



Development of an energy consumption benchmarking model for platinum group metal comminution circuits

E van Moerkerken

 **[orcid.org/ 0000-0002-7355-4296](https://orcid.org/0000-0002-7355-4296)**

Dissertation accepted in fulfilment of the requirements for the degree *Master of Engineering in Electrical and Electronic Engineering* at the North-West University

Supervisor:	Dr B Pascoe
Co-supervisor:	Dr JF van Rensburg

Graduation:	August 2024
-------------	-------------

ACKNOWLEDGEMENTS

I would like to express my appreciation and gratitude to everyone involved in conducting the research. A special thanks to:

- The Heavenly Father who gave me the ability and strength to accomplish this feat. “I can do all things through Christ which strengthens me” – Phillipians 4:13
- To my parents who have supported me through my educational career and for always believing in me, even when I did not.
- My girlfriend for her patience and understanding during the research period. Your constant reassurance was invaluable.
- Prof. Edward Matthews and ETA Operations (Pt) Ltd. for enabling me to conduct the research and further my academic career.

ABSTRACT

The mining sector plays a pivotal role in bolstering South Africa's GDP. Nonetheless, it has encountered numerous obstacles of late, most notably a marked surge in energy costs. The extraction of platinum group metals (PGMs) is particularly energy-intensive, relying heavily on comminution circuits for processing and electric furnaces for smelting.

Yet, electric furnaces must operate at full power to reach the necessary temperatures, complicating efforts to enhance energy efficiency. Nevertheless, comminution circuits possess various adjustable variables that offer opportunities for optimising energy use. The primary challenge lies in enhancing the circuits' energy efficiency without compromising production goals or altering the size of the product particles.

Hence, this study aimed to develop an energy benchmarking model for comminution circuits at PGM concentrators, employing metrics and data analysis to evaluate the performance of these circuits. Two energy benchmarking models were developed: an average benchmark model and a frontier benchmark model. Single or multivariable regression functions can be used to predict the effect of given variables on the energy consumption of the comminution circuits. These functions were developed for the critical milling circuit components, including pre-milling, primary milling, and secondary milling.

The above measures resulted in an average benchmark model that constructed through ordinary least squares regression. In contrast, the resulting frontier benchmark model employs corrected ordinary least squares and stochastic frontier analysis derived from the average model. These benchmarks facilitate the creation of a streamlined scoring system, assigning a performance score to each milling circuit relative to others. This approach enables a performance ranking and ongoing monitoring of the circuit's efficiency. The scoring system indicated that the comminution circuits performed between 98–114% of their expected performance, and 73-127% of the frontier performance.

Furthermore, the developed score-based method ranked the performance of the circuits with an 92% accuracy, when compared to the current intensity benchmarking. However, the score-based method aims to address the shortcomings of the current intensity benchmarking method. By spotlighting inefficient sections and technologies within circuits, it narrows down the areas requiring scrutiny for inefficiencies. This targeted approach allows for strategic enhancements to the circuits, resulting in significant energy savings, lowered carbon emissions, and diminished energy intensity.

Keywords: platinum group metals; concentrators; benchmarking; energy efficiency; comminution circuit

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	I
ABSTRACT	II
TABLE OF CONTENTS	IV
LIST OF FIGURES	VII
LIST OF TABLES	X
NOMENCLATURE	XIII
LIST OF ABBREVIATIONS	XIV
CHAPTER 1 PGM COMMINUTION CIRCUITS AND ENERGY BENCHMARKING	1
1.1 Introduction to mining in South Africa	1
1.2 PGM processing	3
1.3 Energy consumption of PGM comminution circuits	3
1.4 Benchmarking methods	7
1.5 Previous studies on energy benchmarking	11
1.6 Problem statement	22
1.7 Need for study, aim and objectives	23
1.8 Research methodology	25
1.9 Dissertation layout	26

CHAPTER 2	METHODOLOGY TO DESIGN BENCHMARKING MODEL	27
2.1	Introduction.....	27
2.2	Step 1: Data collection	27
2.3	Step 2: Pre-processing data	28
2.4	Step 3: Obtain prediction equations	29
2.5	Step 4: Validate predicted results	34
2.6	Step 5: Development of a scoring method.....	36
2.7	Step 6: Monthly benchmarking	39
2.8	Step 7: Comparison with other methods.....	39
2.9	Method verification.....	40
2.10	Conclusion	41
CHAPTER 3	BENCHMARKING MODEL VERIFICATION, APPLICATION, AND VALIDATION.....	42
3.1	Introduction.....	42
3.2	Step 1: Data collection	43
3.3	Step 2: Pre-processing data	50
3.4	Step 3: Obtain prediction equations	50
3.5	Step 4: Validate predicted results	61
3.6	Step 5: Application of the scoring method	67
3.7	Step 6: Monthly benchmarking	69
3.8	Step 7: Comparison with other methods.....	85
3.9	Study validation	87

3.10	Conclusion	88
CHAPTER 4 CONCLUSION		90
4.1	Overview	90
4.2	Method review	91
4.3	Limitations and recommendations	91
BIBLIOGRAPHY		93
APPENDIX A		100

LIST OF FIGURES

Figure 1: Employment by the platinum mining industry versus the platinum price	2
Figure 2: Typical PGM processing flowchart [2]	3
Figure 3: Basic crushing circuit layout [18]	6
Figure 4: Ordinary least-square linear regression.....	8
Figure 5: Corrected ordinary least-square linear regression.....	9
Figure 6: Stochastic frontier analysis	10
Figure 7: Data envelope analysis.....	10
Figure 8: Benchmarking results of Ballantyne and Powell's study [44].....	20
Figure 9: Conversion of COLS-to-SFA (adapted from [33]).....	33
Figure 10: Regression of corrected percentage versus initial percentage.....	38
Figure 11: Score-based benchmarking procedure [33].....	39
Figure 12: Score-based benchmarking framework	42
Figure 13: Pre-milling power consumption versus ore throughput.....	51
Figure 14: Primary mill power consumption versus ore feed	52
Figure 15: Primary mill power consumption versus mill weight	53
Figure 16: Primary mill power consumption versus feed water	53
Figure 17: Primary mill power consumption versus ball load.....	54
Figure 18: Secondary mill power consumption versus ore throughput.....	56
Figure 19: Secondary mill power consumption versus mill weight.....	56
Figure 20: Percentage-corrected versus percentage initial – average primary milling model ...	68
Figure 21: Pre-milling average function MAPE	71

Figure 22: Pre-milling SFA function MAPE	72
Figure 23: Primary milling average function MAPE	73
Figure 24: Primary milling SFA function MAPE	74
Figure 25: Secondary milling average function MAPE.....	75
Figure 26: Secondary milling SFA function MAPE.....	76
Figure 27: Pre-milling – percentage corrected values (average benchmarking)	79
Figure 28: Pre-milling – percentage corrected values (SFA benchmarking)	80
Figure 29: Primary milling – percentage corrected values (average benchmarking).....	81
Figure 30: Primary milling – percentage corrected values (SFA benchmarking).....	82
Figure 31: Secondary milling – percentage corrected values (average benchmarking).....	83
Figure 32: Secondary milling – percentage-corrected values (SFA benchmarking).....	83
Figure 33: Pre-milling COLS function	100
Figure 34: Pre-milling SFA function	100
Figure 35: Primary mill ore feed COLS function	101
Figure 36: Primary mill ore feed SFA function	101
Figure 37: Primary mill weight COLS function	102
Figure 38: Primary mill weight SFA function.....	102
Figure 39: Primary mill feed water COLS function.....	103
Figure 40: Primary mill feed water SFA function.....	103
Figure 41: Secondary mill feed COLS function.....	104
Figure 42: Secondary mill feed SFA function.....	104
Figure 43: Secondary mill weight COLS function	105

Figure 44: Secondary mill weight SFA function	105
---	-----

LIST OF TABLES

Table 1: Comparison of RoM and SAG mill operating parameters, adapted from [12].....	5
Table 2: Review methodology based on the method developed by Winchester <i>et al.</i> [36]	12
Table 3: State-of-the-art matrix	24
Table 4: Monthly intensity of mill power consumption and weight	29
Table 5: LINEST function array (adapted from [33])	30
Table 6: Normalisation category ranges	37
Table 7: Performance categories for variable ranges of regressions	38
Table 8: Corrected percentage calculation	39
Table 9: Method verification	40
Table 10: Pre-milling data availability for each circuit	43
Table 11: Milling data availability for each circuit.....	44
Table 12: Pre-milling equipment of the various case study circuits	45
Table 13: Pre-milling circuit throughput	46
Table 14: Milling equipment of the various case study circuits.....	47
Table 15: Primary mill variables.....	49
Table 16: Secondary mill variables	49
Table 17: Pre-milling circuit array data from LINEST	52
Table 18: Primary mill ore feed – LINEST data array	55
Table 19: Primary mill weight – LINEST data array	55
Table 20: Primary mill feed water – LINEST data array.....	55
Table 21: Secondary mill throughput array data from LINEST	57

Table 22: Secondary mill weight data array from LINEST	57
Table 23: Comminution circuit average function summary	59
Table 24: Comminution circuit best practice (SFA) function summary	61
Table 25: Pre-milling average function MAPE validation	62
Table 26: Pre-milling average function validation	62
Table 27: Pre-milling SFA function MAPE validation	63
Table 28: Pre-milling SFA function validation	63
Table 29: Primary milling – average function MAPE validation	64
Table 30: Primary milling – average function validation.....	64
Table 31: Primary milling – SFA function MAPE validation	65
Table 32: Primary milling SFA function validation.....	65
Table 33: Secondary mill – average function MAPE validation	66
Table 34: Secondary milling average function validation.....	66
Table 35: Secondary mill – SFA function MAPE validation	66
Table 36: Secondary milling SFA function validation.....	67
Table 37: Percentage-normalised calculation based on average function for each section	68
Table 38: Percentage-normalised calculation based on the SFA function for each section	69
Table 39: Pre-milling monthly average power consumption prediction and actual	70
Table 40: Pre-milling monthly SFA power consumption prediction and actual	71
Table 41: Primary milling monthly average power consumption prediction and actual	72
Table 42: Primary milling monthly SFA power consumption prediction and actual	74
Table 43: Secondary milling monthly average power consumption prediction and actual	75

Table 44: Secondary milling monthly SFA power consumption prediction and actual.....	76
Table 45: Pre-milling average model %initial.....	77
Table 46: Primary milling average model %initial	78
Table 47: Secondary milling average model %initial	78
Table 48: Average %corrected average benchmarking of milling circuits	84
Table 49: Average %corrected SFA benchmarking of milling circuits	85
Table 50: Ranking of circuits using the two different methods.....	86
Table 51: Ranking of circuits using the average and SFA benchmarking methods	86
Table 52: Validation of study.....	87
Table 53: Pre-milling average model %initial and %corrected score.....	106
Table 54: Pre-milling SFA model %initial and %corrected score.....	106
Table 55: Primary milling average model %initial and %corrected score	106
Table 56: Primary milling SFA model %initial and %corrected score	107
Table 57: Secondary milling average model %initial and %corrected score	107
Table 58: Secondary milling SFA model %initial and %corrected score	108
Table 59: Pre-milling SFA model %initial.....	108
Table 60: Primary milling SFA model %initial	109
Table 61: Secondary milling SFA model %initial	109

NOMENCLATURE

Symbol	Description	Units
t	Mass	Tonne
kW	Power	Kilowatt
kWh	Energy	Kilowatt-hour
l/s	Flow rate	Litres per second
m ³	Volume	Cubic metre
m ³ /h	Flow rate	Cubic metre per hour
MW	Power	Megawatt
MWh	Energy	Megawatt-hour

LIST OF ABBREVIATIONS

AG	Autogenous
AR	Attainable region
BEF	Benchmarking energy factor
BWi	Bond work index
COLS	Corrected ordinary least square
DEA	Data envelopment analysis
EM	Engineering model
GDP	Gross domestic product
GHG	Greenhouse gases
HPGR	High-pressure grinding roller
MAE	Mean absolute error
MAPE	Mean average percentage error
MBE	Mean bias error
OECD	Organisation for Economic Co-operation and Development
OLS	Ordinary least square
PGM	Platinum group metals
RoM	Run-of-mine
SAG	Semi-autogenous
SFA	Stochastic frontier analysis

SMD	Stirred media detritors
SVM	Support vector machines
UFG	Ultra-fine grinding
UG2	Upper group 2

CHAPTER 1 PGM COMMINUTION CIRCUITS AND ENERGY BENCHMARKING

1.1 Introduction to mining in South Africa

The mining industry is one of the largest industries in South Africa, accounting for 7.7% of the country's GDP in 2021, with the platinum group metals (PGMs) mining sector playing a significant role in this contribution. South Africa is the world's largest producer of the six PGMs, namely platinum, palladium, rhodium, ruthenium, osmium, and iridium. In 2021, the sector employed 171 568 people, representing approximately 37% of the total mining workforce. Nonetheless, the industry is confronted with numerous challenges and threats, both locally and globally¹.

The global pandemic, COVID-19, resulted in a global financial crisis led by severe limitations placed on production and trade. South Africa utilises only a small fraction of its mined materials domestically, heavily relying on exports to major manufacturing and production nations, notably China. In 2021, the country exported metals valued at approximately \$1,96 billion to these international markets².

Furthermore, the price of platinum experienced a significant decline, dropping from 1 719 \$/oz in 2011 to as low as 845 \$/oz in 2023³. Notably, the sector's employment levels do not expand in response to rising profit margins but rather in reaction to increased demand for platinum and the corresponding need to enhance the liberation rate, as illustrated in Figure 1 below. Consequently, one of the only ways mining operations can improve their profit margins, especially in periods of high demand for the metal, is by decreasing their operating or input costs⁴.

¹ Minerals Council South Africa, 'Facts and Figures 2021', 2012.

² www.wits.worldbank.org/CountryProfile/en/Country/ZAF/Year/2021/TradeFlow/Export/Partner/all/Product/72-83_Metals

³ Market Insider, Platinum price, Market insider, 06/01/2023, <https://markets.businessinsider.com/commodities/platinum-price>

⁴ PWC, Highlighting trends in the South African mining industry, 5th ed. 2013. [Online]. Available: www.pwc.co.za/mining

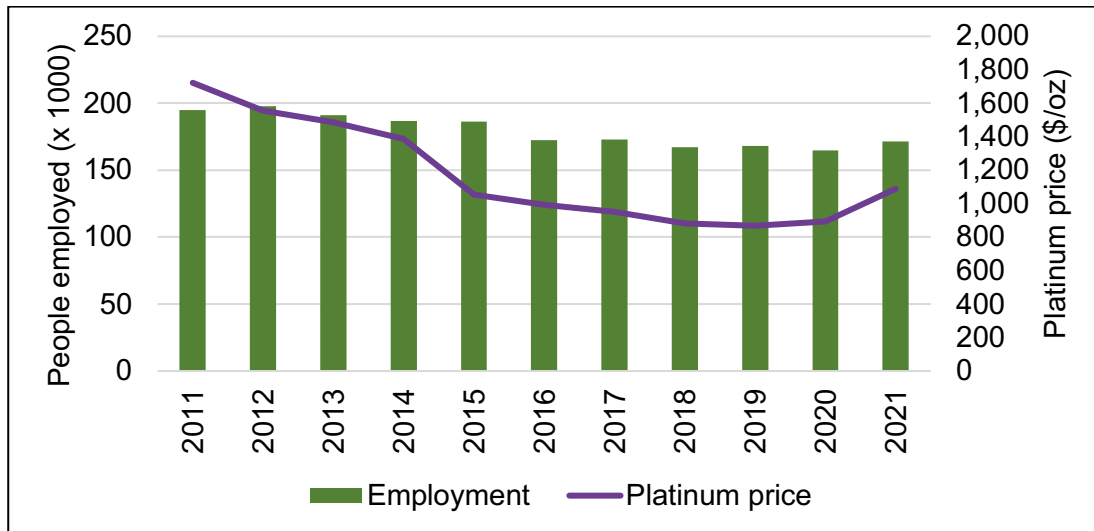


Figure 1: Employment by the platinum mining industry versus the platinum price⁵

In 2021, the input costs of the mining industry increased by 10%⁶, primarily due to rising electricity costs. Eskom, South Africa's energy supplier, has significantly increased energy tariffs, culminating in a combined increase of 653% from 2007 to 2022, whilst inflation for the equivalent period was merely 129%⁷. Moreover, the substantial energy consumption associated with mining operations result in considerable greenhouse gas (GHG) emissions.

The total Scope 1 and 2 emissions amounted to 1,545 billion tonnes of CO₂-equivalent gases in 2023, corresponding to an intensity of approximately 0,145 t/CO₂eq/t ore milled. Fossil fuel emissions (Scope 1) account for only 5% of these emissions, while the vast majority, 95%, stems from purchased energy predominantly produced using coal⁸. Consequently, the mining sector is actively exploring the adoption of renewable energy sources as a means to reduce both operating expenses and environmental impact⁹.

While transitioning to renewable energy sources marks a positive stride, the initial costs of these installations are high, and the technology's efficiency remains somewhat constrained. Therefore, it is essential to minimise the demand for these systems to the greatest extent feasible, aiming to lower both the expenses and environmental footprint of the installations.

⁵ Minerals Council South Africa, 'Facts and Figures 2021', 2021.

⁶ PWC, Highlighting trends in the South African mining industry, 5th ed. 2013. [Online]. Available: www.pwc.co.za/mining

⁷ Sean Moolman, '2022 update: Eskom tariff increases vs inflation since 1988 (with projections to 2024)', *Power Optimal*, Aug. 01, 2022

⁸ Northam Platinum Ltd., 'Northam Sustainable Development Report 2023', Johannesburg, 2023.

⁹ E. Bellini, 'South African platinum mining company wants to deploy 75MW of hybrid wind-solar', *PV Magazine*, Mar. 15, 2022.

1.2 PGM processing

The substantial costs associated with renewable energy solutions renders their application to platinum processing plants impractical, given the energy-intensive nature of PGM processing. The South African mining industry accounts for approximately 15% of the nation's generated electricity¹⁰, of which platinum mining constitutes 14-30% [1]. Generally, 66% of the energy is consumed to process the extracted ore, which involve concentrating, smelting, base metal refining, and precious metal refining [2], as shown in Figure 2 below. The concentrators and smelters are the highest energy consumers, with a consumption of approximately 26% and 28%, respectively [2].



Figure 2: Typical PGM processing flowchart [2]

The concentration plants are responsible for the breakage of ore and the separation of waste rock. This consumption results from the grinding devices used in the comminution circuits of the concentrators, which consume approximately 97% of the concentrators' total energy [3]. Comminution circuits are equipment that mill, grind and break ore as a physical treatment method [4].

1.3 Energy consumption of PGM comminution circuits

Several factors affect the energy consumption of a comminution circuit, potentially leading to inefficiencies in the circuit. The choice of milling technology stands out as a significant determinant [5], [6]. The selection process for milling technology prioritises its effectiveness in achieving the desired particle size, with energy efficiency serving as the secondary consideration. A thorough analysis of the diverse technologies is essential to grasp the rationale behind their selection in comminution circuits, and to identify possible modifications that could enhance the circuit's performance.

¹⁰ Eskom Holdings Limited, 'The energy efficiency series towards an energy efficient mining sector', Johannesburg, 2010.

1.3.1 Tumbling mills

Tumbling mills are large cylindrical drums that rotate, tumbling the ore within the drum. Ball mills are the mining industry's most used tumbling milling technology [7]. They grind the material using grinding media that is fed to the mill. Typically, this medium is composed of ceramic or steel, with diameters varying between 10 mm and 125 mm and are produced in various shapes, such as spherical balls, semi-ellipsoids, and cubes [8].

The cylindrical drum is often tapered at one end and is typically loaded with grinding media-to-ore ratio of 20% to 40%, in line with standard practice. The mill's operational speed is maintained between 70% and 80% of its critical speed – the threshold at which the grinding media align with the drum's rotation due to centrifugal force [9]. Consequently, the mill never reaches its critical speed, as grinding, and crushing inefficiencies would be compromised by the reduced impact energy between the grinding media and the ore.

Rod mills are like ball mills, except that rod mills use rods as grinding media, filling 30% to 40% of the mill volume in contrast with balls. However, their application in practice at PGM concentrators is uncommon, owing to their less efficient breakage performance compared to ball mills [10].

Autogenous (AG) mills are less common in PGM comminution circuits than other technologies. Constructed with diameters ranging from 2 m to 10 m and featuring short cylindrical lengths, these mills are equipped with lifting bars. The design principle behind these mills is that ore will be broken down by being lifted to a significant height by the bars and then falling onto the remaining charge below. This process of breakdown leads to additional attrition and abrasion between the smaller particles and the grinding bodies. The absence of sufficient grinding media extends the time the charge remains in the mill before achieving the desired product size [10].

Semi-autogenous (SAG) mills mitigate this drawback by incorporating grinding media, similar to that used in ball mills, which significantly enhances their processing capacity [11] and facilitates the production of smaller product sizes [12]. SAG mills feature larger diameters and shorter lengths compared to ball mills, exhibiting typical aspect ratios (diameter to length) ranging from 0.7 to 2 [10]. AG mills are operated at 80% of their critical speed, whereas SAG mills usually function at a critical speed lower than 80% to improve grinding efficiency. However, there is a notable operational distinction between SAG mills in Australia and North America and those in Southern Africa.

Southern African mills are often operated in a run-of-mine mode [13]. This mode of operation excludes a pre-milling or crushing circuit, feeding ore from the mine directly to the mill, hence the name run-of-mine. Run-of-mine mills can take the shape of any of the above mill types, although most are designed similarly to a ball mill with an aspect ratio of approximately 0.5 to 0.8, with 5-m to 12-m internal lengths. The constructions that distinguish the mills and the ore feed methods result in significantly different operating parameters. Table 1 below summarises some of the critical differences.

Table 1: Comparison of RoM and SAG mill operating parameters, adapted from [11]

Parameter	AG/SAG	RoM
Speed (% critical)	70-80	75-90
Charge filling (% total mill volume)	20-30	35-45
Ball filling (% total mill volume)	4-15	15-35
Recycle stream	Trommel oversize and pebble crusher	Cyclone under-flow/fine screen
Application	Primary grind	Single stage to final product

1.3.2 Non-tumbling mills

Non-tumbling mills include stirred, high-pressure grinding rollers (HPGR) and Isa mills. The latter two are the most energy-efficient and are thus used for ultra-fine grinding [14]. HPGRs employ two rollers between which the ore is fed, applying high pressure that leads to less “wasted” energy, such as instances where grinding media fail to hit the ore or where friction is not effectively utilised [14].

1.3.3 Pre-milling treatment

Ensuring the mill is fed with high-quality ore of the correct size is crucial for optimising the duration the charge spends in the mill, which can enhance throughput and potentially reduce operational time. This is achieved by improving blasting or ore pre-treatment measures, such as pre-concentrating and electro-magnetic sorting or crushing [15].

Crushing circuits are generally less energy-intensive than mills and are used to break down the ore before the milling circuit. They typically consist of a screen that sorts smaller ore particles from larger ones that require crushing with a crusher. Gyratory, jaw, and cone crushers [16], [17], [18] are the most common crushing technologies. Figure 3 illustrates the basic layout of a crushing circuit with any of the three mentioned crushing technologies in place of the jaw crusher.

Jaw crushers are commonly employed as primary crushers, operating at intensities of 0.1 kWh/t to 0.2 kWh/t due to the large throughput [19]. Post-primary crushing, or secondary crushing, is predominantly carried out using cone crushers, which accept feed sizes ranging from 150 mm to 350 mm and reduce them to 25-mm to 60-mm product sizes [20].

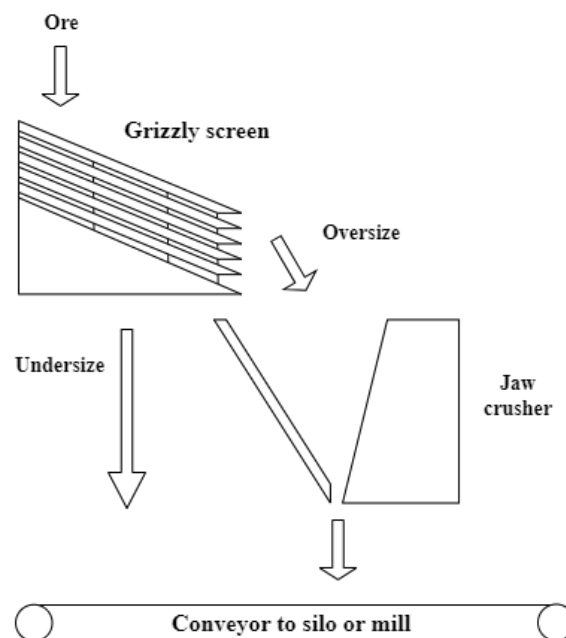


Figure 3: Basic crushing circuit layout [17]

The product size of all the crushers can be achieved either as a design specification of the machine or by adjusting mechanical parts to meet the required size. This ensures that the ore that passes through the circuit requires approximately the same amount of time to grind down to the optimal size for the floating circuit. Instead of crushers, certain operations may use a rod mill or HPGR [21] to achieve the required coarse size for the milling circuit, depending on the circuit configuration.

1.3.4 Milling circuit control variables

Several variables of a milling circuit are controlled to improve its performance and recovery. Mulenga and Chimunani [9] and Metzger *et al.* [22] used a chemical engineering method referred

to as the attainable region (AR) to determine the optimal residence time of a ball mill. The AR method utilises several variables to determine the "ideal" practice and obtain the optimal residence time for ore in a ball mill and the energy consumption [9], [22]. This can hence be described as an optimisation method.

1.3.5 Grinding media

The composition and size of the grinding media are crucial for ore breakage and, consequently, influence the residence time and the milling circuit's energy efficiency. As previously indicated, a substantial portion of energy is used by the grinding media when the grinding media strike the mill's lining instead of the ore. Simulations have corroborated this [9], demonstrating that smaller media sizes enhance ore breakage and reduce residence time through better utilisation of impact energy. This finding has also been confirmed in other studies [23], [24].

1.4 Benchmarking methods

Benchmarking is a systematic and invaluable process used across various industries for evaluating the performance of systems and processes and identifying areas for improvement. Furthermore, benchmarking can help engineers gain insights into best practices and involves measuring and comparing the system or process performance metrics [25]. The primary goal of benchmarking is to enhance efficiency, which can be achieved using an average and best practice benchmark as reference points.

1.4.1 Average benchmarking

The purpose of an average benchmarking model is to compare the average performance of similar entities or systems [26]. The ordinary least square (OLS) method is a linear regression-based technique that can approximate an average function, making it an ideal mathematical tool for analysing the average performance of entities, as shown in Figure 4. This can be applied to the variables applicable to the benchmarked entity, such as energy consumption, production, and other performance indicators.

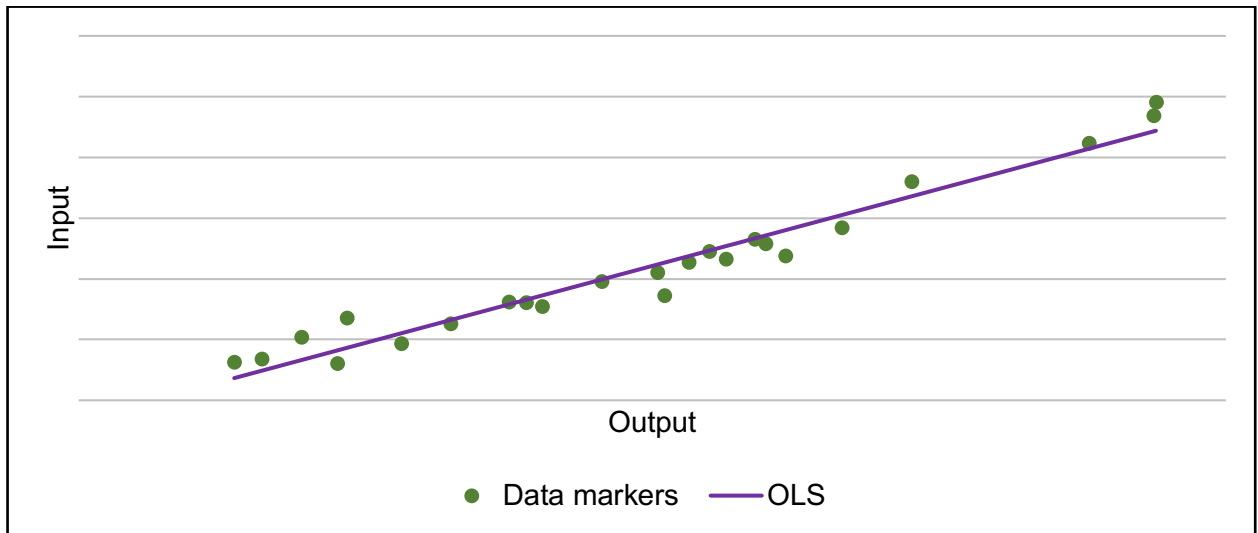


Figure 4: Ordinary least-square linear regression

1.4.2 Frontier benchmarking

Frontier benchmarking is primarily achieved through three methods; these are corrected ordinary least squares (COLS), stochastic frontier analysis (SFA), and data envelope analysis (DEA) [26]. The selected method depends significantly on data availability and whether a representative or accurate best practice performance measurement is required [27].

The COLS method is an improved analysis method of the OLS method that obtains a representative result of an entity's performance [27]. It identifies best practice (frontier) or error data markers based on their position under a regression line [28], as seen in Figure 5. Incorporating outliers or erroneous data markers may distort the outcomes, leading to the creation of misleading benchmarks.

