Chapter 4

Mathematical model

This chapter details the formulation and evaluation of the basic PON planning model. Design considerations, including model type and complexity are discussed before detailing the objective and constraint formulation process. The solution results are verified through hand-analysis before revealing that, for its fast execution, the basic model overestimates deployment cost considerably.

4.1 Design motivation

Building on the background of Chapter 2: Technical Background and Chapter 3: Modelling and modelling techniques, a basic mathematical model of the PON topology will now be implemented and tested. Before describing the actual model however, the model design considerations will be presented, to illustrate the modelling process as it progresses.

4.1.1 Model considerations

Firstly, the aim of the basic model is to capture the basic topology of a PON in an ILP, solve it to optimality for a number of simple datasets and verify that the model produces feasible results. The basic model will then be used to determine the problem environment and serve as a basis for any further refinements made in Chapter 5: Refined mathematical model.

To determine if a connection is to be made between an ONU and a splitter, or if a specific splitter is used, binary decision variables will be used. These integer variables contain a simple *yes* or *no* value with regard to usage. Since the model will only contain integer variables, the model is formulated as an ILP.

The objective function, or the function which determines the apparent value of the current solution, can be formulated to optimize for a number of criteria. Even though some authors maximize for fiber reach [49], minimize for fiber length [2] or minimize the amount of splitters used, a cost minimization provides a number of benefits. By formulating the objective function as the total cost of deploying the PON, a weighted sum of values can be optimized, resulting in a more accurate and versatile model. Furthermore, since most SPs are interested in minimizing their CAPEX per PON deployment, a cost-based objective function provides direct correlation to the real world.

4.1.2 Model complexity

Since the aim of the basic model is to only capture the basic PON topology, the model complexity will be kept to a minimum. This includes the following assumptions:

1. **Every ONU needs to be connected** - This eliminates coverage complexity and simplifies model definition.
2. **Only one CO exists** - This reduces complexity as the number of COs is reduced to a constant.

3. **Fiber cost per unit of distance is constant** - Eliminates economies of scale type costs which would result in non-linear link cost and a non-linear model.

4. **Trenching cost per unit of distance is constant** - As with fiber costs, it avoids a non-linear model.

5. **OLTs, ONUs and splitters have constant costs** - Reduces complexity through exclusion of economies of scale effects in the model which would result in non-linearity.

6. **All splitters are the same / All ONUs are the same** - A single type of equipment reduces the number of allocation configurations.

7. **No civil restrictions on trenching** - Assuming a trench can be made anywhere allows the use of simple distance functions between points.

With these assumptions, the model complexity is drastically reduced, although it is important to note that the overall complexity is still *very* high, with even the basic model being classified as NP-hard [2]. This is also due to the fact that since one of the sub-problems of the PON problem is already NP-complete, the total problem must be *at least* as difficult [28, 45, 51].

4.2 Basic model

4.2.1 Sets

For the basic model, two input sets are required, detailing the topological information of the PON.

- **U** - Contains a set of all possible locations for ONUs in the form of Cartesian coordinates. The index j is used to indicate the coordinates of the j-th ONU. $\mathbf{U} \neq \varnothing$.

- **S** - The set of all possible locations for splitters in the model. Like **U**, this is a set of Cartesian coordinates. For this set, the index i is used to indicate the i-th splitter location. $\mathbf{S} \neq \varnothing$

4.2.2 Variables

In keeping with the most basic model idea, only two variables are used, both representing usage of some resource.

- ψ_i - Binary variable array used to indicate usage of the i-th splitter, $i \in \mathbf{S}$. The variable takes on a value of 1 if the i-th splitter is used and 0 if it's unused.
- ϕ_{ij} - Binary matrix used to define the usage of a link between the i-th splitter and the j-th ONU, $i \in \mathbf{S}, j \in \mathbf{U}$. If the two are connected, the variable takes on a value of 1. If unused, the variable is 0.

4.2.3 Parameters

To solve the basic model, a set of parameters are needed to further specify the PON environment.

- C_{OLT} - The fixed OLT cost incurred for each PON, i.e. the cost of setting up the required PON equipment at the CO.
- c_S - The cost incurred to deploy a single splitter.
- c_{ONU} - The cost to deploy a single ONU.
- γ - The average cost of trenching a single meter for the deployment of subterranean optic fibers.
- κ^S_{ONU} - The maximum number of ONUs that can connect to each splitter. For this model, we assume that the splitters are of the same type.

- σ_{fiber} - The cost of a meter of fiber between the CO, splitters and ONUs.
- ξ - A large number used to define the relationship between ψ_i and ϕ_{ij}. The value must be larger than κ^S_{ONU}. For simplicity, a value of 10^5 is used.
- x^{CO}, y^{CO} - The Cartesian coordinates of the CO.
- x^S_i, y^S_i - The Cartesian coordinates of the i-th splitter. $i \in \mathbf{S}$.
- x^{ONU}_j, y^{ONU}_j - The Cartesian coordinates of the j-th ONU. $j \in \mathbf{U}$.
- ℓ^{CO}_i - The Manhattan distance in meters between the i-th splitter and the CO. $i \in \mathbf{S}$.
- ℓ_{ij} - The Manhattan distance in meters between the i-th splitter and the j-th ONU. $i \in \mathbf{S}, j \in \mathbf{U}$.

