

Characterization of spatial resolution through Fourier ring correlation in digital neutron radiography

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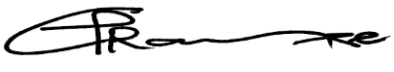
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Declaration

I, Pelonomi Cynthia Ramatlhatse, herein declare that the work presented in this mini-dissertation: “Characterization of spatial resolution through Fourier ring correlation in digital neutron radiography” is my original work and has not been submitted to any other Institution for examination. I further declare that all material used in this dissertation has been fully acknowledged correctly in the references.

Signature: 

Date: 21 November 2019

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Firstly, I would like to give all the glory and thanks to the Almighty God for all He has enabled and helped me to do and achieved thus far on my academic journey.

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Dedication

I would like to dedicate this work to everyone who believed in me, to everyone who think that they cannot make it this far, it's possible only if you put your mind and heart to it. Perseverance and dedication is all that it takes.

Abstract

Spatial resolution is a very important parameter in any imaging system, from medical to industrial even in astronomy and it should be considered every time images are analysed or even before processing. The aim of this work was to characterize spatial resolution in digital neutron radiography using the proposed method of Fourier Ring Correlation (FRC), a method which was mainly developed for cryo-microscopy imaging. This proposed method was adopted in this work for determination of spatial resolution. The Modulation Transfer Function (MTF) also known as Spatial Frequency Response (SFR) is the method used in digital neutron radiography to determine the spatial resolution of an image. The reason for adopting the FRC method was to determine if this proposed method is suitable to be applied in digital neutron radiography to find the spatial resolution of images. The current method used in neutron radiography, which is the standard method MTF/SFR, uses an edge analysis which has been observed to be a complex technique to find the spatial resolution.

The results were achieved using MATLAB. Different MATLAB functions were used for FRC and MTF/SFR. The FRC results were compared with that of the standard method in neutron radiography MTF/SFR. This comparisons were made at different Object to Detector Distance (ODD). The percentage differences of FRC and MTF/SFR at different ODD's were 88.24%, 3.57%, 3.70% and 26.67%. The results yielded a higher percentage difference than expected, especially the 88.24% is quite high. FRC can be applied in neutron radiography to determine the spatial resolution as the results from different ODD's have shown a percentage differences less than 5%. The conclusion from this study is that FRC method can be used to determine spatial resolution in neutron radiography but filtering needs to be done as part of image analysis. Although it is recommended, in future studies, apodization and filtering methods should be taken into consideration for better spatial resolution.

Keywords:

Fourier Ring Correlation (FRC), neutron radiography, spatial resolution, Modulated Transfer Function (MTF), Spatial Frequency Response (SFR).

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List of Abbreviations

CCD	Coupled Charge Detector
Cryo-EM	Cryo-Electron Microscopy
dpi	dots per inch
ESF	Edge Spread Function
FRC	Fourier Ring Correlation
FWHM	Full Width Half Maximum
GUI	Graphical User Interface
ICON	Imaging with Cold Neutrons
ISO	International Organization for Standard
IMAT	Imaging and Material Science
LSF	Line Spread Function
MATLAB	Matrix laboratory
MTF	Modulation Transfer Function
NEUTRA	Neutron Transmission Radiography
ODD	Object-to-Detector-Distance
PSI	Paul Scherrer Institute
SAR	Synthetic Aperture Radar
SD	Standard Deviation
SFR	Spatial Frequency Response
SID	Source-to-Image-Distance
SINQ	Spallation Neutron Source
SNR	Signal-to-Noise-Ratio
SwissFEL	Swiss Free-Electron X-ray Laser
SpS	Swiss Muon Source

TOF

Time of Flight

UWB

Ultra-wideband

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Chapter 1: Introduction

1.1 Background

In the late 1920's, Rutherford and Chadwick were working on atomic disintegration, when they noticed that a proton is not the only particle in the nucleus, they found that the atomic number was less than the atomic mass (meaning that the number of protons is equivalent to the positive charge of the atom) in the nucleus (Chadwick, 1932). Rutherford made a postulate that there could be an unknown particle without a charge; he named the unknown particle "neutron". Chadwick did not rest his mind about the unknown particle. In the early 1930's, two German physicists, Walther Bothe and Herbert Becker performed an experiment that showed that beryllium, fluorine, lithium and boron atoms have a great penetrating power when bombarded with an alpha particle from polonium and they assumed that it was gamma (γ) rays because at that time, previous research had shown that γ -rays have the greatest radiation penetrating power. Not so long after the findings of the two German physicists, Irene Curie conducted a research on the absorption of secondary radiation of Be and Li and showed that Be and Li have the ability to penetrate through a material easier than the γ -rays could not. Chadwick was working on the same experiment Curie performed but he was examining the properties of radiation in Be when he discovered that the γ -rays hypothesis is not correct. He concluded that the unknown particle consists of a mass slightly greater than the mass of a proton and has no electrical charge and is more penetrative than γ -rays. He named the particle neutron just as Rutherford had named it (Nesvizhevsky and Villain, 2017) and he published his findings before more research followed on neutrons.

Neutrons are part of the nucleus and are usually characterised as fast, epithermal, thermal and cold neutrons. Their energies and scattering characters determines their characteristics or classification (Hawkesworth and Walker, 1981, Bayon et al., 1992). Neutrons are nuclear particles and are more penetrating as compared to electrons and protons, because they have no charge (Van Rooyen, 2006). Neutrons are electrically neutral particles and indirectly ionising, they interact directly with the nucleus and usually interact with the nuclei in reactions such as (n, p) , (n, α) , (n, np) .

1.1.1 Neutron interaction with matter

The elastic scattering (n, n) reaction produces a low energy neutrons from fast emitted neutron source. Neutrons collide with a nucleus and transfer its energy to the nucleus by means of losing kinetic energy (E_K). The fraction of its initial energy depends on the angle it hits the nucleus. When neutrons collides with a light nucleus, they loses more kinetic energy compared to when they collides with a heavy nucleus.

Inelastic scattering (n, n) or (n, n, γ) reaction is more like elastic scattering but when the neutrons collide with the nucleus, they have enough kinetic energy to stimulate the nucleus into its excited state. When neutrons are absorbed by the nucleus, gamma-photons are usually emitted when the excited product nucleus returns to its ground energy state. At high incident energies, neutrons are capable of producing charged particles such as protons and alphas by nuclear reaction that happens with nuclei (Alaa Eldin, 2011, Van Rooyen, 2006).

Neutron capture, (n, γ) reaction, is another type of nuclear reaction which occurs when an atomic nucleus collides with one or more neutrons and combine to form a heavier nucleus. The multiple or compound nucleus decays by release of gamma ray (Stacey, 2001).

Charged particle emission (n, p) and (n, α) reactions, neutrons may be absorbed or scattered depending on which reaction took place. The reactions are either exothermic (release) or endothermic (absorb) (Lamarsh and Baratta, 2001).

Fission reaction (n, f) happens when a heavy nucleus splits into two unequal parts, each with a different mass. Fission reaction usually takes place in nuclear research reactors or nuclear power plants. Uranium (^{235}U) and Plutonium (^{239}Pu) undergoes fission after absorption of neutrons.

Table 1-1: Summary of types of nuclear reactions

Type of reaction	Formula	Example
(n, n)	${}^A_ZX + {}^1_0n \rightarrow {}^A_ZX + {}^1_0n$	${}^{59}_{27}\text{Co} + {}^1_0n \rightarrow {}^{59}_{27}\text{Co} + {}^1_0n$
(n, γ)	${}^A_ZX + {}^1_0n \rightarrow {}^{A+1}_ZX + {}^0_0\gamma$	${}^{59}_{27}\text{Co} + {}^1_0n \rightarrow {}^{60}_{27}\text{Co} + {}^0_0\gamma$
(n, p)	${}^A_ZX + {}^1_0n \rightarrow {}^{A-1}_{Z-1}X + {}^1_1\beta$	${}^{59}_{27}\text{Co} + {}^1_0n \rightarrow {}^{59}_{26}\text{Fe} + {}^1_1\beta$
(n, α)	${}^A_ZX + {}^1_0n \rightarrow {}^{(A+1)-4}_{Z-2}X + {}^4_2\text{He}$	${}^{59}_{27}\text{Co} + {}^1_0n \rightarrow {}^{56}_{25}\text{Mn} + {}^4_2\text{He}$

(n, f)	${}^A_ZX + {}^1_0n \rightarrow {}^A_ZX + {}^1_0n$	${}^{235}_{92}U + {}^1_0n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3({}^1_0n)$
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1.1.2 Neutron radiography

After the discovery of neutrons between the years 1935 to 1938, two German scientists started working with neutrons in a laboratory from a small neutron generator to produce neutron radiographs. Research reactors then became available in the 1950's; Thewlis and Derbyshire made a demonstration that neutron radiographs can be produced by utilising thermal neutrons beam from the nuclear research reactors. The application of neutron radiography started expanding rapidly since then (Chankow, 2012).

Neutron radiography is a process of imaging using neutrons. It is a non-destructive imaging technique (Lehmann et al., 2014), where neutrons are used to produce radiographs. It is the process whereby neutrons are passed through an object to produce visible image of the material making up its internal contents. The neutron radiography digital imaging technique is similar to the X-ray radiography principle. Unlike X-rays, neutrons do not interact with electrons, their interaction is not determined or dependent on the atomic number (Z) of the sample material (Koerner et al., 2000) but the size of the nucleus. Neutrons interact with the material by absorption, scattering or transmission (Lehmann et al., 2010). Both neutrons and X-rays are determined by their respective attenuation cross-section as described by the Beer-Lambert law in Eq.1.

$$I = I_0 \lambda e^{-\mu \lambda d} \quad \text{Eq.1}$$

where, I_0 and I are the incident and transmitted beam intensities for a given wavelength λ , μ is the attenuation coefficient and d is the thickness of the absorber (Sun et al., 2017). The Beer-Lambert law describes the absorbance and transmission of photons through a material. The law describes the primary photons, photons which have not yet interacted with any material. When photons pass through a material, they may interact via absorption or scattering. The attenuation coefficient also known as the linear attenuation coefficient μ is dependent on the density of the material (Dance et al., 2014). Figure 1-1 schematically shows the attenuation principle (absorption or scattering).

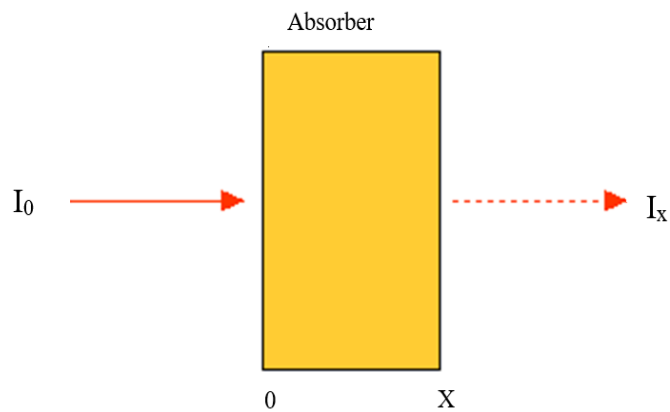


Figure 1-1: Schematic sample of radiation attenuation by the material (Maher, 2016).

The attenuation coefficient for each element is different for neutrons and for X-rays (Van Rooyen, 2006). The reason for this is that the X-ray attenuation increases with atomic number of each element while the neutrons attenuation is randomly as a function of the atomic number because neutrons interact directly with the nucleus. All radiographic techniques are based on the same general principle that radiation is attenuated as it passes through matter, whether making use of X-rays, gamma-rays or neutrons, the basic principle is still the same (Koerner et al., 2000).

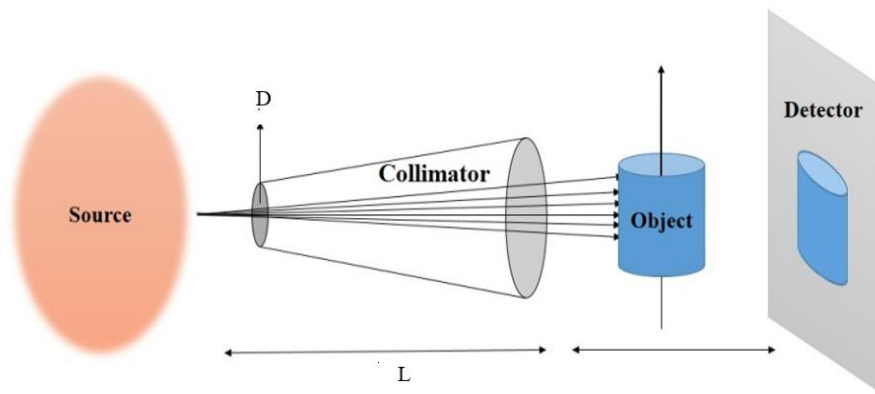


Figure 1-2: Simple set-up of neutron radiography.

A simple experimental setup, Figure 1-2 of a neutron radiography system consisting of a neutron source, a collimator that contains a small pinhole “D” that allow neutrons to pass through an object and a detector (Koerner et al., 2000). An object is placed between the collimator exit and a detector that records a two dimensional image and contains information about the composition and structure of an object being analysed. The neutrons from the neutron source pass through the small pinhole of a collimator guided by the beam to hit an object being analysed. Neutrons penetrate an object to different degrees, which are dependent on the elements making up the sample. After attenuation, the neutrons which are transmitted, are detected by the detector system that captures and forms images of the object that are displayed on the computer for analysis.

Neutrons are either scattered or absorbed by the atomic nuclei (Alam et al., 2006). They have the greatest penetrating ability of the four types (alphas, betas, gammas and neutrons) of radiation and the most difficult to shield (Alaa Eldin, 2011, Van Rooyen, 2006). Different neutron sources available worldwide for neutron radiography include nuclear reactors, particle accelerators and radioisotopes. Their source intensity, properties and energies (fast, thermal and cold) differs accordingly. In neutron radiography, neutron beam characteristics are of great importance and include: collimation, which is generally expressed as the beam tube length to diameter ratio (L/D) and it is the main factor influencing the image quality; the beam intensity, which is the controlling factor for exposure time and beam quality, is important for non-neutron background in radiographs. In all the three categories of neutron sources, the nuclear research reactor is the best

source of neutrons because of the higher neutron flux profile, and they give better radiographs (Hawkesworth and Walker, 1981).

Image quality is a concern in neutron radiography, and in order to obtain a high quality radiographs, neutron beam and detector must have certain characteristics. Characteristics of digital images include resolution, spatial resolution, pixel size, voxels, contrast, and sharpness.

1.1.3 Spatial resolution

Spatial resolution determines how close two features of an object are and how proximate lines can be to each other and still be visibly resolved in an image. Resolution is measured in different ways and its units are described by their physical size (expressed as pixels per unit distance, dots per inch (dpi), lines per mm and lines per inches) to their over-all sizes. The common units in spatial resolution are pixels per unit distance and dpi (Ball and Price, 1995). A two-dimensional image has a three-dimension, namely the width, height and a gray-scale. The height and the width are classified as spatial hence the word spatial resolution (Bushberg et al., 2002). However, in some texts they omit the word spatial and refer just to resolution. Spatial resolution also refers to the number of independent pixel per unit length. The higher the resolution, the more detailed the image is. In most cases, spatial resolution is lost when two objects becomes close together and appear as one, then the image is not visible enough for interpretation. In digital imaging, the resolution depends on characteristics of the system creating the image not just the number of pixels in an image. The optical pixel size and image resolution are not equivalent in reality even though they are often used interchangeably. For instance, the image with a small pixel size does not literally mean that the image has high resolution or vice versa. The resolution of the system is understood in order to know how the edges in an image are blurred. Resolution limit explains exactly how much information makes up an image.

Factors affecting image quality include spatial resolution, noise and contrast. Noise in radiography is defined as any uncertainties that do not correspond to similarities in the radiation attenuation of an object to be imaged whereas contrast is defined as the differences in radiographic density and is the ratio of radiation intensities transmitted in different areas of a radiograph (Alaa Eldin, 2011). Factors affecting spatial resolution include detector blur, geometric un-sharpness and pixel size. Detector blur is produced when neutrons are converted into light on the scintillator screen; when

neutrons enter the scintillator screen guided by the beam line they are converted into light (to enable the camera to interpret them). The light gets divergent and blurriness gets formed. Geometric un-sharpness or penumbra refers to the loss of detail in an image caused by the increasing size of the focal spot of the radiographic equipment, object-film distance and object-focal spot distance are the main factors controlling geometric un-sharpness (Fitzgerald and Francisco, 1947). This occurs because the radiation does originate over an area and not from a single point (Smith, 1999). The maximum un-sharpness or penumbra may be calculated using Eq.2, where the f is the source focal spot size, b is the distance from the object to the detector and a is the distance from the source focal spot to the object,

$$U_g = f \times b/a. \quad \text{Eq.2}$$

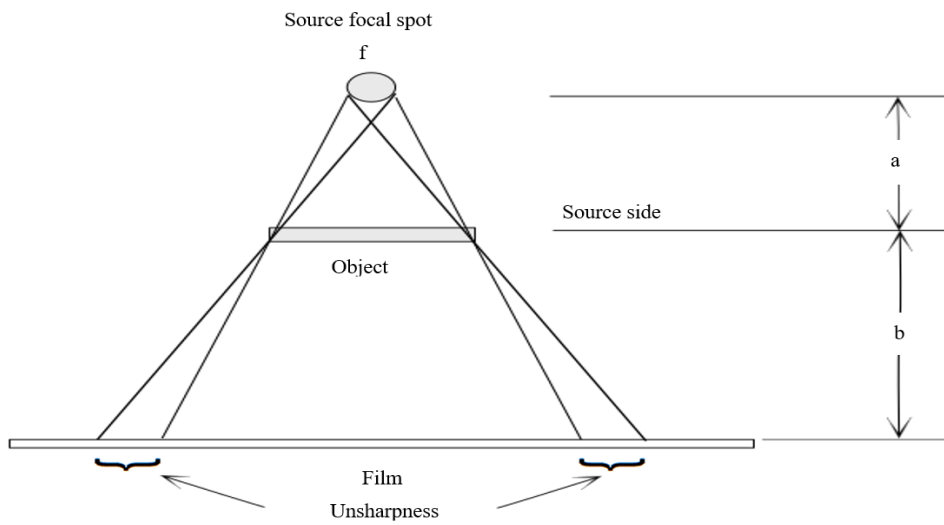


Figure 1-3: Geometric un-sharpness (Molina, 2016).

Figure 1-3 with a point source, an object and a film. The path of the radiation from each edge of the source to each edge of the feature of the focal spot, the locations where this radiation will expose the film and the density profile across the film. The radiation hitting the film originates from the source focal spot and hence a geometric un-sharpness is produced in the image at the edges where the film is located, because as the size of a source focal spot increases, the amount of

geometric un-sharpness also increases. Therefore, to avoid such increase in blurriness of an image, one must consider the use of small source focal spot as possible. Geometric un-sharpness is controlled by the source size, source to object distance and object to detector distance (Smith, 1999). Application of spatial resolution is widely used in the printing industry; for instance, the newspaper company will use about 75 dpi to print their paper. The higher the dpi, the more quality the printout will be. In general, the size of an object does not matter in spatial resolution (Gonzalez and Woods, 2002). Unser defines resolution as the spatial frequency at which ring-shaped samplings of the two Fourier transforms register negligible cross-correlation (Unser et al., 1987).

