STRATEGIES USED BY GRADE 6 LEARNERS WHEN SOLVING MATHEMATICS STORY PROBLEMS

Percy Sepeng
percy.sepeng@nwu.ac.za
North West University

Nothile Kunene
nothile.kunene@yahoo.com
North West University

Abstract

The study reported in this article sought to investigate factors that affect learners' academic achievement in Grade 6 mathematics word problems. Furthermore, the article discusses errors made by the learners when they solve word problems. The study used document analysis in a form of learners' written work (or test) in order to collect quantitative data. The results obtained from a test consisting of six word problem tasks showed that learners struggled with realistic considerations of problem statement as well as with making meaning of situations embedded in the task. In brief, reading instructions aloud repeatedly and explaining key mathematical concepts have emerged as key strategies in understanding and solving word problems in mathematics. The study therefore argues that it does not matter how complex or easy a word problem appears to be; what is important is the ability of the brain to connect the mathematics embedded in a problem statement with real-life situations in order to make meaning of the world.

Keywords: Story problems; problem-solving; academic achievement; realistic considerations; meaning making.

Introduction

Mathematics is essential in almost every field of learning. Story problems (traditionally referred to as 'word problems') have been viewed by people in the field as a way to be used globally by curriculum policy designers to connect informal out-of-school mathematics knowledge with formal written primary and high school mathematics. However, such a call for bridging the gap between informal and formal knowledge of mathematics has an effect on teacher development across the South African primary schooling system. Recently the

Department of Basic Education called on primary school teachers to put more emphasis on teaching learners how to solve and make sense of contextual story problems. In a report presented by the Department of Basic Education (2011b) on Grade 6 learners' ability to solve story problems, it was reported that learners experienced solving story problems as the most difficult skill to master.

One of the annual benchmark tests administered to primary school learners in South Africa is the Annual National Assessment in English and mathematics. The Annual National Assessment is a systemic evaluation conducted every year in Grades 3, 6 and 9.The Progression Requirement, as per the Department of Education; Foundation and Intermediate Phases Continuous Assessment and Policy Statement [CAPS] Orientation (2012), stipulates that in a primary school, with specific reference to the Foundation and Intermediate Phase, a learner must attain a minimum mark of level three at 40% in mathematics in order for him or her to progress to the next class. This implies that a failure to secure level three in mathematics means that the learner will not be able to progress to the next level. According to the analysis of the Annual National Assessment results for 2011 and 2012, the performance of the learners where the study was conducted indicated that the school obtained a pass percentage rate of 0.45% (English) and 0.46% (mathematics).

Jan and Rodrigues (2012) argue that further research should be undertaken to determine which factors influence the ability of students to comprehend word problems, in order to find the best strategies for solving word problems meaningfully. The Department of Basic Education was concerned about poor learner performance in mathematics, more especially regarding solving word problems (KwaZulu-Natal Department of Education: Mathematics Booklet 3: LO1 & 4, n.d.). Learners' work revealed that the poor performance appeared to be due to the difficulties they experienced when answering word problems. In discussions with fellow teachers who taught mathematics in the intermediate phase, issues with regard to word problem solving seemed to be a main cause of poor performance in Grade 6 mathematics.

According to Macmillan and Schumacher (2010, p.3), educational research is very important in teaching and learning processes as 'educators are constantly trying to understand educational processes and must make professional decisions'. Thus, the study discussed in this article was aimed at improving educational practices towards creating favourable teaching and learning platforms in teaching and acquiring the mathematical skills needed for solving story problems. The study also sought to recommend effective pedagogical strategies that could be used by teachers to reduce and remedy the number of mathematical errors made by Grade 6 learners in solving story problems. The potential significance of the research was to provide the policy designers with empirical evidence obtained from a broader understanding of the factors that affect the academic achievement of Grade 6 mathematics learners in solving story problems.

Research problem

According to the Department of Basic Education report (2011b), learners attained a 9% score nationally in solving word problems. One of the authors argued elsewhere (Sepeng, 2013, 2014) that a key skill that learners need in solving story problems is to identify the problem and find functions that link several entities. It is therefore against this background that the study reported in this article explored the factors that affect learners' academic achievements in Grade 6 mathematics story problems within the context of a rural school. In exploring what really constitutes a story problem, factors that affect mathematics achievement within the context of story problems and primary schooling are discussed in this article. Issues that are associated with motivation, language and academic expectations as well as those linked to solving story problems in general are outlined. The theories used to frame the study reported here are discussed in brief. In addition, the authors present the methodological tools employed in the study, reports the findings, provides a discussion of results, and then comes to a conclusion. Finally, recommendations are made for mathematics practitioners at various levels of basic education in South Africa.

