An Application of Real Options in Valuation under Uncertainty

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Dedicated to my TWO BOYS,

Kelvin and Emile

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Abstract

This thesis aims to illustrate how real options embedded in business concerns may be identified and quantified for valuation purposes. Traditionally, Discounted Cash Flow (DCF) and Net Present Value (NPV) techniques are central to valuation under uncertainty. However, option pricing-theory, applied to *real* or *non-financial* assets, is introduced as a means of bridging the gap between real world valuation practicalities and standard theory. This central theme is complemented by the valuation of the process patent and plant breeder rights held by Peppadew International (Pty) Ltd. The real option valuation is conducted in conjunction with an independent valuation of Peppadew by the accounting firm KPMG. Keeping the options analysis in line with the generally accepted and well-understood NPV methodology, renders it intuitively understandable and acceptable to managers and investors alike.

The first part of the study illustrates that standard NPV analysis alone is an inadequate valuation tool in the presence of real assets. Valuing the process patent and plant breeder rights of Peppadew International by means of a real option analysis highlights the fact that some inherent value remains unaccounted for by traditional methods. The company was undervalued when applying the Discounted Cash Flow model only - not because the expected cash flows were too low, but simply because the model ignores the options that the company has, via its patents and breeder rights, to increase future investment and take advantage of business success. Specifically, the real option analysis of Peppadew demonstrates that uncertainty can create value.

The second part of this study illustrates how the results from the real options analysis of Peppadew International may be applied to engineer an enhanced funding strategy for the company. Specific requirements are set forth by a large governmental lender and these requirements are met through a uniquely structured putable bond for Peppadew.

The findings of this study emphasise that strategists, analysts and valuation experts can no longer overlook real options as an analysis tool. Identifying real options not only adds substantial economic value but also introduces a paradigm shift in understanding flexibility, i.e. the ability to successfully adapt to unforeseen changes as uncertainty unfolds. The overall result of this work is a clear and logic demonstration of how real option analysis is an extension (to non-financial assets) of the ways in which financial markets value options on stocks or shares.

Uittreksel

Hierdie tesis illustreer hoe reële opsies onderliggend aan besigheidsbesluite geïdentifiseer en gekwantifiseer kan word. Evaluering van besighiedsbesluite wanneer daar 'n groot mate van onsekerheid random kontantvloeie bestaan, word tradisioneel behartig met standaard verdiskonteerings tegnieke. Opsie teorie word voorgestel as 'n tegniek om praktiese realiteite met standard tegnieke te versoen. Die valuasie van patente- en verbouingsregte wat die maatskappy Peppadew Internasionaal Beperk besit, bring hierdie sentrale tema na vore. Die reële opsie valuasie geskied hand aan hand met 'n onafhanklike valuasie van die maatskappy saamgestel deur die rekenmeestersfirma, KPMG. Die opsie analise tegniek kan intuïtief verstaanbaar gemaak word, vir beide bestuurders en beleggers, deur dit te koppel aan bekende verdïskonterings tegnieke.

Die eerste gedeelte van die studie illustreer dat standard verdiskonterings tegnieke nie omvattend genoeg is wanneer reële bates geprys word nie. Die evaluering van Peppadew Internasionaal se patent- en verbouingsregte aan die hand van 'n unieke reële opsie model, bring die feit na vore dat verdiskonteringstegnieke inherente waarde nie in berekening bring nie. Tradisionele modelle kan nie die opsionaliteit, wat onlosmaaklik verbind is aan die maatskappy se patente- en verbouingsregte, kwantitatief in ag neem nie. Gelvolglik word die maatskappy se waarde te laag beraam. Reële opsie analise bring ook die feit dat onsekerheid waarde kan toevoeg, na vore.

Die tweede gedeelte van die studie behandel die toepassing van die reële opsie benadering op 'n gestruktureerde funderingsoefening vir Peppadew Internasionaal. Die resultate benadruk die feit dat stratege, analiste en waardasie deskundiges nie kan bekostig om hierdie nuwe tegniek te ignoreer nie. Indentifisering van reële opsies wys nie net aansienlike ekonomiese waarde uit nie, maar dit lei ook tot 'n paradigmaskuif in die verstaan van hoe inherente buigsaamheid en aanpasbaarheid in berekening gebring kan word.

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CHAPTER 1
INTRODUCTION AND OVERVIEW

> "The more options you have to evaluate, the more data you have to consider and the more unprecedented the challenges you face, the less you should rely on instinct and the more on reason and analysis".
>
> Eric Bonabeau – Chief Scientist, Icosystem.
> Cambridge, Massachusetts.

1.1 Introduction

Both corporate practitioners and academics have realized that standard discounted cash flow (DCF) techniques alone often undervalue investment opportunities. DCF forecasts the future and then uses a risk-adjusted discount rate to account for the error in estimation. This simplistic model fails to address two critical aspects in business, namely managerial flexibility and strategic interactions. The result is a discrepancy between traditional finance theory and corporate reality. Experienced managers often cushion investment criteria to accommodate operating flexibility and strategic considerations they believe to be as important as direct cash flows. A survey conducted in May 2002 by the US executive search firm Christian and Timbers, revealed that 45% of corporate executives in the USA rely more on instinct or "gut feel" than on facts and figures when running their businesses. (Harvard Business Review, 2003). This is mainly due to the inability of traditional finance tools to incorporate proactive flexibility when valuing a project or business.

Dixit and Pindyck (Dixit and Pindyck, 1994) come to the conclusion that the orthodox theory of investments has not recognized the important qualitative and quantitative implications of the interaction between *irreversibility, uncertainty* and *investment timing.* They argue that most capital investments are irreversible to some extent. The initial cost of the investment is sunk and cannot be fully recovered if things turn out worse than expected. This is especially so when expenditures are firm or industry specific. An example often cited is the cost of advertising. Since advertising usually targets a specific market with a specific product, advertising costs are deemed irreversible. The company placing the advertisement hopes to recover advertising costs through sales of its product *in the future.* Consider also the purchase price of a brand new vehicle. A portion of the total cost is the premium the buyer pays for being the first owner of the vehicle. That cost is fully sunk since the next owner cannot be the first owner and will consequently not pay the premium no matter how good the condition of the vehicle is. Vehicles are considered to be depreciating assets and consequently only a portion of the purchase price can be recovered on the secondhand market. The exact amount is uncertain. This uncertainty is a function of economic factors affecting supply and demand. Thus, the *timing* of a vehicle purchase and/or sale is closely linked to the economic *uncertainty* and cost *irreversibility* implicit in the "investment". Investment in capital budgeting projects, infrastructure developments, natural resources, information and bio-technology, Research and Development, brands, licenses and guarantees etc. is not dissimilar to these two examples.

Most investors recognise the value of investing in stages rather than all at once. The ability to stage investment payments profoundly affects an investor's risk profile. He/she can now wait for the arrival of new information regarding a project before making the decision to invest. In addition to staged investments, the investor may obtain the right to choose at each stage whether he/she wants to continue to invest or not. If things turn out as expected, investment continues. Should conditions become unfavourable, the investor may decide to discontinue further investment and quickly cut losses. Many business decisions thus display option-like characteristics. This insight has paved the way for an "options way of thinking" to supplement standard valuation techniques. Although there is always value in flexibility, it may not, however, always be beneficial to delay investment. The threat of competition in fast-growing industries, like computer technology, implies that business and investment decisions are "now-or-never". Waiting to see what a competitor does often results in significant market loss rather than adding value.

Identifying added value, over and above cash flow projections, is what distinguishes the real options approach from traditional valuation methods. The discounted cash flow value of a young startup firm in a very large market, for instance, may not reflect the possibility, small though it may be, that this firm may break away from the pack and become the next Microsoft Corporation. In the same way, a firm with a patent or a license on a product may be undervalued by a discounted cash flow model because the expected cash flows do not consider the possibility of market dominance through sustained competitive advantage. Discounted cash flows generally understate value, not because they are too low (these cash flows simply reflect the probability of success), but because they ignore the options that firms have to invest in the future and to take advantage of unexpected successes in their businesses. Real options re-direct the thinking process towards what constitutes business value.

Being able to identify where opportunity and flexibility in a capital budgeting project really lies distinguishes the experienced manager/investor from the inexperienced one. This ability is partly linked to intuition. Admittedly, intuition has its place in decision-making, but anyone who believes that intuition is a substitute for reason is mistaken. In fact, intuition is probably most valuable to a firefighter in a burning building or a soldier on a battlefield. A corporate executive faced with a pressing decision to invest millions in a new product for a rapidly changing market cannot assess complexity by intuition only. The average day trader will confirm that making a quick, intuitive decision that turns out well is simply luck - it does not constitute insight or superior knowledge – and that sooner or later luck runs out. The options pricing techniques and applications presented in this thesis will be informative to all "gut feel" managers and investors who want to add quantitative analysis to their already well-developed business intuition.

1.2 Overview

Chapter 2 discusses traditional methods of asset valuation under uncertainty. These methods weigh costs against expected future cash flows. The rate at which cash flows are discounted to the present is referred to as the cost of capital. The cost of capital is central to standard valuation techniques and its calculation, using the Capital Asset Pricing Model, receives attention in this chapter. Popular alternatives to the CAPM are also discussed briefly.

Chapter 3 highlights some problems encountered when static models are used in uncertain investment environments. A few simple examples are used to demonstrate how expectation theory leads to errors in project valuation. A number of popular alternatives which portray cash flows as random quantities, are then introduced. However, most of these fail in one way or another to overcome the problem of finding the "correct" discount rate for a series of estimated or simulated cash flows. The question of how to account for flexibility in strategy and management under uncertainty thus remains unanswered.

Chapter 4 introduces the concept of real options and begins to familiarise the reader with it as an analysis tool. A number of real option examples are given from the literature to further aid understanding of this new way of thinking, illustrating how real option analysis bridges the gap between theory and corporate reality. Real options are then related in detail to financial options, which, in turn, links the theory to the financial markets and establishes a basis for the use of modeling techniques.

It is of the utmost importance to understand the various option-pricing methodologies that may be used to price real options. **Chapter 5** relates the necessary theory regarding the classical closed form analytical option pricing model, derived by Black and Scholes. This model has been successfully applied to many real option problems and forms the basis of the valuation of the real assets of Peppadew International (Pty.) Ltd. later in the study. A correspondence is drawn between each of the six input variables to the Black-

Scholes model and those required for a real option analysis. Chapter 5 concludes with a discussion of both the justification and limitations of the options analogy.

Chapter 6 introduces a numerical solution to real option pricing and focuses in particular on the multiplicative binomial process. This is a very popular technique for more complicated real options problems and thus warrants an introductory discussion. In addition to the valuation using the Black-Scholes model, Peppadew's real assets are also valued using a binomial tree. The latter valuation is attached as an Appendix for perusal by interested readers. The tree method comes to the fore again when a funding strategy for Peppadew is engineered in Chapter 9.

Chapter 7 focuses on some pitfalls of applying real options analysis. Like any new technique, it can easily be misused and its results interpreted incorrectly if it is not clearly understood within the context of a specific problem setting. Especially in the booming information technology sector, analysts and company executives have incorrectly used the real options argument to justify paying large premiums over discounted cash flow values for technology stocks and acquisitions. The error is usually only discovered when it is too late.

Chapter 8 introduces the reader to Peppadew International (Pty) Ltd - its unique product, its operating environment, its business model and the company structure. The most valuable assets Peppadew have are the plant breeder rights and process patent that allow them sole legal rights to grow, process and distribute their product commercially for the next ten years. It is argued that these real assets may be more accurately valued using real options analysis, rather than attempting to incorporate their valuation into a standard NPV analysis. The real options analysis is undertaken parallel with a valuation of Peppadew done by auditing firm KPMG. Where traditional methods assume that the value of the patents and breeder rights are contained within management estimation of cash flows (incurring all the errors of using expectation theory discussed in Chapter 3), the real options way of thinking allows for an explicit and easily quantifiable valuation based on option mathematics. The result illustrates that NPV analysis alone underestimates the true value of Peppadew's assets.

Chapter 9 engineers a borrowing strategy for the company now that its valuation has been revised. In particular, the focus is on structuring a debt security with a certain amount of protection granted to investors in the event of adverse business conditions. When structuring debt issues of this nature, the credit risk of the issuer plays a central role. Peppadew is, however, a privately held, non-listed entity. This circumstance usually complicates the calculation of default probabilities for credit spread estimates. The real options analysis in Chapter 8 not only delivers an enhanced valuation of company assets, but also leads to a measure of the company's business and industry risk. These aspects, together with Peppadew's liabilities, can be used to determine both the default probability and the implied credit rating of the company. A binomial interest rate tree is used to value a putable bond for Peppadew. Finally, the thesis concludes with a summary of, and recommendations for, the use and applications of real options, in particular for the Peppadew Company.

Chapter 10 is an overall conclusion of the real options methodology, its application and place in future research.

CHAPTER 2
TRADITIONAL APPROACHED TO CAPITAL BUDGETING

"The route to success is to put *more* money at risk."

Judy Lewent
Chief Financial Officer
Merck & Co. Inc.

This chapter reviews a few essential concepts surrounding traditional approaches to capital budgeting. In particular, the concept of net present value (NPV) is explored in the context of value maximization as the primary financial objective of a company. Other valuation methods such as payback period, accounting rate of return and internal rate of return are acknowledged by standard finance text to be inferior to NPV.

2.1 Discounted Cash Flows and Net Present Value

Discounting is the process of calculating the present value of future cash flows. The value today of an asset that is expected to generate a stream of cash flows C_i over a number of periods i, when the opportunity cost of investing in period i is r_i, is given by

$$PV = \sum_i \frac{C_i}{(1+r_i)^i}. \qquad (2.1)$$

An *expected* payoff implies a realistic forecast, i.e. neither optimistic nor pessimistic. Experienced managers attempting to make unbiased forecasts are, on average, correct. This means that even if their cash flow projections sometimes turn out to be high and at other times low, errors will average out over many projects.

Equation (2.1) represents the present value of the sum of a series of discounted cash flows and is referred to as the discounted cash flow (DCF) formula. DCF is important because it allows the values of future cash flows to be adjusted to a common time origin. This in turn allows all the values to be summed so that the total present value of a series of cash flows, to be received at many different times, can be calculated. Alternative investments, which have different time patterns and money flow, may be compared in this way. The *net* present value (NPV) of a project takes into account any investment capital I, and is given by

$$NPV = -I + \sum_i \frac{C_i}{(1+r_i)^i}. \qquad (2.2)$$

This formulation ignores the effect of inflation on interest rates. This has become common practice in countries where inflation is low. In countries where inflation reaches 100% per annum (for example in Zimbabwe and a number of other third world economies) Brealey and Myers (Brealey and Myers, 1992) recommend that the effect of such extreme inflation on interest rates be taken into account.

In today's corporate environment, NPV analysis (and its close relative, the internal rate of return, or IRR) is at the heart of most capital budgeting and valuation activities. IRR is defined as that discount rate which equates NPV to zero. Although its use is popular with some practitioners, problems with IRR are most obvious when the term structure of interest rates is not flat. If short term interest rates differ from long term rates, consecutive cash flows are discounted at a different cost of capital. Setting NPV equal to zero would mean computing a complex weighted average of separate discount rates in order to obtain a figure comparable to IRR. Brealey and Meyers (Brealey and Meyers, 1992) point out that IRR, whether derived from single or multiple rates, is void of any simple economic interpretation.

When executives evaluate a potential investment, whether it is to build a new plant, enter a new market or acquire a company, they weigh its costs against all expected future cash flows in the manner described by Equation (2.2). The standard investment rule is simply to invest when NPV is greater than or equal to zero. That is, invest if a project today is worth more than it is going to cost. There are two basic principles of finance at work in discounted cash flow calculations. The first one is that *a unit of currency today is worth more than the same unit of currency tomorrow.* Today's capital can be invested immediately and begin to earn interest. This is why the present value of a delayed payment is calculated by multiplying the expected cash flow C_i by a discount factor $(1+ r_i)^{-1}$, which is less than unity. The opportunity cost of investing, r_i, is the rate of return demanded for delayed payments. This leads to the second principle of finance: *a safe investment is worth more than a risky one.* Rational investors avoid risk whenever they can, without sacrificing return. An investment in a risky new project adds value only if its expected return is higher than what investors could expect from equally risky investments in the capital markets.

2.2 The Cost of Capital

The rates, r_i at which cash flows are discounted within a NPV analysis is also referred to as the *cost of capital.* The cost of capital is central to all NPV calculations. It is chosen to reflect the riskiness of the project and should, in theory, equal the rate of return of equivalent investment alternatives in the capital market. The choice of discount rate has a significant effect on value estimates of a project or company. Equation (2.1) infers that the higher the discount rate, the lower the present value of cash flows will be and vice versa. If a very high cost of capital is used when evaluating projects, potentially valuable opportunities will be rejected. Competitors and corporate raiders usually benefit from such oversight. Conversely, setting the discount rate too low guarantees that resources will be committed to a project that will erode profitability and destroy shareholder value.

It is important to bear in mind the distinction between the cost of equity capital and company cost of capital. *Company cost of capital* is defined as the expected return on a portfolio consisting of all the existing securities held by a company. Many companies estimate the rate of return required by investors in their securities and then use this company cost of capital to discount the cash flows of new projects. This is acceptable only if a new project has similar risk characteristics to that of the firm as a whole. If, for example, a company is contemplating expansion of its *existing* line of business, expected future cash flows may correctly be discounted at the company cost of capital. If new projects are deemed more (or less) risky than a company's existing business, company cost of capital is not the correct discount rate to apply. New projects should, in principle, be evaluated at their own cost of capital. The "true" cost of capital under project uncertainty depends largely on the use to which the capital will be put.

The remainder of this chapter describes how the cost of capital is traditionally calculated. The focus is on the popular but controversial Capital Asset Pricing Model (CAPM). A 2001 survey of financial practice conducted by J. Graham and C. Harvey (Graham and Harvey, 2001) in the United States found that 74% of US firms always, or almost always, use the CAPM to estimate the project cost of capital. Enhancements and alternatives to the CAPM are also discussed below.

2.3 The Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) is widely used in the corporate environment to estimate the return investors require from capital investments. The founding principles of the relationship between risk and reward were stated by Harry Markowitz in 1952. Defusco *et al.* (Defusco *et al.*, 2001), define the CAPM as follows

$$Expected\,return = E(r_p) = r_f + \beta[E(r_m) - r_f] \tag{2.3}$$

where

r_f = risk free rate of return. This is typically the yield on government- treasuries or bonds;

r_m = risky market return;

r_p = return on project p.

$E(r_m)$ = expected return from a market portfolio of common stocks.

A broad, value weighted stock index such as the *Top40* Index may be used to represent the market as a whole. The *Top40* Index consists of the forty largest market capitalisation stocks listed on the Johannesburg Stock Exchange (JSE). In practice, the market portfolio may be observed but is not directly tradeable.

The factor $(r_m - r_f)$ is termed the *market risk premium*. It defines the return investors require from a risky market portfolio, over and above Treasury bill- or government bond (risk free) returns. It is an important issue since investors do not take risk "for fun" but require sufficient compensation for risks taken. The market risk premium represents the price of stock market risk. Beta is a measure of a project's sensitivity to general market movement and is defined as

$$\beta = \frac{Cov(r_p, r_m)}{Var(r_m)} \tag{2.4}$$

It reflects the degree to which a project has historically moved up or down with the market in general. If there is no historical data, a proxy may be used to estimate beta. A proxy is a substitute asset that is used to represent the project being analysed. It usually has certain desirable characteristics, such as being listed on a securities exchange with a long history of price movements, absent or unobservable in the current project.

The market portfolio itself has a beta of one since

$$\frac{Cov(r_m, r_m)}{Var(r_m)} = \frac{Var(r_m)}{Var(r_m)} = 1 \tag{2.5}$$

A project with a beta of one represents average market sensitivity and will be expected to earn the market risk premium exactly. A beta greater (smaller) than unity indicates greater (smaller) than average market risk and earns a higher (lower) expected reward, according to the CAPM. Beta is (thus) simply the linear regression coefficient which predicts returns on the individual asset from returns on the market.

The CAPM relates expected return to *market risk* only. Here, risk is defined as uncertainty with respect to the outcomes of future events. The risk associated with any project may be regarded as either private or market related. Private risk is particular to a specific project or business and encompasses issues such as the risk of the CEO of a new project resigning before completion, mismanagement of project funds, bottlenecks in a development process for new technology, changes in legislation affecting natural resource developments and many more. Market risk is associated with market-wide variations and the effect that shifts in the economy will have on a project's profitability. If the market as a whole is the perfectly diversified portfolio, then investors can eliminate private risk by holding the market portfolio. The only risk remaining in a fully diversified portfolio is market risk. Since investors are only concerned with risk that they cannot "get rid of", the CAPM offers a means of measuring and quantifying this risk.

The fundamental idea underlying the Capital Asset Pricing Model is that a project's expected risk premium should increase proportionally to its sensitivity to overall market movement.

2.4 Alternatives for the Capital Asset Pricing Model

Once the beta for a specific project and the market risk premium for stocks have been calculated, the expected return required by investors is obtained from Equation (2.3). This value is the discount rate used in the NPV formula in Equation (2.2).

The calculation and application of beta in the CAPM is (and has been since its introduction) a contentious issue in the market place. Richard Grinold, in his article "Is Beta Dead Again?" (Grinold, 1993) states the following:

"The old, classic CAPM says that beta will extract the market's excess return, leaving only residuals whose expected return is not explained by other factors. Actual market data, however, suggest that this CAPM view of beta may not be correct."

There is thus empirical evidence that the basic CAPM cannot fit real-world data without some form of enrichment. Business and academia have engineered a number of alternative approaches in an effort to determine which discount rate ought to be applied in any specific NPV analysis. These include:

1. **Arbitrage Pricing Theory (APT)**

   In contrast to capital asset pricing theory, which begins with an analysis of how investors construct efficient portfolios, APT starts by assuming that the return on each stock or project depends partly on macroeconomic factors and partly on noise. It states that assets should be priced to prevent arbitrage. Noise in this context refers to risks and events that are unique to a company or project. (See DeFusco *et al*, 2001).

2. **Fama-French Three Factor Model**

   Research conducted by Eugene Fama and KE French in 1995 showed that the stocks of small firms and those with high book-to-market ratios provided above-average returns. They found evidence that these factors are related to company profitability. This meant that there appears to be certain risk factors that are omitted by the CAPM in its simplest form. (See Fama and French, 1995).

3. **Regression Models**

   Models of this type relate actual historical returns on stocks to observable and measurable characteristics of a firm, such as market capitalisation. (See DeFusco *et al*, 2001).

4. **Market-derived Capital Pricing Model (MCPM)**

A recent development by a team of businessmen, consultants and professors, this model is based on the traded prices of equity options on a company's shares rather than on historical data as in the case of the CAPM. Their research finds that discount rates given by their MCPM are more realistic than rates generated by the CAPM, especially from the perspective of corporate investors. MCPM is a total return measure which has the advantage of being based on forward-looking market expectations. This is helpful since these are the same investor expectations that are built into a company's current stock price. (See McNulty *et al.*, 2002).

2.5 Summary

In the absence of flexibility, discounted cash flow (DCF) is a simple and effective way of comparing the value of sums of money that arise in different time periods. Typically, all future cash flows are reduced to their equivalent values as at the present time. Summation of discounted investment or project cash flows leads to a net present value (NPV). The basis of this methodology lies in investors' demand for compensation for any time delay in receiving a return on their investments. This holds true even if the return were risk-free. Investors also want compensation for any unpredictability in the size of the return. Capital markets reflect all these aspects by pricing assets in a given risk class so that they offer a standard rate of return over time. The Capital Asset Pricing Model (CAPM) is a simple but effective model for measuring risk and for relating the required rate of return to the degree of risk. There is, however, evidence that the basic CAPM cannot fit real-world data without some enrichment. Consequently, a number of alternatives and enhanced models have become popular.

CHAPTER 3

THE FAILURE OF DISCOUNTED CASH FLOW METHODS

> "There is no certainty in life; only opportunity."
>
> Mark Twain.

Discounted cash flow techniques were originally developed to value passive investment in bonds and stocks. They were thus predicated on the implicit assumption of passive management. In the business world today, however, the focus has shifted to active, hands-on management of projects with an emphasis on the value of intangible strategic assets. This chapter highlights the paradigm shift that has lead to a closer look at DCF and NPV as valuation techniques in uncertain investment environments where managerial flexibility plays an ever greater role.

3.1 NPV under Uncertainty

Uncertainty is an inescapable part of life. As rational beings, people attempt to understand and resolve as much of the uncertainty surrounding their future as possible. We plan our days and schedule our time in an effort to manage what the future may bring. Insurance on homes and health buys protection against the unexpected; life is generally full of risks. Because investment always looks to the future, its outcome is uncertain. NPV analysis, as described in Section 2.1, is an attempt to understand and model (at least some of) the future cash flow uncertainties surrounding capital investment. Equation 2.2 portrays investment as a continuous operation until the end of a project venture. It assumes a fixed, multi-year investment model against fixed expectations of annual returns. Multi-year investments are, of course, re-analysed periodically. Nevertheless, one-time decisions are taken on the basis of a static investment plan. A static model narrows the overall project vision, making it very hard for managers and investors to change course as project uncertainty unfolds with the passage of time.

In static models, determining a project's cost of capital is by no means an easy task because it is not determined by a hard and fast rule. Discount rates are often set without calculating beta from the Capital Asset Pricing Model. If assets are not publicly traded and there is no recorded history of prices, managers revert to judgment for an estimate of the discount rate. The observation that beta can shift over time, since some capital investment projects are safer in maturity than in youth, has led to the application of variable discount rates over the life of a project. Refer Fama, 1977. The use of a constant discount rate assumes that project risk does not change over time. Using a single discount rate also implies that larger adjustments than necessary may be made for risk from later cash flows. But unpredictable fluctuations in interest rates pose another set of problems - an *increase* in the expected value of future project cash flows. Consider the following example, from Dixit and Pindyck (Dixit and Pindyck,1994), of an investment that yields a perpetuity paying R1,00 per annum. The present value of this perpetuity at interest rate r is $\frac{1.00}{r}$. If r is 10%, then the PV of the perpetuity is $\frac{1.00}{0.10} = R10.00$. However, interest rates are usually uncertain. Suppose then that r is equal to 5% or 15%, each with probability 0,5. Then the expected value of the interest rate is (still)

$$E(r) = (0.5 \times 0.05) + (0.5 \times 0.15) = 0.10 = 10\%,$$

as above. However, the expected value of the perpetuity is now

$$0.5 \times \left(\frac{1.00}{0.05}\right) + 0.5 \times \left(\frac{1.00}{0.15}\right) = 0.1333 = R13.33$$

which is greater than R10,00. This is because the present value of a series of future cash flows is a *convex* function of interest rates (i.e. the higher the interest rate, the lower the NPV and vice versa) so that, by Jensen's Inequality, the *average of the present values corresponding to a number of interest rates cannot be less than the present value for the* *average interest rate.* Figure 3.1 illustrates the convexity of discounted cash flows as a function of interest rates by means of a generic example.

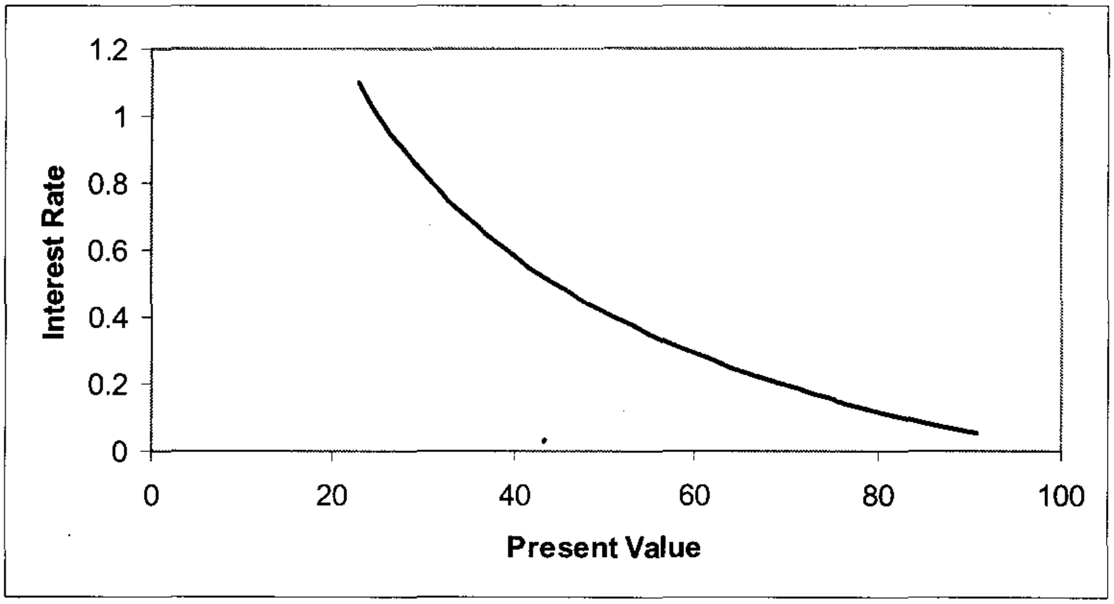
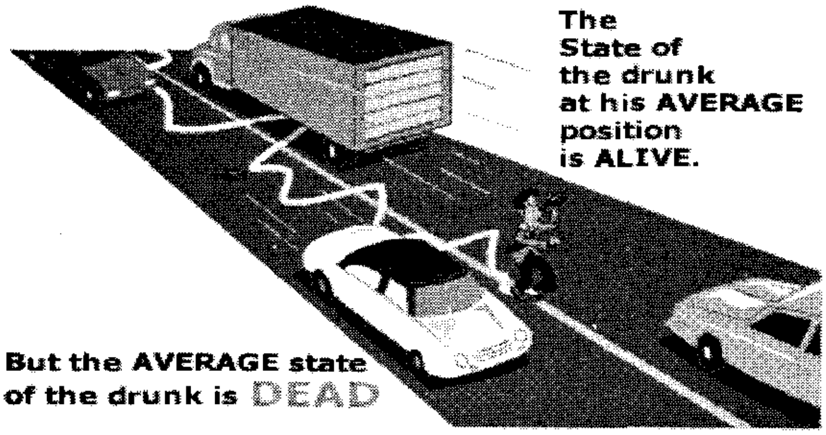


Figure 3.1

A generic example illustrating that present value is a convex function of interest rates. (A future cash flow of 100 was discounted at interest rates ranging from 0.5% to 100% over a fixed two year period).

A Comical Illustration of the Flaw of Averages.



Jensen's Inequality states that if x is a random variable and $f(x)$ is a convex function of x, then $E[f(x)] \geq f[E(x)]$ with equality guaranteed only if $f(x)$ is a linear function of x. Since the fair price of a project is represented by the expected value of a function of uncertain variables and not by the function of the expected values of the uncertain variables themselves, Jensen's Inequality implies that average values of uncertain inputs used by management when estimating future cash flows for a project will not result in average outputs.

There is an additional danger to using an expected value pricing technique such as NPV when estimating a project's value. Probability theory states that if an experiment is repeated *many times over*, the expected value will be the probability weighted average of the possible outcomes. However, in a *once off* experiment, Baxter and Rennie (Baxter and Rennie, 1997) point out that the concept of expected value pricing is hard to grasp and consequently easily misinterpreted. They illustrate with the following example: consider the tossing of a coin where R1.00 is gained for heads and nothing for tails. The expected gain(, after repeating the experiment many times,) is

$$R1.00 \times P_{head} + 0 \times P_{tails} = (R1.00 \times 0.5) + (0 \times 0.5) = 0.5.$$

However, if the coin is tossed only once, the chances of winning 50 cents is zero. Real life projects are similar to one-time coin tossing games; there is usually only one opportunity. The result is that expected value pricing may not accurately estimate the value of an asset or project.

All of the above issues relating to discounted cash flows, NPV and IRR analysis are hotly debated in the literature. Both academics and practitioners have strived to overcome the fundamental shortcomings of NPV - and in particular the formulation's failure to portray cash flows as random rather than static - with techniques such as sensitivity analysis, scenario analysis, simulation and decision tree analysis. These methods aim to bound uncertainty of cash inflows and outflows during the life of a project.

- **Sensitivity analysis** expresses cash flows in terms of key project variables and then calculates the consequences of misestimating the variables. It is sometimes called a "what-if" analysis since the impact on NPV (or IRR) is determined for a stated variation in each key variable with other variables held constant. It is useful in identifying the crucial variables that contribute most to the cash flows of a project. The greatest drawback of this relatively simple method is that it considers the effect on NPV of only one variable at a time, ignoring combinations of errors in many variables simultaneously. Examining the effect of each variable individually is meaningless when there are interdependencies amongst the different variables. Trigeorgis (Trigeorgis, 2002) also finds that if estimates of variables are serially correlated over time, a forecast error in one year may propagate higher errors in subsequent years. This will have a cumulative impact on NPV.

- **Scenario analysis** examines a project under alternative scenarios and is thus an improvement over sensitivity analysis if variables are interdependent. A limited number of different but consistent (with reference to dependencies) combinations of variables are considered to give an estimate of future revenue and costs. This method recognizes that uncertainty exists, but fails to capture the variance across the different scenarios. In this respect, it does not offer comprehensive managerial guidance.

- **Simulation** is probably the best technique for considering the impact of all possible combinations of variables on NPV. It attempts to imitate real-world scenarios by using a mathematical model to capture the important functional characteristics of the project as it evolves through time. Sensitivity analysis usually precedes simulation in order to establish the crucial primary variables driving cash flows. Probability distributions for these variables are then estimated while single point estimates suffice for all others. The distributions may be estimated from historical data (if available), from the historical data of a proxy (in the event of a variable having no history itself) or it may be subjectively chosen under certain criteria. To deal with dependencies between variables, conditional probability distributions may be specified in the same manner. From each of the distributions of the primary variables, a random sample is drawn. For every sample drawn (and for each of the secondary variable point estimates), the net cash flows for each period are calculated. After repeating this process a large number of times, the probability distribution of the project's cash flows in total can be generated. From this, the expected value of cash flows and the appropriate risk-adjusted discount rates can be derived and used to calculate an expected NPV. Simulation is thus used as an aid to implementing NPV. "Just as shaking a ladder helps one to assess the risks of climbing it, Monte Carlo simulation allows one to experiment with a proposed strategy before actually implementing it." (Refer www.stanford.edu)

Simulation can handle complex decision problems involving a large number of interacting variables under uncertainty. Even though the complexities of probability distribution estimation for interdependencies across time and amongst different variables may render this technique less intuitive to managers and investors, it remains the primary practical approach to valuation. It cannot, however, handle distributional asymmetries well and is limited in dealing with *options* and other free boundary problems.

- **Decision Tree Analysis** (DTA) is another method that attempts to account for uncertainty in project valuation. A decision tree is a sequence of decision- and chance nodes, ending in a terminal node. A node indicates a point where a decision must be made and branches emanating from the nodes represent the options available to the decision-maker. In this way, all possible (and mutually independent) alternative managerial decisions are recognized and mapped. The consequence of each consecutive action depends on some uncertain future event or state of nature which management can describe probabilistically based on past information. The tree is solved backwards in a roll-back type of procedure. All the NPV values calculated at the previous (although chronologically following) stages are multiplied with their respective probabilities of occurrence. The *expected risk-adjusted NPV* at each stage is the sum of each of these probability weighted NPVs. Management will choose the alternative at each node that maximizes the risk-adjusted expected NPV.

Decision Tree Analysis is well suited for analyzing sequential investment decisions when uncertainty is resolved at discrete points in time. It graphically illustrates the interdependencies between immediate and consecutive decisions. DTA accommodates the flexibility to abandon a project at certain discrete points in time based on the expectation of cash flows and their probabilistic estimates quantified at the outset of the project. But decision trees rely on NPV calculation inputs and in this sense share the same constraints under uncertainty as does NPV analysis in general:

- The problem of finding the proper discount rate remains. Once again, the use of a constant discount rate presumes that risk per period is constant. Variable discount rates over the life of the project would more accurately reflect the riskiness of cash flows relative to their position in the tree.

- Chance events do not simply occur at a few discrete points in time - the resolution of uncertainty may be continuous. The literature suggests that a continuous-time version of decision tree analysis might be preferable in real-world problems.

Decision trees can easily become large and unmanageable as the number of decisions, outcome variables and states to consider for each variable increases. In jest, many authors consequently refer to this technique as "decision-bush analysis".

Naturally, these techniques are often used in conjunction with one another in order to capture uncertainty in changing market conditions over time. In addition to these four methods popularly used to overcome the shortcomings of the standard discounted cash flow methodology when valuing a project, the following techniques have been applied by a number of researchers with varying degrees of success:

1. Risk-adjusted Discount Rate Method for Multi-Period Problems.
   (Trigeorgis, 2002)

2. Dynamic Optimization under Uncertainty.
   (Dixit and Pindyck, 1984)

3. Sequential Investment Analysis.
   (Bar-Ilan and Strange, 1992)

4. Incremental Investment and Capacity Choice.
   (Jorgenson, 1963)

3.2 NPV and Flexibility

The only serious shortcoming of the NPV methodology is its inability to account for the *flexibility in strategy and management* available to decision makers as the future unfolds. The techniques mentioned in the second chapter of this thesis do not satisfactorily account for the changing levels of risk as projects or investments progress.

"...many managers seem to understand that there is something wrong with the simple NPV rule as it is taught – that there is value to waiting for more information, and that this value is not reflected in the standard calculation. In fact, managers often require than a NPV be more than merely positive. It may be that managers understand that a company's options are valuable, and that it is desirable to keep these options open." (Dixit and Pindyck, 1994).

NPV, refer Equation (2.2), makes implicit assumptions concerning an expected scenario of cash flows and presumes management's passive commitment to an operating strategy. It imposes a fixed path on a business or project's future development without taking into account any form of managerial flexibility. The concise Oxford Dictionary defines flexibility as "the ability to bend and change shape without breaking". Flexibility in project and business management takes its cue from this definition. It refers to the ability of managers to steer a project successfully through changing market, economic, political and company specific conditions until completion. Managers know that things change all the time and that actual cash flows will most likely differ significantly from what was expected at the outset. In fact, all good managers have the ability to capitalise on favourable opportunities and be proactive in mitigating losses. Projects may, for example, be expanded or contracted as demand and supply dictates. Initial operation, exploration, production or investment may be deferred if the current economic environment is unfavourable. Operation may even be shut down temporarily or abandoned permanently for salvage value. The list of possibilities in project management is endless. The point is that management's ability to *"do something"* if and when the need arises, is equivalent to having a number of *options* when steering a project to successful completion. Options of this nature are extremely valuable. They may act as protection on the downside of project uncertainty, while offering upside benefits through flexible adjustment to altered market conditions. Since standard NPV methodology does not account for the flexibility to make decisions in the future, it systematically under valuates projects. Figure 3.2 illustrates how uncertainty affects value from a NPV and real options points of view.

