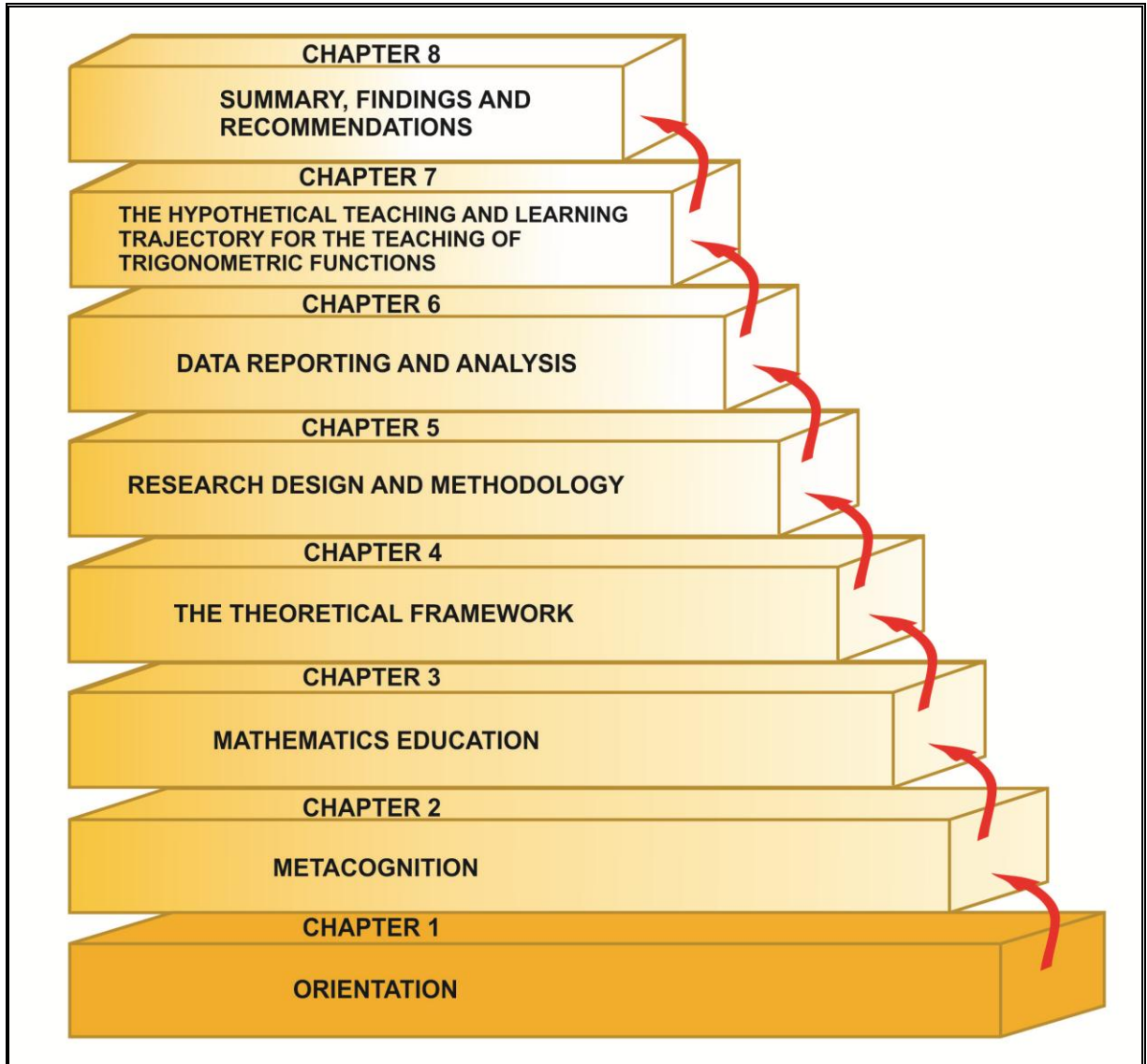
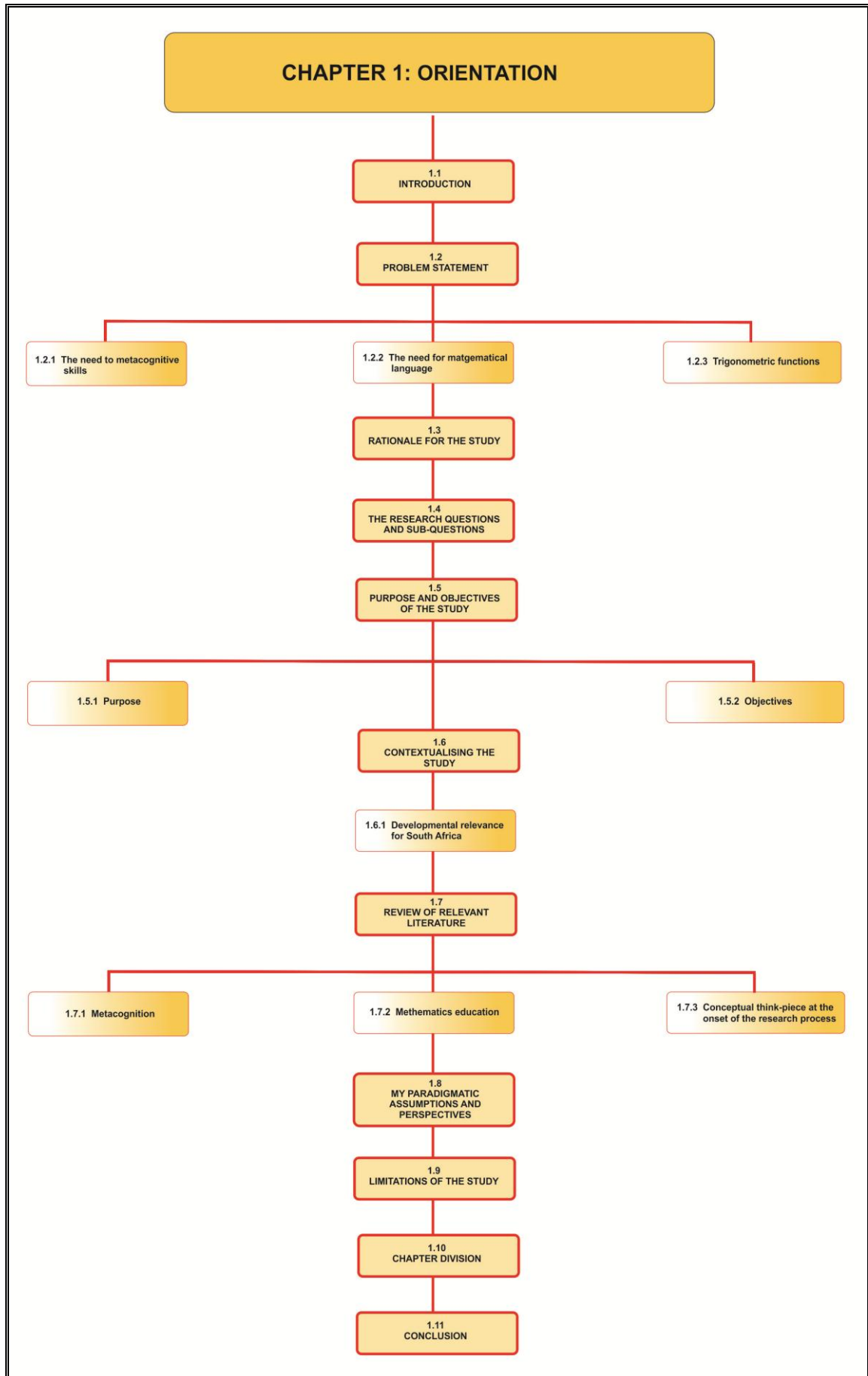