4.2.4 Objective function

The model aims to minimize the total cost of fiber deployment given sets **U** and **S**. The objective function can then be seen as the sum of all the deployment costs as follows:

$$C_{total} = C_{OLT} + C_{splitter} + C_{ONU} + C_{fiber} \tag{4.1}$$

The first term, C_{OLT}, or the OLT cost, is already given in the parameter list, so it is seen as a constant. Next, the splitter cost, $C_{splitter}$, can be determined by multiplying the number of used splitters with the cost per splitter, c_S. Since ψ_i will take on a value of 1 if a splitter is used, the total number of splitters can be determined by summing all the ψ_i, or $\sum_{i \in \mathbf{S}} \psi_i$. Now the splitter cost can be written:

$$C_{splitter} = \sum_{i \in \mathbf{S}} c_S \psi_i \tag{4.2}$$

The total ONU cost, C_{ONU}, can be determined through the same principle of multiplying the total number of used ONUs with the cost per unit, c_{ONU}. Since all ONUs are used however, the number of ONUs is simply specified as $|\mathbf{U}|$. Therefore:

$$C_{ONU} = c_{ONU} \times |\mathbf{U}| \tag{4.3}$$

Finally, we need to determine the total cost of laying fibers, or C_{fiber}. It is evident that the cost per meter of fiber deployment is the sum of the trenching cost per meter, γ, and the fiber cost per meter, σ_{fiber}. Now, to determine the total length of fiber deployed, the lengths of fiber between CO and splitters, and splitters and ONUs must be calculated independently.

Recall that ψ_i is used to specify whether a specific splitter is used. Since all splitters need to be connected to the CO, ψ_i can be seen as a variable specifying the usage of a fiber between the CO and *i*-th splitter. Furthermore, the distance between the CO and the *i*-th SP is already calculated as ℓ_i^{CO}. Therefore, the *i*-th splitter uses a length of fiber equal to $\ell_i^{CO}\psi_i$. Summing for all splitters, the total fiber length between CO and splitters can be determined:

$$\ell_{fiber}^{CO-SP} = \sum_{i \in \mathbf{S}} \ell_i^{CO} \psi_i \tag{4.4}$$

Next, recall that ϕ_{ij} determines the usage of a link between the *i*-th splitter and the *j*-th ONU. Once again, the distance between these, ℓ_{ij}, is already calculated. By summing for all ONUs, $j \in \mathbf{U}$, the total fiber used to connect the *i*-th splitter's allocated ONUs is given as $\sum_{j \in \mathbf{U}} \ell_{ij}\phi_{ij}$. Finally, summing over all splitters, the total length of fiber between all splitters and ONUs can be determined:

$$\ell_{fiber}^{SP-ONU} = \sum_{i \in \mathbf{S}} \sum_{j \in \mathbf{U}} \ell_{ij} \phi_{ij} \tag{4.5}$$

Now that the total fiber length is known, the total cost can be determined by multiplying with the total cost per meter:

$$C_{fiber} = (\gamma + \sigma_{fiber}) \times \Big[\sum_{i \in \mathbf{S}} \ell_i^{CO} \psi_i + \sum_{i \in \mathbf{S}} \sum_{j \in \mathbf{U}} \ell_{ij} \phi_{ij} \Big] \tag{4.6}$$

Substituting (4.2), (4.3) and (4.6) in (4.1) the objective function can be defined:

$$C_{total} = C_{OLT} + \sum_{i \in \mathbf{S}} c_S \psi_i + c_{ONU} |\mathbf{U}| + (\gamma + \sigma_{fiber}) \Big(\sum_{i \in \mathbf{S}} \ell_i^{CO} \psi_i + \sum_{i \in \mathbf{S}} \sum_{j \in \mathbf{U}} \ell_{ij} \phi_{ij} \Big) \tag{4.7}$$

4.2.5 Constraints

Now that the objective function is defined, a number of constraints need to be defined before the basic model can be completed. Whereas the objective function is used to determine the cost of the PON, the constraints enforce the correct PON topology. This topology can be summarized using a number of basic rules:

1. *All splitters must connect to the CO.*
2. *Each ONU must connect to one and only one splitter.*
3. *If a link exists between a ONU and a splitter, that splitter must be active.*
4. *Each splitter can only connect to a limited number of ONUs.*

Using these rules, a set of constraints can be formulated to describe the PON topology. Firstly, since rule 1 mentions only splitters, it does not need to be defined explicitly, with all the necessary information contained within the usage of the splitter ψ_i. Thus, if a splitter is marked as used, i.e. $\psi_i = 1$ for some $i \in \mathbf{S}$, it follows implicitly that there must exist a link between splitter i and the CO.

Rule 2 however, does not pertain to only ONUs. To formulate this rule as a constraint, the variable ϕ_{ij}, which indicates the usage of a link between the i-th splitter and j-th

ONU, can be used, since summing ϕ_{ij} for all splitters $i \in \mathbf{S}$ reveals the total number of splitters connected to the j-th ONU. Therefore, if we constrain this summation to be equal to 1, it will ensure one and only one splitter is associated with the j-th ONU. Finally, to ensure this rule is enforced for *all* ONUs, it is specified for all $j \in \mathbf{U}$ as done in equation (4.8) below.

$$\sum_{i \in \mathbf{S}} \phi_{ij} = 1, \quad \forall j \in \mathbf{U} \tag{4.8}$$

Formulation of rule 3 requires a modelling technique used to model *if-then* relations:

Proposition 4.1 (If-then modelling). *Let* $A \leq A_{MAX}$, $A \in \mathbb{R}$ *and* $B \in \{0, 1\}$. *To model an if-then relation of the form:* $B = 1$ *if* $A > 0$, *a constraint can be formulated as*

$$A \leq \Delta.B, \quad \Delta > A_{MAX} \tag{4.9}$$

Proof. Proposition 4.1 can be proven easily by noting that for all values of $A > 0$, B must equal 1 to ensure the inequality holds, while for values of $A \leq 0$, B can be either 0 or 1 while still satisfying the constraint. □

