

Chapter 8

A Hybrid Model for Cosmic Ray Modulation

8.1 Introduction

In order to model and interpret Voyager CR observations in the heliosheath (and beyond in the local ISM), a realistic approach to modulation is required, i.e. the complex nature of the plasma flow and heliospheric geometry in this region must be taken into account. As demonstrated in Chapter 3, analytical descriptions of a realistic heliospheric geometry and plasma flow are impossible, and the need has arisen to follow a hybrid modelling approach. In this chapter, such a model is presented, where the output from the MHD model, discussed in Chapter 3, is coupled to an SDE based modulation model, where the CRs are treated essentially as test particles. Similar models were presented by *Ball et al.* [2005], *Florinski and Pogorelov* [2009] and *Luo et al.* [2013]. After results from this model have been shown and discussed, the model is applied to the study of GCR modulation beyond the HP. Although the V1 crossing of the HP is not as yet officially confirmed, various CR measurements [e.g. *Webber et al.*, 2012a] indicate that the spacecraft may have already crossed into the ISM, making a study of the modulation of GCRs beyond the HP a very pertinent subject. This chapter thus aims to answer the following question: After crossing the HP, will V1 measure the pristine LIS for GCRs, or will it continue to observe some modulation and a small but persistent positive radial intensity gradient?

Details of this work are presented by *Strauss et al.* [2013b].

8.2 Towards a Hybrid Modulation Model for Cosmic Rays

Output from the 3D MHD model described in Chapter 3 serves as the plasma background on which the SDE based CR modulation model is solved. In order to facilitate this coupling process, the TPE is also solved in Cartesian coordinates [see also *Kopp et al.*, 2012]

$$\begin{aligned} d\vec{r} &= (\nabla \cdot \mathbf{K} - \vec{u} - \vec{v}_d) ds + \mathbf{C} \cdot \vec{w} \\ dP &= \frac{P}{3} (\nabla \cdot \vec{u}) ds, \end{aligned} \quad (8.1)$$

with $2\mathbf{K} = \mathbf{C} \cdot \mathbf{C}^T$ and $\vec{r} = \{\vec{x}, \vec{y}, \vec{z}\}$. Again, time backwards integration is performed, where a pseudo-particle is integrated from an initial *observational* point until the boundary of the computational domain of the MHD model (which also forms the integration domain of the SDE model) is reached. At this position, a cube with volume $(800 \text{ AU})^3$, the LIS for GCRs is specified as a boundary condition. Because the SDE approach is grid-free and the MHD quantities are specified on a grid, the plasma quantities must be interpolated to the present position of a pseudo-particle during its integration process. Here, a cubical tri-linear interpolation is performed. The following MHD calculated quantities serve as input to the SDE model and describe the associated processes: Convection due to $\vec{u}$ (both in magnitude and direction), energy losses through $\nabla \cdot \vec{u}$ (discussed in more detail below) and drifts through $\nabla \times \vec{B}/|\vec{B}|^2$ (also discussed below). The diffusion tensor is, however, described in a phenomenological fashion, although $|\vec{B}|$ influences the magnitude of the diffusion coefficients and $\vec{B}/|\vec{B}|$ their orientation. At present, a clear theoretical description of CR scattering and underlying turbulence beyond the TS is lacking, although recent progress has been made by e.g. *Zank et al.* [2012]. The parallel mean free path is assumed to be given by

$$\lambda_{||} = \lambda_0 \frac{B_0}{|\vec{B}|} \left[\frac{P}{P_0} \right]^\delta, \quad (8.2)$$

with $P_0 = 1 \text{ GV}$ and δ a parameter determining the rigidity dependence,

$$\delta = \begin{cases} 1 & : P > P_0 \\ 1/3 & : P \leq P_0 \end{cases}, \quad (8.3)$$

based on the results of *Teufel and Schlickeiser* [2003]. Similarly,

$$B_0 = \begin{cases} 5 \text{ nT} & : \text{Inside HP} \\ |\vec{B}| & : \text{Beyond HP} \end{cases} \quad (8.4)$$

and

$$\lambda_0 = \begin{cases} 0.1 \text{ AU} & : \text{Inside HP} \\ 10^5 \text{ AU} & : \text{Beyond HP} \end{cases}, \quad (8.5)$$

so that the ISM diffusion coefficients have no spatial dependence. Furthermore, isotropic perpendicular diffusion is assumed

$$\lambda_{\perp 1,2} = \eta \lambda_{||}, \quad (8.6)$$

with η a constant.

As already discussed in Chapter 2, the diffusion tensor is defined in terms of $\vec{B}$ -aligned coordinates and must be transformed to a Cartesian coordinate system before integration can be performed [see also *Effenberger et al.*, 2012]. Here, $\mathbf{K}$ becomes

$$\begin{aligned}
 \kappa_{xx} &= \kappa_{||} \sin^2 \Phi_2 \cos^2 \Phi_1 + \kappa_{\perp 1} \sin^2 \Phi_2 \sin^2 \Phi_1 + \kappa_{\perp 2} \cos^2 \Phi_1 \cos^2 \Phi_2 \\
 \kappa_{xy} &= \kappa_{||} \cos \Phi_2 \cos^2 \Phi_1 \sin \Phi_2 + \kappa_{\perp 1} \cos \Phi_2 \sin^2 \Phi_1 \sin \Phi_2 - \kappa_{\perp 2} \sin \Phi_2 \cos^2 \Phi_1 \cos \Phi_2 \\
 \kappa_{xz} &= \kappa_{||} \sin \Phi_1 \sin \Phi_2 \cos \Phi_1 - \kappa_{\perp 1} \cos \Phi_1 \sin \Phi_2 \sin \Phi_1 \\
 \kappa_{yy} &= \kappa_{||} \cos^2 \Phi_2 \cos^2 \Phi_1 + \kappa_{\perp 1} \cos^2 \Phi_2 \sin^2 \Phi_1 + \kappa_{\perp 2} \sin^2 \Phi_2 \cos^2 \Phi_1 \\
 \kappa_{yz} &= \kappa_{||} \sin \Phi_1 \cos \Phi_1 \cos \Phi_2 - \kappa_{\perp 1} \cos \Phi_1 \cos \Phi_2 \sin \Phi_1 \\
 \kappa_{zz} &= \kappa_{||} \sin^2 \Phi_1 + \kappa_{\perp 1} \cos^2 \Phi_1
 \end{aligned} \tag{8.7}$$

with $\kappa_{yx} = \kappa_{xy}$, $\kappa_{zx} = \kappa_{xz}$ and $\kappa_{zy} = \kappa_{yz}$. The angles Φ_1 and Φ_2 are defined by

$$\sin \Phi_1 = \frac{B_z}{|\vec{B}|} \tag{8.8}$$

and

$$\tan \Phi_2 = \frac{B_x}{B_y} \tag{8.9}$$

and calculated from the MHD output. In the equatorial plane, for Parker HMF, $B_z = 0$, $B_x, B_y \neq 0$ and $\Phi_1 = 0 \Rightarrow \sin \Phi_1 = 0, \cos \Phi_1 = 1$, then

$$\kappa_{xx} = \kappa_{||} \sin^2 \Phi_2 + \kappa_{\perp 2} \cos^2 \Phi_2 \tag{8.10}$$

$$\kappa_{xy} = \kappa_{||} \cos \Phi_2 \sin \Phi_2 - \kappa_{\perp 2} \sin \Phi_2 \cos \Phi_2$$

$$\kappa_{xz} = 0$$

$$\kappa_{yy} = \kappa_{||} \cos^2 \Phi_2 + \kappa_{\perp 2} \sin^2 \Phi_2$$

$$\kappa_{yz} = 0$$

$$\kappa_{zz} = \kappa_{\perp 1} \tag{8.11}$$

so that $\kappa_{\perp 1}$ is directed in the $\mathbf{z}$ direction, while $\kappa_{\perp 2}$ lies in the $\mathbf{xy}$ -plane, completing the orthogonal coordinate system.

