

# Base Bleed projectile simulation for long range large caliber artillery modeling

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Dissertation submitted in partial fulfilment of the requirements  
for the degree *Master of Science in Mechanical Engineering*  
at the North-West University

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Graduation ceremony: May 2019

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## **ACKNOWLEDGEMENTS**

This work acknowledges the contribution of my employers that provided the research resources and assistance to complete this work. Also, I should mention my high regards to my mentor Mr Louis du Plessis that assisted me a lot and guided me through this path.

## ABSTRACT

**KEYWORDS:** Trajectory model, Artillery, Base Bleed, Base Burn, Injection rate, projectile, firing table, large calibre gun, ballistic coefficient, ballistics, Drag Coefficient, Axisymmetric body Aerodynamic.

Base Bleed projectile (BB) are artillery projectiles that use base exhaust ejections to reduce the drag that the projectile encounter during its flight. This function is beneficial in extending the range of the artillery projectiles. 20% to 30% drag reduction can be achieved using base bleed unit by relieving between 0.3 - 0.85 of the total base drag force. This can extend the artillery range 30%.

To estimate the performance the North Atlantic Treaty Organisation (NATO) established STANAG a Standard Agreement (STANAG) to deal with the organisation standards. For the base bleed simulation problem (STANAG 4355) recommend two methods to be used to determine the base bleed projectile trajectory. These two methods take into consideration several fitting parameters to establish a model to calculate the base bleed projectile trajectory. These parameters need to be adjusted for every firing condition, which make the model expensive to model base bleed behaviour in real life conditions as it requires scaling parameters to be determined experimentally. This is a proposal for a research to develop a new model that can predict base bleed projectiles trajectory with fair accuracy and can predict these trajectories in various firing conditions without the need to obtain specific scaling factors in every specific case. The new modified model will be validated using published work and test data that can be obtain form available resources.

(STANAG 4355) has recommended two methods to be used in Base Bleed projectile trajectory calculation. The two methods recommend in STANAG 4355 require considered amounts of fitting parameters. These parameters must be determined from test results and this have a big cost implication. The STANAG models requires fitting parameters for grain temperature and launch elevation to adjust predicted base bleed effect.

The objective of this thesis is to establish an alternative trajectory model for the base bleed projectile, based on principle physics and experimental base bleed performance. The new model suggested by this study is validated using test data.

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## NOMENCLATURE

|                       |                                                                                 |
|-----------------------|---------------------------------------------------------------------------------|
| $\alpha$              | Speed of sound in air                                                           |
| $A_e$                 | Exit of jet                                                                     |
| $BB$                  | Base Bleed                                                                      |
| $\overrightarrow{BB}$ | Acceleration due to drag reduction of Base Bleed motor                          |
| $C$                   | Ballistic coefficient                                                           |
| $C_T$                 | Effective nozzle thrust coefficient                                             |
| $C_{D_0}$             | Zero yaw drag coefficient                                                       |
| $C_{D_{\alpha^2}}$    | Quadratic drag force coefficient                                                |
| $C_{D_{0T}}$          | Zero yaw drag coefficient (thrust on)                                           |
| $C_{d\ orif}$         | Friction of orifice vent edge surface                                           |
| $C_{L_0}$             | Zero yaw Lift coefficient                                                       |
| $C_{L_{\alpha^3}}$    | Cubic lift force coefficient                                                    |
| $C_{M_{\alpha}}^0$    | Overtuning coincident moment for initial projectile configuration               |
| $C_{mag-f}$           | Magnus force coefficient                                                        |
| $C_{x_{BB}}$          | Drag reduction coefficient during base-burn motor burning                       |
| $C_{d0.BB}$           | Zero yaw drag coefficient with functioning base bleed motor                     |
| $\overrightarrow{D}$  | Acceleration due to drag reduction of Base Bleed motor                          |
| $d$                   | Reference diameter of projectile                                                |
| $d_b$                 | Diameter of projectile base                                                     |
| $d_e$                 | Diameter of injector exit                                                       |
| $e$                   | Base of natural logarithms                                                      |
| $f_{\text{initial}}$  | Increase in propellant burn rate due to initial condition                       |
| $f_D$                 | Drag calibration factor                                                         |
| $f_L$                 | Lift calibration factor                                                         |
| $f_T$                 | Thrust calibration factor                                                       |
| $f(i_{BB}, MT)$       | Base-burn calibration factor                                                    |
| $f(I)$                | Function of injection calibration parameter                                     |
| $f(MT)$               | Calibration factor for combustion rate as function of fuel temperature          |
| $\vec{g}$             | Acceleration due to gravity vector                                              |
| $g(P)$                | Combustion rate as function of atmospheric pressure                             |
| $\vec{H}$             | Total angular momentum vector                                                   |
| $i_{BB}$              | Fitting factor to adjust the drag reduction as a function of quadrant elevation |
| $I$                   | Base-burn motor fuel injection parameter                                        |
| $I_0$                 | Base-burn motor fuel injection parameter for optimum efficiency                 |
| $I_{sp}$              | Specific impulse                                                                |

|                |                                                                  |
|----------------|------------------------------------------------------------------|
| $I_x$          | Specific axial moment of inertia of the projectile               |
| $I_{x_0}$      | Initial axial moment of inertia of the projectile                |
| $I_{x_B}$      | Axial moment of inertia of the projectile at burnout             |
| $K_{SR}$       | Correction factor for base injection pressure relieve            |
| $K_{d_b,Inj}$  | Correction for $(C_T/C_{D_0T})$                                  |
| $K_{bT}$       | Correction for nozzle thrust coefficient to                      |
| $K(p)$         | Axial spin burning rate factor                                   |
| $n$            | Number of moles                                                  |
| $m$            | Projectile mass                                                  |
| $m_0$          | Fused projectile initial mass                                    |
| $m_C$          | Fused projectile burnout mass                                    |
| $m_{CB}$       | Fused projectile fuel burnt mass                                 |
| $m_{CB0}$      | Fused projectile fuel burnt in the barrel mass                   |
| $m_{DI}$       | Fused projectile ignition delay element mass                     |
| $m_f$          | Projectile fuel mass                                             |
| $\dot{m}_f$    | Mass flow rate of the motor fuel                                 |
| $MT$           | Temperature of motor fuel                                        |
| $M$            | Mach number                                                      |
| $M_\infty$     | Free stream Mach number                                          |
| $M_{Inj}$      | Base injection Mach number                                       |
| $p$            | Spin rate                                                        |
| $P$            | Air pressure                                                     |
| $P_\infty$     | Air free stream pressure                                         |
| $P_C$          | Base bleed unit chamber pressure                                 |
| $P_b$          | Base pressure                                                    |
| $P_r$          | Reference air pressure for standard thrust                       |
| $\dot{Q}$      | Volumetric flow rate                                             |
| $Q_D$          | Yaw drag factor                                                  |
| $R$            | Gas universal constant                                           |
| $S_c$          | Area of combustion at time t                                     |
| $T$            | Time of flight                                                   |
| $T_c$          | Chamber temperature                                              |
| $T_{Igniter}$  | Increase in chamber temperature due to the igniter effect        |
| $t^*$          | Pseudo machine time for mapping thrust at non-standard condition |
| $t_s$          | Time of rocket burnout                                           |
| $t_{timestep}$ | Specific integration time step                                   |
| $t_{BST}$      | Standard time of rocket motor burnout                            |
| $t_{DI}$       | Time of rocket motor ignition delay                              |

|                              |                                                                                             |
|------------------------------|---------------------------------------------------------------------------------------------|
| $t_{DIST}$                   | Standard time of rocket motor ignition delay                                                |
| $T_F$                        | Thrust factor                                                                               |
| $T_R$                        | Thrust produced by rocket motor at time t                                                   |
| $T_{ST}$                     | Standard thrust as function of burning time                                                 |
| $T^0$                        | Effective thrust                                                                            |
| $T_{Ch}$                     | Base burn unit chamber temperature                                                          |
| $\dot{\vec{u}}$              | Rate of change in projectile vector                                                         |
| $V_c$                        | Total combustion rate of base burn fuel                                                     |
| $V_{C_0}$                    | Progressive combustion rate of base burn fuel on standard burner                            |
| $X_{CG}$                     | Distance of centre of mass from projectile nose at time t                                   |
| $X_{CG_0}$                   | Initial distance of centre of mass from projectile nose                                     |
| $X_{CG_B}$                   | Distance of centre of mass from projectile nose at burnout                                  |
| $Y$                          | Gas expansion factor                                                                        |
| $\vec{x}$                    | Projectile directional unit vector                                                          |
| $\alpha$                     | Total angle of attack                                                                       |
| $\alpha_e$                   | Yaw of repose approximation angle                                                           |
| $\Lambda$                    | Acceleration due to Coriolis effect                                                         |
| $\beta$                      | Orifice diameter over base diameter                                                         |
| $\beta T$                    | Base-burn motor temperature fuel burning coefficient                                        |
| $\gamma_{air}$               | Heat capacity ratio for air at standard conditions                                          |
| $\gamma_{inj}$               | Heat capacity ratio for injected gases                                                      |
| $n$                          | Exponent of burning versus pressure formula                                                 |
| $\rho$                       | Air density                                                                                 |
| $\rho_p$                     | Density of base-burn fuel                                                                   |
| $\rho_{inj}$                 | Density of base-burn injected gases                                                         |
| $K$                          | Constant of burning versus pressure formula                                                 |
| $v$                          | Velocity of air relative to projectile                                                      |
| $\vec{v}$                    | Velocity of air relative to projectile                                                      |
| $\frac{\delta BP}{\delta I}$ | Change in non-dimensional base pressure for a change in base-burn motor injection parameter |
| <i>NATO</i>                  | North Atlantic Treaty Organization                                                          |
| <i>STANAG</i>                | NATO standard agreements                                                                    |
| <i>USYD</i>                  | The University of Sydney                                                                    |

# CHAPTER 1 INTRODUCTION

## 1.1 Background

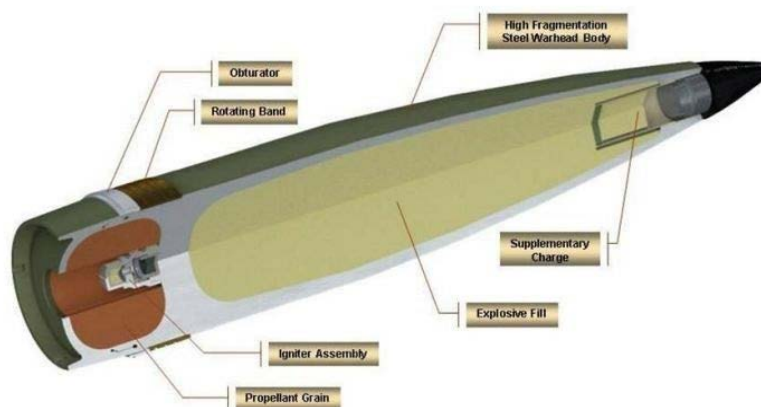
Base Bleed (BB) projectiles are projectiles fitted with a base bleed motor mechanism connected to the base of the projectile. The BB mechanism injects gas into the low-pressure wake region trailing the projectile, thereby relieving some of the base drag. Drag reduction is achieved by relieving the drag caused by low pressure in the base region.

The BB attachment has improved the range performance of heavy artillery projectiles significantly. The base bleed artillery projectile trajectory model requires many fitting parameters to achieve the accuracy required for range-tables.

The lack of precision emanates from the difficulty of prediction base bleed performance under different conditions. The BB motor can face high variation in atmospheric conditions during its trajectory that can affect the flow out of the BB unit. Propellant temperature and projectile spin rate can affect the BB unit injection rate. That variation in the flow out of the BB unit can develop into choked flow which complicate flow prediction out of BB unit.

The available published model for the base bleed trajectory model is using scaling factors to predict the base bleed behaviour under different firing conditions such as elevation, base bleed projectile temperature and the injection rate.

The NATO standard agreement STANAG 4355 established two methods to predict both rocket assisted and base bleed projectile trajectories. The two methods assume a component added to the modified point mass trajectory model to predict the acceleration that the projectile will experience. This model can be improved by a model that presents the physical functioning of the base bleed unit more accurately. The goal of this research is to improve the existing model by using published works and experimental data to obtain the trends of the scaling factors that are used in published models. Also, to exploit the possibility of manipulating these scaling factors and utilising new ways to model these factors.



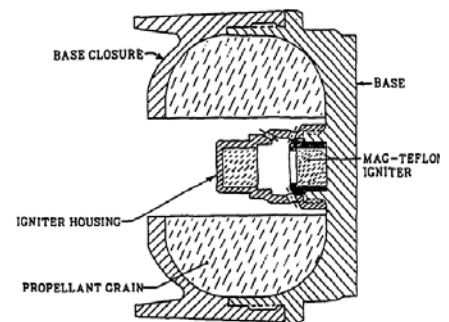
**Figure 1-1:** 155mm projectile fitted with BB unit

### 1.1.1 Base bleed motor function

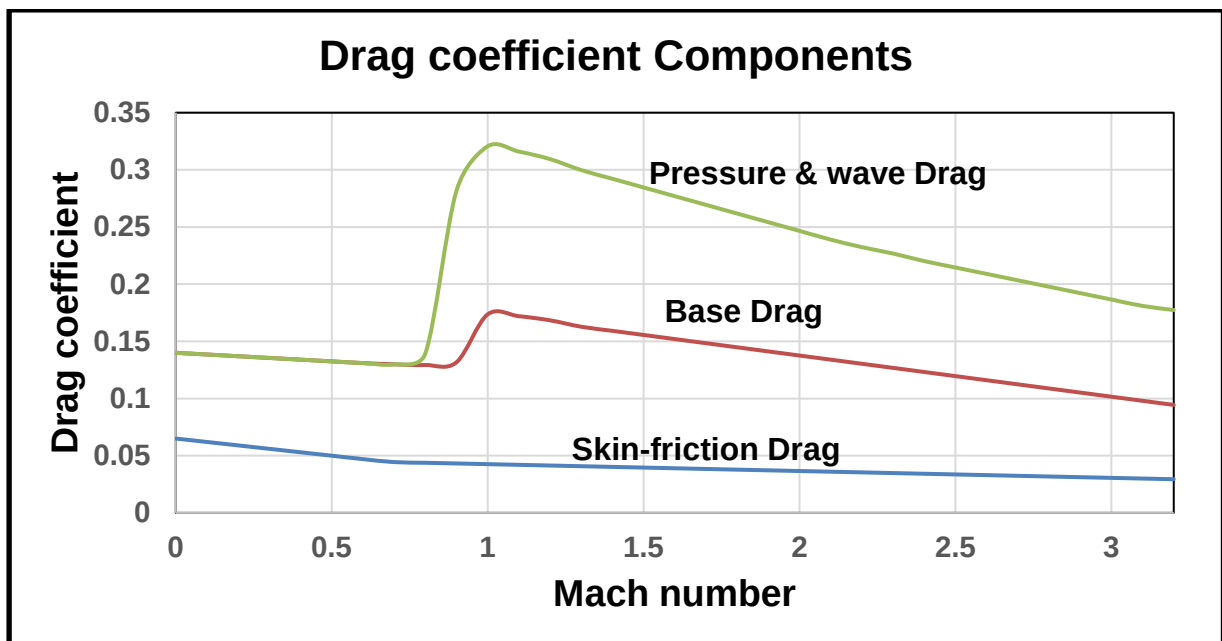
Base bleed projectiles were developed in the 1980s to improve artillery projectile range. The idea behind the method of improving artillery ammunition range, is to reduce the air drag that the projectile is subjected to. The drag reduction is achieved by exhaust gasses ejecting into the base region. The ejection relieves base pressure drop, eventually reducing the base vacuum effect drag. The injection rate must have a precise rate to obtain the optimum performance of the base bleed system.

The base bleed unit consists of an enclosure, grain of chemical propellant, and igniter. The base bleed motor functions as follows:

After the firing of the projectile the process ignites the igniter composition. It prevents extinction of the grain during the pressure drop at muzzle exit. The igniter composition initiates the propellant that ejects gases at specific rates to reduce base drag at the base of the projectile figure 1-3.



**Figure 1-2:** Base bleed unit configuration layout



**Figure 1-3:** Base, pressure, wave, and skin friction drags of total drag

### 1.1.2 BB published model

The existing base bleed simulation model is insufficient because it requires many scaling factors to simulate every specific firing case. For every base bleed projectile, some parameter needs to be scaled. The injection factor, the base bleed propellant temperature and injection rate as function of elevation need to be scaled for that specific case.

Specifically, the STANAG 4355 simulation model that is used to simulate base bleed behaviour suggest two methods to model the trajectory, adding and modifying acceleration terms in the modified point mass model. The component of base bleed is either added as force component to the modified point mass model or as a component of drag.

$$\vec{F} = m\vec{u} = \vec{DF} + \vec{LF} + \vec{MF} + \vec{PDF} + \vec{TF} + m\vec{g} + m\vec{\Lambda} \quad (\text{eq. 1})$$

Where  $\vec{F}$  the force vector acting on the point mass (centre of gravity) of the projectile, it is represented as summation of acceleration vectors of drag force ( $\vec{DF}$ ), Lift force ( $\vec{LF}$ ), Magnus force ( $\vec{MF}$ ), Pitch damping force ( $\vec{PDF}$ ), thrust force ( $\vec{TF}$ ), acceleration due to gravity force ( $m\vec{g}$ ), and Coriolis Effect force ( $m\vec{\Lambda}$ ). Each component is defined by the relative scaling criteria which is govern by the projectile shape, the aerodynamic characteristic of the projectile and the position of the projectile in the reference frame (which is earth reference frame).