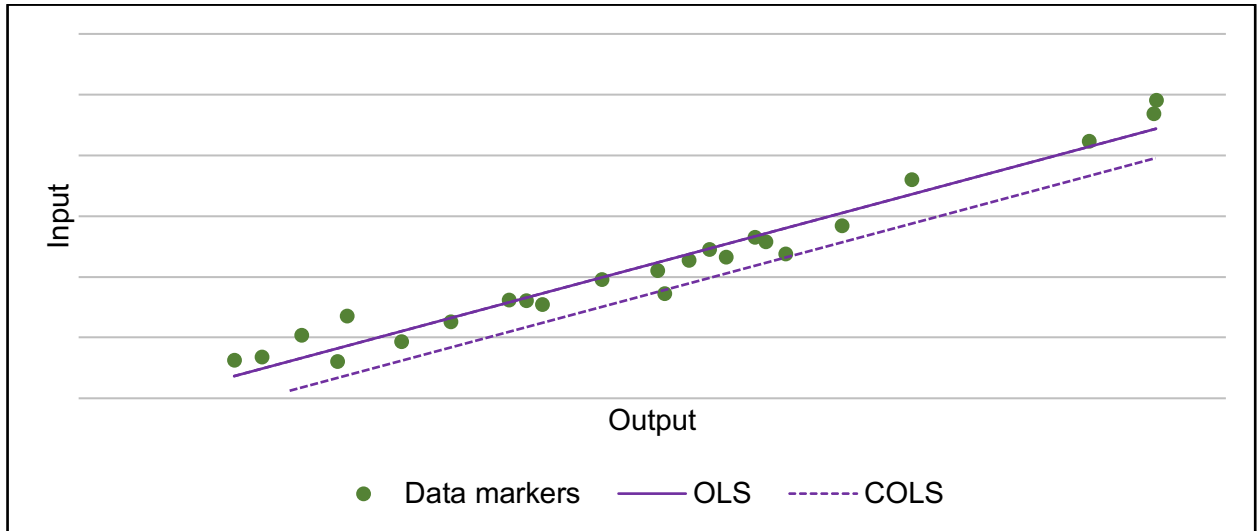


Figure 5: Corrected ordinary least-square linear regression

The SFA method eliminates the risk of skewed analysis by excluding errors and outliers and thereby increasing the function's accuracy. The SFA method can be used to benchmark different units or systems by comparing their performance to the estimate efficient frontier. This is achieved by recognising an absolute error pattern in the data and mitigating the effect of errors and outliers, as shown in Figure 6, thus producing a more accurate estimation of frontier entities [26]. Comparing the produced results to the most efficient frontier marker, produces an accurate representation of the system performance to the best possible performance, maximum output achieved by the minimum input.

The method assumes that the relationship between the output and input function is linear and parametric as with the other linear regression methods. Additionally, it is also assumed that the errors are uniformly distributed and that observed data points consist of both efficient and inefficient (stochastic) components¹¹. Furthermore, it is assumed that the data is parametric, uniformly distributed¹² and the measurement errors are included in the error term.

The main benefit of this method is that it identifies inefficiencies and the cause of the inefficiencies. This is critical to the study, as the correction of the inefficiencies are as vital as identifying them. Identifying inefficiencies in each circuit and comparing the operations of similar circuits, can potentially enable changes to improve the performance of all the circuits.

¹¹ Wallstreetmojo Team, "Stochastic Frontier Analysis," Wallstreetmojo. Accessed: May 25, 2024. [Online]. Available: <https://www.wallstreetmojo.com/stochastic-frontier-analysis/#>

¹² T. Hoskin, "Parametric and Nonparametric: Demystifying the Terms."

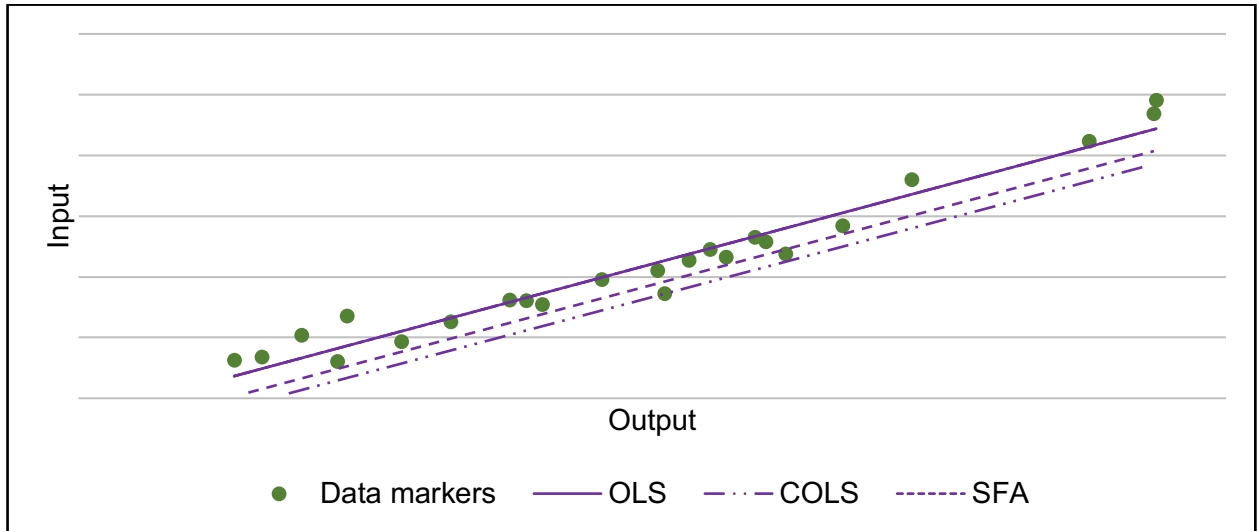


Figure 6: Stochastic frontier analysis

Unlike the other methods, DEA is a computational benchmarking method rather than an estimation method. DEA solves for the most efficient input-to-output data markers using linear programming [29]. Instead of determining a straight-line function to match the data markers as efficiently as possible, DEA develops an "envelope", as seen in Figure 7, through the most efficient data markers [30].

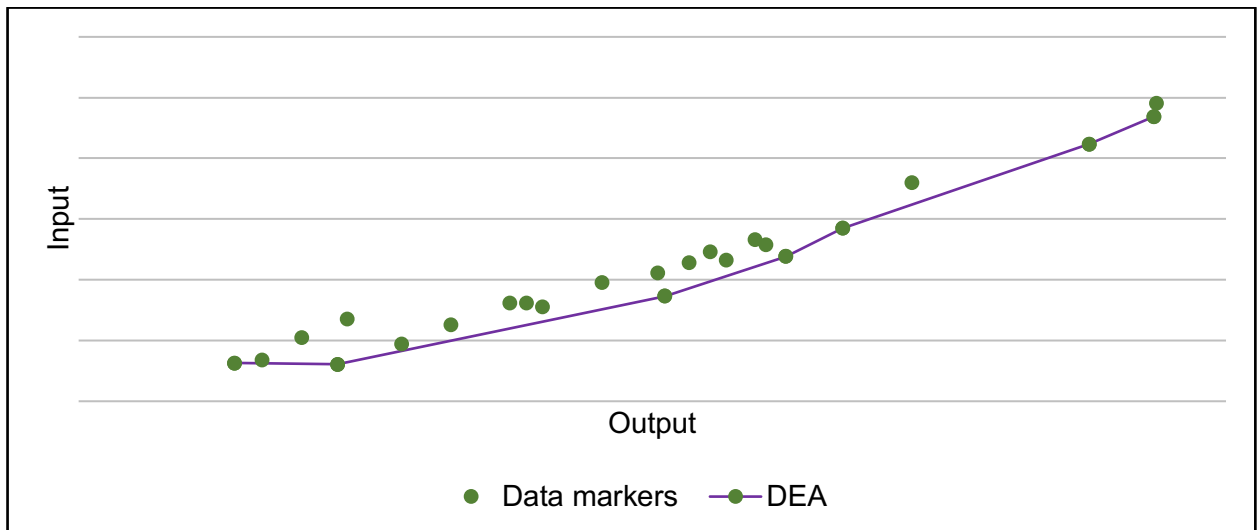


Figure 7: Data envelope analysis

While valuable, however, the DEA method is more susceptible to the influence of outliers compared to SFA; as such the method requires additional processing to eliminate any outliers from the dataset and ensure that the envelope is not skewed [31]. Despite this, these methods are often applied to energy benchmarking models for their straightforwardness in helping to identify areas for improvement [32].

Areas of inefficiency are typically represented by data points that reside above the OLS regression line, indicating underperformance relative to the dataset's average. In contrast, the frontier methods indicate the best practice performance that can be achieved through improved practices. Nonetheless, as previously mentioned, outliers can skew the results, leading to a misrepresented frontier. Consequently, the SFA method is favoured over the COLS and DEA because of its enhanced resilience in managing outliers, making it a more reliable method for identifying true performance frontiers [26].

1.5 Previous studies on energy benchmarking

Energy benchmarking is a process of measuring the energy performance of a sector or individual plant against a “standard” metric that is considered as the “optimal” performance relative to the sample group [33]. There is great benefit in understanding energy consumption patterns, identifying inefficiencies, estimating potential energy reduction, and improving practices and policies to reduce energy consumption [34]. Benchmarking methods may draw upon a variety of foundations, including concepts, processes, plants, and products. Typically, a benchmarking approach employs one of these elements to facilitate the comparison of two variables within the given system or process.

Several studies have investigated the application of energy benchmarking at the industrial or commercial level, making it an acceptable method to apply to practical comminution circuits. Nonetheless, for the development of an accurate model, it is essential to critically review existing literature methodologies. Including of all relevant studies for such a purpose necessitates a pragmatic strategy.

1.5.1 Review methodology

The review is divided into six stages based on the methodology of Winchester *et al.* [35]. The method implements a systematic approach to refine the scope of literature relevant to the proposed research topic, as summarised in Table 2 below.

Table 2: Review methodology based on the method developed by Winchester *et al.* [35]

Stages	Description
1. Research topic	Development of an energy benchmarking model for platinum group metal comminution circuits
2. Keywords and search terms	Benchmarking, energy benchmarking, comminution circuit, platinum group metals
3. Sources of information	Google Scholar, IEEE Xplore, Science Direct, ResearchGate, MDPI, Sabinet, Journals, Textbooks
4. Collection of documents	The information is collected from one PhD thesis and 15 journal articles.
5. Systemisation and evaluation	Relevant information regarding the keywords and topic are compared and assessed.
6. Documentation of literature review	Critical information is summarised and discussed.

The first stage in research is to conduct an initial inquiry to identify gaps in the literature that would guide the research field and define the study's topic. The relevant topics for this study included benchmarking, specifically regarding energy, comminution circuits, and branching into PGM comminution circuits, which were used as keywords to search databases for relevant studies. Subsequently, the relevant information was extracted and is hence analysed, compared, critically reviewed, and summarised in the following sections.

1.5.2 Studies on commercial and industrial energy benchmarking

Siderius *et al.* [34]

Summary: Siderius *et al.* discussed the international benchmarking processes for product efficiency policymakers. The study examined the difference between benchmarking and mapping, where benchmarking serves to analyse comparable data on energy consumption and energy-related aspects of a product for a country or region. Mapping is defined as the process of collecting data on energy consumption and energy-related factors of a single product or category of product over several years to highlight the development of energy-related elements [34].

Conclusion: The study analysed the energy consumption of household appliances, mapping and benchmarking individual products such as freezers, refrigerator-freezer combinations, washing

machines, and dryers. The products and categories, such as cooling appliances, were benchmarked internationally, while several individual products were mapped over several years.

Lee, 2010 [30]

Summary: Lee produced a novel index-total performance benchmarking model to determine the energy used for building cooling purposes. The study utilised the ratio of the total heat-transfer coefficient to the energy efficiency of the air conditioning system. However, as noted in the study, it is crucial that this ratio must accurately indicate performance as it does not account for the required ventilation. Thus, DEA was implemented to examine the ventilation and non-ventilation factors and thereby benchmark the system's efficiency accurately.

Conclusion: The model was developed using the climate data to determine the cooling degree hour, which then served as the basis for a linear regression model correlating the building's energy consumption (dependent variable) to the cooling degree hour (independent variable). Subsequently, factors such as the building's floor area and the number of occupants were considered. These parameters were incorporated into the DEA to develop the benchmarking model.

Shabunko et al. 2014 [27]

Summary: The study used a few conventional, bottom-up methods, OLS, support vector machines (SVM), and engineering modelling (EM) to develop a building benchmarking model. The study highlighted the variance in complexity and accuracy. The EM utilised local weather data, including wind and sunlight hours, compared to the OLS, which only used linear multivariable regression. The SVM was highly effective for solving non-linear problems and only required a small training set.

Conclusion: Based on the produced coefficient of determination (R^2), the study concluded that a polynomial least-square fit is more effective than a linear square fit. Contrary to the simplicity of the linear regression method, the SVM requires exceptional detail when optimising the kernel parameters. The EM can provide a detailed model with energy consumption profiles but requires complex datasets.

Jamasb et al. 2000 [26]

Summary: The study discusses several benchmarking methods and techniques. The study aimed to determine the best benchmarking process for an incentive regulation method to promote efficiency. The method promotes improved efficiency by rewarding good performance relative to a pre-defined benchmark.

Conclusion: The study discussed the implications of the methods used to determine the pre-defined mark. The methods included two benchmarking types, namely, frontier benchmarking methods and average or mean benchmarking methods. The frontier benchmarking techniques include DEA, COLS and SFA; the average or mean benchmarking is the standard OLS method.

Pavlova *et al.* 2023 [36]

Summary: Pavlova *et al.* [36] employed similar OLS methods akin to those mentioned earlier to benchmark the energy intensity of small water resource recovery facilities, employing an intensity ratio of kWh/m³ for this purpose. This benchmarking method pinpointed critical factors related to energy intensity, such as the facility type, supplemental energy usage for sludge treatment, average flow rate, percentage of designed flow rate, climate control, effluent, and influent.

Furthermore, the analysis underscored variations in the energy efficiency models of different facility sizes and indicated that regulatory changes could improve the energy efficiency of these facilities. It also brought to light the disparities in energy efficiency between facilities located in different states. Furthermore, despite overlooking some variables that could account for variability in energy consumption, the models could still provide a benchmark for comparison for other water resource recovery facilities.

Lessons learnt: The study's method is simple to replicate by substituting key variables pertinent to comminution circuits, thereby yielding valuable outcomes for these circuits, including a benchmarking model and a basis for simple comparison.

Chan *et al.* 2014 [37]

Summary: In 2014, Chan *et al.* [37] identified the need for energy benchmarking to estimate energy efficiency. The increasing demand for energy in Taiwan due to rapid economic development necessitated an evaluation of the energy efficiency of the country's most energy-intensive industries. Additionally, the study conducted a benchmarking of the carbon emissions from these industries, enabling comparisons both within the sector and within industries in other countries.

The study used the straightforward specific energy consumption and carbon emission intensity methods as defined in Equation 1 and Equation 2 below to determine the potential achievable efficiency. The findings indicate that the improvements to the blast furnace-basic oxygen furnace could enhance efficiency by 28% based on best practice technology and that pulping efficiency could also be improved by approximately 33% by implementing the best available technology.

Equation 1: Specific energy consumption calculation

$$SEC = \frac{\text{energy used}}{\sum \text{products produced}}$$

Equation 2: Carbon emissions intensity calculation

$$CO_2 = \frac{CO_2}{\sum \text{products produced}}$$

Conclusion: The study produces an average benchmark for various industries and highlights the importance of benchmarking practices and implemented technologies. The simple method of benchmarking the industries and their implemented technologies are suggested to hold significant potential for energy efficiency improvements within various industries.

Shortcomings: The improvements are not validated or motivated by the implementation of specific energy efficient equipment. It is only stated that there is potential based on the benchmarking of the systems within the industries.

Inglesi-Lotz et al. 2014 [38]

Summary: In 2014, Inglesi-Lotz *et al.* [38] proposed a benchmark-and-trade system for improving energy efficiency in South Africa. The system operates under the principle that users can sell or buy credits depending on how their performance measures up against the industry-specific benchmark. In this context, a credit represents the percentage difference between the user's consumption and the established benchmark.

If the user is below benchmark consumption, the user is a seller of credits, and the user is a buyer if the opposite is true. The credits are based on three standards, which are related to the three benchmarking standards of the Organisation for Economic Co-operation and Development (OECD):

1. Twenty (20) times the OECD electricity intensity
2. Ten (10) times the OECD electricity intensity
3. OECD electricity intensity

As expected, the standard used for the benchmark will significantly affect the trade of the credits, with several industries buying credits if standard 3 is applied.

Shortcomings: The study aimed to employ a standardised benchmark to incentivise energy saving practices. Its scope was confined to evaluating the cost-saving potential of this system relative to the proposed carbon tax approach. Instead of creating a benchmarking model, the study examined the extent to which an existing model could serve as a reference point that would encourage users to sustain energy consumption below designated standards.

Sardeshpande *et al.* 2007 [39]

Summary: In 2007, Sardeshpande *et al.* [39] challenged the conventional approach to energy benchmarking by using actual data instead of that generated from statistical analysis, setting their work apart from other studies. Specifically, the study used energy and mass balances to develop a simulation model of the energy consumption of a glass furnace based on actual data that could identify inefficiencies in furnaces. This adjustment facilitated enhancements to the model's parameters, potentially increasing the energy efficiency of the simulated model. Theoretically, these improvements could be replicated in a practical setting to create an efficient real-world system.

Lessons learnt: While this benchmarking approach yields more detailed outcomes, its development is complex. Despite this complexity, it proves advantageous, especially when combined with a conventional statistical analysis model. This synergy enhances the value of comparing both the developed model and the practical system against industry standards, ultimately offering a superior indicator of energy consumption relative to industry norms.

Ke *et al.* 2013 [33]

Summary: Ke *et al.*'s [33] 2013 study focused on developing a process-based energy benchmarking method. This method was used to evaluate intricate, interconnected systems and therein identify potential energy savings. The study also compared more traditional, product-based benchmarking to the process-based one.

Conclusion: The results of the study suggest that while process-based benchmarking offers greater accuracy, its complexity is heightened by the extensive data needed to construct the model. Conversely, product-based benchmarking is simpler to implement and demands considerably less data.

Shortcomings: The complex nature of the system was a challenge to incorporate into the benchmarking method, which entailed having to break the systems down into several subsystems. Additionally, the great variability introduced by the industrial systems' various processes made obtaining accurate and exact or specific data challenging.

Lessons learnt: These challenges could be mitigated using approximation, linearisation, normalisation, and mathematical transformation, thereby overcoming the shortcomings of process-based benchmarking methods.

1.5.3 Studies on mining energy benchmarking

Cilliers, 2015 [32]

Summary: Cilliers conducted a study in 2015 on the energy consumption of the most energy-intensive processes of deep-level mining, focusing specifically on the gold mines in South Africa [32]. The study identified relationships between several variables related to the energy-intensive system and its energy consumption. The specific focus of the study was on the following systems and variables that directly correlated to their energy consumption:

- Ventilation
 - Number of tonnes mined
 - Mine depth
 - Ambient conditions
 - Geographical area
- Refrigeration
 - Number of tonnes mined
 - Mine depth
 - Ambient conditions
 - Geographical area
- Compressed air
 - Number of tonnes mined
 - Mined depth
 - Ambient conditions
- Dewatering
 - Number of tonnes mined
 - Mined depth
 - Amount of fissure water
- Hoisting
 - Number of tonnes mined
 - Mined depth

Using the relationships between the systems' energy consumption and the variables that affect them, Cilliers developed an average and best-practice benchmarking model. These models were applied to case-study mines to predict the respective systems' energy consumption based on the input of the respective parameters.

Conclusion: The models were able to effectively predict the energy consumption of the various systems and were further implemented to forecast energy costs and develop highly accurate utility budgets.

Caromba *et al.* 2023 [40]

Summary: Caromba *et al.* [40] used clustering techniques with statistical methods to benchmark the energy consumption of deep-level mines. This method was compared to the traditional benchmarking method of using energy intensities. The clustering methods include k-means, bisecting k-means and hierarchical. A simple OLS regression was performed on each cluster to develop the equations and predict the systems' energy consumption, based on the input variables from the mining shafts.

Shortcomings: Despite the improved benchmarking benefits of the implemented clustering techniques, several techniques were excluded from the study which might have further improved the benchmarking model.

Wang *et al.* 2016 [41]

Summary: Wang *et al.* [41] developed an energy efficiency benchmarking system based on two indicators. The product-based indicator focused on the energy efficiency of the total energy consumed by all energy sources to produce the coal. Contrary to this, the process-based indicator provided insight into the efficiency of the total energy consumption and the total workload in coal production.

The process-based indicator can be broken down into the main energy-consuming systems to provide deeper insight into the energy consumed by each system, justifying the results of the product-based indicator.

Lessons learnt: Implementing both indicators will produce an improved holistic perspective of the mine's energy efficiency. However, as both indicators were developed using statistical and mathematical methods, this increased the system's complexity.

1.5.4 Studies on comminution circuit energy benchmarking

Ballantyne *et al.* 2019 [42]

Summary: Ballantyne *et al.* [42] developed a benchmarking methodology that employs energy curves to highlight the variations in energy intensity across different operations. The methodology benchmarks case studies using four critical parameters: bond work index (BWi), operating work index, specific energy, and copper-equivalent specific energy. The objective was to determine the most effective parameter for benchmarking comminution circuits with which to explore the correlation between the results obtained.

Conclusion: According to the study's findings, the BWi and the operating work index had similar results. However, results varied considerably under specific energy, implying that the aim of the model will predict the most suitable parameter.

Lesson learnt: If the purpose is to compare the performance of energy consumption, the specific energy parameter would be most appropriate. The method can easily be replicated for PGM plants if the relevant data is available.

Ballantyne *et al.* 2014 [43]

Summary: Ballantyne *et al.* [43] implemented the analysis method to benchmark mills' energy consumption in copper and gold processing plants. The study primarily focused on Australian mines. The scope of the comminution circuit included the grinding and crushing equipment, although data on fine grinding and crushing was limited. The study utilised an intensity factor (MWh/oz) to compare the performance of the circuits of gold mines concerning their annual production (Moz), as seen in Figure 8. To gain a clearer understanding of how inefficiencies affect the circuit, energy cost was factored into the analysis. This method was then applied to comminution circuits in copper mining operations.

Shortcoming: The study only covered the scope of copper and gold mines, not any PGM mines. Furthermore, the study's focus was limited to the overall performance of the circuit and as such neglected the effects of implemented technologies or practices.

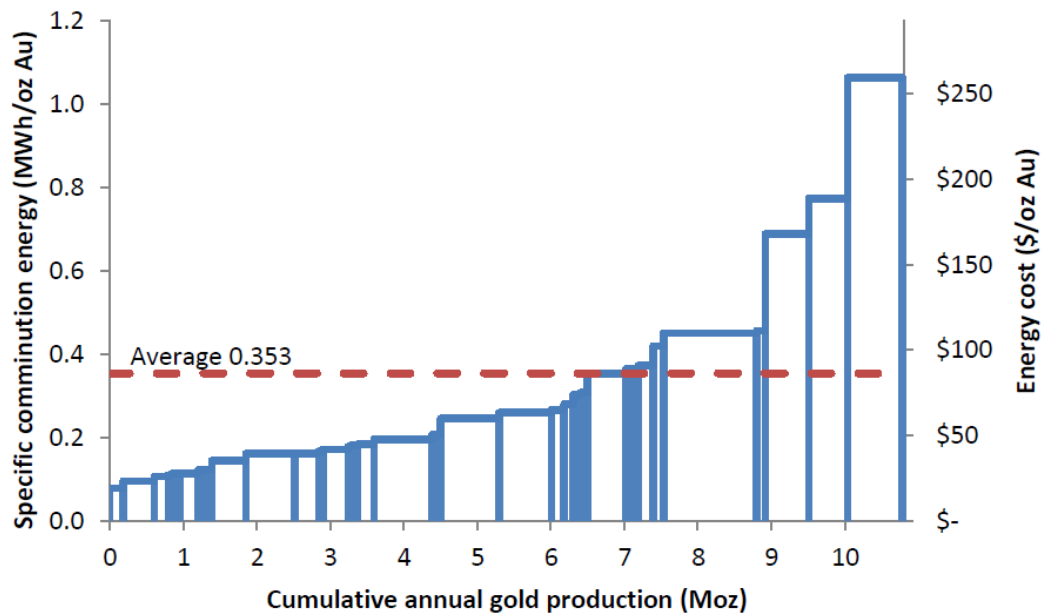


Figure 8: Benchmarking results of Ballantyne and Powell's study [43]

Nadolski *et al.* 2019 [44]

Summary: Nadolski *et al.* [44] implemented engineering methods to measure the performance of the existing comminution circuit at Red Chris mine, which processes copper and gold. The study calibrated circuit models from data collected through surveys. These models were developed, calibrated, and simulated using JKSimMet, which simulates models based on the Morell method [45].

Conclusion: The simulation results, combined with the circuit's actual performance, enabled the identification of energy inefficiencies, prompting further investigations to ascertain their causes. Nadolski *et al.* also used the model they developed to modify the circuit and simulate other operational scenarios to explore alternative energy- and production-efficient operating philosophies.

Nadolski *et al.* 2015 [46]

Summary: The study was based on the work by Shi *et al.* [35] for developing an energy-breakage model, as shown in

Equation 3. The study used the model for its intended purpose, which is to determine the energy required to break the ore. Based on the minimum practical energy needed for ore breakage, a benchmark energy factor (BEF) was developed by comparing the energy required for ore breakage and the expected energy consumption.

Equation 3: Benchmark energy factor expression

$$BEF = \frac{\text{specific energy}}{\text{minimum practical energy}}$$

This benchmark energy factor enables the inclusion of ore hardness levels that would otherwise have skewed the general specific energy. Therefore, the benchmarking energy factor yields results that are more representative of the circuit's performance concerning its ore. However, a significant limitation of this approach is the need for comprehensive practical energy data to finalise the benchmarking model. Such data may not always be accessible, and its accuracy must be consistently verified in consideration of the differences between the ore from different mines. Moreover, the minimum practical energy requirements can vary across different mines, posing a challenge in accurately assessing the circuit's performance relative to the ore type. This challenge is further compounded when ore from multiple mines are processed concurrently.

Lessons learnt: The factor facilitates the comparison of the communication equipment in use. The study used the factor to assess the BEF of the existing equipment and the circuit, establishing a baseline for the analysis. Furthermore, by leveraging performance factors of alternative technologies available on the market, the study recommended modifications to the circuit through the introduction of new equipment and a revised layout.

1.5.5 Summary of literature

Benchmarking methods serve as powerful tools for measuring and comparing the performance of systems and processes. However, the method selected to develop a model is critical, impacting both the accuracy of the model and its complexity. The literature reveals a spectrum of model development approaches, ranging from straightforward statistical methods to intricate engineering techniques such as simulations and mathematical models. Each chosen method brings with it distinct limitations and advantages.

Statistical methods are simple to develop and can be readily applied to systems for relatively accurate performance comparisons. The primary limitation of statistical methods lies in their dependence on linear relationships between variables for the development of the initial model. However, if such linear relationships exist among the variables, average benchmarking can be effectively conducted through linear regression analysis. Meanwhile, frontier benchmarking models can be created using COLS, DEA and SFA methods.

In contrast, engineering methods produce more accurate system models, enabling more dependable comparisons across different systems. However, techniques such as state-vector machines necessitate specific parameters to train the kernel for accurate model development, a process that demands considerable time and resources. Moreover, simulation or engineering models require detailed, complex datasets with numerous data points to precisely replicate a system's performance. While such a model might accurately depict a practical system, creating multiple models for the comparison of various systems is impractical. Thus, striking a balance between complexity and accuracy is crucial, with priority given to the most critical aspect.

1.6 Problem statement

The problem is that comminution circuits are operated in an inefficient manner, where significant improvements can be made.

Thus the need is to develop a simple and cost-effective method which can be used to identify and improve inefficiencies within these circuits.

The PGM mining sector in South Africa is a major consumer of the country's electricity. Challenges faced by the national energy supplier in generating power have led to significant increases in energy tariffs, which in turn raise operating costs. This impact is particularly pronounced in the operational expenses of concentrating plants, attributed to the energy demands of inherently inefficient comminution circuits [6]. Additionally, the pace at which newer, more efficient technologies are developed is sluggish. Therefore, enhancing operational practices emerges as one of the most effective strategies for improving the efficiency of these circuits.

The literature suggests that control variables within a system can be tweaked until the optimal point of energy efficiency is reached. However, this approach is not feasible for practical systems, as any modifications might compromise production efficiency. Consequently, these adjustments are typically tested under laboratory conditions, which do not always accurately reflect real-world system performance.

The core issue is the inherent inefficiency of current comminution circuits, which proves challenging to overcome. Hence, there's a pressing need to identify and spotlight inefficient circuits and adopt more efficient operational practices that either reduce energy consumption or enhance energy intensity. Nonetheless, the prevailing benchmarking methodology assesses the performance of the concentrating plant as a whole rather than focusing on specific circuits or their segments. This makes it challenging to pinpoint inefficient sections or ascertain the root causes of inefficiency within a circuit.

1.7 Need for study, aim and objectives

Numerous studies have investigated different benchmarking methods as tools to pinpoint areas for potential efficiency enhancements. Table 3 summarises the reviewed literature and the particular areas of focus. Yet, only a small number of these studies were specifically aimed at or developed energy benchmarking models for comminution circuits. Furthermore, none of the studies examined have specifically considered PGM concentrating plants, highlighting a notable research gap in benchmarking methodologies for comminution circuits in PGM concentrating settings.

The state-of-the-art matrix presented in Table 3 highlights the research gap concerning comminution circuits in PGM concentrator plants. The comminution circuits' notably high energy consumption and complexity necessitates a comprehensive understanding of the variables and factors impacting their performance.

A pivotal factor is the choice of technologies within these circuits, which greatly affect energy consumption, throughput, and product size – fundamental performance indicators for comminution circuits. Despite the significance of these technologies, there exists a pressing need for a more thorough evaluation of comminution circuit performance. General assessments of circuit performance fail to identify the exact causes of their efficiency or lack thereof relative to others, underscoring the necessity for a more detailed analysis.

Table 3: State-of-the-art matrix

Studies	Energy management	Benchmarking	Statistical methods	Engineering methods	Comminution	PGM plant
[26], [30], [32], [36]	✓	✓	✓	×	×	×
[27], [33], [39], [40] [41]	✓	✓	✓	✓	×	×
[37], [38]	✓	✓	×	×	×	×
[42], [46]	✓	✓	✓	✓	✓	×
[43]	✓	✓	✓	×	✓	×
[34]	×	✓	×	×	×	×
[44]	✓	×	×	✓	✓	×

Included in study	✓
Excluded in study	×

Aim: Drawing from the preceding discussions, this study's primary objective is to develop and apply both average and frontier benchmarking methods to the comminution circuits of PGM concentrators. These benchmarking methods are aimed at evaluating circuit performance and differentiating between efficient and inefficient practices. Additionally, this study seeks to assess the effectiveness and accuracy of score-based benchmarking in identifying inefficiencies in comparison with the traditional intensity benchmarking approach. To fulfil the study's aim, specific objectives have been established.

Study objectives:

Objective 1: Develop a new average benchmarking model for pre-milling circuits of PGM concentrators.

Objective 2: Develop a new best-performance benchmarking model for pre-milling circuits of PGM concentrators.

Objective 3: Develop a new average benchmarking model for primary milling circuits of PGM concentrators.

Objective 4: Develop a new best-performance benchmarking model for primary milling circuits of PGM concentrators.

Objective 5: Develop a new average benchmarking model for secondary milling circuits of PGM concentrators.

Objective 6: Develop a new best-performance benchmarking model for secondary milling circuits of PGM concentrators.

Objective 7: Develop a score-based benchmarking method for PGM comminution circuits.

Objective 8: Compare the score-based benchmarking method to the current one used.

