

Diagnostic mathematics assessment: The impact of the GIST model on learners with learning barriers in mathematics

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TO WHOM IT MAY CONCERN

Editing of the thesis of Rantopo David Sekao

I hereby declare that I have edited the thesis for the degree Doctor of Philosophy entitled *Diagnostic mathematics assessment: The impact of the GIST model on learners with learning barriers in mathematics*. I have suggested several changes to his work, but am in no position to know whether they have been followed. I can therefore take no responsibility for errors that might have slipped through.

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PREFACE AND ACKNOWLEDGEMENTS

The study was conducted after realising the prevalence of anecdotal evidence that mathematics teachers were struggling with the implementation of diagnostic assessment. Effective use of diagnostic assessment could enhance the effective teaching and learning of mathematics particularly in the General Education and Training band. The four sampled schools were from the previously disadvantaged communities and this availed a fertile environment in which to test the GIST model. Numerous challenges were encountered during the research; nonetheless the research was conducted successfully. The biggest challenge was the difficulty to secure the appointments to conduct the lesson observations. This could be attributed to two reasons: *firstly* it was evident that some teachers were not at ease to allow an *unfamiliar person* into their classes to observe their lessons and share their experiences, and *secondly* teachers were faced with uncertainties emanating from the merger of middle schools and high schools after their schools were incorporated into Gauteng province from the North West province. Notwithstanding the challenges encountered, the assistance of all the people is acknowledged, particularly:

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- God Almighty for giving me strength and health.

ABSTRACT

Assessment, as an integral part of teaching and learning, gained unprecedented prominence in the curriculum in South Africa post 1994. When the new curriculum was introduced, it was assumed that teachers would effortlessly adapt their teaching and assessment practices, and swiftly implement the curriculum. Fourteen years after the inception of the new curriculum, majority of teachers are still grappling with issues of assessment. Previously, there was an exclusive bias towards summative assessment, which is mainly learning product-orientated and less or no focus on the other assessment typologies such as diagnostic and formative assessment, which are learning process-orientated. Of these typologies, diagnostic assessment is not being used maximally to enhance mathematics learning and inform the nature of the interventions to attend to learners' needs. The study focused on diagnostic assessment by investigating the impact of a particular model, GIST model, on the learning barriers and learner achievement in mathematics among the grade 9 learners. The investigation of the impact of the GIST model was done through the experimental design in four schools with class sizes of $n > 40$. Data were collected quantitatively through Study Orientation Questionnaire (SOM) and Mathematics Achievement Test (MAT) as well as qualitatively through interviews, observations and document analysis. The *t*-test and the analysis of covariance (ANCOVA) revealed that the GIST model improved the learner achievement practically significantly ($d = 0.79$). However, the GIST model could not mitigate the learning barriers and improve correlations between SOM and MAT. The study, however, does find grounds to conclude that the latter findings can be attributed to teachers' lacking understanding and implementation of diagnostic assessment, in particular the GIST components. Hence, certain recommendations are posed with regard to the applicable training of teachers in order to empower them to effectively utilize diagnostic assessment and to guide learners in overcoming learning barriers in mathematics.

Key words: diagnostic assessment, group intervention strategy, learning barriers, problem solving behaviour, mathematics anxiety, mathematics study environment, attitudes towards mathematics, mathematics achievement.

OPSOMMING

Assessering, as 'n integrale deel van onderrig en leer, het sedert 1994 ongekende prominensie in die Suid-Afrikaanse skoolkurrikulum verwerf. Toe die nuwe kurrikulum geïmplementeer is, is dit stilswyend aanvaar dat onderwysers hulle onderrig- en assesseringpraktyke sonder moeite sou aanpas en die kurrikulum gemaklik sou implementeer. Veertien jaar na die aanvanklike implementering van die nuwe kurrikulum sukkel onderwysers egter steeds om sin te maak van sekere kwessies rakende assessering, waarvan diagnostiese assessering maar een is. Voorheen was daar 'n duidelike voorkeur jeens summatiewe assessering wat hoofsaaklik leerproduk-gerig was, en min of geen aandag is aan ander assesseringstipologieë gespandeer soos diagnostiese en formatiewe assessering, wat leerproses-gerig is. Van hierdie tipologieë word diagnostiese assessering in die besonder nie optimaal benut om wiskundeleer to bevorder nie en om die aard van intervensies om wiskundeleerders se behoeftes te bevredig te bepaal nie. Hierdie studie fokus op diagnostiese assessering deur ondersoek in te stel na die invloed van 'n spesifieke model, naamlik die "Group Intervention Strategy"- of "GIST"-model, op die leerhindernisse en -prestasie in wiskunde by graad 9-leerders. Die ondersoek na die invloed van die GIST-model is eksperimenteel in vier skole met klasgroottes van meer as 40 uitgevoer. Kwantitatiewe data is met behulp van die gestandaardiseerde "Studie-oriëntasie in Wiskunde"-vraelys (SOM) en self-ontwikkelde wiskundeprestasietoetse (MAT) ingesamel, terwyl kwalitatiewe data by wyse van onderhoude, waarneming en dokumentontleding ingesamel is. Die uitkoms van t-toetse en kovariansie-ontleding (ANCOVA) toon dat implementering van die "GIST"-model prakties betekenisvol ($d = 0,79$) tot die verbetering van wiskundeprestasie bygedra het, maar dat daar geen betekenisvolle verbetering ten opsigte van die oorkoming van leerhindernisse of die korrelasie tussen SOM en MAT aangetoon kon word nie. Die studie vind wel gronde om te stel dat laasgenoemde bevindings aan onderwysers se gebrekkige begrip en implementering van diagnostiese assessering, in die besonder van die "GIST"-komponente, toegeskryf kan word. Bepaalde aanbevelings word dan ook met betrekking tot die toepaslike opleiding van wiskunde-onderwysers gemaak ten einde hulle in staat te stel om effektief diagnosties te assesser en leerders in die oorkoming van leerhindernisse in wiskunde by te staan.

Sleuteltermes: leerhindernisse, wiskundeprestasie, probleemoplosgedrag, wiskunde-angs, studie-omgewing, wiskundehouding, diagnostiese assessering.

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LIST OF ACRONYMS AND ABBREVIATIONS

ANCOVA	Analysis of covariance
C	Control group
C2005	Curriculum 2005
CGI	Cognitively-guided instruction
CL	Cooperative Learning
DoE	Department of education
E	Experimental group
ELRC	Education Labour Relations Council
FET	Further Education and Training
GDE	Gauteng Department of Education
GET	General Education and Training
GIST	Group Intervention Strategy
MA	Mathematics Anxiety
MAT	Mathematics Achievement Test
NCS	National curriculum Statement
NWDE	North West Education Department
OBA	Outcomes-based assessment
OBE	Outcomes-based education
PBL	Problem-based learning
PCA	Problem-centred approach
PCI	Problem-centred instruction
PCL	Problem-centred learning
PSB	Problem-solving behaviour
RME	Realistic Mathematics Education
RMI	Realistic Mathematics Instruction
SA	Study attitudes
SH	Study habits
SM	Study milieu
SOM	Study Orientation in Mathematics
STAD	Student Team Achievement Division
TAI	Team Assisted Instruction
TIMSS	Trends in Mathematics and Science Study
ZPD	Zone of proximal development

CHAPTER

1

INTRODUCTION, RESEARCH PROBLEM, AIMS & PLAN OF RESEARCH

The GIST model is designed to offer intervention to learners who experience Mathematics-learning barriers in large classes or where one-to-one intervention processes are impracticable
(Schmidt, 1993:135).

CHAPTER 1

INTRODUCTION, RESEARCH PROBLEM, AIMS AND PLAN OF RESEARCH



INTRODUCTION



REVIEW OF LITERATURE AND PROBLEM STATEMENT



HYPOTHESES



METHOD OF RESEARCH



CHAPTER FRAMEWORK



SIGNIFICANCE OF THE STUDY

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1.1. Introduction

The dawn of the South African democratic dispensation in 1994 necessitated a complete overhaul of the education system – from reception to higher education level (Graven, 2002:21). The complete education overhaul led to the birth of Curriculum 2005 (C2005) in 1997 whose philosophical foundation is pedigreed on outcomes-based education (OBE) (Botha, 2000:136). C2005 was later revised due to numerous factors that do not form the focus of this study and it became known as the National Curriculum Statement¹ (NCS). C2005, as is the case with NCS, was premised upon the view that every learner can learn and therefore the inclusion of learners with special educational needs should also be given attention (Graven, 2002:21).

With the introduction of the new curriculum in South Africa, assessment gained prominence and has since been regarded integral part of teaching and learning as is the case with globally. Prior to the new curriculum in South Africa, assessment was biased towards examination-driven summative assessment which mainly tested knowledge acquisition. However, **baseline, diagnostic, formative** and **alternative** assessment practices (DoE, 2001:5) began to receive more attention as part of curriculum reform post 1994. In particular, diagnostic assessment laid the foundation for what became known as the principle of the Inclusive Education model², which essentially asserts that barriers to learning amongst learners should be identified and interventions be made within the mainstream classes (DoE, 2002a:17). This suggests that learners who experience barriers to learning should not be discriminated against by isolating them into the so-called special schools at the margins of the education system.

While diagnostic assessment cuts across the subjects in the schooling system, lack of proficiency among teachers to use it to identify and rectify (through any appropriate intervention) learning barriers in among mathematics learners may be responsible for the low self-concept (Howie, 2001:94) and poor performance in the subject (Maree, Prinsloo & Claasen, 1997:3). Although learning barriers are multifaceted in nature (Westwood, 2003:6; DoE, 2002a:17), the focus of this research is on emotional and behavioural learning barriers in mathematics, henceforth called negative mathematics disposition. Further, the research aims at testing the proposed **Group Intervention Strategy (GIST model)** (*see* Figure 4.1) in the hope that it will offer assistance towards improving learners' negative mathematics

¹ To avoid a possible confusion, the difference between the National Curriculum Statement (NCS) and Curriculum and Assessment Policy Statement (CAPS) needs to be clarified. The NCS is the actual curriculum of South Africa which came into being after the review of Curriculum 2005. The CAPS is the re-packaged NCS in a simpler and user-friendly format to make it easy for the teachers to implement. The implementation of the CAPS will take place in 2012 and does not cause any upset to this study.

² Inclusive Education model is contained in Education White Paper 6 and was launched in 2001 by the then Ministry of Education. It departs from the constitutional obligation to ensure that all learners have equal access to education.

dispositions and provide mathematics teachers with the required skills to identify and manage learners who experience barriers to the learning of mathematics.

Two different theoretical perspectives have informed the GIST model: the *developmental* perspective and the *motivational* perspective (Bennett, 1994:51). The former departs from the notion that through social interaction (with other learners) the learner acquires a framework for integrating experience and learns how to negotiate meaning (Bennett, 1994:51). The latter focuses primarily on the achievement rewards under which members of the group operate. The GIST model is designed to offer intervention to learners who experience mathematics-learning barriers in large classes or where one-to-one intervention processes are impracticable (Schmidt, 2003:135).

1.2. Review of literature and problem statement

According to the DoE (2001:5) and McIntosh (1997), assessment in mathematics refers to the process of gathering evidence of the learners' performance with regard to knowledge of, ability to use and disposition towards mathematics and making inferences from that evidence for a variety of reasons. In the context of mathematics, the definition of assessment does not only focus on the measurement of the cognitive or academic achievements (i.e. formative and/or summative assessment) for the purpose of promoting learners from one grade to the other, but most importantly, it also focuses on the identification of negative mathematics dispositions that may serve as learning barriers towards effective mathematics teaching and learning.

The learning barriers that fall within the domain of the mathematics disposition, and are the foci of this research are: mathematics anxiety, negative attitude towards mathematics, poor problem-solving behaviour, non-conducive mathematics study environment and inappropriate mathematics study habits (Maree, Prinsloo & Claasen, 1997:3). These factors have been positively correlated with mathematics achievement (Higbee & Thomas, 1999).

The definition above seems to suggest that the teacher's knowledge of learners' disposition towards mathematics helps him/her (the teacher) to make inferences for a variety of reasons of which the two fundamental ones are: modification or restructuring of the teaching-learning strategies (McIntosh, 1997; Steele & Steele, 2003:622), and the development of the appropriate assessment tasks informed by a particular learning barrier(s) (Walther-Thomas, Korinek, McLaughlin & Williams, 2000:235). It should

further be highlighted that in most situations where mathematics learning barriers have been diagnosed among the learners, appropriate teaching-learning interventions should be invoked in order to minimise the impact thereof (DoE, 2002c:9). Therefore, diagnosis and intervention may not be divorced from each other; instead, the latter complements and qualifies the former.

The National Curriculum Statement (NCS) emphasises the principle of inclusivity³ in mathematics classes (DoE, 2002b:2). It requires mathematics teachers to effectively use diagnostic assessment methods in their mathematics classes and to be efficient in developing intervention strategies to address the diagnosed mathematics learning barrier(s). However, large mathematics classes seem to pose a hindrance towards effective individual diagnosis and intervention. This is against the background that, firstly, the envisaged teacher-learner ratio of 1:35 in ordinary public schools (Howie, 2001:113) is still unattainable in most instances (Sekao, 2004: 38), and secondly, about 20% of the learner population experience learning barriers in their scholastic career (Westwood, 2003:5). For the purpose of this study the teacher-learner ratio refers to the average number of learners (per school) the teacher is faced with during teacher-learner interaction in a mathematics class, and not necessarily the national average teacher-learner ratio, which may be deceitful at school level.

For mathematics classes in particular, the effectiveness of diagnostic assessment in large class contexts is made even more challenging by a well intended declaration of mathematics as a compulsory subject across all grades in South African schools (DoE, 2002d:69), and the introduction of the Inclusive Education model (DoE, 2003a:7). Mathematics, unlike other learning areas/subjects, is characterised by, or is susceptible to possible misconceptions and/or mystifications due to a variety of factors such as beliefs, perceptions and philosophy of both teachers and learners about it (mathematics) (Cangelosi, 2003:133). By declaring mathematics a compulsory subject for learners and introducing the Inclusive Education model implies that the number of learners with learning barriers in mathematics is expected to increase in the mainstream classes. The possible consequence is that individual diagnosis and intervention will become more difficult to carry out. To redress the above concerns requires an appropriate intervention strategy, such as the Group Intervention Strategy (GIST), which is presumed to be best adapted for large mathematics class contexts.

Lack of proficiency amongst mathematics teachers with regard to the theory and practice of diagnostic assessment is another possible cause of non-implementation of this type of assessment. For instance, negative attitudes towards mathematics may pose a hindrance towards achievement in the subject and

³ The principle of inclusivity refers to Inclusive Education model defined on p3.

teachers have not been equipped with the skills to effectively deal with attitudinal barriers in mathematics (Ruggiero, 1998:9). Nitko (2001:293) agrees that “unless [the teacher] knows or can hypothesise why the students cannot perform a learning target, [the teacher] will likely be at a loss as to how to focus [her/his] remedial teaching”. Botha (2000:139) reiterates the importance of taking the teachers onboard by asserting that lack of knowledge of appropriate intervention strategies by teachers exacerbates the situation because identifying a mathematics learning barrier without providing a possible remedy does not help the affected learner. The identification and curbing of mathematics learning barriers during the early years of schooling (DoE, 2002a:191) are reliant on the training of teachers to acquire the diagnostic expertise. Lack of such expertise among mathematics teachers may jeopardise the well-intended vision of making every South African citizen mathematically literate in future.

Ruggiero (1998:14) emphasises that the most effective approach to deal with obstructive attitudes is when learners understand them (attitudes) by analysing their (learners’) behaviours and evaluating their (learners’) beliefs. It is assumed, from this assertion, that for learners to effectively deal with elements that pose barriers to the learning of mathematics, the teacher should know the characteristics of such elements and their prospective symptoms. Put synoptically, it is assumed that a mathematics teacher cannot diagnose a mathematics learning barrier she/he does not have conceptual knowledge of or information about. The mathematics learning barriers or obstructive elements referred to within the confines of this study are: mathematics anxiety, negative attitude towards mathematics, poor problem-solving behaviour, non-conducive mathematics study environment and inappropriate mathematics study habits (Maree *et al.*, 1997:3).

The above-mentioned problems prompted the researcher to investigate how a specific intervention model (the so-called GIST model) can be used to redress negative mathematics dispositions in large classes. The GIST model aims to achieve the following:

- helping teachers to effectively manage learners with mathematics learning barriers (particularly negative mathematics disposition). Westwood (2003:13) emphasises that if learning barriers among learners are not effectively managed, they may impact negatively on teachers’ attitudes and motivation towards the learners and mathematics as a subject;
- alleviating mathematics anxiety among learners; and
- inculcating self-confidence and positive attitudes in mathematics.

In general, the critical research question to be investigated in this study is: What is the application potential of the GIST model for desensitising grade 9 learners with a negative mathematics disposition? In particular the following four research questions will be investigated:

Question 1: How does the GIST model influence grade 9 learners' negative mathematics disposition such as anxiety, attitudes, study environment, study habits, and problem-solving in large classes?

Question 2: What factors hamper educators from using diagnostic and formative assessment practices in mathematics classes?

Question 3: How does the GIST model influence learners' mathematics academic achievement?

Question 4: How do learners' mathematics achievements and dispositions correlate?

1.3. Hypotheses

The following research hypotheses were tested in the research:

H₀₁: The application of the GIST model influences the mathematics dispositions of grade 9 learners.

H₀₂: The application of the GIST model influences the mathematics academic achievements of grade 9 learners.

H₀₃: There is a positive correlation between learners' mathematics academic achievements and learners' mathematics dispositions.

1.4. Method of research

1.4.1. Literature review

An intensive and comprehensive review of the relevant literature was done in order to analyse and discuss the effect of the implementation of the GIST model on mathematics achievement and mathematics disposition of grade 9 learners. The following key words were used in EBSCOHost, NEXUS and DIALOG database searches: diagnostic (and) formative assessment, intervention strategy (and) mathematics (and) anxiety/fear, intervention strategy (and) mathematics (and) attitudes, intervention strategy (and) mathematics (and) problem-solving, mathematics (and) disposition (and) academic achievement, mathematics (and) barriers (and) academic achievement.

1.4.2. Experimental design

The combined quantitative-qualitative research approach was used. Regarding the **quantitative research method**, the study involves an empirical assessment of the GIST model as a possible intervention strategy for mitigating a negative mathematics disposition. The experimental design was used (*see* Table 1.1) and will be discussed in detail in chapter 5 (*see* §5.5.1.1).

Table 1.1 The layout of the nonrandomised control group pretest-posttest design

Group	Pre-test	Treatment	Post-test
E	Y_1	X	Y_2
C	Y_1	-	Y_2

Regarding the **qualitative research method**, a particular interactive *mode of enquiry*, namely a **multi-site case study** (McMillan & Schumacher, 2000:11) was used. Case study is credited for affording the researcher an opportunity to interact face-to-face with the participants in their (participants') natural settings or environment to collect data, (McMillan & Schumacher, 2000:255). Further, a case study can be used trans-paradigmatically, i.e. across qualitative and quantitative methods (VanWynsberghe & Khan, 2007:4; Luck, Jackson & Usher, 2006:105). **Interviews, observations** and **document analysis** which are the most commonly used techniques of data collection in case studies (Hancock & Algozzine, 2006:16; Creswell, 2007:73; McMillan & Schumacher, 2001:41; Leedy & Ormrod, 2001:149; Ary, Jacobs, Razavieh & Sorensen, 2006:458) were conducted.

1.4.3. Study population and sample

- Study population: Grade 9 learners in Tshwane West district of the Gauteng Department of Education whose class sizes are large ($n \approx 40$).
- Sample: Four middle schools were sampled using random sampling technique from the Tshwane West district and the participating classes were selected based on availability.

1.4.4. Measuring instruments

1.4.4.1. Standardised questionnaire

The quantitative data were collected using the Study Orientation in Mathematics (SOM) questionnaire (Maree, 1996) to measure the mathematics disposition or mathematics learning barriers of learners in all the groups. The results of the research were processed by the Statistical Consultation Services of the North West University (Potchefstroom campus).

1.4.4.2. Self-constructed Mathematics Achievement Test

The quantitative data regarding mathematics achievement levels of all learners in all the groups were collected using the mathematics Achievement Test (*see* Appendix D). The Mathematics Achievement Test was used as a pre-test and post-test to measure the learners' mathematics achievement levels before and after the intervention. The specific focus of the Mathematics Achievement Test was on patterns, functions and algebra because they are the main focus of the Senior Phase (grades 7, 8 and 9) mathematics as they are collectively allocated approximately 35% of the total teaching time allocated to mathematics (DoE, 2003b:21). Three practicing mathematics teachers who currently teach mathematics in grade 9, and three mathematics curriculum specialists (also in the senior phase and currently employed by Gauteng Department of Education) moderated the mathematics achievement test to ensure its **content** and **face validity**.

1.4.4.3. Interviews, observations and document analysis

The qualitative data were collected through interviews to verify learners' quantitative responses regarding the barriers they experience. Learners and educators were **interviewed** (*see* Appendix B for learner interview) to gather information about the mathematics learning barriers experienced by learners. The interview protocol was used to interview learners. **Observation** protocol (*see* Appendix A) was also used to record observed trends during mathematics lessons as well as to observe the implementation of the GIST model. Further, the learners' and teachers' portfolios were

analysed through **document analysis** to gather additional data on the nature of feedback exchanged between learners and their teacher.

1.4.5. Data analyses

Descriptive and interpretive techniques (i.e. means, medians and standard deviations) were used to describe and analyse the changes (quantitative data) within the groups. Inferential statistical techniques (i.e. *t*-test, *F*-values and ANCOVA) were used to compare the results and to analyse differences with regard to the respective groups' mathematics disposition and achievement.

Qualitative analyses depended on the description of recorded data and were done verbatim. Assistance of the Statistical Advisory Services of the North West University was sought in the planning and execution of the project, as well as the processing of data.

1.4.6. Research procedure

Permission to use the above research population was sought from the Department of Education of the North West Province (*see* Appendix F) and from the principals of the sampled schools (*see* Appendix G). It is probably paramount to highlight that the schools that were used in the study were part of the North West Department of Education when permission was sought. However, when the empirical processes were implemented, the same schools were incorporated into the Gauteng Department of Education, due to the re-demarcation of provincial boundaries.

The teachers of the experimental group were trained in the application of the GIST model and data collection in the form of observation. Pre-test, intervention and post-test were administered. The assistance of the statistician from the Statistical Consultation Services of the North West University (Potchefstroom campus) was sought for data analyses. As much as possible, the meetings and training sessions with the teachers and the completion of the tests and questionnaires were done in a manner that minimised the disruption of classes.

1.4.7. Research ethics observed

All participants (teachers and learners) were informed about the purpose of the research. They were given assurances about the confidentiality of the results and anonymity of their participation in the research. Prospective participants were informed in order to encourage free choice of participation, while the assurances of confidentiality and anonymity were aimed at protecting the participants from the general reading public who might be able to identify them. Learners were therefore allocated numbers to identify them instead of using their real names.

1.5. Chapter framework

Chapter 1: Introduction, research problem, aims and plan of research

A brief literature review and description of the statement of the problem are presented. Further, the chapter offers a broad blue-print regarding the research method, an outline of the other chapters and the significance of the study.

Chapter 2: Learner-based teaching approaches: prospects to mitigate the impact of learning barriers in mathematics

The chapter focuses primarily on two critical issues gleaned from literature: **teaching-learning approaches** and their prospects of enhancing mathematics achievement, and **barriers** that may impede effective learning of mathematics. Regarding the former, the chapter outlines the rationale for choosing the particular teaching-learning approaches, the constructivist bases of the selected teaching-learning approaches, and the actual literature review regarding the selected teaching-learning approaches. Regarding the latter, the chapter offers definitions of the inhibitors, their impact on the teaching-learning of mathematics, and how to mitigate their negative impact on the learning of mathematics. Further, the implied connections between the **learning barriers** and the selected **mathematics teaching approaches** are elicited.

Chapter 3: Assessment and the learning of school mathematics

The chapter focuses on definition, authenticity, and purposes of assessment of school mathematics. Further, two types of synergies are elicited: between different assessment purposes in mathematics, and between mathematics assessment purposes and mathematics teaching-learning approaches.

Chapter 4: *An exposition of the GIST model*

The chapter presents a comprehensive structure of the GIST model, and a description thereof.

Chapter 5: *Method of research*

Elaborate account of the following methodological issues is provided:

Research setting, research aims, research hypotheses, research method, study population and sample, research procedure, data analyses and research ethics.

Chapter 6: *Research findings and discussions*

The chapter is divided into two main sections: findings and discussions. Research findings derived through quantitative and qualitative data collection techniques are presented in the form of written texts, graphs and tables. The findings are discussed to make deductions on their implications for the teaching and learning of school mathematics as well as their impact on the hypotheses and research questions.

Chapter 7: *Recommendations and conclusions*

Recommendations are made with regard to the findings and future studies, limitations are presented with regard to the manner in which the research was carried out, and conclusions are made regarding the achievements of the research.

1.6. Significance of the study

1.6.1. Implications

If the research hypotheses are accepted, the GIST model may offer a significant contribution to resolve the problem of using diagnostic assessment in mathematics in large classes in South Africa. Further, the application of the GIST model may have the following positive implications:

- learners' self-confidence about mathematics may be enhanced;
- the problems of learners experiencing mathematics learning barriers (such as poor problem-solving behaviour, mathematics anxiety, negative mathematics study attitudes, non-conducive mathematics study environment and inappropriate mathematics study habits) may be adequately addressed;

- mathematics teachers may begin to acquire conceptual knowledge and understanding of mathematics learning barriers focused on in this study;
- mathematics teachers may begin to use diagnostic assessment with confidence and may be encouraged to explore other intervention strategies; and
- mathematics teachers may begin to realise the value of formative assessment especially if it is properly harmonised with diagnostic assessment.
- mathematics teachers may cope with the Inclusive Education model with relatively more confidence and not confine it (Inclusive Education model) only to permanent **neurological** and **physical disabilities** but also to **learning difficulties** that may be rectifiable.

1.6.2. Application

Although the research was conducted within the confines of the experimental case study among grade 9 learners of the Tshwane West district, the results may be beneficial to mathematics teachers and learners in the other grades and districts. Furthermore, the GIST model of mathematics diagnostic assessment may be of use in other subjects or learning areas where large classes pose a hindrance to the effective use of diagnostic assessment. It should, however, be borne in mind that the diagnostic instruments that were used in this study are peculiar to mathematics. Therefore, while the structural layout of the GIST model may be applied in other learning areas, diagnostic tools appropriate to the learning areas should be used. The same is applicable to the teaching and learning methodologies.

CHAPTER

2

LEARNER-BASED TEACHING APPROACHES:

Prospects to mitigate the impact of learning barriers in mathematics

The barriers in the learning of mathematics may not be addressed effectively without simultaneously invoking appropriate teaching approaches of mathematics
(Gersten, Jordan & Flojo, 2005:300)

CHAPTER 2

LEARNER-BASED TEACHING APPROACHES: Prospects to mitigate the impact of learning barriers in mathematics



INTRODUCTION



BARRIERS POSING POTENTIAL MATHEMATICS DIFFICULTIES FOR
LEARNERS



MATHEMATICS LEARNER-BASED TEACHING PERSPECTIVES



SYNERGIES BETWEEN THE INSTRUCTIONAL APPROACHES



APPARENT INTERPLAY BETWEEN THE TEACHING APPROACHES
AND THE LEARNING BARRIERS



CONCLUSION

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2.1. Introduction

While it is acknowledged that teaching does not always yield learning (Fenstermacher, 1986:39; Donald, Lazarus & Lolwana, 2006:89), the chapter departs from the point of view that in instances where teaching yields learning, the **barriers in the learning of mathematics** may not be addressed effectively without simultaneously using **appropriate teaching approaches** of mathematics (Gersten, Jordan & Flojo, 2005:300; Bryant, 2005:343). Essentially Denvir *et al.* (1982:21) (cited by Barnes (2005:44)) attest to the assertion that inappropriate teaching methods contribute significantly to low attainment in mathematics. There are, therefore, essentially two areas of foci to be explored in this chapter, namely the **learning barriers** that have a potential to give rise to mathematics difficulty (Gersten, Jordan & Flojo, 2005:294) (*see* §2.2), and the **teaching-learning approaches or perspectives** which have the potential to redress mathematics difficulties (*see* §2.3).

The primary purpose of this chapter is to offer a critical study of documented literature on the **teaching and learning** of school mathematics and the **challenges** associated with the latter. In recognition of the enormous scope of entities that characterises the literature on *teaching and learning* as well as their *associated challenges* the focus of this chapter will be confined to: *firstly*, the elicitation of fundamental characteristics of selected five factors that can gravely **bar learners from learning mathematics effectively**. In the process of the literature study, the interplay or interconnection between the barriers towards the effective learning of school mathematics will also be elicited; *secondly*, the selected four **learner-based mathematics teaching approaches or perspectives** which are in line with the underpinnings of the South African curriculum. The constructivist bases of the four learner-based teaching approaches will be elicited as well as illustrating the interconnection between them; and *lastly*, the illustration of the implicit **interplay** between the teaching approaches and the learning barriers. The accentuation of the three main domains of this chapter emanates from their fundamental role in the constitution of the GIST model (*see* Chapter 3).

It is essential to alleviate the potential confusion that may stem from the concepts **mathematics difficulty** (which constitute the focus of this study) and **mathematics disability**. While Fletcher (2005:308) uses the two concepts interchangeably as though they are synonymous, numerous researchers have identified a clear distinction between them (Bryant, 2005:343; Hanley, 2005:347; Mazzocco, 2005:321; Fuchs 2005:351; Van Kraayenoord & Elkins, 2004:34). mathematics difficulties are generally not stable over time (Hanley, 2005:347; Van Kraayenoord & Elkins, 2004:34), i.e. they

can be remedied if their potential causes are diagnosed, using appropriate diagnostic tools and relevant interventions or treatment is given (Bryant, 2005:343; Hanley, 2005:347). Therefore, without early identification (and intervention) of mathematics difficulty, learners will be deprived of a high level of a development of mathematics proficiency (Bryant, 2005:340).

Mathematics disability, contrary to mathematics difficulty, is a long term chronic, pervasive and severe mathematics difficulty that cannot respond to a validated intervention process from which learners exhibiting mathematics difficulties benefit (Fuchs, 2005:351; Hanley, 2005:348; Van Kraayenoord & Elkins, 2004:35). Mazzocco (2005:321) provides a more comprehensive definition of mathematics disability by referring to it as a presumed biological (neuro-biological or genetic) mathematics disability. Based on the two definitions, it could be deduced that mathematics difficulty can be remedied while mathematics disability can be managed. In this study, therefore, the focus is on mathematics **difficulty** and will be used interchangeably with mathematics **barriers** to refer to the elements that have the potential to impede the effective learning of mathematics.

2.2. Barriers posing potential mathematics difficulties for learners

Barriers to the learning of school mathematics are multi-dimensional in nature: some are dispositional in nature (the focus of this study) while others are content deficient orientated. The assessment thereof through the appropriate diagnostic assessment measures cannot be over-emphasised (Kutz, 1991:277). Knowing learners' outlook or disposition about mathematics, asserts Kutz (1991:277), affords the teacher the opportunity to be informed about, *inter alia*, learners':

- flexibility in exploring mathematical ideas and trying alternative methods in solving problems;
- confidence and willingness to persevere in solving mathematical tasks;
- interest, curiosity and inventiveness in doing mathematics;
- inclination to use meta-cognitive skills; and
- productive disposition (habitual inclination to value and appreciate the role of mathematics).

Maree *et al.* (1997:1) agree that knowing learners' mathematics outlook "... offers ... mathematics teachers ... more information on their pupils than merely the information ... on cognitive achievement". It would therefore appear that the benefits of knowing mathematics learners' disposition outlined by

Kutz (1991:277) characterise the elements of mathematical disposition referred to by Maree *et al.* (1997:1) as learners' Study Orientation in mathematics (SOM) that include attitudes, problem-solving behaviour, anxiety, study milieu and study habits.

2.2.1. Attitudes towards mathematics

2.2.1.1. Definition

The World Book Dictionary (1995) defines "attitude" as a way of thinking, acting or feeling or behaving towards a situation or cause. The Dictionary of Psychology (Corsini, 2002:76) asserts that attitudes are characterised by "... cognitive, emotive and behavioural components which combine to convey a positive, negative, or neutral response". The definition of attitudes which emphasises **thought processes** is influenced profoundly by psychological perspectives (Ruggiero, 1998:10). However,, the definition that emphasises **belief** emanates from a philosophical perspective i.e. an attitude is a habitual emotional response driven by belief (Ruggiero, 1998:13). McLeod (1992:581) extends the definition by combining psychological and philosophical perspectives through which he argues that attitudes involve either positive or negative feelings which dispose learners towards certain behaviours as a result of what they think and what they believe in. Attitude towards mathematics, therefore, would manifest itself through learners' cognitive processes, beliefs, actions during problem-solving, affective and behavioural patterns they exhibit during the teaching and learning of mathematics. Kutz (1991:277) attests: "Positive attitudes include the attributes of confidence, perseverance, interest, reflectiveness, valuing and appreciation as part of ... mathematical disposition". Holmes (1995:36) agrees that positive or negative attitudes towards mathematics have an impact on the learners' mathematics problem-solving behaviour: the prevalence of positive attitudes gives mathematics learners confidence to solve mathematics problems, and perseverance or motivation to solve even very difficult mathematical problems. The opposite is true for the prevalence of negative attitudes. This suggests that there is a close connection between attitudes towards mathematics, mathematics problem-solving behaviour and motivation to do mathematics. McLeod (1992:581) concurs that attitudes towards mathematics include **liking** mathematics and **believing** (i.e. motivation) that the study of mathematics is valuable.

2.2.1.2. Impact of attitudes on mathematics teaching and learning

The manner in which attitudes towards mathematics develop may result in catastrophic consequences with regard to the learning of mathematics. For instance, McLeod (1992:581) distinguishes two ways through which attitudes develop: *firstly*, through automatizing of repeated emotional reaction to mathematics, and *secondly*, through assigning an already existing attitude to a new but related task. Automatization of repeated emotional reaction is apparent if a learner is exposed to recurring negative experiences with, for instance, the solution of algebraic equations. The emotional impact associated with the negative experience will become automatized over a period of time. The automatized emotional reaction will therefore never trigger any acute physiological arousal – hence the emotional reaction towards algebraic equations becomes superficially stable. However, if appropriate intervention measures are not administered, attitude towards algebraic equations may lead to attitudes towards the solution of geometric equations because both are equations. The already existing attitudes towards algebraic equations are therefore transferred or assigned to the solution of geometric equations, hence the second type of development of attitudes according to McLeod (1992:581).

Motivation, which has been positively correlated with attitudes towards mathematics, has the potential to impact negatively (if non-existent) on the learning of school mathematics. Holmes (1995:4) has categorised motivation under **intrinsic** and **extrinsic** motivation. The former refers to the inspiration to attain self-set goals, while the latter refers to the enthusiasm to attain goals set by others. While both forms of motivation are essential in the teaching and learning of mathematics, intrinsic motivation seems to play a major role in learners' mathematics learning (Holmes, 1995:4) as it fosters meta-cognitive skills. Holmes (1995:4) further argues that when mathematics teachers provide mathematical challenges appropriate to the learners' level of development, and they ensure learners' perseverance by giving them support, they (learners) will become autonomous, responsible and confident mathematics problem solvers.

Achievement and motivation have each been found to have a reciprocal effect on interest (which is associated with attitude) in mathematics (Bester & Buhdal, 1995:26). This suggests that when interest (which is a trait of attitude) is maximised, achievement and motivation will also be maximised and vice versa. Therefore, a negative attitude towards the learning of mathematics may also have a negative outcome regarding achievement in and motivation to do mathematics.

2.2.1.3. Fostering positive attitudes in mathematics class

The critical question that is likely to emerge from the above assertions would be: how should teachers help learners who are experiencing negative attitudes towards mathematics? Two instructional trajectories may be of assistance: *firstly*, engaging learners in discussions that will help to examine their attitudes, and *secondly*, by manipulating the classroom environment/setting to enable learners to articulate and communicate their thinking and to take control of their learning (Holmes, 1995:36). The two suggested instructional trajectories have the following implications for mathematics teaching and learning: *Firstly*, when learners are given the opportunity to talk about their attitudes towards mathematics, teachers begin to acquire comprehensive information and understanding about the leverage of learners' inhibitors. Similarly, learners begin to understand in perspective the nature of their attitudes and possible mechanisms to address such attitudinal deficiencies.

Secondly, classroom environment refers to numerous variables (including the mathematics teaching approaches) within the classroom that will have an impact on the teaching of mathematics. It is therefore imperative that this trajectory of attempting to foster positive attitudes towards mathematics requires of the teacher the ability to make the appropriate selection and use of mathematics teaching approaches. If the assertions by Holmes (1995:36) are anything to go by, then the interplay between cognitively guided instruction (CGI), problem centred instruction (PCI), realistic mathematics education (RME) and cooperative learning (CL) can enhance positive attitudes towards mathematics. The four approaches will receive more attention later in this chapter. However, these approaches in the teaching and learning of mathematics offer the following attitudes-related benefits:

CGI – enhances self-confidence about mathematics and meta-cognitive skills; affords learners the freedom to use their own problem-solving strategies in mathematics.

PCI – solving meaningful and practical mathematics problems.

CL – involves learners in their learning through active interaction with peers.

RME – emphasises the connection of mathematics with real life situations.

2.2.2. Problem-solving behaviour

2.2.2.1. Definition

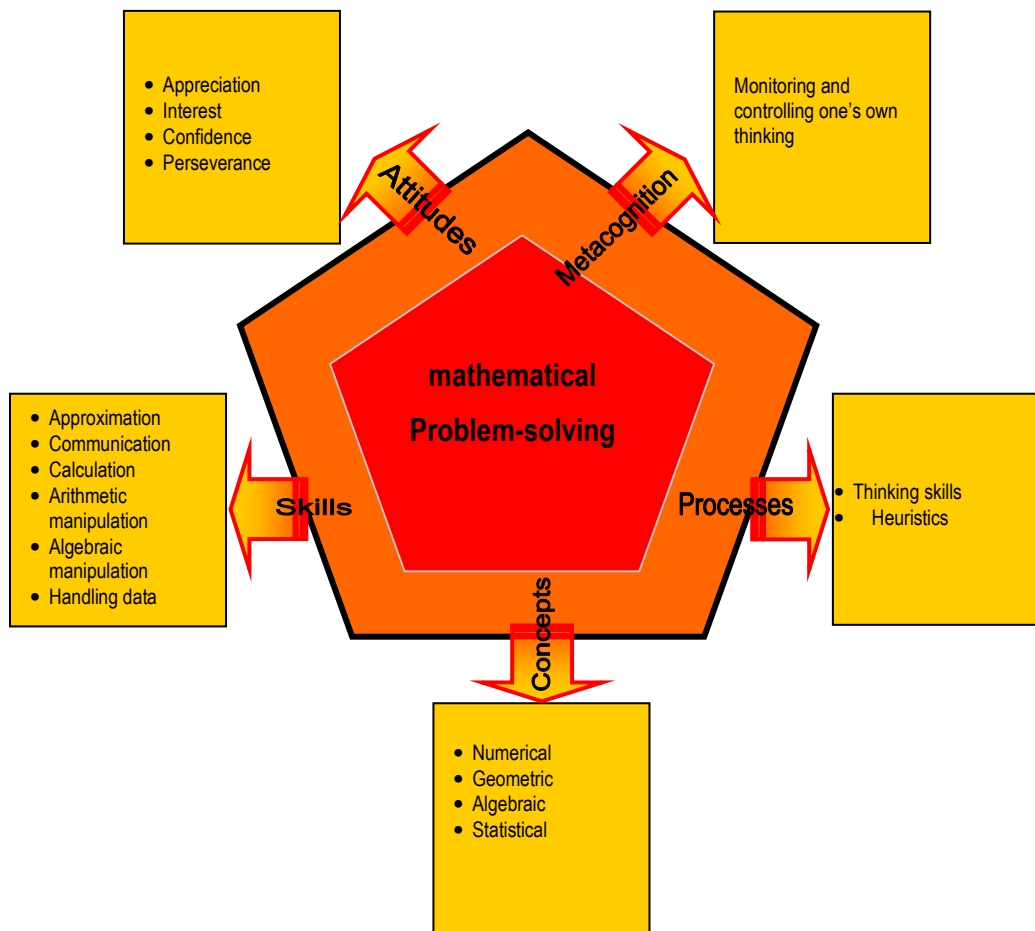
Presumably the most intelligible way to define **problem-solving behaviour** is to distinguish between behaviour and problem-solving. **Behaviour** is generally defined as covert or overt actions, reactions, and interactions in response to stimuli including conscious or unconscious processes (Corsini, 2002:99). Orton (2001:601) defines **problem-solving** as “the incorporation of novel problems or situations that require students to use previously acquired knowledge and expertise in an intelligent and insightful way in order to arrive at a solution or conclusion”. **Problem-solving behaviour** would therefore refer to covert or overt procedural approach through which people attempt to find solutions of the problems they face. De Landsheere (1997:807) concurs that in the context of learning, problem-solving behaviour is the ability of a learner “to select relevant principles and sequencing them into an unencountered problem situation for which the relevant principles are not specified”. The key words emanating from the definitions alluded to earlier are: ***‘actions, reactions, and interactions’***, ***‘select relevant principles and sequencing them’***; and ***‘novel problems or situations’***. In the context of mathematics therefore problem-solving behaviour would refer to the ability of a learner to select mathematics algorithms from the plethora of mathematics algorithms the learner is familiar with, and apply them in solving unfamiliar, but realistic/meaningful problems mathematically. It would therefore appear that the selection of appropriate problem-solving strategies, and the proper orchestration and application thereof qualify positive problem-solving behaviour.

The idea that problem-solving should be central to mathematics instruction at all scholastic grades permeates, and is well articulated in, the mathematics national curriculum statement in South Africa (DoE, 2002e:5). If properly orchestrated and effectively implemented, this idea will supposedly transform mathematics achievement in South Africa as is already happening in Singapore (Ho & Hedberg, 2005:238). According to Figure 2.1, if problem-solving is central to the teaching and learning of school mathematics, other necessary aspects of learning such as meta-cognition, skills, attitudes, concepts and thinking processes will 'automatically' be achieved. However, the fundamentals regarding the constituents of problem-solving in mathematics need to be examined first.

More than a decade ago, Shuell (1990:102) proffered a comprehensive understanding of mathematics problem-solving: "... *a goal directed activity* that requires *active* search for and *generation* of possible *alternative actions and decision making* as to which course of action to follow next". What emerges from this definition is a conspicuous presence of meta-cognitive skills (i.e. 'generating alternative actions' portrays monitoring one's actions) which are perceived as being fundamental (*see* Figure 2.1) in the problem-solving processes (Eizenberg & Zaslavsky, 2003:391; Leikin & Kawass, 2005:255; Ho & Hedberg, 2005:238; Cifarelli & Cai, 2005:304).

Concurrence of Ho & Hedberg (2005:239) with Shuell (1990:102), as illustrated in Figure 2.1, further demonstrates **attitudes** and **skills** through active search and generation of alternative actions. Further, the meta-cognitive skills, which are implied in mathematics problem-solving, include the ability to monitor and regulate one's own cognition and behaviour, and validate one's solutions. Mathematics problem-solving involves the application of relevant and mathematically authentic procedures and thinking processes in real life contexts (Ho & Hedberg, 2005:238).

Figure 2.1 Framework of the Singapore mathematics Program
(Ho & Hedberg, 2005:239)



2.2.2.2. Impact of problem-solving behaviour on mathematics teaching and learning

Problem-solving in mathematics is inherently a frustrating activity, which requires willingness, persistence, motivation and interest to engage in it to attain a problem-solving goal (Kutz, 1991:92). However, mathematical problem-solving exposes learners to the realities of putting mathematical theory into practice. Kutz (1991:93) has identified two categories of problems that can be used by teachers during mathematics teaching in order to translate mathematical theory into practice: **Process problems** which mainly focus on the process of solving non-routine mathematical problems which have analogous real life activities, and **puzzle problems** whose primary role is to provide challenge and enjoyment, are not algorithmic in nature and therefore have no analogous real life activity. It is assumed that the knowledge of and the ability to use the two categories of mathematical problems will demonstrate to learners that

mathematics can be applied to real life situations (*process problems*) and can also be used for recreational purposes (*puzzle problems*). Process problems seem to enhance meaningful learning of mathematics, while puzzle problems seem to enhance interest and motivation in the learning of mathematics.