# CHAPTER 1: ORIENTATION

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## 1.1 INTRODUCTION

In this chapter I am introducing my research study – the metacognitive skills and mathematical language used by Mathematics teachers in Trigonometry teaching – in order to create a space and situate it within the broader field of Mathematics education. I am starting this chapter by putting forward the problem statement (§1.2) and reasons why the study is necessary (§1.3), followed by the actual research question (§1.4) and what I wanted to accomplish with this research in the form of the purpose and objectives (§1.5). Next I am contextualizing the study (§1.6) and show the developmental relevance for South Africa. A short review of the relevant literature (§1.7) follows, then I am explaining the conceptual think-piece at the onset of this study. My paradigmatic assumptions are then addressed (§1.8), followed by some limitations of the study (§1.9) before I finally focus on the chapter division (§1.10) within this thesis and the conclusion of this chapter.

## 1.2 PROBLEM STATEMENT

The failure of the education system to achieve dramatic improvement in the quality of learning and teaching in South African schools was noted as one of the key challenges facing Teacher Education and Development (TED). This challenge, among others, was the main reason why a new strengthened and integrated plan, the Integrated Strategic Planning Framework 2011-2025 (ISPF) for Teacher Education and Development had to be developed at the Declaration of the Teacher Development Summit held in 2009 (Department of Basic Education (DBE), 2011b). Teachers play a central role in the education system in any country. South Africa is no exception. In a review of the literature on teacher demand, supply and quality in South Africa by the Centre for Development and Enterprise (CDE), Roger Deacon, an independent education researcher, points out that:

*Research has shown consistently that many South African teachers fall well short of national and international norms and standards in respect of subject knowledge, teaching methods and productivity (Centre for Development and Enterprise, 2010, p. 11).*

Well-known columnist and renowned academic, Jonathan Jansen, agrees with this notion when he unequivocally stated that: “a large percentage of teachers in the final grades of school do not have enough subject matter knowledge or knowledge of teaching to prepare students adequately for their final examinations. There needed be a basic meeting of prerequisites for quality schooling, such as quality textbooks and quality teachers” (Gernetzky, 2012, p. 2). The 2011 Annual National Assessment (ANA) figures indicated that the national average in performance in Mathematics was only 30 per cent while the National

Senior Certificate (NSC) results in Mathematics have declined in recent years (Centre for Development and Enterprise, 2010). Jansen warns further that it is dangerous to just accept these low pass rates (Jansen, 2011). Learners and parents are losing hope and are giving up on our Mathematics teachers (Jansen, 2011). It seems that the quality of the Mathematics teacher is paramount to the enhanced performance of South African Mathematics learners in order to reverse the dismal situation at results level. But, not all teachers are poor teachers. There are good teachers who honour their profession with hard work and persistence. Teachers with these qualities, unfortunately, are the exception and not the rule. Reality is that South Africa need more and better Mathematics teachers if it wants to deliver on the goals set in the ISPF by 2025.

