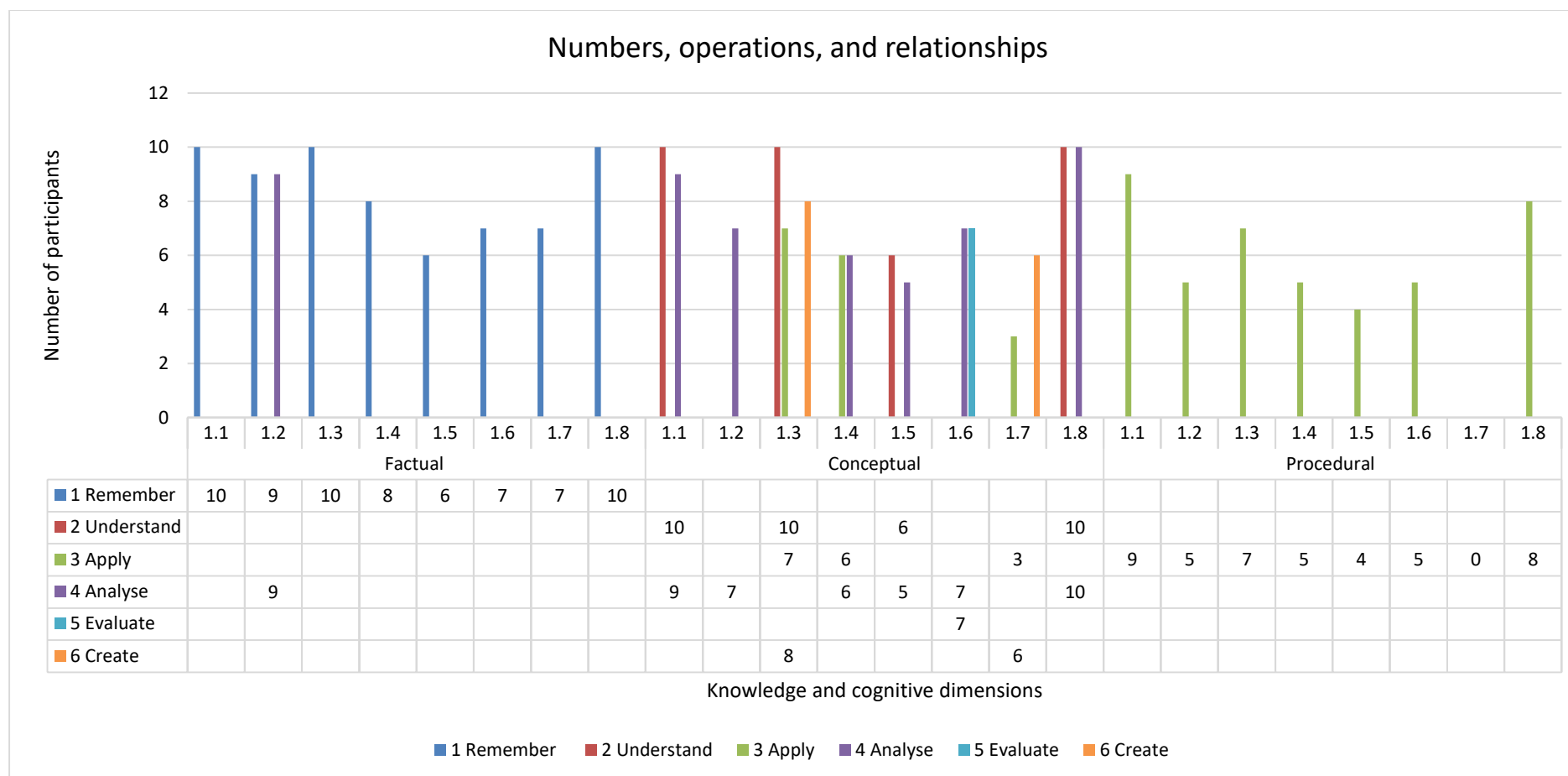
In summary, important observations that emanate from the analysis (cf. Addendum I3) concerning numbers, operations, and relations, Question 1, are the following:

- The most successful questions seemed to be Question 1.1 and 1.8. In Question 1.1 only three participants did not succeed in the procedural knowledge dimension (participants 6, 11, and 12 (Question 1.1), participants 3, 5, and 12 (Question 1.8). Participants 6, 11, and 12 were also not successful in the conceptual knowledge dimension for Question 1.1 - participants 6 and 12 with understanding, and participants 6, 11, and 12 with analysis. Only participants 6 and 12 were not successful in the factual knowledge dimension of Question 1.1. In Question 1.8 ten of the twelve participants were successful in the factual knowledge dimension (participants 3 and 12 were not successful). The same ten participants were successful in the conceptual knowledge dimension, but only nine participants were successful in the procedural knowledge dimension (participants 3, 5, and 12 were not successful).

- Six participants were unsuccessful in the factual knowledge dimension of Question 1.5. The same participants (1, 2, 3, 4, 7, and 12) were unsuccessful in demonstrating understanding and analysing as part of conceptual knowledge for Question 1.5, with the addition of Participant 11 who was also unsuccessful with analysis as part of conceptual knowledge. Participant 12 left out all the questions and did not attempt to answer any question in Question 1.
- Four participants were unsuccessful in the factual knowledge dimension for Question 1.4 (participants 3, 5, 6, and 12). The same four participants were unsuccessful in the conceptual knowledge dimension with analysis of information, with the addition of participants 2 and 4. All the participants who were unsuccessful in the factual and conceptual knowledge dimensions were also unsuccessful in the procedural knowledge dimension with the addition of participant 7 who was also unsuccessful in the procedural knowledge dimension. Mathematical operations (order of calculations, addition, and subtraction) seemed to be the main problem in Question 1.4.
- Seemingly the most difficult question in the procedural knowledge dimension was Question 1.7. Five participants were unsuccessful in remembering the factual knowledge for Question 1.7 and only three participants (1, 2 and 9) were successful in carrying out both cognitive process dimensions to demonstrate conceptual knowledge, namely apply and create. None of the participants were successful in the procedural knowledge dimension for Question 1.7.
- Question 1.6 seemed to be problematic in the procedural knowledge dimension too. Seven of the participants were unsuccessful in the procedural knowledge dimension for Question 1.6 (participants 2, 5, 6, 7, 8, 10, and 12). Participants 5, 6, 7, 8, and 12 were also unsuccessful in both of the cognitive processes within the conceptual knowledge dimension of Question 1.6 (analyse and evaluate). Participants 5, 6, 7, 8, and 12 were unsuccessful with remembering factual knowledge.
- The lack of factual knowledge in some cases seemed to have influenced the conceptual and procedural knowledge dimensions. For example, participant 6 was successful in the factual knowledge dimension for Question 1.5 and 1.8, and also successful in the conceptual and procedural dimensions for those two questions. Participants 6 and 12 were unsuccessful in the factual knowledge dimension for Question 1.1 and also unsuccessful in the conceptual and procedural knowledge dimensions of the question.
- Success in the factual knowledge dimension however, did not necessarily guarantee success in the conceptual and procedural knowledge dimensions. Question 1.7 could be cited as an example here.

From Table 4.5, it is clear that the test average for Question 1 was only 49,58%. Two participants, namely participants 1 and 9 had the highest scores for the test with 17 out of 20. Participant 12 had the lowest test score for the Question 1, namely 0% and in total four participants scored below 50% for Question 1 (Participants 5, 6, 7, and 12).

Graph 4.14 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 1.



Graph 4.14: Question 1 – numbers, operations, and relationships

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required in a specific knowledge dimension.

The researcher observes the following trends from Graph 4.14:

- From Graph 4.14, it seems that the participants were mostly successful in the factual knowledge dimension, with the lowest success in the factual knowledge dimension observed for Questions 1.4, 1.5, 1.6, and 1.7. In Question 1.5 participants seemed to have trouble with the order of calculations, in Question 1.6 participants made mistakes with the operations or place values, and in Question 1.7 participants made mistakes with the simplification of fractions or operations. In all cases, the mistakes seemed to relate to mathematical operations.
- The conceptual knowledge dimension seems relatively successful also, except for Questions 1.4, 1.5, and 1.7, which could be due to the factual knowledge also being the least successful for these questions. Application and analysis as part of conceptual knowledge could be regarded as the most troublesome cognitive processes for the participants.
- The application of procedural knowledge seems to be the least successful of the three knowledge dimensions. This seems to be due to repeated calculation errors throughout Question 1. This could be due to a lack of success within the factual and conceptual knowledge dimensions (Rittle-Johnson et al., 2015; Schoenfeld, 1992) (cf. 2.3.2.3). Where factual knowledge seemed to be solid (Questions 1.1, 1.3, 1.8), the conceptual and procedural knowledge dimensions seemingly benefitted as well. However, where factual knowledge seemed lacking (Questions 1.4, 1.5, 1.6, 1.7), the participants seemed less successful in the conceptual and procedural knowledge dimensions.

The following section pays attention to the data analysis for Question 2 (patterns, functions, and algebra).

4.3.5.2 Data analysis: Question 2 – patterns, functions, and algebra

Table 4.33 reflects the assessment for Question 2.

Table 4.33: Data analysis: Intermediate Phase Question 2 – patterns, functions, and algebra

Knowledge dimension		Cognitive process dimension					
		1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
A Factual	2.1	X 2 3 5 6 7 9 10					
	2.2	X 1 2 3 4 5 6 7 8 9 10					
	2.3	X 1 2 3 4 5 6 7 8 9 10 12					
B Conceptual	2.1		X 2 3 5 6 7 9 10	X 2 3 5 6 7 9 10	X 2 3 5 6 7 9 10		
	2.2			X 1 2 3 4 5 6 7 8 9 10	X 1 2 3 4 5 6 7 8 9 10		
	2.3		X 1 2 3 4 5 6 7 8 9 10 12		X 1 2 3 4 5 6 7 8 9 10 12		
C Procedural	2.1			X 2 3 5 6 7 9 10			
	2.2			X 1 2 3 4 5 6 7 8 9 10			
	2.3			X 1 2 3 4 5 6 7 8 9 10 12			

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.34 (see Addendum I3, p. 177).

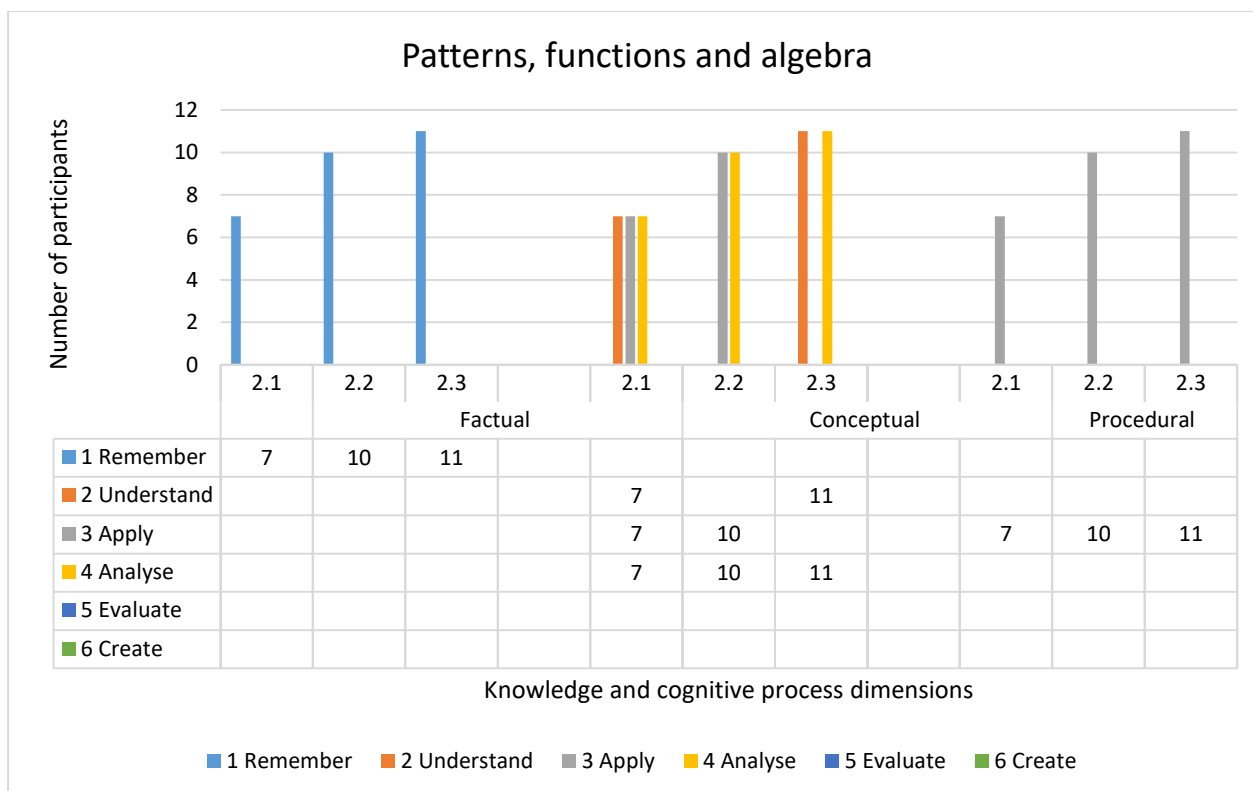
Patterns, functions, and algebra is a topic that requires the knowledge of numeric and geometric patterns (investigate and extend patterns and describe general rules, etcetera) input and output values (flow diagram and tables, etcetera), equivalent forms, and number sentences.

In summary, important observations that emanate from the analysis (cf. Addendum I3) concerning patterns, functions, and algebra (Question 2) are the following:

- Participant 11 was the only participant who did not attempt to answer any of the questions and was therefore the only one that was unsuccessful in all the knowledge and cognitive process dimensions of Question 2.
- Participant 12 was only successful in answering Question 2.3, and also did not attempt to answer Question 2.1 or 2.2.
- Question 2.1 seemed to be the most difficult if judged by the overall answers from the participants. Question 2.1 required the application of three cognitive processes to demonstrate conceptual understanding, whereas the other questions only required the application of two cognitive processes as part of the conceptual knowledge dimension.

Table 4.5 indicates the question average for Question 2 is 75%, which is the second-highest question average for the Intermediate Phase test. Participants 2, 3, 5, 6, 7, 9, and 10 all scored 100% for the question. The lowest question scores were obtained by participants 11 and 12, with 0% and 20% for Question 2.

Graph 4.15 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 2.



Graph 4.15: Question 2 – patterns, functions, and algebra

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required

The researcher observes the following trends from the graph:

- The success in all the knowledge and cognitive process dimensions for Question 2.1 seemed to be lower than that of Question 2.2 and 2.3. It could be as a result of Question 2.1 having less success in the factual knowledge dimension.
- Question 2.3 appeared to be the most successful, with only one participant being unsuccessful with application in the procedural knowledge dimension. Question 2.3 was less challenging than Questions 2.1 and 2.2, and participants could be successful by just counting the matches.
-

In the next section, the researcher analyses the data obtained for Question 3 (measurement).

4.3.5.3 Data analysis: Question 3 – measurement

Table 4.35 reflects the assessment for Question 3.

Table 4.35: Data analysis: Intermediate Phase Question 3 – measurement

	Knowledge dimension		Cognitive process dimension					
			1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Q3	A Factual	3.1.1 & 3.1.2	X 3 5 6 8 9 10					
		3.2	X 1 2 3 4 5 6 7 8 9 10 12					
	B Conceptual	3.1.1 & 3.1.2		X 3 5 6 8 9 10				
		3.2		X 1 2 3 4 5 6 7 8 9 10 12				X 1 2 3 4 5 7 8 9 10 12
	C Procedural	3.1.1 & 3.1.2			X 3 5 6 8 9 10	X 5 6 9 10		
		3.2			X 1 2 3 4 5 7 8 9 10 12	X 1 2 3 4 5 7 8 9 10 12		

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows below in Table 4.36 (see Addendum I3, p. 179).

Measurement is a topic that requires the knowledge of properties of 2D and 3D shapes, characteristics of shapes and objects, angles, symmetry, transformations, viewing objects (position and views, and movements).

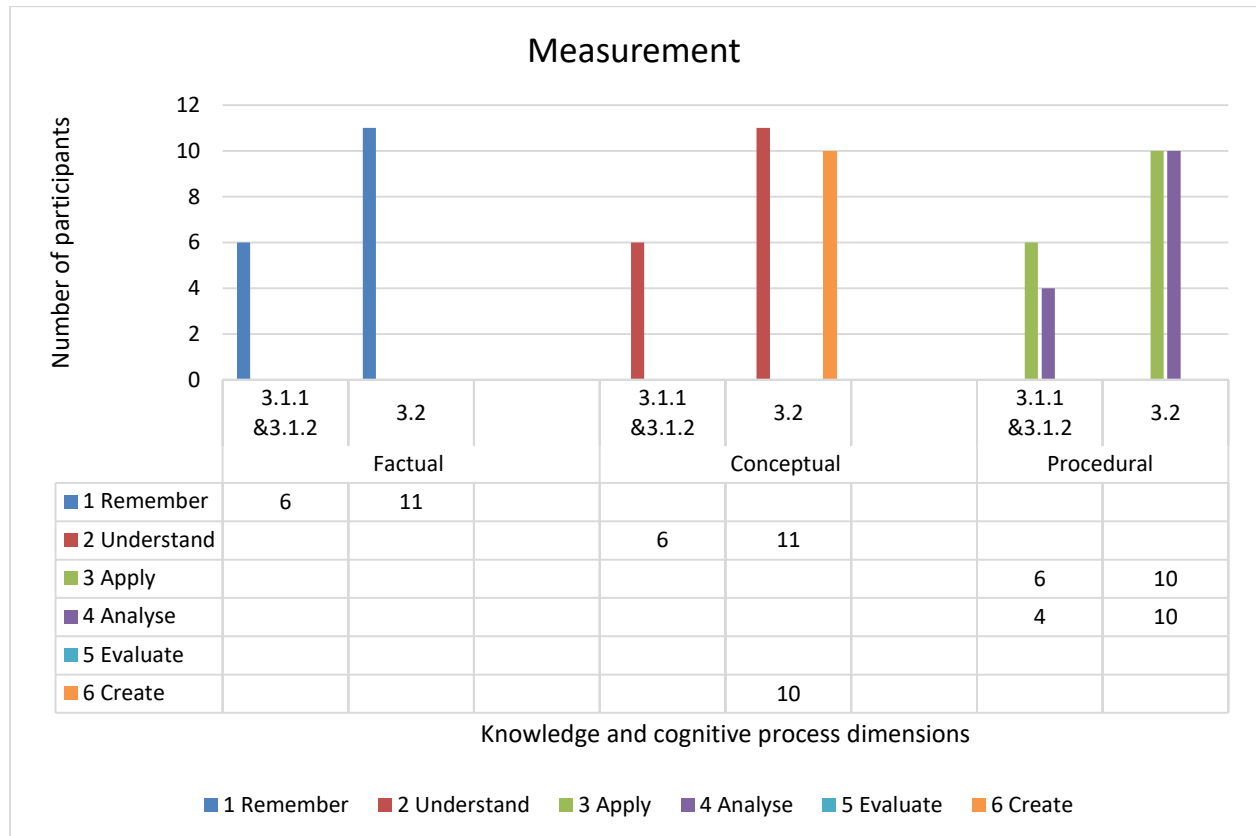
Questions 3.1.1 and 3.1.2 relate to the same measurement of time images of two clocks and their analyses are therefore grouped.

In summary, important observations that emanate from the analysis (cf. Addendum I3) concerning measurement (Question 3) are the following:

- If the participants could not correctly answer both the questions, the enacting of cognitive processes as part of the procedural knowledge dimension was affected. The procedural knowledge dimension required participants to calculate (Question 3.1.1) and compare (Question 3.1.2) the time differences.
- Most of the participants got the correct answer for Question 3.2, with the exception of participants 6 and 11. Participant 6 seemed to have a gap in the conceptual knowledge dimension as participant 6 did not know how to convert between litre and millilitre and could not plan the conversion correctly. Participant 11 did not attempt to provide an answer to the question.

- According to Table 4.5, the question average for Question 3 is 52,77%. Three participants scored 100% for Question 3 (Participants 5, 9 and 10), and one participant scored 0% (participant 11).

The results of the participants across the knowledge and cognitive process dimensions for Question 3 are visually displayed in Graph 4.16.



Graph 4.16: Question 3 – measurement

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required in a specific knowledge dimension

The researcher observes the following trends from Graph 4.16:

- The participants seemingly found Question 3.2 easier than 3.1.1 and 3.1.2, as they were more successful in answering Question 3.2 correctly. It could also be that they were able to apply the cognitive processes in the context of Question 3.2 better. Question 3.2 was a simple conversion of milliliter to liter question, but Question 3.1 consisted of two questions related to the comparison and difference in times between different countries.

- Only six participants were successful in remembering the factual knowledge, demonstrated conceptual understanding, and could enact application as part of procedural knowledge in the context of answering Question 3.1.1. In addition, only four participants were successful with analysis that formed part of procedural knowledge for Question 3.1.1. The lack of success in the procedural knowledge dimension might be a result of limited factual and conceptual knowledge.

The following section pays attention to the data analysis for Question 4 (space and shape).

4.3.5.4 Data analysis: Question 4 – space and shape

The assessment for Question 4 is shown in Table 4.37.

Table 4.37: Data analysis: Intermediate Phase Question 4 – space and shape

	Knowledge dimension		Cognitive process dimension					
			1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
Q4	A Factual	4.1	X 1 2 5 6 7 8 9 10					
		4.2	X 2 4 5 7 8 9 10 12		X 2 5 7 12			
	B Conceptual	4.1		X 2 5 6 7 8 9 10				X 2 5 6 7 9
		4.2		X 2 4 5 7 8 9 10 12		X 2 5 7 12		
	C Procedural	4.1			X 2 5 6 9			
		4.2			X 12			

To support the analysis of the data, a summary of the expectations that participants had to meet concerning knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.38 (see Addendum I3, p. 181).

Space and shape is a topic that requires knowledge of length, units, mass, capacity or volume, time, and temperature.

Question 4.1

In the factual knowledge dimension of Question 4.1, there are four different terms listed under the cognitive process remember, namely: rectangle, square, perimeter, and fence as a perimeter. If the participants recalled one of these four terms, remember in the factual knowledge dimension would be indicated as successful in Table 4.37 (cf. Limitations – 5.7.2). If one or more of the four terms were not recalled correctly, it was indicated in the data analysis (see Addendum I3).

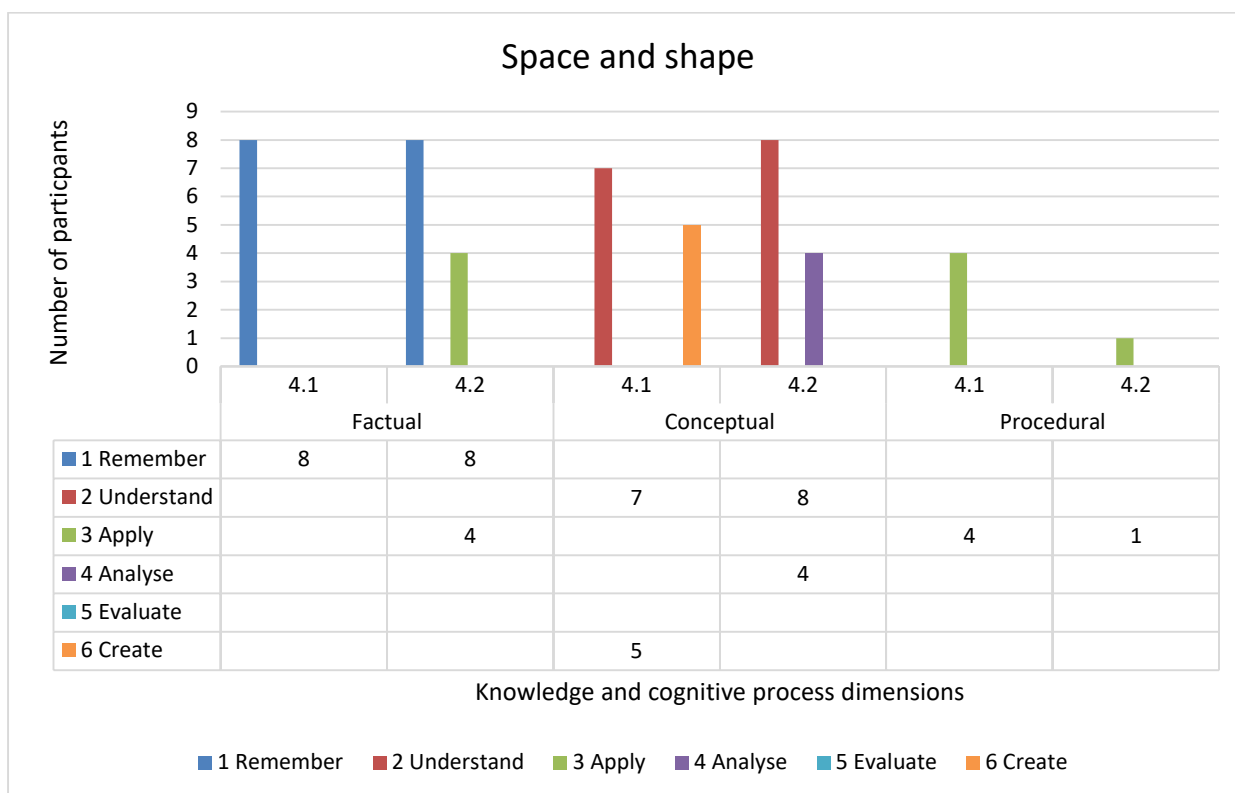
Question 4.2

Question 4.2 requires participants to count the number of vertices, edges, and faces in the pentagonal prism provided. The factual knowledge indicates four terms under the cognitive process remember, namely: pentagonal prism, vertices, edges, and faces. If participants recalled any one of the four terms, remember in Table 2.27 would have been indicated as successful (cf. Limitations – 5.7.2). If a participant could not remember any of the four terms, it is indicated in the data analysis (see Addendum I3).

In summary, important observations that emanate from the analysis concerning space and shape (Question 4) are the following:

- Participants 2, 5, 6, and 9 were the only participants who were successful in carrying out all the cognitive processes related to Question 4.1. Participants 1, 3, 4, 7, 8, 10, 11, and 12 were all unsuccessful in finding the answer to Questions 4.1. Participant 11 did not attempt to answer the question.
- Only participant 12 was successful in enacting all the cognitive processes related to Question 4.2. Again, participant 11 did not attempt to answer the question.
- Overall, only a few participants were successful in answering the questions related to space and shape. Only four participants provided the correct answer to Question 4.1, and only one participant answered Question 4.2 correctly. Question 4.2 might have been slightly more difficult for the participants than Question 4.1. Question 4.2 required the participants to be able to visualise a 3D object and to recall the terminology: faces, vertexes and edges. It seemed like some participants confused the terminology while others counted incorrectly. This observation of the researcher could imply that the participants could not visualise the object.
- According to Table 4.5 the question average for Question 4 is 38,33%, which is the lowest test score average for one of the subject content areas in the Intermediate Phase test. Participants 9 and 12 both scored 60% for Question 4. The lowest scores were obtained by participants 3 and 11, namely 0%. The highest scores were obtained by participants 2 and 5, with 80%. Eight participants scored below 50% for Question 4, implying then that only four participants scored above 50% for Question 4.

Graph 4.17 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 4.



Graph 4.17: Question 4: Space and shape

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required in a specific knowledge dimension

From Graph 4.17, the researcher observes the following trends:

- The majority of the participants seemed to be the most successful with remembering in the factual knowledge dimension for both questions, except for application of factual knowledge for Question 4.2. Also, understanding of knowledge in the conceptual knowledge dimension revealed some success for the majority of the participants.
- It appears as if the participants were mostly unsuccessful in executing the higher-order cognitive processes that involved analysis as part of demonstrating conceptual knowledge for Question 4.2, and with application as evidence of procedural knowledge for Questions 4.1, and 4.2, and with creating to demonstrate conceptual knowledge in Question 4.1.

In contrast to the previous questions, and the literature, it appeared that strong factual and conceptual knowledge did not contribute to success in the procedural knowledge dimension. Factual knowledge in Table 4.38 indicated multiple terms, and as stated in the analysis, not

all terms were recalled by the participants, which possibly contributed to the unsuccessfulness in the procedural dimension.

The following section summarises the data analysis for Question 5 (data handling).

4.3.5.5 Data analysis: Question 5 – data handling

The assessment for Question 5 is reflected in Table 4.39.

Table 4.39: Data analysis: Intermediate Phase Question 5 – data handling

Knowledge dimension	Cognitive process dimension					
	1 Remember	2 Understand	3 Apply	4 Analyse	5 Evaluate	6 Create
A Factual	X 1 2 3 4 5 6 7 8 9 10 12					
B Conceptual		X 1 2 3 4 5 6 7 8 9 10 12		X 1 2 3 4 5 6 7 8 9 10 12		
C Procedural			X 1 2 3 4 5 6 7 8 9 10 12			

To support the analysis of the data, a summary of the expectations that participants had to meet relevant to knowledge dimensions and cognitive process dimensions in answering the question, follows in Table 4.40 (see Addendum I3, p. 183).

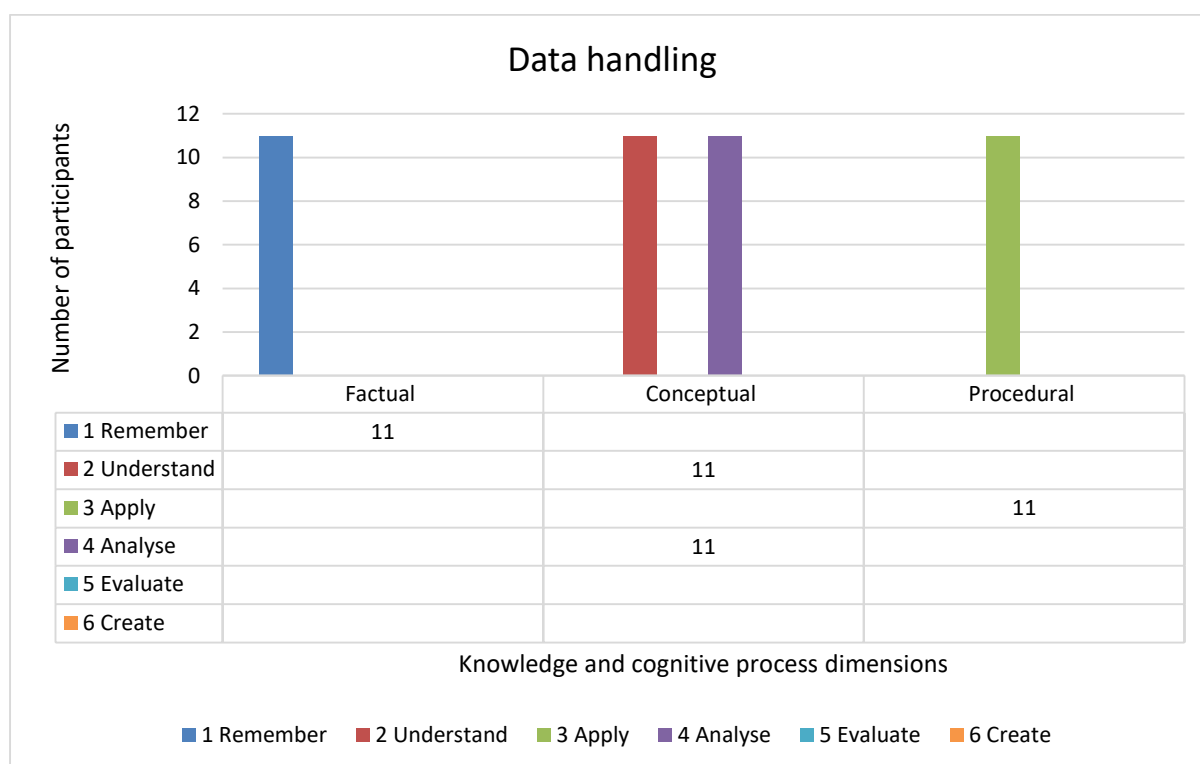
Data handling is a topic that requires the knowledge of collecting and organising data, representing data, analysing, interpreting, and reporting data and probability.

Question 5

Question 5 consists of five very low-level statistics questions all relating to the same pie chart image, and were therefore assessed as one question. In summary, important observations that emanate from the analysis (cf. Addendum I3) of data handling (Question 5) are the following:

- It can be concluded that all the participants, except 11, were successful in all the knowledge and cognitive process dimensions. Participant 6 only made a calculation error with Question 5.3, which is just one of the five sub-questions of Question 5, whereas, participant 11, did not attempt to answer Question 5 as a whole and was thus considered unsuccessful.
- The question average for Question 5 is 90,27% (Table 4.5), which is the highest question average obtained for a subject content area in the Intermediate Phase test. Ten of the 12 participants scored 100% for Question 5. Participant 11 scored 0% for Question 5, and participant 6 lost one point for Question 5.

Graph 4.18 visually displays the results of the participants across the knowledge and cognitive process dimensions for Question 5.



Graph 4.18: Question 5 – data handling

*If no values are displayed in some of the blocks on the data table, it implies that a particular cognitive process dimension was not required

The researcher observes the following trends from Graph 4.18.