Using this concept, a constraint can be formulated which ensures that if a link exists from any ONU to the i-th splitter, the i-th splitter will be set as used. Firstly, the number of links to any given splitter can be calculated through the use of the variable ϕ_{ij} by summing over all ONUs, or all $j \in \mathbf{U}$ as follows:

$$COUNT_i^{ONU} = \sum_{j \in \mathbf{U}} \phi_{ij} \tag{4.10}$$

with $COUNT_i^{ONU}$ the number of ONUs connected to the i-th splitter. Since rule 3 applies to all splitters, the constraint is specified for all $i \in \mathbf{S}$. Then, given a large number $\xi \geq \kappa_{ONU}^{S} = |\mathbf{U}|$, rule 3 can be formulated as constraint 4.11 using proposition 4.1. Even though the value of ψ_i cannot be ensured when $COUNT_i^{ONU} = 0$, the cost involved with setting $\psi_i = 1$ is reflected in the objective function in equation (4.2), avoiding unnecessary splitter usage when the model is minimized.

$$\sum_{j \in \mathbf{U}} \phi_{ij} \leq \xi \psi_i, \quad \forall i \in \mathbf{S} \tag{4.11}$$

Since the number of ONUs connected to the i-th splitter is already defined in equation (4.10), rule 4 can be easily modelled by ensuring that $COUNT_i^{ONU}$ never exceeds the parameter κ_{ONU}^S, as done in equation (4.12).

$$\sum_{j \in \mathbf{U}} \phi_{ij} \leq \kappa_{ONU}^S, \quad \forall i \in \mathbf{S} \tag{4.12}$$

4.2.6 ILP model

Putting together the objective function in (4.7) and the constraints defined in section 4.2.5, an ILP model can be defined as follows:

Minimize

$$C_{OLT} + \sum_{i \in \mathbf{S}} c_S \psi_i + c_{ONU}|\mathbf{U}| + (\gamma + \sigma_{fiber})\Big(\sum_{i \in \mathbf{S}} \ell_i^{CO} \psi_i + \sum_{i \in \mathbf{S}} \sum_{j \in \mathbf{U}} \ell_{ij} \phi_{ij} \Big) \tag{4.13}$$

subject to

$$\begin{aligned}
\sum_{i \in \mathbf{S}} \phi_{ij} &= 1, & \forall j \in \mathbf{U} \\
\sum_{j \in \mathbf{U}} \phi_{ij} &\leq \xi \psi_i, & \forall i \in \mathbf{S} \\
\sum_{j \in \mathbf{U}} \phi_{ij} &\leq \kappa_{ONU}^S, & \forall i \in \mathbf{S} \\
\psi_i &\in \{0,1\}, & \forall i \in \mathbf{S} \\
\phi_{ij} &\in \{0,1\}, & \forall i \in \mathbf{S}, \forall j \in \mathbf{U}
\end{aligned}$$

This set of equations now specifies all the information needed to model and optimize the basic underlying structure of the PON, given a set of input data.

4.3 Methodology

To effectively evaluate the basic model, a testing methodology is required, stating the specific environment in which the tests are conducted, what parameters are used and how the results will be interpreted. This section now outlines these requirements, describing the input datasets, parameters and result interpretation.

4.3.1 Input datasets

Density scenarios

The input data for the basic model is generated randomly from a uniform distribution, i.e. every number has an equal probability of being chosen. Since the basic model will not be used to draw practical conclusions, random data will suffice for verification and will keep the complexity low by allowing the use of distance functions.

Datasets are created using MATLAB [14], by generating random (x, y) coordinates for splitters and ONUs according to common scenarios. COs are placed in the middle of the dataset to avoid overinflated costs arising from edge placement. The authors of [12] propose two different sized scenarios, each with three distribution densities as shown in table 4.1. For scenario 1, a small area of 0.5km^2 is populated at city, town and suburban area densities between 300 and 5000 nodes per km^2. Scenario 2 then expands on the same densities but for a much larger area of 4.7km^2.

To determine the number of splitter locations to generate, the minimum split ratio is fixed at 1:16. Thus, one splitter location is generated for every 16 nodes. The distance between every pair of Cartesian coordinates (x, y) is calculated through the use of the

Table 4.1: Dataset scenarios for basic model

	Scenario 1			Scenario 2		
Population	City	Town	Suburban	City	Town	Suburban
Nodes	2500	400	150	23500	3760	1410
Area (km^2)	0.5	0.5	0.5	4.7	4.7	4.7
Nodes per km^2	5000	800	300	5000	800	300
Splitters (1:16)	156	25	9	1469	235	88

Manhattan or zig-zag distance, which is defined as $\ell_{0-1} = |x_0 - x_1| + |y_0 - y_1|$. This distance provides the total distance between two points, moving only in the horizontal or vertical. Since this captures accurate distances in a grid-like neighbourhood where cables are routed next to streets, Manhattan distances are preferred to Euclidian (*as the crow flies*) distances. Therefore, to populate the parameters ℓ_i^{CO} and ℓ_{ij}, equations (4.14) and (4.15) are used.

$$\ell_i^{CO} = |x^{CO} - x_i^S| + |y^{CO} - y_i^S| \qquad \forall i \in \mathbf{S} \tag{4.14}$$

$$\ell_{ij} = |x_i^S - x_j^{ONU}| + |y_i^S - y_j^{ONU}| \quad \forall i \in \mathbf{S}, \forall j \in \mathbf{U} \tag{4.15}$$

with (x^{CO}, y^{CO}), (x_i^S, y_i^S) and (x_j^{ONU}, y_j^{ONU}) representing the Cartesian coordinates for the CO, the *i*-th splitter and the *j*-th ONU respectively.