2.2.2.3. Fostering effective problem-solving behaviour in mathematics class

Some of the difficulties associated with mathematics problem-solving emanate primarily from **poor meta-cognitive skills**, which are considered fundamental for successful problem-solving (Eizenberg & Zaslavsky, 2003:390). Further, from the study they conducted about cooperative problem-solving in combinatorics, Eizenberg and Zaslavsky, (2003:395) established the positive effects of cooperative problem-solving in mathematics.

Kutz (1991:96) attests to the notion of frequently employing cooperative problem-solving strategy to foster effective problem-solving behaviour. However, Holmes (1995:36) proffers the other problem-solving fostering trajectory, i.e. engaging learners in **discussions** that will help them examine and explore ways of changing their poor problem-solving behaviours. Amalgamating the three points of view may enhance even more effective ways of fostering problem-solving behaviours, i.e. **meta-cognitive skills** that are implied and elaborated in Polya's heuristics for problem-solving (Holmes, 1995:37; Van De Walle, 1997:50; Ho & Hedberg, 2005:250), **cooperative problem-solving**, and **discussions** based on the problems of problem-solving. These three dimensions give a clearer picture about the teaching approaches, and/or strategies/heuristics to be employed in fostering effective problem-solving behaviours in mathematics.

2.2.3. Mathematics anxiety

2.2.3.1. Definition

Mathematics anxiety is counted among serious deterrent for children to learn mathematics in all levels of schooling (Geist, 2010:24). In order to explore the concept *mathematics anxiety* deeper, its definition needs to be examined first. The World Book Dictionary (1995) defines anxiety as 'uneasy **thoughts** or fears about what may happen'. Further, fear is defined as the **emotion** or condition of being afraid. In addition, the Oxford Dictionary of English (2005) defines anxiety as a nervous disorder associated with excessive uneasiness and apprehension, typically with compulsive **behaviour** or panic. Corsini (2002:58) asserts that anxiety is often a response to an unidentified threat which may emanate from internal conflicts. By implication, it seems that anxiety is an uneasy **thought** process that evokes a fearful **emotional** state that will culminate into exhibition of certain negative **physical/behavioural** reaction. It is therefore befitting to define mathematics anxiety as a mental, emotional and physical reaction to mathematical thought processes and problem-solving, often caused by negative childhood mathematical experiences (Arem, 1993:1). The key elements of mathematics anxiety are therefore physical, cognitive and behavioural in nature. Corsini (2002:76) also identifies physical, cognitive and behavioural connotations/nuances regarding attitudes towards mathematics.

Further, negative beliefs about mathematics are perceived by Hackworth (1992:4) as the primary source of mathematics anxiety. The same construct was identified by Ruggiero (1998:13) and Ashcraft (2002:181) as the other source of negative attitude towards mathematics. The prevalence of these characteristics in both mathematics anxiety and attitudes towards mathematics suggests a relationship between the two constructs, namely mathematics anxiety and attitudes towards mathematics. Chinn (2009:62) reveals that mathematics anxiety has the potential to impact negatively on learners' attitude to learn mathematics. Opolot-Okurut (2005:167) supports this assertion through a study conducted among grade 9 learners in Uganda.

2.2.3.2. Possible causes of mathematics anxiety

The definitional aspects of mathematics anxiety namely cognitive, behavioural and physical will form the point of departure regarding the possible causes of mathematics anxiety. Mitchell (1987:15) argues that mathematics anxiety is primarily caused by a combination of physical, cognitive and psycho-behavioural components. This view seem to offer a comprehensive and elaborate account of what constitutes mathematics anxiety especially due to its emphasis on the connection between physical, cognitive and psycho-behavioural aspects. Based on the definition of mathematics anxiety provided by Arem (1993:1) (*see* §2.2.3.1) there seems to be a significant degree of concurrence with the aforesaid view by Mitchell (1987). Further, Hackworth (1992:10) departs from the viewpoint that mathematics anxiety is learned while Donald, Lazarus and Lolwana (2006:312) and Tooke (cited by Malinsky, Ross, Pannells & McJunkin 2006:274) assert that teachers contribute enormously towards the development of mathematics anxiety among learners primarily through their 'poor' teaching methods. Furner and Duffy (2002:68) identify with Hackworth's view by citing overt behaviours exhibited by teachers such as being hostile, exhibiting gender bias, an uncaring attitude, and embarrassing learners in the presence of their peers. However, the views of Furner and Duffy (2002:68) and Hackworth (1992:10) will be infused and implied in the discussion based on the views of Mitchell (1987:15) and Arem (1993:1).

Physical aspects of mathematics anxiety: These are the hormonal, chemical and muscular changes the body exhibits as it attempts to cope with fear of mathematics. The physical manifestation of mathematics anxiety includes, but is not limited to, muscle tension, 'butterflies', perspiration, sweaty hands and feet, and rapid heartbeat (Malinsky *et al.*, 2006:274). These bodily changes occur in response to a perceived threat, which in this context is a fear of mathematics (Mitchell, 1987:16). The ***fight-or-flight*** response, for instance, enables a person to fight back or run away from a threatening situation (such as mathematics in this context) (Mitchell, 1987:16). However, because the fear of mathematics cannot be 'fought' or 'run away' from, learners are caged in a midst situation of looking stupid, experiencing humiliation and losing self-esteem (Mitchell, 1987:16). The prevalence of these anxiety indicators, which underlie mathematics anxiety, often delays learners' cognitive functioning,

motivation to do mathematics and the development of self-confidence (Maree *et al.*, 1997:7).

The aforementioned physical aspects are connected to the cognitive and behavioural aspects in the following manner:

- Inability to run away from or fight back at the fear of mathematics causes learners to exhibit repetitive behaviour such as nail biting, playing with objects and inability to speak fluently (Maree *et al.*, 1997:7); and
- Due to the fear of appearing very stupid in mathematics class, learners' meta-cognitive skills are dreadfully affected, i.e. they are unable to monitor their own learning of mathematics, cannot take risks of applying their own approaches to solve mathematical problems and their cognitive functioning is generally preoccupied or overwhelmed by the fear of mathematics (Maree *et al.*, 1997:7).

Behavioural aspects of mathematics anxiety: Some mathematics learners exhibit strange/inappropriate behaviour when exposed to the context involving mathematics. Mitchell (1987:57) articulates the inappropriate behavioural patterns associated with mathematics anxiety thus:

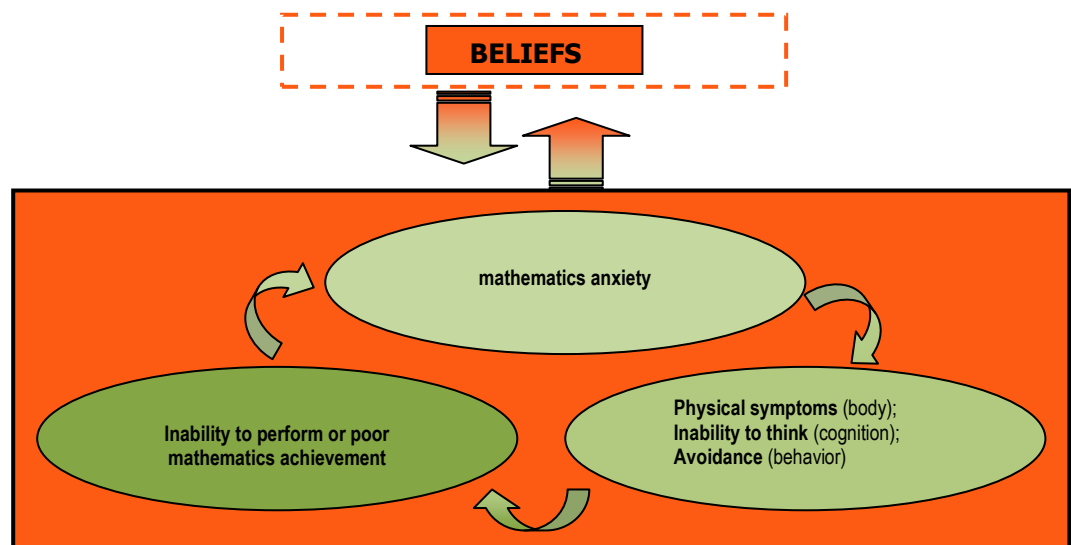
... one would not expect to elicit the vigilance and fight-or-flight behaviour by mere sight of a math classroom, teacher, or a piece of paper with Math Test written across the top. Nor would one expect to be stricken with panic when handed a check for a group having lunch together and asked to figure out how much each person owes on the bill, ...

Cognitive aspect of mathematics anxiety: mathematics anxiety can also be exacerbated by the thoughts of what is likely to happen, e.g. thoughts of appearing foolish when asking or answering a question, consequently a feeling of degradation (Mitchell, 1987:29). In reference to mathematics, the negative mental picture or self-talk created in the learner's mind regarding mathematics has a potential to produce negative emotional reaction, hence emotion is the by-product of cognition and feeling is the by-product of thinking (Mitchell, 1987:29). The intensity of these negative cognitive

processes will give rise to: **physical/bodily** manifestations such as sweating, exaggerated need to visit the toilet, sweaty hands and feet, and rapid heartbeat (Maree *et al.*, 1997:7); and **behavioural** manifestation such as withdrawal or avoidance. A modified version of mathematics anxiety cycle is illustrated by Mitchell (1987:33) in Figure 2.2.

According to the mathematics anxiety cycle, negative beliefs about mathematics (perceived by Hackworth (1992:4) as primary source of mathematics anxiety) are built from past experiences and this is indicative of the likelihood of the presence of emotional or physical threat. The mathematics anxiety sufferers internalize and visualize the threats (or use threatening self-talk) to portray their negative beliefs about mathematics. In turn, their ability to think clearly is compromised and this reinforces their belief that they are incapable in mathematics. The resulting behaviour is characterized by withdrawal and inability to perform or extreme underperformance in mathematics becomes dominant.

Figure 2.2 The mathematics anxiety cycle (Mitchell, 1987:33)



Chinn (2004:107) has further identified more factors contributing to mathematics anxiety which appear to be cognitively based:

- The abstract nature of mathematics.
- Inappropriate instruction.
- Badly designed work tasks.
- The pressure of having to do mathematics quickly.
- Constant under-achievement.
- A poor understanding of mathematics.

A critical examination of the afore-mentioned factors (Chinn, 2004:107) seems to further point to the fairly widespread assertion that mathematics teachers contribute significantly towards learners' mathematics anxiety (Hackworth, 1992:10; Donald *et al.*, 2006:312; Geist, 2010:28; Tooke, cited by Malinsky *et al.*, 2006:274; Furner & Duffy, 2002:68; Reusser, 2000:18). The assertion is evident in the first four factors, namely **the abstract nature of mathematics; inappropriate instruction; badly designed work tasks; the pressure of having to do mathematics quickly**, which are arguably reliant on the teacher's competencies regarding the teaching, learning and assessment of mathematics.

2.2.3.3. Impact of mathematics anxiety on mathematics teaching and learning

Furner and Duffy (2002:68) have documented their views regarding mathematics anxiety whose primary aim is to render it (mathematics anxiety) inoperative in mathematics classes. They assert that poor performance in mathematics is linked to an increase in mathematics anxiety. Mathematics anxiety interferes with the manipulation of numbers and mathematics problem-solving within a variety of everyday life and academic situations (Furner & Duffy, 2002:68). The assertion that mathematics anxiety can be taught and learned (Hackworth, 1992:8; Furner & Duffy, 2002:68) is a cause for concern. Actually, significant mathematics anxiety sufferers acquired it from the mathematics classroom and more specifically from villain teachers (Hackworth, 1992:9). Conscious or unconscious mathematics anxiety sufferers within the teacher population are likely to transfer it to their learners, and it becomes a cyclic process. This can have enormous negative impact on learning and achievement because the teacher 'reinforces' it at all times.

2.2.3.4. Mitigating mathematics anxiety

Hackworth (1992:10) provides four essential steps in an effort to mitigate mathematics anxiety among learners:

Develop an understanding of how mathematics anxiety is created and applying that understanding to the unique experiences of the anxiety sufferer.

Knowing what mathematics anxiety is (definition), how it develops (causes) and what available remedial options are there (solutions) is the starting point towards the reduction of mathematics anxiety (Hackworth, 1992:11). The mathematics teacher who knows the dynamics of mathematics anxiety and its implications to the learning of mathematics will help learners acquire the same knowledge. By implication, the teacher who is more knowledgeable about mathematics anxiety will be in the position to diagnose learners (Furner & Duffy, 2002:69) and help them through a self-desensitisation process. Systematic desensitisation has been found to be most effective in mitigating the intensity of mathematics anxiety (McLeod, 1992:584). However, what remains a key factor is that learners must know what they are suffering from in order to enhance ownership of the remedial programme of mitigating mathematics anxiety.

Become acquainted with learning theory and the qualities of excellent instruction:

The mathematics classroom is an uncomfortable environment for any learner experiencing mathematics anxiety, therefore learners need to be informed about the instructional processes that will make the mathematics classroom environment positive (Hackworth, 1992:11). This suggests that anxiety remediation should be integrated in the process of teaching and learning of mathematics. Learners should be informed, for instance, that they will engage in **cooperative learning** activities, focusing mainly on **meaningful problems** taken from **realistic** experiences and are allowed the freedom to express their **cognition** when solving such problems. The idea is to provide the best practices for teaching mathematics in order to lessen mathematics anxiety (Furner & Duffy, 2002:69).

The following are some of the best practices for teaching mathematics (Furner & Duffy, 2002:69):

- Engaging learners in discussions through cooperative group work.
- Make learning mathematics realistic.
- Use justification of thinking.
- Use problem-solving approaches.

The best practices for the teaching and learning of mathematics suggested by Furner and Duffy (2002:69) provide important information regarding the choice and identification of the instructional approach to employ in mathematics classes. The aforementioned best practices may be linked to **cooperative learning** perspective, **realistic mathematics** instruction, **cognitively guided** perspective, and **problem centred** perspective (*see* §2.3). Further, Tobias (1987:7) focuses mainly on meta-cognitive skills as a primary mitigating aspect of mathematics anxiety. These skills include self-monitoring, active thinking and self-mastery. However, these meta-cognitive skills are some of the characteristics of cognitively guided instruction (Fennema, Carpenter & Peterson, 1991:32) (*see* §2.3.3.2). Being *acquainted with learning theory and the qualities of excellent instruction* puts an enormous task/responsibility on mathematics teachers to identify appropriate instructional approaches to mitigate learning problems associated with mathematics.

Acquire the four necessary learning strategies for studying mathematics:

Hackworth (1992:64) puts more emphasis on the following four essential strategies for the learning of mathematics: **Study environment** – entails physical attributes such as proper lighting, mild temperature in the mathematics class, low noise level. The relaxed atmosphere lowers the psychological attributes such as fear and anxiety and enhances high level of cognition.

Appropriate treatment of **mathematics information** – the emphasis is on the appropriate structuring of the questions during mathematics assessment in order to seek declarative, procedural and/or conditional mathematics knowledge from learners. Inappropriate questioning techniques may further increase the level of mathematics

anxiety among learners. Declarative knowledge (acquired from the “What?” questions) is about the meaning and definition of mathematical concepts, procedural knowledge (acquired from the “How?” questions) is about the process of problem-solving, and conditional knowledge (acquired from the “Why?” questions) is about justification of the action taken in the process of solving mathematical problems. Clarity with regard to the knowledge required from learners will maximise the chances of mitigating mathematics anxiety. Further, Hackworth (1992:68) emphasizes the importance of appropriately defining mathematical concepts as a strategy to minimize mathematics anxiety. Knowing and understanding mathematical concepts accurately is a requisite to identifying the processes needed to solve problems.

Learners’ **minds should be kept active** – good learning of mathematics occurs when the student is thinking about mathematics (Hackworth, 1992:70). As highlighted earlier (*see* §2.2.2.2), meaningful and interesting mathematical problems have a potential to enhance active thinking and time spent on-task in a quest to find the solution. Cooperative learning methods and cognitively guided instruction also play a meaningful role in this regard (*see* §2.3.2.3 and §2.3.3.3). Through these instructional approaches, learners are encouraged to try new avenues and in the process they are encouraged to monitor their progress and validate their mathematical procedures and their answers (Tobias, 1987: 8).

Self-monitoring and evaluation – this is a meta-cognitive skill that enables learners to constantly regulate their mathematics learning processes and to make judgments about their own progress in the learning of mathematics. According to Tobias (1987:9), the essential aspect of mathematics anxiety therapy is self-monitoring. During the learning of mathematics, learners should not only focus on the knowledge of mathematics, but also on the knowledge of how well the mathematics has been learned (Hackworth, 1992:75). For instance, solving a mathematics problem (knowledge of mathematics) but validating the answer (self-evaluation) is even more important.

Find methods of decreasing the counter-productive reactions to anxiety:

This strategy primarily focuses on physical relaxation when the level of anxiety increases. Research has revealed that addressing the physical symptoms of mathematics anxiety through calming has the potential to minimise psychological and emotional manifestations of mathematics anxiety (Hackworth, 1992:81; Davidson & Levitov, 1993:60).

2.2.4. Study milieu

2.2.4.1 Definition

There are two primary environments in which mathematics is studied: the classroom and the learner's home (Maree *et al.*, 1997:9). The former will be the main focus in this study because it is the environment in which the GIST model is tested. Corsini (2002:596) defines milieu as general surrounding or environment. In psychology milieu refers to "... the particular climate of the home and character of the neighbourhood ...” (Corsini, 2002:596).

Generally, study environment or milieu therefore “refers to the ... **social** and **physical** environment from which learners come” (Moodaley *et al.*, 2006:637). While this definition captures two important aspects of the environment namely **social** environment and **physical** environment of the learners' homes, it disregards the social and physical environment of the school, specifically the mathematics classroom as cited by Donald *et al.* (2006:141). Arguably, studying does not only take place at home, but in the classroom too. Therefore, whether the environment in which mathematics is being studied or learnt is the classroom or learner's home, the physical attributes such as proper lighting, mild temperature in the mathematics class and low noise level should be prevalent, as they also constitute the study milieu (Hackworth, 1992:64). Kellough and Kellough (2007:115) argue that classrooms that are pleasant, positive and supportive enhance learning. This argument suggests a third aspect of the classroom environment, namely **emotional contentment** of learners.

Further, in the context of this study, cooperative learning dictates the classroom environment socially and partly physically (Donald *et al.*, 2006:141), **socially** because it affords maximum support from other learners in terms of sharing mathematical problem-solving strategies, and assisting one another in terms of understanding mathematical concepts. The social classroom environment finds its roots on what Aldridge, Fraser and Sebela (2004:245) call constructivist learning environment which views the teacher as facilitator and provider of learning experience, and learners as active participants in constructing their own mathematical meanings. The **partly physical** attributes of the classroom environment offered by cooperative learning focuses only on the seating pattern in the classroom. The arrangement of tables and chairs to accommodate small-group work enhances face-to-face interaction among the group members.

2.2.4.2 Different aspects of the classroom as a study environment

Donald *et al.* (2006:141) have presented some useful information about the classroom as a study environment under the following aspects: **physical, instructional and social aspects**. The three aspects are essentially interlinked; subsequently it is not practical to address them individually and in isolation. Donald *et al.* (2006:141) argue that the physical space in the classroom needs to be adequate for the number of learners it can accommodate. Adequate space makes it possible for mathematics teaching approaches such as cooperative learning to be implemented with negligible constraints. A high level of interaction (learner-teacher interaction and/or learner-learner interaction) is enhanced when the classroom space is adequate. Conversely, if the number of learners in a class far exceeds the number of learners a classroom can accommodate, effective cooperation is compromised (Sekao, 2004:5). However, this assertion does **not** in any manner suggest that a large number of learners accommodated in a big classroom cannot be taught mathematics effectively (Donald *et al.*, 2006:142). A proper orchestration of cooperative learning can yield effective teaching of large classes (Holderness, 2003:4; Jacobs & Inn, 2003:142).

2.2.4.3 Impact of study milieu on mathematics teaching and learning

Research findings have revealed sufficient evidence on the social benefits of cooperative learning regarding the learning of school mathematics (Johnson & Johnson, 1990:108; Landsberg, 2007:72). The social benefits include but are not limited: accepting and supporting each other; listening to one another; and contributing mathematical viewpoints without fear.

Hackworth (1992:64) suggests that the physical attributes such as proper lighting, mild temperature in mathematics class and low noise level enhance a relaxed atmosphere in the mathematics study environment. In turn, a relaxed classroom environment lowers the psychological attributes such as mathematics anxiety and enhances a high level of cognition (Hackworth, 1992:64). Maree *et al.* (1997:28) agree – the SOM questionnaire has revealed a significant correlation coefficient of 0.722 between the study milieu and mathematics anxiety. To a reasonably large extent, the correlation between the two aforementioned fields is attributed to a non-stimulating mathematics learning environment (Maree *et al.*, 1997:29).

2.2.4.4 Fostering positive study milieu in a mathematics class

The research-based information in §2.2.4.1 and §2.2.4.2 provides ideas on how to foster a positive study milieu in mathematics classes. This section therefore departs from the notion that the ability to define ‘study milieu’ and knowing its impact on the study of mathematics provides mathematics teachers with ideas to mitigate its impact.

The following ideas that emanate from the definition and impact of study milieu are suggested:

- The use of cooperative small groups in the learning of mathematics has a potential to enhance a non-threatening classroom environment which in turn enhances social, cognitive, behavioural and affective skills (Whicker *et al.*, 1997:43; Kellough & Kellough, 2007:337).

- Mitigating mathematics anxiety (*see* §2.2.3.4) may also help to create or enhance positive mathematics learning environment. This is against the background that there is a significant positive correlation between the study milieu and mathematics anxiety (Maree *et al.*, 1997:29).
- Improving the physical classroom attributes such as proper lighting, mild temperature in mathematics class, and low noise level enhance a relaxed atmosphere in the mathematics study environment (Hackworth, 1992:64).

2.2.5. Study habit

2.2.5.1. Definition

Habit is defined as a particular way of acting or doing things, following a certain custom (The World Book Dictionary, 1995). Ruggiero (1998:12) defines habits from contemporary psychological perspective by asserting that habits are external, physical patterns of response. Moodaley *et al.* (2006:635) describe study habits as a series of acquired, consistent and effective study tendencies or methods. Study habit in mathematics context, therefore, refers to a particular manner of studying mathematics. Mathematics study habits include time management, willingness to do mathematics consistently, and the use of effective study methods such as note taking, attentiveness and understanding the task instructions (Maree *et al.*, 1997:8; Davidson & Lavitov, 1993:63). Based on the definitions of [study] **attitudes** (§2.2.1.1) and [study] **habits**, there seem to be a tacit suggestion that one's attitudes (thoughts and beliefs) about the learning of mathematics determines habitual mathematics study tendencies one is likely to employ. This assumption further suggests that a learner who has a negative outlook about the learning of mathematics is likely not to invest quality time in learning mathematics.

2.2.5.2. Impact of study habits on mathematics teaching and learning

Maree *et al.* (1997) have confirmed the relationship between study habits and study attitudes as speculated earlier in the definitions of the two concepts (*see* §2.2.51 and §2.2.1.1 respectively). Their study reveals a significant inter-correlation between study habits and attitudes (correlation coefficient = 0.733) and study habits and problem-

solving behaviour (correlation coefficient = 0.601) (Maree *et al.*, 1997:29). The correlation suggests that the negative impacts associated with attitudes on the learning of mathematics (§2.2.1.2) and mathematics problem-solving behaviour (§2.2.2.2) may also lead to the negative study habits regarding mathematics. Further, learners who exhibit poor study habits in mathematics have a tendency of not managing their time efficiently (Maree *et al.*, 1997:8). This is evident from their inability to promptly complete and submit the given mathematics tasks and generally keep up with the time demands in mathematics. Based on the correlation mathematics study habits have with mathematics study attitudes and mathematics problem-solving behaviour, a reasonable assumption could be made that poor study habits may lead to low achievement in mathematics.

2.2.5.3. Fostering positive mathematics study habits

Fostering mathematics study habits may not be an isolated endeavour because of its relation with study attitudes and problem-solving behaviour. The significant correlations of mathematics study habits with problem-solving behaviour and attitude towards mathematics (Maree *et al.*, 1997:29) suggest that fostering effective problem-solving behaviour and positive attitudes towards mathematics *inter alia* may improve learners' mathematics study habits. Further, the use of teaching approaches such as RME (for enhancing interest and motivation, and attaching value to mathematics), CGI (for enhancing confidence and perseverance when doing Math), PCI (for enhancing meaningfulness and interest in mathematics) and/or CL (for enhancing confidence and motivation) is presumed effective in changing learners' study habits. A proper integration and orchestration of all or some of the four aforementioned mathematics teaching approaches has a potential to improve learners' study attitudes towards mathematics and problem-solving behaviour, and subsequently study habits.

2.3. Mathematics learner-based teaching perspectives

There is an assortment of mathematics instructional perspectives, which inform mathematics instructional approaches. Only four mathematics instructional perspectives will constitute the focus of this study, namely: teaching and learning mathematics based on **realistic mathematics education philosophy (RME)** henceforth called **realistic mathematics instruction (RMI)**; **cognitively-guided instruction (CGI)**, **problem-centred instruction (PCI)** and **cooperative learning (CL)**. While the constructivist perspective has also been reflected upon, it will not be the main focus of this study. The primary purpose of reflecting on the constructivist perspective is to highlight its key tenets that influence the apparent bases with which **RMI**, **CGI** and **CL** are characterised especially in the context of mathematics problem-solving. The supposition is based on the following three types of constructivism respectively: **exogenous**, **endogenous** and **dialectical constructivism** (*see* §2.3.1).

2.3.1. Rationale of the chosen instructional perspectives

Reusser (2000:18) has documented the significant role played by the instructional approaches on the teaching and learning of mathematics. An outstanding assumption Reusser (2000:18) makes is that effective teaching methods “...should be considered a key factor in the prevention and remediation of a wide class of mathematical learning difficulties”.

It is therefore essential to justify the choice of RMI, CGI, PCI and CL as a focus of this study. There are fundamentally three propelling reasons for choosing the four instructional perspectives from the plethora of other perspectives:

- They exhibit underpinning characteristics that seem to suggest the potential to redress mathematics difficulties associated with the five learning barriers discussed in §2.2.
- While the instructional approaches to deliver mathematics curriculum are not clearly articulated in the National Curriculum Statement for mathematics (DoE, 2002), their relevance to serve the purpose of delivering mathematics curriculum efficiently is implied (DoE, 2002:4). Their implied traits can be traced in the *Definition, Purpose, and Unique features and scope* of mathematics curriculum (DoE, 2002:4).

- The philosophical perspective of OBE emphasises, *inter alia*, the potential learning benefits when learners work together to share ideas in order to develop a deep conceptual understanding of mathematics and achieve the pre-determined outcomes (Aldridge, Fraser & Sebela, 2004:245). The four aforementioned teaching approaches seem to exhibit suitable characteristics aligned to the OBE philosophy.
- They emphasise learner-centeredness, consequently they can be classified as ‘new’ mathematics teaching-learning perspectives *vis-a-vis* the ‘new-old’ picture of classroom dichotomy that was intended at distinguishing between pre- and post-1994 classroom practices in South Africa (Brodie & Pournara, 2005:31).

2.3.2. Constructivist perspective

The theory of origin of intellect or cognition was mainly explored by a cognitive psychologist, Piaget. Piaget’s theory “... has been called *interactionist* and *constructivist* [because] the organism inherits a genetic programme that gradually provides the biological equipment necessary for constructing a stable internal structure out of its experience with its environment” (Piaget, 1967). Constructivist perspective, therefore, departs from the point of view that learning takes place through personal construction of knowledge or conceptual structures (Nickson, 2000:4; Ernest, 2001:146). The role of constructivism in the teaching and learning of school mathematics emphasises the way in which learners construct their mathematical knowledge. The establishment of constructivism as a dominant theory or perspective of the learning of school mathematics was profoundly influenced by Piagetian cognitive psychology (Nickson, 2000:1; Ernest, 2001:146). The notion of personal construction of knowledge further suggests that knowledge construction starts from within the human cognition, hence endogenous constructivism (Harris & Graham, 1994:235).

Social-psychological perspective on learning (influenced largely by the Vygotskian Theory on acquisition of knowledge) introduced a new but extended version of the construction of knowledge. Vygotskian social-constructivism argues that the participation of an individual in shared social practices precedes the individual’s conceptual structures (Ernest, 2001:147). Neil (2001:553) presents Vygotsky’s assertion quite elaborately: “Vygotsky ... believed that

cognitive growth occurs through social interactions and conceived of the mind as a product of social life". In principle the two paradigms, namely **cognitive constructivism** and **social constructivism**, recognise the essence of construction of mathematical concepts and the presence of environment within the learning context.

Bryant (1995:132) compares Piaget's and Vygotsky's views about learning respectively as follows:

Children are born bereft of intellectual mechanisms but before they can interact and understand their environment, they have to acquire for **themselves** a set of **intellectual mechanisms** that will enable them to understand the environment around them.

versus

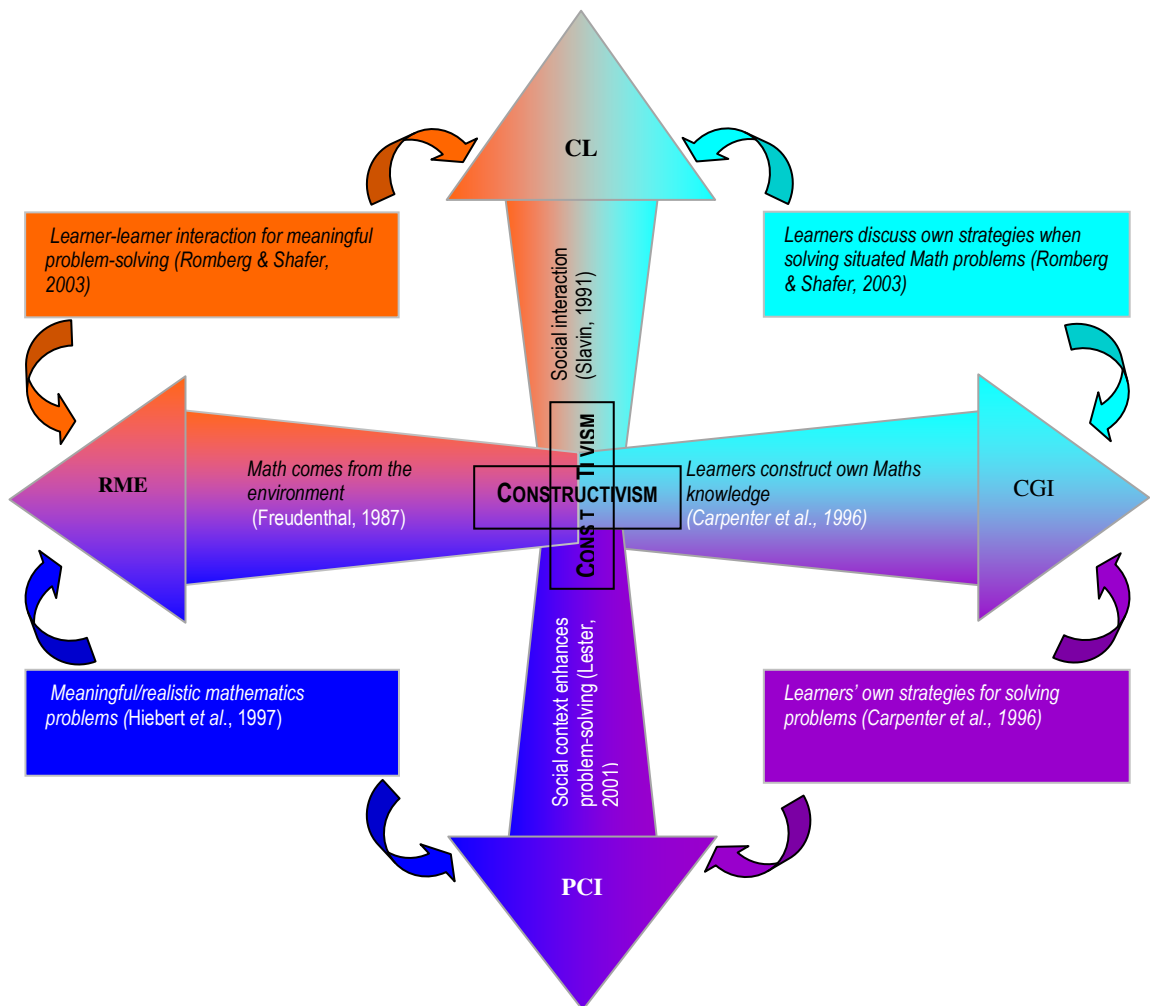
Children are born with considerable intellectual abilities but it is the **social environment** (intellectual mechanisms) with which the child interacts that enhances their intellectual development.

The difference between Piagetian and Vygotskian views seems to be located in the "**proximal locus**" of 'construction': the *former* asserts that construction of mathematical concepts originates from within the **learner's cognitive experiences** while the *latter* regards the **social experience** as the source of learner construction of mathematical concepts. Vygotsky (1978:87) explains the influence of **social experience** in terms of the zone of proximal development (ZPD) which essentially suggests that 'what a child can do with assistance today, s/he will be able to do alone tomorrow'.

Based on the aforementioned assertions, it would appear that constructivist perspective has profoundly influenced cooperative learning approaches whose essential elements are, among others, **individual accountability** and **social interactions** (more on this in §2.3.3). Further, it would appear that cognitively guided instruction (Fennema, Carpenter & Peterson, 1991), realistic mathematics education (Freudenthal, 1987) and problem centred instruction (Dolmans *et al.*, 2005) also have the traits of constructivism (*see* Figure 2.3). However, the following typologies of constructivist perspective provide essential information about the *sub-philosophies* that emanate from the evolution of constructivism.

Exogenous constructivism (rooted in the philosophy of *realism*) emphasizes the impact of the external environmental realities on knowledge construction by learners. One's mental structures reflect the organization of the environment or of the world. The reciprocal interaction with the environment plays a major role (Harris & Graham, 1994:234). Within mathematical context, exogenous constructivism can be exemplified by mathematical modelling in which learners engage in *horizontal* and *vertical mathematization* processes (Gravemeijer (1994:83), i.e. realistic mathematics activities or exercises that reflect the meaningfulness of the environment and translate them into mathematics.

Figure 2.3 Constructivist traits in mathematics teaching-learning approaches



Dialectical constructivism (also called *social constructivism*) regards knowledge construction as a characteristic of social intersection of people, interactions that involve sharing, comparing and debating among learners and mentors. Because of intense social interaction, the social environment plays a meaningful role as a platform on which learners give ideas to and solicit the same from other learners. Further, the social environment within which learning takes place enhances contextualization of learning. In essence, dialectical constructivism encourages cooperative learning and not individualized learning. The above assertions provide the notion that dialectical constructivism inculcates the exogenous and endogenous constructivist perspectives because of the recognition of the role of the environment and the learner's innate construction of knowledge respectively (Harris & Graham, 1994:236). Cooperative learning seems to find its pedigree from the dialectical constructivism, particularly due to the prevalence of the common distinguishing characteristic of social interactions and individual accountability. The perspective of cooperative learning is further explored in the next paragraphs.

Endogenous constructivism (also called *cognitive constructivism*) departs from the view that construction of knowledge starts from within the human cognition. The individual internal construction of knowledge is stimulated by internal cognitive conflict as the learner strives to resolve the mental disequilibrium, i.e. creating new understanding out of existing ones. The environment provides very little or no informational input for the construction of new knowledge (Harris & Graham, 1994:235).

2.3.3. Cooperative learning perspective

2.3.3.1. Definition

Cooperative learning gained its gradual popularity in early 1970s and it has since been one of the most often implemented learning approaches (Slavin 1989/90:52, 1999:74; Lazarowitz & Hertz- Lazarowitz, 2003:453). Cooperative learning is influenced by and is based on the theories such as behaviourism, social psychology, cognitive psychology and instructional theory (*see* Table 2.1).

Table 2.1 Theories informing Cooperative learning methods
(Lazarowitz & Herzt- Lazarowitz, 2003:453)

Theory	Method(s)	Main area of foci
Social psychology	Jigsaw; Expert jigsaw; Cooperative structure; Learning together.	Division of tasks; Specialisation; Interdependence; Individual and group products.
Behaviourism	Student Team Achievement Division (STAD); Team Assisted Instruction (TAI)	Enhancing motivation by rewards; Cooperation within teams; Competition between teams.
Cognitive psychology	Collaboration; Reciprocal teaching/tutoring	Non-structured interaction; Teacher's role as a resource person; Individual products; Scaffolding and construction of students learning.
Instructional psychology	Cooperative classroom contexts: six mirrors of the classroom	Contextualising six dimensions in the classroom, gradual implementation of cooperative learning across methods, coordinating individual, group and whole-class instruction.

There are various but complementing views from the repertoire of definitions of cooperative learning perspective. The following definitions, which epitomize such complementary trait, are informed by some of the theories alluded to in Table 2.2:

Cooperative learning is an instructional use of small heterogeneous groups of students who work together to maximize their own and each other's learning (Vaughan, 2002:359; Johnson, Johnson & Holubec, 1993:5).

Students working together in groups that are small enough so that each individual can contribute to a task that has been clearly defined and is intended to be dealt with collectively without the direct supervision of the teacher (Cohen, 1994 quoted by Watson and Chick, 2001:130).

Cooperative learning approaches are forms of active learning that engage students in working and learning together in small groups, typically with two to five members (Davidson, Reynolds & Rogers, 2001:1).

Cooperative learning refers to a set of instructional methods in which students work in small, mixed-ability learning groups. The groups usually have four members – one high

achiever, two average achievers, and one low achiever (Slavin, 1991:3; Artzt & Newman, 1991:14).

From these definitions it is evident that cooperative learning may not be equated with an ordinary group work. There are certain determining characteristics that will provide a measure of guarantee of effective cooperation. Some of these characteristics, derived from the afore-mentioned definitions, include: heterogeneity; group size; working together; active participation by individual members and learning goal/task. Primarily two determinants of successful cooperation are **group goal** and **individual accountability** (Slavin, 1991:6). According to Slavin, learners learn effectively in small cooperative groups if they know the prospective outcome and/or the potential benefit of the learning process and each takes equal responsibility of ensuring the attainment of a prospective outcome. However, other proponents of Cooperative learning perspective such as Johnson, Johnson & Holubec (1993:7) and Vaughan (2002:359) argue that without **positive interdependence** there is no cooperative learning. Johnson and Johnson (1994:66) have identified five essential determinants of effective cooperation, viz. **positive interdependence, individual accountability, face-to-face interaction, social skills** and **group processing**. The next paragraph provides an elaborate account of the impact associated with each element with regard to the teaching and learning of school mathematics.

2.3.3.2. Essential elements of Cooperative learning

Positive interdependence: Positive interdependence (Johnson, Johnson & Holubec, 1993:7; Vaughan, 2002:359) and individual accountability (Slavin, 1991:6) seem to account significantly for the attainment of cooperative goal structures. However,, positive interdependence seems to surpass the essentiality of individual accountability regarding the attainment of cooperative goals (Johnson, Johnson & Holubec, 1993:7; Vaughan, 2002:359) because the mathematical learning effort exerted by individual learners will determine the success of the group. This suggests that learners within a particular cooperative group depend on one another for a common course (Vaughan, 2002:359), which, in this context, is to achieve the group goal. The impact of positive interdependence in the learning of school mathematics can be demonstrated during group investigative or problem-solving activities.

Individual accountability: While cooperative learning emphasizes the importance of group work, each learner has to be accountable for his/her learning in order to avoid *free riding*. In essence cooperative learning aims at making each learner a very strong individual in terms of learning (Johnson & Johnson, 1999:71). Learners are likely to become accountable for their mathematical learning when they are individually committed to achieving the group goal and they are clear about their learning goal (Johnson, Johnson & Holubec, 1993:7). DoE (2007:3) supports this assertion by arguing that learners learn mathematics successfully when the intended learning outcomes are clearly presented before the teaching-learning process unfolds. Clarity of the learning goal enhances focused learning, and subsequently individual accountability.

Face-to-face interaction: The idea of face-to-face interaction is presumably having two dimensional perspectives: *Firstly*, during cooperative learning process learners should verbally assist, encourage, guide and support one another in their endeavour to learn the given task (Vaughan, 2004:359). In the process they orally explain, elaborate and argue about the given learning tasks in order to establish connections between present and previous knowledge. *Secondly*, the World Book Dictionary (1995) defines “*face-to-face*” as “*in direct contact*” which ideally has a connection with the seating pattern in cooperative learning context. It has to be borne in mind that the concept “*face-to-face*” qualifies the seating pattern of a cooperative small group i.e. learners have to face one another for *promotive* interaction. Therefore assistance, encouragement, guidance and support learners offer to one another (Vaughan, 2004:359) should be executed within a group setting where learners are in direct eye contact with one another and no other group setting.

Social skills: Johnson & Johnson (1994:68) strongly argue that by merely placing socially unskilled learners in a group does not guarantee that they will be able to cooperate effectively. The critical issue worth enquiring about would be what constitutes social skills. Leadership, decision-making, trust-building, communication, conflict management, listening, and tolerance are some of the common social skills required for effective cooperation to be attained (Johnson & Johnson, 1994:68).

Group processing: According to Vaughan (2002:359), group processing is an exercise by which learners reflect and evaluate the process of cooperative group work. Group processing occurs when members of a group discuss how well they are working, what group decisions are helpful and what adaptations are needed to maximise group efficiency. The characteristics of group processing resonate with the characteristics of meta-cognitive skills (e.g. monitoring and evaluation) and meta-cognitive beliefs (e.g. efficacy) as explained by Desoete, Roeyers and De Clercq (2004:55). Group processing has the following benefits (Johnson & Johnson, 1991:51):

- It enables learners to focus on maintaining good working relationships.
- It facilitates the earning of cooperative skills.
- It ensures that members receive feedback on their participation.
- It provides means to celebrate the success of the group and reinforce positive behaviour among the group members.

2.3.3.3. Mathematics learning benefits of cooperative learning

Since 1997 when the new curriculum was officially initiated, South Africa seemed to have declared socio-constructivist approach, particularly cooperative learning approach, a central approach to teaching and learning. This is evident in the fact that South African curriculum envisioned learners who can "... work effectively with others as members of teams, groups ..." (DoE, 2002b:11). In mathematics, the emphasis on cooperation has also been entrenched in the assessment standards as a policy obligation (DoE, 2002e). The entrenchment of cooperative learning is indicative of the potential academic, cognitive, psychological mathematics learning benefits it can offer.

Seemingly mathematics learning benefits emanating from the use of cooperative learning are quite diverse, thus the focus of the benefits will be restricted to **motivational, cognitive, academic achievement** and **psychological health** benefits. Each of these mathematical learning benefits of cooperative learning perspective influences the others; therefore their overlapping characteristic is indicative of such influence (Johnson & Johnson, 1994:69).

Motivational benefits: Motivational perspective of cooperative learning recognizes the impact of rewards or goal structures during the learning of mathematics (Slavin, 1995:16). Ideally there are three types of goal structures: *cooperative* – where individual goal oriented efforts contribute to others’ goal attainment; *competitive* reward - where individual goal oriented efforts frustrate others’ goal attainment; and *individualistic* - where individual goal oriented efforts have no effect on others’ goal attainment (Slavin, 1995:16). Research has revealed that cooperative goal structure has outperformed competitive and individual goal structures regarding the optimization of motivation to learn mathematics (Johnson & Johnson, 1994:69). The social aspect of the learning of mathematics, which is incorporated into Cooperative learning perspective, seems to have a motivational impact because the collective success of a group is a requisite for the individual goal attainment (Bennett, 1994:52).

Further, the study conducted by Vaughan (2002:363) in which the effects of cooperative learning on mathematics achievement and attitude (which correlates with motivation) were examined, revealed that learners derived substantial attitudinal benefits from cooperative learning. The study also revealed that learners of colour attained more attitudinal benefits than their white counterparts when exposed to cooperative learning instruction (Vaughan, 2002:363).

Cognitive benefits (referred to by Bennett (1994:51) as developmental benefits): If the learning of mathematics can be a human activity (DoE, 2002:4), it is argued that the learning of mathematics can be a social activity too (Davidson, Reynolds & Rogers, 2001:2) because human beings are inherently social beings. When learners learn mathematics together in small cooperative groups, they are afforded the opportunity to use informal language that is readily understood by their peers to explain mathematical concepts especially during the problem-solving process. Cooperative learning affords learners to operate within one another’s zone proximal of development, which is defined as “the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers” (Vygotsky (1978) cited by Kutnick and Rogers, 1994:17). The freedom to express their mathematical ideas among themselves enables learners to use variety of approaches, and in the process they seem to feel free to ask their peers questions

than they would not ask of their teacher. Cognitive benefits of cooperative learning correlate with the **dialectical constructivist view** discussed earlier (*see* §2.3.2).