The drag component is defined as follows:

$$\frac{\vec{DF}}{m} = - \left( \frac{\pi \rho i d^2}{8m} \right) \left( C_{D_0} + C_{D_{\alpha^2}}(\alpha)^2 \right) v \vec{v} \quad (\text{eq.2})$$

Where ( $i$ ) is a fitting factor for the shape of the projectile and can be taken as function of elevation for specific projectile. Moreover, lift force acceleration is given as follows:

$$\frac{\vec{LF}}{m} = \left( \frac{\pi \rho d^2 f_L}{8m} \right) \left( C_{L_0} + C_{L_{\alpha^2}}(\alpha)^2 \right) (v^2 \vec{x} - (\vec{v} \cdot \vec{x}) \vec{v}) \quad (\text{eq.3})$$

Magnus force acceleration also is represented as follows:

$$\frac{\vec{MF}}{m} = - \left( \frac{\pi \rho d^3 C_{mag-f}}{I_x 8m} \right) (\vec{H} \cdot \vec{x}) (\vec{x} \times \vec{v}) \quad (\text{eq.4})$$

Acceleration due to pitch damping force is:

$$\frac{\vec{PDF}}{m} = \left( \frac{\pi \rho d^3 (C_{N_q} + C_{N_{\dot{\alpha}}})}{I_Y 8m} \right) v (\vec{H} \cdot \vec{x}) \quad (\text{eq.5})$$

Acceleration due to thrust force is:

$$\frac{\vec{T}\vec{F}}{m} = \frac{(f_L \dot{m}_f I_{sp} + (P_r - P) A_e) \vec{x}}{m} \quad (\text{eq.6})$$

The force of gravity can be taken as constant values or using several models as specified in (STANAG Section I-C). Moreover, Coriolis Effect force is an independent force that has a small force component that effect projectile trajectory and can be calculated in several ways, or as specified in (STANAG Section I-C). Coriolis Effect vector depends on earth latitude and earth rotation.

(STANAG 4355) have two methods to simulate base bleed projectile as two components added to the previous model. The first method adds base bleed component ( $\vec{BB}$ ), and defined as follow:

$$\vec{BB} = \left[ \frac{(\pi/8) \rho d^2 v^2 C_{x_{bb}} f(I) f(i_{bb}, MT)}{m} \right] \left( \frac{\vec{v} \cos \alpha_e}{v} + \vec{\alpha}_e \right) \quad (\text{eq.7})$$

The second method, the modifies the drag coefficient at zero yaw to be:

$$C_{D_0} = C_{D_0} - f(i_{BB, MT}) \left[ \frac{I * \left( \frac{\delta BP}{\delta I} \right)}{(\gamma/2) M^2 (d/d_b)^2} \right] \quad (\text{eq.8})$$

In the two methods, some factors need to be scaled for every firing altitude ( $alt$ ) and base bleed unit temperature, and it depends on other factor like the spin rate ( $\Omega$ ). These factors are scaled as function of Quadrant elevation ( $QE$ ), and base unit temperature ( $MT$ ), and Mach Number ( $M$ ).

## 1.2 Problem statement

As it can be seen from the published literature, base bleed models require scaling factors that have to be found experimentally. The factors that need to be scaled are ( $i_{BB}$ ) base injection fitting factor, and adjusted function for Base Burn factor  $f(i_{BB}, MT)$ , and the base change in pressure with respect to change in injection rate ( $(\frac{\delta BP}{\delta I})_I$ ). These factors are given as follow:

$$\left( \frac{\delta BP}{\delta I} \right)_I = a_0 + a_1 M + a_2 M^2 + a_3 M^3 + a_4 M^4 \quad (\text{eq.9})$$

The injection factor and function of base bleed factors are adjusted as follows:

$$i_{BB(MT=21)} = a_0 + a_1 QE + a_2 QE^2 + a_3 QE^3 + a_4 QE^4 \quad (\text{eq.10})$$

And

$$f(i_{BB,MT}) = i_{BB(MT=21)} + b_1(MT - 21) + b_2(MT - 21)^2 + b_3(MT - 21)^3 + b_4(MT - 21)^4 \quad (\text{eq.11})$$

For every case solving for  $a_0, a_1, a_2 \dots$  etc. and  $b_1, b_2, b_3 \dots$  etc. is needed. These factors need to be solved for each specific firing individually. To find these scaling parameters experimentally is expensive, making the model tedious and inconvenient to use. The research proposed is to solve or simplify this problem.

Therefore, the problem to be solved is creating a new solution method for modelling base bleed projectile trajectory. This can be achieved by either the generation of alternatives to the scaling factors in equations (9), (10) and (11), or by identifying a new solution model that can be generalised for every base bleed firing situation or range of situation under certain conditions.

The solution may require either altering the equation (1) by adding base bleed factor ( $\overrightarrow{BB}$ ), or modifying Drag Force component (eq. 2), or Thrust Force component (eq. 6), or a combination of these solutions.

### 1.3 Objectives

The aim of the proposed research is to obtain an alternative model for simulating base bleed projectile motion. Predicting base bleed behaviour is affected because of changes in base temperature, pressure, spin rate and the design of base bleed unit. From these principles a base bleed model will be constructed to predict the drag coefficient, or a base bleed force component ( $\overrightarrow{BB}$ ). The model is designed to be used in any trajectory model by altering these factors in any given model. The mathematical model that is based on aerodynamic and physics can be used to construct a computer code that can give the drag coefficient.

### 1.4 Method of investigation

Data will be gathered from previous published works to optimise the new model. The method includes:

- Qualitative data from open literature.
- Quantitative test data is obtained from the industry to validate the solution.
- Creating a new model starting from the basic of Modified Point Mass model.
- Test data obtained through collaboration with manufacturers of base bleed projectiles.
- Data quality tested using statistical and numerical method to check the validity of that data.



## **1.5 Limitations of the study**

This study produces a new base bleed model - alternative to STANAG 4355 - that can be used to model the base bleed projectile trajectory. The aim is to model the trajectories of several firing initial-conditions.

The initial projectile temperature that is considered will be between 63° C and - 46° C. The initial firing altitude will be between sea level and 1000 meter elevation.

Moreover, the model proposed will consider base bleed effect through drag reduction. The base bleed may in some exceptional cases give an impulse that is considered in the new model. The model only considers modelling a single axisymmetric injection hole in the projectile base with slow subsonic, transonic, and supersonic injections velocity. The modelling of supersonic and transonic ejection may need some correction.

## **1.6 Contributions of this study**

To give an alternative scientific base bleed trajectory model for the base bleed projectile. The model can be use in producing more accurate firing table with minimal test data. Minimising the need for testing will reduce both costs and saves time in a highly competitive and demanding industry.

## **1.7 Summary**

The study looked into several base bleed projectile modelling aspects with the intention to obtain a simpler and more scientifically explained trajectory model. The topics that are covered by the study include:

- Propellant burning inside the base bleed unit.
- Injection rate and its effect on drag coefficient.
- Trajectory and drag modelling.
- Other aspects of base bleed trajectory modelling.

All the above are surveyed in the literature review that follows.

## **CHAPTER 2 LITERATURE SURVEY**

### **2.1 Introduction**

The published literature reveals many ways to simulate the performance of base bleed projectile. The published work gives good ideas about trends and limitations of specific models that are critical for simulation and modelling of base bleed projectile trajectory. This can help in the reduction of variables in the model used to simulate base bleed flight.

#### **2.1.1 STANAG 4355 third edition (NATO Standard Agreement, 2010)**

The NATO standard agreement provides a detailed description of the standard modified point mass model for artillery trajectory simulation. This model provides two accepted standard models to dictate base bleed effects:

- Method 1: well known as the French model, which is based on D. Chargelegue and M. T. Couloumy work cited in (Kuo & Fleming, 1988).
- Method 2: known as the USA model, based on the work done by Ballistic Research Laboratory (BRL).

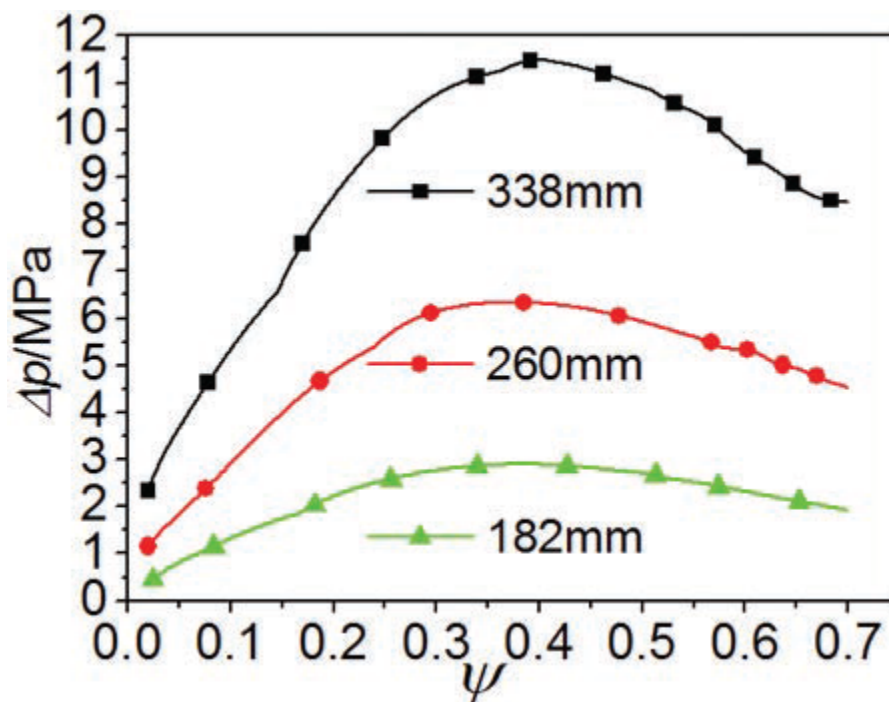
#### **2.1.2 Propellant burning inside the base bleed unit**

Propellant inside the base bleed unit burns in unique burning conditions that differs from other internal ballistic propellant burning such as solid fuel rocket burning and gun propellant burning. The propellant inside the base bleed unit faces high pressure in the gun barrel, and then it burns at lowered pressures with igniter assist (for 3.5 seconds according to Danberg (1990)) that increases temperature and burn rate. After that the propellant continue to burn at the lowered pressure along the trajectory. The main physical characteristics that affect the burn rate along with the chamber pressure, are spin rate and the grain temperature as mentioned by Danberg (1990).

De Yong and Smit (1991) mentioned in their work that the igniter composition burns at 3700° K. That burning continue for short time after the projectile exits the gun muzzle as shown by Danberg (1990). The purpose of the igniter is to ensure the initiation propellant burning at the early stages of flight. The igniter accelerates the burning rate of the propellant at the top layers of the propellant composition resulting in higher injection rate.

The propellant encounter lowered pressures as a result of base drag at high velocities and elevation that the projectile experience during its trajectory. Schoyer and Korting (1986) study gives a good indication of burning rate as a function of pressure. Burn rate is taken usually as power function of surrounding pressure in most literature. Miller and Holmes (1987) gave a good correlation, that they gained through experimental results, for the burn rate as function of pressure.

Results of bench tests for propellant burning inside spinning base bleed units, tested for several spin rates as published by Kayser, and his associate (1988) and illustrated in figure 2-2. It shows a progressive chamber pressure increase with increased spin rate which indicate erosive burning related to increase in spin. This correlates well with the results of erosive burning shown in Zhaom and Zhang (2017), and there is similarity in the burning rate in the static base bleed unit test show in figure 2-2 and the results of Zhaom and Zhang (2017) as in figure 2-1.



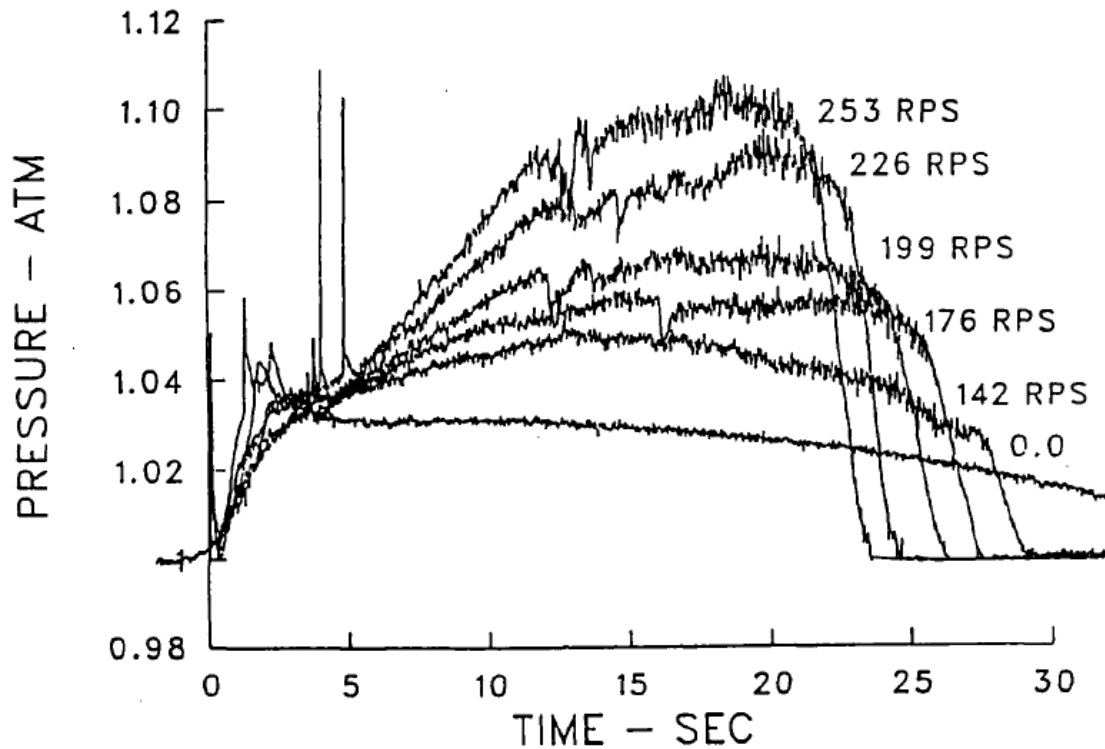
**Figure 2-1:** Maximum pressure difference versus propellant burned fraction ( $\psi$ ) of different grain length (Zhaom & Zhang, January 2017)

### 2.1.3 Injection rate and its effect on drag coefficient

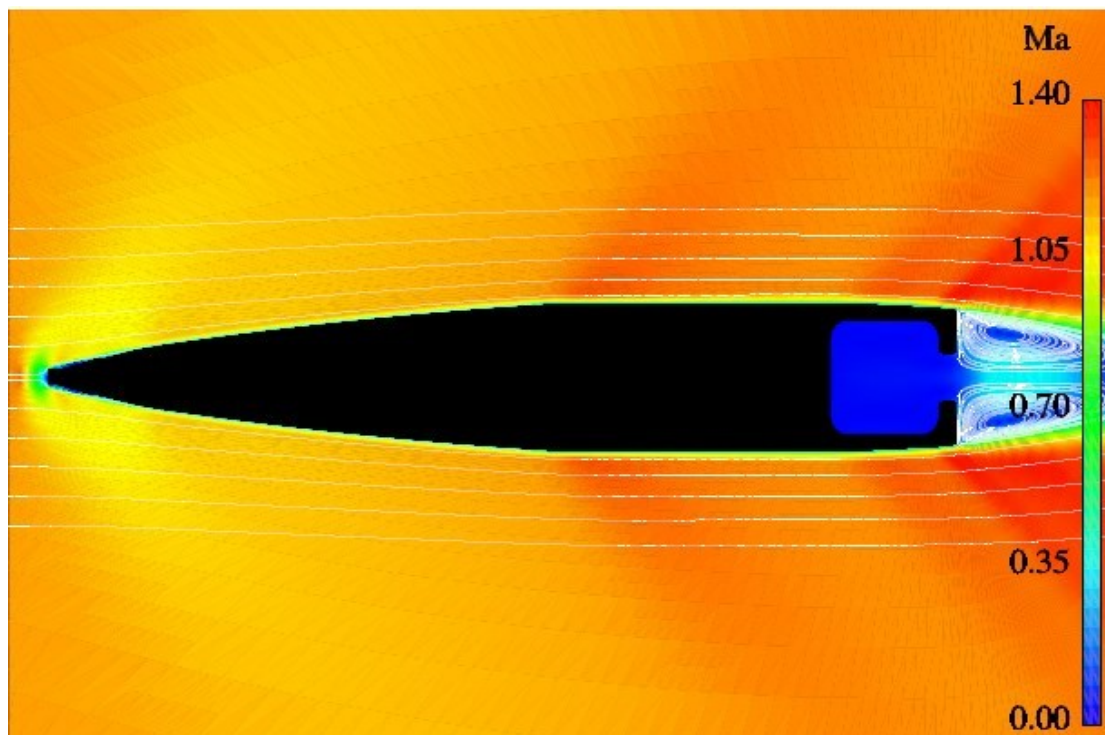
In his thesis Kaurinkoski (2000) published a good analysis of the performance of long-rang artillery projectile, with a base bleed unit. This thesis looks at exit propellant after burn reaction and the effects of it as it increases gas expansion and injection effect on projectile base pressure

(outside the base bleed unit chamber). Kaurinkoski uses multi-block Navier–Stokes solver to model base bleed functioning.

The publication provides various simulations of the projectile flight characteristics under changing circumstances including change in base bleed unit temperature, pressure and injection rate. Also, he indicates the effect of axial rotation rate on the base bleed performance and simulate all of that using CFD (Computational Fluid Dynamic).