The drift velocity is calculated from the MHD output, by numerically evaluating the quantity

$$\vec{v}_d \frac{3}{\beta P} = \nabla \times \frac{\vec{B}}{|\vec{B}|^2}, \tag{8.12}$$

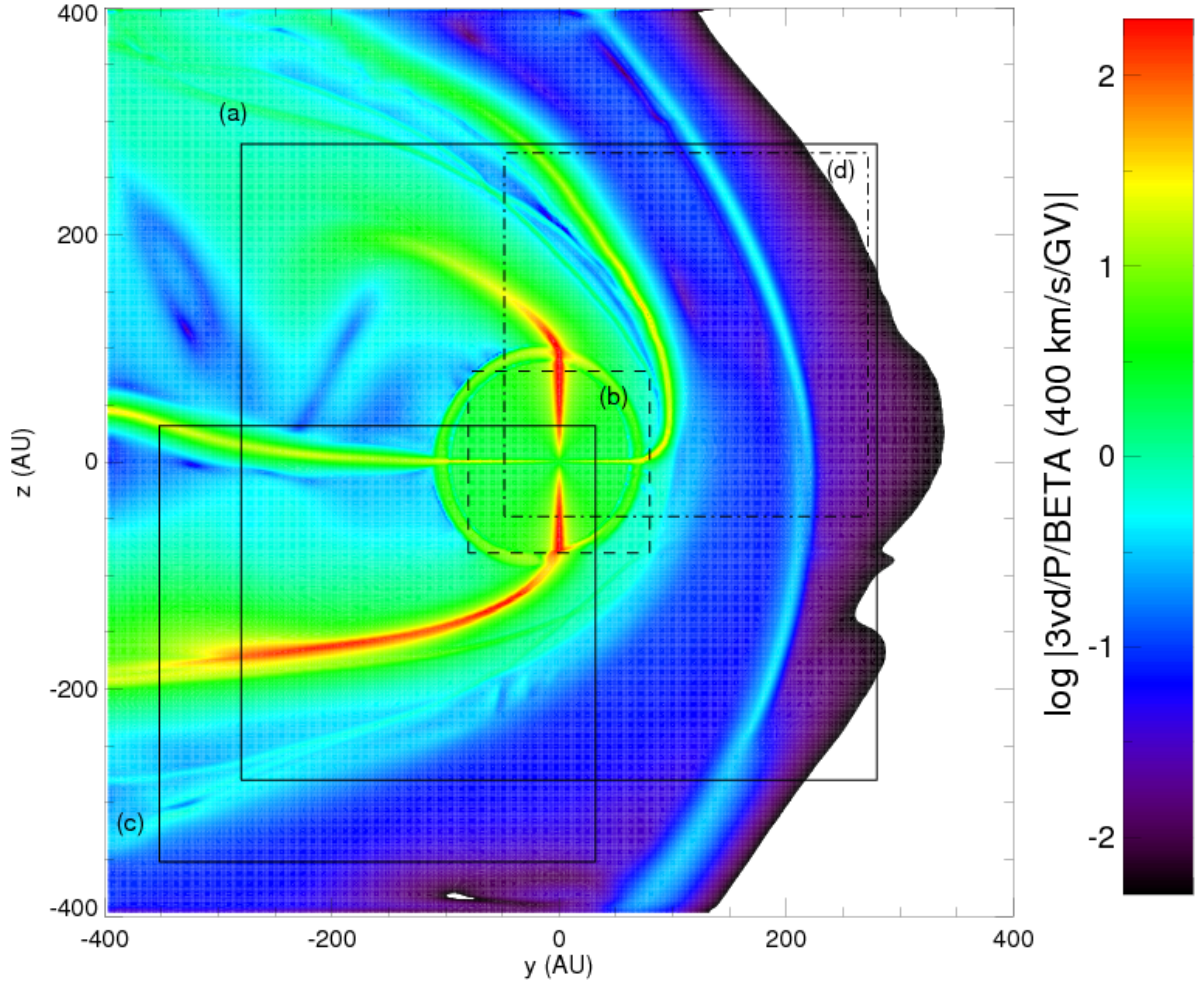


Figure 8.1: The magnitude of $\nabla \times \vec{B}/|\vec{B}|^2 = 3\vec{v}_d/\beta P$, as calculated from the MHD model, shown in the meridional plane of the heliosphere. The different labelled boxes are discussed in Fig. 8.2. Note the large drift speeds along the flat HCS and in the polar regions inside the TS.

where P is particle rigidity.

The magnitude of this, which is proportional to $\vec{v}_d$ at a given energy, is shown in Fig. 8.1 as a contour plot in the meridional plane of the heliosphere. The drift speed is largest at the HCS regions and in the polar regions. Note that the polar outflow from the Sun, and embedded quasi-open HMF lines, form *polar plumes* extending towards the tail of the heliosphere, with very large values of $\vec{v}_d$ in these regions. Outside the TS regions, the drift speed is generally small and will have no, or very little, effect on CR modulation. In the vicinity of the BS however, $\vec{v}_d$ is again large. This too should have little/no effect on CR transport, due to the large diffusion coefficients assumed in this region. To illustrate the direction of $\vec{v}_d$, four boxes are selected (as indicated in the figure) and the resulting flow lines of $\vec{v}_d$ shown in Fig. 8.2. Panel (b) shows a region inside the TS, illustrating the well-known Parkerian drift motion near the Sun for positive CR in the $A > 0$ drift cycle: These particles drift from the polar regions, down to the HCS regions and along the HCS towards the TS. This direction changes in the inner

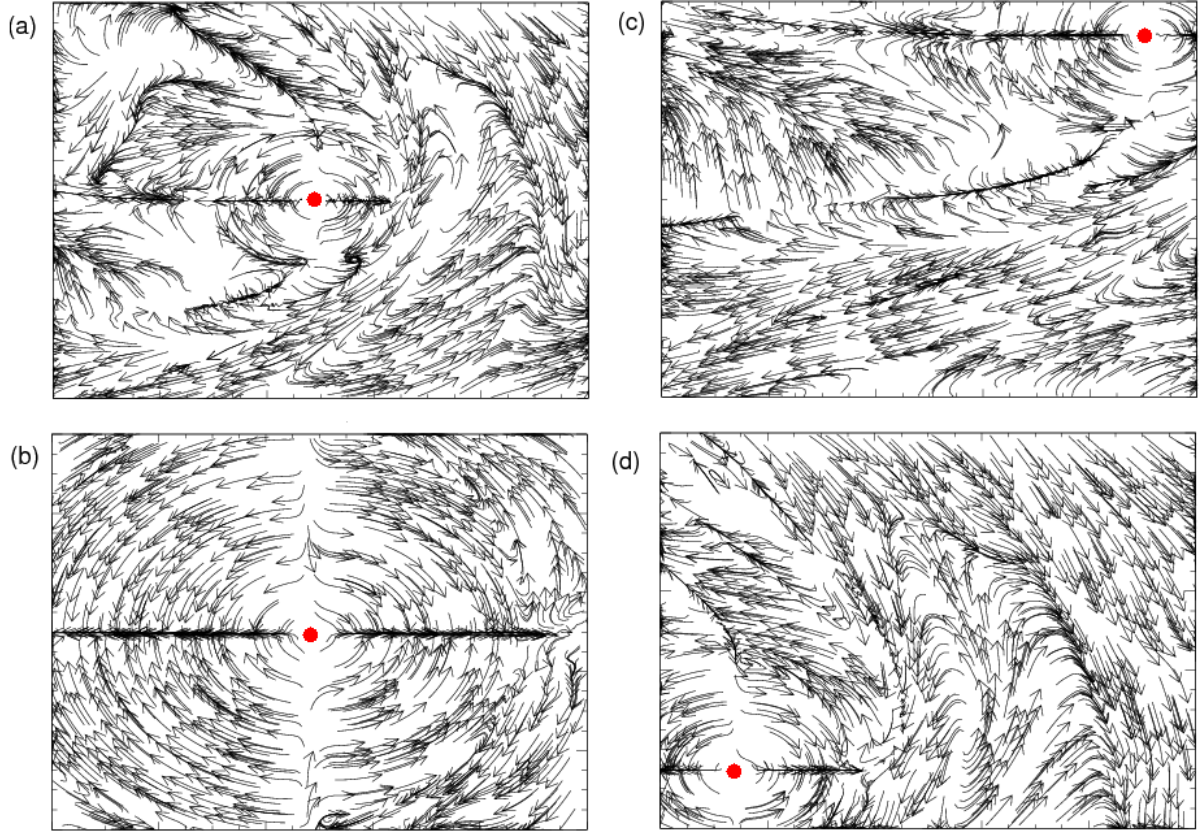


Figure 8.2: The direction of $\vec{v}_d$ (for positive CRs in the $A > 0$ drift cycle) for the four regions shown in Fig. 8.1. Note that these flow lines are normalized to the same length and do not represent the magnitude of $\vec{v}_d$. Panel (b) essentially shows the drift speed inside the TS and corresponds to the classical Parker HMF drift field as discussed by Jokipii and Kopriva [1979]. The red dots indicate the position of the Sun.

heliosheath (due to the change in the gradient of $\vec{B}$), so that the CRs again drift to the polar regions from the HCS regions - see also Panels (a) and (d). This is in accordance with the condition that $\nabla \cdot \vec{v}_d = 0$. Panel (a), where the global drift flow lines are shown, illustrates the complex behaviour of $\vec{v}_d$ in an MHD simulated heliospheric environment.