All the above reports and studies on Mathematics education boil down to a need for the connection of thorough Mathematics content knowledge with an in-depth understanding of how specific content is learned and subsequently the ways in which this content needs to be taught to learners at their different levels in the school curriculum for more effective teaching. The thought-provoking debate around what it is that is needed in Mathematics classrooms for effective teaching and learning warrants yet another study. Studies from extant literature (Lai, 2011; Van der Walt, 2006; Van der Walt & Maree, 2007) indicate metacognitive skills as an important role player in effective Mathematics teaching and learning. However, previous studies in this area concentrated only on teacher metacognition in Mathematics teaching without taking into account the role that mathematical language plays in this process. Thinking, after all, requires a language first and foremost. Scholars (Blessman & Myszcza, 2001; Gifford & Gore, 2008; Larson, 2007; Lovelace & Stewart, 2009; Stahl & Fairbanks, 1986; Van der Walt, 2006) all agree that language and concept knowledge are detrimental factors in the understanding of Mathematics.

From my perspective and experiences, and based on the literature, it appears as if the metacognitive skills and the mathematical language used by Mathematics teachers might be fundamental to ensure more effective teaching. This warrants a study in which explicit focus is placed not only on the metacognitive skills of the teacher, but also the mathematical language he/she is using. For this reason this study focused on the teaching of one specific area in Mathematics teaching, viz. trigonometric functions, to focus not only on the specific way Mathematics teachers are using their metacognitive skills, but also how they are communicating their thinking and understanding to the learners.

### 1.2.1 The need for metacognitive skills

Metacognition is a concept that has been widely researched, but has been done so mostly in the context of the learner or the student *per se*. Limited research has been done in the field of teacher metacognition (Ader, 2013; Kozulin, 2005; McElvany, 2009; Wilson & Bai, 2010; Zohar, 1999):

*I have identified a lack of emphasis on the teachers' role and teaching practices within the efforts to incorporate metacognition into Mathematics classrooms (Ader, 2013, p. 7).*

Research in teacher metacognition is imperative as it provides valuable insight into exactly what and how teachers are thinking about their teaching practices. According to Hartman (2002b) metacognition in teaching is about knowing what instructional strategies are in a teachers repertoire, when and why to use them and exactly how to apply them. It is not only needed for the effective planning of lessons, but also for “switching gears” during a lesson (Hartman, 2002b). Results from a study conducted by Winne and Marx (1982) indicated that teachers were relatively unsuccessful in setting objectives and engaging students productively with well-defined tasks. In other countries, like Singapore, the revised curriculum retained metacognition as one of the key components essential for the development of mathematical problem solving (Wong, 2002). Extensive review into the field of metacognition reveals that metacognition helps students with improved learning. It develops higher order thinking and improves the mental approach to problem solving. Kramarski and Mevarech (2003) discovered in a comparative study that learners exposed to metacognitive instruction outperformed learners with respect to a number of reflective tasks in Mathematics. But, metacognition can also help a teacher to consolidate the learning of his/her learners (Atkhar, Zafar, & Afzal, 2010). In fact, contemporary teacher educators argue that effective Mathematics teachers should reflect on what they are doing when they are teaching and what are the most effective ways in which it can be done (Wong, 2002). Ideally teachers need to teach thinking more effectively and systematically and not leave it to chance that learners will, in the process and as part of the lesson, learn about their own thinking (Pierce & Fontaine, 2009). Not only should teachers therefore teach metacognitive skills, but also create opportunities for learners to apply and practise metacognitive skills themselves (Van der Walt, 2006; Van der Walt & Maree, 2007).

Kozulin (2005) explored the cognitive and metacognitive skills of teachers engaged in cognitive program training. Comparing the cognitive and metacognitive skills of teachers with the cognitive skills of students receiving this type of program, Kozulin (2005: 3) maintain that there is not much known about the teacher's metacognitive performance:

*Little is known of teachers' acquisition of IE problem solving skills and even less of their metacognitive performance associated with this acquisition.*

If one considers the teacher as an example for his/her learners in modelling good metacognitive practices, then the teacher's own metacognitive performance within the class can contribute significantly to the learner's ability to think about his/her thinking.

### **1.2.2 The need for mathematical language**

Mathematical language, on the other hand, is becoming increasingly important in Mathematics instruction and learning (Beck & McKeown, 2007; Pierce & Fontaine, 2009). Not only is mathematical language imperative in providing feedback to learners regarding their success in attaining the goals they want to accomplish, but mathematical language is also important for communication and thinking in the Mathematics classroom (Thompson & Chappell, 2007). Thompson (2010) suggests that Mathematics teachers should attend a Mathematics-specific content reading course that could help them determine how literacy strategies integrate with Mathematics content. In culturally and linguistically diverse South Africa, the problem of mathematical language becomes bigger (Setati, 2005, 2011). This persistent and recurrent problem is exacerbated in rural township schools where the mathematical language is even more restricted due to poverty and non-exposure to the language outside of the school environment (Adler, 2001; Adler, Setati, Reed, & Bapoo, 2002; Kassiem, 2004; Laridon, Mosimege, & Mogari, 2005; Ministry of Education, 2005; Muller, 2013; Setati, Molefe, & Langa, 2008; Van der Walt, Maree, & Ellis, 2008). Studies in this regard for example reveal that there exist a close correlation between poor Mathematics attainment and language skills in South Africa (Simkins & Paterson, 2005) and that language-related factors in particular should be taken into account for effective Mathematics teaching and learning (Kassiem, 2004; Van der Walt et al., 2008). Metacognitive skills and mathematical language might therefore be the perfect match for enhancing effective teaching and learning in Mathematics classrooms as Thurston (1994, p. 168) advocates:

*We mathematicians need to put far greater effort into communicating mathematical ideas. To accomplish this, we need to pay much more attention to communicating not just our definitions, theorems, and proofs, but also our ways of thinking. We need to appreciate the value of different ways of thinking about the same mathematical structure. We need to focus far more energy on understanding and explaining the basic mental infrastructure of mathematics—with consequently less energy on the most recent results. This entails developing mathematical language that is effective for the radical purpose of conveying ideas to people who don't already know them.*

Teachers need to convey their teaching and instruction either verbally or written in clear and understandable mathematical language instruction. If they are unable or incapable to convey mathematical curriculum content, two-directional communication and thinking will be hampered, and effective teaching and learning in the Mathematics classroom will be hindered (Hartman, 2002b).