**Figure 2-2:** Chamber pressure versus time for several spin rates (Kayser, et al., November 1988)



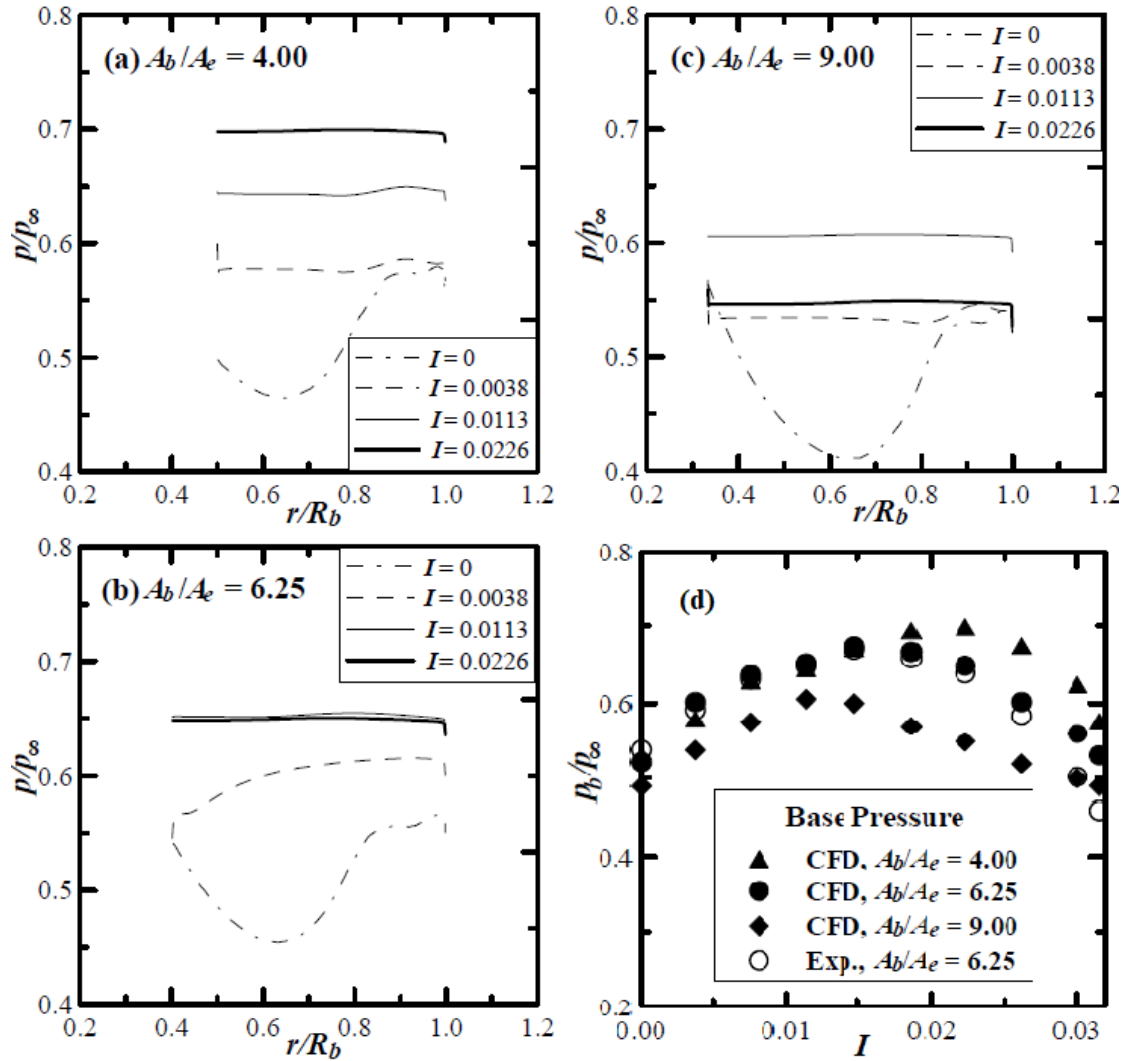
**Figure 2-3:** (Kaurinkoski, 2000) CFD analysis  $C_p$  against Mach number

The report of Kayser (1975) shows results for dynamic tests that measure the based pressure, and the chamber pressure and gave trajectory results of the projectile that is tested. This data was captured during the destructive tests, using onboard sensors. The results can be used in characterising and for model verification.

#### **2.1.4 Effect of base injection diameter on drag reduction**

The base injection diameter can affect the base drag during injection phase. This is because the base drag is the result of integration of pressure coefficient over the base surface. With base injection, the pressure reduction caused by drag is relieved over the orifice injection opening but not over the rest of the base area. This will cause reduction in drag with increased injection diameter for any injection rate (Lee, et al., 2004).

As the injection rate increases out of the base bleed unit, the exhaust gas velocity causes a suction effect on the back of the orifice plate (Anon., 2018). This cause a force that depends on the exposed back plate area and the resultant coefficient of pressure. Lee and his associates (2004) show how injection diameter to base diameter can affect coefficient of pressure on the base of the projectile for several injection rates as shown in figure 2-4.



**Figure 2-4:** (a), (b) and (c) is showing coefficient of pressure along the base diameter for several base area to orifice vent area ratios; while (d) shows pressure ratio against variable injection rate for several base area ratios. All these data are taken at Mach 2.47 (Lee, et al., 2004)

### 2.1.5 Trajectory and drag modelling of Base Bleed projectile

The report by (DANBERG, 1990) shows the variation of several factors against mass flow rate of the base propellant. It gives base pressure as a function of injection ratio. This can simplify the construction of a new model. Reduction of pressure ratio with raising injection rate over some critical injection rates as shown in figure 2-4, is not considered in the model suggested by that paper.

Schilling (1986) gave a simple and scientifically sound explanation for the base bleed functioning. It discusses various factors that affect base bleed projectile trajectory, including Prandtl-Mayer expansion fan that he uses to drive a virtual wake tail length from. This is done to understand the limitation of effective injection.

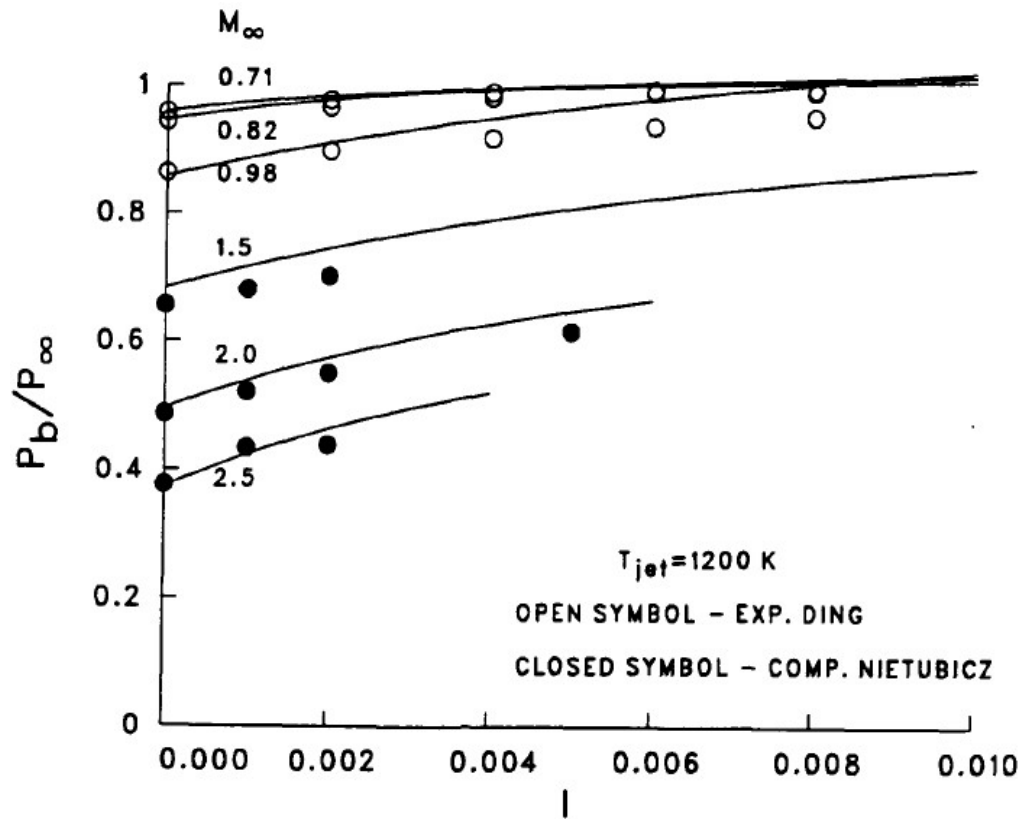


Figure 2-5: Injection rate as function of base pressure for some Mach numbers (DANBERG, 1990)

## 2.2 Conclusion

The literature surveyed gives a good background of the problem. This information is essential to understand the problem and optimise a new solution. The solution method should be based on these previous works that give a guide line for the solution method. The data given in literature can be used to optimise and validate the new solution.

## CHAPTER 3 RESEARCH METHODOLOGY

### 3.1 Introduction

The aim of this research is to obtain a practical and applicable model that can simulate base burn projectile trajectory. The model needs to be compatible with the existing point mass model (PMM), modified point mass model (MPMM) and five degree of freedom (5-DoF) trajectory models. On that basis, the model that is constructed should model the drag reduction obtained from base bleed and the change in projectile mass, so that it can be fitted within the existing trajectory models.

The model should be more scientifically justifiable than the existing models in NATO standard agreement (STANAG, 2009). The model will predict the injection rate using several well establish parameters like ambient pressure, base bleed unit temperature and base bleed motor spin rate. From the injection rate the drag reduction of the base bleed projectile can be predicted using some fluid mechanics equation. The injection model should be built on a propellant burning model that take progressive burning phenomena into consideration. Also, another consideration is the after-burn reaction that increases the volume of base injection outside the base bleed motor.

### 3.2 Theoretical method

STANAG 4355 provides two NATO standard models to predict base bleed effect. The French model represents the base bleed effect as an effective change in drag as shown in eq. 12:

$$\overline{BB} = \left[ \frac{(\pi/8)\rho d^2 v^2 C_{x_{bb}} f(I) f(i_{bb}, MT)}{m} \right] \left( \frac{\vec{v} \cos \alpha_e}{v} + \vec{\alpha}_e \right) \quad (\text{eq.12})$$

In this model the burn rate is described as a function of temperature ( $MT$ ), chamber pressure ( $P_C$ ), and spin rate ( $p$ ):

$$V_C = V_{c0} f(MT) g(P_C) K(p) \quad (\text{eq.13})$$

The amount of gas generated through burning is then calculated as:

$$\text{Flux} = \text{Burn rate} \times \text{Burn Area}$$



The other model provided by Ballistic Research Laboratory in the United States shown in SANAG (2009:D-1) does not give a simulation model for propellant burn. Mass flux is obtained from the scaling propellant burn static test. This model does, however, provide a model to predict the effect base injection on the base pressure as shown in eq. 14:

$$C_{d_{0.BB}} = C_{D_0} - f(i_{bb}, MT) \left[ \frac{I^* \left( \frac{\delta BP}{\delta I} \right)}{(Y/2) M^2 (d/d_b)^2} \right] \quad (\text{eq.14})$$

The method uses a function of the slope of rate of change in base pressure as function of Injection rate  $\left( \frac{\delta BP}{\delta I} \right)$ .

To model the injection of gases through the base bleed unit over the orifice opening in the base of the projectile the principle of fluid mechanics should be applied.

For a fluid passing through an orifice as in figure 3-1. The expansion factor (Y) according to Zucrow and Hoffman (1976) is given by:

$$\dot{m} = \frac{P_C A_e Y}{\sqrt{\gamma_{inj} R T}} \quad (\text{eq.15})$$

And the expansion factor is:

$$Y = \left\{ \frac{2\gamma_{inj}^2}{\gamma_{inj}-1} \left( \frac{P_b}{P_C} \right)^{2/\gamma_{inj}} \left[ 1 - \left( \frac{P_b}{P_C} \right)^{\gamma_{inj}-1/\gamma_{inj}} \right] \right\}^{1/2} \quad (\text{eq.16})$$

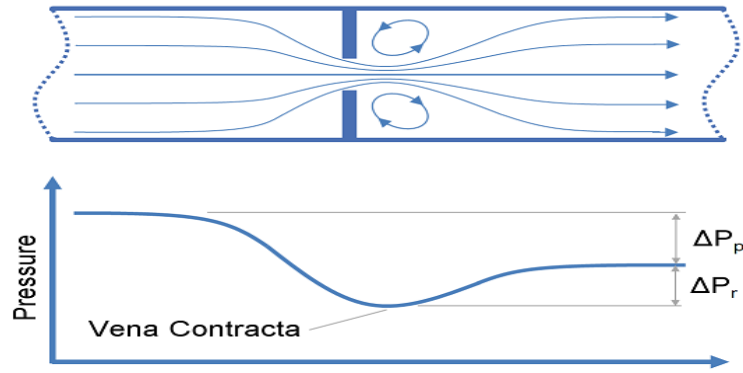
The density of injected gases ( $\rho_{inj}$ ) can be obtained using ideal gas law:

$$\rho_{inj} = \left( \frac{n P_C}{R T_{Ch}} \right) \quad (\text{eq.17})$$

And from that the volumetric flow rate can be obtained:

$$\dot{Q} = \frac{\dot{m}_f}{\rho_{inj}} \quad (\text{eq.18})$$

The volumetric flow rate ( $\dot{Q}$ ) can be obtained from the mass flow rate ( $\dot{m}$ ). Whereas the mass flow rate is obtained using the estimate of the propellant burn rate.

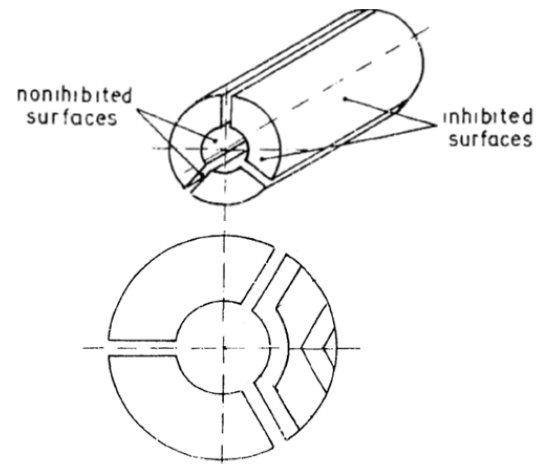


**Figure 3-1:** Fluid flow through an orifice

The burning rate of propellant is obtained from the integration over the area of the burned surface. The configuration of the propellant grain that is considered is tubular grain with three slots (s shown in figure 3-2. The surface integration is modelled according to the grain shape.

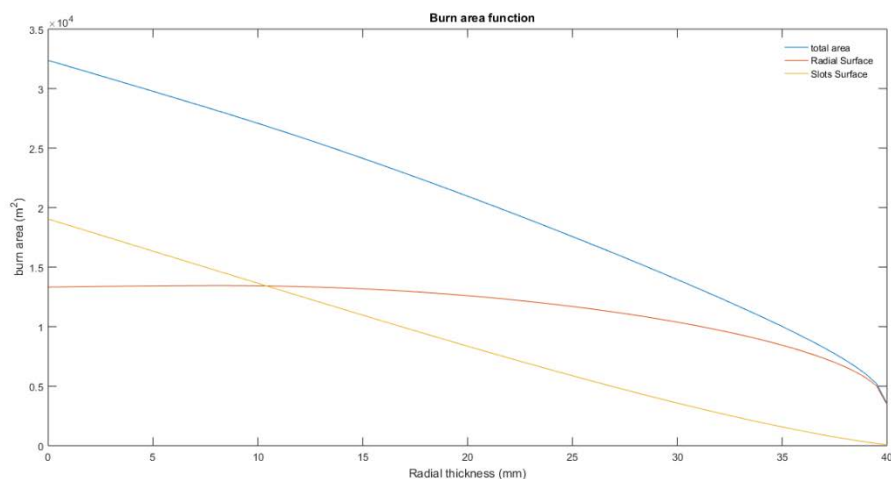
Another consideration is progressive burning of propellant inside the base bleed motor. Static test conducted by Kayser and associates (1990) shows the chamber pressure variation with spin rate as shown in figure 2-2.

Modelling the progressive spin rate tend to have a burn rate like what is shown by Zhaom and Zhang (2017) illustrated in figure 2-1. From that the profile of progressive propellant burn rate ( $V_{c0}$ ) amplitude through burn diameter can be assumed for the change in spin rate.



**Figure 3-2:** Propellant grain shape

An important consideration is the relieve of base pressure drop due to base injection. Change in base pressure to free stream pressure ratio is governed by base injection as illustrated in figure 3-4.



**Figure 3-3:** Propellant burned area against burned surface thickness

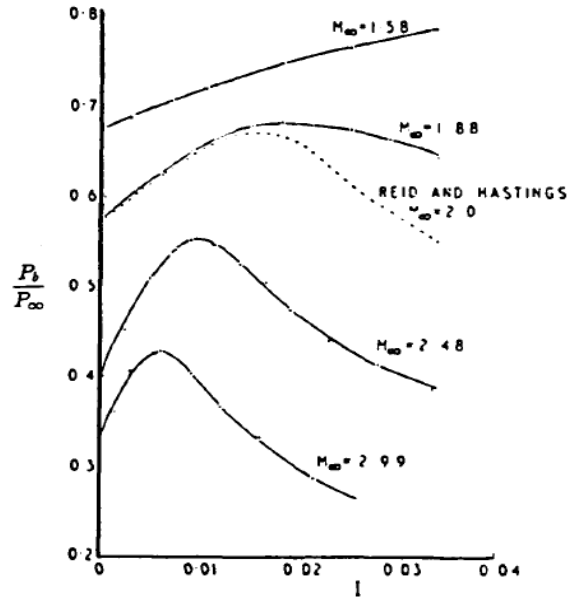


Figure 3-4: Base pressure ratio as function of injection for several Mach numbers (Waliskog, June, 1953)

An assumption to obtain pressure relieve in base region is taken from Danberg (1990). That assumption from rocket theory can be used to determine base pressure:

$$\frac{P_b}{P_\infty} = \left( \frac{P_b}{P_\infty} \right)_{I=0} + \left( \frac{\sigma I}{1+2.6*\sigma I} \right) \quad (\text{eq.19})$$

Where  $\sigma = \frac{\delta(\frac{P_b}{P_\infty})}{\delta I}$  that rate of change is described by Danberg (1990). It suggests the experimental value of (1500° K) for the temperature of base bleed injection. The  $\left( \frac{\delta(\frac{P_b}{P_\infty})}{\delta I} \right)$  as in figure below:

