



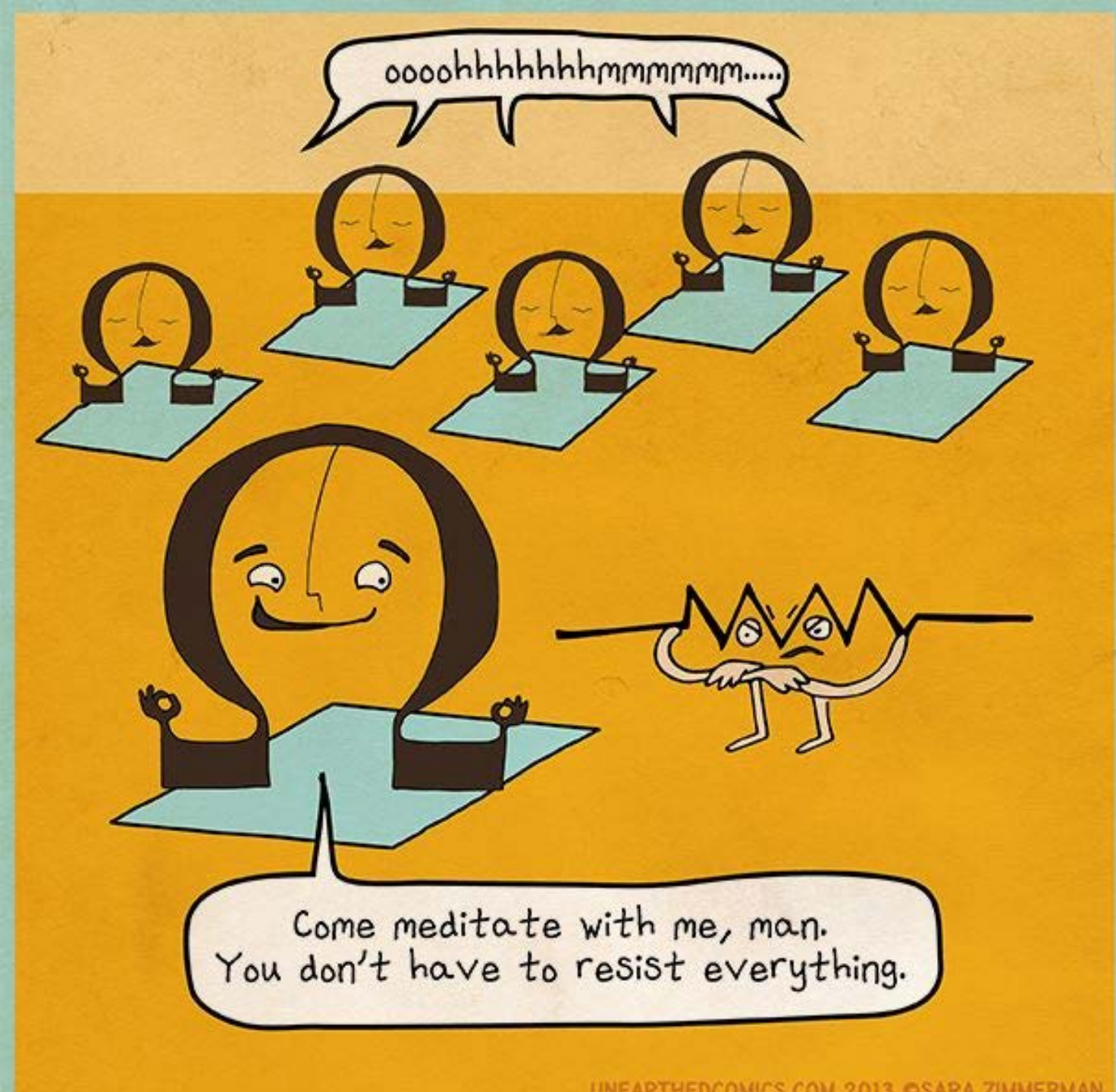
The day the current disappeared

Johan.Rens@nwu.ac.za

3 May 2019

C

- Finding current: What did
- Why the Polish knight war
- That is why the current we
- Until sanity was restored b
- Application in impedance-



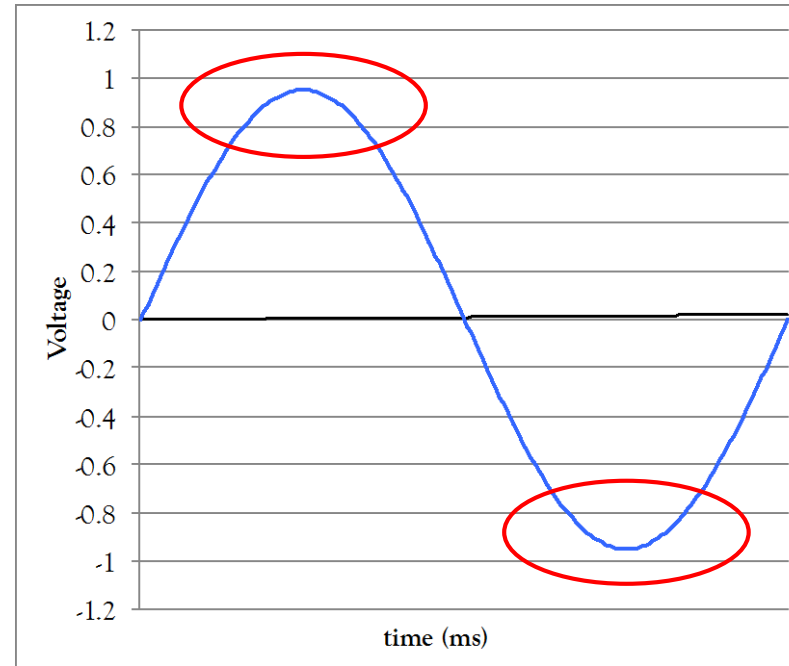
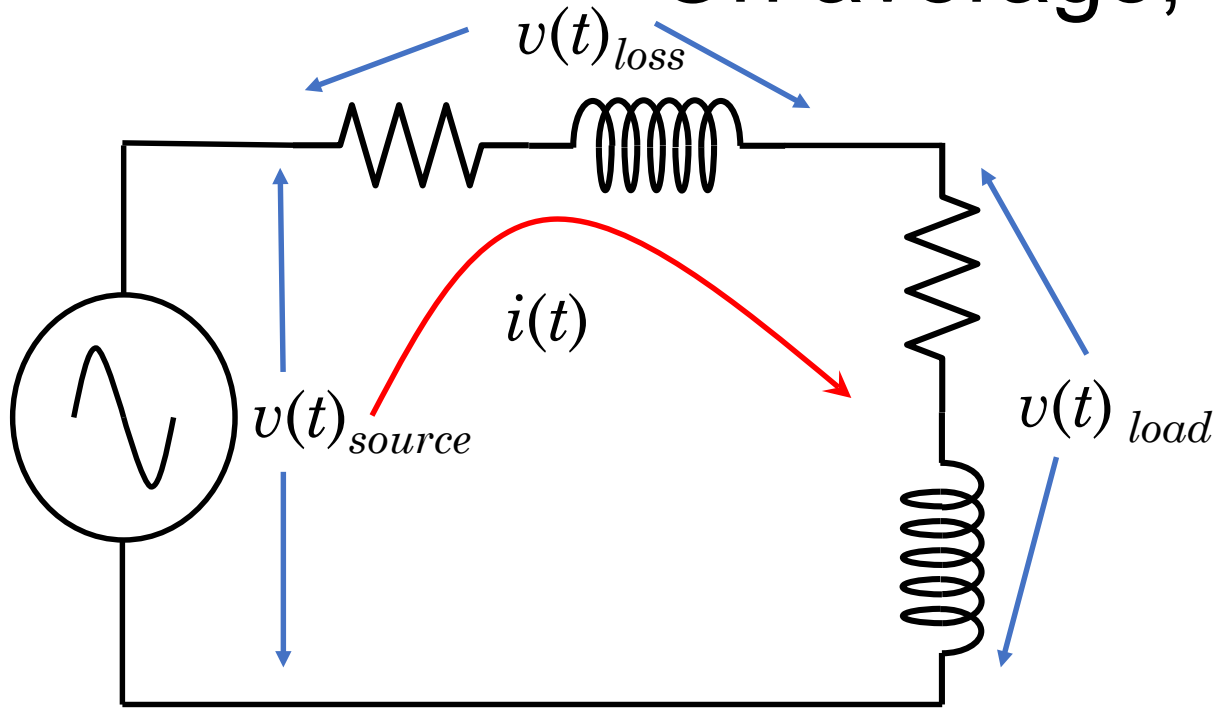
A brief history of nearly every electron

- Electricity as a matter of physics
- Found application during 1800s to do “something”, mostly to be useful
- Moving electrically charged particles
- Mostly done in an alternating current
- How well it is done governed by Ohm's Law
- And that misery made the current

Resistance is futile



On average, the voltage remains shockingly

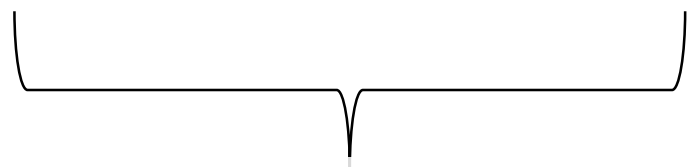


$$p(t) = v(t) * i(t)$$

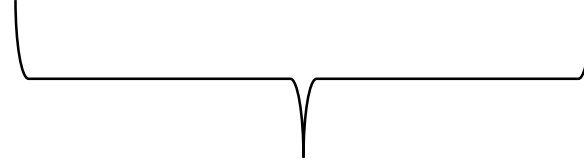
$$= V_{max} * I_{max} * \cos(\omega t + \delta) \cos(\omega t + \beta)$$

$$= \frac{1}{2} \sqrt{2} V_{rms} \sqrt{2} I_{rms} \cos(\delta - \beta) + V_{rms} I_{rms} \cos(\delta - \beta) \cos[2(\omega t + \delta)] + V_{rms} I_{rms} \sin(\delta - \beta) \sin[2(\omega t + \delta)]$$

$$= V_{RMS} I_{RMS} \cos(\delta - \beta) \{1 + \cos[2(\omega t + \delta)]\} + V_{RMS} I_{RMS} \sin(\delta - \beta) \sin[2(\omega t + \delta)]$$

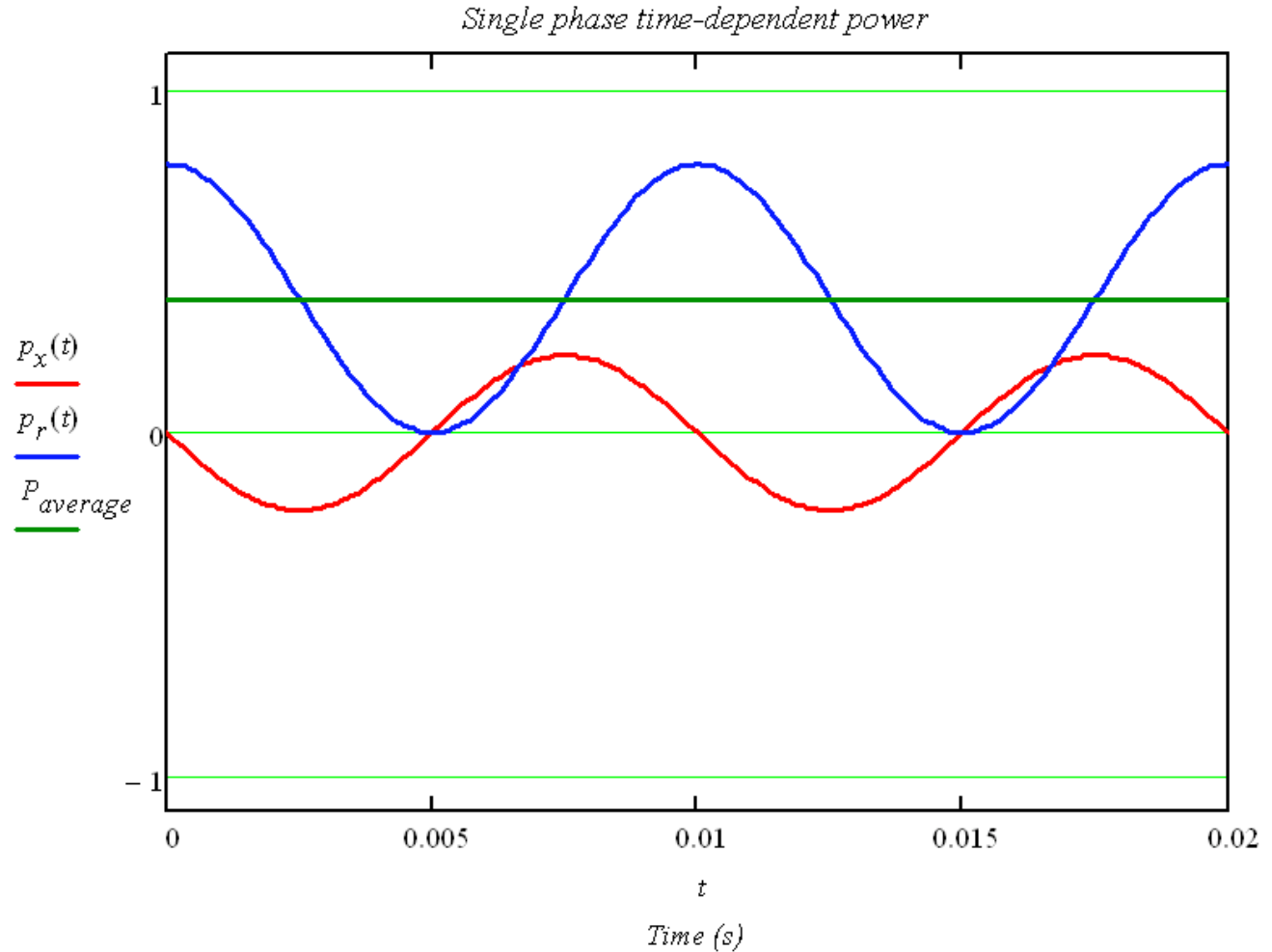
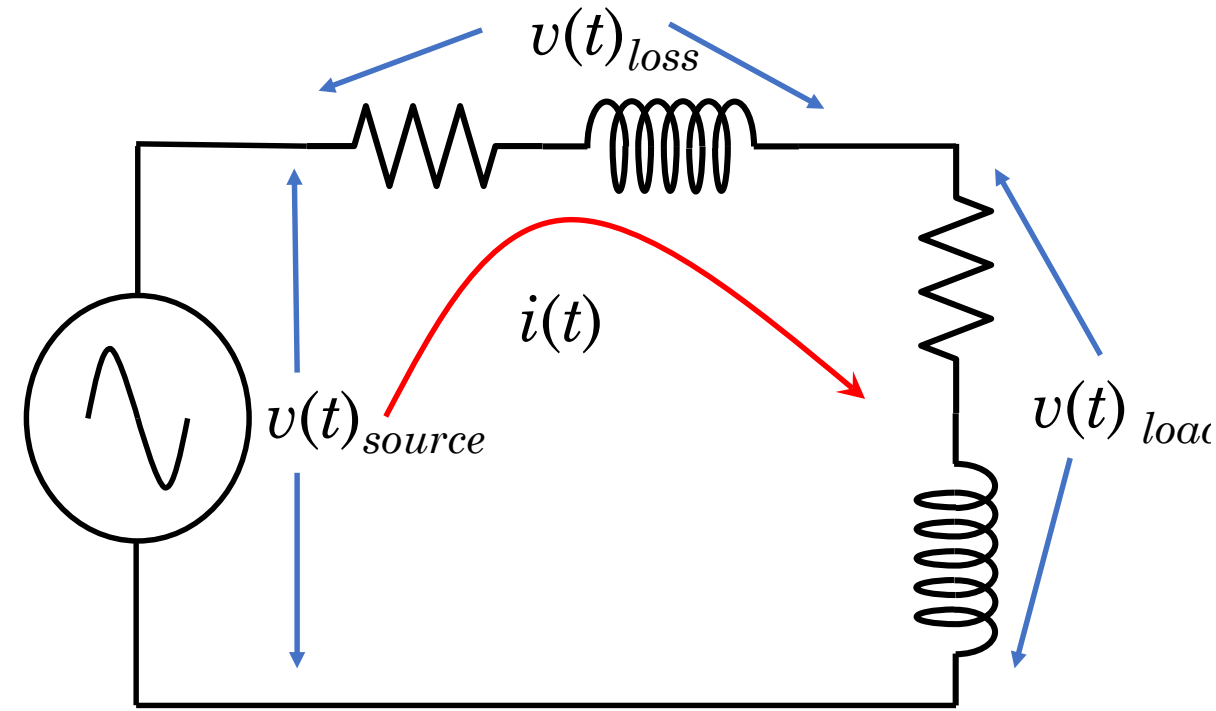