The investment rule for NPV analysis is to invest *immediately* if NPV > 0. I.e. invest *immediately* when the value of a unit of capital is at least as large as its purchase and installation cost. It is a now or never decision. However, much of the uncertainty surrounding new projects is resolved over time. So there must be at least some benefit to waiting for the arrival of new information (market or project specific) before a final, irreversible investment decision is made. The ability to delay investment for a while is a *real option* investors may have- and a valuable one at that. NPV analysis in its standard form does not consider such investment flexibility. As a result, investors are either robbed of this benefit or it is given without considering its true worth.

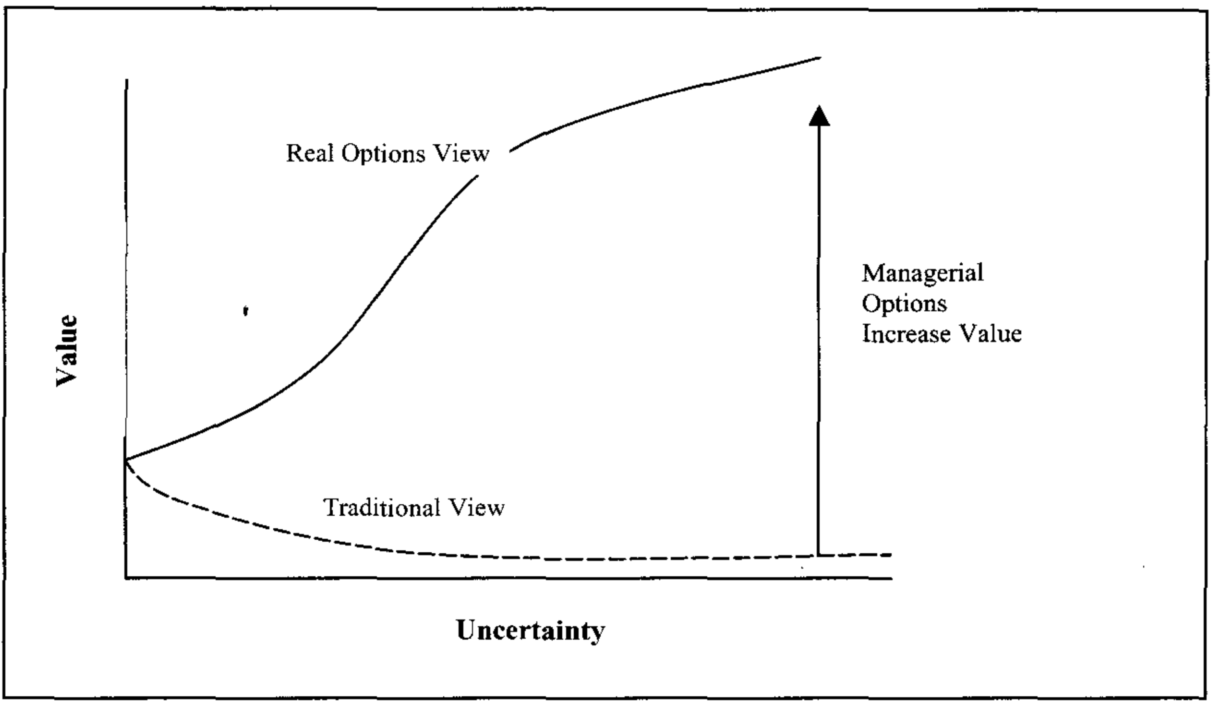


Figure 3.2

How Uncertainty affects Value.

Source: "Real Options: Managing Strategic Investment in an Uncertain World". Amram and Kulatilaka, 1999.

Naturally, not all projects derive value from delay. There may be strategic considerations that make immediate investment imperative in order to preempt potential competitors and establish market dominance. In most cases, however, delay is both feasible and extremely valuable.

3.3 Summary

Traditional NPV analysis is unable to capture the value of operating flexibility properly, mainly because of its dependence on expected future events that are uncertain at the time of an initial investment decision. The possibility of a company's management taking action as project uncertainty unfolds over time, results in investment opportunities that are not symmetric by nature. Although a number of attempts have been made to overcome this fundamental shortcoming of all DCF-type approaches, operating flexibility may be effectively accounted for by visualizing discretionary investment opportunities as options on real assets or as real options. Chapter 4 will now aim to familiarize the reader with the concept of a real option.

CHAPTER 4
REAL OPTIONS

> "From a little spark may burst a mighty flame....."
>
> Dante Alighieri.

This chapter introduces the concept of real options and begins to familiarise the reader with it as an analysis tool. A number of real option examples are given from the literature to further aid understanding of this new way of thinking, illustrating how real option analysis bridges the gap between theory and corporate reality. Real options are then related in detail to financial options, which, in turn, links the theory to the financial markets and establishes a basis for the use of modeling techniques.

The term "real option" was first used by Steward Myers, a MIT professor who introduced this new way of thinking in his popular 1984 publication, "Finance Theory and Financial Strategy", (Myers, 1984). Since then, academics have published widely on the subject. In particular, Avinash Dixit and Robert Pindyck (Dixit and Pindyck, 1994) published a book exploring most of the mathematics necessary to understand and successfully apply investment under uncertainty. Lenos Trigeorgis (Trigeorgis, 2002) is generally considered to be at the helm of new real option developments and regularly organizes academic conferences on real options. Refer www.realoptions.com. Martha Amram and Prof Nalin Kulatilika are also well-known authors who aim to make the insights of real options accessible to the business manager in general. The demand for real option knowledge is mainly driven by business management's need to position a company in such a way that benefit can be derived from uncertainty. It allows management to communicate a company's strategic flexibility internally and to the financial markets as a whole.

4.1 Real Options

Plagued by the shortcomings of traditional capital budgeting tools, academics, project managers and investors have started to change their way of thinking about uncertainty, flexibility and risk when valuing capital budgeting projects. Taking their cue from the seminal work of Fischer Black and Myron Scholes on the pricing of financial options, the real options approach was developed as an extension of financial option theory. A company evaluating an existing asset or potential investment is in much the same position as the holder of a financial option written on stocks, bonds or commodities. The holder of a financial call option on the price of oil may exercise the option if the oil price rises above a pre-agreed level, but will not do so if the price falls. Similarly, the owner of a marginally profitable oil field has the right to exploit it should oil prices rise, but has no obligation to do so if prices slump. The future value of such a real investment opportunity may thus be determined in a similar way to financial options.

As the term indicates, real options are options on real or non-financial assets. Real assets include, inter alia,

- The expected cash flow of a start-up venture;
- Intellectual capital and the ability of a good management team to steer a project to successful completion;
- Natural mineral resources;
- Licenses;
- Guarantees;
- Leases;
- Patents;
- Property and commercial rights.

Real assets typically either refer directly to, or have a significant impact on, the gross value of the operating cash flows of a project. In fact, profitable business ventures exist because they hold some kind of valuable real asset which is exploited and marketed in the correct way. In project management and capital budgeting, strategic flexibility under market uncertainty is an extremely valuable asset, highly prized by investors. It is easy to see why – when real money is on the line, an adaptable strategy is better than a rigid one.

In real options analysis, a real call option offers the freedom, in future, to spend money in order to *acquire* assets in the best way at that time. A real put option represents the freedom, in future to *dispose of* an asset in the best way at that time (i.e. scrap an initiative, sell a going concern etc). The most extreme put option under limited liability is the option to declare insolvency. Options confer a right, but no obligation, on the holder to make a decision.

The key difference between a financial option and a real option is that *a decision about a financial option cannot change the value of a business itself.* The activities and profitability of a company are not influenced to any extent by trade in financial options (assuming, of course, a reasonably efficient market structure). However, real options involve a claim on real economic resources like time and money and can thus alter a company's resources, profitability and competitive advantage. Therefore, a company should actively manage their real options. Howell (2001) considers a company that holds a real option to invest in technology in the future. If it exercises this option at the "wrong" time, the company has not only lost part of the value of its real option but has also spent money investing at a sub-optimal time. Such losses, Howell argues, will inevitably be reflected in the company's share price. Consider also the following examples, adapted from Howell (2001), of business decisions that can be influenced by real options:

- The sequence of stages by which to expand or shrink operating capacity;
- The decision to buy into or make a new product;
- The price at which to accept a long-term fixed-price contract for an input or output whose market price is variable (e.g. oil, gold);
- How to compare and value leases, brands and patents (i.e. deals which constrain the activity of participants and competitors);
  (Refer to the Case Study of this Thesis)
- When to cease operating an asset and when to reactivate it;
- When and how to exit from owning and/or operating an asset;
- The maximum investment to make in a research project;
- How to design government policies and incentives that do not hamper business and entrepreneurial activities within an economy.

These examples suggest how real options can provide management with valuable operating flexibility and strategic adaptability. Many real options occur naturally in valuation and investment opportunities. Growth options, for example, are found in all infrastructure-based and high technology industries. These are industries with multi-product generations where rapid growth implies success. Other types of real options may be built in as part of the strategic planning process. Table 4.1, adapted from Trigeorgis (Trigeorgis, 2002:2-4), gives a comprehensive overview of common real options.

When financial option theory is applied to real options in a business situation, the discipline of the financial markets is brought to internal strategic investment decisions. This ensures that the real option approach is aligned with financial market valuations. The link is important from a management and investor perspective. Even though financial market valuations may produce a myriad transaction prices, the prices of mis-valued assets are swiftly corrected by the market as a whole. Thus, if real option valuations are kept in line with market valuations, mis-pricing will be limited. Since managers are often rewarded with stock options linked to growth in the value of a firm, alignment of strategic business decisions with financial market valuations is important. A model which is clearly linked and anchored to pricing in the financial markets, enables managers to communicate confidently with investors and analysts. Most small companies as well as high-risk start-up ventures must, for example, approach the capital markets periodically during their early growth phases. Although full disclosure of developments and alliances is usually required, the value of such firms lies in their *ability to grow and capture market share*. Communicating to the market a paradigm shift from static expected cash flows to the *real* value of the firm through an option to grow is doomed unless it is intuitively appealing and comprehensible. Real options analysis creates an integrated framework that addresses decisions under uncertainty. It reverses all the rigid assumptions of DCF by recognising that

- the future cannot be forecasted perfectly (only today's market state is known) and
- the use of the risk-free rate of interest is more appropriate than trying to estimate a risk-adjusted discount rate for every project.

Insight into the nature of options and the mechanics of option pricing explains to a large extent why the *actual* investment behaviour of companies differs so much from the methods taught in business schools. Real options bring the theory closer to reality by addressing the option-like characteristics of investment opportunities. A company has to bring together new non-standard and non-traded combinations of real resources in order to effect a "deal" between itself and the outside world.

4.2 Financial Option Theory and Real Options

A financial option gives the holder the right, without any obligation, to buy or to sell a specific financial asset by paying a predetermined price on or before a specified date. This right, which the option buyer obtains, is valuable and comes at a mathematically calculated premium, payable at inception of the contract. The right to buy is known as a *call option* and the right to sell, a put option. Assets underlying financial options are tradable securities like common stock, government bonds, corporate bonds, stock indexes and bond indexes. The exercise (or strike) price is the predetermined price that the holder of the option will pay for a call option or receive for a *put option* (is the exercise or strike price). The maturity or expiration date is the date on which payment is made (call option) or received (put option). The values of call options increase with favourable movements in the underlying asset – they become more valuable on the upside. Put options, on the other hand, function like insurance. They pay off when the value of the underlying asset drops (in value). Options are like a two-edged sword: used on their own, they provide a means of speculating about the directional view of the underlying asset. But used in conjunction with the underlying asset, options reduce risk by hedging against fluctuations in the value of the underlying asset. An option, call or put, which allows the holder to exercise on one specific date only, is known as a European option. An option that may be exercised at any time on or before the expiration date is known as an *American option*. Table 4.2 describes the equivalence between financial- and real options.

Table 4.1

Common Real Options

CATEGORY	DESCRIPTION	APPLICATION	REFERENCES
Option to defer	Management holds a lease on (or an option to buy) valuable land/resources. They can wait x years to see if output prices justify constructing a building or a plant or developing a field.	All natural resource extraction industries, real estate development, farming and certain commercial products.	McDonald and Siegel 1986, Paddock et al. 1988, Tourinho 1979, Titman 1985, Ingersoll and Ross 1992.
Time-to-build option (staged investment)	Staging investment as a series of outlays creates the option to abandon the enterprise in midstream if new information is unfavourable. Each stage can be viewed as an option on the value of subsequent stages and valued as a compound option.	All R&D-intensive industries, especially pharmaceuticals; long-development capital intensive projects (e.g. large-scale construction or energy generating plants); start-up ventures.	Majd and Pindyck 1987, Carr 1988, Trigeorgis 1993.
Option to alter operating scale (e.g. to expand, to contract, to shut down and restart)	If market conditions are more favourable than expected the firm can expand the scale of production or accelerate resource utilization. Conversely, if conditions are less favourable than expected, it can reduce the scale of operations. In extreme cases, production may be halted and restarted.	Natural-resource industries (e.g. mining); facilities planning and construction in cyclical industries; fashion apparel; consumer goods; commercial real estate.	Trigeorgis and Mason 1987, Pindyck 1988, McDonald and Siegel 1985, Brennan and Scwartz 1985.
Option to abandon	If market conditions decline severely, management can abandon current operations permanently and realize the resale value of capital equipment and other assets on secondhand markets.	Capital-intensive industries(e.g. airlines, railroads);financial services;new-product introductions in uncertain markets.	Myers and Majd 1990.

Option to switch(e.g. outputs or inputs)	If prices or demand change, management can change the output mix of the facility (product flexibility). Alternatively, the same outputs can be produced using different types of inputs (process flexibility)	Output shifts: Any good sought in small batches or subject to volatile demand(e.g. consumer electronics); toys; speciality paper; machine parts; autos. Inputs shifts: All feedstock-dependent facilities; electric power; chemicals; crop switching; sourcing.	Margarbe 1978, Kensinger 1987, Kulatilaka 1988, Kulatilaka and Trigeorgis 1994.
Growth options	An early investment(e.g. R&D, lease on undeveloped land or oil reserves, strategic acquisition, information network) is a prerequisite in a chain of interrelated projects, opening up future growth opportunities(e.g. new product or process, oil reserves, access to new markets, strengthening of core capabilities). Like inter-project compound options.	All infrastructure-based or strategic industries – especially high tech, R&D and industries with multiple product generations or applications(e.g. computers, pharmaceuticals);multinational operations; strategic acquisitions	Myers 1977, Brealey and Myers 1991, Kester 1984,1993, Trogeorgos 1988, Pindyck 1988, Chung and Charoenwong 1991.
Multiple interacting options.	Real-life projects often involve a collection of various options. Upward potential enhancing and downward protection options are present in combination. Their combined value may differ from the sum of their separate values, i.e they interact. They may also interact with financial flexibility options.	Real-life projects in most industries listed above.	Trigeorgis 1993, Brennan and Scwarz 1985, Kulatilaka 1994.

Source: Trigeorgis, 2002.

Table 4.2

Describing Financial - and Real Options.

Adapted from "Real Options-Evaluating Corporate Investment Opportunities in a Dynamic World", Sydney Howell *et al*, 2001.

In Financial Markets	In Real Asset Markets
Purchase a call option on a share from a trader. It gives the right to buy the share at a fixed exercise price.	Spend money to buy, or create, the right or power to acquire some business asset e.g. buy a license or brand, or fund new research. This allows acquisition of a business asset, provided an investment of known cost is made (exercise price).
(When profitable) pay the exercise price to the trader who sold the option.	(If expected to be profitable) make the required investment to acquire the "real" asset, e.g. equipment, advertising etc.
In return for paying the exercise price, obtain stock whose current value is based on its future stream of expected dividend income.	In return for paying the exercise price of the investment, obtain a new business asset whose current value is based on its future stream of expected net cash flows.
Only pay the exercise price if the value of the share in the financial market at the time is at least as high as the exercise price.	Only make the investment (exercising the real option) if the future net cash flows of the business asset that will be acquired (discounted at an acceptable rate) are at least as big as the investment cost to be paid for the asset.
The return on the stock follows a random walk.	In some real options, the decision whether to invest in the underlying asset or not will be noticed by other players and will itself change the value of the asset.

The core element of an option's value (whether it be financial or real) is the beneficial asymmetry derived from the right to exercise only when it is in the holder's interest to do so - with no obligation to exercise if conditions are unfavourable. Options thus enable investors to control risk by limiting losses while magnifying upside potential. Figure 4.1 demonstrates this concept. The curved payoff profile of an option to invest is compared with the straight line payoff profile of a typical immediate NPV decision. The horizontal axis shows the possible expected present values of a project's net operating revenues. The vertical axis shows the resulting net present value of the project, i.e. the difference between the present values of all operating revenues and the initial investment cost. Recognizing and quantifying the option to defer investment prevents the payoff profile of the investment decision from becoming negative. Downside risk is mitigated because management now have the option to cut operations before losses are incurred.

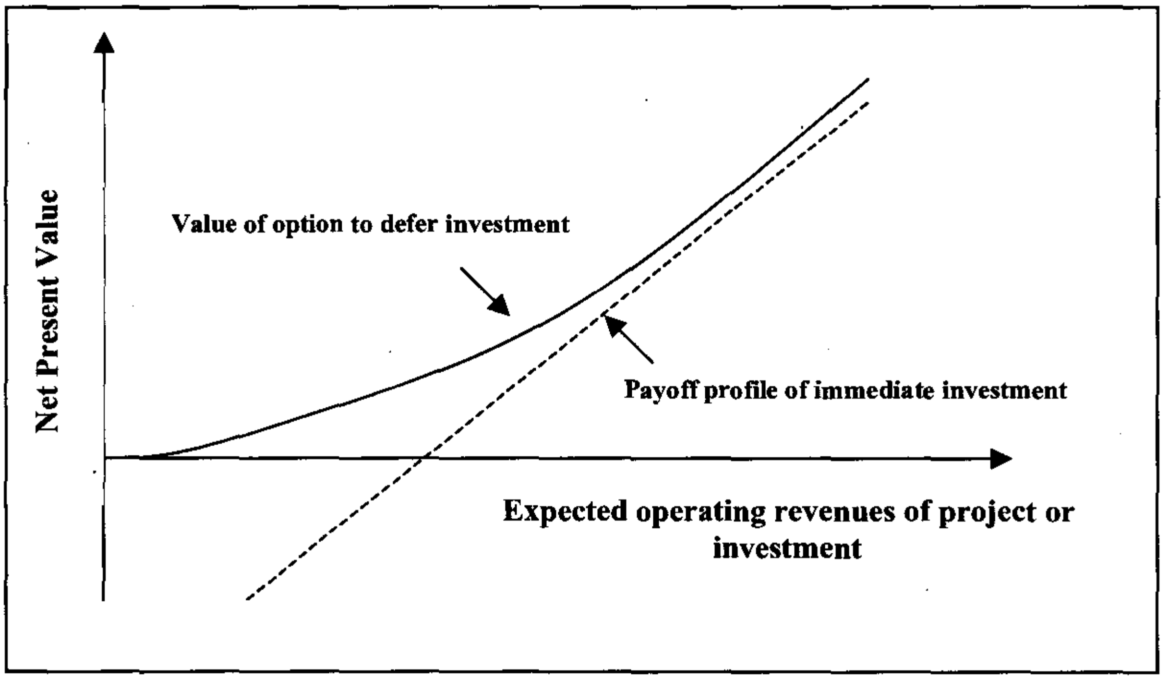


Figure 4.1

Payoff Profile of a Real Investment Call Option versus that of a Standard NPV Decision.

Source: Damodaran, 2001.

The pricing of derivative securities in general, and options in particular, rests on the notion that securities with exactly the same risk-return profiles ought to be identically priced. In technical terms, this is called the arbitrage pricing argument. Arbitrage refers to any riskless profit (involving no net investment) that may be realised from trading the same securities in different markets. Suppose one can construct a portfolio by buying a particular number, N, of shares of the underlying asset. In order to make this purchase, one borrows an amount, A, at the riskless rate of interest. In any small interval of time, the future returns of this portfolio would exactly replicate the future returns of the option. Such a portfolio is thus referred to as a *replicating portfolio*. The number of shares in the replicating portfolio will need to be adjusted every time a price change is observed in the market – only then can it remain an appropriately levered position in the underlying asset. Since the option and the replicating portfolio are equivalent in terms of payoff profile and risk distribution, they must have the same price to avoid arbitrage opportunities. This is commonly referred to as the *Law of One Price*. It follows that the value of the option is then determined by calculating the cost of the replicating portfolio. Arbitrage pricing is of inestimable value when pricing complex contingent claims, financial or real.

Dixit and Pindyck (Dixit and Pindyck, 1994) use this risk-less arbitrage argument to show that real options can be priced if some *equivalent traded security,* whose risk "spans" or "tracks" the uncertainty in the real asset, may be identified. The use of arbitrage arguments in the valuation of real options was a breakthrough because it overcame the valuation difficulties of traditional methods, namely:

- It avoids the pitfalls of expected value pricing techniques. Arbitrage pricing suggests the same price for the same risk and so enforces the fair price of an asset.

- The arbitrage condition is independent of an investor's attitude toward risk (i.e. the price of the security is determined by the non-arbitrage condition only). This allows the risk-adjusted rate of return to be replaced by the risk free rate of interest, rendering it free from the discount rate problem discussed in the first chapter of this thesis.

4.3 Modeling Techniques for Real Option Problems

Unlike financial options, real options are generally not specified in a contract and must be identified and defined through analysis and judgment. A mathematical framework representing the underlying stochastic process, the payoff function and decision rules must be established before the solution method and "option calculator" is chosen. An "option calculator" refers to the specific mathematical techniques used in the different solution methods for option valuation. Three approaches to (real or financial) option valuation will be discussed in sub-Sections 4.2.1 to 4.2.3 below. These are partial differential equations, dynamic programming and simulation. Figure 4.3 below summarises the implementation process and tools for option valuation.

4.3.1 The Partial Differential Equation Approach.

This approach to valuing options involves solving a partial differential equation (PDE) which equates the change in option value to the change in the value of a tracking or replicating portfolio. Boundary conditions specify the value of a particular option at known points (e.g. at expiry of the option) and at the extremes (e.g. the upper boundary where the value of the underlying asset is large and the lower boundary where the value of the underlying asset tends to zero). If the option value can be written as an explicit function of the inputs, an analytical solution to the PDE exists. The Black-Scholes partial differential equation is a point in case. It can be solved with a particular set of boundary conditions to give the arbitrage prices of European call and put options.

If an option's value V is a function of the underlying asset S and the passage of time t, then a graph which shows how option value varies as a result of changes in these two factors is referred to as the valuation surface, $V(S,t)$. See Figure 4.2 for an illustration. Howell (Howell, 2001) explains, by means of an example, that the Black-Scholes PDE controls only the local variations of the valuation surface *V(S,t)* due to small variations in stock price, *S*, and time, *t*. If one imagines the valuation surface as a piece of fabric, the Black-Scholes equation gives the laws by which that fabric can or cannot flex. Since the surface is twice differentiable in *S* (refer to Equation 5.12 below), the fabric cannot have discontinuities or tears. In economic terms, this feature prevents arbitrage, since similar conditions of *S* and *t* do not lead to dissimilar values of V. The Black-Scholes equation leaves the "fabric" of the value surface free to assume infinitely many "flexings". In order to make the valuation surface have a unique shape, one must pin the fabric down along certain boundaries. Refer Figure 4.2. This is done by imposing (sufficient) boundary conditions. Too few boundary conditions will leave the surface undefined (fabric flapping in the wind) with no unique value for given *S* and *t*. Too many boundary conditions will at best include redundant conditions and at worst, some contradictory ones which jointly may permit no solution at all. In mathematical terms, a well-posed problem has just enough boundary conditions to ensure a unique solution.

Although the Black-Scholes model cannot be applied in the valuation of *all* financial and real options, it is arguably the easiest and fastest way to value European options. Chapter 5 is dedicated to a description of its origin and mechanics. However, when an analytical solution to the PDE cannot be obtained, numerical solutions offer an alternative. The PDE is converted into a set of equations that must hold over short intervals of time. Computational algorithms are then used to solve for the specific option value that solves the equations simultaneously. The most widely used numerical solutions to a PDE are the (implicit and explicit) finite difference methods. In this approach, a grid covering the entire range of values for the underlying asset during the life of the option is set up.

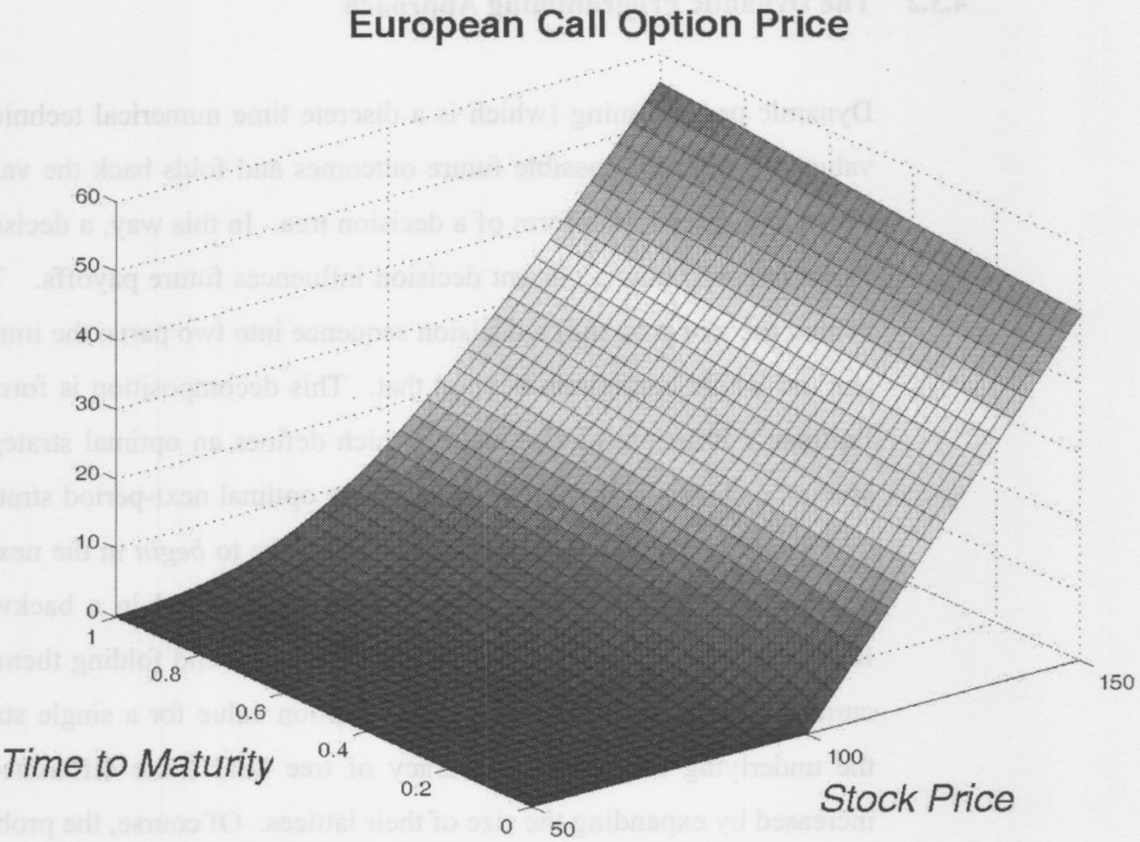


Figure 4.2

European Call Option Value as a Function of Stock Price and Time.

Source : Prof M.F Kruger

For each point in the grid, a set of equations is solved to obtain a value of the option. Calculations begin at option expiry and then proceed backward in time (along the time dimension). Finite difference methods calculate option prices for many starting prices of the underlying asset simultaneously. Although not (directly) applied in (the specific case study of) this thesis, implicit and explicit finite difference methods are important in many real option applications. Consequently, they are discussed further in Appendix I.

4.3.2 The Dynamic Programming Approach

Dynamic programming (which is a discrete time numerical technique for option valuation) lays out possible future outcomes and folds back the value of optimal future strategies in the form of a decision tree. In this way, a decision tree solves the problem of how a current decision influences future payoffs. The basic idea behind the tree is to split a decision sequence into two parts; the immediate period and the whole continuum beyond that. This decomposition is formally stated in Bellman's Principle of Optimality which defines an optimal strategy as follows: given the choice of the initial strategy, the optimal next-period strategy is the one that would be chosen if the entire analysis were to *begin* in the next period. The problem of determining an optimal strategy is solved in a backward recursive fashion, discounting future values and cash flows and folding them back into the current decision. A tree thus gives an option value for a single starting price of the underlying asset. The accuracy of tree (and finite difference) methods is increased by expanding the size of their lattices. Of course, the problem of having to work with enormous trees must be circumvented. The subject of enhanced convergence of standard lattice methods for option pricing is well documented in the literature. It will not receive detailed attention in this thesis.

The most popular dynamic programming model for solving financial or real options is the binomial model. Binomial tree methods are rooted in the one-dimensional random walk model. The lattice structure affords a strong intuition about factors influencing option value. It handles various real assets and real option features transparently and is generally more instructive than analytical solutions. The binomial model is given further attention in Chapter 6. An application to the case study is included for interested readers and may be found on the CD:

File Name : Peppadew International
Sheet Name : Binomial Tree.

4.3.3. Simulation Models

These models simulate thousands of possible paths of evolution of the underlying asset from the present to the final date of the option. The simulation process attempts to imitate a real-world decision setting by using a mathematical model to capture the important characteristics of a project as it evolves through time. The optimal outcome or "strategy" is determined for each simulation run and the payoff calculated. The value of the option is the average of these payoffs, discounted back to the present. The commonly used Monte Carlo simulation method directly approximates the behaviour of a stochastic process (e.g. an option's value) by drawing a large number of random trials of the stochastic behaviour in question, and observing the effects on option value. Monte Carlo simulation in particular, is more intuitive than finite difference methods when valuing complex options. It can handle both complicated decision rules and complex relationships between the option value and the underlying real asset. Consequently, exotic and American style options are commonly valued using Monte Carlo simulations. Refer Longstaff and Schwartz (2001).

Chapter 5 will focus solely on the Black-Scholes closed-form analytical solution to the price of any European-style derivative asset paying dividens. It is arguably the quickest and easiest model to implement, once the underlying mathematics is understood. It cannot, however, handle exotic and American options well. The real assets of Peppadew International will be valued using the Black-Scholes model – not because of its relative simplicity, but because it is well suited to the structure of and information available for the case study.

A number of option calculators are available for different solution methods. Each will provide values consistent with financial market valuations if guided by the real options approach.

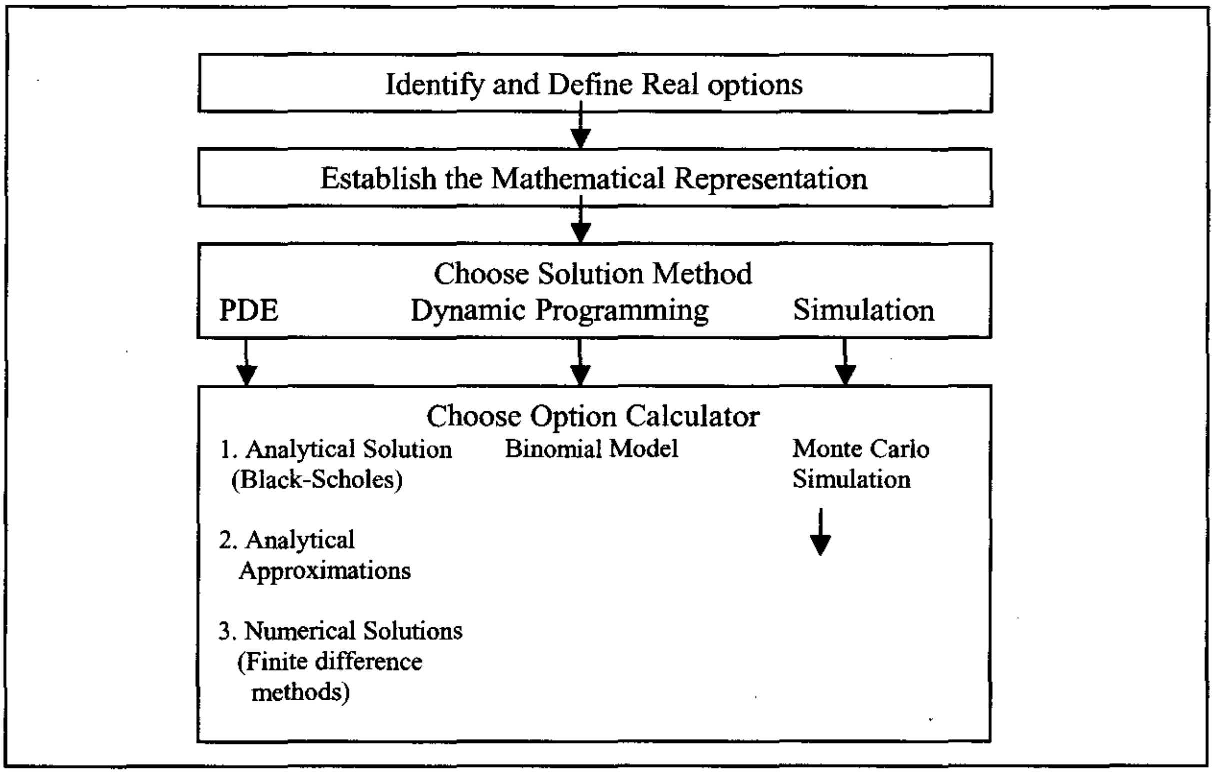


Figure 4.3

Solution Methods and Option Calculators. *Source*: Amram and Kulatilaka, 1999.

4.4 Summary

Real options analysis allows flexibility in the decision making process to be quantified. Consequently, it has become an important tool for many types of business and investment decisions which hinge on change, adaptability and operating efficiency. Real options analysis is essentially an extension of the ways in which financial markets value uncertainty and as such may be valued using well-known mathematical modeling techniques.

CHAPTER 5

BLACK SCHOLES AND REAL OPTIONS

"Chance favours the prepared mind."

Louis Pasteur.

This chapter deals heuristically with continuous processes, leading up to the option pricing model derived by Fischer Black and Myron Scholes. This model is of particular interest in real options analysis. It offers a closed-form analytical solution that is accurate (for sound, quantifiable inputs) and intuitively easy to understand, for the price of financial and real options. In particular, it is applied as the model for valuation in the real options analysis of Chapter 8 of this thesis.

5.1 Continuous Processes

Even though markets do not trade continuously, continuous processes are used to model market prices solely for mathematical convenience without loss of generality. In this context, continuous processes refer to those that are continuous in both time and space. A step process like a birth- and death process, for example, is continuous in time but discrete in space. Over time, with each recorded birth the process will jump up and with each death it will jump back down again. Trading in modern financial markets has become so dynamic that trades are executed in very short time intervals at almost any time of the day or night. Continuous models are thus a reasonable approximation of real markets. A continuous process may be described by the following three basic principles:

(1) Its value can change at any time and from moment to moment.

(2) Actual values can be expressed in arbitrarily fine fractions, i.e. any real number can be taken as a value.

(3) The process changes continuously which means that it does not make instantaneous jumps. If the value of the process changes from 2 to 2.05, it must have passed through all the values in between, albeit quickly.

Intuitively, the prices of market securities seem to behave this way. i.e. they display continuous-time behaviour. But markets are also assumed to be continuous in space. As far back as Bachelier in 1900, academics have compared the price movement of financial market securities to one particular continuous-time process called **Brownian motion**.

5.1.1 Brownian motion

In 1827, the botanist Robert Brown observed and described the erratic motion of gas particles suspended in fluid. Later, in 1905, Albert Einstein proposed a mathematical theory of Brownian motion. Brownian motion was further developed and made more rigorous by Norbert Wiener in 1923.

Define a binomial process $W_n(t)$, for *n* a positive integer and

(i) $W_n(0)=0$

(ii) layer spacing $1/n$

(iii) up and down jumps equal and of size $1/\sqrt{n}$

(iv) a measure $\mathbb{P}$, which describes up and down probabilities; let these be equal to 0.5 everywhere.

Under these conditions, the process $W_n(t)$ is called a random walk and depicted in Figure 5.1. If $X_1, X_2, \ldots$ is a sequence of independent binomial random variables equal to +1 or -1 with equal probability, then the value of W_n at the *i*-th step is defined by

$$W_n(i/n)=W_n((i-1)/n)+X_i\left(1/\sqrt{n}\right)\ ;\ i\geq 1.$$

The first two steps of the process is illustrated in Figure 5.1. As n increases, the distribution of W_n at time 1 tends towards the unit normal $N(0,1)$ since $W_n(1)$ is the sum of n IID random variables, each with zero mean and variance $\frac{1}{n}$. The value of $W_n(t)$ is then the same as $W_n(t) = \sqrt{t}\left(\sum_{i=1}^{n} X_i / \sqrt{nt}\right)$. By the central limit theorem, $\sum_{i=1}^{nt} X_i / \sqrt{nt}$ tends towards a normal $N(0,1)$ random variable. This means that the distribution of $W_n(t)$ tends to a normal $N(0,t)$. All of the t marginal distributions tend to the same underlying normal structure. The next step in each random walk W_n is independent both of where it currently is as well as all previous movements up to the present time. The distribution of W_n converges toward a process known as Brownian motion. Formally, the process $W = (W_t : t \geq 0)$ is a $\mathbb{P}$- Brownian motion if and only if

(i) W_t is continuous and $W_0 = 0$. Although W is continuous everywhere, it is nowhere differentiable. The function is so irregular that it is not smooth anywhere. Such a function changes its shape in the neighbourhood of any point in a completely non-predictable way.

(ii) The value of W_t is $N(0,t)$ distributed under $\mathbb{P}$.

