

An investigation into the occurrence of ferroresonance in small distribution transformers in South Africa

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FOREWORD AND ACKNOWLEDGEMENTS

Ed Catmull, author of “Creativity, Inc.” and founder of Pixar describes scientific research in the following way: “When you embark on an experiment, you don’t know if you will achieve a breakthrough. Chances are, you won’t. Nevertheless, you may stumble on a piece of the puzzle along the way – a glimpse, if you will, into the unknown.” I hope and trust that this thesis will be one of the pieces of the ferroresonance puzzle.

The single greatest frustration of the two years of my Master’s study was an inability to replicate the results found in academic journal articles. It would often happen that I would read a source of literature, believe that it could be useful and then be unable to reproduce the results due to a lack of data or insufficient explanation.

I hope that this Master’s thesis will not be a standalone document that earns me a degree, but will be a stepping stone in finding solutions to some of the problems we have in South Africa. To this end, I would not want someone else to experience the same frustrations I did when they read through my work. I have, therefore, tried to go to extensive lengths to list my approach to the various problems, the assumptions I made and the reasons for my procedures. I have also tried to provide the reader with all the data that will be required to replicate my results with ease. I believe that if scientific research is to continue making progress, then the struggles endured to achieve certain results should not be repeated. Rather, researchers should be able to build upon each other’s work. This then sometimes requires an author to not only list his successes, but also his failed attempts. However, listing all of these struggles and approaches contributes to lengthy discussions. This is the number one factor in the length of my thesis. I hope that readers of this document will endure its discussions with an understanding of my intent.

I would like to acknowledge my shortcomings, which have been made evident in the last two years and give thanks to my Lord and Saviour who often provided me with breakthroughs when I was at my wit’s end. Nobel Prize winner, Daniel Kahneman, makes the following claim in his book *Thinking, Fast and Slow*: “...luck plays a large role in every story of success; it is almost always easy to identify a small change in the story that would have turned a remarkable achievement into a mediocre outcome.” I wholeheartedly agree with this statement, though I do not allocate small changes to luck. Rather, I believe that those small changes are controlled by God, as is also suggested in Proverbs 16:9. All credit for breakthroughs achieved in this study should be credited to God and not to me.

I would like to extend a special word of thanks to my family: to my father (Christiaan), mother (Erika) and brother Josua who supported me and endured my frustrated rants during Covid-19.

Also, to my brother Petrus, whose high standards have set the bar for me to follow. A special shoutout to his wife, Ciska, for comforting me when I thought I struggled more than my brother did.

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In an academic sense, I would like to express a great deal of gratitude towards Mrs. Anriëtte Pretorius of the NWU who regularly helped me to find publications. To Mr. Paddy Padayachee of Revive Electrical Transformers for providing me with the design data required for my study and a transformer that I could use for testing. To Dr. Henry Geldenhuys for his contributions in making this study possible. To the South African Institute of Electrical Engineers (SAIEE), and particularly Mrs. Gerda Geyer, for providing me with the funds to study for two years.

A special word of thanks to Mr. Reigh Walling, Dr. Jean Leite and Mr. Roman Szewczyk. It is my experience that authors of academic literature do not often respond to emails regarding their work. These gentlemen not only responded to my queries but were patient enough to help me achieve significant breakthroughs.

Finally, my most sincere gratitude to my study leader, Professor Jan de Kock for his invaluable contributions on a personal and academic level. For the last two years, I have gotten to know a new side of him that I never saw at the undergraduate level and which I greatly appreciate. I believe that Professor de Kock maintains one of the highest standards at the NWU Faculty of Engineering and I can only hope that this study does him proud.

ABSTRACT

In 2019, an article published by Geldenhuys and Padayachee reported unexpected overvoltages during the commissioning phase of distribution transformers. Consequently, resonance was reported as the cause of overvoltages on the secondary winding when the expected voltage was 0 V. Recent changes to the SANS 780 standard mandated a reduction in distribution transformer core losses, which made the overvoltages even worse.

Geldenhuys and Padayachee claim to have only scratched the surface of the problem. A hypothesis was developed that the unexpected overvoltages could be caused by ferroresonance and a research study was launched to investigate the resonance phenomenon found in 32 kVA, 22/0.4 kV distribution transformers complying with the new SANS 780 standard of 2019.

Ferroresonance is a nonlinear form of resonance involving a saturable inductor and a capacitor. Ferroresonance was first identified and described in 1914. Consequently, a vast amount of resources is available on the topic. Despite extensive studies in this field, ferroresonance is still poorly understood in the scientific community. This work provides a lengthy critical review of the literature on this topic.

The literature on ferroresonance proves that the initiation of ferroresonance is strongly dependent on a wide variety of factors. One of these is remnant flux. The remnant flux dependence makes the modelling of ferroresonance particularly difficult. Accurate ferroresonance studies require a detailed hysteresis model which can store remnant flux upon de-energization. Consequently this work also contains a detailed study on hysteresis, which could be a study all on its own. The most prominent hysteresis models from this study are the Jiles-Atherton, Preisach and Tellinen models. The Tellinen model is the most elementary of these models and its implementation is straightforward. The Preisach model is the most complex and also the most accurate. The Jiles-Atherton model is most often used in literature since it is less involved than the Preisach model while still retaining the ability to model minor loops and remnant flux. Chapter 3 develops Tellinen and Jiles-Atherton hysteresis models and replicates results found in the literature.

An important outcome in the study of ferroresonance proves that the π -transformer model is superior to the traditional T-model for the modelling of ferroresonance. Chapter 3 of this report replicates results found in literature and builds on the π -model to develop an accurate representation of the transformer being studied. The π -model is developed even further by considering not only the standard Open-Circuit (OC) and Short-Circuit (SC) transformer tests, but also the Sweep Frequency Response Analysis (SFRA) of the transformer. The SFRA allows the user to define a parallel capacitance across the magnetization branch of the transformer model,

which significantly improves its frequency behaviour up to 10 kHz. These capacitances are typically neglected for a lack of transformer design data. Chapter 3 also contains a detailed discussion on the use of peak vs Root Mean Squared (RMS) data points for simulation and the effect that high harmonic content can have on the modelling of a nonlinear inductor.

Despite replicating hysteretic results in the literature, neither the Tellinen model nor the Jiles-Atherton model could accurately represent the transformer being studied. High harmonic content during the OC test presented numerous hurdles which could not be overcome. Numerical errors limited the use of the Tellinen model, while parameter determination was problematic for the Jiles-Atherton model. This is despite the use of Particle Swarm Optimization (PSO) to determine the 5 parameters of the Jiles-Atherton model.

The developed π -model is used in Chapter 4 and Chapter 5 to conduct switching studies of the single-phase transformer in a power system. The overvoltages reported by Geldenhuys and Padayachee are replicated and independent studies show that the transformer is susceptible to ferroresonance. Bifurcation diagrams are used to investigate the effect of a variation in parameters on the ferroresonance of the transformer. However, simulations prove beyond doubt that ferroresonance is not the cause of the overvoltages in the power system. Two main reasons can be provided for this conclusion: 1) low line-to-ground voltage during single-phase switching and 2) low internal capacitance of the transformer.

Chapter 5 proceeds to discuss three different mitigation strategies to reduce the overvoltages initiated by single-phase switching. The report concludes by making recommendations for future improvements, such as alternative cost functions for the PSO algorithm. This will allow hysteresis and remnant flux to also be taken into consideration. Several appendices provide proofs of errors in literature, MATLAB code, simulation data points and an article presented by the author at the 2021 South African Universities Power Engineering Conference.

Keywords: magnetic saturation, ferroresonance, distribution transformers, wide frequency band transformer models, switching transients, residual flux, remnant flux, hysteresis, Jiles-Atherton, Tellinen

Research outputs: an article was published in the proceedings of the SAUPEC 2021 conference (Ferroresonance: A Review of the Phenomenon and Its Effects) (see APPENDIX G).

DECLARATION

I, Simeon Jedidja Kruger, declare that this dissertation is a presentation of my original work, conducted under the supervision of Prof J A de Kock.

Whenever contributions of others are involved, every effort is made to indicate this clearly, with due reference to the literature.

No part of this work has been submitted in the past or is being submitted, for a degree or examination at any other university or course.

Signed on this 13th day of February 2022.

.....

ACRONYMS

ANSI	American National Standards Institute
ASC	Arc Suppression Coil
ATP	Alternative Transients Program
CCVT	Coupling Capacitor Voltage Transformer
DC	Direct Current
DE	Differential Equation
DSF	Domain-Size Function
EHV	Extra High Voltage
EMTDC	Electromagnetic Transients Including DC
EMTP	Electromagnetic Transients Program
FFT	Fast Fourier Transform
FRA	Frequency Response Analysis
FSC	Ferroresonance Suppression Circuit
GIC	Geomagnetic Induced Currents
HV	High Voltage
KVL	Kirchhoff's Voltage Law
JA	Jiles-Atherton
LV	Low Voltage
MOV	Metal-Oxide Varistor
MV	Medium Voltage
MSE	Mean Squared Error

NGR	Neutral Grounding Resistor
N4L	Newtons4th Ltd
OC	Open-circuit
PTC	Positive Temperature Coefficient
RK	Runge-Kutta
RMS	Root-Mean-Square
PSO	Particle Swarm Optimisation
SC	Short-Circuit
SFRA	Sweep Frequency Response Analysis
STC	Saturable Transformer Component
VA	Volt-Ampère
VT	Voltage Transformer

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CHAPTER 1 INTRODUCTION TO RESEARCH

1.1 Problem Background

This study involves an investigation into the resonance of small distribution transformers. The transformers are rated at 32 kVA and steps the voltage down from 22 kV to 484 V.

The study was initiated by an article published by Geldenhuys and Padayachee [1] in October 2019. In this article, a resonance phenomenon is discussed where distribution transformers fail during the commissioning period of the transformers in the Eskom grid. It is hypothesized by Geldenhuys and Padayachee [1] that the resonance is initiated by drop-out fuses in one phase, which leads to resonance between the magnetization circuit of the transformer and the stray capacitance between the high voltage (HV) and low voltage (LV) windings of the transformer. When only a single phase of the grid is disconnected, unexpected high voltages are seen on the secondary side of the transformer. These voltages are caused by resonance at a frequency close to 50 Hz. Paired with the arcing of a partially open fuse link, the resonance is capable of damaging the insulation of the transformer [1].

Geldenhuys and Padayachee [1] furthermore point out that the effects of the resonance overvoltages are enhanced by reduced losses in transformer designs. This, in turn, increases the quality factor of the resonance. Recent changes to the SANS 780 [2] standard in South Africa makes low loss cores in distribution transformers a requirement. These changes are well-motivated by discussions provided by Short [3], where studies have proven that distribution transformers are the cause of 55% of distribution losses. Furthermore, 86% of these losses introduced by the distribution transformers are caused by no-load (iron) losses. A reduction of the core losses, therefore, poses significant economic advantages. Geldenhuys and Padayachee [1], however, show that the changes imposed by the 2019 SANS standard [2], as opposed to the 2009 standard, have increased the resonance overvoltages from 12 kV to 22 kV. The use of amorphous cores results in an overvoltage of 35 kV at the fundamental frequency of 50 Hz.

This study, therefore, aims to investigate this resonance in the distribution transformers in greater detail. It is focused on single-phase distribution transformers, which step the voltage down from medium voltage (MV) to LV.

It is important to note that the nature of the resonance within the windings is not yet fully understood. The study aims to develop a better understanding of this unknown. As such, ferroresonance cannot be excluded from the list of possible causes. For this reason, ferroresonance will be the topic of a detailed discussion in section 2.2. Since the nature of the

resonance has not been fully identified, all possibilities need to be investigated. Ferroresonance, being a special form of resonance, is particularly difficult to model and is also considered very likely.

It should also be noted that distribution transformer manufacturers have recently claimed to have solved this problem – the resonance phenomenon has become less prominent. However, it is suspected that their approach to the solution was based purely on a technical change without detailed theoretical knowledge of why the solution is effective. The study, therefore, not only aims to understand the phenomenon itself, but to provide sound engineering solutions to the problem.

Single-phase distribution transformers either have a shell-type or a core-type design [3]. In a core type design, the windings are wound around each leg of a rectangular core and the windings envelop the core [4]. In a shell-type transformer, the core envelopes the windings, where both the *HV* and *LV* windings are wound around a common leg of the core [4]. The transformer being investigated in this report is of the shell type.

Short [3] states that core type transformers require the interlacing of windings. Shell-type transformers do not have this requirement, and since this reduces the design complexity, non-interlaced windings are typically used for shell-type transformers. The interlacing of windings, however, reduces the reactance of the transformer by a factor of up to 4. Higher reactance can, therefore, be expected in the transformers of interest to this study.

The shell-type core of the transformer is obtained by placing two square cores together and winding the *HV* and *LV* windings around the centre leg. This means that the centre leg of the transformer core is twice as wide as the outer legs. The *LV* winding is wound closest to the core.

An important feature of the transformer is that the *LV* winding has a centre tap that is connected to ground. The rated voltage across the two outer terminals is 484 V. The grounding of the centre tap allows the transformer to feed two different distribution networks independently with a 180° phase shift.

1.2 Problem Statement

The above considerations lead to the definition of the following problem statement:

This study aims to investigate the resonance phenomenon found in 32 kVA, 22/0.484 kV distribution transformers complying with the new SANS 780 standard of 2019 [2]. The nature of the resonance needs to be identified and an accurate simulation of the problem needs to be developed. Following identification of the problem at hand, mitigation techniques need to be identified, which can mitigate the resonance and prevent transformer

failure. The study is conducted under the hypothesis that overvoltages are caused by ferroresonance.

1.3 Issues to be Addressed

From the problem statement and the problem background, it is clear that the first issue to be addressed is the theory of resonance. Since the nature of the resonance has not yet been identified, a thorough understanding of this phenomenon is required, this includes an understanding of ferroresonance, as the possibility exists that ferroresonance could be the cause of transformer failures. The possibility of ferroresonance is strongly motivated in section 2.2 since the conditions described by Geldenhuys and Padayachee [1] comply with all the requirements for ferroresonance.

The problem statement also makes it clear that there is a simulation requirement associated with this study. This simulation requires a high-frequency model of the transformer at hand. To this end, the standard T-model will not be sufficient, as is also pointed out by Geldenhuys and Padayachee [1]. The simulation model naturally needs to be an accurate depiction of the real transformer for frequencies up to at least 10 kHz.

The simulation specifically needs to address frequency-dependent components, which are typically ignored in steady-state analyses. This is, firstly, based on the fact that the reactance of inductors and capacitors changes with a change in frequency. This phenomenon may lead to resonance. More importantly, under the assumption that ferroresonance is the cause of transformer failure, the transformer core will enter into saturation. This implies that hysteresis will become important. Literature shows that hysteresis and eddy currents are critical to ferroresonance modelling. It is also shown in section 2.2.3 that ferroresonance is strongly dependent on remnant flux. It is, therefore, important that the simulation model not only simulate hysteresis accurately but also be capable of considering remnant flux. This will ensure that, should ferroresonance be the cause of transformer failure, the model will be sufficiently accurate.

The study also entails a practical component involving measurements. Firstly, these measurements are required for accurate modelling of the system. Such measurements will include the determination of core loss resistance, magnetizing reactance, winding resistance, transformer capacitance etc. Secondly, measurements will also be required for verification of model accuracy, where the output of the simulation should be similar to the output of the actual transformer. For this purpose, a sweep frequency response analysis (SFRA) of the transformer and the model will be valuable.

Two more issues, which need to be addressed, can be identified from the problem statement. The first is that the nature of the resonance needs to be identified. Assuming that the simulation model is sufficiently accurate, this should be possible by varying the parameters of the model to investigate their influence. The second requirement is that of a mitigation technique to be used to ensure that the resonance is avoided in future. This mitigation technique should be sensible. Resonance and ferroresonance mitigation techniques from literature are discussed in sections 2.1.3 and 2.5 respectively.

1.4 Project Scope

The scope of this report correlates to the issues to be addressed, which were mentioned in section 1.3. It should be noted that the scope of this report is limited to single-phase supplied transformers. Three-phase transformers are, therefore, outside the scope of this study. However, the literature study in Chapter 2 will not exclude literature on three-phase systems. Literature on this subject is generally focused on three-phase systems and can, therefore, provide a valuable understanding of the resonance phenomena.

The transformers in question are limited to small distribution transformers, where the rated load does not exceed 100 kVA. The transformer of interest for this report has a rated capacity of 32 kVA, but Geldenhuys and Padaychee [1] include transformers with a rated capacity of 100 kVA in their discussion. The size of these transformers, therefore, make them comparable to voltage transformers (VTs) and hence, VT discussions in literature are particularly relevant.

The study is conducted under the hypothesis that ferroresonance is the cause of the overvoltages, as suggested by the title of this report. Ferroresonance literature will, therefore, receive more emphasis in Chapter 2. It is also assumed that a thorough understanding of ferroresonance will ensure that one understands resonance in general.

Implementation of the proposed solutions and mitigation strategies are beyond the scope of this study. Implementation of the proposed solutions requires access to an Eskom network, which was not possible during the study.

1.5 Methodology

A broad overview of the methodology of this study is presented in Fig. 1-1. It should be noted that several review opportunities are presented in this methodology, as is also evident in the figure, these include ensuring that the hysteresis model is accurate, that the transformer model is functional and that the frequency response of the model matches that of the actual transformer.

The author would like to highlight step number 4, where the hysteresis model is developed. Based on the literature, hysteresis modelling is expected to be the most difficult aspect of the simulation, and therefore, receives a dedicated step.

Step 6 involves developing a high-frequency transformer model. This includes the simulation of eddy currents and the skin effect. Transformer models are discussed in section 2.6. Transformer modelling with emphasis on ferroresonance is discussed in section 2.7. The modelling of frequency-dependent phenomena, such as the skin effect, are discussed in section 2.11.

The simulations in this study will be done in the MATLAB® Simulink™ environment since hysteresis modelling will require the solution of differential equations (*DEs*), which are not standard in programs such as LTSpice®, and *EMTP* (Electromagnetic Transients Program).

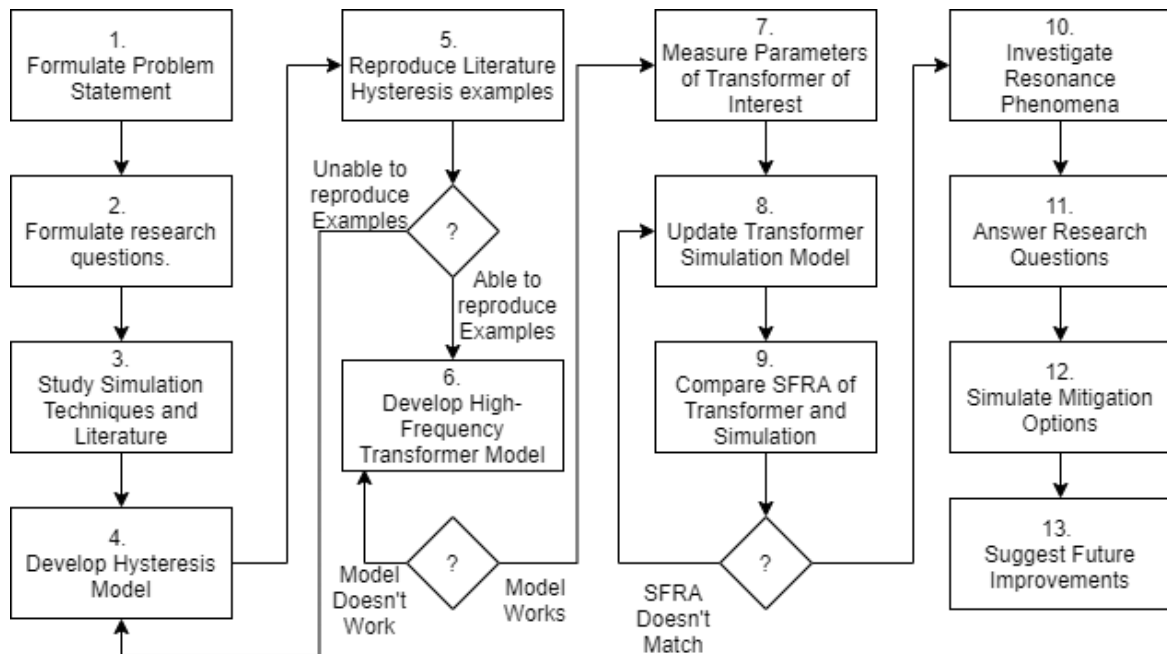


Fig. 1-1: The research methodology

1.6 Dissertation Outline

This thesis is divided into five chapters. Chapter 1 has discussed the problem, providing a thorough motivation of the need for the research, as well as a breakdown of the problem. The problem statement was summarized in section 1.2, while section 1.3 discussed several issues, which need to be addressed. While the issues were not phrased as questions, they do indicate the research questions, which need answering. Section 1.4 provided a more thorough breakdown of the scope of the project, detailing specific aspects being excluded from the study. A breakdown of the planned methodology was then presented in section 1.5.

Chapter 2 provides a detailed breakdown of the literature relevant to this study. The literature study starts with a fundamental discussion of resonance in section 2.1. Section 2.2 then discusses ferroresonance in great detail, since section 1.4 expressly stated that the assumption is made that ferroresonance is the cause of transformer failure. Following the ferroresonance discussion, a detailed breakdown of simulation models and techniques is presented in sections 2.7 through 2.13. These include discussions of transformer modelling, hysteresis, eddy currents, inrush current and the effects of capacitance.

Chapter 3 will focus on the transformer model being developed. This includes developing a transformer model from standard inductors resistors and capacitors, as well as the development of hysteresis models. The models developed in Chapter 3 are supported by the replication of results found in the literature. Chapter 3 will also reflect on the ability of the models to replicate measured results, such as the open-circuit and short-circuit test

Chapter 4 presents ferroresonance simulation using the model developed in Chapter 3. It also presents the modelling of switching transients within an actual power system, where the overvoltages measured by Geldenhuys and Padayachee [1] are replicated. Chapter 4 is mainly focused on simulation data and does not draw specific conclusions on the implications of the simulations.

Chapter 5 presents conclusions based on the simulations of Chapter 4. It answers the research question and reflects on the nature of the resonance. It also presents several mitigation strategies and their effects. It then concludes with recommendations for future work.

Several appendices are attached to the end of this report. These appendices include proofs of errors found in literature, MATLAB programs of hysteresis models, data points relative to simulations and a conference paper published and presented by the author at the South African Universities Power Engineering Conference in 2021.

CHAPTER 2: CRITICAL REVIEW OF THE LITERATURE

Considering the unclear definition of the cause of the resonance found in Chapter 1, there exists a possibility whereby ferroresonance could be initiated within the transformers. Ferroresonance, being a specific form of resonance, will be discussed in detail in this chapter. This detailed discussion will include the definition of ferroresonance, critical parameters upon which it is dependent and topologies that have proven to be particularly susceptible. While the author acknowledges that ferroresonance is not necessarily the cause of the transformer damage, it is assumed that a proper understanding of ferroresonance will result in a sufficient understanding of resonance as well. Furthermore, despite being defined in literature since 1914 [5], ferroresonance is a phenomenon that is both difficult to understand and to model. As such, in the case where the cause of transformer destruction is indeed ferroresonance, a proper understanding of this nonlinear phenomenon is essential.

It was found from the literature that ferroresonance is highly dependent on remnant flux. As such, the modelling of hysteresis is critical to the accurate modelling of ferroresonance. Hysteresis is, therefore, discussed in detail in this chapter, along with other frequency-dependent components, such as eddy currents, the skin effect and the proximity effect. Several hysteresis models are discussed and a comparison is drawn between these models.

In addition to the hysteresis models, it was found that the classical T-model used to simulate the steady-state operation of a transformer is not sufficient for switching transients. As such, several other transformer modelling approaches are also presented here, along with a guideline on how to model a distribution system.

2.1 Resonance

Cigre [6] introduces resonance by stating that it typically occurs in an electrical circuit that periodically transforms energy from an electric field to a magnetic field and vice versa. While this is not specific to transformers, a transformer within a distribution system falls within this category. The result is an oscillation with the so-called natural frequency of the circuit, given that either electrical or magnetic energy was delivered to the circuit. A switching-transient is a perfect example of the energy which is required to initiate this process. The oscillation of the circuit is a result of the exchange of energy between the capacitance and the inductance of the circuit [6].

With regards to the effects of resonance and ferroresonance, Cigre [6] points out that there are several drawbacks to these undesired occurrences, these include a threat to the integrity of the power system, surge arrester damage, unaccounted costs to electric power utilities, distorted and

harmonic waveforms and increased insulation ageing. Overvoltages can cause failures to the transformer and equipment connected to the transformer, e.g. bushings, cable terminations and surge arresters.

While not explicitly stated earlier, the interaction of the magnetic and electric energy required for resonance implies a requirement for both inductors and capacitors within a circuit. The transformer serves as the source of inductance, while distribution lines and underground cables and the transformer itself serve as sources of capacitance. This will be discussed in greater detail in section 2.14. This interaction between the inductance and capacitance constitutes the interchange between magnetic and electric energy, which leads to resonant oscillations as discussed earlier.

Cigre [6] explicitly points out that it is possible to obtain both series and parallel resonance, each with its distinctive characteristics. Elementary circuit diagrams for a parallel and a series resonant circuit are shown in Fig. 2-1 and Fig. 2-2 respectively. There is a frequency where the reactance of the inductor is equal to the reactance of the capacitor. Due to the nature of the equations which describe the reactance, and the laws of the complex numbering system, this implies that the reactances of each element will negate each other – effectively leaving only the resistance to limit the circuit current. The frequency where the two reactances become equal to each other is known as the resonant frequency. Provided that the parameters have equal values in both topologies, the resonant frequency will be the same and is given by (2-1).

$$f_n = \frac{1}{2\pi\sqrt{L \cdot C}} \quad (2-1)$$

Furthermore, a characteristic of parallel circuits is that the equivalent circuit impedance is generally low. However, since the reactance of the capacitor and inductor negate each other at resonance, the parallel connection is eliminated, which means the effective impedance seen by the source is increased. The total impedance of the system for a parallel circuit is, therefore, a maximum at resonance. The opposite is true for series resonant circuits.

Real systems, however, very rarely consist of connections that are purely series or purely parallel. They often consist of complex combinations of both of these. This leads to the possibility of the system resonating at two distinct frequencies. It becomes possible for the circuit to then have a resonant frequency for the series Thevenin equivalent where it is excited by a voltage source. In the same manner, it is also possible for the system to have a unique and different parallel resonant

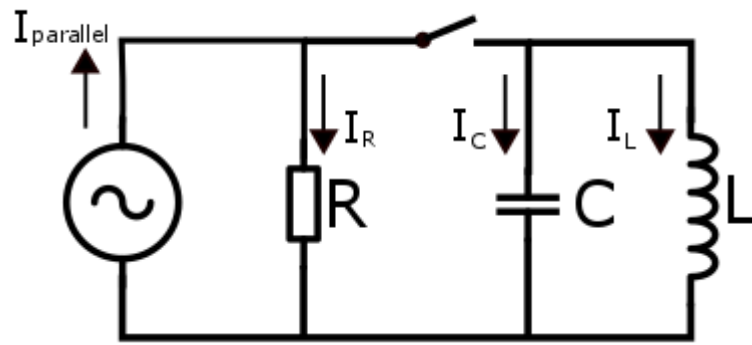


Fig. 2-1: A parallel resonant circuit

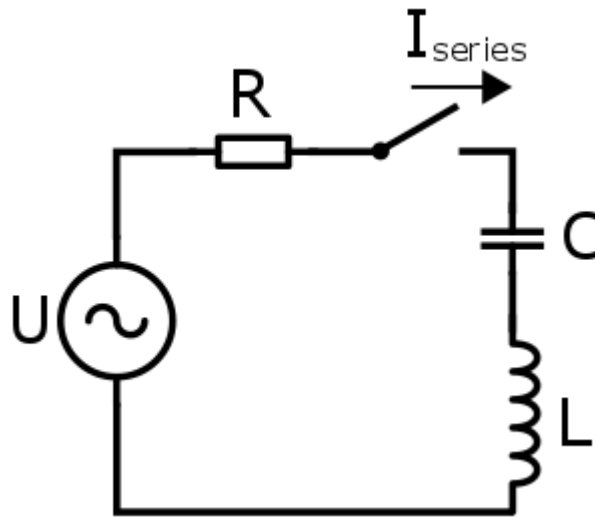


Fig. 2-2: A series resonant circuit

frequency, which can be excited by a current source. Cigre [6] presents an example of an elementary series and parallel combination, along with the equations which can be used to calculate the resonant frequencies. For now, it should suffice to take note of the fact that for a real system, there exists more than one unique resonant frequency. It is for this reason that Bagheri et al. [7] speaks about multiple resonance frequencies in various frequency ranges when referring to transformers. The winding resistance and leakage inductance of the transformer form a series circuit. The magnetization branch of the transformer and the shunt capacitance of transmission lines form parallel circuits. The combination of these elements in a power system, therefore, presents a complex circuit that can have multiple resonance frequencies.

Based on the discussions of [6] on the combination of series and parallel circuits, one might be tempted to conclude that a distribution system will have two distinct natural/resonant frequencies – one for the series equivalent circuit and one for the parallel equivalent circuit. However, it will also be found in section 2.11 that the impedance of a transformer is frequency-dependent due to effects such as eddy currents, the skin effect, the proximity effect and core saturation [8]. This

means that not only does there exist two resonant frequencies due to the combination of series and parallel connections, but these resonant frequencies can vary based on the frequency of the source voltage. Bagheri *et al.* [7] proceed to indicate which elements of the transformer tend to dominate the resonance behaviour in various frequency ranges. The low-frequency behaviour of the transformer is dominated by the self-inductance of the transformer windings. In the mid-frequency range, the mutual inductances of the transformer tend to play an important role. As the frequency continues to increase, however, the inductive reactance becomes large enough to be neglected and the capacitive reactance becomes increasingly important [7].

This introduction to resonance and ferroresonance presents a compelling argument as to why this study into the resonance phenomena is required. It is claimed by the authors of [6] that there are several factors, which are shifting the natural frequency of modern systems closer to the fundamental frequency of 50 Hz, these factors, as listed by [6], include:

- a High capacity corridors added to transfer large amounts of power;
- b New pylon designs for overhead transmission lines;
- c Smaller structures and reduced clearances above the ground;
- d Upgraded existing corridors for operation at higher voltages, and
- e Increase in the use of underground cable circuits.

Couple that with the findings of Geldenhuys and Padayachee [1], and it is clear that the recent change in the SANS 780 standard [2], as well as recent developments in the field of electrical engineering, is presenting resonance problems, which have not been encountered before. There is also a consensus in the literature that the reduction in core losses is particularly problematic for the initiation of ferroresonance, which is why ferroresonance is discussed in detail in section 2.2.

2.1.1 Quality Factor

Resonance is described through a parameter which is referred to as the quality factor. The quality factor of the distribution transformers tested in [1] is discussed by Geldenhuys and Padayachee, where it is explicitly pointed out that the reduction of core losses leads to an increased quality factor. The quality factor increases dramatically from the SANS 780-2009 standard to the SANS 780-2019 standard [1]. The quality factor is even more prominent for an amorphous core, as pointed out in [1, Fig. 7].

The definition of the quality factor is more clear in [5], where it is referred to by Baitch as an indication of the voltage magnitude across the capacitor and the inductor. The equation for the quality factor in the case of a simple series resonant circuit is given by (2-2)

$$K = \frac{L\omega}{R} = \frac{1}{RC\omega} \quad (2-2)$$

where:

- K = the quality factor;
- L = the equivalent inductance;
- ω = the angular frequency, and
- C = the equivalent capacitance.

The voltage, as a function of the quality factor, may be several orders of magnitude greater than the applied voltage [5]. The quality factor, therefore, serves as an indication of the factor by which the voltage magnitude has increased due to the energy oscillation between the capacitor and the inductor and the reduced equivalent impedance of the system.

2.1.2 Resonance Analysis

Transformer modelling and analysis are discussed in great detail in section 2.6. Furthermore, a detailed discussion of modelling for ferroresonance identification is presented in section 2.5. As such, resonance analysis will not be discussed in extensive detail here.

It is, however, worth noting that [9] is focused on the simulation of energization transients in power systems. Here it is explicitly pointed out that key factors for the analysis of resonant harmonic overvoltages include:

- a The resonant frequency of the network;
- b The system damping (network losses and load);
- c The voltage level at the transformer terminals;
- d The saturation characteristics and air-core inductance;
- e The residual flux in the transformer core, and
- f The closing time of the circuit breaker pole.

Factors, which contribute to a higher level of resonance overvoltage, according to [9], are:

- a Transformers with higher ratings being energized;
- b Lower values of source fault levels;
- c Longer circuit lengths;
- d Reduced load in the system;
- e Operating voltages above 1.0 p.u.;
- f Lower knee point voltage/flux designs, and
- g A variation of the turns ratio due to a change in the tap position.

2.1.3 Resonance Mitigation

Section 2.2 will discuss the mitigation of ferroresonance in more elaborate detail. Cigre [6] and Cigre [9] provide indications of how to mitigate resonance, but one should bear in mind that these discussions are focused on three-phase systems. Nevertheless, the theory behind the mitigation of three-phase resonance is worth mentioning and could potentially lead to a solution for the single-phase case. The guidelines provided by [6] and [9] are therefore summarized here.

Cigre [6] states that phase transposition, in certain cases, can assist in the reduction of resonant overvoltages. In three-phase systems, it is often found that de-energizing one of the phases can cause resonance and ferroresonance. Examples of this are found in [3], [10], [11], [12], [13] and [14]. Cigre [6] gives several examples of how one or two de-energized phases leads to resonance and ferroresonance in the transformer through the inter-phase capacitance. Staggered energization of the various phases can lead to the same result. With regards to the mitigation of these resonant cases, Cigre [6] provides several potential solutions. One should bear in mind, however, that these solutions are not exclusive to single-phase systems and may even include systems where more than one three-phase circuit is close to another. This is referred to as a multi-circuit right of way. Passive mitigation techniques suggested by [6, Table 4-5] include:

- a Avoiding shunt compensation degrees which lead to resonance;
- b Detuning zero-sequence resonances using neutral reactors;
- c Damping zero-sequence resonances using neutral resistors;
- d Ungrounding the neutral point of shunt reactors;
- e Selecting shunt reactors appropriately based on interphase magnetic coupling;
- f Circuit transposition, and
- g Avoiding the use of single-phase switching.

Shunt compensation degrees, as a rule of thumb, should be kept below 60%. Cigre [6] reports that high overvoltages can be experienced when there is a multi-circuit right of way and the shunt compensation exceeds this level. If multi-circuit rights of way are not being considered, then the shunt compensation should be kept below 70%.

Neutral reactors assist in adjusting the ratio of the positive-sequence to zero-sequence capacitance. This ratio is very important. In addition to adjusting this ratio, neutral reactors also assist in reducing secondary arc currents [15]. The minimum value of the neutral reactor should be such that the equivalent line-to-ground impedance becomes capacitive. Cigre [6], Knudsen [15] and Carlsson [16] present methods whereby the value of the neutral reactor inductance can be optimized. However, the equations used in these approaches are for three-phase systems.

While it is important to take note of this theory, the equations will not be discussed in further detail, since the scope of simulation is limited to single-phase systems.

Neutral grounding (earthing) resistors (*NGRs* or *NERs*) are used in what Shirakawa *et al.* [17] refer to as non-effectively grounded systems. These are typically for systems with nominal voltages from 11 kV to 154 kV¹. The purpose of the *NGR* includes suppressing fault currents, detection of relay operating current in single-line-to-ground faults and suppression of abnormal voltages [17]. Selkirk, Savostianik and Crawford [18] add that the *NGR* also limits point-of-fault damage, reduces the risk of an arc flash, provides continuity of service during ground faults and provides for sufficient current to detect ground faults. It should also be noted that Hajivar, Mortavazi and Saniei [19] specifically discuss the value of *NGRs* in the mitigation of the zero-sequence current components. Inrush currents are also transient conditions in power systems, which is why the use of an *NGR* can be considered for mitigation of resonance. This correlates with [6], which claims that the neutral resistor adds damping to the resonant circuit.

Point number (d) mentioned that resonance can be mitigated by ungrounding the neutral point of shunt reactors. This is a debatable mitigation technique. García *et al.* [20] report resonant overvoltages due to grounded systems. However, their employed solution did not involve ungrounding the system. Instead, it involved a high resistance grounding transformer. Furthermore, both [21] and [22] confirm that ungrounded systems lead to ferroresonance. Finally, [23] reports that the voltage oscillation magnitude was greater for shunt reactors with ungrounded neutrals than was the case for shunt reactors with grounded neutrals. Lee, Chen and Hsu [23] furthermore state that shunt reactors with grounded neutrals are more effective in eliminating re-ignition overvoltages. Based on these discussions and the danger of ferroresonance initiation, ungrounding the neutral point of shunt reactors should be avoided.

There is a difference between a neutral grounding reactor and a shunt reactor used to control voltage tip up on overhead lines. The neutral grounding reactor is typically used to connect the star point of a generator to ground. The shunt reactors connected to overhead lines compensate for overvoltages served by long lines during low-load periods. They may also have a neutral grounding reactor connecting their star point to ground. Cigre [6] does not differentiate between these two. However, based on the discussion, one must assume that Cigre [6] is referring to the shunt reactors used for voltage tip-up.

Shunt reactor selection based on the interphase magnetic coupling is beyond the scope of this report since magnetic coupling between phases does not exist for a single-phase transformer.

¹ Note that this classification includes power system voltage levels of interest for this report.

However, one should take note of this mitigation technique when dealing with three-phase systems.

Line transposition is an expensive mitigation technique but can be used to reduce the resonant peaks in power systems [24]. More specifically, the transposition of lines, which run parallel to each other, has interesting effects on resonance. Unenergized lines may form a resonant circuit due to coupling from nearby lines, which are energized. If only the unenergized line is transposed, the number of resonant peaks on the outer phases is reduced from three to two. However, when only the energized circuit is transposed, the resonant voltage levels are reduced considerably [24].

With regards to the avoidance of single-phase switching, this is a suggestion that is often made in literature e.g. [5], [10], [25], [26] and [27]. Three-phase switchgear, according to Dugan [25] includes reclosers and sectionalisers at the riser pole. Furthermore, while [11] and [12] do not explicitly recommend three-phase switchgear as mitigation options, they do point out that ferroresonance becomes highly likely when only a single phase is switched in a three-phase system. This serves as added motivation for the use of three-phase switchgear. However, the economic feasibility for this type of solution is a big concern, especially when dealing with smaller distribution generators [25]. Dugan [25] claims that when economic considerations prevent the use of three-phase switchgear, line fuses are inserted in series with generators, but that is not an ideal solution.

In addition to the solutions presented above, Cigre [6] also speaks about modifying circuit geometry to adjust the $\frac{C_0}{C_+}$ ratio, where C_0 represents the zero-sequence capacitance and C_+ represents the positive sequence capacitance. While zero-sequence components are not applicable to single-phase systems, it should be noted that the ratio of the zero-sequence capacitance to the positive sequence capacitance is the most critical parameter affecting resonance in shunt reactors [6].

Active mitigation techniques of resonance are also discussed in [6], these include:

- a Grounding switches, which by-pass shunt reactors upon line de-energization;
- b Installation of circuit-breakers that disconnect shunt reactors when the line is de-energized;
- c Shunt reactors with tapped windings and on-load tap changers;
- d Circuit breaker failure and pole discrepancy protection schemes, and
- e Fast fault clearance.

Based on all of these mitigation techniques, it is clear that shunt reactors play an incredibly important role in resonance mitigation.

Cigre [9] claims that an increase in the number of generators in the system leads to a higher resonance frequency. While this does not apply to distribution systems, the rationale behind the claim is valid. An increase in the number of generators leads to a decrease in the overall system inductance. This leads to a higher resonant frequency [9]. It can, therefore, be noted that decreased overall inductance in the distribution system will tend to shift the resonant frequency away from the fundamental frequency.

2.2 Ferroresonance

Ferroresonance is a special case of resonance, which involves a nonlinear inductor. This section of the literature study serves to discuss this phenomenon in more detail.² Ferroresonance is a strong contender for being the cause of the transformer damage. Since ferroresonance is a special form of resonance, it is argued that a thorough understanding of ferroresonance will enhance one's understanding of resonance as well.

It will also be evident from this section that ferroresonance is a complex phenomenon due to its nonlinear nature. It then becomes necessary to use more involved simulation techniques to obtain an accurate ferroresonance model. For instance, the simulation of ferroresonance requires a more detailed model of the magnetization characteristic of the transformer core than other cases of resonance. Piecewise linear models are not sufficient [28]. Under this assumption, and assuming that ferroresonance could be the cause of the transformer destruction, it makes sense to study ferroresonance in more detail and construct a transformer model which is capable of simulating ferroresonance, as this will most likely yield a more accurate and trustworthy model, even if it turns out that ferroresonance is not the cause of the failures.

The likelihood of ferroresonance can be strongly motivated based on the discussion provided by [29] and [30]. Walling, Barker, Compton and Zimmerman [30] claim that the critical capacitance required to sustain ferroresonance is directly related to the core loss of the transformer. Walling, Hartana and Ros [29] elaborate on this by stating that recent reports indicate ferroresonance in 15 kV transformer banks since distribution transformers have become more efficient. Recall that Section 1.1 referred to the SANS 780 standard of 2019, which dictates that core losses in distribution transformers need to be reduced. Geldenhuys and Padaychee [1] also discussed the capacitance between the *HV* and *LV* winding and pointed out that this constitutes the resonance

² At this stage the author would like to remind the reader that the nature of the resonance within the transformer is not yet defined.

path. Finally, Walling *et al.* [29] state that ferroresonance is reported for 15 kV voltage levels, where the transformers of interest for this study are at a level of 22 kV. It will be found in this literature study that an increase in voltage leads to a higher probability of ferroresonance. Hence, one can conclude that there is a strong likelihood of ferroresonance within the transformers of interest. This suspicion is further confirmed by [31], where they claim that ferroresonance in distribution systems usually occurs as a result of single-pole switching of one or two phases supplying single-phase or three-phase line-to-line connected transformers. The distribution system in question complies with this unfavourable switching topology.

2.2.1 Theory

As stated in the introductory paragraph of this section, ferroresonance involves a nonlinear inductor. Ferroresonance is also commonly referred to as “Neutral Instability” or “Voltage Displacement” by Cigre [6]. Baitch [5] claims that ferroresonance was described in the literature as early as 1914. Cigre [6] states that, in very simple terms, ferroresonance is a nonlinear oscillation caused by the interaction of an iron-core with a capacitance. The iron core is considered nonlinear because its inductance depends on the magnetic flux in the transformer. It is well-known that the graph of magnetic flux density versus magnetic field intensity (also known as the B-H curve) of a transformer has a linear and a saturated region. If the magnetic flux density in the iron core surpasses the knee point of the saturation curve, the inductance of the transformer changes. Based on the discussion of resonance in section 2.1, this would then imply that the resonance frequency of the overall system can also change.