Fig. 8.3 shows $|\nabla \cdot \vec{u}|$, again calculated from the MHD output, in the meridional plane of the heliosphere. Blue indicates regions where $\nabla \cdot \vec{u} > 0$, with CRs propagating through these regions losing energy adiabatically, while the red regions (where $\nabla \cdot \vec{u} < 0$) indicate energy gains and possible acceleration zones. Outside of the TS, the flow can generally be considered to be incompressible with $\nabla \cdot \vec{u} \sim 0$. At the TS, the very large negative value of $\nabla \cdot \vec{u}$ is used to simulate diffusive shock (Fermi I) acceleration occurring here. Although Fermi I acceleration is fundamentally an energy gain mechanism where particles gain energy through collisions with scattering centres located on both sides of a shock, here, where the CR distribution function is modelled, Fermi I acceleration occurs due to a compression at the TS, resulting essentially in an adiabatic heating effect. Although the modelled TS is a strong shock with $s \sim 4$, it should be kept in mind that the TS structure is very diffuse, i.e. spatially smeared-out, leading to an

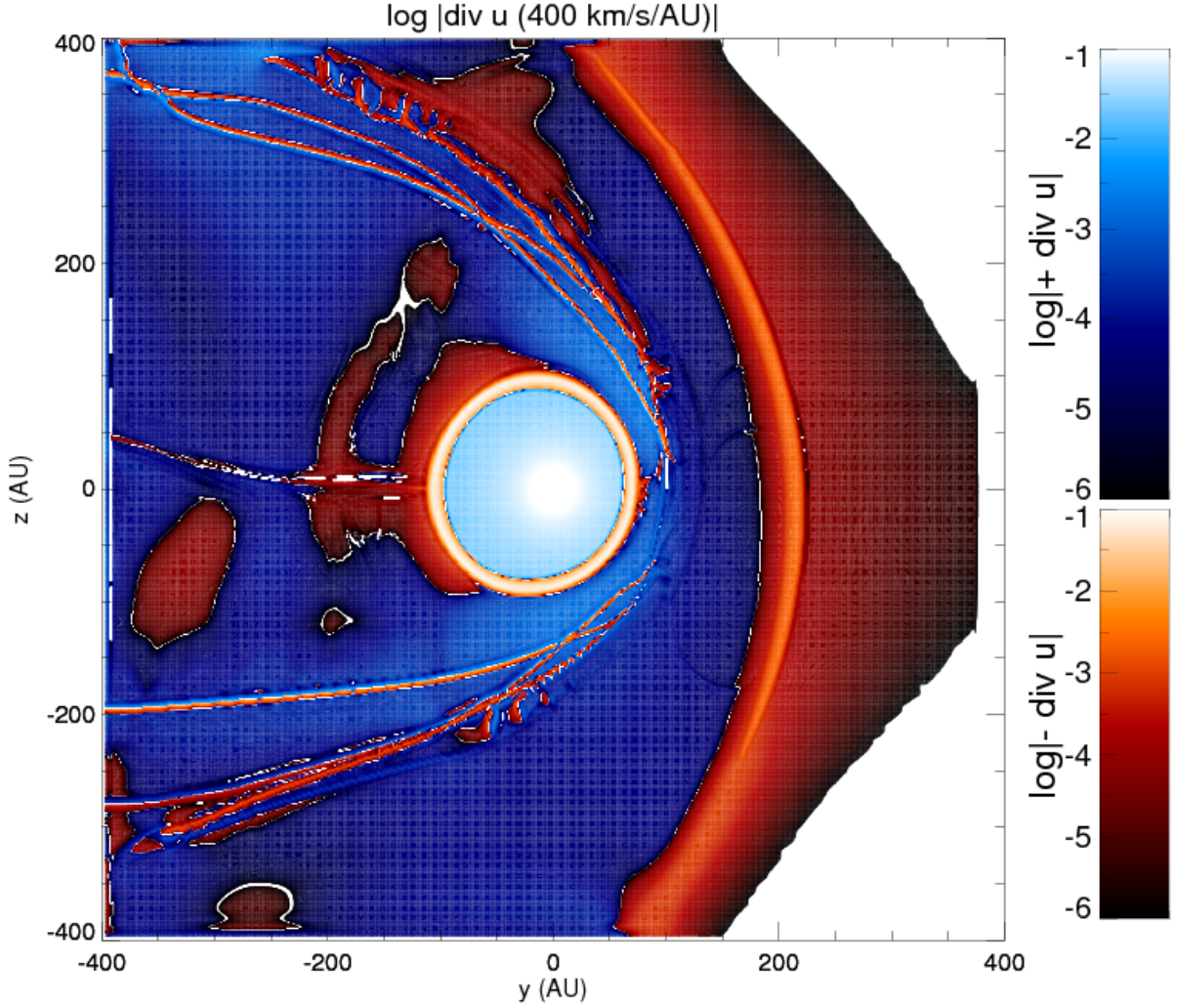


Figure 8.3: The magnitude of $\nabla \cdot \vec{u}$ (blue indicating negative values and red positive values) in the meridional plane of the heliosphere. The Sun is located at the origin, with the light blue region corresponding to supersonic solar wind flow and the TS region indicated by the large negative value of $\nabla \cdot \vec{u}$ (i.e. the light red regions).

effective shock thickness of ~ 10 AU. This is due to the nature of the MHD solver used and the subsequent finite size of the computational cells. This shock will therefore not accelerate particles nearly as efficiently as a discontinuous TS with the same compression ratio [see e.g. *Krülls and Achterberg, 1994*]. Moreover, the inclusion of Fermi I acceleration into the modulation model, puts a restriction on the numerical time step used in the SDE integration scheme. During the integration process, the average spatial integration step (the first-order moment of the relevant SDE) must be small enough for the pseudo-particles to sufficiently sample the acceleration region [*Krülls and Achterberg, 1994; Achterberg and Schure, 2011*]. Similarly, two very different transport regions are modelled here: The heliosphere where the diffusion coefficients are relatively small compared to the extremely large diffusion coefficients assumed in the ISM. This leads to a second restriction on the second-order moment of the spatial SDEs. The numerical time step is thus confined by using

$$\Delta s = \min \left\{ \frac{L}{\max |(\nabla \cdot \mathbf{K}, \vec{u}, \vec{v}_d)|}, \frac{L^2}{\max(\mathbf{K})} \right\}, \quad (8.13)$$

with $\max(\mathbf{K}) = \kappa_{||}$ and $L = 1$ AU. The pseudo-particles will therefore never (on average) propagate more than L in a single time step.