Verification dataset

To verify the basic model output, a very small dataset, *VeriNet*, is generated as specified in table 4.2. This is a large enough dataset to illustrate the model operation, while still being feasible to calculate by hand, since β splitters and α ONUs can be connected in β^α different configurations *. *VeriNet* therefore has 2^5, or 32, possible configurations. Furthermore, the dataset is produced non-randomly to better illustrate the verification

*It is interesting to note that given this formula, even the smallest dataset in scenario 1 has $\approx 10^{146}$ possible configurations, more than the number of atoms in the observable universe ($\approx 10^{80}$), making it impossible to enumerate them all.

process and can be seen in figure 4.1.

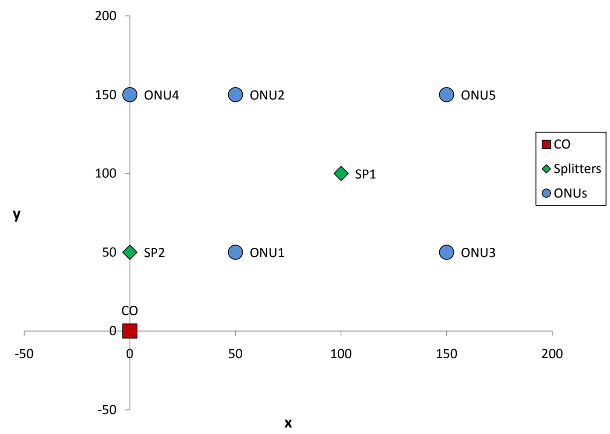


Figure 4.1: *VeriNet* dataset

Table 4.2: *VeriNet* dataset parameters

Parameter	Value
Area (km^2)	0.023
Nodes	5
Splitters	2

4.3.2 Parameters

The basic model is solved using design parameters as outlined in table 4.3. A computational time limit of one hour is used for this model, as more accurate solutions will not benefit further model refinements. Unit cost values are considered good estimates based on similar studies [2] and data obtained from industry sources, although relative cost comparisons are sufficient at this stage of the model.

4.3.3 Result interpretation

With the aim of the basic model being the determination of the PON planning problem environment, providing a basis for future refinements, the results will be interpreted by focussing on the following:

Table 4.3: Basic model design parameters

Model parameter	Symbol	Unit	Value
Fixed OLT setup cost	C_{OLT}	Rand (R)	10,000
Splitter unit cost	c_S	R	6,000
ONU unit cost	c_{ONU}	R	200
Fiber cost	σ_{fiber}	R/m	100
Trenching cost	γ	R/m	300
Max ONUs per splitter	κ^S_{ONU}		64
Max running time		min	60

- **Numerical feasibility** - The numerical outputs of the model, including but not limited to deployment costs and average split ratios, need to correspond to expected outputs, being evaluated in terms of absolute value as well as distribution of values between different datasets.

- **Topological accuracy** - The way in which the PON is connected in the model solution, or the solution topology, needs to correspond to expectations of a typical PON deployment topology before the basic model can be considered feasible.

- **Performance** - Since the basic model is design specifically with low complexity in mind, computational performance needs to be high enough to allow for further refinements, which will undoubtedly increase model complexity.

Given this set of evaluation points, the basic model solutions for the density and verification scenarios can now be analysed.

4.4 Results and analysis

The model was implemented in `C++` using the Concert extension of IBM ILOG CPLEX [13] and the Digia Qt 4.8.0 framework [52]. All tests were run on an Intel Core i7 processor running at 2.67 GHz with 16 GiB main memory and a 64 GiB page file placed on an Intel Solid-State Drive (SSD), for a total of 80 GiB usable memory.

4.4.1 Density scenarios

The model and computational results are given in tables 4.4 and 4.5, while the plotted output of the different density scenarios are given in figures 4.4 and 4.5.

Numerical results

It should be noted that the largest dataset, the city density dataset of scenario 2, proved impossible to solve with the hardware used for a 1:16 minimum average split ratio, with CPLEX giving an *out of memory* error after consuming 64,691 MiB of memory. Given the trend of increasing average split ratios as density increases as noted for the other results, the minimum split ratio for the city dataset was increased to 1:32, effectively halving the amount of splitters to 734. This allowed the application to solve the reduced dataset, denoted with an asterisk (*), to at least a degree of optimality.

Table 4.4: Basic model scenario results

		Scenario 1			Scenario 2		
Result	Unit	City	Town	Suburban	City (*)	Town	Suburban
Total cost	R (mil)	53.78	17.78	9.83	817.12	211.99	117.58
Cost per ONU	R (1000)	21.51	44.42	65.49	34.77	56.38	83.39
Splitters used		88	20	8	709	119	50
Avg. split ratio		28.41	20	18.75	33.14	31.60	28.2
Optimality gap	%	0	0	0	12.5	0	0

Another noteworthy result is that of the cost per ONU, showing a definite decrease with an increase in population density, as would be expected; more ONUs in closer proximity to potential splitter locations will require less fiber. This phenomena is also apparent in the average split ratio, with higher density datasets requiring less splitters for a given number of ONUs.