### **1.2.3 Trigonometric functions**

Trigonometric functions were chosen in this study as an appropriate topic to focus on the metacognitive skills and mathematical language that teachers are using. This study inquired into the use of trigonometric functions to enhance the more effective teaching of trigonometric functions. The reasons were threefold:

1. Firstly the literature on Trigonometry is not only limited, but studies reveal that learners' understanding of Trigonometry is most of the times fragmented (Demir, 2012).
2. Secondly, according to (Markel, 2010), even good Mathematics learners with excellent Mathematics backgrounds and aptitudes draw blank moments on the most fundamental trigonometric concepts.
3. More importantly, trigonometric functions in the newly implemented Curriculum and Assessment Policy Statements (CAPS) document (Department of Basic Education (DBE), 2011a) is swamped with particular mathematical concepts which is only introduced for the first time in grade 10 (Department of Basic Education (DBE), 2011a).

## **1.3 RATIONALE FOR THE STUDY**

The Mathematics education research arena is proliferated with literature on mathematical language, thinking theories and models (Bruning, Schraw, & Ronning, 2004; Cross & Paris, 1988; Flavell, 1976; Fox & Riconscente, 2008; Gernetsky, 2011). Although numerous studies investigated metacognition in learners, only few of the studies focused on the metacognition of the teacher. McElvany (2009) referred to enquiries into teacher metacognition as a new area of metacognition research. It appears as if little is known about the teacher's metacognitive skills. To add onto this, not much scholarly work of an interventionist nature is available on mathematical language and metacognitive skills for more effective teaching and learning in the Mathematics classroom. Ader (2013) identified a lack of emphasis on the role of the teacher in metacognition in Mathematics classrooms. In order to address this paucity in literature, I have embarked on a design-based research study in which six Mathematics

teachers teaching in the Further Education and Training (FET) band in two previously disadvantaged township schools, together with their grade 10 learners and five colleagues from the Mathematics subject group of the university where I am employed, were invited to participate in my study. Together we attempted to determine how teachers are using their metacognitive skills and mathematical language in Mathematics teaching and in particular when teachers are teaching trigonometric functions. It is envisaged that this study contributed to a mind shift of those Mathematics teachers involved in the study. This shift entailed a rethinking of the Mathematics teaching and contributed to the teachers' professional development. It is lastly hoped that through this practice of rethinking a culture of reflection in Mathematics teaching and learning will materialize.

## **1.4 THE RESEARCH QUESTION AND SUB-QUESTIONS**

The central research question this study seeks to address is: What are the characteristics of an in-service arrangement that facilitates the implementation of lesson activities focussing on the metacognitive skills and mathematical language of Mathematics teachers for the teaching of trigonometric functions in township schools in the Dr Kenneth Kaunda District? The focus is therefore on *how* mathematical language and metacognitive skills can be used to enhance the teaching of trigonometric functions and its application in solving problems.

The following sub-questions were developed to refine the main question and are applicable to Mathematics teachers and their learners in the FET band of previously disadvantaged township secondary schools in the Dr Kenneth Kaunda District:

1. How do Mathematics teachers apply metacognitive skills and mathematical language in the teaching of trigonometric functions?
2. Which challenges do Mathematics teachers face in the teaching of trigonometric functions?
3. How can the teaching of trigonometric functions be improved when focussing on the metacognitive skills and mathematical language used by Mathematics teachers?

## **1.5 PURPOSE AND OBJECTIVES OF THE STUDY**

### **1.5.1 Purpose**

The main purpose of this study was to inquire into the methods how mathematical language and metacognitive skills can be applied to facilitate effective teaching and learning of

trigonometric functions in the FET phase of township schools in the Kenneth Kaunda District in the North West Province. This study thus elucidated methods on how the metacognitive skills of teachers can be developed or enhanced by focusing specifically on mathematical language through lesson study.

### **1.5.2 Objectives**

The specific objectives of the study are:

- To understand how Mathematics teachers apply metacognitive skills and mathematical language in the teaching and learning of trigonometric functions in order to develop a metacognitive performance profile for each grade 10 teacher;
- To identify what challenges Mathematics teachers face in the teaching and learning of trigonometric functions;
- To develop a hypothetical teaching and learning trajectory for trigonometric functions in Grade 10.

## **1.6 CONTEXTUALISING THE STUDY**

This study was embedded in a bigger project between South Africa and the Netherlands which aimed to interrogate metacognition and lesson study in urban and township schools in the North-West and Eastern Cape provinces. Three universities were involved; two universities in South Africa and a Netherland university .The project started in January 2011 and ended in April 2013. Table 1.1 gives an indication of how the study progressed within the project.