- It seems that the participants were consistently successful in answering Question 5, except for one participant. Although Question 5 consisted of five separate, but related questions, the overall trend seems to be one of success in all three knowledge dimensions.

4.3.5.6 A synthesis: overall conclusion for the Intermediate Phase

When comparing the average results obtained by the participants for the various questions (cf. Table 4.5), and considering the analyses for each of the questions (cf. Tables 4.31, 4.33, 4.35, 4.37, 4.39; Graphs 4.14 - 4.18) the following trends are observed:

- The participants obtained the highest average for Question 5 (data handling), namely 90,27%.

- The second and third highest averages were obtained for Question 2 (patterns, functions, and algebra) and Question 3 (measurement), with 75% and 52,77%, respectively.
- The lowest averages were obtained for Question 1 (numbers, operations, and relationships), with 49,58%, and Question 4 (measurement (geometry)), with 38,33%.

Combining the average results with the analyses of the participants' performance concerning the knowledge and cognitive process dimensions for each of the questions (cf. Addendum I3), the following can be concluded:

- The highest average obtained for Question 5 (data handling), could be linked to the fact that the cognitive process of remembering factual knowledge was relatively well developed, which contributed to having success in understanding and analysing to demonstrate conceptual and applying procedural knowledge.
- For Questions 2 and 3, it is clear that where factual knowledge appeared to be strong (Question 2.2, 2.3, and 3.2), the participants' also achieved success with enacting the cognitive processes relevant to the conceptual and procedural knowledge dimensions. However, where the factual knowledge seemed to be less solid, or absent (Question 2.1, 3.1.1, and 3.1.2), the participants seemed to find the carrying out of cognitive processes related to conceptual and procedural knowledge as demanding.
- The low average achieved for Question 1, could be attributed to the problems that the participants experienced with the execution of cognitive processes (apply, analyse, evaluate and create) to demonstrate conceptual and procedural knowledge.
- Question 4 had the lowest average, which could be linked to the unsuccessfulness with the application of factual, conceptual and procedural knowledge in the context of both Questions 4.1 and 4.2.

From the data analysis across the three teaching phases, the researcher concludes that the amount of factual knowledge reflected in the extent to which participants were successful in the conceptual and procedural knowledge dimensions (Eisenhart et al., 1993; Rittle-Johnson et al., 2015) (cf. 2.3.2.2, 2.3.2.3). If the participants had gaps in their factual knowledge, it seemed to have a snowball effect on the conceptual and the procedural knowledge dimensions (Eisenhart et al., 1993; Rittle-Johnson et al., 2015) (cf. 2.3.2.2, 2.3.2.3). For example: **FET Phase:** (Questions 3.2 (numbers, patterns, sequences and series), 4.2 (finance, growth, and decay), and 5.2 (functions) (cf. Graphs 4.4, 4.5, 4.6). **Senior Phase:** Questions 1.2.2 (patterns, functions, and algebra) (cf. Graphs 4.9). **Intermediate Phase:** Questions 1.4, 1.5, 1.6 (numbers, operations, and relationships), 2.1 (patterns, functions, and

algebra), 3.1.1, and 3.1.2 (measurement), 4.2 (space and shape (geometry) (Graphs, 4.14, 4.15, 4.16, 4.17). Aligned to the literature, the data seem to indicate that the knowledge dimensions are connected and influence each other (Baker et al., 2012, Rittle-Johnson et al., 2015) (cf. 2.3.2.2)

The data support the arguments of Rittle-Johnson et al. (2015) and Schoenfeld (1992) (cf. 2.3.2.3) that solid factual and conceptual knowledge enable the development of procedural knowledge. One could argue that a strong link exists between the three knowledge dimensions (Baker et al., 2012; Lauritzen, 2013; Porter, 1989; Rittle-Johnson et al. 2015; Stols, 2013) (cf. 2.3.2.2, 2.3.2.3). It might also be that during training conceptual knowledge is not singled out as a knowledge dimension that needs to be developed, which hampers the understanding of content (Stols, 2013) (cf. 2.3.2.3) and the use of procedures (Crooks & Alibali, 2014; Rittle-Johnson & Alibali, 1999) (cf. 2.3.2.5).

In contrast to the aforementioned, the researcher however also observed that successful factual and conceptual knowledge often had a limited influence on success in the conceptual and/or procedural knowledge dimension. For example: **FET Phase:** Questions 1 (differential calculus), and 6.1 (probability) (cf. Graphs 4.2, 4.7). **Senior Phase:** Questions 1.2.2 (patterns, functions, and algebra), Questions 3.2 (space and shape) (cf. Graphs 4.9, 4.11). **Intermediate Phase:** Questions 4.1, 4.2, space and shape (geometry) (cf. Graph 4.17).

Furthermore, the deficiencies noted concerning the effective application of cognitive process dimensions, such as to analyse, apply, create and evaluate could imply that the application of thinking processes is not well developed. This observation supports the literature where it is indicated that the more developed thinking processes are, the better the students will be able to make sense of knowledge (Brownlee & Berthelsen, 2008) (cf. 2.2.2).

The information sheet handed to participants to aid the recall of knowledge they might have forgotten, was not used by all the participants. The researcher wishes to mention that only a few participants used the information sheet to assist them in answering the questions:

- Numbers, operations, and relationships: used by five participants.
- Patterns, functions, and algebra: used by two participants.
- Measurement: used by three participants.
- Space and shape: used by one participant.
- Data handling: used by one participant.

Section 4.4 pays attention to the analysis and interpretation of the qualitative data obtained from the student participants.

4.4 QUALITATIVE DATA ANALYSIS AND INTERPRETATION: STUDENT PARTICIPANTS

The tests completed by the student participants comprised several open questions concerning metacognition. The data obtained for each of these aspects are analysed and interpreted in the following sections. Some of the most important examples to illustrate a possible lack or presence of metacognitive skills are presented as part of the discussion in Addendum J.

4.4.1 Student perceptions: the metacognitive knowledge dimension

The extent to which the research participants employed metacognitive strategies to enhance the quality of their work, was established using their perceptions to open questions that were included in the various mathematics tests. Participants were requested to reflect (in writing) on the following:

- how they plan before they start working on a mathematics task;
- how they monitor themselves while working on a mathematics task to make sure they are on the right track; and
- how they evaluate the outcome of their mathematics task after completion.

In the context of the study, the focus was on the metacognitive skills that the participants have available to self-regulate their actions and behaviour whilst working on a mathematics task, which could also include answering test questions. The researcher wanted to establish if the participants are aware of their thinking processes, to plan their work, eliminate careless mistakes, or to select more effective procedures to solve problems, and to evaluate the quality of their work (Al-Kayyali & Al-Haq, 2018; Flavell; 1979; Schneider & Artelt; 2010) (cf. 2.3.2.4).

Tables 4.41 – 4.43 (cf. Addendum J, p. 185-187) display the written information obtained from the participants about the metacognitive skills they perceive to use during the completion of mathematics tasks. Colour coding is used to identify similarities among the perceptions of the participants, which guided the interpretation of the data.

The researcher concludes the following about the metacognitive skills of the participants for planning, monitoring and evaluating work.

4.4.1.1 Planning

According to the participants, the most frequently used strategies to guide planning involved the following.

- FET Phase – Consulting the CAPS document (participants 2, 3, 4, and 5), and establishing prior knowledge (participants 3 and 5). The participants seemingly understood the question concerning the planning incorrectly as the work they have to do as a teacher before presenting a lesson to learners, and not concerning their work as a student when completing a mathematics task, although the question was clearly phrased. Only participants 1, 6, and 7 provided relevant answers which referred to reading work with understanding before starting (participant 1), considering the time in relation to the amount of work to be completed (participant 6), and being aware of the requirements of a task as part of a checklist (participant 7) (Bannert & Reimann, 2012; Cazan, 2012; Zumbrunn et al., 2011) (cf. 2.3.2.4).
- Senior Phase – Similar to the FET phase, many participants interpreted the question to be about their planning as a teacher and cited consulting the CAPS document for content to cover (participants 3, 5, and 7), determining learners' strengths and weaknesses and prior knowledge, and consulting textbooks (participants 2, 5, 6, and 7) as part of planning. Three participants mentioned relevant strategies for planning the completion of their own work, namely the understanding of task expectations and requirements and the importance of gaining knowledge to complete a task as important aspects for planning (participants 1 and 4).
- Intermediate Phase – Only two of the participants incorrectly focused on their planning task as a teacher and cited preparing for and completing lesson plans, as well as consulting the CAPS document as part of planning (participants 8, 12). As part of personal planning to complete tasks the participants mentioned the following relevant planning strategies, namely to consider prior knowledge (participant 1), to consider the relevancy and appropriateness of work (participant 4), making sure about what is expected (participant 9), to identify areas or concepts that are difficult (participant 11), and to secure resources and the best study material (participant 6, 10, and 7).

What the researcher finds troublesome, is that the participants seem to lack knowledge about important strategies to plan their work, such as setting clear goals (Bannert & Reimann, 2012) (cf. 2.3.2.4), as setting goals was not mentioned by any of the participants. However, it is encouraging that FET participants acknowledged obstacles that may hamper success (Zumbrunn et al., 2011) (cf. 2.3.2.4) as part of planning. In this regard, participant referred to reading work with understanding before starting (participant 1), considering the time

concerning the amount of work to be completed (participant 6) (Cazan, 2012) (cf. 2.3.2.4), and being aware of the requirements of a task as part of a checklist (participant 7) (Hattie & Donoghue, 2016) (cf. 2.3.2.4). Similarly, Senior Phase participants mentioned the understanding of task expectations and requirements, as well as knowledge to complete a task as important for planning (participants, 1, and 4). Also, Intermediate Phase participants mentioned considering prior knowledge (participant 1), considering the relevancy and appropriateness of work (participant 4), making sure about what is expected (participant 9), identifying areas or concepts that are difficult (participant 11), and securing resources and the best study material (participant 6, 10, and 7), as important for planning.

Planning requires different approaches to solve a problem if a student engages in the planning phase of metacognition (cf. 2.3.4.2). It is said that problem-solving is the base of mathematical content and therefore plays a big role in metacognition (and vice-versa) (Flavell, 1985; Schoenfeld, 2008; Umaru, 2010) (cf. 2.3.2.4). Problem-solving can however only be developed if the learners already have solid conceptual understanding, and if they know how to transition the concepts to procedures (Du Plooy & Long, 2014) (cf. 2.3.2.4). The absence of also applying proper planning strategies during the completion of a test, could have hampered the success of the participants.

The researcher acknowledges that good metacognitive planning skills alone might not always guarantee success, as a solid conceptual understanding, and knowing how to transition the concepts to procedures are also required to achieve success in mathematics. (Du Plooy & Long, 2014) (cf. 2.3.2.4). It is a concern to the researcher that many of the participants might not know how to plan their work to ensure successful completion of their work which could obstruct their chances to be successful (Bannert & Reimann, 2012) (cf. 2.3.2.4), as was evident in the tests.

4.4.1.2 Monitoring

During monitoring, one makes sure that one is making progress towards achieving goals, and to make changes to one's way of work if progress is not made (Schunk, 2005) (cf. 2.3.2.4). In the context of the research, many of the participants seemed to lack the skill to monitor their work towards making successful progress.

According to the participants, the most frequently used strategies to guide monitoring involved the following.

- FET Phase – Important monitoring strategies such as the following were mentioned: read the questions (participants 1 and 6), keeping track (participant 3), not losing focus

(participant 7), work with a checklist containing requirements to meet (participant 7) (Cazan, 2012) (cf. 2.3.2.4), reflecting on work (participant 2), and comparing work against textbooks (participant 4), the CAPS document (participants 3 and 4), and stated outcomes (participant 5).

- Senior Phase – Some of the participants cited important strategies for monitoring work: checking for understanding by reading questions again, checking work against guidelines and plans (participants 1, 4, 5, and 7), reflecting on objectives (participant 6), and reviewing work (participant 2). Participant 3 possibly understood the question concerning monitoring one's work as a teacher, and therefore cited consulting the CAPS and compiling notes to explain work as a strategy which did not relate to what the researcher expected the participants to answer.
- Intermediate Phase – Meaningful monitoring strategies were cited: checking answers for correctness (participants 1, 6, and 10), working backwards over own work (participant 11), having others check one's work (participant 11), reflection by asking oneself questions about work to establish if one is answering what is supposed to be answered, and reflecting about past errors to ensure that they are not made in future (participants 4, 7) (Paris & Paris, 2010; Zimmerman & Schunk, 2011) (cf. 2.3.2.4). Participants 2 and 3 also check themselves as teachers against the set lesson plan, application of real-life contexts, the CAPS (Participant 1) and the marking of their learner's work (participant 12), which were not relevant answers to the question posed.

The researcher believes that many unnecessary calculation errors could have been avoided if the participants were more effective at monitoring their work. Incomplete work, calculation errors, and unanswered questions could be tied to a lack of metacognitive skills to monitor work. If students are aware of their thinking processes, careless, common mistakes could be avoided (Flavell; 1979; Schneider & Artelt; 2010) (cf. 2.3.2.4). However, as with the metacognitive skill of planning, errors could also be due to a lack of factual and conceptual knowledge (Lam, 2017) (cf. 2.3.4.2; 2.4.2). It was difficult to establish the successfulness of monitoring, as this was not purposively assessed, and having answers correct could not conclusively be cited as evidence of effective monitoring, as participants might have also guessed the correct answers.

4.4.1.3 Evaluating

Evaluating involves assessing how successful one was in attaining goals (Paris & Paris, 2010; Zimmerman & Schunk, 2011; Yeager et al., 2014) (cf. 2.3.2.4).

According to the participants, the most frequently used strategies to guide evaluating involved the following.

- FET Phase – Many participants mentioned relevant strategies, such as reflecting on task/activity outcomes in comparison to the objectives that had to be achieved (participant 2), doing research and involving peers in evaluation (participant 6), making use of a memorandum (participant 7), and checking for relevancy between the outcomes and the question that had to be answered (participant 1). Again, some of the participants incorrectly interpreted the question in relation to evaluating their teaching activities and mentioned that the CAPS document will be consulted and that learners' understanding of work could be evaluated with homework and tests.
- Senior Phase – Three participants mentioned relevant strategies for evaluating work, such as employing resources to check work (participant 1), rechecking work for logic (participant 4), and checking work against evaluation forms (Participant 7) (supposedly evaluation criteria). Many of the participants incorrectly connected the question to their teaching practice and mentioned the following: using questioning to test learners' understanding, and to compare learners against the class average (participant 2), providing well-constructed notes to learners (participant 3), checking teaching plans against the CAPS (participant 5), and comparing learners with the standards specified in the CAPS (participant 6).
- Intermediate Phase – Applicable strategies to evaluate work was cited by the Intermediate Phase participants. The participants mentioned the following strategies: using rubrics to self-check work (participants 4, 9), checking work with a calculator or asking a friend to check work (participant 10), reading work (Hartwig & Dunlosky, 2012) (cf. 2.3.2.4) and deciding if a simpler method to complete the work should not be used (participant 11), redoing work to check if the same answers are achieved the second time around (participant 1), comparing work to the work of friends (participant 5), comparing work to a memorandum (participant 6), and using a tick list to verify what needs to be achieved (participant 7) (Cazan, 2012) (cf. 2.3.2.4). Participants 2, 3, 8, and 12 incorrectly understood evaluating work in the context of teaching and therefore cited giving learners informal tests and activities, asking questions, and reflecting on work with learners as evaluation strategies.

The researcher finds it troublesome that many participants appear not to possess good evaluation skills which are important to determine the successfulness of achieving goals and to modify strategies to be more successful in future tasks (Paris & Paris, 2010; Zimmerman & Schunk, 2011, Yeager et al., 2014) (cf. 2.3.4). Successful evaluation of work was seen as

looking at answers in a logical way to determine if they are correct. The fact that many participants left questions unanswered or left out steps that had to form part of an answer could suggest that the participants possibly also possess poor evaluation skills. Although a lack of evaluation skills could have contributed to not checking answers for correctness, the researcher is aware that poor factual and conceptual knowledge can also influence the success of task completion and the inability to self-regulate (Crooks & Alibali, 2014) (cf. 2.3.2.4).

In summary, many of the participants seemed to have very limited depth and breadth of content knowledge in their respective teaching phases, which could have contributed to the lack of metacognitive actions during the completion of the test. In particular, the important role of conceptual knowledge for mathematical reasoning (Crooks & Alibali, 2014) (cf. 2.3.2.4), and possessing depth and breadth of content knowledge, should not be underestimated for the application of metacognitive skills in mathematics (Al-Kayyali & Al-Haq, 2018; Flavell, 1971) (cf. 2.3.2.4).

Solid conceptual knowledge will aid learners when evaluating the reliability of their answers or procedures, which again brings to light the important role of conceptual knowledge for mathematical reasoning (Crooks & Alibali, 2014) (cf. 2.3.2.4). Metacognition is the key when it comes to checking a final answer (Schneider & Artelt; 2010; Verschaffel; 1999) (cf. 2.3.2.4), but conceptual knowledge is needed to know what to look for or how to determine the reliability of an answer (Brownell in Crooks & Alibali, 2014) (cf. 2.3.2.4).

Many of the participants across the three phases seemed to have an understanding of what planning, monitoring, and evaluating as part of self-regulatory action entails, and could suggest some valid strategies that would enhance the planning, monitoring, and evaluation of their work. However, it seems that these strategies are not applied automatically to their work, given the many unnecessary mistakes that were made that influenced the success of the participants' achievement in the tests. The researcher, therefore, argues that the participants seemingly do not purposively think about their thinking when involved in mathematics tasks or activities (Schneider & Artelt, 2010) (cf. 2.3.2.4).

It also appears to the researcher, that the participants seemingly do not apply a purposeful questioning nature to the execution of their work, which aids self-regulatory action about the quality of work (Ertmer & Newby, 1996) (cf. 2.3.2.4). It could be that lecturers do not create teaching environments where questioning is encouraged that would promote the application of metacognitive strategies for self-regulatory action (Department of Basic Education, 2018) (cf. 2.3.2.4).

The following section explores the participants' reflections about their preparedness to teach and the availability of teaching strategies to teach subject content.

4.4.2 Student perceptions: preparedness to teach and availability of teaching strategies

As part of the tests that the participants wrote, they were requested to reflect about their preparedness to teach the different subject content areas, and the strategies they have available to teach the different subject content areas, on a five-point semantic differential scale (1 = well prepared; 5 = not well prepared). It was important for the researcher that the participants would indicate that they regard themselves as well-prepared (scale 1) to teach the various content areas. However, the researcher considered the responses on scales 1 and 2 to make final interpretations about preparedness.

4.4.2.1 Preparedness to teach

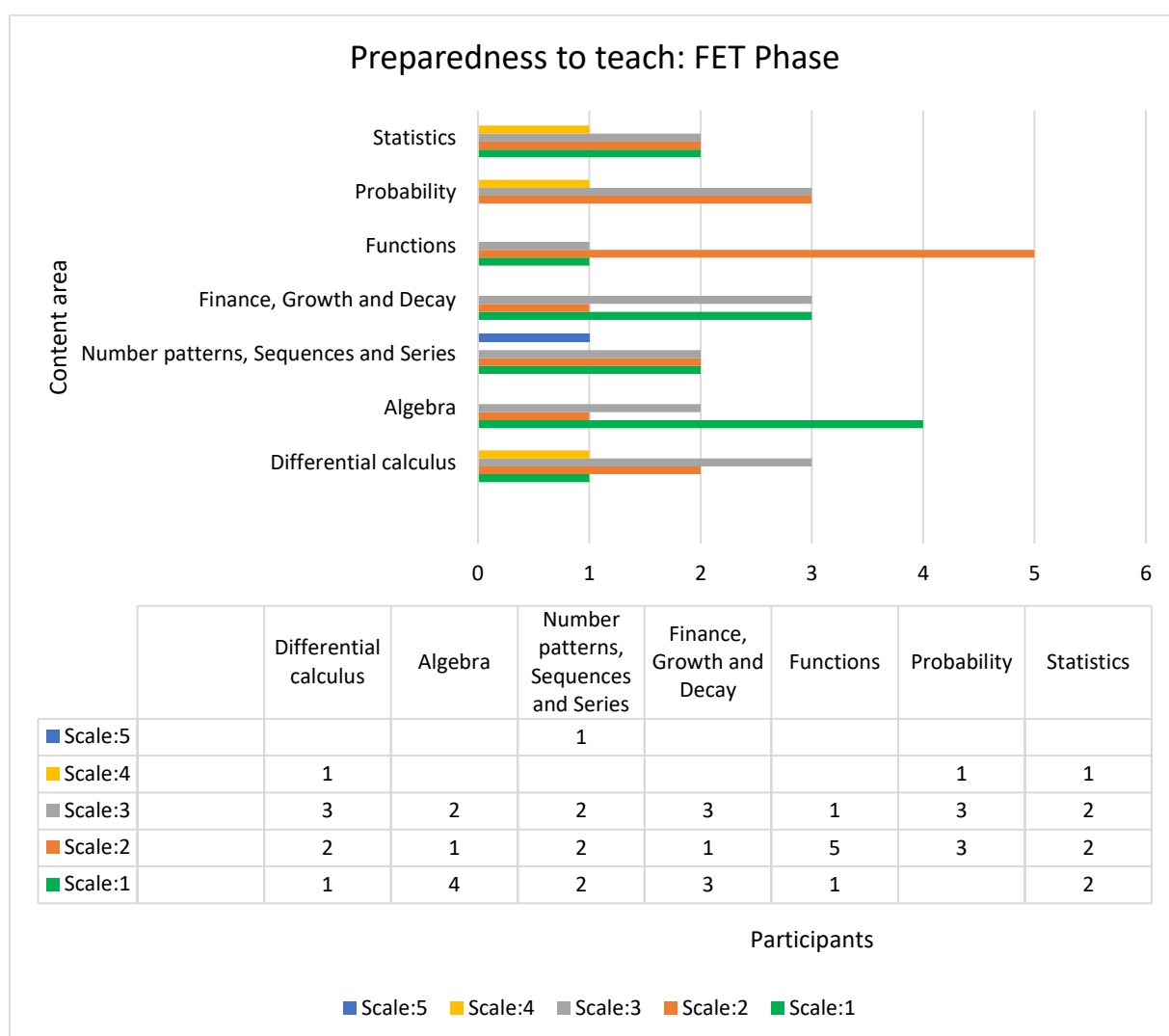
Table 4.44 summarises the frequency data concerning the FET, Senior and Intermediate Phase participants' perceptions about their preparedness to teach the various content areas on a five-point scale.

Table 4.44: Participant perceptions: preparedness to teach content areas

FET Phase	<i>f</i> 1	<i>f</i> 2	<i>f</i> 3	<i>f</i> 4	<i>f</i> 5	Total
Differential calculus	1	2	3	1		7
Algebra and equations	4	1	2			7
Number patterns, sequences, and series	2	2	2		1	7
Finance, growth, and decay	3	1	3			7
Functions	1	5	1			7
Probability		3	3	1		7
Statistics	2	2	2	1		7
Senior Phase	1	2	3	4	5	
Numbers, operations, and relationships	4	3				7
Patterns, functions, and algebra	5		2			7
Space and shape		5	1	1		7
Measurement	4		2	1		7
Data handling	3	2	1	1		7
Intermediate Phase	1	2	3	4	5	
Numbers, operations, and relationships	6	3	2		1	12
Patterns, functions, and algebra	3	4	2	3		12
Space and Shape	3	5	3		1	12
Measurement	2	9		1		12
Data handling	6	4	1	1		12

Scale interpretation: 1 = well prepared; 5 = not well prepared

The information in Table 4.44 is visually presented in Graphs 4.19 – 4.21.



Graph 4.19: Preparedness to teach – FET Phase

It was important for the researcher that all participants would have perceived themselves as well-prepared (scale 1) concerning all the content areas. According to Graph 4.19, and considering all the scale 1 and 2 responses, the content areas for which most of the participants seem to be the best prepared are **functions** (six participants), **algebra and equations** (five participants), followed by **finance, growth, and decay** (four participants), **numbers, patterns, and sequences** (4 participants), and **statistics** (four participants). The lowest responses on scales 1 and 2 were received for **calculus** and **probability** with 3 participants each. Although only one participant indicated to be well-prepared (scale 1) to teach **functions**, five participants perceived their preparedness **at scale 2**, and no participant perceived their **preparedness lower than** level 3, thus on scale 4 or 5. Only for one content area, number patterns, sequences, and series, one participant assessed preparedness to teach the subject content at the lowest scale, scale 5 (not well-prepared).

The majority of the participants appeared to be the least prepared for teaching **probability**, followed by **differential calculus**. With regard to probability, none of the participants perceived themselves to be at scale 1 (well-prepared). It is worth mentioning that none of the participants perceived themselves not to be well-prepared (scale 5) to teach any of the content areas, except for numbers, sequences, and series.

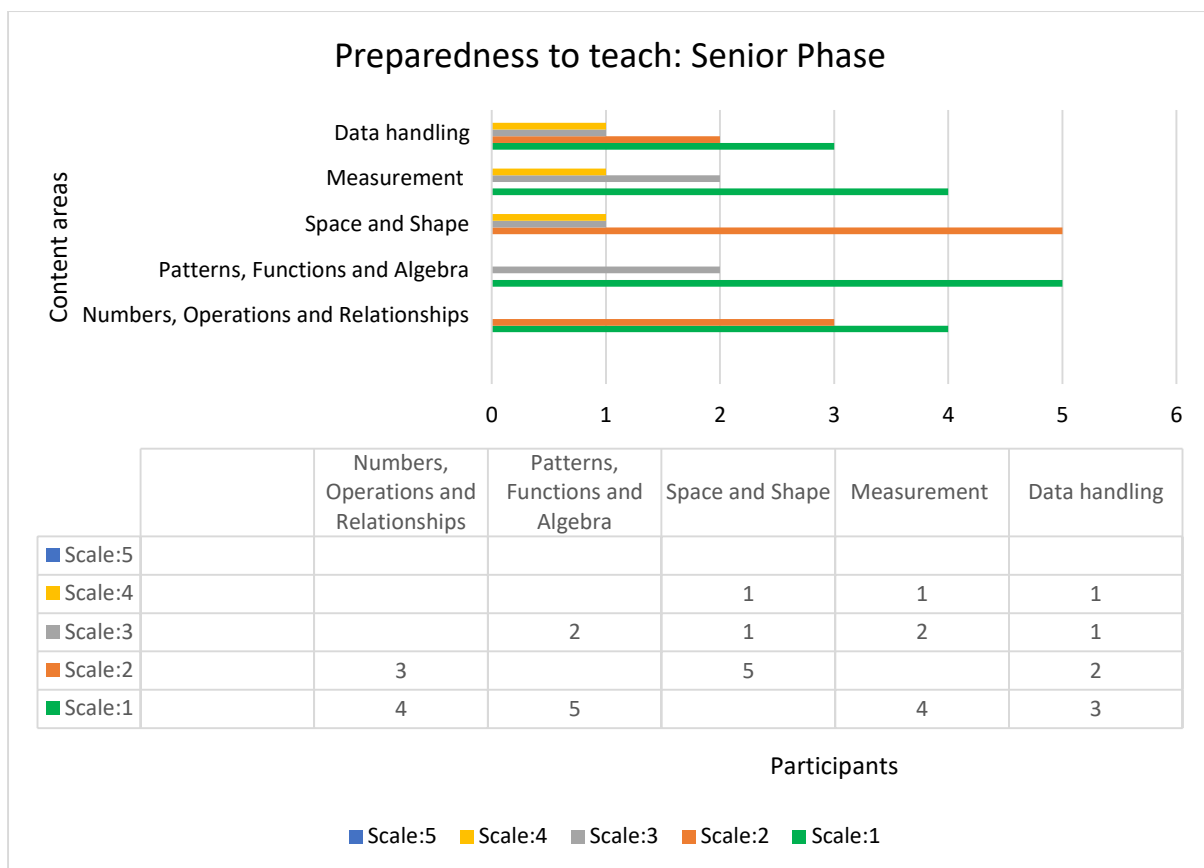
The test results Table 4.2 display that the lowest scores were achieved in Question 1 and 6 (0%); differential calculus and probability. The results obtained for statistics were also low, namely 26,19%. The participants' perceptions about their preparedness to teach differential calculus, probability, and statistics, thus corroborates the test results, although the scale 2 responses observed for differential calculus, probability and statistics might be unrealistic given the poor test averages.

The test results for algebra and equations (33,92%), and number patterns, sequences, and series (28,57%) (cf. Table 4.2) do not support the perceptions of being well-prepared according to Graph 4.19. The results obtained for finance, growth, and decay (20,63%) and functions (59,52%) (cf. Table 4.2), were not exceptional, and therefore do not align with the participants' perceptions about their preparedness concerning these content areas as being good.

The researcher finds it disturbing that the perceptions of the participants about being well-prepared to teach certain subject content appear to be unrealistic when compared to the actual results obtained in the test. Also, the results obtained for the FET test combined with the perceptions about preparedness to teach the content areas, are disconcerting, considering that the participants are final-year pre-service teachers who are about to enter the profession with limited content knowledge.

A number of problems linked to the teacher-training curriculum (cf. 2.3.5) that could have influenced the findings are reported in section 5.4.2 (cf. Finding 9).

The following section pays attention to the preparedness of Senior Phase participants to teach the Senior Phase content areas. Graph 4.20 displays the results obtained for the perceptions of the Senior Phase participants (cf. Table 4.44).



Graph 4.20: Preparedness to teach – Senior Phase

According to Graph 4.20, and according to the perceptions classified as scales 1 and 2 (**well-prepared**), **numbers, operations, and relationships** followed by **patterns, functions, and algebra** seem to be the subject content areas for which the participants feel the best prepared. For numbers, operations, and relationships, the seven participants indicated their level of preparedness on scale 1 (four participants) and on scale 2 (three participants). For patterns, functions, and algebra, preparedness was perceived on scale 1 (five participants) and scale 3 (2 participants) respectively. This is followed by **data handling** (three participants on scale 1 and 2 participants on scale 1), **measurement** (four participants on scale 1), and **space and shape** (five participants on scale 2). Although **space and shape (geometry)** had many participants at scale 2 (five participants), not one of the participants seemed to regard themselves as completely well-prepared (scale 1) to teach space and shape. None of the participants perceived themselves not to be well-prepared (scale 5) to teach any of the subject content areas, which is noteworthy.

According to Table 4.3, Question 3 – space and shape (geometry) only had a question average of 61,03% which could correspond slightly with the data in Graph 4.20 as 5 participants indicated their preparedness to be at scale 2. None of the participants perceived

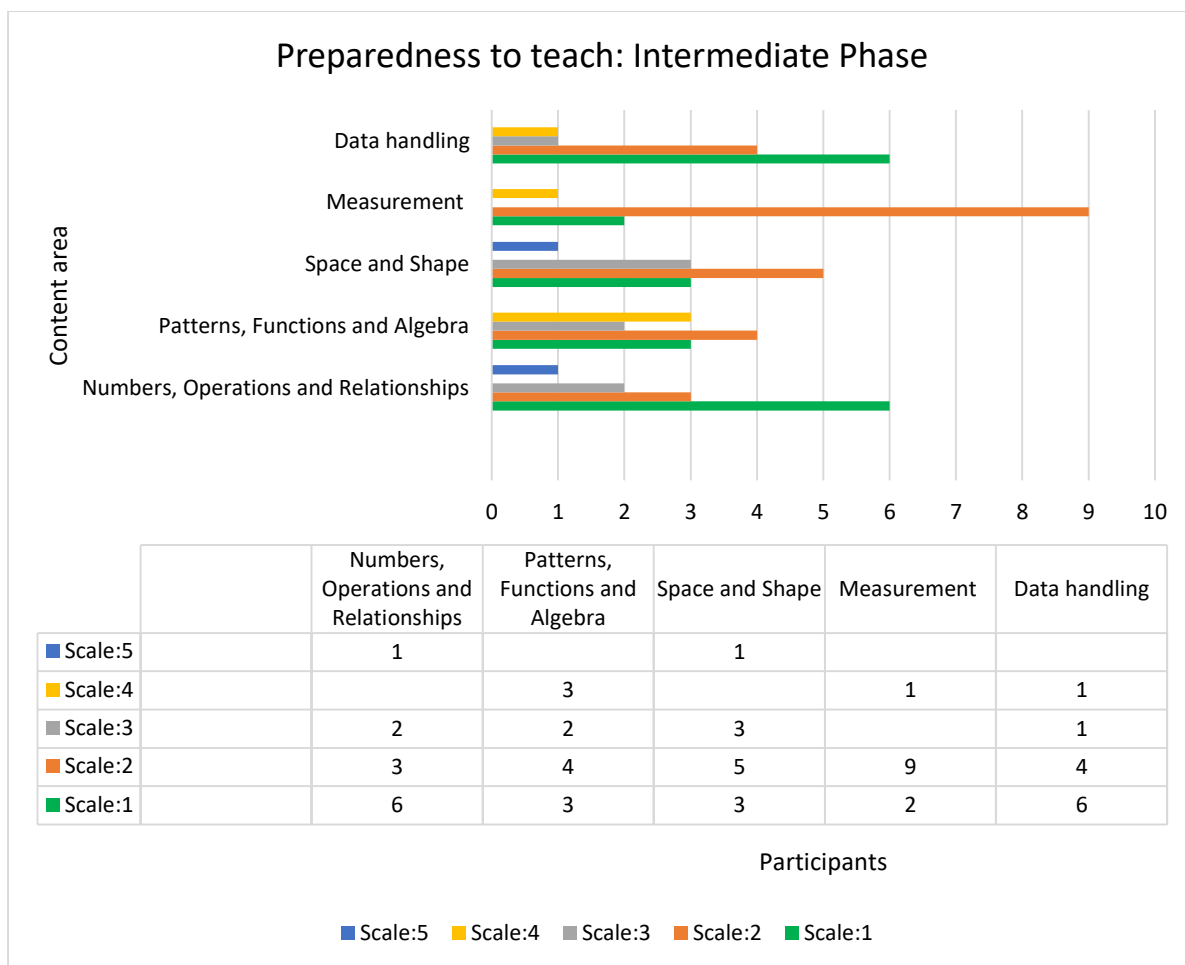
themselves as being well-prepared (scale 1) to teach space and shape, which indicates that gaps in subject content knowledge possibly exist. The highest test averages were obtained for patterns, functions, and algebra (85,71%), and numbers, operations, and relationships (96,10%) respectively. Both these averages are corroborated by the perceptions of the participants who feel that they are best prepared for these two subject content areas. The lowest test average was obtained for data handling (58,92%) (cf. Table 4.3), which does not entirely align with the perceptions presented in Graph 4.20 where three participants seemingly perceive themselves to be well-prepared and two participants perceiving their preparedness at scale 2.

