Investigating Grade 7 mathematics learners' self-directed learning through problem-based tasks

ST Kashango

orcid.org/0000-0002-6782-6750

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Supervisor: Prof. H.M. Havenga

Co-supervisor: Dr T. Jordaan

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Student number: 23283815

DECLARATION

I, the undersigned, hereby declare that the work contained in this dissertation is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

30 November 2022

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Finally, I would like to thank my relatives, friends, and colleagues for supporting me. Be blessed all.

DEDICATION

This dissertation is dedicated to my family.

ABSTRACT

Mathematics is an essential subject since it helps learners to develop an understanding of spatial concepts and relationships and develops their ability to apply mathematics in the contexts of everyday situations. Mathematics teaching also involves developing knowledge and skills that will help learners not only to succeed in the classroom, but also to empower them in becoming productive citizens, who are able to apply the acquired knowledge and skills in the real-world context.

The aim of the study was to investigate Grade 7 mathematics learners' self-directed learning through problem-based tasks. Problem-based learning (PBL) is an approach where active learning is driven by solving real-world problems. This study used a qualitative research methodology. In total, 36 learners participated in this research. Two PBL tasks were assigned to the learners to be completed in groups. Data collection involved learners' mathematics worksheets, reflective sheets as well as semi-structured interviews with randomly selected learners. Data were manually coded and analysed, and several themes emerged from the findings.

The findings of this study showed that after the implementing PBL tasks, learners developed certain skills that are essential for self-directed learning (SDL). It is recommended that PBL should be implemented in the teaching and learning of Gr 7 mathematics learners. Early implementation of PBL can have a positive, lasting effect on learners' SDL, which in turn enhances mathematics performance.

KEY TERMS

Gr 7 learners, mathematics, problem-based learning, self-directed learning, tasks.

TABLE OF CONTENTS

DECLARATION i
ACKNOWLEDGEMENTS ii
DEDICATION iii
ABSTRACT iv
LIST OF FIGURES xii
LIST OF ABBREVIATIONS xiii
CHAPTER 1 1
INTRODUCTION AND PROBLEM STATEMENT 1
1.1 Introduction and problem statement 1
1.2 Research questions and aims 3
1.3 Conceptual-theoretical framework 6
1.3.1 Mathematics as a school subject 7
1.3.2 Problem-based learning 8
1.3.3 Self-directed learning 9
1.4 Research approach 11
1.4.1 Research methodology and design 11
1.4.2 Population and sample 13
1.4.3 Data collection 13
1.4.4 Data analysis 13
1.4.5 Role of the researcher 14
1.4.6 Trustworthiness 14
1.4.7 Ethical considerations 15
1.4.8 Significance of the study 15
1.5 Chapters and time frame 16
1.6 Summary of the chapter 17
CHAPTER 2 18
MATHEMATICS LEARNING: OVERVIEW AND DEVELOPMENT OF ESSENTIAL KNOWLEDGE AND SKILLS 18
2.1 Introduction 18
2.2 Mathematics in the Namibian education system 19
2.3 The school subject mathematics 21
2.4 Challenges in mathematics 23
2.4.1 Mother tongue 23
2.4.2 Attitude towards mathematics 26

2.4.3 Parental role and gender stereotypes 27
2.4.4 Learning and teaching support materials (LTSM) 28
2.4.5 Teachers' knowledge, skills and beliefs 29
2.4.6 Teaching and learning strategies 31
2.5 The learning of mathematics 36
2.5.1 Frameworks, models, and taxonomies of learning 36
2.5.2 Approaches or steps to solve mathematics problems 38
2.5.2.1 Reasons why learners struggle to develop HOT 39
2.5.2.2 Role of the teacher in fostering HOTS 40
2.5.2.3 Learners' development of HOT in mathematics 41
2.5.2.4 Benefits of developing HOT in learners 42
2.5.3 The role of metacognition in mathematics 43
2.5.4 Motivation to learn mathematics 46
2.6 Problem-based learning in mathematics 47
2.6.1 The origin of problem-based learning 47
2.6.2 A taxonomy of problem-based models 49
2.6.3 The problem in problem-based learning 49
2.6.5 Group work and cooperation in problem-based learning 52
2.6.6 Problem-based tasks 54
2.6.7 Planning problem-based tasks for mathematics 56
2.7 Self-directed learning 59
2.7.1 Characteristics of a self-directed learner 59
2.7.2 Factors influencing self-directed learning 61
2.7.2.1 Internal factors 61
2.7.2.2 External factors 61
2.7.3. The mathematics teacher's role during the facilitation of self-directed learning 62
2.7.4 Importance of developing self-directed learners 64
2.7.5. Problem-based tasks and the development of self-directed learning 65
2.8 Summary of the chapter 67
CHAPTER 3 69
EMPIRICAL DESIGN AND METHODOLOGY 69
3.1 Introduction 69
3.2 Research paradigm 69
3.3 Research design and methodology 69

3.4 Research context, population, and sample 72
3.5 Data collection procedures 74
3.5.1 PBL tasks 74
3.5.2 Mathematics worksheets 76
3.5.3 Reflective sheets 76
3.5.4 Semi-structured interviews 78
3.6 Data analysis procedures 78
3.7 Ethical aspects 83
3.8 The role of the researcher 83
3.9 Trustworthiness of qualitative data 84
3.9.1 Credibility 84
3.9.2 Transferability 85
3.9.3 Dependability 85
3.9.4 Confirmability 85
3.9.5 Triangulation 86
3.10 Summary of the chapter 87
CHAPTER 4 88
DATA ANALYSIS AND DISCUSSION 88
4.1 Introduction 88
4.2 Qualitative data analysis 88
4.3 THEME 1: INITIAL CHALLENGES EXPERIENCED WITH PROBLEM-BASED MATHEMATICS TAKS 90
4.3.1.1 Uncertainty about the task and the language barrier 90
4.3.1.2 Insufficient subject knowledge 92
4.3.1.3 Lack of resources 95
4.3.2 Sub-theme 1.2: Attitude towards group work 96
4.4 THEME 2: LEARNERS' EXPERIENCES WHEN SOLVING MATHEMATICS PROBLEMS 98
4.4.1 Sub-theme 2.1: Problem-solving activities 98
4.4.1.1 How learners solved problems 98
4.4.1.2 Problem-solving experiences 99
4.4.1.3 Reflective thinking in solving problems 101
4.4.2 Sub-theme 2.2: Group work and collaboration 103
4.4.2.1 Group communication and participation 103
4.4.2.2 Sharing of learning materials 103

4.4.2.3 Peer learning and support 104
4.4.2.4 Pace of problem solving and task completion 106
4.4.2.5 Member contribution and knowledge construction 107
4.4.2.6 Peer assessment and evaluation 108
4.5 THEME 3: DEVELOPMENT OF ESSENTIAL KNOWLEDGE AND SKILLS .. 110
4.5.1 Sub-theme 3.1: Development of mathematics skills 110
4.5.1.1 Clarification and mathematics concepts 110
4.5.1.2 Communication of gathered information 111
4.5.1.3 Mathematics skill development 112
4.5.1.4 Higher-order thinking (HOT) 113
4.5.1.5 Reflective thinking in mathematics 114
4.5.2 Sub-theme 3.2: Development of self-directed learning abilities 115
4.5.2.1 Setting of learning goals and determining learning needs 115
4.5.2.2 Learning responsibility and persistence 117
4.5.2.3 Self-reflection on learning experiences 118
4.6 Comparison before and after the intervention regarding learners' group work and responsible learning 119
4.7 Summary of the chapter 123
CHAPTER 5 124
CONCLUSIONS AND RECOMMENDATIONS 124
5.1 Introduction 124
5.2 Summary of chapters 124
5.3 RESEARCH FINDINGS 125
5.3.1 Grade 7 Mathematics, problem-based learning, and self-directed learning 125
5.3.2 Problem-based tasks that can be used in Grade 7 Mathematics to develop self-directed learning 127
5.3.3 Experiences of Grade 7 learners when solving Mathematics problems 128
5.4 Development of Grade 7 learners' self-directed learning skills 130
5.5 CONCLUSION 134
5.6 LIMITATIONS 134
5.7 RECOMMENDATIONS 135
5.8 FINAL REMARKS AND PERSONAL REFLECTION 136
References 137
ADDENDUM A 176
ADDENDUM B 177

ADDENDUM C 178
ADDENDUM D 179
ADDENDUM E 180
ADDENDUM F 181
ADDENDUM G 183
ADDENDUM H 185
ADDENDUM I 186
ADDENDUM J 189

LIST OF TABLES

Table 1.1 Description of concepts related to this study 5
Table 1.2 Time frame in terms of the chapters 16
Table 2.1 Teacher-centred approach and learner-centred approach 33
Table 2.2 Teacher-centred vs learner-centred teaching and learning strategies in mathematics 34
Table 2.3 Bloom's revised taxonomy for mathematics 37
Table 2.4 Examples of metacognitive questioning in mathematics 45
Table 2.5 The adapted seven steps in PBL 51
Table 2.6 A short lesson plan employing PBL 58
Table 2.7 Six Bloom's revised taxonomy levels for Grade 7 mathematics 66
Table 3.1 Feedback from Participant 4 before and after intervention 81
Table 3.2 Emerged categories from the data 82
Table 3.3 Emerging themes 82
Table 4.1 The delineation of themes based on the qualitative study 89
Table 4.2 Theme 1 and its related sub-themes 90
Table 4.3 Uncertainty about the task and language barrier 91
Table 4.4 Learners' comments regarding insufficient subject knowledge 92
Table 4.5 Learners' mathematical thinking as based on the cognitive domain of Bloom's taxonomy 94
Table 4.6 Learners' comments regarding the lack of resources 96
Table 4.7 Learners' comments regarding their attitude towards group work when solving mathematical problems 97
Table 4.8 Theme 2 and its related sub-themes 98
Table 4.9 Examples of how learners solved mathematical problems 99
Table 4.10 Learners' comments about their problem-solving experiences 100
Table 4.11 Learners' comments about reflective thinking in solving problems 101
Table 4.12 Examples of metacognitive and reflective thinking 102
Table 4.13 Learners' comments regarding group communication and participation 103
Table 4.14 Learners' comments about sharing learning materials 104
Table 4.15 Learners' comments about peer learning and support 105
Table 4.16 Some examples of group cooperation in PBL tasks 106
Table 4.17 Learners' comments regarding the pace of problem solving and task

completion 107
Table 4.18 Learners' comments regarding member contribution and knowledge construction 108
Table 4.19 Learners' comments about peer assessment and evaluation 109
Table 4.20 Theme 3 and its related sub-themes……...110
Table 4.21 Learners' clarification of mathematical concepts..... 111
Table 4.22 Learners' comments about communication of gathered information . 111
Table 4.23 Mathematical skills developed..... 112
Table 4.24 Learners' higher-order thinking (HOT)……113
Table 4.25 Learners' reflective thinking in mathematics..... 114
Table 4.26 Setting of learning goals and determine learning needs……..116
Table 4.27 Learners' comments on learning responsibility and persistence 117
Table 4.28 Self-reflection on experiences 118
Table 4.29 Learners' coded examples regarding their mathematical experiences 120

LIST OF FIGURES

Figure 1.1 The Venn diagram of all aspects essential to this study…… 10
Figure 1.2 Paradigm, methodology and research design in this study 12
Figure 2.1 Overview of Chapter 2..... 18
Figure 2.2 Challenges in mathematics 23
Figure 2.3 Oshikwanyama vs. English mathematical concept 25
Figure 2.4 COVID-19 cases, recovery, and death in Khomas Region..... 29
Figure 2.5 Examples of higher-order thinking in Grade 7 mathematics..... 39
Figure 2.6 Metacognitive thinking in Grade 7 mathematics..... 45
Figure 2.7 Timeline of key events in the development of PBL..... 48
Figure 2.8 Six representative PBL models in the PBL taxonomy of Barrow 49
Figure 2.9 Example of a mathematics problem 55
Figure 2.10 Calculate the area of a composite shape 60
Figure 3.1 Overview of the empirical research 71
Figure 3.2 How participants were randomly selected online 74
Figure 3.3 A completed reflective sheet of Participant 27 77
Figure 3.4 Example of P4's completed worksheet and coding of reflective sheet task (W1)..... 81
Figure 5.1 Visual representation of the findings 133

LIST OF ABBREVIATIONS

COVID-19: Coronavirus disease 2019

CPBL: Cooperative problem-based learning

DNEA: Department of National Examination and Assessment

FET: Further Education and Training

HOT(S): Higher-order thinking (skills)

ICT: Information and Communication Technology

JP: Junior Primary

JSC: Junior Secondary Certificate

LTSM: Learning and teaching support material

NCBE: Namibian National Curriculum for Basic Education

NCTM: National Council for Mathematics Teachers

NIED: National Institute for Educational Development

NWU: North-West University

NWU-EMELTEN-REC: North-West University Education, Management, and Economic Sciences, Law, Theology, Engineering and Natural Sciences Research Ethics Committee of the North-West University

PBL: Problem-based learning

PCK: Pedagogical content knowledge

SDL: Self-directed learning

SEO: Senior education officer

SGB: School governing body

SP: Senior Primary

CHAPTER 1

INTRODUCTION AND PROBLEM STATEMENT

1.1 Introduction and problem statement

Mathematical thinking is considered crucial to solving problems, thinking critically, and developing a repertoire of abilities with the aim of succeeding in life. Part of this repertoire is the ability to control one's own thinking, identify gaps, develop solutions, and be a self-directed learner. Self-directed learning (SDL) is a process that requires learners to manage their learning activities, develop solutions, evaluate their learning outcomes, and personally pursue knowledge and skills (Abubakar & Arshad, 2015; Knowles, 1975). SDL can be determined by the degree of responsibility that a learner accepts for his or her own learning (Yang & Jiang, 2014).

When learners develop SDL skills, they view problems as challenges, they are motivated, curious, enjoy learning, they are persistent, independent, and goal-oriented (Lai, 2017, Merriam *et al*., 2007). SDL is crucial because learners can transfer their learning, in terms of knowledge and skills, from one situation to another (Winne, 2016). A further development of SDL is beneficial as learners may have greater control over their learning processes than the learners that are not self-directed, improve their beliefs about their abilities to solve problems effectively and increase their motivation to learn (Nasri *et al*., 2020).

Therefore, in developing the ability to take ownership and responsibility for their own learning, learners may develop SDL skills for future demands (Knowles, 1975). SDL skills such as motivation and monitoring are also essential in solving mathematics problems (Tekkol & Demirel, 2018). Motivated learners are confident about their ability to solve mathematical problems by exploring new concepts and persist even if they are not immediately successful (National Council for Teachers of Mathematics [NCTM], 2013). In the process of problem solving, learners need to monitor their work to determine their progress (Guzman, 2018; Ortiz, 2016). Self-directed learners can analyse a problem, activate prior knowledge, set goals for the learning process, employ relevant strategies, make judgements about their ability to complete the task

successfully and reflect on the learning process to solve a mathematical problem (Bishara, 2021).

Unfortunately, several learners experience difficulty in managing their thought processes when working on mathematics tasks, because they do not properly read questions with understanding, and they do not understand mathematical concepts (Mulwa, 2015). Hasan (2019) and Xenofontos and Andrews (2014) agree and emphasise that learners experience challenges in solving mathematical problems due to their conceptualisation thereof. In other words, learners do not understand the problem due to translations (mother tongue and language differences) and as a result they have difficulty in solving such problems. Hasan (2019) and Emanuel *et al.* (2021) further identified mathematical errors, teaching and learning experiences, interpretation and translation activities, and the use of transitional representations as other contributing factors to Grade 7 learners' inability to solve mathematical problems.

Some learners are not able to solve problems because they do not understand them or misinterpret mathematical ideas and are not able to address problems that require higher-order thinking (HOT) as they lack fundamental abilities such as logical thinking, critical thinking, and reasoning skills (Tajudin & Chinnappan, 2017). Unfortunately, most learners do not use the skills that can help them solve problems that require HOT from previous grades. They do not try to seek for guidance or assistance when they do not understand the problem and unfortunately, they rather solve it for the sake of completing it (Kleopas, 2020). These learners do not take responsibility for their own learning (Massonnie *et al.,* 2022).

It is therefore essential that Grade 7 learners develop SDL skills to enable them to solve mathematical problems effectively, to improve their performance and to develop fundamental knowledge and skills as required in the following grades. In addition, learners need to apply certain skills while working on problems, which are essential for lifelong learning such as critical thinking, research, self-management and self-regulation, motivation, and social and communication skills (McGarrah, 2015). Moreover, learners must apply relevant learning strategies to assist them in their learning process to develop SDL abilities when solving mathematical problems.

Although research has been conducted on SDL in the field of mathematics, there is a gap regarding Grade 7 learners' responsibility and accountability regarding learning and working on mathematics tasks as essential skills that are required to succeed. Furthermore, research is required to determine whether the Grade 7 mathematics learners are prepared for the future to take responsibility for their own learning. It is also imperative that learners develop SDL abilities in the Intermediate Phase, which serves as a foundation for complex learning that follows in the Further Education and Training (FET) Phase.

As an experienced Grade 7 mathematics teacher, the researcher realised that learners are too dependent on the teacher, often expecting the teacher to tell them what to do, instead of providing opportunities to develop essential skills and solve problems themselves. Furthermore, these Grade 7 learners are resistant towards taking responsibility for their own learning activities, as they did not develop such a culture of ownership from previous grades. Therefore, the rationale for this study was based on the following:

- The concern that learners in the Grade 7 mathematics class are dependent on the teacher and do not take responsibility for their own learning;
- Learners experience several challenges regarding the learning of mathematics; and
- A gap exists in research regarding the application of relevant teaching and learning strategies (such as problem-based learning (PBL)), to enable Grade 7 learners to develop SDL abilities when solving mathematical problems.

1.2 Research questions and aims

The following main research question was formulated to guide the study: **How can problem-based tasks contribute to the development of Grade 7 mathematics learners' self-directed learning?**

The secondary questions were:

1. What does Grade 7 mathematics, problem-based learning, and self-directed learning entail?

2. What problem-based tasks can be used in Grade 7 mathematics to help learners become self-directed?
3. What are the experiences of Grade 7 learners when solving mathematics problems?

The first and second questions are addressed in the theoretical overview, while the third question is answered from the findings of the empirical research.

The main aim was to investigate how problem-based tasks can contribute to the development of Grade 7 mathematics learners' self-directed learning?

The sub-aims were:

1. To investigate what Grade 7 mathematics, problem-based learning and self-directed learning entail;

2. To investigate what problem-based tasks can be used in Grade 7 mathematics to help learners become self-directed;

3. To explore what the experiences of Grade 7 learners are when solving mathematics problems.

Some concepts related to this study namely mathematics, mathematics teaching and learning, self-directed learning and problem-based learning are outlined in Table 1.1.

Table 1.1: Description of concepts related to this study

Terminology	Description	References
Mathematics	Mathematics deals with the logic of shapes, quantity, and arrangement. Mathematics is a science of the logical study of numbers, shapes, arrangements, quantities, measures, and other related concepts. Mathematics seeks understanding of the patterns and structures of constructs of the human mind. Mathematics is used to deal with and make sense of real-life phenomena.	Hom (2013) and Yadav (2017)
Mathematics teaching and learning	Mathematics teaching "involves creating mathematical classroom discourse, developing classroom norms that support opportunities for mathematical learning and building relationships that support mathematical learning". Mathematics learning "involves observing, representing, and investigating patterns and quantitative relationships in social and physical phenomena and between mathematical objects themselves. Through these processes, new mathematical ideas and insights are generated".	Franke *et al*., (2007: 226) NIED (2015: 1)
Self-directed learning (SDL)	SDL is defined as a "process where individuals manage their own learning, identify learning needs, formulate learning goals, implement appropriate learning strategies, apply the required resources, and evaluate their learning outcomes".	Knowles, (1975:18)
Problem-based learning (PBL)	PBL is "an instructional learner-centred approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to an open-ended problem".	Savery, (2015:4)

For the literature review, terms such as **mathematics, mathematics teaching and learning, problem-based learning** and **self-directed learning** were used to search for related resources as well as useful information to guide the study. In addition, various resources were used such as Google Scholar, EBSCOhost and the NWU (SDL) eFundi site.

1.3 Conceptual-theoretical framework

The social-constructivist paradigm directed the study as Grade 7 learners in a social setting must construct their own knowledge through interacting with one another and their environment to make meaning of what they are learning via addressing the PBL tasks. Social constructivism views learning as a process where learners collaborate by engaging in class activities for meaningful learning to take place (Akpan *et al.,* 2020). Social-constructivism seeks to understand the world in which individuals live and construct their own meanings that are in harmony with their experiences and such meanings are formed through interaction with others (Carmona-Medeiro & Cardeñoso Domingo*,* 2021; Creswell, 2013). Social constructivism portrays the idea that learning occurs when the learners discover knowledge through experiencing things and reflecting on those experiences (Adom *et al*., 2016). Bozkurt (2017) further states that social interaction in joint activities provides learners with the acquisition and construction of knowledge. Bozkurt (2017) points out that in social constructivism, both social interaction and individual meaning make up pivotal and crucial parts of learning. This is supported by Apriliyanto *et al.* (2018), who mention that social interaction is an important aspect of learning mathematics, because learners can express their own thoughts to encourage reflection on the knowledge they have.

Mathematics is particularly dependent on the emphasis and exposure to social interaction between the teacher and the learner and amongst the learners themselves (Csachová & Jurečková, 2020). That means, the more learners interchange ideas with other learners as well as with teachers, the more mathematics become real to them, and they learn better. Productive interactions between teacher and learners and between learners with the content may contribute to the learning potentials Dorimana *et al.* (2022).

Various constructs such as mathematics learning, self-directed learning and problem-based learning are outlined in more detail in the next section.

1.3.1 Mathematics as a school subject

Mathematics is an essential subject, as it helps learners develop an understanding of spatial concepts and relationships and develops crucial abilities to apply mathematics in the contexts of everyday situations (Becker, 2017). Mathematics develops an understanding of subject-specific principles, provides opportunities to analyse problems logically and affords the mathematical background required for further study in this or related subjects (Namibian National Institute for Educational Development [NIED], 2015).

The National Council of Teachers of Mathematics (NCTM) (2016) contends that, to teach mathematics is to inspire learners to look beyond the textbook, to become problem solvers and critical thinkers. Mathematics teaching involves the creation of an opportunity to learn by providing a supportive learning environment, giving relevant mathematical challenges, and nurturing processes and strategies that foster learning (Khan *et al.*, 2016). Mathematics teaching also involves learners to develop knowledge and acquire skills that will help them not only to succeed in the classroom, but also to empower them to become productive citizens, able to apply the acquired knowledge and skills in the real-world environment (Garrett, 2012). Moreover, early proper learning of mathematics skills is important, as it may be the strongest predictor of future mathematical performance (Harris & Petersen, 2017; Singh, 2013). Madosi *et al.* (2020) highlight that mathematics teachers must support learners for effective learning to enable them to be dedicated, manage their own thinking, demonstrate understanding of mathematics concepts, address authentic problems, use various ways to solve problems and apply their thinking to mathematical proofs.

Learning should help learners to develop meaningful connections between mathematical ideas and concepts in real life, since these are essential to solve open-ended problems (Altay *et al*., 2017; Menanti *et al.*, 2018). In addition, mathematics learning in early childhood helps learners to develop dispositions such as persistence and dedication, which contribute to their future success (Clements, 2017). Madosi *et al.* (2020) and Seah *et al.* (2016) point out that working hard and putting in effort when doing mathematics, may cognitively affect learners' levels of reasoning, intellectual capacity, motivation, and determination to learn mathematics. Furthermore, Madosi *et*

al. (2020) believe that if learners are supported, they can appreciate what they learn, be encouraged and benefit from it during the teaching and learning of mathematics. Therefore, what learners value as important in the learning of mathematics could have a direct effect on their learning of the subject (Seah *et al.*, 2016).

Unfortunately, not all learners understand mathematics due to various reasons, including different cognitive and affective factors, among others (Gafoor & Kurukkan, 2015). Learners' beliefs regarding mathematics and the learning thereof can prevent them from understanding the subject content. For example, learners do not necessarily understand the language (concepts and terminology) of mathematics in order to follow the correct procedures to solve problems (Jourdain & Sharma, 2016). Furthermore, many learners are unable to reason logically, and they cannot see the connections between mathematical ideas, which is an important factor in understanding mathematics (Ndiung & Nendi, 2018).

As a result, learners need to take responsibility for their own learning if they want to succeed in the subject. When learners are responsible for their own learning, they take full charge of the learning process, make time for doing their work, investigate, research, and evaluate to draw conclusions from the learning material (Mohamad *et al.,* 2020).

1.3.2 Problem-based learning

Problem-based learning (PBL) is a teaching and learning strategy that enables learners to learn while engaging actively with meaningful problems. For Yew and Goh (2016), PBL is learner-centred as learners develop essential knowledge and skills through the solving of an open-ended problem or addressing a challenge (Mustaffa *et al*., 2015). According to Golightly (2014), PBL is an approach that can help foster learners' SDL skills and encourage them to take responsibility for their own learning. PBL mainly involves learners working together, where group members have multiple opportunities to contribute to the conversation with the aim of solving authentic problems, addressing a question of inquiry and/or developing an artefact (Allard, 2017). By applying PBL, learners can assist each other in understanding and the

development of HOT (Mustaffa *et al*., 2015). Knowledge constructed in this way is valuable as it is retained longer and enables learners to conceptualise knowledge and solve problems in real life (Wooldridge, 2016).

Problem-based tasks refer to specific tasks that provide an opportunity for learners to build conceptual understanding when collaborating on solving an open-ended problem or addressing a task (Aziza, 2017) (See 2.6.6). Solving such problems contribute to a conceptual understanding of mathematics (Bingolbali & Bingolbali, 2021). When learners solve problems in different ways (e.g., drawing pictures, writing algorithms, or using manipulative), they make connections between various skills to help them, not only to think of what works in mathematics, but also how and why it works (Tamer, 2015).

Successful application of PBL depends on how the teacher deals with the challenges regarding the instruction thereof. Consequently, detailed preparation on the part of the teacher is required.

1.3.3 Self-directed learning

SDL is a process where learners are personally involved in setting goals and managing their own thinking and learning activities and this is discussed in this section. Knowles (1975) refers to SDL as a process in which individuals take initiative to assess one's own needs, create goals, and seek out appropriate strategies on their own. Hofmeyer (2016) considers self-directed learners as motivated; they have the desire to achieve and search for new knowledge. Self-directed learners can assess their learning needs, plan effectively, manage their time, give an evaluation of the literature resources as well as a critical evaluation of their own success (El-Nabawi & Shaalan, 2019).

Such learners also demonstrate a greater awareness of their responsibility by making their own learning meaningful (Abdullah, 2001). A further advantage of SDL is that it encourages learners to have control over their learning, thus improving their beliefs about their effectiveness (Hofmeyer, 2016; Van de Walle *et al*., 2013). Guglielmino (2013) points out other benefits of SDL such as increased confidence regarding own learning ability and developing life-long learning.

Hence, SDL is essential in mathematics to develop learners who can monitor their own thinking, reasoning, and are able to see applications of mathematics concepts (Khoo, 2018). Self-directed learners are goal oriented, self-confident, and independent. Such learners are willing to try new things, they are curious, view problems as challenges and have a desire to change, to make learning meaningful, and to monitor themselves (Gündüz, & Selvi, 2016). In other words, to develop as self-directed learners in mathematics, they must be actively involved in their learning, communicate effectively, address possible challenges, and apply a variety of problem-solving skills (Adelabu *et al.*, 2021).

The use of active teaching and learning approaches such as PBL, may assist in the development of SDL (Wijnia *et al.,* 2014). Figure 1.1 illustrates an integration of the three main aspects of this study.

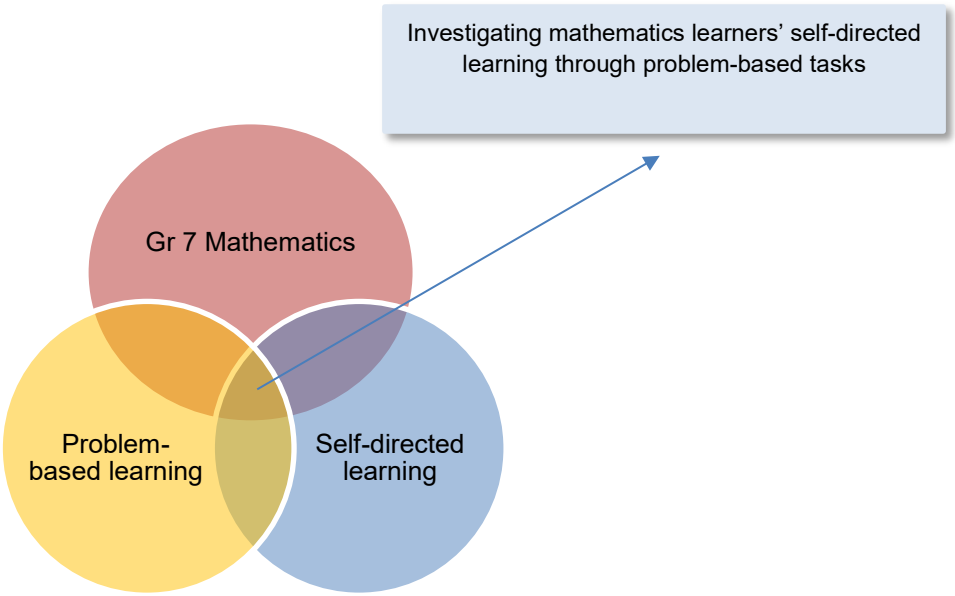


Figure 1.1: Venn diagram of all aspects essential to this study

1.4 Research approach

The next section introduces the research approach, methodology and design as well as ethical considerations and trustworthiness.

1.4.1 Research methodology and design

To investigate the Grade 7 mathematics learners' SDL, the researcher used a generic qualitative research methodology. Such a methodology explores people's opinions, attitudes, and beliefs and seeks how they construct or make meaning of their world and their experiences (Adom *et al*., 2016; Creswell, 2018; Percy *et al*., 2015). A qualitative research design was applicable to this study as it explored environments where learners worked together in groups on open-ended problems in mathematics. Grade 7 learners also completed an open-ended questionnaire in order to investigate how mathematical problem-based tasks can contribute to the development of SDL abilities in Grade 7. Figure 1.2 outlines the research paradigm, methodology, and design.

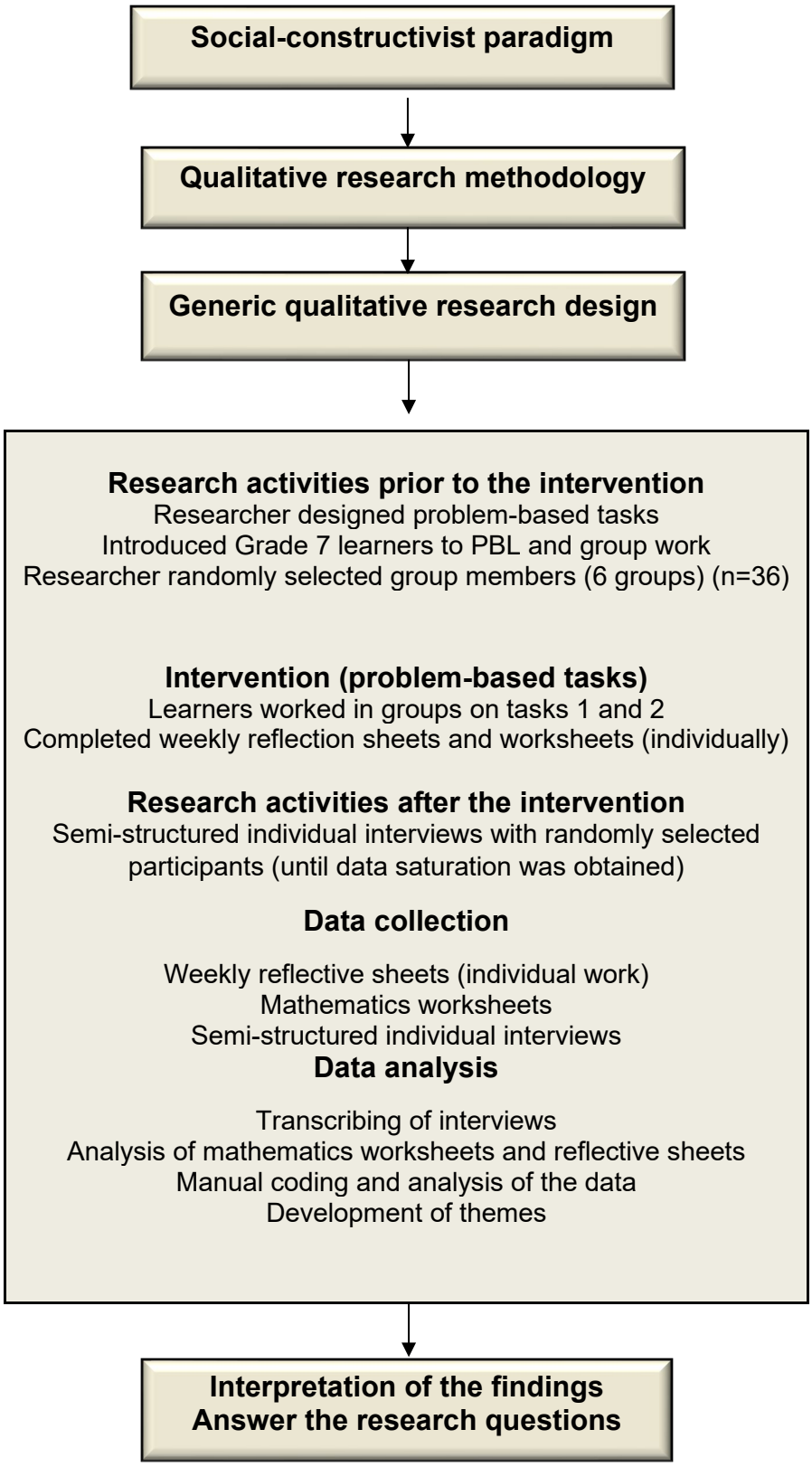


Figure 1.2: Paradigm, methodology and research design in this study

1.4.2 Population and sample

A population involves all the individuals of interest, generally a large collection of individuals or objects that have common characteristics that are the focus of the scientific inquiry or who share some basic attributes of interest (Asiamah *et al.*, 2017). In this study, the population involved 71 Grade 7 learners in a Namibian school. Creswell (2018) views a sample as a subdivision of the population for the purpose of making generalisations about the target population. The sample for this study was 36 Grade 7 mathematics learners, who were conveniently selected on the basis that they all had Grade 7 mathematics as a subject and were readily available to the researcher for the study. Learners worked in six groups of six members on problem-based tasks and the teacher randomly selected the members for each group.

