Liquidity, banking and financial crises

By

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Preface

One of the contributions that was made by the NWU-MC to the activities of the stochastic analysis community has been the establishment of an active Finance, Modelling and Optimization Research Group (FMORG) that has an interest in institutional finance. In particular, FMORG has made contributions about modelling, optimization, regulation and risk management in insurance and banking. Students who have participated in projects in this programme under Prof Petersen's supervision, are the following:

Level	Student	Graduation	Title
MSc	T Bosch	May 2003 Cum Laude	Controllability of HJMM Interest Rate Models
MSc	CH Fouche	May 2006 Cum Laude	Continuous-Time Stochastic Modelling of Capital Adequacy Ratios for Banks
MSc	MP Mulaudzi	May 2008 Cum Laude	A Decision-making Problem in the Banking Industry
PhD	CH Fouche	May 2008	Dynamic Modelling of Banking Activities
PhD	F Gideon	Sept. 2008	Optimal Provisioning for Deposit Withdrawals and Loan Losses in the Banking Industry
MSc	MC Senosi	May 2009 S2A3 Winner for NWU	Discrete Dynamics of Bank Credit and Capital and their Cyclicality
PhD	T Bosch	May 2009	Management and Auditing of Bank Assets and Capital
PhD	BA Tau	May 2009	Bank Loan Pricing and Profitability and Their Connections with Basel II and the Subprime Mortgage Crisis
PhD	MP Mulaudzi	May 2010	The Subprime Mortgage Crisis: Asset Securitization and Interbank Lending
MSc	B de Waal	May 2011 Cum Laude	Stochastic Optimization of Subprime Residential Mortgage Loan Funding and its Risks
PhD	MC Senosi	May 2011	Discrete-Time Modelling of Subprime Mortgage Credit
PhD	S Thomas	May 2011	Residential Mortgage Loan Securitization and The Subprime Crisis
MComm	C Scheepers	2011 Onwards	The Impact of the Global Financial Crisis on the South African Steel Industry
MComm	G Mah	2012 Onwards	Sovereign Debt
MComm	C Meniago	2012 Onwards	An Econometric Analysis of the Impact of the Global Financial Crisis on Household Indebtedness in South Africa
MComm	M Mubanga	2012 Onwards	Shocks to Mortgages and Related Debt Obligations
PhD	CO Miruka	2012 Onwards	Sovereign Guarantees in South Africa
PhD	I Mongale	2011 Onwards	An Analysis of the Impact of the Global Financial Crisis on Savings in South Africa
PhD	F Louw	2011 Onwards	Monetary Policy Transmission Mechanisms in South Africa
Postdoc	J Mukuddem-Petersen	2006-2009	Finance, Risk and Banking
Postdoc	T Bosch	2010	Finance, Risk and Banking
Postdoc	M Agaze Dessi	2011	Business Incubation
Postdoc	S Thomas	2011-2012	Subprime Mortgage Models

Table 1: Postgraduate and postdoctoral supervision

Declaration

I declare that the research that is presented in this thesis is my own unaided work, apart from the assistance that has already been acknowledged. It is being submitted in partial fulfilment of the requirements for the degree Philosophiae Doctor in Economics at the Mafikeng Campus of the North-West University. It has not been submitted before to any other University for any degree or examination.

Nobody, including Prof Mark A Petersen, Prof Janine Mukuddem-Petersen and Dr Thahir Bosch, but myself is responsible for the final version of this thesis.

Signature..................................

Date...27-03-2013...

Executive Summary

In recent times, the frequency of financial crises has increased dramatically. In the last five years, for instance, the subprime mortgage and global financial crises have affected financial markets throughout the world. Problems with liquidity, mortgage funding, valuation and subprime mortgage design have been common to all of these crises and will be studied in this thesis. The researcher's results on these problems are collected in articles [1] (see Chapter 2), [5] (see Chapter 3), [3] (see Chapter 4), [4] (see Chapter 5) and [2] (see Chapter 6) that are briefly described below.

Basel III attempts to raise the quality, consistency and transparency of the regulatory capital base in order to enhance the risk coverage of Basel II. In Chapter 2, the researchers explore the way in which these new Basel liquidity standards, as encapsulated in the liquid coverage ratio (LCR) and net stable funding ratio (NSFR), could be effectively implemented in mitigating liquidity problems by considering a solution paradigm for proposed Basel III liquidity regulation. It has been found that higher LCRs mitigate liquidity risk and the NSFR limits an over-reliance on short-term wholesale funding. However, the overall impression of the researchers is that Basel III has not fully addressed the factors that were responsible for the crises and the fundamental problems of previous Basel regulation.

Chapter 3 asserts that the subprime mortgage crisis (SMC) is an ongoing housing and financial crisis that was triggered by a marked increase in mortgage delinquencies and foreclosures in the US. Since it became apparent in 2007, it has had major adverse consequences for banks and financial markets around the globe. In the research, an originator's nonlinear stochastic optimal control problem that is related to choices regarding deposit inflow rates and marketable securities allocation is examined. Here, the primary aim is to minimize liquidity risk- more specifically, funding and credit crunch risk. In this regard, the researchers consider two reference processes, namely the deposit reference process and the residential mortgage loan reference process. This enables them to specify optimal deposit inflows as well as optimal marketable securities allocation by using actuarial cost methods to establish an ideal level of subprime mortgage extension. In their research, relationships are established in order to construct a stochastic continuous-time banking model to determine a solution for this optimal control problem which is driven by geometric Brownian motion.

In Chapter 4, an important aspect of the subprime mortgage crisis is discussed, namely subprime mortgage design in both a theoretical- and numerical-quantitative framework. Such design utilizes real financial market interest rates, securitization structuring and mortgage pricing to explain the economic mechanism behind the recent crisis. In particular, the researchers model mortgages that are able to fully amortize, voluntarily prepay (involving prepayment and possibly refinancing) or default. It is found that mortgage refinancing is curtailed by high loan-to-value ratios due to house price depreciation, while low loan-to-value ratios increase mortgagor house equity. Furthermore, an optimal originator valuation problem under mortgage origination is solved. In this case, optimal mortgage value and rates, as well as profit, are computed. The paper supports the view that the subprime mortgage crisis was partially caused by the intricacy of design of subprime mortgages that led to information problems (asymmetry, contagion, inefficiency and loss), valuation opaqueness and ineffective risk mitigation.

The researchers provide a stochastic model for a global liquidity standard that is proposed by Basel III banking regulation in Chapter 5. In particular, the focus is on the modelling of the liquidity coverage ratio (LCR) that is defined by the stochastic dynamics of the quotient of high-quality liquid assets (HQLAs) to net cash outflows (NCOs). This model enables the researchers to solve a nonlinear optimal stochastic LCR problem with quadratic cost where LCRs are used as one of the metrics in ratio analysis to measure bank liquidity. The main novelty is the introduction of an LCR reference process with respect to which optimal liquidity provisioning and HQLA allocation are characterized. The researchers also introduce the notion of an adjustment to the rate of liquidity provisioning per unit of the bank's NCOs for deficit that, under certain circumstances, can be related to a bank bailout rate in a theoretical-quantitative framework. Finally, numerical-quantitative results concerning LCRs and their connections with HQLAs and NCOs are provided to supplement the previous analysis.

Numerical results, involving new Basel III liquidity regulation, are obtained in Chapter 6. More specifically, the net stable funding ratio is computed in accordance with the prescripts of the proposed banking rules. In this regard, the researchers investigate the effects of shareholder cash-flow rights on the aforementioned funding ratio and a non-Basel III liquidity coverage ratio for certain developing countries during the period Q1:2005 to Q4:2009. The study finds that the funding ratio appears to have satisfied Basel III minimum liquidity standards during this period and that more concentrated cash-flow rights result in improved liquidity.

Keywords: Bank Bailouts, Basel III, Credit Risk, Deposits, Global Financial Crisis, House Equity, Liquid Assets, Liquidity Coverage Ratio, Liquidity Coverage Ratio Reference Process, Liquidity Risk, Loan-to-Value Ratio, Marketable Securities, Mortgages, Mortgage Funding, Mortgage Rate, Net Cash Outflow, Net Stable Funding Ratio, Prepayment, Procyclicality, Refinancing, Subprime Mortgage Crisis, Subprime Mortgage Insurance.

Samevatting

In die afgelope tyd het die frekwensie van finansiële krisisse dramaties gestyg. In die laaste vyf jaar, byvoorbeeld, het die subprima-verband en wêreldwye finansiële krisisse finansiële markte regoor die wêreld geraak. Probleme met likiditeit, verbandbefondsing, waardasie en subprima-verbandontwerp was algemeen aan al hierdie krisisse en sal in hierdie proefskrif bestudeer word. Die navorser se resultate op hierdie probleme word in artikels [1] (sien Hoofstuk 2), [5] (sien Hoofstuk 3), [3] (sien Hoofstuk 4), [4] (sien Hoofstuk 5) en [2] (sien Hoofstuk 6) bespreek en word kortliks hieronder beskryf.

Basel III poog om die kwaliteit, konsekwentheid en deursigtigheid van die regulerende kapitaalbasis te verbeter ten einde die risikodekking van Basel II te verbeter. In Hoofstuk 2 ondersoek die navorsers hoe hierdie nuwe Basel-likiditeitstandaarde, soos saamgevat deur die likiditeitdekkingsverhouding en netto stabiele befondsingsverhouding, effektief geïmplementeer kan word in die tempering van likiditeitsprobleme deur die oorweging van 'n oplossingparadigma vir voorgestelde Basel III-likiditeitregulasie. Daar is gevind dat hoër likiditeitdekkingsverhoudings likiditeitsrisiko versag en dat die netto stabiele befondsingsverhouding 'n oorafhanklikheid op korttermyn-groothandelbefondsing beperk. Die algehele indruk van die navorsers is egter dat die Basel III nie die faktore wat verantwoordelik was vir die krisisse en die fundamentele probleme met vorige Basel-regulasie ten volle aanspreek nie.

Die bydrae in Hoofstuk 3 beweer dat die subprima-verbandkrisis 'n deurlopende behuisings- en finansiële krisis is wat veroorsaak is deur 'n merkbare toename in agterstallige verbande en die oproeping van verbande in die VSA. Nadat dit in 2007 duidelik geword het, het dit belangrike nadelige gevolge vir die banke en finansiële markte regoor die wêreld gehad, nadat dit duidelik geword het in 2007. In die navorsing word 'n bank se nie-lineêre stogastiese optimale beheerprobleem ondersoek met betrekking tot keuses rakende deposito-invloei en bemarkbare sekuriteite-allokasie. Die primêre doel is om likiditeitsrisiko, meer spesifiek befondsings- en kredietkrisisrisiko, tot die minimum te beperk. In hierdie verband oorweeg die navorsers twee verwysingsprosesse, naamlik die deposito- en die residensiële verbandlening-verwysingsproses. Dit stel hulle in staat om optimale deposito-invloei, asook optimale bemarkbare sekuriteite-allokasie, te bepaal deur van aktuariële kostemetodes gebruik te maak om 'n ideale vlak van subprima-verbanduitreiking vas te stel. In hulle navorsing word verhoudings vasgestel om 'n stogastiese bankmodel in kontinue-tyd te bou ten einde 'n oplossing te bepaal vir hierdie optimale beheerprobleem wat deur 'n geometriese Brownse beweging gedryf word.

In Hoofstuk 4 word 'n belangrike aspek van die subprima-verbandkrisis, naamlik subprima-verbandontwerp, in beide 'n teoretiese en numeriese kwantitatiewe raamwerk bespreek. Sodanige ontwerp maak gebruik van reële finansiële markrentekoerse, sekuritering, strukturering en verbandprysing om die ekonomiese meganisme agter die onlangse krisis te verduidelik. Die navorsers modelleer in die

besonder verbande wat in staat is om ten volle te amortiseer, vrywillig vooraf te betaal (voorafbetaling en moontlik herfinansiering) of wan te betaal. Daar word gevind dat die verbandherfinansiering ingekort word deur hoë lening-tot-waarde-verhoudings as gevolg van 'n afname in huispryse, terwyl lae lening-tot-waarde-verhoudings huisverbandaandele verhoog. Verder word 'n optimale bankwaardasieprobleem in verband met verbanduitbreiding opgelos. In hierdie geval word optimale verbandwaarde en rentekoerse, asook wins, bereken. Die studie ondersteun die siening dat die subprima-verbandkrisis gedeeltelik veroorsaak is deur die kompleksiteit van die ontwerp van subprima-verbande wat gelei het tot inligtingsprobleme (asimmetrie, besmetting, ondoeltreffendheid en verlies), waardasie-ondeursigtigheid en ondoeltreffende risikoversagtings.

Die navorsers ondersoek 'n stogastiese model vir 'n globale likiditeitstandaard soos deur die Basel III-bankregulasie in Hoofstuk 5 voorgestel. In die besonder is die fokus op die modellering van die likiditeitdekkingsverhouding wat deur die stogastiese dinamika van die kwosiënt van hoë-gehalte likiede bates tot netto kontantuitvloei gedefinieer word. Hierdie model stel die navorsers in staat om 'n nie-lineêre optimale stogastiese likiditeitdekkingsverhouding-probleem met kwadratiese koste op te los, waar die likiditeitdekkingsverhouding een van die statistieke is wat gebruik word in verhoudingsanalise om banklikiditeit te meet. Die vernaamste nuutheid is die bekendstelling van 'n likiditeitdekkingsverhouding-verwysingsproses met betrekking tot optimale likiditeitvoorsiening en hoë-gehalte likiede batetoekenning. Die navorsers stel ook die moontlikheid bekend van 'n aanpassing aan die tempo van likiditeitvoorsiening per eenheid van die bank se netto kontantuitvloei vir die tekortkoming wat onder sekere omstandighede verwant is aan 'n bankuitbetalingskoers in 'n teoreties-kwantitatiewe raamwerk.

In Hoofstuk 6 verkry ons numeriese resultate, insluitend resultate vir die nuwe Basel III-likiditeitregulasie. Meer spesifiek, die netto stabiele befondsingsverhouding word in ooreenstemming met die voorskrifte van die voorgestelde bankregulasies bereken. In hierdie verband ondersoek die navorsers die gevolge van aandeelhouer-kontantvloeiregte op die bogenoemde befondsingsverhouding en 'n nie-Basel III-likiditeitdekkingsverhouding vir sekere ontwikkelende lande gedurende die tydperk K1:2005 tot K4:2009. Die studie bevind dat die befondsingsverhouding gedurende hirdie tydperk aan Basel III-minimum likiditeitstandaarde blyk te voldoen. Daar word afgelei dat meer gekonsentreerde kontantvloeiregte verbeterde likiditeit veroorsaak.

Sleutelwoorde: Bankuitbetalings, Basel III, Bemarkbare Sekuriteite, Deposito's, Globale Finansiële Krisis, Herfinansiering, Huisaandeel, Kredietkrisis, Likiditeitdekkingsverhouding, Likiditeitdekkingsverhouding-verwysingsproses, Likiditeitsrisiko, Likiede Bates, Netto Kontantuitvloei, Netto Stabiele Befondsingsverhouding, Pro-siklies, Subprima-verbandkrisis, Subprima-verbandversekering, Verbandbefondsing, Verbande, Verbandrentekoers, Verband-tot-Waarde Verhouding, Voorafbetaling.

Glossary

Bank bailouts are financial assistance or rescue from the government, stakeholder or other entities to prevent a financial institution from bankruptcy and total liquidation.

Basel III is a comprehensive set of rules and measures for banking regulation, supervision and risk management. The Basel Committee on Banking Supervision (BCBS) developed an international framework for risk measurements, standards and monitoring. In response to the global financial crisis, banks should always have a 30-day liquidity cover for emergency situations and meet certain capital requirements in order to minimize systemic risk that is caused by financial shocks.

Cost of mortgages is the interest cost that a bank must pay for the use of funds to originate mortgages.

Credit risk involves a bank's risk of loss from a mortgagor or special purpose vehicle that does not make scheduled payments and its securitization equivalent. This risk category generally includes counterparty risk that, in the researchers' case, is the risk that a subprime agent does not fulfil its obligations on a bond, credit derivative or insurance contract.

Deposits refer to the amount of money that is placed into a bank by a depositor and that gains interest in return. This deposited amount is a liability that is owed by the bank. In this study, deposits include both demand and time deposits. Demand deposits are the larger part of an originator's money supply which are payable immediately on request, while time deposits are money deposits which can only be withdrawn after a preset fixed time period.

Global financial crisis refers to the worldwide economic crisis which became apparent in 2007. The global financial crisis was caused by, amongst other things, the downturn in the US housing market, risky lending and borrowing practices, inaccurate credit ratings, as well as excessive individual and corporate debt levels.

House equity refers to the current market value of the house less the outstanding mortgage payments on this property.

Liquid assets refer to assets that can be converted into cash quickly with a minimal impact on the price of the assets. This means that there should be enough buyers to absorb the assets that are sold on the market. In this study, the stock of high-quality liquid assets is constituted by cash, central bank reserves, marketable securities and government/central bank debt that has been issued.

Liquidity coverage ratio (LCR) refers to the stock of high-quality liquid assets in regard to the net cash outflow over a 30-day period. This ratio measures the banking system's liquidity position that allows the assessment of a bank's capacity to ensure the coverage of some of its more immediate liabilities and that also identifies the amount of high-quality liquid assets and institution holds that can be used to offset the net cash outflows that it would encounter under a short-term stress scenario.

Liquidity coverage ratio (LCR) reference process refers to a structure of optimal control laws with respect to optimal liquidity provisioning and high-quality liquid asset (HQLA) allocation.

Liquidity risk refers to the risk of an asset that cannot be converted quickly enough to prevent a financial loss. Liquidity risk arises from situations in which a banking agent who is interested in selling (buying) residential mortgage products cannot do it, because nobody in the market wants to buy (sell) those residential mortgage products.

Loan-to-value ratio refers to the ratio of a mortgage with respect to the value of the house.

Marketable securities represent claims on originator claims that are guaranteed by sovereigns, central banks, non-central government public sector entities, the Bank for International Settlements (BIS), the International Monetary Fund (IMF), the European Commission or multilateral development banks. These claims are assigned a 0% risk-weight under the Basel II standardized approach. Also, deep repo markets should exist for these securities and they are not issued by banks or other financial service entities.

Mortgages refer to subprime mortgage loans in this study. A subprime mortgage is a mortgage that is extended to mortgagors who do not qualify for market interest rates because of their poor credit history. The term *subprime* refers to mortgagors who are less likely to repay mortgages and who do not qualify for prime interest rates; high interest rates are therefor charged. A subprime mortgage is worse from an originator's view because it is in the riskiest category of mortgages, with high default rates. In general, a subprime mortgage loan is subprime if

1. the mortgagor has a poor credit history;
2. it is extended by an originator who specializes in high-cost subprime mortgage loans;
3. it is part of a reference subprime mortgage loan portfolio which is traded on secondary markets; or
4. it is issued to a mortgagor with a prime credit history, but is a subprime-only contract type, for example a 2/28 hybrid mortgage.

Mortgage funding refers to funds that are generated from marketable securities investments and deposit inflows to fund mortgage extension.

Mortgage rate is the rate of interest on a mortgage loan.

Net cash outflow refers to the total expected cash outflow minus total expected cash inflow for the ensuing 30 calendar days. A negative net cash outflow results in debt. Cash outflows are constituted by retail deposits, unsecured wholesale funding, secured funding and additional liabilities. Cash inflows are made up of receivable amounts from retail and wholesale counterparties, receivables in respect of repo and reverse repo transactions that are backed by illiquid assets and securities lending/borrowing transactions where illiquid assets are borrowed, as well as other cash inflows.

Net stable funding ratio (NSFR) refers to a long-term ratio that measures how much stable funding a bank has to hold in order to endure a year-long liquidity crisis.

Prepayment is the act of paying a mortgage in full before it is due to be paid.

Procyclicality refers to a positive correlation between an economic indicator and the economy. In this case, the researchers study the relationship between credit ratings, as well as mortgage losses and subprime mortgage insurance premium rates that are procyclical.

Refinancing: A refinancing mortgage undergoes a revision of its payment schedule or involves the replacement of an older mortgage by a new mortgage, offering better terms.

Subprime mortgage crisis refers to the ongoing housing and financial crisis during 2007 and 2008 that has been characterized by major losses due to the exposure to subprime financial products. This crisis was triggered by the downturn in the US housing market, inaccurate credit ratings, excessive individual and corporate debt levels, marked increase in mortgage delinquencies as well as foreclosures in the US. It has had major adverse consequences for banks and financial markets around the globe.

Subprime mortgage insurance is insurance on credit default swaps and other subprime mortgage products that are sold by monoline insurers. In case of a default, the monoline insurer will pay out the value of the loss that has been incurred. The problem was that insurers assigned contracts to other insurers to mitigate default risk to other parties, as seen during the subprime mortgage crisis.

Abbreviations

ARM - Adjustable-rate mortgage
ASF - Available stable funding
BCBS - Basel Committee on Banking Supervision
BIS - Bank for International Settlements
CDO - Collateralized debt obligation
CDS - Credit default swap
CoVAR - Covariance
DSGE - Dynamic stochastic general equilibrium
ECAI - External Credit Assessment Institution
GDP - Gross domestic product
HJBE - Hamilton-Jacobi-Bellman equation
HQLA - High-quality liquid asset
IMF - International Monetary Fund
L1A - Level 1 asset
L2A - Level 2 asset
LCR - Liquidity coverage ratio
LIBOR - London interbank offered rate
LTVR - Loan-to-value ratio
NCO - Net cash outflow
NSFR - Net stable funding ratio
RMBS - Residential mortgage-backed security
RSF - Required stable funding

SCFR - Shareholder cash-flow rights

SMC - Subprime mortgage crisis

SMI - Subprime mortgage insurance

T1K - Tier 1 capital

T2K - Tier 2 capital

VaR - Value at risk

VECM - Vector error correction model

Basic notations

B - Marketable securities

B^* - Optimal marketable securities

$\widetilde{B}$ - Risky marketable securities

B - Borrowings

b - Borel-measurable function

b^i - Total earnings of the i-th marketable securities class

b^r - Cost function

C - Insurer payments via the protection leg

$\mathcal{C}$ - Credit rating

$C(S(\mathcal{C}))$ - Subprime mortgage insurance protection leg payments

c - Deposit inflow rate

c^* - Optimal deposit inflow rate

c^B - Costs for holding marketable securities

c^{B} - Cost of borrowing

c^D - Marginal cost of deposits

c^{dw} - Deadweight cost of total capital

c^l - Cost of liquidation

$\bar{c}^{M\omega}$ - Average weighted cost of loans

c^p - Prepayment cost

D - Deposits

D (Ch 5) - Differential operator

D^a - Additional deposits

D^r - Deposit reference process

d - Number of dividends

ΔF - Depreciation of fixed assets

δ - Stochastic discount factor

E - Equity

$\mathbf{E}$ - Conditional expectation

E^c - Common equity

E^p - Preferred equity

E^r - Retained earnings
e (Ch 3) - Volatility of stipulated levels of subprime mortgages
e (Ch 5) - Outflows per monetary unit of the bank's net cash outflows
F - Fixed assets
$\mathcal{F}$ - Right continuous filtration
$F(\cdot)$ - Cumulative distribution of the shock to mortgages
f - Probability distribution
f^f - Fractions of mortgages that refinance
f^M - Fraction of the face value of an originator's subprime mortgages
f^S - Fractions of mortgages that default
$f(u)$ - Probability density function
$\mathcal{G}$ - Class of admissible control laws
g - Control law
g^* - Optimal control law
Γ - Subprime mortgage mass
γ - Factor
H - House prices
h - Investment return on bank's high-quality liquid assets
i - Increase of net cash outflow before outflows per monetary unit of net cash outflow
J (Ch 4) - Originator's performance criterion
J (Ch 5) - Cost function
J^* - Optimal cost function
K - Bank capital
k - Constant
κ - Weighting factor
L - Loan-to-value ratio
L^* - Optimal loan-to-value ratio
l (Ch 3) - Mortgage origination rate
l (Ch 4) - Lagrangian multiplier
l (Ch 5) - Liquidity coverage ratio
l^r - Liquidity coverage ratio reference process
Λ - Market value of all loans
M - Subprime mortgages
M^* - Optimal subprime mortgages
M^r - Mortgage reference process
M^u - Unfunded subprime mortgages
N - Net cash flow
n - Number of originator shares
O - Subordinate debt
ω^B - Risk weights related to the originator's risky marketable securities
$\omega(\mathcal{C})$ - Risk weights related to the originator's mortgages at face value

P - Distribution function

$\mathbf{P}$ - Probability

$P^{\widehat{\mathrm{T}}}$ - Cost of treasuries

P^{T} - Provisions against deposit withdrawals

P_v - Percentage of subprime mortgages to be originated

p - Density function

$p^i(\mathcal{C})$ - Subprime mortgage insurance premium rate

Π - Profit

Π^p - Present value of future profits from additional loans, based on current loans

π (Ch 3) - Marketable securities allocation

π (Ch 4) - Probability

π (Ch 5) - Investment strategy

$\widetilde{\pi}$ - Marketable securities allocation strategy

$\widetilde{\pi}^*$ - Optimal marketable securities allocation strategy

π^{k*} - Optimal high-quality liquid asset allocation

q - Riccati equation

R - Recovery amount

R - Return on reserves

r^a - Rate of actualization

r^B - Rate of return from marketable securities

$r^{\tilde{B}}$ - Rate of return from risky marketable securities

r^{B} - Borrower rate

r^D - Rate of return from deposits

r^d - Discounted rate of interest

r^e - Rate of outflows per monetary unit of the bank's net cash outflows

r^f - Forecasting rate of net cash outflow

r^h - Rate of return on high-quality liquid assets

r^i - Rate of net cash outflows increase before outflows

r^L - London interbank borrowing rate

r^Λ - Loan rate

r^M - Mortgage rate

r^{M*} - Optimal mortgage rate

r^O - Subordinate debt rate

r^p - Penalty rate

r^ψ - Market-based step-up rate

r^R - Recovery rate

r^{T} - Rate of return from treasuries

r^ε - Teaser rates

r^y - Rate of asset returns

ρ - Risk premium

S - Mortgage losses

σ^e - Volatility in the change per net cash outflow unit
σ^h - Volatility in the rate of high-quality liquid assets returns
σ^i - Volatility in the net cash outflow increase before outflow
σ^Λ - A random exogenous shock to the demand for loans
σ^M - Random shock to mortgages
Σ^y - Matrix of high-quality liquid asset returns
T - Riskless assets (treasuries)
t_0 - Beginning of the period
t_1 - End of the period
u - Unanticipated withdrawals
u^1 - Normal rate of liquidity provisioning
u^2 - Adjustment to the rate of liquidity provisioning
u^{2*} - Optimal bailout rate
u^3 - Liquidity provisioning rate
V - Objective function
V - Value function
V^* - Optimal objective function
v - Arbitrary instant of time
ϱ - Risk premium
W - Standard Brownian motion
x (Ch 4)- Deposit withdrawals
x (Ch 5) -Liquidity coverage ratio
x^1 - Dynamics of high-quality liquid asset
x^2 - Dynamics of net cash outflow
x^r - Liquidity coverage ratio reference process
$\widetilde{\xi}$ - Market price of risk
y - Return per high-quality liquid asset unit
y^k - High-quality liquid asset returns in the k-th asset class per unit of the k-th class

List of figures

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Co-authors and contributions

Co-authors and the researcher's contributions:

1. **Type:** Article 1

   Title: An overview of Basel III and liquidity

   Main author: De Waal B

   Authors: De Waal B, Petersen MA, Hlatshwayo LNP & Mukuddem-Petersen J

   Journal: Bulletin of economic research

   doi: Submitted

   My contribution: Lead author

2. **Type:** Article 2

   Title: Subprime mortgage funding and liquidity risk

   Main author: Petersen MA

   Authors: Petersen MA, De Waal B, Mukuddem-Petersen J & Mulaudzi MP

   Journal: Quantitative finance

   doi: 10.1080/14697688.2011.637076, 2012

   My contribution: Joint lead author

3. **Type:** Article 3

   Title: Optimal originator valuation and the global financial crisis

   Main author: Petersen MA

   Authors: Petersen MA, Mukuddem-Petersen J, Thomas S & De Waal B

   Journal: Optimal control applications and methods

   doi: 10.1002/oca.2022, appeared electronically in 2012

   My contribution: Joint lead author

4. **Type:** Article 4

   Title: Liquidity coverage ratios

Main author: Petersen MA

Authors: Petersen MA, De Waal B, Mukuddem-Petersen J & Hlatshwayo LNP

Journal: Mathematical finance

doi: Submitted

My contribution: Joint lead author

5. **Type:** Article 5

   Title: A note on Basel III and liquidity

   Main author: De Waal B

   Authors: De Waal B, Petersen MA, Hlatshwayo LNP & Mukuddem-Petersen J

   Journal: Applied economic letters

   doi: 10.1080/13504851.2012.744130, accepted

   My contribution: Lead author

Supported letters from the editors

07-Oct-2012

Dear Mrs. de Waal

Your manuscript entitled "An Overview of Basel III and Liquidity" by de Waal, Bernadine; Petersen, Mark; Hlatshwayo, Lungile; Mukudden-Petersen, Janine, has been successfully submitted online and is presently being given full consideration for publication in the Bulletin of Economic Research.

Co-authors: Please contact the Editorial Office as soon as possible if you disagree with being listed as a co-author for this manuscript.

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Thank you for submitting your manuscript to the Bulletin of Economic Research.

Sincerely,
Bulletin of Economic Research Editorial Office

Figure 1: Figure An overview of Basel III and liquidity

13-Oct-2011

Dear Miss De Waal,

It is a pleasure to accept your manuscript entitled "Subprime Residential Mortgage Funding and Liquidity Risk" for publication in Quantitative Finance. This is a conditional accept and the Editors ask that you cover any outstanding comments from the reviewer(s), included here for your reference, before sending through the final files for publication.

Could you now please send to me the final version of your paper in either Word or Latex form? If sending in Latex form, please do also provide a PDF version. If your paper contains any figures please also send them to me as EPS files in black and white and, if available, in colour as well. Once I have these, your paper shall be forwarded to our production department who will be your main point of contact from then on. Please feel free, however, to contact me if I can be of any further assistance.

Thank you for your fine contribution. On behalf of the Editors of Quantitative Finance, we look forward to your continued contributions to the Journal.

Sincerely,
Samantha Hutton
Publishing Editor, Quantitative Finance
quant@tandf.co.uk

Figure 2: Figure Subprime mortgage funding and liquidity risk

From: <OCAproofs@wiley.com>
To: <Mark.Petersen@nwu.ac.za>
Cc:
Date: Tue, 31 Jan 2012 09:40:00 -0000
Subject: Status update to your article in Optimal Control Applications and Methods - OCA-2022
Dear Prof Dr Mark Petersen,

Journal: Optimal Control Applications and Methods

Article title: "Optimal Originator Valuation and the Global Financial Crisis"

Your article is currently at the following stage of production:

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To view the status of your article online please visit: http://authorservices.wiley.com/bauthor/default.asp?email=Mark.Petersen@nwu.ac.za and log in using your username (e-mail address) and password.

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Figure 3: Figure Optimal originator valuation and the global financial crisis

From: dbm@rhsmith.umd.edu
To: Mark.Petersen@nwu.ac.za
Cc:
Date: Thu, 19 Jul 2012 09:19:01 -0400 (EDT)
Subject: Mathematical Finance - Manuscript ID MAFI-2012-0148
19-Jul-2012

Dear Prof. Petersen:

Your manuscript entitled "Liquidity Coverage Ratios" has been successfully submitted online and is presently being given full consideration for publication in Mathematical Finance.

Your manuscript ID is MAFI-2012-0148.

Please mention the above manuscript ID in all future correspondence or when calling the office for questions. If there are any changes in your street address or e-mail address, please log in to Manuscript Central at http://mc.manuscriptcentral.com/mafi and edit your user information as appropriate.

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Figure 4: Figure Liquidity coverage ratios

AppliedEconomics@warwick.ac.uk

<AppliedEconomics@warwick.ac.uk> Tue, Oct 23, 2012 at 1:48 PM To: dinkedewaal@gmail.com

Reply | Reply to all | Forward | Print | Delete | Show original

Dear Professor De Waal

AEL-2012-0523
A Note on Basel III and Liquidity

I am pleased to accept your manuscript for publication in Applied Economics Letters.

Applied Economics Letters asks the opinion of two referees who are experts in the relevant field of research. The paper is also read by one of the editors. If both of the referees and an editor concur in their view, their decision is final. We consult a third referee if there is a difference of opinion. In order to expedite the proceedings, which is one of the objectives of a letters journal, we do not require a full report on the paper from the referees. The choice of referees and the fact that we require a consensus view between the referees and an editor ensures, however, that the process is as fair as possible.

If your contact details are different to those in the accepted manuscript, please contact Sophie Shanahan with the details.

Sophie.Shanahan@tandf.co.uk

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This e-mail message is an official confirmation that your article has been accepted for publication. It is also the only confirmation of the acceptance of your paper that you will receive. In particular, we are unable to provide hard copies of acceptance letters by mail or fax and, because of the pressures on the time of our staff, we cannot enter into any further correspondence on the issue of formal acceptance letters.

Thank you again for submitting your article. It is a very fine piece of work and we look forward to seeing it in print.

Kind regards

Mark Taylor

Editor
Applied Economics Letters

Figure 5: Figure A note on Basel III and liquidity

Co-author permission letters

NORTH-WEST UNIVERSITY
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2012-10-08

Ms. Bernadine de Waal
PhD Economics Student
Department of Economics
North-West University (Mafikeng Campus)

Dear Prof. Mark Petersen

Intention: Request for permission

Request for permission to include the following papers, of which you were a co-author, in my doctoral thesis to be submitted in partial fulfillment of the requirements for the degree Philosophiae Doctor in Economics at the Mafikeng Campus of the North West University.

"An Overview of Basel III and Liquidity", Bulletin of Economic Research.

"A note on Basel III and liquidity", Applied Economic Letters.

"Optimal Originator Valuation and the Global Financial Crisis", Optimal Control Applications and Methods.

"Liquidity coverage ratios", Mathematical Finance.

"Subprime mortgage funding and liquidity risk", Quantitative Finance.

Please do not hesitate to contact me if you have any queries.

Yours sincerely,

Bernadine de Waal (Student Number: 20230257)

MPetersen

Prof. Mark A. Petersen (Supervisor)

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2012-10-08

Ms. Bernadine de Waal
PhD Economics Student
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Dear Prof. Janine Mukuddem-Petersen

Intention: Request for permission

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Yours sincerely,

Bernadine de Waal (Student Number: 20230257)

Prof. Janine Mukuddem-Petersen (Co-Supervisor)

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2012-10-08

Ms. Bernadine de Waal
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Dear Dr. Soby Thomas

Intention: Request for permission

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"Optimal Originator Valuation and the Global Financial Crisis", Optimal Control Applications and Methods.

Please do not hesitate to contact me if you have any queries.

Yours sincerely,

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Dr. Soby Thomas

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2012-10-08

Ms. Bernadine de Waal
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Dear Dr. Mmboniseni Mulaudzi

Intention: Request for permission

Request for permission to include the following paper, of which you were a co-author, in my doctoral thesis to be submitted in partial fulfilment of the requirements for the degree Philosophiae Doctor in Economics at the Mafikeng Campus of the North West University.

"Subprime mortgage funding and liquidity risk", Quantitative Finance.

Please do not hesitate to contact me if you have any queries.

Yours sincerely,

Bernadine de Waal (Student Number: 20230257)

Mulaudzi MP

Dr. Mmboniseni Mulaudzi

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2012-10-08

Ms. Bernadine de Waal
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Dear Ms. Lungile Hlatshwayo

Intention: Request for permission

Request for permission to include the following papers, of which you were a co-author, in my doctoral thesis to be submitted in partial fulfillment of the requirements for the degree Philosophiae Doctor in Economics at the Mafikeng Campus of the North West University.

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"A note on Basel III and liquidity", Applied Economic Letters.

"Liquidity coverage ratios", Mathematical Finance.

Please do not hesitate to contact me if you have any queries.

Yours sincerely,

Bernadine de Waal (Student Number: 20230257)

Hlatshwayo

Ms. Lungile Hlatshwayo

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[1] De Waal B., Petersen M.A., Hlatshwayo L.N.P., Mukuddem-Petersen J. (2012). An Overview of Basel III and Liquidity. *Bulletin of Economic Research.* Submitted.

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[5] Petersen M.A., De Waal B., Mukuddem-Petersen J., Mulaudzi M.P. (2012). Subprime mortgage funding and liquidity risk. *Quantitative Finance.* doi: 10.1080/14697688.2011.637076. Appeared electronically in 2012.

Contents

Chapter 1

INTRODUCTION

> As calamitous as the subprime blowup seems, it is only the beginning. The credit bubble spawned abuses throughout the system. Subprime lending just happened to be the most egregious of the lot, and thus the first to have the cockroaches scurrying out in plain view. The housing market will collapse. New-home construction will collapse. Consumer pocketbooks will be pinched. The consumer spending binge will be over. The US economy will enter a recession.
>
> – Eric Sprott (Sprott Asset Management), 2007.

> These days America is looking like the Bernie Madoff of economies: For many years it was held in respect, even awe, but it turns out to have been a fraud all along.
>
> – Prof Paul Krugman (2008 Nobel Memorial Prize Laureate in Economic Sciences, Princeton University, US), 2009.

During the global financial crisis, banks were under severe pressure to maintain adequate liquidity. In general, empirical evidence shows that banks with sufficient liquidity can meet their payment obligations, while banks with low liquidity are not able to do that. The global financial crisis highlighted the fact that liquidity risk can proliferate quickly with funding sources dissipating and with concerns about asset valuation, capital adequacy realizing and subprime mortgage design which involves subprime mortgage extension, prepayment and refinancing, credit risk mitigation, subprime mortgage insurance, house prices, house equity and loan-to-value ratio, as well as procyclicality. Such subprime design utilizes real financial market interest rates, securitization structuring and mortgage pricing to explain the economic mechanism behind the recent crisis. In this regard, important issues have to be accountant for, such as moral hazard in expanding mortgage portfolios, incomplete information among market players about their counterparties, myopia in decision making in the subprime mortgage market and monetary policy incentives that are boosting the growth of the subprime market.

During this period, large amounts of deposits flowed into the US banking system which funded mortgages in a low interest rate environment. Consequently, credit was easy to obtain, thus boosting housing and credit markets. Originators passed on mortgages via securitization to shift credit risk to investors in risky marketable securities such as residential mortgage-backed securities (RMBSs) and collateralized debt obligations (CDOs). As a consequence, mortgage innovation rose dramatically. The proceeds of mortgage origination were invested in marketable securities for higher yields than earnings on treasuries. In turn, the funds from deposits and marketable securities were then used for funding new subprime mortgages. The sharp rise in delinquencies and foreclosures of subprime mortgages had a major adverse impact on the liquidity of financial institutions and markets around the globe.

The three recent financial crises, namely the subprime mortgage crises, global financial crisis and sovereign debt crisis, were also characterized by too-big-to-fail institutions that suffered from a lack of high-quality liquid assets (HQLAs). The realization of such liquidity risk led to credit crunches and had deleterious effects on financial markets globally.

This situation underscores the important relationship between funding risk (involving the raising of funds to bankroll asset holdings) and market liquidity (involving the efficient conversion of assets into liquid funds at a given price). During these financial crises, banks experienced difficulties due to lapses in principles of liquidity risk management. In response to this, the Basel Committee on Banking Supervision (BCBS) published banking regulations (see [4]) in 2008 – commonly known as "Sound Principles" – that provided detailed guidance on risk management and supervision of funding liquidity risk. Subsequently, [3] proposed via Basel III regulation that banks should always have liquidity cover for stress scenarios. Although pre-Basel III regulation establishes procedures for assessing credit, market and operational risk, it does not provide effective protocols for managing liquidity and systemic risks. The drafting of Basel III represents an effort to address the latter (see, for instance, [2], [1] and [4]). The net stable funding ratio (NSFR) is mandated by proposed Basel III regulation as a measure of bank capital stability (see, for instance, [3] and [4]).

In response to this, among other things, the Basel Committee on Banking Supervision (BCBS) is proposing that banks should always have a 30-day liquidity cover for stress scenarios. As far as Basel III liquidity proposals are concerned, the BCBS is suggesting a liquidity coverage ratio (LCR) that is defined as

$$\mathrm{LCR} = \frac{\text{High-quality liquid assets (HQLAs)}}{\text{30-day net cash outflows (NCOs)}} \geq 1.$$

These ratios measure the banking system's liquidity position that allows the assessment of a bank's capacity to ensure the coverage of some of its more immediate liabilities with highly available assets and also identifies the amount of unencumbered, high-quality liquid assets and institution holds that can be used to offset the net cash outflows it would encounter under a short-term stress scenario, specified by supervisors, including both specific and systemic shocks.

1.1 Methods

According to Protter [6], **stochastic calculus** is a branch of mathematics that operates on stochastic processes. It allows a consistent theory of integration to be defined for integrals of stochastic processes with respect to stochastic processes. It is used to model systems that behave randomly. A stochastic process, or sometimes random process, is a collection of random variables; it is often used to represent the evolution of some random value or system over time. This is the probabilistic counterpart to a deterministic process (or deterministic system). Instead of describing a process which can only evolve in one way (as in the case, for example, of solutions of an ordinary differential equation), in a stochastic or random process, there is some indeterminacy; even if the initial condition (or starting point) is known, there are several (often infinitely many) directions in which the process may evolve.

Time can be discrete, for example $t = 1, 2, 3, \ldots$, or continuous, $t \geq 0$. Stochastic calculus deals with functions of time t, $0 \leq t \leq T$. Calculus is more suited for continuous time processes. At any time t, the observation is described by a random variable which is denoted by X_t or $X(t)$. A stochastic process $\{X(t)\}$ is frequently denoted by X or, with a slight abuse of notation, also by $X(t)$ (see also [5] for more information on stochastic calculus).

A function g is called continuous at the point $t = t_0$ if the increment of g over small intervals is small:

$$\Delta g(t) = g(t) - g(t_0) \to 0 \text{ as } \Delta t - t_0 \to 0.$$

If g is continuous at every point of its domain of definition, it is simply called continuous. g is called differentiable at the point $t = t_0$ if, at that point,

$$\lim_{\Delta t \to 0} \frac{\Delta g(t)}{\Delta t} = C.$$

This constant C is denoted by $g'(t_0)$. If g is differentiable at every point of its domain, it is called differentiable.

In 1828, botanist R. Brown described the **Brownian motion** of a pollen particle that is suspended in fluid. It was observed that a particle moved in an irregular, random fashion. Albert Einstein, in 1905, argued that the movement is due to bombardment of the particle by the molecules of the fluid, and obtained the equations for Brownian motion. In 1900, L Bachelier used the Brownian motion as a model for movement of stock prices in his mathematical theory of speculation. The mathematical foundation for Brownian motion as a stochastic process was laid by N Wiener in 1931, and this process is also called the Wiener process. The Brownian motion process $B(t)$ serves as a basic model for the cumulative effect of pure noise. If $B(t)$ denotes the position of a particle at time t, then the displacement $B(t) - B(0)$ is the effect of the purely random bombardment by the molecules of the fluid, or the effect of noise over time t.

The Brownian motion $\{B(t)\}$ is a stochastic process with the following properties:

1. (Independence of increments) $B(t) - B(s)$, for $t > s$ is independent of the past, that is of $B_u,\ 0 \leq u \leq s$, or of $\mathtt{F_s}$, the θ field generated by $B(u),\ u \leq s$.
2. (Normal increments) $B(t) - B(s)$ has a normal distribution with mean 0 and variance $t - s$. This implies (taking $s = 0$) that $B(t) - B(0)$ has $N(0, t)$ distribution.
3. (Continuity of paths) $B(t),\ t \geq 0$ are continuous functions of date t.

Stochastic differential equations: If $x(t)$ is a differentiable function that is defined for $t \geq 0$, $u(x, t)$ is a function of x and t and the following relation is satisfied for all $t,\ 0 \leq t \leq T$

$$\frac{dx(t)}{dt} = x'(t) = u(x(t), t) \text{ and } x(0) = x_0,$$

then $x(t)$ is a solution of the ordinary differential equation with the initial condition x_0. Usually the requirement that $x'(t)$ is continuous is added. The above equation can be written in other forms:

$$dx(t) = u(x(t), t)dt$$

and (by continuity of $x'(t)$)

$$x(t) = x(0) + \int_0^t u(x(s), s)ds.$$

1.2 Research questions and outline of the thesis

In this section, the main research questions are stated and an outline of the thesis is provided.

1.2.1 Research questions

The main aim of this research was to develop optimal banking models and to consider their relationships with financial crises. In particular, this modelling aims to improve the understanding of these subprime activities. An additional objective is to use the principles of stochastic control and other methods to implement optimal preventative strategies that offset the deleterious effects of this subprime design and risk mitigation.

Question a (Quantitative methods involving Basel III liquidity regulation) What quantitative methods can be used to study Basel III liquidity standards? (See Section 2.2, Chapter 2.)

Question b (Potential benefits of Basel III liquidity regulation) What are the potential benefits that are associated with Basel III liquidity proposals in terms of liquidity standards, crisis prevention, financial markets and institutions, liquidity risk, as well as the broader economy? (See Section 2.3, Chapter 2.)

Question c (Potential challenges of Basel III liquidity regulation) In terms of liquidity standards, crisis prevention, financial markets and institutions, liquidity risk as well as the broader economy, what are the potential challenges that are associated with Basel III liquidity proposals? (See Section 2.4, Chapter 2.)

Question d (An optimal originator liquidity risk management question) What should the optimal levels of originator deposit inflows and marketable security returns be in order to reach the optimal specified level for mortgage origination via the financing spread method? (See Section 3.3.2, Chapter 3.)

Question e (Optimal liquidity and the subprime mortgage crisis) What is the connection between the solution to Question d and the subprime mortgage crisis? (See Section 3.3, Chapter 3.)

Question f (Subprime mortgage design) Can subprime mortgages be designed that are able to fully amortize, voluntarily prepay or involuntarily prepay (default)? (See Section 4.2, Chapter 4.)

Question g (Modelling of house equity) Can house equity via loan-to-value ratios be modelled in a subprime framework? (See Section 4.2, Chapter 4.)

Question h (Modelling of subprime risk and profit) Can a discrete-time model be constructed for subprime risk and profit, incorporating costs of funds and subprime mortgage insurance, as well as mortgage losses? (See Section 4.3.1, Chapter 4.)

Question i (Optimal originator valuation problem under subprime mortgages) In order to obtain an optimal originator valuation with mortgages at face value, which decisions regarding mortgage rates, deposits and treasuries must be made? (See Theorem 4.3.8 in Subsection 4.3.2.2, Chapter 4.)

Question j (Liquidity and financial crises) What does empirical data reveal about the relationship between liquidity and financial crises? (See Section 5.2, Chapter 5.)

Question k (Bank liquidity coverage ratio modelling) How can bank liquidity coverage ratios via stochastic means be modelled? (See Section 5.2, Chapter 5.)

Question l (Bank liquidity coverage ratio optimization) Which decisions about the bailout rate and high-quality liquid asset allocation should be made in order to attain an optimal liquidity coverage ratio? (See Theorem 5.3.3 in Section 5.3, Chapter 5.)