(ii) Increment $W_{s+t} - W_s$ is $N(0,t)$ distributed under $\mathbb{P}$ and is independent of $\mathcal{F}_s$, the history of the processes' behaviour as it unfolds up to time s. $\mathcal{F}_s$ is called a *filtration.*

Brownian motion will eventually reach every real value no matter how large or small. It will eventually also return again to zero. Once the process hits a value, it immediately hits it infinitely often and then again from time-to-time in the future. Brownian motion is self-similar (fractal) and thus scale invariant: no matter on what scale it is examined, it looks the same. Figure 5.2 illustrates Brownian motion. Brownian motion is essentially a one-dimensional Gaussian process, often called a Wiener process. Brownian Motion, as is, is not sufficiently flexible for modeling global stock price behaviour. Even though Brownian motion wanders, it has mean zero while stocks usually grow at some rate, driven (at least) by inflation. Brownian motion may also be too noisy (or not noisy enough) when compared to stock price movements. Remembering that Brownian motion can go negative but stock prices never can, an exponential function may be introduced. Incorporating a growth factor μ and a noise factor σ, results in an adjusted process of the form $S_t = \exp(\mu t + \sigma W_t)$. This is called exponential Brownian motion or geometric Brownian motion with drift. It is commonly accepted that most asset prices can be modeled by solutions of stochastic differential equations which are driven by Brownian motion. Brownian motion effectively models the fluctuations of financial markets, i.e independent movements up and down on disjoint time intervals. It is these fluctuations which best represent market information.

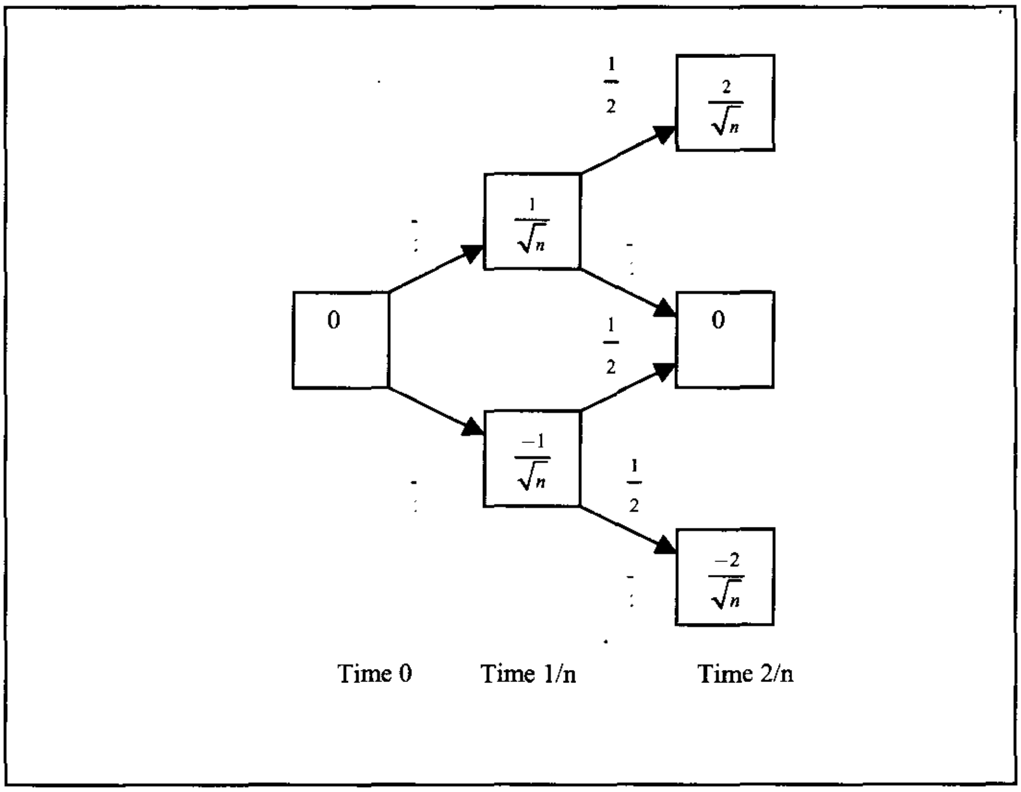


Figure 5.1

The first two time steps of random walk W_n.

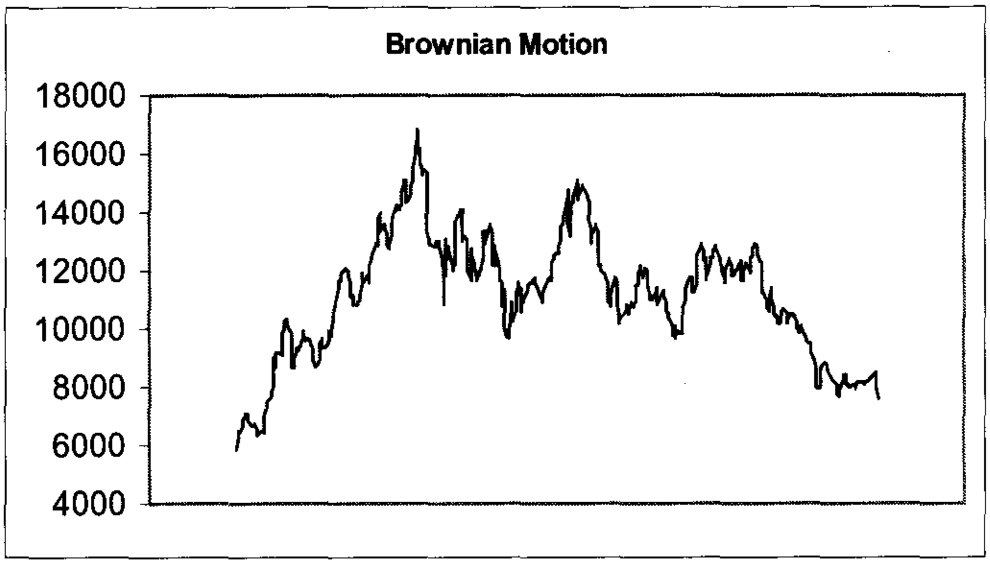


Figure 5.2

The jagged, self-similar path of Brownian Motion.

5.1.2 An Ito Processes

An Ito process is a special kind of stochastic process, X, which is defined as a continuous process $(X_t : t \geq 0)$ such that X_t can be written as $X_t = X_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s$, where μ and σ are random $\mathcal{F}$-previsible processes such that $\int_0^t \left(\sigma_s^2 + |\mu_s|\right) ds$ is finite for all times *t*, with probability 1. The differential from of this equation can be written $dX_t = \mu_t dt + \sigma_t dW_t$. A $\mathcal{F}$-previsible process is a process whose value at any given time *i*, is dependent only on the history of the process up to time $i-1$, namely $\mathcal{F}_i$ -1. Generally, it is not sensible to define the value that a previsible process has at time zero.

Differentiable functions are built from straight line segments and Newtonian calculus is the formal recognition of this. The self-similarity of Brownian motion, however, means that smaller segments are themselves Brownian motion and do not reduce to straight lines at microscopic level. The unbounded variation and non-differentiability of Brownian motion paths are the major reasons for the failure of classical differentiation methods when applied to these paths. Fortunately, general random processes for stock market behaviour can be built from both straight line segments and small segments of Brownian motion. A stochastic process, X, will have both a Newtonian term based on dt (a time interval of infinitesimal length) and a Brownian motion term based on the infinitesimal increment of W, dW_t. Incorporating the drift and noise into X, the infinitesimal change of X_t is

$$dX_t = \mu_t dt + \sigma_t dW_t \tag{5.1}$$

Equation 5.1 is called a stochastic differential. The drift μ_t and noise σ_t can depend on the time but they can also be random and depend on values that X (or W) took, up until time t. Such processes whose values at time t can depend on the history $\mathcal{F}_t$, but not on the future, are called adapted processes to the filtration $\mathcal{F}$ of the Brownian motion W. σ_t is the volatility of the process X at time t and μ_t is the drift. In the special case when μ and σ depend on W only through X_t, such that $\sigma_t = \sigma(X_t, t)$, where $\sigma(x,t)$ is a deterministic function, the equation

$$dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dW_t \tag{5.2}$$

is called a stochastic differential equation (SDE) for X. An SDE may not have a solution, and if it does it is often times not unique. Intuitive integration is not enough for determining the solution to SDEs. It requires a departure from Newtonian differentials formalised by the Chinese mathematician Kazuhito Ito in 1951 and referred to as Ito's rule or Ito's lemma.

In stochastic environments, a formal notion of derivatives does not exist. Shocks to asset prices are assumed to be unpredictable and in continuous time they become too erratic. The resulting asset prices may be continuous but they are not smooth. Stochastic differentials need to be used in place of derivatives. Ito's rule provides an analytical formula that simplifies handling stochastic differentials and leads to explicit computations.

The stochastic version of the chain rule of differentiation, namely

$$\frac{dF(S_t,t)}{dt}=F_s\frac{dS_t}{dt}+F$$

for a function $F(S_t,t)$ which depends on two variables S_t and t, is known as Ito's lemma. The lemma states that if *X* is a stochastic process satisfying $dX_t=\mu_t dt+\sigma_t dW_t$, and *f* is some deterministic, twice continuously differentiable function, then $Y_t=f(t,X_t)$ is also a stochastic function given by

$$dY_t=\left(\sigma_t f_x(t,X_t)\right)dW_t+\left(\mu_t f_t(t,X_t)+\frac{\sigma_t^2}{2}f_{xx}(t,X_t)\right)dt \qquad (5.3)$$

The lemma is used primarily to generate SDEs from a functional expression for a process. For the exponential Brownian motion process

$$X_t=\exp(\mu t+\sigma W_t)$$

let

$$Y_t=\mu t+\sigma W_t$$

and let *g* be the exponential function $g(t,x)=e^x$. Then Y_t can be differentiated to obtain

$$dY_t=\mu dt+\sigma dW_t.$$

Furthermore $X_t=g(t,Y_t)$ so that Ito's lemma results in

$$dX_t=\sigma g_x(t,Y_t)dW_t+\left(\mu g_t(t,Y_t)+\frac{\sigma^2}{2}g_{xx}(t,Y_t)\right)dt \qquad (5.4)$$

From the exponential function $X_t=g(t,Y_t)=g_x(t,Y_t)=g_{xx}(t,Y_t)$ and $g_t(t,Y_t)=0$ so that Equation 5.4 may be rewritten as $dX_t=X_t\left(\sigma dW_t+\left(\mu+\frac{\sigma^2}{2}\right)dt\right)$. The variable σ is the log-volatility of the process because it is the volatility of $\ln X_t$. It is, however, abbreviated to simply volatility notwithstanding the term's existing definition.

Ito's formula thus provides a tool for obtaining stochastic differentials for functions of random processes. But it can also be applied in the reverse situation, i.e. to convert SDEs to processes. Some SDEs can be solved with Ito but others may prove to be too complex. Surprisingly also, even though Ito's formula was introduced as a tool to deal with stochastic differentials, the lemma is useful in evaluating Ito integrals. The reason for this is that stochastic calculus is different from ordinary calculus where integral and derivative are separately defined and then related by the fundamental theorem of calculus. The differential notation of stochastic calculus is really a short hand for Ito integrals over small time intervals.

5.2 The Change of the Underlying Probability Measure

Ito's lemma is probably the most important tool for manipulating stochastic processes and in particular the manipulation of differentials of Brownian motion. W_t, as defined above, is not strictly Brownian motion *per se*, but rather a Brownian motion process with respect to some *measure* $\mathbb{P}$. The stochastic differential of Equation 5.2 thus describes the behaviour of the process X with respect to the measure $\mathbb{P}$ that makes W_t a Brownian motion. Essentially, the main idea of the change of measure consists of introducing a new probability measure via a so-called density function which is in general not a probability density function.

Brownian motion changes in "easy and pleasant" ways under changes in measure. Baxter and Rennie (Baxter and Rennie, 1996: 63) illustrate that all that measure changes on Brownian motion can do is to change the drift. By extension through the differentials of Brownian motion, mapping of stochastic differentials under $\mathbb{P}$ to stochastic differentials under some other measure $\mathbb{Q}$ is natural and "pleasing". The Cameron-Martin-Girsanov theorem formally states that if W_t is a $\mathbb{P}$-Brownian motion and γ_t is a $\mathcal{F}_T$-previsible process satisfying the bounding condition

$$\mathrm{E}\left[\exp\left(\frac{1}{2}\int_0^T \gamma_t^2 dt\right)\right] < \infty .$$

This is the expectation under the original measure, $\mathbb{P}$. Then there exists a measure $\mathbb{Q}$ such that

(i) $\mathbb{Q}$ is equivalent to $\mathbb{P}$. If $\mathbb{P}$ is absolutely continuous with respect to $\mathbb{Q}$ and $\mathbb{Q}$ is absolutely continuous with respect to $\mathbb{P}$, then $\mathbb{P}$ and $\mathbb{Q}$ are equivalent probability measures.

(ii) $$d\mathbb{Q}/d\mathbb{P} = \exp\left(-\int_0^T \gamma_t dW_t - \frac{1}{2}\int_0^T \gamma_t^2 dt\right)$$

(iii) $\tilde{W}_t = W_t + \int_0^t \gamma_s ds$ is a $\mathbb{Q}$-Brownian motion.

W_t is thus a drifting $\mathbb{Q}$-Brownian motion with drift $-\gamma_t$ at time t. The Cameron-Martin-Girsanov theorem is a powerful tool for eliminating the drift term in a stochastic differential equation.

5.3 Martingale Representation Theorem

Martingale theory classifies observed time series according to the way they "trend". A stochastic process behaves like a martingale if its trajectories display no discernable trends or periodicities. This is the fundamental characteristic of martingales. Formally then, stochastic process M_t is a martingale with respect to measure $\mathbb{P}$ is and only if

(i) $\mathrm{E}(|M_t|) < \infty$ for all t

(ii) $\mathrm{E}(M_t|\mathcal{F}_s) = M_s$ for all $s \leq t$

A martingale is a stochastic process whose expected future value, conditional on the present state and all history, is merely equal to its present value. Its future movements are thus completely unpredictable, given the information set $\mathcal{F}_t$. Suppose that M_t is a $\mathbb{Q}$-martingale process, whose volatility σ_t satisfies the additional condition that it is always non-zero. Then, if N_t is any other $\mathbb{Q}$-martingale, there exists an $\mathcal{F}$-previsible process ϕ such that $\int_0^T \phi_t^2 \sigma_t^2 dt < \infty$ with probability one, N can be written as $N_t = N_0 + \int_0^t \phi_s dM_s$. ϕ will be unique. This implies that if there is a measure $\mathbb{Q}$ under which M_t is a $\mathbb{Q}$-martingale, then any other $\mathbb{Q}$-martingale can be represented in terms of M_t. ϕ_t is the ratio of their respective volatilities.

A martingale is not expected to drift upward or downward. If a process X_t is a $\mathbb{P}$-martingale, then with W_t a $\mathbb{P}$-Brownian motion, there is an $\mathcal{F}$-previsible process ϕ_t such that $X_t = X_0 + \int_0^t \phi_s dW_s$. This is the integral form of the increment $dX_t = \phi_t dW$ which has no drift term. Brownian motion is a martingale.

Stock prices are not completely unpredictable and display clearly recognizable long-and short term "trends". Stock prices are thus likely to be sub- or super-martingales. A process that, on average, increases is called a sub-martingale whereas a process which decreases on average is a super-martingale. The Girsanov theorem discussed in Section 5.2 above enables an arbitrary process to be converted into a martingale. A sample path of a martingale may still contain patterns that *look* like short-lived trends but these up and down trends are completely random and do not have any systematic character. Martingale theory is very rich and provides a stable environment for the analysis of stochastic variables in continuous time, imperative in the modeling of asset price behaviour.

5.3.1 A Financial Model

Ito's lemma, the *Cameron-Martin-Girsanov* theorem and the *Martingale Representation* theorem allow financial models to be constructed. The simplest but most popular model is the Black-Scholes model for the pricing of derivative assets. This model has its roots in Brownian motion. Fischer Black and Myron Scholes received the Nobel Prize in Economics for their breakthrough in this area. By means of an introduction to the Black-Scholes model, a number of technical definitions must be given:

(a) A Portfolio

Suppose that there is a market consisting of one random security and a riskless cash account or bond. A portfolio (ϕ, ψ) is then a pair of processes ϕ_t and ψ_t which describe respectively the number of units of security and of the bond held at time *t*. The processes can take positive or negative values (allowing short selling of the stock or bond). The security component of the portfolio, ϕ should be $\mathcal{F}$-previsible, depending only on information up to time t but not on t itself. Baxter and Rennie (1996) give an intuitive way to think about previsibility in this regard: if ϕ is left-continuous (i.e. ϕ_s tends to ϕ_t as s tends upward to t from below), then it means that ϕ is previsible. However, if ϕ were only right-continuous (ϕ_s tends to ϕ_t only as s tends downwards to t from above), then ϕ need not be previsible.

(b) Self-financing Strategies

The description (ϕ_t, ψ_t) is a dynamic strategy which details the amount of each component to be held in the portfolio at each instant of time t. In particular, financially self-financing strategies play a role in the derivation of the Black-Scholes model. A portfolio is self-financing if and only if the change in its value over an infinitesimal time period depends only on the change in the value of the assets and not on any injection of cash. In differential form, this means that if (ϕ_t, ψ_t) is a portfolio with stock price S_t and bond price B_t, then (ϕ_t, ψ_t) is self–financing if and only if

$$dV_t = \phi_t dS_t + \psi_t dB_t.$$

(c) Replicating Strategy

Keeping with the assumption of a market consisting of a riskless bond B and a risky security S which has volatility σ, suppose also that there is a claim C_T on events up to time T. A replicating strategy for C_T is a self-financing portfolio (ϕ, ψ) such that

$$\int_0^T \sigma_t^2 \phi_t^2 dt < \infty \text{ and } V_T = \phi_T S_T + \psi_T B_T = C_T.$$

The claim C_T gives the value of some derivative at time T. Thus, if there exists a replicating strategy (ϕ_t, ψ_t), then the price of C_T at time t will be $V_t = \phi S_t + \psi_t B_t$. If the price at time t were lower, an arbitrage opportunity will exist: a market player could buy one unit of the derivative at time t and sell ϕ_t units of S and ψ_t units of B against it. This short position in the portfolio (ϕ, ψ) will then be held until maturity of the derivative at time T. Because (ϕ, ψ) is self-financing and the portfolio is worth C_T at time T with certainty, the long derivative position and the short portfolio will cancel out exactly at time T. The profit realized by the incorrect pricing at time t is guaranteed without any market risk. The arbitrage works in exactly the same way but in reverse order if the derivative price is higher than V_t. Replicating strategies (*if* they exist and can be identified) are important because they imply the price of the claim C_T everywhere, and not just at payoff time T.

Figure 5.3 summarises the flow of application of these technical definitions as the first steps in understanding the Black-Scholes Model.

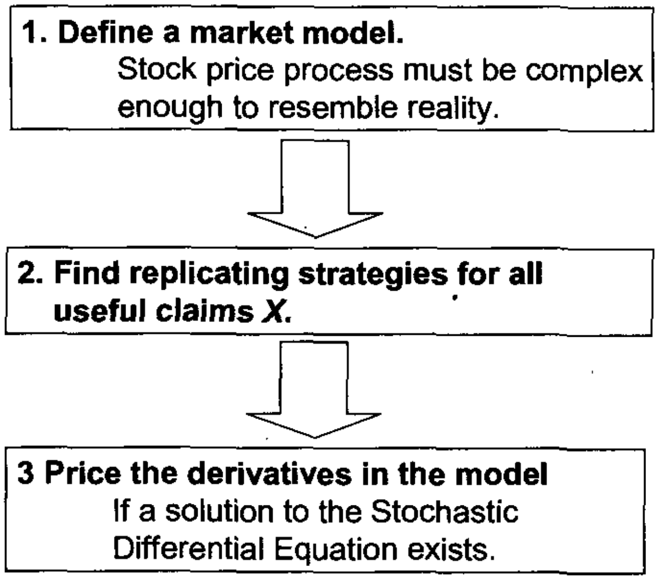


Figure 5.3

A Flow Diagram of Construction Strategies.

5.4 The Black-Scholes Model

Black and Scholes realized that the Brownian motion stock model (discussed in Section 5.1.1) can be changed into a martingale using the Cameron-Martin-Girsanov theorem. Then the martingale representation theorem can be applied to create a replicating strategy for any derivative claim C_T. Ito's lemma lays down the rules for the manipulation of the resultant stochastic integrals.

The basic Black-Scholes model posits the existence of a deterministic r, μ and σ such that the bond price B_t and the stock price follow

$$\begin{aligned} B_t &= \exp(rt) \\ S_t &= S_0 \exp(\mu t + \sigma W_t) \end{aligned}$$

where r is the riskless interest rate, σ is the stock volatility and μ is the drift of the stock. It is assumed that there are no transaction costs and that both instruments are freely and instantaneously tradable at the quoted price. The model for the behaviour of the stock, namely exponential Brownian motion, is simple enough to allow identification of replicating strategies but complex enough to be a plausible match for the real world.

For simplicity, an assumption of zero interest rates is made. For an arbitrary claim C_T, the payoff profile of which is known at time T, a replicating strategy (ϕ_t, ψ_t) is sought. Three steps must be followed in order to find a replicating strategy for C_T.

1. Find a measure $\mathbb{Q}$ under which S_t is a martingale.

A stochastic differential equation (SDE) for S_t must be found before the Cameron-Martin-Girsanov (C-M-R) theorem can be applied and before it is possible to ascertain whether S_t is a $\mathbb{Q}$-martingale for a given $\mathbb{Q}$. The stock price follows an exponential

Brownian motion, $S_t = S_0 \exp(\mu t + \sigma W_t)$. This means that the logarithm of the stock price, $Y_t = \log(S_t) = \mu t + \sigma W_t$ is simple Brownian motion with drift. The SDE for Y_t is $dY_t = \mu dt + \sigma dW_t$. Ito's lemma makes it possible to write down the SDE for $S_t = \exp(Y_t)$ as

$$dS_t = \left(\mu + \frac{\sigma^2}{2}\right) S_t dt + \sigma S_t dW_t. \qquad (5.5)$$

For S_t to be a martingale, there can be no drift in the SDE of Equation (5.5). Letting γ_t be a process with constant value $\gamma = \dfrac{\mu + \dfrac{\sigma^2}{2}}{\sigma}$, the C-M-R theorem says that there is a measure $\mathbb{Q}$ such that $W_t = W_t + \gamma t$ is $\mathbb{Q}$-Brownian motion. Substituting into Equation (5.5), the SDE becomes

$$dS_t = \sigma S_t d\tilde{W}_t \qquad (5.6)$$

which displays no drift so that S_t could be a $\mathbb{Q}$-martingale. Since $\mathrm{E}\left(\exp\left(\frac{1}{2}\int_0^T \sigma_s^2 ds\right)\right) < \infty$ for constant σ, S_t must indeed be a martingale and $\mathbb{Q}$ is the martingale measure for S_t.

2. Form the process $E_t = \mathrm{E}_{\mathbb{Q}}(X \mid \mathcal{F}_t)$

For any claim X depending only on the events up to time T, the process $E_t = \mathrm{E}_{\mathbb{Q}}(X \mid \mathcal{F}_t)$ is a $\mathbb{Q}$-martingale under the constraint that $\mathrm{E}_{\mathbb{Q}}(|X|) < \infty$.

3. Find a previsible process ϕ_t, such that $dE_t = \phi_t dS_t$

Since there is a $\mathbb{Q}$ under which both E_t and S_t martingales, the martingale representation theorem can be invoked: there exists a previsible process ϕ_t which constructs

$E_t = \mathrm{E}_{\mathbb{Q}}(X \mid \mathcal{F}_t)$ out of S_t. The volatility of S_t, σS_t is positive because both σ and S_t are always constant. Thus

$$E_t = \mathrm{E}_{\mathbb{Q}}(X \mid \mathcal{F}_t) = \mathrm{E}_{\mathbb{Q}}(X) + \int_0^t \phi_s dS_s \tag{5.7}$$

and $dE_t = \phi_t dS_t$. It thus follows from the martingale representation that given a $\mathbb{Q}$ that makes S_t a $\mathbb{Q}$-martingale with positive volatility, $dE_t = \phi_t dS_t$ for some ϕ_t.

An applicable replicating strategy, (ϕ_t, ψ_t) for the claim, C_T, is to hold ϕ_t units of stock at time *t* and to hold $\psi_t = E_t - \phi_t S_t$ units of a bond at time *t* since the portfolio must be worth E_t for all *t*. The value of the portfolio at time *t* is $V_t = \phi_t S_t + \psi_t B_t = E_t$ because the bond B_t is always equal to 1. This means that $dV_t = dE_t$. From the martingale representation theorem, $dE_t = \phi_t dS_t$. $dB_t = 0$ so that $dV_t = \phi_t dS_t + \psi_t dB_t$, which is the self-financing condition. The terminal value of the strategy V_t is $E_T = C_T$ which means that there exists a replicating strategy for the claim C_T at all times. There is thus an arbitrage price (or a fair price free of arbitrage) for C_T. In particular, at time zero the arbitrage price of C_T is the value of portfolio (ϕ_t, ψ_t) at time zero which is E_0 or $\mathrm{E}_{\mathbb{Q}}(C_T)$. Thus, the price of the claim C_T is its expected value under the measure that makes the stock process S_t a martingale. It is important to remember that this expectation is not the expectation of the claim with respect to the real measure of S_t, i.e. the measure which makes S_t exponential Brownian motion with drift μ and volatility σ. All the latter expectation gives is a long- term average of the claim's payout. It does not give a price for C_T. Derivative prices for a particular claim can be determined by calculating the expected value of the claim under the martingale measure $\mathbb{Q}$.

Interested readers are referred to Baxter and Rennie (Baxter and Rennie, 1996) for the equivalent three step procedure for finding a replicating strategy when interest rates are not zero. In this case, the measure $\mathbb{Q}$ is not the measure which makes the stock a martingale, but the measure which makes the discounted stock a martingale. An arbitrage price of the claim is the expectation under $\mathbb{Q}$ of the discounted claim.

In summary then, the Black-Scholes model for a continuously tradable stock S_t and bond B_t assumes that, for constant parameters r, μ and σ, stock prices can be represented as $S_t = S_0 \exp(\sigma W_t + \mu t)$ with $B_t = \exp(rt)$. All claims C_T, with maturity time T have associated replicating strategies (ϕ_t, ψ_t). The arbitrage price of such a claim is given by

$$V_t = B_t \mathrm{E}_{\mathbb{Q}}\left(C_t B_T^{-1} \mid \mathcal{F}_t\right) = exp(-r\,(\mathrm{T}-t))\,\mathrm{E}_{\mathbb{Q}}(C_T \mid \mathcal{F}_t)$$

where $\mathbb{Q}$ is the martingale measure for the discounted stock $B_t^{-1} S_t$.

5.5 Call and Put Options

A European call option is the right, but not the obligation, to purchase a unit of stock for a predetermined price, *k* at a particular time, *T*. *k* is referred to as the strike price of the option. The value of this contingent claim is $\max(S_T, 0) = (S_T - k)^+$ at time *T*. In order to value this claim at $t = 0$, the value of a replicating strategy, V_0, at $t = 0$ must be found. This is given by $e^{-rT}\mathrm{E}_{\mathbb{Q}}\left((S_T - k)^+\right)$ where $\mathbb{Q}$ is the martingale measure for $B_t^{-1} S_t$. The value $(S_T - k)^+$ only depends on the stock price at expiry time *T*. In order to calculate the expectation of this claim, the marginal distribution of S_T under $\mathbb{Q}$ must be determined. Considering the process S_t written in terms of $\mathbb{Q}$-Brownian motion $\tilde{W}_t$, $d(\log S_t) = \sigma d\tilde{W}_t + \left(r - \frac{\sigma^2}{2}\right) dt$. Letting the stock price at time zero be represented by s results in $\log S_t = \log s + \sigma \tilde{W}_t + \left(r - \frac{\sigma^2}{2}\right) t$ and thus $S_t = s \times \exp\left(\sigma \tilde{W}_t + \left(r - \frac{\sigma^2}{2}\right) t\right)$. The marginal distribution of S_T is given by *s* times the exponential of a normal distribution with mean $\left(r-\frac{\sigma^2}{2}\right)T$ and variance σ^2T. Define $Z\sim N\left(-\frac{\sigma^2}{2}T,\sigma^2T\right)$ so that $S_T=s\times e^{(Z+rT)}$. Then the claim can be written as the expectation

$$e^{-rT}\mathrm{E}\left(\left(se^{(Z+rT)}-k\right)^{+}\right)=\frac{1}{\sqrt{2\pi\sigma^2T}}\int_{\log\left(\frac{k}{s}\right)-rT}^{\infty}\left(se^{x}-ke^{-rT}\right)\exp\left(-\frac{\left(x+\frac{1}{2}\sigma^2T\right)^2}{2\sigma^2T}\right)dx.$$

Per usual $\Phi(x)=(2\pi)^{-\frac{1}{2}}\int_{-\infty}^{x}\exp\left(-\frac{y^2}{2}\right)dy$ is the probability that a variable which is $N(0,1)$ distributed has a value less than x. Then

$$V_0=V(s,T)=s\Phi\left(\frac{\log\frac{s}{k}+\left(r+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)-ke^{-rT}\Phi\left(\frac{\log\frac{s}{k}+\left(r-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

Which is the Black-Scholes formula for the value of an European call option. Interested readers can refer to Mikosch (Mikosch, 1998) for a derivation of the price of a put option. Naturally, put-call parity may also be used.

5.6 Dividends

In reality, companies often pay dividends and stock cannot always be treated as a pure asset. If the stock price S_t follows the Black-Scholes model $S_t=S_0\exp(\mu t+\sigma W_t)$ and B_t is a cash bond $B_t=\exp(rt)$ for a fixed rate *r*, then a (continuous) dividend payment of $\delta S_t dt$ is made in the time interval *dt* for a constant of proportionality, δ. A stock bought for S_0 at time 0 is worth more than just the price of the stock at the time of sale, *t*. Total accumulated dividends must also be considered. Under the Black-Scholes model, dividend accumulation will depend on the price path the stock has followed up to time *t*. The process S_t must thus be translated to determine a new process involving cash payments and which is tradable. Baxter and Rennie (Baxter and Rennie, 1996) propose a portfolio which initially consists of one unit of stock costing S_0. Each time a dividend is paid, the cash amount received is immediately re-invested to purchase more of the same stock. The infinitesimal payout is $\delta S_t dt$ per unit of stock, which will purchase δdt more units of stock. At time t, the number of stock units held in the portfolio will be $\exp(\delta t)$ and the portfolio will be worth $\tilde{S}_t = S_0 \exp\left((\mu+\delta)t+\sigma W_t\right)$.

Under the assumption that dividend payments are a constant proportion of the stock price, a portfolio of stock and a bond, (ϕ_t, ψ_t) can be rewritten as a portfolio of the *re-invested* stock and a bond, $\left(\tilde{\phi}_t, \psi_t\right)$, where $\tilde{\phi}_t = e^{-\delta t}\phi_t$ and "adjusted" value $V_t = \phi_t S_t + \psi_t B_t = \tilde{\phi}_t \tilde{S}_t + \psi_t B_t$. The self-financing equation retains the familiar form $dV_t = \tilde{\phi}_t d\tilde{S}_t + \psi_t dB_t$. Translating the discounted reinvested stock $\tilde{Z}_t = B_t^{-1}\tilde{S}_t$ into a martingale, $\tilde{Z}_t$ has stochastic differential equation of the form

$$d\tilde{Z}_t = \tilde{Z}_t\left(\left(\mu+\delta+\frac{\sigma^2}{2}-r\right)dt+\sigma dW_t\right)$$

The measure $\mathbb{Q}$ under which $\tilde{W}_t = W_t + \frac{\left(\mu+\delta+\frac{1}{2}\sigma^2-r\right)t}{\sigma}$ is Brownian motion, is such that $d\tilde{Z}_t = \sigma\tilde{Z}_t d\tilde{W}_t$. The martingale representation theorem is used to construct a strategy to hedge a claim *X*, maturing at date *T*. I.e. there exists a previsible process $\tilde{\phi}_t$ such that $E_t = \mathrm{E}_{\mathbb{Q}}\,(B_T^{-1}X \mid \mathcal{F}_t) = \mathrm{E}_{\mathbb{Q}}\left(B_T^{-1}X\right)+\int_0^t \tilde{\phi}_s d\tilde{Z}_s$. Holding ϕ_t units of the *translated* asset $\tilde{S}_t$ and $\psi_t = E_t - \tilde{\phi}_t\tilde{Z}_t$ units of the cash bond is equivalent to holding $\phi_t = e^{\delta t}\tilde{\phi}_t$ units of stock S_t and the same ψ_t units of the bond B_t. Consequently, $S_t = S_0 \exp\left(\left(r-\delta-\frac{1}{2}\sigma^2\right)t+\sigma\tilde{W}_t\right)$ is log-normally distributed under the martingale measure $\mathbb{Q}$.

5.7 Justification of the Options Analogy

Given the above discussion of the pricing of financial assets, the question arises as to whether these techniques can be justifiably applied to the valuation of real options for capital budgeting purposes. Financial options were valued on the basis of no-arbitrage market equilibrium, using portfolios of traded securities to replicate the payoff profiles. In capital budgeting, however, projects are not traded assets. Trigeorgis (2001) points out that DCF approaches attempt to determine what an asset or project would be worth *if* it were traded. To do this, a traded twin security (with the same risk characteristics as the project) is identified and *its* expected rate of return is used as the appropriate discount rate for future expected cash flows of the project. The project's covariability with the market is thus estimated from the prices of one or more twin securities using the CAPM. All that is needed for the options analogy to hold is the *existence of sufficient substitutes*. Assuming the existence of a substitute security, returns on a real option can, in principle, be replicated by purchasing shares of the twin security and borrowing at the risk free rate. The fair value of the option on a non-traded project must then be equal to the no-arbitrage value of the option on its twin security. Non-traded real assets may, however, earn a rate of return below that of comparable traded securities to compensate for lack of liquidity and transparency.

5.8 Limitations of the Options Analogy

Although the analogy between real and financial options is close, it is not exact. Trigeorgis (Trigeorgis, 2001) lists the main differences under the following categories:

1. Ownership and Competition

A standard call option gives the holder an exclusive right to decide when and whether to exercise the option – there is no competition for the underlying asset. Some real options also provide holders with exclusive rights to exercise, uninhibited by competitive threats. All investment opportunities with high barriers to competitor entry are examples of real proprietary options; patents for developing a product that has no close substitutes, unique know-how of a technological process etc. However, certain types of investment opportunities may be jointly held by more than a single competitor. These real options are shared since they can be exercised by any one of a number of participants in the industry. Examples include the opportunity to introduce a new product unprotected by patents, licenses and guarantees or the opportunity to penetrate a new geographical market without barriers to competitive entry. The case study of this thesis focuses further on the issue of exclusive and non-exclusive ownership.

2. Non-tradability and Preemption

In efficient markets, both exchange-traded and over-the-counter (OTC) options can be freely traded with minimal costs. Real options and their underlying investment projects cannot be traded in such a manner. Although selected real options (such as investment opportunities related to patents and licensing agreements) may be traded, both the opportunity and financial costs may be prohibitive. Instead, real options may be left to expire worthless if a project is abandoned before the end of its life. Certain real options cannot be sold at all, especially if they are shared. For example, a company holding a real option shared by competitors cannot easily avoid anticipated losses in its value (often due to competitive entry) by simply selling the option. The only viable protection against such value loss is early investment on the company's part – it must preempt competitors by exercising its rights first. If the company anticipates an increase in demand (and hence increased competitor activity) it may immediately expand its production capacity in order to preempt the competitor. In the absence of such competition, however, it might be preferable to wait until the arrival of new information resolves uncertainty in product demand.

3. Strategic Interdependencies

Analogous to a compound financial option where the exercise of one option leads to another later maturing option kicking in, when certain types of real options are exercised the result is another discretionary investment opportunity. Their payoff is, in essence, another option. For example, an investment in R&D, a lease on an undeveloped tract of land with potential natural resource reserves, or the acquisition of an unrelated business entity is sometimes undertaken primarily for the new opportunity that may be realized, rather than for the sake of immediate cash flows. This type of real option is a compound option.

If the necessary adjustments can be made for ownership and competition issues, non-tradability, preemption and strategic interdependencies in real options if and when they exist, there appears to be no conceptual problem(s) in valuing real options in exactly the same way financial options are valued.

5.9 Real Options and the Black-Scholes Formulation

Armed with the Black-Scholes option(s) pricing methodology discussed above, the challenge now lies in constructing a synthetic option that captures the real strategic opportunities in a project. *The first step is to identify the real option. The second step is to establish a correspondence between the project's characteristics and the five variables (S, X, σ, t and r) that determine the value of a financial option on a stock.* Timothy Luehrman of the American Graduate School of International Management points out that the option we synthesize in this way is not a perfect substitute for the real opportunity, but it is indeed informative in its design and application. (Harvard Business Review, 1998) The third and last step is to *price the option.*

5.9.1 The Correspondence between Financial and Real Options

- **The Underlying Asset and the Spot Price**

  A real call option confers upon the holder the right, but not the obligation to either take an action (i.e. defer an investment, expand / contract production, abandon a project etc) *or* to acquire the operating assets of a business. These actions or acquisitions represent the underlying assets of a real option. In the case of stock options, the stock price, *S*, is simply the market's estimate of the present value of all future cash flows. The real option(s) equivalent is thus the present value of cash inflows expected from the investment opportunity on which the option is purchased. The holder of a financial option on a single stock cannot readily influence the stock price in order to move it closer or further away from the strike price of the option. Even though the option's value is derived from the company's value, decisions about the option(s) will not change overall company value. The sheer size and liquidity of stock markets in general prevent this. With real options, however, the present value of expected cash inflows can be manipulated. This is simply due to the fact that many business decisions have a logical structure which is very similar to a financial option. For example, management of a company may decide to build a factory in order to produce a new brand of product. The costs involved in this project may be known and stable but demand for the brand itself will vary in a random way. The company effectively has a call option on *whether* and *when* to commence building the factory. The decision about this real option will alter the company's resources and value. Once production of the new brand commences, management has a certain degree of control over the "underlying asset" – they are able to increase revenues by, for example, raising prices or producing more of the product. If market reception of the brand is less than enthusiastic, company resources and value will be affected in an adverse manner. However, management will still have a degree of control because they may cut production or, if necessary, close the factory.