1.4.3 Data collection

Weekly reflective sheets, mathematics worksheets, and semi-structured interviews were used to gather the data (Figure 1.2). Maree (2016) describes an interview as a two-way conversation in which the interviewer asks specific questions with the aim to collect data and to learn about ideas and opinions of the participants in a particular research context. The Grade 7 learners (class 7B) were invited to take part in the research. Twelve learners were randomly selected for individual interviews until data saturation was obtained (See Addendum J).

1.4.4 Data analysis

The data were analysed using the thematic analysis method. Nowell *et al.* (2017) refer to thematic analysis as a qualitative data analysis method that can be used over a wide range of epistemologies and research questions. It comprises identifying, analysing, organising, describing, and reporting themes as based on the data set (Maguire & Delahunt, 2017). The approach of Saldaña (2016:105) was followed namely ‘in vivo’ coding (also known as ‘verbatim coding’) where the code relates to a word, sentence or short paragraph as mentioned by the Grade 7 participants themselves (See Chapter 3). Themes that resulted from the data analysis were interpreted to answer the research questions.

1.4.5 Role of the researcher

The nature of qualitative research resulted in the interaction between the researcher and the participants, thus making the researcher part of the research process and working with participants in a particular context (Maree, 2016). The teacher (researcher) planned the teaching and learning context and facilitated the learning activities. The teacher also guided the PBL process, facilitated learners and assessed their problem-based tasks. In addition, the researcher acted as an interviewer and gathered the data after the intervention. Other duties were to record the data in the specific context and to analyse the interview scripts and related documents (See 1.4.3) to make sense thereof with the aim of understanding and interpreting, and subsequently answering the research questions. Maree (2016) warns researchers to be conscious of their own biases and Sandman (2016) suggests that the researcher needs to scrutinise him- or herself as well as the sources of information, for opposing views and evidence. The researcher avoided being overconfident about opinions both old and new as well as assessing the quality of information sources without favouritism.

1.4.6 Trustworthiness

The trustworthiness of a qualitative study is presented among others by credibility (truth of the findings), dependability (findings are consistent), confirmability (participants' experiences and not researcher bias), and transferability (applicable in other contexts), (Elo *et al*., 2014; Guba & Lincoln, 1985). To ensure credibility in this study, the researcher honestly dealt with the data obtained from all the participants without changing any response to fit the researcher's desires. Only willing learners from Grade 7 participated in this study. Individuals were given the opportunity to participate willingly through informed assent as well as their parents' consent (See Chapter 3). The researcher had follow-up interview sessions as needed to clarify some aspects and to ensure dependability. In this study, dependability was ensured when the research had a prolonged engagement, with the teacher's lasting presence in class and interviews at the school. Conformability was addressed as outlined by Sandman (2016), namely that the researcher needed to scrutinize herself, the sources of information as well as opposing views and evidence. The researcher invested sufficient time in becoming familiar with the setting and context, getting tested for

misinformation and building trust. Being in the class helped the researcher to spot out those characteristics and elements that are most crucial to the problem or issue being studied, on which the researcher focused in detail. The researcher also used a thick description so that the behaviours and experiences could become meaningful to the reader (Chapter 4).

1.4.7 Ethical considerations

The researcher first received approval from the Scientific Committee of the Research Unit for Self-directed Learning (See Addendum A) as well as ethical clearance from the North-West University Education, Management, and Economic Sciences, Law, Theology, Engineering and Natural Sciences Research Ethics Committee of the North-West University (NWU-EMELTEN-REC) (See Addendum B). Thereafter, the researcher obtained a letter of permission from the Inspector of Education of the Etayi circuit in the Omusati region in Namibia (the Director of Education of Omusati region in the absence of the inspector) (See Addendum C). The researcher further obtained a permission letter from the school principal and the School Governing Body (SGB) to conduct the study (See Addendum D).

An independent person (a mathematics teacher) obtained informed assent from the participants and informed consent from the participants' parents or guardians (See Addenda E). The informed consent letter explained that participating in the study would be voluntarily and that learners were free to withdraw from the study at any time without penalty. The researcher assured learners' confidentiality and privacy. Furthermore, pseudonyms were used when referring to learners. In addition, all printed copies of data were locked up in a cabinet and electronic versions were password-protected on the computer. All data will be destroyed after seven years as required.

1.4.8 Significance of the study

The findings of this study might be helpful in structuring guidelines for teaching mathematics in this phase (the Senior Primary Phase) as well as in mathematics curriculum development and content allocation of the mentioned phase. Learners will benefit if all mathematics teachers work toward contributing to learners' SDL, which can help them own their learning, remain focused on their learning, improve their academic performance (De La Cruz, 2018), and develop as life-long learners. In

addition, the study will contribute to the Research Unit for SDL regarding learners' development of SDL in the Senior Primary Phase (See Chapter 5).

1.5 Chapters and time frame

The dissertation consists of five chapters. The first chapter presents the background and rationale of the research (introduction to the study and the research problem statement). The second chapter focuses on the overview of literature and concepts utilised in the study. These are mathematics, Mathematics teaching and learning, SDL, and PBL. In the third chapter, the empirical design and methodology, study context, and data collection and analysis methods are explained in detail. In the fourth chapter, the researcher discusses the analysis of the data and interprets the research findings while the last chapter presents the conclusion and recommendations. The table below indicates the time frame for the study. The time frame in terms of the chapters is shown in Table 1.2.

Table 1.2: Time frame in terms of the chapters

	Sept 2018	Dec 2018	Jan 2019	Sept *2020	Dec *2021	April 2022	May 2022	July 2022	Nov2022
Chapter 1									
Chapter 2									
Chapter 3									
Chapter 4									
Chapter 5									
Finalise									

*** COVID-19 circumstances had an impact on this study**

1.6 Summary of the chapter

This chapter outlined essential aspects of this study such as mathematics teaching and learning, SDL, and PBL. The research questions and aims were outlined as well as the research approach, methodology, trustworthiness, and the ethical considerations were briefly mentioned. The next chapter provides an overview of mathematics learning and skill development.

CHAPTER 2

MATHEMATICS LEARNING: OVERVIEW AND DEVELOPMENT OF ESSENTIAL KNOWLEDGE AND SKILLS

2.1 Introduction

This chapter presents an overview regarding mathematics learning as well as the development of essential knowledge and skills. In particular, the first and second research aims are addressed, namely, to investigate what Grade 7 mathematics, PBL, and SDL entail and to investigate problem-based tasks that can be used in Grade 7 mathematics to help learners become self-directed learners. An overview of chapter 2 is presented visually in Figure 2.1.

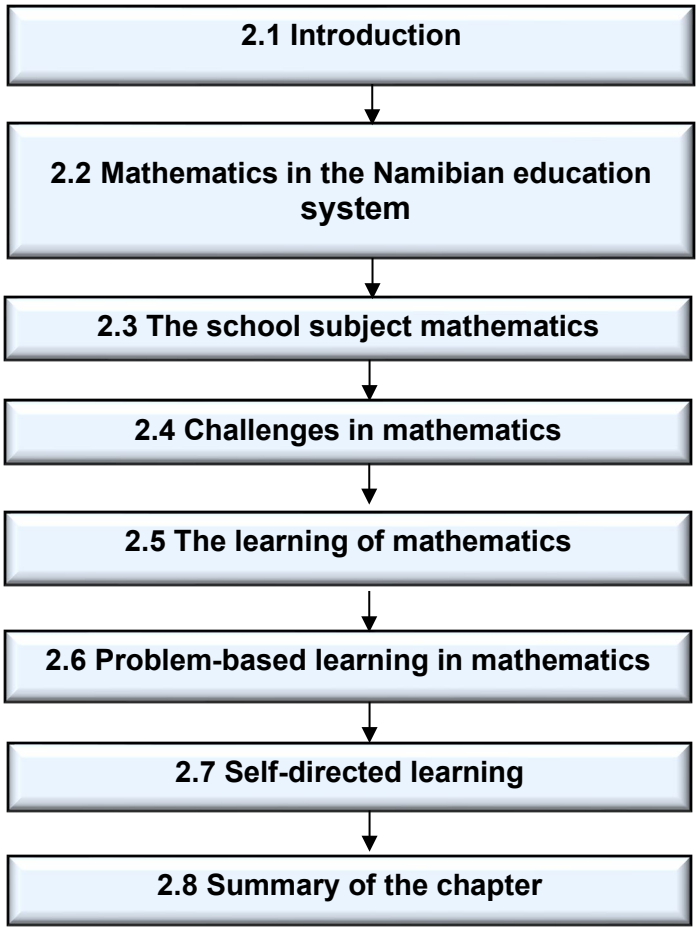


Figure 2.1: Overview of Chapter 2

2.2 Mathematics in the Namibian education system

The Basic Education system in Namibia is divided into four phases, namely Pre-primary (Grade 0), Junior Primary (Grades 1-3), Senior Primary (Grades 4-7), Junior Secondary Phase (Grades 8-9), and Senior Secondary Phase (Grades 10-12) (NNCBE, 2016). In Namibia, mathematics is offered in all state schools as well as private schools and it is considered a promotional compulsory subject from Grade 1 to Grade 10.

According to the previous and revised promotion curriculum guide for Junior and Senior Primary Grades, a learner is expected to obtain an E-grade (40%) or higher in mathematics to be promoted to the next grade (Department of Education, 2014). This requirement was also applicable to Grades 8–10 mathematics in the previous curriculum.

The Namibian Mathematics Syllabus (the revised curriculum which was implemented in 2015) provides teachers with a framework for teaching to deal with the numeracy demands of the modern world. Thus, the ability to interpret and make sense of mathematical concepts and competencies is crucial to solve problems, address challenges and to improve their own lives (Department of Education, 2015).

According to the Junior Secondary Phase Mathematics Syllabus, Grades 8 and 9 (2015:2), school mathematics is important since it promotes the following objectives in the curriculum:

- "develops learners' mathematical knowledge and skills in a way that encourages confidence and provides satisfaction and enjoyment;
- develops a feel for number and measurement, to carry out calculations, and to understand the significance of the results obtained;
- develops an understanding of spatial concepts and relationships;
- develops learners' ability to apply mathematics, in the contexts of everyday situations and of other subjects that they may be studying;
- develops an understanding of mathematical principles;
- develops learners' ability to analyse problems logically, recognise when and how a situation *may* be represented mathematically, identify and interpret

relevant factors, and where necessary, select an appropriate mathematical method to solve the problem;

- uses mathematics as a means of communication with emphasis on the use of clear expression;
- appreciates patterns and relationships in mathematics;
- produces and appreciates imaginative and creative work arising from mathematical ideas; and
- acquires the mathematical background necessary for further study in this or related subjects."

In addition, the Namibian Department of National Examination and Assessment (DNEA, 2010) and the Junior Secondary Certificate (JSC) Exam Results (2018) indicate that JSC (Grade 10) mathematics performance is improving at a very low rate. In the Senior Secondary Examination (Grade 12), the performance is still poor, leading to the question of what should be done, since mathematics is not really a problem in the Junior Primary Phase (JP) (Grades 0–3).

2.3 The school subject mathematics

Mathematics is a science of the logical study of numbers, shapes, arrangements, quantities, measures, and other related concepts (Yadav, 2017). According to Khan *et al.* (2016:13), “Mathematics deals with the logic of shapes, quantity, and arrangement and it refers to a study of logical and philosophical notions”. Hence, “people can make conjectures and test them using means such as classifying, enumerating, ordering and embedding, and build them into generalisations” (Hom, 2013:23). Therefore, mathematics provides access to view the world through numbers, measurement, shapes, and statistics that are essential and informative.

Learners who complete Grade 12 (with mathematics as an elective) are expected to be numerate and the study of mathematics at the Junior Secondary level contributes to the learners’ ability to think logically, work systematically and accurately, and to solve real-world problems. This means that the curriculum caters for various learner abilities, including those who will continue their studies in mathematics or related disciplines in the future (Department of Education, 2015).

Although mathematics is essential, it is regarded as a problematic subject since learners often fail and most of them refuse to choose it in Grades 11 and 12 because it is considered complicated. In line with this, Gomez-Chacon (2017) argues that negative attitudes towards mathematics seem to be a source of challenges in the teaching and learning thereof. In other words, negative attitudes and views towards mathematics are factors that contribute to the failure in this subject. This means that when learners have a negative perception towards mathematics such as perceiving mathematics as too difficult and challenging, they will quickly lose interest and motivation to learn the subject. As a result, they cannot perform well in solving mathematics problems. Winne (2016) asserts that the main factors specified by learners as their challenges are teacher behaviours, practices, and characteristics. For example, learners feel concerned when their teachers are teaching too fast and have unpleasant characteristics. Consequently, learners are afraid to ask for assistance when they experience problems with mathematics. Thus, the strict and decisive teachers trigger the feeling of restlessness in mathematics among learners (Winne, 2016). Gafoor and Kurukkan (2015) mention that mathematics is considered difficult due to preserved beliefs about the subject, aversive teaching styles, difficulty

regarding following instructions and remembering strategies to solve problems. These problems, according to Gomez-Chacon (2017), are because most learners use rote learning. Ahmed and Ahmed (2017) consider rote learning as the memorisation of information based on repetition. When rote memorisation is applied in mathematics, it is not considered higher-level thinking. In rote learning, the learners' creativity is suppressed, and they do not learn how to think, analyse, or solve mathematical problems (Gafoor & Kurukkan, 2015; Mardiyana *et al*., 2018).

Sa'ad *et al*. (2014) and Mabena *et al*. (2021) are of the opinion that low performance in mathematics in the higher grades is a result of learners' inability to translate, transform, and perform arithmetic operations, while Emanuel *et al.* (2021) identify teachers' inability to master the curriculum (most of the teachers at higher phases, i.e. Grades 4–10 are young, novice, and not properly mentored) as another factor to poor performance (also see 2.4).

2.4 Challenges in mathematics

Many learners are faced by challenges that affect their understanding of mathematics. Figure 2.2 presents each of these challenges and they are discussed in more detail below.

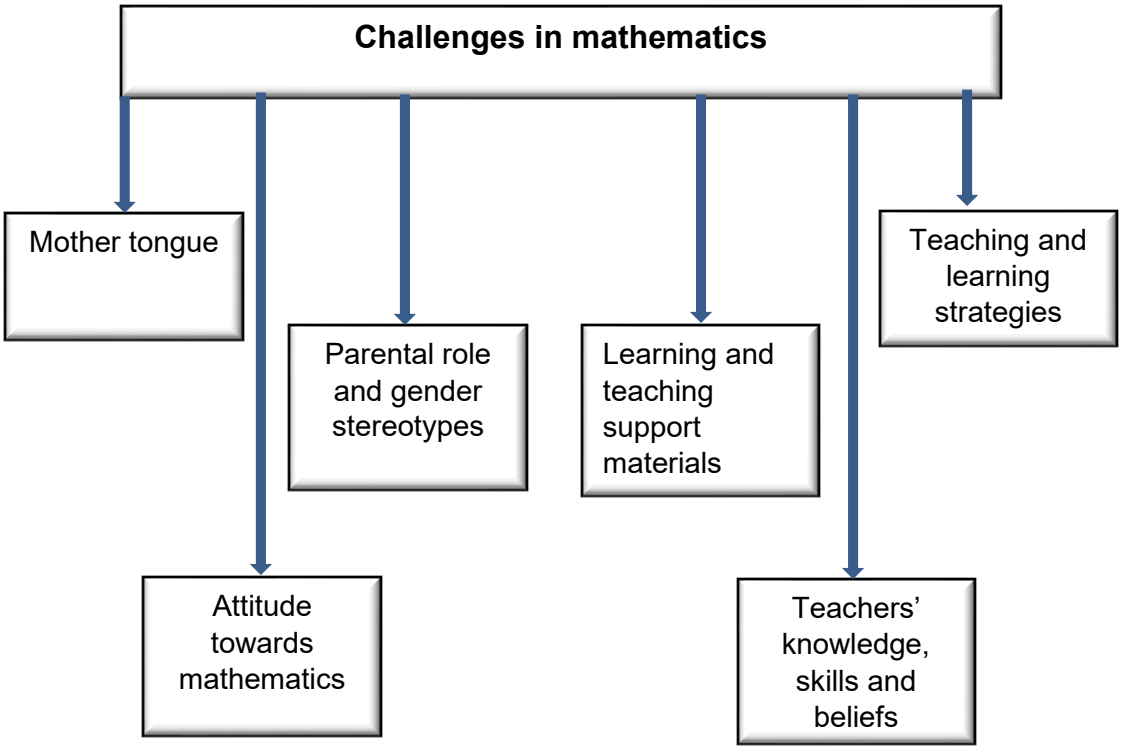


Figure 2.2: Challenges in mathematics

2.4.1 Mother tongue

The Namibian Pre-primary Teachers' Manual (Department of Education, 2010), the Junior Primary Phase Syllabus (2014) and the Language Policy (2003) clearly state that the mother tongue of a learner should be used as the medium of instruction up to Grade 3. The Namibian National Curriculum for Basic Education (NNCBE) (2016) further states that the medium of learning is the predominant local language, unless there are 35 learners (teacher-learner ratio is met) to constitute a class in their mother tongue. However, during Grade 4, learners will learn mathematics in English as a medium of instruction for the first time (Kaputu, 2016). Grades 1–3 are considered as the Junior Primary Phase, with Grade 4 being a transitional grade before the learner starts with the Senior Primary Phase of Grades 5–7, where they are taught through the medium of English in all subjects (Kaputu, 2016). Even though English is offered

from Grade 1, learners will be more fluent and confident when they learn through their mother tongue when starting the pre-primary school (Department of Education, 2015). There are fourteen languages used in Namibia as medium of instruction (Department of Education, 2016).

Although learners in the Junior Primary Phase enjoy learning mathematics in their mother tongue, many teachers in the Senior Primary Phase experience problems with the learners when entering the phase in Grade 4. Consequently, Mabena *et al*. (2021) learnt it in their mother tongue in the previous three years and these indigenous languages do not have all the terms and concepts that English offers. While some learners enjoy and perform well in mathematics, others do not enjoy the subject, nor do they perform well, and they experience the English language as a major challenge (Hamukwaya & Haser, 2021).

Learners find it challenging to make a connection between mathematical terminology in their mother tongue and English mathematical terminology. Seid (2017) states that learners who are taught in their mother tongue (in the early grades) experience the disadvantages of transition to English instruction in a more severe way than those who use English as medium of instruction. Sibomana (2020) concurs with Ssentanda (2014) that this transition is challenging due to learners' limited proficiency in English, because mother tongue impedes the second language acquisition and learning. This is the case with many Grade 7 learners due to the language barrier which occurs (when transition is not properly handled) during the transition to Grade 4 when they move from their mother tongue to English as a medium of instruction (Department of Education, 2015). For example, under rounding and estimation, the Oshikwanyama Grade 3 Syllabus refers to "kokwela nokweeta eenomola nomilongo" (Department of Education, 2014:23), while the English version of the same grade refers to "round off to the nearest 10". The Grade 4 mathematics syllabus indicates "round numbers up to 10 000 to the nearest power of 10" (Department of Education, 2015:11). Learners who have been using the term "kokwela", will be exposed to something new by using the term "round". Figure 2.3 shows Oshikwanyama (mother tongue) vs English mathematical concepts.

Kokwela nokweeta 25 nomilongo – Round 25 off to the nearest 10 = 30

Figure 2.3 Oshikwanyama vs. English mathematical concept

The sudden transition in early-exit grades does not allow learners to develop adequate cognitive, linguistic, and academic skills in their mother tongues prior to the change (Bukhari, 2022). Therefore, transfer of cognitive and academic competencies from the mother tongue to the second language is effective only when the learners have acquired adequate linguistic and academic competency in their mother tongues and are able to make the connection between the two languages (Boateng, 2019). Consequently, learning mathematics in their mother tongue makes it difficult for learners from the Senior Primary Phase onwards if the transition is not properly handled. Carter *et al.* (2020) support this by pointing out that learners do not perform well due to poor transition from local languages to English. As a result, language is vital in the learning of mathematics. Kaputu (2016) argues that learners entering the Senior Primary Phase should start using English as a medium of instruction and make less use of their mother tongue.

Hence, this is where mathematics terminology causes confusion, because even teachers in the Senior Phase are unfamiliar with the terms which learners used in their mother tongue in the previous grades. While teachers use indigenous languages, the curriculum does not allow them to make use or mention basic English concepts in the Junior Phase to facilitate learners' transition to English as required in the Senior Phase. Despite that, teachers are forced to translate the lessons from English to the local language, which is almost impossible if the teacher is not familiar with the local language (Erling *et al.*, 2016). Steyn (2017) concurs that changing the language of learning and teaching (LoLT) seems to be the contributing factor to the problem of transition. Therefore, this problem can be resolved if the Ministry of Education introduces English as the medium of instruction for Grades 1–3 (Junior Phase) or use English in the teaching and learning of mathematics in the Junior Phase in Namibia.

2.4.2 Attitude towards mathematics

A sense of fear and failure regarding mathematics among most learners as well as a negative perception and low self-regulation (one's ability to control his/her energy states, emotions, thoughts, and behaviours in proper way), can also challenge the way in which a learner can learn mathematics (Gomez-Chacon, 2017). Negative experiences in class or doing badly in mathematics due to poor basic skills (e.g., multiplication) can also affect how a learner feels about and learns mathematics (Cheng, 2016). Poor basic skills are a result of a poor foundation in mathematics: a learner is not acquainted with proper skills from the beginning and as a result, the learner's attitude toward mathematics will become a challenge to the way he or she learns the subject (Cheng, 2016). For example, learners fail to add, multiply, and divide numbers, but they have done and covered it in the previous grades, and this leads to certain learning gaps when approaching new challenges such as adding, subtracting, and dividing decimal numbers or whole numbers with more digits.

Tawfik and Lilly (2015) emphasise that the way learners think about mathematics in their everyday lives also influences their attitude towards learning mathematics. This is because behaviours are influenced by each one's characteristics and experiences related to how individual sees his/her academic performances. In addition, Gomez-Chacon (2017) states that a learner's beliefs about mathematics can determine how one chooses to approach a problem, which strategies will be used, and how he or she will solve the problem. Such learners' attitudes are influenced by several factors including the school, peer, home environment and society (Mazana *et al.*, 2019). However, Naungayan (2021) concludes that some learners have a positive attitude towards the subject, reducing the fear of mathematics as a subject. In harmony with this, Hwang and Son (2021) also reveal that learners have developed positive attitude and beliefs toward mathematics.

2.4.3 Parental role and gender stereotypes

Several learners experience a lack of parental support as another challenge in the understanding and learning of mathematics (Department of Education, 2010). According to Andamon and Tan (2018), mathematics learning and a sense of mathematical identity are particularly influenced by learners' involvement in the school, the learning environment at home and the community. This means that while it is accepted that teachers make a significant difference in class, it is also recognised that parents have their own expertise and unique knowledge about their children. Hence, parental involvement is crucial for all the grades in mathematics learning. Rikso (2015) states that the negative attitudes of parents could also contribute to the learners' views regarding mathematics. For example, if parents always emphasise the difficulty of mathematics and fail to keep their children motivated, this may cause learners to develop the same negative perceptions about mathematics.

Farjadpour and Fresneda-Portillo (2019) believe that siblings can influence each other's performance in mathematics either positively or negatively. In line with this, Đurišić and Bunijevac (2017) argue that from birth, parents are engaged in the informal teaching and learning of their children, thus playing a major role in the educational success of the learners. As a result, children acquire skills such as language learning, early number conceptualisation and mathematical abilities from parents prior to participation in formal educational settings. Despite this, the school and the home do not always operate in harmony with each other. Importantly, an increasing pool of research evidence demonstrates that effective partnerships between parents and schools can result in better outcomes for learners in mathematics (Dietze & Kashin, 2016; Durisic & Bunijevac, 2017; Povey *et al.*, 2016). Therefore, parents can assist learners by discussing how to solve a problem and where possible, by providing learning materials related to the subject (e.g., buy them mathematics sets, workbooks as well as make copies of previous question papers) (Sonnenschein *et al.*, 2022). In addition, parents' support was of particular importance, especially during the COVID-19 pandemic when they were required to help their children at home.

Parents' educational background, socioeconomic status, and motivation can influence the role they play in their children's learning (Ayob & Yasin, 2017). Parental gender stereotypes can challenge the way their children learn mathematics, as parents often

expect that their daughters will struggle more with mathematics than their sons (Casad *et al*., 2015). Consequently, boys develop higher competencies in male-typed domains such as mathematics, while girls develop higher competencies and values in domains such as language and arts (Hine *et al*., 2018). Therefore, Andamon and Tan (2018) denote that such gender differences perpetuated at home in mathematics competence beliefs and values, may lead to gender differences in mathematics achievement in the long term.

2.4.4 Learning and teaching support materials (LTSM)

Cheng (2016) asserts that the teaching and learning support materials may either be not enough for all learners or may not be suitable for the content to be learnt. For instance, if there are not enough textbooks for all learners, they must share, which makes it difficult for other learners to have access to the textbooks when at home. Sometimes the textbooks available to the learners do not have appropriate learning content, syllabus outcomes are not addressed, and no clear explanations of essential mathematics principles are provided or given (Ahmadi & Derakhshan, 2016). This can make it difficult for learners to study on their own while at home (Winne, 2016). As a result, teachers are responsible for creating teaching and learning aids for their learners that are based on their experience, related to the learning content, and appropriate for the topic (Kumar, 2017). The teacher may encourage learners to make use of free and ready materials from their environment such as a fish or jam tin to calculate the area, circumference, and volume of a cylinder. Learners can also use chalk boxes and other boxes in rectangular and square dimensions to calculate the volume of a cube and a cuboid. In addition, learners can also use local newspapers such as The Namibian and Namibian Sun to interpret the statistical graphs on the increase of COVID-19 cases, vaccinations as well as deaths. Figure 2.4 indicates the number of cases, recoveries, and deaths of the Khomas Region and Windhoek (a highly populated city), compared to the total rates of infection, recoveries, and deaths in the whole of Namibia (2021).

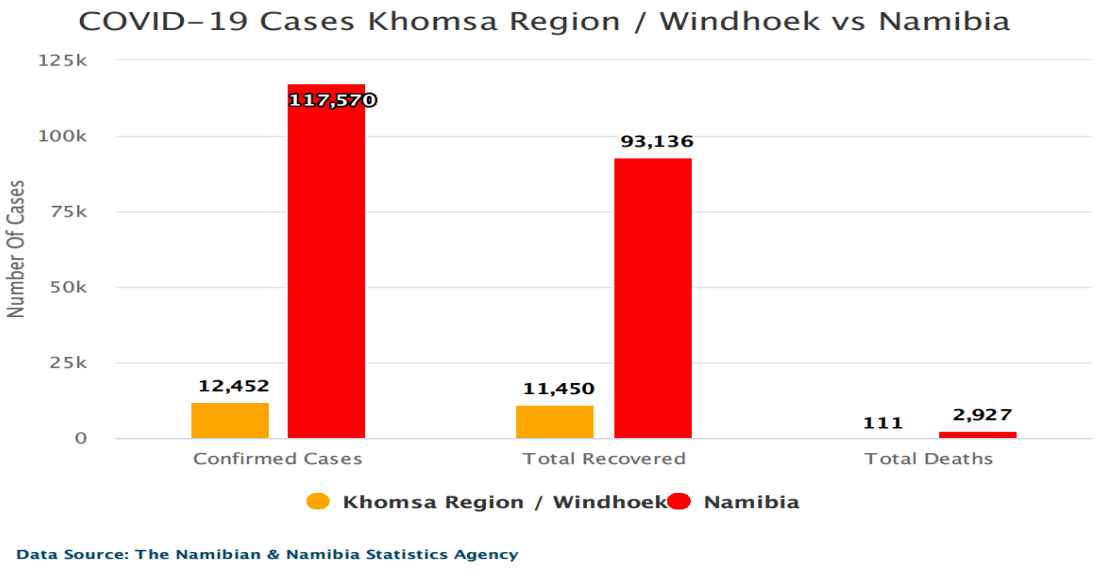


Figure 2.4: COVID-19 cases, recovery, and death in Khomas Region vs the entire country (Namibia)

2.4.5 Teachers' knowledge, skills and beliefs

A lack of teachers' abilities and support in the teaching of mathematics poses a challenge in the understanding thereof (Department of Education, 2014). Teachers who have received training to teach mathematics find it difficult, since they do not have the required mathematical knowledge and skills to guide the learners effectively. In addition, some teachers do not have pedagogical content knowledge (PCK) for effective teaching of the subject. PCK involves subject-specific content and relevant teaching and learning approaches to facilitate learners in developing essential subject-related skills for the future (Ayob & Yasin, 2017; Özdemir & Soylu, 2017). In other words, PCK is the result of the integration between the understanding of teaching material (subject matter or content knowledge) and understanding the way of education (pedagogical knowledge) that the teacher must possess to guide the learning process effectively (Maryono *et al.*, 2017). Ilyas (2015) elaborates and describes PCK as the type of knowledge that integrates content knowledge and pedagogical knowledge to understand how a topic is organised, presented, and adapted with a particular context in mind. Therefore, PCK is a way of knowing how to present the topic effectively to promote the understanding and learning thereof and to reduce misconceptions and problems that learners may experience.

Berry *et al.* (2016) pointed out that one of the components that lies at the heart of PCK is the knowledge of useful forms of presenting and formulating specific subject matter. Hence, if a teacher lacks PCK, he or she is not capable of preparing a good lesson and provide for relevant classroom activities as required. In addition, learners will not necessarily grasp the mathematical concepts, and this can lead to poor performance. Norton (2019) emphasises that teachers' mathematical PCK has become a key area of focus for the improvement of learners' academic outcomes. This is in harmony with Gess-Newsome (2013) who suggests that mathematics teachers' content knowledge significantly and positively predicts student achievement. Teachers' PCK can enhance the teaching and learning of mathematics, and this can boost up learners' performance. Teachers with well-developed PCK should be capable of implementing different ways of learning, for example predicting certain misconceptions, choosing relevant teaching and learning strategies, and using multiple ways of presenting the concepts, to enhance meaningful learning (Moh'd *et al.*, 2021). According to Cueto *et al.* (2017), teachers with well-established PCK are likely to have learners with higher scores in mathematics. Mathematics teachers with a well-developed PCK will be more suitable for effectively transferring their mathematics knowledge and skills to the learners (Callingham *et al.*, 2016).

The teachers' knowledge, skills, and beliefs can also determine the way in which learners learn mathematics (Ali & Jameel, 2016). This is in harmony with Sanchal and Sharma (2017), who state that what the teachers believe, how they behave as well as the teaching strategies they use have a major influence on the learners' attitudes toward learning mathematics. For example, if teachers believe that mathematics is only for intelligent learners and that some learners are unable to understand mathematics, such teachers may limit the support they render to such learners. Winne (2016) says the amount of confidence and support the teacher gives to the learners is influential to the way they learn and how they behave toward mathematics. Ransome *et al.* (2016) assert that the teachers' beliefs, feelings, and attitudes towards mathematics can be resembled by learners' feelings and expectations, and the perceived usefulness of mathematics achievement. The enthusiasm of the teacher can strongly convince learners that he or she is competent in teaching mathematics and this may result in a positive attitude towards learning mathematics.