Question m (Bank liquidity coverage ratio reference processes) How a reference processes for liquidity coverage ratios be determined? (See Section 5.3.2, Chapter 5.)

Question n (Influence of shareholder cash-flow rights on Basel III liquidity) How does shareholder cash-flow rights influence bank liquidity in the proposed Basel III paradigm? (See Section 6.3, Chapter 6.)

1.2.2 Outline of the thesis

The current chapter is introductory in nature. The literature review follows to give important literature that is related to this study (see Chapter 2). The remaining chapters of this thesis are structured as follows:

The thesis describes the most important aspects of the SMC, namely liquidity and mortgage funding (see Chapter 3), subprime mortgage design and originator valuation (see Chapter 4), liquidity coverage ratios (see Chapter 5) and Basel III liquidity regulation (see Chapter 6). Chapter 7 contains concluding remarks and highlights some possible topics for future research.

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Chapter 2

LITERATURE REVIEW

Type:	Article 1
Title:	An overview of Basel III and liquidity
Main Author:	De Waal B
Authors:	De Waal B, Petersen MA, Hlatshwayo LNP and Mukuddem-Petersen J
Journal:	Bulletin of economic research
doi:	Submitted

An overview of Basel III and liquidity

By

Bernadine De Waal [1], Mark A. Petersen [2], Lungile NP Hlatshwayo [3] and Janine Mukuddem-Petersen [4]

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Abstract: Some financial experts have blamed deficiencies in Basel II capital regulation for the exacerbation of financial crises in 2007 to 2012. In response to this, new proposals in the form of Basel III attempt to raise the quality, consistency and transparency of the regulatory capital base in order to enhance the risk coverage of Basel II via appropriate ratios. In this article, the authors explore how the new Basel liquidity standards, as encapsulated by the liquid coverage ratio (LCR) and net stable funding ratio (NSFR), could be effectively implemented in mitigating liquidity problems. In order to partially accomplish this, they consider a solution paradigm for proposed Basel III liquidity regulation. The authors also consider the potential benefits and challenges of this regulation in terms of liquidity standards, crisis prevention, financial markets and institutions, liquidity and other risks, as well as the broader economy. For instance, conclusions are that higher LCRs mitigate liquidity risk and the NSFR limits an over-reliance on short-term wholesale funding. However, their overall impression is that Basel III has not fully addressed the factors that were responsible for the crises and the fundamental problems with previous Basel regulation.

Key Words: Basel III, Liquidity.

JEL: G13; G32.

Glossary of Basel III liquidity abbreviations and definitions

In this glossary, abbreviations and definitions for Basel III liquidity are represented.

Basel Committee on Banking Supervision (BCBS): The BCBS consists of senior representatives of bank supervisory authorities and central banks. It usually meets at the Bank for International Settlements (BIS) in Basel, Switzerland, where its permanent Secretariat is located.

Credit default swap (CDS): A CDS is a financial swap agreement that the seller of the CDS will compensate the buyer in the event of a loan default or other credit event. The buyer of the CDS makes a series of payments (the CDS 'fee' or 'spread') to the seller and, in exchange, receives a payoff if the loan defaults.

Dynamic stochastic general equilibrium (DSGE): DSGE modelling is a branch of applied general equilibrium theory that is influential in contemporary macroeconomics. The DSGE methodology attempts to explain aggregate economic phenomena, such as economic growth, business cycles and the effects of monetary and fiscal policy on the basis of macroeconomic models that are derived from microeconomic principles.

External Credit Assessment Institution (ECAI): An ECAI must satisfy each of the following six criteria: *Objectivity:* The methodology for assigning credit assessments must be rigorous, systematic and subject to some form of validation that is based on historical experience. *Independence:*

An ECAI should be independent and should not be subject to political or economic pressures that may influence the rating. *International access/Transparency:* The individual assessments should be available to both domestic and foreign institutions with legitimate interests and at equivalent terms. In addition, the general methodology that is used by the ECAI should be publicly available. *Disclosure:* An ECAI should disclose the following information: its assessment methodologies, including the definition of default, the time horizon and the meaning of each rating, the actual default rates that are experienced in each assessment category and the transitions of the assessments, for example the likelihood of AA ratings becoming A over time. *Resources:* An ECAI should have sufficient resources to carry out high-quality credit assessments. These resources should allow for substantial ongoing contact with senior and operational levels within the entities that are assessed in order to add value to the credit assessments. *Credibility:* To some extent, credibility is derived from the criteria above. In addition, the reliance on an ECAI's external credit assessments by independent parties (investors, insurers, trading partners) is evidence of the credibility of the assessments of an ECAI.

High-quality liquid asset (HQLA): In order to qualify as an HQLA, assets should satisfy the following characteristics: *Low credit and market risk:* These assets tend to have higher liquidity. Low credit and market risk lower duration, volatility, inflation risk and foreign exchange risk, thus enhancing the asset's liquidity. *Ease and certainty of valuation:* The asset price calculation formula of HQLAs should be simple and not depend on strong assumptions. This increases the asset's liquidity among the market participants. *Low correlation with risky assets; Listed on a Developed and Recognized Exchange Market:* Being listed increases asset transparency. *Eligible by central bank for intraday liquidity needs:* Central bank eligibility increases a bank's confidence in holding the asset that could be used during stress scenarios. *An unencumbered asset:* Assets should not be pledged as collateral, apart from some exceptions.

Incremental risk charge: The incremental risk charge set forth below is intended to address a number of perceived shortcomings in the current 99%/10-day value-at-risk (VaR) framework. Foremost, the current VaR framework ignores differences in the underlying liquidity of trading book positions. In addition, these VaR calculations are typically based on a 99%/one-day VaR which is scaled up to 10 days. Consequently, the VaR capital charge may not fully reflect large daily losses that occur less frequently than two to three times per year, as well as the potential for large cumulative price movements over periods of several weeks or months.

Level 1 asset (L1A): L1As include cash, central bank reserves and marketable securities that are issued by entities which are assigned a 0% risk weight under the Basel II standardized approach; financial institutions and their affiliated entities are excluded from this category. However, non-0% risk-weighted sovereigns and central banks are included if the securities are issued in the currency where the bank incurs liquidity risk. Marketable securities must meet criteria of HQLAs, as exposed in BCBS documents. Among criteria that are relevant for investment banks, these securities have to be listed on a developed and recognized exchange market, and when there is flight to quality, there is an extensive demand on them (see, for instance, [14]).

Level 2 asset (L2A): L2As are included in the HQLAs up to two-thirds of the total amount. In addition, a haircut of at least 15% is applied to the current market value of the securities that are included in this category. Marketable securities that are issued by an entity which was assigned a 20% risk-weight under Basel II can be incorporated here; financial institutions are excluded from these entities. The price of these securities must not have declined more than 10% over 30 days during a liquidity stress period. Additional marketable securities can be included if they are issued by an entity that is rated at least AA and not issued by a financial institution (including the bank itself) or one of its affiliates, and the price remains stable during liquidity stress (not declining by more than 10% in the same conditions of stress - see, for instance, [14]).

Liquidity: The capability of banks to fund increases in assets and meet obligations as they come due, without incurring unacceptable losses.

Liquidity coverage ratio (LCR): The LCR is meant to ensure that the firm has an adequate amount of unencumbered HQLAs to envelop a short-term crisis, based on a predetermined set of net cash inflows and outflows that are recognized by the BCBS.

Management information system: A system that is maintained by the bank in order to manage information.

Minimum reserves requirement: The minimum reserve requirement or cash reserve ratio is a central bank regulation that sets the minimum reserves of customer deposits and notes that each commercial bank must hold (rather than lend out). It is normally in the form of cash that is stored physically in a bank vault (vault cash) or central bank deposits.

Net cash outflow (NCO): The total expected cash outflow minus total expected cash inflow for the ensuing 30 calendar days.

Net stable funding ratio (NSFR): A long-term ratio that measures how much stable funding a bank has to hold in order to endure a year-long liquidity crisis.

Off-balance sheet: An off-balance sheet item usually means an asset, debt or financing activity that is not on the bank's balance sheet. In the financial statements of banks, off-balance sheet volumes are listed in three summary lines of acceptance, documentary credits and guarantees. Here, acceptances (reported off-balance sheet items) are the total amount that the bank accepts to pay, usually under international trade finance arrangements where reported off-balance sheet. Finally, guarantees are the total amount that is guaranteed by the bank.

Public sector entity: The key characteristics of public sector entities are public accountability, multiple objectives, rights, powers and responsibilities (constitutional or devolved), lack of equity ownership, operating and financial frameworks that are set by legislation, the importance of the budget, governance structures, nature of resources and non-exchange transactions.

Required stable funding (RSF): The sum of the value of the assets that are held and funded by the institution, multiplied by a specific required stable funding factor that is assigned to each particular asset type.

Tier 1 capital (T1K): T1K consists mainly of common shares and retained earnings. This standard is reinforced through a set of principles that can also be tailored to the context of non-joint stock companies to ensure that they hold comparable levels of high-quality T1K. Deductions from capital and prudential filters have been harmonized internationally and generally applied at the level of common equity or its equivalent in the case of non-joint stock companies. The remainder of the T1K base must be comprised of instruments that are subordinated, have fully discretionary noncumulative dividends or coupons and have neither a maturity date nor an incentive to redeem. Innovative hybrid capital instruments with an incentive to redeem through features such as step-up clauses, currently limited to 15% of the T1K base, will be phased out.

Tier 2 capital (T2K): T2K consists of the sum of the following elements: instruments issued by the bank that meet the criteria for inclusion in T2K (and are not included in T1K), stock surplus (share premium), resulting from the issue of instruments that are included in T2K, instruments that are issued by consolidated subsidiaries of the bank and are held by third parties that meet the criteria for inclusion in T2K and are not included in T1K, and certain loan loss provisions and regulatory adjustments that are applied in the calculation of T2K.

Value at Risk (VaR): VaR is a widely used risk measure of the risk of loss on a specific portfolio of financial assets. For a given portfolio, probability and time horizon, VaR is defined as a threshold value such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value (assuming normal markets and no trading in the portfolio) is the given probability level.

Vector error correction model (VECM): VECM can lead to a better understanding of the nature of any non-stationarity among the different component series and can also improve longer-term forecasting over an unconstrained model.

2.1 Introduction

The financial crisis that began in mid 2007 re-emphasized the importance of liquidity to financial market and banking sector functioning. Prior to the turmoil, financial markets were buoyant and funding was readily available at low cost. The reversal in market conditions demonstrated how quickly liquidity can evaporate and that illiquidity can last for an extended period of time. The banking system came under severe stress, which necessitated central bank action to support both the functioning of money markets and, in a few cases, individual institutions.

Because banks are at the centre of the credit intermediation process, a robust banking sector is needed for sustainable economic growth (see, for instance, [5]). On 12 September 2010, the Basel Committee on Banking Supervision (BCBS) announced a strengthening of existing rules (see, for instance, [37]). In this regard, Basel III is a new regulatory standard on bank capital adequacy, stress testing (see, for instance, [51]) and market liquidity risk, devised by the BCBS in 2010-11 (see, for instance, [2]). The objective of the BCBS's reform package is to improve the banking sector's ability to absorb shocks arising from financial and economic stress, whatever the source, thus reducing the risk of spill-over from the financial sector to the real economy. The core of the Basel III reform agenda is focused on liquidity and leverage standards (see, for instance, [33]). The new liquidity ratios ensure that adequate funding is maintained in case of a crisis. It builds on previous Basel regulations and was developed in response to deficiencies in financial regulation that were exposed by the subprime mortgage, global financial and ongoing Eurozone sovereign debt crises (see, for instance, [18], [19] and [31]).

In Basel III, the maintenance of the global liquidity standards, the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR) underlie liquidity management and the role of banks in maturity transformations (ultimately triggering banking crises). The LCR standard imposes a requirement that banks maintain an adequate level of

> unencumbered HQLAs that can be converted to cash to meet its liquidity needs for a 30-calendar day time horizon under severe liquidity stress conditions specified by supervisors.

The 30-day LCR is intended to promote short-term resilience to potential liquidity disruptions. On the other hand, the NSFR liquidity standard is designed to

> promote longer-term funding of the assets and activities of banking organizations by establishing a minimum acceptable amount of stable funding based on the liquidity of an institutions assets and activities over a one-year horizon.

The NSFR is a longer-term structural ratio to address liquidity mismatches and provide incentives for banks to use stable sources to fund their activities. The liquidity standard also outlines a set of standard liquidity monitoring metrics to improve cross-order supervisory consistency. These are a common set of monitoring metrics to assist supervisors in identifying and analyzing liquidity risk trends at the bank and at system-wide level in order to better anticipate risks from systemic disruptions.

In this article, potential benefits and challenges in proposed Basel III rules will be considered from the perspective of liquidity standards, crisis prevention, financial markets and institutions, risk and central banking. It will, however, be highlighted in subsequent sections of the paper that the two new Basel liquidity standards will probably not achieve their desired objectives where such standards are not coupled with leverage ratios (see, for instance, [11], [14] and [33]).

2.1.1 Preliminaries about Basel III liquidity

During the early 'liquidity phase' of the financial crisis, many banks, despite adequate capital levels, still experienced difficulties, because they did not manage their liquidity in a prudent manner. The crisis drove home again the importance of liquidity to the proper functioning of financial markets and the banking sector. Prior to the crisis, asset markets were buoyant and funding was readily available at low cost.

2.1.1.1 Global liquidity standards

The difficulties that were experienced by some banks were due to lapses in basic principles of liquidity risk management. In response, as the foundation of its liquidity framework, the BCBS published in 2008 "Principles for Sound Liquidity Risk Management and Supervision", known as "Sound Principles" for short (see [9] for more details). These principles provide detailed guidance on the management and supervision of liquidity risk and are intended to promote improved liquidity risk management in the case of full implementation by banks and supervisors. As such, the BCBS coordinates followups by supervisors to ensure that banks adhere to "Sound Principles" (see [9] for more details).

To complement these principles, the BCBS has further strengthened its liquidity framework by developing two minimum standards for funding liquidity. The two standards have separate, but complementing, objectives. These are outlined below:

(a) LCR: Ensures banks have sufficient liquidity to survive a short-term 30-day stress scenario.

(b) NSFR: Ensures that banks have sufficient stable funds to survive a long-term stress scenario lasting for one year. An additional component of the liquidity framework is a set of monitoring metrics to improve cross-border supervisory consistency.

2.1.1.2 The liquidity coverage ratio (LCR)

For the LCR liquidity standard, an adequate level of unencumbered HQLAs that can be liquidized during stress scenarios has to be maintained. These assets have to endorse net cash outflow for the ensuing 30 calendar days. The LCR formula is given by

$$\text{LCR} = \frac{\text{Total stock of high-quality liquid assets (HQLAs)}}{\text{Total net cash outflows (NCOs) over the next 30 calendar days}} \geq 1.$$

High-quality liquid assets (HQLAs)

According to the LCR standard, banks must hold a stock of unencumbered HQLAs. These assets are described below.

HQLA characteristics

In order to qualify as an HQLA, assets should satisfy the following characteristics:

a. HQLAs should have low credit and market risk associated with them. In general, such assets tend to have higher liquidity. In addition, low credit and market risk lower duration, volatility, inflation risk and foreign exchange risk that enhance liquidity.

b. HQLA valuation should be easy and certain. The HQLA price calculation should be simple and not depend on strong assumptions. This increases the asset's liquidity in relation to market participants.

c. HQLAs should have low correlation with risky assets.

d. HQLAs should be listed on developed and recognized exchange markets. It can be shown that such listings increase asset transparency.

e. HQLAs should be eligible with the central bank for intraday liquidity needs. Such eligibility increases a bank's confidence in holding the asset that could be used during stress scenarios.

f. HQLAs should be unencumbered and as such should not be pledged as collateral, apart from exceptional circumstances.

HQLA categorization

The assets included in the stock are categorized as follows:

a. **Level 1 assets (L1As):** The stock of HQLAs can be constituted by an unlimited pool of L1As. These assets are not subject to any haircut under the LCR. However, supervisors may wish to subject these assets to a haircut, depending on the features of the particular HQLA. L1As consist of the following assets:

   i. Cash

   ii. Central bank reserves: These reserves are determined by the extent to which they can be drawn during stress scenarios.

iii. Marketable securities with the following characteristics:

   - Representing claims on or claims guaranteed sovereigns, central banks, non-central government public sector entities, Bank for International Settlements (BIS), the International Monetary Fund (IMF), the European Commission or multilateral development banks.
   - 0% risk weight according to the Basel II standardized approach for credit risk.

iv. Non-0% risk-weighted sovereigns, sovereign or central bank debt securities that are issued in domestic currencies by the sovereign or the country's central bank.

v. Non-0% risk-weighted sovereigns, domestic sovereign or central bank debt securities that are issued in foreign currencies to the extent that holding of such debt matches the currency needs of the bank's operations in that jurisdiction.

b. **Level 2 assets (L2As):** L2As must not exceed more than 40% of total stock of HQLAs. A minimum of 15% haircut is also applied to the current market value of L2As. Symbolically, this means that

$$\begin{aligned}\text{Market value of L2As} &\leq 0.4 \times \text{market value of high-quality liquid asset total stock}\\ 0.15 &\leq \text{haircut applied to L2A current market value.}\end{aligned}$$

i. L1As that are generated by secured funding transactions (or collateral swaps), maturing within 30 days.

ii. Corporate and covered bonds with the following characteristics:

   - Not issued by any financial institutions or bank itself or respective affiliated entities.
   - Rated AA or higher by ECAI.

iii. Marketable securities with the following characteristics:

   - Representing claims on or claims guaranteed sovereigns, central banks, non-central government public sector entities, BIS, IMF, the European Commission or multilateral development banks.
   - A maximum of 20% risk weight according to the Basel II standardized approach for credit risk.

Total net cash outflow

Total net cash outflow is defined as the total expected cash outflow minus total expected cash inflow for the ensuing 30 calendar days. While calculating total net cash outflow, total expected cash inflow is considered up to an aggregate cap of 75% of total expected cash outflow. Symbolically, this means that

$$\text{Total net cash outflows over the next 30 calendar days} \tag{2.1}$$
$$= \text{expected outflows} - \min\left[\text{ expected inflows; 75\% of expected outflows }\right].$$

Cash outflow

The total expected cash outflow is calculated as the sum of outstanding liabilities after multiplying it with an expected liability runoff during stress scenarios. Cash outflows are categorized in the following way:

a. **Retail deposits runoffs:** Retail deposits are defined as deposits from an individual natural person. Deposits from legal entities, sole proprietorships or partnerships are captured in wholesale deposit categories. Retail deposits that are subject to the LCR liquidity standard include demand and term deposits.

b. **Unsecured wholesale funding runoffs:** Unsecured wholesale funding that is subject to the LCR liquidity standard is defined as those liabilities and general obligations that are raised from non-natural persons (for example legal entities, including sole proprietorships and partnerships) and are not collateralized.

c. **Secured wholesale funding:** Secured wholesale funding that is subject to LCR is defined as those liabilities and general obligations that are collateralized by legal right to specifically designated assets that are owned by the borrowing institution in the case of bankruptcy, insolvency, liquidation or resolution.

Cash inflow

Total expected cash inflows are calculated by multiplying the outstanding balances of various categories of contractual receivables by the rates at which they are expected to flow in under the stress scenario. In order to prevent banks from relying solely on these inflows for its liquidity, an upper cap of 75% of total expected cash outflows is set. This ensures that banks hold a minimum stock of HQLAs, equal to 25% of cash outflows. Symbolically, this means that

$$\text{Total expected cash inflows} \leq 0.75 \times \text{total expected cash outflows.} \tag{2.2}$$

Cash inflows may be categorized in the following manner:

a. **Reverse repo and securities borrowings**

Basel III on the short position		
Maturing reverse repos backed by the following asset category:	**Inflow rate (if collateral is not used to cover short positions):**	**Inflow rate (if collateral is used to cover short positions):**
L1As	0%	0%
L2As	15%	0%
All other collateral	100%	0%

Table 2.1: Basel III on the short position

b. **Credit or liquidity facility (Drawdown rate = 0%):** Credit, liquidity or any other contingent funding that banks hold at other institutions for its own purpose is assumed to be unable to be drawn down in stress scenarios.

c. **Inflows by counterparty:** While considering loan payment as inflow, only fully performing loans are included.

   i. **Retail and small business inflow (Inflow rate = 50%):** This category assumes that banks will receive 100% of performing contractual inflows from retail and small business customers. However, banks will also extend 50% of these inflows towards loans to retail and business customers.

   ii. **Other wholesale inflow:** This category assumes that inflow is the same as above. However, in this category, banks will only extend 50% of these inflows towards non-financial wholesale counterparties. In this case, it means that

      - 100% inflow rate is from financial institution counterparties, and
      - 50% inflow rate is from non-financial wholesale counterparties.

d. **Operational deposits (Inflow rate = 0%):** Deposits that are held at other institutions for operational purposes are assumed to remain with the institution.

e. **Other cash inflows:** All the remaining cash inflows that are not categorized here.

   i. **Derivatives receivable (Inflow rate = 100%):** Derivatives receivable and payable are taken on a net basis. If receivables exist, they will have a 100% inflow rate.

   ii. **Contractual cash inflows:** All the remaining contractual inflows fall into this category. The value of the corresponding inflow rate is at the discretion of the supervisor.

2.1.1.3 Net stable funding ratio (NSFR)

The NSFR is the quotient of the amount of available stable funding and required stable funding over a 1-year stress period. Stable funding is defined as the type of equity and liability financing expected from reliable sources during a stress scenario. It is important to note that in order to avoid reliance on central banks, funding from such banks is not considered in the evaluation of the NSFR liquidity standard. This standard is required to be greater than 100% to ensure that the available funding meets the required funding over the evaluated period. Thus, it means that

$$\text{NSFR} = \frac{\text{Available stable funding (ASF)}}{\text{Required stable funding (RSF)}} \geq 1.$$

Available stable funding (ASF)

Here, ASF is defined as the total amount of bank capital, preferred stock with maturity ≥ 1 year, liabilities with effective maturities ≥ 1 year, demand deposits and/or term deposits with maturities < 1 year and wholesale funding with maturities < 1 year. In order to determine the actual ASF, the aforementioned capital and liability types have to be multiplied by a specific ASF factor that is assigned to each type.

Required stable funding (RSF)

Required stable funding (RSF) is defined as the sum of the value of assets held and funded by the bank, multiplied by a specific RSF factor that is assigned to each particular asset type.

Net stable funding ratio (NSFR) summary table

In the following Table 2.2, a summary of the ASF and RSF components of the NSFR, together with their multiplication factors is represented.

Summary table of the net stable funding ratio			
Available stable funding (sources)		**Required stable funding (uses)**	
Item	**ASF Factor**	**Item**	**RSF Factor**
• T1K & T2K instruments • Other preferred shares & capital instruments in excess of T2K, allowable amount having an effective maturity of 1 yr or > 1 yr • Other liabilities with an effective maturity of 1 yr or > 1 yr	100%	• Cash • Short-term unsecured actively-traded instruments (< 1 yr) • Securities with exactly offsetting reverse repo • Securities with remaining maturity < 1 yr • Non-renewable loans to financials with remaining maturity < 1 yr	0%
• Stable deposits of retail and small business customers (non-maturity or residual maturity < 1 yr)	90%	• Debt issued or guaranteed by sovereigns, central banks, BIS, IMF, European Commission, non-central government, multilateral development banks with a 0% risk weight under Basel II standardized approach	5%
• Less stable deposits of retail and small business customers (non-maturity or residual maturity < 1 yr)	80%	• Unencumbered non-financial senior unsecured corporate bonds & covered bonds rated at least AA-, & debt that is issued by sovereigns, central banks, & public sector entities with a risk-weighting of 20%; maturity ≥ 1 yr	20%
• Wholesale funding provided by non-financial corporate customers, sovereign central banks, multilateral development banks and public sector entities (non-maturity or residual maturity < 1 yr)	50%	• Unencumbered listed equity securities or non-financial senior unsecured corporate bonds (or covered bonds) rated from A+ to A-, maturity ≥ 1 yr • Gold • Loans to non-financial corporate clients, sovereigns, central banks, & public sector entities with a maturity < 1 yr	50%
• All other liabilities and equity not included above	0%	• Unencumbered residential mortgages of any maturity and other unencumbered loans, excluding loans to financial institutions with a remaining maturity of 1 yr or > 1 yr that would qualify for the 35% or lower risk weight under Basel II standardized approach for credit risk	65%
		• Other loans to retail clients and small businesses having a maturity < 1 yr	85%
		• All other assets off-balance sheet exposures	100%
		• Undrawn amount of committed credit & liquidity facilities	5%
		• Other contingent funding obligations	national supervisory discretion

Table 2.2: Summary table of the net stable funding ratio

2.1.2 Literature review of Basel III and liquidity

In this subsection, the authors survey the existing literature about bank liquidity and its connections with regulation (see, for instance, [35]).

Although the "Sound Principles" in [9] focus on liquidity risk management at medium and large complex banks, they have broad applicability to all types of banks. The implementation of these principles by both banks and supervisors was tailored to the size, nature of business and complexity of banking activities. Since the "Sound Principles" were laid down, guidance for supervisors has been augmented substantially. In particular, the proposed Basel III liquidity regulation that is explained in [11] and [14] has added a great deal. These prescripts emphasize the importance of supervisors assessing the adequacy of a bank's liquidity and the associated risk management framework. It also suggests steps that supervisors should take if these are deemed inadequate. The BCBS expects banks and supervisors to implement the revised principles promptly and thoroughly and that the BCBS will actively review progress in implementation (see, for instance, [11] and [14]).

2.1.2.1 Literature review of benefits that are associated with Basel III liquidity regulation

Literature that is related to the potential benefits of the proposed Basel III liquidity regulation in terms of liquidity standards, crisis prevention, financial markets and institutions, liquidity and other risks, as well as the broader economy, are reviewed in this subsection.

As the authors have mentioned before, Basel III regulates that banks are obligated to comply with two liquidity standards, namely the LCR and NSFR, to be effectively implemented in mitigating crises (see, for instance, [19] and [31]). The LCR is intended to promote resilience to potential liquidity disruptions over a 30-calendar day horizon. It will help ensure that global banks have sufficient unencumbered HQLAs to offset the net cash outflows it could experience under an acute short-term stress scenario (see, for instance, [11] and [14]). The specified scenario is built upon circumstances that have been experienced in the global financial crisis that began in 2007 and entails both institution-specific and systemic shocks (see, for instance, [30]). On the other hand, the NSFR aims to limit over-reliance on short-term wholesale funding during times of buoyant market liquidity and encourages better assessment of liquidity risk across all on- and off-balance sheet items. The paper [3] suggests that the timetable for the introduction of Basel III liquidity standards is flexible in that national discretion can be used (see, for instance, [13]). This paper also suggests that imposing high liquidity standards make it unlikely that banks will readily engage in excessive credit extension so that the solvency of the entire banking system will be questioned. This will protect banks against systemic externalities and consequent financial crises. The paper [20] finds incentives to hold a greater stock of HQLAs, but also to provide incentives to increase the cyclical variation in reserves (see also [22] and [29] for comments on procyclicality reduction). The contribution [24] claims that banks build bigger capital buffers when they are exposed to higher illiquidity. Their empirical investigation supports the need to implement minimum liquidity ratios. [47] suggests that banks should accumulate HQLA buffers that should provide absolute insurance against small shocks. The paper [56] suggests maximum net benefits when banks meet the Basel III long-term liquidity requirements.

During the most severe period of the financial crisis, the market lost confidence in the solvency and liquidity of many banks. The weaknesses in the banking sector were rapidly transmitted to the rest of the financial system and real economy, resulting in a massive contraction of liquidity and credit availability. Ultimately, the public sector had to step in with unprecedented injections of liquidity, capital support and guarantees, exposing taxpayers to large losses. In response to this, during February 2008, the BCBS published [9]. The working paper [52] suggests that proposed Basel

III liquidity standards constitute a cornerstone of the international regulatory reaction to financial crises. According to [10], in response to the market failures that have been revealed during crises, the BCBS has introduced a number of fundamental reforms to the international regulatory framework. The reforms strengthen bank level or micro-prudential regulation, which will help raise the resilience of individual banking institutions to periods of stress. Moreover, high liquidity requirements lead to lower cyclical fluctuations that follow supply or demand shocks (see, for instance, [3]). Some potential benefits that may be associated with Basel III liquidity regulation, outlined in [7], [34] and [55], suggest that the probability of a crisis occurring is based on the statistical relationship between the incidence of crises and aggregate liquidity data. Moreover, [22] and [29] suggest that liquidity standards enable banks to resist short-term disruptions, while capital buffers dampen cyclicality of minimum capital requirement.

As far as benefits of proposed Basel III regulation on financial markets are concerned, [32] emphasizes that financial markets expect banks to maintain liquidity buffers in excess of the actual requirements. Enhanced liquidity regulation reflects the fact that a more robust banking system would be less prone to crises, which can impose large losses in terms of forgone output. Despite the potential negative side-effects of the liquidity standards on monetary policy implementation, there is broad agreement that the Basel III liquidity standards constitute an important improvement on financial regulation. The purpose of the non-standard monetary policy tool that is suggested by Basel III is to ensure that banks build up sufficient liquidity buffers on their own to meet cash-flow obligations (see, for instance, [53]). At the same time, the banking system as a whole accumulates a stock of HQLAs to safeguard financial stability. [3] shows that Basel III will force banks to shift their business model from liability management, in which business decisions are made about asset volumes, with the financing found in short-term wholesale markets as necessary, to asset management, in which asset volumes are constrained by the availability of funding. Furthermore, there is a need for financing of risky exposures if a fall in economic growth is to be avoided. The paper [43] claims that Basel III provides liquidity to banks for longer periods of maturity and allows intervention in selected credit markets. A bank and its supervisors should consider the bank's role in the financial sectors of the jurisdictions in which it operates and the bank's systemic importance in those sectors (see, for instance, [30]).

According to [51], a liquidity model should include a submodel to assess risks that rise from maturity transformation and rollover risks. Here, liquidity buffers should cover small refinancing needs because of its limited size. In the management of liquidity risk, a bank can combine liquidity buffers and transparency to hedge small and large refinancing needs. A bank that can assert its solvency will be able to attract external refinancing. Paper [43] suggests that Basel III allows central banks to be in charge of overseeing systemic risk which places them in a position to focus on system wide hazards. Basel III also makes allowances for central bank reserves that serve as a means whereby commercial banks manage their liquidity risk (see, for instance, [43]). It also assists supervisors in informing the central bank of their judgements.

Paper [27] establishes a framework to estimate the effects of higher bank capital and liquidity on output and bank profitability. To estimate the long-term cost effect of higher bank capital and liquidity requirements, [27] uses a vector error correction model (VECM) to estimate the long-run relationships among a small set of variables for the UK. This analysis focuses on the long run effects on interest rates, lending, gross domestic product (GDP) and bank profitability of the Basel III requirements. It establishes a framework to estimate the effects of higher bank capital and liquidity on output and bank profitability.

2.1.2.2 Literature review of challenges that are associated with Basel III liquidity regulation

In this subsection, literature that is related to the potential challenges of proposed Basel III liquidity regulation in terms of liquidity standards, crisis prevention, financial markets and institutions, liquidity and other risks, as well as the broader economy, are reviewed.

According to [1], the definition of eligible HQLAs should be broadened to give banks more scope to hold liquidity in the form of commercial claims and central banks should clarify under which circumstances they will provide emergency liquidity assistance. The banks also face the challenge of interfacing or merging current risk and finance systems to meet the new Basel III LCR ratio standards. The main shortcoming of the NSFR is that it depends on banks and supervisors' capacity to model investor behaviour. According to [4], 1% increase in the LCR causes a median 0.08% decline in the level of steady-state output. [6] suggests that there is a need for a standard NSFR. This can be achieved through banks' increase lending spreads. [6] also outlines a methodology for mapping the possible impact of higher liquidity requirements on banks' lending spreads. These mappings provide a useful tool to study the impact of regulatory changes on the cost of credit and the real economy. Paper [22] suggests that leverage and liquidity ratio interaction is not clear and that these led to noise as a result of conservative adjustments. LCR buffer eligibility and haircut rules rely on external ratings, Basel classification of counterparties and standardized credit risk weights. Furthermore, [3] suggests that Basel III enables markets to anticipate the required changes and penalizes banks that fall substantially short of the LCR and NSFR liquidity requirements. [47] suggests that liquidity and transparency are costly hedges and most of the cost is borne by the bank's shareholders. Maintaining liquidity buffers is costly because of the low return on generic highly liquid assets; lower return accrues to the shareholders as residual claimants.

As crisis prevention is concerned, [3] suggests that a challenge that is faced by banks is to consolidate clean exposures, liabilities, counterparties and market data in a centralized risk data warehouse. Moreover, authorities need to clarify how liquidity will be obtained in times of crisis. Bank regulators need to ensure that the rules are flexible enough to allow banks to access market liquidity. [43] suggests that the 2007 financial crisis was caused by excessive on- and off-balance sheet leverage, which was accompanied by a gradual erosion of the level and quality of the capital base, as well as insufficient liquidity buffers. The crisis was further amplified by a procyclical deleveraging process and systemic effects. [47] claims that liquidity buffers provide complete insurance against small shocks. Paper [48] calibrates the costs for the Eurozone crisis from higher liquidity by using a DSGE model, including financial frictions and a banking sector.

When the focus is on challenges in financial markets, [23] shows that market frictions are significant for financial shock effects and the properties of optimal monetary and fiscal policy. Working paper [51] suggests that many of the most exposed banks did not have an adequate framework during the crisis that satisfactorily accounted for the liquidity risks that were posed by individual products and business lines, and therefore incentives at the business level were misaligned with the overall risk tolerance of the bank. Many banks had not considered the amount of liquidity they might need to satisfy contingent obligations, either contractual or non-contractual, as they viewed funding of these obligations to be highly unlikely.

As far as challenges that are related to financial institutions are concerned, [3] suggests that bank balance sheet shifts will also require substantial portfolio adjustments amongst long-term institutional investors, from short-term to long-term debt and from debt to equity. Paper [36] presents a model which does not directly incorporate a bank's balance sheet conditions and income statement as input variables such as lending spreads. Aggregate liquidity can be seen as the rate of change of the aggregate balance sheet of financial intermediaries (see, for instance, [1]). [41] suggests that, due to post-crisis trends, long-term trends toward local banking are likely to accelerate, especially if liquidity regulations are applied locally. In [3], there is a suggestion that Basel III will force banks to shift their business model from liability management to asset management in which asset volumes are constrained by the availability of funding. Furthermore, new liquidity rules could create new unintended systemic risks. [56] claims that the main channel through which liquidity regulation changes affect economic activity is via an increase in the cost of banking intermediation. Banks tend to increase lending rates to compensate for the cost of holding more liquidity. Working paper [24] suggests that the determinants of bank capital buffers have neglected the role of liquidity and that banks do not strengthen their capital buffer when they face higher illiquidity. The aim of this study is to understand the relationship between bank capital buffers and liquidity. Furthermore, there are questions as to whether banks maintain or strengthen their capital buffer when faced with lower liquidity. [37] also suggests that liquidity standard compliance has a small effect on output. This means that economic costs of stronger liquidity requirements are small and negligible if compared with the benefits that are reaped from reducing the frequency of systemic crises. [39] concludes that there is a need to control bank liquidity, debt, capital and profitability financial ratios better. According to [39], Basel III has laid down suitable recommendations, but they are insufficient. [42] concludes that global liquidity provisioning is highly procyclical. The burden of adjustment thus rests on monetary policy implementation in the various currency areas rather than on fundamental changes to the standards themselves (see, for instance, [53]). In [50], the challenges facing the BCBS with regards to liquidity in the failure to factor in the role of central banks as lenders of last resort are demonstrated.

In [19], it is shown that liquidity risk should be priced into liquidity and transparency instruments in such a way that buying exposure to the same default risk is identically priced. Moreover, [3] claims that new Basel liquidity rules could create new, unintended systemic risks. [43] portrays a liquidity shortfall that is reflected by on- and off-balance sheet maturity mismatches. This paper also accentuates the importance of the role that is assumed by central banks in the funding of bank balance sheets. [27] and [55] outline the advantage of the error correction model which helps to disentangle loan demand and supply factors. [28] claims that most of the familiar events in the financial crisis are more linked to movements in liquidity risk than in credit risk (compare with [8]). [38] provides formal empirical support to the BCBS's proposal to penalize excessive exposures to liquidity risk by showing that short-term wholesale funding, a variable capturing interconnectedness, makes a significant contribution to systemic risk. In [54], the impact is explored of new rules that will require banks to hold capital reserves against market, credit and liquidity risk in their trading books. It is found that the new requirement for incremental credit risks, the incremental risk charge, may be substantial when compared with old regulatory capital levels.

As far as the broader economy is concerned, [42] concludes that global liquidity provision is highly procyclical. This paper then proceeds to review the approaches that have been adopted to assess the GDP loss that is associated with a specific banking crisis. The staff report [4] claims that each 1% increase in the LCR causes a median 0.08% decline in the level of steady-state output, relative to the baseline (see also [16]). As mentioned before, [27] focuses on effects of proposed Basel III requirements on interest rates, credit availability, GDP and bank profitability.

2.1.3 Main questions and article outline

In this subsection, the authors specify the main questions that have been solved and outline the article.

2.1.3.1 Main questions

The main questions that have been addressed in this article can be stated as follows:

Question 2.1.1 (Quantitative methods involving Basel III liquidity regulation) *What quantitative methods can be used to study Basel III liquidity standards? (See Section 2.2.)*

Question 2.1.2 (Potential benefits of Basel III liquidity regulation) *What are the potential benefits that are associated with Basel III liquidity proposals in terms of liquidity standards, crisis prevention, financial markets and institutions, liquidity risk, as well as the broader economy? (See Section 2.3.)*

Question 2.1.3 (Potential challenges of Basel III liquidity regulation) *In terms of liquidity standards, crisis prevention, financial markets and institutions, liquidity risk, as well as the broader* *economy, what are the potential challenges that are associated with Basel III liquidity proposals? (See Section 2.4.)*

2.1.3.2 Article outline

The current section is preliminary in nature. Section 2.2 presents quantitative methods involving proposed Basel III liquidity regulation. Sections 2.3 and 2.4 discuss how this regulation relates to potential benefits and challenges, respectively. These sections also demonstrate how Basel III requirements propose to address problems that are related to liquidity during possible stress scenarios. Finally, a conclusion and topics for possible future research are provided.

2.2 Quantitative methods involving Basel III liquidity standards

In this section, the authors consider a summary of the paragraphs that are related to the metrics that involve Basel III liquidity, a survey of quantitative methods for studying new liquidity regulation and a model for Basel III liquidity regulation.

2.2.1 Survey of quantitative methods for studying Basel III liquidity

In this subsection, the authors consider a summary of the survey of quantitative methods for studying Basel III liquidity.

Survey of quantitative methods for studying Basel III liquidity			
Article	**Models**	**Countries**	**Information**
[4]	DSGE, VECM	Eurozone, US, UK and Italy	Liquidity
[19]	Time-varying VAR	Eurozone	Liquidity
[20]	DSGE	Eurozone	Liquidity
[21]	Regression analysis	East Asia	Liquidity
[24]	Simultaneous equations framework	US & Eurozone	Liquidity
[27]	VECM	US	Liquidity
[28]	Dynamic factor model	US & Eurozone	Liquidity
[32]	General equilibrium model	Sweden	Liquidity
[38]	CoVAR approach	Selection of 18 countries	Liquidity
[39]		UK & Australia	Liquidity
[45]	Structural model regression	US,. UK, Germany Switzerland & Japan	Liquidity
[49]	VaR framework	Selected emerging markets	Liquidity
[51]	Stress testing	US & Eurozone	Liquidity
[53]	Liquidity stress testing	The Netherlands	Liquidity
[54]	VaR and stressed VaR approaches	US	Liquidity
[56]	VECM	UK	Liquidity

Table 2.3: Survey of quantitative methods for studying Basel III liquidity

2.2.2 Models for proposed Basel III liquidity regulation

There are basically two types of models for determining a model for new Basel III liquidity regulation. They may be listed as follow (with examples):

a. **Numerical quantitative modelling types:** DSGE; VECM; VAR; regression analysis; CoVAR; dynamic factor; VaR and stressed VaR.

b. **Theoretical quantitative modelling types:** Differential equations; simultaneous equations; stochastic analysis and systems theory.

2.2.2.1 A model for proposed Basel III liquidity regulation

A model for proposed Basel III liquidity regulation has to satisfy certain minimum requirements. These requirements are outlined below.

The model's supervisory approach to liquidity measurement and monitoring must be supplemented by detailed assessments of other aspects of the bank's liquidity risk management framework in accordance with the BCBS's "Sound Principles" (see [9] for more details). **Paragraph 5** from [9] states that a bank should have a sound process for identifying, measuring, monitoring and controlling liquidity risk. This process should include a robust framework for comprehensively projecting cash flows arising from assets, liabilities and off-balance sheet items over an appropriate set of time horizons. Any model should be flexible and satisfy the above principles.

Regulatory liquidity risk reports will have to be produced at least monthly with the ability, when required by regulators, to be delivered weekly or even daily. In this case, the model requires robust, automated reporting solutions. The next challenge would be in enumerating all the qualifying bank assets, funds and behavioural cash flows according to the various scenarios from the available management information system. In this regard, it is noted that LCR buffer eligibility, haircut rules and run-off rates, as well as NSFR, ASF and RSF factors also rely on external ratings. Also of importance is the Basel classification of counterparties and standardized credit risk weights, jurisdiction, currency and information which are usually only available in risk specific systems and not in treasury systems (see [46] for more detail).

2.2.2.2 Model design

Based on the identified challenges in developing a liquidity model, the design should include the enumeration of data (data submodel), evaluation of Basel III liquidity regulation (core submodel) and a user-interface for the supervisor to monitor the model (user-interface submodel). These proposed model elements are presented in the following diagram:

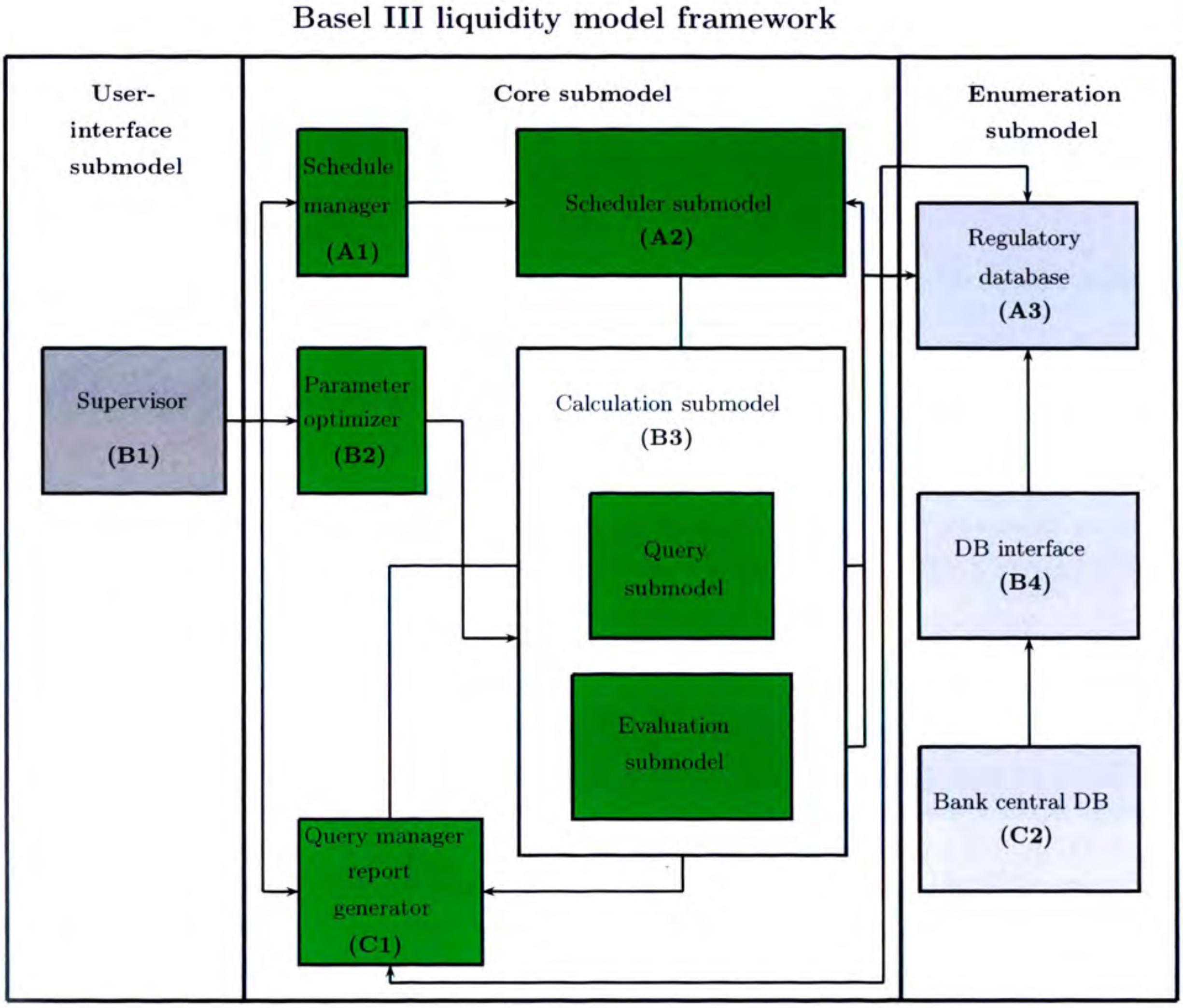

Figure 2.1: Basel III liquidity model framework (Source: [46])

2.2.2.3 Enumeration submodel

The enumeration submodel contains the database (DB) components that are needed for liquidity measurement and monitoring. The purpose of this submodel is to provide a framework that is independent of the third-party solution being employed by the bank for developing its management information system. The submodel also integrates the current solution and the bank's existing third-party solution.

2.2.2.4 Core submodel

The core submodel integrates all the functional components of the model, thus acting as an application layer. This submodel is responsible for operations like scheduling, evaluating and reporting.

Scheduler submodel

The scheduler submodel schedules the regular systematic evaluation of proposed Basel III liquidity standards. According to [46], this submodel may follow sliding window architecture[5].

Calculation submodel

The role of the calculation submodel[6] is to determine the liquidity standards by retrieving all required data, applying all applicable haircuts and finally, obtaining the result. This result is then forwarded to the database for storage and subsequently to the report generator in order to generate reports.

Query manager

A manual request for liquidity standard evaluation is channelled through the query manager. The supervisor interacts directly with this manager using a user-interface. The query manager selects either the standard or date by which the standard has to be evaluated. After accepting the date and standard type, the query manager checks the regulatory database against the date to find the selected standard that was calculated at a later date. If it exists, it returns the earliest ratio after the date or else it passes the date and standard type to the calculation submodel to perform a new calculation. This step of bypassing is done only if the date is entered manually. By default, the query manager uses the current system date.

Report generator

Reporting[7] is an integral part of any corporate process. Such reporting can be implemented by using any available third-party software. Reports are generated immediately after the execution of the calculation submodel, listing all the entities that are considered by such a submodel for stan-

[5]Sliding window is a classical architecture that is followed in computer networks for reliable delivery of data-packets. Such architecture is appropriate because of analogies between the problem statements.

