

**TEACHING AND LEARNING QUADRATIC EQUATIONS
THROUGH A PROBLEM-CENTRED APPROACH: A CASE OF
GRADE 11 CLASSROOM IN CAPRICORN DISTRICT OF
LIMPOPO PROVINCE**

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DECLARATION

I declare that, "Teaching and learning quadratic equations through a problem-centred approach: A case of Grade 11 classroom in Capricorn District of Limpopo Province", is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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DATE

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DEDICATION

This work is dedicated to my sons

FADIPE BOLAJI JUBRIL

and

FADIPE BIMBOLA AKIM

ABBREVIATIONS

AEC	Australian Education Council
CAPS	Curriculum and Assessment Policy Statement
DoE	Department of Basic Education
FET	Further Education and Training
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
NWU	North West University
PCTL	Problem-centred teaching and learning
SDL	Self-directed learning
TIMSS	Trends in Mathematics and Science Survey
ZPD	Zone of Proximal Development

ABSTRACT

Worldwide the teaching and learning of mathematics pose a great challenge to mathematics teachers as learners' performance in the subject leave much to be desired. This is particularly the case in South Africa, where there has been a great disparity in the development of teachers in the past. Extensive research has shown that many teachers in South Africa are under-qualified, especially in the teaching of mathematics at secondary schools. The performance of mathematics and science learners is particularly low in South Africa. The study investigated the benefits of using problem-centred approach in the teaching and learning of quadratic equations in grade-11 classroom using a mixed method approach. Learners were given learning activities on quadratic problems to carry out as part of their normal classroom mathematics' lessons. Data were collected in three stages: pre-intervention, which involved a quantitative approach, pretest and qualitative, questionnaire; during intervention, which included a qualitative approach, video recording and questioning, and learners' journals; post-intervention; quantitative; post-test and qualitative; questionnaire. The responses of the learners were analysed during each of the above stages. The scripts were reviewed based on four problem-solving stages adopted from George Polya (1945) viz.: understanding the problem, devising the plan, carrying out the plan, and looking back. It became evident from the findings of the study that before the intervention, learners had no understanding of problem-solving abilities and they were able to develop these abilities during the intervention and after the intervention, the learners have developed the necessary skills needed in problem-solving in learning quadratic equations. A total of 20 learners participated in the study from a secondary school in the Capricorn District of Limpopo Province. The study adhered to ethical principles and applied several techniques to enhance the validity/trustworthiness of the findings. The study found that learners benefitted tremendously from the problem-centred approach of teaching and learning. To this end, various recommendations were made. Recommendations for further study were also highlighted and the limitations of this research pointed out.

KEYWORDS: problem-centred teaching and learning approach, active learning, quadratic equations, problem solving, traditional teaching approach.

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CHAPTER 1

INTRODUCTION, BACKGROUND AND RATIONALE

1.1 INTRODUCTIONS

Currently, certain educationists conceive of learners as “architects building their own knowledge structures” (Wang, Heartel & Walberg, 1993:264). Until recently, learners were seen as being passive in their own learning but this view has changed to that of being actively involved in the construction of their own learning. For example, the New Zealand Curriculum statement states that, “when learners participate actively in the process of learning mathematics rather than accepting rules and procedures from their teacher, mathematics is learnt effectively,” (Ministry of Education 1992:18). Also, a National Statement on Mathematics for Australian Schools claims, “it is generally believed and accepted that active learning leads to a productive learning on the part of the learners” (Australian Education Council, 1990:16). Here in South Africa, the Curriculum and Assessment Policy Statement for Mathematics (CAPS) envisaged learners who are able to think creatively and critically and using these skills in decisions to identify and solve given problems and secondly, learners who are able to establish the world as a set of connected systems through their understanding of mathematics by acknowledging the fact that problem solving as a context does not exist in isolation (Department of Basic Education, 2011:4).

Problem solving affords learners opportunities to have control and freedom over the organisation of their own learning activities in a dynamic learning environment (Nardos, 2000:87). Problem-centred learning approach is one of those learning strategies that actively involve learners in their own learning (Von Glasersveld, 1995:120). However, the researcher finds it both important and necessary to investigate further the potential gains of using the problem-centred approach as a learning strategy. Therefore, this study explored the benefits of using problem-centred teaching and learning approach in teaching and learning of quadratic equations in the Grade 11 classroom.

In his work, Killen (2000:220) defined problem-centred learning as a technique for teaching through problem-solving. Others have meanwhile argued that problem-centred learning involves the use of problem-solving to help learners learn other concepts (for example, Murray, Olivier & Human, 1998; Hmelo-silver, 2004: 236). They further argued that problem-centred approach is one of the methods of instruction that affords learners comprehensive understanding of a given

task. Problem-centred teaching and learning approach also involves learning authentic mathematics that can be applied to solve real life situations (Van de Walle, Karp & Bay-Williams, 2013:32) and “is driven by problem solving” (Human, 1992:16).

Problem-centred learning approach is not simply an application of the existing knowledge and abilities; it is the mechanism by which all new knowledge and skills are obtained and this is built around problems that afford learners the opportunity to heighten their knowledge and skills as outlined in mathematics curriculum (Killen, 2007:219). Problem-centred learning nurtures learners through the provision of real problems in order to be actively involved in their learning, therefore, responsible for the construction of their own knowledge while at the same time developing various strategies in solving problems (Hmelo & Ferrari, 1997; Kolodner, Hmelo & Narayanan, 1996).

When mathematical problems are posed to learners without prescribing a definite heuristic, learners tend to solve the problem in multiple ways, thereby evoking their sense of mathematical thinking and creativity, which ultimately leads to effective learning (Killen, 2007:219), because the construction of mathematical knowledge is supported when a problem-centred approach is used. Learners are therefore able to connect different concepts and ideas to one another. Solving problems in different ways characterises the creativity of mathematical thought. Quadratic equations are one of the key topics in mathematics, throughout the Further Education and Training Phase. This topic carries great weight as far as content areas are concerned in Paper 1 of the NCS CAPS document for Grade 11, as indicated in the table below.

Table 1.1 *Weighting of topics in Grade 11 Mathematics adapted from CAPS Mathematics document for FET.*

Mathematics paper 1

Descriptions	Grade 10	Grade 11	Grade 12
Algebra Equations (& inequalities)	30	45	25
Patterns & sequence	15	25	25
Finance & Growth	10	---	---
Finance, Growth & Decay	---	15	15
Function & Graph	30	45	35
Differential Calculus	---	---	35
Probability	15	20	15
Total	100	150	150

Engaging learners in their own studies appears to be one of the most effective ways for them to learn mathematics meaningfully (Brown, 1995). Most content areas of mathematics in Grade 11 classrooms involve the solving of equations and, if learners are able to understand quadratic equations quite well, they will not have difficulties in mastering similar equations in any learning area of mathematics. Therefore, it is most important and necessary to use a teaching and learning approach, which is more effective in learning and understanding quadratic equations, where problem-centred learning promotes learners' involvement (Von Glasersveld, 1995:120), which may lead to effective learning.

1.2 PROBLEM STATEMENT AND RESEARCH QUESTION

Problem-centred learning in an academic field involves being presented with a situation that requires a resolution (Snodgrass, 1988). In other words, problem-solvers are given an intelligible space to understand what the problem requires of them to do, devise a way (or method) to solve the problem by selecting from previously learnt tools the best and most effective tool (Sepeng, 2014). He further argued that once problem-solvers are content with the solution, they would have to evaluate effectiveness of the solution by applying it in several other situations.

Kadel (1992:7) argued that problem solving in mathematics is acquired in a problem-centred teaching and learning through exploration and uncovering by the learners. In addition, the learners are expected to use their own imaginations through the gained experiences when confronted with new or similar situation, this practice advances their maturation in problem solving skills in mathematics (Kadel, 1992:4).

It is against this background that the proposed study sought to investigate the benefits (or lack thereof) of using problem-centred learning as an approach in the teaching and learning of quadratic equations in Grade 11 classroom. This study investigated the benefits of a problem-centred approach as a strategy for teaching and learning quadratic equations in a Grade 11 classroom. The main research question is:

What are the benefits of using problem-centred approach in teaching and learning of quadratic equations in Grade 11 classroom?

Apart from this main question the following sub-questions were also answered by the study.

The sub- questions are as follows:

- 1. What is the nature of the effects on grade 11 learners' performance when using problem-centred approach in teaching quadratic equations?*
- 2. What is the correlation between problem-centred approach of teaching and problem-solving abilities of learners when they engage with quadratic equations?*

1.3 AIMS OF THE RESEARCH

The aim of the study was to explore the benefits of using problem-centred approach in the teaching and learning of quadratic equations in the Grade 11 classroom, and to investigate the nature of the effect on Grade 11 learners' performance using problem-centred approach in teaching quadratic equations. In addition to this, it is necessary to establish the correlation between problem-centred approach of teaching and learning and problem-solving abilities of learners when they engage with quadratic equations.

1.4 THE SIGNIFICANCE OF THE STUDY

One of the key aims of education is for learners to develop cognition and problem-solving skills which might be applied in the subject, other fields of studies and in their daily lives (Wessels & Kwari, 2003:74-75).

The aspirations of the department of basic education are to equip learners with meaningful and purposeful knowledge, skills, and values so that they can contribute positively to the society and also, to develop problem solving skills for better performances in mathematics, (Curriculum and Assessment Policy Statement, 2011:4). The department of basic education advocates problem-centred approach as a teaching approach through which the knowledge, skills and values acquired from the classroom can be purposefully applied in the outside world (Wessels & Kwari, 2003:75).

The research by Murray, Olivier and Human (1998) on teaching and learning mathematics in South African schools also supported the suggestion of the National Council of Teachers of Mathematics (NCTM 1991a; NCTM 1991b) that the use of problem-centred teaching should be encouraged in schools for the improvement in, and better understanding of mathematics. The intention of the study therefore was to investigate the benefits of problem-centred learning approach as a strategy in the teaching and learning of quadratic equations in Grade 11 classroom as compared to the present traditional teaching approach which the Grade 11 mathematics teacher uses as a teaching strategy. Furthermore, though there have been many studies on problem-centred learning, to the knowledge of this researcher, there are presently no existing studies that investigate the use of problem-centred learning approach for the teaching of quadratic equations in South African Grade 11 classrooms. The gains may also be applied to

other topics, and eventually other subjects and this may have a positive effect on the learners' studies through tertiary education and beyond.

1.5 THEORETICAL FRAMEWORK

This study is framed on George Polya's theory which takes mathematical problem solving as being presented with written-out problems, which requires one to interpret, devise the solution method, follow mathematical procedures to achieve the results and then analyse the results. It is through the steps outlined that this study turned to adopt the theory, because it addresses problems related to mathematical problem-solving. Primary sources of data in this study are quadratic equations activities that, according to Polya (1945), require heuristic strategies theory for better solution. In this theory, he (Polya) initially noticed that learners do not know how to solve problems, and he further emphasised that the difficulty was not associated with learners not knowing mathematics, but rather, with lacking the ability to guide their thought processes along fruitful channels. The claim is that this is still a challenge to most learners studying problem solving, and has resulted in a series of widely accepted heuristics strategies, which he developed to address the challenge.

The theory requires of learners to understand the given problem, devise an action plan to solve it, carry out the plan using the identified variable, and finally interpret the results. The aspirations of this theory seem to be in line with what the study envisages, namely, that a learner is able to understand what the root is of a given problem, to interpret the given data and the conditions attached to the problem, and then proceeding to the next stage, which is also what the theory is advocating.

- **UNDERSTANDING THE GIVEN PROBLEM**

To understand the given problem, ask:

What is the problem about?

What are the data that will be used to make sense of the problem?

What is the condition under which the problem can be solved?

- **DEVISING THE ACTION PLAN**

Need to find if the identified data and the unknown variable are interconnected.

If an immediate connection cannot be found between the variables identified you may seek for an alternate process of finding the solution.

Finally, a plan for the process of solution should be obtained.

- **CARRYING OUT THE PLAN.**

Execute the plan of your solution and check each step.

- **LOOKING BACK.**

Critically examine the solution you obtained and check for any error(s).

It is through the understanding of connections between the data and the unknown in a given problem that one can arrive at a relevant plan for the problem. The quadratic equations problems require learners to come up with plans for the solutions to the problems. The implementation stage of Polya's model asks learners to implement the proposed plan and check each step as procedures unfold. Finally, learners are expected to interpret the results and check as to whether they make sense of the problems. These also constitute expectations from everyday mathematics problem-solving lessons, and this is why the theory is deemed fit for the study. Mathematical problem-solving as used in this study refers to a classroom situation, wherein one is presented with a written-out problem in which a learner is required to interpret, devise the solution plan, apply the plan through mathematical procedures, and finally analyse the results.

1.6 REASERCH DESIGN AND METHODOLOGY

A research design is a plan or approach consisting of the processes and assumptions involved in the selection of participants or subjects, the techniques for data gathering, and the analysis of data (Maree, 2010).

1.6.1 Research design

The study focused on Grade 11 mathematics classroom, making this investigation a case study. The mixed-method research design was used in the study as it incorporates both quantitative and qualitative approaches. McMillan and Schumacher (2014:33) define mixed method design as a process that involves collection and analysis of both quantitative and qualitative data separately and then “mixing” both data in a single study and, for the complete understanding and interpretation of a research problem., both quantitative and qualitative data are combined, the importance of mixing the data is to complement each other so that the researcher is able to integrate the strengths of each other to the study’s advantage (Creswell, 2002). When more than one research method is used in a study, it is known as *triangulation* (Cohen & Manion, 2007). It involves the collection of qualitative and quantitative data at about the same time, so that the strengths of one method compliment the impuissance of the other for the provision of a more comprehensive set of data. The triangulation design is used because the strengths of each approach can make provisions for, not only a more complete result but also one that is more logical theoretically (McMillan & Schumacher, 2014:33).

1.6.2 Research site

The study was conducted at a high school in Limpopo Province, South Africa. The choice of the school was for convenience and ease of access, because I teach in the same school. It is located in a village *Makgato* in the Capricorn District of Limpopo Province about 50km from Polokwane, the capital city of Limpopo. As a typical rural school, the total enrollment in 2016 was 348 learners (Grade 8–12).

1.6.3 Participants

There are 20 learners in the mathematics classroom in Grade-11 of the school, all are Black Africans, eight are males and 12 females, and they are all residents of Makgato, where the school is located. The participants’ selection was based on convenience sampling (using available subjects) as this is the only available class of Grade 11 in the school. The Grade 11 class at the school comprised mixed ability learners, with all levels of performance represented (Level 1-7), and the home language spoken is Sepedi.

1.6.4 Data Collection

The techniques for generating, collecting and selecting data are all the processes involved in data collection. All these processes have a broad effect on the final data collected for the purposes of studies. The researcher's theoretical perceptions, informed by the problem and purpose of the study, and by the sample selected, according to Meriam (1995), determine the technique used for data collection and the specific information regarded as data in a study. This therefore justifies the learning activities that were used as tools for data collection in this study, which were deemed suitable for the problem identified.

1.6.4.1 Quantitative data

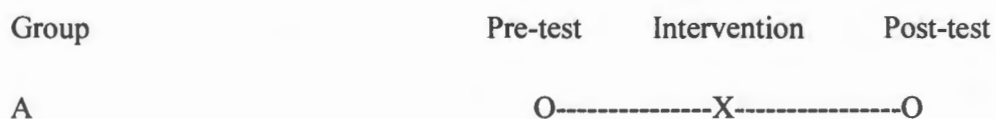
The processes of collecting, analysing, interpreting, and writing the results of a study using quantitative data is, according to Creswell (2009), quantitative method. He (Creswell) further argues that quantitative methods are used in survey and experimental studies. For this study, the tools used for the collection of quantitative data are pre-test and post-test.

1.6.4.1.1 Pre-test/Post-test

A test was administered as written work on quadratic equations, which served as the pre-test before the intervention for all the learners who participated in the study. The intervention was teaching the Grade 11 classroom quadratic equations using the problem-centred teaching and learning approach. After the intervention, a written work on quadratic equations was administered as the post-test. The importance of the post-test was to verify the effect(s) of the intervention. The design is the single-group pretest-posttest design below.

Figure 1.1

Single-Group Pretest-posttest Design



This study employed the single-group pretest-posttest design; the only groups in the study is the Grade 11 mathematics learners as participants who were given a pretest (O), then the treatment (X), and then the posttest (O) after the treatment. The pretest and the posttest were the same, but administered at different times. The aim was to compare the results of the pretest and posttest whether there were any improvements in the two results.

1.6.4.2 Qualitative data

Qualitative approaches to data collection, analysis, interpretation, and report writing differ from the quantitative approaches. Qualitative data collection uses different tools, for example, interview and observation for data collection purposes (Creswell, 2009). In this study, questionnaire, video recording and questioning and learners' journals were used as qualitative tools for the collection of qualitative data.

1.6.4.2.1 Questionnaire

A questionnaire was one of the tools used to gather qualitative data with the purpose of understanding the challenges or/and the benefits of using problem-centred approach in the learning of quadratic equations. A questionnaire as a research tool is very economical, has the same sets of questions for all the participants, and anonymity is assured (McMillan & Schumacher, 2014:194). The questionnaire afforded the researcher the opportunity to clarify and distinguish between different kinds of obstacles that the participants were required to master before they could overwhelmingly benefit from the problem-centred learning approach and also, to acquire problem solving skills in mathematics, the items in the questionnaire were mostly closed items. The questionnaire, which was developed and used by Chirinda (2013), was used with minor modifications to suite the present study. It addressed the following:

1. The benefits of using problem-centred learning approach.
2. The challenges learners have when problem-centred teaching and learning approach is used in teaching and learning of quadratic equations in mathematics.

1.6.4.2.2 Video Recording and Questioning

During the intervention, the learners were observed by the researcher during the process of problem-solving using video recorder, the observations allowed the researcher to clearly understand the interactions among the learners, the types of questions that came forth. The video recording was analysed after every intervention in order to record the observations in the observation comments card. The researcher extensively questioned learners while moving around the class, and while they (learners) solved the given quadratic problems. After interventions, findings were recorded briefly and immediately. Questioning aroused logical thinking of learners about mathematics and this helped the researcher to measure the learners' mathematical problem solving abilities. Video recording and questioning were used, being one of the best methods for measuring problem-solving goals (Wheatley, 1991:6). Both the problem-solving observation comment card and problem observation rating scale was used to record the results and the findings.

1.6.4.2.3 Learners' Journals

It was requested from the learners by the researcher that they (learners) should write summary reports of their experiences on the problem solving procedures they had completed during the intervention in their journals. The researcher asked some focus questions in order to enable the learners to remember and describe how the given problems would have been solved differently. Learners' journals were used as these allowed the researcher to have more specific information of each learner in the acquisition of problem solving skills and approaches (Wheatley, 1991:23).

1.6.5 Data Analysis

Creswell (2009) explains data analysis as a series of connected processes that involve data preparation, conducting different kinds of analysis, comprehension and understanding of the data, representation of the data, and finally, making sensible interpretation of those data which may either be text or image data. The separate analysis of the quantitative data and qualitative data using quantitative methods and qualitative methods respectively and then combining the two



sets of data for the interpretations of the research results is data analysis in a mixed method research design (Creswell & Plano Clark, 2011:211).

1.6.5.1 Quantitative data analysis

In this study, single sample t-test was used as the statistical procedure to analyse the group mean. Hence, this allowed the researcher to compare the differences of the pretest mean scores to those of the posttest (McMillan & Schumacher, 2014:325).

1.6.5.2 Qualitative data analysis

The organisation and categorisation of data into different identified categories allows establishing the relationships or patterns among those categories (McMillan & Schumacher, 2014:395). It is an inductive procedure that involves coding, categorising and interpretation of data to provide explanation of a phenomenon (McMillan & Schumacher, 2014:395). Qualitative data were used to answer the following research sub-question, “what are the benefits of using problem-centred approach in teaching and learning of quadratic equations in Grade 11 classroom”; the answers to this question were used to substantiate the quantitative results.

For the qualitative data to become well manageable and meaningful, the organisation and reduction of the content of qualitative data by the researcher was guided by the following processes as cited in Creswell (2009).