For the past years, Namibia experienced a crisis with a lack of qualified teachers. Therefore, the Department of Education was forced to employ under-qualified teachers especially at the Junior and Senior Primary Phases as well as the Junior Secondary Phases. This crisis had a negative impact, not just on the way learners learned mathematics, but also on their performance. The Ministry of Education, in partnership with different higher education institutions in the country offers distance-learning programmes for different courses, diplomas and degrees in education to address the situation of unqualified teachers. In addition, bursaries are offered to people to pursue their higher education in mathematics. In the Omusati region, workshops are frequently offered to equip the mathematics teachers with new teaching approaches and address the poor performance of mathematics learners.

2.4.6 Teaching and learning strategies

Teaching and learning strategies refer to methods used to assist learners in understanding the desired course content and enable them to develop achievable goals in the future (Cheng, 2016). This is supported by Tachie (2019), who refers to teaching and learning strategies as the ways, behaviours or actions that teachers and learners use to teach and learn mathematics and solve mathematical problems in a more successful and enjoyable way. Therefore, learners will clearly understand beforehand what they are going to learn from the problem if they are able to set goals for themselves (Mahajan & Singh, 2017). This means that appropriate teaching and learning strategies must be selected to enhance the effective learning of mathematics.

Consequently, Sinay and Nahornick (2016) state that effective teaching and learning strategies should lead to teaching for conceptual understanding, teaching reasoning skills, and promoting problem-solving in mathematics. They further claim that appropriate strategies should promote teaching and learning with technology, develop mathematics skills and reduce mathematics anxiety. Similarly, teaching and learning strategies should assist in building positive attitudes toward mathematics and support conceptual thinking. Suitable teaching strategies such as the cooperative learning

strategy or problem-based learning can promote the learning of subjects (Klang *et al.,* 2020; Wegner *et al.*, 2013).

However, some teachers still teach mathematics using direct teaching strategies such as teacher-centred rather than learner-centred approaches (Gomez-Chacon, 2017). These two types of approaches are very distinctive in terms of the roles of the teacher and learners, the types of instruction and activities as well as the usage of materials (Markina & Mollá, 2022). Teacher-centred approaches have a negative effect on the way in which learners learn, since they are not actively involved in different classroom activities, unlike in learner-centred approaches (Gomez-Chacon, 2017). Cheng (2016) argues that a learner-centred approach is when the learners are the target of the learning process, actively involved in class, and do most of the work and talking to achieve a common academic goal. Thus, it is seen that learners who are actively involved in their learning are more motivated and develop essential skills such as critical thinking and problem solving, rather than learning in a teacher-centred approach (Justino & Rafael, 2021). As a result, learners are not passive recipients of information, but rather active agents engaging in constructing their own knowledge (Ayob & Yasin, 2017). Therefore, in a learner-centred classroom, the learner is encouraged to think beyond what is presented, and further explore the concepts, new ways of thinking and solving problems. In the following example a Grade 7 problem is formulated using a teacher-centred approach and a learner-centred approach (Table 2.1).

Table 2.1: Teacher-centred and learner-centred approach to mathematics

Teacher-centred approach
Anna bought school shoes for N$300.00. When she got home, she realised that they do not fit therefore she sells the shoes to a friend for N$450.00. Calculate the percentage profit she made. First, the teacher gives an example on the board where learners follow to use the same strategy to solve the next problem. Learners have no chance to think, but rather memorise the method. Such knowledge acquired by learners is transferred from the source (teacher) to the recipients (learners) and can be forgotten.
Learner-centred approach
Anna bought school shoes for N$300.00. When she got home, she realised that they do not fit therefore she sells the shoes to a friend for N$450.00. The teacher will ask probing questions to encourage the use of pre-knowledge as well as thinking ability as follows: a) Considering the cost price and selling price, did Anna make profit or loss? b) How much profit or loss did she make? c) Express the profit or loss made as a percentage. Here the teacher requests learners to decide on the strategy they can use to execute the task, then give feedback to the class on how they went about solving the problem. Learners can discuss the questions because such questions encourage learners' thinking and help them to form part of the discussion and contribute to the solution. Knowledge is constructed by learners themselves and it can be easily remembered.

Unfortunately, many teachers do not have knowledge about learner-centred approaches and how to use these in the mathematics classroom. As a result, they prefer using teacher-centred strategies. Table 2.2 shows the application of teacher-centred approaches versus learner-centred approaches or strategies in mathematics.

Table 2.2 Teacher-centred vs learner-centred teaching and learning strategies in mathematics

Strategy	Application in mathematics	Reference
1. Teacher-centred strategy		
Direct teaching	The teacher explains what types of numbers are and learners memorise it without understanding.	Emaliana (2017)
Lecture	The teacher lists the prime numbers so that learners memorise it without understanding.	Afurobi *et al.* (2015)
Recitation	Learners are provided with a table of multiples from one to 12 and keep repeating them to memorise all of them.	Wibowo *et al*. (2018)
Demonstration	The teacher demonstrates how to find the missing dimension when area and sufficient information about a cuboid are given, and learners memorise using the same route as the teacher.	Basheer *et al.* (2016)
Drill and practice	The teacher explains the formula for calculating the area of a rectangle and most of the learners memorise the formula only.	Rathakrishnan *et al*. (2018)
2. Learner-centred strategy		
Problem-based learning	A problem with no fixed solution is assigned to the learners in groups. They do research and present their findings. For example, learners can collect empty boxes and tins to calculate the volume of these solids, learners can determine the number of tins that can fit in the box using the volume of the box or determine the height or radius of the tin by measuring the dimensions of a box and calculate its volume.	Heuchemer *et al.* (2020)
Cooperative learning	Learners work together in small groups and challenge problems, e.g. to explore the properties of 3D shapes. They are responsible for and learn from each other. Each member contributes to the group.	Nagaraju (2013). Johnson & Johnson (2018)
Inquiry-based learning	The teacher uses questions, problems, or scenarios to assist learners through individual thought and investigation. Learners may pose questions to the peers presenting. How did you find that? Or why do you think so? To learn angle properties, after discovering that the opposite angles in a parallelogram are equal, the teacher or learners may ask peers why or how they found the answer.	Gholam (2019)
Project-based learning	Learners may select a topic in mathematics, learners can make their own containers of different capacity values from	Serin (2019)

	various recycle materials then present to the class to show to convert between specific metric units of capacity and determine suitable everyday life items that can be handled in each container. Project-based learning involves projects where learners work together to solve the problem and develop essential knowledge and skills	
Gamification	Teacher or learner designs and brings tools and elements to class to learn about a topic. Colour blocks can be used to learn about area, volume, or sequences. The competition can be between learners or between learners and the teacher where each part is awarded points according to certain requirements	Sánchez *et al*. (2020)
Play-based learning	Learners prepare what they are going to do during the lesson and present it in the form of a play. To learn about money, finance, and rates learners can make a 'shop' with items of different mass and capacity so that others can buy the preferred items by determining the ones that are best value for money. Learners can decide whether they should buy 500 g of washing powder for N$15 or 2 kg for N$50.	Kinasih & Hardiani (2020)
Question and answering	Guiding and probing questions are used to promote HOTs. For example, learners plan and present a lesson on algebraic equations to solve an equation of $t + 7 = 6$. During the presentation of their solution of $t = -1$, others can pose questions such as how and why the answer is –1. Further explanations can lead to a deeper understanding.	Kojo *et al.* (2018)
Critical discussion	Learners can answer some of the questions during a discussion. 'Have we found all the possibilities?' 'How do we know?' 'Have you thought of another way this could be done?' 'Do you think we found the best solution?' In a discussion session where learners are learning the properties of 3D shapes, specifically a sphere, how many faces, vertices, and edges can you identify? Learners can answer questions such as 'Have we found all possibilities?' 'How do we know?'	
Guided discovery	The teacher assigns a task such as 'Find the total surface area (TSA) of a cuboid', the teacher guides learners by asking relevant questions until learners find a solution.	Muhali *et al.* (2021)
Self-assessment	The teacher designs a rubric with the goals for learners to assess themselves on a certain topic. as a means of enhance their own learning.	Barana *et al*. (2022)

2.5 The learning of mathematics

This section elaborates on the models of learning and the relevance thereof in mathematics. The roles of metacognition as well as motivation are also discussed.

2.5.1 Frameworks, models, and taxonomies of learning

Several frameworks and taxonomies exist that can be used to outline the cognitive demands regarding the learning of mathematics. For example, Anderson's adaptive control of thought model (ACT) is a theory of the mechanisms that outlines cognitive abilities (Ritter *et al.*, 2017).

Another framework adopted for this study, is Bloom's revised taxonomy (BRT) (Sobral, 2021). Based on the BRT, learners need to demonstrate the six levels of cognitive thinking beginning with the lowest level and progressing to the highest level. These levels are presented in Table 2.3 below.

Table 2.3: Bloom's revised taxonomy for mathematics (as adapted from Ekren and Keskin, 2017)

Level	Descriptive verbs for each level	Grade 7 topic	Examples of tasks
Remember	Recall information, recognise, list, describe, retrieve, name, find	Measurement capacity (Senior Phase syllabus)	Identify the units from the list for mass, capacity, volume, and length.
Understand	Explain ideas or concepts, summarise, and interpret	Measurement capacity (SP syllabus)	Convert units of capacity and volume by calculation.
Apply	Use information in another context, implement, and execute	Measurement capacity and volume (SP syllabus)	Convert units of capacity and volume using empty bottles, cans, and other items that can hold a liquid. Learners should discover the number of 30 ml that can fit into $30\ cm^3$ etc.
Analyse	Break a problem into parts to explore understanding thereof	Measurement -volume (SP syllabus)	Collect empty boxes and tins to calculate the volume of these solids or learners can determine the number of tins that can fit in the box using the volume of the box or determine the height or radius of the tin by measuring the dimensions of a box and calculate its volume.
Evaluate	Justify a decision, critique, and judge	Measurement -time (SP syllabus)	Learners compare various time zones in the world and explain their decision.
Create	Create new ideas or ways of viewing things, design, and construct new knowledge	Measurement -capacity (SP syllabus)	Learners use various materials to come up with containers of different capacity, then do the conversion between units of capacity using the containers they made to support their solutions.

The curriculum emphasises the level of learners' abilities to be taken into consideration as well as the learner-centred approach as a teaching and learning method (National Institute for Educational Development, 2014). Several abilities are outlined in most taxonomies. For instance, teachers are expected to provide learning experiences that are appropriate to the learning phase and actively engage learners in the development of mathematical knowledge by exploring, discussing, describing and demonstrating (Mason, 2017). However, the author decided to focus on the Bloom's revised

taxonomy since the Namibian curriculum for Grade 7 emphasises this taxonomy in terms of the curriculum objectives as well as the competencies and specific objectives (Mathematics Syllabus Grades 4–7 NIED, 2014).

2.5.2 Approaches or steps to solve mathematics problems

A mathematical problem-solving strategy can help learners to develop the skill to solve real-life problems by enhancing their thinking ability (Gurat, 2018; Guzman, 2018). Sepriyanti *et al*. (2020) relate to four steps to mathematical problem-solving which are called the Polya problem-solving steps and they are: understanding the problems, devising a plan, carry out the plan, and checking answers. These are steps that learners need to follow while solving problems in mathematics and be confident in the solutions obtained.

The teaching and learning of mathematics must promote HOT. HOT can be conceptualised as a complex way of thinking that often generates multiple solutions (Sumitra *et al*., 2020). The Bloom's revised taxonomy views analysis, evaluation, and creation as examples of HOTS. Such a mode of thinking involves uncertainty, the application of multiple criteria as well as reflection and self-regulation (Sumitra *et al*., 2020). In other words, HOT requires learners to use information and ideas in ways that transform their meaning. For example, transformation occurs when learners combine facts and ideas to synthesise, generalise, explain, hypothesise, or arrive at some conclusion or interpretation (Mason, 2017). In Grade 7, the mathematics assessment component (as referred to assessment objective B and C in the Namibian Senior Primary Mathematics Syllabus) that promotes HOT are the following: analysis, abstraction, synthesis, and presentation (Department of Education NIED, 2015). Learners in Grade 7 need to apply analysis while learning algebraic expressions, and learners are expected to represent arithmetic operations in algebraic notation. For example, John thinks of a number, he adds two to a number and doubles the result. Let the number be x, represent the operation in algebraic notation. Figure 2.5 shows examples of HOT in Grade 7 mathematics (the arrow indicates the direction of HOT)

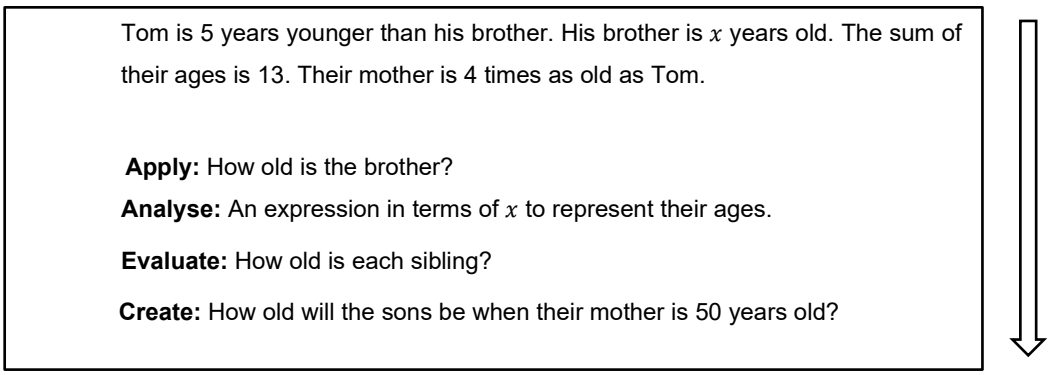


Figure 2.5 Examples of HOT in Grade 7 mathematics

2.5.2.1 Reasons why learners struggle to develop HOT

Learners experience challenges in developing HOTS and difficulties in addressing mathematical problems, due to several reasons (Hasan, 2019). In harmony with that, Bhattacharya and Mohalik (2021) regard teacher factors that are influential toward the development of learners' HOTS as the following:

- teachers' knowledge about HOTS;
- teachers' self-efficacy;
- teachers' teaching and learning strategies; and
- the teachers' evaluation process.

Furthermore, Gafoor and Kurukkan (2015) identify attitudes towards mathematics, negative emotions associated with mathematics, personal beliefs, and their experiences with mathematics as factors causing learners to struggle and consequently perform poorly.

Teacher-centred approach (See 2.4.5). Way (2014) lists some reasons why learners struggle to develop HOT, namely the teaching approach used does not allow learners to express themselves, a lack of facilitation by the teacher, and a failure to stimulate the learners' thinking ability (See 2.4.5). Therefore, teachers need to ask questions that encourage learners to use HOTS in mathematics. For example, a Grade 7 mathematics question that involves HOT is as follows: There are chickens and calves in the farmer's paddock. If there are 24 legs altogether, how many of each animal could be there? Some teachers believe that when transferring knowledge, there is no need to consider learners' existing knowledge or allow them to create new knowledge used in different circumstances which strengthens the foundation for learning

(Geofrey, 2021). Therefore, according to Firdaus *et al*. (2015), learning must start from the simple content to the complex and learners are expected to use their prior knowledge to make sense of more complex or challenging problems that require critical thinking. Some learners struggle to develop HOT simply because some teachers' knowledge about HOT is still insufficient (Afifah & Retnawati, 2019). This has an influence impact on how the teachers teach HOTS to learners.

Material usage and learner guidance: Another reason why learners struggle with the development of HOTS is that some teachers fail to integrate certain important features in the teaching and learning process (such as concepts, schemas, metaphors, visualisation and inference) and lack clear explanations, and the use of such materials (Hasan, 2019). Teachers fail to plan tasks that help learners to make a connection between the task and the real-life problem. As a result, such learners will not necessarily apply HOTS. Ortiz (2016) and Afifah and Retnawati (2019) claim that teachers experience a challenge in developing and delivering HOTS based on learning materials, which can result in learners using low order thinking skills resulting in learners struggling with challenging tasks. For example: when learners are required to add or subtract, a teacher may present the learning activity as [15 + 10 – 8]. Such an activity does not require a learner to apply HOTS. Rather, the activity must promote the application of skills such as analyse, evaluate, and create. The activity must encourage the use of reasoning, creative, and critical thinking (Birgili, 2015) to enable learners to develop HOTS (Ariska *et al*., 2020).

2.5.2.2 Role of the teacher in fostering HOTS

Teachers should help learners to develop questioning skills that stimulate deep thinking and encourage critical thinking to assist learners in developing HOTS (Chinedu & Kamin, 2015). For example, the teacher needs to prepare problems that encourage learners to apply the higher levels of thinking. Unfortunately, problems that demand the excursion of knowledge and existing skills, do not always promote a learner to think critically. White *et al.* (2016) suggest that teachers use representations to engage learners in mathematical thinking by encouraging them to justify and use those representations to scaffold their thinking processes. In the recommendation of

Chinedu and Kamin (2015), they suggest teaching for the development of HOTS as follows:

1) Support learners to keep track of their thinking and engage in a purposive and conscious reflection, and evaluation of their thinking;

2) Critically engage learners in analysis, evaluation, and creation of something new and insightful;

3) Employ instructional teaching methods such as problem-based learning to provide opportunities for group thinking and learning; and

4) Engage learners in brainstorming activities to support the development of ideas and solution generation.

Tanujaya *et al.* (2017) concur that teachers make use of open-ended questions in the classroom to develop HOT in learners. In addition, Gupta and Mishra (2021) assert that teachers need to be proficient in the assessment of HOTS in mathematics in order to help learners develop these skills. In harmony with this, Gupta and Mishra (2021) urge teachers to use standard-based assessments to help foster the development of HOTS, as these assessments include open-ended items, teach learners to give detailed answers, and motivate them to draw conclusions. Consequently, all teachers need to understand HOTS, otherwise learners might miss the development thereof in some grades. Teachers who know how to cultivate the thinking habits of learners and provide the opportunity to challenge them, will enrich the learners with knowledge and the development of HOT (Cope, 2014).

Cheng (2016) argues that in helping learners to become constructors of knowledge, the teacher's main task is to create activities or learning environments that allow learners opportunities to engage in HOT. This is in harmony with Retnawati *et al*. (2018) who mention that the outcome of learners' HOTS could be achieved by the active role of teachers in planning, implementing, and evaluating HOTS-oriented learning. Teachers should not assess the product (answer), but rather the process (how to reach the answer).

2.5.2.3 Learners' development of HOT in mathematics

Strategies to develop HOTS are the following (Chinedu & Kamin, 2015):

1) understand learners' own strengths and weaknesses.
2) discuss a particular concept and understand how such a concept differs from other concepts. In doing so, learners can develop their analytical ability, which is an essential component of HOT.
3) learn how to categorise concepts according to concrete, abstract, verbal, non-verbal or a process. In this way, learners develop the skill of aligning their thinking that goes beyond mere memorising.
4) move from concrete to abstract thinking. If a learner is able to state an abstract concept in terms of everyday practical applications, then he or she understands and will be able to make inferences from what has been learnt to solve new problems.
5) learners develop HOTS through drawing useful conclusions as based on evidence or facts. When learners connect concepts, it helps them to develop the ability to make logical conclusions upon examining the presented information; and
6) when learners carry out brainstorming, it allows for the development of possible solutions and a higher level of thinking.

These strategies are supported by Gupta and Mishra (2021), who assert several ways to develop HOTS among learners, like conducting brainstorming sessions, encouraging questioning, emphasising project-based, and activity-based learning. Lee (2021) emphasises that reflective thinking is a mental activity from which mathematical understanding emerges. Lee (2021) further states that, through multiple solutions, learners are encouraged to solve problems in different ways, reflect on their thinking, and make connections among different solutions with the aim of enhancing conceptual understanding.

2.5.2.4 Benefits of developing HOT in learners

The importance of HOT in learning mathematics is that it can improve the critical thinking skills of learners as well as their ability to solve problems (Sa'dijah *et al.*, 2021). According to Widana (2017), learners who develop HOTS can transfer learned concepts and skills to different and new contexts. Such learners can look for connections between a variety of information, judge ideas, and apply information to solve challenging problems. As a result, if learners think critically, they demonstrate advancement of their educational goals (Chinedu & Kamin, 2015). In addition, HOT

can enhance learners' academic achievement which ought to result in better performance in mathematics as a result of using skills such as analysis, evaluation, and creation (Chinedu & Kamin, 2015; Tanujaya *et al*. 2017). In harmony with that, Purnama and Nurdianingsih (2019) assert that learners developing HOTS will be more successful, as it helps them to think critically and innovatively, and become independent problem-solvers in the future compared to those having lower order thinking skills (LOTS) (Chun & Yen, 2019). This is supported by Singh and Marappan (2020), who recognise the development of skills such as critical thinking, collaboration learning, creativity and innovation, problem-solving, independence, and the willingness to try new things because of HOT.

HOT can also enhance learners' questioning skills, reasoning skills, and communication skills. Because of HOTS, learners can connect the small pieces of information into one entity and be able to transfer acquired knowledge and skills to a new situation (Widana, 2017).

2.5.3 The role of metacognition in mathematics

Metacognition is a skill that involves an awareness of one's thinking and having the ability to regulate one's cognitive processes (Lai, 2017). Winne (2016) views metacognition as an individual's knowledge of his or her own thinking as well as knowledge about the nature of cognitive tasks, and of strategies for addressing such tasks. Moreover, metacognition also includes control, monitoring, and self-regulation of one's own cognitive activities (Alzahrani, 2017).

Metacognition has two constituent parts as mentioned by Flavell (1979) and Cheng (2016). The first part is metacognitive knowledge, which refers to the *declarative knowledge* regarding the interplay between personal and task characteristics, and relevant strategies in learning performance (knowledge about cognition). Metacognitive knowledge comprises explicit, conscious, and factual knowledge about the person, task, and strategy. The second part of metacognition is metacognitive skills, which refers to the *procedural knowledge* that is required for the actual regulation of, and control over one's learning performance (planning, monitoring, and evaluation) (Cheng, 2016). Metacognitive monitoring and evaluation facilitate learners to avoid or correct errors during the mathematics problem-solving process, detect the

progression made and compare the answer against the problem statement (Johnson, 2014). According to Lai (2017), monitoring refers to the ability to be aware of and guide one's overall participation, and performance carefully. On the other hand, evaluation refers to the ability to assess specific processes and products involved in learning (Tachie, 2019).

Metacognition is essential in succeeding in mathematics (Alzahrani, 2017). Cheng (2016) states that metacognitive activities in mathematics focus on understanding the problem, establishing the relationship between prior and new knowledge, using appropriate strategies to solve a problem, and reflecting on the processes and solution(s). Hence, learner success in solving the problem depends on the consciousness of thinking. According to Gomez-Chacon (2017), problem solving is seen as a process of finding ways to overcome problems. Therefore, the success of solving mathematics problems is not only supported by cognitive processes, but also by the learners' ability to manage their thinking in solving such problems. In addition, Gomez-Chacon (2017) argues that metacognition emphasises the importance of controlling the cognitive mind during problem solving and assist learners in understanding mathematics concepts. Consequently, Winne (2016) argues that mathematics teachers can incorporate learners' active reflection throughout the learning process.

Effective metacognitive strategies include the following as mentioned by Way (2014). First, questioning by the teacher – "What are you working on?", "Why are you working on it?", and "How does it help you?" are questions that the mathematics teacher can ask while learners are solving problems. Second, self-assessment – learners reflect on their learning and determine how well they have learned something. Third, selecting strategies – learners decide which learning strategies are appropriate for a given mathematical task. The fourth strategy involves using directed thinking – learners choose to follow a specific line of thinking. Fifth, using discourse – learners discuss ideas with each other and with the mathematics teacher. Then, critiquing – learners provide feedback to other learners about their work in a positive, constructive manner. From there: revising – learners revise their work after they have received feedback from the mathematics teacher. The last one is evaluating work – learners determine their strengths and weaknesses by reviewing their work. Table 2.4 gives examples of

metacognitive strategies in mathematics as based on Winne (2016), whereas Figure 2.6 presents an example of metacognitive questioning in Grade 7 mathematics.

Table 2.4. Examples of metacognitive questioning in mathematics

Type of questions	Examples
1. Why and how questions	How do you find a prime factor of a given number? Why is it a prime factor?
2. Self-reflection	Identify learning needs
3. Appropriate learning strategy	Set learning goals, explore
4. Directed thinking	Questions of why and how
5. Discourse	Discussion, effective questioning, and solve a problem
6. Critiquing	Learners do presentations of their work
7. Revising	Learners go through the feedback to avoid future repetition of the same error.
8. Evaluation	Learners think of their strength, how they can work on their weakness, and improve.

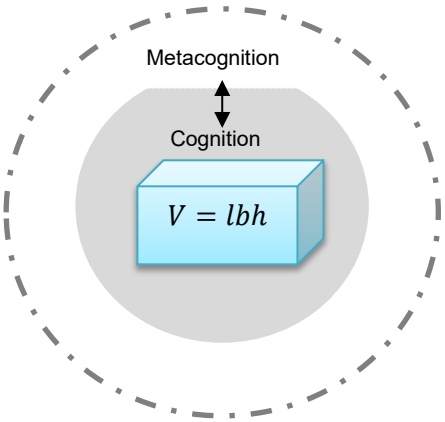


Figure 2.6: Metacognitive thinking in Grade 7 mathematics (volume of cuboid)

Grade 7 learners are expected to write the formula for calculating the volume of 3D solids such as a cube and cuboid in algebraic notation. The following is an example: A rectangular box has a length of y cm, the breadth is 3 times less than the length. Write down the formulae for calculating the volume of this box if its height is 4.5 cm^3.

2.5.4 Motivation to learn mathematics

Motivation refers to reasons that support a certain action or behaviours that are characterised by willingness and volition, meaning that it is the attribute that makes us to do or not to do something (Lai, 2017). Intrinsic and extrinsic motivation are distinguished. Legault (2020) contends that intrinsic motivation is the driving force of doing something for its own sake or to engage in behaviours that are inherently satisfying. On the other hand, extrinsic motivation refers to the pursuit of a goal or to perform an action that is fundamentally dependent on the achievement of an outcome that is separable from the action itself. Thus, motivation may be manifested in cognition, emotion and/or behaviour (Johnson, 2014). For example, a learner's motivation to get a good grade in mathematics may result in happiness (emotion), if he or she scores high marks in a test. It may also be manifested in studying for a mathematics test (behaviour) and in new learning (cognition) when studying for the test.

Fuqoha *et al.* (2018) mention that motivation for learning is influenced by various aspects. Some examples are the learning and teaching support materials (LTSM), teaching and learning strategies (e.g., problem-based learning (See Table 2.2)), previous achievements, and the learners' background on a specific topic or subject. Fuqoha *et al.* (2018) emphasise that appropriate LTSM can assist learners to improve their understanding and achievement because of an increase in the learning. For example, using the appropriate media (charts and work cards) may increase the achievement of mathematics learners.

In contrast, Schunk *et al.* (2014) stipulate that learners who are not motivated, do not usually put much effort into their learning. Such learners do not organise their learning resources (Okongo *et al*., 2015), nor do they monitor their level of understanding or seek for assistance when they are struggling with what is being taught, since they do not believe that they are capable of learning. Educators believe that motivation can

influence what learners learn, and how well they acquire skills, strategies, and behaviours that they learned before (Schunk *et al*., 2014). According to Lai (2017), motivating learners to be enthusiastically receptive is one of the most important aspects of mathematics teaching and a critical aspect that contributes to curriculum attainment. Hence, effective teachers mainly focus their attention on the least interested learners as well as the least motivated ones (Varghese & Aneesa, 2021).

2.6 Problem-based learning in mathematics

Teaching goes hand in hand with learning and during the teaching process suitable strategies should be employed for effective learning to take place. This section highlights problem-based learning as a strategy that can be used in the teaching and learning of mathematics.

Problem-based learning (PBL) is "an instructional learner-centred approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem" (Savery, 2015:7). Yew and Goh (2016) refer to PBL as a pedagogical approach that enables learners to learn from each other, while actively dealing with meaningful problems. According to Golightly (2014), PBL is a teaching and learning strategy that can help foster learners' self-directed learning skills and encourage them to take responsibility for their own learning. By applying PBL, learners may help one another towards the construction of knowledge, in-depth understanding, and the development of HOT as they work in groups (Mustaffa *et al*., 2015). In PBL the teacher is the facilitator of learning while the learners work together in groups to solve an open-ended problem (Leary *et al*., 2022).

2.6.1 The origin of problem-based learning

According to Servant (2016), PBL originated in the late 1960s at McMaster University in Canada in the Faculty of Medical Sciences where Dr Evans and his team started with the preparation of PBL in 1965 and the implementation was done in 1969 (Servant, 2019). PBL then came to Maastricht through an unplanned encounter at Michigan State University between Evans and the founding Dean of the Maastricht University (in the Netherlands) Faculty of Medicine, Dr Harmen Tiddens (Servant, 2019). Subsequently, PBL made its way from McMaster and Maastricht to two new university centres which were scheduled to open in Denmark (in 1972 in Roskilde and

the other in 1974 in Aalborg) (Hansen, 1997). Barrows made an outstanding contribution the PBL approach (De Graaff & Kolmos, 2007; Servant, 2016; 2018). Figure 2.7 shows the timeline of key events in PBL.

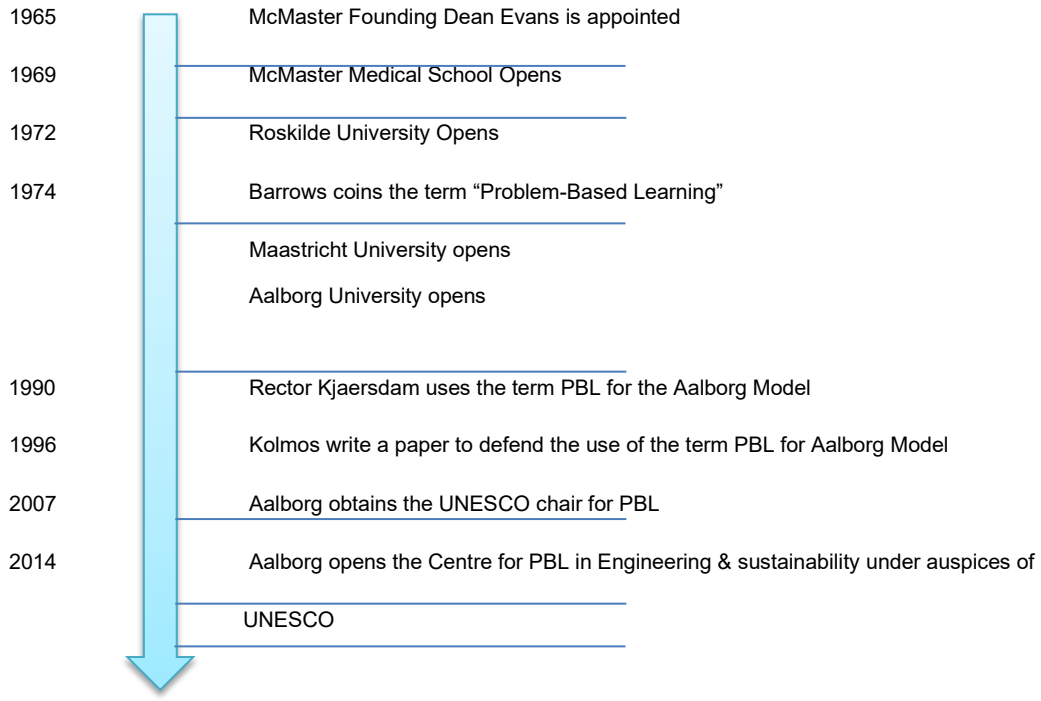


Figure 2.7: Timeline of key events in the development of PBL (Adapted from Servant, 2020)

The use of the PBL approach became widely spread in many parts of the world. In support of this, Savery (2015) states that the use of PBL expanded in elementary schools, middle schools, high schools, and universities worldwide.

2.6.2 A taxonomy of problem-based models

There are six PBL models mentioned by Hung (2011), namely lecturer-based with problem-based solving activities model, the case-based learning model, the project-based learning model, the anchored instruction model, the hybrid-PBL model (hPBL) and the pure PBL model, as presented in Figure 2.8.