**Table 1.1: Schedule of collaborating activities within the study**

| Activity                                                                     | Date                 | Attended by...                                                                                                                                                                                  | Aim                                                                                                                                                                                              |
|------------------------------------------------------------------------------|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Start-up workshop of project                                                 | March 2011           | Project leader (study leader)<br>Members within the project from North-West University<br>Other 3 members within the project from Walter Sisulu University<br>Researcher                        | Introduction to project<br>Shared understanding of: <ul style="list-style-type: none"> <li>• Metacognition</li> <li>• Lesson study</li> <li>• Design-based research</li> </ul>                   |
| Invitation meeting to participate in the study                               | November 2011        | Researcher and 6 teachers from two township schools                                                                                                                                             | Informing possible participant teachers of the proposed study and invitation to take part in the study.                                                                                          |
| First project meeting                                                        | October 2011         | Project leader (study leader)<br>Members within the project from North-West University<br>Other 3 members within the project from Walter Sisulu University<br>Netherlands partner<br>Researcher | Common understanding of project and different studies within project.<br>Designing of questionnaire                                                                                              |
| Individual interviews                                                        | February- April 2012 | Study leader and participating teachers                                                                                                                                                         | Deeper understanding of : <ul style="list-style-type: none"> <li>• teacher as person and attitudes towards Mathematics</li> </ul>                                                                |
| Project meeting                                                              | May 2012             | Project leader (study leader)<br>Members within the project from North-West University<br>Researcher                                                                                            | Discussion of design based research based on report back from study leader who attended conference on design-based research in the Netherlands                                                   |
| 2 Lesson observations                                                        | June 2012            | Researcher<br>Colleague within subject group<br>Two of the participating teachers<br>Learners                                                                                                   | Observing and video tapping the presented lesson<br>Establishing metacognitive performance profile of teachers                                                                                   |
| Project meeting                                                              | June 2012            | Project leader (study leader)<br>Members within the project from North-West University<br>Other 3 members within the project from Walter Sisulu University<br>Netherlands partner<br>Researcher | Data-analysis: Discussion of data from questionnaire and lesson observation<br>Dissemination: Planning presentation of studies within project at conference (AMESA 2012)                         |
| First workshop within the study (Trig assessment task completed by teachers) | August 2012          | Researcher<br>Colleague from Mathematics subject group<br>Participating teachers                                                                                                                | Metacognition<br>Mathematical language<br>Discussion of video-taped lesson as first step towards lesson study<br>Completion of trigonometric task to establish metacognitive performance profile |
| Project meeting                                                              | August 2012          | Researcher<br>Colleagues from subject group within Education Sciences faculty and from Natural Sciences faculty.                                                                                | Data analysis: videotaped lesson<br>Discussion of problem areas in Trigonometry                                                                                                                  |
| Assessment of learners (Trigonometric task)                                  | August 2012          | Researcher<br>Participating teachers                                                                                                                                                            | To establish the extent to which the learners of participating teachers are using metacognitive skills and mathematical language                                                                 |
| Workshop                                                                     | September 2012       | Researcher<br>Participating teachers<br>Colleague from Mathematics subject group                                                                                                                | Planning the study lesson                                                                                                                                                                        |

| Activity                                | Date          | Attended by...                                                                                                                                                                                                                                                     | Aim                                                                                                 |
|-----------------------------------------|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|
| Project meeting                         | October 2012  | Project leader (study leader)<br>Members within the project from North-West University<br>Other 3 members within the project from Walter Sisulu University<br>Netherlands partner<br>Researcher                                                                    | Data-analysis:                                                                                      |
| Focus group discussion                  | February 2013 | Project leader (study leader)<br>Colleagues from Mathematics subject group from the North-West University<br>Participant teachers<br>Researcher                                                                                                                    | Adapted lesson study                                                                                |
| Lesson observation                      | March 2013    | Participant teachers<br>Researcher<br>Learners                                                                                                                                                                                                                     | Adapted lesson study                                                                                |
| Focus group discussion                  | March 2013    | Colleagues from Mathematics subject group from the North-West University<br>Participant teachers<br>Researcher                                                                                                                                                     | Adapted lesson study                                                                                |
| Closing project workshop and conference | April 2013    | Project leader (study leader)<br>Members within the project from North-West University<br>Other 3 members within the project from Walter Sisulu University<br>Netherlands partner<br>Researcher (not attending because of RCI workshop happening at the same time) | Dissemination of results within the study<br>Presentation of paper from study<br>Closing of project |
| Lesson planning                         | May 2013      | Participant teachers<br>Researcher                                                                                                                                                                                                                                 | Adapted lesson study                                                                                |
| Lesson observation                      | May 2013      | Participant teacher<br>Researcher<br>Learners                                                                                                                                                                                                                      | Adapted lesson study                                                                                |
| Focus group discussion                  | May 2013      | Project leader (study leader)<br>Colleagues from Mathematics subject group from the North-West University<br>Participant teachers<br>Researcher                                                                                                                    | Adapted lesson study                                                                                |

### **1.6.1 Developmental relevance for South Africa**

#### **(i) Contribution to Epistemological knowledge**

With the study it is envisaged that South African teachers will gain more insight on the importance of metacognitive skills and effective mathematical language usage for the enhancement of conceptualization in Mathematics teaching and learning in South African schools.

#### **(ii) Contribution to Methodological knowledge**

The use of design-based research as emerging research methodology for reflective practice of South African Mathematics teachers broadened both the mathematical content knowledge and pedagogical knowledge of the teachers while addressing the need to introduce South African teachers to and engage them in adapted lesson study. The merging of the design-based research approach with adapted lesson study brought about teacher-lecturer collaboration and in this way brought together theory and practice. This study also illustrated the affordances of complexity theory in design-based research studies.