Resistive component of load



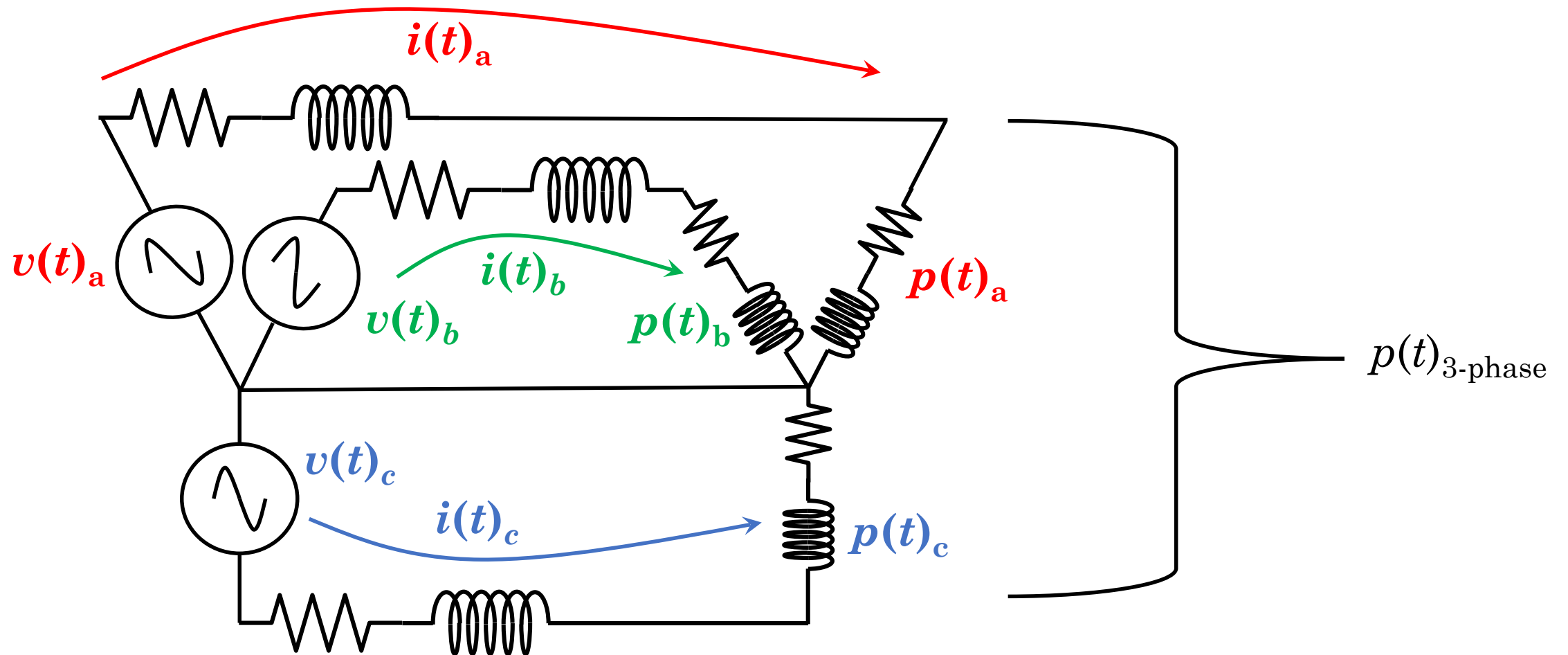
Reactive component: L, C or both

That is why that single-phase generator remained so



Real work done: $P_{average} = V_{RMS} I_{RMS} \cos(\delta - \beta)$

That is why we have 3-phase electricity



The War of Currents winner : George Westinghouse; 1886

$$v_a(t) = \sqrt{2}V_{LN} \cos(\omega t + \delta)$$

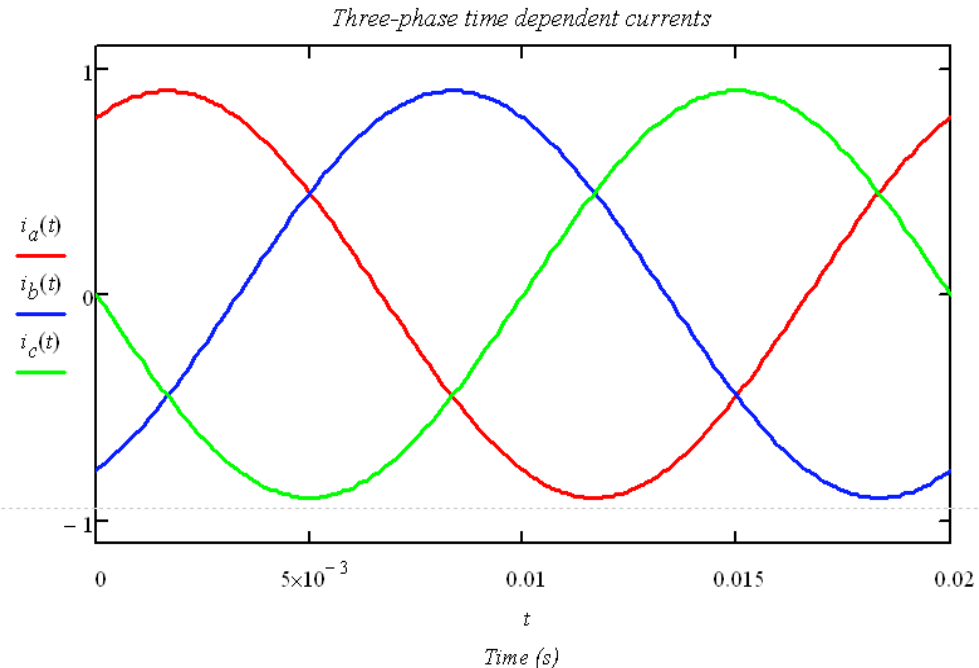
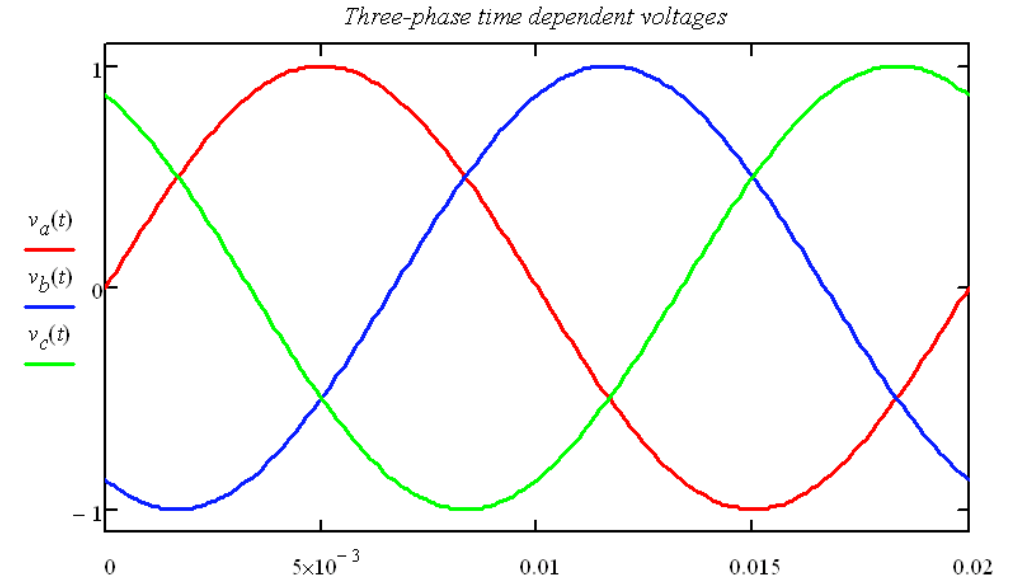
$$v_b(t) = \sqrt{2}V_{LN} \cos(\omega t + \delta - 120^\circ)$$

$$v_c(t) = \sqrt{2}V_{LN} \cos(\omega t + \delta - 240^\circ)$$

$$i_a(t) = \sqrt{2}I_L \cos(\omega t + \beta)$$

$$i_b(t) = \sqrt{2}I_L \cos(\omega t + \beta - 120^\circ)$$

$$i_c(t) = \sqrt{2}I_L \cos(\omega t + \beta - 240^\circ)$$



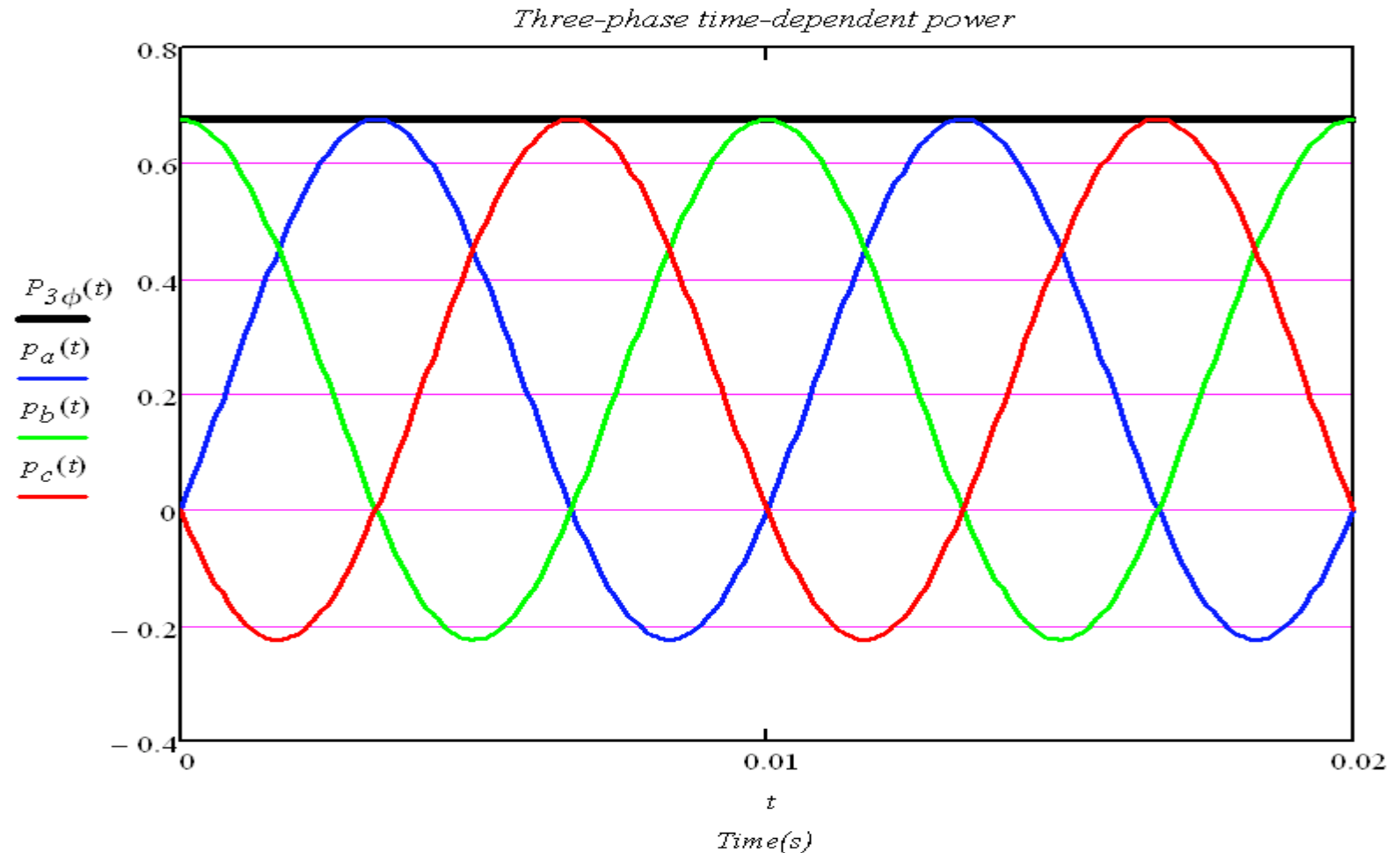
The War of Currents winner : George Westinghouse; 1886

$$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t)$$

$$= 3V_{LN}I_L \cos(\delta - \beta) + V_{LN}I_L \cos(\delta - \beta) + V_{LN}I_L [\cos(\delta - \beta) + V_{LN}I_L \cos(2\omega t + \delta + \beta) + \dots$$