Cigre [6] proceeds to state that when the core is driven into saturation and oscillates with the circuit capacitance, severe overvoltages can result in the system. In certain cases, overcurrents are also seen in the system, and if the energy supplied to the system is sufficient, it can overcome the small amount of damping present in the system. In such a situation, the oscillations may be sustained indefinitely.

Horak [10] phrases the definition of ferroresonance as a set of electric circuit conditions where an inductor oscillates in and out of its saturation region and where the oscillations are with the local capacitance. This concept of the inductor oscillating in and out of its saturation region was not explicitly pointed out by [6] but is key to a more detailed understanding of ferroresonance in further discussions. Horak [10] continues to add that sometimes ferroresonance also describes a system where a transformer does not technically enter saturation, but due to the cancelling of the voltage drops across the inductance and capacitance, there is an increase in current and voltages. It is the opinion of the author, however, that this constitutes the typical definition of resonance provided in section 2.1. Furthermore, based on definitions by various other sources, including [5], [8], [26],

[32], [33], [34], [35], and [36], ferroresonance requires the presence of a nonlinear inductance. The inductance of a transformer only becomes nonlinear at the knee point where saturation occurs and hence it can be concluded that the saturation of the transformer is a requirement for the resonance to be classified as ferroresonance. Kobet [32] states that the saturation voltage is important and claims that ferroresonance becomes more likely when the operating voltage approaches the saturation voltage.

The requirement of core saturation for the presence of ferroresonance can furthermore be motivated by the explanation provided by Short [3]. A simplified ferroresonant circuit, as presented by Short [3], is shown in Fig. 2-3. The core-loss resistance is represented by R_C and the magnetizing reactance of the transformer is represented by X_M . Short [3] explicitly represents the magnetizing reactance as a switched reactance. The same figure and discussion are also found in [37]. In the linear region of the transformer's magnetization curve (see Fig. 2-4), the magnetizing inductance of the transformer is high. Upon saturation, however, the inductance of the transformer is drastically reduced to the air-core inductance. The switch is, therefore, closed upon saturation and the charge, which was built up in the capacitor, is dumped into the system. This causes a voltage overshoot. When the transformer moves out of saturation, a charge is once again stored in the capacitor, but the polarity is reversed. This process is repeated every half cycle [3]. If, however, the losses in the circuit are large enough (core losses or resistive loads), the charge on the capacitor drains and ferroresonance is avoided. This explains why ferroresonance becomes increasingly likely with reduced core losses since most transformers have an average load of approximately 30% of transformer rating [3].

Walling *et al.* [38] describe the same process as discussed by Short [3]. They [38] state that the overvoltages appearing on the open-phases of a three-phase system are a result of transformer excitation current pulses, which charge the capacitance of the system. This charging of the equivalent capacitor is first in one polarity and then in the other. The excitation current pulses are caused by the saturation of the core, which in turn is caused by the charge being stored in the capacitor during the saturation pulses. This causes the resonant exchange of energy between the inductor and capacitor, as discussed in section 2.1. It is important to note, however, that the capacitor only discharges every time the core enters saturation. This has an effect on the voltage waveforms measured across the capacitor.

Tanggawelu *et al.* [8], Baitch [5], Erbay [11], Ferracci [26] and Kováč *et al.* [36] add to the requirements of ferroresonance by stating that it involves not only the interaction of the nonlinear inductance with the capacitance but also requires a voltage source and low losses. The presence of a voltage source is typically assumed. The requirement for low losses is particularly important when one considers that [6] claimed the oscillations will continue indefinitely if the supplied energy

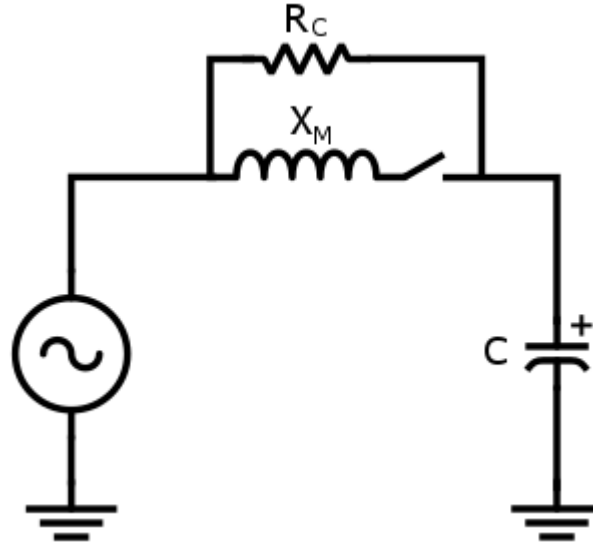


Fig. 2-3: The ferroresonant circuit as seen in [3, Fig. 4.26]

is sufficient to overcome the load. The supplied energy will only overcome the load if the losses are low. It, therefore, makes sense to also conclude that increased load on the secondary side of the transformer has the potential to mitigate ferroresonance, which is indeed a mitigation technique suggested by [13], [39] and [40]. Mitigation techniques will be discussed in more detail in section 2.5.

A typical example of a lightly loaded transformer, as given by Baitch [5], is that of a voltage transformer (VT). Since VTs are used for measurement, they very rarely have a large load connected to their secondary, which makes them prone to ferroresonance. The capacitance required for ferroresonance to take place can be provided by long transmission lines, capacitor banks and circuit breaker capacitances [5]. Horak [10], Price [13], and Moses and Masoum [33] confirm that circuit breaker grading capacitance can indeed be a source of capacitance for ferroresonance. This is a well-discussed topic in ferroresonance literature. However, circuit breaker capacitance only becomes relevant in systems exceeding 132 kV [41], which falls beyond the scope of this discussion. Therefore, circuit breaker capacitances will not be considered in this report.

When discussing the definition of ferroresonance, Moses and Masoum [33] claim that it is a phenomenon with a low probability of occurrence. However, the effects of ferroresonance can be catastrophic.

One of the main features of ferroresonance, according to Ferracci [26], is that there is more than one stable steady-state response for the circuit. It is for this reason that Erbay [11] refers to ferroresonance as a jump resonance. Jump resonance is defined as follows by Erbay [11]: “Jump

resonance refers to a condition in a sinusoidally excited system: if an incremental change in amplitude or frequency of the input to the system or in the magnitude of one of the parameters of the system causes a sudden jump in signal amplitude somewhere in the system, jump resonance is said to have occurred.” This jump resonance is characteristic of a transformer moving in and out of saturation.

To understand this jump phenomenon of ferroresonance, as well as the claim made by Ferracci [26] regarding the multiple stable steady states of the system, it is worth looking at the magnetization characteristic of a transformer and how it interacts with the system capacitance. Fig. 2-4 shows the typical anhysteretic curve as also depicted in [13, Fig. 1]. The anhysteretic curve is described by [42] as a graph of the magnetization (M) and the internal field (H) after a degaussing process. In effect, this means that the AC signal has decayed to zero. Jiles [43] states that the anhysteretic magnetization at a given field H correlates with the global minimum energy level for that specific field. In other words, the anhysteretic magnetization curve is the ideal curve in the case where no remnant magnetism is evident and no hysteresis takes place.

One will notice that figure Fig. 2-4 refers to the magnetizing inductance and the air core inductance. These are two important parameters, as they form the impedance of the system, which will play a role in resonance and ferroresonance. One should note that the air-core inductance is considerably lower than the magnetizing inductance, as is evident by the slopes of the two individual inductances. The air core inductance has approximately the same value as the leakage inductance of the transformer [3]. It should furthermore be noted that, although the axes represent voltage and current in Fig. 2-4, these can be interchanged with magnetic flux density B and magnetic field intensity H , which will then constitute the well-known B-H curve. This is based on the well-known relations between magnetic flux density and voltage, and magnetic field intensity and current as described in [4].

Price’s [13] definition of the jump phenomenon extends from the anhysteretic curve to its interaction with the system capacitance. Consider the circuit shown in Fig. 2-5. The circuit is very similar to the series resonant circuit seen in Fig. 2-2, but the inductance is specifically represented by a transformer. Also note the annotations X_C , X_M , V_S , V_C and V_M , which represent the capacitive impedance, inductive impedance, source voltage, capacitor voltage and transformer voltage respectively.

The interaction of the magnetization curve of the transformer with the source voltage and the capacitor voltage is shown in Fig. 2-6, which is also found in [13, Fig. 11]. Price [13] presents the best discussion of the important points regarding this magnetization curve and its interaction with

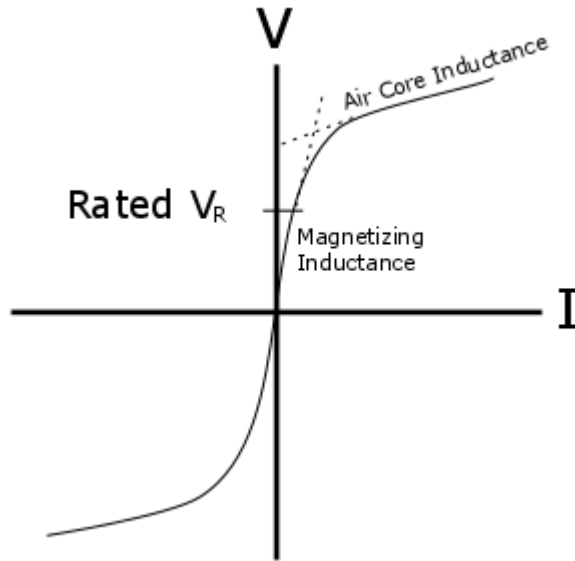


Fig. 2-4: The anhysteretic magnetization curve

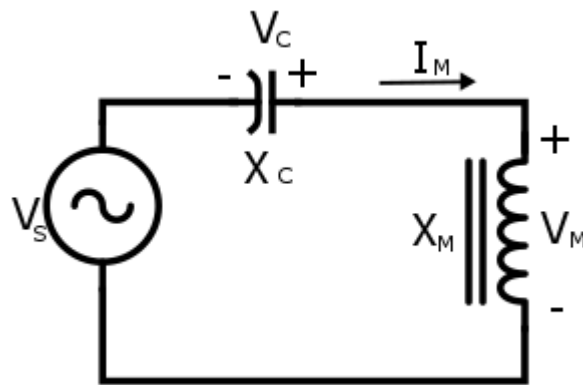


Fig. 2-5: An elementary ferroresonant circuit for the magnetization curve discussion

the system capacitance and source voltage. Additionally, [5], [6], [10], [44] and [45] also provide some valuable insights.

As is evident in Fig. 2-6, there are three points where the equations for the transformer voltage intersect. These equations represent simultaneous equations, which serve as solutions for the circuit shown in Fig. 2-5. Alternatively, [5] states that the three points represent the three situations where KVL (Kirchoff's Voltage Law) is satisfied.

Point 1 represents the steady-state solution of the circuit, where the transformer has not moved into saturation [13]. Point 2 represents the stable ferroresonant state and confirms the claim made earlier that ferroresonance requires the transformer to operate in a saturated region, contrary to the claim made by Horak [10].

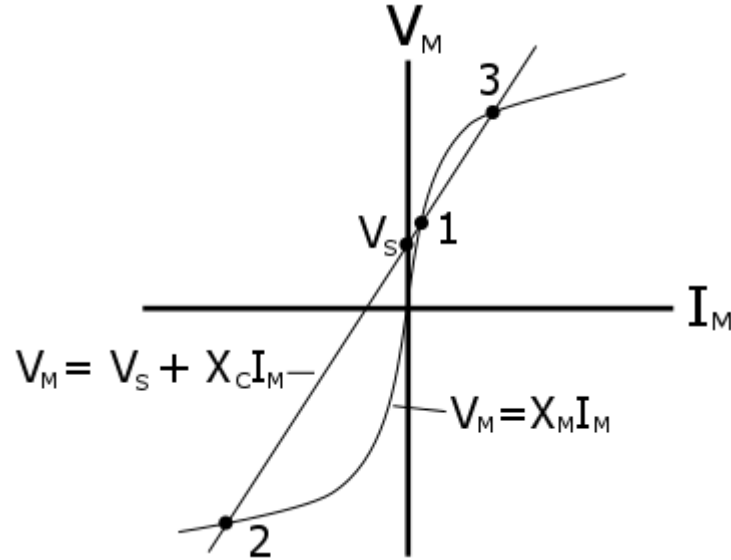


Fig. 2-6: The ferroresonant interaction of the magnetization curve with the system capacitance and the source voltage

Point 3 is also a ferroresonant state, but it is an unstable state and will never be realized. Price [13] presents the following motivation for this claim: a small increase in current at point 3 will lead to an increase in voltage. This, in turn, leads to an increase in current, which is in the direction of the original change. Therefore, the current will avalanche away from point 3. Similarly, a small decrease in current leads to a decrease in voltage. This, in turn, reduces the current even further, which means the current will, once again, avalanche away from point 3. Tsao and Ning [45] describe this process as a violation of the equilibrium theory. Consequently, the point is unstable and unattainable in a steady state. It should be noted, however, that Santoso, Dugan and Nedwick [27] claim that the system may pass through point 2 briefly during a transient.

Regarding the voltage at point 1, both Baitch [5] and Tsao and Ning [45] agree that the largest voltage is across the inductor and is equal to the sum of the voltage of the source and the voltage of the capacitor. At point 2, however, the largest voltage falls across the capacitor. This implies a danger to electrical equipment [5]. Horak [10] claims that the voltages relative to the ground are also higher at point 2.

Another characteristic of point 2 is that it can be self-sustaining. Once the ferroresonant state has been achieved, Horak [10] claims that the source voltage can be reduced without a transition back to the steady-state point seen at point 1. Horak [10] claims that even when the source voltage approaches zero, there is no reason for the operating point to transition beyond the intersection of the two graphs. This is due to the lack of a burdening load and correlates to one of the requirements of ferroresonance, namely low losses. Cigre [6] elaborates by stating that even if

the source voltage is completely removed, there is still a voltage drop across the capacitor, which can be sufficient to self-sustain the ferroresonant state.

What is important to note from this discussion is that there are two stable states in which the system can operate. An increase in the magnetic flux beyond the knee point of the B-H curve moves the transformer into saturation and this causes a jump from one stable state to another. It is for this reason that Ferracci [26] claims multiple steady-state responses and Erbay [11] refers to ferroresonance as a jump resonance. It should be noted that Kieny [46] defines this multiplicity of solutions as a fundamental requirement for a transient to be classified as ferroresonance. This is in addition to those requirements mentioned earlier.

Variation of the $V_M = V_S + I_M X_C$ line leads to a further understanding of the ferroresonance phenomenon. The line can be varied by either varying the source voltage V_S or changing the capacitance, which will lead to a change in X_C . Variation of the source voltage is shown in Fig. 2-7 and variation of the system capacitance is shown in Fig. 2-8. These changes and their impact are best described by Valverde, Mazon, Zamora and Buigues [44].

Fig. 2-7 provides an interesting insight into the impact of a higher voltage on the ferroresonance phenomenon. When the source voltage is varied, the $V_M = V_S + I_M X_C$ line moves parallel to itself. When the source voltage becomes sufficiently high, there is only one solution to the KVL equation, which corresponds to the ferroresonant state. This would imply that higher voltage levels lead to an increased risk of ferroresonance. This conclusion is supported by [6], [8] and [36]. It should furthermore be noted that Santoso, Dugan and Nedwick [27] state that the resulting current of a jump into the third quadrant, as indicated by the blue line in Fig. 2-7, could be so great that the voltage would drop again. This would lead to the solution point jumping between point 1 and point 2 as indicated in Fig. 2-6. Indeed, such instances have been confirmed in actual systems and result in random and unpredictable voltage waveforms [27].

The impact of a change in system capacitance is shown in Fig. 2-8. When the system capacitance changes and the source voltage remains constant, the $V_M = V_S + I_M X_C$ lines rotate about the point where the line crosses the y-axis. This corresponds to the point indicated as V_S in Fig. 2-8.

Since the capacitance of a system is below the denominator in the calculation of capacitive reactance, a decrease in system capacitance leads to an increase in system reactance. This shows in the increased gradient of the blue and green lines in Fig. 2-8. When the gradient of the $V_M = V_S + I_M X_C$ line is increased sufficiently, there is also only one solution to the KVL loop of the system. This single solution correlates to a ferroresonant solution. An increase in the system capacitance leads to a decrease in the line gradient, which means that there can be a non-

ferroresonant solution to the system. One should bear in mind, however, that even when the system capacitance is increased and the gradient of the line is reduced, there are still multiple solutions to the KVL loops, which means that ferroresonance is still possible. This simply means

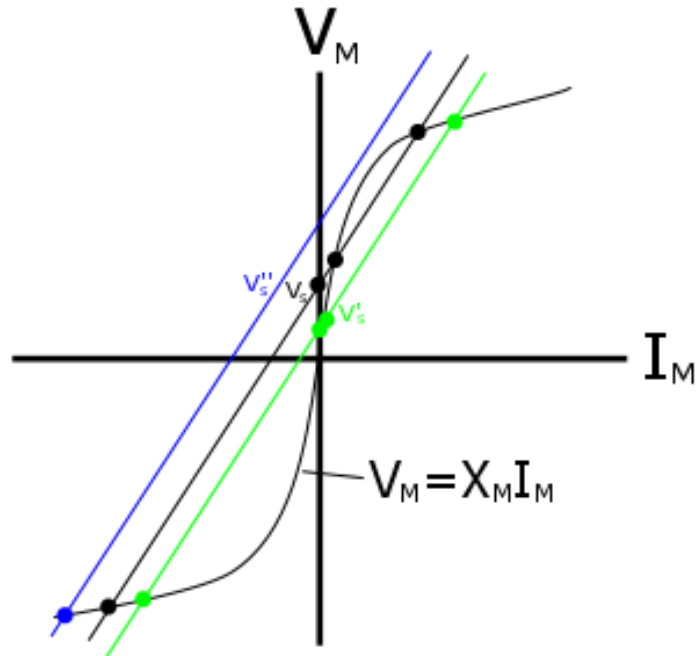


Fig. 2-7: The impact of a source voltage variation on the ferroresonant states

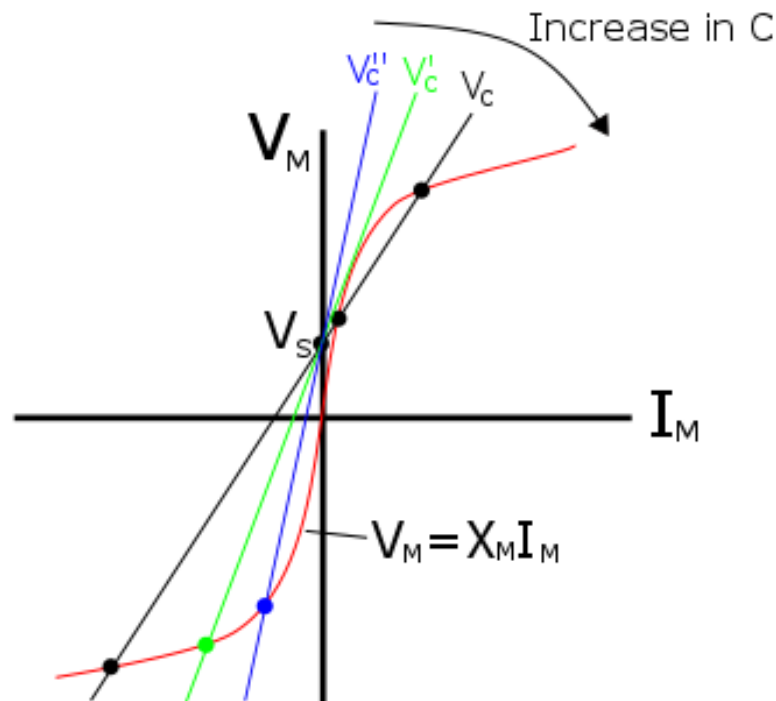


Fig. 2-8: The impact of a capacitance change on the ferroresonant states

that the ferroresonant state indicated by point 2 in Fig. 2-6 is not the only solution to the system. There are several other factors, which can still increase the likelihood of ferroresonance, such as the remnant flux, the source voltage and the losses in the system. These dependencies are discussed in greater detail in section 2.2.3.

Regarding the decrease of capacitance and the increase of the gradient of the $V_M = V_S + I_M X_C$ line, one can also see that a decrease in the capacitance moves the stable ferroresonant point (point 2 in Fig. 2-8) closer to the linear region of the magnetization curve. The possibility, therefore, exists that for small enough capacitance, the stable ferroresonant point may move back into the linear region. In such cases, there will still be only one solution to the KVL loop, but the solution will not be ferroresonant in nature. It would, therefore, seem as though there is a small range of capacitances for which ferroresonance is guaranteed, but a decrease in capacitance beyond the minimum of that range will result in non-ferroresonant behaviour. This, however, is not confirmed in the literature.

Cigre [6] also highlights the effect of damping on ferroresonance initiation. When damping elements are introduced to the ferroresonance circuit of Fig. 2-5, the KVL equation becomes:

$$V_S = R \cdot I + j(X_M(I) - X_C) \cdot I \quad (2-3)$$

which can be manipulated to yield:

$$|V_M(I) - X_C \cdot I| = \sqrt{V_S^2 - (R \cdot I)^2} \quad (2-4)$$

A graph can once again be drawn to illustrate the solutions to (2-4). This graph is shown in Fig. 2-9. The left-hand side of (2-4) is represented by the blue graph, while the red ellipse represents the right-hand side of (2-4). The circle in Fig. 2-9 corresponds to solution point 1. The triangle corresponds to solution point 3 and the star to the right represents solution point 2. This numbering convention is used to ensure that solution point number 3 is still the unstable ferroresonance point. Solution points number 1 and 2 are stable, where solution point number 1 is the non-ferroresonant solution.

The ellipse in Fig. 2-9 is dependent on the value of the damping resistance R . When the damping resistance is increased, the ellipse tends to cross the x-axis for lower values of current. When the damping resistance becomes sufficiently large, the stable and unstable ferroresonant states are avoided altogether. This is shown graphically in Fig. 2-10.

The change in the ellipse seen in Fig. 2-10 is purely based on a change in resistance. This highlights two important points: 1) sufficient resistance eliminates ferroresonance modes and 2)

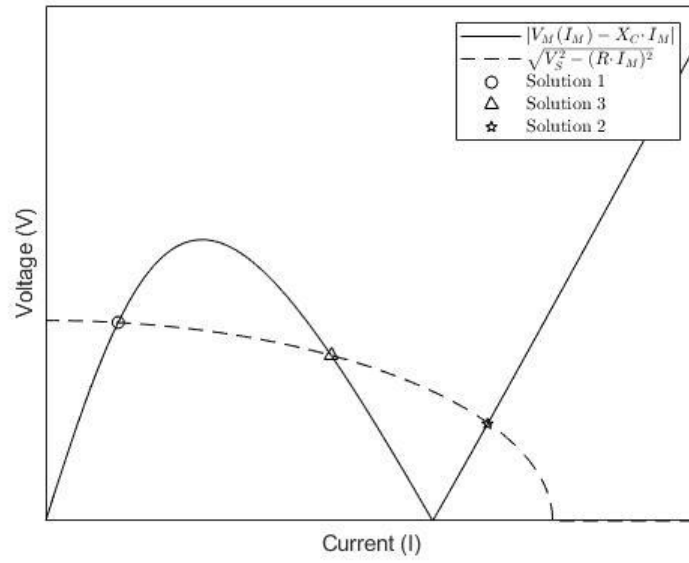


Fig. 2-9: The effect of a small damping resistance on the ferroresonant solutions of (2-4) as seen in [6, Fig. 2-24]

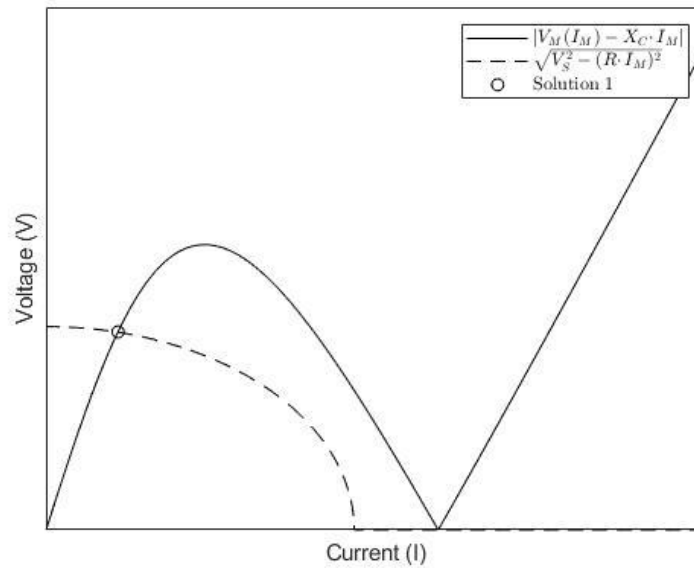


Fig. 2-10: The effect of increased resistance on the ferroresonant points based on (2-4), as seen in [6]

analysis of ferroresonant modes based purely on the magnetization characteristic of the transformer and the capacitance of the system, as seen in Fig. 2-6 through Fig. 2-8 is inaccurate.

Most discussions on ferroresonance point out that there are four main modes of ferroresonance, as first pointed out by Ferracci [26]. These modes are fundamental ferroresonance, sub-harmonic ferroresonance, quasi-periodic ferroresonance and chaotic ferroresonance. The distinction between the various ferroresonance modes is based on the waveforms caused by the

ferroresonance. These four modes are also confirmed by [5], [6], [47] and [48]. Reference [49] presents a more elaborate list of classifications, but this is uncommon. The four modes introduced by Ferracci [26] suffice for all other discussions encountered in literature.

Fundamental and sub-harmonic ferroresonance are examples of periodic ferroresonance, as claimed by Cigre [6]. While the waveforms are distorted by harmonics, there is a definite indication of periodicity. Fundamental ferroresonance is characterized by the fact that the waveforms have a frequency equal to that of the source. Sub-harmonic ferroresonance, on the other hand, has a frequency equal to the fundamental frequency divided by some integer n . The integer is then often used to refer to the sub-harmonic degree [5].

Quasi-periodic ferroresonance is characterized by multiple frequencies. Both [6] and [5] claim that the frequency spectrum is discontinuous. The waveforms, therefore, seem to transition from one frequency to another without stabilising into any specific one [6]. Chaotic ferroresonance is highly irregular and unpredictable. There is a continuous frequency spectrum, which is characterized by what literature refers to as a “strange attractor.” The strange attractor, however, becomes more relevant in a digital simulation where the frequency diagram is plotted on Poincaré maps [6]. Examples of waveforms demonstrating all four modes of ferroresonance can be found in [5], [13] and [26] (see Fig. 2-11).

It should be noted that [48] predicts only the periodic (fundamental and sub-harmonic) forms of ferroresonance in low-voltage systems. Cazecu *et al.* [48] suggest that the cause of this is the reduced levels of capacitance and voltage levels. This was also confirmed by practical measurements, where neither the quasi-periodic nor chaotic ferroresonance mode was evident. Cigre [6] claims that chaotic ferroresonance has never even been reported in an *EHV* (Extra High Voltage) substation to date. While it has been predicted through simulation, this was only possible when unrealistic values for source voltage and circuit capacitance were used. Radmanesh, Rostami and Abassi [50] confirm that their simulations only show chaotic ferroresonance in *VTs* under unrealistic conditions, but they did find that quasi-periodic ferroresonance in a *VT* was possible for realistic values of system parameters.

Ferroresonance is classified as a low-frequency phenomenon by [6], [51] and [52]. Both [51] and [52] state that the frequency spectrum rarely exceeds 2 kHz. This makes sense, because most forms of ferroresonance, especially in low voltage systems, are of the fundamental and sub-harmonic type. Reference [6] presents a formula whereby the frequency of ferroresonance can be predicted. The formula is given in (2-5):

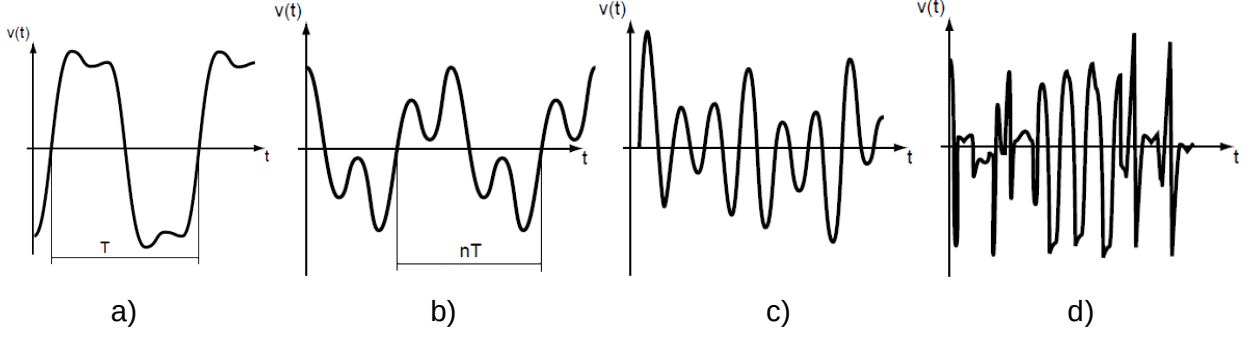


Fig. 2-11: The ferroresonance modes as seen in [26, Fig. 4]: a) fundamental b) subharmonic c) quasi-periodic d) chaotic

$$T_{ferro} = 4 \cdot \frac{\phi_{sat}}{U_0} + 2\pi \cdot \sqrt{L_{sat} \cdot C} \quad (2-5)$$

where:

- T_{ferro} = the ferroresonant period;
- ϕ_{sat} = the saturation magnetic flux;
- U_0 = the initial capacitor charge;
- L_{sat} = the saturated inductance, and
- C = the system capacitance.

This period, which is calculated according to (2-5), can also indicate whether one can expect fundamental or sub-harmonic ferroresonance. Cigre [6] claims that if the ferroresonant frequency is greater than the source frequency, then the frequency may “lock” onto the source frequency, resulting in fundamental ferroresonance. If, however, the ferroresonant frequency is less than the source frequency, the waveforms will most likely lock onto an odd sub-multiple of the source frequency and this will lead to sub-harmonic ferroresonance.

Some visual and audible indications may indicate ferroresonance. Reference [6] claims that repeated core saturation can result in flux moving through the tank and other metallic parts. This, in turn, can result in the charring and bubbling of the transformer paint. It is also the experience of Santoso, Dugan and Nedwick [27] that there are audible indications of ferroresonance in transformers. These audible indicators are described by [27] as “likened to shaking a bucket of bolts, whining, or to a chorus of a thousand hammers pounding on the transformer from within.” This is caused by the magnetostriction of the steel and the vibration of the core laminations [6]. It should be noted, however, that literature does not classify this definitive audible sound as a requirement of ferroresonance, since this is not reported by many authors. As such, it is assumed that a lack of such audible indicators does not form a ground for the elimination of ferroresonance as a potential cause.

2.2.2 Effects/Symptoms of Ferroresonance

The impact that ferroresonance can have on a power system is diverse. For instance, Walling *et al.* [30] claim that despite severe ferroresonance exposure, no damage was suffered by the nine transformer units being tested. Mork [14], on the other hand, reports 72 voltage transformers from the same manufacturer, failed after being exposed to ferroresonance for three minutes. Reference [8] claims that one of the symptoms of ferroresonance can be transformer heating. This is confirmed by [11] and [34]. Tanggawelu, Mukerjee and Ariffin [8] even claim that this can lead to thermal and insulation breakdown. Reference [30], on the other hand, claims that exciting currents, even during severe ferroresonance, are small in comparison to the rated currents of transformers and, hence, there is no risk of thermal winding overload. Cigre [6] claimed in 2014 that no power transformer failures due to ferroresonance had been reported to date, though sustained ferroresonance did contribute to accelerated ageing.

Nevertheless, despite some contradictions between various articles, the following list of symptoms/effects of ferroresonance can be compiled based on [8], [11], [30], [34], [47] and [53]:

- a Random overvoltages for short cycles;
- b Significant continuous overvoltages and overcurrents;
- c Jump resonance and several stable states;
- d High and sustained oscillating phase-to-phase and phase-to-ground overvoltages;
- e Sustained levels of distortion to both current and voltage waveforms;
- f Transformer heating;
- g Loud noise due to magnetostriction;
- h Electrical equipment damage;
- i Thermal/insulation breakdown;
- j Misoperation of protective devices (e.g. circuit breakers nuisance tripping);
- k Flicker;
- l Displacement of neutral point voltage;
- m Damage to electrical equipment (capacitor banks, VTs etc.);
- n Zero-sequence overcurrent, and, and
- o Misoperation of single-phase earth fault protection devices.

It should also be noted that Walling, Hartana and Ros [29] claim that ferroresonance may result in voltages being supplied to the transformer windings, which are more severe than the voltages applied during tests complying with the ANSI (American National Standards Institute) standards.

2.2.3 Ferroresonance Dependencies

Ferroresonance and its initiation are highly dependent on several factors. Based on [6], [8] and [36], these include:

- a The capacitance in the circuit;
- b The initial charge of the capacitor;
- c The source voltage magnitude and frequency;
- d The losses in the circuit;
- e The initial magnetic flux in the transformer iron core;
- f The time of switching, and
- g The saturation levels of the transformer core.

This section of the report aims to discuss these dependencies and the role they play in the ferroresonance phenomena.

The influence of capacitance on ferroresonance was pointed out in section 2.2.1 but deserves a more elaborate discussion. According to Mork [54], a list of capacitive sources in power systems include:

- a Series compensated lines;
- b Shunt capacitor banks;
- c Underground cables;
- d Systems grounded only via stray, zero sequence capacitance;
- e Grading capacitors on circuit breakers, and
- f Generator surge capacitors.

Walling *et al.* [30] claim that transformer phase-to-ground capacitance can also play a very important role in determining ferroresonance susceptibility, especially at 25 kV and 35 kV voltage levels. In this regard, the distributed winding capacitance is particularly important. Walling *et al.* [30] report a case where overvoltages of 1.47 p.u. were measured on 34.5 kV transformers. In another example, overvoltages of 1.29 p.u. were measured on 24.9 kV transformers. In both these cases, the overvoltages were solely due to transformer capacitances. Tygai and Besarab [55] add that inter-phase capacitance can have a significant effect on ferroresonance behaviour and can increase ferroresonant current by 10% to 25%.

Rezaei-Zare *et al.* [52] make an important statement regarding winding capacitances. They [52] state that the winding capacitances of transformers are generally small when compared to the shunt capacitances of long transmission lines. This means that the transmission line mainly determines the capacitive characteristics of the power system when the circuit breaker is closed.

However, the final mode of ferroresonance is determined by the circuit breaker grading capacitance and the winding capacitance in conjunction with the core losses. Therefore, after the circuit breaker has been opened, the winding capacitance becomes very important. The importance of winding capacitance becomes even more prominent when bearing in mind that circuit breaker grading capacitors are being ignored here due to low voltage levels. Martinez *et al.* [56] claim that winding and bushing capacitances can be critical to slow transients such as ferroresonance and other resonance phenomena. Chiesa and Høidalen [57] claim that the estimation of transformer capacitances is important for extending model validity at high frequencies and correct prediction of residual flux. Residual flux, in turn, is one of the factors upon which ferroresonance is highly dependent.

The influence of the initial charge of the capacitor can be explained as follows: let us consider the typical ferroresonant circuit seen in Fig. 2-5, taking note of the polarity of the voltage across the capacitor V_C . Assuming a voltage rise to be negative, Kirchhoff's voltage law dictates that:

$$\begin{aligned} -V_S - V_C + V_M &= 0 \\ \therefore V_M &= V_S + V_C \end{aligned}$$

Now, suppose that at the time of switching, the system voltage assumes a maximum positive value, but the charge across the capacitor is at a maximum negative value. The polarity of the capacitive voltage V_C will change, which means that the KVL solution to the system will become:

$$V_M = V_S - V_C \quad (2-6)$$

A phasor diagram of (2-6) is shown in Fig. 2-12. The voltage across the transformer, denoted V_M , is small in comparison to the rated source voltage. From the magnetization curve seen in Fig. 2-4, it should be clear that when the transformer voltage is small, the transformer is operating in its linear region. Therefore, if the initial charge of the capacitor is such that the capacitive voltage opposes the source voltage, the risk of ferroresonance is reduced.

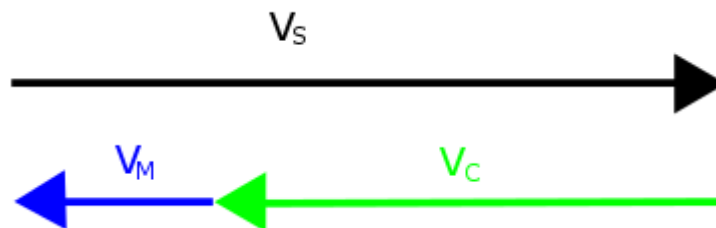


Fig. 2-12: The phasor diagram for a ferroresonant circuit with a negative initial capacitive charge

However, consider the case where the charge of the capacitor is positive and the voltage across the capacitor complements the voltage being applied to the circuit. Assuming the worst case scenario whereby, at the time of switching, the source voltage is at its peak value and the capacitive voltage also has a value equal to the peak of the source voltage, the voltage being applied to the transformer will assume a value of twice the peak source voltage. This concept is demonstrated in phasor format in Fig. 2-13. Transformers are not designed to operate at such voltages, which means that there is a strong likelihood of core saturation, which may lead to ferroresonance. It is for this reason that ferroresonance is dependent on the initial charge of the capacitor.

Fig. 2-7 in section 2.2.1 discussed the influence of source voltage on the excitation curve. Furthermore, [8] points out that transformer saturation is frequency-dependent. It is for this reason that [8] also claims that ferroresonance is dependent on both the magnitude and the frequency of the source voltage.

Section 2.2.1 discussed the effect of low losses and their influence on ferroresonance. Continuous improvement of ferromagnetic materials allows for reduced core losses, which is desirable from an economic point of view. However, this increases the risk of ferroresonance [6]. Ribas *et al.* [51] mention that studies conducted using the Preisach theory of hysteresis prove that transformer cores with lower magnetic losses are also more susceptible to chaotic ferroresonance. Walling *et al.* [30] claim that core losses are the critical factor influencing ferroresonance susceptibility in grounded wye-wye transformers with five-legged cores. Short [3] also points out that when core losses are reduced, as is the case with amorphous cores, ferroresonance occurs for lower levels of system capacitance.

The initial magnetic flux is important and has two effects on ferroresonance [40]: it changes the length of transient build-up into ferroresonance and increases the probability of core saturation. When a high remnant flux is left in the transformer core when it is disconnected, the total flux in the core can exceed the designed knee point when the transformer is brought back into operation.

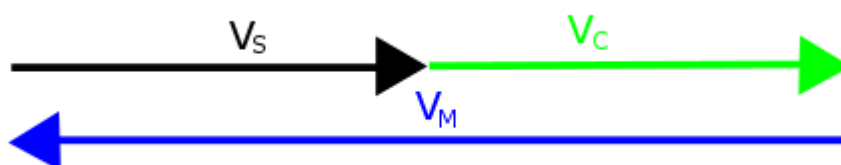


Fig. 2-13. The phasor diagram for a ferroresonant circuit with a positive initial charge on the capacitor

This correlates to the final point made earlier, namely that ferroresonance is also dependent on the saturation levels of the transformer. If the transformer tends to saturate at lower levels of magnetic flux, ferroresonance is more likely. This is demonstrated in the example provided by Mork [14], where 72 transformers were destroyed by ferroresonance due to poor design.

The time of switching influences the value of the applied voltage when the transformer is brought into commission. This influences the flux being applied to the transformer core. It also plays a role in the phasor diagrams shown in Fig. 2-12 and Fig. 2-13. Cigre [9] and Young, Schmid and Fergestad [40] state that energization at the point where the waveform is crossing the x-axis results in a maximum flux-linkage. This means that energization at the voltage zero-crossing is potentially dangerous, as it can drive the transformer into saturation. Energization at the peak voltage, on the other hand, is also dangerous, since it can result in a total voltage of twice the peak voltage being applied to the transformer, provided the charge on the capacitor is favourable for such a condition.

Regarding switching in general, Young, Schmid and Short [40] found that sustained overvoltage was always bigger when a switch was opened, than was the case when a switch was closed for identical capacitance values. They [40] hypothesize that this is caused by a trapped charge on the capacitor when the line is opened, which results in higher voltages across the transformer winding. Walling *et al.* [30], however, contradict this claim by Young, Schmid and Short [40] by claiming that opening and closing events produced overvoltages of similar magnitude, except when large amounts of capacitance were involved. Walling *et al.* [30] found that the most severe overvoltages were observed when one phase remained open after switching, e.g. when the first phase was opened or the next-to-last phase was closed. However, this claim does not apply to a single-phase transformer.

Even more interesting is the results of the study conducted by Mork and Stuehm [58], where they found that ferroresonant transformers in the field tended to make more noise than transformers tested in the lab. It was also found that the dampening of ferroresonance in field transformers required larger loads. The conclusion was made that the source impedance and the type of capacitance could determine whether or not transformers were damaged since the capacitance in field transformers were supplied by underground cables. Capacitance in the lab was supplied by fixed capacitors. It should be noted, however, that the waveforms measured on both sets of transformers were similar [58].

Based on this discussion of the dependencies, it should be clear that ferroresonance is dependent on a wide range of factors. This makes ferroresonance an unpredictable phenomenon and difficult to model. The initial charge on capacitors and residual flux in the transformer core cannot be

predicted with certainty. It is for this reason that [40] repeated their experiments 25 to 50 times for each value of cable capacitance. This also explains why bifurcation diagrams are commonly used to analyse ferroresonance. Bifurcation diagrams will be discussed in section 2.7.3.

2.3 Resonance vs. Ferroresonance

At this stage, it should be clear to the reader that there are several differences between a ferroresonant and a resonant linear circuit. Some of these differences are pointed out by Baitch [5] and Ferracci [26]. Firstly, ferroresonance is possible for a wide range of capacitance in ferroresonant circuits. This is due to the nonlinear nature of the transformer inductance. The frequency of the voltage and current waveforms during ferroresonance may differ from the source frequency, as was made clear by the discussion on ferroresonant modes. There is more than one stable state possible for ferroresonance, as was made clear by the magnetization curve. Small changes in parameter values may result in a jump from one stable state to another, which justifies its classification as a “jump resonance.”