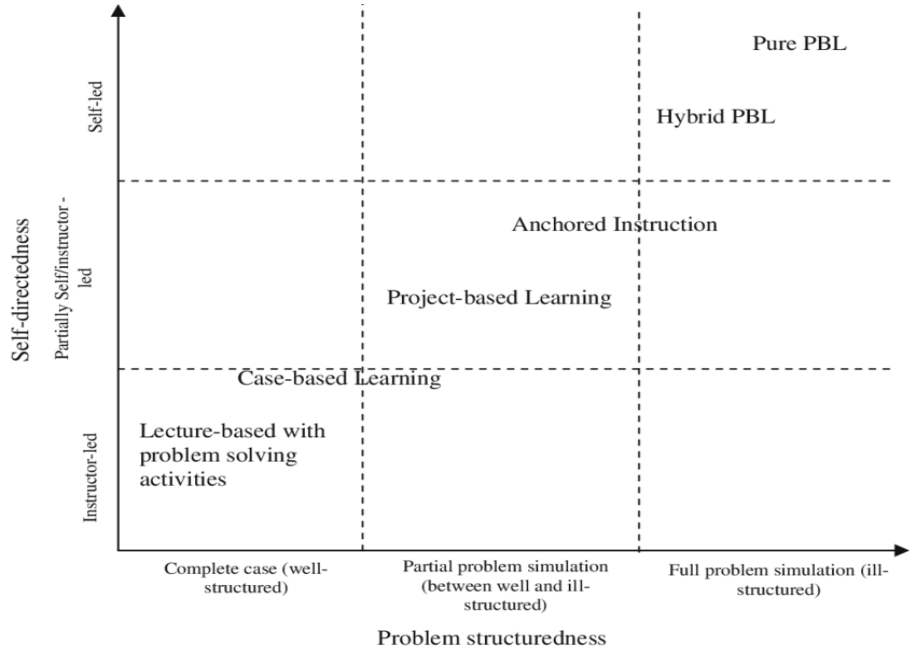


Figure 2.8: **Six representative PBL models in the PBL taxonomy of Barrow (Hung, 2011)**

Although several PBL models are used, the [1]PBL tasks that were applied in this study are comprised of some components, namely identification of learning needs, allocation of time for small group learner-centred sessions, group presentations and the methods of learners' assessment.

2.6.3 The problem in problem-based learning

PBL differs from problem-solving in such a way that problem-solving usually consists of finding solutions to conventional problems that contain a description of given facts (Cheng, 2016). In the PBL approach, the problem is usually a scenario, a challenging event in daily life or an ill-structured problem that that drives the learning activities and is addressed in small groups (Wijnia *et al.*, 2014). Such a problem may not have an

[1] Although *PBL tasks* were used in this study, they are not specifically associated with a specific PBL model, as outlined by Hung (2011).

obvious solution and there are several ways to solve such a problem (Srikan *et al*., 2021). In PBL, learners must suggest a tentative solution, and explain processes and underlying principles. This ought to promote their thinking ability (Cheng, 2016; Wijna, 2016).

For a problem to be considered a PBL problem, depends on the extent to which the problem is familiar to learners (prior knowledge), interests learners, promotes collaborative learning, and requires critical thinking, reasoning and consequently promote HOTS (Sockalingam *et al.*, 2012).

2.6.4 Problem-based learning steps

When learners engage in PBL tasks, there are certain steps that they must follow to solve the task. The “Seven Jump” approach was developed by Schmidt and Moust (2000:21) (Servant-Miklos, 2019). The steps are outlined in Table 2.5. (The application in Grade 7 mathematics is given in brackets).

Table 2.5: The adapted seven steps in PBL (Maurer & Neuhold, 2012)

Steps	Activity	Expectation in details
1	Clarification of terms and concepts	Ask for explanation of words or concepts that are not understood (e.g. terms and concepts: renovate or any other term learners do not understand)
2	Formulation of problem statement	What is the problem to be solved? Formulate the problem statement i.e., what is it about? Why is it relevant? (Formulation: calculate the rectangle, square, and the cost price)
3	Brainstorm ideas	Collection of ideas, potential explanations regarding the problem statement. The idea is that the group determines what they know (what questions can we answer) (e.g., What is the area of the floor? What is the area of a carpet? How many carpets can cover the floor?)
4	Categorising and structuring	Keywords are put into similar categories (according to question type: why, how, what consequences etc.) (e.g., categories are areas of each shape and the price)
5	Formulation of learning objectives	Formulate questions (e.g., "look for x") (e.g., how many carpets can cover the classroom floor? Objective: to be able to calculate the area of a rectangle, a carpet, and floor.)
6	Research for resources	Look for additional sources, read about the topic and prepare to the formulated learning objectives (dictionary, Grade 7 textbooks, and supportive resources).
7	Feedback	Learners report on how they addressed the learning objectives, compared results and exchanged ideas.
	Reflection and evaluation	Self-assessment of learning process (How did I do?), peer assessment, especially in roles of peer and presentation; Also, if there is time the group should evaluate their pre-discussion. (Did we ask the correct kind of questions? Should we change anything in our inquiry process?)

In this study, the researcher applied all the PBL steps as outlined in Table 2.3. Although some scholars (Zwaal & Otting, 2014) refer to eight or nine PBL steps (Mustajab *et a*l., 2020), the abovementioned seven steps were used in Grade 7 mathematics tasks. The seven steps are short and concise and therefore used because learners have not been introduced to PBL before. It is essential that learners understand all the terms used in tasks and that every group member evaluates information collected by their peers. Members make explanations during the collection of ideas. The group determines what knowledge they lack and formulate learning goals

accordingly (Xia, 2017). To gather relevant information, group members must do research and then prepare to report to the whole class (Markula & Aksela, 2022).

Some advantages of PBL are the following: it promotes the solving of open-ended problems, enhances independent learning, provides opportunities for group collaboration and the development of essential communication skills (Chang, 2022). Furthermore, PBL proves to be highly beneficial for learners, as it encourages their critical thinking and innovation (Samo *et al.,* 2017; Tadesse, 2022). In PBL, learners are encouraged to work as a group to find solutions to real-world problems, which enables them to use the acquired knowledge to solve such problems in the future (Guido, 2016; Siew *et al.*, 2017).

2.6.5 Group work and cooperation in problem-based learning

The development of learning in small groups has occurred because it promotes peer learning, as group members assist each other (Rajabzadeh *et al*., 2022). Furthermore, Rajabzadeh *et al*. (2022) point out that learners may grow in several important areas including knowledge development, thinking skills, social skills, and course satisfaction. Downton *et al.* (2017) further state that in PBL each group consists of members with several abilities and interests: the members meet, have critical discussions, and work together to solve a problem. Such a teaching and learning strategy helps learners to learn from each other as well as to cooperate with their peers.

Campos (2017) asserts that in PBL, problems are carefully constructed and presented to small groups who are responsible for discussing these problems and providing tentative explanations for the problem being described. The facilitator (teacher) is responsible for the distribution of tasks to be executed, while each group deals with specific responsibilities such as organisation, discussions, and reports (Downton *et al.*, 2017). The value of the solution obtained by each group is addressed during the group presentations, as they suggest solutions to the problem (Markula & Aksela, 2022).

Although working in groups fosters a better understanding of the content as learners receive frequent assistance from the members, learners may also face challenges

while working in a group (Ferdous & Karim, 2019). Davidson *et al.* (2014) identify some challenges as poor interpersonal skills. Group members may possess low self-monitoring skills and poor effective verbal communication. Eventually, the group work may explicate that an individual may suffer from the pressure to conform to the opinion of the majority. It may direct him or her to agree to an impractical decision or solution to avoid conflict (Davidson *et al.*, 2014). Conversely, Ferdous and Karim (2019) stress that the powerful individuals dominate in discussions, thereby influencing group decisions and sometimes group members exert pressure on others to accomplish the task.

Although some groups have challenges in working together, others may cooperate according to the social interdependence theory and principles of group cooperation (Johnson & Johnson, 2014). Cooperative learning is a teaching and learning strategy that makes use of small groups in such a way that learners work together to maximise their learning (Mahmoud, 2014). Cooperative learning requires learners to work in small groups, usually from two to five learners, to make individualised face-to-face interactions between members and reflect on executed tasks, and possibly, to determine whether the goal has been achieved (Buchs & Butera, 2015; Slavin, 2014). As based on the views of Johnson and Johnson (2014), the principles of cooperative learning are the following: positive interdependence, group and personal responsibility, promotive interaction, social group skills, and group processing.

Positive interdependence means that all learners in a group must understand what they need to do in any given activity and that they need to depend on each other to do it: either all of them will succeed, or they will fail to do so (Pablo & Vargas, 2018). Regarding group and personal responsibility, learners must have a clear idea of what is expected from them and be aware that his or her performance will be assessed by other members of the group (Buchs *et al*., 2021). This means that there are individual and collective accountabilities, and they have different roles in the group. Promotive interaction allows learners to support and encourage one another (Karmina *et al.,* 2021). Social group skills allow learners to learn how to work as a group, how to deal with discrepancies, and how to negotiate with each other to make decisions that benefit the whole group (Johnson & Johnson, 2014). Finally, during group processing,

the teacher must allow learners to assess their own work to identify problems encountered and find ways to minimise them in future (Pablo & Vargas, 2018).

However, cooperative problem-based learning (CPBL) is essential when learners work together on a PBL task (Mohd-Yusof *et al*., 2011; Samsuri *et al.*, 2017). CPBL outlines principles for group work where all learners are responsible for specific PBL activities to ensure that the group will succeed (Havenga & Swart, 2022; Samsuri *et al.,* 2017).

The group aims to engage in a real task, which requires contributions from all members rather than a single individual (Gillies, 2017). Hautemo (2017) stresses that the use of cooperative learning allows learners to work closely with one another and with the help from more knowledgeable individuals, they can potentially gain knowledge (Farzaneh & Nejadansari, 2014). Thus, cooperative learning requires teachers to introduce and structure both positive interdependence and individual responsibility or accountability (Baloche & Brody, 2015).

2.6.6 Problem-based tasks

According to Cheng (2016), a problem-based task is challenging and does not offer an immediate solution. For example, in mathematics a problem-based task might be that you must pack small cubical boxes (A) into another big box (B). Give the possible volume of the unoccupied space in the big box (B), if any. Each learner can find a possible solution. This means that a given problem might be problematic for many learners. For example, consider the question: “You have two-thirds of an apple to divide among your friends, what fraction each friend will receive? Prove your answer with a diagram”. This question can be problematic to some learners who do not understand how fractions can be divided by whole numbers. This could be a result of a language barrier during the transition grade or a lack of knowledge to answer mathematical problems. Seid (2017) states that learners who learn in their mother tongue (in the early grades) experience the disadvantages of transition to English instruction in a more severe way (See 2.4.1).

In problem-based tasks, the problems must be challenging, open-ended, and direct learning activities to develop HOT (Munroe, 2015). In other words, there could be 1)

more than one solution to the problem, 2) the solution is not obvious, and 3) solving a problem requires HOT (See 2.5.2). Accordingly, Lai (2017) asserts that problem-based tasks require learners to apply their current understanding and skills to new contexts. For example, learners entering Grade 7 must have prior knowledge of dividing fractions diagrammatically to address a problem that could be solved through division of a fraction by a whole number and reciprocal of the second fraction.

Lai (2017) argues that ill-structured problem-based tasks provide multiple opportunities for learners to engage in problem solving, ensuring that all learners have access to the same concepts. White *et al*. (2016) claim that when learners solve the problems in different ways, including drawing pictures, writing algorithms, and using manipulatives, they make connections between a variety of models that all accurately illustrate the underlying mathematical knowledge. For example, when learners are asked to calculate the area of a trapezium, (Figure 2.9 shows 2D shapes given that each square represent 1 cm^2), they can solve the problem in different ways. Some of them can draw squares to obtain the answer while others can use the formula to find a solution.

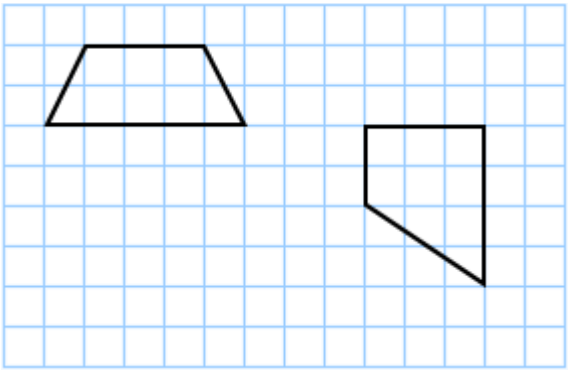


Figure 2.9: **Example of a mathematics problem**

Ali (2019) states that PBL tasks should require learners to engage in active learning and such tasks must meet the following principles such as independent learning, learning must happen in a group but not in isolation, and the teacher must play the role of the facilitator. Ali (2019) further states that all groups must participate equally, learners learn about motivation, group work, problem-solving, and engagement with

the task and the materials (such as data, photographs, articles) that can be used to solve the problem as a self-directed learning disposition.

Therefore, an ill-structured real-life problem that requires the application of prior knowledge and requires learners to contact multiple resources best describes a PBL task. Such planned tasks must promote metacognitive skills if these tasks are aimed to increase learners' self-directed learning (SDL) (See 2.7.3)

2.6.7 Planning problem-based tasks for mathematics

When planning mathematical problem-solving tasks, teachers are required to engage in all aspects of the planning process to assist learners (Ayyildiz & Tarhan, 2015). Planning does not only refer to the time teachers spend on preparing and designing activities for learners (Knellopoulou & Darra, 2018). During this process, teachers make decisions about a variety of teaching aspects such as clear problem formulation, relevant teaching and learning strategies, appropriate teaching and learning materials, and learners' level of understanding (learning pace) that will ultimately shape opportunities to learn (Sumitra *et al*., 2020).

During the planning of activities, teachers need to formulate appropriate questions that guide the teaching and learning events without giving away too much information, at the same time encouraging learners to take risks to participate in these activities (Suwartono & Oktavia, 2019). In addition, teachers need to anticipate possible misunderstandings to respond appropriately and assist learners in successfully solving the problem (Sumitra *et al*., 2020). Therefore, teachers need to plan for challenges that learners may experience and be prepared to assist, and guide them towards finding the solution.

Hew *et al.* (2016) point out different task requirements that a teacher should consider when planning to help learners to become self-directed. Such tasks must activate prior knowledge and identify gaps between what the learners already know, and the new learning goals (Twafik *et al*., 2021). Khodaei *et al.* (2022) suggest that explicit development of metacognitive skills such as metacognitive awareness of learners can increase self-confidence, motivation, and improve their level of self-directed learning

because metacognition skills are part of SDL. Kim *et al*. (2019) highlighted that PBL improves understanding of content knowledge and self-directed learning. For this reason, the PBL tasks should enhance learners' autonomy so that they can take the initiative in their learning process, which is a characteristic of SDL. Kim *et al*. (2019) further state that tasks should involve searching for relevant information using various resources, and through discussions with their (learners) small group to evaluate the usefulness of their collected information. In their small groups, members can determine whether additional information is needed to support or repeat the process until the group arrives at a refined problem solution. White *et al*. (2016) indicate that the use of representations (including physical, verbal, and written symbols) can have a positive effect on the teaching and learning of mathematics, by supporting the communication of mathematical ideas, the understanding of concepts, and the solving of problems.

The teacher identifies a topic and finds a problem related to that specific topic that he or she wants the learners to work on (learning objective). Plan the lesson where learners are going to start working on that problem. The teacher can divide learners in groups and present the problem to the class. Ensure that the learners are aware of what is expected from them. Table 2.6 shows a shortened plan for a lesson where PBL task is to be executed.

Table 2.6: A short lesson plan employing PBL

<table>
<tr><td colspan="2">Topic: Volume of a cuboid
Introduction: indicate the topic, expectation and test the pre-knowledge</td></tr>
<tr><td><u>Teacher's activities:</u>
-Enable learners to work in small groups.

-Facilitate learners group activities. Provide opportunities four group reflection and give feedback.</td><td><u>Learners' activities:</u>
-Learners discuss and clarify mathematics concepts, brainstorm ideas, and set clear learning objectives.
-Discuss the problem and search for relevant resources.
Critically discuss possible solutions, reflect on their thinking, and evaluate their activities
(Also see PBL steps, Table 2.5)</td></tr>
<tr><td colspan="2">Consolidation:
-Teacher and learners review the process to reinforce learning through questions and answers.
-Allow learners to reflect on their thinking.</td></tr>
<tr><td colspan="2">Conclusion
-Teacher sums up the learning content and highlights the learning objectives.</td></tr>
</table>

Learners learn with a little help of their teacher who is facilitating their work rather than spoon feeding them with the knowledge. Learners follow the PBL steps during the process of executing the task and this involves consulting different resources to gather necessary information and to solve the problem in the end. This problem requires learners to use different ways to get to the solution. Learners will need to report to the class about their findings and be ready to defend their information when questions arise from the class. At the end, learners must reflect about how the lesson went as part of the knowledge they gained.

2.7 Self-directed learning

Knowles (1975:18) refers to self-directed learning (SDL) as “a process in which individuals take the initiative, with or without the help of others, in diagnosing their learning needs, formulating learning goals, identifying human and material resources for learning, choosing and implementing appropriate learning strategies, and evaluating learning outcomes”. This is in harmony with Olivier and Wentworth (2021) who refer to SDL as a learner’s ability to manage his or her own learning process by perceiving oneself as the source of actions, decisions, and responsibilities towards one’s own lifelong learning. Hofmeyer (2016) regards self-directed learners as motivated by internal incentives such as the desire to achieve, the satisfaction of accomplishments and the need for specific knowledge. Self-directed learners can assess their learning needs by determining the gap between what they know and what they are learning, plan effectively, manage their time by setting priorities and give an evaluation of the literature resources to complete the task (Karataş *et al.,* 2021). Learners who are self-directed can learn independently and are able to critically evaluate their own success (Maphalala, 2021).

2.7.1 Characteristics of a self-directed learner

Characteristics of self-directed learning are best described by the kind of learners developed through SDL. Knowles (1975) identifies the following characteristics of self-directed learners such as they set clear goals for themselves, manage their learning in line with goals and plans, they monitor their learning process, and evaluate the outcomes thereof. Furthermore, Gündüz and Selvi (2016) as well as Lai (2017) describe self-directed learners as learners who are autonomous, self-motivated, persistent, independent, view problems as challenges, enjoy learning, are open to learning, curious, willing to learn, value learning, are self-controlled, and who take the initiative to learn.

Moreover, Mentz *et al.* (2019) recognise some other characteristics of self-directed learners such as a tendency to view problems as challenges, a strong desire to learn or change, the ability to organise their time and set an appropriate pace for learning, self-confident, and they can find joy in learning. Such learners prefer active participation, have knowledge of a variety of potential learning resources and the

ability to use them, and they can select and use relevant learning strategies. Singh (2016) states some of the characteristics such as they are intrinsically motivated, they possess metacognitive skills, they feel supported, and they experience improved performance.

Learners who are self-directed can control and regulate their thinking during the learning activities and learning experiences (Ayyildiz & Tarhan, 2015). Self-directed learners demonstrate a greater awareness of their responsibility by making learning meaningful to them (Cheng, 2016).

A self-directed learner is a good role model for learners who are weaker or less self-directed. Some learners may feel a great need to be guided by others in acquiring self-directed learning skills, attitudes and facilitating their learning process (Gündüz & Selvi, 2016). As weaker learners observe the self-directed learner carrying out their tasks effectively, less self-directed learners can take courage or imitate the self-directed one (Khalid *et al.*, 2020). As a result, self-directed learners help the weaker or less self-directed learners to become more self-directed too. Teachers can therefore spend more time with learners who need additional support. For example, in mathematics, a teacher may introduce a task to the whole class about calculating the area of composite figures (See Figure 2.10). Instead of spending the entire lesson giving attention to the whole class, he or she can focus more on the learners who need additional support. In fact, learners who are more self-directed can assist other learners.

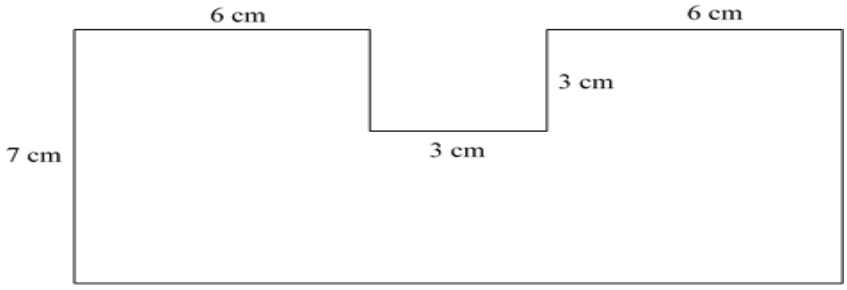


Figure 2.10: Calculate the area of a composite shape

The findings of Boyer *et al.* (2014) indicate SDL can lead to improved performance, confidence, ability, and self-efficacy. A self-directed learner must believe that he or she is in control of the learning process and that the outcomes of the events result

primarily from his or her own behaviour (Avsec & Savec, 2022). At the same time, the learner's belief about his or her capability in carrying out the tasks successfully contributes to the development of SDL skills.

2.7.2 Factors influencing self-directed learning

SDL is influenced by internal as well as external factors (Lai, 2017). Each factor is briefly outlined below.

2.7.2.1 Internal factors

Such factors comprise personal characteristics of the learner such as learner motivation, goal orientation, self-efficacy, and locus of control, which determine the way in which learners approach a task as well as their persistence and initiative in completing the task (Cheng, 2016). The personal characteristics, like the attitudes and motivation of the learner have an impact on the learner's SDL (Hammarlund *et al.*, 2015). The learner's attitude toward learning influences the development of self-directedness. In the same manner, motivation can affect the way in which the learner learns and the self-directedness of such a learner. Hammarlund *et al.* (2015) note that intrinsic motivation influences SDL (See 2.5.4).

A lack of motivation presents a challenge to SDL. Cheng (2016) contends that motivation is needed both to embark on SDL as well as to ensure that learners complete the process of attaining that skill. Boyer *et al*. (2014) further suggest that some learners rarely put effort into trying anything for which they do not get academic credit. This means that if the learners are poorly motivated the process of developing SDL is deterred. Khalid *et al.* (2020) report that one of the challenges for the development of SDL is self-motivation, which refers to a lack of internal motivation.

2.7.2.2 External factors

External factors include the support network, the learning environment or social settings, the role of the teacher, the curriculum, the availability of resources and usage (learning facilities), and the use of mathematical tasks (Leatemia *et al.*, 2016).

The learning context refers to the social and psychological environment of learning and these determinants of learning can affect a learner's achievement and attitudes, thus affecting the learner's SDL (Alli, 2019). Ramli *et al.* (2018) mention that academic environment in terms of learning facilities and academic atmosphere experienced by learners are factors that influence SDL. In addition, school management, classroom organisation, and the teaching qualities of the teacher have an influence on learners' behaviour. Classrooms must be designed in such a way that understanding is the primary goal of teaching (Breed, 2013). Gremmen *et al.* (2016) claim that classroom arrangements influence classroom climate and learners' relationships with each other, the learning process, and affect learners' attitudes towards school. This involves arranging seats in small groups to promote learners' cooperation, interaction and ensuring that the teacher can move through the classroom to access, engage with, and monitor learning and behaviour (Gremmen *et al.*, 2016; Poole *et al.*, 2019) more efficiently.

Learners' SDL is also affecting teachers and parents (Nyambe *et al.*, 2016). For example, if parents are positively involved in the school activities of their children, the mathematics performance of those children can be improved (Crowley, 2015) (See 2.4.2 and 2.4.3). Effective partnerships between parents and schools can result in better outcomes for learners in mathematics (Durisic & Bunijevac, 2017; Dietze & Kashin, 2016; Povey *et al.,* 2016). In the mathematics classroom, learners interact with their peers and teachers and these interactions influence their SDL ability (Lai, 2017). In harmony with that, Moradi (2018) claims that classroom management is crucial and of significant importance for teachers to assist learners in developing SDL. Abdelkarim and Abuiyada (2016) report that peer teaching is helpful, because the fast learners can help slow peers to master the concepts related to the course and confidently express their ideas (See 2.6.5).

2.7.3. The mathematics teacher's role during the facilitation of self-directed learning

Tjakradidjaja *et al.* (2016) suggest that the roles of the teacher in SDL should be a facilitator, a motivator, and a role model. Furthermore, a teacher needs to be competent, be concerned about the learners, be creative, and well qualified to facilitate for SDL (Tjakradidjaja *et al.*, 2016). The teacher must value and care about the

learners and assist and guide all of them. The teacher's role should be clearly seen to direct, encourage and minimise the distance between him- or herself and the learners in the learning process.

Similarly, Du Toit-Brits (2019) recommends the role of the teacher, namely transforming their learning environments into SDL supportive environments by practising good teaching such as motivating learners to learn. Teachers must have a longing to share their love of the subject with learners, encourage independence in learning and implement teaching approaches that necessitate learners to learn actively by taking responsibility for their own and co-operative learning (Du Toit-Brits, 2019).

Another role of the teacher is to develop problems based on real-life experience, in the mathematics classroom that learners can solve, which can help learners to develop thinking ability (Bishara, 2022). The problems must be related to what learners see, hear or do in everyday life, so that it can be easier for them to make the connection between what they know and what they are about to learn. The task must not puzzle the learners, because they do not know what the problem is about.

Ohashi (2018) urges that teachers make use of autonomous learning and demonstrates the potential they have to influence learners' out-of-class actions through in-class guidance and support SDL development. Erturan and Jansen (2015) emphasise the important task of the teachers of bridging the existing gap within teaching and learning to enable learners to realise the learning needs and learning goals. Breed (2013) argues that teachers are responsible for teaching learners explicitly about SDL skills to reduce the likelihood of missing the development thereof in learners. For these reasons, mathematics teachers should understand their roles, not only as educators, but also as coaches (Cheng, 2016). Lai (2017) emphasises the importance of socio-cultural and real-life mathematics teaching and learning to enhance the development of learners' SDL skills.

Therefore, teachers need to play their role of facilitating SDL, because it is important to develop self-directed learners. This is discussed in the next sub-section.

2.7.4 Importance of developing self-directed learners

SDL learners can transfer their learning, in terms of knowledge and skills, from one situation to another (Winne, 2016). A further development of self-directed learners is encouraging, namely that learners may have greater control over their learning, improve their beliefs about their abilities to solve problems effectively and increase their motivation to learn (Crowley, 2015). The development and strengthening of SDL characteristics in learners may differ depending on the attitudes, skills, and behaviours that the learners seek to acquire (Johnson, 2014). The development of a life-long learning attitude involves the acquisition of new skills such as information gathering by the learners and the development of certain specific attitudes and notions about learning (Ayyildiz & Tarhan, 2015). This means that learners are expected to initiate their own learning through SDL. According to Gomez-Chacon (2017) initiating SDL includes scheduling learning time, identifying necessities, and formulating learning goals. Gomez-Chacon (2017) further states that for some to initiate SDL, he or she must be able to identify relevant resources for learning, select appropriate strategies and evaluate learning outcomes, as important aspects in learning mathematics. Hence, learners will put in effort, as they are the ones who design their own learning, thus optimal resources will be used to achieve their learning goals. Furthermore, using metacognitive skills such as planning, monitoring, and evaluation, learners use new knowledge and extract knowledge they already have into solving various problems in mathematics (Winne, 2016). Monitoring and evaluation of the thinking process creates an awareness of what is being thought about. In other words, this enables learners to question themselves regarding their actions during the learning process.

Learners develop critical thinking by asking questions, looking critically at solutions and reaching conclusions regarding learning issues (Abubakar & Arshad, 2015). All these form part of characteristics that contribute to the development of SDL.

During the academic year 2020, schools were closed to curb the spread of COVID-19, as these are public spaces with high movement and concentrated populations (See 2.4.6). This meant that learners had to stay home, and teachers had to deliberately plan how they could best teach their learners whilst at home (Bayham & Fenichel, 2020). Mahlaba (2020) discusses the importance of learners being self-directed during learning while at home, since those who are self-directed are proficient in their manipulation and application of mathematics in a variety of problem-solving

contexts. Self-directed learners tend to be ethical, they do not cheat in assessments, because their aim is not to simply to pass a certain subject, but to obtain extensive conceptual understanding of the subject, which will allow them to prosper in the future, if faced with similar problems (Mahlaba, 2020). Self-directed learners are likely to meet their academic requirements, therefore, it is of importance to develop learners' SDL and prepare them for unpredictable circumstances, like those of COVID-19.

2.7.5. Problem-based tasks and the development of self-directed learning

To enable learners to develop skills that are essential for SDL, the completed mathematical PBL tasks must meet certain requirements. The teacher is responsible for planning tasks that promote HOTS and acts as a facilitator to scaffold learners during the teaching and learning process (Alli, 2019). When learners are exposed to tasks that promote critical and reflective thinking (Muin et al., 2017), HOT, goal setting, determining of learning needs, independence, persistence, and curiosity, they are likely to develop and nurture such skills (Maphalala, 2021). PBL tasks require learners to engage in active learning, especially in groups so that learners can develop skills that can promote cooperation and collaboration during task execution and the solving of problems (Singh & Marappan, 2020).

When teachers plan tasks that are required to promote HOTS, it becomes easier for learners to deal with such tasks in future (Gupta & Mishra, 2021). The implementation of PBL tasks enables learners to develop the ability to set goals and determine learning needs before them attempting to solve any task and think of the possible resources to use to address the task (Mahajan & Singh, 2017). Through PBL tasks, learners can cultivate the quality of persistence because they are willing to complete the task that is always assigned to them (Lai, 2017). Learners become independent of the teacher because they are determined and eager to complete the tasks on their own.

When learners work on tasks that require the activation of prior knowledge and making connection between what they know and what they are about to learn, it enables them to do the same in the future and by so doing, they become self-directed (Wang *et al.,* 2017). As a result, their development of metacognitive skills, which contributes to their metacognitive awareness, can be enhanced, and thus improves their level of SDL. By

searching for relevant information, using various resources (Gomez-Chacon, 2017), and through discussions with their small group to evaluate the usefulness of their collected information pertaining the task, the development of learners' SDL skills is enhanced (Mohamad *et al*., 2020). Table 2.7 presents some examples of PBL tasks regarding mathematics Grade 7 content.

Table 2.7 Some examples of PBL tasks in mathematics Grade 7

PBL topic Grade 7	PBL tasks description	Motivation
1. Time (Time-zone)	Learners will be expected to compare time in various areas of the globe. They will do so in each group by drawing the continents and their countries on graph paper. Once they have done this, they design the paper into a globe, then compare the time of different cities in the world using their own globes.	This problem is a real-life event, it requires learners to use their existing knowledge about time, it enhances critical thinking, because they will need to think of time ahead or time behind a certain city keeping in mind the Greenwich Meridian.
2. Mass	Learners will need to make items from waste with different mass sizes (weigh the items), and then convert the mass of those items to different units of mass.	This task will promote HOTS such as evaluation. Learners will judge the mass of the items and the answers they converted.
3. Money and finance	Each group will have a shop, where items are sold with percentage profit, loss or discount.	Learners tend to be careful and ensure they get the right answer as they are afraid to pay more

	Learners (customers) will be expected to work out the selling or cost price themselves.	money or give more change. Learners use various ways to get the solution to the problem at hand.
4. Geometry (order of line symmetry)	Learners design shapes of their choice and decide on the number of lines of symmetry and order of rotational symmetry.	This task promotes HOTS because it involves learners create any shape they can think of and analyse those shapes to identify the symmetry properties.

These problems are good for Grade 7 mathematics, because the solutions are not obvious, meaning that it will require learners to work and think critically. The solutions to such problems are not the same, each group can get an answer different from the other group and each group may use a different method to find the solution. All examples are based on real-world problems, so learners can employ their prior knowledge to make sense of what they are learning to create their own knowledge.

2.8 Summary of the chapter

This chapter presented the conceptual-theoretical framework of the study. The study context and background were explained as well as mathematics as a school subject and how learners can solve real-life problems in mathematics. Furthermore, this chapter highlighted the challenges that learners experience when learning mathematics with understanding. The implementation of PBL as an approach to improve learners' SDL was also discussed as well as the characteristics and

importance of SDL. Chapter 3 focuses on the empirical design and methodology of this study.

CHAPTER 3

EMPIRICAL DESIGN AND METHODOLOGY

3.1 Introduction

This chapter outlines the paradigm, research design and methodology as well as participant details, data gathering and analysis procedures, ethical aspects, trustworthiness, and related details.

3.2 Research paradigm

Creswell (2018) states that the term 'paradigm' originated from the Greek word '*paradeigma*' which means 'pattern'. Furthermore, Kamal (2019) says that a paradigm generates a brief guideline of how research is conducted, the strategy that is followed as well as appropriate data collection techniques, and data analysis methods.

This study employs a social-constructivist paradigm. Creswell (2018) describes social constructivism as a paradigm where people work together to construct their own understanding and knowledge of the world through experiencing things and reflecting on those experiences. In addition, social constructivists are of the opinion that people tend to develop biased meanings of their experiences. These subjective meanings, according to David and Amey (2019), lead researchers to look for the complexity and depth of different views. Social constructivism implies that learners, through interaction with other learners, construct new knowledge. Social constructivism is applicable to this study, because learners worked together in groups and they interacted socially with each other within their groups to construct new knowledge and solve mathematics problems.

3.3 Research design and methodology

The researcher used a generic qualitative research design and adopted a qualitative methodology to acquire a holistic picture of the study (Kahlke, 2014). A qualitative methodology explores meaning, for example people's ideas and opinions regarding their experiences of things in real-world settings (Creswell, 2018).