The overall test average for the Senior Phase test was 76,49% (cf. Table 4.3), which is much higher compared to the 26,31% (cf. Table 4.2) for the FET Phase test. Taking into account that the same participants wrote both tests, one can conclude that the participants' content knowledge concerning the Senior Phase appears to be more solid than their subject content knowledge for the FET Phase.

The Senior Phase participants do not follow a specific Senior Phase curriculum during their training but attend lectures with the FET participants (cf. Table 2.8). However, according to the test results (cf. Table 4.3), this arrangement apparently did not pose a severe limitation concerning the quality of the subject content knowledge for the participants who will specifically teach in the Senior Phase, although test averages for space and shape (geometry), measurement and data handling could improve.

The content area space and shape in the Senior Phase is not adequately covered in the module Euclidean and spherical geometry presented at FET level, as the module does not focus on topics dealt with in space and shape (cf. 2.3.5). Moderate averages in the Senior Phase test were obtained for Question 3 (space and shape (geometry) (61,03%)), and Question 4 (measurement) (64,28%), which could probably be linked to the limited focus on these subject content areas during the FET training. Also, not one of the participants perceived themselves to be well-prepared (scale 1) to teach space and shape, thus confirming the test results.

The following section elaborates on the perceptions of the Intermediate Phase participants (cf. Table 4.44, Addendum J) concerning their preparedness to teach the Intermediate Phase subject content areas. Graph 4.21 displays the data obtained from the Intermediate Phase participants.



Graph 4.21: Preparedness to teach – Intermediate Phase

Based on the data presented in Graph 4.21, and considering scale 1 and 2 perceptions, **measurement** seems to be the content area for which the participants feel the best prepared (11 participants), followed by **data handling** (ten participants), then **numbers, operations, and relationships** (nine participants), **space and shape (geometry)** (eight participants), and then **patterns, functions, and algebra** (seven participants). Space and shape (geometry) as well as numbers, operations, and relationships each had one participant who perceived not to be prepared at all to teach the content area (scale 5). In all instances, more than half of the participants perceived their preparedness to teach the subject content areas at scale 1 and 2.

According to the data in Table 4.5, numbers, operations, and relationships only had an average result (49,58%), which does not align well with the participants' perceptions of being well-prepared to teach the content. However, the participant's perceptions about being well-prepared to teach data handling are supported by the average test result of 90,27%, which was the highest for the Intermediate Phase test. The participants' perceptions concerning their preparedness to teach patterns, functions, and algebra, also support their test results for which

a good average of 75% (the second-highest average for the Intermediate Phase test) was achieved. The lowest test average was obtained for space and shape (geometry) (38,33%). This low average does not concur with the perceptions of the participants concerning their preparedness to teach the content. The majority of the participants (9), perceived their preparedness to teach measurement at scale 2. However, the test result in measurement only revealed an average of 52,77%. The participants' perceptions about their preparedness thus do not correlate well with their performance in measurement. Except for data handling, and patterns, functions, and algebra, the researcher concludes that the participants do not have a realistic perception of their preparedness levels to teach the various subject content areas.

The average obtained for the Intermediate Phase test was only 57,90%, which possibly indicates that the participants are not entirely well-prepared to teach all the subject content areas, in particular numbers, operations, and relationships (49,48%), space and shape (geometry) (38,33%), and measurement (52,77%). A limitation that the teacher-training curriculum could have posed concerning the preparedness and achievement of the Intermediate Phase participants is that they only start their subject content training in the second year of study (cf. 2.3.5). Starting subject content training from the first year of study might contribute to developing more solid subject content knowledge.

In overview, the following main trends emerged from the data obtained concerning the participants' preparedness to teach the various subject content areas.

- FET Phase – In ordering the preparedness of the participants to teach the subject content areas, the majority of the participants perceived themselves to be best prepared to teach functions, algebra and equations, finance, growth and decay, number patterns, sequences, and series as well as statistics.
- Senior Phase – In ordering the preparedness of the participants to teach the subject content areas, the majority of the participants perceived themselves to best prepared to teach numbers, operations, and relationships, and patterns functions and algebra.
- Intermediate Phase – In ordering the preparedness of the participants to teach the subject content areas, the majority perceived themselves to be best prepared to teach measurement, data handling, and numbers, operations, and relationships.

Based on the data, the researcher believes that it is reasonable to conclude that the pre-service teachers lack solid subject content knowledge in a number of the subject content areas (Avalos et al., 2010; Blignaut et al., 2013; Malaty, 2004; Mij & Makgato 2006; Sorto, 2010). The findings in relation to the FET participants specifically support the research of Adler (2005), Blignaut et al. (2013), Kanyongo and Brown (2013), and Plotz et al. (2012) who add

that teachers specifically lack content knowledge in the field of statistics. Also, the research of Tsanwani and Sinthumule (2015) highlighted problems with teachers' understanding of the subject content in differential calculus, which is confirmed by this study (cf. 1.1).

The findings support the concern of Blignaut et al. (2013) and Siyepu (2013), that pre-service teachers do not perform well, but still graduate and become teachers, and contribute to the problem of poor mathematics performance at the school-level (cf. 1.1). It could be argued that the lack of solid content knowledge among the participants could result in the participants being ineffective teachers. In this regard Ball et al. (2008), and Tsanwani and Sinthumule (2015), argue that there is a relationship between a teacher's mathematical knowledge and their ability to be effective teachers (cf. 2.3.4.1).

In the next section, the researcher explores the perceptions of the participants concerning the availability of teaching strategies that they have to teach the various subject content areas.

4.4.2.2 Availability of teaching strategies to teach the various subject content areas

Table 4.45 below summarises the frequency data obtained for the participants' perceptions about the availability of teaching strategies to teach the various subject content areas on a five-point semantic scale (1 = a wide variety of strategies; 5 = limited strategies).

Table 4.45: Perceptions: availability of teaching strategies to teach subject content areas

FET Phase	<i>f</i> 1	<i>f</i> 2	<i>f</i> 3	<i>f</i> 4	<i>f</i> 5	Total	Missing
Differential calculus	1	1	2		2	6	1
Algebra and equations	2	2	2			6	1
Number patterns, sequences, and series	1	2	2		1	6	1
Finance, growth, and decay	1	1	3	1		6	1
Functions	1	2	3			6	1
Probability		2	4			6	1
Statistics		3	2		1	6	1
Senior Phase	Scale:1	Scale:2	Scale:3	Scale:4	Scale:5		
Numbers, operations, and relationships	1	3	1	1		6	1
Patterns, functions, and algebra	2	1	2	1		6	1
Space and shape		2	3	1		6	1
Measurement	1	2	2	1		6	1
Data handling		2	2	1		5	2

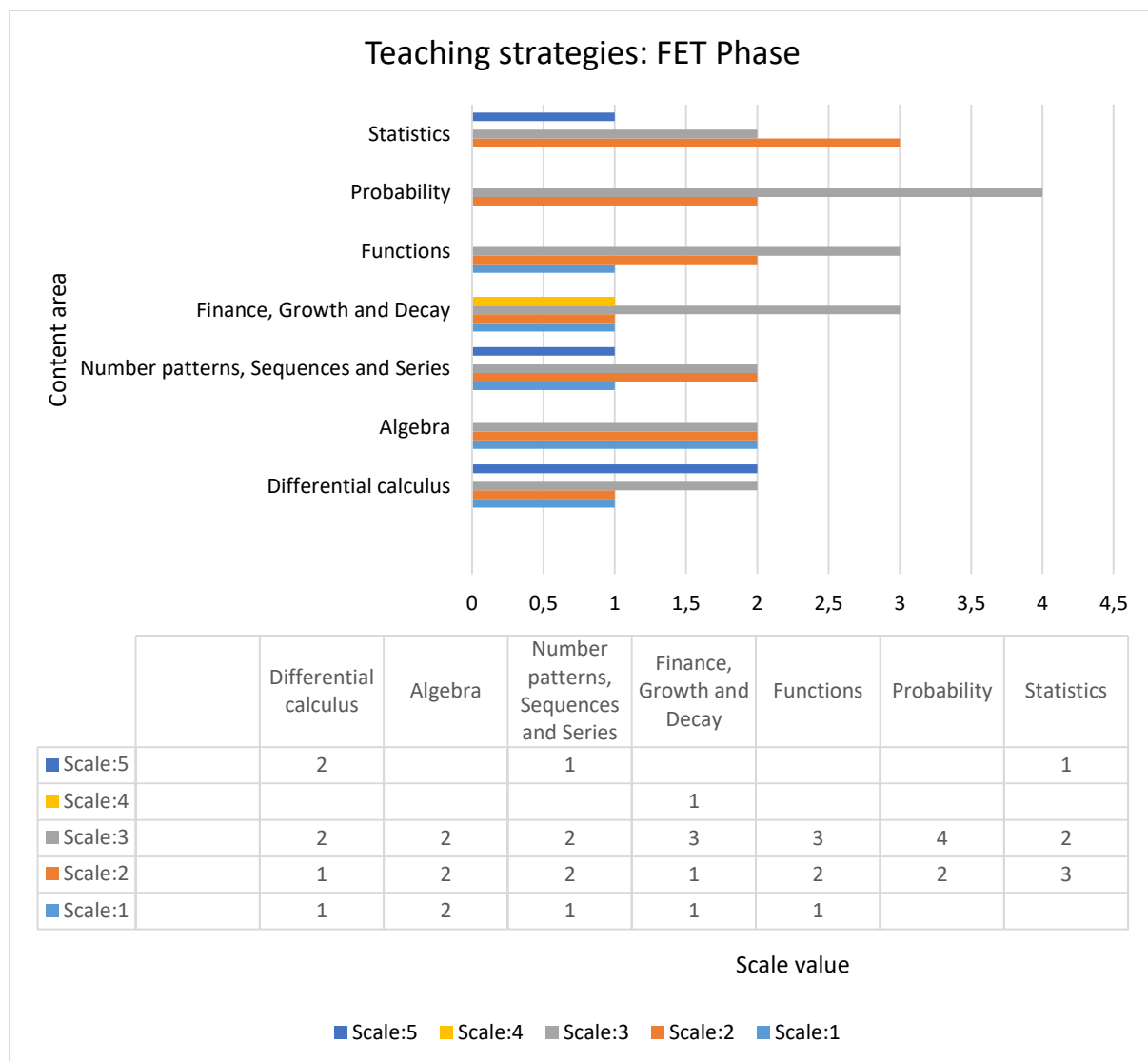
Intermediate Phase	Scale:1	Scale:2	Scale:3	Scale:4	Scale:5		
Numbers, operations, and Relationships	2	6	3	1		12	
Patterns, functions, and algebra	1	4	4	1	2	12	
Space and shape	2	4	4		2	12	
Measurement	4	4	3		1	12	
Data handling	3	3	5		1	12	

Scale interpretation: 1 = a wide variety of strategies; 5 = limited strategies

The data in the column with the heading “missing” refers to the number of participants who did not respond to a question.

Graphs 4.22 – 4.24 visually display the data presented in Table 4.45 above. Ideally, participants had to indicate that they possess a wide variety of strategies (scale 1) to teach different subject content areas. However, the researcher considered the scale 1 and 2 responses of the participants in making conclusions about the data.

Graph 4.22 displays the data for perceptions about the availability of teaching strategies (cf. Table 4.45) for the FET Phase.



Graph 4.22: Availability of teaching strategies – FET Phase

Graph 4.22 reveals a gloomy picture, as very few participants appear to possess a variety of strategies (scale 1) to teach the different FET subject content areas.

The subject content areas for which very limited teaching strategies are available appear to be **differential calculus** and **finance, growth and decay** if the lower scales of 4 and 5 are also used as points of reference. Two participants indicated on scale 5, that they possess limited teaching strategies for differential calculus, whereas for finance, growth and decay three participants indicated scale 3 and one indicated scale 4. The participants also appear to have limited strategies to teach the subject content **probability** and **statistics** if scale 1 is used as a point of reference. None of the participants perceived that they possess a variety of strategies (scale 1) to teach probability and statistics. The majority of the participants perceived the strategies they possess to teach probability on scale 3 (four participants), and

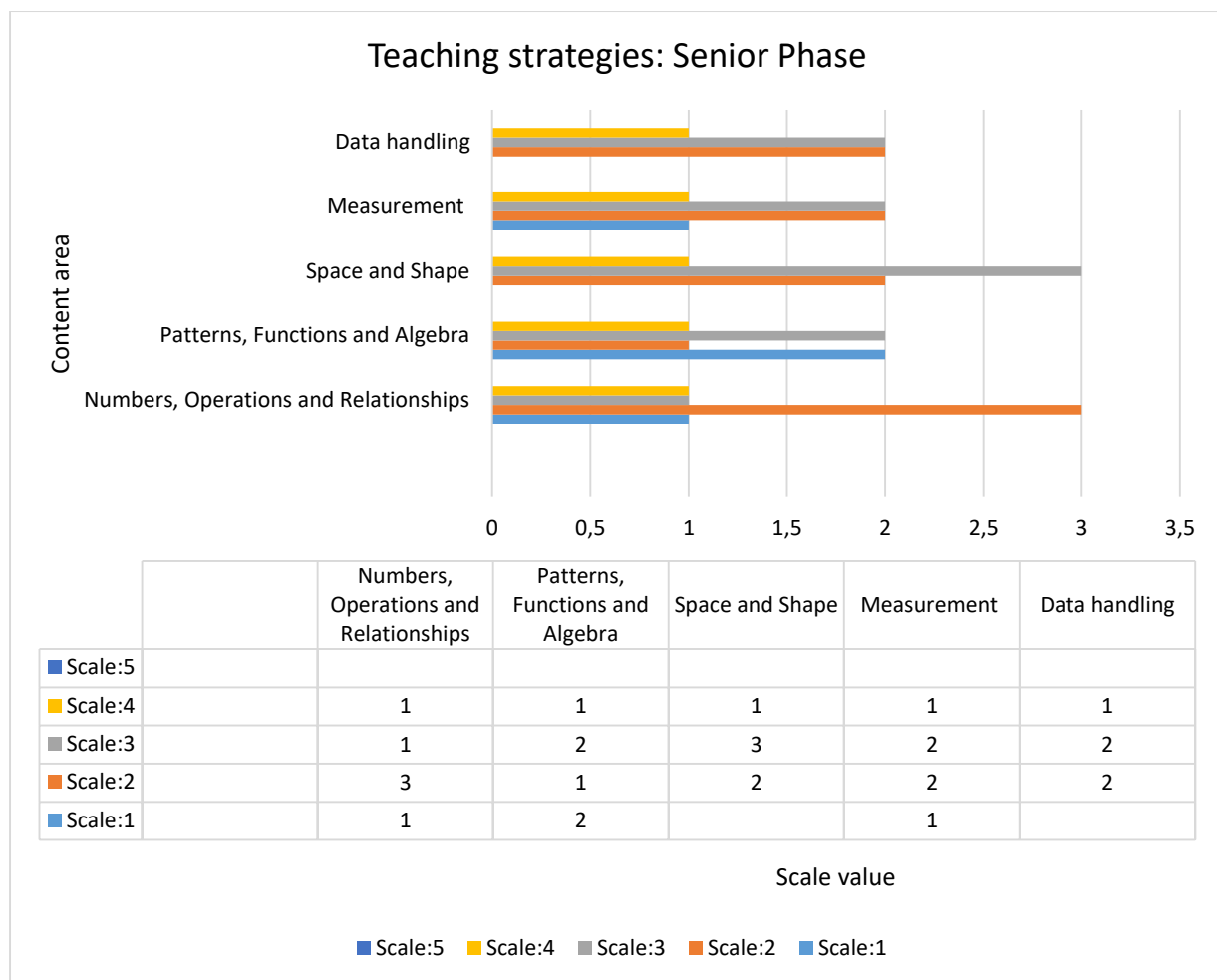
for statistics, one participant seems to have very limited strategies (scale 5) to teach statistics. Probability and statistics were the two subject content areas for which the participants also obtained the lowest results in the test, with 0% and 26,19% average respectively. It also seems that the participants' perceptions about their poor preparedness to teach probability and statistics (cf. Graph 4.19), align well with their perceptions about having limited strategies to teach the two subject content areas.

Participants indicated that they possess the most strategies for teaching **algebra and equations** (four participants at scales 1 and 2), **functions** and **number patterns, sequences and series** (both with three participants at scales 1 and 2), statistics (and finance, growth, and decay (four participants at scales 1 and 2). Number patterns, sequences, and series also had one participant on scale 5 (limited strategies). The participants obtained the highest average score in functions (59,52%), which seems to be supported by their perceptions about being prepared to teach the content and that they have strategies available to teach the content, although the average is not very high. This perception according to the researcher would however not be realistic, as a 59,52% average cannot be considered as being well-prepared.

Differential calculus and **finance, growth, and decay** could also be viewed as subject content areas for which participants appear to have limited strategies to teach, with one participant each on scales 1 and 2 (a wide variety of strategies). Also, differential calculus had two participants on scale 5 (limited strategies), and finance, growth, and decay had one participant on scale 4. For these two subject content areas, the participants also obtained poor average results; 0% for differential calculus and 20,63% for finance, growth, and decay. For differential calculus, the participants also indicated that they perceive themselves as not being well-prepared to teach the content (cf. Graph 4.19). This observation therefore supports their perception of not having sufficient strategies to teach the subject content area differential calculus.

It appears as if there could be a link between the participants' performance in the FET test (cf. Table 4.2), their perceptions about their preparedness to teach (cf. Graph 4.19), and the availability of strategies they have to teach certain subject content areas (cf. Graph 4.22), in particular, probability, statistics, differential calculus, and functions.

The next section explores the perceptions of the Senior Phase participants concerning the teaching strategies to teach the Senior Phase subject content areas. Graph 4.23 displays the data obtained (cf. Table 4.45) for the Senior Phase participants.



Graph 4.23: Availability of teaching strategies – Senior Phase

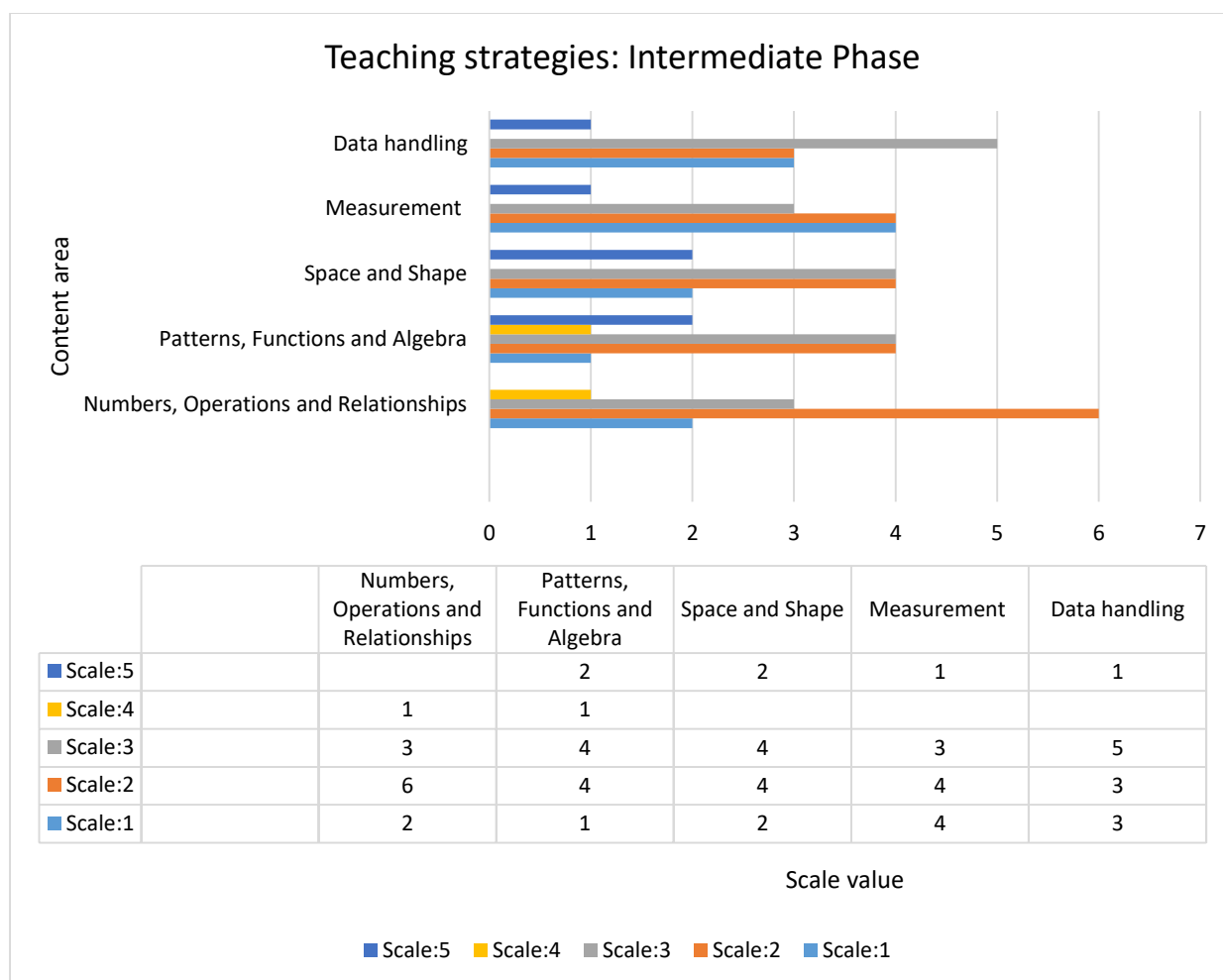
According to Graph 4.23, the subject content areas for which the participants appear to have the most strategies available to teach are, **number patterns, operations, and relationships** (four participants on scales 1 and 2), **patterns, functions, and algebra** (three participants on scales 1 and 2), as well as **measurement** (three participants on scales 1 and 2). No participants indicated that they have a variety of teaching strategies available (scale 1) to teach **space and shape (geometry)** and **data handling**. Space and shape (geometry) and data handling however had two participants on scale 2 respectively. The majority of the participants (three participants) perceived their teaching strategies to teach space and shape on scale 3. For data handling, the participants mainly perceived their availability of teaching strategies on scales 2 and 3 (two participants respectively). For all of the subject content areas one participant indicated the availability of teaching strategies on scale 4.

Similar to the FET phase, it appears as if there could be a link between the participants' performance in the Senior Phase test (cf. Table 4.3), their perceptions about their

preparedness to teach (cf. Graph 4.20), and the availability of strategies they have to teach certain subject content areas (cf. Graph 4.23), for example:

- Number patterns, operations, and relationships – The participants scored an average of 96,10%, the majority (4 participants) perceived themselves to be well-prepared to teach the content area, and to have strategies available to teach the content area.
- Patterns, functions, and algebra – A test average of 85,71% was achieved by the participants, and three of the seven participants perceived themselves to be well-prepared to teach the content area.
- Space and shape (geometry) had a moderate average result (61,03%) and the majority of the participants did not perceive themselves as well-prepared to teach the content. Additionally, Graph 4.23 displays that the participants have limited strategies available to teach the content.

The next section interprets the perceptions of the participants in the Intermediate Phase (cf. Table 4.45) concerning the availability of teaching strategies to teach the Intermediate Phase content. Graph 4.24 displays the data obtained for the Intermediate Phase participants.



Graph 4.24: Availability of teaching strategies – Intermediate Phase

Graph 4.24 indicates that **numbers, operations, and relationships** together with **measurement** had most of the participants in total on scales 1 and 2 combined (eight participants respectively), who indicated that they perceive to have a variety of strategies to teach the subject content. In the following places, are **space and shape (geometry)** (six participants at scales 1 and 2), **data handling** with six participants in total on scales 1 to 2 combined, and **patterns, functions, and algebra** with five participants in total on scales 1 and 2 combined. Patterns, functions, and algebra, and space and shape (geometry) also had two participants each who indicated that they have limited strategies (scale 5) to teach the content. Similarly, one participant indicated limited strategies to teach measurement and data handling, respectively (scale 5). The researcher observed that a number of participants (between three and five) perceived their strategies to teach the subject content knowledge at scale 3.

In the context of the Intermediate Phase participants, perceptions that pointed to not being well prepared to teach subject content (cf. Graph 4.21), seems to align with the perceptions

about not having a too wide variety of strategies to teach a particular subject content area, and poor performance in the test, for example, **space and shape (geometry)**, for which a 38,33% average was obtained (cf. Table 4.5). Although good test results were obtained for **data handling** for which an average of 90,27% was achieved (cf. Table 4.5, Graph 4.24), the availability of teaching strategies to teach the content effectively seems questionable.

Exceptions are also identified, namely **numbers, operations, and relationships** as well as **measurement** for which only 49,58% and 52,77% were obtained respectively in the test, but the majority of the participants perceived themselves to be well-prepared to teach the content and to have strategies available to teach the content (cf. Table 4.5, Graph 4.24).

In sum, the data obtained for the FET, Senior and Intermediate Phases concerns the researcher, as the participants do not seem to possess solid pedagogical content knowledge for teaching all the subject content areas in their respective phases. Given the possible link between subject content knowledge and pedagogical knowledge that is underscored in the literature (Brijlall & Isaac, 2011; Kanyongo & Brown, 2013) (cf. 1.1), it seems reasonable to argue that if teachers do not improve their subject content knowledge, they will not be able to improve their pedagogical knowledge and therefore be ineffective mathematics teachers (Ball et al., 2005; Ball et al., 2008; Crooks & Alibali, 2014; Hill et al., 2008; Long et al., 2014; Ma, 1999; Rittle-Johnson et al., 2015; Shulman, 1998; Stols, 2013; Tsanwani & Sinthumule, 2015) (cf. 2.3.2.2, 2.3.4.1). Subject content knowledge only partially contributes to a teacher's efficacy. Higher achievement in mathematics is strongly related to teachers' pedagogical efficacy within the classroom (Fung et al., 2017; Mij & Makgato, 2006) (cf. 1.1). Therefore, Plotz et al. (2012) argue that teachers need more pedagogical knowledge than content knowledge to be effective at teaching mathematics (cf. 1.1). If learner performance in mathematics needs to improve, teachers not only need stronger mathematical knowledge, but also pedagogical content knowledge (Fuentes et al., 2014; Kanyongo, & Brown, 2013) (cf. 1.1).

In overview, the participants across the three phases seem to lack teaching strategies to teach all the subject content areas effectively. The Intermediate Phase participants appeared to have the most strategies available to teach four of the five subject content areas. Across the three teaching phases, the subject content areas for which participants perceived to have the most strategies available to teach were the following:

- FET Phase: algebra and equations, functions, and number patterns, sequences, and series.

- Senior Phase: number patterns, operations, and relationships, patterns, functions, and algebra, and measurement.
- Intermediate Phase: numbers, operations, and relationships, measurement, space and shape, and data handling.

In the next section, the researcher explores the participants' perceptions regarding the strengths of their training.

4.4.3 Student perceptions: strengths of teacher training

Table 4.46 (see Addendum J, p. 189) summarises the written reflections concerning the perceptions of the FET Phase participants regarding the strengths of their training. Colour coding of the various perceptions enabled the researcher to cluster similar perceptions together.

The participants had diverse perceptions about the strengths of their training. It was encouraging that four of the seven participants mentioned subject content as a strength, linked to lecturers who are well-prepared and had good content knowledge and that the content topics were dealt with in-depth and their scope of content thus broadened (participants 2, 4, 5, and 7). Comparing the viewpoint of the participants to the test results (cf. Table 4.2) the four participants did not excel in any of the subject content areas and obtained very low averages, which raises questions about the solidity of the content knowledge. This is acknowledged in a contradictory statement by participant 7 who added that the content and the teaching strategies appeared to be fragile.

Another positive strength is that participants seem to be challenged to apply cognitive processes such as analysis and identification (participant 1), and providing their own opinions (participant 7).

Participant 7 added that the training has a practical focus, which could be considered as a strength. This perception is however not supported by participant 3 who argues for more practical teaching exposure.

It seems the training encourages the development of attitudes and dispositions, such as enthusiasm, mentioned by participant 6.

Four of the seven participants indicated that the subject content knowledge of the teacher training was a strength, but if the results of the FET test are considered (cf. Table 4.2) then it seems like none of the participants can be considered to have solid subject content

knowledge. It might be assumed that either the content was too difficult for the participants or that they did not have the necessary potential to master the content.

Some of the participants might have underperformed in Grade 12, but were allowed into the teacher-training programme regardless of their baseline knowledge. The highest test score for an individual FET participant was 36,84% (participant 2) which is very low and does not provide evidence of someone with solid content knowledge. Since four of the participants see the content knowledge as a strength, the participants might assume that they are indeed well-qualified to teach mathematics, which by the results (cf. Table 4.2) is not the case. The data presented in Graph 4.19 supports this argument as the participants' perceptions about their preparedness to teach the subject content areas did not align well with the poor results obtained in the FET test (cf. Table 4.2).

Table 4.47 (see Addendum J, p. 190), summarises the perceptions of the Senior Phase participants about the strengths of their training.

The same participants who wrote the FET test also wrote the Senior Phase test. Similar to the FET phase participants, the Senior Phase participants perceived the content as a strength, and mentioned that they gained the necessary knowledge and understanding although they were exposed to FET Phase content only, and not trained in the content for Senior Phase. According to the participants, they received in-depth and good coverage of subject content (participants 2, 5), and learned a lot more about mathematics (participants 3, 4, 6, 7), in particular algebra (participant 1), functions, algebra, number patterns and geometry (participant 2). In addition, the content went beyond the school curriculum (participant 4). The responses of participants 1 and 2, support the data presented in Graph 4.23, where participants indicated to have the most strategies available to teach, among others, numbers, operations, and relationships, and patterns, functions, and algebra. The participants also obtained high average test results for patterns, functions, and algebra (85,71%) and numbers, operations, and relationships (96,10%) (cf. Table 4.3).

It is not surprising, that the subject content for data handling, for which a test average of only 58,92% was achieved, is not regarded as a strength by the participants. The participants also appeared not to have a variety of strategies available to teach data handling (cf. Graph 4.23).

Participants 5 and 7 indicated practical teaching as a strength, and participants 4 and 7 mentioned the teaching strategies that they acquired as a strength of the teacher-training programme. The latter observation does not correspond with the limited number of strategies the participants appear to have to teach the subject content areas (cf. Graph 4.23).

Several dispositional and attitudinal aspects were also highlighted as strengths of the teacher-training programme by participant 3, namely love and joy for the subject, confidence in work, and developing to become a reflective problem solver.

The support received from and the experience of the lecturers and school mentors was also cited as strengths (participants 3, 7).

Only two of the seven participants scored above 90% average for the Senior Phase test and could be considered to have solid content knowledge (participants 3, 6). These two participants might also be effective mathematics teachers as high level of content knowledge, among others, is linked to teaching effectiveness in mathematics (Ball et al., 2008; Boyd, 2011; Livy & Vale, 2011; Schoenfeld & Kilpatrick, 2008; Shulman, 1998; Stols, 2013) (cf. 1.1, 1.6.2.1, 2.3.4.1)

One participant scored 78,94% average (participant 4) and all the other participants scored below 80%, with the lowest test average being 59, 21% (Table 4.3). As most of the participants scored below 80% for the test, the researcher believes that most of the participants do not have solid content knowledge, or at least not the amount of content knowledge that they assumed they obtained from the teacher training. By not realising the gaps in their content knowledge the participants might not pursue improvement and therefore might be less effective teachers in some or all subject content areas (Boyd, 2011; Livy & Vale, 2011) (cf. 1.1, 2.3.4.1).

The perceptions of the Intermediate Phase participants about the strengths of their training are reported in **Table 4.48** (cf. Addendum J, p. 191).

The Intermediate Phase participants mentioned the following as strengths of their training.