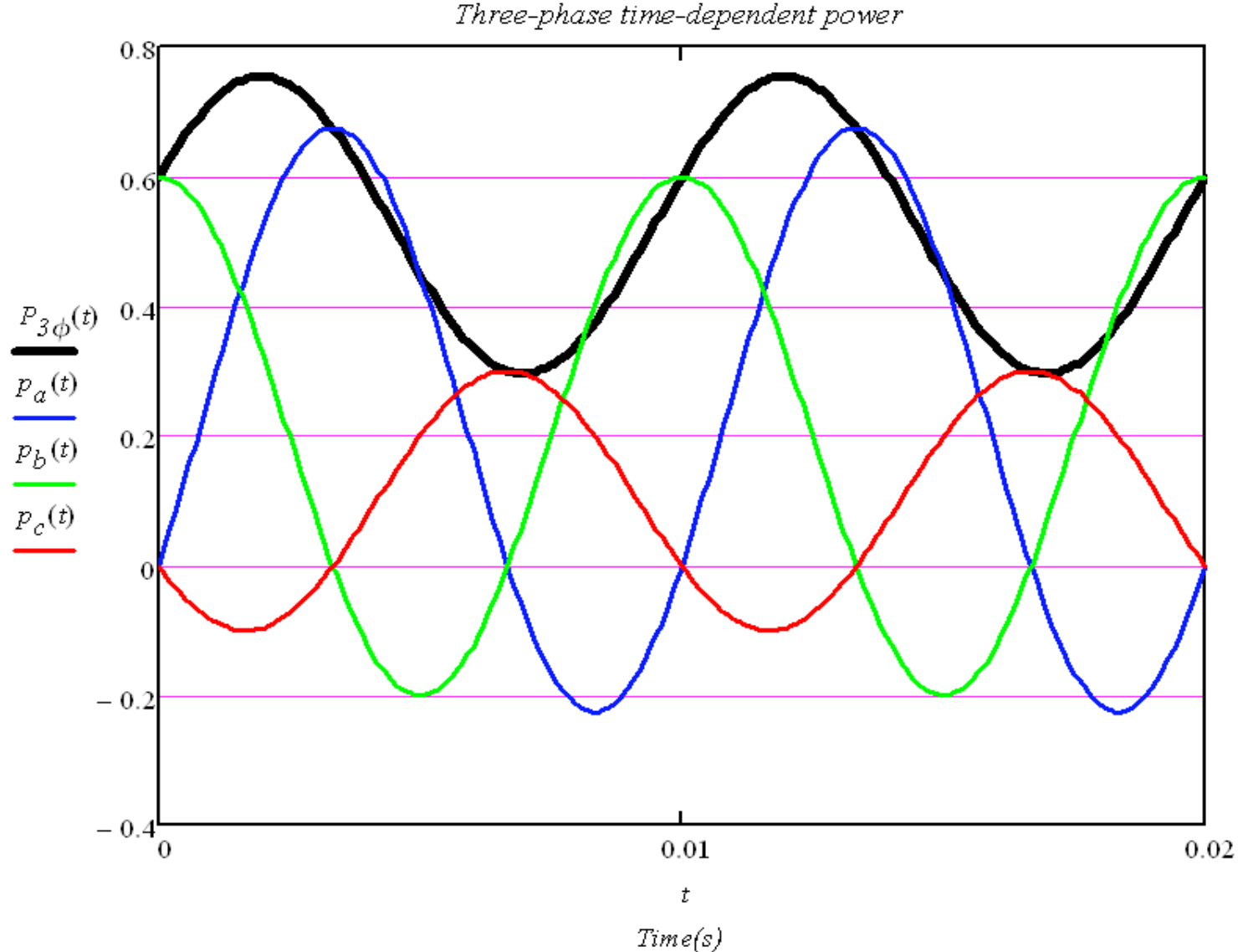
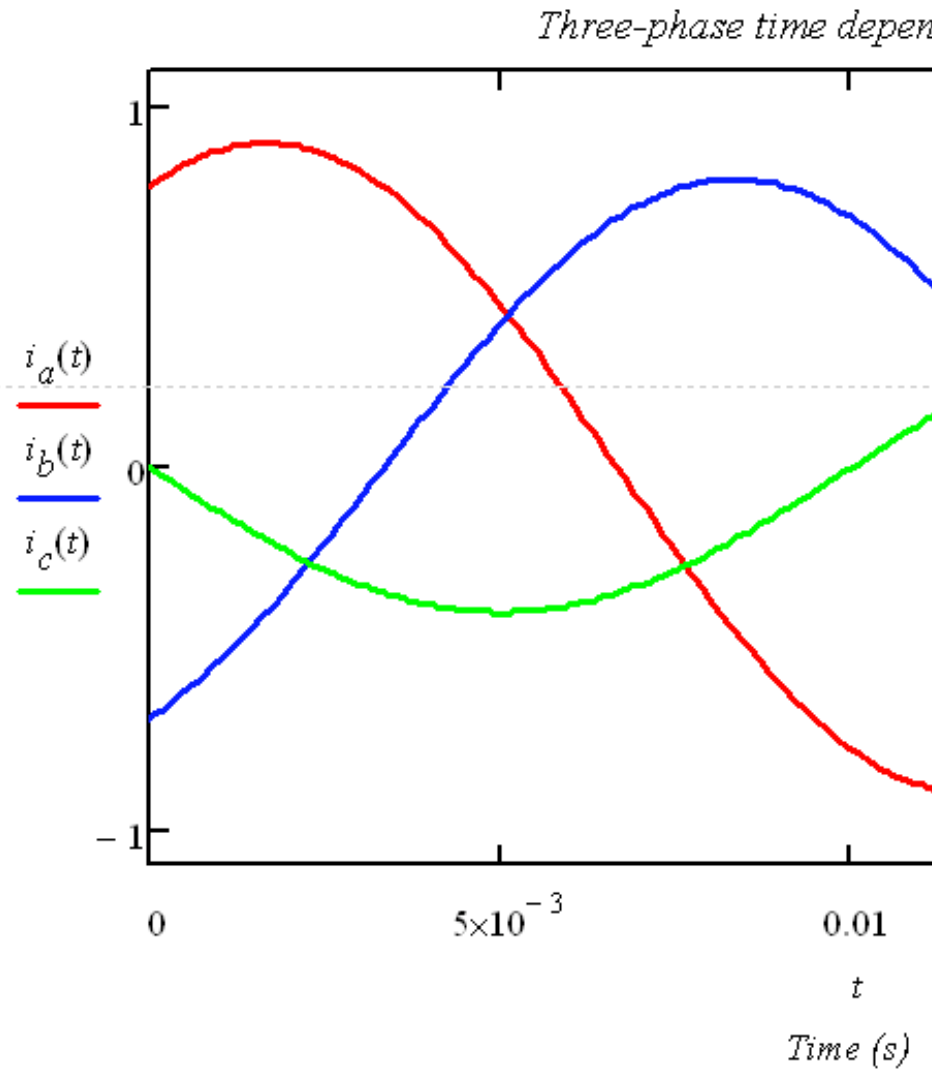
$$\dots + \cos(2\omega t + \delta + \beta - 240^\circ) + \cos(\delta - \beta) + V_{LN}I_L \cos(2\omega t + \delta + \beta + 240^\circ)]$$

$$= 3V_{LN}I_L \cos(\delta - \beta) = P_{3\phi}$$



And if those voltages are no longer symmetrical and those currents unbalanced?

The time-independent feature of three-phase energy transfer is lost.....



It get worse when the load is non-linear (and supply voltage non-sinusoidal)....

- What does non-linear loading mea
- More or less the modern way of co
- Such as in a LED lamp, a laptop cl
- Nowadays, MW's and kV's
- The energy transfer is impacted
- We are used to:

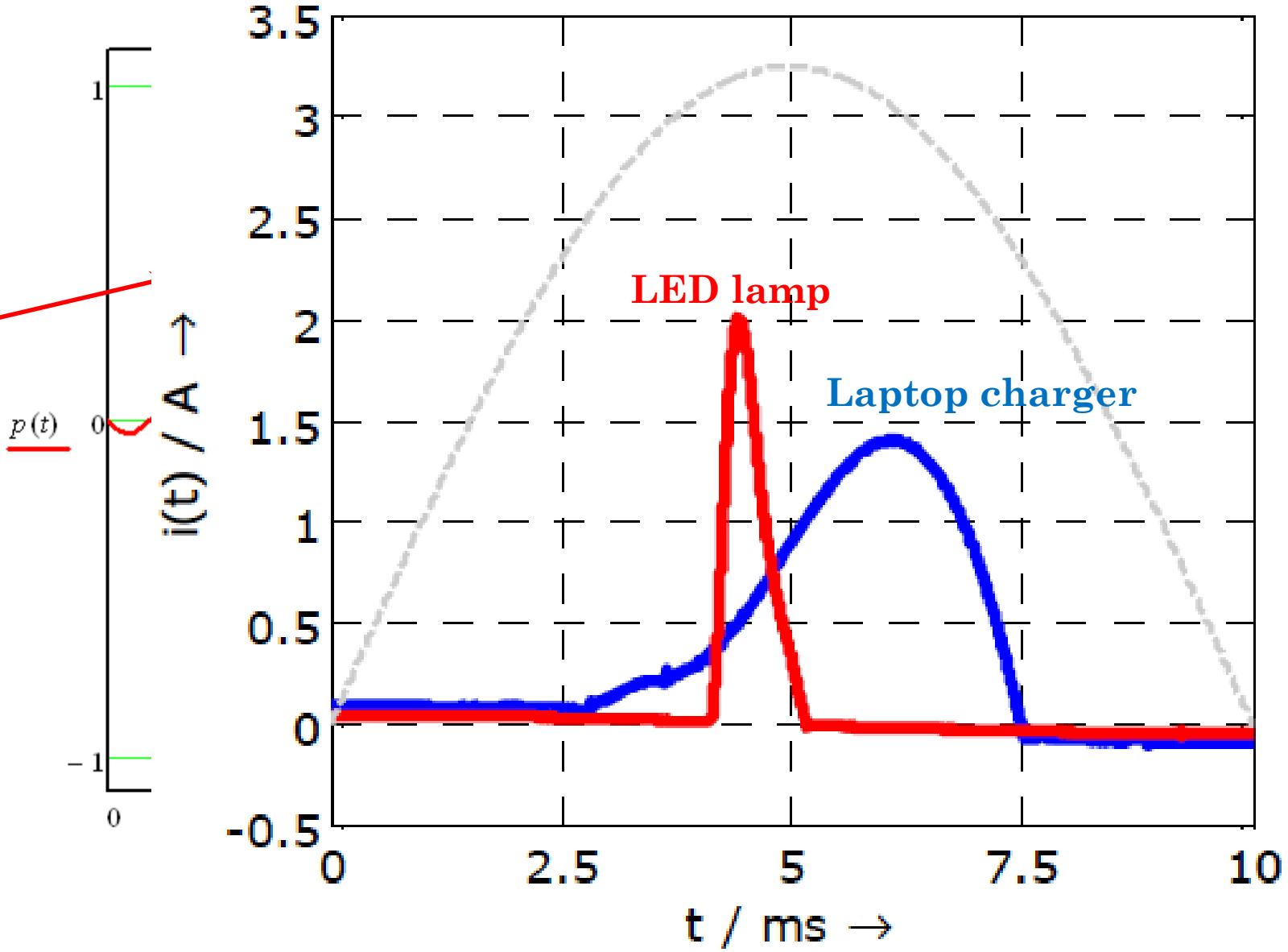
$$S = P + jQ$$

$$\therefore S^2 = P^2 + Q^2$$

Not anymore....

$$S^2 > P^2 + Q^2$$

$$D^2 = S^2 - P^2 - Q^2$$



Now this is where the current got lost!

- Distortion power?
- What is that?
- Something missing in power theory?
- Budeanu defined the concept of distortion power – as “distortion” is the cause
- Many contributions to power theory followed
- Modern power phenomena required new definition
- Engineers have to design, specify and operate power system in which energy phenomena has physical interpretation
- Why is the term “distortion power” then still in use?
- Discussion became intense.....