Academic achievement: According to Slavin (1995:95), reviewers of cooperative learning literature agree that cooperative learning improves academic achievement significantly in mathematics, especially with regard to mathematics computations, concepts and problem-solving. Cooperative learning setting has been positively associated with promotion of greater achievement than competitive and individualized learning setting (Kutnick & Rogers, 1994:4). The study conducted by Vaughan (2002:362) in which the effects of cooperative learning on mathematics achievement and attitudes were examined, also revealed a statistically significant difference between the pre-test and the post-test. Further, the study revealed that learners acknowledged and supported cooperative learning as a preferred learning approach in mathematics. Kramarski, Mevarech and Arami (2002:226) have investigated the effects of cooperative meta-cognitive instruction on mathematics problem-solving. They have found that while cooperative learning has the potential to improve mathematical achievement because it provides a natural setting for learners to supply explanation and elaborate their reasoning, mathematics achievement is even more enhanced when meta-cognition is embedded in cooperative learning approach. By implication, the combination of cooperative learning and cognitively guided instruction can enhance mathematics achievement because the latter is characterised by meta-cognitive skills (*see* §2.3.4.2).

Psychological health benefits: In chapter 1 (*see* §3) reference was made to the entrenchment of the Inclusive Education model which encourages educational mainstreaming (DoE, 2003a:7) in the current South African curriculum. Mainstreaming is a corrective measure for the psychological damage caused by isolating learners with learning barriers from the so-called “normal” learners and providing them with specialised education aimed at enhancing academic growth, and emotional and social development (Slavin, 1995:54). For mainstreaming to fulfil its potential to improve learners’ psychological health and academic growth, more specifically in mathematics classes, there is a need for relevant instructional methods (Slavin, 1995:55).

Cooperative learning is an appropriate method to promote inclusion philosophy because of its advocacy for heterogeneity in a mathematics classroom (Augustine, Gruber and Hanson, 1991:46; Slavin, 1991:87). There is documented evidence regarding the improvement of psychological health among learners who experience a variety of learning barriers in mathematics such as attitudes, anxiety, low self-esteem, poor study habits, low achievers and gifted learners (Augustine, Gruber and Hanson, 1991:47).

2.3.3.4. Potential limitations of cooperative learning perspective

Like many teaching and learning perspective, cooperative learning is not panacea, as a result it is characterised by certain limitations that are summarised below.

- *'Free rider'* effect – where some group members do the bulk of the work while others do not contribute anything (Slavin, 1995:19).
- Teachers' proficiency constraints regarding cooperative group dynamics in mathematics class may adversely affect the learners' effective cooperation (Kutnick & Rogers, 1994:7). The knowledge of the elements of cooperative learning and the skills of forming and managing cooperative groups (specifically considering group size and heterogeneity) remain the critical requisites of effective cooperative learning.
- Teachers are prone to eliminating or suppressing learners' academic controversies or intellectual disagreements that are likely to erupt from cooperative groups, subsequently depriving learners of the memorable learning experiences (Johnson & Johnson, 1994:71). The importance of the memorable learning experience emanating from academic controversies is seen against the background that argumentation and consensus are some of the social skills associated with cooperative learning.
- Large cooperative group size (arguably seven and more) has the potential to compromise face-to-face interaction and subsequently lead to the emergence of sub-groups (Jaques & Salmon, 2007:10).

2.3.4. Cognitively Guided Instruction

2.3.4.1 Definition

Cognitively Guided Instruction, henceforth referred to as CGI, focuses primarily on the development of learners' mathematical thinking processes (Franke & Kazemi, 2001:102). During the teaching of mathematics using CGI, the focus is on what the learners think and do in the learning process and the primary role of the teacher is to facilitate the learners' active mental involvement (Fennema, Carpenter and Peterson, 1991:27). It is quite appropriate, therefore, to define CGI as the instruction that "...focuses on facilitating the child's thinking and learning in mathematics by building instruction on what the learners already know" (Fennema *et al.* (1991:31).

2.3.4.2 Essential elements of CGI perspective

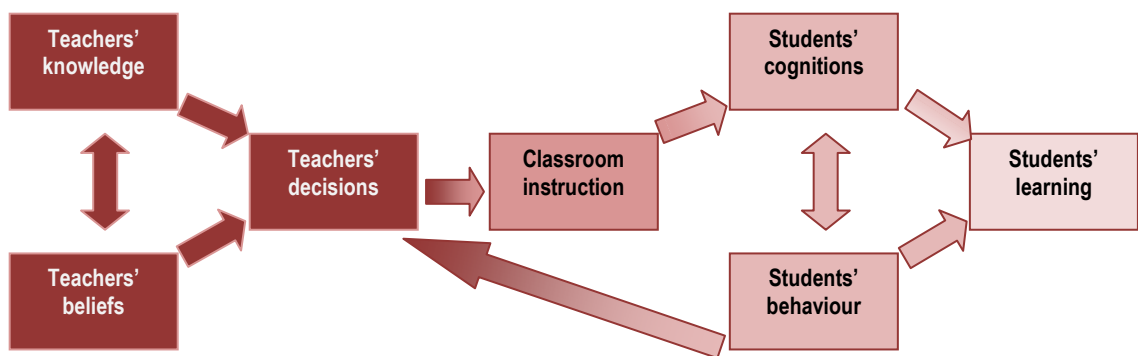
Carpenter, Fennema and Franke (1996:3) argue that while CGI emphasizes the development of learners' mathematical thinking processes, it is the knowledge and beliefs possessed by the teacher with regard to the learners' mathematical thinking processes that is paramount. Franke and Kazemi (2001:102) confirm the assertion in their study whose primary aim is to consolidate the development of learners' mathematical thinking and the teaching of mathematics.

For instance, if mathematics teachers believe that mathematics learning is mainly about learning facts and procedures quickly and efficiently, or that only certain students need and can learn mathematics, or that the learning of mathematics is genetically motivated (Hiebert *et al.*, 1997:xiv), then the development of mathematical understanding will be compromised.

The study conducted by Fennema, Carpenter and Peterson (1991:31) which explores the hypothesis about how teachers' knowledge and beliefs influence learning, has resulted in the synoptic version of the components of CGI presented in figure 2.4. Fennema, Carpenter and Peterson (1991:31) have categorized **pedagogical content knowledge** into the knowledge of: ways of representing and explaining mathematics to

make it comprehensible; and learners' mathematical thinking, especially with regard to conceptions, misconceptions and preconceptions which normally pose a hindrance to the effective learning of mathematics. Sherin (2002:121) concurs by arguing that, in addition to understanding the relevant mathematical facts and concepts teachers need to understand how to teach a particular mathematics topic. If mathematics teachers possess this knowledge, they are likely to structure their mathematics lessons and assessment in a manner that will encourage learners to freely and openly express their mathematical thoughts.

Figure 2.4 Components of CGI model (Fennema *et al.*, 1991)



Connected to the content knowledge are the **teachers' beliefs** about learners' mathematical learning processes. Fennema, Carpenter and Peterson (1991:31) argue that CGI teachers should believe that:

- learners construct their own mathematical knowledge. This view is shared by socio-constructivists (*see* §2.3.1.2) and proponents of Cooperative learning perspective (*see* §2.3.2.3);
- instruction should facilitate learners' construction of mathematical knowledge. The same view is implicit in constructivist philosophy (Ernest, 2001:147);
- mathematical skills should be taught in relation to understanding and problem-solving; and
- learners' natural development of mathematical ideas should provide the basis for sequencing mathematics topics for instruction.

The teacher's pedagogical content knowledge (mathematical content and learners' mathematical thinking processes) and beliefs influence to a large extent the teacher's decisions regarding the nature of the **teaching practices** to be employed in the classroom (Franke & Kazemi, 2001:103). CGI enhances self-reliance, confidence in learning mathematics and the belief that they (learners) are responsible for their own mathematics learning (Fennema, Carpenter & Peterson, 1991:32). These are the characteristics of self-regulated learning and metacognition.

2.3.4.3 Mathematics learning benefits of CGI

Franke & Kazemi (2001:104) have conducted a study whose primary goal was to share with teachers their research-based knowledge about the development of learners' mathematical thinking and establish the impact that knowledge would have on learners' learning of mathematics. The following findings were revealed:

- Grade 1 learners could solve word problems normally reserved for grade 3 learners.
- During the process of solving word problems, learners used a variety of strategies.
- Instead of providing answers to their learners, teachers learnt the skills of eliciting learners' mathematical explanations in order to understand their cognitive processes. Listening to the learners provided teachers with an added benefit of engaging in practical enquiry i.e. learning how learners learn mathematics.
- CGI learners reported being more confident and better able to understand mathematics. Confidence has been credited for its potential to mitigate mathematics anxiety (*see* §2.2.3.4).

2.3.4.4 Potential limitations of CGI perspective

Research on the impact of teachers' knowledge of learners' cognitive processes on mathematics learning appears to have enhanced the effective teaching and learning of mathematics in recent years. However, it has not provided clear shortcomings of CGI with regard to the teaching and learning of mathematics.

The potential limitations of CGI enlisted in the next paragraph are, therefore, based on the researchers' personal account, hence anecdotal account of potential limitations of CGI. Nonetheless, these potential limitations of CGI, however anecdotal, are implied in the documented empirical studies and are the result of the critique thereof.

- There appears to be consensus that CGI is centred on the knowledge possessed by teachers regarding the learners' cognitive processes as they learn mathematics (Fennema, Carpenter & Peterson, 1991:31; Carpenter, Fennema & Franke, 1996:5; Nieuwoudt, 2000:35; Franke & Kazemi, 2001:102; Koehler & Grouws, 1992:119). The prerequisite for effective use of CGI is that the teacher must know how learners learn mathematics i.e. in terms of thinking processes. The primary focus is therefore on the mathematics teacher and later on the learner. If this prerequisite may be missed, CGI is likely not to yield the expected results.
- Fennema *et al.* (1991:32) have listed the teachers' beliefs in four continua that reflect opposite belief positions (*see* Table 2.2). They further purport that the teachers who hold beliefs that are situated closer to the left of each continuum are compatible with CGI perspective while those who hold the belief that are situated towards the right of each continuum "are more traditional". This suggests, therefore, that CGI is tailor-made for teachers whose beliefs are situated closer to the left of each continuum and not the so-called traditional mathematics teachers.

Table 2.2 Continua of teacher's beliefs (Fennema et al., 1991:32)

	<i>Left of the continuum</i>	<i>Right of the continuum</i>
Continuum 1	Children construct their own knowledge	Children receive knowledge
Continuum 2	Skills should be taught in relationship to understanding and problem-solving	Skills should be taught in isolation
Continuum 3	Within a given culture, learners' natural development of mathematical ideas should provide the basis for sequencing topics for instruction	Formal mathematics should provide the basis for sequencing topics for instruction
Continuum 4	Instruction should facilitate learners' construction of knowledge	Teachers should present knowledge

2.3.5. Problem-centred instruction

2.3.5.1 Definition

Prior to a focus on the actual definition of problem centred approach, there seems to be a need to clarify a possible confusion that may prevail as a result of other different names given to this approach. For instance, **problem-centred approach (PCA)** (Hiebert *et al.*, 1997:115; Nieuwoudt, 2000:25; Ridlon, 2009:188); **problem-based learning (PBL)** (Dolmans *et al.*, 2005:732); **problem-centred learning (PCL)** (Ridlon, 2009:188) and **problem-centred instruction (PCI)** (Lubienski, 1999). Ideally all these different names refer to the same perspective of mathematics teaching and learning. What seems to be different is the point of reference i.e. some scholars name it from the point of reference of learners e.g. PBL and PCL, while others name it from instructional point of view which includes both teaching and learning e.g. PCA and PCI. Problem-centred instruction (PCI) has been adopted in this study for the purpose of consistency and is used to refer to any of the above-mentioned views.

Hiebert *et al.* (1997:115) define problem-centred approach as a teaching-learning approach in which learners are presented with meaningful and interesting mathematical problems that cannot be solved using routine procedures or drilled responses. Nieuwoudt (2000:25) also emphasises the meaningful nature of mathematical problems as a hallmark for effective teaching and learning of mathematics when using problem centred approach. Trafton, Midgett and Joyner (2001) define PCI as an approach to teaching and learning mathematics using interesting and well-selected problems that will enhance the emergence of new ideas, techniques and mathematical relationships. The other essential component of problem-centred approach is that the teacher does not prescribe the method to be used. However, learners have to employ their own strategies without compromising the mathematical logic when they discuss, critique, explain and justify their solutions (Hiebert *et al.*, 1997:115).

2.3.5.2 Essential elements of PCI

The following features characterise PCI (Hiebert *et al.*, 1997:125): the nature of the task, the role of the teacher and the social culture of the classroom.

The nature of the task: The task has to possess a degree of meaningfulness and mathematical value. Maree *et al.* (1997:9) argue that names and life styles in word problems that do not come from the learner's field of experience confuse learners and negatively affect their (learners') performance in mathematics. In other words, the abstract nature of mathematics, and badly designed mathematics tasks have a potential to confuse learners (Chinn, 2004:107). The tools of solving the given task must therefore be within learners reach and learners have to possess some skills with which they can solve the task.

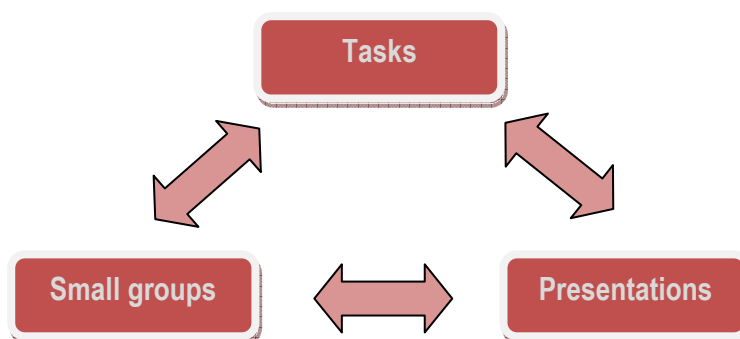
The role of the teacher: The selection of mathematical problems or tasks by the teacher should primarily be informed by the learners' level of cognitive development. Chinn (2004:107) concurs by arguing that the curriculum that does not take into account the range of learners to whom it is targeted is also likely to promote mathematics anxiety among the learners. Teachers should be able to sequence mathematical tasks in order to afford learners the opportunity to construct mathematical relationships that built on each other over time. The teacher' role is dependent upon the intended learning outcome (Lubienski, 1999). If the learning goal is to enable learners to learn *about* problem-solving, the teacher's role will be to focus on specific problem-solving strategies and skills. If the learning goal is to enable learners to learn mathematics *for* problem-solving, the teacher's role is to select key mathematical ideas and skills that learners can apply in problem situations. Further, if the learning outcome is to enable learners to learn *through* problem-solving, the teacher's role will essentially be to carefully select and analyse problems (with plenty mathematics ideas and concepts) through which learners can learn.

The social culture of the classroom: Nieuwoudt (2000:25) recognises the social-constructivist as a pedigree of PCI. The social interaction enables learners to learn mathematics through their own and other learners' construction of mathematical knowledge (Hiebert *et al.*, 1997:126). Cooperative group work has a potential to

stimulate learners interactions that have a positive effect on learning such as discussions, explanations, posing and answering questions related to a variety of mathematical concepts (Dolmans *et al.*, 2005:734).

Ridlon (2009:195) presents a slightly modified version of PCI where the role of the teacher is viewed as over-arching in terms of selecting the appropriate tasks, engaging learners in social interactions (small group discussions) and facilitating the presentations by learners (*see* Figure 2.5).

Figure 2.5 Problem-centred learning model (Ridlon, 2009:195)



2.3.5.3 Mathematics learning benefits of PCI

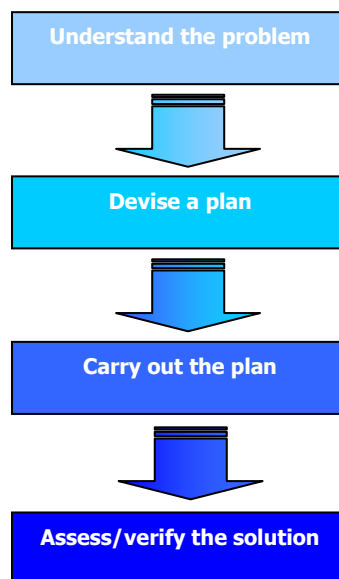
One of the unique features of the teaching and learning of mathematics in South Africa is problem-solving (DoE, 2002:5). Mathematics problem-solving skills gained from this unique feature afford learners the ability to:

- make sense of the problem especially if the problem is informed by real life experiences of learners;
- analyse and synthesise the problem i.e. respectively view the problem from its individual components and consolidate the components in order to view it from the whole;
- determine and execute the appropriate problem-solving strategy which is mathematically valid; and
- validate and interpret the solutions appropriate to the context.

Critical analyses of the mathematics learning benefits of PCI as outlined by DoE (2002:5) show an alignment with the two main strategies of problem-solving: Polya's linear problem-solving framework (Polya, 1945:6) and cyclic problem-solving framework (Fernandez, Hadaway & Wilson, 1994:196). The linear framework of problem-solving depicts problem-solving as a linear process involving a series of steps to be completed to arrive at a correct answer (Fernandez, Hadaway & Wilson, 1994:196) (*see* Figure 2.6).

The linear framework of problem-solving seems to imply that as a learner gets involved in problem-solving activity, there is no likelihood of moving back and forth within any two steps in order to validate the next step. For instance, it seems to suggest that making an appropriate plan implies proper understanding of the problem while disregarding the idea that proper understanding of the problem also implies making an appropriate plan.

Figure 2.6 Linear problem-solving framework

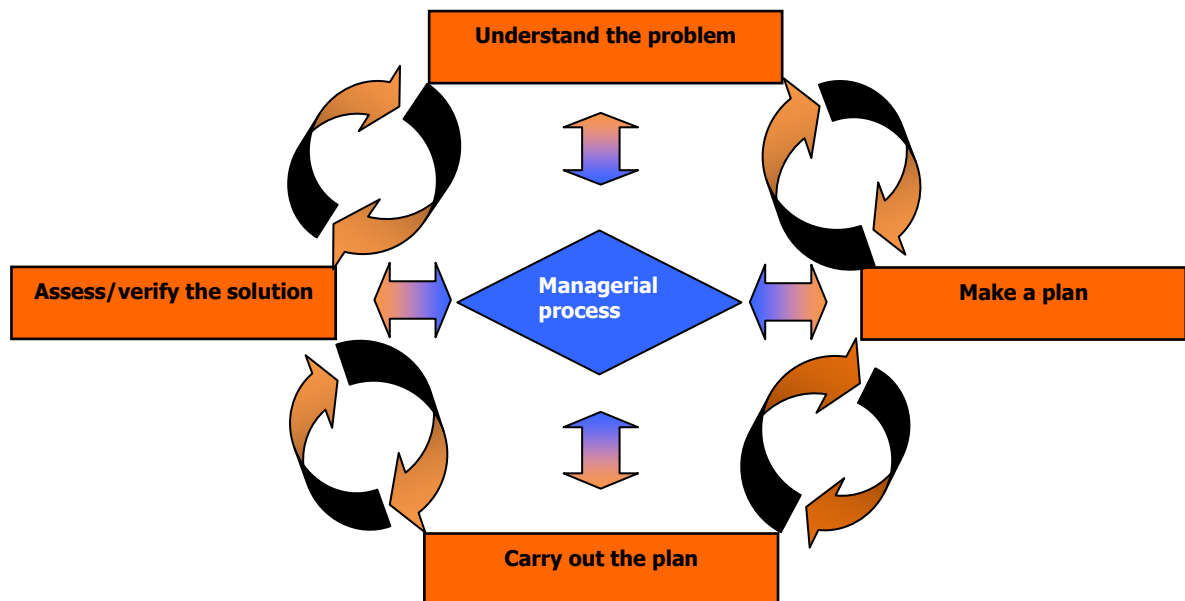


The cyclic framework (*see* Figure 2.7) offers a more elaborate account of the formulations of problem-solving framework for two reasons: it recognises the idea that one is likely to move back and forth between any two steps and cyclic within all the steps, and it captures the managerial processes of self-monitoring, self-regulation and

self-assessment (collectively constituting meta-cognitive skills) (Fernandez, Hadaway & Wilson, 1994:196). Synoptically, the cyclic framework suggests that as one moves back and forth between the steps and within the framework there are numerous meta-cognitive skills implicit in that movement.

The aforementioned benefits associated with PCI emanate also from teaching *about*, *for* and *via* problem-solving (Campbell & Bamberger, 1993:3). *Firstly*, teaching *about* problem-solving focuses primarily on the instruction that equips learners with the strategies/guidelines to solve mathematical problems. *Secondly*, teaching *for* problem-solving implies the application of mathematical concepts and skills in real-life contexts. *Thirdly*, teaching *via* problem-solving refers to the use of a problem as a means of learning new mathematical ideas and for connecting new and already constructed mathematical ideas.

Figure 2.7 Dynamic and cyclic problem-solving framework
(Fernandez, Hadaway & Wilson, 1994:196)



Campbell & Bamberger (1993:4) further emphasise that solving carefully constructed problems that embody essential aspects of mathematics content is a path through which learners learn mathematics concepts, connect mathematical ideas and develop mathematical skills. This assertion is in line with the features of PCI identified by Hiebert *et al.* (1997:125) (*see* §2.3.4.1).

Ruggiero (1998:19) purports that PCI has the potential to mitigate negative attitudes learners possess regarding mathematics. Ruggiero (1998) asserts that learners' attitudes are likely to change positively when they (learners) are allowed to grapple with carefully chosen mathematical problems instead of being told how to solve them (mathematical problems). It is assumed that CGI may also impact positively in mitigating the problem associated with attitudes in the learning of school mathematics due to the similar characteristic it shares with PCI, namely allowing learners to use their own strategies to solve mathematics problems.

2.3.5.4 Potential limitations of PCI

- PCI emphasises learners' personal construction of mathematical knowledge using own strategies during the process of problem-solving (Nieuwoudt, 2000:25). However, in the process of teaching and learning, mathematics teachers may be tempted to prescribe the use of a particular approach especially when learners struggle in search of “mathematically valid” strategies (Hiebert *et al.*, 1997:164). Dolmans *et al.* (2005:735) have also noted that too prescriptive, directive and/or dominant teachers are a possible impediment on effective PCI implementation.
- PCI discourages continual interruptions and interference by the teacher when learners engage in problem-solving activity because constructing mathematical knowledge and reaching true understanding are personal processes that are sensitive to interruptions (Hiebert *et al.*, 1997:125). However, even the teachers who ascribe the success of PCI to this philosophy often tend to do the opposite (Hiebert *et al.*, 1997:125).
- The real-life context advocated in “teaching *for* problem-solving” by Campbell & Bamberger (1993:3) still remains a huge concern in the teaching of mathematics – learners' life experiences are virtually different because of different backgrounds. Holmes (1995:457) agrees that rural and urban learners have different life experiences; nonetheless they both come to school with a repertoire of knowledge that is useful in subsequent mathematics learning.

2.3.6. Realistic mathematics education perspective

2.3.6.1 Definition

Before focusing on the actual definition of realistic mathematics education perspective, it will presumably be helpful to clarify the potential confusion that may emanate from RME perspective and RME teaching methods. RME teaching methods are not clearly articulated; nonetheless they seem to be implied in the RME philosophy. For instance, Van den Heuvel-Panhuizen (1996:11) emphasises that the process of mathematization (horizontal and vertical) is one of the significant features that characterises RME teaching methods. Therefore, in this study the use of the concept **realistic mathematics instruction (RMI)** is adopted to denote mathematics instruction based on RME philosophy.

The most appropriate way of extracting the fundamentals of RMI from RME is supposedly to differentiate between **RME philosophy** and the **information processing** approach. **Information processing approach** views mathematics as a ready-made system with general applicability, and during mathematics instruction these formal ready-made systems are merely broken down into learning procedures for application (Gravemeijer, 1994:91). Learners are therefore not afforded the opportunity to apply informal procedures to solve mathematical problems. They operate within the parameters of certain predetermined procedures where particular algorithms are used. **RME** departs from the view that learning mathematics is doing mathematics – hence a human activity (Gravemeijer, 1994:91). The key issues defining RME are therefore realistic contexts and mathematizing. In short, the effective learning of mathematics is underpinned by the **translation of real life problems into mathematical problems** and **using informal procedures** (as a starting point) to solve the problems (Gravemeijer, 1994:91) without compromising the mathematical validity. It is therefore assumed that the '*operationalisation*' of the RME philosophy (underpinned by its essential elements in §2.3.6.2) translates to RMI rationally because the RME philosophy ought to dictate the classroom practice (RMI).

2.3.6.2 Essential elements of RME (RMI)

Gravemeijer (1994:90) identifies the essential elements of RME as *Guided reinvention and progressive mathematization*, *Didactical phenomenology* and *Learners' self-developed models*. Van den Heuvel-Panhuizen (1996:11) refers to them as *Levels of understanding*, *Link to reality and the focus on application*, and *Students' own activities and contributions* respectively. The next paragraphs provide an abridged but detailed account of the three elements according to Gravemeijer (1994).

Guided reinvention and progressive mathematizing: Primarily this principle refers to the learning of mathematics through the process similar to the process by which mathematics was invented. mathematics was invented by initially using informal procedures to solve problems and later progressing towards more formal procedures of problem-solving (Gravemeijer, 1994:90). Through progressive mathematization learners are able to translate real-life non-mathematical or mathematically under-developed problems into mathematical problems (horizontal mathematization), and employ mathematically valid (however informal) procedures to solve such problems (vertical mathematization) and translating it back to its original real life context (horizontal mathematization) (Gravemeijer, 1994:91; Nieuwoudt, 2000:33; Van den Heuvel-Panhuizen, 1996:10).

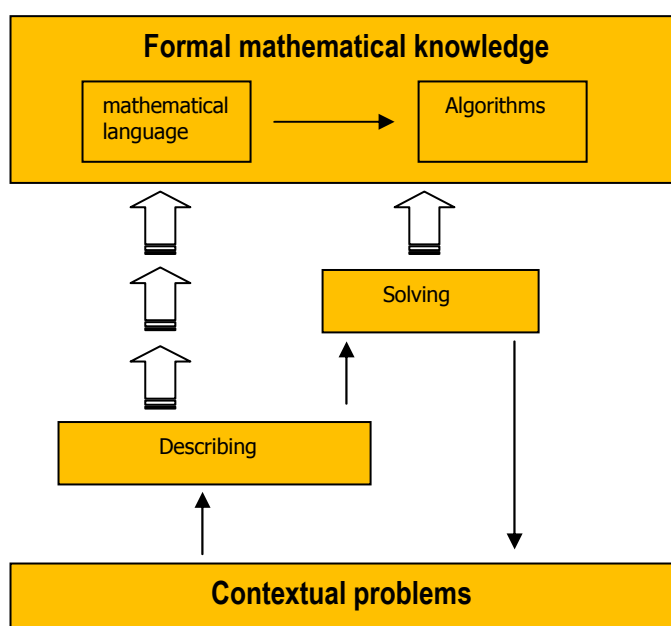
By implication, horizontal mathematization involves moving from the *world of life* into the *world of symbols* while vertical mathematization involves moving *within the world of symbols*, within which mathematical reinvention takes place. Figure 2.8 provides a synoptic version of mathematization and reinvention according to Gravemeijer (1994:94).

Didactical phenomenology: This element focuses primarily on the need for the investigation of the contexts within which the given mathematical topic is to be applied. The aim is to establish and analyse the kind of applications to be anticipated in the instruction and the suitability of such contexts to enhance effective progressive mathematization. Van den Heuvel-Panhuizen (1996:12) concurs by emphasising that during instruction one must start with the identification of rich contexts that demand

mathematical organization (i.e. contexts that can be mathematized) and establish whether the context will contribute to putting this mathematization into practice.

The main point is to determine which phenomena contributes to particular mathematical concepts and whether these phenomena are accessible to students for the teaching and learning of mathematics (Van den Heuvel-Panhuizen, 1996:12) – hence didactical phenomenology.

Figure 2.8 **Mathematization and reinvention** (Gravemeijer, 1994:94)



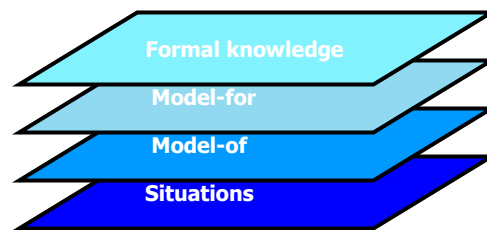
Self-developed models: The use of the word model in realistic mathematics context refers to mathematical manipulatives, diagrams, situation, a scheme, a description or a way of noting (Gravemeijer, 1994:101). The informal situated knowledge possessed by the learner is recognised and they (learners) are encouraged to model it – in fact models are inspired by informal strategies in realistic mathematics context (Gravemeijer, 1994:101). The following four levels of mathematics modelling are distinguished:

- Situations – use of situational knowledge and strategies to solve the problem.
- Model *of* – modelling the situation using paper and pencil.
- Model *for* – mathematical focus dominates the reference to context.

- Formal mathematics – the focus is on working with conventional mathematics procedures.

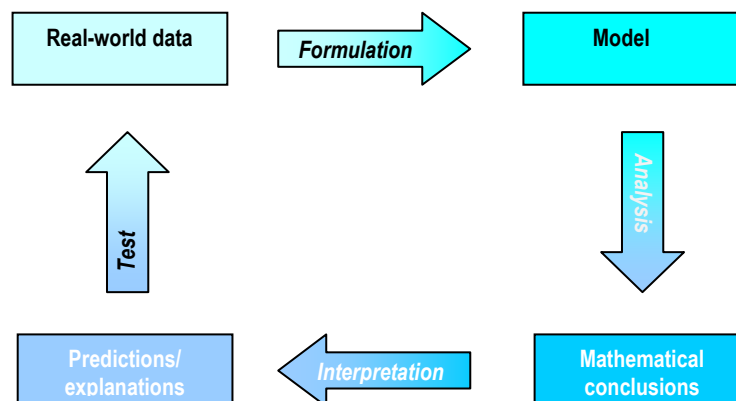
Contrary to the information processing of mathematics teaching and learning approach (*see* §2.3.5.1), a realistic approach has adopted the bottom-up approach regarding modelling – learners construct models themselves based on the real-life situation (they model their own understanding of the problem) in order to develop formal mathematical knowledge (Gravemeijer, 1994:100). However, the mathematics modelling process suggested by Gravemeijer (1994:101) appears to be linear (*see* Figure 2.9) and as a result does not make provision for the validation of the model and modification thereof if needed.

Figure 2.9 Levels of mathematics modelling (Gravemeijer, 1994:101)



Giordano *et al.* (2009:1) provide a more comprehensive and dynamic modelling process (*see* Figure 2.10) whose cyclic structure allows for predicted mathematical interpretations to be tested in real world context. The cyclic modelling process therefore makes provision for validation of the mathematical conclusions.

Figure 2.10 Modelling process (Giordano *et al.*, 2009:1)



2.3.6.3 Mathematics benefits of RMI

- According to Gravemeijer (1994:83), Mathematization is an essential feature of realistic mathematics instruction for two reasons: *firstly*, it enables learners the opportunity to apply mathematics in solving real-life problems. Consequently learners view mathematics as a subject that is not rigid and confined to the classroom. *Secondly*, learners are encouraged to become mathematics re-inventors because they start by informally solving real-life problems without undermining mathematical validity, and later formalising their findings by way of ‘axiomatising’. This is the process by which mathematics was invented. In short, learners are encouraged to use their own strategies to solve real-life problems and later formalise them to become mathematically sound.
- The idea of level-raising, which forms the basis of progressive mathematization, enables learners to progress swiftly from one level of mathematical proficiency to the next (Gravemeijer, 1994:83). The lower level of mathematical proficiency is normally a prerequisite for the envisaged mathematical proficiency in the next level (Van den Heuvel-Panhuizen, 1996:12).

The notion of level-raising as defined by Van den Heuvel-Panhuizen (1996) and Gravemeijer (1994) is informed by the Van Hiele theory of mathematical development (Davey & Holliday, 1992:26; Presmeg, 1991:9) which postulates that the lower level of mathematical development or proficiency later becomes the object of analysis on a higher level (Nieuwoudt, 2000:35).

- Barnes (2005:52) has documented a theoretical account of the potential RMI benefits to improve learner attainment. Barnes (2005:52) cites the following global trends in mathematics education as the main rationale for using RMI theory to teach ‘*low attainers*’ in mathematics: the **arithmetical trend** in which puzzles and games are used to learn basic operations; **structural trend** in which number lines, grids and diagrams

are used to teach basic operations and enhancement of mathematical reasoning; and the **empirical trend** in which the teaching of mathematics is connected to actuality.

2.3.6.4 Potential limitations of RMI

Realistic mathematics instruction, as the name implies, emphasises the use of real-life contexts as a basis for the teaching and learning of mathematics. However, critics of real-life contexts argue that the phenomenon 'real-life context' is relative and subjective. It is based on the life experiences of individuals; as a result one learner's real-life context may not necessarily be another learner's context. It seems to be a common practice also that mathematics teachers normally use the contexts with which they (teachers) are familiar while it may not be the case with learners. It is therefore against this background that Gravemeijer (1994:90) emphasizes the importance of didactical phenomenology according to which the real-life context must be carefully and thoroughly investigated and analyzed to enhance progressive mathematization.

2.4. Synergies between the teaching approaches

In an endeavour to demonstrate the synergies between the four instructional approaches that inform the GIST model of this study, it would probably be appropriate to depart from the **zone of proximal development (ZPD)** delineated by Vygotsky (1978:86) as:

... the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers.

A thorough probing of Vygotsky's ZPD sheds some light on the collective effectiveness of problem-centred instruction, cooperative learning and cognitively-guided instruction. The definition of ZPD implies that learners' cognitive level of problem-solving increases when they work with other learners (peer interaction) than when they work as individuals. It is therefore assumed that the teacher's main role is to ensure the prevalence of the environment that will maximise the learner's potential in the learning of school mathematics. A proper orchestration of **cognitively-guided instruct, cooperative**

learning and **problem-centred instruction** in which mathematics is taught and learnt within a **realistic context** is likely to enhance achievement.

Johnson and Johnson (1990:108) also demonstrate the synergy between problem-solving and cooperation among learners by arguing that ‘mathematics problem-solving is an interpersonal enterprise’. It would perhaps be essentially appropriate to suggest that mathematics may be taught effectively to maximise learning when teachers efficiently navigate from one teaching-learning approach to the other within the same mathematics lesson. Synoptically, when learners are engaged in the process of solving realistic problems (traits of PCI and RMI) in the context of group work (trait of cooperative learning), the teacher should establish how they (learners) construct their own mathematical knowledge (trait of CGI).

The synergies depicted in this section suggest that the teaching of mathematics can be enhanced by navigating between the teaching approaches. Further, the synergies suggest that no particular teaching approach is a panacea. The nature of the afore-mentioned mathematics teaching perspectives together with their potential mathematics learning benefits documented in the earlier paragraphs can, hypothetically, assist to mitigate the impact of mathematics learning barriers. However, it is further assumed that the requisite orchestration of the four afore-mentioned mathematics teaching approaches should be appropriately carried out.

2.5. Apparent interplay between the teaching approaches and the learning barriers

The preceding paragraphs have offered in-depth analyses of some of the **learning barriers** that pose potential difficulties for the learning of mathematics, and the **instructional perspectives** presumed to enhance effective teaching and learning of school mathematics and mitigate the intensity of the learning barriers. In some instances the symptoms and causes of the learning barriers were explored, and also the characteristics, benefits and shortcomings of mathematics instructional perspectives were critiqued. It is therefore against this background that the synoptic account of the presumed relevance of **cooperative learning, problem centred instruction, cognitively guided instruction** and **realistic mathematics instruction** is presented in Table 2.3. In other words, the researcher aims to elicit the direct connection between the characteristics of the four instructional approaches in mathematics and the symptoms of the five learning barriers dominantly prevalent in mathematics classes. Mathematics

learning barriers are presented on the left and instructional perspectives are on the right (differentiated by the two colours) of Table 2.3.

Table 2.3 Presumed connections between mathematics barriers and teaching approaches

MATHEMATICS LEARNING BARRIERS			INSTRUCTIONAL PERSPECTIVES	
<i>Nature of mathematics difficulty</i>	<i>Symptoms</i>	<i>Inter-correlation with other mathematic difficulties</i>	<i>Presumed appropriate instructional perspective</i>	<i>Rationale and relevance</i>
Negative attitudes about mathematics	Low self-confidence; uninterested in mathematics; disliking mathematics; unmotivated to learn mathematics; lack of perseverance; attaches no value in mathematics	Study habits; Problem-solving behaviour	Cooperative learning	Positive interdependence and social skills enhance self-confidence and motivation.
			Cognitively guided instruction	Enhances confidence and perseverance when doing mathematics.
			Realistic mathematics instruction	Enhances interest and attaches value in mathematics due to <i>mathematization</i> process used in solving real-life problems
Poor problem-solving behaviour	In ability to: self-monitor, self-evaluate, self-regulate and plan; Uninformed decision making	Study habits Study attitudes	Problem centred approach	Enables learners to acquire and use meta-cognitive skills, and make informed decisions.
			Cognitively guided instruction	Enables learners to construct their own mathematical knowledge and use variety of strategies in problem-solving.
mathematics anxiety	Muscle tension; perspiration; clammy hands and feet; rapid heartbeat; low self-esteem; nail biting; poor meta-cognitive skills	Study milieu	Cooperative learning	Small group setting provides a relaxed and stimulating learning environment to counter fear and hostile milieu
			Problem centred approach	Emphasis on meaningful and interesting mathematics reduces fear associated with abstract and uninteresting mathematics problems
Poor study milieu	Problems with reading, hearing, seeing due to high noise level and improper lighting in the study environment; inability to identify with fairly common contexts used in word problems due to lack of exposure	Mathematics anxiety	Cooperative learning	Small group setting provides a relaxed and stimulating learning environment to counter fear and hostile milieu
			Problem centred approach	Focus on meaningful and realistic contexts increases learners mathematical experiences by exploring their immediate environment/context
			Realistic mathematics instruction	
Poor study habits	Poor planning; unwillingness to do Math consistently; poor time management skill; inattentiveness; no effort to understand task instructions	Problem-solving behaviour; Study attitudes	Cognitively guided instruction	Enhances confidence and perseverance when doing mathematics.
			Problem centred instruction	Emphasizes understanding and analysis of the problem before solving it. It is therefore presumed that if mathematics task/problem is understood elements such as attentiveness, willingness to do mathematics and time management will improve.

Conclusion

Firstly mathematics learning barriers are multi-faceted in nature and they include mathematics anxiety, non-conducive study environment, poor problem-solving behaviour, negative attitudes towards mathematics and ineffective study habits. This chapter has shown that the aforementioned barriers are interrelated. This interrelatedness is particularly evident in the overlap among the symptoms, i.e. low mathematics achievement, and on the approaches employed to mitigate their intensity (Hackworth, 1992; Davidson & Lavitov, 1993; Tobias, 1987; Mitchell, 1987; Maree *et al.*, 1997). The overlap tends to make it difficult to discuss the learning barriers in isolation. Therefore, hypothetically, a group of learners each suffering from one of these mathematics barriers can be remedied simultaneously (in a small group setting).

Secondly it has been inferred through the critical review of literature regarding the four mathematics teaching-learning perspectives (namely PCI, RME, CL and CGI) that there appears to be tacit link amongst these perspectives. The synergy between the teaching-learning perspectives is evident in Vygotsky's ZPD. The power of synergy prevalent in PCI, RME, CL and CGI suggests further that the navigation between the instructional approaches in a mathematics class is vital if high level of mathematics conceptualization and problem-solving skills are to be attained. Further there seem to be a link, however tacit, between the four aforementioned mathematics teaching-learning perspectives and the mathematics learning barriers referred to earlier. Nonetheless, the practical impact of the four mathematics teaching-learning perspectives on the learning barriers is the subject of further investigation.

The next chapter will focus on assessment in mathematics; however the main focus will be on diagnostic, summative and formative assessment in mathematics because they form the bases of the GIST model.

CHAPTER

3

ASSESSMENT AND THE TEACHING AND LEARNING OF SCHOOL MATHEMATICS

Donald et al. (2006:101) trace diagnostic assessment processes from constructivist theory as the tracking of the progress of the individual learner in their growth of understanding in order for the teacher to connect with their (learners') ZPD during the lesson mediation.

CHAPTER 3

ASSESSMENT AND THE LEARNING OF SCHOOL MATHEMATICS



INTRODUCTION



DEFINITION OF THE CONCEPT 'ASSESSMENT'



AUTHENTICITY OF ASSESSMENT



PURPOSES OF ASSESSMENT



SYNERGIES BETWEEN DIFFERENT ASSESSMENT PURPOSES



SYNERGIES BETWEEN ASSESSMENT PURPOSES AND THE
TEACHING APPROACHES



CONCLUSION

Heading	Table	Figure
3.1. Introduction		
3.2. Definition of the concept ‘assessment’		
3.3. Authenticity of assessment		
3.4. Purposes of assessment		
3.4.1. Summative assessment (assessment of learning)		
3.4.1.1. Definition		
3.4.1.2. Positive attributes of summative assessment on the teaching and learning of mathematics		
3.4.1.3. Limitations of summative assessment on the teaching and learning of mathematics		
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3.4.3.3. Challenges regarding diagnostic assessment on the teaching and learning of mathematics		
3.5. Synergies between different assessment purposes		Figure 3.1: Synergy between formative and summative assessment in mathematics Figure 3.2: Formative assessment system
3.6. Synergies between assessment purposes and teaching approaches		
3.7. Conclusion		

3.1. Introduction

When the outcomes-based education (OBE) policy (with its associated outcomes-based assessment (OBA) policy) was introduced in South Africa, it was assumed that teachers would effortlessly and swiftly adapt their teaching and assessment practices accordingly (Vandeyar, 2005:461) in mathematics classes. However, a change in assessment policy seemed to have been overwhelmed by challenges that deterred smooth implementation. Black and Wiliam (1998) concur that the changing of national, provincial or district assessment policies has a potential to leave teachers confused. This confusion, emanating from the change in assessment policy, may lead to disastrous consequence of inappropriate mediation of policy by teachers (Vandeyar, 2005:461).

While there may be other reasons for teachers not to cope with complex assessment policy changes, it is assumed that the following changes exacerbated the situation: *firstly*, the change requires greater emphasis on continuous assessment and a focus on demonstrable performances (Vandeyar, 2005:462); *secondly*, greater emphasis is on a shift from exclusively norm-referenced summative form of assessment (that teachers were more used to), to criterion-referenced formative assessment (Vandeyar, 2005:462).

The current curriculum in South Africa articulates the different types of assessment for implementation in schools, namely summative, formative, diagnostic, and baseline (DoE, 2001:5). This chapter will only focus on the first three because they form the basis of this study. As it unfolds, the chapter will sequentially focus on: **the definition of the concept ‘assessment’; authenticity of assessment**, critical review of literature will be offered on the characteristics of **summative, formative and diagnostic assessments; synergies between different assessments purposes**; and of critical importance will be the demonstration of **synergies between mathematics assessment purposes and mathematics teaching approaches** discussed in Chapter 2. In the process, justification of the relevance of the three afore-mentioned types of assessment in the teaching and learning of school mathematics will be explored. The chapter will then end with a conclusion that offers a synopsis of the researcher’s views regarding the latest trends in assessment of school mathematics.

3.2. Definition of the concept ‘assessment’

In recent years assessment practices within the classroom environment have been characterised by a gradual shift away from the practices of assessing the outcome of learning (assessment *of* learning) traditionally conceptualised as summative assessment (Wininger, 2005:164; Even & Wallach, 2004:485) towards the practices of assessing the process of learning (assessment *for* learning) (Vandeyar, 2005:462; Black & Wiliam, 1998). This contention suggests a shift from one assessment **purpose** to the other. The argument therefore authenticates the need to redefine the concept *assessment* in terms of its focus or purpose (Cowie, 2005:137). Harlen (2005:207) concurs that, in a general sense, assessment involves the process of collecting and interpreting evidence for a *particular purpose*, and making decisions about what is relevant evidence and communicating that evidence to the intended users. DoE (2002a:93) states it more concisely:

Assessment ... is a continuous, planned process of gathering information about the performance of learners measured against assessment standards of the Learning Outcome. It requires clearly-defined criteria and a variety of appropriate strategies to enable teachers to give constructive feedback to learners and to report to parents and other interested people.