1.1.4 Fourier ring/shell correlation

Fourier Shell Correlation (FSC) was introduced in 1982 by van Heel; however, its metrics was only introduced in 1986 by van Heel. FSC was originally applied to cryo-electron microscopy (cryo-EM) and electron microscopy (EM) of structural biology. FSC is a mathematical method used for calculating resolution of an image. It measures the cross correlation between two, three dimensions (3-D) whereas Fourier Ring Correlation (FRC) measures cross correlation between two, two dimensions (2-D). FSC is also known as the spatial frequency correlation. FRC assesses resemblance of two independent reconstruction of the same object to regulate threshold resolution in frequency space. It measures the standardized cross correlation to estimate the resolution of an image in Fourier domain over the corresponding rings and varies in thickness and radii (Saxton and Baumeister, 1982). The FRC curve is plotted as a function of spatial frequency in Fourier space, which is computed from two images having the same data set (Nieuwenhuizen et al., 2013). The resolution (limiting resolution) is estimated at the intersection of threshold and FRC curve. It has been shown that FRC may be applied to microscopy and nanoscopy data and supplies a software tool that is easy to use, which interprets positional information in description films tested to their actual resolution (Saxton and Baumeister, 1982).

FSC/FRC has now made its way in other fields, such as X-ray tomography, X-ray crystallography, planetary science and super resolution microscopy. Basically, FSC/FRC metrics are applicable to all 3-D and/or 2-D data (Van Heel and Schatz, 2017).

1.2 Problem statement and motivation

Fourier Ring Correlation (FRC) has been applied in microscopy and nanoscopy (Nieuwenhuizen et al., 2013, van Heel and Schatz, 2005) to analyse the threshold criterion that is suitable for determining spatial resolution limit. Nieuwenhuizen et al., 2013 have worked with FRC to measure resolution limit in super resolution microscopy and different nanoscopy methods. In neutron radiography, the method used to determine the spatial resolution of images is the Modulated Transfer Function (MTF) or Spatial Frequency Response (SFR) (Cao and Biegalski, 2007), which uses Fourier analysis of an edge image. The edge analysis is a complex method to determine the spatial resolution of an image hence this study seek to adapt FRC as a method to characterize spatial resolution in digital neutron radiography. The FRC method has not been applied to neutron radiography to determine the spatial resolution. Since digital neutron imaging is developing towards imaging in the micron and submicron range, there is a need to develop tools to assess spatial resolution in this range.

1.3 Research aim and objectives

The aim of this study was to establish the suitability of the FRC to characterise spatial resolution in digital neutron radiography, especially in the micron and submicron range of digital neutron radiography. The research objectives were to:

- Formulate the experimental protocol to analyse digital neutron radiographs on which FRC assessment can be conducted.
- Assess the algorithm and mathematics of FRC in assessing spatial resolution in digital neutron radiography.
- Adapt and apply FRC to evaluate spatial resolution in digital neutron radiography.
- Apply the MTF/SFR standard method to evaluate the spatial resolution.
- Compare FRC results to the standard method, MTF/SFR.

Chapter 2: Literature review

2.1 Introduction

In this chapter, literature relevant to FRC techniques and spatial resolution are discussed. The methods and techniques of finding the image resolution in digital neutron radiography are also discussed. The Edge Spread Function (ESF), Line Spread Function (LSF), Modulation Transfer Function (MTF) and ISO 12233 standards are discussed on how to determine the spatial resolution, as well as filtering methods used in neutron radiography.

2.2 Resolution

Resolution estimation methods are not all based on data consistency in Fourier space using the quantitative measure (Liao and Frank, 2010). These methods have been applied in data images and diffraction patterns from electron microscopy, X-ray diffraction, optical microscopy, single particle reconstructions and nanoscopy (Radebe, 2017). These methods can also be used in digital neutron radiography and tomography. But should be slightly modified in order to characterize the output data.

Resolution criteria have been limited to a value of λ/NA , where λ is the wavelength of light and NA is the numerical aperture of image lens. This resolution was captured by Rayleigh and Sparrow on the conventional law of optical imaging science. Abbe and Nyquist (1984) have placed these criteria on solid foundations and they defined resolution as “*the inverse of spatial bandwidth of the imaging system*”. To produce super-resolution images, the resolution depends on numerous factors including the fundamental spatial structure of the sample and the wide-ranging data processing necessary to produce a final super-resolution image (Nieuwenhuizen, 2016).

In resolution test, data consistency is needed, usually, this is done by splitting the full data set into two half datasets and comparing the resulting average data by calculating two reconstructions of the same object independently (Liao and Frank, 2010, van Heel and Schatz, 2005). The consistency of the resulting data set is compared as a function of frequency space (van Heel and Schatz, 2005) over rings or shells (reconstruction assessment is 3D) with a cumulative radius in Fourier domain (Liao and Frank, 2010) to determine the resolution threshold (Banterle et al.,

2013). Another method, is by calculating the cross-correlation between nearby voxels in Fourier space (Liao and Frank, 2010).

In cryo-electron microscopy (cryo-EM), FRC is usually used to evaluate single-particle reconstructions of macromolecular multiplexes (Nieuwenhuizen et al., 2013). To compute FRC resolution in cryo-EM, Nieuwenhuizen, 2016, divided the set of super-resolution images into two equally independent subsets that constitute of single-emitter localizations and produced two sub-images. Nieuwenhuizen et al. 2016 analysed different threshold criteria in cryo-EM and the findings resulted in fixed threshold value approximated to 0.143 which was found to be appropriate for microscopy images (Nieuwenhuizen, 2016). In cryo-EM, the single particle reconstruction is usually measured in FSC (3D of FRC) and that is, the resolution is measured over shells in Fourier Space as a function of spatial frequency (Diebolder et al., 2015).

2.3 Spatial resolution

Velo et.al, 2017, determined the spatial resolution of Gamma cameras used in medical imaging for acquiring medical images with precise quantitative figures. They defined the spatial resolution of gamma camera as the capacity of the overall camera system to precisely determine the location of a gamma-ray on the X-Y plane with and without the scatter. Spatial resolution can be defined as the ability to differentiate between two points, the higher the spatial resolution, the smaller the distance between the two points. In the gamma cameras, it is believed that the spatial resolution is related to the effectiveness and linearity of the collimator and its photomultiplier tube. In order to quantify the spatial resolution, they determined the Full-width at half maximum (FWHM) of the LSF from the acquired line source images and MTF was calculated. MTF was calculated by Fourier transformation of the LSF. To determine the spatial resolution after data acquisition and processing, a MATLAB program was used to analyse the data and the same data was analysed using the standard processing system. The Graphical User Interface (GUI) as a MATLAB program was developed to evaluate the spatial resolution of the gamma camera. The FWHM results from the standard method (data processing system) and the GUI program were assessed and compared. It was determined that the spatial resolution calculated from the MATLAB was lower than the one from the standard method by 1.24 % (Velo and Zakaria, 2017).

The differences between the two programs were analysed and it was found that the standard program was not installed with software that could determine the MTF. So the GUI results were considered to be relevant for the spatial resolution. In their work to determine the spatial resolution they strongly believed that the GUI program was reliable to use because it required less time for data analysis, minimises human interface error in analysis because of the fact that the calculations are performed automatically by the software. The benefits of using the GUI program are that the line source images can either be obtained from actual or simulated gamma camera, the program is faster, reliable, and cost-effective and not to forget that it is easier to use to analyse the spatial resolution of the line source image in gamma cameras (Velo and Zakaria, 2017).

2.4 ISO 12233

The International Organisation for Standard (ISO) 12233 is the standard method created to evaluate electronic still images. This standard describes the test charts, terminologies and test methods for performing resolution measurements for both digital and analogue electronic still images (ISO12233, 2014). The standard introduced the Spatial Frequency Response (SFR) and it was the first to address the performance of the digital camera (Burns and Williams, 2008). The SFR can be derived from the edge analysis or periodic signals; however, ISO introduces SFR based on the edge analysis in a digital image. ISO 12233 defines SFR as a "multi-value metric that measures contrast loss as a function of spatial frequency." SFR is measured based on the analysis of an edge device and this analysis is done on digital images containing high-quality edge features. The ESF is determined from the data set of an edge profile, and the LSF is calculated from the first derivation of the estimated ESF (Burns and Williams, 2008) (ISO 12233). SFR is, in other words, the normalised signal modulation function of spatial frequency and basically the MTF. The following subsection discusses the MTF using different test methods. The Edge is the common method used to find MTF/SFR.

2.5 Modulation transfer function

Modulation Transfer Function (MTF), also known as Spatial Frequency Response (SFR) is a well-established factor to determine the estimation and evaluation of imaging system resolution (Cao and Biegalski, 2007). It's a standard method used in the radiography imaging system (X-ray) to

determine the spatial resolution of an imaging system. MTF is defined as a simple measure of an imaging system able to reproduce an image contrast and describe the signal transfer of the system with varying spatial frequencies (Buhr et al., 2003, Cao and Biegalski, 2007, Samei and Flynn, 1997). Spatial frequency is similar to frequency of sound and it is measured in cycles per unit length (millimetres or inches) instead of cycles per unit time (seconds) also known as Hertz which is the measure of frequency of sound. MTF is useful when it comes to evaluating the quality of an image in different imaging systems (Velo and Zakaria, 2017). It is measured in line pairs per millimetre (lp/mm) and cycles per millimetre (cy/mm).

There are three methods which have been proposed to calculate MTF which are slit, edge or bar pattern (Buhr et al., 2003). The methods that are commonly used in the radiographic system are the edge and the slit method because of their benefits. The bar pattern method has been considered to be challenging due to difficulties in determining the modulations in digital bar pattern images (Buhr et al., 2003).

2.5.1 Bar pattern method

The bar pattern method uses a bar pattern test object, made with high atomic number metal available in different frequency ranges and multiple thicknesses. The bar pattern test object is placed on the detector to get the radiographs. After image acquisition, the radiographs of the bar pattern are processed to attain the square-wave response function at each of the spatial frequencies of the pattern. This is done by averaging the data of the related bar patterns. To obtain MTF using this method, it is mathematically deduced from the square-wave response function of the averaged bar pattern data. The advantages of this method include absolute ease, quick implementation and conceptual simplicity. The disadvantages of the method are low accuracy, noise and coarse sampling curve of the MTF (Samei, 2003).

2.5.2 Slit method

The slit test method has been used for many years as the traditional method to measure the MTF. Usually, the slit test is performed with an object made with two thick lead metal pieces. This two lead metal pieces are placed at a distance from each other in a way that they form an opening with a width of about ten microns (Samei, 2003). When using the slit method, there are a few things to

keep in mind in order to acquire radiographs. Precise fabrication and alignment of the system in the radiation beam is needed, high radiation exposure in order to allow sufficient transmission through the narrow slit and correct slit width are required (Samei and Flynn, 1997). Even though high radiation exposure is needed, it is often the main resolution limiting factor (Diebold et al., 2012). The image data is averaged to form an LSF. Using the obtained LSF, Fourier transform is applied to the averaged data to deduce the MTF. The benefits of this method include high precision at high spatial frequencies and the major disadvantage is correct alignment and manufacturing of the slit device, which is time-consuming and complicated to take precise measurements.

2.5.3 Edge method

The edge method as mentioned, is commonly used in digital radiographic systems (Samei, 2003). In this method, the edge test device and an opaque object with a straight edge are used to measure the ESF of the radiographic system (Samei and Flynn, 1997). The edge device is made of a thin metal foil, preferably a high atomic number material like lead, tungsten and platinum-iridium alloys (Samei, 2003). In the Samei et al. 1997 experiment, an edge test was performed with a $5 \times 10 \text{ cm}^2$, $250 \text{ }\mu\text{m}$ lead foil. The lead foil was used as the attenuating material on a 115 kV polychromatic X-ray beam. The lead foil was laminated between two acrylic thick slabs of about 1 mm, this was placed in a holder in order to station and align the edge in the X-ray beam. The holder was made of two 6.35 mm lucite frames and had adjustable screws to easily tilt the edge device without dislocating it. Figure 2-1 summarises the three alignment steps.

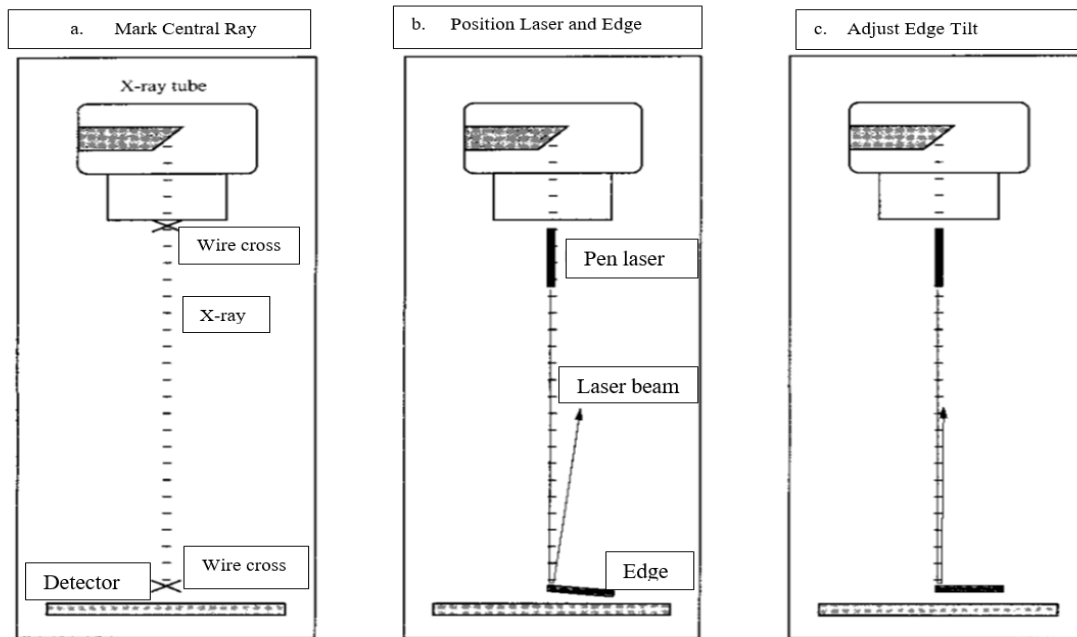


Figure 2-1: Alignment steps for the edge test method (Samei and Flynn, 1997)

On the X-ray beam, the central axis was identified and marked with metal marker on the collimator front. In order to ensure that the central axis has not interpreted a radiograph was already taken. The edge device on the holder was oriented on the receptor, with its centre intersecting on the central axis of the X-ray beam. Depending on the direction in which the MTF was measured, the edge was placed both horizontally and vertically with an angle varying between 1° and 6° . In the second step, Figure 2-1b, a laser pointer was oriented on the collimator face with the source of the laser on the central axis of the X-ray beam. The distance from the laser beam and the centre of the edge was 1-2 mm. The reflection of the laser beam on the edge surface was identified and finally, on Figure 2-1c, which was the last step, the edge device was tilted using adjustable screws on the holder until the reflection spot corresponds with the laser beam. These three steps were done with precaution in order to get the right alignment of the edge device. A radiograph of a properly aligned edge device was taken. The focal spot blur was reduced by using a long source-to-image-distance (SID), a small focal spot, a short distance between the detector and the edge device. The digital image data of the radiograph was computed and processed to get the pre-sampled MTF which is obtained using $8 \times 8 \text{ cm}^2$ sub-region radiographs containing data from the transferred radiographs of the edge device. Samei et al, 1997, discusses the method used to get the computed ESF and

calculation method to obtain a differentiated LSF and finally, Fourier transform the LSF to get pre-sampled MTF (Samei and Flynn, 1997).

Buhr et al. 2003 also shows the results of ESF measured with an edge method slightly angulated with a detector grid and the ESF is generated. The ESF is differentiated to obtain the Line Spread Function (LSF) and MTF is calculated by a Fourier transformation of the LSF (Buhr et al., 2003), (Samei and Flynn, 1997). The benefit of the edge device includes high precision at very low spatial frequencies, precise alignment and it is not complex to use

The slit and edge methods have been compared, with the edge method showing favourable results when determining the MTF at low spatial frequency whereas the slit method shows more accurate results when determining MTF at a high spatial frequency (Buhr et al., 2003). MTF basically is the plot of the ratio of the output-to-input modulation transfer as a function of spatial frequency, measured in cycles/mm (Kerr, 2010). If the plot yields a higher MTF, there is a better chance of having good resolution and sharpness of an image (Samei, 2003)

Many authors have used a similar method to get the pre-sampled MTF curve using the edge test method. However, they have applied different techniques to get to the MTF curve.

2.6 FRC/FSC theoretical behaviour

FRC is a mathematical function that calculates the correlation quantities between two-dimensional volumes of images in Fourier space (van Heel et al., 1982). Both FSC and FRC have the same metrics; the only difference is that FSC measures correlation in 3-D objects and FRC in 2-D objects.

$$FSC/FRC(r_i) = \frac{\sum_{r \in r_i} P_1(r) * P_2(r)^*}{\sqrt{\sum_{r \in r_i} P_1^2(r) * \sum_{r \in r_i} P_2^2(r)}} \quad \text{Eq.3}$$

Eq. 3 is the FSC/FRC formula that assesses the resemblances between two independent reconstructions of the same object in frequency space in order to find the spatial resolution (Banterle et al., 2013). FRC measures the normalised cross-correlation between two images over resultant rings in spatial frequency (van Heel & Schatz, 2005). Eq. 3 describes the FRC/FSC

between two images in Fourier space where the summation are over the volume of radius r in rings (or shells in 3D space) of each volume, $P_1(r)$ is complex structure factor at radius r in volume 1 and $P_2(r)^*$ is the complex conjugate with radius r in volume 2 (Diebolder et al., 2015). The denominator in this equation is a normalising factor. Two images are used because the objective is to determine which image has the better spatial resolution keeping in mind that the two images being used are two independent reconstructions imaged at the same position.

Van Heel et.al, 2005, discussed two FSC threshold curves, the sigma factor curve, and the bit-based threshold curve. The sigma curve has different resolution criterion curve that has been used and tested, the 2σ , 3σ , 5σ , 0.5 and 0.143. Various authors have used the resolution criterion to apply it in different applications. However, recent studies show that the used criterion in conjunction with FRC curves is the 0.5 threshold criterion. The 2σ threshold criterion was initially introduced by Saxon and Baumeister in 1982 related to the definition of spatial frequency (van Heel and Schatz, 2005). The 0.143 value was proposed to be the realistic value by Rosenthal et al., but van Heel et al., 2017, have criticised their finding. This is because they believe that the fixed threshold value 0.143 is incorrect, as signal and noise cross terms were omitted. Eq.4 and Eq.5 will give a short description from van Heel et al., 2005.

$$P_1(r) \approx S(r) + N_1(r) \quad \text{Eq.4}$$

$$P_2(r) \approx S(r) + N_2(r). \quad \text{Eq.5}$$

Eq.4 and Eq.5 are two volumes containing the same data signal (S) and different random noise (N). Using these two equations to find the cross-correlation between signal and noise, we have un-normalized FRC, un-normalized means that the denominator of Eq.3 is not included, then Eq.6 yields the cross-correlation for the un-normalized FRC as;

$$FRC_u(r_i) \approx \sum_{r \in r_i} P_1(r) \cdot P_2(r). \quad \text{Eq.6}$$

Substituting Eq.4 and Eq.5 into Eq.6 gives:

$$FRC_u(r_i) \approx \sum_{r \in r_i} S^2(r) + \sum_{r \in r_i} S(r) \cdot N_1(r) + \sum_{r \in r_i} N_2(r) \cdot S(r) +$$

$$\sum_{r \in r_i} N_1(r) \cdot N_2(r). \quad \text{Eq.7}$$

The correlation between signal and noise is important when studying FRC. Many authors have in their papers assumed that the middle term of Eq.7 is uncorrelated, meaning that it equals to zero and falls off the equation. Van Heel et al., 2005, have emphasized that the cross-term is important and should not be left out as the uncorrelated signal and noise does not mean that the middle vector product is zero. The middle term should not be omitted during the derivations. Omitting it will lead to incorrect results as this will mean that the correlation between signal and noise is orthogonal (van Heel and Schatz, 2005, van Heel and Schatz, 2017).