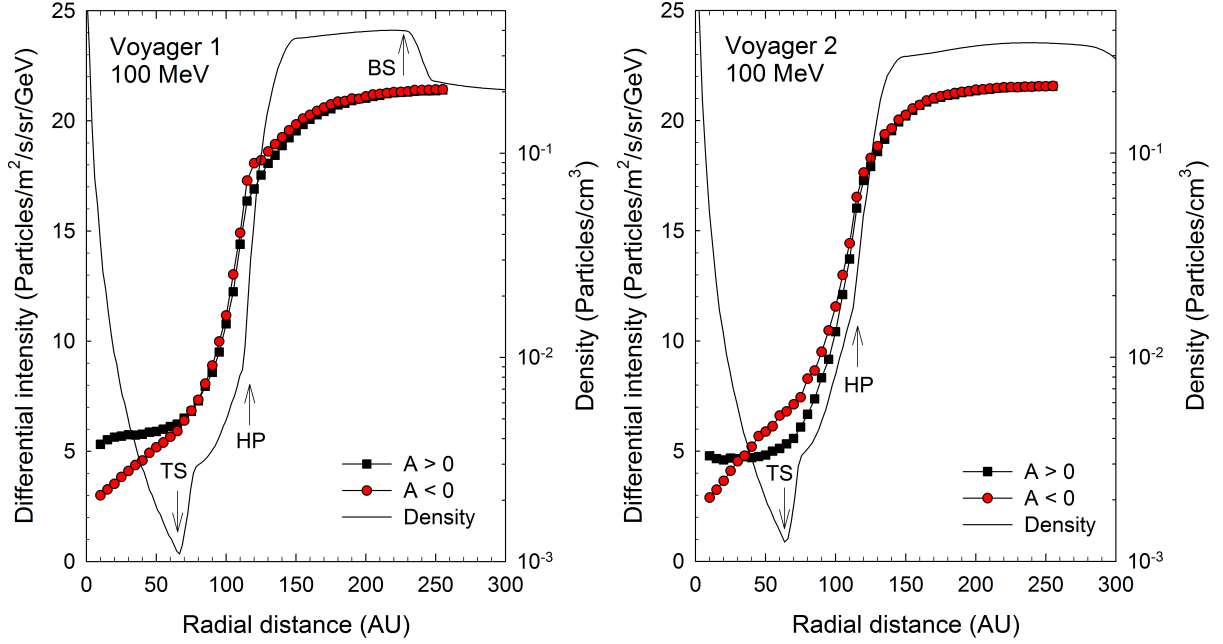


Figure 8.4: Computed 100 MeV proton intensity along the V1 (left panel) and the V2 (right panel) trajectories for the $A > 0$ (black squares) and the $A < 0$ (red circles) drift cycles. The computed radial profile of ρ (solid line) along these trajectories is added in order to distinguish the different heliospheric regions.

8.3 General Results

Fig. 8.4 shows results from the hybrid modulation model. Here, the 100 MeV proton intensity is shown along the V1 (left panel) and V2 (right panel) trajectories for the $A > 0$ (black squares) and the $A < 0$ (red circle) drift cycles. In order to differentiate the different heliospheric regions, the solid lines show the plasma density along these trajectories in order to distinguish the different heliospheric regions. In the coordinate system used, the V1 trajectory is approximated as a straight line, inclined 35° N of the equatorial plane and 5° towards $-x$. V2 is inclined 30° S and 45° towards x . The transport of CRs can be classified into three distinct regions by the radial intensity gradient: Inside the TS, a small gradient is evident, which increases in the inner heliosheath. Beyond the HP, a small gradient is again present; an effect discussed in the next section. For V1, the maximum drift effects (i.e. the maximum intensity difference between successive drift cycles) occur inside the TS, with a larger gradient but lower intensities in the $A < 0$ drift cycle. The behaviour is similar along the V2 trajectory, although

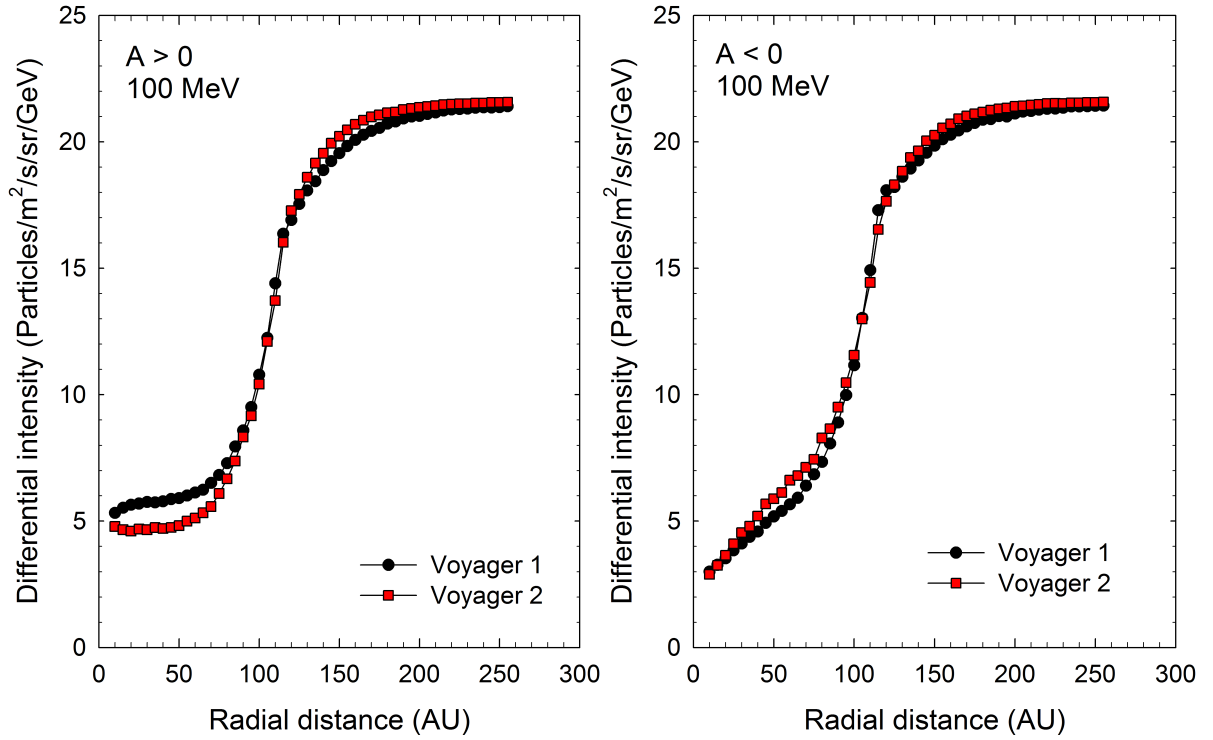


Figure 8.5: Similar to Fig. 8.4, but here the modelled intensities along the two spacecraft trajectories are directly compared for the $A > 0$ (left panel) and the $A < 0$ (right panel) drift cycles.

the radial intensity profiles cross at ~ 35 AU, so that the $A < 0$ intensities are generally higher than those of the $A > 0$ cycle in the outer heliosphere. The average radial gradient in the inner heliosheath is computed to be $\sim 4\%/AU$ which is consistent with Voyager observations; *Webber et al.* [2012b] report an average gradient of $\sim 1 - 3\%/AU$ for > 200 MeV protons, although the observations also illustrate the temporal nature of modulation in this region which is not included in this study.

In Fig. 8.5, the computed intensities (shown in the previous graph) along the two spacecraft trajectories are directly compared for the $A > 0$ (left panel) and the $A < 0$ (right panel) drift cycles. Generally, the modulation along the different trajectories are compatible, with very similar intensities and radial profiles found. Studies by e.g. *Manuel et al.* [2011] and *Luo et al.* [2013] have also found the intensities along these two trajectories to be comparable, while the Voyager GCR nuclei measurements of *Webber et al.* [2012c] also support this result. It should, however, be kept in mind that Fig. 8.5 shows the computed radial intensity as a function of radial distance for a fixed time, and not taking temporal variations of the plasma environment into account, while the Voyager measurements are inherently always a temporal intensity profile.