1.8 Research methodology

Based on the quantitative nature of this study and its application in a real-life setting, the methodology is structured around two approaches: quantitative analysis [47] and case study research [48].

The quantitative approach has three fundamental principles: data collection, data analysis, and making predictions. Additionally, this approach can be categorised into four distinct types [47]:

1. Survey
2. Correlational
3. Experimental
4. Casual-comparative

The study hence uses a quantitative approach in the form of a correlation analysis to investigate the relationship between two variables within a real-life context or a case-study scenario. Linear regression is identified as the optimal method for assessing the impact of the predictor (independent) variable on the outcome (dependent) variable in actual comminution circuits.

Given that the quantitative analysis occurs within a real-life context, it necessitates the adoption of a case-study methodology. This approach is employed to gain a comprehensive understanding of the process or system by closely examining its operation [48].

The case-study methodology is divided into three primary categories: intrinsic, instrumental, and collective. For this study, the collective case study approach is employed, focusing on the examination of multiple case studies. This strategy was chosen to gain a deeper insight into the performance characteristics of PGM comminution circuits.

1.9 Dissertation layout

Chapter 1 delved into the pivotal role of the mining industry within South Africa, particularly emphasising the substantial energy demands of processing plants. It highlighted concentrator plants as notably energy-intensive, primarily due to the extensive comminution circuits tasked with ore fragmentation. A thorough review of the literature examined various milling technologies and optimisation strategies. Moreover, the chapter explored the application of diverse benchmarking methodologies in the mining sector. Stemming from this exploration, a new area of research emerged: energy benchmarking for comminution circuits. The chapter concluded by summarising the literature reviewed, underscoring the rationale of the study, articulating the problem statement, and outlining the research objectives.

Chapter 2 presents the methodology design that was developed to address the problem statement and achieve the objectives of the study. It systematically describes the process of developing both the average and frontier benchmarking methods. Additionally, the chapter examines the formulation of the scoring method in detail.

Chapter 3 discusses the implementation of the formulated benchmarking methods on specific comminution circuits and the technologies employed within them. It showcases how score-based benchmarking identified the most efficient segments and overall performance of circuits. The evaluation involved scoring the circuits based on their effectiveness in pre-milling, primary milling, and secondary milling stages. The operational performance of these circuits was established as a benchmark for enhancing the efficiency of other circuits.

Chapter 4 serves as the concluding chapter of the study, providing a summary of the dissertation. It emphasises the key findings and outlines recommendations for future research, thereby drawing the chapter to a close.

CHAPTER 2 METHODOLOGY TO DESIGN BENCHMARKING MODEL

2.1 Introduction

Reflecting the research's dual quantitative and case study approach, it is essential to integrate respective approaches in constructing the methodology for this study. From the literature emerged specific criteria essential for the development of a statistical benchmarking model. These criteria are summarised as follows:

Criteria	Source
1. Collect data	[32], [40], [49]
2. Pre-process data	[32], [50], [51]
3. Determine prediction equations	[26], [32], [26], [52]
4. Validate results using statistical methods	[53], [54]
5. Employ scoring method to statistical benchmarking method	[32], [40], [46]
6. Benchmark monthly entries	[32], [55], [56]
7. Compare scoring-based benchmarks with other methods	[42], [43], [46], [57]

A structured approach that integrates the necessary criteria and research methodologies outlined is adopted. This ensures the fulfilment of all required criteria for the development of a benchmarking model applicable to studies of similar scope.

2.2 Step 1: Data collection

The first criterion involves collecting data on variables that influence the circuit's energy consumption, as identified in the literature, as well as the energy consumption data itself. Thus, the first step entails compiling this data into a format that is digitally accessible and compatible with analysis via third-party software.

To reduce the variability observed over shorter periods, data is extracted as monthly averages from the supervisory control and data acquisition (SCADA) system [32]. It should be noted that

utilising data from a broader timeframe will facilitate a more effective benchmarking against other circuits [40], [49].

2.3 Step 2: Pre-processing data

The datasets include information on power consumption, ore feed, feed water flow, and mill weight. This information facilitates an understanding of the circuits and supports the derivation of expected values, which are crucial for establishing outlier boundaries. Excluding these outliers from the correlation analysis can enhance the precision of the analysis. Outliers can be readily detected by examining the deviation of the intensities from the mean intensity. The intensity of the respective variables is computed as depicted in Equation 4 [32].

Equation 4: Calculation of intensities

$$i \left(\frac{kW}{unit} \right) = \frac{power (kW)}{variable (unit)}$$

With:

i	= intensity (kW/unit)
v	= variable (feed, feed water, weight of mill or ball load)
p	= mill power consumption

Employing intensity metrics enhances the precision of outlier prediction by facilitating a comparison between two variables expected to have a direct correlation, rather than relying on a single variable for outlier identification. The computed intensities undergo evaluation to determine a z-score, representing the deviation of the intensities from the mean. This calculation can be performed as detailed in Equation 5 [50]. Outliers are defined by a z-score that falls below -3 or exceeds 3, indicating x values situated more than three standard deviations away from the mean [51].

Equation 5: Calculation of z-score

$$z = \frac{x - \mu}{\sigma}$$

With: z = z - score
 x = x - value
 μ = mean
 σ = standard deviation

Upon computing the intensity, as depicted in Equation 4, and the subsequent z-score outlined in Equation 5, it becomes evident that the dataset presented in Table 4 below contains no outliers. Consequently, all data points are deemed valid for inclusion in the analysis, ensuring that the results are not skewed by anomalous values.

Table 4: Monthly intensity of mill power consumption and weight

Month	Weight (tonne)	Power (kW)	Intensity (kW/tonne)	z-score
1	460	3 290	7	-1.1
2	461	4 995	11	0.4
3	462	4 984	11	0.4
4	469	5 138	11	0.5
5	466	5 050	11	0.4
6	486	1 776	4	-2.6

While the variation may appear minimal in comparison to other data points within the table, incorporating all circuits into the regression analysis could lead these points to skew the model, potentially leading to an inaccurate interpretation of the correlation derived from the regression analysis. Hence, identifying and eliminating any outliers is crucial to maintaining the integrity of the analysis.

2.4 Step 3: Obtain prediction equations

To formulate precise predictive equations, it is imperative to establish a definitive relationship between the dependent and independent variables. Therefore, the correlation between the variables needs to be confirmed prior to deriving the average and frontier equations.

2.4.1 Verify correlations

The third criterion involves establishing the predictive equations for both average and frontier analyses. To accomplish this, a demonstrable relationship between the dependent variable (power consumption) and the independent variable(s) is required. Such relationships are confirmed through regression analysis, conducted using third-party software such as Microsoft Excel ®.

The expected linear relationship suggests that simple linear regression is the most appropriate method of analysis, as illustrated in Equation 6. The validity of the linear regression correlations is ascertained using Excel's LINEST and FDIST functions. The statistical details derived from the linear regression analysis via the Excel LINEST function are displayed in Table 5 [32] as an array.

Table 5: LINEST function array (adapted from [32])

	A	B	C	D	E	F
1	m_n	m_{n-1}	...	m_2	m_1	b
2	se_n	se_{n-1}	...	se_2	se_1	se_2
3	R^2	se_y				
4	F	d_f				
5	SS_{reg}	SS_{resid}				

This array is distilled into a linear equation that can be generated by Microsoft Excel ®, although the coefficients within the array remain essential for confirming the correlation. The equation of the line can be formulated as a straightforward linear equation, depicted in Equation 6 below. The other coefficients found in the array play a crucial role in the deeper examination of the regression equation and its associated errors.

Equation 6: Linear regression function

$$y = mx + c$$

With:

y	= y-axis variable
m	= gradient
x	= x-axis variable
c	= y-axis intercept

The correlation between the two variables must be confirmed with the coefficient of determination (R^2). A value of 1 represents a perfect linear relationship between the two variables. Suppose the

correlation proves true for an R^2 value close to 1, then F-statistic (F) and degrees of freedom (df) must be considered to confirm that the correlation did not occur by chance. This is done by calculating the right-tailed *F-probability distribution (degree of diversity)* using Excel's FDIST function [26], [52].

Given the y-value's estimated nature within the function, it inherently carries an associated error termed as the standard error of the y-estimate (se_y). The regression function plays a crucial role in evaluating the significance of the y-estimate's error, taking into account the inputs and whether they are single or multiple variables.

Regression analysis can be expanded to a multivariable regression function that follows the same analytical steps but incorporates more than two variables. This approach is valuable for assessing the correlation among three or more input variables and their impact on the dependent variable (y-variable). The formulation of this function is presented in Equation 7. Conversely, a modified version of the simple regression analysis can offer insights into optimal practices or frontier performance.

Equation 7: Multivariable regression function

$y = a + m_1x_1 + m_2x_2 + m_3x_3 + \dots + m_nx_n$		
With:	y	= y-axis variable
	a	= constant or intercept
	m_n	= gradient for x_n
	x_n	= x-axis variable

Once the relationships between the dependent and independent variables are confirmed from the literature, the regression functions above are utilised to develop the respective prediction equations, average, and frontier.

2.4.2 Average function

The average function is developed using the OLS regression analysis. The linear function, derived from regression analysis, confirms the existence of strong correlations among the variables. It is important to acknowledge that the pre-milling circuit features just one independent variable: the ore feed. Consequently, a single-variable analysis suffices for the development of the function representing average performance.

For both primary and secondary mills, numerous input variables must be taken into account. Thus, those variables demonstrating a significant correlation to the mill's power consumption are utilised to construct an average performance function. This function is derived from a multivariable regression model, acknowledging that multiple input variables influence power consumption. However, rather than employing the conventional multivariable regression analysis of Microsoft Excel ®, the analysis proceeds by conducting a single linear regression analysis for each variable and then averaging these to ascertain the coefficients for the multivariable function, as detailed in Equation 8.

Equation 8: Adapted multivariable function

$$y = \left(\frac{1}{n}\right) \times (a_1 + a_2 + a_3 + a_n + m_1x_1 + m_2x_2 + m_3x_3 + \dots + m_nx_n)$$

With:	y	= y-axis variable
	n	= number of variables
	a_n	= constant or intercept of variable x_n
	m_n	= gradient for x_n
	x_n	= x-axis variable

2.4.3 Frontier function

The developed benchmark model introduced in Section 2.4.2 serves as a general tool for comparison, being an average model. In contrast, this section focuses on crafting a best-practice benchmark model that sets a performance target for each circuit. The development of this model mirrors the process used for the average function, employing both single- and multivariable regression functions. The distinct approach here involves refining the function, derived from the frontier method, to align with the regression data more accurately.

Frontier or best practice benchmarking can be executed using either the COLS or the SFA method. The suitability of one method over the other hinges on the specifics of the data and the quantity of variables involved. For instance, for crushing circuits, it is imperative that the power requirement is 0 kW when the feed amounts to 0 tonnes, a principle that similarly applies to milling circuits. Nonetheless, given that the mass of the mill invariably exceeds 0 tonnes, a nonzero power consumption is anticipated even when all other variables are null.

The COLS function is extrapolated from the linear regression plot. The data point representing "best performance" is used to determine the y-intercept of the COLS, leveraging the average

function's known gradient alongside the selected data point's y- and x-variable values. Consequently, the intercept can be determined as depicted in Equation 9.

Equation 9: Calculation of variable y-intercept

$c = y - mx$			
With:	c	=	y-axis intercept
	m	=	gradient
	x	=	x-axis variable
	y	=	y-axis variable

Once the y-intercept is calculated, the function can be rewritten in its original form, and the resulting expression is the COLS function.

Furthermore, as previously mentioned, practical constraints invalidate a negative y-intercept. To address this, the SFA function adjusts by assigning a zero value to the intercept of variables that cannot take negative values. Furthermore, the constant for any variable that, in practice, must assume positive values is determined by taking the target value and multiplying it by the gradient of the respective variable, as illustrated in Figure 9.

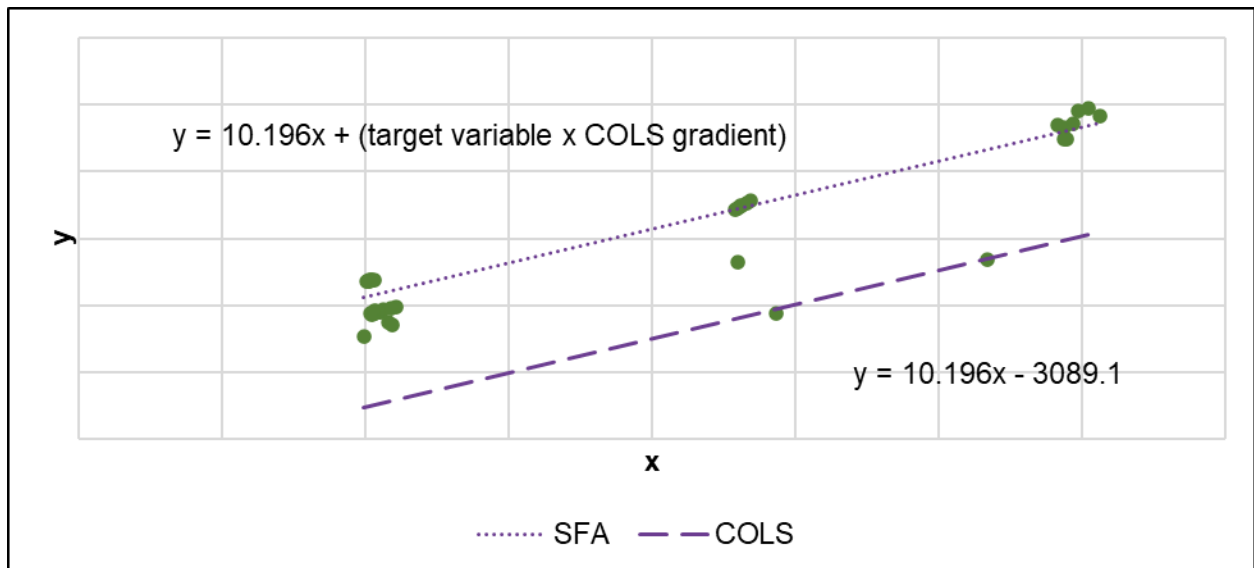


Figure 9: Conversion of COLS-to-SFA (adapted from [32])

Therefore, the SFA function can be expressed as shown in Equation 10. The constants are separated from the gradients, as the number of gradients and non-zero constants could differ. As a result, the gradients and constants must be averaged separately.

Equation 10: SFA function

$$y = \left(\frac{1}{k}\right)(b_1 + b_2 + b_3 + \dots + b_k) + \left(\frac{1}{n}\right) \times (m_1x_1 + m_2x_2 + m_3x_3 + \dots + m_nx_n)$$

With:	y	= y-axis variable
	b_k	= non-zero target constant of variable x_n
	m_n	= gradient for x_n
	x_n	= x-axis variable

The forecasted power requirements can be calculated by inserting the specific values of the x-variables and constants into the average and frontier functions. The outcomes represent the anticipated and best-practice performance levels for the circuit sections. Nevertheless, to verify the accuracy of the developed functions, it is essential to validate their outputs against the actual operational data from the circuits.

2.5 Step 4: Validate predicted results

Both the average and frontier functions are developed and verified using statistical methods. Therefore, their accuracy in predicting the power consumption of comminution circuit sections in the various case studies must be validated by comparing the predicted results of the average model function to the actual results.

The discrepancy between actual and predicted outcomes is assessed using the root-mean-square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). These metrics are chosen for their distinct characteristics: The RMSE calculates the mean standard error between the predicted and actual results across the dataset, thereby emphasising the variance of the prediction in comparison to the actual outcome, expressed in the same units as the results.

MAE is similar to RMSE, with the significant difference being that MAE uses an absolute value, thus removing all the negative values without squaring them. Mean-bias error (MBE) captures the average deviations between two datasets while considering the actual data values.

Equation 11: Mean bias error formula

$$MBE = \left(\frac{1}{n}\right) \sum_{t=1}^n (F_t - A_t)$$

With: MBE = mean bias error
 n = number of observations
 A_t = actual value
 F_t = forecast value

The derived value signals whether the prediction constitutes an underestimation, indicated by a negative value, or an overestimation, marked by a positive value [54]. A result of zero implies that the prediction is error-free, aligning it with the principles of MAPE.

The MAPE assesses the absolute percentage error between each predicted and actual value, facilitating the identification of predictions that deviate significantly from the actual data. This metric is particularly useful for determining whether a specific value skews the average MAPE. Additionally, it can highlight circuits that are underperforming relative to expectations, as evidenced by discrepancies between predicted and actual values.

Equation 12: Mean absolute percentage error

$$M = \left(\frac{1}{n}\right) \sum_{t=1}^n \left| \frac{A_t - F_t}{A_t} \right|$$

With: M = mean absolute percentage error
 n = number of iterations for summation
 A_t = actual value
 F_t = forecast value

A notable limitation of this method lies in its handling of intermittent data, specifically zeros and negative values [53]. However, given that the methodology involves processing data to yield monthly averages, the occurrence of zero values is unlikely.

2.6 Step 5: Development of a scoring method

This section focuses on the creation of a scoring technique designed to explicitly reflect circuit efficiency and pinpoint areas for enhancement. This technique can provide plant personnel with a uncomplex means to assess the circuit's performance through generated scores. Caromba [40] compared the actual energy consumption of a circuit directly to the benchmark energy consumption by calculating the ratio between the two and expressing them as a score. Similarly, Nadolski [46] employed a ratio that contrasted the specific energy consumption against the minimum practical energy.

Consequently, the initial score of the circuit section, denoted as %initial, is determined by employing a ratio similar to that depicted in Equation 13. This involves calculating the predicted power consumption as a proportion of the actual power consumption. Typically, the predicted power consumption is anticipated to be lower than the actual, particularly in the context of frontier analysis, resulting in scores below 100.

Equation 13: Initial percentage calculation

$$\%_{initial} = \frac{P_{model}}{P_{actual}}$$

With:

$\%_{initial}$	= Initial benchmarking score (%)
P_{model}	= Required power (kW)
P_{actual}	= Actual power (kW)

Nonetheless, an essential element generated by the regression model that can enhance the precision of both the functions and the scoring method is the standard error of the y-estimate (se_y), resulting from the variance in the data and produced by the LINEST function. Therefore, the most effective approach to factor in this error is by normalising the functions to account for the standard error.

The formulation of the normalisation equation begins with the initial score, which excludes the standard error. The adjusted score, denoted as %corrected, is derived using the se_y associated with each circuit section function. The outcomes of the initial score classify the performance of each circuit under underperformance, normal performance, and overperformance categories.

Prior to incorporating the standard error, the measure of error must be transformed into a percentage error to facilitate easier comparison and calculation. This conversion is accomplished

by determining the ratio of the produced standard error to the model's average predicted power, as illustrated in Equation 14. The resulting ratio is identified as the percentage error (%error).

Equation 14: Percentage error calculation

$$\%error = \frac{se_y}{P_{model_{ave}}}$$

With:

- $\%error$ = Percentage error of se_y (%)
- se_y = Standard error of y-estimate (kW)
- $P_{model_{ave}}$ = Average required power (kW)

Where se_y is calculated using the predicted values of the model and the following equation, as shown in Equation 15, the same predicted values are used to calculate the standard error of the average predicted values.

Equation 15: Standard error of y-estimate calculation

$$se_y = \frac{\sigma}{\sqrt{n}}$$

With:

- se_y = Standard error of y-estimate (kW)
- σ = Standard deviation
- n = Number of samples

Once the %error is determined, the three performance categories can be defined numerically, as shown in Table 6 below [32].

Table 6: Normalisation category ranges

Performance category	%initial range
Under	0 to (100 - %error)
Normal	(100 - %error) to (100 + %error)
Over	(100% + %error) to 200

Now that the %initial is calculated and a category can be selected, the %corrected can be calculated. The same method develops the %corrected functions as the %initial categories. Considering the available data and the model's development and application, a 10% deviation

from the actual value is deemed acceptable. Consequently, to formulate the %corrected ranges, a 10% margin is added to the %initial range. This adjustment creates a permissible performance range, ensuring that circuits slightly above or below expected performance are not disqualified. It results in the normalisation of %initial scores that fall within the %error range to within 10% of the anticipated or benchmark value.

This 10% range is also applied to the categories of underperformance and overperformance. It is important to note that a score of 100% is used to signify circuit sections that perform on par with the benchmark. As a result, underperformance is within the range of 0% to 90% and overperformance 110% to 200%, as summarised in Table 7.

Table 7: Performance categories for variable ranges of regressions

Underperformance (f1)		Normal performance (f2)		Over performance (f3)	
%initial	%corrected	%initial	%corrected	%initial	%corrected
0	0	$100 - \%_{\text{error}}$	90	$100 + \%_{\text{error}}$	110
$100 - \%_{\text{error}}$	90	$100 + \%_{\text{error}}$	110	200	200

The %initial and %corrected are plotted against one another to derive the three functions for the respective performance categories, as shown below in Figure 10. Using the functions obtained, f_1 , f_2 and f_3 , the %normalised can be calculated.

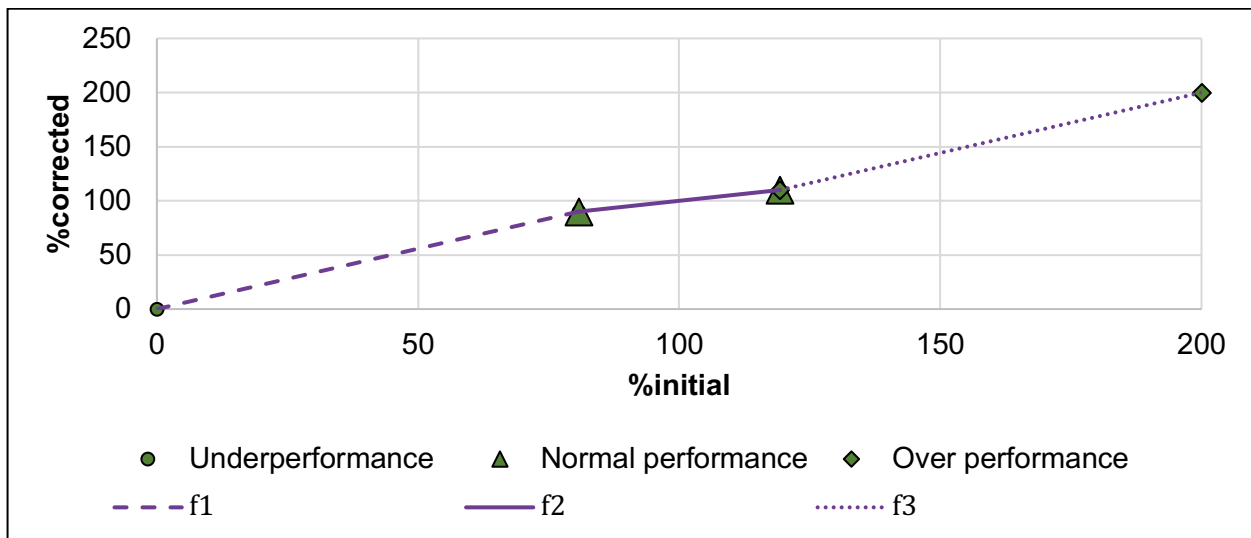


Figure 10: Regression of corrected percentage versus initial percentage

The criteria used to determine which function to use are shown in Table 8. Each function corresponds to a specific criterion and performance category, which will change according to the varying %error.

Table 8: Corrected percentage calculation

Performance category	Criteria	Function used
Under	$\%_{\text{initial}} < (100 - \%_{\text{error}})$	f_1
Normal	$(100 - \%_{\text{error}}) < \%_{\text{initial}} < (100 + \%_{\text{error}})$	f_2
Over	$\%_{\text{initial}} > (100\% + \%_{\text{error}})$	f_3

2.7 Step 6: Monthly benchmarking

Scoring-based benchmarking is developed from the results of statistical benchmarking. The first step of the benchmarking procedure and the case-study research method is identifying the process or system to be investigated and the investigation period [48]. The second step is statistical benchmarking, performed using the developed average and best practice models based on the regression models [55], [56].

Following the conventional statistical benchmarking method, the score-based benchmarking method calculates the initial and corrected scores [32]. The circuits are benchmarked according to their %corrected scores for the improved accuracy, including the %error of the estimation brings, thereby normalising all the results for fair comparison. Furthermore, this measure categorises the performance of each section and circuit into the three performance categories. The procedure can be broken down into steps summarised in Figure 11.

**Figure 11: Score-based benchmarking procedure [32]**

2.8 Step 7: Comparison with other methods

The final step involves assessing the score-based benchmarking alongside traditional methods and those documented in the literature, addressing the last criterion. The existing method incorporates intensities as a benchmarking metric. The newly introduced score-based benchmarking requires evaluation through a comparison of its outcomes to those generated by conventional methods, using the same dataset for consistency.

The score-based method is juxtaposed with the prevailing practice of intensity benchmarking, noting and assessing the differences between the two. This comparison focuses on each method's efficacy in pinpointing the highest and lowest performing circuits.

2.9 Method verification

Table 9 verifies the methodology development against the steps followed in the literature to successfully develop an energy benchmarking method. The method was adapted to produce a score-based method for its application on PGM comminution circuits.

Table 9: Method verification

Step	Criteria	Source	How?
1.	Collect data	[32], [40], [49]	The monthly average data is retrieved from the processing plants' SCADA system.
2.	Pre-process data	[32], [50], [51]	Outliers are identified with a z-score greater than 3 or less than -3 and removed.
3.	Determine prediction equations	[26], [32], [26], [52]	Linear regression analysis produces the prediction equations once the relationship between the variables has been verified.
4.	Validate results using statistical methods.	[53], [54]	The results are validated using four statistical methods, including MAPE, RMSE, MAE, and MBE.
5.	Employ scoring method to statistical benchmarking method	[32], [40], [46]	The initial score is calculated as a ratio of the predicted model power to the actual power. This is corrected by introducing the standard error of the regression analysis to produce an improved, normalised score.
6.	Benchmark monthly entries	[32], [55], [56]	The benchmarking is divided into four steps: identifying the circuit and period to benchmark. Benchmarking using statistical methods and applying the scoring-benchmarking method to the statistical method.

7.	Compare scoring-based benchmarks with other methods.	[42], [43], [46], [57]	The score-based benchmarking results are compared to the results of the conventional benchmarking method.
----	--	---------------------------	---

2.10 Conclusion

This chapter has outlined the methodology for creating the score-based benchmarking approach. In alignment with the objectives, it is necessary to construct both average and frontier methods for the three segments of the comminution circuits: pre-milling, primary milling, and secondary milling.

In step, the derived equations are utilised to forecast the average and frontier power consumption of the circuit sections. This initial comparison between calculated and actual consumption lays the groundwork for the score-based benchmarking approach. Importantly, the completion of the six equations fulfils objectives 1-6, as stated in Section 1.7, by serving as the models for each respective analysis.

However, the results obtained from step 3 must be used to develop the scoring-based method in step 5. The completion of step 5, as discussed in this chapter, fully completes objective 7. The remaining objective, objective 8, is reached in Chapter 3 when the methodology is applied to the case studies.

CHAPTER 3 BENCHMARKING MODEL VERIFICATION, APPLICATION, AND VALIDATION

3.1 Introduction

The benchmarking model methodology was developed according to the criteria summarised in Table 9 and were discussed in detail in the previous chapter. The developed method is outlined in the steps in Figure 12 below.

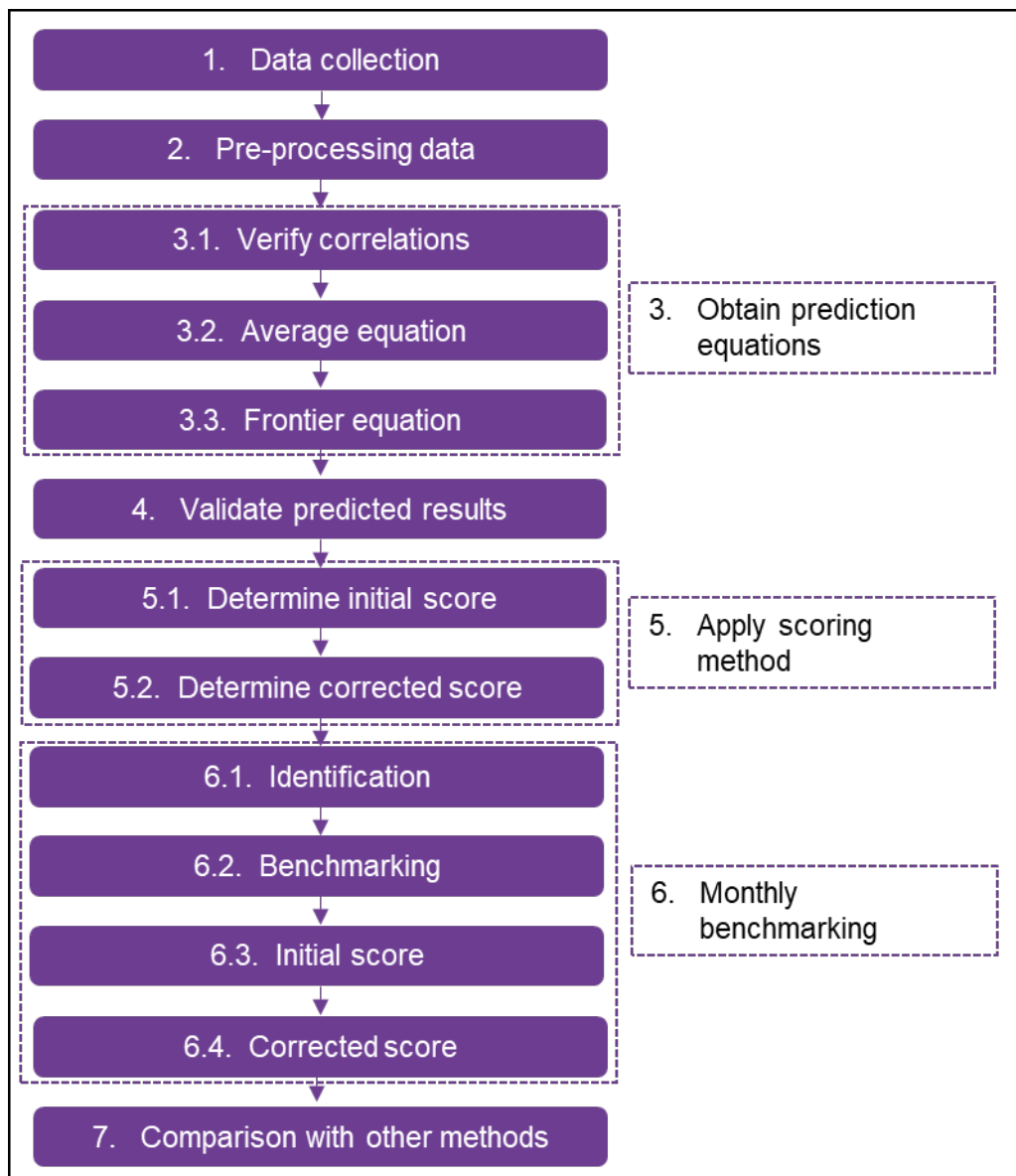


Figure 12: Score-based benchmarking framework

This chapter outlines the application of the developed method across the different sections of the comminution circuits. Reflecting previous discussions, the study employs a comprehensive, quantitative methodology. Consequently, the sections of the comminution circuits from all included case studies are examined and assessed.