A threshold curve is introduced as a measure to give a quantitative estimation of the image resolution from FRC. The threshold curve is chosen as a logical expression independent of the reconstruction data for the expected FRC images with a signal-to-noise ratio that is constant in Fourier space. The $\frac{1}{2}$ bit criterion was introduced in the work of Villa-Comamala et al., 2011 as a measure that produced results similar to the resolution estimations used for X-ray crystallography (Vila-Comamala et al., 2011).

The 2 sigma threshold criterion was applied by Banterle et al., 2013, 70 rings were assigned into the images after they were Fourier transformed, in order to have that 2 sigma cut-off frequency, the pixel size was adjusted and the cut-off was found to be between ring 20th and 30th (Banterle et al., 2013). In most cases, radiation damage is often the main factor when it comes to the resolution limit (Diebolder et al., 2015).

FRC plot is a one-dimensional curve, approaching zero at high spatial frequencies and fluctuating around the zero mark (Van Heel and Schatz, 2017). Filtering is applied to the FRC curve if data does not give the expected FRC curve, this may be caused by under-sampling or over-sampling. Apodization is one type of filtering that is applicable to FRC to give the expected FRC curve.

2.7 Filtering methods

The world of digital imaging is evolving and the method of recording or transmitting these digital images is usually affected by many factors including faulty detectors, inaccuracy or even quality of the system. Filters are invented for such cases so that we can have good filtered images, without

losing the important details of an image. Filtering is a method or technique used to modify an image or hide the unwanted artefacts in an image. This technique may also be used to enhance some features of an image such as edges, brightness, colour, contrast and increase or decrease pixel size. Different methods of filtering are used in different fields such as medical, industrial and photography. They either use filtering in real space or Fourier space.

Filters may be categorised in two classes, linear and non-linear filters. Linear filters contain an arrangement of pixel values of an input image, the sum, and averages whereas non-linear filters consist of combinations of median, minimum and maximum. Linear filter may be demonstrated as convolution and can be analysed in Fourier space. The convolution theorem can function both in real and Fourier space. The convolution operator in real space may be expressed as in Eq.8 and in Fourier space may be expressed as in Eq.9.

$$g(i,j) = f(i,j) \odot h(i,j) \quad \text{Eq.8}$$

$f(i,j)$ and $h(i,j)$, being the input image and filter function respectively, and $g(i,j)$ being the output filtered image.

$$g(k,l) = \mathcal{F}^{-1}\{F(k,l)H(k,l)\} \quad \text{Eq.9}$$

where; $g(k,l)$ is the output or filtered image, and $F(k,l)H(k,l)$, are both inverse Fourier transforms of input image of Eq.8 and filter function of Eq.9, respectively. The following form part of the filters in Fourier space.

Low-pass filter is a linear type of filter, which works in such a way that it allows low frequency to pass un-attenuated and reducing or even completely blocking high frequencies from passing. The high frequency that is attenuated may be usable to decrease the random noise in the input image. The low spatial frequency in an image will be used to verify the characteristics of an image; this blurs the image and diminishes the image edges, sharpness and noise associated with high spatial frequencies.

High-pass filter is the opposite of low-pass filter; only high spatial frequencies are allowed to pass whereas low spatial frequencies are attenuated. The high-pass filter improves the edges in images. These improvement effects are associated with un-attenuated high frequencies and thus vary in real space and in Fourier space.

Band pass filter is the type of filter that can pass low-pass and high-pass frequencies. But only to a certain frequency range.

2.7.1 Apodization

Apodization may be defined as an ideal window function used to remove the side-lobes in a signal. In simple terms, it is a filtering function used to remove unwanted artefacts in a signal (Qi et al., 2013). Zhou (2011), defines apodization as the suppression of side-lobes (Zhou, 2011). Traditionally apodization methods have been there and used for adjusting the main-lobes of an imaging signal and reducing the side-lobes (Reeg, 2016). Main-lobe is the highest peak of the signal pattern produced either by the antenna or radiation pattern. Side-lobes are the local peaks in signal patterns lower than the main-lobe. These side lobes in signal processing are usually considered as noise if they are too many to interpret well.

The apodization method was developed by Dolph in 1946. He used Chebyshev polynomials (a sequence of orthogonal polynomials) for a collection of linear antenna that formed its radiation pattern such that the base main-lobe width was achieved for a set maximum side-lobe level. In 1954, Taylor enhanced Dolph's method by improving the distribution function of the linear array and the result located the side-lobes farther away from the main-lobes. Different types of apodization techniques have been tried and tested. T'Hoen (1982), studied these different methods in the search for a better method to apply on linear ultrasound array and the effects each apodization technique has on image quality. From his findings, he concluded that only four apodization techniques such as Hamming, cosine, sine and 10% reduced Gaussian give appropriate results when compared to the mostly used rectangular apodization function (Dolph, 1946, Reeg, 2016, Taylor, 1955, t'Hoen, 1982).

Apodization has been applied in ultrasound images to increase the main-lobe and decrease the side-lobes. Spatial resolution in ultrasound distinguishes between two points at different positions in space, and consists of two components, axial and lateral resolution. It is important to distinguish between two. The axial resolution or longitudinal or linear resolution as referred to in some context; is the resolution of an image parallel in direction to the ultrasound beam. This axial resolution may detail out the smallest tissue layer density and has the ability to detect the smallest anatomical details in an ultrasound imaging system. The axial resolution is dependent on the length

of the pulse and many methods already exist to improve this resolution (Almualimi et al., 2018, Reeg, 2016). However, only a few methods have been established to improve the lateral resolution. Lateral resolution is the ability of the system to differentiate between two points in the perpendicular direction to the ultrasound beam. This resolution is in most cases affected by the width of the ultrasound beam. For instance, if the beam of the ultrasound is wide, the lateral resolution will decrease and this will result in bad quality images (Qasim and Raina).

Reeg, 2006 proposed a method of apodization to improve the lateral resolution and proved that the method was better than rectangular apodization function and has used apodization function to achieve better resolution of the imaging system in ultrasound. This lateral resolution was improved by a factor of 25 times when comparing it with the rectangular apodization method and for this the side-lobes were low. The proposed method by Reeg (2006), was tested using two approaches, the masking and bridging approach of apodization. The masking approach was tested in simulation and subtracted an image with zero mean apodization from a scaled rectangular apodization function. This subtraction was done so that the beam patterns of the zero mean, images' two main-lobes to be masked by the main-lobe of the rectangular images. Masking approach was used by Savoia et al., 2014, and minimal improvements in the lateral resolution were attained; this was with suppression of side-lobes (Reeg, 2016). The bridging approach was also tested using two images. The two images were constructed using zero mean apodization separately. This was because both images were apodized on collection and a constant was added to the apodization function.

In Zhou's 2011 thesis, Fourier transform infrared emission spectroscopy was used to measure spatially resolved flame temperature and species concentration. Apodization function was applied in emission spectra in Fourier space. The type of apodization function applied was boxcar and it was used to find emission spectra in Fourier transform calculations. This is because integration of Fourier transform is done in the time domain from negative infinity to positive infinity and that is not practical, hence the boxcar apodization function was applied (Zhou, 2011).

In Akhter's 2012 thesis, different weighting functions are evaluated to control side-lobes on ultra-wideband (UWB) Synthetic Aperture Radar (SAR) images. SAR is a comprehensible sensor that is capable of producing images of earth's surface with high resolution. The SAR images relate to backscattered electromagnetic energy. The SAR images need to be processed before use, and they

apply two dimensional Fourier transform to the raw images for processing. After Fourier transformation, the output spectrum has main-lobes and side-lobes. SAR images always have side-lobes which severely misrepresent the images. Side-lobes are a concern and need to be dealt with and that's when apodizing function comes in. This technique is well known for reducing side-lobes level but still giving a clear image resolution. Different weighting functions are tested, both linear and non-linear, the Hamming, Hanning, Blackman, and Taylor were used in SAR images to decrease the impulse response (IPR) side-lobes and increases the IPR main-lobe width (Akhter, 2012).

Akhter (2012), proposed a new linear weighting function and applied it to SAR images which then showed that it is better than the traditional apodizing technique when it comes to preserving lower side-lobes. Linear and non-linear apodization techniques may be applied to SAR images and both are capable of decreasing the side-lobes but linear apodization technique decreases the side-lobes and leads to a loss in image resolution whereas on the other method non-linear can decrease the side-lobes without causing any changes to the image resolution. Non-linear apodization technique is not used because Akhter (2012), believes that it is not the best technique to use because it is not easy to find a relation between the apodized image and the real image, and also mentions that for non-linear weighting function two or more weighting functions are required in UWB SAR imaging (Akhter, 2012).

A rectangular approximation, a general apodization technique, is used on two-dimensional weighting functions and applied in a narrow beam and narrow band SAR images. This technique only induces orthogonal side-lobes but in UWB SAR images even non-orthogonal images are induced. In SAR images, where the narrow beam and narrowband spectrum is estimated to a rectangular area, there is a loss in spatial resolution and which is still acceptable. For UWB SAR images this is a different case, the loss in spatial resolution is not acceptable at any level (Akhter, 2012).

The new proposed linear weighting function is,

$$w(k_x, k_r) = \left[0.63 - \alpha_k \cos \left(\frac{2\pi \left(\text{atan} \frac{k_x}{k_r} \right)}{\frac{\phi_o}{2}} \right) - \beta_k \cos \left(\frac{6\pi \left(\text{atan} \frac{k_x}{k_r} \right)}{\frac{\phi_o}{2}} \right) \right]$$

$$\left[0.63 - \alpha_k \cos\left(\frac{2\pi(\sqrt{k_x^2 + k_y^2} - k_c)}{k_{max} - k_{min}}\right) - \beta_k \cos\left(\frac{6\pi(\sqrt{k_x^2 + k_y^2} - k_c)}{k_{max} - k_{min}}\right) \right] \quad \text{Eq.10}$$

where α_k is 0.45,

β_k is 0.002,

k_x is azimuth wave number,

k_r is range wave number,

k_c center wavelength of SAR system,

k_{max} and k_{min} are the wavenumbers that correspond to the lowest and highest signal frequencies.

This linear weighting reduces the side-lobes for both orthogonal and non-orthogonal and still keeps the image resolution unchanged. The spectrum was generated using different angles proportional to the bandwidths and inverse Fourier transform was applied to the spectrum. The loss of image resolution was calculated and the linear weighting function was applied in all directions, azimuth, range, and angular direction. Figure 2-2 to Figure 2-10 shows how Akhter (2012), came to the conclusion that the linear method is best when compared to traditional apodization function (Akhter, 2012).

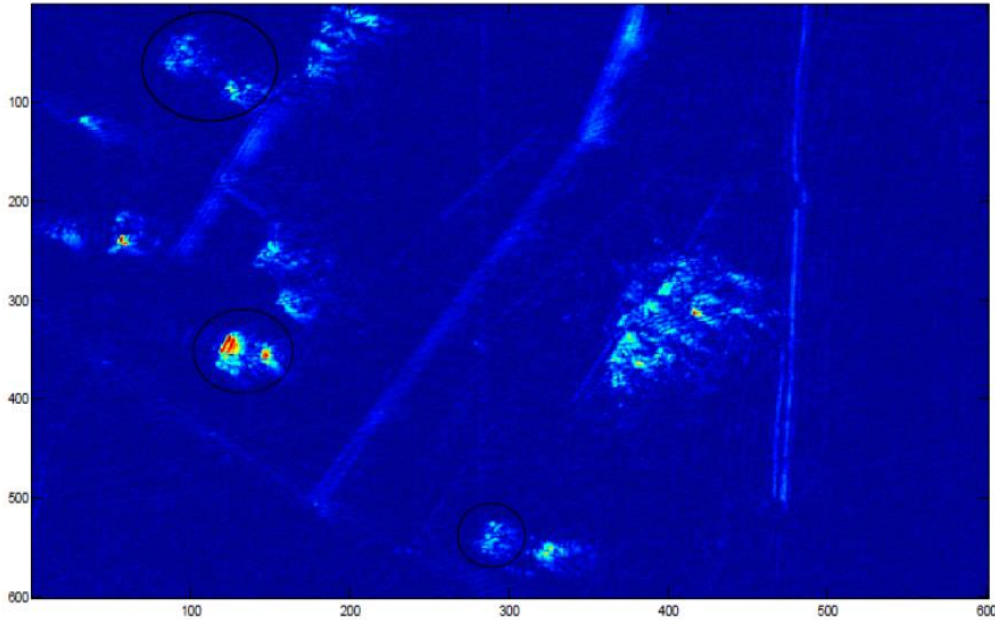


Figure 2-2: Non-apodized real SAR image (Akhter, 2012).

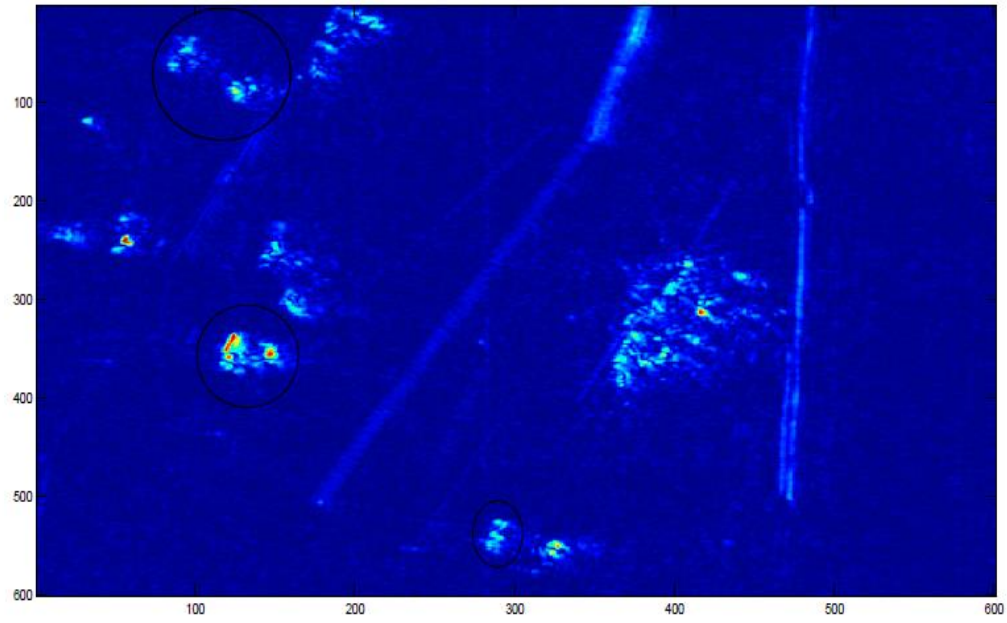


Figure 2-3: Apodized real SAR image by implementing the proposed weighting function (Akhter, 2012).

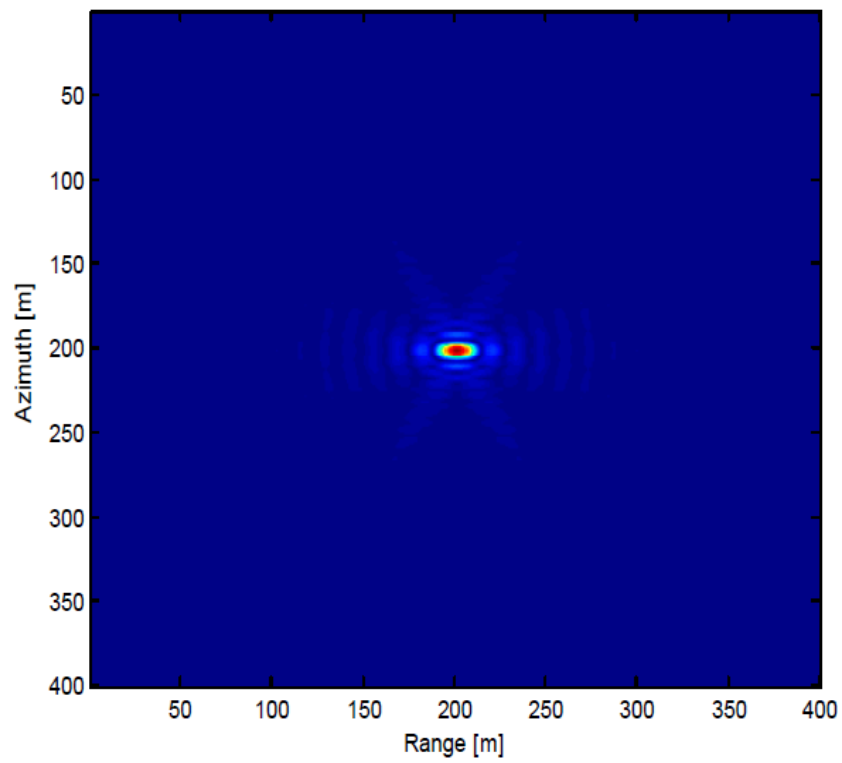


Figure 2-4: SAR non-apodized image with different (10-110) integration angle (Akhter, 2012).

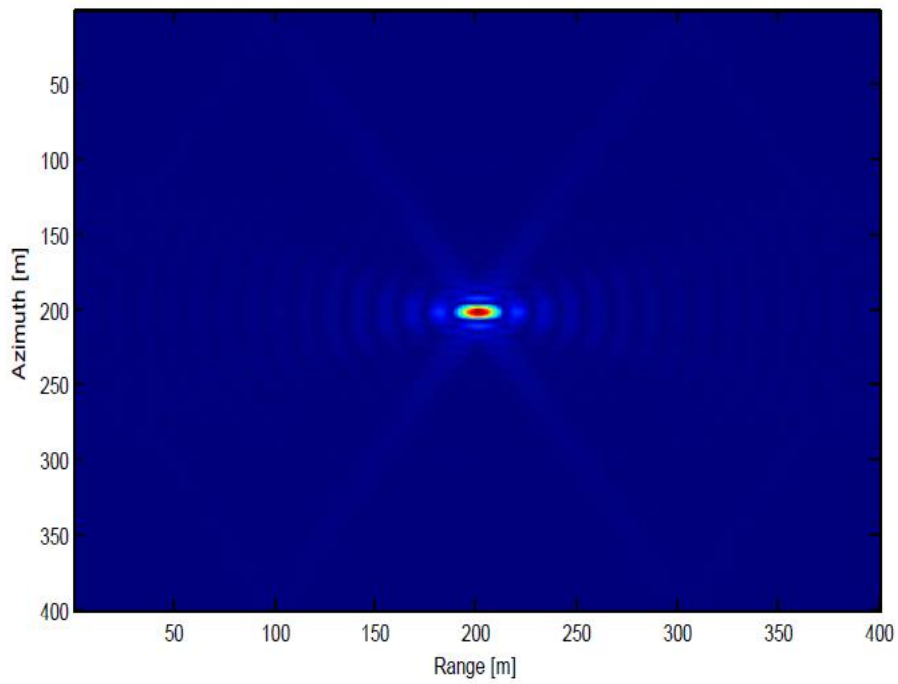


Figure 2-5: Apodized image using rectangular weighting function (Akhter, 2012).

Figure 2-5, the rectangular weighting function was implemented to decrease the orthogonal side-lobes but did not decrease them as expected.