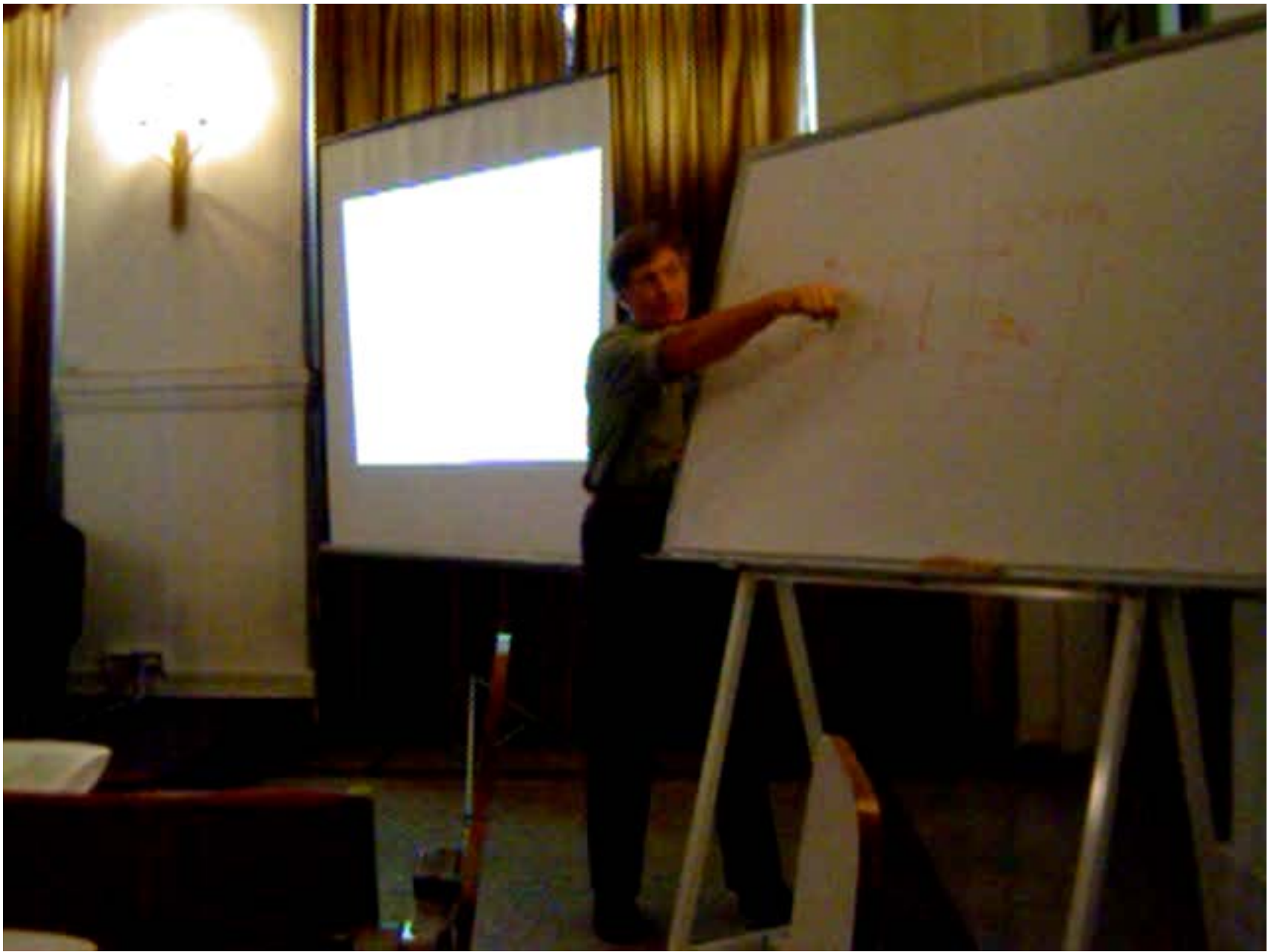
13 21:45



13 20:42

And then the Polish knight
arrived...







What is Wrong Reactive and Distort

LESZEK S.

Shouldn't we ra

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Year

Single Year | Range

1988 | 2017

From: 1988 To: 2017

- Author
- Affiliation
- Publication Title
- Publisher
- Conference Location
- Index Terms

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Sort By: **Relevance**

Investigating the validity of the Czarnecki three phase power definitions

A.P.J. Rens ; P.H. Swart
IEEE AFRICON. 6th Africon Conference in Africa,
Year: 2002 , Volume: 2
Pages: 815 - 821 vol.2

IEEE Conferences
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Closure on "Instantaneous Reactive Power p-q Theory and Power Properties of Three-Phase Systems"

Leszek S. Czarnecki
IEEE Transactions on Power Delivery
Year: 2008 , Volume: 23 , Issue: 3
Pages: 1695 - 1696
Cited by: Papers (8)

IEEE Journals & Magazines
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Closure on "Could Power Properties of Three-Phase Systems Be Described in Terms of Poynting Vector?"

Leszek S. Czarnecki
IEEE Transactions on Power Delivery
Year: 2007 , Volume: 22 , Issue: 2
Pages: 1269 - 1270
Cited by: Papers (2)

IEEE Journals & Magazines
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Instantaneous reactive power p-q theory and power properties of three-phase systems

L.S. Czarnecki
IEEE Transactions on Power Delivery
Year: 2006 , Volume: 21 , Issue: 1
Pages: 362 - 367
Cited by: Papers (103)

But what is the problem?

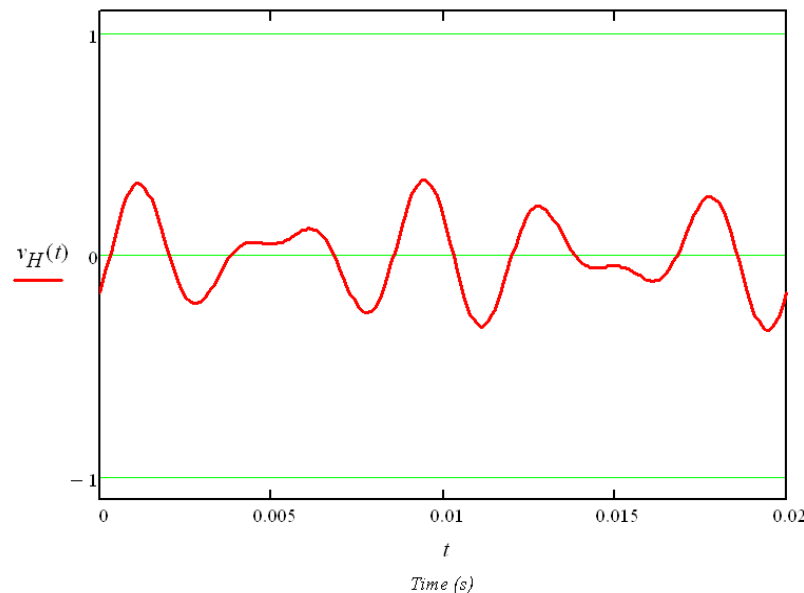
- Steinmetz (1892): Ratio of active to apparent power decrease when waveform becomes more distorted such as electric arc (lighting application).
- Impact of distortion and unbalance new phenomena
- Power factor reduction a concern -
- **Unbalance in loading, asymmetry in supply voltages, AND distortion in voltage and/or current** contributes to the degradation of power factor (the efficiency in the transfer of real energy)
- Classical power theory can only deal with perfectly sinusoidal voltages, perfectly symmetrical between phases and perfectly linear loads that withdraw perfectly balanced currents
- Budeanu (1927) described $S^2 > P^2 + Q^2$ when waveforms are non-sinusoidal
- IEEE-1459-2010 attempted to further practical formulations universally acceptable for engineers to deal with modern power systems

Lyon	1920	Depenbrock	1977-2003
Bucholz	1922	Kusters and Moore	1980
Budeanu	1927	Page	1980
Fryze	1932	Nomowiesjki	1981
Goodhue	1933	Akagi-Nabae	1983.....
Quade	1937	Filipski	1984
Nudelcu	1963	Sawicki	1986
Sharon	1972	Czarnecki	1987-.....
Shepherd-Zakikhani	1972	Enslin	1988
Emanuel	1974	Tenti	1990
Harashima	1976	Ferrero	1991-.....
		Willems	1992-.....
		IEEE, Emanuel	1996-.....

Effective values: nonsinusoidal waveform conditions (1)

- The IEEE 1459-2000 document further practical guidelines on power definitions
- The time-domain or the frequency domain can be used for power definitions
- Focus will be on frequency domain power definitions in this presentation
- A non-sinusoidal, single-phase, time-dependent voltage $v(t)_{1\phi}$ with fixed and repetitive period T is applied to a load - represented as a finite series of harmonic components: $h=1, 5, 7$
- Single phase distortion component of $v(t)_{1\phi}$ can be isolated as $v_H(t)_{1\phi}$:

$$v(t)_{1\phi} \neq \sum_{h=1}^N v(t)_{h\phi} \quad (\neq) \quad \sum_{h=1}^N \sqrt{2} V_h \left(\sin(h\omega t) + \alpha \frac{1}{5} \sin\left(\frac{1}{5} \sin(5\omega t) + \theta_{57}^1\right) \sin\left(\frac{1}{7} \sin(7\omega t) + \theta_7\right) \right)$$

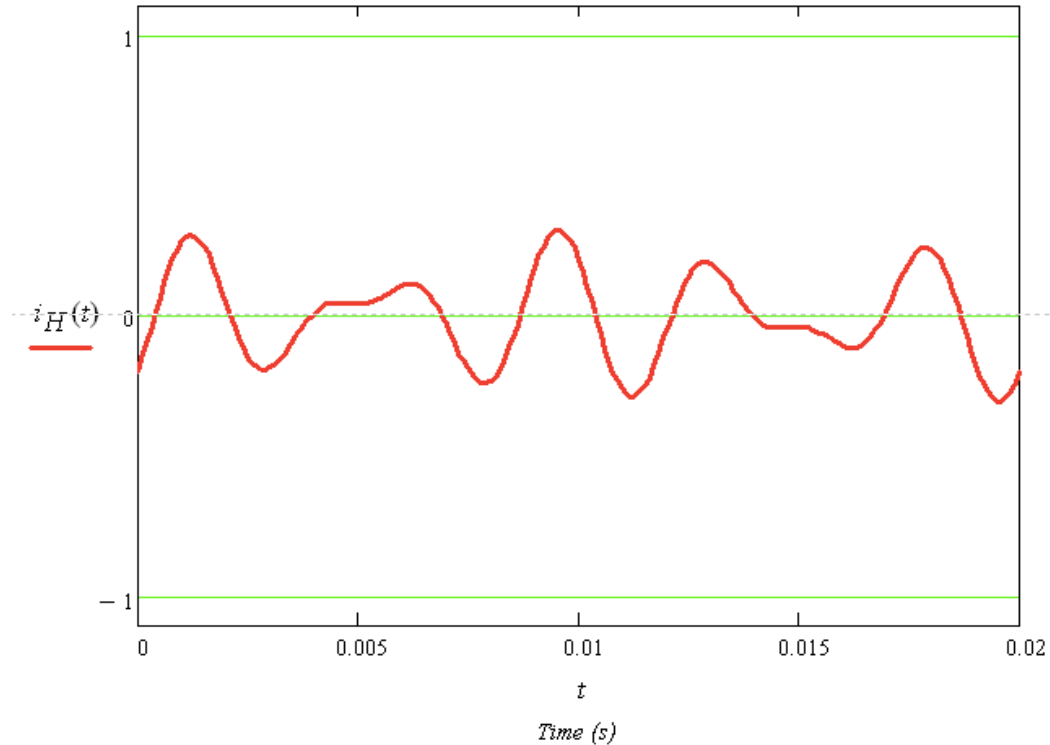


Effective values: nonsinusoidal waveform conditions (2)

- $i(t)_{1\phi}$ when $v(t)_{1\phi}$ is applied to frequency dependent impedance load:

- Distortion components $i_H(t)_{1\phi} = \sum_{h=1}^N i_{hN}(t)_{1\phi} = \sum_{h=1}^N \sqrt{2} I_{h,1\phi} \sin(h\omega_1 t + \beta_h)$

$$i_H(t)_{h \in [1, 5, 7]} = \sum_{h \neq 1} \sqrt{2} I_h \sin(h\omega_1 t + \beta_h)$$



Active Power: nonsinusoidal waveform conditions

- The *effective* or *RMS* values: $V_{1\phi} = \sqrt{\sum_{h=1}^N |V_{h,1\phi}|^2}$; $I_{1\phi} = \sqrt{\sum_{h=1}^N |I_{h,1\phi}|^2}$

- The single phase power: $p_{1\phi}(t) = v(t)_{1\phi} i(t)_{1\phi} = \left(\sum_{h=1}^N v_h(t)_{1\phi} \right) \left(\sum_{h=1}^N i_h(t)_{1\phi} \right)$

- Classical power theory formulates ***Time-dependent Active Power*** (per harmonic order h):

$$p_h(t)_{1\phi} = \operatorname{Re} \left[\sqrt{2} v_h(t)_{1\phi} \right] \operatorname{Re} \left[\sqrt{2} i_h(t)_{1\phi} \right]$$

- The ***Time-dependent Total Active Power*** of a circuit under distorted waveform conditions:

$$p(t)_{1\phi} = \sum_{h=1}^N p_h(t)_{1\phi}$$

- ***Total (or Joint) Average Active Power*** requires integration over a period T :

$$P_{1\phi} = \frac{1}{T} \int_T p(t)_{1\phi} dt = \sum_{h=1}^N V_{h,1\phi} I_{h,1\phi} \cos(\alpha_h - \beta_h)$$

Reactive Power: nonsinusoidal waveform conditions

- Classical power theory formulates ***Time-dependent Reactive Power*** (per harmonic order h):

$$q(t)_h = \operatorname{Re} \left[\sqrt{2} \mathbf{v}(t)_h \right] \operatorname{Im} \left[\sqrt{2} \mathbf{i}(t)_h \right]$$

- The ***Time-dependent Total Reactive Power*** of a circuit under distorted waveform conditions:

$$q(t) = \sum_{h=1}^N q(t)_h$$

Does it make sense?