Opportunities were received to do practical work (participants 1, 6), which included how to do lesson planning (participant 2). The practical teaching sessions and experience of the workplace were regarded as strengths by participants 4 and 5. Another important strength that was mentioned is the exposure to different methods to teach mathematics (participants 2, 3, 6, 7, and 10). This response aligns well with the data reported in Graph 4.24, according to which the majority of the participants appeared to have strategies available to teach four of the five subject content areas, except for patterns, functions, and algebra. In particular, the following teaching strategies experienced by the participants were cited: learning many teaching strategies (participant 10), interaction and engaging with learners using different methods (participant 6) such as, cooperative discussions, demonstrations with teaching aids and charts and real life objects (participants 2, 3, and 7).

Strengths also involved the study material and the good quality training received (participant 4). Lecturers were found to be supportive of problem-solving and equipped participants with more knowledge than required by the phase (participants 4, 11).

One participant cited the increase concerning confidence in teaching the subject as a strength (participant 9).

Considering the test results obtained by the Intermediate Phase participants (cf. Table 4.5), none of the participants scored above 90% average for the test and only two participants scored above 80% average for the test. The researcher thinks that the results in Table 4.5 do not indicate the presence of solid content knowledge. It does not seem like the good training and study material, as well as practical experience and exposure to different teaching strategies mentioned by the participants as strengths, aided the development of solid subject content knowledge. It is interesting that none of the participants explicitly indicated content knowledge as a strength of the training.

In conclusion, the major trends concerning the strengths cited by the participants across the three teaching phases can be grouped into the following categories:

- Strengths concerning subject content.
- Strengths concerning teaching practice.
- Strengths concerning teaching methods and strategies.
- Strengths concerning lecturer support, preparedness, and knowledge.
- Strengths concerning the development of attitudes and dispositions.

The proceeding section pays attention to the weaknesses of the training as perceived by the participants.

4.4.4 Student perceptions: weaknesses of teacher training

A synthesis of the FET Phase participants about the weaknesses they experience in their training is presented in **Table 4.49** (see Addendum J, p. 192). By means of colour coding the researcher clustered similar perceptions together.

The FET participants mentioned the following weaknesses concerning their training:

Participants 2 and 7 mentioned teaching strategies and the practical application of mathematics as weaknesses, respectively. Subject content was indicated as a weakness, concerning the following: Grade 12 content is hardly revised (participant 1), thus possibly leading to forgetting the content cited by participants 4 and 6, the content does not relate to

the school context (participant 7), in particular, the irrelevancy of being taught spherical geometry instead of Euclidean geometry was cited by participant 3 as a weakness. Participant 5 added that because of not engaging fully in the topic of measurement, it becomes a difficult content area to teach.

Based on the responses, it could be concluded that subject content is regarded as a major weakness among the FET Phase participants. The researcher finds the responses of the participants disturbing, as a good understanding of subject content knowledge is required in mathematics to teach for conceptual understanding (Baker et al., 2012; Farooq et al., 2011; Long et al., 2014; Rittle-Johnson et al., 2015; Sorto, 2010; Stols, 2013) (cf. 2.2.3, 2.3.2.2, 2.3.2.3). Furthermore, the participants will not be able to help learners comprehend the content, which they themselves do not understand (Ball et al., 2005; Ball et al., 2008; Blignaut et al., 2013; Tsanwani & Sinthumule, 2015; Shulman, 1998) (cf. 1.1; 2.3.4.1). When the averages of the test results for the FET Phase participants (cf. Table 4.2) are compared to the weaknesses concerning subject content mentioned by the participants, there seems to be correspondence. The participants' performance in the FET test was not good and it seemed like they had very little to no subject content knowledge or recollection of the FET subject content.

The perceptions of the Senior Phase participants concerning the weaknesses in their training are summarised in **Table 4.50** (see Addendum J, p. 193).

Of all three teaching phases, the participants of the Senior Phase cited the most weaknesses concerning their training.

Similar to the FET Phase, subject content, although being presented in a broader manner than what is expected at school-level, was cited as a major weakness (participants 1, 3, 4, and 7), as it does not cover the school content (participant 4, 7). In this regard, the participants mentioned the following: geometry content is not dealt with effectively, and participants 1, 3, 4, and 7 noted that additional learning is needed for them to learn and supplement their subject content to be equipped to teach at school, or they have to depend on their high school knowledge of mathematics to teach. It is encouraging to see that three of the participants (1, 3, and 7) could identify their shortcomings regarding the subject content knowledge of mathematics and acknowledge that they need to improve themselves.

Another weakness relates to the availability of adequate teaching strategies to teach mathematics (participants 2 and 6), and that participants have to, on their own, consider the

use of strategies to teach mathematics (participant 2). Participant 6 commented on teaching strategies being presented on paper (theory), but their practical application for mathematics is not clear. In this regard the participant suggests additional workshops to equip them with relevant teaching strategies.

Curriculum-related weaknesses that were highlighted are a strong theoretical focus and the fact that there is no subject content training in the fourth year of study (participant 7), as well as modules that seem to be impractical, such as first aid, that form part of the curriculum (participant 5). The researcher argues that time spent on this module could, for example, have been used to reinforce the mathematical subject content knowledge or to strengthen the participants' pedagogical content knowledge. Participant 7 also cited an irrelevant lesson planning format as a weakness of their training.

Table **4.51** (see Addendum J, p. 195) presents the weaknesses of the teacher training according to the Intermediate Phase participants.

The following weaknesses emanated from the participants' responses:

The lack of proper didactical teaching, thus acquiring different teaching strategies to teach subject content was mentioned by participants 1, 2, 9, and 11, in particular, strategies to involve learners actively during teaching (participant 2), using calculators in expressions (participant 6), and doing mental math (participant 9).

Participants seemingly expected to receive more guidance during their training concerning classroom-related issues, such as teaching large classes (participant 2), keeping learners' attention and concentration, as well as dealing with discipline (participant 7).

A few weaknesses concerning the lecturers were mentioned. Participants were exposed to many different lecturers with different approaches to teaching, which was regarded as problematic (participant 4). Participant 4 also mentioned a lack of support concerning how to prepare for examinations, and participant 1 felt that too much homework was given.

Curriculum-related weaknesses involved not being trained from the first year of study (participants 5, 10, and 12), and that more training time per week is necessary (participant 10).

A worrisome aspect is that participant 11 indicated that the focus during their training was still on the old curriculum, the National Curriculum Statement (NCS), and not the CAPS.

Finally, the subject content appears to be complex, and different from what is expected at the school-level (participant 5).

In essence, the main weaknesses across the three phases concerned the following areas, and contradict what was indicated as strengths concerning their training (cf. 4.4.3), in particular concerning subject content and the availability of teaching strategies.

- Weaknesses concerning subject content.
- Weaknesses concerning adequate teaching strategies to teach mathematics.
- Weaknesses concerning curriculum-related aspects, such as lecturing time and exposure time to subject content training.

The researcher finds the weaknesses mentioned concerning subject content and teaching strategies unexpected and contradictory given the perceptions that pointed to some strengths concerning subject content and pedagogical strategies (Graphs 4.19 – 4.24).

From the responses, it is however unsettling that the participants' ability to develop solid conceptual and procedural knowledge among learners might be restricted, as this requires adequate and vast pedagogical knowledge as well as efficient subject content knowledge (Department of Basic Education, 2018, Long et al., 2014; Stols, 2013) (cf. 2.3.2.2). Furthermore, teachers need to know the terminology and have detailed knowledge about content before application of knowledge can be dealt with successfully (Ball et al., 2008) (cf. 2.3.2.2). Teachers who in particular lack strategies to teach problem-solving and different methods for problem-solving could lead to deficient procedural knowledge (Crooks & Alibali, 2014) (cf. 2.3.2.3). Teaching for problem-solving was also not mentioned as a strength by the participants.

In the absence of solid evidence that the development of adequate teaching strategies is a strong focus in the teacher-training curriculum, the researcher believes it is fair to argue that the teacher training the participants receive does not seem to combine the teaching of subject content with the development of epistemological beliefs (Schraw & Olafson, 2003) (cf. 2.2.2). This implies that prospective teachers are probably not made aware of the fact that teaching mathematics requires a diverse approach to teaching (Baker et al., 2012; Depaepe et al., 2016) (cf. 2.4.1). Mathematics teachers need to learn that mathematics need to be viewed from a fallibilist or relativist perspective acknowledging that mathematics content is open to

revision, and does not only involve truths that need to be transmitted but also involves discovery or invention of knowledge (Depaepe et al. 2016) (cf. 2.2.4). In particular, teachers need to know about the importance of, and become knowledgeable about epistemological stances in mathematics (Crooks & Alibali, 2014) (cf. 2.3.2.3) as to teach for procedural knowledge requires a dualistic or multiplistic stance.

In the following section, the participants' perceptions about changes to improve the training of mathematics teachers are examined.

4.4.5 Student perceptions: changes to improve teacher training

The participants had to reflect on suggestions to improve the training of mathematics teachers. Tables 4.52 – 4.54 (see Addendum J), present the perceptions of the participants across the different teaching phases. Similarities among the participants' perceptions were indicated by means of colour coding.

The FET Phase participants had diverse perspectives (see **Table 4.52** – Addendum J. p. 196) about how the teacher-training curriculum might be improved. Participants 3 and 7 indicated that to improve the training the mathematics content should be more CAPS-oriented, thus more focused on what is expected at school level. Related to the aforementioned, participant 1 mentioned that content should provide coverage of all the school grades, and the subject content modules need to receive more attention in the teacher-training curriculum (participant 5). More attention should be paid to the development of teaching strategies to teach mathematics, according to participant 2. Participant 4 thought that the lecturing of mathematics should be more interactive and focus on problem-solving and the development of higher-order thinking skills. Participant 6 proposed a better understanding of content during training. It seems as if the methodology modules (modules aimed at the teaching of mathematics) and other modules that form part of the curriculum and are not specifically maths related, should receive less focus (participant 5).

The researcher feels that the suggestions hold merit as the FET test results (Table 4.2) and their perceptions about their preparedness to teach subject content (cf. Table 4.19), suggest that the participants indeed have a lack of solid mathematical content knowledge that could influence their teaching effectiveness. Besides, suggestions concerning a stronger focus on school content align with weaknesses mentioned concerning the content as part of the training that does not relate to the school context (cf. 4.4.4), and that the development of teaching strategies also appears to be underemphasised during training (cf. 4.4.4).

The suggestion of paying more attention to the development of teaching strategies holds merit as the participants would be able to improve their effectiveness as teachers if they had better instructional skills (Blignaut et al., 2013; Kanyongo & Brown, 2013; Plotz et al., 2012; Siyepu, 2013; Stronge, 2018) (cf. 1.1, 2.4). Moreover, the suggestion supports the absence of a variety of teaching strategies to teach the FET subject content (cf. Graph 4.22).

Table 4.53 (see Addendum J, p. 196) summarises the suggested changes to improve teacher training made by the Senior Phase participants. By using colour coding similar perceptions were clustered together.

Similar to the FET Phase participants, the Senior Phase participants also mentioned the following as suggestions to improve teacher training: equipping pre-service teachers with content relevant to the school context, specifically geometry (participants 3, 4, and 7); paying more attention to the teaching of mathematics during subject content training (participant 2), placing a stronger focus on mathematics and doing away with irrelevant modules to enable students to become subject specialists (participant 7), as well as the methodology modules (participant 2). Linked to doing away with the methodology modules, participant 2 suggests that methodology training be linked to what is dealt with in the subject content, and not be presented as separate modules. According to participant 7, a more appropriate format for lesson planning in mathematics also seems essential. It is likely that participant 6 advocates for a stronger focus on teaching practice sessions, as this appears to be a time for learning and improvement. Participant 4 proposes a clearer explication of the differences that previous curricula have in comparison to the CAPS to enable teachers to adapt to curriculum changes.

Table 4.54 (see Addendum J, p. 197) provides an overview of the changes to improve teacher training according to the Intermediate Phase. The similarities observed among the participants' perceptions were grouped by means of colour coding.

The perceptions of the Intermediate Phase participants concerning improvements of the teacher-training curriculum included the following:

Similar to the FET and Senior Phase participants, the participants feel that more focus should be placed on equipping them with teaching strategies (participants 1, 3, 9, and 12), that would enable them to keep learners interested in mathematics (participant 9), and how to correct errors or misconceptions in mathematics (participant 11). Different teaching methods and strategies are important to enable learners to effectively acquire knowledge and to guide learning appropriately (Killen, 2015; Sibuyi, 2012) (cf. 2.4). It was clear from the data presented in Graph 4.21, that possessing a wide variety of teaching strategies to teach the

subject content has possibly not yet become a reality for many of the Intermediate Phase participants.

The curriculum also appears to be overloaded with little time to practise and do mathematics (participants 4, 7). In particular, participant 4 feels that teaching practice already in the first year of training places a lot of pressure on students. According to the researcher, this suggestion does not hold merit in itself but could raise some interesting thoughts. Practical teaching is a good experience for students and the sooner they start, the quicker they will overcome their fear of teaching. However, the teaching practice sessions should ensure that the students are allowed to learn from experienced teachers and should be given clear pointers or guidelines as to what will be expected the first time. To shorten the curriculum would not benefit students as the test results (cf. Table 4.5) already indicate a lack of content knowledge. However, some of the less relevant modules could be reduced and more time allocated to mathematics modules.

Three participants, (participants 3, 5, and 10) suggested that teaching time should be increased to every day and that mathematics should be presented from the first year of study. Additional support from tutors and extra classes to assist with problems also appear to be needed, as indicated by participants 5 and 6. To the best knowledge of the researcher, the university has a tutor system in place which could address this suggestion. Linked to the aforementioned, the development of subject content knowledge would greatly benefit, as the participants did not appear to be extremely well prepared to teach the subject content (cf. Graph 4.21), and their test results overall were mediocre (cf. Table 4.5).

Participant 2 experienced the training as good, and that application of what was acquired during the lectures could be applied during teaching practice. Participant 7 viewed the provisions of resources to the pre-service teachers that could be used with learners during teaching as essential to improving the teacher-training curriculum.

In conclusion, the main trends observed in the suggestions made by the participants across the three phases can be summarised as follows:

- Suggestions to cover the content to be presented at the school-level more thoroughly.
- Suggestions to equip students with strategies to teach mathematics.
- Suggestions to reduce irrelevant modules in the curriculum and to place a stronger focus on mathematics.

The importance of solid subject content and possessing a variety of strategies to teach mathematics cannot be underscored enough. An analysis of the teacher-training curriculum revealed gaps in the training concerning the development of subject content (cf. 2.3.5), pedagogical content knowledge (cf. 2.4.5), which are rightfully mentioned by the participants. The Senior Phase participants do not follow a specific Senior Phase curriculum but attend classes with the FET students. The content area space and shape (geometry) in the Senior Phase is also not adequately covered in the module Euclidean and spherical geometry in the FET phase, and also focuses on content that differs from school content. Also, the Intermediate Phase students only start with mathematics in their second year of study (cf. 2.3.5). Pedagogical content knowledge is included in the teacher-training curriculum only from the second year of study. The pedagogical module outcomes rather emphasise knowledge about the curriculum and lesson planning and presentation and do not have a strong focus on equipping students with appropriate teaching strategies for teaching mathematics (cf. 2.4.5). The researcher concludes that teacher-training curriculum unfortunately does not seem to prepare pre-service mathematics teachers with the required well-rounded subject and pedagogical content knowledge to become effective teachers (Adedoyin, 2011; Avalos et al., 2010) (cf. 1.2).

The researcher also established the participants' reasons for perceiving themselves as good teachers, which is summarised in the next section.

4.4.6 Student perceptions: reasons for being a good teacher

Tables 4.55 – 4.57 (see Addendum J) summarise the participants' reasons for regarding themselves as good teachers. To identify similarities among the participants' perceptions, the researcher made use of colour coding to classify the perceptions of the participants.

The perceptions of the FET participants (see **Table 4.55** – Addendum J, p. 198) concerning the reasons why they view themselves as good teachers can be grouped as follows:

Perceptions concerning skills that would enable them to be good teachers such as the ability to reason well, and having good interpersonal skills (participant 7). Participant 7 seems to be very confident, however, the participant did not seem to have the necessary content knowledge for the FET test as the participant had a low average of 17,54% (cf. Table 4.2). According to the researcher, this perception might be dangerous, as being overly confident can impair the teachers' sense of self-improvement or could limit the teacher's chances of learning from more experienced peers (Crooks & Alibali, 2014; Fuentes et al., 2014; Stronge, 2018, Ünsal et al., 2018) (cf. 2.3.4, 2.4, 2.7, 2.7.2).

Perceptions concerning personal traits, attitudes, and dispositions that would enable them to be good teachers: open to life-long learning, persisting and studying to improve knowledge (participant 1), passion and dedication (participants 3, 4), being hardworking, having a good work ethic, and interested to continue trying to be better and to improve (participants 3, 4, and 5). It is encouraging that the participants seem motivated to improve themselves since that would consequently enable them to help their learners to perform better (Blignaut et al., 2013; Farooq et al., 2011; Siyepu, 2013) (cf. 11), and positively influence the learners' attitudes towards mathematics (Killen, 2010; Stronge, 2018) (cf. 2.4).

Perceptions concerning subject content: participant 2 indicated the possession of sufficient content knowledge. The researcher finds this perception worrying, as the participant only managed to obtain a 36,84% average for the FET test. It seems that the participant does not have a realistic view of the solidity of his/her content knowledge. This could result in lessons where learners are disengaged, due to the teacher being ineffective (Ball et al., 2009; Ball et al., 2008; Livy & Vale, 2011; Schoenfeld & Kilpatrick, 2008; Stylianides & Stylianides, 2010) (cf. 2.3.4.1). Participant 1 also acknowledges the importance of good content knowledge to become a good teacher.

The following section explores the reasons for being a good teacher from the viewpoint of the Senior Phase participants. **Table 4.56** (see Addendum J, p. 199) summarises the participants' viewpoints. Similar perceptions were grouped together by means of colour coding to guide the analysis of the data.

The perceptions of the Senior Phase participants can be categorised as follows:

Perceptions concerning personality traits, attitudes, and dispositions that would enable the participants to become good teachers. The participants mentioned the following: being enthusiastic about teaching mathematics (participant 1), being open-minded and willing to engage with learners and colleagues to seek ways to make maths easier (participant 2), is dedicated to and enjoying mathematics teaching (participant 3), as well as preparing for classes every day (participant 5). Being a teacher requires a lot of hard work and drive. The aforementioned traits, attitudes, and dispositions could contribute to teaching effectiveness and influence learner's attitudes in a positive way towards mathematics (Killen, 2010, Stronge, 2018) (cf. 2.4). Being well-prepared and having well thought through lesson plans would benefit the teachers as being well-prepared could increase their confidence in class and provide learners with quality learning experiences.

Perceptions about ways to approach learners: participant 4 indicated having patience characterises him as good teacher, as well as having concern about learners understanding of mathematics, teaching until learners understand, knowing where learners struggle with mathematics (participant 1), and finding ways to make mathematics easier for learners (participant 2). It is encouraging to see that three of the seven participants care about the learners' understanding of mathematics. Ensuring understanding of mathematics is important for the application of procedural knowledge in mathematics (Du Plooy & Long, 2014; Long et al, 2014; (cf. 2.3.2.5).

Perceptions about subject content: participants 6 and 7 perceived their subject content to contribute to them being good teachers. The reason might hold true for participant 6, who scored an average of 94,73% in the Senior Phase test. However, participant 7 only scored 72,36% for the test (cf. Table 4.3). This observation raises some concern, as the participant does not seem to realise the gaps in the subject content knowledge. By not realising that the gaps exist, the participant might not attempt to supplement the knowledge that the participant already has. A lack in content knowledge might have a negative impact on the teacher's quality of teaching and might impair the transference of knowledge to the learners (Adedoyin, 2011; Avalos et al., 2010; Brodie, 2001; Goos, 2013; Malaty, 2004; Mij & Makgato 2006; Tsanwani & Sinthumule, 2015) (cf. 1.1, 2.3.4.1).

Perceptions about teaching strategies: participant 6 indicated the use of teaching strategies. It will benefit the learners' acquisition of content knowledge if the teacher can effectively apply different teaching strategies (Adedoyin, 2011; Avalos et al., 2010; Fuentes et al., 2014; Malaty, 2004; Mij & Makgato 2006; Plotz et al., 2012; Sibuyi, 2012; Stronge, 2018) (cf. 1.1, 2.4). As only one participant mentioned the use of teaching strategies for being a good teacher, it is likely that the participants might not be knowledgeable about various strategies to teach mathematics, but might just resort to the direct teaching method (Baker et al., 2012; Green & Hood, 2013) (cf. 2.2.3). Direct teaching is effective, but being able to supplement it with other strategies will aid the understanding of content and help learners to develop different skills (Baker et al., 2012; Green & Hood, 2013; Kanyongo & Brown, 2013; Stronge, 2018 (cf. 1.1, 2.2.3; 2.4).

Participant 7 was the only participant who mentioned that effective communication of mathematics content and good interpersonal skills could be regarded as important for effective mathematics teaching.

The next section pays attention to the reasons mentioned by the Intermediate Phase participants for being a good teacher. **Table 4.57** (see Addendum J, p. 200) summarises the participants' perceptions. By means of colour coding similar perceptions were clustered together.

For the Intermediate Phase participants, the reasons that they cited can be clustered as follows:

Perceptions about subject content and pedagogical knowledge: two participants mentioned good content knowledge (participant 2, 4), and two participants mentioned good pedagogical knowledge (participant 1, 2) as reasons to be viewed as good teachers. One participant, participant 12, considered having both content and pedagogical knowledge, as well as practical experience as reasons to be a good teacher. Participant 5 cited the use of teaching aids during the teaching of subject content as a reason to be considered a good teacher. The use of teaching aids might help learners to relate mathematics to real-life situations or problems (Rüütman & Kipper, 2011) (cf. 2.4.2), and understanding complex topics or questions, which will contribute to the development of problem-solving skills (Du Plooy & Long, 2014; Fuentes et al., 2014) (cf. 2.3.2.4; 2.4). In section 4.4.5, some of the Intermediate Phase participants suggested the inclusion of extra classes, additional lecturing time, and an enhanced focus on teaching strategies as a way to improve the teacher training (Participants 3, 5, 6, 12). Therefore, indicating both mathematical content knowledge and teaching strategies as reasons for being good teachers by participants 1, 2, 4, and 12 appear to be contradictory.

Perceptions about positive personality traits, attitudes, and dispositions. The following were cited by the participants: having a love and passion for mathematics (participants 3, 10), never giving up, irrespective of the difficulty of problems (participant 6), persisting to be better (participant 11), loving teaching (participant 8), and always being prepared (participant 9). According to Killen (2010), and Stronge (2018) (cf. 2.4), personality traits, attitudes, and dispositions of the teacher are important, as they could affect effectiveness and influence learners' attitudes towards learning. Preparation also appears to be an important aspect, because, when teachers prepare well, they can establish their strengths and weaknesses concerning a topic before teaching (Bada, 2015) (cf. 2.2) (Crooks & Alibali, 2014, Lauritzen, 2012) (cf. 2.3.2.3), and ensure that they have solid knowledge before teaching the learners. In that way, the learners will be exposed to learning opportunities that could enhance their understanding of what is being taught (Farooq et al., 2011) (cf. 1.1).

It is positive to know that some of the participants are passionate about their future career as a mathematics teacher. Being passionate about mathematics teaching might motivate the teachers to better themselves to provide the best possible education to their learners (Green & Hood, 2013; Killen, 2010; Stronge, 2002) (cf. 2.2.3, 2.4).

Participants 6 and 11 indicated that they are persistent in the pursuit of knowledge. The researcher values this disposition among the participants, as curricula and the skill set that learners need to acquire need to adapt according to the ever-changing expectations of being part of a global society. Teachers, therefore, need to be adaptable and open to pursue relevant knowledge (Baker et al., 2012; Brownlee & Berthelsen, 2008; Green & Hood, 2013; Schoenfeld, 1992; Sing & Khine, 2008) (cf. 2.2, 2.2.3) in order to remain effective teachers (Ball et al., 2009; Long et al., 2014; Stols, 2013; Tsanwani & Sinthumule, 2015 (cf. 2.3.2.2, 2.3.4.1).

Perceptions about working with learners. The participants mentioned the following: being good at working with learners (participant 5) and making mathematics fun (participant 7). It is heart-warming that these participants enjoy working with learners and try to make mathematics enjoyable. The effectiveness of a teacher depends, among others, on the interrelationship between a teacher and learner (Killen, 2015) (cf. 2.4), which is important to nurture positive attitudes towards learning (Killen, 2010; Stronge, 2002) (cf. 2.4).

To conclude, the main trends emanating from the participants' responses across the phase concerning reasons that make them good teachers can be grouped as follows:

- Possessing positive personality traits, dispositions, and attitudes toward teaching mathematics.
- Concern for learners' progress and understanding of mathematics.
- Possessing content knowledge and teaching strategies, which however contradicts the weaknesses concerning the training that were mentioned (cf. 4.4.4).

To establish reasons that might obstruct participants to be good teachers, the researcher also requested the participants to reflect on possible reasons that would not contribute to them being good teachers

4.4.7 Student perceptions: reasons for not being a good teacher

Table 4.58 (see Addendum J, p. 201) reveals the perceptions of the participants in the FET Phase about the reasons that contribute to them not being good teachers. To identify similarities among the participants' perceptions, colour coding was used to group similarities.

Participants 2, 3, 5, and 7 did not answer the question, which might suggest that they perceive themselves to be good teachers. Comparing their previous responses that relate to possessing enough content knowledge (participant 2), being well-prepared and passionate (participant 3), being hardworking and constantly improving on efforts (participant 5), as well as having good reasoning and communication skills (participant 7), confirms their lack of response and the researcher's suggestion that they perceive themselves to be good teachers. The other participants had diverse perceptions.

The response of participant 1 was difficult to understand, but the researcher carefully concludes that the participant might have indicated a lack of content knowledge across the grades in a phase he will teach. Participant 1 was the only participant who specifically indicated the Grade 12 content as a weakness in the FET Phase (cf. 4.4.4), as well as covering the content of all grades in a Phase as part of the changes that could improve the teacher-training curriculum (cf. 4.4.5).

Participant 4 indicated that it is still possible to make mistakes despite being a good teacher. The response that the participant cited for the weaknesses of the teacher training, namely forgetting and not remembering content, possibly contributes to the mentioning of mistakes being made (cf. 4.4.4). The response could also indicate that metacognitive skills for self-regulation might not be well-developed in the participant, hampering self-awareness and self-regulatory action to identify mistakes and knowledge gaps in ones' knowledge base and thinking (Schneider & Artelt, 2010; Yeager et al., 2014; Zimmerman & Schunk, 2011) (cf. 2.3.2.4).

Participant 6 indicated frustration when he lacks understanding of content, and if learners do not understand. Both aspects make the participant feel like a failure as a teacher. According to the researcher, the participant needs to realise that teachers regularly encounter learners who do not understand the content. The participant might struggle with the transfer of knowledge due to a lack of content or pedagogical knowledge, or a lack of experience. The data in Table 4.2, indicate that participant 6 only obtained a 17,54% average for the FET test, which indicates a lack of subject content knowledge. The participant also indicated the

understanding of content as an aspect that needs improvement as part of teacher training (cf. 4.4.5).

Table 4.59 (see Addendum I, p. 201) synthesises the perceptions of the Senior Phase participants about the reasons they feel would contribute to them not being good teachers. Similar perceptions observed among the participants were clustered together by means of colour coding.

Participants 1, 2, 6, and 7 did not answer the question, which possibly implies that they do not regard themselves to be bad teachers, because of being enthusiastic (participant 1), open minded and willing to engage (participant 2), having good content knowledge (participant 6), and being good at communicating mathematics and possessing good interpersonal skills (participant 7) (cf. Table 4.56).

Participant 3 indicated that it is still possible to make a mistake despite being a teacher. Content that is taught incorrectly might make the content increasingly difficult to comprehend for learners and therefore a teacher must be well prepared and skilled in metacognition to keep mistakes to a minimum and to ensure that it will not influence the overall learning experience of the class (Bannert & Reimann, 2012; Flavell, 1979; Paris & Paris, 2010; Schneider & Artelt, 2010; Yeager et al., 2014; Zimmerman & Schunk, 2011) (cf. 2.3.2.4). If participant 3 is well prepared for class and has well developed metacognitive skills, the mistakes should remain minimal and manageable (Flavell, 1979; Schneider & Artelt, 2010) (cf. 2.3.2.4).

Participant 4 indicated that he might not fully understand the high school mathematics content, although the participant obtained an 78,94% average in the Senior Phase test. This response is confirmed by the participant's observation of suggested changes to improve the curriculum, namely presenting content relevant to the school context (cf. 4.4.5), specifically geometry (cf. Table 4.50). The researcher believes that this response is alarming as the participant is a final-year pre-service mathematics teacher, who does not feel equipped to teach mathematics. According to Cazzola (2008), Mij and Makgato (2006), Plotz et al. (2012), Pournara et al. (2015), and Fuentes et al. (2014) (cf. 1.1, 2.4), a lack of content knowledge might disable a teacher to teach learners effectively. The learners might not understand mathematics either as a teacher can only teach what that teacher already knows (Avalos et al., 2010; Blignaut et al., 2013; Pournara et al., 2015; Sorto, 2010; Tsanwani & Sinthumule, 2015) (cf. 1.1, 2.4). This response could imply that four years of teacher training did not adequately prepare participant 4 to be a mathematics teacher. The researcher is concerned that participants with a much lower test average than participant 4 (cf. Table 4.3), seemingly do not have the same concern

about the quality of their subject content as being a reason for them to not be regarded as good teachers.

Participant 5 perceived having patience to enable all learners to grasp content as a drawback, as completing the curriculum involves keeping close track of time. The researcher regards being patient with learners as a noble characteristic, and finds it troublesome that time to complete the curriculum stands in the way of giving all learners a chance to comprehend the subject content.

The perceptions of the Intermediate Phase participants about the reasons that would not make them a good teacher are displayed in **Table 4.60** (see Addendum I, p. 202). With colour coding the researcher clustered the similarities among the participants' responses.

As with the FET and Senior Phase, some participants (1, 2, 3, 6, 8, and 9) did not answer the question, possibly suggesting that they feel to be good teachers. The participants probably feel that their mentioned strengths (cf. Table 4.57) of being able to apply different teaching styles (participant 1), possessing subject content and pedagogical content knowledge (participant 2), loving mathematics (participant 3), being persistent in tackling mathematics problems (participant 6), being prepared (participant 8), and passionate about mathematics (participant 9) would probably suffice to enable them to be good teachers.

It seems as if participant 4 experiences difficulties with the use of technology for teaching. If a teacher can use technology to support teaching and learning, the teacher might be more effective in communicating the content knowledge to the learners in a way that the learners will comprehend (Anderson et al., 2001; Brodie, 2001; Goos, 2013; Park & Oliver, 2008; Plotz et al., 2012) (cf. 1.1, 2.3.1).

Participants 5, 10 and 11 indicated that they find the mathematical content knowledge troublesome, inadequate, and overwhelming. The same participants mentioned a stronger focus on subject content, how to teach it, and how to fix errors (cf. Tables 4.51, 4.54, Addendum J) as possible weaknesses that need to be changed to improve the quality of the teacher-training curriculum. It is good that the participants could identify their shortcomings and they might try to supplement their lack of content knowledge to become better teachers. However, it is a concern that the content knowledge appears to be inadequate as the participants are final-year pre-service mathematics teachers, who will soon enter the profession. The responses of the participants support the argument of Bansilal et al. (2014), Lam (2017), Pournara et al. (2015), and Sorto (2010) (cf. 1.1) who argue that mathematics

teachers who do not acquire specialised subject content, have limited subject content knowledge and do not understand the content they have to teach.

The researcher argues that the response of participant 12 rather refers to teachers in the profession, who have to cope with constant curriculum changes, which affects the quality of their practice.

In sum, the main trends emanating from the responses across the teaching phases for reasons that would contribute to not being a good teacher can be grouped as follows:

- Inadequate subject content knowledge, and a lack of understanding of the subject content knowledge.
- Making mistakes, which could probably be linked to inadequate subject knowledge and a lack of understanding of the subject content knowledge.
- Weighing time concerns regarding completing the curriculum against learner comprehension of subject content.

To make possible links with the test results, the participants were asked to indicate which of the test questions they found the easiest and most difficult. The following section reports on the participants' views about the easiest and most difficult test questions.

4.4.8 Student perceptions: the easiest and most difficult test questions

Tables 4.61 (see Addendum J, p. 203) summarises the FET participants' views on the easiest and most difficult test questions. Similarities among the participants' responses were grouped with different colours.

The participants experienced the easiest and most difficult questions in diverse ways.

The easiest questions

Participants 1 and 6 both indicated Question 6 (statistics) as easy. Looking at the results in Table 4.2, it is surprising that participant 1 indicates statistics as easy, as a score of 0 out of 6 was obtained for the question. Participant 1 was the only participant who indicated Question 3 (number patterns, sequences, and series) as easy. Participant 1 only scored 2 out of 6 for the question, which does not testify to the content being easy for the participant. Participant 6 obtained 3 out of 6 for the statistics question, which does not support the participants' perception of it being easy. Participant 6 also found Question 6 (probability) easy, but obtained 0 out of 6 for the question. The researcher is concerned that the participants do not have a

realistic understanding of the adequacy of their knowledge levels, and possibly do not realise what they can do successfully and not do successfully.