Word problem solving: a conceptual framework

Most learners in South African primary mathematics classrooms struggle with the notion of what constitutes a word problem in the learning and teaching of tasks linked to this topic. Word problems form part of the South African mathematics curriculum and are used as a vehicle to teach learners how to model problems in primary mathematics classrooms (Sepeng, 2014; Sepeng & Webb, 2012), and in the process strengthen their ability to be good problem solvers (Sepeng, 2013, 2014). A review of what it means to solve mathematical word problems within South African classroom contexts formed part of recent reports from studies that explored learners' abilities in solving word problems (see for example Sepeng, 2013, 2014). Such studies were done in relation to other studies elsewhere in the world (e.g. Nesher, 1980; Jamison, 2000; Verschaffel, De Corte & Lasure, 1994; Verschaffel, Van Dooren, Chen, & Stessen, 2009). These researchers agreed that solving word problems in mathematics is part of a unit of the text comprising a question and a language text that is accompanied by an authentic background story and the syntactical and rhetorical structure needs to be explicitly clear to enhance understanding. Other scholars such as Pape and Wang (2003) and Verschaffel, De Corte, Lowyck, Dhert and Vandeput (2000) have alluded to the idea that problem solving begins when the *solver* reads the problem text for the solution process that leads to success as the schemas for problems are activated. In a report presented by the United States National Department of Education (2004, as cited in Bogdan & Biklen, 2007, p. 22), problem solving is defined as:

> an individual's capacity to use cognitive processes to confront and resolve real, cross-disciplinary situations where the solution is not immediately obvious, and where the literacy domains or curricular areas that might be applicable are not within a single domain of mathematics, science, or reading.

Kenneth (1991) defines word problem solving as a process by which the learners experience the power and the usefulness of mathematics in the world around us. According to Kenneth, word problems are sometimes called *story problems* or *verbal problems*. Problem solving was identified by the National Council of Teachers of Mathematics in the 1980s as a priority and a cornerstone for mathematics instruction, and it was argued that this type of problem solving needed to be a focus of mathematics teaching and learning processes (Xin, 2007). Kenneth (1991, p. 5) points out that 'word problems provide a context through which learners practice the algorithms and apply the formulas which they are learning' so that they acquire both *problem-solving* skills and knowledge on how to *make meaning to* (or *make sense of*) word problem solving.

Ilany and Margolin (2010) define a word problem in mathematics as an independent unit that comprises a speech event and a question sentence which is divided into two types according to the topics they relate to. In addition, they argued for two types of word problems: one deals with mathematical relationships between objective sizes and the other with real-life situations. Other reports defined word problems as calculation tasks that are embedded in a text and whose words and structure create a problem (see for example Conrad & Serlin, 2006; Reikeras, 2009). As such, solving word problems seems to present an opportunity for the learners to experience the power and the usefulness of mathematics in the world around us.

Jimenez and Garcia (2002) argue that word problems are a classification of a function of semantic structure that include changing, combining, comparing and equalising. According to these scholars, a problem classified as *change* is characterised by an increase or decrease in the initial quantity, for example: '*Antonio had 18 stickers. His friend Paco gave him six more stickers. How many stickers does Antonio have altogether?*' A typical *combine* problem consists of a static relationship among a particular set and its two disjoint subsets, for example: '*There are 12 sheep in a van, four are black and the rest are white. How many white sheep are there?*' A word problem categorised as *compare* involves a static relationship with a comparison of two distinct, disjoint sets, such as: '*Oscar's bicycle has 14 gears and Anita's bicycle has nine gears. How many fewer gears does Anita's bicycle have than Oscar's?*' Lastly, an *equalise* word problem is a comparison of two disjoints, for example, *'My dress has 12 buttons. If my sister's dress has five buttons more, it will have the same number of buttons as my dress. How many buttons does my sister's dress have?'* Several factors associated with word problem solving are discussed in the next sections.

Language issues in word problem solving

The issue of language use in the teaching and learning of primary mathematics has formed part of current educational debates among commentators in South Africa. These debates are triggered by poor mathematics academic performances of primary school learners. However, it is widely reported that learners' inability to succeed in solving word problems seems to be due to a lack of linguistic knowledge (Jan & Rodrigues, 2012, p.152). In South African primary schools, word problems in arithmetic require the integration of linguistic and arithmetic processing skills (see Sepeng, 2013, 2014). The literature suggests that the majority of teachers are monolingual, and as a result they struggle to respond adequately to the increased linguistic diversity that is found in the classrooms (Botes & Mji, 2010). In addition, these scholars argue that learners who are taught in a non-mother tongue language do not achieve academic excellence due to a 'created artificial linguistic problem' (p. 123). Teachers who are monolingual struggle in the increased linguistic diversity that is found in their classrooms when they have to respond effectively and adequately to learners' difficulties in solving word problems (Botes & Mji, 2010). Research has shown that teaching strategies and/or techniques such as code-switching, translation, and re-voice have a potential draw on and promote the use of the home language of learners to enhance better understanding in mathematics classrooms (Sepeng, 2014).

Mathematics teachers face the challenge of having a dual task: finding themselves teaching both mathematics and English at the same time (Botes & Mji, 2010). Furthermore, Botes and Mji have concluded that the learners encountered challenges such as learning to speak, read and write like mathematicians. According to Ilany and Margolin (2010), the learners appeared to be 'faced with two languages, viz. natural language and mathematical language mixed together' when solving word problems (p.138). In doing so, and learning in a language that is not spoken at home, learners encounter the challenge of using English to make sense of problem statements. According to Reikeras (2009), the language in the word problems is referred to as consistent when the order in which information is given matches the order that problem solvers prefer. Furthermore, Reikeras argues that it becomes inconsistent when there is a mismatch and an inverse relationship placed in the correspondence text.