👉 **It does not make sense!**

- ***Total (or Joint) Average Reactive Power*** requires integration over a period T :

$$Q_B = \frac{1}{T} \int_T q(t) = \frac{1}{T} \int_T \sum_{h=1}^N q(t)_h = \sum_{h=1}^{\infty} V_{h,1\phi} I_{h,1\phi} \sin(\alpha_h - \beta_h)$$

$$p(t)_{1\phi} = v(t)_{1\phi} i(t)_{1\phi}$$

$$= \sum_{h=1}^N v_h(t)_{1\phi} \sum_{h=1}^N i_h(t)_{1\phi}$$

$$p(t)_{1\phi} = \sum_{h=1}^N \sqrt{2}V_{h,1\phi} \sin(h\omega_1 t + \alpha_h) * \sum_{h=1}^N \sqrt{2}I_{h,1\phi} \sin(h\omega_1 t + \beta_h)$$

$$= \sqrt{\left(\sum_{h=1}^N P_{h,1\phi}\right)^2 + \left(\sum_{h=1}^N Q_B\right)^2 + \left(\sum_{h,k=1(h \neq k)}^N |V_{h,1\phi}| |I_{k,1\phi}|\right)^2}$$

$$= P_H + Q_B + D_B$$

Budeanu's Distortion Power

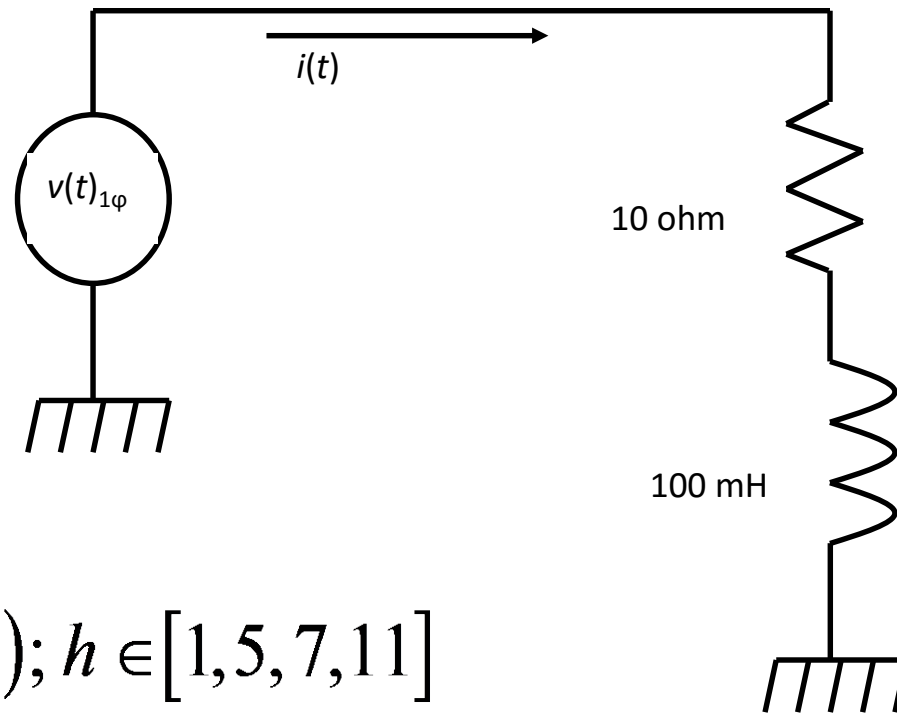
Total Active Power Total Reactive Power (Budeanu's Reactive Power)

What is wrong with Q_B – the Budeanu Reactive Power?

- Physical nature of reactive power follows from the application of field theory (Maxwell's equations)
- Reactive Power not to contribute to real energy transfer
- Physical nature of reactive power - energy accumulation in electric and magnetic fields of reactive components in the load and source
- Results in oscillatory exchange of energy between these reactive components
- Similar explanation assigned to harmonic reactive power Q_h at each harmonic order h
- Is Q_B (Joint/Total Reactive Power) a useful concept?
 - Let's investigate.....

$$Q_B = \frac{1}{T} \int_T q(t) = \frac{1}{T} \int_T \sum_{h=1}^N q(t)_h = \sum_{h=1}^{\infty} V_{h,1\phi} I_{h,1\phi} \sin(\alpha_h - \beta_h)$$

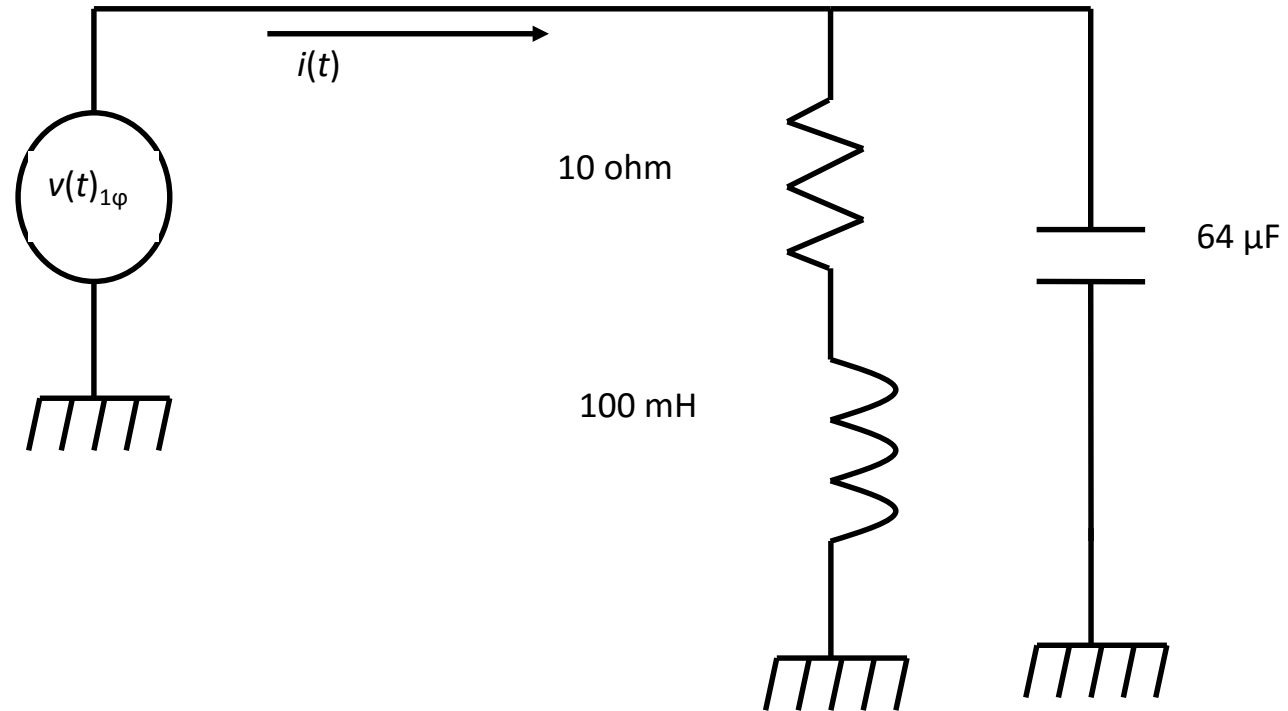
Application of Q_B in power factor correction (1)



$$v(t)_{1\phi} = \frac{100}{h} \sin(2 \cdot \pi \cdot 50 \cdot h \cdot t); h \in [1, 5, 7, 11]$$

- PF = 0.3 based on Q_B

Application of Q_B in power factor correction (2)



- Calculate the capacitance in parallel to compensate Q_B

$$C = \frac{Q_B}{V_1^2 \omega_1 + V_5^2 \omega_5 + V_7^2 \omega_7 + V_{11}^2 \omega_{11}}$$

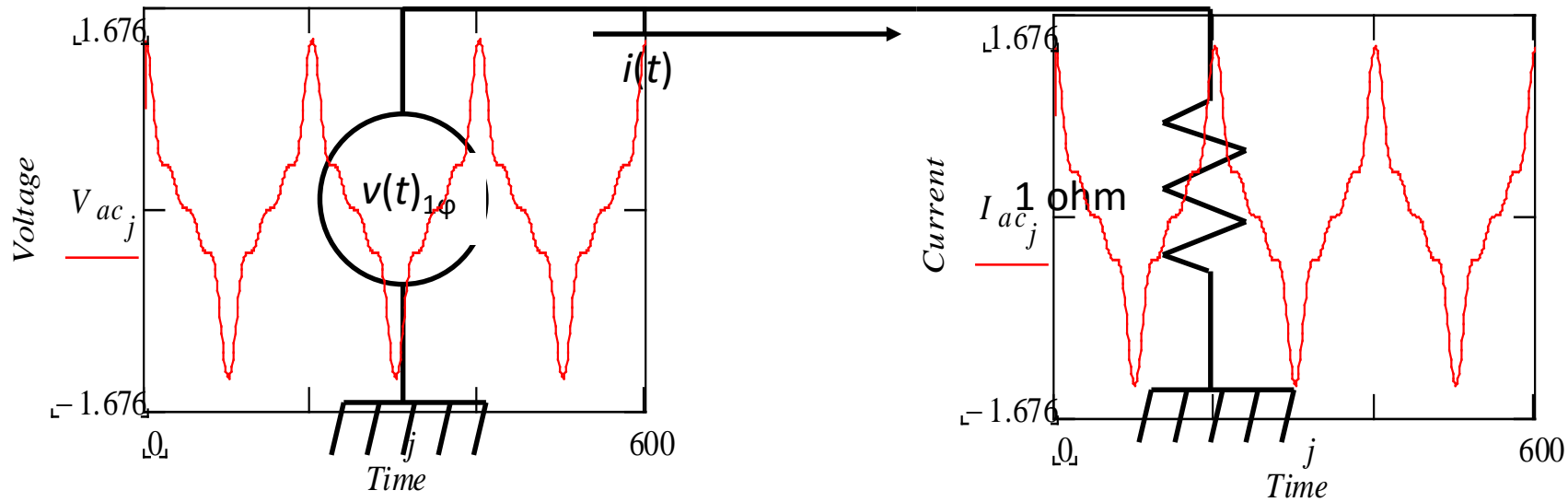
Application of Q_B in power factor correction (3)

Symbol	Without a capacitor	Compensated	Units
$S_{1\phi}$	602	412	VA
$P_{1\phi}$	180.	181	Watt
Q_B	574	4	VAr
D_B	16	370	VAr
PF	0.3	0.4	
C	0	65	μF

- Q_B completely compensated by capacitor, but- power factor of compensated circuit did not change significantly.
- Apparent power is less in compensated circuit but unnecessary loading remains (difference between apparent power and real power).

Remark

- Budeanu's reactive power (Q_B) not useful for power factor compensation.
- Power factor correction results in "distortion power" (D_B) to increase significantly due to increased interaction between uneven harmonic voltage and current components.



$$v(t)_{1\phi} = \frac{1}{h} \sin(2 \cdot \pi \cdot 50 \cdot h \cdot t); h \in [1, 3, 5, 7]$$

- D_B has zero value: waveforms perfectly sinusoidal?
- Both voltage and current are distorted!
- D_B does not relate to degree of waveform distortion.

$S_{1\phi}$	1.194 VA
$P_{1\phi}$	1.194 Watt
Q_B	0 VAr
D_B	0 VAr

Summary: Power theory in a modern power system

- It must, as far as possible consist of a generalisation of the classic single-frequency power theory that has by now been universally accepted.
- It must be as amenable to conventional measurement techniques as possible and require the minimum of sophistication in instrumentation.
- It's different defined components must be relatable to physically observable or ascribable phenomena and not to hypothetical or abstract mathematical definition.
- It must present a suitable basis for quantifiable measurement, control, tariff systems and design.
- It must cater for every conceivable practical situation and never violate circuit laws, regardless of which domain it is transformed into.
- It must be useful to the engineer who has to apply these definitions in design, specification and operation of the power system.