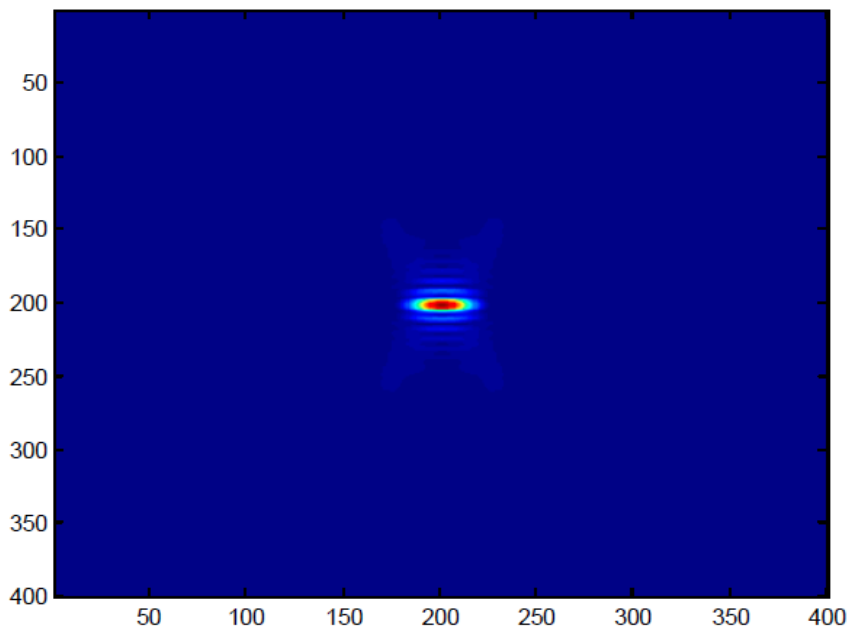


Figure 2-6: Apodized image using Hanning weighting function (Akhter, 2012).

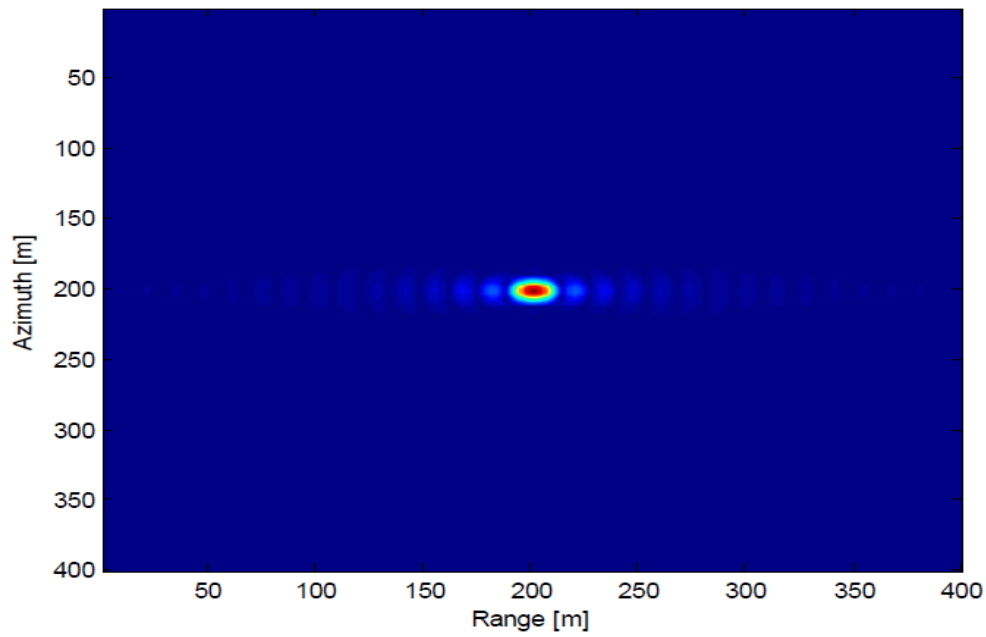


Figure 2-7: Apodized image using Hanning weighting function in the angular direction (Akhter, 2012).

In Figure 2-6 and Figure 2-7, Hanning weighting function was applied in azimuth and angular direction respectively. It was observed that when Hanning weighting function was applied to the azimuth direction, Figure 2-6, 60% of resolution was lost even though the orthogonal side-lobes are adequately decreased. In Figure 2-7, about 45% of resolution was lost with sufficient side-lobes suppression when Hanning weighting function was applied in the angular direction.

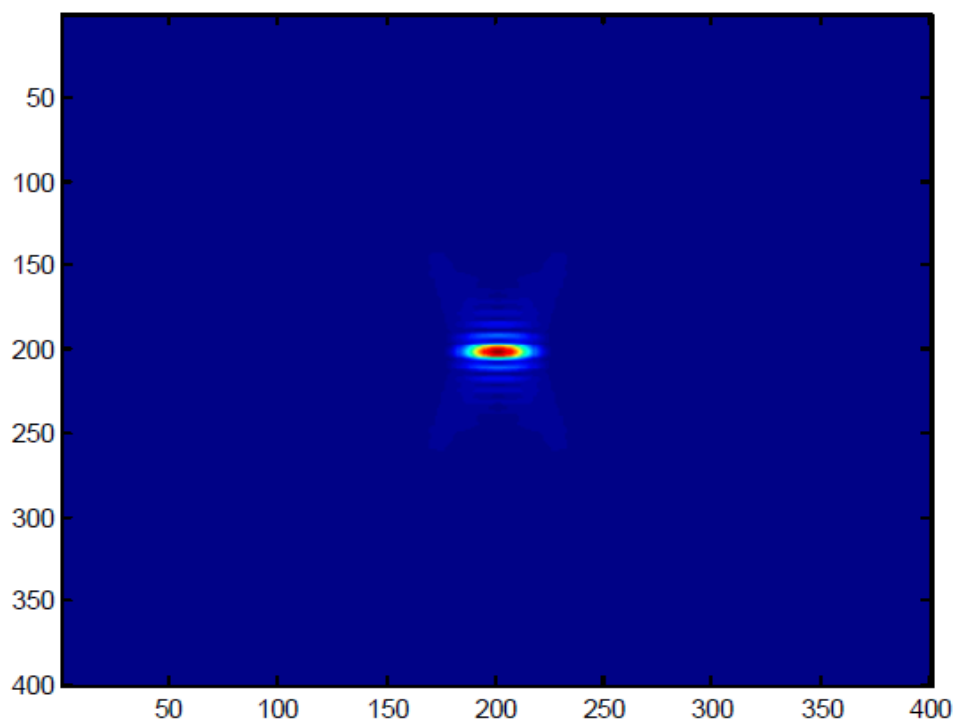


Figure 2-8: Apodized image by Hanning weighting function (Akhter, 2012).

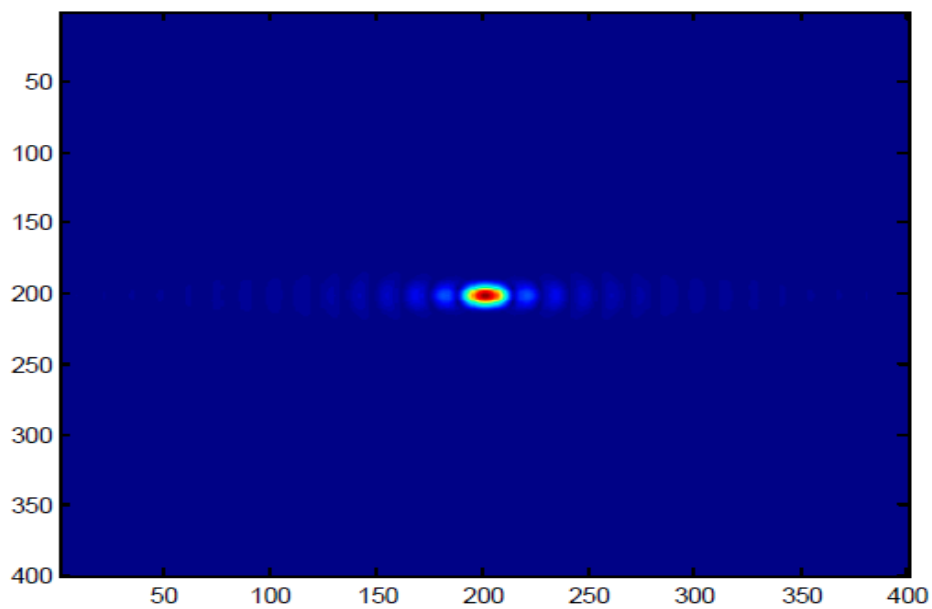


Figure 2-9: Apodized image by Hamming weighting function in the angular direction (Akhter, 2012).

Figure 2-8 and Figure 2-9, the Hamming weighting function was applied in the azimuth direction and angular direction respectively, 45% loss of resolution was calculated on both directions with reduced orthogonal side-lobes.

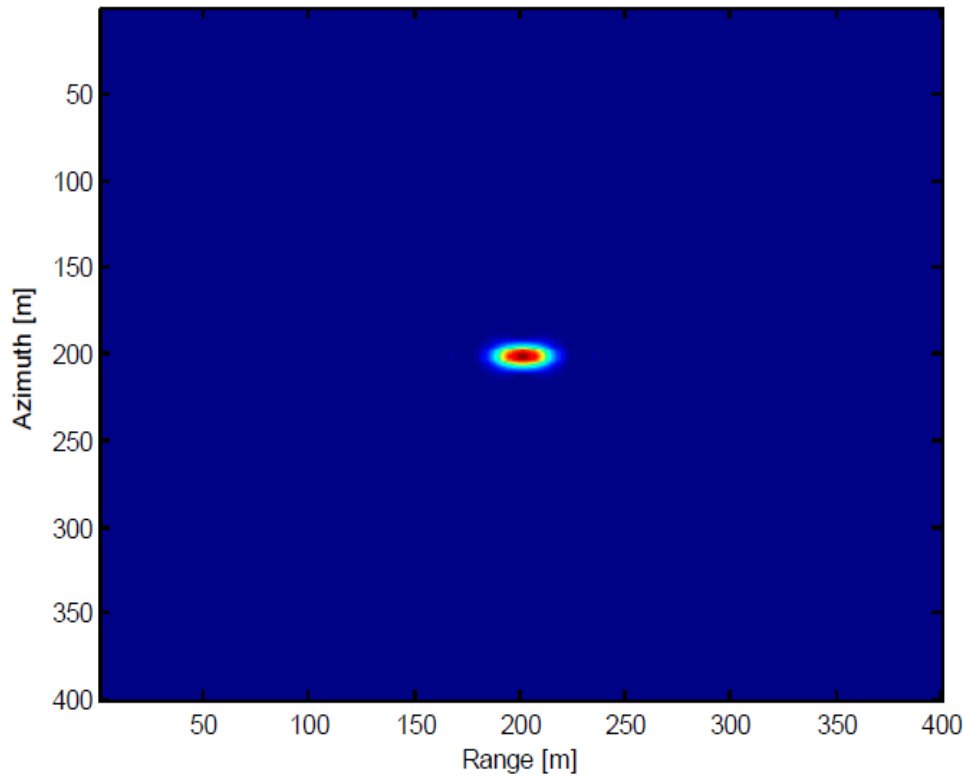


Figure 2-10: SAR apodized image after applying proposed new linear weighting function (Akhter, 2012).

In Figure 2-10, the proposed linear weighting function was applied in azimuth, range and angular directions. The side-lobes in orthogonal and non-orthogonal were reduced and resolution loss was measured to be 33.34% (azimuth direction) and 36.58% (range direction). The resolution loss is better than the other traditional weighting functions. Akhter (2012), mentioned in her thesis that the resolution in angular direction is much less than the two other directions (azimuth and range) and did not state by what percentage was the resolution loss in angular direction. The proposed weighting function was verified to be accurate to use on SAR images because it was proven to reduce side-lobes yet retain spatial resolution on SAR images (Akhter, 2012).

2.7.1.1 Different types of apodizing functions

There are many different types of apodizing functions or weighting functions as referred in most studies. The weighting functions that are classified as the classic functions are Hamming, Hanning, Blackman, and Taylor. These weighting functions are commonly used in apodization technique. The following subsection is the description of each weighting function and its related theory.

2.7.1.1.1 Hamming window

Hamming window is used in many applications, such as speech and sound processing. It is used because of its great ability to reduce side-lobes while retaining the main-lobes (Akhter, 2012). The coefficient of Hamming window is computed from Eq.11.

$$w_{hm}(n) = \alpha - \beta \cos\left(\frac{2\pi n}{M}\right) \quad \text{Eq.11}$$

where α is 0.54 and

β is 0.46.

2.7.1.1.2 Hanning

Hanning window is generally used when doing operational noise and vibration measurements. It is often used with random data because of its moderate impact on frequency resolution and amplitude precision of the resulting frequency spectrum. Eq.12 and Eq.13 are applied in Hanning window operations.

$$w(n) = \frac{1}{2} \left(1 - \cos\left(\frac{2\pi n}{N-1}\right) \right) \quad \text{Eq.12}$$

or

$$w(n) = \sin^2\left(\frac{\pi n}{N-1}\right). \quad \text{Eq.13}$$

2.7.1.1.3 Blackman window

Blackman window uses a cosine function to reduce the side-lobes of the signal, it is defined in Eq.14.

$$w_{blk}(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{N-1}\right) + a_2 \cos\left(\frac{4\pi n}{N-1}\right) \quad \text{Eq.14}$$

where $a_0 = \frac{1-\alpha}{2}$ $a_1 = \frac{1}{2}$ $a_2 = \frac{\alpha}{2}$

for $0 \leq n \leq M$.

2.7.1.1.4 Triangular

$$w(n) = 1 - \left| \frac{n - \frac{N-1}{2}}{\frac{L}{2}} \right| \quad \text{Eq.15}$$

2.7.1.1.5 Cosine

Cosine window functions are commonly used in signal processing. Eq.16 mathematically defines the Cosine window,

$$w(x) = \cos\left(\frac{\pi x}{2a}\right). \quad \text{Eq.16}$$

2.7.1.1.6 Gaussian

Gaussian window is applied in both signal processing and uses a standard deviation and an eigen function of the Fourier transform. Eq.17 is the confined Gaussian window and Eq.18 is the Gaussian window function which uses the standard deviation.

$$w(n) = G(n) - \frac{G\left(-\frac{1}{2}\right)[G(n+N)+G(n-N)]}{G\left(-\frac{1}{2}+N\right)+G\left(-\frac{1}{2}-N\right)} \quad \text{Eq.17}$$

$$G(x) = e^{-\left(\frac{x - \frac{N-1}{2}}{2\sigma_t}\right)^2} \quad \text{Eq.18}$$

for $\sigma_t < 0.14N$,

2.7.2 Filtering using ImageJ

ImageJ is digital image processing and analysis software which was inspired by the National Institute of Health (NIH) in the USA. It is a Java-based image processing software developed by Wayne Rasband. It is an open source and may be used online or offline (Rasband, 2018).

2.7.2.1 Types of filtering in ImageJ

The following are the built-in filtering modes in ImageJ.

2.7.2.1.1 Mean

In the mean filter, also known as box filter, average filter is used to smooth an image to remove all the artefacts contained in the image. It decreases the quantity of intensity variation between one pixel and the next. What the filter does is, it calculates the mean or the average by adding up all the numbers in the data set and dividing by the total; Eq.19 present the mean formula (Paul-Scherrer-Institute, 2019);

$$\mathbf{mean} = \frac{\sum x_i}{n} \quad \mathbf{Eq.19}$$

where x_i is numbers in the individual data set and n represent how many numbers are in the data set. On the software one gets to select radius. With the choice of radius selected, the mean is calculated and each pixel value is replaced by the mean value of its neighbour.

2.7.2.1.2 Median

Median filter is a non-linear filter used to remove noise and smooth an image. Median filter is usually used in real space images and is not only limited to real space images but can also be applied in Fourier space. Median is defined as the number that represents the middle of the distribution. Before calculating the median, the data set needs to be sorted first from smallest to largest and the median value may be determined from the sorted data set. In simple, median is calculated as in Eq.20;

$$\mathbf{median} = \frac{n+1}{2} \quad \mathbf{Eq.20}$$

where n represent the numbers of data sets. Note that for odd and even data set the median is calculated differently. Just like in mean filter, ImageJ gives an option to select the radius, within a specified radius median is calculated and each pixel value is replaced by the median value.

2.7.2.1.3 Gaussian blur

Gaussian blur is a type of filters that blurs the image artefacts and removes noise. It works similar to the mean filter but does not average the neighbouring pixels. Eq.21 represent Gaussian function in one dimension

$$G(x) = ae^{\frac{-x^2}{2\sigma^2}} \quad \text{Eq.21}$$

where $a = \frac{1}{\sqrt{2\pi}\sigma}$ and substituting a gives:

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}} \quad \text{Eq. 22}$$

where σ is the standard deviation, x is the distance from the origin on the horizontal axis. Eq.23 applies for 2D

$$G(x, y) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-x^2+y^2}{2\sigma^2}}. \quad \text{Eq. 23}$$

The idea of Gaussian blur is to remove unwanted details in an image but still keep the edges. The Gaussian blur filter depends on the standard deviation.

2.7.2.1.4 Minimum and maximum filter

Minimum and maximum filter are morphological filters. Morphological filters are non-linear filters which use mathematical operators to detect edges of an image. Morphology is a wide-range imaging process technique that uses operators to transform images based on their structure. Originally, morphology filter was typically applied to binary images, but the application expanded and it is now applied to grayscale images too. They use dilation and erosion operation to detect edges. The morphological filters may be applied in real and Fourier space. Morphological methods

analysis images with a small shape known as structuring element (SE) (Gopal and Svalbe, 2018). Morphological filters are split into two methods which are; minimum and maximum filters. These two filters are opposite of each other, minimum filter is known as erosion and maximum filter as dilation (resizing). The erosion and dilation operators are used to find the minimum and maximum of the pixel values within a specified radius in an image.

Maximum- to calculate dilation of an image (grayscale image in this case), each pixel in an input image is considered. The rudimentary effect of the dilation operator is to enlarge the image pixels especially the foreground pixels. In simple terms, the maximum filter applied on text, the text will gradually become thicker.

2.7.2.1.5 Un-sharp Mask

Un-sharp mask is another form of filtering that sharpens the edges of an image; this is done by removing the un-sharpness or blurredness on the original image and inventing a new form of image from the filtered original image. This type of filter is equivalent to adding a high pass filter to an image. Using un-sharp mask as a filter, there are three settings that needs to be well understood; amount, radius and threshold. Amount controls the magnitude of the contrast that is added to the edges of an image, it determines how much lighter or darker the edges of an image become. Radius influences the size of the edges to be improved. A smaller radius will improve smaller scale details whereas a higher radius will cause halos at the edges of an image. Threshold controls the contrast of an image and determines the minimal brightness change before the filter applies any changes. This setting is also used to sharpen more definite edges while leaving subtler edges untouched.

Chapter 3: Methodology

3.1 Introduction

This chapter focuses on outlining the methodology and the choice of research method for testing the theoretical framework and addresses the research question, objectives and research problem. The research methodology is a key activity that supports the choice of research design and includes a step by step process of how the research was carried out. The research design of choice was quantitative using an experimental approach. Data was obtained from digital neutron radiographs with values ranging from 0 – 65535 for a 16-bit image format obtained from PSI. The images are 2160 by 2560 pixels. Within the quantitative approach, the experiment was conducted to obtain data which was analysed to provide results with regards to spatial resolution.

3.2 Facilities – neutron sources

There are several instruments that can be utilised to obtain neutron radiographs or images. The data was collected from PSI in the Neutron Transmission Radiography (NEUTRA) facility. Figure 3-1 is an aerial view of PSI campus in Switzerland. PSI has four large facilities; the Switzerland X-ray free-electron laser (SwissFEL) which is focused on matters in biology, chemistry and material science. The Swiss Light Source (SLS) is the third generation synchrotron light source at PSI and it is part of the photon science division focusing on research in chemistry, biology and material science for in-depth 3D imaging. The other two large facilities are Spallation neutron source (SINQ) and the Swiss Muon Source (S μ S) (Paul-Scherrer-Institute, 2019). Each large facility has different beam lines for different uses. SINQ is used for neutron radiography and tomography, it has both cold and thermal neutrons. The facilities are discussed in the section 3.2.1.