In a broad sense, there are primarily two main purposes of assessment, namely **for helping/enhancing learning** and **for summarising/measuring learning** (Cowie, 2005:137). This means that the same information or evidence, collected in the same way, would be formative if its purpose is to adapt and improve teaching and learning of mathematics, or summative if used for reporting learners’ level of performance in mathematics (Harlen, 2005:208). Taras (2005:468) further supports the notion of the significance of ‘purpose’ by emphasising that the purpose of assessment does not impinge on the process of assessment but on the goals of assessment. Even and Wallach (2004:484) extend the definition further by asserting that some of the information collected serves the purpose of identifying learners’ disposition towards mathematics. This suggests that assessment does not only serve formative purpose or summative purpose, but diagnostic and baseline purposes too.

Diagnostic assessment – Donald, Lazarus and Lolwana (2006:101) define diagnostic assessment as “tracking the progress of individual students in their growth of understanding”. The definition of diagnostic assessment by Donald, Lazarus and Lolwana (2006:101) seems to epitomize the traits of formative assessment rather than of diagnostic assessment. This assumption is qualified by the definition of formative assessment provided by Wininger (2005:164), Taras (2005:468), Biggs (1998), and Black and Wiliam (1998) whose emphasis is **measurement of learners’ progress, modification of teaching and improvement of learner performance, and feedback** (also *see* §3.4.2.1 & 3.4.2.2). Maree and Erasmus (2006:14) seem to provide a more appropriate definition of diagnostic assessment by asserting that diagnostic assessment is primarily concerned with identification of the nature and cause of teaching-learning problems/barriers that may hinder/impede effective learning performance. Through diagnostic assessment, learners’ difficulties regarding the learning of mathematics are scrutinised so that relevant remedial assistance and guidance is provided (Jennings, Price & Pankhurst, 1999:2; DoE, 2002a:94). Relevant remedial assistance and guidance seem to be located within formative assessment (Wininger, 2005:164; Taras, 2005:468; Biggs, 1998; Black & Wiliam, 1998). In mathematics, diagnostic assessment serves the following two broad intentions (Maree & Erasmus, 2006:14): identification of **cognitive-based** problems such as academic underperformance, and **affective-based** problems such as attitudes and disposition towards mathematics.

Baseline – Normally conducted at the beginning of the lesson to establish prior knowledge possessed by learners (DoE, 2002:94). Its primary role is to assist teachers to make informed decisions when they plan their teaching-learning activities. The line of demarcation between diagnostic assessment purpose and baseline assessment purpose is that the former focuses on the mathematics learning barriers while the latter focuses on mathematics academic prior knowledge possessed by learners.

3.3. Authenticity of assessment

The previous section attempted to offer a generic outlook regarding the definition of assessment. Mention was made of the critical role played by *purpose* in defining assessment. In essence it appears as if any form of assessment is useful only if it serves the purpose for which it is intended. However, assessment whose purpose(s) is/are not premised upon preparing learners to lead life beyond the classroom has been subjected to scholarly criticism (Tanner, 2001). Presumably this is one of the factors that necessitated the call for assessment reform, hence a gradual shift away from the practices of assessing the outcome of learning towards the practices of assessing the process of learning that also informs teaching (Vandeyar, 2005:462; Black & Wiliam, 1998; Tanner, 2001). Arguably this gave

birth to *authentic assessment*, which is defined by Tanner (2001) as the assessment approach that connects the classroom to life beyond the school and in the process advances the quality of teaching and learning. Suurtamm (2004:499) concurs thus:

... assessment that involves students in tasks that are worthwhile, significant, and meaningful and that resemble learning activities. Such assessment activities also encourage risk taking, allow for mathematical communication, and provide the opportunity to demonstrate the application of knowledge in unfamiliar settings.

This suggests that apart from providing the teacher with enlightening information about his/her learners and their academic (in)capabilities, authentic assessment supports and transforms the process of mathematics teaching and learning. Tanner (2001) argues that authentic mathematics assessment is characterised by allowing learners to explain their mathematics solutions. The process of providing verbal explanation to solutions enhances conceptualization of mathematics content.

Based on the definition of authentic assessment above and the characteristic of authentic assessment identified by Tanner (2001), it is evident that authentic assessment is premised upon the synergy between formative and diagnostic assessment. Characteristics of authentic assessment identified by Tanner (2001) and their synergy with formative and diagnostic assessment are explored next.

Authentic assessment is criterion-based: Authentic assessment emphasises the use of assessment criteria whose standards emulate a measuring stick with which learners' actual level of performance is established relative to the ideal or envisioned performance (Tanner, 2001). Implicit in this rationale is that authentic assessment disapproves of a norm-referenced assessment which advocates measuring learners' competencies and achievements by comparing them with other learners (Klecker, 2003:216). Contrary, authentic assessment advocates the idea of measuring learner competencies against the set of criteria, which in South African mathematics context are referred to as assessment standards (DoE, 2002a:93). Mathematics assessment standards (hallmark of criterion-based assessment) have to help teachers to redefine or re-examine mathematics teaching, learning and assessment practices (Darling-Hammond & Falk, 1997). The assertion justifies and qualifies the naming of authentic assessment as performance-based or outcomes-based assessment (Suurtamm, 2004:499).

The criterion-based feature of authentic assessment also characterises formative assessment (Black & Wiliam, 1998). A critical analysis and elaboration of Tanner' criterion-based view suggests a prevalence of similarity with what characterises feedback. In §3.4.2.2 assertion is made by which feedback is seen to be characterised by the gap between the actual performance level and reference performance level (Black & Wiliam, 1998) which Tanner (2001) refers to them as actual products and ideal products respectively. This confirms the synergy between authentic assessment and formative assessment, which may further imply that **formative assessment** characterises **authentic assessment**.

Multiple indicators of quality: This feature primarily departs from the viewpoint that learners' level of mathematics proficiency cannot be decided based on just one mathematics performance (Tanner, 2001). Continuous assessment practice, as advocated in outcomes-based assessment (OBA) philosophy, is based on multiple indicators of mathematical performance before making a final decision about the learners' competence or incompetence in mathematics (DoE, 2002a:93).

Judgement reliability: DoE (2002:94) argues that, in order to enable learners to reach their full potential in mathematics, assessment should be varied in terms of methods and context. This view is in compliance with the principle or philosophy of inclusion which advocates accommodation of diversity through provision of expanded opportunities in teaching and assessment (DoE, 2002a:5). In authentic assessment, reliable judgement is attained when learner differences in terms of background and abilities are accommodated (Tanner, 2001). Conversely, when teachers control mathematics learning and assessment circumstances in order to compare learners' mathematical performances, authenticity is sacrificed. The converse therefore renders assessment 'traditional' i.e. comparing learners' mathematics performance against other learners (Klecker, 2003:216)

3.4. Purposes of assessment

It would appear that a particular type of assessment is named and defined according to the purpose it serves. There are essentially five such purposes of assessment, namely summative, formative, diagnostic, baseline and alternative assessment purposes. Due to the focus of this study and in particular due to the dictates of the GIST model, the first three will be reviewed. However, the review of summative, formative and diagnostic assessment purposes should not be viewed as a means to render the other assessment purposes inconsequential.

3.4.1 Summative assessment (assessment *of* learning)

3.4.1.1 Definition

Wininger (2005:164) defines summative assessment as an assessment practice through which teachers gather information about learner performance for the purpose of assigning grades and reporting achievement. The gathered information may be used for many other varied reasons such as: internal school tracking of learners' progress; informing parents, learners and learners' next teacher of what has been achieved; certification or accreditation of learning by an external body; selection for employment or higher education, and monitoring performance of teachers (Harlen, 2005:208).

3.4.1.2 Positive attributes of summative assessment on the teaching and learning of mathematics

The call for the shift from predominantly norm-referenced summative to criterion-referenced formative assessment purposes does not preclude and disqualify nor render summative assessment insignificant. Besides, research has shown that a critical scrutiny on the differences between formative and summative assessment purposes suggests that the former is an extension of the latter (Taras, 2005:468; *also see* §3.5). If this assumption is anything to go by, then summative assessment has a role to play, However, less significant, in the learning of school mathematics. Different uses of summative assessment pointed out in §3.4.1.1 by Harlen (2005:208) lead us to the effects thereof in the teaching and learning of mathematics. When reporting about learners' mathematics performance is done especially at the end of school calendar, it is done successfully using summative assessment. Essentially summative assessment may be equally motivating to learners as they aspire for high ultimate output in mathematics performance. Presumably, proper orchestration and execution of formative assessment in mathematics is a build-up towards high summative outputs in mathematics performance.

3.4.1.3 Limitations of summative assessment on the teaching and learning of mathematics

Harlen (2005:210) has identified the following shortcomings of summative assessment:

- Summative assessment has the potential to induce mathematics test anxiety which may further result in low performance in the subject.
- Underperformance in mathematics may further impinge on learners' self-esteem.
- Learners' motivation in the learning of mathematics can also be compromised.
- Summative assessment, especially high stakes tests and examinations, has a potential to direct the teacher's focus on the product of learner performance and not process of learner's performance in mathematics. The teaching of school mathematics is therefore likely to become examination oriented, i.e. emphasising transmission of knowledge that will help learners pass mathematics test or examination.
- Judgemental feedback which is mainly associated with summative assessment and comparing learners with other learners, has a negative influence on learners' view of their mathematical capabilities.

3.4.2 Formative assessment (assessment *for* learning)

3.4.2.1 Definition

Wininger (2005:164) defines formative assessment as the measurement of student progress before or during instruction for the specific purpose of modifying teaching and improving learner performance. Donald *et al.* (2006:99) assert that formative assessment helps learners to 'form' or shape their learning development. Formative assessment provides feedback to teachers about the effectiveness of their teaching-learning approaches, and to the learners about their learning expertise as they strive towards attaining the learning goals. Formative assessment is centred on two critical actions sequenced thus (Black & William, 1998): *firstly*, learners' action of being perceptive or conscious of the existence of the gap between a desired goal and her/his present state of mathematical knowledge, understanding and/or skills; *secondly*, the

action taken by the learner and the teacher to minimise or close the gap in order for the learner to attain the intended Mathematical goal. This assertion analyses and qualifies the definition provided by Wininger (2005:164) whose emphasis is **a measurement of learners' progress and modification of teaching and improvement of learner performance**. According to Black and Wiliam's analysis, it is assumed that measuring learner performance makes them (learners) conscious about the gap between their actual and intended performances.

3.4.2.2 Feedback: essential feature of formative assessment

There is a very strong overlap between assessment for learning (formative assessment) and feedback. Actually, feedback qualifies formative assessment – without it (feedback) there is no formative assessment (Taras, 2005:468; Biggs, 1998; Black & Wiliam, 1998). To further authenticate this assertion, reference is made to §3.4.2.2 wherein two core features qualify formative assessment, namely learner consciousness about the existence of a gap, and action to be taken to minimise or close the gap. Therefore, the information about the gap and the use of that information to minimise the gap in some way is referred to as *feedback* (Taras, 2005:470). Further, Black and Wiliam (1998) concur with this underlying principle of formative assessment by identifying and elaborating further on the four critical elements that make a feedback system:

- Data on the actual level of some measurable attribute;
- Data on the reference level of that attribute;
- A mechanism for comparing the two levels, and generating information about the gap between the actual and reference levels; and
- A mechanism by which the information can be used to alter the gap.

The aforementioned elements will be explored further in the next paragraphs. The main focus will be on the impact and relevance of feedback (which forms a critical constituent of formative assessment) on the teaching and learning of school mathematics.

Data on the actual level of some measurable attribute: In mathematics this refers to the information in respect of the actual level of performance by the learner towards the attainment of particular assessment standards⁴. These assessment standards are articulated in mathematics National Curriculum Statement (NCS) (DoE, 2002). The evidence regarding the learners' actual mathematics academic achievement may be collected or measured using tests, class work, home work, assignments, projects and/or investigations (DoE, 2007:30). In order to enhance reliable learner performance, learners should be assessed on regular bases as advocated in formative assessment (Tanner, 2001), hence continuous assessment (DoE, 2007:95).

Data on the reference level of that attribute: Learners' mathematics performance should be measured against certain standards or "yardsticks", i.e. in order to decide whether a learner has achieved or not in mathematics, there ought to be a point of reference or measurable outcome that will inform that decision.

A mechanism for comparing the two levels, and generating information about the gap between the actual and reference levels: mathematics assessment tools or instruments play a fundamental role in this regard. In mathematics the commonly used assessment tools are rubrics, memoranda and check-lists (DoE, 2003b:18). The development of the assessment tools is informed by appropriately interpreted mathematics assessment standards. Mathematics assessment tools therefore serve as a mechanism for comparing and deciding whether a learner's actual mathematics achievement is below or closer to the assessment standards/reference level. Put differently, assessment tools help to measure the size of the gap between the learner's actual level of performance and the reference level of performance.

A mechanism by which the information can be used to alter the gap: This feature serves as a critical element of feedback, and subsequently a fundamental element of authentic formative assessment (Black & Wiliam, 1998). Mathematics tasks are given to learners not only to establish their actual level of mathematics performance or to compare their actual performance with the reference performance (i.e. assessment standards), but most importantly to serve a diagnostic purpose (Black & Wiliam, 1998).

⁴ The National Curriculum Statement is presented in three design features: Critical outcomes (CO's), Learning outcomes (LO's) and Assessment standards (AS's). In mathematics content is entrenched in the AS's. The AS's therefore provide the content to be taught and the minimum level of competency (in terms of content knowledge and skills) to be demonstrated by learners. The mathematics content in the forthcoming CAPS document has been extracted from the AS's in the NCS.

Learners' mathematics written work can provide the teacher with valuable information regarding their (learners') mathematical inadequacies. These [mathematics] diagnostic findings or information need to be communicated to the learners (Harlen, 2005:207) and to advise/guide them on how to overcome their inadequacies and/or to adapt the teaching approaches (Wininger, 2005:164). Ramaprasad (1983), cited by Black and Wiliam (1998), contends that for effective feedback to exist the information about the gap must be used (by the learner, the teacher or both) to alter it (the gap). This contention necessitates the posing of a critical question: When is feedback qualitative or effective? Cowie (2005:139) distinguishes between **evaluative** and **descriptive/informative** feedback: the former uses very short remarks such as 'not good' or 'poor'. As illustration, if the remark is 'poor', learners lose motivation to do mathematics and consign themselves in the 'poor' category of mathematics proficiency with no prospects of improving. Contrary informative feedback is of high quality because learners are given clear indication about their performance i.e. their misunderstanding and how/what they should do to improve their mathematical learning. Quality feedback has been positively correlated to achievement (Black & Wiliam, 1998). Informative or descriptive feedback complies fully with the philosophy of **assessment for learning**, which underpins formative assessment.

3.4.2.3 Positive attributes of formative assessment on the teaching and learning of mathematics

As will be seen in §3.5, formative assessment has a potential to serve a diagnostic purpose (Black & Wiliam, 1998) in the teaching and learning of mathematics, especially if a high level of quality is invested in feedback (DoE, 2002a:95). Constructive and informative feedback, which are crucial elements of formative assessment (DoE, 2002a:95), enable the teacher to modify his/her teaching and improve learner performance (Wininger, 2005:164) in mathematics. This is against the background that, as the teacher employs formative assessment, mathematics academic-related inadequacies are identified within the learner population. Possible re-adjustments in the teaching approaches to address such inadequacies are therefore required.

The mathematics curriculum (DoE, 2002a:95) further outlines other mathematics benefits of formative assessment as follows:

- Formative assessment *supports mathematical growth and development* of learners because they (learners) become actively involved in their learning and assessment of mathematics - they understand the criteria to be used for assessment, they are involved in self-assessment, and reflect on their learning, thereby enhancing their self-esteem with regard to the learning of mathematics.
- It allows for the *integrated assessment* through which more than one mathematics learning outcome can be assessed in one assessment activity or task. Further, it allows for the use of differentiated assessment which helps to cater for the mathematics needs of all the learners.

3.4.2.4 Challenges regarding formative assessment on the teaching and learning of mathematics

The success of formative assessment hinges on the ability of mathematics teachers to interpret mathematics assessment standards because they (assessment standards) are the focal point of teaching, learning and assessment (Darling-Hammond & Falk, 1997). This ability does not seem forthcoming (Vandeyar, 2005:462) because of the complex demands on assessment and also probably because of little emphasis on rethinking classroom practice (Darling-Hammond & Falk, 1997).

Quality feedback, which is also a leverage of formative assessment, appears to be a daunting task for mathematics teachers who have been accustomed to 'traditional' norm-referenced assessment. Feedback in formative assessment is not intended for consumption by learners only, but for teachers too. In addition, feedback should be informative and individualised hence it may be a daunting task if mathematics classes are relatively large.

Another challenge is regarding the alignment of teaching, learning and assessment as implied by formative assessment. Traditionally the teaching and learning of mathematics and mathematics assessment were seen as separate entities. A call for the shift towards formative assessment practices in school mathematics, therefore, suggests a shift towards making assessment an integral part of mathematics teaching and learning (Hattie & Jaeger, 1998; Black & Wiliam, 1998). DoE (2002a:94) puts more emphasis on the following principles of mathematics assessment: transparency and clear focus; integration with teaching and learning of mathematics; predetermination of assessment standards; differentiation in terms of methods and contexts. Based on these principles it is assumed that a shift towards formative assessment will be a difficult task for mathematics teachers.

The other challenge regarding the effective use of feedback emanates from a potential confusion between *feedback* and *guidance*. It seems that most teachers use guidance wherein the so-called corrections are done. Essentially, feedback provides information about how a person did in terms of what s/he attempted i.e. what resulted from one's action, while guidance provides information about how to improve the situation (Wiggins, 1998:51). The proponents of formative assessment therefore advocate that learners' mistakes emanating from solving Mathematical problems should be identified and clarified by the teacher. Subsequently their (learners') attention should be drawn to such mistakes in order for them to correct their 'actions'.

3.4.3 Diagnostic assessment

3.4.3.1 Definition

Diagnostic assessment is primarily concerned with the identification of teaching-learning problems that may hinder effective learning performance (Maree & Erasmus, 2006:14). Darling-Hammond and Falk (1997) assert that "assessment that give rich information about students' approaches to learning ... can help identify ... what kinds of additional resources would be helpful to them". Donald *et al.* (2006:101) trace diagnostic assessment processes from the constructivist theory as the tracking of the progress of the individual learner in their growth of understanding in order for the teachers to connect with their (learners') ZPD during the lesson mediation.

Diagnostic assessment, therefore, can serve as a trigger for the learners' need of additional help in mathematics teaching and learning. Two broad aims are the focus of diagnostic assessment in mathematics: identifications of *cognitive*⁵ and *affective*⁶ problems, referred to by Allal (2002:55) as *capabilities* and *dispositions* respectively. Nitko (2001:293) seems to share Allal's view by asserting that diagnostic assessment serves the purpose of, *firstly*, identifying the learning targets learners did not achieve or master, and *secondly*, suggesting the possible causes of non-achievement of the learning targets. Therefore learners' low level of mathematics performance in South Africa should not be attributed exclusively to cognitive problems (Maree & Erasmus, 2006:2) in which case learners' written work (*inter alia* tests, class work, assignments, examinations) is often used to make judgement about performance. Affective or emotional factors account excessively towards inadequate performance in school mathematics (Maree & Erasmus, 2006:2). Manifestations of affective or emotional Mathematical problems among learners are often observable (Even & Wallach, 2004:468) and they include attitudes towards mathematics, mathematics anxiety, low self-confidence and mathematics study attitudes (Maree *et al.*, 1997:4). Therefore, learners' mathematical problems, cognitive and/or affective, should be identified early and appropriate interventions be instituted (Clarke & Shinn, 2004:235).

3.4.3.2 Positive attributes of diagnostic assessment on the teaching and learning of mathematics

There seems to be a strong connection between the purpose of diagnostic assessment and formative assessment in mathematics. Primarily, both depart from the viewpoint of *identifying/establishing learners' Mathematical inadequacies and taking action to address them* (see §3.4.2.1 and §3.4.3.1). Therefore the positive effects offered by **feedback from formative assessment** are presumably similar to the effects offered by **intervention from diagnostic assessment**. Intervention will receive more attention in §4.3. Nonetheless, diagnostic assessment helps mathematics teachers to know their learners' cognitive level and mathematics disposition in order to adapt their (teachers') teaching and assessment practices (Maree *et al.*, 1997:6).

⁵Cognitive domain of diagnostic assessment could be the identification of a particular mathematics content or learning targets that learners find difficult to understand and the possible causes thereof. Baseline assessment could be administered after identifying the content difficulty to establish the actual gap.

⁶Affective domain of diagnostic assessment is mainly concerned with the learners mathematics dispositions. Factors such as high level of mathematics anxiety and negative attitudes could be symptomatic of negative mathematics dispositions.

Early identification, intervention and constant feedback and monitoring have been correlated positively with mathematics achievement (Clarke & Shinn, 2004:235). Further, if learners know the basis of their mathematical difficulties, they are likely to begin to redress the situation voluntarily. This assumption is supported by the self-regulatory behaviours (self-monitoring and self-assessment) exhibited by learners who are conscious of the gap (and its cause) between their actual mathematics performance and the envisioned level of achievement during the process of formative assessment (Cowie, 2005:138; *also see* §3.4.2.1).

3.4.3.3 Challenges regarding diagnostic assessment on the teaching and learning of mathematics

It is presumed that there are two dimensions or perspectives of diagnosing learners who experience difficulties in mathematics, namely diagnostic assessment as per or infused in formative assessment and diagnostic assessment using standardised testing. The former is primarily reliant on the mathematics teachers' judgement during the formative assessment process wherein observation plays a critical role (Even & Wallach, 2004:486). This is normally given as feedback. This kind of diagnosing learners' mathematics difficulties is susceptible to misdiagnosis because it is very subjective.

Standardised tests, on the other hand, should meet stringent criteria for reliability and validity (Clarke & Shinn, 2004:244). Standardised tests with a low level of reliability are likely to reveal misleading findings which may result in inappropriate intervention. The majority of teachers lack knowledge of the symptoms of mathematics difficulties as this was not integrated in their training as teachers. Lack of such critical knowledge compromises their ability for effective diagnosis or at least their ability to suspect that something beyond mathematics academic underperformance is prevalent which may impact negatively on performance. Resulting from this inadequacy, early identification of learners who are at risk of failing mathematics is virtually impossible (Clarke & Shinn, 2004:244).

3.5. Synergies between different assessment purposes

Numerous authors and reviewers of literature who advocate the reform in classroom assessment have focused their arguments on the call for a shift from assessment *of* learning (summative assessment) towards assessment *for* learning (formative assessment) (Biggs, 1998; Cowie, 2005:137; Harlen, 2005:208; Wininger, 2005:164; Jennings, Price & Pankhurst, 1999:1; Hattie & Jaeger, 1998; Black & Wiliam, 1998). While the call appears legitimate, formative assessment is not inherently absolute. There seem to be a prevalence of complementary tendencies, however tacit, among the assessment purposes. The purpose of this section is to illustrate, through critical analysis of literature, presumed interplay between summative, formative, diagnostic and baseline assessments and also their possible implication for the teaching and learning of mathematics. The illustration does not in any way downplay the primary focus of this study, namely diagnostic and formative assessment.

The definition proffered by Wininger (2005:164) where formative assessment is viewed as the measurement of student progress *before* or *during* instruction for the specific purpose of modifying teaching and improving learner performance, is indicative of the overlap between formative and baseline assessment. A careful analysis of this definition seems to accommodate *baseline assessment*, which by implication, suggests formative assessment that has a tendency to serve a baseline purpose. Baseline assessment is used to establish mathematics prior knowledge possessed by learners. Therefore, it is conducted before the actual teaching takes place (DoE, 2002:94).

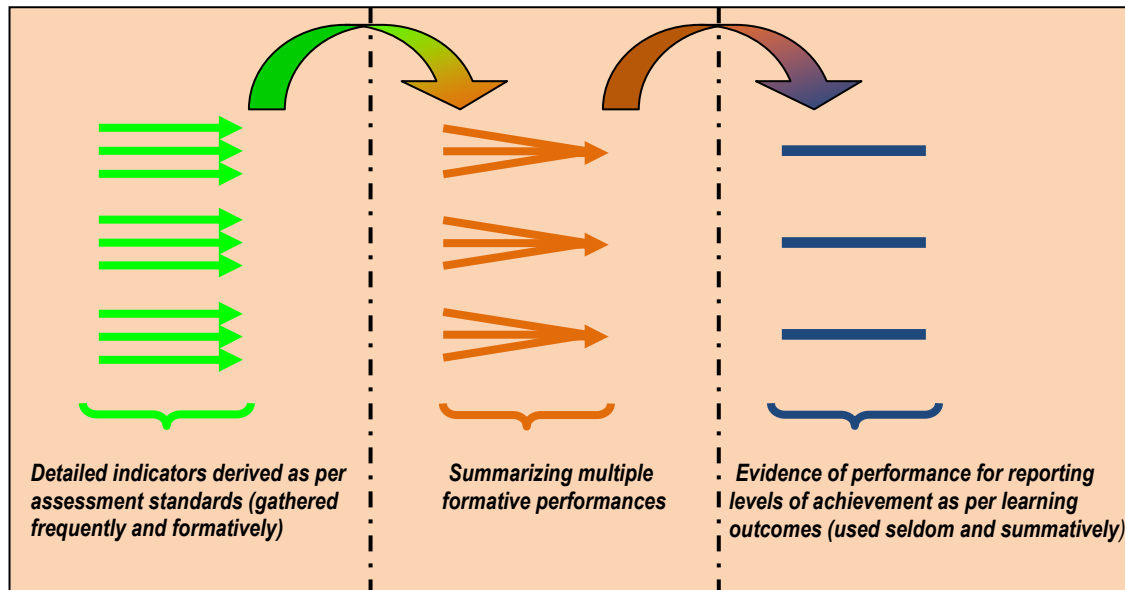
Hattie and Jaeger (1998) have identified five postulates that form the basis of assessment *for* learning or formative assessment. One of the postulates they have identified is that (mathematics) achievement is enhanced as a function of feedback. According to Hattie and Jaeger (1998), feedback provides information about *how* and *why* the student understands and misunderstands (mathematics), and *what* directions the student must take to improve. However, implied in this view is that formative assessment, whose hallmark is feedback (Taras, 2005:468), to a large degree has a potential to exhibit *diagnostic* character (Black & Wiliam, 1998). In mathematics teaching-learning context, diagnostic assessment tries to answer *why* learners perform on a particular level, and it attempts to offer interventions regarding *what* direction learners should follow to improve their mathematics performance (*see* §3.4.3.2). Therefore, while formative assessment is assumed to offer feedback to improve academic achievement in mathematics, diagnostic assessment offers intervention in connection with cognitive and affective mathematics difficulties.

Taras (2005:468) presents a much intelligible and highly structured assertion regarding the connection between *formative assessment* and *summative assessment* thus:

... formative assessment is the same process as summative assessment.... for an assessment to be formative, it requires feedback which indicates the existence of a gap between the actual level of the work being assessed and the required standard. It also requires an indication of how the work can be improved to reach the required standard. It is possible for assessment to be uniquely summative where the assessment stops at the judgement. However, it is not possible for assessment to be uniquely formative without the summative judgement having preceded it.

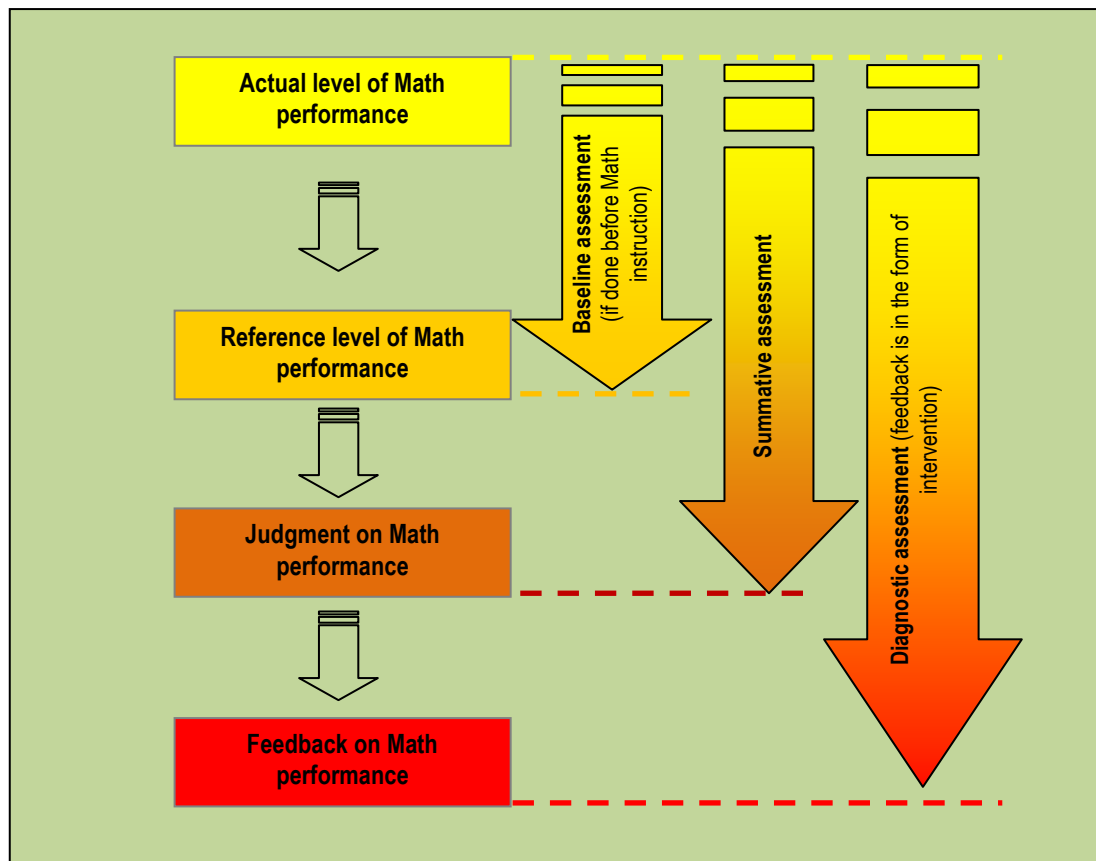
From this assertion, which is also supported by Harlen (2005:208), it can be deduced that formative assessment encompasses summative assessment. The judgement referred to in the above assertion is in respect of the actual mathematical performance by the learner and the reference level of performance. Therefore, a particular assessment is summative if the process of assessment aims at making judgement about learners' mathematical performance without providing informative feedback. In short, formative assessment justifies summative assessment, while summative assessment (as is the case with feedback) qualifies formative assessment. However, Harlen (2005:219) concurs but extends further the assertion by Taras (2005:468). Accordingly, summative assessment precedes formative assessment if the *evidence* of learning is *interpreted*. The interpretation informs mathematics teachers about what to do to further enhance learning. However, if the multiple evidence of learner's mathematics performances used formatively can further be reviewed against broader criteria that define levels or grades, the assessment purpose becomes summative (Harlen, 2005:219). An illustration using the South African context of mathematics teaching and assessment offers better clarity thus: mathematics learners are assessed against mathematics assessment standards (derived from the learning outcomes) and the evidence of performance in a particular assessment standard is gathered. If the evidence of mathematics performance is not interpreted, then assessment is purely summative or else it is formative. Further, a number of mathematics formative assessment performances based on interpreted evidence can be used to make judgements about the attainment of learning outcomes to allocate grades – in this case formative assessment leads to summative judgements (*see* Figure 3.1).

Figure 3.1 Synergy between formative and summative assessment in mathematics
 (adapted from Harlen, 2005:220)



At this point the supposed prevalence of synergy between formative assessment and each of diagnostic assessment, baseline assessment and summative assessment has been demonstrated. However, the area that may still need clarity is the actual location of baseline, diagnostic and summative assessment purposes within the formative assessment system (*see* §3.4.2.2). Figure 3.2 provides a concise account thereof. It is quite befitting and makes academic logic for scholars in the area of classroom assessment to call for a shift towards formative assessment or assessment for learning. This is against the background that formative assessment serves a variety of assessment purposes (Jennings, Price & Parkhurst, 1999:1), which collectively impact positively on the teaching and learning of school mathematics.

Figure 3.2 Formative assessment system



3.6. Synergies between assessment purposes and teaching approaches

A study conducted by Suurtamm (2004:502) to describe and explore mathematics teachers' experiences as they used formative authentic assessment in their mathematics classes revealed synergy between realistic mathematics instruction (RMI) (*see* §2.3.6.3 – Chapter 2) and problem-centred instruction (PCI) (*see* § 2.3.5.2 – Chapter 2). Suurtamm (2004) has this to say about mathematics teachers for using (formative) authentic assessment:

... they recognised the importance of applying mathematics in real contexts, creating connections between mathematics and the student's world, developing problem-solving skills, creating a deeper understanding of mathematics, developing metacognitive skills and encouraging student responsibility for their own learning.

These findings confirm the assertion by Gravemeijer (1994:83) that RMI affords learners the opportunity to apply mathematics in **solving real-life problems**. Consequently, learners view mathematics as a subject that is not rigid and confined to the classroom. Problem-solving skills encapsulated in Polya's problem-solving model (Fernandez, Hadaway & Wilson, 1994:196; *see* §2.3.5.2 – Chapter 2) include but is not limited to the following:

- making sense of the mathematics problem;
- analysing and synthesising the problem;
- determining and executing the appropriate problem-solving strategy; and
- validate and interpret the solutions appropriate to the context.

The synergy between formative assessment and mathematics teaching instructions reiterate the call by Hattie and Jaeger (1998) that there should be interplay between assessment/feedback, teaching and learning. Further, Hattie and Jaeger (1998) contend that feedback provides information about **how** and **why** the student understands and misunderstands (mathematics), and **what** directions the student must take to improve. Darling-Hammond and Falk (1997) share a similar opinion in their study whose primary focus has been to establish how standards and assessment can be used to support student learning. The study argues that for learners to get specific help they need in mathematics, detailed information about **what** students know and **how** they learn should be gathered (Darling-Hammond & Falk, 1997). When mathematics teachers strive to comprehend **how** learners learn mathematics they may do so by using cognitively guided instruction (CGI) which builds mathematics teachers' knowledge of learners' mathematics thinking (*see* §2.3.4.2 – Chapter 2). According to Cowie (2005:138), feedback and formative assessment enhance self-monitoring, self-assessment and self-regulation among mathematics learners. Similar metacognitive effects have been identified by Fennema, Carpenter and Peterson (1991:32) when CGI was used in the teaching and learning of school mathematics. It is therefore assumed that the synergy between formative assessment, diagnostic assessment and CGI can maximise metacognitive skills of mathematics learners even more and enable mathematics teachers to employ informed instructional processes to curb learners' mathematical problems.

3.7. Conclusion

This chapter has proffered a critical review of literature regarding assessment in mathematics. Among other aspects a critical review was done regarding assessment *of* learning and assessment *for* learning which are respectively referred to as summative and formative assessment. Their effects on teaching and learning were also explored and indeed formative assessment seems to surpass other types of assessment in terms of enhancing mathematics teaching, learning and achievement.

A plethora of research has revealed that formative assessment or assessment *for* learning surpasses other types of assessment primarily because: it improves learning; it informs classroom practice; it offers constructive and informative feedback; it enhances understanding of goals and criteria; it fosters motivation; and it enhances prospects for self-assessment in mathematics classes. However, its preference and current dominance of literature within the field of research in assessment do not render it a panacea or absolute.

The synergies between formative assessment and other types of assessment such as diagnostic, summative and baseline within the confines of mathematics assessment were revealed. Based on these synergies it was evident that the formative assessment system encompasses other aforementioned assessment types. The distinguishing feature between all types of assessment is their purposes and not necessarily their process.

Since assessment forms an integral part of teaching and learning of mathematics, mutual relationships between effective teaching-learning approaches in mathematics such as CGI, PCA, RME and CL, and assessment purposes were established. Based on this revelation it is befitting to suggest that teaching, learning and assessment should be integrated intensively at all times whenever the intention is to maximise the level of performance in mathematics and to enable learners to demonstrate their full potential in the subject.

The next chapter (chapter 4), will provide a comprehensive outline of the GIST model. Among other aspects, the chapter will provide an explanation of each stage of the GIST model in a sequential order. Mathematics teachers who implement the GIST model will also be provided with the information regarding the implications of each stage for them (teachers). Further, the four mathematics teaching approaches reviewed in chapter two (namely **cooperative learning**, **cognitively guided instruction**, **problem-centred instruction** and **realistic mathematics instruction**), as well as the **diagnostic** and

formative assessment practices, will feature prominently in the explication of stage three and four respectively. Further, the navigation between the abovementioned four teaching-learning approaches will be illustrated.

CHAPTER

4

THE EXPOSITION OF THE GIST MODEL

“Students need to understand their mathematical difficulties and formulate a plan to overcome them ...”

Perry (2004)

CHAPTER 4

THE EXPOSITION OF THE GIST MODEL



INTRODUCTION



THE STRUCTURE OF GIST MODEL



STAGES OF INTERVENTION



1st Stage:
Diagnostic
assessment



2nd Stage:
Facilitation of the
group
intervention



3rd Stage:
Teaching-learning
process



4th Stage:
Formative
assessment
practices



CONCLUSION

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4.3.	<i>Stages of intervention</i>		
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4.4.	<i>Conclusion</i>		

4.1. Introduction

Before presenting the actual structure of the GIST model of mathematics teaching and learning, it would be beneficial to the implementer of the model (*viz.* mathematics teacher) to be orientated on and be reminded about what informs the GIST model. In the context of this study, the explication of the GIST model indicates how diagnostic assessment was practiced in the participating classes.

Two policy issues and two theoretical perspectives inform the GIST model, namely *Norms and Standards for educators* and *Inclusive Education model* (for the former), and *developmental and motivational perspectives* (Bennett, 1994:51) (for the latter). According to the Norms and Standards for Educators as entrenched in the National Education Policy Act 27 of 1996, educators in South Africa have an obligatory role of being effective *learning mediators, interpreters and designers of learning programmes and materials, pastoral caregivers, and assessors* (ELRC, 2003:A47). The above four roles of educators suggest that educators should: adjust teaching-learning strategies during the mathematics lesson mediation in order to cater for different learning styles and preferences, and to mainstream learners with learning barriers in mathematics; interpret and design learning programmes and materials in cognisance of barriers to learning in mathematics; counsel and/or teach learners in need of assistance regarding learning barriers in mathematics; and make appropriate use of different assessment practices, especially continuous, diagnostic and alternative assessment practices in mathematics (ELRC, 2003:A47) which did not receive adequate attention (if any) in the past.

If the assertion by Naicker (1999:25) from an Inclusive Education point of view is anything to go by, i.e. that every educator should become a “special educator” who will be able to recognize and respond to the diverse needs of learners, then mathematics teachers need to possess appropriate skills to diagnose mathematics learning barriers in order to address them effectively. Nitko (2001:293) puts it emphatically that “unless you [the teacher] know or can hypothesise why the student cannot perform a learning target, you [the teacher] will likely be at a loss as to how to focus your remedial teaching”. The Department of Education concurs with this notion through the introduction of diagnostic and alternative assessment practices (DoE, 2001:5). These two assessment practices require proficiency in identification and remediation of learning barriers in mathematics. The two aforementioned backgrounds are implied in the philosophical backgrounds of outcomes-based education, i.e. all learners can learn, including learners who experience one or the other form of learning barrier.

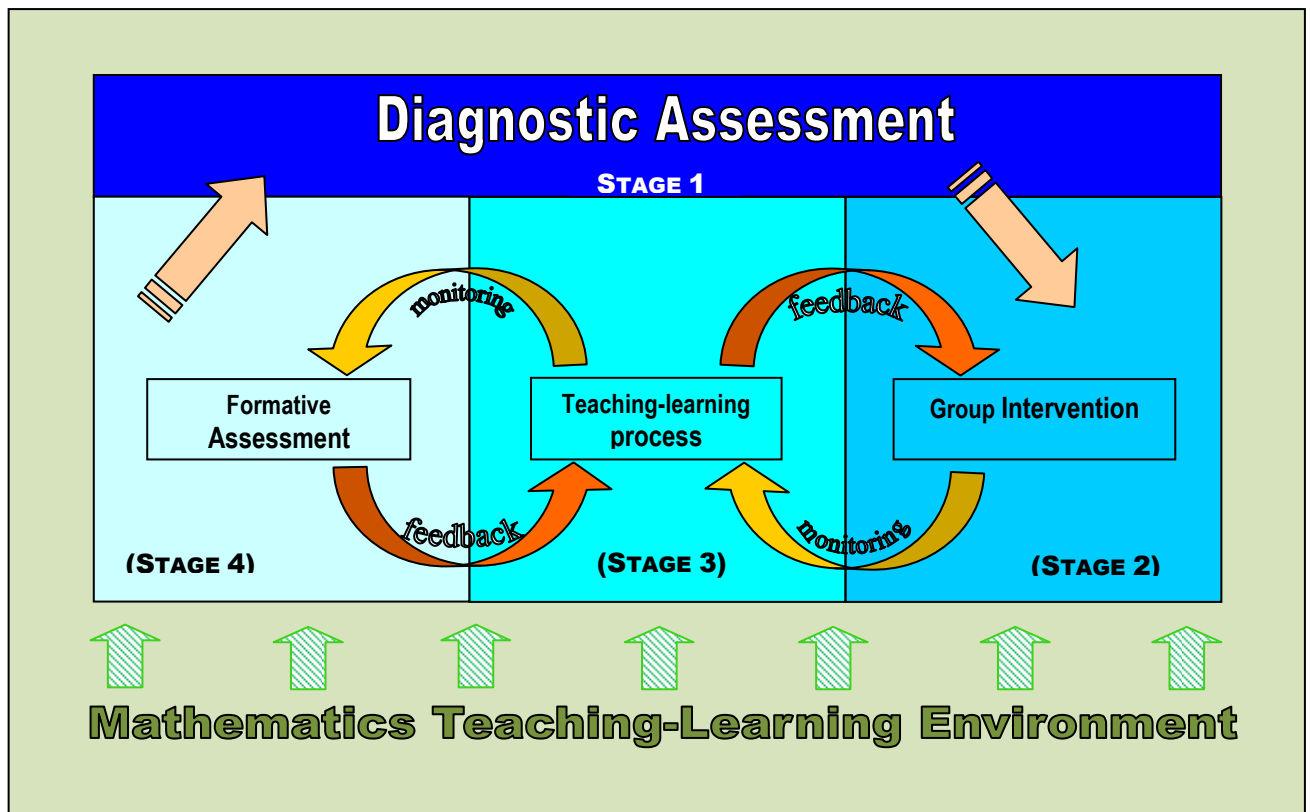
From the theoretical perspective, the *developmental perspective* departs from the notion that through social interaction (with other learners) the learner acquires a framework for integrating experience and learning how to negotiate meaning. *Motivational perspective* focuses primarily on the achievement rewards under which members of the group operate. The GIST model is designed to offer intervention to learners who experience mathematics-learning barriers in a whole class setting where one-to-one intervention processes are impracticable (Schmidt, 2003:147). It is more time-effective since it reaches out to a greater number of mathematics learners, and offers a platform where learners can freely share their mathematics feelings, concerns, attitudes, beliefs and problem-solving skills with one another (Studer, 2005:285).

4.2. The structure of the GIST model

Before the structure of the GIST model can be outlined, it is fundamental to highlight that the implementation of the GIST model takes place within the confines of the normal mathematics class as per school time table. This suggests that learners experiencing mathematics difficulties are not isolated into the so-called remedial class where extra lessons (mistaken for remediation) are offered in the afternoon when other *able-learners* have gone home. The GIST model is instead an integrated approach which is infused in the mathematics lessons as allocated in the school time table.