Furthermore, Creswell (2018:23) states that a “qualitative study is an inquiry process of understanding a social or human problem based on building a complex, holistic picture with words, reporting detailed views of informants and conducted in a natural setting”. Hence, qualitative researchers attempt to collect rich descriptive data in respect of a phenomenon or context with the intention of developing an understanding of what is being observed. It therefore focuses on the views of individuals and groups, their understanding of the world and how to construct meaning from their experiences. In other words, the methods within generic qualitative approaches are generally highly inductive: the use of open codes, categories, and thematic analysis are common (Kennedy, 2016). Details regarding the methodology and research design followed in this study are outlined in Figure 3.1.

Generic qualitative design

Grade 7 learners (n=36)

1. **Introduction to PBL and group work**
   -Introduction of PBL to the learners and assign members to each group.
   -Conduct interviews before the intervention (determine learners' initial views on problem solving and group work) (see Addendum J)

2. **Intervention (PBL tasks and group activities)** (see 3.5.1)
   (Three weeks' lapse between task 1 and task 2)
   -**Task 1** (Group work, see 3.5.1): Data handling
   - Worksheet 1 (learners follow the PBL steps to execute task 1) (Addendum I)
   -Reflective sheet 1 (learners completed the reflective sheet individually after task 1) (Addendum I)

   -**Task 2** (Group work, see 3.5.1): Measurement
   -Worksheet 2 (learners follow the PBL steps to complete task 2)
   -Weekly reflective sheets 2 (groups) (Each learner completed a reflective sheet after the group has done, task 2)
   -Peer assessment (was done by individual learner as part of the reflective sheet)

3. **Interviews after the intervention**
   -Interviewed randomly selected participants (to determine learners' new views on PBL and mathematics).

4. **Data collection**
   -Mathematics worksheets (individual) (see 3.5.2)
   -Reflection sheets (individual) (see 3.5.3)
   -Semi-structured interviews (individual, P1, P4, P9, P10, P12, P14, P19, P21, P27, P30, P31 and P36) (see 3.5.4)

5. **Data analysis**
   -*In vivo* coding
   -Analysis of the data using Excel

P* = participant

Figure 3.1: Overview of the empirical research

3.4 Research context, population, and sample

The selected school in this study is a combined school, which is situated in the Northern part of Namibia in the Omusati region. It is a rural school close to the Angolan border. The school started in 1976 as a primary school offering Grades 1–7. It became a combined school in 2006 when Grade 8 was added and from there Grade 9 and 10 followed. Since the beginning of the school in 1976, mathematics has been offered from Grade 1.

The average number of learners in a classroom is about 40 and the classrooms are overcrowded and insufficient for the number of learners enrolled. The school also has a shortage of chairs and tables. Grade 1–3 learners use Oshikwanyama as the medium of instruction, while Grade 4–10 learners use English as the medium of instruction.

The school has access to electricity and water. However, there is poor internet connection due to the location of the school. Although there is a Subject Senior Education Officer (SEO) for Mathematics in the region, he is not supportive. Furthermore, there is a feeding programme at the school and several learners benefit from this. However, in most cases the supply is not delivered on time.

An additional challenge is that the school is not equipped with technology despite the call in the Namibian National Curriculum for Basic Education (NCBE) to produce an Information and Communication Technology (ICT) society (NCBE, 2016). Unfortunately, the school does not have computers that allow learners to explore and do research. Most of the learners do not have smartphones and those who have phones do not have internet connection at home, or the parents do not have money to buy data. In addition, learners are not allowed to bring cell phones to school and to use the school's internet. To overcome this challenge, the school should buy computers so that learners can do their research when needed or to organise learners to visit the community library (which is better equipped than the schools). However, the library is 20 km from the school.

One of the problems regarding the learning of school mathematics is the number of inexperienced teachers across all phases. Furthermore, learners experience reading problems as they are promoted to the next grade (mostly from Junior Primary phase)

without mastering the reading component in English as medium of instruction. In addition, a lack of teaching and learning resources also contributes to learning problems in mathematics

A population refers to all the individuals or units of interest, generally a collection of individuals that have similar characteristics that are the focus of the scientific inquiry (Creswell, 2018). A sample is a small portion of the total set of objects, events, or persons, which together comprises the subject of the research study (Creswell, 2015). The population of the Grade 7 learners at the school was 71. However, the sample for this study was 36 Grade 7 learners in mathematics, who were conveniently selected because the researcher was their mathematics teacher and this made them readily available. The participants were expected to complete the tasks during school hours and without interrupting other classes. Mathematics class time was used to complete their tasks, thus making them readily available for the study. According to Creswell (2018), convenience sampling is a non-probability sampling method, and it occurs when participants are drawn from the population as they are close to hand. Therefore, the convenience sampling technique allowed the researcher to select the individual participants who would be most likely to be available for this study. Table 3.1 shows the participants and their groups. It also indicates the participants who were involved in semi-structured interviews. These participants were selected using online random number generation to keep the researcher's bias the minimum (htps://www.random.org). The researcher randomly selected six groups of six members each using the same technique used to select participants for semi-structured interviews. An example of the random generator is shown in Figure 3.2.

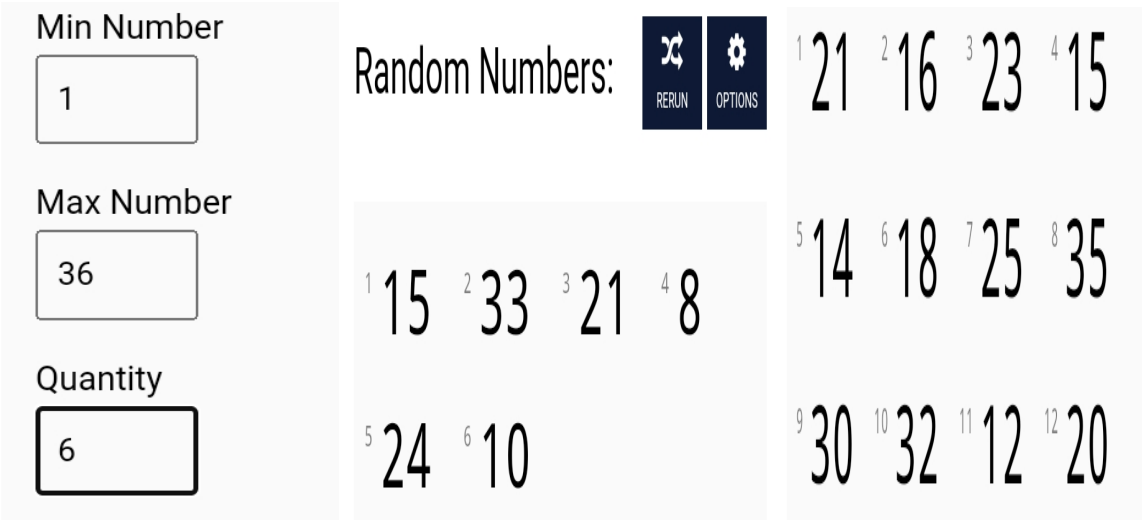


Figure 3.2 shows how participants were randomly selected online

3.5 Data collection procedures

The Grade 7B learners, 18 boys and 18 girls, were selected to participate in the research. However, only 12 learners were randomly selected for individual interviews due to limited time and until data saturation was obtained. Maree (2016) describes an interview as a two-way conversation in which the interviewer asks the participant questions to collect data and to learn about ideas, beliefs, views, opinions, and behaviours of the participants in a research context. Fusch and Ness (2015) point out that conducting interviews is an efficient method to collect data and to obtain data saturation. Learners' weekly reflection sheets, mathematics worksheets and individual semi-structured interviews were used to gather the data (Figure 3.1).

3.5.1 PBL tasks

Learners completed two problem-based tasks in their groups (See Addenda F and G). The groups were set up as follows:

Group 1 = P1 to P6;

Group 2 = P7 to P12;

Group 3 = P13 to P18;

Group 4 = P19 to P24;

Group 5 = P25 to P30; and

Group 6 = P31 to P36.

The learners in their various groups used a variety of mathematical procedures to find reasonable solutions. The PBL activities were divided into tasks in which the learners were expected to record their answers or solutions according to the problem requirements. The two PBL tasks are outlined briefly below.

PBL task 1: Data collection (also see Addendum F)
The owner of the tuck shop at the school wants to make sure that he provides drinks that the learners will buy. However, he is unsure whether he sells the drinks that they prefer. Being a very busy man, he does not have the time or the knowledge to investigate it himself. He decided to ask your mathematics class to assist him in investigating that his tuck shop will in future provide in the demand of the learners. You as a group should investigate the type of drinks that your classmates like most.
Subtask 1: Develop a measuring instrument (questionnaire to find out about the drinks each learner prefers).
Subtask 2: Collect the data over a period of four days.
Subtask 3: Organise the data in tables and present them on various graphs.
Subtask 4: Report the results by means of a group presentation (using flip charts, poster, and the chalk board).

Within each group, the learners were expected to come up with an instrument (namely a questionnaire) to collect data about the type of drinks each learner prefers. Members had to record the finding in a table and present the data using different types of graphs (e.g., two members were constructing a bar graph, the other two a pie chart and the remaining two a pictogram). At the end, the learners compiled a group report, and presented their findings to the class.

PBL task 2: Measurement, area, money and decimals (also see Addendum G)
The School Governing Body plans to renovate various classrooms soon. They have a limited budget and must accurately plan for the cost before the work can begin. They plan to paint the walls and either tile the floors or lay carpets. Your class should advise the school principal on the most cost-effective way to renovate the floor.
Subtask 1: Investigate the size of the room by measuring the building (to determine the area of the floor).
Subtask 2: Determine the number of tiles needed to tile the floor (by calculation).

Subtask 3: Determine the size of the carpet needed to cover the floor (by calculation).
Subtask 4: Compare the cost of the tiles with that of the carpet.
Subtask 5: Determine the amount of paint needed and the cost thereof (by calculation).
Subtask 6: Report the results by means of a group presentation (using flip charts, a poster and the chalk board as the only available resources).

Within each group, the participants were expected to measure the length, height, and the breadth of a specifically assigned build in order to calculate the area of the assigned building respectively. Members calculated the area of the assigned building as well as the area of a tile and a carpet. Participants then determined the cost of tiles and carpets and compared them to determine, which of the materials to cover the floor was the cheapest. Similarly, participants determined how much paint was needed to renovate the walls and the cost involved, then calculated the total costs needed to renovate the classroom. Each group reported by means of a presentation.

3.5.2 Mathematics worksheets

Lee (2021) and Fauziah and Nurita (2019) point out that a worksheet can be useful in many ways, as it can be used to add information and enable learners to monitor the progress of their own learning. Learners completed worksheets (See Addendum H) containing the PBL steps followed during the execution of each task every week after each mathematical PBL task. The first worksheets were completed prior to the PBL intervention on the 17th of June 2019 and another one (post to the PBL intervention) on the 08th of July 2019. Participants completed the worksheets while at school. In the worksheets, learners demonstrated that they could use the PBL steps to solve a mathematical problem. In total, two worksheets were completed individually, one after Task 1 and another after Task 2, based on each PBL task, which helped to demonstrate whether learners could formulate learning objectives, whether they used various resources as well as their critical discussions on mathematics topics.

3.5.3 Reflective sheets

Reflective sheets mirror learners' experiences (Tiessen, 2018). Thus, by encouraging learners to reflect on their activities, they focus both on how and why they undertook specific activities (Johnson, 2014). Gomez-Chacon (2017) points out that the purpose

of reflective sheets is to assist the learner in making connections between theory and practice. In this study, participants completed reflective sheets (See Addendum I) in which they reflected on what they had learned, their degree of success, how they could improve as well as the group interaction with their members. The reflective sheets were also completed on different dates, the first one on the 21st of June 2019 as well as on the 12th of July 2019 respectively. All these activities were done by participants while they were at school. In the reflective sheets, the learners provided a description of the problem and how it was solved, their feelings and reactions, an evaluation of what was good or bad about the experience, and an analysis of their sense-making of the problem situation. Figure 3.3 shows a completed reflective sheet of Participant 27.

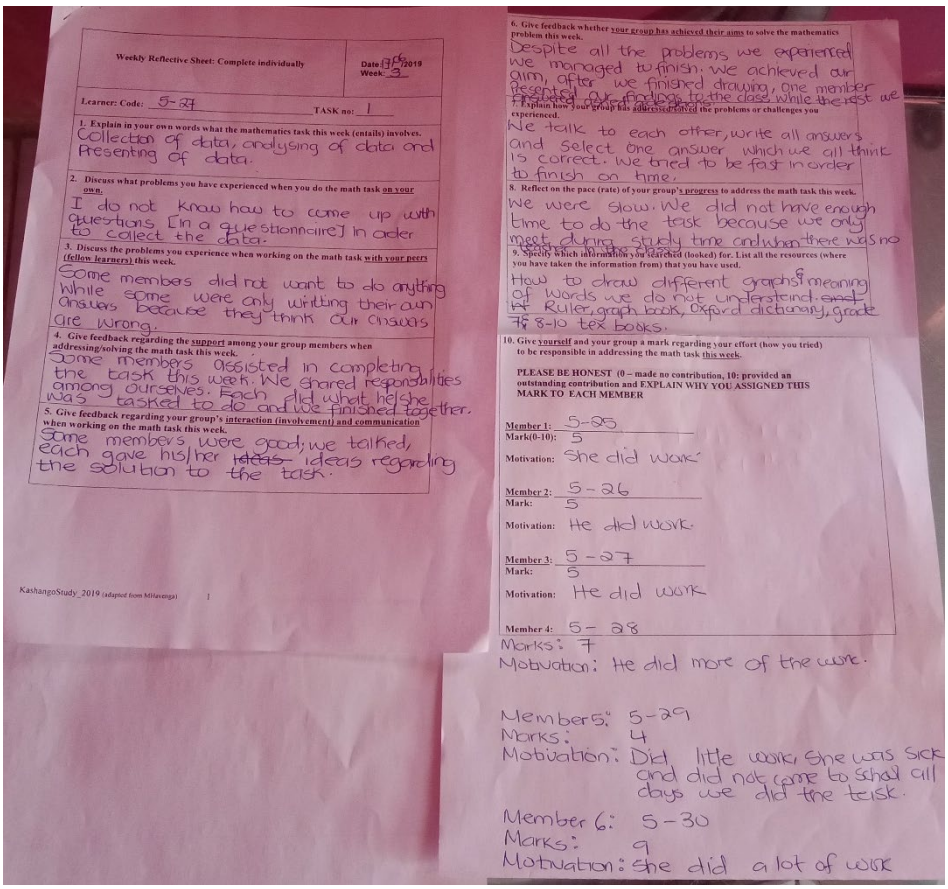


Figure 3.3: A completed reflective sheet of Participant 27

3.5.4 Semi-structured interviews

Interviews are one of the main data gathering methods involved in qualitative research, as they provide in-depth information about the experiences and viewpoints of the participants about a specific phenomenon (Maree, 2016; Ruslin *et al.*, 2022). Interviews were conducted as part of this study to explore and probe participants' responses and to get rich data regarding their experiences and feelings about mathematics. DeJonckheere and Vaughn (2019) claim that semi-structured interviews are often used in qualitative research with the aim to collect open-ended data, to explore participants' thoughts, feelings, and beliefs about a particular topic and to delve deeply into personal and sometimes sensitive issues.

A semi-structured interview protocol (See Addendum J) was formulated and used during all the individual interviews. It was expected of learners to share their experiences on solving mathematics problems and how they went about taking responsibility for their own learning. Interviews were conducted on the school premises, each in one day, the first one (before the PBL intervention) on the 7th of June 2019 and the last one (after the PBL intervention) on the 10th of September 2019. The length of the first interview session varied depending on the interviewee and this gave an average interview length of 4.7 minutes, while the second session was a bit shorter compared to the first one with an average of 3.04 minutes. The interviews took place at the school.

The researcher, who is the mathematics teacher of the participants, used some lessons for research activities and the afternoon study time was also used for interviews and completion of the worksheets. This was done with the aim of not violating the ethical guidelines set by the Regional Director of Education and the SGB (See Addendums C & D) because learners did the work that as part of the learning content for their grade. Due to travelling safety reasons, learners were not supposed to remain after school to carry out research activities.

3.6 Data analysis procedures

Data analysis is the process of examining data with the aim to find out about useful information, arrive at conclusions, and supporting the decision-making process

(Creswell, 2018). In line with this, Maree (2016) argues that data analysis comprises breaking down the data into patterns, trends, and relationships. Hence, the main purpose of data analysis in qualitative research is to comprehend different elements of the data, and relationships between concepts, constructs or variables (Creswell, 2018). Another reason for data analysis is to check if there are patterns or trends that can be identified and isolated, or to indicate specific themes that emerged from the data (Taber, 2013). In this study, the researcher employed an *in vivo* coding approach to analyse the data gathered.

Due to technical challenges with Atlas.ti™, the following steps were used as suggested by the supervisor. 'Track changes' in MS Word were used as a 'tool' to analyse and code the data. The researcher typed the code assigned to each question until all participants were done. Figure 3.4 below shows a completed worksheet and how the researcher coded P4's reflective sheet.

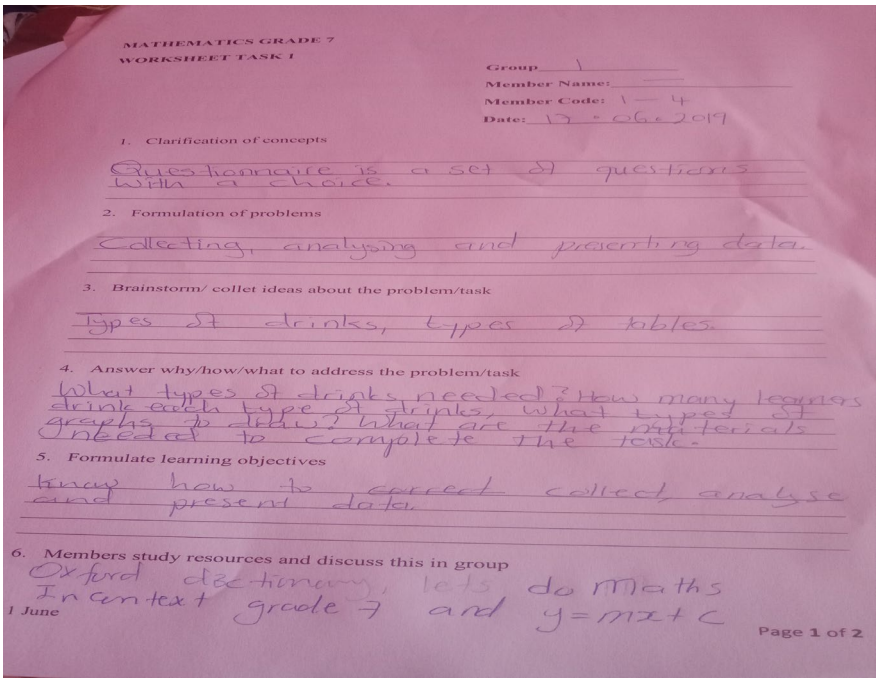
These steps were followed.

1) Highlight and code particular words or sentences or sections in the participant's data using "track changes" in MS Word.
2) Select all codes (comments) in all the track change "balloons" (Ctrl+Shift+End).
3) Copy all the codes and paste it to a new Word file within a table (See Table 3.3). Codes *prior* to the intervention were copied in the column on the left, while codes obtained *after* the intervention were copied on the right.
4) Specific categories and themes emerged from the research, and they were coloured to indicate a particular category or theme (See Figure 3.4).

The following legend was used to assist the researcher:

Legend:

-**W1**: Worksheet 1 (before intervention)
-**W2**: Worksheet 2 (after the intervention)
-**mt**: Mathematics task
-**I1**: Interview 1 (before the intervention)
-**I2**: Interview 2 (after the intervention)



MATHEMATICS GRADE 7
WORKSHEET TASK 1

Group 1
Member Name:
Member Code: 1 – 4
Date: 13 - 06 - 2019

1. Clarification of concepts

Questionnaire is a set of questions with a choice.

2. Formulation of problems

Collecting, analysing and presenting data.

3. Brainstorm/ collet ideas about the problem/task

Types of drinks, types of tables.

4. Answer why/how/what to address the problem/task

What types of drinks needed? How many learners drink each type of drinks, what type of graphs to draw? What are the materials needed to complete the task.

5. Formulate learning objectives

Know how to correct collect analyse and present data.

6. Members study resources and discuss this in group

Oxford dictionary, lets do Maths In context grade 7 and y=mx+c

1 June

Page 1 of 2

P4

Reflective sheet task 1 (Before Intervention)

1. **Explain in your own words what the mathematics task this week (entails) involves.**
   - Collection, analysis and presentation of data.
2. **Discuss what problems you have experienced when you do the math task on your own.**
   - I did not know what to do.
3. **Discuss the problems you experience when working on the math task with your peers (fellow learners) this week.**
   - Some people were laughing at others.
4. **Give feedback regarding the support among your group members when addressing/solving the math task this week.**
   - Only some people worked.
5. **Give feedback regarding your group's interaction (involvement) and communication when working on the math task this week.**
   - Some people worked with their friends only.
6. **Give feedback whether your group has achieved their aims to solve the mathematics problem this week.**
   - Yes.
7. **Explain how your group has addressed/solved the problems or challenges you experienced.**
   - We tell them to work.
8. **Reflect on the pace (rate) of your group's progress to address the math task this week.**
   - It was ok.
9. **Specify which information you searched (looked) for. List all the resources (where you have taken the information from) that you have used.**
   - Calculate angles and check the meaning of words. I used Oxford dictionary, Let's do maths grade 7, Incontext grade 7 and y=mx+c grade 8-10.
10. **Give yourself and your group a mark regarding your effort (how you tried) to be responsible in addressing the math task this week.**

PLEASE BE HONEST (0 – made no contribution, 10: provided an outstanding contribution and EXPLAIN WHY YOU ASSIGNED THIS MARK TO EACH MEMBER

- Member: 1-1 – 9 marks did a lot.
- Member: 1-2 –9 marks also did a lot.
- Member: 1-3 – 4 marks was just laughing at people.
- Member: 1-4 – 9 marks I did a lot.
- Member: 1-5 – 5 marks was sick.
- Member: 1-6 – 9 marks did but not like us

Commented [s1]: W1: mt: involves: collection, analysis and presentation of data.

Commented [s2]: W1: mt: own: do not know what to do

Commented [s3]: W1: mt: peer: some people laugh at other.

Commented [s4]: W1: mt: support: only some supported.

Commented [s5]: W1: mt: interaction: some worked with friends only.

Commented [s6]: W1: mt: achieved aims: yes.

Commented [s7]: W1: mt: how solved: tell them to work

Commented [s8]: W1: mt: progress: it was ok.

Commented [s9]: W1: mt: resources: calculate angles and meaning of words. Dictionary and Grd 7-10 text books.

Commented [s10]: W1: mt: contribution: 1(9), 2(9), 3(4), 4(9), 5(5),6(9)

Figure 3.4: Example of P4's completed worksheet and coding of reflective sheet task 1 (W1)

After all participants' feedback sheets were coded, the researcher selected all the comments in the other balloons. Once the texts were selected in all the comment balloons, the researcher copied all the comments and pasted them into a new Word file. Similarities found in the feedback of both worksheets and reflective sheets were combined to form one idea. For example, learning materials used in executing the tasks and the learning objectives appeared in both sheets which informed the researcher regrading learners' experiences. The worksheets also gave the researcher a broad picture of how participants followed the PBL steps to complete the tasks. Final outputs are shown in Table 3.1 regarding data collected from Participant 4.

Table 3.1: Feedback from Participant 4 before and after intervention

	Before the intervention	After the intervention
P4*	W1: mt: involves: collection, analysis and presentation of data. W1: mt: own: do not know what to do W1: mt: peer: some people laugh at other. W1: mt: support: only some supported. W1: mt: interaction: some worked with friends only. W1: mt: achieved aims: yes. W1: mt: how solved: tell them to work W1: mt: progress: it was ok. W1: mt: resources: calculate angles and meaning of words. Dictionary and G 7-10 text books. W1: mt: contribution: 1(9), 2(9), 3(4), 4(9), 5(5),6(9) I1: challenges: finding answer to question I do not understand I1: solve others: yes, they can tell the answer. I1: Determine needs: yes, my needs to pass Mathematics. I1: goals: yes, my goals to pass Mathematics. I1: learn responsible: yes, I study hard. I1: curious: yes, if I fail I make correction. I1: comparing: yes, best answer help get marks. I1: additional resources: yes, assisted by friends and teachers. I1: evaluate: yes, see if answer correct.	W2: mt: involves: measure, area, money and fractions. W2: mt: own: do not understand task. W2: mt: peer: time was not enough. W2: mt: support: worked together and shared ideas. W2: mt: interaction: we communicated. W2: mt: achived: yes, we achieved. W2: mt: how solved: met when no teacher in class. W2: mt: progress: we were fast. W2: mt: resources: find meaning of words, divide carpet. Dictionary and grd 7-10 textbooks. W2: mt: contribution: each 10/10. I2: challenges: do not use calculator, big numbers, time not enough. I2: solve others: I am not always right, others help. I2: determine needs: yes, difference what I know and what I am learning. I2: goals: yes I know to find goals. I2: learn responsible: yes answer previous question papers and textbooks, do homework no one tell me to do. I2: curious: yes, problems tough but I try. [persistence] I2: compare: yes if many ways I select best answer. I2: additional resources: yes textbooks. I2: evaluate: no others check my answer

I1, I2: where interviews were used as data collection method (**some** participants were interviewed)

Categories were identified and used as indicated in Table 3.2 with the aim of grouping data with the same meaning and making it easy to understand. Each category was given a specific colour to distinguish between different categories.

Table 3.2: Emerged categories from the data

Colour code	Category name
Orange	Challenges
Purple	Progress in activity
Green	Positive experiences
Blue	Resources
Brown	Communication
Red	Contribution
Yellow	Group collaboration
Dark blue	Self-directed learning
Grey	Mathematics tasks
Pink	Other

Some categories were combined to form new themes as well sub-themes thereof as presented in Table 3.3 (See Chapter 4).

Table 3.3: Emerging themes

THEME 1: INITIAL CHALLENGES EXPERIENCED WITH PROBLEM-BASED MATHEMATICS TASKS (§4.3)
Sub-theme 1.1: Initial personal challenges (4.3.1) Sub-theme 1.2: Attitude towards group work (4.3.2)
THEME 2: LEARNERS' EXPERIENCES WHEN SOLVING MATHEMATICS PROBLEMS (§4.4)
Sub-theme 2.1: Problem solving activities (4.4.1) Sub-theme 2.2: Group work and collaboration (4.4.2)
THEME 3: DEVELOPMENT OF ESSENTIAL KNOWLEDGE AND SKILLS (§4.5)
Sub-theme 3.1: Development of mathematics skills (4.5.1)
Sub-theme 3.2: Development of self-directed learning abilities (4.5.2)

3.7 Ethical aspects

The researcher obtained approval from the Scientific Committee of the Research Unit for Self-directed Learning (See Addendum A) and then applied for ethical clearance from the North-West University Education, Management, and Economic Sciences, Law, Theology, Engineering and Natural Sciences Research Ethics Committee of the North-West University (NWU-EMELTEN-REC) (See Ethics approval letter, ADDENDUM B). After that, the researcher obtained a letter of permission from the Inspector of Education of the Etayi circuit in the Omusati region in Namibia (the Director of Education of Omusati region in the absence of the Inspector) (See Addendum C). It was also essential that the researcher obtained a permission letter from the school principal and the School Governing Body (SGB) to conduct the investigation at the school (Addendum D).

Participants were minors (younger than 18 years) and regarded as a vulnerable group and at high risk. The independent mathematics teacher obtained informed assent from the participants and informed consent from their parents or guardians (See Addendum E). The informed consent explained that participation in the study would be voluntarily and that learners were free to withdraw from the study at any time without penalty (See 1.4.7).

3.8 The role of the researcher

The researcher plays a key role in data collection, interpretation, and presentation, thus defining the meaning of data at all stages of the research process (Yoon & Uliassi, 2022). Creswell (2018) states that the role of the qualitative researcher is to collect and generate data from the research participants. The researcher also acts as part of the research process in working with the participants (Maree, 2016). The researcher planned instructions and facilitated the learning activities (learners' reflections).

The researcher interviewed the 12 randomly selected participants and played the role of a facilitator during the PBL tasks. The researcher was also responsible for recording the data for analysis purposes (Creswell, 2018). The researcher analysed the interview scripts and all documents (mathematics worksheets and reflective sheets)

for the purpose of understanding and interpreting, and subsequently to answer the research questions.

3.9 Trustworthiness of qualitative data

Trustworthiness of the findings can be ensured through credibility, transferability, dependability, confirmability, and triangulation (Simons & Copley, 2022).

3.9.1 Credibility

Credibility is the confidence in the 'truth' of the research findings and the context in which the inquiry was carried out (Moon *et al.*, 2016). In line with this, Creswell (2018) mentions that credibility refers to the assurance that a researcher's inferences emanate from the findings and should be presented and outlined in an accurate manner. To ensure credibility in this study, the researcher honestly dealt with the data obtained from all the participants without changing any response to fit the researcher's desires.

Only willing participants from Grade 7 participated in this study. Learners were given the opportunity to participate through informed assent or refuse to participate. This ensured that the data gathering involved only those who were willing to participate and offer their data. Tabler (2013) supports the idea that willing participants are likely to be truthful during data collection as compared to unwilling or forced participants. In addition, participants were required to be honest in all discussions and interview session, thereby maintaining integrity.

The researcher used probes to obtain detailed feedback and iterative questioning. Related questions were rephrased so that falsehoods or inconsistencies could be detected with extreme care or suspicious data could be discarded. This meant that the researcher had to look for any deviant data that did not support the explanation for any behaviour, situation, or event. The researcher traced for some patterns that were completely different from the rest of the study.

3.9.2 Transferability

Transferability involves indicating that the findings are applicable to other contexts (Creswell, 2015). To ensure transferability, the researcher used thick descriptions, comprising not just Gr 7 learners' experiences, but also the context, so that the mathematical experiences become meaningful. A thick description was ensured through suggesting information that exhibited the study context and the data collection techniques used. Several methods of data gathering were used such as reflective sheets, worksheets, and interviews.

3.9.3 Dependability

Dependability shows that the research findings are consistent and could be repeated (Taber, 2013). The researcher repeated the PBL tasks with some of the groups that were seemingly unsure of what they were doing, and the interview sessions to ensure dependability. Group 2 repeated Task 2 because they were not satisfied with their findings, which confirmed dependability of the study. For example, Participant 4 in the interview before and after the PBL intervention, said that he prefers solving problems in groups or with peers, as it results in many ideas. Most of the responses from various participants were consistent, ensuring dependability.

3.9.4 Confirmability

Confirmability measures the degree of neutrality or the extent to which the findings of a study are shaped by the participants and not researcher bias, motivation, or interest (Korstjens & Moser, 2018). Maree (2016) warns researchers to be conscious of their own biases and to design ways to deal with them since bias distorts how we process information. The researcher ensured that the interpretation of the data was not based on her own preferences or viewpoints, but on the opinions of participants. Confirmability was addressed as outlined by Sandman (2016), namely that the researcher needs to scrutinise him- or herself, the sources of information, opposing views and evidence. To ensure confirmability of these research findings, the researcher employed self-description and self-reflection to acknowledge and reduce bias and criticism. Self-description and self-reflection enable researchers to keep their position within the study in mind and how their personal beliefs influence the research findings (Hadi & Closs, 2015). The researcher also recognised and acknowledged

shortcomings and limitations in the methods used in the study and their potential effects.

3.9.5 Triangulation

Triangulation is a research technique that seeks to ensure the credibility and reliability of the study information (Creswell, 2018). In this study, triangulation was employed by upholding honesty, iterative questioning, and through negative case analysis (See 3.7.1) to ensure that all the information in the study was consistent and dependable. *Methodological triangulation* was used, which promotes the use of several data collection methods (Noble & Heale, 2019). Methodological triangulation enables the researcher to review the data collected carefully through different methods to achieve a more accurate and valid estimate of qualitative results for a construct (Korstjens & Moser, 2018). In this study, methodological triangulation was ensured through documents from the literature review, using a series of data collection methods such as semi-structured interviews, weekly reflective sheets, and solutions from mathematical PBL tasks. Data triangulation was also achieved through collecting data from all the participants. Methodological triangulation enables the researcher to review the data collected carefully through different methods to achieve a more accurate and valid estimate of qualitative results for a construct (Korstjens & Moser, 2018).

Creswell (2015) emphasises that it is important to consider triangulation methods as well as member checking strategies between a researcher and the participants to determine how the findings may influence their practice (mathematics learning). Taber (2013) mentions that triangulation is used to capture a holistic view of the phenomenon being studied, leading to a greater understanding of the phenomenon, as each research strategy used contributes to a different piece to the puzzle. Hence, each strategy chosen offers another unique perspective on the phenomenon. Furthermore, Creswell (2015) emphasises that triangulation used for the purpose of completeness means that different results can enrich the explanations for the phenomenon under study.