3.2 Step 1: Data collection



Although the analysis encompasses all comminution circuits, the availability of data for each circuit, as depicted in Table 10 and Table 11 below, is subject to variation owing to differences in circuit layouts and instrumentation quality. Circuit sections lacking instrumentation or possessing malfunctioning instruments, which result in unreliable readings, are omitted from the correlation analysis.

Table 10: Pre-milling data availability for each circuit

Parameter	Circuit A	Circuit B	Circuit C	Circuit D	Circuit E
Energy consumption of crushing equipment	-	-	✓	✓	✓
Throughput	-	-	✓	✓	✓

Table 11: Milling data availability for each circuit

Parameter	Circuit A	Circuit B	Circuit C	Circuit D	Circuit E
Energy consumption of milling equipment	✓	✓	✓	✓	✓
Primary mill ore feed	✓	✓	✓	✓	✓
Secondary mill ore feed	✓	✓	✓	✓	✓
Primary mill weight	✓	✓	×	✓	✓
Secondary mill weight	✓	✓	×	✓	×
Ball load primary mill	✓	✓	×	✓	×
Ball load secondary mill	×	×	×	×	×
Feed water primary mill	✓	✓	✓	✓	✓
Feed water secondary mill	×	×	×	×	×

Data available	✓
No data available	×
Not applicable	-

Data was gathered as monthly averages over a period of six months, spanning from January to June. This period includes months of low production to ensure a comprehensive dataset for developing the functions and models. Utilising monthly averages offers a more accurate representation of circuit performance than shorter by mitigating the impact of outliers and accounting for periods of shutdown.

Therefore, as previously discussed, any outliers with a z-score greater than 3 and less than -3 are excluded from the analysis. The outliers are not substituted for by an average value or the average of a different month, as the aim is to isolate the dataset to the mentioned months and avoid having varying months for each circuit or section of a circuit.

Furthermore, acknowledging the equipment ratings for each circuit is vital to exclude impractical values that exceed these ratings, as such occurrences are unfeasible in practice. Understanding

these ratings aids in detecting performance variations among similar technologies and equipment, offering crucial insights for suggesting enhancements to the circuits.

Pre-milling

Pre-milling encompasses all the processes that prepare the ore before it enters the milling stage. Crushing, a crucial component of pre-milling, forms an integral part of the comminution circuit and, as such, is included in the benchmarking process. As indicated by existing literature, the milling process is significantly influenced by preceding treatments. Specifically, the extent to which the crushing circuit achieves consistent product size directly impacts the efficiency of subsequent breakage within the mill, thereby enhancing throughput.

The pre-milling equipment installed at the various case study plants are summarised in Table 12 below. Neither circuits A nor B has any pre-milling equipment, suggesting that both these plants utilise a run-of-mine approach for their milling circuits.

Table 12: Pre-milling equipment of the various case study circuits

Circuit	Primary	Power rating (kW)	Secondary	Power rating (kW)
C	Grizzly screen 1	37	Vibrating screen 1	2
	Grizzly screen 2	37	Vibrating screen 2	2
	Jaw crusher 1	200	Vibrating screen 3	2
	Jaw crusher 2	200	Vibrating screen 4	2
	NA		Cone crusher 1	160
			Cone crusher 2	160
			Cone crusher 3	160
			Jaw crusher	160
D	Grizzly screen 1	22	NA	
	Grizzly screen 2	22		
	Jaw crusher 1	185		
	Jaw crusher 2	185		
E	Grizzly screen	37	NA	
	Jaw crusher	185		

Moreover, while circuits A and B do not include pre-milling equipment, circuit C utilises both primary and secondary crushing circuits to achieve the necessary product size for its mills. Circuits D and E employ identical equipment with the same power ratings for the screens and

comparable ratings for the crushing machinery. Consequently, it is anticipated that the performance of the equipment across these circuits should be relatively similar.

Additionally, it is noteworthy that circuit A comprises two distinct streams, with one specifically designated for UG2 ore and the other for a mixture of UG2 and Merensky reef ore. As previously indicated, the BWi for these two ore types differs, which implies that the stream processing the mixed ore is likely to require more specific energy. Similarly, circuit C operates with two streams, yet both handle the same mixed ore type.

Among the pre-milling processes, crushing circuits stand out as significantly more energy-intensive compared to other discussed methods. They also represent the sole piece of pre-milling equipment implemented, aside from cyclones. Consequently, crushing circuits become the exclusive focus of the pre-milling method evaluation. The critical variables relating to the considered crushing circuits are summarised as follows:

Dependent variable:

- Power consumption

Independent variable:

- Throughput

The benchmarking model necessitates a correlation between the aforementioned variables. Its primary goal is to evaluate the energy efficiency of the pre-milling circuits. Thus, energy consumption is considered the constant variable, and its correlation with the previously mentioned independent variable must be established.

The only regression analysis for the crushing circuit is the power consumption and the throughput of the crushing circuit. The six-month averages of these two variables for the four respective crushing circuits are summarised in Table 13 below.

Table 13: Pre-milling circuit throughput

Circuit	Feed (t)	Power (kW)
C Primary 1	289	18
C Primary 2	232	34
D	597	99
E	673	72

The throughput of the circuit is measured in tonnes per hour. Similarly, the energy consumption is measured in kW-hour. Therefore, the relationship between the two variables is kW/tonne and is referred to as the power intensity.

Milling

The focus of the milling process analysis encompasses only the primary and secondary mills along with their operational parameters. Although regrind mills are mentioned, they are not investigated in detail due to their limited application to a single circuit. Additionally, the auxiliary equipment associated with the milling circuit is excluded from this analysis.

Milling circuits are notably complex, requiring the consideration of numerous variables to understand the operational parameters of the mills effectively. However, only a select few of these variables significantly influence energy consumption and are pertinent to the mill's grinding efficiency.

To provide context and facilitate further comparison, the design, and operational practices of the milling circuits in the case studies are examined. This discussion offers insight into the technologies employed across different circuits and their anticipated performance outcomes. The technologies implemented in the respective circuits are compiled in Table 14.

Table 14: Milling equipment of the various case study circuits

Circuit	Primary	Power (kW)	Secondary	Power (kW)	Regrind	Power (kW)
A	RoM ball mill mix	3 200	Ball mill mix	3 200		
	RoM ball mill UG2	2 500	Ball mill UG2	2 500		
B	RoM mill	2 500	Ball mill 1	2 500		
C	Ball mill 1	1 150	Ball mill 3	1 800	SMD 1	355
	Ball mill 2	1 150	Ball mill 4	1 800	SMD 2	355
D	Ball mill 1	10 400	Ball mill 2	10 400		
E	Ball mill 1	10 400	Ball mill 1	1 500		
			Ball mill 2	1 500		

All five cases utilise ball mills, as shown in Table 14, with circuits A and B operating them in a run-of-mine mode due to the lack of crushing equipment to initiate the initial ore breakage. The most significant difference in the installed technologies is the power ratings of the equipment. Circuit C utilises smaller rated mills – approximately 10% of the rating of the motors implemented

on circuits D and E. This is due to the extensive pre-milling stage of circuit C and reduced throughput design specification per stream.

Despite the variance in the power ratings, the mills have the same input variables controlled to produce the ideal product size. The controlled input variables investigated to determine their correlation with the energy consumption of the milling equipment are summarised below.

Dependent variable:

- Power consumption

Independent variable:

- Throughput
- Feedwater
- Mill weight
- Ball load

The mill's throughput is measured at the feed side. The weight of the mills is measured with load cells, either under the feed or discharge end or from bearing pressures [58]. Furthermore, as the weight of the mills is measured whether or not the mill is operational, the weight is only considered for operational periods to ensure the data is not skewed.

The feedwater flow is monitored by a flowmeter attached to the mill's pump, which only operates when the mill is active. Conversely, the ball load is regulated through a pre-set set point, with changes in the hopper's weight indicating the ball feed rate to the mill. Table 15 and Table 16 provide a six-month average for each variable, akin to the primary crushing circuit approach. These averaged values are then disaggregated into six distinct monthly figures for subsequent analysis.

Table 15: Primary mill variables

Circuit	Feed (t)	Weight (t)	Feed water (m ³ /h)	Ball load (kg)	Power (kW)
A UG2	180	211	10	2 288	1 828
A Mix	225	203	85	2 185	2 735
B	191	205	12	2 214	1 778
C Stream A	168		41		814
C Stream B	169		41		812
D	607	687	107	7 129	6 839
E	264	467	57	4 865	4 206

The secondary mills are operated and designed similarly to primary mills, as shown in Figure 16 above. However, due to different control methods and input variables, there is less emphasis on monitoring specific variables. Thus, there are more limitations in data availability for the secondary mills when compared to the primary mill. As a result, variables such as the ball load and feed water are neglected for the study of the secondary mills.

Table 16: Secondary mill variables

Circuit	Feed (t)	Weight (t)	Power (kW)
A UG2	388	195	1 692
A Mix	463	212	2 356
B	453	209	1 453
C Stream A	352		1 482
C Stream B	293		1 470
D	1 325	805	8 578
E	480		1 596

Regrind mills, used in a select few circuits to attain finer or ultra-fine grinds (UFG), employ stirred media detritors (SMDs) that share similar input variables as primary and secondary mills. The notable difference is that SMDs use smaller ceramic grinding media to achieve the desired level of fineness. However, given the limited prevalence of regrind mills across circuits and constraints related to data availability, this study does not include them in its analysis.

3.3 Step 2: Pre-processing data



The data is pre-processed to identify any outliers that could skew the models. The intensity of each variable is calculated. The calculated intensities are then used to determine the mean intensity for each variable. The calculated mean intensity is used to calculate the z-score for the variable.

The calculated z-scores fall within the established range of -3 to 3, as previously outlined. Z-scores beyond these limits suggest the necessity of excluding the corresponding data point from the dataset employed in model development.

In this instance, only one outlier was detected within the pre-milling dataset: that of stream A circuit C, which in the month of May yielded a z-score of -3.54. Consequently, this data point is excluded from the dataset prior to confirming the correlations.

3.4 Step 3: Obtain prediction equations



As previously discussed, to develop accurate prediction equations, the relationship between the dependent variable and the independent variables must be proven. Thus, the variables' correlation must be verified before the average and frontier equations can be obtained.

3.4.1 Verification of correlations

The decision to incorporate a variable into the circuit section model development is guided by the outcomes obtained. Typically, a determination coefficient of 0.81 indicates a model with relatively accurate predictive capability. For values ranging from 0.66 to 0.80, the model is considered to have the capacity to categorise values into low, mid, and high segments. A value between 0.50 and 0.65 suggests that over half of the variance in the dependent variable (y) is explained by the variance in the independent variable (x) [59]. The relationship between power consumption and the input variables for the pre-milling, primary, and secondary milling stages are hence analysed to ascertain the determination coefficient for each variable.

Pre-milling

The sole independent variable linked to the power consumption in the primary crushing circuit is the ore throughput, as identified in existing literature. Figure 13 illustrates a notably strong correlation between the power consumption of the crushing circuit and the volume of ore processed, evidenced by an R^2 value of 0.7104. Furthermore, the data reveals two distinct clusters, indicating consistent performance across the different circuits.

Only one circuit features a secondary crushing stage. Given that the ore in this circuit has already undergone a milling process and different technologies are employed, it is not anticipated to mirror the performance of primary crushing circuits. Consequently, the secondary crushing circuit is omitted from this study due to the lack of a comparable benchmark.

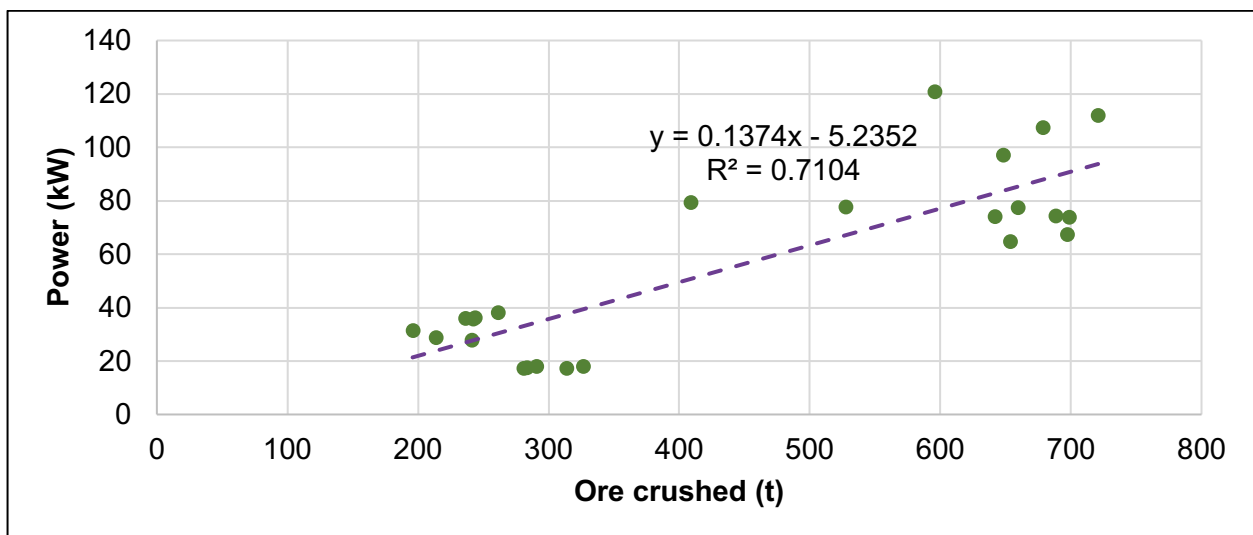


Figure 13: Pre-milling power consumption versus ore throughput

The expected independent variable indeed correlates with power consumption. Yet, to confirm that this correlation is not merely coincidental, each relationship requires validation through statistical analysis, employing tools like the LINEST and FDIST functions, as mentioned earlier.

A strong correlation between ore throughput and power consumption is observed in the primary crushing circuit. This correlation has been statistically validated, showing a probability of 0.109 that the observed R^2 value occurred, with a degree of freedom of 22, as detailed in Table 17 below.

Table 17: Pre-milling circuit array data from LINEST

R^2	0.710
F	53.96
d_f	22
se_y	18.15
R^2_{prob}	0.107

The probability is somewhat higher than anticipated, yet it remains within acceptable bounds. Furthermore, the coefficient of standard error is crucial because it informs the development of the scoring technique. It is imperative that this value be minimised, as it quantifies the estimation's standard error. This principle is equally relevant to both the primary and secondary milling circuits.

Primary milling

As previously highlighted, four variables influence the power consumption in the primary milling circuit, one of which is the ore feed to the mill. Figure 14 below illustrates a significant correlation between the power consumption of the primary mills and their ore feeds, as evidenced by a robust R^2 value of 0.8563. Notably, the y-intercept of this correlation is negative, suggesting an unrealistic scenario of negative power consumption when the ore feed is zero. This anomaly underscores the need to consider additional variables that impact the mills' power consumption. Nonetheless, for all the variables discussed, a positive y-intercept is typically expected.

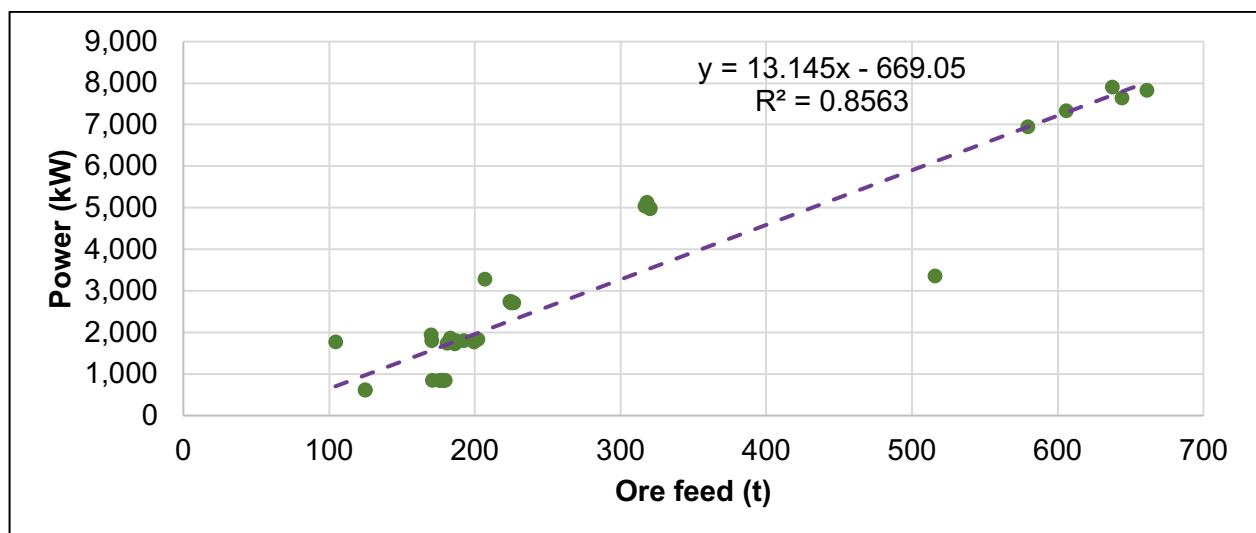


Figure 14: Primary mill power consumption versus ore feed

The mill's weight likely plays the most significant role in the variability of power consumption across mills. This weight encompasses all input variables except for power itself, thus it is

anticipated to exhibit the strongest correlation with power consumption. Figure 15 reveals a robust correlation, with an R^2 value of 0.8020. Moreover, three distinct clusters are identifiable, corresponding to the mills' power ratings. Despite these clusters, their proximity to the trendline indicates a comparable average performance across the groups.

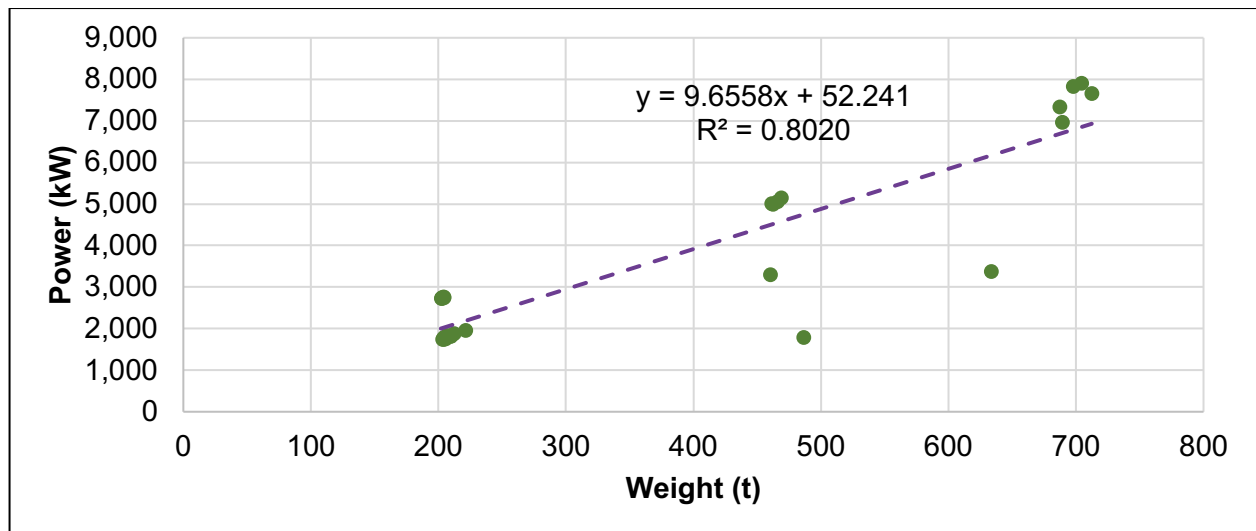


Figure 15: Primary mill power consumption versus mill weight

The influence of feedwater on the mill's power consumption is anticipated to be minor, given that feedwater constitutes a much smaller percentage of the mill's total weight compared to other variables. This expectation is corroborated by a relatively weaker correlation, with an R^2 value of 0.6089. Nonetheless, this correlation is still strong enough to demonstrate a relationship between the two variables, as illustrated in Figure 16 below.

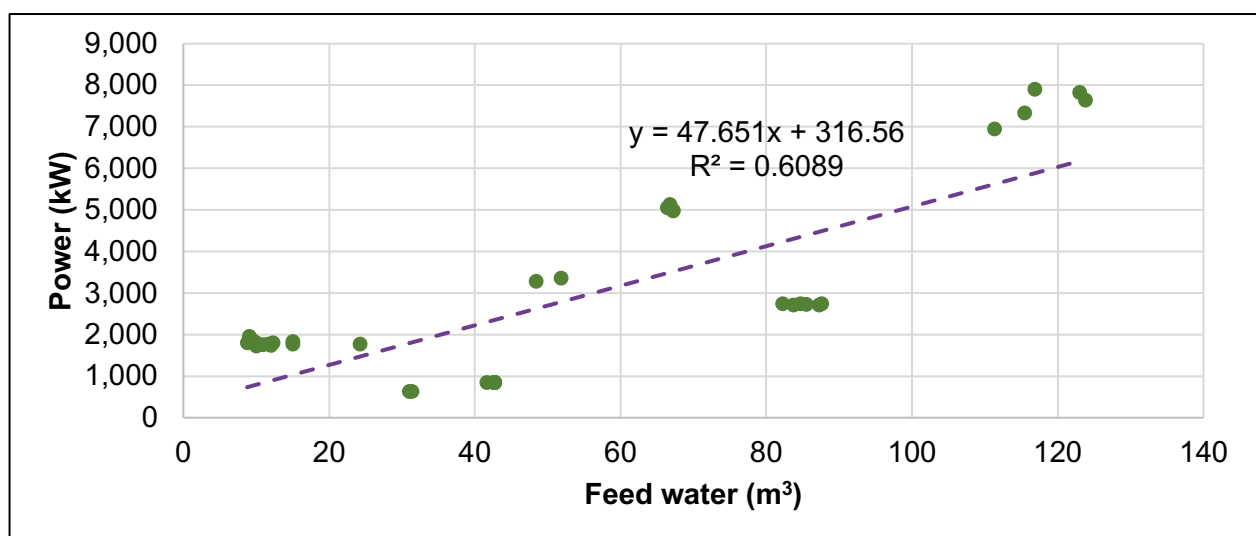


Figure 16: Primary mill power consumption versus feed water

In Figure 17 below, a notable absence of correlation is observed, with an R^2 value of 0.0113, between the power consumption of the primary mill and its ball load. The primary difficulties in establishing a relationship between these two variables lie in the characteristics of the datasets and the minimal impact that the grinding media's weight has on the mill's total weight.

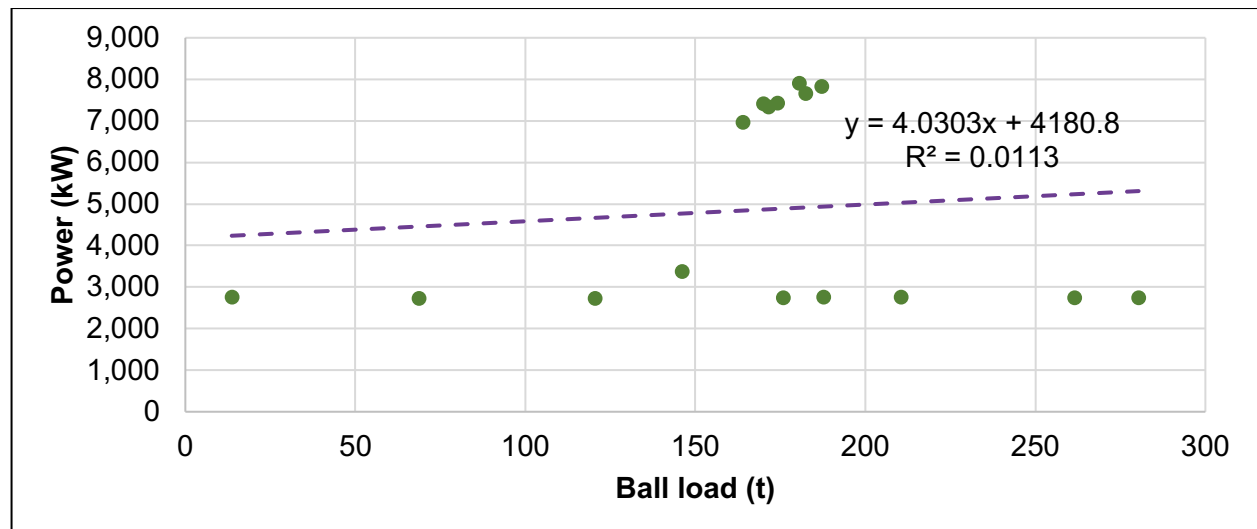


Figure 17: Primary mill power consumption versus ball load

The quantity of media added to the mills is not directly measured; instead, the scale of the media hopper is monitored. While it is possible to estimate the mill load from this data, the hoppers are replenished at random intervals throughout the day. Consequently, to obtain a more accurate measurement, the data collection frequency needs to be intensified to half-hourly intervals. Monthly data points, as currently collected, encompass the total weight of the hopper rather than the specific weight of the load being added to the mill.

Upon enhancing the data resolution, it becomes apparent that a relatively small amount of media, roughly 0.4 kg to 0.7 kg per tonne of ore, was being added. Consequently, within the overall context of the mill's operation, the weight of the grinding media is negligible and challenging to distinguish from the rest of the system's components. Given the weak correlation associated with ball loading, only the feed, mill weight, and feed water are considered in the multivariable regression analysis for constructing the primary milling regression model.

The data arrays generated by the LINEST function for each variable are presented in Table 18, Table 19, and Table 20 below. The degrees of freedom vary because of differences in data availability. Nevertheless, there remains a strong confidence level that the correlations observed between the variables and power consumption are not coincidental, evidenced by the maximum R^2 probability being 0.1.

Table 18: Primary mill ore feed – LINEST data array

R^2	0.8563
F	238.3
d_f	40.00
se_y	834.0
$R^2\ prob$	0.05133

Table 19: Primary mill weight – LINEST data array

R^2	0.8020
F	113.4
d_f	28.00
se_y	968.7
$R^2\ prob$	0.07415

Table 20: Primary mill feed water – LINEST data array

R^2	0.6089
F	62.28
df	40.00
se_y	1376
$R^2\ prob$	0.1002

Secondary milling

Given that secondary milling takes place subsequent to the initial rougher stage, the feed into the secondary mill comprises a slurry, rather than dry ore mixed with process water. Therefore, in analysing the feed of the secondary mill, the unit of measurement is cubic metres (m³) instead of tonnes, which is the case for the primary mill.

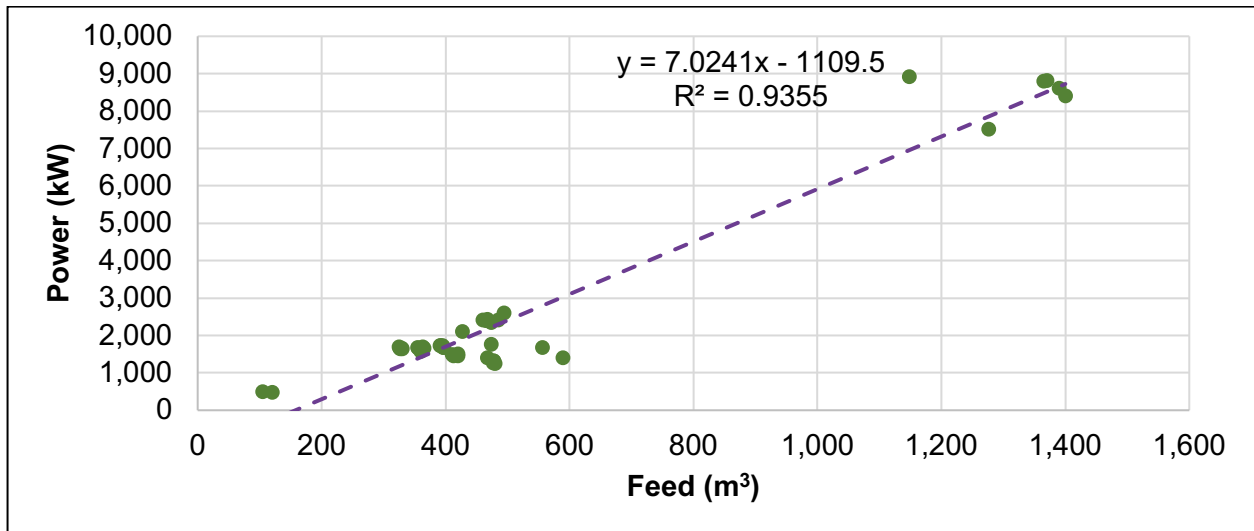


Figure 18: Secondary mill power consumption versus ore throughput

The correlation between feed and power consumption in the secondary mills is stronger than that observed in the primary mills as evidenced by an R^2 value of 0.9355 as seen in Figure 18. Furthermore, the relationship between the mill's weight and its power consumption demonstrates an improvement, with an R^2 value of 0.9840, as shown in Figure 19 below. However, it is important to acknowledge once more that the y-intercept is negative, suggesting an unrealistic scenario of negative power consumption for a mill with zero mass.