- **The Strike Price**

  Spending money to exploit a business opportunity is analogous to exercising a real option. It is the flexibility inherent in the opportunity (i.e. invest now or defer, alter the scale of operation, expand or contract construction and/or investment, shut operations down, abandon project, etc.) which gives the investment option-like characteristics. The amount spent to acquire such flexibility (i.e. the investment cost or capital expenditure) corresponds to the exercise or strike price, X. The net present value of the underlying real asset is thus the present value of the asset's future income stream (net revenues less operating costs), less the exercise price.

  $$NPV = (\textit{present value of assets}) - (\textit{capital expenditure})$$

  Management's NPV calculations already contain the value of S and X necessary for real option valuation. An increase in the exercise price implies that flexibility becomes more expensive, thereby reducing the value of a real call option and increasing the value of the real put.

- **Time to Expiry of the Option**

  The length of time an investor can defer the investment decision without loosing the business opportunity represents the real option's time to expiry, t. It will depend on technology (life cycle of a product), competitive advantage (intensity of competition) and contracts (patents, leases and licenses). Option pricing recognizes that there is value in the right to postpone an investment decision. Payment later is preferable to payment sooner, all else being equal, since the time value of money may be earned on deferred expenditure. In addition, deferment allows the investor to wait for the arrival of new information before making the final decision. If the value of the operating assets an investor intends to obtain increases over time, the investment opportunity is not lost since the option is simply exercised. If value decreases, the investor avoids a poor investment by not exercising the option. Traditional NPV misses the extra value associated with deferral because it assumes that the investment decision is immediate. Options, on the other hand, are all about choice and option pricing technology provides a way to quantify the value of the choice to postpone a real or financial decision. Long-dated options are more valuable than short-term ones since there is more "time" for the holder to benefit from future upside uncertainties in the underlying asset. Some financial options, however, have no fixed maturity date. These are referred to as perpetual options. Except for options to develop land, perpetual options on real assets are rare. Most real investment opportunities are short lived due to competitor action, the speed of innovation and technical change.

Real options can be American or European by nature and sometimes even a mix of the two. Howell (Howell , 2001) notes that investment in the distribution of a (potentially profitable) drug can only commence (by law) once clinical drug trials have been completed. So even though the fixed time delay, before exercise of a real option to invest, gives this option a European flavour, the holder probably also has the flexibility to wait a while and consider the market and strategic implications of the distribution of the drug before making a final decision whether to invest (usually a substantial amount of money) or not. This type of real option thus has both European and deferred American option-type characteristics.

- **Volatility**

The riskiness of a (real) project is reflected by the uncertainty about the value of the project's future cash flows and corresponds to the standard deviation of financial stock returns σ. Having the flexibility to defer, stage, alter the operating scale, abandon or grow projects becomes that much more valuable when the project's outcome is highly uncertain. This is a crucial difference between NPV analysis and option pricing methodology in valuing capital budgeting and investment projects. The more uncertainty surrounding a project's future cash flows, the higher the discount rate used in Equation (2.2).

The higher the discount rate, the lower the NPV of the project and the more likely the project is to be rejected. NPV analysis thus assumes that uncertainty has a negative effect on project value. However, the greater the uncertainty surrounding expected future returns, the higher the chances for realising above average profits *or* losses. An option, with its asymmetric nature, has exposure only to the upside with downside protection. This effectively allows the holder to exercise at a profitable time while backing out when things go wrong. An option (financial or real) will thus be worth more if volatility is expected to be high and less if volatility is low – exactly the opposite to the way NPV analysis handles volatility.

Figures 5.4 (a) and (b) illustrate how an increase in volatility increases the value of an option. In Figure 5.4(a) two possible distributions of outcomes overlay the option profile. When volatility is high, the distribution of outcomes is wider, creating more outcomes with positive payoff. Higher volatility naturally also leads to a higher chance of negative outcomes, but losses are limited by the asymmetric structure of the option. Figure 5.4(b) demonstrates how this one-sided effect increases the value of the option.

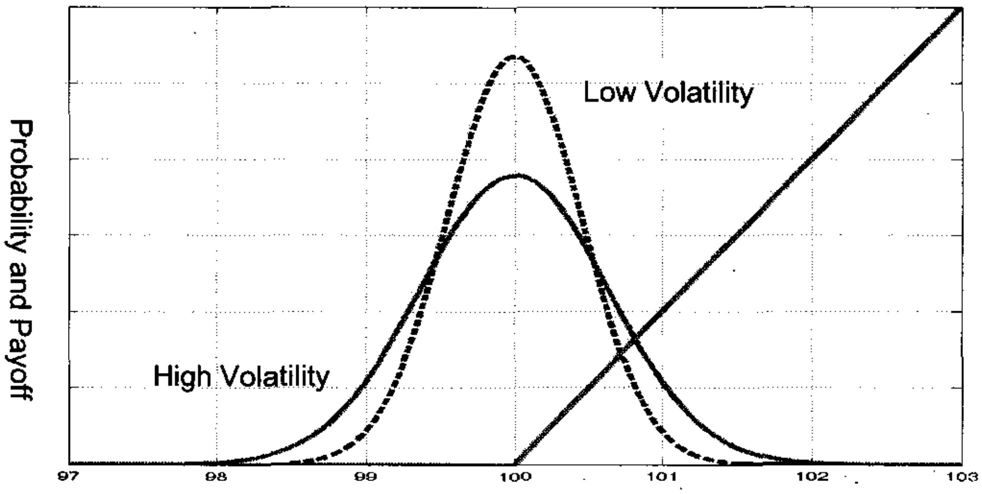


Figure 5.4 (a)

Volatility and Option Value. *Source*: Amram and Kulatilaka (1999)

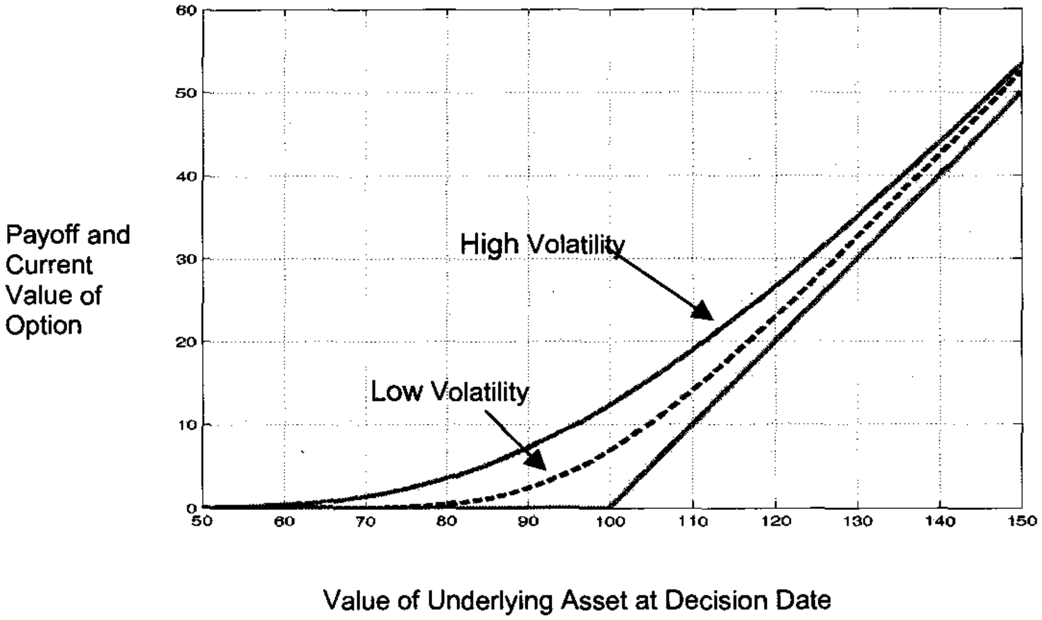


Figure 5.4 (b)

The Payoff Function and the Current Value of the Option.

Source: Amram & Kulatilaka, (1999).

Estimating the volatility of the underlying real asset is challenging when no historical information on the project is available. This hurdle is overcome through the use of approximations. A Monte Carlo simulation of many possible future values of project cash flows may be generated. These values are part of a distribution of outcomes. The expected value of the project cash flows is represented by the mean of the distribution and its standard deviation, measuring the deviation of all generated outcomes about the mean, is an estimate of the project's volatility. Alternatively, the volatility of another, closely related, traded asset or portfolio of assets may be used to span or track the uncertainty in the real asset. This asset is referred to as a *spanning asset*, a *twin security* or a *proxy*. The appropriate use of spanning assets is discussed in detail by Dixit and

Pindyck (Dixit and Pindyck, 1994) and Amram and Kulatilaka (Amram and Kulatilaka, 1999). The case study, Chapters 6 and 7 of this thesis, demonstrates the power of spanning assets in real options pricing. Generally, the challenge of estimating project volatility should be viewed in a positive light. Identifying appropriate twin securities as proxies for unlisted and non-traded project value, forces both management and analysts to think laterally about the uncertainty surrounding a project; which factors drive that uncertainty and how it changes over time.

- **Time Value of Money**

  The time value of money is given by the market risk free rate of return, r_f for both financial and real options. The exact specification of these input values will be unique for each individual project under consideration.

- **Dividends**

  The real options equivalent of dividend paying stock is a profit-generating investment. As soon as investment in a project commences, profits (may) accumulate. Profit will, however, be uncertain and vary according to a random walk. This is equivalent to receiving dividends from an underlying stock asset a soon as an American option to invest is exercised. If investment is delayed, there is thus a tradeoff between gaining more information and losing a profit opportunity. By waiting for the arrival of new information, unfavourable developments may be avoided before the option to invest expires. However, there is a cost to delaying investment in a project, once the net present value turns positive. Rights, licenses and guarantees may, for example, expire after a fixed period of time. Since these were the source of positive NPV, excess profit will begin to disappear as competitors step in. Each period of delayed investment translates into an equivalent loss of cash flows. In the case of a financial American-style call option on a dividend- paying stock, this resembles the trade-off between making capital gains from holding the option,.and making dividend gains from holding the stock through early exercise.

Just like financial options, the intrinsic value of a real option is the maximum of zero and value it would have if it were exercised immediately. The extra value which a real option has until expiry is referred to the time value of the option. The longer the life of an option generally, the greater the time value will be. Time value of a real option is the greatest when the option is at-the-money. When a project has an expected NPV of zero, it is on the borderline of being profitable or not. An option holder derives maximum value for being able to delay an investment at this point. Before expiry, the total value of the real option is the sum of its intrinsic and time value.

Table 5.1 illustrates this mapping of an investment opportunity onto a financial call option. Application of the Black-Scholes pricing methodology to real options not only offers a means of quantifying flexibility and managing uncertainty, but it also identifies the crucial issues for the maximization of option value. In highlighting a project's sensitivity to each variable in its unique contribution to overall project value, the Black-Scholes model is considerably more insightful that simple discounted cash flows analysis.

Investment Opportunity	Call Option	Option Model Variable
Present value of a project's operating assets to be aquired.	Stock price.	S
Expenditure required to acquire project assets.	Exercise price.	X
Length of time decision may be deferred.	Time to expiry.	t
Time value of money.	Risk-free rate of return.	r_f
Riskiness of project assets.	Variance of stock returns	σ^2
Profit generated from an investment.	Dividend.	q

Table 5.1

Linking Real Options to the Black-Scholes Model. *Source*: Luehrman, 1998.

5.10 Summary

Although the Black-Scholes model is a simplification of the actual price dynamics of financial securities, it is intuitive enough for practitioners to be able to use with ease. Its beauty, and, at the same time its disadvantage, is that it requires only a small number of parameters to describe the basic aspects of asset prices and the prices of derivative securities based on those assets. But option valuation does not end with the Black-Scholes model. For complex cases, numerical solutions are used. These methods calculate a large area of the valuation surface and follow backward induction calculations which commence at some known payoff function at expiry of the option. The next chapter discusses the most popular numerical technique for (real and financial) option valuation.

CHAPTER 6
NUMERICAL METHODS

"Never again will a single story be told as though it is the only one"

John Berger.

This chapter introduces a numerical solution to real option pricing and focuses in particular on the multiplicative binomial process. This is a very popular technique for more complicated real options problems and thus warrants an introductory discussion. In addition to the valuation using the Black-Scholes model, Peppadew's real assets are also valued using a binomial tree. The latter valuation is attached as an Appendix for perusal by interested readers. The tree method comes to the fore again when a funding strategy for Peppadew is engineered in Chapter 9.

6.1 Numerical Methods of Option Valuation

Numerical methods are used for valuing real options when analytical solutions are not available. To implement numerical methods for solving the Black-Scholes PDE (with given boundary conditions), the Black-Scholes equation must first be linearised using Taylor's theorem. This means that the curved solution surface is approximated by a piece-wise linear problem. The linear problem is then evaluated over short discrete increments of asset price and time. Numerical integration and finite difference methods follow this route. However, when it is difficult to write down a set of partial differential equations describing the underlying process, the stochastic behaviour of the underlying asset may be simulated directly through different approximations. Various lattice approaches and Monte Carlo simulation are examples hereof. Binomial lattices (easily extended to the trinomial case) and finite difference methods will now be discussed since they are most commonly used for real option analysis.

6.2 The Multiplicative Binomial Process

For the Black-Scholes approximation of Chapter 5, the underlying stock price *S* was described by geometric Brownian motion with drift. This diffusion process can be replicated by a multiplicative binomial process in discrete time. A binomial model of possible movements of *S* must then converge to the continuous log-normal distribution underlying the process of Equation 5.1 as the number of time steps between trades tends to infinity. Mathematically, convergence can be achieved in a number of ways, two of which are popularly used in practice. Either

(1) The risk neutral probabilities of a rise or a fall in the price of *S* can be taken as equal. Formulae are then derived which give unequal proportional price movements or,

(2) The price of *S* can be made to move up or down by the same proportion, in which case formulae are derived that give unequal risk-neutral probabilities of those price movements.

With a sufficiently large number of time steps, these two alternative models converge to deliver a single, correct option value. However, they diverge significantly when using a small number of time steps. The model of unequal probabilities (alternative number two above) was first introduced by Cox, Ross and Rubenstein in 1979. The same principles of arbitrage pricing used for the Black-Scholes approximation apply to the binomial model. It will function in a risk-neutral world where the option value derived will be the same as in the true, risky world. (I.e. an option can be valued using only the risk-free rate). The underlying asset's stochastic process is approximated using basic algebra and probability theory. Once the value of the underlying asset has been modeled, the option may be valued starting at the end of the tree and working backwards in a recursive manner.

For the discrete time process, the underlying stock price, *S*, at the beginning of a given period may increase by a multiplicative factor, *u*, with probability *p* to *Su* or decrease (by a factor *d*) with probability $\left(1-p\right)$ to *Sd* at the end of the period. The proportional upward and downward movements are the "step lengths" in the random walk. The multiples *u* and *d* are chosen such that the price reached after an upward movement followed by a downward movement, *Sud*, is the same as that reached after a downward movement followed by an upward movements, *Sdu*. In the CRR model, this means that $d=\frac{1}{u}$. The tree is then said to recombine, reducing the number of nodes in the overall tree. The present node in the tree has no memory of the particular sequence of upward and downward jumps by which it has been reached. Such trees are computationally elegant to work with even though stock prices are not trees. It is sufficient to remember that there is more than one path to the final nodes.

The stock price *S* follows a stationary multiplicative binomial process described by

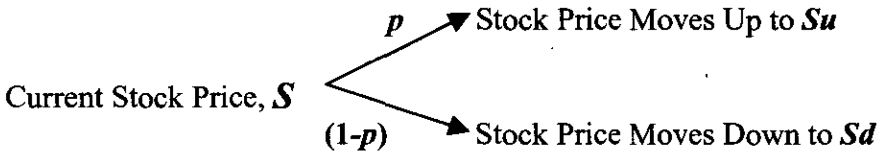


Figure 6.1 illustrates how the binomial model leads to a lattice of stock prices that mimics the underlying stochastic process. This is commonly known as a binomial tree with successive moves or outcome branches called nodes.

Suppose that the life of the option on a non-dividend paying stock is divided into *n* subintervals of length Δt. At time $i\Delta t$, $(i+1)$ stock prices Su^jd^{i-j} for $0\leq i\leq n$ and $0\leq j\leq i$ originate. The second node, $2\Delta t$, in Figure 6.1, has three possible values: $Suu=Su^2$, $Sud=Sdu$ or $Sdd=Sd^2$. The binomial variable in this application is the number of up moves. The sequence $uu=u^2$ has two up moves (successes) out of two jumps. There is only one way to realise two successes in two trails (top most node) and this occurs with probability p^2. The sequence $dd=d^2$ has zero successes with probability $(1-p)^2$.

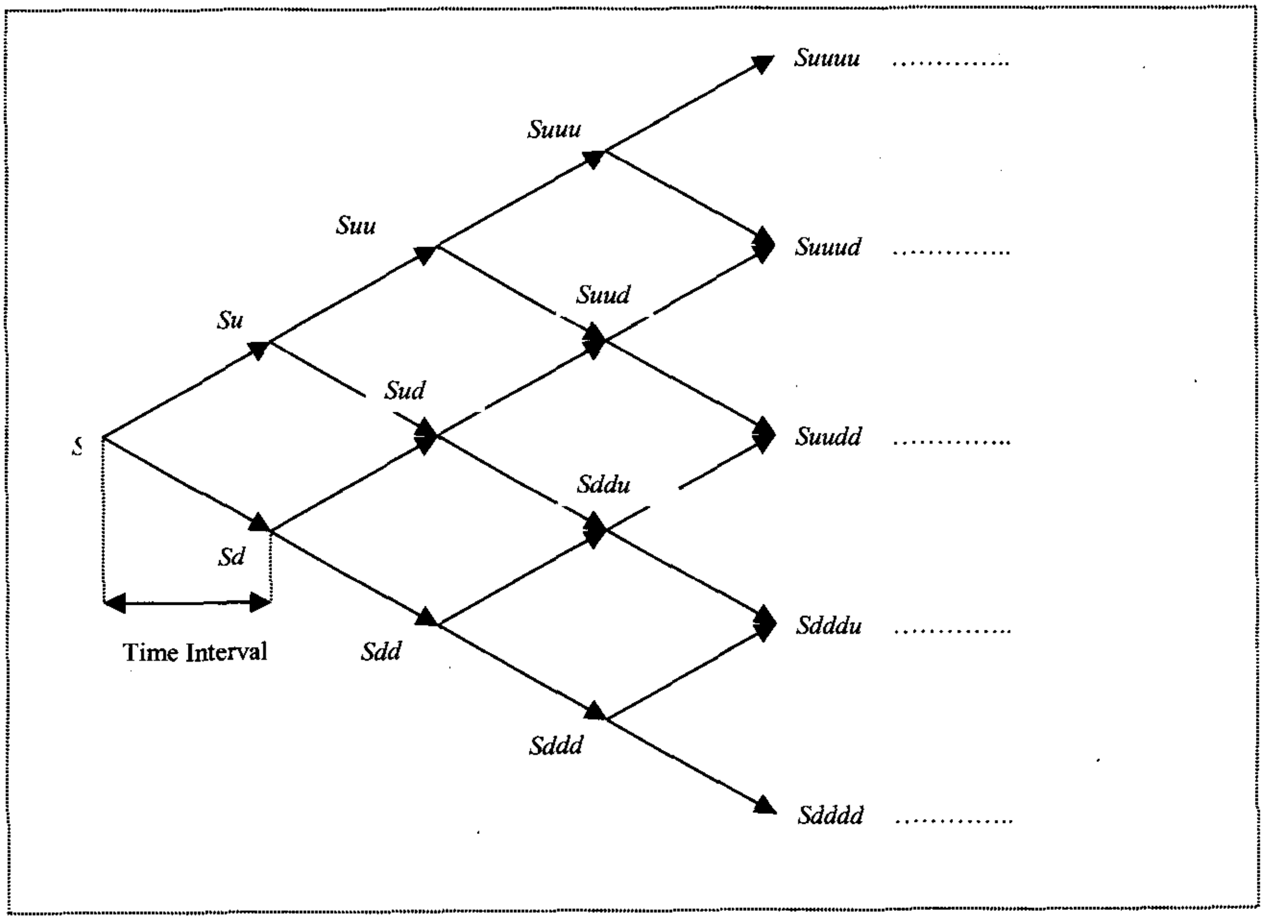


Figure 6.1
Stock Price Movement illustrated as a Binomial Tree.

However, the sequence *ud*=*du* has one up move out of two jumps. The number of ways to realise one success in two trials is given by $\binom{2}{1} = \frac{2!}{1!(2-1)!} = 2$. The graphical representation of the tree confirms this. The probability of achieving one success in two moves is equal to $p(1-p)$.

For all nodes in the tree, the probability that the stock will take j independent upward jumps in i steps, each with risk neutral probability p is given by

$$P(j) = \frac{i!}{j!(i-j)!} p^j (1-p)^{i-j} \qquad (6.2)$$

Note that the stock price itself is not assumed to be a binomial random variable. Rather, it is a function of a binomial variable, of u, d and the initial stock price, S, in a short period of time. The ability to choose the value of these parameters allows for various distributional approximations of stock price behaviour. The parameters u, d and p are chosen in such a way that the expected return from the stock is $\mu\Delta t$ and the variance of return is $\sigma^2\Delta t$, in a small time interval Δt. There is no single correct formula for u, d and p. The popular CRR model, however, suggests

$$u = e^{\sigma\sqrt{\Delta t}}, \qquad d = \frac{1}{u}, \qquad u > 1, \quad d < 1$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{6.3}$$

Using the same notation throughout, r represents the risk-free rate of return. The key to calculating u and d is the standard deviation or volatility of the underlying asset, S. Although it is unknown exactly how asset prices will change, standard deviation gives a good idea of how volatile they will be. As in the continuous time random walk, variance must be linear in time since successive increments of time must produce independent variations in S. That is, S must follow a random walk in order to model prices in a perfectly competitive market. The standard deviation of proportional changes in stock prices grows larger over time, but at a decreasing rate. As an approximation then, the volatility of a stock price is equal to the standard deviation of the continuously compounded return provided by the stock in one year. Thus, the variables u and d represent the continuously compounded rate of return if the stock price moves up and down respectively.

In the continuous-time limit, as the number of periods, n, approaches infinity, the binomial distribution function in Equation 6.2 will converge to the standard normal distribution function. It proves simple to ensure that the discrete binomial distribution, which has only two points, will have the same mean and variance as a chosen normal distribution with mean μ and variance σ^2: only one point at $\mu+\sigma$ and one at $\mu-\sigma$ is needed. The mean of these two points is clearly μ, as required. As for their variance, each of the two points deviates from μ by σ, so that each of their squared deviations from μ is σ^2. This is also the mean of their squared deviations, which is the variance of the two points, as required.

The variance of the binomial model should imitate the variance of the continuous random walk in order to estimate

(1) How wide a range of likely end states it is worth considering in the model, and

(2) How fine the subdivision of the possible value differences ought to be included within this range.

The binomial model thus utilizes the variance to determine the size (and not the probability) of the possible up and down movements of *S* (viewed as multiplications of *S*). Because of the possibility of delta hedging over infinitesimal periods of time, the model becomes indifferent to the actual probabilities that *S* will move either upward or downward at each step; hence the indifference to drift or trend.

6.3 Option Pricing using a Binomial Lattice

Options are evaluated by starting at the end of the tree and working backwards. The mechanics can be visualized as a folding back through the life of the option until the current price can be determined. Let V_{ij} be the value of an option at node (i, j) at time $i\Delta t$ when the stock price is $Su^j d^{i-j}$ for $0 \le i \le n$ and $0 \le j \le i$. As defined above, *j* denotes the number of independent upward jumps the stock will make in *i* steps. Then, at expiration,

$$V_{nj} = \max\left[Su^j d^{n-j} - X, 0\right] \quad \text{for a } \textbf{call option}$$

$$(6.4)$$

$$V_{nj} = \max\left[X - Su^j d^{n-j}, 0\right] \quad \text{for a } \textbf{put option}, \quad 0 \le j \le n.$$

The "probability" of jumping from node (i, j) at time $i\Delta t$ to node $(i+1, j+1)$ at time $(i+1)\Delta t$ is denoted by p and calculated from Equation (6.3). In the same manner, (1- p) is the "probability" of a down jump from node (i, j) at time $i\Delta t$ to node $(i+1, j)$ at time $(i+1)\Delta t$. The literature takes care to stress the fact that if p and (1- p) are interpreted as "risk-neutral probabilities", it is important to keep in mind that these "probabilities" will in general have no relationship to the actual probabilities of upward and downward movements of S. They are simply the probabilities that the option's payoff, after up and down movements of S, would have to have if risk neutral investors (who do not hedge but act on probabilities) are assumed to reach the same valuations as risk-averse investors (who do hedge, but ignore probabilities). Many authors thus prefer to replace the term "risk-neutral probability" with "risk-eliminating weighting".

The value of V_{ij} at node (i, j) is then calculated as the present value of the expected option price in time $(i+1)\Delta t$. That is, the summation of all possible option values at the ultimate node, multiplied by the risk-eliminating weightings and discounted at the (continuously compounded) risk free rate of interest.

$$V_{ij} = e^{-r\Delta t}\left[pV_{i+1,j+1} + (1-p)V_{i+1,j}\right] \tag{6.5}$$

for $\quad 0 \le i \le n-1$ and $0 \le i \le j$.

For American style options, where early exercise is a possibility, the option value at each node is compared with the option's intrinsic value. Per definition, the intrinsic value of an option is the value realized if the option should be exercised immediately. In this event,

$$V_{ij} = \max\left\{X - Su^j d^{i-j}, e^{-r\Delta t}\left[pV_{i+1,j+1} + (1-p)V_{i+1,j}\right]\right\} \tag{6.6}$$

for a put option, since it is never optimal to exercise an American call option early.

The binomial tree approach is invaluable for the pricing of real options since its basic structure allows for testing of contingencies at each node. As such, it is particularly useful in analyzing complex sequential investment decisions when uncertainty is resolved at distinct, discrete points in time. The value of the option can be determined in a relatively straightforward way and the backward calculation captures the effect of early exercise possibilities. At subsequent nodes throughout the tree, managers and investors are afforded a means of continual re-evaluation as the underlying asset evolves and related options values are calculated. The flexibility to exercise an option at any time in order to lock in profit or prevent disaster is a valuable asset in any capital budgeting project.

There is no set rule as to which of the above (or any other) option pricing methodology ought to be applied when valuing real options. The type of real asset and applicable option under consideration, as well as the quality of the data available will determine which methodology would be best suited. This is illustrated in the case study of Chapter 8.

Chapter 9 illustrates the use of a *Binomial Interest Rate Tree* for the purposes of pricing a fixed income security. In much the same way as the tree discussed above is a discrete-time representation of the process followed by a stock price, an interest rate tree is a discrete-time representation of the stochastic process of interest rates. The main difference between interest rate trees and stock price trees lies in the discounting process. In a stock price tree, the discount rate is usually assumed to be the same at each node while the discount factor varies from node to node in an interest rate tree.

6.4 Summary

There are three basic numerical methods for option valuation – Monte Carlo simulation, tree or lattice approaches and finite difference methods – all rooted in the random walk theory. Of these, the tree and finite difference methods are applicable to common real option valuation problems. Trees model the possible future prices of an asset upon which there is an option. Starting from a single present price, it is usually assumed that the asset's price can change in a series of equal time steps. At each step, the price can move only to a set number of new prices; commonly two or three. Option values throughout the tree can be found using the concepts of discounting and probability.

CHAPTER 7

THE PITFALLS OF REAL OPTIONS

"….noise is a fact of life. It does not make valuation less useful...
So take advantage of uncertainty to create value."

Aswath Damodaran

Moving away from traditional analysis to embrace a new train of thought always involves the risk of misappropriation. This chapter aims to highlight some of the risks involved in real options analysis.

7.1 The Pitfalls of Real Options Analysis

The academic literature is full of examples of real options analysis that generate powerful and important but nevertheless counter-intuitive recommendations whenever the future is unforecastable. Refer specifically to Howell *et al* (Howell *et al*, 2001), Trigeorgis (Trigeorgis, 2002) and Damodaran (Damodaran, 2001). Like any new technique, it is sure to be over-hyped, misunderstood and misapplied. It is thus vital to be aware of when real option analysis can and should be used and under which conditions it would be futile.

The most fundamental assumption of real options analysis is that the relevant uncertainties follow a random walk. In general, the less perfectly competitive markets are, the less likely it is that market values will evolve randomly and unaffected by business decisions. Markets should be large and efficient enough to absorb all trading activity smoothly. In the extreme, where the activities of a small number of players can dominate financial markets for extended periods of time (oligopoly), competitive game theory may supplement real options analysis or even replace it altogether. There are special conditions under which game theory prescribes that competitors should deliberately take random decisions, and there are other conditions in which competitive economics derives values very similar to those of standard option theory.

Overestimation of the option due to incorrect estimates of the volatility of the real underlying asset must also be guarded against. This type of mistake can be avoided by fully using market information to arrive at input values for the real options analysis. It is vital to make many sanity checks as the analysis progresses to ensure that the final result is economically rational. Most experienced analysts and business managers will quickly detect a flaw in logic – whether in parameter specification, application or calculation of the real options.

Damodaran (Damodaran, 2001) gives three basic guidelines to be kept in mind when assessing whether an investment creates valuable options that need to be analyzed and valued:

1. The first investment must be a pre- requisite for later investment / expansion.

If this is not so, the question arises as to how necessary the first investment really is. The analysis of the value of a patent or the value of an undeveloped oil reserve as a real option is given as an example. A company cannot generate patents without investing in research. It cannot get rights to an undeveloped oil reserve without bidding for it at a government auction or buying it from another oil company. In these two cases, the initial investment (spending on R&D, bidding at the auction) is required for the firm to have the second option to grow and expand. But investment in a store with an option to expand into the offshore market later, is a different issue altogether. The initial store investment will provide the investor with information about market potential, without which he/she is unwilling to expand into the larger market. However, unlike in the case of the patent and undeveloped reserves, the initial investment is not a pre-requisite for the second investment, though it might be viewed as such. The connection gets even weaker when considering the acquisition of one firm by another with the option to capture larger or related market share. For example, acquiring an internet service provider in order to have a foothold in the internet retailing market, or buying a Brazilian brewery to preserve the option to enter the Brazilian beer market. In such cases, the initial investment is not necessary at all and could cost a lot of money without any derived benefit. I.e. the Brazilian beer market can be entered with a brewer's existing product without incurring the additional costs of actually purchasing another brewery.

2. *A company must have an exclusive right to the later investment/expansion.*

If it does not, the initial investment must at least provide the firm with significant competitive advantages on subsequent investments. The value of the option ultimately derives not from the cash flows generated by the second and subsequent investments, but from the potential for *excess returns* generated by these cash flows. The greater the potential for excess returns on later investments, the greater the value of the option in the first investment. At one extreme, consider investing in research and development to acquire a patent. The patent gives the firm that owns it exclusive rights to produce the product. If the market potential is large, the potential for the excess returns is large and the option to invest is valuable. At the other extreme, the firm might get no competitive advantages on subsequent investments. In this event, it is questionable as to whether there can be any excess returns on these investments. In reality, most investments will fall in the continuum between these two extremes, with greater competitive advantages being associated with higher excess returns and larger option values.

3. *The competitive advantage(s) which a company may have must be sustainable for a period of time.*

In a competitive market place, excess returns attract competitors, and competition drives out excess returns. The more sustainable the competitive advantage(s), the greater will be the value of the options embedded in the initial investment. The sustainability of competitive advantages is a function of two forces. The first is the nature of the competition; all else being equal, competitive advantages fade much more quickly in sectors where there are aggressive competitors. The second is the nature of the competitive advantage . If the resource controlled by the firm is finite and scarce (as is the case with natural resource reserves and vacant land), the competitive advantage is likely to be sustainable for longer periods. Alternatively, if the competitive advantage comes from being the first mover in a market or technological expertise, it will come under assault far sooner. The most direct way of reflecting this in the value of the option is in its life; the life of the option can be set to the period of competitive advantage and only the excess returns earned over this period counts towards the value of the option.

Although carefully constructed real option valuations result in higher and more effective valuations than does traditional NPV, it may still differ significantly from the value the *market* would place upon a business or project at a particular point in time. Examples of this have been noted in the valuation of oil reserves and real estate development projects (Patel *et al*, 1999). These case studies highlight the fact that "rational" investors may assign vastly different values to strategic options than would the analyst who aims to rely on cold facts only. In such cases, the motto that "the market is always right" may be used to double check whether real options analysis is the best tool to use under the given circumstances. Real options analysis is not a self-fulfilling prophecy and it is better to walk away if it fails than to spend huge amounts of money pursuing a lost cause.

7.2 Summary

When real options are used to justify a decision, the justification has to be in more than qualitative terms. Business managers who want to invest in a project with poor returns and/or negative NPV, or who are willing to pay a premium on an acquisition on the basis of a real options analysis, should be required to accurately value such real options and illustrate that the economic benefits exceed the costs. This must be done in the presence of all the difficulties real options analysis presents in terms of parameter estimation and validation. Although there is opposition to this argument, a noisy estimate is better than no estimate at all. The process of quantitatively trying to estimate the value of a real option is the first step to understanding what drives it value.

CHAPTER 8
REAL OPTION VALUATION OF PEPPADEW INTERNATIONAL

> "The process of scientific discovery is, in effect, a continual flight from wonder."
>
> Albert Einstein

8.1 Introduction and Background

Peppadew™ is a sweet piquanté (pronounced pee-caahn-teh) pepper with a very unique taste when processed. It is the first new fruit to be launched in 26 years since the discovery of Kiwi fruit in 1977. Pappadew™ is part of the family *Solanacea*, genus *Capsicum Annuum* and variant *Piquanté*. It has a peppery taste, yet is as sweet and tantalizing as the morning dew.

In 1990, Johan Steenkamp, a businessman and farmer in the Eastern Cape region of South Africa, noticed an unusual bush growing in his garden. It stood chest high and bore bright red fruit which resembled a sweet pepper but was the size of a cherry tomato. It tasted both peppery-sweet and had a very unique flavour. It is only after the fruit has been processed in a selective and secret manner that its distinctive hot-sweet and spicy flavour emerges. This discerns Peppadew™ from other pepper varieties.

In the latter part of 1990, the department of Agriculture in South Africa conducted a worldwide search and determined that the plant had not previously been described. It is believed that the seeds could have possibly been deposited in South Africa by migrating flocks from South America. Johan Steenkamp had the foresight to apply for plant breeder rights within the SADC countries of Southern Africa, the European Union, Australia and the United States. This gave him sole international rights to commercially grow and sell the fruit. He also obtained an international processing patent for the fruit - it is in this form that consumers across the globe are enjoying the unique flavour of

Peppadew™ piquanté peppers. The whole processed Peppadew™ fruit as well as its derivatives has been extremely well received by retail markets locally and abroad. Popularity of the many food and cooking programs and magazines that flood the market currently has aided the tremendous demand for Peppadew™ products.

Figures 8.1 to 8.3 illustrate the flower and fruit of the Peppadew™ plant. Figures 8.4 and 8.5 illustrate the processed fruit as retail products.

Figure 8.1

The Peppadew Flower and Fruit.

Figure 8.2

Harvesting the Crop.

Figure 8.3

Washing Fruit at the Factory

Figure 8.4

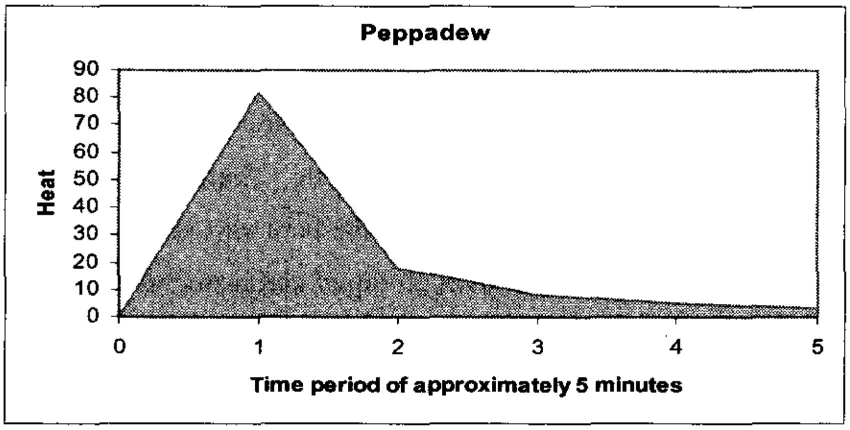
Whole Sweet Piquanté Peppers.

Figure 8.5

A Variety of Peppadew's delicious sauces.

The substance that makes a pepper hot is called *capsaicin* and its highest concentration is found in the ribs of the pepper. It is medically known for increasing human blood circulation and metabolism. The heat of a chili pepper is measured in Scoville units. In 1912, Wilbur Scoville blended ground chili peppers with a sugar-water solution and diluted the mixture until it no longer burnt the mouth. The Scoville unit number was based on how much dilution was needed before the burn was tamed. Today, however, the heat of chili peppers is determined through liquid chromatography. This entails the chemical separation of substances through absorbing materials. Bell peppers (commonly known as green peppers in South Africa) measure zero Scoville units and the fiery Habanero pepper measures roughly 300 000 Scoville units. The heat of a Peppadew™e pepper is around 15 000 Scoville units, approximately half that of the hot Jalapeno chili. With most hot chilies, the tasting of heat in the mouth takes some time. Once in effect, it lingers for a period of time, effectively paralysing the taste buds. Other flavours cannot easily be discerned during this time. The processed Peppadew™ pepper gives a quick taste of heat which dissipates rapidly, leaving a light chili and crisp fruity flavour in the mouth. Figure 8.6 illustrates the heat profile of Peppadew™ piquanté peppers versus a normal chili.

Peppadew™ is a local and international brand name. In addition to South Africa, where major retailers Woolworths and Pick'n Pay are Peppadew™ customers, markets have also been established in the UK, USA and Canada, Japan, Switzerland, Scandinavia, Switzerland, Italy and Germany.



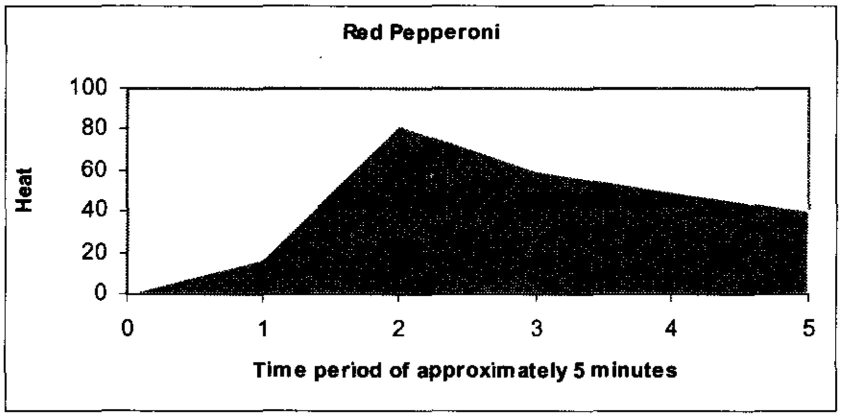


Figure 8.6

Comparing the heat distribution of Peppadew to a standard hot pepper.