3.10 Summary of the chapter

This chapter outlined the paradigm, research design and methodology as well as data gathering, analysis techniques, and related details. The next chapter, Chapter 4, focuses on the data presentation, analysis, interpretation, and discussion of Grade 7 learners' development of self-directed learning through problem-based tasks.

CHAPTER 4

DATA ANALYSIS AND DISCUSSION

4.1 Introduction

The aim of this chapter is to present the qualitative data, discuss the analyses thereof and interpret the research findings. In this chapter, the data generated from semi-structured interviews, mathematical worksheets, and the reflective sheets are presented and discussed. In particular, the third and fourth research aims are addressed, namely, to explore what the experiences of Grade 7 learners are when they solve mathematics problems and to determine how the use of mathematical problem-based tasks can enable Grade 7 learners to develop self-directed learning skills.

The participants in the study were 18 boys and 18 girls with an average of 13 years of age. Most of the participant have a history of poor mathematics performance across all genders. Some of them live in the vicinity of the school while others travel about 5 km to and from school daily. All participants communicate properly in Oshikwanyama, which is also their first language and home language. They rarely communicate in English when they are not at school.

4.2 Qualitative data analysis

The data analysis was done manually (See 3.6) and the findings were organised and presented in themes and sub-themes with accompanying quotations from the participants. Table 4.1 captures the delineation of themes based on the qualitative study.

Table 4.1: The delineation of themes as based on the qualitative study

THEME 1: INITIAL CHALLENGES EXPERIENCED WITH PROBLEM-BASED MATHEMATICS TASKS (§4.3)	
Sub-theme 1.1: Initial personal challenges (4.3.1)	4.3.1.1 Uncertainty about the task and language barrier
	4.3.1.2 Insufficient subject knowledge
	4.3.1.3 Lack of resources
Sub-theme 1.2: Attitude towards group work (4.3.2)	4.3.2 Learners' attitudes towards group work
THEME 2: LEARNERS' EXPERIENCES WHEN SOLVING MATHEMATICS PROBLEMS (§4.4)	
Sub-theme 2.1: Problem solving activities (4.4.1)	4.4.1.1 How learners solved problems
	4.4.1.2 Problem-solving experiences
	4.4.1.3 Reflective thinking in solving problems
Sub-theme 2.2: Group work and collaboration (4.4.2)	4.4.2.1 Group communication and participation
	4.4.2.2 Sharing of learning materials
	4.4.2.3 Peer learning and support
	4.4.2.4 Pace of problem solving and task completion
	4.4.2.5 Member contribution and knowledge construction
	4.4.2.6 Peer assessment and evaluation
THEME 3: DEVELOPMENT OF ESSENTIAL KNOWLEDGE AND SKILLS (§4.5)	
Sub-theme 3.1: Development of mathematics skills (4.5.1)	4.5.1.1Clarification and mathematics concepts
	4.5.1.2 Communication of gathered information
	4.5.1.3 Mathematics skill development
	4.5.1.4 Higher-order thinking (HOT)
	4.5.1.5 Reflective thinking in mathematics
Sub-theme 3.2: Development of self-directed learning abilities (4.5.2)	4.5.2.1 Set of learning goals and determining learning needs
	4.5.2.2 Learning responsibility and persistence
	4.5.2.3 Self-reflection on learning experiences

In the following sub-sections, each theme and its related sub-themes are discussed in detail.

4.3 THEME 1: INITIAL CHALLENGES EXPERIENCED WITH PROBLEM-BASED MATHEMATICS TAKS

Mathematics forms a fundamental part of human knowledge and is essential in real life (See section 2.3). However, not all learners understand mathematics due to various reasons, including different cognitive and affective factors, among others. Grade 7 learners experienced various challenges while solving mathematical problems. Unfortunately, these challenges can affect their ways of understanding mathematics as a subject, which in turn affects the way learners apply the acquired knowledge to solve a real-world problem. To facilitate the discussion of the results regarding Theme 1, the sub-themes delineated in Tables 4.2 will be discussed in more detail.

Table 4.2: Theme 1 and its related sub-themes

THEME 1: INITIAL CHALLENGES EXPERIENCED WITH PROBLEM-BASED MATHEMATICS TASKS (4.3)	
Sub-theme 1.1: Initial personal challenges (4.3.1)	4.3.1.1 Uncertainty about the task and language barrier
	4.3.1.2 Insufficient subject knowledge
	4.3.1.3 Lack of resources
Sub-theme 1.2: Attitude towards group work (4.3.2)	4.3.2 Learners' attitudes towards group work

4.3.1 Sub-theme 1.1: Initial personal challenges

Learners face numerous challenges during the process of learning mathematics and such challenges affect their understanding of the subject in such a way that their performance in mathematics is also affected negatively. Some examples are outlined below.

4.3.1.1 Uncertainty about the task and the language barrier

Table 4.3 shows examples of quotes from participants indicating different challenges they experienced regarding understanding what was expected from them.

The letter "P" is used when referring to the participants, for example P1 refers to participant 1. Note that all quotes were transcribed verbatim.

Table 4.3 Uncertainty about the task and language barrier

Participants	Responses
P1	*I do not know what to do, because I did not understand the problem.* (W1 & I1)
P2	*Is not easy to find the answer, because I do not understand at all.* (W1 & I1)
P3	*To make a questionnaire was challenging … I do not know what to do.* (W1 & I1)
P10	*I do not understand the problem.* (W1 & I1)
P14	*Language is a problem; in most cases I do not understand the problem. I do not know what is being asked.* (I1)
P24	*I have no idea of what is to be done in the task.* (I2)
P27	*I do not understand the method to solve problem.* (I1)
P36	*I have a challenge of answering a question that I do not understand, because of the word problem.* (W1 & I1)

Some participants indicated that they were unsure about the tasks because they did not understand the mathematical problems and consequently did not know what to do, as they were unsure of what was expected from them. P1, 2, 3 and 10 revealed that they did not know what to do, while P14, P24, P27 and P36 mentioned that language was a problem.

The PBL tasks required of Grade 7 learners to compile a questionnaire (Task 1) and a budget to paint the walls of a classroom and renovate the floor (Task 2) (See Addenda F and G). Difficulty in understanding such problems can be a result of a language barrier. Many learners in the Senior Primary Phase experience comprehension and language problems when entering the phase in Grade 4. This is a result of using their mother tongue as a medium of instruction in the previous phase (See section 2.4.1). Steyn (2017) supports this by asserting that learners find it difficult to learn mathematics in English because they have learnt it in their mother tongue during the previous three years and these indigenous languages do not have all the terms and concepts that English offers. Furthermore, Sibomana (2020) also considers transition as a challenge due to learners' limited proficiency in English, since mother-tongue instruction impedes the second language acquisition and learning. Such a

language barrier is carried forward, because teachers are forced to teach in Oshikwanyama (the local mother tongue) until further grades to assist learners to understand. Although not all the Grade 7 learners are having this challenge, most of them do, depending on their backgrounds (where they came from and with whom they live).

4.3.1.2 Insufficient subject knowledge

Learners need to possess subject knowledge to be able to solve mathematical problems. However, findings indicated that most Grade 7 learners have insufficient knowledge about mathematics. Insufficient subject knowledge means that learners are incompetent in terms of mathematics concepts and cannot apply their knowledge. Some examples are outlined in Table 4.4.

Table 4.4 Learners' comments regarding insufficient subject knowledge

Participants	Responses
P4	*I have a problem of dealing with big numbers in a calculation because we do not use a calculator and the time is not always enough.* (W2 & 12)
P9	*It was not easy to measure and calculate number of tiles and carpets.* (W2)
P14	*Another challenge is getting different answers to the same question; it leaves me in a dilemma, not knowing which answer I should take.* (W2)
P24	*I struggle solving challenging problems.* (W1 & I1)
P27	*I struggle to solve [a] complex problem requiring critical thinking in a short time.* (W1)
P36	*I have a challenge of answering difficult questions and more especially word problem[s].* (W1 & I1)

Participants indicated that they lack the knowledge and skills that can help them to solve the mathematics problems successfully. P4, for example, had a problem in doing calculations manually. This is either due to a lack of strategies and insufficient knowledge to do mental calculations. P9 found it difficult to calculate the number of tiles and carpets needed to cover the floor because of a lack of understanding the calculation of an area. P14 had an issue of getting more than one answer to the same problem, because at the end he or she did not know which answer was correct. This indicates an inability regarding deciding and evaluating the answer.

In addition, P24 and P27 struggled to answer questions based on HOT, because such questions require more time to think deeply and critically to solve the problems. Another possible cause of this is a lack of cognitive skills to be used to draw logical conclusions (See 2.5). P36 experienced a problem of answering word problems due to a lack of understanding the problem or due to the learner's inability to translate and write down what is known and what is being asked. Learners were also unable to solve word problems, because of difficulties in determining the formula to solve the problem or even not being able to perform arithmetic operations (See 2.3).

These challenges indicate that many learners did not have the necessary mathematical knowledge, for example to measure and calculate the number of tiles. Moreover, they struggled to analyse the problem. Regarding to Bloom's revised taxonomy, this means that the learners experienced challenges in applying their mathematics knowledge and skills as well as HOT such as analysis, where they were supposed to analyse a problem and break it down into more understandable parts (See 2.5.1 and Table 2.2). For example, they found it challenging to determine and calculate the number of tiles and the area of the carpet needed to cover the floor, because learners were either unable to determine the steps to follow in solving the problem, or to apply arithmetic operations correctly.

Table 4.5 displays some initial challenges with the solving of mathematics problems as related to Bloom's taxonomy (See 2.5.1).

Table 4.5 Learners' mathematical thinking as based on the cognitive domain of Bloom's taxonomy

Levels	Challenges
Remember	*I forget the formulae for calculating the area of a square and rectangle. (P36)* (W2)
Understand	*I have a challenge of answering difficult questions and more especially word problems. (P36)* (W1 & I1)
Apply	*I have a problem of dealing with big numbers in a calculation because we do not use a calculator and the time is not always enough. (P4)* (W2) *Find the number of tiles and carpets needed to cover the classroom floor. (P9)*(W2)
Analyse	*I struggle to solve complex problems requiring critical thinking in a short time. (P27)* (W1)
Evaluate	*Another challenge is getting different answers to the same question, it leaves me in a dilemma, not knowing which answer I should take. (P14)* (W2)
Create	*I do not know how to make a questionnaire. (P5)* (W1) *I do not know how to change the number of learners to angles for me to make a pie chart. (P18)* (W1)

Scholars such as Gupta and Mishra (2021), and Hasan (2019), are of the opinion that learners struggle to develop HOTS when solving mathematics problems, therefore learners should be facilitated in their thinking processes. Such facilitation can be done when the teacher implements a four-step model when assisting learners to increase their HOTS, namely: (1) discussing learners' reasoning; (2) providing feedback and scaffolding; (3) offering feedback to learners' responses with praise along with 'what' questions; and (4) giving suggestions for the learners' improvement transformed into challenges (Ariska *et al*., 2020). Participants also expressed resources being a challenge as outlined in the next sub-section.

4.3.1.3 Lack of resources

Inadequacy of resources is a serious concern, because it can affect the teaching and learning process (See 2.4.6). Some responses are presented in Table 4.6.

Table 4.6: Learners' comments regarding the lack of resources

Participants	***Responses***
P4	*We are not allowed to use a calculator and some numbers are big, therefore we only used pencil and paper method which is a challenge sometimes.* (W2)
P5	*I only asked the teacher the meaning of words and used a grade 7 textbook.* (W1)
P7	*I used the Oxford dictionary and any available textbook from grade 7 to 10.* (W2 & I2)
P16	*There is only one textbook that we share in our group.* (W1)
P21	*The problem is that we only have two textbooks in our group, sometimes it is not easy to use two textbooks in a group.* (W1)
P27	*I searched for how to calculate area of combined shapes from textbooks of Grade 7 to 9.* (W2)

The availability of learning resources can pose a challenge to the way in which learners learn and understand mathematics. P4 and P21 mentioned that some materials and resources were not available for their grade, for example calculators and textbooks. P5 discussed this issue with the teacher, while others mentioned using limited resources such as a textbook, from other grades as well (Grades 8, 9 and 10). Unfortunately, most of the learners do not have a smart phone to search for additional information. In fact, learners are not allowed to bring cell phones to school and use the internet at the school. Most of the learners live with their elderly grandparents, who are not informed about smart phones and the internet, making it difficult to assist their grandchildren with this. Hofmeyer (2016) emphasised the importance of using a variety of resources in the classroom to support the solving of mathematics problems.

4.3.2 Sub-theme 1.2: Attitude towards group work

Different people have different attitudes toward group work and Table 4.7 contains comments about learners' attitude towards group work.

Table 4.7: Learners' comments regarding their attitude towards group work when solving mathematical problems

Participants	Responses
P4	*I am not always right, so in a group others can help you.* (W2 & I2)
P10	*I do not like group work because some members like to copy from others.* (W1 & I1)
P21	*I like to work in a group because other members can bring in their good ideas.* (W1 & I2)
P24	*Group member wastes time and some people do not like other people's ideas.* (W1 & I1)
P27	*I enjoy group work because in a group, work is easier and fast due to many ideas.* (W2 & I2)
P32	*I don't like group work because all group members get marks even those who did not do anything.* (W1 & I1)

P10 expressed his concern about group work because sometimes members do not like to work, they rather wait for others to finish and copy from them and therefore he does not like working in a group. On the same note, P24 does not support group work as not all members work on tasks according to a schedule, and it is very difficult to consider all members' ideas and inputs. P32 also dislikes group work and considers it as an unfair practice because sometimes, at the end of the day, all members get awarded whether they contributed to the solution or not. This can be prevented by applying the principles based on the social interdependence theory, which means that working together will increase the chances of group achievements (Johnson & Johnson, 2014). However, P4, P21 and P27 had different views regarding group work. They preferred working in a group, since there are always different thoughts, and this makes the work bearable and ensures that it is also done fast (See 2.6.5).

Additional experiences were mentioned by Grade 7 learners as set out in the following theme presented in the next section.

4.4 THEME 2: LEARNERS' EXPERIENCES WHEN SOLVING MATHEMATICS PROBLEMS

The Grade 7 learners mentioned their experiences regarding the solving of mathematics problems. To facilitate the discussion of the results regarding Theme 2, the sub-themes delineated in Tables 4.8 to 4.11 will be discussed in more detail.

Table 4.8: Theme 2 and its related sub-themes

THEME 2: LEARNERS' EXPERIENCES WHEN SOLVING MATHEMATICS PROBLEMS (§4.4)	
Sub-theme 2.1: Problem-solving activities (4.4.1)	4.4.1.1 How learners solved problems
	4.4.1.2 Problem-solving experiences
	4.4.1.3 Reflective thinking in solving problems
Sub-theme 2.2: Group work and collaboration (4.4.2)	4.4.2.1 Group communication and participation
	4.4.2.2 Sharing of learning materials
	4.4.2.3 Peer learning and support
	4.4.2.4 Pace of problem solving and task completion
	4.4.2.5 Member contribution and knowledge construction
	4.4.2.6 Peer assessment and evaluation

4.4.1 Sub-theme 2.1: Problem-solving activities

Problem solving is seen as a process of applying relevant steps with the aim of solving a problem (See 2.6.4). This aspect and others are outlined in the following sub-sections.

4.4.1.1 How learners solved problems

Table 4.9 presents learners' responses regarding how they solved the mathematical problems.

Table 4.9: Examples of how learners solved mathematical problems

Participants	Responses
P1	*Other groups borrowed us the measuring tape to measure the length of the classroom when they were not using it.* (W2)
P10	*I establish high expectation[s] and clear goal[s], work hard to achieve.* (W2 & I2)
P11	*We meet every time there was no teacher in class.* (W2)
P23	*I looked for the meaning of words from the dictionary and textbooks.* (W1, W2, I1, I2)
P23	*I searched in the dictionary, Let's do maths grade 7, platinum Grade 7 and asked my friend.* (W1 & I1)
P27	*Some group members help to go ask other groups the type of drinks they prefer.* (W1 & I1)
P36	*I determine my learning needs first.* (W2)

P1 mentioned that they had to measure the length of the classroom to calculate the area thereof, while P10 indicated that he first made sure he set learning goals before trying to work on the task. This is essential because it enables a learner to have a focus point in terms of what he or she wants to achieve after completing the task. P11 also indicated that group members should make efforts to meet whenever they are free to work on their tasks. P27 said that group members had to share responsibilities among themselves so that they can use the time wisely. P36 indicated the importance of determining learning needs first to execute the task correctly. Participants also mentioned that they used several resources to gather relevant information to work on the tasks (See also 2.5.2).

4.4.1.2 Problem-solving experiences

Grade 7 learners had both positive and negative experiences in addressing the tasks. Table 4.10 includes quotes to indicate some of the learners' responses.

Table 4.10: Learners' comments about their problem-solving experiences

Participants	Responses
P4	*I answer [check in] previous question papers and textbooks, I do homework even if no one tell me to do [to prepare].* (W2 & I2)
P7	*Exchanged ideas, thoughts, communicated freely and supported one another.* (W2)
P9	*We measured the length of the classroom and calculated the number of tiles and carpets needed, it was not easy.* (W2)
P10	*We achieved our aims, we managed to solve the problem. [responsible], established high expectation and clear goal[s], worked hard to achieve.* (W2)
P11	*It is important to know what you need to solve … before you start solving the task.* (W1)
P12	*This week everything went well, I did not have any problem.* (W2)
P25	*Some group members were just quiet doing their own things instead [of helping in solving the task].* (W1)
P27	*One group member presented our findings to the class while the rest of us were answering the questions posed by other groups [class].* (W1 & W2)
P36	*I did not experience any problems while working on the tasks.* (W2)

P4 mentioned searching for information from different resources to be able to prepare to work on the task. This is in harmony with P10 and P11 who mentioned that before you start working on a task, it is better to have objectives in mind and be aware of what is needed, as it helps you to be focused. P9 indicated that sharing materials during the task challenged the group because it was time consuming to wait for others to finish. The same participant also experienced a problem in carrying out calculations involved, due to the inability to interpret the problem and think of a possible solution. P12 and P36 mentioned that they did not have any problem while working on the mathematics tasks. This may be because they have now been exposed to PBL and gained some knowledge of how to work in groups and solve mathematical problems. P27 mentioned shared responsibilities between members, which is a good indication of collaboration of members within the same group. However, P25 did not enjoy working on the tasks,

because of group issues such as some members not contributing (See 2.6.5). As a result, it is essential that learners develop good interpersonal skills, self-monitoring abilities, and valuable communication skills. Good interpersonal skills enable learners to work together, share ideas, and learn from each other, thus promoting cooperative and collaborative learning. As learners work together, it promotes the expansion and growth of knowledge among peers (See 2.6.4).

Some learners regarded challenges experienced during the learning process as an obstacle that prevented them from learning or achieving success, while others viewed such challenges as an opportunity to explore and create new knowledge and skills. Lai (2017) mentions that learners can enjoy learning if they are self-directed, while Mabena *et al*. (2021) mention that some learners do not enjoy the subject or perform well due to both teacher and learner-related factors.

4.4.1.3 Reflective thinking in solving problems

Reflection is important in the learning process, as it helps learners to think about what they have done, what they have learned, and how they can apply it in their everyday life. Reflective thinking is also crucial in mathematics (See 2.5.2.3). Table 4.11 consists of examples of learner quotes regarding their reflective thinking in solving mathematical problems.

Table 4.11 Learners' comments about reflective thinking in solving problems

Participants	Responses
P10	*I evaluate my answers to make sure I have it right. (I2)*
P14	*I re-read the question and check if I answered what [was] asked.(I2)*
P19	*I judge the answer and make sure I answered properly. (I2)*
P21	*If I am wrong, I make correction.(I1)*
P24	*If I use different ways and did not get the same answers, I compare them and select the one answer carefully.(I2)*
P27	*Yes, make correction on my own, do homework; study at night and during weekends.(I2)*
P36	*When I get different answers to the same question, I take the best thoughtfully.(I2)*

Participants indicated that they looked back and reflected on what they have done. P10, 14 and 19 mentioned that they evaluated their answers to ensure that they perform better in the future. Others mentioned comparing their answers, which involves reflection in terms of determining what went wrong and what went right, maintaining the strengths and working on the weaknesses to improve (See 2.6.5).

Metacognitive and reflective thinking play a major role in the learning process. Table 4.12 aligns learners' responses with the literature (See 2.5.3) regarding examples of their metacognitive and reflective thinking in the mathematics tasks.

Table 4.12 Examples of metacognitive and reflective thinking

Metacognition and reflection	Responses
Planning	*I establish high expectation and clear goal, work hard to achieve. I take something from Grade 8 and see if I can solve it, if not then I ask for assistance. (P10)* *I ask myself what I need to solve the problem. (P31)* *Yes, I think of what I need to know apart from what I know. (P36)*
Monitoring	*I do my work on my own when no teacher in class. (P11)* *I enjoy challenging problems very much, they help me to learn new things, avoid make noise or go outside to play. (P24)*
Evaluation	*I evaluate by re-reading the question and check if answered what asked. (P14)* *I judge the answer and make sure I answered properly. (P19)*
Reflection	*I feel bad when I do not understand. (P11)* *I make correction on my own. (P27)*

Table 4.12 shows examples of participants' metacognitive thinking. For example, setting goals (P10), monitoring their thought processes (P24), and evaluating their answers (P14 and P19). In addition, P27 reflected on her thinking, identified some issues, and made corrections to ensure she was successful. Such thinking is aligned with Kleden (2015) as he argues that an effective metacognitive strategy for learning mathematics includes planning, monitoring, reflection, and evaluation to succeed.

4.4.2 Sub-theme 2.2: Group work and collaboration

Group work and collaboration are essential in problem-based tasks. This sub-theme outlines learners' experiences and participation.

4.4.2.1 Group communication and participation

An essential aspect of problem-based learning is the ability to work together in groups when solving a problem. Table 4.13 consists of learners' comments regarding their group communication and participation in the mathematics tasks.

Table 4.13: Learners' comments regarding group communication and participation

Participants	Responses
P7	*Our group members communicated freely with one another in order to solve the task.(W2)*
P11	*The interaction was good even though some members did not communicate.(W2)*
P13	*The members' involvement was excellent and we all communicated to each other well.(W1 & W2)*
P25	*Some members interacted, communicated and participated, but some were just quiet doing their own things instead of participating.(W1)*
P27	*I enjoy group work because in a group work is easier and fast due to many ideas.(I2)*

Participant 7 indicated that their group members communicated freely and P13 mentioned that the interaction among group members was good, because most of them had knowledge of working together from the first group task. However, others indicated that some group members were not actively involved. This may be because of such members not being serious or because of other members' bad behaviour, for example your ideas not being considered. Good group communication is essential for the group to succeed (See 2.6.5).

4.4.2.2 Sharing of learning materials

Table 4.14 includes learners' comments regarding the sharing of learning materials.

Table 4.14: Learners' comments about sharing learning materials

Participants	Responses
P1	*Other groups borrowed us the measuring tape to measure when they were not using it.(W2)*
P21	*We share two textbooks that we have in our group.(W1)*
P27	*I borrowed textbooks from other grades.(W2)*

Participants mentioned some challenges with resources. Since the school does not have enough learning materials (See 4.3.1.3), group members had to share the available learning materials to execute the tasks. It was clearly indicated by P1, P21, and P27 that they always had to borrow the materials they wanted to use and could only do so if other learners were not using it. These available materials are only provided by the school and they are not always enough for all learners, since the enrolment numbers are high. This is the reason why the materials were never enough or insufficient (See 2.4.6). This is in harmony with Cheng (2016) asserts that the teaching and learning materials may not be enough for all learners or may not be suit the need as per the curriculum.

4.4.2.3 Peer learning and support

As Grade 7 learners worked on the PBL tasks in the groups, members had experiences regarding the learning and support they received from each other. Table 4.15 contains learners' quotes regarding peer learning and support.

Table 4.15: Learners' comments about peer learning and support

Participants	Responses
P4	*I am not always right, that is why I like working with others as they can help, they can check my answer and help me when needed, which is a good thing.(I2)*
P7	*Our group was very supportive; we supported one another to make sure the task is addressed.(W2)*
P14	*I learn from others and feel safe from making [a] mistake as they help me.(I2)*
P19	*Group members can help you understand the question better, since we can learn from others too.(I2)*
P23	*All members supported the group in order to solve the mathematical problems.(W2)*
P27	*I prefer working with other members, because they might have better understanding, and share responsibilities, one member presented while the rest of the members were answering the questions posed by other groups.(W2)*

Participants indicated that their group members supported each other during the execution of the task. Participant 4 mentioned that other group members could verify his answer because they might understand what he did not understand. Participants 14, 19, and 27 mentioned that they preferred to be assisted by other members of the group to minimise their chances of making mistakes. Participant 7 concurred with P19 and P23 that the members in their group exchanged ideas, communicated freely, and supported each other. Similarly, P27 referred to group members who cooperated and shared the group to succeed. Group work is strongly recommended, due to its impact on the knowledge growth, which is a result of social interaction among learners, and which subsequently promotes peer learning (See 2.6.5).

Although some groups had challenges in working together, others were able to work together according to the principles of group cooperation (Johnson & Johnson, 2014) (See 2.6.5). Table 4.16 displays some examples of learners' cooperation during the PBL intervention when working on mathematics tasks.

Table 4.16: Some examples of group cooperation in PBL tasks

Characteristics of cooperative group work	Responses
Positive interdependence	*I am not always right, that is why I like working with others as they can help, they can check my answer and help me when needed, which is a good thing. (P4)(I2)*
Personal responsibility	*I do my work on my own, my education is [in] my own hands. (P19)(I2)*
Promotive interaction	*I learn from others and feel safe from making mistake[s] as they help me. (P14)(I2)*
Social group skills	*The members' involvement was excellent and we all communicated to each other well. (P13)(W2)*
Group processing	*The group achieved the aims. (P24)(W1)*

Participant 4 mentioned that she does not always get things correct, she might sometimes solve a problem incorrectly and she prefers to work in groups to be assisted by other learners. P19 indicated that he takes responsibility for his own learning by doing his schoolwork on his own, while P14 indicated that he enjoys learning from others since they might have good ideas, which may be better than his own. Participant 13 mentioned that group members were involved and communicated well to find a solution to the task. P24 mentioned that their group managed to address the PBL tasks. Learners developed cooperative learning skills, because they worked closely with one another and through the help from more knowledgeable individuals, they acquired new knowledge. Supporting this, Rajabzadeh *et al*. (2022) point out that learners may grow in several important areas when they learn in groups. Downton *et al.* (2017) further state that each group consists of members with several abilities and interests, which may serve as an advantage, as they may learn from each other when they critically discuss while working together to address a problem.

4.4.2.4 Pace of problem solving and task completion

Table 4.17 includes examples of quotes from participants indicating the pace of problem solving and task completion.

Table 4.17: Learners' comments regarding the pace of problem solving and task completion

Participants	Responses
P12	*We achieved our aim; we managed to solve the problem. (W1 & W2)*
P14	*I don't finish because the time was not enough.(W1 & I2)*
P15	*We took time because the materials were not enough, sharing [consuming] a lot of time.(W2)*
P27	*Our group was a bit slow in solving mathematics tasks because there was not enough time to meet to do the task together. (W2)*
P27	*We finished quickly, fast. (W2)*
P32	*We were fast when it comes to completing the task, we tried our best to meet as often as possible in order to finish on time and I like it.(W2)*
P36	*Our mission is accomplished; we solved the problem, no challenge. (W1)*

The lack of sufficient materials and time for group members to meet affected the pace of some groups in addressing the problem. As P14, P15, and P27 (during mathematics Task 1) indicated, they were a bit slow due to aforementioned reasons. However, some members such as P12, P27 (mathematics task 2), P32 and P36, revealed that their group was fast because the members tried to meet as often as possible to ensure that the task is completed on time. Time management is essential, as it is one of the characteristics of a self-directed learner (See 2.7 and 2.7.1). Learners set priorities, plan, and implement these priorities in the available time (Mathur *et al.*, 2021). Since there was not enough time for learners to meet and work on the tasks, it was important to organise themselves well and set realistic goals (See 2.7.5).

4.4.2.5 Member contribution and knowledge construction

In group work, each member is responsible for contributing toward the completion of the task. Table 4.18 consists of quotes from participants' regarding member contribution and knowledge construction.

Table 4.18: Learners' comments regarding member contribution and knowledge construction

Participants	Responses
P7	*We supported each other, our group exchanged ideas and thoughts in order to solve the task. Each member deserve[s] 10 out of 10. (W2)*
P10	*Yes, there are those good in math they can help by explaining … establish high expectation and clear goal, work hard to achieve.(I2)*
P10	*Sometime there are many ways and get different answers so I compare and select the correct. (I2)*
P14	*I enjoy new challenges, it enable[s] me to think critically.(I2)*
P19	*[Curious] it can bring my subject knowledge to high level.(I2)*
P27	*One member presented while the rest of the members were answering the questions posed by other groups, therefore all of us got 10/10.(W1)*

P7 mentioned that their group members contributed well and therefore everyone deserved good marks. This is in harmony with P19 who indicated that their group members supported each other to get the task completed. P27 also indicated that each member had something to do, so they shared responsibilities among themselves. Group members formed part of the discussion and contributed to the solution. The knowledge constructed by learners during the group discussion should be easily remembered. Knowledge constructed in this way is valuable, as it is retained longer and enables learners to conceptualise knowledge, and solve problems in real life (Benson-O'Connor *et al.*, 2019).

4.4.2.6 Peer assessment and evaluation

Peer assessment and evaluation are very important tools that can help to determine how group members contributed to the group work. Table 4.19 includes examples of quotes from participants regarding peer assessment and evaluation.

Table 4.19: Learners' comments about peer assessment and evaluation

Participants	Responses
P5	*All members made [an] outstanding contribution, each member got 10 out of 10.(W1 & W2)*
P7	*Member 8 did well, he drew (W2)*
P10	*Member 8 did nothing (W1)*
P11	*Some people are not good to work with; they do nothing and deserve no marks.(W1)*
P24	*Group members do not like other people's ideas.(I1)*
P28	*Some members did not want to write other people's work.(W1)*

Participants indicated that some group members did nothing and deserved few or no marks at all. This is clearly seen in P10 and P11 who specifically mentioned that P8 and some others did not contribute to the group. This could be because some members might perceive others as unable of doing something correctly. P24 and P28 indicated that some members did not want to consider others' ideas, because they thought they were wrong. To support this, Davidson *et al.* (2014) pointed out that group work may pose some challenges to members due to poor interpersonal skills, low self-monitoring skills, and some members may suffer pressure from others. However, P5 and P7 had good experiences regarding this. They indicated that all group members deserved good marks because they contributed well toward solving the second mathematical problem (Task 2) and participants completed the responsibilities assigned to them within their groups. This can be clearly seen in the comments of P10 who mentioned that, although P8 could not contribute to the answer, at least he did something, which was to draw graphs. The group benefitted from different members because they had different views and abilities. In harmony with this, Downton *et al.* (2017) state that each group consists of members with several ideas and interests and through critical discussions and working together to address a problem each member can benefit (See 2.6.5). The next section presents a discussion of Theme 3.

4.5 THEME 3: DEVELOPMENT OF ESSENTIAL KNOWLEDGE AND SKILLS

The main objective of teaching and learning mathematics is to help learners develop mathematical knowledge and skills. These skills enable learners to deal with mathematics and other subjects in the classroom and outside the classroom in everyday-life events. To facilitate the discussion of the results regarding Theme 3, the sub-themes delineated in Table 4.20 are outlined in more detail.

Table 4.20: Theme 3 and its related sub-themes

THEME 3: DEVELOPMENT OF ESSENTIAL KNOWLEDGE AND SKILLS (§4.5)	
Sub-theme 3.1: Development of mathematics skills (4.5.1)	4.5.1.1Clarification and mathematics concepts
	4.5.1.2 Communication of gathered information
	4.5.1.3 Mathematics skills development
	4.5.1.4 Higher-order thinking (HOT)
	4.5.1.5 Reflective thinking in mathematics
Sub-theme 3.2: Development of self-directed learning abilities (4.5.2)	4.5.2.1 Setting of learning goals and determining learning needs
	4.5.2.2 Learning responsibility and persistence
	4.5.2.3 Self-reflection on learning experiences

4.5.1 Sub-theme 3.1: Development of mathematics skills

Mathematics is an essential subject, as it helps learners to develop an understanding of spatial concepts and relationships and develops their ability to apply mathematics in the contexts of everyday situations. The mathematical skills developed by learners are the clarification of mathematical concepts, communication of gathered information, drawing of graphs, measuring of lengths, and other calculations involving money and decimal fractions. Some skills that learners have developed are outlined in Tables 4.21 to 4.25.

4.5.1.1 Clarification and mathematics concepts

Table 4.21 includes examples of quotes from participants regarding the clarification of mathematical concepts.