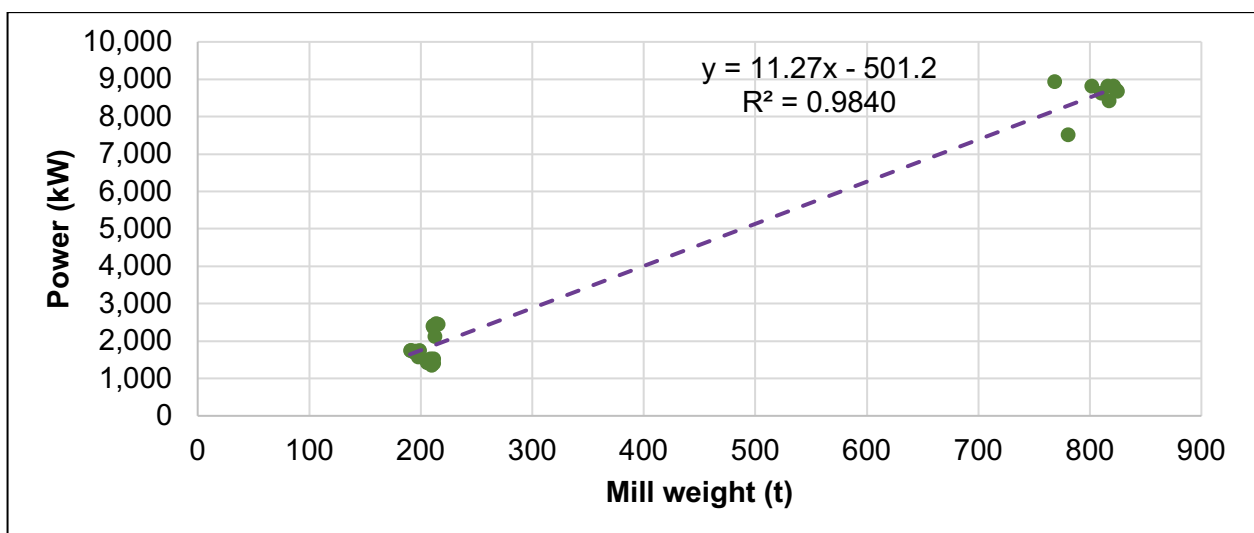


Figure 19: Secondary mill power consumption versus mill weight

The absence of sufficient data on the feed water and ball loading for the secondary mills precluded testing their correlation with power consumption. Consequently, the analysis focused solely on the ore feed and the mill's weight.

Similarly to the primary mills, circuit C lacks available data on the weight of the secondary mills. To enhance the precision of the power consumption prediction, it becomes necessary to formulate a distinct function that predicts power based exclusively on the mill feed. The data arrays for the feed and weight of the mills are provided in Table 21 and Table 22.

Table 21: Secondary mill throughput array data from LINEST

R^2	0.9355
F	580.3
d_f	40.00
se_y	634.4
$R^2 prob$	0.0329

Table 22: Secondary mill weight data array from LINEST

R^2	0.9840
F	1241
d_f	22.00
se_y	403.0
$R^2 prob$	0.0224

The average and frontier functions are derived from the regression equations for each variable. Initially, an average expression is calculated by aggregating all the input variables. Subsequently, this average expression serves as the foundation for constructing the frontier function, employing both COLS and SFA methods. The selection of the most suitable function is based on the analysis of the results obtained.

3.4.2 Average function

Pre-milling

Considering that the correlation between the ore feed and the power consumption is verified and only a single independent variable exists for the system, the primary crushing circuit power requirement can be estimated using Equation 16 below.

Equation 16: Primary crushing circuit power requirement from LINEST

$$P_{crushing} = -5.24 + 0.14(T)$$

With: $P_{crushing}$ = Crushing equipment power requirement (kW)
 T = Tonnes of ore crushed (t)

Primary milling

The function designed to forecast the power consumption of the primary milling circuit, taking into account its input variables, is presented in Equation 17. As indicated, the inclusion of ore feed, feed water, and mill weight is crucial for enhancing the precision of the predicted value. It is also important to emphasise that the y-intercept of this function is negative.

Equation 17: Primary mill power requirement from LINEST

$$P_{prim,mill} = -100.08 + 4.38(T) + 3.22(W) + 15.88(F)$$

With: $P_{prim,mill}$ = Primary mill power requirement (kW)
 T = Ore feed (t)
 W = Weight of mill (t)
 F = Feed water (m³)

Secondary milling

The power requirement for the secondary mill can be calculated using the values derived from the LINEST data arrays, while omitting feed water and ball load, as illustrated in Equation 18 below. This exclusion of feed water and ball load is based on the previously mentioned limitations in data availability.

Equation 18: Secondary mill power requirement from LINEST

$$P_{sec,mill} = -782.06 + 3.51(T) + 5.64(W)$$

With: $P_{sec,mill}$ = Secondary mill power requirement (kW)
 T = Ore feed (t)
 W = Weight of mill (t)

The resulting predictions that resulted from substituting the actual values into the developed and averaged equations are summarised below and compared to the actual power consumption. The functions from each circuit section, to Equation 18, are summarised in Table 23 below. All three sections share ore feed as common independent variable.

Table 23: Comminution circuit average function summary

System	Constant	Gradient		
		Ore feed	Mill weight	Feed water
Primary crushing	-5.24	0.14		
Primary mill	-192.04	4.38	3.22	15.88
Secondary mill	-782.06	3.51	5.64	

3.4.3 Frontier function

As demonstrated in Equation 19, Equation 20 and Equation 21, all yielded a negative y-intercept, which as previously addressed, is not feasible in practical terms. This suggests that the COLS function, while reflective of trends observed in the literature, lacks precision and requires refinement to yield a more accurate frontier function. Consequently, it is necessary to transform the COLS equations into SFA expressions to remove the negative y-intercept, thereby potentially enhancing accuracy.

Equation 19: Primary crushing – COLS best practice

$$P_{COLS_{crush}} = -28.77 + 0.14(T)$$

With: $P_{COLS_{crush}}$ = Best practice benchmark for primary crushing power requirement (kW)

Equation 20: Primary milling – COLS best practice

$$P_{COLS_{pri,mill}} = -2732.70 + 4.38(T) + 3.22(W) + 15.88(F)$$

With: $P_{COLS_{pri,mill}}$ = Best practice benchmark for primary mill power requirement (kW)

Equation 21: Secondary milling – COLS best practice

$$P_{COLS_{sec,mill}} = -1934.65 + 3.51(T) + 5.64(W)$$

With: $P_{COLS_{sec,mill}}$ = Best practice benchmark for secondary mill power requirement (kW)

To accomplish this, a target variable must be established for each circuit section. For the primary crushing circuit, the decision is straightforward: if no ore is fed into the crushing equipment, the circuit has no operational requirement, and the anticipated power consumption should be zero. Consequently, the target variable is designated as zero, ensuring that the y-intercept passes through the origin.

The approach for milling circuits diverges from others because, before any variables are introduced into the mill, the mill itself is activated, and the forces associated with its weight must be counteracted. As such, the minimal operational weight of the mill, approximating a no-load condition, is employed as the target, representing the lowest possible weight for the model. Consequently, for both primary and secondary mills, the target variable is determined by the mill's weight, with all other variables set to zero.

Following this adjustment, SFA functions specific to the primary and secondary milling are developed. However, these functions are not applied to the secondary milling calculations that exclusively consider ore feed.

Equation 22: Primary milling – SFA best practice

$$P_{SFA_{pri,mill}} = -1544.93 + 4.38(T) + 3.22(W) + 15.88(F)$$

With: $P_{SFA_{pri,mill}}$ = Best practice benchmark for primary crushing power requirement (kW)

Equation 23: Secondary milling – SFA best practice

$$P_{SFA_{sec,mill}} = -788.9 + 3.51(T) + 5.64(W)$$

With: $P_{SFA_{sec,mill}}$ = Best practice benchmark for primary crushing power requirement (kW)

Appendix A shows that the COLS functions all result in a negative y-intercept. Therefore, SFA is applied to all the circuit sections, the resulting graphs of which are shown in Appendix A.

The target variable for the primary and secondary mills is the mill weights, which are selected as 400 tonnes based on the average weight data for these circuits. These target variables produce the following best practice equations for the respective sections as summarised below in Table 24.

Table 24: Comminution circuit best practice (SFA) function summary

System	Constant	Gradient		
		Ore feed	Mill weight	Feed water
Primary crushing		0.14		
Primary mill	-966	4.38	3.22	15.88
Secondary mill	-1127	3.51	5.64	

An average and best practice model has now been developed to benchmark the performance of the comminution circuits. In the next step, models' results are validated to confirm their accuracy.

3.5 Step 4: Validate predicted results



The functions that have been developed need to undergo validation against the predicted outcomes derived from both the average and frontier functions. As outlined in Section 2.5, four methods for forecasting error – MAPE, RMSE, MAE, and MBE – are employed to assess and validate the functions' accuracy. The initial evaluation utilises MAPE for each circuit, aiming to pinpoint circuits that do not align well with the model, thereby affecting the model's overall average across all circuits. Subsequently, an examination using the remaining three error metrics – RMSE,

MAE, and MBE – is conducted to ascertain the functions' average performance and identify whether the function tends to under- or overestimate the predicted results.

Pre-milling

Initially, the average function is utilised to facilitate comparison across the circuits. Following that, the accuracy of the model in predicting each circuit's power consumption is assessed using the aforementioned validation techniques.

As illustrated in Table 25 below, the column for predicted power is color-coded based on the comparison between the estimated and actual values: a green highlight indicates the estimated power is lower than the actual, whereas red suggests the opposite.

Moreover, taking the percentage error into account reveals that all circuits exhibit errors exceeding 10%, with circuit C stream A demonstrating an error as significant as 73%. Contrastingly, the error for the second stream within the same circuit is only 22.6%. From these observations, it is apparent that circuits C (stream B), D, and E align more closely with the average model, with circuits C stream A and E outperforming the average expectation.

Table 25: Pre-milling average function MAPE validation

Circuit	Predicted power (kW)	Actual Power (kW)	MAPE (%)
C Stream A	34	20	73.0
C Stream B	27	34	22.6
D	77	99	22.5
E	93	72	28.8
Average	56	56	36.7

The pre-milling function validation in Table 26 below indicates an MBE improvement of 2.3%. The average result of the RMSE and MAE is 16.8, suggesting that the model overestimates the prediction by approximately 17 kW – a significant percentage, depending on the evaluated circuit.

Table 26: Pre-milling average function validation

RMSE	MAE	MBE (%)
17.3	16.3	2.3

Given that the SFA function is designed to forecast optimal (best-practice) results for the pre-milling circuit, a higher MAPE is anticipated. However, as demonstrated in Table 27, the predicted power consumption surpasses the average function's predictions due to the adjustment of the y-

intercept to zero. Consequently, the derived frontier function does not measure frontier performance but rather represents an anticipated performance function predicated on the assumption that no power should be consumed when no ore is being processed.

Table 27: Pre-milling SFA function MAPE validation

Circuit	Predicted power (kW)	Actual Power (kW)	MAPE (%)
C Stream A	42	19	75.0
C Stream B	51	34	38.5
D	126	99	23.7
E	56	72	24.6
Average	67	56	40.5

This observation is reinforced when examining the other validation metrics where both the RMSE and MAE exceed the average function. Furthermore, the bias indicated by the MBE shifts from underestimation to overestimation, showing a significant increase to approximately 22.4%.

Table 28: Pre-milling SFA function validation

RMSE	MAE	MBE (%)
21.0	20.5	22.4

Primary milling

Enhanced outcomes are anticipated for the primary milling circuit, given that the model incorporates multiple variables. The outcomes derived from the average model, constructed via the adjusted multivariable equation, are presented in Table 29.

Table 29: Primary milling – average function MAPE validation

Circuit	Predicted power (kW)	Actual power (kW)	MAPE (%)
A UG2	1 530	1 828	16.3
A Mix	2 891	2 735	5.7
B	1 586	1 778	10.8
C Stream A	1 278	814	56.9
C Stream B	1 286	812	58.3
D	6 472	6 839	5.4
E	3 462	4 206	17.7
Average	2 643	2 716	24.4

The values calculated through Equation 8 closely mirror the actual power consumption, except for notable variances in both streams of circuit C. The predictions for these streams are skewed because they lack a weight variable and are computed using the identical equation.

Considering the average MAPE, the average function's derived accuracy is modest but deemed acceptable [60]. Excluding the two streams of circuit C from the analysis would markedly improve the model's accuracy; nonetheless, the results can still be validated through the RMSE, MAE, and MBE percentages, as detailed in Table 30. The MBE reveals that, on average, the model underestimates power consumption by 3%, equivalent to about 405 kW, as shown by the RMSE and MAE.

Table 30: Primary milling – average function validation

RMSE	MAE	MBE (%)
427	385	-2.7

Similar to the pre-milling SFA model, the expectation is that the model will significantly underestimate actual values, given its objective to establish a best practice or target benchmark for the circuits. As demonstrated in Table 31 below, all actual values were indeed underestimated as anticipated, indicating that none of the circuits is operating at its potential best-practice level.

Table 31: Primary milling – SFA function MAPE validation

Circuit	Predicted power (kW)	Actual power (kW)	MAPE
A UG2	664	1 897	63.6
A Mix	2 025	2 739	26.0
B	721	1 794	59.5
C Stream A	412	645	49.4
C Stream B	421	506	48.2
D	5 606	7 267	18.0
E	2 597	4 932	38.3
Average	1 778	2 826	43.3

This point is highlighted by the MBE, which indicates that the model underestimates circuit power consumption by 43%. This suggests that there is, on average, a potential for improvement in circuit performance by approximately 983 kW, as seen in Table 32 below.

Table 32: Primary milling SFA function validation

RMSE	MAE	MBE (%)
1 028	938	-34.5

Secondary milling

When analysing the secondary milling circuits, it is critical to acknowledge that the model is formulated with a reduced number of variables due to data constraints. Specifically, the model does not account for the weight variable in three out of the seven circuits (C and E), relying solely on ore feed and weight, unlike the primary circuits that also considered feedwater. Despite these limitations, as indicated in Table 33 below, the secondary milling model demonstrates a MAPE improvement of 18% compared to the 24% of the primary model.

Table 33: Secondary mill – average function MAPE validation

Circuit	Predicted power (kW)	Actual power (kW)	MAPE (%)
A UG2	1 663	1 692	1.7
A Mix	2 014	2 356	14.5
B	1 959	1 462	34.0
C Stream A	1 472	1 482	0.6
C Stream B	1 035	1 470	29.6
D	8 367	8 520	1.8
E	2 259	1 596	41.5
Average	2 681	2 654	17.7

A significant improvement is observed in the RMSE and MAE values of the secondary milling average model, with the average of the two values being approximately 343. Additionally, the MBE indicates that the model overestimates the actual values by approximately 1%, implying improved accuracy for the model.

Table 34: Secondary milling average function validation

RMSE	MAE	MBE (%)
381	304	1.0

With the exception of circuit B, the SFA function for the secondary milling circuit tends to underestimate power consumption. Furthermore, the error margin for the streams of circuits C and E has widened, indicating that the absence of weight data has a more pronounced impact on the accuracy of the frontier analysis when compared to the average model.

Table 35: Secondary mill – SFA function MAPE validation

Circuit	Predicted power (kW)	Actual power (kW)	MAPE (%)
A UG2	1 341	1 692	20.7
A Mix	1 692	2 356	28.2
B	1 638	1 462	12.0
C Stream A	355	1 482	76.1
C Stream B	136	1 470	90.8
D	8 046	8 520	5.6
E	557	1 596	65.1
Average	1 966	2 654	42.6

This is further evident when considering the validation methods. The MBE suggests that the model underestimates the actual power consumption by almost 26%, resulting in an underestimation of approximately 789 kW.

Table 36: Secondary milling SFA function validation

RMSE	MAE	MBE (%)
840.9	737.8	-25.9

3.6 Step 5: Application of the scoring method



Now that the models have been developed and validated, it is necessary to employ the scoring method in the system to normalise the results and enhance the accuracy of comparisons. For each circuit section, the three functions – average, frontier, and %corrected – are utilised, with %corrected being calculated from the %initial score.

The initial score is determined by the ratio of predicted to actual power consumption, as outlined in Equation 13. This equation is then applied to the results from the average model for each section of the circuit: pre-milling, primary milling, and secondary milling.

The standard error is calculated using Equation 15 to obtain the corrected score. The standard error is used to derive the three functions defining the performance categories: underperformance, normal performance, and overperformance. The resulting functions are shown Figure 20 below.

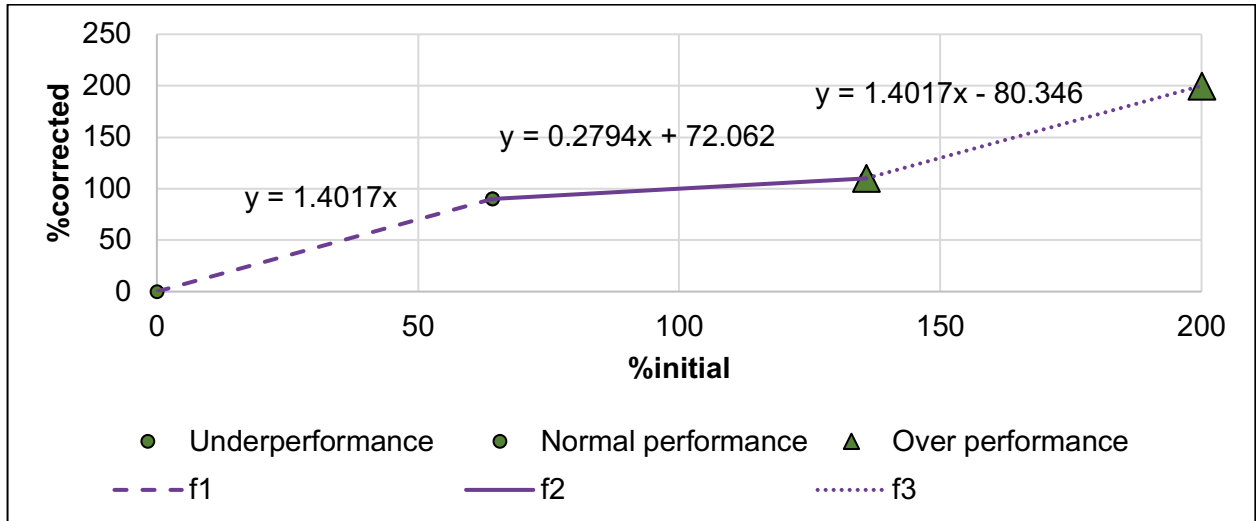


Figure 20: Percentage-corrected versus percentage initial – average primary milling model

Replicating this method for the average and SFA models, the equations and criteria for each section of the circuit are found and summarised below in Table 37 and Table 38, respectively. The corrected functions can now be used to accurately score the performance of a circuit section by including the standard error of the estimations.

Table 37: Percentage-normalised calculation based on average function for each section

Performance category	Criteria	Functions
Pre-milling		
Under	$\%_{\text{initial}} < 70.2$	$\%_{\text{corrected}} = 1.28 \times \%_{\text{initial}}$
Normal	$70.2 < \%_{\text{initial}} < 129.8$	$\%_{\text{corrected}} = (0.34 \times \%_{\text{initial}}) + 66.4$
Over	$\%_{\text{initial}} > 129.8$	$\%_{\text{corrected}} = (1.28 \times \%_{\text{initial}}) - 56.3$
Primary milling		
Under	$\%_{\text{initial}} < 73.0$	$\%_{\text{corrected}} = 1.23 \times \%_{\text{initial}}$
Normal	$73.0 < \%_{\text{initial}} < 127.0$	$\%_{\text{corrected}} = (0.37 \times \%_{\text{initial}}) + 63.0$
Over	$\%_{\text{initial}} > 127.0$	$\%_{\text{corrected}} = (1.23 \times \%_{\text{initial}}) - 46.7$
Secondary milling		
Under	$\%_{\text{initial}} < 85.64$	$\%_{\text{corrected}} = 1.05 \times \%_{\text{initial}}$
Normal	$85.64 < \%_{\text{initial}} < 114.36$	$\%_{\text{corrected}} = (0.70 \times \%_{\text{initial}}) + 30.34$
Over	$\%_{\text{initial}} > 114.36$	$\%_{\text{corrected}} = (1.05 \times \%_{\text{initial}}) - 10.17$

Table 38: Percentage-normalised calculation based on the SFA function for each section

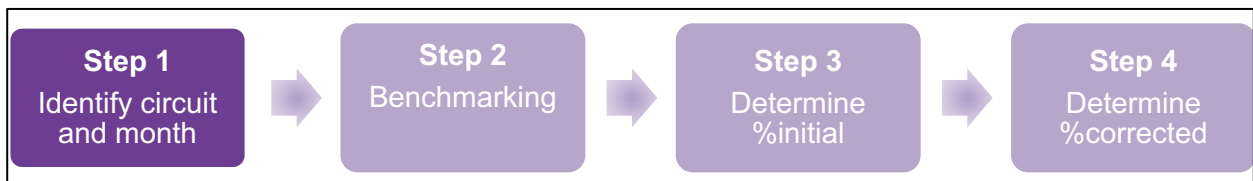
Performance category	Criteria	Functions
Pre-milling		
Under	$\%_{\text{initial}} < 75.5$	$\%_{\text{corrected}} = 1.19 \times \%_{\text{initial}}$
Normal	$74.5 < \%_{\text{initial}} < 124.5$	$\%_{\text{corrected}} = (0.41 \times \%_{\text{initial}}) + 59.19$
Over	$\%_{\text{initial}} > 124.5\%$	$\%_{\text{corrected}} = (1.19 \times \%_{\text{initial}}) - 38.43$
Primary milling		
Under	$\%_{\text{initial}} < 59.81$	$\%_{\text{corrected}} = 1.50 \times \%_{\text{initial}}$
Normal	$59.81 < \%_{\text{initial}} < 1140.19$	$\%_{\text{corrected}} = (0.25 \times \%_{\text{initial}}) + 75.12$
Over	$\%_{\text{initial}} > 140.19$	$\%_{\text{corrected}} = (1.50 \times \%_{\text{initial}}) - 100.97$
Secondary milling		
Under	$\%_{\text{initial}} < 47.09$	$\%_{\text{corrected}} = 1.91 \times \%_{\text{initial}}$
Normal	$47.09 < \%_{\text{initial}} < 152.91$	$\%_{\text{corrected}} = (0.19 \times \%_{\text{initial}}) + 72.06$
Over	$\%_{\text{initial}} > 152.91$	$\%_{\text{corrected}} = (1.91 \times \%_{\text{initial}}) - 80.346$

3.7 Step 6: Monthly benchmarking



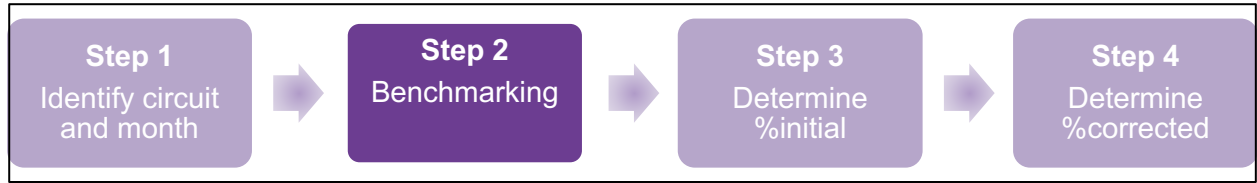
The final step of the methodology is to benchmark the three sections of the circuits and the complete circuits using the four steps discussed in Section 2.7. The first step, “identification”, is the same for all the circuits; the same period is used for analysis, and benchmarking is done for all the circuits.

3.7.1 Identification



The circuits in all the case studies are examined over a period of three months. This three-month period is distinct from the monthly data points initially utilised for the investigation and development of the models. This approach is then consistently applied across all circuit sections and the entire circuit, negating the need to repeat step 1 in this discussion.

3.7.2 Benchmarking



The models are applied to forecast the power consumption for each section of the respective circuit, encompassing both the average and frontier analyses in step 2. Following this, the %initial and %corrected scores are calculated in steps 3 and 4, respectively. Moreover, the combined sections of each circuit are subject to comparative analysis to underscore key differences that may contribute to disparate performance levels.

Pre-milling circuit

The forecasted monthly average power consumption for the three-month period is detailed in Table 39 below. This dataset does not contain any additional outliers, making it suitable for further analysis. The results demonstrate relative consistency and align closely with the actual power consumption figures.

Table 39: Pre-milling monthly average power consumption prediction and actual

Circuit	Month 1		Month 2		Month 3	
	Predicted	Actual	Predicted	Actual	Predicted	Actual
C Stream A	32	31	34	36	39	38
C Stream B	22	33	25	36	41	50
D	75	100	87	100	101	89
E	82	77	86	75	94	79
Average	53	60	58	62	69	64

The model's accuracy is showcased in Figure 21 below. Notably, circuit C stream B exhibits a significant MAPE of up to 31%, in stark contrast to the 3% MAPE of its counterpart in the same circuit. However, given the lower power consumption relative to other circuits, an error margin of 30% or 11 kW is not markedly impactful, especially since the predictions for other circuits are off by a similar magnitude. Consequently, despite the high MAPE, the results are deemed acceptable and suitable for further analysis using the scoring method.

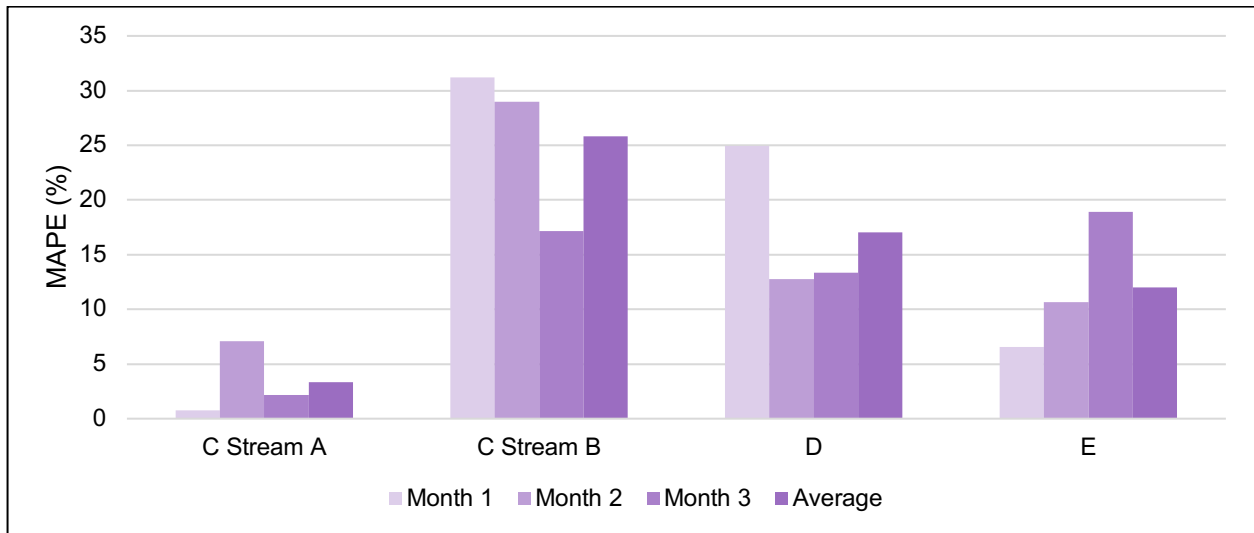


Figure 21: Pre-milling average function MAPE

The frontier analysis of the pre-milling circuit is achieved by correcting the y-intercept to zero, ensuring that the equation predicted a power consumption of 0 kW when no ore was being crushed. As a result, the SFA predictions, as shown in Table 40 below, are greater than both the average predicted and actual power consumption for the same feed.

Table 40: Pre-milling monthly SFA power consumption prediction and actual

Circuit	Month 1		Month 2		Month 3	
	Predicted	Actual	Predicted	Actual	Predicted	Actual
C Stream A	40	31	42	36	47	38
C Stream B	32	33	35	36	49	50
D	81	100	92	100	105	89
E	88	77	92	75	99	79
Average	60	60	65	62	75	64

As anticipated, the model was designed to identify the optimal performance targets for each circuit based on their feed. Nevertheless, the outcomes suggest that the function better reflects anticipated actual performance rather than exemplifying best practice or frontier performance. Thus, the SFA model does not adequately serve the frontier analysis of the pre-milling circuit, and neither does the COLS approach. Therefore, the SFA function is utilised solely as a representative tool.

Figure 22 provides a visual depiction of the MAPE for predictions made by the SFA function. Notably, for circuit C stream B, the MAPE is relatively low, peaking at only 3%, in stark contrast to the 29% observed for its counterpart stream. This indicates that circuit C stream B operates in

a manner more closely aligned with what the model represents. Conversely, circuit C stream A exhibits performance that aligns with the average model but falls short of the performance indicated by the SFA model.

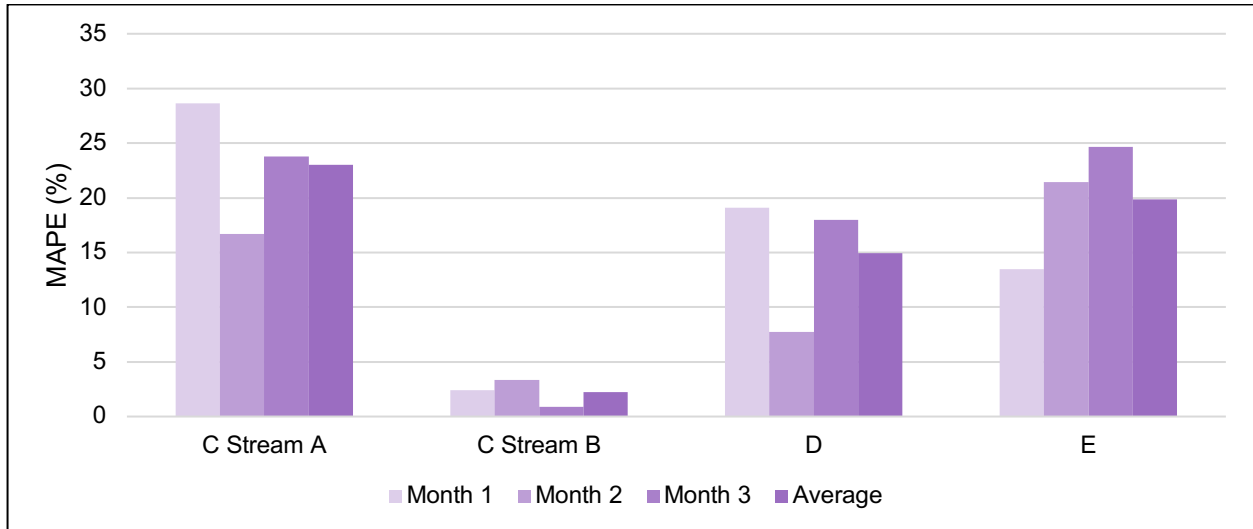


Figure 22: Pre-milling SFA function MAPE

Primary milling

Benchmarking for the primary milling follows the same procedure as that for the pre-milling circuit, yet the outcomes are anticipated to show greater variability due to the higher number of input variables influencing power consumption. The comparison between actual and predicted power consumption for the primary milling, based on the average function, is compiled in Table 41 below. This comparison reveals a markedly higher degree of variance in the results relative to the pre-milling benchmarking, a discrepancy that is particularly evident when examining the MAPE.