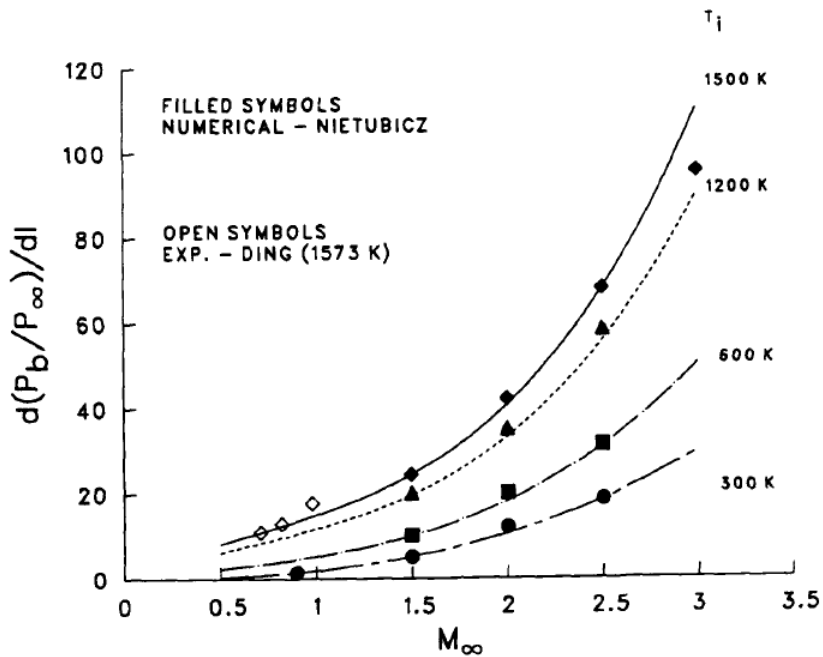
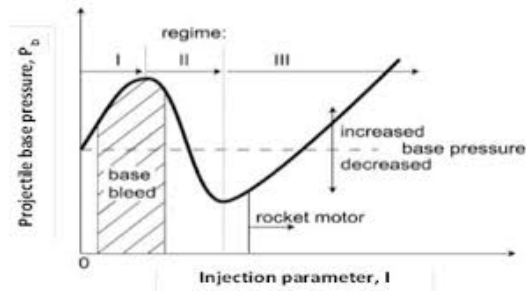


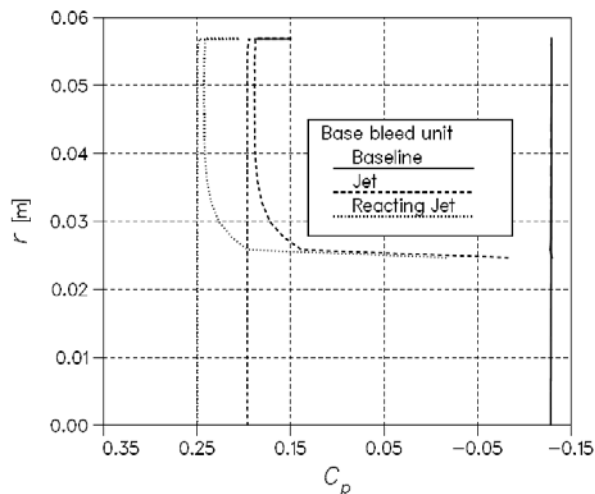
Figure 3-5: Slope of base pressure curve versus Mach number for several temperatures



**Figure 3-6:** Base pressure versus base injection typical profile

There is usually a sweet point where at a base burn unit operate. That point depends on the orifice vent area to base area ratio. If injection rate reaches over that point as shown in figure 3-6 the base pressure will again decrease, and the benefit of base bleed motor will be reduced.

In theory functioning base bleed motor relieve base pressure drop. This pressure-relieve depend on base injection, injected gases expansion, and injected gases after burn. Oxidisation of injection gases contribute to increase in gas temperature and expansion which eventually increases the pressure in the base region as shown in figure 3-7 by Kaurinkoski (2000).



**Figure 3-7:** Pressure coefficient distribution on the surface inside the base bleed unit. Mach number of 1.2, injection rate of 0.0122 and no spin.

### 3.3 Conclusion

The solution will depend on the theoretical solution method that is observed in literature. This will be the base of scientific solution for base bleed projectile trajectory model. The proposed new base bleed model uses trends observed in literature and attempts to couple it with flow dynamic principles.

## CHAPTER 4 PROPOSED NEW BASE BLEED MODEL

### 4.1 Introduction

Based on the theories discussed a scientific solution for specific base bleed projectile configuration is constructed. The solution is found by breaking down the problem to its elemental components. Firstly, it models propellant burning inside the base bleed motor with the initial condition taking into consideration the effects of temperature, chamber pressure, and the spin rate. The propellant burn model takes into consideration the progressive burning effect resulting from the base bleed motor spin. Secondly, exhaust gases injection is modelled based on the burning rate and orifice coefficient of friction and the ambient air pressure. Using all the mentioned factors chamber pressure can be determined and that will be a factor in determining the propellant burning rate. Lastly, the drag reduction due to injected gases expansion and reactions is modelled base on aerodynamic principles.

### 4.2 Modelling propellant burn

The first thing to be modelled is the burn rate as shown is eq. 20. The burn rate is the function of several factors namely the spin rate ( $p$ ), pressure ( $P$ ), and grain temperature ( $MT$ ). The new model needs to accommodate for gun blast and the igniter effect in the initial phase of burning. therefore, the initial condition factor ( $f_{initial}$ ) will be zero after it was used to correct to the initial phase of burning. The spin rate function  $K(p)$  as shown by Schoyer and Korting (1986) will be used to correct the change in burn rate due to spin. Furthermore, the effect of chamber pressure and grain temperature on the burn rate will be taken as  $g(P_c)$  and  $f(MT)$  respectively as according to work by Du Plessis (2004). Eventually the burn rate will be as follows:

$$V_C = V_K f(MT) g(P_c) K(p) + f_{initial} \quad (\text{eq.20})$$

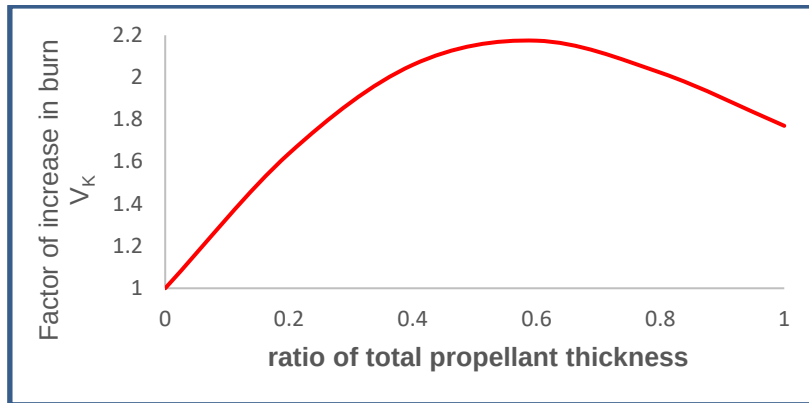
$$f(MT) = e^{0.0035 (MT-21)} \quad (\text{eq.21})$$

$$g(P_c) = 0.9132 P_c^{0.6655} \quad (\text{eq.22})$$

$$K(p) = 0.00023698 (p) + 0.98691 \quad (\text{eq.23})$$

Where  $P_c$  is the chamber pressure. The burn rate is modelled to satisfy data observation in figure 2-2 and firing data. The progressive burn rate is corrected for such conditions using correction factor ( $V_K$ ). The progressive burn factor ( $V_K$ ) is determined using test data as done by Kayser and

associates (1988). The test results which are burnout time and chamber pressure can be used to construct progressive burn profiles for several spin rates. These burn profiles can be used to give an empirical progressive burn profile for propellant thickness as in figure 4-1.



**Figure 4-1:** Progressive burn correction factor ( $V_k$ ) vs proportion of propellant thickness

The ignitor effect of the base bleed unit motor that burn for 3.5 seconds needs to be considered. The ignitor consists of two 6.2 grams pellets, usually made of Magnesium-Teflon composite. Burning temperature of Magnesium-Teflon can reach 3500° K according to de Yong and Smit (1991). The base bleed motor chamber temperature should follow the ideal gas law through an isentropic process. Therefore, the temperature can be modelled using exponential relationship as follow:

$$T_C = 817.33 V_C^{1.5465} + T_{Igniter} \quad (\text{eq.24})$$

The igniter temperature ( $T_{Igniter}$ ) factor is given a value for the initial phase of the trajectory. This factor affects the initial burning of the igniter which lasts for 3.5 seconds after ignition by the barrel blast. The factor is considered in this model to be a fixed temperature (2200° K), that temperature can be modified to satisfy the initial phase of ignitor burning.

During the trajectory, the mass of the propellant burned reduces the projectile weight. This reduction is described mathematically along the trajectory for specific time step ( $t_{timestep}$ ) as noted in the following equation:

$$m = (m_0 + m_f) - (\dot{m}_f \times t_{timestep}) \quad (\text{eq.25})$$

From the chamber temperature ( $T_C$ ) the density of propellant injection gases ( $\rho_{inj}$ ) can be obtained using ideal gas law. Considering the molar mass ( $n$ ) to be 19.6 (*grams/mole*) and the gas constant ( $R = 0.08314472 \frac{L \cdot bar}{mole \cdot K}$ ) as follows:

$$\rho_{inj} = P_C n / T_C R = P_C (19.6) / T_C (0.08314472) \quad (\text{eq.26})$$

### 4.3 Modelling injection and drag

Gas injection exiting of the base bleed motor is described using equations 15,16,17 and 18 in chapter 3. Where the base pressure ratio ( $\frac{P_b}{P_\infty}$ ) can be obtained using the method described in DATCOM (1968) as shown the following table:

**Table 4-1:** base pressure ratio for several Mach numbers

| Mach number            | 0.71   | 0.82   | 0.98   | 1.58   | 1.88   | 2      | 2.48   | 2.99   |
|------------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| $\frac{P_b}{P_\infty}$ | 0.9536 | 0.9353 | 0.8525 | 0.6699 | 0.5745 | 0.5685 | 0.4228 | 0.3282 |

Or alternatively for ( $M > 0.71$ ) using the polynomial that fit the data in table 4-1 as follows:

$$\frac{P_b}{P_\infty} = -0.1489M^6 + 1.6636M^5 - 7.4249M^4 + 16.84M^3 - 20.291M^2 + 11.885M - 1.6743 \quad (\text{eq.27})$$

Injection parameter is obtained from mass flow rate ( $\dot{m}$ ) divided by the free stream flow through an area equal to the base area:

$$\dot{m} = V_C A_{\text{propellant}} \rho_{\text{propellant}} \quad (\text{eq.28})$$

$$I = \left( \frac{\dot{m}}{\rho_{\text{air}} v \pi (d_b/2)^2} \right) \quad (\text{eq.29})$$

The density of the propellant, which is a double based propellant (AP-2) is ( $\rho_{\text{propellant}} = 1532 \frac{\text{g}}{\text{L}}$ ). Whereas ( $A_{\text{propellant}}$ ) is calculated as shown in figure 3-3.

To estimate the base pressure accurately there is a need to consider the compressibility of injected gases. Injections with a high Mach number ( $M_{\text{Inj}} > 0.6$ ) can contribute to reduction of base pressure ( $P_b$ ). The reduction of base pressure over the base surface increases base drag. To have an estimation of base drag due to gas injection ( $C_{d_{b,\text{Inj}}}$ ) some theoretical calculations are used. In Drag Coefficient Prediction resource published by University of Sydney (2015) base drag will be a function of body shape, Reynolds number and some fitting factor for the drag coefficient. The fitting factor ( $f_{b0}$ ) for injection Mach number ( $M_{\text{Inj}}$ ) is given as shown below:

$$f_{b0} = 1 + 215.8(M_{\text{Inj}} - 0.6)^6 \quad (\text{eq.30})$$

For ( $1 > M_{\text{Inj}} > 0.6$ )

$$f_{b0} = 2.0881(M_{\text{Inj}} - 1)^3 - 3.7938(M_{\text{Inj}} - 1)^2 + 1.4618(M_{\text{Inj}} - 1) + 1.883917 \quad (\text{eq.32})$$

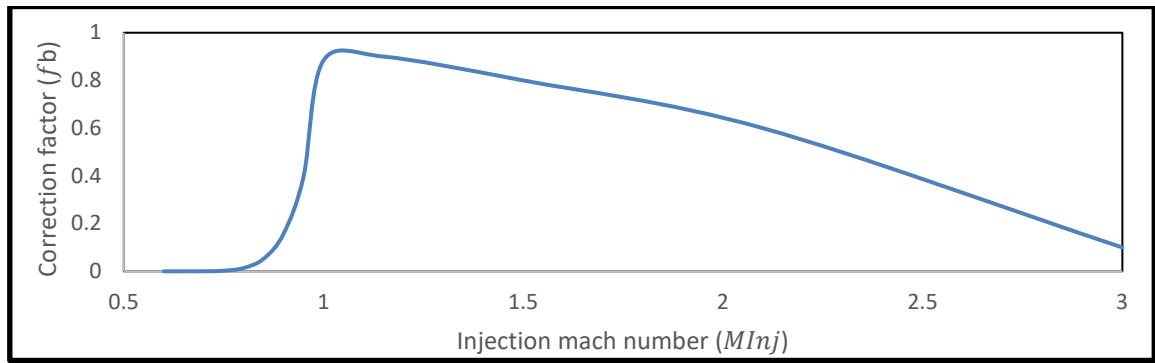
For ( $2 > M_{\text{Inj}} > 1$ )

$$f_{b0} = 0.297(M_{\text{Inj}} - 2)^3 - 0.7937(M_{\text{Inj}} - 2)^2 - 0.1115(M_{\text{Inj}} - 2) + 1.64006 \quad (\text{eq.33})$$

For ( $M_{\text{Inj}} > 2$ )

For convenience purpose base drag due to injection at low Reynolds numbers is taken as constant value ( $C_{d_{b,Inj}} = 0.24$ ) (Hollingshead, Johnson, Barfuss, & Spall, 2011). To compensate to the chocking effect the model needs to consider the compressible flow part of supersonic and transonic injection without the constant part of the equation as in figure 4-2. So, the fitting factor that will be used is ( $f_b$ ), which is ( $f_b = f_{b0} - 1$ ), that will be multiplied by base drag coefficient ( $C_{d_{b,Inj}}$ ).

Equation eq. 19 given by Danberg (1990) is excellent in modelling subsonic injections.



**Figure 4-2:** Base drag due to base injection factor versus injection Mach number

The ratio base pressure divided by static free steam pressure is modelled by the following equation:

$$\left(\frac{P_b}{P_\infty}\right)_{bb} = \left(\frac{P_b}{P_\infty}\right)_{I=0} + K_{SR} \left(\frac{\sigma I}{1+2.6*\sigma I}\right) - K_{d_{b,Inj}} f_b C_{d_{b,Inj}} \frac{(d_b^2 - d_e^2)}{d^2} \frac{\gamma_{Inj}}{2} M_{Inj}^2 + T_f K_{bT} f_b \frac{d_b^2}{d^2} \frac{\gamma_{Inj}}{\gamma_{air}} \frac{P_b}{P_\infty} \frac{M_{Inj}^2}{M_\infty^2} \quad (\text{eq. 34})$$

The second term taken from Danberg (1990) used to model the pressure ratio need correction to higher injection ratios. The correction is needed for spin rate of the projectile and injection gases secondary burning as mention by Kaurinkoski (2000). The correction factor is taken as ( $K_{SR} = 1.47$ ) which is an empirical value that works well with the specific case used in validation. The third term may need some correction of drag for base injection, and that factor is taken as ( $K_{d_{b,Inj}} = 1$ ). The fourth term is describing the impulse resulting from base injection.

There are factors used to adjust the impulse term. The first ( $K_{bT}$ ) is correcting for nozzle thrust coefficient ( $C_T$ ) to drag coefficient ( $C_{D_{0T}}$ ) ratio. These coefficients are approximated as ( $C_T \approx 0.7$ ) for nozzle effectiveness coefficient figure 4-3 and ( $C_{D_{0T}} \approx 0.24$ ) as drag coefficient with thrust on. The above considered, and for simplification purposes, that ratio



will be taken as fixed number ( $K_{bT} = 2.7$ ). The second ( $T_f$ ) is thrust factor which equals one when ( $M_{Inj} > 1$ ) and zero when ( $M_{Inj} < 1$ ). Third factor will be the base pressure fitting factor ( $f_b$ ) that adjust to base pressure ( $P_b$ ) loss due to increase in injection velocity ( $M_{Inj}$ ).

Finally, the drag coefficient is modelled based on base pressure to static pressure ratio as in the next equation:

$$C_{d0.BB} = C_{D0} - \left[ \frac{(P_b/P_\infty)_{bb}}{(\gamma/2)M^2(d/d_b)^2} \right] \quad (\text{eq. 35})$$

Where ( $C_{D0}$ ) is the basic drag coefficient of the projectile.

The model shown will represent the drag coefficient profile of the base bleed projectile with good accuracy satisfying many firing conditions. The equation terms can be modified to model many kinds of projectile due to the clear scientific explanation of each term of the model.

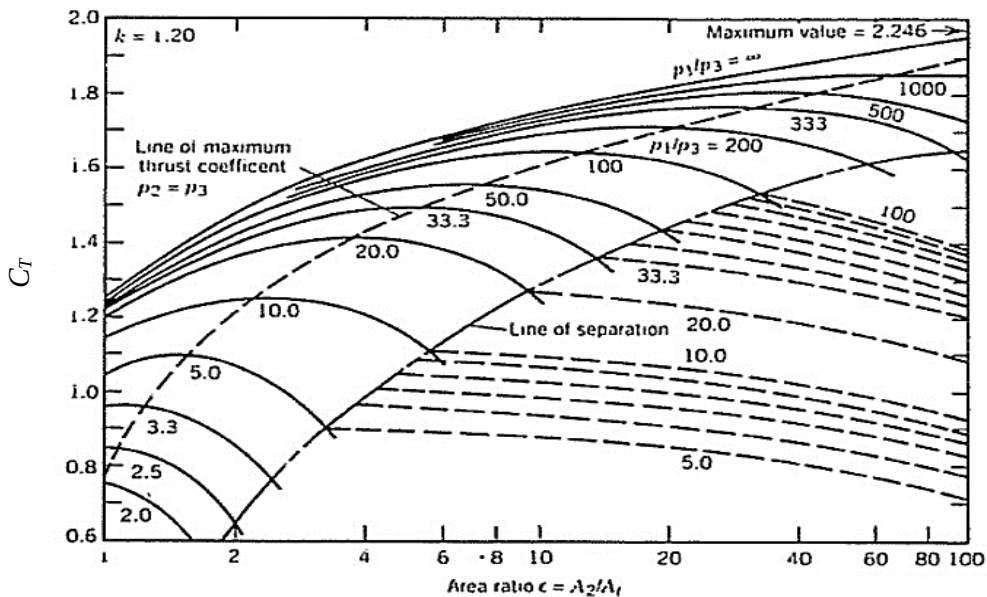


Figure 4-3: Thrust factor versus nozzle throat to exit area ratio for gas heat capacity ratio of (1.2) (Kirk, 2018)

#### 4.4 Conclusion

The new model is aimed to be a simple, reasonable and scientifically justified base bleed trajectory model. This model requires a limited number of parameters to model a specific configuration. The terms that require verification using test data are the correction factors ( $K_{SR}$ ), ( $K_{db,Inj}$ ) and ( $K_{bT}$ ). The term ( $K_{SR}$ ) can be verified using shots with low velocity injection. The two terms ( $K_{db,Inj}$ ) and ( $K_{bT}$ ) can be determined using shots with high velocity injection which happens usually at high trajectory shots, at high altitude.