Importance of mathematical language use and achievement

Other factors arising from the literature are the teachers themselves who use mathematical language carelessly and in a confusing way, such as using the word 'sum' to refer to a calculation other than addition (Haylock & Thangata, 2007). According to Haylock and Thangata, the misuse and degree of inaccuracy in the use of words to refer to mathematical concepts can be a barrier to learners' understanding of these concepts. Both Sepeng and Webb (2012) and Sarmini (2009) emphasise the importance of language as a factor that affects learners in the development of the understanding of word problems and supporting problem solving in mathematics. Moreover, learners are likely to make numerous errors when they encounter inconsistent language problems (Xin, 2007). In the same vein Krick-Morales (2006) argues that '[m]any English Language Learners (ELLs) may have difficulty reading and understanding the written content in a word problem. If a learner is learning English as a second language, he might not yet know key terminology needed to solve the equation' (p. 3).

Classroom practices and errors made by learners when solving word problems

Learners often make errors when trying to solve word problems in mathematics classroom practices. According to Sepeng and Webb (2012), learners need to develop and know the skill of solving word problems so that they will know when and how to apply classroom mathematical knowledge as well as everyday life knowledge. This is confirmed by Russell (2013) who argues that learning how to solve problems in mathematics is to know what to look for. Therefore, a learner should know the procedure, how to apply it, collect the appropriate information, and identify and use the strategy. He emphasises that solving problems in mathematics requires practice and more practice.

Steele (2002) found that learners – more especially the learners who have visual processing difficulties – made errors in their class practices when computing word problems. The following errors and practices occurred most often: they confused numbers, such as 17 and 71; copied inaccurately; worked problems in the wrong direction; reversed negative and positive numbers on a graph; lined up work incorrectly for place value; used the number line in reverse and often lost their places when working. Learners who had difficulty in understanding complex ideas involving graphing inequalities switched the labels on the x- and y-axes. Those learners who had motor processing difficulties had a hard time writing numerals again which led to errors in mathematics assignments and tests. They also made errors in finding common denominators and common multiples. They also found it difficult to memorise the multiplication tables.

In an analysis of errors made by learners in their mathematics classrooms in all four operations (addition, subtraction, multiplication and division) Mundia (2012, pp.353-355) found the following, among other errors: failure to regroup when adding and subtracting; zero difficulties, such as '*0-6 =6';* difficulties with the relationship between units; tens the distinction between and hundreds was not clear; confusing multiplication with addition; confusing division with either addition or multiplication; using or mixing any two of the four process (+, -, x, ÷). Miller (2013) argues that learners tend to think linearly, step-by-step, and try making the numbers and the text to match in the same order. Example: *'Jane had 25 pens and she gave away 15. How many does she have now? Answer: 25-15.'* Sepeng (2013) argues that teachers do not seem to offer the learners enough exercises to enhance the components of problem solving such as decoding and interpreting.

Theoretical Framework

The study discussed in this article is framed by the constructivist philosophy which underpins the view that learners are expected to discuss mathematics with their peers and their teachers in mathematics classrooms (Brenner, 1998). This theory views learning as an effective social interacting process that is undergone by learners and teachers. It takes the learner to collaborate with the adult at the 'zone of proximal development' to accomplish a task. In their teaching and learning interaction, new knowledge is constructed to rethink ideas, to argue, evaluate, share, and examine. In the process the learners come to a better understanding of the conceptual underpinnings of mathematics as they become better problem solvers (Hart, 1999; Brenner, 1998). Learners are expected to gain a greater understanding of the conceptual underpinnings of mathematics and become problem solvers through active discussion with their teacher. The constructive perspective proposes that learners have to be active in constructing their own knowledge and enhance their abilities to interact and engage socially (Woolfolk, 2007).

According to Vygotsky's (1978) social learning and cultural theory, the 'zone of proximal development' is a term used to describe 'the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers' (p.86). Piaget (1978) concurs that the learning of mathematics is a constructive process and much of current research on mathematics problem solving is consistent with these two. The constructivist perspective seems to suggest that any curriculum aimed at promoting mathematical thinking must, by the very nature of the phenomenon, be problem-based. In addition, mathematics knowledge is constructed and relationships are created as hallmarks of solving word problems (Thompson, 1985).

Research Methodology

This section presents the research design used in the study reported in this article. Thereafter, the data generating instrument and sampling techniques are discussed. The section ends by describing issues of validity and the data analysis techniques outlined in this article are presented. The study reported in this article sought to answer the following research questions:

- What are the factors that affect learners' poor academic achievement in Grade 6 mathematics word problems?
- What are the errors made by the learners when they solve word problems?