The structure of the GIST model is categorized into four stages (*see* Figure 4.1), which will be delineated explicitly in the next sub-sections. Each stage comprises a number of steps, making it easier for the mathematics teacher to follow each stage during the implementation process. At the end of each stage there is a discussion with regard to the implications of that stage for the mathematics teacher and/or the learner. The rationale for this is to provide mathematics teachers with a comprehensive explanation of what is expected of them (and their learners) and/or what their (and the learners') roles are. Regarding the learners' roles, it should be highlighted that it is the primary role of the teacher to ensure that learners observe their roles because they (teachers) are the implementers of the GIST model and not the learners. Figure 4.1 provides an abridged illustrated version of the GIST model, which must be interpreted in conjunction with the explication texts in the pages that follow. However, it is particularly important to highlight that the '*mathematics teaching-learning environment*' referred to in the illustration is the environment in which mathematics teaching and learning takes place, e.g. a classroom. To a large extent the mathematics teaching-learning environment referred to in Figure 4.1 is structured in accordance with the dictates or tenets of cooperative learning as outlined in Chapter 2 (§2.3.3.2).

Figure 4.1 The structure of the GIST model



4.3. Stages of intervention

The following four stages of the GIST model have been identified with a sole purpose of helping the mathematics teachers implement the model sequentially. The stages of the GIST model (as indicated in Figure 4.1) are *Diagnostic assessment*, *Facilitation of the group intervention*, *Teaching-learning process*, and *Formative assessment practices*. The proper orchestration and implementation of the four stages will presumably maximise the teaching-learning benefits of mathematics in terms of alleviating the potential learning barriers outlined in Chapter 2 (§2.2).

4.3.1. Stage 1: Diagnostic assessment

4.3.1.1. Abridged orientation

The mathematics curriculum in South Africa is characterised by learning targets or milestones in which learners have to demonstrate competencies at the end of each grade. However, learners often find themselves not being able to achieve the learning targets due to some form of barrier. Early *identification, intervention* and *constant feedback/monitoring* are required as they have been correlated positively with mathematics achievement (Clarke & Shinn, 2004:235). Because of its ability to reveal problems symptomatic of barriers to effective learning performance, diagnostic assessment (*see* §3.4.3.1) should be invoked in order to institute appropriate interventions. The literature evidence pronounced in §3.4.3.2 with regard to the positive attributes of diagnostic assessment suggests, *inter alia*, that teachers too are the beneficiaries of diagnostic assessment because the outcome of diagnosis moulds the teaching practice.

4.3.1.2. Step 1: Determine the appropriate diagnostic assessment tool

The first stage of the GIST model i.e. **Diagnostic assessment** is a fundamentally important part of the model. Correct diagnosis using an appropriate diagnostic assessment tool will lead to the use of relevant intervention strategy. While the first stage remains the most important stage of the GIST model, the interrelatedness of all the stages should not be downgraded or undermined. Put more elaborately, the correct *diagnosis* of a particular mathematics-learning barrier will inform the choice of the *intervention* approach, which in turn will inform the structuring of the *teaching-learning practices*. The teaching–learning practices (within which assessment is integrated) will in turn impact on the *assessment practices* to be employed.

Determining the appropriate diagnostic assessment tool is an exercise that will differ from one mathematics teacher to the other. The choice of the tool is dependent upon the teacher's focus regarding what aspects of mathematical barriers s/he wants to attempt to identify, and on what the available tools have to offer.

Ideally the mathematics teacher should have a variety of diagnostic assessment tools at his disposal, from which a choice can be made. However, the Study Orientation in mathematics (SOM) questionnaire (Maree, 1996) was used in this study because the area of focus is on: **mathematics problem-solving behaviours** (PSB); **mathematics study attitudes** (SA); **mathematics study milieu** (SM); **mathematics anxiety** (MA) and **mathematics study habits** (SH). The Study Orientation in mathematics (SOM) questionnaire (Maree, 1996) addresses all five aspects appropriately. Its associated manual, answer sheet and scoring stencil can respectively be used to administer, collect data and score/analyse data easily. The SOM questionnaire will be explained in detail in Chapter 5 (§5.5.3.1)).

4.3.1.3. Step 2: Create a conducive environment for diagnostic assessment

The environment (i.e. classroom) in which the diagnostic assessment will be administered should have a very welcoming setting. The room should be well ventilated and have enough space for the teacher to move between the tables to offer assistance whenever necessary. The necessary paraphernalia should be provided, such as pencils and answer sheets. The aim is to curb the problem where learners will disrupt the process of administering the diagnostic assessment by requesting to fetch some equipment to use. Most importantly, a “Do not disturb” sign should be displayed on the outside of the door in order to prevent outsiders from moving in and out of the room reserved for testing.

4.3.1.4. Step 3: Administer the diagnostic test

The administration of the diagnostic test enables the teacher to collect information about learners' possible hindrances towards the effective learning of mathematics. The supreme importance of understanding the essential rules and guidelines on the administration of the diagnostic test should not be undermined lest unreliable information/data are collected. In order to curb the possible administering irregularities, the Study Orientation in mathematics questionnaire (SOM) has an associated manual that should be used by the diagnostic assessor (teacher in the context of this study). The manual should be carefully read and comprehended to ensure correct administration of the SOM questionnaire.

However, when there is a prevalence of the lack of comprehension of the questionnaire on the learners' side, the teacher may offer clarity without compromising the purpose of the questionnaire, i.e. without influencing the learner to choose a particular response. The SOM questionnaire is written in English; therefore the likely assistance the teacher can offer to the learners is to explain certain 'difficult-to-comprehend' words in the language that learners can readily understand. The primary aim is to enable learners to make informed decisions when rating their responses as outlined in the SOM questionnaire. Essentially a high level of caution should always be exercised when assisting the learners in order to avoid rendering the diagnostic assessment inconsequential.

4.3.1.5. Step 4: Analyse the results

The analysis of the results is done primarily to assist the teacher to: *firstly*, know the learning barrier (and its intensity) experienced by her/his learners in the mathematics class; *secondly*, make informed decision regarding which barrier should be given more attention when structuring the intervention; and *thirdly*, to inform parents about the main mathematics barrier their child is experiencing for support purposes. The standardised test used for diagnostic purposes, e.g. SOM in this instance, has an accompanying manual which guides the teacher on how to score and analyse the learners' responses. It is therefore important to analyse individual learner's responses in order to make an informed decision about each learner. Each learner's results should be kept in his/her Learner Profile which is normally kept by the school which the learner is attending (DoE, 2007:17). According to the National policy on assessment and qualifications for schools, the contents of the learner profile "...assists teachers in the next grade or at different school to understand the learner better and thereby respond appropriately to the learner" (DoE, 2007:17).

4.3.1.6. Implication of Stage 1 for the mathematics teacher

The crucial aspect implied by the first stage is that mathematics teachers must know, understand and recognise the plethora of learning barriers in mathematics and their possible impact on learner achievement. These prerequisites will enable the teacher to immediately make sense of the prevalent diagnosed mathematics learning barriers.

It will also make it quite easy for him/her to guide learners during facilitation of the second stage of the GIST model. The key issue is that diagnostic assessment in mathematics is dependent upon the general knowledge of what the revelation of diagnosis is, i.e. if the diagnosis reveals mathematics anxiety, the teacher must be knowledgeable about what mathematics anxiety is and what its potential impact can be on the learning of mathematics. Such knowledge will imply proper orchestration and facilitation of the second stage of the GIST model.

4.3.2. Stage 2: Facilitation of the group intervention

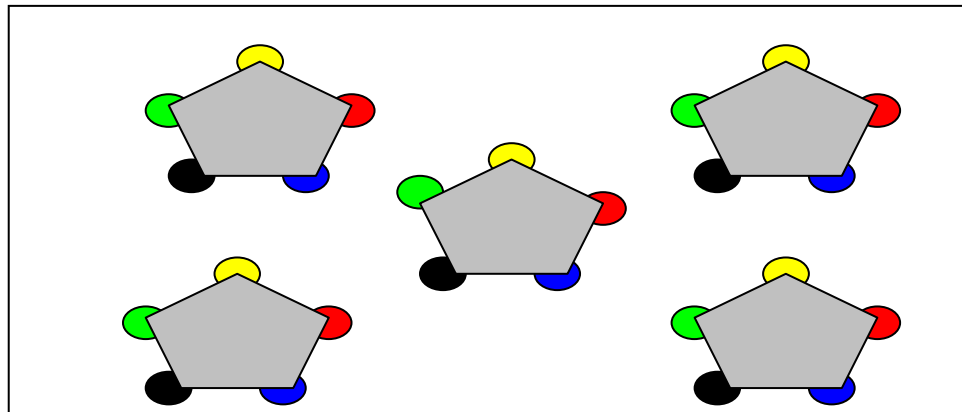
4.3.2.1. Abridged orientation

Large mathematics classes have the potential to overwhelm teachers, consequentially compromising one-to-one interventions for learners who have difficulties with mathematics. When one-to-one intervention is difficult to carry out, group intervention should be resorted to (Schmidt, 2003:135). Facilitation of the group intervention is a fundamental process as it should enable learners to discover themselves with regard to the barriers they experience in mathematics. Therefore, the facilitation of the GIST model will be integrated within the normal teaching-learning activities. However, three hours should be allocated for Stage 2 where intensive discussions will take place interactively (*see* step 1 to step 3 of Stage 2).

4.3.2.2. Step 1: Delineating the purpose of the GIST model

The teacher outlines the purpose of the GIST model i.e. help the learners to primarily understand their difficulties in mathematics and to identify possible solutions to overcome those difficulties. Learners are informed, *inter alia*, that they will be seated in small heterogeneous groups of about five (*see* Figure 4.2). Learners are welcomed and encouraged to interact freely with one another and to express their mathematics concerns openly without fear of being condemned. By so doing they will be in the process of understanding their own general orientation or disposition about mathematics (Berg & Landreth, 1990:14).

Figure 4.2 Group structuring



The teacher then gives a copy of the possible ground rules and primary expectations of the GIST class to all members and explains them (*see* Table 4.1). More rules may be added by learners and the teacher in order to enhance ownership. However, teachers are cautioned not to include rules which are too rigid to compromise maximum freedom to explore and test new mathematics learning behaviours.

Table 4.1 Rules and primary expectation during the intervention
Adapted from Berg, Landreth and Fall (2006;137)

Rule/Primary expectations	Implications for Learners
Listen carefully to other group members	All GIST learners want to be listened to; therefore they must also learn to listen to others when they talk. This enhances respect and allows for practice of tending skills.
Be honest and open in discussing problems	Frank and genuine discussions tend to benefit all group members to a large extent.
Ask questions freely	There is no question regarded as a “stupid” question; probably other members want to know the same thing.
Help other members to participate	If one member appears reluctant to say something (maybe through a non-verbal gesture), encourage him/her to say it.
Be willing to accept other points of view	Hegemony of your views should not be encouraged. Other members’ viewpoint may offer better help than yours.
Keep up with the discussion	Ensure that you are in tune with the flow of the discussion. If it is confusing, seek clarity.

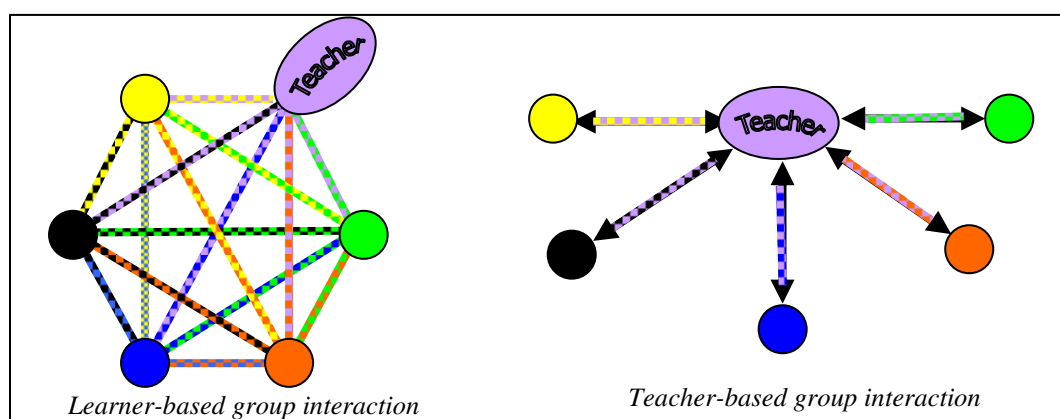
4.3.2.3. Step 2: Understanding the learners' learning barriers.

At this point learners are seated in heterogeneous groups. They are firstly requested to write down (individually for ten minutes) their experiences about the teaching and learning of mathematics, i.e. how/what they feel; what they resort to as a result of those feelings; how do they study under what environment; and how do they solve mathematics problems. They are thereafter requested to share with one another within their groups what they have written down in order to compile one list of experiences for that group. Each group is then afforded about seven minutes to share their written experiences with the rest of the class without discussing them. The teacher summarises each group's experiences by writing them on a chart. The summarised written experiences of each group are then posted on the wall for use in the next session.

The teacher explains the five learning barriers experienced by different groups, i.e. problem-solving behaviour; mathematics anxiety; study habits; study milieu; study attitudes and then classify (with the help of learners) the learners' written experiences according to the mathematics learning barriers. This exercise is quite essential because learners will begin to know and understand what the main cause(s) of their underperformance in mathematics is/are attributed to. The exercise is likely to last the whole duration of the session due to its level of importance.

Towards the end of the session, each learner in the group explains to his/her group members how s/he understands his/her learning barrier in his/her own words. Ruggiero (1998:14) emphasises that the most effective approach to deal with obstructive attitudes is when learners understand them (attitudes) by analysing their behaviours and evaluating their beliefs. Perry (2004) also agrees that "students need to understand their mathematics difficulties and formulate a plan to overcome them...". Holmes (1995:36) concurs that one of the effective ways to assist learners who experience negative attitudes towards mathematics is to engage them in discussions that will help examine their attitudes. At this point the teacher offers periodical assistance to groups without compromising the quality of the learners' interaction, i.e. learner-based group interaction should supersede the teacher-based group interaction (*see* Figure 4.3).

Figure 4.3 Types of group interactions



Each learner is given a task (homework) to identify possible solution(s) to the mathematics learning barrier they experience and to the mathematics barrier experienced by others. This means that since they know what their barriers to the learning of mathematics are, they may come up with the solution for themselves, and how to advise other learners to solve theirs. This is against the assumption that learner A may not experience the same learning barrier as learner B, so learner A can offer advice about the barrier s/he is not experiencing. The idea is that learner A can share with learner B how s/he manages not to have a problem with, for instance, negative attitude (which learner B experiences). The sharing of possible solutions will be reserved for the next session.

4.3.2.4. Step 3: Self-identified solutions

Learners are still seated in their heterogeneous groups and it is assumed that they understand what characterises their respective mathematics learning barriers, and the mathematics learning barriers experienced by other learners. Learners are given about fifteen minutes to reconcile their proposed advice in their groups. Thereafter the groups present their proposed solutions. Each group focuses on one mathematics barrier while the particular learning barrier “sufferers” are listening attentively. The teacher is also a role player regarding the solutions i.e. s/he guides learners by modifying their inputs without making learners feel that their inputs are worthless (if at all); s/he gives advice not put forward by learners. The teacher will then collect all the inputs and post them on the wall so that learners can be referred to them whenever necessary during the actual teaching and learning process (as per Stage 3).

4.3.2.5. Implications of the Stage 2 for the mathematics teacher

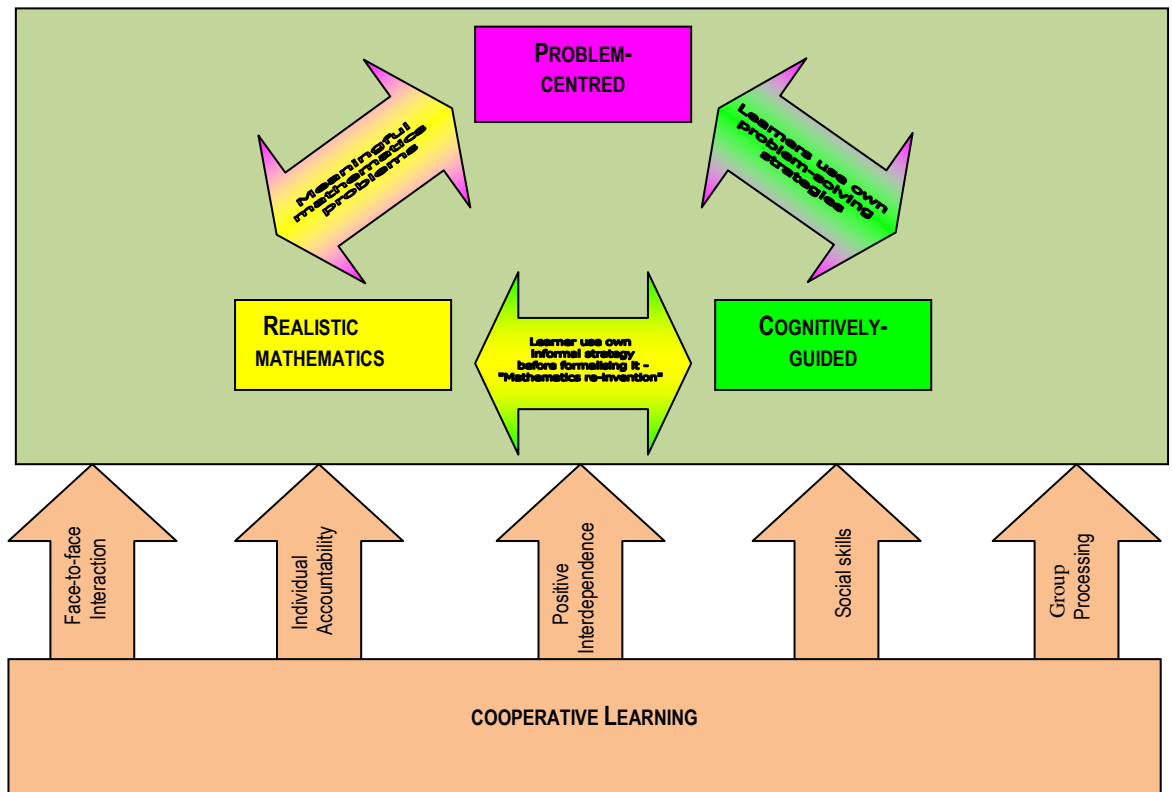
The main intention of the second stage is to allow learners to talk about their barriers to learning in mathematics as part of the desensitisation or 'healing' process. Similarly, learners have to collectively identify the possible solutions for their barriers to mathematics learning in order to enhance ownership of such solutions. This implies that the teacher must be aware of his/her role in the second stage. S/he has to avoid the prescriptive approach of identifying the solutions pertinent to the learning barriers in mathematics lest they (learners) regard those solutions as not theirs but the teacher's. The teacher's role should be dominated by facilitation; hence the caption of stage two should be '**Facilitation of group intervention**'.

4.3.3. Stage 3: Teaching-learning process

4.3.3.1. Abridged orientation

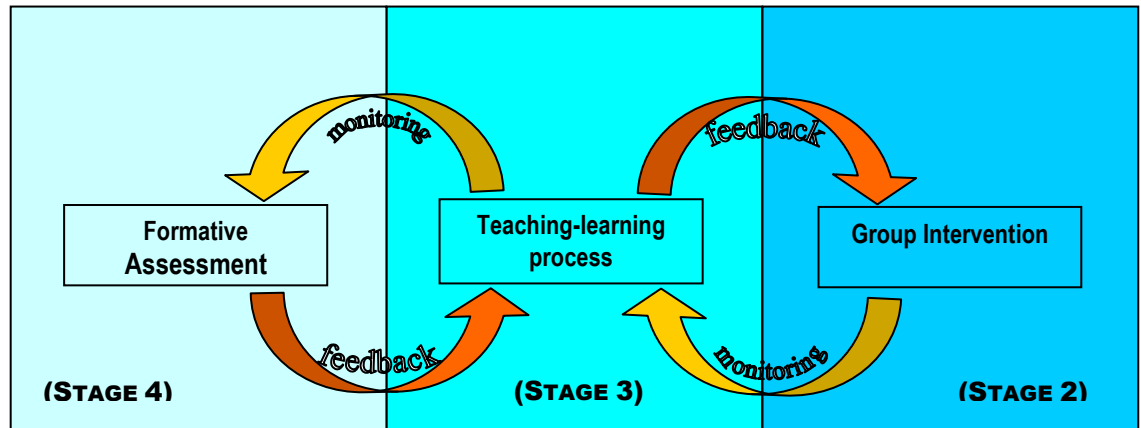
As mentioned earlier in Chapter 1 (§1.1), the main purpose of the GIST model is to mitigate or completely eradicate the learning barriers experienced by learners during the learning of mathematics, particularly where one-to-one intervention processes are impracticable (Schmidt, 2003:135). The effective attainment of this primary goal depends primarily on the choice and correct execution of appropriate teaching-learning approaches by the teacher. No single teaching-learning approach has the exclusive potential to help learners deal with: their (learners') fear of mathematics, problems associated with problem-solving behaviour, negative attitudes towards mathematics, poor study habits and non-conducive study environment. Choosing and being completely loyal to one teaching-learning approach is counter-productive in the teaching process (Even & Tirosh, 2002:233). For these reasons four fundamental teaching-learning approaches were chosen for use in the GIST class to enhance effective teaching and learning of mathematics. The literature study in Chapter 2 (§2.3) pronounced the peculiarities that necessitate the choice of the four teaching and learning approaches that are also briefly articulated in the next paragraphs. However, these teaching-learning approaches should be used in an integrated manner, i.e. interchangeably to maximise effective mathematics learning (*see* Figure 4.4).

Figure 4.4 Navigating between the perspectives



Further, the connectivity between stages 2, 3 and 4 will be clearly demonstrated (see Figure 4.5). During the teaching-learning process (Stage 3) the skills learnt from group intervention (Stage 2) are applied by the learners and the teacher is very conscious about them for effective teaching. The teacher and the learner constantly monitor their application. Continuous assessment practices are used to enhance and monitor the implementation of the acquired skills. Regular feedback, which is the leverage of assessment *for* learning and the basis of assessment in the GIST model, is given to learners regarding the learning of mathematics and/or the skills acquired from the group intervention.

Figure 4.5 Connectivity between the stages of the GIST model



4.3.3.2. Problem-centred instruction (PCI)

The fundamental characteristics of PCI encompass the use of meaningful problems as a context of the teaching and learning of mathematics (Nieuwoudt, 2000:25). GIST learners should be encouraged to use their “own strategies” when solving mathematical problems provided they (own strategies) are mathematically valid. The teacher should inculcate problem-solving strategies such as **Polya’s problem-solving model** and **meta-cognitive processes** (Van de Walle, 1997:50) within the teaching and learning process. Polya’s problem-solving model will guide the GIST learners to maintain focus when solving mathematical problems, and to demonstrate the acquisition of fundamental problem-solving skills such as understanding the problem, making and executing a plan; finding the solution; assessing/validating the solution. It may also be very helpful if the steps of Polya’s problem-solving model are written on the chart for learners’ quick reference (*see* Figures 2.6). However, a more comprehensive version of problem-solving model namely ‘dynamic and cyclic problem-solving framework’ (Fernandez, Hadaway & Wilson, 1994:196) can benefit the teachers and learners (*see* Figure 2.7). Learners should be given the opportunity to use their own understanding/words to interpret each step of the model while the teacher’s role is to help them where necessary. It is quite befitting to highlight that problem-solving in mathematics (and by implication PCI) forms part of the three unique features of teaching and learning of General Education and Training mathematics curriculum.

It is therefore in the best interest of the GIST learners that PCI should form the core of their mathematics instruction as it enhances **problem-solving skill** (which is one of the barriers experienced by GIST learners).

4.3.3.3. *Cognitively Guided Instruction (CGI)*

Fennema *et al.* (1991:31) define CGI as “ ... the instruction that focuses on facilitating the child’s thinking and learning in mathematics by building instruction on what the learners already know”. Further the CGI focuses on the affective or emotional dimensions/aspects of the learning of mathematics, i.e. learners’ feelings about mathematics, the belief that they can understand mathematics and be **self-reliant** when solving mathematics problems. As learners gain confidence, self-reliance and self-regulation (collectively known as metacognition), their poor **study habits** are likely to improve. The relevance of this in the implementation of GIST model is that the CGI teacher will gain knowledge and understanding on what and how the GIST learners are thinking and doing about mathematics. It is this knowledge of learners’ mathematical thinking that can be used by the teacher in a way that has an impact on learners’ learning of mathematics (Tate & Rousseau, 2002:289). Figure 2.4 illustrates the interplay between learners’ mathematical thinking/behavioural/learning processes and the teachers’ decision making processes.

Further, CGI can directly address the problem of **mathematics anxiety, attitude towards mathematics and problem-solving skills** because of its philosophical background of recognising learners’ prior mathematical knowledge and facilitating learners’ thinking processes. Another critical element associated with CGI which learners should demonstrate in mathematics classes is the use of learners’ own strategies in solving mathematics problems. However, in order to encourage learners to use their own strategies teachers should refrain from being too conservative regarding the use of one particular method or strategy of problem-solving.

4.3.3.4. Cooperative learning

The rationale for the preferential choice of cooperative small group work is because groups are natural medium for learning. The **study milieu/environment** conducive for learning mathematics is therefore enhanced through cooperative small groups work. Essentially cooperative learning dictates the preferred learning environment or study milieu in the implementation of the GIST model. Learners will be exposed to an environment where they can learn (from one another) mathematics problem-solving skills, positive attitudes about mathematics and social skills (such as tolerance, listening skills, valuing other people's opinions). As learners interact in such a relaxed atmosphere, and realising that they share common problems, they will begin to freely talk about such problems and their fear/anxiety for mathematics will deteriorate. The emphasis of some of the rules outlined in Table 4.2 directly address the requirements of cooperative group work, because cooperative group work is not about placing learners in groups and expecting them to cooperate. Therefore quality time should be invested in proper group structuring. Essentially the following elements should be prevalent in order to qualify effective cooperation:

Positive interdependence: the mathematical learning effort exerted by individual learner will determine the success of the group. This suggests that learners within a particular cooperative group depend on one another for a common course.

Individual accountability: while cooperative learning emphasizes the importance of group work, each learner has to be accountable or responsible for his/her learning in order to avoid *free riding effect*.

Face-to-face interaction: learners are seated facing one another in an endeavour to *verbally* assist, encourage, guide and support one another as they learn the given mathematics task.

Social skills: Leadership, decision-making, trust-building, communication, conflict management, listening, and tolerance are essential skills for effective Cooperation.

Group processing: during the process of learning learners reflect and evaluate the process of cooperative group work.

In order to enhance positive interdependence in particular, heterogeneous groups are recommended for the purpose of effective mathematics learning (*see* Figure 4.2). In the GIST class heterogeneity should be extended to diversifying learners according to the learning barriers they experience. Further group size should be kept small (preferably five learners per group) in order to make it quite manageable, and to avoid very complex lines of communication among learners. This implies that in a group of five each learner will communicate with four other learners at any given time, while group of eight for instance, implies seven lines of communication per learner. Such complex lines of communication may constitute noise instead of constructive interaction.

4.3.3.5. *Realistic mathematics Instruction*

As far as possible, the teaching of mathematics in a GIST class should be dominated by the realistic contexts of mathematics. Quite often the inability of the learners to realise the realistic nature of mathematics promotes mystifications about mathematics. The relationship of mathematics with what is happening in real life helps learners to change their attitudes about the subject and it also impacts positively on their problem-solving behaviour. Further the dominant factor that promotes mathematics anxiety is the abstract manner in which mathematics is taught, i.e. mathematics becomes such a scary subject because it is not related to anything realistic so it draws a lot from learners' imaginations. Contrary, if mathematics is taught meaningfully by drawing realistic contexts from the learners' environment, mathematics anxiety can be effectively mitigated or completely avoided. The idea is that learners may not be fearful of mathematics because they are able to identify with it – it is readily available in their locality. Further different learners learn differently, as such they interpret, represent and solve the same problem differently. It is against this background that learners will learn the skills of '*mathematization*' (*see* Figure 2.8) which essentially informs mathematical modelling (*see* Figure 2.10).

Mathematical modelling is a mathematical construct designed to study a particular real world system or phenomenon. In other words learners are taught to use mathematical representations to analyze and solve real world problems. Mathematical representations or models are mainly in the form of (but not limited to): **verbal**

representation, equations, symbols, numbers, graphs, pictures, diagrams. There are four broad classifications of mathematical models:

- Verbal representations: words and sentences are used to explain the problem.
- Graphical/Pictorial representations: use of graphs, pictures and diagrams.
- Symbolic representations: use of formulae and equations.
- Tabular/Numeric representations: use tables and numbers.

In order to authenticate or validate the mathematics-curricula-imperatives regarding *mathematization* and mathematics modelling (elements of RMI) reference is made to Learning Outcome 2 which primarily focuses on patterns, functions and algebra (DoE2002e:47)

4.3.3.6. *Implications of the Stage 3 for the mathematics teacher*

Mathematics teachers must ensure proper understanding of the connectivity between the second, third and fourth stages of the GIST model because effective management of that connectivity will determine the positive effects of the model. Further the understanding of and the ability to navigate between **cooperative learning**, **realistic mathematics** teaching and learning, **cognitively guided instruction** and **problem centred** approaches are potentially crucial to the GIST model. While much effort was invested in attempting to outline the dynamics of the four mathematics teaching and learning approaches it may be quite beneficial for the teachers if they can read further about these approaches for more information. The information regarding these approaches in this study constitutes abridged versions.

4.3.4. **Stage 4: Formative assessment practices**

4.3.4.1. *Abridged orientation*

The global call for a shift from assessment *of* learning (summative assessment) towards assessment *for* learning (formative assessment) gained prominence in recent times (*see* §3.5). Jennings, Price and Parkhurst (1999:1) credited formative assessment for its ability to serve variety of assessment purposes such as baseline, diagnostic and summative purposes. Essentially as the teacher assesses learners

formatively s/he is able to: establish the actual level of learners' mathematics performance (baseline); identify the difficulties learners experience in mathematics (diagnostic); and make judgements about learners' performance in mathematics (summative).

4.3.4.2. Implementation of stage four

It is of absolute importance to recognize the notion that assessment in mathematics constitutes an integral part of teaching and learning processes (Van den Heuvel-Panhuizen, 1996:15). However, for the purpose of this study, mathematics teachers are cautioned not to use any item of the pre-tests as an assessment activity during the teaching and learning process because the same will be administered as post-tests. The formative assessment practices employed in the GIST model are to a very large extent informed by two rudiments: *firstly* assessment in the Realistic mathematics Education (RME) context (Van den Heuvel-Panhuizen, 1996:15) which essentially emphasises the notion that assessment "is a meaningful activity" (Freudenthal, 1973, cited by Van den Heuvel-Panhuizen, 1996:15); *secondly* regular feedback should be given (either verbally or in writing) in order to qualify formative assessment (Taras, 2005:468; Biggs, 1998; Black & William, 1998). However, the traits of CGI and PCA will feature prominently within the RME contexts as will be indicated in the RME preferences. In the RME context, assessment is characterized by certain preferences (explained below), which mathematics teachers employing the GIST model should be familiar with, namely

- high priority assigned to **observation** which also features prominently in **CGI** during which learners' cognitive expositions are observed;
- **continuous and integrated** nature of assessment;
- a holistic approach;
- choosing an **open-ended test format** which also features prominently in **PCA** where learners are allowed freedom to express their diverse way of solving the same mathematical problem provided their approaches are Mathematically sound ; and
- a preference for **true application** problems (also a characteristic of **PCA** where meaningful mathematical problems forms a core of activities that require problem-solving skills).

While the GIST model encourages group work during the teaching and learning process of mathematics, group assessment should be used limitedly and individualised activities should be given main focus. The aim is to enable the teacher to *observe* and *establish* individual learner's mathematics learning process and not just the learning product. An added advantage of *high priority attached to observation* for the teacher is to identify possible discontinuity that may occur during mathematics learning process. The discontinuity in the learning process may manifest itself in the form of mathematically unsound short cuts or taking a mathematically uninformed standpoint during the process of learning. The prevalence of such discontinuities will inform the teacher about the lack of a particular concept formations or lack of relevant mathematical insights required during problem-solving processes.

Teachers should properly sequence the content within and across the mathematics topics in order to enhance *continuity and integration* of mathematics concepts in the assessment tasks. Learners experiencing the barriers to learning in mathematics should see mathematics topics as continuous set of activities, i.e. the achievement of one topic should be a prerequisite for the achievement of the other. The starting point, however, should be the integration of teaching, learning and assessment. This suggests that assessment should not be seen as an isolated entity from teaching and learning processes. In short teachers should teach what they will assess or stated differently, they should assess what they have taught. If learners are conscious of this, they are likely to pay a particular attention to whatever takes place in the mathematics class.

Similarly, assessment should be an ongoing process, i.e. it should take place as the teaching and learning process unfolds. This may happen through observing certain nonverbal gestures from learners which may alert the teacher about the prevalence of, for instance, frustrations, need for support, etc. Continuity and integration with regard to teaching, learning and assessment can therefore be effectively used to motivate learners experiencing learning barriers in mathematics.

Assessment should not only focus on skill acquisition such as problem-solving skill, but also attempt to acquire a *holistic* picture about the learner. The GIST model aims to redress variety of mathematics learning barriers (*see* §4.2.1.1). Written academic tests

cannot adequately provide information with regard to aspects such as mathematics attitude, manner of cooperation during group work, emotional aspects such as mathematics anxiety, level of concentration and motivation. However, through observation the teacher is able to assess such aspects much more effectively. The added advantage of the holistic approach to assessment in mathematics is that negative nonverbal reactions from learners can be picked up and addressed immediately as the teaching and learning process unfolds. This is contrary to a test or class work, which will normally be of help to the teacher (and the learner) after it has been marked.

The discussion with regard to holistic approach does not in any manner discourage the use of tests for the measurement of mathematics achievement. However, **open-ended tests/questions** are generally recommended for learners who experience one or more of the mathematics learning barriers mentioned earlier. The aim is to identify or determine learners' insight/skills regarding horizontal *mathematization* (i.e. ability to translate mathematically under developed matter into mathematically symbolic form) and vertical *mathematization* (i.e. ability to compute the symbolic form of mathematics to find solutions). Through open-ended questions learners are offered an opportunity to work out a problem and formulate an answer on their own (Van den Heuvel-Panhuizen, 1996:18). Further, learners are offered an opportunity to express themselves mathematically, using different strategies of problem-solving without following fixed pattern. This *mathematical freedom of expression* affords the teacher an opportunity to identify, if any, mathematically unsound or incorrect statements, which may warrant further instruction. As a result the probability of conjecturing, which is most prevalent in closed questions, is highly mitigated.

True application mathematical problems (which emphasise **meaningful learning context**) allow learners to demonstrate their full potential with regard to analysis, creative thinking, comprehension, informed decision making about the procedure to be followed, and ability to validate their answers. The starting point may be the solution of ordinary word problems, which according to Van den Heuvel-Panhuizen (1996:20) are arithmetic oriented. Later there should be a gradual move towards the solution of true application mathematical problems, which are context oriented. The following two

examples illustrate the difference between arithmetic oriented word problem and context oriented application problems in mathematics respectively:

Example 1 (Arithmetic oriented problem)

If you multiply a certain number by 5, and then subtract 3 the answer is 17. Determine the number.

Example 2 (Context oriented true application problem)

(Laridon et al., 2001:175)

*The farmer wanted to ensure that his livestock had drinking water in a remote part of the farm. So he constructed a wind-driven water pump and a cylindrical cement tank to hold the water. A steady wind pumped water into the tank at a constant rate. The level of the water in the tank **rose** at a **rate of $\frac{2}{3}$ cm per minute**. After **30** minutes of pumping, the level of water in the tank was **35 cm** high.*

- a) *Use the information above to draw a graph showing how the level of the water in the tank went up.*
- b) *What was the level of the water in the tank when the pump started?*
- c) *Find the equation of the graph in question a).*
- d) *If the tank is 195cm high, how long will it take to fill it completely at this rate?*

4.3.4.3. Implications of the stage 4 for the mathematics teacher

The assessment preferences (discussed earlier), which characterise the elements of formative assessment in the GIST model, have a primary implication for the mathematics teacher, namely: the teacher must play a key role in the assessment of learners by investing quality time in the development of effective assessment activities. Observing, administering tests, diagnosing and providing remedial work are parts of effective mathematics teaching (Van den Heuvel-Panhuizen 1996:17). The Math teacher has to be conscious of the view that assessment should not only focus on learners' written responses, but also on learners' non-verbal emotive/affective responses. Such gestures/responses are mainly prevalent during the unfolding of the lesson and can only be picked up by the teacher through observation. This further implies that mathematics teachers must be aware that assessment is an ongoing/continuous process, which is integrated within the teaching and learning process of mathematics. Further mathematics teachers are expected to know that contexts enhance meaningful learning in the teaching and assessment of mathematics. Learners who struggle with the comprehension of mathematics concepts due to one or

more of the factors mentioned in (*see* §4.3.1) are likely to improve if relevant and realistic contexts are used.

The structure of the GIST model is cyclic and therefore suggests that if necessary its implementation may be repeated (preferably the beginning of every quarter). However, when the cycle is repeated Step 1 of Stage 1 should be omitted because the diagnostic tool would already be available; while Steps 2, 3 and 4 of Stage 1 will remain applicable but for the purpose of post testing essentially because testing is administered on the same learners to establish the level of mathematics barriers mentioned in §4.3.1 (Step 1).

4.4. Conclusion

The South African curriculum structure and the Norms and Standards for Educators require educators practising in South Africa to possess remedial skills, which can be invoked when learners experience barriers to learning. The implication for mathematics teachers is that they should possess diagnostic and intervention skills pertinent to mathematics teaching and learning. However, the unrealistic perceptions should be avoided which seem to suggest that mathematics teachers should have skills equal to those possessed by psychologists and other specialised service providers in the medical fields. Basic knowledge of what constitutes mathematics learning barriers and what diagnostic measures to employ is adequate to enable mathematics teachers to effectively reach all learners in a mathematics class. Diagnosis of any deep-rooted barriers of neurological, physical and sensory nature, which ideally fall within the medical area of speciality, should still be left for the relevant specialists to attend to. Mathematics teachers should be very proficient in dealing with mathematics learning barriers, which are pedagogic/didactic in nature, i.e. those that can be redressed by modifying/adapting the mathematics teaching-learning practices. These include, but not limited to, **mathematics anxiety, mathematics study environment, mathematics study habits, problem-solving skills and attitudes towards mathematics**. Further the knowledge of different mathematics teaching-learning approaches such as **problem centred instruction, realistic mathematics teaching, cognitively guided instruction, cooperative learning**, and their associated benefits for learners is imperative for mathematics teachers.

The next chapter will provide a comprehensive account of the research methodologies used to probe the impact of the GIST model on learners' mathematics dispositions and achievement. Essentially the following research elements will be articulated: research aims and methods, the hypotheses to be tested, study population and sample, research procedures and data analyses measures.

CHAPTER

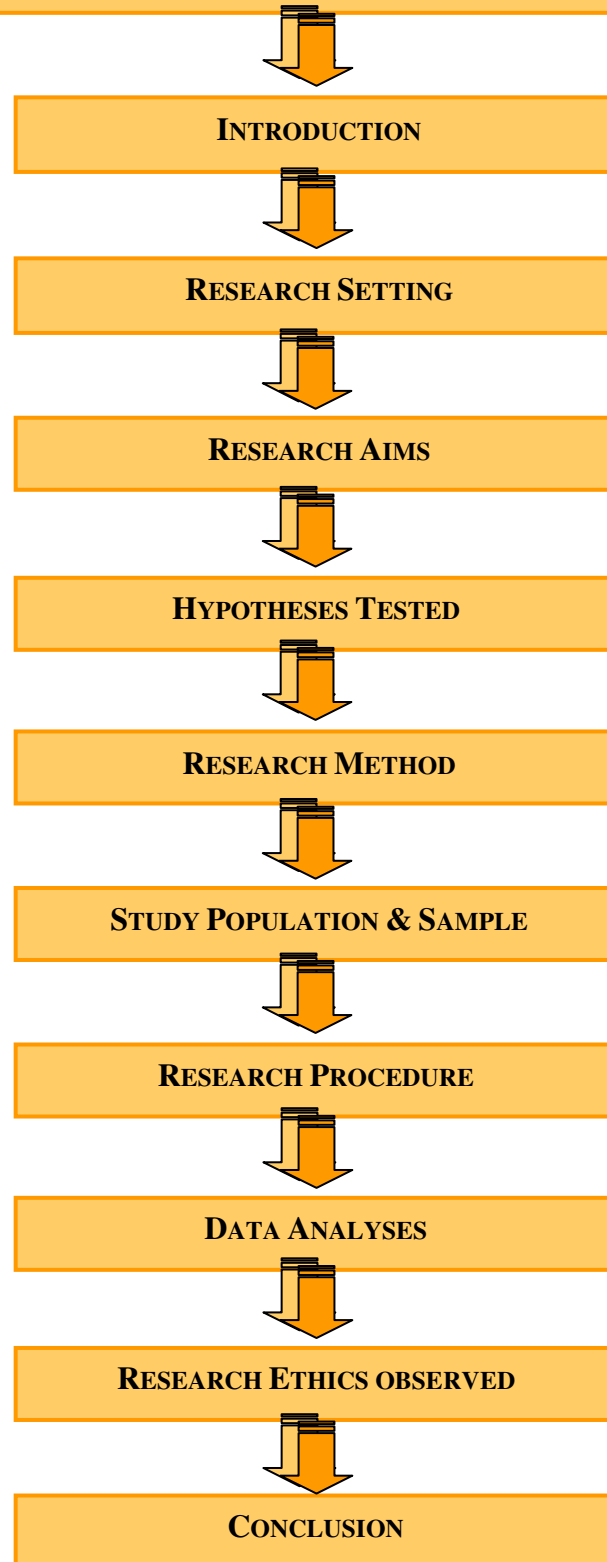
5

METHOD OF RESEARCH

Employing the assortment of data collection techniques (enhanced by using combined research approaches) increases the reliability and authenticity of the results.
(De Vos, 2002:365)

CHAPTER 5

METHOD OF RESEARCH



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5.1. Introduction

With this chapter the researcher aims to offer an elaborate descriptive trajectory of the methodology employed in the study. The combined **quantitative-qualitative** approach (with their associated multiple data collection techniques), referred to by De Vos *et al.* (2005:362) as **methodological triangulation**, was adopted for two primary reasons:

- *firstly* to broaden the horizon from which data were collected; and
- *secondly* to increase the level of confidence that the targeted data are being accurately captured (Berg, 2001:4).

De Vos (2002:365) concurs that employing the assortment of data collection techniques (enhanced by using combined research approaches) increases the reliability and authenticity of the results. **Concurrent triangulation**⁷ was preferred over **explanatory**⁸ and **exploratory**⁹ methods (McMillan & Schumacher, 2006:405; Creswell, 2003:217). The preference of triangulation as a mixed-method design was necessitated by its potential to offset the strengths and weaknesses of each method (McMillan & Schumacher, 2006:405).

5.2. Research setting

The study took place within the classroom setting or environment. The sample was drawn from large mathematics classes ($n \geq 40$) i.e. the class size that exceeds the international teacher-learner ratio of **1:30** (Howie, 2001:100). Cooperative learning perspective dictated the seating arrangement of learners in the GIST class. Some of the dictates of the cooperative learning perspective that characterised the GIST classroom setting are: small group seating pattern; interdependence among learners; and opportunity for teacher-learner and learner-learner interactions. Based on the problem statement (*see* Chapter 1, §1.3) and the aim of the research (*see* §5.3) the intervention was carried out by the practicing mathematics teachers (and not the researcher). The intervention was integrated within the normal teaching-learning time i.e. the time allocated for the teaching and learning of mathematics as per school time table.

⁷ Concurrent triangulation affords equal emphasis on both quantitative and qualitative methods.

⁸ Explanatory triangulation gives more emphasis on quantitative method and less on qualitative method.

⁹ Exploratory triangulation gives more emphasis on qualitative method and less on quantitative method.

5.3. Research aims

Large classes pose a hindrance to the effective teaching and learning (within which assessment is integrated) of school mathematics. Seemingly the main problem associated with large classes particularly in the teaching and learning of mathematics in South Africa emanates from the time allocation. Mathematics has been allocated more teaching-learning time than any learning area in the senior phase (Grades 7-9) i.e. 18% of the total teaching-learning time (DoE, 2002:18). While the time allocated for mathematics is substantially commendable, it has an influence on the frequency of assessment activities or tasks to be given to learners - more time allocation implies more assessment tasks (DoE, 2007:14). This has a profound implication for quality assessment, particularly **formative assessment** and **diagnostic assessment** which respectively require individualised feedback (Taras, 2005:468; Biggs, 1998; Black & Wiliam, 1998) and interventions (Donald *et al.*, 2006:101). It would therefore appear that large class sizes have a potential to compromise: *firstly* the frequency of written activities because more time is spent on marking, *secondly* formative assessment because individualised feedback becomes a laborious exercise, and *thirdly* diagnostic assessment because individualised interventions are practically almost impossible.