- i. Coordination and preparation of the data for analysis was done by the researcher. This involved recording all the observations, field notes and transcribing learners’ journals;
- ii. In order to obtain general information and to reflect on the information gathered for meaningful interpretation, the researcher read through all the data;
- iii. The researcher read through the learners’ journal one after the other in order to transcribe the notes of the learners and thereby gaining useful knowledge about their meaning;
- iv. Step (iii) above was repeated for the transcription of the recorded data of all the participants and the researcher then drew a list of all possible themes and grouped similar themes together;

- v. The arrangement and coding of the themes was done and entered into the appropriate segment of the text in order to check for possible emergence of new categories;
- vi. The researcher marked the themes after the identification of perfect descriptive wording and placed them into categories;
- vii. Decision was made on the abbreviation and numbering of all categories;
- viii. The preliminary analysis was done after the data materials for each category were put together; and
- ix. Then the existing data were re-coded where necessary.

According to Creswell (2009), the above steps are processes of textual data analysis that engage a researcher. As was explained earlier, the researcher acted as an independent qualitative coder, assumed all the responsibilities of independently coding the data with the guidance of the supervisor of the proposed study. These data were transcribed and analysed using Polya's theory that frames the study.

1.6.6 Reliability and validity

The consistency under comparable conditions of the same instrument or closely similar instruments yielding the same or very similar results when administered independently is referred to as reliability (De Vos, 2002:168). McMillan and Schumacher (2014:195) define reliability as "consistency of measurement".

Validity is the magnitude to which meaningful and useful inferences and uses of data based on numerical scores are allowed (McMillan & Schumacher, 2014:116). A research tool that measures exactly what it was designed to measure is said to be valid. The learners' written work (pre-test and post-test) was assessed using an analytic scoring scale in order to ascertain its reliability. Pilot testing of the pre-test and post-test was conducted to ensure their reliability and validity. The questionnaire contains all possible questions in problem solving skills development to ensure content validity. In the study, supervisor was given the research tools for his analysis, opinions and approval before the outset of the intervention.

1.6.7 The Roles of the Researcher

The researcher was a participant in this study. He was responsible for the teaching of the quadratic equations using the problem-centred learning approach for three days in a week (Tuesday, Wednesday and Thursday). The duration of each lesson was an hour, over the course of five weeks. The lessons took place after school hours, on the chosen days. These arrangements catered for those learners who may not want to participate further in the study.

- administering pre-test and post-test;
- administering closed-ended questionnaires;
- questioning of the participants (qualitative questioning);
- analysing a verbatim description of the data; and
- interpretation and triangulation of the data.

1.6.8. Ethical Issues

Ethics are generally considered to deal with beliefs about what is right or wrong, proper or improper, good or bad. Openness and honesty were catered for in this study by informing participants and their parents about the plans and aspirations of the study. The participants, most importantly were assured of confidentiality and anonymity and that their participation was voluntary in the study. It was also conveyed to the participants that they could withdraw from the study at any time if they deemed it necessary, after gaining permission to conduct the study from North West University.

A letter of request was sent to the principal of the school and the school governing board asking for their permission to use the school as a research site.

1.7 CONTRIBUTION OF THE STUDY

The expectations of the study are the contributions of new understandings on the teaching and learning of quadratic equations in other similar schools, and in addition:

- The study may help education department in the provision of necessary review and support in the implementation of problem-centred approaches in South African schools;
- The study may also help teachers in adjusting their teaching methods in quadratic equations if the need be; and
- It may help improve learners' understanding on how active learning approaches could impact positively on their performance.

The study can contribute to the strengthening of theories that focus on the perceptions regarding problem-centred learning approaches.

The study may also serve as a point of departure for further extended research in problem-centred teaching and learning approaches.

1.8 PRELIMINARY STRUCTURE

Chapter 1: Introduction, Background and Rationale

This chapter discussed the introduction and background of the study extensively, the problem statement and aims of the study were presented, and general summary of the research design and methodology, theoretical framework were discussed as well as issues relating to ethics. This chapter also discussed the significance of the research, and research questions were presented. The chapter concluded by presenting the preliminary structures of the study.

Chapter 2: The Literature Review

Chapter two presents the literature review that relates to the varieties of work by other researchers on the teaching and learning of quadratic equations. Some studies were on the difficulties that learners experience in understanding quadratic equations and also basic requirements for the acquisition and development of mathematical problem solving abilities in learners. Problem-solving models were also investigated in this chapter, and this was the stage at which the study aligned itself to one of the models identified. How learners could be assessed in problem-centred teaching and learning environment was also discussed.

Chapter 3: Research Methodology

The research design and also the research methods used to conduct the study were thoroughly explained in this chapter. These include sampling technique, both quantitative and qualitative data collection were discussed, so also were analyses of data, reliability, validity and research ethics.

Chapter 4: Results and Discussion

The results of the findings were presented, analysed and discussed in this chapter. The quantitative results and the qualitative results of the study were separately analysed for better understanding. The quantitative and qualitative results before the intervention, during the intervention and after the intervention were separately compared and analysed.

Chapter 5: Conclusion, Recommendations and Limitations

Summary of the whole study was presented in this chapter. The conclusion established based on the findings from this study was explained exhaustively. Limitations of the current study and the identified areas for possible future studies were recommended.

CHAPTER 2

THE LITERATURE REVIEW

2.1 INTRODUCTION

Variety of skills in problem solving are much valued (Sweller, 1988:257) and learners need to develop diversified problem solving abilities to be efficient and be able to solve mathematical problems effectively. Sweller (1988) further stressed that only if real mathematics problem solving takes place, it is then that learners are able to develop these problem-solving abilities. In a problem-centred teaching and learning (PCTL) environment, it is generally believed and accepted that genuine mathematical problem solving takes place (Sweller, 1988). Problem-solving in an academic field involves being presented with a situation that requires a resolution (Snodgrass, 1988). Hence, for the grade 11 mathematics learners, a PCTL environment was created by the researcher to develop their problem solving abilities in quadratic equations. The main purpose of the PCTL approach as a teaching strategy is to prompt learners towards their active participation in the process of learning as this may help to heighten and nurture mathematical problem solving abilities in the learners. Developing problem-solving abilities for academic success in learners should stretch far beyond school level. According to Kleitman and Stankov (2003:2), learners' ability to identify the types of help they need in managing their problem solving abilities is critical in understanding mathematics. Problem-solving therefore continues to feature in the policy documents of many different educational organisations, both nationally (DoE, 2010:3) and internationally (TIMSS, 2003; NCTM, 1989), and research into mathematical problem-solving has become increasingly more complex than it has been in previous years (Lester & Kehle, 2003:510). Inquiry and decision-making are very important components of problem solving skills (Fortunato, Hecht, Tittle & Alvarez, 1991:38). In general, two types of mathematical problems exist, viz., routine problems and non-routine problems. As part of the scope of mathematics education in South Africa, non-routine problem solving is the application of principles and processes of mathematics in solving unfamiliar mathematical problems (DoE, 2003:10).

Presentation of the literature review on the teaching and learning of quadratic equations in Grade 11 classroom using problem-centred teaching and learning approach is one of the aims of this

chapter. More importantly, the exploration of the basic requirements for the development of mathematical problem solving skills in Grade 11 mathematics learners. It is believed that mathematical problem solving embraces skills that can be applied in everyday life (NCTM, 1980); therefore, teaching and learning in mathematics should focus on mathematical problem solving. However, researchers have different views on the teaching of problem solving. Researchers like Cobb, Wood and Yackel (1991:25) and Murray, Olivier and Human (1998:270) have provided evidence of the benefits of the PCTL approach. The problem-centred teaching and learning approach can be contrasted with the traditional teaching approach; the latter, preponderantly focusing on the mathematical contents memorisation. The literature review, predominantly concentrates on the problem-centred teaching and learning approach, as the vehicle used to explore the Grade 11 learners' mathematical problem solving skills in quadratic equations in this study.

The point of departure of any sequence of instruction should involve situations that are practically real to learners, so that they can instantaneously engage in personally meaningful mathematical activity (Gravemeijer, van den Heuvel & Streefland, 1990). Such problems often involve everyday life settings or fictitious scenarios, although mathematics itself can also serve as a context of interest. Hence, the reflection of such activities should either be real situation from which mathematics has developed historically, or actual phenomena in which further interpretation, study and analysis require the use of mathematics. Many researchers have been prompted to provide tools for teachers to support their learners' problem solving skills due to the importance of problem solving activities for mathematics learners.

In order to learn as well as apply meaningful mathematics, learners need to identify, formulate, and address problems relating to their (learners) thorough understanding of mathematics (NCTM, 2007). Solving problems is not only a goal in mathematics, but also its central methodology. Learners should be afforded a significant amount of effort and encouragement to reflect on their thinking through the provision of frequent opportunities to formulate, contend with, and solve complex authentic problems. Research has shown that logical problems that engage learners and encourage them to wrestle with the mathematics concepts in a meaningful way, can affect their mathematical achievement in positive ways:

Contrary to learners' beliefs that mathematics is arbitrary, they (learners) should know that mathematics can be learned, used appropriately and it is understandable. Abilities to make sense of mathematics, to recognise the rewards and opportunities of doing mathematics require the development of positive attitudes towards mathematics (Kilpatrick & Swafford, 2001:131). Teachers can foster positive attitude towards mathematics by making mathematics more interesting, bringing real life contexts into classrooms.

In essence, the learners must have some real interest in the mathematics that they are doing, and they must feel that they have the necessary tools to finish the problem successfully, and that doing so is valuable. Therefore, this study investigated the benefits of using the problem-centred approach as a learning strategy in the learning of quadratic equations in Grade 11 classroom.

In their studies, Anderson (1998:8) and Cangelosi (1996:31), observe that in many mathematics classrooms, despite encouragements and promotion of contemporary approaches of teaching by educational reforms, traditional methods of teaching still predominate. Many studies claimed that traditional methods of teaching only produce rote learning and passive knowledge that can be used for tests and examination preparations, but not necessarily useful in real life situations (Tynjala, 1999:373).

Problem-centred learning is an approach that promotes active learning and shifts the responsibilities of learning to learners (Hmelo-Silver, 2004).

2.2 PREVIOUS RESEARCHES ON MATHEMATICS PROBLEM-CENTRED TEACHING AND LEARNING

Numerous studies on PCTL have provided evidence of improved performances in learners learning mathematics and sciences in various topics. According to Memory, Yoder and Williams (2003: 69), learners that were thought using PCTL were able to give enhanced presentations on their own, and were better able to apply the skills they learnt in new activities. In their own study, Yen and Lee (2011:139) concluded that problem-solving activities provide the setting for learners to engage in more creative and interactive ways, thereby shifting the focus of the class to a learner centred orientation. In a study assessing the impact of problem-centred learning in a large classroom setting, Klegeris, Bahniwal and Hurren (2013:75) found a 13%

increase in the test scores of those learners who were taught using PCTL approach, when compared to those who did not receive the instructions through PCTL. In a similar study, Patricia (2015:161) concluded that problem-centred learning engages and motivates learners thereby helping them (learners) to develop critical thinking skills and therefore enhancing their performance.

2.2.1. Learners' performances in solving quadratic equations

Researchers, Vaiyavutjamai and Clements (2006) have noted that there is a limited number of studies on teaching and learning of quadratic equations and the most disturbing is the fact that, in the literature, quadratic equations in mathematics education has been given little attention. Evidence to the techniques and methods that learners engage in while solving quadratic equations has been the focus of the limited number of research studies on quadratic equations (Bossé & Nandakumar, 2005); geometric approaches used by learners for solving quadratic equations (Allaire & Bradley, 2001); learners' understanding of and difficulties with solving quadratic equations (Kotsopoulos, 2007; Lima, 2008; Tall, Lima, & Healy, 2014; Vaiyavutjamai, Ellerton, & Clements, 2005; Zakaria & Maat, 2010); the teaching and learning of quadratic equations in classrooms (Olteanu & Holmqvist, 2012; Vaiyavutjamai & Clements, 2006); comparing how quadratic equations are handled in mathematics textbooks in different countries (Sağlam & Alacacı, 2012); and the application of the history of quadratic equations in teacher preparation programmes to highlight prospective teachers' knowledge (Clark, 1997). In general, quadratic equations create challenges in various ways for most learners such as difficulties in algebraic procedures (most importantly in factoring quadratic equations), and the inability to apply meaning to the quadratics, which are some of the identified challenges in the literature. Kotsopoulos (2007) suggests that engaging in factoring quadratics by learners is directly influenced by the learners' ability to recall multiplication. Furthermore, learners who are struggling with multiplication may not be able to solve simple quadratic equations. Using factorisation to solve quadratic equations requires learners to have basic multiplication knowledge since it requires finding factors of the equations, factoring simple quadratics may become quite challenging ($2x + 4$), while more difficult ones – such as $ax^2 + bx + c$, where $a \neq 1$ – becomes incomprehensible. When the leading coefficient in quadratic equations has many pairs of factors, it may become very difficult for the learners to find the factors (Bossé &

Nandakumar, 2005). Lima (2008) and Tall et al. (2014) suggest that learners rather than understanding the mathematical concepts in a given equations, they attach much importance to the methods of solving the equations which only relates to the movement of symbols. This makes it very difficult for learners to solve simple linear equations and quadratic equations become harder. Tall et al. (2014) stress that learners' failure to pay attention to the unknown in an equation is a challenge, they (learners) see quadratic equations as ordinary calculations. For instance, while attempting to solve $m^2 = 9$, some learners applied the exponent associated with the unknown as if it were the coefficient, that is, m^2 equals $2m$, and they (learners) showed a tendency to use the quadratic formula as the only valid method in solving every quadratic equations. Although it is expected that learners should use factorisation for solving equations such as $t^2 - 2t = 0$ and $3k^2 - k = 0$, studies have shown that few learners would do so (Tall et al., 2014). Vaiyavutjamai and Clements (2006) proposed that learners' difficulties with quadratic equations stem from their lack of both implemental understanding and relational understanding associated with solving quadratic equations. They suggest that while teacher-centred instruction with strong emphasis placed on the manipulation of symbols, rather than on the meaning of symbols, increases learner performance regarding solving quadratic equations, their (relational) understanding would still be quite low, and they could develop misconceptions. For example, they found that many learners had an inadequate understanding of the 'null factor' law. In solving $(x-3)(x-5) = 0$, although most learners gave the correct answer; $x = 3$ and $x = 5$, they considered two x s in the equations as representatives of different variables, and thus, they must take different values. That is, when they were asked to check their solutions, they simultaneously substituted $x = 3$ into $(x - 3)$ and $x = 5$ into $(x - 5)$ and found that $0 \cdot 0 = 0$ and in doing so, decided that their solutions were correct. This misconception also appeared in learners' solution of $x^2 - x = 12$.

The results indicate that learners mainly focus on the symbols to find the roots of the equations and perceive the quadratic equations as a calculation without thinking about its meaning (Lima, 2008; Tall et al., 2014). Therefore, since learners memorise the rules, formulas, and algebraic procedures to solve quadratic equations without understanding the meaning, they could not transfer these rules, formulas, and procedures to solve the quadratic equations with non-standard structured properties. They also have a tendency to forget the formula after some time has passed since they learnt it. In addition, learners usually do not think about alternative techniques for

solving quadratic equations in terms of their effectiveness and usefulness in a given context, thereby choosing one over other possible methods (Bossé & Nandakumar, 2005). Similar to other findings regarding word problems in linear equations (Clement & Battista, 1992; Stacey & MacGregor, 2000), the second major observation made from this study is that the quadratic word problems are quite difficult for learners. Consistent with (Cummins, Kintsch, Reusser & Wermer, 1988), they found comprehension of the problem statement to be the central reason for learners' difficulties with the word problems, rather than cognitive challenges in the solution phase of the symbolic equations.

Thus, the cognitive processes involved in forming and solving quadratic equations in different contexts could be of interest for further research. On the other hand, learners' difficulties with quadratics demonstrated in this study, whether in the form of symbolical equations or word problems, could also be explained by learners' reliance on rote thinking and reasoning in mathematics (Lithner, 2008:187). As Lithner (2008) explained, global strategy choices made by learners are guided by reasoning based on established experiences, which also tend to dominate learners' plausible reasoning; however, "this causes problems when the familiar routines do not work for different reasons". Thus, it is important to design teaching and learning environments such as problem-centred teaching and learning that can actively promote creative mathematically founded reasoning over rote learning and imitative reasoning (i.e., memorised and algorithmic reasoning) (Lithner, 2008). Furthermore, teachers should be aware that solving quadratic equations is not based only on procedures and rules and attempt to seek out alternative ways of teaching quadratic equations such as problem-centred teaching and learning. Moreover, learners should also be given opportunities to explore a range of situations in which they are required to construct, interpret, and then solve quadratic equations (Didis & Erbas, 2014: 1149).

The literature review that was conducted addressed the two sub-research questions for this study:

1. What is the nature of effects on Grade 11 learners' performance when using problem-centred approach in teaching quadratic equations?
2. What is the correlation between problem-centred approach of teaching and problem-solving abilities of learners when they engage with quadratic equations?

2.3 PROBLEM-SOLVING MODELS

The first model reviewed is that of Polya (1945), around which other models seem to be centred. The model from his book, '*How to solve it*' is one wherein heuristics and strategies of solving mathematical problems were emphasised, and he explicitly outlines the following captions and explanations of his model as follows.

2.3.1 Polya's problem-solving phases

During the problem solving process, learners should develop a very sound knowledge and understanding of representing and interpreting given information in order to develop effective and efficient skills in mathematics (Department of Education CAPS, 2011:9). NCTM (2000:52) also emphasised that "In all mathematics programmes, problem solving should be an integral part". The problem solving process as outlined by George Polya (1957) has four distinctive but interconnected phases that are parallel in nature.

2.3.1.1 Understanding the problem

Learners should firstly be able to understand what is expected of them on the given task so that they (learners) can extract vital information from the problem and be able to set aside the irrelevant information. They should clearly understand how to formulate the required and necessary information derived from the problem. They should be able to link the familiar situations they have lived before and liken it to the current problem. Finally, they should think of the process that may need the required solution.

2.3.1.2 Devising a plan

This is an important phase of the process; learners must be able to connect the identified variable to make sense of the problem, applying the mathematical knowledge already acquired to the given problem, followed by formulation of the mathematical concepts and procedures to follow (Van de Walle, 1998:41). The vital information extracted from the problem should be remembered and applied to the procedures to solve the problem. The mathematical routine to follow in solving the problem should be set up.

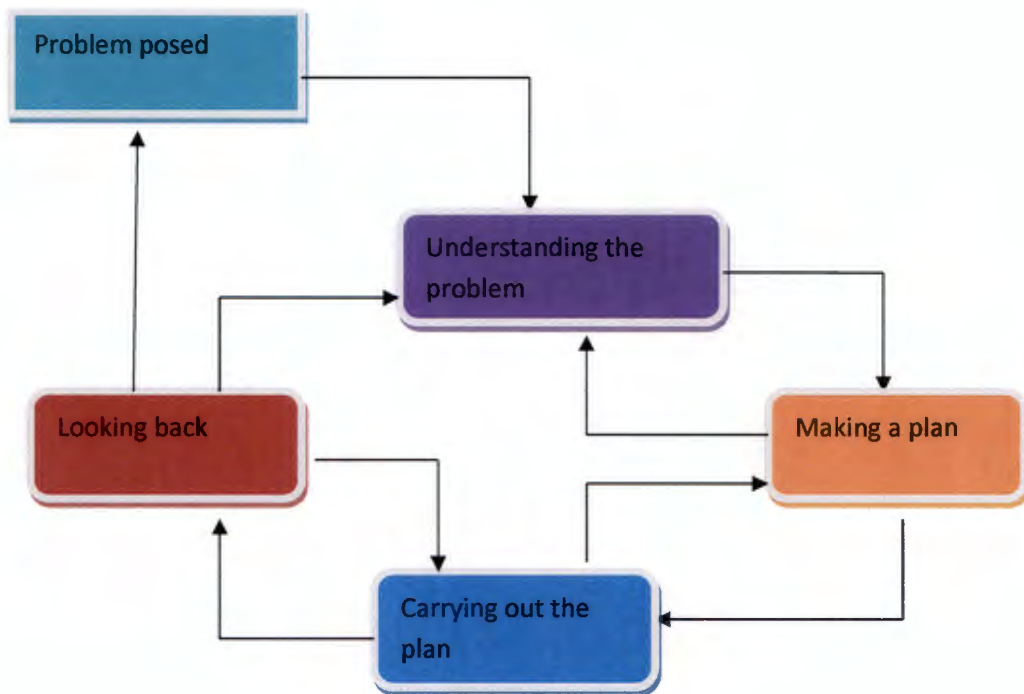
2.3.1.3 Carrying out the plan

The selection of the appropriate steps to follow should be informed by the decisions made during the adoption of the action plan. Self-monitoring of their own progress is very important during this phase and they should (learners) regulate the methods being used (Van de Walle, 1998:41). It is at this phase that the learners make use of the variables identified in order to solve the given problem. If learners could not proceed further from this phase, they should go back to the first stage (understanding the problem) and correctly dissect the problem again and look for a new approach to the problem. Learners should work carefully through the steps of solving the problem. However, failure to proceed from this phase, learners need to go back to stage one and start all over again.