8.2 Company and Ownership Structure

International plant breeder rights for Peppadew™ piquanté peppers are held by Piquanté International Ltd. Piquanté International is registered in the British Virgin Isles and is responsible for maintaining and protecting the intellectual property surrounding Piquanté peppers. Peppadew™ International (Pty) Ltd (referred to as Peppadew) is a South

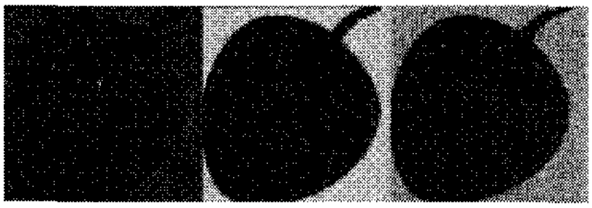
African based company. Worldwide, it alone is licensed to grow and sell piquanté peppers commercially.

Denny Mushrooms (Pty) Ltd, the largest producer of fresh and processed mushrooms in Africa, owns 89% of Peppadew. Denny also holds 50% plus one share of Piquanté International. Founder Johan Steenkamp has retained 10% ownership of Peppadew and 50% minus one share in Piquanté International. Listed in the bibliography are applicable websites for additional information regarding Peppadew™, the company and its product.

8.3 Purpose of the Analysis

Peppadew has exclusive use of the local and international patents, trademarks and plant breeder rights for sweet piquanté peppers via a royalty agreement with Piquanté International. The objective of this study is the valuation of intellectual property held by Peppadew and in particular, the patent and breeder rights. Real options analysis is employed to explicitly and quantitatively value these assets. The real options analysis complements the standard valuation performed for the company by an auditing firm. This offers both analysts and managers fresh insights into the valuation of non-financial assets held by privately owned companies with limited trading history.

Raising funds for working capital purposes is a difficult and time consuming exercise for small, non-investment grade companies like Peppadew. In addition to the lack of interest from large investors, lending rates are often crippling. This study also aims to illustrate that the results and insights obtained from the real options analysis can be used to affect an enhanced borrowing strategy for Peppadew.



8.3.1 Data

In September 2002, the auditing firm KPMG South Africa, undertook a valuation of Peppadew International and Piquanté International. A copy of this valuation report appears in Appendix II, with permission from the CEO of Denny Mushroom, Mr Richard Baker. The principal sources of information are historical financial statements for Peppadew, management accounts and forecasts, and interviews with company directors. (Refer KPMG Report, Appendix II, Page 1, Section 1.2.1). Peppadew reports all figures in ZAR (the South African Rand) and revenue from all the geographical areas is reported together.

Note : This study makes no attempt to justify or repudiate the data and information contained in the KPMG report.

The majority of inputs used for the real option valuation has been directly adapted from the KPMG report. This ensures that the real option analysis remains comparable with the traditional DCF techniques applied by KPMG. Recognising the differences and commonalities of the two approaches adds insight to the analysis, highlighting the effects of uncertainty and its tradeoff with value. The aim is mainly to make the options way of thinking both understandable and acceptable to managers and investors alike.

An information memorandum was prepared for Peppadew by Valbridge Trust in June 2003. This document supplied detail relevant to the case study. Due to its volume, however, it has not been attached as an appendix but may be viewed with the permission of Mr. Richard Baker.

8.4 The Application of Real Options in the Evaluation of Peppadew (Pty)Ltd

It takes a lot of time and careful thinking in order to identify the real options in a project or business. Aswath Damodaran, in his highly acclaimed book, The Dark side of Valuation, reminds the analyst that "*not all projects, businesses and investments have options embedded in them and not all options, even if they do exist, necessarily have value*" (Damodaran, 2001). In the same way as Discounted Cash Flow analysis may underestimate the true value of a project, there is a danger that value may be inflated if real options analysis is performed without an in depth understanding of the problem.

8.4.1. Patents, Plant Breeder Rights and Brand Name

Peppadew has exclusive use of the local and international plant breeder rights, process patent and trademark via a royalty agreement with Piquanté International. This means that Peppadew is the only company that may grow, process and sell sweet piquanté peppers in the world. The following table lists the life of the patent and breeder rights for the main geographical areas targeted by Peppadew:

REGION	**EXPIRY DATE of PATENTS & BREEDER'S RIGHTS**	**REMAINING LIFE OF PATENT & BREEDER'S RIGHT** As of 30 September 2003.
Southern Africa (South Africa, Swaziland, Tanzania, Zambia and Zimbabwe)	14 February 2012	8.38 years
European Union	31 December 2026	22.27 years
USA & Canada	30 September 2013	10 years

The patents and plant breeder rights give Peppadew a *substantial and sustainable competitive advantage*. Not only is the company globally protected from product competition during the life of the patents and breeder rights but it also has time to develop and establish its Peppadew™ brand name.

In a competitive environment, firms with more valuable brand names are either able to charge higher prices for the same product (leading to higher margins) or sell more than their competitors at the same price (leading to higher turnover ratios). Creating a brand name is, however, a difficult and expensive process that may take many years to achieve. Since Peppadew™ is a relatively young brand, an independent brand evaluation will not form part of this analysis. Instead, the assumption is made that management inputs to the discounted cash flow evaluation incorporate the effects of the young brand name.

Competitive advantage from exclusive licensing may not always lead to value enhancement. When a firm is granted these rights by another entity like the government, that entity may, through regulation, preserve the right to control prices charged and margins earned. In these circumstances, firms may actually gain in value by giving up their legal monopolies if they get pricing freedom in return. Specifically, this has become a much debated issue in the South African airline and telecommunications industries. Peppadew, on the other hand, has no external price restrictions imposed on its product. Prices are (and will continue to be) driven largely by supply and demand in the market. Consequently, the patents and plant breeder rights that Peppadew holds are real and valuable assets. *This is the key to a real option valuation of Peppadew.*

Consider a hypothetical company that grows, processes and sells standard hybrid tomatoes. Unlike Peppadew, this company is not a first mover in the market for tomatoes, it does not have a unique product and it faces fearsome competition. With regard to growth and expansion, the company may strategise along the same lines as Peppadew, but it does not have exclusive rights to its product nor any significant competitive advantage over other tomato companies in the market (even though management of the company may view their product as superior). Potential for excess returns is thus limited. The only real asset of value would be a brand name, if it existed.

The competitive advantage Peppadew has is sustainable until expiry of the process patent and plant breeder rights. This translates into a potential for excess returns for the company over an approximate ten year period. (Refer Appendix II, KPMG valuation, Section 2.2.1, page 3). Thereafter, if the market for Peppadew™ peppers has grown significantly, competitors will enter and returns will depend on how well the Peppadew™ brand name has been established, all else being equal.

8.4.2 Valuing the Process Patent and Plant Breeder Rights held by Peppadew(Pty)Ltd

A Growth Option

Peppadew is a small, high-growth firm in a large, evolving market for fusion food. Fusion food is a term used to describe the mingling of cultural dishes to create a new culinary experience. Peppadew™ products have been sold in South Africa since 1996 but accurate figures are only available from October 2000. Retail sales have grown by 52% per annum in volume terms and 64% per annum in Rand terms since January 2001. Management expects net revenue to grow at an average compounded growth rate of 66% per annum for the next four years. This growth assumption is the biggest contributor to the value derived for Peppadew. (Refer Appendix II, KPMG report, Section 1.3.4, page 2.) Clearly, it is the process patent and plant breeder rights which the company holds that allow for such estimates of growth. Thinking in broader terms, Peppadew has a right, but no obligation really, to grow its business and entrench itself in the market for sweet chili peppers without any threat from competitors for roughly the next ten years. One may argue that if a company holds such patents and rights (and pays royalties for it), it will necessarily and immediately apply them to its advantage. This is the static assumption of immediate "investment" that standard NPV analysis makes. Consider, for example, the possibility of a prolonged drought in the southern African region. Patents and rights will not aid the company if the growing pepper fields cannot be watered. In addition, the South African Rand has been appreciating steadily against the US dollar since the beginning of 2002, as illustrated in Figure 8.7. A strengthening currency puts a lot of pressure on exporters like Peppadew. Although these (any many other) risks may be hedged in the financial markets, they highlight the inherent *optionality* that goes hand in hand with real assets such as patent and plant breeder rights. *The company has an option to grow at phenomenal rates due to its competitive advantage, but only if it is economically viable to do so.* The patent and plant breeder rights held by Peppadew may thus be valued as a *growth option.* According to Trigeorgis (Trigeorgis, 2002), early investment is a prerequisite in a growth option. It initiates a link in a chain of interrelated projects, opening up future growth opportunities. It is especially relevant in cases involving strategic acquisitions or multinational operations.

At first glance, one would expect Peppadew's breeder rights and process patent to be valued separately. However, management delivers only *one* set of operational figures that arise from having *both* the breeder rights *and* the patent. Thus, in order to prevent over-estimation and double counting, the real option valuation considers the rights and patent *as one asset.* The result gives a minimum value for the sum of the two.

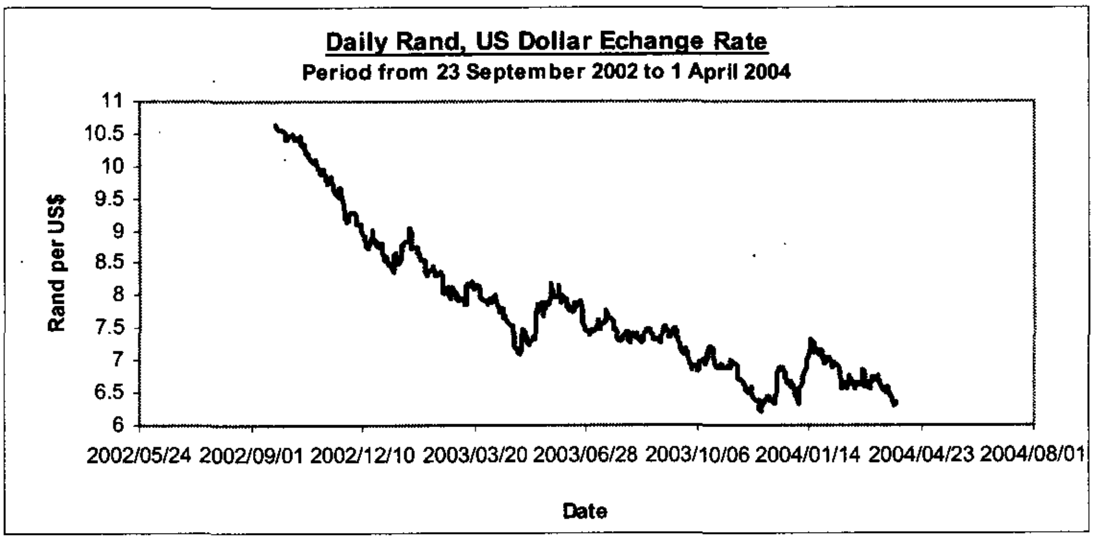


Figure 8.7

The recent strength of the Rand against the US Dollar. *Source*: INET Bridge.

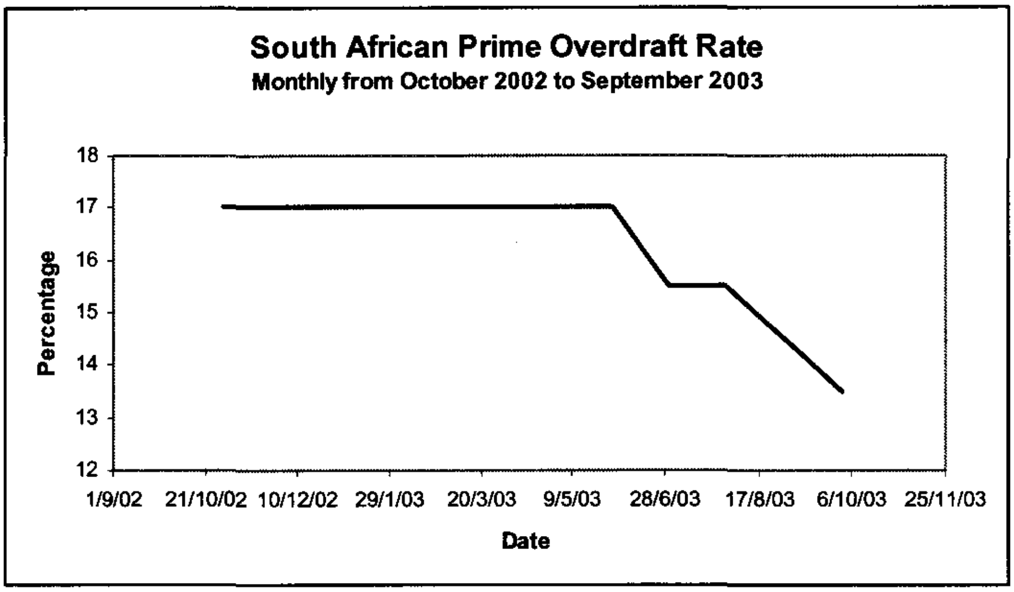


Figure 8.8

South African interest rates between October 2002 and September 2003.

Source: INET Bridge

8.4.3 Methodology of the Analysis

The growth option will be quantified and priced using the closed form analytical solution derived by Black and Scholes and discussed in Chapter 5. Even though analytical solutions are not applicable in many real option cases, it is easily and effectively applied here. A binomial tree will also be constructed to price the same option numerically. This analysis has been included in Appendix IV because of the general popularity of binomial trees. Practitioners may find the tree model intuitively easier to understand and interpret. In addition, trees are almost always used (either directly or indirectly) as an approximation to the continuous Black Scholes case.

8.4.4 Net Present Value Calculation

All figures and calculations for this section appear on the attached CD:

File Name	:	Peppadew International
Sheet Name	:	Peppadew NPV

The discounted cash flow analysis undertaken by KPMG and stated in their report (Refer Appendix II, KPMG Report Annexure I, page 3 of 7), had to be adjusted to take account of the fact that the real options analysis is being done eighteen months later than KPMG undertook the initial valuation. Management of Peppadew has indicated that the estimates as contained in the report remain mostly unchanged, except that figures for the year to 30 September 2003 can no longer be considered as future estimates. They now form part of history. In addition, the growth option does not consider any data post 30 September 2012. (I.e. estimates referred to as "Maintainable" in the KPMG evaluation, Annexure I, 3 of 7, Appendix II of this thesis). The fact that the reported figures are across all geographical areas and that the patent and breeder rights extend over different time frames is not crippling to the analysis. It simply implies that the results from the real option analysis must be interpreted as a minimum value for patent and breeder rights until September 2012. Naturally there will be additional value for the period after September 2012 until the expiry of all the patent and breeder rights. Data this far into the future, however, is not available (and it is debatable whether estimates so far into the future would be of any value).

Table 8.2 depicts the discounted cash flow analysis adapted from the KPMG report. On October 1, 2002, KPMG applied a **24.03%** cost of equity for Peppadew. This was based on the inputs to the Capital Asset Pricing Model as discussed in Section 5.2, page 13 of their report. A risk free rate of 11,67% (compounded semi-annually) was the then prevailing ten year interest rate, implied from the R153 government bond. However, prime lending rates in South Africa have since dropped 350 basis points. Figure 8.8 illustrates. An equivalent adjustment in both the discounted cash flows and the weighted cost of capital is thus warranted. The current nine year swap rate, compounded semi-annually, is 9.70%. This translates into an adjusted weighted average cost of capital of **21.19%**, using the same market risk premium and beta as proposed by KPMG. (Refer page 13 and page 14, Section 5.2 of the KPMG report). Calculations are illustrated in Table 8.1.

Given the above inputs, the net present value of Peppadew's cash flows amounts to **R52 680 000**. This is before the deduction of any liabilities, i.e. shareholder and other loans. (Refer Annexure I, page 3 of 7, for KPMG's concise summary of liabilities.) Keeping in mind the time frame and interest rate adjustments mentioned above, this figure is in line with KPMG's 2003 enterprise valuation of R59 154 000.

Table 8.1

Adjusting the Weighted Average Cost of Capital for Peppadew International.

Date: 30 September 2003

	KPMG	**Adjusted for Real Options Analysis**
Riskfree Rate	11.67 %*	9.70 % NACS*
Risk Premium	7.50 %	Remains unadjusted
Beta	66.90 %	Remains unadjusted
Cost of Equity		
Early growth risk of new product	16.69 %	14.72 % **
Private Company Risk	20.00 %	20.00 %
Cost of Equity	24.03 %	21.19 %
Cost of Debt		
Interest Rate Charged	17.00 %	15.03 %
After tax	11.90 %	10.52 %
Proposed Debt equity Ratio		
Debt	0 %	Remains unchanged
Equity	100%	Remains unchanged
WACC		
Cost of Equity (weighted)	24.03 %	21.19 %
Cost of Debt (weighted)	0 %	0 %

* Nominal Annual Compounded Semi-Annually.

** Riskfree Rate + (Beta * Risk Premium)

Table 8.2

Adjusted NPV Calculations for Peppadew International.

Date: 30 September 2003

	R' 000								
Date	30 Sept 2004	30 Sept 2005	30 Sept 2006	30 Sept 2007	30 Sept 2008	30 Sept 2009	30 Sept 2010	30 Sept 2011	30 Sept 2012
Operating Cash Flows	7 410.00	17 851.00	24 489.00	29 176.00	35 035.00	40 307.00	44 345.00	48 788.00	51 715.00
WACC	21.19%								
Discounted Cash Flows	6 114.20	12 153.63	13 757.40	13 524.24	13 400.19	12 720.70	11 547.74	10 483.03	9 168.79
Total	**102 869.92**								
Capex & Working Capital	(10 500.00)	(12 000.00)	(16 000.00)	(16 000.00)	(15 820.00)	(13 501.00)	(11 170.00)	(11 937.00)	(6 572.00)
Discounted	(8 663.85)	(8 170.05)	(8 988.46)	(7 416.64)	(6 050.83)	(4 260.85)	(2 908.74)	(2 564.89)	(1 165.18)
Total	**(50 189.50)**								
NPV *	**52 680.42**								

* Before liabilities.

8.5. Linking NPV and Option Value

The growth option is similar to a financial call option because the process patent and plant breeder rights effectively give Peppadew the right, but not the obligation, to realise above average revenue growth over a finite period of time. In order to value the growth option using the Black-Scholes model, a correspondence between the financial characteristics of Peppadew and the six call option variables must be established:

1. The underlying asset is the present value of expected operating cash flows over the life of the patent and breeder rights, i.e. the next nine years to September 2012. This value is **R102 869 920**. Expected cash flows were obtained directly from management spreadsheets and are based on the potential market size and profit that the firm can expect to realise (Refer Appendix II, KPMG Report - Annexure I, page 3 of 7). The adjustment to the discount rate was discussed in Section 8.4.4 above.

2. The exercise price is equivalent to the expenditure required to generate these cash flows. I.e. the working capital and capital expenditure requirements of the business over the life of the option. These figures are also directly obtained from the management accounting sheets. (Refer Appendix II, KPMG Report - Annexure I, page 3 of 7). The present value of these costs amounts to **R50 189 500.**

3. The growth option has value only until expiry of the patent and breeder rights. For purposes of this evaluation, maturity of the option is 30 September 2012.

4. The time value of money is depicted by the risk free rate of **9,94%** compounded annually. This is the ruling nine-year swap rate.

5. Uncertainty about the future value of the project's cash flows (i.e. the inherent riskiness of the business) corresponds to the standard deviation of returns of the underlying asset. Since Peppadew is not a traded entity, volatility was determined using the proxies suggested by KPMG. Although any number of other proxies may

have been applied (i.e. sweet bell peppers or olives as suggested in the Valbridge Trust report, individual food companies both locally and offshore or even just an educated guess from a food sector analyst) the aim is to keep the real option valuation in line with the KPMG analysis.

Each of the proxy companies, Anglo Vaal Industries (share code AVI), Crookes Brothers (CKS), Illovo (ILV) and Intertrading (INT), is listed on the Johannesburg Stock Exchange (JSE). These companies are good proxies for Peppadew in terms of their product offering, target market and business model. The following short descriptions of each are adapted from KPMG's valuation report contained in Appendix II, Section 5.2.11, page 15:

- **Anglo Vaal Industries** focuses on a small range of luxury food items not dissimilar to the Peppadew product. However, the company has a holding in a glass manufacturer and thus cannot be used as a proxy on its own. A 10% weighting is assigned to AVI.

- **Crookes Brothers** produces a range of products from sugar cane, fruit, grain and animal husbandry. Because of the focus on primary agricultural products, it also is not a perfect proxy for Peppadew. A 10% weighting is assigned to CKS.

- **Illovo** focuses on a single product (sugar) with a range of value added components. This is more in line with Peppadew and Illovo is assigned a 30% weighting.

- **Intertrading** is an international trading company focusing on the procurement and export of South African fruit, nuts and vegetables. Unlike the other three companies, it is very similar to Peppadew in terms of product and general operating environment. A 50% weighting is assigned to INT.

An estimate of the historical volatility of Peppadew was calculated using the appropriately weighted standard deviation of continuously compounded daily returns of the four proxies over the period from 11 June 2002 to 29 September 2003. Refer Appendix III for data, graphics and calculations. The result is an annualized volatility of **30.49%** for Peppadew. The volatility of "the market" may be used to determine a lower boundary on this estimate. The Top 40 Index, which consists of the forty JSE listed shares with the highest market capitalisation, constitutes a well diversified market portfolio. Calculated in the same way and over the same data period, the volatility of this index is 20.79%. Data and calculations may be viewed on the attached CD:

File Name : Peppadew International
Sheet Name : Data & Vol Estimates.

It seems reasonable that a single stock should be a lot more volatile than a market Index. A quick consensus in the market amongst seasoned equity traders confirmed that 10% above market volatility would be a reasonable level to trade at for an unlisted entity such as Peppadew.

The above methodology is the standard approach to estimating volatility from historical data. It assumes that volatility is constant. In practice, however, the volatility of an asset is a stochastic variable, much like its price is. There are a number of schemes that attempt to keep track of the shifting levels of volatility. The generalized autoregressive conditional heteroskedastic model (GARCH) attempts to capture and weight the long- run average variance rate of a series of price returns. The popular GARCH (1,1) model was fitted to the weighted series of proxy price returns in order to ascertain the presence of a any long-term trend in volatility. It is important to know if and how volatility shifts when applying option pricing models. The results, contained in Appendix IV, confirm that the series does not exhibit any significant shift in volatility. The assumption of a constant volatility may thus safely be made from this point forward.

6. The potential for excess returns exists only during the life of the patent and breeder rights. Thereafter competition will challenge and erode Peppadew's

market dominance. Amongst other factors such as product diversification and marketing, the strength and quality of the brand name will have to carry the company forward from that point onward. Thus, every passing year "costs" the firm one year of patent-protected excess returns. In market terms, the dividend expense is represented by the value that drains away over the duration of the option. If the life of the patent and breeder rights is *n* years, then the "dividend" is defined as $\left(\frac{1}{n}\right)$. In this case, the dividend yield is $\left(\frac{1}{9.01}\right) = 11{,}10\%$. The dividend yield rises with the passing of every year, decreasing the real value of the patent and breeder rights to the company. Refer Damodaran (2001)

Table 8.3 below summarises the correspondence between the six real option variables and their applicable values for Peppadew, and a financial call option.

Variable for Black Scholes Input	**Correspondence to Financial Call Option**	**Value for Peppadew Case Study**
Present Value of company's expected cash inflows	Stock Price (*S*)	R 102 869 920
Expenditure required to generate operating assets	Exercise Price (*X*)	R 50 189 500
Life of patent and breeder rights	Time to expiration (*T-t*)	30 Sept 2003 – 30 Sept 2012 9.01 years
Time Value of money	Risk free rate of return (*r*)	9,94% annual compounding
Riskiness of Peppadew business	Volatility of stock returns (σ)	30,49% annualised
Value Erosion	Dividend Yield (*q*)	11,10%

Table 8.3

The Option Variable inputs for Peppadew International.

8.6 Black- Scholes Valuation

The real call option is priced using the standard Black-Scholes model discussed in Chapter 5, Section 5. From this and the information contained in Table 8.3 above, the following estimates are obtained for the option pricing model:

Variable	Value
d_1	1.1986
$\Phi(d_1)$	0.8847
d_2	-0.4586
$\Phi(d_2)$	0.3232

Substituting these values into the call option formula,

$$C = Se^{-q(T-t)}\Phi(d1) - Xe^{-r(T-t)}\Phi(d2),$$

the value of the patent and plant breeder rights is then calculated to be at least **R26 852 4900.**

A minimum value for Peppadew (before liabilities) is thus the *sum* of the Net Present Value of the business's estimated cash flows (until September 2012) *and* the value added by the patent and breeder rights:

R 52 680 420 + R 26 852 490 = **R 79 532 910**

The enterprise valuation of Peppadew as calculated by the auditing firm may now be compared to the results obtained from the real options analysis:

Original KPMG NPV Valuation	R59 154 000
NPV Valuation after adjustment for time and interest rate decrease	R52 680 420
Real Option Valuation of assets	R26 852 490
Real option Valuation of Peppadew (adjusted for time span and interest rate decrease)	R79 532 910

All figures and calculations appear on the attached CD:

File Name : Peppadew International
Sheet Name : Black Scholes

The NPV analysis alone (after adjustments for time and interest rates) comprises 66.24% of the fair value of Peppadew. The value of the real assets (i.e. the process patent and plant breeder rights) make up the remaining 33.76%. Clearly, by relying on NPV analysis alone, a substantial amount of the company's value remains unaccounted for. Real options analysis has offered a means of capturing and pricing the value of Peppadew's real assets.

8.7 Discussion

The value of Peppadew's patent and breeder rights, explicitly quantified and priced as a real option, remained unaccounted for by the standard NPV valuation as performed by KPMG. The NPV calculation alone failed to take into account this additional source of value because

(1) It does not recognize the inherent optionality which the patent and breeder rights offer Peppadew. This leads to the simplistic assumption that these real assets have already been accounted for in management's estimates of future cash flows. To an extent, one has to acknowledge that they do play a role when future cash flows are estimated. However, it is the options way of thinking which allows real assets to be mathematically quantified in line with management expectations as well as the market in general.

(2) It assumes, by definition, that the fair price of the business is a (convex) function of the expected values of uncertain variables, $Price = f\left[E(x)\right]$. Jensen's Inequality however, highlights the fact that this is a flaw of averages. The correct price is represented by the expected value of the

(convex) function of uncertain variables, $E[f(x)]$, and that $E[f(x)] > f[E(x)]$. Consequently, average values of certain inputs used by management when estimating future cash flows will not result in average outputs. In fact, the result is a systematic under valuation of projects or businesses, especially in the presence of real options.

CHAPTER 9

FINANCIAL ENGINEERING WITH REAL OPTIONS

> "The onus is on us, through hard work....to reach for the stars."
>
> Nelson Mandela.

9.1 Overview

The Peppadew™ pepper has a growth cycle between November and March in the southern hemisphere. This is a period of high intensity and high cost for Peppadew International. At the start of the season, management carefully selects farmers to grow the chili under strict regulations with regard to quality and pest control. Once the fruit has been harvested, it is purchased from the designated farmers at a market-related price. Thereafter, the peppers are transported to the processing factories in Tzaneen in the north-eastern region of South Africa. One of the greatest challenges facing the company is ensuring a sufficient supply of the fruit all year round in order to meet global demand. To this end, farming under controlled conditions in diverse geographical locations is being heavily researched. Existing processing facilities are also being upgraded and expanded while new processing plants will be required in the near future. The company thus faces high working capital requirements.

To date, Peppadew has been financed mainly by its holding company Denny Mushroom. Peppadew now aims to obtain additional funding on a more permanent basis. The natural consideration when raising funds for the working capital requirements of an agricultural-based company offering employment to a large number of farmers and factory workers, is the Industrial Development Corporation of South Africa (the "IDC"). The IDC is a government institution that offers financial assistance to small and medium size enterprises, with an emphasis on black empowerment initiatives. After the fall of the apartheid regime ten years ago in 1994, the new South African government began to focus heavily on job creation with equal opportunities for all races. Companies that create jobs in rural farming areas and in factories where education levels are low and poverty is high, are favoured by government in terms of funding, contracts, tax status etc.. Peppadew could thus look toward the IDC for funding of its working capital requirements. They are, however, not limited to the IDC only. As long as a proposed funding strategy is aligned with the market, the company may approach any of a number of funding entities like banks, venture capitalists, institutional and private investors.

9.1.1 Roadmap for the Construction of a Funding Strategy for Peppadew

The following steps will be followed and explained in Sections 9.2 to 9.8 below, in order to demonstrate the process driving the construction of a funding strategy for Peppadew via an option-embedded debt obligation:

Step 1 - The proposed debt structure, a putable bond, is introduced to the reader.

Step 2 – Valuation principles of the structure are discussed.

Step 3 – The risk that an issuer of debt will not honour promised future payments is referred to as default risk. Default risk will determine at which interest rates an individual issuer (in this case, Peppadew International) may obtain funding in the market relative to issuers deemed to have very low or no default risk at all. This risk-return profile is called the issuer's yield curve and is derived from the Treasury (zero default) yield curve.

Step 4 – The issuer's yield curve allows the term structure of interest rates to be explicitly used in the pricing of the Peppadew putable bond.

9.2 The Structure of a Debt Obligation

Investors are often willing to purchase an equity stake in a private company. The proceeds received from such a deal then provide the necessary funding which the company requires. Investors who make private equity investments are, however, not promised a fixed cash flow but are instead entitled to whatever cash flows remain after other claimants (like creditors) are paid. These cash flows are referred to as residual cash flows. Uncertainty revolves around what these residual cash flows will be, relative to expectations. The actual cash flows can be lower than expected but they can also be much higher. Armed with the enhanced valuation and strategic insights obtained from the real options analysis performed in Chapter 8, private equity funding is one avenue that the management of Peppadew may explore anew.

This chapter of the study, however, probes the concept of *embedding a debt option within the format of a fixed income security. The major impetus in creating a structure of this nature is to illustrate how the results from the real options analysis may be applied to engineer an enhanced funding strategy for the company.*

Fixed interest securities like bonds promise a certain cash flow to the holder in future periods. The risk that these cash flows will not be delivered is called *default risk.* The results from the real options analysis enable an accurate, market related estimate of the default risk of Peppadew to be made - hence, a competitive debt funding strategy may be structured for the company. Even though Peppadew International has tremendous growth prospects due to its sole right to grow and commercially distribute a truly unique food product, it is still a reasonably young company facing many challenges. Consequently, embedding an exit clause in a standard bond issue adds an additional element of security for potential lenders. Such a structure is commonly referred to as a putable bond.

*A **putable bond** is a fixed income security where the investor has the right (but not the obligation) to terminate an investment by selling the security back to the issuer at a predetermined price, on one (or more) predetermined dates prior to maturity of the security.* In return for receiving the right to put the bond back to the issuer before maturity, the investor accepts a lower return on the investment. The issuer receives the benefit embodied in the lower borrowing cost. This is reflected in the option premium received from the sale of the put on its bond. The issuer accepts the interest rate and liquidity (maturity contraction) risk of the structure. This means that if the investor decides to put the bond and cease further investment before expiry of the bond, the issuer is responsible for settlement of the debt as well as refinancing thereof, if necessary, in a changing interest rate environment.

The putable bond to be structured and priced for Peppadew initially will be a **one year, zero-coupon bond, putable at fair value**. Once this structure is in place, variations in coupon, maturity and redemption value may be considered to find a tailored solution to fit the risk profile of any particular investor. In addition, the insights obtained from the bond calculations allow for further pricing of debt and equity structures, or a combination of the two in convertible securities. This in turn may lead to a number of possible real option-based strategies, the pricing of which would previously be hampered by the lack of quantifiable inputs for real assets.

9.3 Valuation Principles of a Putable Bond.

The value of a putable bond is equal to the value of a comparable non-putable bond plus the value of a put option. I.e.

Putable Bond = Non-Putable Bond + Put Option.

or

Put Option = Putable Bond – Non-Putable Bond.

The theory behind the valuation of a standard put option may sound straightforward, but determining the value of a put option embedded in a fixed interest security is not always simple. The biggest stumbling block when considering the use of an option pricing model in the valuation of a put option on a Peppadew bond in particular, is the estimation of the underlying volatility of such a bond. Being an unlisted, non-investment grade entity, Peppadew has no other outstanding bonds which may serve as a starting point for volatility estimates of its debt issues. In addition, there are not many private firm debt issues currently in the South African market which may be used as proxies. A valuation technique that does not rely on an option pricing model is thus most suitable in this specific case.

The valuation framework that can be used to value any bond, *both option free and option embedded*, involves generating a *binomial interest rate tree* based on an issuer's yield curve, an assumed *interest rate generation process* and an assumed *interest rate volatility*. The binomial interest rate tree yields the appropriate volatility-dependent one-period forward rates that should be used to discount the expected cash flows of the bond. Once the tree has been constructed, the value of the bond at each node is calculated in a backward recursive fashion (much like the tree for stock price movements discussed in Chapter 6 of this study). The value of both a standard, non-putable bond as well as an option embedded bond is calculated in this manner, with a few necessary adjustments for the embedded option.

9.4 Valuing an Asset with Default Risk

Inherent in investments with promised cash flows, is the risk that the cash flows may not be delivered. Such cash flows can consequently not be discounted at the riskless rate of interest alone, but must include an appropriate premium to compensate for the risk of default. This premium is called the *default spread*. Default spreads, when added to the riskless rate, yield the interest rates applicable to bonds of companies with specific credit ratings. Default spreads for most *listed* South African companies are routinely calculated by banks as part of their overall credit risk management. Independent rating agencies like

Standard & Poors and Moodys also measure default risk and give companies a rating based on the default risk of their bonds. Low rated firms have more default risk and generally have to pay higher interest rates on their bonds than do highly rated firms. Table 9.1 summarises the ratings used by Standard & Poors to rate U.S. companies. These rating outlines have been accepted (with minor adjustments) and applied in the South African market.

The starting point for valuing a putable bond is thus to determine the appropriate default spread of the issuer. In this case, the issuer is Peppadew International. Being an unlisted entity, however, the appropriate default spread is not widely known in the market. Results from the real options analysis performed in Chapter 8 once again overcomes many obstacles which previously would have made estimates of the company's default spread highly unstable and biased.

RATING	DESCRIPTION
AAA	The highest debt rating assigned. The borrower's capacity to repay debt is extremely strong.
AA	Capacity to repay debt is strong and differs from the highest quality only by a small amount.
A	Has strong capacity to repay. Borrower is susceptible tp adverse effects of changes in circumstances.
BBB	Has adequate capacity, but adverse economic conditions or circumstances are more likely to lead to risk.
BB, B, CCC, CC	Regarded as predominantly speculative, BB being the least speculative and CC the most.
D	In default or with payments in arrears.

Table 9.1

Standard & Poor's Rating Description. Source: *www.standardpoors.com*

9.4.1 The Treasury Yield Curve

A yield curve is a graphical depiction of the relationship between yield and maturity on securities of the same credit risk. The treasury yield curve is constructed from the maturity and observed yield of Treasury securities.

Treasuries reflect the pure effect of maturity alone on yield, given that market participants do not perceive government securities to have any credit risk. The shape of the yield curve does not remain static but changes periodically with shifting interest rate scenarios. Figure 9.1 illustrates the most commonly observed shapes of the yield curve in South Africa.

Figure 9.2 reflects the short end of the curve (up to one year) constructed from forward rates. It is important to highlight the fact that it is irrelevant whether a yield curve is constructed from forward or spot rates, as long as this is taken into account when pricing is done. The interest rate tree constructed in Section 9.7 below will be based on forward rates, as proposed by Fabozzi (1993), rather than spot rates. Applicable forward rates were obtained from South African International Money Brokers. They are one of the largest independent money market brokers in the country. Rates reflected on their trading screen are the best bids and offers available in the market at any point in time. Their service is provided to the market through Reuters SA and all information may be viewed on Reuters screen "IMBA". A printout of this, and all other applicable Reuters screens used throughout this chapter, appears in Appendix IV.

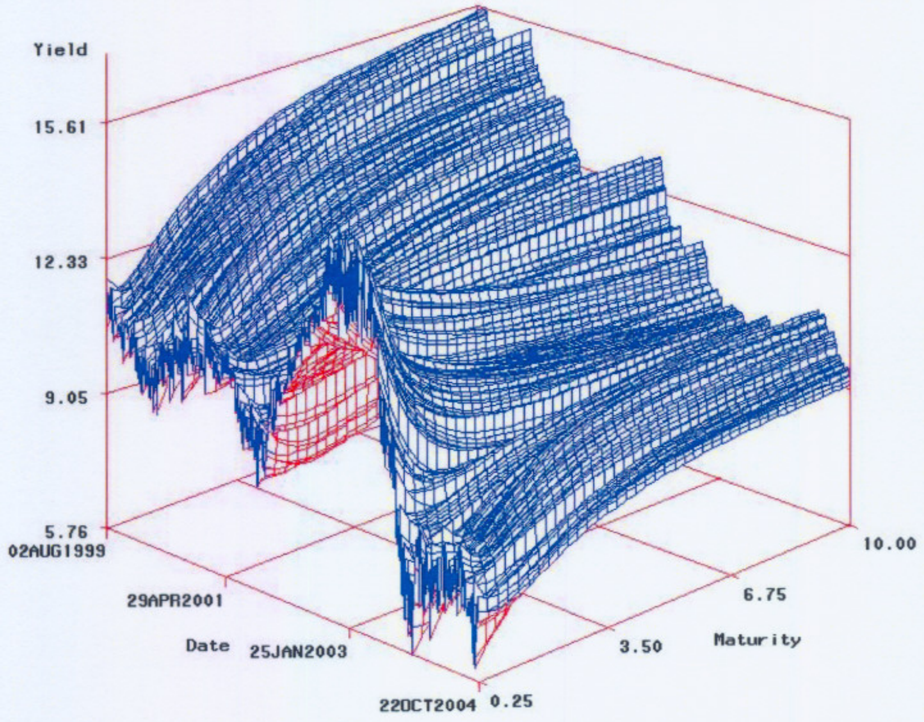


Figure 9.1

The Changing Shape of the South African Yield Curve.