[6]The calculation submodel can be triggered in manual and automated approaches. With regard to the former, the supervisor manually starts the evaluation interacting with the query manager. The supervisor selects either of the standard and a date according to which the standard has to be evaluated. On the other hand, the automated approach is triggered by scheduler submodel. According to algorithm, the scheduler submodel regularly triggers the calculation submodel.

[7]Reporting can be defined as a regular provision of information to decision makers within an organization to support them in their work.

dard evaluation. The report is then updated to the database. These reports provide information to supervisors in order to assist them in decision making.

Parameter optimizer

The parameter optimizer lists every variable that is required for liquidity standard evaluation. These variables include various run-off factors, hair-cuts, currencies et cetera. Although most of these parameters are internationally harmonized, there are some that have to be decided upon by local or national supervisors. Such parameters are not consistent and may be dependent on economic conditions. The parameter optimizer submodel allows flexibility in updating such parameters. Bank supervisors are responsible for monitoring and updating the aforementioned parameters accurately, subject to value and time.

2.2.2.5 User-interface submodel

The user-interface submodel provides a platform for supervisors to fulfil authorized roles.

Supervisors are natural persons or a team of natural persons. The application that is developed will be used solely by supervisors. Supervisors are responsible for updating the parameters according to the socio-economic environment, requesting standard evaluation when required, monitoring the evaluations and reports that are generated to suggest a strategy in stress scenarios, as well as updating the bank database with off-balance sheet entries where necessary. The user-interface submodel includes a portal to perform the above tasks and a security access to avoid unnecessary access.

2.3 Potential benefits of Basel III liquidity requirements

In this subsection, potential benefits of Basel III liquidity requirements are discussed with respect to liquidity standards, crisis prevention, financial markets and institutions, liquidity and other risks, as well as the broader economy. In short, these requirements provide incentives to encourage banks to retain liquidity. The aforementioned incentives should also account adequately for situations where poor liquidity management systems that are supposed to be risk mitigating are implemented.

2.3.1 Benefits that are related to liquidity standards

Moral hazard and regulatory arbitrage were the principal rationales for the proposed Basel III banking regulation. Imposing high liquidity standards make it unlikely that banks will engage readily in excessive credit extension that leads to solvency problems. It will protect banks against systemic externalities and consequent financial crises (see, for instance, [3]). This is the main benefit that is related to liquidity standards.

Working paper [24] claims that banks build larger capital buffers when they are exposed to higher illiquidity. Their empirical investigation supports the need to implement minimum liquidity ratios. Furthermore, [3] suggests that the timetable for the introduction of these ratios could be adjusted, depending on economic developments. Despite the fact that the BCBS has determined dates by which the various aspects of the transition have to be completed, there is national discretion in the speed of transition subject to those end-dates, but note that this might require greater flexibility in the intermediate dates for achieving minimum capital requirements than announced by the BCBS (see, for instance, [13]).

Working paper [47] asserts that banks should accumulate an HQLA buffer or strengthen transparency to communicate solvency. The aforementioned paper claims that liquidity buffers provide complete insurance against small shocks. Moreover, liquidity requirements can compromise banks' transparency choices and increase refinancing risk. To be effective, liquidity requirements should be complemented by measures that increase bank incentives to adopt transparency (see, for instance, [47]). Similarly, [20] finds some rationale for Basel III in providing commercial banks with an incentive to hold a greater HQLA stock such as reserves. The regulation should also provide incentives to increase the cyclical variation in reserve holdings in order to limit excessive procyclical lending to the private sector (see also [22] and [29] for comments on procyclicality reduction).

Contribution [56] estimates the maximum net benefit when banks adhere to the Basel III long-term liquidity requirements. In this regard, the estimated permanent net benefit is larger than the average that the BCBS estimates. In particular, this significant marginal benefit suggests that UK banks need to increase their reliance on common equity in their capital base beyond the level that is required by Basel III. Furthermore, [56] claims that neither the portfolio model nor the stress testing models can assess the impact of liquidity requirements (see also [51]). With this in mind, they estimate the reduced-form relationship between the probability of a banking crisis occurring and UK banks' liquidity ratios (see also [53]).

2.3.2 Benefits that are related to crisis prevention

The liquidity ratio and the redefinition of 'capital' would enable banks to withstand short-term disruptions better. Capital buffers are also expected to dampen cyclicality of the minimum capital requirement (see, for instance, [22] and [29]).

[12] averages across a number of studies of financial crises in many countries, namely that increased capital and liquidity lead to a substantial reduction in the probability of a crisis event. The BCBS conducts two further quantifications of the benefits of higher capital and liquidity requirements that will lead to lower cyclical fluctuations, following supply or demand shocks (see, for instance, [3]). They acknowledge, though, that these further results are subject to several caveats and do not place great stress on them in their final conclusions (see, for instance, [3]).

Some potential benefits that may be associated with Basel III liquidity regulation are outlined in [7], [34] and [55] where reduced-form profit models are adopted for a panel of countries over a period of years. The probability of a crisis occurring is based on the statistical relationship between the incidence of crisis episodes and aggregate liquidity data, as well as other variables that serve as controls. Comparing with [7], [34] uses a general-to-specific approach to choose the preferred specification by considering the substitutability between Japanese banks' liquidity.

2.3.3 Benefits that are related to financial markets and institutions

In this subsection, the potential benefits of the proposed Basel III banking regulation on financial markets and institutions are considered.

2.3.3.1 Benefits that are related to financial markets

[32] claims that financial markets will expect banks to maintain liquidity buffers in excess of the actual requirements. The economic benefits of the enhanced liquidity regulations reflect mainly the fact that a more robust banking system would be less prone to crises that can result in large losses in terms of forgone output. The paper [32] synthesizes the evidence on this effect, and discusses the impact of capital and liquidity regulation on the probability of systemic banking crises occurring.

2.3.3.2 Benefits that are related to financial institutions

One of the elements of the liquidity framework that has been identified by [51] is a model to simulate the impact of bank run scenarios. [3] demonstrates that Basel III will force banks to shift their business model from liability management, in which business decisions are made about asset volumes, with the financing found in short-term wholesale markets as necessary, to asset management, in which asset volumes are constrained by the availability of funding. They find, contrary to what many have argued, that once there is a full adjustment, the costs of credit to low-risk bank borrowers (the majority of customers) will be only moderately affected, but that there will be a reduction in availability and higher cost at the riskier end of the credit spectrum. According to [3], alternative arrangements are, therefore, needed for the financing of risky exposures if a fall in economic growth is to be avoided. In this context, securitization (broadly defined to include all forms of bank-sponsored collateralized instrument, including covered bonds) will be of central importance. Re-establishing securitization markets on a sounder footing appears essential in order to prevent a renewed credit contraction and to help prevent riskier borrowers from being cut off from credit.

Basel III provides liquidity to banks for longer periods of maturity and allows intervention in selected credit markets (see, for instance, [43]). Basel III will not require the sale of government securities, thus putting pressure on banks' liquid assets, since such sales would simply force the banks to exchange one kind of liquid asset (cash) for another (government securities) (see, for instance, [3]). Capital and liquidity requirements can give banks an incentive to securitize relatively safe assets as a means of getting around the limitations that are imposed by capital requirements, the so-called regulatory arbitrage.

Basel III allows for the provision of central bank reserves. This serves as a means whereby commercial banks are able to manage their liquidity risk (see, for instance, [43]). Basel III assists supervisors in informing central banks of their judgement regarding the range of liquidity risks that are faced by firms for which they are responsible (see, for instance, [43]). Moreover, the timing of the withdrawal of the emergency liquidity facilities that are provided by central banks during the crisis might be kept under review. In the UK, decisions on these matters should be made part of the normal monetary policy process that is conducted by the Bank of England Monetary Policy Committee. On the other hand, the purpose of a non-standard monetary policy tool that is suggested by Basel III is to ensure that banks build up sufficient liquidity buffers on their own to meet cash-flow obligations (see, for instance, [53]). At the same time, the banking system as a whole accumulates a stock of liquid reserves to safeguard financial stability (see, for instance, [43]).

2.3.4 Benefits that are related to liquidity and other risks

According to [51], the liquidity framework includes a submodel to assess risks arising from maturity transformation and rollover risks, implemented either in a simplified manner or as a fully-fledged, cash flow-based approach, and a framework to link liquidity and solvency risk.

As ultimate providers of liquidity, Basel III allows central banks to be in charge of overseeing systemic risk, which places them in a position to focus on system-wide risks, as well as to obtain an integrated view of both the individual financial institutions and the financial system as a whole (see, for instance, [43]). A liquidity buffer can only cover small refinancing needs, because its size is limited. A bank can optimally combine liquidity buffers and transparency in its risk management to hedge small refinancing needs fully and hedge large ones partially.

Since risk-taking of financial institutions varies procyclically, Basel III requirements may be adjusted appropriately. Another, less conventional, way is to enhance the ability to communicate solvency information to outsiders. A bank that can assert its solvency will be able to attract external refinancing. While the benefits of higher liquidity standards are very uncertain, these may still be worth pursuing if the costs are comparatively small.

2.3.5 Benefits that are related to the broader economy

Occasional paper [37] claims that compliance with the new Basel III liquidity standards has a small effect on output. The above estimate suggests that the economic costs of stronger liquidity requirements are small and become negligible if compared with the potential benefits that can be reaped from reducing the frequency of systemic crises and the amplitude of boom-bust cycles.

2.4 Potential challenges of Basel III liquidity regulation

It is a commonly held belief that requirements that were proposed by Basel III were insufficient in addressing liquidity and funding problems which arose during the crisis. The importance of enhanced disclosures is also reflected and embodied within the BCBS's second objective in relation to its proposal to strengthen the resilience of the banking sector, that is its endeavours to improve risk management and governance, as well as strengthen banks' transparency and disclosures. The main challenges that are related to the new Basel III liquidity risk requirements are outlined below.

2.4.1 Challenges that are related to liquidity standards

Unfortunately, raising liquidity standards may be costly. Banks may respond to regulatory tightening by passing on additional funding costs to their retail business, by raising lending rates in order to keep the return on equity in line with market valuations and/or by reducing the supply of credit in order to lower the share of risky assets in their balance sheets (see, for instance, [37]). [6] claims that, for the standard NSFR, a bank would need to increase lending spreads by 24 basis points;the article presents a methodology for mapping the possible impact of higher liquidity requirements on bank lending spreads. Such a mapping provides researchers with a useful tool to study the impact of regulatory changes on the cost of credit and the real economy. The mapping that is outlined in this paper provides a starting point for analyzing the impact of higher capital and liquidity requirements on a bank's profitability and lending spreads. As highlighted at the outset, the estimates are sensitive to changes in the balance sheet structure of different banks and their costs of capital. Future research may, therefore, calculate such estimates for different categories of banks (for example regional banks vs. multinationals), based on actual balance sheet data that take the revised capital and liquidity proposals of the BCBS into account. It will be important to compare such estimates to the elasticity of loan demand across countries to better understand the ability of banks to pass on such costs. Such an analysis would also highlight the different macroeconomic impact across countries, as well as the potential spillover (see [6] for more details).

In [22], second-order concerns include the lack of clarity for the interaction between leverage ratios, and liquidity ratios and the noise that is introduced because of some conservative adjustments. LCR buffer eligibility and haircut rules rely on external ratings, Basel classification of counterparties and standardized credit risk weights. The LCR numerators runoff rates, as well as NSFR, ASF and RSF factors, also depend on such information that is usually only available in risk specific systems and not treasury systems.

In Basel III, markets are likely to anticipate the required changes and penalize banks that fall substantially short of the LCR and NSFR liquidity requirements, in other words they may well have to increase the liquidity of their assets and reduce the liquidity of their liabilities well ahead of the announced implementation timetable (see, for instance, [3]).

A bank can combine liquidity buffers and transparency optimally in its risk management to hedge small refinancing needs fully and large ones partially. Yet, liquidity buffers and transparency are strategic substitutes, because for a bank that adopts one hedging instrument, the value of another diminishes. Liquidity and transparency are costly hedges, and most of the cost is borne by the bank's shareholders. Maintaining liquidity buffers is costly because of the low return on generic highly liquid assets; lower return accrues to the shareholders as residual claimants (see, for instance, [47]).

2.4.2 Challenges that are related to crisis prevention

The first challenge banks will face is to consolidate clean exposures, liabilities, counterparties and market data in a centralized risk data warehouse. All portfolios' contractual and behavioural cash flows shall be made available and banks should have the ability to stress those and produce liquidity gap analysis according to various scenarios. [3] asserts that authorities should also clarify how liquidity will be obtained in times of crisis: Bank regulators need to ensure that the rules are flexible enough to allow banks to access market liquidity, either from the sale of assets or through short-term borrowing; the central bank needs to state in what circumstances it would provide emergency liquidity assistance to supplement bank access to market liquidity.

According to [43], one of the main reasons that has caused to the economic and financial crisis which began in 2007 to become so severe was that the banking sectors of many countries had built up excessive on- and off-balance sheet leverage. This was accompanied by a gradual erosion of the level and quality of the capital base. At the same time, many banks were holding insufficient liquidity buffers. The banking system, therefore, was not able to absorb the resulting systemic trading and credit losses, nor could it cope with the re-intermediation of large off-balance sheet exposures that had built up in the shadow banking system. The crisis was further amplified by a procyclical deleveraging process and by the interconnectedness of systemic institutions through an array of complex transactions.

Working paper [47] claims that liquidity buffers provide complete insurance against small shocks. Transparency covers also large shocks, but imperfectly. [48] also calibrates the costs for the Euro-zone crisis from higher liquidity by using a DSGE model, including financial frictions and a banking sector. The paper features banks' balance sheets and credit markets explicitly. It provides a unified framework to analyze how changes in liquidity requirements affect banking conditions (spreads and lending) and output. DSGE models also allow counter-factual policy experiments in a conceptually consistent manner. However, since it is fully calibrated, the estimation process is often daunting. Furthermore, [52] says that the Basel III liquidity standards constitute a cornerstone of the international regulatory reaction to the ongoing economic and financial crisis. While capital regulation at international level dates back to the 1980s, liquidity regulation has remained fragmented across borders.

2.4.3 Challenges that are related to financial markets and institutions

In this subsection, potential challenges that are related to financial markets and institutions are discussed.

2.4.3.1 Challenges that are related to financial markets

[3] claims that the shifts in bank balance sheets will also require substantial portfolio adjustments amongst long-term institutional investors, from short-term to long-term debt and from debt to equity. The associated adjustment of both market prices and required returns can be accommodated, but poses a substantial co-ordination problem and could take a long time. As the article emphasizes, costs are properly understood as the reductions in aggregate supply that come about if liquidity requirements make bank intermediation more costly and hence increase the cost of capital to firms for investment and to households for purchase of durables, notably housing (see, for instance, [3]).

The international regulatory and central banking community has less expertise concerning the potential negative side-effects of international liquidity standards, for example on money markets and monetary policy implementation (see, for instance, [53]). In addition, the mandate of the BCBS and its sub-group, the Working Group on Liquidity, focused on financial stability.

In [19], it is claimed that liquidity risk should be priced into liquidity and transparency instruments in such a way that buying exposure to the same default risk is identically priced. [19] also investigates the potential spill over effects between the credit and liquidity spreads in the Eurozone sovereign bond market and the sovereign credit default swap (CDS) market during the European sovereign debt crisis (see [31] for more information). Similarly, in the current study, it will be investigated such and, in addition to that, reconsider how the BCBS responded to the crisis.

Working paper [23] shows that capital market frictions are significant for financial shock effects and the properties of optimal monetary and fiscal policy. For instance, for shocks that increase banks' demand for liquidity, optimal monetary policy accepts an output contraction, while it would not do it in the absence of the aforementioned frictions (or under suitably conducted fiscal policy). [36] uses semi-structural models; however, these models do not directly incorporate banks' balance sheet conditions and income statements as input variables. Instead, these effects must be incorporated into other variables such as lending spreads. This means that the first step is to map the impact of the higher liquidity requirements on lending spreads. It is, therefore, difficult to estimate the impact of the cost effect of capital (or liquidity) on output. Moreover, the computation of long-term effects is difficult, due to the size of the models and the fact that cost can only be approximated by simulations over a reasonably large number of years (see, for instance, [23]).

The ECB study (see [25]) and the MOC, MPC and BSC letters (see [26]) suppose that banks would face strong disincentives to lend and/or borrow on the unsecured money market. The breadth and depth on unsecured money markets would further shrink and the market would lose its allocation and distribution function for liquidity in the Euro area. The reduction of volumes and tenors in money markets would be stronger than desired and might have negative repercussions on the liquidity risk management, the liquidity risk exposure and the liquidity risk absorption capacity of banks. Banks would further lose confidence in the unsecured money market and in segments of the secured money market. The system of pooling collateral would be under pressure. Banks would prefer an 'ear marking' system which would allow them to single out the assets that are available to cover the LCR (see also [52]).

2.4.3.2 Challenges that are related to financial institutions

As a result of post-crisis regulatory reform, [41] claims that the long-term trend toward local banking is likely to accelerate, especially if liquidity regulations are applied locally. Furthermore, [1] claims that aggregate liquidity can be seen as the rate of change of the aggregate balance sheet of financial intermediaries.

[3] demonstrates that Basel III will force banks to shift their business model from liability management, in which business decisions are made about asset volumes, with the financing found in short-term wholesale markets as necessary, to asset management, in which asset volumes are constrained by the availability of funding. Paper [3] claims that the new liquidity rules could create new, unintended systemic risks. In particular, the proposed definition of eligible liquid assets is dangerously over-concentrated on government debt. It is stated by [3] that, in the light of the experience of the recent crisis, there clearly need to be adequate contingency planning for any future crises. There is a strong case for publication of the plan, including information about the circumstances in which emergency liquidity assistance might be provided.

[56] claims that the main channel through which changes in liquidity regulation affect economic activity is via an increase in the cost of bank intermediation. Banks will increase lending rates to compensate for the cost of holding more capital and liquidity. Owing to imperfect substitutability between bank credit and other forms of market financing, this leads to lower investment and lower output. The computation of the steady-state economic costs of higher liquidity requirements for the level of output is based on a variety of macroeconomic models. The models differ in many respects. Firstly, they refer to different countries or areas. Secondly, some are almost fully estimated, whereas others are entirely calibrated (the values of the coefficients are taken from unrelated, generally microeconomic studies, casting light on the specific parameters). Finally, some models feature explicitly a banking sector and a role for bank capital and liquidity, while others do not (see, for instance, [56]).

[24] studies the determinants of bank capital buffers that neglected the role of liquidity. On the whole, it is found that banks do not strengthen their capital buffer when they face higher illiquidity as defined in the Basel III accords or when they create more liquidity as measured previously, as stressed by the BCBS. Their findings also shed light on the need to clarify further how to define

and measure illiquidity. The aim of [24] is to study the relationship between bank capital buffers and liquidity. Based on previous studies, highlighting that capital and liquidity are presumably jointly determined, the authors consider a simultaneous-equations model to investigate the impact of liquidity on the capital buffer beyond the determinants that are considered in the existing literature. Specifically, [24] questions if banks maintain or strengthen their capital buffer when they face lower liquidity, since regulatory requirements regarding liquidity have not yet been implemented. The main results of the authors show that banks hold lower capital buffers when they create more liquidity, that is when they fund larger portions of illiquid assets with liquid liabilities. By contrast, banks do not build up capital buffer when they face higher illiquidity as defined in the Basel III accords, in other words when they fund illiquid assets more extensively with unstable liabilities. Nevertheless, the definition of 'stable funding' might be adjusted in the US case. By considering an alternative indicator of liquidity that is adjusted for the importance of core deposits for US banks, it is found that, except for the 20 very large institutions, US banks do actually increase their capital buffer when they face higher illiquidity. The findings of the authors support the need to implement minimum liquidity ratios concomitantly to capital ratios as stressed by the BCBS, but they also cast doubts on the accuracy of the current framework. Adding liquidity ratios to capital ratios might be more relevant for the very large systemic institutions than for smaller banks. Moreover, the definition and the measurement of liquidity need to be further clarified under a global regulatory framework. Regulators need to focus on what type of liquid liabilities should be considered as stable for a deeper regulatory definition of the notion of 'core' or 'stable' deposits. Their findings also raise questions regarding the implementation of uniform liquidity requirements to all types of banks if very large institutions either behave differently because of their too-big-to-fail position or are able to manage their liquidity differently (see, for instance, [24]).

[42] concludes that global liquidity provision is highly procyclical. Even when central banks have the ability to create abundant official liquidity, there should be some limits to its support for the financial sector. It then proceeds to review the approaches that have been adopted to assess the GDP loss that is associated with a specific banking crisis. Despite the potential negative side-effects of the standards on monetary policy implementation, there is a broad agreement that the Basel III liquidity standards constitute an important improvement of financial regulation. While they have some conceptual shortcomings, they are broadly in line with a functional approach to liquidity regulation. The burden of adjustment thus rests on monetary policy implementation in the various currency areas rather than on fundamental changes to the standards themselves (see, for instance, [53]). The structural liquidity deficit implies that money market participants demand more central bank reserves on aggregate than are available on the market. In principle, the variation of minimum reserve requirements would be an additional instrument for central banks to manipulate aggregate demand for their reserves and volatility throughout the maintenance period. Minimum reserve requirements are intended to stabilize the demand for central bank money. In [50], the shortfall of the BCBS with regard to liquidity, namely the failure to factor in the role of central banks as lenders of last resort, is demonstrated. Moreover, the aforementioned contribution highlights the BCBS's acknowledgement of the fact that particular runs are less likely for certain institutions in

wich deposit base and deposit insurance exist than is the case for institutions with greater wholesale funding. [50] also adds that the likelihood of runs taking place in the case of banks with a high percentage of liabilities in deposits still exists.

Paper [39] concludes that UK banks need to control their liquidity, debt, capital and profitability financial ratios better. According to [39], Basel III has laid down suitable recommendations, but they are insufficient. In particular, shadow banking should be subject to the same rules and regulations as the banking sector to create a level playing field.

2.4.4 Challenges that are related to liquidity and other risks

Paper [3] claims that the new liquidity rules could create new, unintended systemic risks. In particular, the proposed definition of 'eligible liquid assets' is dangerously over-concentrated on government debt. The definition of this should be broadened to give banks more scope to hold liquidity in the form of commercial claims; central banks should clarify in what circumstances they will provide emergency liquidity assistance.

The next challenge that banks face is interfacing or merging their current risk and finance systems to meet the new Basel III LCR ratio requirements. The funding concentration monitoring requirement will require banks to put a clean hierarchical referential of counterparties in place for consolidating their liabilities. Different LCR ratios will have to be produced per consolidation level and currencies. As is already the case for credit risk rules, international banks will have to cope with various national discretions and local conditions for such new liquidity ratio rules, and will have to generate various kinds of liquidity risk regulatory reporting templates in different electronic formats per jurisdiction.

The liquidity proposal favours more non-risky assets to be maintained. Some countries' sovereign bonds are risky and to treat all government bonds as non-risky may impact on the solvency of banks. This situation may impact on the profitability of banks and consequently their franchise value. The main shortcoming of the net stable funding ratio is also that it depends on the capacity of banks and supervisors to model investor behaviour.

The importance of the link between liquidity risks and systemic risks within the banking sector is highlighted by the consequences that are attributed to the reluctance of banks to retain liquidity, given the cost of holding liquidity. The consequential shortfalls of liquidity, as reflected by on- and off-balance sheet maturity mismatches, accentuate the importance of the role that is assumed by central banks in the funding of bank balance sheets (see, for instance, [43]). Regulatory liquidity risk reports will have to be produced at least monthly with the ability, when required by regulators, to be delivered weekly or even daily. In this regard, banks are responsible to make robust, automated reporting solutions available to meet this need. This may be problematic where a framework does not exist.

Papers [27] and [55] use an error correction model to estimate long-term output reduction that is caused by higher lending spreads, arising from stronger liquidity standards. The main advantage of this approach is that it helps to disentangle loan demand and supply factors in the steady state. Based on aggregated historical data, it can establish a long-run relationship between liquidity and the reduction in output. The main disadvantage is that it does not allow for the conduct of counterfactual experiments such as the introduction of countercyclical capital buffers (see, for instance, [15] and [17]). Also, [28] claims that most of the familiar events in the financial crisis are more linked to movements in liquidity risk than credit risk (compare with [8]).

Paper [38] provides formal empirical support to the BCBS's proposal to penalize excessive exposures to liquidity risk by showing that short-term wholesale funding, a variable capturing interconnectedness, makes a significant contribution to systemic risk. Given the relevance of liquidity strain as a contributing factor to systemic risk, the regulation of systemic risk could be strengthened by giving incentives to disclose contingent short-term liabilities, in particular those related to possible margin calls under credit default swap contracts and repo funding (see, for instance, [38]).

In [54], the impact of new rules that will require banks to hold capital reserves against market, credit and liquidity risk in their trading books are explored. It is found that the new requirement for incremental credit risks, namely the incremental risk charge, may be substantial when compared with old regulatory capital levels. However, stressed VaRs that have been estimated on corporate bond portfolios during the current crisis show that market risk-related losses may be far greater and, depending on the characteristics of the portfolio, may be more than ten times as large as the incremental risk charge. These findings are at odds with evidence that has been presented by the BCBS. The results show a much greater increase in capital, following the introducing of the new rules. It is conjectured that banks may be able to report lower sensitivity to the new regulation because of the risk reductions they may achieve through hedging (see, for instance, [54]). However, if this was the case, one may question the extent to which hedging counterparts may be able to fulfil their contractual obligations in the case of a shock such as the one observed in the recent crisis. If counterparty risk was substantial, then the efficacy of the new bank capital adequacy rules may be greatly impaired by the assumptions that banks make about their ability to manage risk in extreme scenarios. Future research should aim to shed more light on this important issue. Further investigation should also be directed to the influence that the stressed VaR requirement will have on the pricing of tradable assets, as well as its implications for banks' asset allocation strategies. For instance, if (following the introduction of the new rules) trading book instruments like stocks and bonds become too capital intensive relative to bank loans, then banks may prefer to redirect their investments to the latter type of assets (see, for instance, [54]). The result could be a move towards more traditional banking, which would counter the trend that has been observed in the last decades and that culminated in the repeal of the Glass and Steagal act in 1999. The implications for banks' profitability, availability of credit, financial stability and economic growth may be substantial and deserve further research (see, for instance, [54]).

2.4.5 Challenges that are related to the broader economy

The staff report [4] claims that each 1% increase in the LCR causes a median 0.08% decline in the level of steady-state output, relative to the baseline (see also [16]).

Reduced credit availability and higher financing costs could affect household and firm spending. While the regulator has taken this concern into account, planning a long transition period (until 2018), the new rules could cause subdued gross domestic product (GDP) growth in the short to medium term. For each 1% increase in the capital requirement, implemented over an eight-year horizon, the level of GDP relative to the baseline path would decline at trough by 0.00-0.33%, depending on the estimation method (0.03-0.39%, including non-spread effects); the median decline at trough would be 0.12% (0.23%, including non-spread effects). The trough occurs shortly after the end of the transition period; thereafter, output slowly recovers and by the end of 2022 it is above the baseline value. Based on these estimates, the reduction of the annual growth rate of output in the transition period would be in a range of 0.00-0.04% (0.00-0.05%, including non-spread effects). The fall in output is driven for the most part by the slowdown in capital accumulation, which suffers from higher borrowing costs (and credit supply restrictions). Compliance with the new liquidity standards also yields small costs. The additional slowdown in annual GDP growth is estimated to be at most 0.02% (see, for instance, [37]). These results are broadly similar to those shown in [40], derived for the main G20 economies. If banks felt forced by competitors or financial markets to speed up the transition to the new capital standards, the fall in output could be steeper and quicker. Assuming that the transition is completed by the beginning of 2013, GDP would reach a trough in the second half of 2014; for each 1% increase in capital requirements, GDP would slow down by 0.02-0.14% in each year of the 2011-2013 period (see, for instance, [37]). It would subsequently rebound, partly compensating for the previous fall. The long-run costs of achieving a 1% increase in the target capital ratio are also small (slightly less than 0.2% of steady state GDP); those needed to comply with the new liquidity requirements are of similar size. Econometric estimates are typically subject to high uncertainty; those presented in this paper are no exception. The main finding of this paper is nonetheless shared by several other studies, namely that the economic costs of stronger capital and liquidity requirements are not huge and become negligible if compared with the potential benefits that can be reaped from reducing the frequency of systemic crises and the amplitude of boom-bust cycles.

[12] evaluates that if the capital ratio increases by 1% relative to the historical average, the expected net benefits in terms of GDP level would be in a range of 0.20-2.32% (0.25-3.33% if liquidity requirements are also met), depending on whether financial crises are assumed to have a temporary or a permanent effect on output. Even taking the most cautious estimate, the gains undoubtedly outweigh the costs to be paid to achieve a sounder banking system (see, for instance, [37]).

2.5 Conclusions and future directions

In this article, Questions 2.1.1, 2.1.2 and 2.1.3 about the Basel III liquidity regulation and its components, as well as potential benefits and challenges are answered.

Basel III can be seen as an opportunity, as well as a challenge for banks. It can provide a solid foundation for the next developments in the banking sector and it can ensure that past excesses are avoided. Basel III is changing the way that banks address the management of risk and finance. The new regime seeks much greater integration of the finance and risk management functions. However, the adoption of a more rigorous regulatory stance might be hampered by a reliance on multiple data silos and by a separation of powers between those who are responsible for finance and those who manage risk. The new emphasis on risk management that is inherent in Basel III requires the introduction or evolution of a risk management framework that is as robust as the existing finance management infrastructures. As well as being a regulatory regime, Basel III provides in many ways a framework for true enterprise risk management, which involves covering all risks to the business.

As highlighted in a previous paper, "the monitoring of useful data such as market-wide data on asset prices and liquidity, institution-related information such as credit default swap (CDS) spreads and equity prices, additional institution-specific information related to the ability of the institution to fund itself in various wholesale funding markets and the price at which it can do so, will be vital in obtaining a source of instantaneous data on potential liquidity problems".

Until intended leverage ratios are introduced and coupled with the new liquidity standards, nasmely LCR and NSFR requirements, these standards will probably not achieve half their desired effects, since liquid assets could be accumulated under these standards to an extent where they are susceptible to sovereign exposures. This is one reason (amongst many) for concluding that, whilst the BCBS has gone a long way in addressing liquidity risks, its efforts still remain a modest milestone in combating liquidity risks in prudential supervision.

There appears to be consensus that no single tool or measure would have prevented the financial crisis and that an adequate policy response requires a menu of macro- and micro-prudential policy tools. The leverage ratio can be a useful prudential tool, and one that can be relatively easy to implement, for jurisdictions that do not want to rely solely on risk-sensitive capital requirements, though it is no silver bullet. Combining the leverage ratio with Basel-type capital, rules can reduce the risk of excessive leverage building up in individual entities and in the system as a whole.

As the financial crisis showed, however, policy makers need to be cognizant of the inherent limitations and weaknesses of the leverage ratio. The proposals at an international level to supplement risk-based measures with an internationally harmonized and appropriately calibrated leverage ratio are welcome and could lead to their adoption by a wide range of countries in the future. A leverage ratio cannot do the job alone; it needs to be complemented by other prudential tools or measures to ensure a comprehensive picture of the build up of leverage in individual banks or banking groups, as well as in the financial system. Additional measures to provide a comprehensive view of aggregate leverage (including embedded leverage) and to trigger enhanced surveillance by supervisors need to be developed.

The solution paradigm that has been proposed in Subsection 2.2.2 can be used as an extension to the existing banking solution, satisfying Basel II. The proposal achieves a robust framework for liquidity risk measurement and monitoring according to Basel III. The framework provides a dynamic approach to maintain banks' liquidity considering various idiosyncratic and market-wide shocks and thus effecting liquidity. The proposal includes a solution framework to implement new concepts and tools that correspond to Basel III, like the two liquidity standards. It doesn't include Basel III proposals, which improve on the existing Basel II norms. Nevertheless, the framework is flexible to include any further tools or concepts that are yet to be introduced into Basel III. As an example, the framework can include leverage ratio, which is one of the new proposals of Basel III. This tool can be included into the solution at later times, after the BCBS has determined the calculation and calibration details. Based on the proposal made in this paper, a future research or study would be to develop an end-to-end application, implementing the framework for liquidity risk measurement and monitoring in banks and financial institutions. As mentioned, the framework can also be extended with new standards or tools that are yet to be released by BCBS.

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Chapter 3

LIQUIDITY AND MORTGAGE FUNDING

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Subprime mortgage funding and liquidity risk

By

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Abstract: In this article, actuarial methods are used to solve a nonlinear stochastic optimal liquidity risk management problem for subprime originators with deposit inflow rates and marketable securities allocation as controls. The main objective is to minimize liquidity risk in the form of funding and credit crunch risk in an incomplete market. In order to accomplish this, a stochastic model that incorporates originator mortgage and deposit reference processes is constructed. Finally, numerical examples that illustrate the main modelling and optimization features of this article are provided.

Keywords: Subprime Mortgages, Mortgage Funding, Marketable Securities, Deposits, Mortgage and Deposit Reference Processes, Liquidity Risk, Subprime Mortgage Crisis.

3.1 Introduction

Prior to 2007, the booming US house market resulted in large amounts of subprime mortgage loans (simply known as *mortgages* hereafter), involving high risk mortgagors being originated. In this period, large amounts of deposits flowed into the US banking systems which funded mortgages in a low interest rate environment. Consequently, credit was easy to obtain, thus boosting housing and credit markets. Originators passed on mortgages via securitization to shift credit risk to investors in risky marketable securities such as residential mortgage-backed securities (RMBSs) and collateralized debt obligations (CDOs). As a consequence, mortgage innovation rose dramatically (see [6] and [7] for more details). The proceeds of mortgage origination were invested in marketable securities for higher yields than earnings on treasuries. In turn, the funds from deposits and marketable securities were then used for funding new subprime mortgages. Subsequent to 2007, during the subprime mortgage crisis (SMC), there was a sharp rise in delinquencies and foreclosures of such mortgages. This had a major adverse impact on the liquidity of financial institutions and markets around the globe. Figure 3.1 below illustrates some of the issues that have been raised before.

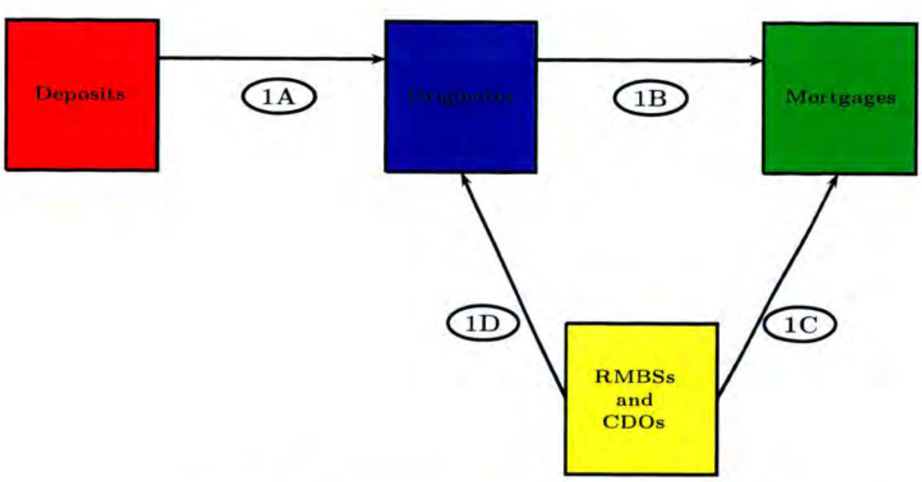


Figure 3.1: Diagrammatic overview of mortgage funding

In Figure 3.1, **1A** represents deposit inflows from investors to the originator. This enables the originator to originate mortgages in **1B**. Next, **1C** indicates that mortgages are securitized into risky marketable securities. Originators use earnings that are received from RMBSs and CDOs (**1D**) to fund new mortgages. This cycle repeats itself with subprime mortgages being funded by deposits and marketable securitie, and vice versa (see, for instance, [6]). During the SMC, this cycle collapsed when mortgagors started to default and securities lost their value. Financial institutions suffered greatly from illiquidity with losses reaching US$ 300 billion by June 2008 (see [7]).

In this article, the authors solve a nonlinear optimal liquidity risk management problem for subprime originators in continuous-time. Subprime mortgages are extended to mortgagors who do not qualify for market interest rates due to factors such as income level, size of the down payment made, credit history and employment status. Some originators regularly evaluate their mortgage origination volumes, as well as levels of deposits and marketable securities. The *actuarial cost method* may be used for this valuation. This method enables the authors to specify an optimal rate for deposit inflows to fund mortgages, referred to as the *deposit reference process*, D^r. Also, a desired rate of mortgage origination, called the *mortgage reference process*, will be denoted by M^r. These levels are chosen optimally in order for acceptable mortgage origination rates to be reached at the end of a specific time period. In this process, it may be that excessive or insufficient mortgage origination will develop. As a result, appropriate methods should be available in order to adjust the rate of deposit and marketable securities inflow so that mortgages can be extended optimally. The *spread method of mortgage funding* is a well-known method for handling such adjustments. This technique allows for unfunded mortgage origination to be spread over a given period that is called the *spread period*.

In the sequel, originators aim to minimize liquidity risk in the form of funding and credit crunch risk in order to ensure originator stability and security. *Liquidity risk* refers to the risk of an originator that is unable to supply sufficient liquid or cash assets to meet depositor and borrower demand. In this article's case, *funding risk* relates to the lack of funds or deposits to finance mortgages, while *credit crunch risk* refers to the risk of tightened mortgage origination and increased credit standards. A measure of funding risk is the deviation of the deposit inflow rate from D^r, which is associated with the stability of funding mortgage origination. The level of unfunded subprime mortgages is an indicator of the extent of credit crunch risk. The SMC significantly disrupted the origination of subprime mortgages, which led to liquidity problems that are related to the inability of originators to obtain funding.

The discussion above has connections with various strands of literature. According to [1], before the SMC, returns from securitization provided a reliable source of funding for mortgages. In this regard, [7] demonstrates that converting illiquid mortgages into liquid marketable securities causes origination to be less sensitive to the cost of fund shocks, because securitization provides additional funds (see, for instance, [6]). As a consequence, securitization causes an increase in mortgage supply and a decrease in liquid securities holdings. In this case, [7] shows that mortgage supply is less sensitive to external sources of funding and therefore reduces the effect monetary authority has on origination. The authors' subsequent findings support the views expressed in [1] and [7]. The impact of speculative mortgage funding on the pricing of subprime mortgages and securities backed by these mortgages is studied in [5]. By contrast, in the current contribution, the authors concentrate on funding sources such as deposits (including demand deposits and interbank borrowing) and proceeds from mortgage securitization that are less speculative by nature (see also [6]). Before the SMC, elevated liquidity in the credit markets facilitated mortgage origination to risky subprime mortgagors, as well as securitization to fund more mortgage origination (see, for instance, [7]). According to [3], securitization altered the originators' liquidity, credit and maturity transformation duties during the SMC. They found that mortgage securitization sheltered originator mortgage supply from liquidity problems. [2] also analyzes how mortgage supply and credit crunch risk interact. They find that originators with lower expected default rates have the ability to originate more mortgages and therefore increase their supply. In principle, securitization is a substitute for an originator's on-balance sheet liquidity and therefore increases liquidity that is related to mortgage origination (see also [6]). In fact, highly liquid mortgage portfolios lessen the impact of shocks to external funding costs. This article's discussion on originator leverage has connections with [8]. Furthermore, [5] investigates how deposit inflows were affected by the SMC. In essence, depositor behaviour resulted in bank runs, illiquidity and originator insolvency (see also [6] and [7]).

3.1.1 Main questions and article outline

In this subsection, the main questions are stated and an outline of the article is provided.

3.1.1.1 Main questions

The authors' main objectives are to determine a solution to an optimal liquidity risk management problem for subprime originators and to consider its connection with the SMC. The aforementioned problem is solved on an infinite time horizon under a positive discount rate. Characterizing deposit inflow and marketable securities returns in an incomplete market will be crucial in achieving the aforementioned. The primary questions are posed below.

Question 3.1.1 (An optimal originator liquidity risk management question): *What should the optimal levels of originator deposit inflows and marketable security returns be in order to reach the optimal specified level for mortgage origination via the financing spread method?*

Question 3.1.2 (Optimal liquidity and the SMC): *What is the connection between the solution to Question 3.1.1 and the SMC?*

3.1.1.2 Outline of the article

The current section is introductory in nature. The remaining sections of this article are structured as follows. In Section 3.2, the authors discuss subprime mortgages and marketable securities, deposits and equity, as well as associated reference processes in Subsections 3.2.1, 3.2.2 and 3.2.3 respectively. Furthermore, the authors solve an optimal originator liquidity risk management problem that is associated with subprime mortgage funding via marketable securities and deposits (see Section 3.3). This includes discussions on mortgage funding and the SMC (see Subsections 3.3.1 to 3.3.4), as well as numerical examples involving mortgage funding (see Subsection 3.3.5). Section 3.4 presents a few concluding remarks and highlights some possible topics for future research.

3.2 Stochastic model for subprime originators

The model that is constructed in this section is related to [7], where originator equity and mortgage origination are aligned. The probability space, $(\Omega, \mathcal{F}, \mathbf{P})$, is considered on a time period $T = [t_0, t_1]$. The one-dimensional Brownian motion, $\{W_t^l\}_{t\geq 0}$, can be associated with the right continuous filtration, $\mathcal{F} = \{\mathcal{F}_t\}_{t\geq 0}$, with probability $\mathbf{P}$ which is measured on Ω. The mortgage parameters that are considered, include subprime mortgages, $M : \Omega \times T \to \mathbb{R}_+$, risky marketable securities, $\widetilde{B} : \Omega \times T \to \mathbb{R}_+$, and riskless Treasuries, $\mathtt{T} : T \to \mathbb{R}_+$, as well as deposits, $D : \Omega \times T \to \mathbb{R}_+$, and equity, $E : \Omega \times T \to \mathbb{R}_+$.

3.2.1 Subprime mortgages and marketable securities

In this subsection, models for subprime mortgages and marketable securities are considered.

3.2.1.1 Subprime mortgages

The stochastic process, $l : \Omega \times T \to \mathbb{R}$, represents the mortgage origination rate, with value l_t at time t. It is acceptable for l to be modelled as a path-continuous scalar Itô process that is defined on $(\Omega, \mathcal{F}, \mathbf{P})$, so that

$$l_t = l_0 + \int_{t_0}^{t_1} c^M(l_s, s)ds + \int_{t_0}^{t_1} \sigma^M(l_s, s)dW_s^l, \ \ t \geq 0, \ l_0 = l_{t_0},$$

where $c^M \in \mathbb{R}$ is the drift coefficient and $\sigma^M \in \mathbb{R}_+$ the volatility of a positive mortgage origination rate. This enables liquidity risk management in an uncertain environment with

$$dl_t = l_t[c^M dt + \sigma^M dW_t^l], \ l_0 = l_{t_0}, \ \ dM_t = l_t dt \tag{3.1}$$

and

$$l_t = l_0 \exp\{\left(c^M - \frac{(\sigma^M)^2}{2}\right)t + \sigma^M W_t^l\}. \tag{3.2}$$

Before the SMC, low credit risk resulted in an increase in mortgage origination. An increase in liquidity and competition in the credit market pressured originators eventually to extend mortgages to subprime mortgagors, which led to a flourishing market for subprime marketable securities. Subsequently, the SMC affected the whole economy with mortgage supply and credit conditions being tightened to preserve regulatory capital reserves. This led to liquidity problems, which negatively affected the mortgage origination rate in (3.1).

3.2.1.2 Marketable securities

Marketable securities are available for funding the origination of subprime mortgages. Here, the originator has $n+1$ categories of marketable securities. Suppose that the $n+1$-th securities class corresponds to riskless treasuries, $\mathtt{T}(t)$, that is deterministic. Also, the risky marketable securities are given by $\widetilde{B}_t^1, \ \ldots, \ \widetilde{B}_t^n$, where $1, \ 2, \ \ldots, \ n$. The dynamics of the treasuries may be represented by

$$d\mathtt{T}(t) = \mathtt{T}(t)r^{\mathtt{T}}dt, \ \mathtt{T}(t) = 1, \tag{3.3}$$

where $r^{\mathtt{T}}$ is the rate of return from treasuries that is described in (3.3). Next, the stochastic dynamics of marketable securities is expressed as

$$d\widetilde{B}_t^i = \widetilde{B}_t^i[b^i dt + \sum_{j=1}^{n} \sigma^{ij} dW_t^j], \ \ \widetilde{B}_0^i = \mathtt{T}, \ \ 1 \le i \le n. \tag{3.4}$$

with $b^i \to \mathbb{R}_+$, $\sigma^{ij} \to \mathbb{R}_+$, $1 \le i, j \le n$. $(W_t^0, \ W_t^1, \ \ldots, \ W_t^n)^T$ is an $n+1$-dimensional vector on $(\Omega, \mathcal{G}, \mathbf{P})$, where the completion of the filtration, $\sigma\{(W_t^0, \ W_t^1, \ \ldots, \ W_t^n)^T\}$, with condition, $0 \le s \le t$, is represented by $\{\mathcal{G}_t\}_{t \ge 0}$. Suppose there exists a correlation of $q^i \in [-1, 1]$ between W^l and W^i for $1 \le i \le n$. As a consequence, the authors have that $\mathbf{E}(W_t^l W_s^i) = q^i \min(t, s)$ for $i = 1, \ \ldots, \ n$ and

$$W_t^l = \sqrt{1 - \widetilde{q}^T \widetilde{q}} W_t^0 + \widetilde{q}^T \widetilde{W}_t, \text{ where } \widetilde{q}_t = (q_t^1, \ \ldots, \ q_t^n)^T \text{ and } \widetilde{W}_t = (W_t^1, \ \ldots, \ W_t^n)^T. \tag{3.5}$$

When $\widetilde{q}^T \widetilde{q} \neq 1$, then the risk associated with subprime mortgage origination cannot disappear by trading in the financial market. If l_t is contingent and $q^i \in [-1, 1]$, then (3.5) follows the market price of risk, $\widetilde{\xi}$, represented by (3.6) when $\widetilde{q}^T \widetilde{q} \neq 1$. Also, the *market price of risk*, $\widetilde{\xi}$, is given by

$$\sigma \widetilde{\xi} = \widetilde{b} - r^{\mathtt{T}} \mathrm{I} \tag{3.6}$$

with $\widetilde{b} = (b^1, \ \ldots, \ b^n)^T$, a column vector, $\overline{1}$, consisting of $1's$ and an invertible matrix, σ. Note that originators bear the risk which is associated with the marketable securities price that is reflected in the risk premium. It is generally described by the *reward-to-risk ratio* of the market portfolio. Furthermore, in the situation described above, the *risk premium* can be computed, ρ^i, on risky marketable securities, i, as

$$\rho^i = \sum_{j=1}^{n} \sigma^{ij} \widetilde{\phi}^j, \tag{3.7}$$

where $\widetilde{\phi} = (\widetilde{\phi}^1, \ \ldots, \ \widetilde{\phi}^n)^T$. It is deduced from (3.6) and (3.7) that

$$b^i = r^{\mathrm{T}} + \sum_{j=1}^{n} \sigma^{ij} \widetilde{\phi}^j. \tag{3.8}$$

Equation (3.8) reflects the fact that returns from risky marketable securities are usually higher than returns from treasuries, so that it is realistic to have $b^i > r^{\mathrm{T}}$ for each $i = 1, \ 2, \ \ldots, \ n$. Also, it is noted that investing in the i-th marketable securities class yields a total return of b^i. In this case, (3.4) will become

$$d\widetilde{B}^i_t = \widetilde{B}^i_t[(r^{\mathrm{T}} + \sum_{j=1}^{n} \sigma^{ij} \widetilde{\phi}^j)dt + \sum_{j=1}^{n} \sigma^{ij} dW^j_t], \quad \widetilde{B}^i_0 = \mathrm{T}, \quad 1 \leq i \leq n.$$

In the following discussion, σ denotes the invertible matrix, (σ^{ij}), which results in the symmetric matrix, $C = \sigma\sigma^T$, being positive definite. Furthermore, π^i_t denotes the value of the marketable securities that are invested in securities i at time t, for $i = 0, \ \ldots, \ n$. In this case,

$$B - \sum_{i=1}^{n} \pi^i$$

represents the value of riskless treasuries. Marketable securities are unbounded with a negative value of π^i, implying that the originator may take part in the short selling of securities. On the other hand, if

$$B - \sum_{i=1}^{n} \pi^i < 0,$$

then the originator may borrow money to invest in securities at rate r^{T}. It is clear that the marketable securities dynamics can be expressed as

$$dB_t = \sum_{i=1}^{n} \pi_t^i \frac{d\widetilde{B}_t^i}{\widetilde{B}_t^i} + \Big(B_t - \sum_{i=1}^{n} \pi_t^i\Big) \frac{d\mathrm{T}(t)}{\mathrm{T}(t)} + [c_t - l_t]dt. \tag{3.9}$$

Finally, the authors have the *portfolio process* or *trading strategy*, $\widetilde{\pi}_t$, given by $\widetilde{\pi}_t = (\pi_t^1, \ \ldots, \ \pi_t^n)^T$. Here, $\widetilde{\pi}_t$ is taken to be an $\mathbb{R}^n$-measurable process that is adapted to $\{\mathcal{G}_t\}_{t\geq 0}$, so that

$$\int_0^\infty (\widetilde{\pi}_s)^T \widetilde{\pi}_s ds < \infty, \text{ a.s.} \tag{3.10}$$

Securitization incentivized originators to operate with low equity levels. Before the SMC, it seemed that originators had less bad mortgages, less risk and higher profits. According to [7], the flaws in the securitization system were hidden because of the increasing house prices, as well as fund inflow from these securities to extend more subprime mortgages.