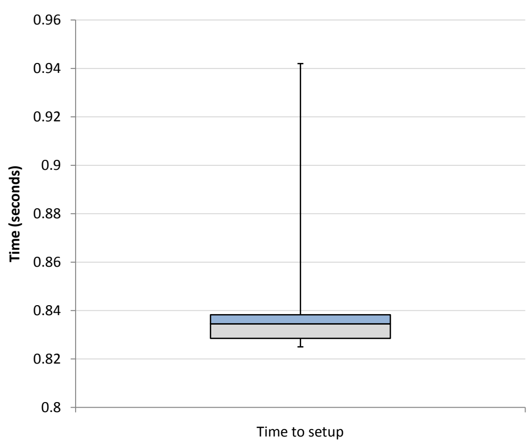
Since none of the solutions required all the available splitters, it can also be deduced that a sufficient number of splitter locations were included in the datasets. Given a real-world scenario however, it might be deemed worthwhile to investigate minimum average split ratios of 1:8 or lower to ensure that this isn't a limiting factor.

Computational results

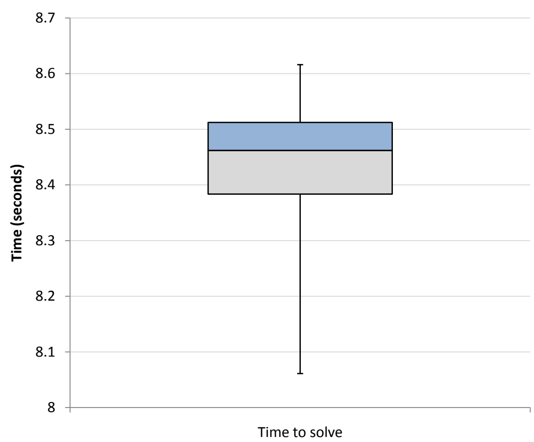
Computational results for the basic model show very fast execution for the low to medium sized datasets, with even the 3760 node dataset being solved to optimality in just under 3 minutes. Complexity increases significantly with the city dataset of scenario 2, with the solver terminating after an hour of computation, producing a feasible solution with an optimality gap of 12.5 %. This optimality gap however, should be interpreted as an absolute maximum distance from optimality as proven by the solver, since the global optimum solution may potentially be much closer to the produced feasible solution.

Table 4.5: Basic model computational results

		Scenario 1			Scenario 2		
Result	Unit	City	Town	Suburban	City (*)	Town	Suburban
Setup time	s	3.2	0.06	0.02	160.43	6.62	0.85
Time to solve	s	52.58	0.42	0.22	3,615	169.88	8.43
CPU time	s	94	3	3	6,452	331	19
Peak memory	MiB	928.14	79.75	66.84	47,955.83	2,107.39	332.61



(a) Setup time



(b) Solution time

Figure 4.2: Setup and solution time boxplots for Scenario 2 - Suburban

To determine the variation in solution and setup time, boxplots of a number of iterations are plotted in figures 4.2a and 4.2b. Setup time over runs are very stable, with quartiles showing low deviations of 1 % from the median of 0.83 seconds. Background applications are likely responsible for the single outlier of 0.94 seconds. A similar trend is found in the solution time boxplot, with most of the observations again staying within 1 % of the 8.46 second median. A linear relation in the log-log plot of solution times given dataset size, figure 4.3, shows exponential growth in solution time as dataset sizes increase.

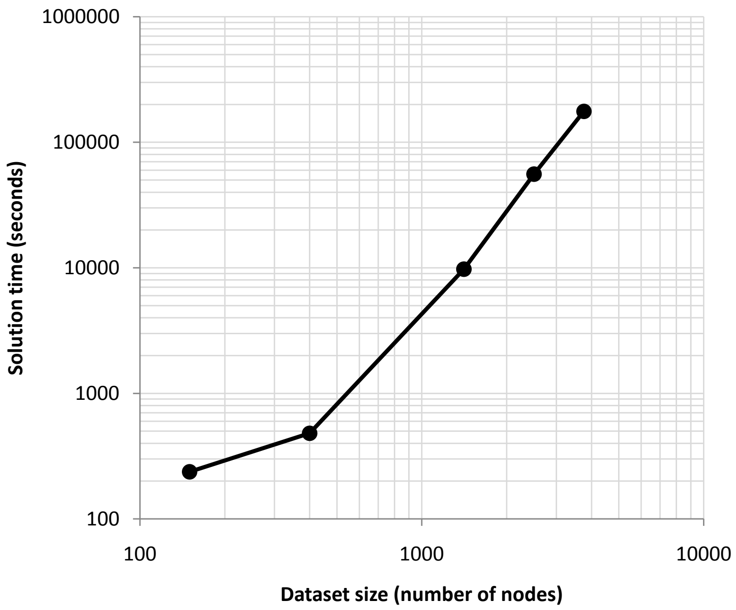


Figure 4.3: Log-log plot of basic model solution time vs dataset size

Topological results

The topological results plotted in figures 4.4 and 4.5, showing the Manhattan distances as stair-plots, illustrates how the CO, splitters and ONUs are connected. The CO is shown as a red square, splitters as green circles and ONUs as blue circles. Unused potential splitter locations are shown as uncoloured or white circles. Black and grey lines represent fibers between the CO and splitters, and splitters and ONUs respectively. Even though the plot for the city dataset of scenario 2 was rendered, the sheer density of nodes at this scale, $\approx$ 2 nodes/mm^2, obstructed the topology in such a way that it would be meaningless to show it on this page.

Analysing the plots, especially the fiber links between the CO and splitters, it is clear that the model overestimates the overall cost of the deployment. For fibers sharing a path for a part of their total length, the cost as calculated in the objective function effectively dictates that each fiber has its own trench, a definite deviation from a real-world deployment.

It is however promising to note that a definite clustering symmetry is evident, with no outlying allocations, allowing for preliminary verification of the model's ability to allocate ONUs to splitters effectively. To prove this, a closer look at *VeriNet* is required.