The GIST model therefore aims to achieve the following:

- Equipping mathematics teachers with the skills of identifying/diagnosing learning difficulties associated with the learning of school mathematics, and employing appropriate intervention to remedy the situation within the large class context;
- Helping teachers to effectively manage and assist learners who present mathematics learning difficulties (particularly negative mathematics disposition). Westwood (2003:13) emphasises that if learning barriers among learners are not effectively managed, they may impact negatively on the teachers' attitudes and motivation towards the learners and mathematics as a subject;
- Minimising or eroding mathematics anxiety among learners within large classes; and
- Enhancing the development of self-confidence, positive study habits, improved problem-solving behaviours and positive attitudes among learners regarding the learning of mathematics.

5.4. Hypotheses tested

The following research hypotheses were tested in the research:

- **H₀₁** – The application of the GIST model influences the mathematics dispositions of grade 9 learners.
- **H₀₂** – The application of the GIST model influences the mathematics achievement of grade 9 learners.
- **H₀₃** – There is a positive correlation between learners' mathematics achievement and learners' mathematics dispositions.

5.5. Research method

The quantitative and qualitative research methods were used to complement each other. Each research method was implemented using a distinct research design, namely experimental research design and interactive modes for quantitative and qualitative research methods respectively. Elaborate descriptions of experimental research design and interactive modes of research are outlined in §5.5.1.

5.5.1. Research designs

5.5.1.1. Quantitative research design

The sampling procedure was initially randomised to sample the four schools while the classes that participated in the study were selected based on availability (see §5.6 for details). For this reason a specific **quasi-experimental design**, namely **nonrandomised control group pretest-posttest design** (Table 5.1) was used in order to investigate cause-and-effect relationship between the intervention (GIST model) and measured outcomes (as per SOM questionnaire and MAT) (McMillan & Schumacher, 2010:22). According to McMillan and Schumacher (2010:22) quasi-experimental design is commonly used in studies that involve classes that are already organised for instructional purposes and are taught by different teachers.

Table 5.1 The layout of the nonrandomised control group pretest-posttest design

<i>Group</i>	<i>Pre-test</i>	<i>Treatment</i>	<i>Post-test</i>
E	Y ₁	X	Y ₂
C	Y ₁	-	Y ₂

E and C respectively refer to the **experimental group** ($n \approx 45$ per school); **control group** ($n \approx 45$ per school). Y₁ and Y₂ represent the dependent variables (i.e. learning barriers and mathematics achievement) before and after the intervention. X denotes the independent variable (i.e. the GIST model which serves as a treatment or intervention).

5.5.1.2. Qualitative research design

McMillan and Schumacher (2000:255) argues that, unlike in quantitative research, qualitative researchers find it difficult to identify and adhere to a precise research design because more often than not the design is emergent as it evolves during the study. However, qualitative researchers always begin the study with some knowledge of what data are required and the procedures to be employed. It is therefore against this background that a particular interactive **mode of enquiry**, namely a **case study**, will be employed. Interactive qualitative modes use face-to-face interaction between the researcher and the participants in their (participants') natural settings or environment to collect data (McMillan & Schumacher, 2001:35; Ary *et al.*, 2006:457).

There seems to be various definitional characteristic of a case study (VanWynsberghe & Khan, 2007:2). The definition of a case study seems to be further diluted by its inconsistent categorization by researchers as a method, comprehensive strategy, design or methodology (VanWynsberghe & Khan, 2007:2; Jones & Lyons, 2004:71). In their attempt to present a more inclusive definition that seems to resolve the conundrum of a case study, VanWynsberghe & Khan (2007:4) define it as “ ... a transparadigmatic and transdisciplinary heuristic that involves the careful delineation of the phenomena for which evidence is being collected”. Essentially Luck *et al.* (2006:105) concur that case studies can be employed across the research methods/paradigms, namely qualitative and quantitative methods. The phenomena for

which evidence is being collected may be events, programs, settings, social groups, individuals, and communities anchored within real life (McMillan & Schumacher, 2000:11; Ary *et al.*, 2006:457; Creswell, 2007:78; Leedy & Ormrod, 2001:149).

McMillan and Schumacher (2000:11) distinguishes between *with-in site* (case study conducted within one research site) and *multisite* (case study conducted in more than one research site). However, whether the case study takes the form of *with-in site* or *multisite* the purpose should be to probe and understand the same phenomenon (McMillan & Schumacher, 2001:398). Case studies may be used to track the changes an individual undergoes as a result of the intervention (Leedy & Ormrod, 2001:149). This assertion, pertaining to the introduction of the intervention, seems to suggest the potential blend between the case study design and the experimental design. For the purpose of this study, qualitative case study involving multiple sites (four schools) was used. The purpose of the multiple sites was to establish how different schools react to the same intervention, namely the GIST model, and probably to acquire more/diverse information regarding the impact of the GIST model on the teaching and learning of mathematics.

There is a plethora of documented agreement among researchers suggesting that **interviews, observations and document analysis** are the most commonly used techniques of data collection in case studies (Hancock & Algozzine, 2006:16; Creswell, 2007:73; McMillan & Schumacher, 2001:41; Leedy & Ormrod, 2001:149; Ary *et al.*, 2006:458). The aforementioned data collection techniques provided more information on how the GIST model influenced learners' dispositions towards mathematics and towards learners' mathematics achievement (*see* Research hypotheses outlined in §5.4). Essentially the case study research 'design' complements the true experimental design to refine the Group Intervention Strategy phenomenon i.e. to establish the conditions and consequences under which its effectiveness can be enhanced (McMillan & Schumacher, 2001:37). This suggests that, while the **pre-test-post-test control group design** provided quantified data about the influence of the GIST model, the **case study** justified such data (through observations, interviews and document analysis) by providing conditions and consequences to refine the GIST model even further.

5.5.2. Data collection

The techniques used to collect data were informed by the research methods that characterise this study, namely quantitative and qualitative methods. A distinction is therefore made between the quantitative data collection techniques and qualitative data collection techniques employed to gather data in the study. Brief indications of which techniques were used and the purposes for which they were used are discussed next.

5.5.2.1. Quantitative data collection techniques

The **standardized** Study Orientation Mathematics (SOM) questionnaire and a **self-constructed** Mathematics Achievement Test (MAT) were used to collect quantitative data (McMillan and Schumacher, 2001:39). The SOM questionnaire was used to collect data with regard to learners' mathematics disposition before and after exposing them to the treatment i.e. GIST model. The areas of learners' mathematics disposition focused on were: mathematics problem-solving behaviours (PSB); mathematics study attitudes (SA); mathematics study milieu (SM); mathematics anxiety (MA) and mathematics study habits (SH). The structure and statistical requisites of the SOM questionnaire are further described in §5.5.3.1.

The self-constructed mathematics achievement test was used to collect data with regard to the performance of learners before and after the treatment. The MAT focused on **patterns, functions and algebra** of the Senior Phase (grades 7 to 9) mathematics (DoE, 2002e:62) primarily because:

- *firstly* it appears to encompass the Mathematical skills required to demonstrate adeptness in the other sections of mathematics within the Senior Phase, and
- *secondly* it provides foundational knowledge and skills required to demonstrate competency in algebra of grade 10 to 12 in the Further Education and Training (FET) band.

The structure, content validity and other domains of the self-constructed mathematics achievement test are further explained in §5.5.3.2.

5.5.2.2. Qualitative data collection methods

Ary, Jacobs and Razavieh (2002:434) recommend the use of multiple sources of data and/or multiple methods of data collection, respectively called **data triangulation** and **methods triangulation**. Triangulation within the qualitative research method assisted the researcher to gain more insight into the GIST model from different viewpoints and to enhance credibility of the findings. Therefore **interviews**, **observation** and **document analysis** were used to gather qualitative data.

Interview

According to Keats (1997:310), interviews yield data rich in detail and quality where individual differences are not submerged, and can be used with other means of data collection. Gay (1990:204) differentiates between and cautions about the structured interview (easy to analyse but defeats the purpose of interview) and unstructured interviews (yield in-depth responses but difficult to quantify). In order to acquire the combined potential benefits of both the structured and unstructured interview, the semi-structured interview (Appendix B) was adopted (Gay, 1990:204). While the interview was conducted within the confines of the items outlined in the interview protocol learners were allowed freedom of expressing their experiences regarding the intervention administered to them.

The semi-structured interview was conducted in a focus group setting (Ary *et al.*, 2002:434; McMillan & Schumacher, 2006:360) comprising ten randomly sampled learners from E. The rationale for sampling the focus group members from E only is that the semi-structured interview focused on the intervention strategy, namely GIST which only E received. The interviews were conducted by the researcher at different venues (schools) on different days. The audiotape recorder was used to record data primarily for two reasons: it is less distracting than taking notes during the interview, and it provides a verbatim record of notes (Ary *et al.*, 2002:434).

Mathematics teachers who taught the experimental group and the control group were interviewed before the implementation of the GIST model to establish and probe:

- their knowledge (and subsequent application) of the **teaching approaches** employed in the GIST model namely cooperative learning, problem-centred instruction, cognitively-guided instruction and instruction based on realistic mathematics education;
- their knowledge (and subsequent application) of **diagnostic and formative** assessment in mathematics; and
- their knowledge (and subsequent curbing) of learners' mathematics problem-solving behaviour, mathematics anxiety, study attitudes towards mathematics, study environment, and mathematics study habits.

Mathematics teachers who implemented the GIST model were interviewed further after the intervention. Their interview was, however, unstructured because: *firstly* they were required to provide further information about the observed trends, and *secondly* they were required to share their experiences specifically about the GIST model. Mathematics teachers were informed before the implementation of the GIST model that they will be interviewed to offer critical analysis, evaluation and assessment of the GIST model to establish its merits and demerits. The primary aim of critiquing the GIST model was to offer information that would be used to streamline its implementation and strengthen its components. The information shared by the teachers during the interview would also assist the researcher in compiling the **limitations** of the study and the **recommendations** for future research.

Observation

Observation data collection technique has a potential to overcome the limitations associated with self-reports (Borg & Gall, 1989:474). In self-reports such as questionnaires, standardised tests and interviews learners are more likely to bias the information about themselves. In this study the focal point of the observation process was mainly informed by the five fields of the SOM questionnaire (*see* §5.3.2.1), i.e. to observe the prevalence of the symptomatic behaviours related to the fields of the SOM questionnaire. The observation protocol was used in this regard (*see* Appendix A). The

lesson observations were also used to ensure that the GIST model was implemented as required. The continuous recording procedure¹⁰ (McMillan & Schumacher, 2006:208) was used to record the data.

Document analysis

Analyses of official documents offered critical information regarding assessment practices by the teachers and learner responses to assessment tasks or activities. Learner portfolios and teacher assessment portfolios were important documents from which information relevant to assessment was contained. The following specific information was analysed in both the teacher assessment portfolio and learners portfolio:

Teacher assessment portfolio: The National Policy on Assessment and Qualifications for Schools requires teachers to develop the learning programmes, work schedules and lesson plans (DoE, 2007:7). Accordingly, assessment should be integrated within the afore-mentioned three-levels of planning (DoE, 2007:7). It is therefore against this background that the teachers' assessment portfolios were analysed to establish their **implications on mathematics learning barriers**. To achieve this, the assessment tasks (e.g. class work, tests) and assessment tools (e.g. memorandum) developed by the teachers were analysed to verify their *relevance to the content* in the Assessment Standards, *application* to real life contexts, and the coverage of *different forms of assessment*. The learners' mark schedules or sheets were also analyzed with the sole purpose of determining the baseline performance of learners as per the assessment tasks developed by teachers for their classes.

Learner portfolio: Some of the assessment items that form part of the learner portfolio are written tests and class work. The learner portfolios were analysed in conjunction with the teacher portfolios to establish: evidence of written *feedback* (by the teacher) emanating from learner responses; whether learners' incorrect responses were noticed and guidance given for redress; evidence of the use of *learners' own strategies* of problem solving other than those in the assessment tools.

¹⁰ Continuous recording procedure involves the recording of all the behaviours of the learners during the observation session in an exploratory manner. In this study the data on learners' symptomatic behaviours relative to the SOM fields, and the teachers' implementation of the GIST model were recorded.

Measures to enhance validity of data

The following measures, suggested by McMillan and Schumacher (2006:324), were followed to ensure the degree to which the interpretations of qualitative data have mutual meanings from the view point of the researcher and the participants:

Multi-method strategy – lesson observations, interviews and document analysis to corroborate and enhance the credibility of the collected data. Multi-method strategy enhances consistency of emergent assertions.

Participant language – as far as possible data collected during interviews were phrased verbatim to avoid falsification of information. However, corrections were effected in instances where grammatical errors were identified from the learner responses. Further participants were requested to review the researcher's synthesis of the interview and observations for accurate portrayal. Denzin (1997:319) refers to this approach or technique of enhancing credibility of the data as **member-check triangulation**.

Mechanically recorded data – A scanning device was used to capture transcripts of the teachers written questions as well as learners' written responses/calculations from their respective portfolios. In particular, the scanned clips of the teachers' and learners' portfolios enabled the researcher to present the actual written work in order to validate the analyses thereof.

5.5.3. Measuring instruments

Data collection techniques were outlined in §5.5.2 which essentially inform the eventual measuring instruments to utilize. This section aims to present comprehensive analyses of all the measuring instruments or tools that were used to measure the key constructs (namely **mathematics dispositions** and **mathematics achievement**) of the hypotheses outlined in §5.4. The measuring instruments used were the SOM questionnaire, mathematics Achievement Test, observation protocol and Interview protocol. The analyses were primarily conducted in terms of the rationale and layout/structure of the measuring instruments. However

in the process of the analysis of the measuring instrument the researcher attempted to outline the measures followed to ensure validity and/or reliability of the tools and/or the data collected using the same tools.

5.5.3.1. *Study Orientation in mathematics (SOM) questionnaire: Rationale and layout.*

The relevance of the SOM questionnaire (Maree, 1996) to the study is primarily due to diagnostic nature on the five mathematics learning barriers this study aimed to explore i.e. mathematics problem-solving behaviours (PSB); mathematics study attitudes (SA); mathematics study milieu (SM); mathematics anxiety (MA) and mathematics study habits (SH). Therefore it can appropriately measure the effects of the GIST model on learners' mathematics disposition (*see* H₀₁). Further, it is effective for use in large sample size and time economic in terms of administering and marking (Ary *et al.*, 2002:217; Maree *et al.*, 1997:3). Its convenience and simplicity in terms of marking emanates from the five point scale by which it is characterised, where learners estimate their response ratings as illustrated in Table 5.2 and Appendix C by shading the block of their choice.

Table 5.2 Response rating of the SOM questionnaire

Rarely	Sometimes	Frequently	Generally	Almost always
R	S	F	G	A
1	2	3	4	5

The SOM questionnaire is constituted by seventy six questions or items distributed across the five fields as shown in Table 5.3. Ary *et al.* (2002:217) are of the opinion that the comparative norms of the standardised tests must be derived, their validity and reliability established and directions for administering and scoring be prescribed.

Table 5.3 Number of items per SOM field

SOM field	Number of questions
Study Attitudes	14
Mathematics Anxiety	18
Study Milieu	17
Problem-solving behaviour	14
Study Habits	13
	76

Validity: Zeller (1997:822) defines validity as the prevalence of isomorphism between the reality that exists and the description of that reality. In establishing the construct validity of the test, the test developer must define concepts, select and construct scales of indicants, and establish correlations among scales (Zeller, 1997:926). According to Maree *et al.* (1997:27), such measures were taken to ensure content and construct validity of the SOM questionnaire. The questionnaire is a product of rigorous literature study and extensive evaluation by various experts who also checked the correlation trends across the fields/items in order to ensure content validity (Maree *et al.*, 1997:27). Further, the constructs (defined by Ary *et al.* (2002:32)) and Gay (1990:31) as abstractions that cannot be observed directly) i.e. SOM fields, which the questionnaire purports to measure, are identified, clearly defined, and their indicant scaled and correlated (Maree *et al.*, 1997:27). It is therefore assumed that the construct validity of the SOM questionnaire is ensured.

Reliability: Ary *et al.* (2002:254) contend that a test is reliable if there is a consistency in the scores obtained by an individual person in repeated measurements. The reliability of a test is expressed as a coefficient called reliability coefficient (r) whose value ranges from 0 to 1. In the case of the SOM questionnaire, the adapted KR-20 formula was used to determine the reliability coefficient (r) and it was found to be ranging between 0,89 and 0.95 for the five fields according to gender, grade and language (Maree *et al.*, 1997:25). The reliability coefficient that ranges from 0.70 to 0.90 is highly satisfactory. The reliability coefficient for the five fields of the SOM questionnaire relative to this study was calculated and will be presented in §6.3.1 in chapter 6.

5.5.3.2. Mathematics achievement test: Rationale and layout

The self-constructed mathematics achievement test (Appendix D) was used as a pre-test and post-test to measure the learners' mathematics proficiency and the possible correlation between the SOM fields and mathematics achievement. This helped the researcher to test H_{02} and H_{03} (see §5.4). There are two primary rationales for using a researcher's self-developed mathematics achievement test: *firstly* it is well adapted, in terms of the **context**, for the schools at which the study is conducted, and

secondly it focuses primarily on patterns, functions and algebra of Grade 9 mathematics (DoE, 2002:62) which:

- encompasses the mathematical skills required to demonstrate adeptness in the other mathematics topics;
- enhance the development of more generalized language of mathematics (DoE, 2003:21);
- provide foundational knowledge (such as patterns, functions, algebra, mathematical modelling) and skills (such as problem-solving, describing, identifying, analyzing, investigating patterns and relationships) required to demonstrate competency in algebra of Grade 10 to 12 in Further Education and Training (FET) band (DoE, 2003:21; 2002:62);
- enhance the development of mathematics cognitive skills such as generalizing, explaining, describing, observing, inferring, justifying, representing, refuting and predicting (DoE, 2002:63); and
- are the main foci of the Senior Phase (Grades 7 to 9) mathematics as they are collectively allocated 35% of the total teaching time allocated for grade 9 mathematics (DoE, 2003:21).

The layout/structure of the mathematics achievement test comprises twenty multiple-choice questions (*see* Appendix D) which were answered on the supplied answer sheet (*see* Appendix E). Learners were only required to make a cross on their chosen answer. The learners' scores were used for the purpose of **descriptive** and **inferential** statistics (*see* §5.8).

The test was given to three mathematics teachers who teach in the Senior Phase (Grades 7 to 9) and three mathematics subject advisors (also in the Senior Phase and currently employed by the Department of Education) to ensure its **content** and **face validity**. The appropriateness of content was checked against the content relevant for grade 9 as outlined in the mathematics curriculum (DoE, 2002e:74).

5.5.3.3. *Interview protocol: rationale and layout*

The interview protocol was administered to the learners only and not to the teachers. The learners' interview took place in a focus group setting. The primary aim of the focus group setting was to enhance interactions among learners. Patton (2002:386) qualifies the benefit of focus group thus: "Interactions among participants enhance data quality because participants tend to provide checks and balances on each other which erode false or extreme views". The questions posed during the focus group interview were not specifically directed at any particular individual, but at the whole group. Learners could therefore differ, agree or even remind one another about their experiences about the intervention they received.

The structure or layout of the interview protocol is illustrated in Appendix B. The interview protocol consists of three questions only which focused on: learners' views about the importance of understanding their difficulty of learning mathematics; their impressions about small group work; and their views about the contributions of real-life problems to the learning of mathematics. There are spaces reserved for recording learners' responses regarding each posed question. The responses to be recorded were the consensus reached by the learners after interacting with one another; however, if the consensus could not be reached the different versions of the responses were recorded.

5.5.3.4. *Observation protocol: rationale and layout*

Creswell (2007:135) defines a protocol in the context of qualitative research, as a form used to record information collected during an observation or interview. The primary rationale for using the observation protocol was, therefore, to record the observed behavioural trends informed by behavioural trends in the SOM questionnaire. The main foci of the observable behavioural trends demonstrated by learners are the tendencies that define or characterise mathematics anxiety, mathematics problem-solving behaviour, attitudes towards mathematics, mathematics study habits and mathematics learning environment (*see* Appendix A). The specific observed tendencies were recorded during the class visit for the duration of the implementation of the GIST model.

The layout of the observation protocol comprises five fields, namely mathematics anxiety, mathematics problem-solving behaviour, attitudes towards mathematics, mathematics study habits and mathematics learning environment. Each field comprises three observable items whose frequency of prevalence was recorded. The items of each field of the observation protocol are informed by the items of the fields of the SOM questionnaire. The observation protocol was given to two experts in the field of educational psychology in order to ensure its construct validity. Further, the assistance of the statistician was sought to establish the correspondence of the constructs and items of the observation protocol with the SOM questionnaire.

5.6. Study population and sample

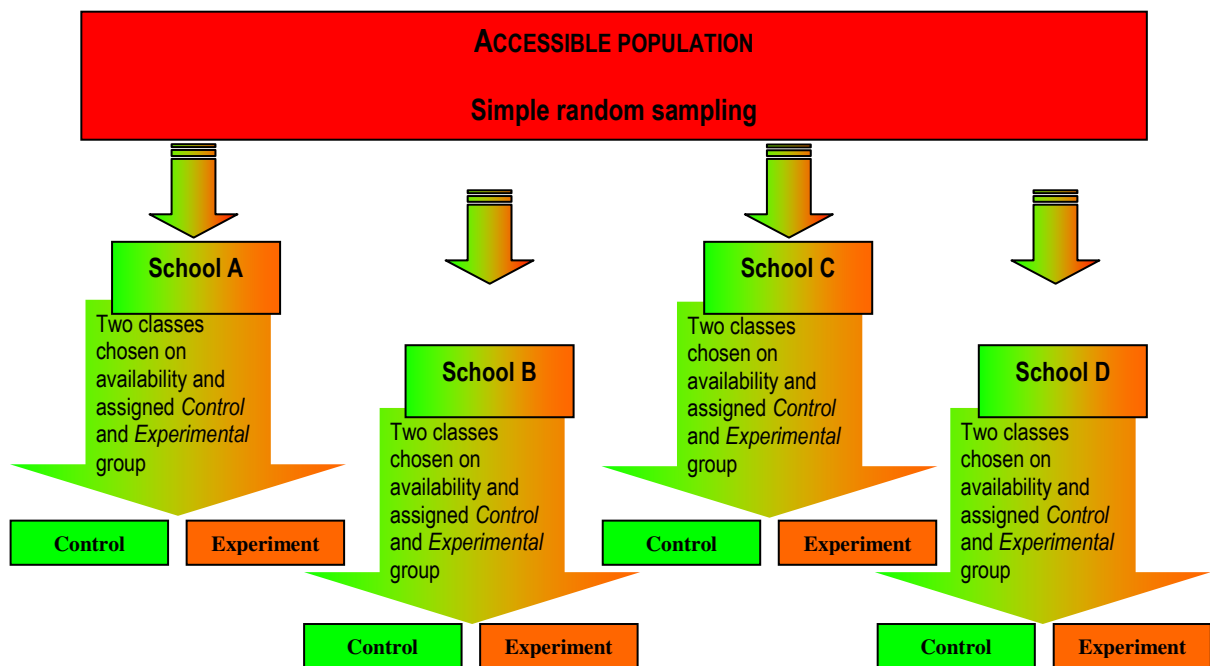
The sampling frames (Ary *et al.*, 2002:381; Ross & Rust, 1997:428) were initially obtained from the Mabopane district from which the total number of schools offering grade 9 and their associated class sizes were established. All the secondary schools in the Mabopane district constituted the study population, while schools that offered grade nine constituted the *target population* of this study (Ary *et al.*, 2002: 164). However, the focus of this study was primarily on grade 9 mathematics classes whose class sizes were large ($n \geq 40$), and they constituted the *accessible population*¹¹ (Ary *et al.*, 2002: 164). **Simple random sampling** was used to draw a sample of four schools from the accessible population. According to Ary *et al.* (2002:167), **simple random sampling** guarantees the probability of all members of the accessible population to be selected without affecting other members' probability of being selected. However, the two classes in each school were selected and assigned control and experimental groups based on availability¹².

While Gay (1990:115) recommends fifteen subjects per group for experimental studies, the sample size in this study was about 333 learners (average of 83 per school, translating to an average of 41 learners for each of the experimental and control group per school).

¹¹ The population from which the sample was drawn.

¹² Randomization at this stage was not possible because some of the schools that participated in the study had only two grade 9 classes, and in the schools where there were more than two grade 9 classes the determining factor was the willingness of the teachers to participate.

Figure 5.1 Sampling procedures



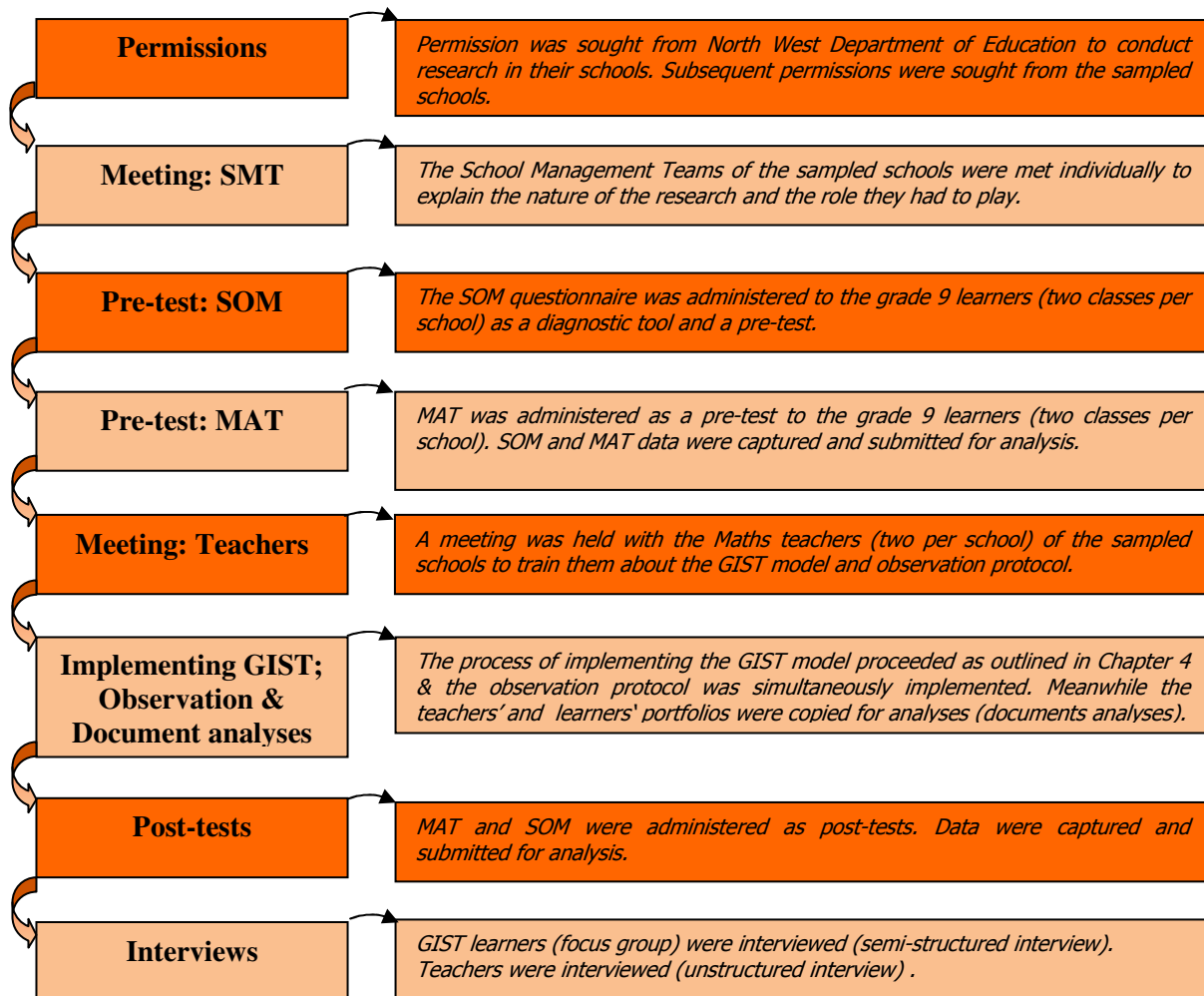
5.7. Research procedure

The sequential procedure of the research unfolded as follows (also *see* Figure 5.2): Permission was sought from the North West Department of Education three months before the research was conducted (*see* Appendix F). Four schools were randomly sampled and permission was sought from the principals of the sampled schools (*see* Appendix G). The principals and mathematics teachers of the randomly sampled schools were informed about the purpose of the study and its prospective benefits for their schools (Maruyama & Deno, 1992:18).

The SOM questionnaire was administered as a pre-test for diagnostic purposes. The MAT was later administered as a pre-test to establish the learners' level of achievement before the intervention. Data collected through the SOM questionnaire and the MAT were captured and submitted to the statisticians for analyses. Mathematics teachers from the four schools were trained (at their different schools) on the application of the GIST model (as presented in chapter 4) and the observation protocol. The training regarding the observation protocol was a rigorous exercise as it involved the simulation of the real observation in the GIST class. The meetings and training sessions with the teachers and the

completion of the SOM questionnaires and answering of the MAT by the learners were done in such a manner that minimal disruption of classes occurred.

Figure 5.2 Synopsis of the research procedure



The processes of administering the intervention, i.e. GIST model, unfolded through experimentation over a period of nine weeks during the third term. The observation protocol was simultaneously used to record observed trends. The SOM questionnaire was administered as post-test to establish the extent to which the intervention had impacted on the learners' disposition towards mathematics. Similarly, MAT was administered as a post-test to establish the degree to which the intervention had impacted on the learners' mathematics achievement. The post-test data were captured and submitted to the statisticians for analyses. Meanwhile the focus groups were selected and the interviews were conducted as per Appendix B.

5.8. Data analyses

McMillan and Schumacher (2001:206) define **descriptive statistics** as a technique that summarises a set of numbers or observations into indices that describe or characterise the data. In this study the summary of quantitative data was done in graphic portrayals before employing other descriptive statistical techniques. Descriptive statistics were used in order to describe the findings with regard to the influence of the GIST model on mathematics learning barriers and on mathematics achievement. The primary descriptive statistics identified by Gay (1990:344), which underpin the analyses of data in this study are: measures of central tendency (to determine the average scores of the groups using mean, mode and median); measure of variability (to indicate the degree of spread of scores using the range and standard deviation); measure of relationship (to indicate the degree of the relationship between two or more group scores using Pearson's correlation coefficient r).

Inferential statistics were also used in order to generalize the study population with regard to the quantitative data (McMillan & Schumacher, 2001:361). The *t*-Test (to compare means of two groups); ANOVA (to compare means of two or more groups); and the standard error of the mean (to indicate the degree to which the sample means would differ if other samples from the same population were used) (Gay, 1990:344) were used for inferential/generalisation purposes. Further, Cohen effect size¹³ was used to assess the magnitude of the difference between the groups and to establish the level of significance of such differences (Cohen, 1988:222; Ary *et al.*, 2002: 151 & 2006:156). Analysis of covariance (ANCOVA) was also conducted to control the possible effect of any extraneous factor that could contaminate the results after the intervention was administered. Ary *et al.* (2006:308) argue that ANCOVA is an effective technique that can be used to control the covariate that might have been prevalent during the pre-test by removing the score associated with it from the post-test.

While descriptive and inferential statistics were both used to analyse **quantitative data**, the analyses of **qualitative data** were dependent on the description of the recorded data obtained from document analyses, interviews and observations. The qualitative data were presented in a narrative structure i.e. where data are presented as quotations of participants' language and field observation notes, documents and interview transcripts are cited as sources (McMillan & Schumacher, 2001:487). Qualitative data were analysed concurrently with data collection to avoid collecting huge amounts of data so that analysis becomes an overwhelming task which may compromise the quality of the work

¹³ Cohen effect size (d) is categorised into three categories: small effect size if $d < 0.3$; medium effect size if $d \geq 0.5$ and large effect size if $d \geq 0.8$ (Cohen, 1988:222).

carried out (Keeves & Sowden, 1997:299) The data analysis **triangulation**, which employs multi-method strategy, was used to analyse qualitative data i.e. observation, interview and document analysis (Ary *et al.*, 2002: 436; McMillan & Schumacher, 2001:408; Borg & Gall, 1989:393). Triangulation affords the researcher the opportunity to corroborate and authenticate the findings, subsequently enhancing credibility of the results and generalisations. Therefore, *open coding*¹⁴ and *axial coding*¹⁵ procedures of analysis were used for the qualitative data analyses (Strauss & Corbin 1998, cited by Leedy & Ormrod (2001:154)).

5.9. Research ethics observed

The assertion by Gudmundsdottir (2001:237) about the importance of being aware of the integral role the ethical issues play in the qualitative research necessitated the observation of the ethical issues. McMillan and Schumacher (2006:333) agree that qualitative research is more prone to personal intrusion than quantitative research, and that credibility of research is enhanced by adherence to research ethics. Research ethics adhered to in this study include:

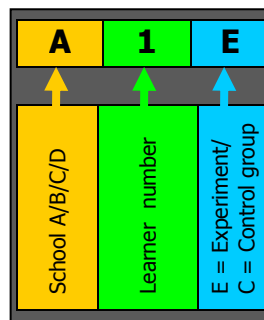
Informed consent – the consent by the School Management Teams was sought after the purpose and process of the research was explained fully to them. A detailed explanation of the research enabled the School Management Teams to make an informed decision whether or not to allow their learners to take part in the study.

Confidentiality and anonymity – Before learners could take the SOM test, they were assured of the confidential nature of the test results, i.e. test results were strictly meant to be used for research purposes and would not even be disclosed to their teachers (Gall *et al.*, 1996:91). For this purpose the learners did not use their real names; instead identification numbers were assigned to them and they (learners) maintained the same numbers during the pre-test and the post-test.

¹⁴ Scrutinising data to establish common attributes and categorising such data accordingly

¹⁵ Establishing the interconnections between categories in order to determine the conditions, contexts and consequences underpinning the dynamics of the GIST model.

The numbers assigned to learners resembled the following design:



Therefore, learner **A1E** refers to a learner in school A, whose number is 1 and is in the experimental group. Similarly, learner **A1C** would refer to the learner in school A whose number is 1 and is in the control group. Learners were numbered chronologically from number one to the last number in each class. This suggests that if there were fifty learners in the control group and forty-five learners in the experimental group of school A, the former were numbered from one to fifty and the latter were numbered from one to forty-five. The same goes for the control and experimental groups of schools B, C and D.

Caring – The research did not have a potential to inflict physical pain on the learners. However, it had a potential to cause emotional pain as it focuses mainly on learning difficulties which are primarily psychological. In order to avert the potential for emotional pain, learners were made aware of the emotional benefits the study had i.e. addressing learning barriers which is an emotional relief in itself. The trust between the researcher and the learners was also maintained in order to enhance openness among the learners.

Feedback – The outcomes of research were made available to the schools that took part in the study as well as to the district office as they have requested. This is against the background that teachers were implementing the GIST model and knowing the research results would help them reflect on the teaching and learning of mathematics for future improvement. However, the results were revealed collectively to individual schools and not per learner to avoid defeating the principle of *confidentiality* and *anonymity*.

5.10. Conclusion

The use of the multiple-methods of research, especially triangulation within and across the research methods, afforded the researcher an opportunity to identify the attributes of the changes in the behavioural patterns of the learners. This suggests that the researcher was able to use interviews, observation and document analysis (qualitative methods) to further explain and make inferences regarding the revelations of the quantitative methods. It is against this background that the use of combined research methods for the purpose of data collection and subsequently data analyses promote credibility with regard to the research findings.

The next chapter (Chapter 6) will present the findings of the **data quantitative** (collected through the SOM questionnaire and MAT) and **qualitative data** (collected through the use of observation protocol, document analysis and interview protocol). Later in the chapter the focus will be on the discussions of the findings and in particular the impact they (findings) had on the hypotheses outlined in §5.4 of the current chapter. Essentially the findings will inform the acceptance or rejection of the hypotheses.

CHAPTER

6

FINDINGS & DISCUSSIONS

Some of the issues learners raised, especially those that relate to teaching, confirm the assertions that teachers contribute enormously towards learners' negative mathematics disposition

(Geist, 2010:28; Hackworth, 1992:10; Donald *et al.*, 2006:312; Furner & Duffy, 2002:68)

CHAPTER 6

FINDINGS & DISCUSSIONS



INTRODUCTION



CASE DESCRIPTION



QUANTITATIVE RESEARCH FINDINGS



QUALITATIVE RESEARCH FINDINGS



DISCUSSIONS



CONCLUSION

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6.1. Introduction

The two main sections of the findings of this study, namely quantitative and qualitative findings, are presented in this chapter. The actual findings are preceded by the case descriptions, namely the schools that constituted the case studies. The description is deemed essential because: *firstly* it provides a context in which the study was conducted; and *secondly* it provides peculiar contextual factors that will assist in the discussion and conclusions drawn from the interpreted data.

6.2. Case description

The descriptions of the four schools are summarised in Table 6.1. The primary focus of the description is on the five key strands named below. The information on learner enrolment was gleaned from each school's class register (also called attendance register) while the time allocated for mathematics was obtained from each school's timetable. The information on teachers' knowledge of diagnostic assessment and learner-centred teaching and learning approaches was obtained from the interviews.

- Location of the school;
- Learner enrolment: to establish the teacher-learner ratio;
- Teaching-learning time allocation: to establish compliance with the departmental policy;
- Knowledge of diagnostic assessment: to specifically establish the teachers' knowledge of the barriers towards the effective learning of mathematics; and
- Knowledge of specific learner centred approaches: to establish the instructional approaches teachers are likely to use in their mathematics classes.

It is evident from the summary on Table 6.1 that the location of all four schools is fairly similar. This is essentially a reflection of settlement dynamics of the district in which the study was conducted. While the average class sizes of the four schools is fairly equal, schools A and C have the highest teacher-learner ratio of 1:105 and 1:90 respectively. In terms of instructional time allocated for mathematics, school B complied with the 18% instructional time stipulated by the Department of Education while schools C and D allocated less time of 16.6% and 15% respectively. Generally the grade 9 mathematics teachers of the four schools did not have any in-depth knowledge and understanding of diagnostic assessment and the learner-centred teaching-learning approaches.

Table 6.1 Description of the schools' contexts

School	Location	Learner enrolment in grade 9	Time allocation (18% prescribed)	Knowledge of Diagnostic assessment	Knowledge of Learner- centred approaches
A	Township. Majority of learners reside in the nearby previously informal settlement.	316 Learners (average class size \approx 50); 3 Mathematics teachers for grade 9 learners. One of them has been declared in excess ¹⁶ .	19% of the total teaching time. 30 Minutes per period.	No knowledge of diagnostic assessment; No knowledge of the mathematics learning barriers.	Vey superficial knowledge; deduced from the descriptive names such as cooperative learning. Learners are seated in pairs-in-rows.
B	Township. Majority of learners reside in the same area in which the school is located.	119 Learners (average class size \approx 40); 2 Mathematics teachers for grade 9.	18% of the total teaching time. 1 Hour per period.	No knowledge of diagnostic assessment; Both teachers have some knowledge of mathematics attitudes and study habits but do not know how to identify them.	Vey superficial knowledge of cooperative learning. Learners are seated in pairs-in-rows.
C	Township. Majority of learners reside in the same area in which the school is located.	269 Learners (average class size \approx 50). 3 Mathematics teachers for grade 9.	16.6% of the total teaching time. 35 Minutes per period.	No knowledge of diagnostic assessment; They both have some idea about mathematics attitudes, study milieu and study habits	Vey superficial knowledge of cooperative learning. Learners are seated in pairs-in-rows.
D	Previously informal settlement. Majority of learners reside in the same area in which the school is located.	290 Learners (average class size \approx 48); 4 Mathematics teachers for grade 9.	15% of the total teaching time. 30 Minutes per period.	No knowledge of diagnostic assessment; The two teachers have considerable knowledge of the mathematics attitudes and study habits but do not know how to identify them.	They have some knowledge of cooperative learning and problem-centred learning. The knowledge was, however, inferred from the descriptive names of the two approaches.

¹⁶ According to the school's post establishment the school is over-staffed by 8 teachers (one of them a grade 9 mathematics teacher). The excess teachers will be transferred to the schools where their services are required.

It emerged from the interviews that the teachers who have shown some knowledge of learner-centred approaches such as cooperative learning merely deduced their meanings from the descriptive nature of their names. When probed further it became evident that their understanding was very superficial.

6.3. Quantitative research findings

The quantitative research findings are based on the data that were collected through the SOM questionnaire and MAT.

6.3.1. Reliability – Cronbach's coefficient alpha

The reliability coefficient was calculated during the standardisation process of the SOM questionnaire using the adapted KR-20 formula and was found to be ranging between 0,89 to 0.95 for the five fields according to gender, grade and language (Maree *et al.*, 1997:25). Since the sample for this study was drawn from one particular district and might not have been as representative as the original SOM sample, there was a need to determine its reliability for this study. For this reason the Cronbach coefficient alpha (α) was calculated and it revealed that the reliability coefficients of the SOM fields range from 0.64 to 0.79, which is satisfactory, notwithstanding the lower levels of reliability compared to the standardised SOM.

6.3.2. *t*-Test between the groups for the pre-test

6.3.2.3. *t*-Test results between the control and experimental groups: MAT pre-test

The *t*-test results presented in Table 6.2 are for the combined control groups ($n = 170$) and experimental groups ($n = 163$) with regard to MAT. The *t*-test was used to establish the degree to which the mean of the control group and that of the experimental group differed before the treatment was introduced. The Cohen Effect size (d) with relevant guidelines for its interpretation (Cohen, 1988:222) as well as the p -value were used to determine the extent to which the

means of the groups differed. The d -values are presented below Table 6.2. The t -test result revealed a difference of no practical significance (small effect, $d \approx 0.2$, and $p > 0.05$).

Table 6.2 Difference of means between the groups – MAT (pre-test)

Group	n	Variable	Mean	SD	t	$P > t $	Effect size (d)
C	170	MAT	4.39	1.81	-0.96	0.3359**	0.10 ⁺
Exp	163	MAT	4.59	1.88			

*significant at 1% level

* $d \approx 0.2 \Rightarrow$ small effect size

**significant at 5% level

** $d \approx 0.5 \Rightarrow$ medium effect size

***significant at 10% level

*** $d \approx 0.8 \Rightarrow$ large effect size

6.3.2.4. t -Test results between the control and experimental groups: SOM pre-test

The t -test results for the combined control groups ($n = 175$) and experiment groups ($n = 166$) relative to the SOM fields are presented in Table 6.3. The t -test procedure was carried out to determine the similarity between the two groups in terms of the five fields of SOM, namely study attitudes, mathematics anxiety, study habits, problem solving behaviour and study milieu. It was desired that the two groups should not show any difference of practical nature before administering the intervention.

Table 6.3 Difference of means between the groups per SOM fields: pre-test

Variable	Group	n	Mean	SD	t	$P > t $	Effect size (d)
Study attitude	C	175	36.72	9.04	-1.84	0.0672*	0.19 ⁺
	E	166	38.46	8.40			
Mathematics anxiety	C	175	35.95	7.40	0.12	0.9076	0.01 ⁺
	E	166	35.86	7.35			
Study habits	C	175	41.15	10.72	-2.07	0.0389**	0.22 ⁺
	E	166	43.55	10.66			
Problem solving behaviour	C	175	39.43	10.58	-0.76	0.4454	0.08 ⁺
	E	166	40.31	10.65			
Study milieu	C	175	34.18	7.02	-0.1	0.9224	0.01 ⁺
	E	166	34.25	7.36			

*significant at 1% level

* $d \approx 0.2 \Rightarrow$ small effect size

**significant at 5% level

** $d \approx 0.5 \Rightarrow$ medium effect size

***significant at 10% level

*** $d \approx 0.8 \Rightarrow$ large effect size

The *t*-test revealed that the means of the control group and the experimental group relative to the SOM fields did not show a difference of practical significance before the intervention. In all the five SOM fields small effect sizes ($d \approx 0.2$) were recorded. The difference between the groups relative to study habit and study attitudes was significant at $p = 0.05$ and $p = 0.1$ respectively.