3.2.2 Deposits and equity

In this subsection, the dynamics of deposits and equity are described.

3.2.2.1 Deposits

In this article, the term 'deposits' include both demand deposits and interbank borrowing. The dynamics of deposits, D, can be expressed as

$$dD_t = c_t dt, \quad D_0 = D_{t_0}, \tag{3.11}$$

with the *deposit inflow rate* denoted by c, $c_0 = c_{t_0}$. The deposit inflow rate is assumed to be a measurable adapted process with respect to the filtration, $\{\mathcal{F}_t\}$, which satisfies

$$\int_0^\infty |c_s| ds < \infty, \text{ a.s.} \tag{3.12}$$

Before the SMC, originators had a strong deposit inflow that funded mortgages. During this crisis, originators' deposits were used for funding subprime mortgages. According to [5], during the SMC, originators hid risky mortgages. However, after their true financial status was revealed, depositors and investors lost their confidence in these originators, and bank runs, insolvency and illiquidity followed. This caused the erosion of originator deposit levels and, as a result, originator deposit inflow for mortgage funding was compromised.

3.2.2.2 Equity

An originator balance sheet approach as in [7] allows the authors to express the dynamics of equity, E_t, as

$$dE_t = \sum_{i=1}^{n} \pi_t^i \frac{d\widetilde{B}_t^i}{\widetilde{B}_t^i} + \left(B_t - \sum_{i=1}^{n} \pi_t^i\right) \frac{d\mathrm{T}(t)}{\mathrm{T}(t)}. \tag{3.13}$$

During the SMC, defaults on subprime mortgage products reduced originator equity levels significantly and left many of them insolvent. Originators that were deemed to be too-big-to-fail acquired equity from private and public sources in order to provide for these losses. In general, originator equity can be equated with net worth. In this regard, the leverage ratio is the assets-to-net worth ratio, while the debt ratio is the liabilities-to-net worth ratio (see, for instance, [8]). This means that, in the authors' case, the leverage and debt ratios can be expressed as

$$\Lambda_t = \frac{M_t + B_t}{E_t} \text{ and } \Delta_t == \frac{D_t}{E_t}, \tag{3.14}$$

respectively. Excessive leverage and debt ratios contributed to the SMC. Although leverage is a valuable tool in managing mortgages and marketable securities, excessive leverage poses huge financial risks. Changes in these assets significantly magnify changes in equity (net worth) for originators that are highly leveraged. Furthermore, in order to maintain the same debt ratio when the value of mortgages and marketable securities decrease, the originator must either raise more equity or liquidate assets.

3.2.3 Mortgage and deposit reference processes

In order to establish an ideal level of subprime mortgage origination, the authors consider two reference processes, namely the mortgage and deposit reference processes. The *discounted rate of interest*, denoted by r^d, is related to originator valuation. Actuarial cost methods are used to determine charges against annual operating techniques, and also as a measure of required levels of originator assets and liabilities at any given time. To the authors' knowledge, this technique has not been used to determine subprime mortgage origination levels before. The unfunded subprime mortgages, M^u, that are related to the individual cost method, are determined by the difference between the originator mortgage reference process, M^r, and the marketable securities that are available for mortgage origination, B. Symbolically, it is

$$M_t^u = M_t^r - B_t. \tag{3.15}$$

Additional deposits, D^a, are used to achieve the required deposit adjustment. In this case,

$$D_t^a = c_t - D_t^r. \tag{3.16}$$

3.2.3.1 Description of mortgage and deposit reference processes

Throughout this subsection, $\mathbf{E}(\cdot|\mathcal{F}_t)$ represents the conditional expectation with respect to $\mathcal{F}_t$ that is associated with $\{W_t^l\}_{t\geq 0}$. The *mortgage reference process* describes the value which is required for an originator to extend subprime mortgages that are determined by actuarial methods. The *deposit reference process* is the amount that is sourced from deposits for funding subprime mortgage origination. For the mortgage reference process,

$$M_t^r = \int_{t_0}^{t_1} \exp\{-r_{t_1-v}^d\} P_v \mathbf{E}(l_{t+t_1-v}|\mathcal{F}_t)dv, \quad P_{t_0} = 0. \tag{3.17}$$

The distribution of deposit accumulation during the mortgage funding period is done according to the distribution function, P, with p as the density function. The value P_v includes the percentage of the subprime mortgages to be originated and the accumulation during this time of funding. Also,

$$D_t^r = \int_{t_0}^{t_1} \exp\{-r_{t_1-v}^d)\} p_v \mathbf{E}(l_{t+t_1-v}|\mathcal{F}_t)dv, \quad p_{t_0} = 0 \tag{3.18}$$

represents the deposit reference process for $t > 0$. To calculate the reference processes given by (3.17) and (3.18), information that is available up to time t is used with respect to the conditional expectation. The Markov property is satisfied, since the assumption is made that l is a diffusion process. Therefore, $\mathbf{E}(\cdot|\mathcal{F}_t)$ equals the conditional expectation at period t in terms of the current value of l. Before the SMC, deposit inflows were strong, which had a positive effect on mortgage funding. In this period, attaining and exceeding the reference processes were relatively easy. During the SMC, deposit inflows plummeted and deposits were withdrawn, which caused illiquidity and the failure of many originators. Attaining the reference processes in this environment were problematic, with deposit inflow and mortgage origination falling short of the mark. It is observed that p_v is zero for $v \leq t_0$ or $v \geq t_1$, since $[t_0, t_1]$ is the support of p. In addition, it follows that

$$P^{'} = p. \tag{3.19}$$

3.2.3.2 Relationship between mortgage and deposit reference processes

In the sequel, r^a represents the rate of actualization. Here, actualization is defined as the present value of a stipulated process level in the future. When r^a is high, the originator will be more concerned about the present value than future values. Relationships between the mortgage and deposit reference processes, M^r and D^r, respectively, as well as the subprime mortgage origination rate, l, are explained by Theorem 3.2.1 below.

Theorem 3.2.1 (Relationship between M^r, D^r and l): *In the case where (3.1) holds, there exist constants Q and Z such that $D^r = Ql$ and $M^r = Zl$. Also, identities*

$$Q = 1 + (c^M - r^a)Z \tag{3.20}$$

and

$$(r^a - c^M)M_t^r + D_t^r - l_t = 0 \tag{3.21}$$

hold for every $t \geq 0$.

Proof 3.2.2 *Since (3.2) holds and l is modelled as a geometric Brownian motion, it follows that*

$$\mathbf{E}(l_{t+t_1-v}|\mathcal{F}_t) = l_0\mathbf{E}\big(\exp\left\{\left(c^M - \frac{(\sigma^M)^2}{2}\right)(t+t_1-v) + \sigma^M W^l_{t+t_1-v}\right\}|\mathcal{F}_t\big) = \exp\{c^M_{t_1-v}\}l_t$$

for every $v \in [t_0, t_1]$ and $t \geq 0$. The mortgage and deposit reference process constants may be given by

$$Z = \int_{t_0}^{t_1} \exp\{(c^M - r^a)(t_1 - v)\}P_v dv$$

and

$$Q = \int_{t_0}^{t_1} \exp\{(c^M - r^a)(t_1 - v)\}p_v dv, \tag{3.22}$$

respectively. This completes the first part of the proof.

In order to show that (3.20) holds, (3.17), (3.19) and (3.22) are considered. Also, from the first part of this proof, as well as (3.1) and (3.18), it follows that (3.21) holds.

The next corollary follows immediately from Theorem 3.2.1.

Corollary 3.2.3 (Relationship between M^u and D^a): *Assume that the hypothesis of Theorem* **3.2.1** *holds. Let constants Z and Q be non-zero, real values, then*

$$D_t^r = SM_t^r \text{ and } D_t^a = S[M_t^u + B_t] - l_t,$$

for $t \to \mathbb{R}$. *Here,* $S = \dfrac{Z}{Q}$ *represents an interest valuation rate in terms of* Q, c^M *and* r^a, *by*

$$S = \frac{1}{Q} + c^M - r^a.$$

In addition,

$$dM_t^r = M_t^r[c^M dt + \sigma^M dW_t^l], \tag{3.23}$$

where $M_t^r = M_0^r = Zm_0$.

3.3 Optimal liquidity risk management

Before the SMC, subprime originators shared the perception that liquidity risk was low. In addition, strong deposit inflows, high liquidity, low inflation and default rates, as well as elevated growth, incentivized greater risk-taking. Subsequently, the bursting of the housing bubble caused interest and default rates on mortgages to increase dramatically. During the SMC, equity and deposit levels plummeted, which resulted in a significant tightening of mortgage supply, as well as elevated credit standards. This resulted in a credit crunch that led to illiquidity. The SMC reinforced the idea that liquidity risk management is crucial in determining the capability of originators to originate mortgages. In this regard, it is important to ensure access to equity to improve liquidity and mortgage supply. In the sequel, these issues are discussed in both a theoretical- and numerical-quantitative framework.

3.3.1 Stochastic dynamics of marketable securities

By using (3.3), (3.4) and (3.9), the dynamics of marketable securities, B, can be expressed as

$$\begin{aligned} dB_t &= \sum_{i=1}^{n} \pi_t^i [b^i dt + \sum_{j=1}^{n} \sigma^{ij} dW_t^j] + (c_t - l_t)dt + \big(B_t - \sum_{i=1}^{n} \pi_t^i\big) r^{\mathrm{T}} dt \\ &= \big(r^{\mathrm{T}} B_t + \sum_{i=1}^{n} \pi_t^i (b^i - r^{\mathrm{T}})\big)dt + (c_t - l_t)dt + \sum_{i=1}^{n}\sum_{j=1}^{n} \pi_t^i \sigma^{ij} dW_t^j, \ B_t = B_0. \end{aligned}$$

This stochastic differential equation is rewritten by applying Theorem 3.2.1 so that

$$dB_t = \big(r^{\mathrm{T}}B_t + \sum_{i=1}^{n} \pi_t^i(b^i - r^{\mathrm{T}})\big)dt + (D_t^a - (c^M - r^a)M_t^r)dt + \sum_{i=1}^{n}\sum_{j=1}^{n} \pi_t^i \sigma^{ij} dW_t^j. \tag{3.24}$$

Here, the expected value of the unfunded subprime mortgages is elevated to zero by choosing additional deposits, D^a. One cannot trade the stipulated subprime mortgages to be originated so that the originator is unable to hedge the risk inherent in these mortgages. As a result, the discussion has to account for market incompleteness. Also, the state variables in (3.24) are marketable securities that are available for mortgage origination and M^r. It is also known that marketable securities allocation, π, and additional deposits, D^a, are the control variables. Originator marketable securities structure and levels changed dramatically during the SMC. Subprime securities, including RMBSs and CDOs, constituted a sizeable portion of the marketable securities sector. The implosion of the market for these securities caused significant originator losses and exacerbated the SMC.

3.3.2 Optimal originator liquidity risk management

In order for an optimal deposit inflow rate, c^*, and marketable securities allocation strategy, $\widetilde{\pi}^*$, to be determined, it is necessary to consider a well-defined objective function, V, with appropriate constraints. It is also important for solutions to the stochastic optimal control problem not to be ambiguous. Here, the control law, g, is chosen to minimize the objective function, $V : \mathcal{G}_{B_0,M^{r0}} \to \mathbb{R}_+$. All these parameters are significant during financial crises such as the SMC, as well as the global financial and debt crises. In this case, $\mathcal{G}_{B_0,M^{r0}}$ is the class of admissible control laws where

$$\begin{aligned}\mathcal{G}_{B_0,M^{r0}} = \{&\text{Markovian stationary } (D^a, \widetilde{\pi}) \text{ adapted to filtration } \{\mathcal{G}_t\}_{t\geq 0},\\ &\text{where both (3.10) and (3.12) hold with (3.23) and (3.24) a}\\ &\mathcal{G}_t\text{ -measurable continuous unique solution}\}.\end{aligned} \tag{3.25}$$

As a consequence, $(D^a, \widetilde{\pi}) \in \mathcal{G}_{B_0,M^{r0}}$ in (5.1) is well-defined. Also, the objective function, $V : \mathcal{G}_{B_0,M^{r0}} \to \mathbb{R}_+$, is given by

$$V(D^a, \widetilde{\pi}) = \mathbf{E}\bigg[\int_{t_0}^{t_1} \kappa(D^a)_s^2 + (1-\kappa)[M_s^r - B_s]^2 ds\bigg]. \tag{3.26}$$

It is noted that κ is a weighting factor with $0 < \kappa \leq 1$. This parameter reflects the relative importance of risks that are associated with deposit inflows and equity. The objective function (3.26) is formulated in order to minimize funding and credit crunch risk. More specifically, the authors want to minimize funding risk (insufficient deposit inflows), given by the cost on D^a, and credit crunch risk, which is reflected by costs on investing in the marketable securities, π. Of course, this depends on factors that are associated with mortgage origination. The stochastic optimal originator liquidity risk management problem can now be stated.

Problem 3.3.1 (Optimal originator liquidity risk management problem): *Suppose that* $\mathcal{G}_{B_0,M^{r0}} \neq \emptyset$. *The system (3.24) is considered for managing originator liquidity risk with control laws,* $\mathcal{G}_{B_0,M^{r0}}$, *stated in (5.1) and objective function,* $V : \mathcal{G}_{B_0,M^{r0}} \to \mathbb{R}_+$, *represented by (3.26). The authors want to solve*

$$\min_{(D^a,\widetilde{\pi})\in\mathcal{G}_{B_0,M^{r0}}} V(D^a,\widetilde{\pi}),$$

in order to determine optimal V^*, *given by*

$$V^* = \min_{(D^a,\widetilde{\pi})\in\mathcal{G}_{B_0,M^{r0}}} V(D^a,\widetilde{\pi}),$$

and the optimal control law,

$$(D^a,\widetilde{\pi})^* = \arg\min_{(D^a,\widetilde{\pi})\in\mathcal{G}_{B_0,M^{r0}}} V(D^a,\widetilde{\pi}) \in \mathcal{G}_{B_0,M^{r0}},$$

if it exists.

3.3.3 Spread method of mortgage funding

In this subsection, the spread method of originator mortgage funding is discussed. A formal characterization of this method follows below.

Definition 3.3.2 (Spread method of mortgage funding implementation): *Suppose that* D^a *and* M^u *are originator additional deposits and unfunded subprime mortgages, respectively. The spread method of originator mortgage funding can be implemented if there exists a constant,* k, *such that*

$$D^a = kM^u.$$

The availability of additional deposits and unfunded mortgages were severely affected by the SMC. Lemma 3.3.3 below provides a condition for the existence of a spread method of mortgage funding.

Lemma 3.3.3 (Existence of a spread method of originator mortgage funding): *Suppose that* σ^M, r^{T}, $\widetilde{q}^T$ *and* $\widetilde{\phi}$ *are characterized by* (3.1), (3.3), (3.5) *and* (3.6), *respectively. Then a rate of actualization,* r^a, *of the form,*

$$r^a = r^{\mathrm{T}} + \sigma^M \widetilde{q}^T \widetilde{\phi}, \tag{3.27}$$

exists and allows a spread method of originator mortgage funding to be implemented.

Proof 3.3.4 *The authors want to attain a specific rate of subprime mortgage origination, l_{t_1}, where $u = t_1$ is the end of the risk period. To be able to achieve this, l should be valued as in Theorem 3.2.1 such that*

$$\mathbf{E}(l_{t+t_1-u}|\mathcal{F}_t) = \exp\{(c^M - r^a)(t_1 - u)\}l_t.$$

This valuation should be insensitive to risk that is associated with mortgage origination. Therefore, in a risk neutral environment,

$$c^M - r^a = -(r^{\mathrm{T}} - \alpha^*) \tag{3.28}$$

is required, where $\alpha^* = c^M - \sigma^M \widetilde{q}^T \widetilde{\phi}$. The expression (3.27) follows from (3.28).

The equality (3.28) is required in order for the valuation of l to be insensitive to risk that is related to mortgage origination levels.

3.3.4 Optimal originator liquidity risk management

In this subsection, the authors consider the main optimal originator liquidity risk management results and the strategies that are associated with them.

3.3.4.1 The main optimal originator liquidity management results

Transversality is an obstacle to overcome when solving the infinite horizon optimal originator liquidity risk management problem. Here, the transversality condition with actualized form,

$$\lim_{t\to\infty} \exp\{k^a t\} h(B_t^*, M_t^r, t) = 0, \tag{3.29}$$

is considered for a candidate solution and feasible state trajectory given by B^* and M^r, respectively. In this case, constant, k^a, has to be carefully chosen (see, for instance, [4] and the references that are contained therein). The analysis is limited to an infinite horizon dynamic optimization problem, with accumulating infinite debt immediately being ruled out. A finite T-period horizon is considered in order to find a first-order condition for $nt+T$. Then the limit of this condition is taken as T goes to infinity in order to obtain the transversality condition. Next, the optimal liquidity risk management result for subprime originators is stated.

Theorem 3.3.5 (Main optimal originator liquidity risk management result): *Suppose that (3.1) and (3.27) in Lemma 3.3.3 hold. Also, assume that r^i represents the strict upper bound for $2c^M + (\sigma^l)^2$ that relates to the rate of impatience of the originator, i.e.,*

$$2c^M + (\sigma^l)^2 < r^i. \tag{3.30}$$

Then the optimal allocation of risky marketable securities is given by

$$\widetilde{\pi}_t^* = [C^{-1}\sigma\widetilde{\xi} + \sigma^l\sigma^{-T}\widetilde{q}](M_t^u + B_t) - C^{-1}\sigma\widetilde{\xi}B_t, \tag{3.31}$$

with M^u, the unfunded subprime mortgages to be extended. Also, the optimal deposit inflow rate is given by

$$c_t^* = D_t^r + \frac{\delta^{aa}}{\kappa}M_t^u, \tag{3.32}$$

where δ^{aa} is the only, non-negative solution to

$$(\delta^{aa})^2 + \kappa(r^i - 2r^{\mathrm{T}} + \widetilde{\xi}^T\widetilde{\xi})\delta^{aa} - \kappa(1-\kappa) = 0, \tag{3.33}$$

under the assumptions made earlier.

Proof 3.3.6 *In order to obtain the dynamic programming equation or Hamilton-Jacobi-Bellman equation (HJBE), the objective function (3.26) and control system (3.24) are considered. In turn, the authors solve the dynamic programming equation by applying standard second order partial differential equation theory. Here, the spread method of originator mortgage funding is employed. Next, it is verified that the transversality condition holds in order to show that the solution of the liquidity risk management problem and the original value function are related. Here, the HJBE for the problem that is stated in Problem 3.3.1 is given by*

$$\begin{aligned} r^iV = \min_{(D^a,\widetilde{\pi})\in\mathcal{G}_{B_0,M^{r0}}} \Big\{ & (r^{\mathrm{T}}B + \widetilde{\pi}^T(\widetilde{b} - r^{\mathrm{T}}\widetilde{1}) + D^a + (c^M - r^a)M^r)V^a \\ & + c^M M^r V^{M^r} + \frac{1}{2}\widetilde{\pi}^T C\widetilde{\pi}V^{aa} + \frac{1}{2}(\sigma^M)^2(M^r)^2V^{M^rM^r} + \sigma^M M^r\widetilde{\pi}^T\sigma\widetilde{q}V^{aM^r} \\ & + \kappa(D^a)^2 + (1-\kappa)(B - M^r)^2 \Big\}. \end{aligned} \tag{3.34}$$

Also, the equation (3.34) can be separated into terms, having a

a. dependence on D^a;

b. dependence on $\widetilde{\pi}$; and

c. dependence on neither D^a nor $\widetilde{\pi}$.

The minimization problem is solved in two separate parts. In order to obtain minimization (i), set the first derivative of

$$D^a(V^a + \kappa D^a)$$

with respect to D^a equal to zero. In this case,

$$V^a + 2\kappa D^a = 0. \tag{3.35}$$

Therefore, the optimal deposit inflow rate is

$$c^*(V^a) = -\frac{V^a}{2\kappa}, \tag{3.36}$$

where a smooth, strictly convex solution V of equation (3.34) exists. Minimization (ii) follows under the same conditions as above, from the optimization of

$$\widetilde{\pi}^T(\widetilde{b} - r^T\overline{1})V^a + \frac{1}{2}\widetilde{\pi}^T C\widetilde{\pi}V^{aa} + \sigma^M M^r \widetilde{\pi}^T \sigma\widetilde{q}V^{aM^r}$$

with respect to $\widetilde{\pi}$. Here, the optimal allocation of marketable securities is

$$\widetilde{\pi}^*(V^a, V^{aa}, V^{M^rM^r}) = -C^{-1}(\widetilde{b} - r^T\overline{1})\frac{V^a}{V^{aa}} - \sigma^M M^r \sigma^{-T}\widetilde{q}\frac{V^{aM^r}}{V^{aa}}. \tag{3.37}$$

Now, the optimal values, c^* and $\widetilde{\pi}^*$, are substituted, that are given by (3.36) and (3.37), respectively, into the right hand side of (3.34) to obtain

$$
\begin{aligned}
&\Big\{r^{\mathrm{T}}B - (\widetilde{b}^T - r^{\mathrm{T}}\overline{1}^T)C^{-T}(\widetilde{b} - r^{\mathrm{T}}\overline{1})\frac{V^a}{V^{aa}} - \overline{q}^T\sigma^{-1}M^r\sigma^M(\widetilde{b} - r^{\mathrm{T}}\overline{1})\frac{V^{aM^r}}{V^{aa}} - \frac{V^a}{2\kappa} + (c^M - r^a)M^r\Big\}V^a \\
&\quad + c^M M^r V^{M^r} + \frac{1}{2}\Big\{(\widetilde{b}^T - r^{\mathrm{T}}\overline{1}^T)C^{-T}(\widetilde{b} - r^{\mathrm{T}}\overline{1})\Big[\frac{V^a}{V^{aa}}\Big]^2 + \widetilde{q}^T\sigma^{-1}c^M M^r(\widetilde{b} - r^{\mathrm{T}}\overline{1})\frac{V^{aM^r}}{V^{aa}}\frac{V^a}{V^{aa}} \\
&\quad + (\widetilde{b}^T - r^{\mathrm{T}}\overline{1}^T)C^{-T}c^M M^r\sigma^{-T}\overline{q}\frac{V^{aM^r}}{V^{aa}}\frac{V^a}{V^{aa}} + \widetilde{q}^T\sigma^{-1}(c^M)^2(M^r)^2\sigma^{-T}\widetilde{q}\Big[\frac{V^{aM^r}}{V^{aa}}\Big]^2\Big\}V^{aa} \\
&\quad + c^M M^r\Big\{ - (\widetilde{b}^T - r^{\mathrm{T}}\overline{1}^T)C^{-T}\frac{V^a}{V^{aa}} - \widetilde{q}^T\sigma^{-1}c^M M^r\frac{V^{aM^r}}{V^{aa}}\Big\}\sigma\widetilde{q}V^{aM^r} \\
&\quad + \frac{1}{2}(c^M)^2(M^r)^2V^{M^rM^r} + \frac{(V^a)^2}{4\kappa} + (1-\kappa)(B^2 - 2BM^r + (M^r)^2). \qquad (3.38)
\end{aligned}
$$

By standard second order partial differential equation theory, the form of (3.38) implies that

$$
V(B, M^r) = \delta^{aa}B^2 + \delta^{M^rM^r}(M^r)^2 + \delta^{aM^r}aM^r \qquad (3.39)
$$

is a quadratic solution of (3.34). It follows that

$$
(\delta^{aa})^2 + \kappa(r^i + \widetilde{\phi}^T\widetilde{\phi} - 2r^{\mathrm{T}})\delta^{aa} - \kappa(1-\kappa) = 0, \qquad (3.40)
$$

$$
\begin{aligned}
&4\kappa(r^i - 2c^M - (\sigma^M)^2)\delta^{M^rM^r}\delta^{aa} + \delta^{aa}(\delta^{aM^r})^2 - 4\kappa(c^M - r^a)\delta^{aM^r}\delta^{aa} \qquad (3.41)\\
&\quad + \kappa((\sigma^M)^2\widetilde{q}^T\widetilde{q} + 2\sigma^M\widetilde{q}^T\widetilde{\phi} - \widetilde{\phi}^T\widetilde{\phi})(\delta^{aM^r})^2 - 4\kappa(1-\kappa)\delta^{aa} = 0
\end{aligned}
$$

and

$$
\begin{aligned}
&-\delta^{aM^r}\delta^{aa} + \kappa(c^M + r^{\mathrm{T}} - r^i - \widetilde{\phi}^T\widetilde{\phi} - \sigma^M\widetilde{q}^T\widetilde{\phi})\delta^{aM^r} \\
&\quad + 2\kappa(c^M - r^a)\delta^{aa} + 2\kappa(1-\kappa) = 0 \qquad (3.42)
\end{aligned}
$$

have solutions δ^{aa}, $\delta^{M^rM^r}$ and δ^{aM^r}, respectively.

Equations (3.40), (3.41) and (3.42) can be solved by applying standard theory. However, a solution procedure is chosen that leads to a spread method of originator mortgage funding. This approach enables the authors to calculate the optimal rate of deposit inflows and rate of actualization, r^a. In particular,

$$
D^a = -\frac{1}{2\kappa}\Big(2\delta^{aa}B + \delta^{aM^r}M^r\Big) = -\frac{1}{2\kappa}\Big((2\delta^{aa} + \delta^{aM^r})B + \delta^{aM^r}M^u\Big) \qquad (3.43)
$$

follows from equations (3.15), (3.36) and (3.37). Also, it is noticed from (3.43) that the additional deposits, D^a, are directly proportional to the unfunded subprime mortgages, M^u, if and only if

$$2\delta^{aa} = -\delta^{aM^r}. \tag{3.44}$$

In order for (3.40) and (3.42) to have a solution, the condition

$$r^a = r^{\mathrm{T}} + \sigma^M \widetilde{q}^T \widetilde{\phi} \tag{3.45}$$

should hold. The inequality

$$\delta^{M^r M^r} = \frac{r^i - 2c^M - (\sigma^M)^2 \widetilde{q}^T \widetilde{q}}{r^i - 2c^M - (\sigma^M)^2} \delta^{aa} \geq \delta^{aa}$$

is obtained by substituting conditions (3.44) and (3.45) into (3.41) and subtracting from (3.40). The authors conclude that the value function, V, is always non-negative.

Next, it is necessary to verify that the transversality condition holds in order to show that the value function from (3.34) is the solution (3.39). Firstly, the optimal values, c^* and $\widetilde{\pi}^*$, are substituted, that are given by (3.36) and (3.37), respectively, into (3.24). In this case, the stochastic dynamics of optimal marketable securities, B^*, is represented by

$$dB_t^* = \Big\{ (r^{\mathrm{T}} - \widetilde{\phi}^T \widetilde{\phi} - \frac{\delta^{aa}}{\kappa}) B_t^* + (-r^{\mathrm{T}} + \widetilde{\phi}^T \widetilde{\phi} + \frac{\delta^{aa}}{\kappa} + c^M) M_t^r \Big\} dt$$
$$+ \Big\{ -\widetilde{\phi}^T B_t^* + (\widetilde{\phi}^T + \sigma \widetilde{q}^T) M_t^r \Big\} dW_t.$$

Secondly, it is observed that V in (3.39) is a homogeneous quadratic polynomial in B and M^r. Therefore, the authors have to calculate the conditional expectation of $(B^*)^2$, $B^* M^r$ and $(M^r)^2$ with respect to $(B_0, M^{r^{\mathrm{T}}})$. The following notation is used in the sequel:

$$f_t = \mathbf{E}_{B_0, M^r 0}(B^* M^r)_t, \; g_t = \mathbf{E}_{B_0, M^r 0}(B^*)_t^2 \text{ and } h_t = \mathbf{E}_{B_0, M^r 0}(M^r)_t^2. \tag{3.46}$$

Furthermore, M^r is modelled as a geometric Brownian motion which results in

$$h_t = (M^{r^{\mathrm{T}}})^2 \exp\{(2c^M + (\sigma^M)^2)t\} \text{ and } \lim_{t \to \infty} \exp\{-r^i t\} h_t = 0$$

if and only if (3.30) holds. In addition, f and g in (3.46) satisfy the linear differential equations

$$f_t' = (r^{\mathrm{T}} - \widetilde{\phi}^T\widetilde{\phi} - \frac{\delta^{aa}}{\kappa} + c^M - \sigma^M\widetilde{q}^T\widetilde{\phi})f_t$$
$$+(-r^{\mathrm{T}} + \widetilde{\phi}^T\widetilde{\phi} + \frac{\delta^{aa}}{\kappa} + c^M - \sigma^M\widetilde{q}^T\widetilde{\phi})h_t, \quad f_0 = B_0 M^{r^{\mathrm{T}}}$$

and

$$g_t' = 2(r^{\mathrm{T}} - \widetilde{\phi}^T\widetilde{\phi} - \frac{\delta^{aa}}{\kappa})g_t$$
$$+2(-r^{\mathrm{T}} + \widetilde{\phi}^T\widetilde{\phi} + \frac{\delta^{aa}}{\kappa} + c^M - \sigma^M\widetilde{q}^T\widetilde{\phi})f_t, \quad g_0 = B_0^2,$$

respectively. In this regard, the authors conclude that

$$f_t = (B_0 - k^1 M^{r^{\mathrm{T}}})M^{r^{\mathrm{T}}}\exp\left\{(r^{\mathrm{T}} - \widetilde{\phi}^T\widetilde{\phi} - \frac{\delta^{aa}}{\kappa} + c^M - \sigma^M\widetilde{q}^T\widetilde{\phi})t\right\} + k^1 h_t, \tag{3.47}$$

with the constant

$$k^1 = \frac{\delta^{aa} + \kappa(-r^{\mathrm{T}} + \widetilde{\phi}^T\widetilde{\phi} + c^M - \sigma^M\widetilde{q}^T\widetilde{\phi})}{\delta^{aa} + \kappa(-r^{\mathrm{T}} + \widetilde{\phi}^T\widetilde{\phi} + c^M - \sigma^M\widetilde{q}^T\widetilde{\phi} + (\sigma^M)^2)}.$$

Therefore,

$$\lim_{t\to\infty}\exp\{-r^i t\}f_t = 0$$

if and only if (3.30) and

$$r^{\mathrm{T}} - \widetilde{\phi}^T\widetilde{\phi} - \frac{\delta^{aa}}{\kappa} + c^M - \sigma^M\widetilde{q}^T\widetilde{\phi} < r^i \tag{3.48}$$

hold simultaneously. Next, the authors conclude that the positive solution of (3.33) implies (3.48) when they estimate the rate of impatience inequality given by (3.30) and δ^{aa} as in

$$-\sigma^M\widetilde{q}^T\widetilde{\phi} \le \sigma^M\overline{1}^T\widetilde{\phi} = \frac{1}{2}(\widetilde{\phi}^T\widetilde{\phi} + (\sigma^M)^2 - (\widetilde{\phi} - \sigma^M\overline{1})^T(\widetilde{\phi} - \sigma^M\overline{1}))$$
$$\le \frac{1}{2}\widetilde{\phi}^T\widetilde{\phi} + \frac{1}{2}(\sigma^M)^2, \tag{3.49}$$

for $-1 \leq q^i \leq 1$. *The authors also have that (3.33) implies that*

$$r^{\mathrm{T}} - \frac{\widetilde{\phi}^T \widetilde{\phi}}{2} - \frac{\delta^{aa}}{\kappa} < \frac{r^i}{2}. \tag{3.50}$$

Also, (3.48) holds when the condition (3.30), as well as inequalities (3.49) and (3.50), is considered. Moreover,

$$g_t = (B_0^2 - 2B_0 M^{r^{\mathrm{T}}} + (2k^1 - k^1 k^2 - k^3)(M^{r^{\mathrm{T}}})^2) \exp\left\{2(r^{\mathrm{T}} - \frac{\widetilde{\phi}^T \widetilde{\phi}}{2} - \frac{\delta^{aa}}{\kappa})t\right\} \\ +2f_t - (2k^1 - k^1 k^2 - k^3)h_t,$$

with constants k^2 *and* k^3. *An important fact is that*

$$\lim_{t \to \infty} \exp\{-r^i t\} g_t = 0$$

is analogous to the expressions that are stated above. Finally, the transversality condition holds and therefore the proof of the theorem is complete.

The next corollary follows immediately and provides a formula for optimal allocation when investing in the i-th risky marketable securities class, $\widetilde{\pi}^{i*}$.

Corollary 3.3.7 (Optimal allocation for the i-th marketable securities class): *Assume that Theorem 3.3.5 holds. Then the optimal allocation for the i-th risky marketable securities class can be represented by*

$$(\widetilde{\pi}_t^i)^* = \overline{e}^i [C^{-1} \sigma \widetilde{\xi} + \sigma^l \sigma^{-T} \widetilde{q}](M_t^u + B_t) - \overline{e}^i C^{-1} \sigma \widetilde{\xi} B_t,$$

where $\overline{e}^i = (0 \ldots 1 \ldots 0)$ *and the unit vector for all* $i = 1, 2, \ldots, n$ *has one in position* i.

The optimal allocation of risky marketable securities, represented by (3.31) in Theorem 3.3.5, can be rewritten as

$$\widetilde{\pi}^* = C^{-1}(\widetilde{b} - r^{\mathrm{T}}\overline{1})M^u + \sigma^M \sigma^{-T} \widetilde{q} M^r. \tag{3.51}$$

The solution to the optimal originator liquidity risk management problem implies a direct relationship between M^u and $\widetilde{\pi}^*$ via a proportional constant which depends on σ, $\widetilde{b}$ and $r^{\mathrm{T}}\overline{1}$. Also, the market price of risk that is associated with this proportionality can be introduced by (3.6). The term $\sigma^l\sigma^{-T}\widetilde{q}M^r$ in (3.51) suggests that the optimal allocation strategy of B depends on uncertain mortgage origination and M^r, how subprime mortgages and the returns on marketable securities correlate, as well as constraints on risks in the model. In fact, the optimal allocation of marketable securities was greatly influenced by the SMC.

3.3.4.2 Optimal allocation strategies for marketable securities

The following strategies regarding marketable securities allocation result from Theorem 3.3.5. These optimal strategies involve borrowing, short selling and overfunding. In order to facilitate a discussion of optimal securities allocation strategies, the authors set

$$J^i = \overline{e}^i C^{-1}\sigma\widetilde{\phi} \text{ and } N^i = \sigma^M \overline{e}^i \sigma^{-T}\widetilde{q}.$$

Firstly, the authors consider an optimal allocation strategy for marketable securities when borrowings are considered. It is noticed that funds are borrowed by the originator to invest in the risky marketable securities i, at a rate $r^{\tilde{B}}$ via the optimal solution (3.31) in Theorem 3.3.5. This implies that $\pi^i > B^*$ when the fund level of securities is below

$$\left\{\frac{J^i + N^i}{1 + J^i}\right\}M^r.$$

Secondly, they consider an optimal allocation strategy when short selling is considered. Short selling occurs when the originator expects marketable securities prices to decrease. The originator then sells these securities at a price with the intention to buy it back at a lesser price in the future. When the i-th marketable securities are above the value

$$\left\{1 + \frac{N^i}{J^i}\right\}M^r,$$

then a short position in securities i will be taken by the originator. In this case, $(\pi^i)^* < 0$.

Finally, they consider an optimal allocation strategy for marketable securities when overfunding occurs, that is $B^* > M^r$. The optimal strategy is still to borrow funds to invest in securities. For this strategy to be feasible, there should be a positive correlation between W^l and W^i for $i = 1,\ 2,\ \ldots,\ n$. Suppose that the fund is invested in just one class of securities and

$$\frac{q\sigma^l}{\sigma} > 1.$$

In this case,

$$\frac{J^i + N^i}{1 + J^i} > 1 \text{ and the inequality } M^r < B^* < \left\{\frac{J^i + N^i}{1 + J^i}\right\} M^r$$

might hold. This suggests that borrowing could occur. It may even be true when overfunding of risky marketable securities occurs. This is a consequence of a higher instantaneous volatility of stipulated levels of subprime mortgages, e, and involves positive correlation between W^i, $i = 1, 2, \ldots, n$ for risky marketable securities. As a result,

$$\mathbf{E}_{B_0, M^{r0}} B_t^* - \mathbf{E}_{B_0, M^{r0}} M_t^r = (B_0 - M^{r^{\mathrm{T}}}) \exp\{(r^{\mathrm{T}} - \widetilde{\xi}^T \widetilde{\xi} - \frac{\delta^{aa}}{\kappa})t\}$$

follows from the proof of Theorem 3.3.5. This implies that if

$$\delta^{aa} > \kappa(r^{\mathrm{T}} - \widetilde{\xi}^T \widetilde{\xi}), \tag{3.52}$$

then $\mathbf{E}_{B_0, M^r 0} B_t^* - \mathbf{E}_{B_0, M^r 0} M_t^r$ converges to zero when time approaches infinity. It is also noticed that inequality (3.52) is automatically satisfied if $r^{\mathrm{T}} \geq r^i + \widetilde{\xi}^T \widetilde{\xi}$. On the other hand, where $r^{\mathrm{T}} < r^i + \widetilde{\xi}^T \widetilde{\xi}$, inequality (3.52) becomes

$$\kappa < \frac{1}{1 + r^{\mathrm{T}}(r^i + \widetilde{\xi}^T \widetilde{\xi} - r^{\mathrm{T}})}. \tag{3.53}$$

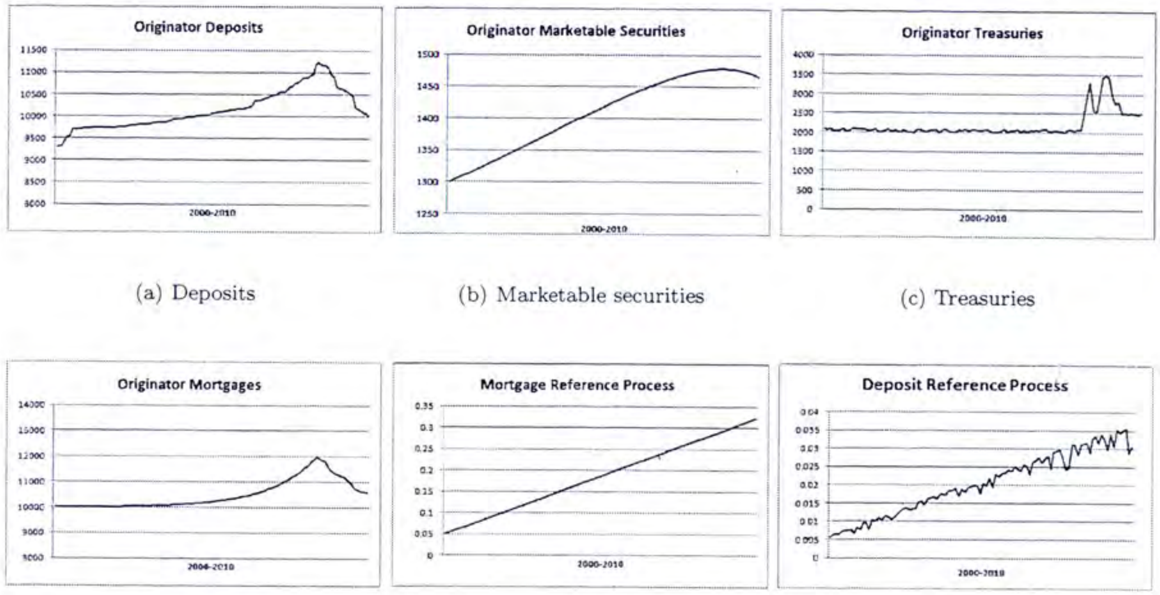
Since $\widetilde{\xi}^T \widetilde{\xi} > 0$, the authors conclude that inequality (3.53) is more restrictive than in the case where investments are fixed. For the deposit inflow rate, c, and the deposit reference process, D^r, it follows that

$$\mathbf{E}_{c_0 D^{r0}} c_t^* - \mathbf{E}_{c_0 D^{r0}} D_t^r$$

behaves the same when time tends to infinity.

3.3.5 Numerical examples involving mortgage funding

In this subsection, numerical quantitative examples are provided, which illustrate the main issues that are raised in this article. Numerical solutions for the dynamics of originator mortgages, treasuries, deposits and marketable securities are obtained by using the stochastic differential equations (3.1), (3.3), (3.11) and (3.24), respectively. This is achieved by means of the numerical Euler-Maruyama method. Originator mortgage and deposit reference processes (as in Corollary 3.2.3 in Section 3.2) are also computed by the use of this method. In the following figures, originator parameter values are generated in order to obtain appropriate solutions:



(a) Deposits (b) Marketable securities (c) Treasuries

(d) Subprime mortgages (e) Mortgage reference process (f) Deposit reference process

Figure 3.2: Dynamics of mortgage funding parameters

Figure 3.2(a) shows that originator deposits increased dramatically as large amounts of deposits for mortgage funding flowed into the US before 2007. This spurred increases in mortgage origination and house prices. Subprime mortgage securitization offered higher returns than the standard case. As a consequence, more and more mortgages were securitized for higher returns. In turn, this funded new subprime mortgage origination. This phenomenon is illustrated via the dramatic increase in originator marketable securities and mortgages before 2007 as seen in Figures 3.2(b) and 3.2(d), respectively. The SMC affected mortgage supply, while credit conditions were tightened after 2007. Investment in marketable securities imploded when mortgages started to default. This caused the value of these securities to plummet. As shown in Figure 3.2(a), depositors withdrew their funds and there were no new funding sources, which resulted in originator failure after 2007. The fact that treasuries are riskless with very stable rates throughout is illustrated in Figure 3.2(c). For the sake of stability, more treasuries were bought after the collapse of the financial market after 2007. Originator mortgage and deposit reference processes are shown in Figures 3.2(e) and 3.2(f), respectively.

3.4 Conclusions and future directions

In this article, the authors attain an optimal level of mortgage origination via deposit inflow rates and marketable securities allocation that ensure originator stability and security (compare with

Question 3.1.1). Furthermore, the relationship between these optimal levels and the SMC (see Question 3.1.2) are considered. It is concluded that there is a positive correlation between both deposits and marketable securities and mortgage origination. Also, the interplay between mortgage and deposit reference processes and origination is described in Theorem 3.2.1. A condition for the spread method of mortgage funding to be implementable is given by Lemma 3.3.3. Furthermore, it is found that liquidity risk mitigation is possible when unfunded mortgages are directly proportional to additional deposits, as described in Theorem 3.3.5. These results are tied to the fact that the SMC was characterized by illiquidity, as well as risk opaqueness. In this period, originators did not have adequate liquidity risk management strategies in place (compare with [7]). In current and subsequent financial crises, the mitigation of such risk should be given more attention.

In future, instead of solving the optimal liquidity risk management problem in a continuous-time context, the authors will introduce more sophisticated models, driven by general Lévy processes. Mortgage and marketable securities cyclicality, as well as originator financial stability, will be included in these models. Furthermore, the authors would like to expand the discussion of the leverage and debt ratios in (3.14). In this regard, they will strive to determine the leverage that balances expected return against liquidity risk, developing an early warning system for crises, as well as investigating the relationship between excessive leverage and debt, and the probability of financial crises occurring.

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Chapter 4

SUBPRIME MORTGAGE DESIGN

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Optimal originator valuation and the global financial crisis

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Abstract: In this paper, subprime mortgage design will be discussed in both a theoretical- and numerical-quantitative framework. In particular, mortgages that are able to fully amortize, voluntarily prepay (involving prepayment and possibly refinancing) or default are modelled. In this regard, it is found that mortgage refinancing is curtailed by high loan-to-value ratios due to house price depreciation, while low loan-to-value ratios increase mortgagor house equity. Furthermore, an optimal originator valuation problem under mortgage origination is solved. In this case, optimal mortgage value and rates, as well as profit, are computed. The paper supports the view that the subprime mortgage crisis was partially caused by the intricacy of design of subprime mortgages that led to information (asymmetry, contagion, inefficiency and loss) problems, valuation opaqueness and ineffective risk mitigation.

Keywords: Subprime Mortgages, Mortgage Rate, Prepayment, Refinancing, Credit Risk, Subprime Mortgage Insurance, House Prices, House Equity, Loan-to-Value Ratio, Procyclicality, Subprime Mortgage Crisis.

4.1 Introduction

The subprime mortgage crisis (SMC) was triggered by a marked increase in mortgage delinquencies and foreclosures in the US. Since it became apparent in 2007, it has had major adverse consequences for banks and financial markets around the globe. This article discusses an important aspect of this crisis, namely subprime mortgage design. Such design utilizes real financial market interest rates, securitization structuring and mortgage pricing to explain the economic mechanism behind the recent crisis. In this regard, there have to be accounted for important issues such as moral hazard in expanding mortgage portfolios, incomplete information among market players about their counterparties, myopia in decision making in the subprime mortgage market and monetary policy incentives boosting the growth of the subprime market. In the discussions, both a theoretical- and numerical-quantitative approach are used.