4.4.2 Verification

Calculated results

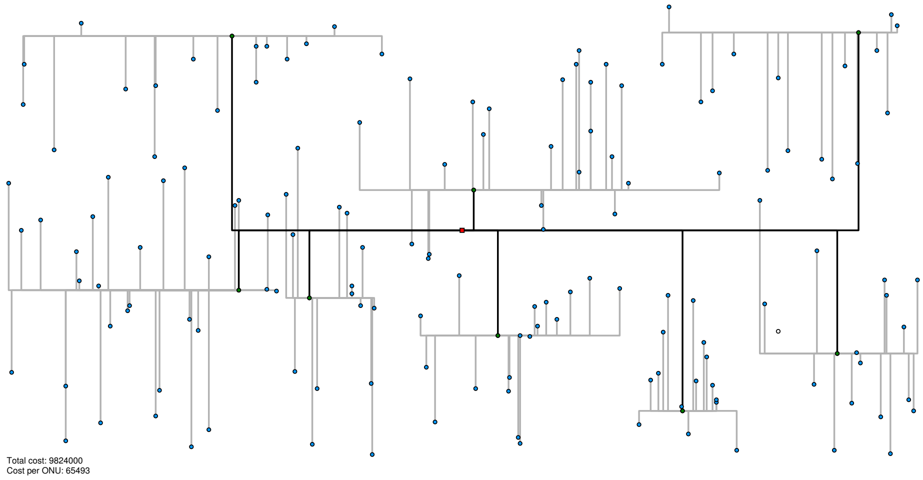
As shown in figure 4.1, the *VeriNet* dataset consists of a CO, 2 splitters and 5 ONUs. Before allocations can be made, the Manhattan distances between CO, splitters and ONUs are calculated using equations (4.14) and (4.15) and tabulated in table 4.6.

Table 4.6: Calculated Manhattan distances between nodes of *VeriNet*

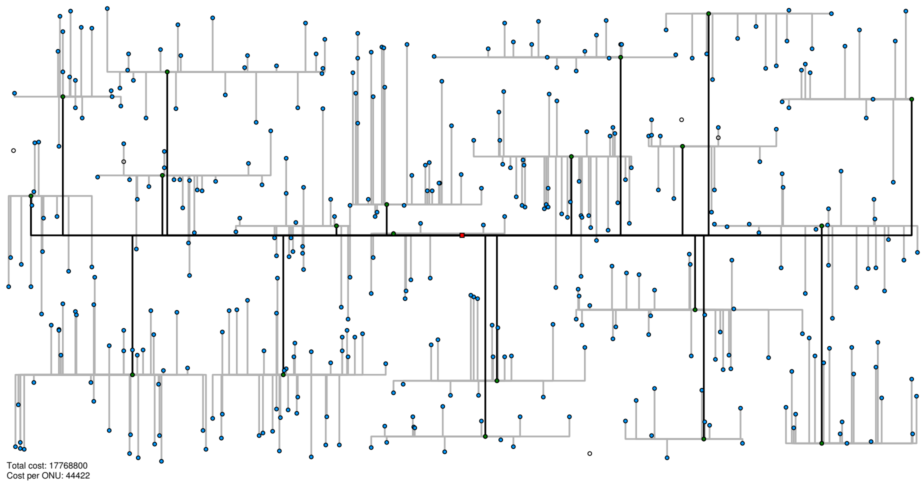
	Splitters		ONUs				
Distance (meters)	SP1	SP2	ONU1	ONU2	ONU3	ONU4	ONU5
CO	200	50	-	-	-	-	-
SP1	-	-	100	100	100	150	100
SP2	-	-	50	150	150	100	250

The 32 different allocations of ONUs to splitters are shown in table 4.7, along with the total fiber distance to connect those ONUs to their respective splitter. Finally, the table shows the breakdown of the fiber, splitter, ONU and OLT costs as well as the final total deployment cost.

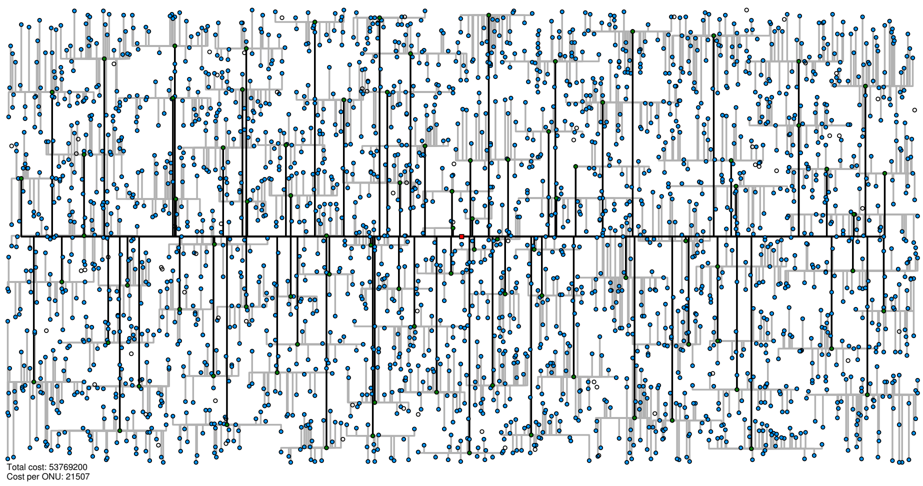
As an example, for configuration 17, ONU4 and ONU5 are allocated to SP1, for a com-



(a) Suburban density (150 nodes)

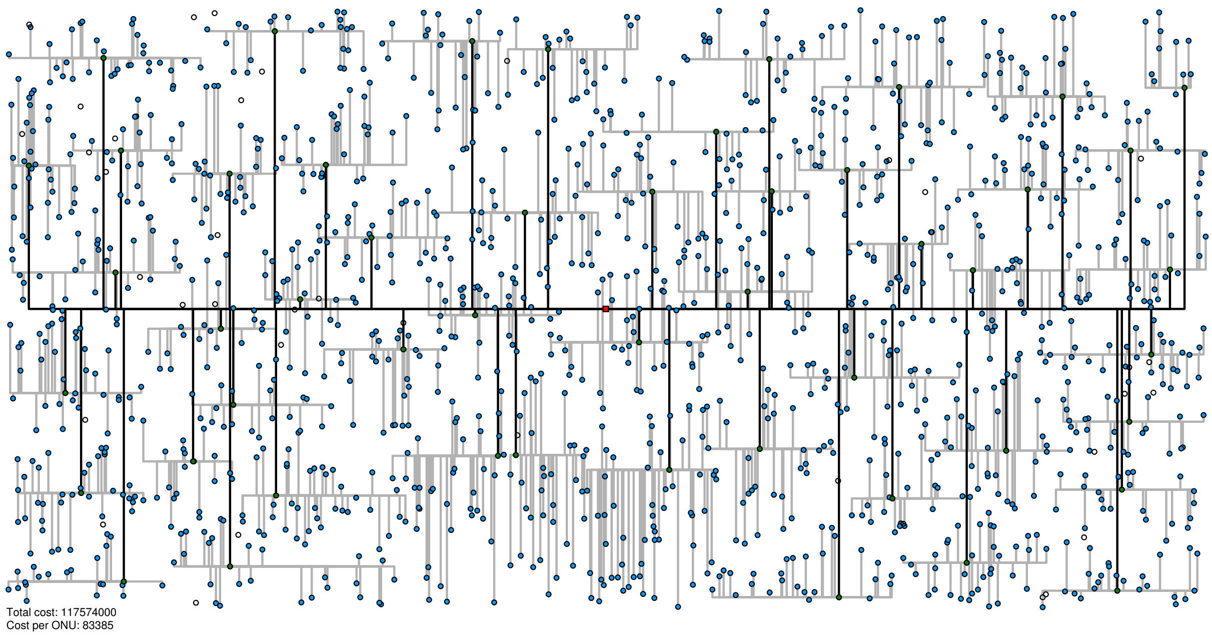


(b) Town density (400 nodes)

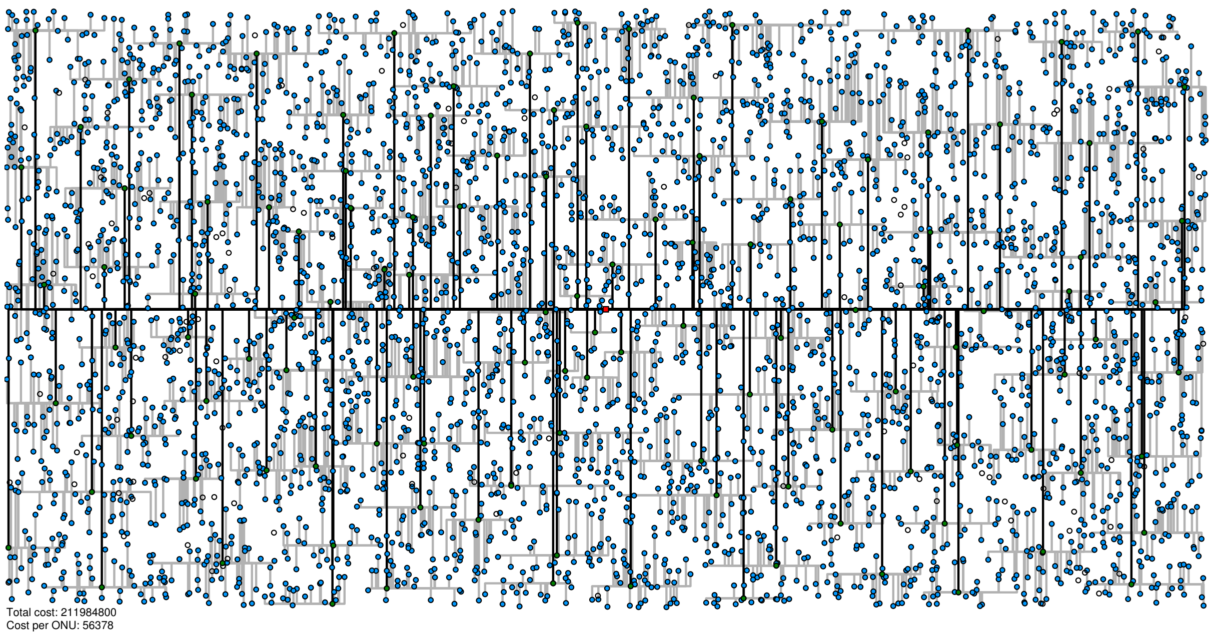


(c) City density (2500 nodes)

Figure 4.4: Basic model results for scenario 1



(a) Suburban density (1410 nodes)



(b) Town density (3760 nodes)

Figure 4.5: Basic model results for scenario 2

bined distance of $100 + 150 = 250$m. Adding this to the similarly calculated fiber length of SP2 of 350 m, the total fiber length is then 600 m. At R 100/m of fiber and R 300/m of trenching, the total fiber cost is R $400/\text{m} \times 600$ m $=$ R $240,000$, as specified in the ONU fiber column. Since SP1 is 200 m and SP2 is 50 m from the CO, the fiber to connect CO to splitters costs $(200 + 50) \times$ R $400/\text{m}$ $=$ R $100,000$. Then, with costs of R 6,000 per splitter, R 200 per ONU and R 10,000 per OLT, we arrive at the total of R 363,000.