Participants 3 and 4 both indicated Question 5 (functions) as easy. Participant 3 scored 100% for Question 5, whereas participant 4 scored 7 out of 12 (58,33%) for the question. Compared to the other questions, functions appear to be the content area in which participant 4 scored the best result (cf. Table 4.2).

Participants 2, 5, and 7 indicated the biographic and or metacognitive sections as the easiest in the test. This response is discouraging, as these two sections did not expect of the participants to apply subject content knowledge and tend to indicate that the sections where subject content knowledge was required, were experienced as challenging. Participant 2 and 7 scored 0% for three of the test questions, which possibly confirm that subject content knowledge was difficult. The data reported for the application of metacognitive skills (cf. 4.4.1) indicated that participants 2, 5, and 7 had an idea of strategies to self-regulate their learning, which is probably why they might have found the section on metacognition easy. The researcher acknowledges that the wording of the question might have been confusing, as it did not specifically ask which question, but which section was the easiest or most difficult. The researcher assumed that the participants would focus on the subject content-related and pedagogical-related questions.

The most difficult questions

The most difficult test sections were finance growth and decay (participants 1, 3, and 4) and four of the seven participants (participants 2, 5, 6, and 7) mentioned that all the test sections were difficult. All four participants achieved very low averages in the FET Phase test (cf. Table 4.2).

Participant 1 scored 3 out of 9 (33,33%), and participants 3 and 4 scored 0 for out of 9, for finance, growth, and decay which confirms their experience of the content area being difficult.

Participants 2, 5, and 7 indicated that all the questions in the test were difficult, which testifies to the very low averages obtained by the participants for the test, namely 36,84%, 29,84, and 17,54% respectively (cf. Table 4.2). Also, it seems that the open questions about metacognition and the qualities of the teacher-training curriculum were also difficult for the participants to answer.

Participant 1 also indicated differential calculus, algebra and equations, functions, and probability as difficult. Participant 1 scored a test average of 28,07%, which indicates that the participant had difficulty with most of the questions (cf. Table 4.2).

Table 4.62 (see Addendum J, p. 203) classifies the easiest and most difficult questions according to the Senior Phase participants' views. Similar perceptions are grouped together by means of colour coding.

The easiest questions

Participant 1 indicated that Question 1 (patterns, functions, and algebra) was the easiest. Participant 1 got 4 out of 8 for this question (Table 4.3), which means that the participant's view does not align completely with the achievement obtained. Participant 1 scored 100% for numbers, operations, and relationships, which appears to be the subject content area the participant found the easiest.

Participants 2, 6, and 7 indicated that all the questions were easy. Their results however revealed that there might be some gaps in their subject content knowledge. Participant 2 had a test average of 69,73%, participant 6 had a test average of 94,73% and Participant 7 had a test average of 72,36% (Table 4.3). The researcher feels that Participant 6 is the only one of the three participants who could have indicated all the questions as easy considering the high test average obtained, namely 94,73% Participants 1, 2, 5, and 7 scored below the overall test average of 76,49% (Table 4.3).

Participants 3 and 5 indicated that Question 4 (measurement) was the easiest question. Participant 3 and 4 both scored 3 out of 4 for measurement, which indicates that the question was probably indeed easy for the participants (cf. Table 4.3).

Participant 4 indicated that Question 1, (numbers, operations, and relationships) was the easiest. Participant 4 scored 100% for Question 1 (cf. Table 4.3), which suggests that the participant indeed found Question 1 easy and probably mastered the subject content knowledge.

The most difficult questions

Participants 1 and 3 indicated space and shape (geometry) (Question 3) as difficult. Participant 1 had 5 out of 11, an average of 45,45% for space and shape (geometry), and participant 3 achieved 11 out of 11 (100%). Based on the results, Question 3 must have been difficult for participant 1, but participant 3's view does not support the actual test results. It could be that because the Senior Phase participants were not exposed to the content space and shape

(geometry) as they completed the FET curriculum (cf. 2.3.5), participant 1 might have felt uneasy about the subject content for space and shape (geometry). Also, participant 5 indicated Euclidean geometry as a difficult section. Euclidean geometry links to the content area space and shape (geometry) (Question 3). Participant 5 scored 4 out of 11 (36,36%) for space and shape (geometry), and it can therefore be assumed that the participant indeed found the question difficult (cf. Table 4.3). The fact that the geometry subject content area is not adequately addressed in the FET curriculum (cf. 2.3.5) possibly contributes to the perceptions of the participants who found this question difficult.

Participant 1 added that the question about measurement was difficult. The participant obtained 0 out of 4 for the question on measurement (cf. Table 4.3), which confirms the perception of the difficulty of the content for this participant.

Participant 4 perceived the question concerning numbers, operations, and relationships as difficult. This response is unexpected as the participant managed to obtain 11 out of 11 for the question (cf. Table 4.3).

Participants 2 and 7 found all the subject content areas easy, although their average results were only 69,73% and 72,36% respectively, which indicates that there are gaps in their subject content knowledge. They also indicated that the section on metacognition was difficult. This section in the test was not directly related to subject content, and the limited knowledge displayed by the participants about metacognition (cf. Table 4.42), support their responses that they found the section on metacognition difficult. The development of metacognitive knowledge, in particular, to self-regulate learning, is not explicitly promoted in the teacher-training curriculum (cf. 2.3.5).

Participant 6 indicated that nothing was difficult. The participant's test average was 94,74% (cf. Table 4.3), which suggests that the participant found all the questions easy, and that subject content knowledge seems to be in place.

Tables 4.63a and 4.63b (cf. Addendum J, p. 204) present the perceptions of the Intermediate Phase participants about the test questions they found the easiest and the most difficult. Colour coding was used to cluster similar responses together.

The easiest questions

Participants 2, 5, 6, and 9 indicated Question 5 (data handling) as the easiest question. Their perceptions are confirmed by their test results. Participants 2, 5, and 9 all scored 100% for Question 5 (cf. Table 4.5), and participant 6 had 5 out of 6 for the question. Question 5 had

the highest question average in the test, 90,27% (Table 4.5). The mentioned participants all obtained full marks for Question 2 (patterns, functions, and algebra) as well, and the researcher finds it strange that this question is also not indicated as easy.

Participants 4 and 10 indicated Question 4 (space, and shape) as the easiest. Participant 4 scored 1 out of 5, and participant 10 scored 2 out of 5 for the question, which suggests that the participants both struggled with space and shape (cf. Table 4.5), and that the question could not have been the easiest for them. The question average was only 38,33%, which means the participants both scored below the question average. Both the participants scored 6 out of 6 for data handling, and the researcher finds it unexpected that data handling is not perceived by the participants as easy. Participant 10 also obtained 5 out of 5 for patterns, function, and algebra and 3 out of 3 for measurement. Both subject content areas were not perceived as being easy by the participant.

Participant 1 indicated that all the questions were easy. Although the participant's average test score (71,79%) is higher than the group average of 57,90%, there might still be gaps in the subject content knowledge of the participant that the participant is not aware of.

Participants 7 and 12 indicated that the biographical questions were the easiest section. This is obvious, as no difficult subject content or challenging thinking tasks were expected in this section. This might be due to the wording of the question that did not specifically ask which question, but which section was the easiest or most difficult.

Participant 3 indicated Question 1 (numbers, operations, and relationships) as the easiest. Participant 3 only obtained 12 out of 20 (60%) for the question. The participant achieved 5 out of 5 for patterns, functions, and algebra, and 6 out of 6 for data handling (cf. Table 4.5). The researcher finds it strange that the participant did not mention these two subject content areas being easy.

Participant 8 indicated that nothing was easy and had a test average of 63,15%. This observation suggests that the participant realises that subject content knowledge is not solid. The overall test average was 57,90%, which means that the participant was still able to score above the test average. The participant however scored 100% for Question 5 (data handling), which could thus have been indicated as easy.

Unfortunately, participant 11 did not answer the question.

The most difficult questions

Participants 1, 7, and 12 indicated that Section C comprising the open questions about metacognition and curriculum-related questions was the most difficult. The researcher thinks that the participants might have never been asked to reflect on their training and requested to voice an opinion, could have made them feel uneasy to share this information with the researcher.

Participants 2, 5, 6, and 10 indicated that Question 1 (numbers, operations, and relationships) was the most difficult. Participant 2 also added that Question 1 was the most time-consuming. Participant 2 scored 10 out of 20 (50%) for Question 1, participant 5 scored 4 out of 20 (20%), participant 6 scored 3 out of 20 (15%), and participant 10 scored 15 out of 20 (75%) (cf. Table 4.5). It is obvious from the question averages of participants 5 and 6 that the participants would have found the question to be difficult. Participant 10's response does not support the results that the participant obtained for this question. Participant 10 had a test score average of 79,48%, and Questions 1 and 4 were the only questions for which the participant did not obtain full marks (cf. Table 4.5). Although obtaining better results for Question 1 than Question 4, participant 10 however perceived Question 1 to be more difficult, probably because Question 1 was weighted heavier, and therefore more challenging to answer.

Participants 4 and 11 indicated Question 2 (patterns, functions, and algebra) as the most difficult question. Participant 4 scored 60% (3 out of 5) for the question, whereas participant 11 scored 0 for the question (Table 4.5). Given the result of participant 11, it is obvious that the participant found Question 2 difficult, and the participant also had a poor test average of 28,94% (Table 4.5).

Participant 9 indicated "mental math" (doing maths without using pencil and paper or a calculator) as the most difficult section of the test. Mental calculations were not expected in the test, but mental operations had to be applied throughout the test to answer the questions as a calculator was prohibited. Participant 9 had a test average of 87,17% (cf. Table 4.5) which suggests that the participant could answer most of the questions without too much difficulty. The participant might likely have had a higher score if the use of a calculator was allowed.

For participant 8, the entire test seemed difficult. Participant 8 obtained an average of 61,53% for the test, and it appears as if Question 1 (numbers, operations, and relationships), and Question 4 (space and shape (geometry)) for which the participant achieved 12 out of 20, and 1 out of 5 respectively, were experienced as the most difficult questions. Participant 8 however achieved 6 out of 6 for data handling.

Participant 3 unfortunately did not answer the question.

In overview, the participants across the three phases had diverse perceptions about the easiest and most difficult questions as summarised in Table 4.64. The number of the participants who experience a subject content area as easy and/or difficult is presented in brackets. A worrying observation made by the researcher is that of the participants' apparent inaccurate perception of what easy and difficult in relation to the subject content areas are. Subject content areas for which good results are achieved are often cited as difficult, and vice versa.

Table 4.64: Summary: the easiest and most difficult questions

	FET Phase	Senior Phase	Intermediate Phase
Easiest	Statistics (2) Functions (2) Metacognition (2) Biographic (2)	All questions (3) Measurement (2) Patterns, functions, and algebra (1) Numbers, operations, and relationships (1)	Data handling (4) Space and shape (2) All questions (1) Numbers, operations, and relationships (1) Nothing (1) Biographic information (2)
Most difficult	All questions (4) Finance, growth, and decay (3)	Space and shape, geometry (3) Metacognition (2) Measurement (1) Nothing (1)	Numbers, operations, and relationships (4) Patterns, functions, and algebra (2) Open questions (3) Everything (1) Mental maths (1)

To establish the pedagogical knowledge of the participants, the participants were asked to reflect on the teaching strategies they would employ to teach the various subject content areas to learners. The next section reveals the participants' responses in this regard.

4.4.9 Student perceptions: teaching strategies used to teach subject content areas

Table 4.65 (see Addendum J, p. 205) reflects the strategies mentioned by the FET participants to teach the various FET subject content areas, and Addendum M1 provides an elaborate analysis of the participants' responses. The section to follow only provides a synthesis of what transpired from the data.

4.4.9.1 Differential calculus (FET Phase)

In section 4.4.2, the majority of the participants perceived differential calculus to be a content area for which they do not have a variety of teaching strategies available, which aligns well with the examples cited in Table 4.65, and the participants' perception of having limited strategies to teach differential calculus (cf. Graph 4.22). Only two of the seven participants perceived the availability of teaching strategies to teach differential calculus at scales 1 and 2 (cf. Graph 4.22).

4.4.9.2 Algebra and equations (FET Phase)

For the topic algebra and equations, the participants all mainly indicated a form of direct teaching. Although four of the seven participants indicated to have a variety teaching strategies available to teach algebra (cf. Graph 4.22), the examples cited above do not convince the researcher that they have any other strategies apart from direct teaching to teach algebra and equations.

4.4.9.3 Number patterns, sequences, and series (FET Phase)

Although the responses indicate that some of the participants have an idea of other teaching strategies apart from direct teaching, the responses in 4.4.2.2, such as peer learning making use of examples and demonstrations, and mind maps (cf. Graph 4.2.2) confirm that the participants do not have a variety of teaching strategies available to teach number patterns, sequences, and series. Only three of the seven participants apparently have a variety of strategies available (cf. Graph 4.22).

4.4.9.4 Finance, growth, and decay (FET Phase)

To teach finance, growth, and decay, the participants mainly indicated two different teaching strategies, namely interactive and direct teaching. Most of the participants indicated using the direct teaching strategy, which supports their perceptions in 4.4.2.2 that indicated a limited availability of strategies to teach finance, growth, and decay (cf. Graph 4.22).

4.4.9.5 Functions (FET Phase)

In general, the participants did not indicate a specific teaching strategy, but vaguely explained a way that they would approach the content area functions. Most of the participants indicated some form of direct teaching as identified above. One participant indicated the use of real-life examples instead of direct teaching. The researcher argues that the participants' responses that indicated the availability of limited teaching strategies they have available to teach

functions (cf. Graph 4.2.2) confirm the limited examples presented in the aforementioned section.

4.4.9.6 Probability (FET Phase)

The researcher cautiously assumes that the participants mainly rely on the use of direct teaching to teach probability. The examples of strategies cited, namely drilling and the use of examples support the participants' perceptions that they do not have a wide variety of strategies available to teach probability (cf. Graph 4.22).

4.4.9.7 Statistics (FET Phase)

The responses obtained for strategies to teach statistics corroborate the perceptions of the participants indicating that they do not have a variety of teaching strategies available to teach statistics (cf. Graph 4.2.2, cf. 4.4.2.2). The examples cited related to drilling, scaffolding, and the use of real-life examples.

In conclusion, the examples of teaching strategies to teach the subject content areas in the FET phase appear to be limited and unsettling, as direct teaching (drilling, providing information and examples to learners, learners practising examples, and demonstrations), appears to be the strategy used by most of the participants. Although limited, there are however encouraging signs of participants who seem to employ indirect (scaffolding) and interactive teaching strategies (discovery, peer learning) and who connect real-life experiences to their teaching. It is also interesting to note, that individual participants mainly mention one strategy, which does not testify to possessing a variety of teaching strategies.

Table 4.66 (see Addendum J, p. 209) categorises the strategies used by the Senior Phase participants to teach the various subject content areas. Colour coding assisted the researcher to group similar perceptions together. Addendum M2 provides an elaborate analysis of the participants' responses. The section to follow only provides a synthesis of what transpired from the data.

4.4.9.8 Patterns, functions, and algebra (Senior Phase)

In section 4.4.2, (cf. Graph 4.23), only three of the seven participants noted that they feel they have a wide variety of strategies to teach patterns, functions, and algebra, given the scale 1 and 2 responses. From the answers provided concerning the explanation of the strategies they use to teach patterns, functions, and algebra, the teaching strategies in deed appear to be limited to direct teaching (drilling, demonstrations) and problem-solving (independent teaching).

4.4.9.9 Numbers, operations, and relationships (Senior Phase)

The teaching strategies mentioned by the participants could be regarded as limited, and support their perceptions of not having a wide variety of strategies available to teach numbers, operations, and relationships (cf. Graph 4.23). The main focus still appears to be on the direct teaching strategy (explanations and demonstrations), with some evidence of employing class discussions and peer learning.

4.4.9.10 Space and shape (geometry) (Senior Phase)

The researcher feels encouraged that the participants seem to have a greater variety of strategies available to teach space and shape (geometry), such as teaching and explaining of information, questioning, discovery, group discussions, using real-life applications and employing FUNZ and geoboard sketch, and do not resort to direct teaching only. It is however strange, that the participants indicated in section 4.4.2.2 (cf. Graph 4.23) that they do not particularly feel they have a wide variety of teaching strategies available to teach space and shape (geometry). None of the participants indicated the availability of their teaching strategies to teach space and shape (geometry) at scale 1 (cf. Graph 4.23).

4.4.9.11 Measurement (Senior Phase)

The researcher concludes that using examples for demonstration purposes, providing presentations and explanations can be clustered as direct teaching strategies. However, positive signs indicate that the participants also choose to incorporate peer teaching and group discussions when teaching measurement, which could be regarded as teaching strategies that encourage greater learner involvement and independent discovery of information. The teaching strategies cited by the participants however do not support their perceptions in section 4.4.2.2 (cf. Graph 4.23), according to which the majority did not perceive to have a variety of strategies available to teach measurement.

4.4.9.12 Data handling (Senior Phase)

To summarise, the teaching strategies mentioned to teach data handling do not convince the researcher that the participants have a variety of strategies available to teach data handling as the focus tends to be on direct teaching strategies (memorising information, demonstrations, and providing information and explanations as well as summaries to learners). There is however an indication of using group work and discussions as well as case studies, which is encouraging. The limited strategies cited, augur well with the participants' perceptions in section 4.4.2.2 (cf. Graph 4.23), where data handling was mentioned as a

content area for which limited teaching strategies are available (no scale 1 responses were reported).

Table 4.67 (see Addendum I, p. 212) groups the teaching strategies mentioned by the Intermediate Phase participants to teach the different subject content areas. Again, colour coding was applied to cluster similar perceptions together. Addendum M3 presents an elaborate overview of the strategies cited by the participants. The section below synthesises the data that emanated from the elaborate overview.

4.4.9.13 Numbers, operations, and relationships (Intermediate Phase)

To conclude, four participants (1, 3, 11, and 12) did not answer the question, which indicates a possible lack of strategies to teach numbers, operations, and relationships. Many of the participants tend to favour direct teaching strategies such as explanations, demonstrations, questions and answers, and practise questions from the textbook. The responses cited, do not entirely corroborate the participants' perceptions (cf. Graph 4.24) according to which eight of the twelve participants at scale 1 and 2 indicated that they do have a variety of strategies available to teach numbers, operations, and relationships.

4.4.9.14 Patterns, functions, and algebra (Intermediate Phase)

In overview, two participants (3 and 11) did not answer the question, and the majority of the participants seem to rely on direct teaching strategies such as demonstrations, explanations, and repetition, and practise to teach patterns, functions, and algebra. This observation supports the participants' perceptions that they do not have a wide variety of strategies available to teach patterns, functions, and algebra (cf. Table 4.2.4). Only five of the twelve participants reported a variety of teaching strategies at scale 1 and 2 (cf. Table 4.24).

4.4.9.15 Measurement (Intermediate Phase)

To sum up, five participants did not specify a teaching strategy therefore by default it is assumed that they would make use of direct teaching as a teaching strategy for the topic, measurement. The majority of the participants indicated the use of direct teaching strategies (demonstrations with practical objects/real-life examples). From the answers provided by the participants, it seems like the availability of teaching strategies were limited to just direct teaching. Therefore, their perceptions of having a variety of strategies available to teach measurement do not correlate with the answer provided by the participants, according to which eight of the twelve participants perceived the availability of teaching strategies to be at scale 1 and 2, reflecting some variety (cf. Graph 4.24).

4.4.9.16 Space and shape (geometry) (Intermediate Phase)

Three participants did not indicate a teaching strategy, and the majority of the participants are likely to use direct teaching to teach space and shape. From the answers provided by the participants it seems like the availability of teaching strategies was very limited and therefore their perceptions about not possessing a wide variety of strategies to teach space and shape (cf. Graph 4.2.4) are supported by the limited strategies that they mentioned. Only six of the twelve participants perceived the availability of their teaching strategies to teach measurement at scale 1 and 2 (cf. Graph 4.24).

4.4.9.17 Data handling (Intermediate Phase)

In sum, two participants (1 and 11), did not indicate a teaching strategy and ten indicated using some form of a direct teaching strategy that relies on demonstration. The strategies cited by the participants are limited and do not confirm their perceptions where six of the twelve participants perceived to have a wide variety of strategies available to teach data handling (cf. Graph 4.24).

In retrospect the qualitative data analysis elicits concern about the two focus points of the research, namely subject content knowledge and pedagogical knowledge. The researcher feels concerned about the preparedness of teachers to teach the various subject content areas, as it became clear that the participants appear not to be well-prepared to teach all the subject content areas in their respective phases (cf. 4.4.2.1). This observation is supported by weaknesses concerning certain subject content knowledge that was mentioned by the participants (cf. 4.4.4) which was also suggested as an aspect that needs improvement in the curriculum (cf. 4.4.5), and as an obstacle that could impact on them being good teachers (cf. 4.4.7).

The effectiveness of the research participants as mathematics teachers is therefore questionable, as an understanding of mathematical content knowledge, and solid content knowledge impacts on teaching effectiveness (Ball et al., 2008, Lam, 2017, Tsanwani & Sinthumule, 2015) (cf. 2.3.4.3). If teachers have limited subject content knowledge, the meaningfulness of knowledge transfer to learners is jeopardised (Plotz et al., 2012) (cf. 2.3.4.2).

The second point of concern is about the teaching strategies mentioned by the participants across the three teaching phases. For several subject content areas, the participants seem to have limited teaching strategies available (cf. Graphs 4.22 – 4.24), which resonates with the lack of having teaching strategies available being mentioned as a weakness in the teacher-

training curriculum (cf. 4.4.4), which, also, as suggested by the participants, should be thought of as a change to improve the teacher-training curriculum (cf. 4.4.5). There appears to be an overreliance on the use of direct teaching, particularly among the Intermediate Phase participants. Teaching strategies, such as explanations, demonstrations, drilling, and using questions and answers seem to be favoured by participants across the three teaching phases (cf. 4.4.9) (cf. Tables 4.65 – 4.67, Addendum J). It is, however, a positive sign, that peer learning and group discussions that promote indirect, independent, interactive, and learner-centred teaching and learning (cf. 2.4.2 – 2.4.5) seem to be utilised by some of the participants. It is however disturbing, that many of the participants often cited no teaching strategies, or just did not answer the questions. In particular, Participant 11 in the Intermediate Phase, seemed to not know any teaching strategy for any of the subject content areas.

If a participant does not know of any specific teaching strategies to make use of for teaching a content area, the participant might make use of the default teaching strategy, namely direct teaching, which is often the teaching strategy they might have been exposed to during their schooling or training. Most teachers teach the way they were taught, namely using direct teaching (Ho et al., 2001; Wang et al., 2008) (cf. 2.4.4). The use of direct teaching strategies focuses on factual information, such as formulas, definitions, or principles and ensures the acquisition of procedural knowledge rather than ensuring conceptual understanding (Baker et al., 2012) (cf. 2.2.3). Problem-solving in mathematics is often seen as a skill to be taught by using direct teaching (Schoenfeld, 1992) (cf. 2.2.3). However, Depaepe et al. (2016) argue problem-solving is not a skill to be taught, but an art to be mastered (Schoenfeld, 1992) (cf. 2.2.3), thus suggesting the application of teaching strategies that would promote learner-centred indirect, independent and interactive teaching and learning (cf. 2.4.2 – 2.4.5). Combining teaching strategies might engage learners in the learning process, and help them retain information, and have a deeper understanding of the concepts related to a topic (Long et al., 2014; Stols, 2013) (cf. 2.3.2.2).

Studies have also confirmed the relationship between a teacher's mathematical content knowledge and their ability to be effective teachers (Ball et al., 2008; Tsanwani & Sinthumule, 2015) (cf. 2.3.4.1). It is worrying that the participants seem to experience gaps in their subject content and pedagogical content knowledge, as effective teachers need efficient knowledge – both pedagogical and content (with the focus on conceptual knowledge) (Long et al., 2014; Stols, 2013) (cf. 2.3.2.2).

The following section explores the data obtained from the mathematics lecturers.

4.5 QUALITATIVE DATA ANALYSIS AND INTERPRETATION: LECTURERS

The following section reports on the data obtained from the written reflections of the four mathematics lecturers who took part in the study. Table 4.68 summarises the biographical data of the lecturers.

4.5.1 Biographic information: lecturers

Question 1 posed to the lecturers gathered information about their qualifications in mathematics, which is presented below.

Table 4.68: Biographic information: lecturers

Lecturer O	Lecturer R	Lecturer F	Lecturer M
BEd (Mathematics) BEd Honours (Mathematics), 2001	BSc 1974 (Mathematics) BEd Honours in Learning and Teaching, 2006 MEd in Learning and Teaching, 2009	BA in Psychology and Law, 2010 BA Honours, 2011 PGCE, 2012 MEd 2017	BSc Ed. 1987 (Natural Science) BEd Honours in Learning and Teaching, 2012 MEd Environmental Management, 2015 Ph.D. Projects and Management, 2018

According to the biographical data, three of the lecturers obtained a master's degree in education, whereas the best-qualified lecturer had a doctorate in education, and the lowest qualification was an Honours degree in education in the field of mathematics. Three of the four lecturers (lecturer O, R, and M) obtained a first bachelor's degree with mathematics or science as a major subject. Lecturers O and M obtained bachelor of education degrees, with majors in mathematics and science respectively, whereas lecturer R obtained a BSc bachelor's degree with a major in mathematics. Although the lecturers seem well-qualified in the field of teaching, the researcher believes it would have been beneficial to the delivery of the teacher-training curriculum if they were all qualified in the field of mathematics. Although it was not explicitly asked, two of the lecturers taught mathematics at school (lecturers O and R), before becoming involved in teacher training. The researcher is aware, that of all the lecturers, lecturer R, has the most experience in teaching mathematics at school-level and as part of teacher training. Nevertheless, it was encouraging to note that the participants perceived their lectures to be well-prepared and experienced and to have good content knowledge, which were cited as some of the strengths of the teacher-training curriculum (cf. 4.4.3).

Question 2 asked lecturers to reflect on the strengths and weaknesses of the mathematics teacher-training curriculum.

4.5.2 Lecturer perceptions: strengths and weaknesses of the mathematics teacher-training curriculum

Table 4.69 (cf. Addendum D, p. 73) summarises the responses of the lecturers regarding the strengths of the teacher-training curriculum. Similar responses were grouped together and guided the analysis of the data.

Lecturers F, R, and M indicated the subject content of the teacher-training curriculum as a strength. Lecturer R indicated that the content extends the knowledge needed for the Grade 12 school curriculum, and added that the extended content might aid in stimulating the interest of students in mathematics.

Lecturer F only indicated the content of the curriculum as a strength of the training, whereas lecturer M added that the content knowledge presented to the participants could be regarded as solid. Lecturer M added that a teacher with solid content knowledge would be able to explore different teaching strategies and that solid content knowledge links with pedagogical knowledge, which is confirmed in the literature (Fung et al., 2017, Mi & Makgato, 2006; Plotz et al., 2012) (cf. 1.1).

Lecturer O viewed small-group teaching as a strength to the teacher-training curriculum. Although the researcher does not think small-group teaching is specifically prescribed by the curriculum, Lecturer O might find the use of this teaching strategy effective in her own classes. Lecturer O indicated that small-group teaching makes it possible for a lecturer to identify areas of struggle, and it also makes it easier for students to ask questions if needed. Additionally, Lecturer O asserted that the university offers teacher training for various phases, and is thus not limited to a particular phase.

Overall, the main strength identified by the lecturers seems to be the subject content knowledge presented as part of the teacher-training curriculum. The content knowledge was linked with increased interest in mathematics and enhancing the ability to teach the content. The researcher finds it surprising that subject content knowledge and limited teaching strategies, were mentioned as some of the weaknesses of the teacher-training curriculum by the student participants (cf. 4.4.4, 4.4.5, 4.4.7), thus contradicting the lecturers' responses.

Table 4.70 (cf. Addendum D, p. 75) reports on the weaknesses of the teacher-training curriculum as perceived by the lecturers.

According to the lecturers, the following could be regarded as weaknesses of the teacher-training curriculum. The lecturers had diverse opinions about the weaknesses of the teacher-

training curriculum. Opposed to the content indicated as a strength in Table 4.69 the lecturers (O and R) viewed the content of the teacher-training curriculum as a weakness. However, according to the lecturers, it seems to be that the limitation concerns only geometry. Lecturer O specified that some of the participants did not do geometry as part of their school curriculum either, and not including the topic in the teacher-training curriculum leaves the participants with little to no content knowledge of geometry. The document analysis conducted by the researcher supports the observation of the two lecturers. The teacher-training curriculum for the FET Phase does not include content on analytical geometry and trigonometry. Besides, the content area space and shape (geometry) in the Senior Phase is not adequately covered in the module Euclidean and spherical geometry, as the topics dealt with in space and shape (geometry) are not focused on in the module (cf. 2.3.5). The Senior Phase participants also do not receive training specifically geared at the Senior Phase but attend classes with the FET Phase participants (cf. 2.3.5).

Lecturer F stipulated the baseline knowledge of the participants as a weakness of the teacher training. Lecturer F did not elaborate on her response, and the researcher assumes that the response refers to gaps in the participants' school-curriculum which places them at a disadvantage when they enrol for the mathematics teacher-training curriculum.

Lecturer R and M both specified the importance of pedagogical knowledge in the teacher-training curriculum and declared it as a weakness. Lecturer R suggested that knowledge regarding the setting and marking of tests and papers is also a weakness. Lecturer M elaborated on the lack of classroom exposure and teaching strategies as a weakness, which was linked to the workplace integrated learning (WIL), or teaching practice sessions of the participants. Increased classroom exposure and working with expert teachers would increase the experience of students in a real-life setting. Large classes seem to hinder the participants to experience meaningful practical teaching, and the lecturer suggests the use of video recordings to make participants aware of their teaching ability.

The researcher's document analysis of the teacher-training curriculum also revealed gaps concerning pedagogical knowledge. The participants seem to have to rely on the generic information about teaching methods and strategies that they receive in the Professional Studies modules, without purposeful exposure to the application of teaching strategies aimed at teaching mathematics subject content (cf. 2.4.6).

Lecturer F specified time as a weakness of the teacher-training curriculum along with the university transformation. The large classes mentioned by Lecturer M and lack of time mentioned by Lecturer F could be connected. Enabling all participants to experience the

practice of teaching in large classes would require a lot of time. If class sizes were reduced, time might not hamper the practice of teaching strategies by the participants. According to the researcher, large class sizes is not a weakness as such of the teacher-training curriculum, but likely an outflow of allowing too many students into the curriculum. Lecturer F was not clear on exactly how the university transformation presents weaknesses to the teacher-training curriculum.

Obtaining practical experience in teaching does not qualify one as being an effective teacher, but experience helps teachers to refine their pedagogical strategies (Killen, 2010; Tsanwani & Sinthumule, 2015) (cf. 2.3.4.1, 2.4). The mentioned limitations around practical experience in the teacher-training curriculum could limit the participants' acquisition of teaching strategies.

Lecturer O indicated that students are allowed to continue with modules in the following study years, even if the student did not pass previous modules. The lack of solid prior knowledge might influence a student's ability to comprehend the new knowledge in the following module, and therefore limit the conceptual understanding of the student. This problem is exacerbated if students lack base-line knowledge when entering university. Blignaut et al. (2013) and Siyepu (2013) argue that students who enter university unprepared, automatically are exposed to the risk of failure (cf. 1.1).

In sum, the lecturers regarded the following as the main weaknesses of the teacher-training curriculum: not including subject content coverage for geometry, a lack of time and teaching large classes, insufficient pedagogical knowledge and exposure to quality practical classroom experiences as weaknesses. The weaknesses cited by the lecturers were also supported by many of the student participants (cf. 4.4.4). Furthermore, the practice of allowing participants to continue with progressively more difficult modules without passing previous modules, poses threats to the solid content knowledge of the participants. Blignaut et al. (2013) and Plotz et al. (2012), argue that teacher-training institutions cannot afford to advance teachers with poor content knowledge as this will exacerbate the problem of poor mathematics performance at the school level (cf. 1.1).

The lecturers' perceptions about the limitations that the teacher-training curriculum poses to their teaching, are synthesised in the following section.

4.5.3 Lecturer perceptions: limitations of the mathematics teacher-training curriculum for teaching students

Table 4.71 (cf. Addendum D, p. 77) summarises the lectures views on the limitations that the teacher-training curriculum poses for teaching students.

According to two of the lecturers (Lecturers O and F), their teaching is prohibited by the teacher-training curriculum in the following ways. Lecturer F stipulated that some of the subject content is irrelevant to some phases, which, according to the researcher, supposedly limits the teaching time for the content that is relevant (Lecturer F previously mentioned time as a weakness in the curriculum). Lecturer O added that the teacher-training curriculum puts pressure on the lecturers to complete the content in a limited amount of time. Due to the time limitation, some topics and or sections of the mathematics content cannot be covered in depth.

Lecturer R suggests that the lecturer's content knowledge and experience in the subject mathematics are very important, and that a curriculum is what the lecturer makes of it. This response suggests that lecturer R feels that a lecturer's knowledge influences the delivery of teaching of content as well as the quality of the learning experiences of the students. The lecturer acknowledges that limitations in the curriculum might occur and that a curriculum should therefore be revised from time to time to improve the learning experiences of the students. It seems as if the opinion of Lecturer R supports the literature where it is argued that that one's personal knowledge influences the teaching of content as well as the learning experience of the students (Depaepe et al., 2016; Schoenfeld, 1992) (cf. 2.2.4), and their effectiveness as teachers (Lodewyk, 2011) (cf. 2.4.2).