Subprime residential mortgage loans (or simply *mortgages* in the sequel) involve the extension of mortgages to mortgagors who do not qualify for market interest rates due to factors such as income level, size of the down payment made, credit history and employment status. Such mortgages are designed in a way that enable mortgagors to repay loans, based on the appreciation of house prices. In reality, few other consumer loan designs have the characteristic that the appreciation of an underlying asset (in this case, house prices) is so closely associated with repayment capacity. In this regard, from a mortgagor's viewpoint, the ability to make timely mortgage repayments is dramatically affected by payment reset. They may also be subjected to higher interest rates, fees or costs for late payments or prepayments. From the originators' perspective, mortgages are more demanding, because they are considered riskier than prime mortgages, with a higher probability of default. In this case, originators justify higher rates and fees to compensate for additional risk. Here, risk and profit are major indicators of financial crises for households, companies and financial institutions. In the sequel, risk and profit (along with regulatory capital) form an important component of originator valuation.

4.1.1 Literature review of subprime mortgage design

The research that is conducted on subprime mortgage design in this article takes place against the backdrop of the SMC and has connections with several strands of existing literature. For the purposes of this study, the selection of literature mainly involves subprime mortgages, prepayment and refinancing, credit risk and subprime mortgage insurance, house prices, house equity and loan-to-value ratio, as well as procyclicality.

In [11], light is shed on typical subprime mortgagors and mortgage design, as well as their historical performance (see also [1] for subprime credit). In particular, [11] considers interest rate reset (from teaser to step-up) attempts to estimate what fraction of resetting mortgages will end up in foreclosure. Teaser rate mortgage products inflated the US home-ownership market artificially. The aforementioned contributions and the description of mortgage design in [11] have strong connections with the subsequent discussions. The issue of teaser to step-up rates is discussed in Subsection 4.2.1. Furthermore, [11] shows that mortgage design has important implications for mortgage pricing. As far as the latter is concerned, it is found in Subsection 4.2.1 that the risk premium is a key to mortgage pricing and had an important role to play in the SMC. In this regard, the risk premium acts as an indicator of perceived credit risk and the likeliness to engage in mortgage securitization. Before the SMC, the average difference between prime and subprime mortgage rates (the *subprime mark-up*) declined quite dramatically (see also [4]).

Article [5] suggests that mortgage losses take place in housing markets with moderately volatile and flat or declining nominal house prices. These mortgages are typically more risky than prime mortgages and are characterized by higher rates of prepayment, delinquency and default, as well as inadequate mortgage insurance (see Subsections 4.2.1 and 4.2.2 for the authors' take on this issue). Working paper [6] quantifies how different determinants contributed to high delinquency and foreclosure rates for vintage 2006 mortgages (see also [4]). Their analysis suggests that different mortgage-level characteristics, as well as low house price appreciation, were quantitatively too small to explain the bad performance of 2006 mortgages. By contrast, the authors' paper investigates what impact the decline in house prices had on house equity and originator profits (see Subsection 4.3.1 and [4] for more details).

Paper [3] examines the choice of subprime mortgagors to extract equity while refinancing, and assesses the prepayment and default performance of these cash-out refinancing loans relative to rate refinancing loans (see also [9] and [10]). In this research, it is investigated whether the mortgage amount or cash that is extracted is a determinant of the incentive to refinance. The relationship between the recovery amount the originator receives in the case of default to house prices and mortgagor collateral is also investigated. Consistent with survey evidence, the propensity to extract equity while refinancing is sensitive to interest rates on other forms of consumer debt. After the loan is originated, [3] indicates that cash-out refinances perform differently from non-cash-out refinances. For example, cash-outs are less likely to default or prepay and the termination of cash-outs is more sensitive to changing interest rates and house prices. In this regard, the loan to value ratio (LTVR) as a measure of the incentive to extract house equity, as well as its relationship with delinquencies, is investigated (compare with [4]).

Subprime lending was procyclical with new mortgage origination in 2008, being significantly below such origination in 2007 (see, for instance, [8]). This period was also typified by accelerated market expansion, deteriorating underwriting standards, declining loan performance and decreasing risk premiums. The book [11] claims that, during the boom, the US built up an excess housing supply, experienced a decline in mortgage standards and had a household sector that was vulnerable to falling housing prices. Some of these outcomes seem to have been driven by tax, legal and regulatory systems that encouraged households to increase their leverage and permitted lenders to enable that development. The credit ratings that accompanied booms and busts are discussed in [12] (see also [11]). Before the SMC, high credit ratings were given by credit rating agencies, thereby financing and exacerbating the housing boom. However, during the SMC, it became clear that the rating process was faulty (see [11] for more information). In this regard, the relationship between credit ratings that are procyclical, as well as mortgage losses and subprime mortgage insurance (SMI) premium rates, is discussed in Subsection 4.3.2 (see also [4]).

4.1.2 Preliminaries about subprime mortgage origination

In this subsection, the authors discuss preliminaries about subprime mortgages, involving originator's balance sheet, credit ratings and SMI, as well as the economy and economic agents.

4.1.2.1 The balance sheet

In period t, the originator's on-balance sheet for the market value of all loans can be identified as

$$\Lambda_t + \widehat{B}_t + \widehat{R}_t + \widehat{\mathtt{T}}_t = \widehat{D}_t + \widehat{\mathtt{B}}_t + \widehat{K}_t; \ \widehat{K}_t = n_t \widehat{E}_{t-1} + \widehat{O}_t, \ \widehat{E}_{t-1} = \widehat{E}^c_{t-1} + \widehat{E}^p_{t-1},$$

where Λ, $\widehat{B}$, $\widehat{R}$, $\widehat{\mathtt{T}}$, $\widehat{D}$, $\widehat{\mathtt{B}}$, $\widehat{E}$, $\widehat{O}$, $\widehat{E}^c_t$ and $\widehat{E}^p_t$ are the market value of all loans, marketable securities, reserves, treasuries, deposits, borrowings, equity, subordinate debt, common equity and preferred equity, respectively. Next, a realistic assumption is made about the relationship between $\widehat{R}$ and $\widehat{D}$.

Assumption 4.1.1 (Reserves and deposits): *Assume that the reserves, $\widehat{R}$, is a factor, γ, of the deposits, $\widehat{D}$, so that $\widehat{R} = \gamma\widehat{D}$.*

Then,

$$\Lambda_t + \widehat{B}_t + \widehat{\mathtt{T}}_t = (1-\gamma)\widehat{D}_t + \widehat{\mathtt{B}}_t + \widehat{K}_t; \widehat{K}_t = n_t\widehat{E}_{t-1} + \widehat{O}_t, \quad \widehat{E}_{t-1} = \widehat{E}^c_{t-1} + \widehat{E}^p_{t-1}. \tag{4.1}$$

Furthermore, in this case, the originator faces a *Hicksian loan demand*, given by

$$\Lambda_t = \lambda_0 - \lambda_1 r^\Lambda_t + \lambda_2 \mathcal{C}_t + \sigma^\Lambda_t \tag{4.2}$$

where r^Λ_t is the loan rate, $\mathcal{C}_t$ is a variable representing the credit rating (as a proxy for the level of macroeconomic activity via procyclicality) and σ^Λ_t represents a random exogenous shock to the demand for loans, which is defined over the interval $[\underline{\Lambda}, \overline{\Lambda}]$. The possibility that mortgagors can default on their principal payment is considered. Moreover, the proportion of defaulted debt is assumed to be a negative function of current credit ratings (macroeconomic conditions), in other words write-offs are higher when credit ratings are low than when they are high. Mortgage defaults translate into an extra cost term in the originator's profit function.

In models presented here, the originator is allowed to hold risky marketable securities (such as residential mortgage-backed securities (RMBSs)) and riskless treasury securities on its balance sheet. Marketable securities, $\widehat{B}$, are important for generating profit, since their returns, r^B, usually exceed the returns from other less risky assets. In the sequel, the all-in cost for holding marketable securities is denoted by c^B_t. Also, the value of *treasuries* in the t-th period is denoted by $\widehat{\mathtt{T}}_t$, and the treasury rate by $r^{\mathtt{T}}$. The originator takes deposits, $\widehat{D}_t$, at a constant *marginal cost*, c^D, that may be associated with cheque clearing and bookkeeping. The following assumption is related to this:

Assumption 4.1.2 (Deposit taking): *It is assumed that deposit taking is continual, even when* $r^D < r^{\mathtt{T}}$.

This assumption ensures that the originator deposits are never zero. The possibility is considered that *unanticipated withdrawals*, u, will occur. The originator holds liquid treasuries in order to make provision for these withdrawals. For the sake of argument, it is also assumed that the *cost of liquidation*, c^l, or additional external funding is a quadratic function of $\mathtt{T}$.

Assumption 4.1.3 (Unanticipated deposit withdrawals): *Assume that u is related to the probability density function, $f(u)$, that is independent of time. Suppose that the unanticipated withdrawals have a uniform distribution with support* $[0, \overline{\overline{D}}]$.

This assumption implies that the *cost of liquidation*, c^l, or additional external funding is a quadratic function of $\mathtt{T}$. In addition, for any t, if $u > \widehat{\mathtt{T}}_t$, then originator assets are liquidated at some *penalty rate*, r^p_t. In this case, the *cost of treasuries* is $P^{\widehat{\mathtt{T}}_t}(\widehat{\mathtt{T}}_t) = r^p_t \int_{\widehat{\mathtt{T}}_t}^{\infty} [u - \widehat{\mathtt{T}}_t] f(u) du = \frac{r^p_t}{2\overline{\overline{D}}}[\overline{\overline{D}} - \widehat{\mathtt{T}}_t]^2$, with $P^{\widehat{\mathtt{T}}_t}(\widehat{\mathtt{T}}_t)$ being the provisions against deposit withdrawals.

In the sequel, the amount that is borrowed from other banks is denoted by $\widehat{\mathtt{B}}$, while the marginal interbank borrowing rate – for instance, the London interbank borrowing rate (LIBOR), r^L, for originators (as in (4.1) below) – and costs are denoted by $r^{\mathtt{B}}$ and $c^{\mathtt{B}}$, respectively. Of course, when the originator borrows from the central bank, then $r^{\mathtt{B}} = r^L$.

4.1.2.2 Credit ratings for subprime mortgages

Concerns about credit ratings have resurfaced during the SMC, where originators and investors have used ratings to determine the risk that is attached to their mortgage investments. For simplicity, the following assumption is made:

Assumption 4.1.4 (Credit rating process): *Suppose that $\mathcal{C} = \{C_t\}_{t\geq 0}$ follows the first-order autoregressive stochastic process $C_{t+1} = \mu_t^C C_t + \sigma_{t+1}^C$, where μ_t^C and σ_{t+1}^C denote the coefficient and zero-mean stochastic shocks to credit rating, respectively:*

The uncertainty in credit ratings before and after the SMC is encapsulated in this assumption.

4.1.2.3 Subprime mortgage insurance

Mortgage insurance contracts involve one of the simplest insurances that can be used to hedge against credit risk. In this case, the following assumption is made:

Assumption 4.1.5 (SMI premium rate): *Assume that the SMI premium rate is of the form*

$$p_t^i(C) = h(\Gamma_t(C), \mathbf{E}[C_t(S)]), \tag{4.3}$$

where Γ, S and C_t denote subprime mortgage mass, losses and insurer payments via the protection leg.

This assumption reflects the fact that credit ratings and expected mortgage losses have a role to play in deciding the size of SMI premiums.

4.1.2.4 The economy, economic agents and equilibrium

Throughout this paper, the authors consider an economy with periods, $t-1$, t and $t+1$, and two main agents, namely the subprime originator and mortgagor. In the sequel, the originator and mortgagor are considered to be risk-neutral with stylized interactions. For setting the scene, the following is assumed:

Assumption 4.1.6 (Oligopolistic market, monopolistic originator and equilibrium): *The originator is assumed to operate in an oligopolistic market for subprime mortgages in which there* *are several originators that charge r^M consistent with monopoly power. An individual originator pre-commits to a mortgage quantity through its capital holding in period t and loan rate competition follows in period $t+1$. To characterize the equilibrium, the behaviour of the originator who operates as a monopolist is analyzed.*

The assumption above implies that competition between monopolistic originators is fixed in an oligopolistic market. The pre-commitment assumption refers to the fact that the period t choice of capital restricts mortgage origination in period $t+1$.

4.1.3 Main questions

In this section, the main problems that have been solved, are identified and an outline of the article is given.

Question 4.1.7 (Subprime mortgage design): *Can subprime mortgages that are able to fully amortize, voluntarily prepay or involuntarily prepay (default) be designed? (See Subsection 4.2.2 of Section 4.2.)*

Question 4.1.8 (Modelling of house equity): *Can house equity via LTVRs in a subprime framework be modelled? (See Subsection 4.2.3 of Section 4.2.)*

Question 4.1.9 (Modelling of subprime risk and profit): *Can a discrete-time model for subprime risk and profit, incorporating costs of funds and SMI, as well as mortgage losses be constructed? (See Subsection 4.3.1 of Section 4.3.)*

Question 4.1.10 (Optimal originator valuation problem under subprime mortgages): *In order to obtain an optimal originator valuation with mortgages at face value, which decisions regarding mortgage rates, deposits and Treasuries must be made? (See Theorem 4.3.8 in Subsection 4.3.2 of Section 4.3.)*

4.2 Subprime mortgage design

The most important design feature of a subprime mortgage is that mortgagors have the option to pay off the loan in its entirety (voluntary prepayment with the option of refinancing).

4.2.1 Subprime mortgage rates

The *mortgage rate*, r_t^M, for profit maximizing originators may be determined as

$$r_t^M = r_t^L + \varrho_t, \tag{4.1}$$

where r_t^L is, for instance, the 6-month LIBOR rate and ϱ_t is the risk premium.

Extending fully amortizing (two-period) mortgages to subprime mortgagors is considered to be too risky, so that a mortgage design with special features is needed. In this regard, reset is related to the severity of payment reset (depending on r^M and r^L), as well as mortgagor's house equity at the time of reset (function of initial LTVR and decreases in house prices, H) and market conditions. Before and during the SMC, for an initial period t, low teaser rates, r^ε, were charged at the beginning of the mortgage term – these are typically fixed for the first two (for 2/28 mortgages) or three (for 3/27 mortgages) years and are lower than what mortgagors would pay for a 30-year FRM. Subsequently, in period $t+1$, higher market-based step-up rates, r^ψ, were charged. As a consequence, for the adjustment periods t and $t+1$, coinciding with r^ε and a subsequent r^ψ, respectively,

$$r^M = \begin{cases} r_t^\epsilon = r_t^L + \varrho_t^\epsilon, & \text{Period } t, \\ r_{t+1}^\psi = r_{t+1}^L + \varrho_{t+1}^\psi, & \text{Period } t+1, \end{cases}$$

where ϱ^ϵ and ϱ^ψ are the risk premiums for r^ϵ and r^ψ, respectively. An estimated one-third of mortgages that were originated between 2004 and 2006 had $r^\varepsilon < 4\%$, which then increased significantly after some initial period. During the SMC, many new mortgagors eventually had trouble making their monthly repayments when H started to decline and r^ε increased to r^ψ. The hypothesis that interest rate resets were central to causing the SMC may be explained as follows (see, for instance, [5]): Originators found mortgages attractive because of high r^ψ. Mortgagors found them attractive because of r_t^ε, but subsequently regretted their decisions when they were obligated to pay r^ψ.

The risk premium, ϱ, that is key to mortgage pricing (see equation (4.1)), had an important role to play in the SMC. The size of ϱ is an indication of the originator's perceived credit risk. Immediately before the SMC, the ϱ that was required for mortgage origination declined, even though conventional wisdom suggested that risk premiums should increase. This trend continued to occur during the SMC, even though mortgagor credit ratings and mortgage quality both declined. Another observation is that if ϱ is very high, then originators are not likely to engage in mortgage securitization. Before the SMC, low risk premiums led to increased mortgage securitization.

4.2.2 Subprime mortgages

In period t, a decomposition of M is considered of the form

$$M_t = (1 - f^f - f^S)M_t + f^f M_t + f^S M_t, \tag{4.2}$$

where f^f and f^S are the fractions of M that refinance and default, respectively. The left-hand side of the decomposition (4.2) is an algebraic sum that enables the authors to separately analyze the refinancing and default components of mortgage face value, $f^f M_t$ and $f^S M_t$, respectively. This is very useful for the subsequent analysis. Also, decompositions with high f^S values have connections with predatory mortgages – mortgages that mortgagors should not take. In this regard, [1] finds that predatory lending is associated with highly collateralized mortgages, inefficient mortgage refinancing, lending without due regard to the ability to pay, prepayment costs, balloon payments and poorly informed mortgagors (see also [10]). In period t, for *net mortgage losses*, S,

$$S_t = (1 - r_t^R) f_t^S M_t, \quad \text{where } R_t = \min[aH_t, \ r_t^R f_t^S M_t], \quad a \in [0,1], \tag{4.3}$$

is the recovery amount with r^R being the recovery rate. Ceteris paribus, S, increases when credit ratings, $\mathcal{C}$, deteriorate according to $\dfrac{\partial S_t}{\partial \mathcal{C}_t} < 0$. It is noted that the above description of S is consistent with empirical evidence that suggests that mortgage losses are correlated with credit ratings under any capital adequacy regime (see, for instance, [11]). It is also noted that the originator receives only part of the collateral value, aH. Before the SMC, mortgage default rates were low and collateral values were high, so that recovery was not problematic. During the SMC, the value of the recovery amount, R, decreased steadily as H and mortgagor collateral declined. By considering (4.2),

$$M_t = m_0 - m_1 r_t^M + m_2 \mathcal{C}_t + \sigma_t^M, \tag{4.4}$$

where $m_0 = f^M \lambda_0$, $m_1 = f^M \lambda_1$ and $m_2 = f^M \lambda_2$ with f^M being the fraction of the face value of originator's subprime mortgages to the mortgage demand of all the originator's mortgages and λ_0, λ_1 and λ_2 are as in (4.2) of Subsection 4.1.2.1. It is noted that M in (4.4) is an increasing function of $\mathcal{C}$ and a decreasing function of r_t^M. As a consequence, M was high before the SMC. During the SMC, the opposite was true, as mortgage credit ratings began to decline significantly. Further, the next assumption is made:

Assumption 4.2.1 (Random shock to mortgages): *Assume that σ_t^M is the random shock to M with support $[\underline{M}, \overline{M}]$ that is independent of an exogenous stochastic variable, x_t, to be characterized below. Also, suppose that M follows the first-order autoregressive stochastic process,*

$$M_{t+1} = \mu_t^M M_t + \sigma_{t+1}^M, \quad \mu_t^M = g(\overline{c}_t^{M\omega}, p^i(\mathcal{C}_t)), \tag{4.5}$$

where $\overline{c}_t^{M\omega}$ and $p^i(\mathcal{C}_t)$ are the average weighted cost of loans and SMI premium rate, respectively. Furthermore, σ_{t+1}^M denotes zero-mean stochastic shocks to the mortgage process.

Equation (4.5) embeds the fact that mortgage losses, S, and SMI premium rates, $p^i(\mathcal{C}_t) \in [0,1]$, (see (4.3)) are correlated with credit rating. In particular, if $\mathcal{C}$ decreases, then p^i increases according to $0 \leq p^i(\mathcal{C}_t) \leq 1, \quad \dfrac{\partial p^i(\mathcal{C}_t)}{\partial \mathcal{C}_t} < 0$ (see, for instance, [2]). Premium, $p^i(\mathcal{C}_t)$, and its interplay with

$C(S(\mathcal{C}_t))$ is important with respect to counterparty risk – the inability of economic agents to fulfil their obligations towards each other. During the SMC, the main cause of systemic contagion was the interconnectedness of the financial system. Asset managers relied on large financial institutions for their funding, asset protection and derivative transactions, resulting in significant counterparty exposure to those institutions. Simultaneously, these institutions had complex balance sheets that led to significant counterparty risk to one another. For instance, when Lehman Brothers and AIG failed, the market was unable to ascertain whether the same fate would befall other large financial institutions, thereby hastening the collapse of the entire financial system. As a consequence, investors decided to move their assets away from institutions that were perceived to be risky. The SMC was deepened by investors that chose to discontinue their funding of such institutions. Counterparty risk became a system-wide problem and ceased only when the US government intervened with capital injections and credit guarantees via, for instance, the Troubled Asset Relief Program (TARP).

4.2.3 Subprime mortgage options and LTVRs

The loan to value ratio (LTVR) that is denoted by L is one of the most important determinants of the riskiness of mortgages. Moreover, in period t, L is defined as

$$L_t = \frac{M_t}{H_t}, \tag{4.6}$$

where M and H are the mortgage and house values, respectively. From the above formulation it is clear that high L results from declining house prices that ultimately curtail mortgage refinancing. This was the case immediately before and during the SMC. On the other hand, low ratios imply favourable house equity for subprime mortgagors. The main objective of the illustration is to demonstrate the effect of LTVRs on subprime mortgage origination and that a mortgage contains an implicit embedded option on H, as well as to determine the originator's expected profit from mortgage origination.

From equation (4.6), it is noted that L at origination is positively correlated with delinquency. L is also a measure of the incentive for mortgagors to extract house equity via cash-out refinancing (see, for instance, [11]). When the house prices declined, subprime mortgagors with high LTVRs were more likely to have a larger M than H. Therefore, such mortgagors are more likely to default than prime mortgagors, thus increasing the originator's exposure to credit risk – the main subprime risk that is associated with the SMC.

4.3 Optimal subprime mortgage design

In this section, a second characterization of the originator's profit under mortgages at face value is provided. The originator's valuation and corresponding balance sheet are also considered, given by

$$M_t + B_t + \mathtt{T}_t = (1-\gamma)D_t + \mathtt{B}_t + K_t, \tag{4.1}$$

where $M = f^M\Lambda$, $B = f^M\widehat{B}$, $\mathtt{T} = f^M\widehat{\mathtt{T}}$, $D = f^M\widehat{D}$, $\mathtt{B} = f^M\widehat{\mathtt{B}}$ and $K = f^M\widehat{K}$, with f^M as before. The rest of the originator's balance sheet items are analogously defined. As a consequence of these relationships, analogues of results for Λ can easily be obtained when attention is restricted to M.

4.3.1 Risk and profit under subprime mortgage origination

In this subsection, the originator's retained earnings and a model for its profit are discussed.

4.3.1.1 A motivating example

In the case of subprime mortgagors that may not have any collateral, the originator may be reluctant to extend a traditional two period – consisting of period t and $t+1$ – mortgage. During period t, H will either increase or decrease. If H increases and mortgagors build equity in their houses, the origination of mortgages may become a viable option. However, originators may not be willing to wager on H and mortgagor repayment behaviour for long periods and, because delinquency and foreclosure are costly, may reserve the right to terminate the mortgage before the end of period $t+1$. If mortgagors extract equity via refinancing, subsequent to an increase in H, then the originator's aforementioned plans may be thwarted. In order to stop the mortgagor from doing this, originators usually incorporate high prepayment costs, c^p, in the mortgage contract[5]. For setting the scene, the following assumption is made:

Assumption 4.3.1 (Mortgage refinancing): *Suppose that r^{ψ} and c^p for period $t+1$ are restrictively high so that the mortgagor must refinance the mortgage or involuntarily prepay at the end of period t. Moreover, suppose that during periods t and $t+1$ there is a π probability of a $\Phi \in [0,1]$ increase in H.*

In period t, the probability of default is $p(r^{\epsilon}_t M_t, L_t)$, where L_t denotes the LTVR. Here, p is increasing in $r^{\epsilon}M$ – implicitly relative to the mortgagor's income – and in L that measures the mortgagor's house equity. For simplicity, the following assumption is made:

Assumption 4.3.2 (Changes in house prices): *Assume that H changes occur an instant before the end of period t, so that it does not affect L_0 or $p(r^{\epsilon}_t M_t, L_t)$ during period t.*

For the sake of illustration, in formula (4.3) for the recovery amount, R_t, the authors set $a = 0.5$. As in [11], the following assumption is also made:

[5] The prepayment cost, c^p, extends the duration of mortgages, while mortgagor documentation status is associated with higher delinquency status.

Assumption 4.3.3 (Constant LTVR): *Assume that the subprime mortgagor applies for a mortgage with face value, M, for a house worth H with $L = 1$.*

It is noted that under the conditions set in the above discussion, the originator breaks even on a one-period mortgage if r^ϵ is such that

$$(1+r_1^\epsilon)\Big(1 - p(r_t^\varepsilon M_t, L_1)\Big)M + R_1 p(r_t^\varepsilon M_t, L_1) - (1+\bar{c}^{M\omega})M_t = 0.$$

A diagrammatic overview of the evolution of L may be represented as follows:

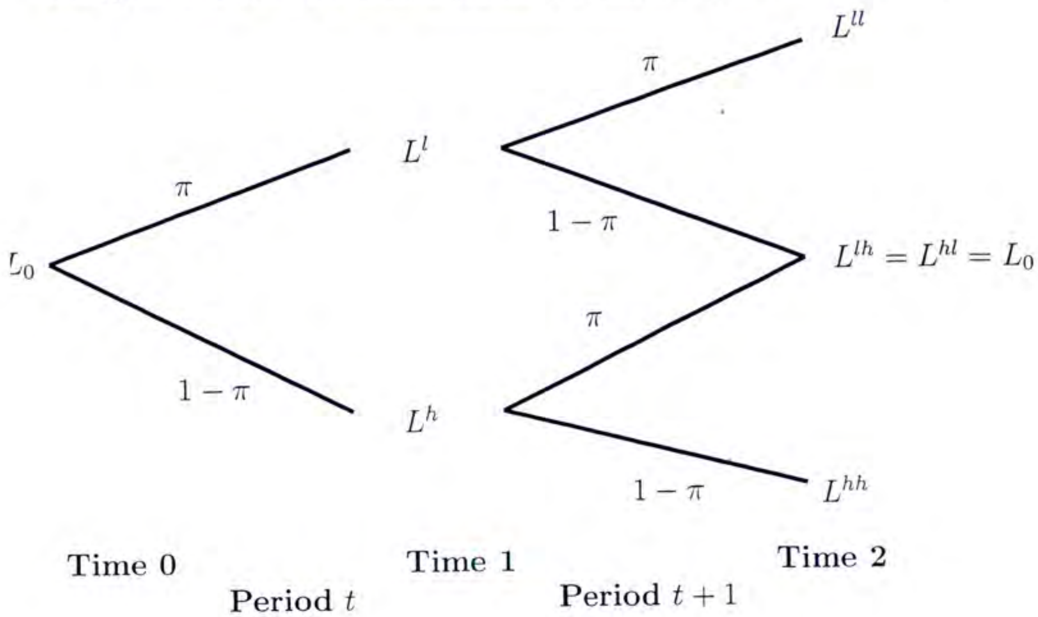

Figure 4.1: Evolution of house prices and the loan-to-value ratio

From Figure 4.1, it is seen that, during period t, if H has fallen, then L will increase to L^h, where the subscript h refers to *higher*, with the mortgagor having negative house equity. In this case, the originator will not be prepared to refinance the mortgage. By contrast, at the beginning of period $t+1$, if H rises, L will fall to L^l, where the subscript l refers to *lower*. In this case, the subprime mortgagor has positive house equity. Here, it is assumed that the initial or any other originator will refinance the mortgage.

The dynamics of H affects the mortgagor's choice to either involuntarily prepay or refinance at the end of period t. By assumption, this price dynamics does not affect $p(r_t^\varepsilon M_t, L_t)$, but rather impacts the recovery amount, R. If involuntary prepayment occurs at the end of period t, then M depends on whether H appreciated or not. In this case, by considering monitoring, transaction, funding and SMI costs, the expected value of mortgages in period t is given by

$$\mathbf{E}(M_t) = \Big(1 - p(r_t^{\varepsilon} M_t, L_0)\Big)(1 + r_t^{\varepsilon})M_t + p(r_t^{\varepsilon} M_t, L_0)\pi R_t^l$$

$$+p(r_t^{\varepsilon} M_t, L_0)(1-\pi)R_t^h - \Big(1 + \bar{c}_t^{M\omega}\Big)M_t,$$

where $R_t^l = \min[0.5(1+\Phi)H_t, M_t]$ in the case where H increases and L decreases. Similarly, $R_t^h = \min[0.5(1+\Phi)H_t, M_t]$ in the case where H decreases and L increases. Also, the superscripts l and h of R refer to L going down (since H increased) and up (since H decreased), respectively. In the sequel, the following is assumed to ensure that the mortgagor has negative house equity when H decreases, and *vice versa*:

Assumption 4.3.4 (Refinancing choices): *Suppose that, if H decreases at the end of period t, the initial or any other originator will not refinance the mortgage. On the other hand, if H increases at the end of period t, it is assumed that the initial originator will be prepared to refinance the mortgage.*

The first part of this assumption ensures that the mortgagor has negative house equity and, by assumption, $p(r_t^{\varepsilon} M_t, L_t)$, is too high for any originator. An increase in H during period t has the following two effects: The mortgagor has positive house equity, which is collateral from the originator's point of view – this makes the originator's recovery amount, R, higher. With all else being equal, with a lower future, L, the probability of default is lower, so the step-up rate, r_{t+1}^{ψ}, may be lower, making the payment lower, which also reduces the probability of default. Throughout this process, the mortgagor may extract equity for consumption.

As for period t, house prices may increase or decrease during period $t+1$. As before, it is assumed that H changes an instant before the end of period $t+1$ and so the change does not affect the probability of default of mortgages during the period, but influences R at the end of period $t+1$. The expected value of the period $t+1$ mortgage is given by

$$\mathbf{E}(M_{t+1}) = \Big(1 - p(r_{t+1}^{\psi} M_{t+1}, L_t^l)\Big)(1 + r_{t+1}^{\psi})M_t$$

$$+p(r_{t+1}^{\psi} M_{t+1}, L_t^l)\pi R_{t+1}^{ll} + p(r_{t+1}^{M} M_{t+1}, L_t^l)(1-\pi)R_{t+1}^{lh} - \Big(1 + \bar{c}_t^{M\omega}\Big)M_t.$$

It is clear that, at the start of period $t+1$, L is now L^h, as H has increased. At the end of period $t+1$, if H decreases, and the mortgagor involuntarily prepays, the originator will recover R_{t+1}^{hl}. On the other hand, the originator will recover R_{t+1}^{hh} if H increases. The above discussion leads to the following statement about the expected profit from subprime mortgage origination: If we neglect to consider discounting and L, the originator's expected profit from mortgage origination during periods t and $t+1$ is $\mathbf{E}(M_t) + \pi\mathbf{E}(M_{t+1})$. Note that the mortgage in period $t+1$ is only extended with probability π if prices increased during period t. At the end of period t, the mortgagor has to either involuntarily prepay or refinance. Similarly, the originator faces a choice which depends on H and leads to optionality in the mortgage. If H increases and L decreases, the originator decides to refinance and opts for $\mathbf{E}(M_{t+1}) = \max[R_t^l, \mathbf{E}(M_{t+1})]$. If H decreases and L increases, the originator decides to opt for the recovery amount and chooses $R_t^h = \max[R_t^h, \mathbf{E}(M_{t+1})]$. It is clear that this optionality is implicit as the strike price is R, which depends on how H performed during period $t+1$. The originator does not take into account costs to the mortgagor from involuntarily prepaying, if there are such costs (see [11] for more details). The main deduction that the authors can make is that the SMC was exacerbated by subprime mortgage design that led to the loss of information and opaqueness. As a consequence, subprime originators could not implement effective risk management policies.

Paper [3] shows that, in the subprime market, lower income and rising H – decreasing L – increase the likelihood of extracting equity while refinancing. The aforementioned contribution produces evidence from individual loan-level data that subprime mortgagors are substituting mortgage debt for credit card debt and car loans. In particular, subprime mortgagors are more likely to extract house equity when the prevailing interest rate on this debt rises. These findings indicate that, in the subprime market, despite being liquidity- and credit-constrained, mortgagors do respond to the relative cost of borrowing and attempt to finance consumption in the cheapest manner possible. In addition, the proportional increase in the probability of default is over 75% higher for cash-outs than for non-cash-outs in response to a 10% reduction in L.

During the SMC, the positive expectation of house prices enhanced all mortgage extension, especially subprime mortgage origination that was accompanied with high credit risk. Therefore, risks, especially credit risk, increased on an originator's balance sheet. Credit risk was shifted to investors around the world via securitization. Higher LTVRs increased mortgagor default risk. The collapse of the house market caused mortgagors to default, as risk grew in the subprime market.

4.3.1.2 Retained earnings under subprime mortgage origination

It is suspected that the originator's profit, Π, is an increasing function of credit ratings, $\mathcal{C}$, so that $\frac{\partial \Pi_t}{\partial \mathcal{C}_t} > 0$. This is connected with procyclicality where profitability increased in the boom prior to the SMC, when credit ratings increased. By contrast, profitability decreased during the SMC because of, among many other factors, an increase in provisioning for bad mortgages. In period t, it is known that *bank profits*, Π_t, are used to meet the originator's obligations that include *dividend payments on equity*, $n_t d_t$, where n_t and d_t are the number of originator shares and their dividends, as well as *interest and principal payments on subordinate debt*, $(1 + r_t^O)O_t$, where O_t is subordinate debt in the case where M is at face value. The *retained earnings*, E_t^r, subsequent to these payments, may be computed by using

$$\Pi_t = E_t^r + n_t d_t + (1 + r_t^O)O_t. \tag{4.2}$$

In standard usage, retained earnings refer to earnings that are not paid out in dividends, interest or taxes. They represent wealth that is accumulating in the bank and should be capitalized in the value of the bank's equity. Retained earnings are also defined to include bank charter value income. Normally, *charter value* refers to the present value of anticipated profits from future lending.

In each period, banks invest in fixed assets (including buildings and equipment) which are denote by F_t. For simplicity, the following is assumed:

Assumption 4.3.5 (Depreciation of fixed assets): *Assume that the originator maintains fixed assets, F_t, throughout its existence so that it must only cover the costs that are related to the* depreciation of fixed assets, ΔF_t.

These activities are financed through the retaining of earnings and the eliciting of additional debt and equity, E_t, so that

$$\Delta F_t = E_t^r + (n_{t+1} - n_t)E_t + O_{t+1}. \tag{4.3}$$

The equations (4.2) and (4.3) can be used to obtain an expression for bank capital of the form

$$K_{t+1} = n_t(d_t + E_t) + (1 + r_t^O)O_t - \Pi_t + \Delta F_t, \tag{4.4}$$

where K_t is defined by $K_t = f^M \widehat{K}_t$, with $\widehat{K}$ being defined as in (4.1).

4.3.1.3 Model for profit under subprime mortgage origination

A model for originator profit at mortgage face value can be built by considering the difference between cash inflow and outflow. For this profit, in period t, *cash inflow* is constituted by returns on risky marketable securities, $r_t^B B_t$, loans, $r_t^M M_t$, treasuries, $r_t^{\mathrm{T}} \mathrm{T}_t$, recovery amount, R_t, SMI protection leg payments, $C(S(\mathcal{C}_t))$, and Π_t^p is the present value of future profits from additional loans that are based on current loans. Furthermore, the authors consider the cost of funds for M, $\bar{c}^{M\omega} M_t$, face value of loans in default, $f^S M_t$, recovery value of loans in default, $r^R M_t$, SMI premium, $p^i(\mathcal{C}_t) M_t$, the all-in cost of holding risky marketable securities, $c_t^B B_t$, interest that is paid to depositors, $r_t^D D_t$, cost of taking deposits, $c^D D_t$, interest that is paid to borrowers, $r_t^{\mathrm{B}} \mathrm{B}_t$, the cost of borrowing, $c^{\mathrm{B}} \mathrm{B}_t$, provisions against deposit withdrawals, $P^{\mathrm{T}}(\mathrm{T}_t)$, and the value of loan losses $S(\mathcal{C}_t)$, as *cash outflow*. Here, r^D and c^D are the deposit rate and marginal cost of deposits, respectively, while r^{B} and c^{B} are the borrower rate and marginal cost of borrowing, respectively. In this case, a discrete-time model for originator profit with all mortgages at market value may be expressed as

$$\begin{aligned}\Pi_t &= \left(r_t^M - \bar{c}_t^{M\omega} - p_t^i + c_t^p f_t^f - (1-r_t^R)f_t^S\right)M_t + C(\mathbf{E}[S(\mathcal{C}_t)]) \qquad (4.5)\\ &\quad + \left(r_t^B - c_t^B\right)B_t + r_t^{\mathrm{T}}\mathrm{T}_t - P^{\mathrm{T}}(\mathrm{T}_t) - \left(r_t^D + c_t^D\right)D_t - \left(r_t^{\mathrm{B}} + c^{\mathrm{B}}\right)\mathrm{B}_t + \Pi_t^p.\end{aligned}$$

Below, it is roughly attempted to associate different risk types to different cash inflow and outflow terms in (4.5). It is noted that the cash inflow term, $r_t^M M_t$, embeds credit and market risk (in particular, interest rate risk), while $c_t^p f_t^f M_t$ can be associated with market risk (in particular, prepayment risk). Also, $(r_t^B - c_t^B)B_t$ mainly embeds market risk. Furthermore, $C(\mathbf{E}[S(\mathcal{C}_t)])$ and Π_t^p involve at least credit risk (particularly, counterparty risk) and market risk (more specifically, interest rate, basis, prepayment, liquidity and price risk), respectively. In (4.5), the cash outflow terms, $\bar{c}_t^{M\omega} M_t$ and $p^i(\mathcal{C}_t)M_t$, carry credit and operational risks. Also, $(1 - r_t^R)f_t^S M_t$ carries credit and market risk (including valuation risk). Deposit withdrawals, $P^{\mathrm{T}}(\mathrm{T}_t)$, carry systemic and market risk. Finally, the term $\left(r_t^{\mathrm{B}} + c^{\mathrm{B}}\right)\mathrm{B}_t$ embeds market and credit risk. In reality, the risks that are associated with each of the cash inflow and outflow terms in (4.5) are more complicated than presented above. For instance, these risks are interrelated and may be strongly correlated with each other. All of the above risk-carrying terms contribute to systemic risk that affects the entire banking system.

4.3.2 Originator valuation under subprime mortgages

In this subsection, net cash flow and optimal originator valuation under subprime mortgages are discussed.

4.3.2.1 Net cash flow under subprime mortgage origination

If the expression for retained earnings that is given by (4.2) is substituted into (4.3), the originator's *net cash flow that is generated* is given by $N_t = \Pi_t - \Delta F_t = n_t d_t + (1 + r_t^O)O_t - K_{t+1} + n_t E_t$, where n_t is the number of originator shares. In addition, the following relationship is found:

Bank value for a shareholder = Net cash flow + ex-dividend bank value.

This translates to the expression $N_t + K_{t+1}$. Furthermore, the stock analyst evaluates the expected future cash flows in j periods, based on a *stochastic discount factor*, $\delta_{t,j}$, such that the value of the bank is

$$N_t + \mathbf{E}_t\left[\sum_{j=1}^{\infty} \delta_{t,j} N_{t+j}\right]. \qquad (4.6)$$

4.3.2.2 Optimal originator valuation under subprime mortgages

In this subsection, the modelling of banking activities from previous discussions is used to solve an optimal originator valuation problem. From [11], the originator's total capital constraint for subprime mortgages at face value is given by

$$K_t = n_t E_{t-1} + O_t \geq \rho\Big[\omega(C_t)M_t + \omega^B B_t + 12.5 f^M (mVaR + 0)\Big], \tag{4.7}$$

where $\omega(C_t)$ and ω^B are the risk-weights that are related to the originator's mortgages at face value and risky marketable securities, respectively, while ρ – Basel II pegs ρ at approximately 0.08 – is the Basel capital regulation ratio of regulatory capital to risk-weighted assets. In order to state an optimal originator valuation problem, it is necessary to assume the following:

Assumption 4.3.6 (Originator's performance criterion): *Suppose that the originator's performance criterion, J, at t is given by*

$$\begin{aligned} J_t &= \Pi_t + l_t\Big[K_t - \rho\Big(\omega(C_t)M_t + \omega^B B_t + 12.5 f^M (mVaR + 0)\Big)\Big] - c_t^{dw}\Big[K_{t+1}\Big] \\ &\quad + \mathbf{E}_t\Big[\delta_{t,1} V\Big(K_{t+1},\ x_{t+1}\Big)\Big], \end{aligned} \tag{4.8}$$

where l_t is the Lagrangian multiplier for the total capital constraint, K_t is defined by $K_t = f^M \widehat{K}_t$ with $\widehat{K}$ being defined as in (4.1), $\mathbf{E}_t[\cdot]$ is the expectation conditional on the bank's information at time t and x_t is the deposit withdrawals in period t with probability distribution $f(x_t)$. Also, c_t^{dw} is the deadweight cost of total capital that consists of common and preferred equity, and subordinate debt, V, is the value function with a discount factor that is denoted by $\delta_{t,1}$.

The optimal originator valuation problem is to maximize the *bank value* that is given by (4.6). The optimal valuation problem can now be stated as follows:

Problem 4.3.7 (Statement of optimal originator valuation problem): *Suppose that the total capital constraint and the performance criterion, J, are given by (4.7) and (4.8), respectively. The optimal originator valuation problem is to maximize the value of the bank, given by (4.6), by choosing the loan rate, deposits and regulatory capital for*

$$V(K_t,\ x_t) = \max_{r_t^M,\ D_t,\ \Pi_t} J_t, \tag{4.9}$$

subject to mortgage, balance sheet, cash flow and financing constraints that are given by (4.4), (4.1), (4.5) and (4.4), respectively.

Next, a solution to Problem 4.3.7 is found when the capital constraint (4.7) holds, as well as when it does not hold. In this regard, the main result can be stated as follows:

Theorem 4.3.8 (Solution to an optimal originator valuation problem (holding)): *Suppose that J and V are given by* (4.8) *and* (4.9), *respectively. When the capital constraint that is given by* (4.7) *holds (i.e., $l_t > 0$), a solution to the optimal originator valuation problem yields an optimal M and r^M of the form*

$$M_t^* = \frac{K_t}{\rho\omega(\mathcal{C}_t)} - \frac{\omega^B B_t + 12.5 f^M(mVaR + \mathtt{0})}{\omega(\mathcal{C}_t)} \tag{4.10}$$

and

$$r_t^{M*} = \frac{1}{m_1}\Big(m_0 + m_2\mathcal{C}_t + \sigma_t^M - M_t^*\Big), \tag{4.11}$$

respectively. In this case, the originator's corresponding optimal deposits, provisions for deposit withdrawals and profits are given by

$$\begin{aligned} D_t^* &= \frac{1}{1-\gamma}\Big(\overline{D} + \frac{\overline{D}}{r_t^p}\Big[r_t^{\mathtt{T}} + (r_t^B - c_t^B) + (r_t^{\mathtt{B}} + c_t^{\mathtt{B}}) - \frac{1}{1-\gamma}(r_t^D + c_t^D)\Big] \\ &\quad + \frac{K_t}{\rho\omega(\mathcal{C}_t)} - \frac{\omega^B B_t + 12.5 f^M(mVaR + \mathtt{0})}{\omega(\mathcal{C}_t)} + B_t - K_t - \mathtt{B}_t\Big), \end{aligned}$$

$$\mathtt{T}_t^* = \overline{D} + \frac{\overline{D}}{r_t^p}\Big[r_t^{\mathtt{T}} + (r_t^B - c_t^B) + (r_t^{\mathtt{B}} + c_t^{\mathtt{B}}) - \frac{1}{1-\gamma}(r_t^D + c_t^D)\Big]$$

and

$$\begin{aligned}
\Pi_t^* &= \left(\frac{K_t}{\rho\omega(\mathcal{C}_t)} - \frac{\omega^B B_t + 12.5 f^M(mVaR + 0)}{\omega(\mathcal{C}_t)}\right) \\
&\left\{\frac{1}{m_1}\left(m_0 - \frac{K_t}{\rho\omega(\mathcal{C}_t)} + \frac{\omega^B B_t + 12.5 f^M(mVaR + 0)}{\omega(\mathcal{C}_t)} + m_2\mathcal{C}_t + \sigma_t^M\right)\right. \\
&\left. -\left(\overline{c}_t^{M\omega} - c_t^p f_t^f + p_t^i + (1 - r_t^R) f_t^S + (r_t^D + c_t^D)\frac{1}{(1-\gamma)}\right)\right\} \qquad (4.12) \\
&-\left((r_t^D + c_t^D)\frac{1}{(1-\gamma)}\right)\left(B_t - K_t - \mathtt{B}_t\right) \\
&+\left(\overline{D} + \frac{\overline{D}}{r_t^P}\left[r_t^{\mathtt{T}} + (r_t^B - c_t^B) + (r_t^{\mathtt{B}} + c_t^{\mathtt{B}}) - \frac{1}{1-\gamma}(r_t^D + c_t^D)\right]\right)\left(r_t^{\mathtt{T}} - (r_t^D + c_t^D)\frac{1}{(1-\gamma)}\right) \\
&+\left(r_t^B - c_t^B\right)B_t - \left(r_t^{\mathtt{B}} + c_t^{\mathtt{B}}\right)\mathtt{B}_t + C(\mathbf{E}[S(\mathcal{C}_t)]) - P^{\mathtt{T}}(\mathtt{T}_t) + \Pi_t^p,
\end{aligned}$$

respectively.