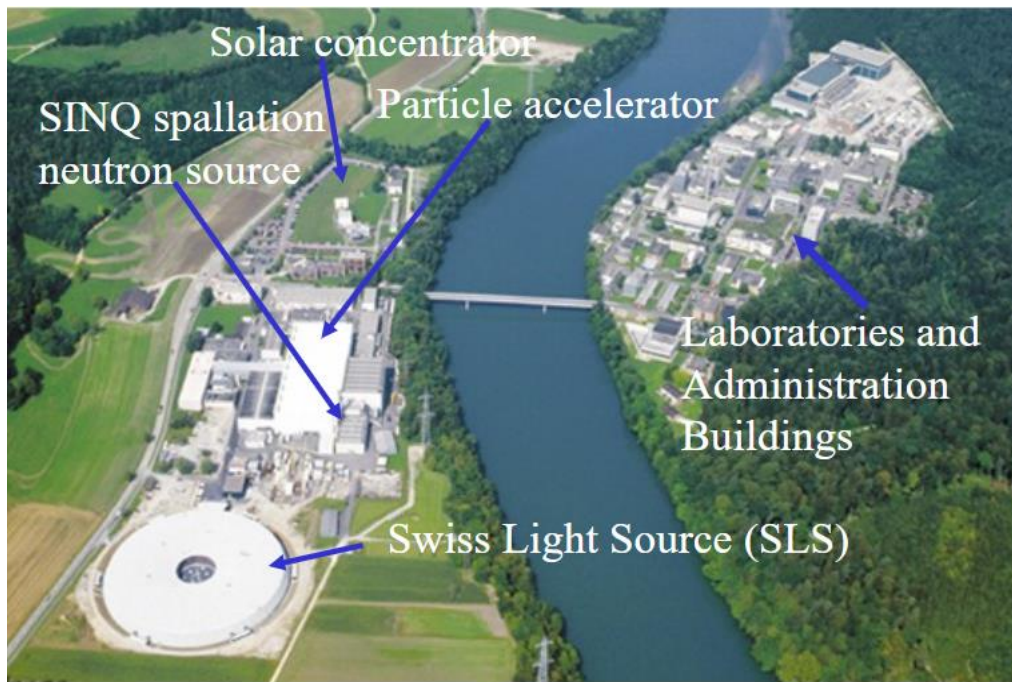


Figure 3-1: PSI campus (Paul-Scherrer-Institute, 2019)

3.2.1 NEUTRA

NEUTRA is thermal neutron radiography and tomography facility at PSI. It is one of the leading radiography facilities in Europe because of its steady-state (Lehmann et al., 2001) proton beam current of $850\mu\text{A}$ and driven by 590MeV energy (Alam et al., 2006). NEUTRA has three key components which are, the working room with a shielded concrete assembly block, the outer and the inner collimator (Lehmann et al., 2001) as shown in Figure 3-2. The inner collimator is located within the target shielding block and the outer collimator is an evacuated divergent aluminum tube. They are described in detail by (Alam et al., 2006).

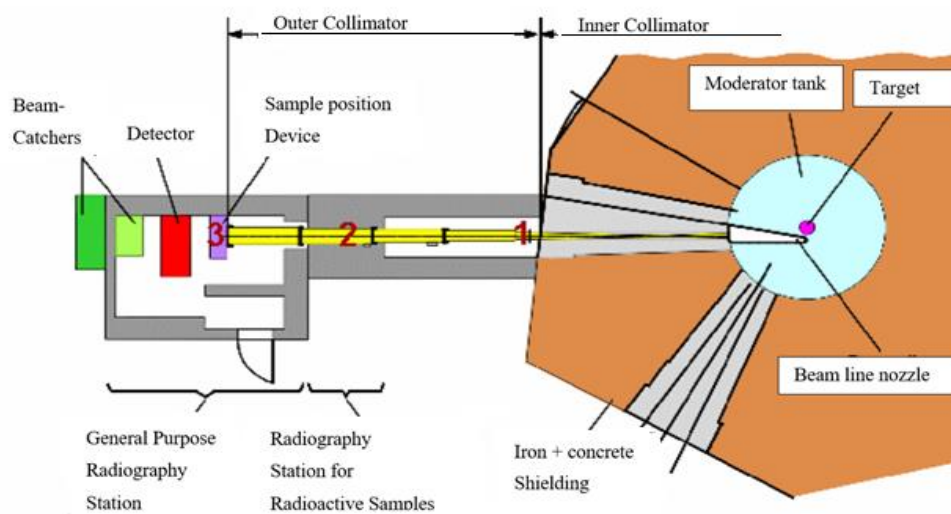


Figure 3-2: Graphic layout of the NEUTRA facility at PSI thermal beam line of SINQ (Lehmann et al., 2003).

The great thing about the outer collimator is that it can be fragmented into two or more parts to perform different investigations at positions of choice under suitable conditions. It has a high neutron flux, which makes it appropriate for irradiation experiments. The properties of the beam positions were verified at the facility by Lehmann et.al, 2001. The L/D verification was done following the same method which was applied by (Kobayashi and Kiyanagi, 1996). The L/D of 500 is advantageous when performing experiments which require a larger distance between the detector and the sample, Table 3-1 and Table 3-2 show properties of the system and different positions at NEUTRA facility that are available to use.

Table 3-1: Properties of the NEUTRA (Lehmann et al., 2001)

System Characteristic	Position number		
	1	2	3
Distance from the target centre (mm)	6404	9876	13131
Distance from the aperture (mm)	3820	7292	10547
Beam diameter (mm)	150	290	400
Neutron flux ($\text{cm}^{-2}\text{s}^{-1}\text{mA}^{-1}$)	1.6×10^7	5×10^6	3×10^6
L/D	200	350	550

Table 3-2: Properties of NEUTRA imaging detector system (Lehmann et al., 2001).

System Characteristic	Imaging Plates	Scintillator & CCD camera	X-ray film & digital transmission scanner
Inherent spatial resolution	50 μm	500 μm	20 μm
Typical exposure time for generation of good images	20 sec	10 sec	5 min
Number of pixels per line	4000 x 8000	512 x 512	9000 x 12000
Detector area	20 cm x 40 cm (given by the IP)	25 cm x 25 cm (given by the scintillator)	18 cm x 24 cm (given by the vacuum cassette & film)
Dynamic image	10^5 (linear)	10^5 (linear)	10^5 (linear)
Applicability for quantitative studies	Yes	Yes	Yes

3.2.2 ICON

Imaging with Cold Neutrons (ICON) is a beamline attached to the cold neutron source of SINQ based in Switzerland, PSI, which started its operation in 2005 and has provided services for both science and industry. The facility as its name suggests it uses cold neutrons for investigations. The objects that are sampled at ICON are often thinner because of the high attenuation coefficient when using cold neutrons. The neutrons generated from this sector are moderated by a liquid deuterium (D_2) tank at a temperature of 25 K.

Cold neutrons also have an advantage for use on several applications including and not limited to neutron imaging, neutron diffraction, material science and archaeology (Kaestner et al., 2011). The advantage of using cold neutrons compared to thermal neutrons of NEUTRA is that it permits diverse investigations and high contrast images can be obtained. It also allows larger macroscopic cross-section for materials as a result of the energy dependence of absorption cross-sections. Table 3-3 summarizes the camera system and Table 3-4 summarizes beam parameters.

Table 3-5 is the resolution and scintillation available at ICON with different experimental positions.

Table 3-3: Properties of camera system used at ICON (Kaestner et al., 2011).

System Property	Andor DW434	Andor DV436	PCO. 4000	PI-MAX 1300
Pixels	1024 x 1024	2048 x 2048	4032 x 2688	1340 x 1300
Dynamic	16 bits	16 bits	14 bits	16 bits
Exp. Pos.	2.3	2	2.3	2.3
Camera box	Midi/maxi	Micro	Micro/midi/maxi	Midi/maxi
Ext. trigger	-	-	Yes	Yes
On-chip memory	-	-	4 GB	-
Photomultiplier	-	-	-	Yes

Table 3-4: Resolution and scintillators accessible at ICON (Kaestner et al., 2011)

Scintillator material	⁶Li		Gadolinium	
Thickness (μm)	50	100	10	20
Resolution (lp/mm at10%MTF)	12.7	11.8	24.6	18.0
Resolution (μm)	39.4	42.4	20.3	27.8
Sensitivity (count/s.mA	226.5	515.8	61.46	95.35

Table 3-5: Resolution and scintillators accessible at ICON (Kaestner et al., 2011).

System Characteristic	Value at Experimental Position									
	2					3				
Aperture diameter (mm)	1	10	20	40	80	1	10	20	40	80
L/D Range	6864	686	343	172	86	12083	1208	604	302	151
Neutron Flux (cm ⁻² s ⁻¹ mA ⁻¹)	-	3.2x 10 ⁶	1.3x 10 ⁷	-	1.4x 10 ⁸	-	-	3.9x 10 ⁶	-	5.2x 10 ⁷
Spectral Resolution (%) at 20 mm aperture diameter	$\Delta\lambda/\lambda = 15$					$\Delta\lambda/\lambda = 15$				
Wavelength range (Å)	2.5 – 9					2.5 – 9				
FOV (mm ²)	27 x 27					120 x 200 except 80 mm aperture				

3.3 Neutron radiography setup

This section discusses the methodology used to analyse the results to obtain spatial resolution in digital neutron radiography using FRC and MTF/SFR methods.

In neutron radiography the method used to obtain radiographs is standard and consists of the following: Image acquisition, pre-processing of images, processing of images, analysis of images and finally the results. Image acquisition is basically the act of retrieving an image from a hardware-based source such as a CCD camera, in order to get the images for analysis. Figure 3-3 shows the basic neutron radiography setup.

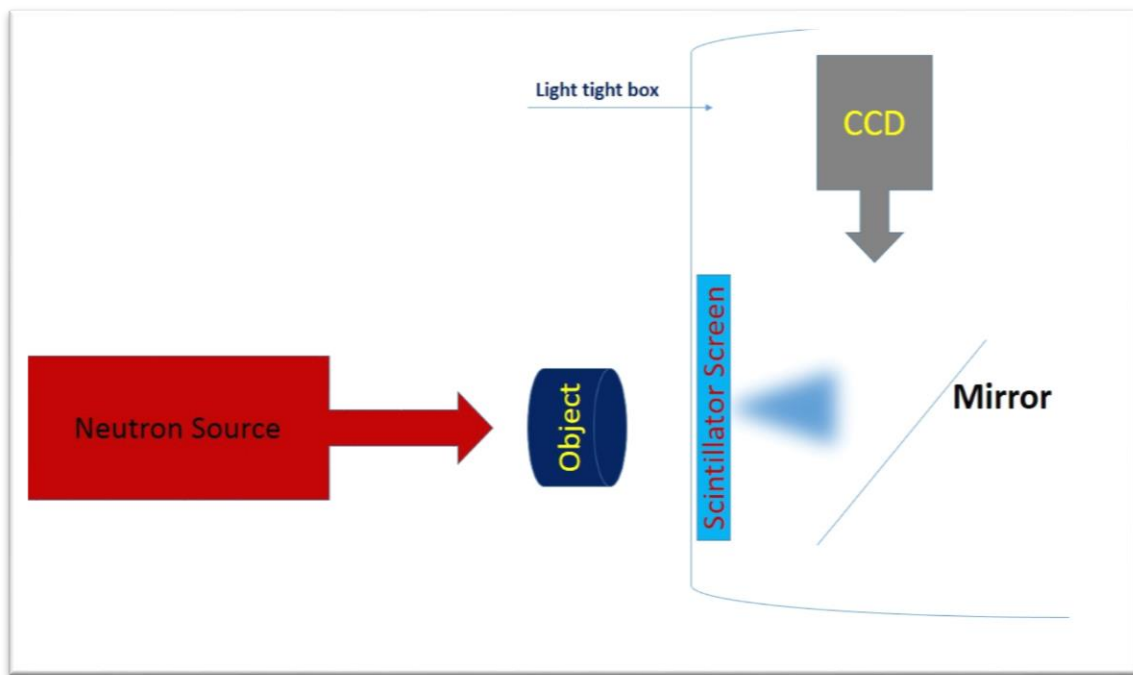


Figure 3-3: Neutron radiography setup.

The research design was quantitative, data used was measured at the thermal neutron imaging instrument, NEUTRA at the SINQ spallation neutron source, PSI. Gadolinium knife edge. Figure 3-4 was used as a test object to acquire images that were used for analyses to obtain the results. The test object consisted of gadolinium sheet fixed in two aluminium layers. The aluminium layers were there to hold and support the Gadolinium sheet. Gadolinium was used because of its greatest ability to absorb neutrons. Gadolinium consists of six stable isotopes, ^{154}Gd , ^{155}Gd , ^{156}Gd , ^{157}Gd , ^{158}Gd and ^{160}Gd with natural abundance of 2.18%, 14.8%, 20.5%, 15.7%, 24.8% and 21.9% respectively and one radioactive isotope ^{152}Gd with half-life of 1.08×10^{14} years. In the nuclear industry, ^{155}Gd and ^{157}Gd are the two stable isotopes used because of their high neutron absorption cross-section. Their absorption cross-sections are 61 000 b and 254 00 b respectively (Power, 2018, Radebe, 2017).

The two aluminium layers used were 2 mm in thickness; this allowed room for the gadolinium sheet to be exposed to the detector. The object were sampled in two dimensional, vertical and horizontal planes. The gadolinium knife edge test object was placed close to the detector, the ODD $\approx$ 0 mm, and the collimation L/D=800 mm with an exposure time of 35 seconds, which was sufficient to provide statistical accuracy. The imaged radiographs have statistical consistency in order to minimise noise. In radiography, noise is the unwanted details in an image that affect its visualization and interpretation (Samei, 2003). Statistics here is described as signal-to-noise-ratio (SNR). The same gadolinium knife edge test object was imaged at different ODD of 12.0 mm, 32.0 mm and 104 mm to determine which ODD gives the best spatial resolution.

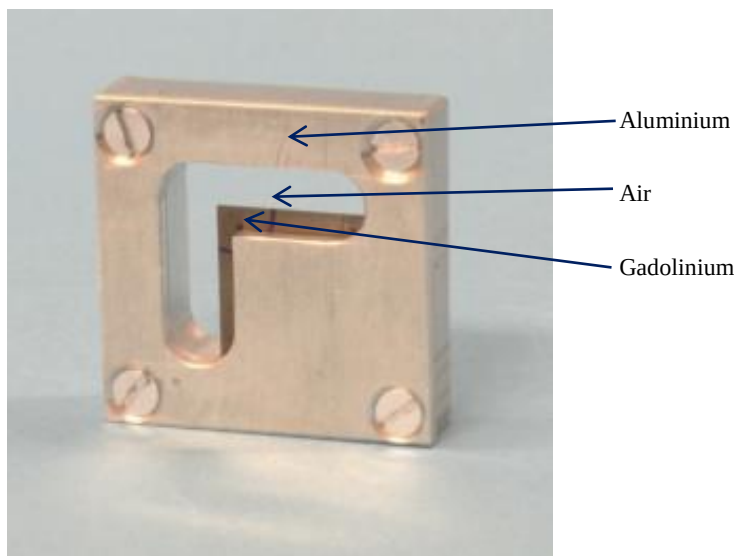


Figure 3-4: Gadolinium knife edge test object (Radebe, 2017).

The gadolinium knife edge was imaged twice at the same distance and properties. The resultant images are shown in Fig 3-5a and Fig 3-5b.

3.4 Determination of spatial resolution

This subsection details out the method to determine the spatial resolution of an image using the proposed method, the FRC method. By definition spatial resolution is the ability of an imaging instrument to be able to differentiate between two features that are close to each other. The test object was imaged twice at the same position, in order to have two images with the same SNR.

Both images were in real space and were analysed in Fourier space. A method on how real space images were analysed in Fourier space follows in section 3.4.1

3.4.1 Matrix Laboratory (MATLAB)

MATLAB is a multi-paradigm numerical coding and programming language. It is a product of the MathWorks, Inc., developed by Cleve Moler in the late 1970's. It is a powerful language for technical computing and an accessible environment where problems and solutions are expressed in mathematical notation. MATLAB has two kinds of program files, scripts and functions. Both program files have .m extension. In these two files, a series of commands can be written which wants to be executed together. MATLAB has an image processing toolbox which allows the user to perform noise reduction, background subtraction, segmentation, image enhancement and geometric transformation. And this can be applied to different image formats as MATLAB is capable of reading numerous image formats (Jalalvand et al., 2019).

3.4.2 Fourier Ring Correlation method

FRC is mainly used to characterise the spatial resolution in nanoscopy, X-ray crystallography, microscopy and diffraction applications. It hasn't been used in digital neutron radiography and X-ray imaging. In this research, the FRC method was proposed to measure spatial resolution with different threshold criterion available.

Two images were acquired in real space using neutron beamline, and were analysed and compared against one another because in nature cross-correlation compares one thing to the other. Therefore, two images were compared to determine the spatial resolution. A mathematical approach Fourier Ring Correlation (FRC) was used to find the resolution of an image. To find the suitable threshold criteria, different threshold criterion including (the σ factor, 2σ , 3σ , $\frac{1}{2}$ bit, 1 bit, 0.143 and 0.5) were tested for the FRC method using a MATLAB program and the suitable threshold criterion was 0.5 threshold, and was used to present the results. The criteria used to determine the suitable threshold was, on the FRC MATLAB code, different threshold criterion were tested. The other criterion could not give the output as close as expected. Literature shows that FRC was applied to assess spatial resolution in microscopy and nanoscopy (Nieuwenhuizen et al., 2013). The FRC formula to be applied, is a criterion on its own, and the criterion work in a way that two images

must be sampled. The FRC equation is mathematically described as in Eq.3. The two images were put as an input in the FRC MATLAB function, the code ran to display the FRC and spatial frequency curve as an output results.

An algorithm of how the FRC MATLAB function calculates FRC and spatial frequency curve to determine the spatial resolution is described in the following steps;

- Two independent reconstructions (image 1 and image 2) of the same object were required.
- The number of rings and image pixel size are also required for the input.
- Apply Fourier transform on each independent reconstruction,
- Assign rings on the two independent reconstructions in the same manner; every ring differs with thickness and radii.
- Image 1 is calculated as it is, the complex conjugate of image 2 is calculated as in Eq.3
- Take ring one of image one and ring one of the complex conjugate of image two, find artefacts (pixels) in each ring and get the products of artefacts in image one and image two and sum up all the artefacts of ring one.
- Do the same for ring two till the n^{th} ring (on image one and image two). To normalise the method, the square of artefacts in ring one of image one is summed up and the square of artefact in ring one of image two are summed up and the product of the two are over the square root in Fourier space. This is done on the denominator of Eq.3.

$$FSC/FRC(r_i) = \frac{\sum_{r \in r_i} P_1(r) * P_2(r)^*}{\sqrt{\sum_{r \in r_i} P_1^2(r) * \sum_{r \in r_i} P_2^2(r)}}, \text{ and}$$

- Plot the FRC and threshold versus spatial frequency curve on the x-y plane.

The FRC measurements are based on statistical accuracy. A resolution limit is determined from the curve, where the FRC and threshold curve meet. Figure 3-5 summarises the workflow used to compute the FRC curve using the FRC MATLAB function to determine spatial resolution. Fig 3-5a and Fig 3-5b are the images of gadolinium knife edge imaged twice at the same position. The two images are Fourier transformed to form Fig 3-5c and Fig 3-5d. These results were plotted on an x-y plane where FRC is on the y-axis and spatial frequency is on the x-axis as in Fig 3-5e. The MTF/SFR method which is the standard method that has been applied in neutron radiography is detailed in section 3.4.3. The FRC results were then compared with the results obtained from the MTF/SFR.