Constructed from Swap Rates.

Source: Prof M.F Kruger

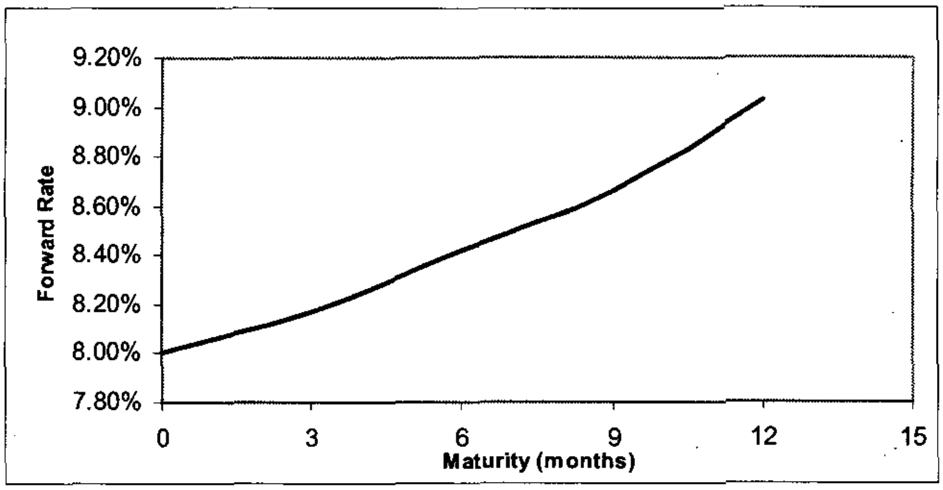


Figure 9.2

Short End of the South African Yield Curve: Spot Forward Rates.

Date : 8 March 2004. *Source*: IMBA and INet

9.4.2 Measuring Default Probability

The methodology introduced by Stephen Kealhofer, Mac McQuown and Oldrich Vasicek (www.kmw.com) to model default risk is applied in this thesis. The options pricing approach to modeling default risk is based on the asset value model originally proposed by Robert Merton (Merton, 1974). The reader's attention is drawn to the fact that the databases collected and developed by KMV for the calculation of default risk were *not* accessed at any time. These databases are only available to fully subscribed customers of KMV and not to independent researchers. Consequently, this thesis makes use of an independent default database for listed companies as compiled by Absa. (Amalgamated Banks of South Africa is one of the four largest commercial and corporate banks in the country). The database remains the intellectual property of Absa.

There are three main elements that determine the default probability of a company:

1. The *market value of assets.* This is a measure of the present value of the future free cash flows produced by the company's assets, discounted back at the appropriate discount rate. It is a measure of the company's prospects and incorporates relevant information about the specific industry and economy in which the company operates.

2. *Volatility of asset returns.* This is a measure of the company's business and industry risk. This estimate reflects the degree of difficulty in forecasting future cash flows due to the uncertainty of the business. Firms in the same industry and of similar size tend to have similar asset volatilities.

3. *Leverage or default point.* A measure of the liabilities due in the event of business distress, i.e. the extent of the company's contractual liabilities. Generally, non-cash and long-term obligations put less financial stress on companies than short term borrowing does.

Asset value, business risk and leverage are combined into a single measure of default risk which measures the number of standard deviations the asset value is away from default. This is referred to as the distance-to-default (DD) and is calculated as

$$\frac{\text{Market value of assets - Default point}}{\text{Market value of assets} \times \text{Asset volatility}}$$

The distance-to-default is thus a measure which accounts for the effects of industry, geography and company size, via the asset value and volatility. The probability of company default can be directly computed from the distance-to-default if the probability distribution of the assets is known, or, equivalently, if the default rate for a given level of distance to default is known. Oldrich Vasicek and Stephen Kealhofer have extended the

Black-Scholes-Merton framework to produce a model of default probability known as the Vasicek-Kealhofer (VK) model. This model assumes that the company's equity is a perpetual option with the default point acting as the absorbing barrier for its asset value. When the asset value hits the default point, the firm is assumed to default. A company defaults when its asset value falls below its liabilities. The probability of default is the likelihood that the market value of a company's assets will be less than the book value of its liabilities by the time the debt matures. Under the Black-Scholes assumption that the market value of a company's underlying assets follows the stochastic process

$$dV_A = \mu V_A dt + \sigma_A V_A dz \tag{9.1}$$

where V_A, dV_A are the company's asset value and change in asset value

μ, σ_A are the company's asset value drift and volatility

dz is a Wiener process,

the probability of default is given by

$$p_t = P\left[V_A^t \leq X_t \,|\, V_A^0 = V_A\right] = P\left[\ln V_A^t \leq \ln X_t \,|\, V_A^0 = V_A\right] \tag{9.2}$$

where p_t is the probability of default at time t

V_A^t is the market value of the company's assets at time t

X_t is the book value of the company's liabilities due at time t.

Equation (9.1) describes the change in the value of the company's assets. Its asset value at time t, V_A^t is thus given by

$$\ln V_A^t = \ln V_A + \left(\mu - \frac{\sigma_A^2}{2}\right)t + \sigma_A \sqrt{t}\varepsilon \tag{9.3}$$

since the asset value of the company is V_A at time 0. μ is the expected return on the company's assets and ε is the random component of the firm's return. The relationship given by Equation (9.3) describes the evolution in the asset value path, as illustrated in Figure 9.3.

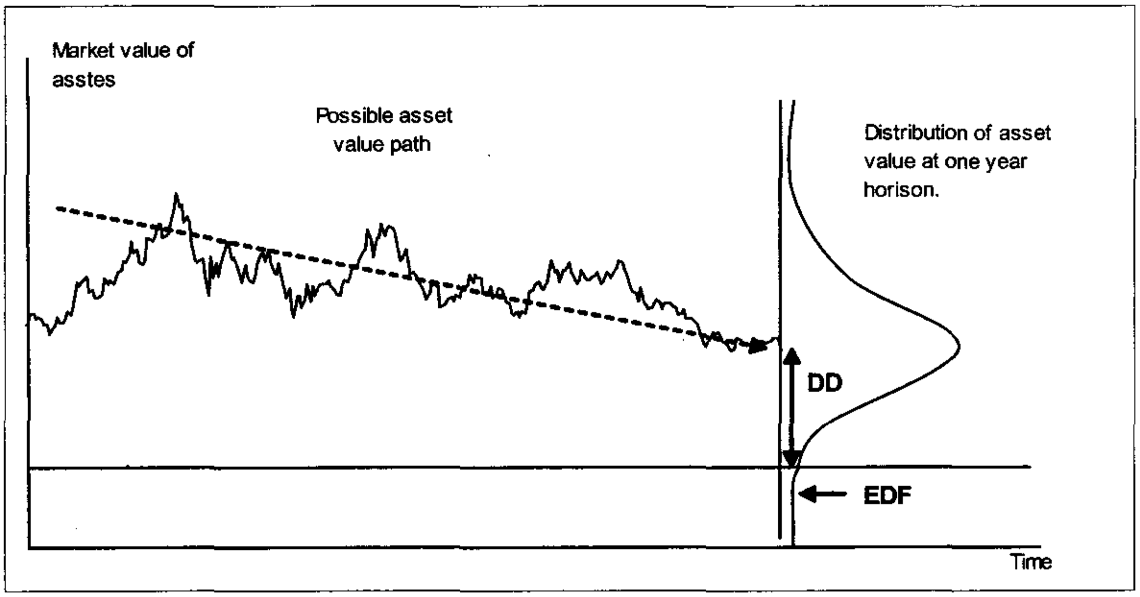


Figure 9.3

A Framework for Credit Measures. *Source*: KMV

Combining Equations (9.2) and (9.3), the probability of default may be written as

$$p_t = P\left[\ln V_A + \left(\mu - \frac{\sigma_A^2}{2}\right)t + \sigma_A\sqrt{t}\varepsilon \le X_t\right] \tag{9.4}$$

or

$$p_t = P\left[-\frac{\frac{\ln(V_A)}{X_t} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}} \ge \varepsilon\right] \tag{9.5}$$

The Black-Scholes model assumes that the random component of the company's asset returns is normally distributed, i.e. $\varepsilon \sim N(0,1)$. The probability of default may thus be expressed in terms of the cumulative normal distribution

$$p_t = N\left[-\frac{\ln\frac{V_A}{X_t} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}}\right] \tag{9.6}$$

Since the distance to default (DD) is simply the number of standard deviations that the company is away from default, it is given by

$$DD = \frac{\ln\frac{V_A}{X_t} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}} \tag{9.7}$$

The normal distribution is in reality a poor choice for defining the probability of default. The reason for this is that the default point is in reality also a random variable. The assumption that the default point is described by the company's liabilities and amortization schedule is flawed. Companies will often adjust their liabilities as they near default; the liabilities of commercial and industrial firms *increase* as they near default while the liabilities of financial institutions mostly *decrease* as they approach default. The difference is usually a reflection of the liquidity of the company's assets and their ability to adjust their leverage as they encounter difficulties.

It is difficult, ex ante, to specify the behaviour of liabilities, which means that the uncertainty in the adjustments in liabilities must be captured elsewhere. This uncertainty is included in the mapping of the DD to the Expected Default Frequency credit measure, described below. The resulting empirical distribution of default rates has much wider tails than the normal distribution.

9.4.3 Expected Default Frequency

A default database is used to derive an empirical distribution relating the distance-to-default to a default probability. Absa has implemented the VK model using their default database in order to calculate an Expected Default Frequency™ (EDF™) credit measure. EDF™ is the probability of default during the forthcoming year for companies with publicly traded equity within each sector of the market. Figure 9.3 is a scatterplot depicting the relationship between the distance-to-default and the Expected Default Frequency™ as calculated by Absa for all South African companies listed in the retail and wholesale sectors of the JSE. It illustrates that the closer a company is to its default point, the higher the expected probability of default.

Once the EDF™ for a company has been determined, it can be mapped onto any rating system (most commonly Standard & Poors or Moodys) to derive the equivalent credit rating of the company. Once the credit rating has been established, the market typically dictates the applicable credit spread for the company's debt.

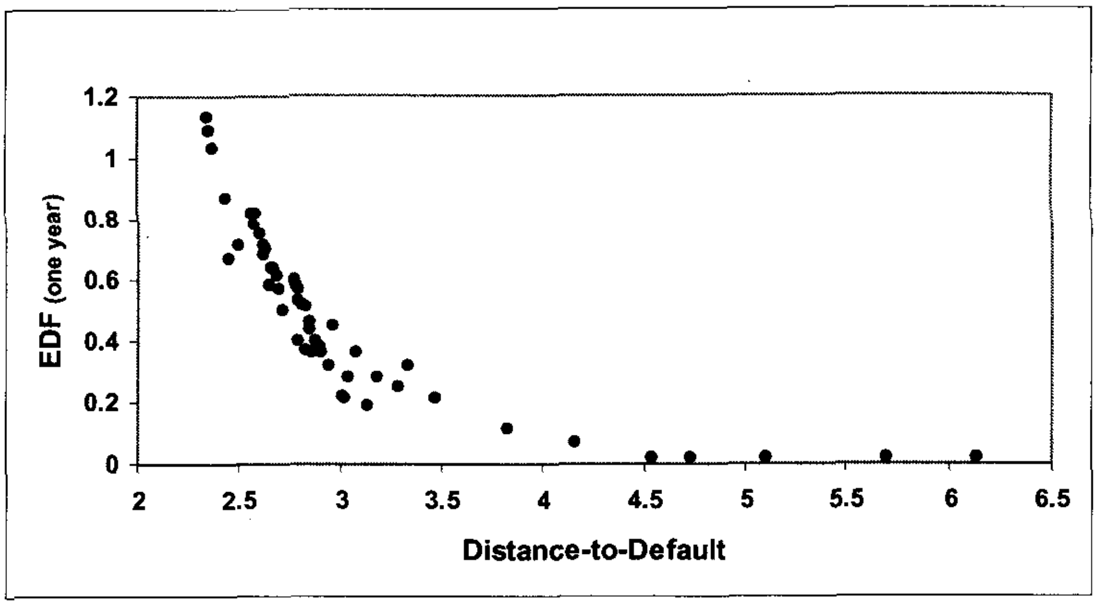


Figure 9.4
The Relationship between Distance-to-Default and Expected Default Frequency for all listed South African companies in the Retail and Wholesale Sectors of the JSE.
Source: Absa

9.5 Estimating the Default Probability of Peppadew (Pty)Ltd.

In order to calculate the default spread applicable to Peppadew, an applicable one year EDF™ measure must first be calculated. This involves the current market value and volatility of the company's assets, determining how far Pappedew is from default (i.e. its DD) and then scaling the distance-to-default to a probability. In Chapter 8, the market value of Peppadew's assets was calculated to be **R 79 505 180** with a volatility of **30.49%**. The default point, i.e. current value of liabilities was obtained directly from the KPMG Valuation Report (Annexure I, page 3 of 7; Appendix II of this thesis) as **R 11 796 000**. The distance to default (in millions of Rand) may thus be calculated as

$$DD = \frac{79{,}505.18 - 11{,}796}{(79{,}505.18)0.3049} = 2.793$$

This implies that Peppadew is currently approximately 2.8 standard deviations away from default. The Absa database cannot be used directly to calculate the EDF™ of Peppadew since the company is unlisted. Consequently, a regression analysis is performed in order to predict the EDF™ of Peppadew from the available data in the retail and wholesale sector of the JSE. If Peppadew were listed, it would be classified in this sector also. Since the relationship between the DD and EDF™ in Figure 9.3 is exponential, a logarithmic transformation is first made before a linear regression model is fit. The independent variable in the regression equation is the distance-to-default (DD) and the dependent variable is the EDF™. Letting

$x_t = DD = 2.793$ and $y_t = EDF$ ™, the regression equation is written as

$$\log\left(\frac{y_t}{1-y_t}\right) = \alpha + \beta x_t + \varepsilon_t \;\; ; \;\; E(\varepsilon_t) = 0$$

The regression results are

$\hat{\alpha} = -0.4489$ with 95% confidence interval (-0.5264 , -0.3715)

$\hat{\beta} = -1.6823$ with 95% confidence interval (-1.7119 , -1.6527).

Thus

$$\hat{y}_t = \frac{\exp(-0.4489 - 0.6823x_t)}{1 + \exp(-0.4489 - 1.6823x_t)}$$

$$= \frac{\exp(-0.4489 - 1.6823(2.793))}{1 + \exp(-0.4489 - 1.6823(2.793))} = 0.0057799$$

The one year EDF™ for Peppadew is estimated to be 0.57799 %.

This estimated EDF™ of Peppadew is now overlaid on the Absa credit rating scale, the basis of which is the Standard and Poor's rating system. Table 9.2 summarises the implied credit rating for a given EDF™ as calculated by Absa. It also gives market related credit spreads for each of the ratings.

An EDF™ of 0.578% for Peppadew implies that Peppadew's debt will trade 150 basis points above the Treasury Yield Curve. With this information, the issuer's yield curve may now be constructed from the Treasury curve. Finally, the debt security is then priced form the issuer's yield curve.

Rating	**EDF™**	**Default Spread** (basis points above Treasury Yield Curve)
AAA	0.02%	45
AA	0.08%	80
AA- / AA+	0.11% to 0.16%	82.5
A	0.22%	85
A-	0.28%	105
BBB+	0.37%	125
BBB	**0.50%**	**150**
BBB- / BB+	0.83% to 1.3%	320*
BB	1.5%	495
BB- / B+	2.6% to 3.4%	525
B	5.12%	575
B-	8.07%	600
CCC+ / CCC	12.70% to 16.70%	655
CC	17.5%	735

Table 9.2

Credit Rating and Default Spread for *One Year* Expected Default Frequency™

Source: Absa (EDF™ Values)
Sanlam Capital Markets (Default Spreads)

* The large jump in default spread between BBB and BBB- indicates the cut-off point between investment grade and non-investment grade debt.

9.6 Constructing the Issuer's Yield Curve

The issuer's yield curve (up to one year maturity) is implied from the treasury yield curve by adding the credit spread of 150 basis points to the risk free rates. If a parallel shift in the issuer's yield curve is assumed, the following is obtained:

Instrument	Yield*	Yield + Default Spread
3 month effective rate	8.00%	9.50%
3 month rate, 3 months from today. (I.e. 3X6 Forward Rate Agreement (FRA))	8.17%	9.67%
3 month rate, 6 months from today. (6X9 FRA)	8.42%	9.92%
3 month rate, 9 months from today. (9X12 FRA)	8.66%	10.16%
3 month rate, 12 months from today. (12X15 FRA)	9.03%	10.53%

Table 9.3

The Short End of Peppadew's Yield Curve

* Effective forward rates quoted by International Money Brokers, Reuters screen code IMBA.

Figure 9.4 depicts the issuer's yield curve relative to the treasury curve. Naturally, the assumption of a parallel shift is only one of many possibilities. In the South African interest rate environment, the shorter end of the curve is often the most volatile. Consequently, any of a number of scenarios could apply. The parallel shift chosen for this study mirrors the general view on short term interest rates.

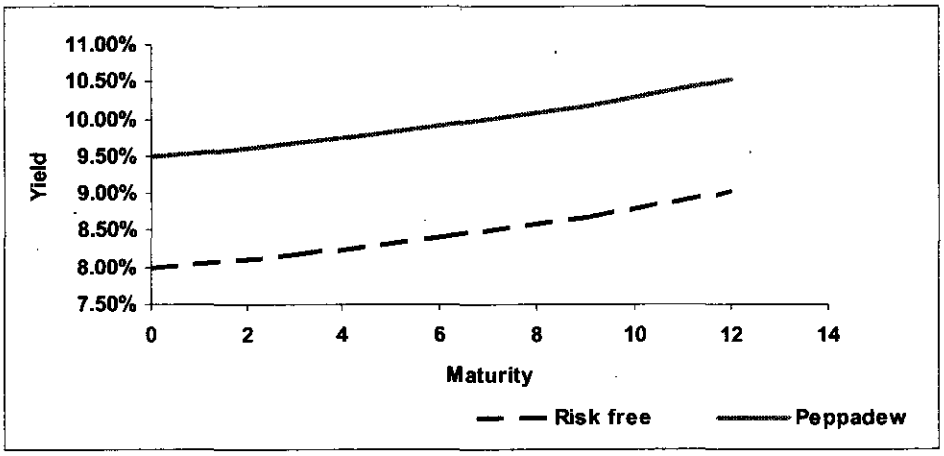


Figure 9.5

Issuer's Yield Curve versus the Treasury Yield Curve.

Now that the applicable credit spread and implied yield curve for Peppadew's debt has been determined, the next step is the pricing of the putable bond structure.

9.7 The Price of an Option-Free Bond

The price of an option-free bond is the present value of the promised cash flows, discounted at the applicable interest rates. Discounting may be done using either spot rates or periodic forward rates. The two methods must yield the same result to prevent riskless arbitrage. To illustrate, the implied spot rates are bootstrapped from the three month forward rates tabulated in Table 9.3 in the usual way. For figures and calculations, refer to the attached CD,

File Name	:	Peppadew International
Sheet Name	:	Interest Rate Tree

For the Peppadew curve, the following is obtained

Period	Three Month Forward Rate	Implied Spot Rate
Today	9.50%	9.50%
Three Months	9.67%	9.58%
Six Months	9.92%	9.70%
Nine Months	10.16%	9.81%
Twelve Months	10.53%	9.96%

Table 9.4

Spot and Forward Rates.

Discounting a zero-coupon, one year par bond at the forward rates results in a present value of

$$\frac{100}{(1+0.950)^{0.25}\times(1+0.967)^{0.25}\times(1+0.992)^{0.25}\times(1+0.1016)^{0.25}\times(1+0.1053)^{0.25}} = 88.814$$

Discounting a zero-coupon, one year par bond at spot rate results in a present value of

$$\frac{100}{(1+0.0996)^{(0.25\times5)}} = \frac{100}{(1+0.0996)^{1.25}} = 88.814\,.$$

The value of a one year, *non-putable*, zero-coupon, par bond for Peppadew is thus R88.81%. This means that for every R88,81 that Peppadew borrows via a standard bond issue, the company will have to repay R100 after one year, at maturity of the bond. This implies an annual 12.595% borrowing rate (or a monthly compounded rate of 11.92%).

The next step is to construct an interest rate tree model from which a *putable* bond may be priced. If Peppadew writes a put option against its own debt, the premium income from the put is incorporated to lower the borrowing cost. But the put option gives the investor the right to cease investment and claim the promised cash flows before maturity of the bond. A lattice approach to pricing option- embedded bonds models possible future interest rate movements and interest rate volatility in a relatively straight forward way, once the issuer's yield curve has been correctly determined. It is also very popular in the industry because of the logical and graphical way a tree represents the possible outcome of events.

9.8 Constructing a Binomial Interest Rate Tree

A binomial interest rate tree model is simply a graphical representation of one-period forward rates over time, based on an assumption about interest rate volatility. Refer Neftci (2000) for a discussion on the Black-Derman-Toy model which is the approach followed here. The tree, Figure 9.5, looks similar to the binomial stock price tree generated in Chapter 6. Each node represents a time period three months later from the node to its left. Moving in the direction of the arrow heads, the different paths that three month forward rates may take over one year are depicted. The tree recombines at each node with the assumption that three month forward rates can take on only to possible values in the next period – each one with equal probability. r_0 is known and represents the root of the tree. It is the current three month rate (or equivalently, the three month forward rate). The consecutive three month forward rates form the bottom branches of the tree. r_d represents the three month forward rate, three months from today, r_{dd} represents the three month forward rate, six months from today, r_{ddd} represents the three month forward rate, nine months from today and r_{dddd} is the three month forward rate, twelve months from today – all of these in the event of a decrease in interest rates. The assumption is made that three month forward rates evolve over time according to a lognormal random walk process. The implied volatility of three month forward rates is directly observable in the market. CEDEF, a large brokerage firm based in Switzerland, actively imply periodic interest rate volatility from South African interest rate derivatives. CEDEF is considered the market leader in this respect. Their services are provided through Reuters and their screen code is CEDEFOPT2 - attached in Appendix IV. Three month forward implied volatilities may be summarized from CEDEFOPT2 as follows:

Instrument	Volatility Bid – Offer
3 X 6 FRA	15% - 18%
6 X 9 FRA	16% - 19%
9 X 12 FRA	17% - 20%
12 X 15 FRA	18% - 21%

Table 9.5

Market Volatility Quotes. *Source*: CEDEF

If σ is the volatility of the applicable three month forward rate and r_d is known, then $r_u = r_d\left(e^{2\sigma}\right)$. In the second time period of the tree, r_{dd} represents the three month forward rate, three months from today if rates fall. Recursively, r_{ud} and r_{uu} may be calculated as

$$r_{ud} = r_{dd}\left(e^{2\sigma}\right)$$
$$r_{uu} = r_{dd}\left(e^{4\sigma}\right)$$

In the same way, the applicable interest rate at each node may be determined until the whole tree has been completed. However, the forward rates at each node must be determined in such a way that they are consistent with the volatility assumptions above as well as the market value of the standard bond calculated in Section 9.7 above. Since there is no simple formula for this, an iterative process must be used to determine the correct forward rates at each node.

An educated guess for the value of $r_1 = r_d$ is initially made. The corresponding three month forward rate in a rising interest rate scenario, r_u, may now be calculated as indicated in the tree. The fundamental rule for valuation is next applied: the bond value is the present value of expected cash flows (which include any coupon payments), weighted by the probability of occurrence, i.e. the assigned 50% probability of rates moving upward and 50% probability of rates moving downward. The present value of the cash flows is calculated at each of the two nodes, using the calculated forward rates, r_u and r_d as the appropriate discount rates. This present value is then compared to the expected fair value of the bond in three months' time. The guestimate of r_d may be considered accurate if these two values are the same. If not, however, that unique value for r_d must be determined iteratively using the same process until the present value of the bond calculated through the tree equals its market value. These steps are repeated at each node until maturity of the bond.

As mentioned before, valuation of the bond via the tree which models interest rates and accounts for interest rate volatility *must* equate to the value under straight forward discounting. If they do not, arbitrageurs will realize a riskless profit by selling the over-priced issue and buying the fair value bond. At the point in time in the tree when the put may be exercised by the investor, the bond value must be changed to reflect the *larger* of the current value (i.e. value if it is not put) or put strike price (the borrowing to be repaid). Figure 9.6 demonstrates the binomial interest rate tree, with applicable bond prices for the standard, one year, zero coupon, par Peppadew bond. Figure 9.7 illustrates the changes for the same issue, but which is now putable by the issuer at R94,00% (R94,00 per R100 nominal) after six months.

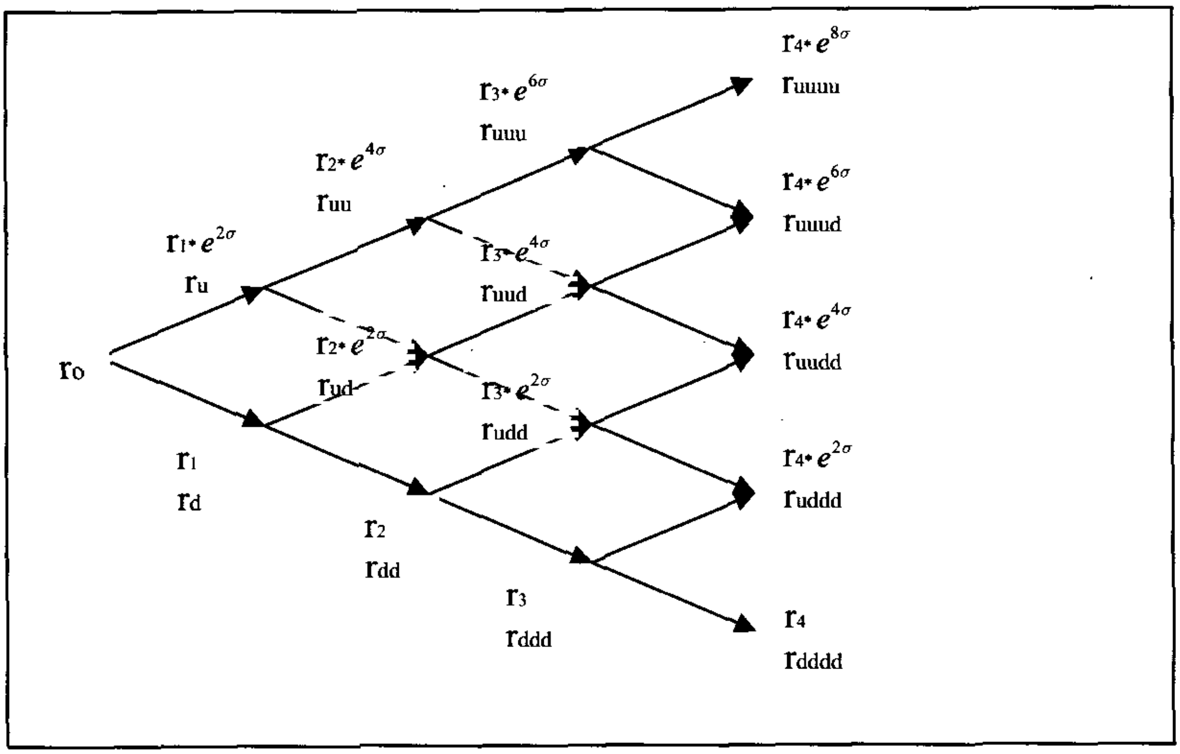


Figure 9.6

One year Binomial Interest Rate Tree with Three Month Forward Rates.

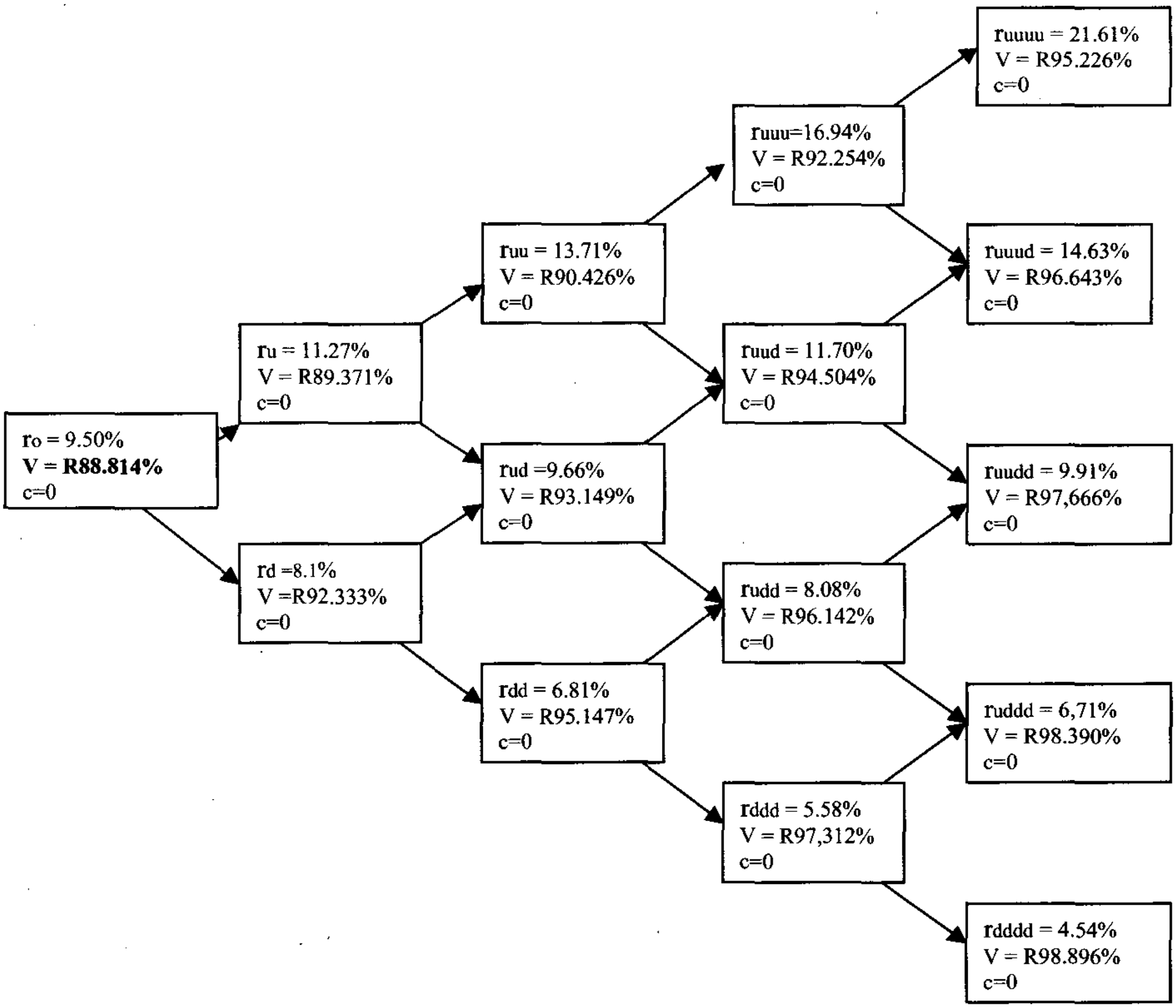


Figure 9.7
Finding the three month forward rates and valuing a standard, zero coupon, par bond issue with one year to maturity.
Issuer: Peppadew International (Pty)Ltd.

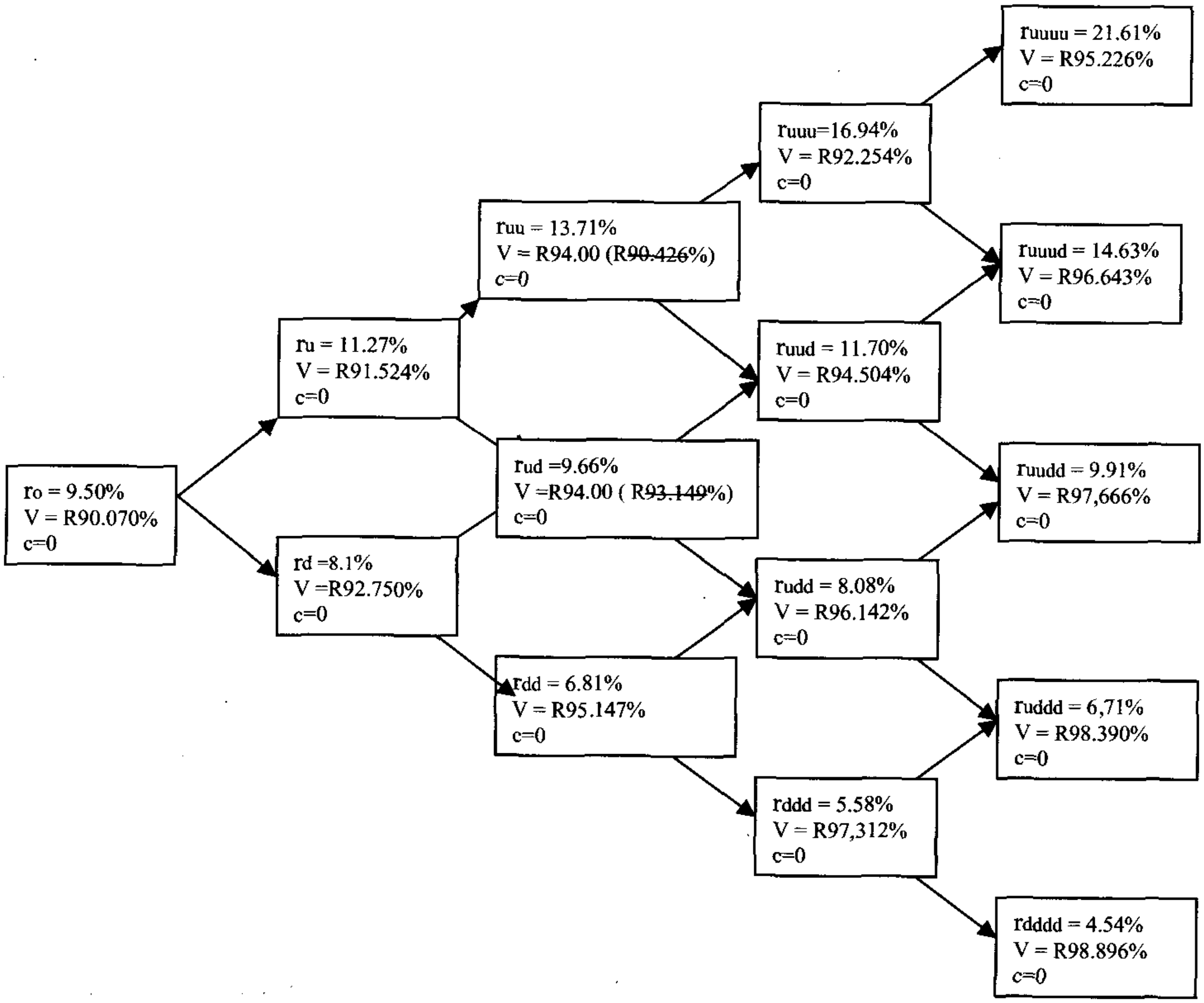


Figure 9.8
Valuing a *putable*, zero-coupon, par bond with one year to maturity which is callable after six months at R94%.
Issuer: Peppadew International (Pty)Ltd

9.9 Interpretation of Results

The path of interest rates modeled in Figure 9.6 is such that the bond value at the root of the tree is identical to the value of the bond when it was discounted at either the spot or three month forward rates in the beginning of Section 9.7. The price reflects the credit risk spread inherent in Peppadew's debt – for every **R88.814** the company borrows through a debt issue, it must repay R100 to investors after one year. The implied one year borrowing rate is 12.595%. There are no interim coupon payments to be made by the issuer to the investor.

Figure 9.7 incorporates into the zero coupon bond issue, a six month European put option sold to the investor by the issuer. The strike of the put option is assumed to be R94.00%. The value of the putable bond is again reflected at the root of the tree, but this time it is **R90.07%**. This new price is the sum of the standard bond issue value and the premium received from the written put option. By writing a R94% strike, six month put option, Peppadew effectively lowers their borrowing cost because for every R90.07 borrowed, the company must

(1) Repay R100 after one year *if the put option is **not** exercised* OR
(2) Repay R94.00 after six month if the investor exercises the option.

The implied borrowing rate for one year *if the option is not exercised* is 11.02%. I.e. 1.575% below that of a standard bond issue. If however, the option is exercised after six months, the implied annual borrowing rate drops to 9.12% ***but*** the issuer faces substantial maturity contraction and interest rate risk. The benefit of writing the option is readily observable in both of the possible outcomes, even though there are risks to be considered.

Table 9.6 contains three different scenarios which aid in understanding the risk-return profile of the putable bond structured for Peppadew international. In scenario one, the fair value of a standard bond after six months is R93.294%. It would thus be senseless to attempt to strike the put below fair value – no rational investor would purchase such an issue. In fact, the most sensible thing to do would probably be to strike the option exactly at fair value. This means that the issuer is prepared to pay back fair value of the bond at maturity of the option, and nothing more. The option premium, however, is at its lowest when the option is struck at fair value. Nevertheless, the issuer still enjoys a favourable borrowing rate for writing an at-the-money put option. What is noticeable from the table is how the borrowing rate decreases significantly with higher strike prices. This is a direct result of the higher premium received for writing an in-the-money put option. In effect, the issuer is agreeing to pay back more than fair value *if the put is exercised.* Peppadew (and any other issuer, for that matter) will only do so if they have real reasons to believe that the option will not be exercised. Otherwise, writing a deep-in-the-money put option against its debt may be extremely risky, bearing in mind the maturity contraction- and interest rate risk which Peppadew will face in the event of exercise against them. From the perspective of the investor, an embedded put option in a bond issue offers protection against increasing interest rates. That is, if interest rates should rise substantially during the life of the put option, new issues at higher interest rates will become more attractive. The investor will then put the original issue back to the issuer in order to invest in higher yielding securities and earn a better return on his/her investment. However, there are indeed many arguments as to why investors in high growth start-up companies may *not* exercise their options when they are deep in-the-money. The chance of converting debt into equity with runaway profits in the event of a listing, is but one of the reasons often debated in the industry.