Lecturer M asserted that the teacher-training curriculum corresponds to the school curriculum, despite lecturers O, R, and F indicating that the teacher-training curriculum does not include subject content for geometry. This observation of Lecturer M is worrying, as it appears as if the lecturer is not knowledgeable about the CAPS curriculum content for mathematics.

To summarise, the main limitations of the teacher-training curriculum posed to the teaching practice of the lecturers are: in-depth training is prohibited and the presentation of irrelevant content in the curriculum limits teaching time. Based on the aforementioned, the researcher concludes that the student participants might lack specialised subject content knowledge that is required to nurture conceptual understanding in learners, rather than just factual and procedural understanding (Fuentes et al., 2014) (cf. 2.4). Besides, time constraints for delivering content knowledge might become a contributing factor to poor academic achievement (Avalos et al., 2010) (cf.1.1).

The lecturers were also requested to reflect on the adequacy of the content and pedagogical knowledge offered in the teacher-training curriculum. Their views are shared in the section below.

4.5.4 Lecturer perceptions: adequacy of content and pedagogical knowledge in the teacher-training curriculum

Table 4.72 (cf. Addendum D, p. 78) presents the lecturers' perceptions on the adequacy of the content and pedagogical knowledge in the teacher-training curriculum.

Lecturer O was the only lecturer to disagree that the teacher-training curriculum provided students with adequate content and pedagogical knowledge. Lecturer O also indicated previously that content coverage was not comprehensive enough (cf. 4.5.2) and that the training lacked depth (cf. 4.5.3). Lecturer O again spoke about the fact that geometry subject content knowledge was not included in the mathematics curriculum, but is included in the curriculum for students who major in mathematical literacy. The response of Lecturer F latches on to that of Lecturer O indicating that the subject content should be more phase specific, probably referring to the fact that there is no Senior Phase-specific training.

Lecturer O further remarked that students were not taught to set examination papers, work on memorandums, or how to mark correctly. Previously, Lecturer R also indicated setting and marking of assessments as a weakness in the teacher-training curriculum. Lecturer M stipulated that the teacher-training curriculum offers opportunities for blended learning (an approach that combines online material and opportunities with traditional classroom teaching) (Hew & Cheung, 2014), and self-directed learning (an approach that allows students to take initiative in identifying their learning needs formulating own learning goals, identifying learning resources, implement their choice of learning strategies, and evaluating the outcome of their learning) (Knowles, 1975). The deficiencies observed concerning the participants' application of metacognitive skills (cf. 4.2.1), however, prove that self-directed learning needs stronger encouragement.

Concerning the teaching strategies that the lectures employ, Lecturer R commented on the importance of using inductive teaching strategies that shape comprehension. Lecturer O mentioned something very important, namely that a teaching strategy should align to a specific topic, for example, for geometry, discovery and problem-solving seem applicable teaching strategies, and for functions, explanation and independent problem-solving are suggested.

The strategies proposed by Lecturer M seem to resemble direct teaching, as the emphasis is on explanations with examples, explanations of strategies to solve problems, and application of knowledge with the assistance of the lecture after explanations. Lecturer M also mentioned the use of collaboration for solving difficult problems, which according to the researcher is a promising approach towards teaching mathematics.

Even though Lecturers R, F, and M indicated that the teacher-training curriculum provides adequate content and pedagogical knowledge, their answers indicated otherwise. None of them provided any indication of a variety of teaching strategies used during the training. The adequacy of the teacher-training curriculum according to Lecturers R, F, and M, is also not supported by the research participants who cited several limitations concerning subject content and pedagogical knowledge (cf. 4.4.2.2, 4.4.4, 4.4.5, 4.4.9).

In a nutshell, from the lecturers' perceptions about the adequacy of subject content and pedagogical knowledge the researcher concludes the following. Subject content knowledge does not seem to be adequate, and the exposure to a variety of teaching strategies is limited. Unfortunately, this observation brings to question the lecturers own pedagogical knowledge and application skills, along with their effectiveness in teaching for conceptual understanding (Brodie, 2001; Park & Oliver, 2008; Plotz et al., 2012; Umaru, 2010) (cf. 1.6.2.2, 2.3.2.4).

Based on the aforementioned the researcher suspects that the pre-service teachers might not be made aware of the importance of epistemological beliefs for teaching subject content. The researcher is also not sure what the lecturers' epistemological beliefs about teaching mathematics entail, and whether they make their students aware of the importance of epistemological beliefs (Killen, 2010; Schmidt, 1999; Tsanwani & Sinthumule, 2015) (cf. 2.4), given the limited strategies mentioned that they use in class during teaching, as well as the limited strategies that the student participants perceived to have for teaching the various subject content areas (cf. 4.4.9). The only lecturer that possibly considers the nature of the subject content during teaching and aligns this with specific pedagogical strategies is Lecturer O, who cited specific approaches for teaching trigonometry and functions (cf. 4.5.4).

The researcher also acknowledges that the teaching of mathematics at the school level might have caused gaps in the baseline knowledge of the pre-service teachers (mentioned by Lecturer F, cf. 4.5.2), which caused obstacles for the acquisition of subject content knowledge during their training. The researcher supports her argument by referring to the analysis of the CAPS (cf. 2.3.5), according to which no clear distinction between the acquisition of factual, conceptual, and procedural knowledge is made, that the development of metacognitive knowledge is not explicitly promoted, and that teachers are not encouraged to develop cognitive processes in conjunction with subject content (cf. Addendum E, cf. 2.3.5). It is thus questionable whether the student participants acquired knowledge related to all the knowledge dimensions. All of the aforementioned knowledge dimensions are implied in the CAPS curriculum and the teacher-training curriculum, but the researcher believes if pre-service teachers are not explicitly made aware of the various dimensions of knowledge that learners have to acquire, the development of some of the dimensions might fall by the roadside.

In the final section of the chapter, the researcher derives preliminary conclusions based on the data, by triangulating the quantitative and qualitative data collected during the study.

4.6 TRIANGULATING QUANTITATIVE AND QUALITATIVE DATA

The researcher derives the following preliminary conclusions based on the participants' test data, curriculum analyses, and written reflections of the student participants and lecturers. The researcher included quantitative and qualitative data collection methods to check for the consistency of findings, to confirm findings, and to enhance the understanding of the studied phenomenon concerning whether pre-service mathematics teachers meet the subject content knowledge and pedagogical content knowledge expectations required of teachers.

Table 4.73 provides an overview of how the data were integrated to support the confirmation of initial findings (cf. Chapter 5) about whether the participants meet the competency expectations concerning subject content and pedagogical content knowledge to effectively teach mathematics.

Table 4.73: Triangulation of data

Data collection instruments					
Competency expectations	Student tests Subject content questions	Document analysis: curricula	Student tests with open questions	Student tests with closed Likert scale questions	Lecturer questionnaire with open questions
Subject content knowledge	Factual knowledge and cognitive process dimensions (cf. test, section B; Addendum C)	Factual knowledge and cognitive process dimensions (cf. Addendums E, F; cf. 2.3.5, 2.4.6)	Strengths and weaknesses of the curriculum (cf. 4.4.3, 4.4.4)	Preparedness to teach subject content (cf. 4.4.2.1)	Strengths and weaknesses of the curriculum (cf. 4.5.1, 4.5.2)
	Conceptual knowledge and cognitive process dimensions (cf. test, section B; Addendum C)	Conceptual knowledge and cognitive process dimensions (cf. Addendums, E, F; cf. 2.3.5, 2.4.6)	Changes to improve curriculum (cf. 4.4.5)		Limitations of the curriculum for teaching (cf. 4.5.3)
	Procedural knowledge and cognitive process dimensions (cf. test, section B, Addendum C)	Procedural knowledge and cognitive process dimensions (cf. Addendums E, F; cf. 2.3.5, 2.4.6).	Reasons for being a good/not so good teacher (cf. 4.4.6, 4.4.7)		Adequacy of subject content and pedagogical knowledge (cf. 4.5.4)
			Easiest and most difficult question (cf. 4.4.8)		
		Metacognitive knowledge (cf. 2.3.5)	Metacognitive strategies (cf. Test, section C; cf. 4.4.1) (cf. Addendum J)		
Pedagogical knowledge	Pedagogical (cf. test, section B; Addendum C)	Pedagogical content knowledge (cf. 2.4.6; Addendum F)	Pedagogical strategies used to teach subject content areas (cf. 4.4.9)	Availability of a variety of teaching strategies (cf. 4.4.4.2)	Strengths and weaknesses of the curriculum (cf. 4.5.1, 4.5.2)
			Strengths and weaknesses of the curriculum (cf. 4.4.3, 4.4.4)		
			Changes to improve curriculum (cf. 4.4.5)		Limitations of the curriculum for teaching (cf. 4.5.3)
			Reasons for being a good/not so good teacher (cf. 4.4.6, 4.4.7)		Adequacy of subject content and pedagogical knowledge (cf. 4.5.4)

To avoid duplication of information, the triangulation of data is incorporated in the discussion of the findings of the study (cf. 5.4).

To conclude the chapter, the final section, section 4.7, summarises the gist of the data analyses and interpretations presented in the chapter.

4.7 CHAPTER SUMMARY

From the quantitative data collected with the student tests that focused on subject content knowledge (factual, conceptual, procedural, and metacognitive) (cf. 4.3.3.1 – 4.3.3.3), the following transpired. The participants obtained low test averages for all the FET subject content areas (cf. Table 4.2), suggesting that subject content knowledge is limited (cf. Graphs 4.19 – 4.21). The subject content of the Senior Phase participants seemed to be more solid, as positive test results were revealed. The test results of the Intermediate Phase participants did not reveal solid knowledge for all the subject content areas. Across the three phases where factual knowledge and related cognitive process dimensions appeared to be relatively well developed, success in the application of cognitive process dimensions in the conceptual and procedural knowledge dimensions benefitted.

The quantitative semantic-scale data (cf. 4.4.2), and the written reflection data collected from the student participants as part of the tests (cf. 4.4.9), alerted the researcher to the limited teaching strategies used by the FET participants, which could mainly be classified as direct teaching strategies, namely: drilling, providing information and examples to learners, learners practising examples, and the use of demonstrations. The strategies mentioned by the Senior Phase participants to teach the various subject content areas also mainly revolved around the use of direct teaching strategies, such as demonstrations, explanations, providing information and summaries to students, and drilling. The Intermediate Phase participants' perceptions confirmed the possession of a very limited range of teaching strategies that mainly focused on the direct teaching of content, using demonstrations, explanations, repetition, and practise (cf. 4.4.9).

The qualitative data collected through written reflections (cf. 4.4.1, 4.4.4 – 4.4.8) as part of the student tests mainly related to the teacher-training curriculum and confirmed strengths concerning teaching practice, teaching methods and strategies, good lecturer support, lecturers who are prepared and knowledgeable, and the development of attitudes and dispositions during the training (cf. 4.4.3). Weaknesses of the curriculum pertained to subject content, not having adequate teaching strategies to teach mathematics, as well as curriculum-related aspects, such as limited lecturing time and exposure time to subject content training (cf. 4.4.4). Some suggestions were made to improve the teacher-training curriculum and included suggestions such as covering the school-level content more thoroughly, equipping students with strategies to teach

mathematics, and reducing irrelevant modules in the curriculum (cf. 4.4.5). Many participants regarded themselves as good teachers (cf. 4.4.6) because of the positive personality traits, dispositions, and attitudes toward teaching mathematics they possess, having a concern for learners' progress and understanding of mathematics, and possessing content knowledge and teaching strategies (which however contradicts the weaknesses concerning the training that were mentioned) (cf. 4.4.4). The gaps identified in the subject content knowledge based on the test results, were some of the reasons mentioned that would not enable the participants to be good teachers (cf. 4.4.7).

The qualitative data collected through written reflections by the lecturers vouches for the subject content knowledge presented as part of the teacher-training curriculum being a strength, however, acknowledging limitations concerning the exclusion of geometry as part of the curriculum and mentioning a lack of time and the teaching of large classes that possibly reduce the chances of pre-service teachers to obtain solid, in-depth content knowledge (cf. 4.5.2). Insufficient pedagogical knowledge and exposure to quality practical classroom experience as well as allowing participants who failed modules to continue with progressively more difficult modules, were cited as weaknesses (cf. 4.5.2). A lack of in-depth training and irrelevant content in the curriculum that limit teaching time were mentioned as the main limitations posed to the teaching efforts of the lecturers (cf. 4.5.3). Subject content knowledge does not seem to be adequate, and the exposure to a variety of teaching strategies limited (cf. 4.5.4), according to the lecturers.

The qualitative data collected through document analyses (cf. 2.3.5, 2.6.4) (Addendum E, Addendum F), provided evidence of the following: the CAPS does not clearly distinguish between the development of factual, conceptual, procedural, and metacognitive knowledge, although an evaluation of the CAPS outcomes for the different subject content areas in mathematics supported the development of facts, concepts, procedures and strategic competence, which entails metacognition. The CAPS outcomes focus on the development of various cognitive processes, but the importance of, and "how" these processes should be developed in conjunction with subject content knowledge, appear elusive.

Similarly, the teacher-training curriculum contains outcomes that emphasise the development of cognitive processes in each of the mathematics subject content modules, but how these processes should be developed in conjunction with mathematical subject content knowledge is not clear. A clear distinction between factual, conceptual, and procedural subject content knowledge is also not present, with no mention of how metacognitive knowledge could be developed. The Senior Phase pre-service teachers receive FET training, questioning the solidity

of their training to become Senior Phase teachers. Also, the subject content concerning geometry is not presented adequately as part of the FET training (cf. Addendum F).

In overview, the data collected through the various data collection instruments guide the researcher to believe that the pre-service teachers who were involved in the research do not fully meet the expectations concerning subject content and pedagogical knowledge to become effective mathematics teachers.

The final chapter, Chapter 5, presents the findings, recommendations, and conclusion of the study.

CHAPTER 5: SUMMARY FINDINGS AND RECOMMENDATIONS

5.1 INTRODUCTION

The study was aimed at determining to what extent the teacher-training curriculum for the Intermediate (Gr. 4-6), Senior (Gr. 7-9) and the FET (Gr. 10-12) phases at the NWU, Vanderbijlpark Campus, prepare mathematics teachers to meet the competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge foregrounded in the CAPS for teaching mathematics (cf. 1.4.1). Subsequently, the study also makes recommendations to strengthen, adapt or enrich the current mathematics teacher-training curriculum used at the NWU and possibly other universities (cf. 1.4.1). These recommendations could assist in ensuring that teachers meet the competency expectations for subject content knowledge and pedagogical content knowledge.

The aim and objectives that were formulated at the beginning of the study are re-assessed in this chapter to determine whether they were achieved. It is important that the literature study and the data gathered, answered the primary and secondary questions which steered the study. This chapter addresses the following aspects.

- 5.2 AN OVERVIEW OF THE STUDY**
- 5.3 FINDINGS FROM THE LITERATURE REVIEW**
- 5.4 FINDINGS OF THE EMPIRICAL STUDY**
- 5.5 FINDINGS CONCERNING THE AIM AND OBJECTIVES OF THE STUDY**
- 5.6 RECOMMENDATIONS TO STRENGTHEN THE TEACHER-TRAINING CURRICULUM**
- 5.7 LIMITATIONS OF THE STUDY**
- 5.8 SUGGESTIONS FOR FURTHER RESEARCH**
- 5.9 CONTRIBUTION OF THE STUDY**
- 5.10 CONCLUSION**

5.2 AN OVERVIEW OF THE STUDY

This section provides a brief overview of the gist of each chapter.

- **Chapter 1**

Chapter 1 presented the research problem on which the research focused and clarified the theoretical and conceptual frameworks that guided the execution of the study. Teacher training seemingly does not prepare teachers to meet the competency expectations concerning mathematical subject content knowledge and pedagogical knowledge that in turn impact on their

ability to teach mathematics effectively (Adedoyin, 2011; Avalos et al., 2010; Blömeke et al., 2011; Bowie & Reed, 2016; Farooq et al., 2011; Fung et al., 2017; Jojo, 2019; Kanyongo & Brown, 2013; Malaty, 2004; May, 2017; Mij & Makgato 2006; Plotz et al., 2012; Taylor, 2018; Venkat & Spaul, 2015) (cf. 1.1). The study employed the theoretical framework of Hill et al. (2008) to conceptualize the competency expectations concerning teachers' subject content knowledge in mathematics and pedagogical content knowledge for teaching mathematics (cf. 1.5.1). Subject content knowledge involves the following dimensions: common content knowledge, specialised content knowledge, horizon content knowledge (cf. 1.5.1). Pedagogical content knowledge involves knowledge about how learners learn, employing strategies to teach subject content, and knowledge about lesson planning to achieve learning outcomes (Hill et al., 2008) (cf. 1.5.1).

The revised taxonomy of Bloom (Krathwohl, 2002) assisted the researcher in understanding and describing the competency expectations concerning common subject content knowledge, as factual, conceptual, procedural and metacognitive knowledge dimensions that have to be developed in conjunction with cognitive process dimensions (remember, understand, analyse, apply, evaluate, and create) to ensure solid subject content knowledge (cf. 1.5.1).

- **Chapter 2**

Chapter 2 expanded on understanding the competency expectations concerning subject content knowledge and pedagogical content knowledge. The importance of upholding epistemological beliefs during the teaching of mathematics subject content was highlighted. Epistemological beliefs direct a teacher's view of the nature of subject content knowledge and how it may be acquired best (Green & Hood, 2013; Lodewyk, 2015; Schraw & Olafson, 2003) (cf. 2.2.1). The specific epistemological beliefs related to mathematics teaching were aligned to the general epistemological beliefs, namely the absolutist or realist (knowledge is either true or false, and only one truth exists) (cf. 2.2.5), and the fallibilist or relativist and contextualist epistemological beliefs (Blömeke et al., 2008; Depaepe et al., 2016) (knowledge comprises multiple truths depending on the context or perspectives of a person) (Brownlee et al., 2016; Green & Hood, 2013; Pynn, 2016; Stoel et al., 2017) (cf. 2.2.5).

The major findings from the literature review concerning the competency expectations for subject content knowledge and pedagogical knowledge are presented as part of Section 5.3 in order to avoid duplication of information.

- **Chapter 3**

This chapter explained and motivated the choice of the research methodology that was employed in the study. A concurrent triangulation mixed methods design with pragmatism as the research

paradigm was selected for the study (Creswell, 2014) (cf. 3.4). The study included both quantitative and qualitative data collection methods to derive the findings (cf. 3.5). As part of the quantitative study, the researcher used a non-experimental, descriptive, survey research strategy (Delport & Roestenburg, 2011; Lodico et al., 2010; Maree & Pietersen, 2016a) (cf. 3.4.2.1), and for the qualitative part of the study, critical educational research was used (Hancock et al., 2009) (cf. 3.4.2.2). The data were collected using various data collection methods to gather information from 4th year pre-service mathematics teachers (n = 19) and mathematics lecturers (n = 4) about the subject content knowledge and pedagogical content knowledge of the pre-service teachers, as well as the qualities of the teacher-training curriculum (cf.3.5). Lastly, the qualitative data collection also included a document analysis of the NWU teacher-training curriculum against the CAPS to establish alignment concerning the competency expectations for mathematics teachers (cf. 3.5.2).

- **Chapter 4**

In this chapter the data obtained from the test and the interview questions were analysed and interpreted. The essence of this chapter is presented as the findings that emanated from the research in Section 5.4 of this chapter in order to avoid duplication of information.

The next section elucidates the findings that were retrieved from the literature review.

5.3 FINDINGS FROM THE LITERATURE REVIEW

This section presents the main findings concerning competency expectations about subject content knowledge and pedagogical content knowledge that were derived from the literature review. It was imperative to establish whether the teacher-training curriculum meets the competency expectations in the CAPS, since Feza (2014) argues that curriculum reform is not aligned with adequate teacher preparation.

5.3.1 Findings concerning subject content knowledge

Aligned to the competency expectations for teaching mathematics in South Africa according to the Department of Basic Education (2018) (cf. Figure 2.2), Bloom's revised taxonomy (Anderson et al., 2001) (cf. 2.3.1.1) guided the researcher's understanding of subject content knowledge in the context of the study as follows:

Concerning the dimensions of knowledge, the following could be viewed as important findings that emanated from the literature review.

- **Factual knowledge** covers basic theory, rules, algorithms and terminology which is a fundamental part of succeeding in mathematics (Ball et al., 2008; Kahan et al., 2003; Löwe & Müller, 2010). Without knowing the theory and terminology, the application content will become increasingly difficult (cf. 2.3.2.1).
- **Conceptual knowledge** is important for knowing why a certain procedure/concept is necessary and to understand how a procedure/concept is applied to problem-solving (Crooks & Alibali, 2014; Lauritzen, 2012; Rittle-Johnson et al., 2015) (cf. 2.3.2.2). Conceptual knowledge therefore implies the understanding and interpretation of mathematics, and not the application of knowledge (Lauritzen, 2012; Rittle-Johnson et al., 2015; Stols, 2013) (cf. 2.3.2.2).
- **Procedural knowledge** is the knowledge of identifying a problem, and knowing how to correctly solve it (Department of Basic Education, 2018; Eisenhart et al., 1993; Haapasalo, 2003; Rittle-Johnson et al., 2015; Star & Stylianides, 2013) (cf. 2.3.2.3).
- **Metacognitive knowledge** shows that one is able to intentionally and purposefully assess the quality of one's thinking (Al-Kayyali & Al-Haq, 2018; Schneider & Artelt, 2010) (cf. 2.3.2.4). It involves reflecting about planning, monitoring, and evaluating learning with the aim to increase successfulness and enhance performance (Bannert & Reimann, 2012; Cazan, 2012; Zumbunn et al., 2011) (cf. 2.3.2.4).

5.3.2 Findings concerning subject content knowledge and cognitive process dimensions

The teaching of the various knowledge dimensions is always accompanied by the infusion of various cognitive processes within the subject content knowledge, namely: remember, understand, apply, analyse, and create (Krathwohl, 2002) (cf. 2.3.3). In this regard, the CAPS refers to the cognitive processes as knowledge, routine procedures, complex procedures, and problem-solving (Department of Basic Education, 2011a, b, c) (cf. 2.3.3).

5.3.3 Findings concerning pedagogical content knowledge

Pedagogical content knowledge comprises three components, that emphasise the interrelatedness of teacher and subject content, of teacher and learner, and of learner and content (Killen, 2010; Sibuyi, 2012; Stronge, 2018; Tsanwani & Sinthumule, 2015; Ünsal, et al., 2018) (cf. 2.4). More specific, the components refer to the following:

- knowledge about how learners learn certain subject content;
- knowledge about how to employ different methods and strategies to enable the acquisition of knowledge; and
- knowledge for planning teaching to achieve outcomes, and assessing what is taught (Hill et al., 2008) (cf. 2.4).

Knowledge about how to employ different teaching methods and strategies during teaching, which was the focus in this study, includes four main categories namely: direct teaching, indirect teaching, independent teaching and interactive teaching (cf. 2.4).

Learning in mathematics is enhanced when learners are challenged to link mathematical concepts for deeper understanding (Green & Hood, 2013) (cf. 2.2). Effective mathematics teachers incorporate higher-order thinking strategies during teaching, for example, the use of inquiry-based questions or real-life problems, and a variety of activities and strategies to engage students in group discussions (Brodie & Molefe, 2010; Kunene & Sepeng, 2018; Sepeng & Webb, 2012; Stronge, 2018; Venkat, 2013) (cf. 1.1, 2.4). Problem-based learning that focuses on application is argued to make mathematics more understandable, but seems to be underscored in mathematics classrooms (Bansilal et al., 2014; Cazzola, 2008; Luneta, 2018) (cf. 1.1), probably because teachers have not experienced its application and do not understand the application of problem-based learning (Cazzola, 2008) (cf. 1.1). The Department of Basic Education (2018) advocates for a learning-centred classroom where interaction between teachers and learners is central (cf. 2.4.5).

5.3.4 Findings concerning the relationship between the different knowledge dimensions

The interrelatedness of the various knowledge dimensions appears to be crucial for being successful in all the knowledge dimensions (cf. 2.3.2.5). Solid factual knowledge promotes the understanding of conceptual knowledge, and conceptual knowledge influences the effective application of procedures (Crooks & Alibali, 2014; Kahan et al., 2003; Long et al., 2014; Löwe & Müller, 2010; Rittle-Johnson et al., 2015) (cf. 2.3.2.5).

5.3.5 Findings concerning the relationship between subject content knowledge, pedagogical knowledge, and epistemological beliefs

The teaching of mathematics should be approached from a variety of epistemological beliefs, as the nature of mathematical subject content knowledge is multi-dimensional (Blömeke et al., 2008; Depaepe et al., 2016) (cf. 2.2.5). Considering different epistemological beliefs can help teachers to teach for factual acquisition, conceptual understanding and procedural application (Blömeke et al., 2008; Plotz et al., 2012) (cf. 2.1), and not only as fixed truths that have to be transmitted (Depaepe et al., 2016) (cf. 2.2.5).

5.3.6 Findings concerning the competency expectations for mathematics teachers in the CAPS and teacher-training curriculum

A comparison was made between the CAPS and teacher-training curriculum to establish alignment between the two curricula concerning competency expectations for subject content knowledge (cf. 2.3.5) and pedagogical subject knowledge (cf. 2.4.6) that teachers have to acquire to enable them to become effective teachers. The CAPS was regarded as the guideline, as the CAPS guides teaching and learning in South African classrooms. A succinct overview of the main findings emanating from the comparison include the following:

- Both curricula focus on the development of factual, conceptual, and procedural knowledge, though not clearly distinguishing between the three dimensions.
- Both curricula do not explicitly provide guidance about how metacognitive knowledge could be developed.
- Both curricula do not explicitly promote the development of cognitive process dimensions in conjunction with subject content knowledge, although the learning outcomes to be achieved bear evidence of numerous cognitive processes that have to be acquired within the subject content.
- Both curricula are vague in terms of teaching strategies to best teach mathematics subject content. However, the Department of Basic Education (2018) specifies the importance of creating a learning-centred classroom that supports active and interactive learning (cf. Figure 2.2, cf. 2.4.4).
- The analysis of the teacher-training curriculum revealed gaps in the preparation of mathematics teachers who need well-rounded subject content and pedagogical content knowledge to become effective teachers (Adedoyin, 2011; Avalos et al., 2010; Blömeke et al., 2011; Ikeda, 2013; Luneta, 2018) (cf. 1.1, 2.3.5, 2.4.6).

5.3.7 Findings concerning the attributes of teachers' mathematical subject content knowledge

The mathematical subject content knowledge that a qualified teacher needs to possess should be characterised by a number of attributes:

- Common or basic subject content knowledge that could be classified as factual, conceptual, and procedural knowledge concerning the grade they have to teach (Ball et al., 2008; Holmes, 2012) (cf. 2.3.4.1).
- Specialised subject content knowledge that comprises depth (saturated knowledge of topic within a specific content area) and breadth (knowledge of a variety of topics with content areas) (Green & Hood, 2013) (cf. 2.3.4.2). Depth is also intertwined with

progression of knowledge across the grades within a teaching phase (Department of Basic Education, 2011a, b, c) (cf. 2.3.4.2). Specialised content knowledge enables a teacher to nurture conceptual understanding in learners, rather than just focusing on the acquisition of factual knowledge and procedural understanding (Fuentes et al., 2014; Kanyongo & Brown, 2013) (cf. 2.3.4.2).

- Horizon subject content knowledge refers to being aware of the interrelatedness between mathematical topics that might not be addressed in the curriculum (Ball & Bass, 2009; Ball et al., 2008) (cf. 2.3.4.3).

The following section presents the findings that emanated from the empirical study.

5.4 FINDINGS FROM THE EMPIRICAL STUDY

The data collection focused on two aspects, namely competency expectations of subject content knowledge and pedagogical knowledge. The main findings integrate the data obtained from the various data collection instruments and are briefly presented below.

5.4.1 Findings concerning biographic information

According to Table 4.1, the majority of the participants across the three phases were representative of the age groups 20-22 years. An equal number of male and female participants in the FET and Senior Phases were included in the sample, and the Intermediate Phase participants were mainly female participants. Most of the FET and Senior Phase participants, namely three, were involved in weekly extra mathematics classes at school, whereas the majority of the Intermediate Phase participants, namely ten, were not involved in any extra mathematics classes at school, thus not exposed to applying mathematics subject content knowledge wider than their daily teaching, that, according to the researcher, could assist in reinforcing subject content knowledge.

The participants completed their matric at 14 different public schools, of which six were township schools. No private schools were involved in the study. Participants also attended seven of the so-called "model C" schools, a term which is still commonly used to describe former whites-only government schools.

The mathematics lecturers (cf. 4.5.1) were all female, well-qualified, and experienced in the field of teaching. The researcher is of the opinion that it would however have been beneficial to teacher-training if they were all qualified in the field of mathematics.

5.4.2 Findings concerning competency expectations for subject content knowledge of the pre-service teachers

Finding 1: the participants in the Senior Phase obtained better average test results overall, than the FET and Intermediate Phase participants. The total test averages for the participants were as follows: FET (26.31%), Senior (76.49%), and Intermediate (57.90%) (cf. Tables 4.2, 4.3, 4.5). All of the participants therefore seem to lack solid content knowledge, which is evident in the following:

- The FET participants obtained low test averages for all subject content areas, suggesting that subject content knowledge is limited, with averages for subject content areas ranging between the highest, 59.52% for Question 5 (functions), and 0% for Question 6 (differential calculus). In general, it appeared that where application of cognitive process dimensions in the factual knowledge dimension were well-developed, it contributed to success in the application of cognitive process dimensions in the conceptual and procedural knowledge dimensions (cf. Finding 4).
- The subject content of the Senior Phase participants seemed to be more solid, and they obtained encouraging results, with test averages ranging between the highest, 96.10%, for Question 2 (numbers, operations, and relationships), and the lowest, 58.92%, for Question 5 (data handling).

Finding 2: compared to their actual test results, the FET and Intermediate Phase participants seem to have an unrealistic perception of their levels of content knowledge.

- The test results obtained by the Intermediate Phase participants for the various subject content area did not reveal solid knowledge for all the subject content areas. The participants obtained the highest average for Question 5 (data handling), namely 90.27%, and the lowest averages for Question 4 (space and shape (geometry)), with 38.33%, and Question 1 (numbers, operations, and relationships), with 49.58%.
- Similar to the FET Phase participants, the Intermediate Phase participants also appeared to have an unrealistic view of their preparedness to teach subject content. Most of the participants perceived to be best prepared to teach measurement, data handling, and numbers, operations, and relationships. (cf. Graph 4.21). For the mentioned content areas, test averages of 52.77%, 90.27%, and 49.58% were obtained respectively. Only concerning data handling (90.27%) the participants' view could be regarded as realistic, as they obtained the best test average for data handling. The researcher finds it strange that the participants did not view themselves to be well-prepared to teach patterns,

functions, and algebra for which the second highest test average, namely 75%, was obtained.

- Linked to the previous observation, in response to their preparedness to teach mathematics (cf. 4.4.2), most of the FET Phase participants perceived themselves to be prepared to teach functions, algebra and equations, finance, growth and decay, number patterns, sequences, and series as well as statistics (cf. Graph 4.19), which do not completely correspond with their aforementioned test averages. Also, the test averages obtained for the content areas mentioned by the participants do not reveal solid content knowledge: algebra and equations (28,57%), finance, growth, and decay (20,63%), number patterns, sequences, and series (28,57%), and statistics (26,19%) (cf. Table 4.2).

Finding 3: in comparison to their test results, the Senior Phase participants seemed to have a more realistic view of their preparedness to teach subject content.

- The Senior Phase participants viewed themselves to be the best prepared to teach numbers, operations and relationships, and patterns, functions, and patterns, functions, and algebra. Test averages of 96,10%, and 85,71% were obtained for these two content areas respectively (cf. Table 4.3). As the participants obtained the best averages for these two content areas, it seems as if their perceptions regarding preparedness to teach these two content area could be regarded as realistic, although some gaps in the subject content knowledge appear to be evident (cf. 4.3.3.2).

Finding 4: Across the three phases (FET, Senior, and Intermediate) where factual knowledge appeared to be relatively well developed, success was observed in the conceptual and procedural knowledge dimensions as well (cf. 4.3.3 – 4.3.5):

- FET Phase: for example, Questions 3.1 (numbers, operations, and relationships), 4.1 (finance, growth, and decay), 5.1, and 5.3 (functions).
- Senior Phase: for example, Questions 1.1, 1.2.1 (patterns, functions, and algebra), 2 (numbers, operations, and relationships), and 3.1, space and shape (geometry), and 4.1 (measurement) (cf. Graphs 4.9, 4.10, 4.11, 4.12).
- Intermediate Phase: for example, Questions 1.1, 1.3, 1.8 (numbers, operations, and relationships), 2.2, 2.3 (patterns, functions, and algebra), 3.2 (measurement), and 5 (data handling) (Graphs 4.14, 4.15, 4.16, 4.18).