2.3.1.4 Looking back

Being able to finally find the correct answer to the given problem does not necessary mean the problem is considered “solved” (Van de Walle, 1998:41). If learners are able to find the correct answer, they should reflect on the procedure or the solution process and again, learners should try other solution methods and check if it will lead to the same answer. If learners are unable to get the right answers, they should go back again to the first stage (understanding the given problem) and any necessary assistance should be given to learners who are grappling with the problem.

Figure 2.1 Polya's four-phase process of problem-solving. Adapted from Wilson, Fernandez and Hadaway (1993:25).



2.3.2 Hmelo-Silver Model

This model conforms to that of Polya's. Below are the steps identified in Hmelo-Silver's model (Hmelo-Silver, 2004):

- presenting the problem scenario;
- identify given facts (then formulate and analyse the problem);
- generate a hypothesis in line with the facts identified;
- identify knowledge deficiencies (self-directed learning);
- apply the acquired new knowledge;

- abstract; and
- evaluate.

When learners are presented with a problem scenario, they try to make sense of the problem scenario by articulating and analysing the problem in order to identify the relevant variables or facts. This assists the learners to represent the problem. As they have better understanding of the problem, hypothesis can then be generated for possible solution. In self-directed learning (SDL), learners try the process of solution by making sense of the identified facts or variables, if they succeed in getting the correct answers, they should try other methods and check if they will succeed, if the learners do not have enough relevant facts (Identifying knowledge insufficiency), they should start again from the problem scenario and re-examine the given problem. During the self-directed learning (SDL), learners are able to focus more on the process towards the solution than on the envisaged answer (Hmelo-silver, 2004). Hmelo-Silver (2004) identifies the following during (SDL): knowledge deficiencies, application of new knowledge to the problem; assessing the hypotheses formulated in line with the acquired knowledge, reflecting on the new knowledge developed at the completion of the problem. Reflection on the new knowledge gained is the most important aspect of the step in mathematics and science learning, as this helps the learners to apply the new knowledge in other mathematical problems. The teacher's role is to assist the learners to develop the skills required for mathematical problem solving. The skills necessary for lifelong learning are acquired through self-directed learning because they (learners) can control and manage the process of solution towards their learning goals. This also helps learners to be independent.

Hmelo-Silver (2004) further identified five goals that are developed in learners through problem-centred learning, these are:

1. acquisition of flexible knowledge;
2. develop effective problem solving skills;
3. self-directed learning skills towards their learning goals;
4. effective and efficient collaboration skills developed; and
5. intrinsic motivation for better performances.

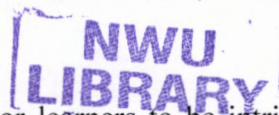
Learners constructing extended and flexible knowledge involve incorporating information from various knowledge bases (Hmelo-Silver, 2004). The gained knowledge should be conditioned in such a way that it can be applied in other situations. In encouraging learners to acquire such flexible knowledge in problem-solving abilities, the contexts that require such skills should be embedded into the learning programme. In developing effective and efficient problem solving skills, learners should be able to monitor their progress, control the process of solution, improve their reasoning and be able to assess the achievement of their goals (Schoenfeld, 1985).

The type of skills gained during self-directed learning (SDL) encourage independent learning as well as resilience and there are variety of sub-skills that are also gained (Hmelo & Lin, 2000; Zimmerman, 2002).

First, learners need to understand what they know or don't know about the given problem. Second, they should be able to set their achievable goal within the time given and further identify more useful information about the problem. Third, learners are able to plan and select the most appropriate process towards the solution. Finally, they execute their plans and are able to control and assess whether the set goal is achieved.

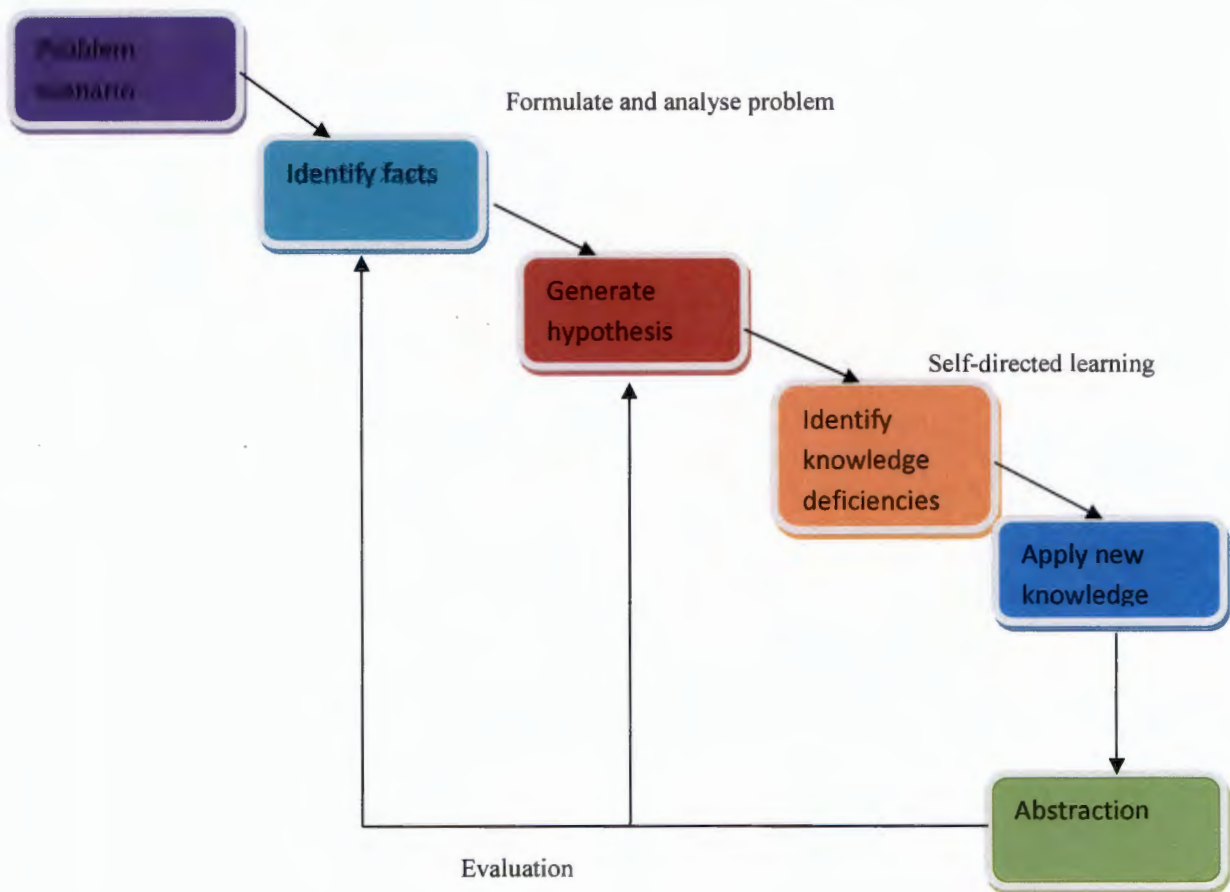
The fourth goal is that of collaborator; learners are able to function properly in a group; able to negotiate the actions taken by the group, and reaching agreement with the group (Barron, 2002). These tasks require all members of the group to be engaged with each other through effective communication so that they can freely exchange vital ideas and information (Cohen, 1994; Wenger, 1998).

Assisting learners to have intrinsic motivation is the last goal. For learners to be intrinsically motivated, they must willingly work on the assigned tasks with interest and not because of any external rewards. This means, the given problem should be real, interesting and at least a little bit challenging. The given problem should be easy to understand, clearly stated, and free of errors. Tasks given to learners should afford them the opportunities to apply the acquired knowledge in solving complex problems. In traditional classrooms, rewards are given to learners based on comparative achievements and this does not necessarily motivate other learners because they see it as competition, rewards then should be structured in such a way that learners are rewarded on their individual understanding of the learning contexts (Ames, 1992; Biggs, 1985; Ramsden in



Hmelo-Silver, 2004:241). It is very important that the less skilled learners be part of a research study in a problem-centred teaching and learning to understand how to achieve these goals. Problem-centred learning assists learners in developing flexible understanding and also as an instructional approach, affords learners the opportunities to acquire lifelong learning skills as evidence suggests (Hmelo-Silver, 2004).

Figure 2.2 Hmelo-Silver problem-centred leaning cycle. Source: Hmelo-Silver, (2004:237)



To further explore the components of the problem-centred teaching and learning process, the following sections concentrate on the role of the problem, the role of the facilitator, collaborative learning and reflection in problem-centred teaching and learning.

2.3.3 COMPONENTS OF THE PROBLEM-CENTRED TEACHING AND LEARNING

In this section, the roles of the given problem and that of the facilitator, collaborative learning and reflection on PCTL are discussed.

2.3.3.1 The role of the problem

From research point of view and more of practical experience, problem-centred teaching and learning has made critical footsteps in the identification of a good problem and its characteristics (Barrows & Kelson, 1995; Kolodner, Hmelo & Narayanan, 1996) adapted from Hmelo-Silver (2004: 246). For a given problem to support flexible thinking and intrinsic motivation, problems need to be authentic, open-ended and challenging (Hmelo-Silver, 2004). A good problem affords learners the opportunities to assess the effectiveness of the applied knowledge, their way of reasoning, and approaches to learning adopted. A good problem promotes hypothesis formulation and logical reasoning. A good problem should satisfy learners' needs to learn, to know and to motivate. The problem should allow learners to formulate hypotheses and to be able to defend the hypotheses to their other members. Solutions to a good problem should contain interconnected pieces, in order to foster development, understanding and heighten knowledge construction for future learning (Hmelo-Silver, 2004: 247). A problem that engages learners in their learning process based on prior understanding is a good problem. Good problems often require various logical ways of solutions. When knowledge is gathered from various sources, learners' communication skills are enhanced, extended knowledge is built, hence, a good problem should afford learners to search extensively for solutions (Hmelo-Silver, 2004:246).

2.3.3.2 The role of the facilitator

Formulation of a good problem is a necessary condition for effective PCTL. To make PCTL function very well, the role of a facilitator is important. The expertise of a facilitator should be visible through problem solving in PCTL (Collins, Brown, & Newman, 1989). Rather than a professional in only the context, a teacher/facilitator should formulate good approach for learning and thinking as an expert learner in PCTL. Learners should be supported through the use of good questioning strategies and coaching (Hmelo-Silver, 2004:246). Teachers gradually and progressively withdraw their supports, as learners become more experienced and finally can adopt the role of a facilitator in PCTL. The responsibility of the facilitator includes managing

and controlling the learners through different phases of the PCTL. This managing should encourage and assure the learners of externalising their thinking (Hmelo-Silver, 2002; Koschmann, Myers, Feltovich & Barrows, 1994). In PCTL, the teacher/facilitator: (i) encourage justification of learners' thinking by guiding thinking skills development; and (ii) asking appropriate questions from each learner to externalize individual-reflection. The teacher/facilitator as a player has an important role in modelling the skills needed for problem-solving and SDL for the self-assessment in learners' reasoning and understanding in PCTL. Though, support is withdrawn as the learners acquire experience with the PCTL method, Hmelo-Silver (2002) agrees that monitoring the learners about decision making in the process of PCTL still rests on the facilitator. Directly supporting several goals of PCTL is still continued by the facilitator. For example, modelling learners' problem-solving and SDL processes, helping learners learn to collaborate with one another very well (Barrows, 2000). It is assumed that construction of exile knowledge is better achieved by the learners if there is support on process of learning and collaboration from the facilitator (Hmelo-Silver, 2002). Facilitation is not an easy task; a facilitator who is used to a smaller class with few learners may not be able to control and monitor a large class with many learners. A facilitator should know when to direct an appropriate question to learners, when the learners do not comprehend, and when the process of PCTL is not producing the desired results (Hmelo-Silver, 2004: 246). In a study of an expert PCTL facilitator, Hmelo-Silver (2002) found that he accomplished his role largely through meta-cognitive questioning, and questioning that focused learners' attention and evoked causal explanations. The role of the facilitator is extremely important in modelling thinking skills and providing meta-cognitive scaffolding.

2.3.3.3 Collaborative Learning in PCTL

One of the main features of PCTL collaborative group is solving problems. Problems that will be very difficult for individual learners are distributed among members of the group and this allows learners that are more gifted to share their knowledge with the less gifted learners in the group. It is assumed in PCTL that members of the group can take advantage of knowledge distribution through participation of the members in the group (Pea, 1993; Salomon, 1993), through distributed expertise which helps the group to tackle problems that each learner may find

difficult to solve as an individual. In PCTL, distributed expertise is a very important and relevant impression because when learners break down a given task into different components, each learner becomes an expert in the given aspect of the problem. In their studies (Blumenfeld, Krajcik, Mark & Soloway, 1996; Brown, 1995; Goldman, Hmelo, Voss, Vye & Williams, 1997) it is suggested that high order thinking, problem solving skills and knowledge construction are heightened in group discussions in a PCTL environment. Furthermore, Barrows (2000) affirms that as learners work together in a group, knowledge is constructed and shared and that it is the role of the facilitator to ensure the participation of all the members of the group in the debate or discussion. There are other strategies to use when a facilitator is not available, O'Donnell (1999), Herrenkohl and Palinscar (1999) suggest reciprocal teaching; a situation whereby a learner from the group will assume the role of a facilitator and that it has been successful in K-16 learners. These roles, most importantly are to ensure that all learners in the group participate actively and engaged in the construction of the shared knowledge (Hmelo-Silver, 2002).

2.3.3.4 Reflection on PCTL

For the construction of purposeful and usable knowledge, it is essential that there should be reflection between problem-solving and learning in a PCTL environment (Salomon & Perkins, 1989). When learners reflect on what they have learnt, it assists them (learners) to establish if there is any relationship between what they have learnt and the targeted goals. Hence, each problem-solving task should not be seen as an end but rather a means to achieve a learning goal (Bereiter & Scardamalia, 1989). How does reflection assist learners? (a) It helps to relate the newly acquired knowledge to the prior understanding brought to the classroom; and (b) to better understand the application of problem solving approaches in order to enhance their learning (Hmelo-Silver, 2004:247). Reflection should be incorporated throughout PCTL sessions, when completing and after completion of a problem, so that learners can reflect on the process of learning, how they have learnt, what they have learnt, how they have participated in their groups, how effectively their learning has been directed and the knowledge acquired. Making these inferences allows them to understand the skills and knowledge gained during the process and how they can apply the gained knowledge in different situations (Hmelo-Silver & Barrow, 2002). Learners can develop such understanding if they reflect on their knowledge and approaches. In the PCTL environment, reflection as a thinking process assists learners to identify

possible gaps in their thinking; and the application of the gained knowledge and various skills to new situations (Hmelo-Silver, 2004). Studies show that the application of recently acquired knowledge and skills to a new and relevant situation is always difficult for learners most especially if they do not have the in-depth understanding of the new tasks (Gick & Holyoak, 1980, 1983; Novick & Holyoak in Hmelo-silver, 2004:247). The probability of new knowledge transfer and application to a relevant situation is influenced by reflection (Salomon & Perkins, 1989).

2.4 ZONE OF PROXIMAL DEVELOPMENT (ZPD)

The zone of proximal development according to Vygotsky (1978), is the level at which a learner needs an appropriate assistance in order to solve a given problem (Woolfolk, 2007:622). This assistance or support is rendered by a person who is currently above such level in knowledge; it may be a teacher, parent or peer learner. This type of support in learning and solving problem is referred to as “scaffolding” (Wood, Bruner & Ross in Woolfolk, 2007:48). Scaffolding comes in different forms; it could be encouragement, reminder, or motivation as long as it promotes learners’ knowledge to a higher cognitive level (Woolfolk, 2007:620). When learners presently within the zone of proximal development are assisted or supported, they are afforded the opportunity of heightened knowledge acquisition in problem solving abilities in mathematics.

2.5 ENHANCING LEARNERS’ THINKING

Killen (1996:244-5) stressed that developing learners’ focusing, organising, information-gathering, evaluating, analysing and integrating abilities improves their ways of thinking and problem solving abilities.

2.5.1 Focusing skills

When learners are focused on a given task and concentrate on the problem, they (learners) will be able to think better on the process of solution thereby, promoting a better output in their performances.

2.5.2 Information- gathering skills

It is critical that learners identify the relevant and purposeful information in a given problem; teachers should help learners in acquiring this skill.

2.5.3 Organising skills

Ability to organise data in a useful way will help to enhance the learners' understanding of the given problem. Information should be organised in a way that will be easy to interpret.

2.5.4 Analysing and integrating skills

After gathering the necessary and useful information, the analysis of such information and the integration of the data into the formulated plan of solution should be done. The learners should be able to link their prior knowledge to the existing problem in order to have viable solution.

2.5.5 Evaluating skills

Assessment is an integral part of learning (Tynjala, 1999), learners should be aware that assessment and evaluation should be an ongoing process in order to maximise the benefits of assessments in their learning. Learners should be taught the criteria for evaluation of useful information.

2.6 WHY SHOULD PCTL BE USED AS A TEACHING STRATEGY?

The effective use of problem-centred teaching and learning has numerous advantages compared to using direct teaching strategy, and one of these advantages is the active and purposeful engagement of learners, thereby, developing their thinking and reasoning skills (Albanese & Mitchell, 1993). Higher levels of understanding and skills development are evoked when the given problems are engaging and difficult in PCTL than in direct instructions. Critical thinking skills and the ability to apply the gained experience to a new situation are developed in PCTL if learners are "mindful of what they are doing" (Marshall, 2003:1).

Problem-centred teaching develops learners' ability to make informed judgments and emphasises the importance of being able to explain and justify those judgments (Hmelo-silver, 2004).

Moreover, engaging learners in problem-centredness can give the teacher a better understanding of the abilities and talents of the learners.

PCTL can create three conditions that assist in subsequent retrieval and appropriate use of new information: activation of prior knowledge: similarity between the contexts in which information is learnt and later applied: an opportunity to elaborate on that information (Schmidt & Moust, 2000)

In addition to these advantages, one can achieve all the benefits of small-group work and co-operative learning by having learners' work collaboratively on problems.

2.7 SOME LIMITATIONS OF USING PBCL AS A TEACHING STRATEGY

One needs to be aware of the following limitations when using problem-centred learning approach.

Successful problem-centred lesson requires a lot of preparation in the sense that each problem needs to be carefully structured to produce specific learning outcome. Unless learners understand why they are attempting to solve a particular problem, they may not learn what they are supposed to learn. For example, they may focus on the strategy and not on the principle. If learners believe they are not capable of solving the problem, they may be reluctant to engage with it. Learners who are accustomed to their teacher being the main source of knowledge and information may feel very uncomfortable with the self-directed learning necessary for problem solving (Schmidt, De Volder, De Grave, Moust & Patel, 1989). When learners are working in groups, it is easy for the less able learners to be dominated by the confident learners. This can be minimised by proper monitoring of each group during class sessions.

2.8 ASSESSMENTS IN PCTL

Brown and Knight (1994) define assessment as a process of gathering, recording, analysing, interpreting and documenting in measurable terms learners' knowledge, skills, attitudes and beliefs. According to the definition above, assessment is an ongoing process with the purpose of providing an understanding of the level at which learners understand what they are learning to both the teacher and the learners.

Assessment is not a stand-alone process but an integral part of learning and should not only be a process of examining learners at the end of their course or programme of learning (Tynjala 1999:365). Tynjala (1999) further points out that the focus of assessment should be both on the outcomes and the process of learning in a problem-centred teaching and learning approach. Assessment should improve learners' skills for knowledge acquisition (Biggs 1996). Tynjala (1999:428) argues that when self-assessment and peer review are made as an inherent part of the process of learning, the cognitive abilities of learners are heightened. Assessment in the PCTL approach seeks to assist both the teacher and the learners in decision making about the strengths and weaknesses of the teaching approach, guide and monitor the progress of learners, check their understanding of the lessons, measure their achievements and promotes the learning process so as to determine the kind of qualitative changes in learners' knowledge (Brown & Knight, 1994). The Department of Basic Education CAPS (2011:154) defines assessment as "an ongoing process of identifying, gathering and interpreting information regarding the performance of learners, using various forms of assessment." Assessment should be continuous, occur in nearly all lessons, and should assess what ideas learners bring to a task, how they learn and what processes they use. The Department of Basic Education CAPS (2011) further stresses that continuous assessment of learners' work not only heightens their learning experience, but also assists them to achieve the minimum performance level required in mathematics for promotion purposes.