Proof. An immediate consequence of the prerequisite that the capital constraint (4.7) holds, is that M is closely related to the capital adequacy constraint and is given by (4.10). Also, the dependence of changes in r^M on credit ratings may be fixed as $\frac{\partial r_t^{M*}}{\partial \mathcal{C}_t} = \frac{m_2}{m_1}$. Equation (4.10) follows from (4.7) and the fact that the capital constraint holds. This also leads to equality in (4.7). In (4.11), the originator's optimal value for M_t is substituted into the mortgage constraint (4.4) to obtain its optimal default rate. Optimal $\mathtt{T}_t$ is obtained by using the following steps: Firstly, rewrite (4.1) in terms of deposits so that $D_t = \frac{\mathtt{T}_t + M_t + B_t - K_t - \mathtt{B}_t}{1-\gamma}$. Next, the first-order conditions are given by

$$\frac{\partial \Pi_t}{\partial r_t^M}\left[1 + c_t^{dw} - \mathbf{E}_t\left\{\int_{\underline{M}}^{\overline{M}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^M)\right\}\right] + l_t\rho m_1\omega(\mathcal{C}_t) = 0, \tag{4.13}$$

$$\frac{\partial \Pi_t}{\partial D_t}\left[1 + c_t^{dw} - \mathbf{E}_t\left\{\int_{\underline{M}}^{\overline{M}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^M)\right\}\right] = 0, \tag{4.14}$$

$$\rho\left[\omega(\mathcal{C}_t)M_t + \omega^B B_t + 12.5 f^M(mVaR + 0)\right] \leq K_t \text{ and} \tag{4.15}$$

$$-c_t^{dw} + \mathbf{E}_t\left\{\int_{\underline{M}}^{\overline{M}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^M)\right\} = 0. \tag{4.16}$$

Here, $F(\cdot)$ is the cumulative distribution of the shock to M. Using (4.16), it is seen that (4.14) becomes $\frac{\partial \Pi_t}{\partial D_t} = 0$. Looking at the form of Π_t, given in (4.5), and the equation

$$P^{\mathrm{T}_t}(\mathrm{T}_t) = \frac{r_t^p}{2\overline{D}}[\overline{D} - \mathrm{T}_t]^2, \tag{4.17}$$

it follows that

$$\begin{aligned}\Pi_t &= \left(r_t^M - \bar{c}_t^{M\omega} - p_t^i + c_t^p f_t^f - (1 - r_t^R) f_t^S\right) M_t + C(\mathbf{E}[S(\mathcal{C}_t)]) + \left(r_t^B - c_t^B\right) B_t \\ &\quad + r_t^{\mathrm{T}} \mathrm{T}_t - \frac{r_t^p}{2\overline{D}}[\overline{D} - \mathrm{T}_t]^2 - (r_t^D + c_t^D) D_t - \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right) \mathrm{B}_t + \Pi_t^p.\end{aligned} \tag{4.18}$$

Finding the partial derivatives of profit, Π_t, with respect to deposits, D_t, it follows that:

$$\begin{aligned}\frac{\partial \Pi_t}{\partial D_t} &= (1-\gamma)\left[r_t^{\mathrm{T}} + \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right) + \frac{r_t^p}{\overline{D}_t}(\overline{D}_t - \mathrm{T}_t) + (r_t^B - c_t^B)\right] \\ &\quad -(r_t^D + c_t^D).\end{aligned} \tag{4.19}$$

This would then give the authors an optimal value for D_t. Using (4.1) and all originators optimal values that have been calculated to date, optimal deposits can be found, and the same goes for optimal profits. $\square$

During the SMC, uncertainty in financial markets spread to other financial role players, increasing the counterparty risk which caused interest rates to increase. Refinancing became almost impossible and default rates exploded as profits declined (compare with the formula for profit, (4.12)). All these operations embed systemic risk which finally caused the whole financial system to collapse. In order for the originator to fulfil its primary role as a subprime lender, (4.10) should satisfy the condition

$$\frac{K_t}{\rho\omega(\mathcal{C}_t)} > \frac{\omega^B B_t + 12.5 f^M (mVaR + \mathrm{O})}{\omega(\mathcal{C}_t)}.$$

In other words, optimal mortgages, M^*, should have a positive value. A similar comment can be made about the optimal mortgage rate, r^{M*} in (4.11). It is ironic that the SMC occurred despite the fact that most originators complied with the capital adequacy standards that are prescribed in Theorem 4.3.8.

In the case where the capital constraint (4.7) does not hold, the following corollary follows directly:

Corollary 4.3.9 (Solution to optimal originator valuation problem (not holding)): *Suppose that J and V are given by (4.8) and (4.9), respectively. When the capital constraint (4.7) does not hold (i.e., $l_t = 0$), a solution to the optimal originator valuation problem, stated in Problem*

4.3.7, *yields optimal* M *and* r^M *of the form*

$$\begin{aligned} M_t^{n*} &= \frac{1}{2}(m_0 + m_2 C_t + \sigma_t^M) \\ &\quad -\frac{m_1}{2}\left[\overline{c}_t^{M\omega} + p_t^i + (1 - r_t^R) f_t^S + \frac{1}{1-\gamma}(r_t^D + c_t^D) - c_t^p f_t^f\right] \end{aligned} \tag{4.20}$$

and

$$\begin{aligned} r_t^{M^{n*}} &= \frac{1}{2m_1}(m_0 + m_2 C_t + \sigma_t^M) \\ &\quad -\frac{1}{2}\left[\overline{c}_t^{M\omega} + p_t^i + (1 - r_t^R) f_t^S + \frac{1}{1-\gamma}(r_t^D + c_t^D) - c_t^p f_t^f\right], \end{aligned} \tag{4.21}$$

respectively. In this case, the corresponding $\mathtt{T}_t$*, deposits and profits are given by*

$$\mathtt{T}_t^{n*} = \overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathtt{T}} + (r_t^B - c_t^B) + (r_t^{\mathtt{B}} + c_t^{\mathtt{B}}) - \frac{1}{1-\gamma}(r_t^D + c_t^D)\right],$$

$$\begin{aligned} D_t^{n*} &= \frac{1}{1-\gamma}\bigg(\overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathtt{T}} + (r_t^B - c_t^B) + (r_t^{\mathtt{B}} + c_t^{\mathtt{B}}) - \frac{1}{1-\gamma}(r_t^D + c_t^D)\right] \\ &\quad + M_t^{n*} + B_t - K_t - \mathtt{B}_t\bigg) \end{aligned}$$

and

$$
\begin{aligned}
\Pi_t^{n*} &= \left(\frac{1}{2}(m_0 + m_2\mathcal{C}_t\right.\\
&\left. +\sigma_t^M) - \frac{m_1}{2}\left[\overline{c}_t^{M\omega} + p_t^i + (1 - r_t^R)f_t^S + \frac{1}{1-\gamma}(r_t^D + c_t^D) - c_t^p f_t^f\right]\right)\\
&\left\{\frac{1}{2m_1}(m_0 + m_2\mathcal{C}_t + \sigma_t^M)\right.\\
&\left. -\frac{1}{2}\left[\overline{c}_t^{M\omega} + p_t^i + (1 - r_t^R)f_t^S + \frac{1}{1-\gamma}(r_t^D + c_t^D) - c_t^p f_t^f\right]\right\}\\
&-\left((r_t^D + c_t^D)\frac{1}{1-\gamma}\right)\left(B_t - K_t - \mathrm{B}_t\right) + \left(\overline{D}\right.\\
&\left. +\frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}} + (r_t^B - c_t^B) + (r_t^{\mathrm{B}} + c_t^{\mathrm{B}}) - \frac{1}{1-\gamma}(r_t^D + c_t^D)\right]\right)\\
&\left(r_t^{\mathrm{T}} - (r_t^D + c_t^D)\frac{1}{1-\gamma}\right) + \left(r_t^B - c_t^B\right)B_t - \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right)\mathrm{B}_t\\
&+C(\mathbf{E}[S(\mathcal{C}_t)]) - P^{\mathrm{T}}(\mathrm{T}_t) + \Pi_t^p,
\end{aligned}
$$

respectively.

Proof. For the situation where capital constraint (4.7) does not hold (i.e., $l_t = 0$), using equation (4.16) and the fact that $l_t = 0$, it can be seen that (4.14) becomes $\frac{\partial \Pi_t}{\partial r_t^M} = 0$. As in the proof of Theorem 4.3.8, looking at the form of Π_t that is given in (4.5) and (4.17), equation (4.18) is found. Therefore,

$$
\begin{aligned}
\frac{\partial \Pi_t}{\partial r_t^M} &= M_t - m_1\left(r_t^M - \overline{c}_t^{M\omega} - p_t^i + c_t^p f_t^f - (1 - r_t^R)f_t^S\right) \qquad (4.22)\\
&+m_1(r_t^{\mathrm{T}} + (r_t^{\mathrm{B}} + c_t^{\mathrm{B}}) + (r_t^B - c_t^B)) + \frac{r_t^p}{\overline{D}}(\overline{D} - \mathrm{T}_t)m_1 = 0.
\end{aligned}
$$

Substituting (4.19) into (4.22) and using (4.4) would give an optimal mortgage and its rate, given by (4.20) and (4.21), respectively. Furthermore, the originator's optimal deposit, deposit withdrawals and profits can be found. □

To derive equations (4.13) to (4.16), equation (4.9) is rewritten to become

$$V(K_t, x_t) = \max_{r_t^M,\, D_t,\, \Pi_t} \Big\{ \Pi_t + l_t \Big[K_t - \rho\Big(\omega(\mathcal{C}_t)M_t + \omega^B B_t + 12.5 f^M (mVaR + 0)\Big)\Big] \\ - c_t^{dw}\Big[K_{t+1}\Big] + \mathbf{E}_t\Big[\delta_{t,1} V\Big(K_{t+1},\ x_{t+1}\Big)\Big]\Big\}, \tag{4.23}$$

but

$$\Pi_t = n_t(d_t + E_t) - K_{t+1} + \Delta F_t. \tag{4.24}$$

By substituting the mortgage constraint (4.4) and equation (4.24), equation (4.23) becomes

$$V(K_t, x_t) = \max_{r_t^M,\, D_t,\, \Pi_t} \Big\{ \Big(n_t(d_t + E_t) - K_{t+1} + \Delta F_t \Big) \\ + l_t \Big[K_t - \rho \Big[\omega(\mathcal{C}_t)\Big(m_0 - m_1 r_t^M + m_2 \mathcal{C}_t + \sigma_t^M \Big) + \omega^B B_t + 12.5 f^M (mVaR + 0)\Big]\Big] \\ - c_t^{dw}\Big[K_{t+1}\Big] + \mathbf{E}_t\Big[\delta_{t,1} V\Big(K_{t+1},\ x_{t+1}\Big)\Big]\Big\}. \tag{4.25}$$

Finding the partial derivative of originator value in (4.25) with respect to the capital constraint, K_{t+1}, it follows that:

$$\frac{\partial V}{\partial K_{t+1}} = -1 - c_t^{dw} + \mathbf{E}_t\Big[\int_{\underline{M}}^{\overline{M}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF\Big(\sigma_{t+1}^M\Big)\Big].$$

□

4.3.3 Optimal originator valuation and LTVRs

In this subsection, the results from Subsection 4.3.2 are connected with LTVRs, mortgage rates and prepayment costs. The following corollary about LTVRs follows immediately from Theorem 4.3.8:

Corollary 4.3.10 (LTVRs from an optimal originator valuation problem): *Suppose that the hypothesis of Theorem 4.3.8 holds and that the LTVR is given by* (4.6). *When* $l_t > 0$, *then the LTVR, corresponding to the optimal mortgages from* (4.10), *is given by*

$$L_t^* = \frac{K_t}{\rho\omega(\mathcal{C}_t)H_t} - \frac{\omega^B B_t + 12.5 f^M (mVaR + 0)}{\omega(\mathcal{C}_t)H_t}. \tag{4.26}$$

On the other hand, when $l_t = 0$, then the LTVR, corresponding to the optimal mortgages from (4.20), *is given by*

$$
\begin{aligned}
L_t^{n*} &= \frac{1}{2H_t}(m_0 + m_2 C_t + \sigma_t^M) - \frac{m_1}{2H_t} \\
&\left[\overline{c}_t^{M\omega} + p_t^i + (1 - r_t^R) f_t^S + \frac{1}{1-\gamma}(r_t^D + c_t^D) - c_t^p f_t^f\right].
\end{aligned}
\tag{4.27}
$$

Proof. The proof follows from the hypotheses of Theorem 4.3.8 and Corollary 4.3.9. □

From (4.26) it is clear that, *ceteris paribus*, optimal LTVR, L^*, will be favourable when ρ, H and ω are high and K is low. Before the SMC, conditions that were conducive to high H and low K prevailed, while ρ was also relatively high. During the SMC, ω increased steadily, although its influence on L is unclear. On the other hand, when $l_t = 0$, then the L that is corresponding to M^* is given by (4.27). This formula for L^* has a more complicated relationship with its constituting components than was the case in (4.26).

Paper [7] provides a relationship between the originator's mortgage rate, r^M, the LTVR, L, and prepayment cost, c^p, by means of the simultaneous equations model,

$$
\begin{aligned}
r_t^M &= \alpha^0 L_t + \alpha^1 c_t^p + \alpha^2 X_t + \alpha^3 Z_t^{r^M} + u_t \\
L_t &= \beta^1 r_t^M + \beta^2 X_t + \beta^3 Z_t^L + v_t \\
c_t^p &= \gamma^1 r_t^M + \gamma^2 X_t + \gamma^3 Z_t^{c^p} + w_t.
\end{aligned}
\tag{4.28}
$$

Mortgagors typically have a choice of r^M and L, while the choice of c^p triggers an adjustment to r^M. Thus, L and c^p are endogenous variables in the r^M-equation. There is no reason to believe that L and c^p are simultaneously determined. Therefore, c^p does not appear in the L-equation and L does not make an appearance in the c^p-equation. From [7], matrix X comprises exogenous explanatory variables such as mortgage characteristics (owner occupied, mortgage purpose, documentation requirements); mortgagor characteristics (income and Fair Isaac Corporation (FICO) score); and distribution channel (broker origination). The last term in each equation Z^{r^M}, Z^L or Z^{c^p} comprises the instruments that are excluded from either of the other equations. [7] points out that the model is a simplification, with other terms such as type of interest rate, the term to maturity and distribution channel possibly also being endogenous. Nevertheless, by consideration of simultaneity in the choice of r^M and c^p, it is possible to address the issue of possible bias in estimates of the effect of c^p on r^M.

For the first equation in (4.28), r^M may be used to explain ϱ that is primarily used to price mortgages and that reflects an opportunity cost of production of such mortgages. The prime rate is not widely used as an index rate for adjustable-rate mortgages (ARMs) or hybrid ARMs. It is an administered rate that changes relatively infrequently and is influenced by many considerations other than the cost of funds (see [7] and the references contained therein). As such, the prime rate is not very responsive to changes in market rates and is largely uncorrelated with mortgagors' decisions to choose a mortgage with or without a c^p. For the second equation in (4.28), explaining L, [7] uses mortgagor age and H in mortgagor's zip-code area as instruments. The use of these variables as instruments is motivated by observations that older households tend to have higher wealth than younger households, which may make them less likely to seek a large mortgage relative to H, and that wealthier mortgagors tend to choose higher value properties than less wealthy mortgagors (see [7]). These values would not be expected to be correlated with mortgagor choices for ϱ or c^p. For the last equation in (4.28), explaining the choice of c^p, the share of mortgagors that recently moved into residential areas and a dummy variable, indicating whether mortgagor's state passed a law that restricts c^p, is utilized. A high share of mortgagors that recently moved is an indication of high turnover in the local real estate market, which may lessen demand for mortgages with c^p. This indicator would be uncorrelated with r^M or L. State laws, restricting c^p directly, affect the mortgage supply with c^p.

The next result follows from Theorem 4.3.8 and relates to the mortgage rate, r^M, prepayment cost, c^p, and LTVR, L.

Corollary 4.3.11 (Optimal LTVR and prepayment cost): *Suppose that the hypothesis of Theorem 4.3.8, as well as (4.6) and (4.28), holds. In this case, when $l_t = 0$, then the LTVR, corresponding to the optimal mortgages from (4.20), and the optimal prepayment cost are given by*

$$\begin{aligned} L_t^* &= \frac{\beta^1}{2m_1}(m_0 + m_2C_t + \sigma_t^M) + \frac{\beta^1}{2}\left[(1 - r_t^R)f_t^S + \frac{1}{1-\gamma}(r_t^D + c_t^D)\right. \\ &\quad \left. -(\overline{c}_t^{M\omega} + p_t^i + c_t^p f_t^f)\right] + \beta^2 X_t + \beta^3 Z_t^L + v_t \end{aligned}$$

and

$$\begin{aligned} c_t^{p*} &= \frac{\gamma^1}{2m_1}(m_0 + m_2C_t + \sigma_t^M) + \frac{1}{2}\left[(1 - r_t^R)f_t^S + \frac{1}{1-\gamma}(r_t^D + c_t^D)\right. \\ &\quad \left. -(\overline{c}_t^{M\omega} + p_t^i + c_t^p f_t^f)\right] + \gamma^2 X_t + \gamma^3 Z_t^{c^p} + w_t, \end{aligned}$$

respectively.

4.4 Originator valuation example

In this section, choices for subprime mortgage origination, risk, profit and valuation parameters are made by considering data from the Federal Reserve Bank of St Louis database and the Financial Service Research Program's subprime mortgage database for selected periods before and during the crisis.

4.4.1 Choices of subprime mortgage origination parameters

In Table 4.1 below, choices for subprime mortgage origination, profit and valuation parameters are made.

Parameter	Period t	Period $t+1$	Parameter	Period t	Period $t+1$
M	\$ 10 000	\$ 12 000	m^0,	\$ 5 000	\$ 5 000
ω^M	0.5	0.5	m^1	\$ 5 000	\$ 5 000
r^M	0.051	0.082	m^2	\$ 5 000	\$ 5 000
$\bar{c}^{M\omega}$	0.0414	0.0414	C	0.5	0.5
p^i	0.01	0.01	a	0.05	0.05
c^P	0.05	0.05	$C(\mathbf{E}[S(C_t)])$	400	400
f^f	0.01	0.01	r^P	0.1	0.1
r^R	0.5	0.5	K	\$ 500	\$ 650
f^S	0.15	0.25	O	\$ 150	\$ 150
H	\$ 11 000	\$ 11 500	r^{O}	0.101	0.101
Λ	\$ 50 000	\$ 63 157.89	E	\$ 250	\$ 250
ρ	0.08	0.08	E^P	\$ 150	\$ 150
C	\$ 1 000	\$ 1 000	E^c	\$ 100	\$ 100
r^B	0.105	0.105	r^{B}	0.1	0.1
c^B	0.101	0.101	c^{B}	0.09	0.09
B	\$ 1 300	\$ 1 500	B	\$ 5 200	\$ 6 200
ω^B	0.5	0.5	Π^P	\$ 6 000	\$ 6 000
r^{T}	0.036	0.04	D	\$ 9 300	\$ 11 100
T	\$ 2 000	\$ 2 000	r^D	0.105	0.105
O	150	150	r^O	0.101	0.101
n	1.75	2	r^L	0.02	0.05
d	5.4	5.4	f^M	0.08	0.19
$mVar$	400	400	$\omega(C)$	0.5	0.5
P^{T}	\$ 800	\$ 1 200	c^D	0.101	0.101

Table 4.1: Choices of subprime mortgage origination parameters

4.4.2 Computation of subprime mortgage origination parameters

Important equations are computed by using the values from Table 4.1. For $\widehat{R}_t = \gamma \widehat{D}_t$, equation (4.1) is used to compute $\gamma_t = 0.1828$ for the fraction of subprime mortgages, $M_t = 10000$. Also, the equity in (4.2) in period $t-1$ is $\widehat{E}_{t-1} = (500 - 150)/1.75 = 200$. In this case, the risk premium from equation (4.1) is calculated as $\varrho_t = 0.031$.

Net subprime residential mortgage loan losses in equation (4.3) become $S_t = 750$, where $R_t = \min[550,\ 750] = 550$. In period t, the random shock to M, denoted by σ_t^M in equation (4.4), is given by $\sigma_t^M = 2755$. The random shock, σ_{t+1}^M, in period $t+1$ is given by $\sigma_{t+1}^M = 4910$, so that μ_t^M in equation (4.5) is $\mu_t^M = 0.709$. The left-hand side of (4.1) is $M_t + B_t + \mathrm{T}_t = 10000 + 1300 + 2000 = 13300$, and also the right-hand side $(1-\gamma)D_t + B_t + K_t = (1-0.1828)\times 9300 + 5200 + 500 = 13300$. The retained earnings in equation (4.2) is $E_t^r = 1849.8$. Also, the depreciation of fixed assets, ΔF_t, in equation (4.3) is $\Delta F_t = 2062.3$. From equation (4.4), it follows that $K_{t+1} = n_t(d_t + E_t) + (1 + r_t^O)O_t - \Pi_t + \Delta F_t = 650$. The originator's profit at market value in (4.5) is given by $\Pi_{t+1} = 2024.4$. The originator's total capital constraint for subprime residential mortgage loans at face value is given by (4.7), namely

$$K_t = 500 \geq 0.08\big[0.5\times 10000 + 0.5\times 1300 + 12.5\times 0.08\times(400+150)\big] = 496.$$

Furthermore, the originator's optimal mortgages from (4.10) is computed as $M_t^* = 10100$, with the optimal mortgage loan rate in equation (4.11) being given by $r_t^{M^*} = 0.031$. The originator's corresponding optimal deposits, provisions for deposit withdrawals and profits in period t in (4.12) are given by $D_t^* = 15842.55$, $T_t^* = 7246.53$ and

$$\begin{aligned}
\Pi_t^* &= 10100\times\Bigg[\frac{1}{5000}(5000-10100+5000\times 0.5+2755)-\\
&(0.0414-0.05\times 0.01+0.01+(1-0.5)\times 0.15+(0.105+0.101)\frac{1}{1-0.1828})\Bigg]-\\
&\left((0.105+0.101)\frac{1}{1-0.1828}\right)\left(1300-500-5200\right)+7246.53\times\\
&\left(0.036-(0.105+0.101)\frac{1}{1-0.1828}\right)+(0.105-0.101)\times 1300-(0.1+0.09)\times 5200+\\
&400-800+6000 = 656.02,
\end{aligned}$$

respectively.

When the capital constraint (4.7) does not hold, then the solutions for originator's optimal mortgage loans (4.20) and mortgage loan rate (4.21) are given by $M_t^{n^*} = 4182.55$ and $r_t^{M^{n^*}} = 0.8365$, respectively. In this case, corresponding optimal deposits, provisions for deposit withdrawals and profits are given by $D_t^{n^*} = 8601.42$, $T_t^{n^*} = 7246.53$ and $\Pi_t^{n^*} = 7659.22$, respectively. The LTVR, given by equation (4.6) in period t, is given by $L_t = 0.909$. However, the optimal LTVR (4.26) is $L_t^{1*} = 0.9182$. On the other hand, when $l_t = 0$, then (4.27) becomes $L_t^{n*} = 0.3802$. When $X_t = 0.5$ is chosen, $Z_t^{r^M} = Z_t^L = Z_t^{c^p} = 0.02$ and $\alpha^0 = \alpha^1 = \alpha^2 = \alpha^3 = \beta^1 = \beta^2 = \beta^3 = \gamma^1 = \gamma^2 = \gamma^3 = 0.01$, the values from the simultaneous equations model (4.28) are $u_t = 0.03621$, $v_t = 0.90329$ and $w_t = 0.04429$. Therefore, L_t^* and c_t^{p*} in Corollary (4.3.11) are given by $L_t^* = 0.9201$ and $c_t^{p*} = 0.1973$, respectively.

The following values are obtained in period $t+1$: The factor $\gamma_{t+1} = 0.2207$, while the equity in (4.2) in period t is $\widehat{E}_t = (650-150)/2 = 250$. The risk premium in period $t+1$ is $\varrho_{t+1} = 0.082 - 0.05 = 0.032$. Net subprime mortgage loan losses in equation (4.3) become $S_{t+1} = 1500$, where $R_{t+1} = 575$. In period $t+1$, $\sigma_{t+1}^M = 4910$. That the left- and right-hand side of (4.1) correspond to 15500. Retained earnings, $E^r_{t+1} = 518.65$, is given by equation (4.2) and the originator's profit at market value in (4.5) is given by $\Pi_{t+1} = 694.6$. The originator's optimal mortgages (4.10) and optimal mortgage loan rate (4.11) are $M^*_{t+1} = 12137.5$ and $r^{M^*}_{t+1} = 0.0545$, respectively.

Furthermore, the originator's corresponding optimal deposits, provisions for deposit withdrawals and profits (4.12) are given by $D^*_{t+1} = 18631.83$, $T^*_{t+1} = 7732.28$ and $\Pi^*_{t+1} = -974.36$, respectively. When the capital constraint (4.7) does not hold, then $M^{n^*}_{t+1} = 5104.4$ and $r^{M^{n^*}}_{t+1} = 1.0209$, respectively. In this case, corresponding optimal deposits, provisions for deposit withdrawals and profits are given by $D^{n^*}_{t+1} = 9606.93$, $T^{n^*}_{t+1} = 7732.28$ and $\Pi^{n^*}_{t+1} = 8918.64$, respectively.

The LTVR that is given by equation (4.6) in period $t+1$ is calculated as $L_{t+1} = 1.043$. However, the optimal LTVR (4.26) is $L^{1*}_{t+1} = 1.0554$. On the other hand, when $l_{t+1} = 0$, then (4.27) becomes $L^{n*}_{t+1} = 0.4439$. When $X_{t+1} = 0.5$ is chosen, $Z^{r^M}_{t+1} = Z^L_{t+1} = Z^{c^p}_{t+1} = 0.02$ and $\alpha^0 = \alpha^1 = \alpha^2 = \alpha^3 = \beta^1 = \beta^2 = \beta^3 = \gamma^1 = \gamma^2 = \gamma^3 = 0.01$, the following is obtained: $u_{t+1} = 0.06587$, $v_{t+1} = 1.03698$ and $w_{t+1} = 0.04398$. Therefore, L^*_{t+1} and c^{p*}_t in Corollary (4.3.11) are $L^*_{t+1} = 1.0563$ and $c^{p*}_{t+1} = 0.2303$, respectively.

A summary of computed subprime mortgage origination parameters is provided in Table 4.2 below to give a better understanding of what has happened before and during the SMC.

Parameter	Value	Parameter	Value	Parameter	Value
γ_t	0.1828	γ_{t+1}	0.2207	μ^M_t	0.709
$\widehat{E}_{t-1}$	\$ 200	$\widehat{E}_t$	\$ 250	u_t	0.03621
ϱ_t	0.031	ϱ_{t+1}	0.032	u_{t+1}	0.06587
S_t	750	S_{t+1}	1 500	v_t	0.90329
R_t	550	R_{t+1}	575	v_{t+1}	1.03698
σ^M_t	2 755	σ^M_{t+1}	4 910	w_t	0.04429
Π_t	\$ 2 024.4	Π_{t+1}	\$ 694.6	ΔF_t	2 062.3
E^r_t	\$ 1 849.8	E^r_{t+1}	\$ 518.65	w_{t+1}	0.04398
M^*_t	\$ 10 100	M^*_{t+1}	\$ 12 137.5	$r^{M^*}_t$	0.031
D^*_t	\$ 15 842.55	D^*_{t+1}	\$ 18 631.83	$r^{M^*}_{t+1}$	0.0545
T^*_t	\$ 7 246.53	T^*_{t+1}	\$ 7 732.28	Π^*_t	\$ 656.02
$M^{n^*}_t$	\$ 4 182.55	$M^{n^*}_{t+1}$	\$ 5 104.4	Π^*_{t+1}	- \$ 974.36
$r^{M^{n^*}}_t$	0.8365	$r^{M^{n^*}}_{t+1}$	1.0209	$T^{n^*}_t$	\$ 7 246.53
$D^{n^*}_t$	\$ 8 601.42	$D^{n^*}_{t+1}$	\$ 9 606.93	$T^{n^*}_{t+1}$	\$ 7 732.28
$\Pi^{n^*}_t$	\$ 7 659.22	$\Pi^{n^*}_{t+1}$	\$ 8 918.64	L_t	0.909
L_{t+1}	1.043	L^{1*}_t	0.9182	L^{1*}_{t+1}	1.0554
L^{n*}_t	0.3802	L^{n*}_{t+1}	0.4439	L^*_t	0.9201
L^*_{t+1}	1.0563	c^{p*}_t	0.1973	c^{p*}_{t+1}	0.2303

Table 4.2: Computed subprime mortgage origination parameters

4.5 Conclusions

In this paper, subprime mortgages that are able to fully amortize, voluntarily prepay or default (see Question 4.1.7) were discussed. House equity via LTVR in a subprime framework (see Question

4.1.8) was also modelled. Moreover, a discrete-time model was investigated for subprime risk and profit, incorporating costs of funds and SMI, as well as mortgage losses (see Question 4.1.9). Finally, in order to obtain an optimal originator valuation with mortgages at face value, decisions are made regarding mortgage rates, deposits and treasuries (see Question 4.1.10).

In general, the authors' contention is that the SMC was partly caused by the intricacy of design of subprime mortgages that led to information (asymmetry, contagion, inefficiency and loss) problems, valuation opaqueness and ineffective risk mitigation. During this process, declining H – inversely related to L – curtailed mortgage refinancing. This sensitivity to H has widespread implications. The authors also concur with the literature that suggests that subprime mortgages are the first link of an interwoven chain that includes structured mortgage products. If mortgages are dysfunctional, the quality of the notes in the rest of the chain will be compromised. From the findings, it was observed that high LTVRs curtailed mortgage refinancing due to house price depreciation, while low LTVRs increased the house equity of mortgagors.

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Chapter 5

LIQUIDITY MODELLING

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Article 4

Liquidity coverage ratios

By

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Abstract: The authors provide a stochastic model for a global liquidity standard that has been proposed by Basel III banking regulation. In particular, the focus is on the modelling of the liquidity coverage ratio (LCR) that is defined as the quotient of high-quality liquid assets (HQLAs) to net cash outflows (NCOs). This model enables the solving of a nonlinear optimal stochastic LCR problem with quadratic cost. In this regard, an LCR reference process is discussed with respect to which optimal liquidity provisioning and HQLA allocation are characterized. Also, the notion is introduced of an adjustment to the rate of liquidity provisioning per unit of the bank's NCOs for deficit that, under certain circumstances, can be interpreted as a bank bailout rate. Finally, numerical-quantitative results are provided about LCRs and their connections with HQLAs and NCOs to supplement the previous analysis.

Keywords: Basel III, High-Quality Liquid Asset (HQLA), Net Cash Outflow (NCO), Liquidity Coverage Ratio (LCR), Bank Bailouts.

5.1 Introduction

High-quality liquid assets (HQLAs) provide banks with opportunities to operate optimally (see, for instance, [5] and [6]). By contrast, the three recent financial crises, namely the subprime mortgage and global financial crises, as well as the sovereign debt crisis, were characterized by too-big-to fail institutions that suffered from a lack of HQLAs (see, for instance, [9]). The realization of such liquidity risk led to credit crunches and had deleterious effects on financial markets globally. In response to this, among other things, the Basel Committee on Banking Supervision (BCBS) is proposing that banks should always have a 30-day liquidity cover for stress scenarios (see, for instance, [1], [2], [3] and [4]).

As far as Basel III liquidity proposals are concerned, the BCBS is suggesting a liquidity coverage ratio (LCR), defined as

$$\text{LCR} = \frac{\text{High-Quality Liquid Assets (HQLAs)}}{\text{30-Day Net Cash Outflows (NCOs)}} \geq 1.$$

Here, the HQLA and NCO values are related to those that would arise during shock scenarios, a deposit run-off and/or a 3-notch credit rating downgrade. In order to qualify as an HQLA, these assets should satisfy the following characteristics: They should have low credit and market risk, resulting in lower duration, volatility, inflation risk and foreign exchange risk, thus enhancing the asset's liquidity. The HQLA pricing formula should be simple and not depend on strong assumptions. This increases the asset's liquidity among market participants. HQLAs should have low correlation with risky assets. They should be listed on a developed and recognized exchange market, thus increasing transparency. These assets should be central bank eligible for intraday liquidity needs that increase the confidence in HQLA holding that is used during stress scenarios. The asset should be unencumbered and not be pledged as collateral, apart from some exceptional cases. On the other hand, cash outflows are based on the modelling of funding run-offs such as stable and less stable deposits, unsecured wholesale funding and secured (collateralized) funding run-off. As far as cash inflows are concerned, supervisors and banks need to ensure that there is no concentration or dependence on a few sources and on fully performing assets. No credit facilities that are extended to the bank can be included as inflow. NCOs are the total expected cash outflow minus total expected cash inflow for the ensuing 30 calendar days.

The authors' contribution has strong connections with [8] and [9]. In the former, actuarial methods were used to solve a nonlinear stochastic optimal liquidity risk management problem with deposit inflow rates and marketable securities allocation as controls. The main objective in [8] is to minimize liquidity risk in the form of funding and credit crunch risk in an incomplete market. In order to accomplish this, a stochastic model is constructed that incorporates mortgage and deposit reference processes. However, the current article is an improvement on [8] in that bank balance sheet features play a more prominent role (see Sections 5.2, 5.3 and 5.6). In order to construct their LCR model, the authors take into account results that have been obtained in [9] in a discrete-time framework (see also [6]). Their contribution is distinct from the aforementioned in the following respects: Firstly, their analysis has a heavy reliance on the derivation of a stochastic model for LCR dynamics that depends mainly on the liquidity provisioning rate, HQLA returns and NCO outflows. Secondly, the authors obtain an analytic solution of a particular type to their stochastic bank LCR problem (with a quadratic objective function) that they pose. Finally, the optimal choices for the bailout and asset allocation are both expressed in terms of an LCR reference process. To their knowledge, such processes have not been considered for LCRs before.

In short, in this article, the authors study the LCR global liquidity standard (see, for instance, [5] and [9]) with at least two parameters that are able to be controlled, namely the liquidity provisioning rate and HQLA allocation. The main motivation for studying LCR dynamics is to show that, in principle, banks are able to control their liquidity via an appropriate provisioning strategy. This should ensure that the said ratio does not move below an acceptable level. The control theoretic liquidity problem is to meet LCR targets with as little additional liquidity provisioning (essentially corresponding to bank bailouts in their article) as possible and with optimal HQLA allocation. As they have mentioned before, the additional provisioning may arise from an injection from bank bailouts. In summary, in Sections 5.2 and 5.3, the theoretical-quantitative model demonstrates what banks were supposed to do, but did not do before the financial crisis. In Section 5.6, numerical results for LCRs are provided that clearly demonstrate that the financial crisis resulted from both problematic liquidity structures and unexpected cash outflows, as well as the irrational behaviour of banks.

5.2 A liquidity coverage ratio model

In this section, the authors model HQLAs, NCOs and LCR stochastically. This is important for solving the optimal LCR control problem that is outlined subsequently in Section 5.3.

5.2.1 Description of the liquidity coverage ratio model

Before the financial crisis, banks were prosperous, with high liquidity provisioning rates, low interest rates and soaring cash outflows. This was followed by the collapse of the housing market, exploding default rates and the effects thereafter. The following assumption is made to set the space and time index that the authors consider in their LCR model:

Assumption 5.2.1 (Filtered probability space and time index) *Throughout, the authors assume that they are working with a filtered probability space* $(\Omega,\ \mathcal{F},\ \mathbf{P})$ *with filtration,* $\{\mathcal{F}_t\}_{t\geq 0}$, *on a time index set* $T = [t_0, t_1]$.

Furthermore, it is possible to produce a system of stochastic differential equations that provide information about HQLA value at time t, denoted by $x^1 : \Omega \times T \to \mathbb{R}^+$. The assets that are included in the stock of HQLAs are Level 1 assets (L1As) and Level 2 assets (L2As). HQLAs can be constituted by an unlimited pool of L1As. These assets are not subject to any haircut under the LCR. However, supervisors may wish to subject these assets to a haircut-based. L1As are limited to cash, central bank reserves, marketable securities, non-0% risk-weighted sovereigns, sovereign or central bank debt securities that are issued in domestic currencies by the sovereign or central bank in the country and non-0% risk-weighted sovereigns, domestic sovereign or central bank debt securities that are issued in foreign currencies, to the extent that holding of such debt matches the currency needs of the bank's operations in that jurisdiction. L2As must not exceed more than 40% of total stock of HQLAs. Also, a minimum of 15% haircut is applied to the current market value of L2As. Such assets consist of L1As that are generated by secured funding transactions (or collateral swaps), maturing within 30 days, corporate and covered bonds, and marketable securities. It is inefficient for banks to invest all in riskier or riskless HQLAs with asset allocation being important. In this regard, it is necessary to make the following assumption to distinguish between riskier and riskless HQLAs for future computations: HQLAs, x_t^1, are stochastic in nature, because, in part, it depends on the stochastic rates of return on L1As and L2As (see [8] for more details). NCOs at time t, x_t^2, with $x^2 : \Omega \times T \to \mathbb{R}^+$ are stochastic, because its value has a reliance on random cash in- and outflows, as well as liquidity provisioning. Furthermore, for $x : \Omega \times T \to \mathbb{R}^2$, the notation x_t is used to denote

$$x_t = \begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix}$$

and represent the LCR with $l : \Omega \times T \to \mathbb{R}^+$ by

$$l_t = x_t^1 / x_t^2. \tag{5.1}$$

It is important for banks that l_t in (5.1) has to be sufficiently high to ensure high LCRs. Obviously, low values of l_t indicate that the bank has low liquidity and is at high risk of causing a credit crunch.

Bank liquidity has a heavy reliance on liquidity provisioning rates. Roughly speaking, this rate should be reduced for high LCRs and increased beyond the normal rate when bank LCRs are too low. In the sequel, the stochastic process, $u^1 : \Omega \times T \to \mathbb{R}^+$, is the *normal rate of liquidity provisioning per monetary unit of the bank's NCOs* whose value at time t is denoted by u_t^1. In this case, $u_t^1 dt$ turns out to be the normal liquidity provisioning rate per unit of the bank's NCOs over the time period $(t, t + dt)$. A notion that is related to this is the *adjustment to the rate of liquidity provisioning per monetary unit of the bank's NCOs for surplus or deficit*, $u^2 : \Omega \times T \to \mathbb{R}^+$, that will in closed loop be made dependent on the LCR. In the case of deficit, during stress scenarios, this adjustment rate can correspond to a bailout rate. Here, the amount of surplus or deficit is reliant on the excess of HQLAs over NCOs. The sum of u^1 and u^2 is denoted by the *liquidity provisioning rate*, $u^3 : \Omega \times T \to \mathbb{R}^+$, that is

$$u_t^3 = u_t^1 + u_t^2, \text{ for all } t. \tag{5.2}$$

The following assumption is made in order to model the LCR in a stochastic framework:

Assumption 5.2.2 (Liquidity provisioning rate) *The liquidity provisioning rate, u^3, is predictable with respect to $\{\mathcal{F}_t\}_{t\geq 0}$ and provides the authors with a means of controlling bank LCR dynamics (see (5.2) for more details).*

The closed loop system will be defined such that Assumption 5.2.2 is met, as will be seen in the sequel. In times of deficit, for (5.2), the bailout rate, u^2, should be chosen sufficiently large in order to guarantee bank liquidity. In reality, bailouts are subject to more stringent conditions.

Before and during the financial crisis, the LCR decreased significantly as a consequence of rising NCOs. During this period, extensive cash outflows took place. Banks bargained on continued growth in the financial markets (see, for instance, [9]). The dynamics of the *outflows per monetary unit of the bank's NCOs*, $e : \Omega \times T \to \mathbb{R}$, is given by

$$de_t = r_t^e dt + \sigma_t^e dW_t^e, \quad e(t_0) = e_0, \tag{5.3}$$

where e_t is the outflows per NCO monetary unit, $r^e : T \to \mathbb{R}$ is the rate of outflows per monetary unit of the bank's NCOs, the scalar, $\sigma^e : T \to \mathbb{R}$, is the volatility in the outflows per NCO unit and $W^e : \Omega \times T \to \mathbb{R}$ is standard Brownian motion. Furthermore, the following is considered:

$$dh_t = r_t^h dt + \sigma_t^h dW_t^h, \quad h(t_0) = h_0, \tag{5.4}$$

where the stochastic process, $h : \Omega \times T \to \mathbb{R}^+$, is the *investment return on bank HQLAs per monetary unit of HQLAs*, $r^h \to \mathbb{R}^+$ is the rate of HQLA return per HQLA unit, the scalar, $\sigma^h : T \to \mathbb{R}$, is the volatility in the rate of HQLA returns and $W^h : \Omega \times T \to \mathbb{R}$ is standard Brownian motion. Before the financial crisis, riskier HQLA returns were much higher than those of riskless reserves, making the former a more attractive, but much riskier investment.

Assumption 5.2.3 (HQLA classes) *Suppose from the outset that bank HQLAs can be classified into $n+1$ asset classes. One of these HQLAs is risk free (like central bank reserves), while the HQLAs $1,\ 2,\ \ldots,\ n$ have some risk that is associated with them, as permitted by definition.*

Riskier HQLAs evolve continuously in time and are modelled by using an n-dimensional Brownian motion. In this multidimensional context, the *investment returns on bank HQLAs in the k-th HQLA per monetary unit of the k-th HQLA* are denoted by $y_t^k,\ \ k \in \mathbb{N}_n = \{0, 1, 2, \ldots, n\}$ where $y : \Omega \times T \to \mathbb{R}^{n+1}$. Thus, the return per HQLA unit may be represented by

$$y = (\mathtt{R}(t),\ y_t^1,\ \ldots,\ y_t^n),$$

where $\mathtt{R}(t)t$ represents the return on reserves and $y_t^1,\ \ldots,\ y_t^n$ represent riskier HQLA returns. Furthermore, y can be modelled as

$$dy_t = r_t^y dt + \Sigma_t^y dW_t^y, \quad y(t_0) = y_0, \tag{5.5}$$

where $r^y : T \to \mathbb{R}^{n+1}$ denotes the rate of asset returns, $\Sigma_t^y \in \mathbb{R}^{(n+1)\times n}$ is a matrix of HQLA returns and $W^y : \Omega \times T \to \mathbb{R}^n$ is standard Brownian motion. The investment strategy, $\pi : T \to \mathbb{R}^{n+1}$, is assumed to be outside the simplex

$$S = \{\pi \in \mathtt{R}^{n+1} : \pi = (\pi^0,\ \ldots,\ \pi^n)^T, \pi^0 + \ldots + \pi^n = 1,\ \pi^0 \geq 0, \ldots \pi^n \geq 0\}.$$

In this case, short selling is possible. The *investment return on bank HQLAs* is then $h : \Omega \times R \to \mathbb{R}^+$, where the dynamics of h can be written as

$$dh_t = \pi_t^T dy_t = \pi_t^T r_t^y dt + \pi_t^T \Sigma_t^y dW_t^y.$$

This notation can be simplified as follows:

$$
\begin{aligned}
r^{\mathrm{R}}(t) &= r^{y^0}(t), \;\; r^{\mathrm{R}} : T \to \mathbb{R}^+, \text{ the rate of return on riskless assets,} \\
r_t^y &= (r^{\mathrm{R}}(t), \widetilde{r_t^y}^T + r^{\mathrm{R}}(t)1_n)^T, \; \widetilde{r}^y : T \to \mathbb{R}^n, \\
\pi_t &= (\pi_t^0, \widetilde{\pi}_t^T)^T = (\pi_t^0, \pi_t^1, \ldots, \pi_t^k)^T, \; \widetilde{\pi} : T \to \mathbb{R}^k, \\
\Sigma_t^y &= \begin{pmatrix} 0 & \ldots & 0 \\ & \widetilde{\Sigma}_t^y & \end{pmatrix}, \; \widetilde{\Sigma}_t^y \in \mathbb{R}^{n \times n}, \\
\widetilde{C}_t &= \widetilde{\Sigma}_t^y \widetilde{\Sigma_t^y}^T. \text{ Then,} \qquad\qquad (5.6) \\
\pi_t^T r_t^y &= \pi_t^0 r^{\mathrm{R}}(t) + \widetilde{\pi}_t^{jT} \widetilde{r}_t^y + \widetilde{\pi}_t^{jT} r^{\mathrm{R}}(t) 1_n = r^{\mathrm{R}}(t) + \widetilde{\pi}_t^T \widetilde{r}_t^y, \\
\pi_t^T \Sigma_t^y dW_t^y &= \widetilde{\pi}_t^T \widetilde{\Sigma}_t^y dW_t^y, \\
dh_t &= [r^{\mathrm{R}}(t) + \widetilde{\pi}_t^T \widetilde{r}_t^y]dt + \widetilde{\pi}_t^T \widetilde{\Sigma}_t^y dW_t^y; \; h(t_0) = h_0.
\end{aligned}
$$

Next, take $i : \Omega \times T \to \mathbb{R}^+$ as the *increase of NCOs before outflows per monetary unit of NCOs*, $r^i : T \to \mathbb{R}^+$ is the rate of NCO increase before outflows per monetary unit of NCOs, the scalar, $\sigma^i \in \mathbb{R}$, is the volatility in the NCO increase before outflows per monetary unit of NCOs and $W^i : \Omega \times T \to \mathbb{R}$ represents standard Brownian motion. Then, set

$$
di_t = r_t^i dt + \sigma^i dW_t^i, \;\; i(t_0) = i_0. \qquad (5.7)
$$

The stochastic process, i_t, in (5.7) may typically originate from NCOs that have recently taken place or instability in the value of pre-existing NCOs.

Two approaches can be chosen from when modelling the LCR in a stochastic setting. The first is a realistic model that incorporates the most important aspects of the LCR, like HQLAs and NCOs. Alternatively, a simple model can be developed which acts as a proxy for something more realistic that emphasizes features that are specific to the particular study. In this situation, the latter option is chosen, with the model for HQLAs, x^1, and NCOs, x^2, and their relationship being derived as

$$
\begin{aligned}
dx_t^1 &= x_t^1 dh_t + x_t^2 u_t^3 dt - x_t^2 de_t \\
&= [r^{\mathrm{R}}(t)x_t^1 + x_t^1 \widetilde{\pi}_t^T \widetilde{r}_t^y + x_t^2 u_t^1 + x_t^2 u_t^2 - x_t^2 r_t^e]dt \qquad (5.8) \\
&\quad + [x_t^1 \widetilde{\pi}_t^T \widetilde{\Sigma}_t^y dW_t^y - x_t^2 \sigma^e dW_t^e],
\end{aligned}
$$

$$
\begin{aligned}
dx_t^2 &= x_t^2 di_t - x_t^2 de_t \\
&= x_t^2 [r_t^i dt + \sigma^i dW_t^i] - x_t^2 [r_t^e dt + \sigma^e dW_t^e] \qquad (5.9) \\
&= x_t^2 [r_t^i - r_t^e]dt + x_t^2 [\sigma^i dW_t^i - \sigma^e dW_t^e].
\end{aligned}
$$

The stochastic differential equations (5.8) and (5.9) may be rewritten into matrix-vector form in the following way in Definition 5.2.4:

Definition 5.2.4 (Stochastic system for the LCR model) *Define the* stochastic system for the LCR model *as*

$$dx_t = A_t x_t dt + N(x_t)u_t dt + a_t dt + S(x_t, u_t)dW_t, \tag{5.10}$$

with the various terms in this stochastic differential equation being

$$\begin{aligned}
u_t &= \begin{bmatrix} u_t^2 \\ \widetilde{\pi}_t \end{bmatrix}, \quad u : \Omega \times T \to \mathbb{R}^{n+1}, \\
A_t &= \begin{bmatrix} r^{\mathrm{R}}(t) & -r_t^e \\ 0 & r_t^i - r_t^e \end{bmatrix}, \\
N(x_t) &= \begin{bmatrix} x_t^2 & x_t^1 \widetilde{r}_t^{y^T} \\ 0 & 0 \end{bmatrix}, \quad a_t = \begin{bmatrix} x_t^2 u_t^1 \\ 0 \end{bmatrix}, \\
S(x_t, u_t) &= \begin{bmatrix} x_t^1 \widetilde{\pi}_t^T \widetilde{\Sigma}_t^y & -x_t^2 \sigma^e & 0 \\ 0 & -x_t^2 \sigma^e & x_t^2 \sigma^i \end{bmatrix}, \\
W_t &= \begin{bmatrix} W_t^y \\ W_t^e \\ W_t^i \end{bmatrix},
\end{aligned}$$

where W_t^y, W_t^e and W_t^i are mutually (stochastically) independent standard Brownian motions. It is assumed that for all $t \in T$, $\sigma_t^e > 0$, $\sigma_t^i > 0$ and $\widetilde{C}_t > 0$. Often, the time argument of the functions σ^e and σ^i are omitted.