The validity of the new base bleed model is verified in the next chapter with several case studies.

## CHAPTER 5 MODEL VERIFICATION AND VALIDATION

### 5.1 Introduction

Test data is gathered from a base bleed projectile manufacturer. The test data obtained is Doppler radar data for base bleed projectile shots fired at two different altitudes some of which are fired at sea level altitude and others are fired at 1050 m altitude. The projectiles used in that test are preconditioned at temperatures of (+63 °C, +21 °C and -46 °C) as required by NATO standard agreement. Shots are fired at several charge increments that gives specific muzzle velocity for each increment. The shots are fired at various gun elevations. Data for two to five shots with the same preconditioning temperature, gun elevation, charge increment and altitude from sea level were obtained.

Drag coefficient, muzzle velocity and other telemetric data of each group of shots are obtained on the field and compared to the new constructed model to validate the results. The new model is also compared with the drag reduction term given in eq. 19.

### 5.2 Radar data comparison to the model

The doppler radar data is compared to the new model and eq. 19 given by Danberg (1990). The new model is illustrated in red while the model given by Danberg (1990) will be in blue and the test group will be illustrated in black lines. The data is modelled without consideration of yaw induced drag coefficients - ( $C_{d\alpha}$ ) and ( $C_{L\alpha}$ ) - that appear in the radar data. Therefore, the simulated drag is expected to be lower than the measured total drag which includes yaw induced effects. Plots comparing radar data and base bleed models are shown as follow:

## 5.2.1 Shots at sea level elevation

### 5.2.1.1 Trajectory at 19.7 degrees

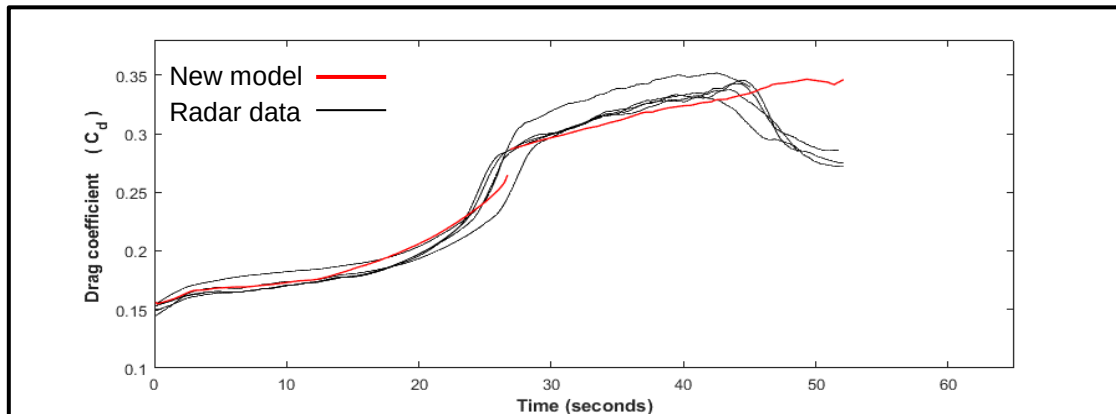


Figure 5-1: Shots with (954.2 m/s) average muzzle velocity, preconditioned at + 21 °C, fired at an angle of 19.7 degree from sea level

### 5.2.1.2 Trajectories at 30.9 degree

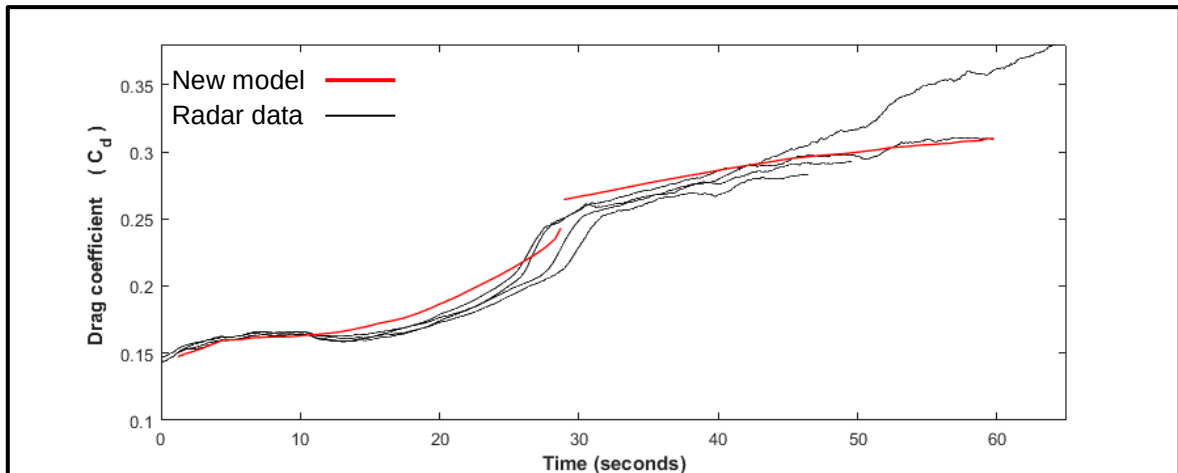


Figure 5-2: Shots with (998.2 m/s) average muzzle velocity, preconditioned at + 63 °C, fired at an angle of 30.9 degree from sea level

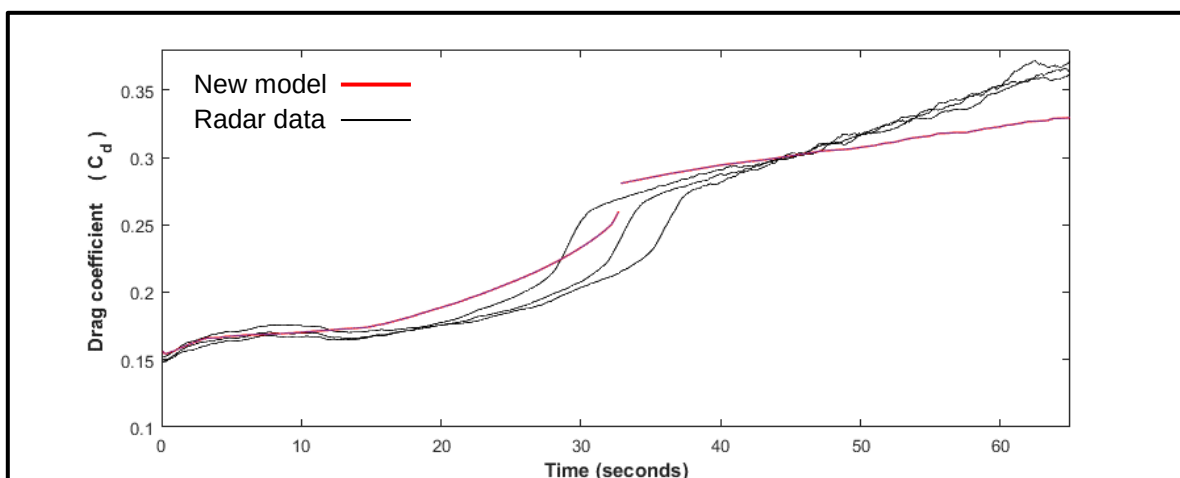
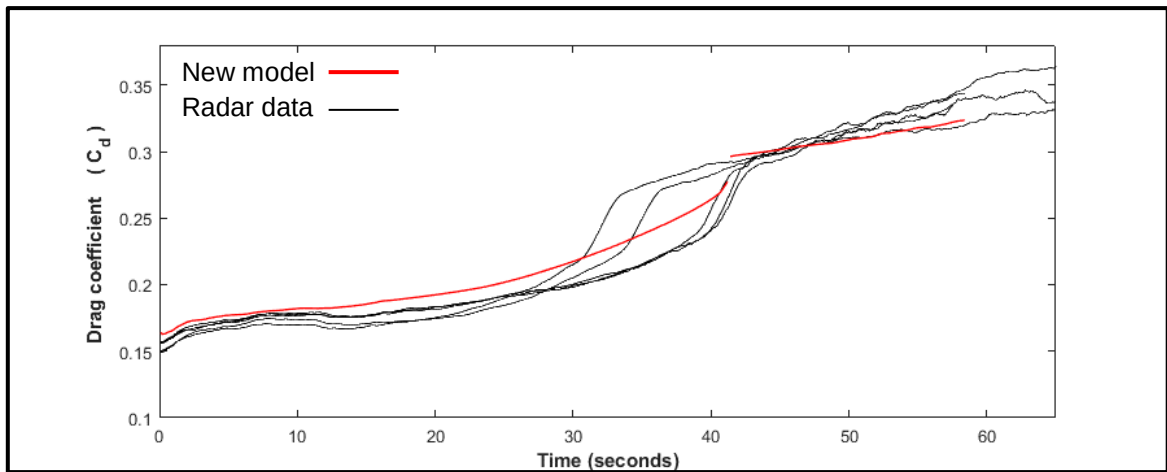
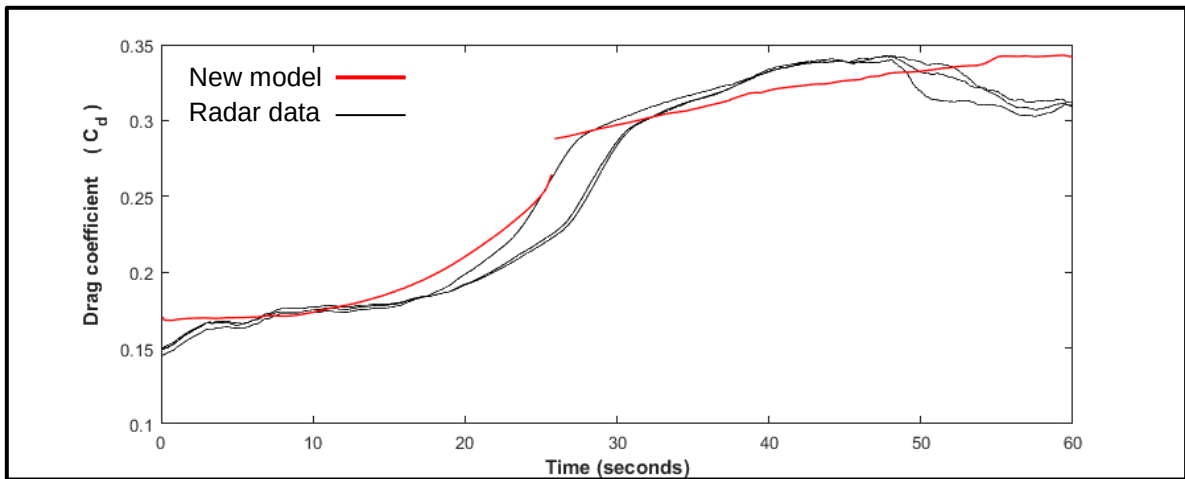


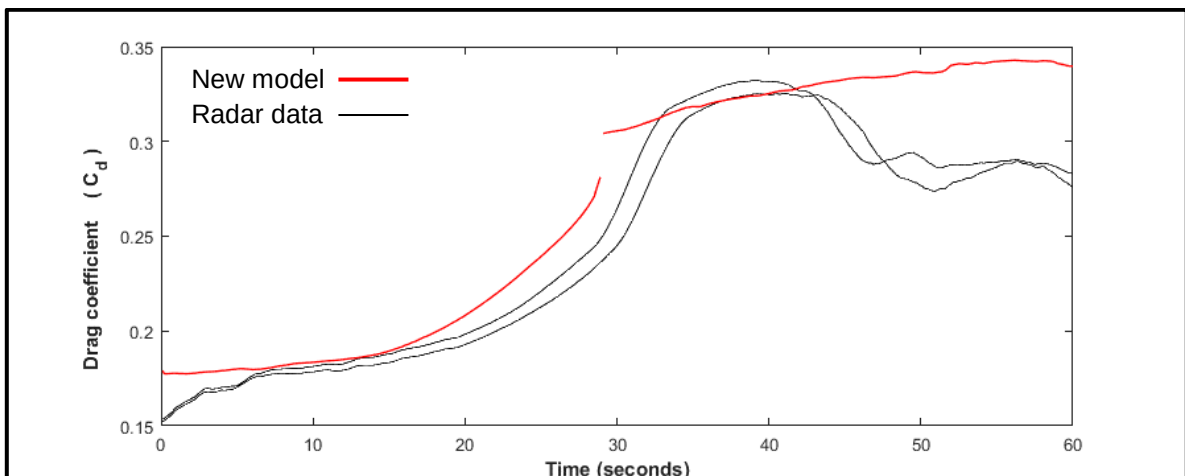
Figure 5-3: Shots with (953.8 m/s) average muzzle velocity, preconditioned at + 21 °C, fired at an angle of 30.9 degree from sea level



**Figure 5-4:** Shots with (937.4 m/s) average muzzle velocity, preconditioned at - 46 °C, fired at an angle of 30.9 degree from sea level

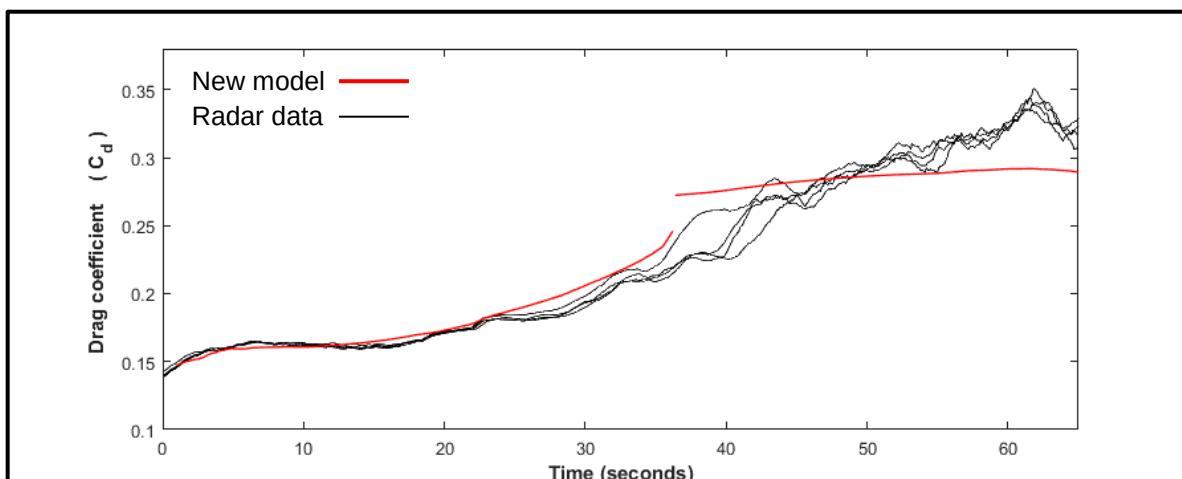


**Figure 5-5:** Shots with (862.5 m/s) average muzzle velocity, preconditioned at + 63 °C, fired at an angle of 30.9 degree from sea level

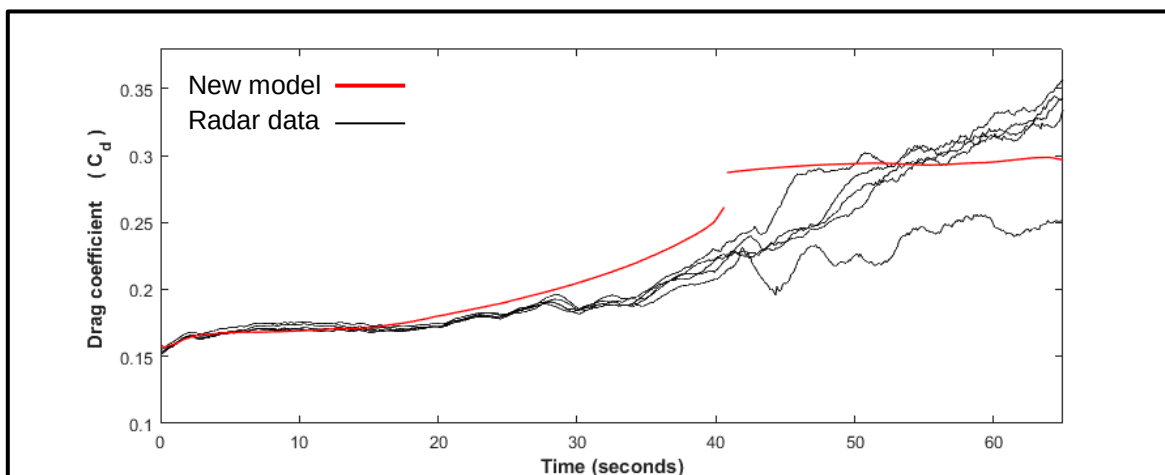


**Figure 5-6:** Shots with (812.8 m/s) average muzzle velocity, preconditioned at + 21 °C, fired at an angle of 30.9 degree from sea level