The traditional methods of mathematics instruction as suggested in the review of literature do not promote construction of learners' own knowledge but as mere recipients of knowledge from their teacher, therefore, leave "most learners believing that mathematics is mystical and incomprehensible" (Van de Walle, 2004:37). However, the PCTL instructional approach, places problem solving as the point of departure for learning, thereby putting learners in the active role of problem solving and knowledge construction (Hmelo-Silver, 2004). Therefore, conceptual understanding is the main focus of PCTL rather than procedural knowledge. Through the engagement and exploration of problems, basic skills in mathematical problem solving are expected to be acquired by the learners (Marsall, 2003). Furthermore, the process and commitment to problem solving should be valued by learners.

From the literature review it was revealed that the aim and purpose of any teaching strategy employed by the teacher should be to improve and heighten learners' mathematical reasoning and thinking, therefore, teachers should plan and structure their lessons in such a way that these could be achieved.

2.9 SUMMARY

Chapter Two discussed the past similar studies on quadratic equations, and discussed in this chapter was Polya's theory and related theory of Hmelo-Silver. This chapter also compared the traditional teaching methods with the problem-centred teaching and learning approach. Assessment in (PCTL) environment was discussed.

The next chapter presents methodology and the research design used in this study including the research tools.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

In the previous chapter, the literature review of this study was discussed. In this chapter, the research design and methodology, the procedures embarked on for data collection are thoroughly discussed. Much emphasis is on the data collections, the processing of the collected data and also all the necessary measures to ensure reliability and validity of the data collected are presented and, lastly, the issue of ethics is discussed.

3.2 RESEARCH PARADIGM

Research is a process that involves the systematic and logical collection and analysis of data for some specific purpose. This is an inquiry procedure of making informed decisions based either on one of the following methodological paradigms or on both, the qualitative approach or quantitative approach. The word 'paradigm', as used in this context, according to Maree (2010), it is a set of assumptions or beliefs about some aspects of reality, on how we view such reality.

3.2.1 Logical positivist and post-positivist paradigms

Logical positivism emphasised that there is a single reality within known probability, objectivity, empiricism, and numbers (McMillian & Suhumacher 2014:13; Creswell, 2009). He (Creswell) further argued that post-positivists hold a deterministic philosophy, that the outcomes or effects are probably determined by causes. Thus, the post-positivists believe that the causes that influence outcomes in studies need to be identified and assessed as found in such studies. Post-positivists also believe that to test ideas, it should be reduced to a discrete set, such as the variables that comprise hypotheses and research questions. In a post positivist lens, the knowledge developed is based on careful observation and measurement of the objective reality that exists 'out there in the world'. Thus, paramount for a post-positivist is developing numeric measures of observations and studying the behaviour of individuals. Finally, in order to better understand the world, the laws or theories that govern the world need to be verified by testing those laws or theories and refine them if need be. Thus, the accepted approach to research by

post-positivists, in the scientific method is that an individual begins with a theory, followed by collection of data that either supports or opposes the theory, after which necessary revisions are made before any additional tests if necessary. The following are the assumptions made by Phillips and Burbules (2000) about the post-positivist paradigm, adapted from Creswell (2009):

1. As far as knowledge is hypothetical, determination of absolute truth is hardly possible. Thus, the imperfection and fallibility of evidence established in research. Hence, researchers only indicate a failure to reject a hypothesis but not to prove it.
2. When research is conducted, claims are made and these claims are either refined for other claims or abandoned. For example, most quantitative research starts with testing a theory.
3. The collection of information on instruments based on measures completed by the participants, or by observations recorded by the researcher, the collected data, evidence and rational consideration, in practice, shape knowledge.
4. When a research is conducted, the essence of such study is to explain a phenomenon by developing relevant, true statements that explain such concerns or one that can describe the causal relationships of interest. The relationships among variables are posed as questions or hypotheses in quantitative studies.
5. For a competent inquiry, objectivity is essential, methods and conclusions are examined for bias by researchers. In quantitative studies, for example, standard of validity and reliability are important.

3.2.2 Interpretivists/constructivists paradigm

Interpretive/constructivist researchers believe in the existence of multiple socially constructed realities (unlike post-positivism, which postulates a single reality) while maintaining systematic procedure. The researcher's professional judgments and perspective are considered, rather than objectivity, in the interpretation of data (McMillan & Suhumacher, 2014, Sepeng, 2010). For the

interpretivists, there is more emphasis on values and contexts than there is on data (Creswell, 2009; McMillian & Suhumacher, 2014).

3.2.3 Transformative paradigm

In this paradigm, emphasis is on social, political, cultural, gender and ethnic factors as the substantial contribution to the design and interpretation of studies (Mertens, Blodsoe, Sullivan & Wilson, 2010). This worldview understands humans as engaging with their own world based on their historical and social perspectives and making sense of it, where the culture we are all born into bestowed the meaning of the world upon us. According to Crotty (1998), through visiting context, and gathering information personally, the qualitative researcher seeks to understand this context or setting of the participant. They also interpret what they find, and this interpretation is shaped by the researcher's own experiences and background. The social interaction with human community is the basic generation of meaning. The process, which is largely inductive, in qualitative research is the inquirer generating meaning from the data collected in the field (Creswell, 2009, 2012).

3.2.4 Pragmatic paradigm

In this paradigm, it is the belief that practical thinking and common sense are used rather than the scientific method which on its own is not sufficient to determine the best approach (that is, whether quantitative, qualitative or other), and this depends on the purpose of the study and more importantly on contextual factors (McMillian & Suhumacher, 2014). According to Creswell (2009), in a mixed method study, the collection and analysis of data is in various approaches as the researcher does not subscribe to only one way because pragmatists do not see the world as an absolute unity (e.g., quantitative or qualitative). Similarly, pragmatic researchers are against the positivist position that through a single scientific method, the truth about the real world can be accessed (Mertens in McMillan & Suhumacher, 2014:343). The pragmatist researchers, based on the intended outcomes look for the "what and how" to research (McMillan & Suhumacher, 2014). The rationale for mixing both quantitative and qualitative data should be established by the mixed method researcher in the first place.

Many researchers comprehend pragmatic worldview as the paradigm of mixed-methods research (Creswell & Plano Clark, 2011:41-43). Hence, the researcher of this study identified himself

with a pragmatic worldview. The researcher emphasised the outcomes of the research instead of focusing on the research methods, multiple methods of data collection were used to answer the research questions, while, without compromising the research standards abide by ethical issues at the same time, the creation of a problem-centred teaching and learning environment by the researcher with the intention of answering the research questions by all means practically available making pragmatic paradigm integral part of this study. Therefore, it was the aim of this study to explore the benefits of using problem-centred approach in the teaching and learning of quadratic equations in a Grade 11 classroom, using mixed-methods research, and for this reason, both positivist and interpretivist paradigms seem to be appropriate frameworks within which to show the intent, need and prospect of the study. For better understanding of the issue being investigated several approaches and methods were used (Cohen, Marion & Morrison, 2003:31-34).

3.3 QUALITATIVE METHODS

Assumptions mark the beginning of a qualitative research, followed by the paradigm, and if necessary, the use of theoretical lens, and lastly, the study of research problems inquiring into individual or groups ascribe to a social or human problem (McMillian & Suhumacher, 2014:344). Assumptions and worldview that are used in qualitative studies are different from those of quantitative research, but also different among variations of qualitative studies. Qualitative research assumes that multiple realities are socially constructed through individual and collective perceptions of the same situation and this is based more on constructionism (Creswell, 2009). According to McMillian and Suhumacher (2014) and Creswell (2009), the main characteristics of qualitative research are as follows:

- natural setting: study of behaviour as it occurs naturally;
- context sensitivity: consideration of situation factors;
- direct data collection: researcher collects data directly from the source;
- rich narrative description: detailed narrative that provides in-depth understanding of behaviour;
- process preference: focus on why and how behaviour occurs;

- inductive data analysis: generalizations are induced from synthesizing gathered information;
- participant view: focus on participants' understanding, descriptions, labels and meanings; and
- emergent design: the design evolves and changes as the study takes place.

In this study, information was gathered by actually engaging or interacting with, talking directly to the participants, and seeing them behave and act within their context, which Creswell (2009) refers to as a major characteristic of qualitative research. In the natural setting, the researcher had face-to-face interaction with the participants over time.

In the research literature (Cohen, Manion & Morrison, 2000; Leedy & Ormrod, 2005 and Creswell, 2007, 2009) six types of qualitative research designs are often discussed, namely, conceptual studies, historical research, action research, case study research, ethnography and grounded theory.

Conceptual research aims to add to the existing body of knowledge by engaging critically on the understanding of concepts based on secondary sources., whereas historical research is a systematic process that describes, analyses and interprets the past, using information selected from sources that relate to the topic under review (Creswell, 2009:13).

Case study design is, according to Bromley (in Creswell, 2009:13), a design that aims to describe and explain the phenomenon of interest with a systematic inquiry into an event or a set of related event, and it is very similar to action research, which draws attention to collaborative or participative dimensions, and to the focus on practical problems experienced by participants, for which a practical solution is sought.

Ethnography is associated with social interactions and cultures. The focusing on social systems and cultural heritage is ethnography in the field of anthropology.

Action research draws attention to collaborative or participative dimensions and to the focus on practical problems experienced by participants for which the practical solution is sought. As a research design, it often utilises both quantitative and qualitative data, but they focus more on procedures useful in addressing practical problems in schools and in classrooms (Creswell,

2011). It is a systematic procedure, undertaken by teachers or other individuals in the education setting to gather information about and subsequently improve the ways their particular educational setting operates, their teaching and their learners' teaching (Mills, 2010).

Action research begins with problem identification and then collection of data with the use of different data collection techniques, followed by data analysis and taking action to solve the problem and finally assessing the results of the intervention (Maree, 2010).

Grounded theory is a strategy of inquiry in which the researcher derives a general abstract theory of a process action or interaction grounded in the views of participants (Creswell, 2007, 2009 & 2012). This process involves using multiple stages of data collection and the refinement and interrelationship or categories of information (Charmaz, 2006, Strauss & Corbin, 1990, 1998). Two primary characteristics of this design are the constant comparison of data with emerging categories and theoretical sampling of different groups to maximise the similarities and the differences of information.

Phenomenological research is a method of inquiry and also a philosophy that studies a small number of subjects for a long period of time in order to develop relationship of meaning (Moustakas in Creswell, 2009:13). The researcher, in order to understand the lived experiences of the participants in the study sets aside his or her own experiences (Nieswiadomy in Creswell, 2009:13).



3.4 QUANTITATIVE METHODS

Objectivity in measurement and description of situations is much emphasised in quantitative research designs (McMillian & Suhmacher, 2014). Statistics are used in the determination of the existence of a relationship between two or more variables in quantitative research (McMillian & Suhmacher, 2014; Creswell, 2009). Positivist paradigm uses quantitative approaches in which science quantitatively measures independent facts about a single apprehensible reality (Tashakkori & Teddlie, 2003; Creswell, 2009). Therefore, the objective of the quantitative aspects of this study is to provide data for triangulation with the qualitative data generated in order to attempt to answer the main research-question:

What are the benefits of using problem-centred approach in teaching and learning of quadratic equations in the Grade 11 classroom?

3.5 MIXED METHODS

In mixed methods procedures, both quantitative and qualitative methods are employed. Creswell and Plano Clark (2011:5) state that in a single or series of studies, the mixing of both quantitative and qualitative approaches in various stages of the research process concentrating much more on collection and analysis of data is referred to as mixed method research design. For the intent of this study, the characteristics of the mixed methods research design were adopted from Creswell and Plano Clark (2011:5) who list that in a mixed methods research design the researcher takes following actions:

- collection and analysis of both qualitative and quantitative data;
- integration of the two forms of data simultaneously by merging them, consecutively by having one built on the other or imbedding one with the other;
- giving priority to one or both forms of data;
- using these procedures in a single study or multiple stages of a programme of study;
- frames these procedures within the worldviews and the necessary theoretical lenses; and
- combining the procedures into specific research designs that direct the plan for conducting the study.

3.5.1 Sequential design

According to Creswell (2009:207; MacMillan & Schumacher, 2014:431), in this type of design, the researcher seeks to elaborate on or expand on the findings of one method with another method. It may involve beginning with a qualitative interview for exploratory purposes and following up with a quantitative, survey method with a large sample so that the researcher can generalise results to a population. Alternatively, the study may begin with a quantitative method in which a theory or concept is tested, followed by a qualitative method involving detailed exploration with a few cases or individuals. Figure 3.1 and Figure 3.2 below show both the explanatory and exploratory design, respectively.

Figure 3.1 *Sequential explanatory design*



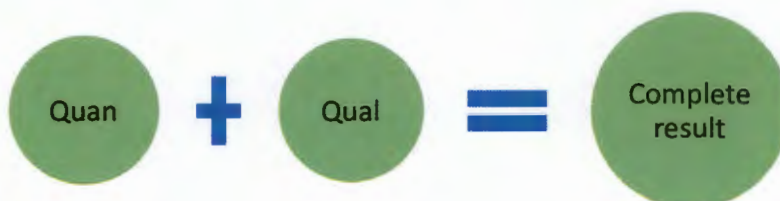
Figure 3.2 *Sequential exploratory design*



3.5.2 Triangulation design

In triangulation design the researcher merges quantitative and qualitative data so as to provide a comprehensive analysis of the research problem. Both forms of data are collected at the same time by the researcher who then integrates the information in the interpretation of the overall results (Creswell, 2009:207; MacMillan & Suhumacher, 2014:431). The researcher may also embed one smaller form of data to another larger data collection in order to analyse different types of questions in this design (the qualitative addresses the process while the quantitative the outcomes). The figure below shows the triangulation design layout, where 'Quan' and 'Qual' respectively stand for quantitative data and qualitative data.

Figure 3.3 *Triangulation design*



3.6 RESEARCH DESIGN

This is the process that provides the overall framework and the procedure for data collection. It is the blue print of the whole study. It gives the elaborated steps of the study and provides guidelines for systematic sampling techniques, the sample size, instruments and data gathering decisions from broad assumptions, to elaborated methods of data analysis (Creswell, 2009:116-118). A mixed methods approach was followed in this study. Mixed methods research design engages a multiple set of approaches, where all approaches are valuable and have something to contribute to understanding. When multiple approaches are used in a study, they lead to a better understanding of the issue being investigated (Cohen, Marion & Morrison, 2003:31-34). Hence, a mixed-method approach using both quantitative and qualitative investigation was employed. Both quantitative and qualitative approaches complement each other and the restrictions of one approach can be counterbalance by the rewards of another (Creswell, 2009:204-226). Cohen, Marion and Morrison (2003:24-28) state that “ when both qualitative and quantitative data are used, researchers are able to simultaneously make generalizations [sic] about a population from the results of a sample and also gain a deeper understanding of the phenomena of interest.” Creswell and Plano Clark (2011) further stress that a better understanding of the research problem is gained and it provides more evidence for the study when quantitative and qualitative approaches are combined rather than each approach on its own. A more comprehensive account to the research question is provided using mixed method research approach, since it annexes the strengths of both methods Therefore, this study used a mixed-methods design in order to take the advantage of the strengths of each approach, so as to deepen the research findings. The researcher used the mixed method in order to annex the strengths of both qualitative and quantitative for more detailed findings as (He) the researcher thought that one data source may be insufficient to outline the benefit of using problem-centred teaching and learning approach in teaching quadratic equations in the Grade 11 classroom. The chosen approach (mixed method) assisted the researcher tremendously in the provision of answers to questions that could not have been answered by either qualitative or quantitative research alone in this study. The other reason for combining both quantitative and qualitative data is to better understand this research problem by converging both quantitative (broad numeric trends) and qualitative (detailed views) data (Creswell, 2009:123).

For example, in reading through the learners' journals, it was easy for the researcher to gain insight into the reasoning and thought processes of learners which quantitative research method alone could not have afforded the researcher this opportunity and the collected data would have been insufficient. Lastly, heightening the credibility of the findings was important to the researcher; therefore, this was achieved using the mixed method design. Creswell and Plano Clark (2011:62) advocate that incorporating different research methods in a study is more likely to produce better results in terms of quality and measure. Creswell and Plano Clark (2011:13-16) pointed out that there are also challenges when mixed-methods research design is used.

One of the challenges identified is that it requires extended time, effort and resources on the part of researchers. To overcome this, enough resources were made available and problems with time were considered into the programme; also identified as a challenge in the mixed method design was the question of skills for undertaking the design. This was subdued, by extended acquaintance to both qualitative and quantitative methods by the researcher before the study began. Creswell and Plano Clark (2011:68-104) name six major types of mixed-methods research design: the explanatory sequential design; exploratory sequential design; convergent design; embedded design; transformative design; and multiphase design. The convergent research design was employed for this study.

3.6.1 Design type

This study used the convergent design as the researcher collected and analysed both quantitative and qualitative data during the same stage of the research process.

3.6.1.1 Convergent design

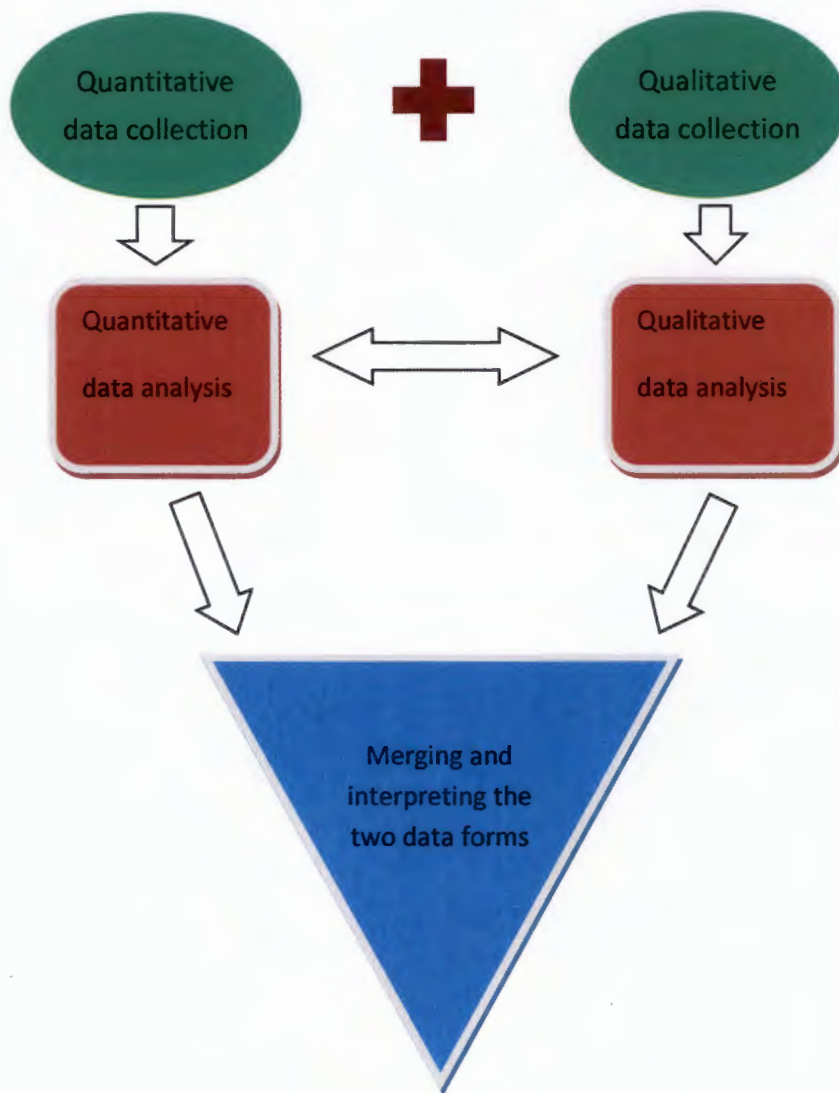
The convergent design is a procedure that involves collection and analysis of both qualitative and quantitative data separately and afterwards merging the two sets of results and interpreting the merged results (Creswell & Plano Clark, 2011:77). McMillan and Schumacher's (2006:404) definition of convergent design agrees with the above view. It states that the collection of both qualitative and quantitative data simultaneously by the researcher merges both set of data followed by interpretation of the findings in order to deeply understand the research question. The same priority is always given to both quantitative and qualitative data collected, allowing for the combination of the strengths of both methods by the researcher. As discussed above, in this

study, the collection of both qualitative and quantitative data was done simultaneously by the researcher and both sets of data were given priority at every phase of the process, then the two sets of data were merged for the final interpretation of the findings. The convergent notation for this study is quantitative + qualitative = complete understanding (Creswell & Plano Clark 2011:77-78).