Rewrite (5.10) as follows:

$$\begin{aligned}
N(x_t)u_t &:= \begin{bmatrix} x_t^2 \\ 0 \end{bmatrix} u_t^2 + \begin{bmatrix} x_t^1 \widetilde{r}_t^{y^T} \\ 0 \end{bmatrix} \widetilde{\pi}_t \\
&:= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_t u_t^3 + \sum_{m=1}^{n} \begin{bmatrix} x_t^1 \widetilde{r}_t^{y,m} \\ 0 \end{bmatrix} \widetilde{\pi}_t^m \\
&:= B_0 x_t u_t^0 + \sum_{m=1}^{n} \begin{bmatrix} \widetilde{r}_t^{y,m} & 0 \\ 0 & 0 \end{bmatrix} x_t \widetilde{\pi}_t^m \\
&:= \sum_{m=0}^{n} [B_m x_t] u_t^m
\end{aligned}$$

and

$$
\begin{aligned}
S(x_t, u_t)dW_t &= \begin{bmatrix} [\widetilde{\pi}_t^T \widetilde{C}_t \widetilde{\pi}_t]^{1/2} & 0 \\ 0 & 0 \end{bmatrix} x_t dW_t^1 + \\
&\quad + \begin{bmatrix} 0 & -\sigma^e \\ 0 & -\sigma^e \end{bmatrix} x_t dW_t^2 + \begin{bmatrix} 0 & 0 \\ 0 & \sigma^i \end{bmatrix} x_t dW_t^3 \\
&= \sum_{j=1}^{3} [M^{jj}(u_t) x_t] dW_t^{jj},
\end{aligned}
$$

where B and M are only used for notational purposes to simplify the equations. From the stochastic system that is given by (5.10), it is clear that $u = (u^2, \widetilde{\pi})$ only affects the stochastic differential equation of x_t^1, but not that of x_t^2. In particular, for (5.10) it follows that $\widetilde{\pi}$ affects the variance of x_t^1 and the drift of x_t^1 via the term $x_t^1 \widetilde{r_t^y}^T \widetilde{\pi}_t$. On the other hand, u^2 only affects the drift of x_t^1. Then (5.10) becomes

$$
dx_t = A_t x_t dt + \sum_{j=0}^{n} [B^j x_t] u_t^j dt + a_t dt + \sum_{j=1}^{3} [M^j(u_t) x_t] dW_t^{jj}. \tag{5.11}
$$

5.2.2 Description of the simplified LCR model

The model can be simplified if attention is restricted to the system with the LCR, as stated earlier, denoted in this section by $x_t = x_t^1 / x_t^2$.

Definition 5.2.5 (Stochastic model for a simplified LCR) *Define the* simplified LCR system *by the stochastic differential equation*

$$
\begin{aligned}
dx_t &= x_t [r^{\mathrm{R}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2 + \widetilde{r_t^y}^T \widetilde{\pi}_t] dt \\
&\quad + [u_t^1 + u_t^2 - r_t^e - (\sigma^e)^2] dt \\
&\quad + [(\sigma^e)^2 (1 - x_t)^2 + (\sigma^i)^2 x_t^2 + x_t^2 \widetilde{\pi}_t^T \widetilde{C}_t \widetilde{\pi}_t]^{1/2} d\overline{W}_t, \quad x(t_0) = x_0.
\end{aligned} \tag{5.12}
$$

The model is derived as follows: The starting point is the two-dimensional stochastic differential equation for $x = (x^1, x^2)^T$, as in the equations (5.8) and (5.9). Next, determine

$$
\begin{aligned}
d(x_t^2)^{-1} &= -(x_t^2)^{-2}dx_t^2 + \frac{1}{2}2(x^2)_t^{-3}d < x^2, x^2 > t \\
&= [-(x^2)_t^{-1}(r_t^i - r_t^e) + (x_t^2)^{-1}((\sigma^e)^2 + (\sigma^i)^2)]dt \\
&\quad -(x_t^2)^{-1}\begin{bmatrix} 0 & -\sigma^e & \sigma^i \end{bmatrix} dW_t, \\
dx_t &= x_t^1 d(x_t^2)^{-1} + (x_t^2)^{-1}dx_t^1 + d < x^1, (x^2)^{-1} > t \\
&= [r^{\mathrm{R}}(t)x_t - r_t^e + u_t^1 + u_t^2 + x_t\widetilde{r}_t^y\widetilde{\pi}_t \\
&\quad -x_t[r_t^i - r_t^e] + x_t((\sigma^e)^2 + (\sigma^i)^2) - (\sigma^e)^2]dt \\
&\quad + \begin{pmatrix} x_t\widetilde{\pi}_t^T\widetilde{\Sigma}_t^y & -\sigma^e(1-x_t) & -\sigma^i x_t \end{pmatrix} dW_t \\
&= x_t[r^{\mathrm{R}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2 + \widetilde{r_t^y}^T\widetilde{\pi}_t]dt + \\
&\quad +[u_t^1 + u_t^2 - r_t^e - (\sigma^e)^2]dt \\
&\quad + \left[(\sigma^e)^2(1-x_t)^2 + (\sigma^i)^2(x_t)^2 + (x_t)^2\widetilde{\pi}_t^T\widetilde{C}_t\widetilde{\pi}_t\right]^{1/2} d\overline{W}_t,
\end{aligned}
$$

for a stochastic process $\overline{W} : \Omega \times T \to \mathbb{R}$ which is a standard Brownian motion. Note that, in the drift of the stochastic differential equation (5.12), the term

$$-r_t^e + x_t r_t^e = -r_t^e(x_t - 1)$$

appears because it models the effect of depreciation of both HQLAs and NCOs. Similarly, the term $-(\sigma^e)^2 + x_t(\sigma^e)^2 = (\sigma^e)^2(x_t - 1)$ appears.

The predictions that were made by a previously constructed model are consistent with the empirical evidence in contributions such as [5] and [6]. For instance, in much the same way as the authors do, [6] describes how NCOs affect LCRs. On the other hand, to the best of the authors' knowledge, the modelling that is related to collateral and LCR reference processes (see Section 5.3 for a comprehensive discussion) have not been tested in the literature before.

5.3 Optimal bank liquidity coverage ratios

In order for a bank to determine an optimal bank bailout rate (seen as an adjustment to the normal provisioning rate) and HQLA allocation strategy, it is imperative that a well-defined objective function with appropriate constraints is considered. The choice has to be carefully made in order to avoid ambiguous solutions to the stochastic control problem.

5.3.1 The optimal bank LCR problem

In their contribution, the authors choose to determine a control law $g(t, x_t)$ that minimizes the cost function $J : \mathcal{G}_A \to \mathbb{R}^+$, where $\mathcal{G}_A$ is the class of admissible control laws

$$\begin{aligned} \mathcal{G}_A &= \{g : T \times \mathcal{X} \to \mathcal{U} | g \text{ Borel measurable} \\ &\quad \text{and there exists a unique solution to the closed-loop system}\}, \end{aligned} \tag{5.1}$$

with the closed-loop system for $g \in \mathcal{G}_A$ being given by

$$\begin{aligned} dx_t &= A_t x_t dt + \sum_{j=0}^{n} B^j x_t g^j(t, x_t) dt + a_t dt \\ &\quad + \sum_{j=1}^{3} M^{jj}(g(t, x_t)) x_t dW_t^{jj}, \quad x(t_0) = x_0. \end{aligned} \tag{5.2}$$

Furthermore, the cost function, $J : \mathcal{G}_A \to \mathbb{R}^+$, of the LCR problem is given by

$$\begin{aligned} J(g) &= \mathbf{E}\Big[\int_{t_0}^{t_1} \exp(-r^f(l - t_0)) b(l, x_l, g(l, x_l)) dl \\ &\quad + \exp(-r^f(t_1 - t_0)) b^1(x(t_1)) \Big], \end{aligned} \tag{5.3}$$

where $g \in \mathcal{G}_A$, $T = [t_0, t_1]$ and $b^1 : \mathcal{X} \to \mathbb{R}^+$ is a Borel-measurable function. Furthermore, $b : T \times \mathcal{X} \times \mathcal{U} \to \mathbb{R}^+$ is formulated as

$$b(t, x, u) = b^2(u^2) + b^3(x^1/x^2),$$

for $b^2 : \mathcal{U}_2 \to \mathbb{R}^+$ and $b^3 : \mathbb{R}^+ \to \mathbb{R}^+$. Also, $r^f \in \mathbb{R}$ is called the *NCO forecasting rate*. The functions b^1, b^2 and b^3 are selected below, where various choices are considered. In order to clarify the stochastic problem, the following assumption should be made:

Assumption 5.3.1 (Admissible class of control laws) *Assume that $\mathcal{G}_A \neq \emptyset$.*

The authors are now in a position to state the stochastic optimal control problem for a continuous-time LCR model that they solve. The said problem may be formulated as follows:

Problem 5.3.2 (Optimal bank LCR problem) *Consider the stochastic control system* (5.2) *for the LCR problem with the admissible class of control laws, $\mathcal{G}_A$, given by* (5.1) *and the cost function, $J : \mathcal{G}_A \to \mathbb{R}^+$, given by* (5.3). *Solve*

$$\inf_{g\in\mathcal{G}_A} J(g),$$

which amounts to determining the value J^*, given by

$$J^* = \inf_{g\in\mathcal{G}_A} J(g)$$

and the optimal control law, g^*, if it exists,

$$g^* = \arg\min_{g\in\mathcal{G}_A} J(g) \in \mathcal{G}_A.$$

5.3.2 Optimal bank LCRs in the simplified case

Consider the simplified system (5.12) for the LCR problem with the admissible class of control laws, $\mathcal{G}_A$, given by (5.1), but with $\mathcal{X} = \mathbb{R}$. In this section, the authors have to solve

$$\begin{aligned} & \inf_{g\in\mathcal{G}_A} J(g), \\ J^* &= \inf_{g\in\mathcal{G}_A} J(g), \qquad (5.4) \\ J(g) &= \mathbf{E}\Big[\int_{t_0}^{t_1} \exp(-r^f(l-t_0))[b^2(u_t^2)+b^3(x_t)]dt \\ &\quad + \exp(-r^f(t_1-t_0))b^1(x(t_1))\Big], \qquad (5.5) \end{aligned}$$

where $b^1 : \mathbb{R} \to \mathbb{R}^+$, $b^2 : \mathbb{R} \to \mathbb{R}^+$ and $b^3 : \mathbb{R}^+ \to \mathbb{R}^+$ are all Borel-measurable functions. For the simplified case, the optimal cost function (5.4) should be determined with the simplified cost function, $J(g)$, given by (5.5). In this case, assumptions have to be made in order to find a solution for the optimal cost function, J^*. Next, an important result in a stochastic framework is stated.

Theorem 5.3.3 (Optimal bank LCRs in the simplified case)
Suppose that g^{2*} and g^{3*} are the components of the optimal control law, g^*, that deal with the optimal bailout rate, u^{2*}, and optimal HQLA allocation, π^{k*}, respectively. Consider the nonlinear optimal stochastic control problem for the simplified LCR system (5.12) that is formulated in Problem 5.3.2. Suppose that the following assumptions hold:

a. The cost function is assumed to satisfy

$$
\begin{aligned}
& b^2(u^2) \in C^2(\mathbb{R}), \\
& \lim_{u^2 \to -\infty} D_{u^2} b^2(u^2) = -\infty, \quad \lim_{u^2 \to +\infty} D_{u^2} b^2(u^2) = +\infty, \\
& D_{u^2 u^2} b^2(u^2) > 0, \ \forall u^2 \in \mathbb{R},
\end{aligned} \tag{5.6}
$$

with the differential operator, D, which is applied in this case to function b^2.

b. *There exists a function $v : T \times \mathbb{R} \to \mathbb{R}$, $v \in C^{1,2}(T \times \mathcal{X})$, which is a solution of the partial differential equation given by*

$$
\begin{aligned}
0 &= D_t v(t,x) + \frac{1}{2}[(\sigma^e)^2(1-x)^2 + (\sigma^i)^2(x_t)^2] D_{xx} v(t,x) \\
&\quad + x(r^{\mathrm{R}}(t) + r^e_t - r^i_t + (\sigma^e)^2 + (\sigma^i)^2) D_x v(t,x) \\
&\quad + [u^1_t - r^e_t - (\sigma^e)^2] D_x v(t,x) \\
&\quad + u^{2^*}_t D_x v(t,x) + \exp(-r^f(t-t_0)) b^2(u^{2^*}_t) \\
&\quad + \exp(-r^f(t-t_0)) b^3(x) - \frac{[D_x v(t,x)]^2}{2 D_{xx} v(t,x)} \widetilde{r}^{y\,T}_t \widetilde{C}_t^{-1} \widetilde{r}^y_t, && (5.7) \\
v(t_1, x) &= \exp(-r^f(t_1 - t_0)) b^1(x), && (5.8)
\end{aligned}
$$

where u^{2*} is the unique solution of the equation

$$
0 = D_x v(t,x) + \exp(-r^f(t-t_0)) D_{u^2} b^2(u^2_t). \tag{5.9}
$$

Then the optimal control law is

$$
\begin{aligned}
g^{2*}(t,x) &= u^{2^*}, \ g^{2^*} : T \times \mathcal{X} \to \mathbb{R}^+, && (5.10) \\
& \quad \textit{with } u^{2*} \in \mathcal{U}_2 \textit{ the unique solution of the equation (5.9)} \\
\widetilde{\pi}^* &= -\frac{D_x v(t,x)}{x D_{xx} v(t,x)} \widetilde{C}_t^{-1} \widetilde{r}^y_t, && (5.11) \\
g^{3,k*}(t,x) &= \min\{1, \max\{0, \widetilde{\pi}^{k*}\}\}, g^{3,k*} : T \times \mathcal{X} \to \mathbb{R}. && (5.12)
\end{aligned}
$$

Furthermore, the value of the problem is

$$
J^* = J(g^*) = \mathbf{E}[v(t, x_0)]. \tag{5.13}
$$

Proof. It will be proven that the infimization in the dynamic programming equation can be achieved and that there exists a solution to the dynamic programming equation.

Step 1: Recall from optimal stochastic control theory that the dynamic programming equation for the optimal control problem for $w : T \times \mathbb{R} \to \mathbb{R},\ w \in C^{1,2}(T \times \mathcal{X})$, is given by

$$\begin{aligned} 0 &= D_t w(t,x) + \\ &+ \inf_{u^2 \in \mathbb{R},\ \tilde{\pi} \in [0,1]} \Big[\frac{1}{2} [(\sigma^e)^2 (1-x)^2 + (\sigma^i)^2 (x)^2 + (x)^2 \tilde{\pi}^T \widetilde{C}_t \tilde{\pi}] D_{xx} w(t,x) \\ &+ [(r^{\mathrm{T}}(t) + r^e_t - r^i_t + (\sigma^e)^2 + (\sigma^i)^2 + \tilde{r^y_t}^T \tilde{\pi}) x] D_x w(t,x) \\ &\quad + [u^1_t + u^2 - r^e_t - (\sigma^e)^2] D_x w(t,x) \\ &+ \exp(-r^f (t - t_0)) [b^2(u^2) + b^3(x)] \Big], \qquad (5.14) \\ w(t_1, x) &= \exp(-r^f (t_1 - t_0)) b^1(x). \qquad (5.15) \end{aligned}$$

Note that this dynamic programming equation separates additively into terms

(a) depending on u^2,

(b) depending on $\tilde{\pi}$ and

(c) not depending on either of these variables.

The infimization is therefore decomposed into two infimizations.

Step 2: The infimizations are calculated as follows with the function v:

$$\begin{aligned} &\inf_{u^2 \in \mathbb{R}} H_2(t,x,u^2), \\ H_2(t,x,u) &= u^2 D_x v(t,x) + \exp(-r^f (t - t_0)) b^2(u^2), \qquad (5.16) \\ D_{u^2} H_2(t,x,u) &= D_x v(t,x) + \exp(-r^f (t - t_0)) D_{u^2} b^2(u^2) = 0. \qquad (5.17) \end{aligned}$$

Because of Assumption (1), equation (5.17) for all $(t,x) \in T \times \mathcal{X}$ has a unique solution $u^{2^*} \in \mathcal{U} = \mathbb{R}$ and

$$\inf_{u^2 \in \mathbb{R}} H_2(t,x,u^2) = H_2(t,x,u^{2^*}) = u^{2^*} D_x v(t,x) + \exp(-r^f (t - t_0)) b^2(u^{2^*}). \qquad (5.18)$$

Define the function $g^2(t,x)^* = u^{2^*}$, $g^{2^*} : T \times \mathcal{X} \to \mathbb{R}$. It follows from Assumption (1) that g^{2^*} is Borel-measurable. Consider the infimization problem

$$\inf_{\tilde{\pi}\in \mathbb{R}^k} H_3(t,x,\tilde{\pi}), \tag{5.19}$$

$$\begin{aligned} H_3(t,x,\tilde{\pi}) &= \frac{1}{2}[\tilde{\pi}_t^T \widetilde{C}_t \tilde{\pi}_t](x)^2 D_{xx}v(t,x) + \tilde{r_t^y}^T \tilde{\pi} x D_x v(t,x) \qquad (5.20)\\ &= \frac{1}{2}\left(\tilde{\pi}_t + (\frac{x[D_x v(t,x)]}{(x)^2 D_{xx}v(t,x)})\widetilde{C}_t^{-1}\tilde{r}_t^y\right)^T \times \\ &\quad \times \tilde{C}_t(x)^2 D_{xx}v(t,x)\left((\tilde{\pi}_t + (\frac{x[D_x v(t,x)]}{(x)^2 D_{xx}v(t,x)})\widetilde{C}_t^{-1}\tilde{r}_t^y\right) \\ &\quad -\frac{1}{2}(\frac{[xD_x v(t,x)]^2}{(x)^2 D_{xx}v(t,x)})\tilde{r_t^y}^T \widetilde{C}_t^{-1}\tilde{r}_t^y, \\ &\quad \widetilde{C}_t(x)^2 D_{xx}v(t,x) > 0,\ \forall x \in \mathbb{R}\backslash\{0\}. \qquad (5.21) \end{aligned}$$

Hence, it follows that:

$$\begin{aligned} \widetilde{\pi^k}^* &= \min\{1, \max\{0, -\frac{D_x v(t,x)}{xD_{xx}v(t,x)}\widetilde{C}_t^{-1}\tilde{r}_t^y\}\}, \qquad (5.22)\\ g^{3,k}(t,x)^* &= \begin{cases} \tilde{\pi}^*, & \text{if } \sum_{i=1}^k \tilde{\pi^i}^* \in [0,1], \\ \frac{\tilde{\pi}^*}{\sum_{j=1}^k \tilde{\pi^j}^*}, & \text{if } \sum_{i=1}^k \tilde{\pi^i}^* > 1 \end{cases} \quad \forall k \in \mathbb{Z}_n, \qquad (5.23)\\ H_3(t,x,\tilde{\pi}^*) &= -\frac{[D_x v(t,x)]^2}{2D_{xx}v(t,x)}\tilde{r_t^y}^T \widetilde{C}_t^{-1}\tilde{r}_t^y. \qquad (5.24) \end{aligned}$$

If, for a $k \in \mathbb{Z}_n$, $\tilde{\pi^k}^* \notin [0,1]$, then the dynamic programming equation (5.7) has to be modified with the difference of the terms that are obtained with and without the constraint. This is not indicated in the current paper.

Step 3: Using the infima that has been obtained in Step 2, the dynamic programming equation for v, Equation (5.7), can now be rewritten. The resulting equation is

$$\begin{aligned} 0 &= D_t v(t,x) + \inf_{u^2\in \mathbb{R}, \tilde{\pi}\in[0,1]}\left\{\frac{1}{2}[(\sigma^e)^2(1-x)^2 + (\sigma^i)^2(x)^2 + (x)^2(\tilde{\pi}^T\widetilde{C}_t\tilde{\pi})]D_{xx}v(t,x)\right. \\ &\quad +[r^{\mathrm{T}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2 + \tilde{r_t^y}^T\tilde{\pi}]x \\ &\quad +[u_t^1 + u^2 - r_t^e - (\sigma^e)^2]D_x v(t,x) \qquad (5.25)\\ &\quad \left. + \exp(-r^f(t-t_0))[b^2(u^2) + b^3(x)]\right\}, \\ v(t_1,x) &= \exp(-r^f(t_1-t_0))b^1(x). \qquad (5.26) \end{aligned}$$

Thus, v, whose existence is assumed by Condition (2), is a solution of the dynamic programming equation. It follows then from optimal stochastic control that g^{2^*} and g^{3^*} are the optimal control laws and that the value is given by $J^* = J(g^*) = \mathbf{E}[v(t_0,x_0)]$. □

It is of interest to choose particular cost functions for which an analytic solution can be obtained for the value function and for the control laws. The following theorem provides the optimal control laws for a particular choice of cost functions:

Theorem 5.3.4 (Optimal bank LCRs with quadratic cost functions): *Consider the nonlinear optimal stochastic control problem for the simplified LCR system* (5.12) *that has been formulated in Problem* 5.3.2. *Consider the cost function*

$$\begin{aligned} J(g) &= \mathbf{E}\Big[\int_{t_0}^{t_1} \exp(-r^f(l-t_0))[\frac{1}{2}c^2((u^2)^2(l)) + \frac{1}{2}c^3(x_t - l^r)^2]dl \\ &\quad + \frac{1}{2}c^1(x(t_1) - l^r)^2 \exp(-r^f(t_1 - t_0))\Big]. \end{aligned} \tag{5.27}$$

It is assumed that the cost function satisfies

$$\begin{aligned} b^1(x) &= \frac{1}{2}c^1(x-l^r)^2, \;\; c^1 \in (0,\infty), \\ b^2(u^2) &= \frac{1}{2}c^2(u^2)^2, \;\; c^2 \in (0,\infty), && (5.28) \\ b^3(x) &= \frac{1}{2}c^3(x-l^r)^2, \;\; c^3 \in (0,\infty), && (5.29) \\ & l^r \in \mathbb{R}, \text{ called the reference value of the LCR.} \end{aligned}$$

Define the first-order ordinary differential equation

$$\begin{aligned} -\dot{q}_t &= -(q_t)^2/c^2 + c^3 + q_t 2(r^{\mathrm{R}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2) \\ &\quad + q_t[-r^f - \widetilde{r_t^y}^T \widetilde{C}_t^{-1} \widetilde{r}_t^y + (\sigma^e)^2 + (\sigma^i)^2], \;\; q_{t_1} = c^1, && (5.30) \\ -\dot{x^r}_t &= -c^3(x_t^r - l^r)/q_t - x_t^r[r^{\mathrm{R}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2] \\ &\quad - [u_t^1 - r_t^e - (\sigma^e)^2] - (x_t^r - 1)((\sigma^e)^2 + (\sigma^i)^2) - (\sigma^i)^2, \;\; x^r(t_1) = l^r, && (5.31) \\ -\dot{l}_t &= -r^f l_t + c^3(x_t^r - l^r)^2 - q_t(\sigma^e)^2(x_t^r - 1)^2 - q_t(\sigma^i)^2(x_t^r)^2, \\ &\quad l(t_1) = 0. && (5.32) \end{aligned}$$

The function $x^r : T \to \mathbb{R}$ *will be called the* LCR reference (process) function. *Then,*

(a) there exist solutions to the ordinary differential equations (5.30), (5.31) *and* (5.32)*; moreover, for all* $t \in T$, $q_t > 0$;

(b) the optimal control laws are

$$\begin{aligned}
u^{2*}_t &= -(x - x^r_t)q_t/c^2, && (5.33)\\
g^{2^*}(t,x) &= u^{2^*},\ g^{2^*} : T \times \mathcal{X} \to \mathbb{R}^+, \\
\widetilde{\pi}^*_t &= -\frac{(x - x^r_t)}{x}\widetilde{C}_t^{-1}\widetilde{r}^y_t,\ g^{3^*} : T \times \mathcal{X} \to \mathbb{R}^k, && (5.34)\\
g^{3,k*}(t,x) &= \begin{cases} \widetilde{\pi}^{k*}, & \textit{if } \widetilde{\pi}^{k*} \in [0,1], \\ \min\{1, \max\{0, \widetilde{\pi}^{k*}_t\}\}; & \textit{otherwise}, \end{cases} \quad \forall k \in \mathbb{Z}^n. && (5.35)
\end{aligned}$$

(c) the value function and the value of the problem are

$$\begin{aligned}
v(t,x) &= \exp(-r^f(t_1 - t_0))[\frac{1}{2}(x - x^r_t)^2 q_t + \frac{1}{2}l_t], && (5.36)\\
J^* &= J(g^*) = \mathbf{E}[v(t_0, x_0)]. && (5.37)
\end{aligned}$$

Proof. (a) The ordinary differential equation (5.30) is of the Riccati type. It follows from [10, Theorem 37, p. 364] that a unique solution to this equation exists.

The second statement requires a more comprehensive argument. Firstly, rewrite the Riccati differential equation (5.30) in the form

$$-\dot{q}_t = -b(q_t)^2 + c + 2a_t q_t,\ \ q_{t_1} = c^1,\ \ b = 1/c^2, c = c^3 \in (0,\infty),\ a : T \to \mathbb{R}.$$

Transform the equation according to

$$\begin{aligned}
q^1_{t_1} &= q(t_1 - t),\ \ a^1_t = a(t_1 - t),\ \ q^1 : [0, t_1 - t_0] \to \mathbb{R},\\
\dot{q}^1_t &= -\dot{q}(t_1 - t) = -b(q^1_t)^2 + c + 2a^1_t q^1_t,\ q^1_{t_0} = c^1,\\
0 &= \dot{q}^1_t + b(q^1_t)^2 - c - 2a^1_t q_t,\ q^1_0 = c^1.
\end{aligned}$$

Consider the second Riccati differential equation

$$-\dot{q}^2_t = -\bar{b}(q^2_t)^2 + \bar{c},\ \ q^2_{t_1} = c^1;\ \ \bar{b}, \bar{c} \in (0,\infty).$$

The latter Riccati differential equation is time-invariant. The associated first-order linear system with system matrices $(0, \sqrt(b), \sqrt(c))$ are both controllable and observable. It then follows from a result for this Riccati differential equation that $q^2_t > 0$ for all $t \in T = [0, t_1 - t_0]$.

Next, the result [7, Lemma 5.1] on the comparison of solutions of two Riccati differential equations is invoked. The lemma states that for all $t \in [t_1 - t_0]$, $q^1_t \geq q^2_t > 0$ if

$$b \in (0,\infty), \quad \begin{pmatrix} -\bar{c} & 0 \\ 0 & \bar{b} \end{pmatrix} \geq \begin{pmatrix} -c & -a_t^1 \\ -a_t^1 & b \end{pmatrix}, \; \forall t \in [0, t_1 - t_0].$$

The matrix inequality is met if, and only if,

$$\begin{aligned} & b \in (0,\infty), \; (\bar{b} - b) \geq 0, \; (c - \bar{c}) \geq 0, \\ (a_t)^2 \;\; \leq \;\; & (c - \bar{c})(\bar{b} - b) = (c^3 - \bar{c}^3)(\frac{1}{\bar{c}^2} - \frac{1}{c^2}). \end{aligned}$$

By assumption, all functions in a, and hence those of $a_t^1 = a(t_1 - t)$, are bounded on the bounded interval $[0, t_1 - t_0]$. Hence, there exists a constant $c^4 \in (0,\infty)$ such that $(a_t^1)^2 < c^4$. Then one can determine real numbers $\bar{b}, \bar{c} \in (0,\infty)$ such that

$$\begin{aligned} & c - \bar{c} = c^3 - \bar{c}^3 \geq 0, \; \bar{b} - b = 1/\bar{c}^2 - 1/c^2 \geq 0, \\ (a_t)^2 \;\; \leq \;\; & c^4 \leq (c - \bar{c})(\bar{b} - b) = (c^3 - \bar{c}^3)(1/\bar{c}^2 - 1/c^2), \end{aligned}$$

for example by choosing $\bar{c}^2, \bar{c}^3 \in (0,\infty)$, both very small. Thus, the inequalities are satisfied and for all $t \in [0, t_1 - t_0]$, it follows that $q(t_1 - t) = q_t^1 \geq q_t^2 > 0$.

The ordinary differential equation (5.31) is linear, hence it follows from a theorem for this type of differential equation that a unique solution exists.

(b, c) The cost function b^2 satisfies the conditions of Theorem 5.3.3. It will be proven that the function (5.36) is a solution of the partial differential equation (5.7, 5.8) of Theorem 5.3.3.

Firstly, note that

$$\begin{aligned} v(t,x) &= \exp(-r^f(t - t_0))[\frac{1}{2}(x - x_t^r)^2 q_t + \frac{1}{2} l_t], \\ D_t v(t,x) &= -r^f v(t,x) + \exp(-r^f(t - t_0))[-(x - x_t^r)\dot{x^r}_t q_t + \frac{1}{2}(x - x_t^r)^2 \dot{q}_t + \frac{1}{2}\dot{l}_t], \\ D_x v(t,x) &= \exp(-r^f(t - t_0))(x - x_t^r) q_t, \\ D_{xx} v(t,x) &= \exp(-r^f(t - t_0)) q_t. \end{aligned}$$

The optimal control laws are calculated according to the formulas of the previous theorem, where

$$
\begin{aligned}
0 &= D_x v(t,x) + \exp(-r^f(t-t_0)) D_{u^2} b^2(u^2) \\
&= \exp(-r^f(t-t_0))[(x-x_t^r)q_t + c^2 u^2], \\
u^{2^*} &= -(x-x_t^r)q_t/c^2, \\
\tilde{\pi}^* &= -\frac{(x-x_t^r)q_t}{x q_t} \tilde{C}_t^{-1} \tilde{r}_t^y.
\end{aligned}
$$

From the partial differential equation (5.7) and the terminal condition (5.8) follow that

$$
\begin{aligned}
& q_{t_1} = c^1,\ x^r(t_1) = l^r,\ l(t_1) = 0, \\
v(t_1, x) &= \exp(-r^f(t_1-t_0))[\frac{1}{2}(x - x^r(t_1))^2 q_{t_1} + \frac{1}{2} l(t_1)] \\
&= \exp(-r^f(t_1-t_0))\frac{1}{2}c^1(x-l^r)^2 = \exp(-r^f(t_1-t_0)) b^1(x),
\end{aligned}
$$

$$
\begin{aligned}
& \exp(+r^f(t-t_0)) \Big[D_t v(t,x) + \frac{1}{2}[(\sigma^e)^2(1-x)^2 + (\sigma^i)^2(x)^2] D_{xx} v(t,x) \\
& + x[r^{\mathrm{T}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2] D_x v(t,x) \\
& + [u_t^1 - r_t^e - (\sigma^e)^2] D_x v(t,x) \\
& + u^{2^*} D_x v(t,x) + \exp(-r^f(t-t_0)) b^2(u^{2^*}) \\
& + \exp(-r^f(t-t_0)) b^3(x) - \frac{1}{2} \frac{(D_x v(t,x))^2}{D_{xx} v(t,x)} \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y \Big] \\
= & -r^f \frac{1}{2}(x-x_t^r)^2 q_t - \frac{1}{2} r^f l_t - (x-x_t^r) q_t \dot{x^r}_t \\
& + \frac{1}{2}(x-x_t^r)^2 \dot{q}_t + \frac{1}{2}\dot{l}_t \\
& + \frac{1}{2}[(\sigma^e)^2(1-x)^2 + (\sigma^i)^2(x)^2] q_t \\
& + (x-x_t^r) q_t [x(r^{\mathrm{T}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2) + u_t^1 - r_t^e - (\sigma^e)^2] \\
& - \frac{1}{2}(x-x_t^r)^2 (q_t)^2/c^2 \\
& + \frac{1}{2}c^3(x-l^r)^2 - \frac{1}{2}(x-x_t^r)^2 q_t \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y
\end{aligned}
$$

$$
\begin{aligned}
= \quad & \frac{1}{2}(x)^2 \left[-r^f q_t + \dot{q}_t + (\sigma^e)^2 q_t + (\sigma^i)^2 q_t \right. \\
& \quad + 2q_t[r^{\mathrm{T}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2] - (q_t)^2/c^2 + c^3 \\
& \quad \left. - q_t \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y \right] \\
& + x \left[+r^f x_t^r q_t - q_t \dot{x^r}_t - x_t^r \dot{q}_t - (\sigma^e)^2 q_t \right. \\
& \quad - x_t^r q_t [r^{\mathrm{T}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2] + q_t[u_t^1 - r_t^e - (\sigma^e)^2] \\
& \quad \left. + x_t^r (q_t)^2/c^2 - c^3 l^r + x_t^r q_t \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y \right] \\
& + \frac{1}{2}(x)^0 \left[-r^f (x_t^r)^2 q_t + 2x_t^r q_t \dot{x^r}_t + (x_t^r)^2 \dot{q}_t - r^f l_t + (\sigma^e)^2 q_t \right. \\
& \quad - 2x_t^r q_t [u_t^1 - r_t^e - (\sigma^e)^2] - (x_t^r)^2 (q_t)^2/c^2 + c^3 (l^r)^2 \\
& \quad \left. - (x_t^r)^2 q_t \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y + \dot{l}_t \right].
\end{aligned}
$$

It will be proven that the terms at the powers of the indeterminate x are zero, thus at $(x)^2$, $(x)^1$ and $(x)^0$. The term at $(x)^2$ is zero due to the differential equation for q. The term at $(x)^1$ equals

$$
\begin{aligned}
& -q_t \dot{x^r}_t - x_t^r \dot{q}_t + r^f x_t^r q_t - 2(\sigma^e)^2 q_t \\
& -x_t^r q_t [r^{\mathrm{T}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2] + q_t[u_t^1 - r_t^e - (\sigma^e)^2] \\
& +x_t^r (q_t)^2/c^2 - c^3 l^r + x_t^r q_t \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y \\
= \quad & -q_t \dot{x^r}_t \\
& +x_t^r [-(q_t)^2/c^2 + c^3 + 2q_t (r^{\mathrm{T}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2) \\
& \quad + q_t((\sigma^e)^2 + (\sigma^i)^2 - r^f - \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y)] \\
& -x_t^r q_t [(r^{\mathrm{T}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2) - r^f - \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y] \\
& +q[-(\sigma^e)^2 + (u_t^1 - r_t^e - (\sigma^e)^2)] + x_t^r (q_t)^2/c^2 - c^3 l^r \\
= \quad & q_t[-\dot{x^r}_t + c^3 (x_t^r - l^r)/q_t \\
& \quad + x_t^r [r^{\mathrm{T}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2] \\
& +(x_t^r - 1)((\sigma^e)^2 + (\sigma^i)^2) + (\sigma^i)^2 + (u_t^1 - r_t^e - (\sigma^e)^2) \\
= \quad & 0.
\end{aligned}
$$

Similarly, the term of $(x)^0$ equals

$$
\begin{aligned}
& \dot{l}_t - r^f l_t - r^f (x_t^r)^2 q_t \\
& +2x_t^r[q_t \dot{x^r}_t + x_t^r \dot{q}_t] - (x_t^r)^2 \dot{q}_t + (\sigma^e)^2 q_t \\
& -2x_t^r q_t [u_t^1 - r_t^e - (\sigma^e)^2] + c^3 (l^r)^2 - (x_t^r)^2 (q_t)^2/c^2 \\
& -(x_t^r)^2 q_t \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y \\
= \; & \text{(using the term with } (x)^1) \\
& \dot{l}_t - r^f l_t - r^f (x_t^r)^2 q_t + (\sigma^e)^2 q_t - 2x_t^r q_t [u_t^1 - r_t^e - (\sigma^e)^2] + c^3 (l^r)^2 \\
& -(x_t^r)^2 (q_t)^2/c^2 - (x_t^r)^2 q_t \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y \\
& +2r^f (x_t^r)^2 q_t - 2x_t^r q_t (\sigma^e)^2 - 2(x_t^r)^2 q_t [r^{\mathrm{T}} + r^e - r^i + (\sigma^e)^2 + (\sigma^i)^2] \\
& +2x_t^r q_t [u_t^1 - r_t^e - (\sigma^e)^2] - c^3 l^r 2 x_t^r \\
& +2(x_t^r)^2 (q_t)^2/c^2 + 2(x_t^r)^2 q_t \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y \\
& -(x_t^r)^2 (q_t)^2/c^2 + c^3 (x_t^r)^2 + (x_t^r)^2 q_t 2 (r^{\mathrm{T}} + r^e - r^i + (\sigma^e)^2 + (\sigma^i)^2) \\
& +(x_t^r)^2 q_t (-r^f + (\sigma^e)^2 + (\sigma^i)^2 - \tilde{r_t^y}^T \tilde{C}_t^{-1} \tilde{r}_t^y) \\
= \; & \dot{l}_t - r^f l_t + c^3 (x_t^r - l^r)^2 - (\sigma^e)^2 q_t (x_t^r - 1)^2 - (\sigma^i)^2 q_t (x_t^r)^2 \\
= \; & 0.
\end{aligned}
$$

It then follows from Theorem 5.3.3 that the indicated control laws are the optimal ones. □

5.3.3 Discussion of the cost function and control laws

In this subsection, the cost function and control laws that have been introduced in Subsections 5.3.1 and 5.3.2 are discussed.

5.3.3.1 Discussion of the cost function

For the cost on the extra liquidity contributions, the function (5.28) is considered. Here, two cases can be considered.

Case 1 ($u^2 > 0$): If $u^2 > 0$, then the bank may contemplate encouraging extra liquidity contributions. However, the cost function should be structured such that too high extra liquidity payments are penalized, hence $u^2 > 0$ should imply that $b^2(u^2) > 0$.

Case 2 ($u^2 < 0$): If $u^2 < 0$, then liquidity contributions should be reversed. In general, it seems best not to penalize this process. Another option is to restrict the input variable, u^2, to the set $\mathbb{R}^+$.

For Theorem 5.3.4, the cost function, $b^2(u^2) = \frac{1}{2}c^2(u^2)^2$, given by (5.28), have been selected. This penalizes both positive and negative values of u^2 equally. The only reason for doing so is that an analytic solution of the value function can then be obtained.

The reference process, l^r, may take the value 0.025 that is the threshold for the LCR standard. The cost on meeting shareholder NCO costs will be encoded in an LCR cost. If the LCR, $x > l^r$, is strictly larger than a set value, l^r, then there should be a strictly negative cost. If, on the other hand, $x < l^r$, then there may be a positive cost.

The authors have selected the cost function, $b^3(x) = \frac{1}{2}c^3(x - l^r)^2$, in Theorem 5.3.4 that is given by (5.29). This is also done to obtain an analytic solution of the value function and that case is interesting in itself. Another cost function that is considered, is

$$b^3(x) = c^3[\exp(x - l^r) + (l^r - x) - 1],$$

which is strictly convex and asymmetric in x with respect to the value l^r. For this cost function, costs with $x > l^r$ are penalized lower than those with $x < l^r$. This seems realistic. Another cost function that is considered, is to keep $b^3(x) = 0$ for $x < l^r$.

5.3.3.2 Discussion of the control laws

An interpretation of the control laws that are given by (5.33) and (5.34) follows. In times of low liquidity, the extra contribution rate, u^{2^*}, is proportional to the difference between the LCR, x, and its reference process, x^r. The proportionality factor is q_t/c^2, which depends on the relative ratio of the cost function on u^2 and the deviation from the reference ratio, $(x - x^r)$. The property that the control law is symmetric in x with respect to the reference process, x^r, is a direct consequence of the cost function, $b^r(x) = \frac{1}{2}c^3(x - x^r)^2$, being symmetric with respect to $(x - x^r)$.

The optimal portfolio distribution is proportional to the relative difference between the LCR and its reference process, $(x - x_t^r)/x$. This seems natural. The proportionality factor is $\widetilde{C}_t^{-1}\widetilde{r}_t^y$, which represents the relative rates of HQLA return, multiplied with the inverse of the corresponding variances. It is surprising that the control law has this structure. Apparently, the optimal control law is not to liquidate (i.) all HQLAs with the highest liquidity contribution rate, then (ii.) the HQLAs with the next to highest liquidity contribution rate et cetera. The proportion of all HQLAs depends on the relative weighting in $\widetilde{C}_t^{-1}\widetilde{r}_t^y$ and not on the deviation $(x - x_t^r)$.

The novel structure of the optimal control law is the LCR reference process, $x^r : T \to \mathbb{R}$. The differential equation for this reference function is given by (5.31). This equation is new for the area of LCR control and deserves, therefore, discussion. The differential equation has several terms on its right-hand side which will be discussed separately. Consider the term

$$u_t^1 - r_t^e - (\sigma^e)^2.$$

This represents the difference between normal liquidity contribution rate per monetary unit of the bank's NCOs, where r_t^e is the rate of outflow per monetary unit of NCOs. Note that if $[u_t^1 - r_t^e - (\sigma^e)^2] > 0$, then the reference LCR function can be increasing in time, due to this inequality, so that, for $t > t_1$, it is found that $x_t < l^r$. The term, $c^3(x_t^r - l^r)/q_t$, models that if the reference LCR function is smaller than l^r, then the function has to increase in time. The quotient, c^3/q_t, is a weighting term which accounts for the running costs and for the effect of the solution of the Riccati differential equation. The term,

$$x_t^r[r^{\mathrm{R}}(t) + r_t^e - r_t^i + (\sigma^e)^2 + (\sigma^i)^2],$$

accounts for two effects. The difference, $r_t^e - r_t^i$, is the net effect of the rate of NCO per monetary unit of the bank's NCO's, r^e, and the rate of NCO increase before outflows per monetary unit of NCOs, r_t^i. The term, $r^0(t) + (\sigma^e)^2 + (\sigma^i)^2$, is the effect of NCO increase due to the riskless HQLAs and the variance of riskier HQLAs. The last term,

$$(x_t^r - 1)((\sigma^e)^2 + (\sigma^i)^2) - (\sigma^i)^2,$$

accounts for the effect that capital outflow has on both HQLAs and NCOs. More information is obtained by streamlining the ordinary differential equation for x^r. In order to accomplish this, it is necessary to assume the following:

Assumption 5.3.5 (Liquidity parameters): *Assume that the parameters of the problem are all time-invariant and also that q has become constant with value q^0.*

Then the differential equation for x^r can be rewritten as

$$\begin{aligned}
-\dot{x^r}_t &= -k(x_t^r - m),\ x^r(t_1) = l^r, \\
k &= (r^{\mathrm{R}} + r^e - r^i + 2((\sigma^e)^2 + (\sigma^i)^2)) + c^3/q^0, \\
m &= \frac{l^r c^3/q^0 - (u^1 - r^e - (\sigma^e)^2) + (\sigma^e)^2}{(r^{\mathrm{R}} + r^e - r^i + 2((\sigma^e)^2 + (\sigma^i)^2)) + c^3/q^0}.
\end{aligned}$$

Because the finite horizon is an artificial phenomenon to make the optimal stochastic control problem tractable, it is of interest to consider the long-term behaviour of the LCR reference trajectory, x^r. If the values of the parameters are such that $k > 0$, then the differential equation with the terminal condition is stable. If this condition holds, then $\lim_{t\downarrow 0} q_t = q^0$ and $\lim_{t\downarrow 0} x_t^r = m$ where the down arrow prescribes to start at t_1 and to let t decrease to 0. Depending on the value of m, the control law, for a time very far away from the terminal time, becomes

$$
\begin{aligned}
u_t^{2*} &= -(x_t - m)q^0/c^2 = \begin{cases} > 0, & \text{if } x_t < m, \\ < 0, & \text{if } x_t > m, \end{cases} \\
\pi_t^* &= -\frac{(x_t - m)}{x_t}\widetilde{C}\widetilde{r}^j = \begin{cases} > 0, & \text{if } x_t < m, \\ < 0, & \text{if } x_t > m,\ \text{if } \pi^* < 0 \text{ then set } \pi^* = 0. \end{cases}
\end{aligned}
$$

The interpretation of the two cases follows below:

Case 1 ($x_t > m$): Here, the LCR, x, is too high. This is penalized by the cost function, hence the control law prescribes not to invest in riskier HQLAs and to hand back any surplus to the shareholders. This can be seen in Figures 5.1 and 5.2 that are presented in the ensuing Section 5.4. If the LCR, x, is above a certain reference value (for instance, $l^r = 1.05$), the proportion that has been invested in risky HQLAs is zero and the extra liquidity contribution is negative. The payback advice is due to the quadratic cost function which was selected to make the solution analytically tractable. An increase in the liquidity contribution will increase NCOs that, in turn, will lower the LCR.

Case 2 ($x_t < m$): In this case, the LCR, x, is too low. The cost function penalizes and the control law prescribes to invest more in riskier HQLAs and to request additional liquidity contributions, because both will lead to a higher LCR in the long run. Again this can be seen in Figures 5.1 and 5.2 that are presented in the ensuing Section 5.4. If the LCR, x, is below a certain reference value (for instance, $l^r = 1.05$), the proportion that has been invested in riskier HQLAs is higher and the extra liquidity contribution is positive. Note that it follows from the formula of m that

$$
m = \frac{l^r c^3/q^0 - (u^1 - r^e - (\sigma^e)^2) + (\sigma^e)^2}{(r^{\mathrm{R}} + r^e - r^i + 2((\sigma^e)^2 + (\sigma^i)^2)) + c^3/q^0}.
$$

An expression for the difference, $m - l^r$, is not obvious and depends on many parameters of the problem, as it should.

5.4 Numerical results involving liquidity

In this section, further insight into LCR dynamics is obtained by considering several numerical examples and trajectories of the LCR reference process, x^r. Here, LCR values are scaled down by a factor of 40 for ease of presentation. In particular, a simulation is provided of the LCR that is given in equation (5.12). Consider the LCR, discussed in previous sections, with constant rate and variance functions, and with the following parameter values:

Liquidity parameter values									
r^0	r^e	r^i	σ^e	σ^i	u^1	u^2	c^2	c^3	l^r
0.030	0.025	0.025	0.002	0.002	0.025	0.000	1.000	5.000	1.050

Table 5.1: Liquidity parameter values

5.4.1 Values of q and m for different values of r^0 and l^r

The following table provides values for q^0 and m for a small range of values of r^0 and l^r.

Values of q and m for different values of r^0 and l^r					
		l^r			
r^0	q	1.01	1.03	1.05	1.07
0.01	2.229	1.0055	1.0254	1.0453	1.0652
0.02	2.239	1.0010	1.0209	1.0407	1.0605
0.03	2.249	0.9965	1.0163	1.0360	1.0551
0.04	2.259	0.9921	1.0117	1.0314	1.0510

Table 5.2: Values of q and m for different values of r^0 and l^r

The following conclusions can be drawn from Table 5.2: For a fixed value of l^r, the value of the cost function for the LCR, the value of m is strictly less than that of l^r. For a fixed value of l^r, the value of m decreases if the rate of the riskless HQLA, r^0, increases. The LCR that needs to be held in focus for a long horizon can be less than the value of l^r that appears in the cost function.

In Figures 5.1 and 5.2 below, the authors present LCR dynamics in the form of a trajectory that is derived from (5.12).