Research design

A research design is the general plan in terms of how the research is set up, what happens to the subjects, and which methods of data collection are used (Landsberg, Kruger, & Swart, 2011). Babbie, Mouton, Vorster and Prozesky (2001, p. 279) define a research design as 'the road map or blueprint according to how one intends to conduct a research and achieve their research goals and objectives'. A document analysis in a form of learners' written work was done in order to gather quantitative information. A convenient sample of 30 Grade 6 learners was given a test consisting of the six word problem tasks (WPT) to solve. The test measured learners' abilities to solve word problems, marked by the researcher, and was then immediately analysed using the theories adopted for the study.

Sampling

Sampling is a process of identifying relevant participants, subjects or people who are rich informants from which data can be collected. The researcher used purposive sampling as one type of non-probability sampling in order to gain access to the individuals and groups being studied. The rationale of using the purposeful sampling method was to obtain detailed information from the participants in order to maximise the range of specific information that can be obtained (Macmillan & Schumacher, 2010).

In the study discussed in this article, a convenient sample of 30 Grade 6 learners (10% of the learners) participated in the study. These learners were purposefully identified using the convenience sampling from the first and second terms' results as they were readily available. The teachers' assessment record books were used to identify the learners. These learners were at the intermediate exit grade and they underwent the ANA formal examination or assessment. All 30 learners wrote a prepared task on solving word problems. All these learners used English, a foreign language, as a language of teaching and learning and they were all isiZulu home language speakers.

Data collection instrument

A test was used as the data collection strategy in the study reported here used. The test, consisting of six WPTs, was administered to 30 participating Grade 6 learners. The WPTs were selected from a range of previous ANA question papers and all the selected tasks were in line with the standards of the Grade 6 curriculum with localised contexts. Sepeng and Sigola (2013) point out that the new South African Curriculum Statement (CAPS) places emphasis on teaching of problem solving as a key strategy that allows for using everyday contexts to teach and learn mathematics with understanding. The selected WPTs formed part of the topics already covered in the current academic year as per their work schedule, which meant that the WPTs were familiar questions to the participating learners. The WPTs covered aspects relating to addition, subtraction, multiplication, subtraction with units, discounts, discounted amounts and subtraction, division, multiplication, and subtraction. The aspects covered the four basic operations that learners in this grade should be able to perform. Learners were advised to work individually so that the individual performance of each learner could be measured.

The results of the WPTs assisted the researcher in understanding the learners' ways of solving problems and in measuring their problem-solving abilities. The learners' responses in the form of 30 answer booklets were marked and the learners' errors were analysed in order to identify patterns of strategies used. In addition, the test measured the learners' ability to solve word problems.

Validity

Validity within the quantitative approach means the degree of trustworthiness of findings in the conclusions drawn (Macmillan & Schumacher, 2010). In order to ensure a high degree of trustworthiness of the findings in the study reported here, the following data collection strategies were employed;

- *Population external validity*: the subjects that were used had variables such as age, sex and ability;
- *Negative or discrepant data*: the researcher actively searched for, recorded, analysed, and reported negative cases or discrepant data that were an exception to patterns or that modified the patterns found in data; and
- The task to be written by learners which is part of the instrumentation was taken from previous papers that are aligned with the standards of the Annual National Assessments (ANA). This is a Grade 6 curriculum with localised

contexts. The questions were administered to learners of the same grade over time.

Data analysis

The quantitative data were initially examined and organised according to categories, using a schema that was an elaboration of the classification schema developed by Verschaffel et al. (1994). The results from the test administered were presented in percentages to show the attainment level. The test was coded to realistic reaction (RR), no reaction (NR), and other reaction (OR) as adopted from the work of Sepeng and Webb (2012). It was also presented in a bar graph and interpreted against the pass mark in mathematics. The marks were first converted to percentages to find out how many learners had attained marks less than 40% and 40% and above. A 40% mark qualifies a learner to progress to the next grade. Quantitatively, descriptive statistics were used to present the data: the frequency distribution of scores (Macmillan & Schumacher, 2010).

Ethical considerations

Bogdan and Biklen (2007) argue that official guidelines of ethics in research are to ensure that informants enter research projects voluntarily, and understand the nature of the study and the obligations that are involved. Informants should not be exposed to risks that are greater than the gains they might derive.

Permission to carry out the research at the selected school was requested from the Department of Education and the participating primary school. The researcher obtained consent from the parents to work with their children in the study.

The researcher employed the following strategies to support the ethical approaches to fieldwork:

- The researcher let the participants know that participation was voluntary and withdrawal without reprisal was accepted.
- The researcher honoured the participants' privacy.
- Unless otherwise agreed to, the informants' identities were protected so that the information collected did not embarrass or in any other way harm them.

- The informants were treated with respect and their cooperation in the research was sought.
- In negotiating permission to do the study, it was made clear to those with whom negotiations were undertaken what the terms of the agreement were, and the researcher abided by that contract.
- The researcher told the truth when writing up and reporting the findings (Bogdan & Biklen, 2007, pp. 49-50; Saritas & Akdemir, 2009).