3.6.1.2 Why the convergent design?

Morse (1991:122) states that in order to better understand the research problem the convergent design is used “to obtain different but completing data on the same topic”. The researcher is of the same opinion and felt that collecting and analysing both quantitative and qualitative data at the same time will enhance better understanding in the exploration of the benefits of using PCTL approach in teaching quadratic equations in Grade 11 classroom(s). Hence, the researcher chose the convergent design. During the PCTL lessons, video observation and questioning were carried out by the researcher while the learners were involved in lengthy discussions (qualitative) and simultaneously solving the given mathematical task (quantitative) in their groups. It was easy for the researcher to compare the quantitative statistical results with qualitative findings and this allowed the elaboration of a well substantiated conclusions (Creswell & Plano Clark, 2011:77) about the benefits of using PCTL approach to teach quadratic equations in Grade 11 classroom(s). The figure below shows the detailed convergent design.

Figure 3.4 *Convergent design*



3.6.1.3 The programme of intervention

From the beginning to the end of the intervention, the researcher acted as a facilitator for the Grade 11 mathematics class. This is because there is only one Grade 11 mathematics class in the school, so the researcher is the only teacher that teaches Grade 11 and secondly, many teachers may not be familiar with the PCTL approach as a teaching strategy in South Africa. In all the PCTL sessions throughout the intervention, the researcher assumed the role of a facilitator. The researcher intervened during the sessions with relevant questions to deeply understand the

reasoning behind the learners' process of problem solving. As the facilitator, the researcher managed and controlled class discussions while maintaining a spirit of inquiry and critical reasoning during problem-solving sessions. At the beginning, before the process of intervention the questionnaire and the pre-test were administered to all the participants, while the post-test and the same questionnaire were administered at the end of the intervention. The rationale behind the second administration of the questionnaire and the post-test was to know whether they have benefitted from the intervention. The session started by the detailed explanation of the meaning of problem-centred teaching and learning approach (PCTL) to the learners and what it entails, as presented by the Grade 11 mathematics CAPS documents. A friendly classroom environment was created in which social interaction was highly valued by the researcher. It is believed that learners are able to focus on a given task in a friendly environment where they are free to discuss with each other on a given problem (Hmelo-Silver, 2004). Throughout the entire sessions of the intervention, Polya's four-phase problem solving process was used because this is the theory that frames this study (see Figure 2.1). It covers:

1. problem posing;
2. understanding the problem;
3. making a plan;
4. carrying out the plan and;
5. looking back

There were no formal lessons during the problem-solving session as the researcher acted as a facilitator. After contending with unfamiliar problems, each group leader was requested to come forward and explain the group work on the chalk board to all the learners, the facilitator advised them to critique each solution on the board and they may suggest alternative process of solutions. The researcher controlled, managed and guided all the discussions by asking relevant questions, making sure that all the learners understood the given problems before proceeding to any other necessary problems.

A learner-led group was employed in all the sessions where the learners were randomly divided into five groups of four learners in each group, the groups are A, B, C, D and E. On each day, the researcher worked with each group for about ten minutes, questioning them in the process. After each lesson, a follow-up work was assigned to each group by the researcher for discussion during

the following session. Decoding of the video recording took place after the lesson for each day in order to record and analyse the observation in the observation comment cards.

The following steps were taken in order to enhance the development of the learners' mathematical problem-solving skills.

1. Learners were urged to identify any similarities or differences between the previously solved problems and the new problems given;
2. Learners were encouraged to solve given problems using different approaches and if possible compare the effectiveness of those approaches;
3. They were encouraged to incorporate all the problem solving skills discussed in (Chapter 2, Section 5);
4. To develop mathematical problem solving skills, learners were encouraged to generate questions on their own as this gives them authority over the problems (Van de Walle, 2004:37); and
5. Before they responded to any questions asked, learners were given enough time to reflect on their answers. This afforded them the opportunity to think deeply about their problem solving skills.

3.6.2 Participants

The participants in this study were 20 Grade 11 learners from a public high school in a rural part of Capricorn district, about 50km from Polokwane, the capital city of Limpopo Province. The participants' ages ranged from 16 to 21 years. Furthermore, all of the participating learners are Black African, with eight males and 12 females in mathematics class. However, the participants of the study cannot be considered as representative of all Grade 11 learners in South Africa, as certain high schools are better-equipped than others in terms of resources. Therefore, the results of this study ought to be interpreted accordingly.

3.6.3 Data generating instrument

The official language of teaching and learning in South Africa (English) was chosen as the preferred language for this study. During the process of data collection, the participants had already studied quadratic equations in about 15 lessons at the same grade level. The researcher was expected to teach the subject through factorisation, the quadratic formula, and completing square methods. In addition, the curriculum emphasised teaching symbolic and tabular representations, as well as the graphical representations of quadratic equations with two unknowns. Moreover, although the curriculum guidelines explicitly emphasised problem-solving with the use of quadratic equations, it suggests using word problems to find roots and solution sets of quadratic equations. Furthermore, the mathematics textbooks used in schools emphasise symbolic approaches regarding quadratic equations, such as factorisation, the quadratic formula, and completing the square.

3.6.3.1 Quantitative data collection (instrumentation)

For quantitative data collection, both the pre-test and post-test were used as the data collection instruments (see Appendices D and E). The procedures taken for the collection of the data are discussed in detail in the following section.

3.6.3.1.1 Pre-test and Post-test

For this study, data collecting tools such as, pre-test and post-test on quadratic equations was used as quantitative instruments. A test was administered to all the participants (Grade 11 mathematics class in the school) at the same time of the same day before the intervention (for the pre-test, see Appendix D), and after the intervention (for the post-test, see Appendix E). The researcher monitored invigilation sessions by availing himself to the classroom throughout the period of writing the tests. Learners' works were marked using a marking guideline. These items were developed by the researcher; the tools were given to the supervisor for moderation. The final instrument with 16 items covered three sub-topics: factorisation, completing the square, and quadratic formula. The sub-topic of factorisation has four questions, four questions for completing the square, and four questions for quadratic formula and four word problems on quadratic equations. The word problems covered all the three sub-topics.

3.6.3.2 Qualitative data collection (instrumentation)

During each and every problem solving session in the PCTL environment, relevant questions were asked by the researcher from each group, to assess and evaluate the development of the mathematical solving skills in every learner. The responses and findings of those questions were recorded immediately in the problem-solving comment card (see Appendix H).

It was requested of the learners to write reports instantaneously in their journals after the lesson on their experience in the problem solving completed during the intervention; this was intended to make sure that no vital information was lost. Those reports were read and analysed after each session by the researcher in order to gain more insight into what transpired during the lesson and more importantly if learners were really developing the necessary and targeted mathematical problem solving skills.

3.6.3.2.1 Questionnaire

Statements or questions are often used in questionnaires to obtain information or data from the participants by responding to the statements or the questions for a purpose (McMillan & Schumacher, 2006:194). The main purpose of the questionnaire was to inform the researcher about the present impediments that may hinder the learners from benefitting from the PCTL approach and more importantly how this could be overcome in this study. The questionnaire was used to answer the research sub-question 1, as to what the effects on Grade 11 learners' performance were when using problem-centred approach in teaching quadratic equations. The questionnaire used in this study was developed and used in a similar study (Chirinda, 2013). The questionnaire consisted of 54 different questions, divided into five different sections with section A having three questions requiring learners to fill in biographical details about their gender, age and the name of their school. The other four sections, B addressed the general attitude of the learners towards mathematics; C addressed self-confidence; D, perseverance with respect to problem solving processes; and lastly, E their willingness to engage in problem solving activities in quadratic equations.

Learners were required to indicate their responses on a five-point Likert scale in which 1 = strongly disagree, 2 = disagree, 3 = undecided, 4 = agree and 5 = strongly agree. Babbie

(2010:179) and McMillan and Schumacher (2006:198-199) stated the advantages of using Likert scale as follows:

1. it is flexible;
2. it is generally very simple to use and understand; and
3. easy establishment of the intensities between items.

The administration of the questionnaire to all the participants was done at the commencement and also at the end of the intervention. Simplicity and clarity of the language was ensured in all the instructions and the contents were explained to the learners before they completed it. They were also informed of their freedom to inquire more about any aspect of the questionnaire for clarity. The whole exercise of completing the questionnaire took an average of 30 minutes for each learner. The researcher checked each questionnaire to ensure its completion and the questionnaires were all collected.

3.6.3.2.2 Video recording and questioning

The researcher was responsible for the video recordings; this was done purposely to make sure that the learners did not feel threatened because using someone unfamiliar might cause anxiety. The researcher regularly moved quietly around the classroom so as not to disturb the peaceful atmosphere of the learning environment and while questioning learners in each group as they solved the given problems. The use of video recording was to make sure that all that transpired, for example, the interactions within each group during the problem solving sessions were recorded for later analysis. With the use video recording and questioning as qualitative tools, the researcher was able to see how the learners developed their problem solving skills in quadratic equations through the interaction within and among the groups. Hence, vital information was obtained about how learners perceived the process of problem solving as expressed in their actions and their thoughts (McMillan & Schumacher, 2006:347). McMillan and Schumacher (2006) further stress that the observational method is crucial as this allows the researcher to see and hear all that transpires in the classroom. The learners were asked some thought evoking questions in order to assess and evaluate the mathematical problem solving skills development in the learners. Some of these questions are:

- what came to your mind when you were confronted with the problem?

- What are the factors that you think could assist you in understanding the problem?
- which approaches have you used in solving the problem?
- what next would you do if your chosen strategy failed?
- are you able to get the right answer to the question?
- how are you sure you have the right response?
- do you know any other approach that can be used? and
- what do you think about your live experience with this problem?

Questioning as a qualitative tool used in this study has assisted the researcher to deeply understand how and what learners thought of quadratic equations problems and their reasons behind each and every process of solutions they employed in a problem-centred teaching and learning environment. It was easy to assess and evaluate each learner at a time, therefore very flexible.

3.6.3.2.3 Learners' journals

During each PCTL session, each and every learner was asked to write a brief but detailed report in their journals on a problem-solving experience they had completed on a regular basis, during the intervention. This included how they solved the quadratic problems given to them during each intervention. The researcher used learner journals because of being "direct source of information from the learners." (Wheatley, 1991:23).

3.6.3.3 The recording techniques

The researcher mounted a video recorder in the classroom with the assistance of a teacher in the school. The purpose of the recorder was to record the involvement of the participants while they solved given problems on quadratic equations and this was done after each intervention. Learners' actions, how they performed during mathematical problem solving sessions and the interpretation of these by the researcher were all included in the recording. These were recorded in the problem-solving comment card, and the problem-solving observation rating scale (see appendices N and P). The recording scales were based on Sections 2.5.1-2.5.6 of Chapter 2, namely, focusing, information-gathering, organising, evaluating, analysing and integrating skills

that learners should acquire and was developed by the researcher. The researcher carefully studied all the actions observed in the video recording for later analysis. Learners were asked various questions on the problem they were given to solve. For every learner, the recorded data were interpreted and recorded in the problem-solving rating scale.

3.6.3.4 Pilot testing the instruments

The questionnaire was tested at a neighbouring school, as well as pre- and post-tests; there were 26 Grade 11 mathematics learners at the school. Babbie (2010:98 & 233) points out how important and critical it is to pilot test an instrument because it improves the reliability of such an instrument. He argues further, that pilot testing affords the researcher to be able to identify any error that can affect the study to be detected earlier. Clarity of the language used and appropriateness of statements in the questionnaire and also legibility and clarity of both the pre-test and post-test were pilot tested.

The feedback provided by the respondents was used to correct, amend, simplify and clarify some of the items. The pilot testing process was managed and controlled by the researcher. The final version of the tools was presented to the study supervisor for assessments and it was approved.

3.6.4 Data analysis

The process that involves making sense out of collected data, for the purpose of interpreting, merging and reasserting the participants actions and responses, is called data analysis. Data analysis is the breaking up of collected data into manageable patterns (Mouton, 2001:108). Provision of answers to research questions requires in-depth analysis of the collected data and this study is not different. The collected data were thoroughly analysed in order to answer all the research questions.

Similar steps are taken by most researchers for both qualitative and quantitative data analysis and these steps are outlined in Creswell and Plano Clark (2011:204) as follows:

- developing the data for analysis;
- translating the data for analysis; and
- data validation and interpreting the findings.

3.6.4.1 Quantitative data analysis

The pre-test and the post-test were administered to all the learners by the mathematics teacher, as well as researcher and with assistance from a teacher to invigilate the proceedings. It was during their normal class time, and they were given 1 hour 30 minutes to complete each test. The quantitative data analysis consisted of descriptive statistical analysis (McMillan & Schumacher, 2006:153). Descriptive statistics transform a set of observations into indices that characterise the data and thus are used to summarise and coordinate observations (McMillan & Schumacher, 2006:150). For this study, data from the pre-test and post-test in this study were analysed using descriptive statistics and this includes means, standard deviations and dependent sample *t*-tests.

The researcher also categorised learners' performance in solving quadratic equations with respect to their use of methods (that is, quadratic formula, factoring method, completing square method, and square root property) to solve quadratic equations. Learners' answers were coded and categorised with respect to their comprehension of the problem statement, setting up the correct relationship and formulating quadratic equations, and solving the formulated quadratic equations. Correct, incorrect, incomplete, and blank solutions were some major codes that evolved from the data.

3.6.4.2 Qualitative data analysis

An inductive procedure whereby data are organised into different themes or categories and then identification of any relationship among the themes is referred to as qualitative data analysis (McMillan & Schumacher, 2006:364). According to Creswell and Plano Clark (2011:208), qualitative data analysis involves "coding of the collected data, then dividing the coded text into units, marking each unit and then grouping the codes into themes." In this study, as soon as data collection began, so also was the qualitative analysis and it was an ongoing process. Analysis of the findings from the recorded video and responses from learners during questioning were recorded on the problem-solving comment card and problem-solving rating scale. Learners' responses as written down in their journals were also analysed by categorising the mathematical problem-solving skills that were identifiable from the performances.

3.6.5 Mixed-methods data analysis

McMillan and Schumacher (2006) pointed out that in a mixed-methods data analysis, the quantitative data are analysed separately from the qualitative data and the two results are then merged and interpreted in order to answer the research question. A mixed methods data analysis takes place when analytic techniques are applied to both quantitative and qualitative data, as well as to the integration of the two forms of data concurrently and sequentially in a study (Creswell & Plano Clark, 2011:212). In this study, both quantitative and qualitative data were collected concurrently by the researcher, the findings were separately analysed and the two sets of data were merged and this was followed by the interpretations of the results and then conclusions. As suggested by Creswell and Plano Clark (2011:215-216), the steps below should be taken when convergent research design data analysis is used in a study, and this was the case in this study;

- concurrent collection of the two sets of data (quantitative and qualitative data);
- separate analysis of quantitative data and qualitative data using quantitative methods and qualitative methods respectively;
- analysis of both the quantitative data and the qualitative data were done by comparing the merged data side-by-side (Creswell & Plano Clark 2011:223); and
- the researcher used the interpreted results of the merged data to answer the main research question.

3.7 Test for validity and reliability of measuring instruments

The critical questions that required validating were: were all the aspects of quadratic equations covered in the tasks (the pre-test and the post-test); and, do the assessment tasks measure the aspects that were assumed central to the learning of quadratic equations?

3.7.1 Reliability

The degree to which data collection tool consistently measures that which it is designed to measure is referred to as reliability. This refers to the extent to which the data collection tool gives similar results and conclusions even if administered to a different group of participants

under different conditions such as time and venue. This also implies that if the same research is done again under similar conditions, the researcher will obtain the same or very similar results.

When the observations from participants are consistently stable and stable over time, it is referred to as qualitative reliability (Creswell & Plano Clark, 2011:211). To address this issue, the questionnaire used in this study was used in similar study (Chirinda, 2013) and the reliability coefficients was 0.82 (see Table 3.1), any instrument with reliability above 0.7 is regarded as a good instruments (McMillan & Schumacher, 2006:183; 186-187). McMillan and Schumacher (2006) go on to state that the Cronbach alpha is generally the most appropriate type of reliability for questionnaires, in which there is a range of possible answers for each item and this was the case for the questionnaire used in this study. The Spearman-Brown formula (McMillan & Schumacher 2006:185) was used to calculate the reliability coefficients of the, pre- and post-tests. The Spearman-Brown coefficients for these instruments were generally above 0.70, which is generally acceptable for these kinds of instruments.

Table 3.1 *Reliability of instruments*

Instrument	Recording technique	Reliability of instrument	Value of reliability of instrument
Questionnaire	Likert scale	Cronbach's alpha	0.82
Video recording and questioning	Comment card and rating scale	Visual evidence	-----
Pre-test	Analytic scoring scale	Spearman-Brown	0.77
Post-test	Analytic scoring scale	Sperman-Brown	0.77

3.7.2 Validity

Validity means to ascertain the quality of the data collected, the results presented and the interpretations of the results (Creswell & Plano Clark 2011:210). Validity assesses and evaluates whether the study measures that which it was intended to measure, or if the research results and findings are trustable. McMillan and Schumacher (2006:134) concur with this when they state that validity refers to the truthfulness of findings and conclusions. It is important that procedures to ensure the validity of the data, along with their results and their interpretations, are utilised.

3.7.2.1 Quantitative validity

Quantitative validity is the quality of the results that can be obtained from data collection tools and also the conclusion of the quantitative analyses (Creswell & Plano Clark, 2011:210). This includes external validity, content validity, criterion-related validity and construct validity.

External validity refers to the generalisability of the results, that is, the results and findings can be generalised to a larger population. This might be a threat because the chosen school is a rural school with little resources, compared to other urban schools in South Africa.

Content validity is concerned with whether the data collection tools represent all items. For this study, regarding content validity, the mathematical problem solving skills, Polya's four-phase problem solving stages and other critical factors explored in the literature review were all represented by the items in the different sections of the questionnaire and the learners' journal focus questions.

Criterion-related validity refers to whether the scores relate to some form of external standard, such as scores on a similar instrument. For this study, the criteria set in the teachers' guide for Grade 11 Mathematics were used as a reference for criterion-related validity.

Construct validity is concerned with whether data collection tools measure what they intend to measure. To strengthen construct validity of this study, various methods of assessment, that is, written work and observation and questioning, were used to test learners' achievement and performance in quadratic equations.

Internal validity is the extent to which the researcher can reach a conclusion that there is a “cause and effect relationship among variables” (Creswell & Plano Clark, 2011:211). For this study, the selection threat to internal validity was not a factor, as there is no comparison group. There were no extraneous events that occurred during the data collection period, and this led the researcher to conclude that there was no known historical threat to internal validity for this study.

3.7.2.2 Qualitative validity

Qualitative validity deals with the accuracy, credibility or trustworthiness of the researcher’s own account with the responses from the participants. Internal validity in qualitative research checks whether researchers observe what they think they observe and actually hear what they think they hear. In this research, it was essential that the researcher understood learners’ responses during the analysis of video observation, as well as questioning. To enhance qualitative validity, the researcher employed triangulation and “member checks” (Creswell & Plano Clark, 2011:211; McMillan & Schumacher, 2006:324). Member checks imply that the researcher went back to participants after the completion of the study to ask participants whether the findings were truly what they experienced during the data collection process. However, no changes were provided by the participants to the data that were presented to them during the member checks process.

3.8 ETHICAL CONSIDERATIONS

From a moral perspective, ethics address the wrongs or rights and, that is, the responsibility of the researcher to protect the welfare of the participants against any harm (McMillan & Schumacher, 2006:142). For this research, all these precautions were taken into consideration before the data collection process.

3.8.1 Obtaining informed consent

Informed consent means willingness of the participants to participate in a study after being informed of their roles and responsibilities without being forced to do so. According to Wiesma and Jurs (2009:456), prior to a study, the participants should be informed of their roles in the study, the research aims and possibly any risks associated with the research and they should sign a consent form before participating in such study. Before embarking on this study, a letter was

written by the researcher to the school principal and school governing board requesting their permission to use the school as the research site. The approval to use the school was given to the researcher by the school principal and the school governing board. However, the name of the school remains anonymous (see Appendix B). Also the participants and their parents or guardians were given similar letters requesting the participation of the learners and stating the main purpose of the study. The participants voluntarily agreed to participate in the study after understanding the content of the letters and it was signed by their parents or guardians (see Appendix C).