6.3.3. The dependent *t*-test within the groups for the difference between the pre- and post-test

6.3.3.1. The dependent *t*-test results within the **control groups**: difference between pre- and post-test

The dependent *t*-test was done to determine the difference in the pre- and post-test means within the control group (all schools combined, $n = 321$) for MAT post-test ($n = 153$) and SOM post-test ($n = 168$). The difference in the means of SOM was calculated per SOM field. The results presented in Table 6.4 reveal the mean difference of no practical significance ($d \approx 0.2$) within the groups for MAT pre- and post-tests and SOM fields pre- and post-tests.

Table 6.4 Difference of means within the control groups: post-pre-test

Difference (post-test – pre-test)	<i>n</i>	Mean	DF	SD	SE	Effect size (<i>d</i>)
MAT	153	0.54	152	2.62	0.21	0.21 ⁺
Study attitude	168	-0.40	167	7.03	0.54	-0.06 ⁺
Mathematics anxiety	168	-0.70	167	6.37	0.49	-0.11 ⁺
Study habits	168	-0.05	167	8.70	0.67	-0.01 ⁺
Problem solving behaviour	168	0.91	167	8.59	0.66	0.11 ⁺
Study milieu	168	-1.16	167	6.20	0.48	-0.19 ⁺

*significant at 1% level

**significant at 5% level

***significant at 10% level

⁺ $d \approx 0.2 \Rightarrow$ small effect size

⁺⁺ $d \approx 0.5 \Rightarrow$ medium effect size

⁺⁺⁺ $d \approx 0.8 \Rightarrow$ large effect size

6.3.3.2. The dependent *t*-test results within the **experimental groups**: difference between pre- and post-test

The mean procedure was also done to determine the difference in the means within the experimental group (all schools combined, $n = 311$). The experimental group ($n = 158$) registered an improved performance in MAT post-test when compared to the same group's performance in MAT pre-test. This is evident in the statistically significant mean difference of large effect size ($d = 0.79$). The difference of means within the experimental group for the SOM fields has no practical significance ($d \approx 0.2$, small effect size).

Table 6.5 Difference of means within the experimental groups: post-pre-test

Difference (post-test – pre-test)	<i>n</i>	Mean	DF	SD	SE	Effect size (<i>d</i>)
MAT	158	2.62	157	3.33	0.27	0.79 ^{**}
Study attitude	153	-0.94	152	6.67	0.54	-0.14 ⁺
Mathematics anxiety	153	0.15	152	7.17	0.58	0.02 ⁺
Study habits	153	-0.87	152	8.27	0.67	-0.11 ⁺
Problem solving behaviour	153	0.46	152	8.65	0.70	0.05 ⁺
Study milieu	153	-0.67	152	6.86	0.55	-0.10 ⁺

*significant at 1% level

* $d \approx 0.2 \Rightarrow$ small effect size

**significant at 5% level

** $d \approx 0.5 \Rightarrow$ medium effect size

***significant at 10% level

*** $d \approx 0.8 \Rightarrow$ large effect size

A more intensive analysis of the mean difference within the experimental group was carried out to gain more insight in the individual school's performance regarding MAT. The results of the paired *t*-test in Table 6.6 suggest that the performances of E₂ and E₄ are practically significant relative to MAT (large effect size, $d \approx 0.8$). The findings have also revealed a statistically significant performance for E₁ and E₃ (medium effect size, $d \approx 0.5$).

Table 6.6 Paired *t*-test within the groups regarding MAT

Group	<i>n</i>	Mean difference	SD	SE	Effect size (<i>d</i>)	Cohen's category
E ₁	34	2.09	2.96	0.51	0.71**	medium
E ₂	34	2.65	3.09	0.53	0.86***	large
E ₃	42	2.05	3.11	0.48	0.66**	medium
E ₄	48	3.48	3.81	0.55	0.91***	large

**d* ≈ 0.2 ⇒ small effect size

***d* ≈ 0.5 ⇒ medium effect size

****d* ≈ 0.8 ⇒ large effect size

6.3.4. Analysis of covariance (ANCOVA): post-test

ANCOVA was conducted to control extraneous variables (covariates) that might have been prevalent during the pre-test and could contaminate the results of the post-test. In this study the covariate was the pre-test scores. The root mean square error (RMSE) signifies the approximation for the standard deviation of data.

Table 6.7 Adjusted post-test means: ANCOVA

Variable	<i>n</i>	Control	Experiment	RMSE	P-value	<i>d</i> -Value
MAT	311	4.80	7.28	2.15	0.019	1.15***
SA	321	37.09	37.27	6.14	0.030	0.03*
MA	321	35.50	36.19	6.06	0.483	0.05*
SH	321	42.08	42.07	7.85	0.043	0.01*
PSB	321	40.69	40.68	7.75	0.381	0.001*
SM	321	33.26	33.80	5.77	0.346	0.095*

**d* ≈ 0.2 ⇒ small effect size

***d* ≈ 0.5 ⇒ medium effect size

****d* ≈ 0.8 ⇒ large effect size

After the possible influence of the pre-test was controlled by adjusting the post-test means, there is no variation between the pre-test and post-test means for the SOM

fields. None of the d -values for the SOM fields show a difference of practical significance. However there is a difference of statistical significance at 5% - level for SA ($p = 0.030$) and SH ($p = 0.043$).

6.3.5. Correlation between the SOM fields and MAT

The Pearson Correlation coefficient (r) was used to determine the correlation between the SOM fields and MAT before and after the treatment. According to McMillan and Schumacher (2001:230) the value of r ranges from -1 to $+1$ and it implies strong negative correlation when $r = -1$; no correlation when $r = 0$; and strong positive correlation when $r = +1$.

The data in Table 6.8 show the correlation of the pre-SOM (control and experiment) and MAT (before and after the intervention). Generally there appears to be no correlation between the SOM fields and the MAT before and after the treatment in the control group and the experimental group ($r \approx 0$ in all the cases).

Table 6.8 Correlation between SOM (pre-test) and MAT (pre and post test) for control group and experimental group

	Control group (pre-SOM) $n = 175$					Experimental group (pre-SOM) $n = 166$				
	SA	MA	SH	PSB	SM	SA	MA	SH	PSB	SM
Pre MAT ($n=170$)	0.0701	0.035	0.042	0.001	0.125	0.027	0.000	0.015	0.030	0.001
Post MAT ($n=153$)	-0.062	-0.122	-0.024	-0.043	-0.038	0.091	-0.058	0.124	0.080	0.056

The data in Table 6.9 show the correlation of the post-SOM (control and experiment) and MAT (pre and post). None of the correlations were practically significant.

Table 6.9 Correlation between SOM (post-test) and MAT (pre and post test) for control group and experimental group

	Control group (post-SOM) <i>n</i> = 168					Experimental group (post-SOM) <i>n</i> = 153				
	SA	MA	SH	PSB	SM	SA	MA	SH	PSB	SM
Pre MAT (<i>n</i> = 163)	-0.016	0.047	0.002	-0.006	0.060	0.033	0.012	0.076	0.035	0.126
Post MAT (<i>n</i> = 158)	-0.050	-0.180	-0.095	-0.061	-0.016	0.064	-0.055	0.111	0.054	0.031

6.4. Qualitative research findings

The qualitative research findings presented in the next paragraphs were acquired from the document analysis, lesson observations and interviews.

6.4.1. Document analysis

Three documents that relate to assessment were analysed prior to the intervention. The fundamental aim was to establish the state of assessment regarding the setting of questions, and marking as well as the nature of feedback given to learners in each school. These data would help in the training of teachers of the experimental groups and to strengthen the implementation of the GIST model. *Firstly* the analyses of the teachers' portfolios were aimed at establishing whether:

- the assessment tasks focus on the **Assessment Standards**¹⁷. This is essential because the Assessment Standards provide mathematics content (what used to be called mathematics 'syllabus') for each grade as well as the skills to be acquired by the learners. Learner achievement in

¹⁷ The Assessment Standards give an indication of the mathematics content to be taught and assessed.

mathematics is primarily a demonstration of attainment of mathematics Assessment Standards (in which the skills and content knowledge are entrenched); and

- the assessment tasks focus on the **application of mathematics in real life context**.

Secondly the analysis of the learners' portfolios was intended to establish whether:

- teachers gave written **constructive feedback** as they mark the learners' written work because constructive feedback is the foundation of formative assessment (Taras, 2005:468; Biggs, 1998; Black & Wiliam, 1998) and it has been positively correlated to achievement (Black & Wiliam, 1998); and
- learners are credited for using their **own methods of problem solving** other than those in the teachers' marking guides/memoranda.

Lastly, the analysis of the mark schedules was intended to establish the average learner performance against the 7-point national coding system as outlined in the National Policy on Assessment for grades R to 9 (DoE 2007:13)(*see* Table 6.10). The mark schedules contain the record of learners' performance as per the assessment conducted by the teachers during the school year. Essentially the idea was to obtain a broad picture about the mathematics landscape of each school without comparing them with each other because the assessment was the discretion of the teachers in each school.

Table 6.10 National codes for recording and reporting learner performance

(DoE, 2007:13)

Rating code	Description of competence	Percentage
7	Outstanding achievement	80-100
6	Meritorious achievement	70-79
5	Substantial achievement	60-69
4	Adequate achievement	50-59
3	Moderate achievement	40-49
2	Elementary achievement	30-39
1	Not achieved achievement	0-29

6.4.1.1. School A

Teacher assessment portfolios

Assessment Standards: The revelation emanating from the analysis of the teachers' assessment portfolios points to the challenges regarding assessment (particularly appropriate interpretation of the assessment standards). The content of the assessment tasks does not match the content in the assessment standards that the teacher intended to assess. For instance, one of the tasks given to the learners focused on the calculation of the mean, median, mode and range. To be more precise the question was stated as follows:

*“Given the set of data, **find** the (i) mean (ii) median (iii) mode (iv) range”.*

Based on the teachers' memorandum it is assumed that **'find'** implied **'calculate'**.

Among the assessment standards listed in this assessment task were the following:

Assessment standard 9.5.1 which states:

'... the learner poses questions relating to human rights, social, economic, environmental and political issues in South Africa' (DoE 2002:89).

Assessment standard 9.5.4 which states:

'... the learner draws a variety of graphs by hand/technology to display and interpret data including: ...' (DoE 2002:91).

It is quite noticeable that the content embedded in assessment standards 9.5.1 and 9.5.4 do not correlate with the contents of the question posed by the teachers as cited above. The two assessment standards focus respectively on **posing of (research) questions** and **drawing of graphs** which cannot mean calculating the mean, median, mode or range.

Application: None of the assessment tasks analysed showed any evidence of the use of real life contexts to apply to mathematics concepts learnt. The assessment activity that involved data handling, which can almost always be drawn from real life scenarios, involved the use of a set of data that learners had to manipulate to determine measures of central tendency and dispersion.

Forms of Assessment: In terms of the forms of assessment, class work and tests were the most prominent. This practice defeated the purpose for which a variety of the forms of assessment is designed to achieve, i.e. accommodating the learners' different learning styles. This finding was anticipated given that the prior interview of the teachers revealed the lack of knowledge of learner-centred approaches. Essentially, mathematics investigations and projects require learners' ingenuity, use of own strategies and often require learner-learner collaboration to enhance mathematics performance.

Learner assessment portfolios

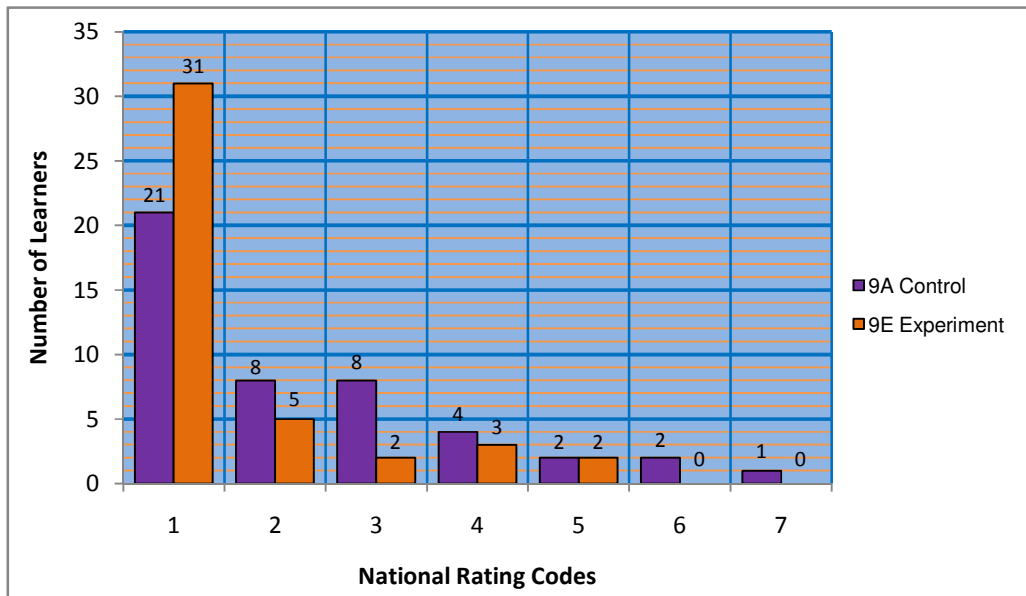
Use of learners' own methods of solving problems: In an attempt to solve the activities given by the teachers, all the learners used the same approach of arriving at the solution (whether correct or incorrect). The similarity in the approach is indicative of, and may be attributed to, the 'transmission' type of teaching that dominated the mathematics classes. Learners were not encouraged to use their own methods of solving mathematics problems.

Constructive feedback: The evidence of written feedback could not be revealed from the learners' written work that was analysed. Only a wrong (×) mark was given without any written justification and guidance for future improvement.

Mark schedules

The findings of the analysis of the learner performance as per mark schedule are presented in Figure 6.1. The findings revealed that thirty-one learners (approximately 72%) of the experimental group learners were performing at level 1 compared to only twenty-one learners (52%) of the control group learners. There were no learners performing at level 6 and 7 in the experimental group. It is evident that in terms of the national codes the control group performed better than the experimental group.

Figure 6.1 Performances in mathematics per National Rating Codes: School A



6.4.1.2. School B

Teacher assessment portfolios

Assessment Standards: Generally the assessment tasks are aligned to the Assessment Standards. Even though the teachers demonstrated a commendable practice of addressing the Assessment Standards, a discrepancy was identified

where the tests given to the learners by both teachers were similar but the memoranda differed. The question was stated thus:

Simplify: $(2a^2b^3)^3 \times (3ab^3)^2$

This question was allocated three marks by both teachers. The actual transcripts of the memoranda (indicating the answers and the distribution of marks) of the two teachers are shown in Figure 6.2. Teacher A and Teacher B respectively refer to the teacher of the control class and experimental class.

Figure 6.2 Memoranda transcripts (School B)

Teacher A	Teacher B
$(2a^2b^3)^3 \times (3ab^3)^2$ $= 2^{1 \times 3} a^{2 \times 3} b^{3 \times 3} \cdot 3^{1 \times 2} a^{1 \times 2} b^{3 \times 2}$ $= 2^3 a^6 b^9 \cdot 3^2 a^2 b^6$ $= 72 a^{6+2} b^{9+6}$ $= 72 a^8 b^3$ <div style="text-align: right; border: 1px solid black; padding: 2px; width: fit-content; margin-left: auto;">3 marks for the answer only</div>	$(2a^2b^3)^3 \times (3ab^3)^2$ $= 8a^6b^9 \times 9a^2b^6$ $= 72a^{6+2}b^{9+6}$ $= 72a^8b^{15}$ <div style="text-align: right; border: 1px solid black; padding: 2px; width: fit-content; margin-left: auto;">3 marks for each step</div>

There are fundamentally two major differences between the two memoranda: *firstly* there is a discrepancy regarding the allocation of marks. Teacher A allocated three marks for the correct answer, presumably one mark for '72', one mark for a^4 and another one mark for b^8 . Contrary to Teacher A, Teacher B allocated one mark for each of the last three steps. *Secondly* there is a computational discrepancy where Teacher A applied an inappropriate law of exponent in the second last step and as a result obtained an incorrect answer to the original exercise.

Application: The application of mathematics remains unattended to as is the case with school A. All the activities, other than those used in the transcript, did not show any evidence of real life scenarios. At the time of the analysis the concept 'equations' which offers abundant opportunities for application had already been covered. However, the teachers followed the conventional route of solving equations in which alphabets are variables, e.g. solve for y in $y = 2x + 1$ if $x = 2$

Variety of forms of assessment: Learners were not exposed to different forms of assessment. The assessment was dominated by tests and class work. Assignments, mathematical investigations and projects were not catered for at all. The assessment tools (marking guides) were exclusively memoranda and they were mainly suitable for the use in class works and tests. Rubrics and check lists which are mainly used in investigations and projects were not used.

Learner assessment portfolios

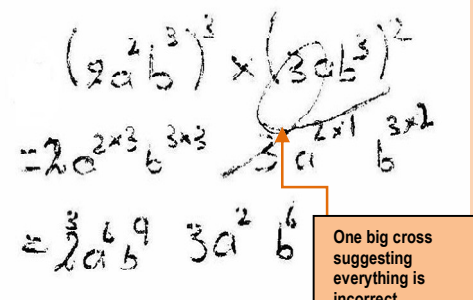
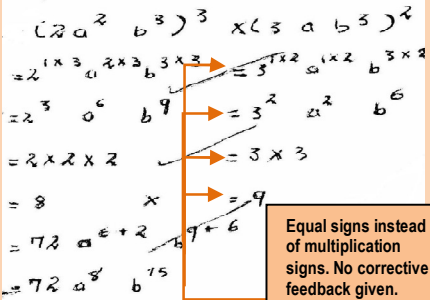
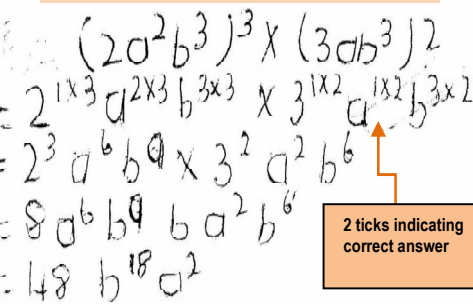
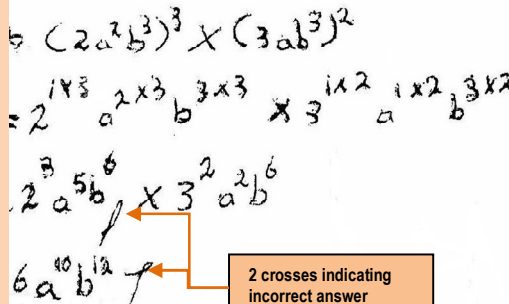
Use of learners' own methods of solving problems: Similar approaches were used to solve the mathematics problems in all the activities that were analysed. It would appear that learners relied only on the methods given to them by their teachers, and the teachers did not encourage their learners to explore other avenues of problem solving. However, this was not surprising given that the appropriate forms of assessment (mathematics investigations and projects) which provide the opportunity to explore other methods were neglected.

Constructive feedback: Written constructive feedback was not given. Like many other activities analysed, it is evident that teachers marked wrong the learner responses without a comment or guide on how they (learners) should do the activity differently. Some of the learner responses from the two classes regarding the question which stated: **Simplify** $(2a^2b^3)^3 \times (3ab^3)^2$ are shown in Figure 6.3.

The computation carried out by Learner 1 from Teacher A's class is not completely wrong. The appropriate law was applied in simplifying the variables while the same law of indices was not applied in simplifying the coefficients. This learner seems to have the knowledge or at least the idea about a particular law of exponents. However, the teacher marked him/her wrong possibly because he (the teacher) was guided by his incorrect memorandum.

Similarly, the computation carried out by Learner 2 from Teacher B' class shows two equal signs in each step. However, the teacher marked only the answers on the left-hand side of the second equal sign in each step. It is, however, evident that when Learner 3 was solving the exercise s/he thought of adding the exponents of $a^{2 \times 3}$ and $b^{3 \times 3}$ in $(2a^2b^3)^3$ as shown in Step 3.

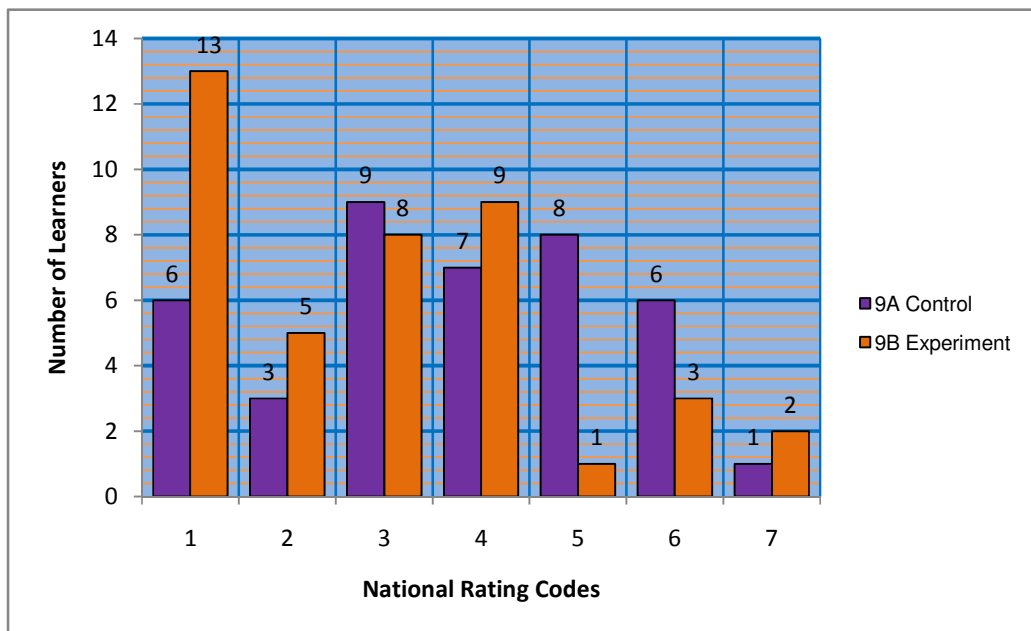
Figure 6.3 Learner responses (School B)

<p>Learner 1 (from Teacher A's class)</p>  <p>One big cross suggesting everything is incorrect</p>	<p>Learner 2 (from Teacher B's class)</p>  <p>Equal signs instead of multiplication signs. No corrective feedback given.</p>
<p>Learner 3 (from Teacher A's class)</p>  <p>2 ticks indicating correct answer</p>	<p>Learner 4 (from Teacher B's class)</p>  <p>2 crosses indicating incorrect answer</p>

Mark Schedules

The mark schedule of the summative assessment conducted by the teachers before the study was carried out was analysed (see Figure 6.4). The mark schedules of the two classes revealed that the experimental group (grade 9B) had a greater number of learners whose average mathematics performance was in Level 1 (from 0% to 29%) and Level 2 (from 30% to 39%) than was the case with the control group (grade 9A). However, a substantial number of the control group performed at Level 5 (60% to 69%) and Level 6 (70% to 79%) when compared to their counterparts in the experimental group.

Figure 6.4 Performances in mathematics per National Rating Codes: School B



6.4.1.3. School C

Teacher assessment portfolios

Assessment Standards: The assessment tasks covered the mathematics content embedded in Assessment Standards. However, the occurrence similar to that in school B regarding the mistakes in the memorandum was prevalent in school C. The incorrect memorandum was used to mark the learners' assessment tasks (see Figure 6.5).

Application: Application of mathematics was not catered for in the assessment tasks. Conventional mathematics problems which directly focus on mathematics content were given.

Figure 6.5 Question and its memorandum

The figure is divided into two main sections. The top section, titled "The teacher's actual question", contains the handwritten text: "Subtract $(10m^3 - 10m^2 - 10m)$ from $(m^3 + m^2 + m)$ ". The bottom section, titled "The teacher's actual memorandum", shows a handwritten subtraction problem. The top line is $10m^3 - 10m^2 - 10m$ with a horizontal line underneath. The second line is $-m^3 + m^2 + m$ with a checkmark to its right. A horizontal line separates this from the result $9m^3 - 11m^2 - 9m$, which also has a checkmark to its right. A bracket on the right side of the top two lines points to a text box that says "Ordering of the expressions is incorrect". At the bottom right, a text box says "Incorrect answers resulting from wrong order of the expression based on the question", with three arrows pointing upwards to the terms $9m^3$, $-11m^2$, and $-9m$ in the result line.

Variety of forms of assessment: Tests were predominantly used while investigations, projects and class work were not used altogether. The major finding regarding the activities constituting the tests was that they focused only on the polynomials (multiplication, addition, subtraction and division), bar graphs, data handling and finance mathematics. It was evident that the teacher would not be able to cover the bulk of the outstanding content prescribed for the grade 9 learners.

Learner assessment portfolios

Use of learners' own methods of solving problems: Similar to the two previous schools, learners in school C used the same methods to solve the given mathematics activities. Presumably the method they used in each activity was dictated to by the teacher.

Constructive feedback: The sample of the learners' portfolios that was analysed did not reflect any developmental feedback. Learners were marked either wrong (×) or right (✓) and in most instances only the correct answer(s) was/were recognised and not the process of solving a given mathematics task. In order to validate this finding, the actual learner responses (see Figure 6.6) to the question posed in Figure 6.10 bear testimony.

The responses of Learner 1 and Learner 3 are mathematically rational in terms of the interpretation of the question, although the responses contradict the teacher's (incorrect) memorandum. It is argued that Learner 1 should have been given a mark for the correct representation of the question and maybe a mark for the second and the third terms of the answer. Instead, Learner 1 was 'treated' like Learner 2 whose interpretation of the question was as wrong as the teacher's memorandum. No feedback was given to Learner 1 and Learner 2 even though they got 'everything' wrong according to the teacher.

According to the teacher's memorandum, the solution to the question (however wrong) was allocated three marks. Learner three was, however, given three marks for the solution. If the teacher's memorandum was used as a marking guide, then the solution of Learner 3 should not be correct.

Figure 6.6 Learner responses

Learner 1

$$\begin{array}{r} M^3 + m^2 + m \\ - 10M^3 - 10m^2 - 10m \\ \hline M^3 + m^2 + m \\ + 10M^3 + 10m^2 + 10m \\ \hline 11M^3 + 11m^2 + 11m \end{array}$$

Expressions correctly ordered compared the teacher's incorrect memo. Signs correctly changes except $10m^3$.

Two answers are correct.

The learner was marked wrong.

Learner 2

Subtract: $(10m^3 - 10m^2 - 10m)$
from $m^3 + m^2 + m$

$$\begin{array}{r} = 10m^3 - 10m^2 - 10m \\ = m^3 + m^2 + m \\ = 9m^3 - 9m^2 + 9m \end{array}$$

Expressions ordered the same as the teacher's memo, learner not credited.

Signs are incorrect, no feedback was given.

Incorrect use of the equal sign, but no feedback given.

Learner 3

$$\begin{array}{r} (m^3 + m^2 + m) - (10m^3 - 10m^2 - 10m) \\ m^3 + m^2 + m - 10m^3 + 10m^2 + 10m \\ = 9m^3 + 11m^2 + 11m \end{array}$$

Expressions ordered correctly but different from the teacher's memo. Same teacher marked it correct.

$9m^3$ is wrong but was marked correct.

In some instances a similar question was posed by both teachers in their different classes. Two learners each from the two classes responded in the same manner but they were marked differently (see Figure 6.7).

Figure 6.7 Discrepancies in marking

Teacher's Question: What is the difference between $(a^2 + 2a + 3)$ and $(2a^2 + a + 2)$?

Response: Learner from Teacher A class

$$\begin{aligned} & (a^2 + 2a + 3) - (2a^2 + a + 2) \\ &= a^2 + 2a + 3 - 2a^2 - a - 2 \\ &= \underline{a^2 + a + 1} \end{aligned}$$

Response: Learner from Teacher B class

What is the difference between $(a^2 + 2a + 3)$ and $(2a^2 + a + 2)$

$$\begin{aligned} &= (a^2 + 2a + 3) - (2a^2 + a + 2) \\ &= a^2 + 2a + 3 - 2a^2 - a - 2 \\ &= \underline{a^2 + a + 1} \end{aligned}$$

Learners responded similarly but marked differently although a^2 is incorrect.

It would appear that the two responses are similar but the learner from Teacher B class was penalised for writing a^2 in the answer. A feedback was again not given by the teacher regarding the answer presumed wrong. One learner from Teacher B responded to the question in Figure 6.5 this way:

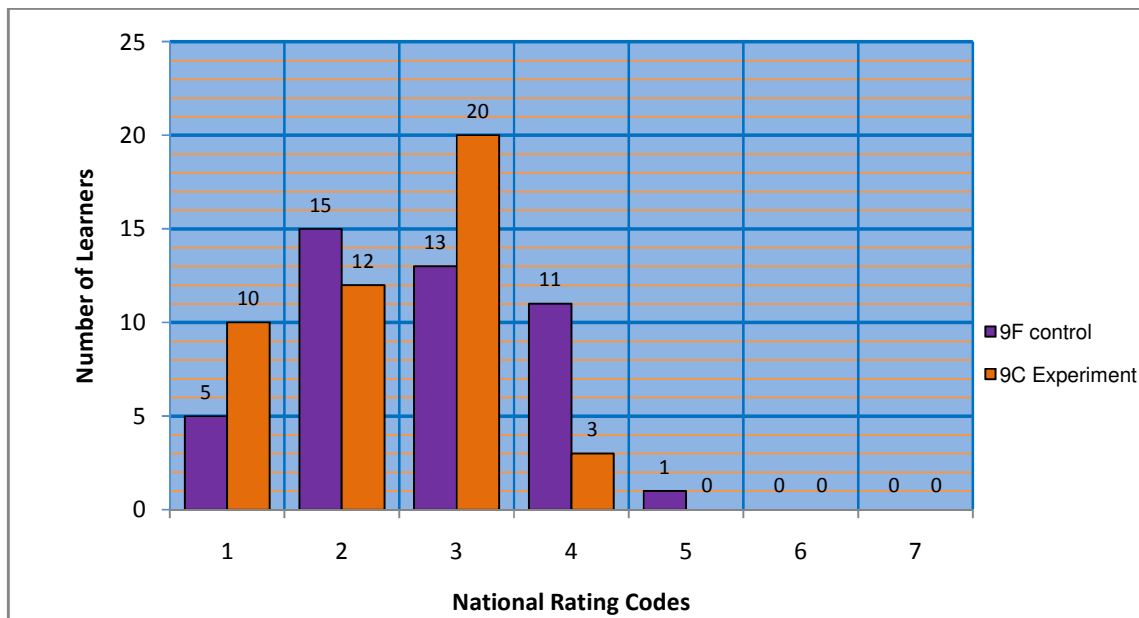
The is no difference between this term because
The are like term

Probably the learner was implying that “There is no difference between these expressions because they have like terms”. Nonetheless, it is evident that the learner did not understand the concept of ‘*difference*’ in mathematical context. However, there was no evidence that the teacher gave any guidance/feedback to the learner about the meaning of ‘*difference*’.

Mark schedules

The analysis of the learners’ recorded marks revealed the achievement levels presented in Figure 6.8. From the graph it is evident that the majority of learners perform below level 4 (50%). The majority of learners ($n = 20$, approximately 40%) in the experimental group perform at level 3 while 29% ($n = 13$) of the learners in the control group perform at the same level. However, more learners in the control group ($n = 11$) than learners in the experimental group ($n = 3$) perform at level 4. There are no learners performing at level 6 and 7 in either group. In terms of the quality of performance in general, the control group performs better than the experimental group.

Figure 6.8 Performances in mathematics per National Rating Codes: School C



6.4.1.4. School D

Teacher assessment portfolios

Assessment Standards: Of the four schools, School D was more advanced in terms of managing the assessment portfolio. The assessment tasks as well as their marking guides focused on the Assessment Standards they purport to assess. Essentially it is evident that the two teachers understood the mathematics Learning Outcomes and Assessment Standards fairly well.

There was also an indication of the presence of strong collaboration among the mathematics teachers. This was confirmed by the availability of the common assessment programme (for the activities that are recorded, i.e. formal assessment activities). It also emerged that the mathematics Head of Department moderates all the assessment tasks/activities before they are given to the learners.

Application: At the time of the analysis of the teachers' assessment portfolios, mathematics topics such as data handling, equations and expressions were already covered. These topics can generally be taught by drawing from real life scenarios; however, the two teachers did not take advantage of that possibility.

Variety of forms of assessment: A notable shortfall in the teacher portfolio in School D was, however, the hegemony enjoyed by class work and tests with no attention given to mathematics projects, investigations and assessment at all. This was a trend across all the schools that took part in this study. Once again the learners who would perform to their full potential when working in groups or pairs to discuss, investigate, analyse and present reports were marginalised. Mathematics investigations and projects are best suited to provide these proficiencies.

Learner assessment portfolios

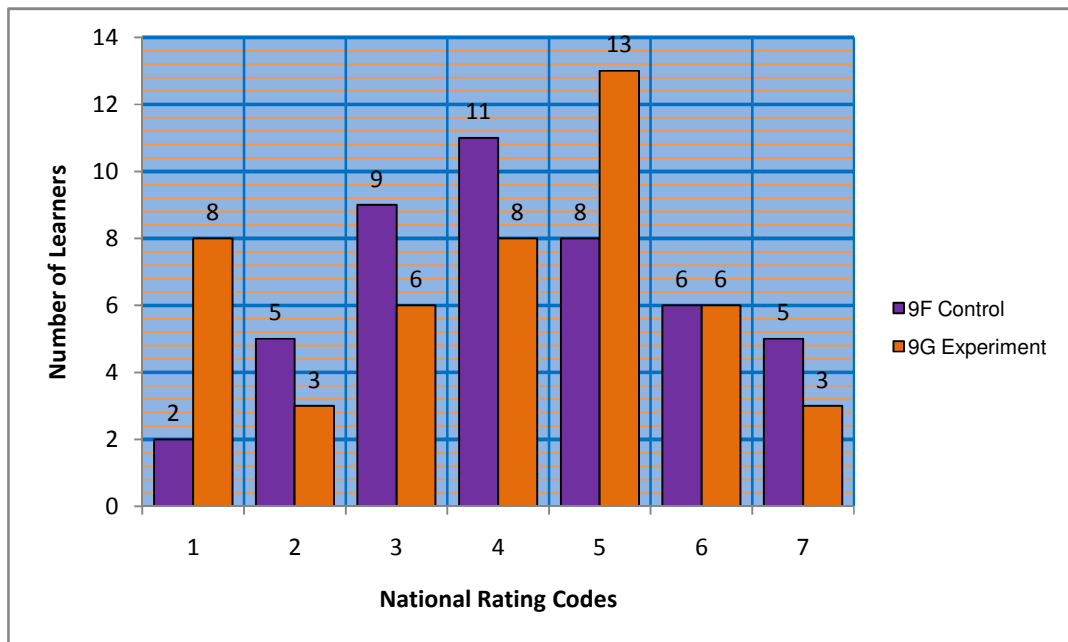
Use of learners' own methods of solving problems: There is a general impression from the analysis of learner assessment portfolios that learners answered questions using fairly similar approaches. This finding suggests that learners are not encouraged to use different methods and strategies to solve mathematics problems. This is possibly also encouraged by not exposing learners to mathematics investigations and projects.

Constructive feedback: Written feedback was not given, especially when the learners did not perform well. However, when the matter was taken up with the teachers, they indicated that when they have time they normally engage in face-to-face verbal interaction with the learners to address their mathematics deficiencies. The evidence of this was confirmed by the responses of learners during the focus group interview (*see* §6.4.3.1).

Mark schedules

Figure 6.9 shows the general performance of learners before their school took part in the implementation of the GIST model. There were 47 learner entries in the experimental group and 46 in the control group in the mark schedule at the time of analysis. Except for two extremes, the spread of the learner performance relative to the national coding system was fairly balanced for the control group and the experimental group. For instance, a notable extreme comparison was approximately 17% ($n = 8$) of learners in the experimental group performing at level 1 while approximately 4% ($n = 2$) of the control group learners performed at the same level. More learners in the experimental group (app.28%) than the control group (approximately 17%) performed at level 5. Other than the performance at level 5, the quality of learner performance in the control group was higher than in the experimental group.

Figure 6.9 Performances in mathematics per National Rating Codes: School D



6.4.2. Lesson observations

The entire process of the study was outlined by the researcher during the first encounter with the teachers. The intentions to conduct lesson observations were clearly spelt out. It was evident that a significant number of teachers were very reluctant to grant unreserved permission for the researcher to conduct the lesson observations. Therefore the intended frequency of class visits to conduct lesson observation could not be adhered to and was scaled down (*see* Table 6.11). However, the instances where lesson observations were conducted using the observation protocol (Appendix A) revealed the findings in the next paragraphs. The first class visit (henceforth called pre-GIST lesson observation) in all four schools was primarily used to observe the classroom learning environment or physical characteristics just before the implementation of the GIST model. The rationale was to have an impression of how the experimental class compared with the control class before it (experimental class) was transformed to suit the requirements of the GIST model. Subsequent visits (henceforth called during-GIST lesson observations) were aimed at

observing and documenting the discussions among learners on the barriers they experience and providing possible solutions towards their mitigation. These discussions focus on steps 2 and 3 of stage 2 of the GIST model (§ 4.3.2 in Chapter 4). Further the other lesson observations focused on ensuring that the GIST model was implemented correctly.

Table 6.11 Frequency of lesson observations per school

Lesson observation	School A		School B		School C		School D	
	Control	Experiment	Control	Experiment	Control	Experiment	Control	Experiment
Intended frequency	3	10	3	10	3	10	3	10
Actual frequency	1	5	3	8	1	7	3	9

6.4.2.1. School A

It is essential to highlight that, although the experimental group teacher appeared to have no qualms about the lesson observation, it was always a struggle to secure an appointment for the lesson observation as was the case with the control group teacher. The reasons such as “*I will be marking learners’ books during the mathematics period*” were cited. Only five lessons were observed in the experimental group and one in the control group. Essentially school A was the most uncooperative in terms of securing appointments to conduct interviews and lesson observation.

Pre-GIST lesson observation: control and experimental groups

The classrooms are very big and learners were properly accommodated, ideal for group work. Learners were seated in rows and columns (two learners shared a rectangular table).

During-GIST lesson observation: experimental group

Learner 1: I prefer to use a calculator than solving mentally.

Solution: ***We should not be allowed to bring the calculators in class.***

Learner 2: It is a problem for me to write while the teacher is writing on the chalkboard.

Solution: ***The teacher must give us the last five minutes of the lesson to write the notes.***

Learner 3: I do my home work in class because I do not have space at home to do it.

Solution: ***I think it is ok to do it in class, but the teacher does not allow it.***

Learner 4: Some of the methods the teacher uses are too complicated and I don't like using them.

Solution: ***We should be taught simple methods.***

Learner 5: If I don't get a correct answer I write any answer.

Solution: ***I do not have a solution.***

6.4.2.2. School B

School B was the second most cooperative school in this study. The teachers were always available for any interactive session the researcher wanted to have with them or their learners. It was always very easy to secure appointments with them.

Pre-GIST lesson observation: control and experimental groups

The classrooms were very clean and well ventilated, making them ideal for the diagnostic testing as well as facilitation of group intervention (Stage 2 of the GIST model). Although the number of learners in a class exceeded forty, learners were seated comfortably in pairs. There was enough space between the rows and behind every desk.

During-GIST lesson observation: experimental group

Learner 1: I cannot concentrate in class because it is too noisy.

Solution: ***We have to reduce the noise level in class so that other learners can study.***

Learner 2: mathematics rules are too difficult to understand.

Solution: ***Maybe our teacher can help us with the simple maths rules.***

Learner 3: Sometimes our teacher gives certain learners a correct mark and gives others a wrong mark even if the answers are the same.

Solution: ***The teacher should be fair when marking our books.***

Learner 4: Our teacher laughs at us when we give a wrong answer to a question. I feel like a joke especially when other learners also laugh at me. I have decided not to try any question in class.

Solution: ***Our teacher must help us not laugh at us or make jokes about our wrong answers.***

Learner 5: I do not like maths because the methods used to solve problems are too complicated.

Solution: ***I will not choose maths next year.***

6.4.2.3. School C

Like school A, it was not easy to secure an appointment with the teachers in school C. Each lesson observation session always took place after numerous attempts were made. The challenges were mainly management-orientated and beyond the experimental teacher's jurisdiction. The experimental group teacher in school C was always willing to cooperate. Ultimately seven lesson observation sessions were conducted with the experimental class and only one session with the control class.

Pre-GIST lesson observation: control and experimental groups

The learning environment was very conducive for learning in terms of cleanliness, ventilation, and the learners were very free to engage with the teachers in class.

Compared to the other three schools, school C was the cleanest school inside and outside the classroom.

During-GIST lesson observation: experimental group

Learner 1: I get very angry if I cannot correctly use the method the teacher used.

Solution: *I think I must practice the method more and more.*

Learner 2: I struggle to understand the instructions of the maths questions.

Solution: *We must discuss the instructions before writing.*

Learner 3: I depend too much on the calculator that I do not realise the wrong answer my calculator gives.

Solution: *I think I must try to stop using a calculator.*

Learner 4: The teacher uses different methods, this confuses me.

Solution: *I have no solution.*

Learner 5: It is easy to understand maths in class but when I get home to do my homework I do not understand anything.

Solution: *Maybe we should be allowed to discuss the homework activities in class so that when I get home I know what I must do.*

Learner 6: Sometimes I feel confident that I did it [mathematics exercise] correctly but when we solve it in class I get it wrong.

Solution: *I need to practice more maths exercises.*

Learner 7: The teacher takes too long to mark and return our books.

Solution: *If he can return them quickly before we forget what we wrote about.*

6.4.2.4. School D

Pre-GIST lesson observation: control and experimental groups

The control group and experimental group classrooms were similar in terms of structure. Both classes are well ventilated. The class sizes are fairly equal, with $n = 47$ and $n = 46$ respectively for the experimental group and control group. Learners are too many for the classroom space/area as a result they are seated in pairs and in rows and columns facing the chalkboard. Some learners are seated

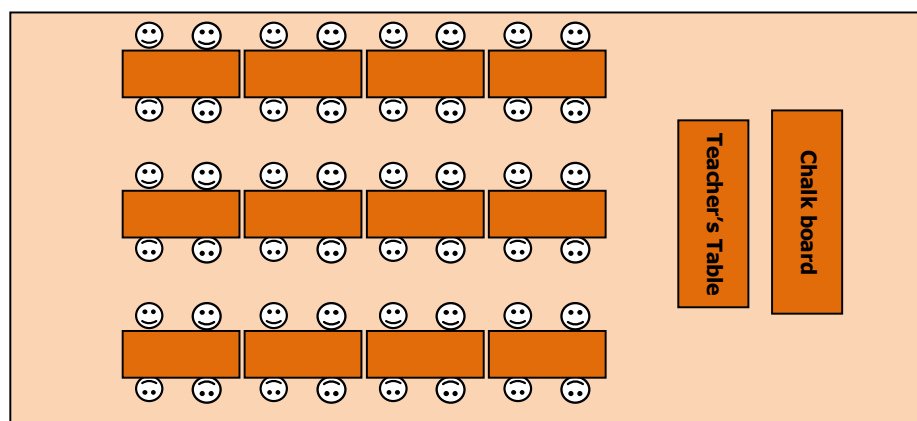
on old school desks while others are seated on chairs and rectangular tables. All classrooms are supplied with electricity.

The teaching process is fairly interactive but learners are mainly made to drill the methods of solving mathematics problems. The control group teacher frequently reminded the learners that they should ask questions when they did not understand. However, during that lesson none of the learners asked questions to seek for clarity. In both classes learners and teachers frequently switched from Setswana to English and *vice versa* during the lesson. Learners were not encouraged to work with one another, especially in pairs.

During-GIST lesson observation: experimental group

The seating arrangement was changed to be appropriate for group work. The group seating arrangement resembled a boardroom pattern (*see* Figure 6.10) rather than the desired illustration in Figure 4.2 (Chapter 4). The desired arrangement was constrained by the small classroom area/space and the rectangular desks; however, the heterogeneity of the groups was observed. There were spaces between each group, but not big enough to enable the teachers to move freely in-between. Like in the other three schools the teacher started by explaining the purpose of the session as per GIST model to the learners (*see* § 4.3.2) and the ground rules were set.

Figure 6.10 Seating arrangement



The most defining period was the feedback from the groups as per Step 3 of Stage 2 of the GIST model (*see* § 4.3.2 Chapter 4). Some of the learners' mathematics barriers and the possible solutions were:

Learner 1: It is very difficult for me to change from one maths topic to the other.