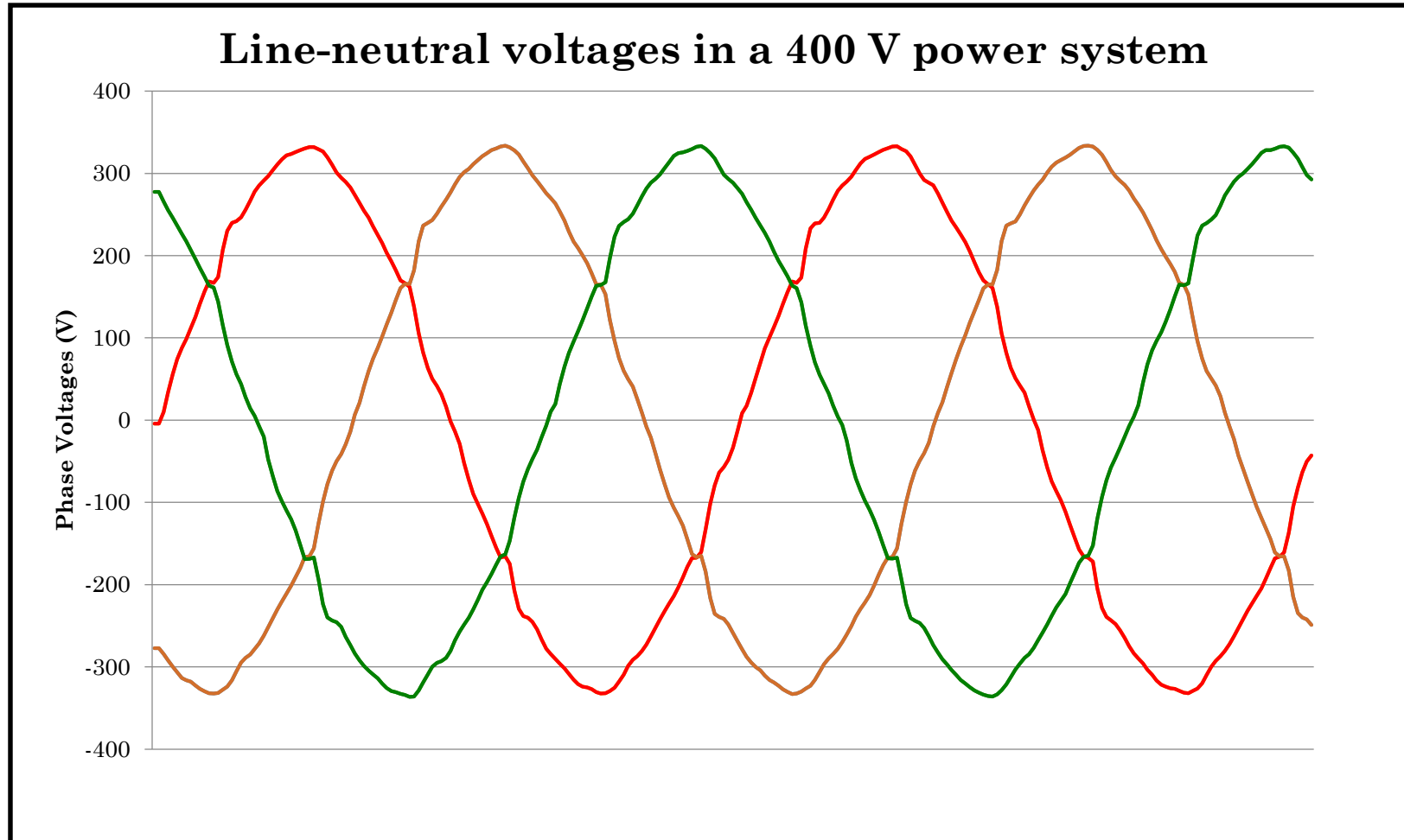
Modern Power Theory

2. The IEEE 1459 made easy

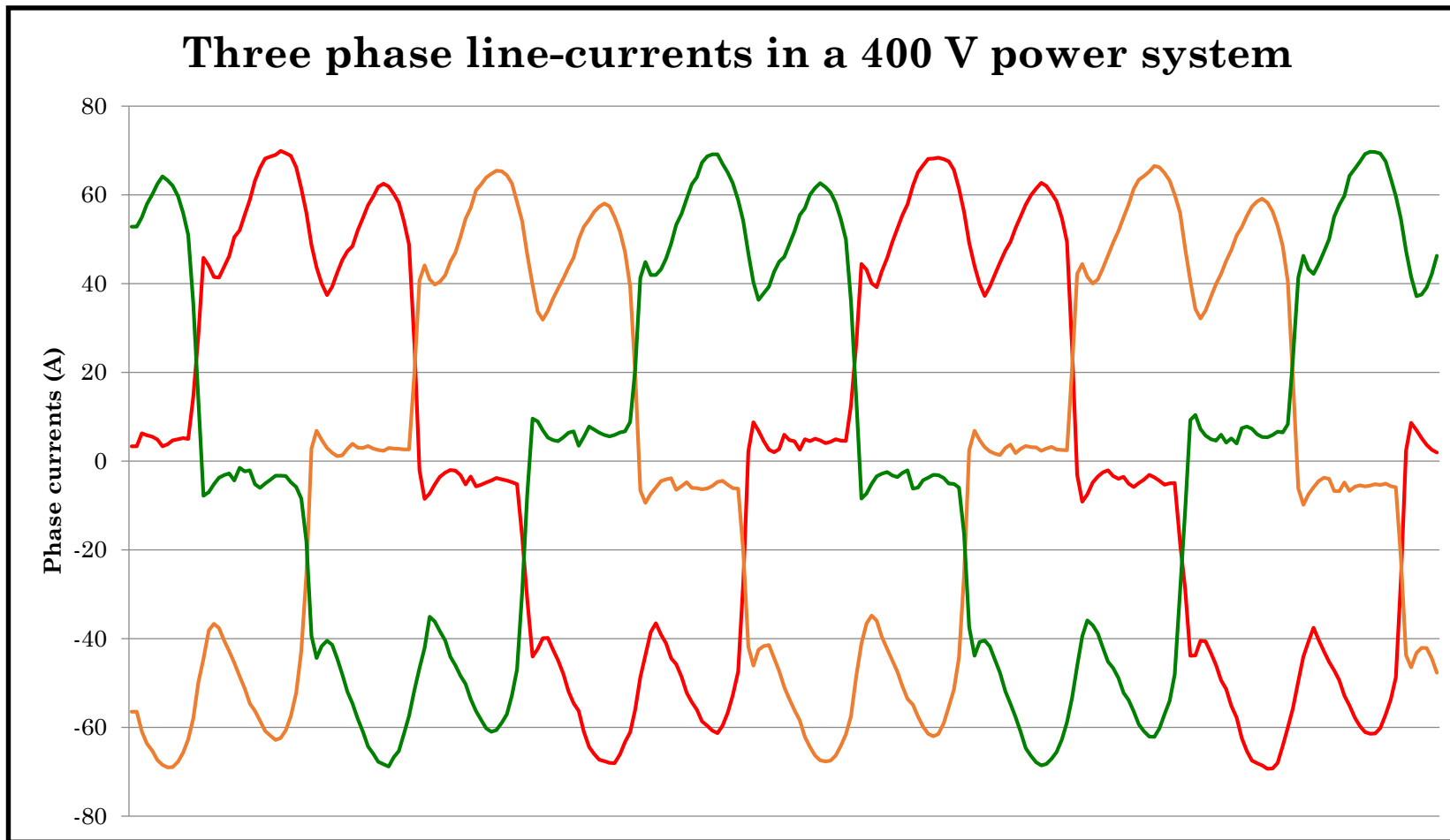


What is a modern Power System? (1)

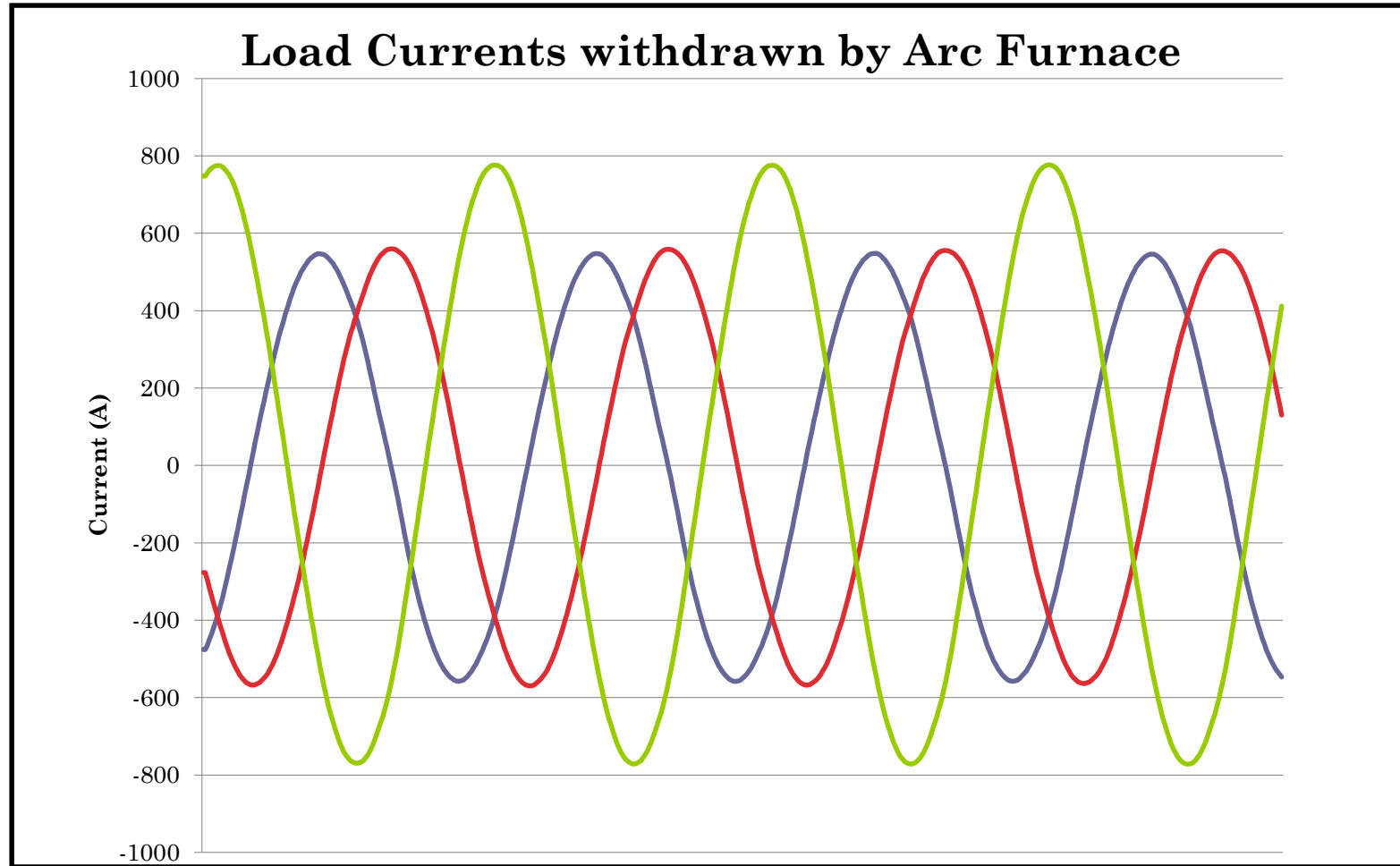
Condition 1: Non-sinusoidal supply voltages at a PCC.....



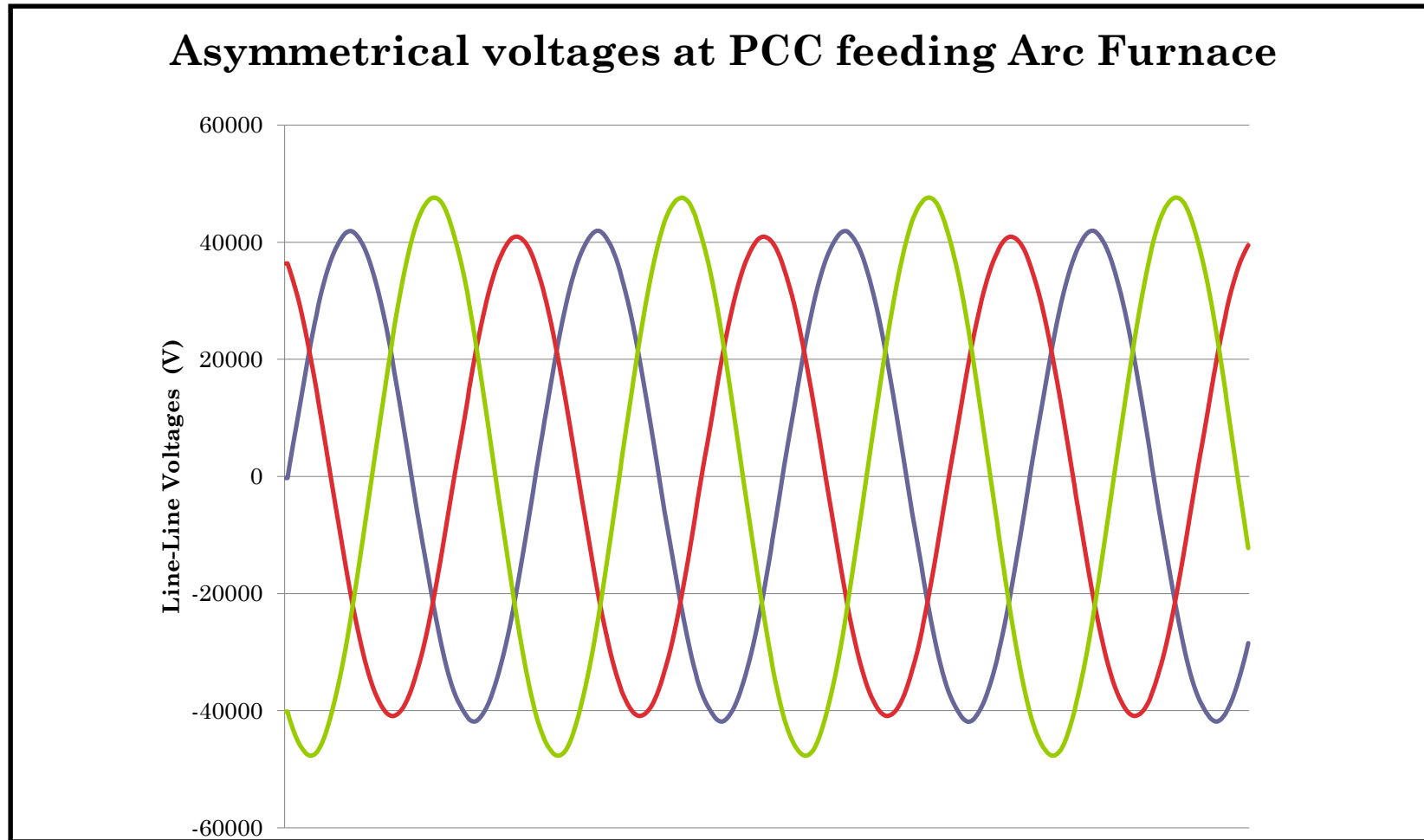
Condition 2: Nonlinear load current



Condition 3: Unbalance in loading...



Condition 4: Asymmetrical supply voltages...



- A Three-phase power system under non-sinusoidal waveform conditions, unbalanced loading and asymmetrical supply voltages is considered.
- Sinusoidal, balanced and single-phase power system operation is easily introduced as a simplification

Define nonsinusoidal line-neutral voltages and line currents:

$$v_a(t) = \sqrt{2} \sum_{h \neq 0}^{\infty} V_{a,h} \sin(h\omega t + \alpha_{a,h})$$

$$v_b(t) = \sqrt{2} \sum_{h \neq 0}^{\infty} V_{b,h} \sin(h\omega t + \alpha_{a,h} - 120^\circ h)$$

$$v_c(t) = \sqrt{2} \sum_{h \neq 0}^{\infty} V_{c,h} \sin(h\omega t + \alpha_{c,h} + 120^\circ h)$$

$$i_a(t) = I_{a0} + \sqrt{2} \sum_{h \neq 0}^{\infty} I_{a,h} \sin(h\omega t + \beta_{a,h})$$

$$i_b(t) = I_{b0} + \sqrt{2} \sum_{h \neq 0}^{\infty} I_{b,h} \sin(h\omega t + \beta_{a,h} - 120^\circ h)$$

$$i_c(t) = I_{c0} + \sqrt{2} \sum_{h \neq 0}^{\infty} I_{c,h} \sin(h\omega t + \beta_{c,h} + 120^\circ h)$$

- DC components in voltage, (V_{a0} , V_{b0} and V_{c0}) **should always be zero**.
- DC values in line currents (I_{a0} , I_{b0} and I_{c0}) **could be nonzero** depending on nature of load.
- The *RMS line-neutral voltage* V_a and RMS line current I_a (similarly for phase b and c) are related to harmonic components by:

$$V_a^2 = V_{a,1}^2 + \sum_{h \neq 2}^{\infty} V_{a,h}^2 = \underbrace{V_{a,1}^2}_{\text{fundamental}} + \underbrace{V_{aH}^2}_{\text{harmonics}} \quad (V_{aH}^2 = \sum_{h \neq 2}^{\infty} V_{a,h}^2)$$

$$I_a^2 = I_{a,1}^2 + \sum_{h \neq 2}^{\infty} I_{a,h}^2 = I_{a,1}^2 + I_{aH}^2 \quad (I_{aH}^2 = \sum_{h \neq 2}^{\infty} I_{a,h}^2)$$

- Fundamental frequency and the nonfundamental frequency (harmonic frequencies grouped)

👉 What is the “effect” of three-phase voltages and three-phase currents in terms of the three-phase power system?