#### **(iii) Contribution to Theoretical knowledge**

Except for the development of a hypothetical teaching and learning trajectory (HTLT) for the teaching of trigonometric functions, this study also contributes to the growing body of knowledge in complexity theory.

## **1.7 REVIEW OF RELEVANT LITERATURE**

In order to link previous research and theory in the area of metacognition and mathematical language in Mathematics teaching with each other, a computer literature search of PsycINFO and ERIC databases for the timeframe mainly from 2003 to 2013 was conducted. Terms and phrases like teacher metacognition, Mathematics teaching, cognition, mathematical language, teacher change, professional development, metacognitive skills, lesson study and Trigonometry teaching was used. However, the search also included seminal works published prior to 2003. The focal topics of the literature review centered around two broad issues: Metacognition and Mathematics Education.

### 1.7.1 Metacognition

Metacognition is defined most simply as “thinking about thinking” (Lai, 2011). The very first studies on metacognition were initiated in the early 1970s by Ann Brown and John Flavell. It was then mostly developmental in nature:

*Metacognition refers to one’s knowledge concerning one’s own cognitive processes and products or anything related to them, e.g. the learning-relevant properties of information or data (Flavell, 1976, p. 232).*

According to Lai (2011), cognition (knowledge about it and how to monitor it) is a paramount part of metacognition (Flavell, 1979; Paris & Winograd, 1990; Schraw, Crippen, & Hartley, 2006; Schraw & Moshman, 1995; Whitebread et al., 2009). Various models to explain the metacognitive process exist in the literature. The work of Piaget (Piaget, 1957; Vygotsky, 1978) and his concept of “concrete operational thought” and the work of Vygotsky, (1978) and his concept of the “zone of proximal development” and “language of thought” are only two examples of forerunners to the phenomenon metacognition that were discussed in the study. As mentioned earlier (§1.2.1) studies in metacognition concentrated mainly on the metacognition of learners, but because the main purpose of this study was to understand and find methods in which the metacognitive skills of Mathematics teachers can be utilized in Mathematics teaching, teaching *with* metacognition and teaching *for* metacognition were distinguished in the literature review and particularly the development of metacognitive skills were addressed.

Metacognition brings to the learning process an awareness of the process of learning, the structures involved in that process, the use of skills and the monitoring of those activities as a way of improving the educational process. But it is not that simple: Developing the skills for applying metacognition is not clear-cut. Lin, Schwartz, and Hatano (2005) argue that conventional metacognitive instruction falls short when it comes to the challenges teachers often face, and they agree with Bransford, Brown, and Cocking (2000); Darling-Hammond and Bransford (2005) that each teacher’s situation is different and is influenced by his/her environment. (Lin et al., 2005). Therefore they are of the opinion that adaptive metacognition, rather than conventional metacognition will be of more value in effective teaching ((Lin et al., 2005; Swartz & Parks, 1994). The development of metacognitive skills will thus to a large extent depend on the unique situation of the teacher. A review of the literature regarding metacognitive skills raised the following critical questions:

- (i) **What are metacognitive skills?** Swartz and Perkins (1990, p. 34) refer to metacognitive skills as “a crosscutting super ordinate kind of thinking which indicates “one’s

knowledge about, awareness of, and control over one's own mind and thinking". On the other hand, Lee, Chang, and Lee (2001, p. 22) see metacognitive skills as

*the individual's declarative knowledge and procedural knowledge about his or her cognitive processes as well as self-regulatory procedures, including monitoring and "on-line" decision making.*

In order to understand this definition one should distinguish between declarative knowledge and procedural knowledge: Declarative knowledge is factual knowledge and information that a person knows, while procedural knowledge, on the other hand, is knowing how to perform certain activities (Bruning et al., 2004). Schraw et al. (2006) also distinguish conditional cognitive knowledge, which is knowledge of why and when to use a given strategy.

(ii) **Are metacognitive skills teachable?** Data from a longitudinal study conducted by Desoete (2007) seem to suggest that metacognitive skills cannot be assumed to develop from experiences in the Mathematics classroom. However, Atkhar et al. (2010) are of the opinion that educators need psychological knowledge of metacognitive processes, how they develop and how metacognitive skills can be fostered if educators want to teach metacognitive skills to their learners. In the study conducted by McElvany (2009), the teachers' ratings of the relevance for teaching of knowledge about metacognition reveal that teachers do not seem to attribute as much relevance to knowledge about metacognition as to other areas of knowledge. The teacher should model using good metacognitive skills in order for learners to follow in developing metacognitive skills to achieve effective Mathematics teaching and learning because learners acquire metacognition through the implicit socialization with experts such as parents and teachers (Veenman, Van Hout-Wolters, & Afflerbach, 2006).