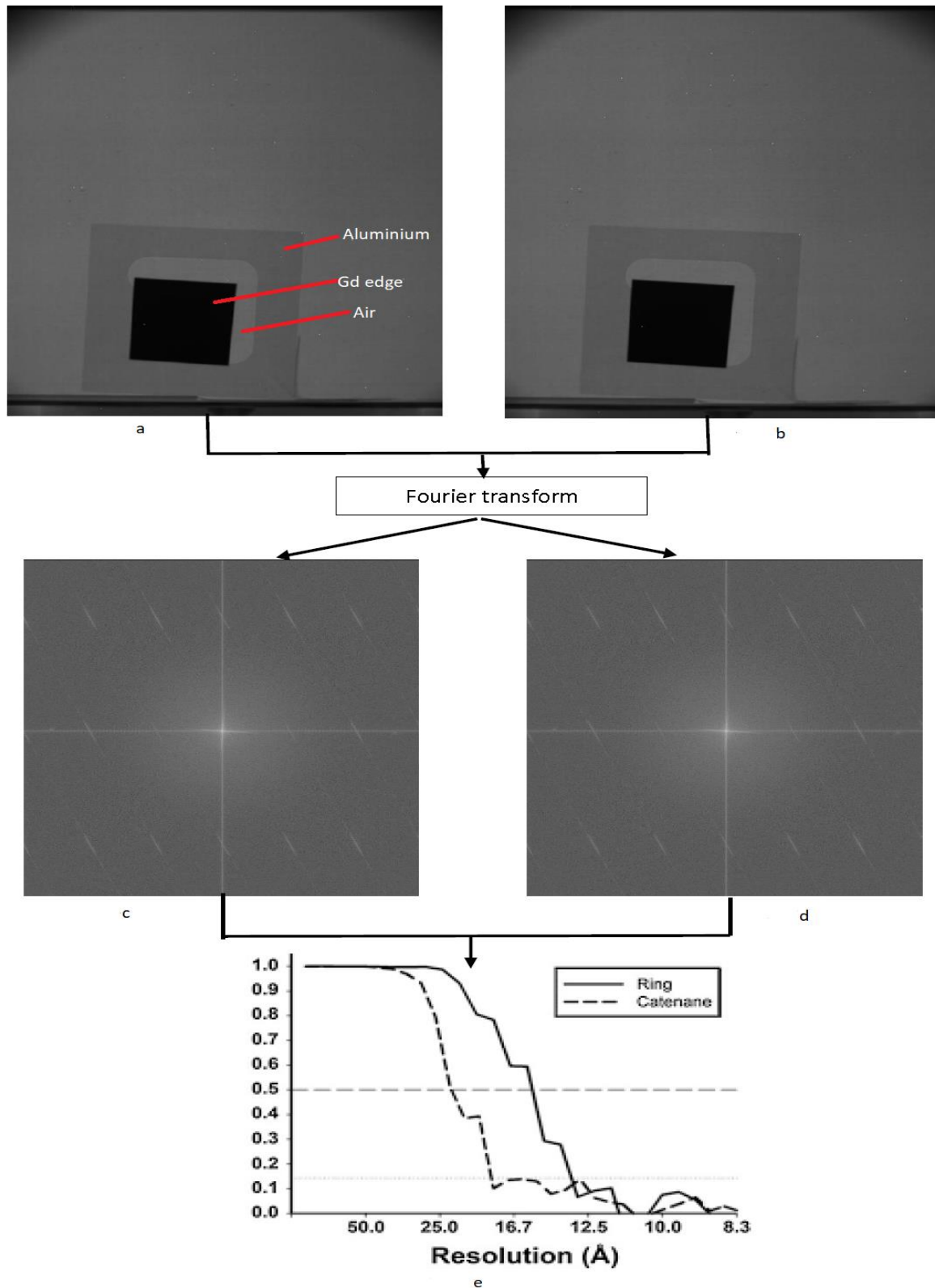


Figure 3-5: Computational steps on how to compute FSC/FRC. Fig 3-5e is the FRC/Resolution curve (Radebe, 2017).

3.4.3 Modulation Transfer Function (MTF) method

To compute the spatial resolution using the standard method MTF/SFR, *sfrmat* was used as a MATLAB function to provide an MTF/SFR and ESF from digital images containing slanted-edge features. *Sfrmat* is a spatial frequency response MATLAB function developed by Peter Burns in order to obtain the MTF curve from an edge image (Burns, 2009). The precise edge-gradient algorithm follows the same standard of the ISO 12233, which was developed for resolution measurements of the electronic still images (Burns, 2003).

An *sfrmat* on MATLAB was launched, once opened, the run button on the MATLAB toolbar was selected to run the script, the slanted-edge analysis window appeared to select an image to be analysed. The Gadolinium edge image was selected (same image as in Figure 3-5a and Figure 3-5b). Only images with the following extensions can be used (Dicom, JPEG, GIF, BMP, RVG and RAW) otherwise the script won't run. A display of data sampling window in pixel/inch (ppi) or millimetre (mm) appeared, this was for selecting the correct sampling data that the user want to be sampled. To select the region of interest (ROI) on gadolinium knife edge images, 50% of the Gadolinium edge and 50% of air was selected in an upright rectangular shape because if incorrectly selected the *sfrmat* function interpret the results incorrectly. The ROI was selected as shown in Figure 3-6.

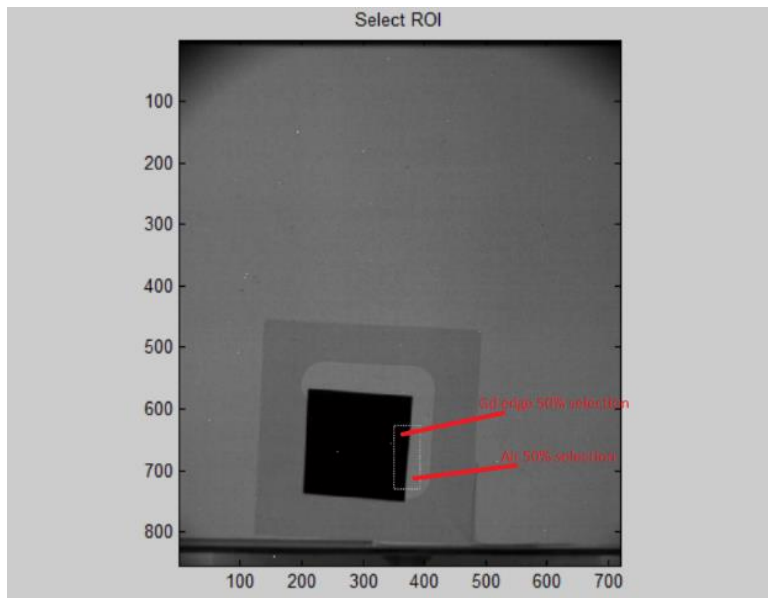


Figure 3-6: Region of interest selection on Gadolinium edge with 50% Gadolinium edge selection and 50% air selection.

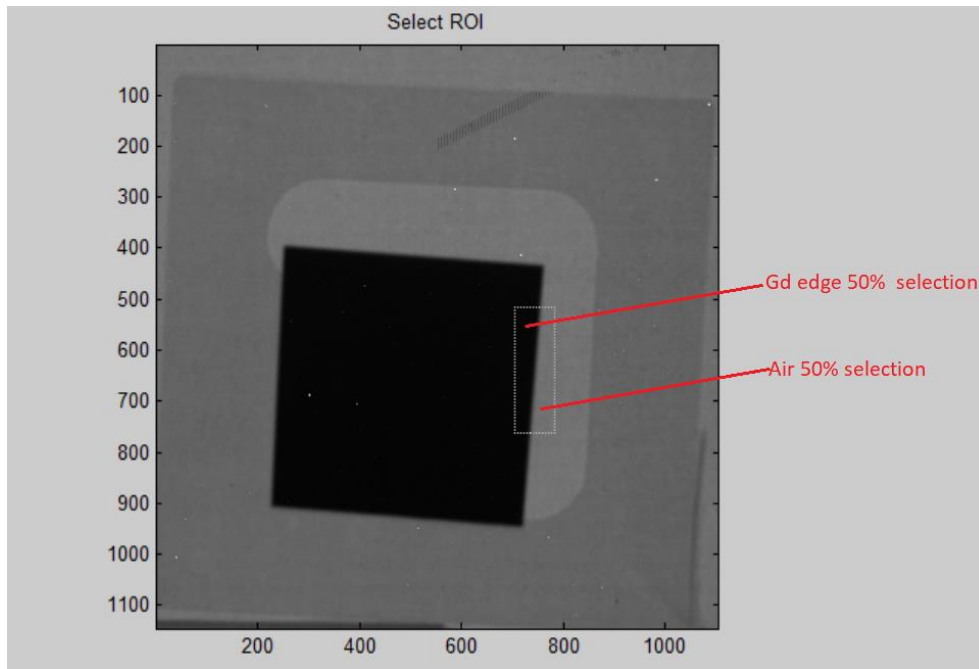


Figure 3-7: Cropped Gadolinium knife edge image to display ROI with 50% selection on Gadolinium edge and 50% air selection.

After the selection of the ROI, the Edge Spread Function ESF and MTF/SFR graphs were automatically displayed as an output as in Figure 3-8 and Figure 3-10 and the output data points were saved as an MS EXCEL spreadsheet. These MS EXCEL data points were used to plot the Line Spread Function (LSF). The LSF was obtained by differentiating the ESF output.

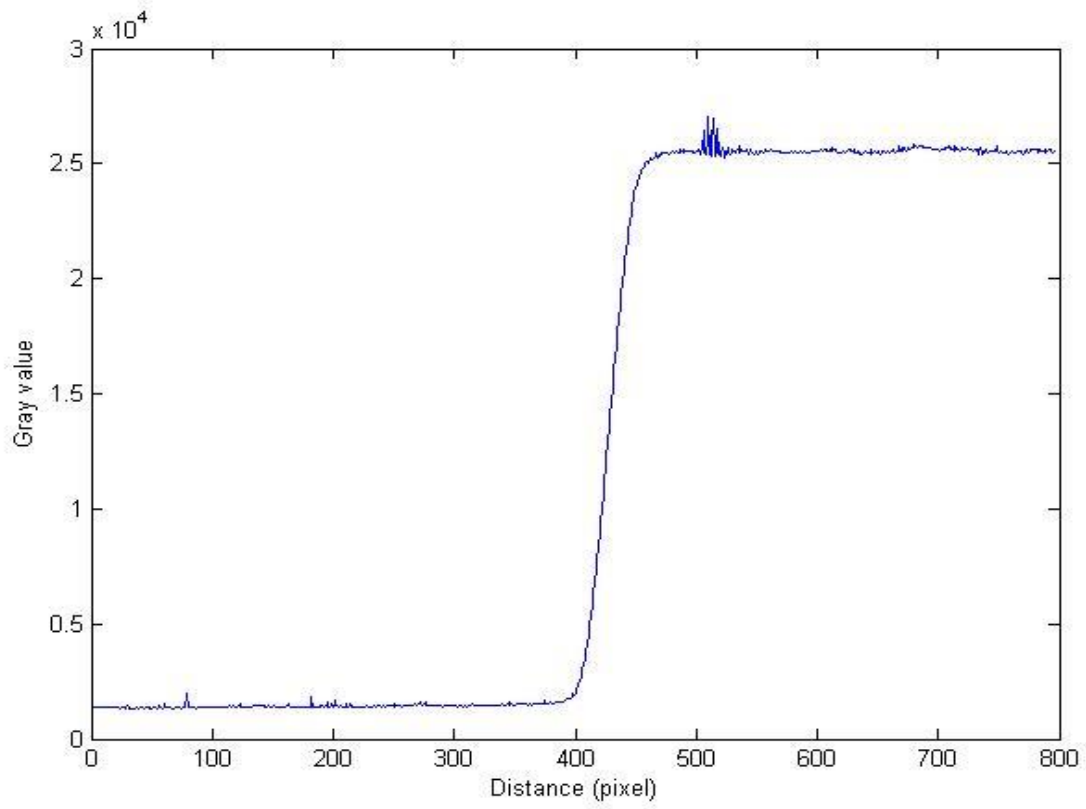


Figure 3-8: Edge Spread Function (ESF) curve of Gadolinium knife edge test object.

The ESF output saved as MS EXCEL data points was used to derive Line Spread Function (LSF). The LSF is equal to the first derivative of ESF which is differentiated using numerical method as shown in equation Eq.24

$$LSF = \frac{ESF_{i+1} - ESF_i}{2(x_{i+1} - x_i)} \quad \text{Eq. 24}$$

where $i = 1, 2, 3 \dots n-1$ and x_i is the position of the i^{th} pixel. Figure 3-9 shows the LSF output.

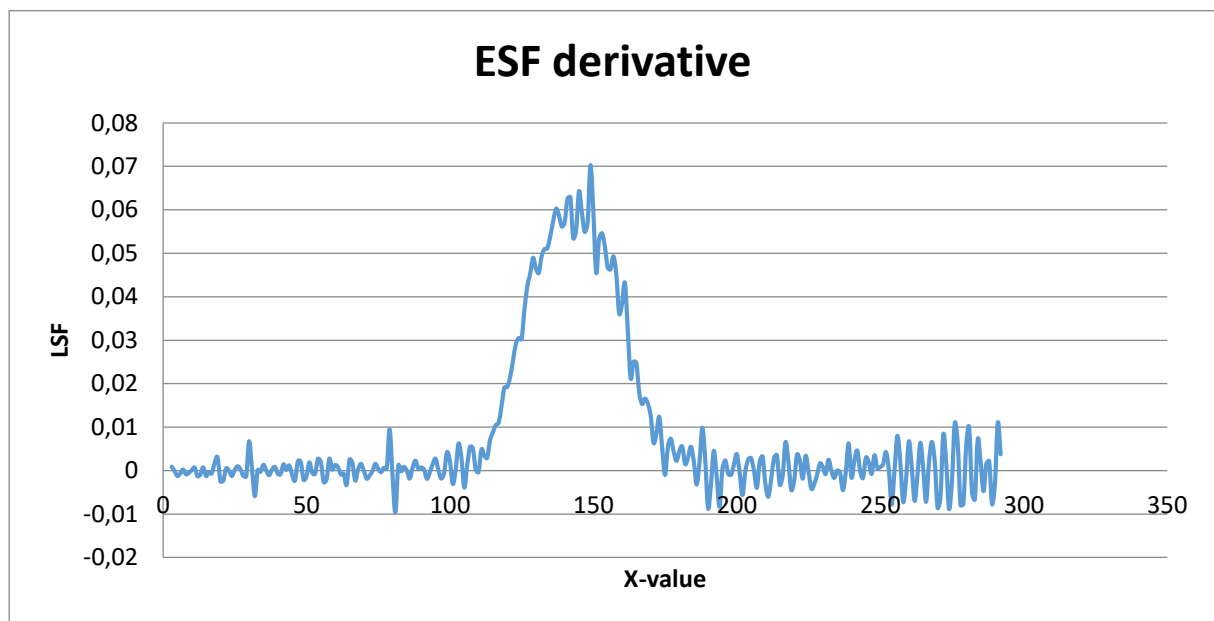


Figure 3-9: Line Spread Function which is the ESF derivative.

LSF can also generate an MTF curve by Fourier transforming the LSF data point.

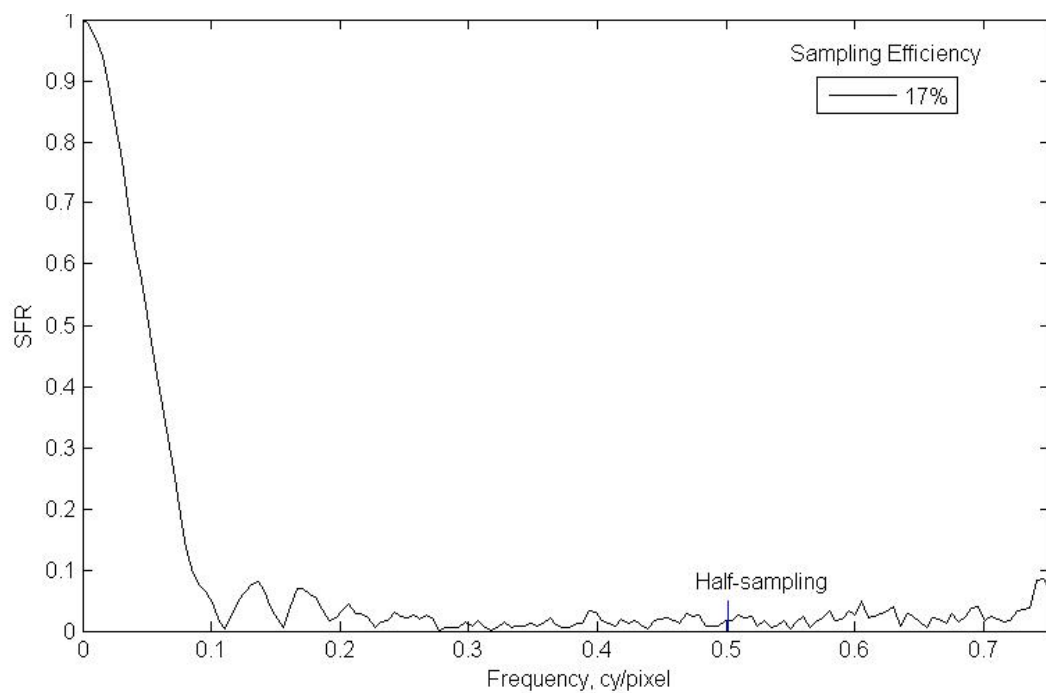


Figure 3-10: MTF/SFR curve with sampling efficiency of 17%.

Chapter 4: Results and discussion

4.1 Introduction

This chapter discusses the results obtained using the methods described in chapter 3 and has three sections detailing out the results. Section 4.2 of this chapter focusses on FRC results using FRC script as an input in MATLAB whereas section 4.3 focusses on the results obtained using the standard method MTF. Section 4.4 is the comparison of the two methods, the proposed FRC method in neutron radiography and the standard method used in neutron radiography MTF/SFR. The remainder of this chapter summarises the results and the aim of this study.

4.2 Fourier Ring Correlation results

The edge images used on fig 3-5a and fig 3-5b were used to compute the results. These two images (fig 3-5a and fig 3-5b) are the original images as obtained from the NEUTRA facility before any image analysis. The FRC MATLAB function was ran using two input images to produce the FRC curve as a function of spatial frequency. To give a measureable estimate of the image resolution from the FRC, a threshold criteria was used. The FRC curve produced has two curves in one Cartesian plane, the threshold curve and the FRC curve. This is shown in Figure 4-1. The FRC curve and the threshold curve are used to characterize a resolution limit of images.

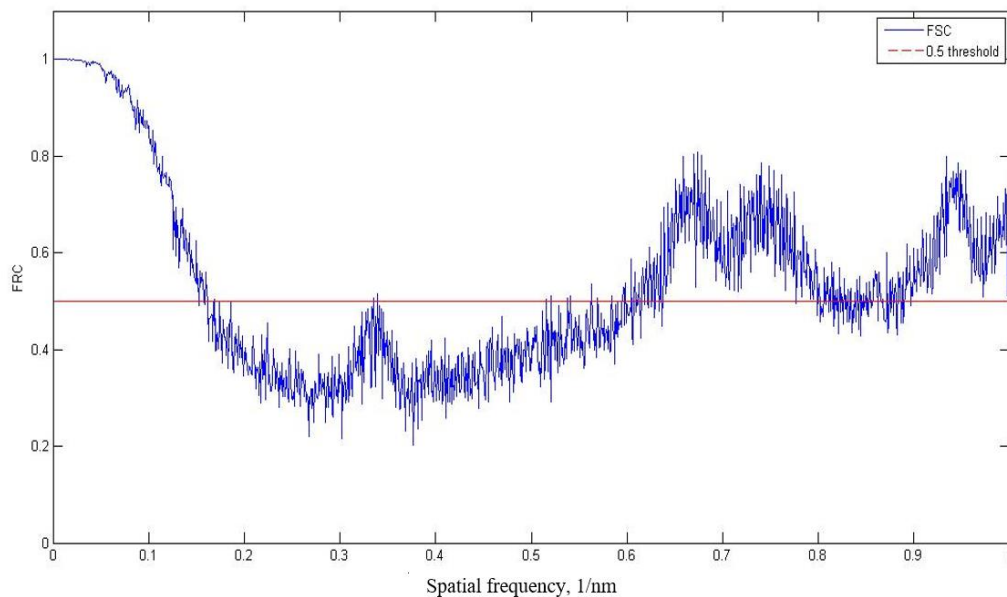


Figure 4-1: FRC and spatial frequency curve.