Table 41: Primary milling monthly average power consumption prediction and actual

Circuit	Month 1		Month 2		Month 3	
	Predicted	Actual	Predicted	Actual	Predicted	Actual
A UG2	1 499	1 953	1 460	1 812	1 544	1 927
A Mix	2 846	2 745	2 784	2 746	2 811	2 728
B	1 538	1 752	1 548	1 808	1 541	1 822
C Stream A	275	232	1 310	846	1 356	856
C Stream B	277	219	693	457	1 376	843
D	6 442	7 400	6 428	7 420	6 614	6 982
E	3 847	4 894	3 855	4 858	3 921	5 046
Average	2 389	2 742	2 583	2 849	2 738	2 886

The MAPEs for all the predicted values listed in Table 41 are visualised in Figure 23. This graph delineates the variability in prediction errors across different circuits and months under evaluation. On average, the MAPE for all circuits stands at approximately 23%. Notably, the model achieves exceptional accuracy for circuit A Mix, with a MAPE of 3%, indicating that this circuit most closely aligns with the average expected performance. Conversely, the model exhibits less precision for both streams of circuit C, a discrepancy likely attributable to the absence of mill weight data for this circuit, as indicated in previous analyses. Interestingly, for month 1, the error percentage is roughly half of that for months 2 and 3, suggesting that the model performs more effectively at lower feeds when no mill weight data is available.

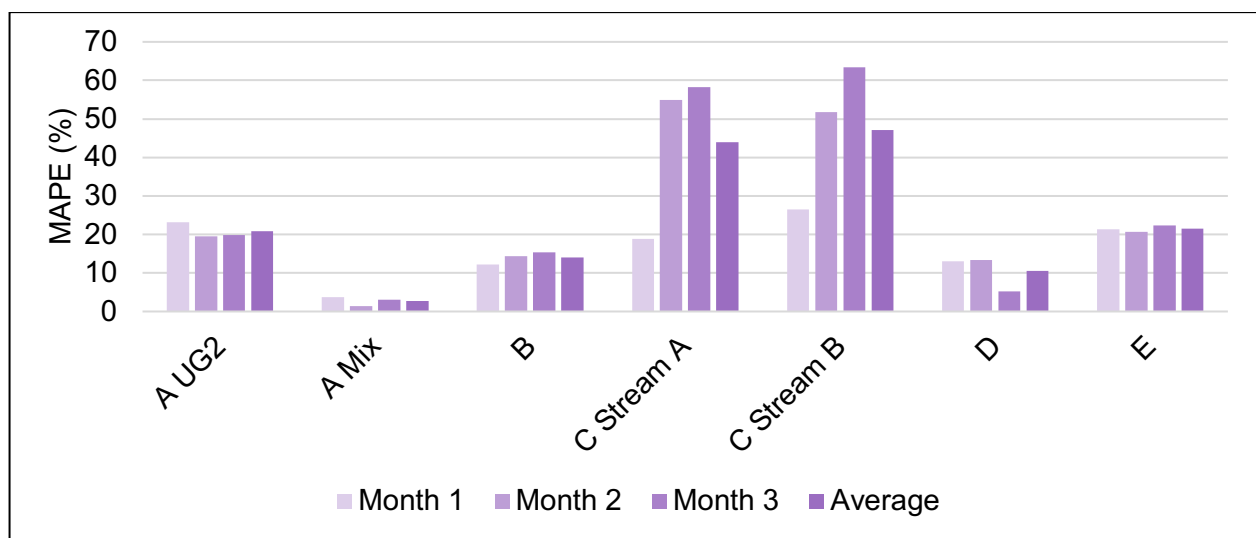


Figure 23: Primary milling average function MAPE

Furthermore, when considering the frontier analysis of the primary milling circuits, it can be seen in Table 42 below that the model underestimates the predicted power for all the circuits, as expected, except for both streams of circuit C.

Table 42: Primary milling monthly SFA power consumption prediction and actual

	Month 1		Month 2		Month 3	
Circuit	Predicted	Actual	Predicted	Actual	Predicted	Actual
A UG2	634	1 953	594	1 812	678	1 927
A Mix	1 980	2 745	1 918	2 746	1 945	2 728
B	673	1 752	683	1 808	676	1 822
C Stream A	376	232	1 410	846	1 456	856
C Stream B	377	219	793	457	1 476	843
D	5 576	7 400	5 563	7 420	5 749	6 982
E	2 981	4 894	2 990	4 858	3 055	5 046
Average	1 800	2 742	1 993	2 849	2 148	2 886

The adjustment of the y-intercept to account for the mill's weight, aimed at deriving a more representative function, results in a considerable increase in the percentage error. As illustrated in Figure 24 below, this adjustment leads to only circuits A Mix and D maintaining an average error below 30%.

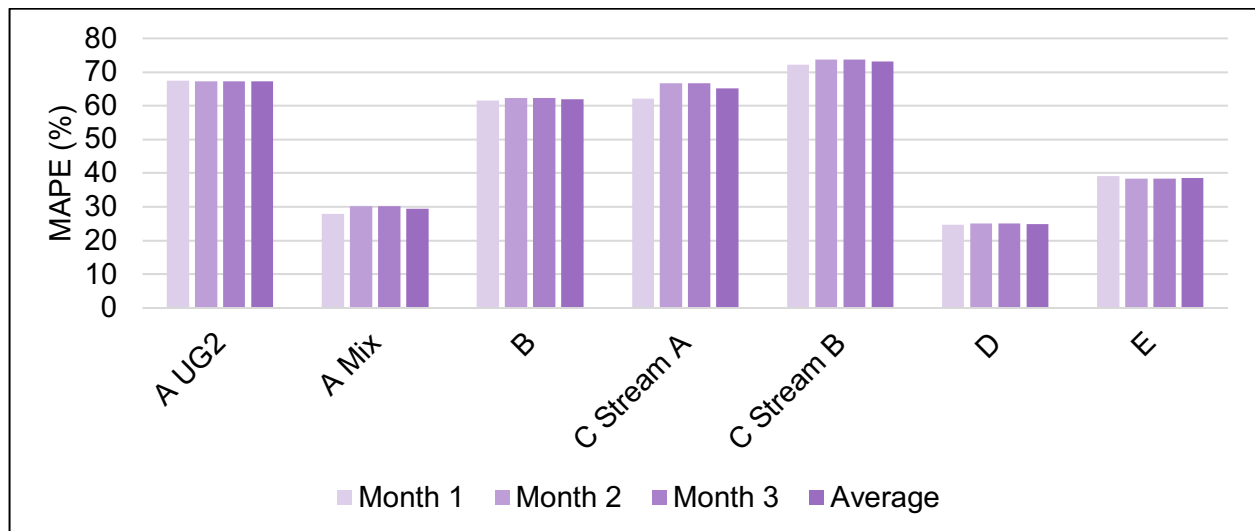


Figure 24: Primary milling SFA function MAPE

Secondary milling

The benchmarking approach used for primary milling is also applied to secondary milling; however, the number of considered variables is reduced because feedwater is excluded owing to data unavailability.

Table 43 demonstrates that the average prediction model for the secondary milling accurately forecasts power consumption in a consistent manner. Nonetheless, the model substantially

overestimates the actual consumption for circuit B, suggesting that this circuit outperforms the average across all circuits. This inference is supported by the consistent overperformance of circuit B observed over a three-month period.

Table 43: Secondary milling monthly average power consumption prediction and actual

	Month 1		Month 2		Month 3	
Circuit	Predicted	Actual	Predicted	Actual	Predicted	Actual
A UG2	1 693	1 708	1 671	1 748	1 664	1 739
A Mix	2 019	2 463	1 357	1 409	2 075	2 456
B	1 864	1 509	1 733	1 344	1 842	1 518
C Stream A	1 410	1 683	1 408	1 641	1 593	1 627
C Stream B	1 329	1 672	1 272	1 632	1 311	1 579
D	7 913	8 822	8 471	8 683	8 521	8 719
E	2 211	2 607	2 234	2 610	2 247	2 230
Average	2 705	2 976	2 652	2 743	2 834	2 940

This is highlighted graphically in the MAPE plot for the secondary milling circuits, as presented in Figure 25. It shows the MAPE for circuit B as exceeding twice the model's average MAPE of 12.7%. In contrast to the higher MAPE observed for circuit B, circuit D closely aligns with the anticipated average performance, demonstrating a MAPE of 5%, positioning it as the second-closest performer after circuit B.

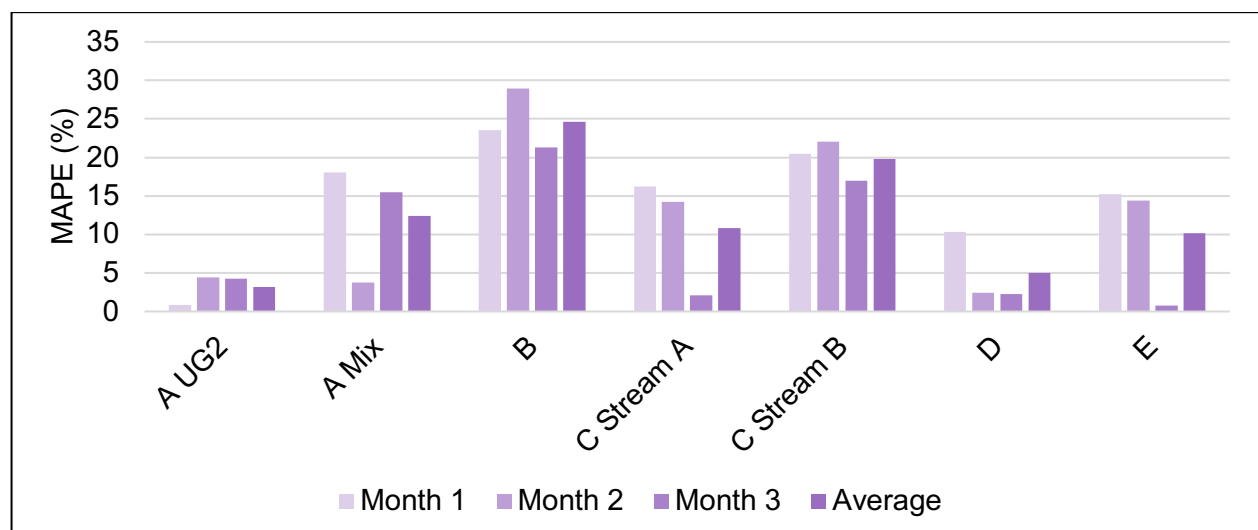


Figure 25: Secondary milling average function MAPE

The majority of the predictions resulting from the SFA model fall below the actual values, with five notable exceptions outlined in Table 44, three of which pertain to circuit B. This suggests that the

secondary milling performance of circuit B does not meet the expected standards set for secondary milling circuits. Moreover, given its superior performance relative to the average, circuit B is inferred to be the highest-performing secondary circuit among those studied. Additionally, the data reveals that circuits A, UG2, and E exceeded the frontier benchmarks in months 1 and 3, indicating their potential for even greater operation efficiency.

Table 44: Secondary milling monthly SFA power consumption prediction and actual

	Month 1		Month 2		Month 3	
Circuit	Predicted	Actual	Predicted	Actual	Predicted	Actual
A UG2	1 742	1 708	1 720	1 748	1 713	1 739
A Mix	2 070	2 463	1 403	1 409	2 127	2 456
B	1 914	1 509	1 783	1 344	1 892	1 518
C Stream A	1 546	1 683	1 544	1 641	951	1 627
C Stream B	1 466	1 672	1 408	1 632	809	1 579
D	8 006	8 822	8 602	8 683	8 619	8 719
E	2 354	2 607	2 401	2 610	2 390	2 230
Average	2 728	2 924	2 694	2 724	2 643	2 838

The error between the predicted and actual values is displayed in Figure 26 below. The error was expected to be more significant on average as the model represents a more frontier-based practice; however, the resulting average MAPE is 14.5%, indicating that the secondary circuits are not underperforming to the degree expected.

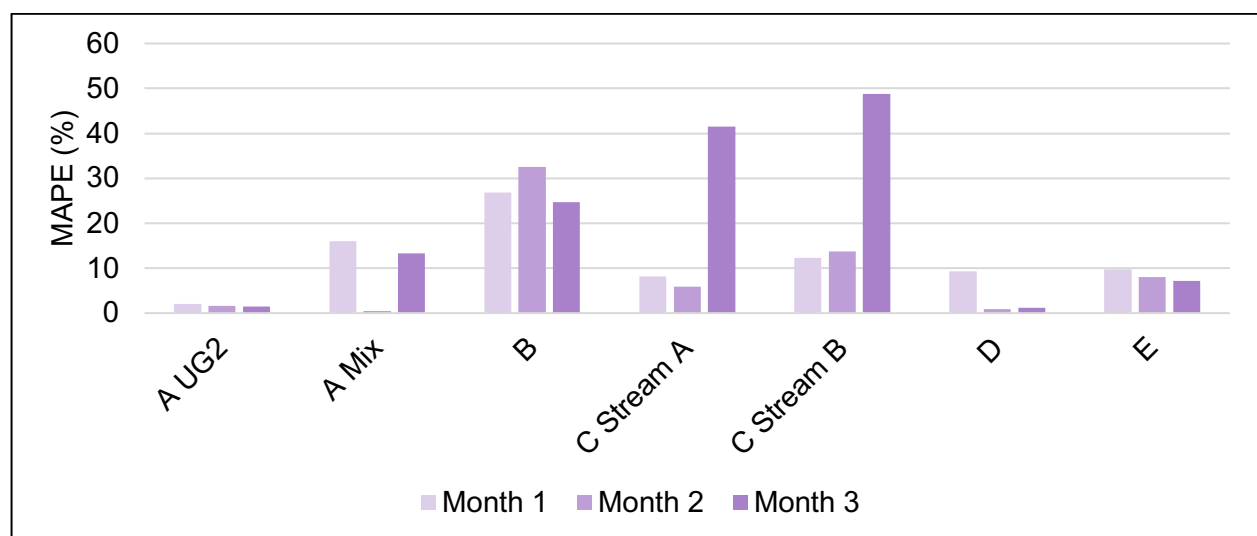
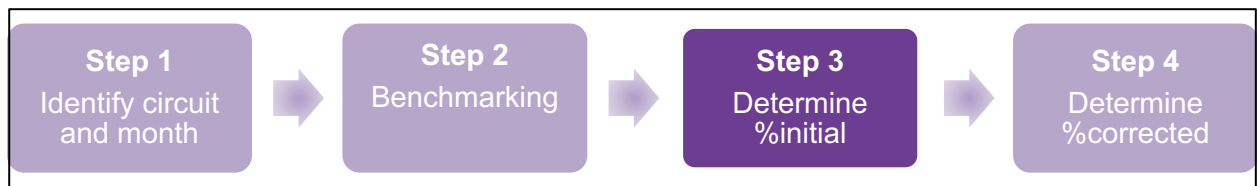


Figure 26: Secondary milling SFA function MAPE

3.7.3 Determine initial score



As discussed in Section 2.6, the %initial must be calculated before the score can be corrected by including the sey. Therefore, the %initial is calculated for the three milling sections to be converted to %corrected values for an improved comparison of the circuit's performance. As a result, the %initial values are only briefly discussed.

Pre-milling

Upon examining the %initial values for circuits equipped with pre-milling infrastructure, it is observed that, on average, all circuits exhibit performance below that of the average model when the standard error of the estimation is not accounted for, with the exception of circuit D. Circuit C stream B performs consistently below the average model across all three months, suggesting that it operates with less efficiency. In contrast, the performance of the remaining circuits suggests a potential for greater operational efficiency.

Table 45: Pre-milling average model %initial

Circuit	Month 1	Month 2	Month 3	Average
C Stream A	101	93	102	99
C Stream B	69	71	83	74
D	75	87	113	92
E	107	115	119	113

Primary milling

Table 46 illustrates that the primary milling circuits generally exhibit consistent performance, with the majority operating near the anticipated average model. Notably, circuit C surpasses the average performance across the circuits, but this is observed only in months 2 and 3. This improvement aligns with the throughput for this stream doubling from month 1 to months 2 and 3, indicating that both the model and the circuits achieve enhanced performance at higher throughputs.

Table 46: Primary milling average model %initial

Circuit	Month 1	Month 2	Month 3	Average
A UG2	86	90	89	88
A Mix	109	107	108	108
B	97	95	94	95
C Stream A	79	145	149	125
C Stream B	85	133	154	124
D	97	97	105	100
E	88	88	87	87

Secondary milling

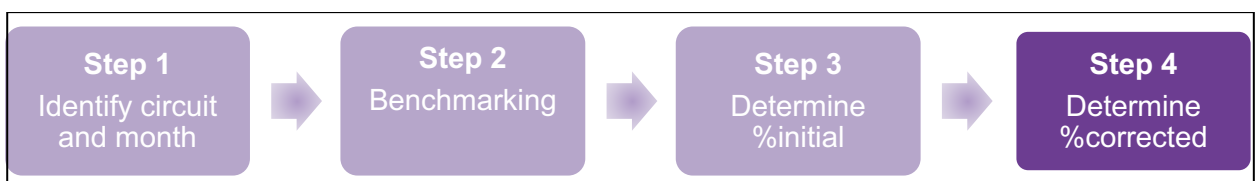
Comparatively fewer secondary milling circuits than primary circuits exceeded the performance predicted by the average model, with only circuits A UG2 and B achieving %initial scores of 119 and 125, respectively. This indicates a greater opportunity for improvements on the secondary circuits based on the %initial analysis.

Table 47: Secondary milling average model %initial

Circuit	Month 1	Month 2	Month 3	Average
A UG2	98	148	111	119
A Mix	78	114	80	91
B	121	132	121	125
C Stream A	93	95	108	99
C Stream B	89	87	93	90
D	92	92	101	95
E	85	86	101	90

The same is done for the SFA function of all three sections, and the resulting values are tabulated in Appendix A.

3.7.4 Determine corrected score



Pre-milling

The pre-milling section's evaluation leverages the %corrected values calculated in Table 37. This facilitates straightforward comparison of the three circuits equipped with pre-milling infrastructure, enabling the identification of circuits with potential for improvement through the examination of corrected values, as depicted in Figure 27 below.

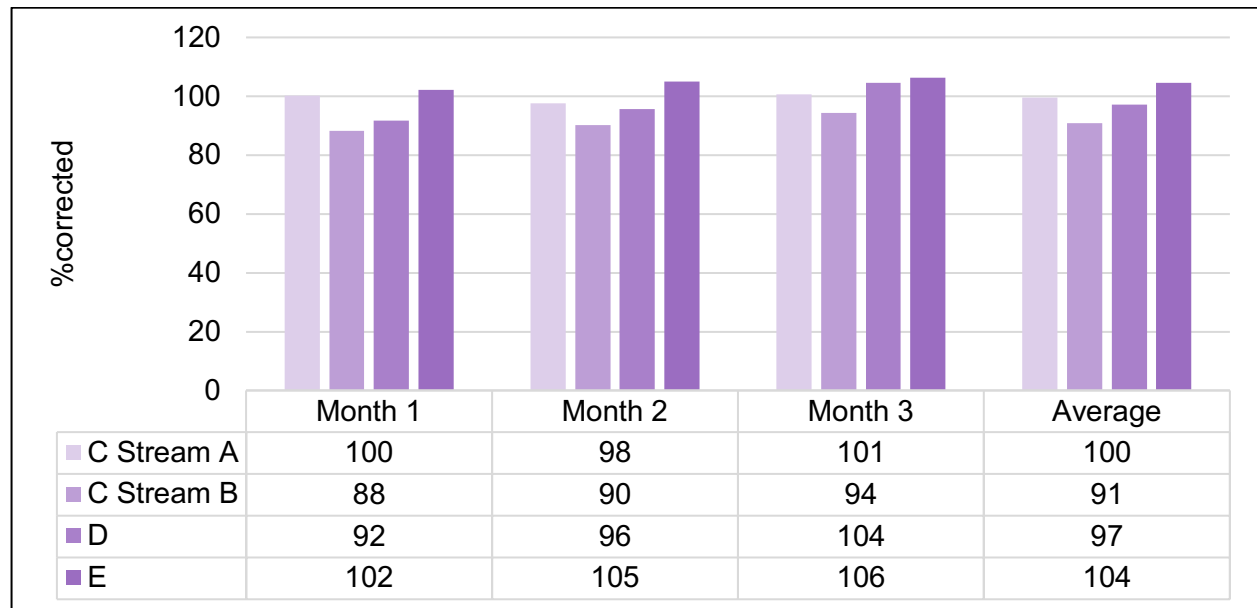


Figure 27: Pre-milling – percentage corrected values (average benchmarking)

Figure 27 demonstrates that the operational consistency of the circuits from month to month is relatively stable. Notably, Circuit D shows a significant improvement in month 3 relative to earlier months. Furthermore, Circuit E demonstrates superior average performance compared to its counterparts, while circuit C stream B is identified as the poorest performer on average. Given these results, Circuit E emerges as the top-performing pre-milling circuit, exceeding the average performance. This makes it a prime candidate for further analysis aimed at enhancing the performance of the other underperforming circuits.

The results above show that **objective 1, to develop a new average benchmarking model for pre-milling circuits of PGM concentrators, has been completed.**

The outcomes of the average benchmarking can be cross-verified through the SFA model, which, while not providing a frontier analysis, more accurately reflects actual performance. Contrary to the average model, inverse results are anticipated here. This delineation indicates that the underperforming circuits are closer to the representative benchmark in terms of higher power

consumption, whereas circuits that perform better – and thus, are more distant from the representative benchmark – achieve lower scores compared to their counterparts.

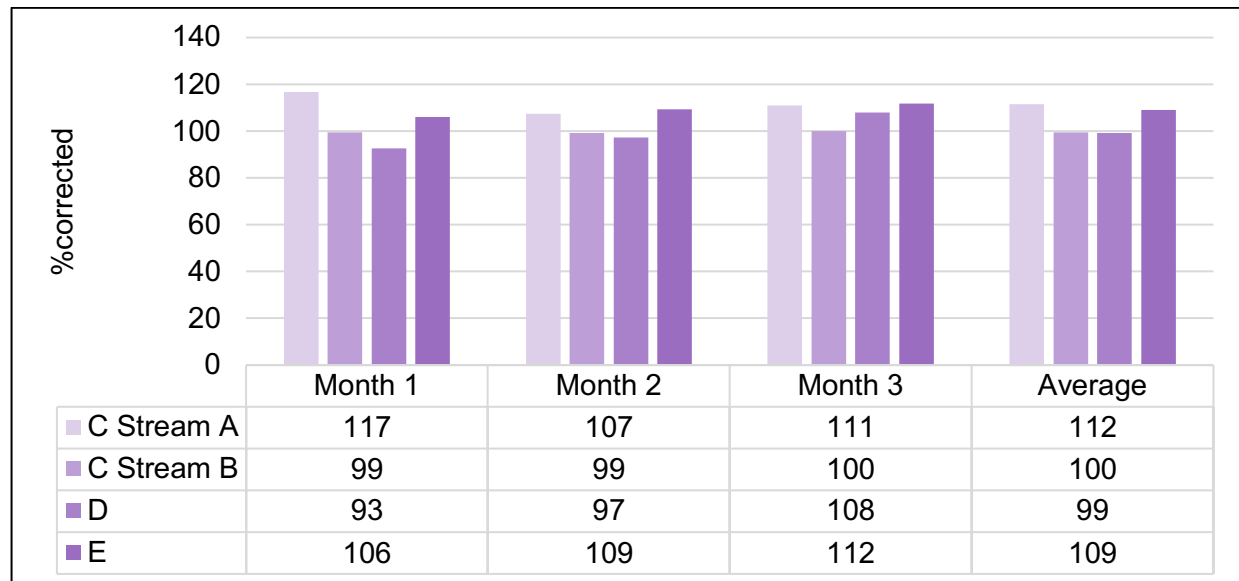


Figure 28: Pre-milling – percentage corrected values (SFA benchmarking)

Considering Figure 28 above, circuit C stream A emerges as the top performer among the pre-milling circuits, achieving an average corrected percentage of 112%. Following closely is circuit E, which was identified as the top performer in the average model analysis. Despite this, circuit E maintains its status as the leading circuit. This distinction arises because the SFA function, with its elimination of the y-intercept, more accurately depicts actual performance rather than an idealised frontier performance.

The results above show that **objective 2, to develop a new frontier benchmarking model for pre-milling circuits of PGM concentrators, has only been partially completed.** The SFA function only provides a representative, expected performance, not a best-practice performance, as stated it would in the objective.

Primary milling

The %corrected values derived from the average model for the primary milling circuits are summarised in Figure 29 below. Similar to the observations in the pre-milling analysis, the primary milling circuits exhibit comparable performance levels post-normalisation. Aside from circuit C, which stands out significantly, the remaining circuits perform marginally below the average, with circuit A Mix maintaining a slight but consistent advantage over the average. In contrast, circuits A, UG2, and E rank as the least efficient among the primary milling circuits.

The results below show that **objective 3, to develop a new average benchmarking model for primary milling circuits of PGM concentrators, has been completed.**

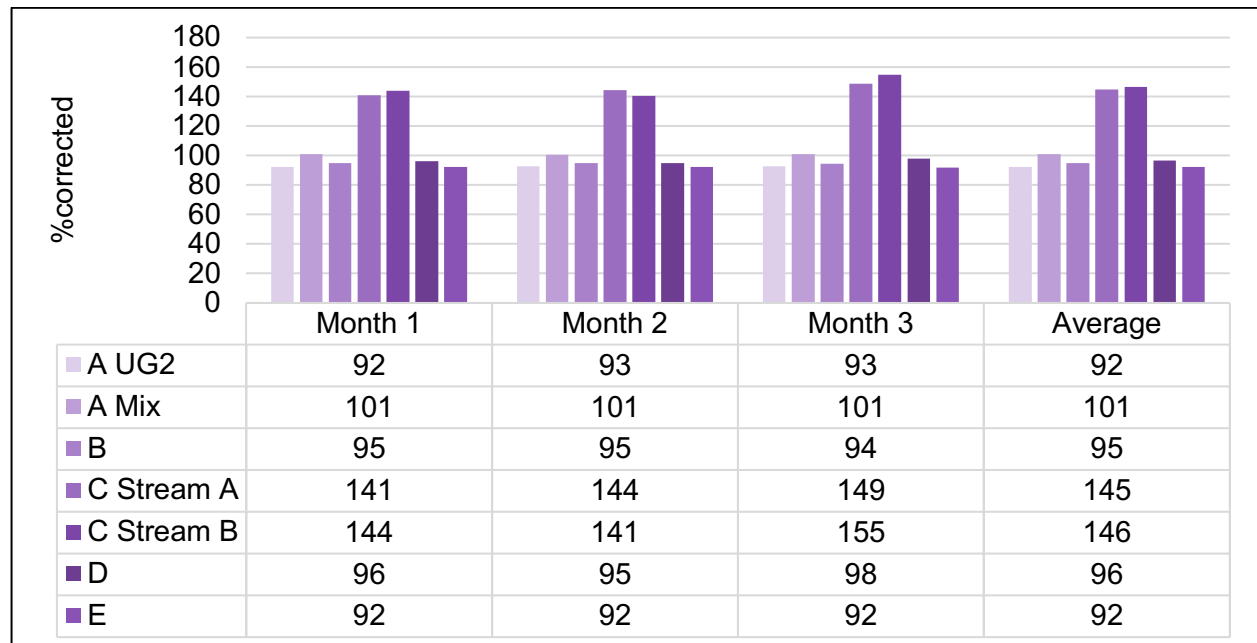


Figure 29: Primary milling – percentage corrected values (average benchmarking)

The %corrected values from the SFA are presented in Figure 30. These findings align with the analysis from the average model, indicating that the SFA model more accurately forecasts the frontier performance, especially when compared to the pre-milling circuit model. Consequently, there is a significantly wider spread in the %corrected scores among the primary milling circuits, with circuit C's score surpassing 200%. This substantial variation can be attributed to the absence of weight data for the mills in those two streams, leading to a drastic skew in the data. As previously noted, the impact of this absence was reduced in month 1 due to the lower feed to the circuits.

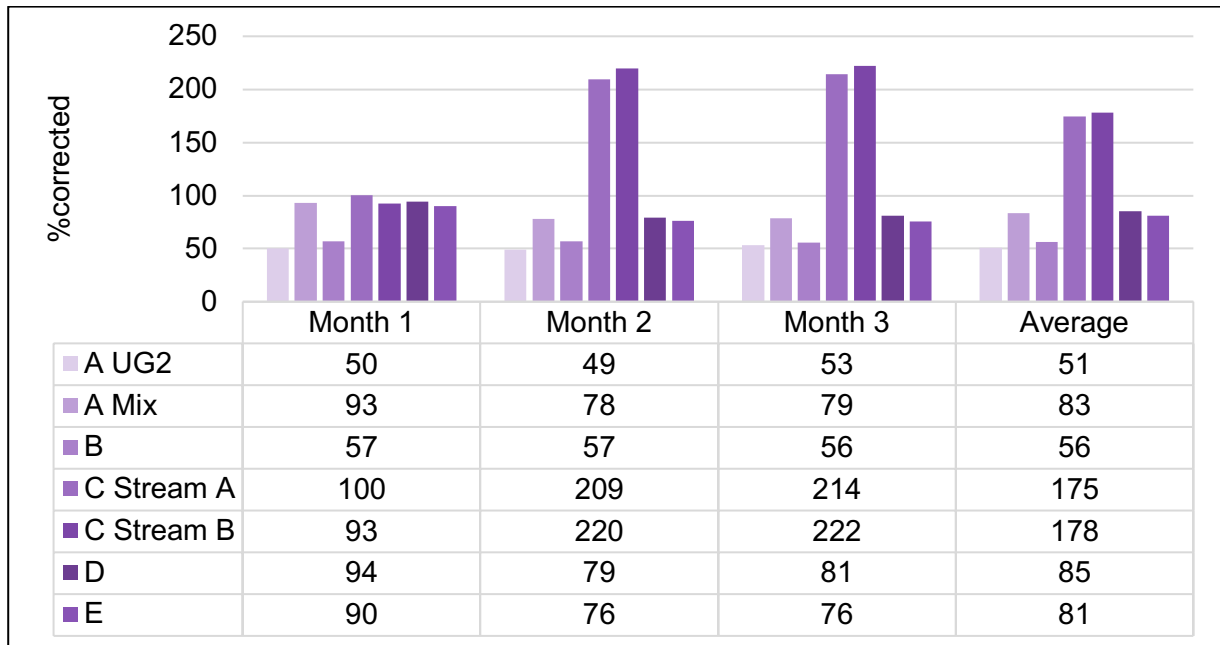


Figure 30: Primary milling – percentage corrected values (SFA benchmarking)

The results above show that **objective 4, to develop a new frontier benchmarking model for primary milling circuits of PGM concentrators, has only been partially completed**. The SFA function provides a frontier analysis, but the benchmark accuracy is unreliable due to the effect of missing data on the overall model.

Secondary milling

In contrast to the primary milling circuits, only circuit B outperforms the average among the secondary milling circuits, as evidenced by the %corrected results depicted in Figure 31 below. Notably, circuit C exhibits a marked decline in performance relative to its primary milling counterpart. Nonetheless, across all circuits, monthly performances remain consistent, exhibiting only minor variances.