Results

As noted earlier, a formal written test consisting of six WPTs was administered to the learners. The WPTs assisted the researcher in understanding learners' ways of solving problems as well as their problem-solving abilities. The six WPTs formed part of the topics already covered in the current academic year as per their work schedule. In other words, all the WPTs were familiar to the participating learners. Learners were allowed to work individually so that the individual performance of each one could be measured. The learners' responses gathered from the 30 scripts were marked using a marking guideline that was validated by all Grade 6 mathematics teachers in the participating school as well as those teaching in neighbouring schools. The errors made by the learners were analysed within a framework of theories and literature used in this study, with the primary aim of identifying patterns of approaches or strategies employed during the problem-solving process. A further aim was to determine how WPTs were solved. The data gathered from the test provided quantitative information on learners' academic performances.

Descriptive analysis of the word problem tasks

The WPT covered aspects relating to addition, subtraction, multiplication, subtraction with units, discounts, discounted amounts and subtraction, division, multiplication and subtraction.

As discussed earlier, the six WPTs from the written test were coded using a schema that was an elaboration of the classification scheme developed by Verschaffel et al. (1994). The classification scheme comprised 14 categories, which were reduced to the three general categories discussed below.

• *Realistic reaction* (*RR*): This category covered all cases where a learner either gave the (most) correct numerical solution that also took into account the real-world aspects of the problem context, as well as cases where there was a clear indication that the learner tried to take into account those real-world aspects, without giving the mathematical and situational (most) accurate numerical answer.

• *No reaction* (*NR*): In this category all the cases were included where there was no indication that the learner was aware of the realistic modelling difficulty; for example, incorrect or inappropriate responses and computational errors. This category also provided a measure of the problem-solving performance of the learner.

• *Other reaction* (*OR*): In this category all cases were included where a learner did not provide a numerical response and did not give any written comment that indicated that the learner was aware of the realistic modelling difficulty that prevented him or her from answering the problem (Sepeng, 2010).

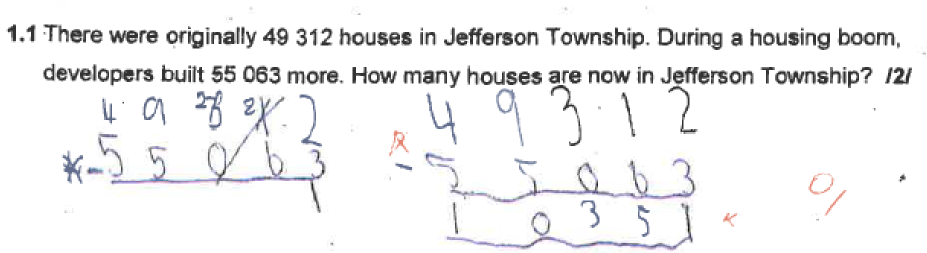
The researcher used the data depicted in Table 1 to discuss the learners' responses to the word problem solving on each task. Descriptive analyses were done using the frequencies (*f*) and percentages.

Table 1: Summary of the test analyses

	RR		**NR**		**OR**	
	f	%	*f*	%	*f*	%
WPT 1	27	90	3	10	0	0
WPT 2	5	17	25	83	0	0
WPT 3	4	13	26	87	0	0
WPT 4	18	60	12	40	0	0
WPT 5	10	33	20	67	0	0
WPT 6	7	23	23	77	0	0
Mean		39		61		0

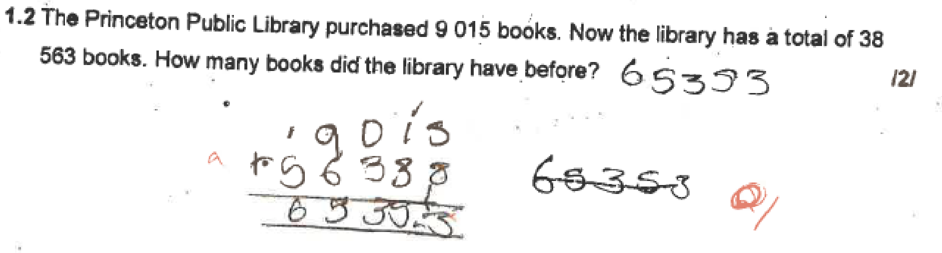
WPT 1: There were originally 49 312 houses in Jefferson Township. During a housing boom, developers built 55 063 more. How many houses are now in Jefferson Township?

In WPT 1, learners were expected to add the number of houses in Jefferson township; 49 312 before the boom and 55 063 after the boom, in showing a real-life situation in a word problem statement. Question 1 of the task was based on addition. Out of the 30 learners, only 90% were grouped under the category realistic reaction (RR). They gave the most correct numerical solutions. Ten per cent of the learners fell into the no reaction (NR) category, since they gave incorrect or inappropriate responses, or computation errors. The word ‘more’ in mathematical terms usually means that the total number increases, so the addition (+) sign is used and as such the addition computation is applied. An analysis from the table shows that 90% (27) of the responses were mathematically correct, while only 10% (3) were incorrect. Analysis of the responses shows that the 90% learners were able to identify and relate the word ‘more’ to addition in order to compute the task and obtain a correct or partially correct answer. Only 10% of the learners could not take the real-life world into consideration. They could not carry over from the units to the tens, could not add properly or even used the wrong operation, a subtraction (-) instead of an addition operation. The analysis shows that all learners attempted to respond to the task, whether correctly or incorrectly. The learners could not re-group the tens, carry the one ten from the units and add it to the tens to make it ten tens instead of the nine tens registered. This study concurs with Mundia’s (2012) finding that the error analysis was made by learners who failed to regroup when adding and subtracting and had difficulty with the relationship between units, tens, and hundreds. The extract below is an example of such a case for WPT 1, due to the inability to use the incorrect operation.