3.8.2 Voluntary participation

The learners were all informed about the main purpose of the study; they were also informed that their participation was voluntary. The researcher also emphasised that any participants who may not want to continue with the study were free to withdraw at any time they deemed fit and that there was no consequence if they did. The researcher was fair, sincere and open to all the participants about their participation in the study.



3.8.3 Confidentiality and anonymity

Confidentiality is the act of information retention within oneself. This is very important in any study; the identity of the participants should be between the researcher and the participants. Anonymity means unknown, the names of the participants should not be dissected to any other person outside the study (Wiersma & Jurs, 2009:458). The participants were assured of confidentiality, and that their responses would be used purposely for this study. Necessary efforts were made to maintain complete participant confidentiality and anonymity. Fictitious names were ascribed to school, principal, and participant names. McMillan and Schumacher (2006:334) point out that the participants in a study should not be known by any external person.

3.9 SUMMARY

In this chapter, most of the discussions were based on the research methodology. The convergent design which was the mixed method design used for the study was thoroughly dealt with. The rationale for employing the mixed methods convergent design was explained. In order to ensure high degree of validity and reliability, different data collection tools were used in the study.

Chapter four deals with the presentation of the findings and the analyses of both the qualitative and quantitative data.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 INTRODUCTION

The primary aim of this study was to provide answers to the research questions outlined in Chapter 1. Participants were exposed to learning activities in quadratic equations in which the problem-centred approach of teaching and learning was used, as explained in Chapter 3. In addition, I gave an account of how the data emanating from the interventions were collected and analysed. This chapter presents the findings, interpretation and discussion in relation to the research questions, informed by the adapted theory. In doing so, the quantitative data collected through pre-test and post-test are presented using tables and graphs and also the qualitative data collected by questionnaire, video recording and questioning and learners' journals which complimented the quantitative data.

4.2 QUANTITATIVE DATA ANALYSIS (Pre intervention)

A test was administered to the grade 11 mathematics learners (pre-test) to enable the researcher to know the current level of understanding of quadratic equations of the learners before the intervention (problem-centred teaching and learning)

4.2.1 Pre-test

As noted earlier in the previous chapter, the pre-test consisted of 16 items, divided into three sub-topics: factorisation, completing the square, and quadratic formula. The subtopic of factorisation had four questions, four questions for completing the square, four questions for quadratic formula, and four word problems on quadratic equations which is a mixture of all the three sub-topics (Appendix D). Garegae's (2007) analytic framework was used to classify the overall learners' responses into four categories, viz., Correct Response (CR), Incorrect Response (InC), Incomplete Response (IC) and Empty Response (ER). The marking of the questions was informed by the framework adopted by this study (Appendix G). The pre-test was used as a baseline assessment in order to measure the current level of understanding of Grade 11 learners in quadratic equations.

4.2.1.1 Factoring method: Question 1

There are four questions in (Q1). The following themes were used for each category of the learners' responses:

Correct (CR): *findings via factorisation and connections between the roots*

Incomplete (IC): *factorising but not being able to find the result*

Incorrect (InC): *incorrect factorisation*

Empty (ER): *no attempt made*

The results of the pre-test on question 1 are displayed in the table below.

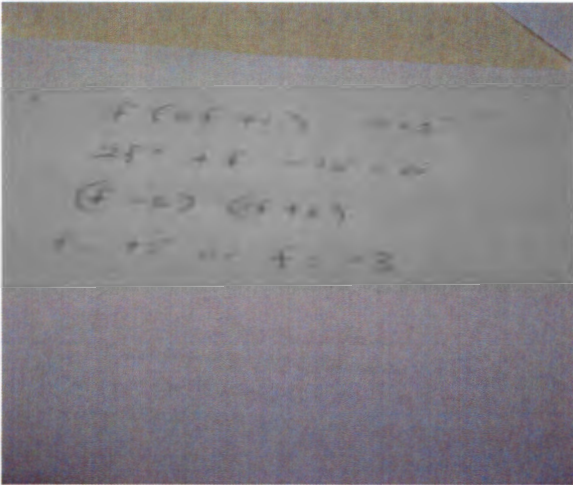
Table 4.1

Percentages (and absolute numbers) of correct response (CR), incomplete response (IC), incorrect response (InC) and empty response (ER) on Question 1 items for pre-test.

QUESTION 1	Items	CR	IC	InC	ER
(N=20)	i	5% (1)	70% (14)	25% (5)	0% (0)
	ii	5% (1)	75% (15)	20% (4)	0% (0)
	iii	0% (0)	50% (10)	50% (10)	0% (0)
	iv	0% (0)	55% (11)	30% (6)	0% (0)

The quantitative analysis of the data revealed that only 10% (2) of the learners (N=20) solved only half of the items in Q1 on factorisation correctly. The researcher observed that learners employed different approaches to factorisation depending on the kind or structure of the quadratic equations to be solved, and thus experienced difficulties in different stages of the process. Certain errors made by the learners revealed that they had factorised the quadratic equations into two linear factors incorrectly, and determined the roots incorrectly because they had made false guesses while using the cross-multiplication method. Some of the learners' working is displayed below.

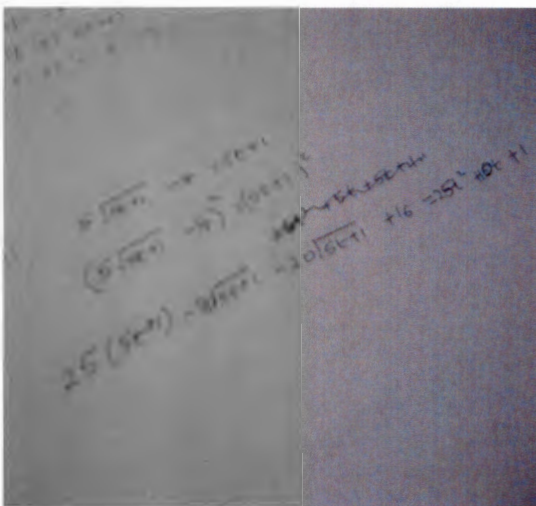
Item 1: Learner 9 response to question 1.i



Handwritten mathematical work on a piece of paper. The text is as follows:

$$f(f+1) = 12$$
$$2f^2 + f - 12 = 0$$
$$(f-3)(2f+4)$$
$$f-3 = 0 \quad f = -3$$

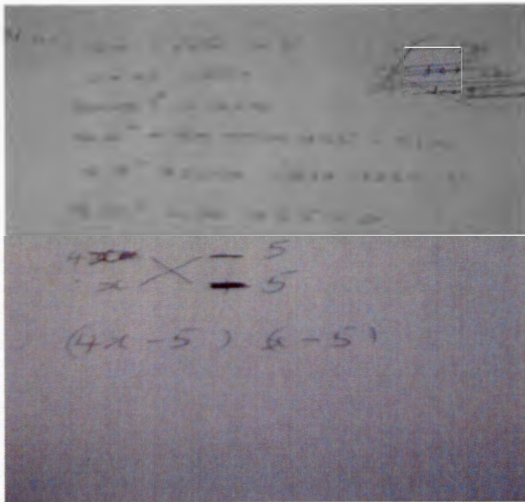
Item 2: Learner 3 response to question 1.ii



Handwritten mathematical work on a piece of paper. The text is as follows:

$$25(x^2) - 10x + 1 = 0$$
$$(5x-1)^2 = 0$$
$$5x-1 = 0 \quad 5x = 1 \quad x = \frac{1}{5}$$

Item 3: Learner 11 response to question 1.iii



The most common error emerged when learners used a cross-multiplication method, based on a kind of “guess-and-check” approach, while factorising the quadratic equations. For example, learners guessed the factors of the constant term 15, in Q1.i, incorrectly as +5 and -3 instead of -5 and +3, respectively. For Q1.ii, most of the learners did not collect like terms together before squaring both sides and this led to incorrect solutions. Similarly, the same mistake was repeated for Q1.iv. The data revealed that some of the learners did not correctly judge whether the quadratic equations to be solved was factorable over some domain, such as rational numbers. For example, some learners attempted to factor the equations $4x^2 - x + 25$ in Q1.iii, although they are not factorable. The attempted solutions of the learners showed that most of them understand the question, but some learners are unable to devise a plan of solving the problem while those that had plans were unable to carry out the plans, which appeared to suggest that the learners lacked problem-solving skills.

4.2.1.2 Completing the square: Question 2

The following categories were used as a classification of the learners’ responses in Question 2:

Correct (CR): *able to find the correct roots using completing the square*

Incomplete (IC): *using the method but not being able to complete the procedure*

Incorrect (InC): *using the method but not able to get the correct roots*

Empty (ER): *no attempt to solve the equations using completing the square*

The results of the pre-test on question 2 are displayed in the table below.

Table 4.2

Percentages (and absolute numbers) of correct response (CR), incomplete response (IC), incorrect response (InC) and empty response (ER) on Question 2 items for pre-test

QUESTION 2	Items	CR	IC	InC	ER
(N=20)	i	0% (0)	20% (4)	60% (12)	20% (4)
	li	0% (0)	20% (4)	65% (13)	15% (3)
	iii	0% (0)	30% (6)	40% (8)	30% (6)
	Iv	0% (0)	15% (3)	40% (8)	35% (7)

Only few learners applied the *completing the square* method in approaching Question 2 items. In fact, none of the learners solved the quadratic equations correctly with this method. The analysis of learners' responses in the pre-test showed that the majority of the learners attempted to use either *factorisation* or *quadratic formula* methods, even when the instruction clearly stated the use of completing the square. One of the fundamental reasons for learners' preference to use the quadratic formula and factorisation may be explained by the fact that the learners in this study lacked sufficient algebraic and arithmetic abilities to complete operations with the necessary fractional and radical expressions to efficiently use the complete the square technique, as indicated by Bossé and Nandakumar (2005). These results also justified the fact that the learners lack the necessary problem-solving abilities. Hence, it was difficult for them (learners) to devise a plan of solving the equations and so could not carry out any actions in order to solve the questions correctly.

4.2.1.3 Using algebraic formula: Question 3

The following categories were used as a classification of the learners' responses in Question 3:

Correct (CR): *solution with correct discriminant and roots.*

Incomplete (IC): *able to state the discriminant correctly, but unable to successfully complete the procedure.*

Incorrect (InC): *stating the discriminant incorrectly or Stating the discriminant correctly, but making procedural mistake or finding the discriminant correctly, but unable to reach the correct result.*

Empty (ER): *no attempt.*

The results of the pre-test on question 3 are displayed in the table below.

Table 4.3

Percentages (and absolute numbers) of correct response (CR), incomplete response (IC), incorrect response (InC) and empty response (ER) on Question 3 items for pre-test

QUESTION 3	Item	CR	IC	InC	ER
(N=20)	i	75% (15)	10% (2)	15% (3)	0% (0)
	ii	60% (12)	20% (4)	20% (4)	0% (0)
	iii	60% (12)	25% (5)	15% (3)	5% (1)
	iv	65% (13)	25% (5)	5% (1)	5% (1)

The data showed that learners encountered the following challenges while applying the quadratic formula to find the roots of the quadratic equations:

- (i) Learners computed the discriminant incorrectly because of calculation errors.
- (ii) Learners computed the discriminant correctly, but applied the quadratic formula incorrectly, since they seemed to have forgotten it.

Learners computed the discriminant incorrectly but they applied the quadratic formula correctly. Particularly, for Questions 3 (ii-iv), where the learners mostly used the quadratic formula to find the roots, learners' incorrect solutions were mainly based on either the incorrect calculation of the discriminant or incorrect use of the quadratic formula. Most of the learners were not able to solve Q3 (ii) quadratic equations correctly, because they made calculation errors while they were finding the discriminant of the quadratic equations. On the other hand, most of the learners calculated the discriminant correctly in Q3 (iii & iv) and used the correct form of the quadratic formula. For example, many of the learners remembered the quadratic formula and applied it correctly. Essentially, these results indicate that learners may have blindly memorised the rules and applied them without enough thought. Though, despite these mistakes most learners

performed better in this question (Q3) than any other question, because this did not necessarily involve problem-solving skills, because there was a formula given to the learners, to which they merely applied the rules, substituting the values into the formula.

4.2.1.4 Question4: Using Quadratic Equations to Formulate and Solve Word Problems

Correct (CR): *able to solve the word problems using any of the methods (factorisation, completing the square or quadratic formula).*

Incomplete (IC): *unable to solve the word problems using any of the methods (factorisation, completing the square or quadratic formula)*

Incorrect (InC): *incorrectly writing the equations and not able to state the correlations between variables correctly.*

Empty (ER): *no attempt.*

The results of the pre-test in question 4 are displayed in the table below.

Table 4.4

Percentages (and absolute numbers) of Correct Response (CR), Incomplete Response (IC), Incorrect Response (InC) and Empty Response (ER) on question 4 items for pre-test.

QUESTION 4	Items	CR	IC	InC	ER
(N=20)	i	0% (0)	5% (1)	5% (1)	90% (18)
	ii	0% (0)	0% (0)	0% (0)	100% (20)
	iii	0% (0)	0% (0)	0% (0)	100% (20)
	iv	0% (0)	0% (0)	0% (0)	100% (20)

None of the learners were able to solve any question correctly in Q4. The learners' solutions indicated that their performance differed depending on the structural characteristics of the word problems. The large majority of the learners (90%), either did not formulate the quadratic equations to represent the information given in the problem, or did not solve them correctly. Furthermore, the rate of empty responses was highest in (Q4.ii, Q4.iii and 4.iv) problems, where the learners were usually unable to set up the correct quadratic equations or did not solve the equations in any way conclusively. On the other hand, the learners who attempted (Q4.i) were unable to provide a reasonable procedure; learners could not construct the algebraic relationships

and formulate the quadratic equations correctly. The learners' solutions revealed that, since a large proportion of learners either did not comprehend the text in the word problems, particularly in the fourth problem, or they could not formulate the related equations as required of them. These suggest a lack of understanding of the question, where the learners could not devise any plan for the solution. Hence, they lacked problem-solving skills.

Table 4.5

The percentage learners' scores in the pre-test, the mean and the standard deviation (SD)

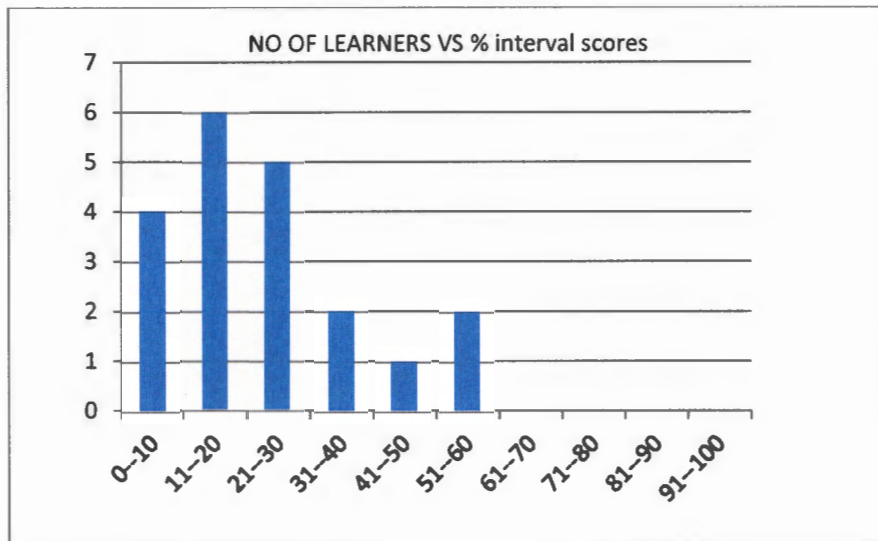
Pre-test					Mean	SD																
N= 20	35	21	20	24	06	42	52	52	21	06	18	27	18	18	04	11	38	11	0	27	22.6	14.9
					Min	Max																
					0	52																

The final results of the pre-test are shown in the above Table 4.5 with the mean score of 22.6. The analysis of the pre-test shows that the lowest score obtained before the intervention is 0 while 52 was the highest score obtained. This indicates the current level of achievement before the intervention, and as reported earlier, the learners had completed the topic in about 15 lessons in the present Grade 11 class with their teacher.

The graph below (Figure 4.1) shows the graphical representation of the learners' percentage score intervals.

Figure 4.1

Graph showing the no. of learners against the percentage scores interval of the pre-test.



The above graph shows that most learners scored between 11%-30%, 11 out of 20 learners (55% of the learners), and two out of 20 learners (10% of the learners) scored above average 50, with all the scores below 60 percent. As explained during the analysis of each question, the summary of the pre-test shows that the learners thought that quadratic equations are about procedure and memorising, and this was the case in Q4, where almost all the participants were unable to interpret the question. Killen (1996:244) has meanwhile argued that learners need to develop focusing skills, information-gathering skills, organising skills, analysing and integrating skills, and finally, evaluating skills, in order to be good problem-solvers, and therefore, these results show that the participants lack problem-solving skills.

4.3 QUALITATIVE DATA ANALYSIS (Pre-intervention)

The qualitative phase of these mixed-methods was purposefully designed to investigate the effect of the problem-centred teaching and learning approach of teaching on Grade 11 learners' performance and achievement in quadratic equations. The qualitative data were collected and recorded in the form of questionnaires, questioning on video, and learners' journals. The use of descriptive statistics was relevant for this phase of mixed methods design. Descriptive statistics, according to McMillian and Schumacher (2006:150), referred to as summary statistics transform a set of observations into indices that describe the data. Data from the questionnaires, video recording and questioning were tabulated for statistical analysis. Descriptive statistics including means, standard deviations (SD) and overall responses were then analysed.

4.3.1 Questionnaire

As reported earlier, the questionnaire (see Appendix F) was administered to all the participants at the beginning as well as at the end of intervention. Section A of the questionnaire was about gender and age and the results are shown in Table 4.1 below.

Section B: General attitude of learners towards mathematics.

Section C: Willingness of learners to engage in problem solving activities in quadratic equations.

Section D: Learners perseverance during problem solving process in quadratic equations.

Section E: Learners self-confidence with respect to problem solving.

Table 4.6

Pre-intervention response to the questionnaire showing age and sex of the learners in section A, the scores for Sections B, C, D and E with the mean and standard deviation (SD) for each section were recorded and tabulated below

QUESTIONNAIRE	SECTION A	SECTION B	SECTION C	SECTION D	SECTION E
Pre-intervention	AGE & SEX				
N=20	15 F	24	19	15	22
	16 F	30	26	14	20
	16 F	44	32	16	19
	16 M	32	28	13	25
	16 M	20	30	7	36
	17 F	24	35	10	27
	17 F	36	22	8	22
	17 F	38	24	9	19
	17 M	42	24	9	23
	17 M	20	30	13	25
	18 F	32	32	8	23
	18 F	28	18	17	36
	18 M	26	18	19	31
	18 M	31	16	20	20
	19 F	21	20	21	25
	19 M	44	46	25	16
	20 F	56	58	24	19
	20 M	58	33	20	23
	21 F	33	20	8	30
	21 F	24	26	10	22
MEAN		33.2	27.9	14.3	24.2
SD		5.6	10.8	9.9	5.3

4.3.2 Video recording and questioning (During intervention)

The video recording and questioning data supported the fact that forming quadratic equations was quite challenging for learners, due to their difficulty in comprehending the problem statement, especially Questions 2 and 4, before the intervention. The following questions were posed to some of the learners at the beginning of the intervention by the researcher (R) and the responses from some of the learners (L) follows each question:

- Researcher: *What was it that caused you difficulty in the fourth question?*
- Learner 3: *Obviously, I read the question many times but it was very difficult to understand, then I read it again, and I still did not understand it.*
- R: *Was the problem too long? Or you did not understand what you read?*
- L9: *Both; due to the length of the problem and the reading comprehension. Difficulty in setting up the correct relationship. [sic]*

The data revealed that certain learners understood the problem statement; however, they did not know how to represent the given information as quadratic equations. The learners were able to gradually develop interest in problem solving approach as the intervention continued, and this reflected in the final performance. During the video recording and questioning, the researcher filled in the problem-solving rating scale for all learners. This data was recorded in the frequency tables. Learners were gradually acquiring problem solving skills in quadratic equations; and had developed them by the end of the intervention as the rating shows.