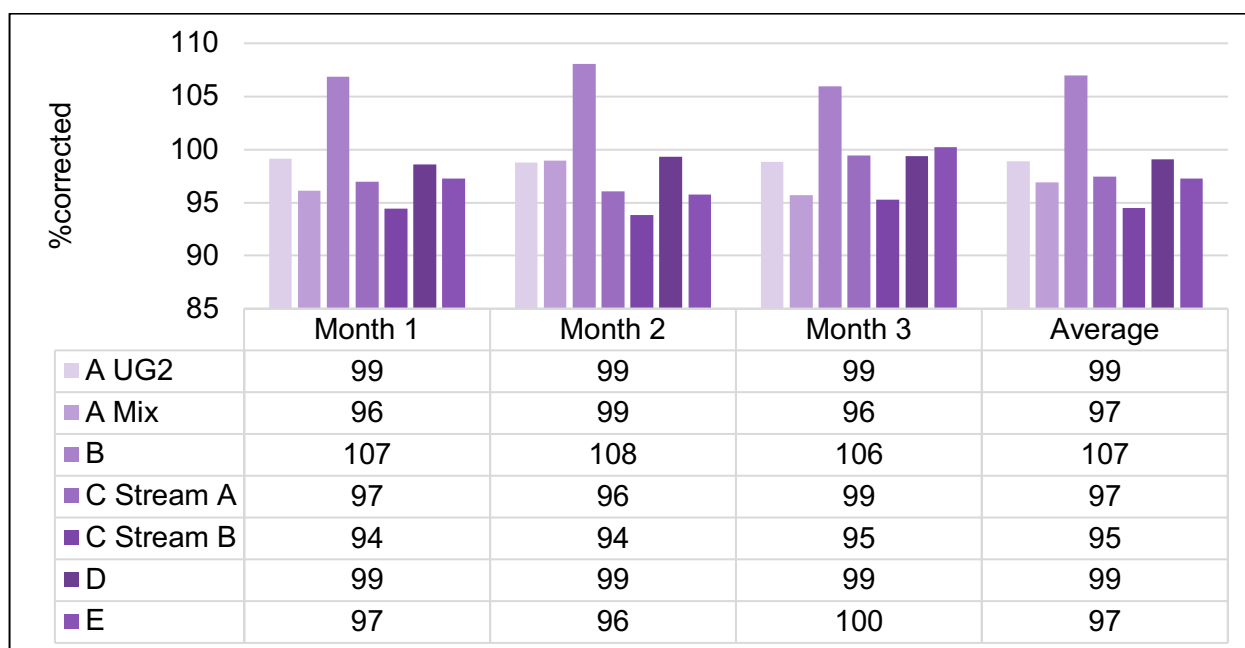


Figure 31: Secondary milling – percentage corrected values (average benchmarking)

Furthermore, it is simple to evaluate and compare the relative performances between each circuits. It can be concluded that **objective 5, to develop a new average benchmarking model for secondary milling circuits of PGM concentrators, has been completed.**

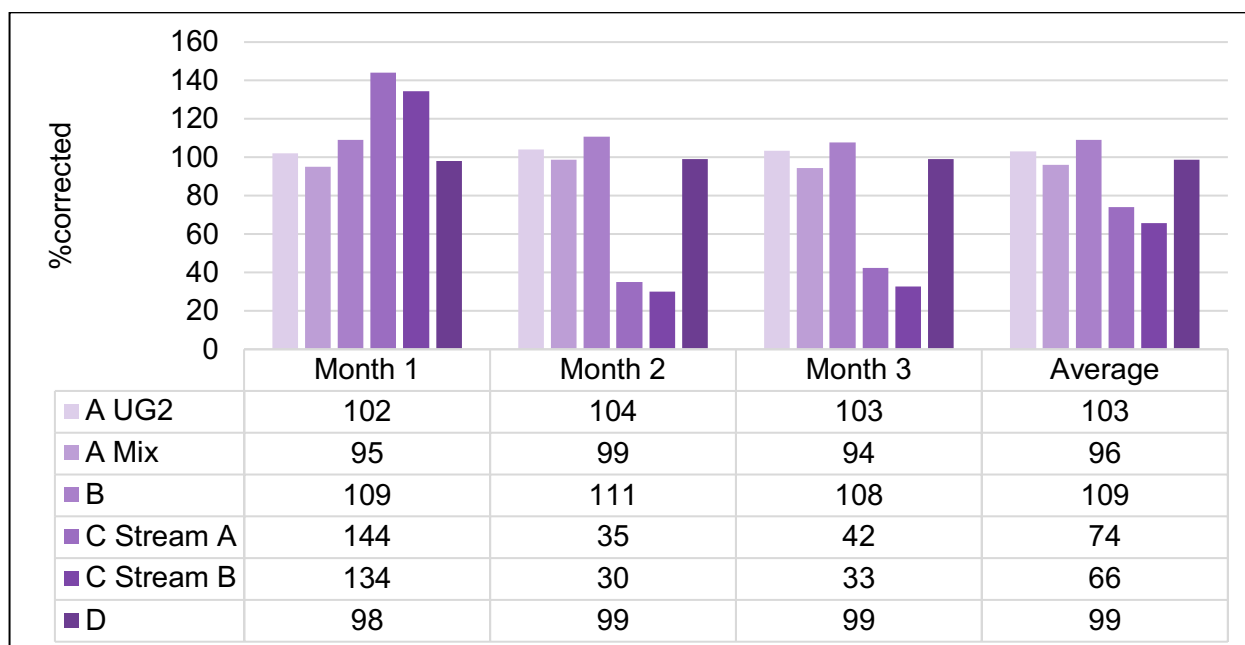


Figure 32: Secondary milling – percentage-corrected values (SFA benchmarking)

Moreover, the SFA yields results that closely mirror those of the average model, particularly identifying circuits A UG2 and B as the top two performers in both analyses. Consequently, these

circuits stand out as exemplary in terms of efficiency, warranting further investigation into their practices and equipment. Such an inquiry could provide valuable insights into potential improvements for the other circuits. Therefore, **objective 6, to develop a new frontier benchmarking model for PGM concentrators' secondary milling circuits, has been completed.**

Having analysed the individual sections and assessed their performance, a comprehensive benchmarking of the circuits is now necessary for a broader perspective. This benchmarking process utilises the average %corrected values from each circuit section, offering a holistic overview of both the performance of each section and the circuit as a whole. This approach facilitates a more effective comparison of the circuits and helps identify and prioritise which ones require efficiency enhancements. Table 48 below summarises the outcomes of the average model analysis that were attained by averaging the three-month results for each circuit section and computing an overall performance metric based on the aggregated section data.

Table 48: Average %corrected average benchmarking of milling circuits

Circuit	Pre	Primary	Secondary	Average
A UG2		97	99	98
A Mix		101	97	99
B		95	107	101
C Stream A	100	145	97	114
C Stream B	91	146	95	111
D	97	96	99	98
E	104	92	97	98

The table identifies both the high-performing and underperforming sections within each circuit. The best-performing section is highlighted green, while the worst-performing section is highlighted in red. The gradient colours from green-to-red indicate the performance compared to the best and worst performing sections. Considering the scale and the pre-milling sections, circuit E ranks as the most efficient, while circuit C stream B is identified as the least efficient. These circuits, therefore, warrant further analysis to elucidate the causes behind their performance discrepancies.

This evaluation approach is similarly applied to the primary and secondary milling circuits. Notably, the primary milling sections of circuit C demonstrate superior performance compared to both the average and other circuits, and a similar methodology is adopted for the secondary milling section of circuit B.

Additionally, Table 49 consolidates the outcomes from the SFA analysis for each circuit section, calculating an overall average performance for each. The comparison between the average and SFA models reveals discrepancies in the results.

Table 49: Average %corrected SFA benchmarking of milling circuits

Circuit	Pre	Primary	Secondary	Average
A UG2		51	94	73
A Mix		83	93	88
B		56	101	79
C Stream A	112	175	95	127
C Stream B	100	178	94	124
D	99	85	98	94
E	109	81	97	96

The rankings derived from the SFA benchmarking align closely with those from the average benchmarking approach, with a notable deviation occurring with circuit B. Additionally, it is important to acknowledge that the pre-milling SFA does not serve as a frontier benchmark and may influence the overall SFA score for circuits incorporating pre-milling processes. As highlighted earlier, the existing approach to benchmarking comminution circuits revolves around their energy intensity per tonne processed, offering a straightforward and reliable method despite its shortcomings.

3.8 Step 7: Comparison with other methods



As mentioned earlier, the prevailing practice for benchmarking comminution circuits relies on assessing their energy intensity per tonne processed, a methodology that is both uncomplicated and precise. Despite certain limitations, it is crucial to compare and evaluate the outcomes from this conventional approach to those derived from the score-based method to gauge the latter's efficacy. As depicted in Table 50, energy intensities are determined for the entire plant rather than for individual streams or circuits. Consequently, an average corrected score is computed and ranked for direct comparison against the energy intensity figures furnished by the processing plants over the investigation period.

Table 50: Ranking of circuits using the two different methods

Circuit	Corrected	Corrected average	Intensity	Difference
A UG2	5	3	2	1
A Mix	4			
B	3	2	3	1
C Stream A	1	1	1	0
C Stream B	2			
D	5	4	4	0
E	6	5	5	0
Average difference			0.4	
Correct prediction			92%	

As seen in Table 50 above, there is minimal variance in the ranking of the circuits. The intensity method ranks circuit A second, while the newly developed method ranks it third on average for the two streams. The variance is likely due to the intensity method being based on the entire concentrator plant, which is another limiting factor of this method.

Table 51: Ranking of circuits using the average and SFA benchmarking methods

Circuit	Average	SFA	Difference
A UG2	5	7	2
A Mix	4	5	1
B	3	6	3
C Stream A	1	1	0
C Stream B	2	2	0
D	5	3	2
E	6	4	2
Average difference		1.4	
Correct prediction		80%	

The variability in outcomes underscores the substantial advantage of employing both benchmarking methods to analyse the circuits. The discrepancy between the average and potential performance highlights the room for enhancement. Therefore, even if a circuit outperforms the average, it does not negate the possibility for further efficiency optimisation of either the specific section or the entire circuit. **Objective 7, to compare the score-based benchmarking method to the current one used, has been completed.**

3.9 Study validation

The results of the study are summarised in Table 52 below. The results are used to validate the study by evaluating the completion of the objectives discussed in Chapter 1.7. The main objective is the study's aim, which has been completed.

Table 52: Validation of study

Objective	Chapter(s)	How?
1.	3.7.4	The average benchmarking model for the pre-milling circuit produced an average MAPE of 15% and RMSPE of 4%.
2.	3.7.4	Partially completed as the SFA model has a MAPE of 15% and RMSPE of 4% but is above the average benchmark due to the model being more representative of true operation.
3.	3.7.4	The average benchmarking model for the primary milling circuit produced an average MAPE of 25% and RMSPE of 23%.
4.	3.7.4	The SFA benchmarking model for the secondary milling circuit produced an average MAPE of 36% and RMSPE of 34%.
5.	3.7.4	The average benchmarking model for the primary milling circuit produced an average MAPE of 12% and RMSPE of 17%.
6.	3.7.4	The SFA benchmarking model for the secondary milling circuit produced an average MAPE of 16% and RMSPE of 21%.
7.	2	A score-based benchmarking model was developed for PGM comminution circuits.
8.	3.8	Comparing the ranking of circuits based on the current method with the score-based benchmarking method.

According to the literature, a MAPE below 10% and RMSE below 15% are considered acceptable thresholds [61], [62]. Consequently, despite the fulfilment of all objectives, there is room to refine the models to diminish the prediction errors' percentage. Nonetheless, when juxtaposed with traditional intensity benchmarking, the models accurately predict the circuits' rankings with a 92% precision. Thus, despite the prediction models' relative imprecision, the benchmarking methodology itself demonstrates robust accuracy.

The robustness of the methodology will allow it to be replicated for comminution circuits of other metals. However, it will not produce the required results when applying it to varying metal comminution circuits. The accuracy of the model is limited to the application of the model to circuits of similar use and specifications. Variances in performance and their comparisons are likely to occur if the type of reef processed, technology implemented or size of technology. Despite that, the model is will still produce representative performance. It is however recommended, that for the most accurate comparison, circuits of similar nature and design are applied to the model. Therefore, developed model must be applied discretely to other PGM comminution circuits for direct comparison of performance.

3.10 Conclusion

In this chapter, the methodology introduced in Chapter 2 was applied to a condensed dataset spanning three months, in contrast to the six months utilised for the initial development of the models. This application facilitated the effective benchmarking of the various circuit sections, thereby fulfilling all eight objectives set forth in this study.

Objectives 1, 3, and 5 were achieved through the benchmarking of the pre-milling, primary milling, and secondary milling circuits using the average benchmarking model devised in Chapter 2. This model pinpointed the less efficient sections by comparing them against the average performance of counterparts, fulfilling its intended purpose. Moreover, by leveraging the average corrected values, it was possible to benchmark the complete circuits against each other, akin to the assessment typically conducted through circuit intensity evaluation. Nonetheless, it is important to acknowledge that both methodologies have their respective advantages and limitations.

Objectives 2, 4, and 6 were met by employing the frontier models developed for benchmarking the three circuit sections. The pre-milling model did not yield the anticipated frontier benchmark, instead offering a representation more aligned with actual performance. The models for the other sections functioned as expected, effectively highlighting the most efficient circuits.

The score-based method, formulated in Chapter 2, was applied to the case studies, offering a more nuanced insight into performance variability. The evaluation of this method's results against those of the traditional benchmarking validated the score-based approach, thus completing objective 8 in Section 3.8.

While the score-based method unveils potential cost and operational advantages, it is imperative to note that the benefits stem not from the model's application per se but from addressing the inefficiencies it uncovers. It proves more effective than current methods at identifying specific

inefficiencies within circuits, which broadens the scope for targeted improvements. Unlike traditional approaches that merely identify inefficient circuits, the score-based method dissects the circuit into sections, pinpointing underperforming areas within otherwise efficient circuits. This precision reduces the resources required for identifying and rectifying inefficiencies.

Addressing these inefficiencies through the adoption of practices from more efficient sections has the potential to drive operational enhancements.

CHAPTER 4 CONCLUSION

4.1 Overview

South Africa is endowed with a wealth of minerals and metals, making mining and mineral processing vital to the nation's economy and its GDP. Yet, the mining sector has encountered several challenges recently, including falling metal prices, the impact of global pandemics, and escalating operational expenses. These issues have led to decreased production and narrowing profit margins, compelling South African mining companies to seek ways to cut their operational costs.

In light of these challenges, this study was designed to identify potential inefficiencies in one of the most energy-intensive systems in metal liberation from ore: the comminution circuits. Through benchmarking methods, the study aimed to compare the operational performance of various case study circuits against each other.

A benchmarking model for comminution circuits was developed, drawing on methodologies found in existing literature. This research successfully established three average benchmarking models for pre-milling, primary milling, and secondary milling circuits, fulfilling objectives 1, 3, and 5 as outlined in Section 1.7. While objective 2 was partially achieved, as detailed in Section 3.7.4, objectives 4 and 6 were fully met, with suggestions for further improvement of the model. Application of these benchmarking models illuminated the performance discrepancies across different circuit sections and entire circuits.

Sections or circuits exhibiting significant deviation from the average model underwent further analysis to explore their operational practices and identify reasons behind performance variance. This inquiry facilitated a comparative analysis between the score-based benchmarking method and the existing intensity method, accomplishing the study's final objective, objective 8, as discussed in Section 3.8.

The score-based method yielded results that were comparable to those obtained using the straightforward intensity benchmarking method, ranking circuits by efficiency with 92% accuracy. Although the model's precision may be slightly reduced, it effectively pinpoints the most inefficient circuit sections. This approach ensures that the exceptional performance of any single section does not distort the overall performance evaluation, unlike the intensity method.

Moreover, the intensity figures can be affected by outliers resulting from data inaccuracies, plant shutdowns, or faulty instrumentation. The score-based method addresses these issues by pre-processing and normalising all data points to ensure equitable comparison. Despite the added complexity in method development, the implementation of the score-based approach remains straightforward.

4.2 Method review

The method was developed in Chapter 2 based on findings in Chapter 1. The developed method was applied to five concentrator plants with seven comminution circuits. The flowchart in Figure 12 can be followed systematically and implemented on various platinum comminution circuits to identify inefficiencies within the circuit.

4.3 Limitations and recommendations

The model devised is specifically tailored for platinum comminution circuits, reflecting the particular practices and technologies employed within these systems. Consequently, it is not directly transferable to comminution circuits processing other metals. Nonetheless, the underlying methodology offers a foundation for developing both average and frontier score-based benchmarking models applicable to such circuits.

While the method has been successfully established, there is room for enhancing the accuracy of the models to gain deeper insights into circuit performance. Expanding the dataset and incorporating more variables could potentially improve model precision. Yet, to avoid adding undue complexity, the impact of any additional variable on power consumption should be clearly established.

The utility of a score-based benchmarking method hinges on the feasibility of implementing changes to augment circuit efficiency. Given the intricate nature and cost sensitivity of these circuits, even minor modifications could disrupt operations and impact production. Therefore, any proposed alterations should first undergo simulation using the simulation software packages discussed earlier. This precautionary step allows for the assessment of potential changes in a risk-managed environment, yielding relatively reliable outcomes.

Incorporating metrics such as breakage rates or product sizes following each milling stage into the benchmarking model could offer further refinement. This addition would enhance circuit comparisons by accounting for variations in energy consumption versus product quality,

potentially highlighting more cost-effective operations based on the liberation and cost-per-tonne ratio.

Moreover, the model presents an opportunity to delve deeper into milling practices; by isolating energy consumption for benchmarking purposes, the model narrows its focus. A broader examination of control strategies, grinding media, and other operational practices could shed light on optimal combinations of energy use and product quality, offering valuable insights for operational improvements.

BIBLIOGRAPHY

- [1] Z. Botha, 'The Mining Industry and the Energy Crisis', *The Journal of Southern African Institute of Mining and Metallurgy*, vol. 122, pp. v–vi, 2022.
- [2] B. Buchspies, L. Thormann, C. Mbohwa, and M. Kaltschmitt, 'PGE production in Southern Africa, part II: Environmental aspects', *Minerals*, vol. 7, no. 11, Nov. 2017, doi: 10.3390/min7110225.
- [3] G. R. Ballantyne, M. S. Powell, and M. Tiang, 'Proportion of energy attributable to comminution', in *Proceedings of the 11th Australasian Institute of Mining and Metallurgy Mill Operator's Conference*, 2012, pp. 25–30.
- [4] Multotec, 'Comminution', Multotec. Accessed: Oct. 18, 2023. [Online]. Available: <https://www.multotec.com/en/comminution>
- [5] F. Shi, R. Morrison, A. Cervellin, F. Burns, and F. Musa, 'Comparison of energy efficiency between ball mills and stirred mills in coarse grinding', *Miner Eng*, vol. 22, no. 7, pp. 673–680, 2009, doi: <https://doi.org/10.1016/j.mineng.2008.12.002>.
- [6] D. W. Fuerstenau and A. Z. Abouzeid, 'The energy efficiency of ball milling in comminution', *Int J Miner Process*, vol. 67, no. 1–4, pp. 161–185, Nov. 2002, doi: 10.1016/S0301-7516(02)00039-X.
- [7] O. D. Neikov, 'Mechanical Crushing and Grinding', *Handbook of Non-Ferrous Metal Powders: Technologies and Applications*, pp. 47–62, Jan. 2009, doi: 10.1016/B978-1-85617-422-0.00002-1.
- [8] N. Hlabangana, E. Nhira, N. Masayile, P. M. Tembo, and G. Danha, 'A fundamental investigation on the breakage of a bed of PGM ore particles: An attainable region approach, part 2', *Powder Technol*, vol. 346, pp. 326–331, Mar. 2019, doi: 10.1016/j.powtec.2019.02.005.
- [9] F. K. Mulenga and N. Chimwani, 'Introduction to the use of the attainable region method in determining the optimal residence time of a ball mill', *Int J Miner Process*, vol. 125, pp. 39–50, 2013, doi: 10.1016/j.minpro.2013.09.007.
- [10] S. Medina and Sifter International, 'Mineral Processing (Practical Action Technical Brief)', Appropedia.

- [11] A. K. Moaveni, M. Noaparast, and B. Rezai, 'A Method to Determine the Ball Filling, in Miduk Copper Concentrator SAG Mill', *International Journal of Mining and Geo-Engineering*, vol. 46, no. 1, pp. 15–19, Jun. 2012.
- [12] M. S. Powell, A. N. Mainza, M. H. Hilden, and M. Yahyaei, 'Full pre-crush to SAG mills-the case for changing this practice', in *The 6th International Conference on Semi-Autogenous and High Pressure Grinding Technology, Canadian Institute of Mining and Metallurgy (CIM), Vancouver, BC, Canada*, 2015.
- [13] M. S. Powell, S. Morrell, and S. Latchireddi, 'Developments in the understanding of South African style SAG mills', *Miner Eng*, vol. 14, no. 10, pp. 1143–1153, Oct. 2001, doi: 10.1016/S0892-6875(01)00132-7.
- [14] A. Z. M. Abouzeid and D. W. Fuerstenau, 'Grinding of mineral mixtures in high-pressure grinding rolls', *Int J Miner Process*, vol. 93, no. 1, pp. 59–65, Sep. 2009, doi: 10.1016/j.minpro.2009.05.008.
- [15] L. Buckingham, J.-F. Dupont, J. Stieger, B. Blain, and C. Brits, 'Improving energy efficiency in Barrick grinding circuits', in *Proceedings of the V International Conference on Autogenous and Semiautogenous Grinding. Vancouver, Canada*, 2011.
- [16] C. Young, 'Cone Crushers', in *SME Mineral Processing and Extractive Metallurgy Handbook*, vol. 1 & 2, 2019, pp. 406–407.
- [17] C. Young, 'Gyratory Crushers', in *SME Mineral Processing and Extractive Metallurgy Handbook*, vol. 1 & 2, 2019, pp. 391–397.
- [18] C. A. Young, 'Jaw Crushers', in *SME Mineral Processing and Extractive Metallurgy Handbook*, vol. 1 & 2, R. C. Dunne, K. S. K. Kwatra, and C. A. Young, Eds., SME, 2019, pp. 367–374.
- [19] G. Liu, T. Ichinose, A. Tokumaru, and S. Owada, 'Surface-grinding kinetics for the concentration of PGMs from spent automobile catalysts by Attritor surface grinding', *Mater Trans*, vol. 55, no. 6, pp. 978–985, 2014, doi: 10.2320/matertrans.M2014082.
- [20] M. Kleinhans, 'Excellence breeds success: commodity: Gold & PGMs', *Inside Mining*, vol. 9, no. 4, pp. 12–13, 2016, doi: 10.10520/EJC187606.

- [21] L. Liu, Q. Tan, L. Liu, W. Li, and L. Lv, 'Comparison of grinding characteristics in high-pressure grinding roller (HPGR) and cone crusher (CC)', *Physicochemical Problems of Mineral Processing*, vol. 53, no. 2, pp. 1009–1022, 2017, doi: 10.5277/ppmp170226.
- [22] M. J. Metzger, D. Glasser, B. Hausberger, D. Hildebrandt, and B. J. Glasser, 'Use of the attainable region analysis to optimize particle breakage in a ball mill', *Chem Eng Sci*, vol. 64, no. 17, pp. 3766–3777, Sep. 2009, doi: 10.1016/J.CES.2009.05.012.
- [23] K. Kabezya and H. Motjotji, 'The Effect of Ball Size Diameter on Milling Performance', *Journal of Material Science & Engineering*, vol. 04, no. 01, 2014, doi: 10.4172/2169-0022.1000149.
- [24] N. S. Lameck, K. K. Kiangi, and M. H. Moys, 'Effects of grinding media shapes on load behaviour and mill power in a dry ball mill', *Miner Eng*, vol. 19, no. 13, pp. 1357–1361, Nov. 2006, doi: 10.1016/J.MINENG.2006.01.005.
- [25] L. H. Long, 'Benchmarking Management Practices and Manufacturing Performance of Manufacturing Companies in Penang', MBA, 2005.
- [26] T. Jamasb and M. Pollitt, 'Benchmarking and regulation: international electricity experience', *Util Policy*, vol. 9, no. 3, pp. 107–130, Sep. 2000, doi: 10.1016/S0957-1787(01)00010-8.
- [27] V. Shabunko, C. M. Lim, S. Brahim, and S. Mathew, 'Developing building benchmarking for Brunei Darussalam', *Energy Build*, vol. 85, pp. 79–85, Dec. 2014, doi: 10.1016/J.ENBUILD.2014.08.047.
- [28] G. Shuttleworth, 'Benchmarking of electricity networks: Practical problems with its use for regulation', *Util Policy*, vol. 13, no. 4, pp. 310–317, Dec. 2005, doi: 10.1016/J.JUP.2005.01.002.
- [29] N. Adler, V. Liebert, and E. Yazhensky, 'Benchmarking airports from a managerial perspective', *Omega (Westport)*, vol. 41, no. 2, pp. 442–458, Apr. 2013, doi: 10.1016/J.OMEGA.2012.02.004.

- [30] W. S. Lee, 'Benchmarking the energy performance for cooling purposes in buildings using a novel index-total performance of energy for cooling purposes', *Energy*, vol. 35, no. 1, pp. 50–54, Jan. 2010, doi: 10.1016/J.ENERGY.2009.08.026.
- [31] L. Simar and V. Zelenyuk, 'Stochastic FDH/DEA estimators for Frontier Analysis', *Kyiv School of Economics, Discussion Papers*, vol. 36, Jan. 2008, doi: 10.1007/s11123-010-0170-6.
- [32] C. Cilliers, 'Benchmarking electricity use of deep-level mines', Ph. D, North-West University, Potchefstroom, 2015.
- [33] J. Ke, L. Price, M. McNeil, N. Z. Khanna, and N. Zhou, 'Analysis and practices of energy benchmarking for industry from the perspective of systems engineering', *Energy*, vol. 54, pp. 32–44, Jun. 2013, doi: 10.1016/J.ENERGY.2013.03.018.
- [34] H. P. Siderius, S. Jeffcott, and K. Blok, 'International benchmarking: Supplying the information for product efficiency policy makers', *Energy Policy*, vol. 45, pp. 389–398, Jun. 2012, doi: 10.1016/J.ENPOL.2012.02.047.
- [35] C. L. Winchester and M. Salji, 'Writing a literature review', *J Clin Urol*, vol. 9, no. 5, pp. 308–312, Sep. 2016, doi: 10.1177/2051415816650133.
- [36] A. Pavlova and T. Sikorskaya, 'Benchmarking Intensity', *Rev Financ Stud*, vol. 36, no. 3, pp. 859–903, Mar. 2023, doi: 10.1093/rfs/hhac055.
- [37] D. Y. L. Chan, C. F. Huang, W. C. Lin, and G. B. Hong, 'Energy efficiency benchmarking of energy-intensive industries in Taiwan', *Energy Convers Manag*, vol. 77, pp. 216–220, Jan. 2014, doi: 10.1016/J.ENCONMAN.2013.09.027.
- [38] R. Inglesi-Lotz and J. N. Blignaut, 'Improving the electricity efficiency in South Africa through a benchmark-and-trade system', *Renewable and Sustainable Energy Reviews*, vol. 30, pp. 833–840, Feb. 2014, doi: 10.1016/J.RSER.2013.11.028.
- [39] V. Sardeshpande, U. N. Gaitonde, and R. Banerjee, 'Model based energy benchmarking for glass furnace', *Energy Convers Manag*, vol. 48, no. 10, pp. 2718–2738, Oct. 2007, doi: 10.1016/J.ENCONMAN.2007.04.013.

- [40] C. Caromba, C. Schutte, and J. van Laar, 'Application of Clustering Techniques for Improved Energy Benchmarking on Deep-Level Mines', *Energies (Basel)*, vol. 16, no. 19, 2023, doi: 10.3390/en16196879.
- [41] N. Wang, Z. Wen, M. Liu, and J. Guo, 'Constructing an energy efficiency benchmarking system for coal production', *Appl Energy*, vol. 169, pp. 301–308, May 2016, doi: 10.1016/J.APENERGY.2016.02.030.
- [42] G. Ballantyne and A. Giblett, 'Benchmarking Comminution Circuit Performance for Sustained Improvement', in *SAG CONFERENCE*, Vancouver, 2019, pp. 1–16.
- [43] G. R. Ballantyne and M. S. Powell, 'Benchmarking comminution energy consumption for the processing of copper and gold ores', *Miner Eng*, vol. 65, pp. 109–114, Oct. 2014, doi: 10.1016/j.mineng.2014.05.017.
- [44] S. Nadolski, R. Haycock, K. Li, S. Seiler, and A. Kumar, 'A Summary Of Energy Efficiency Opportunities For The Red Chris Comminution Circuits', *International Semi-Autogenous Grinding And High Pressure Grinding Roll Technology 2019At: Vancouver, Canada, September 2019*, 2019.
- [45] S. Morrell, M. Daniel, and J. Burke, 'Morrell method for determining comminution circuit specific energy and assessing energy utilization efficiency of existing circuits', *Global Mining Guidelines Group (GMG)*, Aug. 2016.
- [46] S. Nadolski, B. Klein, D. Gong, Z. Davaanyam, and A. Cooper, 'Development and application of an energy benchmarking model for mineral comminution', in *Proceedings International Conference on Autogenous and Semiautogenous Grinding Technology, Vancouver, Canada*, 2015.
- [47] S. Sukamolson, 'Fundamentals of quantitative research', *Language Institute Chulalongkorn University*, vol. 1, no. 3, pp. 1–20, 2007.
- [48] S. Crowe, K. Cresswell, A. Robertson, G. Hubby, A. Avery, and A. Sheikh, 'The case study approach', *BMC Med Res Methodol*, vol. 11, no. 1, p. 100, 2011, doi: 10.1186/1471-2288-11-100.
- [49] C. E. Alvarez, L. L. Motta, and L. C. P. da Silva, 'An Energy Performance Benchmarking of office buildings: A Data Mining Approach', in *2020 IEEE International Smart Cities Conference (ISC2)*, 2020, pp. 1–8.