Finding 5: Where the factual knowledge seemed to be less solid, or absent the participants in all three phases seemed to find the conceptual and procedural knowledge dimensions and related cognitive process dimensions demanding:

- FET Phase: For example, (Questions 3.2 (numbers, patterns, sequences and series), 4.2 (finance, growth, and decay), and 5.2 (functions) (cf. Graphs 4.4, 4.5, 4.6).
- Senior Phase: Questions 1.2.2 (patterns, functions, and algebra) (cf. Graph 4.9).
- Intermediate Phase: Questions 1.4, 1.5, 1.6 (numbers, operations, and relationships), 2.1 (patterns, functions, and algebra), 3.1.1, and 3.1.2 (measurement), 4.2 (space and shape (geometry) (Graphs, 4.14, 4.15, 4.16, 4.17).

Finding 6: Often, successful factual and conceptual knowledge seemed to have a limited influence on being successful in the conceptual and/or procedural knowledge dimension:

- FET Phase: for example, Questions 1 (differential calculus), and 6.1 (probability) (cf. Graphs 4.2, 4.7).
- Senior Phase: for example, Questions 1.2.2 (patterns, functions, and algebra), and 3.2 (space and shape) (cf. Graphs 4.9, 4.11).
- Intermediate Phase: for example, Questions 4.1, 4.2, space and shape (geometry) (cf. Graph 4.17).

Finding 7: Subject content knowledge was cited as a weakness in the teacher-training curriculum by all the participants and the mathematics lecturers.

- In this regard participants felt that the subject content they were taught is complex and does not relate to the school context. In particular reference was made to Grade 12 subject content being weak, the lack of content about geometry, that no subject content training is provided in the fourth year of study, and that irrelevant modules in the curriculum reduce time that could have been used to place a stronger focus on mathematics subject content (cf. 4.4.4). Aligned to the aforementioned weaknesses, improving subject content knowledge was therefore highlighted as a suggestion to improve the teacher-training curriculum (cf. 4.4.5), and also noted as a reason for not to be regarded as a good teacher (cf. 4.4.7).
- Mathematics lecturers perceived the subject content knowledge to be a weakness in the teacher-training curriculum, and not adequate (cf. 4.5.2, 4.5.4). The mathematics lecturers argued for improvement in the teacher-training curriculum by citing that content needs to be presented more in-depth and thoroughly, with a focus on school-level content (cf. 4.5.2, 4.5.3). Similar to the participants, they mentioned that irrelevant modules in the curriculum need to be reduced, and a stronger focus placed on mathematics subject content. Not including subject content coverage for geometry, a lack of time, and teaching large classes, pose limitations to students to obtain solid content knowledge. The practice of

allowing participants to continue with progressively more difficult modules without passing previous modules, also poses threats to acquiring solid content knowledge (cf. 4.5.2).

- The fact that participants experienced some of the content areas as difficult, could be an indication that there might be gaps in their subject content knowledge (cf. 4.4.8, Tables 4.61 - 4.63). The Senior Phase in particular mentioned the questions concerning space and shape, and measurement to be difficult, which possibly links with the gap in the teacher-training curriculum that does not focus on geometry for the Senior Phase.

Finding 8: Limited subject content knowledge is mentioned as a reason that would disqualify participants to be viewed as good teachers.

- Inadequate subject content knowledge, a lack of understanding of the subject content knowledge, and making mistakes, were presented by the participants as reasons that would disable them to be good teachers (cf. 4.4.7). This perception is supported by the poor test results obtained by the FET participants for differential calculus and statistics (cf. 4.3.3).
- The participants' perceptions do not reveal that they view themselves as being prepared to teach all the subject content areas (cf. 4.4.2.1). Also, the responses about the easiest and most difficult sections in the test (cf. 4.4.8) attest to gaps in subject content knowledge of the participants, as well as the test results (cf. 4.3.3 – 4.3.5).

Finding 9: The teacher-training curriculum poses limitations to the acquisition of solid subject content knowledge.

The comparison of the CAPS (cf. 2.3.5) and teaching-training curriculum (cf. 2.4.6) support the findings that point to limitations in the teacher-training curriculum concerning subject content knowledge.

- Probability and statistics which appear to be two problem content areas in the FET Phase (cf. Table 4.2) share a one-semester module (8 credits each; 80 notional hours each) in the participants' first year of study, which could imply that time might be limited to prepare teachers with solid knowledge to teach these content areas (cf. Table 2.8). Additionally, the participants might have forgotten the subject content taught in their first year of study.
- Numbers, patterns, and sequences, and finance growth and decay (FET Phase), are presented as sections in one 16-credit semester module ($\pm$ 80 notional hours each) in the second year of study, thus receiving limited teaching time (cf. Table 2.8) This could have contributed to participants not acquiring solid subject content knowledge as reflected in the test results of 28,57%, and 20,63% respectively.

- Algebra and equations (FET Phase) are taught as part of one 16-credit semester module together with numbers, patterns, and sequences as well as finance, growth, and decay in the second year of study, and in one 16-credit module in the third year of study, thus receiving more teaching time than the other content areas (cf. Table 2.8). Compared to all the other content areas, algebra seems to receive the most teaching time. Nevertheless, the additional time did not seem to have influenced the participants to achieve better results as compared to the other content areas for which less teaching time was allocated. Low average results were achieved for algebra and equations (33,92%).
- The content area functions (FET Phase), that was also presented in a one-semester module (16 credits, 160 notional hours) in the participants' first year of study (cf. Table 2.8), only revealed a moderate achievement (59,52%) (cf. Table 4.2). Seeing that content that was addressed in the participants' first year of study, could have implied that they might have forgotten some of the content by their fourth year of study.
- Differential calculus (FET Phase) which is presented as part of a one-semester module (16 credits) in the third year of study (cf. Table 2.8), revealed disappointing results. The test average was 0%, which implied that compared to the results for probability and statistics for which less teaching time was allocated in the curriculum, the participants' performance in differential calculus was extremely deficient.
- The MATE 121 module which is divided between probability and statistics, needs to receive greater coverage and weighting within the curriculum, as the FET phase participants scored 0% as a question average for probability, and only 26,19% average.
- The FET curriculum does not include content on analytical geometry and trigonometry, and the module content of Euclidean and spherical geometry differs from school content. The Senior and FET phases do combined training, and although the researcher initially assumed that the Senior Phase pre-service teachers might have limitations to their content knowledge due to attending mainly the FET classes (cf. 2.3.5), they seem to possess more solid Senior Phase subject content knowledge than the FET participants (cf. Tables 4.2, 4.3).
- The Senior Phase subject content was likely well-anchored at school-level, or the FET training might be more Senior Phase focused. Some of the lowest averages in the Senior Phase test were obtained for Question 3 (space and shape (geometry)) (61,03%), and Question 4 (measurement) (64,28%), which could probably be because of the limited focus on these content areas during the FET training.
- The Intermediate Phase participants only start their subject content training in the second year of study and only have content training until the third year (cf. 2.3.5), which could hamper the acquisition of solid subject content knowledge. Examining the average results

obtained by the Intermediate Phase participants, pointed to gaps in their subject content knowledge (cf. Table 4.5).

Main trends emanating from the written reflections of the participants about the teacher-training curriculum support the test results that point to gaps in subject content knowledge as follows:

- The main strength identified by the lecturers seems to be the subject content knowledge presented as part of the teacher-training curriculum, in particular, mention is made of increased interest in mathematics and enhancing the ability to teach the content (cf. 4.5.2). The perceptions of the lecturers contradict the student participants' test results that do not reveal solid subject content knowledge (cf. 4.3.3.1 – 4.3.3.3) and do not confirm the participants' observation of subject content knowledge being a weakness in the curriculum (cf. 4.4.4), the participants' viewpoint that a lack of understanding of the subject content knowledge could be seen as a reason for them not being regarded as good teachers (cf. 4.4.7), and the indication given by the participants that they experience many subject content areas to be difficult (cf. 4.4.8).
- It is positive that the lecturers mentioned the coverage of geometry in the curriculum as a major weakness (cf. 4.5.2), thus supporting the participants' perceptions (cf. 4.4.4, 4.4.5). Although subject content knowledge was viewed as a strength by the lecturers, they did acknowledge that the training lacked depth, and that irrelevant content poses limitations to the curriculum (cf. 4.5.3). The latter observation confirms the perceptions of the student participants who suggested among others, the following changes to the curriculum: the presentation of content relevant to school, doing away with irrelevant methodology modules, and providing content coverage of all grades (cf. 4.4.5).
- Another aspect that could possibly contribute to subject content knowledge being limited is mentioned by the lecturers, namely allowing participants to continue with progressively more difficult modules without passing previous modules (cf. 4.5.2). According to the researcher, such an arrangement could pose threats to the acquisition of solid content knowledge.
- From the lecturers' responses, the researcher carefully concludes that subject content knowledge could not be regarded as adequate, in particular concerning the subject content knowledge for geometry in the Senior Phase and FET Phase (cf. 4.5.4)

Finding 10: Specific problems in relation to subject content areas were observed.

Main problems observed in the FET Phase subject content (cf. 4.3.3) that emanated from the researcher's analysis of the test responses were the following (cf. Addendums G1 – G3).

- Differential calculus: the participants could not make connections between and across content areas and could not apply, analyse, evaluate and create information across all the knowledge dimensions (cf. Table 4.8, Addendum I1, p. 116).
- Number patterns, sequences, and series: participants did not identify the geometric sequence formula explicitly in Question 3.1, thus not demonstrating proof of conceptual understanding (cf. Table 4.12, Addendum I1, p.123). In Question 3.2 the participants could not make the connections between and across topics.
- Algebra and equations: conceptual understanding appeared to be troublesome (cf. Table 4.10, Addendum I1, p. 119), which possibly indicates gaps in the factual knowledge dimension.
- Finance, growth, and decay: the participants might have had limited exposure to questions containing two graphs on one plane, and not enough integrated knowledge that hampered their understanding (cf. Table 4.14, Addendum I1, p.127).
- Functions: solid factual and conceptual content knowledge appeared limited and contributed to being unsuccessful in the procedural knowledge dimension (cf. Table 4.16, Addendum I1, p.133).
- Probability: many participants did not attempt to answer the probability question, thus suggesting that the participants experienced problems with the application of cognitive processes in all the knowledge dimensions (cf. Table 4.18, Addendum I1, p.140).
- Statistics: gaps in the factual, conceptual and procedural knowledge dimensions were observed (cf. Table 4.20, Addendum I1, p.142).

Main problems observed in the Senior Phase subject content knowledge (cf. 4.3.4):

- Numbers, operations, and relationships: sign errors seemed to be the main problem in answering the questions (cf. Table 4.24, Addendum I2, p.152).
- Patterns, functions, and algebra: most of the participants were successful in the factual knowledge dimension which contributed to success in the conceptual and procedural dimensions (cf. Table 4.22 Addendum I2, p.146). The biggest problem occurred because some participants only identified the first concept that needed to be factorised and not the second concept.
- Space and shape: participants seemed to find the understanding and analysis of information in the conceptual knowledge dimension as troublesome, as well as application and evaluation to demonstrate procedural knowledge (cf. Table 4.26, Addendum I2, p. 156). A common notation problem was identified because the participants did not start the solution of Question 3.2 by identifying which two triangles had to be investigated for congruency.

- Measurement: all the participants seemed successful at applying circle area formulas (cf. Table 4.28, Addendum I2, p. 160). Common mistakes made by participants were the notation and meaning of “in terms of” and adding the units to the final answer. Three participants calculated the final answer instead of leaving it in terms of π and three left out the unit.
- Data handling: the participants who could not recall the factual knowledge (identify the mode, identify the middle value- median) were less successful in calculating or finding the correct values (organise values in chronological order to find the middle value – median) (cf. Table 4.30, Addendum I2, p.163).

Main problems observed in the Intermediate Phase subject content knowledge (cf. 4.3.5):

- Numbers, operations and relationships: procedural knowledge related to mathematical operations were problematic (cf. Table 4.32, Addendum I3, p. 165).
- Patterns, functions, and algebra: participants seemed to be overall successful, except for failing to recall some of the factual knowledge related to terminology (cf. Table 4.34, Addendum I3, p.177).
- Measurement: participants seemed to have trouble with recalling some of the factual knowledge (Question 3.1) (cf. Table 4.36, Addendum I3, p.179) which contributed to being unsuccessful in answering the question. Most of the participants were successful with Question 3.2.
- Space and shape: the participants probably miscounted one, two or all of the indicated “vertex, edge and face” in the table, which could be due to a lack of factual knowledge (knowing what each term represents) (cf. Table 4.38, Addendum I3, p.181).
- Data handling (cf. Table 4.40, Addendum I3, p. 183): all participants were successful, except for one participant who did not attempt a solution.

Finding 11: Subject content knowledge seems to be taught without any link to the simultaneous development of cognitive process dimensions (cf. 4.3.3 – 4.3).

According to the aforementioned findings for the participants who took part in the research, the researcher argues that the participants appear to not fully meet the competency expectations concerning subject content knowledge, as gaps in subject content knowledge and cognitive process dimensions seem to be evident. In this regard the following connections with the literature are established:

The absence of well-developed cognitive process dimensions could have obstructed the understanding of content knowledge (Brownlee & Berthelsen, 2008) (cf. 2.2.2). Besides, the

different knowledge dimensions and cognitive process dimensions might not always be developed in conjunction with one another (Krathwohl, 2002) (cf. 2.3.3) among the participants who took part in the research. Although mention is made in the CAPS of the following cognitive levels that need to guide learner assessment across all teaching phases at school-level: knowledge, routine procedures, complex procedures, and problem-solving (cf. Table 2.8), the CAPS does not communicate the importance of ensuring that cognitive processes are developed in conjunction with subject content knowledge explicitly to teachers.

The teacher-training curriculum provides coverage of all the content areas specified in the CAPS (excluding analytic geometry and trigonometry) for the FET Phase (cf. Table 2.8), and all the module outcomes also emphasise the development of the cognitive process dimensions according to the revised taxonomy of Bloom (cf. Addendum F). However, similar to the CAPS curriculum there is no explicit indication of how cognitive processes are developed in conjunction with content knowledge. Teachers must acknowledge success in subject content knowledge as a combination of content knowledge and cognitive processes (Department of Basic Education, 2011a, b, c; Krathwohl, 2002) (cf. 2.3.2, 2.3.3). A lack of emphasis on the explicit development of cognitive processes in conjunction with subject content knowledge possibly emanating from the participants' school years and continued during their training could have contributed to the unsuccessfulness of the student participants in some of the subject content areas.

A lack of well-developed dispositions could have influenced the poor results obtained by many of the research participants (Maree, Louw, & Millard, 2004; Cobb and Hodge, as well as Ames and Archer, cited by Gresalfi & Cobb, 2006). Several thinking dispositions that enhance the quality of work, such as having an open mind and appreciation of new ideas, self-confidence, scepticism, evidence and logic, the consideration of alternatives, inquisitiveness, perseverance, and a tendency to reflect on learning (metacognition), play an important role in mathematics (Kloppers & Grosser, 2014) (cf. 2.3.2.4).

The CAPS and the teacher-training curriculum underscore the progressive development of knowledge. However, a limitation observed for both the CAPS and teacher-training curriculum is that they do not clearly distinguish between factual, conceptual, and procedural knowledge. In her own analyses of the CAPS, the researcher grouped the CAPS outcomes and module outcomes of the teacher-training curriculum to reflect factual, conceptual, and procedural knowledge (See the researcher's analysis in Addendums E and F). According to the researcher it is important to make student teachers explicitly aware of the different knowledge dimensions and their relatedness in mathematics, to ensure that all the knowledge dimensions are acquired effectively.

The effectiveness of the pre-teachers at teaching mathematics, in particular the FET and Intermediate Phase participants, seems questionable, due to a possible lack of solid and specialised subject content knowledge (Avalos et al., 2010; Fung et al. 2017; Kanyongo & Brown, 2013; Lam, 2017; Plotz et al., 2012; Tsanwani & Sinthumule, 2015) (cf. 1.1, 2.3.4.3). The Senior Phase participants seem to possess a more solid understanding of their mathematical subject content knowledge (cf. 2.3.4.1) and, could therefore, according to Shulman (1998), be regarded as becoming more effective teachers than the FET and Intermediate Phase participants (cf. 2.3.4.1).

The findings specifically support the research of Adler (2005), Blignaut et al. (2013), Kanyongo and Brown (2013), and Plotz et al. (2012) who add that teachers (in this regard, the FET participants) specifically lack subject content knowledge in the field of statistics. Also, the research of Reddy et al. (2014) and Tsanwani and Sinthumule (2015) highlighted problems with teachers' understanding of the subject content in differential calculus, which is confirmed by this study (cf. 1.1). Additionally, the findings support the concern of Blignaut et al. (2013), Mahlobo (2015), and Siyepu (2013), that pre-service teachers do not perform well, but still graduate and become teachers, and contribute to the problem of poor mathematics performance at the school-level (cf. 1.1).

5.4.3 Findings concerning competency expectations for metacognitive knowledge of the pre-service teachers

Finding 1: Many of the participants appeared to be unsuccessful with the application of metacognitive self-regulatory action during the completion of the test due to the unnecessary errors made, and therefore do not seem to meet the competency expectations concerning this the self-regulatory aspect of metacognitive knowledge (cf. 4.4.1).

Finding 2: There were participants across the three phases who seemed to have an understanding of what planning, monitoring, and evaluating as part of self-regulatory action entail (cf. 4.4.1, Tables 4.41 – 4.43). For planning, relevant strategies such as considering and understanding task expectations and requirements, as well as considering prior knowledge were cited, which the researcher finds encouraging (cf. 4.4.1). As part of monitoring, keeping track with checklists containing task requirements or guidelines, comparing work against stated outcomes, reviewing work, and reflecting on the task objectives were mentioned (cf. 4.4.1). To evaluate, participants mentioned among others, checking answers with others, asking questions to verify if an answer corresponds with what was asked, which seemed appropriate evaluation strategies (cf. 4.4.1). The mentioned strategies could be regarded as valid strategies that would enhance

the planning, monitoring, and evaluation (Bannert & Reimann, 2012; Cazan, 2012; Flavell 1985; Paris & Paris, 2010; Zumbrunn et al., 2011) (cf. 2.3.2.4). Nevertheless, it seems to the researcher that the mentioned strategies are not applied automatically (Schneider & Artelt, 2010) (cf. 2.3.2.4), given the many unnecessary mistakes that were made that influenced the success of the participants' achievement in the tests (cf. 4.4.1, Tables 4.41 – 4.43, Addendum J, p. 185-187).

Finding 3: The “how” of metacognitive development seems vague in the CAPS (cf. 2.3.2.4, Addendum E), and therefore possibly not nurtured during teaching. Also, the teacher-training curriculum seems unclear about the development of metacognition and its importance for successful learning (cf. 2.3.5, Addendum F). The researcher could not find any evidence of how metacognitive development is addressed in any of the modules that form part of the teacher-training curriculum for mathematics.

Owing to an apparent lack of explicit focus on the development of metacognitive self-regulation at school-level (cf. Addendum E), and during teacher-training (cf. Addendum F) (cf. 2.3.2.4), participants could be regarded as deprived of opportunities to become skilled at metacognitive self-regulation.

The findings indicate that the participants do not fully meet the competency expectations concerning metacognitive self-regulation that mathematics teachers need to possess. The researcher integrates the findings and the literature below.

Because many of the participants seemed to have very limited depth and breadth of content knowledge in their respective teaching phases (cf. 4.3.3.1 – 4.3.3.3) they were therefore possibly ineffective at being successful with the application of metacognitive self-regulatory actions during the completion of the test (Al-Kayyali & Al-Haq, 2018; Crooks & Alibali, 2014; Flavell, 1971) (cf. 2.3.2.4). It also appears to the researcher, that the participants seemingly do not apply a purposeful questioning nature to the quality of the completion and outcome of their work (Ertmer & Newby, 1996) (cf. 2.3.2.4). It could be that the mathematics lecturers do not create teaching environments where questioning is encouraged (cf. Table 2.5) that would promote the application of metacognitive strategies for self-regulatory action (Department of Basic Education, 2018) (cf. 2.3.2.4).

5.4.4 Findings in relation to the competency expectations concerning pedagogical content knowledge of the pre-service teachers

Finding 1: The availability of strategies to teach mathematics subject content seem insufficient.

- One of the weaknesses cited by the student participants is that they do not have adequate strategies to teach mathematics (cf. 4.4.4). They therefore propose as a suggestion to improve the teacher-training curriculum that they need to be equipped with strategies aimed at teaching mathematics (cf. 4.4.5). The lecturers too indicated insufficient pedagogical knowledge and exposure to practical classroom experiences (cf. 4.5.2), as well as exposure to a variety of teaching strategies as weaknesses and limitations in the teacher-training curriculum (cf. 4.5.3). Furthermore, according to the lecturers, the teacher-training curriculum limits teaching time (cf. 4.5.3) which, according to the researcher, poses threats to the acquisition of solid subject content and pedagogical content knowledge.

Finding 2: There appears to be an overreliance on the use of direct teaching.

- Teaching strategies such as explanations, demonstrations, drilling, and using questions and answers seem to be mostly used by participants across the three teaching phases (cf. Addendum M1 – M3). The researcher, therefore, finds it contradictory that the majority of the FET participants indicated to have a wide variety of strategies available to teach algebra and equations, functions, and number patterns, sequences, and series (cf. Graph 4.22).
- The strategies mentioned by the Senior Phase participants to teach the various subject content areas rather seem limited, and mainly revolve around the use of direct teaching strategies such as demonstrations, explanations, providing information and summaries to students, and drilling (cf. Addendum M2). However, although limited, positive signs are indicating the use of problem-solving, peer learning, discovery learning, group work and group discussions, case studies, and real-life applications. The participants indicated having limited strategies available to teach space and shape and data handling (cf. Graph 4.23). A closer look at their perceptions about examples of strategies to teach data handling, revealed the opposite, as the use of real-life examples, group work, discussions, case studies were combined with direct teaching and providing of information and summaries to learners.
- The examples of teaching strategies cited by the Intermediate Phase participants confirmed the possession of a very limited range of teaching strategies that mainly focus on the direct teaching of content, demonstrations, explanations, repetition, and practise (cf. Addendum M3). These observations contradict the perceptions of the majority of participants who expressed the availability of a wide range of teaching strategies to teach at least four of the five subject content areas (cf. Graph 4.24).

- Many of the participants often could not present any example of a teaching strategy to use to teach a content area, or just did not answer the questions (cf. Addendum J; Tables 4.65 – 4.67, p. 205-212).

Finding 3: The CAPS and teacher-training curriculum do not provide specialised mathematics-focused pedagogical guidance or training. Both curricula present scanty information about teaching strategies to best teach mathematics (cf. 2.3.5, 2.4.6). Pre-service teachers have to mainly rely on the generic information about teaching strategies that they receive as part of their training in the Professional Studies modules (cf. 2.4.6). The researcher believes that the limited exposure to pedagogical content knowledge might be the reason for the participants having a limited range of teaching strategies available (cf. 4.4.9).

Finding 4: The responses of the lecturers concerning the strategies they employ to teach mathematics did not convincingly indicate the use of a large variety of strategies (cf. 4.5.4). An absence of pedagogical modules in the curriculum to equip pre-service teachers with pedagogical knowledge, as well as subject content, and lecturers who also appear to have limited teaching strategies to teach mathematics, pose a double disadvantage to the pre-service teachers concerning the acquisition of meaningful pedagogical content knowledge. Evidence of direct teaching and collaboration was observed in the data as well as suggestions for more inductive teaching strategies and applying strategies that align with the subject topics to be taught (cf. 4.5).

Based on the findings, the researcher argues that the pre-service teachers do not comply with the competency expectations concerning pedagogical content knowledge that mathematics teachers need to possess. The researcher integrates the findings with the literature in the following ways:

Teachers need teaching strategies to nurture all the dimensions of knowledge, and not for example only focus on factual and procedural understanding (Fuentes et al., 2014) (cf. 2.4). The overuse of direct teaching favours factual and procedural knowledge development (Killen, 2015; Markusic, 2012) (cf. 2.4.1). Participants need to acquire indirect and independent teaching strategies to encourage learners to discover information and do investigations independently, to apply information (Burden & Byrd, 2010; Killen, 2015; Lodewyk, 2011; Philpott, 2009) (cf. 2.4.2, 2.4.3), and to develop higher-order thinking (cf. 5.3.3). If only direct teaching is applied, learners will not be challenged to link mathematical concepts to gain a deeper understanding (Green & Hood, 2013) (cf. 2.2). It could be, as argued by Depaepe et al. (2016), Ho et al. (2008), Plotz et al. (2012), Sing and Khine (2008), and Wang et al. (2008), that the pre-service teachers tend to teach the way they were taught, by making use of direct teaching and explanations on the board

without any interactive teaching that encourages critical thinking (cf. 1.1, 2.4.4). Also, Baker et al. (2012) assert that teachers are trained to teach mathematics using direct instruction (cf. 2.2.4).

5.4.5 Additional findings

A number of additional and worthy findings emanated from the research:

Finding 1: It is encouraging to note that the reasons for being regarded as a good teacher (cf. 4.4.6) referred to possessing positive personality traits, dispositions, and attitudes (enthusiasm, love, confidence, joy) toward teaching mathematics, as well as having a concern for learners' progress in and understanding of mathematics. The researcher feels that although dispositional aspects were not the focus of the study, and not explicitly addressed in the literature review as part of the competency expectations that directly influence mathematics achievement, they should be regarded as important as they could possibly contribute to influencing learners' attitudes towards learning mathematics (Stronge, 2018) (cf. 2.4).

Finding 2: It is encouraging that the participants mentioned that they receive more knowledge than what is required for their teaching phase, and that the knowledge goes beyond the curriculum (cf. 4.4.3), which possibly reflects attributes of breadth (specialised subject content knowledge) in the subject content knowledge of the lecturers (Green & Hood, 2013) (cf. 2.3.4.2). Similarly, the lecturers also noted that the subject content extends Grade 12 content (cf. 4.5.1).

Finding 3: Another positive finding is that the mathematics lecturers appear to be well-prepared, knowledgeable and supportive (cf. 4.4.3).

Finding 4: A discouraging finding that can be deduced from the data is that the participants do not seem to view their teaching from a variety of epistemological beliefs, given the strong focus on direct teaching methods and strategies (cf. 4.4.9) that support a realist approach and foregrounds factual transmission during teaching. Absolutist (realist) and fallibilist (relativist or contextualist) epistemological beliefs need to frame the teaching of mathematics in order to ensure that factual, conceptual, and procedural knowledge dimensions are developed (Depaepe et al., 2016) (cf. Table 2.1) (2.4.4).

5.4.6 Contradictory findings

The following contradictory findings were derived from the data:

Finding 1: The student participants noted that reasons for being regarded as a good teacher (cf. 4.4.6) could include their content knowledge and teaching strategies, which contradicts the weaknesses concerning the training that they mentioned (cf. 4.4.4), as well as their limited

preparedness to teach subject content (cf. 4.4.2.1), and the gaps identified in the subject content knowledge based on the test results (cf. 4.3.3). Also, some of the lecturers viewed the subject content to be solid and a strength in the curriculum (cf. 4.5.1), which could not be confirmed by the test results. Although some of the participants mentioned their teaching strategies as reasons for being regarded as good teachers (cf. 4.4.6), their perceptions contradict the weaknesses concerning the availability of adequate teaching strategies that were mentioned (cf. 4.4.9).

In the following section the researcher reflects on the aim and objectives of the study to establish if they were achieved.

5.5 FINDINGS CONCERNING THE AIM AND OBJECTIVES OF THE STUDY

The aim of the study was to determine to what extent the teacher-training curriculum for the Intermediate (Gr. 4–6), Senior (Gr. 7–9) and the FET (Gr. 10–12) phases at the NWU Vanderbijlpark Campus prepares mathematics teachers to meet the competency expectations concerning mathematical subject content knowledge and pedagogical content knowledge that is expected in the CAPS for teaching mathematics.

The aim was operationalised in the following objectives:

Objective 1: To identify what mathematical subject content knowledge should form part of a teacher-training curriculum. This objective was achieved with a literature review and by qualitatively comparing the subject content knowledge foregrounded in the CAPS with the subject content knowledge specified in the teacher-training curriculum.

- The researcher established that subject content knowledge should comprise the following dimensions: factual, conceptual, procedural, and metacognitive (Krathwohl, 2002 (cf. 2.3.2), and should be developed in conjunction with cognitive process dimensions, such as remember, understand, analyse, apply, create, and evaluate (Krathwohl, 2002) (cf. 2.3.3, 5.3.1).
- Both the CAPS and teacher-training curriculum cover the content areas that mathematics teachers need to teach as follows. However, the teacher-training curriculum for the FET phase and the CAPS do not align completely (cf. 2.3.5, Table 2.8).
- Neither the CAPS nor the teacher-training curriculum distinguish clearly between the teaching of the various knowledge dimensions (factual, conceptual, procedural, metacognitive), and therefore do not emphasise the importance to teachers of developing all the knowledge dimensions in unison, although the knowledge dimensions are visible in the outcomes that the learners at school-level (CAPS) (cf. Addendum E) and in the module outcomes of the teacher-training curriculum) (cf. Addendum F, 5.4.2 (Finding 11)).

- The importance of developing cognitive process dimensions in conjunction with the various knowledge dimensions, is also not communicated explicitly in the CAPS and teacher-training curriculum, although visible in the outcomes to be achieved by learners at school, and by the students at university level (cf. Addendums E, F, 5.3.2, 5.4.2 (Finding 11)).

Objective 2: To establish what pedagogical content knowledge should form part of a teacher-training curriculum in mathematics. This objective was achieved with a literature review and by qualitatively comparing the pedagogical content knowledge foregrounded in the CAPS with the pedagogical content knowledge specified in the teacher-training curriculum.

- In this study, the focus was on strategies that the participants employ to best teach mathematics subject content. The literature review revealed that a variety of pedagogical strategies (direct, indirect, independent, and interactive) (cf. 2.4), is required for effective mathematics teaching (Fung et al., 2017; Kanyongo & Brown, 2013; Plotz et al., 2012) (cf. 1.1), such as inquiry-based strategies (Brodie & Molefe, 2010; Kilpatrick et al., 2001), problem-based learning (Bansilal et al., 2014; Cazzola, 2008), discussions (Kunene & Sepeng, 2018; Venkat, 2013), modelling (Durandt & Jacobs, 2013; Ikeda, 2013), and cooperative learning (Spangenberg, 2018) (cf. 1.1). Also, the Department of Basic Education (2018) and Stronge (2018) support a learning-centred classroom where learners are actively involved in learning and inquiry, discover information on their own and in group-based activities (cf. 2.4, 2.4.5, 5.3.3).
- Neither the CAPS, nor the teacher-training curriculum provide clear guidance towards employing specific teaching strategies to teach mathematics content. Although a learning-centred pedagogy is suggested by the Department of Basic Education (2018), mere examples of how to create a learning-centred classroom are provided (cf. 2.4.5) without practical demonstration at the hand of subject content. (cf. 5.4.4, Finding 3)).

Objective 3: To evaluate to what extent the teacher-training curriculum at the NWU, Vanderbijlpark Campus, prepare teachers to meet competency expectations for concerning mathematical subject content knowledge and pedagogical content knowledge. This objective was achieved by integrating quantitative and qualitative data to derive findings.

The findings are reported as part of 5.4.2, 5.4.3, and 5.4.4, and provided evidence supporting the achievement of the objective.

Objective 4: To determine how qualified and experienced lecturers who train pre-service mathematics teachers at the NWU, Vanderbijlpark Campus are, to prepare students to meet the competency expectations concerning subject content and pedagogical content knowledge. This

objective was achieved by obtaining lecturers responses in relation to their qualifications in a questionnaire (cf. 5.4.1).

Objective 5: To explore how pre-service mathematics teachers and mathematics lecturers involved in teacher training at the NWU Vanderbijlpark campus perceive the quality of the mathematics teacher-training curriculum. This objective was achieved by exploring qualitative written reflection data obtained from lecturers and pre-service teachers in response to open items in a questionnaire (lecturers) and open items as part of the student tests (pre-service teachers). The research findings that support the achievement of this objective are reported in 5.4.2 (Findings 7, 8, and 9), 5.4.3 (Finding 2), and 5.4.4 (Findings 1, 2, and 4).

The researcher believes that all the objectives were achieved in the study, and carefully concludes that the aim of the study was achieved. The pre-service mathematics teachers who took part in the study, do not fully comply with the competency expectations concerning subject content and pedagogical content knowledge to become effective mathematics teachers. In this regard, the final objective of the study concerns the following.

Objective 6: To make curriculum-related recommendations that could strengthen, adapt, or enrich the current pre-service mathematics teacher-training curriculum used at the NWU Vanderbijlpark Campus to ensure that teachers meet the expectations for competency concerning mathematical subject content and pedagogical content knowledge. This objective was achieved by suggesting recommendations in relation to the findings obtained with the quantitative and qualitative data collection.

The gaps identified in the participants' subject content knowledge and pedagogical knowledge, as well as the restrictions observed in the teacher-training curriculum, enabled the researcher to suggest recommendations that could specifically enhance teacher-training at the university that formed part of the study.

5.6 RECOMMENDATIONS TO STRENGTHEN THE TEACHER-TRAINING CURRICULUM

Aligned with the findings reported in the Section 5.4 the researcher presents the following recommendations

5.6.1 Recommendations concerning lecturers (Finding 4) (cf. 5.4.4)

Recommendation 1: As the CAPS and the teacher-training curriculum do not provide clear guidance to lecturers concerning the development of subject content knowledge as facts, concepts and procedures in conjunction with cognitive process dimensions, lecturers would need

support in this regard to ensure that all competency expectations for subject content knowledge are met during teacher training.