The left panel of Fig. 8.6 shows energy spectra along the V1 trajectory at three different radial positions for both drift cycles. Note that the 75 AU and 120 AU cases correspond to the radial

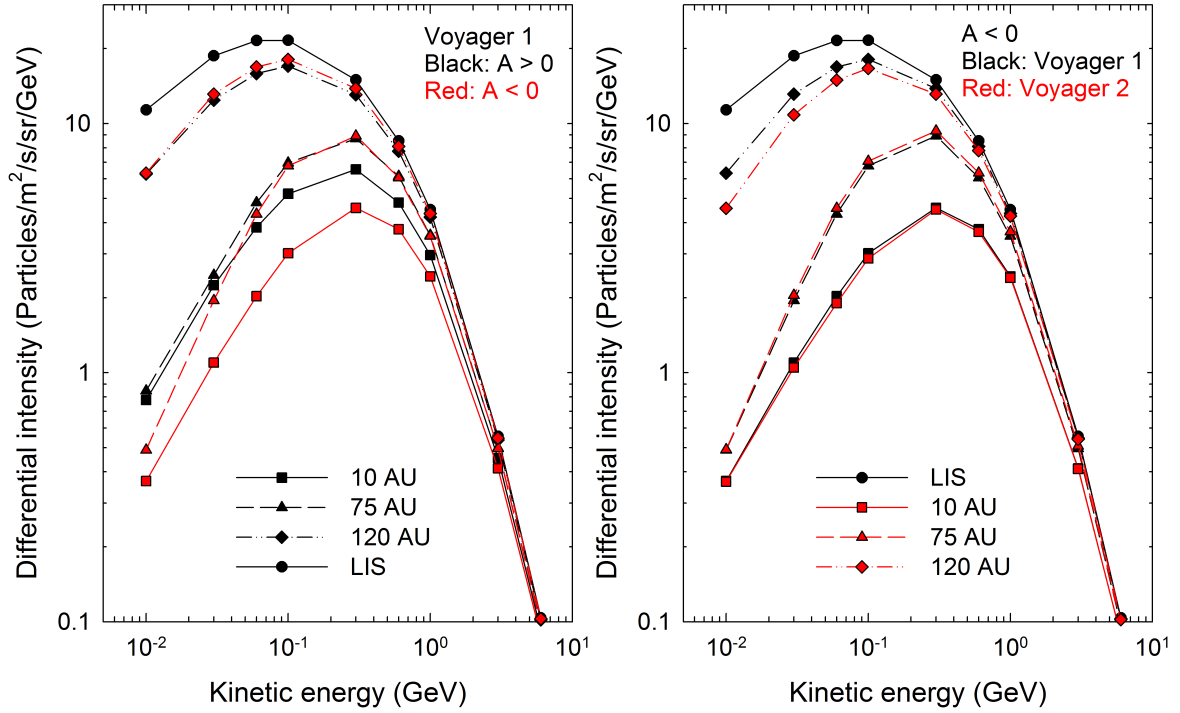


Figure 8.6: The left panel shows computed energy spectra at different radial distances for the $A > 0$ (black lines) and the $A < 0$ (red lines) drift cycles along the V1 trajectory. In the right panel, energy spectra for the $A < 0$ drift cycle are compared along the V1 (black lines) and V2 (red lines) trajectories.

positions of the TS and the HP respectively. In the inner heliosphere, the well-known drift effects are prominent, with the $A > 0$ spectrum well above that of the modelled $A < 0$ spectrum. The drift effects however diminish with increasing radial distance, so that the intensities at the TS and the HP become comparable for both drift cycles.

In the right panel of Fig. 8.6, the energy spectra, at different radial positions, are compared along the two spacecraft trajectories for the $A < 0$ cycle. As already seen, the differences between the intensities along the two trajectories are generally small, with the biggest difference occurring at ~ 300 MeV at the TS regions and at low energies (~ 10 MeV) near the HP.

Fig. 8.7 shows the computed 100 MeV proton intensity in the meridional plane of the heliosphere for the $A > 0$ (top panel) and $A < 0$ (bottom panel) drift cycles as a contour plot. The approximate positions of the TS and HP are indicated by the symbols, while a projection of the V1 and V2 trajectories are shown as the solid lines. The figure illustrates the expected latitudinal proton distribution in different drift cycles in the inner heliosphere: A negative latitudinal gradient (in terms of the polar angle) is seen in the $A > 0$ drift cycle, while this becomes positive in the $A < 0$ drift cycle [see also *Strauss and Potgieter, 2010*]. Note that the high intensities in the tail regions of the heliosphere close to the computational boundary are artificial and caused by the fact that the MHD model does not resolve the entire heliospheric tail, with the result that the LIS is essentially specified within the HP regions.

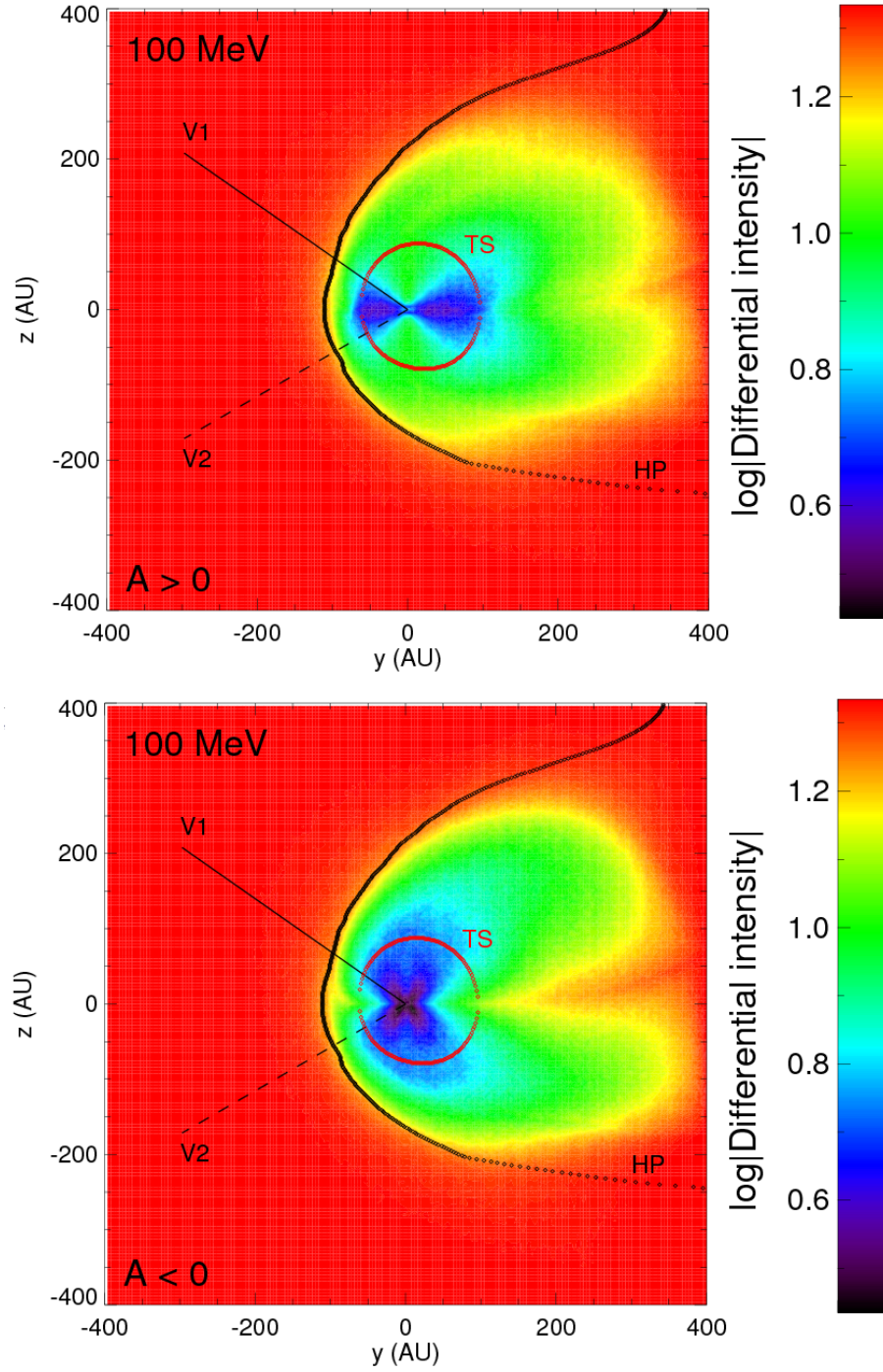


Figure 8.7: The 100 MeV proton intensity in the meridional plane of the heliosphere for the $A > 0$ (top panel) and the $A < 0$ (bottom panel) drift cycles. The red and black symbols (diamonds) show the approximate position of the TS and HP, while the solid and dashed lines show a projection of the V1 and V2 trajectories.