Resonance is not dependent on the saturation of an inductor. This means that there is only a single capacitance value where resonance will occur. This also implies that the resonant frequency can be calculated and remains constant. Resonance does not have a multiplicity of solutions – there is only a single steady-state solution. This also means that small changes in parameters do not initiate changes in the steady-state of the circuit.

2.4 Ferroresonance Causes and Susceptible Topologies

The causes of ferroresonance should be clear from the theory discussion found in section 2.2.1. This section serves to summarize the causes and elaborate on some of the concepts. It is also true that some transformer topologies are more susceptible to ferroresonance than others. These topologies mostly hold a reference to three-phase systems and will be discussed briefly as well.

Since ferroresonance requires the saturation of the transformer, the cause of ferroresonance needs to initiate this saturation requirement. Steady-state conditions are, in general, not sufficient to drive the core into saturation. Causes listed by Ferracci [26] include: transients, lightning overvoltages, energizing and de-energizing of transformers and faults. The most common cause found in literature is the opening of a circuit breaker or the clearance of a fault. The initial conditions at the time of occurrence then determine the mode of ferroresonance. Section 2.2.3 discusses the dependencies of ferroresonance in more detail. Erbay [11] confirms the list of causes presented by Ferracci [26] and also adds that phase-to-ground and phase-to-phase faults can also cause ferroresonance.

Andrei and Halley [59] present an interesting energy approach to the determination of whether a system is susceptible to ferroresonance or not. They [59] derive an equation, which describes the maximum energy that the voltage source can supply to the transformer within a single cycle. They then proceed to calculate the energy, which the transformer will receive in a single cycle if it is in the ferroresonant state. If the maximum energy that the source can supply to the transformer exceeds that of the energy, which the transformer will receive in the ferroresonant state, then the transformer can be driven into ferroresonance by a switching transient such as de-energization. The derivation of these equations can be found in [59].

Based on this calculation, Andrei and Halley [59] make two conclusions. The first of these is that if a substation has a low K_C ratio, then VT ferroresonance is unlikely. The K_C ratio is a ratio of the equivalent grading capacitance to the sum of station bus-to-ground capacitance and the grading capacitance. In mathematical terms it can be written as [59]:

$$K_C = \frac{C_b}{C + C_b} \quad (2-7)$$

where

- C_b = the grading capacitance, and
- C = the station bus-to-ground capacitance.

A low K_C ratio means that the grading capacitance is small in comparison to the bus-to-ground capacitance of the station. As more lines are connected to the station and more grading capacitances are introduced to the system, the K_C ratio increases and the susceptibility of ferroresonance will increase. Station bus-to-ground capacitance includes bus conductors, support insulators and VT high voltage windings and bushings.

The second conclusion made by Andrei and Halley [59] is that voltage transformers with high saturation factors, K_S , are less likely to be driven into saturation. The saturation factor is a ratio of the frequency of the system and the saturation flux to the peak system voltage. In mathematical terms, this means that:

$$K_S = \frac{\omega \phi_S}{\sqrt{2} V_{sy}} \quad (2-8)$$

where

- ω = the angular frequency of the source;
- ϕ_S = the saturation flux, and
- V_{sy} = the system root mean squared (RMS) voltage.

Short [3] also presents an energy approach to determining whether or not ferroresonance can occur. Short [3] claims that if the capacitive reactive power per phase, denoted B_C and measured in vars, is larger than the core loss of the transformer, then ferroresonance can occur. In mathematical terms, this means that:

$$B_C \geq P_{NL}$$

The capacitive reactive power on one phase depends on the voltage and the capacitance and is given by (2-9):

$$B_C = \frac{V_{kv}^2}{3} 2\pi f C \quad (2-9)$$

where

V_{kv} = the rated line-to-line voltage, measured in kV;

f = the frequency in Hz, and

C = the capacitance from one phase to ground, measured in μF .

Note, however, that this criterion does not guarantee ferroresonance. It only indicates a possibility of ferroresonance. The Southern California Edison Company found that ferroresonance did occur when [3]

$$B_C \geq 1.27 P_{NL}$$

In addition to criteria presented by [59] for susceptible transformers, literature also presents several topologies, which are particularly susceptible. Short, Burke and Mancao [60] state that the most common cause of ferroresonance in distribution systems is when single-phase switching takes place in a three-phase system, which is being fed by an underground cable. A typical diagram of such a ferroresonant circuit is shown in [3, Fig. 4.24]. This figure is also shown in Fig. 2-14 of this report. Note that this is a common diagram in literature, which can also be seen in references [10], [11], [12], [13] and [14], to name a few. One should also note that ferroresonance can also occur in cases where only a single phase is opened, as opposed to the two phases seen in Fig. 2-14. An example of this can be seen in [10, Figure 2a]. This discussion regarding single-phase switching of three-phase systems once again correlates to the discussion provided by Geldenhuys and Padayachee [1], where they discuss the initiation of resonance in the transformers in question whenever a single phase is opened. A diagram of this single-phase switching in the South African network can be seen in [1, Figure 1.3].

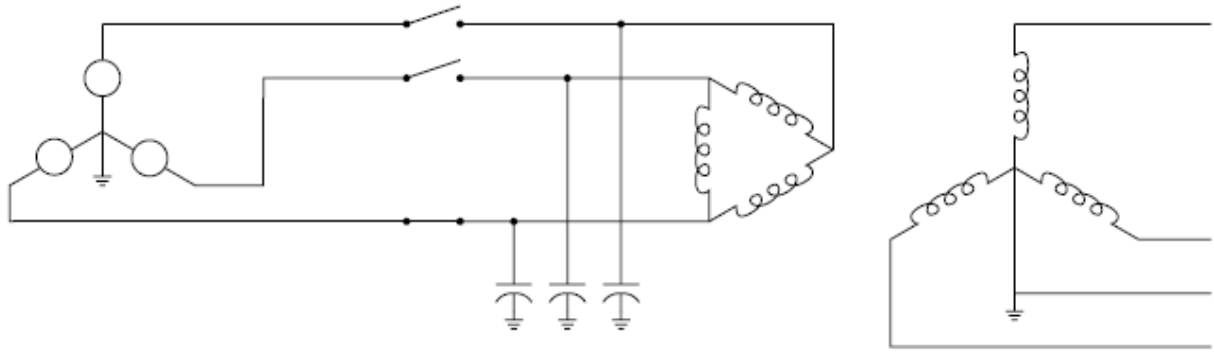


Fig. 2-14: A typical single-phase switching ferroresonant topology as given by Short [3, Fig. 4.24]

Short [3] and Horak [10] also point out that transformers with ungrounded primaries, such as delta and floating-wye primary connections, are susceptible to ferroresonance and experience the highest magnitude of overvoltages. This list includes:

- a Delta/grounded wye connections;
- b Floating wye/delta connections;
- c Delta/delta connections;
- d Open delta/open delta connections, and
- e Delta/floating wye connections.

Short [3] adds that ferroresonance can also occur in single-phase transformers, which are connected phase-to-phase³. Horak [10], Hassan, Vaziri and Vadhva [31] and Bohmann, McDaniel and Stanek [61] confirm this statement. If one of the phases is then opened remotely or opened at the transformer, then ferroresonance is also possible. It is interesting to note that ferroresonance was detected in single-phase transformers, which were switched when connected to relatively short cable lengths [3].

Examples of single-phase ferroresonant transformers provided by Short [3] include 25 kVA and 50 kVA transformers resonating with 30.48 m (100 ft.) cables at a voltage level of 12 kV, as well as 75 kVA units resonating with 45.72 m (150 ft.) cables. Longer cables tended to produce higher overvoltages during ferroresonance. Short [3] proceeds to point out that ferroresonance was generally avoided if the sum of the core losses and load losses of the transformer exceeded 1.15 times the capacitive reactive power per phase. Mathematically, this means that ferroresonance was avoided when:

$$P_{NL} + P_L > 1.15B_C$$

³ Single-phase transformers are connected between phases in South Africa, since South Africa does not use a 4-wire distribution system.

Other single-phase ferroresonant transformers include a *VT*, which is connected between a phase and earth on an isolated neutral system [26].

Additional risky configurations and situations are pointed out by [6], [10] and [13]. These risky configurations are not confined to specific topologies but are rather a confirmation of the theory discussed in section 2.2.1, these include:

- a Unbalanced switching in three-phase systems;
- b Voltage transformers connected to circuit breaker grading capacitances;
- c Coupling capacitor voltage transformers (*CCVT*);
- d Auxiliary transformers;
- e Transformers connected to de-energized and ungrounded lines in parallel with energized lines;
- f Lightly loaded transformers energized via a cable or long line, and
- g Transformers connected to series compensated lines.

The *CCVT* is a capacitive voltage divider with a transformer connected to the lower capacitor [6]. Its very nature, therefore, makes it highly susceptible to ferroresonance. Price [13] claims that *CCVT* manufacturers provide various designs of ferroresonance suppression circuits (*FSC*) to be connected to the secondary of the transformer, owing to its susceptible nature. A typical *CCVT* configuration is shown in Fig. 2-15. The tuning reactor is typically used to improve voltage regulation. The *FSC* is also shown on the secondary.

With regards to ferroresonant auxiliary transformers, Price [13] mentions that auxiliary transformers connected to *VT* and *CCVT* secondaries are typically susceptible. This makes sense because the *VT* and *CCVT* themselves are also susceptible. Price [13] recommends that the *VT* saturates before the auxiliary transformer.

Series compensation is used in long transmission lines to improve voltage regulation. If, however, the load on the secondary side of the transformer is rejected by opening the circuit breaker, the transformer can operate in its saturated region. The added capacitance in the series compensated line can then initiate ferroresonance [6].

From the list of susceptible three-phase topologies provided earlier, it would seem as though transformer configurations with grounded primaries are immune to ferroresonance. In general, this is a good assumption [30], [32], [40]. However, Short [3] adds that even transformers with grounded primaries are susceptible to ferroresonance when the windings are on a common core,

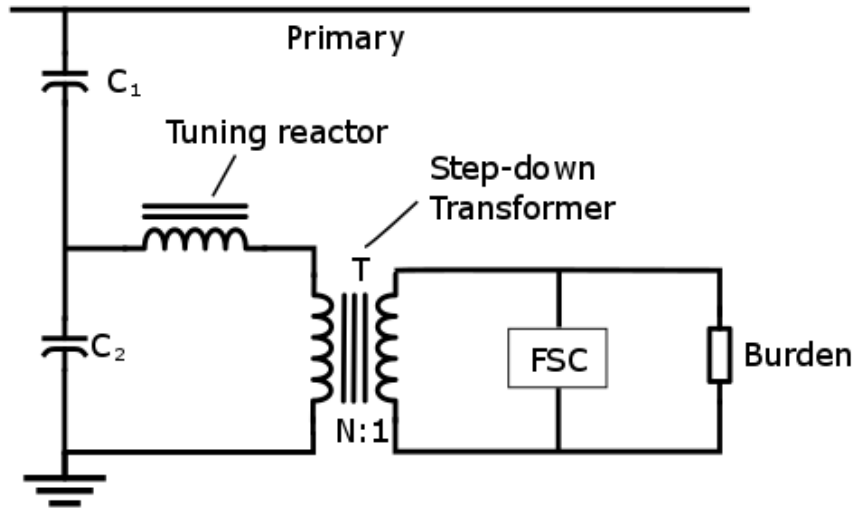


Fig. 2-15: The equivalent circuit of a CCVT as seen in [13, Fig. 21(a)]

such as in the five-legged wound core topology⁴. The ferroresonance, however, is not as severe in such cases. Horak [10] adds that when grounded-wye transformers are connected to ungrounded capacitor banks, single-phase switching can also cause ferroresonance. If, however, the three-phase system consists of three single-phase units and there is no phase-to-phase capacitance, as is the case with underground distribution, the grounded-wye configuration is immune to ferroresonance [30]. If the secondary windings of grounded wye transformers have ungrounded capacitors, a series resonant circuit can also exist and lead to ferroresonance [3].

Kobet [32] conducted a series of simulations to determine whether ferroresonance would occur on a 13.2 kV system. Through the course of his simulations, several interesting conclusions were made. Firstly, he proved that a delta ungrounded system is immune to ferroresonance when zig-zag grounding transformers were connected. He also found that connecting 100% of the thermal load to the VT secondary eliminates ferroresonance, but claims that measurement inaccuracy would be unacceptable. Finally, he also noted that connecting 50 kvar capacitors from each phase-to-ground was a valid solution for ferroresonance prevention.

Other configurations provided by Short [3], which prove to be immune to ferroresonance, are the open wye/open delta configuration, triplex construction units and line-to-ground connected single-phase units.

⁴ Treanor [62] defines a wound core as "...made by winding a long ribbon of magnetic steel in a tight spiral on a mandrel into the form of a heavy-walled circular cylinder"

2.5 Ferroresonance Mitigation

Section 2.4 has shown that only a few transformer topologies have proven to be immune to ferroresonance. As such, it is also necessary to consider mitigation techniques, which can reduce the damage that ferroresonance does to the power system.

The most common mitigation technique discussed in the literature involves the increase of the load in the circuit to damp out the oscillations. This addresses one of the key requirements of ferroresonance, namely low losses. References [5], [10], [11], [13], [32], [39], [40], [47] and [63] mention this mitigation technique.

Kobet [32] states that loading a VT secondary to 30% of its thermal rating is effective in damping out ferroresonance overvoltages. Note that this is in stark contrast to the 100% mentioned in [32]. Baitech [5] claims that a load of 3% of the rated load of the transformer should be sufficient to suppress the overvoltages. Customer connected loads could, therefore, be sufficient, but the use of portable resistors for this purpose is not practical. Walling [37] states that the amount of load required to eliminate or attenuate ferroresonance is related to the amount of capacitance in the circuit, but it will always be a few per cent of the transformer rating in practical situations.

Erbay [11] claims that the type of connection used when connecting the damping resistor varies depending on the ferroresonance mode. He [11] claims that recent literature is focused on the calculation of the damping resistor values through the use of the Harmonic Balance Method. He presents an equation for calculating the value of the damping resistor for a voltage transformer with only one secondary winding, where the resistance is dependent on the source voltage, the rated output of the transformer and the required power for accurate measurement. This equation is provided in (2-10)

$$R = \frac{U_s^2}{k \cdot P_t - P_m} \quad (2-10)$$

where

- U_s = the rated secondary voltage (V);
- k = the factor between 0.25 and 1 to take errors and service conditions into account;
- P_t = the voltage transformer's rated output (VA), and
- P_m = the power required for accurate measurement (VA).

If the VT has two secondary windings, a damping resistor in an open-delta configuration is particularly advantageous. This is confirmed by Horak [10]. Such a secondary winding is only

operational during unbalanced conditions and does not circulate current under steady-state conditions. In such a case, the value of the damping resistor is determined by the thermal burden of the VT and can be calculated using (2-11):

$$R = \frac{3\sqrt{3}U_S^2}{P_e} \quad (2-11)$$

where

U_S = the rated voltage of the tertiary windings (V), and

P_e = the rated thermal burden⁵ of the tertiary winding (VA).

It should be noted that these equations presented by Erbay [11] are empirical. However, Erbay [11] concludes that these empirical approaches are less accurate than the harmonic balance method, as they do not provide sufficient damping for all sub-harmonic modes of ferroresonance. Resistance values calculated with the use of the harmonic balance method differed from those found by empirical methods and succeeded in damping sub-harmonic ferroresonance.

Horak [10] presents two alternative approaches to calculating the value of the open-delta resistance. The first method involves the determination of the rated current of the VT and sizing the resistance such that the rated current flows through the resistor in a line-to-ground fault. The second method applies to three-phase VTs, where the resistor is sized in such a manner that rated transformer apparent power is applied to the VT during a line-to-ground fault. It should be noted, however, that this will result in a current higher than rated current to flow through the transformer, but Horak [10] claims that this should be satisfactory for events of short duration.

It is interesting to note, regarding open-delta suppression resistance, that Solak and Rebizant [47] claim that sub-harmonic modes of ferroresonance are the most challenging to suppress. They [47] also present two equations, which can be used to calculate the value of the damping resistor. These equations are shown in (2-11) and (2-12). Between (2-11) and (2-12), (2-11) presented better results.

(2-12) describes the value of the damping resistor

$$R_{d1} = \frac{100L_{ms}}{N^2} \quad (2-12)$$

⁵ Thermal burden is defined by [11] as “the apparent power that a voltage transformer can supply without exceeding thermal constraints.”

where

- R_{d1} = the value of the damping resistance;
- L_{ms} = the VT saturation inductance, measured in mH, and
- N = the VT turns ratio.

Price [13] also confirms that increasing the circuit resistance will eliminate operation in the ferroresonant state. However, Price [13] adds that the resistance should be added in parallel with the transformer magnetizing inductance since it is impractical to add a load in series without changing the operation of the circuit. Note that, in this discussion, Price [13] is referring to a simplified ferroresonant circuit as seen in Fig. 2-5. The magnetizing inductance is, therefore, not represented as in the typical T-model used for steady-state transformer analysis⁶, but rather, the equivalent circuit can be seen in Fig. 2-16. Following a detailed mathematical derivation, which can be found in [13], Price derives an equation whereby the maximum resistance to be inserted in parallel with the magnetizing reactance is calculated. While the derivation is beyond the scope of this report, the equation is shown in (2-13).

$$R_{Max} = \frac{X_C X_{M2}}{X_C - X_{M2}} \sqrt{\left(\frac{X_C V_{sat}}{X_{M2} V_S}\right)^2 - 1} \quad (2-13)$$

where

- R_{Max} = the maximum parallel resistance that can be inserted into the circuit;
- X_C = the equivalent circuit capacitance;
- X_{M2} = the air-core reactance of the transformer;

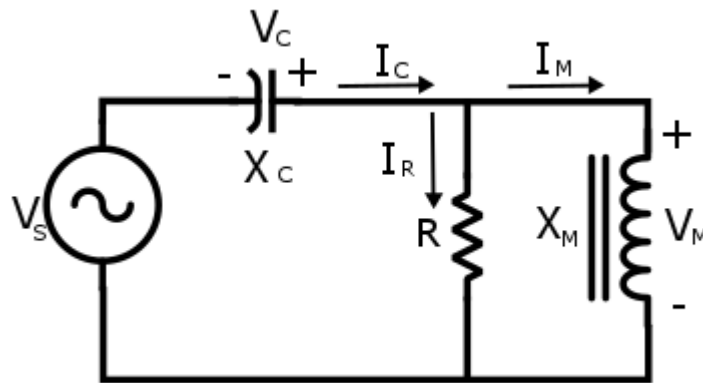


Fig. 2-16: A ferroresonant mitigation load added in parallel with the magnetization reactance of the transformer as seen in [13, Fig. 15(a)]

⁶ See section 2.6.3 for a discussion of the standard T-model.

V_{sat} = the saturation voltage of the transformer, and
 V_S = the source voltage.

Any resistance value below that of R_{max} , which is inserted into the circuit as shown in Fig. 2-16, will eliminate ferroresonance. However, this solution is not practical. A parallel resistor can be connected across the magnetizing inductance in a simple ferroresonant circuit and in simulation. In a real transformer, however, the resistor can only be connected across the terminals of the transformer. Such a connection is not the equivalent of connecting a resistor in parallel with the saturable inductor.

Piasecki *et al.* [64] also discuss the use of a resistor in the open-delta auxiliary winding of a voltage transformer. However, they caution that with lower core losses, the required resistance value is very small. Consequently, the power dissipation requirements become unreasonable. The alternative suggested by Piasecki *et al.* [64] is to make use of a positive temperature coefficient (PTC) thermistor connected in series with the resistor. The small resistance value of the PTC thermistor assists in damping out ferroresonance within a few cycles. However, if an earth-fault causes a prolonged zero-sequence current in the open-delta winding, the thermistor undergoes a transition into a high-ohmic state, which limits the current.

Price [13], Burgos *et al.* [39] and Solak and Rebizant [47] mention that the secondary resistance may be switched upon ferroresonance detection. Price [13], for instance, describes a saturating reactor circuit, which switches the damping resistance on when secondary overvoltages and excessive flux become a problem. However, [13], [39] and [47] all caution that opening the switch may introduce a new transient, which initiates ferroresonance again. Recall that section 2.2.3 mentioned that ferroresonance is dependent on the time of switching. The same principle applies to the opening of the switch, which controls the damping resistance. As such, Burgos *et al.* [39] claim that the use of a switchable damping resistor is not advisable. The advantage of the switchable resistor, however, is that the VT is only thermally stressed for short periods [47].

Bergos *et al.* [39] also prove through simulation that a secondary resistance value, which is too low, can cause more pronounced transients and may also lead to the unexpected reappearance of ferroresonance. As such, Bergos *et al.* [39] claim that alternative solutions to the damping resistor are required for ferroresonance mitigation.

A common concern with the use of resistance as a mitigation technique is that fuse operation may initiate unanticipated ferroresonance and that it may take an extended time before the condition is corrected. Walling [37], however, claims that most transformers are at least lightly loaded, and since such a small load is often required to mitigate ferroresonance, this is not a cause for

concern. An exception to this is when the transformer is used for completely on/off loads, such as irrigation pumps.

Walling [37] also addresses a few other scenarios related to fuse operation. Fuse operation in underground systems, according to Walling [37] is generally caused by a fault, which is permanent. If the fault is on the transformer side of the cable, then the capacitance of the cable is reduced significantly and ferroresonance will not be initiated. If the fuse is on a riser pole, it may fail during heavy loading, which means that the load will also dampen out the ferroresonance. However, if the transformer is part of an overhead bank, then the fuse may operate due to animal faults or lightning currents. In such cases, unloaded transformers may still be susceptible.

According to Price [13], ferroresonance mitigation can be classified into four main approaches: correction of voltage unbalance conditions, change of transformer magnetic designs, damping resistances and detuning of the circuit. Therefore, alternatives to the damping resistance techniques discussed thus far, have been investigated.

Solak and Rebizant [47] specifically looked at the use of a Petersen coil to ground a system. In their simulations, they found that ferroresonance was particularly difficult to initiate when Petersen coils were present in the system. Petersen coils are also referred to as Arc Suppression Coils (ASCs). O'Donovan, Cowhey and Barry [65] claim that ASCs are the most common neutral treatment used throughout Europe. They are defined as high impedance grounding systems, whereby a reactor is connected between the neutral point of the system and ground. In the case of a single line-to-ground fault, the inductive current of the ASC neutralizes the capacitance to ground of the healthy phases. The inductance can also be varied by changing the air gap of the iron core in the magnetic circuit of the Petersen coil [65]. Sumner [66] states that ASCs are designed to saturate at a voltage slightly above the system phase voltage, which assists in keeping resonance within safe limits.

ASCs are not used in South Africa. The South African power systems make use of *NGRs/NERs* to ground the system and control the earth fault current in *MV* applications. This dampens the system for many forms of resonance.

Bayona *et al.* [67] also discuss a similar concept but simply refer to the use of shunt connected saturable reactors. However, [67] claims that the use of such reactors with *VTs* results in added losses and can lead to *VT* losses. Instead, Bayona *et al.* [67] suggest the use of a flyback converter operating in the critical conduction mode to function as a resistor emulator. They conclude that this concept has proven to be a fast and effective way to mitigate ferroresonance.

Damstra [68] has applied one of the concepts discussed in section 2.2.1, namely that an increase in capacitance reduces the risk of ferroresonance. Damstra [68] claims that applying capacitors to the low-voltage side of the transformer can suppress the symptoms of ferroresonance since the low-voltage side capacitance can be transformed to the high-voltage side, which will lead to an increase in the system capacitance. It should be noted that Geldenhuys and Padayachee [1] also investigated a similar mitigation technique for the resonance being studied, but claim that adding such a large capacitor to the secondary of the transformer is unacceptable for several reasons.

Baith [5] suggests limiting the cable length connected to the transformer. It is assumed that the rationale behind this suggestion is to limit the capacitance of the system to such an extent that there will only be one solution to the KVL loop in Fig. 2-5 and that the gradient of the $V_M = V_s + I_M X_C$ line will be small enough that the solution will be non-ferroresonant in nature. This, however, is not a viable solution, since the capacitance needs to be limited to a very small value, resulting in a maximum cable length of 31 m in some cases [5]. Horak [10] also suggests limiting cable length, especially when single-phase switching is employed. Hopkinson [69], [70], Prusty and Panda [71] all provide guidelines for limiting cable length based on the transformer size. It should be noted, however, that Walling *et al.* [38] state that critical cable lengths are highly dependent on the no-load losses of transformers. Since distribution transformer designs are continuously improved, no-load losses are a fraction of the values they once were, and hence, the critical cable lengths for modern transformers are considerably shorter than suggested by long-standing guidelines such as those seen in [69], [70] and [71]. Finally, Walling *et al.* [29] also conclude that transformers have sufficient internal capacitance to “self-sustain” ferroresonance. This internal capacitance is mostly due to the capacitance between the HV and LV winding, which also confirms the suspicions of Geldenhuys and Padayachee [1] discussed in section 1.1.

Baith [5] also provides some interesting insights into switching protocols, which can help to avoid ferroresonance. He claims that providing switches at the transformer terminals, as opposed to having tail-ended transformers controlled by drop-out fuses will reduce the risk of ferroresonance. This suggestion is reiterated by Horak [10], who motivates it by stating that this ensures that transformers are not energized by long un-energized lines in parallel with other circuits. Baith [5] also says that one should avoid switching both a cable and a transformer at the same time, as this can cause ferroresonance.

Zhou and Yin [72] suggest the use of a bidirectional thyristor in the open-delta winding of the transformer as an effective ferroresonance mitigation technique. The thyristors are then controlled by a microprocessor, which detects overvoltages in the open-delta winding. Erbay [11] makes

reference to this suggestion and claims that it is theoretically superior to the static resistance damping techniques presented earlier.

Chiesa *et al.* [73] provide some interesting insights into the impact of shunt capacitance on the residual flux of the transformer, which is one of the parameters that have a strong influence on ferroresonance susceptibility. Chiesa *et al.* [73] claim that increased shunt capacitance in a system leads to longer ringdown transients. This, in turn, leads to lower values of residual flux in the transformer. It can, therefore, be concluded that increasing the shunt capacitance of the system can assist in reducing the likelihood of ferroresonance since lower levels of residual flux imply that the likelihood of transformer saturation is also lower. Chiesa *et al.* [73] caution, however, that longer ringdown transients may also drive the core into saturation. It is for this reason that Chiesa expressly states in [74] that capacitances need to be included if a model is to be capable of calculating residual flux.

Short [3] presents a summarized list of some of the solutions to ferroresonance and includes:

- a Using a transformer with higher losses;
- b Using three-phase switchgear instead of a single-phase switching device;
- c Switching at the transformer, as opposed to switching at the riser pole;
- d Using transformer connections, which are not susceptible to ferroresonance, and
- e Limiting the switching of transformers to cases where the capacitive reactive power of the system is less than the no-load losses of the transformer

Solution (a) is not sensible in this case, as it goes against what the changes in the SANS 780 standard attempt to achieve. The problem being investigated was initiated by the fact that the no-load losses of distribution transformers need to be reduced. Increasing the no-load losses is, therefore, not a sensible solution.

Solution (b) was also discussed in section 2.1.3 as a resonance mitigation technique. However, it was pointed out in section 2.1.3 that three-phase switchgear is not economically feasible, especially in distribution systems with lower voltage levels. This solution is, therefore, also not feasible.

Solution (c) correlates to the discussion on switching protocols presented earlier and supports the statements made by Baitch [5] and Horak [10]. The rationale behind this suggestion involves decreasing the capacitance of the system.

Solution (d) is not sensible in this case, as transformer topologies refer to three-phase connections of transformers. This suggestion by Short follows his lengthy discussion on transformer topologies, which are not susceptible to ferroresonance. However, since single-phase

distribution transformers are being considered here, the mitigation technique becomes irrelevant for this study.

Solution (e) is also not feasible, since it does not make sense to limit switching in such a manner. Recall from section 1.1 that the resonance overvoltages were measured during the commissioning phase of the transformers. This means that switching between the transformers is unavoidable. Furthermore, this suggestion assumes that one has control of all switching procedures, which is not necessarily the case. Dugan [25], for instance, reports a case of ferroresonance caused by an automobile accident resulting in an open conductor fault on an overhead line. These events cannot be controlled and hence, a more reliable solution, which will account for unexpected switching procedures, should be employed.

Walling *et al.* [29], [38] discuss the use of metal-oxide varistors (*MOV*) (a type of surge arrester) and the impact that they may have on ferroresonance. Short, Burke and Mancao [60] investigated the use of *MOVs* using digital simulation. From their results, they concluded that arresters are capable of clipping ferroresonant overvoltages without destroying themselves in some cases. However, Short, Burke and Mancao [60] explicitly highlight that arresters are not capable of knocking the system out of ferroresonance. Walling *et al.* [38], on the other hand, conducted experiments where *MOVs* were connected to amorphous core transformers. During their experiments, they found that the transformers remained in a ferroresonant state in the absence of arresters. However, with arresters present, the same circuit displayed dropout from the ferroresonant state. These results obtained by Walling *et al.* [38] confirm that *MOVs* possess the ability to eliminate ferroresonance. Furthermore, even if they do not eliminate the ferroresonant state, both simulations by Short [60] and physical tests by Walling *et al.* [38] confirm that *MOVs* can safely limit ferroresonant overvoltages.

Concerns regarding metal oxide arresters during ferroresonance were pointed out by Sakshuag, Burke and Kresge [75], where they suspected that the current conducted by a metal oxide arrester exposed to ferroresonance overvoltages of 2 p.u. would be up to a few hundred A. However, the tests conducted by Walling *et al.* [38] proved that this is not the case. This is because ferroresonance is described by [29] as a “weak source.” Walling *et al.* also make this claim in [38], as does Short in [3] and [60]. Short elaborates on this term in [3] by stating that ferroresonance is classified as weak since the overvoltages collapse once the arrester starts conducting. The result of this collapse is that the current conducted by the arrester is considerably less than expected. This was proved by Walling *et al.* [38], where an expected current of 6 kA based on the overvoltage was only measured as 400 mA in the presence of the arrester. The conclusion is drawn by Walling *et al.* [38] that the ferroresonant overvoltages in a power system in the absence of an arrester cannot be used to predict the arrester discharge currents. Another

concern regarding metal oxide surge arresters is that the threshold voltage of the arrester may be set to a higher value than the overvoltage caused by ferroresonance, as pointed out by Kováč *et al.* [76].

Thermal runaway of metal oxide arresters is still a concern for ferroresonance, as is pointed out by Walling *et al.* [38], especially for porcelain riser pole arresters. However, it was also found by both [29] and [38] that liquid-immersed arresters can survive indefinite exposure to ferroresonant overvoltages without experiencing thermal runaway.

Pavlas and Javora [77] recently discussed the use of the VT Guard Pro/VT Guard Pro-D to eliminate ferroresonance in VTs. This device is an implementation of the open-delta resistor discussed by Piasecki *et al.* [64]. The problem with the open-delta resistor is the size of the resistor. It requires a small resistor to damp out ferroresonant oscillations. This small resistance can cause thermal damage to the VT during network asymmetry. The VT Guard Pro replaces the conventional linear resistor with a nonlinear resistor. When a zero-sequence voltage is measured across the VT Guard Pro, the device has a very high resistance, which does not drain power from the VT. A predefined threshold value is set for the VT Guard Pro and can be varied by the user. When the zero-sequence voltage exceeds the threshold value for a defined duration, the VT Guard Pro is activated and the ferroresonant oscillations are damped out with a low ohmic burden. The VT Guard Pro-D is more sophisticated and has added diagnostic features for its internal circuits.

One final important note made by Walling *et al.* [38] is that amorphous cores do not add a thermal duty to their arresters. This would imply that metal oxide arresters are a viable solution to ferroresonance mitigation, even for transformers with reduced core losses, such as those being investigated by this report.

Recall from section 2.4 that Short [3] claimed that five-legged core constructions are susceptible to ferroresonance. This is also confirmed by Walling [37]. It is for this reason that Walling [37] suggests the use of triplex core construction as a potential mitigation technique. This mitigation technique, however, is not relevant to single-phase systems.

2.6 Transformer Modelling

Modelling the distribution transformer is perhaps the most important task of the entire project. Transformer modelling encompasses most of the topics discussed further in this literature review, including the modelling of hysteresis and frequency-dependent components. The focus of this thesis is on the ferroresonance of the transformer and, therefore, accurate transformer modelling is crucial.

The most important components to be modelled will be determined by the overall intent of the model. Models for high-frequency transients will differ from models used for steady-state analyses. Chiesa [74] claims that at low frequencies between *DC* (Direct Current) and a few kHz, transformer modelling is particularly difficult. This is because both frequency- and voltage-dependent nonlinear effects need to be accounted for simultaneously.

It should, therefore, be clear that transformer modelling is an important and complicated process. Cigre [9] claims that an exact representation of a transformer is more complex than any other network component. The ensuing subsections will discuss various topics related to the modelling of transformers.

2.6.1 Modelling Basics

Martinez and Mork [79] state that a transformer model can be separated into two parts: representation of the windings and representation of the iron core. The windings display linear behaviour. The core is nonlinear. Both the windings and the core are frequency dependent. It is important to note that Martinez and Mork [79] point out that core representation is critical for ferroresonance simulations. This section will look at some of the basics of transformer modelling.

Important Parameters

Based on the work of [9], [79] and [80], a list can be made of the most important aspects of transformer modelling for low- and mid-frequency transients, including:

- a Accurate representation of the core and its topology;
- b Inclusion of hysteresis/saturation effects;
- c Frequency-dependent winding losses and frequency-dependent iron core losses;
- d Careful representation & calculation of leakage inductance;
- e Inclusion of coil capacitances for accurate simulation of some transients;
- f Winding resistance;
- g Air-core inductance;
- h Residual flux and flux initialization;
- i Magnetic phase coupling;
- j Zero-sequence impedance, and
- k The piecewise-linear curve characterization of the nonlinear inductance and resistance.

The core topology is particularly relevant for three-phase transformers e.g. three-limb vs five-limb transformers because this can influence the magnetic path of zero-sequence flux. Frequency-dependent losses include eddy currents, which are discussed in greater detail in section 2.11.1. Hysteresis and saturation are discussed in detail in section 2.10. Leakage inductance and

windings resistance are typically included in the standard transformer model. Coil capacitances are not included in the standard transformer model but are very important. Core losses typically consist of hysteresis and eddy current losses, which have already been mentioned. Residual flux importance was discussed in section 2.2.3. Magnetic phase coupling and zero-sequence impedance only become relevant when dealing with three-phase transformers.

The importance of piecewise-curve characterizations is debatable. Abdull Halim, Dao, Phung and Fletcher [80] claim that this is one of the challenges of transformer modelling. However, section 2.7.2 warned against the inaccuracies of piecewise-linear models. Based on other literature referenced in this report, including [28], [78] and [81], a strong argument can be made to avoid piecewise linear models for ferroresonance simulation.

Abdull Halim *et al.* [80] claim that transients between 50 Hz and 20 kHz are classified as slow transients. This includes the frequency range of ferroresonance. For these types of transients, the SC impedance data, the saturation data and all losses are very important.

Losses

A lot of emphasis has been placed on the importance of losses in transformer modelling. Transformer losses can be subdivided into no-load losses and load losses. No-load losses are a very important component of transformer losses and are also referred to as excitation losses [82]. No-load losses can be broken down into three components: the hysteresis loss, the classical eddy current loss and the anomalous loss. No-load losses are measured in W/kg and are measured for a specific lamination thickness, frequency and peak flux density [83]. The no-load losses include the core loss, the dielectric loss, the conductor loss in the winding due to excitation currents and the conductor loss due to circulating currents in the parallel windings [82].

The core loss is the main component of the no-load losses. It is dependent on the magnitude, frequency and waveform of the test voltage [82].

The no-load losses are rather sensitive. Not only do they vary with temperature but they are also very sensitive to waveform distortion. Other design-related parameters, which can also influence the no-load losses, are:

- a The type and thickness of the core steel;
- b The geometry of the core joints, and
- c The core flux density

Furthermore, even in transformers with the same design, the following factors may also cause variance in no-load losses:

- a The variability in the core steel characteristics;
- b The mechanical stresses induced during manufacturing, and
- c The variation in joint gaps.

Akcay and Ece [83] state that in addition to the three types of no-load losses in a transformer (hysteresis, eddy current and anomalous), there is an additional loss in the core due to the “building factor,” which may cause an increase of up to 10% in core losses. This is generally a factor of the grade of the core material, core geometry, core sheet stacking pattern and quality of core construction.

Load losses form the other major component of transformer losses. Load losses are composed of copper losses in the windings and stray load losses [82]. Stray losses are caused by the electromagnetic flux in the windings, core, clamps, magnetic shields, enclosures and tank walls. Since stray losses are also found within windings, [82] subdivides stray losses into winding stray losses and stray losses in other components. Winding stray losses are defined as the losses due to eddy currents in the windings and the losses due to circulating currents between strands or parallel winding circuits [82].

No-load losses are measured using the open-circuit (OC) test. Load losses are measured using the short-circuit (SC) test. Both these tests are well-described by Guru [84].

Frequencies of Interest

It was stated earlier that the frequency of interest will determine the nature of the model being developed. Cigre [9] claims that for transformer energization studies, the frequency of interest ranges from DC to 1 kHz. Martinez-Velasco [82] provides a breakdown of important parameters for various transients. This breakdown is also given in Table 2-1.

Table 2-1: A breakdown of transformer modelling guidelines as seen in [82, Table 4.1]

Parameter/Effect	Low-Frequency Transients	Slow Front Transients	Fast Front Transients	Very Fast Transients
SC-Impedance	Very Important	Very Important	Important	Negligible
Saturation	Very Important	Very Important	Negligible	Negligible
Iron Losses	Important	Important	Negligible	Negligible
Eddy Currents	Very Important	Important	Negligible	Negligible
Capacitive Coupling	Negligible	Important	Very Important	Very Important

Martinez-Velasco [82] does not indicate the frequency ranges of the various classifications. However, Cigre [85] provides a table that indicates the frequency ranges of these transients. This table is repeated in Table 2-2. There is some overlapping between the four groups. Cigre [85] and Table 2-3 also indicate the types of transients, which fall within the various classifications.

In Table 2-1, the parameters that are marked as “very important” should be taken into account very carefully for all studies. Parameters that are indicated as “important” should be represented as accurately as possible. If the parameters are unknown, they can be estimated, but a sensitivity analysis should be done to ensure that they are not too sensitive to minor changes [86].

It is interesting to note that Table 2-1, describes capacitive coupling as negligible for low-frequency transients. Rezaei-Zare *et al.* [52] contradict this by saying that for a low-frequency phenomenon like ferroresonance, series inductive impedance in a power system is negligible when compared to the capacitive impedance. This means that the system can be effectively modelled by its power frequency capacitive characteristics. It should also be kept in mind that it was Rezaei-Zare *et al.* [52] who claimed that ferroresonance is classified as a low-frequency phenomenon since the frequency rarely exceeds 2 kHz.

Table 2-2: A breakdown of the frequency ranges as seen in [85, Table 2]

Group	Frequency Ranges for Representation	Shape Designation	Representation mainly for
1	0.1 Hz – 3 kHz	Low-Frequency Oscillations	Temporary Overvoltages
2.	50/60 Hz – 20 kHz	Slow Front Surges	Switching Overvoltages
3.	10 kHz – 3 MHz	Fast Front Surges	Lightning Overvoltages
4.	100 kHz – 50 MHz	Very Fast Front Surges	Restrike Overvoltages

Table 2-3: A breakdown of various transients and their associated frequencies as seen in [85, Table 1]

Transient	Frequency Range
Transformer Energization Ferroresonance	0.1 Hz – 1 kHz
Load Rejection	0.1 Hz – 3 kHz

Fault Clearing	50/60 Hz – 3 kHz
Fault Initiation	50/60 Hz – 20 kHz
Line Energization	50/60 Hz – 20 kHz
Line Reclosing	50/60 Hz – 20 kHz
Transient Recovery Voltage	
Terminal Faults	50/60 Hz – 20 kHz
Short line faults	50/60 Hz – 100 kHz
Multiple restrikes of Circuit Breaker	10 kHz – 1 MHz
Lightning surges, Substation Faults	10 kHz – 3 MHz
Disconnecter switching (single restrike) and faults in GIS	100 kHz – 50 MHz

Commonly Available Parameters

Transformer modelling is often complicated by a lack of data. Martinez-Velasco [82] says that geometrical data is only available to transformer manufacturers and this is considered proprietary. It is sometimes possible to estimate the parameters needed to make a transformer model accurate, but this requires knowledge of internal construction details. This means that sometimes a less accurate model must be chosen so that known parameters can be used. Martinez-Velasco [82] claims that this is perhaps the greatest challenge for the computation of transients.

Martinez *et al.* [56] claim that the following data is usually available for any transformer:

- a The power rating (S);
- b The voltage rating (V_{primary} , $V_{\text{secondary}}$);
- c The excitation current (I_{exc});
- d The excitation voltage (V_{exc});
- e The excitation losses (P_{exc});
- f The short circuit current (I_{sh});
- g The short circuit voltage (V_{sh}), and
- h The short circuit losses (P_{sh}).

Martinez *et al.* [56] also state that in some cases, the saturation curves and the capacitances may also be known, but this is not always available. Short [3] points out that transformer units with a rated power of less than 50 kVA have impedances of less than 2%. This implies that those

transformers have better voltage regulation and less voltage flicker for fluctuating loads. But it also implies increased fault currents on the secondary side of the transformer, which in turn have a greater impact on the primary side.

Short [3] discusses the impedance of single phase distribution transformers. However, the guidelines provided by Short differ from the requirements of the SANS 780 standard [2]. The SANS 780 standard dictates that the impedance of distribution transformers with a rated power of less than 200 kVA should be between 3.0% and 4.5%.

Core Representation

Core losses are traditionally modelled as a linear resistance. An example of this is shown in the parallel resistor R_c of Fig. 2-19. However, since the core losses are dependent on both hysteresis and eddy currents, the core losses differ for various levels of excitation. Using a constant value equal to the core losses at rated voltage is generally a good estimate. Furthermore, the importance of accurate core loss modelling is determined by whether the transformer is a single-phase or three-phase transformer. According to Iravani *et al.* [87], single-phase transformers show low sensitivity to variations in the value of the core loss resistor. However, three-phase transformers are highly sensitive to these variations.

Martinez *et al.* [56] claim that core losses are critically important to low-current phenomena such as ferroresonance. An accurate representation of the transformer core may require a good hysteresis model. However, Martinez *et al.* [56] claim that hysteresis losses are smaller than eddy current losses in modern transformers. Furthermore, at rated voltage, the winding losses are much smaller than the core losses, but above rated voltage, the winding losses can form a significant part of the total excitation losses.

For an accurate core representation, Martinez *et al.* [56] suggest obtaining excitation measurements at several voltage levels. This will assist in defining an accurate resistance to represent core losses. Martinez *et al.* [56] caution that unless it is accurately implemented, a nonlinear resistance model can have serious limitations. This is because hysteresis losses are dependent on the maximum flux and not the voltage. Furthermore, matching the losses at nominal frequency does not imply that the correct flux-current trajectory is followed.

Transformer Capacitance

Section 2.5 mentioned that transformer internal capacitance may be sufficient to self-sustain ferroresonance. Hopkinson [69] provides some guidelines on the source of transformer capacitances and how they can be modelled.

According to Hopkinson [69], transformer internal capacitance originates from three sources: capacitance between each winding and ground, the capacitance between individual windings and capacitance between the turns of each winding. Simulation of these capacitances can easily be achieved by making use of lumped capacitances connected to the winding terminals and ground. However, the interwinding capacitances are not represented in the simulation model. These lumped capacitances are a function of the voltage along and within the windings. Hopkinson [69] also claims that capacitances can be calculated by simple geometric relations using field theory. However, one should bear in mind that geometric relations of windings may also be considered proprietary, which complicates the process of calculating capacitance.

Section 2.6.3 discusses transformer models found in the literature. Circuit diagrams in that section will provide a better breakdown of how transformer capacitances are typically added to a transformer model.

2.6.2 Parameter Determination

Given the importance of transformer modelling, it is worth discussing literature on the determination of transformer parameters. Martinez-Velasco [82] states that there are two basic methodologies for determining transformer parameters:

- a Experimental determination through laboratory tests, and
- b Through electromagnetic field simulations using topological information and design data.

Inductance

The magnetizing and leakage inductance parameters are of paramount importance to produce correct transformer model performance [88]. These parameters can be determined using the OC test and the impedance test – both of which are described in the IEEE C57.12.91-1995 standard [88]. Note that the impedance test is more commonly known as the short-circuit (SC) test. Both the OC and SC tests are also described by Guru [84].

The OC test involves energizing the *LV* winding with the rated voltage and leaving the *HV* winding open-circuit – hence the name. The *HV* terminal voltage and the *LV* current are measured. Guru [84] explains that the test is conducted on the *LV* side because it is safer and there is a greater likelihood that a source of sufficient magnitude will be available in a test facility. The core loss can be measured during the OC test, which allows for the calculation of the magnetizing inductance and a constant core loss resistance.

For the impedance/SC test, the *HV* winding is energized and the *LV* winding is short-circuited. The voltage on the *HV* side is varied until rated current is measured. This typically occurs between

4% and 20% of rated voltage. Guru [84] advises that the voltage be applied on the *HV* side for safety reasons since rated current is to be measured. The SC test ensures a proper simulation of the leakage flux pattern and is, therefore, used to determine the winding resistance and the leakage reactance.

From the results of the OC and SC tests, the following parameters can be calculated [88]:

$$R_1 + R'_2 = \frac{P_{sc}}{I_{sc}} \quad (2-14)$$

$$L_s = \frac{1}{2\pi f} \sqrt{\left(\frac{V_{sc}}{I_{sc}}\right)^2 - (R_1 + R'_2)^2} \quad (2-15)$$

$$R_m = \frac{(V_{oc} - R_1 I_{oc})^2}{P_{oc}} \quad (2-16)$$

$$L_m = \frac{(V_{oc} - R_1 I_{oc})^2}{2\pi f Q_{oc}} \quad (2-17)$$

where

- R_1 = the primary ac resistance;
- R'_2 = the secondary ac resistance referenced to the primary side;
- $R_1 + R'_2$ = the total series ac resistance;
- P_{sc} = the active power from SC test;
- P_{oc} = the active power from OC test;
- Q_{oc} = the reactive power from OC test;
- V_{oc} = the *RMS* OC voltage;
- I_{oc} = the *RMS* OC current;
- V_{sc} = the *RMS* SC voltage;
- I_{sc} = the *RMS* SC current;
- L_s = the total series leakage inductance;
- R_m = the magnetizing resistance or core loss resistance;
- L_m = the magnetizing inductance, and
- f = the nominal frequency.

The series inductance and resistance are often divided equally between the primary and secondary sides on a per-unit basis. However, the ratio may also be determined by the ratio of the *DC* resistances of the primary and secondary windings. This is the approach used by [88].

Mork *et al.* [86] explain that the leakage inductance can also be calculated based on transformer design information that is given by

$$X_{HL} = \frac{\omega \mu_0 N^2}{h} \cdot \left(\frac{L_{mt,H} b_H}{3} + \frac{L_{mt,L} b_L}{3} + L_{mt,2} a_2 \right) \quad (2-18)$$

where

- a = the insulation thickness between coils;
- b = the coil thickness;
- $L_{mt} = 2\pi r_{mt}$ for a circular coil;
- r_{mt} = the mean turn length of the respective winding;
- f = the frequency;
- μ_0 = the permeability of air;
- h = the height of the winding, and
- N = the number of turns of *HV* or *LV* winding, depending on which side the leakage reactance is referred to.

Avendano, Mork, Ishchenko and Gonzalez [89] prove that the equation presented in (2-21) is for a cylindrical winding configuration. For a pancake configuration, the leakage inductance equation changes to

$$X_{HL} = \frac{\omega \mu_0 N^2 L_{mt}}{2nW} \cdot \left(\frac{b_H}{6} + \frac{b_L}{6} + a_2 \right) \quad (2-19)$$

where

- n = the number of coils composing the high voltage winding per phase and
- W = the width of the pancake coil.

Avendano, Mork, Ishchenko and Gonzalez [89] also state that the leakage inductance between windings and the core becomes important when the core is under heavy saturation because the additional inductance path has a greater influence on the magnetization currents. However, these leakage inductances are not accounted for in typical factory tests. The values of these leakage inductances are calculated as a proportion of the SC reactance between the innermost winding and the next closest winding to the core. If the transformer has only two windings, this is between the high- and low-voltage windings. Assuming that the low-voltage winding is closest to the core, the leakage inductance is given by

$$X_{LC} \approx K_1 \cdot X_{HL} \quad (2-20)$$

and

$$X_{HC} = X_{LC} + X_{HL} = (K_1 + 1) \cdot X_{HL} = K_2 \cdot X_{HL} \quad (2-21)$$

where X_{LC} and X_{HC} are the per-phase reactances between an infinitely thin “coil” at the surface of the core and the low- and high-voltage windings respectively. The values of K_1 and K_2 are dependent on the winding configuration e.g. cylindrical or pancake. If design dimensions are known, K_1 can be determined as the ratio of the insulation thickness between coils. For a cylindrical configuration, this amounts to $K_1 = \frac{a_1}{a_2}$, where a_1 is the insulation thickness between the core and its closest winding and a_2 is the insulation thickness between the two windings. If this design data is unknown, Avendano, Mork, Ishchenko and Gonzalez [89] claim that literature has calculated the factors to be $K_1 = 0.5$ and $K_2 = 1.5$ for a cylindrical winding configuration.

The inductance of air is often neglected in transformer models. However, Jazebi and de León claim in both [90] and [91] that when the transformer core enters deep saturation, the relative permeability becomes the same as that of air. This, in turn, affects the distribution pattern and magnitude of the magnetic flux in the air. Therefore, modelling the air as an inductance becomes important in such cases. Piecewise approximations can be used for this purpose, where the inductance is zero in normal conditions and has a constant slope line in deep saturation.

Jazebi, de León and Farazmand [92] present a method to measure the air-core inductance of the transformer. This is motivated by their claim that the air-core inductance is the most important parameter to be modelled for transients involving deep saturation, such as inrush current and ferroresonance. Cigre [9] describes the air-core inductance as the self-inductance of the coil in the absence of an iron-core. This means that the air-core inductance is the inductance of the winding when the core is completely saturated. Jazebi, de León and Farazmand [92] also claim that their proposed method does not need demagnetization of the core or zero-crossing switching, which is a significant advantage. Another advantage is that iron losses are eliminated due to the DC excitation, which is not the case with the standard OC test.

The method proposed by Jazebi, de León and Farazmand [92] makes use of a dc-biased ac source. Alternatively, a dc voltage source such as a simple static uncontrolled rectifier with ripple is also sufficient. The dc component forces the transformer into saturation with a small voltage and the ac component sweeps around the dc operating point. The test setup, as seen in [92, Fig. 2], is shown in Fig. 2-17. The resistor R_v controls the current and protects the source when transformers have low winding resistance. The winding and source resistances are eliminated from the measurement since the voltage is measured on the secondary side. A fast Fourier transform is used to extract the fundamental voltage and current and the air-core inductance is calculated using

$$L_{Air} = \frac{V_{2f}}{2\pi f n I_{1f}} \quad (2-22)$$

where

- L_{air} = the air-core inductance;
- V_{2f} = the fundamental secondary voltage;
- f = the fundamental frequency, and
- I_{1f} = the fundamental primary current.

A more elaborate discussion of this experiment can be found in [92].

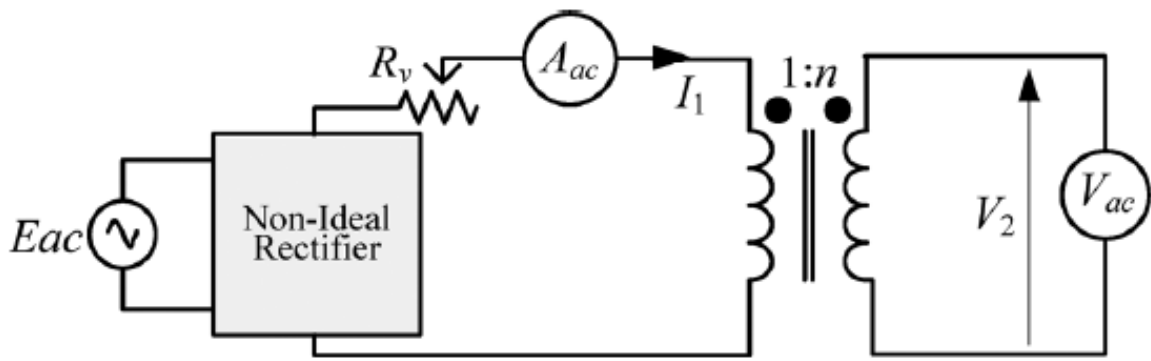


Fig. 2-17: A circuit diagram of the air-core inductance determination setup, obtained from [92, Fig. 2]

Capacitance

Cigre [9] claims that the representation of stray capacitance is only marginally important for the study of low-frequency transients such as transformer energization. However, it becomes very important if de-energization transients are simulated to calculate residual flux. This also makes it particularly important for ferroresonance. Finally, Cigre [9] also claims that stray capacitances influence the OC test and the excitation characteristics. However, Cigre [9] is not clear on what is meant by “stray capacitance.” In some cases, the term is used to refer to stray capacitance when a circuit breaker is open. In other cases, it seems to refer to transformer capacitances.

Mork *et al.* [86] also emphasise that capacitance effects may be significant and should be included in a transformer model. However, Mork *et al.* [86] specifically refer to the capacitance between the HV and LV windings and capacitances between the windings and the core. A capacitance exists between the HV and LV windings because the windings are separated by oil and paper, which effectively forms a parallel plate capacitor. Assuming that the LV winding is closest to the core, this capacitance is connected from the outside of the LV winding to the inside of the HV winding. Mork *et al.* [86] also add that there are capacitances between the HV and LV windings

and the tank. These capacitances include the effect of the bushings. However, there is no capacitance between the core and the tank because both of them are grounded. Fig. 2-18 shows a breakdown of all these capacitances. Note that the capacitances are divided by two to partially distribute their effects. It is also possible to have capacitances between the *HV* windings of the phases of a three-phase transformer.

Hopkinson [69] presents a step-by-step method to measure the capacitances of the transformer:

- a Connect both ends of the primary to ensure an equipotential surface;
- b Connect both ends of the secondary to ensure an equipotential surface;
- c Connect the core to the tank;
- d Connect (a) and (b) and measure the capacitance from the primary and secondary to ground (core and tank) with a capacitance bridge; the value obtained is C_{HL-G} ;
- e Connect (b) and (c) and measure the capacitance from the secondary core and tank to the primary; the value obtained is C_{H-LG} , and
- f Connect (a) and (c) and measure the capacitance from the primary core and tank to the secondary; the value obtained is C_{L-HG} .

In item (d), C_{H-L} is short-circuited so that C_{HL-G} yields $C_{HL-G} = C_{H-G} + C_{L-G}$. In item (e), C_{L-G} is short-circuited so that $C_{H-LG} = C_{H-G} + C_{H-L}$. In item (f), C_{H-G} is short-circuited so that $C_{L-HG} = C_{H-L} + C_{L-G}$. Solving these equations yields values for C_{H-G} , C_{L-G} and C_{H-L} :

$$\begin{aligned} C_{H-G} &= \frac{1}{2}(C_{H-LG} - C_{L-HG} + C_{HL-G}) \\ C_{L-G} &= \frac{1}{2}(-C_{H-LG} + C_{L-HG} + C_{HL-G}) \\ C_{H-L} &= \frac{1}{2}(C_{H-LG} + C_{L-HG} - C_{HL-G}) \end{aligned} \quad (2-23)$$

If transformer design data is available, it is once again possible to calculate the capacitances shown in Fig. 2-18. Mork *et al.* [86] state that for a core-type transformer with cylindrical windings, winding-to-winding and winding-to-core capacitance can be calculated with:

$$C = \frac{2\pi\epsilon_0\epsilon_r \cdot h}{\ln\left(\frac{d_2}{d_1}\right)} \quad (2-24)$$

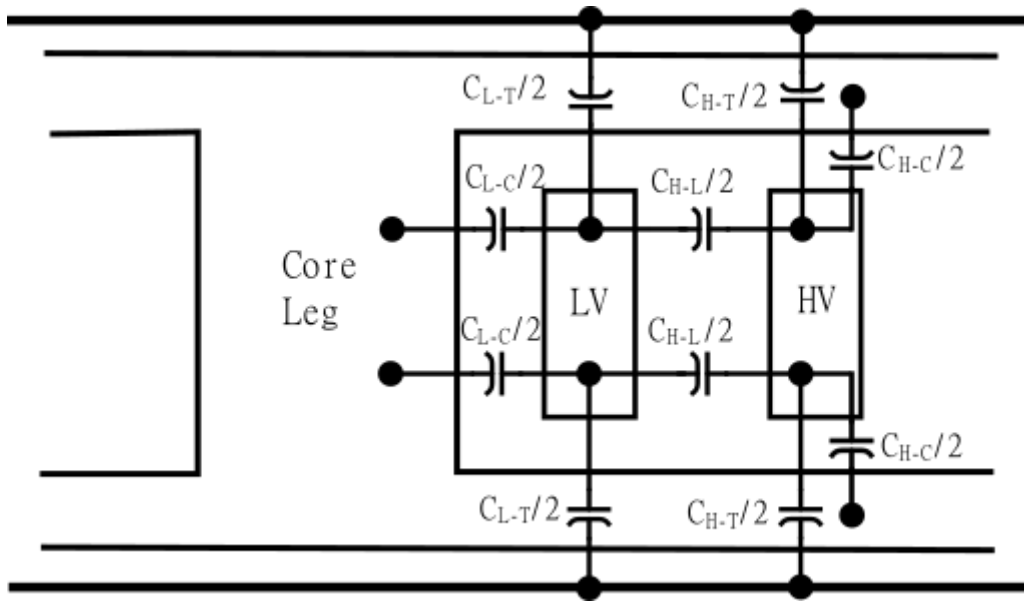


Fig. 2-18: The capacitances between the windings, tank and core as suggested by Mork *et al.* [86, Fig. 10]

where

- ϵ_0 = the permittivity of free space;
- ϵ_r = the relative permittivity of the dielectric;
- h = the height of the winding;
- d_1 = the inner diameter of *HV* winding, and
- d_2 = the outer diameter of *LV* winding.

If the capacitance is between the winding and the core, then d_2 becomes the external diameter of the core leg and d_1 is the inner diameter of the *LV* winding.

For shell-type transformers using interleaved pancake windings, (2-24) can still be used to calculate the capacitance between the winding and the core. However, the parallel plate equation must be used for the capacitance between the windings.

2.6.3 Models in Literature

Section 2.6 has been dedicated to the topic of transformer modelling. It started by providing a thorough motivation for the need for an accurate transformer model. It then proceeded to discuss the basics of transformer modelling and transformer parameter determination. This last subsection will discuss the various transformer models, which can be found in the literature.

Semlyen and de León [93] explain that there are five mainstream models used for transformer modelling. The first of these is *modelling based on self and mutual inductances*. This is based on very accurate equations which can be used to calculate self and mutual inductance. However,

the presence of an iron core leads to ill-conditioned equations. This is solved by subtracting a common flux from the self and mutual inductances to ensure that their numerical values are not so close to each other. Semlyen and de León [93] state that this approach is equivalent to the direct use of leakage inductances.

The second mainstream model mentioned by Semlyen and de León [93] is *modelling based on leakage inductances*. These models adequately represent the leakage inductance of the transformer, which leads to a good representation of load and short circuit conditions. However, the core of the transformer is not well modelled in this type of model.

The third type of mainstream model is *modelling based on the principle of duality*. In this case, the core of the transformer is modelled accurately at the cost of accurate leakage inductance. Semlyen and de León [93] state that this type of model has recently been used for the modelling of highly saturated conditions.

The fourth type of model is *modelling based on measurements*. This is based on performing transformer tests in the frequency domain or in the time domain. The drawback of this type of modelling is that accuracy cannot be guaranteed for transformers that have not been tested. Semlyen and de León [93] add that some general trends can be inferred from the trends based on design, size, manufacturer etc. but accuracy cannot be assured.

The final type of model discussed by Semlyen and de León [93] is *analyses based on electromagnetic fields*. This is commonly used by designers of large transformers for the calculation of design parameters. The technique of finite elements is the most common analysis technique in this category. It is also generally accepted that three-dimensional field analyses are required. These methods are not used for transient simulation since they require expensive simulations.

The most common transformer model is the *T* equivalent circuit [94], [95]. A circuit diagram of the T-model is shown in Fig. 2-19. This is the standard model used in introductory textbooks such as Guru [84]. In Fig. 2-19, R_1 and R_2 represent the winding losses of the primary and secondary windings respectively. R_c represents the core losses of the transformer. L_m represents the magnetizing inductance and may also sometimes be nonlinear to consider saturation effects. L_1 and L_2 represent the leakage reactances of the windings. The T-model is used for steady-state analyses where transformer internal capacitances do not play a significant role. Therefore, there are no capacitances in Fig. 2-19

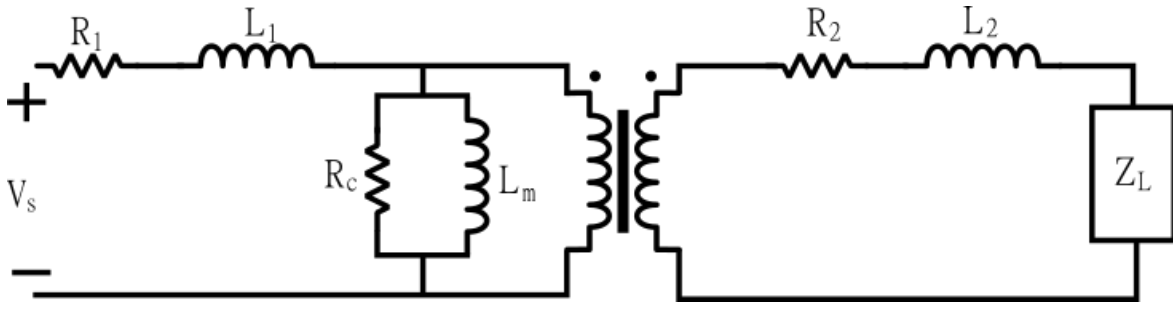


Fig. 2-19: The circuit diagram of a transformer T-model

Cazacu, Petrescu and Ionita [48] used a modified version of the T-model, also referred to as the Steinmetz T equivalent circuit, to model ferroresonance. Their [48] circuit is shown in Fig. 2-20. In this case, the network capacitance is added in series to the winding resistance. One also notes that only the primary winding resistance and leakage inductance is shown because the secondary of the transformer is left open. The magnetizing inductance is represented by a nonlinear inductor depicting the relationship between flux and current. The H/B Brauer function is used to describe this relationship between flux and current. Using systematic parametric and initial condition studies, Cazacu, Petrescu and Ionita [48] could establish that only fundamental and subharmonic ferroresonance could occur.

Zirka *et al.* [94] present a couple of arguments against the use of the classical T-model. Firstly, there is no correspondence between the classical T-scheme parameters and the physical arrangement of the windings. This is confirmed by [82]. Furthermore, the T-scheme cannot reproduce the fact that the flux density in the yoke of the transformer reaches substantially lower levels than the limbs of the transformer during saturation.

Martinez and Mork [79] provide an alternative breakdown of existing transformer models to that presented by [93]. According to [79], there are three main approaches to transformer modelling. These approaches are matrix representation (BCTRAN models), saturable transformer components (STC models) and topology-based models.

The matrix representation, also referred to as a BCTRAN model, involves steady-state matrix equations of the form

$$[V] = [Z][I] \quad (2-25)$$

which is referred to as the branch impedance matrix. Cigre [9] describes the BCTRAN model as an n-phase model that is capable of considering inter-winding coupling. It uses coupled RL or RL¹ matrices to represent SC impedances between windings, load losses at rated frequency and linear inductive magnetization in special cases [9]. However, this is only valid for steady-state

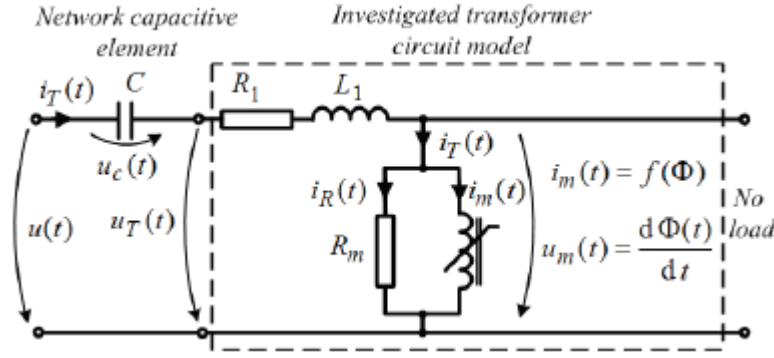


Fig. 2-20: The Steinmetz T-model used for ferroresonance simulation as seen in [48, Fig. 1]

conditions. When transients are being evaluated, the branch impedance matrix needs to be rewritten to:

$$[v] = [R][i] + [L][di/dt] \quad (2-26)$$

where $[R]$ and $j\omega[L]$ are the real and imaginary components of $[Z]$ respectively. Phase-to-phase coupling and terminal characteristics are considered in this approach. Differences in core and winding topology are not taken into consideration because the same mathematical treatment is given to all the transformers.

Small excitation currents and short-circuit impedances, which are particularly important to transfer characteristics, may be problematic in this type of matrix equation. The problem is solved by making use of the admittance matrix $[Y]$ as opposed to the impedance matrix $[Z]$. Equation (2-25) then becomes

$$[I] = [Y][V] \quad (2-27)$$

where $[Y]$ always exists and can be obtained directly from the SC test. It still holds that $[Y]$ must be broken up into resistive and inductive components for transient studies. Martinez and Mork [79] also state that the transformer can be described by the following equation:

$$[di/dt] = [L]^{-1}[v] - [L]^{-1}[R][i] \quad (2-28)$$

Martinez and Mork [79] state that all the models are linear. Saturation and hysteresis may need to be included in the models for transient studies, which would make the models nonlinear. They [79] also add that the excitation currents may be linearized and left in the matrix but this may become inaccurate when the core saturates. Alternatively, the excitation may be left out of the matrix and attached externally. Such an approach is also mentioned by Cigre [9] and Borrill [96]. This is not necessarily topologically correct but is often accurate enough. Cigre also cautions that when the core is externally attached to the innermost winding, it should be connected before the

winding resistance to account for inrush current damping. Martinez and Mork [79] conclude by stating that although the matrix approach is theoretically only accurate for nominal frequency, it is adequate up to a frequency of 1 kHz. This is also confirmed by [56].

The second type of model mentioned by Martinez and Mork [79] is the saturable transformer component (*STC*) model. Cigre [9] states that it is a nonlinear version of the classical Steinmetz model. Recall that Cazacu, Petrescu and Ionita [48] used a modified version of the Steinmetz T-model, which was also shown in Fig. 2-20. The *STC* model is described by Cigre [9] as a two- and three-winding single-phase transformer model. It is based on a star-circuit connection [9], [56], [79]. An example of such a star-circuit connection is shown in Fig. 2-21.

Cigre [9] states that the *STC* model represents the SC impedance between the windings, the load and magnetization losses and a nonlinear magnetizing inductor. The nonlinear magnetizing inductor is attached at the star point. Cigre also advises that if the transformer only has two windings, the core representation can be attached to the winding closest to the core. However, when the transformer has three windings, it becomes mandatory to attach it to the winding closest to the core. This is because the star point voltage has no relation to core flux and attaching the nonlinear inductor at the winding closest to the core prevents numerical instability.

The input data to the *STC* model is the R-L values of each star branch, the turn's ratio and the parameters for the magnetizing branch of the core [79]. The primary branch of the model is treated as an uncoupled R-L branch. All other windings are treated as two-winding transformers [9].

Cigre [9] states that the BCTRAN model can also be used for single-phase transformers and should yield the same results as the *STC* model. Furthermore, the *STC* model is limited to three windings, since the star circuit is not valid for more than three elements [79], [96]. The BCTRAN model does not have this limitation [9]. Another drawback of the *STC* model is that the connection of the core model to the star-point is not topologically correct [79].

References [9], [56] and [79] all agree that the *STC* model can be extended to a three-phase model through the addition of a zero-sequence reluctance parameter. However, the application then becomes limited and Cigre [9] cautions that magnetic coupling is then ignored, making it inaccurate for unbalanced operation.

The final type of transformer model mentioned by Martinez and Mork [79] is the topology based model. According to Martinez-Velasco [82], topology-based models are superior to matrix approximations for the study of low-frequency electromagnetic transients. This is mainly because the location of the core equivalent in a matrix approximation is arbitrary. This may lead to incorrect results, especially for transients where transformer saturation is important. Zirka *et al.* [94] note

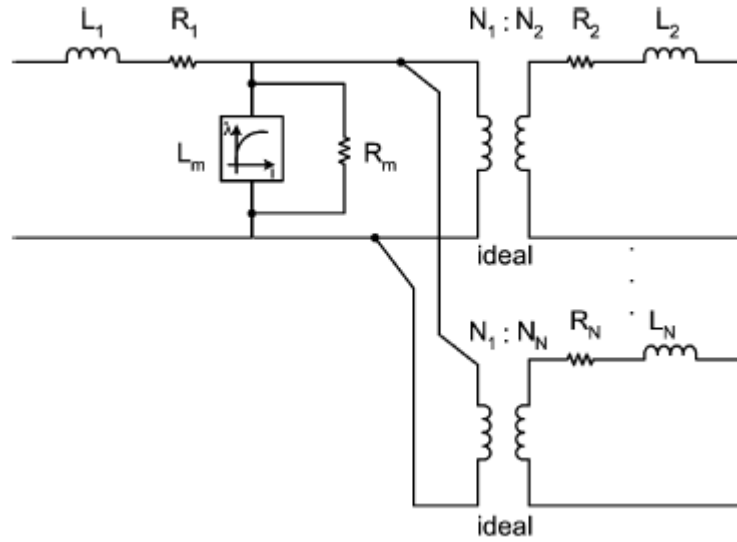


Fig. 2-21: The star-circuit connection of an STC model found in [79, Fig. 2]

that topologically correct models specifically focus on the magnetic fluxes in the core and the windings. This means that it is clear where the core should be connected in the circuit [82]. Therefore, the most important difference between matrix-based approximations and topological models is how the core and the windings are assembled.

Despite the accurate representation of the transformer core, there are some drawbacks to topology-based models in general, which should also be noted. Zirka *et al.* [50] state that less attention is paid to the fluxes, which close in the nonmagnetic space outside of the windings of the transformer. Jazebi *et al.* [97] state that these “air” flux paths represent fringing magnetic fluxes and they flow roughly parallel to the iron branches. This means that the magnetic flux not only flows in the iron core but also in the surrounding media. According to Jazebi *et al.* [97], these fringing magnetic flux elements are significant during saturation and important to predict low-frequency transients involving deep saturation of the core.

Zirka *et al.* [50] make an important note on their topological model's ability to model inrush currents. Analytical evaluation of inrush currents requires knowledge of the air-core inductance of the excited winding. For the construction of a reliable reversible topological model, knowledge of the core geometry is also critical in addition to knowledge of the air-core inductance. Standard no-load and short-circuit tests are insufficient in this regard. This means that the modelling of inrush currents is impossible without detailed knowledge of the core geometry, which is not always available to the engineer. One can furthermore conclude that if this applies for inrush current modelling, it will also apply for the modelling of ferroresonance.

Under the classification of topology-based models, Martinez and Mork [79] define two subcategories: geometric models and models based on the principle of duality. Geometrical models consider the core topology but they have a mathematical description. They are based on the formulation [56], [79]:

$$[v] = [R][i] + [d\lambda/dt] \quad (2-29)$$

Martinez and Mork [56], [79] state that the coupling between the magnetic and electrical quantities takes the core topology into account. Martinez and Mork [79] also provide a list of models proposed in the literature, which can be classified as geometric models.

Models based on the principle of duality are discussed more frequently in literature. Martinez and Mork [79] state that such an approach results in a model that includes the effect of saturation in each leg of the core. The model also considers interphase magnetic coupling as well as leakage effects. Jazebi *et al.* [97] state that the principle of duality is recognized as a physically correct technique to obtain critical models of power transformers.

Jazebi *et al.* [97] indicate two advantages of models based on the principle of duality. The first of these is that a physical magnetic circuit of electromagnetic devices can be converted to its electric equivalent. The electric equivalent is suitable for simulation in electromagnetic transient programs with standard circuit elements. A second advantage of the approach is that it can describe the distribution of magnetic flux in the core of the transformer. This provides critical information on the electromagnetic behaviour of the transformer elements. Martinez and Mork [79] also add that the model derived from the principle of duality is independent of the winding configuration.

One of the disadvantages of a duality based model, according to Mork *et al.* [86], is the lack of a detailed leakage inductance representation in the short-circuit model. They [86] claim that this shortcoming can be addressed by using an inverse inductance matrix for the leakage inductances. Martinez and Mork [79] add that the winding resistance, core losses and capacitive coupling effects are not obtained from the principle of duality. However, [56] states that these elements can simply be added externally to the electrical equivalent circuit.

A few guidelines exist for modelling with the principle of duality. The first step is to derive a magnetic equivalent circuit for the transformer. Martinez and Mork [56], [79] provide a guideline for how this can be achieved. Windings are converted to MMF sources, leakage paths become linear reluctances and magnetic cores become saturable reluctances. Once this magnetic circuit has been constructed, the magnetic circuit is converted into an electrical equivalent circuit. Jazebi *et al.* [97] provide the following guidelines for how this can be achieved:

- a Reluctances are transformed into inductances;

- b Nodes become loops;
- c Loops become nodes, and
- d MMF sources (current-controlled voltage sources) become current-controlled current sources.

Martinez and Mork [79] explain this concept by stating that the mesh and node equations of the magnetic equivalent circuit are the node and mesh equations of the electrical equivalent circuit respectively.

Martinez and Mork [79] furthermore indicate that the current sources are replaced with ideal transformers to make the model more practical. They [79] state that this helps to ensure primary-to-secondary isolation, it isolates the core and it preserves the winding ratio. Jazebi *et al.* [97] elaborate on the list of advantages of ideal transformers. Firstly, the inductors of the dual circuit need to be referred to a common number of turns. Jazebi *et al.* [97] state that usually this is selected as $N=1$. This means that ideal transformers are required to reflect the correct equivalent inductance to all terminals of the transformer and maintain proper voltage conversion. Ideal transformers are also required for some three-phase transformers to maintain accuracy. Thirdly, the ideal transformers facilitate the series and parallel connections of different windings, which becomes important for mid-frequency models. Finally, the inductors describing the magnetic core and the resistors/capacitors describing the effect of electric charges are completely different. The ideal transformers isolate the electrical parts from the magnetic parts [97].

An example of a single-phase shell-type transformer model based on the principle of duality is shown in Fig. 2-22. This model was derived by Martinez and Mork [79]. The core parameters are referenced to the LV winding. The capacitances and winding resistances are connected externally to the coupling transformers. This holds the advantage that the core functions are independent of winding configuration [79].

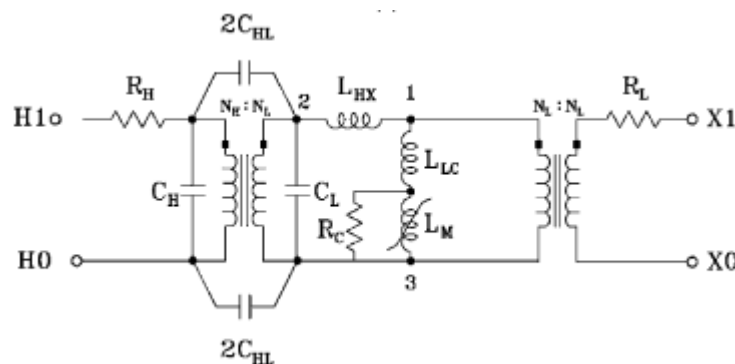


Fig. 2-22: A single-phase transformer model based on the principle of duality [79, Fig. 3b]

The principle of duality has been used to derive a model which has become known in the literature as the π -model. The π -model is strikingly similar to the T-model. However, Jazebi *et al.* [95] compared the performance of the π -model to the T-model for transient simulations and found the π -model to be vastly superior – especially for ferroresonance modelling. The π -model is also discussed by [82], [88], [97] and [98]. A schematic of the π -model is shown in Fig. 2-23.

The parameters of the π -model are determined from the standard OC and SC tests, as was the case with the T-model [82], [95]. Martinez-Velasco [82] explain that the internal information of the transformer can be used to separate the magnetizing effects into the two branches. However, if internal information is not known, one can simply double the magnetizing parameters [82]. The relationship between the parameters of the T-model and the π -model without any knowledge of internal construction is given by:

$$\begin{aligned} L_{sh} &= L_H + L_L \\ R_C &= R_y = 2R_m \\ L_C &= L_y = 2L_m \end{aligned} \quad (2-30)$$

Jazebi *et al.* compared the π -model to the T-model with a focus on ferroresonance simulation in [95]. Farazmand, de León and Joseph [88] compared these two models with a focus on inrush current modelling. In both cases, it was found that the π -model was superior.

For the ferroresonance tests completed by Jazebi *et al.* [95], the secondary of the transformer was left open and a series capacitor was connected to the primary winding. The core was demagnetized before the experimental tests and the capacitor was discharged before each experiment. The nonlinearities of the core were represented with piecewise linear inductor models – not detailed hysteresis models. It was found that the maximum voltage simulated by the π -model was within 5% of the experimental results. The oscillation frequency and the current waveforms also matched those of the experiment. The T-model, on the other hand, predicted maximum voltages that were 44% higher than the experimental results with noticeable differences between the waveforms. Furthermore, the T-model predicted ferroresonance even when the

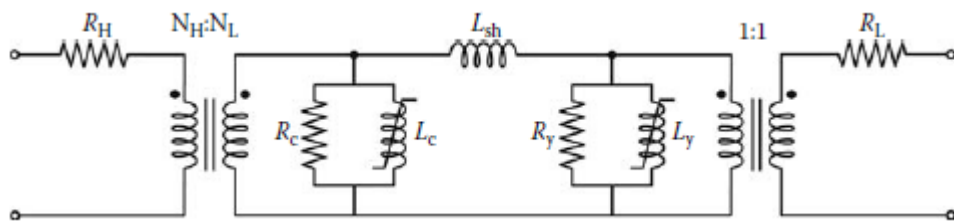


Fig. 2-23: A π -shaped model schematic representation as seen in [82, Fig. 4.6]

experimental results showed none. The π -model did not suffer from this drawback. Based on these results, Jazebi *et al.* [95] conclude that the π -model is superior to the T-model for ferroresonance simulation.

In their π -model, Jazebi *et al.* [95] make use of nonlinear inductors to represent the saturation of the core. The input to the nonlinear inductor in both MATLAB and ATPDraw is a list of data points that depicts the nonlinear relationship between flux-linkage and current. Since the π -model contains two nonlinear inductors, in comparison to the single inductor used in the T-model, the data points need to be modified for use in the π -model. Equation (2-30) stated that the magnetizing inductance needs to be doubled to be used in the π -model. This can be achieved by doubling the flux linkage or by halving the current. Borrill [96] states that it is preferable to halve the current, rather than doubling the flux or flux linkage. This is because the flux is determined by the source, while the current is determined by the circuit. Furthermore, the shunt inductor needs to draw half the current. Investigation of [88, Table VI] and [88, Table VII] will show that this was also the approach employed by Jazebi *et al.* in [95].

The reader should also note that [88] and [95] contain an error with regards to the flux-linkage/current data points used for simulations. This error is discussed in greater detail in section 3.1.

With regards to inrush current modelling, the π -model also showed better results. Farazmund, de León and Joseph [88] state that the T-model can cause errors of up to 75% in inrush current modelling. The π -circuit, on the other hand, predicts the inrush current within a few per cent. They [88] hypothesize that the inaccuracy of the T-model is caused by the primary winding leakage inductance, which limits the circulation of current to the magnetizing branch. One notes in Fig. 2-23 that the primary branch only contains the winding resistance. The leakage reactance is connected between the two sections of the core. In the case of the T-model (Fig. 2-19), the leakage inductance is connected before the magnetizing branch. This leakage inductance prevents large currents from being drawn. In the π -model, the path of the inrush current is open to one of the magnetizing branches, which makes it more accurate.

Farazmand, de León and Joseph [88] also investigated the impact of distributing the magnetizing impedance unevenly between the two magnetizing branches of the π -model. They found that when the distribution is 50% between each magnetizing branch, the error of the simulation was approximately 10.9%. The best accuracy was achieved with a distribution of 32.5%, where the error became 0%.

Corea-Araujo *et al.* [98] also compared the π -model to the T-model for ferroresonance simulation but with a focus on including hysteresis in the magnetizing branches. They [98] also conclude that the π -model is more accurate for ferroresonance simulation than the T-model. Furthermore, they claim that hysteresis effects significantly improve results. In some cases, the simulation was only accurate when hysteresis was included [98].

Jazebi, de León, Farazmand and Deswal [91] developed an improvement on the π -model. The improvement was based on the claim that the π -model transient behaviour varies depending on the winding being energized. This means that the air-core inductance of the model needs to be varied depending on whether the transformer is being energized from the *HV* or *LV* winding. In their improved model, which they term a reversible model⁷, this variation of the air-core inductance is no longer necessary and the results seem to indicate more accurate simulations.

This reversible model developed by [91] is based on the measurement of the air-core inductance discussed at the end of section 2.6.2. The test setup for this experiment was shown in Fig. 2-17. Having determined the air-core inductance as seen from the primary and secondary windings, the following equations become relevant:

$$L_{air-in} = \frac{L_{1-sat}(L_s + L_{2-sat})}{L_{1-sat} + L_{2-sat} + L_s} \quad (2-31)$$

$$L_{air-out} = \frac{L_{2-sat}(L_s + L_{1-sat})}{L_{1-sat} + L_{2-sat} + L_s} \quad (2-32)$$

where

L_{air-in} = the air-core inductance from the inner winding;

$L_{air-out}$ = the air-core inductance from the outer winding;

L_{1-sat} = the slope of the deep saturation part of hysteresis branch corresponding to the inner winding;

L_{2-sat} = the slope of the deep saturation part of hysteresis branch corresponding to the outer winding, and

L_s = the leakage inductance determined from the SC test.

Combining these two equations yields a quadratic equation. The solution to the quadratic equation will always be real and is given by

⁷ Note that this is the same reversible model discussed in Section 2.7.2, which discusses specific models focused on ferroresonance.

$$L_{2-sat} = \left(\frac{L_s}{L_{air-out} - L_{air-in} - L_s} \right) \times \left(L_s - 2L_{air-out} \pm \sqrt{L_s^2 + 4L_{air-in}L_{air-out}} \right) \quad (2-33)$$

When L_{2-sat} is determined, L_{1-sat} can be determined from (2-31). These parameters are then used to define the slope of the hysteresis curve from the last point to infinity.

Jazebi, de León, Farazmand and Deswal [91] prove that this new slope in the deep-saturation region of the transformer allows for better simulation of peak inrush currents when compared to the conventional π -model. However, it should also be noted that the determination of the air-core inductance, as described in section 2.6.2, is a complicated process requiring specialised equipment. The parameters cannot be determined from the standard OC and SC tests. The argument can therefore be made that the marginal improvement provided by this new model does not warrant the effort of the additional tests.

Mork *et al.* [86] developed an alternative model based on both the matrix representation of transformers and the principle of duality. This model is referred to as the Hybrid transformer model and is available in standard electromagnetic transient programs such as *ATP*. Mork *et al.* [86] claim that the model is suitable for frequencies up to 5 kHz. The Hybrid model uses the strength of the inverse inductance matrix to represent leakage inductances while using the principle of duality to maintain a topologically correct core representation. The model also includes frequency-dependent winding resistances and capacitive effects, which are attached externally to the model.

Vaessen [99] discusses a completely different approach to transformer modelling based on the principle of modal analysis. Modal analysis is also discussed in [100] and [101]. However, this approach will not be discussed in detail in this literature for a couple of reasons. Firstly, Vaessen [99] describes the model as a high-frequency model. Recall that ferroresonance is a low-frequency phenomenon. Furthermore, Vaessen [99] also states that the single-phase model derived by this approach does not consider the saturation effects of the transformer core. Throughout this literature study, the importance of hysteresis and saturation for ferroresonance modelling has been emphasised. Finally, the tests required to model a transformer using modal analysis are specialised tests – transformer data cannot be obtained from the standard OC and SC tests.

To conclude this chapter on transformer models in the literature, a final note should be made on transformer capacitances. Several literature sources, including [10], [79], [82] and [98], include the following sources of capacitance in their model: capacitance between the *HV* and *LV* winding, capacitance between the *HV* winding and ground and capacitance between the *LV* winding and ground. A circuit diagram of the T-model with these capacitances is shown in Fig. 2-24. Fig. 2-25 shows the π -model with externally connected capacitances. The model in Fig. 2-25 is the most

physical representation of a transformer yet available [82]. It makes use of Cauer circuits⁸ to represent the frequency dependency of eddy current effects in both the windings and the core. It also contains nonlinear inductances to represent saturation and hysteresis and the capacitances mentioned earlier.

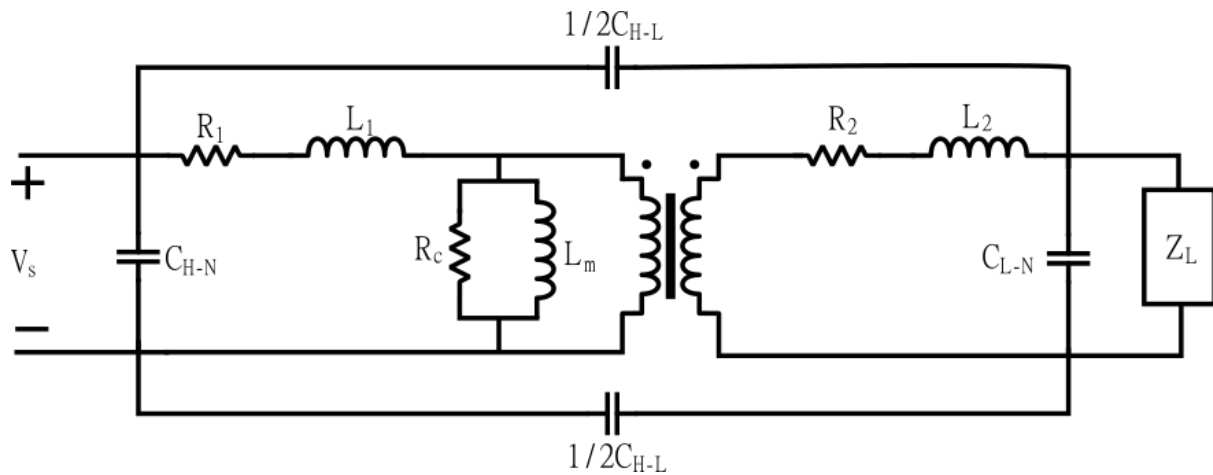


Fig. 2-24: The circuit diagram of a T-model with transformer capacitances

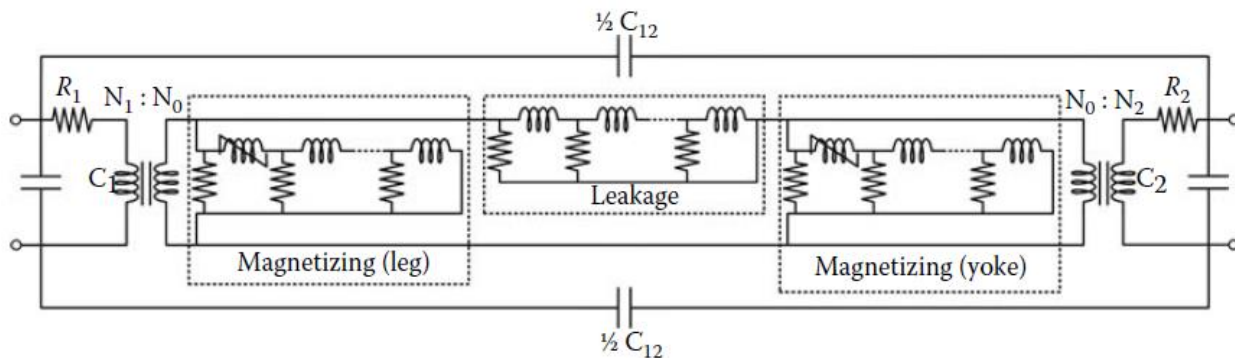


Fig. 2-25: A detailed π -model with transformer capacitances as seen in [82, Fig. 4.38]

2.7 Ferroresonance Simulation

Under the hypothesis that ferroresonance is the cause of transformer failure, it is important to look at simulation and modelling techniques, which are focused on ferroresonance. Section 2.6 discussed transformer modelling in general, but this section focuses on simulation techniques, which are particularly relevant to ferroresonance.

⁸ Cauer circuits are discussed in Section 2.11.4

The difficulty of modelling ferroresonance is that the ferroresonant frequency becomes a moving target. The change in system inductance implies that there is a range of capacitive values for which ferroresonance can occur.

The nonlinear nature of ferroresonance makes analytical solutions particularly difficult. Though Cigre [6] does present a few analytical solutions, it claims that time-domain digital simulation has proven most effective. Cigre [6] also claims that analytical solutions used for ferroresonance analysis differ from those used for linear resonance.

This section of the report will firstly discuss a few general modelling guidelines, these include important parameters, which need to be modelled. These important parameters relate to section 2.2.3, where ferroresonance dependencies were discussed. Thereafter, the report will look at a few mathematical techniques, which have been used in the past. It will also discuss specific techniques frequently mentioned in the literature, such as bifurcation diagrams, in more detail.

2.7.1 General Analysis

Mork [54] provides some guidelines for ferroresonance modelling in his introduction to ferroresonance. He claims that important power system components, which need to be modelled, include:

- a The steady-state Thevenin equivalent of source;
- b RLC coupled-pi models for lines and cables;
- c Shunt and series capacitances, and
- d Stray capacitances.

Mork [54] adds that for long lines, the coupled-pi models need to be cascaded. He also indicates that the stray capacitances being referred to here are interwinding and winding-ground capacitances. Iravani *et al.* [87] confirm this but add that transformer bushing and busbar capacitances may also play a role. Above all else, Mork [54] states that the transformer model used should be of the correct topology and should include core saturation and core losses. Iravani *et al.* [87], once again, confirms that the transformer model is the most important and add that modelling results are most sensitive to magnetic saturation and core loss. With regards to the source, Iravani *et al.* [87] claim that source representation is not critical unless the source contains nonlinearities. It is for this reason that the Thevenin-equivalent should suffice.

The importance of core loss is also highlighted by Walling, Barker, Compton and Zimmerman in [30]. In their abstract, they claim that ferroresonant overvoltages are directly related to the ratio of capacitance susceptance over core losses. The core loss, however, needs to be accurately modelled. An attempt was made by [30] to model secondary loading as added core-loss to the

transformer. However, this resulted in inconsistent results. Walling, Barker, Compton and Zimmerman hypothesize that this is because the secondary loads they added to the transformer were entirely ohmic, while hysteresis has an effect on core loss and that effect differs from a purely ohmic load. This highlights the importance of accurate core-loss measurement – especially the effect of hysteresis.

The importance of hysteresis modelling is also highlighted by [78] and [102]. Both claim that the hysteresis loop determines the stability domain of ferroresonance modes. Lamba, Grinfield, McKee and Simpson [78] did tests whereby the width of the hysteresis loop was varied. While reducing the width, they found that ferroresonance was initiated at lower levels of excitation. Rezaei-Zare, Iravani and Sanaye-Pasand [102] claim that if the hysteresis loop is not accurately modelled according to the measured hysteresis data, the model will not be able to accurately model overvoltages, nor will it identify all modes of ferroresonance. Lamba, Grinfield, McKee and Simpson [78] reach the same conclusion. They proceed to claim that even when piecewise-linear inductors are used in parallel with resistors to simulate core losses, the results are not as accurate as when hysteresis is properly modelled. It is for this reason that the introduction to this literature study claimed that hysteresis modelling is so important. This is also why hysteresis and hysteretic models receive a dedicated discussion in section 2.10.

Hysteresis modelling is closely related to the saturation characteristics of the transformer core. Jazebi and de León [90] claim that the saturation inductance and the winding resistance are the dominant parameters for transients, these transients include inrush currents, geomagnetic-induced currents and ferroresonance. They proceed to claim that this influences the transient requirements for different windings. Their discussion, however, is focused on multi-winding transformers. Multi-winding transformers do not apply to this study but the importance of accurate modelling of saturation inductance and winding resistance is clear from their discussion.

The introduction to this section on ferroresonance simulation mentioned that simulation of ferroresonance is particularly difficult due to its nonlinear nature. Nevertheless, several mathematical and analytical tools have been used to model and detect ferroresonance more accurately. Moses and Masoum [33] presents a summary of some of these tools, which includes:

- a Flouquet Theory;
- b Continuation Methods;
- c Lyapunov Stability Analysis;
- d Galerkin Method;
- e Fractal Mathematics;
- f Phase-plane and Poincaré Maps, and
- g Bifurcation Theory.

Lyuonov analyses are also mentioned by Mork and Stuehm [58] and Bo and Tiecheng [103]. Bo and Tiecheng [103] state the Lyuonov indexes are used in nonlinear dynamics to describe the distance between two points in the phase space. This is done as time evolves. The phase space holds a close reference to Poincaré maps and phase plane diagrams discussed in sections 2.7.4 and 2.7.5 respectively. The mathematical definition of the Lyuonov index is:

$$\lambda = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \left| \frac{\Delta x(t)}{\Delta x(0)} \right| \quad (2-34)$$

where λ represents the Lyuonov index and Δx represents a change in the coordinates of points on the phase-plane.

Of the seven tools mentioned by [33], the Galerkin Method, Phase-plane diagram, Poincaré Map and Bifurcation diagram are most commonly found in the literature. Ferracci [26] confirms the use of continuation methods, the Galerkin method, Poincaré maps and bifurcation methods. Additionally, Ferracci [26] also lists the following useful methods:

- a Analog simulation;
- b Time-domain digital simulation;
- c Perturbation methods, and
- d Isolats.

Analog simulation is not a sensible method, since it is difficult to model a real case with any accuracy [26]. This is also confirmed by Cigre [6]. Time-domain digital solutions are claimed to be the most powerful by Cigre [6], but Ferracci [26] cautions that they may not be realistic. This is because ferroresonance is highly sensitive to initial conditions, which means that a study must be conducted for each possible combination of parameters. It is for this reason that other methods, such as the Bifurcation diagram, are used to detect ferroresonant transitions as a certain parameter is slowly varied. Following such identification of critical parameters, the digital simulation is useful in confirming results and specifying waveforms.

The continuation method is classified as a tool of bifurcation theory. It describes a numerical method whereby the influence of a certain parameter is investigated by slowly varying that parameter and observing its impact. Mork [54], for instance, used this method to investigate the impact of capacitance values on the initiation of ferroresonance.

Bifurcation diagrams, Poincaré maps, phase-plane diagrams and the Galerkin method will be discussed in sections 2.7.3, 2.7.4, 2.7.5 and 2.7.6 respectively.

The perturbation method involves a simplification of the system by ignoring certain parameters, such as losses. Once the resulting simplified equations have been solved, the parameters are reintroduced and the system is solved around the limits of the previously obtained solutions. Ferracci [26] claims that this is particularly useful in conjunction with the continuation method.

Isolats involve the continuation of isolated curves obtained employing bifurcation theory – hence the name. Ferracci [26] claims that the perturbation method works well in solving these types of problems. Thereafter, the continuation method is used on the isolated curves. It can, therefore, be seen that Isolats do not involve an independent mathematical technique, but rather a combination of those techniques already mentioned. It should further be clear from the preceding discussions that the continuation method is a branch of bifurcation theory, the perturbation method involves the continuation method and Isolats involve a combination of both. Bifurcation theory, therefore, forms a core of many of the methods listed thus far – hence its regular appearance in other literature.

Bifurcation diagrams are also mentioned by Tanggawelu, Mukerjee and Ariffin [8], as are S-curves. These are used to verify the existence of ferroresonance. Thereafter, [8] claims that phase-plane portraits are used to verify the type of nonlinearity. Tanggawelu, Mukerjee and Ariffin [8] do not elaborate on what they mean by the “type of nonlinearity,” but it is assumed that this refers to the ferroresonance mode (fundamental, sub-harmonic etc.) since the phase-plane diagram differs for different modes.

Tanggawelu, Mukerjee and Ariffin [8] also refer to a mathematical technique not mentioned before – wavelet analysis. Wavelet analyses are used to identify the dominant frequencies of the ferroresonance waveforms. This is similar to the use of a Fast Fourier Transform (*FFT*). However, [8] claims that wavelet analysis has two advantages over the use of an *FFT*: It preserves time information and is not limited to only periodic functions. Sherbain Osman and El-Hag [63] also make use of wavelet analysis. The advantages listed by [63] confirm those of [8]. More importantly, Sherbain, Osman and El-Hag [63] made use of wavelet transforms and an artificial neural network to distinguish ferroresonance from other transient effects, such as capacitor switching.

Kong, Zhang and Bo [53] also confirm the use of Fourier Transforms and Wavelet Analyses as potential methods of ferroresonance identification. However, they also highlight that fundamental ferroresonance has a frequency equal to that of the power frequency, which makes it difficult to distinguish from a single-phase earth fault. As such, [53] presents the following list of mathematical methods/algorithms employed in literature in an attempt to improve ferroresonance detection:

- a Negative Selection Algorithm;
- b Hilbert-Huang Transform;
- c Fractal Theory;
- d S Transform Matrix;
- e Genetic Algorithm, and
- f Particle Swarm Optimization Algorithm.

It should be noted, however, that Kong, Zhang and Bo [53] claim that these methods require a computer with strong calculation capability and a large number of sample data. Therefore, Kong, Zhang and Bo [53] suggest using the excitation characteristics of the transformer to determine the saturation degree of the core. Transformer cores, which have gone into ferroresonance, should saturate more deeply than those in steady-state or those experiencing a single-phase-to-earth fault.

Irrespective of the mathematical model used to detect and simulate ferroresonance, literature makes it very clear that ferroresonance is highly sensitive to initial conditions. As such, Cigre [6] highlights the importance of conducting a large number of simulations using a particular topology to gain confidence in that particular scheme. This principle was also applied by [40], where they ran the simulation 25 to 50 times.

2.7.2 Models in Literature

Literature also contains a few models, which have been developed for transient simulation. This subsection serves to highlight some of these models.

Horak [10] claims that numerous sources of literature have used the fundamental frequency linear circuit approach to modelling ferroresonance over the years. It is based on finding a maximum capacitive reactance X_C based on the inductive reactance X_M of the transformer. The intent is then to limit the values of capacitive voltage and inductive voltage to some maximum by defining a minimum ratio of X_C/X_M . However, there are several drawbacks to this approach. The first is that it involves excessive simplification of a very non-linear phenomenon. The second is that transformer designs have seen a lot of changes since the technique has been proven.

A convincing argument is presented by Walling [37] as to why such approaches should not be employed to evaluate ferroresonance. Firstly, it is common practice to define the inductive reactance X_M of the transformer as the ratio of the rated voltage to the excitation current at rated voltage. However, Walling [37] claims that distribution transformers show some saturation even below rated voltage, which has a significant effect on calculations. This simplified definition of X_M results in a value substantially less than the actual, unsaturated reactance of the transformer core.

Furthermore, Walling [37] also claims that measurements of X_M are greatly affected by the coil capacitance connected to the transformer. The coil capacitance has the effect of compensating for the magnetizing reactance, which means that the measured value of X_M is substantially higher than the actual value. Moreover, Walling [37] claims that it is virtually impossible to accurately measure the magnetizing reactance of a completed transformer. Finally, when Walling [37] attempted to calculate the critical capacitance based on an X_C/X_M ratio, there was a very weak correlation between the reality and the predicted values – roughly 50%. However, when the prediction of critical capacitance was made based on the core losses, the correlation was close to 97%. This provides convincing evidence that critical capacitance values are not determined by the ratio of system capacitance to magnetizing reactance, but by the ratio of system capacitance to core losses. For this reason, fundamental frequency linear circuit analysis, as described by Horak [10], is not a good ferroresonance modelling technique.

Horak [10] also discusses a dual-state X_M analysis, which he claims is a better depiction of reality. In essence, it involves defining the impedance of the transformer core as a piecewise-linear function, where there are two values for X_M – a saturated and an unsaturated value. It should be noted, however, that Horak [10] does not apply these techniques to model an actual transformer. Rather, he uses these two techniques to present the reader with a better understanding of the ferroresonance phenomenon.

Jazebi and de León [90] present what they refer to as a reversible model to simulate low-frequency transients, including ferroresonance. The term “reversible” is defined in [94] as a model, which is valid independent of whether the *HV* or *LV* winding is excited. This model derived by Jazebi and de León [90] is based on the application of the principle of duality. The principle of duality is discussed in section 2.6. Note furthermore that the model derived by Jazebi and de León [90] is for a transformer with n -windings. However, it is based on the models derived in [104] and [105].

The leakage inductances of the transformer are represented by linear inductors and mutual couplings [90]. The mutual inductances compensate for missing linkage flux between the windings. The importance of mutual inductances is discussed in [105]. Traditional models made use of negative inductances, as opposed to mutual inductances. These negative inductances were not realizable and were also the cause of numerical oscillations [105]. The advantage of mutual inductances is that they are related to physical flux paths in the transformer window and they are readily implemented in simulation programs, such as *EMTP*. Álvarez *et al.* [104] also claim that the circuit elements can be determined in two ways: by observing the behaviour of the magnetic field in the transformer window and by performing short-circuit tests at the transformer terminals.

It should be noted that iron core losses in [90] are modelled by hysteretic inductors. This is one of the claimed advantages of the model – it can be built with circuit elements, which are all available in the *EMTP*. This hysteretic magnetizing inductor is connected in parallel with a core loss resistor, as demonstrated in Fig. 2-26. However, the reader should note that several other sources of literature have cautioned that the use of such nonlinear inductors in an attempt to model hysteresis is not accurate. Zou *et al.* [106] specifically highlight the inaccuracy of the renowned Type-96 hysteretic inductor found in *EMTP*. This inaccuracy is confirmed by Rezaei-Zare and Iravani [28]. They claim that the inaccuracy of the model is due to its piecewise-linear nature. Changing the operating branch from an unsaturated to a saturated value can mimic the effect of switching the inductance on and off. This adds stress to the model and causes errors in simulation. This implies that either the modelling of hysteresis is not as critical for ferroresonance simulation as claimed by literature or that the results of the model presented by [90] are not as accurate as claimed.

Equations are presented in [90] for the calculation of the leakage inductances of the various windings, their mutual coupling, the core loss resistances and also the hysteretic inductance of the inductors used to simulate hysteresis. However, it is considered beyond the scope of this report to present all of these equations. For a more detailed discussion of this reversible model, refer to [90]. The full circuit used to model the multi-winding transformer, as also seen in [90, Fig. 5] is shown in Fig. 2-26.

Zou *et al.* [106] present a basic ferroresonant circuit, along with its reduction to a Thevenin equivalent circuit. Recall that Mork [54] highlighted the Thevenin equivalent circuit as one of the important parameters for ferroresonance simulation. The basic ferroresonant circuit and its reduced Thevenin equivalent, as seen in [106, Fig. 1], is shown in Fig. 2-27. Note that the Thevenin equivalent voltage also changed based on the value of the capacitors. The voltage E_b of the basic ferroresonant circuit is given by $E_b = E_s \sin \omega t$. The Thevenin equivalent voltage E_{Th} is given by $E_{Th} = \frac{C_{series}}{C_{series} + C_{shunt}} E_s \sin \omega t$. For this specific example, C_{series} represents the circuit breaker grading capacitance. However, circuit breaker grading capacitance is not relevant to this study, since grading capacitances are not required for circuit breakers below 132 kV [41]. Nevertheless, it can be argued that the series capacitance of the underground cables and overhead lines of the system can contribute to this series capacitance. The shunt capacitance C_{shunt} in Fig. 2-27 represents the total phase-to-earth capacitance of the entire system, including that of the transformer windings. Finally, the behaviour of the inductor is described by the value of the current flowing through the inductor, i_L . This current is a function of the flux linkage of the transformer and hence $i_L = f(\phi_L)$. Zou *et al.* [106] claim that the nonlinear inductor makes the entire system nonlinear and is incredibly important in determining the ferroresonant mode.

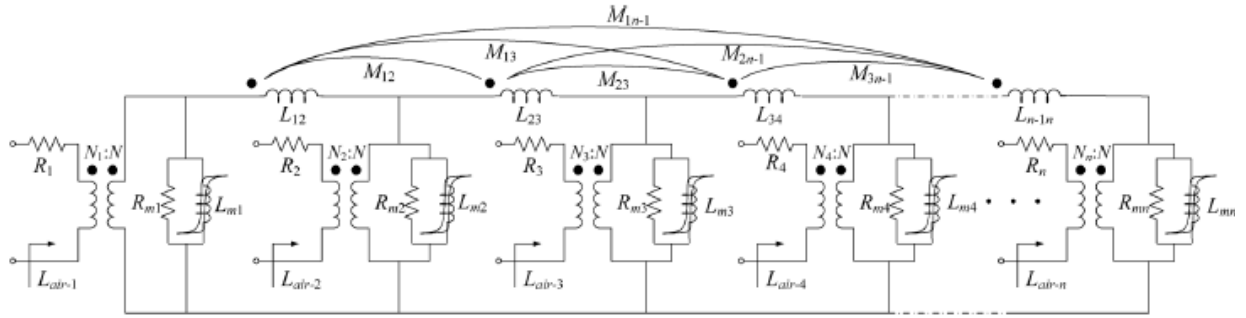


Fig. 2-26: A reversible multiwinding transformer model found in [90, Fig. 5]

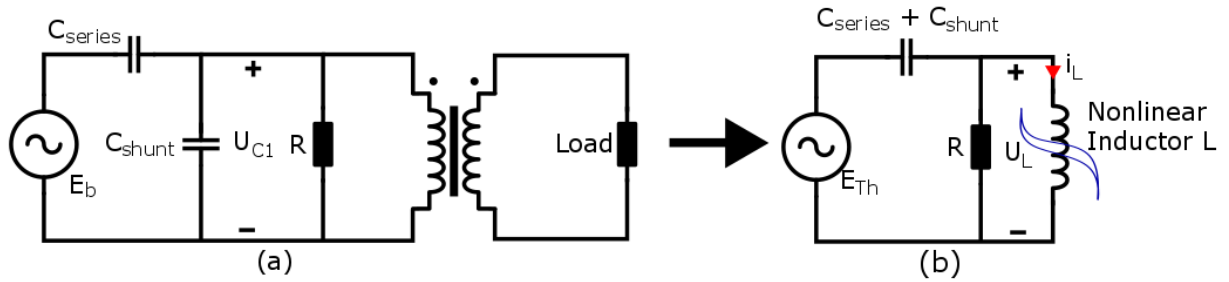


Fig. 2-27: The basic ferroresonant circuit and the reduced Thevenin equivalent as seen in [106, Fig. 1]. a) Basic ferroresonant circuit; b) Reduced Thevenin equivalent

Kong, Zhang and Bo [53] present another transformer model, which also makes use of a nonlinear inductor. The model is focused on ferroresonance simulation in a three-phase 10 kV system. Four different transmission lines were connected to the same bus, with a potential transformer connected to the bus. A single phase of the transformer model is shown in Fig. 2-28. This model is very similar to the standard T-model discussed in section 2.6, though the nonlinear inductor is not in parallel with the core loss resistance. The leakage inductances on the secondary side of the potential transformer, R_2 and L_2 , are referred to the primary side by the square of the winding

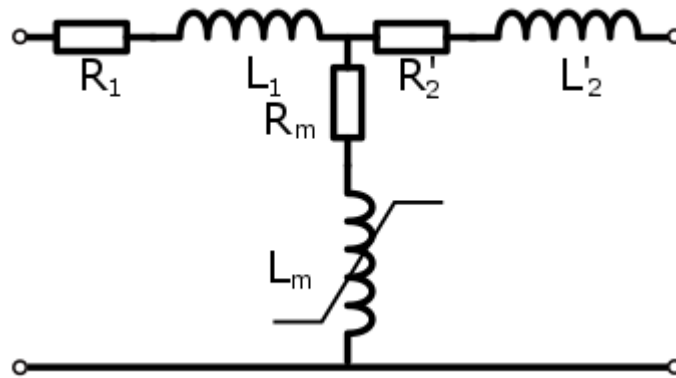


Fig. 2-28: A single-phase potential transformer model as seen in [53, Fig. 6]

ratio. The secondary side of the potential transformer is left open since potential transformers are typically lightly loaded.

Ferroresonance, in this case, is simulated by initiating a single-phase-to-earth fault on one of the lines and then removing it. The capacitance of the system is varied by changing the length of the lines. The lines are modelled with the help of the Bergeron model⁹.

It should be noted, however, that the focus of [53] is not to determine whether or not ferroresonance will occur. Rather, [53] aims to identify ferroresonance based on the excitation characteristics of the potential transformer. More importantly, the aim of [53] is also to distinguish ferroresonance from a single-phase-to-earth fault. This is done by calculating the excitation inductance of the transformer based on the excitation voltage and the rate of change of the excitation current. The excitation inductance of the transformer is then compared to some threshold inductance value and the ratio of these two values indicates whether the core has saturated. If there is an indication of core saturation, then the overvoltages are due to ferroresonance.

Stosur *et al.* [107] present a ferroresonance model for an *HV* Inductive *VT*, which was developed by making use of the *ATP* (Alternative Transients Program) environment. Since the focus is placed on an *HV* system, the circuit breaker grading capacitance becomes relevant. The authors of [107] also added several other elements to make the simulation more accurate, these included:

- a Adding a series resistor for winding resistance;
- b Adding a series linear inductor for winding leakage reactance;
- c Adding a parallel capacitance for equivalent winding capacitance;
- d Making use of the TYPE-98 nonlinear inductor model for magnetization characteristics;
- e Using a linear resistor to model core losses based on experimental measurements, and
- f Removing all load on the secondary side of the *VT* to account for the worst-case scenario.

Based on the elements added to the *ATP* model, it is clear that the model is not novel. It can be found that the series winding resistance, series leakage reactances and parallel core loss resistances are found in the typical T-model used for steady-state analyses [4]. The parallel capacitance accounting for winding capacitance and the use of the TYPE-98 inductor, however, is unique. The *ATP* circuit, as seen in [107, Fig. 5], is shown in Fig. 2-29. Stosur *et al.* [107] used this model to vary the circuit breaker capacitance and the source voltage to determine parameter values, which could potentially cause ferroresonance.

⁹ The Bergeron model is discussed in section 2.14.

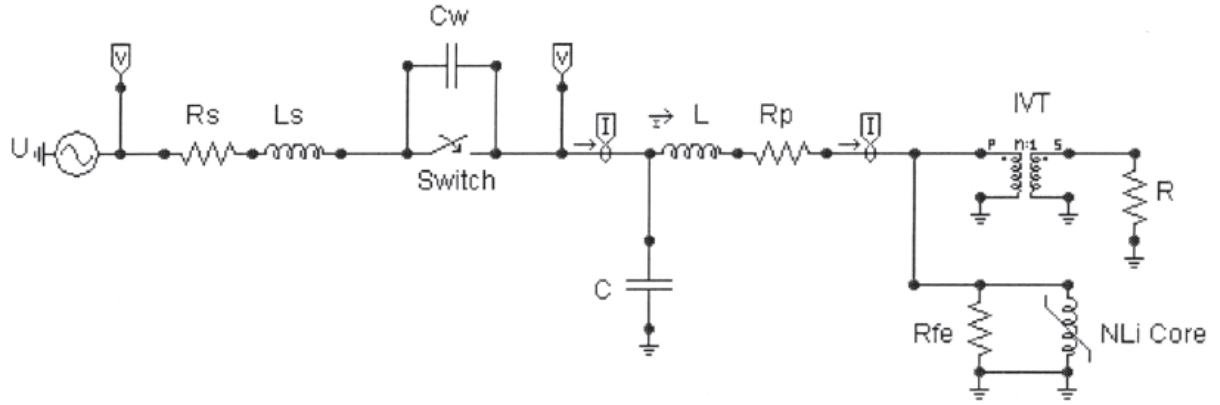


Fig. 2-29: An ATP/EMTP model used for ferroresonance simulation, as seen in [107, Fig. 5]

Burgos *et al.* [39] make use of an ATP model very similar to that used by [107]. However, Burgos *et al.* [39] make use of a variable core-loss resistor, which is dependent on the flux. This is based on measurements. In their model, they [39] also investigate the impact of circuit breaker capacitance on the initiation of ferroresonance. Burgos *et al.* [39] also developed the same ferroresonance model in a MATLAB environment. The MATLAB model can be seen in [39, Fig. 15].

Cazacu, Ionita and Petrescu [34] present a model, which is similar to that seen in [106]. However, they [34] elaborate by also presenting the differential equations, which need to be solved. The iron core, as was the case with the circuit seen in Fig. 2-27, consists of a core loss resistor and a nonlinear inductor. This can be seen in Fig. 2-30, which is a replica of [34, Fig. 2]. Additionally, Cazacu, Ionita and Petrescu [34] also add a resistor, R_p , which models the network series losses. The parameter L_p is representative of the leakage inductance of the transformer, though it can also take into account the inductive effects of cables. The network capacitance is represented by a single equivalent capacitor, C . The blue dotted line around the core loss resistance and magnetizing inductance indicate the nonlinear representation of the iron core. Based on this circuit, the following differential equations can be defined:

$$\begin{aligned}
 \frac{di_T(t)}{dt} &= \frac{1}{L_p} [e(t) - u_c(t) - (R_m + R_p)i_T(t) + R_m i_m(t)] \\
 \frac{d\phi(t)}{dt} &= R_m [i_T(t) - i_m(t)] \\
 \frac{du_c(t)}{dt} &= \frac{1}{C} i_T(t) \\
 e(t) &= E\sqrt{2} \sin(\omega t + \varphi) \\
 i_m(t) &= \gamma \phi(t) + \xi \phi^{11}(t)
 \end{aligned}
 \tag{2-35}$$

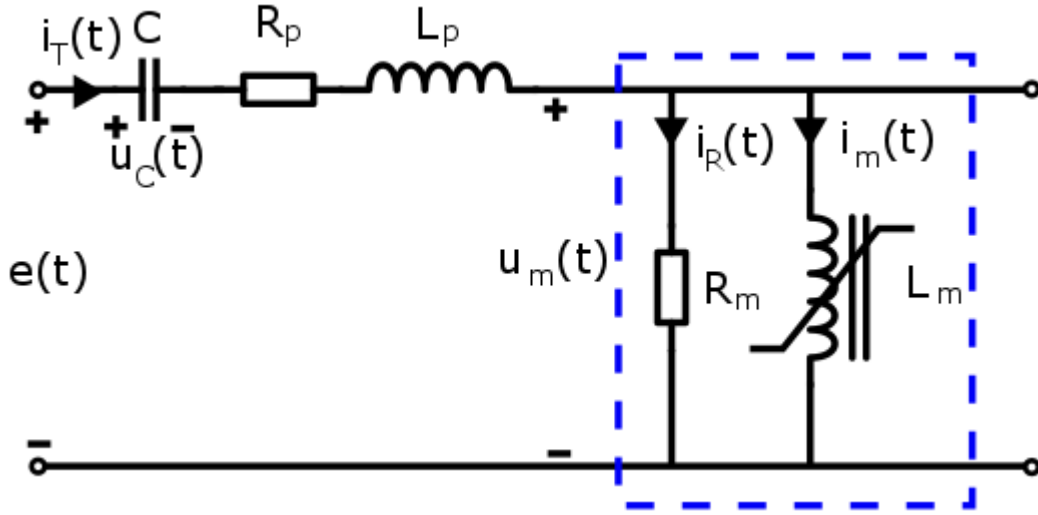


Fig. 2-30: A ferroresonant simulation circuit as seen in [34, Fig. 2]

where:

- $i_T(t)$ = the total current;
- $e(t)$ = the supply sinusoidal voltage;
- $u_C(t)$ = the voltage on the network capacitance;
- $i_m(t)$ = the magnetizing current through the nonlinear inductance L_m ;
- $\phi(t)$ = the core magnetic flux as a function of time;
- L_p = the leakage inductance and cable inductive effects;
- R_m = the core loss resistance;
- R_p = the network and series losses, and
- γ & ξ = parameters based on transformer manufacturer data.

One will note that the magnetizing current $i_m(t)$ is a function of the parameters γ and ξ . This is indicative of the fact that the magnetizing current is fitted to match the core magnetic properties (the B-H loop).

Cazacu, Ionita and Petrescu [34] claim that the parameters used in their model are based on basic data. From their process flow diagram in [34, Fig. 3], the device electrical data and magnetic core characteristics are based on the rated data of the device and the measured magnetic (B-H) properties. The rated data refers to the standard data available from the transformer rating plate and the short-circuit and open-circuit tests. Having obtained this data, as well as the network resistance capacitance and inductance, the differential equations in (2-35) are solved with a Runge-Kutta-Fehlberg method. Cazacu, Ionita and Petrescu [34] also make use of phase-plane diagrams and Poincaré maps to further analyse the ferroresonance phenomenon.

In [48], Cazacu, Petrescu and Ionita conducted another investigation into the ferroresonance of toroidal transformers. The model used for their simulations is identical to that of Fig. 2-30, but a different representation of the magnetizing current was used. The alternative representation of the current is shown in (2-36). The values of α , β and γ are obtained with a curve fitting procedure of the core characteristics.

$$i_m(t) = \alpha\phi(t) + \beta\phi(t) \exp[\gamma\phi^2(t)] \quad (2-36)$$

The rest of the differential equations remain the same. It should furthermore be noted that in [48], Cazacu, Petrescu and Ionita use the Rosenbruck method to solve the differential equations, as opposed to the Runge-Kutta-Fehlberg method in [34].

Mork [108] presents a detailed three-phase model for a five-legged transformer, which was implemented with the help of EMTP. Due to the added complexity of the three phases, this model will not be discussed in detail. However, it is worth noting that he makes use of the TYPE-93 inductor available in EMTP to model the core nonlinearity.

In the examples provided in this section, it can be seen that a piecewise linear approximation is often used to model the core losses of the transformer. In most cases, the piecewise linear inductor was placed in parallel with a core-loss resistor, though [53] placed them in series. However, the importance of an accurate hysteresis model has been emphasised throughout this report. These models may create the perception that even with simplified inductor models, ferroresonance can be modelled accurately. However, [28], [87] and [109] emphasise the importance of an accurate transformer core model. Rezaei-Zare and Iravani [28] claim that this is in contrast to the popular belief that a single-valued approach to magnetization modelling is sufficient for ferroresonance modelling. They [28] claim that an accurate core model, which takes hysteresis into account, is a necessity for two reasons: 1) asymmetric excitation conditions make single-valued approaches inaccurate and 2) specific transients are highly dependent on hysteresis minor loops and dynamic core inductance. Mousavi and Engdahl [110] confirm this second claim regarding the minor loops. They [110] state that the ideal hysteresis model for ferroresonance and inrush current modelling should be able to model both symmetric and asymmetric minor loops.

Gaudreau *et al.* [111] focus on the value of the core loss resistance and claim that this, too, is not a fixed value. Their discussion agrees with the findings of Burgos *et al.* [39], where a variable core loss resistor was used based on the flux in the core. Gaudreau *et al.* [111] develop a model, which calculates the magnetizing resistance of the transformer based on the instantaneous flux in the core. The results of their simulation model prove that the resistance is not constant, since their

instantaneous flux approach leads to an overall transformer core model, which produces accurate hysteresis and coercivity. It is also found by Gaudreau *et al.* [111] that the core loss resistance decreases by a factor of 10 between an applied voltage of 1 p.u. and 1.4 p.u. Furthermore, weak inrush current and overexcitation results in lower values of core loss resistance. This indicates that modelling the core loss resistance by a single resistance value based on rated measurements is inaccurate. However, it should also be noted that Rezaei-Zare and Iravani [28] claim that even this approach is inaccurate for asymmetric hysteresis loops. De León and Semlyen claim that the core-loss resistance is frequency-dependent [112].

Based on these discussions by [28], [87], [109] and [111], it is clear that transformer core modelling for ferroresonance requires careful consideration. It also leads one to the conclusion that the models in this section are not necessarily sufficiently accurate, despite their claimed results. Rezaei-Zare and Iravani are adamant that hysteresis loops significantly impact ferroresonance studies and that single-valued magnetization characteristics are not sufficient. They make this claim in [28], [102] and [109].

2.7.3 Bifurcation Diagrams

Bifurcation theory in general is defined as “a branch of applied mathematics concerned with the structural stability of ordinary differential equations under the variation of one or more parameters” [113]. Corea-Araujo *et al.* [114] motivate the use of bifurcation diagrams on the basis of simulation time. Parametric studies can involve thousands of simulations, which take a significant amount of time. Bifurcation diagrams reduce this simulation time by recording all the locations of abrupt changes in the final operating state of a system as one or more parameters are varied very slowly. To emphasize this slow variation of parameters, Corea-Araujo *et al.* [114] define the variation as “quasi-static.”

The definition of a bifurcation diagram provided by Mork and Stuehm [58] differs from that of Corea-Araujo *et al.* [114]. Mork and Stuehm [58] claim that bifurcation diagrams are developed by plotting magnitudes taken from a family of Poincaré sections as the system parameter is varied. This is confirmed by Sakarung [115]. Poincaré maps are defined in section 2.7.4. Mork and Stuehm [58] add that single-valued areas on these diagrams are indicative of period one ferroresonance. Dual valued diagrams indicate period two ferroresonance. Blurred areas indicate chaotic ferroresonance.

The term “bifurcation diagram” is based on the definition of a bifurcation. A bifurcation, or period-doubling, is defined by Mork and Stuehm [58] as “a transition from period one to period two.” However, Mork and Stuehm [58] also point out that the term bifurcation can also refer to transitions

from one mode to another in a general sense when a system parameter is varied. It should also be noted that a transition into chaotic ferroresonance mode may be referred to as a “catastrophe.”

Kieny [46] provides yet another definition of bifurcation. However, to understand his definition, one must first understand the concept of a fixed point, which involves a discussion of Poincaré mapping. Assume that the state variables of a system are represented on an x-y plane and that the point M represents the initial condition of the system. The state variables change over time and a plot of these state variables changing is termed the trajectory of the system. This means that the point on the trajectory at time $t=0$ is $M(0)$.

At the instant P, where P is equal to the period of the supply voltage, the state of the system is given by $M(P)$. When the mapping of the points in the phase-plane relates $M(0)$ to $M(P)$, then this process is referred to as Poincaré mapping or return mapping and this is designated by $T[M(0)] = M(P)$.

A fixed point is then defined by Kieny [46] as a point where $T(X) = X$. If the state of the system is periodic, then the trajectory of the system is a closed curve and there exists a single fixed point in the system. Cigre [6] confirms this and refers to such a closed curve as a cycle. Kieny [46] states that the position of the fixed point is determined by the values of the parameters. If the parameters are varied, then the fixed point can be displaced¹⁰. Kieny [46] then further adds that if a parameter is varied beyond a critical value, then the nature and number of fixed points may be modified – this is referred to as a bifurcation. It should also be clear from this discussion that if the number of fixed points increase, then the solution to the system is no longer periodic, which correlates to those ferroresonance modes that are not fundamental. Kieny [46] elaborates on this theory of bifurcations by stating that two types of bifurcations exist: the limit point and the Hopf bifurcation.

The limit point is also referred to as a reversal point or a saddle-node bifurcation. Suppose that a diagram is drawn where the amplitude of a parameter (e.g. capacitance of system) is on the x-axis and the amplitude of a state variable (e.g. transformer overvoltage) is on the y-axis. Each steady-state solution for a given value of the parameter on the x-axis results in a point on the graph. If the parameter is varied, then a curve will be drawn and this curve is known as a bifurcation diagram. The bifurcation diagram may look like the graph seen in Fig. 2-31, which is the graph seen in [46, Fig. 7]. Kieny [46] points out that there may exist one or three solutions to the system, as also seen in the graph. The points where two solutions meet, designated P_1 and

¹⁰ Note that the definitions of bifurcation diagrams provided earlier all related to variations of parameters. This correlates to Kieny's [46] discussion here.

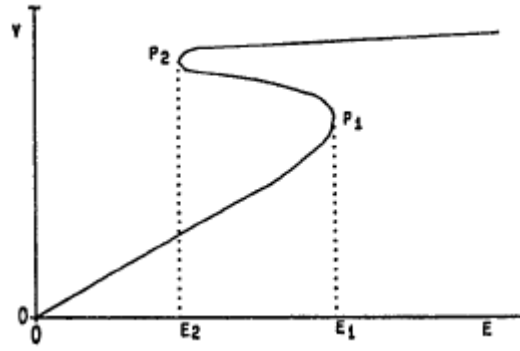


Fig. 2-31: A typical S-curve obtained in a bifurcation diagram, as seen in [46, Fig. 7]

P_2 in Fig. 2-31, are defined as limit points. It should be noted that this type of S-shaped diagram is common in ferroresonance literature and can also be seen in [11], [26], [113], [116], [117] and [118]. Jacobson, Lehn and Menzies [117] state that at the turning point of a saddle-node bifurcation, one eigenvalue of the linearized circuit attempts to move into the right half of the complex plane. It should also be noted that a single solution to the system (left of P_2 or right of P_1) does not necessarily imply that the system is not ferroresonant [11]. It can be seen from Fig. 2-31 that if the transformer overvoltage is on the y-axis, then solutions to the right of P_1 could indicate high overvoltages, which are still indicative of ferroresonance. This means that a ferroresonant solution is the only solution to the system.

The second type of bifurcation mentioned by Kieny [46] is the Hopf bifurcation. This is also sometimes referred to as a bifurcation towards the torus. This happens when a stable fixed point becomes unstable and the solution becomes pseudo-periodic (as opposed to periodic). For a more detailed discussion of stable and unstable fixed points, refer to [46]. It should be noted that Kieny [46] points out that Hopf bifurcations have not come to light in the past. It is also the experience of the author of this paper that Hopf bifurcations are not commonly discussed in ferroresonance literature. Wörnle, Harrison and Zhou [113] briefly mention Hopf bifurcations and state that they correspond to the onset of quasi-periodic ferroresonance. This could explain why Hopf bifurcations are not commonly discussed, as the reader is reminded that fundamental and sub-harmonic ferroresonance are the most common forms of ferroresonance in distribution systems [48].

Jacobson, Lehn and Menzies [117] state that there are two ways through which bifurcation diagrams can be drawn. The first is based on the principle of continuation, which was discussed in Section 2.7.1. This involves a numerical investigation into the impact of a parameter. The second method involves experimentation or time-domain simulations. Diagrams generated by this second method are referred to as brute-force bifurcation diagrams. Jacobson, Lehn and Menzies

[117] state that the process used in obtaining brute-force bifurcation diagrams is sometimes referred to as Poincaré sampling, which correlates to the discussions of Mork and Stuehm [58].

References [58], [87], [108], [114] and [119] refer to a bifurcation diagram as a “road map,” which indicates the potential modes of ferroresonance in the system. Based on the output of the bifurcation diagram, *EMTP* or other simulation packages can then be used to perform individual simulations at fixed parameter values to determine the maximum peak voltages. Mork and Stuehm [58] claim that the initial conditions of the simulation can then be varied until the engineer is satisfied that the simulations have yielded all the probable modes of ferroresonance. It is important to note, however, that Mork and Stuehm [58] prove that the parameter needs to be varied in both directions. They claim that this is because mode transitions are hysteretic in the sense that their occurrence may vary depending on whether the parameter is being increased or decreased. Though Mork and Stuehm [58] specifically refer to capacitance when they make this claim, it is the opinion of the author that this will hold for any state variable being investigated. Aside from the S-shaped curve shown in Fig. 2-31, a typical bifurcation diagram is also shown in Fig. 2-32. In this instance [54], the capacitance of the system was varied and bifurcations in system voltage were detected.

Mork makes three additional comments regarding bifurcation diagrams in [108]. Firstly, bifurcation diagrams provide an overview of most ferroresonant behaviours possible in a system. However, they do not necessarily indicate all ferroresonant modes. Secondly, bifurcation diagrams prove that it is not possible to linearly extrapolate cable capacitance values and predict ferroresonant behaviours. Finally, both bifurcation diagrams and Poincaré maps require long simulation times, since one point is taken every 50 Hz or 60 Hz cycle. The diagrams plotted in [108] contained roughly 8000 points each, which corresponds to simulation times of 130 s. Mork also emphasises in [54] the importance of an adequate ferroresonance model for drawing bifurcation diagrams.

Abdi, Abbasi and Moradi [49] present an interesting discussion on ferroresonance boundaries and ferroresonant turning points. Recall that Mork [58] said that the parameter of interest needs to be increased and decreased, since system behaviour may differ for the two cases. Abdi, Abbasi and Moradi [49] confirm this. In their example, the system voltage was varied upwards and a ferroresonant jump was detected at 0.8 p.u. If the system voltage was decreased, the ferroresonant state would remain until the system voltage reached 0.1 p.u., before the system would move out of ferroresonance. Based on these different turning points, system boundaries for ferroresonance can be defined. However, these system boundaries are not absolute and transparent. Beyond the first turning point ($V_s = 0.8$ p.u.), ferroresonance is guaranteed irrespective of the initial conditions. Below the second turning point, ferroresonance will not occur.

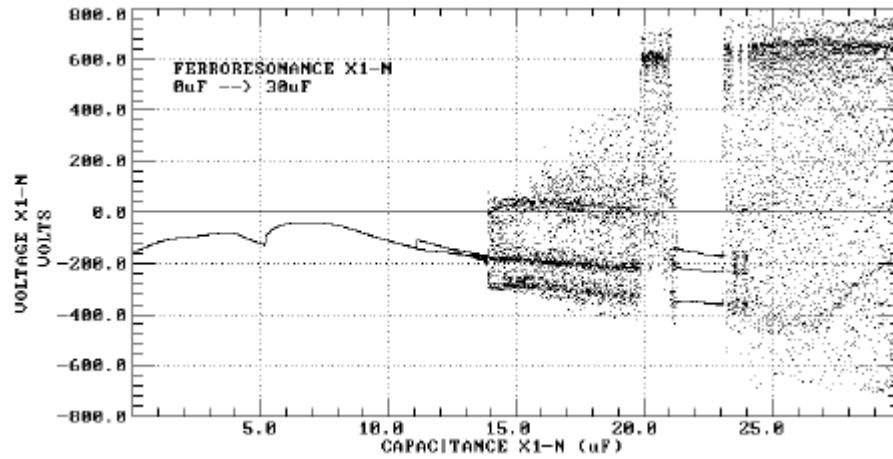


Fig. 2-32: A typical bifurcation diagram found in [54], where the capacitance of the system was varied¹¹

However, between the boundaries of the two turning points, the initiation of ferroresonance depends on the initial conditions of the system.

Bifurcation diagrams discussed thus far were only plots in a two-dimensional plane. Corea-Araujo *et al.* [119] branch out on this concept by presenting a way to draw 3-D bifurcation diagrams. This is done by performing time-domain simulations of the system and then doing frequency-domain sampling of the same output. This enables the engineer to vary two system parameters and investigate their impact on ferroresonance. For each value given to the second parameter, the algorithm stacks a plane on the three-dimensional figure. Corea-Araujo *et al.* [119] claim that the resulting map is easy to read, where straight lines represent a fundamental solution and broken lines represent a transition from period-2 to the chaotic mode. A MATLAB algorithm for drawing such a 3D diagram is provided in [119, Fig. 1]. Corea-Araujo *et al.* [119] claim that the strength of this diagram is not only in its ability to predict ferroresonant modes but also in providing the design-engineer with safe parameters for new systems.

2.7.4 Poincaré Maps

Poincaré mapping was mentioned briefly in section 2.7.3 when the definition of a limit point, as presented by Kieny [46], was discussed. Mork and Stuehm [58] also discuss Poincaré sections. They claim that one point per period is sampled from the phase-plane trajectory. It should, therefore, be clear that bifurcation diagrams are related to Poincaré maps and Poincaré maps are related to phase-plane trajectories, which are discussed in section 2.7.5.

¹¹ The poor quality of this image is caused by the poor quality of the image in the original source [54], which could not be improved.

In their discussion of Poincaré maps, Mork and Stuehm [58] refer to fractals. A thorough understanding of fractals requires an in-depth discussion of chaos theory, which is considered beyond the scope of this report. A simple definition for fractals can be found in [120], where a fractal is described as the complex geometric structure generated by a differential dynamical system. Emin *et al.* [120] elaborate by stating that there are three fractal dimension methods: the capacity dimension method, the information dimension method and the correlation dimension method. Mork and Stuehm [58] claim that fractal dimensions and Lyapunov exponents are useful in categorizing chaotic responses. Emin *et al.* [120] state that fractal dimensions are used to qualitatively specify whether the behaviour of a system is chaotic or not.

Cigre [6] states the Poincaré section is used to simplify the phase space diagram of a complicated system. When the discrete points of the phase space trajectory are recorded at distinct time intervals, a Poincaré section is created. The distinct time interval is typically taken as the fundamental frequency of the system. Consequently, when the frequency of the ferroresonance is fundamental, only a single point will be seen on the Poincaré section, as was the case with bifurcation diagrams. A sub-third harmonic will produce three points and chaotic ferroresonance will result in a random set of points in a particular region of the plane. It would therefore seem as though the Poincaré map and the bifurcation diagram are very similar. The bifurcation diagram is, in fact, a collection of Poincaré sections, each calculated for a different value of a particular parameter [115], [121]. The Poincaré section simplifies the phase-plane diagram to a certain extent, but it still maintains the essential features of the dynamics of the system [6].

Corea-Araujo *et al.* [114] state that the Poincaré map can be used to determine the ferroresonance mode with high precision when the bifurcation diagram is unclear. More specifically, the Poincaré map is useful in distinguishing quasi-periodic from chaotic ferroresonance. In other words, when the bifurcation diagram does not indicate the ferroresonance mode, a Poincaré map can be drawn from a time-domain simulation to determine the signal's exact mode.

A Poincaré section of a transformer in fundamental resonance is shown in Fig. 2-33, which can also be found in [103, Fig. 8]. As expected, the Poincaré section contains only a single point, since the ferroresonant frequency is equal to that of the forcing function. One should also note that the Poincaré section for a transformer in normal operation will be similar, as demonstrated in [103, Fig. 4]. The main difference is the location of the single point on the Poincaré map. In this case, the flux has exceeded 2 p.u. and one would expect core saturation. In the case of normal operation seen in [103, Fig. 4], the flux is less than 0.4 p.u. The phase-plane diagrams for the two cases, however, are distinctly different as seen in [103, Fig. 3] and [103, Fig. 7]. These phase-plane diagrams are also shown in Fig. 2-36 and Fig. 2-37 in section 2.7.5.

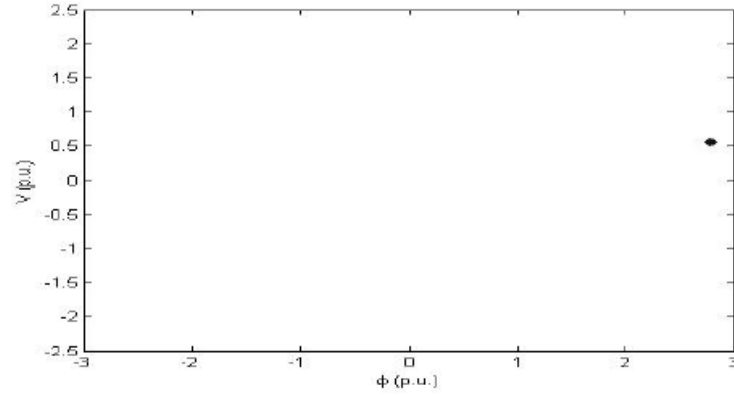


Fig. 2-33: A fundamental ferroresonance poincaré section as seen in [103, Fig. 8]

Fig. 2-34 shows the Poincaré section of sub-harmonic resonance with a frequency of $\frac{1}{2}$ of the fundamental frequency [103, Fig. 12]. In this figure, one can see the two points of sub-harmonic ferroresonance on the Poincaré map. The phase-plane trajectory of sub-harmonic ferroresonance is shown in Fig. 2-38. An example of the Poincaré section of chaotic ferroresonance is shown in Fig. 2-35, with the corresponding phase-plane trajectory shown in Fig. 2-39.

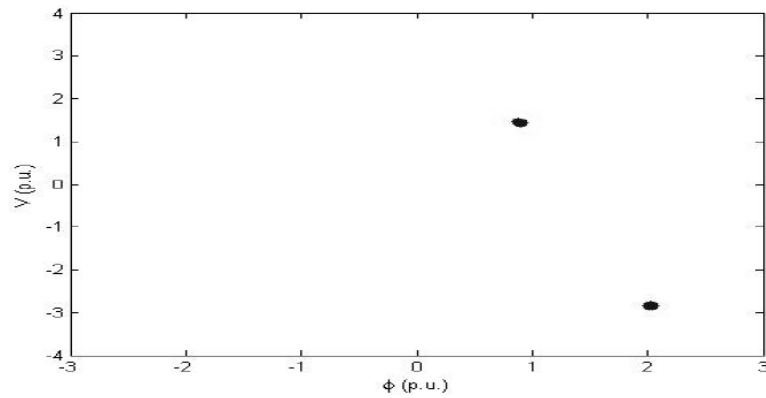


Fig. 2-34: A poincaré section of $1/2$ sub-harmonic ferroresonance as seen in [103, Fig. 12]

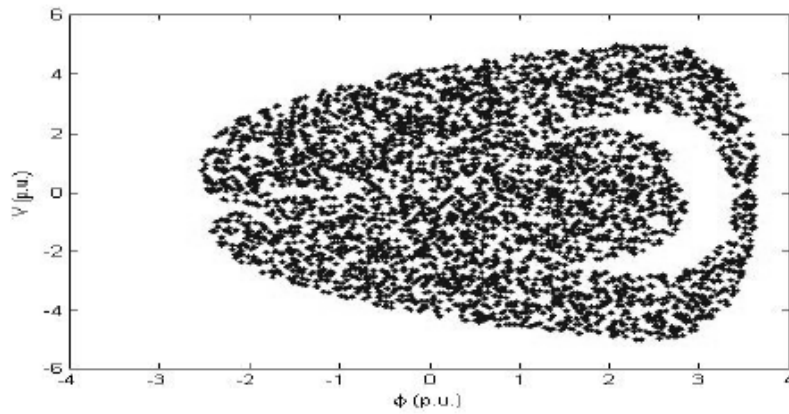


Fig. 2-35: A poincaré section of chaotic ferroresonance as seen in [103, Fig. 16]

2.7.5 Phase-Plane Diagram

The phase-plane diagram is the basis of the Poincaré section, as has become evident in section 2.7.5. Mork and Stuehm [58] describe the phase-plane diagram as a plot of the magnitude of a waveform vs. its slope. The trajectory that it forms is then a unique signature of the waveform itself. If the waveform is periodic, then the phase-plane trajectory will close back on itself. Mork and Stuehm [58] furthermore claim that superimposing phase-plane trajectories onto each other is more efficient for waveform comparison than superimposing the waveforms themselves.

The concept of a trajectory was also discussed by Kieny [46] in section 2.7.3. Kieny [46] referred to the trajectory of the system as a plot of the state variables of a system on an x-y plane as they change over time. This then led to Kieny's discussion of Poincaré maps, and since phase-plane diagrams are the basis of Poincaré maps, it would seem as though Kieny's [46] overall definition of the concept of a phase-plane diagram may differ from that of Mork and Stuehm [58]. It should also be noted that the definitions of phase space, as provided by Bo and Tiecheng [103] and Cigre [6], correlate more closely to that of Kieny [46] than that of Mork and Stuehm [58].

The phase-plane diagram of a transformer in normal operation is shown in Fig. 2-36. The trajectory of Fig. 2-36 closes onto itself, as predicted by theory. The phase-plane diagram developed by Erbay [11, Fig. 7.5] for normal operation has an identical shape.

Fig. 2-37 shows the phase-plane trajectory of a transformer in a fundamental ferroresonant state. This is the phase-plane diagram that was also used to develop the Poincaré map in Fig. 2-33. Though the shape of the trajectories between Fig. 2-36 and Fig. 2-37 differ, it is clear that the trajectories close on themselves in both cases and are still periodic. An identically shaped phase-plane diagram can be seen in [11, Fig. 7.9].

Fig. 2-38 shows the phase-plane diagram of a transformer in period 2 sub-harmonic ferroresonance. Period 2 sub-harmonics imply that there is a strong oscillation with a frequency equal to half of the fundamental frequency. The phase-plane diagram closes on itself, but it contains a point of intersection [103]. The Poincaré map correlating to Fig. 2-38 can be seen in Fig. 2-34. Erbay also shows a period 2 sub-harmonic ferroresonant phase-plane diagram in [11, Fig. 7.13], but the shape of the trajectory differs in this case. Erbay [11] points out that the diagram shows two closed trajectories, which indicates the subharmonic content of the signal. Fig. 2-38 also clearly has two closed trajectories.

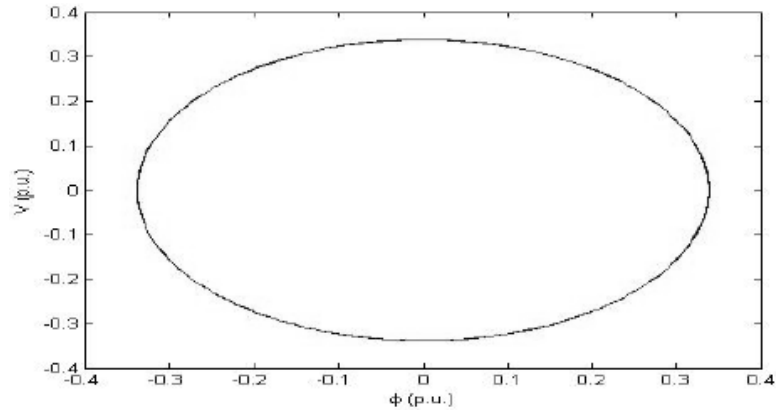


Fig. 2-36: A phase-plane diagram of a transformer in normal operation, as seen in [103, Fig. 3]

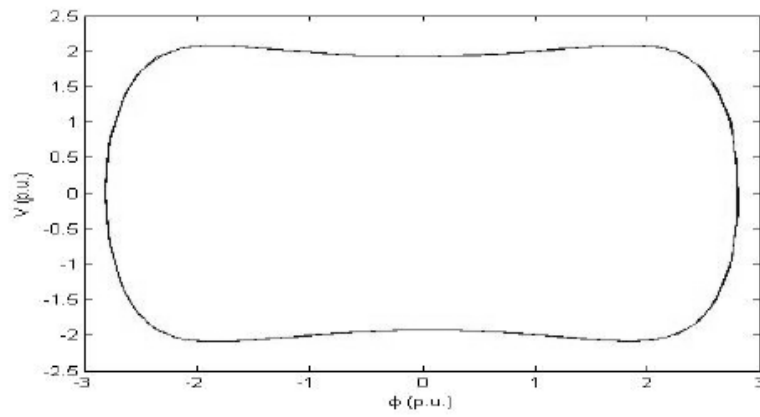


Fig. 2-37: A phase-plane diagram for fundamental ferroresonance, as seen in [103, Fig. 7]

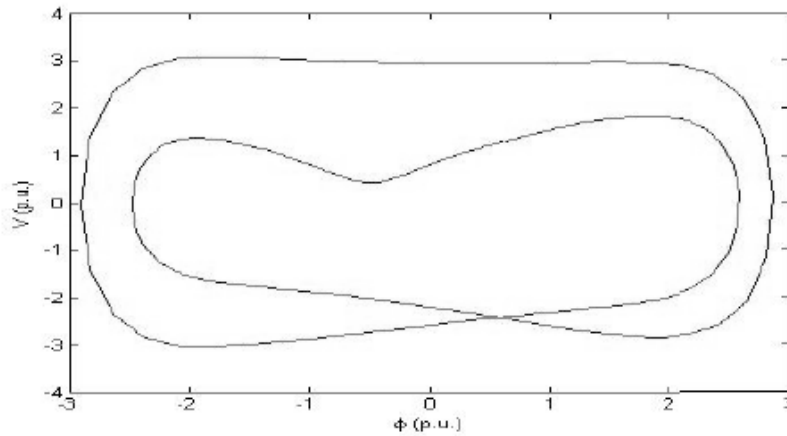


Fig. 2-38: A phase-plane diagram of period 2 sub-harmonic ferroresonance, as seen in [103, Fig. 11]

Fig. 2-39 shows the phase-plane diagram used to draw the Poincaré map seen in Fig. 2-35. This phase-plane diagram shows evidence of chaotic ferroresonance. The phase-plane trajectory does not close on itself [103]. It is for this reason that Mork and Stuehm [58] claim that phase-plane trajectories are useless for analysing chaotic ferroresonance. In such cases, the Poincaré

map is more insightful. Cigre [6] briefly discusses the complex trajectory of a chaotic solution, which never closes on itself. This complex trajectory is known as a strange attractor.

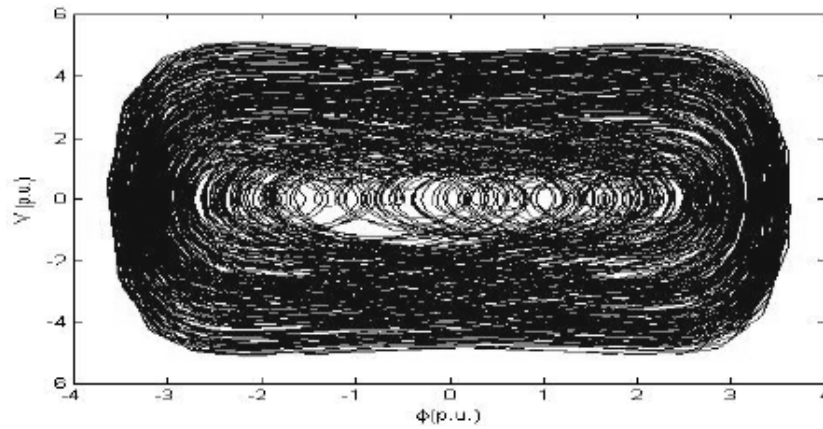


Fig. 2-39: A phase-plane diagram of chaotic ferroresonance, as seen in [103, Fig. 15]

2.7.6 Galerkin Method

The Galerkin method is described as an analytical method, which aims to find an approximate solution to an equation by decomposing the unknown and the equation on the same base [118]. Amar and Dhifaoui [122] state that the approximate solution is found by minimizing the error associated with the solution. It is particularly useful in analysing periodic solutions, which means that it will be useful in analysing fundamental and sub-harmonic ferroresonance.¹² Amar and Dhifaoui [122] describe the Galerkin method, in conjunction with the pseudo-arclength continuation method, as a useful tool for the computation of bifurcation lines similar to that seen in Fig. 2-31. The need for the pseudo-arclength continuation method will be motivated shortly.

Kieny, Le Roy and Sbai [118] motivate the use of the Galerkin method based on two drawbacks of time-domain simulations. Firstly, time-domain simulations establish results for a very specific set of conditions and these results cannot be generalized. This is a well-known disadvantage, which was also used to motivate the use of bifurcation diagrams. Secondly, time-domain simulations take a long time due to the simulation of transients before the convergence of the system into a ferroresonant state [118]. The Galerkin method, on the other hand, directly derives the steady-state of the system without a need to compute the transients. This is especially true for periodic solutions, which is why they are useful for fundamental and sub-harmonic ferroresonance.

¹² Recall that quasi-periodic and chaotic ferroresonance is uncommon in distribution systems.

Since the main focus of the Galerkin method in most cases is to simulate periodic ferroresonance, the approximate solution it generates is well-represented by a Fourier series. Kieny, Le Roy and Sbai [118] state that this leads to a system of non-linear equations where the unknowns are Fourier coefficients.

Literature on the Galerkin method regarding ferroresonance is not common. However, it should be noted that Kim and Perkins [123] use references to the Galerkin method and the harmonic balance method interchangeably, though their discussion does not pertain to ferroresonance or electrical systems. Nevertheless, based on their interchangeable reference to the Harmonic Balance method and the Galerkin method with regards to dynamical systems, it can be argued that the Harmonic balance method is very closely related to the Galerkin method. Gilmore and Steer [124] confirm this argument by claiming that harmonic balance methods are variations of Galerkin's method, where they are applied to nonlinear circuits. This means that the work of Erbay [11] on the harmonic balance method is particularly relevant to this discussion.

Erbay [11] states that the basis of the harmonic balance method is to substitute an unknown of a non-linear, ordinary differential equation (DE) with an assumed solution. This allows the user to find an approximate solution to the DE .¹³ Erbay furthermore states that the assumed solution may be defined as a Fourier series, which includes the forcing frequency. Gilmore and Steer [124], however, point out that linear contributions are computed in the frequency domain and nonlinear contributions are calculated in the time domain. This seems to contradict the work of Erbay [11]. It should furthermore be noted that Erbay [11] refers to an analytical and numerical harmonic balance method.

Regarding the analytical harmonic balance method, Erbay [11] presents a lengthy derivation of the equations used to solve an example system. This derivation is not presented here. However, using this analytical harmonic balance method, Erbay [11] draws S-shaped curves similar to that seen in Fig. 2-31 for the ferroresonant circuit. Erbay [11] proceeds to vary circuit parameters and investigate their influence on the S-shaped curve. The limit points are determined for each case.

The numerical balance method follows the same logic as the analytical balance method – it utilizes a Fourier series to solve the circuit. However, the equations used in this case are different from those used in the analytical harmonic balance method. It should also be noted that the numerical harmonic balance method relies on the circuit being simplified into a linear Thevenin and a non-linear component. In this sense, the numerical harmonic balance method is more similar to the Galerkin method described by Kieny, Le Roy and Sbai [118] than the analytical

¹³ Recall that the Galerkin method also pertained to obtaining approximate solutions.

harmonic balance method was. A system of equations is derived based on this Thevenin approximation to the circuit, which can be solved with the use of the Levenberg-Marquardt algorithm in MATHCAD.

The numerical harmonic balance method, and so too the Galerkin method, cannot draw continuous curves without the help of a continuation method. Both numerical approaches develop a problem at the turning point of the S-shaped curve since the slope of the tangent to the S-shaped curve at the turning point is infinite. In such an instance, the Jacobian of the matrix becomes singular and the numerical method cannot converge to a solution. Kieny, Le Roy and Sbai [118] and Amar and Dhifaoui [122] suggest the use of a pseudo-arclength method to solve the convergence problem. Erbay [11], however, makes use of a hyper-sphere method for its simplicity. A MATHCAD algorithm describing how the hyper-sphere method can be implemented in conjunction with the numerical harmonic balance method is shown in [11, Fig. 9.5]. Readers are referred to [11] for more details on the equations used in the numerical harmonic balance method. The Galerkin method and the pseudo-arclength method are discussed in [118], [122] and [125]

The use of the Galerkin method with the pseudo-arclength continuation method presents interesting insights into the stability of solutions. Amar and Dhifaoui [125] show that the line segment connecting the limit points of the S-shaped curve is representative of an unstable solution to the system. This corresponds to the line segment connecting P_1 and P_2 in Fig. 2-31. The Galerkin method itself, however, does not provide information on the stability of a solution. But with the continuation technique, if a limit point is reached and the solution was initially stable, then the solution becomes unstable until the next limit point is reached. If another limit point is reached from an unstable solution, the next solution will once again be stable.

2.7.7 Summary

Based on the foregoing discussions of ferroresonance simulation techniques summarized in Section 2.7, it should be clear that there are many tools at the researcher's disposal for the effective analysis of ferroresonance. In essence, these tools aim to address the limitations of time-domain simulations, especially concerning simulation time. The most prominent of these is the bifurcation diagram, which aims to identify the limit points where ferroresonant transitions may occur. However, the bifurcation diagram is based on the Poincaré map, which, in turn, is dependent on the phase-plane diagram. It was also found that the Galerkin method or harmonic balance method is useful in drawing the bifurcation diagram, especially with the aid of a continuation technique such as the pseudo-arclength method.

2.8 Distribution System Modelling Guidelines

Cigre provides some useful guidelines on power system modelling in general in [6] and [9]. The guidelines are focused on the amount of detail that needs to be included in a power system model to accurately simulate transients. The level of sophistication of the power system model is closely related to the transformer model itself.

The first modelling guideline relates to the study zone of interest. Cigre [9] states that it is common practice to divide the distribution system into a study zone and an external system that encompasses the rest of the system. In the study zone, the system must be modelled in greater detail.

For the rest of the system which is not modelled in such detail, Cigre [9] advises that a network equivalent can be used. The network equivalent must reproduce the frequency response of the network in the range of inrush current frequencies.

It is difficult to determine how far the study zone should be extended before a network equivalent is used. Cigre [9] cannot provide an exact distance. However, they suggest increasing the distance of the study zone both horizontally and vertically until the results no longer change in a significant manner.

Cigre [9] states that sympathetic interaction occurs when a transformer is energized in the presence of another energized transformer. The inrush transient caused by the energization of the transformer can cause a new transient between the existing and newly energized transformer.

Cigre [9] cautions that shunt capacitor banks close to the substation should be modelled. Furthermore, the shunt capacitance of distribution cables of significant length should be added to the equivalent shunt capacitance of these shunt capacitor banks.

In [6], Cigre provides a few more guidelines specific to resonance and ferroresonance. They [6] claims that ferroresonance is a localised phenomenon and therefore a large network model is not necessary. The network model only requires the main elements. The non-linear reactances, capacitances and voltage source are considered to be the main elements [6]. The feeding network can simply be represented as a Thevenin equivalent at power frequency.

Cigre [6] also says that an accurate representation of the line parameters at the resonant and near-resonant frequencies is essential. Among these parameters, the circuit capacitances are the most important. This means that accurate geometrical data of the conductors needs to be obtained. Cigre [6] also advises that phase transpositions need to be explicitly represented.

In general, the frequency dependency of the overhead line parameters is not essential. To this end, the Bergeron model¹⁴ yields reasonable results when its parameters are calculated at the power frequency. This also holds for ferroresonance, provided that the ferroresonance is not initiated by a switching transient. However, if ferroresonance is initiated by a switching transient, then frequency dependency becomes important. For these types of switching transients, circuits that are not involved in the switching can be represented by lumped or distributed models. If the inter-circuit capacitance is evident, both parallel circuits need to be represented in detail. Series capacitors in the power system can be modelled as lumped elements between line sections.

For transformers, Cigre [6] suggests using three-phase transformer models for power transformers and single-phase transformer models for voltage transformers. Recall, however, that this study is focused on single-phase distribution transformers and hence a three-phase model is not warranted. Stray, bushing and inter-winding capacitances can be modelled as lumped elements in parallel with the appropriate winding [6]. The nonlinear behaviour of the core is also particularly important. Cigre [6] states that Preisach¹⁵ hysteresis models have been used in rare cases to model core losses but this is particularly difficult due to a lack of data. However, the need for a Preisach model emphasizes how important it is to represent saturation data. Other hysteresis models require less complex measurements. Hysteresis will be discussed in greater detail in section 2.10. An accurate hysteresis model is warranted by the fact that a nonlinear resistor has serious limitations in representing core losses. A linear resistor is the most common method used to represent core losses but its value is calculated at rated voltage. Iravani *et al.* [87] claim that this approach yields reasonable results. They [87] also state that single-phase transformers show a low sensitivity for large changes in the core loss resistor. Three-phase transformers, on the other hand, show a high sensitivity to changes in the core loss resistance.

Other substation equipment is not necessarily critical to a ferroresonance model. The main component to be modelled accurately is the capacitance.

2.9 Geomagnetic Induced Currents (GIC)

Geomagnetic induced currents are a phenomenon often encountered in the literature related to the modelling of transformers. Geomagnetic induced currents are caused by electromagnetic storms on the sun [126]. Transmission lines running from east to west are primarily affected. The phenomenon causes a DC potential on the surface of the earth [90]. This causes the transformer neutral to be biased by a DC voltage. de Kock [126] adds that *GIC* is only relevant to long transmission lines. *GIC* will not be considered in this thesis for two reasons: 1) the power system

¹⁴ The Bergeron model is discussed in section 2.14.

¹⁵ The Preisach hysteresis model is discussed in section 2.10.18.

being considered consists of short transmission lines and 2) the primary windings of transformers in the power system in question are connected in delta.

2.10 Hysteresis

The importance of accurate transformer core modelling has been emphasised throughout this report. Section 2.7.2 particularly highlighted the inaccuracy of piecewise linear inductor models used in literature. The main contributors to core losses are hysteresis and eddy currents [135]. Preceding discussions have also indicated that reduced core losses have a significant impact on the onset of ferroresonance. Short [3] claims that amorphous core materials have lower hysteresis losses and lower eddy current losses due to their high resistivity and thin laminations. It is, therefore, important that hysteresis and the magnetization characteristic of a transformer be accurately modelled when ferroresonance is being considered. Rezaei-Zare and Iravani [28] state that hysteresis loops significantly influence transformer ferroresonance behaviour. Chiesa [74] states that hysteresis is important for low current ferroresonance behaviour with series capacitance. However, he also adds that hysteresis can be neglected for high current ferroresonance resulting from excessive shunt capacitance.

The field of hysteresis, however, warrants its own independent study. Various models have been developed in an attempt to represent hysteretic behaviour in iron cores. A detailed discussion of all of the models is beyond the scope of this report. Instead, a brief review of the models will be presented without providing too much detail on their associated equations. Three models found in the literature will receive special emphasis, these are the Tellinen model (for its simplicity), the Jiles-Atherton model (for being used most often in the literature) and the Preisach model (for being the most accurate). Before these hysteresis models are discussed, a breakdown of the theory of hysteresis is provided in the next section.

2.10.1 Theory of Hysteresis

Liorzou, Phelps and Atherton [127] claim that hysteresis is caused by three main types of phenomena: interaction between magnetic domains, anisotropy and internal frictional type pinning forces. These pinning forces are caused by interstitials, dislocations and impurities [127]. This list is confirmed by Corea-Araujo *et al.* [98].

The definition provided by Takács [128] provides valuable insight into hysteresis. He claims that ferromagnetic substances are generally solids, which are characterized by the Curie Point and the atoms, which have large dipole moments. When subjected to an external magnetic field, these dipole moments (also referred to as domains) tend to align themselves in the direction of the magnetic field. This results in a larger volume of dipoles taking on the direction of this external

field. When the external magnetic field is removed, not all of these dipoles will return to their original orientation but will remain aligned in a certain direction. The overall state of the magnetic material is then different and this magnetic history property is called hysteresis.

The concept of magnetic dipoles or magnetic domains is well-described by Preisach's theory of hysteresis. While Preisach's theory as a potential hysteresis model will be discussed in section 2.10.18, it is worth considering the theory here for a better understanding of hysteresis and some of its effects. Mayergoyz [129] describes Preisach's theory as a phenomenological model of magnetics. He presents a magnetic definition, as well as a mathematical definition of the model. He then proceeds to present a geometrical interpretation of the model as well. This geometric interpretation will be presented here to enhance the understanding of hysteresis.

In the Preisach model, it is assumed that the magnetic material consists of a range of domains with rectangular hysteresis loops (see Fig. 2-40). This rectangular hysteresis loop means that the magnetic domain can only assume one of two orientations: H_d or H_u , which correspond to an upward or downward orientation respectively. If the magnetic field has a negative value of less than $-m_s$, then the magnetic domain has an orientation of H_d . If the magnetic field has a positive value of greater than $+m_s$, then the magnetic domain has an orientation of H_u . It can be seen that m_s is the switching value for the domain. For any value of magnetization between $+m_s$ and $-m_s$, the orientation of the domain depends on its history. This magnetic domain can be likened to a two-position relay with up and down positions.

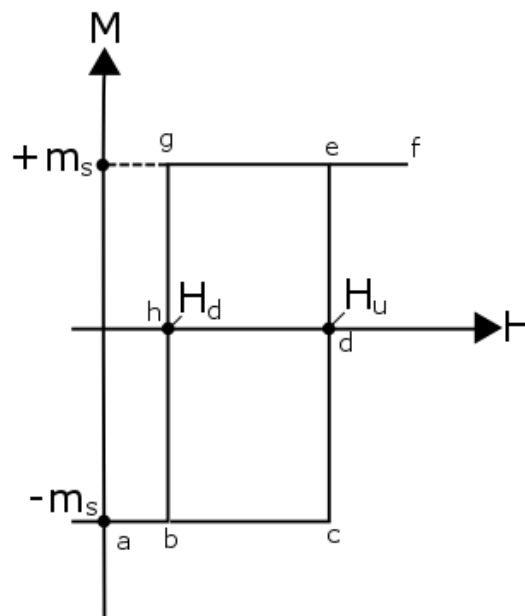


Fig. 2-40: A Preisach magnetic domain, as seen in [129, Fig. 1.3]

Assume that the magnetization of the domain is initially at the negative saturation value, $-m_s$. If the magnetization is then monotonically increased, the branch $abcdef$ in Fig. 2-40 is followed and the domain will have an upward orientation. If the magnetic field is then monotonically decreased, the descending branch $feghba$ will be followed and the domain will assume a downward orientation.

The history of the magnetic domain is important. Assume that the magnetic domain initially has a downward orientation and the magnetization is increased. The domain will maintain its downward orientation until the magnetization reaches m_s , and only then will it assume an upward orientation. The same applies to a domain with an initial upward orientation and a decreasing magnetic field.

Now, consider a collection of magnetic domains, which have different switching values m_s . This means that the switching values of all the magnetic domains cannot be equal, because then the magnetic orientation of the entire magnetic core would switch upward and downward simultaneously. The collection of all of these magnetic domains can be represented by a right triangle, whose hypotenuse is a part of the line $\alpha = \beta$ (see Fig. 2-41). The vertices of this triangle correspond to lines with coordinates α_0 and β_0 . These vertices represent the saturation values of the magnetic core. In other words, if a magnetic field is applied to the core with a magnetization value of greater than α_0 , then all the domains will assume an upward position and the core is fully saturated in that direction. Similarly, if the magnetic field strength assumes a value less than β_0 , then all the magnetic domains will assume a downward position and the core is fully saturated.

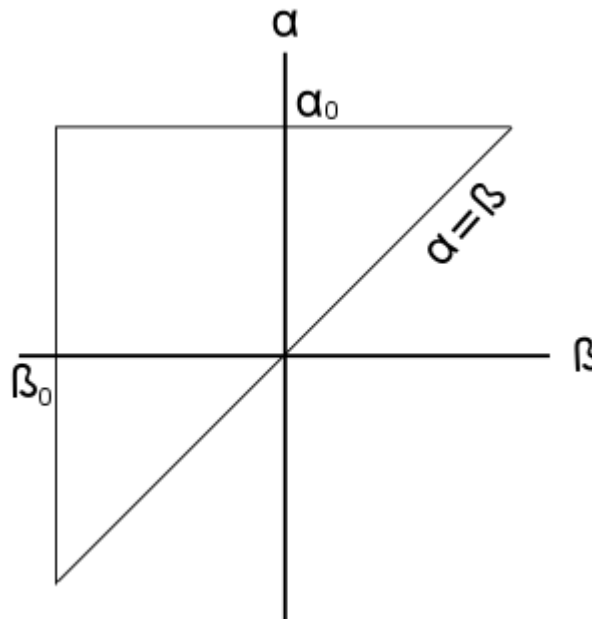


Fig. 2-41: A Preisach hysteresis representative triangle, as seen in [129]

Assume that the magnetic field is initially negative with a value less than β_0 and that it is monotonically increasing. This increase in the magnetic field reaches a maximum value, denoted u_1 , which has a value less than α_0 . As the magnetic field is increased, all magnetic domains with a positive switching value m_s less than the strength of the magnetic field will rotate and assume an upward position. This is shown graphically in Fig. 2-42, where all magnetic domains below the blue line are switched to an upward state. In other words, the monotonically increasing magnetic field can be likened to a horizontal line moving upwards from the bottom of the triangle. It switches all magnetic domains with a switching value of m_s less than its magnetic field strength, into an upward position. Since the maximum value of this field, u_1 , is less than α_0 , the core is not fully saturated.

Suppose that a monotonically decreasing magnetic field is now applied to the core after u_1 . This decreasing magnetic field reaches a maximum value, denoted u_2 , which is less than β_0 . All magnetic domains, which were switched into an upward state by u_1 , but have a negative switching value $-m_s$ less than u_2 , will be switched back into the downward state. Those magnetic domains above the u_1 line in Fig. 2-42 remained in the downward state and are now still unchanged. The decreasing magnetic field can be likened to a vertical line, starting at the right of the triangle and moving towards the β_0 line. It switches all magnetic domains with a switching value $-m_s$ greater than its current magnetic field strength into the downward position. This is shown graphically in Fig. 2-43. All magnetic domains situated in the area denoted “A” are in an upward state, while all magnetic domains situated in “B” are in a downward state.

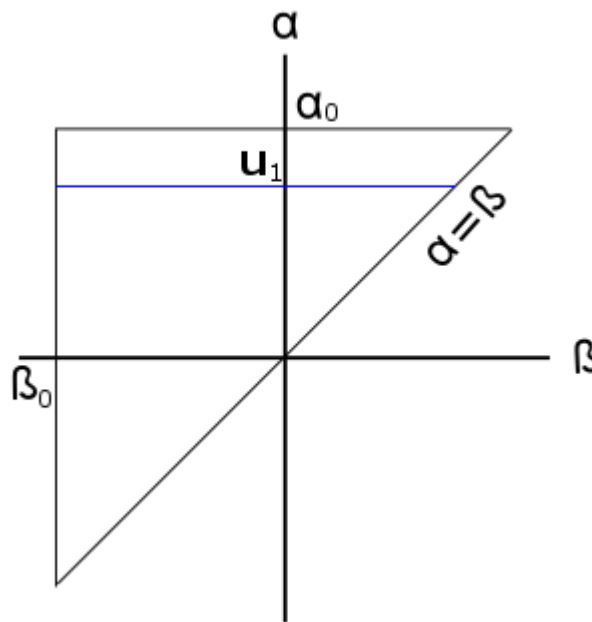


Fig. 2-42: A monotonically increasing magnetic field U_1 on the Preisach hysteresis triangle [129]

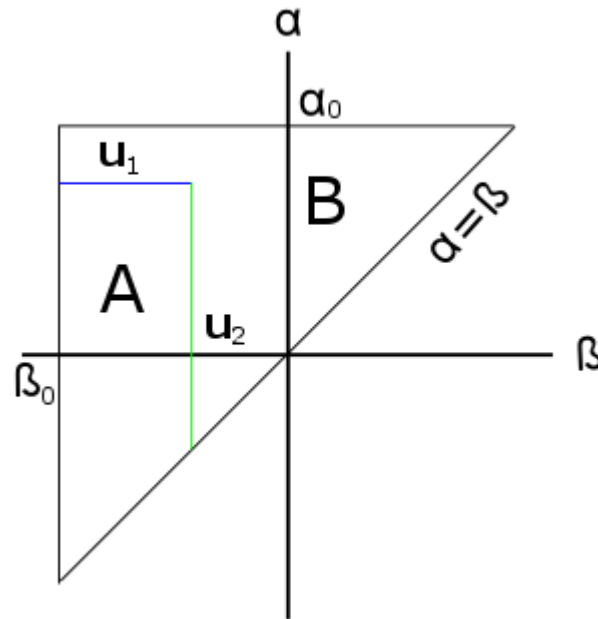


Fig. 2-43: A monotonically decreasing magnetic field U_2 applied to hysteresis triangle, following U_1 [129]

A monotonically increasing magnetic field can now be applied to the core again. A process similar to that described for u_1 will take place. Assume that the maximum value of this third magnetic field, u_3 , is less than u_1 . The effect on the hysteresis triangle is shown in Fig. 2-44. A range of increasing and decreasing magnetic fields can then be applied in succession and assuming that the amplitude is slightly smaller in each case; the resulting triangle will be similar to that seen in Fig. 2-45.

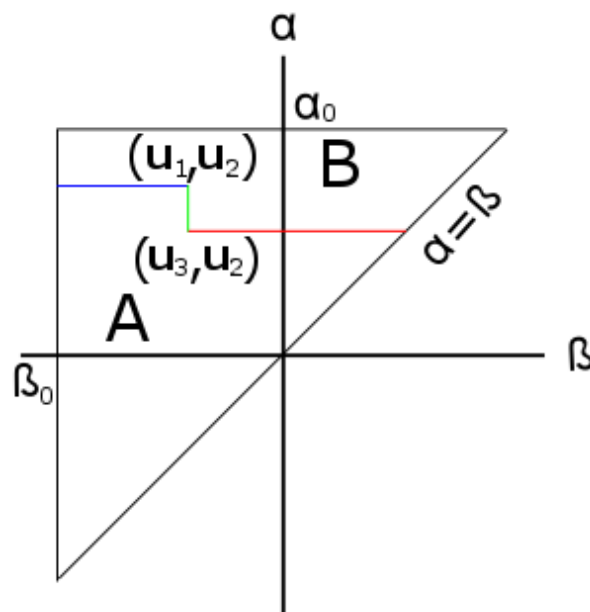


Fig. 2-44: A hysteresis triangle following three separate monotonic magnetic fields [129]

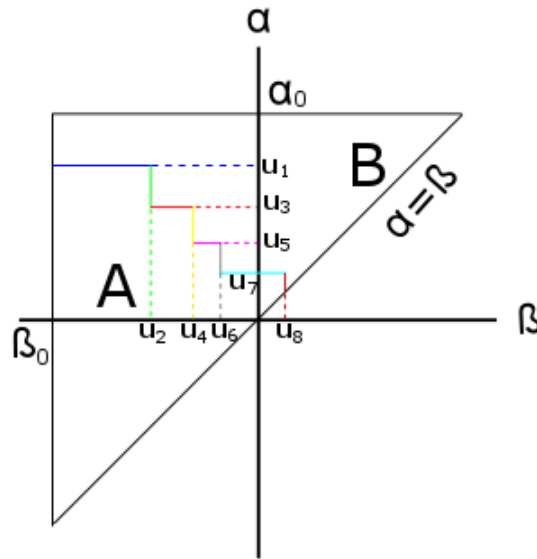


Fig. 2-45: A hysteresis triangle following 7 monotonic magnetic fields [129]

From this discussion, it should be clear that in this Preisach representation of hysteresis, there is a memory mechanism involved with hysteresis. The Preisach model retains a memory of the vertices of the past magnetic fields and this memory property leads to a staircase interface, as seen in Fig. 2-45. Though this memory property may be considered a property of the Preisach model, the presence of residual flux in transformer cores is indicative of the fact that this is a good depiction of the reality of hysteresis. Magnetic domains maintain their orientation following magnetization – in the absence of this property, hysteresis would not exist.