The FRC curve generated from MATLAB utilised Eq.3 with two slanted gadolinium edge images as in the input. The resulting FRC and threshold curves were evaluated to determine the spatial resolution of the images. For Figure 4-1, the 0.5 threshold criterion was used to measure the spatial resolution of an image. According to literature, there are different threshold criterion for FRC method being the σ factor, 2σ , 3σ , $\frac{1}{2}$ bit, 1 bit, 0.143 and 0.5 criterion. The 0.5 threshold criterion was used in this study because it was fitted to be the one to give the solution for the resolution limit. Figure 4-2 show the solution of the spatial resolution.

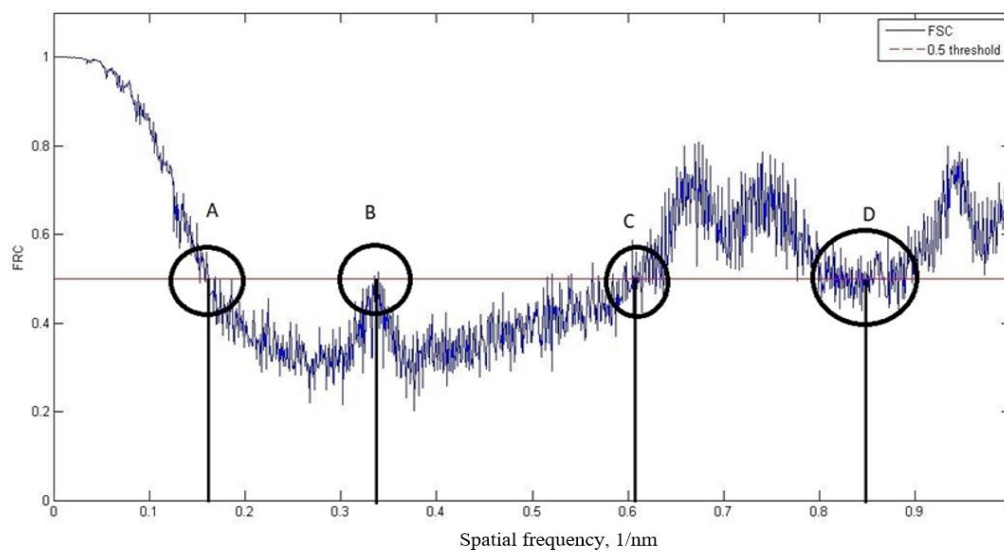


Figure 4-2: FRC curve and spatial resolution solutions at approximately 0 mm ODD.

According to Figure 4-2, the two curves are equal in four positions at 0.5 threshold criterion cut-off; therefore there are solutions at four positions. According to literature, the FRC curve approaches unity and decays to zero (Banterle et al., 2013), before it approaches zero it cuts-off the threshold curve. That is where the spatial resolution result is recorded. For this case where there are four possible solutions, the results for the spatial resolution was recorded at position **A** in Figure 4-2 because it is where the two curves are equal after the FRC curve approached unity. The spatial resolution at position **A** is approximately 0.16 spatial frequency, nm^{-1} . FRC is measured in nanometres (nm) or angstrom ($\text{\AA}$). Here the FRC curve do not decay to zero as expected, instead it floats. This is because of unfiltered noise in the input images. Images were not filtered because

the integrity of the spatial resolution was to be lost as filtering may alter the spatial resolution of images.

4.3 Modulation Transfer Function

The FRC spatial resolution results were compared with the MTF or SFR, this was because the MTF method is the established metric to characterise spatial resolution performance in radiography system especially the X-ray radiography community and is used as a standard method to evaluating the spatial resolution.

The MTF/SFR curve, Figure 4-3 was obtained from the *sfrmat* MATLAB Function. This curve can also be attained using the LSF output by Fourier transforming the output to get the MTF curve.

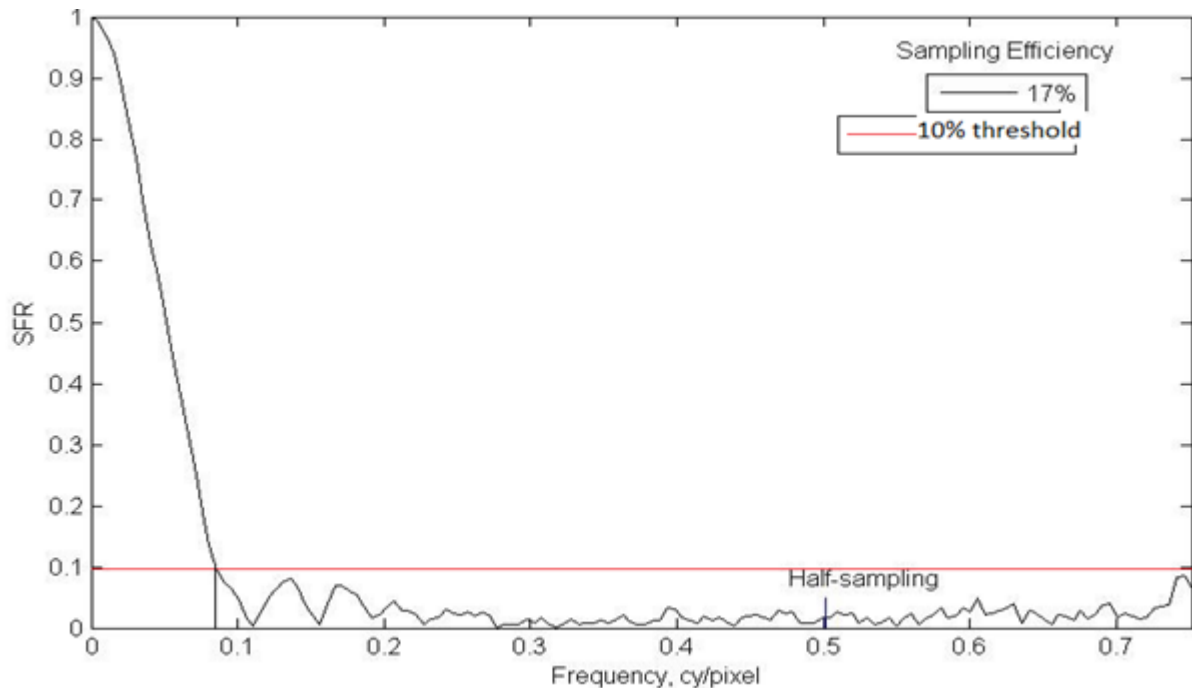


Figure 4-3: MTF/SFR curve with the threshold curve to determine the spatial resolution.

The SFR and frequency curve in Figure 4-3: was obtained from running an *sfrmat* MATLAB function. The SFR curve is expected to approach zero and float on the x-axis but never to touch it. The SFR decaying to zero at 0.5 frequency (half-sampling), would yield good image resolution; as the higher the MTF/SFR value the better the spatial resolution. The FRC results were compared with the MTF/SFR method to evaluate the spatial resolution in digital neutron radiography. In

Figure 4-3 the MTF/SFR curve decays to zero approximately at 0.085 frequency, cy/pixel because of the unfiltered noise on the images used. In Figure 4-3 the spatial resolution solution is observed approximately at 0.085 frequency, cy/pixel at 10% threshold and 17% sampling efficiency.

Sampling efficiency is determined by the limiting resolution (also known as spatial resolution) and the sampling resolution. A limiting resolution is the cut-off between the MTF/SFR and the frequency (measured in cy/mm or cy/pixel), meaning where the two curves intersect. The sampling resolution is defined by the half-sampling frequency. Half-sampling frequency is the halfway of the frequency on an x-axis plane. It is measured in cy/pixel. Sampling efficiency is defined mathematically as in Eq.25.

$$E_v = \frac{\text{limiting resolution}}{\text{sampling resolution}} \quad \text{Eq.25}$$

where E_v , is the sampling efficiency measured in percentage. Sampling efficiency is the results of sampling frequency (number of pixels per spatial unit) which estimates the good or bad of image quality and differentiating between a sharp and blurred image. An image with a good resolution has a sampling efficiency of 80% and above (Burns and Williams, 2008).

4.4 Comparisons of the FRC method with the standard method (MTF/SFR)

In Figure 4-2 the spatial resolution for FRC is observed to be approximately at 0.16 spatial frequency, nm^{-1} . And the spatial resolution in figure 4-3 is observed at 0.085 frequency, cy/pixel. The spatial resolution from FRC is twice the one from the MTF/SFR at ODD of approximately 0 mm.

Using Eq.25 to calculate the limiting resolution of MTF/SFR curve at approximately 0 mm ODD yields

$$\text{limiting resolution} = E_v \times \text{sampling resolution}$$

$$\text{limiting resolution} = 0.17 \times 0.5$$

$$\text{limiting resolution} = 0.085$$

4.4.1 FRC and MTF/SFR comparisons at different object-detector-distance.

The same test object of gadolinium knife edge was imaged at different ODDs. The results are graphically presented. Different ODDs measured in millimetres at 12, 32 and 104. The results for the spatial resolution on Figure 4-4, figure 4-5, Figure 4-6, figure 4-7, Figure 4-8 and figure 4-9 are presented.

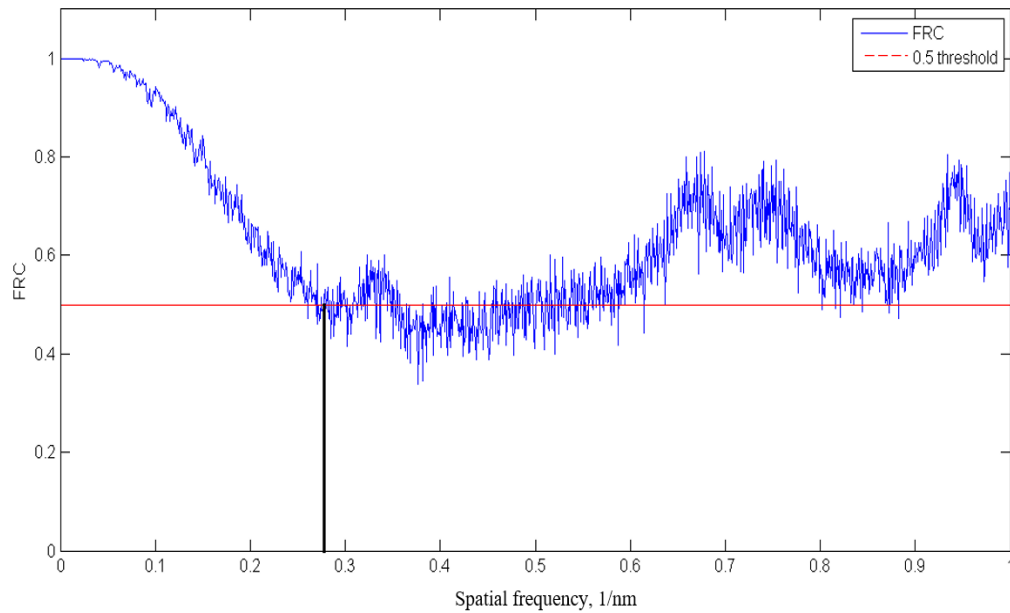


Figure 4-4: FRC curve at 12 mm ODD.

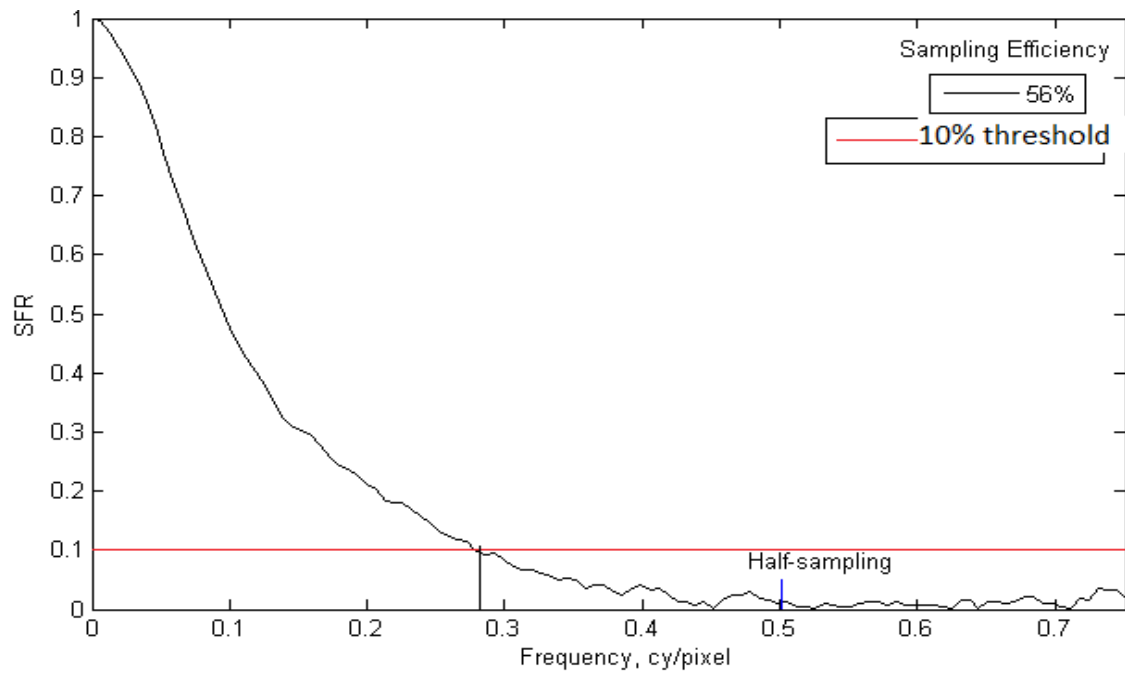


Figure 4-5: MTF/SFR curve at 12 mm ODD.

$$\text{limiting resolution} = E_V \times \text{sampling resolution}$$

$$\text{limiting resolution} = 0.56 \times 0.5$$

$$\text{limiting resolution} = 0.28$$

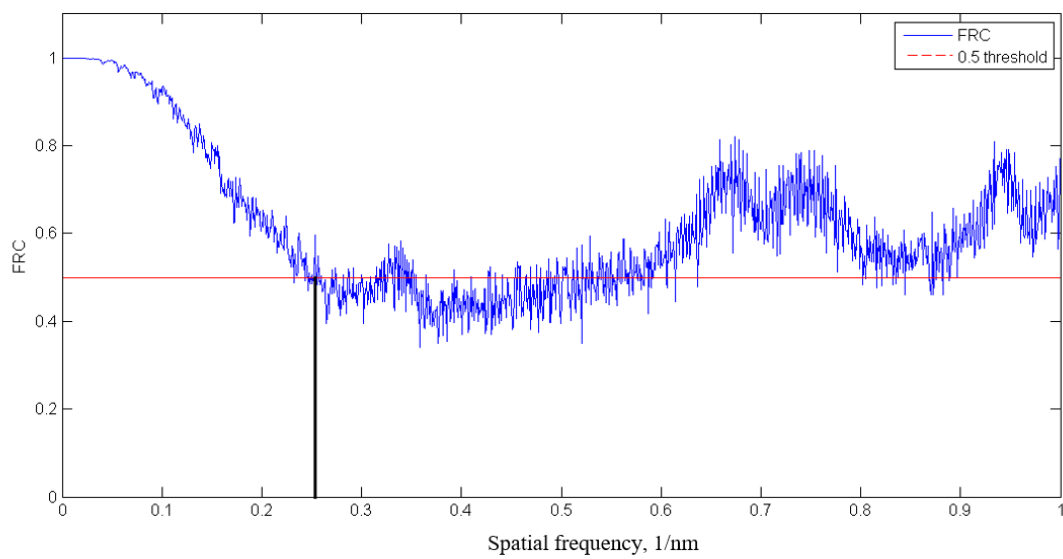


Figure 4-6: FRC curve at 32 mm ODD.

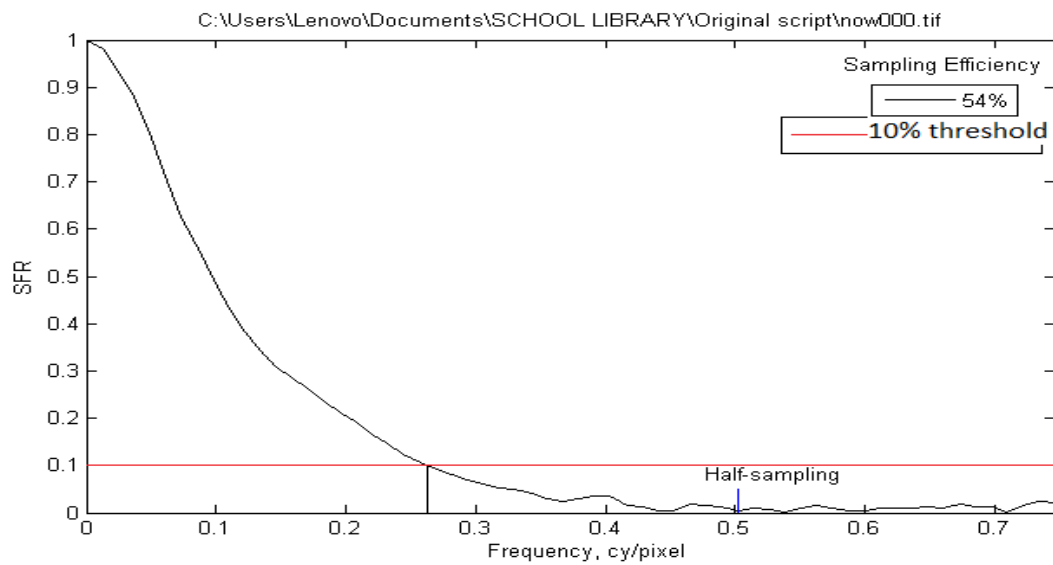


Figure 4-7: MTF/SFR curve at 32 mm ODD.

The spatial resolution on figure 4-6 is observed at approximately 0.25 nm^{-1}

$$\text{limiting resolution} = E_v \times \text{sampling resolution}$$

$$\text{limiting resolution} = 0.54 \times 0.5$$

$$\text{limiting resolution} = 0.27$$

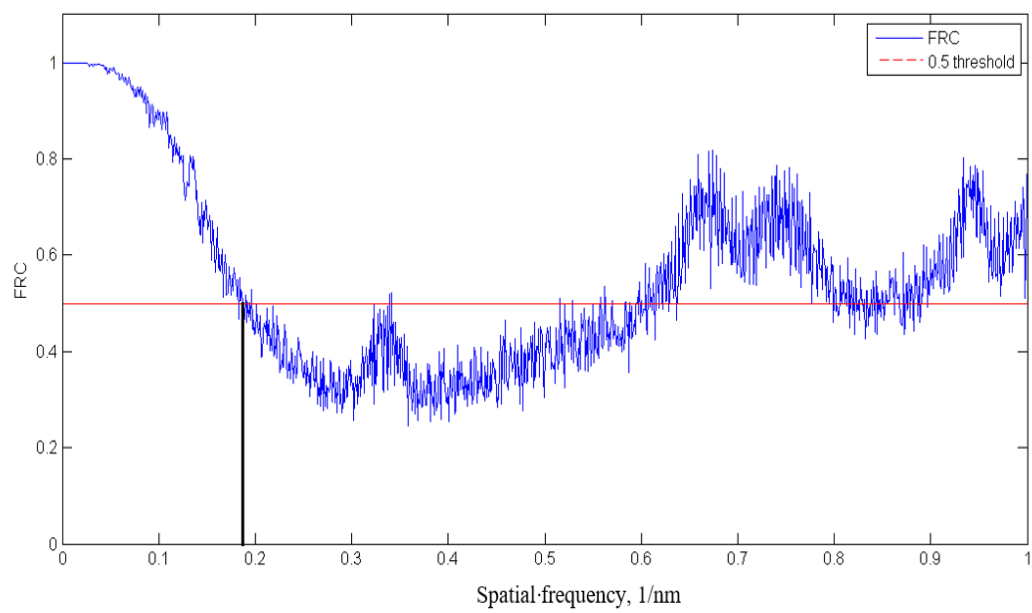


Figure 4-8: FRC curve at 104 mm ODD.