In Fig. 8.8, the so-called *exit position* of 100 MeV protons, released time backwards from V1's position at the HP, is shown for the $A > 0$ (top panel) and $A < 0$ (bottom panel) drift cycles. The boundaries of the computational cube are unfolded, so that panel (ii) points towards the ISM inflow direction, while panel (vi) points towards the heliospheric tail regions. The red

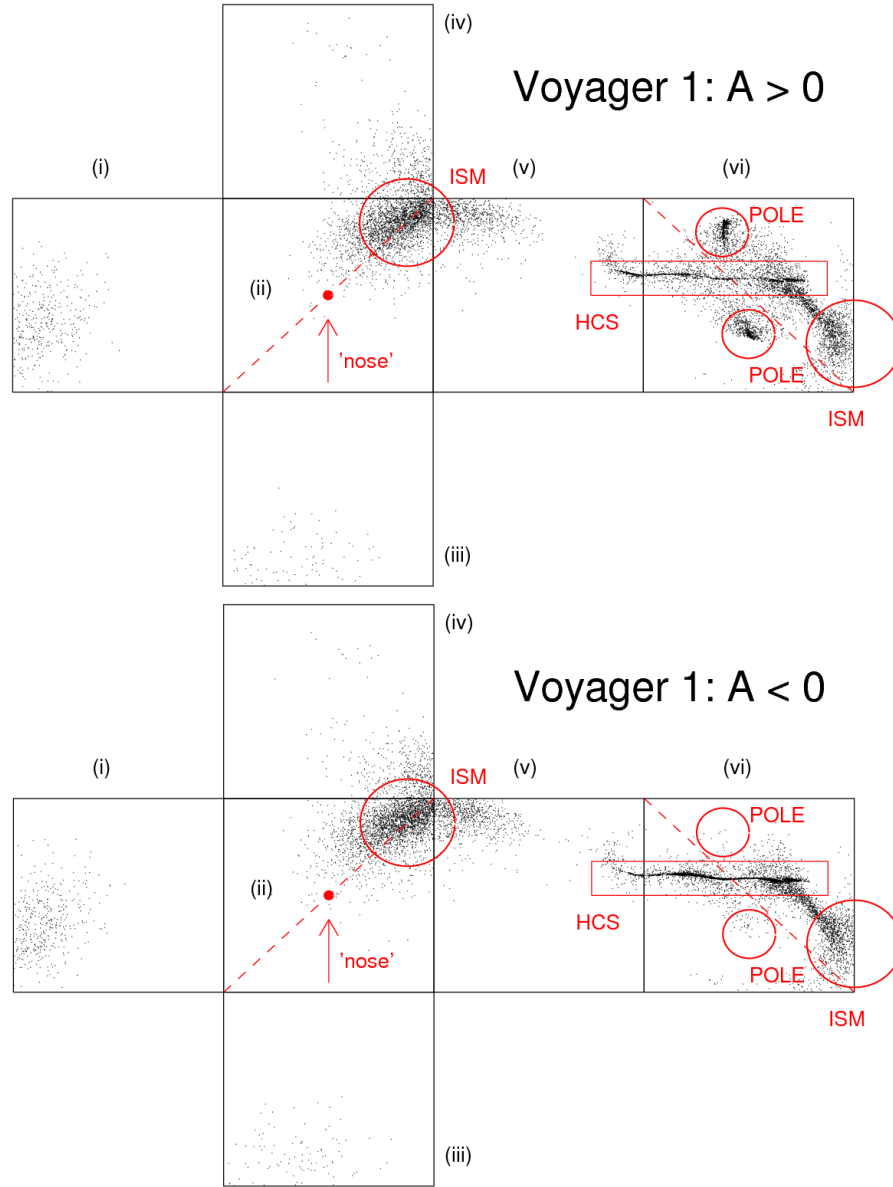


Figure 8.8: The exit position of 100 MeV protons at V1's position at the HP. The top panel is for the $A > 0$ drift cycle, and the bottom panel for the $A < 0$ cycle. The boundaries of the cubical computational region is unfolded so that panel (ii) points in the nose direction of the heliosphere (i.e. towards ISM inflow) and panel (vi) towards the heliospheric tail regions. The labelling is discussed in the text.

dashed line in panel (ii) shows a projection of the unperturbed ISM $\vec{B}$, while the red circles, labelled *ISM*, show the approximate position where ISM $\vec{B}$ lines, that connect with V1's position directly beyond the HP, intersect with the computational boundaries. It is clear that a significant fraction of the CRs reach V1's position at the HP by propagating from the ISM (although they may enter/exit the HP multiple times as discussed in the next section), while a large portion can also propagate very effectively from the tail to the nose regions. What is of interest, is the fact that, even at this position at the HP, drift effects are still evident: In the $A > 0$ cycle, the CRs drift inwards from the polar regions (the red circles labelled *POLE* show the position

where the polar plumes exit the computational region), while drifting along the HCS regions (the red box labelled *HCS*) in the $A < 0$ cycle, to reach the position of V1 at the HP.

8.4 Cosmic Ray Modulation Beyond the Heliopause

In this section, the possibility of CR modulation occurring beyond the HP is investigated. *Scherer et al.* [2011] have shown that GCRs can be modulated in the outer heliosphere because of their mobility across the HP: After entering the heliosphere, a GCR can spend a significant amount of time propagating towards the inner heliosphere, while continuously being cooled adiabatically, whereafter it may escape back across the HP before being *observed* in the outer heliosheath. Furthermore, such a CR can enter and exit the heliosphere multiple times, making this a significant modulation effect [see also *Herbst et al.*, 2012]. In contrast to the simplified spherical symmetric model used by *Scherer et al.* [2011], a much more realistic modulation scenario is presented here by making use of the hybrid modulation model discussed in the previous section. Note that the results of this section neglect drifts, as the previous section showed that the effect thereof beyond the HP is negligible.

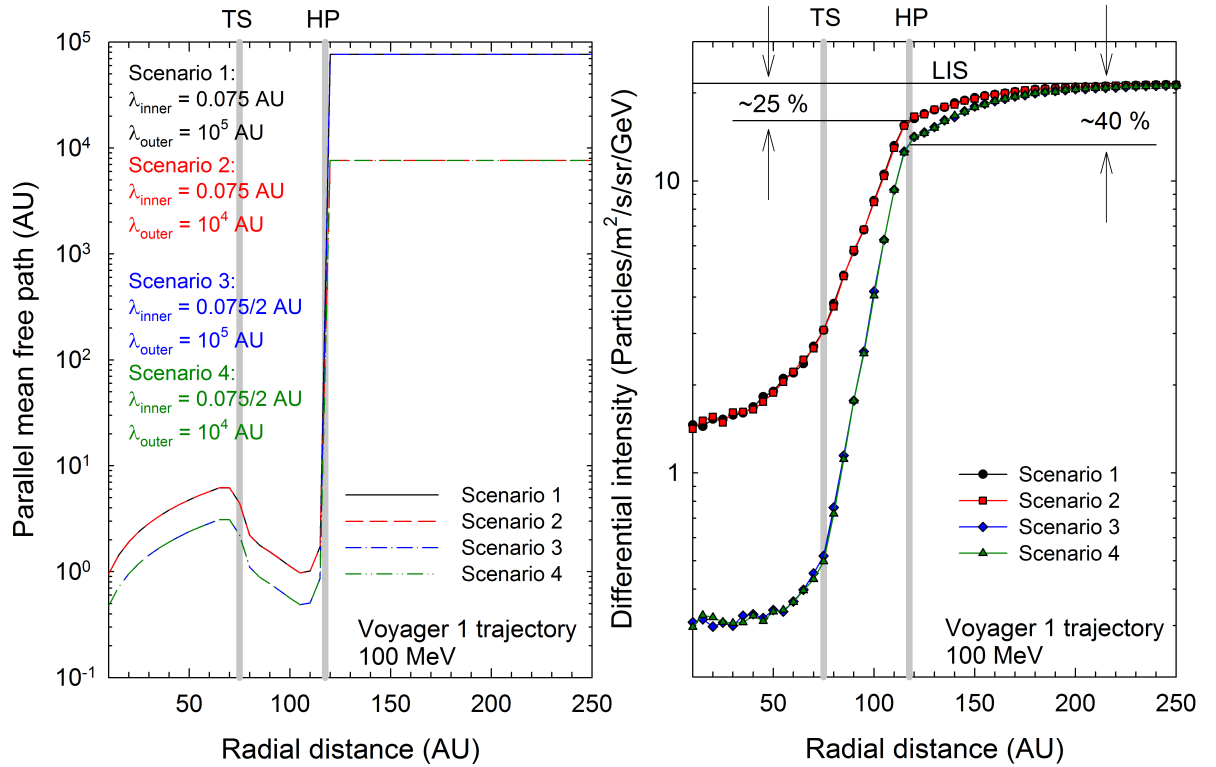


Figure 8.9: The left panel shows different assumptions of $\lambda_{||}$ at an energy of 100 MeV along the V1 trajectory. The TS and HP positions are indicated by the vertical grey lines. The right panel shows the resulting CR proton differential intensity.