“Effective” voltage and **“effective” current** to represent the state of the three-phase power system:

$$V_e^2 = V_{e1}^2 + V_{eH}^2$$

$$I_e^2 = I_{e1}^2 + I_{eH}^2$$

“Effective” voltage and **“effective” current** in a 3-wire three-phase power system:

$$V_e = \sqrt{\frac{V_a^2 + V_b^2 + V_c^2}{3}}$$

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}}$$

☞ If an artificial neutral point in a **3-wire three-phase system** is not used to find the line-neutral voltage values, the *effective three-phase voltage* can be calculated from the **RMS phase-phase voltage** values as:

$$V_e = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{9}}$$

- The **fundamental frequency components** of the *effective voltage* and *current* in a **3-wire three-phase power system**:

$$V_{e,1} = \sqrt{\frac{V_{ab,1}^2 + V_{bc,1}^2 + V_{ca,1}^2}{9}}$$

$$I_{e,1} = \sqrt{\frac{I_{a,1}^2 + I_{b,1}^2 + I_{c,1}^2}{3}}$$

• **Non-fundamental frequency** components of the *effective voltage* and *current* in a 3-wire three-phase power system:

• **Non-fundamental frequency** components relates to the harmonic components (line-neutral voltages assumed):

$$V_{eH} = \sqrt{\frac{V_{abH}^2 + V_{bcH}^2 + V_{caH}^2}{9}}$$

$$I_{eH} = \sqrt{\frac{I_{aH}^2 + I_{bH}^2 + I_{cH}^2}{3}}$$

$$V_{eH} = \sqrt{\frac{\sum_{h \neq 1}^{\infty} (V_{a,h}^2 + V_{b,h}^2 + V_{c,h}^2)}{3}}$$

$$I_{eH} = \sqrt{\frac{\sum_{h \neq 1}^{\infty} (I_{a,h}^2 + I_{b,h}^2 + I_{c,h}^2)}{3}}$$

- **Unbalanced** condition in a **4-wire three-phase power** system requires reformulation of the *effective voltage* and *current* :

$$V_e = \sqrt{\frac{1}{18} \left[3(V_a^2 + V_b^2 + V_c^2) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2 \right]}$$

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2}{3}}$$

- The **fundamental frequency** components of the *effective voltage* and *current* in a **4-wire three-phase power system**:

$$V_{e,1} = \sqrt{\frac{1}{18} \left[3(V_{a,1}^2 + V_{b,1}^2 + V_{c,1}^2) + V_{ab,1}^2 + V_{bc,1}^2 + V_{ca,1}^2 \right]}$$

$$I_{e,1} = \sqrt{\frac{I_{a,1}^2 + I_{b,1}^2 + I_{c,1}^2 + I_{n,1}^2}{3}}$$

- The **non-fundamental** frequency components of the *effective voltage* and *current* in a **4-wire three-phase power system**:

$$V_{eH} = \sqrt{\frac{1}{18} \left[3(V_{aH}^2 + V_{bH}^2 + V_{cH}^2) + V_{abH}^2 + V_{bcH}^2 + V_{caH}^2 \right]}$$
$$I_{eH} = \sqrt{\frac{I_{aH}^2 + I_{bH}^2 + I_{cH}^2 + I_{nH}^2}{3}}$$



👉 Implementation straightforward with modern digital instrumentation

- **Arithmetic** apparent power S_A :

- The Budeanu power definition for single-phase power systems are applied per phase

$$S_a = \sqrt{P_a^2 + Q_{Ba}^2 + D_{Ba}^2}$$

$$S_b = \sqrt{P_b^2 + Q_{Bb}^2 + D_{Bb}^2}$$

$$S_c = \sqrt{P_c^2 + Q_{Bc}^2 + D_{Bc}^2}$$

$$S_A = S_a + S_b + S_c$$

- **Vector** apparent power S_V :

- Active, Budeanu's reactive and distortion power is summated over all three phases:

$$P_{\Sigma,3\phi} = P_a + P_b + P_c$$

$$Q_B = Q_{Ba} + Q_{Bb} + Q_{Bc}$$

$$D_B = D_{Ba} + D_{Bb} + D_{Bc}$$

$$S_V = \sqrt{P_{\Sigma,3\phi}^2 + Q_B^2 + D_B^2}$$

And the effective apparent power

The *three-phase or system effective apparent power* S_e can be written in terms of the contribution of all harmonic components:

$$S_e^2 = (V_e I_e)^2 = (V_{e,1} I_{e,1})^2 + (V_{e,1} I_{eH})^2 + (V_{eH} I_{e,1})^2 + (V_{eH} I_{eH})^2$$

The components in the *system effective apparent power* S_e can be grouped in the **fundamental and nonfundamental** frequency voltage and current components:

$$S_e^2 = S_{e,1}^2 + S_{eN}^2$$

The *non-fundamental frequency apparent power* S_{eN} consist of 3 “distortion” components:

$$\begin{aligned} S_{eN}^2 &= (V_{e,1} I_{eH})^2 + (V_{eH} I_{e,1})^2 + (V_{eH} I_{eH})^2 \\ &= D_{eI}^2 + D_{eV}^2 + D_{eH}^2 \end{aligned}$$

And the “distortion” powers are.....

$$\begin{aligned} S_{eN}^2 &= (V_{e,1} I_{eH})^2 + (V_{eH} I_{e,1})^2 + (V_{eH} I_{eH})^2 \\ &= D_{eI}^2 + D_{eV}^2 + D_{eH}^2 \end{aligned}$$

$V_{e,1} I_{eH}$: The **current distortion power**, D_{eI}

$V_{eH} I_{e,1}$: The **voltage distortion power**, D_{eV}

$V_{eH} I_{eH}$: The **harmonic distortion power**, D_{eH}

An **effective harmonic apparent power** S_{eH} is defined:

$$S_{eH}^2 = P_{H,3\phi}^2 + D_{eH}^2$$

The Total Three-Phase Active Power

The ***Joint (Total) Harmonic Active Power*** of three-phase power system:

$$\begin{aligned} P_{H,3\phi} &= \operatorname{Re} \sum_{h=1}^N \left(V_{ah} \mathbf{I}_{ah}^* + V_{bh} \mathbf{I}_{bh}^* + V_{ch} \mathbf{I}_{ch}^* \right) \\ &= \sum_{h=1}^N P_{h,3\phi} \end{aligned}$$

The ***Joint Active Power*** of three-phase power system:

$$\begin{aligned} P_{3\phi} &= \left[\mathbf{v}(t)_{3\phi}, \mathbf{i}(t)_{3\phi} \right] \\ &= \operatorname{Re} \sum_{h=1}^N \left(V_{ah} \mathbf{I}_{ah}^* + V_{bh} \mathbf{I}_{bh}^* + V_{ch} \mathbf{I}_{ch}^* \right) \\ &= \sum_{h=1}^N P_{h,3\phi} \end{aligned}$$

Quantification on the level of distortion is done with ***three-phase effective values***:

$$VTHD_e = \frac{V_{eH}}{V_{e,1}}$$

$$ITHD_e = \frac{I_{e,H}}{I_{e,1}}$$

👉 **Voltage total harmonic distortion factor: $VTHD_e$**

👉 **Current total harmonic distortion factor: $ITHD_e$**

👉 The IEEE 1459-2000 write these symbols as THD_{eV} and THD_{eI}

A shortcut to S_{eN} :

$$S_{eN} = S_{e1} \sqrt{VTHD_e + ITHD_e + (VTHD_e ITHD_e)^2}$$

And to D_{eI} ; D_{eV} ; D_{eH} (The “distortion” powers):

$$D_{eI} = S_{e1} ITHD_e$$

$$D_{eV} = S_{e1} VTHD_e$$

$$D_{eH} = S_{e1} ITHD_e VTHD_e$$

To do what with?



Harmonic pollution (S_{eN}/S_{e1}):

$$\left(\frac{S_{eN}}{S_{e1}}\right)^2 = \left(ITHD_e\right)^2 + \left(VTHD_e\right)^2 + \left(ITHD_e VTHD_e\right)^2$$

Unbalance pollution:

👉 The positive sequence voltage (V_1^+) and current (I_1^+) in the three-phase fundamental frequency components have to be found.....

$$S_1^+ = 3V_1^+ I_1^+$$

Then calculate:
$$S_{u1} = \sqrt{S_{e1}^2 - (S_1^+)^2}$$

The “unbalance pollution”:
$$UnbalancePollution(\%) = \frac{S_{u1}}{S_{e1}} * 100$$

👉 “Unbalance pollution” includes both the effect on loading unbalance and voltage asymmetry

☞ The positive sequence components in the three-phase fundamental frequency voltages and currents:

- ✓ The 50 Hz positive sequence active power: $P_1^+ = 3V_1^+ I_1^+ \cos \theta_1^+$
- ✓ The 50 Hz positive sequence reactive power: $Q_1^+ = 3V_1^+ I_1^+ \sin \theta_1^+$
- ✓ The 50 Hz positive sequence apparent power: $(S_1^+)^2 = (P_1^+)^2 + (Q_1^+)^2$

And power factor?

Various options exist....

Different answers to the same question!

Different apparent powers have been formulated:

- Arithmetic apparent power, S_A
- Vector apparent power, S_V
- Three-phase effective apparent power, S_e
- Positive sequence apparent power, S_1^+

Different apparent power factors have been formulated:

- Arithmetic apparent power factor
- Vector apparent power factor
- Three-phase effective apparent power factor
- Positive sequence apparent power factor

$$PF_A = \frac{P}{S_A}$$

$$PF_V = \frac{P}{S_V}$$

$$PF_e = \frac{P_{\Sigma,3\phi}}{S_e} = \frac{P_{1,3\phi} + P_{H,3\phi}}{S_e}$$

$$PF^+ = \frac{P_1^+}{S_1^+}$$

If the waveforms are sinusoidal, the loading is in perfect balance and the supply voltages are in perfect symmetry, then:

$$PF_A = PF_V = PF_e = PF^+$$

In a practical power system with distorted waveforms, unbalanced loading and asymmetrical supply voltages:

$$PF_e < PF_A < PF_V$$

- PF_e reflects the impact of harmonics and asymmetrical waveforms the best.

👉 The smallest numerical value – regulatory application

👉 What power factor formulation does your instrument use?

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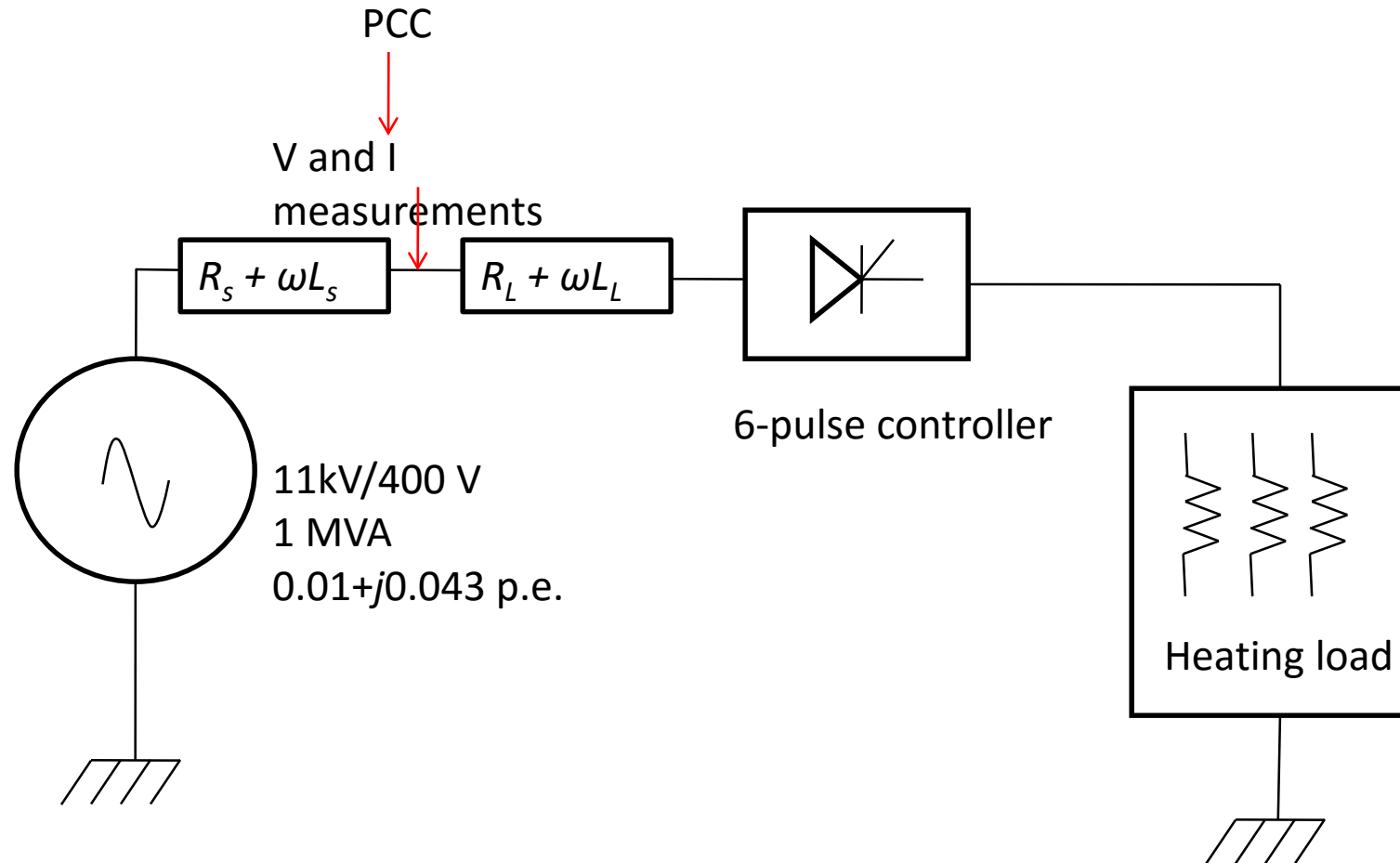