A putable bond does not necessarily have to be a zero-coupon bond by nature. The size and frequency of coupon payments will usually be negotiated to meet the needs of both issuer and investor. For Peppadew, Scenario 2 and Scenario 3 of Table 9.6 illustrate how the value added by the put option decreases as the coupon rate increases. The coupon is assumed to be paid every three months as a percentage of the nominal amount. The higher the periodic coupon rate, the sooner the outstanding debt will be repaid and the less risk taken by the investor up to the put date (the first date at which he may demand repayment of the debt). The value of an exit clause after six months will thus steadily decrease as the coupon size and/or frequency increases until the putable bond offers no more benefit than a high coupon, non-putable bond.

Table 9.6

Scenario 1

Risk-return profile of zero coupon, one year bond, putable after six months.

Issuer: Peppadew International (Pty) Ltd

Coupon	0%
Nominal / Face Value of Bond	R100
Standard Bond Price	R 88.814%
Fair Value of Standard Bond after Six Months	R 93.294%
Prime Lending Rate (March 2004)	11.50% Nominal Annual Compounded Monthly (NACM)

Put Strike (R%)	**Price of corresponding Putable Bond** (R%)	**Implied Put Value** (R%)	**Implied Annual Borrowing Rate** *if issuer is put.*	NACM	**Implied Annual Borrowing Rate** *If issuer is NOT put.*	NACM
93.294	89.5650	0.7510	8.681 %	8.35 %	11.65 %	11.07 %
94	90.0702	1.2562	9.115 %	8.76 %	11.02 %	10.50 %
95	90.7858	1.9718	9.725 %	9.32 %	10.15 %	9.71 %
96	91.7059	2.8919	9.814 %	9.40 %	9.04 %	8.69 %
97	92.6612	3.8472	9.814 %	9.40 %	7.92 %	7.65 %
98	93.6165	4.8025	9.814 %	9.40 %	6.82 %	6.61 %
99	94.5717	5.7577	9.814 %	9.40 %	5.74 %	5.59 %
100	95.5270	6.7130	9.814 %	9.40 %	4.68 %	4.58 %

Table 9.6 (continued)

Scenario 2

Risk-return profile of 0.5% coupon, one year bond, putable after six months.

Frequency of coupon payments: Three monthly.

Issuer: Peppadew International (Pty) Ltd

Coupon		0.5%				
Nominal / Face Value of Bond		R100				
Standard Bond Price		R 91.146%				
Fair Value of Standard Bond after Six Months		R 94.727%				
Prime Lending Rate (March 2004)		11.50% Nominal Annual Compounded Monthly (NACM)				
Put Strike (R%)	**Price of corresponding Putable Bond** (R%)	**Implied Put Value** (R%)	**Implied Annual Borrowing Rate** *if issuer is put.*	NACM	**Implied Annual Borrowing Rate** *If issuer is NOT put.*	NACM
94.7272	91.7161	0.3970	5.51 %	5.38 %	9.24 %	8.87 %
95	91.7810	0.6349	7.26 %	7.03 %	8.96 %	8.61 %
96	92.4579	1.3119	7.96 %	7.68 %	8.16 %	7.87 %
97	93.1735	2.0275	8.56 %	8.24 %	7.33 %	7.09 %
98	94.1010	2.9550	8.64 %	8.31 %	6.27 %	6.10 %
99	95.0605	3.9145	8.64 %	8.31 %	5.20 %	5.08 %
100	96.0200	4.8740	8.64 %	8.31 %	4.14 %	4.07 %

Table 9.6 (continued)

Scenario 3

Risk-return profile of 1% coupon, one year bond, putable after six months.

Frequency of coupon payments: Three monthly.

Issuer: Peppadew International (Pty) Ltd

Coupon	1.00 %
Nominal / Face Value of Bond	R100
Standard Bond Price	R 93.478 %
Fair Value of Standard Bond after Six Months	R 96.160 %
Prime Lending Rate (March 2004)	11.50% Nominal Annual Compounded Monthly (NACM)

Put Strike (R%)	**Price of corresponding Putable Bond** (R%)	**Implied Put Value** (R%)	**Implied Annual Borrowing Rate** *if issuer is put.*	NACM	**Implied Annual Borrowing Rate** *If issuer is NOT put.*	NACM
96.160	93.9361	0.4199	4.58 %	4.49 %	6.50 %	6.31 %
97	94.1360	0.6579	6.27 %	6.10 %	6.23 %	6.06 %
98	94.8456	1.3676	6.88 %	6.67 %	5.43 %	5.30 %
99	95.5612	2.0832	7.46 %	7.22 %	4.65 %	4.55 %
100	96.5046	3.0266	7.51 %	7.26 %	3.62 %	3.56 %

9.10 Summary and Conclusion

The analysis of Chapter 8 served not only to highlight the enhanced valuation of Peppadew when the company's real assets are valued as growth options, but the insights gained enabled a debt funding structure to be engineered for the company. It is possible to price a market related putable bond issue for an unlisted, privately held company like Peppadew if the volatility of its assets can quantitatively be estimated. For Peppadew, this was achieved via the real options analysis by means of carefully selected business proxies. As a result, all the necessary inputs for the determination of Peppadew's credit default spread were calculated with relative ease and accuracy. Knowledge of the Treasury yield curve and the company's default spread lead to the construction of a yield curve for Peppadew's debt. From the company's yield curve, a debt security of almost any nature can quite effectively be priced via a lattice model. The result is that management of Peppadew may now consider debt financing as an option when deciding how to meet its capital requirements. From a broader perspective then, Chapter 8 and 9 highlight the fact that unlisted, privately owned companies of worth need not be excluded in debt origination and structured finance schemes. If quantitative methods of evaluation are employed, they can be become a new asset class worth taking notice of.

CHAPTER 10
CONCLUSION

> "We live not only our own lives but also the life of our time."
>
> Sir Laurens van der Post.

10.1 Conclusions

Financial markets are constantly evolving and players are forced to remain abreast if they wish to survive....such is "the life of our time". Recall the wave of privatisations, mergers and acquisitions, derivative innovations, financial engineering, securitisation etc. over the last ten years. Financial markets are becoming more and more sophisticated, driven primarily by innovation. And innovation derives not from knowing the right answers, but from understanding the right questions. When making business decisions of a strategic nature (often times based on the valuation of intangible assets), the real options approach offers a disciplined way of thinking about the "right questions". These questions typically revolve around uncertainty – and in particular, the consequences of uncertainty. Although often times misunderstood and even feared, it is uncertainty which leads to opportunity. And where there is opportunity, there are always "options" - to expand, to grow, to change, to act or to wait. Business managers know they have these options. They know also that to survive in a fast changing market place, they have to be flexible in their approach to business decisions and the uncertainty surrounding any such decisions. Real option analysis not only stresses the economic value of flexibility embedded in business activities, but directly offers a means of pricing and trading such flexibility.

Financial option mathematics forms the framework for all real investment decisions; integrating capital budgeting and strategic planning with long-term value maximization. Since the options approach is based on (arbitrage free) market equilibrium, it offers a consistent approach which is well suited to the analysis of non-financial, real assets, opportunities and investments. The ability to evaluate all real projects (normal or strategic) and all investments (financial and non-financial) based on a single, consistent options-based methodology has lead to a new era in the evolution valuation, capital budgeting and corporate finance.

10.2 Contribitions

For Peppadew International, the use of real options analysis enabled the company to explicitly value their real assets. This, together with the traditional NPV analysis resulted in a more accurate estimation of the total economic worth of the firm. It is referred to as an "expanded" or "strategic" NPV. Higher net worth implies an improved credit rating which in turn allows the company to access funds at competitive rates.

The rapid pace of financial innovation will certainly continue to expand the applicability and reliability of the real options paradigm. The more uncertainty business management faces, the more they are beginning to change the way they think. Higher uncertainty (in any economic or financial variable) does not necessarily make an investment opportunity less desirable, contrary to popular belief. In fact, discrepancies between finance theory and corporate reality highlight the importance of active managerial flexibility in adapting to uncertainty – this flexibility is now being viewed as real options with a broad scope of applications. Option valuation appears to offer a unifying evaluation framework for all real investment decisions by integrating capital budgeting and strategic planning under the umbrella of long-term value maximization.

Small and medium sized enterprises in particular, may benefit considerably by taking a strategic view of their business to determine if real options analysis can contribute to valuation and/or decision making processes. For large, listed organisations, the application of real options analysis lies mainly in disciplined decision making - enabling managers to make investment decisions that lead to greater shareholder value.

10.3 Future Directions

There remains a lot of scope for further application of real options to business decisions under uncertainty. The real life implementation issues of real options may be developed and documented comprehensively so that generic option-based and user friendly software may become available to managers. Further research onto how real options may also be applied to the valuation of flexibility in related areas such as competitive bidding, information technology, global energy and R&D, is also recommended. Research into the potential to evaluate all real projects (normal and strategic) and all financial instruments through a single, consistent option-based methodology has far-reaching implications for the field of corporate finance as a whole.

APPENDIX I

Finite Difference Methods

Finite difference methods are standard numerical procedures for solving complex option problems when analytic solutions are not available. In contrast to the tree model discussed in Chapter 6, which allows for easy visualization of the process of asset price movements over time, finite difference methods work directly from the Black-Scholes partial differential equation (or some variant of it) plus suitably chosen boundary conditions. Imagine a three dimensional graph where option values are measured vertically versus asset price measured on the horizontal axis. Time is represented on the second horizontal axis. The option values, which are as yet unknown, except at the boundaries of the graph (i.e. along the line representing expiry of the option), form a surface above the asset and time axis. The shape of this surface conforms to the partial differential equation that approximates a specific derivative. Sometimes it is possible to find analytical equations representing the surface to infinitesimally small detail. If not, the curved surface can be approximated by a mesh of points separated by finite "distances". Hence the name "finite differences". The shape of the mesh approximating the surface can then be calculated in a backward recursive manner by working inward from the boundaries where option values are known and using the relationships demanded by the partial differential equation. Like tree models, the fineness chosen for the mesh determines the accuracy of the option valuation.

The value of any option or contingent claim, *f*, on an underlying asset, *S*, must satisfy the fundamental partial differential equation

$$rf = \frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \qquad \text{(A)}$$

where r is the market risk free rate of return. For a simple extention of the PDE to include dividends, see Trigeorgis (2002) and John C. Hull (2002). The no-dividend scenario will be discussed here as a base case.

Choosing a finite number, *N*, of equally spaced time intervals, Δt, between the current time, zero, and maturity of the option, *T*, allows for *N+1* time points $0, \Delta t, 2\Delta t, \ldots\ldots T$ with $\Delta t = \frac{T}{N}$. In the same way, a finite number of stock prices are also chosen. Letting $S_{\max}$ be the stock price when the put has no value and defining $\Delta S = \frac{S_{\max}}{M}$, results in a total of *M+1* stock prices $0, \Delta S, 2\Delta S, \ldots\ldots S_{\max}$. One of these is assumed to be the current stock price. The grid consists of $(N+1)(M+1)$ points. Point (*i, j*) corresponds to time $i\Delta t$ and stock price $j\Delta S$. f_{ij} represents the option value at the point (*i, j*). **The objective is now to construct approximations of the partial derivatives $\frac{\partial f}{\partial t}, \frac{\partial f}{\partial S}$ and $\frac{\partial^2 f}{\partial S^2}$ of Equation (A).** In general, the two most popular approximations are the implicit and explicit difference methods.

Implicit Finite Differences.

At the point (*i, j*) on the grid above, the partial derivative, $\frac{\partial f}{\partial S}$, may be approximated as a *forward difference equation*

$$\frac{\partial f}{\partial S} = \frac{f_{i,j+1} - f_{ij}}{\Delta S} \qquad \text{(B)}$$

or by a *backward difference equation*

$$\frac{\partial f}{\partial S} = \frac{f_{i,j} - f_{i,j-1}}{\Delta S} \qquad \text{(C)}$$

In the implicit difference method, a symmetric approximation of $\frac{\partial f}{\partial S}$ is obtained at point (*i, j*) using the average of both Equation (C) and Equation (D):

$$\frac{\partial f}{\partial S}=\frac{f_{i,j+1}-f_{i,j-1}}{2\Delta S}\cdot \qquad \text{(D)}$$

For the partial derivative of the option, *f*, to time *t*, a forward difference approximation at *i* is suggested by Hull (1993). The value of *f* at time $j\Delta t$ is related to its value at time $(j+1)\Delta t$ by

$$\frac{\partial f}{\partial t}=\frac{f_{i+1,j}-f_{ij}}{\Delta t} \qquad \text{(E)}$$

In the same way as Equation (C) is the backward difference equation for $\frac{\partial f}{\partial S}$ at the point (*i,j*), the backward difference at point (*i*, *j*+1) is $\frac{f_{i,j+1}-f_{ij}}{\Delta S}$. Applying both these approximations results in a finite difference approximation for the second order partial derivative $\frac{\partial^2 f}{\partial S^2}$. I. e.

$$\begin{aligned}\frac{\partial^2 f}{\partial S^2}&=\frac{1}{\Delta S}\times\left(\frac{f_{i,j+1}-f_{ij}}{\Delta S}-\frac{f_{ij}-f_{i,j-1}}{\Delta S}\right)\\&=\frac{f_{i,j+1}+f_{i,j-1}-2f_{ij}}{\Delta S^2} \qquad \text{(F)}\end{aligned}$$

Approximations for each of the partial derivatives in Equation (A) have now been determined. Substituting Equations (D), (E) and (F) into Equation (A) and letting $S=j\Delta S$, gives

$$rf_{ij}=\frac{f_{i+1,j}-f_{ij}}{\Delta t}+r(j\Delta S)\frac{f_{i,j+1}-f_{i,j-1}}{2\Delta S}+\frac{1}{2}\sigma^2(j\Delta S)^2\frac{f_{i,j+1}+f_{i,j-1}-2f_{ij}}{\Delta S^2} \qquad \text{(G)}$$

for $i=1,2\ldots\ldots N\text{-}1$ and $j=1,2\ldots\ldots M\text{-}1$.

Equation (G) may be simplified and expressed as

$$f_{i+1,j} = a_j f_{i,j-1} + b_j f_{ij} + c_j f_{i,j+1} \tag{H}$$

where

$$a_j = \frac{1}{2} rj\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t = -\frac{1}{2} j\Delta t\left(r + \sigma^2 j\right)$$

$$b_j = 1 + \sigma^2 j^2 \Delta t + r\Delta t$$

$$c_j = -\frac{1}{2} rj\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t = -\frac{1}{2} j\Delta t\left(r + \sigma^2 j\right).$$

The system is solved recursively from the terminal option condition at $i = N$ and moving backward, one step at a time to the beginning at $i = 0$. The methodology is similar to the binomial tree approach discussed in Chapter 6. Note that Equation (H) expresses the relationship among three different and unknown option values, $f_{i,j-1}$, f_{ij} and $f_{i,j+1}$ at time *i*, and one known value, $f_{i+1,j}$, at time (*i*+1). a_j, b_j and c_j may, in fact, be viewed as the transitions probabilities to jump from state *j* at time *i* to state *j*+1, state *j* or state *j*-1 at time *i*+1, respectively.

For an American put option, Equation (A) is subject to one terminal condition and two boundary conditions. The value of the put at time *T* is $\max[X - S_T, 0]$ when S_T is the stock price at expiry time *T*. Thus

$$f_{Nj} = \max[X - j\Delta S, 0] \qquad \text{for } j = 0, 1 \ldots\ldots M \tag{I}$$

When the stock price is zero, the value of the put is equal to the strike price, *X*. I.e.

$$f_{i0} = X \qquad \text{for } i = 0,\ 1 \ldots\ldots N \tag{J}$$

As the stock price increases, the value of the option decreases so that

$$f_{iM} = 0 \qquad \text{for } i = 0, 1 \ldots\ldots N \qquad \text{(K)}$$

The three conditions described by Equations (I), (J) and (K) define the value of the put option at S = $S_{\max}$, $S = 0$ and $t = T$ along three edges of the grid. Equation (H) must now be employed to determine the value of f along the left side of the grid. I.e.the value of the option must be determined for each change in stock price, $S = j\Delta S$. For the last time interval before expiry, $T - \Delta t$, where $i = N$-1, Equation (H) may be written as

$$f_{Nj} = a_j f_{N-1,j-1} + b_j f_{N-1,j} + c_j f_{N-1,j+1} \qquad \text{for } j = 1, 2 \ldots\ldots M\text{-}1 \qquad \text{(L)}$$

A useful rule of thumb when considering the size of stock price changes and time intervals, is that $i\Delta t, \Delta t, (i+1)\Delta t$ ΔS should be kept proportional to $\sqrt{\Delta t}$.

Equation (L) represents $M - 1$ simultaneous equations that must be solved for $M - 1$ unknowns, $f_{N-1,1}, f_{N-1,2} \ldots\ldots f_{N-1,M-1}$. This does not involve inverting a matrix. As mentioned above, f_{Nj} is known from Equation (I). From Equations (J) and (K), $f_{N-1,0} = X$ and $f_{N-1,M} = 0$. The first equation in (L) may be used to express $f_{N-1,2}$ in terms of $f_{N-1,1}$; the second equation in turn may be used to express $f_{N-1,3}$ in terms of $f_{N-1,1}$ and so on. The final equation then provides a value for $f_{N-1,1}$. This value is then used to determine the other $f_{N-1,j}$. After the equations have been solved, each value of $f_{N-1,j}$ is compared to the value of early exercise, $X - j\Delta S$. If $f_{N-1,j} < X - j\Delta S$, then early exercise of the option is optimal at time $T - \Delta t$. $f_{N-1,j}$ is then set equal to $X - j\Delta S$.

The nodes corresponding to time $T - i\Delta t$, i = 2, 3……. N-2, are handled in the same way. Eventually this iterative process yields the beginning option values, $f_{01,}f_{02,}f_{03,}.....f_{0,M-1}$. Given the current value of S and time to maturity of the option, one of these is the American put option price of interest.

The beauty of the implicit finite difference method is its robustness. It always converges to the solution of the differential equation as ΔS and Δt approach zero. However, it is laborious in that $M-1$ simultaneous equations relating three different option values at time i to one value at time $i+1$, must be solved. For many practitioners and academics, the answer to this dilemma lies in the explicit finite difference method.

Explicit Finite Differences.

The implicit difference method may be simplified if the values of $\frac{\partial f}{\partial S}$ and $\frac{\partial^2 f}{\partial S^2}$ at the point (i,j) are assumed to be the same as at the point (i+1,j). This is a reasonable assumption since the time increment, Δt, is usually small. Substitution into both Equations (D) and (F) yields

$$\frac{\partial f}{\partial S} = \frac{f_{i+1,j+1} - f_{i+1,j-1}}{2\Delta S} \quad \text{and} \quad \frac{\partial^2 f}{\partial S^2} = \frac{f_{i+1,j+1} + f_{i+1,j-1} - 2f_{i+1,j}}{\Delta S^2}$$

Equation (H) then becomes

$$f_{ij} = a_j^* f_{i+1,j-1} + b_j^* f_{i+1,j} + c_j^* f_{i+1,j+1} \qquad \text{(M)}$$

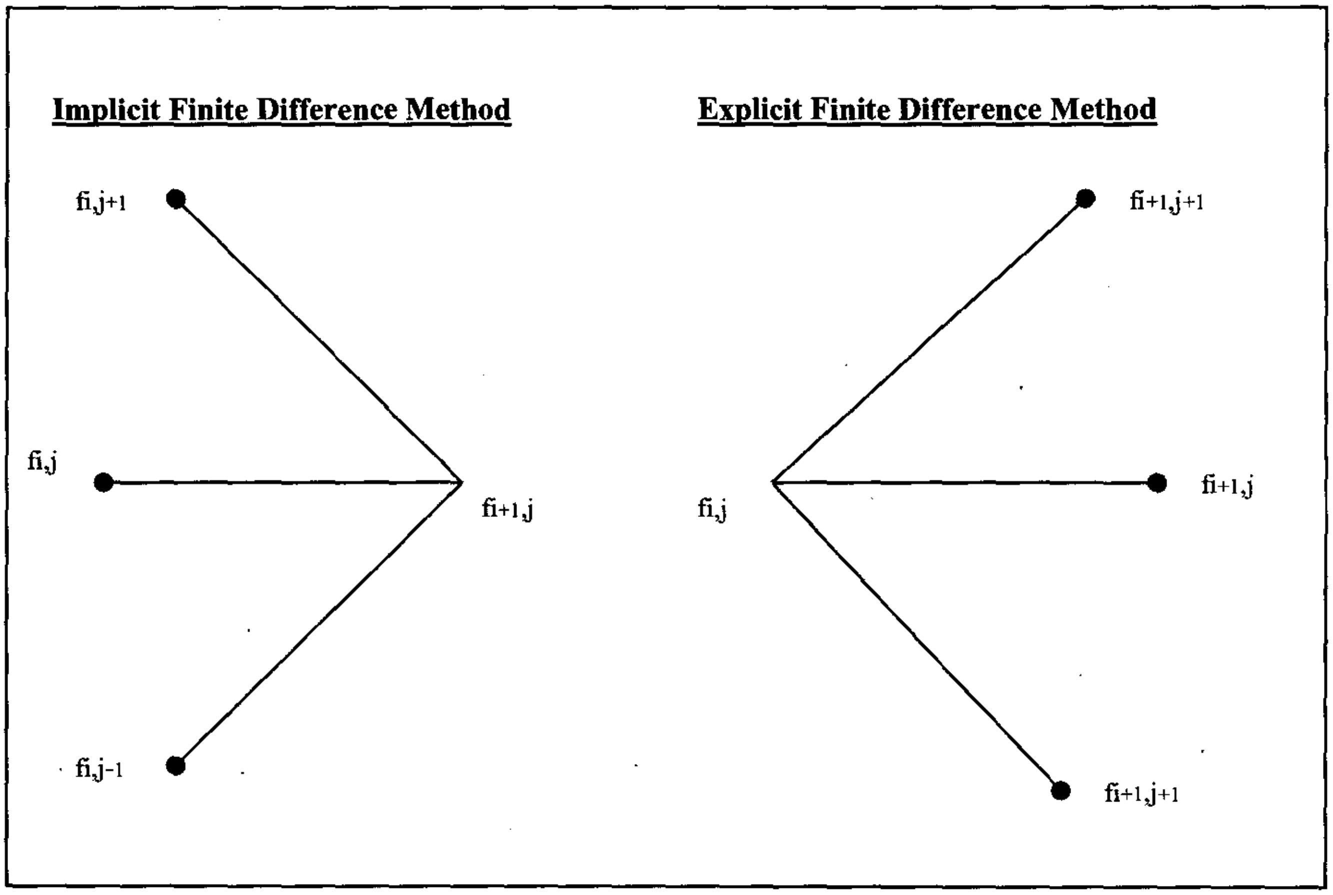
where

$$a_j^* = \frac{1}{1+r\Delta t}\left(-\frac{1}{2}rj\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$

$$b_j^* = \frac{1}{1+r\Delta t}\left(1 - \sigma^2 j^2 \Delta t\right)$$

$$c_j^* = \frac{1}{1+r\Delta t}\left(\frac{1}{2}rj\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$

Equation (M) gives the relationship between one (unknown) value of the option, f_{ij}, at time $i\Delta t$ and three different (but known) values of the option, $f_{i+1,j-1}, f_{i+1,j}, f_{i+1,j+1}$, at subsequent time $(i+1)\Delta t$. All the data is known (or has been derived) from the previous step in the backward iterative process. Thus, contrary to the implicit difference scheme, only one equation need be solved at a time. In this regard, the explicit finite-difference method is similar to the lattice approach discussed in Chapter 6. The figure below illustrates the difference between the implicit and explicit finite difference methods.



The coefficients, a_j^*, b_j^*, c_j^*, contain the actual risk-neutral probabilities of the system which may be interpreted as follows :

1. $-\frac{1}{2}rj\Delta t+\frac{1}{2}\sigma^2 j^2\Delta t$ is the probability of the state variable (or stock price), S, decreasing from $j\Delta S$ to $(j-1)\Delta S$ in time Δt.

2. $1-\sigma^2 j^2 \Delta t$ is the probability of S remaining unchanged at $j\Delta S$ in time Δt.

3. $\frac{1}{2} rj\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t$ is the probability of S increasing from $j\Delta S$ to $(j+1)\Delta S$ in time Δt.

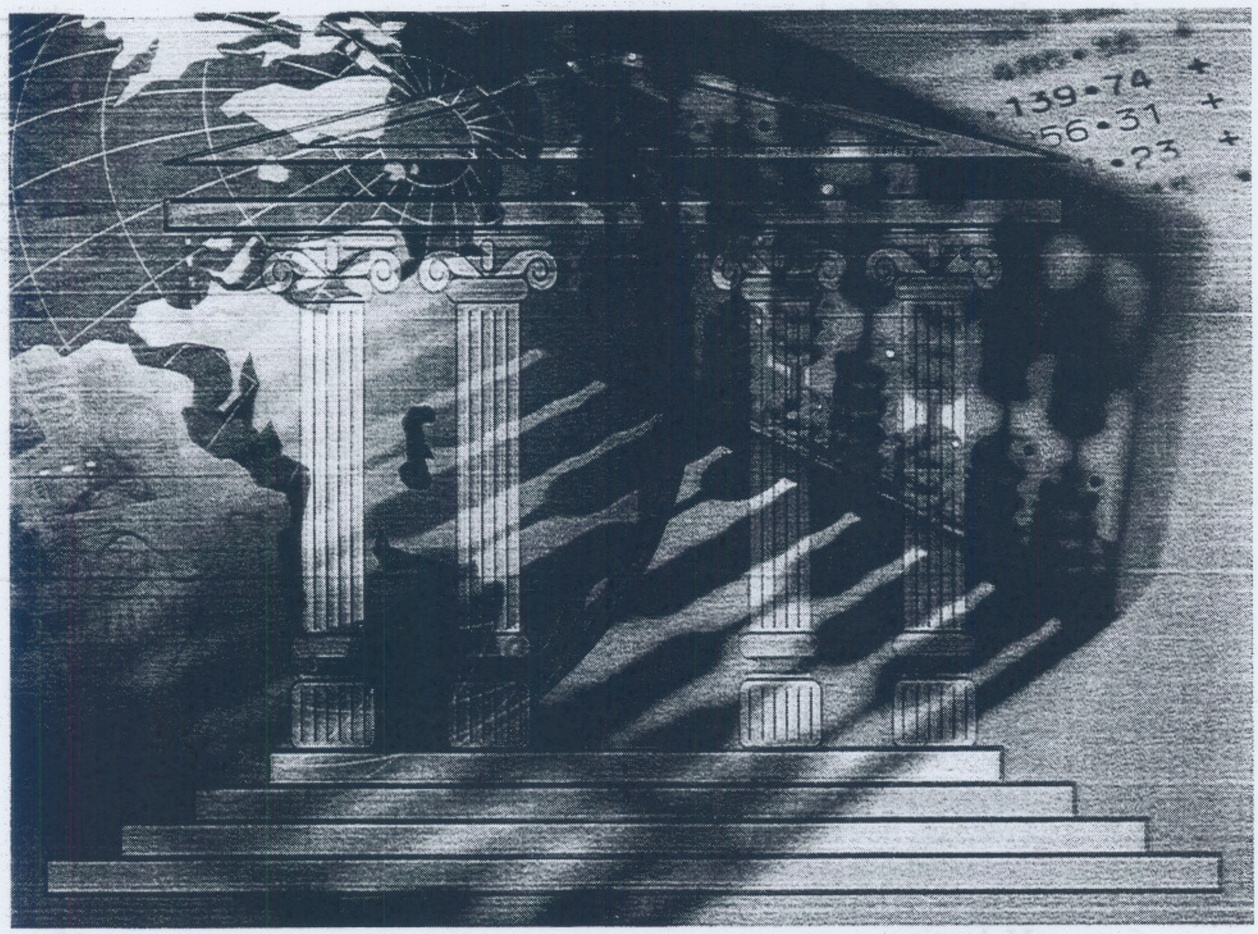
These probabilities sum to unity and they should all be positive. If these two conditions do no hold, instability problems may be experienced for large values of j. A logarithmic transformation of S is often applied in order to improve stability and efficiency.

In view of the above, the explicit finite difference expression in Equation (M) essentially states that the current option value is obtained from the expected future option values one period hence, discounted back one period at the riskless rate of interest. I.e. using the probabilities in a trinomial lattice, the system moves from the expected value of f at time $(i+1)\Delta t$ to the value of f at time $i\Delta t$.

Although the explicit difference method is less complicated than its implicit twin, it does not always produce results that converge to the solution of the differential equation. This problem can however, be overcome in a number of ways. Hull and White show how this problem may be overcome in "Valuing Derivative Securities using the Explicit Finite Difference Method." (Hull and White, 1990)

APPENDIX II

KPMG Valuation Report of Peppadew International, Pty, Ltd



Peppadew International (Pty) Ltd and Piquante International

Valuation

September 2002

This report contains 22 pages

Corporate Financial Services

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1 Introduction

1.1 Purpose of the document

1.1.1 In accordance with our terms of reference, an independent valuation of Peppadew International (Pty) Ltd ("Peppadew") and Piquante International ("Piquante") is required for the purpose of providing the directors of Denny Mushrooms (Pty) Ltd ("Denny's") with an independent value of Denny's interest in the companies as at 1 October 2002 if disposed as a going concern on a willing buyer and willing seller basis.

1.2 Sources of information

1.2.1 The principal sources of information used in preparing this report are as follows:

- Historic financial statements for Peppadew for the periods 30 June 2000 and 30 September 2001;
- Management accounts for the eleven months ended 31 August 2002 and an estimate of the September numbers for Peppadew;
- Forecasts for Peppadew and Piquante as provided by management of Peppadew;
- Interviews with Reilly Keen (financial director- Denny and Peppadew), Richard Baker (chairman- Denny), and Phil Ovens (managing director- Peppadew);
- Research on similar local and international companies; and
- A prior information memorandum prepared by Peppadew management.

1.3 Limitations on scope

1.3.1 We have relied upon the sources of information referred to above and the representations provided in undertaking the valuation. Except where specifically stated, we have not sought to establish the reliability of those sources. We have however reviewed the information and have sought explanations for key trends and salient features identified by us. We have also satisfied ourselves, as far as possible, that the information presented is consistent with other information obtained by us in the course of the work undertaken to prepare this report.

1.3.2 We draw your attention to the fact that the valuation does not constitute an audit or due diligence.

1.3.3 The valuation is a function of the assumptions incorporated within the valuation methodology. In particular, the valuation assumes that the future projections as indicated by management are realisable or, as a minimum, understood by a potential investor who has a similar outlook of the projected level of cash flow. Given the limited available history of the companies, it is important to note that reliance has been placed on managements forecasts being realistic.

1.3.4 Net revenue has been assumed to grow at an average compounded growth of 66% p.a. for the next five years. This growth assumption is the biggest contributor to the value derived for Peppadew and Piquante. Any variation in the growth rate of future revenues will have a material impact on the value of these operations (refer to section 7 for reasonableness tests)

1.3.5 Limited information was made available in connection with Piquante. We have therefore relied heavily on management for the level of historic activity and the agreements that exist.

1.3.6 The true value negotiated between parties on a willing buyer willing seller basis may differ from this value as it is dependent upon considerations, including but not limited to, relative positions of strength, emotive issues, differing views of trading projections, growth potential, different assessments of risk, human resource issues, warranty conditions, etc., all of which can only be determined through a process of negotiation.

1.4 Confidentiality

1.4.1 This report is provided on the basis that it is for the information of Peppadew and its directors only and that it will not be quoted or referred to, in whole or in part, without our prior written consent.

2 Industry environment

2.1 Overview

2.1.1 Sweet piquante peppers are the first new fruit to be launched in 26 years. The main focus of Peppadew is the contracting of farmers and the processing and marketing of these sweet piquante peppers to a worldwide market, which meets the worldwide trend to spicy foods and eating away from home or easier food preparation at home.

2.2 Industry competitiveness

2.2.1 Peppadew currently operate in the pickle and condiments sector of the food market. However, as apposed to other fruit companies, Peppadew have exclusive use of the local and international patents, trademarks and plant breeding rights for sweet piquante peppers via a royalty agreement with Piquante. It is estimated that these rights will prevail for at least the next 10 years during which time the company intends establishing a worldwide brand which will act as a barrier to entry.

2.2.2 Due to the intellectual property rights that have been registered worldwide there are therefore no direct competitors for the fruit. There are also no direct substitutes. They compete with other peppers and spicy foods but have the advantage of a unique taste that differentiates them from the rest of the market.

2.3 Customer strength

2.3.1 Peppadew have a number of high profile local and international customers. These include:

- South Africa- listed with every major retailer including Woolworths and Pick & Pay;
- UK- Majority of retailers including Sainsburys, Tesco, Asda, Waitrose and Coscos;
- US- High profile up market stores like Wegmans, Kings, and Food Emporium. Also working with Pizza Hut and Domino Pizza.

2.3.2 Other countries where a market has already been established include Japan, Scandinavia, Canada, Switzerland, Italy and Germany. New markets being established at present include Holland, Mexico, New Zealand, Australia and Dubai.

2.3.3 Each international market has a chief agent/ distributor who facilitates the placing of goods in the retail outlets. Due to the broad geographic focus, no market is currently key. It is therefore felt that no customer has excessive strength and limited risks are associated with the customers.

2.4 Supplier strength

2.4.1 Peppadew has control of the international plant breeder rights and as such control their own supply by outsourcing the farming. However, the concern is that as this is a new plant, there is limited knowledge on the ideal growing conditions. As a result it is currently difficult to forecast the yields that will be achieved and the variability of supply exposes Peppadew to the risk of not meeting demand or having an oversupply.

2.4.2 Although suppliers do not pose a risk, an irregular supply of the fruit does pose a potential risk. The risks are reduced as a result of the short time it takes for the plant to grow from a seed to a fruit producing plant. At present the plant grows over a four month period and produces fruit for a further four months. They are therefore able to estimate potential oversupply and make some adjustments in the medium term to the next harvest.

2.4.3 Peppadew are endeavouring to further address this risk by commissioning studies of the ideal growing conditions and assessing the possibility of utilising intensive farming for part of their supply.

3 Company overview

3.1 Company backgrounds

3.1.1 Sweet piquante peppers were discovered by Johan Steenkamp at his Plettenburg Bay holiday home. Extensive research of the plant led him to discover that the plant was a new species that enabled him to name the plant and to secure breeders rights. He also developed a secret recipe to process the fruit that effectively removed the extreme heat from the fruit. He invented the brand Peppadew and followed this by registering the plant breeders rights and patent of the process internationally as well as the trademark.

3.1.2 The production facilities were housed in Peppadew Tzaneen (Pty) Ltd (now Peppadew International (Pty) Ltd) and the intellectual property rights are housed in Piquante International, which earns 3% royalty on the invoiced value of Peppadew sales.

3.1.3 He initially received funding from a Scandinavian company. In July 2000 Denny Mushrooms acquired a shareholding in Peppadew International and Piquante International and have since been providing the majority of the funding and management expertise for ongoing operations.

3.1.4 The current shareholders of Peppadew International are as follows:

Denny Mushrooms	89%
Johan Steenkamp	10%
Other	1%

3.1.5 The current shareholders of Piquante International are:

Denny Mushroom	50% + 1 share
Others	50%

3.2 Products and services

3.2.1 The current focus is to provide limited products to a broad international market. This involves the supply of the fruit only. Once the brand is established Peppadew will market other products such as pastes and spices. This valuation has not considered the markets for these additional products due to the uncertainty attached to these markets and the lack of information available on the potential markets.

3.3 Commercial agreements or alliances

3.3.1 Peppadew has the perpetual exclusive rights to the use of the fruit and the patents registered within Piquante. As the controlling shareholder of Piqante , Denny's also have direct control over the intellectual property. No other significant or material commercial agreements or alliances exist.

3.4 Historic results

3.4.1 Set out below is a summary of the income statements up to the EBIT (earning before interest and tax) level of Peppadew for the financial years ended 28 February 1999 to 30 September 2002 No formal financials were provided to us for Piquante International.

Table 1: Historical and forecast income statement for Peppadew

	Audited	*Audited*	*Unaudited Actuals and Forecast*
	16 months	*15 months*	*12 months*
	30 June 00	*30 Sept 01*	*31 Sept 02*
	R'000	*R'000*	*R'000*
Net invoice value	N/A	N/A	20 904
Sales deductions	N/A	N/A	(2 766)
Net Revenue	6 437	11 390	18 138
Cost of sales	(2 565)	(7 005)	(15 066)
Gross profit	3 872	4 385	3 072
Direct marketing	0	(966)	(4 437)
Operating expenses	(10 643)	(10 625)	(6 189)
Earnings before interest and tax (EBIT)	(6 771)	(7 206)	(7 554)
Key performance indicators			
Gross profit %	60%	38%	15%
Net revenue growth rate (annualised)	N/A	89%	99%
Direct marketing as a % of net invoice value	0%	8%	21%
Operating costs as % of net invoice value	165%	93%	30%
EBIT to net invoiced value	(105%)	(63%)	(36%)

Source: Audited annual financial statements and management accounts

Revenue

3.4.2 Since Denny Mushrooms became involved, they have managed to grow net revenues at over 80% a year due largely to their marketing expertise and contacts.

Gross Profit

3.4.3 The Gross profit percentage has dropped due to the learning curve in automating some of the processes to handle the packing of products for the food service market. It is believed that the problems identified are well on their way to being solved.

Direct Marketing

3.4.4 Direct marketing has exceeded the accepted norm of 15% in the food industry. This is due to the focus on breaking into new international markets and the costs associated with this. It is anticipated that the higher than normal levels will continue for a short period after which it will reduce to more acceptable levels.

Operating expenses

3.4.5 The operating expenses have been better controlled as the increased revenues have absorbed some of the excess capacity. Furthermore the recent absolute drop in costs is due to a focus on costs and some forex gains that have been included in the operating expenses.

4 Basis of valuation

4.1 Methodology

4.1.1 There is no single standard or specific formula available to determine the fair market value of a company's shares but rather a variety of approaches, the most common being:

- Discounted cash flow;
- Capitalisation of maintainable earnings at a fair price earnings ratio;
- Capitalisation of future dividends at a fair dividend yield; and
- Net asset value.

4.1.2 The above valuation methodologies have been summarised below:

- The discounted cash flow methodology is based on the anticipated future cash flows of the business. These projected future cash flows are discounted together with the value of the company in perpetuity, at the company's weighted average cost of capital, taking into account the risks associated with the business;
- The capitalisation of maintainable earnings methodology is based on the assumption that an investor, in purchasing a business, is acquiring a future earnings stream. The expected future maintainable earnings are capitalised at an appropriate price-earnings ratio, having regards to the risks and growth potential of the business being valued;
- The capitalisation of future dividends at a fair dividend yield methodology is used to value a minority interest, which interest has no influence on the business's earnings or its declared dividend. The value of the minority holding is determined by capitalising the anticipated dividends at an appropriate dividend yield; and
- The net asset value methodology is useful in valuing non-trading businesses which are primarily asset based e.g. property and commodity companies. It is also a useful methodology when the "going concern" assumption is no longer applicable to the entity being valued.