The left panel of Fig. 8.9 shows the assumed $\lambda_{||}$ as a function of radial distance along the V1

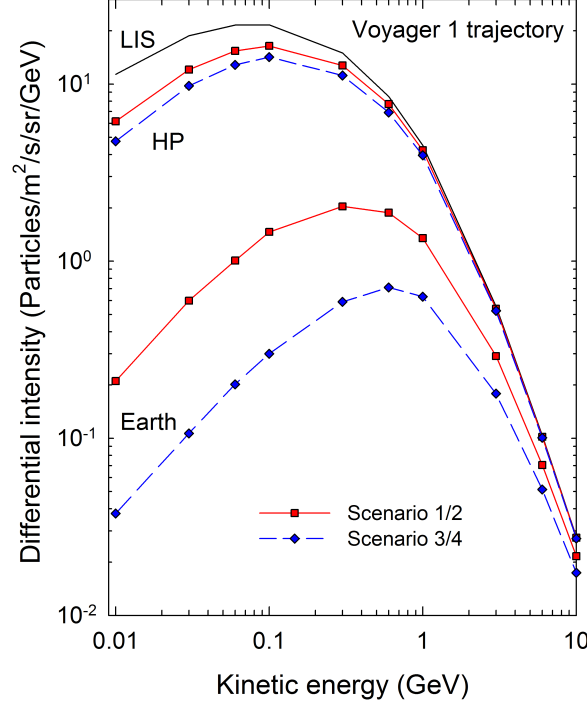


Figure 8.10: Computed CR proton energy spectra at the HP (top two coloured curves) and at Earth (two bottom curves) with respect to the unmodulated LIS. The red and blue lines correspond to two different assumptions of $\lambda_{||}$ inside of the HP (see Fig. 8.9) representing solar minimum and maximum modulation conditions.

trajectory using Eq. 8.2, while the right panel shows the resulting CR differential intensity at 100 MeV. The approximate positions of the TS and HP are indicated by the vertical grey lines. Results are shown for 4 scenarios of λ_0 as indicated on the figure, with $\eta = 0.02$. For scenario 1, $\lambda_0 = 0.075$ AU inside the heliosphere and $\lambda_0 = 10^5$ AU in the outer heliosheath. Note that $\lambda_0 = 10^5$ AU gives $\kappa_{||} \approx 1.5 \times 10^{28} \text{ cm}^2 \cdot \text{s}^{-1}$; a value used often in GCR propagation models [e.g. *Shalchi and Büsching, 2010*]. A small but positive radial intensity gradient is present in the outer heliosheath which eventually approaches zero. The results of *Scherer et al. [2011]* are therefore confirmed, with a $\sim 25\%$ decrease in CR intensity in the outer heliosheath, while contradicting the earlier suggestion made by *Jokipii [2001]* that very little or no CR modulation should occur beyond the HP. It is, however, well known that GCR propagation models, producing as output the different LIS, do not take local ISM effects into account [e.g. *Potgieter, 2008a*]. It is thus unlikely that V1 will encounter the LIS just beyond the HP. Scenario 2 is similar to scenario 1, except that $\lambda_0 = 10^4$ AU in the outer heliosheath. A striking result is that there is no change in the CR flux; CR modulation beyond the HP is therefore not determined by the value of the individual diffusion coefficients in this region. Scenarios 3 and 4 are similar to those previously described, with the value of λ_0 inside of the HP decreased by a factor of 2, producing a large increase in modulation in all regions considered. This concurs with the conclusions made by *Scherer et al. [2011]*: CRs are modulated in the outer heliosheath because

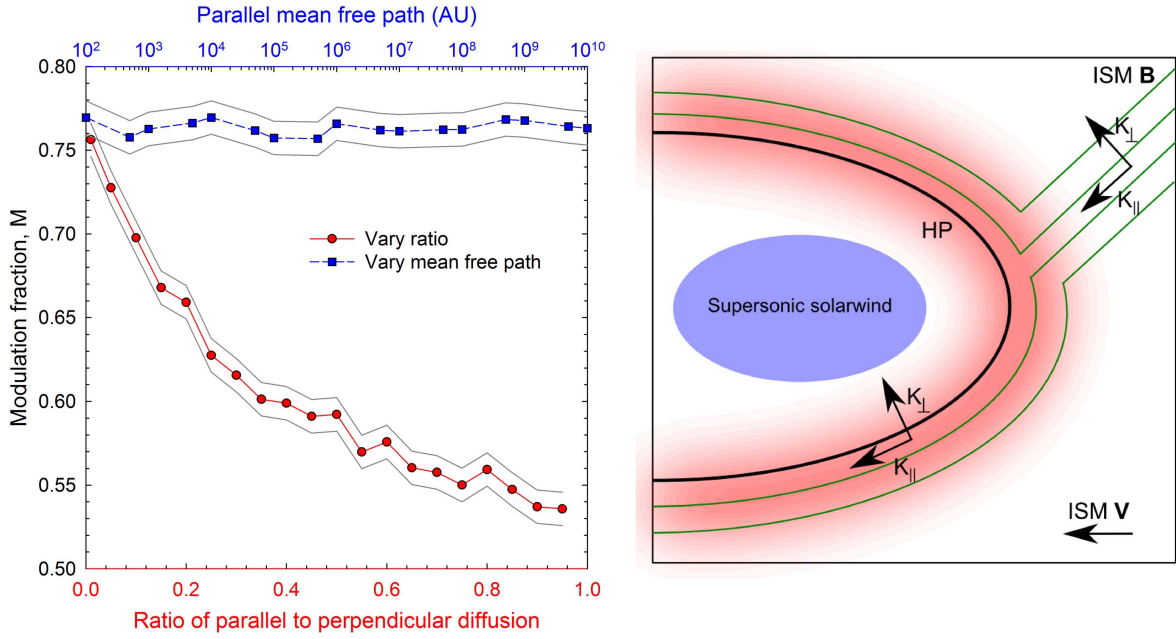


Figure 8.11: The left panel shows the CR intensity at 100 MeV, directly at the HP, normalized to LIS levels. The black squares are results when λ_0 in the outer heliosheath is changed, while $\eta = 0.02$ is kept constant. The red circles show the resulting intensity when η is changed, while $\lambda_0 = 10^5$ AU is held constant in the outer heliosphere. The grey lines show a 1% uncertainty in the results due to the stochastic nature of the numerical scheme, where 10 000 sample trajectories were solved. The right panel shows a schematic of the ISMF geometry close to the HP with an illustration of the parallel and perpendicular diffusive directions.

of their mobility across the HP, entering and exiting the HP regions multiple times and cooled adiabatically while propagating in the supersonic solar wind regions. If increased modulation is experienced inside the HP, more modulation will also occur in the outer heliosheath.

In Fig. 8.10 energy spectra are shown at Earth and at the HP for two choices of λ_0 inside the heliosphere (i.e. inside the HP). The blue dashed lines correspond to the case when λ_0 is reduced by a factor of 2 relative to those for the red curves. For this case, the CR intensity is lower at Earth and at the HP. These two choices of λ_0 can be considered to correspond to solar minimum (red curves) and maximum (blue curves) modulation conditions. The intensity of CRs beyond the HP thus exhibits a solar cycle dependence, with $\sim 15\%$ reduction at 100 MeV at the HP from solar minimum to maximum conditions.