Solution: *The teacher should show us the connection between the topics.*

Learner 2: Maths exercises especially in an examination or test make me fearful because I never know which method does the teacher want me to use.

Solution: *I don't know what to say, but maybe our teacher must tell us which method to use.*

Learner 3: The teacher is sometimes impatient with me.

Solution: *She must give us more time to solve the question.*

Learner 4: Other learners laugh at me when I give a wrong answer.

Solution: *They should stop laughing at us because we are all here to learn.*

Learner 5: My parents are always fighting at home and I always think about them in class.

Solution: *I don't know.*

Learner 6: Maths is not important. For instance if you ask somebody about x^2 they will say: "you cannot go to the shop and ask for x^2 it is not there in life".

Solution: *We should learn real maths that uses numbers and not alphabets.*

Learner 7: My parents do not know mathematics and they cannot help me at home.

Solution: *I think we must do more maths at school than at home because at school the teacher can help us.*

Learner 8: Maths *ga o tlwaelege* (Response in Setswana¹⁸ which **translates: you will never get used to mathematics**), it is unlike the other subjects. You will never say you know it. Every day you bring new topics in class.

Solution: *I don't know what to say.*

Learner 9: I like to use my pen to write things that are not related to maths during the maths period.

Solution: *I don't know what I can do because I tried to stop but every time I find myself doing it again.*

¹⁸ Setswana is one of the 11 official languages spoken in South Africa.

6.4.3. Interviews

6.4.3.1. Learners' interview

Learners were interviewed in a focus group setting. The interview process was semi-structured; however, the interview schedule (*see* Appendix B) was used. The focus groups were constituted by the learners from RE only because the semi-structured interview focused on the GIST model which was only implemented on RE. It should however be highlighted that each school had its focus group and the interviews took place separately at each school. Each school was requested to provide a venue within the school campus where potential disturbances were minimal. The interview schedule comprised of three questions:

Question 2(a) focused on the importance (or not) of knowing and understanding the mathematics learning barriers.

Question 2(b) focused on liking/disliking cooperative small group work.

Question 2(c) focused on the effect of solving real-life problems in the learning of mathematics.

Condensed response to Question 2(a): Learners indicated that they found the session on 'Understanding the learners' learning barriers' (*see* §4.3.2 – Step 2 of the GIST model) very helpful. However, at the beginning they felt uneasy because other learners would laugh when they presented their barriers openly. Some of the responses are listed below:

- I initially thought that things such as boredom during mathematics lessons were normal until I realised that boredom could be linked to the attitude I had towards mathematics.
- Our teacher became more supportive after we discussed our barriers in the classroom.

- After we have answered mathematics activities she explains the mistakes we have made by writing in our activity books or calls us to explain verbally.
- Our teacher has taught us to read the instructions of each question carefully so as to understand the problem and select an appropriate method to solve it.

Condensed response to Question 2(b): Learners were quite vocal regarding their evaluation of the group work. Some of the things they liked about group work were that “Learners help one another” and they “Understand better when peers explain”. In all four schools the majority of girls asserted their liking of cooperative group work.

- Sharing of ideas and different methods on solving mathematics problems;
- Slow learners benefit;
- Fulfilment derived from assisting other learners; and
- Development of friendships among learners.

The following issues emerged regarding the learners’ dislike of group work.

- Some learners do not participate; one learner indicated that she feels used because she always offers solutions;
- Information is dominantly provided by the same learners;
- Arguments about which answer to take;
- Doing other things that do not focus on mathematics;
- Capable learners dominate the discussion; and
- Small groups of about five members are better than a bigger group because we are able to listen to one other.

Learners generally agreed that the benefits of group work outweigh the limitations and they would always prefer to work in small groups.

Condensed response to Question 2(c): One learner indicated that “Steps of mathematics are difficult” and this was supported by all the learners in the focused group. When this statement was explored further, learners explained that they always struggle to “understand” and “remember” the methods used to solve mathematics exercises. However, they indicated that in some instances their teacher uses real life examples to teach topics such as data handling and “We enjoy it”. One learner said: “The teacher makes mathematics very practical”. It emerged from further interrogation regarding this statement that the learner became content when the topic taught in class was applied using real life scenarios.

The steps in the problem-solving framework featured prominently in the learner responses from schools C and D. They indicated that their teachers always reminded them to understand the problem by discussing it in groups, devise a plan by choosing the appropriate method, use the method agreed upon and verify the answer. One learner indicated that they can now readily follow the steps when a task is given to them.

6.4.3.2. Teachers' interview

Mathematics teachers of the **experimental** and **control groups** were interviewed before the intervention to probe the following factors (also *see* Table 6.1) that are linked to the GIST model:

- Their knowledge (and subsequent use) of the **teaching approaches** employed in the GIST model.
- Their knowledge (and subsequent use) of **diagnostic and formative** assessment in mathematics. Further **research question 2** would be probed i.e. *‘What factors hamper educators from using diagnostic and alternative assessment practices in mathematics classes?’*

- Their knowledge (and subsequent curbing) of learners' **barriers** to the learning of mathematics.

Two primary rationale were served by the teachers' interview: *firstly* to dispel or confirm a possibility that either E and C teachers would be advantaged by virtue of being more knowledgeable about the aspects of the GIST model before its implementation; and *secondly* to enable the researcher to acquire some of the information used to put together the **case description** of each school (*see* Table 6.1). The responses of teachers summarised in Table 6.1 imply that their knowledge of the teaching approaches, diagnostic assessment and learning barriers in mathematics were fairly similar.

The teachers of E were further interviewed after the implementation of the GIST model to share their experiences about or evaluations of the GIST model after its implementation. The aspects of the GIST model that gained prominence in terms of the responses were "group work" and the "steps used during problem-solving". Of the four schools the teachers of school D showed the highest passion regarding the CL and PCI.

The teachers in the four schools indicated that they previously did not give constructive feedback to the learners and since they started giving feedback, learners began showing interest in their work. Again this was more evident in school D as it emerged strongly during the learners' focus group interview. One teacher (from school C) indicated that since he started using the GIST model he began to pay attention to details when marking the work of the learners. He indicated that he derived interest in knowing how "...each learner approaches the solution of the given task". By implication, attention has shifted from the product to the process of learning.

6.5. Discussions

The quantitative and qualitative data that were collected and analysed revealed critical findings that will constitute the focus of the discussions. While the discussions of the findings of the Mathematics Achievement Test (MAT) and Study Orientation in Mathematics (SOM) will be fore-grounded, focus will also be on the discussions of the findings gleaned from the qualitative data. The discussions will also highlight the implications emerging from the quantitative and qualitative findings on MAT and SOM. Essentially, the emerged implications will be consolidated to draw conclusions on hypotheses tested and to answer the research questions referred to in Chapter 1 (§1.2 and §1.3).

6.5.1. Diagnostic Assessment in mathematics : realities and misleading notions

The fundamental question to be answered regarding diagnostic assessment in mathematics is, “What factors hamper educators from using diagnostic assessment in mathematics classes?” The answer to this question is packaged in the form of assessment realities as per document analysis and the misnomer or misleading notions regarding the underpinnings of diagnostic assessment based on the interview with the teachers. As a way to preface the discussion on the question posed above, the assertion by Maree and Erasmus (2005:14) that, broadly, diagnostic assessment encapsulates diagnosis of cognitive-based problems such as academic underperformance, and affective-based problems such as attitudes and disposition towards mathematics need to be reiterated. Essentially, diagnostic assessment assists teachers to adapt their teaching and assessment to respond to the learners’ identified needs (Maree *et al.*, 1997:6).

It is argued that the most readily available diagnostic tool teachers can use to diagnose content-related difficulties among learners is the written work (mainly in the form of a class work). The findings gleaned from the document analysis point to the prevalence of unbearable realities teachers are still grappling with regarding the use diagnostic assessment in mathematics. The lack of feedback, or most crucially constructive feedback, inconsistencies in the marking of learners’ activities as well as the use of wrong

memoranda are the indicators of lack of understanding of diagnostic assessment. These aspects have a potential to contribute to the learners' underperformance in mathematics as well as perpetuating negative mathematics disposition among learners. This was confirmed by the low levels of underperformances in the school-based assessment (marks recorded in the mark schedules) as well as learners' responses during the interviews before the implementation of the GIST model. The low levels of performance in pre-MAT also bear testimony to the assertion. This is a cause for concern because assessment is regarded as the integral part of teaching and learning. It is through assessment that teachers are able to establish the degree to which learners are advancing the achievement of learning goals.

The underlying cause of the difficulty among teachers to use diagnostic assessment is that this type of assessment was never given much needed focus as is the case with summative and only recently formative assessments. Therefore teachers do not know diagnostic assessment (*see* Table 6.1) as well as its potential to optimise teaching and learning. The teaching and learning opportunities that presented themselves in the form of mistakes committed by learners (*see* Learner assessment portfolios in §6.4.1) were never exploited by the teachers to enhance their teaching, and establish the causes of non-mastery of mathematics algorithms as well as mathematics content.

It emerged from the interviews conducted with the teachers to collect information on 'Case description' (*see* §6.2 and Table 6.1) that there is a misnomer regarding the diagnostic assessment particularly in the context of affective-based barriers to learning such as mathematics anxiety and attitudes. Teachers who participated in this study from all four schools admitted that they were never trained in the use of diagnostic assessment. Lack of knowledge about diagnostic assessment and how to use it to identify learners' barriers to learning are cited as the key factors that inhibit mathematics teachers to implement diagnostic assessment in schools.

The other factor is the lack of knowledge of affective-based barriers such as mathematics anxiety and attitudes. It is argued that teachers' proficiency to use diagnostic assessment

is not sufficient to assist learners who experience barriers to learn mathematics unless it is coupled with the knowledge of affective-based barriers. Knowing the affective-based barriers and how they manifest themselves is fundamental because teachers can easily identify the manifestations and formally confirm their prevalence through an appropriate diagnostic test as articulated in Step 1 of Stage 1 (§ 4.3.1).

6.5.2. Learners' mathematics achievement: the consequence of the GIST model

The quantitative findings in this study have revealed a practically significant improvement (large effect size, $d = 0.79$) in the learner performance (experimental group) in the Mathematics Achievement Test. The question that emerges is “How does the GIST model influence learners' mathematics academic achievement?” In other words, is the improvement in learner performance attributed to the orchestration of all four stages of the GIST model or to only some of the stages? In an attempt to answer these questions the point of departure would be to highlight that, central to the teaching profession, is the teaching and learning process. If this argument is anything to go by, it is imperative that teachers will be inclined to focus on Stage 3 of the GIST model, namely “teaching-learning processes”. The inclination towards Stage 3 of the GIST model emerged prominently during the lesson observations in the experimental groups of the four schools that participated notwithstanding the researcher's emphasis on the navigation between stages 2, 3 and 4. In Stage 3 the most dominantly used teaching approaches were CL and PCI, primarily because of the evidently similar characteristics pertaining to the use of group work (*see* §2.3.3.2 that addresses the ‘Essential elements of Cooperative learning’ and §2.3.5.5 that addresses the ‘Essential elements of PCI’).

The aspects of Stage 3 of the GIST model featured prominently in the learners' responses during the focus group interviews. They generally appreciated the ‘different’ ways in which the teachers taught them “...since you started to visit our class”. Learners, especially in school D, knew and understood the four steps of the problem-solving framework and they would readily employ the steps when given any task to solve. It would appear that learners

and teachers of the experimental groups had a strong interest in and were more comfortable to use CL and PCI.

Given the scenarios portrayed above, the GIST model influenced learners' mathematics achievement; however, without underplaying the contributions of the other stages of the GIST model, it would be reasonable to primarily attribute the improved learner achievement to Stage 3 of the GIST model.

6.5.3. Learners' mathematics disposition: the effect of the GIST model

The *t*-test results revealed no difference between the means of the control group and experimental group for pre-SOM and post-SOM in all the five fields ($d < 0.3$). The interview conducted before the introduction of the GIST model revealed that the issues pertaining to the learners' mathematics disposition were quite novel for the teachers (*see* Table 6.1). This was also evident during the training of the teachers on the implementation of the GIST model. The GIST model did not have an influence on learners' mathematics disposition (*see* Research question 1).

Nonetheless, there are two attributes that may have accounted for the lack of improvement in the learners' mathematics dispositions, with specific reference to anxiety, attitudes, study environment, study habits, and problem-solving behaviour. *Firstly* it was evident that more time was required than budgeted for to train the teachers, given that the elements of mathematics dispositions listed above were new to them. *Secondly* teachers were biased towards the teaching-learning process (Stage 3 of the GIST model) and they gave very little attention to the aspects entrenched in group intervention (Stage 2 of the GIST model). The two attributes do not necessarily suggest that if they are mitigated the GIST model would have had a positive impact on learners' mathematics disposition; however, the impact of their mitigation may need to be probed further.

Learners seemed very enthusiastic to share with one another the barriers they experience in mathematics. Some of the issues they raised, especially those that relate to teaching, confirm the assertions that teachers contribute enormously towards learners' negative mathematics disposition (Geist, 2010:28; Hackworth, 1992:10; Donald *et al.*, 2006:312; Furner & Duffy, 2002:68). The study was intended to assist the learners to cope with the barriers in mathematics using their teachers; however, it was not envisaged that teachers too needed assistance because they were cited by the learners as being part of the 'barriers'. Therefore, the lack of improvement in the learners' mathematics dispositions is acceptable in this study. The GIST model needs to be adapted to include the intervention programme for teachers to deal with 'the teacher as a barrier' before using the teacher as an 'intervener'.

6.5.4. Correlations between mathematics achievement and dispositions

During the implementation of the GIST model it emerged that the teachers of all the four experimental groups were biased towards the Stage 3 (teaching-learning processes) of the GIST model. The biasness towards Stage 3 was predictable for two reasons. *Firstly* issues of pedagogy are central to the teaching profession and the teachers are likely to be more attracted to Stage 3 than to Stages 1, 2 and 4 of the GIST model. As indicated earlier in §6.5.3, the dominant use of CL and PCI might have contributed to the improved correlations (notwithstanding lack of statistical significance – *see* Figure 6.5) between study attitudes, study habits and problem-solving behaviour relative to the post test.

Secondly the revelation that before the implementation of the GIST model, teachers had superficial knowledge of the learner-centred approaches that is central to the implementation of the GIST model and to the teaching of mathematics in recent years. In the context of this study, the lack of correlation of statistical significance between the MAT and SOM is therefore attributed to the biasness towards Stage 3 and little attention given to the other stages of the GIST model.

It is further argued that the attitudes of learners towards mathematics do not necessarily correlate with enhanced achievement. This assertion is confirmed by the TIMSS report which revealed that South Africa's learners have higher levels of attitudes towards mathematics although they performed poorly compared with their international counterparts (Reddy, Kanjee, Diedericks & Winnaar, 2006:115). Inversely, learners from countries such as Singapore performed extremely well although their attitudes towards mathematics were low.

6.6. Conclusion

The following hypotheses that this study aimed to test were presented in Chapter 1:

H₀₁: The application of the GIST model influences the mathematics dispositions of grade 9 learners.

H₀₂: The application of the GIST model influences the mathematics academic achievements of grade 9 learners.

H₀₃: There is a positive correlation between learners' mathematics academic achievements and learners' mathematics dispositions.

The analysis of quantitative data gleaned from the SOM questionnaire and MAT as well as the qualitative data gleaned from interviews, observations and document analysis assisted with the decision to accept or reject the afore-stated hypotheses.

H₀₁ – the discussion in §6.5.3 suggested possible attributes to the lack of improved learners' dispositions towards mathematics. The primary attribute is centred on the assertions by various scholars that teachers contribute enormously towards learners' negative mathematics disposition. The responses of learners during the interviews as well as the quantitative data gleaned from the SOM questionnaire revealed that the application of the GIST model does not influence the mathematics dispositions of grade 9 learners, hence the rejection of **H₀₁**.

H₀₂ – the difference between the means of the experimental and control groups regarding MAT after the post-test were adjusted using ANCOVA to control the influence of the pre-test in the post-test data. The post-test adjusted mean was practically significant ($d = 0.79$, large effect size). It is therefore concluded that the application of the GIST model had influenced the learners' mathematics achievement positively.

H₀₃ – while there was an indication of enhanced correlations between mathematics achievement and certain SOM fields, there was no suggestion that the correlations were statistically significant. Essentially there is no correlation between learners' mathematics academic achievements and learners' mathematics dispositions, hence **H₀₃** is rejected.

CHAPTER

7

CONCLUSIONS AND RECOMMENDATIONS

This chapter presents concluding commentary with specific focus on three areas, namely, **summary of the research, limitations of the study and recommendations for future research.**

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS



INTRODUCTION



SUMMARY OF RESEARCH



LIMITATIONS OF THE STUDY



RECOMMENDATIONS FOR FUTURE RESEARCH



CONCLUSION

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7.2.5. Research findings and discussions		
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7.1. Introduction

The conceptualization of this study, including the coinage of the topic, was initially informed by the anecdotal evidence suggesting that diagnostic assessment is not used adequately in mathematics classes in the General Education and Training band (GET) of schooling in South Africa. Empirical investigation was carried out to determine the existence of any substantial evidence in this regard by using the GIST model, MAT and SOM questionnaire to test a set of three hypotheses. This chapter, therefore, presents concluding commentary with specific focus on three areas, namely **summary of the research, limitations of the study** and **recommendations for future research**.

7.2. Summary of the research

7.2.1. Problem statement

Since the inception of the post-1994 curriculum in South Africa, assessment in mathematics, as is the case with other subjects, has attracted more attention and has since been regarded as an integral part of teaching and learning. Other types of assessment, such as diagnostic and formative assessments, began to gain prominence as is the case with summative assessment. Diagnostic assessment, in particular, requires that teachers should identify the barriers that inhibit effective learning of mathematics and develop appropriate interventions to address them. However, appropriate skills to handle issues of diagnostic assessment are lacking among teachers. The situation is made more difficult by large class sizes South Africa is confronted with. Large class sizes are a reality in most of the schools in the country, and they make it difficult for the possibility of one-to-one (teacher-to-learner) intervention to address mathematics learning barriers. The Inclusive Education model makes it an obligation to focus on diagnostic assessment. Given all these challenges, the GIST model provides a viable solution to use diagnostic assessment in a group setting. While further research needs to be done to improve on the GIST model, this study has revealed that it has the potential to mitigate mathematics barriers to learning as well as to improve on learner performance in mathematics.

7.2.2. Review of the literature

The review of literature was done on **learner-based teaching approaches** and on **assessment and the teaching and learning of school mathematics**. These two areas form the basis of the GIST model. Availability of extensive research on the two areas assisted the researcher to acquire more insight into the underpinnings of mathematics teaching, learning and assessment.

7.2.3. The GIST model

The GIST model was conceptualised and tested in four schools to investigate its impact on the learners who experience mathematics learning barriers as well as its impact on mathematics achievement. Central to the framework of the GIST model is diagnostic assessment. However, it is designed to offer intervention to learners who experience mathematics-learning barriers in a whole class setting where one-to-one intervention processes are impracticable. The GIST model is characterised by the following four stages which should be implemented in a cyclic manner: **Diagnostic assessment, Facilitation of the group intervention, Teaching-learning process, and Formative assessment practices.**

7.2.4. Method of research (chapter 5)

The combined quantitative and qualitative method enabled the researcher to have a balanced view on issues related to diagnostic assessment that would have otherwise been impossible to reveal if one method was used. What appeared to be the highlight were the discrepancies revealed in the document analysis with regard to the teachers' memoranda, learners' responses and the actual marking of learner activities. It is virtually impossible that the qualitative method could have revealed these findings. However, the quantitative method, particularly the use of the correlation coefficient, was very helpful to reveal the lack of correlation between the MAT and SOM fields which the qualitative methods could not reveal.

In this study four schools were sampled from Tshwane West education district of the Gauteng province. The sample was drawn from large mathematics classes ($n \geq 40$) i.e. the class size that exceeds the international teacher-learner ratio of 1:30. The experimental design was employed to test the effect of the GIST model on achievement and learning barriers. Two classes per were assigned an experimental and a control group respectively.

Data collection was done in two ways: quantitatively, using the MAT and SOM questionnaire as data collection instruments; and qualitatively, using interviews, document analysis and observations. This approach was used to corroborate and authenticate the collected data with a view to enhance the credibility of the findings. Descriptive and inferential statistical techniques were used to analyse quantitative data while the analysis of qualitative data was done in a narrative form.

7.2.5. Research findings and discussions (chapter 6)

The MAT and SOM questionnaires were administered as pre-tests and post-tests. The t -test statistical analysis was done to determine the mean difference between the control group ($n = 175$) and the experimental group ($n = 166$). The difference between the means was not practically significant ($d \approx 0.2$) and it was therefore concluded that the groups were comparatively similar before the application of the GIST model. The analysis of covariance (ANCOVA) was used to analyse the post-test data and the adjusted mean scores ($n = 311$ for MAT, and $n = 321$ for SOM fields) revealed a difference of statistical significance between the control group and the experimental group relative to mathematics achievement ($d \approx 0.5$, medium effect). However, there was a small difference of no practical significance between the control group and the experimental group with respect to the SOM fields ($d \approx 0.2$, small effect size). The correlations between mathematics achievement and the SOM fields were not practically significant ($d \approx 0.2$, small effect). Given these scenarios, H_{01} was rejected, H_{02} was accepted and H_{03} was rejected.

7.3. Limitations of the study

Generally, the teachers of the four schools did not approve of bringing the second observer into their classes. They alleged that that would create an unfriendly classroom environment for their learners i.e. the presence of two unfamiliar people in a classroom would not be desirable for learning.

The study was initially intended to take place in the schools in the North West Department of Education (NWDE). However, after the permission was granted by the NWDE, the provincial re-demarcation of boundaries took place with the result that the schools that were sampled for the research were incorporated into Gauteng Department of Education (GDE). The implementation of the GIST model took place during the time when teachers were going through a transition in the *modus operandi* (NWDE to GDE). The incorporation into GDE came with the merger¹⁹ of the former middle schools (grades 7 to 9) and high schools (grades 10 to 12). The middle school concept was not in line with schooling classification of GDE and the merger put uncertainties on the teachers regarding the possibilities of being transferred to other schools. The merging process happened during the period of the research and the teachers' inability to honour appointments with the researcher (in some instances) was attributed to the uncertainties emanating from the merging process.

It was assumed that teachers would have a fair knowledge and understanding of the issues that constitute barriers to the learning of mathematics, particularly in the GET band. It was evident during the interactions with the teachers that the assumption was incorrect and more time was needed for intensive training. Time was therefore a constraint in this study.

The GIST model, in its current format, was used in the whole class context where the responses of the learners regarding the fields of SOM were averaged. Practically, teachers would not know which group of learners, within a larger group in a class, experienced a certain barrier (defined in terms of the SOM fields in this context).

¹⁹ The typologies of schools in Gauteng province differ from those in North West province. North West province has primary schools (grades 1 to 6), middle schools (grades 7 to 9) and high schools (grades 10 to 12). In Gauteng province there are no middle schools. The primary schools cover grades 1 to 7, and high schools cover grades 8 to 12.

7.4. Recommendations

7.4.1. Teacher training

The emergence of a lack of knowledge of, as well as the skills to implement diagnostic assessment among mathematics teachers calls for the adaptation of the GIST model to include teacher training. Teacher training in this context does not refer to the training that was done for the purpose of the research, but training to ensure that teachers acquire the requisite skills necessary for the use of diagnostic assessment. Diagnostic assessment, by its nature, requires knowledge of potential barriers to learning in mathematics so that teachers can recognise the symptoms thereof among the learners. Perhaps a study needs to be conducted to investigate whether thorough teacher development on diagnostic assessment could have an influence on learner achievement in mathematics. Essentially the recommended hypothesis in this regard would be: the teacher's knowledge of diagnostic assessment influences mathematics achievement.

7.4.2. Recommendations for future studies

Further, research needs to be conducted on the use of the GIST model in much smaller groups than a classroom context. The GIST model could start by identifying learners' barriers and group them in small homogeneous groups according to the barrier they experience and investigate the impact of this model on that barrier. The current version of the GIST model encourages heterogeneous groups in terms of the barriers experienced by learners and it does not seem to have had a significant impact on the mitigation of the intensity of the barriers. The recommended hypothesis in this regard could be: The homogeneous group in the use of the GIST model mitigates the intensity of the common barrier in the homogeneous group.

7.5. General conclusion

This study has contributed to the many possible solutions to the problem of mathematics learning and teaching within mathematics education environment. The findings are not conclusive, but merely the beginning of exploration by researchers in mathematics education. The problem of large class sizes is one of the biggest challenges confronting South Africa and different models that can promote effective

teaching and assessment in such conditions should be explored. The effective use of diagnostic assessment in the context of large classes, which the GIST model has attempted to address, requires more research attention.

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APPENDIX A: OBSERVATION PROTOCOL

School: Observation Number: Learners observed: Date/Time (Duration):

Construct	Construct descriptors	Brief explanation of the observation
Mathematics anxiety	Learner(s) ask(s) to visit toilet during mathematics lessons	
	Learner(s) nervously play(s) with a ruler, pen etc during math lessons	
	Learner(s) nervously speak(s) indistinctly when math question are asked	
Problem solving behaviour	Learners use other languages than English to interpret and understand the math problem	
	Learner(s) translate(s) the problem (verbally or written) into mathematics (i.e. modelling of the problem)	
	Learners validate/evaluate their solutions by substituting in the original statement	
Attitudes towards mathematics	Learners promptly and enthusiastically want to react to the wrong answers provided by other learner(s)	
	Prevalence of persistence, motivation and confidence when solving mathematics tasks	
	Learner(s) willingly help(s) other group(s) without being asked by the teacher	
Study/learning habits	Learner(s) ask(s) questions to clarify mathematics concepts	
	Note-taking takes place every time the teacher explains some concepts	
	Learner(s) pose(s) questions to ensure that they understand instructions	
Study/learning environment	The teacher encourages all learners to ask questions if they do not understand certain mathematics concepts	
	The classroom has windows and/or air conditioner for ventilation	
	Learners are allowed to use other languages than English to explain mathematics concepts	
	Small cooperative group work is encouraged	

APPENDIX B: INTERVIEW SCHEDULE

1. Introduction

(Establish Rapport – greet the learners by shaking their hands)

My name is Your teacher and I have agreed that I should interview you so that I can understand better what you have gained or not gained from the intervention sessions you have attended.

(Outline the purpose of the interview)

I would like to ask you some questions about the experiences you have gained during the sessions facilitated by your mathematics teacher.

(Motivation)

- The information you are going to provide will not be used to influence your results for promotion purposes.
- Instead it will hopefully help to reshape the group intervention strategy for effective teaching and learning of mathematics in future.
- There is therefore no correct or wrong answer, and please feel very free to respond to the questions.

(Time Line)

The interview should take about 40 minutes. Hope you are ready to respond to the questions?

2. Actual semi-structured interview

- a) In one of your mathematics lessons you were mainly focusing on the process of understanding your barrier to the learning of mathematics, i.e. you focused on mathematics anxiety, attitudes, study environment, study habits and problem solving behaviour. Do you think it is important to know and understand the barrier that affects you in mathematics? Explain.

(Follow up questions will enable the researcher to establish whether or not the understanding math anxiety, problem solving behaviour, attitudes towards math, study habits and study environment assists with 'self-desensitization')

- b) During your mathematics lessons you were mainly working in small groups in which you shared information with other learners. Share with us what you liked or did not like about **small group work**. Always give reasons for your response.

(GIST is about 'group intervention'; therefore the researcher aims to establish the degree of success of group work among learners)

Liked	Disliked

- c) Your teachers made every effort to give you mathematics exercises that were meaningful to you. This means that the exercises were aimed at solving **real-life problems**. How did the exercises contribute to your learning of mathematics?

(Follow up questions will enable the researcher to establish the impact of solving of real-life mathematics problems on learners' math anxiety, problem solving behaviour, attitudes towards math, study habits and study environment).

3. Closing

(Summarize and Maintain Rapport)

- I appreciate the time you took for this interview. Is there anything else you think would be helpful for me to know so that I can effectively modify the GIST model?

- I wish you success in your studies especially in the learning of mathematics.

APPENDIX C: ANSWER SHEET - SOM QUESTIONNAIRE

- i. Answer all the questions.
- ii. There is no correct or wrong answer. Always give your honest responses.
- iii. Your responses will be kept confidential and will be used strictly for the research purpose.
- iv. Decide how you feel about each statement. Use a pencil to make a cross on the code that indicates your response. The codes are described below:

1 = rarely
2 = sometimes
3 = frequently
4 = generally
5 = always

Example: Statement 1 in the questionnaire says: “I enjoy solving maths problems”. If you *frequently* enjoy solving maths problems make a cross on 3 like this:

1	2	3	4	5
---	---	--------------	---	---

- v. Read each statement carefully and make sure you understand it before answering.
- vi. **PLEASE DO NOT MAKE ANY MARKS ON THE QUESTIONNAIRE**

EARNER NUMBER:

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SCHOOL:

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1	1	2	3	4	5
2	1	2	3	4	5
3	1	2	3	4	5
4	1	2	3	4	5
5	1	2	3	4	5
6	1	2	3	4	5
7	1	2	3	4	5
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60	1	2	3	4	5

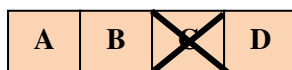
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67	1	2	3	4	5
68	1	2	3	4	5
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72	1	2	3	4	5
73	1	2	3	4	5
74	1	2	3	4	5
75	1	2	3	4	5
76	1	2	3	4	5

APPENDIX D: MATHEMATICS ACHIEVEMENT TEST (MAT)

Important information and instructions

1. This test is strictly intended for research purposes. The scores you obtain will **NOT** form part of your **CASS mark** at your school.
2. Use the answer sheet provided to answer all the questions.
3. The duration of this test is one hour.
4. **Section A** of the paper consists of multiple choice questions.
5. One of the four answers in each question of **Section A** is correct. Choose the answer you think is correct and make a cross on the letter representing that answer. Example: if you think the answer represented by a letter C is correct, make a cross on C like this.



6. Submit your rough work/calculations for questions **18; 19 and 20** together with the answer sheet.
7. **Section B** does **NOT** comprise of multiple choice questions; therefore show **ALL YOUR CALCULATIONS** and submit them on the answer sheet provided.

QUESTIONS

1. There are different kinds of mathematical patterns around us. Some of them are written as number sequences. What will be the **fourth term** in the following sequence? **1; 4; 9;; 25**
- A. 14
B. 15
C. 20
D. 16

2. If the exchange rate for the South African and the US dollar is R7/\$, which one of the following rand-dollar relationships is correct?

A.

R	7	8.75	10.50
\$	1	1.25	1.50

B.

\$	7	8.75	10.50
R	1	1.25	1.50

C.

R	7	87.5	105.0
\$	1	1.25	1.50

D.

\$	7	8.75	10.50
R	1	12.5	15.0

3. Solve the following equation and choose the correct value of x from the answers given below:




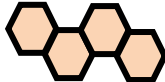
$$4(x - 1) = 2(x + 1)$$

- A. 5
B. 3
C. 4
D. 6
4. The factors of $4a^2 - 16b^2$ are

- A. $(2a - 8b)(2a - 8b)$
B. $(2a + 8b)(2a + 8b)$
C. $(2a + 4b)(2a - 4b)$
D. $(4a + 2b)(4a - 2b)$

Use the following information to answer questions 5 and 6:

Hexagonal floor tiles measuring **1 meter per side** are arranged in the pattern as illustrated below. The perimeter of each hexagonal tile is **6 meters**.

			
1 hexagonal tile	2 hexagonal tiles	3 hexagonal tiles	4 hexagonal tiles
Perimeter = 6 meters	Perimeter = 10 meters	Perimeter = 14 meters	Perimeter =

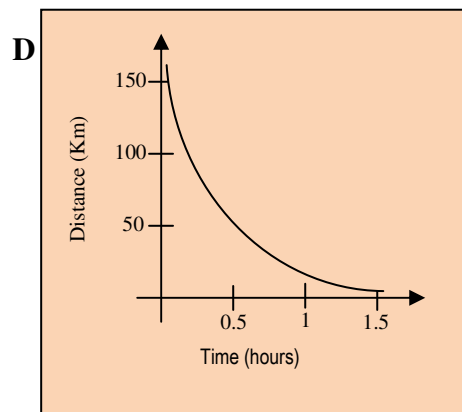
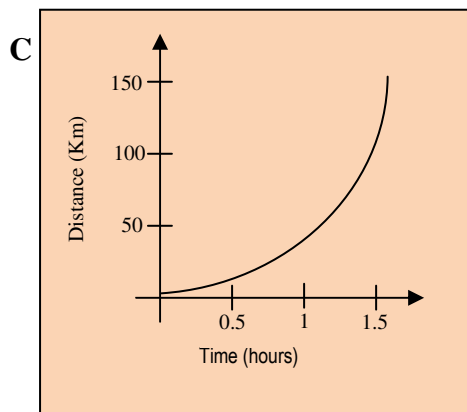
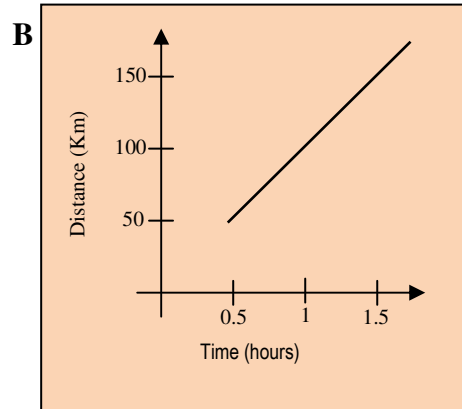
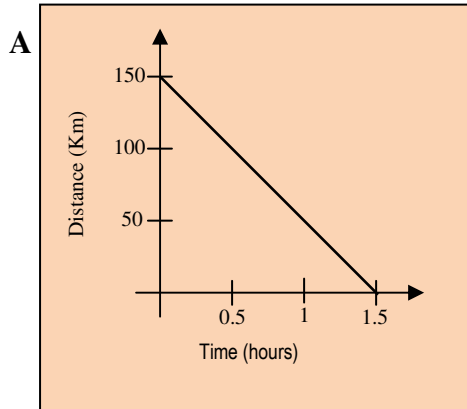
5. The perimeter of **four** hexagonal tiles joined together is:
- 24 meters
 - 18 meters
 - 4 meters
 - 14 meters
6. The general rule or equation you can use to calculate the perimeter (p) of the n^{th} number of hexagonal tiles joined together is:
- $p = 2n - 4$
 - $p = 2n + 4$
 - $p = 4n - 2$
 - $p = 4n + 2$
7. When $(x + y)(n - m)$ is simplified using a distributive law the answer is
- $xn + xm + yn + ym$
 - $xn - xm - yn - ym$
 - $nx + ny - mx - my$
 - $nx - ny - mx + my$
8. Which values of b will make the following equation true? $b^2 + 5b + 6 = 0$
- 2 and 3
 - 2 and 3
 - 5 and 6
 - 5 and -6
9. Which one of the following descriptions fits the number sequence: -8; -6; -4; -2; (**Note that the sequence is from left to right**)
- The next number in the sequence is smaller than the previous number by two
 - The sequence decreases by two from left to right.
 - Two is subtracted to obtain the next number.
 - Two is added to obtain the next number.

Use the following information to answer questions 10 and 11:

A motor car driver drives at a constant speed of 100km/h . The relationship between distance (d) and time (t) is tabulated as follows:

d	50	100	150
t	0.5	1	1.5

10. Which one of the following graphs represents the speed of the car according to the relationship described above?



11. The words 'constant speed' inform us that

- A. The x -intercept of the graph is 100.
- B. The y -intercept of the graph is 100.
- C. The gradient or slope of the graph changes every time.
- D. The gradient or the slope of the graph is the same throughout.

12. The simplified form of $\frac{4a^6+6a^3-10}{-2a}$ is

- A. $-2a^5 - 3a^2 + 5a$
- B. $-2a^7 - 3a^4 + 5a^2$
- C. $2a^5 - 3a^2 + 5$
- D. $2a^5 + 3a^2 - 5$

13. The highest common factor in $2a^4 - 4x^2$ is

- A. $2x^4$
- B. $-2x^4$
- C. $2x^2$
- D. $-2x^2$

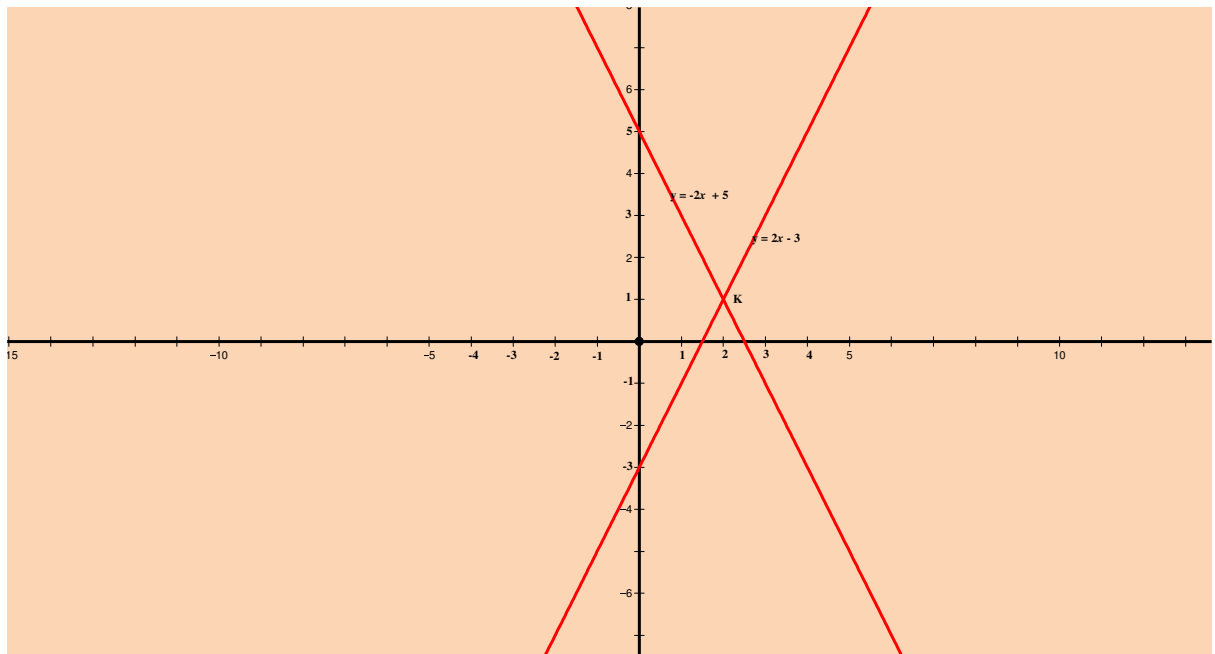
14. The simplified form of $2x^2(x - 2x^2)$ is

- A. $-4x^4 + 2x^3$
- B. $-2x^3 + 4x^4$
- C. $-3x^3 - 4x^2$
- D. $3x^3 + 4x^2$

15. When $3x^2 - 2x + 2$ is subtracted from $5x^2 + 4x + 2$ the answer is

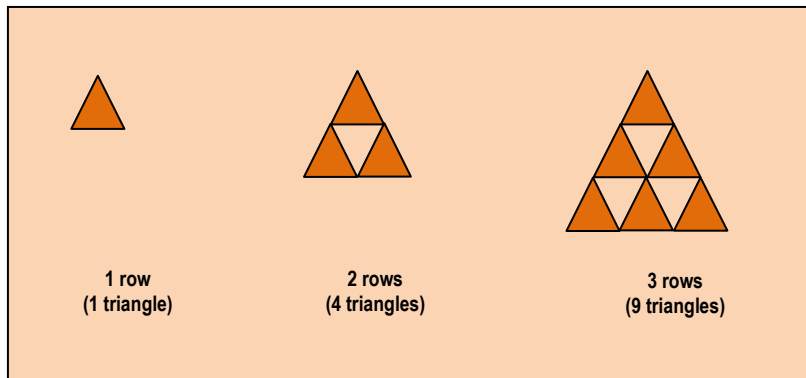
- A. $8x^2 + 2x + 4$
- B. $2x^2 + 6x + 4$
- C. $2x^2 + 6x$
- D. $2x^2 - 6x$

16. The graphs represented by $y = 2x - 3$ and $y = -2x + 5$ intersect at point **K** as illustrated below.
The coordinates of the point of intersection (**K**) are



- A. (2; 1)
- B. (1; 2)
- C. (-2; -1)
- D. (-1; -2)

17. The following pattern shows the relationship between the number of rows (r) and the number of triangles (t). Which rule defines the geometric pattern illustrated below?



- A. $t = 2r$
B. $t = r$
C. $t = r^2$
D. $t = \frac{1}{2}r$

Use the linear equation $y = 3x + 2$ to answer questions 18; 19 and 20

18. Calculate the y –intercept and choose one of the following answers:

- A. 2
B. 3
C. -3
D. no correct answer

19. If $y = 5$ calculate the value of x and choose one of the following answers:

- A. 2
B. 3
C. 1
D. no correct answer

20. Does the point given by the coordinates $(1; 3)$ lie on the linear graph $y = 3x + 2$?
Use calculations to prove your answer.

- A. Yes
B. No
C. Only 3 because there is 3 in $y = 3x + 2$
D. no correct answer

APPENDIX E: ANSWER SHEET: MAT

1

A	B	C	D
---	---	---	---

2

A	B	C	D
---	---	---	---

3

A	B	C	D
---	---	---	---

4

A	B	C	D
---	---	---	---

5

A	B	C	D
---	---	---	---

6

A	B	C	D
---	---	---	---

7

A	B	C	D
---	---	---	---

8

A	B	C	D
---	---	---	---

9

A	B	C	D
---	---	---	---

10

A	B	C	D
---	---	---	---

11

A	B	C	D
---	---	---	---

12

A	B	C	D
---	---	---	---

13

A	B	C	D
---	---	---	---

14

A	B	C	D
---	---	---	---

15

A	B	C	D
---	---	---	---

16

A	B	C	D
---	---	---	---

17

A	B	C	D
---	---	---	---

18

A	B	C	D
---	---	---	---

19

A	B	C	D
---	---	---	---

20

A	B	C	D
---	---	---	---

APPENDIX F: LETTER OF REQUEST – DISTRICT MANAGER

P.O.Box
Tramshed
0126
13 February 2007

District Manager
Mabopane

REQUEST: PERMISSION TO CONDUCT RESEARCH

This communiqué serves to request for your permission to conduct my research project in your district/region. I am currently doing **PhD in Mathematics Education** and the following are some of my particulars:

Institution: North West University
Student Number:

Theses title: *Diagnostic mathematics assessment: the impact of the GIST model on learners with learning barriers in mathematics*

Nature of the study:

If permission is granted, four randomly sampled schools offering grade 9 will be used for a period of two months, ideally from April to June of 2007. The study involves piloting a **Group Intervention Strategy (GIST)** with the hope of mitigating the problems associated with under-performance in mathematics such as mathematics anxiety, negative attitudes towards maths, poor study habits in maths, and poor problem solving behaviour in maths. Diagnostic assessment will be conducted among all the grade 9 learners in each of the four schools, thereafter only two classes per school will be for piloting the GIST model.

Further be assured that the study will be conducted such that very minimal class disruptions are maintained and high research ethics are upheld.

I appreciate your anticipated positive response in this regard

Yours faithfully

R.D.Sekao
Cell:
E-Mail:

APPENDIX G: LETTER OF REQUEST - PRINCIPAL

P.O. Box
Tramshed
0126
19 March 2007

The Principal
..... Middle School

REQUEST: PERMISSION TO CONDUCT RESEARCH

This communiqué serves to request for your permission to conduct my research project in your school. Your school is among those that were sampled for research purposes. The office of *PSS Sub-directorate*, on behalf of the *Regional Manager*, has granted me permission in this regard (the attached letter serves as a proof).

I am currently doing **PhD in Mathematics Education** and the following are some of my particulars:

Institution: North West University
Student Number:

Theses title: *Diagnostic mathematics assessment: the impact of the GIST model on learners with learning barriers in mathematics*

The study involves piloting a **Group Intervention Strategy** (GIST) with the hope of mitigating the problems associated with under-performance in mathematics such as *mathematics anxiety, negative attitudes towards maths, poor study habits in maths, and poor problem solving behaviour in maths*.

Be assured that the study will be conducted such that very minimal class disruptions prevail and high research ethics are upheld. More information will be divulged during the meeting to be scheduled with the Site Manager/Deputy Principal after permission is granted.

I appreciate your anticipated positive response in this regard.

Yours faithfully

R.D.Sekao
Cell:
E-Mail: