

Model misspecification in Financial Risk Management

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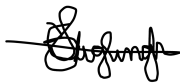
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Declaration

I hereby declare that the research work **Model misspecification in Financial Risk Management** is of my own originality. It has not been submitted to any other university or institution for examination purposes. All the sources consulted in this work are fully acknowledged and are completely found in the reference section. Furthermore, this research work is submitted in partial fulfilment of the requirements for the degree Master of Science at the Centre for Business Mathematics and Informatics, North-West University (Potchefstroom campus).



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Date

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Executive Summary

Model risk has become more relevant over the past few decades. A few reasons that attribute to the rise in awareness and interest in model risk include the requirement of advanced risk models by financial sector authorities, the increasing complexity of models deployed, and the vast range of risk types that models impact. Model risk occurs when a financial model is performing inadequately given the data, such that it leads to inappropriate results. Model misspecification is one component of model risk, which refers to violation of necessary model assumptions. A variety of financial risk categories are impact by model misspecification. Among these financial risks are market risk which is the risk arising from the fluctuations of financial asset prices. This study aims to evaluate the impact of model misspecification in Financial Risk Management with our primary focus being on market risk. We identify Generalized Autoregressive Conditional Heteroscedastic (GARCH) as our modelling technique because it is useful in assessing risk and expected returns for assets that display clustered periods of volatility in returns.

GARCH is a statistical modelling technique used to assess, analyse and forecast the volatility of returns on financial time series data. GARCH models have become significant tools in the assessment of time series data, largely the traditional normal distribution of GARCH models, because of its ease of use in practice. However, it is proven that high frequency financial data have heavy tails leading to the resulting estimates being inefficient. The Student-t and General error (GED) distributions are more capable of representing these financial series. In this study, we consider the GARCH (1,1) with Normal, Student-t and GED innovations and varying sample sizes. We conduct a series of simulations for the GARCH (1,1) model with the error terms following a Normal distribution. The return series of Samsung electronics daily stock prices, Bitcoin-USD daily cryptocurrency and Moody's seasoned Aaa corporate bond yield (BAAA) are fitted to the GARCH (1,1) with Normal, Student-t and GED innovations. We investigate if these models are subject to model misspecification

when the error terms do not assume similar distributions as the simulated data and real data innovations.

Keywords: Model risk, Model misspecification, GARCH, Value-at-Risk, Expected shortfall .

List of Acronyms

ADF	- Augmented Dickey-Fuller
AIC	- Akaike Information Criterion
ARMA	- Autoregressive moving average
BAAA	- Moody's seasoned Aaa corporate bond yield
BIC	- Bayesian Information Criterion
CBOE	- Chicago Board Options Exchange
CGARCH	- Component Generalized Autoregressive Conditional Heteroskedastic
DM	- Deutsche Mark
EGARCH	- Exponential GARCH
ES	- Expected shortfall
FEVDs	- Forecast error variance decompositions
FIGARCH	- Fractional Integrated Generalized Autoregressive Conditional Heteroskedastic

FRED	- Federal Reserve Bank of St. Louis
GARCH	- Generalized Auto Regressive Conditional Heteroskedasticity
GARCHM	- GARCH in the Mean
GED	- Generalized Error Distributions
IBM	- International Business Machines Corporation
IGARCH	- Integrated GARCH
iid	- independent and identically distributed
JB	- Jarque-Bera
LM	- Lagrange multiplier
MAE	- Mean Absolute Error
MAPE	- Mean absolute percentage error
MAPFE	- Mean Absolute Percentage Forecast Error
MLE	- Maximum likelihood estimation
MSE	- Mean squared error

PP	- Phillips Perron
QQ-plot	- Quantile-Quantile plots
RC	- Reality check
RMSE	- Root Mean Squared Error
RMSFE	- Root Mean Squares Forecast Error
S&P	- Standard and Poor's
SPA	- Superior predictive ability
TGARCH	- Threshold GARCH
USD/GBP	- U.S. Dollar /British Pound
VaR	- Value-at-Risk
VIX	- CBOE Volatility Index
VXN	- CBOE Nasdaq Volatility Index

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1. Introduction

1.1 Background

Financial institutions are known for dealing with financial services and monetary transactions such as equity, foreign exchange, commodity trading, etc. They are constantly developing advanced strategies to manage risk in order to grow and protect their finances. Recognizing and characterizing the risks the company faces, evaluating their extent, utilizing a variety of strategies to mitigate them, and saving money for expected losses are all critical components of a risk management strategy (Zhang, 2017a). Financial institutions have been utilizing mathematical sciences techniques for risk modelling, in order to help them in tasks such as the identification, assessment, mitigation, monitoring and reporting of risks. According to Long et al. (2020), the main financial risks faced by most financial institutions are market risk, credit risk and operational risk .

Fabozzi et al. (2010) defines market risk as the threat arising due to fluctuations in financial asset prices. In other words, market risk affects any institution with a portfolio of financial assets (Lobato et al., 2007). Market risk is measured using probabilistic models that reflect the statistical features of price changes in financial assets. Some examples of statistical features are aspects such as autocorrelation, time-dependent volatility, and extreme events. The Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) model is often used for the analysis of financial assets because it can successfully predict time-varying and clustered volatility. GARCH models are originally built on the assumption that financial time series have a Normal (Gaussian) distribution. However, financial time series are rarely Gaussian. There are significant evidences that support the fact that financial time series are mostly leptokurtic and heavy-tailed (Bollerslev, 1987). To address this issue and capture the heavy-tails of financial time series Bollerslev (1987) and Nelson (1991)

proposed the Student-t distribution and Generalized Error Distributions (GED), respectively. In this study the GARCH process exists with the assumption that return innovations follow the Normal, Student-t and GED.

Recently, a great deal of attention has been given by academic researchers and practitioners to model risk. [Danielsson et al. \(2016\)](#) considered the evaluation of model risk on the prediction models of market risk and systematic risk. [Kellner and Rösch \(2016\)](#) analysed how Value-at-Risk (VaR) and Expected shortfall (ES) react to different types of model risk. [Lazar and Zhang \(2019\)](#) suggested model risk of Expected shortfall to be measured as the optimal correction required to pass several ES back tests. The development of robust and accurate models is of great importance because this can decrease the likelihood of suffering large financial losses and can result in competitive advantage for financial institutions. According to [Barrieu and Scandolo \(2015\)](#) the specification of a model is an important step when quantifying financial risks to which an institution is exposed to. The hazard of working with an inappropriate model is known as model risk ([Barrieu and Scandolo, 2015](#)). Model risk consists of several components. These components include parameter uncertainty, incorrect model implementation and model misspecification ([Kellner and Rösch, 2016](#)). In this study our primary focus is model misspecification.

Model misspecification occurs when a model is in error due to the model being unable to account for everything it should. Misspecification refers to several and varied conditions under which the assumptions of the model are not met ([McNeil et al., 2015](#)). The process of ensuring that a model meets its intended goal, known as model validation, is of great interest since it helps us to determine if a model is correctly specified. Decision-making within financial institutions is based on the results of analysis produced by utilising the models hence measurement errors could lead to errors in the required capital buffers. These errors originate from different sources such as uncertainty, misspecification and flawed calibration. They can be due to violation of key modelling assumptions (i.e. misspecification) or to wrong estimates of key parameters (i.e. flawed calibration).

Ben-Akiva and Lerman (2018), make the statement that models are theoretical idealisations rather than a practical reality. This implies that the representation of events for example returns on an investment, in models will unavoidably vary from the actual individual events. According to Williams and Ortúzar (1982) any deviation from the actual decision-making process can be classified as model misspecification, which might result from errors made in assuming that the model accurately represents the true data generating process. Examples include the exclusion of variables, failure to assess the correlations between variables, or the incorrect assumption of the error distributions. The evaluation of model misspecification is necessary to ensure that a model is capable of correctly resolving the issue faced.

1.2 Problem Statement

Financial risk management models are used for a wide range of tasks which include risk measurement, determining the sufficiency of capital and reserves, estimating exposures and managing as well as protecting assets of clients (Federal Reserve, 2011). Hence, the use of correctly specified models are crucial for institution to avoid suffering huge losses. Examples of financial institutions that suffered extensive losses due to faulty models, mentioned by Gibson et al. (1998) include: i) Merrill Lynch's pricing biases that led to a loss of \$70 million (Borodovsky and Lore, 2000), ii) JP Morgan's loss of \$200million in the mortgage-backed securities market in 1992 as a result of improper modelling of prepayments, iii) NatWest Markets' mispricing on sterling interest rate options in 1997 that cost them £90 million. Thus, using the correct model and assuring that the assumptions of the model being used are met, is of utmost importance.

According to Seitshiro and Mashele (2020), model risk occurs because of inappropriateness of modelling. This means that a model will not be able to solve the problem at hand, for example give high accuracy model predictions. This study will evaluate

model misspecification in Financial Risk Management, specifically focusing on market risk modelling. In market risk modelling of the financial time series the spread of the time series changes not only with time but also intensely in some periods. This scenario exhibits volatility clustering characteristics by time. This means that the spread or variance of the financial time series is relatively small in some certain time periods, while it turns to be relatively large in another time period. Therefore, the error term value distribution reveals leptokurtosis characteristics, which imply that the probability near mean and tail is larger than that of the normal distribution, while the probability of the rest is smaller than that of the normal distribution. This phenomenon illustrates model misspecification of the error terms assumed distribution.

The analysis focuses on the impact model misspecification has on the GARCH models with Normal, Student-t and GED distributions. These models would be considered misspecified in an event that the distribution of the innovations from the given GARCH model are assumed to follow a certain given distribution, whereas there exist a true distribution which is different from the assumed distribution (Richmond and Horowitz, 2015). The analysis is performed on the normal distribution simulated dataset and empirical data by making use of a variety of model performance methods and measures.

1.3 Motivation

Day-to-day financial decision-making procedures in the financial sector rely largely on mathematical science quantification and financial models. Decisions based on incorrectly designed or implemented models, or even decisions influenced by improper use or understanding of models can negatively impact financial institutions in a number of ways. This can result in financial losses, reputational damage, and managerial decisions that are not optimal. Identifying the misspecification errors made in the market

risk model building process can help financial modellers, traders and investors in making informed decisions which prevents the occurrence of extreme financial losses. The GARCH model was chosen as our modelling technique due to its ability to capture the time varying heteroskedasticity and volatility clustering characteristics of financial time series.

1.4 Aim of the Study

This study aims to evaluate the impact of model misspecification in Financial Risk Management. Financial Risk Management may include credit risk, or operational risk however they are not considered in this study. This study's primary focus is on market risk because of volatile price movements which are inherent in financial markets. We investigate the GARCH modelling technique by analysing if there are any violations made on the model assumptions and what impact these violations have on the model. Model assumptions are key conditions that must be met before we can draw conclusions from model estimates or use a model to make a prediction. By making use of several model performance tests and performance measures, we will be able to quantify the impact of model misspecification on the models' prediction accuracy and performance.

1.5 Objectives of the Study

The primary objective of this research is to evaluate model misspecification during the model building process under market risk modelling. The following secondary objectives were formulated to assist in achieving the primary objective:

- Explore the GARCH model with error terms assuming Normal, Student-t and

Generalized Error distributions.

- Identify several model performance methods as well as performance measures, to help us in determining the possible impact of model misspecification on the models' selection and performance.
- Evaluate how violation of model assumptions impact the prediction accuracy and performance of a model.

1.6 Method of Investigation:

We consider the GARCH (1,1) with Normal, Student-t and GED innovations and varying sample sizes. We conduct a series of simulations for the GARCH (1,1) model assuming the error terms follow a Normal distribution. Empirical results are based on the daily closing prices of Bitcoin cryptocurrency and Samsung Electronics stocks, over the period from 17 September 2014 to 29 July 2021 and from 26 May 2011 to 30 July 2021 respectively. These datasets are retrieved from yahoo finance at <https://finance.yahoo.com/>. We also consider Moody's seasoned Aaa corporate bond yield (BAAA) over the period 23 September 1997 to 9 May 2008 with the dataset retrieved from the Federal Reserve bank of St. Louis (FRED) at <https://www.stlouisfed.org/>. Quantitative analysis is performed by use of RStudio.

1.7 Dissertation Overview

The dissertation is divided into the following chapters:

Chapter 1: This chapter served as an introduction to the study. It outlines the background and research problem. It provides a clear and conceptual overview of the research and the topics that will be discussed at length.

Chapter 2: This chapter primarily focus on the literature of model misspecification in market risk modelling. The chapter discusses model risk, model misspecification, market risk and the modelling technique used namely the GARCH model. The risk measures of market risk, Value-at-Risk (VaR) and Expected shortfall (ES) are also presented in this chapter. We outline some advantages and drawbacks concerning the GARCH modelling technique.

Chapter 3: The nature and the scope of the methodology applied in the study is presented in this chapter. The mathematical expressions of the modelling technique, risk measures, model performance measures and selection criteria measures used in the study are described.

Chapter 4: This chapter presents the results and findings on the simulated and empirical datasets. The discussion on the results is based on the outlook of model misspecification and the impact it has on the models performance.

Chapter 5: This chapter concludes the study by providing a summary of the major findings and recommendations for future possible studies based on the findings from the study.

2. Literature Review

2.1 Introduction

This chapter provides an overview on relevant literature and information on model misspecification in financial risk management with main focus being on market risk. In Section 2.2 we provide an overview of model risk followed with a discussion on model misspecification which is a component of model risk in Section 2.3. Literature related to market risk is reviewed in Section 2.4. Lastly, the risk measures Value-at-Risk (VaR) and Expected shortfall (ES) as well as the GARCH modelling technique are discussed in Sections 2.5 and 2.6, respectively.

2.2 Model risk

The [Federal Reserve \(2011\)](#) defines a financial model as a quantitative system or procedure that processes input data into quantitative estimates using statistical, financial, or mathematical theories and techniques. A financial model comprises of the following three components:

- An information input component: it provides the model with assumptions and data;
- A processing component: it converts data into estimations;
- A reporting component: it translates the statistical estimates into meaningful business data.

The attention is given to most of the relevant components for model application, thus

reducing inherent complexities of the model. Models are simplified representations of observed features, values, and occurrences in the real world.

Model risk, which is the possibility of adverse implications resulting from decisions based on inaccurate or misused model outputs, is invariably present when financial models are used (Federal Reserve, 2011). Model risk occurs because either the model may be misspecified, it may be used incorrectly or it may have fundamental errors and give inaccurate results when it is compared to the design objective and intended uses.

Various studies have been done investigating the impact of model risk in the financial sector. For instance, the study by Jorion (1996) demonstrates the extent to which estimation risk affects Value-at-Risk (VaR) and provides ideas on how to evaluate quantiles. They reckon that using a multiple of the sample standard deviation to estimate quantiles is considerably better than using sample quantiles directly. They conclude that recognizing estimating errors can lead to more accurate measurement techniques. Model risk exists due to estimation risk being present in Jorion's study. Hence, the development of robust risk management functions and correct implementation thereof is essential.

The paper of Green and Figlewski (1999) considers the impact of model risk on a financial institution's basic writing of options. They use data from 1976 to 1996 to perform an empirical simulation with and without hedging. The options are appraised at model values using standard Black–Scholes valuation formulas. The root mean square error is used as forecast accuracy measure with results showing surprisingly large forecasting errors. They discovered that model risk in trading and hedging derivatives is quite large and concluded that delta hedging based on valuation models applied with forecasted volatilities is the only practical trading and risk management technique for most institutional options writers.

Berkowitz and O'Brien (2002) evaluated the accuracy of Value-at-Risk models at

commercial banks. According to their findings, the banks' 99th percentile VaR forecasts are conservative and, in some cases, highly incorrect. They concluded that the banks' VaR forecasts could not surpass estimates based solely on an ARMA (1,1) and GARCH (1,1) model of the banks' daily Profit and Loss in terms of forecast accuracy and violation size. They suggested that future studies use density forecast evaluation methodologies because considerably more might be learned about the empirical performance of internal valuation models if density predictions were reported.

Kerkhof et al. (2010) suggest a method for calculating capital reserves that takes model risk into consideration. They concentrate on the total model risk (constructed by the three components: estimation risk, misspecification risk and identification risk) associated with econometric modelling uncertainties. Empirical analysis was performed using the S&P 500 index and the USD/GBP exchange rate time series data. The GARCH (1,1) and Gaussian independent and identically distributed (iid) models were chosen as modelling techniques. The outcomes of the models results indicated that when determining the 1% Value-at-Risk at a 95% confidence level for estimate risk, roughly half of the regulatory capital imposed by the Basel Committee can be explained by considering estimation and misspecification risk.

Glasserman and Xu (2014) developed a general approach and particular tools for quantifying model risk and limiting the impact of model error. They generated a simple description of the worst-case deviation from a baseline nominal model by using relative entropy as a measure of 'distance' across stochastic models, defining the worst-case deviation by an exponential change of measure. Using simulation to implement this representation, they bound the influence of model error over several values of relative entropy with a fraction of the computational time required to run the baseline nominal model alone. They demonstrate how researchers and practitioners can include new information into their analysis to set limits on the underlying model's moments and other auxiliary functions, or to leave certain marginal distributions constant. Model error in mean variance portfolio optimization, conditional Value-at-Risk, the Gaussian copula model of portfolio credit risk, delta hedging, and credit

valuation adjustment are all evaluated using these tools. They found that a change in parameters within the baseline nominal model appears to be significantly different from the worst case model deviation.

Jokhadze and Schmidt (2020) look into model risk in the area of financial risk measurement and contingent claim pricing. The modelling technique considered is the dynamic conditional correlation GARCH (1,1) model with multivariate skew normal and multivariate skew Student-t distributions innovations. To determine the distribution of the model from the data they made use of reversible jump Markov chain Monte Carlo simulation methods. The results obtained showed that risk measures exposed to the tail distribution of financial positions turn out to be particularly sensitive to model risk.

Model risk increases when the model becomes more complicated, inputs and assumptions become more unclear, the model is used more widely, and the potential impact develops. Model risk cannot be eliminated, but it can be efficiently managed (Federal Reserve, 2011). However, in order to efficiently manage model risk, we need to ensure that we know and understand the components it consists of. The next section presents model misspecification which is a component of model risk.

2.3 Model misspecification

Any model is just an estimate of reality. This infers that we unavoidably encounter misspecified models. White (1982) states that, misspecified models tend to have biased coefficients and error terms and can also have biased parameter estimations. The problems of parameter estimation and statistical model inference are important in the presence of misspecified model as it could ultimately lead to incorrect decisions being made. Despite the researcher's best efforts, a certain level of model misspecification may be unavoidable, necessitating the use of robust methods that address the assump-

tion of correct specification (Golden et al., 2019). The three model misspecification types are distinguished by Inoue et al. (2020) as due to:

- erroneous serial correlation structures;
- incorrect assumption of constant asset returns;
- measurement error

Several research studies have been done over the years on model misspecification with different methods being developed and suggested with the purpose of identifying the presence of misspecification or one of its components in a model. Winship and Western (2016) presented a Bayesian approach, incorporating uncertainty about the model specification into the standard errors and thus into the confidence intervals. They show how this methodology could be applied in two empirical instances where multicollinearity is a concern. The method proposed here could be broadly used to investigate the sensitivity of identification limits in simultaneous equation models or latent class analysis.

Bonhomme and Weidner (2018) present a framework for the estimation and inference for when a model may be specified incorrectly. They make use of a local asymptotic technique in which the sample size is used to index the degree of misspecification. The method used is based on a one-step change of the initial estimate using a min-max mean squared error criteria allowing for systematic sensitivity analysis when the parameter of interest is partially or irregularly identified. Their framework is inspired by Hansen and Sargent (2001) research on robust decision-making in the face of ambiguity and uncertainty. Bonhomme and Weidner (2018) considered parametric models and semi-parametric likelihood models with a mixture structure and suggest that future studies should focus on extending the approach's applicability to other semi-parametric models.

Inoue et al. (2020) propose an empirical method for discovering and recognizing misspecification of structural economic models. Their method formalizes the typical practice of including "shocks" in the model and uses forecast error variance decomposition and marginal likelihood analysis to identify potential misspecification. The margins are viewed as a measure of Model misspecification, with each margin assessing the extent of misspecification in a specific area of the model individually. The forecast error variance decompositions (FEVDs) and marginal likelihood comparisons are used to determine where and how large the misspecification is. The Monte Carlo simulations performed showed that their method is capable of accurately identifying the source of the misspecification.

Cerreia-Vioglio et al. (2020) extend decision theory to confront model misspecification concerns. Their analysis expands on the research of Hansen and Sargent (2020) by providing an axiomatic foundation for a related decision theory as well as a representation of the implied preferences that can be used to guide applications. They examine Model misspecification formally by proposing a potentially extensive set of probability distributions that illustrate various representations of the data without formal substantive reason. As a result, they recover a variation representation of preferences, which incorporates penalization based on discrepancy measurements between "unstructured alternatives" and the set of structured probability models.

The problem faced by a financial institution may be addressed inappropriately if no actions are taken to address model misspecification. Hence, proper methods should be implemented during the modelling process to identify and address model misspecification. The Federal Reserve (2011) states that robust model development, implementation, application and validation are critical elements to ensure risks are managed properly.

Risk measures and pricing models are based on assumptions about the probability distributions, of the underlying financial positions. Those assumptions are subject to misspecification. This study investigates Model misspecification in the context of

financial risk management under market risk modelling. A discussion on market risk modelling is presented in the next section.

2.4 Market risk modelling

Market risk, also referred to as systematic risk, is the likelihood that the value of an investment may drop which is related to unfavourable price fluctuations that influence the market as a whole (Margaret and Kevin, 2010). According to Milanova (2010), the most common elements associated with market risk include currency exchange rates, interest rates, commodity and equity prices, and the expenses of trade portfolio investments. Market risk is defined by Basel (2006), as the risk of losses in on- and off-balance sheet positions due to market price changes. It is tied to equity and interest rate risk for trading book instruments, as well as foreign exchange and commodities risk across the bank. The interest in forecasting and stress testing of market risk instruments has increased in recent years. Research include that of Giot (2002), who by use of the CBOE Volatility Index(VIX) or CBOE Nasdaq Volatility Index(VXN), RiskMetrics and GARCH type models evaluate the efficiency, information content, and unbiasedness of volatility of market risk forecasts. The study of Aydın and Küçüközmen (2010) involves determining the best-performing risk model and the most appropriate volatility procedure. Sogiakas (2017) examines the use of asymmetric models and skewed distributions while applying the Value-at-Risk method to manage market risk, with empirical results reflecting that assuming skewed distributions when modelling market risk provides more accurate estimates than assuming symmetric distributions.

Regulators have put more scrutiny on the way firms translate supervisory scenarios into significant market factors. They frequently require innovative econometric procedures and extensive model validation, which numerous organizations experience difficulty providing in a timely, transparent, and auditable manner. Financial institu-

tions are expected to track and disclose the degree of market risk in order to impose a capital requirement. The sum of the capital charges for market risks estimated using the standardised technique or the measure of market risk produced from an internal model-based approach, or a mixture of the two approaches, must be used to calculate the minimum capital required for market risk (Basel, 2006).

The standardized methodology employs a “building-block” approach, in which specific risk and overall market risk associated with debt and equity positions are assessed separately. In other words, as stated by Basel Committee on Banking Supervision (2011b) the minimum capital requirement is separated into two capital charges that are calculated independently: The capital charge for each security’s specific risk, regardless of its position (whether it is long or short); and a capital charge to offset long and short positions in various instruments or securities for general market risk. The internal model-based method, on the other hand, permits banks to employ risk measures derived from their internal risk management models provided that

- There are rules for stress testing and identifying the right set of market risk elements;
- There are qualitative guidelines for internal control of model use and quantitative standards that specify the use of common statistical parameters for risk measurement;
- There are proper validation procedures in place for external oversight of model use;
- General criteria for risk management adequacy have been established;
- The bank using a mixture of models and the standardised approach adheres to all the rules stipulated.

The most extensively used risk measures are the Value-at-Risk (VaR) and Expected shortfall (ES), which are used by financial institutions to define and quantify market

risk and distribute capital (Kellner and Rösch, 2016). We discuss these risk measures in the next section.

2.5 Value-at-Risk (VaR) and Expected shortfall (ES)

A risk measure is used to assess how much of an asset (traditionally money) should be kept in reserve. The goal of this reserve is for the regulator to accept the risks that financial institutions, such as banks and insurance firms, take. Value-at-Risk (VaR) and Expected Shortfall (ES) are two popular methods used for measuring market risk. Market risk is the hazard that movements in financial markets, such as equities, bonds, and commodity markets, will have a negative influence on the value of assets (Huang, 2014).

In statistical terms, VaR is a quantile of the conditional distribution of the returns on the portfolio given agent's information set. VaR modelling is a statistical risk management method that quantifies a portfolio's possible loss and the likelihood of that loss occurring. According to Giot (2005) Value-at-Risk is a quantitative instrument that assesses the potential loss that a trader or bank can incur over a particular time period and for a given asset portfolio. Hendricks and Hirtle (1997) define Value-at-Risk models as models that produce an estimate of the maximum amount that the bank can lose on a particular portfolio over a given holding period with a certain degree of confidence level. Saunders and Lange (1997) state that this technique is increasingly being utilized by banks and regulators around the world to predict potential losses associated with financial asset trading, i.e. as a tool to quantify and predict market risk. VaR is an easy and simple approach to describe the size of the portfolio's anticipated losses. VaR approaches are considered flexible as according to Dowd (2007), they can handle a large range of different distributions and

can be used to solve a wide range of risk issues. [Kellner and Rösch \(2016\)](#) state that although VaR suffers with model uncertainties, statistical mistakes, and model risk, it might be regarded as the most widely recognized and used risk measure. Despite being one of the most widely used risk measures, VaR is not coherent. In other words, VaR is not sub-additive, hence it does not reward diversification. This is not the case for ES.

ES is closely related to the VaR since it estimates the predicted loss value when VaR's violation has occurred. [Kheir \(2019\)](#) defines ES as a risk measure that calculates the likelihood that a portfolio would lose a certain percentage or more in returns by averaging out the sampled returns below a given quantile of data. ES is considered an alternative coherent risk measure used in place of VaR. A risk measure is considered coherent if it satisfies the four axioms set by [Artzner et al. \(1999\)](#):

- Axiom 1 –Translation invariance: The risk measure should demonstrate that the capital required supports the loss's perceived variability rather than its expected amount. When a fixed sum is added or subtracted from a loss, the capital (the amount in excess of the expected loss) remains unchanged.
- Axiom 2 –Subadditivity: There should be a diversification benefit from compounding loss distributions. The risk measure of the compounded distribution should not surpass the sum of the risk measures of the individual distributions, even if the distributions are hundred percent correlated.
- Axiom 3 –Positive homogeneity: The risk measure should show that the capital required to sustain “ T ” identical losses should be equivalent to “ T ” times the capital required to support one loss
- Axiom 4 –Monotonicity: The risk measure should show that the capital required to sustain a smaller loss (with the same distribution) should be less than the capital required to support a larger loss.

ES is considered a coherent risk measure because it satisfies these four axioms hence it is able to generate a diversification benefit (Tasche, 2002). Degiannakis et al. (2013) consider ES a better risk measure since it is more useful in “tail-risk” situations compared to VaR. The Basel III Committee has also recommended the replacement of VaR with ES for the internal model-based approach and recommends use of the 97.5% confidence level for the ES (Stavroyiannis, 2017).

Despite the fact that risk measures are becoming more popular, the possibility of adverse implications due to inaccurate or misused model outputs is of great concern. This is, because we neither know the “true” underlying model or the “true” value for the risk measure itself (in terms of statistical parameters), model assumptions have to be specified and parameters for the model have to be estimated in order to derive estimates for the risk measure. This leads to different sources of model risk such as model misspecification or estimation errors.

The most popular parametric VaR and ES models are those derived from traditional location-scale models such as autoregressive moving average (ARMA) and generalized autoregressive conditional heteroscedastic (GARCH) models. GARCH and its variants comprise a broad set of volatility models. According to Orhan and Köksal (2012) early calculations made use of the variance–covariance approach that assumed normality. Such models are abandoned since normality was not approved analytically because many returns have heavy tails. One remedy was to use the historical simulation approach. This approach could not survive long, because the method to calculate VaR and ES must be able to handle clustering of volatility (Orhan and Köksal, 2012). Orhan and Köksal (2012) state that standard deviation and the distribution – as well as the parameters and the critical value of the distribution – are the key constituents of the VaR and ES risk measures. According to Nwogugu (2006), GARCH models have proven themselves as the best methods to return the standard deviation and the GARCH estimates are known to be sensitive to the magnitude of the standard deviation and the distribution used. Researchers used the traditional normal distribution of GARCH models for a long time because of its ease of use in practice. However,

it is proven that high frequency financial data have heavy tails and Student's t and GED distributions are addressed to be more capable of representing these series.

2.6 Forecasting Volatility using GARCH (1,1)

The ARCH model have been a typical method for analysing the volatility of financial time series over the last few decades. Engle (1982) proposed this framework, which Bollerslev (1986) later expanded to the GARCH model. The GARCH model is a statistical model that is used to assess a range of financial data. The GARCH model is well-known for predicting time-varying and predictable volatility. According to Hansen et al. (2012) the standard GARCH models utilise daily squared returns to obtain information about the present level of volatility, which is then used to construct expectations about the volatility of the next period. The GARCH process exists with the assumption that return innovations follow the Normal, Student-t and Generalized Error Distributions (GED). Originally, GARCH models are built on the assumption that financial time series have a Normal (Gaussian) distribution. However, financial time series are heavy-tailed in most cases (Bollerslev, 1987). Hence the Student-t distribution and the GED distribution were proposed by Bollerslev (1987) and Nelson (1991) respectively, to address this issue by capturing the heavy-tailed behaviour of financial time series.

Hansen and Lunde (2005) make a comparison of 330 ARCH-type models ability to predict the conditional variance in an out-of-sample setting. The out-of-sample performance of the volatility models was assessed using six loss functions, with realized variance being utilized to create an estimate of unobserved conditional variance. To compare the 330 volatility models to the GARCH (1,1), they utilized the test for superior predictive ability (SPA) and the reality check (RC) for data snooping. The daily returns of IBM and the daily exchange rate data of DM in dollars was used to perform the model comparisons. Findings showed no evidence of the GARCH (1,1)

being outperformed by the other models when the models are tested using exchange rate data. However, in the analysis of IBM stock returns they obtained substantial evidence that the GARCH (1,1) performs poorly when compared to models that can account for the leverage effect. This according to Hansen and Lunde (2005) suggests that good out-of-sample performance necessitates a specification that can support a leverage effect.

The study of Vee et al. (2011) use the GARCH (1,1) with Generalized Error Distribution (GED) and Student-t distribution to assess forecasts for the US dollar and Mauritian Rupee. The accuracy measures they made use of were the Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE). These measures were calculated using daily data and the difference between the daily data and the estimated forecast. The results reveal that both models perform well in forecasting out-of-sample volatility. However, the model with GED errors had a modest advantage over the model with Student-t errors. The study suggested future studies to consider the aspect of skewness that was not addressed in their paper. In other words, GARCH (1,1) models with skewed Student-t or Skewed GED distributions.

Yaya et al. (2014) evaluated the effect of misspecification of correct sampling probability distribution of GARCH processes under the three distributions (i.e., Normal, Student-t and GED). They performed Monte Carlo simulation on the GARCH model using the three distributions and estimation of the model was based on the three distributions in each sample. The estimated model's performance was evaluated using parameter estimates and forecast performance. The forecast evaluation criteria they considered were the Root Mean Squares Forecast Error (RMSFE), Mean Absolute Error (MAE), Mean Absolute Percentage Forecast Error (MAPFE) and Theil Inequality. The results showed that the GARCH (1,1) with GED assumed distribution outperformed the Normal and Student-t distributions in terms of parameter estimates and forecast. The study recommended future studies to make use of higher-order models, and in that case, more advanced software should be considered for simulations to avoid convergence issues.

The study performed by Grek (2014) focused on the forecasting accuracy of heteroscedastic models namely the Autoregressive conditional heteroskedasticity model (ARCH), ARCH with Student-t distribution, Generalized autoregressive conditional heteroskedasticity model (GARCH), GARCH with Student-t distribution, Integrated GARCH (IGARCH), Exponential GARCH (EGARCH), GARCH in the Mean (GARCHM) and Threshold GARCH model (TGARCH). They made use of the daily closing prices for the stocks Ericsson, H&M and Investor during a ten-year period from 31st of March 2004 to 31st of March 2014. The MSE (Mean squared error) was used to determine which model has the best forecast accuracy. They found that when the continuously compounded rate of return shows weak ARCH effect (due to volatility clustering), the GARCH (1,1) with Student-t innovation performed better than any of the other ARCH- and GARCH models at forecasting. However, when the rate of return has strong ARCH effect, the EGARCH (1,1) is superior to the other models.

The work of Sharma et al. (2020) compares linear (symmetric) and non-linear (asymmetric) Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models for forecasting the volatility of the financial markets of the top five major emerging countries, namely China, India, Indonesia, Brazil, and Mexico. The volatility of the financial markets of the five countries was empirically investigated using GARCH (1,1), Exponential generalized autoregressive conditional heteroscedasticity (E-GARCH (1,1)), and Threshold Generalized autoregressive conditional heteroscedasticity (T-GARCH (1,1)) volatility models. Due to the effect of leverage being minimal, their results identified the GARCH (1,1) model superior to non-linear GARCH models for forecasting volatility. Sharma et al. (2020) suggested that the use of multivariate GARCH models to analyse the diversification opportunities among emerging countries, should be considered for future studies.

We find that the distribution of the error terms has a significant impact on the results from the research studies discussed above. In this study we want to identify model misspecification of the GARCH models with Normal, Student-t and GED distributions by analysis performed on the estimation of the key parameters of the

model. Hence identifying the presence of misspecification by evaluating the parameter estimates accuracy, efficiency, consistency and biasness.

2.7 Chapter summary

The purpose of this chapter was to present the theoretical framework, empirical literature, and various research approaches used by others for model risk, model misspecification, market risk, VaR and ES, and the identified modelling technique the GARCH (1,1) model. The following chapter presents a detailed review on the methods used in this study.

3. Research methodology

3.1 Introduction

In this chapter we focus on the methods used to achieve the study's objectives highlighted in Chapter 1. We review the procedures and techniques employed in the data analysis. The GARCH (1,1) with Normal, Student-t and GED innovations was used in this study because it is good at modelling volatility and extreme values. The sections of this chapter are constructed as follows: A discussion on how and why we transform the asset prices into returns follows in Section 3.2. In Section 3.3 we discuss the preliminary data analysis. A description on the normality and stationarity tests is presented in Section 3.4 and 3.5, respectively. In Section 3.6 we provide a review on the GARCH modelling technique. A discussion on the maximum likelihood estimation method follows in Section 3.7. The innovations from the Normal, Student-t and GED are reviewed in Section 3.8. A discussion on the Auto Regressive Conditional Heteroscedasticity (ARCH) effects tests is given in Section 3.9. In Sections 3.10, 3.11 and 3.12 we provide an overview on the model selection criteria, accuracy measures and risk measures respectively.

3.2 Asset returns

The asset prices are transformed into asset returns in this study. This is done because the returns of an asset provide a comprehensive and scale-free overview of the investment and due to returns' attractive statistical properties it makes the return series easier to handle (Campbell et al., 1998).

3.2.1 Simple returns

The simple returns , R_t , of an assest as stated by Tsay (2005) is calculated as follows:

$$1 + R_t = \frac{P_t}{P_{t-1}}, \quad (3.2.1)$$

where P_t is the closing price of the asset at time index t and P_{t-1} is the closing price of the asset at time index $t - 1$.

3.2.2 Continuously compounded asset returns

We transform our asset prices into the continuously compounded returns by taking the simple return's natural logarithm. As a result, the daily returns r_t are computed as:

$$r_t = \ln(1 + R_t) \quad (3.2.2)$$

$$= \ln\left(\frac{P_t}{P_{t-1}}\right)$$

$$= \ln(P_t) - \ln(P_{t-1})$$

Applying the natural exponential function on both sides of Equation 3.2.2 results in the continuously compounded returns given as:

$$e^{r_t} = \frac{P_t}{P_{t-1}}. \quad (3.2.3)$$

Therefore, the current prices of an asset may be given as:

$$P_t = e^{r_t} P_{t-1}. \quad (3.2.4)$$

3.2.3 Stylized facts for asset returns

Through investigations of many markets and instruments, there are common denominators found among their properties, known as stylized facts. According to Cont (2001a) asset returns have the following stylized statistical properties:

Table 3.1: Stylized facts for asset returns

Statistical property	Explanation of property
Absence of autocorrelations	Over consecutive time intervals, there is little to no similarity between a time series and a lag version of itself, except for very small intraday time scales.
Heavy tails	Finite tail index displays a power-law or Pareto-like tail for the unconditional distribution of asset returns with the normal distribution and stable laws with infinite variance excluded. However, establishing the exact shape of the tails is challenging.
Gain/loss asymmetry	Large downward changes in asset prices and asset index values are observed, but rarely similarly large upward moves.
Aggregational Gaussianity	The distributional features of asset returns resemble a normal distribution more and more if the time scale over which asset returns are evaluated is increased.
Intermittency	The presence of irregular bursts in the time series of a variety of volatility estimators shows that returns are highly volatile.
Volatility clustering	Various measures of volatility show a positive autocorrelation, over the course of several days, indicating that high-volatility occurrences tend to cluster in time. In other words, large changes tend to be followed by large changes, and small changes tend to be followed by small changes.
Conditional heavy tails	After accounting for volatility clustering by use of GARCH-type models, the residual time series still displays heavy tails. These tails, are however not as heavy as the tails of the unconditional distribution of returns.
Slow decay of autocorrelation in absolute/ squared returns	The autocorrelation function (ACF) of absolute or squared returns decays slowly as the time lag increases.
Leverage effect	The majority of measures of an asset's volatility have a negative relationship with its returns.
Volume/volatility correlation	Correlation exists between trading volume and all volatility measures.
Asymmetry in time scales	Fine-scale volatility is better predicted by coarse-grained volatility than vice versa.

In this study, model misspecification may be observed through the violation of the above mentioned properties for the asset returns.

3.3 Preliminary data analysis

Preliminary data analysis is done to determine some key features of the data. This is done by use of descriptive statistics and basic plots such as Histogram charts and Quantile-Quantile plots (QQ-plots). Prior to undergoing further statistical data analysis, preliminary data analysis is required since it helps us to characterize the behavior of the observed data.

3.4 Normality test

We perform normality tests in order to determine if our return series is normally distributed. Normality of the return series in this study is formally tested by use of the Shapiro-Wilk test and Jarque-Bera (JB) tests. The null and alternative hypotheses for both test are defined as follows:

H_0 : The distribution of the series is normal

H_a : The distribution of the series is not normal

The Jarque-Bera (JB) and Shapiro-Wilk tests are illustrated in Section 3.4.1 and Section 3.4.2 respectively. Both the tests have the decision rule that the null hypothesis is rejected if the p-value obtained for the test statistic is less than the significant level of $\alpha=5\%$.

3.4.1 Jarque-Bera (JB)

The JB test formulated by [Jarque and Bera \(1987\)](#) derives tests for normality from observations and regression residuals using the Lagrange multiplier approach on the Pearson family of distributions. This test analyses how well the skewness and kurtosis of the data matches that of a Normal distribution. The JB test statistic is given as:

$$JB = \frac{T}{6} \left((S)^2 + \frac{(K - 3)^2}{4} \right), \quad (3.4.1)$$

where

$$S = \frac{\frac{1}{T} \sum_{t=1}^T (r_t)^3}{\left(\frac{1}{T} \sum_{t=1}^T (r_t)^2 \right)^{\frac{3}{2}}}, \quad (3.4.2)$$

$$K = \frac{\frac{1}{T} \sum_{t=1}^T (r_t)^4}{\left[\frac{1}{T} \sum_{t=1}^T (r_t)^2 \right]^2}, \quad (3.4.3)$$

T is the sample size, S and K the skewness and the sample kurtosis respectively. r_t is the t^{th} order statistic.

3.4.2 Shapiro-Wilk test

The Shapiro-Wilk test proposed by [Shapiro and Wilk \(1965\)](#) is one of the most widely used tests for normality. The correlation between data and corresponding normal scores forms the basis of the Shapiro-Wilk test. It is especially powerful when applied to smaller sample sizes as it has more power to detect non-normality. The Shapiro-Wilk test statistic for normality is defined as:

$$W = \frac{\left(\sum_{t=1}^T a_t r_t \right)^2}{\sum_{t=1}^T (r_t)^2}, \quad (3.4.4)$$

where r_t is the t^{th} order statistic and a_t is given by:

$$a_t = \frac{m^T V^{-1}}{\sqrt{m^T V^{-1} V^{-1} m}}, \quad (3.4.5)$$

where $m = (m_1, \dots, m_n)^T$ are known as the expected values of the order statistics of independent and identically distributed (i.i.d) random variables and V is the covariance matrix of those order statistics.

3.5 Test for Stationarity

Many time series techniques make the assumption that a series is generated by a stationary process. In other words, the mean, variance and autocorrelation of the series generated remain constant over time. A time series r_t , is said to be weakly stationary if the following properties hold:

1. $E(r_t) = \mu_t$ for all t
2. $Var(r_t) = E[(r_t - \mu_t)^2] = \sigma^2$
3. $Cov(r_t, r_{t-k}) = E[(r_t - \mu_t)(r_{t-k} - \mu_t)] = \gamma_k$ for all t and any k ,

with γ_k being the covariance at lag k between r_t and r_{t-k} . The variance of the time series r_t is obtained as γ_0 , that is if $k = 0$ (Tsay, 2005).

Stationarity tests are performed in order to avoid spurious results (Hill et al., 2018). Stationarity of a series can be determined by looking at time series plots, identifying if any trends or seasonality is present in the series. Formal tests can also be performed on the series to determine stationarity. In this study the Augmented Dickey-Fuller (ADF) and Phillips Perron (PP) tests will be used and are discussed in Sections 3.5.1 and 3.5.2.

3.5.1 Augmented Dickey-Fuller (ADF) test

The ADF test was developed by [Dickey and Fuller \(1979\)](#). The purpose of the ADF test is to see if the unit root is still present in the return series. The series is said to be stationary if the unit root does not exist in the return series. [Yunita \(2016\)](#) and [Santos \(2019\)](#) both considers the ADF test as a good measure in assesing the stationarity of a series. The ADF model is used to test the following hypotheses:

H_0 : The series is non-stationary.

H_a : The series is stationary.

If we do not reject the H_0 it implies that the series is non-stationary or contains a unit root. We do not reject the null hypothesis if the p-value of the test displays a higher value at a significance level of 5%.

3.5.2 Phillips Perron (PP) test

An alternative stationarity test is the PP test developed by [Phillips and Perron \(1988\)](#). The estimating scheme for the PP test is the same as for the ADF test. Although these tests are similar, they differ in the way they deal with heteroscedascity and autocorrelation. Some of the advantages of the PP test is that it is extremely powerful when used for large sample sizes and it corrects heteroskedasticity in the error terms. The PP test makes use of non-parametric methods and disregards serial correlation ([Santos, 2019](#)). The hypothesis procedures for the PP test is the same as the ADF test. The calculated p-value of PP is then compared with the significance level of 5%. If the calculated p-value is less than the significance level, we reject the null hypothesis that the series have a unit root, therefore confirming that the series is stationary.

3.6 Specification of the modelling techniques

In this section we first review the Auto Regressive Conditional Heteroskedasticity (ARCH) model. We do this in order to justify why we chose the Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) as our modelling technique. The ARCH and GARCH models are discussed in Sections 3.6.1 and 3.6.2 respectively.

3.6.1 The Auto Regressive Conditional Heteroskedasticity (ARCH) model

The ARCH model, developed by Engle (1982), is straightforward and easy to use. It accounts for clustered errors and nonlinearities. The ARCH model is expressed as follows:

$$r_t = \mu_t + a_t, \quad (3.6.1)$$

$$a_t = \sigma_t \epsilon_t, \quad (3.6.2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2, \quad (3.6.3)$$

$$\alpha_0 > 0, \alpha_1 > 0,$$

where α_0 and α_1 are the parameters of the conditional variance. The mean of r_t is denoted by μ_t , σ_t^2 represents the volatility of today, a_{t-1}^2 is the value of the time series of yesterday, and ϵ_t is the random error of today. According to Matei (2009) the “random coefficient problem” is a feature of ARCH models. In other words, the power of forecast changes from one period to another. However, ARCH is not without some weaknesses. The ARCH weaknesses is listed by Tsay (2005) as follows:

- Positive and negative shocks are assumed to have similar effects on volatility.

- The ARCH model is restrictive. For instance, in an ARCH (1, 1) model, α_1^2 must be in the interval $[0, \frac{1}{3}]$.
- Because ARCH models respond slowly to large isolated shocks to the return series, they are likely to overpredict volatility.

The GARCH model was introduced in order to overcome these weaknesses.

3.6.2 The Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) model

The GARCH model was proposed by [Bollerslev \(1986\)](#) which is an extension of the ARCH model. The GARCH model is a common statistical modelling tool used for predicting the volatility of financial asset returns because it allows for conditional variance to be dependent upon previous own lags ([Xiao et al., 2009](#)). The GARCH (1,1) model is specified as follows:

$$r_t = \mu_t + a_t, \quad (3.6.4)$$

$$a_t = \sigma_t \epsilon_t, \epsilon_t \sim N(0, 1) \text{ or } t(0, 1, \nu) \text{ or } GED(0, 1, \kappa), \quad (3.6.5)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad (3.6.6)$$

$$\alpha_0 > 0, \alpha_1 > 0, \beta_1 > 0, (\alpha_1 + \beta_1) < 1,$$

where α_0 , α_1 and β_1 are the parameters of the conditional variance. The degrees of freedom is denoted by ν , κ denotes the shape of the distribution and σ_{t-1}^2 is considered as the volatility of yesterday. The constraints on the parameters α_0 , α_1 and β_1 , assure that volatility (σ_t^2) is always positive and stationary. The sum, $\alpha_1 + \beta_1$ measures how fast large volatilities decay after a shock, which is known as volatility

persistence. The ϵ_t known as the innovation distribution is a sequence of i.i.d random variables.

3.7 Parameter estimation

Parameter estimation comprises of calculating the parameters for a distribution using sample data. According to Myung (2003) we can assess how closely the specified model is to the observed data. The method of parameter estimation is used to assess the goodness of fit, that is, by determining the parameter values of a model that best fits the data. The estimation techniques used are referred to as estimators. Maximum likelihood estimation (MLE) is the estimator used in this study. MLE is a popular method of parameter estimation in statistics that is used in a variety of statistical modelling techniques, particularly non-linear modelling using non-normal data (Myung, 2003).

3.7.1 Maximum likelihood estimation

Casella and Berger (2002) state that “maximum likelihood estimation is the most natural and popular way to estimate parameters where the parameter values that are most likely to generate the observed data are chosen”. MLE is the basis for many statistical inference methods. In other words, the MLE is required for several inference methods such as those given by Myung (2003) in the chi-square test, the G-square test, Bayesian methods, inference with missing data, modelling of random effects, and many model selection criteria such as Akaike information criterion (Akaike, 1973) and the Bayesian information criteria (Schwarz, 1978).

Suppose a sample $r_1, \dots, r_T$ has been obtained from a probability model specified by density function $f(r; \alpha_0, \alpha_1, \beta_1)$ depending on parameter(s) α_0, α_1 and β_1 . The

maximum likelihood estimate is generated as follows:

We first obtain the likelihood function, $L(\alpha_0, \alpha_1, \beta_1)$, where

$$L(\alpha_0, \alpha_1, \beta_1) = \prod_{t=1}^T f(r_t; \alpha_0, \alpha_1, \beta_1), \quad (3.7.1)$$

that is, the product of the T density function terms (where the t^{th} term is the density function evaluated at r_t) viewed as a function of the parameters, α_0, α_1 and β_1 .

After writing the likelihood function we must get the natural logarithmic function of the likelihood

$$l_T(\alpha_0, \alpha_1, \beta_1) = \ln \prod_{t=1}^T f(r_t; \alpha_0, \alpha_1, \beta_1) = \sum_{t=1}^T \ln f(r_t; \alpha_0, \alpha_1, \beta_1). \quad (3.7.2)$$

We then continue with differentiation in order to get the values for $\hat{\alpha}_0, \hat{\alpha}_1$ and $\hat{\beta}_1$ by which $l_T(\alpha_0, \alpha_1, \beta_1)$ is maximized.

$$\frac{\partial}{\partial_{\alpha_0, \alpha_1, \beta_1}} [l_T(\alpha_0, \alpha_1, \beta_1)] = 0. \quad (3.7.3)$$

Finally we need to check that the estimates, $\hat{\alpha}_0, \hat{\alpha}_1$ and $\hat{\beta}_1$ obtained corresponds to a maximum in the (log) likelihood function by evaluating the second derivative of $l_T(\alpha_0, \alpha_1, \beta_1)$ with respect to the parameters (α_0, α_1 and β_1). In the single parameter case such as α_0 if the second derivative of the log-likelihood is negative at $\alpha_0 = E[\hat{\alpha}_0]$, then $E[\hat{\alpha}_0]$ is confirmed as the MLE of α_0 .

3.8 Innovations of GARCH (1,1) model

It is often assumed that the distribution of innovation follows a normal distribution. However, this is not always the case as financial time series are often leptokurtic. As a result we make use of the Student-t and GED distributions because they take into account the leptokurtic behaviour of financial time series data. The conditional density and log-likelihood function of ϵ_t under the Normal, Student-t and GED distributions are discussed as follows.

3.8.1 The Standardized Normal Distribution

$$f(\epsilon_t) = \frac{1}{\sqrt{2\pi\sigma_t^2}} e^{-0.5\left(\frac{\epsilon_t^2}{\sigma_t^2}\right)}, -\infty < \epsilon_t < \infty \quad (3.8.1)$$

where σ_t^2 is the variance of the innovations. The corresponding log-likelihood function is given by:

$$l_T(\epsilon_t) = -0.5 \left[T \times \ln(2\pi) + \sum_{t=1}^T \ln(\sigma_t^2) + \sum_{t=1}^T \frac{\epsilon_t^2}{\sigma_t^2} \right], \quad (3.8.2)$$

where T is the sample size.

3.8.2 The Standardized Student-t distribution

$$f(\epsilon_t, \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\pi(\nu-2)\sigma_t^2}} \left[1 + \frac{\epsilon_t^2}{(\nu-2)\sigma_t^2} \right]^{-\frac{\nu+1}{2}}, -\infty < \epsilon_t < \infty \quad (3.8.3)$$

where ν is the degrees of freedom ($\nu > 2$) also known as the shape parameter and Γ is the gamma function. The Student-t distribution is symmetric around zero. The Student-t distribution converges to the Normal distribution as the shape parameter

approaches infinity. The log-likelihood function for the Student-t distribution can be written as:

$$l_T(\epsilon_t, \nu) = T \left[\ln \Gamma \left(\frac{\nu+1}{2} \right) - \ln \Gamma \left(\frac{\nu}{2} \right) - 0,5 \ln [\pi(\nu-2)] \right] - 0,5 \sum_{t=1}^T \left[\ln(\sigma_t^2) - (1+\nu) \ln \left(1 + \frac{\epsilon_t^2}{(\nu-2)\sigma_t^2} \right) \right]. \quad (3.8.4)$$

3.8.3 The Standardized GED

$$f(\epsilon_t, \kappa) = \frac{\kappa e^{-0,5 \left| \frac{\epsilon_t}{\lambda \sigma_t} \right|^\kappa}}{\lambda 2^{(\kappa+1/\kappa)} \Gamma(1/\kappa)}, -\infty < \epsilon_t < \infty \quad \text{and} \quad \kappa > 0 \quad (3.8.5)$$

where

$$\lambda = \left[\frac{\Gamma(1/\kappa)}{2^{(2/\kappa)} \Gamma(3/\kappa)} \right]^{0,5}.$$

Depending on the degrees of freedom the GED is a symmetric distribution that can be both leptokurtic and platykurtic. The GED has thicker tails than the Normal distribution when $0 < \kappa < 2$ (leptokurtic) and has thinner tails than the normal distribution at $\kappa > 2$ (platykurtic). At $\kappa = 2$ the GED distribution becomes the standard Normal distribution and as κ approaches infinity the GED distribution becomes a Uniform distribution on the interval $[-\sqrt{3}, \sqrt{3}]$. The log-likelihood function for the GED can be given by:

$$l_T(\epsilon_t, \kappa) = \sum_{t=1}^T \left[\ln \left(\frac{\kappa}{\lambda} \right) - 0,5 \left| \frac{\epsilon_t}{\lambda} \right|^\kappa - (1 + \kappa^{-1}) \ln(2) - \ln \Gamma \left(\frac{1}{\kappa} \right) - 0,5 \ln(\sigma_t^2) \right]. \quad (3.8.6)$$

3.9 Auto regressive conditional heteroscedasticity (ARCH) effects

The presence of stationarity and ARCH effects in the residual return series should be evaluated before applying GARCH models to returns series. To test the presence of ARCH effects in the return series data the Ljung-Box and Lagrange multiplier (LM) tests are used. Discussion on these test follows.

3.9.1 The Ljung-Box test

The Ljung-box test formulated by [Box and Pierce \(1970\)](#) is used to determine the presence of serial autocorrelation up to a given lag k . The Ljung-Box hypotheses test is given by:

H_0 : The series data do not exhibit serial autocorrelation.

H_a : The series data exhibit serial autocorrelation.

The formula of the Ljung-box test statistic is given by:

$$Q = T(T+2) \sum_{k=1}^s \frac{\hat{\rho}^2(k)}{T-k}, \quad (3.9.1)$$

where T is the sample size, s is the number of autocorrelation lags and $\hat{\rho}(k)$ is known as the estimated autocorrelation of the series at lag k . ρ_k is defined as:

$$\rho_k = \frac{\sum_{t=1}^T (r_t - \bar{r})(r_{t-k} - \bar{r})}{\sum_{t=1}^T (r_{t-k} - \bar{r})^2},$$

where $\bar{r}$ is the mean of the series. We reject the null hypothesis if the calculated p-value is less than the significance level of 5 % which imply that the return series data display serial autocorrelation.

3.9.2 The Lagrange multiplier test

It is necessary to assess the residuals for the presence of Auto Regressive Conditional Heteroscedasticity (ARCH) effects before applying GARCH models on the returns series data. This is done by use of the LM test defined as:

H_0 : No ARCH effects in series.

H_a : Presence of ARCH effects in series.

With the test statistic used given by:

$$LM = T.r^2 \sim \chi^2(p), \quad (3.9.2)$$

where T is the total number of observations and r^2 is the coefficient of determination. The number of restrictions imposed by the null hypothesis is defined by p .

3.10 Model selection criteria

Model selection criteria are rules that we use to ensure that, based on the observed data, we select the most appropriate statistical model from a group of candidate models. The Akaike Information Criterion (AIC) by Akaike (1973) and the Bayesian Information Criterion (BIC) by Schwarz (1978) are the most generally and widely used criteria methods. According to Christensen and Angeles (2018) the AIC is

efficient because it penalises the model for the higher number of parameters included. The BIC, on the other hand, is consistent because when the sample size increases, the criterion selects a finite-dimension model as long as it is among the candidate models (Christensen and Angeles, 2018). The model with the lowest AIC and BIC is preferred. These methods are defined as follows:

$$AIC = -2l_T + 2p, \quad (3.10.1)$$

$$BIC = -2l_T + p\ln(T), \quad (3.10.2)$$

where l_T is the log-likelihood of the estimated model, p is the total number of parameters that are estimated in the model and T is the sample size.

3.11 Model accuracy evaluation

Accuracy measures are used to evaluate the performance of models. The accuracy can be measured using the Mean squared error (MSE), Root mean squared error (RMSE), Mean absolute error (MAE), and Mean absolute percentage error (MAPE). Discussion on accuracy measures are given as follows.

3.11.1 Mean squared error (MSE)

According to Schluchter (2005) the MSE is a measure for both the accuracy and precision of an estimator. The accuracy (bias) of an estimator is determined by how much the expected value deviates systematically from the true value. The precision (variance) indicates how much it varies about its expected value due to sampling variability (Schluchter, 2005). Since the MSE penalizes large forecast errors more harshly than other commonly used accuracy measures, it is seen to be the most appropriate method to determine which methods avoid large errors (Lim and Sek,

2013). The MSE is calculated as follows:

$$MSE = T^{-1} \sum_{t=1}^T (r_t - \hat{r}_t)^2, \quad (3.11.1)$$

where the number of days observed is represented by T . The observed actual value at time t is represented by r_t and $\hat{r}_t$ is the estimated value at time t . The smaller the error the better the accuracy.

3.11.2 Root mean squared error (RMSE)

The RMSE is a statistic that measures the degree of uncertainty in a prediction. Despite being affected by outliers more than other measures, the RMSE and MSE are commonly used in statistical modelling (Bratu, 2012). The RMSE can be expressed as follows:

$$RMSE = \left[T^{-1} \sum_{t=1}^T (r_t - \hat{r}_t)^2 \right]^{0.5}, \quad (3.11.2)$$

we see that the RMSE is computed by the square root of mean square errors.

3.11.3 Mean absolute error (MAE)

The arithmetic mean of absolute errors is computed by the MAE. According to Chen et al. (2017), when there are outliers in the data set, the MAE provides biased results. This is one of the drawbacks of using the MAE. The MAE can be calculated using the following expression:

$$MAE = T^{-1} \sum_{t=1}^T |r_t - \hat{r}_t|. \quad (3.11.3)$$

3.11.4 Mean absolute percentage error (MAPE)

According to [Kim and Kim \(2016\)](#), because of its scale independence and interpretability, the MAPE is one of the most extensively used measures of accuracy. However, MAPE produces infinite or undefined values for zero or close to zero actual values ([Kim and Kim, 2016](#)). This is a significant drawback of MAPE. The expression of MAPE is as follows:

$$MAPE = T^{-1} \sum_{t=1}^T \left| \frac{r_t - \hat{r}_t}{r_t} \right| \times 100, \quad (3.11.4)$$

where $\left| \frac{r_t - \hat{r}_t}{r_t} \right| \times 100$ defines the absolute percentage error calculated on the fitted values for a particular forecasting method. We made use of the MSE and RMSE accuracy measures in our data analysis. In Chapter 4 we will focus on the MSE and RMSE. The risk measures are discussed in the next section.

3.12 Risk measures: Value-at-Risk (VaR) and Expected Shortfall (ES)

The Value-at-Risk (VaR) metric is arguably the most extensively used statistic for calculating market risk. Consider a portfolio of risky assets with the cumulative distribution function of the corresponding loss function given by $F_L(l) = P(L \leq l)$. [McNeil et al. \(2015\)](#) state that the VaR can be thought of as the portfolio's maximum loss which is not exceeded with a given high probability. The VaR of a portfolio is given by:

$$VaR_\alpha = \inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} = \inf\{F_L(l) \geq \alpha\}, \quad (3.12.1)$$

with $\alpha \in (0, 1)$. The VaR is known as a quantile of a loss distribution with the size of losses that exceeds the threshold being unknown. This is a major drawback of VaR hence ES was introduced.

The ES takes the size of the tail loss into consideration by overcoming the difficulty faced when using the VaR. The ES is defined as follows:

$$ES_\alpha = \frac{1}{1 - \alpha} \int_\alpha^1 VaR_u(L) du, \quad (3.12.2)$$

with L representing the loss and confidence interval $\alpha \in (0, 1)$. [Ladokhin \(2009\)](#) states that we might think of ES as a loss L that occurs when the VaR is exceeded. The ES can thus also be expressed by:

$$ES_\alpha = E(L|L \geq VaR_\alpha). \quad (3.12.3)$$

3.13 Chapter summary

In this chapter we discussed the methodology. The method for carrying out preliminary data analysis to be presented in Chapter 4 is discussed. We presented the Stationary tests, Normality tests, and test for ARCH effects. We illustrated the mathematical expressions of the ARCH model and its extensions known as the GARCH(1,1) model. We also presented the mathematical expressions for the GARCH (1,1) model innovations namely the Normal, Student-t and GED. We discussed the AIC and BIC model selection criteria. Accuracy performance methods were reviewed such as the MSE, RMSE, MAE and MAPE. The two risk measuring techniques used for measuring market risk namely the VaR and ES were presented in this chapter. The data analysis and results obtained based on the methodology presented in this chapter is provided next in Chapter 4.

4. Results and findings

4.1 Introduction

In this chapter we present the analysis for our simulated and empirical results. The aim of the study is to identify possible errors made during the model building and selection process that can result in model misspecification. To achieve this, the statistical methods discussed in Chapter 3 were performed on the simulated and empirical datasets. The data analysis is done using RStudio with the packages fGarch by [Wuertz et al. \(2013\)](#), fBasics by [Wuertz et al. \(2017\)](#), tseries by [Trapletti et al. \(2020\)](#) and quantmod by [Ryan et al. \(2020\)](#). We make use of RStudio because it is an open source, simple to use, and has excellent graphical displays [Tsay \(2005\)](#).

The outline of the chapter is as follows: In Section 4.2 the simulated results and analysis for three different scenarios are presented. We discuss the results obtained for Samsung electronics series, Bitcoin cryptocurrency series and Moody's seasoned Aaa corporate bond yield (BAAA) series in Section 4.3. We close off the chapter with a summary in Section 4.4.

4.2 Simulated Results

We conducted a series of simulations for the GARCH (1,1) for varying sample sizes. The true innovation distribution for the simulation studies of the three scenarios were generated from the Standard Normal distribution, i.e. $r_t \sim N(0, 1)$. We generated different initial values for $\alpha_0, \alpha_1, \beta_1$. The different initial values used for each scenario and each model are summarised in Table [4.1](#).

For Scenario 1, the parameters α_0 and α_1 had a constant increment of 0,1 and 0,2

Table 4.1: Initial parameter values

Scenario	Model	α_0	α_1	β_1
Scenario 1	1	0,1	0,1	0,8
	2	0,2	0,3	0,6
	3	0,3	0,5	0,4
	4	0,4	0,7	0,2
Scenario 2	1	0,4	0,8	0,1
	2	0,3	0,6	0,3
	3	0,2	0,4	0,5
	4	0,1	0,2	0,7
Scenario 3	1	0,15	0,15	0,15
	2	0,25	0,25	0,25
	3	0,35	0,35	0,35
	4	0,45	0,45	0,45

Notes: This table represents the initialised values used for each scenario and each model

respectively and β_1 was assigned a constant decrease of 0,2. For Scenario 2, the parameter β_1 had a constant increment of 0,2. The parameters α_0 and α_1 were assigned a constant decrease of 0,1 and 0,2 respectively. Lastly, for Scenario 3 we initialised the parameters α_0 , α_1 and β_1 to be tied with a constant increment of 0,1. Different sample sizes were generated with $T = \{10, 50, 100, 200, 500, 1000, 5000, 10000\}$. The simulated data were then fitted to a GARCH (1,1) model with innovations following Normal, Student-t and GED distributions, respectively. The results in Tables 4.4 to 4.31 shows the estimated parameters with their related standard errors (SE) in brackets. The volatility persistence measure considered in the analysis is the sum of the parameters, $\alpha_1 + \beta_1$. Violation of the volatility persistence assumption, that is $(\alpha_1 + \beta_1) < 1$, is highlited in red in these tables. The goodness of fit measure considered is the log-likelihood. The model selection criteria used are the AIC and BIC. All the latter measures are presented in the tables below. The tables also display the results obtained for the MSE and RMSE as well as our risk measures, the Value-at-Risk and Expected shortfall calculated at 95% confidence level only.

4.2.1 Simulation study: Scenario 1

Four different models were simulated from a GARCH (1,1) process with different parameter initial values of α_0 , α_1 , β_1 and sample sizes as illustrated in Table 4.1. Discussion on the descriptive statistics obtained for Scenario 1 follows.

Descriptive statistics for Scenario 1

Table 4.2 displays the descriptive statistics obtained for the four models of Scenario 1 for sample sizes 50, 500 and 5000. We performed the analysis on different ranges of sample sizes to determine if normality is present throughout the samples of Scenario 1.

For Model 1 we see that both the skewness and kurtosis values for the sample sizes are close to zero indicating that the returns are normally distributed. In Appendix Table A.1 and Table A.2 which presents the QQ-plots for Scenario 1, we also observe that the returns of Model 1 for the selected sample sizes are normally distributed since most of the data points fall approximately along the reference line with a few data points diverting from the straight line.

For Model 2, the values of skewness for the three sample sizes are close to zero however we see that this is not the case for the kurtosis values. For example at sample size 5000 we obtained a large kurtosis value of 36,27, this suggest that our returns could be leptokurtic for sample size 5000. In other words, the return series has heavier tails than that of a Normal distribution. However, when looking at Appendix Table A.1 and Table A.2 the QQ-plots for Scenario 1 of Model 2 for sample sizes 50, 500 and 5000, we see that the returns appear to be normally distributed with only a few data points diverting from the reference line.

Similar to Model 2, we observe that the skewness values of Model 3 are close to zero.

The kurtosis for sample sizes 500 and 5000 are 6,56 and 11,6 respectively, suggesting that the returns for these sample sizes could be leptokurtic. This is also seen in Appendix Table A.1 and Table A.2 of the QQ-plots for Scenario 1 of Model 3 for sample sizes 500 and 5000 where we observe heavy tails in both the QQ-plots.

For Model 4, the distribution of the returns for the three sample sizes are not normally distributed. When looking at sample size 5000 we obtained a value of $-1,06$ for the skewness indicating that the returns are skewed to the left. This could be due to a few low valued outliers. The kurtosis is 12,47 indicating that the returns are leptokurtic. The QQ-plots in Appendix Table A.1 and Table A.2 for Scenario 1 of Model 4 for all three samples also do not show normality for the returns.

The Shapiro-Wilk and Jarque-Bera (JB) normality test are used to formally test if the returns of the three sample sizes are normally distributed.

Table 4.2: Descriptive statistics of Scenario 1 return series

	T	mean	standard deviation	median	min	max	skewness	kurtosis
Model 1	50	-0,030000	0,890000	-0,050000	-2,120000	2,100000	0,010000	-0,370000
	500	0,040000	0,920000	0,050000	-2,550000	3,090000	-0,090000	-0,110000
	5000	0,020000	0,970000	0,010000	-3,850000	3,600000	0,000000	0,230000
Model 2	50	0,100000	1,030000	0,360000	-2,360000	2,080000	-0,200000	-0,680000
	500	-0,090000	1,250000	0,010000	-6,450000	3,680000	-0,280000	1,550000
	5000	-0,020000	1,720000	0,000000	-21,330000	23,090000	0,580000	36,270000
Model 3	50	-0,120000	1,200000	-0,120000	-2,980000	3,430000	0,280000	1,080000
	500	-0,120000	1,670000	-0,110000	-9,240000	10,720000	-0,170000	6,560000
	5000	-0,010000	1,670000	0,000000	-20,180000	14,620000	-0,300000	11,600000
Model 4	50	0,110000	1,330000	-0,020000	-4,910000	3,910000	-0,550000	3,630000
	500	-0,040000	1,430000	-0,040000	-4,910000	7,170000	0,020000	1,560000
	5000	-0,040000	1,440000	0,000000	-17,700000	9,760000	-1,060000	12,470000

Normality test for Scenario 1

To support the conclusions drawn from the descriptive statistics we formally test normality of the four models for sample sizes 50, 500 and 5000. The results obtained for the Jarque-Bera test and Shapiro-Wilk test are displayed in Table 4.3. Both tests rejected the normality of the return series at a 5% significance level for Model 1 at sample size 5000, Model 2 at sample sizes 500 and 5000, Model 3 at sample sizes 500 and 5000 and Model 4 for all three sample sizes.

Table 4.3: Normality tests of Scenario 1 return series

		Jarque-Bera (JB)		Shapiro-Wilk test	
	T	JB	p-value	W	p-value
Model 1	50	0,142540	0,931200	0,995440	0,999500
	500	0,894750	0,639300	0,997030	0,500500
	5000	11,315000	0,003491	0,999200	0,021290
Model 2	50	1,055500	0,589900	0,978100	0,475000
	500	57,749000	0,000000	0,984640	0,000039
	5000	274519,000000	0,000000	0,788540	0,000000
Model 3	50	3,960500	0,138000	0,965550	0,151800
	500	910,270000	0,000000	0,920280	0,000000
	5000	28134,000000	0,000000	0,914790	0,000000
Model 4	50	34,493000	0,000000	0,903170	0,000613
	500	51,674000	0,000000	0,984590	0,000038
	5000	33373,000000	0,000000	0,921980	0,000000

GARCH model selection process for Scenario 1

According to Table 4.4 to 4.11, there is consistency in the SE which is a measure of estimating efficiency of the parameters for all three distributions at the same sample size. However, we note that the GED produced smaller SE values than the Student-t distribution. This is also evident as we observe that when the sample size increase, the GED estimates are similar to the estimates of the Normal distribution. In terms of the model selection criteria, the GED outperforms the Student-t distribution because it produces smaller AIC and BIC values that are close to those of the Normal distribution. When we look at our accuracy measures and risk measures we note that the values obtained for all three distributions at the same sample size are consistently close to each other.

To illustrate model misspecification, we consider examples for all three scenarios. The following example is applicable to all the models of the three scenarios:

An analyst was given the task to estimate the VaR and ES. He had to simulate 100 data points using a GARCH (1,1) process with the true innovation following a Normal distribution. The initial parameter values for α_0 , α_1 and β_1 were set to 0,3, 0,5 and 0,4 respectively. The 100 simulated data points were then fitted into a GARCH (1,1) model with innovations following Normal, Student-t and GED distributions. He did not evaluate the model selection criteria and progressed straight to identifying the best model by use of the accuracy measures. He obtained the results displayed in Table 4.9 for $T = 50$. He identified the GARCH (1,1) model with GED innovations as the best suited model as it produced the lowest MSE (2,694992) and RMSE (1,641643) values. The VaR and ES values he obtained were 1,703613 and 1,695468 respectively. However, one cannot identify the presence of model misspecification by merely looking at the accuracy measures and risk measures. We can expect model specification errors to be present in the model of the analyst because he bypassed one of the important steps during the model building process which is analysis of the goodness-of-fit and model selection. This example is applicable to all three scenarios and their respective

models.

Scenario 1 example: The analyst was given a task to identify the model that best describes 50 simulated data points. He had to simulate 50 data points using a GARCH (1,1) process with the true innovation following a Normal distribution. The initial parameter values for α_0 , α_1 and β_1 were set to 0,3; 0,5 and 0,4 respectively. The 50 simulated data points were then fitted into a GARCH (1,1) model with innovations following Normal, Student-t and GED distributions. The output he obtained is that of Table 4.8 for $T = 50$. After analysing the output, the Student-t distribution produced the lowest AIC (3,174037) and BIC (3,326999) values. The optimal model the analyst identified was the GARCH (1,1) model with Student-t innovations by application of the model selection criteria. This is model misspecification because the inappropriate innovation distribution is assumed. When we observe Table 4.8, we note that for all samples sizes excluding $n = 50$ the Student-t performs the worst amongst the three distributions. Whereas the Normal distribution was consistent in producing the lowest AIC and BIC values which imply that the GARCH model with Normally distributed error terms is the best suited model.

Table 4.4: Scenario 1 Model 1: $\sigma_t^2 = 0, 1 + 0, 1a^2_{t-1} + 0, 8\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 9$	Log likelihood	AIC	BIC
Normal	10	0,092797 (0,312300)	0,000000 (NaN)	0,792543 (0,509600)	0,792543	-10,387690	-	-
	50	0,247205 (NaN)	0,000000 (NaN)	0,683000 (NaN)	0,683000	-64,543000	2,741720	2,894682
	100	0,515391 (0,243710) +	0,313620 (0,171160) -	0,218490 (0,250100)	0,532110	-142,382500	2,927651	3,031857
	200	0,551310 (0,276030) +	0,241840 (0,115070) +	0,219900 (0,296190)	0,461740	-281,256800	2,852568	2,918535
	500	0,280581 (0,128830) +	0,192765 (0,070340) +	0,478952 (0,183970) +	0,671717	-658,115800	2,648463	2,682180
	1000	0,266805 (0,125850) +	0,175380 (0,053660) +	0,490860 (0,192250) +	0,666240	-1287,413000	2,582825	2,602456
	5000	0,076052 (0,015230) +	0,079810 (0,010950) +	0,838810 (0,023180) +	0,918620	-6835,653000	2,735861	2,741075
	10000	0,084648 (0,011625) +	0,083991 (0,007755) +	0,830173 (0,016455) +	0,914164	-13925,400000	2,785880	2,788764
	10	0,498260 (NaN)	0,000000 (0,298900)	0,000000 (NaN)	0,000000	-10,492690	-	-
	50	0,672763 (0,508600)	0,219563 (0,363600)	0,000000 (8,069000)	0,219563	-64,720050	2,788802	2,980004
Student-t	100	0,553297 (0,274740) +	0,353272 (0,202200) -	0,202750 (0,256300)	0,556022	-143,339700	2,966794	3,097052
	200	0,597735 (0,333200) -	0,265164 (0,135430) -	0,209946 (0,332220)	0,475110	-283,492100	2,884921	2,967379
	500	0,289317 (0,155560) -	0,197990 (0,080630) +	0,500066 (0,206780) +	0,698056	-663,669500	2,674678	2,716824
	1000	0,257727 (0,141860) -	0,176616 (0,061980) +	0,534789 (0,204610) +	0,711405	-1298,200000	2,606400	2,630939
	5000	0,081890 (0,018530) +	0,084449 (0,013050) +	0,836469 (0,026660) +	0,920918	-6862,990000	2,747196	2,753713
	10000	0,096489 (0,014543) +	0,088165 (0,009244) +	0,822432 (0,019272) +	0,910597	-13972,050000	2,795410	2,799015
	10	0,310868 (0,802300)	0,000000 (NaN)	0,345600 (1,489000)	0,345600	-10,384230	-	-
	50	0,259707 (NaN)	0,000000 (NaN)	0,667073 (NaN)	0,667073	-64,520510	2,780820	2,972023
	100	0,514520 (0,238940) +	0,309120 (0,166260) -	0,222655 (0,247130)	0,531775	-142,341600	2,946832	3,077091
	200	0,547422 (0,256370) +	0,239650 (0,108330) +	0,226142 (0,275200)	0,465792	-280,949600	2,859496	2,941954
GED	500	0,284677 (0,116350) +	0,197720 (0,066270) +	0,469630 (0,166230) +	0,667350	-656,621700	2,646487	2,688633
	1000	0,277384 (0,116370) +	0,180498 (0,050030) +	0,472655 (0,176830) +	0,653153	-1284,796000	2,579592	2,604131
	5000	0,076030 (0,015200) +	0,079815 (0,010920) +	0,838836 (0,023130) +	0,918651	-6835,645000	2,736258	2,742775
	10000	0,084918 (0,011736) +	0,083987 (0,007809) +	0,829889 (0,016596) +	0,913876	-13925,210000	2,786042	2,789647
	10000	0,084918 (0,011736) +	0,083987 (0,007809) +	0,829889 (0,016596) +	0,913876	-13925,210000	2,786042	2,789647

Notes: This table presents the estimates of Scenario 1 Model 1 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " . "

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.5: Scenario 1 Model 1 return series Accuracy measures and Risk measures results

<i>Distribution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,467719	0,683996	1,296421	1,625764
Student-t		0,468267	0,684300	1,859441	2,331814
GED		0,469505	0,685131	1,645228	2,063182
Normal	50	0,774041	0,879796	1,592831	1,997474
Student-t		0,775059	0,880375	1,859441	2,331814
GED		0,774045	0,879798	1,599678	2,006061
Normal	100	1,088684	1,043400	1,763661	2,211702
Student-t		1,088690	1,043403	1,776496	2,227797
GED		1,088686	1,043401	1,762170	2,209833
Normal	200	1,020705	1,010299	1,771083	2,221010
Student-t		1,020927	1,010409	1,783580	2,236681
GED		1,020647	1,010271	1,768651	2,217960
Normal	500	0,845546	0,919536	1,607942	2,016424
Student-t		0,845721	0,919631	1,615798	2,026275
GED		0,845441	0,919478	1,612297	2,021886
Normal	1000	0,795551	0,891937	1,589727	1,993582
Student-t		0,795544	0,891933	1,581344	1,983069
GED		0,795551	0,891937	1,600716	2,007363
Normal	5000	0,932484	0,965652	1,424945	1,786939
Student-t		0,932478	0,965649	1,453551	1,822812
GED		0,932485	0,965652	1,424914	1,786899
Normal	10000	0,982052	0,990985	1,445607	1,812849
Student-t		0,982044	0,990981	1,480290	1,856344
GED		0,982051	0,990985	1,445961	1,813294

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

Table 4.6: Scenario 1 Model 2: $\sigma_t^2 = 0, 2 + 0, 3a^2_{t-1} + 0, 6\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 9$	Log likelihood	AIC	BIC
Normal	10	0,010567 (5,584000)	0,000000 (2,076000)	0,999999 (4,037000)	0,999999	-15,072110	-	-
	50	0,030394 (0,113100)	0,000000 (NaN)	0,973356 (NaN)	0,973356	-72,035600	3,041424	3,194386
	100	0,337239 (0,302450)	0,117460 (0,122990)	0,608080 (0,276460) +	0,725540	-151,425900	3,108517	3,212724
	200	0,188907 (0,104230) -	0,203089 (0,091280) +	0,661254 (0,126880) +	0,864343	-303,794900	3,077949	3,143916
	500	0,235710 (0,082880) +	0,328460 (0,074580) +	0,530400 (0,091810) +	0,858860	-775,677500	3,118710	3,152427
	1000	0,250588 (0,065500) +	0,273907 (0,047630) +	0,570504 (0,068860) +	0,844411	-1566,367000	3,140734	3,160365
	5000	0,186149 (0,019675) +	0,287085 (0,019708) +	0,615691 (0,022762) +	0,902776	-8045,231000	3,219692	3,224960
	10000	0,195478 (0,014640) +	0,292343 (0,014310) +	0,604074 (0,016860) +	0,896417	-16018,170000	3,204435	3,207319
	10	0,041899 (2,521000)	0,000000 (NaN)	1,000000 (2,483000) +	1,000000	-15,360760	-	-
	50	0,048275 (0,136900)	0,000000 (NaN)	0,962590 (NaN)	0,962590	-72,806500	3,112260	3,303462
Student-t	100	0,380179 (0,364950)	0,111464 (0,135180)	0,611736 (0,296710) +	0,723200	-152,889400	3,157788	3,288046
	200	0,176107 (0,123220)	0,193886 (0,102850) -	0,699870 (0,142820) +	0,893756	-305,807400	3,108074	3,190532
	500	0,241219 (0,095270) +	0,339037 (0,088100) +	0,546555 (0,100000) +	0,885592	-780,622900	3,142492	3,184638
	1000	0,273102 (0,081940) +	0,287574 (0,058180) +	0,566611 (0,081620) +	0,854185	-1573,665000	3,157330	3,181869
	5000	0,197582 (0,023408) +	0,303939 (0,023573) +	0,613669 (0,025556) +	0,917608	-8074,697000	3,231897	3,238396
	10000	0,205062 (0,017300) +	0,309223 (0,017050) +	0,604201 (0,018810) +	0,913424	-16078,450000	3,216689	3,220295
	10	1,103100 (0,771600)	0,000000 (NaN)	0,000000 (0,219800)	0,000000	-13,707760	-	-
	50	0,047949 (0,090690)	0,000000 (NaN)	0,957198 (NaN)	0,957198 -	71,303360	3,052134	3,243337
	100	0,325748 (0,261280)	0,131951 (0,117220)	0,605605 (0,240930) +	0,737556	-150,586200	3,111725	3,241983
	200	0,196608 (0,097550) +	0,212583 (0,089150) +	0,647040 (0,119580) +	0,859623	-303,439900	3,084399	3,166857
GED	500	0,238595 (0,079250) +	0,332419 (0,070950) +	0,525230 (0,087600) +	0,857649	-774,979800	3,119919	3,162065
	1000	0,248880 (0,063560) +	0,273870 (0,046440) +	0,571840 (0,066840) +	0,845710	-1566,181000	3,142361	3,166900
	5000	0,186098 (0,019569) +	0,287009 (0,019601) +	0,615786 (0,022642) +	0,902795	-8045,173000	3,220069	3,226586
	10000	0,195530 (0,014520) +	0,292300 (0,014200) +	0,604070 (0,016730) +	0,896370	-16017,900000	3,204581	3,208186
	10000	0,195530 (0,014520) +	0,292300 (0,014200) +	0,604070 (0,016730) +	0,896370	-16017,900000	3,204581	3,208186

Notes: This table presents the estimates of Scenario 1 Model 2 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " .",

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.7: Scenario 1 Model 2 return series Accuracy measures and Risk measures results

<i>Distribution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	1,194859	1,093096	1,875804	2,352334
Student-t		1,195371	1,093330	2,089909	2,620831
GED		1,198735	1,094868	1,567499	1,965707
Normal	50	1,045574	1,022533	1,733396	2,173749
Student-t		1,045688	1,022589	1,832514	2,298046
GED		1,048407	1,023918	1,727860	2,166806
Normal	100	1,227576	1,107960	1,743083	2,185896
Student-t		1,229423	1,108793	1,789933	2,244649
GED		1,226470	1,107461	1,753570	2,199048
Normal	200	1,227576	1,107960	1,803644	2,261843
Student-t		1,316663	1,147459	1,879127	2,356501
GED		1,314334	1,146444	1,822013	2,284878
Normal	500	1,563971	1,250588	1,795657	2,251827
Student-t		1,564241	1,250696	1,842502	2,310572
GED		1,563676	1,250470	1,794588	2,250486
Normal	1000	1,555632	1,247250	1,811135	2,271237
Student-t		1,555614	1,247243	1,843429	2,311735
GED		1,555613	1,247242	1,812106	2,272454
Normal	5000	2,973603	1,724414	1,846973	2,316178
Student-t		2,973657	1,724430	1,881078	2,358947
GED		2,973600	1,724413	1,847039	2,316262
Normal	10000	2,449448	1,565071	1,841678	2,309539
Student-t		2,449452	1,565073	1,877114	2,353977
GED		2,449448	1,565071	1,841699	2,309565

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

Table 4.8: Scenario 1 Model 3: $\sigma_t^2 = 0, 3 + 0, 5a^2_{t-1} + 0, 4\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 9$	Log likelihood	AIC	BIC
Normal	10	0,000000 (0,570200)	0,000000 (0,262300)	0,957920 (0,088500)	0,957920	-10,105930	-	-
	50	0,355595 (0,212110) -	0,908470 (0,567220)	0,099880 (0,233830)	1,008350	-75,350940	3,174037	3,326999
	100	0,176660 (0,112450)	0,591960 (0,198480) +	0,433380 (0,114230) +	1,025340	-174,131300	3,562626	3,666833
	200	0,132731 (0,056190) +	0,726780 (0,168860) +	0,379410 (0,082660) +	1,106190	-325,710400	3,297104	3,363071
	500	0,199371 (0,048990) +	0,541618 (0,084920) +	0,430570 (0,061050) +	0,972188	-814,478900	3,273916	3,307632
	1000	0,234100 (0,039710) +	0,571080 (0,059930) +	0,390790 (0,041180) +	0,961870	-1622,476000	3,252952	3,272583
	5000	0,282421 (0,021652) +	0,512348 (0,026020) +	0,404431 (0,020987) +	0,916779	-8301,752000	3,322301	3,327515
	10000	0,291385 (0,016129) +	0,503095 (0,018866) +	0,399800 (0,015961) +	0,902895	-16403,060000	3,281411	3,284296
	10	0,356143 (0,746200)	0,000000 (NaN)	0,274409 (1,342000)	0,274409	-10,320510	-	-
	50	0,371950 (0,268300)	0,636810 (0,534800)	0,230110 (0,315900)	0,866920	-75,505420	3,174037	3,326999
Student-t	100	0,211810 (0,141300)	0,598000 (0,225400) +	0,439980 (0,124700) +	1,037980	-175,200000	3,604000	3,734259
	200	0,154351 (0,071310) +	0,691809 (0,190020) +	0,400210 (0,093180) +	1,092019	-326,825000	3,318250	3,400708
	500	0,211163 (0,057500) +	0,566390 (0,100150) +	0,429000 (0,066460) +	0,995390	-816,651100	3,286604	3,328750
	1000	0,251439 (0,047340) +	0,595540 (0,071070) +	0,390030 (0,045710) +	0,985570	-1628,511000	3,267022	3,291561
	5000	0,293727 (0,025399) +	0,546251 (0,031364) +	0,406134 (0,023220) +	0,952385	-8338,962000	3,337585	3,344102
	10000	0,303475 (0,018956) +	0,530203 (0,022519) +	0,403371 (0,017714) +	0,933574	-16468,990000	3,294799	3,298404
	10	0,000000 (0,216600)	0,000000 (0,276800)	0,938830 (NaN)	0,938830	-9,027367	-	-
	50	0,351430 (0,199280) -	0,967760 (0,661110)	0,081150 (0,211370)	1,048910	-75,340620	3,213625	3,404827
	100	0,165450 (0,100870)	0,616560 (0,194820) +	0,426530 (0,106840) +	1,043090	-173,848800	3,576977	3,707235
	200	0,133698 (0,057600) +	0,721260 (0,173890) +	0,381240 (0,084590) +	1,102500	-325,699500	3,306995	3,389453
GED	500	0,199755 (0,049660) +	0,541780 (0,086040) +	0,430187 (0,061760) +	0,971967	-814,441100	3,277764	3,319910
	1000	0,234410 (0,040060) +	0,570870 (0,060380) +	0,390680 (0,041490) +	0,961550	-1622,454000	3,254907	3,279446
	5000	0,283039 (0,021223) +	0,511903 (0,025430) +	0,404275 (0,020548) +	0,916178	-8300,768000	3,322307	3,328824
	10000	0,291607 (0,015976) +	0,503143 (0,018674) +	0,399615 (0,015806) +	0,902758	-16402,630000	3,281526	3,285131
	10	0,000000 (0,216600)	0,000000 (0,276800)	0,938830 (NaN)	0,938830	-9,027367	-	-
	50	0,351430 (0,199280) -	0,967760 (0,661110)	0,081150 (0,211370)	1,048910	-75,340620	3,213625	3,404827
	100	0,165450 (0,100870)	0,616560 (0,194820) +	0,426530 (0,106840) +	1,043090	-173,848800	3,576977	3,707235
	200	0,133698 (0,057600) +	0,721260 (0,173890) +	0,381240 (0,084590) +	1,102500	-325,699500	3,306995	3,389453
	500	0,199755 (0,049660) +	0,541780 (0,086040) +	0,430187 (0,061760) +	0,971967	-814,441100	3,277764	3,319910
	1000	0,234410 (0,040060) +	0,570870 (0,060380) +	0,390680 (0,041490) +	0,961550	-1622,454000	3,254907	3,279446

Notes: This table presents the estimates of Scenario 1 Model 3 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " . "

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.9: Scenario 1 Model 3 return series Accuracy measures and Risk measures results

<i>Distrbution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,454196	0,673941	0,926173	1,161458
Student-t		0,456382	0,675561	1,396431	1,751181
GED		0,556693	0,746119	0,941220	1,180328
Normal	50	1,429124	1,195460	1,449464	1,817686
Student-t		1,422457	1,192668	1,431796	1,795530
GED		1,429694	1,195698	1,453929	1,823285
Normal	100	2,695029	1,641654	1,358499	1,703613
Student-t		2,695120	1,641682	1,388372	1,741075
GED		2,694992	1,641643	1,352005	1,695468
Normal	200	3,524664	1,877409	1,342294	1,683290
Student-t		3,523373	1,877065	1,353281	1,697069
GED		3,524540	1,877376	1,342279	1,683272
Normal	500	2,797728	1,672641	1,373990	1,723039
Student-t		2,796331	1,672223	1,383886	1,735449
GED		2,797573	1,672595	1,373971	1,723014
Normal	1000	2,647067	1,626981	1,396007	1,750649
Student-t		2,646583	1,626832	1,408025	1,765720
GED		2,647028	1,626969	1,396035	1,750684
Normal	5000	2,803683	1,674420	1,425397	1,787505
Student-t		2,803682	1,674420	1,438386	1,803794
GED		2,803683	1,674420	1,425670	1,787847
Normal	10000	2,759487	1,661170	1,427368	1,789977
Student-t		2,759492	1,661172	1,440637	1,806617
GED		2,759486	1,661170	1,427420	1,790042

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

Table 4.10: Scenario 1 Model 4: $\sigma_t^2 = 0, 4 + 0, 7a^2_{t-1} + 0, 2\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$ (NaN)	$\hat{\alpha}_1$ (NaN)	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 9$	Log likelihood	AIC	BIC
Normal	10	0,162582 (NaN)	0,000000 (NaN)	0,603552 (NaN)	0,603852	-9,563746	-	-
	50	0,224580 (0,109670) +	0,941600 (0,357910) +	0,067410 (0,096360)	1,009010	-67,083050	2,843322	2,996284
	100	0,322830 (0,113400) +	0,642820 (0,204100) +	0,112460 (0,124410)	0,755280	-133,187800	2,743757	2,847964
	200	0,357727 (0,107101) +	0,826388 (0,192555) +	0,099095 (0,097869)	0,925483	-304,966600	3,089666	3,155632
	500	0,503060 (0,107130) +	0,636050 (0,104330) +	0,185120 (0,078480) +	0,821170	-821,988300	3,303953	3,337670
	1000	0,474416 (0,066630) +	0,598250 (0,071300) +	0,220777 (0,049540) +	0,819027	-1624,039000	3,256078	3,275709
	5000	0,393393 (0,024594) +	0,619081 (0,031132) +	0,227612 (0,022139) +	0,846693	-7821,344000	3,130138	3,135352
	10000	0,364866 (0,015704) +	0,654666 (0,022210) +	0,223918 (0,014589) +	0,878584	-15639,200000	3,128640	3,131524
	10	0,204300 (0,827200)	0,000000 (0,911100)	0,575680 (2,003000)	0,575680	-9,855103	-	-
	50	0,254580 (0,133570) -	0,977233 (0,416630) +	0,070335 (0,103750)	1,047568	-67,772280	2,910891	3,102093
Student-t	100	0,363892 (0,140530) +	0,657000 (0,240010) +	0,114060 (0,138060)	0,771060	-134,499700	2,789993	2,920252
	200	0,406843 (0,130130) +	0,882412 (0,224220) +	0,090430 (0,104100)	0,972842	-307,617900	3,126179	3,208637
	500	0,552161 (0,133010) +	0,707430 (0,127870) +	0,159728 (0,088730) -	0,867158	-826,107600	3,324430	3,366576
	1000	0,509670 (0,081170) +	0,625220 (0,084950) +	0,220270 (0,056550) +	0,845490	-1630,975000	3,271949	3,296488
	5000	0,415632 (0,029523) +	0,647854 (0,037284) +	0,229167 (0,024960) +	0,877021	-7850,881000	3,142353	3,148870
	10000	0,383514 (0,018875) +	0,684813 (0,026614) +	0,227318 (0,016533) +	0,912131	-15698,970000	3,140793	3,144398
	10	0,000000 (0,165500)	0,000000 (0,443600)	0,968740 (NaN)	0,968740	-7,811540	-	-
	50	0,212114 (0,095760) +	1,000000 (0,351080) +	0,064277 (0,086460)	1,064277	-66,803040	2,872122	3,063324
	100	0,309720 (0,098610) +	0,680850 (0,196830) +	0,111470 (0,108280)	0,792320	-132,724600	2,754493	2,884751
	200	0,341086 (0,094710) +	0,838310 (0,176840) +	0,106080 (0,089490)	0,944390	-303,646500	3,086465	3,168923
GED	500	0,497741 (0,101070) +	0,627835 (0,099110) +	0,192720 (0,075030) +	0,820555	-821,434500	3,305738	3,347884
	1000	0,472324 (0,064690) +	0,599468 (0,069550) +	0,221470 (0,048150) +	0,820938	-1623,740000	3,257480	3,282018
	5000	0,393319 (0,024448) +	0,619252 (0,030959) +	0,227576 (0,022009) +	0,846828	-7821,277000	3,130511	3,137028
	10000	0,364894 (0,015598) +	0,654868 (0,022072) +	0,223793 (0,014492) +	0,875661	-15639,030000	3,128806	3,132411

Notes: This table presents the estimates of Scenario 1 Model 4 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " "

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.11: Scenario 1 Model 4 return series Accuracy measures and Risk measures results

<i>Distrbution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,396661	0,629810	0,824288	1,033691
Student-t		0,397133	0,630185	0,871364	1,092725
GED		0,407454	0,638321	0,882073	1,106155
Normal	50	1,728544	1,314741	0,350034	0,439572
Student-t		1,728772	1,314828	0,371178	0,465472
GED		1,728576	1,314753	0,341212	0,427893
Normal	100	1,272154	1,127898	0,437266	0,548349
Student-t		1,273814	1,128633	0,460959	0,578062
GED		1,270659	1,127235	0,430164	0,539443
Normal	200	1,794276	1,339506	0,420716	0,527594
Student-t		1,797349	1,340652	0,426409	0,534735
GED		1,791525	1,338479	0,424923	0,532870
Normal	500	2,038833	1,427877	0,642683	0,805951
Student-t		2,038986	1,427931	0,618684	0,775854
GED		2,038763	1,427852	0,654403	0,820648
Normal	1000	1,951624	1,397005	0,692769	0,868762
Student-t		1,951443	1,396941	0,713749	0,895071
GED		1,951649	1,397014	0,693147	0,869235
Normal	5000	2,064067	1,436686	0,656318	0,823049
Student-t		2,064251	1,436750	0,676656	0,848554
GED		2,064058	1,436683	0,656197	0,822898
Normal	10000	2,438642	1,561615	0,640499	0,803212
Student-t		2,438676	1,561626	0,661972	0,830139
GED		2,438640	1,561614	0,640304	0,802967

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

4.2.2 Simulation study: Scenario 2

Four different models were simulated from a GARCH (1,1) process with different varying initial values of α_0 , α_1 , β_1 and sample sizes as illustrated in Table 4.1. The descriptive analysis results discussion follows next.

Descriptive statistics for Scenario 2

The descriptive statistics obtained for the four models of Scenario 2 for sample sizes 50, 500 and 5000 are presented in Table 4.12.

For Model 1, the values of skewness for the three sample sizes are close to zero. The kurtosis values are only zero for sample size 50 with sample size 500 and 5000 having values of 9,66 and 15,48 respectively. The kurtosis values for sample sizes 500 and 5000 suggests that the return series for these sample sizes are leptokurtic. We also observe heavy tails for sample sizes 500 and 5000 in the QQ-plots presented in Appendix Table A.3 and Table A.4 for Model 1 supporting the findings of the kurtosis values. We can conclude that only the returns series of sample size 50 appears to be normally distributed.

We make the same conclusion for Model 2 as made by Model 1 with the exception of sample size 500. The value obtained for kurtosis for sample size 500 is 1,33. This suggest that the return series of sample size 500 is platykurtic. This implies that the distribution has thinner tails than the Normal distribution.

For Model 3, the distrubtion of the returns for the three sample sizes are not normally distributed. When looking at sample size 50 we obtained a value of $-1,77$ for the skewness indicating that the returns are skewed to the left. The kurtosis is 7,25 indicating that the returns are leptokurtic. The QQ-plots presented in Appendix Table A.3 and Table A.4 for Scenario 2 of Model 3 for all three samples also does not

show normality for the returns with heavy tails being apparent.

For Model 4 we see that both the skewness and kurtosis values for the sample sizes 50 and 500 are close to zero indicating that the return series for these sample sizes are normally distributed. However, for sample size 5000 we observe a value of 1,09 for the kurtosis suggesting that the returns for sample size 5000 is platykurtic. We can also observe non normality from the QQ-plot for sample size 5000 presented in Appendix Table A.3 and Table A.4.

Table 4.12: Descriptive statistics of Scenario 2 return series

	T	mean	standard deviation	median	min	max	skewness	kurtosis
Model 1	50	-0,030000	1,140000	-0,080000	-2,160000	2,740000	0,290000	-0,250000
	500	0,070000	1,720000	0,110000	-8,320000	12,080000	0,350000	9,660000
	5000	0,000000	1,630000	0,010000	-18,980000	15,000000	-0,370000	15,480000
Model 2	50	-0,030000	0,790000	0,130000	-2,430000	1,680000	-0,680000	0,960000
	500	-0,080000	1,140000	0,010000	-4,980000	4,160000	-0,210000	1,330000
	5000	0,030000	1,370000	0,020000	-11,210000	10,150000	-0,060000	4,660000
Model 3	50	-0,130000	1,260000	0,060000	-5,930000	2,840000	-1,770000	7,250000
	500	0,050000	1,280000	0,040000	-5,930000	7,810000	0,580000	4,310000
	5000	0,010000	1,260000	0,020000	-6,910000	7,810000	-0,060000	2,250000
Model 4	50	0,240000	1,080000	0,110000	-2,080000	2,270000	-0,010000	-0,470000
	500	0,040000	0,920000	-0,010000	-2,990000	3,810000	0,300000	0,870000
	5000	-0,010000	0,990000	-0,010000	-4,330000	4,630000	0,030000	1,090000

Normality test for Scenario 2

We formally test for normality of the returns for sample sizes 50, 500 and 5000 by use of the Jarque-Bera and Shapiro-Wilk normality tests. The results obtained are presented in Table 4.13. Both tests at 5% significance level confirmed normality of the return series only at sample size 50 for Model 1 and Model 4.

Table 4.13: Normality tests of Scenario 2 return series

		Jarque-Bera (JB)		Shapiro-Wilk test	
	T	JB	p-value	W	p-value
Model 1	50	0,767860	0,681200	0,973760	0,326900
	500	1974,900000	0,000000	0,876480	0,000000
	5000	50076,000000	0,000000	0,858470	0,000000
Model 2	50	6,705800	0,034980	0,937850	0,011080
	500	41,811000	0,000000	0,977990	0,000000
	5000	4541,300000	0,000000	0,953470	0,000000
Model 3	50	150,370000	0,000000	0,842430	0,000010
	500	420,100000	0,000000	0,952330	0,000000
	5000	1059,100000	0,000000	0,979530	0,000000
Model 4	50	0,278290	0,870100	0,975000	0,364900
	500	23,783000	0,000007	0,995054	0,002702
	5000	248,630000	0,000000	0,992490	0,000000

GARCH model selection process for Scenario 2

After analysing the results of Scenario 2 we note that similar conclusions are observed as in Scenario 1.

Scenario 2 example: An analyst was given the task to identify the model that best describes the data. He had to simulate 200 data points using a GARCH (1,1) process with true innovation following a Normal distribution. The initial parameters values for α_0 , α_1 and β_1 were set to 0,4, 0,8 and 0,1 respectively. The 200 simulated data points were then fitted to a GARCH (1,1) model with innovations following Normal, Student-t and GED distributions. The output he obtained is that of Table 4.14 for $T = 200$. After analysing the data, the GED produced the lowest AIC (3, 159033) and

BIC (3, 241491) values. The optimal model the analyst identified was the GARCH (1,1) model with GED innovations by application of the model selection criteria. This is Model misspecification because the inappropriate innovation distribution which is the GED was assumed. The Normal distribution on the other hand was consistent in producing the lowest AIC and BIC values for most of the sample sizes. This imply that the GARCH (1,1) model with Normally distributed error terms is the best suited model.

Table 4.14: Scenario 2 Model 1: $\sigma_t^2 = 0, 4 + 0, 8a^2_{t-1} + 0, 1\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$ (NaN)	$\hat{\alpha}_1$ (NaN)	$\hat{\beta}_1$ (NaN)	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 9$	Log likelihood	AIC	BIC
Normal	10	0,025554 (NaN)	0,000000 (NaN)	0,999999 (NaN)	0,999999	-13,866420	-	-
	50	0,069456 (0,081630)	0,515407 (0,264500) -	0,540755 (0,180190) +	1,056162	-72,701390	3,068056	3,221018
	100	0,115749 (0,103680)	0,510339 (0,176850) +	0,485907 (0,149740) +	0,996246	-160,540000	3,290801	3,395007
	200	0,247275 (0,096650) +	0,695554 (0,161720) +	0,263967 (0,103220) +	0,959521	-315,953100	3,199531	3,265497
	500	0,289580 (0,049870) +	0,862270 (0,105790) +	0,133940 (0,041510) +	0,996210	-773,664900	3,110660	3,144377
	1000	0,307837 (0,039870) +	0,767896 (0,072410) +	0,159965 (0,038110) +	0,927861	-1492,411000	2,992822	3,012453
	5000	0,385671 (0,020360) +	0,783784 (0,033350) +	0,117030 (0,016890) +	0,900814	-7679,978000	3,073591	3,078805
	10000	0,404524 (0,014636) +	0,781107 (0,023438) +	0,101465 (0,011416) +	0,882572	-15287,450000	3,058290	3,061175
	10	0,047098 (NaN)	0,000000 (NaN)	1,000000 (NaN)	1,000000	-14,004570	-	-
	50	0,074210 (0,091010)	0,583241 (0,308150) -	0,525148 (0,179780) +	1,108389	-73,181230	3,127249	3,318452
Student-t	100	0,109635 (0,108610)	0,581054 (0,208280) +	0,491489 (0,142930) +	1,072543	-162,345000	3,346900	3,477159
	200	0,275388 (0,118730) +	0,780160 (0,200870) +	0,260689 (0,114720) +	1,040849	-319,966300	3,249663	3,332121
	500	0,311044 (0,061200) +	0,955256 (0,133060) +	0,128593 (0,046460) +	1,083849	-780,530600	3,142123	3,184269
	1000	0,326373 (0,047310) +	0,836034 (0,089830) +	0,151640 (0,041550) +	0,987674	-1500,362000	3,010724	3,035262
	5000	0,407249 (0,024180) +	0,841135 (0,040720) +	0,112930 (0,018410) +	0,954065	-7715,366000	3,088147	3,094664
	10000	0,433149 (0,017677) +	0,834639 (0,028634) +	0,096122 (0,012615) +	0,930761	-15364,960000	3,073992	3,077597
	10	0,487740 (NaN)	0,403350 (0,332100)	0,000000 (NaN)	0,403350	-12,153700	-	-
	50	0,068406 (0,079560)	0,493040 (0,263990) -	0,555081 (0,187970) +	1,048121	-72,644570	3,105783	3,296985
	100	0,172440 (0,117230)	0,470830 (0,168260) +	0,468660 (0,169670) +	0,939490	-158,397600	3,267953	3,398211
	200	0,248605 (0,073970) +	0,682522 (0,121060) +	0,274408 (0,075110) +	0,956930	-310,903300	3,159033	3,241491
GED	500	0,291586 (0,043950) +	0,848169 (0,091630) +	0,138463 (0,036590) +	0,986632	-770,705700	3,102823	3,144969
	1000	0,307414 (0,038740) +	0,764332 (0,070000) +	0,161988 (0,037270) +	0,926320	-1492,041000	2,994082	3,018621
	5000	0,385657 (0,020000) +	0,782661 (0,032700) +	0,117540 (0,016640) +	0,900201	-7679,267000	3,073707	3,080224
	10000	0,403815 (0,014246) +	0,780369 (0,022782) +	0,102359 (0,011155) +	0,882728	-15284,640000	3,057928	3,061534

Notes: This table presents the estimates of Scenario 2 Model 1 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " . "

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.15: Scenario 2 Model 1 return series Accuracy measures and Risk measures results

<i>Distrbution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,950909	0,975146	1,806686	2,265657
Student-t		0,945739	0,977107	1,964012	2,462951
GED		0,997402	0,998700	1,467983	1,840910
Normal	50	1,281752	1,132145	1,999636	2,507625
Student-t		1,281720	1,132131	2,061134	2,584745
GED		1,281821	1,132175	1,998008	2,505583
Normal	100	2,178775	1,476068	1,863937	2,337453
Student-t		2,181385	1,476951	1,955029	2,451685
GED		2,169392	1,472886	1,793665	2,249329
Normal	200	2,031697	1,425376	1,708396	2,142397
Student-t		2,030331	1,424897	1,746069	2,189641
GED		2,034984	1,426528	1,723780	2,161689
Normal	500	2,967337	1,722596	1,618979	2,030265
Student-t		2,967392	1,722612	1,635328	2,050767
GED		2,967494	1,722641	1,622146	2,034236
Normal	1000	2,337071	1,528748	1,626946	2,040256
Student-t		2,337076	1,528750	1,637988	2,054103
GED		2,337071	1,528748	1,628196	2,041823
Normal	5000	2,669952	1,633999	1,596345	2,001881
Student-t		2,669952	1,633999	1,604976	2,012705
GED		2,669952	1,633999	1,596719	2,002350
Normal	10000	2,558577	1,599555	1,581043	1,982692
Student-t		2,558576	1,599555	1,586347	1,989344
GED		2,558579	1,599556	1,581887	1,983750

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

Table 4.16: Scenario 2 Model 2: $\sigma_t^2 = 0, 3 + 0, 6a^2_{t-1} + 0, 3\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 9$	Log likelihood	AIC	BIC
Normal	10	0,000000 (0,250800)	0,000000 (NaN)	0,951560 (0,499200) -	0,951560	-10,644900	-	-
	50	0,179555 (0,108700) -	0,999999 (0,979300)	0,000000 (0,190100)	0,999999	-52,650510	2,660200	2,418982
	100	0,201709 (0,089420) +	1,000000 (0,323270) +	0,068132 (0,089890)	1,068132	-132,694500	2,733890	2,838097
	200	0,214664 (0,065190) +	0,749565 (0,171970) +	0,233006 (0,069400) +	0,982571	-279,975900	2,839759	2,905726
	500	0,211842 (0,047410) +	0,552970 (0,095710) +	0,361016 (0,065930) +	0,913986	-711,551800	2,862207	2,895924
	1000	0,267285 (0,039463) +	0,524822 (0,065870) +	0,335900 (0,050198) +	0,860722	-1458,271000	2,924543	2,944174
	5000	0,342668 (0,022485) +	0,588066 (0,030212) +	0,272092 (0,022026) +	0,860158	-7706,531000	3,084212	3,089426
	10000	0,296643 (0,014557) +	0,606511 (0,021634) +	0,299939 (0,015438) +	0,906450	-15551,630000	3,111126	3,114010
	10	0,039068 (0,262600)	0,000000 (NaN)	0,871215 (0,529000) -	0,871215	-10,565730	-	-
	50	0,258798 (0,326700)	0,000000 (NaN)	0,808478 (NaN)	0,808478	-56,205300	2,448212	2,639414
Student-t	100	0,199730 (0,100350) +	1,000000 (0,392290) +	0,073740 (0,091630)	1,073740	-130,364700	2,707294	2,837553
	200	0,213837 (0,073140) +	0,853723 (0,219410) +	0,210183 (0,077400) +	1,063906	-280,436400	2,854364	2,936822
	500	0,216896 (0,053130) +	0,587310 (0,114920) +	0,353890 (0,073620) +	0,941200	-711,998600	2,867994	2,910140
	1000	0,275755 (0,045972) +	0,563523 (0,079001) +	0,327573 (0,055832) +	0,891096	-1460,723000	2,931446	2,955984
	5000	0,350438 (0,026140) +	0,621159 (0,035936) +	0,275349 (0,024419) +	0,896508	-7726,189000	3,092476	3,098993
	10000	0,308276 (0,017103) +	0,636768 (0,025697) +	0,303783 (0,017109) +	0,940551	-15606,820000	3,122365	3,125970
	10	0,002457 (0,269100)	0,000000 (NaN)	0,948072 (0,545900) -	0,948072	-10,632200	-	-
	50	0,202227 (NaN)	0,999999 (NaN)	0,000000 (NaN)	0,999999	-51,881600	2,275264	2,466466
	100	0,197020 (0,099780) +	1,000000 (0,388310) +	0,072696 (0,100650)	1,072696	-130,568200	2,711365	2,841623
	200	0,212547 (0,068780) +	0,775527 (0,191470) +	0,224438 (0,074120) +	0,999965	-279,620900	2,846209	2,928667
GED	500	0,211984 (0,051450) +	0,560156 (0,107340) +	0,358513 (0,072640) +	0,918669	-709,799500	2,859198	2,901344
	1000	0,268661 (0,042406) +	0,531702 (0,071433) +	0,331490 (0,053621) +	0,863192	-1456,315000	2,922631	2,947170
	5000	0,342118 (0,022854) +	0,588461 (0,030749) +	0,272324 (0,022394) +	0,860785	-7705,867000	3,084347	3,090864
	10000	0,296630 (0,014569) +	0,606503 (0,021651) +	0,299954 (0,015451) +	0,906457	-15551,630000	3,111326	3,114931
	10	0,002457 (0,269100)	0,000000 (NaN)	0,948072 (0,545900) -	0,948072	-10,632200	-	-

Notes: This table presents the estimates of Scenario 2 Model 2 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " . "

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.17: Scenario 2 Model 2 return series Accuracy measures and Risk measures results

<i>Distrbution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal Student-t GED	10	0,152745	0,716062	0,924654	1,159553
		0,152745	0,716062	0,988144	1,239172
		0,152745	0,716062	0,936515	1,174427
Normal Student-t GED	50	0,613579	0,783313	1,031589	1,293665
		0,630809	0,794235	1,777500	2,229056
		0,621171	0,788144	1,031589	1,293655
Normal Student-t GED	100	1,177081	1,084934	1,099391	1,378681
		1,179873	1,086219	1,107230	1,388512
		1,179583	1,086086	1,105652	1,386532
Normal Student-t GED	200	1,346203	1,160260	1,209893	1,517254
		1,347184	1,160683	1,216974	1,526134
		1,347051	1,160625	1,210575	1,518109
Normal Student-t GED	500	1,293273	1,137222	1,223466	1,534275
		1,294009	1,137545	1,238384	1,552984
		1,294363	1,137701	1,231336	1,544146
Normal Student-t GED	1000	1,436997	1,198748	1,245119	1,561430
		1,437238	1,198849	1,258873	1,578677
		1,437283	1,198867	1,247653	1,564607
Normal Student-t GED	5000	1,888606	1,374266	1,247442	1,564343
		1,888623	1,374272	1,263517	1,584502
		1,888606	1,374266	1,247558	1,564488
Normal Student-t GED	10000	2,237610	1,495864	1,260648	1,580903
		2,237597	1,495860	1,280269	1,605509
		2,237610	1,495864	1,260657	1,580914

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

Table 4.18: Scenario 2 Model 3: $\sigma_t^2 = 0, 2 + 0, 4a^2_{t-1} + 0, 5\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 9$	Log likelihood	AIC	BIC
Normal	10	0,481951 (NaN)	0,929926 (0,403900) -	0,000000 (NaN)	0,929926	-19,887180	-	-
	50	0,170590 (0,102930) -	0,654030 (0,249030) +	0,209880 (0,158760)	0,863910	-62,402060	2,656082	2,809044
	100	0,203164 (0,110700) -	0,616840 (0,217940) +	0,276810 (0,127720) +	0,893650	-132,950200	2,739003	2,843210
	200	0,242169 (0,098420) +	0,556803 (0,135550) +	0,298847 (0,100770) +	0,855650	-278,871800	2,828718	2,894685
	500	0,194691 (0,062940) +	0,364556 (0,078220) +	0,514910 (0,091110) +	0,879466	-746,031200	3,000125	3,033842
	1000	0,184655 (0,040100) +	0,365904 (0,051470) +	0,512002 (0,059180) +	0,877906	-1459,600000	2,927199	2,946830
	5000	0,215452 (0,020098) +	0,375994 (0,023593) +	0,501022 (0,025163) +	0,877016	-7586,379000	3,036152	3,041365
	10000	0,201043 (0,012818) +	0,369597 (0,015947) +	0,525005 (0,016125) +	0,894602	-15489,940000	3,098789	3,101673
Student-t	10	0,640303 (0,317300) +	0,999999 (0,572900) -	0,000000 (NaN)	0,999999	-20,093850	-	-
	50	0,195769 (0,126060)	0,669326 (0,299640) +	0,225886 (0,174650)	0,895212	-63,277010	2,731080	2,922283
	100	0,212476 (0,120570) -	0,609648 (0,244340) +	0,305222 (0,137130) +	0,914870	-133,751900	2,775038	2,905297
	200	0,264870 (0,113630) +	0,563530 (0,163920) +	0,314610 (0,108920) +	0,878140	-281,165200	2,861652	2,944110
	500	0,198150 (0,075370) +	0,367700 (0,095400) +	0,531530 (0,107200) +	0,899230	-748,756200	3,015025	3,057171
	1000	0,184217 (0,046100) +	0,371053 (0,061920) +	0,532345 (0,066900) +	0,903398	-1466,116000	2,942232	2,966771
	5000	0,228869 (0,024302) +	0,391717 (0,028421) +	0,503998 (0,020955) +	0,895715	-7619,821000	3,049929	3,056446
	10000	0,213531 (0,015341) +	0,392352 (0,019253) +	0,522673 (0,018270) +	0,915025	-15553,250000	3,111651	3,115256
GED	10	0,067427 (NaN)	0,955668 (0,228800) +	0,000000 (NaN)	0,955668	-18,186530	-	-
	50	0,144040 (0,082260) -	0,842420 (0,222730) +	0,167900 (0,123060)	1,010320	-60,685340	2,627414	2,818616
	100	0,200189 (0,107690) -	0,635689 (0,217440) +	0,269426 (0,123460) +	0,905115	-132,864900	2,757299	2,887557
	200	0,230370 (0,089270) +	0,586820 (0,128700) +	0,292800 (0,092010) +	0,879620	-278,013800	2,830138	2,912596
	500	0,194690 (0,062780) +	0,364730 (0,078110) +	0,514780 (0,090890) +	0,879510	-746,030100	3,004120	3,046266
	1000	0,185390 (0,039630) +	0,367100 (0,050780) +	0,510440 (0,058370) +	0,877540	-1459,510000	2,929020	2,953559
	5000	0,215266 (0,019676) +	0,376418 (0,023122) +	0,500888 (0,024605) +	0,877306	-7585,597000	3,036239	3,042756
	10000	0,200961 (0,012682) +	0,369479 (0,015952) +	0,525154 (0,015952) +	0,894633	-15489,510000	3,098902	3,102507

Notes: This table presents the estimates of Scenario 2 Model 3 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " .",

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.19: Scenario 2 Model 3 return series Accuracy measures and Risk measures results

<i>Distrbution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	5,772409	2,402584	0,690119	0,865436
Student-t		5,790236	2,406291	0,690119	0,865436
GED		5,812020	2,410813	0,690119	0,865436
Normal	50	1,566204	1,251481	0,816070	1,023385
Student-t		1,568302	1,252319	0,838594	1,051631
GED		1,569999	1,252996	0,814661	1,021617
Normal	100	1,242719	1,114773	0,894779	1,122090
Student-t		1,244748	1,115683	0,932687	1,169627
GED		1,242282	1,114577	0,891766	1,118310
Normal	200	1,285178	1,133657	0,933740	1,170948
Student-t		1,282893	1,132649	0,967713	1,213550
GED		1,290126	1,135837	0,934819	1,172301
Normal	500	1,650584	1,284751	1,337885	1,677762
Student-t		1,649402	1,284291	1,416458	1,776296
GED		1,650620	1,284765	1,337596	1,677399
Normal	1000	1,587943	1,260136	1,319288	1,654441
Student-t		1,587513	1,259965	1,411941	1,770631
GED		1,588038	1,260174	1,315122	1,649216
Normal	5000	1,581220	1,257466	1,312998	1,646553
Student-t		1,581196	1,257456	1,352764	1,696421
GED		1,581228	1,257469	1,312743	1,646233
Normal	10000	2,305971	1,518542	1,392056	1,745694
Student-t		2,305971	1,518542	1,418831	1,779272
GED		2,304971	1,518542	1,392504	1,746257

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

Table 4.20: Scenario 2 Model 4: $\sigma_t^2 = 0, 1 + 0, 2a^2_{t-1} + 0, 7\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 9$	Log likelihood	AIC	BIC
Normal	10	0,110310 (0,117200)	0,999999 (0,201100) +	0,000000 (NaN)	0,999999	-10,401260	-	-
	50	0,874652 (NaN)	0,242201 (0,198900)	0,000000 (NaN)	0,242201	-73,629130	3,105165	3,258127
	100	0,238549 (0,156100)	0,320789 (0,173380) -	0,535464 (0,182510) +	0,856253	-154,136500	3,162730	3,266937
	200	0,136945 (0,078644) -	0,251706 (0,098257) +	0,631323 (0,120275) +	0,883029	-277,368200	2,813682	2,879648
	500	0,113336 (0,050036) +	0,151893 (0,046157) +	0,708672 (0,089285) +	0,860565	-641,409800	2,581639	2,615356
	1000	0,155377 (0,045323) +	0,159885 (0,033681) +	0,651681 (0,073258) +	0,811566	-1295,572000	2,599143	2,618774
	5000	0,106941 (0,013145) +	0,206872 (0,016284) +	0,685870 (0,023093) +	0,892742	-6698,517000	2,681007	2,686221
	10000	0,107093 (0,009291) +	0,198739 (0,011599) +	0,691813 (0,016779) +	0,890552	-13381,810000	2,677163	2,680047
Student-t	10	0,000000 (0,311600)	0,083626 (0,274300)	1,000000 (NaN)	1,083626	-11,696350	-	-
	50	0,938320 (NaN)	0,273430 (0,301700)	0,000000 (NaN)	0,273430	-74,329530	3,173181	3,364383
	100	0,208563 (0,154750)	0,347566 (0,183240) -	0,563654 (0,179320) +	0,911220	-154,631400	3,192627	3,322886
	200	0,131258 (0,081810)	0,269770 (0,113080) +	0,639750 (0,127280) +	0,909520	-278,070000	2,830700	2,913158
	500	0,117834 (0,057246) +	0,157645 (0,054059) +	0,716300 (0,096331) +	0,873945	-646,043400	2,604173	2,646320
	1000	0,163399 (0,052250) +	0,166662 (0,040800) +	0,657370 (0,080060) +	0,824032	-1304,309000	2,618618	2,643157
	5000	0,111786 (0,015525) +	0,217319 (0,019649) +	0,687882 (0,026026) +	0,905201	-6730,989000	2,694396	2,700913
	10000	0,110499 (0,010898) +	0,212107 (0,013990) +	0,692364 (0,018826) +	0,904471	-13440,180000	2,689036	2,692641
GED	10	0,242150 (0,146100)	0,386200 (NaN)	0,000000 (NaN)	0,386200	-8,770309	-	-
	50	0,889622 (NaN)	0,235513 (0,146700)	0,000000 (NaN)	0,235513	-73,420050	3,136802	3,328004
	100	0,242814 (0,160490)	0,318320 (0,173430) -	0,533620 (0,183070) +	0,851940	-154,128500	3,182571	3,312829
	200	0,135664 (0,079125) -	0,252841 (0,100046) +	0,631901 (0,121638) +	0,884742	-277,338300	2,823383	2,905841
	500	0,113703 (0,047932) +	0,152568 (0,043873) +	0,707612 (0,085279) +	0,860180	-640,795500	2,583182	2,625328
	1000	0,155453 (0,043534) +	0,160476 (0,031882) +	0,651057 (0,070156) +	0,811533	-1294,502000	2,599004	2,623543
	5000	0,106935 (0,013086) +	0,206899 (0,016207) +	0,685855 (0,022985) +	0,892754	-6698,479000	2,681392	2,687909
	10000	0,107093 (0,009291) +	0,198739 (0,011600) +	0,691814 (0,016780) +	0,890553	-13381,810000	2,677363	2,680968

Notes: This table presents the estimates of Scenario 2 Model 4 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " . "

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.21: Scenario 2 Model 4 return series Accuracy measures and Risk measures results

<i>Distrbution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,595908	0,771951	0,186734	0,234173
Student-t		0,597691	0,773105	1,663464	2,086051
GED		0,596410	0,772276	0,186734	0,234173
Normal	50	1,140549	1,067965	0,186735	0,234173
Student-t		1,142642	1,068944	0,186735	0,234173
GED		1,137739	1,066648	0,186735	0,234173
Normal	100	1,571160	1,253459	1,904692	2,388560
Student-t		1,580537	1,257194	2,054306	2,576183
GED		1,569426	1,252767	1,893254	2,374217
Normal	200	1,132809	1,064335	1,979762	2,482701
Student-t		1,134515	1,065136	2,069226	2,594893
GED		1,133117	1,064480	1,985175	2,489489
Normal	500	0,852429	0,923271	1,831934	2,297319
Student-t		0,852536	0,923329	1,897242	2,379218
GED		0,852264	0,923182	1,830284	2,295250
Normal	1000	0,850358	0,922149	1,740987	2,183268
Student-t		0,850459	0,922203	1,800744	2,258206
GED		0,850273	0,922103	1,739324	2,181182
Normal	5000	0,977200	0,988535	1,979844	2,482805
Student-t		0,977200	0,988535	2,036475	2,553821
GED		0,977200	0,988535	1,979868	2,482834
Normal	10000	0,975325	0,987585	1,968470	2,468541
Student-t		0,975340	0,987593	2,031190	2,547190
GED		0,975325	0,987585	1,968472	2,468543

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

4.2.3 Simulation study: Scenario 3

Four different models were simulated from a GARCH (1,1) process with different varying initial values of $\alpha_0, \alpha_1, \beta_1$ and sample sizes as illustrated in Table 4.1. Discussion on the results obtained for the descriptive statistics follows.

Descriptive statistics for Scenario 3

In Table 4.22 the descriptive statistics obtained for the four models of Scenario 3 for sample sizes 50, 500 and 5000 are presented.

For Model 1 and Model 2 we observe that both the skewness and kurtosis values for the sample sizes are close to zero, which indicate that the return series are normally distributed. In Appendix Table A.5 and Table A.6 which presents the QQ-plots for Scenario 3, we also observe that the returns of Model 1 and Model 2 for the selected sample sizes are normally distributed with most of the data points falling approximately along the reference line with only a few data points diverting from the straight line.

We observe that the skewness values of Model 3 are all close to zero. The kurtosis for sample sizes 5000 was 2,380000, suggesting that the returns for these sample sizes could be platykurtic. This is also seen in Appendix Table A.5 and Table A.6 the QQ-plots of Model 3 for sample sizes 5000 where non normality is observed in the QQ-plot.

For Model 4, only the returns of sample size 5000 appears to be non-normal as it has a kurtosis value of 11,340000 indicating that the returns are leptokurtic. Heavy tails are also apparent in the QQ-plot presented by Appendix Table A.5 and Table A.6 for Model 4 at sample size 5000.

Table 4.22: Descriptive statistics of Scenario 3 return series

	T	mean	standard deviation	median	min	max	skewness	kurtosis
Model 1	50	-0,090000	0,460000	-0,080000	-1,300000	0,740000	-0,330000	-0,160000
	500	-0,010000	0,450000	0,000000	-1,300000	1,290000 -	0,060000	-0,190000
	5000	-0,010000	0,460000	-0,020000	-1,880000	2,170000	0,080000	0,240000
Model 2	50	-0,060000	0,620000	-0,090000	-1,400000	1,190000	-0,120000	-0,500000
	500	-0,010000	0,760000	0,000000	-2,460000	5,170000	-0,040000	0,590000
	5000	0,000000	0,710000	0,000000	3,580000	3,220000	0,020000	0,480000
Model 3	50	0,150000	1,120000	0,050000	-2,030000	2,500000	0,130000	-0,760000
	500	0,050000	1,100000	0,060000	-3,440000	3,810000	-0,030000	-0,050000
	5000	0,020000	1,100000	0,020000	-7,480000	8,460000	0,100000	2,380000
Model 4	50	-0,250000	1,450000	-0,040000	-3,070000	3,880000	0,330000	0,080000
	500	-0,100000	1,490000	-0,010000	-5,220000	3,880000	-0,310000	0,340000
	5000	-0,010000	2,240000	-0,020000	-24,260000	17,760000	-0,030000	11,340000

Normality test for Scenario 3

The Scenario 3 results obtained for the Jarque-Bera test and Shapiro-Wilk test are presented in Table 4.23. Both tests rejected the normality of the return series at 5% significance level for Model 1 at sample size 5000, Model 2 at sample sizes 500 and 5000, Model 3 at sample size 5000 and Model 4 at sample sizes 500 and 5000.

Table 4.23: Normality tests of Scenario 3 return series

		Jarque-Bera (JB)		Shapiro-Wilk test	
	T	JB	p-value	W	p-value
Model 1	50	0,982670	0,611800	0,980870	0,589200
	500	0,945690	0,623200	0,997560	0,682500
	5000	16,651000	0,000242	0,998850	0,001383
Model 2	50	0,442210	0,801600	0,985590	0,796600
	500	7,669800	0,021600	0,993650	0,034100
	5000	48,830000	0,000000	0,998010	0,000005
Model 3	50	1,082400	0,582100	0,982950	0,681100
	500	0,133010	0,935700	0,998890	0,991300
	5000	1188,100000	0,000000	0,988330	0,000506
Model 4	50	1,044600	0,593200	0,977500	0,452000
	500	40,534000	0,005160	0,992350	0,011510
	5000	26798,000000	0,000000	0,893410	0,000000

GARCH model selection process for Scenario 3

After analysing the results of Scenario 3 we note that similar conclusions are observed as in Scenarios 1 and 2.

Scenario 3 example: An analyst was given the task to identify the model that best describes the data. He had to simulate data for $T = 500$. He simulated the data points using a GARCH (1,1) process with true innovation following a Normal distribution. The initial parameters values for α_0 , α_1 and β_1 were set to 0,35, 0,35 and 0,35 respectively. The simulated data points were then fitted to a GARCH (1,1) model with innovations following Normal, Student-t and GED distributions. The output he obtained is that of Table 4.28 for sample size 500. After analysing the data, the GED

distribution produced the lowest AIC (2,941499) and BIC (2,983646) values. The optimal model the analyst identified was the GARCH (1,1) model with GED. This is Model misspecification because the inappropriate innovation distribution is assumed. When we observe Table 4.28, we note the Normal distribution was consistent in producing the lowest AIC and BIC values for majority of the sample sizes. This imply that the GARCH (1,1) model with error terms following a Standard Normal distributed error terms describes the data the best.

Table 4.24: Scenario 3 Model 1: $\sigma_t^2 = 0, 15 + 0, 15a^2_{t-1} + 0, 15\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 3$	Log likelihood	AIC	BIC
Normal	10	0,003904 (NaN)	0,000000 (NaN)	0,999999 (NaN)	0,999999	-4,402468	-	-
	50	0,000000 (0,022980)	0,000000 (0,084590)	0,995190 (0,188100) +	0,995190	-31,899160	1,435966	1,588928
	100	0,029198 (NaN)	0,000000 (NaN)	0,869088 (NaN)	0,869088	-67,523660	1,430473	1,534680
	200	0,005182 (0,007462)	0,000000 (0,039020)	0,978255 (0,051770) +	0,978255	-138,259000	1,422590	1,488556
	500	0,180444 (0,060520)	0,122992 (0,065850) +	0,000000 (0,304500) -	0,122992	-311,399700	1,261599	1,295316
	1000	0,185482 (0,058250) +	0,154213 (0,042930) +	0,000000 (0,264000)	0,154213	-650,702800	1,309406	1,329037
	5000	0,148854 (0,019557) +	0,145803 (0,019743) +	0,150799 (0,095645)	0,296602	-3165,739000	1,267895	1,273109
	10000	0,153500 (0,013412) +	0,147771 (0,013922) +	0,111817 (0,065879) -	0,259588	-6230,293000	1,246859	1,249743
	10	0,604646 (9,998000)	0,999999 (4,896000)	0,000000 (3,463000)	0,999999	-2,504949	-	-
	50	0,222085 (0,061420) +	0,000000 (0,105700)	0,000000 (NaN)	0,000000	-32,389770	1,495591	1,686793
Student-t	100	0,212725 (0,183200)	0,138089 (0,309400)	0,000000 (0,948400)	0,138089	-68,354610	1,467092	1,597351
	200	0,217958 (0,174000)	0,130864 (0,203000)	0,000000 (0,839600)	0,130864	-139,071800	1,440718	1,523176
	500	0,190730 (0,073450) +	0,142680 (0,081920) +	0,000000 (0,352000)	0,142680	-316,031500	1,284126	1,326272
	1000	0,195875 (0,070860) +	0,155794 (0,051010) +	0,000000 (0,309400)	0,155794	-655,656600	1,321313	1,345852
	5000	0,162049 (0,025572) +	0,145947 (0,023357) +	0,130133 (0,118266)	0,276080	-3190,960000	1,278384	1,284901
	10000	0,166665 (0,016718) +	0,154274 (0,016633) +	0,090292 (0,078296)	0,244566	-6289,728000	1,258946	1,262551
	10	0,064992 (0,113900)	0,000000 (NaN)	0,380639 (0,998500)	0,380639	-2,575706	-	-
	50	0,000000 (0,028550)	0,000000 (0,166300)	0,995020 (0,310300) +	0,995020	-31,889420	1,475577	1,666779
	100	0,034881 (NaN)	0,000000 (NaN)	0,844136 (NaN)	0,844136	-67,282330	1,445647	1,575905
	200	0,005089 (0,007036)	0,000000 (0,004279)	0,978663 (0,052220) +	0,978663	-138,185000	1,431850	1,514308
GED	500	0,181022 (0,057680) +	0,119738 (0,062220) -	0,000000 (0,289300)	0,119738	-310,929100	1,263717	1,305863
	1000	0,185557 (0,059270) +	0,153794 (0,043530) +	0,000000 (0,268900)	0,153794	-650,630500	1,311261	1,335800
	5000	0,149358 (0,020108) +	0,145260 (0,020127) +	0,148867 (0,098273)	0,294127	-3165,050000	1,268020	1,274537
	10000	0,153516 (0,013426) +	0,147766 (0,013932) +	0,111744 (0,065947) -	0,259510	-6230,291000	1,247058	1,250663

Notes: This table presents the estimates of Scenario 3 Model 1 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " . "

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.25: Scenario 3 Model 1 return series Accuracy measures and Risk measures results

<i>Distrbution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,143777	0,379179	0,703299	0,881965
Student-t		0,150629	0,388109	0,501651	0,629091
GED		0,152150	0,390064	0,513743	0,644254
Normal	50	0,211691	0,460099	0,670188	0,840443
Student-t		0,211805	0,460223	0,501651	0,629091
GED		0,211704	0,460113	0,667234	0,836738
Normal	100	0,225976	0,475369	0,746582	0,936244
Student-t		0,226268	0,475677	0,501651	0,629091
GED		0,226006	0,475401	0,741843	0,930301
Normal	200	0,233368	0,483082	0,797504	1,000102
Student-t		0,233411	0,483126	0,501651	0,629091
GED		0,233368	0,483082	0,797986	1,000706
Normal	500	0,205719	0,453564	0,501651	0,629091
Student-t		0,205726	0,453570	0,501651	0,629091
GED		0,205718	0,453561	0,501651	0,629091
Normal	1000	0,219789	0,468817	0,501651	0,629091
Student-t		0,219764	0,468789	0,501651	0,629091
GED		0,219784	0,468811	0,501651	0,629091
Normal	5000	0,211736	0,460148	0,605155	0,758888
Student-t		0,211737	0,460149	0,595113	0,746296
GED		0,211735	0,460147	0,603709	0,757075
Normal	10000	0,207400	0,455413	0,580783	0,728326
Student-t		0,207400	0,455413	0,570519	0,715454
GED		0,207400	0,455413	0,580735	0,728265

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

Table 4.26: Scenario 3 Model 2: $\sigma_t^2 = 0, 25 + 0, 25a^2_{t-1} + 0, 25\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 5$	Log likelihood	AIC	BIC
Normal	10	0,005803 (0,613400)	0,000000 (0,696100)	0,999999 (2,135000)	0,999999	-9,877080	-	-
	50	0,275412 (0,181200)	0,262966 (0,201700)	0,000000 (0,497600)	0,262966	-45,194610	1,967784	2,120746
	100	0,281026 (0,111000) +	0,297961 (0,140000) +	0,062138 (0,252520)	0,360099	-97,413720	2,028274	2,132481
	200	0,317318 (NaN)	0,461747 (NaN)	0,000000 (NaN)	0,461747	-213,775100	2,177775	2,243717
	500	0,373014 (0,110984) +	0,333003 (0,076469) +	0,026868 (0,190106)	0,359871	-551,672300	2,222689	2,256406
	1000	0,347806 (0,076380) +	0,284860 (0,051690) +	0,066540 (0,150020)	0,351400	-1073,642000	2,155285	2,174916
	5000	0,309389 (0,028127) +	0,246154 (0,022516) +	0,147755 (0,058976) +	0,393909	-5284,055000	2,115222	2,120436
	10000	0,292051 (0,018336) +	0,255502 (0,016318) +	0,178716 (0,039240) +	0,434218	-10589,790000	2,118759	2,121643
	10	0,014823 (1,094000)	0,000000 (1,190000)	1,000000 (3,328000)	1,000000	-10,088450	-	-
	50	0,302729 (0,210000)	0,291588 (0,246800)	0,000000 (0,509400)	0,291588	-46,086250	2,043450	2,234652
Student-t	100	0,271590 (0,129580) +	0,317545 (0,168340) -	0,123578 (0,282300)	0,441123	-98,270690	2,065414	2,195672
	200	0,335170 (0,175600) -	0,494850 (0,173600) +	0,000000 (0,338600)	0,494850	-215,334800	2,203348	2,285806
	500	0,388271 (0,133103) +	0,343403 (0,089139) +	0,031001 (0,221142)	0,374404	-553,523000	2,234092	2,276238
	1000	0,344828 (0,088000) +	0,305892 (0,061580) +	0,089154 (0,168630)	0,395046	-1075,329000	2,160658	2,185197
	5000	0,334480 (0,033362) +	0,261301 (0,026860) +	0,127489 (0,065656) -	0,388790	-5308,801000	2,125520	2,132038
	10000	0,312010 (0,021806) +	0,265577 (0,019280) +	0,168215 (0,044169) +	0,433792	-10630,400000	2,127080	2,130685
	10	0,419040 (0,561500)	0,000000 (NaN)	0,000000 (1,339000)	0,000000	-8,868364	-	-
	50	0,282398 (0,174600)	0,269567 (0,157600) -	0,000000 (0,488600)	0,269567	-44,332960	1,973318	2,164521
	100	0,288707 (0,107300) +	0,299272 (0,133810) +	0,042799 (0,241760)	0,342071	-97,321250	2,046425	2,176684
	200	0,317283 (NaN)	0,461822 (0,030710) +	0,000000 (NaN)	0,461822	-213,729100	2,187291	2,269749
GED	500	0,372569 (0,113702) +	0,331919 (0,078499) +	0,028385 (0,195251)	0,360304	-551,502800	2,226011	2,268157
	1000	0,344684 (0,078920) +	0,286704 (0,054030) +	0,071320 (0,155440)	0,358024	-1072,879000	2,155757	2,180296
	5000	0,309389 (0,028136) +	0,246152 (0,022517) +	0,147754 (0,058998) +	0,393906	-5284,055000	2,115622	2,122139
	10000	0,292322 (0,018526) +	0,255424 (0,016485) +	0,178220 (0,039636) +	0,433644	-10589,330000	2,118866	2,122471

Notes: This table presents the estimates of Scenario 3 Model 2 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " . "

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.27: Scenario 3 Model 2 return series Accuracy measures and Risk measures results

<i>Distrbution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,423775	0,650979	1,141728	1,431772
Student-t		0,424505	0,651541	1,244810	1,561042
GED		0,423576	0,650828	1,560614	1,957073
Normal	50	0,383756	0,619481	1,560614	1,957073
Student-t		0,382674	0,618607	1,560614	1,957073
GED		0,390879	0,625203	1,560614	1,957073
Normal	100	0,454491	0,674159	1,529055	1,917497
Student-t		0,454368	0,674068	1,498804	1,879560
GED		0,454536	0,674193	1,539006	1,929975
Normal	200	0,571366	0,755887	1,560614	1,957073
Student-t		0,571471	0,755957	1,560614	1,957073
GED		0,571388	0,755902	1,560614	1,957073
Normal	500	0,574698	0,758088	1,548739	1,942182
Student-t		0,574925	0,758224	1,547407	1,940511
GED		0,574756	0,758127	1,548076	1,941349
Normal	1000	0,534485	0,731085	1,530301	1,919059
Student-t		0,534483	0,731083	1,520463	1,906721
GED		0,534483	0,731083	1,528025	1,916205
Normal	5000	0,510322	0,714369	1,491824	1,870807
Student-t		0,510320	0,714367	1,503722	1,885729
GED		0,510322	0,714369	1,491824	1,870807
Normal	10000	0,515772	0,718172	1,476098	1,851086
Student-t		0,157670	0,718169	1,484176	1,861216
GED		0,515771	0,718172	1,476353	1,851406

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

Table 4.28: Scenario 3 Model 3: $\sigma_t^2 = 0, 35 + 0, 35a^2_{t-1} + 0, 35\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 7$	Log likelihood	AIC	BIC
Normal	10	0,138372 (NaN)	0,000000 (NaN)	0,999999 (NaN)	0,999999	-13,240910	-	-
	50	0,825418 (NaN)	0,333645 (0,189900) -	0,000000 (NaN)	0,333645	-74,491060	3,139643	3,292604
	100	0,749293 (NaN)	0,380996 (0,117500) +	0,000000 (NaN)	0,380996	-146,117300	3,002345	3,106552
	200	0,791765 (0,525100)	0,260851 (0,159000)	0,000000 (0,580900)	0,260851	-285,638700	2,896387	2,962353
	500	0,534827 (0,146540) +	0,343052 (0,082060) +	0,219806 (0,146230)	0,562858	-734,642700	2,954571	2,988288
	1000	0,420565 (0,080990) +	0,355664 (0,057370) +	0,297129 (0,088860) +	0,652793	-1448,564000	2,905127	2,924758
	5000	0,405224 (0,035462) +	0,340622 (0,024620) +	0,319877 (0,039625) +	0,660499	-7214,731000	2,887492	2,892706
	10000	0,384839 (0,022725) +	0,356562 (0,017753) +	0,319078 (0,026558) +	0,675640	-14299,980000	2,860795	2,863679
	10	0,028440 (NaN)	0,000000 (NaN)	1,000000 (NaN)	1,000000	-13,458980	-	-
	50	0,904890 (NaN)	0,388590 (0,231400) -	0,000000 (NaN)	0,388590	-75,516730	3,220669	3,411872
Student-t	100	0,790850 (NaN)	0,429600 (0,119900) +	0,000000 (NaN)	0,429600	-147,256600	3,045133	3,175391
	200	0,842890 (0,561200)	0,287390 (0,180700)	0,000000 (0,574300)	0,287390	-287,816600	2,928166	3,010624
	500	0,598485 (0,189500) +	0,374001 (0,099550) +	0,208273 (0,171950)	0,582274	-742,327200	2,989309	3,031455
	1000	0,427056 (0,094130) +	0,390492 (0,069480) +	0,311948 (0,097230) +	0,702440	-1459,154000	2,928309	2,952847
	5000	0,414669 (0,040490) +	0,365709 (0,029610) +	0,332653 (0,042770) +	0,698362	-7257,425000	2,904970	2,911487
	10000	0,400705 (0,026600) +	0,377941 (0,021220) +	0,323398 (0,029400) +	0,701339	-14363,750000	2,873749	2,877354
	10	0,014559 (0,816400)	0,000000 (0,375400)	0,999999 (1,164000)	0,999999	-12,457050	-	-
	50	0,864685 (NaN)	0,304529 (0,151300) +	0,000000 (NaN)	0,304529	-73,123700	3,124948	3,316150
	100	0,415740 (0,548650)	0,328290 (0,155130) +	0,340920 (0,502070)	0,669210	-146,271600	3,025432	3,155691
	200	0,793513 (0,520500)	0,259392 (0,152600) -	0,000000 (0,575500)	0,259392	-285,340400	2,903404	2,985862
GED	500	0,522874 (0,118270) +	0,344912 (0,070270) +	0,230284 (0,118120) -	0,575196	-730,374900	2,941499	2,983646
	1000	0,430638 (0,076860) +	0,352669 (0,053410) +	0,289975 (0,083560) +	0,642644	-1446,841000	2,903682	2,928221
	5000	0,409306 (0,034449) +	0,339958 (0,023599) +	0,316453 (0,038366) +	0,656411	-7211,319000	2,886528	2,893045
	10000	0,385215 (0,022481) +	0,356518 (0,017540) +	0,318740 (0,026265) +	0,675258	-14299,390000	2,860877	2,864483

Notes: This table presents the estimates of Scenario 3 Model 3 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " . "

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.29: Scenario 3 Model 3 return series Accuracy measures and Risk measures results

<i>Distrbution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,83253	0,91243	1,62074	2,03248
Student-t		0,83508	0,91383	1,74034	2,18246
GED		0,84300	0,91815	1,63545	2,05092
Normal	50	1,22927	1,10873	2,90347	3,64107
Student-t		1,22942	1,10879	2,90347	3,64107
GED		1,22982	1,10897	2,90347	3,64107
Normal	100	1,18357	1,08792	2,90347	3,64107
Student-t		1,18447	1,08833	2,90347	3,64107
GED		1,18392	1,08808	2,51305	3,15147
Normal	200	1,06669	1,03281	2,90347	3,64107
Student-t		1,06698	1,03295	2,90347	3,64107
GED		1,06657	1,03275	2,90347	3,64107
Normal	500	1,20215	1,09643	2,65612	3,33089
Student-t		1,20229	1,09649	2,67809	3,35844
GED		1,20202	1,09637	2,64319	3,31467
Normal	1000	1,19230	1,09192	2,55023	3,19809
Student-t		1,19230	1,09193	2,53817	3,18296
GED		1,19231	1,09193	2,55996	3,21029
Normal	5000	1,21073	1,10033	2,51923	3,15921
Student-t		1,21068	1,10031	2,50972	3,14729
GED		1,21075	1,10034	2,52387	3,16504
Normal	10000	1,21897	1,10407	2,51541	3,15443
Student-t		1,21896	1,10406	2,51599	3,15515
GED		1,21897	1,10407	2,51587	3,15501

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

Table 4.30: Scenario 3 Model 4: $\sigma_t^2 = 0, 45 + 0, 45a^2_{t-1} + 0, 45\sigma^2_{t-1}$

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 = 0, 9$	Log likelihood	AIC	BIC
Normal	10	0,00000 (4,267000)	0,00000 (NaN)	0,989000 (NaN)	0,989000	-22,056400	-	-
	50	0,253970 (0,241400)	0,252530 (0,196600)	0,652810 (0,237800) +	0,905340	-86,399350	3,615974	3,768936
	100	0,190332 (0,197330)	0,136314 (0,099570)	0,759501 (0,179300) +	0,895815	-166,990100	3,419801	3,524008
	200	0,402620 (0,278900)	0,232390 (0,130630) -	0,584050 (0,219390) +	0,816440	-350,759100	3,547591	3,613558
	500	0,481638 (0,167750) +	0,247441 (0,065020) +	0,547331 (0,103460) +	0,794772	-890,148400	3,576594	3,610310
	1000	0,336722 (0,070063) +	0,397053 (0,052119) +	0,527693 (0,049079) +	0,924746	-1867,971000	3,743942	3,763573
	5000	0,426818 (0,035810) +	0,469335 (0,025740) +	0,450906 (0,022610) +	0,920241	-9593,075000	3,838830	3,844044
	10000	0,459529 (0,027240) +	0,452537 (0,018090) +	0,445750 (0,016610) +	0,898287	-18962,960000	3,793392	3,796276
Student-t	10	4,258953 (1,165000)	0,000000 (1,831000)	0,237535 (2,044000)	0,237535	-22,326550	-	-
	50	0,244520 (0,269700)	0,274350 (0,236300)	0,658090 (0,261800) +	0,932440	-86,662920	3,666517	3,857719
	100	0,179935 (0,221090)	0,147051 (0,114490)	0,771465 (0,187880) +	0,918516	-167,891500	3,457831	3,588089
	200	0,485840 (0,327680)	0,268700 (0,151260) -	0,538970 (0,230140) +	0,807670	-352,435300	3,574353	3,656811
	500	0,503539 (0,191190) +	0,258253 (0,074690) +	0,549777 (0,111360) +	0,808030	-892,899900	3,591600	3,633746
	1000	0,348329 (0,081401) +	0,408226 (0,060410) +	0,535547 (0,053383) +	0,943773	-1873,165000	3,756331	3,780870
	5000	0,452680 (0,043190) +	0,504865 (0,031440) +	0,444832 (0,025680) +	0,949697	-9625,663000	3,852265	3,858782
	10000	0,487550 (0,032750) +	0,474566 (0,021650) +	0,445936 (0,018800) +	0,920502	-19023,310000	3,805662	3,809268
GED	10	0,000000 (6,640000)	0,000000 (NaN)	0,972300 (1,085000)	0,972300	-20,650130	-	-
	50	0,250933 (0,243500)	0,266301 (0,215400)	0,643972 (0,242800) +	0,910273	-86,357600	3,654304	3,845506
	100	0,191791 (0,198290)	0,136022 (0,099020)	0,758820 (0,179040) +	0,894842	-166,988200	3,439764	3,570023
	200	0,394810 (0,275480)	0,228660 (0,128320)	- 0,591100 (0,217970) +	0,819760	-350,722500	3,557225	3,639683
	500	0,480410 (0,170140) +	0,247700 (0,066140) +	0,547760 (0,104860) +	0,795460	-890,070500	3,580282	3,622428
	1000	0,336004 (0,070915) +	0,396632 (0,052750) +	0,528302 (0,049657) +	0,924934	-1867,899000	3,745799	3,770337
	5000	0,426792 (0,035350) +	0,468709 (0,025400) +	0,451302 (0,022320) +	0,920011	-9592,772000	3,839109	3,845626
	10000	0,459462 (0,027110) +	0,452597 (0,018010) +	0,445740 (0,016530) +	0,898337	-18962,880000	3,793577	3,797182

Notes: This table presents the estimates of Scenario 3 Model 4 returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " . "

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.31: Scenario 3 Model 4 return series Accuracy measures and Risk measures results

<i>Distribution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	4,833960	2,198627	3,427199	4,297845
Student-t		4,838660	2,199696	3,869897	4,853007
GED		4,950758	2,225030	3,201143	4,014362
Normal	50	2,087250	1,444732	3,356140	4,208735
Student-t		2,082985	1,443255	3,421483	4,290678
GED		2,097433	1,446179	3,374213	4,231399
Normal	100	1,762377	1,327546	3,138110	3,935316
Student-t		1,760864	1,326975	3,223692	4,042640
GED		1,762426	1,327564	3,136347	3,933106
Normal	200	2,124865	1,457692	3,297585	4,135304
Student-t		2,124259	1,457484	3,366945	4,222285
GED		2,124946	1,457719	3,292448	4,128863
Normal	500	2,231507	1,493823	3,343731	4,193174
Student-t		2,232838	1,494268	3,375866	4,233473
GED		2,231879	1,493947	3,344115	3,193655
Normal	1000	3,811126	1,952211	3,498536	4,387306
Student-t		3,811139	1,952214	3,532440	4,429822
GED		3,811126	1,952211	3,498465	4,387217
Normal	5000	5,025017	2,241655	3,537316	4,435938
Student-t		5,025049	2,241662	3,574947	4,483128
GED		5,025012	2,241654	3,537021	4,435568
Normal	10000	4,327607	2,080290	3,527086	4,423109
Student-t		4,327562	2,080279	3,557351	4,461062
GED		4,327610	2,080291	3,527128	4,423161

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

The following remarks are made on the simulated results for the GARCH (1,1) model with Normal, Student-t and GED innovations assumed respectively. The quantile-quantile plots of the simulated returns presented in Appendix Table A.1 to Table A.6 show that returns shy away from normality with increase in α_1 and decrease in β_1 . The steepness of the reference line is also affected by changes in the initialised parameters.

When the true innovation distribution is Normal, the GARCH (1,1) model with GED consistently outperforms the GARCH (1,1) model with Student-t distribution. We based this conclusion on the AIC and BIC values obtained. In other words, the GED had lower AIC and BIC values in comparison to the Student-t distribution. We also see that when the true innovation distribution is Normal, the GARCH (1,1) model with GED produces similar results to the true model innovation, particularly for large sample sizes. The results obtained for all three scenarios show that when we draw data from a Normal distribution it does not show much volatility. We also found that when data is simulated from a GARCH (1,1) model assuming that the error terms follows a Normal distribution then Student-t and GED can be fitted to the data. We expect that when we fit a GARCH (1,1) model assuming that the error terms follow Normal, Student-t and GED distributions respectively, similar results will be obtained for our empirical dataset if the innovations are Normally distributed. We also note the importance of the goodness of fit and model selection criteria as one of the critical initial steps in the model building process because it helps us to minimize the likelihood of misspecifying a model.

4.3 Empirical results

We made use of the daily closing prices of Bitcoin cryptocurrency and Samsung Electronics stocks as our real time data set, over the period of 17 September 2014 to 29 July 2021 and of 26 May 2011 to 30 July 2021, respectively. The financial time

series used was retrieved from yahoo finance. We also made use of a third real dataset, Moody's seasoned Aaa corporate bond yield (BAAA) over the period 23 September 1997 to 9 May 2008. This dataset was retrieved from the Federal Reserve bank of St. Louis (FRED). We wanted to include a real dataset that has normally distributed returns to confirm the findings of the simulated returns. Hence, we made use of the BAAA dataset over the specific time period mentioned above.

The return series of the real datasets were calculated using the continuously compounded rate of return. That is by using Equation 3.2.2. We perform this transformation on the closing prices in order to make the series stationary. The transformation of the closing prices and yields makes it possible for us to assess the forecast accuracy of the GARCH models. The closing prices and yields of the three real dataset as well as their return series are presented in Figure 4.1-4.6. The return series of Samsung Electronics, Bitcoin and BAAA were fitted into a GARCH (1,1) model with Normal, Student-t and GED, respectively. The results presented in the tables shows the estimated parameters with their related standard errors (SE) in brackets. The volatility persistence measure which is the sum of the parameters given as $\alpha_1 + \beta_1$, goodness of fit measure which is the log-likelihood and model selection criteria namely the AIC and BIC are also presented in the tables. The tables also display the results obtained for the MSE and RMSE as well as our risk measures, the Value-at-Risk and Expected shortfall which were calculated at a 95% confidence level.

4.3.1 Samsung Electronics stock's results

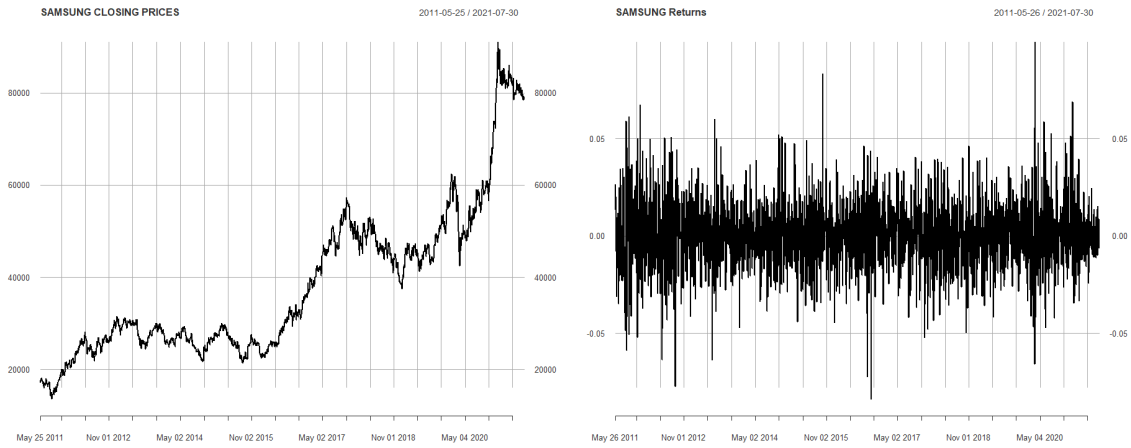


Figure 4.1: Closing stock prices and return series of Samsung Electronics

Descriptive Statistics

Table 4.32 shows that Samsung Electronics has a skewness value close to zero indicating that the distribution of its returns are fairly symmetric. The kurtosis of 1,77 which is less than 3 indicates that Samsung electronics return series is platykurtic. This is also apparent in Figure 4.2, the QQ-plot of Samsung electronics returns where we observe quite a few data points diverting from the straight line.

Table 4.32: Descriptive statistics of Samsung Electronics return series

	T	mean	standard deviation	median	min	max	skewness	kurtosis
Samsung electronics	2500	0,000000	0,020000	0,000000	-0,080000	0,100000	0,060000	1,770000

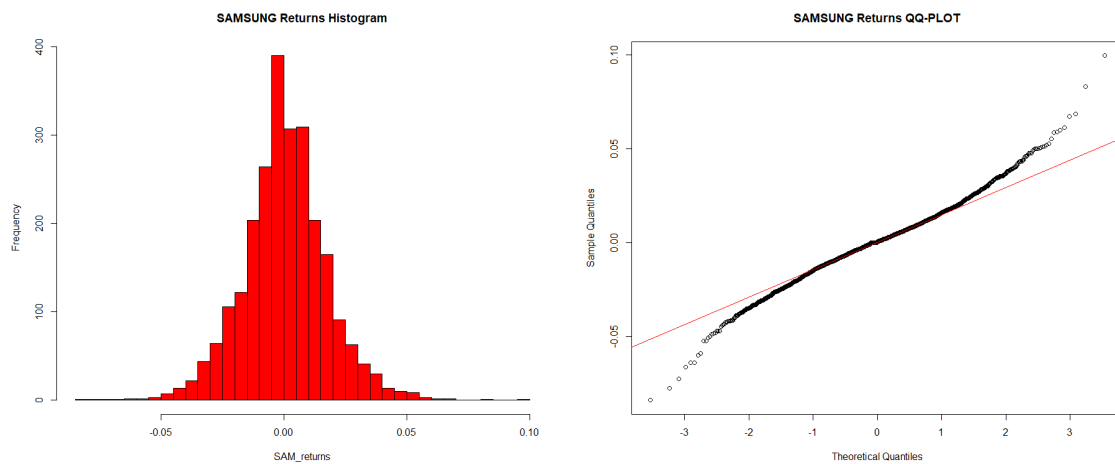


Figure 4.2: Histogram and Quantile-Quantile plot of Samsung Electronics returns

Normality test

Normality is formally tested for Samsung Electronics by use of the Jarque-Bera test and Shapiro-Wilk tests. Samsung Electronics returns do not follow a normal distribution. This is confirmed from the Jarque-Bera and Shapiro-Wilk probability values in Table 4.33 as the hypothesis tests rejected the normality of the Samsung return series at 5% significance level.

Table 4.33: Normality tests of Samsung Electronics return series

	Jarque-Bera (JB)		Shapiro-Wilk test	
	JB	p-value	W	p-value
Samsung electronics	329,720000	0,000000	0,984710	0,000000

Stationarity test

We formally tested for stationarity of the Samsung returns series by use of the ADF and PP tests. The results obtained for both tests are presented in Table 4.34. The probability values of both tests confirms stationarity of the Samsung return series by testing at a 5% significance level.

Table 4.34: Stationarity tests of Samsung Electronics return series

	Augmented Dickey-Fuller (ADF)	Phillips Perron (PP)
p-value	0,010000	0,010000

ARCH effects test

Before we estimate the parameters of the GARCH models, it is important that we first detect the presence of ARCH disturbances in time series data. We tested for ARCH effects in the return series by making use of the Ljung-Box and LM tests with the results obtained presented in Table 4.35. We obtained a p-value of 0,004474 for the Ljung-Box test hence rejecting the null hypothesis at a 5% significance level and concluding that serial autocorrelation exist in the return series. The p-value obtained for the LM test was less than 0,05 hence we also reject the null hypothesis and conclude that ARCH effect is present in the return series of Samsung Electronics. In other words, the returns of Samsung Electronics are heteroscedastic.

Table 4.35: ARCH effects test of Samsung Electronics return series

	Ljung-Box Test		LM Arch test	
Return series	Q	p-value	LM	p-value
Samsung electronics	25,500000	0,004474	3154,000000	0,000000

GARCH model selection process

Similar to Scenario 1's simulated data, Samsung Electronics returns series have small $\hat{\alpha}_1$ values with large $\hat{\beta}_1$ values seen in Table 4.36. There is consistency in the SE of the estimated parameters for all three innovations at the same sample sizes. However, the GED produced relatively smaller SE values compared to the competing distributions. The results show that the time series stationarity condition holds for all three distributions, i.e. $\hat{\alpha}_1 + \hat{\beta}_1 < 1$. For sample sizes larger than 500 the GARCH (1,1) with GED innovations is identified as the best model as it produced the highest log-likelihood values and lowest AIC and BIC values. The GARCH (1,1) with GED innovations describes Samsung Electronics return series the best. This is consistent with the results obtained from the descriptive statistics, because the GED distribution is a symmetric distribution which can be platykurtic.

However, if we based our model selection solely on our accuracy measures presented in Table 4.37, and used the MSE as our main selection measure we would have identified the GARCH (1,1) with Normal innovations as our optimal model because this model consistently produced the lowest MSE for all the sample sizes excluding $T = 100$. This is model misspecification because from the analysis we performed confirmed that Samsung returns are not normally distributed and has thinner tails than that of the Normal distribution. Hence, the Normal distribution does not capture all the features of Samsung returns. The main reason for building a market risk model

is to predict the VaR and ES thus if the distributional properties of the returns are not captured appropriately it can result in the underestimation of these risk measures. This highlights the importance of identifying the distributional properties of the returns in order to ensure that the model chosen upholds the distributional properties and in turn minimize the likelihood of model misspecification.

Table 4.36: Empirical results: Samsung Electronics return series

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 < 1$	Log likelihood	AIC	BIC
Normal	10	0,00000 (NaN)	0,00000 (NaN)	0,975480 (NaN)	0,975480	34,896250	-	-
	50	0,000012 (0,000009)	0,000000 (NaN)	0,792570 (NaN)	0,792570	173,106700	-6,764269	-6,611308
	100	0,000055 (NaN)	0,311820 (NaN)	0,000000 (NaN)	0,311820	332,901800	-6,578035	-6,473828
	200	0,000000 (0,000002)	0,072561 (0,031070) +	0,922570 (0,030020) +	0,995131	575,846100	-5,718461	-5,652495
	500	0,000002 (0,000002)	0,078830 (0,020130) +	0,916650 (0,021940) +	0,995480	1355,578000	-5,406311	-5,372594
	1000	0,000002 (0,000002)	0,047540 (0,009490) +	0,944580 (0,011870) +	0,992120	2694,591000	-5,381183	-5,361552
	2000	0,000005 (0,000002) +	0,036626 (0,006807) +	0,943870 (0,011030) +	0,980496	5405,382000	-5,401382	-5,390180
Student-t	2500	0,000005 (0,000002) +	0,038435 (0,006306) +	0,944270 (0,009722) +	0,982705	6670,077000	-5,332862	-5,323543
	10	0,000000 (NaN)	0,000000 (NaN)	0,951180 (NaN)	0,951180	35,595920	-	-
	50	0,000001 (0,000014)	0,000000 (NaN)	0,985060 (0,190600) +	0,985060	173,693300	-6,747731	-6,556529
	100	0,000057 (NaN)	0,359930 (NaN)	0,000000 (NaN)	0,359930	332,261100	-6,545221	-6,414963
	200	0,000002 (0,000003)	0,065403 (0,036690) -	0,923770 (0,038270) +	0,989173	579,266500	-5,742665	-5,660207
	500	0,000005 (0,000004)	0,094813 (0,030220) +	0,890370 (0,035440) +	0,985183	1360,767000	-5,423069	-5,380922
	1000	0,000004 (0,000003)	0,054045 (0,014250) +	0,933710 (0,018680) +	0,987755	2705,129000	-5,400259	-5,375720
GED	2000	0,000007 (0,000003) +	0,036095 (0,009536) +	0,936790 (0,017480) +	0,972885	5439,307000	-5,434307	-5,420304
	2500	0,000006 (0,000002) +	0,039118 (0,008281) +	0,941830 (0,013020) +	0,980948	6707,091000	-5,361673	-5,350025
	10	0,000000 (0,000022)	0,000000 (NaN)	0,925420 (NaN)	0,925420	36,313640	-	-
	50	0,000009 (0,000013)	0,000000 (NaN)	0,852920 (0,113000) +	0,852920	173,341300	-6,733653	-6,542451
	100	0,000056 (NaN)	0,307170 (NaN)	0,000000 (NaN)	0,307170	332,954200	-6,559084	-6,428825
	200	0,000007 (0,000002)	0,066140 (0,035110) -	0,924730 (0,036130) +	0,990870	579,213700	-5,742137	-5,659679
	500	0,000003 (0,000004)	0,084446 (0,026740) +	0,905200 (0,031220) +	0,989646	1360,993000	-5,423974	-5,381828
	1000	0,000003 (0,000003)	0,049946 (0,012610) +	0,939070 (0,016470) +	0,989016	2709,146000	-5,408292	-5,383753
	2000	0,000007 (0,000003) +	0,035653 (0,008846) +	0,939580 (0,015860) +	0,975233	5443,485000	-5,438485	-5,424483
	2500	0,000006 (0,000002) +	0,038252 (0,007915) +	0,942800 (0,012590) +	0,981052	6711,864000	-5,365491	-5,353843

Notes: This table presents the estimates of Samsung electronics returns fitted by the GARCH (1,1) model with

Normal, Student-t and GED. Values in the parentheses are the corresponding standard error. " + ", " - ", " .",

denotes significance at 1%, 5% and 10% levels respectively.

Table 4.37: Samsung Electronics return series Accuracy measures and Risk measures results

<i>Distribution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,005040	0,741903	0,010771	0,013507
Student-t		0,006297	0,793535	0,010177	0,012762
GED		0,007413	0,860985	0,009677	0,012135
Normal	50	0,005762	0,759048	0,012248	0,015360
Student-t		0,005800	0,761400	0,013427	0,016838
GED		0,005778	0,760143	0,012422	0,015578
Normal	100	0,008051	0,897276	0,010444	0,013097
Student-t		0,008045	0,896951	0,010444	0,013097
GED		0,008056	0,897566	0,010444	0,013097
Normal	200	0,022644	1,504800	0,013080	0,016403
Student-t		0,022814	1,510415	0,013705	0,017186
GED		0,022812	1,151037	0,013350	0,016742
Normal	500	0,030891	1,757594	0,014472	0,018148
Student-t		0,030906	1,758000	0,015679	0,019663
GED		0,030914	1,758235	0,151646	0,019017
Normal	1000	0,028886	1,699580	0,016028	0,020100
Student-t		0,028886	1,699577	0,016956	0,021263
GED		0,028887	1,699614	0,016613	0,020834
Normal	2000	0,027458	1,657046	0,018739	0,023499
Student-t		0,027458	1,657035	0,019925	0,024987
GED		0,027458	1,657040	0,019469	0,024415
Normal	2500	0,029675	1,722644	0,018781	0,023552
Student-t		0,029675	1,722645	0,019126	0,023984
GED		0,029675	1,722644	0,019018	0,023850

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

4.3.2 Bitcoin-USD cryptocurrency results

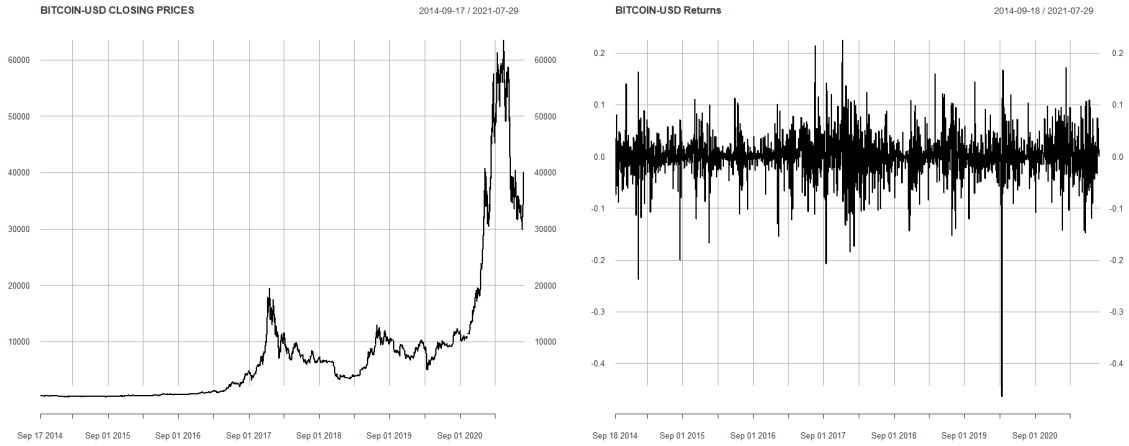


Figure 4.3: Closing prices and returns of Bitcoin-USD

Descriptive Statistics

Table 4.38, the descriptive statistics table of Bitcoin returns show that the distribution of Bitcoin returns are non-symmetric. This is supported by the results obtained for the skewness of Bitcoin returns. The value of the skewness is -0.82 , indicating that the returns from Bitcoin is negatively skewed. We also observe this skewness when looking at the Figure 4.4 which is the histogram of Bitcoin returns series. The kurtosis of Bitcoin returns series of 11.50 is greater than 3 which indicates that the distribution of returns is leptokurtic. We observe the presence of heavy tails from Figure 4.4 the QQ-plot of Bitcoin return series. Heavy tails are also observed in Appendix A.7 and A.8 which presents the quantile-quantile plots of Bitcoin returns for all sample sizes considered.

Table 4.38: Descriptive statistics of Bitcoin cryptocurrency return series

	T	mean	standard deviation	median	min	max	skewness	kurtosis
Bitcoin/USD	2500	0,000000	0,040000	0,000000	-0,460000	0,230000	-0,820000	11,550000

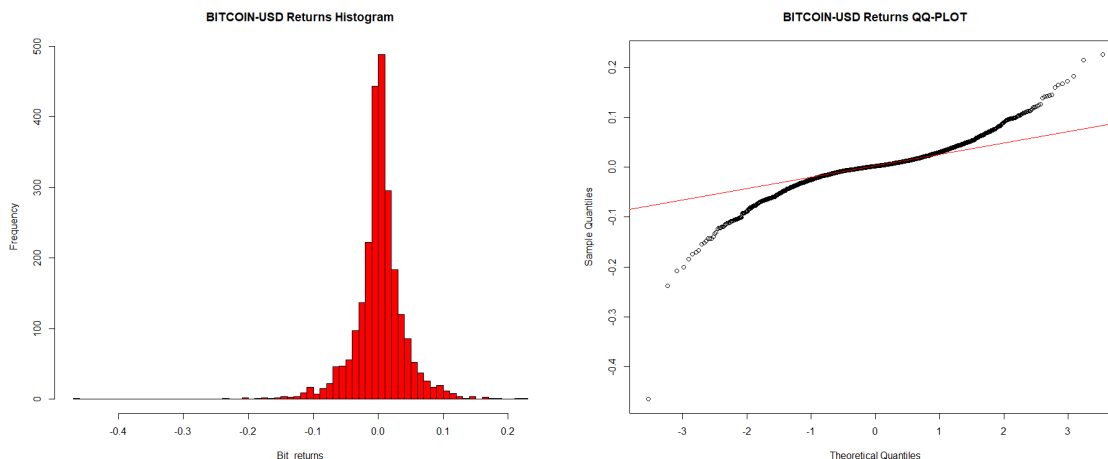


Figure 4.4: Histogram and Quantile-Quantile plot of Bitcoin-USD returns

Normality test

To support the conclusions drawn from the descriptive statistics we formally test normality for Bitcoin returns series. The Jarque-Bera test and Shapiro-Wilk tests were used. Results obtained are presented in Table 4.39. Both tests rejected the normality of Bitcoin returns at 5% significance level.

Table 4.39: Normality tests of Bitcoin cryptocurrency return series

	Jarque-Bera (JB)		Shapiro-Wilk test	
	JB	p-value	W	p-value
Bitcoin/USD	14195,000000	0,000000	0,899470	0,000000

Stationarity test

By use of ADF and PP tests we formally test for stationarity of the Bitcoin returns and present the results obtained in Table 4.40. Both tests rejected non-stationarity of Bitcoin return series at 5% significance level. We can therefore conclude that the return series of Bitcoin is stationary.

Table 4.40: Stationarity tests of Bitcoin cryptocurrency return series

	Augmented Dickey-Fuller (ADF)	Phillips Perron (PP)
p-value	0,010000	0,010000

ARCH effects

In Table 4.41 we present the results obtained for the Ljung-Box and LM tests. We obtained a p-value of 0,011890 for the Ljung-Box test hence rejecting the null hypothesis at a 5% significance level and concluding that serial autocorrelation exist in the return series. The p-value obtained for the LM test was less than 0,05 hence we reject the null hypothesis and conclude that ARCH effect is present in the return series of Bitcoin.

Table 4.41: ARCH effects test of Bitcoin cryptocurrency return series

	Ljung-Box Test		LM Arch test	
Return series	Q	p-value	LM	p-value
Bitcoin/USD	22,705000	0,011890	12545,000000	0,000000

GARCH model selection process

In Table 4.42 results show that the Bitcoin return series resemble Scenario 1's simulated data as well with small $\hat{\alpha}_1$ values paired with large $\hat{\beta}_1$ values. We observe that for sample sizes larger than 200 the GED failed to produce any significant estimates hence accuracy and risk measures presented in Table 4.43 could not be calculated for the GED. Using the Log-likelihood and model selection criteria the results indicate that the GARCH (1,1) model with Student-t distribution is the optimal model to describe the volatility of the Bitcoin return series with sample sizes of 200 to 2500. The Student-t distribution consistently produced the largest log-likelihood and smallest AIC and BIC values. The SE of the parameter estimates for the Student-t distribution are relatively small, which indicates efficient parameter estimation. However, once the sample size is greater than or equal to 500 the volatility persistence measure exceeds the bound of being less than one. This is of great concern as one of the key assumptions is being violated. In other words, the time series stationarity condition does not hold. We can therefore conclude that there is model misspecification present since the optimal model chosen does not uphold all the model specifications.

In Table 4.43 we see that there is inconsistency as to which distribution produced the lowest accuracy measures throughout the sample sizes. If we based our model selection solely on the accuracy measures obtained for $T = 200$. The GARCH (1,1) with GED innovations would have been identified as the optimal model since it pro-

duced the lowest MSE (0,00212819) and RMSE (0,04613235) values for sample size 200. However, from the analysis performed the GARCH (1,1) model with Student-t distribution was identified as the model that best describes the return series of Bitcoin. Hence, model misspecification would have occurred because the inappropriate innovation would have been used to predict the VaR and ES

Table 4.42: Empirical results:Bitcoin return series

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 < 1$	Log likelihood	AIC	BIC
Normal	10	0,000000 (NaN)	0,202170 (0,427500)	0,893940 (NaN)	1,096110	23,567850	-	-
	50	0,000000 (0,000027)	0,000000 (NaN)	0,990780 (NaN)	0,990780	90,302540	-3,452102	-3,299140
	100	0,000098 (0,000169)	0,038030 (0,038730)	0,916731 (0,082010) +	0,954761	161,747800	-3,154957	-3,050750
	200	0,000091 (0,000066)	0,033292 (0,023290)	0,920790 (0,035040) +	0,954082	332,903100	-3,289031	-3,223064
	500	0,000044 (0,000016) -	0,047913 (0,01430) +	0,918040 (0,018760) +	0,965953	877,599900	-3,494399	-3,460683
	1000	0,000146 (0,000031) +	0,113047 (0,026060) +	0,812889 (0,030140) +	0,925936	1824,079000	-3,640158	-3,620527
	2000	0,000065 (0,000010) +	0,116194 (0,020330) +	0,822080 (0,017750) +	0,984020	3779,669000	-3,775669	-3,764467
	2500	0,000070 (0,000011) +	0,140940 (0,015960) +	0,831470 (0,015680) +	0,972410	4754,742000	-3,800593	-3,791275
Student-t	10	0,000000 (0,000281)	0,089503 (0,361900)	1,000000 (NaN)	1,089503	23,375740	-	-
	50	0,000000 (NaN)	0,000000 (NaN)	0,991870 (NaN)	0,991870	90,630060	-3,425202	-3,234000
	100	0,000112 (0,000192)	0,053317 (0,054160)	0,903722 (0,089829) +	0,957039	163,876100	-3,177522	-3,047263
	200	0,000080 (0,000077)	0,042904 (0,029950)	0,919720 (0,043080) +	0,962624	339,543200	-3,345432	-3,262974
	500	0,000023 (0,000020)	0,105340 (0,041580) +	0,910910 (0,022980) +	1,016250	959,210100	-3,816841	-3,774694
	1000	0,000060 (0,000095)	0,431030 (0,653500)	0,911710 (0,018110) +	1,342740	1999,320000	-3,988640	-3,964101
	2000	0,000005 (0,000005)	0,209710 (0,058640) +	0,896090 (0,014160) +	1,105800	4089,112000	-4,084112	-4,070110
	2500	0,000011 (0,000006) -	0,216090 (0,050090) +	0,879830 (0,014500) +	1,095920	5148,471000	-4,114776	-4,103128
GED	10	0,000335 (0,000526)	0,248970 (0,345200)	0,000000 (1,523000)	0,248970	25,440740	-	-
	50	0,000000 (NaN)	0,000000 (NaN)	0,991300 (NaN)	0,991300	90,794590	-3,431784	-3,240581
	100	0,000090 (0,000178)	0,042754 (0,046890)	0,916566 (0,089390) +	0,959320	164,527800	-3,190557	-3,060298
	200	0,002133 (NaN)	0,000000 (0,026870)	0,000000 (NaN)	0,000000	339,373500	-3,343735	-3,261277
	500	0,002133 (NaN)	0,000000 (0,026870)	0,000000 (NaN)	0,000000	339,373500	-3,343735	-3,261277
	1000	0,002133 (NaN)	0,000000 (0,026870)	0,000000 (NaN)	0,000000	339,373500	-3,343735	-3,261277
	2000	0,002133 (NaN)	0,000000 (0,026870)	0,000000 (NaN)	0,000000	339,373500	-3,343735	-3,261277
	2500	0,002133 (NaN)	0,000000 (0,026870)	0,000000 (NaN)	0,000000	339,373500	-3,343735	-3,261277

Notes: This table presents the estimates of Bitcoin returns fitted by the GARCH (1,1) model with Normal,

Student-t and GED. Values in the parentheses are the corresponding standard error. "+", "-", "." denotes significance at 1%, 5% and 10% levels respectively.

Table 4.43: Bitcoin return series Accuracy measures and Risk measures results

<i>Distribution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,051218	2,328475	0,044058	0,055251
Student-t		0,054181	2,327690	0,050012	0,062717
GED		0,054438	2,333188	0,023904	0,029976
Normal	50	0,163147	4,039145	0,052509	0,065848
Student-t		0,163358	4,041758	0,054038	0,067765
GED		0,163411	4,042417	0,053196	0,066709
Normal	100	0,234015	4,837513	0,067387	0,084506
Student-t		0,234195	4,839366	0,068879	0,086377
GED		0,234494	4,842453	0,066457	0,083340
Normal	200	0,212855	4,613620	0,065470	0,082103
Student-t		0,212831	4,613355	0,065425	0,082046
GED		0,212819	4,613235	0,023904	0,029976
Normal	500	0,197928	4,448906	0,056670	0,071066
Student-t		0,197931	4,448942	0,066184	0,082997
GED		-	-	-	-
Normal	1000	0,164267	4,052988	0,060127	0,075402
Student-t		0,164161	4,051674	0,132575	0,166255
GED		-	-	-	-
Normal	2000	0,162575	4,032052	0,060630	0,076032
Student-t		0,162592	4,032270	0,080720	0,101226
GED		-	-	-	-
Normal	2500	0,156420	3,954997	0,059990	0,075230
Student-t		0,156414	3,954918	0,077119	0,096711
GED		-	-	-	-

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

4.3.3 BAAA yield results

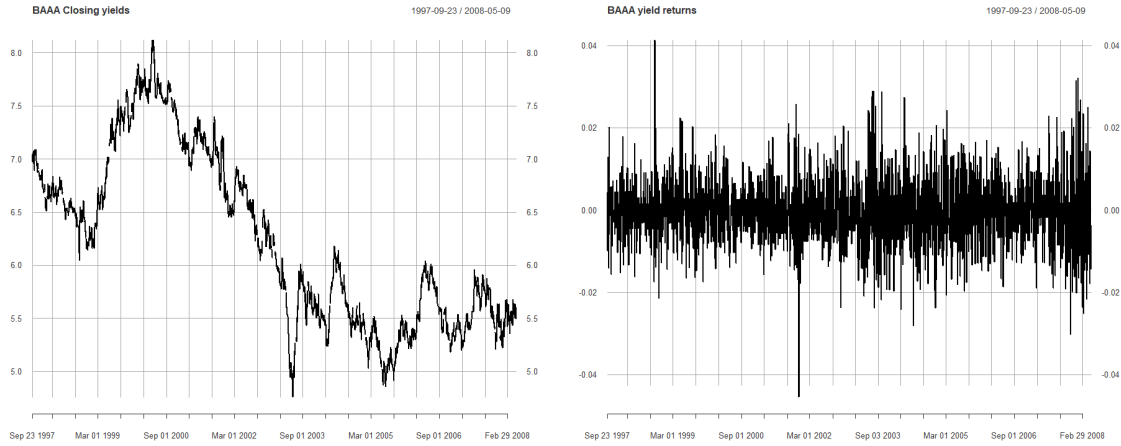


Figure 4.5: Closing yields and yield returns of BAAA

Descriptive Statistics

Table 4.44 shows that BAAA yield returns has skewness and kurtosis values close to zero. Indicating that the distribution of its yield returns are fairly symmetric and may have thinner tails than the Normal distribution. However Figure 4.6 the QQ-plot of BAAA yield returns shows most of the data points fall along the reference line with a few data points diverting from the straight line. We also observe normality of the returns in Appendix Table A.7 and Table A.8, the quantile-quantile plots of BAAA at all sample sizes considered. Indicating that the yield returns are normally distributed. We confirm this by formally testing the yield return series by use of Jarque-Bera test and Shapiro-Wilk tests.

Table 4.44: Descriptive statistics of BAAA yield return series

	T	mean	standard deviation	median	min	max	skewness	kurtosis
BAAA	2500	0,000000	0,010000	0,000000	-0,020000	0,020000	0,060000	-0,010000

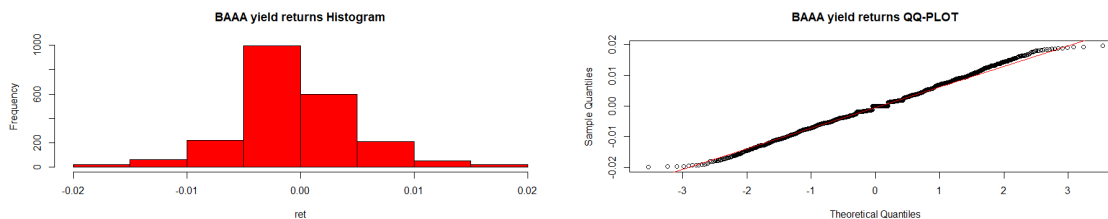


Figure 4.6: Histogram and Quantile-Quantile plot of BAAA yield return series

Normality test

BAAA yield return series show that the Jarque-Bera probability value in Table 4.45 is greater than 5%. Thus, we do not reject the null hypothesis at a 5% significance level and conclude that BAAA yield return series follow a normal distribution. Supporting the conclusion made from Figure 4.6, the QQ-plot of BAAA yield returns.

Table 4.45: Normality tests of BAAA yield return series

	Jarque-Bera (JB)		Shapiro-Wilk test	
	JB	p-value	W	p-value
BAAA	1,6097000	0,447100	0,996930	0,000064

Stationarity test

Formal test for stationarity of the BAAA yield returns are conducted by using ADF and PP tests. The results obtained for both tests are presented in Table 4.46. Both hypothesis tests rejected non-stationarity of BAAA yield return series at 5% significance level. We can therefor conclude that the yield return series of BAAA is stationary.

Table 4.46: Stationarity tests of BAAA yield return series

	Augmented Dickey-Fuller (ADF)	Phillips Perron (PP)
p-value	0,010000	0,010000

ARCH effects

From the results presented in Table 4.47, we found that no serial autocorrelation exists for the BAAA yield returns since we fail to reject the null hypothesis at a 5% significance level. However, BAAA yield return series are heteroscedastic since the p-value obtained for the LM test was less than 0,05 hence we reject the null hypothesis of no presence of ARCH effects.

Table 4.47: ARCH effects test of BAAA yield return series

	Ljung-Box Test		LM Arch test	
Return series	Q	p-value	LM	p-value
BAAA	16,916000	0,076250	1213,100000	0,000000

GARCH model selection process

The output obtained for BAAA yield returns displayed in Table 4.48 are largely similar to the simulated returns of Scenario 1. In other words, the yield return series have small $\hat{\alpha}_1$ values with large $\hat{\beta}_1$ values with varying sample sizes. Also, according to Table 4.48, the BIC model selection criterion is consistent in producing the smallest values for the Normal distribution. This is also the case for the simulated return series of Scenario 1. Table 4.48 show that the time series stationarity condition holds for all three distributions, i.e. $\hat{\alpha}_1 + \hat{\beta}_1 < 1$. Using the BIC criterion as our key model selection measure, the GARCH (1,1) with Normal innovations is identified as the best model to describe the yield returns of BAAA. However, if we only consider the GARCH (1,1) with Normal, Student-t and GED innovations at sample size 200 the inappropriate distribution for the error terms would have been assumed, that is the GED. This contradicts what we see in Table 4.48. Thus, the Normal distribution consistently produced the smallest BIC values among the three distributions. This is model misspecification since returns that are normally distributed cannot be best described by a GED. Moreover, in Table 4.48 results the Student-t distribution performed poorly in comparison to the Normal and GED, producing the highest BIC values.

In Table 4.49, we observe that the risk measure values within each sample size are relatively close to each other. Similar to Table 4.48, we see that there is inconsis-

tency as to which distribution produced the lowest accuracy measures throughout the sample sizes. If we again were to base our model selection solely on the accuracy measures only consider $T = 100$. The GARCH (1,1) with Student-t innovations would have been identified as the optimal model since it produced the lowest MSE (0,00003594) and RMSE (0,00557675) values for sample size 100. However, from the analysis performed the GARCH (1,1) model with Normal error terms was identified as the model that best describes the return series of BAAA. Hence, model misspecification would have occurred because the Student-t distribution does not capture all the distributional properties of the Normal distribution.

Table 4.48: Empirical results:BAAA yield return series

<i>Distribution</i>	<i>T</i>	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_1 + \hat{\beta}_1 < 1$	Log likelihood	AIC	BIC
Normal	10	0,00000 (NaN)	0,00000 (NaN)	0,959330 (NaN)	0,959330	35,031900	-	-
	50	0,00000 (NaN)	0,00000 (NaN)	0,988200 (NaN)	0,988200	193,306600	-7,572265	-7,419303
	100	0,000004 (0,000010)	0,024159 (0,057990)	0,843100 (0,319000)	0,867259	369,927800	-7,318555	-7,214348
	200	0,000003 (0,000005)	0,016135 (0,032830)	0,883390 (0,181100)	0,899525	754,229100	-7,502291	-7,436325
	500	0,000011 (0,000006) +	0,075276 (0,043600) -	0,624210 (0,167200) +	0,699486	1839,462000	-7,341848	-7,308131
	1000	0,000006 (0,000003) -	0,098018 (0,032660) +	0,756090 (0,098110) +	0,854108	3691,923000	-7,375847	-7,356216
	2000	0,000000 (0,000000) +	0,031954 (0,007505) +	0,957820 (0,010470) +	0,989774	7150,957000	-7,146957	-7,135755
Student-t	2500	0,000000 (0,000000) +	0,031207 (0,006431) +	0,959840 (0,008787) +	0,991047	8899,864000	-7,116691	-7,107373
	10	0,000049 (0,000061)	0,000000 (NaN)	0,162500 (0,714300)	0,162500	34,798390	-	-
	50	0,000000 (NaN)	0,000000 (NaN)	0,988250 (NaN)	0,988250	194,169100	-7,566764	-7,375561
	100	0,000005 (0,000019)	0,034151 (0,083060)	0,807130 (0,580800)	0,841281	370,585000	-7,311699	-7,181441
	200	0,000002 (0,000005)	0,020316 (0,037780)	0,897630 (0,187000) +	0,917946	756,935700	-7,519357	-7,436900
	500	0,000010 (0,000005) -	0,096872 (0,053320) -	0,639500 (0,158800) +	0,736372	1839,028000	-7,336113	-7,293967
	1000	0,000005 (0,000003) -	0,113670 (0,039410) +	0,754230 (0,103500) +	0,867900	3691,795000	-7,373591	-7,349052
GED	2000	0,000000 (0,000000) +	0,039522 (0,009806)	0,954130 (0,012030) +	0,993652	7141,531000	-7,136531	-7,122528
	2500	0,000000 (0,000000) -	0,037729 (0,008185) +	0,957400 (0,009813) +	0,995129	8887,543000	-7,106034	-7,094386
	10	0,000042 (0,000053)	0,000000 (NaN)	0,232200 (0,649400)	0,232200	34,931740	-	-
	50	0,000000 (NaN)	0,000000 (NaN)	0,988330 (NaN)	0,988330	194,451900	-7,578077	-7,386875
	100	0,000006 (0,000033)	0,027240 (0,098400)	0,802510 (0,980500)	0,829750	371,249400	-7,324988	-7,194729
	200	0,000003 (0,000006)	0,015561 (0,038730)	0,896850 (0,221000) +	0,912411	757,957500	-7,529575	-7,447117
	500	0,000011 (0,000006) -	0,082396 (0,048410) -	0,630780 (0,169500) +	0,713176	1840,658000	-7,342633	-7,300487
	1000	0,000005 (0,000003) -	0,101750 (0,035320) +	0,754930 (0,102700) +	0,856680	3693,603000	-7,377205	-7,352666
	2000	0,000000 (0,000000) +	0,032360 (0,007692) +	0,957520 (0,010640) +	0,989880	7151,084000	-7,132082	-7,132082
	2500	0,000000 (0,000000) +	0,031569 (0,006580) +	0,959600 (0,008915) +	0,991169	8900,022000	-7,116018	-7,104370

Notes: This table presents the estimates of BAAA returns fitted by the GARCH (1,1) model with Normal,

Student-t and GED. Values in the parentheses are the corresponding standard error. "+", "-", "." denotes significance at 1%, 5% and 10% levels respectively.

Table 4.49: BAAA yield return series Accuracy measures and Risk measures results

<i>Distribution</i>	<i>T</i>	Accuracy measures $\times 10^{-2}$		Risk measures	
		MSE	RMSE	VaR	ES
Normal	10	0,005438	0,737429	0,009759	0,012238
Student-t		0,005430	0,736889	0,008248	0,010344
GED		0,005430	0,736889	0,008553	0,010726
Normal	50	0,002695	0,519117	0,006357	0,007972
Student-t		0,002698	0,519400	0,006367	0,007985
GED		0,002697	0,519313	0,006379	0,008000
Normal	100	0,003595	0,599556	0,011051	0,013859
Student-t		0,003594	0,599523	0,011336	0,014216
GED		0,003600	0,600018	0,011435	0,014340
Normal	200	0,003110	0,557675	0,008160	0,010233
Student-t		0,003111	0,557747	0,008116	0,010178
GED		0,003118	0,558353	0,008240	0,010333
Normal	500	0,003768	0,613808	0,011492	0,014412
Student-t		0,003768	0,613815	0,011920	0,014949
GED		0,003768	0,613827	0,011617	0,014568
Normal	1000	0,003755	0,612806	0,016127	0,020224
Student-t		0,003756	0,612829	0,016604	0,020822
GED		0,003755	0,612803	0,016224	0,020346
Normal	2000	0,004764	0,690198	0,011634	0,014590
Student-t		0,004764	0,690215	0,011955	0,014992
GED		0,004764	0,690195	0,011638	0,014594
Normal	2500	0,004927	0,701923	0,014382	0,018036
Student-t		0,004927	0,701919	0,014963	0,018764
GED		0,004927	0,701920	0,014298	0,018055

Notes: The bold values represent the lowest MSE and RMSE values paired with the highest VaR and ES values for the particular sample size.

4.4 Chapter Summary

Chapter 4 presented the results obtained for the three simulated scenarios and three real datasets after application of the methods discussed in Chapter 3. We considered the GARCH (1,1) with Normal, Student-t and GED innovations and varying sample sizes. We conducted a series of simulations for the GARCH (1,1) model assuming the error terms follow a Normal distribution. We found that when the true innovation distribution is Normal, the GARCH (1,1) model with GED produces similar results to the true model innovation, particularly for large sample sizes. The results revealed that if the innovation assumed does not follow a similar distribution to that of the return series model misspecification will be present.

The return series of Samsung Electronics daily stock prices, Bitcoin-USD daily cryptocurrency and Moody's seasoned Aaa corporate bond yield (BAAA) were fitted to the GARCH (1,1) with Normal, Student-t and GED innovations. Overall results revealed that these models are subject to model misspecification if the error terms do not assume similar distributions as real data.

We found for both the simulated and empirical results, that one cannot identify the presence of model misspecification by merely looking at the accuracy measures and risk measures. This emphasizes the importance of the Box-Jenkins five-step process for identifying, selecting, and assessing models. The steps given by [Box et al. \(1994\)](#) we applied as follows:

1. The stationarity of the time series were established by use of the ADF and PP tests.
2. The GARCH(1,1) model with Normal, Student-t and GED innovations was identified as the stationary models for the data.
3. The model parameters were estimated by use of the MLE. The model was specified by identifying the significant parameters and applying the AIC and BIC model selection criteria.

4. Goodness-of-fit checks were conducted to ensure the model describes the data adequately using the log-likelihood.
5. After choosing a model and checking its fit, model accuracy was examined using the accuracy measures (MSE and RMSE) and risk measures (VaR and ES).

The results revealed that if the model chosen does not describe your data adequately, model misspecification will certainly be present. This in turn will result in inaccurate predictions of the risk measures. Presented in the next chapter are the conclusions and recommendations of the study.

5. Conclusions and Recommendations

5.1 Introduction

Financial models which are potentially not well-suited to account for all non-stationary returns of the asset prices is a great concern. This study evaluated the impact of model misspecification in Financial Risk Management with the primary focus being on market risk. The outline of this chapter is discussed as follows: We discuss how the secondary objectives were achieved in Section 5.2. This follows with a discuss on the limitations of the study and recommendation for future studies in Section 5.3.

5.2 Objectives and Conclusions

The primary objective of this research was to evaluate model misspecification during the model building process under market risk modelling. In order to help achieve this objective we formulated secondary objectives. A discussion on the achievement of these objectives follows.

5.2.1 Objective 1: Explore the GARCH model with error terms assuming Normal, Student-t and Generalized Error distributions

Exploration of the GARCH (1,1) model with Normal, Student-t and GED distributed innovations were done using simulations. We simulated several GARCH (1,1) models

varying the distribution of the error term from Normal, Student-t and GED. The return series simulated from the GARCH (1,1) model was set to be Normally distributed. In order for us to determine how the simulated GARCH (1,1) model with Normal distribution behaves we varied the initial parameters and performed simulations from different sample sizes. We concluded that the GARCH (1,1) with Normal distributed error terms as the most appropriate model for the simulated scenarios. We expected this outcome since the return series were simulated using Normally distributed innovations. Furthermore, we also conclude that the GED outperforms the Student-t distribution when considering estimation efficiency, goodness of fit and the model selection criteria of the simulated returns.

We also considered three real datasets, namely: Samsung electronics daily stock prices, Bitcoin-USD daily cryptocurrency and Moody's seasoned Aaa corporate bond yield (BAAA). To assess the GARCH models we calculated the return series of each dataset. The return series were tested for normality and only the BAAA yield returns followed a Normal distribution. The distribution of Bitcoin return series had heavy tails (leptokurtic) and the distribution of Samsung returns had thinner tails than the Normal distribution (platykurtic). In the paper of Gyamerah (2019) where the volatility of Bitcoin returns were evaluated, they also found Bitcoin return series to be skewed and heavy tailed.

5.2.2 Objective 2: Identify several model performance methods as well as performance measures, to help us in determining the possible impact of model misspecification on the models' selection and performance

We identified the log-likelihood as our goodness-of fit measure and used the AIC and BIC for model selection. The summary findings of the simulations gave the following. The model misspecification for GARCH (1,1) model at sample size 50 with

Student-t distribution is found to be appropriate in **Scenario 1 example**. This is mainly because it was inconsistent for the analyst decision. In **Scenario 2 example**, we concluded that the inconsistency of the model selection criteria namely the AIC and BIC led to model misspecification for the GARCH (1,1) model at sample size 200 with an inappropriate GED. Model misspecification was also found in **Scenario 3 example** with an analyst identifying an inappropriate model, the GARCH (1,1) with GED innovations at sample size 500, as the model that best describes the return series.

The GARCH (1,1) with Normal, Student-t and GED innovations and varying sample sizes were fitted to each real datasets' return series. The BIC was used as the key model selection measure. Samsung electronics optimal model identified was the GARCH (1,1) with GED. Bitcoin return series was best described by the GARCH (1,1) with Student-t. However, the time series stationarity condition did not hold for sample sizes larger than 500 indicating the presence of Model misspecification. The GARCH (1,1) model with Normally distributed error terms was identified as the optimal model for the BAAA return series. The analysis on the results showed that the distribution of the return series influences the decision of which innovation is most appropriate to use.

5.2.3 Objective 3: Evaluate how violation of model assumptions impact the prediction accuracy and model performance

Evaluation on the models prediction accuracy and performance in the presence of model misspecification was done using the accuracy measures (MSE and RMSE) and risk measures (VaR and ES). The analysis of the accuracy and risk measures revealed the importance of goodness-of-fit checks to ensure the model describes your data adequately. The results showed that the best GARCH model can still be subjected

to model misspecification. Hence one should ensure that the data has a distribution similar to the distribution assumed by the models error terms. We conclude that if the model chosen does not describe your data adequately, model misspecification will certainly be present. This in turn will result in the model under performing and producing inaccurate predictions

5.3 Limitations of the study and future studies

This study only focused on the GARCH (1,1) with Normal, Student-t and GED innovations. The study limited the order of the GARCH models and did not consider higher orders than the GARCH (1,1). We also did not consider other types of Generalized Auto Regressive Conditional Heteroskedastic models. Few examples of models to consider for future studies may include models such as the Asymmetric Power Generalized Auto Regressive Conditional Heteroskedastic (APARCH), the Component Generalized Auto Regressive Conditional Heteroskedastic (CGARCH), the Exponential Generalized Auto Regressive Conditional Heteroskedastic (EGARCH), the Fractional Integrated Generalized Auto Regressive Conditional Heteroskedastic (FIGARCH) and the Integrated Generalized Auto Regressive Conditional Heteroskedastic (IGARCH). The study limited the innovation of the simulated returns to being Normally distributed only. Hence, simulating returns assuming the error terms follows a Student-t, GED or any other distribution could also be considered with the latter GARCH type models for future studies. The study limited the distributions of the innovations to be Normal, Student-t and GED only. Therefore, future studies could consider other possible distributions for the innovations. This research could be expanded by taking non-parametric GARCH models into consideration since no distribution assumptions will be made similar to the work done by [Er and Fidan \(2013\)](#), and comparing the outcomes to that of the parametric GARCH models.

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Appendix A. QQ-plots of Simulated and Empirical returns

Table A.1: Scenario 1 QQ-plots

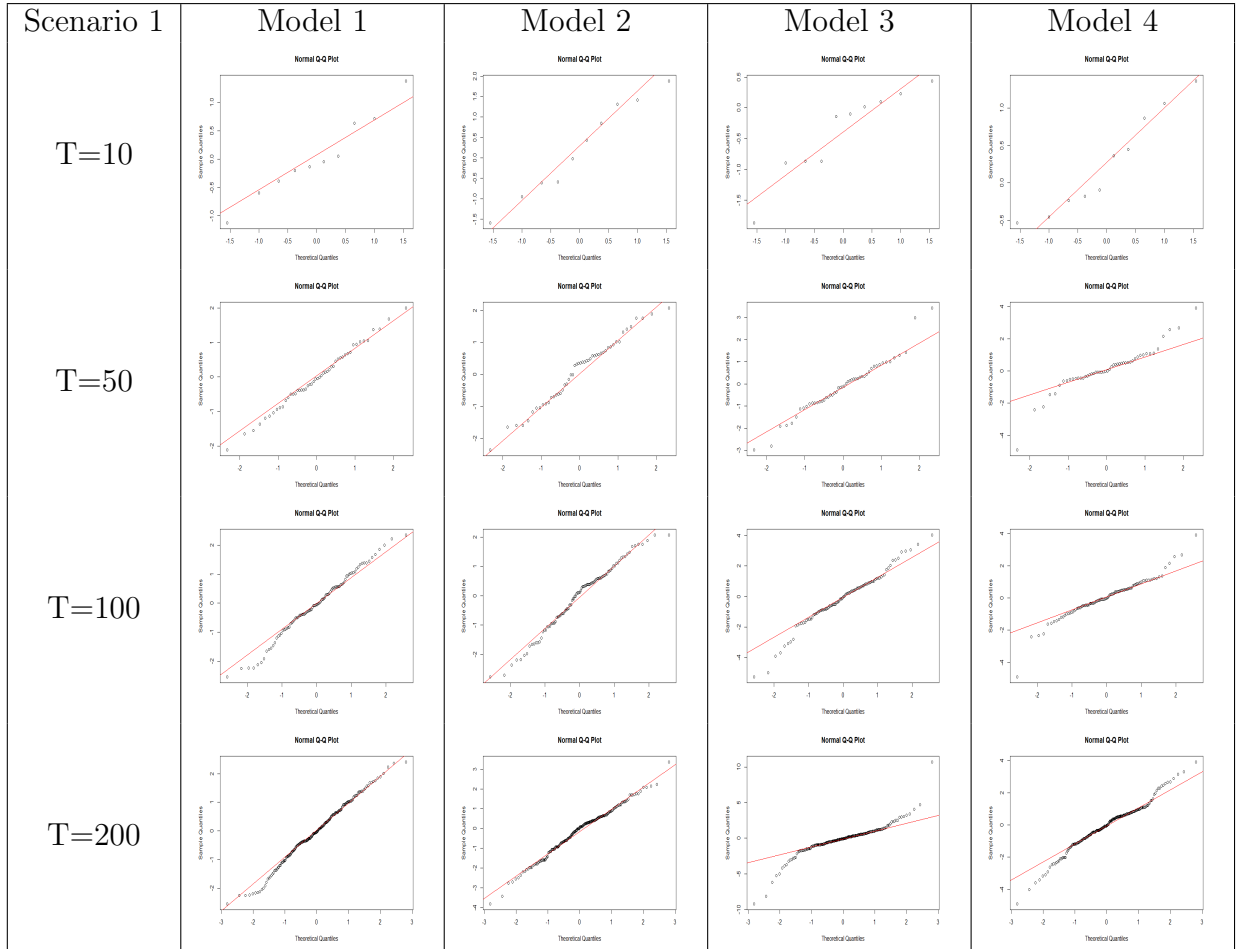


Table A.2: Scenario 1 QQ-plots (continue)

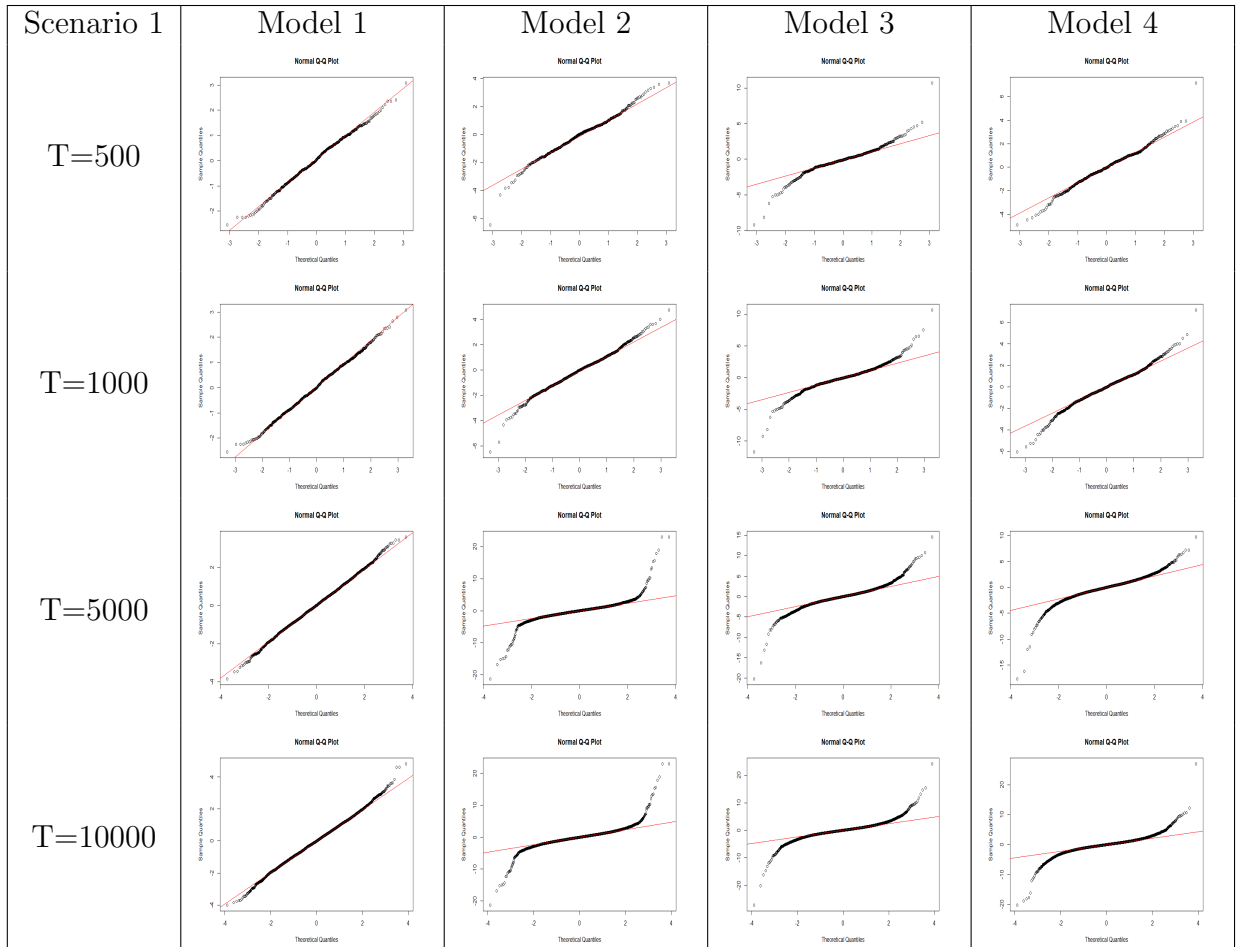


Table A.3: Scenario 2 QQ-plots

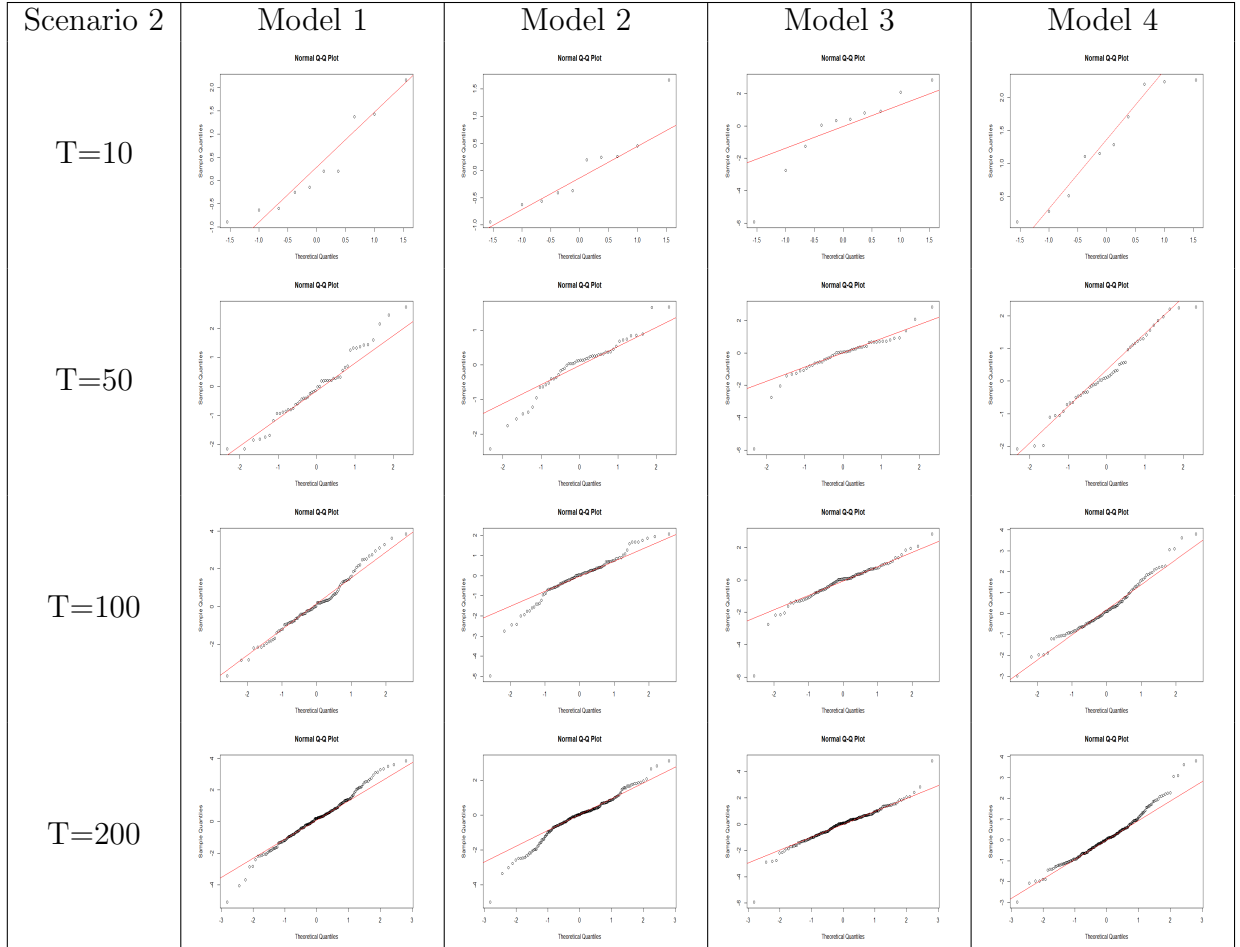


Table A.4: Scenario 2 QQ-plots (continue)