For Fig. 8.11, the curious feature that CR modulation in the outer heliosheath is independent on the magnitude of $\lambda_{\parallel}$ in this region is investigated further. The results are expressed in terms of the modulation fraction M , which is the CR intensity at the HP (at V1's position) normalized to the LIS value at 100 MeV. Note that $M \leq 1$, where $M = 1$ corresponds to a *no-modulation* scenario. Moreover, the value of $1/M$ gives an indication of the level of modulation, i.e. a larger value of $1/M$ indicates more modulation (lower CR intensities). The black squares show the results when λ_0 is changed in the hybrid model, while $\eta = 0.02$ is kept constant. It is clear

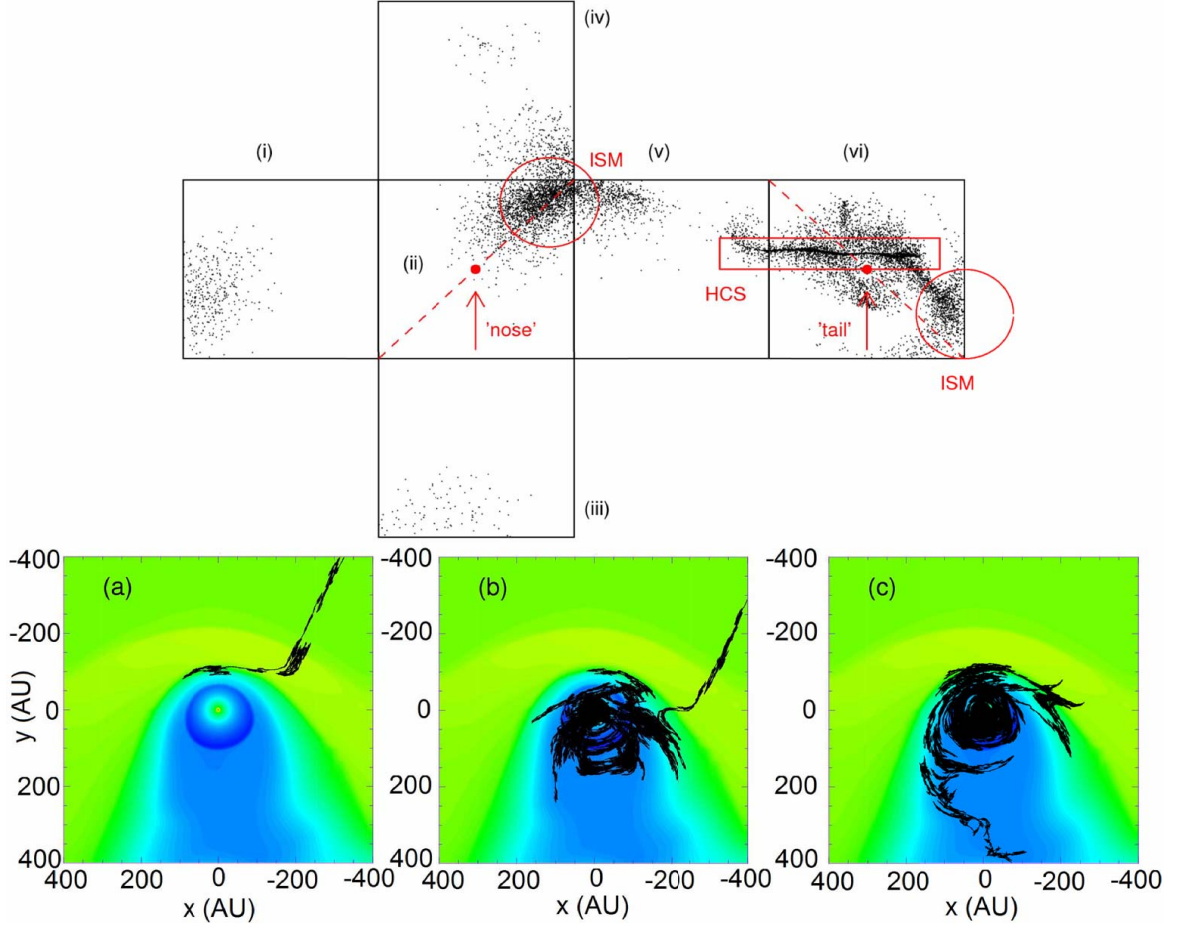


Figure 8.12: The top panel shows the position where CRs, with an energy of 100 MeV, enter the unfolded computational cube, before being *observed* at V1's position at the HP. The bottom panel shows pseudo-particle traces, illustrating the behaviour of CRs in the hybrid modelling approach, projected onto the ecliptic plane. The heliospheric geometry is shown here in terms of density with the ISM moving downwards.

that M is independent of these changes. The red circles show the results when η is changed, while $\lambda_0 = 10^5$ AU is kept constant. For this scenario a strong dependence of M is found. The right panel of Fig. 8.11 explains this result further: From the MHD computations shown in Chapter 3 (especially Fig. 3.8), it is clear that the ISM $\vec{B}$ lines close to the HP are draped along the HP surface, so that they are essentially parallel to the HP. For CRs to enter the heliosphere and be modulated, they thus have to diffuse perpendicular to the ISMF lines. If they are unable to do so, no modulation will be observed in this region, irrespective of the value of $\lambda_{||}$. The dependence on η can be summarized as

$$\frac{1}{M} \propto \lambda_{\perp} \tau. \quad (8.14)$$

The amount of modulation should be proportional to both τ (the time CRs spend close to the HP; the red shaded part in the figure) and $\lambda_{\perp}$ (noting again that CRs can only enter the HP by cross-field diffusion). Furthermore, with $\tau \propto 1/\lambda_{||}$ if $\lambda_{\perp} \ll \lambda_{||}$ (see chapter 5), and $\lambda_{\perp} = \eta \lambda_{||}$, it

is found that

$$M \propto \frac{1}{\eta}, \quad (8.15)$$

which is approximately the behaviour evident in Fig. 8.11.

The top panel of Fig. 8.12 is similar to Fig. 8.8 and shows the position at the sides (boundaries) of the computational cube, where CRs with an energy of 100 MeV enter the computational volume in order to reach the position of the V1 spacecraft at the HP. Again, a significant fraction of the CRs, reaching V1's position at the HP, enter from the heliospheric tail regions (the red rectangle). This can be understood by noting, from the results in Chapter 3, that the HMF spiral is stretched far out in the tail regions of the heliosphere (while being compressed in the nose regions), allowing effective parallel diffusion of CRs from the tail to the nose regions of the heliosphere. The bottom panel of Fig. 8.12 shows pseudo-particle traces (projected onto the ecliptic plane), illustrating the behaviour of CRs discussed above: Case (a) shows CRs diffusing efficiently from the ISM to V1's position at the HP, undergoing almost no modulation – the classical no-modulation paradigm. Case (b) shows a similar scenario, but with CRs entering the heliosphere, undergoing modulation (especially energy losses) after escaping the heliosphere before being *observed* at the HP. Case (c) shows CRs entering the tail regions of the heliosphere before propagating to the nose regions.

8.5 Summary and Conclusions

A new hybrid CR modulation model is introduced, where the TPE is solved using an MHD simulated heliospheric environment instead of assuming a pre-described simplistic heliospheric geometry. The general characteristics of the model were shown and discussed, after which the model was applied to CR modulation beyond the HP and selected results presented. The modelling results suggest that GCR modulation continues beyond the HP, so that V1 will continue to measure a small but positive CR gradient well into the outer heliosheath. The average radial intensity gradient in the outer heliosheath is computed to be $\sim 0.2 - 0.4 \text{ \% / AU}$, which is significantly smaller than in other regions of the heliosphere. A more detailed parameter study, followed by a detailed comparison with V1 observations in this region, might, however, change these results quantitatively. It was shown that CR modulation in this region may exhibit measurable solar-cycle related changes, due to variations in CR transport conditions within the HP. Due to the draping of IMF lines across the HP, CR transport beyond the HP is described mainly by the ratio of parallel to perpendicular transport. This effect illustrates the importance of the $\vec{B}$ topology used in a CR modulation model and the advantage of using this hybrid CR model, as this unexpected and overlooked effect would not be evident in simpler modelling approaches.