1.1 There were originally 49 312 houses in Jefferson Township. During a housing boom, developers built 55 063 more. How many houses are now in Jefferson Township? **/2/**

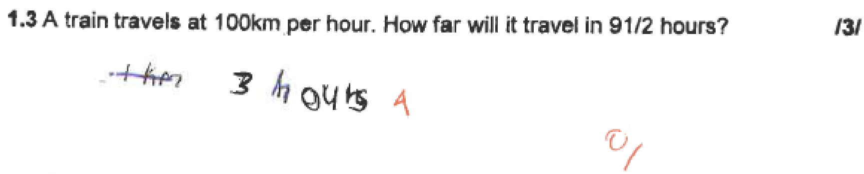
WPT 2: The Princeton Public Library purchased 9 015 books. Now the library has a total of 38 563 books. How many books did the library have before?

WPT 2 required the learners to get the difference of the number of books at the Princeton Public Library by subtracting the number of books that were purchased from the current number of books. As in WPT 1, in WPT 2, the learners were supposed to take real-life situations into account and to understand the phrase ‘how many … have before’. In order to make sense of the problem statement, learners had to use the mathematical language merged with the language of learning and teaching. Table 1 indicates that 17% (5) of the responses were mathematically correct or partially correct and 83% (25) were inaccurate. From the analysis of the learners’ responses to this task, it is evident that most of the learners used addition instead of subtraction, most put the digits in the wrong place value and a few did not put in any operation. The most obvious error made in this category was the wrong placements of digits to be subtracted. Digits were placed in the wrong place value. This was coupled with the wrong operation used. The extract below shows an incorrect response that is the result of a combination of failure to align digits in their correct place value, numbers written wrongly and the use of the incorrect operation (+) instead of a subtraction (-), which is failure to take real life into consideration:


1.2 The Princeton Public Library purchased 9 015 books. Now the library has a total of 38 563 books. How many books did the library have before? **/2/**

WPT 3: A train travels at 100 km per hour. How far will it travel in $9\frac{1}{2}$ hours?

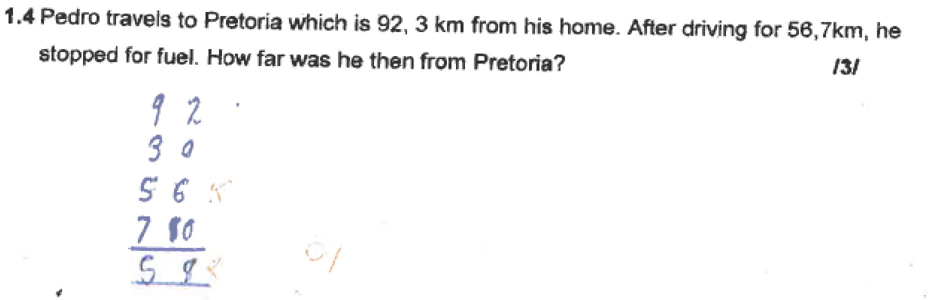
As shown in Table 1, 13% (4) of the respondents gave accurate answers and 87% (26) gave inaccurate answers. Learners had to multiply the speed by the number of hours in order to get the distance. They were also expected to give the units of distance which was kilometres (km). Learners worked out the answers without understanding and thus gave a logically incorrect answer. It seemed that the learners were not sure of the computation technique they needed to use. The language barrier was prominent in this task. It also seems that learners used whatever operation came to their minds first or they did guesswork. Most of the learners used a plus (+) sign instead of the multiplication (x) sign in order to get a product. In some responses no operations were given. In other responses they gave either wrong units or no units at all. The following is an example of a response with no computation, but only an answer with wrong units:


1.3 A train travels at 100km per hour. How far will it travel in 91/2 hours? **/3/**

WPT 4: Pedro travels to Pretoria which is 92.3 km from his home. After driving for 56.7km, he stopped for fuel. How far was he then from Pretoria?

The results provided in Table 1 show that 60% (18) of the learners gave correct or partially correct answers and 40% (12) gave wrong answers. WPT 4 required the learners to subtract the two figures and give units of km as part of the answer.
This task appeared similar to WPT 2 as it used the same operation. Learners had to understand the mathematical language and the language of instruction in order for their schema to use the correct operation.