Modern Power Theory

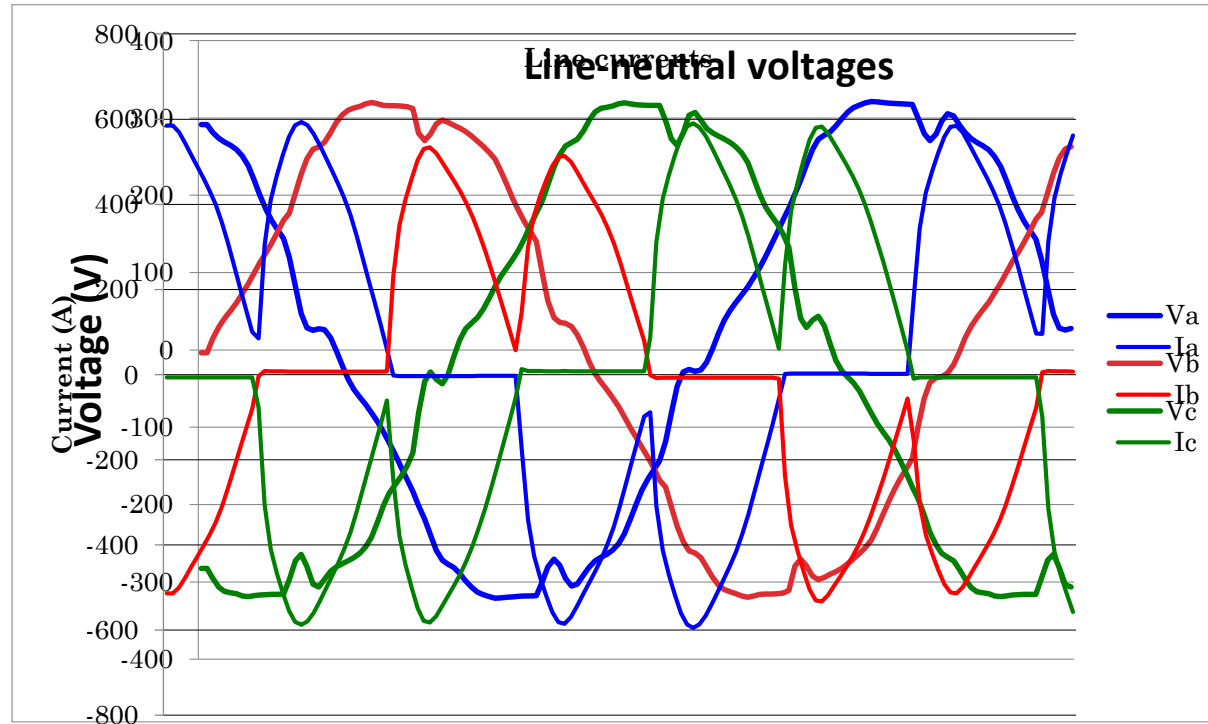
3. Case Study



Application of IEEE 1459-2000 - practical power system



Voltage and Current waveforms



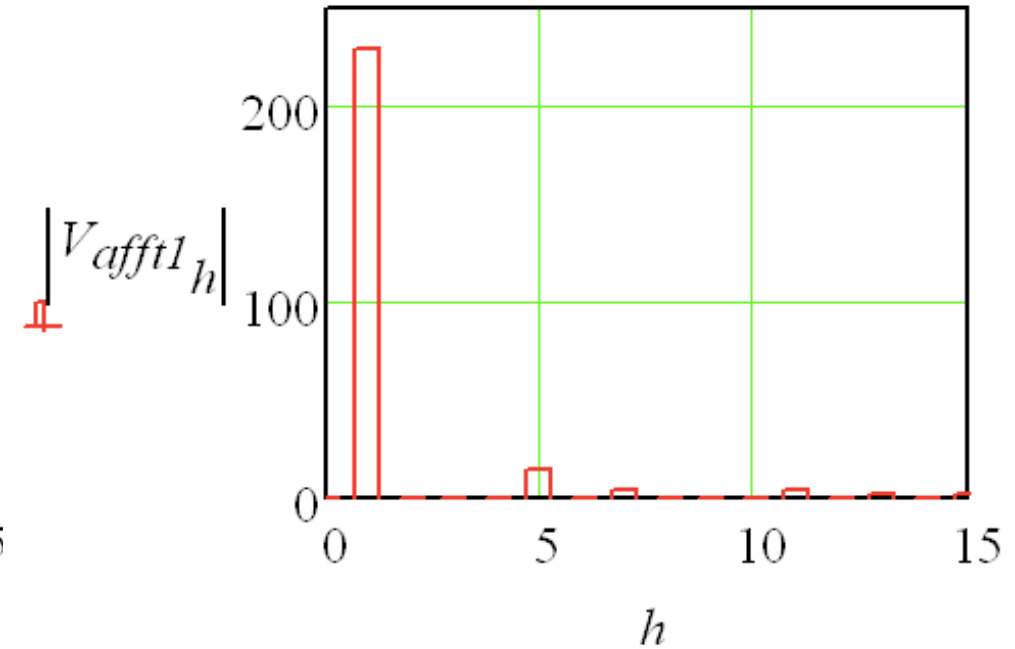
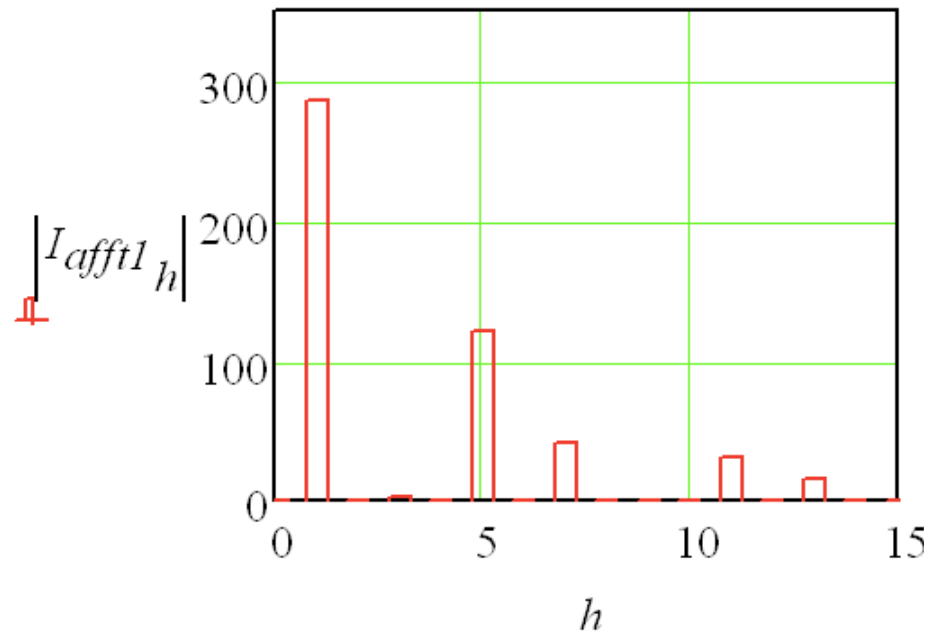
- Line-neutral voltages



- Line Currents



The IEEE 1459 power definitions require translating time-domain waveforms to the frequency domain by means of the Fourier transform:



The level of distortion requires quantification:

- $ITHD_e = 48.3\%$
- $VTHD_e = 7.5\%$

- The **non-fundamental** frequency components of the *effective voltage* and *current*:

$$V_{eH} = \sqrt{\frac{1}{18} \left[3(V_{aH}^2 + V_{bH}^2 + V_{cH}^2) + V_{abH}^2 + V_{bcH}^2 + V_{caH}^2 \right]}$$

$$= 17.2 \text{ V}$$

$$I_{eH} = \sqrt{\frac{I_{aH}^2 + I_{bH}^2 + I_{cH}^2 + I_{nH}^2}{3}}$$

$$= 145.8 \text{ A}$$

- The **fundamental** frequency components of the *effective voltage* and *current*:

$$V_{e,1} = \sqrt{\frac{1}{18} \left[3(V_{a,1}^2 + V_{b,1}^2 + V_{c,1}^2) + V_{ab,1}^2 + V_{bc,1}^2 + V_{ca,1}^2 \right]}$$

$$= 227.6 \text{ V}$$

$$I_{e,1} = \sqrt{\frac{I_{a,1}^2 + I_{b,1}^2 + I_{c,1}^2 + I_{n,1}^2}{3}}$$

$$= 301.5 \text{ A}$$

The *effective voltage and current* :

$$V_e = \sqrt{V_{e1}^2 + V_{eH}^2}$$
$$= 228.3 \text{ V}$$

$$I_e = \sqrt{I_{e1}^2 + I_{eH}^2}$$
$$= 335 \text{ A}$$

The RMS line-neutral voltages per phase:

$$V_a = \sqrt{\sum_{h=1}^{\infty} V_{a,h}^2} = 229.2 \text{ V}$$

$$V_b = 227.7 \text{ V}$$

$$V_c = 227.8 \text{ V}$$

The RMS line currents per phase:

$$I_a = \sqrt{\sum_{h=1}^{\infty} I_{a,h}^2} = 315.4 \text{ A}$$

$$I_b = 342.9 \text{ A}$$

$$I_c = 345.8 \text{ A}$$

The RMS neutral current:

$$I_n = 8.1 \text{ A}$$

The powers

Effective apparent power: $S_{e1} = 229.4 \text{ KVA}$

Arithmetic apparent power: $S_A = 199.7 \text{ KVA}$

Vector apparent power: $S_V = 199.7 \text{ KVA}$

Joint harmonic active power:

$$P_{H,3\phi} = \text{Re} \sum_{h \neq 1}^N \left(V_{ah} I_{ah}^* + V_{bh} I_{bh}^* + V_{ch} I_{ch}^* \right) = \sum_{h \neq 1}^N P_{h,3\phi} = -387 \text{ W}$$

Joint (three-phase) active power:

$$P_{3\phi} = \left[\mathbf{v}(t)_{3\phi}, \mathbf{i}(t)_{3\phi} \right] = \text{Re} \sum_{h=1}^N \left(V_{ah} I_{ah}^* + V_{bh} I_{bh}^* + V_{ch} I_{ch}^* \right) = \sum_{h=1}^N P_{h,3\phi} = 92.9 \text{ kW}$$

Current distortion power: $D_{eI} = S_{e1} ITHD_e = 99.9 \text{ kVar}$

Voltage distortion power: $D_{eV} = S_{e1} VTHD_e = 15.5 \text{ kVar}$

Harmonic distortion power: $D_{eH} = S_{e1} ITHD_e VTHD_e = 7.5 \text{ kVar}$

And the power factors....

The arithmetic power factor: 0.465 p.u.

The vector power factor: 0.465 p.u.

The effective power factor: 0.405 p.u

And the “pollution” factors....

Harmonic pollution:
$$\frac{S_{eN}}{S_{e1}}(\%) = \sqrt{(ITHD_e)^2 + (VTHD_e)^2 + (ITHD_e VTHD_e)^2} * 100$$
$$= 3.6\%$$

Unbalance pollution:
$$S_{u1} = \sqrt{S_{e1}^2 - (S_1^+)^2} = 9.1 \text{ KVA}$$
$$\therefore \text{UnbalancePollution}(\%) = \frac{S_{u1}}{S_{e1}} * 100 = 4.4\%$$

Unbalance factors:
$$V_{UB} = \frac{V_2}{V_1} = 0.4\%$$

$$I_{UB} = \frac{I_2}{I_1} = 3\%$$

☞ The three-phase effective power factor is numerically the smallest if unbalance and/or waveform distortion exist.

☞ It is a helpful reflection on the impact of nonsinusoidal waveforms, unbalanced loading and asymmetrical supply voltages from the point of view from operating such power system.

☞ It is possible to isolate the contribution of harmonics to useless power by means of voltage distortion power (D_V), current distortion power (D_I) and harmonic distortion power (D_H).

☞ The three-phase effective distortion index for voltage ($VTHD_e$) and current ($ITHD_e$) furthers straightforward calculation of distortion powers.

☞ The contribution to apparent power by both voltage asymmetry and unbalance in loading, was shown to be significant even with “low” values in V_{UB} and I_{UB} .

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Modern Power Theory

Conclusion on Power Theory



- The classical power theory explains phenomena in a power system with a sound physical explanation but is inadequate when waveforms are non-sinusoidal, loading is unbalanced and voltage waveforms are asymmetrical.
- Numerous approaches to power theory exist which are formulated in either the time- or frequency domain.
- New contributions are forthcoming as inadequacies are better understood.
- In general, a power theory has to:
 - ✓ Conform to Electrical Network Laws
 - ✓ Explain Physical Phenomena
 - ✓ Be Measurable
 - ✓ Enables Compensation

👉 The IEEE 1459-2000 is a practical approach to the operation of a power system