- [50] R. E. Shiffler, 'Maximum Z Scores and Outliers', *Am Stat*, vol. 42, no. 1, pp. 79–80, Feb. 1988, doi: 10.1080/00031305.1988.10475530.
- [51] R. S. Witte and J. S. Witte, *Statistics*. John Wiley & Sons, 2017. Accessed: Nov. 13, 2023. [Online]. Available: <https://books.google.co.za/books?id=KcxjDwAAQBAJ>
- [52] P. Borah, M. K. Singh, and S. Mahapatra, 'Estimation of degree-days for different climatic zones of North-East India', *Sustain Cities Soc*, vol. 14, no. 1, pp. 70–81, Feb. 2015, doi: 10.1016/J.SCS.2014.08.001.
- [53] A. Davydenko and R. Fildes, 'Forecast error measures: critical review and practical recommendations', *Business forecasting: Practical problems and solutions*, vol. 34, 2016.
- [54] K. Bakirci, 'Evaluation of models for prediction of diffuse solar radiation and comparison with satellite values', *J Clean Prod*, vol. 374, p. 133892, 2022.
- [55] Y. Zhou, C. Lork, W.-T. Li, C. Yuen, and Y. M. Keow, 'Benchmarking air-conditioning energy performance of residential rooms based on regression and clustering techniques', *Appl Energy*, vol. 253, p. 113548, 2019, doi: <https://doi.org/10.1016/j.apenergy.2019.113548>.
- [56] J. Roth and R. Rajagopal, 'Benchmarking building energy efficiency using quantile regression', *Energy*, vol. 152, pp. 866–876, 2018, doi: <https://doi.org/10.1016/j.energy.2018.02.108>.
- [57] G. R. Ballantyne, A. Mainza, and M. S. Powell, 'Using comminution energy intensity curves to assess efficiency of gold processing circuits'. [Online]. Available: www.ceecthefuture.org
- [58] A. P. van der Westhuizen and M. S. Powell, 'Milling curves as a tool for characterising SAG mill performance', in *Proceedings of SAG*, 2006, pp. 217–232.
- [59] A. Man, C. Chaichana, S. Wicharuck, and D. Rinchumphu, 'Predicting sunlight availability for vertical shelves using simulation', *IOP Conf Ser Earth Environ Sci*, vol. 1094, p. 012011, Sep. 2022, doi: 10.1088/1755-1315/1094/1/012011.
- [60] D. A. Swanson, 'On the relationship among values of the same summary measure of error when used across multiple characteristics at the same point in time: an

examination of MALPE and MAPE', *Review of Economics and Finance*, vol. 5, no. 1, 2015.

- [61] J. der Bijl, 'Sustainable DSM on deep mine refrigeration systems: A novel approach', Ph. D, North-West University, Potchefstroom, 2007.
- [62] S. J. Cloete, D. F. Le Roux, and R. T. Bührmann, 'Reducing compressed air wastage by installing new technology in underground mines', in *2013 Proceedings of the 10th Industrial and Commercial Use of Energy Conference*, IEEE, 2013, pp. 1–6.

APPENDIX A

Appendix A contains all the figures and tables excluded from Chapter 3 due to the relevance of the information in the chapter itself. However, this information is critical for the complete regression analysis and the justification for excluding certain variables and methods.

Step 2: Verification of correlations

Figure 33 and Figure 34 below show the difference in the COLS and SFA linear equations for the ore crushed in the pre-milling circuits. The correction of the SFA model is clear due to the y-intercept passing through the origin. Furthermore, the most “efficient” point determines the corrected ordinary least squares equation.

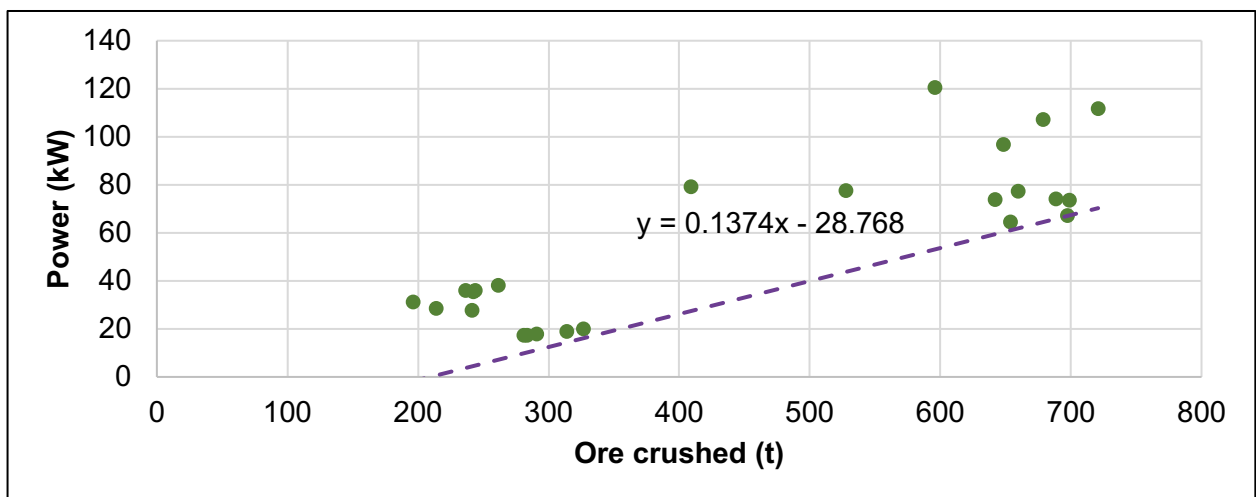


Figure 33: Pre-milling COLS function

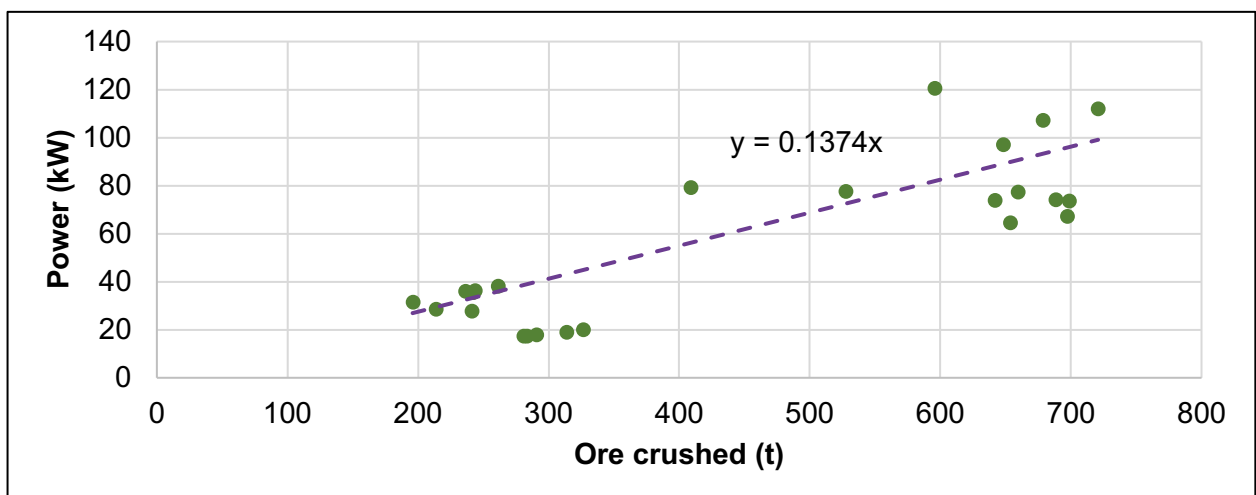
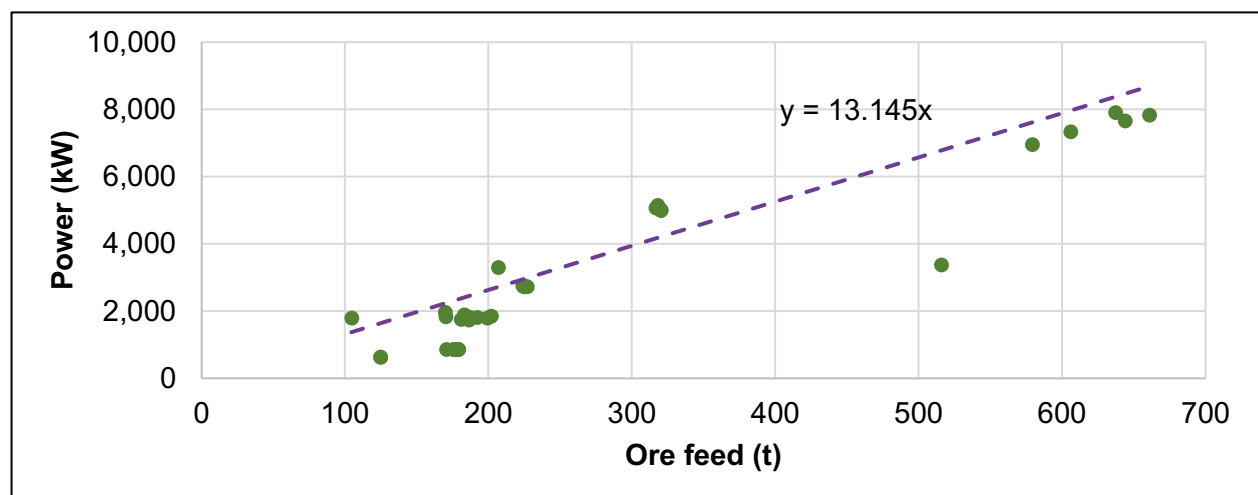


Figure 34: Pre-milling SFA function

A scatter plot showing the relationship between Ore feed (t) on the x-axis and Power (kW) on the y-axis. The x-axis ranges from 0 to 700 with major ticks every 100 units. The y-axis ranges from 0 to 9,000 with major ticks every 1,000 units. There are 18 data points plotted as green circles. A dashed purple line represents the linear regression fit, with the equation $y = 13.145x - 3417.8$ displayed on the plot.

Ore feed (t)	Power (kW)
110	1800
130	600
170	1900
170	800
180	1800
180	900
190	1700
200	1800
210	3300
230	2700
320	5000
520	3400
580	6900
610	7300
640	7600
650	7900
660	7800

The SFA function largely corrects this by altering the y-intercept point through the origin, producing a linear equation that closely passes most data points and is not skewed by a single point, as seen in Figure 36.



101

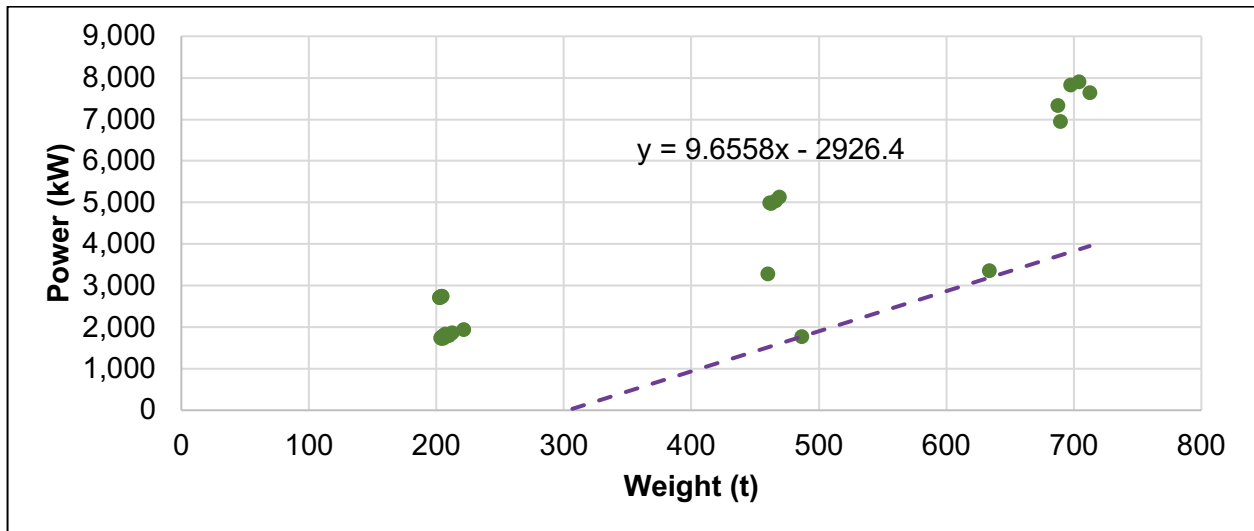


Figure 37: Primary mill weight COLS function

Unlike the COLS function, the SFA function passes through several data points, as the weight of the mills strongly correlates with the power consumption of the mills. Furthermore, it is not expected for the y-intercept to be zero as the mill will never weigh zero tonnes. Therefore, the SFA function has a positive intercept, as seen in Figure 38.

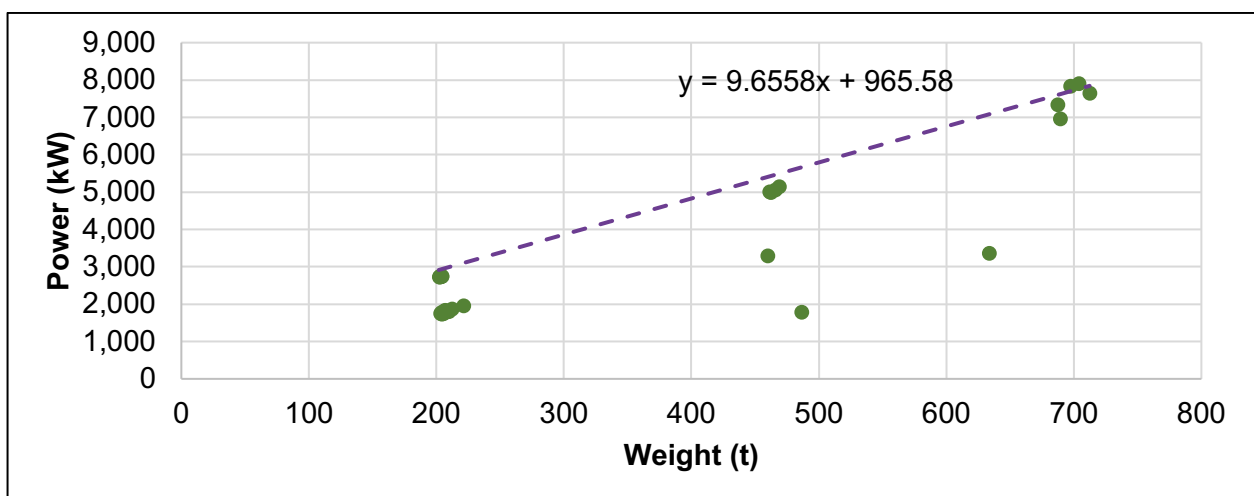


Figure 38: Primary mill weight SFA function

The feed water data points are scattered. However, two clusters can be seen as the most efficient practice. This is highlighted by the COLS function passing through both clusters, as seen in Figure 39. Despite that, the y-intercept is negative, which once again is corrected by the SFA function.

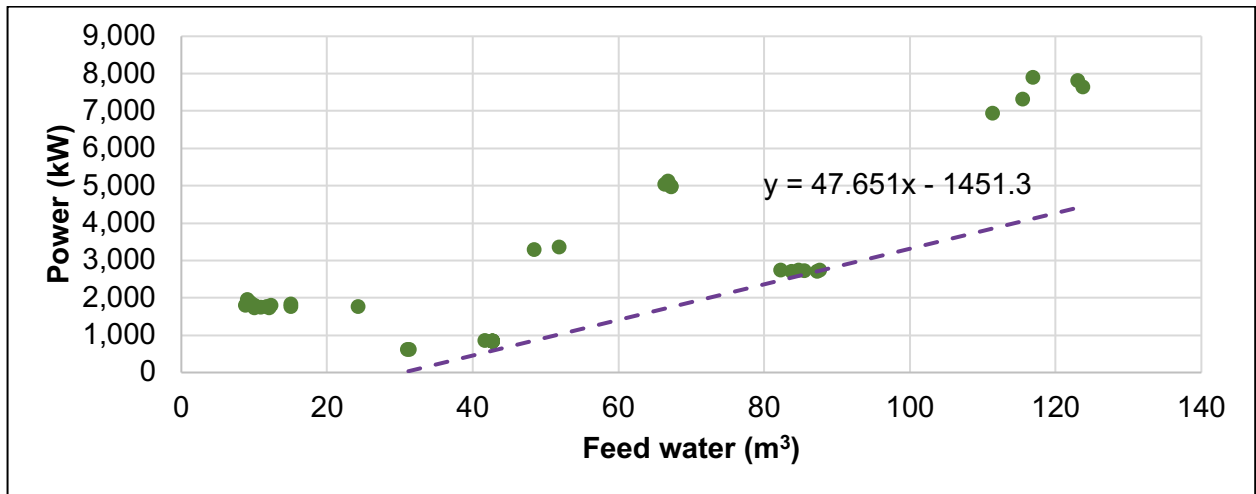


Figure 39: Primary mill feed water COLS function

As seen in Figure 40 below, the SFA passes through the centre of all the data points, producing a more representative performance function instead of the best practice performance as expected from the SFA function.

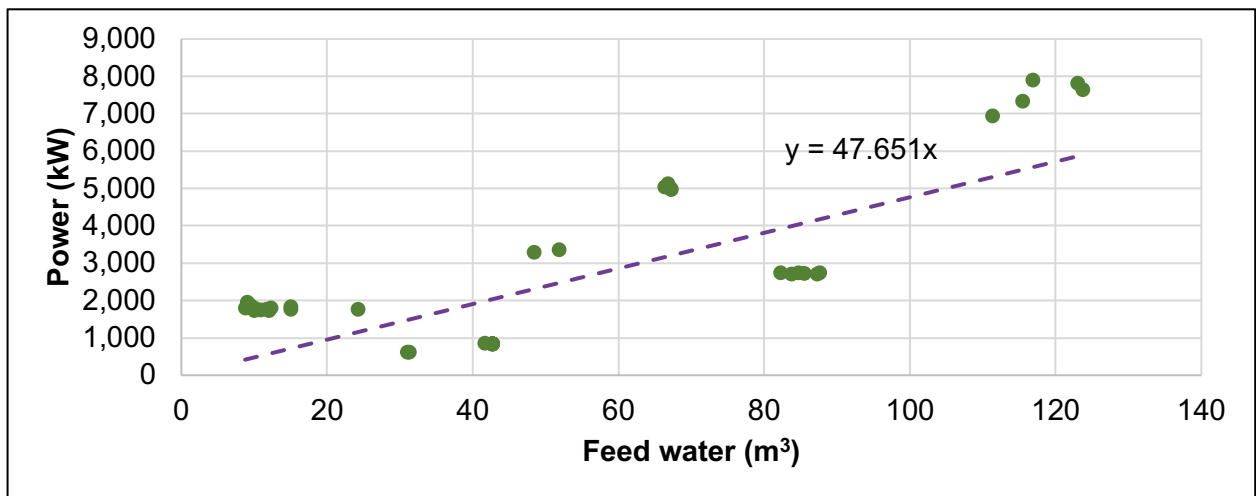


Figure 40: Primary mill feed water SFA function

The COLS function for the secondary mill feed is more accurate as the data points are closer than the other variables. However, the "efficient" data point used for the COLS function results in a negative intercept, as seen in Figure 41 below.

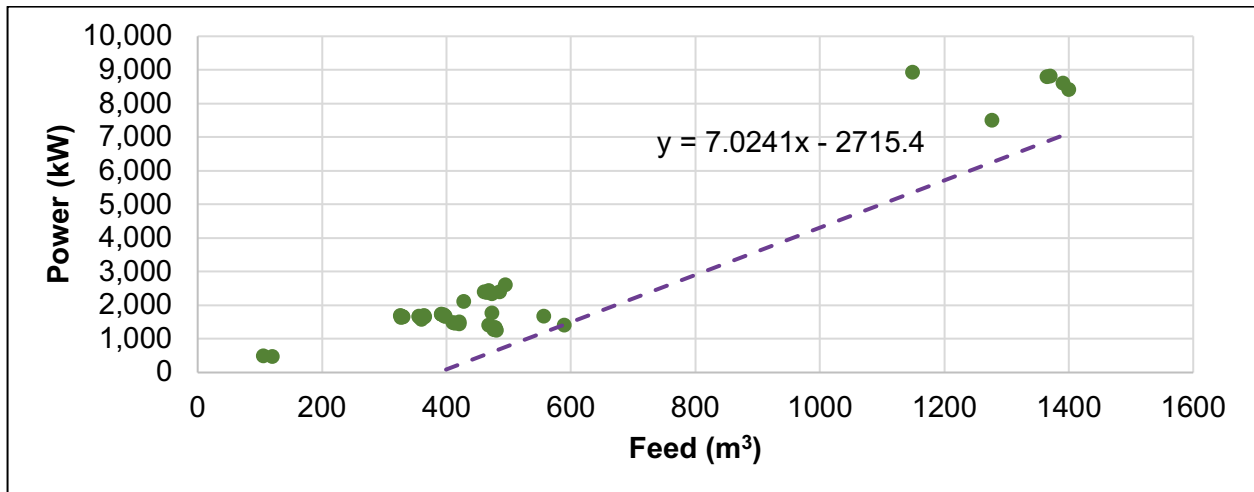


Figure 41: Secondary mill feed COLS function

The correction of the negative intercept by the SFA results in overestimating the average performance, producing a more representative function and not a frontier practice. Therefore, the function is inaccurate for the average and best practice benchmark, as shown in Figure 42.

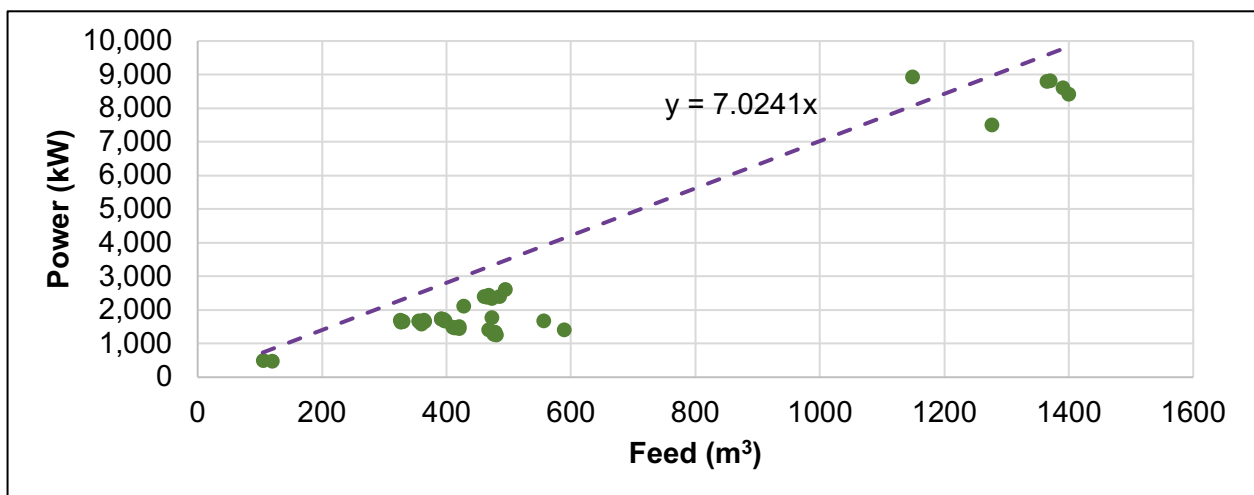


Figure 42: Secondary mill feed SFA function

As with the primary mills, the weight of the secondary mills is clustered together. Therefore, The COLS function is a more accurate best practice benchmark than the SFA function, which corrects the negative y-intercept, as seen below in Figure 43 and Figure 44, respectively.

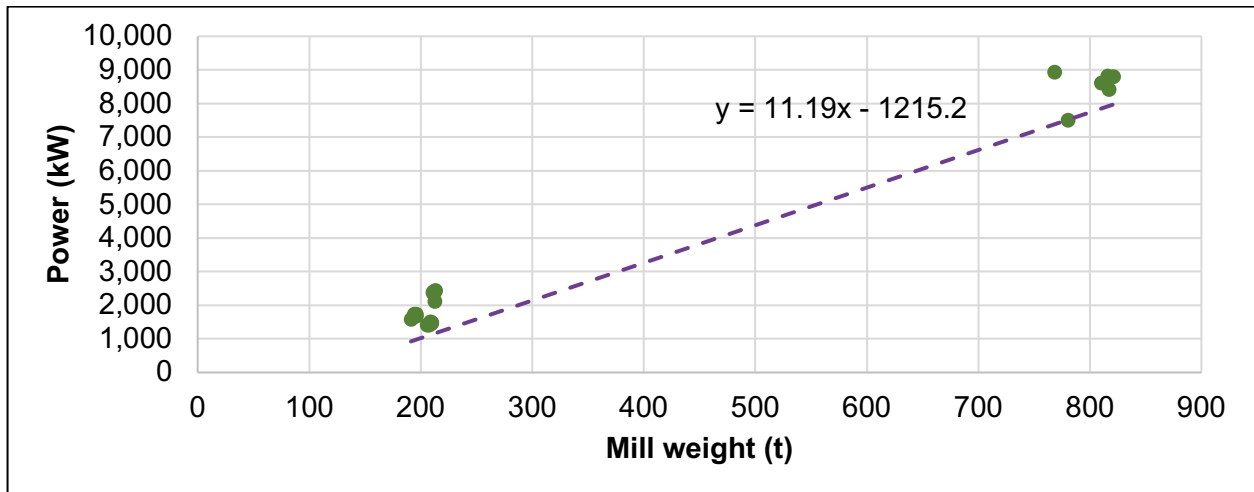


Figure 43: Secondary mill weight COLS function

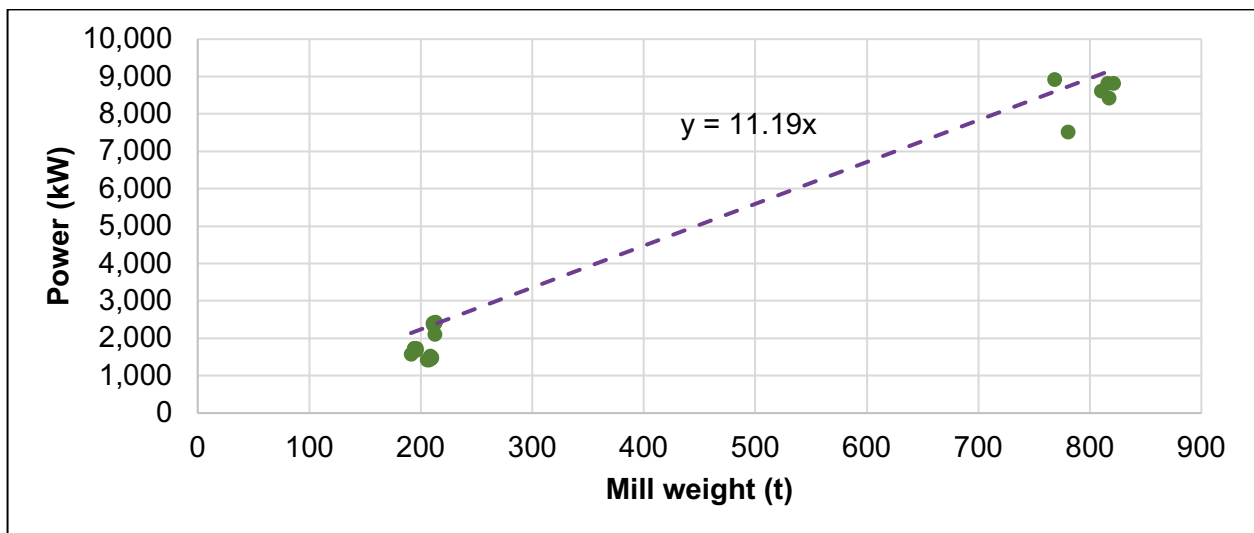


Figure 44: Secondary mill weight SFA function

Step 3: Initial and corrected scoring

Considering Table 53, the %initial and %corrected values are summarized for the average model of the pre-milling circuit. From the table, the best-performing circuit is circuit C, stream A, with scores well above 100.

Table 53: Pre-milling average model %initial and %corrected score

Circuit	%initial	%corrected
C Stream A	179	166
C Stream B	77	95
D	100	100
E	82	84
Average	109	111

This is confirmed by the scores of the SFA model, as shown in Table 54, which produces scores above 200, circuit C stream B underperforming compared to the other circuits.

Table 54: Pre-milling SFA model %initial and %corrected score

Circuit	%initial	%corrected
C Stream A	207	207
C Stream B	93	96
D	106	105
E	129	121
Average	134	133

The %initial and %corrected scores of the primary milling circuits, shown in Table 55 and Table 56, indicate that the circuits perform similarly. However, both streams of circuit C outperform the other circuits. This is contrary to the results of the SFA model, which suggests that circuits A mix and D are the best-performing circuits compared to the SFA. However, this is due to the SFA model's representative nature and not the intended best practice.

Table 55: Primary milling average model %initial and %corrected score

Circuit	%initial	%corrected
A UG2	84	94
A Mix	106	102
B	89	96
C Stream A	157	147
C Stream B	158	149
D	95	98
E	82	93
Average	110	111

Table 56: Primary milling SFA model %initial and %corrected score

Circuit	%initial	%corrected
A UG2	36	45
A Mix	74	90
B	41	50
C Stream A	51	62
C Stream B	52	64
D	82	93
E	62	76
Average	57	69

The same trend can be seen in Table 57 and Table 58 with the secondary milling circuits. However, circuit B is the best-performing average and SFA model circuit. The SFA model produces a slightly better frontier analysis than the primary circuits.

Table 57: Secondary milling average model %initial and %corrected score

Circuit	%initial	%corrected
A UG2	98	100
A Mix	85	96
B	134	109
C Stream A	99	100
C Stream B	70	92
D	98	99
E	142	118
Average	104	102

Table 58: Secondary milling SFA model %initial and %corrected score

Circuit	%initial	%corrected
A UG2	79	94
A Mix	72	92
B	112	103
C Stream A	24	34
C Stream B	9	13
D	94	98
E	35	49
Average	61	69

Table 59 to Table 60 summarise the calculated %initial scores used to determine the %corrected scores for the three months being benchmarked of each section of the circuit. The pre-milling circuit produces the expected results for the SFA model with only two scores above 100.

Table 59: Pre-milling SFA model %initial

Circuit	Month 1	Month 2	Month 3	Average
C Stream A	130	118	125	124
C Stream B	99	98	100	99
D	82	93	119	98
E	115	123	126	121

Considering Table 60 below, both streams of Circuit C score well above 200. That is due to the circuits performing well below the average, and the SFA is above the average, increasing the difference between the SFA and actual performance.

Table 60: Primary milling SFA model %initial

Circuit	Month 1	Month 2	Month 3	Average
A UG2	106	112	110	109
A Mix	123	121	123	122
B	120	117	116	117
C Stream A	252	193	196	213
C Stream B	267	220	201	229
D	102	102	111	105
E	96	96	95	95

Contrary to this, the secondary milling circuit model produces scores well under 100, indicating that even with the more representative model of the SFA, the circuits “underperform”, as seen in Table 61. However, as discussed, this is not the case.

Table 61: Secondary milling SFA model %initial

Circuit	Month 1	Month 2	Month 3	Average
A UG2	80	77	77	78
A Mix	69	73	71	71
B	102	105	100	102
C Stream A	75	77	83	78
C Stream B	73	73	77	74
D	86	94	94	91
E	64	64	75	68