What the researcher noted from the learners' responses was that most of them again added instead of subtracting. Some of the learners used altogether wrong figures to compute their tasks. The following response is evidence of the latter:


1.4 Pedro travels to Pretoria which is 92, 3 km from his home. After driving for 56,7km, he stopped for fuel. How far was he then from Pretoria? /3/

9 2
3 0
5 6
7 10
5 9

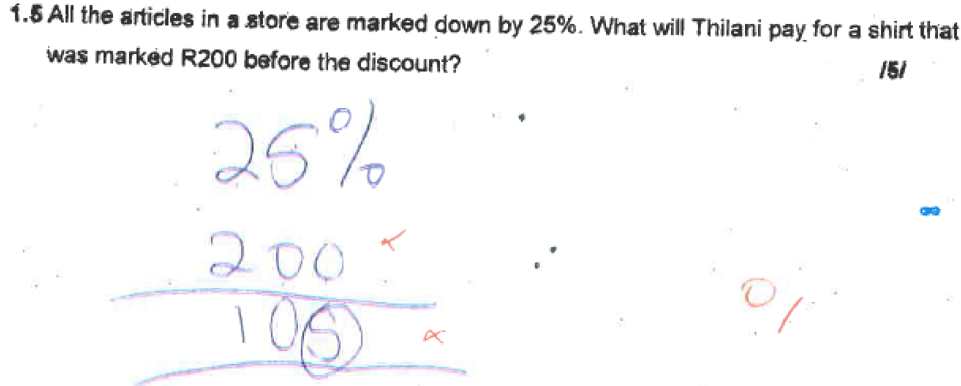
WPT 5: All the articles in a store are marked down by 25%. What will Thilani pay for a shirt that was marked R200 before the discount?

Data from the results in this question indicate that about two-thirds of the learners obtained computationally wrong responses and situationally inappropriate solutions.

Out of the 33% (10) learners who took into account the real-life aspects, about 80% got incomplete and partially correct answers. Some of the learners had no discount amount computed and others had no marked down solutions.

This task was a two-step task. It needed the learners to understand and identify the operations embedded in the text. Learners first had to find a discount, by getting 25% of R200, thereafter subtract the answer obtained from the initial amount of R200 in order to get the discounted figure. Learners failed to show awareness of the modelling aspect of the task.

Such a task shows the measure of the problem-solving performance of the learners, when they eventually give inappropriate responses. The extract below shows a learner's response as a result of lack of inability to model a word problem:


1.5 All the articles in a store are marked down by 25%. What will Thilani pay for a shirt that was marked R200 before the discount? /5/

25%

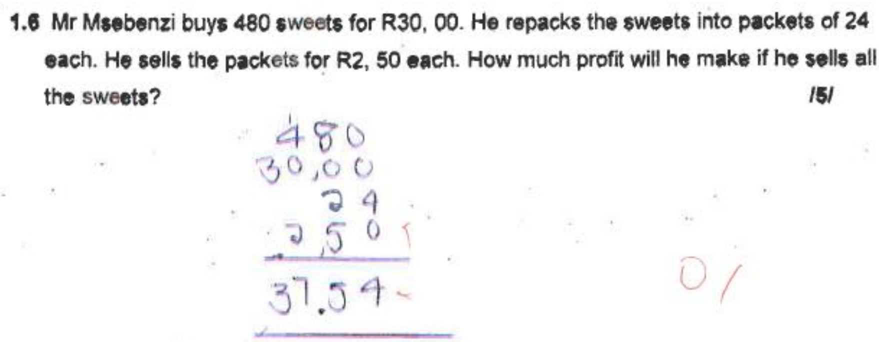
200

105

0/

WPT 6: Mr Msebenzi buys 480 sweets for R30.00. He repacks the sweets into packets of 24 each. He sells the packets for R2.50 each. How much profit will he make if he sells all the sweets?

WPT 6 seemed to be one of the most challenging tasks in the grade. It needed the learners to show – as in WPT 5 – an indication of the realistic modelling aspect. It also needed the learners to use their schema to engage the correct operation in this task. It is a multi-step task that required the learners to divide the sweets by the number of packets, take the quotient and multiply it by R2.50 to get the total amount of the sales. Finally they had to subtract the two amounts in order to get the profit made. In this task, 23% (7) learners arrived at partially correct solutions. Not a single learner got the whole question correct. Altogether 77% could not understand the task, as shown by the incorrect solutions. The following is an extract showing the response of a learner who showed no indication of the realistic modelling of the word problem:


1.6 Mr Msebenzi buys 480 sweets for R30, 00. He repacks the sweets into packets of 24 each. He sells the packets for R2, 50 each. How much profit will he make if he sells all the sweets? /5/

480

30,00

24

2,50

37,54

0/

Discussion

An analysis of the data illustrated that learners experienced difficulties in solving the word problems. It seemed that they encountered difficulties in understanding and interpreting the tasks. Some of the difficulties might have been caused by the nature of calculation tasks that are embedded in text and whose constituent words and structure create problems during word problem solving **(**Reikeras, 2009). The learners could also not make sense of the mathematical word problems that dealt with mathematical relationships between objective sizes and those that dealt with real-life situations (Ilany & Margolin, 2010; Sepeng, 2014). The inability of learners to interpret WPTs successfully appeared to be caused by a failure to apply the correct operation needed for the task.