### **1.7.2 Mathematics education**

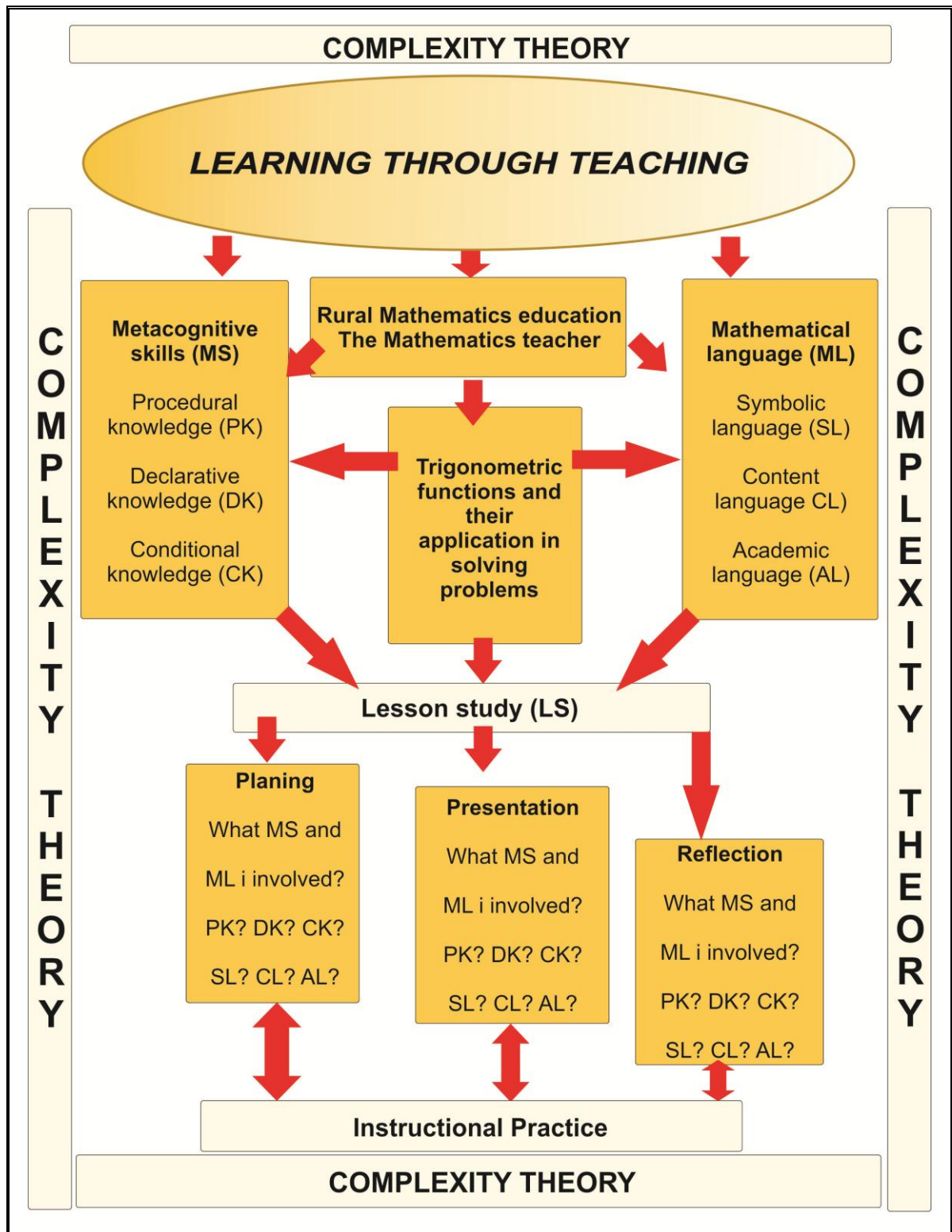
**Most South African Mathematics teachers do not teach Mathematics effectively.** This is one of the conclusions any person will come to when reading the latest report (Centre for Development and Enterprise, 2010, p. 5) on the quality and quantity of South Africa's teachers. This report asserts that:

*While their formal qualifications have improved, existing teachers spend too little time in the classroom, and many teach badly when they do.*

This statement concurs with observations made in another study involving grade six Mathematics learners in North-West schools by Professor Carnoy from the Stanford University and his team. In a report by the Human Sciences Research Centre (Human Sciences Research Council (HSRC) & Education Policy Consortium (EPC), 2005), they found that teachers did not even teach 60 per cent of the lessons they were scheduled to teach. There seems to be a lack of “a culture of learning” evident in most South African schools which is reported by Howie, van Staden, Draper, and Zimmerman (2010). Understanding the structure of teachers’ knowledge and how it is developed is described by many researchers as a complex endeavour (Artzt & Armour-Thomas, 2002; Leiken, 2005; Leikin & Rota, 2006; Leikin & Zazkis, 2010). Several Mathematics cycle teaching models were discussed in finer detail in the study. More specifically, the teaching of Trigonometry is instrumental in this study and is discussed by synthesising existing studies relating to Trigonometry teaching and learning that have already been done (§3.9). Many teachers and providers of professional development globally have become interested in lesson study, a form of professional development typically used in Japanese elementary schools (Stigler & Hiebert, 1998).

### **1.7.3 Conceptual think-piece at the onset of the research process**

This section addresses the main concepts and the theoretical underpinnings and how they seemed to relate to each other at the initial stages of the study.



**Figure 1.1: Conceptual think-piece at the onset of the research process**

As can be seen in Figure 1.1 the mathematical language and metacognitive skills are the two main concepts which were interrogated through the lens of complexity theory.

## 1.8 MY PARADIGMATIC ASSUMPTIONS AND PERSPECTIVES

According to Campbell (2001), orientations in the philosophy of science refer to worldviews and ways of thinking related to our understanding of the nature of knowledge (epistemology) and reality (ontology) and usually form the basis of the methodological choices we make in research. It is imperative for us to establish how do we know and how do we think. (Creswell, 2009) uses the term “worldview” as a basic set of rules that guide the researcher’s actions in a study. Burrell and Morgan (1994) assert that it is important for a theorist to understand the assumptions on which his/her own perspective is based in order to understand alternative points of view.