Table 4.7

The problem-solving observation rating scale frequency table.

N= 20	19/09/16	26/09/16	3/10/16	7/10/16	17/10/16
Learners that understand the given problem	1	5	8	13	19
Learners stuck with the problem	16	12	7	2	0
Learners that understand conditions and variable in the problem	0	6	8	15	19
Learners who were able identify relevant data needed to solve a problem	0	3	8	17	19
Learners that used different strategies when needed	0	0	4	12	18
Learners that were able to correct answers in terms of data in the problem	0	8	12	16	19
Learners that checked and reflect on their solution	0	6	13	17	19
Learners who examined their solution	0	5	13	17	19
Learners who show willingness to engage in problem solving activities	2	7	12	18	20

Learners who demonstrated self confidence	2	7	16	18	20
Learners who persevered during the problem solving process	1	7	16	18	20

4.3.3 Learners' journals: During intervention

The learners (L) were told to keep a personal journal during the intervention for the purpose of recording their experiences during the intervention. The following statements are adapted from some of the learners' journal.

L2: *I answered the pre-test very badly because of all those questions are not easy. I thought they are supposed to be simple quadratic equations. [sic]*

L1: *In Question One and Two, I didn't remember how to factories and completing the square procedure. It was difficulty. [sic]*

L15: *I thought I know quadratic equations, but I can see that I don't really understand the procedure very well, especially questions on factorisation, completing the square and word problems. [sic]*

L6: *I don't understand how we going to work in groups, because I am not used to working with other learners, and I don't think it will work for me but may be as time goes on, it will make sense to me.*

As the intervention progressed, these are what some of the learners wrote in their journals:

L6: *supposing we can learn all other topics and even subjects using the same problem-centred approach, no learners will fail.*

L8: *Now I can boldly say, 'I am interested in learning mathematics'.*

L7: *I am really improving in the way I solve mathematical problems now, I know how to solve problems by breaking them into smaller pieces.*

By the end of the intervention, the following statements have been adapted from the learners' journal:

L20: *I have really enjoyed the lessons; I thought I will never understand quadratic equations.*
[sic]

L11: *why wouldn't other teachers use this problem-centred approach, it is very helpful.*

L17: *I have never being so interested in a topic.*

L2: *I didn't know that mathematics is so interesting.* [sic]

4.4 QUALITATIVE DATA ANALYSIS (Post-intervention)

After the intervention, qualitative data were collected again with the same questionnaire used before the beginning of the research in order to evaluate the effect(s) of the intervention of problem-centred teaching and learning.

4.4.1 Questionnaire

At the end of the intervention, the same questionnaire that was given to Grade 11 learners before the intervention was given again after the intervention. This was done in order to measure the effect of the intervention on the participants. This gave the researcher an idea of the experience of the learners during the intervention.

Table 4.8

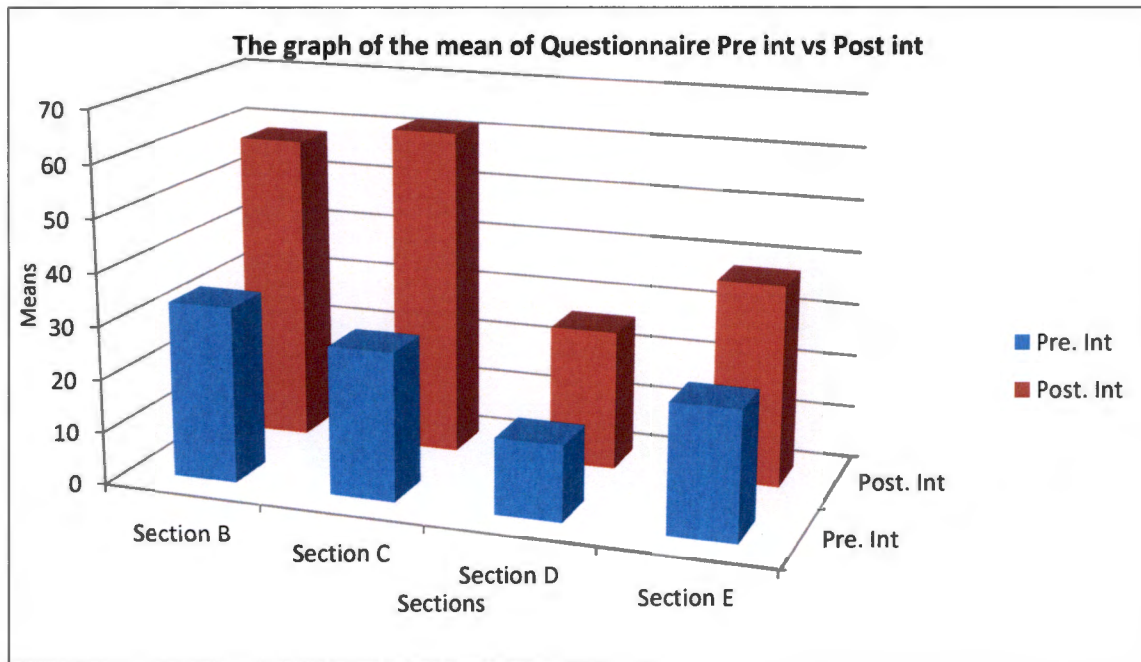
Post-intervention response to the questionnaire showing age and sex of the learner, the mean and the standard deviation (SD) for each section

QUESTIONNAIRE	SECTION A	SECTION B	SECTION C	SECTION D	SECTION E
Post-intervention	AGE & SEX				
N=20	15 F	54	66	29	38
	16 F	45	75	30	39
	16 F	65	65	31	42
	16 M	44	58	27	42
	16 M	46	57	29	38
	17 F	56	63	31	41
	17 F	65	59	25	37
	17 F	69	64	28	35
	17 M	71	66	23	29
	17 M	65	76	25	38
	18 F	70	69	21	40
	18 F	66	68	26	41
	18 M	61	62	25	39
	18 M	57	58	28	32
	19 F	49	52	21	37
	19 M	56	46	20	39
	20 F	63	59	28	41
	20 M	44	61	27	38
	21 F	62	57	21	36
	21 F	56	54	29	36
MEAN		58.2	61.8	26.3	37.9
SD		8.7	7.1	3.4	3.2

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Figure 4.2

The graph of the mean values of pre-intervention against mean values of post-intervention of the questionnaire responses



The graph above shows the data gathered from the questionnaire before and after the intervention indicating various levels from Section B to E. These sections are defined below:

Section B: General attitude towards mathematics.

Section C: Willingness to engage in problem solving activities in quadratic equations.

Section D: Perseverance during problem solving process in quadratic equations.

Section E: Self-confidence with respect to problem solving.

For example, Section B of the questionnaire covered the general attitude towards mathematics, whereby the results show that the mean for the pre-intervention responses and post-intervention responses is 33.2 and 58.2, respectively. To test the null hypothesis for sections B, C, D and E, the t-test was run:

H₀: There is no statistically significant difference between the mean scores of the pre-intervention and the post-intervention.

For Section B, the dependent sample *t*-test statistic of (9.294) with a degree of freedom of 19 the critical *t*-value for the test is statistically significant at level $p < .001$. This implies that there was statistically significant difference in the mean scores between the pre-intervention and the post-intervention, and that therefore, the null hypothesis was rejected.

Section C, which explores the willingness of the learners to engage in problem-solving activities in quadratic equations, the *t*-test statistic of ($t=11.15$) at level $p < .001$, section D ($t=7.61$) at level $p < .001$, which covered perseverance during the problem-solving process in quadratic equations, and lastly, section E ($t = 9.81$), which explored the learners' self-confidence, with respect to problem solving, and when the *t*-test of all these sections (C, D, E) were run to test the null hypothesis, it was found that there is a statistically significant difference between the means of the pre-intervention responses and post-intervention responses, hence, the null hypothesis was rejected.

4.5 QUANTITATIVE DATA ANALYSIS (post-intervention)

At the end of the intervention (PCTL), the Grade 11 learners were given another test (post-test) in order to assess the effect of the intervention.

4.5.1 Post-test

A post-test in quadratic equations was administered to the Grade 11 mathematics learners immediately after the intervention. The purpose of administering post-test was to measure the impact of the intervention on the participants. The analyses of the results obtained from the post-test are displayed in the tables below. The themes used in each question in the post-test are the same as those used in the pre-test.

4.5.1.1 Question 1 (factorisation)

The results of the post-test on question 1 are displayed in the table below:

Table 4.9

Percentages (and absolute numbers) of Correct Response (CR), Incorrect Response (IR), Incomplete Response (InR) and Empty Response (ER) on Question 1 items for the post-test.

	Items	CR	IC	InC	ER
Question 1					
(N=20)	i	75% (15)	15% (3)	10% (2)	0% (0)
	ii	80% (16)	20% (4)	0% (0)	0% (0)
	iii	80% (16)	20% (4)	0% (0)	0% (0)
	iv	70% (14)	20% (4)	10% (2)	0% (0)

The data illustrated in the pre-test indicated that only two of the 20 learners were able to solve Question 1 correctly (see Table 4.1), but the results of the post-test show tremendous improvements. This can be attributed to the effects of the intervention, where 16 learners, 80% of the participants solved Question 1 correctly, where there were no learners that gave an empty response. The learners were able to solve the quadratic problem due to the benefits acquired during the problem-centred teaching and learning approach, which drives the developments of mathematical problem solving skills of Grade 11 learners in quadratic equations in this study.

4.5.1.2 Question 2 (Completing the square)

The results of the post-test on Question 2 are displayed in the table below.

Table 4.10

Percentages (and absolute numbers) of Correct Response (CR), Incorrect Response (IR), Incomplete Response (InR) and Empty Response (ER) on Question 2 items for post-test.

Question 2	Items	CR	IC	InC	ER
(N=20)	i	60% (12)	30% (6)	10% (2)	0% (0)
	ii	55% (11)	30% (6)	15% (3)	0% (0)
	iii	50% (10)	30% (6)	20% (4)	0% (0)
	iv	50% (10)	30% (6)	20% (4)	0% (0)

The same improved results were experienced in Question 2, in which at least 50% of the learners gave the correct response (CR), and there were no empty responses in the post-test, but in the results of pre-test, there were up to seven learners who gave empty response (ER) in Question 2. (iv). This again proved the benefits of the intervention on the performances of the learners.

4.5.1.3 Question 3 using the algebraic formulae

The results of the post-test on Question 3 are displayed in the table below.

Table 4.11

Percentages (and absolute numbers) of Correct Response (CR), Incorrect Response (IR), Incomplete Response (InR) and Empty Response (ER) on Question 3 items for post-test.

Question 3	Items	CR	IC	InC	ER
(N=20)	i	85% (17)	10% (2)	5% (1)	0% (0)
	ii	80% (16)	10% (2)	10% (2)	0% (0)
	iii	75% (15)	15% (3)	10% (2)	0% (0)
	iv	75% (15)	15% (3)	10% (2)	0% (0)

In Question 3, there is an improvement in the results of post-test as compared to pre-test, in this question the learners were given the algebraic formulae to use. They were only requested to substitute in the values of the variables into the formulae; this question does not necessarily require problem-solving skills. That could be the reason whereby the performance in the pre-test is similar to the results of the post-test.

4.5.1.4 Question 4: word problem on quadratic equations

The results of the post-test on Question 4 are displayed in the table below.

Table 4.12

Percentages (and absolute numbers) of Correct Response (CR), Incorrect Response (IR), Incomplete Response (InR) and Empty Response (ER) on Question 4 items for post-test.

Question 4	Items	CR	IC	InC	ER
(N=20)	i	75% (15)	10% (2)	15% (3)	10% (0)
	ii	60% (12)	25% (5)	15% (3)	10% (0)
	iii	40% (8)	30% (6)	30% (6)	10% (0)
	iv	40% (8)	30% (6)	30% (6)	10% (0)

The results of the post-test for Question 4 items show that learners performed very well when compared with the results of the pre-test, about 75% of the learners were able to solve Question 4(i) correctly (CR), but in the pre-test, no learners (0%) were able to give the correct response. Also, there was no empty response in the results of the post-test, but in the pre-test, most of the learners' responses were empty for all the four items. These results also show that the learners' were able to acquire the required problem-solving skills through the intervention and used these skills efficiently.

4.5.1.5 Post-test results

The general performance in the post-test results shows overall improvements as compared to the results of the pre-test, as shown in Table 4.13 below. The minimum score is 27%, and the maximum score is 83 percent. The mean score of 51,2 was recorded for the post-test. Only 2 learners (10% of the learners) scored below 30 percent. Eleven learners (55% of the learners) scored well above 50% in the post-test, and seven learners (35% of the learners) scored over 30% to below 50 percent.

Table 4.13

The percentage scores of the learners in post-test, the mean and the standard deviation (SD)

Post-test	Mean	SD
(N= 20) 62 56 45 58 43 72 78 83 56 43 46 56 52 59 35 29 66 38 27 37	51.2	16.2
	Min	Max
	27	83

Figure 4.3

The graph of the no of learners against the percentage scores interval of the post-test

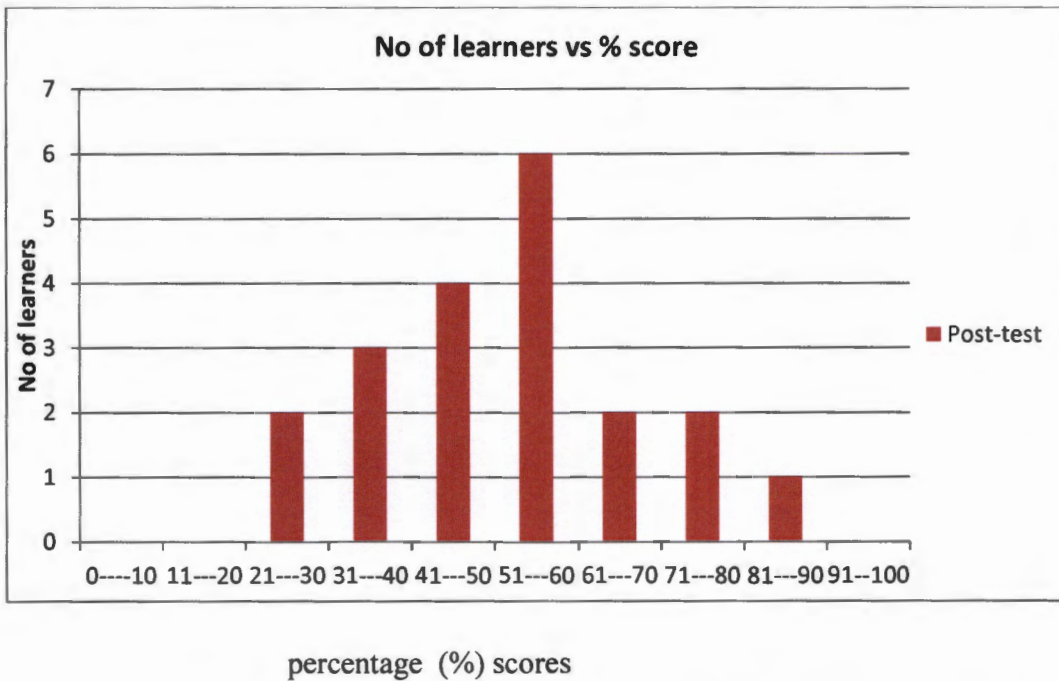


Figure 4.3 above shows the graphical representations of the interval of scores from the post-test results, 0% learners scored less than 20% of the total marks (100) in the post-test. Eighteen out of 20 learners (90% of the learners) scored above the pass mark of 30% in mathematics in the NSC (National Senior Certificates) in the post-test. The general improvements in the post-test

can be attributed to the positive effect of the intervention. Hence, learners benefitted from the intervention.

The table below (Table 4.14) shows the comparison of the pre-test and post-test results.

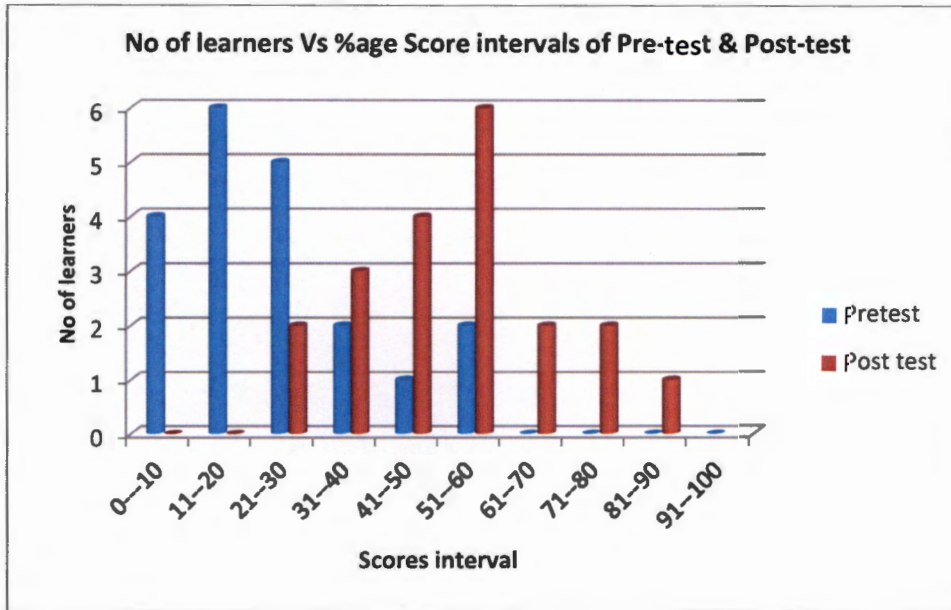
Table 4.14

Percentage scores intervals of the pre-test and post-test and the number of learners.

Score interval	Pre-test N=20	Post-test N=20
0----10	4	0
11----20	6	0
21----30	5	2
31----40	2	3
41----50	1	4
51----60	2	6
61----70	0	2
71----80	0	2
81----90	0	1
91----100	0	0

Figure 4.4

The graph of the no of learners against the percentage scores interval of pre-test and post-test



The above graph clearly shows that before the intervention, the performance of the learners in the pre-test was very low, for instance, in the interval (0-10) there are four learners, whereas the minimum score interval for the post-test is between (21-30) with three learners in the interval. Moreover, in the interval (51-60), there are only two learners, in the pre-test and six learners in the post-test, and this is the highest interval in the pre-test. In the post-test, the highest score interval is (81-90) with one learner. All the above comparison proves that the learners have benefitted from the problem-centred approach of teaching and learning explored in this study. The mean score of the post-test is higher than that of the pre-test. The minimum and maximum scores for the post-test are higher than the minimum and maximum scores for the pre-test. Therefore, it is very important to determine the significant difference in the mean scores of the groups using statistical techniques. A t-test for dependent data was run to test the null hypothesis:

H₀: There is no statistically significant difference between the mean scores of the pre-test and the post-test.

The dependent sample *t*-test statistic of (18.28) with a degree of freedom of 19 the critical *t* value for a two-tailed test is statistically significant at level $p < .001$. This implies that there was a statistically significant difference in the mean scores, between the pre-test and the post-test. Hence, the null hypothesis was rejected.

4.6 DISCUSSION

Creswell and Plano Clark (2011:223-232) outline three options when quantitative results and qualitative findings are merged: (a) comparing the findings side-by-side in a discussion; (b) joint display of comparisons in the results; (c) interpreting the transformed data in the results. Side-by-side comparison for merged data analysis was employed in this study. The researcher compared the presented quantitative and qualitative results side-by-side in the analysis of the findings. The detailed analysis of the findings was driven and conveyed by the merged results. Records from learner journals, video recording and questioning revealed the facts that learners were acquiring mathematical problem solving abilities needed to solve mathematical problems progressively during the intervention, and by the end of the intervention they had developed these skills. It was very clear that learners were able to understand and interpret the given problems as the intervention progressed, they were able to extract the relevant variables, learners were able to select appropriate strategies and correctly implement such strategies for the process of solution, they were able to solve the given problems using appropriate methods of solution and lastly, learners were able to reflect on the methods used and steps taken to solve the problems. It was observed through the recorded video by the researcher that learners were conscious of the procedures involved in problem solving during the problem-solving sessions. The transcribed data from the recorded video and learners' journals revealed that learners had become conscious of both how and why they were solving a given problem. The learners acquired various mathematical problem solving abilities necessary to solve quadratic equations by the end of the intervention. This confirmed what the researcher concluded from the questionnaire, journal entries, and video recording and questioning, that Grade 11 mathematics learners had developed mathematical problem solving skills at the end of the intervention. This was evident in the overall performance after the intervention, in the post-test, the questionnaire responses, the data transcribed from recorded video, and the learners' own journals, where the results show that learners had benefitted from problem-centred teaching and learning approach and that this suggested a positive impact of the intervention on their performance and achievements in Grade 11 learning of quadratic equations.