Factors that affect learners' mathematics academic achievement

The other findings that emerged in the study reported here were issues regarding factors that affect the learners' academic achievement in mathematics. These factors could have been caused by issues such as the linguistic diversity in the classroom and the home language of individual learners, school level factors, learners' conceptions and perceptions, factors at a teacher level as well as the inability of learners to apply problem-solving skills as well as those associated with sense-making (Sepeng, 2013; Loder-Symonds, 2012; Hodge, Riccomini, Buford, & Herbst, 2006; Steele, 2002; Botes & Mji, 2010; Saritas & Akdemir, 2009; Mundia, 2012). The results reported in this article seem to suggest that learners have difficulty in solving word problems in arithmetic that require the integration of linguistic and arithmetic processing skills. Such difficulty might have been due to a lack of linguistic knowledge. Similar to findings reported by Botes and Mji (2010), the participating learners were taught in a language that is not spoken by many at home; consequently they appeared to achieve poorly.

Most of the learners' written work suggested that they struggled with word problems and frequently 'shut down' mentally (Steele, 2002) The findings showed that the learners found the combination of reading, writing, reasoning, and mathematical skills required in word problems extremely complex (Steele, 2002). In so doing, learners struggled to show any reasoning skill when applying mathematical operations when they attempted to solve WPTs. In view of this problem, it is crucial that learners are involved in specific activities to assist them in constructing mathematic knowledge and that relationships should be created as these are hallmarks of solving word problems (Thompson, 1985). In addition, teachers should create and foster a learning environment in which their learners can construct the mathematical and linguistic knowledge necessary to solve story problems (Josephine, 1999).

Factors that are associated with word problem solving

The findings of this study show that the learners were faced with the difficulty of two languages mixed together: their first language and mathematical language. The learners could not make sense of the words and phrases used and in the process they made numerous errors when they encountered language problems (Sarmini, 2009). The comprehension of the text in word problems is necessary as that is not only the means to convey information but it is also used to interpret the event and phenomenon in a way that provokes the thinking of the learners (Jan & Rodrigues, 2012). The findings from this study show that because of a language barrier the learners could not comprehend the text and could therefore not understand what they were required to do.

The data reported in this article demonstrated, as Aksu (2001) claimed, that learners' performance declines significantly in all four operations when the operations are presented in the form of word (or story) problems. The greatest decline was in the multiplication operation. In this study it was found that 13% of the respondents got multiplication task (WPT 3) solutions correct, which was the lowest in all six tasks. WPT 5, with 33% and WPT 6 with 23% correct responses, also had multiplication computation that the learners needed to do. Learners had difficulty in identifying and interpreting the terms and words in the word problem. This finding is similar to Sepeng's (2014) findings from a study conducted among Grade 9 learners. In WPT 2, learners could not identify that the phrase 'how many … have before' required a subtraction operation. Learners demonstrated a lack of representations based on their real-world knowledge, in particular as a factor that influences learners' performance in solving word problems (Voyer, 2010).

Conclusion and recommendations

The purpose of the research presented in this article was to understand how Grade 6 learners solve mathematical word problems. In the research word problem-solving tasks (WPTs) were used to make meaning of the nature of errors made by learners when they solve word problems. Factors that are associated with learners' academic achievement were also explored in relation to learners' abilities to make sense of word problems. The relevant literature referred to in this article confirms that learners find it very challenging to understand and effectively solve word problems in mathematics because they are taught in English, their second language or in some cases even a foreign language (see Sepeng & Webb, 2012; Botes & Mji, 2010; Krick-Morales, 2006). Furthermore, other reports suggested that when the solution of mathematical problems is accompanied by text and symbols, the learners are faced with two languages mixed together, namely their 'natural' language and mathematical language (Kenneth, 1992; Ilany & Margolin, 2010).

The research findings outlined in this article seem to suggest that the mathematics academic achievements of participating learners in Grade 6 classrooms in solving word problems are affected by a number of variables. The key variables identified in this article are the importance of the correct mathematical language use, text comprehension and terminology, understanding operations embedded in the text, understanding mathematical symbols and concepts, clarity of the vocabulary, mathematical knowledge, and the structure of the word problem. Furthermore, factors such as reading, understanding, sense-making and problem solving emerged as primary predictors of academic achievement.

In word problems two languages are mixed together: the 'natural' language and the mathematical language that need to be dissected as problems are solved. For this reason, in order for policy makers to generate achievable strategies in solving word problems, and in the process improve learners' academic achievement in Grade 6 classrooms, it is important to conduct further research on how learners solve word problems. It is therefore recommended that teacher development activities should focus on assisting teachers on how to teach mathematical word problems using learners' everyday situations. In addition, mathematics teachers in the primary schools should assist learners on how to apply arithmetic operations to real-life situations to get rid of the negative influence of the stereotypical belief held by many people that mathematics is a difficult subject.

The research reported in this article contributes more knowledge (in South Africa) to existing literature on issues of how Grade 6 learners solve mathematical word problems by exploring factors that affect learners' academic performance in word problem solving. However, the findings reported in this article may not be applicable to other contexts that are different from those associated with the research site and participants. It does not matter how complex or easy a word problem appears to be; what is important is the ability of the brain to connect the mathematics embedded in a problem statement with real-life situations in order to make meaning of the world.

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