Mayergoyz [129] points out another property related to this memory mechanism. Suppose one has two different magnetic cores subjected to a different history of magnetic fields. However, at some point in time, the inputs to these cores are cyclically varied between the same consecutive maximum values u_+ and u_- (see Fig. 2-46). From these two separate hysteresis triangles, one can see that the magnetic orientation of the core as a whole differs in the two cases based on their history. This means that the hysteresis of a core is not only dependent on the exciting waveform, but also the history of the magnetic material. Mayergoyz [129] points out that the excitation signal applied to these cores will generate minor hysteresis loops. Minor loops will be discussed in more detail at a later stage.

There is an exception to the memory property, which also needs to be discussed. This is referred to as the wiping out property and is consistent with experimental facts. The wiping out property is phrased by Mayergoyz [129] as follows: “Each local input maximum wipes out the vertices of $L(t)$ whose α -coordinates are below this maximum, and each local minimum wipes out the vertices whose β -coordinates are above this minimum.”

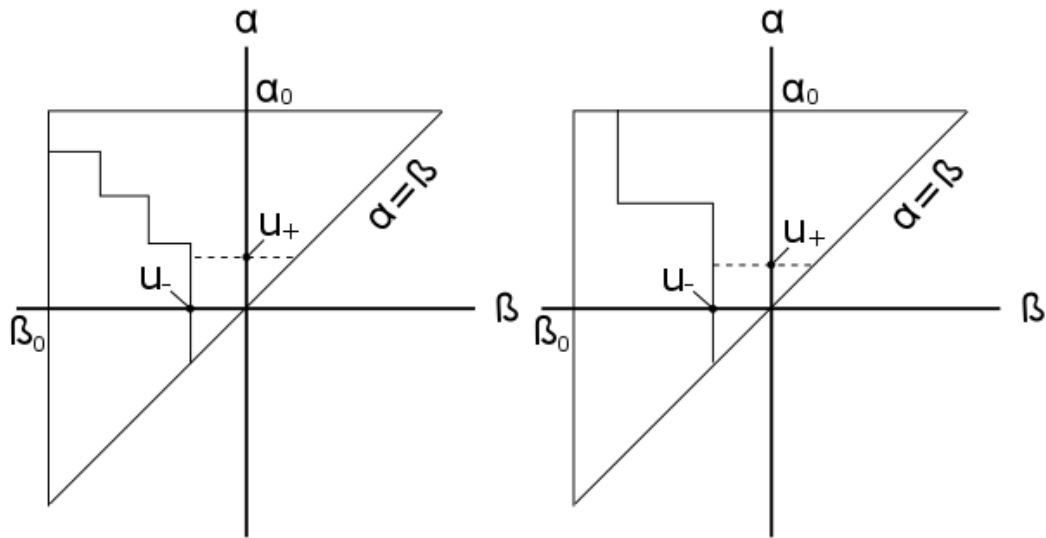


Fig. 2-46: Two hysteresis triangles under the same cyclic magnetic field, but with different magnetization histories [129]

The wiping out property can also be shown geometrically with the help of hysteresis triangles. Consider once again a core, which has been subjected to a range of increasing and decreasing magnetic fields, with a resulting hysteresis triangle as shown in Fig. 2-45. Now suppose that a monotonically increasing magnetic field is applied to this core with a maximum input u_9 , which is greater than u_3 . The resulting hysteresis triangle is seen in Fig. 2-47. Those vertices with an α -coordinate less than u_9 have been wiped out. The history of the magnetic fields, which caused those vertices, is no longer relevant. The same wiping-out property will apply for decreasing magnetic fields of sufficient magnitude.

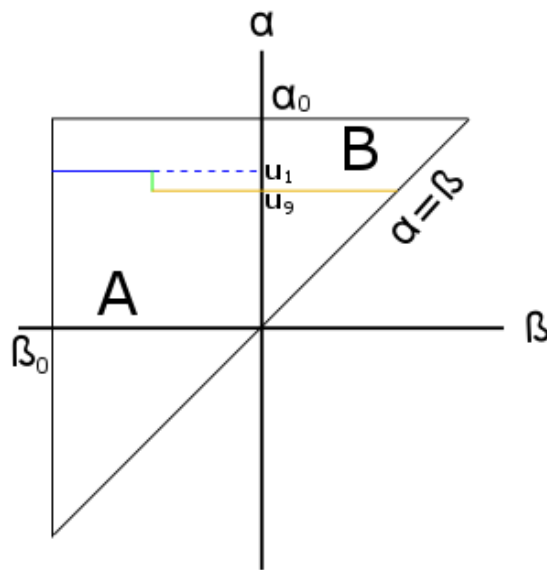


Fig. 2-47: A demonstration of the effect of the wiping-out property on the hysteresis triangle [129]

An important aspect of hysteresis theory is that of major and minor loops. Corea-Araujo *et al.* [98] state that three types of hysteresis loops can be observed: major hysteresis loops, symmetric minor hysteresis loops and asymmetric minor hysteresis loops. Liorzou, Phelps and Atherton [127] define the major loop as “the largest possible ones where the ends of the loop enter into ‘technical saturation.’” All other loops, which do not conform to this definition, are considered to be minor hysteresis loops. Takács [128] defines a minor loop as a collective name for loops that have at least one extreme differing from that of the major loop. Jiles [130] defines a minor loop as “any excursion for which either the range of magnetization ΔM or the range of magnetic field ΔH is not symmetric about zero.” Chan *et al.* [131] present a slightly alternative definition for a major loop. They claim that the major loop encompasses all other B-H loops and is obtained when the current in the windings is sufficient to saturate the core.

Chan *et al.* [131] state that the major hysteresis loop is characterized by three parameters: the coercivity H_c , the remnant flux B_r and the saturation flux B_s . The coercivity is the point where the hysteresis loop intersects the positive H-axis. Halim *et al.* [80] describe coercivity as the required magnetic field to remove the residual magnetism. The remnant flux is the point where the hysteresis loop intersects with the positive B-axis. This corresponds to the amount of flux left in the core after the current has decayed to zero [80]. The saturation flux is represented by the maximum flux density as the limit of the magnetic field reaches infinitely large values. These concepts are demonstrated more clearly in Fig. 2-48.

Liorzou, Phelps and Atherton [127] make a distinction between symmetric and asymmetric minor loops. Rezaei-Zare *et al.* [109] refer to the symmetric minor loop as being symmetric about the origin of the flux-current (or the B-H) coordinates. Rezaei-Zare *et al.* [109] state that under normal operating conditions, the hysteresis of a core takes the form of a symmetric minor loop.

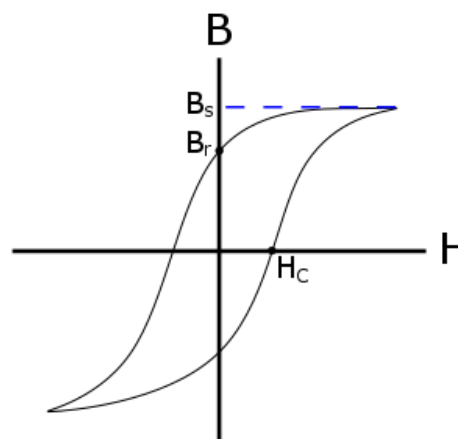


Fig. 2-48: A major hysteresis loop with indications of saturation flux, coercivity and remnant flux

The concept of a minor hysteresis loop can also be explained with the help of the Preisach hysteresis triangles seen in Fig. 2-41 through Fig. 2-47. Consider, for example, the cyclic magnetic fields applied to some core, as shown in Fig. 2-46. Since the maximum values of the magnetic field do not exceed α_0 or β_0 , the core does not become saturated. Since the history of these two cores differs, the minor hysteresis loops will not be identical. Nevertheless, Mayergoyz [129] states that in this case, the minor loops will be congruent, since the excitation signals are identical. This principle is demonstrated in Fig. 2-49, where the position of the minor loop in the overall B-H plane is determined by its past magnetization history. The magnetization history determines the orientation of the magnetic domains which are not being switched by the cyclic magnetic field. Liorzou, Phelps and Atherton [127] state that this magnetic history property makes for a general difficulty in modelling the magnetization curves. Each of the magnetic field values is related to an infinity of possible magnetizations based on the core's history. In other words, if the hysteresis model is to be able to model core behaviour accurately, it must be able to draw both the major hysteresis loop and all the associated minor loops.

An example of a major loop, an asymmetric minor loop and a symmetric minor loop, as found in [127, Fig. 1], can be seen in Fig. 2-50.

From the hysteresis triangles, one can also conclude that if an increasing magnetic field is applied to the core, such that its maximum value exceeds α_0 , then all magnetic domains will be aligned in the upward position, the core will be saturated and the upper half of the major hysteresis loop will be traced. If the magnetic field is then decreased to a maximum value below that of β_0 and the process is repeated, an entire major hysteresis loop can be drawn.

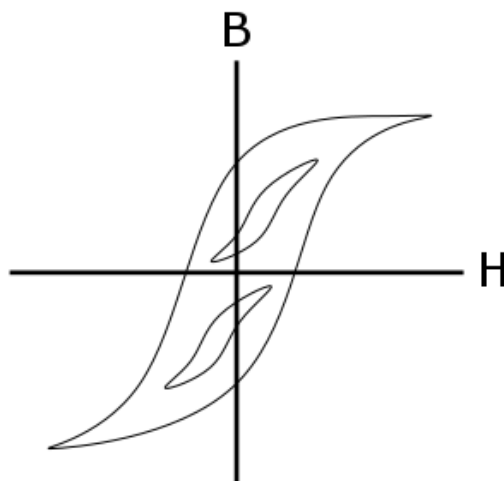


Fig. 2-49: An example of a major hysteresis loop with two congruent minor loops based on similar inputs, but different past magnetization histories

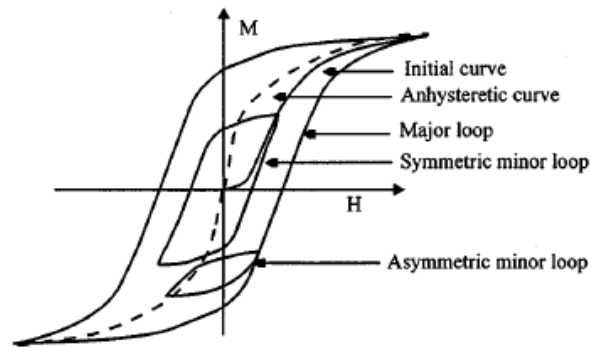


Fig. 2-50: A diagram showing a major hysteretic loop, a symmetric hysteretic loop and an asymmetric hysteretic loop, as seen in [127, Fig. 1]

Liorzou, Phelps and Atherton [127] state that the anhysteretic curve cannot be measured directly. Instead, each point of the anhysteretic curve needs to be plotted in a process similar to that of core demagnetization. A slowly decaying, low frequency, ac magnetic field must be superimposed upon a constant *DC* bias field. This is confirmed by Jiles and Atherton [43]. For that particular *DC* field, the measured point then corresponds to the global energy minimum. This curve is important because once it has been obtained, the core model is completed by simply adding the effects of hysteresis.

2.10.2 Importance of Hysteresis

Section 2.10.1 discussed the theory of hysteresis as a phenomenon. It is also important to consider the impact that hysteresis can have on transformers. Section 2.7.2 briefly mentioned the importance of hysteresis in transformer modelling. Neglecting hysteresis can result in inaccurate representations of core loss and magnetization characteristics [109], [132]. De León and Semlyen [112] claim that this is because hysteresis can have a significant effect on the damping of system transients. This is confirmed by Talukdar and Bailey [133]. Additionally, Talukdar and Bailey [133] also state that hysteresis, along with the de-energization conditions, determines the residual flux in the core. Recall that Section 2.2.3 pointed out that ferroresonance is highly dependent on the residual flux. Chiesa and Høidalen [57] also emphasize the importance of proper hysteresis modelling by stating that the use of true hysteresis models is of great importance for the determination of residual flux. This allows the model to automatically initialize itself upon re-energization following a disconnection transient. Common R-L representations fail to take residual flux into account since energy cannot be stored in the core.

Hysteresis can also influence the ferroresonance mode. Lamba, Grinfield, McKee and Simpson [78], showed that subharmonic ferroresonance was never initiated when a single-valued core

inductor and parallel core loss resistor were used to model core losses. This implies that the area of the hysteresis loop is important. Lamba, Grinfield, McKee and Simpson [78] also present a very interesting conclusion from their simulations – changing the magnetization characteristics of the core in such a manner that core losses are reduced may increase the stability region of ferroresonant solutions. Rezaei-Zare *et al.* [52] elaborate on this concept by stating that a reduction of the hysteresis area also increases the probability of chaotic ferroresonance. This is caused by a change in the variation of the core inductance and a reduction of the core losses. This is particularly important since this study is focused on the impact that reduced core losses mandated by the SANS 780 standard may have on the initiation of ferroresonance. If transformer designs changes and reduced core losses change the magnetization characteristics, and this, in turn, increases ferroresonance stability regions, then this may explain the observed transformer overvoltages.

Section 2.10.1 defined the major and minor hysteresis loops. Rezaei-Zare *et al.* [109] state that minor loops are often ignored in ferroresonance analysis. Those models which do take minor loops into account, scale the minor loops based on the shape of the major loop. This is referred to as path tracing by Chiesa and Høidalen [134]. Though the model is criticized for its simplicity, it is capable of taking residual flux into account and it requires a limited amount of input data. However, Rezaei-Zare *et al.* [109] state that this approach does not reflect the behaviour of the core. They add that the experimental results have shown that minor loops are not congruent to each other, nor are they congruent to the major loop. It is for this reason that both major and minor loops need to be accurately represented by the hysteresis model for proper ferroresonance simulation.

One other note of caution should also be made regarding hysteresis modelling. The addition of hysteresis models will not always guarantee a more accurate simulation. The hysteresis model itself needs to be accurate. Rezaei-Zare *et al.* [109] claim that improper representation of hysteresis effects may result in a more inaccurate simulation than the case where hysteresis is ignored completely.

Hysteresis modelling requirements are not as strenuous when ferroresonance is not a concern [56]. Martinez *et al.* [56] claim that most modellers are unduly concerned with saturation characteristics. Saturation characteristics are only critical in transients with a high flux value. The most important parameters are the slope upon saturation and the zero-current intercept. Martinez *et al.* [56] also add that the location of the saturation representation within the model topology is important. For simulations that do not consider ferroresonance, a piecewise-linear approach with 2 slopes is generally sufficient. Martinez *et al.* [56] claim that increasing the number of slopes does not significantly influence the accuracy.

2.10.3 Hysteresis Losses

The power loss caused by hysteresis is dependent on several factors including the frequency, the volume of the core and the magnetic flux density. The power lost due to hysteresis is described mathematically by Short [3]:

$$P_h \propto V_e f B^{1.6}$$

where

- V_e = the volume of the core;
- f = the frequency;
- B = the maximum flux density, and
- P_h = the hysteresis losses.

Chandrasena *et al.* [135] also claim that hysteresis is influenced by frequency. Zirka *et al.* [136], on the other hand, claim that it is generally accepted that the hysteresis loss is quasi-static and, thus, frequency independent. Chiesa and Høidalen [134] and Cigre [9] also claim that hysteresis is frequency independent.

2.10.4 Hysteresis and Magnetization Measurements

Before discussing the hysteresis models seen in literature, it is important to firstly consider the measurement of hysteresis and a core's magnetization curve.

De León, Farazmand and Joseph [88] state that the deep saturation region is of paramount importance for the modelling of inrush currents. It can then be assumed that it will also be important for ferroresonance. However, de León, Farazmand and Joseph [88] also add that in their case it was not possible to measure the deep saturation characteristics due to the high power requirements of such measurements, since the transformer needs to be driven into saturation. Therefore, de León, Farazmand and Joseph [88] made use of Finite Element Analysis to calculate the air-core inductance.

Martinez *et al.* [56] also emphasise the importance of saturation but claim that this data can be obtained from tests, from manufacturer's curves or transformer geometry. If the saturation characteristics are estimated from test data or manufacturer curves, there are a few factors, which need to be taken into account. Firstly, the excitation current includes both core losses and magnetizing components. Secondly, manufacturers provide *RMS* currents, as opposed to crest current values. Thirdly, winding capacitance can significantly influence low-current data. Finally, hysteresis biases the saturation curve.

Martinez *et al.* [56] also provide a few suggestions for compensation of these effects. The loss component can be extracted from the excitation for each current point using (2-37)

$$I_m = \sqrt{I_{exc}^2 - \left(\frac{P_{exc}}{V_{exc}}\right)^2} \quad (2-37)$$

The *RMS* values of the voltage-current curve can be converted to an instantaneous flux-current relationship. To this end, the SATURATION supporting routine in *EMTP* programs is particularly useful. Finally, Martinez *et al.* [56] state that one should compensate for the effect of winding capacitance, though no indication is given of how this is done. When winding capacitance dominates the magnetizing reactance, the resulting hysteresis loop takes on a “cobra” shape (see Fig. 2-51).

Rezaei-Zare *et al.* present a method for determining the magnetization characteristics of a transformer in [109]. Their method requires two capacitive dividers, one resistive divider and one current shunt. The capacitive dividers, with effective capacitance C_{m1} and C_{m2} respectively, are used to measure the terminal voltage and source voltage on the *HV* side. The resistive voltage divider is used to measure the voltage on the *LV* side. Rezaei-Zare *et al.* [109] explicitly point out that the resistance of the voltage divider must be high enough so as not to load the transformer. If the transformer is loaded, ferroresonance may not be initiated or may be damped out. In their case, the total resistance was several hundred kilo-ohms.

The current shunt resistor is connected in series with the primary winding and has a low resistance value. The range suggested by [109] is between a few ohms and a few tens of ohms and will be determined by the magnetizing impedance. The hysteresis loop is drawn based on the time

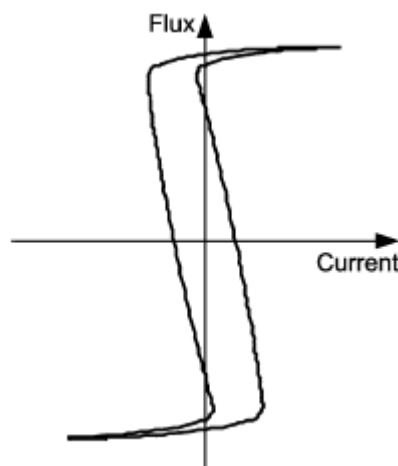


Fig. 2-51: A cobra-like hysteresis curve due to winding capacitance, as seen in [56, Fig. 6]

Thanawarithorn and Aree [137] provide an alternative method of deriving the B-H curve by measuring the flux-current relationship with an RC integrator circuit (see Fig. 2-53). The flux is determined by the RC integrator on the secondary side. Thanawarithorn and Aree [137] state that the relation between the output voltage, v_o , and the secondary voltage, v_s , is given by:

Based on Faraday's law and the relationship between primary windings and the winding ratio, one can rewrite the equation in the form

By making the flux the topic of the equation:

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Thanawarithorn and Aree [137] conclude that since the mutual flux ϕ_m is equal to the product of the primary windings N_p with the flux ϕ_m , the mutual flux can be calculated using (2-41). Therefore, if the excitation current is measured on the primary side and the output voltage of the RC circuit is measured on the secondary side, a flux current ($\phi_m - i_{exc}$) graph can be drawn, based on.

$$\phi_m = a(RC)v_o \quad (2-41)$$

An alternative integrator circuit is described by Nilsson and Riedel [138]. In their integrator circuit, an operational amplifier is used to determine the integral of the input voltage. The output of the operational amplifier is then proportional to the integral of the input voltage.

Thanawarithorn and Aree [137] use this method of drawing flux-current graphs to support the use of the Takács hysteresis model. They show in [137, Fig. 4] that the measured flux-current loop is well-fitted by the modified Takács model. However, Rezaei-Zare and Iravani [28] caution against this type of comparison for model verification. They state that visual alignment of the magnetization characteristics cannot guarantee model accuracy during transients. This is because dynamic core inductance agreements cannot be recognized properly by this method. Therefore, Rezaei-Zare and Iravani [137] claim that in addition to aligning the magnetization characteristics, the derivatives of these curves should also be matched, as this is a more accurate depiction of dynamic inductance.

Having measured the magnetization characteristics of the magnetic core, one can move on to discussing hysteresis models in more detail. The rest of this section on hysteresis will focus on hysteresis models found in the literature. Kachniarz and Szewczyk [139] divides existing hysteresis models into two main categories. The first group is referred to as physical models. These models include the Jiles-Atherton, Harrison and Preisach models. They attempt to connect the physical properties of the material to its magnetic behaviour. However, these models are the most complex and require a lot of computing power for their calculations.

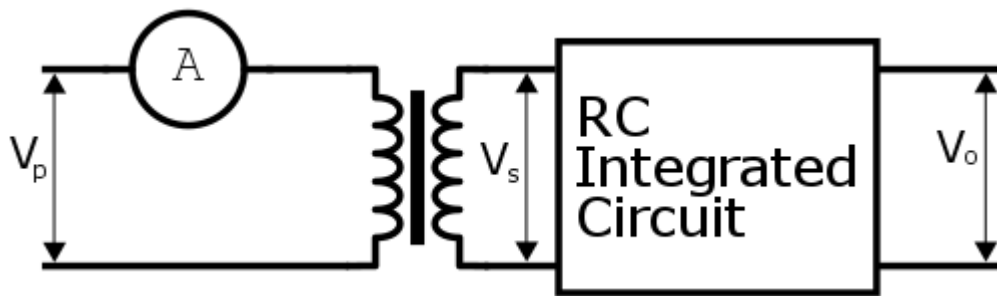


Fig. 2-53: An experimental setup for measuring the flux-current relationship with an RC integrator circuit, as seen in [137, Fig. 3]

The second category mentioned by Kachniarz and Szewczyk [139] is that of simplified technical models. These models make use of simplified mathematical functions to represent hysteresis, without taking any physical properties into account. Kachniarz and Szewczyk [139] state that hyperbolic or logarithmic curves are typically utilized. It should be noted, however, that since these models ignore the physical properties of the material, they cannot be generalized. Though this property simplifies the calculations, it also means that these types of models are only applied for simple technical applications.

Mousavi and Engdahl [110] present five sets of criteria for the ideal hysteresis model to simulate inrush current, ferroresonance and *DC* magnetization:

- a The model must be able to accurately predict symmetric and asymmetric minor loops and must also be able to model the saturated region;
- b The model's input should be magnetization and the output should be the magnetic field;
- c The model should be based on a direct mathematical relationship and not some complex algorithm;
- d Model parameters should be easily determined based on measurement data that is easily accessible, and
- e The model should be fast, easy to implement and should not be memory intensive.

2.10.5 R-L Representation

Hysteresis representation utilizing an inductor and a resistor has previously been discussed in this report. More specifically, section 2.7.2 spoke about ferroresonance models in the literature, where it was seen that many ferroresonance models make use of parallel combinations of resistors and inductors. In some cases, the resistor was constant and in other cases, the resistor was dependent on flux. Nevertheless, a thorough motivation was also provided as to why this type of modelling is insufficient.

The model proposed by Martinez and Mork [79] makes use of a nonlinear inductor and a nonlinear resistor in parallel. Chiesa and Høidalen [134] state that these types of components are typically represented by piecewise linear curves. Their piecewise-linear approximations are obtained by direct matrix change or by the compensation method. Additionally, there is also a capacitor in parallel with the resistor and the inductor in the model of Martinez and Mork [79]. This capacitor models the turn-to-turn winding capacitance. This means that the magnetization is represented by three parallel legs (see Fig. 2-54). The no-load losses are represented by the nonlinear resistance and the hysteresis, eddy current and anomalous losses are represented by the nonlinear inductor. Chiesa and Høidalen [134] also make use of such a model to divide the losses.

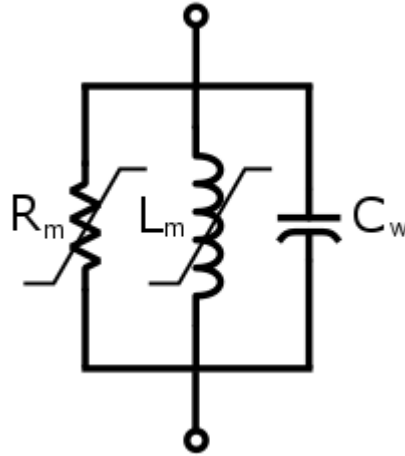


Fig. 2-54: An R-L Representation for core saturation, as seen in [79, Fig. 6]

Chiesa and Høidalen [134] discuss the use of both a linear and a nonlinear resistor in parallel with the inductor. For the linear case, the resistance value is fixed to equal the losses at rated values. The nonlinear resistor is defined by a V-I curve. The differential voltage then tends to decrease as the voltage increases [134]. Pérez-Rojas [140] also makes use of a linear resistor in parallel with a nonlinear inductor. However, he represents the saturation characteristic of the core with the help of arctangent functions.

The advantages and disadvantages of an R-L representation of hysteresis is summarized in Table 2-4.

Table 2-4: The advantages and disadvantages of an R-L representation of hysteresis

Advantages	Disadvantages
The model's simplicity.	The model's failure to represent remnant magnetization.
The model is easy to implement.	The model does not provide proper frequency dependency.
The model only relies on standard available components available in simulation software.	The model may result in a bobbling effect of the hysteresis loop (see Fig. 2-55).
Arctangent functions render a model which is not computationally intensive.	Linear resistors overestimate losses at lower levels of magnetization and underestimate losses at high levels of excitation.
A nonlinear inductor in parallel with a constant resistor is accurate to within 5% up to 3 kHz.	Nonlinear resistors underestimate core losses in the knee area of the curve.

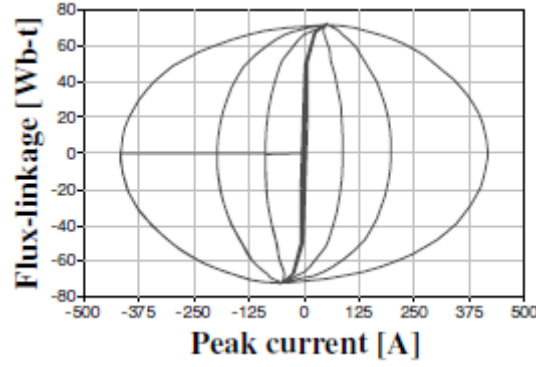


Fig. 2-55: A bobbled hysteresis loop due to a nonlinear resistance representation of the core loss, as seen in [134, Fig. 3(b)]

2.10.6 Polynomial and Piecewise Approximations

The inaccuracy of piecewise linear approximations to magnetization characteristics has been repeatedly emphasised in this report. This section will briefly discuss some of the polynomial functions and piecewise linear approximations, which can be found in the literature.

Two-term polynomials are commonly found in literature and are used to represent the relationship between magnetic flux density and magnetic field intensity. Examples of two-term polynomials can be found in [79], [141] and [142]. Martinez and Mork [79] use a two-term polynomial to relate the current to magnetic flux linkage λ . Jacobson, Lehn and Menzies [141] use a polynomial of the 13th order to represent the relationship between field intensity H and flux density B . The coefficients of the polynomial are determined by matching the curve to manufacturer data. Rezaei-Zare and Iravani [142] also use a polynomial to relate current to flux linkage. Their discussion supports the work of Jacobson, Lehn and Menzies [141]. Rezaei-Zare and Iravani [142] state that lower-order polynomials (5th to 7th order) are suggested for VTs and higher-order polynomials (11th to 13th order) are suggested for power transformers.

Cazacu, Petrescu and Ioniță [48] make use of a Brauer function to represent the magnetization characteristic. The Brauer function also only has two terms but is represented by

$$H = aB + bBe^{cB^2} \quad (2-42)$$

where parameters a , b and c are determined through a fitting procedure.

Chandrasena *et al.* [81] caution against the use of piecewise linear solutions, especially with regards to remanence. They claim that for short time simulations, these models may appear to take remanence into account because of the system time-constant. However, over time scales of

seconds, these models allow the flux to decay to zero, proving that they do not simulate remanence accurately.

2.10.7 B-Spline Curves

Dolinar, Pihler and Grčar [143] present an interesting approach to hysteresis modelling through the use of B-spline curves. The advantage of this model is that it is capable of simulating minor loops, and though only inrush current is simulated in [143], the results are accurate.

The model divides the hysteresis loop into five different segments (see Fig. 2-56). In the figure, one can see that the magnetization/anhyseretic curve is represented by segment 1. It is also interesting to note that [143] draws an H-B curve, rather than a B-H curve.

The approach employed by Dolinar, Pihler and Grčar [143] is to model sections 4 and 5 with parabolas, while sections 1 through 3 are modelled with the help of B-spline curves of the fourth-order. The B-spline curves are twice differentiable.

To explain the concept of the B-spline curve and the minor loop, Dolinar, Pihler and Grčar [143] establish the convention that $+B_m$ and $-B_m$ represent the flux densities of the last reversal point. Furthermore, $+B_{max}$ and $-B_{max}$ both represent saturation points. If the reversal points $\pm B_m$ are less than $\pm B_{max}$, then a minor loop is created. Dolinar, Pihler and Grčar [143] claim that these minor loops lie within the major loop and are similar to it. This is in contrast to the claims made by Rezaei-Zare *et al.* [109] regarding the asymmetry of minor loops.

The area of the hysteresis loop represents the hysteresis loss per cycle. Common calculus knowledge dictates that integration over the curve will provide the area under the curve. The scaling of minor loops is based on this principle. Dolinar, Pihler and Grčar [143] also state that the hysteresis loss per cubic meter at a constant frequency is defined by

$$P_{Fe} = 2W_v f \quad (2-43)$$

where

- P_{Fe} = the hysteresis power loss
- W_v = the area under the hysteresis curve, and
- f = the frequency.

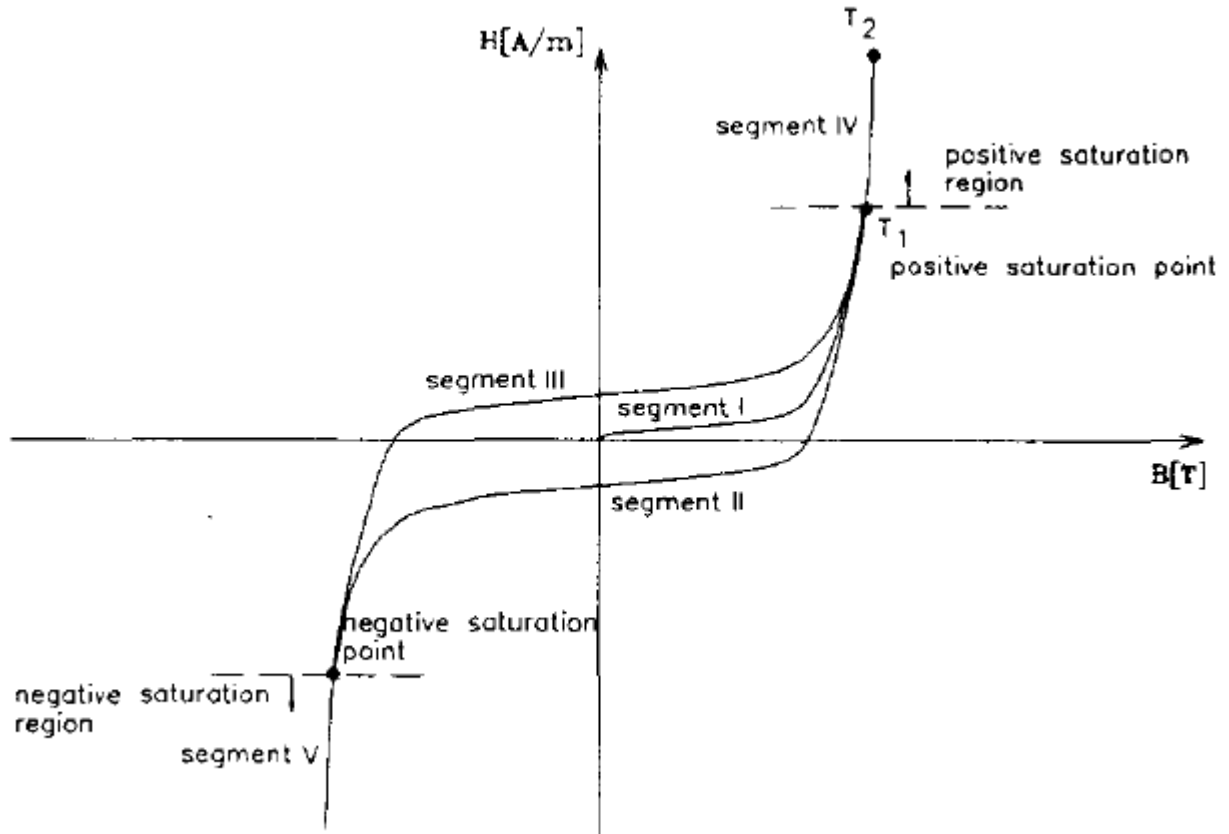


Fig. 2-56: An H-B Curve showing the five segments of a hysteresis curve, as seen in [143, Fig. 2]¹⁶

Dolinar, Pihler and Grčar [143] add that steel core manufacturers often provide the hysteresis power loss as a function of flux density in a graph. An example of such a graph can be seen in [143, Fig. 5]. The minor hysteresis loop can, therefore, be scaled such that the area under the loop corresponds to the hysteresis loss for a given flux density as defined by the manufacturer.

The area of the minor loop will be smaller than the area of the major loop. Dolinar, Pihler and Grčar [143] define a factor that quantifies the difference between the area of the minor loop and the area of the major loop. This factor, denoted ΔW_p , is used to define a change in the magnetic field intensity (ΔH). This change in the magnetic field intensity determines the factor whereby the major hysteresis loop is scaled. Dolinar, Pihler and Grčar [143] conclude that by selecting ΔB appropriately, an adequate number of points can be determined for the minor hysteresis loop. These points are then used to determine the equation of the B-spline curve based on the instantaneous flux density. The advantages and disadvantages of the B-spline curve hysteresis model are given in Table 2-5.

¹⁶ This figure was taken from [143], which was published in 1993. An attempt was made to improve the quality of the image, but the poor quality of the image in the original article could not be overcome.

Table 2-5: The advantages and disadvantages of a B-spline approach to hysteresis modelling

Advantages	Disadvantages
Can model minor loops.	The scaling of the major loop will not yield asymmetrical minor loops.
Accurately replicated current results in [143].	The model requires a graph of the power loss per cubic meter from the manufacturer, which could be proprietary.

2.10.8 Least-Squares Numerical Model

Akçay and Gökhan [83] present a numerical approach to modelling hysteresis loops based on minimizing the least-squares error of measured hysteresis loops. They provide a cost function, along with constraints, which are solved using the Levenberg-Marquardt and trust-region reflective Newton methods. The *lsqcurvefit* command available in the optimization toolbox of MATLAB works well for this.

The equations presented by Akçay and Gökhan [83] are not repeated here. A list of advantages and disadvantages is shown in Table 2-6.

Table 2-6: The advantages and disadvantages of the least-squares numerical model

Advantage	Disadvantages
Requires estimation of fewer parameters than polynomial and spline approximations.	A measurement set is required for each level of flux density that requires the modelling of a hysteresis loop.
	The model cannot be generalized.
	Not practical for ferroresonance modelling.

2.10.9 Zou Model

Section 2.10.5 spoke about the use of an arctangent model to represent the nonlinear inductor in an R-L representation of hysteresis. However, Pérez-Rojas [140] did not compare the model to any experimental measurements, nor did he prove its ability to model minor loops.

Zou *et al.* [106] elaborate on the arctangent model in an attempt to model minor loops as well. Though they claim to modify the equations presented by Pérez-Rojas [140], the equations presented in [106] are not representative of those seen in [140].

Zou *et al.* [106] begin by stating that the *VT* hysteresis loop can be defined as:

$$\phi(i) = B \cdot \text{atan}(W \cdot i) + A \cdot i \quad (2-44)$$

where A, B and W are determined by experimental fitting. However, Zou *et al.* [106] state that this is insufficient to model hysteresis loops accurately. Therefore, the ascending leg of the hysteresis loop is obtained by shifting (2-44) to the right by a factor of C. Similarly, the descending leg of the hysteresis loop is obtained by shifting (2-44) to the left by a factor of C. Factor C then controls the width of the hysteresis loop. Zou *et al.* [106] also propose an improvement to the model of [140] using piecewise linearization of segments of the hysteresis loop. Asymptotes are used to ensure a smooth saturation region.

To model the minor loop, Zou *et al.* [106] propose scaling and compressing the major loop. An example of a major and minor loop is, as presented by [106, Fig. 2(b)], is shown in Fig. 2-57. This figure will also be used to explain how minor loops are scaled from major loops.

Zou *et al.* [106] claim that if a reversal point is found at *a*, the transition curve *ab* will intersect the major loop at the saturation coordinates designated (i_s, ϕ_s) in Fig. 2-57. Curve *ab* is then given by

$$\phi_1 = \phi_r - d(\phi) \quad (2-45)$$

In this case, ϕ_r represents the ascending major loop and $d(\phi)$ represents the displacement between the ascending major loop and curve *ab*. Due to the complexity of ferroresonance, $d(\phi)$ is well-represented by a second-order polynomial: Zou *et al.* [106] present limits that can be used to calculate the coefficients of the polynomial. These equations are not presented in this report.

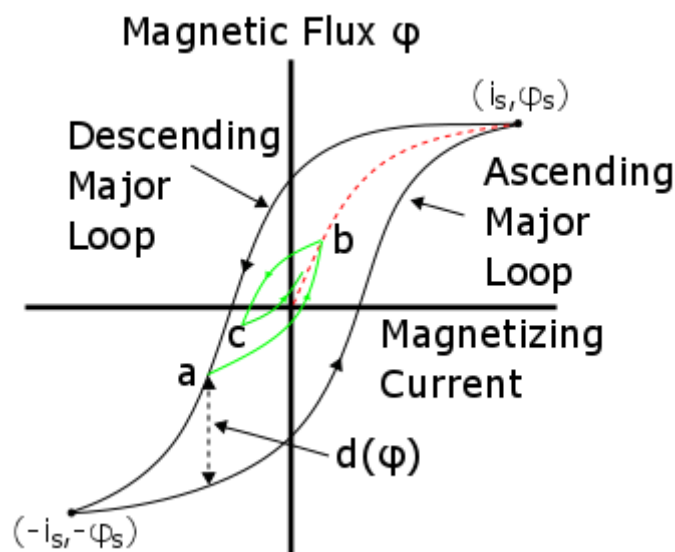


Fig. 2-57: A major and minor hysteresis loop demonstrating the principles of Zou's model [106, Fig. 2(b)]

The reader should note that the model proposed by Zou *et al.* [106] is not titled the Zou model as suggested by the heading of this subsection. The authors do not provide a title for their model. Therefore, for the sake of clarity, it is labelled the Zou model in this report based on the surname of the first author.

The advantages of the Zou model, according to [106], are:

- a The model has better stability and convergence than the Jiles-Atherton and Preisach models;
- b The model is capable of simulating both the voltage and the current waveforms of all periodic ferroresonance modes;
- c The model shows a better representation of the major and minor hysteresis loops than the EMTP-Type 96 model;
- d There are fewer parameters to be determined than for other models;
- e The model has improved calculation speed, and
- f The model is sufficiently accurate for both transients and steady-state simulations.

2.10.10 Takács Model

Takács [128] presented a hyperbolic tangent approach to modelling hysteresis as an alternative to the Langevin and Brillouin functions. This hyperbolic tangent approach was termed the Takács model by Nová, Havlíček and Zemánek [144] in 2013. Hence, the model will be referred to as the Takács model in this report.

The Takács model shows striking similarities to the Langevin and Brillouin functions according to [128]. The equations of the model will not be presented in this report, but Mészáros [145] points out that the equation consists of a linear and a hyperbolic part, which represents the reversible and irreversible magnetization respectively. However, Takács [128] says that for problems that do not involve deep saturation, the linear component can be ignored.

Takács [128] states that by appropriate selection of appropriate constants, most hysteresis loops can be fitted by the proposed hyperbolic function. As was the case with the Zou model, a coefficient is used to shift the mathematical function horizontally and vertically to form a hysteresis loop. Takács [128] presents several permutations to his original equations, which take various phenomena into account. These include DC bias, hysteretic minor loops and remnant flux.

The model provided by Takács in [128] can be considered the classical Takács model and is often referenced in other literature. However, Takács later elaborated on this model in [146] by also taking into account the effects of temperature and internal molecular interaction. The molecular

interaction is taken into account by considering an effective magnetic field, H_{eff} . This effective field is influenced by the coercive field and a mean-field interaction constant.

The advantages of the Takács model, as listed by [137] and [144] are presented in Table 2-7. The disadvantages listed in Table 2-7 are based on [137], [144], [145], [147] and [148].

The lack of eddy current consideration was improved by Chwastek [147]. The unsatisfactory replication of experimental results was addressed by [137] and [148]. The reversible magnetization problem was addressed by Mészáros [145]. The slow magnetization problem was addressed by Nová, Havlíček and Zemánek [144].

Table 2-7: The advantages and disadvantages of the traditional Takács model

Advantages	Disadvantages
The model is transparent and flexible.	The model does not take eddy currents into account.
The parameters of the model have a physical meaning, which helps to understand the magnetization sub-process.	The traditional model's replication of experimental hysteresis loops is unsatisfactory.
The model can be applied to both hard and soft magnetic materials.	When dealing with soft magnetic materials and high magnetic fields, the model is not accurate enough to model reversible magnetization.
	The fundamental model is only suitable for slow magnetization.