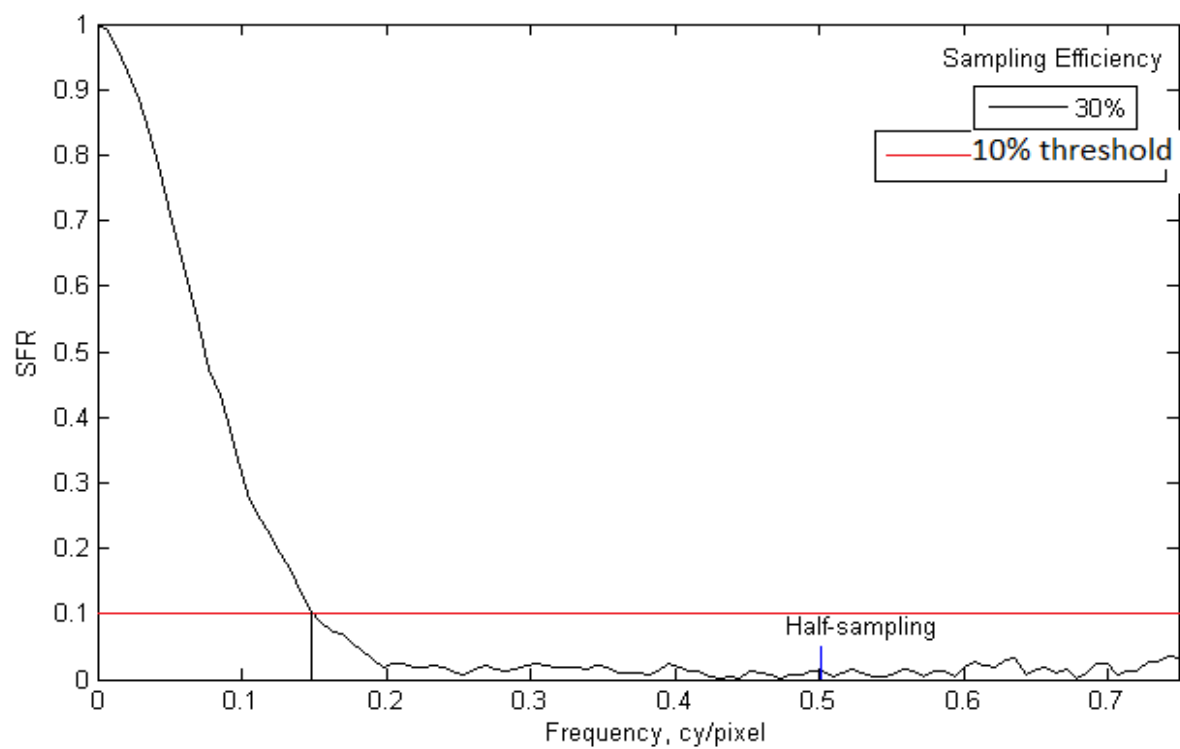


Figure 4-9: MTF/SFR curve at 104 mm ODD

The spatial resolution for sampling efficiency of 30% at 104 mm ODD is calculated as,

$$\text{limiting resolution} = E_v \times \text{sampling resolution}$$

$$\text{limiting resolution} = 0.30 \times 0.5$$

$$\text{limiting resolution} = 0.15$$

Table 4-1 summarizes the results of spatial resolution measured using two methods compared at different ODD. The MTF/SFR curves with x-axis measurements are for frequency in cy/pixel. Burns et.al, 2008, explains that MTF/SFR is a function of spatial frequency and that spatial frequency when expressed in pixel, 1 cy/mm is 0.5 cy/pixel. FRC as a function of spatial frequency and measured in nm^{-1} . Comparing two measurements, cy/mm with nm^{-1} . Both methods are function of frequency. This shows that the spatial resolution of an FRC can be compared with that of MTF/SFR.

Table 4-1: Summary of FRC and MTF/SFR results at different ODD

ODD (mm)	FRC	MTF/SFR	% difference
0.0	0.16	0.085	88.24
12	0.29	0.28	3.57
32	0.26	0.27	3.70
104	0.19	0.15	26.67

The percentage difference is made on the ratio of FRC vs MTF/SFR using Eq.26;

$$\left(\left(\frac{\text{FRC}}{\text{MTF/SFR}} \right) - 1 \right) \times 100\% \quad \text{Eq.26}$$

CHAPTER 5: Conclusion and recommendation

5.1 Conclusion

In this study the scope to characterize spatial resolution using the adapted method from cryo-electron microscopy, FRC was evaluated using digital neutron radiographs. The method was evaluated on digital neutron radiographs obtained at NUETRA facility, PSI at different ODDs.

Determining the spatial resolution of neutron radiographs was achieved using FRC mathematical algorithm on the FRC MATLAB script (Vila-Comamala et al., 2011). MATLAB was a very useful tool to find the spatial resolution of an image, it was used to compute both results for FRC and MTF/SFR. MTF/SFR was applied on the images obtained at different ODDs to evaluate the spatial resolution. The methods (FRC and MTF/SFR) were compared to find the spatial resolution. It showed that the spatial resolution at 0 mm ODD was twice than expected, whereas at 12 mm, 32 mm and 104 mm ODDs yielded approximately the same results.

The spatial resolution values obtained at different ODDs were compared using the percentage difference shows that the two methods are comparable even though the 0 mm ODD spatial resolution was not as expected. The highest percentage difference calculated was 88.24%, this is the spatial resolution difference at approximately 0 mm ODD. From this it can be concluded that imaging the test object closer to the detector is not ideal. Even though the FRC curves do not decay to zero as expected from the literature and as the MTF curves, this comparisons shows that the FRC method can be adapted to assess and evaluate the spatial resolution in digital neutron radiography. The FRC curves do not decay to zero as expected because of the unfiltered noise in the images as SNR was not evaluated when analysing the images. Reason for not evaluating the SNR was the limitation of the FRC MATLAB function.

5.2 Recommendation

For future work the study needs to look at apodization method to filter out the noise on the original images before analysis. The SNR needs to be evaluated in order for the FRC curves to decay to zero. Different filtering methods also need to be assessed to determine which one is best suited to determine the spatial resolution in digital neutron radiography. It is recommended that FRC

method to be applied with filtering before it can be adapted to be used in digital neutron radiography to determine the spatial resolution. The current edge analysis of the MTF/SFR is complex and sensitive in terms of finding the best edge angle.

References

- AKHTER, H. 2012. Evaluation of weighting functions for Sidelobe control in UWB SAR image processing. Master of Science, Electrical Engineering department, Blekinge Institute of Technology. Sweden.
- ALAA ELDIN, M. T. 2011. Neutron Radiography Master of science, Nuclear Engineering department, Alexandria University. Egypt
- ALAM, M. K., LEHMANN, E., VONTOBEL, P. & KHAN, M. A. Study on Water Uptake Behaviour of Jute Reinforced Polymer Composites Using NEUTRA Facility at SINQ. In: ARIF, M. & R., G. D., eds. Proceedings of the 8th World Conference in Neutron Radiography 2006 Gaithersburg, Maryland, USA. DEStech Publications, Inc., 639.
- ALMUALIMI, M., WILLE, M.-L. & LANGTON, C. M. 2018. Potential for ultrasound transit time spectroscopy to improve axial resolution. *Applied Acoustics*, 133, 91-96.
- BALL, J. L. & PRICE, T. 1995. *Chesney's Radiography Imaging*, , Oxford, Wiley Blackwell Publishing.
- BANTERLE, N., BUI, K. H., LEMKE, E. A. & BECK, M. 2013. Fourier ring correlation as a resolution criterion for super-resolution microscopy. *J Struct Biol*, 183, 363-367.
- BAYON, G., DOMANUS, J. C., GREIM, L., HARMS, A. A., LEEFLANG, H. P., MARKGRAF, J. F. W. & TAYLOR, D. J. 1992. *Practical Neutron Radiography* Kluwer Academic Publishers.
- BUHR, E., GÜNTHER-KOHFAHL, S. & NEITZEL, U. Simple method for modulation transfer function determination of digital imaging detectors from edge images. In: M. J. YAFFE & L. E. ANTONUK, eds. *Proceedings of SPIE*, 2003. 877-884.
- BURNS, P. D. 2003. *sfrmat 2.0 User's Guide*.
- BURNS, P. D. 2009. MATLAB Function: sfrmat: Slanted-edge analysis for digital camera and scanner evaluation.
- BURNS, P. D. & WILLIAMS, D. 2008. Sampling Efficiency in Digital Camera Performance Standards. *Proc. SPIE*, 6808.
- BUSHBERG, J. T., SEIBERT, J. A., LEIDHOLDT, E. M. & BOONE, J. M. 2002. *The essential physics for medical imaging* Philadelphia, USA, Lippincott Williams and Wilkins
- CAO, R. L. & BIEGALSKI, S. R. 2007. The measurement of the presampled MTF of a high spatial resolution neutron imaging system. *Nuclear Instruments and Methods in Physics Research, Accelerators, Spectrometers, Detectors and Associated Equipment*, 582, 621-628.

- CHADWICK, J. 1932. Possible Existence of a Neutron
<https://www.atticusrarebooks.com/pages/books/468/james-chadwick/possible-existence-of-a-neutron-in-nature-vol-129-january-to-june-1932-chadwick-asserts-possible> [Online]. [Accessed April 2018].
- CHANKOW, N. 2012. Neutron Radiography. In: OMAR, D. M. (ed.) Nondestructive Testing Methods and New Applications. InTech.
- DANCE, D. R., CHRISTOFIDES, S., MAIDMENT, A. D. A., MCLEAN, I. D. & NG, K. H. 2014. IAEA-Handbook
- Diagnostic Radiology Physics: A Handbook for Teachers and Students, Vienna, Marketing and sales unit, Publishing section international atomic energy agency Vienna international centre.
- DIEBOLDER, C. A., FAAS, F. G., KOSTER, A. J. & KONING, R. I. 2015. Conical Fourier shell correlation applied to electron tomograms. *J Struct Biol*, 190, 215-23.
- DOLPH, C. L. 1946. A current distribution for broadside arrays which optimizes the relationship between beam width and side-lobe level. *Proc. IRE*, 34.
- FITZGERALD, G. M. & FRANCISCO, S. 1947. The factors that control geometric unsharpness. *The journal of the American dental association*, 34, 1-5.
- GONZALEZ, R. C. & WOODS, R. E. 2002. *Digital Image Processing*, USA, Pearson Prentice Hall. 23-26
- GOPAL, P. & SVALBE, I. 2018. Spatial domain morphological filtering for interpolation of the Fourier domain. *Pattern Recognition Letters*, 116, 107-113.
- HAWKESWORTH, M. R. & WALKER, J. 1981. Basic principles of thermal neutron radiography. In: BARTON, J. P. & HARDT, P. V. D., eds. *Proceedings of the first world conference in Neutron Radiography, 1981 San Diego, California, USA*. USA: D. Reidel Publishing Company 5-19
- ISO12233 2014. ISO 12233:2014 Photography – Electronic still picture imaging – Resolution and spatial frequency responses. Tech. rep., International Organization for Standardization, Geneva, Switzerland (2014).
- JALALVAND, A. R., ROUSHANI, M., GOICOECHEA, H. C., RUTHLEGDE, D. N. & G., H.-W. 2019. MATLAB in electrochemistry: A review. *Talanta*, 194, 205-225.
- KAESTNER, A. P., HARTMANN, S., KUHNE, G., GRUZWEIG, C., JOSIC, L., SCHMID, L. & LEHMANN, E. H. 2011. The ICON beamline – A facility for cold neutron imaging at SINQ. *Nuclear instruments and Methods in Physics Research Section A*, 659, 387-393.

- KERR, D. A. 2010. Determining MTF with a Slant Edge Target.
- KOBAYASHI, H. & KIYANAGI, Y. 1996. Basic concept and characteristics of beam quality indicator for continuum neutron spectra. Nucl Instrum and Methods A377, 52-57.
- KOERNER, S., LEHMANN, E. H. & VONTOBEL, P. 2000. Design and optimization of a CCD-neutron radiography detector. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, 158-164.
- LAMARSH, J. R. & BARATTA, A. J. 2001. Interaction of radiation with matter Introduction to Nuclear Engineering. 3 ed. New Jersey, USA: Prentice Hall
- LEHMANN, E., VONTOBEL, P. & HERMANN, A. 2003. Non-destructive analysis of nuclear fuel by means of thermal and cold neutrons. Nucl Instruments and Methods in Physics Research, 515, 745-759.
- LEHMANN, E. H., KAESTNER, A. P., JOSIC, L., HARTMANN, S. & MANNES, D. Imaging with Cold Neutrons. In: DE BEER, F. C. & FRANKLYN, C. B., eds. Proceedings of the 9th World Conference on Neutron Radiography "The Big 5 on Neutron Radiography", 2010 Kwa-Maritane, Pilansberg National Park, South Africa 161-165.
- LEHMANN, E. H., MORGAN, M. & PEETERMANS, S. 2014. Neutron optics Requirements for Neutron Imaging Techniques. Journal of Physics: Conference Series, 528, 01, 20-28.
- LEHMANN, E. H., VONTOBEL, P. & WIEZEL, L. 2001. Properties of the Radiography Facility Neutra at Sinq and Its Potential for Use as European Reference Facility. Nondestructive Testing and Evaluation, 16, 191-202.
- LIAO, H. Y. & FRANK, J. 2010. Definition and estimation of resolution in single-particle reconstructions. Structure, 18, 768-75.
- MAHER, K. 2016. Basic Physics of nuclear medicine/attenuation of gamma-rays [Online]. Wikibooks https://en.wikibooks.org/wiki/Basic_Physics_of_Nuclear_Medicine/Attenuation_of_Gamma-Rays. [Accessed July 2018].
- MOLINA, C. 2016. ASME V Radiology Introduction, <https://apiexam.com/asme-v-radiology-introduction/> [Online]. [Accessed August 2018].
- NESVIZHEVSKY, V. & VILLAIN, J. 2017. The discovery of the neutron and its consequences (1930–1940). Comptes Rendus Physique. Vol 18, Issues 9, 592-600.
- NIEUWENHUIZEN, R. P. 2016. QUANTITATIVE IMAGE ANALYSIS FOR SINGLEMOLECULE LOCALIZATION MICROSCOPY. PhD, Delft University of Technology.

- NIEUWENHUIZEN, R. P., LIDKE, K. A., BATES, M., PUIG, D. L., GRUNWALD, D., STALLINGA, S. & RIEGER, B. 2013. Measuring image resolution in optical nanoscopy. *Nat Methods*, 10, 557-62.
- PAUL-SCHERRER-INSTITUTE Website. 2019. Facilities and Instruments, <https://www.psi.ch/en/science/facilities-and-instruments> [Online]. Switzerland: PSI. [Accessed June 2019 2019].
- POWER, N. 2018. Gadolinium. <https://www.nuclear-power.net/glossary/gadolinium/> [Online]. Available: <https://www.nuclear-power.net/glossary/gadolinium/> [Accessed 15 July 2018 2018].
- QASIM, A. & RAINA, A. Principles of echocardiography [Online]. Available: <http://www.echocardiographer.org/Echo%20Physics/Axresolution.html> [Accessed 18 July 2018 2018].
- QI, Y., LI, H., WILLS, R. H., PEREZ-HURTADO, P., YU, X., KILGOUR, D. P., BARROW, M. P., LIN, C. & O'CONNOR, P. B. 2013. Absorption-mode Fourier transform mass spectrometry: the effects of apodization and phasing on modified protein spectra. *J Am Soc Mass Spectrom*, 24, 828-34.
- RADEBE, M. J. 2017. Simplified spatial resolution criterion to obtain the MTF curve from edge analysis in the real space.
- RASBAND, W. 2018. ImageJ. <https://en.wikipedia.org/wiki/ImageJ> [Online]. [Accessed May 2018].
- REEG, J. R. 2016. NULL SUBTRACTION IMAGING TECHNIQUE FOR BIOMEDICAL ULTRASOUND IMAGING. Master, University of Illinois at Urbana-Champaign. Urbana, Illinois.
- SAMEI, E. 2003. Performance of Digital Radiographic Detectors: Quantification and Assessment Methods. PhD, Duke University Medical Center, Durham.
- SAMEI, E. & FLYNN, M. J. 1997. A method of measuring MTF of digital radiographic system. *Assoc. Medical Physics*, 25, 102-113.
- SAXTON, W. O. & BAUMEISTER, W. 1982. The correlation averaging of a regularly arranged bacterial cell envelope protein. *Journal of Microscopy*, 27, 127-138.
- SMITH, S. W. 1999. Theory of spatial resolution. *The Science and Engineer's Guide to Digital Signal Processing*. California Technical publishing.
- STACEY, W. M. 2001. Neutron nuclear reactions. *Nuclear Reactor Physics*. A Wiley-Interscience Publication.
- SUN, F., MARKÖTTER, H., MANKE, I., HILGER, A., ALRWASHDEH, S. S., KARDJILOV, N. & BANHART, J. 2017. Complementary X-ray and neutron radiography study of the initial lithiation process in lithium-ion batteries containing silicon electrodes. *Applied Surface Science*, 399, 359-366.
- T'HOEN, P. J. 1982. Aperture apodization to reduce the off-axis intensity of the pulsed-mode directivity function of linear arrays. *Ultrasonics*, 20, 231-236.

- TAYLOR, T. T. 1955. Design of line-source antennas for narrow beamwidth and low side lobes. IRE Trans. Antennas Propag., 3.
- UNSER, M., TRUS, B. L. & STEVEN, A. C. 1987. A new resolution criterion based on spectral signal-to-noise ratios. Ultramicroscopy, 23, 39-52.
- VAN HEEL, M., KEEGSTRA, W., SCHUTTER, W. & VAN BRUGGEN, E. F. J. 1982. Anthropod hemocyanin structures studied by image analysis. In: WOOD, E. J. (ed.) Life Chemistry Reports, Suppl.1 The structure & function of invertebrate respiratory protein
- VAN HEEL, M. & SCHATZ, M. 2005. Fourier shell correlation threshold criteria. J Struct Biol, 151, 250-62.
- VAN HEEL, M. & SCHATZ, M. 2017. Reassessing the revolution's resolutions.
- VAN ROOYEN, T. J. 2006. The Science of Radiation Protection. Somerset West, Western Cape, S.A.: iThemba LABS
- VELO, P. & ZAKARIA, A. 2017. Determining Spatial Resolution of Gamma Cameras Using MATLAB. Journal of Medical Imaging and Radiation Sciences, 48, 39-42.
- VILA-COMAMALA, J., DIAZ, A., GUIZAR-SICAIROS, M., MANTION, A., KEWISH, C. M., MENZEL, A., BUNK, O. & DAVID, C. 2011. Characterization of high-resolution diffractive X-ray optics by ptychographic coherent diffractive imaging. OPTICS EXPRESS, 19.
- ZHOU, B. 2011. Spatially resolved temperature and concentration determination in flames using FTIR emission spectroscopy combined with HITRAN/HITEMP database. Master, Department of Physics, Lund University, Lund Sweden