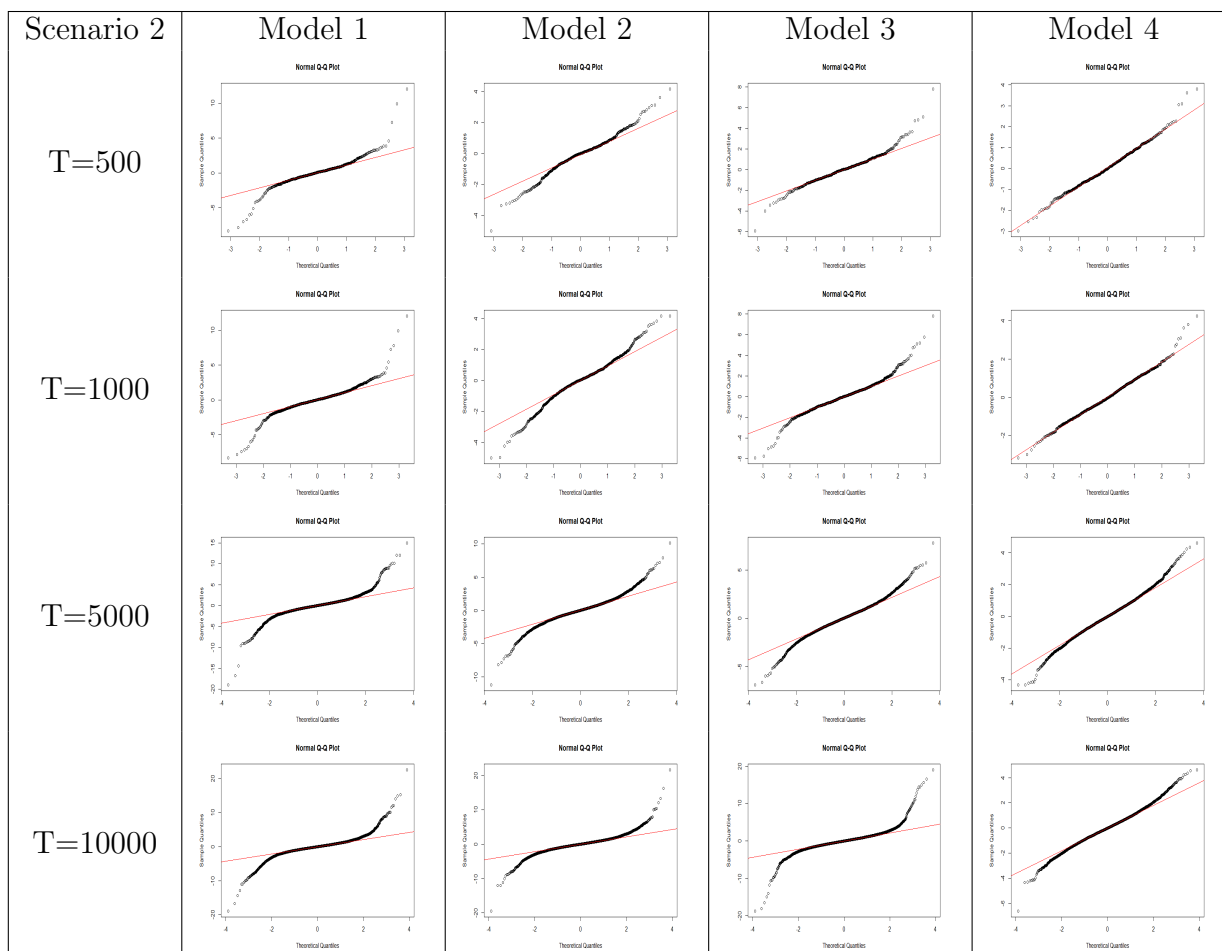


Table A.5: Scenario 3 QQ-plots

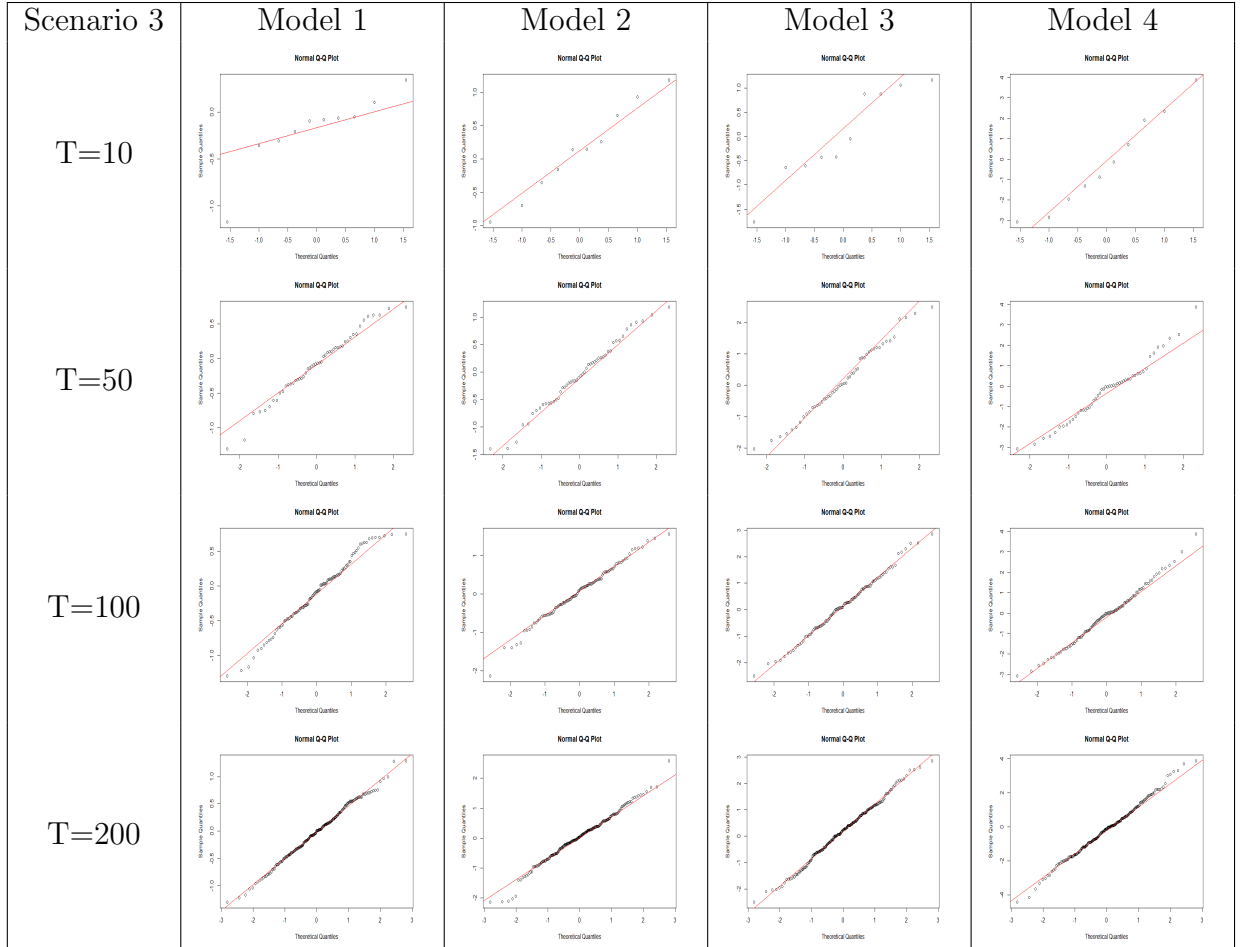


Table A.6: Scenario 3 QQ-plots (continue)

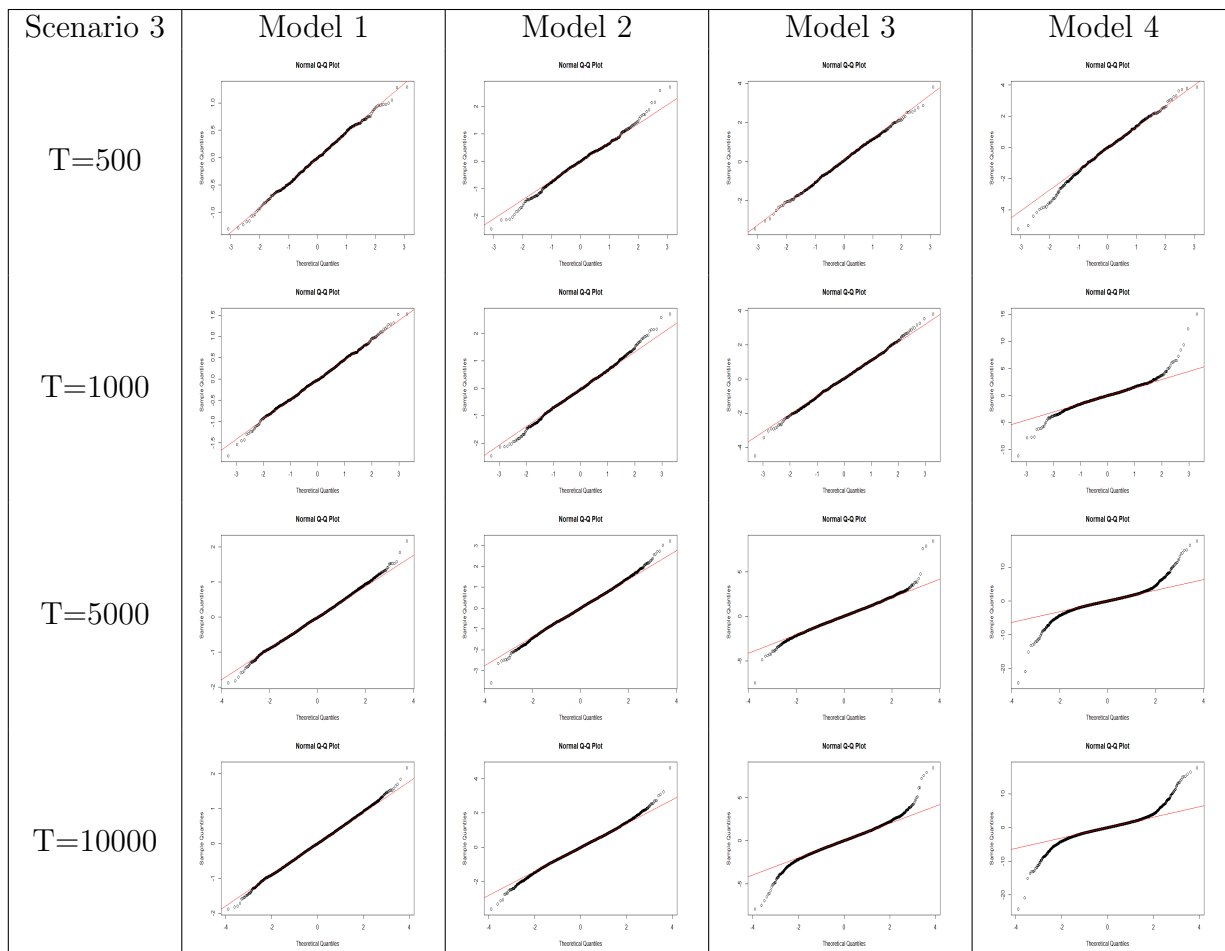


Table A.7: Empirical results QQ-plots

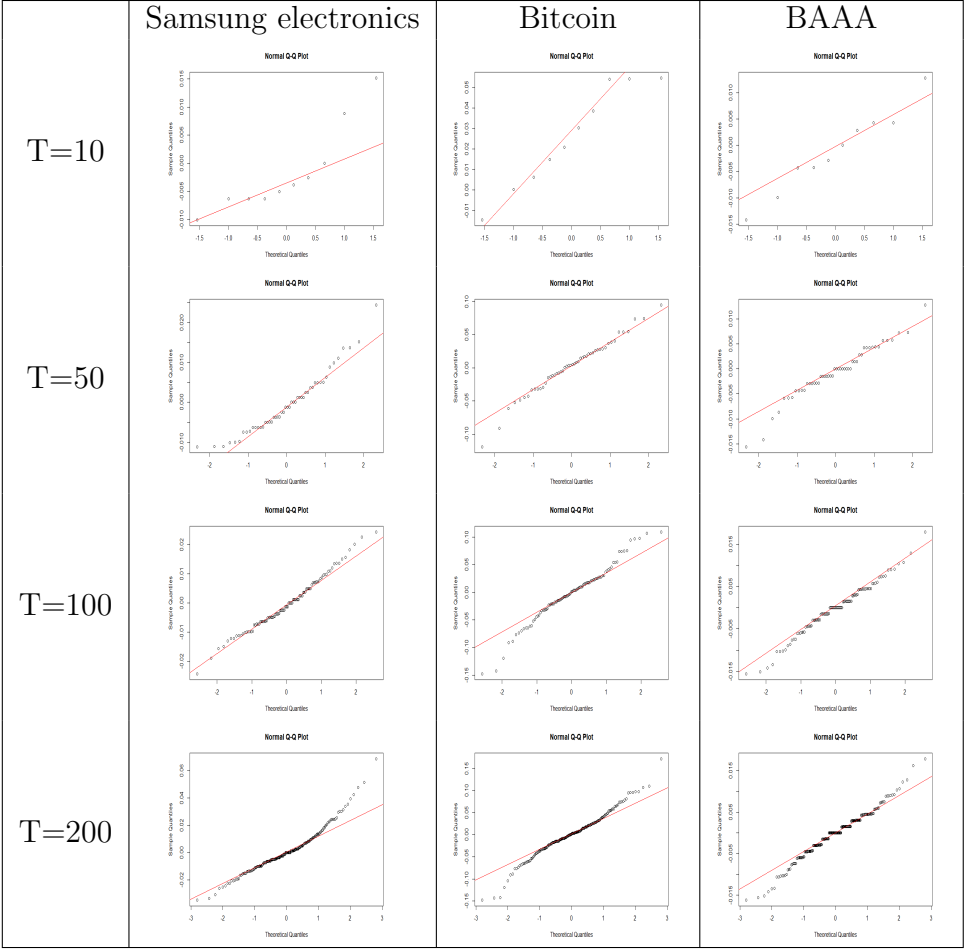
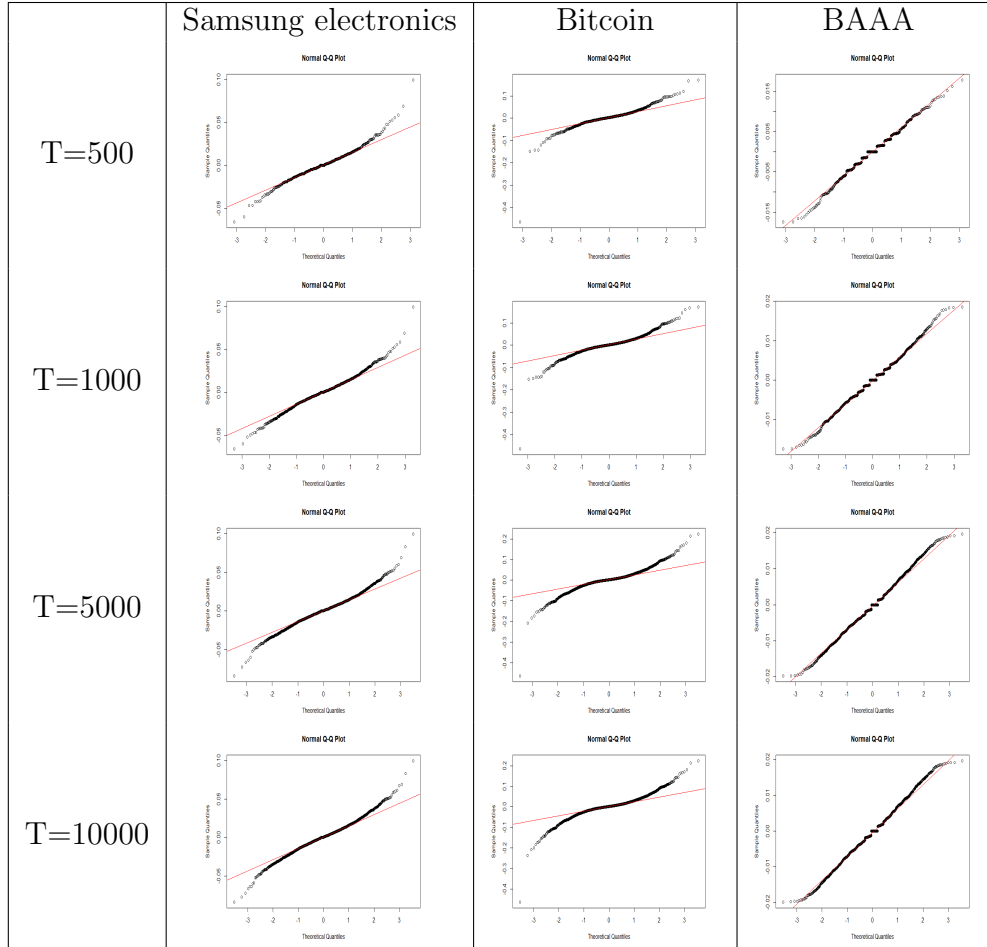


Table A.8: Empirical results QQ-plots (continue)



Appendix B. RCode

```
#Simulations for each scenario was done as follows:
#SCENARIO 1
#MODEL1
spec = garchSpec(model = list(omega = 0.1, alpha = 0.1, beta = 0.8))
s1m1= garchSim(spec, n = 10000)
#plot(s1m1)
write.csv(x = s1m1, file = "s1m1.csv", row.names = TRUE)
#Changes for each Scenarios model were made by
changing the initial values of the parameters omega, alpha and beta.

#n=50
x1m1 <- read.csv("s1m1.csv", colClasses = c("Date", NA)) # importing
the particular model of the scenario #
x1m1 <- x1m1[c(9951:10000),] #changes were implemented here
according to each sample size#
print(x1m1)

#plot of returns
ts <- xts(x1m1$garch, order.by = x1m1$time)
plot(ts, main = "Returns for T=50", xlab = "Date", ylab = "returns")

#qqplot of simulated returns
qqnorm(ts)
qqline(ts, col="red")
#DESCRIPTIVE STATS
describe(x1m1$garch) #changed x1m1 with the model 1,2,3 or 4 of Scenario 1,2,3#
```

#NORMALITY TEST

```
jarque.bera.test(x1m1$garch) #changed x1m1 with the model 1,2,3
or 4 of Scenario 1,2,3#
```

```
shapiro.test(as.vector(x1m1$garch[c(1:50)])) #changed x1m1 with the model 1,2,3
or 4 of Scenario 1,2,3#
```

#STATIONARITY TEST

```
options(warn=-1)
```

```
adf.test(x1m1$garch) #changed x1m1 with the model 1,2,3
or 4 of Scenario 1,2,3#
```

```
pp.test(x1m1$garch) #changed x1m1 with the model 1,2,3
or 4 of Scenario 1,2,3#
```

```
#fit simulation and real datasets to normal (ng), student-t (tg)
and GED (gg) innovations
```

```
ng <- garchFit(formula = ~ garch(1,1), data = x1m1$garch,
trace = TRUE) #Replaced
```

```
x1m1 with the model 1,2,3 or 4 of Scenario 1,2,3
for simulations or replace it with x for Samsung
and bitcoin or ret for BAAA #
```

```
# Following code of accuracy and risk measures were used for both
the simulations and real datasets
```

```
summary(ng)
```

```
#Standardize the residuals
```

```
ng_res = ng@residuals
```

```
ng_res
```

```

ng_sigma = ng@sigma.t#conditional standard deviation
ng_sigma

ng_st_res = ng_res/ng_sigma
ng_st_res

#rest = residuals(ng,standardize = TRUE)
#rest

#MODEL ACCURACY MEASURES
mse = mean(ng_res^2)
mse

rmse =sqrt(mse)
rmse

#RISK MEASURES
n=length(ng_st_res)
n
par = coef(ng)[4]
par
sigma_t = as.numeric(ng_sigma[n])
e_t = as.numeric( x1m1$garch[n]) #Replaced x1m1 with the model 1,2,3
or 4 of Scenario 1,2,3 for simulations or replace it
with x for Samsung and bitcoin or ret for BAAA #
sigma_t
e_t
fvariance=par*(sigma_t^2)+(1-par)*(e_t^2)

```

```

fvariance
ell=1
ell
vol_forecast = replicate(ell,0)
vol_forecast

k=c(1:ell)
k
sqrt_k = sqrt(k)
sqrt_k
vol_forecast=sqrt_k*sqrt(fvariance)
vol_forecast

p=0.05
VaR_k=qnorm(1-p)*vol_forecast
ES_k=dnorm(qnorm(1-p))/p*vol_forecast
invest = 1
Var_invest_k = invest*VaR_k
ES_invest_k=invest*ES_k
VaR_dat = data.frame(k,Var_invest_k)
ES_dat = data.frame(k,ES_invest_k)
VaR_dat
ES_dat

#STUDENT-T
tg <- garchFit(formula = ~ garch(1,1), data = x1m1$garch,
trace = TRUE, cond.dist = "std") #Replaced
  x1m1 with the model 1,2,3 or 4 of Scenario 1,2,3
  for simulations or replace it with x for Samsung

```

```

    and bitcoin or ret for BAAA #
summary(tg)
#Standardize the residuals
tg_res = residuals(tg)
tg_res
tg_sigma =tg@sigma.t #conditional standard deviation
tg_sigma
tg_st_res = tg_res/tg_sigma
tg_st_res

#MODEL ACCURACY MEASURES
mse = mean(tg_res^2)
mse

rmse =sqrt(mse)
rmse

#RISK MEASURES
n=length(tg_st_res)
par = coef(tg)[4]
par
sigma_t = as.numeric(tg_sigma[n])
e_t = as.numeric( x1m1$garch[n]) #Replaced
    x1m1 with the model 1,2,3 or 4 of Scenario
    1,2,3 for simulations or replace it with x for Samsung
    and bitcoin or ret for BAAA #

sigma_t
e_t

```

```

fvariance=par*(sigma_t^2)+(1-par)*(e_t^2)
fvariance
ell=1
ell
vol_forecast = replicate(ell,0)
vol_forecast

k=c(1:ell)
k
sqrt_k = sqrt(k)
sqrt_k
vol_forecast=sqrt_k*sqrt(fvariance)
vol_forecast

p=0.05
VaR_k=qnorm(1-p)*vol_forecast
ES_k=dnorm(qnorm(1-p))/p*vol_forecast
invest = 1
Var_invest_k = invest*VaR_k
ES_invest_k=invest*ES_k
VaR_dat = data.frame(k,Var_invest_k)
ES_dat = data.frame(k,ES_invest_k)
VaR_dat
ES_dat

#GED
gg <- garchFit(formula = ~ garch(1,1), data = x1m1$garch,
trace = TRUE, cond.dist = "ged") #Replaced
x1m1 with the model 1,2,3 or 4 of
  Scenario 1,2,3 for simulations or replace it with

```

```

x for Samsung and bitcoin or ret for BAAA #
summary(gg)

#Standardize the residuals
gg_res = residuals(gg)
gg_res
gg_sigma = gg@sigma.t #conditional standard deviation
gg_sigma
gg_st_res = gg_res/ng_sigma
gg_st_res

#MODEL ACCURACY MEASURES
mse = mean(gg_res^2)
mse

rmse =sqrt(mse)
rmse

#RISK MEASURES
n=length(gg_st_res)
par = coef(gg)[4]
par
sigma_t = as.numeric(gg_sigma[n])
e_t = as.numeric( x1m1$garch[n]) #Replaced
x1m1 with the model 1,2,3 or 4 of Scenario 1,2,3
for simulations or replace
it with x for Samsung and bitcoin or ret for BAAA #
sigma_t
e_t

```

```

fvariance=par*(sigma_t^2)+(1-par)*(e_t^2)
fvariance
ell=1
ell
vol_forecast = replicate(ell,0)
vol_forecast

k=c(1:ell)
k
sqrt_k = sqrt(k)
sqrt_k
vol_forecast=sqrt_k*sqrt(fvariance)
vol_forecast

p=0.05
VaR_k=qnorm(1-p)*vol_forecast
ES_k=dnorm(qnorm(1-p))/p*vol_forecast
invest = 1
Var_invest_k = invest*VaR_k
ES_invest_k=invest*ES_k
VaR_dat = data.frame(k,Var_invest_k)
ES_dat = data.frame(k,ES_invest_k)
VaR_dat
ES_dat
#Extraction of the real datasets
#Samsung
getSymbols("005930.KS",from="2011-05-25",
to="2021-07-31",src="yahoo",periodicity="daily")
#Bitcoin

```

```

getSymbols("BTC-USD",from="2014-09-17",
to="2021-07-31",src="yahoo",periodicity="daily")
#BAAA
F <- getSymbols("DAAA",from="1990-01-01",
to="2018-01-23",src="FRED")

#TRANSFORM PRICES INTO RETURNS : SAMSUNG
SAMr <- CalculateReturns(SAMT, method = c("log"))
SAM_returns <- na.omit(SAMr)
SAM_returns
length (SAM_returns)
#length= 2500
plot(SAM_returns, main="SAMSUNG Returns")

#TRANSFORM PRICES INTO RETURNS: BITCOIN
Busdr <- CalculateReturns(Busd, method = c("log"))
Bit_returns <- na.omit(Busdr)
length (Bit_returns)
Bit_returns
plot(Bit_returns, main="BITCOIN-USD Returns")
#TRANSFORM PRICES INTO RETURNS: BAAA
X = DAAA
length (X)
plot(X,main="BAAA Closing yields")
Xr <- diff(log(X), lag=1)
Xrr <- na.omit(Xr)
length (Xrr)
plot(Xrr,main="BAAA yield returns")

```

#Used same code for all three datasets with changes made as indicated.

```

#QQ-PLOT OF RETURNS
qqnorm(SAM_returns, main="SAMSUNG Returns QQ-PLOT")
qqline(SAM_returns, col="red", main="SAMSUNG Returns QQ-PLOT")

#HISTOGRAM OF RETURNS
ret_hist <- hist(SAM_returns, breaks=50,col='red',
main="SAMSUNG Returns Histogram")

#DESCRIPTIVE STATS
describe(SAM_returns)

#NORMALITY TEST
jarque.bera.test(SAM_returns)
shapiro.test(as.vector(SAM_returns[1:2500]))

#STATIONARITY TEST
options(warn=-1)
adf.test(SAM_returns)
pp.test(SAM_returns)

#ARCH TEST
Box.test(SAM_returns, lag = 10, type = "Ljung-Box")

Lm.test(SAM_returns,lag.max = 10,alpha = 0.05)
#arch.test(SAM_returns,arch = "box",alpha = 0.05)
#arch.test(SAM_returns,arch = "Lm",alpha = 0.05)

# Changed SAM_returns to Bit_returns for bitcoin data and to ret for BAAA

#EXTRACTING RETURNS

```

```

#n=10
getSymbols("005930.KS",from="2021-07-16",
to="2021-07-31",src="yahoo",periodicity="daily")
#extracting returns using the appropriate code for real
  dataset working with,
  using periods according to sample size
#'BTC-USD'
'005930.KS'
length('005930.KS')
x11 <- '005930.KS'[c(1:11),c(4)] #changed the column size
  to particular sample size working with
x11

rxx <- CalculateReturns(x11, method = c("log"))
rx = na.omit(rxx)
length(rx)

x <- rx[c(1:10),c(1)] #changed the column size to particular
sample size working with
length(x)
print(x)
plot(x)
#QQ-PLOT OF RETURNS
qqnorm(x)
qqline(x, col="red")

#fitting returns to the garch(1,1) models and
calculating the accuracy measures were done
using above code with changes implemented as made in the comments

```