Recommendation 2: Lecturers might be products of teacher training that limited exposure to a variety of pedagogical strategies to teach mathematics, and emphasised direct teaching (Depaepe et al., 2016) (cf. 2.2.4). The lecturers' responses concerning the teaching strategies that they use did not convince the researcher that they possess a variety of teaching strategies to teach mathematics (cf. 4.5.4). It is therefore advisable that mathematics lecturers receive professional development training to acquire teaching strategies that would enable them to equip pre-service teachers with applicable strategies to teach mathematics, in particular the following: inquiry-based strategies, problem-based learning, discussion, cooperative learning and modelling (cf. 5.5, Objective 2).

5.6.2 Recommendations concerning subject content knowledge (factual, conceptual, and procedural) of pre-service teachers (Findings 1, 4, 5, 6, 7, 8) (cf. 5.4.2)

As the researcher observed limitations concerning the pre-service teachers' factual, conceptual, procedural, and metacognitive knowledge, the following recommendations address the various knowledge dimensions

Recommendation 1: Factual knowledge is the knowledge of facts like terminology, rules and algorithms (Kahan et al., 2003; Löwe & Müller, 2010) (cf. 2.3.2.5). As factual knowledge forms the basic foundation of all mathematical content knowledge it needs to be ensured that pre-service teachers successfully acquire all the factual knowledge concerning each subject content area. As problems with base-line knowledge of the student teachers on entering their training were mentioned by the mathematics lecturers (cf. 4.5.2), a pre-test could be administered to determine the baseline knowledge of the prospective teachers, before entering the training programme. The applicants can then be either accepted or denied entry into the programme. Alternatively, given the shortage of mathematics teachers in South Africa, the applicants who do not meet the factual knowledge requirements could be required to enrol for a training programme designed by the university that will address all the factual knowledge required to be enrolled for the teacher-training programme. The university can determine the criteria for successful completion of the training programme.

Recommendation 2: Factual knowledge of participants should be determined at the end of each semester during their teacher-training by means of a standardised test. If the participants do not meet the requirements, a remedial programme during a holiday period could be offered after which an additional standardised test is administered. If the pre-service teacher does not meet the requirements of the factual knowledge test per study year or semester, the pre-service teacher

should not be allowed to advance to the mathematical content module/s of a following semester or year. Advancing a student with lacking baseline factual knowledge would not benefit the student. It would limit the student to develop deeper conceptual understanding.

Recommendation 3: Depaepe et al. (2016) state that experience as a mathematics teacher can help teachers to teach for conceptual understanding and procedural application (cf. 2.3.2.3). The pre-service teachers might not be able to increase the time spent at a school for practical experience, but the university could consider offering free mathematics classes to school learners in need. There are many underfunded schools where extra mathematics lessons are desperately needed. The university could start an after-school programme under the supervision and guidance of the mathematics lecturers on a rotation basis, and assign pre-service teachers to the programme. The programme could assist the pre-service teachers in developing their conceptual understanding by means of preparing for practical teaching experiences (with the guidance of lecturers) that could benefit learners in need. Such a programme could also ensure that the university might get recognition from the Department of Basic Education for making a contribution for improving the quality of mathematics performance among learners.

Recommendation 4: Pre-service teachers should be provided with examples of how to teach for conceptual development and understanding. Lessons that focus on problem-based learning (Boyd, 2011; Cazzola, 2008) (cf. 1.1) would help the development of conceptual understanding of the pre-service teachers by bridging the gap between theory and practice. In order for lectures to consider using a problem-based approach during teaching, they might have to also first experience and understand the application of problem-based learning before they will be able to teach the students its application (Cazzola, 2008; Plotz et al., 2012) (cf. 1.1).

Recommendation 5: As the CAPS and the teacher-training curriculum do not emphasise the integrated nature of the various subject knowledge dimensions, the researcher believes that the various knowledge dimensions might be viewed in isolation (Kahan et al., 2003; Löwe & Müller, 2010) (cf. 2.3.2.5). Specifically, the interrelationship between conceptual and procedural knowledge needs to be strengthened (Crooks & Alibali, 2014; Rittle-Johnson et al., 2015) (cf. 2.3.2.3). The more developed the conceptual knowledge is the deeper understanding of procedural knowledge the pre-service teachers will have (Eisenhart et al., 1993; Rittle-Johnson et al., 2015) (cf. 2.3.2.3). Problem-solving is central to mathematics, and to be an effective problem-solver a pre-service teacher needs a multidimensional understanding of mathematical procedures, which hinges on conceptual knowledge (Du Plooy & Long, 2014) (cf. 2.3.2.4).

Recommendation 6: The assessment of the pre-service teachers' subject content knowledge during their training might have to focus on a clearer and in-depth separated assessment of their

conceptual and procedural knowledge to identify gaps and to ensure that both dimensions are developed adequately. In turn, the pre-service teachers need to be made aware of their assessments in both of these dimensions, as well as of the importance of being successful in both of these knowledge dimensions. Reporting the assessment results for both of these knowledge dimensions to the pre-service teachers would enable them to attend to specific gaps in their knowledge. The assessment of conceptual knowledge should emphasise the correctness of procedures, definitions of explanations of concepts, and knowing different methods to solve problems (Rittle-Johnson et al., 2015). The assessment of procedural knowledge should highlight the importance of correct answers or the correct application of specific procedures when answering questions (Crooks & Alibali, 2014; Star & Stylianides, 2013). The researcher believes that this recommendation could avoid a superficial understanding of mathematics subject content knowledge.

5.6.3 Recommendations concerning the development of metacognitive knowledge (Findings 1, 2) (cf. 5.4.3)

Recommendation 1: Metacognitive reflection on a solution to a problem is only successful if one has sufficient content knowledge to know what is right (or wrong) or to identify different problem-solving methods that would lead to similar, correct solutions (Al-Kayyali & Al Haq, 2018; Flavell, 1971 (cf. 2.3.2.4). Bearing in mind the gaps observed in the subject content knowledge of the participants, the researcher suggests that the subject content modules in the curriculum should be phase specific, and ensure the depth and breadth of the subject content knowledge of the pre-service teachers to make it possible for them to engage in effective metacognitive reflection.

Recommendation 2: Lecturers could prompt reflection during learning or task completion by means of purposive questioning (cf. Table 2.5) (Ertmer & Newby, 1996), to enable pre-service teachers to adapt to a working routine that automatically includes metacognitive self-regulation. Besides, it would be important that the lecturers explain the theory behind metacognitive self-regulatory involvement in learning (planning, monitoring and evaluating learning) and demonstrate strategies to self-regulate learning to the pre-service teachers to make them realise the importance of metacognitive involvement in achieving successful learning (Alexander et al., 2003; Kron-Sperl, et al., 2008; Schneider & Artelt, 2020; Umaru, 2010) (cf. 2.3.2.4). Also, the researcher suggests that the guided application of strategies to self-regulate learning would be important to assist the pre-service teachers to become skilled at self-regulation so that they will be able to demonstrate self-regulated action to the learners they will teach. Teaching pre-service teachers to purposefully reflect on answers, would help them to develop a sense of metacognitive self-regulation (Lauritzen, 2012) (cf. 2.3.2.3, 2.3.2.4). Also, reflection about which procedures would be most effective to apply during problem-solving would nurture the development of

metacognitive ability (Crooks & Alibali, 2014) (cf. 2.3.2.3). If teachers' metacognitive self-regulation is unsuccessful, how will they assess and reflect on their learners; cognitive capabilities, self-efficacy, motivation, knowledge to be able to complete tasks, and misunderstandings (Crescenzi, 2016), as well as to evaluate if teaching goals were achieved (Gravett & De Beer, 2015).

5.6.4 Recommendations to foster realistic perceptions about competency among mathematics pre-service teachers (Findings 2, 3) (cf. 5.4.2)

Recommendation 1: The reason for the unrealistic perceptions among the pre-service teachers about subject content competency should be established, as unrealistic perceptions about competency could create a false confidence in one's abilities to teach effectively. One could have a look at the participants' assessments and achievements in mathematics across their years of study to establish where the unrealistic perceptions stem from.

5.6.5 Recommendations to addressing specific problems related to subject content areas (Finding 10) (cf. 5.4.2)

Recommendation 1: Some of the main problems observed concerning the subject content in the FET phase that need to receive attention are: making connections between and across content areas (integrated knowledge) (differential calculus, numbers, patterns, sequences, and series, finance, growth, and decay), demonstrating proof of conceptual understanding (number, patterns, sequences, and series), and ensuring that factual knowledge in all content areas are solid (functions, statistics, algebra). Moreover, the content area probability proved to be extremely troublesome, as none of the participants scored any points for the question on probability, and would require intensive attention during training (cf. 4.3.3.1).

Recommendation 2: Some of the main problems observed concerning the subject content in the Senior Phase that need to receive attention are: sign errors (numbers, operations, and relationships), not identifying the second concept during factorisation (patterns, functions, and algebra), notation and meaning of "in terms of" and adding units to the final answer (measurement), as well as not being able to recall factual knowledge (data handling) (cf. 4.3.3.2).

Recommendation 3: Main problems noted among the Intermediate Phase participants that could benefit from additional attention are: procedural knowledge concerning mathematical operations (numbers, operations, and relationships), and a lack of factual knowledge (patterns, functions, and algebra, measurement, space and shape) (cf. 4.3.3.3).

5.6.6 Recommendations for developing cognitive process dimensions in conjunction with subject content knowledge (Finding 11) (cf. 5.4.2)

Recommendation 1: The researcher suggests that pre-service teachers need to be made aware of the cognitive process dimensions that play a role in the acquisition of subject content knowledge (cf. Table 2.3) and be reminded of these processes when they are engaged in mathematics tasks. It would also be important to ensure that exposure to engagement in the various cognitive process dimensions are balanced according to guidelines provided by the CAPS curriculum (cf. Table 2.8) to enable students to acquire all the cognitive process dimensions in relation to all the knowledge dimensions.

5.6.7 Recommendations concerning pedagogical content knowledge (Findings, 1, 2) (cf. 5.4.4)

Recommendation 1: The researcher disagrees with the approach followed in the teacher-training curriculum concerning the development of pedagogical knowledge, because subject content and pedagogical knowledge should be taught in conjunction to enable a teacher to enhance teaching effectiveness (Ball et al., 2007; Blömeke et al., 2011, Killen, 2015; Pimentel et al., 2007; Sibuyi, 2012) (cf. 1.1, 2.4). Therefore, it seems that pedagogical modules integrated with subject content modules need to be included in all four years of the teacher-training curriculum for all the teaching phases, and more opportunities for practical teaching experiences included in the curriculum where pre-service teachers get the opportunity to apply teaching strategies.

Recommendation 2: The addition of mathematics teaching-phase specific pedagogical modules throughout the course of the four-year teacher-training programme could ensure that the pre-service teachers receive greater exposure to various applicable pedagogical methods and strategies for teaching mathematics. The four major teaching approaches: direct, indirect, independent and interactive all play a very important role in the teaching and learning of mathematics, but need to be customized to the mathematics context. In particular, exposure to acquiring teaching strategies that would promote independent, indirect, and interactive learning (cf. 2.4.2 – 2.4.4) would enable teachers to create a learning-centred teaching environment (cf. 2.4) as proposed by the Department of Basic Education (2018) (cf. 2.4.5). Only theoretical knowledge of the various approaches will not be effective in the pedagogical development of the pre-service teachers; therefore, the pre-service teachers should be required to practically develop their pedagogical skills. The pedagogical modules should pay attention to the theory and the application of various teaching methods and strategies, and the pre-service teachers need to design and present various lessons that focus on the development of factual, conceptual, and procedural knowledge to explore the value of different teaching strategies suitable to teach the

various knowledge dimensions. The pre-service teachers should be encouraged to use different teaching strategies, as the change in teaching strategies and being open-minded to change the application of teaching strategies require time and is not easy (Wang et al., 2008) (cf. 2.4.4). The presentation of the pedagogical modules therefore need to provide consistent support to students to move towards sophisticated teaching strategies (Wang et al., 2008) (cf. 2.4.4), to avoid that students teach the way they were taught, namely by mainly using direct teaching strategies (Ho et al., 2001; Sing & Khine, 2008; Wang et al., 2008) (cf. 2.4.4).

5.6.8 Recommendations to address limitations in the teacher-training curriculum: subject content knowledge and cognitive process dimensions (Findings 9, 11, cf. 5.4.2; Finding 3, cf. 5.4.3), and pedagogical content knowledge (Finding 3, cf. 5.4.4).

To address some of the limitations observed in the teacher-training curriculum (cf. 2.3.5, 2.4.6) the following recommendations are made. These recommendations support two important arguments in the literature, namely the argument of Loyiso and Olubenga (2019) that there is an urgent need to build the capacity for ongoing professional development of mathematics teachers, and the argument of Zazkis and Leikin (2010) that a more connected relationship between subject content knowledge acquisition and the knowledge of teaching, should be established.

Recommendation 1: The FET curriculum should include subject content knowledge concerning analytical geometry and trigonometry and the content for Euclidean and spherical geometry should be aligned to what is expected at school level (Reddy et al., 2014; Taylor, 2018; Zazkis & Leikin 2010 (cf. 1.1). Greater coverage concerning the subject content for differential calculus, probability and statistics should be considered (Reddy et al., 2014; Tsanwani & Sinthumule, 2015) (cf. 1.1), as these content areas are covered in one semester module only in the teacher-training curriculum. Teachers need solid subject content knowledge which plays an important role in a teacher's ability to teach effectively (Ball et al., 2008; Kanyongo & Brown, 2013; Plotz et al., 2012; Tsanwani & Sinthumule, 2015) (cf. 1.1, 2.4).

Recommendation 2: Although the Senior Phase participants performed the best among all the participants who took part in the research, the researcher feels it would be advisable to present a specialised Senior Phase curriculum to the participants who wish to register for the Senior Phase mathematics curriculum. This would enable the participants to acquire subject content knowledge specific to the Senior Phase, in particular, for geometry for which content coverage appears to be completely inadequate, and to avoid unpreparedness of teachers due to poor subject content knowledge (Blignaut et al., 2013; Mahlobo, 2015; Plotz et al., 2012; Siyepu, 2013) (cf. 1.1).

Recommendation 3: To enable the Intermediate Phase participants to receive more solid subject content knowledge, the researcher recommends that the subject content training for the

Intermediate Phase commences during their first year of study and not only from their second year of study. In line with the aforementioned, it should also be considered to present subject content training for all the teaching phases to continue across the four years of training to ensure in-depth coverage of the subject content knowledge, and avoid the possible acquisition of compressed knowledge (Kanyongo & Brown, 2013, Plotz et al., 2012) (cf. 1.1). Also, students should not be allowed to continue with modules in a proceeding semester or study year if the previous modules were not successfully completed. This will avoid that teachers with poor content knowledge enter the field of teaching (Blignaut et al. 2013) (cf. 1.1).

Recommendation 4: The importance of metacognitive knowledge and its application should receive explicit focus in the teacher-training curriculum. In the study, the focus was only on the participants' strategies to self-regulate learning, namely: to plan, monitor, and evaluate their work to ensure success. However, metacognition comprises more than that. Students also need to learn how to understand their own abilities, know when and how to apply strategies to solve problems, be able to identify obstacles in their environments that hamper learning success, know how to find resources to assist them when obstacles are encountered, and be able to change their approach to learning if it does not bring success (Crescenzi, 2016; Finn & Metcalfe, 2013; Paris & Paris, 2010; Roebbers, 2017; Schunk, 2005, Sperling et al., 2004; Zumbrunn et al., 2011) (cf. 2.3.2.4). The researcher therefore suggests that the teacher-training curriculum be enriched with units that address the theory and practice of self-regulation in the various study guides that focus on subject content knowledge across the four years of study to enable the pre-service teachers to acquire metacognitive knowledge about the how of self-regulation.

Recommendation 5: The teacher-training curriculum should provide explicit guidance to pre-service teachers about how to infuse the development of cognitive process dimensions in the teaching of subject content, and not leave the development of cognitive process dimensions to chance (cf. 2.3.5). Student teachers need to know that achieving success in mathematics requires the application of various cognitive process dimensions (Department of Basic Education, 2011a, b, c; Krathwohl, 2002) (cf. 2.3.2, 2.3.3).

Recommendation 6: The teacher-training curriculum needs to provide more specialised mathematics-focused pedagogical content training across the four years of study, instead of the two one-semester modules presented to each of the teaching phases. Students cannot rely on generic pedagogical content modules only to become effective at teaching mathematics. Mathematics teachers need to acquire a variety of teaching strategies to enable the development of all the subject content knowledge dimensions (factual, conceptual, and procedural) (Fuentes et al., 2014) (cf. 2.4). In particular, a stronger emphasis should be placed on equipping students with the skill to apply inquiry-based strategies and problem-based learning that could ensure

better understanding of mathematics (Brodie & Molefe, 2010; Bansilal et al., 2014, Cazzola, 2008) (cf. 1.1).

Recommendation 7: The researcher feels it is necessary to also make the Department of Basic Education aware of limitations in the CAPS curriculum concerning a clearer identification of the different knowledge dimensions, the importance of developing cognitive process dimensions in collaboration with subject content. Additionally, clearer guidelines need to be presented about appropriate teaching strategies to teach mathematics subject content. Paying attention to these aspects could enable teacher-training institutions to better align their training with the competency expectations in the CAPS which guides teaching and learning in South African classrooms.

5.6.9 Recommendations to build on the strengths observed in the teacher-training curriculum (Finding 1, 2, 3) (cf. 5.4.5)

The researcher feels that some of the strengths observed in the teacher-training curriculum (cf. 4.4.3 4.5.1) should not go unnoticed and needs to be nurtured and encouraged during the training of students.

Recommendation 1: The development of positive dispositions and attitudes, such as enthusiasm, joy, confidence, and love to teach mathematics during the training that emanated from the data (cf. 4.4.3) are commendable and need to be fostered. Although the subject content knowledge and pedagogical content knowledge of a teacher contributes to a great extent to teaching effectiveness, personality traits, attitudes and dispositions of a teacher could affect the learners' attitudes towards learning (Stronge, 2018) (cf. 2.4), which in the case of the participants who took part in the research, seem to be a powerful asset.

Recommendation 2: The mathematics lecturers were commended for their support and understanding when students experience problems, being experienced staff, and possessing adequate subject knowledge. The researcher views the attributes of the lecturers as admirable and proposes that the type of support that the pre-service teachers receive be continued, as the mentioned attributes possibly fostered the positive attitudes and dispositions mentioned earlier, among the pre-service teachers.

5.6.10 Recommendations for creating awareness of the importance of framing teaching within a variety of epistemological beliefs (Finding 4) (cf. 5.4.5).

Although epistemological beliefs were not regarded as a competency expectation in the study, the researcher believes that the absence of an epistemological belief that directs one's teaching practice, may pose limitations concerning one's view about the nature of subject knowledge and

how it may be best acquired. In particular, it was observed that the pre-service teachers seem to possess limited teaching strategies (cf. 4.4.9), which could be evidence of not teaching with an epistemological belief in mind. The researcher wishes to propose that pre-service teacher training places emphasis on, and provides guidance to, students in exploring and acquiring different epistemological beliefs applicable to the teaching of mathematics (cf. 2.2.4).

Recommendation 1: Students need to be made aware of the fact that multiplistic epistemological beliefs to teach for factual, conceptual, and procedural development of subject content knowledge (cf. Table 2.1) are required for teaching mathematics. The teaching of mathematics needs to be approached from a belief that acknowledges more than one truth, and an understanding that many different methods can lead to the same solutions or solve problems (Crooks & Alibali, 2014) (cf. 2.3.2.3). Students' epistemological beliefs could be developed and enhanced through implicit and/or explicit instruction (Holliday, 2006; Soulios & Psillos, 2016) (cf. 2.2.4). Information on the importance, theory and application of various epistemological beliefs in mathematics could be included in both the subject content and pedagogical content modules, as epistemological beliefs concern the nature of the subject as well as how it could be best taught (Depaepe et al., 2016; Schraw & Olafson, 2003) (cf. 2.2.3). It is important that pre-service teachers understand that the focus of a classroom needs to move from teacher to learner to ultimately where the focus is on indirect and independent learning (cf. Figure 2.1) (Green & Hood, 2013; Sing & Khine, 2008) (cf. 2.2.3).

The researcher's study was approved during 2016, and during 2017 major revisions of the mathematics teacher-training curriculum at NWU were undertaken. The researcher however commenced with her study, given the following reasons:

- Many mathematics pre-service teachers prior to 2017 received their qualifications at NWU based on the curriculum that formed part of the study. Gaps in the student participants' subject content and pedagogical knowledge are evident from the data presented. Therefore, the researcher argues that there are students in the field who require support to enhance their subject content and pedagogical content knowledge. In this regard, the recommendations made could sensitise schools and the Department of Basic Education, that professional development of in-service teachers (maybe not only NWU graduates), who might lack solid subject content knowledge and pedagogical content knowledge due to limitations in the teacher-training curriculum, need prominent attention.
- The researcher's recommendations could also be useful in future reviews of curriculum for mathematics teacher training. As the findings of the researcher's study was not available to the staff involved in the restructuring of the NWU mathematics curriculum, findings might be used to assess to what extent the new curriculum addresses the

limitations and gaps identified with the present study, in order to ensure that the competency expectations for mathematics teachers are fully met in the new curriculum.

The researcher presents a succinct overview of the changes made to the mathematics teacher-training curriculum as part of Addendum N.

5.7 LIMITATIONS OF THE STUDY

5.7.1 Limitations: aim and objectives, research design, data collection, sample

- The current sample was a convenient sample, drawn from NWU Vanderbijlpark Campus mathematics pre-service teachers and lecturers only, and the findings cannot be generalised to other pre-service teachers or universities.
- The researcher focused on mathematics subject content knowledge and pedagogical knowledge of final year pre-service mathematics teachers in the Intermediate, Senior and FET phases only. A study including pre-service teachers in the first, second, and third year of study could provide greater insight into the subject content knowledge and pedagogical content knowledge of pre-service teachers.
- In the tests, the researcher focused on the subject content knowledge of the highest grade in each of the teaching phases. A study that examines the development of subject content knowledge in relation to each of the grades in a particular teaching phase could point a researcher to gaps in the subject content knowledge of pre-service teachers across a teaching phase. Gaps in subject content knowledge in the lower grades in a teaching phase might explain gaps in the subject content knowledge in higher grades in a particular phase.
- The use of interviews instead of open-ended questions would have been more ideal in understanding the student participants' and lecturers' perceptions and reasoning about the teacher-training curriculum. The use of interviews would also have enabled the researcher to gain more accurate information about the pre-service teachers' metacognitive ability to self-regulate and the teaching strategies they employ. Interviews would have enabled the researcher to prompt for more information and to ask for practical examples that would have enriched the qualitative data. The written responses of the participants were often limited, and it appeared as if some of the student participants had difficulty in transferring their thoughts into writing.
- As a novice researcher, the researcher acknowledges that she could have better assessed the knowledge and cognitive process dimensions in the tests to gain deeper insight and more accurate data. The researcher presents some of the limitations she observed with the collection and analysis of the test data.

- The five-point differential semantic scale used for data collection could have been more specific and descriptive in guiding participants to provide more meaningful responses.
- The pre-service teachers might have forgotten subject content knowledge that was done in their previous years of study, which could have contributed to gaps observed in their subject content knowledge. The researcher tried to assist the pre-service teachers in this regard by supplying an information sheet with the tests. However, not many of the participants made use of this information sheet (c. 4.3.3.8, 4.3.4.6, 4.3.5.6). The aforementioned might suggest that increased attention should be paid to the consolidation and retention of subject content knowledge during teacher-training. It should also be acknowledged that probability and statistics might not have been part of the school curriculum of some of the participants, which could have contributed to a lack in baseline subject content knowledge.

5.7.2 Possible limitations: collection and analysis of test data

The researcher acknowledges that the data analysis in some instances, might not have provided a true reflection of the participants' subject content knowledge, therefore affecting the validity of the findings. Nevertheless, the data provides initial information about gaps in the participants' subject content and pedagogical content knowledge that might obstruct their becoming effective teachers.

Often, evidence of factual knowledge required of participants to recall more than one concept/fact/aspect. The researcher argued that if participants could recall one of the concepts/facts/aspects it would have been unfair to indicate that they were entirely unsuccessful, and therefore assessed the participants as successful in the factual knowledge dimension. The researcher acknowledges, that it might have been more meaningful to assess the factual knowledge on a scale for strong evidence/moderate evidence/minimal evidence of factual knowledge. However, due to the large volumes of data, the interpretation would have been complicated with the finer divisions of knowledge. Interpretations and findings about the participants meeting expectations concerning factual knowledge are therefore made with caution.

The researcher presents examples of the aforementioned below.

Possible limitations regarding the assessment of factual knowledge dimension: participants were often expected to recall a number of concepts (three to four concepts, definitions or formula) as part of factual knowledge, for example: **Senior Phase**, Question 3 (space and shape)), 5 (data handling), **Intermediate Phase**, Questions 2.1 (patterns, functions, and algebra), 4.1, 4.2 (space and shape)), and **FET Phase** Question 2.1 (algebra), Question 3.2 (number

patterns, sequences, and series), Questions 4, 5, 6 and 7 (finance, growth, and decay, functions, probability, and statistics). Thus, if one of the concepts/aspects were recalled successfully the participants were indicated as successful in the factual knowledge dimension recall, and received an X as part of the knowledge dimension. In Question 3.1 for the FET Phase, participants had to satisfy one of the three requirements to be regarded as successful in procedural knowledge.

Possible limitations regarding the assessment of procedural knowledge: In Question 2.2 (algebra) of the FET Phase a participant rounded the final answer to 3, but in the standardised memorandum of the Grade 12 examination and in school mathematics it is required to provide two decimals unless required to round off. Therefore, the participants were not regarded as successful in the procedural knowledge dimension despite having the correct steps (cf. 4.3.3.2).

In the FET test, if a participant substituted incorrectly into a formula, although using algebraic skills correctly the participants were not viewed as successful in the procedural knowledge dimension (cf. 4.3.3.4). The procedural knowledge dimension was only considered successful when a correct answer was calculated (Crooks & Alibali, 2014; Rittle-Johnson et al., 2015) (cf. 2.3.2.3).

In the Senior Phase test, Question 1.2.1 (patterns, functions, and algebra) (cf. 4.3.4.1), participant 4 added an additional step (after writing down the correct answer). Due to writing down the correct answer the participant was still indicated as successful in the procedural knowledge dimension, but in actual fact the participant might have had insufficient conceptual and procedural knowledge due to not realising that the answer was already calculated.

The researcher acknowledges that the assessment of the aforementioned examples could be problematic, thus not providing a true reflection of procedural knowledge. It might have been more meaningful to also assess the procedural knowledge on a more detailed scale for correctness of procedural application.

Some participants in the FET and Senior Phase test did not make use of equal signs below each other when making use of expressions. Therefore, it is considered a notation error. However, the participants still computed the correct answers in most cases, therefore the notation error did not influence the answers to the questions (cf. 4.3.4.1), and the participants were not viewed as unsuccessful

In the Senior Phase test, Question 3.2 (space and shape), some of the participants did not start the solution by identifying which two triangles were investigated for congruency. Although the notation error did not influence the answer, it is of importance for all mathematicians not to make this error. When two triangles are compared to test for congruency, it is important to clearly specify which two triangles are used for the comparison. The lack of specification did not influence the solution, as there were only two triangles (cf. 4.3.4.3). The researcher acknowledges that this assessment could be problematic, and it might have been more meaningful to have added notation to the assessment of the conceptual and procedural knowledge.

In Question 4.2 (measurement) of the Senior Phase the participants were required to leave the final answer in terms of π and to add a unit. Some participants made use of the correct computation skills to find a final answer, but due to either not leaving the answer in terms π or not adding the unit caused them to be unsuccessful in the procedural knowledge dimension (cf. 4.3.4.4).

More information would be required of lecturers to make conclusive conclusions about their subject content knowledge and experience to train pre-service mathematics teachers effectively.

The next section focuses on suggestions for further research.

5.8 SUGGESTIONS FOR FURTHER RESEARCH

. The following suggestions are made for further research.

- It would be interesting if other universities could analyse their curriculum to ensure that pre-service mathematics teachers are equipped with the necessary subject content and pedagogical knowledge to teach mathematics effectively. A comparative study between universities would provide a great opportunity for universities to learn from one another and to embrace and incorporate strengths across curriculum into teacher-training curriculum for training mathematics teachers.
- The Department of Basic Education needs to design and present professional development programmes to address the subject content knowledge and pedagogical knowledge gaps of the pre-service mathematics teachers that have already graduated from the teacher-training programme that was assessed in this study. Research can be conducted to assess the successfulness of the professional development programmes for enhancing subject content and pedagogical content knowledge.
- Further studies could focus on the factual subject content knowledge of pre-service teachers as a prerequisite to develop metacognition. This study showed that a lack of solid

factual knowledge possibly influenced the application of metacognition in all the phases of this study (Intermediate, Senior and FET).

- Further studies could look into the presence and development of epistemological beliefs among pre-service mathematics teachers at South African universities, as epistemological beliefs directly influence the way subject content is viewed and taught (cf. 2.2).
- The researcher became aware that there might be additional factors, that could influence the effectiveness of mathematics teaching apart from complying with competency expectations concerning solid subject content and pedagogical content knowledge. Research could establish the extent to which depth and breadth of teachers' subject content knowledge (Ball et al., 2008; Ünsal et al., 2018), teachers' understanding of how learners learn and what they find difficult or easy to learn, the preconceived ideas learners have about content (Killen, 2015), and the importance of positive personality traits, attributes and dispositions among teachers (Stronge, 2018) (cf. 2.4), contribute to effective mathematics teaching.
- Research could be conducted to establish how well-developed the dispositions (open-mindedness, self-confidence, inquisitiveness, perseverance, etcetera) of pre-service teachers are, as well-developed dispositions have a strong influence on academic performance in mathematics (Kloppers & Grosser, 2014) (cf. 2.3.2.4).
- Research to establish the experience and level of subject content knowledge of lecturers involved in the training of mathematics lecturers could provide interesting information to support conclusions made about pre-service teachers' level of subject content and pedagogical content knowledge.
- In-depth scrutiny of the content of the various modules that form part of the teacher-training curriculum would be necessary to make final conclusions about the quality of the subject content and pedagogical content knowledge of pre-service teachers.

Although the researcher is aware of the limitations involved in the research, the research makes an initial contribution to the theory and practice of mathematics teaching, in particular for the university where the research was conducted, which is discussed in the following section.

5.9 CONTRIBUTION OF THE STUDY

5.9.1 Contribution to theory

The research provides an original way of conceptualising the various dimensions of subject content knowledge in mathematics that need to function in unison. Furthermore, the research emphasises the importance of developing subject content knowledge in conjunction with cognitive

process dimensions, and includes metacognition as an important prerequisite and dimension in the subject content knowledge mathematics teachers have to acquire (cf. 2.3).

5.9.2 Contribution to practice

Based on the research findings, the pre-service mathematics teachers and their lecturers could become aware of the lack of subject content and pedagogical content knowledge among pre-service teachers. Moreover, the limitations and gaps in the teacher-training curriculum employed at the university prior to 2017 that contributed to the apparent gaps in subject content and pedagogical content knowledge, were outlined and could be utilised in streamlining future curriculum initiatives. Being aware of their own shortcomings the pre-service teachers might try to improve their subject content knowledge by doing self-study or attending professional development courses. Mathematics lecturers could also consider purposeful attempts to enhance the observed gaps in the subject content and pedagogical content knowledge of the pre-service teachers.

This study did not set out to evaluate the institution where the research was conducted, but to focus attention to some limitations in the mathematics teacher-training curriculum at NWU prior to 2017, for which recommendations were made. The identified limitations and recommendations could guide future attempts that aim to review mathematics curriculum at NWU, or at any other university involved with the training of mathematics teachers to ensure that mathematics teachers meet competency expectations concerning subject content and pedagogical content knowledge.

5.10 CONCLUSION

Learner achievement in mathematics is a direct result of teaching quality (Taylor, 2008), and the problem of teaching quality needs to be addressed at pre-service and in-service level (Sorto, 2010; Taylor, 2008; Volmink, cited by BusinessTech, 2020). The pool of students who complete school and pursue a study in the field of mathematics is small and not strong (Shay 2019). Efforts at primary and secondary school-level need to strengthen in order to grow the pool of matriculants who enter for studies at universities, for which teachers with solid subject content and pedagogical content knowledge are required. According to Shay (2019), the responsibility for strengthening and growing a pool of qualified graduates who enter the field of mathematics teaching lies with higher education who has failed to graduate some of the strongest learners in the pool of mathematics students that the South African schooling system offered between 2015–2017. Higher education is therefore faced with the challenge and responsibility to rethink the quality of teacher-training curriculum, in order to produce teachers who fully meet the competency expectations for subject content and pedagogical content knowledge that would bring

extraordinary change to the quality of mathematics teaching in South Africa, and end the teacher-learner-teacher conundrum of ineffective mathematics teaching (Maseko, 2015).

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