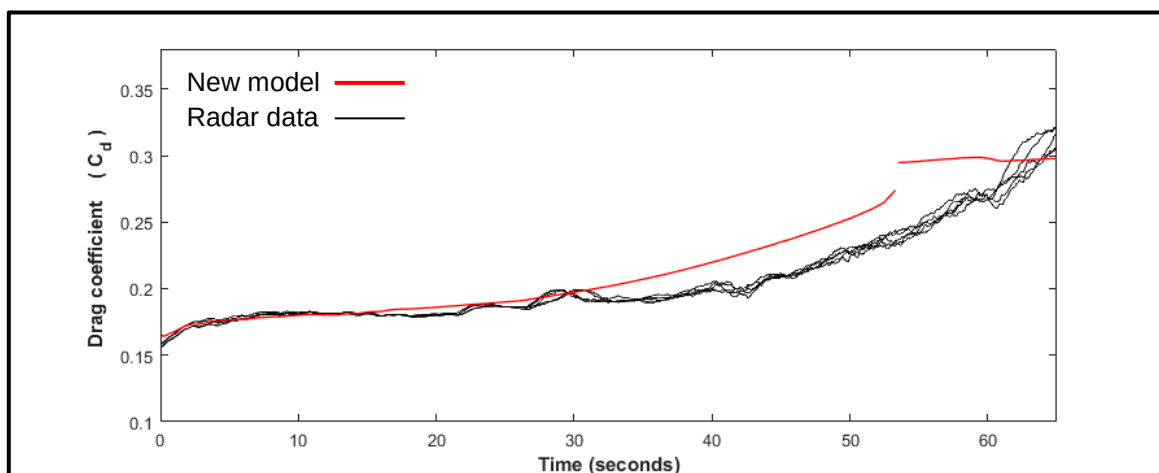
### 5.2.1.3 Trajectories at 42.2 degree



**Figure 5-7:** Shots with (998.1 m/s) average muzzle velocity, preconditioned at + 63 °C, fired at an angle of 42.2 degree from sea level



**Figure 5-8:** Shots with (954.4 m/s) average muzzle velocity, preconditioned at + 21 °C, fired at an angle of 42.2 degree from sea level



**Figure 5-9:** Shots with (932.4 m/s) average muzzle velocity, preconditioned at - 46 °C, fired at an angle of 42.2 degree from sea level

#### 5.2.1.4 Trajectories at 53.5 degree

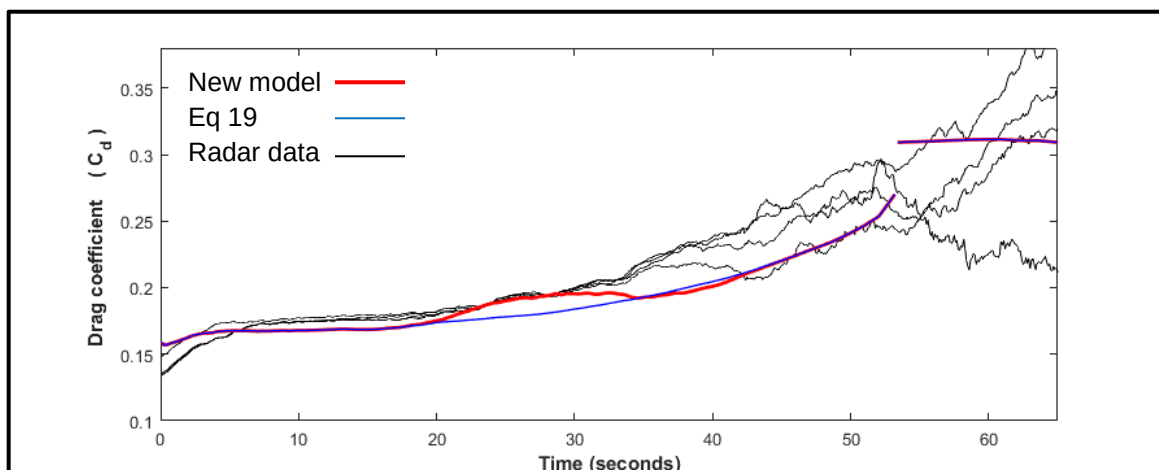


Figure 5-10: Shots with (953.4 m/s) average muzzle velocity, preconditioned at + 21 °C, fired at an angle of 53.5 degree from sea level

#### 5.2.1.5 Trajectories at 64 degree

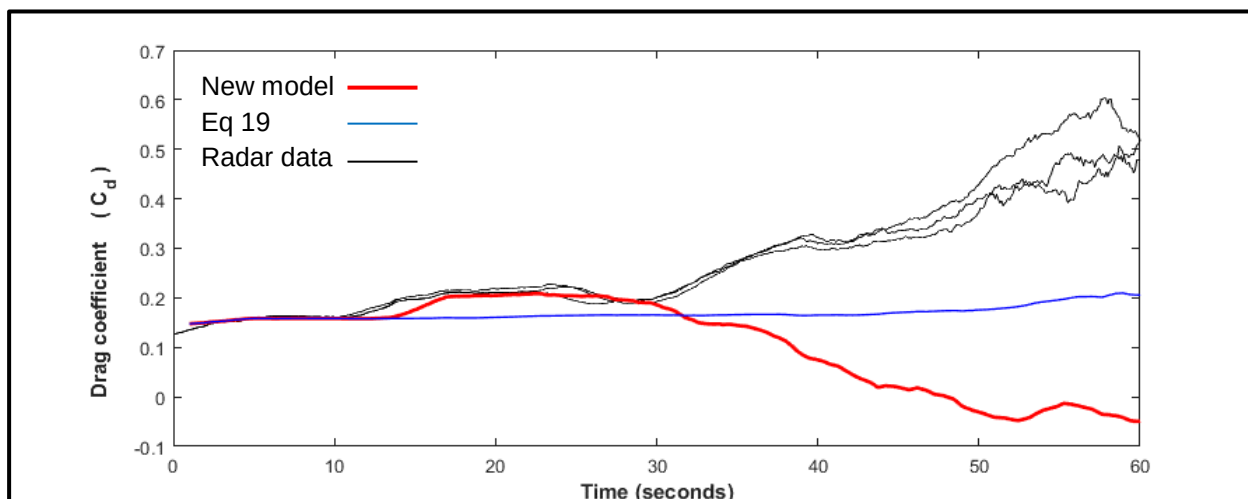
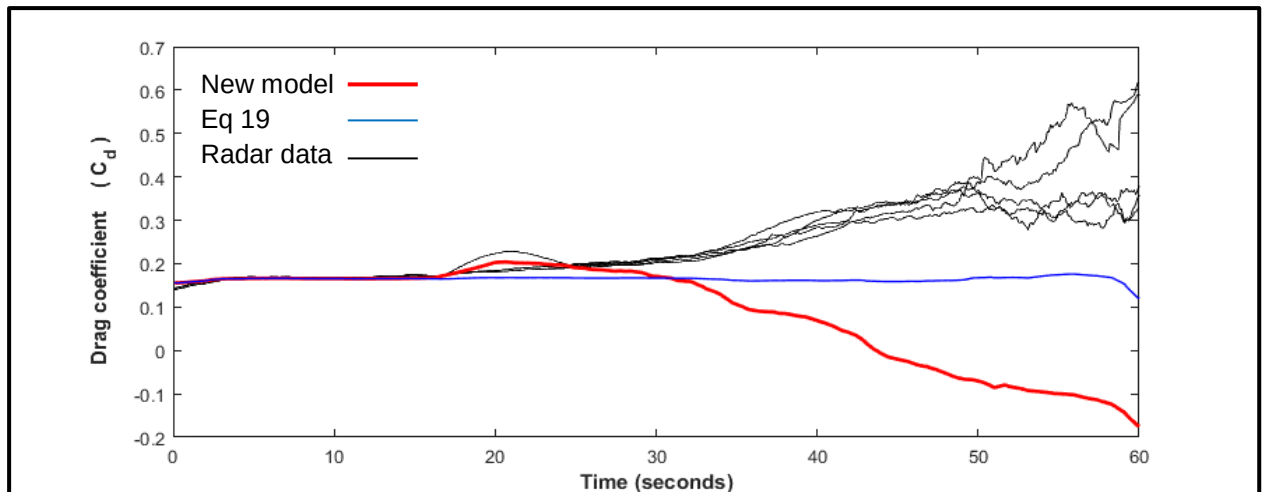
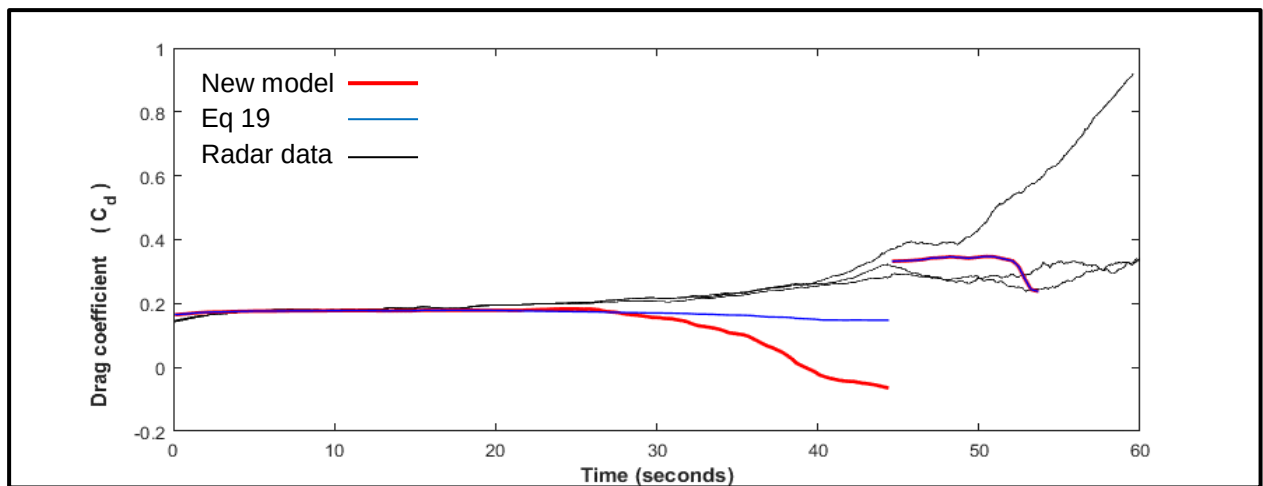


Figure 5-11: Shots with (999.8 m/s) average muzzle velocity, preconditioned at + 63 °C, fired at an angle of 64 degree from sea level



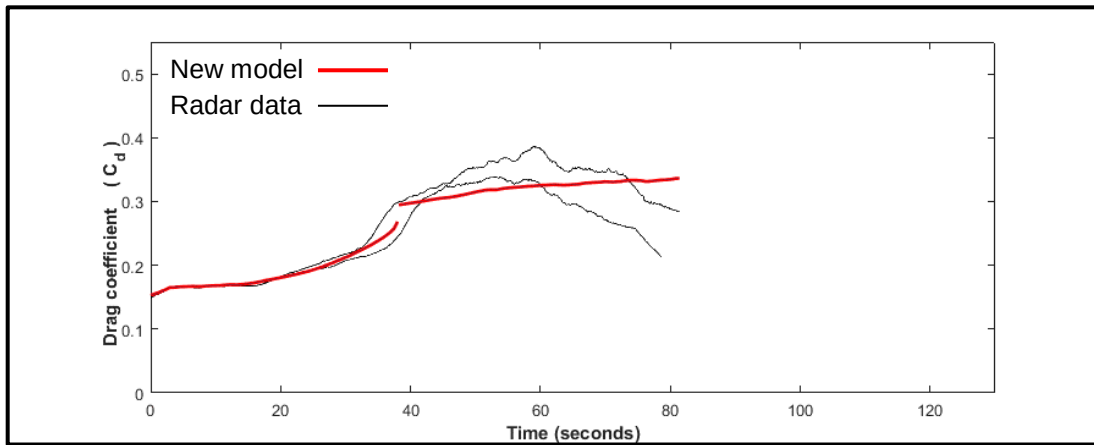
**Figure 5-12:** Shots with (957.3 m/s) average muzzle velocity, preconditioned at + 21 ° C, fired at an angle of 64 degree from sea level



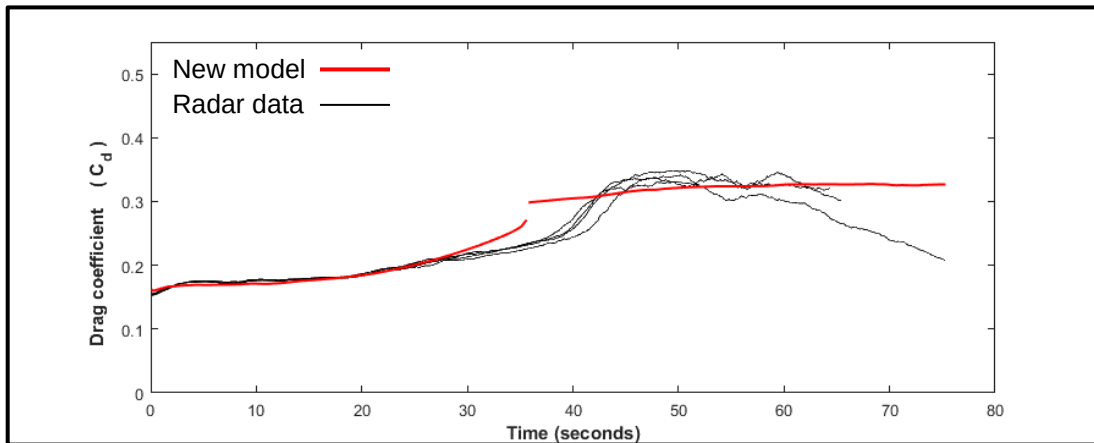
**Figure 5-13:** Shots with (934.6 m/s) average muzzle velocity, preconditioned at - 46 ° C, fired at an angle of 64 degree from sea level

## 5.2.2 Shots at 1050 m elevation from sea level

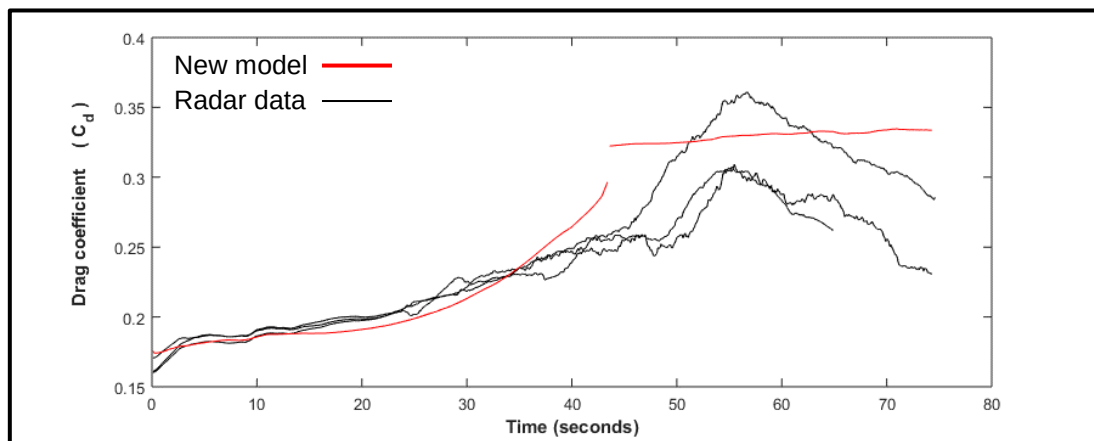
### 5.2.2.6 Trajectory at 33.75 degrees



**Figure 5-14:** Shots with (988.77 m/s) average muzzle velocity, preconditioned at + 63 °C, fired at an angle of 33.75 degree from 1050-meter elevation

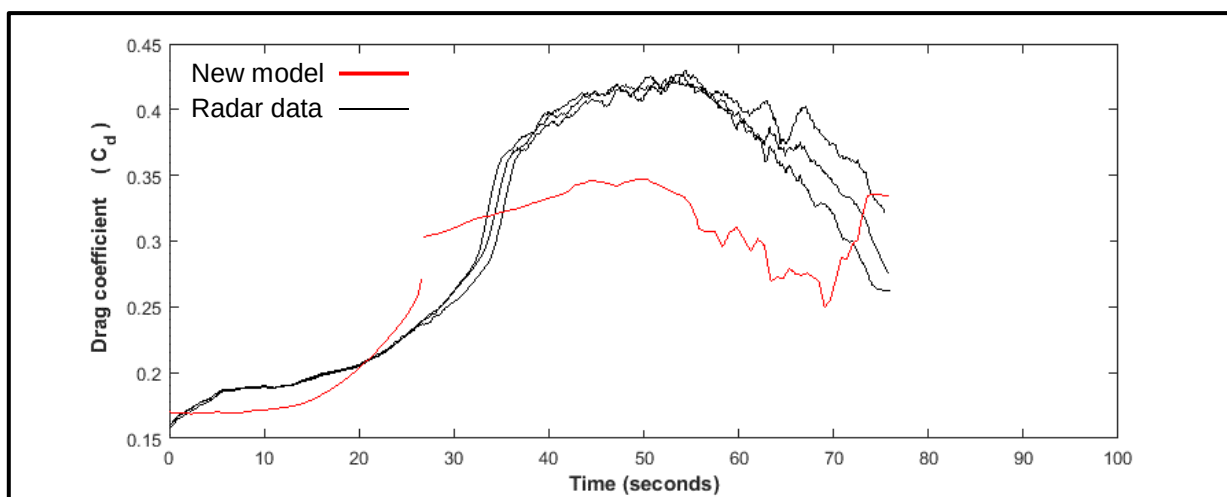


**Figure 5-15:** Shots with (961.52 m/s) average muzzle velocity, preconditioned at +21 C, fired at an angle of 33.75 degree from 1050-meter elevation