Of all the configurations, number 9 has the minimum total cost and therefore the optimum objective value is R 303,000, with ONUs 2, 3 and 5 connected to splitter 1 and ONUs 1 and 4 connected to splitter 2.

Table 4.7: Total cost for all *VeriNet* configurations

	SP1		SP2		Cost (R thousand)					
#	ONUs	Fiber (m)	ONUs	Fiber (m)	ONU fiber	SPs	ONUs	OLT	SP fiber	Total
1	1,2,3,4,5	550	-	-	220	6	1	10	80	317
2	2,3,4,5	450	1	50	200	12	1	10	100	323
3	1,3,4,5	450	2	150	240	12	1	10	100	363
4	1,2,4,5	450	3	150	240	12	1	10	100	363
5	1,2,3,5	400	4	100	200	12	1	10	100	323
6	1,2,3,4	450	5	250	280	12	1	10	100	403
7	3,4,5	350	1,2	200	220	12	1	10	100	343
8	2,4,5	350	1,3	200	220	12	1	10	100	343
9	2,3,5	300	1,4	150	180	12	1	10	100	303
10	2,3,4	350	1,5	300	260	12	1	10	100	383
11	1,4,5	350	2,3	300	260	12	1	10	100	383
12	1,3,5	300	2,4	250	220	12	1	10	100	343
13	1,3,4	350	2,5	400	300	12	1	10	100	423
14	1,2,5	300	3,4	250	220	12	1	10	100	343
15	1,2,4	350	3,5	400	300	12	1	10	100	423
16	1,2,3	300	4,5	350	260	12	1	10	100	383
17	4,5	250	1,2,3	350	240	12	1	10	100	363
18	3,5	200	1,2,4	300	200	12	1	10	100	323
19	3,4	250	1,2,5	450	280	12	1	10	100	403
20	2,5	200	1,3,4	300	200	12	1	10	100	323
21	2,4	250	1,3,5	450	280	12	1	10	100	403
22	2,3	200	1,4,5	400	240	12	1	10	100	363
23	1,5	200	2,3,4	400	240	12	1	10	100	363
24	1,4	250	2,3,5	550	320	12	1	10	100	443
25	1,3	200	2,4,5	500	280	12	1	10	100	403
26	1,2	200	3,4,5	500	280	12	1	10	100	403
27	5	100	1,2,3,4	450	220	12	1	10	100	343
28	4	150	1,2,3,5	600	300	12	1	10	100	423
29	3	100	1,2,4,5	550	260	12	1	10	100	383
30	2	100	1,3,4,5	550	260	12	1	10	100	383
31	1	100	2,3,4,5	650	300	12	1	10	100	423
32	-	-	1,2,3,4,5	700	280	6	1	10	80	317

Model results

For the small dataset, *VeriNet*, the application provided the connection configuration as illustrated in figure 4.6 for a total cost of R 303,000.

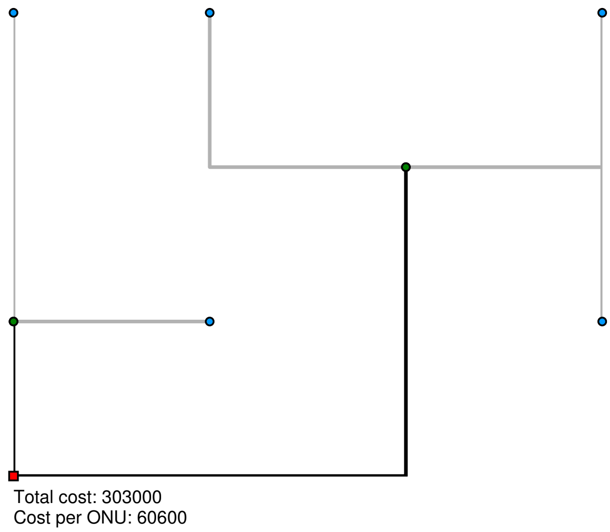


Figure 4.6: *VeriNet* optimal model result

Comparing this with the *VeriNet* layout in figure 4.1, it can be seen that ONUs 2, 3 and 5 are connected to splitter 1, while ONUs 1 and 4 are connected to splitter 2. Since the optimum objective value as given by the model matches the calculated optimum, the model produces an accurate objective value and minimization is done correctly. Finally, since the topology given by the model matches the associated configuration to produce the optimum calculated objective value, the model can be considered correct.

4.5 Conclusion

In this chapter, the design process for the basic model was outlined, with detailed considerations for the model complexity, the objective function and constraints. After the sets, parameters and variables were given, an ILP model of the PON planning problem was formulated, keeping complexity low.

Testing the model with six datasets with different densities and sizes showed promising initial results, with good computational performance for all but the largest dataset and realistic and accurate topology outputs. Exponential growth in solution time as dataset sizes increase is unfortunate but not unexpected given the problem complexity. To verify the model, the output was compared with a small hand-calculated dataset solution, *VeriNet*, which showed identical topology allocation and optimum objective value.

Although the model showed the ability to allocate ONUs to splitters effectively, it was noted that the model greatly overestimates the overall deployment cost, with no concept of fiber duct sharing. The use of uniformly generated data also did not allow for any practical conclusions to be made in terms of node clustering or practical complications. For this reason, the next chapter aims to refine the basic model presented to have better correlation with real-world deployments.