4.2 Basis of valuation

4.2.1 The primary valuation technique adopted in undertaking this valuation has been the discounted cash flow methodology, assuming a willing buyer and willing seller, and *based on anticipated future profits and cash flows* (as per our letter of engagement).

4.2.2 As there are currently no positive earnings and therefore no dividends we have not used either capitalization method. Due to the size of the shareholders loans, the net asset value is also meaningless and has therefore been ignored.

5 Discounted free cash flows

5.1 Free cash flows

5.1.1 The future forecast cash flows to be discounted are the "free cash flows" which is the cash available from a business after all internal funding requirements necessary to maintain a required rate of growth have been met. Free cash flow is defined as the cash available to all providers of finance (shareholders and lenders to the company) and is discounted at the weighted average cost of capital to determine the present value of the future forecast free cash flows.

5.1.2 Free cash flows = NOPAT - I

where

NOPAT (net operating profit after tax) is the pre-interest, pre-non operating and extraordinary items, after-tax earnings of the business.

I (investment) is the investment in operating assets (net of depreciation) required to maintain and grow the business. This includes the flow of funds as a result of changes in working capital and investment in capital equipment.

Forecast free cash flows- Peppadew

5.1.3 Set out in Annexure I are the projected free cash flows of Peppadew for the years ending 30 September 2003 to 30 September 2012. The forecast cash flows have been prepared by Peppadew management. These forecasts and the assumptions underlying these forecasts have been discussed with management and compared with historical cash flows generated during the periods ended 30 June 2000, 30 September 2001 and 30 September 2002.

5.1.4 The summarised forecast income statements of Peppadew for the years ending 30 September 2003 to 30 September 2007 are presented below. For presentation purposes a further five forecast years are included in Annexure I.

Table 2: Summarised income statements of Peppadew

	Forecast *30 Sept 03* *R'000*	*Forecast* *30 Sept 04* *R'000*	*Forecast* *30 Sept 05* *R'000*	*Forecast* *30 Sept 06* *R'000*	*Forecast* *30 Sept 07* *R'000*
Net invoiced value	41 098	80 149	140 247	200 553	260 718
Sales deductions	(3 141)	(7 384)	(12 923)	(18 481)	(24 024)
Net Revenue	37 957	72 765	127 324	182 072	236 694
Cost of sales	(23 148)	(44 193)	(74 366)	(105 987)	(139 448)
Gross profit	14 809	28 572	52 958	76 085	97 246
Direct marketing	*(5 588)*	*(10 000)*	*(18 000)*	*(22 000)*	*(25 000)*
Operating expenses	*(8 429)*	*(12 162)*	*(17 857)*	*(23 949)*	*(30 443)*
Earnings before interest and tax (EBIT)	792	6 410	17 101	30 137	41 803
Key performance indicators					
Gross profit %	36%	36%	38%	38%	37%
Net revenue growth rate (annualised)	N/A	92%	75%	43%	30%
Direct marketing as a % of invoice value	14%	12%	13%	11%	10%
Operating costs as % of turnover	21%	15%	13%	12%	12%
EBIT to revenue	2%	8%	12%	15%	16%

Source: Management forecasts

Revenue

5.1.5 The first five years are expected to have significant revenue growth as new markets are entered and each market grows. This is expected to taper off gradually until a steady growth rate of 6% is achieved in 2012.

Gross Profit

5.1.6 Cost of sales is determined based on an expected gross profit percentage of between 35 and 40%. This does not take into account the potential increase in gross profit margin due to Peppadew costs being incurred in declining South African rands while receiving hard currencies. Furthermore, exported fruit produces a higher margin and as the sales mix moves more towards export sales, the average gross margin should improve.

Direct marketing

5.1.7 Direct marketing is expected to drop as a percentage of revenue to a more acceptable level of around 10% as the markets become more established.

Operating expenses

5.1.8 Additional costs have been allocated to Peppadew for expenses that Denny incurs on their behalf but which are not charged to Peppadew at present. This relates to the use of Denny's IT, debt collection, financial director and system support facilities.

Forecast free cash flows- Piquante

5.1.9 Set out in Annexure I are the projected free cash flows of Piquante for the years ending 30 September 2003 to 30 September 2012. The forecast cash flows have been prepared by Peppadew management. These forecasts and the assumptions underlying these forecasts have been discussed with management. Historical information was not made available to us and we have therefore been unable to make comparisons. This forecast is a function of the Peppadew forecasts, given that Piquante earns a 3% royalty on Peppadew turnover.

5.1.10 The summarised forecast income statements of Piquante for the years ending 30 September 2003 to 30 September 2007 are presented below. For presentation purposes a further five forecast years are included in Annexure I.

Table 3: Summarised income statements of Piquante

	Forecast 30 Sept 03 R'000	*Forecast 30 Sept 04 R'000*	*Forecast 30 Sept 05 R'000*	*Forecast 30 Sept 06 R'000*	*Forecast 30 Sept 07 R'000*
Royalty	1 233	2 404	4 207	6 017	7 822
Expenses	(350)	(420)	(504)	(605)	(726)
Earnings before interest and tax (EBIT)	883	1 984	3 703	5 412	7 096

Source: Management forecasts

Royalty

5.1.11 At present the royalty agreement results in a payment of 3% of all invoiced sales from Peppadew to Piquante. We have therefore correlated Piquante's royalty revenue to be in line with Peppadew's revenue projections.

Expenses

5.1.12 The only costs to be borne by Piquante relate to the renewal of the intellectual property rights, defence of these property rights and some administration costs involved in running the company. The expenses are being grown at 20% per annum to take into account increased protection of the intellectual property as the product becomes more widely known.

Overall assumptions

5.1.13 We have assumed that a potential investor in Peppadew and Piquante has a similar outlook of the assumptions used to be able to place reliance on the projected earnings/ cash flows.

5.1.14 Since the forecast is based on assumptions concerning future events, actual results may vary from the forecast presented and variations may be material. Accordingly, we express no opinion on whether or not the forecasts will be achieved.

Taxation

5.1.15 Peppadew currently has a significant assessed loss that is anticipated to be in the region of R34 million. We have taken this into account in the forecast cash flows in order to determine the point at which taxes become payable. Thereafter an effective tax rate of 32% has been assumed.

5.1.16 Piquante is registered offshore and as such is expected to have a low tax rate. However, as the effective management of Piquante is based in South Africa, we have assumed that Piquante will be taxed in line with South African tax legislation at an effective tax rate of 32%.

Perpetuity value

5.1.17 The calculated perpetuity value for Peppadew assumes that the company will continue to grow at 6% p.a. (nominal) and that depreciation approximates future capex requirements. The growth rate is considered feasible as management believe that the main source of earnings will be in hard currencies which should continue to appreciate against the South African rand.

Shareholders loans

5.1.18 Shareholder loans currently comprise over 90% of the funding of Peppadew with the majority being funded by Denny Mushrooms. As there are no fixed terms of repayments and interest is charged but capitalised, we feel that the shareholder loans represent quasi equity and we have therefore valued these loans separately.

5.1.19 In order to ascertain their value we have assumed that the free cash flow required/ generated by Peppadew will by funded by/ repaid to the shareholders as and when it is earned, and have excluded interest from the calculation. We have then discounted the resultant cash flows at the weighted average cost of capital of Peppadew.

5.1.20 In order to apportion the loans between the various shareholders we have used the ratio that will be applicable after taking into account the additional funds required for the next two years. As Denny Mushrooms will provide all of this funding the current split in shareholder loans will not be maintained. Detailed calculation are included in Annexure I.

5.2 Weighted average cost of capital

5.2.1 A discount rate to determine the present value of the stream of future cash flows has been based on the company's Weighted Average Cost of Capital (WACC). A company's WACC is derived from the cost of equity and the after tax cost of debt in proportions to the long term target capital structure of the company.

5.2.2 For the purposes of these valuations we have assumed that as the risks with Peppadew and Piquante relate to the same product they should be valued using the same WACC. Similarly, the repayment of the shareholder loans has been valued using the WACC rate.

Cost of equity

5.2.3 The cost of equity capital (C) is determined by applying the Capital Asset Pricing Model, which is

$$C = R_f + \beta (R_m - R_f)$$

where

R_f is the risk free rate

R_m is the return on the market

$R_m - R_f$ is the market risk premium

β is the risk index or beta co-efficient.

Risk free rate (R_f)

5.2.4 A risk free rate of 11.67% has been assumed, based on the prevailing return on long-term government stock (R153) as at 1 October 2002. The R153 has been selected as it is a long term Government Bond with high liquidity and also attracts overseas investors.

Market risk premium (R_m - R_f)

5.2.5 A market risk premium is the premium over and above the risk free rate appropriate for investing in an average listed company. This premium is based on empirical studies, which compare the return on the All Share Index ("ALSI") to the yield on long term government bonds.

5.2.6 Aswath Damodaran in his book Investment Valuation states a market risk premium of 7,5% is applicable for emerging markets without political risk. Stern Stewart and Co. believe a market risk premium of 6% (UK and USA research) is correct for whichever country one researches, as the "country risk" is included in the risk free rate. In a survey published by PWC in February 2000 it was shown that the majority of valuation practitioners in South Africa use between 5% and 8% as a market premium. The debate of what the exact South African market risk premium should be has not been concluded. For the purposes of this report and in line with the majority of KPMG's valuations and based on KPMG's own research, a market premium of 7.5% has been used.

Risk index or Beta co-efficient

5.2.7 The beta co-efficient is a measure of the volatility of the company's share price relative to the overall stock market. A higher beta co-efficient indicates a more volatile share price and therefore a perceived higher investment risk. The beta co-efficient of a company is a function of its business risk index (determined by operating risk, strategic risk, asset management, size and diversity) and its financial risk index (determined by the level of gearing in a company). For the purpose of a valuation the beta co-efficient of a listed company is usually derived from the company's historic share price movement relative to the market.

5.2.8 Since Peppadew is an unlisted entity there is no share price history on which to base the Beta co-efficient calculation. The Beta co-efficients of companies involved in comparable activities listed on the Johannesburg Stock Exchange are used as a proxy.

5.2.9 The results of regressions of share prices of the companies listed below, against the JSE Actuaries All Share Index as performed by the Department of Statistical Services at the University of Cape Town for the five year period ended June 2002, using monthly data, are given below.

Table 4:Comparative company beta's

	Beta	*Weighting*	*Result*
AVI	0.65	10%	0.065
Crookes Brothers	0.35	10%	0.035
Illovo	0.63	30%	0.189
Intertrading	0.76	50%	0.38
Average			0.669

Financial Risk Service for Johannesburg Stock Exchange Shares, a quarterly bulletin published by the University of Cape Town

5.2.10 The above comparable companies are used and weighted as follows:

AVI

5.2.11 AVI focuses on a smaller range of products that fall into the more luxury type food items similar to Peppadew. The exception here is their interests in Consol glass. It has therefore only been weighted at 10%.

Crookes Brothers

5.2.12 Crookes Brothers produces primary agricultural products in sugar cane, fruit, grain and animal husbandry. Due to the focus on primary agricultural products we have only applied a 10% weighting.

Illovo

5.2.13 Illovo focuses on a single product and some value added components. Due to the similar focus on a single product we have weighted this Beta at 30%.

Intertrading

5.2.14 Intertrading is an international trading company with a focus on the procurement and export of South African fruit. Due to its similarity with Peppadew's focus we have applied a weighting for Intertrading of 50%.

Resultant Beta

5.2.15 Based on our experience and given the information shown above we estimate that an acceptable Beta for Peppadew and Piquante is 0.669 being the weighted average of the above companies.

Additional risk factors

5.2.16 It is normal for a private company to trade at a discount to listed companies mainly as a result of the restrictions on the marketability and transferability of the shares. We have therefore applied a 20% discount for Peppadew not being a listed company.

5.2.17 In addition, the following factors need to be taken into consideration as these are unique to the business being valued (i.e. not shared by the companies listed above).

- This is a newly discovered fruit with no prior history in terms of the growth of the plant. There are therefore risks associated with ensuring a constant reliable supply to meet the demand. Either over or under supply could expose Peppadew to financial distress; and
- Although the initial signs are promising, the demand for the fruit is still largely unknown with limited historical information available.

5.2.18 As a result of the above risks, we feel that the cost of equity should have a further 20% premium attached to it.

Cost of equity

5.2.19 Taking the above into consideration, a cost of equity of 24.03% is considered reasonable for Peppadew.

After tax cost of debt

5.2.20 At present Denny's has access to funds at a rate of prime which implies an after tax rate of 11.9%. With the backing of its holding company, it is felt that Peppadew will be able to borrow money at the same rate.

Capital structure

5.2.21 Denny Mushroom currently operates with a 50:50 debt equity ratio. It is felt that once Peppadew reaches a stable state they will be able to revert to this level. However, at this stage the risks associated with debt would be high and it is unlikely that this ratio will be used in the foreseeable future. Therefore we have used a 100% equity weighting to calculate the WACC rate.

Summary

5.2.22 Based on the above we have effectively used the cost of equity of 24.03% to discount the future cash flows.

6 Valuation summary

6.1.1 Based on the forecast assumptions and the calculation's set out in annexure I, the valuation of Denny's interest in Peppadew and Piquante amounts to approximately R58.2 million and is summarised below.

Table 5: Summary of valuation

		R'000
Peppadew		
Discounted value of free cash flows at 1 October 2002		59 154
Less: debt at valuation date		(11 795)
Value of equity		47 359
Denny Mushroom stake	*89%*	*42 149*
Piquante		
Discounted value of free cash flows at 1 October 2002		17 785
Less: debt at valuation date		0
Value of equity		17 785
Denny Mushroom stake	*50%*	*8 892*
Therefore Denny Mushroom total interest		
Peppadew		42 149
Piquante		8 892
Shareholder loans		7 139
Total		58 180

Source: KPMG calculations

6.1.2 Notwithstanding this valuation, we stress that the valuation would be materially different in the event that one or more of the fundamental assumptions discussed earlier do not prove to be valid. In particular the turnaround from historical losses to projected profits is a critical assumption.

6.1.3 The above valuation is a function of the assumptions incorporated within the valuation methodology. In particular, the valuation assumes that the future projections as indicated by management are realisable or, as a minimum, understood by a potential investor who has a similar outlook of the projected level of cash flow.

6.1.4 The true value negotiated between parties on a willing buyer willing seller basis may differ from this value as it is dependent upon considerations, including but not limited to, relative positions of strength, emotive issues, differing views of trading projections, growth potential, different assessments of risk, human resource issues, warranty conditions, etc., all of which can only be determined through a process of negotiation.

7 Reasonableness tests

7.1 Sensitivity analysis

7.1.1 The most important value driver of Denny's interest is the discount rate used to discount the future cash flows. Due to the turnaround projections and a new worldwide product, the discount rate could be very subjective and could range from a low of 20% for a defensive food company to 35% for a company in its early growth phase.

7.1.2 We have therefore provided a range of discount rates and the resultant values that would be generated. Set out below is a sensitivity analysis of Denny's interest in Peppadew and Piquante

Table 6: Sensitivity analysis on the discount rate

Discount rate used	*Value of Denny's interest* *R'000*
20.00%	80 922
24.03%	58 180
25.00%	53 814
30.00%	36 122
35.00%	24 186

Source: KPMG valuation

7.1.3 The above calculation highlights the sensitivity of the value to changes in the discount rate used.

Annexure I

Assumptions
Peppadew Valuation

01-Oct-02		16 mths 30-Jun-00	15 mths 30-Sep-01	12 mths 30-Sep-02	12 mths 30-Sep-03	12 mths 30-Sep-04	12 mths 30-Sep-05	12 mths 30-Sep-06	12 mths 30-Sep-07	12 mths 30-Sep-08	12 mths 30-Sep-09	12 mths 30-Sep-10	12 mths 30-Sep-11	12 mths 30-Sep-12	Maintainable
MACRO ECONOMIC ASSUMPTIONS															
RSA Inflation					6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%
Effective Tax Rate	32%														
REVENUE															
Net invoiced Sales					41 098	80 149	140 247	200 553	260 718	312 862	359 791	395 770	435 347	461 468	489 156
Revenue growth post Peppadew forecasts						*95%*	*75%*	*43%*	*30%*	*20%*	*15%*	*10%*	*10%*	*6%*	*6%*
Sales Deductions															
Rebates					510	1 618	2 831	4 049	5 263	6 316	7 263	7 989	8 788	9 315	9 874
Other					276	1 050	1 838	2 629	3 417	4 100	4 715	5 187	5 706	6 048	6 411
Transport					2 055	4 007	7 013	10 028	13 037	15 644	17 991	19 790	21 769	23 075	24 460
Damages					300	709	1 241	1 775	2 307	2 768	3 184	3 502	3 852	4 083	4 328
COSTS															
Total Cost of Sales- Materials					17 082	35 701	62 477	89 343	116 146	139 375	160 281	176 310	193 941	205 577	217 912
% Growth						*109%*	*75%*	*43%*	*30%*	*20%*	*15%*	*10%*	*10%*	*6%*	*6%*
Total Cost of Sales- Manufacturing exp					6 877	9 628	13 479	18 870	26 419	31 703	36 458	40 104	44 114	46 761	49 567
% Growth						*40%*	*40%*	*40%*	*40%*	*20%*	*15%*	*10%*	*10%*	*6%*	*6%*
Direct marketing					5 588	10 000	18 000	22 000	25 000	30 000	34 500	37 950	41 745	44 250	46 905
% Growth						*79%*	*80%*	*22%*	*14%*	*20%*	*15%*	*10%*	*10%*	*6%*	*6%*
Selling and Distribution					2 322	4 528	7 924	11 331	14 730	17 676	20 327	22 360	24 596	26 072	27 636
% Growth						*95%*	*75%*	*43%*	*30%*	*20%*	*15%*	*10%*	*10%*	*6%*	*6%*
Marketing					198	228	262	301	346	415	477	525	578	612	649
% Growth						*15%*	*15%*	*15%*	*15%*	*20%*	*15%*	*10%*	*10%*	*6%*	*6%*
General and Administration					4 445	5 897	8 382	10 995	14 011	16 813	19 335	21 269	23 396	24 799	26 287
% Growth						*33%*	*42%*	*31%*	*27%*	*20%*	*15%*	*10%*	*10%*	*6%*	*6%*
Royalty Costs (Incl in G&A already)	3.0%				1 233	2 404	4 207	6 017	7 822	9 386	10 794	11 873	13 060	13 844	14 675
Notional charge from Denny to Peppadew					480	509	539	572	606	642	681	722	765	811	860
CAPEX REQUIREMENTS															
Plant and equipment					2 500	4 000	3 000	5 000	5 000	3 500	3 500	3 500	3 500	1 004	1 064
Depreciation included in peppadew forecasts of overheads				718	811	1 136	1 590	2 226	3 117	3 740	4 301	4 731	5 205	5 517	5 848
Growth applied to overheads					*13.0%*	*40.0%*	*40.0%*	*40.0%*	*40.0%*	*20%*	*15%*	*10%*	*10%*	*6%*	*6%*
Depreciation included in forecast cash flow statement	6%				984	1 000	750	750	750	795	843	893	947	1 004	1 064

Income Statement Peppadew Valuation 01-Oct-02	16 mths 30-Jun-00 Audited	15 mths 30-Sep-01 Audited	12 mths 30-Sep-02 Forecast	12 mths 30-Sep-03	12 mths 30-Sep-04	12 mths 30-Sep-05	12 mths 30-Sep-06	12 mths 30-Sep-07	12 mths 30-Sep-08	12 mths 30-Sep-09	12 mths 30-Sep-10	12 mths 30-Sep-11	12 mths 30-Sep-12	Maintainable
Net Revenue	6 437	11 390	18 138	37 957	72 765	127 324	182 072	236 694	284 033	326 638	359 301	395 232	418 946	444 082
Net invoice value			20 904	41 098	80 149	140 247	200 553	260 718	312 862	359 791	395 770	435 347	461 468	489 156
Sales Deductions			(2 766)	(3 141)	(7 384)	(12 923)	(18 481)	(24 024)	(28 829)	(33 153)	(36 468)	(40 115)	(42 522)	(45 074)
Rebates			(422)	(510)	(1 618)	(2 831)	(4 049)	(5 263)	(6 316)	(7 263)	(7 989)	(8 788)	(9 315)	(9 874)
Other			(274)	(276)	(1 050)	(1 838)	(2 629)	(3 417)	(4 100)	(4 715)	(5 187)	(5 706)	(6 048)	(6 411)
Transport			(1 885)	(2 055)	(4 007)	(7 013)	(10 028)	(13 037)	(15 644)	(17 991)	(19 790)	(21 769)	(23 075)	(24 460)
Damages			(185)	(300)	(709)	(1 241)	(1 775)	(2 307)	(2 768)	(3 184)	(3 502)	(3 852)	(4 083)	(4 328)
Total Cost of sales	(2 565)	(7 005)	(15 066)	(23 148)	(44 193)	(74 366)	(105 987)	(139 448)	(167 338)	(192 438)	(211 682)	(232 850)	(246 821)	(261 631)
Cost of materials			(9 719)	(17 082)	(35 701)	(62 477)	(89 343)	(116 146)	(139 375)	(160 281)	(176 310)	(193 941)	(205 577)	(217 912)
Cost of sales overheads			(6 065)	(6 877)	(9 628)	(13 479)	(18 870)	(26 419)	(31 703)	(36 458)	(40 104)	(44 114)	(46 761)	(49 567)
remove Depreciation from overheads (grown based on % increase)			718	811	1 136	1 590	2 226	3 117	3 740	4 301	4 731	5 205	5 517	5 848
Gross Profit	3 872	4 385	3 072	14 809	28 572	52 958	76 085	97 246	116 695	134 199	147 619	162 381	172 124	182 451
Gross Profit Margin	*60.2%*	*38.5%*	*14.7%*	*36.0%*	*35.6%*	*37.8%*	*37.9%*	*37.3%*	*37.3%*	*37.3%*	*37.3%*	*37.3%*	*37.3%*	*37.3%*
Direct marketing	0	(966)	(4 437)	(5 588)	(10 000)	(18 000)	(22 000)	(25 000)	(30 000)	(34 500)	(37 950)	(41 745)	(44 250)	(46 905)
Selling and Administration Expenses	(10 643)	(10 625)	(6 189)	(8 429)	(12 162)	(17 857)	(23 949)	(30 443)	(36 342)	(41 664)	(45 769)	(50 281)	(53 298)	(56 496)
Depreciation (per cash flow statement)			(718)	(984)	(1 000)	(750)	(750)	(750)	(795)	(843)	(893)	(947)	(1 004)	(1 064)
Selling and Distribution			(1 886)	(2 322)	(4 528)	(7 924)	(11 331)	(14 730)	(17 676)	(20 327)	(22 360)	(24 596)	(26 072)	(27 636)
Marketing			(347)	(198)	(228)	(262)	(301)	(346)	(415)	(477)	(525)	(578)	(612)	(649)
R&D														
General and admin			(3 238)	(4 445)	(5 897)	(8 382)	(10 995)	(14 011)	(16 813)	(19 335)	(21 269)	(23 396)	(24 799)	(26 287)
Notional allocated charge from Denney for staff				(480)	(509)	(539)	(572)	(606)	(642)	(681)	(722)	(765)	(811)	(860)
Operating profit/ loss before financing (EBIT)	(6 771)	(7 206)	(7 554)	792	6 410	17 101	30 137	41 803	50 353	58 036	63 900	70 355	74 576	79 051
Percentage of Sales	*-105.2%*	*-63.3%*	*-36.1%*	*1.9%*	*8.0%*	*12.2%*	*15.0%*	*16.0%*	*16.1%*	*16.1%*	*16.1%*	*16.2%*	*16.2%*	*16.2%*

Free Cash Flow **Peppadew Valuation** **01-Oct-02**		**16 mths** **30-Jun-00**	**15 mths** **30-Sep-01**	**12 mths** **30-Sep-02**	**12 mths** **30-Sep-03** **1**	**12 mths** **30-Sep-04** **2**	**12 mths** **30-Sep-05** **3**	**12 mths** **30-Sep-06** **4**	**12 mths** **30-Sep-07** **5**	**12 mths** **30-Sep-08** **6**	**12 mths** **30-Sep-09** **7**	**12 mths** **30-Sep-10** **8**	**12 mths** **30-Sep-11** **9**	**12 mths** **30-Sep-12** **10**	**Maintainable** **10**
EBIT					792	6 410	17 101	30 137	41 803	50 353	58 036	63 900	70 355	74 576	79 051
Add back non cash items															
Depreciation					984	1 000	750	750	750	795	843	893	947	1 004	1 064
Tax					0	0	0	(6 397)	(13 377)	(16 113)	(18 571)	(20 448)	(22 514)	(23 864)	(25 296)
Assessed Loss					(34 448)	(33 656)	(27 246)	(10 145)	0	0	0	0	0	0	0
Profits					792	6 410	17 101	30 137	41 803	50 353	58 036	63 900	70 355	74 576	79 051
Assessed Loss					(33 656)	(27 246)	(10 145)	0	0	0	0	0	0	0	0
Operating Cash flows					1 776	7 410	17 851	24 489	29 176	35 035	40 307	44 345	48 788	51 715	54 818
% Increase						*317%*	*141%*	*37%*	*19%*	*20%*	*15%*	*10%*	*10%*	*6%*	*6%*
Working Capital Requirements					(9 781)	(6 500)	(9 000)	(11 000)	(11 000)	(12 320)	(10 004)	(7 670)	(8 437)	(5 568)	(5 902)
Stock					(8 480)	(2 500)	(5 000)	(6 000)	(6 000)	(9 210)	(6 328)	(4 851)	(5 336)	(3 522)	(3 733)
Debtors					(1 750)	(5 000)	(5 000)	(6 000)	(6 000)	(9 544)	(5 786)	(4 436)	(4 879)	(3 220)	(3 414)
Creditors					449	1 000	1 000	1 000	1 000	6 433	2 109	1 617	1 779	1 174	1 244
Capex Requirements					(2 500)	(4 000)	(3 000)	(5 000)	(5 000)	(3 500)	(3 500)	(3 500)	(3 500)	(1 004)	(1 064)
Free Cash Flow					(10 505)	(3 090)	5 851	8 489	13 176	19 215	26 803	33 176	36 851	45 143	47 852
% Increase						*-71%*	*-289%*	*45%*	*55%*	*46%*	*39%*	*24%*	*11%*	*23%*	*6%*
Perpetual Growth	6%														265 402
PV Factor					0.81	0.65	0.52	0.42	0.34	0.27	0.22	0.18	0.14	0.12	0.12
Discounted Cash Flow Wacc=	24.03% 24.03%				(8 469)	(2 009)	3 067	3 587	4 489	5 278	5 936	5 924	5 305	5 240	30 806

Peppadew Enterprise Value			59 154
less Liabilities			
Shareholder Loans- Denny	☑ Future SH Loans (2004)		(3 942)
Shareholder Loans- Inter co Denny			(3 197)
Shareholder Loans- Other			(907)
Other Loans (face value)			(3 750)
Peppadew 100% Equity Value			47 359
Denny Mushroom Interests			
Denny Mushroom Equity Share- Peppadew		89.0%	42 149
Denny Mushroom Shareholder Loans- Peppadew			3 942
Denny Mushroom Inter Company Loans- Peppadew			3 197
Denny Mushroom Equity Share- Piquante		50.0%	8 892
			58 180

Sensitivity Table		**Total Interest**
		58 180
WACC	20.0%	80 922
	24.0%	58 180
	25.0%	53 814
	27.5%	44 071
	30.0%	36 122
	32.5%	29 589
	35.0%	24 186

Shareholder Loans			16 mths	15 mths	12 mths	12 mths	12 mths	12 mths	12 mths	12 mths	12 mths	12 mths	12 mths	12 mths	12 mths	
Peppadew Valuation			30-Jun-00	30-Sep-01	30-Sep-02	30-Sep-03	30-Sep-04	30-Sep-05	30-Sep-06	30-Sep-07	30-Sep-08	30-Sep-09	30-Sep-10	30-Sep-11	30-Sep-12	**Maintainable**
	01-Oct-02					1	2	3	4	5	6	7	8	9	10	10
Free cash flow available to repay loans						(10 505)	(3 090)	5 851	8 489	13 176	19 215	26 803	33 176	36 851	45 143	47 852
Shareholders Loans Balance O/S					42 630	53 135	56 225	50 374	41 884	28 709	9 494	(17 309)	(50 485)	(87 336)	(132 479)	(180 331)
Therefore, Free Cash flows						(10 505)	(3 090)	5 851	8 489	13 176	19 215	9 494				
Free Cash Flow						(10 505)	(3 090)	5 851	8 489	13 176	19 215	9 494	0	0	0	0
% Increase							*-71%*	*-289%*	*45%*	*55%*	*46%*	*-51%*	*-100%*	*#DIV/0!*	*#DIV/0!*	*#DIV/0!*
Perpetual Growth		6%														0
PV Factor						0.81	0.65	0.52	0.42	0.34	0.27	0.22	0.18	0.14	0.12	0.12
Discounted Cash Flow	Wacc=	24.0%				(8 469)	(2 009)	3 067	3 587	4 489	5 278	2 103	0	0	0	0
Shareholder Loan Value				8 045												

Split between Denny and other shareholders

	Face Value 30 Sept 2002	% of Total Shareholder loans 2002	Apportionment based 2002 split	Face Value 30 Sept 2004	% of Total Shareholder loans 2004	Apportionment based 2004 split
Shareholder Loans- Denny	(15 543)	36%	(2 933)	(26 021)	49%	(3 942)
Shareholder Loans- Inter co Denny	(21 102)	50%	(3 982)	(21 102)	40%	(3 197)
Shareholder Loans- Other	(5 985)	14%	(1 130)	(5 985)	11%	(907)
	(42 630)		(8 045)	(53 108)		(8 045)

Peppadew WACC

Cost of Equity

Risk free rate (R153 as at 1 October 2002)	11.67%
Market Premium	7.50%

Weighted Average Beta

	Beta's	Weight	Beta
Anglovaal (AVI)	0.65	10%	0.065
Crookes Brothers (	0.35	10%	0.035
Intertrading (ITR)	0.76	50%	0.38
Illovo (ILV)	0.63	30%	0.189
	0.5975	100%	0.669

Cost of Equity	16.7%
Early growth risk of new product	20.0%
Private company risks	20.0%
Cost of Equity	24.0%

Cost of Debt

Rate charged	17.0%
After tax	11.9%

Proposed Debt Equity Ratio

Debt	0%
Equity	100%

WACC

Cost of Equity- weighted	24.0%
Cost of Debt- weighted	0.0%
	24.0%

APPENDIX III

GARCH(1,1) Model for the Estimation of Historical Volatility

The general form of the model, $\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$, is applied with $u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$, the continuously compounded return on day *i*, *i*= 1,2.....*m*. σ_n is defined as the volatility of a market variable on day *n*, as estimated at the end of day *n*-1. The square of the volatility on day *n*, σ_n^2, is the variance rate. S_i is the value of the market variable at the end of day *i*.

Trial estimates of the GARCH(1,1) parameters : $\alpha = 0.028550$, $\beta = 0$, $\omega = 0.00037$

Maximum value of the maximum likelihood function, $\sum_{i=1}^{m}\left[-\ln\left(\sigma_i^2\right) - \frac{u_i^2}{\sigma_i^2}\right] = 2{,}239.95$

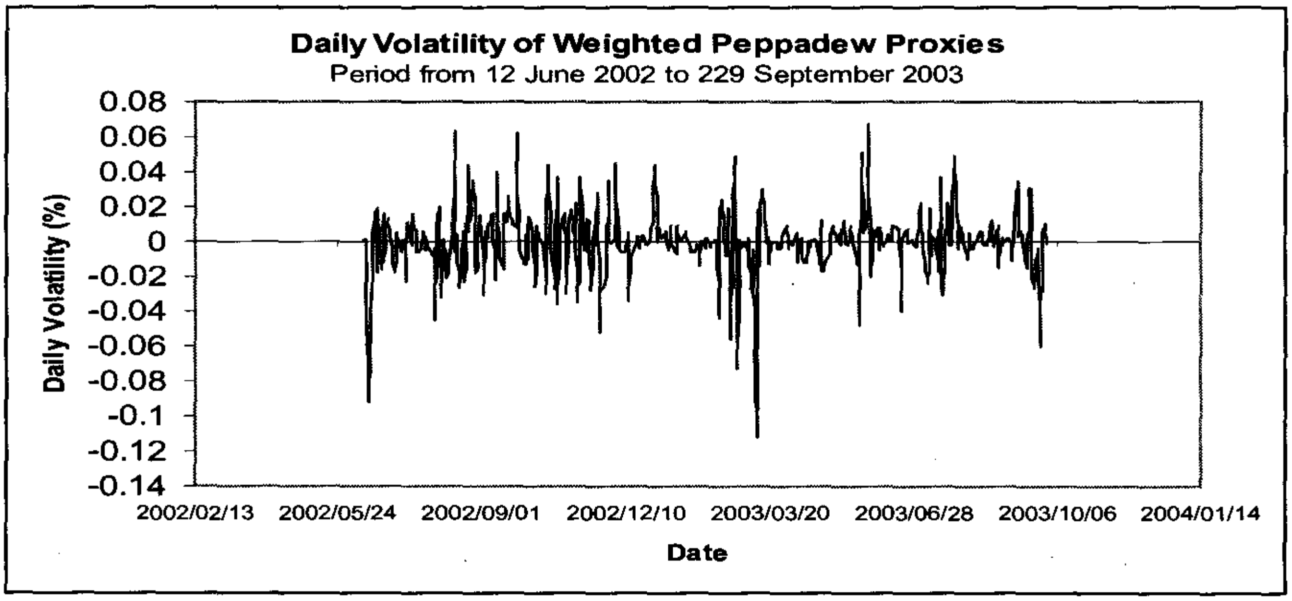
The long-term variance rate is $\frac{\omega}{1-\alpha-\beta} = 0.00038087$

The long-term volatility is $\sqrt{0.00038087}$ or 1.95% per day.

This translates into a $1.95\% * \sqrt{250} = 30.86\%$ annualised volatility, in line with the value obtained using the standard (homoskedastic) approach to estimating volatility from historical data. There does not appear to be any significant shift in volatility during the period over which the analysis was conducted.

The illustration below is a graphical representation of the daily volatility of the proxy companies used to estimate the volatility of Peppadew. For data and calculations, refer to the attached CD,

File Name	:	Peppadew International
Sheet name	:	Garch Model



APPENDIX IV

Market Forward Rates and Volatility Quotes.

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Printed By Reuters                                             Monday, 8 March 2004 16:21:3
09:51 08MAR04   CEDEF OPTIONS NYON +41 22 994 1632   CH02387      CEDEFOPT02
  SOUTH AFRICAN RAND INTEREST RATE OPTIONS
ATM  3M OPT    6M OPT    12M OPT   2YR OPT   3YR OPT   4YR OPT    5YR OPT
   SWAPTION VOLATILITIES
1Y  21.0 23.0  20.5 22.5  18.8 20.8  15.7 18.7  15.0 18.0  14.3 17.3  13.3 16.3
2Y  20.2 22.2  19.8 21.8  18.3 20.3  15.7 18.7  14.7 17.7  13.8 16.8  13.0 16.0
3Y  19.3 21.3  19.0 21.0  17.8 19.8  15.5 18.5  14.2 17.2  13.5 16.5  12.7 15.7
4Y  18.4 20.4  18.4 20.4  17.6 19.6  15.0 18.0  13.8 16.8  13.2 16.2  12.5 15.5
5Y  17.8 19.8  18.0 20.0  17.3 19.3  14.7 17.7  13.5 16.5  13.0 16.0  12.4 15.4
6Y  17.3 19.3  17.6 19.6  17.0 19.0  14.4 17.4  13.4 16.4  12.9 15.9  12.5 15.5
7Y  16.9 18.9  17.1 19.1  16.6 18.6  14.2 17.2  13.4 16.4  13.0 16.0  12.6 15.6
8Y  16.5 18.5  16.7 18.7  16.3 18.3  14.1 17.1  13.5 16.5  13.1 16.1  12.7 15.7
9Y  16.2 18.2  16.4 18.4  16.0 18.0  14.1 17.1  13.6 16.6  13.2 16.2  12.8 15.8
10Y 16.0 18.0  16.2 18.2  15.9 17.9  14.1 17.1  13.7 16.7  13.3 16.3  12.9 15.9

1Y  16.5 19.5      IRG VOL  3's 6's   15.0/18.0
2Y  17.0 20.0               6's 9's   16.0/19.0
3Y  16.7 19.7               9's 12's  17.0/20.0
4Y  16.0 19.0  CAPS/FLOORSS   0
5Y  15.5 18.5  VS 3M JIBARR   0
6Y  15.0 18.0               0
7Y  14.7 17.7               0
8Y  14.5 17.5               0
9Y  14.2 17.2               0
10Y 14.0 17.0               0
```

```
Printed By Reuters :                                           Monday, 8 March 2004 17:12:
1512 SA INTERNATIONAL MONEY BROKERS-DERIVATIVES 838-3512   IMBA
   VVOLB BID F OFF  VOL LST      BID   OFF
  1X4 50  8.02  8.14  50      1YR 0     0    DATE  8/3/04
  2X5 50  8.07  8.19  50      2YR 0     0    JIBAR 8
  3X6 50  8.11  8.23  50      3YR 0     0    BA    7.844
  4X7 50  8.16  8.28  50      4YR 0     0    1MTH  7.775
  5X8 50  8.24  8.36  50      5YR 0     0    O/N
  6X9 50  8.36  8.48  50      6YR 0     0    BRIAN 838-3724
 7X10 50  8.42  8.54  50       7YR 0     0    DAVE  838-3512
 8X11 50  8.51  8.63  50       8YR 0     0    ALAN  833-2284
 9X12 50  8.60  8.72  50  75  9YR 0      0     EOIN  833-2286
12X15 50  8.97  9.09  50      10YR 0     0          838-3512
15X18 50  9.28  9.40  50       R150 0          RODS  SEE
18X21 50  9.54  9.66  50       R153 0          IMBB
```

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