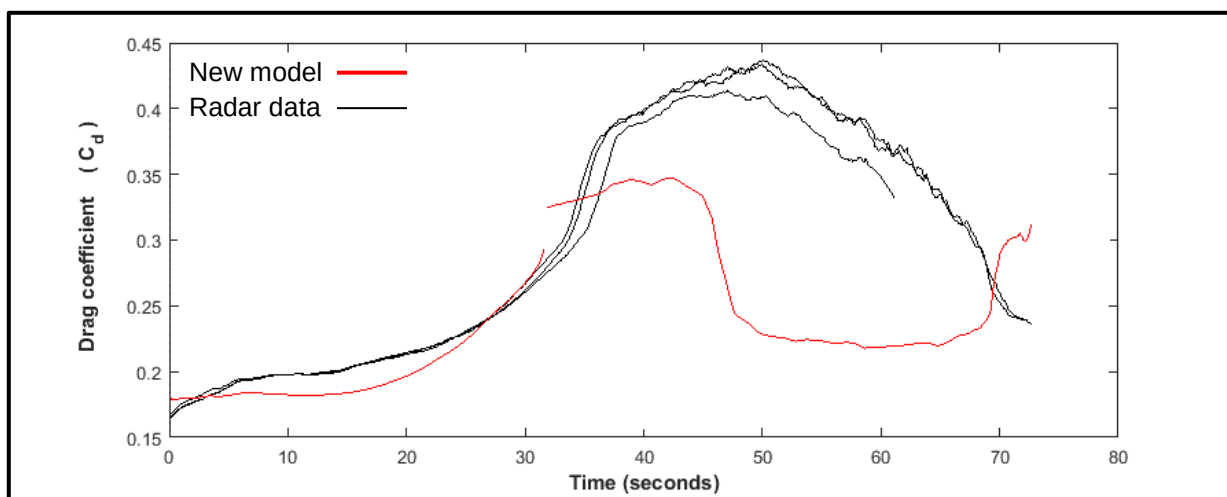


**Figure 5-16:** Shots with (901.06 m/s) average muzzle velocity, preconditioned at - 46 °C, fired at an angle of 33.75 degree from 1050-meter elevation

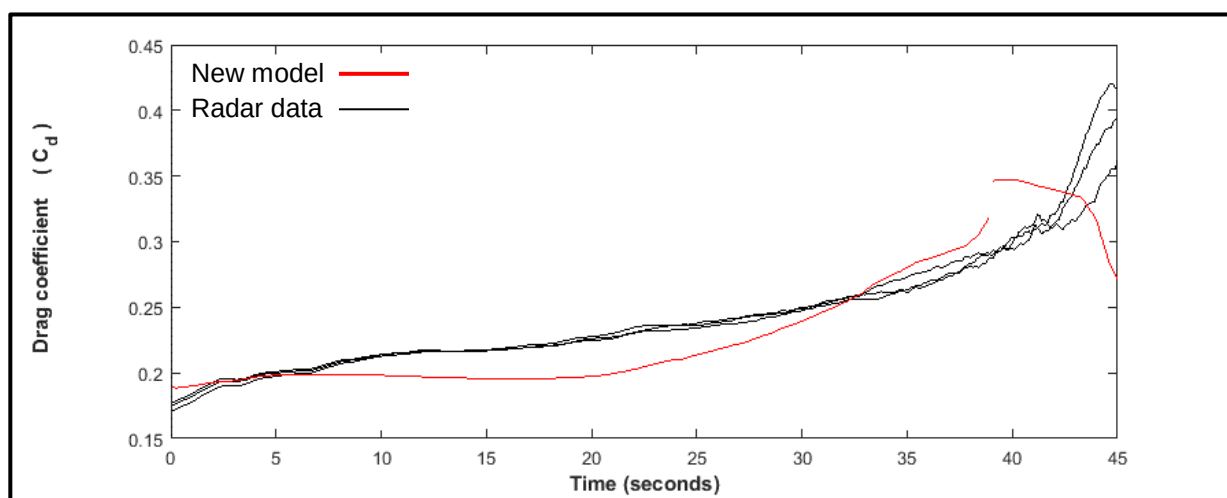




**Figure 5-17:** Shots with (855.5 m/s) average muzzle velocity, preconditioned at + 63 °C, fired at an angle of 33.75 degree from 1050-meter elevation

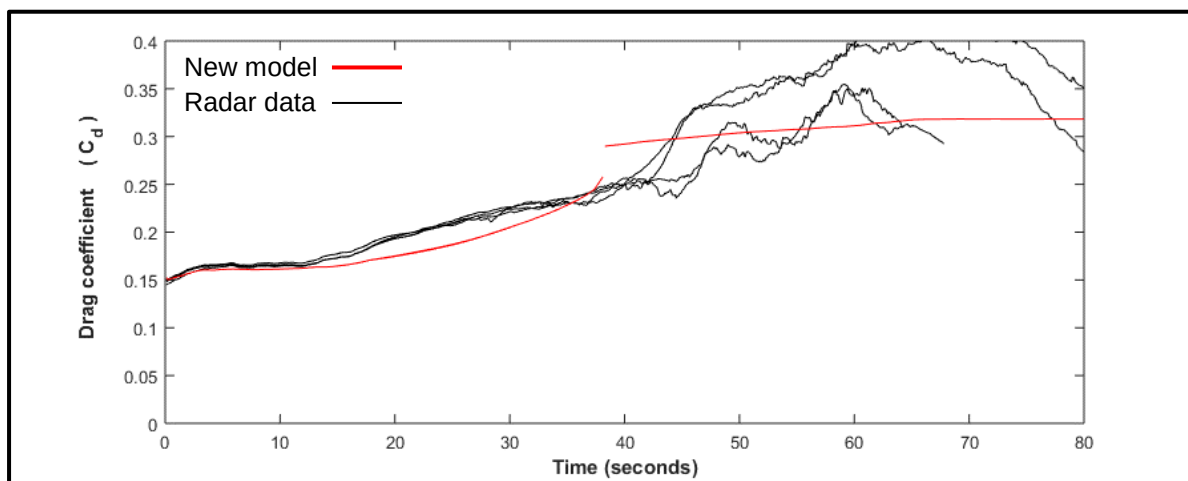


**Figure 5-18:** Shots with (807.57 m/s) average muzzle velocity, preconditioned at + 21 °C, fired at an angle of 33.75 degree from 1050-meter elevation

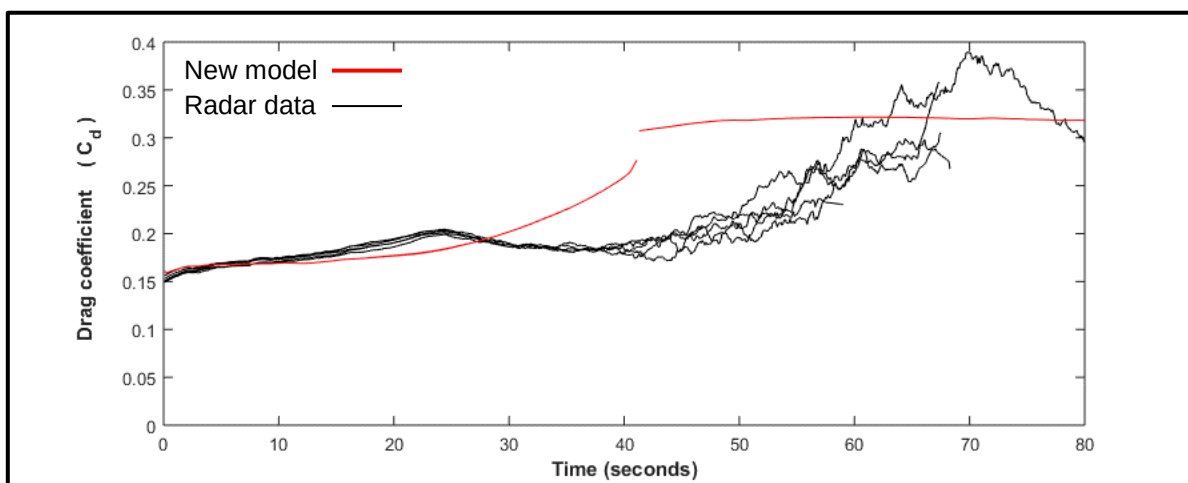


**Figure 5-19:** Shots with (771.74 m/s) average muzzle velocity, preconditioned at - 46 °C, fired at an angle of 33.75 degree from 1050-meter elevation

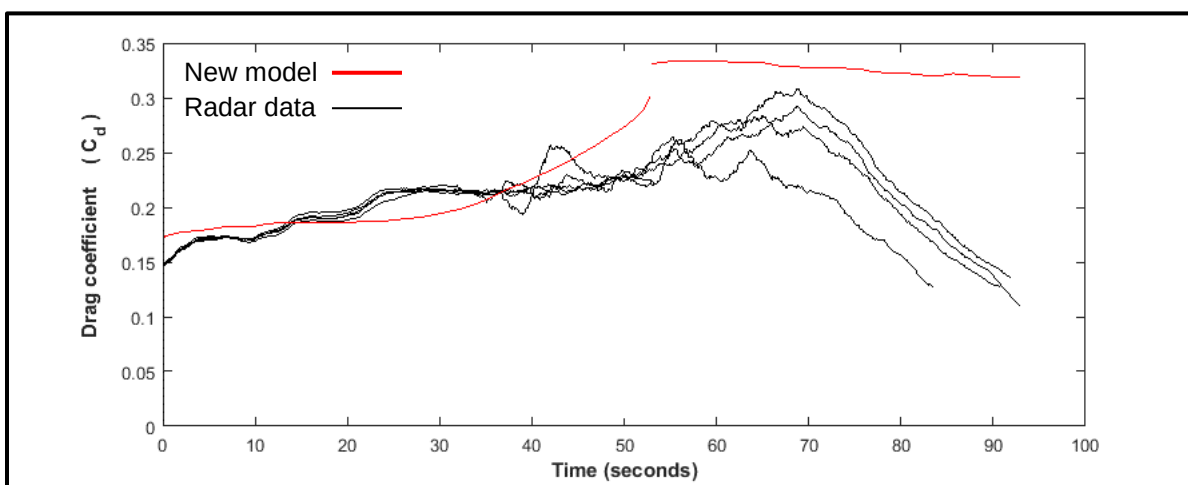
### 5.2.2.7 Trajectory at 42.2 degrees



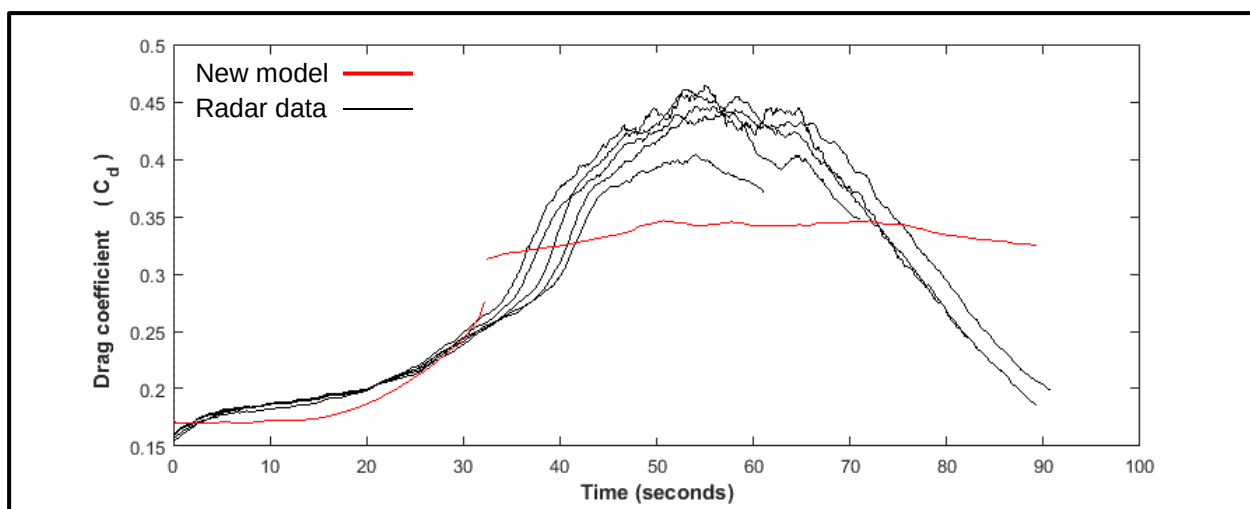
**Figure 5-20:** Shots with (988.28 m/s) average muzzle velocity, preconditioned at + 63 ° C, fired at an angle of 42.2 degree from 1050-meter elevation



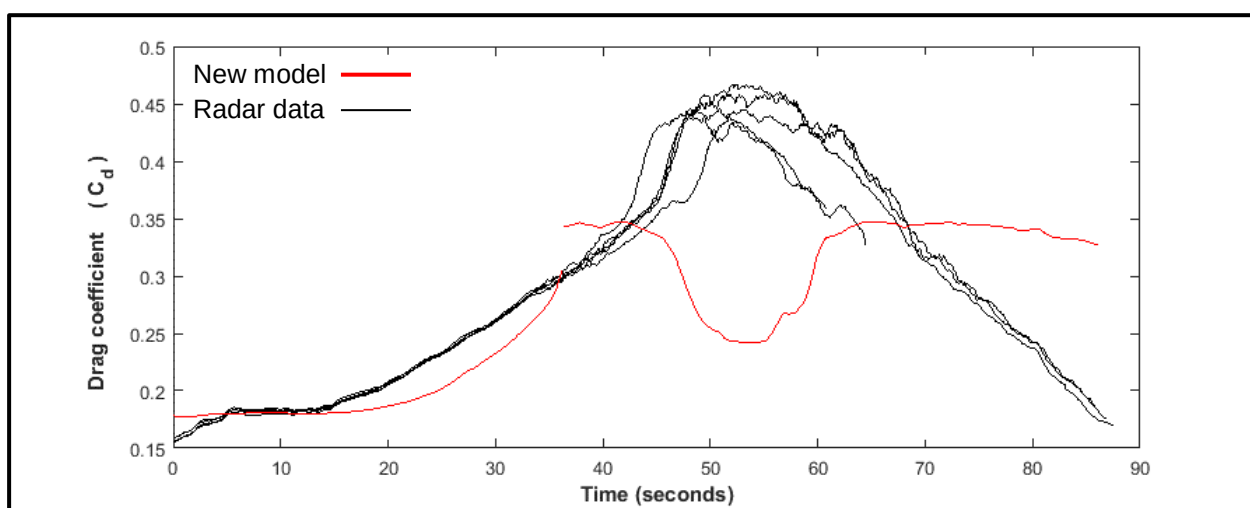
**Figure 5-21:** Shots with (955 m/s) average muzzle velocity, preconditioned at + 21 ° C, fired at an angle of 42.2 degree from 1050-meter elevation



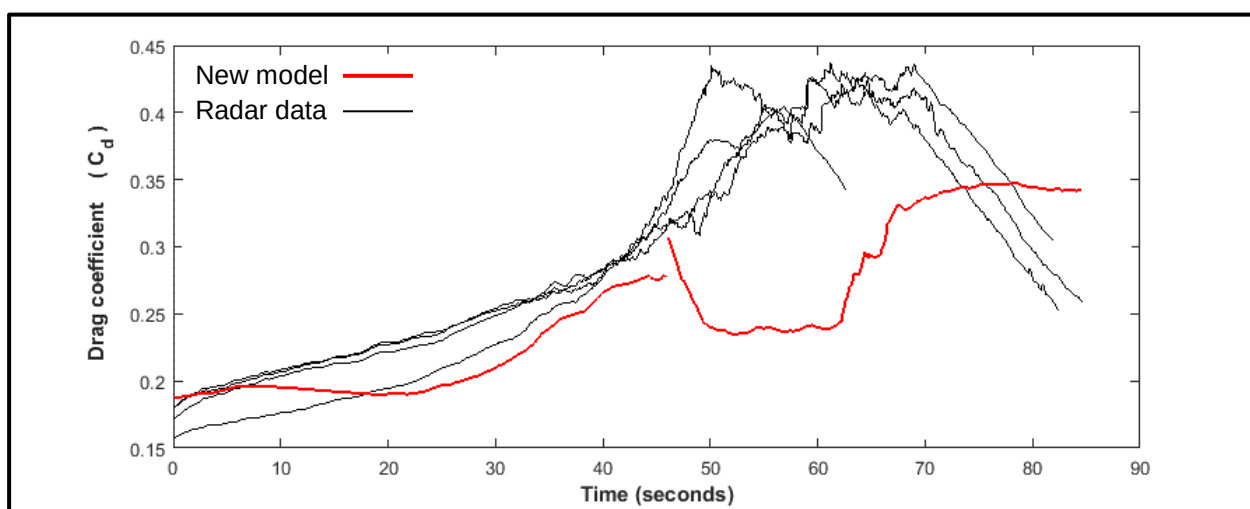
**Figure 5-22:** Shots with (901.17 m/s) average muzzle velocity, preconditioned at - 46 ° C, fired at an angle of 42.2 degree from 1050-meter elevation



**Figure 5-23:** Shots with (859.92 m/s) average muzzle velocity, preconditioned at + 63 °C, fired at an angle of 42.2 degree from 1050-meter elevation

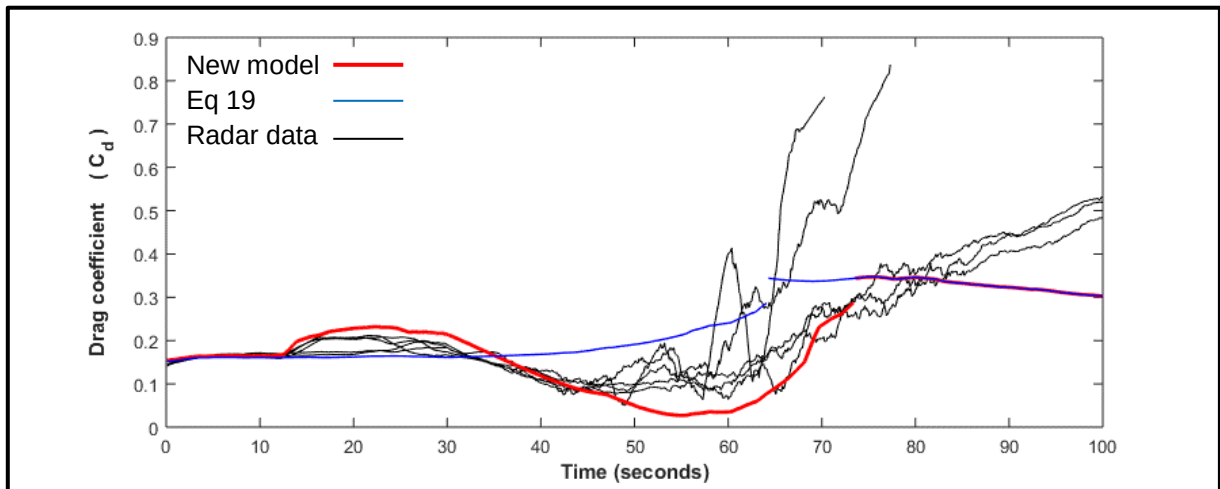


**Figure 5-24:** Shots with (813.09 m/s) average muzzle velocity, preconditioned at + 21 °C, fired at an angle of 42.2 degree from 1050-meter elevation