Having been involved in Mathematics nearly all my life, first as learner taking mathematics (and loving it), later specializing in mathematics, then as mathematics teacher and now as lecturer in mathematics for education, I have always been aware of the constant unwavering problem in Mathematics education: that of the poor mathematics results. Creswell (2009) is of the opinion that a researcher’s worldview is shaped by her/his discipline area, the beliefs of advisors and past research experiences. I guess my involvement with mathematics education, and my wanting to make a difference, even if it can help only one mathematics learner to understand even a small part of mathematics better, to a large extent influenced my pragmatic mindset. What works? How does it work? When does it work? Why does it work? These are all questions that occupy my mind on a daily basis. However, Mathematics teaching is complex, and thus understanding the particular relationship between the logical structure of mathematics subject matter and the psychological structure of mathematical thinking has evaded educational theorists for ages (Campbell, 2001). I support Campbell (2001) in asking the following question: Should the teaching of mathematics conform to the logic of the subject matter, or should the subject matter conform to the psychology of the learner (or the teacher)? Both Piaget (1957) and Dewey (1990) mention the interplay between the logical and psychological aspects when mathematics is taught to learners. This interplay informed my assumptions and perspectives and formed the basis of the methodological choices which I have made in this study.

Having said that, Sloane (2006, p. 29) argues that “Design is based on pragmatism as the underlying epistemological notion”. Pragmatism has its origin in the late nineteenth century with Charles Sanders Pierce and refers to those definitions, accounts and theories embedded in truth with the emphasis on inquiry as a process resulting in a product of truth, which can take the form of a belief, certainty or knowledge (Peirce, 1878). This idea was later echoed by James (2000, p. 197) who believed that pragmatism is all about “that the truth of an idea needs to be tested to prove its validity”. It was Powell (2001) who argued for a

somewhat different focus than *truth*, that of solving a problem. Powell (2001, p. 884) claims that:

*To a pragmatist, the mandate of science is not to find truth or reality, the existence of which are perpetually in dispute, but to facilitate human problem-solving.*

It is this same human problem-solving that I am embracing in my inquiry as truth can never be seen as absolute but one can always try to solve a problem, to improve a situation. Therefore, to attempt to solve the problem of how mathematics teachers might teach trigonometric functions using their metacognitive skills and mathematical language more effectively, I used a design-based approach in this study.

## **1.9 LIMITATIONS OF THE STUDY**

The following factors had a limiting effect on the study:

- Generalisability is limited to only participant township schools.
- Lack of the establishment of teacher proficiency.
- Teachers' limited ability of critical thinking and reflection of own work was challenging.
- Language was a barrier. The language of learning and teaching (LoLT) is English and all of the teacher participants and their learners were Setswana mother tongue speakers.
- Researcher was to a large extent dependent on participant teachers as well as the participating lecturers for data. Although the teachers and lecturers did their best to attend to the activities of lesson observation and focus group discussions within the study, they were not always available due to commitments at their workplace.

## **1.10 CHAPTER DIVISION**

A synopsis of how the chapters of the thesis are organised is given in this section:

### **CHAPTER 1: ORIENTATION**

In this first chapter I attempted to create a research space and situated the study within the inquiry fields of Mathematics Education and Metacognition.

## **CHAPTER 2: METACOGNITION**

Chapter two forms the first part of the literature review and focuses mainly on metacognition and definitions, theories and models underpinning metacognition with a particular focus on the metacognitive skills of the teacher and metacognitive instruction.

## **CHAPTER 3: MATHEMATICS EDUCATION**

This second part of the literature review offers a report on literature pertaining to the past and current state of Mathematics Education in South Africa as well as the challenges surrounding the teaching and learning of Mathematics teaching and learning in schools, with special reference to the teaching and learning of Trigonometry and the role of mathematical language in Mathematics education. Literature about lesson study and possible adaptations thereof is also included in this part. Literature on Mathematics education and in particular Trigonometry teaching led to the identification of guiding principles for the introductory teaching of trigonometric functions which were used as prototype to develop the hypothetical teaching and learning trajectory in this design-based research study.

## **CHAPTER 4: THE THEORETICAL FRAMEWORK**

This chapter is addressing the theoretical underpinnings of the main concepts in the study and the framework that were used to analyze the data that were collected.

## **CHAPTER 5: RESEARCH DESIGN AND METHODOLOGY**

This chapter outlines the research approach and in particular the way research has been applied in this study in an attempt to address the research questions.

## **CHAPTER 6: DATA REPORTING AND ANALYSIS**

This chapter gives an analytical account of the data that were collected.

## **CHAPTER 7: THE HYPOTHETICAL TEACHING AND LEARNING TRAJECTORY FOR TRIGONOMETRIC FUNCTIONS**

This chapter reports on and analyses the implementation of the hypothetical teaching and learning trajectory for the teaching of trigonometric functions.

## **CHAPTER 8: SUMMARY, FINDINGS AND RECOMMENDATIONS**

This last chapter sums up and brings together the literature review and the empirical data within the entire study and suggests possible further research endeavors.

## **1.11 CONCLUSION**

This chapter provided an orientation to the thesis. The chapter also offered a rationale as to why it was necessary to focus on the metacognitive skills and mathematical language usage by Mathematics teachers. The main concepts and theories were explained and diagrammatically presented. Some ethical issues and limitations of the study were also mentioned. Next the main concepts and theories in Metacognition underpinning the problem will be addressed.