4.7 SUMMARY

In this chapter, the collected data were analysed, and the results were presented and interpreted. This was followed by the deductions and explanations made through the interpreted results. The final chapter, presents the overall summaries and conclusions. This is followed by recommendations, the limitations of the study, and the possibilities for further research.

CHAPTER FIVE

CONCLUSIONS, RECOMMENDATIONS AND LIMITATIONS

5.1 INTRODUCTION

Chapter 4 presented the results of this study and a discussion of the results. The current chapter, which is the final chapter of this study, presents the conclusion of the entire research and this is in line with the results discussed in the previous chapter, recommendations for future studies are outlined as well as the limitations of the current study.

5.2 RESEARCH QUESTIONS

The main purpose of this research was to explore the benefits of using problem-centred teaching-learning approach in the teaching and learning of quadratic equations in a Grade 11 classroom(s), where the main research question below was posed:

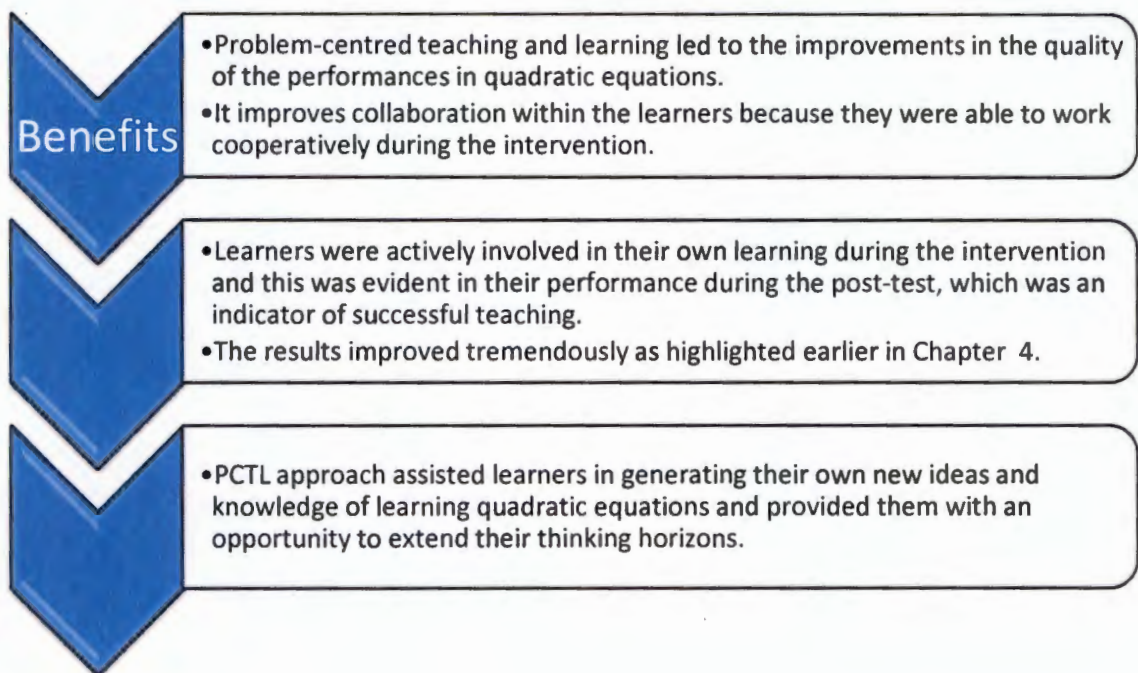
What are the benefits of using problem-centred approach in teaching and learning of quadratic equations in Grade 11 classroom?

In search of answers to this primary question, the study set out in a high school at Makgato in Capricorn District in Limpopo province of South Africa. The data were collected from 20 learners in a Grade 11 classroom. Using a mixed-methods design, the qualitative data were mainly gathered through questionnaires, video recorded questioning, and learners' journals. The values of the mean of the response before intervention and after the intervention were compared using dependent sample t -test, of which the results were outlined in Chapter 4. The quantitative data obtained through pre-test and post-test in this same chapter were analysed by comparing the mean scores of both pre-test and post-test scores using dependent sample t -test. These comparisons led to the testing of the null hypotheses. The results from the null hypothesis indicated that there were differences in the means compared, learners performed better in the post-test which suggested that the intervention had impacted positively on the learning of quadratic equations in the Grade 11 mathematics classroom.

The results and findings of the study suggested that there were benefits when problem-centred approach was used in teaching and learning of quadratic equations in Grade 11 classroom(s). These benefits are outlined in the diagramme below:

Figure 5.1

The list of the benefits of problem-centred approach for the teaching and learning of quadratic equations in a Grade 11 mathematics classroom.



5.2.1 Research sub-questions

The following sub-questions were posed:

1. What is the nature of the effects on Grade 11 learners' performance when using problem-centred approach in teaching quadratic equations?

There was a clear answer to this question from the post-intervention results of both qualitative and quantitative data, the learners' performance increased and most importantly the learners were able to show better understanding in their different approaches in answering the post-test.

2. What is the correlation between problem-centred approach and learners' problem-solving abilities when they engaged with quadratic equations?

The findings from the video recording and questioning, combined with data transcribed from the learners' journals within the discussion, serve as answers to the research sub-questions. Each theme is explored quantitatively and qualitatively and from the results, a problem-centred approach improved learners' problem-solving abilities when engaged with quadratic equations.

5.3 RECOMMENDATIONS

The following constitute the main recommendations emanating from the results of this investigation.

There is a need for organisation of professional development centres by government-funded initiatives for educators to enhance their knowledge and skills necessary for successful implementation of problem-centred teaching and learning approaches. Professional development activities such as training, workshops, as well as all other forms of supports and guidance. The curriculum emphasises that learners should be prepared for the world of work and lifelong learning which involves teaching skills to analyse problems and be able to extract relevant information from any given task, therefore, problem-centred teaching and learning is recommended as an approach that delivers such needs. This study also recommends that teaching materials should be written in such a way that they involve activities to process new materials that link what learners knows already. Tasks should be authentic, set in a meaningful context, and related to the real world. They should not just involve repeating facts, as this causes 'surface' learning. As learners' learning will involve errors, tasks should offer opportunities for self-assessment, correction, peer discussion, and educator's feedback.

Further research is also recommended to verify the findings of the current study in order to strengthen this contribution towards the development of sound research data, based on problem-centred learning approach used in this study.

5.4 LIMITATIONS OF THE STUDY

There are limitations that are acknowledged in the consideration of results in all research. This study is not an exception. The main limitations of this study are the type of school (rural area); the size of sample (20 learners); and limited sources of information (quadratic equations in Grade 11 classroom). A greater sample spread over a wider demographic area may possibly enhance insight and enable greater generalisation. Furthermore, the sampled school being a public owned and not a private school may have influenced the findings.

5.5 CONCLUSION

The teaching and learning of mathematics poses a serious challenge to educators, parents, policy makers most especially in South Africa. This study has, to some meaningful extent, proved that problem-centred teaching and learning approach can help in improving learning achievement and performance in the topic. Though the study explored a specific topic (quadratic equations), the findings in this study justify further research in other topics in mathematics using the same teaching and learning approach. Post-intervention responses revealed the impact of exposing learners to problem-solving strategies in the last chapter. Learners were encouraged to engage with the problem by breaking the problem into meaningful parts for better understanding; they seemed to be gaining a better insight into the problems as a whole. This intervention even assisted learners in planning for the solution as the second stage for problem-solving in the adopted theory. It is, therefore, recommended that learners be encouraged to engage with the problem for better understanding, before embarking on the solution. The plans that were generated after intervention were relevant to problems in question. In most cases, plans were accounted for. It is through intervention that learners also started justifying each domain used in the mathematical equations developed. They also refrained from their norm of directly picking up numbers from mathematical word problems into equations. Learners could now realise the connections between the data and the unknown, which is subsequently followed by the application of operational processes.

The looking-back stage of problem solving seemed to be heavily impacted through exposure to problem-solving approaches, because learners were now able to substitute their calculated values in planned equations to check whether answers are relevant to questions. Still missing is the second stage of looking back, which, according to the argument I present here, appears not to be fully developed, and is the stage whereby one is required to check the answer against the context within which the problem is given. Most responses can be found not to capture that stage. It is further recommended that solutions to problems must, at all times, be interpreted against the planned approach for solution and the problem as a whole.

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APPENDIX A

ETHICAL CERTIFICATE

APPENDIX B

Letter to School principal and School governing board inviting the grade 11 mathematics learners to participate in the research

Contact: 0723854129

fadipebanji@gmail.com

The Principal

Dear Sir

INVITING GRADE 11 MATHEMATICS LEARNERS TO PARTICIPATE IN A RESEARCH PROJECT

I am currently enrolled for the degree M.Ed (Maths/Sc) at the North-West University Mahikeng. The purpose of my study is to explore the benefits of problem-centred approach of teaching and learning quadratic equations in the Grade 11 classroom. The Mathematics results and achievement of learners in the Limpopo Province is generally a matter of concern.

To complete this study successfully, I hereby request your permission for the following:

- (i) To administer questionnaires among learners in grade 11 mathematics class.
- (ii) To conduct Pretest and Posttest on quadratic equations

Permission will also be requested from the Ethics Committee of the North-West University. The information regarding learners and the school will be kept confidential and they will by no means be mentioned in the research. All respondents and participants will be kept anonymous. Feedback will be given by means of published results available to the school. Arrangement may also be made on request to discuss my findings with parents.

Kind Regards

Fadipe M.B.

APPENDIX C
LETTER OF CONSENT

Dear Mr/Mrs

INVITING YOUR CHILD TO PARTICIPATE IN A RESEARCH PROJECT

I am currently busy with research focusing on the benefits of using problem-centred teaching and learning approach in teaching quadratic equations in Grade 11 classrooms.

The Mathematics results and achievement of learners in the Limpopo Province are generally a matter of concern especially in algebra (quadratic equations), which is one of the reasons for this study.

Learners who participate in this study will be requested to solve one Mathematics problem in quadratic equations and complete two questionnaires. During the study, the learners will be observed and the observation will be recorded in order to understand how they solve given quadratic equations problems. Afterwards the learners will be able to record their experience in learners' journal describing, in words, what each did or thought when solving the problem.

Participation in the project is free willingly and each learner is allowed to withdraw from the study at any time.

Your permission as parent is therefore necessary to allow your child to participate in the study. Information gathered will be regarded as confidential and learners will be kept anonymous.

The research will be conducted during the month of August 2016.

Feedback will be provided by arrangement with the researcher.

Thanking you kindly,

Fadipe M.B. Cell: 0723854129.

E-mail: fadipebanji@gmail.com

Permission I have read the above and understand the nature of the research. I understand that by giving permission, the researcher will conduct the study in a professional manner and will consider the child's human rights. I understand that I can contact the researcher or any queries regarding the study.

Undersigning, I hereby give my child permission to participate in the above study.

Name and surname of learner: _____

Grade: _____ Contact number: _____ Parent's /

Guardian's signature: _____

Participant signature _____ Date: _____

APPENDIX D

PRE TEST

QUADRATIC EQUATIONS (Grade 11) 1HR 45MIN

INSTRUCTIONS

ANSWER ALL QUESTIONS

START EACH QUESTION ON A NEW PAGE

WRITE CLEARLY AND LEGIBLY

1. Use factorisation method for these questions.

i. $f(2f + 1) = 15$

ii. $5\sqrt{5t+1} - 4 = 5t + 1$

iii. $2x = \sqrt{21x - 5}$

iv. $p^2 - 5pq + 4q^2$

(20 marks)

2. Solve the following equations using completing the square method

i. $x(x - 9) + 14 = 0$

ii. $2t^2 - 6t - 10 = 0$

iii. solve for p in terms of b : $p^2 + 6p + b$

iv. $k - 3/k = 2$

(20 marks)

3. Solve the following quadratic equations using quadratic formula

- i. $5t^2 + 3t - 3 = 0$
- ii. $2p(2p + 1) = 2$
- iii. $9(k^2 - 1) = 7k$
- iv. $8(1 - 4g^2) + 24g = 0$ (16 marks)

4. Solve the following quadratic equations.

- i. A stone is thrown vertically upwards and its height (in metres) above the ground at time t (in seconds) is given by :
 $H(t) = 35 - 5t^2 + 30t$.
Find its initial height above the ground. (5 marks)
- ii. An equations of the form $ax^2 + bx + c = 0$ is written on the board. Mpho and Lerato copy it down incorrectly. Mpho has a mistake in the constant term and obtains the solutions -4 and 2 . Lerato has a mistake in the coefficient of x and obtains the solutions 1 and -15 . Determine the correct equations that was on the board. (12 marks)
- iii. Jacob has played a few games of pin bowling. In the third game, Jacob scored 80 more than in the second game. In the first game Jacob scored 110 less than the third game. His total score for the first two games was 208 . If he wants an average score of 146 , what must he score on the fourth game. (13 marks)
- iv. a. The equations $ax^2 + bx + c = 0$ has roots $x = 2/3$ and $x = -4$. Find one set of possible values for a, b, c . (7 marks)

b. The two roots of the equations $4x^2 + px - 9 = 0$ differ by 5. Calculate the value of p.

(7 marks)

100 marks

APPENDIX E

POST-TEST

QUADRATIC EQUATIONS (Grade 11) 1HR 45MIN

INSTRUCTIONS

ANSWER ALL QUESTIONS

START EACH QUESTION ON A NEW PAGE

WRITE CLEARLY AND LEGIBLY

1. Use factorisation method for these questions.

- v. $f(2f + 1) = 15$
- vi. $5\sqrt{5t+1} - 4 = 5t + 1$
- vii. $2x = \sqrt{21x - 5}$
- viii. $p^2 - 5pq + 4q^2$ (20 marks)

2. Solve the following equations using completing the square method

- v. $x(x - 9) + 14 = 0$
- vi. $2t^2 - 6t - 10 = 0$
- vii. solve for p in terms of b : $p^2 + 6p + b$
- viii. $k - 3/k = 2$ (20 marks)

3. Solve the following quadratic equations using quadratic formula

- v. $5t^2 + 3t - 3 = 0$
- vi. $2p(2p + 1) = 2$
- vii. $9(k^2 - 1) = 7k$
- viii. $8(1 - 4g^2) + 24g = 0$ (16 marks)

4. Solve the following quadratic equations.

- v. A stone is thrown vertically upwards and its height (in metres) above the ground at time t (in seconds) is given by :
 $H(t) = 35 - 5t^2 + 30t$.
Find its initial height above the ground. (5 marks)
- vi. An equations of the form $ax^2 + bx + c = 0$ is written on the board. Mpho and Lerato copy it down incorrectly. Mpho has a mistake in the constant term and obtains the solutions -4 and 2 . Lerato has a mistake in the coefficient of x and obtains the solutions 1 and -15 . Determine the correct equations that was on the board. (12 marks)
- vii. Jacob has played a few games of pin bowling. In the third game, Jacob scored 80 more than in the second game. In the first game Jacob scored 110 less than the third game. His total score for the first two games was 208 . If he wants an average score of 146 , what must he score on the fourth game. (13 marks)
- viii. a. The equations $ax^2 + bx + c = 0$ has roots $x = \frac{2}{3}$ and $x = -4$. Find one set of possible values for a, b, c . (7marks)

b. The two roots of the equations $4x^2 + px - 9 = 0$ differ by 5. Calculate the value of p.

(7 marks)

100 marks

APPENDIX F
QUESTIONNAIRE

SECTION A: Write answers in the spaces provided

1. Name of school? _____
2. What is your gender? _____
3. How old are you? _____

Section B, C, D, and E, are about how you feel about learning and studying mathematics and how you feel about problems solving in mathematics

Read the statements carefully and mark one of the most appropriate choices for each item on the answer sheet:

5. Completely Agree
4. Agree
3. Undecided
2. Disagree
1. Completely Disagree

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SECTION B: Attitude towards Mathematics

		1	2	3	4	5
1	Mathematics is the subject that I like.					
2	I look forward to my mathematics lessons.					
3	I do mathematics because I enjoy it.					
4	I am interested in the things that I learn in mathematics.					
5	If there are no mathematics classes, being a student will be more enjoyable					
6	I like discussing mathematics with my friends.					
7	I wish there were more mathematics classes a week					
8	I would not get bored if I study mathematics for years.					
9	I have always believed that mathematics is one of my best subjects.					
10	Among all the lessons, mathematics is most unlikable.					
11	I learn mathematics quickly.					
12	Making an effort in mathematics are worth it because it will help in the work that I want to do later.					
13	Mathematics is an important subject for me because I need it for what I want to study later on.					
14	I will learn many things in mathematics that will help me get a job.					

15	Do you think quadratic equations is a difficult topic.					
16	Do you always score above average in any tests on quadratic equations					
	SECTION C : Willingness to engage in Problem Solving Activities	1	2	3	4	5
1	I will try almost any mathematics problem.					
2	It is no fun to try and solve problems.					
3	I like to try challenging problems					
4	I do not like to try problems that are hard to understand.					
5	I like to try to solve problems.					
6	One learns mathematics best by memorising facts and procedures.					
7	I try to understand the problem solving process instead of just getting answers to the problems					
8	I solve the problems the way the teacher shows me and do not think up of my own ways					
9	I try to find different ways to solve problems.					
10	Mathematics is about inventing new ideas.					
11	When I am confronted with quadratics problems, I can usually find several solutions.					
12	If I am engaged with a difficult quadratic problem, I can usually think of a strategy to use.					
13	The teacher must always show me which method to use to given mathematics					
14	I am willing to try a different problem solving approach when my first attempt fails.					
15	I feel the most important thing in quadratics problem solving is getting the correct answer.					
16	When I have finished working on a problem, I look back to see whether my answer makes sense.					
17	With my level of resourcefulness, I can solve quadratic equations that I am not familiar with					
18	I try to explain my ideas to other learners					
	SECTION D: Perseverance during the problem solving process.					
1	With perseverance and determination, I can solve challenging mathematics problems.					
2	I do not stop working on a problem until I get a solution.					
3	I put down any answer just to finish a problem.					
4	When I do not get the right answer right away, I give up.					
5	I work for a long time on a problem.					
6	I keep on working on a problem until I get it right.					
7	I give up on challenging problems right away.					
	SECTION E: Self confidence with respect to problem solving.					
1	I get nervous doing mathematics problems.					
2	My ideas about how to solve problems are not as good as other learners' ideas					
3	I can only do problems everyone else can do.					
4	Problems solving makes me feel uncomfortable.					
5	I am sure I can solve most quadratic problems.					
6	I am better than many learners in solving quadratic equations					
7	I need someone to help me work on quadratic equations.					
8	I can solve most hard quadratic equations.					
9	Most quadratic equations are too hard for me to solve.					
10	I am a good problem solver.					

APPENDIX G

The Analytic scoring scale

UNDERSTAND THE PROBLEM	0	Do not understand the problem
	1	Part of the problem misinterpreted/ misunderstood
	2	Completely understand the problem
PLANNING FOR SOLUTION	0	Inappropriate plan
	1	Partially correct plan
	2	Correct solution due to proper planning
GETTING A SOLUTION	0	Incorrect solution
	1	Partially correct answer due to computational error
	2	Correct solution

APPENDIX H

The Problem-solving Observation Comment Card.

Learner _____

Date _____

Comments (Examples of what was written by the researcher when observing and questioning learners).

- ✓ Able to understand the given question, look for pattern if any.
- ✓ Able to devise meaningful steps towards solution.
- ✓ Keep trying even when in trouble finding the right steps towards solution
- ✓ Need help in finding the right solution
- ✓ Able to explain solution to other learners.

APPENDIX I

The Problem-solving Observation Rating Scale

Learner _____

Date _____

Always Sometimes Never

1. understand the given problem	_____	_____	_____
2. stuck with the problem	_____	_____	_____
3. understand conditions and variable in a problem	_____	_____	_____
4. able to identify relevant data needed to solve a problem	_____	_____	_____
5. use different strategies when needed	_____	_____	_____
6. correct answers in terms of the data in the problem	_____	_____	_____
7. check and evaluate the reasonableness of the solution	_____	_____	_____
8. could describe or analyse solution	_____	_____	_____
9. show willingness to engage in problem solving activities	_____	_____	_____
10. demonstrate self-confidence	_____	_____	_____
11. persevered during problem solving activities	_____	_____	_____

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