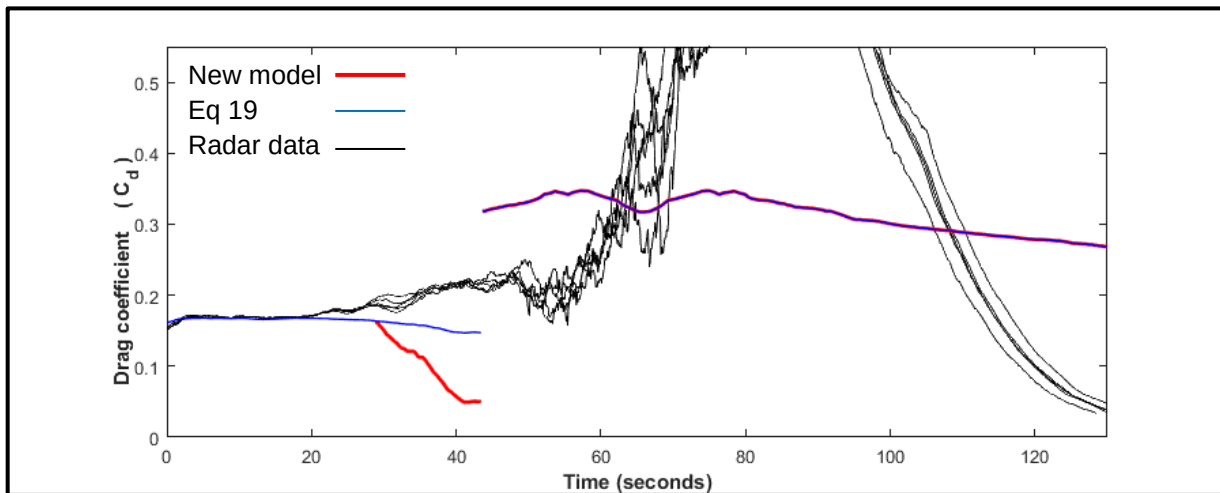


**Figure 5-25:** Shots with (790.03 m/s) average muzzle velocity, preconditioned at - 46 °C, fired at an angle of 42.2 degree from 1050-meter elevation

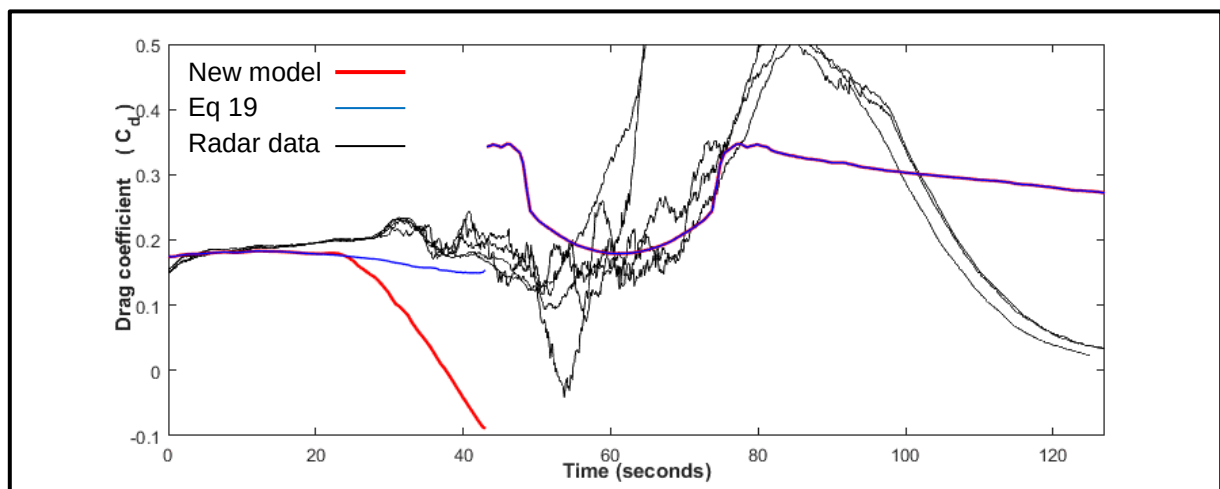
### 5.2.2.8 Trajectory at 64 degrees



**Figure 5-26:** Shots with (986.29 m/s) average muzzle velocity, preconditioned at + 63 °C, fired at an angle of 64 degree from 1050-meter elevation



**Figure 5-27:** Shots with (943.43 m/s) average muzzle velocity, preconditioned at + 21 °C, fired at an angle of 64 degree from 1050-meter elevation



**Figure 5-28:** Shots with (900.9 m/s) average muzzle velocity, preconditioned at - 46 °C, fired at an angle of 64 degree from 1050-meter elevation

### **5.3 Discussing of radar data and models**

The Doppler radar data shows good indication of drag coefficient during projectile flight. Some drag coefficient values need to be explained. Sometimes it shows values that are lower than minimum value (at zero yaw) for a given moment in flight which cannot be explained by the base bleed function. In other moments in flight, it shows very high values that cannot be explained by the projectile yaw or base bleed function. These observations can be explained by data method of integration that gives positive slopes instead of negative slopes. Another explanation is over amplification of the coefficient due to integration time step and method of integration used to obtain the drag coefficient.

The two models that are used to model the drag coefficient are based on the propellant burning and gas injection models discussed in Chapter 4. The two models use the base drag reduction model mentioned by Danberg (1990) and presented in eq. 19. The suggested base drag reduction model shown in eq. 34 and eq. 35 predicts an increase in drag due to the compressibility of injected gases and thrust force caused by supersonic gas injection.

The suggested model shows good prediction of the drag coefficient and burn-time in most cases. That confirms the accuracy of the burn and injection models as suggested. The cases illustrated in figures 5-10 and 5-26 indicate that the suggested model give accurate description base drag reduction, drag due to gas injection and impulse caused by supersonic injection. Cases like 5-11, 5-12, 5-13, 5-27 and 5-28 can be explained better with the suggested model. In these cases, the drag and impulse caused by base bleed injection need further investigation.

#### **5.3.1 Model limitation**

As the test data show the new suggested model is most accurate at high muzzle velocity shot. The model is less accurate with lower muzzle velocities. The igniter effect is approximated with empirical values and it is not modelled accurately. The model accuracy is limited in the following aspects:

- The lower muzzle velocity shots.
- The igniter effect and the initial phase of base bleed propellant burning.

## 5.4 Conclusion

The suggested new model gives a more accurate and scientific explanation of base bleed projectile trajectory performance. It suggests propellant burn model, gas injection model, and a base drag relieve model. In addition, it models supersonic injection that is responsible for increase in drag with increased injection and impulse resulting from supersonic injections. This model needs fewer tests than existing STANAG models. These tests are needed to scale limited number of correction factors. Mainly testing high velocity injections that occurs at high trajectories.

### 5.4.1 Future work proposal

Some limitations of this suggested model that need to be improved upon:

- Modelling of flow transition from subsonic to fully developed shock wave alongside the base bleed injector.
- Using of a uniform term like the Mach number of injected gases and Mach number of air to model the coefficient of pressure over the base.
- Giving better explanation for the initial phase of the base bleed functioning mainly the ignitor effect and the barrel blast effect.
- Better modelling of nozzle efficiency and drag due injection coefficient ( $C_{d_{b,Inj}}$ ).

The new suggested model could be a helpful tool for designing new projectiles that can make use of the base bleed performance and rocket performance using the same injection hole. These projectiles could reach longer ranges than base bleed projectiles using well designed nozzles. These projectiles could fill the gap between the base bleed projectiles and rocket assisted projectiles.

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## ANNEXURES (MATLAB CODE)

This MATLAB code used to model the base bleed performance:

```
function [Cdvalue,BBTime,CBdeltaPtest,massratetest,injection,Bbpressure,...
    Bcpressure,bbdensity,brtemper,Qratetest,MINj,Psidratio] = BBNmodel ...
    ( alpha,pressure, spinrate, Graintemp, ~, time,timestep,BBMach,...
    DensityAir,speedofsound,propthick,prophight,prophightfl,Machspeed,...
    Dragcoeff,CAlp)

% Cdvalue: is the resultant BB drag coefficient
% BBTime: Time of BB functioning
% CBdeltaP: Chamber to base pressure ratio
% massrate: Propellant mass flow rate
% injection: Injection rate
% Bbpressure: BB base pressure during flight (without functioning BB)
% Bcpressure: BB chamber pressure
% bbdensity: Gas density in side BB chamber
% brtemper: Propellant burn temperature
% Qrate: Volumetric injection flow rate
% MINj: injection Mach number for the flow over the orifice
% Y: Gas expansion factor
% alpha: Heat capacity ratio for propellant gases
% pressure: Static pressure
% spinrate: spin rate
% Graintemp: Precondition grain temperture
% time: total time of flight
% timestep: time step
% BBMach: Projectile Mach number
% DensityAir: Density of air
% speedofsound: Speed of sound
% propthick: Propellant burned thickness
% prophight: Propellant hight for a given burned thickness
% prophightfl: Propellant remnant volume for given burned thickness
% Dragcoeff: Drag coefficient for the projectile without BB
% Machspeed: Mach number for the drag coefficient matrix
%CAlp: Spin damping coefficient

% Initial conditions
% Sigma is the ratio d(P_b/P_0)/DI mentioned in (eq. 19)
SigmaI = 0;
% n is matrix counter
    n = 1;
    % (brpressure, brGraintemp, brspinrate) burn rate factors
brpressure = (0.9132*(pressure(n)/100000)^0.6655);
brGraintemp = exp(0.0035*(Graintemp-21));
brspinrate = 0.00023698*(spinrate+(0))+0.98691;
SumDburnrate = (1+(0))*(brpressure*brGraintemp* brspinrate);
BBdeltaP = 0;

Bcpressure = pressure(n)/100000;
Machnumber = [0.71 0.82 0.98 1.58 1.88 2 2.48 2.99];
%Base pressure to static pressure ratio values for several Mach number
dBpconst = [0.9536 0.9353 0.8525 0.6699 0.5745 .5685 0.4228 0.3282];
%CorrMofP is dBpconst for given Mach number
    CorrMofP = interp1(Machnumber,dBpconst,BBMach(n));
Bbpressure = (CorrMofP+BBdeltaP)*(pressure(n)/100000);
```

```

%beta: The orifice opening diameter to base diameter
beta = (22/65);
% if there is a given trajectory Matrix Msize is the matrix size
[Msize,~] = size(time);
%brigniter: Iginter burn temperature
brigniter (Msize) = zeros;

brigniter (1:35) = 2200;

CBdeltaPtest (Msize) = zeros;
% if the trajectory model uses a loop the while loop can be changed to if
%loop
%Sumburnrate: Total burn diameter at any given moment
while SumDburnrate < 40

    CorrMofP = interp1(Machnumber,dBpconst,BBMach(n));

    Bbpressure (n)= (CorrMofP+BBdeltaP)*(pressure(n)/1000000);

    brpressure = (0.9132*(Bbpressure(n))^0.6655);

    brGraintemp = exp(0.0035*(Graintemp-21));

    brspinrate = 0.00023698*(spinrate)+0.98691;

% The spin rate is use in the loop when needed
spinrate = spinrate+(Assegaispindamping (CAIp,DensityAir(n),...
speedofsound(n),BBMach(n),.155,spinrate)*0.1/0.1444);

% The term SumB is used to correct for progressive burn rate
    SumB = SumDburnrate/40;
    Dburnrate(n) = (1+1.15*( 2.8587*SumB^4-5.0027*SumB^3-0.1787*SumB^2+...
    2.9927*SumB))*(brpressure*brGraintemp* brspinrate);
    SumDburnrate = timestep*Dburnrate(n)+SumDburnrate;
    Columheight = interp1 (propthick,prophight,SumDburnrate);

    ACylinder = 0.5*2*(Columheight*(2*pi*(25+SumDburnrate)- ...
    (3*(25+SumDburnrate )^2*asin((2*SumDburnrate +3)/(2*(25+SumDburnrate...
    )))))/1000000;

    ASides = 0.5*6*interp1(propthick,prophightfl,SumDburnrate)/1000000;
% devided by 1000000 to convert to meter squired.
    Aburntotal = ((ACylinder + ASides));
    massrate = ...
    1.532*(((Aburntotal*Dburnrate(n)))-(6*Dburnrate(n)*Columheight)/1000000);

    massratetest(n)= massrate;

    brtemper(n) = (817.33*Dburnrate(n)^1.5465)+brigniter(n);

% R = 0.083144, molar mass of exhust gases = 19.6
    Bdensity = Bcpressure(n)*19.6/((brtemper(n))*0.08314472);
    Qrate = massrate/Bdensity;
    Qratetest(n)=Qrate;
% Injection mach number using Ideal gas law

```

```
bbdensity(n) = Bdensity;
```

```
Psibyratio = [0.00,0.16,0.23,0.28,0.32,0.36,0.40,0.43,0.46,0.49,0.52,0.55,...
0.58,0.60,0.63,0.65,0.68,0.70,0.73,0.75,0.77,0.80,0.82,0.84,0.86,0.89,...
0.91,0.93,0.95,0.97,1.00,1.02,1.04,1.06,1.08,1.11,1.13,1.15,1.17,1.20,...
1.22,1.24,1.27,1.29,1.31,1.34,1.36,1.39,1.41,1.44,1.46,1.49,1.51,1.54,...
1.57,1.59,1.62,1.65,1.68,1.71,1.74,1.77,1.80,1.83,1.86,1.90,1.93,1.97,...
2.00,2.04,2.08,2.12,2.16,2.21,2.25,2.30,2.34,2.39,2.45,2.50,2.56,2.62,...
2.68,2.75,2.83,2.90,2.99,3.08,3.18,3.28,3.40,3.54,3.69,3.86,4.07,4.32,...
4.64,5.06,5.70,6.89];
```

```
ptoverP = [1,0.99,0.98,0.97,0.96,0.95,0.94,0.93,0.92,0.91,0.9,0.89,0.88,...
0.87,0.86,0.85,0.84,0.83,0.82,0.81,0.8,0.79,0.78,0.77,0.76,0.75,0.74,...
0.73,0.72,0.71,0.7,0.69,0.68,0.67,0.66,0.65,0.64,0.63,0.62,0.61,0.6,...
0.59,0.58,0.57,0.56,0.55,0.54,0.53,0.52,0.51,0.5,0.49,0.48,0.47,0.46,...
0.45,0.44,0.43,0.42,0.41,0.4,0.39,0.38,0.37,0.36,0.35,0.34,0.33,0.32,...
0.31,0.3,0.29,0.28,0.27,0.26,0.25,0.24,0.23,0.22,0.21,0.2,0.19,0.18,...
0.17,0.16,0.15,0.14,0.13,0.12,0.11,0.1,0.09,0.08,0.07,0.06,0.05,0.04,...
0.03,0.02,0.01];
```

```
% change in pressure due to flux (R = 8134, molar mass of exhaust gases ...
% = 19.6 ; Psidratio: is the flux factor mulitibplied by chamber pressuree to
% ambient pressure ratio (ptbyP) as (eq 15) and (eq 16)
```

```
Psidratio (n) =
(massrate*(brtemper(n)*alpha*8134.489/19.6)^.5)/((pi*0.0225^2)*Bbpressure(n)*
100000);
```

```
ptbyP (n) = interp1(Psibyratio,ptoverP,Psidratio (n));
```

```
Bcpressure(n+1) = pressure(n)/100000/ptbyP (n);
```

```
% Gas law
```

```
MInj(n) = (((100000* ptbyP (n)^(1/alpha))/pressure(n))*Qrate...
/(pi*0.0225^2)/(brtemper(n)*alpha*8134.489/19.6)^.5)^.5;
```

```
%Injection rate fitting
```

```
Inj = ((massrate)/(DensityAir(n)*BBMach(n)*speedofsound(n)*pi*0.065^2));
```

```
% Sigma is the ratio d(P_b/P_0)/DI; T_jet: is the temperature of ...
% injected gases
```

```
T_jet = [1500 1200 600 300];
```

```
Sigma(1) = 10.25992*BBMach(n)^2.167378;
```

```
Sigma(2) = 8.010123*BBMach(n)^2.207893;
```

```
Sigma(3) = 3.977058*BBMach(n)^2.321928;
```

```
Sigma(4) = 2.040912*BBMach(n)^2.440573;
```

```
SigmaI = interp1(T_jet,Sigma,1500);%brtemper(n)
```

```
% BetaIP injection for base pressure = (15.1-46.3*(BBMach(n)-1.71)
```

```
Pbofratio = ((SigmaI*Inj)/(1+(2.6)*SigmaI*Inj));
```

```
Injection(n) = Inj;
```

```

% CDInj: Base drag factor due to Injection f_b

MachInj = [ 0.0 0.6 0.7 0.8 0.9 1 1.15 1.2 1.5 2.1 3 5];
CDInj = [ 0.0 0.0 0.0001 0.1 0.4 0.883919298 0.8 0.75 0.55 0.37
0.20 0.15];

%CDInj = [ 0.0 0.0 0.0001 0.014 0.16 0.88 0.883919298 0.9 0.7 0.4 0.35];

BBCDrag = interp1(MachInj,CDInj,MInj(n));

Cdvalue(n) = interp1(Machspeed,Dragcoeff,BBMach(n))...
-((Pbofratio)*1.47/BBMach(n)^2)...
+(alpha*BBCDrag/155^2)*(((130^2-45^2)*(0.24/2)*((MInj(n)^2)))...
-(3/1.1)*(45^2/1.4)*((100000*Bcpresure(n))/(pressure(n)))...
*((MInj(n)^2)/(BBMach(n))^2));

injection(n) = Inj;

n = n+1;
BBTime(n) = timestep*(n-1);

% Breaking while loop if neccesary
%(Injection neverstops throughout the flight)
if n == Msize
    break
end
end

Cdvalue(n:Msize) = interp1(Machspeed,Dragcoeff,BBMach(n:Msize));

%plot (time(1:Msize),Cdvalue(1:Msize) )
%aero(:,2)= Cdvalue(1:50:250)';
%aero(:,1)= BBMach(1:50:250);

```

**LAST UPDATED: 12 MARCH 2019**