Table 4.21: Learners' clarification of mathematical concepts

Participants	Responses
P2	*I search for details on how to calculate the area. (W2)*
P7	*I searched for information about writing the number of learners as degrees (angles) in order to draw the pie chart.(W1)*
P13	*I looked for graph-related information.(W1)*
P15	*I look for meaning of the word questionnaire and other information about different graphs.(W1)*
P27	*I asked the teacher to help me to calculate the area of combined shapes.(W2 & I2)*

Participants indicated that they searched for various information to be able to clarify mathematical terminologies involved in the tasks that they did not understand. These terminologies included terms related to area (P2), degrees (P27) and, graphs (P7, P13, and P15). To gather relevant information, group members had to do research and report their results to the whole class. This requires from learners to clearly understand the terminology themselves before presenting their results to the class (See 2.6).

4.5.1.2 Communication of gathered information

Information that was gathered by learners needs to be presented and communicated in in an understandable way. Table 4.22 includes examples of quotes from participants regarding their communication.

Table 4.22: Learners' comments about communication of gathered information

Participants	Responses
P11	*We draw the graphs.* (W1)
P27	*One group member presented the finding.(W1)*

P11 indicated that the information gathered by groups was presented in the various graphs and after each group was done, one member from every group presented their finding to the class (P27). Communication of mathematical ideas can be in the form of pictures, or graphs can promote the understanding of concepts and solving of problems. Furthermore, discussing and presenting a group's solution promote communication skills and critical dialogue about the subject, since there is a chance of discussing and listening (See 2.6.6.).

4.5.1.3 Mathematics skill development

Communication is not the only skill that learners need to develop and enhance. Table 4.23 consists of quotes from learners regarding mathematical skills development.

Table 4.23: Mathematical skills development

Participants	Responses
P5	*We measured the length of the classroom and calculated its area.* (W2)
P6	*We found out the number of 2 ℓ drums of paint that can paint the wall.* (W2)
P12	*We calculated the number of tiles and carpets needed to cover the floor.* (W2)
P17	*Made a questionnaire. (W1)*
P18	*We drew the graphs such as a pie chart.* (W1)
P22	*Know how to calculate amount of money needed to renovate the classroom. (W2)*

P5 mentioned that the mathematical skills they developed were measuring in units of length as well as calculating the area of a rectangular shape and a square. P6, P12, and P22 indicated that they did some calculations involving whole numbers (the number of tiles, carpets and a 2 ℓ tin of paint), decimal fractions (area of the floor and wall) as well as calculations that involved money (computation). Furthermore, P17 indicated that he learned to compile a questionnaire (communication skills), while P18 mentioned that he learned to draw different types of graphs (presentation skills). From

these examples, it is essential that Grade 7 learners develop mathematical skills and be able to apply such abilities in the classroom as well as the real-world context (Vale *et al.*, 2019). The learning of such mathematical skills is important, as it may be the strongest predictor of future mathematical success (Harris & Petersen, 2017).

4.5.1.4 Higher-order thinking (HOT)

As discussed in Chapter 2 (See 2.5.2), it is crucial that learners develop HOT abilities in mathematics, which enable them to solve real-world problems. Therefore, the learning activity must encourage the use of reasoning, creative, and critical thinking skills. Table 4.24 consists of some quotes regarding such thinking abilities.

Table 4.24: Learners' HOT

Participants	Responses
P14	*I enjoy new challenges; it enables me to think deeply.* (I2)
P19	*Challenges can bring my subject knowledge to a high level.* (I2)
P24	*I struggle solving challenging problems.* (W1 & I1)
P27	*I struggle to solve complex problems requiring critical thinking in a short time.* (W1)
P36	*I have a challenge of answering difficult questions and more especially word problem.* (W1 & I1)

Participants 24, 27, and 36 indicated that they struggled with problems that require HOT or problems that demand critical thinking, which could be a result of factors associated with the teacher, for instance learners were not used to working on tasks that require HOTS. This is in accordance with Retnawati *et al.* (2018), who asserted that some learners struggle to develop HOT. Reasons for this could be the factors related to the teacher that may influence the development of learners' HOTS. Such factors include the teacher's knowledge about HOTS, the teacher's self-efficacy, the teacher's teaching and learning strategy, and the teacher's evaluation process (Bhattacharya & Mohalik 2021). Firdaus *et al*. (2015) emphasise that learning should start from simple content and develop into complex content, so that learners can use their prior knowledge to make sense of more complex or challenging problems that require critical thinking to enhance their thinking ability. However, P14 and P19

believed that they enjoy such problems, because they keep them busy and enhance their thinking ability.

4.5.1.5 Reflective thinking in mathematics

Reflective thinking is significantly related to mathematical problem-solving skills. It involves reflection, which is the process of internal evaluation and exploration of problems by connecting with related experiences (Rahmi *et al.,* 2020). Table 4.25 displays some feedback from participants that indicate learners' reflective thinking in mathematics.

Table 4.25: Learners' reflective thinking in mathematics

Participants	Responses
P10	*I evaluate my answers to make sure [I] have it right.* (I1)
P14	*Problem tells me what to know. When I am done, I re-read the question and check if [I] answered what was asked.* (I2)
P19	*I judge the answer and make sure I answered properly.* (I1 & I2)
P21	*The learning goals help me to know what is expected from me in terms of competencies.* (I2)
P24	*If I use different ways and did not get same answers, I compare them and select carefully.* (I2)
P27	*I ask myself what I need in order to solve the problem … double check if correct* .(I2)
P31	*Yes, I think of what I need to know apart from what I know.* (I1)

P14, P21, P27, P31, and P36 mentioned some examples of reflection and emphasised that such thinking helps them make connections between what they already know and what they need to learn, and it enables them to apply their prior knowledge. This is an indication that they can plan properly and monitor their thinking while solving mathematics problems. P10, P14, and P19 showed that learners also looked back to their solutions, thought about what happened, and what went wrong to avoid repeating the same mistakes in the future. Similarly, P24 mentioned the importance of comparing multiple answers of the same question to make an informed choice. Such

activities are essential in mathematics since they enhance critical thinking and the solving of problems.

Among the qualities of a self-directed learner, Knowles (1975) also included evaluation of the outcomes of one's own learning. Learners are expected to make judgements about their abilities to complete the task successfully and reflect on the learning process to solve a mathematical problem (Xenofontos & Andrews, 2014). When learners are responsible for their own learning, they evaluate solutions to make a meaningful conclusion of what is being learned. This is important, as reflection plays a crucial role in learners' mathematical thinking. Lee (2021) asserts that reflective thinking is one of five forms of mental activity from which mathematical understanding emerges. Lee (2021) further states that, through multiple solutions, learners are encouraged to solve problems in different ways, to reflect on what they have done during the solution processes, and to build connections between the different solutions and these connections significantly influence learners' conceptual understanding (see 2.5.2.3). The next section presents sub-theme 3.2 regarding self-directed learning.

4.5.2 Sub-theme 3.2: Development of self-directed learning abilities

To enhance Grade 7 learners' SDL, learners had mathematical problem-based tasks to solve. SDL is important, because when learners are self-directed, they are likely to perform well, not only academically, but also in general. SDL was outlined in Chapter 2 and requires learners to be personally involved in their learning processes. Some examples of SDL abilities are outlined below.

4.5.2.1 Setting of learning goals and determining learning needs

Learning goals are set to achieve a specific outcome. Table 4.26 contains examples of quotes from participants indicating the setting of learning goals and determining learning needs.

Table 4.26: Setting of learning goals and determining learning needs

Participants	Responses
P10	*I establish high expectation[s] and clear goal[s], work hard to achieve.* (I1)
P14	*Problem tells you what to know, [I consider that as my goal].* (I1)
P21	*Goals is what [is] expected from me in terms of competencies, [I therefore set that as my goal].* (I2)
P24	*My learning goals is what the problem wants me to learn and know.* (W1 & I1)
P30	*[I think of] what I need to know apart from what I know.* (I2)
P36	*The question describes learning goals, so I determine my learning needs first.* (W1 & I1)

Learning goals enable learners to remain focused on what is expected from them. Participants indicated that they set goals for themselves and work hard to achieve these goals. Some participants considered what the tasks required from them and set those expectations as their goals (P10, P21, P24, and P36). According to P30, a learning need is a gap between what a learner already knows and what he or she needs to know. Participant 36 also stated that he knows how to determine his learning needs. When learners can realise the learning needs, they can see the gap in their learning process, and this ought to assist them to focus on what they need to learn and know. It is therefore essential that learners set their own learning objectives before approaching a task. This will enable them to focus on what they need to learn and know.

Erturan and Jansen (2015) emphasise the important task of the teacher to bridge the existing gap within teaching and learning to enable learners to realise their learning needs and to achieve their learning goals. Therefore, learners will clearly understand beforehand what they are going to learn from the problem if they are able to set goals for themselves (Mahajan & Singh, 2017) (See 2.6.3.1, 2.7 and 2.7.3).

4.5.2.2 Learning responsibility and persistence

The primary purpose of this study was to investigate learners' SDL to take responsibility for their own learning. Table 4.27 contains some examples of participants' responsibility in learning and persistence.

Table 4.27: Learners' comments on learning responsibility and persistence

Participants	Responses
P4	*I do my homework; no one tells me to do. Some mathematical problems might be tough, but I try them until I get the answer.* (I2)
P10	*It is my responsibility to study hard, I take something from Grade 8 and see if can solve, if not I ask for assistance.* (I2)
P11	*I do not need to wait for someone to tell me. I take my book and study.* (I1)
P14	*I learn to be fast, understand question on my own and work without somebody. I prefer challenging problems, because they keep me busy until I get the answers.* (I2)
P21	*I check what to revise when there is no teacher in class.* (I1 & I2)
P24	*I practice even not told by teacher.* (I2)
P27	*I make corrections on my own, do homework; study at night and during weekends.* (I2)
P36	*I take ownership of my own study, I do not rely on someone.* (I2)

Participants indicated that they take responsibility for their own learning, because in most cases they do their schoolwork without somebody reminding them to do so. P4, P10, P11, P24, and P36 mentioned that they work on their own, even if no one tells them to do so. P21 and P27 mentioned that they are also responsible for their own learning because they use extra time such as weekends and evenings to do their schoolwork. Furthermore, P4 indicated persistence because he tried to solve the problem until he obtained an answer. This is in harmony with P14 as she mentioned that she kept herself busy with a challenging problem and in the process of finding a solution she learned a lot. Learners' responsibility and experience regarding their learning are crucial, as these aspects can influence the way in which they learn and enable them to address more challenging problems.

Mentz *et al.* (2019) and Du Toit-Brits (2019) elaborate on some of the characteristics of self-directed learners as persistence and a sense of responsibility for one's own learning. Cheng (2016) mentions persistence among other qualities of SDL as key to completing the task. When learners spend more time on the task with the aim of completing it, they possess self-directed skills. Self-directed learners persist in achieving task completion, even if they are not immediately successful, which is a good indication of SDL development (NCTM, 2013). As a learner spends more time on the task, such a learner may become involved and as a result, he or she will be willing to learn even more (Sanchal & Sharma, 2017). Shifting responsibilities to learners may enable them to identify additional resources that will assist them in the learning process and the development of new knowledge on which deeper learning can be based (Barton & Novak, 2017) (See 2.7.1).

4.5.2.3 Self-reflection on learning experiences

Table 4.28 presents learners' comments regarding self-reflection on their experiences.

Table 4.28: Self-reflection on experiences

Participants	Responses
P10	*Sometimes there are many ways to solve a problem and you may get different answers, so I compare them and select the correct.* (I2)
P11	*If I do not understand, I feel bad. Challenges make me tired, but are important. If you get many answers to one question you have to be careful because it confuses people.* (I2)
P14	*I enjoy new challenges; it enables me think critical.* (I2)
P19	*Challenges can bring my subject knowledge to high level.* (I2)
P21	*Challenges keeps me busy trying to get answer and If I do not understand I do research. If I am wrong, I make correction.* (I2)
P24	*I enjoy challenging problems very much, they help me to learn new things, avoid make noise or go outside to play.* (I2)
P27	*Challenges help me think fast and be able to handle complex problem and improve.* (I2)
P36	*I want to try whatever comes my way to expand my mathematical knowledge.* (I2)

Although some learners may view challenges as a threat and find it overwhelming, others may experience it as an opportunity to create new knowledge and develop essential skills. P11 indicated that she felt bad about mathematical challenges, especially when she did not understand the problem or when she obtained many possible answers, because at the end she did not know which answer to choose. This was also the case with P10, P11, and P19, because with many solutions to a problem, you must always compare them and select one answer, which is a challenge. P21 felt that getting a wrong answer and waiting for the teacher to give the feedback, worked well for him. This can be a sign of laziness and a possible lack of self-trust and confidence. Some participants regarded challenges as a good opportunity to expand their knowledge, because solving challenges involves doing research, which may result in knowledge construction. Khalid *et al*. (2020) as well as Ngololo and Kandjabanga (2021) support this in that learners take responsibility to adjust their learning process and goals. Responsibility is important in the learning of mathematics, because if one is not responsible, he or she will not make time for learning, nor reflect on what has happened. By doing so will likely negatively impact mathematics performance.

When learners develop SDL skills, they tend to be curious and willing to try new things, view problems as challenges, and enjoy learning as mentioned by Lai (2017). On the other hand, learners who are not self-directed, do not really enjoy the subject (mathematics), and do not perform well as outlined by Ukobizaba *et al.* (2021) and Hamukwaya and Haser (2021). Stupel and Ben-Chaim (2017) claim that it is not easy to choose between multiple solutions, especially when learners are uncertain which solution is correct. Learners are confused when they are uncertain about the solution or when group members suggest a different answer (See 2.5.2.3).

4.6 Comparison before and after the intervention regarding learners' group work and responsible learning

Table 4.29 displays some examples of learners' experiences as based on their completion of the questionnaires before and after the intervention.

Table 4.29: Learners' coded examples regarding their mathematics experiences

Learner's coded examples regarding their mathematics experiences		
Participant	**Before the intervention**	**After the intervention**
P1	W1: mt: Only some members assisted and wrote.	W2: mt: We supported the group by working together, some do this, others do that and give ideas, was nice.
P4	I1: Yes, my learning responsibilities is that I study hard.	I2; Yes, I take the responsibility of my learning by answering previous question papers and questions in textbooks. I also do my homework even if no one tell me to do.
P5	W1: mt: The teacher helps us to solve the group issues of not cooperating by telling us to work together.	W2: mt: We solved the issue of not having enough time by using the study and free time.
P6	W1: mt: We were very slow because we did not understand the problem.	W2: mt: This week we were fast, because all members cooperated.
P10	I1: I do not like group work, because members like to copy answers from others	I2: I prefer working in groups because there are those good in maths, they help me by explaining
P13	W1: mt: I had a problem with one member who did not want to work in a group.	W2: mt: The problem was that other group members did not want to share the measuring tape.
P14	I1: If I did not get the answer to a challenging problem I wait to be assisted, from there I know.	I2: Yes, enjoy new challenges because it enable me [to] think critical.
P19	W1: mt: The problem I had was to come up with questions in a questionnaire.	W2: mt: I did not experience any problem this week.

P21	No, I do not know what to learn.	Yes, I formulate my learning goals because the goals what is expected from me to know in terms of {competences}.
P24	I1: Yes, I do evaluate the solutions to the problems but not always.	I2: Yes, it is important to evaluate the answer to check for correctness by double check and judge it.
P25	I1: We told those who did not want to work that they will not get marks.	I2: When there are those who do not want to work, we always reminded ourselves about the aims and meaning of group work.
P27	I1: The problem was that some members were writing their own answers only.	I2: I did not experience any problem, this time we worked together very well.
P30	I1 & mt1: No if I do not understand I wait for the teacher to give the answers, then I make correction.	I2 & mt: Yes, I search or look in different textbooks, dictionary, and use other materials such math sets, etc.
P31	I1: No, try answer question in best way.	I2: Yes, what I need to know apart from what I know.
P36	I1: No, if I get different answers to the same question I don't compare, it confuses (leave me in dilemma) me because I do not know which one I should take.	I2: Yes, if I get different answers to the same question, I compare the answers and take one thoughtfully.

It is clear from Table 4.26 that learners made certain improvements considering their responses before and after the PBL intervention. For example, P1, P13, and P27 mentioned challenges they experienced while working in their groups such as that some group members did not support the group in solving the mathematical problem, some did not want to consider other members' ideas, while some group members did not want to share the materials. This is supported by Alfares (2017), who mentions

that learners' behaviours can result in uncooperative work in groups. However, after the PBL intervention, the same participants stated that members supported the group, they worked together, and they even divided responsibilities among themselves. P1 further mentioned that the group members shared some ideas. Similarly, before the intervention, P6 mentioned that their group's pace in addressing the task was very slow, but after the intervention, they progressed faster because all members cooperated. When Alfares (2017) relates to the advantages of group work, he mentions that group work can enhance the motivation and courage of learners to participate, and allow them to be more independent, because they are encouraged to speak during the group presentation or during peer teaching.

P4 knew that it was his responsibility to learn but could not identify how to do so. After the PBL intervention, he clearly stated that he did not wait for someone to tell him to study, but instead made time to answer previous question papers and to do other schoolwork. In harmony with this, Yang and Jiang (2014) defined SDL in terms of the amount of responsibility a learner accepts for his or her own learning. There is always a challenge to make time to meet regularly face-to-face on the group work as mentioned by Roychowdhury *et al.* (2020). Before the PBL intervention, participants indicated some challenges with group work. In contrast, after the intervention, Grade 7 learners mitigated this challenge by meeting during a free lesson (when a teacher did not turn up for a lesson) or during study time without involving the teacher as indicated by P5.

Before the intervention some learners did not like working in groups, but after the intervention, they seemed to prefer working in groups as mentioned by P10, considering the contribution of ideas from the group members who might have better ideas. In addition, P14 mentioned that she did not enjoy challenging problems, neither did she persist with the task before the intervention. She rather waited for the feedback from the teacher and made corrections. After the intervention, the same participant mentioned that she enjoyed new challenges, because it kept her busy and enabled her to think critically. Before the intervention, P19 had a challenge in solving a specific problem, but after the PBL tasks, he indicated that he no longer experiences problems. Participants 21 and 31 did not know how to formulate learning goals, however after the intervention indicated that they were able to do so. Similarly, P24 specified before

the intervention, that he did not always evaluate the solutions, but after the intervention, he clearly understand the importance of evaluating solutions to a problem to check for correctness and to make changes where needed. Participant 25 indicated they had some arguments about marks in the group, however after the intervention, members made it a goal to work on the task by reminding each other of the purpose of group work. Moreover, Participant 30 mentioned that she did not use various materials such as textbooks, however, after the intervention she employed several resources. From the findings it is evident that the Gr 7 learners developed essential knowledge and skills to solve real-world problems

4.7 Summary of the chapter

In this chapter, the findings were presented, discussed, and interpreted. Initially, most of the Grade 7 learners did not possess the skills that would enable them to solve real-world mathematical problems, improve their performance, and become self-directed learners. It was therefore necessary to intervene as a teacher and a researcher by employing PBL tasks to assist the learners. They completed two tasks, a weekly reflective sheet after each task and purposefully selected learners participated in semi-structured interviews. These activities generated the data from which the three themes emerged. These themes are: the initial challenges of the learners, learners' experience while working on PBL tasks, and the development of essential knowledge and skills. As a result of learners' involvement with PBL tasks, they ought to develop certain SDL skills which are vital for the learning of mathematics.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

This chapter aims to present the conclusion drawn from the data obtained and to make recommendations for future research. The chapter commences with a brief overview of the previous chapters.

5.2 Summary of chapters

Chapter 1 introduced this study by referring to the background and rationale for this research. It also provided an overview of essential aspects such as mathematics teaching and learning, SDL, and PBL. The research questions and aims were formulated, and the research approach, methodology, trustworthiness, and ethical considerations were briefly mentioned.

Chapter 2 provided a conceptual-theoretical framework regarding mathematics learning as well as the development of essential knowledge and skills. In particular, the first and second research aims were addressed, namely 1) to investigate what Grade 7 mathematics, problem-based learning, and self-directed learning entail, and 2) to investigate which problem-based tasks can be used in Grade 7 mathematics to help learners become self-directed.

Chapter 3 outlined the empirical research. The paradigm, research design and methodology, the population and sample as well as data gathering and analysis procedures, and related details such as ethical aspects, the researcher's role, and trustworthiness were discussed in this chapter.

Chapter 4 presented and discussed the research findings within three broad themes namely:

- Theme 1: Initial challenges experienced with problem-based mathematics tasks;

- Theme 2: Learners' experiences when solving mathematics problems; and
- Theme 3: Development of essential knowledge and skills.

The next section answers the research sub-questions and then the main research question.

5.3 RESEARCH FINDINGS

This section indicates how the findings discussed in Chapter 4 relate to the sub-questions and the main research question that guided the study. First, the three sub-questions are addressed, followed by the main research question.

5.3.1 GRADE 7 MATHEMATICS, PROBLEM-BASED LEARNING, AND SELF-DIRECTED LEARNING

Research sub-question 1:
What does Grade 7 mathematics, problem-based-learning and self-directed learning entail?

Grade 7 mathematics is a continuation of preceding grades (4, 5, and 6) and forms a fundamental basis for the grades ahead (the secondary grades). The Grade 7 mathematics curriculum provides teachers with a framework for supporting learners to deal with the numeracy demands to meet future challenges. The curriculum aims to develop learners' abilities in terms of the conceptual understanding of the topics, provides for active involvement in solving problems and enables learners to apply such problems to new contexts. Thus, the ability to understand, interpret, and make sense of everyday situations in mathematical concepts and competencies is crucial for solving problems and to improving their lives (See 2.2).

It is expected of learners to attain several competencies such as the understanding of mathematics concepts and relationships, and using expressions (See 2.2), carrying out specific calculations, applying logical thinking, and making interpretations. The main topics are the following: data handling, whole numbers, common and decimal

fractions, money and finance, measure, graphs and functions, and basic algebra (NIED, SP Mathematics Syllabus, 2015) (See Addendums G & H).

Therefore, Grade 7 mathematics covers a range of expectations for learners to develop an understanding of spatial concepts, relationships, apply mathematics in the contexts of everyday situations and related subjects as well as to use mathematics as a means of communication with emphasis on the use of clear expressions (See 2.2 and 2.5.2).

PBL focuses on addressing open-ended and real-world problems in a particular context that do not offer an immediate solution. It is a learner-centred approach and provides an opportunity for learners to work together in small group where all the members are responsible for solving the problem. The role of the teacher in PBL is to facilitate and scaffold the learning process (See 2.5.2). PBL requires learners to search for relevant information and apply their knowledge with the aim of constructing new knowledge (See 2.6.3). The problem is formulated in such a way that there could be more than one solution, the solution is not obvious and the solving of such a problem requires the integration of HOTS such as decision making, creativity as well as analytical, critical, and reflective thinking (2.6.3). Learners are also expected to be involved in PBL activities (or steps) to achieve their goal. Moreover, PBL tasks promote group collaboration, as learners interact with peers with different levels of understanding (See 2.6.5). In addition, PBL provides for opportunities to enhance learners' self-directed learning (See 2.7).

SDL is a process that requires learners to be responsible for managing their own learning activities, developing possible solutions, evaluating their learning outcomes, and personally pursuing knowledge and skills (See 2.7). SDL is therefore an essential skill and crucial process that provides learners with opportunities to prioritise their learning needs, to do effective planning and time management, select relevant resources, and evaluate their own learning abilities. Consequently, such learners have a greater degree of control over their learning processes, improve their beliefs about their abilities to solve problems, and increase their motivation to learn. They are independent learners, monitor their own learning processes, persist in learning, and are curious to solve challenging real-world problems (See 2.7.1).

5.3.2 PROBLEM-BASED TASKS THAT CAN BE USED IN GRADE 7 MATHEMATICS TO DEVELOP SELF-DIRECTED LEARNING

Research sub-question 2:
What problem-based tasks can be used in Grade 7 mathematics to help learners become self-directed?

Ill-structured and open-ended tasks that do not offer an immediate solution, but rather provide an opportunity for learners to work together responsibly, search and evaluate various resources, and apply their knowledge to new contexts can assist Grade 7 mathematics learners in building a conceptual understanding of the subject and developing self-directed learning abilities.

In particular, the teacher must ask probing questions to facilitate learning and provide for the development of HOTS (See 2.5.2.2). Such tasks must require learners to apply their current understanding and skills to new contexts by engaging in active learning. This helps learners construct meaningful learning, as the tasks require them to make connections between their existing knowledge and new knowledge that has been acquired. In addition, the tasks should allow for solving real-world problems (e.g. renovate a room), motivate learners, and promote independent and group learning. Tasks should also involve metacognitive thinking where learners must reflect, monitor their thought processes, and evaluate the answers (See 2.6.6).

An example of such a PBL task for Grade 7 mathematics is the following. Learners can solve problems about time zones, where they are expected to compare the time in various areas on the globe. Learners can pretend that they are living in several cities and compare or convert their actuals times. This is a real-life problem that requires learners to use their existing knowledge about time and it promotes critical thinking because they will have to compare and evaluate different time zones (high level thinking on Bloom's taxonomy) (See section 2.5.1). Learners need to be curious, persist in solving such a problem and develop responsible, and independent learning. Consequently, solving such problems may develop SDL abilities. Therefore, the tasks must improve understanding of content knowledge by enhancing learners' autonomy

as well as critical and reflective thinking, so that they can take the initiative in their learning process, which is characteristic of SDL (See 2.7.1).

The next section answers sub-questions 3, based on the research findings.

5.3.3 EXPERIENCES OF GRADE 7 LEARNERS WHEN SOLVING MATHEMATICS PROBLEMS

Research sub-question 3:
What are the experiences of Grade 7 learners when solving mathematics problems?

Grade 7 learners had several experiences regarding the solving of mathematical problems in this study. Findings indicate that most of the participants experienced challenges before the implementation of PBL. Participants mentioned that they were unsure about the tasks due to a lack of understanding of the mathematical problems, which was due to *language barrier* (See 2.4.1 and 4.3.1.1), as learners had difficulty in understanding the problem (See 2.4.1). This is also aligned with the findings of Emanuel *et al.* (2021). In addition, the study results signify that learners were challenged by *insufficient knowledge* about mathematics (prior to the intervention). This means that learners were incompetent in understanding some of mathematics concepts (See 2.4.7) and they lacked the knowledge and skills to solve the problems successfully, because they were not able to reason or think critically. These challenges indicate that some of learners did not have the essential mathematical knowledge at their disposal (especially before the PBL intervention), for example to measure and calculate the number of tiles, because such knowledge was not inculcated into them (See 4.3.1.2).

The availability of *learning resources* was also a challenge to learn and understand the subject. Some participants mentioned that using limited resources such as a textbook from other grades as well as not having a smart phone to enable them to search for additional information (See 2.4.7 and 4.3.1.3), contributed to their failure.

Furthermore, participants expressed their concern about 1) some group members who laughed at others' contributions, 2) some preferred to work with friends, 3) while others did not consider the ideas of the group and 4) some members did not make time for group work, as they preferred to do things on their own (See 4.3.2) (Theme 1).

However, some had positive experiences, especially after the intervention (Table 4.29). Group work *and collaboration are* essential in developing skills for future demands. Group work can enhance peer learning while learners solve mathematical problems together. Different ideas from group members are combined to address the problem. For instance, some participants mentioned that they prefer and enjoy group work since there are many members with different levels of understanding and ideas, and this promote peer learning (See 4.4.2.3). Responsibilities can be shared among group members and that makes the work easier and faster, unlike when the task is done individually (See 4.3.2). Members also learn how to critically think about the solutions, which encourages reflection and evaluation.

An increase in collaboration between group members was evident from some responses. This is an indication that participants also implemented some cooperative principles such as enhancing each other's learning (See 2.6.5). Responsibilities were shared among members to contribute to the success of the group (Theme 2). It has been proven that setting of learning goals is part of the SDL characteristics promoted by most learners as also aligned with the findings of Mahajan and Singh (2017) (See 2.7.1). This is supported by learners who developed time management skills through the implementation of PBL tasks. SDL learners regard challenging problems as an opportunity to learn and expand knowledge (P36) (Adelabu, 2021) (See 1.3.3).

5.4 DEVELOPMENT OF GRADE 7 LEARNERS' SELF-DIRECTED LEARNING SKILLS

The three research sub-questions discussed in 5.3 were formulated to help address the main research question below that guided the study (See 1.2).

MAIN RESEARCH QUESTION
How can problem-based tasks contribute to the development of Grade 7 mathematics learners' self-directed learning?

Participants were expected to demonstrate skills such as taking ownership of their own learning, setting learning goals and identifying learning needs, using resources to search for relevant information, appreciating group work and peer learning, evaluating learning outcomes, and applying reflective skills to solve real-world mathematics problems (See 2.7.1). It was clear that participants became time conscious to the extent that they had to sacrifice of their free time and focus on learning. The implementation of PBL enabled participants to develop responsible learning and essential SDL skills. This was evident in the participants' responses, as they indicated that they could reflect on their learning by looking back at what happened to make changes where needed and avoid repeating the same mistake in future (See 4.4.1.3). Reflection is important since it gives learners an opportunity to express their thoughts and monitor their thinking processes. Through PBL, participants had an opportunity to develop a love for mathematics, as the tasks required of them to actively participate and manage their own learning. This motivated them and offered relevant opportunities to learn more and expand knowledge on how to solve problems such as analysing data and drawing graphs (See 4.5.1.4). The tasks triggered curiosity in learners by making them eager to analyse data and come to some conclusions (See 1.3.1). PBL tasks enabled learners to do additional work even if they were not instructed to do so.

The PBL tasks required participants to apply HOTS. After the intervention they did not experience problems working on the task, unlike before the intervention. This was proved by the findings, that when learners are exposed to HOTS, it becomes easier for them to deal with such tasks in future. The implementation of PBL tasks enabled

learners to develop the ability to set goals and determine learning needs and think of the possible resources to use to address the task (See 4.5.2.1). Through PBL tasks learners were able to cultivate the characteristic of persistence because they were willing to do their best. Learners became independent and they were determined to complete the tasks, even before the directive from the teacher. Consequently, the PBL tasks helped learners to develop such essential qualities (See 4.5.2.2).

When learners are exposed to tasks that promote critical and reflective thinking, HOT, goal setting, determination of learning needs, independent learning, persistence, and the development of curiosity, they are likely to become more self-directed in their learning (See 2.7.1).

The results of this study indicated that the application of problem-based tasks in mathematics contributed to the development of Grade 7 learners' SDL due to the development the following skills. Learners were able to take responsibility for their own learning and manage their learning processes. They enjoyed challenging problems (e.g., to renovate a room), formulated learning goals, and persisted in their learning. Learners were involved in active learning and most indicated that working together in groups was valuable in addressing the PBL tasks. Through searching for relevant information using various resources and through discussions with their peers (evaluated the usefulness of the collected information), their SDL skills were enhanced (see 5.3.1 and 5.3.2) as aligned with El-Nabawi and Shaalan (2019). It is essential that the teacher facilitates active learning, scaffolds the development of HOTS and assists in fostering SDL skills. PBL tasks required the activation of prior knowledge and the making of connections between what learners knew and what they were about to learn, enabling them to do the same in the future and thus becoming more self-directed.

The implementation of PBL tasks enabled learners to cultivate responsibility in terms of setting goals and be willing to work hard to complete the task. The development of metacognitive skills contributed to learners' awareness and level of self-directed in learning. This is supported by Adelabu (2021) when listing the characteristics of a self-directed learner. After the implementation of PBL, learners started valuing group work.

The Gr 7 learners also engaged in activities that helped them participate equally in their groups, boosted their motivation, and increased their love for group work and problem-solving.

Figure 5.1 shows an integration of the findings of this study where PBL provided opportunities to develop SDL skills in Grade 7 mathematics learners.