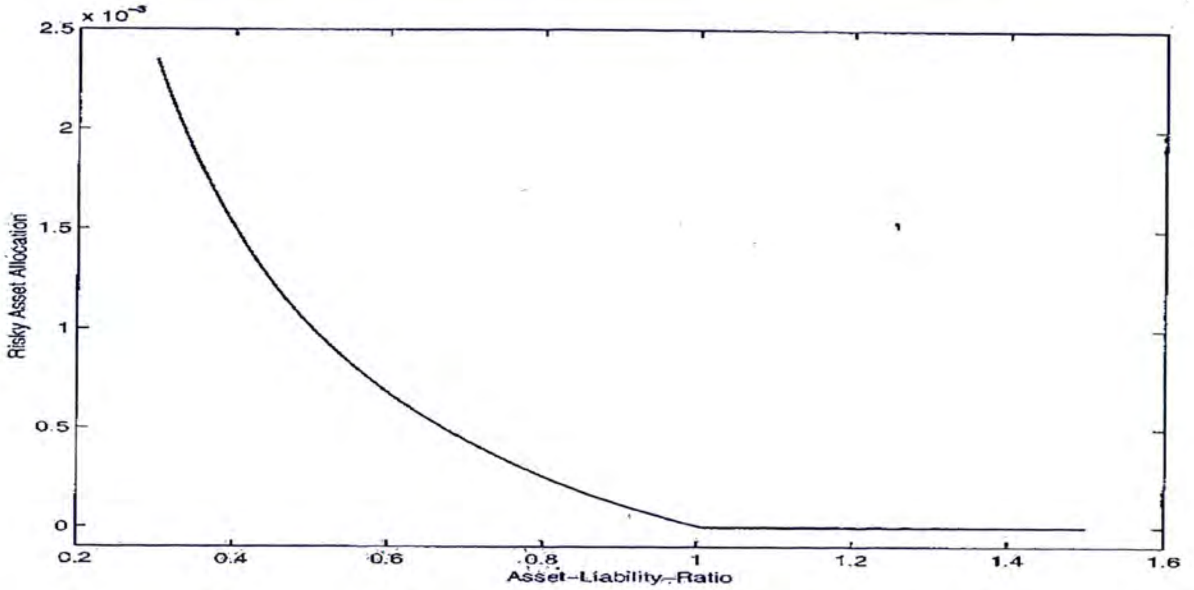

Figure 5.1: Risky HQLA allocation vs. liquidity ratio

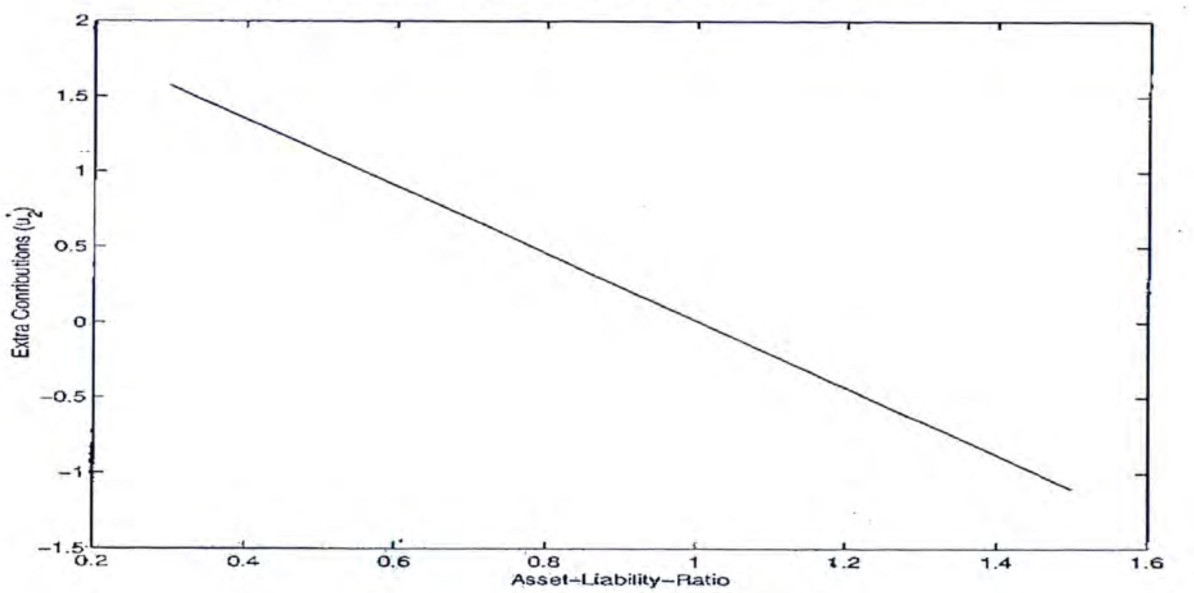

Figure 5.2: Extra liquidity contributions vs. liquidity coverage ratio

5.4.2 Trajectories of the LCR reference function, x^r, and of m for $l^r = 1.05$

In Figure 5.3, a numerical solution shows the time trajectory of the LCR reference function, x^r, and m for the values of $l^r = 1.05$, $t_1 = 10$ and $r^0 = \{0.01,\ 0.02,\ 0.03,\ 0.04\}$.

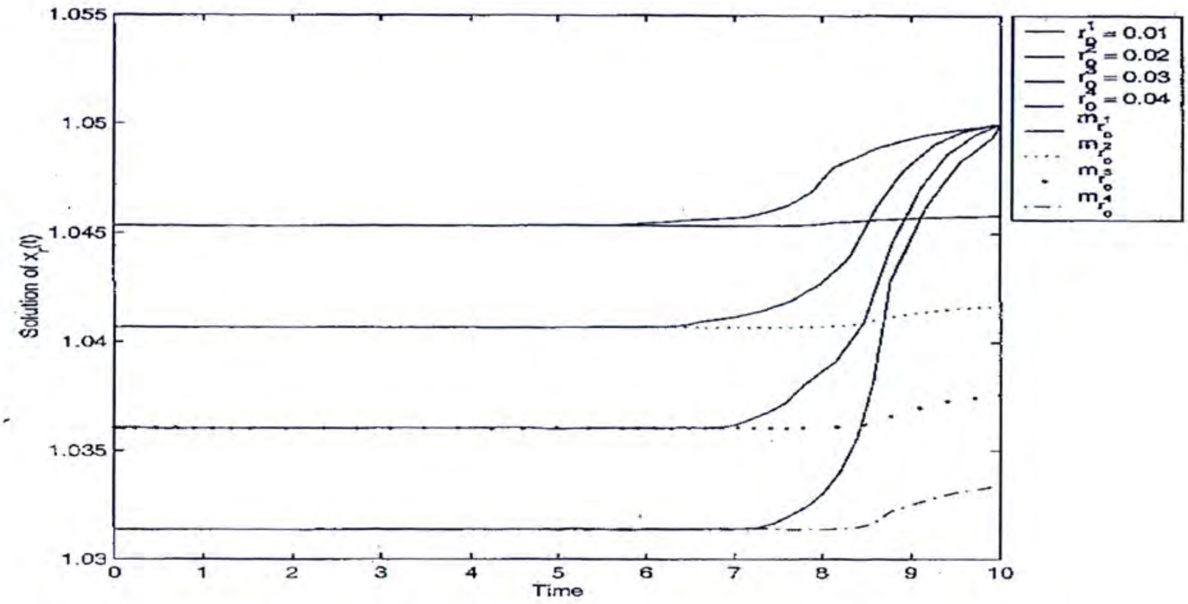

Figure 5.3: Trajectories of the LCR reference function and of m for $l^r = 1.05$

Figure 5.3 shows that the LCR reference function over time is less than the l^r value.

5.4.3 Simulated LCR, x_t, and extra liquidity contribution rate; $l^r = 1.05$, $r^0 = 0.01$

Figure 5.4 presents a simulated LCR, x_t, and superimposes the dynamics of the extra liquidity contribution rate, using the LCR reference function, x^r, with $l^r = 1.05$ and $r^0 = 0.01$.

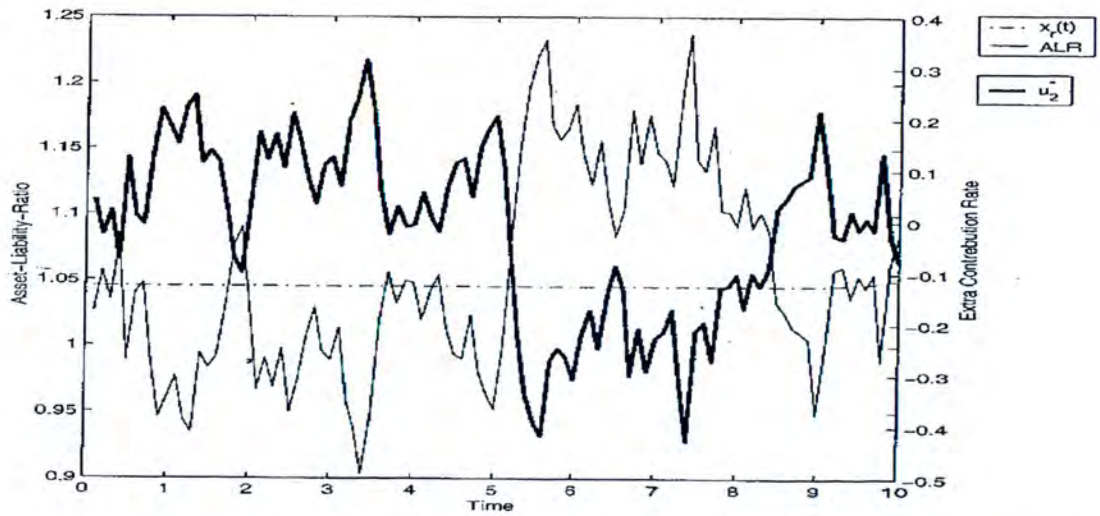

Figure 5.4: Simulated LCR, x_t, and extra liquidity contribution rate; $l^r = 1.05$, $r^0 = 0.01$

Figure 5.4 shows that when $l^r \geq x^r$, the extra liquidity contribution is negative. Similarly, when $l^r \leq x^r$, the extra liquidity contribution is positive.

5.4.4 Simulated LCR, x_t, and risky HQLA allocation; $l^r = 1.05,\ r^0 = 0.01$

Figure 5.5 presents a simulated LCR, x_t, and superimposes the dynamics of the risky HQLA allocation, using the LCR reference function, x^r, with $l^r = 1.05$ and $r^0 = 0.01$.

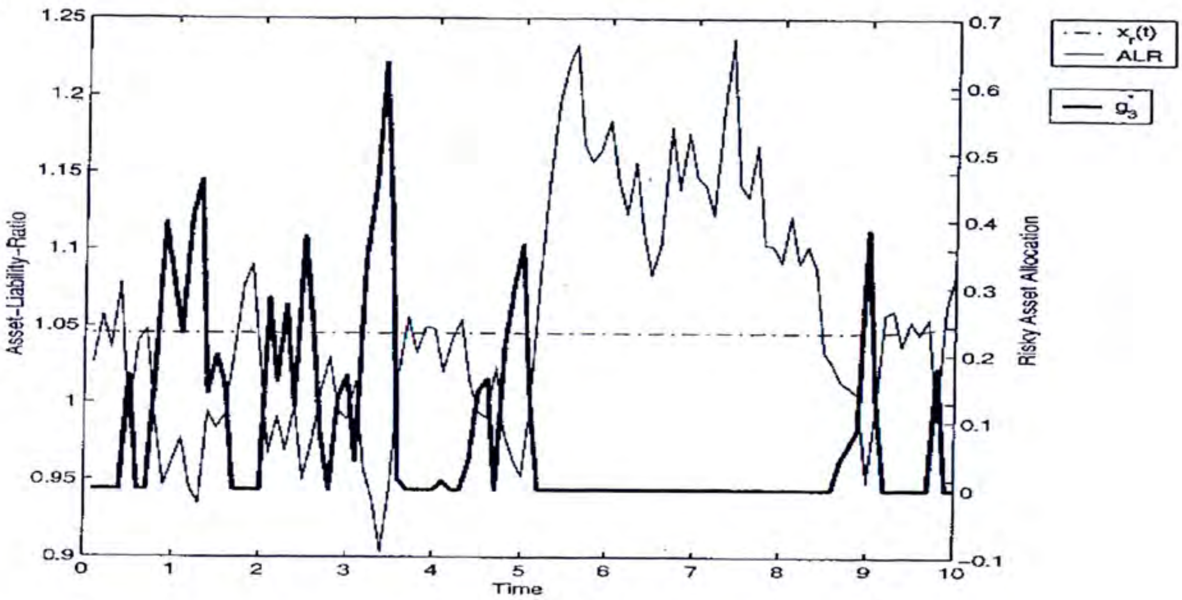

Figure 5.5: Simulated LCR, x_t, and riskier HQLA allocation; $l^r = 1.05,\ r^0 = 0.01$

Figure 5.5 shows that when $l^r \geq x^r$, the riskier HQLA allocation is zero. Similarly, when $l^r \leq x^r$, the riskier HQLA allocation is higher.

5.4.5 Simulated LCR, x_t, using control laws; $l^r = 1.05,\ r^0 = 0.01$

Figure 5.6 presents a simulated LCR, x_t, using the control law with $l^r = 1.05$ and $r^0 = 0.01$.

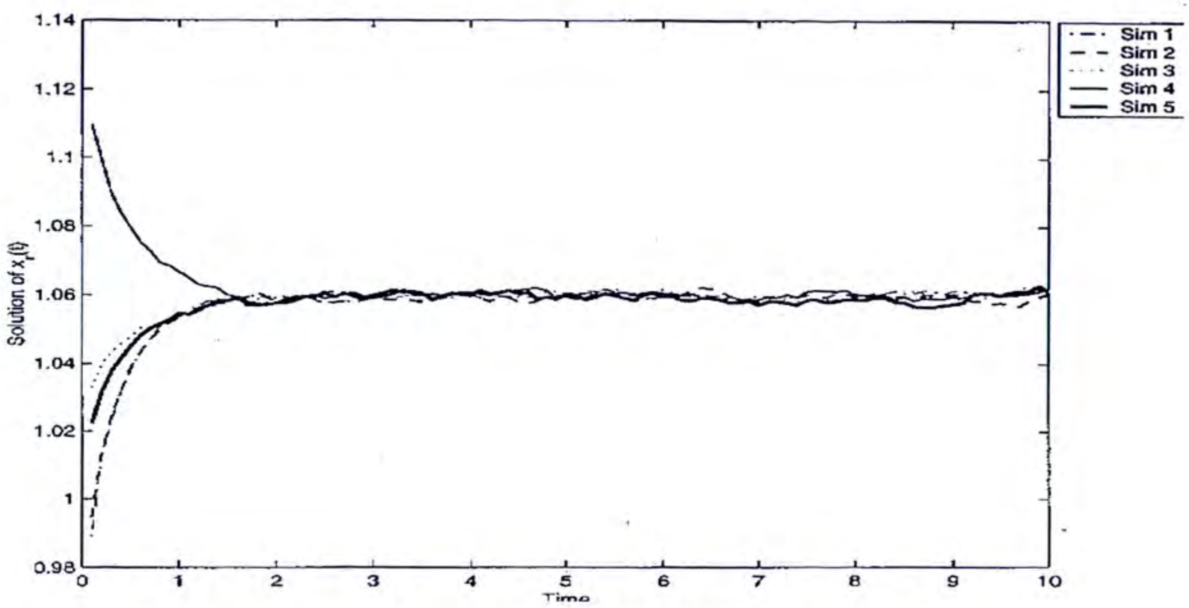

Figure 5.6: Simulated LCR, x_t, using control laws; $l^r = 1.05$, $r^0 = 0.01$

Figure 5.6 shows that if the bank uses a control law as above with l^r instead of m, then the fund will generate a higher LCR, x_t, than required. It can be seen that the simulated LCR, x_t, using the control law, is just above $l^r = 1.05$.

5.5 Conclusions and future directions

One of the main contributions of the article is the way in which the LCR dynamics model is constructed by using stochastic techniques. This model depends on HQLAs and NCOs, as well as the liquidity provisioning rate. The authors believe that this is an addition to pre-existing literature, because it captures some of the uncertainty that is associated with LCR variables. In this regard, they provide a theoretical-quantitative modelling framework for the establishing of bank LCR reference processes and the making of decisions about liquidity provisioning rates and asset allocation. Furthermore, they have obtained an analytic solution to an optimal bank LCR problem with a quadratic objective function. In principle, this solution can assist in managing LCRs. Here, liquidity provisioning and HQLA allocation are expressed in terms of a reference process. To the knowledge of the authors, such processes have not been considered for LCRs before. This article makes a clear connection between liquidity and financial crises in a numerical-quantitative framework.

Specific open questions that arise out of the discussion in Section 5.3 are given below. The LCR has some limitations regarding the characterization of banks' liquidity positions. Therefore, complementary Basel III ratios such as the net stable funding ratio (NSFR) should be considered for a more complete analysis. This should take the structure of the short-term HQLAs and NCOs of residual maturities into account. There are several other directions in which the results that have been obtained in this article can possibly be extended. These include addressing further risk issues, as well as improvements in the LCR modelling procedure. In this regard, instead of using a continuous-time stochastic model in order to solve an optimal bank LCR problem, the authors would like to construct a more sophisticated model with jump-diffusion processes (compare with [8]). A study of information asymmetry during the financial crisis should also be interesting.

The authors have already made several contributions in support of the endeavours that were outlined in the previous paragraph. For instance, their article [8] deals with issues that are related to liquidity risk and the financial crisis. Also, the role of information asymmetry in a subprime context is related to the main hypothesis of the book [9].

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5.6 Appendix: Numerical results for LCRs

In this section, numerical-quantitative results are provided for LCRs and their connections with HQLAs and NCOs to supplement the theoretical-quantitative treatment in Sections 5.2 and 5.3. More precisely, the authors discuss an LCR simulation (see Subsection 5.6.1 for a precise discussion), as well as bank (see Subsection 5.6.2 for more details) and bond (see Subsection 5.6.3) level liquidity data. In both cases, they conclude that low liquidity coincided with periods of financial crises.

5.6.1 Appendix A: LCR simulation

In this subsection, a simulation is provided of the LCR dynamics that are given in equation (5.12).

5.6.1.1 Appendix A1: LCR simulation parameters

In this sub-subsection, parameters and values for a numerical simulation are provided. The parameters and their corresponding values for the simulation are shown below.

Parameter	Value	Parameter	Value	Parameter	Value
C	\$ 1 000	r^C	0.06	r^e	0.07
r^i	0.02	σ^e	1.7	σ^i	1.9
$\overline{r}^y$	0.05	$\tilde{\overline{\pi}}$	0.4	u^1	0.03
u^2	0.01	$\tilde{C}$	750	$\overline{W}$	0.01

Table 5.3: Choices of liquidity coverage ratio parameters

5.6.1.2 Appendix A2: LCR dynamics

In Figure 5.6.1.2 below, LCR dynamics are presented in the form of a trajectory that has been derived from (5.12).

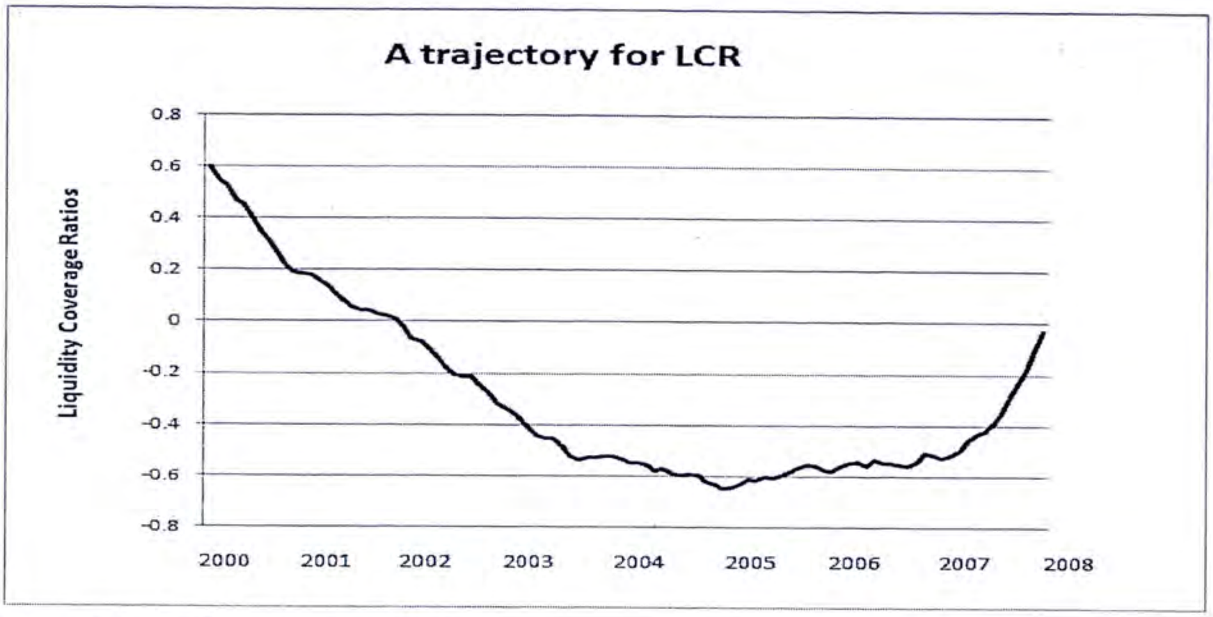

Figure 5.6.1.2 LCR trajectory for HQLAs

5.6.1.3 Appendix A3: Features of the LCR trajectory

Figure 5.6.1.2 shows the simulated LCR trajectory, corresponding to low liquidity assets, where chosen bank parameter values are collected in Table 5.3. The number of jumps of the trajectory was limited to 1001, with the initial values for l fixed at 20. The simulation seems to support the subsequent view in Subsection 5.6.2 (see, for instance, Table 5.4) that financial crises coincide with periods of low liquidity.

As is known, banks manage their liquidity by offsetting NCOs via HQLAs. It is actually the diversification of the banks' HQLAs and NCOs that expose them to liquidity shocks. Here, ratio analysis (in the form of the LCR) is used to manage liquidity risk, relating to various components in the bank's balance sheets. In Figure 5.6.1.2, it is observed that between $t = 2000$ and $t = 2005$, there was a significant decrease in the trajectory, which shows that either HQLAs declined or NCOs increased.

There was also an increase between $t = 2005$ and $t = 2007$, which suggests that either HQLAs increased or NCOs decreased. There was an even sharper increase subsequent to $t = 2007$, which comes as somewhat of a surprise. In order to mitigate the aforementioned increase in liquidity risk, banks can use several facilities, such as repurchase agreements, to secure more funding. However, a significant increase was recorded between $t = 2005$ and $t = 2008$, with the trend showing that banks had more HQLAs on their books. If $l > 0$, it demonstrates that the bank has kept a high HQLA volume, which might stem from prudent liquidity risk management. In order for banks to improve liquidity, they may use debt securities that allow savings from non-financial private sectors, a good network of branches and other competitive strategies.

5.6.2 Appendix B: Bank level liquidity data

In this subsection, quarterly call report data is considered on HQLAs, semi-liquid assets, illiquid assets and illiquid guarantees from selected US commercial banks from Q3:2005 to Q4:2008. The authors consider data involving 9 067 distinct banks, yielding 453 579 bank-quarter observations over the aforementioned period.

Bank level liquidity (in $ billions)

QUARTERS	HQLAs	Semi-liquid assets	Illiquid assets	Illiquid guarantees
Q3:2005	80	220	100	170
Q4:2005	30	200	200	220
Q1:2006	70	220	280	300
Q2:2006	150	210	380	490
Q3:2006	110	200	410	530
Q4:2006	180	230	490	620
Q1:2007	190	220	500	710
Q2:2007	200	230	600	900
Q3:2007	310	215	710	980
Q4:2007	270	230	800	1 010
Q1:2008	250	210	820	1 015
Q2:2008	240	220	830	1 005
Q3:2008	260	230	810	995
Q4:2008	310	250	780	990

Table 5.4: Bank level liquidity (in $ billions)

In Table 5.4, the authors note that HQLAs decreased during the financial crisis after a steady increase before this period. As liquidity creation procedures (like bailouts) became effective in Q3:2008 and Q4:2008, the HQLA volume increased. By comparison, the semi-liquid assets fluctuated throughout the period. On the other hand, the illiquid assets increased until Q3:2008 and then decreased as bailouts came into effect. During the financial crisis, illiquid guarantee dynamics had a negative correlation with those of HQLAs.

5.6.3 Appendix C: Bond level liquidity data

In this subsection, the authors explore whether the effect of illiquidity is stronger during times of financial crises. The results for the crisis period are presented and they are compared with those for the period with normal bond market conditions. Firstly, evidence is provided from the descriptive statistics of the key variables for the two subperiods and then the main conclusions are drawn, based on this. The analysis of the averages of the variables in these subperiods allows the authors to gain some important insights into the causes of the variation (see Table 5.5). In this regard, Table 5.5 presents the mean and standard deviation for bond characteristics (amount issued, coupon, maturity and age), trading activity variables (traded volume, number of trades and time interval between trades) and liquidity measures (Amihud, price dispersion, Roll and zero-return measure). The data set consists of more than 23 000 US corporate bonds that were traded over the period Q3:2005 to Q4:2008.

Bond level liquidity				
	Mean		Standard deviation	
	Non-crisis	Crisis	Non-crisis	Crisis
Amount issued (bln)	0.45	0.54	0.03	0.06
Coupon (%)	6.24	6.23	0.05	0.05
Maturity (yr)	7.75	8.31	0.20	0.18
Age (yr)	4.36	4.76	0.12	0.14
Volume (mln)	1.44	1.53	0.22	0.32
Trades	4.06	5.33	0.24	1.31
Trade interval (dy)	3.38	3.37	0.49	0.48
Amihud (bp per mln)	53.21	89.20	7.42	35.75
Price dispersion (bp)	39.75	70.02	1.83	21.84
Roll (bp)	142.82	209.77	10.57	52.34
Zero-return (%)	0.02	0.03	0.01	0.01

Table 5.5: Bond level liquidity

Table 5.5 clearly supports the view that all liquidity measures indicated lower liquidity levels during the financial crisis. As an example, the authors considered the average price dispersion measure and found that its value is higher during the crisis in comparison with the non-crisis period. With regard to the trading activity variables, they found that the average daily volume and the trade interval at the bond level stay approximately constant. However, the number of trades increased during the financial crisis. These results are consistent with the level of market-wide trading activity, where they found that, during the financial crisis, trading took place in fewer bonds with an increased number of smaller-sized trades.

Chapter 6

LIQUIDITY AND BASEL III

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A note on Basel III and liquidity

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Abstract: In this paper, the authors obtain numerical results, involving new Basel III liquidity regulation. More specifically, they compute the net stable funding ratio in accordance with the prescripts of the proposed banking rules. In this regard, the effects of shareholder cash flow rights are investigated on the aforementioned funding ratio and on a non-Basel III liquidity coverage ratio for certain developing countries during the period Q1:2005 to Q4:2009. The study finds that the funding ratio appears to have satisfied Basel III minimum liquidity standards during this period. Also, it is concluded that more concentrated cash flow rights result in improved liquidity.

Keywords: Basel III, Net Stable Funding Ratio (NSFR), Liquidity Coverage Ratio (LCR).

JEL: G13; G32.

6.1 Introduction

During the financial crisis that began in mid 2007, banks experienced difficulties due to lapses in principles of liquidity risk management. In response to this, as the foundation of its liquidity framework, the Basel Committee on Banking Supervision (BCBS) published [3] – commonly known as 'Sound Principles' – in 2008. This document provided detailed guidance on risk management and the supervision of funding liquidity risk. Subsequently, [1] proposed via Basel III regulation that banks should always have liquidity cover for stress scenarios. This document further strengthens the BCBS's liquidity framework by introducing two minimum standards for funding liquidity. These standards have separate, but complementary objectives. Firstly, the liquidity coverage ratio (LCR) standard[5] strives to ensure that banks have sufficient liquidity to survive a short-term stress scenario, lasting for one month. Secondly, the net stable funding ratio (NSFR) ensures that banks have sufficient stable funds to survive a long-term stress scenario, lasting for one year. The NSFR is a structural ratio that is designed to address liquidity mismatches and provide incentives for banks to use stable sources to fund their activities.

The NSFR is mandated by proposed Basel III regulation as a measure of bank capital stability (see, for instance, [1] and [3]). Although Basel III implementation is to commence in 2013, the authors calculate the degree of capital stability that would have been experienced if such regulation was in effect (see, for instance, [6]). The NSFR is defined as

$$\text{NSFR} = \frac{\text{Available stable funding (ASF)}}{\text{Required stable funding (RSF)}} \geq 1.$$

The authors' contribution has connections with [4], [5] and [6]. In the former, actuarial methods are used to discuss liquidity risk management, focussing on cash inflows and securities allocation.

[5]At present, Basel III LCR data is not freely available, which makes this liquidity standard difficult to compute. Instead, for the purposes of this study, the authors consider the LCR to be the ratio of current assets to current liabilities.

The main objective in [4] is to minimize liquidity risk in the form of funding and credit crunch risk in an incomplete market (see also [5] and [6]). In order to accomplish this, a stochastic model that incorporates reference processes is constructed. However, the current article is an improvement on [4] in that it complies with Basel III liquidity regulation that is related to NSFRs (see Sections 6.2 and 6.3 below). The main question that is addressed in this paper can be stated as follows:

Question 6.1.1 (Influence of SCFR on Basel III liquidity) *How does shareholder cash flow rights (SCFR) influence bank liquidity in the proposed Basel III paradigm? (See Section 6.3.)*

In particular, in a Basel III paradigm, the subsequent analysis assists in providing insights into the likely features of capital stability as it is affected by shareholder cash flow rights (SCFR) that are defined as the cash flow rights of the major bank shareholder.

6.2 Data and methodology

In this section, the authors consider the data and methodology that are used to probe the influence that SCFR has on bank liquidity and capital stability in a proposed Basel III paradigm.

6.2.1 Data

The data for SCFR, NSFRs and LCRs mainly come from Bankscope. In particular, the NSFR calculation is based on the BCBS quantitative study [2]. The total bank sample consists of 58 active commercial banks from Brazil, Russia, India, China and South Africa[6].

6.2.1.1 Descriptive statistics

The descriptive statistics of the SCFR, (Basel III) NSFR and (non-Basel III) LCR are given below.

Descriptive statistics for liquidity					
Parameter	Mean	Median	S.D.	25th Prcntl	75th Prcntl
SCFR	0.317	0.238	0.244	0.102	0.275
NSFR	3.530	1.967	2.958	1.223	6.123
LCR	0.194	0.178	0.120	0.101	0.550

Table 6.1: Descriptive statistics for liquidity

From Table 6.1, it is clear that the NSFR for most banks seem to have satisfied the Basel III minimum liquidity standard. On the other hand, in the absence of empirical evidence, it is hard to conclude that the non-Basel LCR had complied with these standards.

[6]These countries are otherwise known as the BRICS countries.

6.2.2 Methodology

The liquidity model framework methodology, outlined in [5] is used. Such a framework should include data enumeration (data submodel), evaluation of Basel III liquidity regulation (core submodel) and a supervisor user-interface for monitoring the model (user-interface submodel). These proposed model framework components are presented in the following diagram:

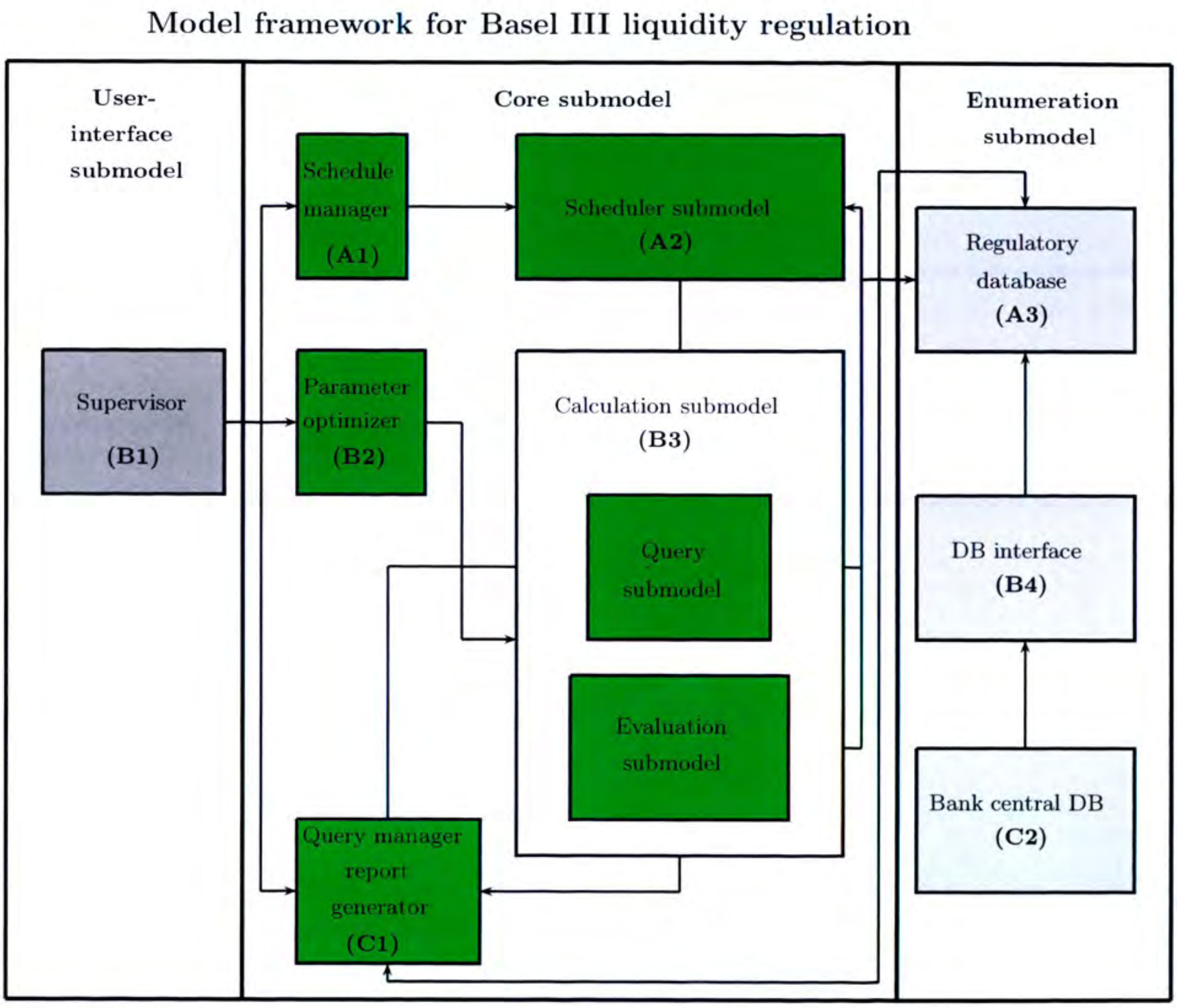

Figure 6.1: Model framework for Basel III liquidity regulation; source: [6]

The enumeration submodel contains the database components that are needed for liquidity measurement and monitoring. The purpose of this submodel is to provide a framework, independent of outsider frameworks, that is employed by banks for developing their management information system. The submodel also integrates banks' current model framework with existing outsider frameworks. The components of the enumeration submodel are the regulatory database, database interface and the bank's central database. The core submodel integrates all the functional components of the model and acts as an application layer. This submodel is responsible for operations like scheduling, evaluating, reporting et cetera. The user-interface submodel provides a platform for supervisors to fulfil authorized roles. The application that is developed will be solely used by supervisors. They are responsible for updating the parameters according to the socio-economic environment, requesting standard evaluation when required. Supervisors also monitor evaluations and reports that are generated in order to suggest stress scenarios strategies, as well as update the bank database with off-balance sheet entries where necessary. The user-interface submodel includes a portal to perform the tasks above and a security access to avoid unnecessary access.

6.3 Results and discussion

In this section, liquidity correlation (see Subsection 6.3.1) and regression analysis (see Subsections 6.3.2 and 6.3.3) are discussed in order to elucidate the relationships between SCFR, NSFR and LCR.

6.3.1 Liquidity correlation analysis

In this subsection, correlation is considered for the aforementioned liquidity parameters for the period Q1:2005 to Q4:2009. In Table 6.2 below, *** indicates a statistical significance of 1%.

Liquidity correlation analysis			
Variable	SCFR	NSFR	LCR
SCFR	1.000		
NSFR	-0.098	1.000	
LCR	0.313***	-0.369***	1.000

Table 6.2: Liquidity correlation analysis

Table 6.2 shows the correlation matrix for SCFR, NSFR and LCR. SCFR is not significantly related to capital stability, as indicated by the NSFR. It is also positively and significantly correlated with the non-Basel III LCR. This implies that higher SCFR concentration promotes more liquidity. Although these results may be useful, they do not control for a number of factors that influence risk-taking. The authors control for a number of other variables in Section 6.3.2 below.

6.3.2 Liquidity regression analysis

In this subsection, the liquidity regression is considered for period Q1:2005 to Q4:2009. In Table 6.2, ** and *** indicate statistical significance at the 5% and 1%, respectively.

Liquidity regression analysis		
Dependent variable	**NSFR**	**LCR**
Constant	35.915***	0.3413
	(5.15)	(0.95)
SCFR	0.648***	0.1096**
	(0.44)	(2.14)
Year dummies included	Yes	Yes
Adjusted R^2	37.47%	30.64%

Table 6.3: Liquidity regression analysis

Table 6.3 presents liquidity regression results. The standard errors are adjusted to account for bank-level clustering. SCFR displays a positive and significant coefficient. Next, the authors proxy for capital stability by computing the NSFR according to Basel III and include it as a dependent variable, while the coefficient of SCFR concentration is not significant. If implemented in the sample period, the requirement for capital stability that is based on Basel III would not have been affected by SCFR concentration. Finally, Table 6.3 considers the LCR as a dependent variable. The SCFR coefficient is positive and significant, suggesting that more concentrated SCFR enhances bank liquidity. It is also worth noting that the regressions explain 37.47% and 30.64% of the variation for the NSFR and LCR, respectively, as indicated by the R^2s. This level is within the normal range for this analysis type.

6.3.3 Financial crisis regression analysis

The banking crisis initiated in late 2008 and continued into 2009 and later. Thus, 2009 was the first full year when banks felt the impact of the financial crisis and will be classified as the crisis period. A dichotomous variable is constructed which is equal to one for 2009 and zero otherwise. This variable is labelled as 'financial crisis' (FC) and has a relationship with SCFR concentration. The coefficient of this variable would reveal the impact of SCFR concentration on bank capitalization before versus after the financial crisis. In Table 6.4, ** and *** indicate statistical significance at the 5% and 1%, respectively.

Financial crisis regression analysis		
Dependent variable	**NSFR**	**LCR**
Constant	36.161***	0.3026
	(5.28)	(0.83)
SCFR x FC	0.901	-0.025
	(0.84)	(-0.74)
SCFR	0.489	0.107**
	(0.33)	(2.07)
FC	-0.101	0.013
	(-0.24)	(0.78)
Adjusted R^2	37.49%	29.37%

Table 6.4: Financial crisis regression analysis

The regression results are displayed in Table 6.4. Clearly, the coefficient of FC is positive and significant. For NSFR and LCR, the coefficient for the relationship with SCFR is not significant. As a consequence, it appears that the financial crisis does not change the way in which SCFR influences capital stability and bank liquidity, although it seems to contribute to bank capitalization.

6.4 Conclusions and future directions

This paper explores the relationships between SCFR and capital stability, as well as liquidity via the NSFR and LCR, respectively. The results suggest that, as SCFR concentration rises, bank liquidity increases in a statistically and economically significant manner. Furthermore, the authors hypothetically explore the impact of Basel III via the NSFR for sample banks. The evidence suggests that capital stability will be related to SCFR concentration. At lower levels of SCFR concentration, concentrated SCFR diminishes capital stability. On the other hand, at higher levels, concentrated SCFR enhances capital stability. The results provide insights into how Basel III liquidity regulation might be applied in future.

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Chapter 7

CONCLUSIONS AND FUTURE RESEARCH

7.1 Discussions and concluding remarks

The global financial crisis was caused by, amongst other things, the downturn in the US housing market, risky lending and borrowing practices, inaccurate credit ratings, as well as excessive individual and corporate debt levels. As a consequence, banks were under severe pressure to maintain adequate liquidity. The financial crises have led to the renewed interest in the modelling of financial economic systems and liquidity risk.

The thesis describes some of the most important aspects of the financial crisis such as liquidity and mortgage funding, subprime mortgage design and valuation, liquidity coverage ratios and Basel III liquidity regulation. This research has the potential to contribute to the economic development of the South African economy, as well as economies worldwide. Throughout their contribution, the authors considered optimization models in order to chart such fundamental subprime issues, namely optimal level of mortgage extension, optimal profit and valuation models, optimal liquidity coverage ratios, relationships between cash-flow rights and liquidity, as well as optimal liquidity provisioning and high-quality liquid asset (HQLA) allocation.

The subprime mortgage crisis (SMC) was characterized by illiquidity, as well as risk opaqueness. In this period, originators did not have adequate liquidity risk management strategies in place. In Chapter 3, liquidity risk in the form of funding and credit crunch risk is specifically investigated. The eruption of the SMC significantly tightened the loan supply and increased credit standards to preserve regulatory capital reserves and liquidity problems which affected the availability of mortgage loans. In order to find the ideal level of mortgage origination, the funding- and credit crunch risk of an originator was minimized. Relationships were established in order to construct a stochastic continuous-time model for the purpose of attaining an optimal level of mortgage origination via optimal portfolio choices regarding deposit inflow rates and marketable securities allocation that ensure originator stability and security. The conclusion is that there is a positive correlation between both deposits and marketable securities and mortgage origination. Furthermore, the authors find that liquidity risk mitigation is possible when unfunded mortgages are directly proportional to additional deposits (see Theorem 3.3.5, Chapter 3).

Mortgage delinquencies and foreclosures in the US was a common occurrence during the SMC. Chapter 4 discusses an important aspect of this subprime crisis, namely subprime mortgage design. In this regard, the authors focussed on originator valuation with mortgages at face value, subprime mortgages that are able to fully amortize, voluntarily prepay or default, house equity models via loan-to-value ratios (LTVRs) in a subprime framework and a discrete-time model for subprime risk and profit, incorporating costs of funds and subprime mortgage insurance (SMI), as well as mortgage losses. In general, the contention is that the SMC was partly caused by the intricacy of design of subprime mortgages that led to information problems, valuation opaqueness and ineffective risk mitigation. During this process, declining house prices curtailed mortgage refinancing. This sensitivity to house prices has widespread implications. The authors also concur with the literature that suggests that subprime mortgages are the first link of an interwoven chain that includes structured mortgage products. If mortgages are dysfunctional, the quality of the notes in the rest of the chain will be compromised. From their findings, the authors observe that high LTVRs curtailed mortgage refinancing due to house price depreciation, while low LTVRs increase the house equity of mortgagors.

The mitigation of credit risk via securitization reduced capital requirements during the subprime financial crisis. This caused low liquidity levels in financial institutions. The lack of liquidity led to credit crunches and had deleterious effects on financial markets globally. In Chapter 5, a clear connection is made between liquidity and financial crises in a numerical-quantitative framework. One of the main contributions of this chapter is the way the liquidity coverage ratio (LCR) dynamic model is constructed by using stochastic techniques. This model depends on high-quality liquid assets (HQLAs), net cash outflows (NCOs), as well as the liquidity provisioning rate, and captures some of the uncertainty associated with LCR variables. In this regard, a theoretical-quantitative modelling framework is provided for establishing bank LCR reference processes and the making of decisions about liquidity provisioning rates and asset allocation. Here, liquidity provisioning and HQLA allocation are expressed in terms of a reference process. In principle, these reference processes can assist in finding a solution in managing LCRs optimally by means of a quadratic objective function. To the authors' knowledge, such processes have not been considered for LCRs before.

Chapter 6 explores relationships between shareholder cash-flow rights (SCFR), capital stability and liquidity via the net stable funding ratio (NSFR) and the liquidity coverage ratio (LCR), respectively. The results of liquidity correlation, liquidity regression and financial crisis regression analysis suggest that, as SCFR concentration rises, bank liquidity increases in a statistically and economically significant manner. Furthermore, the impact of Basel III via the NSFR for sample banks is hypothetically explored. The evidence suggests that capital stability will be related to SCFR concentration. At lower levels of SCFR concentration, concentrated SCFR diminishes capital stability. On the other hand, at higher levels, concentrated SCFR enhances capital stability. It also appears that the financial crisis does not change the way in which SCFR influences capital stability and bank liquidity, although it seems to contribute to bank capitalization.

7.2 Recommendations and future research

Observing the extent of the subprime mortgage crisis in the US and the global financial crisis that followed, another important issue is whether this turmoil and its magnitude were anticipated by role players in the financial system. In this regard, data analysis suggests that some market participants were likely aware of an impending market correction. Furthermore, the following viewpoint has to be investigated: The effect of the SMC in the US is unique, since it is large and devastating, and has led to global financial turmoil. However, neither the origin of this crisis nor the way it has played out was unique at all. In fact, it seems to have followed the classic lending boom-and-bust scenario that has been observed historically in many countries. Before the SMC, there was a conventional belief that a market as relatively small as the US subprime mortgage market (about 16% of all US mortgage debt in 2008) could not cause significant problems in wider markets, even if it were to crash completely. However, now a severe ongoing crisis is experienced that has affected the real economies of many countries in the world, causing recessions, banking and financial failure and a credit crunch, rippling out from failures in the subprime market. In future, the authors would like to investigate the mechanism that was involved in causing this knock-on effect. It is anticipated that the solution to this problem lies in the complexity of the market for securities, backed by subprime mortgages.

The main thrust of future mathematically rigorous research should involve more sophisticated models of banking items, driven by Lévy processes (see, for instance, Protter in [4, Chapter I, Section 4]). Such processes have an advantage over the more traditional modelling tools such as Brownian motion in that they describe the non-continuous evolution of the value of economic and financial items more accurately. For instance, because the behaviour of bank mortgages, profit, deposit inflow, cash in- and outflow et cetera are characterized by jumps, the representation of the dynamics of these items by means of Lévy processes is more realistic. As a result of this, recent research has striven to replace the discrete, as well as the existing Brownian motion-based bank models (see, for instance, [1], [2], [3] and [5]) by systems driven by more general processes. From a numerical point of view, the authors plan to validate the aforementioned models by analyzing specific real-world data in an empirically sound manner.

More specifically, the authors would like to include mortgage and marketable securities cyclicality, as well as originator financial stability in the mortgage origination models, and expand the discussion of the leverage and debt ratios in Chapter 3. In this regard, they will strive to determine the leverage that balances expected returns against liquidity risk, develop an early warning system for crises, as well as investigate the relationship between excessive leverage and debt, and the probability of financial crises occurring. In addition, they would like to highlight the link between cyclicality, financial stability and regulation as it pertains to financial crises.

A specific activity in Chapter 4 will be to determine whether subprime mortgages were extended to all classes of mortgagors, and not only to those with impaired credit. A mortgage can be labelled 'subprime' not only because of mortgagor characteristics, but also because of the type of originator that originated it, features of the mortgage product itself or how it was securitized. In this regard, the authors would like to investigate whether subprime mortgages were also extended to mortgagors without impaired credit and the impact of this.

The liquidity coverage ratio (LCR) has some limitations regarding the characterization of banks' liquidity positions in Chapter 5. Therefore, complementary Basel III ratios such as the net stable funding ratio (NSFR) should be considered for a more complete analysis. This should take the structure of the short-term, high-quality liquid assets (HQLAs) and net cash outflows (NCOs) of residual maturities into account (see Chapter 6 for more details).

There are several other directions in which the results that have been obtained in this chapter can possibly be extended. These include addressing further risk issues, as well as improvements in the LCR modelling procedure. A study of information asymmetry during the financial crisis should be interesting.

The results provided in Chapter 6 give insights into how Basel III liquidity regulation might be applied in future. Research that has been done in this paper has the potential to contribute to the Basel III liquidity regulation worldwide.

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