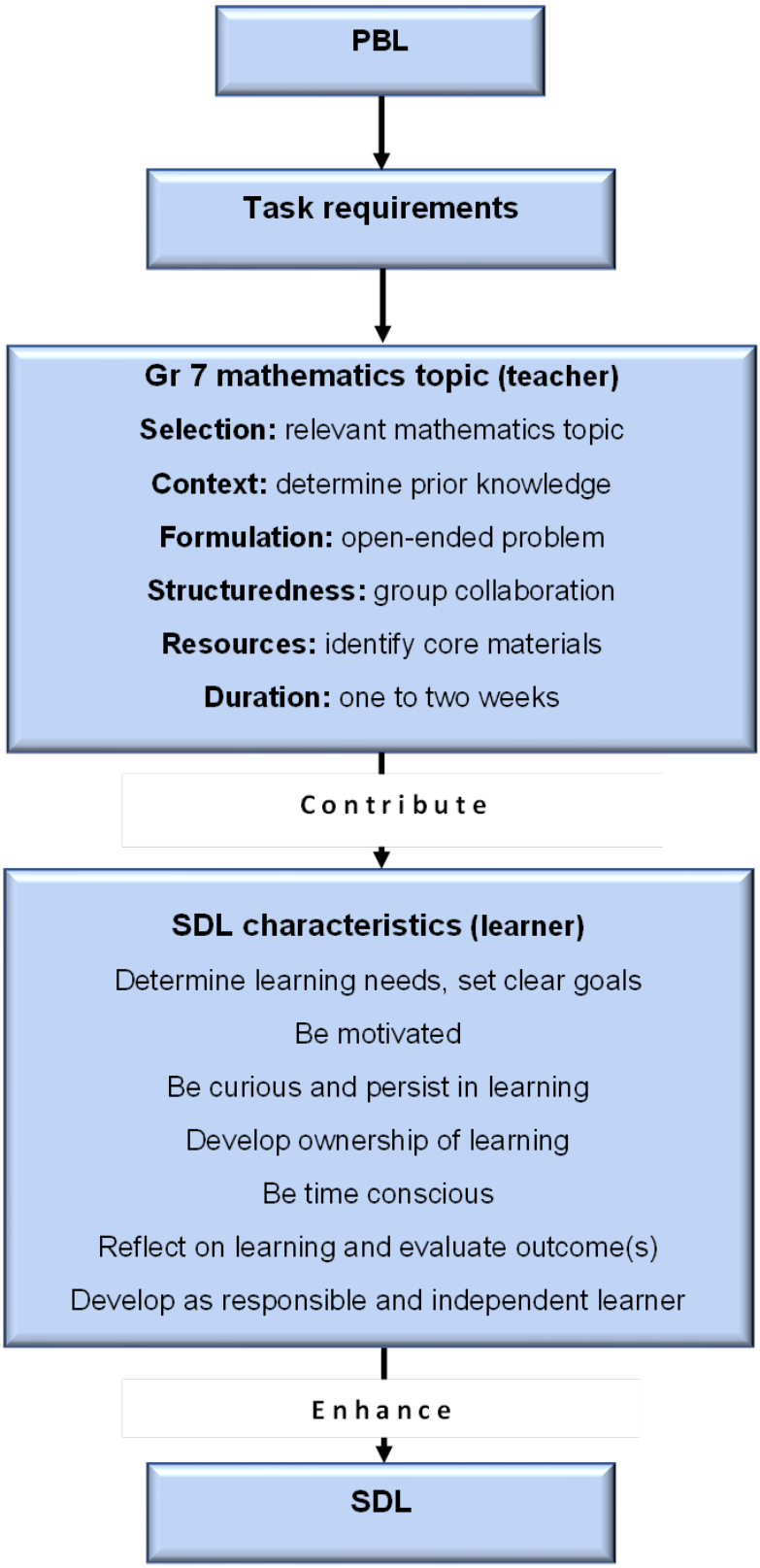


Figure 5.1: A visual representation of the findings

5.5 CONCLUSION

This research explored how mathematical problem-based tasks can contribute to the development of Grade 7 learners' self-directed learning skills. The implementation of PBL in Grade 7 not only supports the learning of the subject but findings have also proven it as an effective teaching and learning strategy that can help learners develop essential skills (such as HOTS) for dealing with several problems in real-life situations. Through mathematical PBL tasks, learners can construct their own knowledge as they interact with peers and their environment to address a real-world problem. The implementation of mathematical PBL tasks in Grade 7 also emphasised the role of the teacher as facilitator of the teaching and learning activities, as he/she needs to plan for active learner involvement in such tasks (individuals and within groups) and provides for essential skill development.

The implementation of PBL in mathematics promoted several SDL skills in participants. Some of these skills are time management, self-control, independence, willingness to learn, persistence, the ability to work in groups and collaborate with peers, reflective, and critical thinking. These skills are crucial in the learning of mathematics, as they can improve learners' performance in this subject. An advantage of SDL is that it encourages learners to have greater control over their learning, improves their beliefs about their effectiveness and increases their motivation to learn mathematics.

The findings might give insight to curriculum developers in the Ministry of Education, Arts and Culture on how to provide guidelines that best suit the teaching of mathematics in the Senior Primary Phase, considering the time allocated to the content of the mentioned phase. The findings also contributed to the existing knowledge regarding SDL and PBL.

5.6 LIMITATIONS

Despite all efforts to ensure the trustworthiness and validity of the results, certain limitations were encountered which make generalisation problematic. Some of these limitations were that the sample size was small, all participants were from the same economic, literacy, racial, and cultural background. As the study aimed at investigating how mathematical problem-based tasks can contribute to the development of Grade

7 learners' self-directed learning skills, these skills were not tested before the study commenced. The researcher's inexperience of planning mathematical PBL tasks and personal perceptions of the interpretation of qualitative data was initially another limitation. Based on this study, some recommendations are presented in the next section.

5.7 RECOMMENDATIONS

Based on this study, the following recommendations are made:

5.7.1 Recommendations regarding the implementation of PBL tasks to enhance mathematics skill development

PBL tasks allow for integrated theoretical and practical mathematics knowledge and skills when solving a real-world problem such as renovating a room. Gr 7 learners developed essential skills for the future such as calculating the area of combined shapes, preparing a questionnaire, drawing different graphs to represent data, and consequently developed HOTS in mathematics. In addition, reflection is essential when solving mathematics problems where learners are able to evaluate their answers, develop different solutions to the same problem and critically reflect on their thinking. It is therefore essential to make provision for such thinking when planning PBL tasks.

5.7.2 Recommendations regarding the implementation of PBL tasks to enhance self-directed learning skills and for future research

Participants set specific goals and remained focused on what was expected from them. They were persistent, reflected on their thinking and focused on what they needed to learn. Learners were responsible for individual and group activities, and this is essential to succeed in solving open-ended mathematics problems in PBL contexts. Some participants indicated that they preferred challenging problems and took ownership for their leaning. It is an important recommendation to plan for such thinking. As a result, learners can become more responsible for their learning, remain focused on their learning processes, improve their academic performance, and develop as self-directed learners. This study recommends further research regarding the use of problem-based tasks by teachers in Grade 7 mathematics to enhance

learners' SDL. This will reduce their workload by facilitating and focus on providing guidance, feedback, and scaffolding, and create a more collaborative and learner-cantered environment rather than directing the learning process.

5.8 FINAL REMARKS AND PERSONAL REFLECTION

The study journey was a learning opportunity as I acquired more knowledge, not just regarding SDL, but also interpersonal skills, and how to be determined and persist despite challenges. I have learnt to be patient with learners by adhering to the ethics of research. Not only did I grow academically and professionally, but it also brought me joy and peace of mind, as I observed learners, some of them developing while others enhancing their self-directed learning skills while working on PBL tasks. I realised that as a teacher, there is much I need to do to assist my learners to become proficient in mathematics. Through the implementation of PBL tasks, Gr 7 learners can become responsible for managing their own learning processes and develop as self-directed mathematics learners.

"Every problem is a gift. Without them we wouldn't grow"
Tony Robbins

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ADDENDUM A

Faculty of Education

MINUTES: SDL SCIENTIFIC COMMITTEE MEETING

Date of meeting: **12 November 2018**

Time of meeting **14:00**

Meeting room: **Potchefstroom Building: C6-299E**

1 Opening, welcome and personalia

Prof E Mentz welcomed all the committee members to the meeting.

Attendance: Prof E Mentz (chair), Profs JJJ de Beer, C du Toit-Brits, HM Havenga, JAK Olivier, CP van der Westhuizen.

Minute keeper: Ms S Turkstra.

Apology: Profs WT Dudu, MS van der Walt, Dr R Bailey.

Personalia:

It is with sadness that we that we express our heartfelt condolences to Prof Kobus Lombard's family.

1.1 Approval of research proposals that have previously been referred back

1.1.1 Student	**Ms ST Kashango (23283815)**
Degree	**Mathematics Education**
Title	Investigating grade 7 Mathematics learners' self-directed learning through problem-based tasks
Supervisor	Prof HM Havenga
Co-supervisor	Dr T Jordaan
Critical readers	**Prof JJJ de Beer** **Prof MS van der Walt**
Risk level	**High risk - 1**
Previous decision	**Referred back – major changes** The student should refer to the recommendations of the Scientific Committee as well as the track changes provided and **re-submit** with a **change log** for approval.
Decision	**Approved – minor changes** The student should refer to the recommendations of the Scientific Committee as well as the track changes provided.

Original details: (21830399) S:\SNTC\Faculty of Education\Minutes\2018\SDL\2018-11-12_Scientific Committee\Minutes-SDL Scientific Committee Meeting.docm
11 November 2018

File reference: 9.4

ADDENDUM B

Private Bag X1290, Potchefstroom
South Africa 2520

Tel: 018 299-1111/2222
Fax: 018 299-4910
Web: http://www.nwu.ac.za

Research Ethics Regulatory Committee
Tel: 018 299-4849
Email: nkosinathi.machine@nwu.ac.za

09 April 2019

ETHICS APPROVAL LETTER OF STUDY

Based on approval by the **North-West University Education, Management and Economic Sciences, Law, Theology, Engineering, and Natural Sciences Research Ethics Committee (NWU-EMELTEN-REC)** on 09/04/2019, the North-West University Education, Management and Economic Sciences, Law, Theology, Engineering, and Natural Sciences Research Ethics Committee hereby **approves** your study as indicated below. This implies that the North-West University Research Ethics Regulatory Committee (NWU-RERC) grants its permission that, provided the special conditions specified below are met and pending any other authorisation that may be necessary, the study may be initiated, using the ethics number below.

Study title: Investigating grade 7 Mathematics learners' self-directed learning through problem-based tasks.
Study Leader/Supervisor (Principal Investigator)/Researcher: Prof HM Havenga
Student: Ms. ST Kashango

Ethics number: NWU-00068-19-A2
(Institution – Study Number – Year – Status)

Status: S = Submission; R = Re-Submission; P = Provisional Authorisation; A = Authorisation

Application Type: Single Study
Commencement date: 09/04/2019
Expiry date: 08/04/2020
Risk Category: Greater than minimal but provides prospect of direct benefit

Approval of the study is initially provided for a year, after which continuation of the study is dependent on receipt and review of the annual (or as otherwise stipulated) monitoring report and the concomitant issuing of a letter of continuation.

Special in process conditions of the research for approval (if applicable):

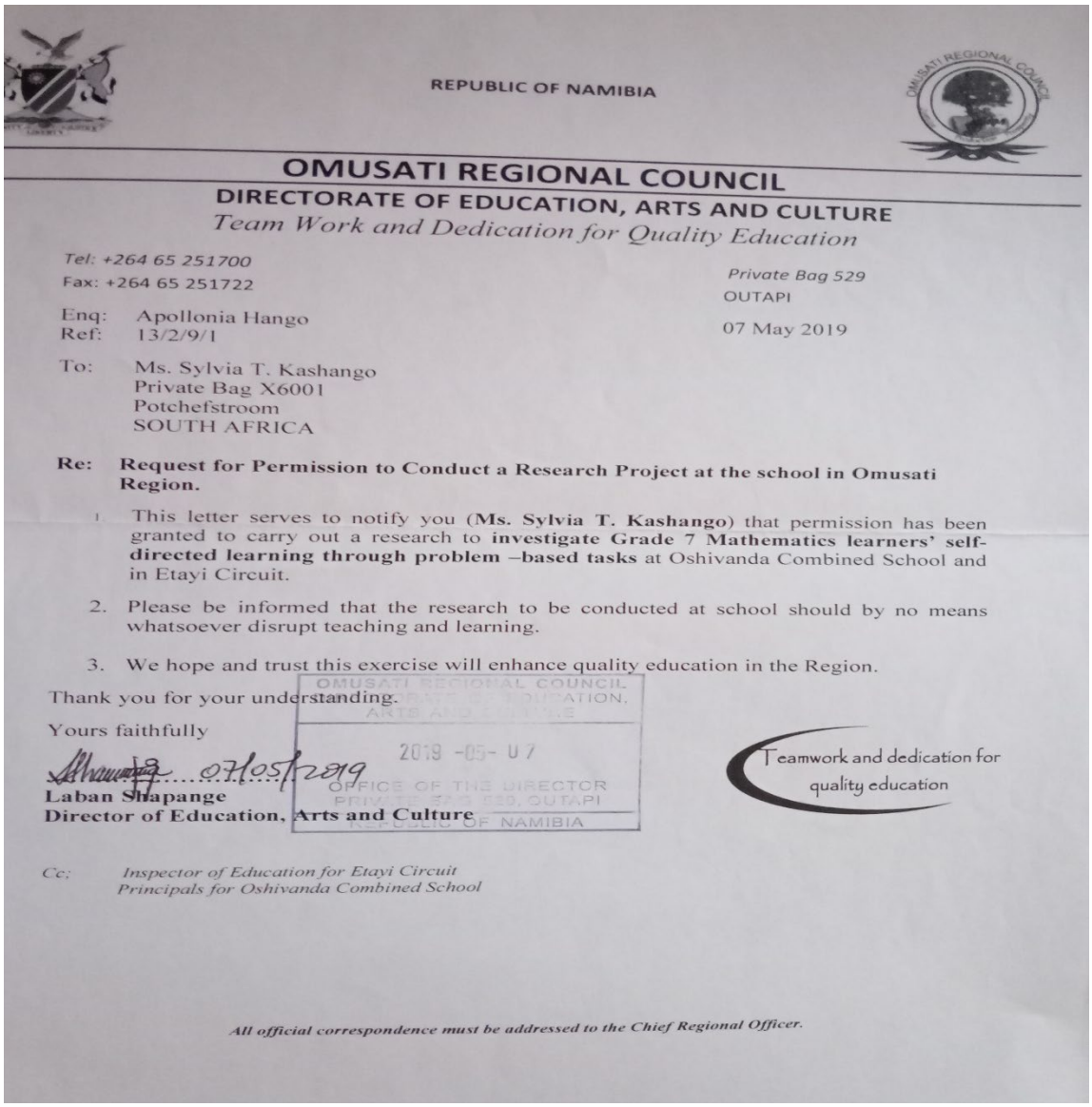
-

General conditions:

While this ethics approval is subject to all declarations, undertakings and agreements incorporated and signed in the application form, the following general terms and conditions will apply:

- *The study leader/supervisor (principle investigator)/researcher must report in the prescribed format to the NWU-EMELTEN-REC:*
  - *annually (or as otherwise requested) on the monitoring of the study, whereby a letter of continuation will be provided, and upon completion of the study; and*
  - *without any delay in case of any adverse event or incident (or any matter that interrupts sound ethical principles) during the course of the study.*
- *The approval applies strictly to the proposal as stipulated in the application form. Should any amendments to the proposal be deemed necessary during the course of the study, the study leader/researcher must apply for approval of these amendments at the NWU-EMELTEN-REC, prior to implementation. Should there be any deviations from the study proposal without the necessary approval of such amendments, the ethics approval is immediately and automatically forfeited.*
- *Annually a number of studies may be randomly selected for an external audit.*
- *The date of approval indicates the first date that the study may be started.*
- *In the interest of ethical responsibility, the NWU-RERC and NWU-EMELTEN-REC reserves the right to:*
  - *request access to any information or data at any time during the course or after completion of the study;*

ADDENDUM C



REPUBLIC OF NAMIBIA

OMUSATI REGIONAL COUNCIL

DIRECTORATE OF EDUCATION, ARTS AND CULTURE

Team Work and Dedication for Quality Education

Tel: +264 65 251700
Fax: +264 65 251722

Private Bag 529
OUTAPI

07 May 2019

Enq: Apollonia Hango
Ref: 13/2/9/1

To: Ms. Sylvia T. Kashango
Private Bag X6001
Potchefstroom
SOUTH AFRICA

Re: Request for Permission to Conduct a Research Project at the school in Omusati Region.

1. This letter serves to notify you (**Ms. Sylvia T. Kashango**) that permission has been granted to carry out a research to **investigate Grade 7 Mathematics learners' self-directed learning through problem –based tasks** at Oshivanda Combined School and in Etayi Circuit.
2. Please be informed that the research to be conducted at school should by no means whatsoever disrupt teaching and learning.
3. We hope and trust this exercise will enhance quality education in the Region.

Thank you for your understanding.

Yours faithfully

07/05/2019

Laban Shapange
Director of Education, Arts and Culture

OMUSATI REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE
2019 -05- 07
OFFICE OF THE DIRECTOR
PRIVATE BAG 529, OUTAPI
REPUBLIC OF NAMIBIA

Teamwork and dedication for quality education

Cc; Inspector of Education for Etayi Circuit
Principals for Oshivanda Combined School

All official correspondence must be addressed to the Chief Regional Officer.

ADDENDUM D

Ministry of Education

Oshivanda Combined School

Inspectorate Division: Etayi Circuit

Teamwork and Dedication for quality education

Enq: Ms. Nakale
Cell: 0812354264

P.O.Box 2153
Oshakati

Dear Mrs Kashango

Re: Request for permission to conduct a research project at Oshivanda Combined School.

I am writing this in response to your letter dated the 09.05.2019 in which you requested for permission to conduct research at our school.

I am pleased to inform you that your request was discussed in the school board meeting of the 14.05.2019 and authorization has been granted in this regard.

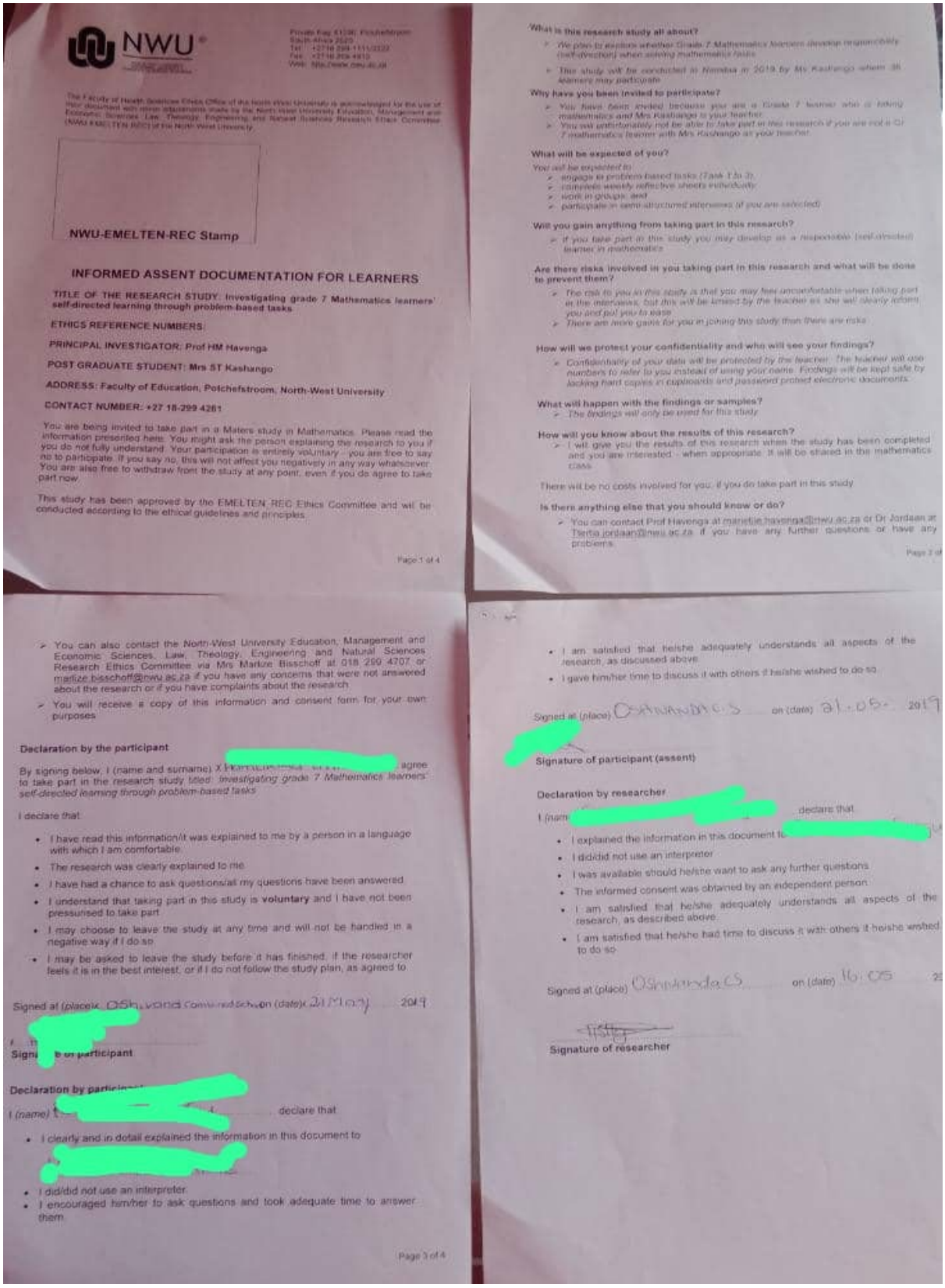
Let me wish you the best of luck during this exercise and I am hoping that the outcome of this exercise will not only be beneficial to yourself as a researcher but also to the entire learner community.

Regards

.....
Protasius Linongaba
School board Chairperson

MINISTRY OF EDUCATION
Office of the Principal
Oshivanda Combined School
Etayi Circuit
2019 -05- 16
Omusati Region
P.O. Box 2153
Oshakati
REPUBLIC OF NAMIBIA

ADDENDUM E



NWU

NWU-EMELTEN-REC Stamp

INFORMED ASSENT DOCUMENTATION FOR LEARNERS

TITLE OF THE RESEARCH STUDY: Investigating grade 7 Mathematics learners' self-directed learning through problem-based tasks

ETHICS REFERENCE NUMBERS:

PRINCIPAL INVESTIGATOR: Prof HM Havenga

POST GRADUATE STUDENT: Mrs ST Kashango

ADDRESS: Faculty of Education, Potchefstroom, North-West University

CONTACT NUMBER: +27 18-299 4261

You are being invited to take part in a Maters study in Mathematics. Please read the information presented here. You might ask the person explaining the research to you if you do not fully understand. Your participation is entirely voluntary – you are free to say no to participate. If you say no, this will not affect you negatively in any way whatsoever. You are also free to withdraw from the study at any point, even if you do agree to take part now.

This study has been approved by the EMELTEN REC Ethics Committee and will be conducted according to the ethical guidelines and principles.

Page 1 of 4

What is this research study all about?

- *We plan to explore whether Grade 7 Mathematics learners develop responsibility (self-direction) when solving mathematics tasks.*
- *This study will be conducted in Namibia in 2019 by Mrs Kashango where 36 learners may participate.*

Why have you been invited to participate?

- *You have been invited because you are a Grade 7 learner who is taking mathematics and Mrs Kashango is your teacher.*
- *You will unfortunately not be able to take part in this research if you are not a Gr 7 mathematics learner with Mrs Kashango as your teacher.*

What will be expected of you?

You will be expected to:

- *engage in problem based tasks (Task 1 to 3);*
- *complete weekly reflective sheets individually;*
- *work in groups; and*
- *participate in semi-structured interviews (if you are selected).*

Will you gain anything from taking part in this research?

- *If you take part in this study you may develop as a responsible (self-directed) learner in mathematics.*

Are there risks involved in you taking part in this research and what will be done to prevent them?

- *The risk to you in this study is that you may feel uncomfortable when taking part in the interviews, but this will be limited by the teacher as she will clearly inform you and put you to ease.*
- *There are more gains for you in joining this study than there are risks.*

How will we protect your confidentiality and who will see your findings?

- *Confidentiality of your data will be protected by the teacher. The teacher will use numbers to refer to you instead of using your name. Findings will be kept safe by locking hard copies in cupboards and password protect electronic documents.*

What will happen with the findings or samples?

- *The findings will only be used for this study.*

How will you know about the results of this research?

- *I will give you the results of this research when the study has been completed and you are interested - when appropriate. It will be shared in the mathematics class.*

There will be no costs involved for you, if you do take part in this study.

Is there anything else that you should know or do?

- *You can contact Prof Havenga at marietjie.havenga@nwu.ac.za or Dr Jordaan at Tertia.jordaan@nwu.ac.za if you have any further questions or have any problems.*

Page 2 of

- You can also contact the North-West University Education, Management and Economic Sciences, Law, Theology, Engineering and Natural Sciences Research Ethics Committee via Mrs Marlize Bisschoff at 018 299 4707 or marlize.bisschoff@nwu.ac.za if you have any concerns that were not answered about the research or if you have complaints about the research.
- You will receive a copy of this information and consent form for your own purposes.

Declaration by the participant

By signing below, I (name and surname) [redacted] agree to take part in the research study titled: *Investigating grade 7 Mathematics learners' self-directed learning through problem-based tasks*.

I declare that:

- I have read this information/it was explained to me by a person in a language with which I am comfortable.
- The research was clearly explained to me.
- I have had a chance to ask questions/all my questions have been answered.
- I understand that taking part in this study is **voluntary** and I have not been pressurised to take part.
- I may choose to leave the study at any time and will not be handled in a negative way if I do so.
- I may be asked to leave the study before it has finished, if the researcher feels it is in the best interest, or if I do not follow the study plan, as agreed to.

Signed at (place) Oshivandi Combined School on (date) 21 May 2019

[redacted]

Signature of participant

Declaration by participant

I (name) [redacted] declare that:

- I clearly and in detail explained the information in this document to [redacted]
- I did/did not use an interpreter.
- I encouraged him/her to ask questions and took adequate time to answer them.

Page 3 of 4

- I am satisfied that he/she adequately understands all aspects of the research, as discussed above.
- I gave him/her time to discuss it with others if he/she wished to do so.

Signed at (place) Oshivandi C S on (date) 21-05-2019

[redacted]

Signature of participant (assent)

Declaration by researcher

I (name) [redacted] declare that:

- I explained the information in this document to [redacted]
- I did/did not use an interpreter.
- I was available should he/she want to ask any further questions.
- The informed consent was obtained by an independent person.
- I am satisfied that he/she adequately understands all aspects of the research, as described above.
- I am satisfied that he/she had time to discuss it with others if he/she wished to do so.

Signed at (place) Oshivandi C S on (date) 16-05-2019

Signature of researcher

ADDENDUM F

PBL task 1

Data collection – group work (6 members)

The owner of the school's tuck shop wants to make sure that he provides drinks that the learners will buy. He is unsure whether he sells the drinks that they prefer. Being a very busy man, he does not have the time or the knowledge to investigate it himself. He decided to ask your mathematics class to assist him in investigating that his tuck shop will in future provide in the demand of the learners.

You as a group should investigate the type of drinks that your classmates like most. Think of **ten drinks** (non-alcoholic drinks) that they might like, that can be sold in the tuck shop. For example: Coke, Pepsi, Tea, Coffee, Milk, Juice, Fanta orange, Fanta pineapple and Sprite.

Subtask 1: Develop a measuring instrument

A group needs to compile a **questionnaire** that can be used to get the answers regarding the type of drinks that group members prefer.

Subtask 2: Collect the data

Use your group's questionnaire and ask each group member to indicate their choice.

Subtask 3: Organize and present the data

Record each learner's choices in a **table** and a **tally chart.**

Represent the data on a:

- Bar chart
- Pie chart
- Pictogram

NB: Two members should construct a bar graph, other two members do a pie chart and the remaining two members do a pictogram.

Subtask 4: Report the results

Compile a report of your results that will assist the tuck shop owner to order and sell the appropriate drinks in future.

Subtask 5: Learners complete reflecting weekly sheets individually

Complete the reflecting weekly sheets individually.

Subtask 6: Semi-structured interviews

Semi-structured interviews will be conducted with randomly selected participants.

ADDENDUM G

PBL Task 2

Measurement, area, money, and decimals

The School Governing Body plans to renovate the various classrooms in the nearby future. They have a restricted budget and must plan the cost accurately before the work can begin. They plan to paint the walls and either tile the floors or lay a carpet. Your class must advise the school principal on the most cost-effective way to renovate the floor.

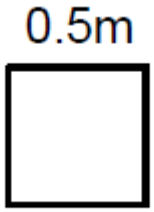
Subtask 1: Investigate the size of the room

Each group selects a **different classroom**.

Draw a plan of the floor. Include the correct measurements (length, width).

Subtask 2: Determine the area using tiles

The tiles are squares with a length of 0.5 m. They cost N$5.00 each. If a tile cannot completely fit part of the floor space, professional help is required to cut the tile to fit. Learners need to calculate this as well. Calculate the number of tiles needed to cover the floor and the cost to cover the floor.



Subtask 3: Determine the area of the floor using a carpet

The carpet has a standard width of 4 meters. Learners need to determine the length of the carpet themselves. The carpet costs N$12.00 per square meter. A loose carpet section of 0.5 m by 1m for patchwork is available at N$6.00 per piece. Calculate the carpet or parts thereof and the cost to cover the floor.

.

Subtask 4: Compare the cost of the tiles and the carpet

Determine the cheapest option to cover the floor. Explain clearly and mathematically your choice of the most cost-effective way to renovate the floor.

Subtask 5: Determine the amount of paint needed and the cost

The wall needs to be painted. Measure the correct height of the wall. There are only 2L of paint available in the shop. Determine the area that will be covered by one drum of paint and the cost of the paint needed if a 2L of paint cost N$105.00. Determine the total budget for renovating both the floor and the wall.

Subtask 6: Report the results

Compile a report of your results

Subtask 7: Complete the reflective weekly sheets

ADDENDUM H

MATHEMATICS GRADE 7

WORKSHEET

Group_____

Member Code:_____

Date:_____

1. Clarification of concepts

_____
_____
_____
_____

2. Formulation of problems

_____
_____
_____
_____

3. Brainstorm/ collect ideas about the problem/task

_____
_____
_____

4. Answer why/how/what to address the problem/task

_____
_____
_____

5. Formulate learning objectives

_____
_____
_____
_____

6. Members study resources and discuss this in group

_____
_____
_____

7. Learners do the mathematics task in phases

ADDENDUM I

Weekly Reflective Sheet: Complete individually	Date: _ /_ /2019 Week:_____
Learner: Code: _____TASK no: _____	
1. Explain in your own words what the mathematics task this week involves.	
2. Discuss what problems you have experienced when you do the mathematics task <u>on your own.</u>	
3. Discuss the problems you experience when working on the mathematics task <u>with your peers (fellow learners)</u> this week.	
4. Give feedback regarding the <u>support</u> among your group members when addressing/solving the mathematics task this week.	
5. Give feedback regarding your group's <u>interaction (involvement) and communication</u> when working on the mathematics task this week.	

6. Give feedback whether <u>your group has achieved their aims</u> to solve the mathematics problem this week.
7. Explain how your group has <u>addressed/solved</u> the problems or challenges you experienced.
8. Reflect on the pace (rate) of your group's <u>progress</u> to address the mathematics task this week.
9. Specify which information you searched (looked) for. List all the resources (where you have taken the information from) that you have used.

10. Give <u>yourself</u> and your group a mark regarding your effort (how you tried) to be responsible in addressing the math task <u>this week</u>.

PLEASE BE HONEST (0 – made no contribution, 10: provided an outstanding contribution and EXPLAIN WHY YOU ASSIGNED THIS MARK TO EACH MEMBER

<u>Member 1</u>: _____

Mark(0-10):

Motivation:

<u>Member 2</u>: _____

Mark:

Motivation:

<u>Member 3</u>: _____

Mark:

Motivation:

<u>Member 4</u>: _____

Mark:

Motivation:

Member 5:_____

Mark:

Motivation:

Member 6:_____

Mark:

Motivation:

ADDENDUM J

SEMI-STRUCTURED INTERVIEW PROTOCOL

Code: _____

Date: _____

1. What challenges do you experience when working on mathematical problems on your own?
   .....
   .....
   .....
   .....

2. Do you prefer solving mathematical problems with others? Motivate your answer.
   .....
   .....
   .....
   .....

3. Do you determine your learning needs in mathematics before solving a problem? Motivate please.
   .....
   .....
   .....
   .....

4. Do you formulate learning goals in mathematics? Motivate please.
   .....
   .....
   .....
   .....

5. Do you take responsibility for your own learning in mathematics? Motivate please.
   .....
   .....
   .....
   .....

6. Are you curious to solve challenging mathematics problems? Motivate please.
   .....
   .....
   .....
   .....

7. Are you comparing different types of solutions to select the best answer? Motivate please.
   .....
   .....
   .....
   .....

8. Do you often use additional resources to assist you in solving mathematics problems? Motivate please.
   .....
   .....
   .....
   .....

9. Do you evaluate your answer to a problem? Motivate please.

   ……
   ……
   ……

Certificate for editing work done

26 November 2022

To whom it may concern

This serves to declare that the **dissertation** by **Sylvia Kashango** was duly edited for language usage, spelling and referencing. Track changes were used, and it was the responsibility of the author to effect changes and finalise the reference list.

Title of the work: **Investigating Grade 7 Mathematics learners' self-directed learning through problem-based tasks**

Yours truly

Juanita du Toit

076 821 2588

juanitadutoit5@gmail.com

TurnItIn report: 26 Nov 2022

10113924:SYLVIA_KASHANGO_26_OCT_Fin_TurnItIN3.docx

ORIGINALITY REPORT

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