2.10.11 Harrison Model

The Harrison model is another one of the more recently developed models, as opposed to the Jiles-Atherton model and the Preisach model. It differs from the JA model, Preisach model and Stoner-Wohlfarth model in the sense that it is based on a quantum-physics picture of atomic moments. The most basic Harrison model also only has three inputs: The Curie temperature, the saturation magnetization and the domain coefficient.

Harrison [159] starts his derivation of the model by stating that the ferromagnet consists of a three-dimensional collection of magnetic domains. The vectors of these domains will then point in the same direction under some external magnetic field. If the ferromagnet is completely

unmagnetized, the orientation of the magnetic vectors will be random and the macroscopic average of these domains will cancel. Harrison [159] states that between the magnetic domains are “Bloch walls,” which are approximately 100 atoms thick. The moment-vector directions of these walls will change sequentially from the preferred direction of one magnetic domain to that of another. When a magnetic field, H , is applied to the material, the direction of the vectors of the Bloch wall will favour that of the magnetic domains pointing in the direction of H . This results in a macroscopic magnetization, M , of the material. The magnetization of the material is defined as the vector sum of all the atomic moments, m , per unit volume. This is considered part of the reversible magnetization. However, as the magnetic field continues to increase, the process becomes irreversible and the magnetization eventually reaches its saturation value, M_s .

The model involves the averaging of macro magnetic approximations to the concept of a magnetic dipole [159]. The average of these approximations is taken across a magnetic domain. The model also applies to systems of ferromagnetic domains. However, it is not concerned with micromagnetic mechanisms, nor does it consider how micromagnetic domains are formed in the first place.

The model aims to present a mathematical function, which relates the atomic and domain movements to the macroscopic observations. Since the magnetization is caused by a collection of magnetic moments, m , the total magnetic moment can be expressed as

$$\mathbf{m}_{sum} = \sum_{i=1}^{\beta} \mathbf{m}_i \quad (2-46)$$

In this equation, the moments $\mathbf{m}_i$ are all collinear and the parameter β represents the domain coefficient. The domain coefficient is representative of the average of the number of atomic moments that are aligned in a head-to-toe fashion over a given space. This is closely related to the size of the domain under consideration. Harrison [159] states that in ferromagnetic materials, the sum of $\mathbf{m}_i$ is closely approximated by $\mathbf{m}_B$, which is the Bohr magneton with a value of $9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2$. The sum of the magnetic moments is then reduced to

$$\mathbf{m}_{sum} = \beta \mathbf{m}_B \quad (2-47)$$

where $\beta \mathbf{m}_B$ represents a new vector quantity and is termed a “super-magneton.” The value of β will vary from very small to very large during magnetization. However, Harrison [159] also adds that for static hysteresis loops, a constant value of β can yield accurate results. Harrison [159] claims that for the Harrison model, the super-magneton is the quantum particle upon which the effective field acts. The effective field is represented by $H_{eff} = H + \alpha M$, where H_{eff} is the

effective field, H is the magnetic field, α is the mean field parameter and M is the magnetization. From the Brillouin function¹⁷, and taking the effective field into account:

$$\frac{M}{M_s} = \tanh \left[\frac{\mu_0 \beta m_B}{kT} (H + \alpha M) \right] \quad (2-48)$$

where

$$\begin{aligned} \mu_0 &= \text{the vacuum permeability} = 4\pi \times 10^{-7} \text{ H/m} \\ k &= \text{the Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J/K, and} \\ T &= \text{the absolute temperature.} \end{aligned}$$

There exists a relationship between the domain coefficient β and the Curie temperature T_C . The Curie temperature represents a critical temperature where ferromagnetic behaviour occurs. An increase beyond the Curie temperature means that ferromagnetic behaviour will no longer take place in the material.

Following a couple of derivations, Harrison [159] defines a second form of his model:

$$\frac{H}{H_{ci}} = \frac{t \cdot \operatorname{arctanh} \left(\frac{M}{M_s} \right) - \frac{M}{M_s}}{t \cdot A - \tanh(A)} \quad (2-49)$$

This second form of the Harrison model only depends on the intrinsic coercive field (H_{ci}), the saturation magnetization (M_s), the Curie temperature (T_C) and the actual temperature (T).

The result of (2-49) is important, as it permits the addition of a pinning coefficient as an elaboration to the standard Harrison model. The pinning coefficient represents an attempt to model domain-wall effects not considered by the standard model. These include nucleation, pinning and unpinning [159]. Suppose that (2-49) is represented by $x_h = \frac{H}{H_{ci}} = \operatorname{hyst}(y, t)$, then the modified Harrison model is given by

$$x_{hm} = \operatorname{hyst}(y, t) + \gamma \cdot \operatorname{arcl}(y) \quad (2-50)$$

where $\operatorname{arcl}(y)$ represents the inverse Langevin function and γ is the pinning coefficient. The impact of the addition of this domain-wall effect can be seen in Fig. 2-58. It is clear from the figure that in the absence of the domain-wall pinning parameter, the hysteresis loop is incorrect. Magnetic domains do not instantaneously jump as depicted by the saturation characteristic of Fig. 2-58(a). Harrison [159] points out that the addition of the pinning effect does not influence the

¹⁷ The Brillouin function for ferromagnetic materials is $\frac{M}{M_s} = \tanh \left(\frac{\mu_0 m_B H}{kT} \right)$ [159].

coercive points of the hysteresis loop, it only influences the shape. In other words, the addition of the pinning effect can be interpreted as a correction of the gradient of the hysteresis loop, as is also evident in Fig. 2-58(b). The value of the pinning coefficient is also determined by the coercive point of the loop.

Based on the fact that the pinning effect only influences the shape of the hysteresis loop, Harrison [159] claims that domain-wall pinning is not the cause of hysteresis. This is in contrast to the Jiles-Atherton model, where domain-wall motion is considered the major cause of magnetization.

Harrison presents another addition to his model in [150]. Recall that when the initial model was discussed, Harrison [159] said that a constant value of β can yield accurate results for static hysteresis loops. However, the value of β should vary with magnetization. In [150], Harrison presents two methods to make the domain coefficient β dependent on the effective field. This is done through the use of what Harrison [150] refers to as a domain-size function (*DSF*).

The introduction of a variable domain coefficient means that there is no longer a simple expression relating β to the Curie temperature [150]. This also means that the measured Curie temperature can no longer be used as a direct input to the function. The new Harrison model, taking the variable domain coefficient into account, is given by

$$\frac{M}{M_s} = \tanh \left[\frac{\mu_0 m_B H_{eff}}{kT} \beta(H_{eff}) \right] \quad (2-51)$$

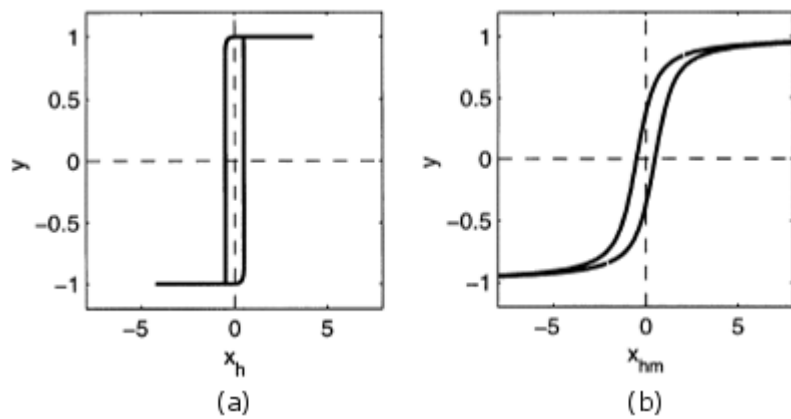


Fig. 2-58: The Harrison model as seen in [159, Fig. 5] a) without domain wall pinning and b) with domain wall pinning

Table 2-8: The advantages and disadvantages of the Harrison hysteresis model

Advantages	Disadvantages
The model is entirely algebraic (no numerical integration required).	Classified as a technical model, which requires high computing power. The literature does not discuss the ability of the Harrison model to generate minor loops.
Produces both exterior loops and initial magnetization curves.	
Uses magnetization (or magnetic flux density) as an independent parameter and calculates the magnetic field intensity.	
The model applied to monocrystals, amorphous materials, grain-orientated materials, polycrystalline materials and ferrimagnetics.	

2.10.12 Rayleigh Model

The Rayleigh model attempts to describe the relationship between magnetic flux density and magnetic field intensity with the help of a parabolic model [139]. It has the advantage of being strongly correlated to the physical properties of the material, which makes it more accurate. It also has a low computational requirement. However, Kachniarz and Szewczyk [139] state that it is only valid for low values of magnetization field. When the magnetization field becomes high, the parabolic relationship between B and H is lost. Kachniarz and Szewczyk [139] state that in many cases, the magnetizing field must be lower than the saturation coercive field. As such, the Rayleigh model will not be discussed in any further detail in this report, since it will not be relevant for ferroresonance analysis. Interested readers are referred to the work of Kachniarz and Szewczyk [139] for further details involving this model.

2.10.13 Chua-Type Model

The Chua-Type model is closely related to the Preisach and the Rayleigh models [151]. The Preisach model is discussed in more detail in section 2.10.18. Since the Rayleigh model was only briefly mentioned in section 2.10.12, this discussion on the Chua-Type model will be very brief. The Chua-type model divides the magnetization characteristics into two major properties, i.e. the saturation and hysteretic properties. The saturation property is represented in terms of

permeability and the hysteretic property is represented by a B-H relationship. The equations of the Chua-type model can be found in [151], [152] and [153].

The basis of the Chua-type model lies in its assumption that the flux linkage and current trajectory is unique to the last point where the time derivative of flux changed sign [151], [153]. Saito *et al.* also claim in [152] that the Chua-type model shows that hysteresis is a result of the magnetic aftereffect. Saito *et al.* prove in [153] that the Chua-type model is capable of modelling the magnetization characteristics, frequency dependency, minor loops, aftereffect and ferroresonance.

The work of Saito *et al.* in [151], [152] and [153] was completed between 1986 and 1989. However, the Chua-type model was also used more recently in 2004 by Endo *et al.* [155] to analyse ferroresonance and in 2017 by Miyata, Yamaura and Okamoto [154]. Endo *et al.* [152] make an interesting claim regarding the Chua-type model, namely that the parameters of the Chua-type model are not influenced by past magnetization histories. This is in contradiction to the Preisach model. This makes for an interesting discussion regarding whether or not the Chua-type model can be considered accurate for ferroresonance modelling. The importance of residual flux has been thoroughly emphasised in this report. Residual flux is a product of magnetization history. The question, then, is whether the hysteresis loop generated by the Chua-type model is dependent on magnetization history. If the magnetization history is not taken into account by the hysteresis model, then the author of this report would like to argue that the Chua-type model is insufficient for ferroresonance modelling. If, however, only the parameters are independent of magnetization history but the loop itself takes the history into account, the Chua-type model holds a tremendous advantage over other models, since the parameters of the model only need to be calculated once.

2.10.14 Tellinen's Model

There is some discrepancy in the literature regarding the importance of hysteresis. Some sources of literature claim that hysteresis is critical to modelling ferroresonance. Other sources of literature claim that piecewise linear approximations are sufficient. Still, other sources of literature say that the hysteresis model must be able to retain residual flux. The Tellinen hysteresis model is one of the most elementary hysteresis models. It allows for simple and easy implementation. Consequently, it can serve as a reference for the importance of highly accurate hysteresis models. Therefore, the Tellinen model is described in more detail than other models mentioned thus far.

Tellinen [156] presented a model in 1998 based on a distribution of the domain-wall movements. These domain-wall movements are referred to as Barkhausen jumps [156]. A slope function ρ is

defined for the rising and the falling legs of the hysteresis loops. These two slope functions are symmetrical and represent the domain-wall distributions. These slope functions are then used for computing the magnetization process.

Based on the slope functions, which are discussed in more detail in [156], the following equations can be defined to calculate the magnetic flux density of the hysteresis loop:

$$\begin{aligned}\frac{db}{dh} &= \mu_0 + \frac{b_{h-}(h) - b}{b_{h-}(h) - b_{h+}(h)} \left[\frac{db_{h+}}{dh}(h) - \mu_0 \right], \quad dh > 0 \\ \frac{db}{dh} &= \mu_0 + \frac{b - b_{h+}(h)}{b_{h-}(h) - b_{h+}(h)} \left[\frac{db_{h-}}{dh}(h) - \mu_0 \right], \quad dh < 0\end{aligned}\quad (2-52)$$

Here, μ_0 represents the vacuum permeability. The discussion of Tellinen [156] also makes it clear that b_{h+} represents the ascending leg of the hysteresis loop and b_{h-} represents the descending loop. However, Tellinen [156] does not provide a clear indication of what the parameter b represents. One is left to assume that it is some form of magnetic flux density, but it is not clear whether this parameter is measured or calculated from the equation. Tellinen only indicates in [156, Fig. 3] that b assumes some value between b_{h+} and b_{h-} .

For verification purposes, Tellinen [156] converted (2-52) slightly, because it is easy to measure the voltage waveform. This means that the variation in the flux density as a function of time is known, thus transforming (2-52) into

$$\begin{aligned}\frac{dh}{dt} &= \frac{1}{\mu_0 + \frac{b_{h-}(h) - b}{b_{h-}(h) - b_{h+}(h)} \left(\frac{db_{h+}}{dh}(h) - \mu_0 \right)} \cdot \frac{db}{dt}, \quad \frac{db}{dt} > 0 \\ \frac{dh}{dt} &= \frac{1}{\mu_0 + \frac{b - b_{h+}(h)}{b_{h-}(h) - b_{h+}(h)} \left(\frac{db_{h-}}{dh}(h) - \mu_0 \right)} \cdot \frac{db}{dt}, \quad \frac{db}{dt} < 0\end{aligned}\quad (2-53)$$

Tellinen [156] also adds that if dynamic hysteretic effects need to be considered, then the effects of eddy currents also need to be taken into account. The dynamic field intensity is given by

$$h_d = h(b) + \sigma_e \frac{db}{dt} \quad (2-54)$$

where σ_e is the eddy-current factor for a single lamination. Tellinen [156] adds that more information on the determination of this eddy-current factor can be found in [157].

Moses and Masoum [33] specifically used Tellinen's model to simulate ferroresonance. They found it easier to express the equations in terms of flux linkage λ and magnetising current i_m , which changes the equations to:

$$\begin{aligned}\frac{di_m}{dt} &= \frac{d\lambda}{dt} \cdot \frac{1}{L_0 + \frac{\lambda_-(i_m) - \lambda(i_m)}{\lambda_-(i_m) - \lambda_+(i_m)} \left(\frac{d\lambda_+(i_m)}{di_m} - L_0 \right)}, \quad \frac{d\lambda}{dt} > 0 \\ \frac{di_m}{dt} &= \frac{d\lambda}{dt} \cdot \frac{1}{L_0 + \frac{\lambda(i_m) - \lambda_+(i_m)}{\lambda_-(i_m) - \lambda_+(i_m)} \left(\frac{d\lambda_-(i_m)}{di_m} - L_0 \right)}, \quad \frac{d\lambda}{dt} < 0.\end{aligned}\quad (2-55)$$

The parameter L_0 represents the fully saturated inductance of the transformer. Moses and Masoum [33] start by calculating a nonlinear function to fit the hysteresis loop. This nonlinear function is given by

$$f(i_m) = \text{sgn}(i_m) \cdot \alpha \log_{10}(\beta |i_m| + 1) \quad (2-56)$$

and the ascending and descending legs of the hysteresis loop are given by shifting this function to the right or the left by a factor of σ . In other words, the ascending function is given by

$$\lambda_+(i_m) = f(i_m - \sigma) \quad (2-57)$$

and the descending function is given by

$$\lambda_-(i_m) = f(i_m + \sigma) \quad (2-58)$$

Moses and Masoum [33] also give an equation for the derivative of flux to magnetising current:

$$\frac{d\lambda_{\pm}(i_m)}{di_m} = \frac{\alpha\beta}{\beta|i_m| \mp \sigma + 1} \quad (2-59)$$

In both (2-56) and (2-59), the parameters α and β are determined through the help of a fitting process to the measured hysteresis loop.

The nonlinear hysteresis effect is implemented in a ferroresonant circuit by Moses and Masoum [33] through the use of a controlled voltage source. This voltage source generates an induced voltage across the transformer windings, which is based on (2-55). The transformer then draws a nonlinear current.

The advantages and disadvantages of the Tellinen model are shown in Table 2-9.

Table 2-9: The advantages and disadvantages of the Tellinen hysteresis model

Advantages	Disadvantages
Simple implementation.	Does consider past magnetization.
Easy parameter determination.	Minor loops are inaccurate for excitation levels below 0.3 p.u.

Ascending and descending loop symmetry implies that only one segment of the hysteresis loop needs to be measured.	Certain ferroresonance oscillations could not be replicated due to ill-conditioned residual flux and capacitive charge.
Has been used for ferroresonance analysis.	

2.10.15 Electromagnetic Transient Models

Section 2.7.2 spoke about a few R-L hysteretic representations used in simulation programs. More specifically, Zou *et al.* [106] spoke about the TYPE-96 hysteretic model, Stosur *et al.* [107] spoke about the TYPE-98 model and Mork [108] spoke about the TYPE-93 model. This section of the report will focus on some of these hysteretic models and their underlying principles as discussed by the IEEE Power System Relaying Committee [158]. They are referred to as Electromagnetic Transient Models because they are found as elements within the Electromagnetic Transients Program (*EMTP*) and other similar programs (*ATP*, *EMTDC* etc.).

The IEEE Power System Relaying Committee [158] start by stating that programs such as *EMTP* and *ATP* support two different types of models, i.e. true nonlinear models and pseudo-nonlinear models. The TYPE-93 model is an example of a true nonlinear model and the TYPE-96 and TYPE-98 models are examples of pseudo-nonlinear models.

For true nonlinear models, the nonlinearity is explicitly defined as a nonlinear function [158]. Flux is, therefore, defined as a nonlinear function of current in some cases ($\psi = f(i)$). Both *ATP* and *EMTP* then solve the nonlinear function through the use of a Newton-Raphson numerical formula.

For the pseudo-nonlinear case, magnetization is defined through the use of a range of piece-wise linear functions. The piecewise linear segments are represented by the program as a resistor in parallel with a current source. The segment used by the program is determined by monitoring the flux in the inductor. The segment being used is only changed once the program has started to operate outside the limits of a given linear segment. For this reason, small time steps are suggested by [158].

The *EMTDC* model consists of a nonlinear inductor and a nonlinear resistor, which are in parallel [158]. These branches then represent the magnetising current and the hysteretic current. For the magnetising current, the mathematical representation is a power series. For the hysteretic current, the mathematical representation is an exponential equation with a coefficient referred to as the Steinmetz coefficient.

The power series coefficients are determined from saturation curves and this is done through an iterative fitting process [158]. The resulting B-H curve is smooth and similar to that found in *EMTP/ATP*.

The IEEE Power System Relaying Committee [158] also discuss the use of conductance as opposed to impedance for inductors and capacitors. However, the inversion of matrices for this purpose and the re-evaluation of these matrices with each time step has a significant impact on simulation time. For a more detailed discussion of this topic, as well as the *EMTDC* model, the reader is referred to the work of [158].

2.10.16 Stoner-Wohlfarth Model

Liorzou, Phelps and Atherton [127] and Chandrasena *et al.* [81] state that four hysteresis models are now considered “classical.” The Stoner-Wohlfarth model is one of these four models. Per Kachniarz and Szewczyk [139], Harrison [159] also provides two hysteresis model classifications, i.e. physical models and empirical fitting procedures. The physical models are computationally intensive, while the empirical models have a flimsy correlation to the underlying physics. However, Harrison [159] adds that there are some models, which lie in-between these two extremes. The Stoner-Wohlfarth model is one such model. Harrison [159] describes the model as one which considers all possible energies on a microscopic level and then finds the minimum-energy configuration.

Atherton and Beattie [149] state that the theory is based on the rotation of single-domain particles for their easy axes. This results in the magnetic domains taking up a minimum-energy position, as also referred to by Harrison [159]. The model assumes that the magnetic core consists of many small particles. These particles are anisotropic¹⁸ – a property caused by either stress, crystal structure or particle shape [149]. Liorzou, Phelps and Atherton [127] describe this property as “an assembly of non-interacting single-domain particles with uniaxial anisotropy.” Liorzou, Phelps and Atherton [127] also confirm the three causes of this anisotropy as being crystal structure, shape or stress.

The model assumes that each particle is uniformly magnetized to saturation. This leads to a single magnetic domain with a moment m , which is free to rotate in any direction. Atherton and Beattie [149] then further point out that quantum exchange forces and dipole-dipole forces are ignored.

¹⁸ Magnetic anisotropy is defined by [143] as the property whereby the magnetic properties of single crystals of iron depend on the direction in which they are measured.

Liorzou, Phelps and Atherton [127] also say that pinning effects are typically ignored in the model. They propose a solution to this in [127], which will be discussed shortly.

The model is based on the assumption that the magnetic particles rotate such that the minimum energy orientation is achieved [149]. Two energy minima can be defined for the magnetic particle (see Fig. 2-59). For small magnetic fields, the magnetization of the particle is only adjusted from the easy axis¹⁹. There also exist two energy minima, m_1 and m_2 , as depicted in Fig. 2-59. It should be noted that Liorzou, Phelps and Atherton [127] present a similar diagram and discussion, but in their case, the lines m_1 and m_2 represent the particle magnetization vector itself and not the energy minima. However, one may argue that the magnetization vector represents energy minima since the model claims that the particle will assume an orientation of minimum energy. For this discussion, the convention used by Atherton and Beattie [149] will be used – namely that m_1 and m_2 represent energy minima.

When the magnetic field applied to the particle changes, the positions of the energy minima change. If the magnetic field is kept below a critical value, the change in the energy minima is reversible. However, when the magnetic field exceeds some critical value H_c , one of the energy minima becomes unstable [149]. The magnetization of the particle then undergoes a sudden change to the other energy minimum. This new minimum is referred to as the global energy minimum and a switch to this global energy minimum is irreversible. This means that even when the magnetic field is reduced, the particle will not return to its original orientation. Liorzou, Phelps and Atherton [127] claim that the global energy minima referred to by [149] is a new energy minimum and that magnetization changes following the transition will once again be reversible. Since irreversible magnetization is the cause of hysteresis, the transition from one energy minima to another leads to hysteresis

One of the disadvantages of the Stoner-Wohlfarth theory, as pointed out by Harrison [159], is that it only considers single-domain particles that do not interact. This problem is addressed by Atherton and Beattie [149] through the use of a mean field. The mean-field theory was proposed by Jiles and Atherton [43] and is also found in the Jiles-Atherton hysteresis model discussed in Section 2.10.19. The theory proposes that the effective magnetic field is given by $H_{eff} = H + \alpha M$, where M is the total magnetization of the sample and α is the mean field interaction constant [127]. Atherton and Beattie [149] state that micromagnetic models are more precise than the mean-field theory, but computation time is a concern for such models. This makes the mean-field

¹⁹ For a better understanding of the easy axis of a magnetic material, refer to [159]

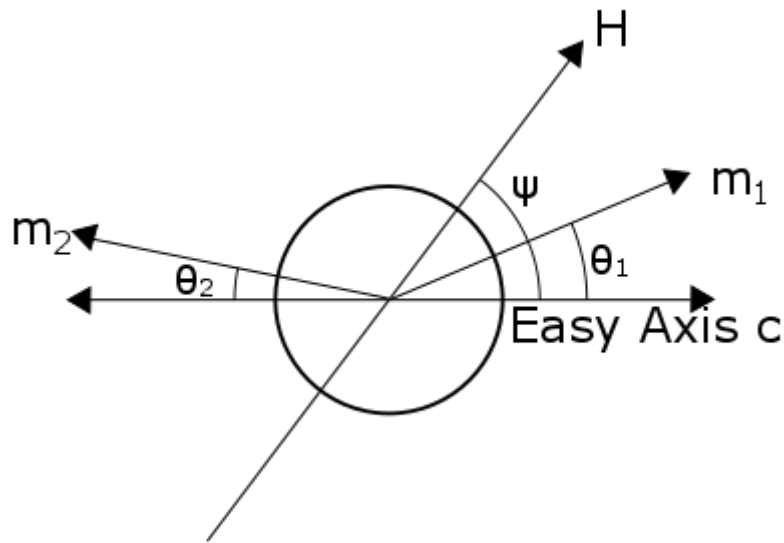


Fig. 2-59: A demonstration of a Stoner-Wohlfarth particle and minimum energy levels, as seen in [149]

theory a good compromise. Other advantages and disadvantages of the Stoner-Wohlfarth model are given in Table 2-10.

Liorzou, Phelps and Atherton [127] suggest a further improvement to the classical model to address some of the disadvantages in Table 2-10, especially the lack of consideration of pinning effects. They motivate this based on the fact that the rotation of particles is liable to be impeded by lattice defects and residual stresses. This means that the torque exerted on the particles need to overcome resistive pinning torques generated by the defects to orient the particles. This pinning torque is taken into consideration by making the pinning torque proportional to the bulk magnetization using a proportionality coefficient. The proportionality coefficient is denoted p . However, Liorzou, Phelps and Atherton [127] do not provide a further indication of how this parameter is brought into the equations nor how to solve the equations for hysteresis. They only point out that the model is now determined by six parameters.

Table 2-10: The advantages and disadvantages of the Stoner-Wohlfarth model

Advantages	Disadvantages
A single mechanism generates both reversible and irreversible magnetization.	The model only considers single domain particles that do not interact.
The model easily accounts for the preferential orientation of crystals of the material.	Each particle must consist of a single, uniformly magnetized domain.

The model is computationally simple.	Pinning effects are ignored.
The model is naturally three-dimensional.	Hysteresis is introduced only through anisotropy.
The model accounts for anisotropy.	

2.10.17 Globus Model

The Globus model is the second “classical” model discussed by Liorzou, Phelps and Atherton [127]. The other two “classical” models are the Preisach and Jiles-Atherton models, which are discussed in sections 2.10.18 and 2.10.19 respectively.

Harrison [159] describes the Globus model briefly by stating that it assumes the domain walls are pinned at grain boundaries. They are then also capable of deforming elastically under the influence of a magnetic field. This represents domain-wall energy and contains components of both reversible and irreversible magnetizations. However, Harrison [159] states that the model is restricted to “polycrystalline soft ferrite materials that are of high quality on the microscopic scale.” The model also operates under the hypothesis that the entire sample can be represented by a single spherical grain.

Jiles and Atherton [43] refer to the Globus model as being a good qualitative explanation of the general shape of ferromagnetic and ferrimagnetic hysteresis cores. They also refer to the pinning effect on grain boundaries but add that this is caused by some friction force. The reversible component of hysteresis under a magnetic field is caused by domain wall bulging, while the irreversible component is caused by domain wall displacement.

Liorzou, Phelps and Atherton [127] state that the magnetic field required to move a flat domain wall is determined by three different types of energy. The sum of the energies then gives the magnetic field. The first energy is a function of the friction at the domain wall grain boundary and is proportional to the wall circumference. The second energy is a function of the wall energy and varies with the domain wall surface. The third energy term represents the magnetostatic energy, which works like a magnetic pressure on the wall.

Liorzou, Phelps and Atherton [127] proceed to discuss the parameters upon which the model is dependent, as well as how they can be measured. However, they conclude that the model is not yet useful for engineering purposes and that further development is required. For this reason, the model will not be discussed in any further detail. Recent developments in hysteresis literature are

not focused on addressing the model's shortcomings. A breakdown of some of the key features of the four classical models, including the Globus model, is given in Table 2-13.

2.10.18 Preisach Model

Tellinen [156] states that the Preisach model is the most well-known of all the hysteresis models. The Preisach model was used to discuss the theory of hysteresis in section 2.10.1. This section of the report will, therefore, serve as an elaboration on those concepts already discussed in section 2.10.1.

The Preisach model discussion presented in section 2.10.1 was a geometrical interpretation of the Preisach model, as presented by Mayergoyz [129]. This geometrical interpretation is also found in [127] and [160]. The geometrical interpretation of the Preisach model is easier to understand, but it should be noted that the Preisach model can be explained mathematically as well. To understand the mathematical description of the model, it is necessary to revisit the concept of the magnetic dipole presented by Preisach. This dipole is shown in Fig. 2-40. However, an updated magnetic dipole, as seen in [161, Fig. 1], is shown in Fig. 2-60. In this figure, two important parameters are indicated, namely the coercivity h_c and the field h_m caused by neighbouring dipoles. The reference to coercivity is ambiguous, as Coulson, Slater and Simpson [161] seem to make a distinction between the coercive force of the major B-H loop (denoted H_c) and the coercivity h_c . The coercive force of the major B-H loop was defined in section 2.10.1, but the coercivity h_c is an undefined term. One can assume that the coercivity h_c represents half the width of the magnetic dipole, but there is also a discrepancy in the literature regarding this width since [52, Fig. 2] and [162, Fig. 1] indicate that the width of the magnetic dipole equals only h_c . Coulson, Slater and Simpson [161] claim that the width of the magnetic dipole is $2h_c$, as also indicated in Fig. 2-60. This is also confirmed by Liorzou, Phelps and Atherton [127]. For this discussion, it will be assumed that [127] and [161] are correct, i.e. the width of the magnetic dipole is equal to $2h_c$.

The parameters h_c and h_m are statistical [162]. A distribution function can be defined, which gives the number of dipoles between (h_c, h_m) and (h_c+dh_c, h_m+dh_m) . This distribution function is given by [162] as:

$$dn = \gamma(h_c, h_m) dh_c dh_m \quad (2-60)$$

In this distribution function, $\gamma(h_c, h_m)$ represents the probability density function. It has two properties: it assumes a value of 0 for $h_c < 0$ and it has symmetry so that $\gamma(h_c, h_m) = \gamma(h_c, -h_m)$.

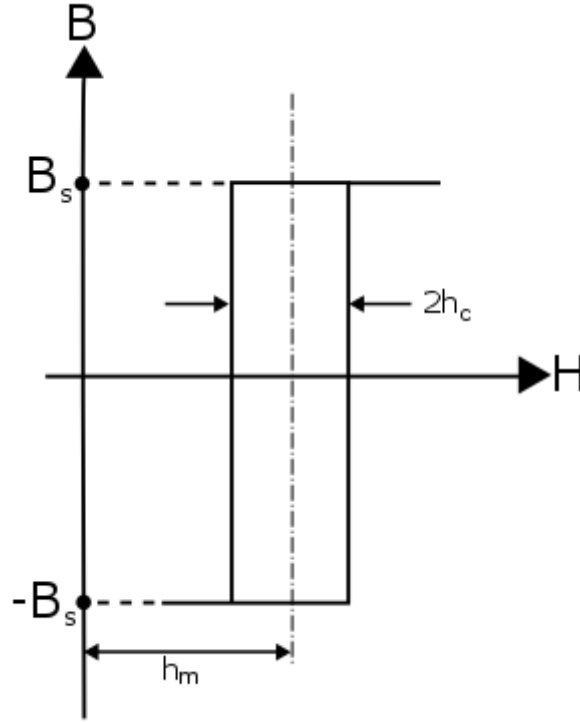


Fig. 2-60: An alternative Preisach magnetic dipole with statistical parameters, as seen in [161, Fig. 1]

The magnetic flux density B is determined by the sum of the upward and downward states of all these magnetic dipoles. This can be expressed in integral format. The area of integration is related to the geometrical interpretation of the Preisach model. Naidu [162] and Coulson, Slater and Simpson [161] present alternative geometrical interpretations to those presented by Mayergoyz [129]. Nevertheless, the magnetic flux density is given by [161] as:

$$B = 2B_s \iint_R \gamma(h_c, h_m) dh_c dh_m \quad (2-61)$$

Coulson, Slater and Simpson [161] also present an equation for the $\gamma(h_c, h_m)$ distribution function. It was found that the equation

$$2B_s \gamma(h_c, h_m) = ah_c^n e^{-p_1 h_c} e^{-p_2 |h_m|} \quad (2-62)$$

yielded a good approximation, where a , p_1 and p_2 are coefficients and n is a positive integer. Coulson, Slater and Simpson [161] also found that it is necessary to modify (2-61) to:

$$B = \mu_0 H + 2B_s \iint \gamma(h_c, h_m) dh_c dh_m \quad (2-63)$$

to represent the real magnetic characteristic more accurately. This presents the user with a convenient definition of the coefficient a in (2-62). Coulson, Slater and Simpson [161] also derive

equations for the remanence and the coercive force of the outer loop. However, they point out that the approximation of (2-62) is only valid for soft magnetic materials. For hard magnetic materials, the equation needs to be extended to a series of terms. Interested readers are referred to [161, Eq. 10] for more information in this regard.

Vajda and Della Torre [163] claimed that the distribution function $\gamma(h_c, h_m)$ took the form of a Gaussian function in 1991. Liorzou, Phelps and Atherton [127] referenced this equation but modified it into a different form in 2000. Rezaei-Zare *et al.* [52] in turn reference the work of [127] but ignored the saturation magnetization component altogether in 2007. The final distribution function, as presented by Rezaei-Zare *et al.* is given by:

$$\gamma(h_c, h_m) = \frac{1}{2\pi\sigma_c\sigma_m} \exp\left[-\frac{(h_c - \bar{h}_c)^2}{2\sigma_c^2}\right] \exp\left[-\frac{h_m^2}{2\sigma_m^2}\right] \quad (2-64)$$

where

- $\bar{h}_c$ = the average of the coercive fields;
- σ_c = the distribution width of the coercive field, and
- σ_m = the distribution width of the applied field.

It is worth noting that both Mayergoyz [129] and Hui and Zhu [160] present an alternative distribution function for the magnetic dipole, which in turn changes the double integral. The distribution function presented by [129] and [160] is a function of the switching characteristics α and β . Recall from section 2.10.1 that these represented the upward and downward switching fields respectively for a given magnetic dipole. An applied field greater than α will result in the magnetic dipole rotating to the upward direction and an applied field less than β will result in a downward rotation of the magnetic field. The new distribution function is then given as $\mu(\alpha, \beta)$. The integral to determine the magnetic flux density is given by

$$B = \iint_S \mu(\alpha, \beta) \gamma_{\alpha\beta}(H) d\alpha d\beta \quad (2-65)$$

where $\gamma_{\alpha\beta}(H)$ assumes a value of +1 when the dipole has an upward orientation and -1 when the dipole has a downward orientation. This means that the double integral can be broken up over the two segments of the Preisach hysteresis triangle seen in Fig. 2-45. If an upward orientation of magnetic dipoles is assumed to be positive and a downward orientation is assumed to be negative, the flux density of the material is given by:

$$B = \iint_A \mu(\alpha, \beta) d\alpha d\beta - \iint_B \mu(\alpha, \beta) d\alpha d\beta \quad (2-66)$$

One should also take note that Hui and Zhu [160] present the magnetic flux as only a single integral. This is assumed to be a mistake since the equation is identical to that presented by Mayergoyz [129] in all other aspects. Furthermore, equations presented in [161] and [162] also take the form of a double integral.

Mousavi and Engdahl [110] present an equation very similar to (2-65). However, the integral yields the magnetization M of the material rather than the magnetic flux density B . All other parameters in the equation are identical. One, therefore, has to assume that the density function $\mu(\alpha, \beta)$ used in [110] differs from that used in [160] and [129] since magnetic flux density is not directly proportional to magnetization. Rather, the relationship between flux density and magnetization is given by [51] as:

$$B = \mu_0(H + M) \quad (2-67)$$

Simulation time and complexity is a concern with the Preisach model. In comparing the four classical hysteresis models, Liorzou, Phelps and Atherton [127] state that the Preisach model has the longest computational time. Mousavi and Khayatian [164] state that inverse models can be used to reduce the complexity of the Preisach model. Mousavi and Engdahl [110] employed such an inverse model and claimed that the simulation speed became comparable to that of the JA model. The need for an inverse model is also motivated by the fact that electromagnetic transients, such as inrush current, DC magnetization and ferroresonance require a model, which uses magnetization M as input and yields the field H as output.

Several sources of literature on the Preisach model are focused on its ability to simulate minor hysteresis loops, these include [110], [160], [161], [162], [165] and [166]. The minor loop is closely related to the concept of a principal trajectory and a reversal point. Two principal trajectories are shown in Fig. 2-61. This is a replica of [162, Fig. 4a]. The upward trajectory is denoted $B_{H_1}^u(H)$ and the downward principal trajectory is denoted $B_{H_2}^d(H)$. A principal trajectory starts on a reversal point on the main hysteresis loop and then moves to positive or negative saturation. From Fig. 2-61 one can conclude that the saturation point towards which the principal trajectory curve moves is dependent on whether the reversal point is on the ascending or descending leg of the limiting hysteresis curve. This is not explicitly stated by Naidu [162].

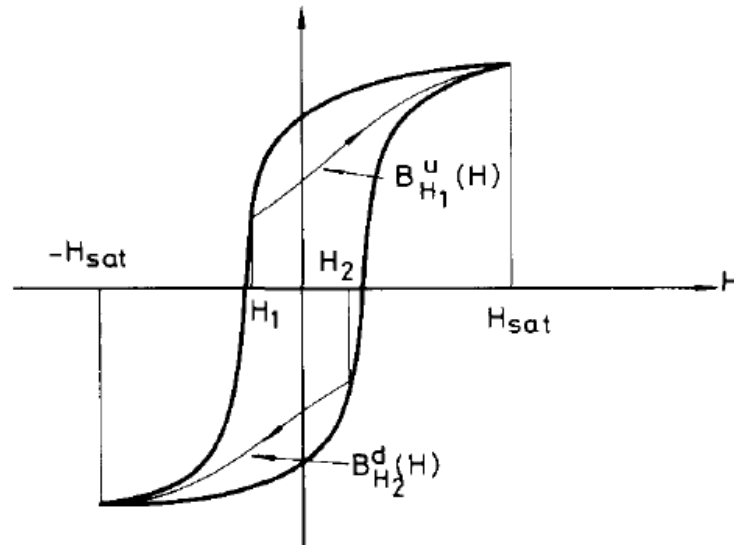


Fig. 2-61: A hysteresis loop showing two principal trajectories, as seen in [162, Fig. 4a]

The concept of a reversal point is also not explicitly defined in any of the abovementioned sources on minor loops. It is assumed that the reversal point represents the point at which the change in magnetic field intensity ΔH switches signs, e.g. from positive to negative or from negative to positive.

To demonstrate the concept of a minor loop and its relation to the principal trajectory, consider Fig. 2-62, which can also be found in [162, Fig. 5a]. Assume that the magnetic material is firstly taken into negative saturation. Thereafter, at some point on the ascending hysteresis loop, the field is reversed. This leads to the reversal point labelled as point 2. The field is decreased and the minor hysteresis loop follows the trajectory of 2-3-4. The applied field is then once again reversed at 4, after which it follows the path 4-5-2 back to the initial reversal point. Thereafter, the major hysteresis loop will be followed for a further increase in the magnetic field.

It is not made clear in Fig. 2-62, but the path 4-5-2 is parallel to the principal trajectory denoted 4'-5'-2'. Naidu [162] proves this with the help of the triangular Preisach diagram. This concept is also affirmed by Mousavi and Engdahl [110]. In other words, the minor loop will be parallel to any reversal curve or principal trajectory, where the reversal points are on a vertical line. It is also important to take note of the fact that once the minor loop is completed, the hysteresis diagram will follow the major loop again. This means that the wiping-out property discussed in section 2.10.1 has taken effect. It also means that the reversal point designated as point 2 in Fig. 2-62 no longer has any effect. Mousavi and Engdahl [110] state that the magnetic memory of the

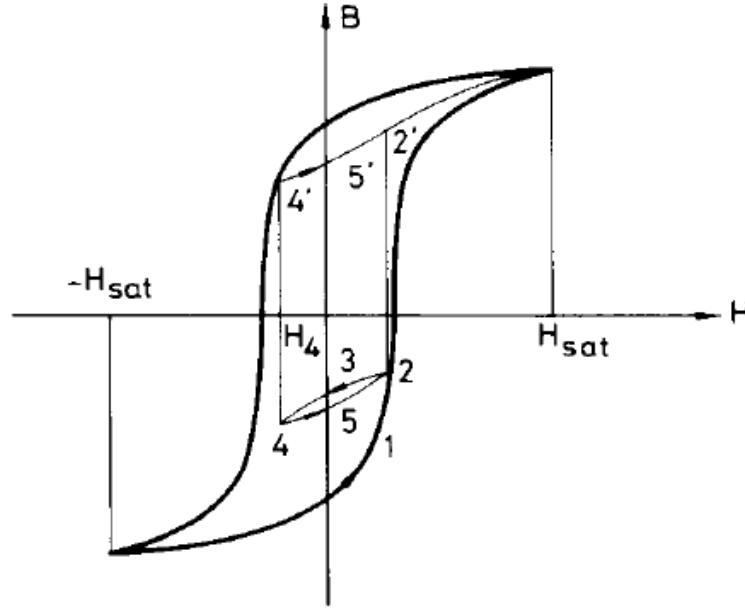


Fig. 2-62: A hysteresis loop showing the correlation between the principal trajectory and a minor loop, found in [162, Fig. 5a]

material is, therefore, only dependent on the last reversal point of an unclosed hysteresis loop. Algorithms implementing the Preisach model make use of a stack to store the coordinates of the reversal points [160], [162], [165]. Whenever a minor loop is closed, the last two reversal points are dropped from the stack, since they no longer influence the magnetic history [110], [166].

The statistical nature of the Preisach model, together with the long simulation times, can be intimidating. However, Hui and Zhu [160] claim that the statistical nature of the equations can easily be implemented in software and the graphical approach can be used to apply the Preisach model. In such a case, only the limiting major B-H loop is required as input, which can be determined through experimentation. A detailed discussion is presented by Hui and Zhu [160] for interested readers, but the essence of their approach can be summarised in the following equations²⁰:

$$\begin{aligned} B(H) &= B(H_n) - 2T(H_n, H) \text{ for } \Delta H > 0 \\ B(H) &= B(H_n) + 2T(H, H_n) \text{ for } \Delta H < 0 \end{aligned} \quad (2-68)$$

$$T(\alpha, \beta) = \frac{B_u(\alpha) - B_d(\beta)}{2} + F(\alpha)F(-\beta) \quad (2-69)$$

²⁰ The author of this report has found that there is a logic error in the equations of Hui and Zhu [160]. This error is discussed in detail in Appendix A.

$$F(\alpha) = \frac{B_d(\alpha) - B_u(\alpha)}{2\sqrt{[B_d(\alpha)]}} \text{ for } \alpha \geq 0 \quad (2-70)$$

$$F(\alpha) = \sqrt{[B_d(-\alpha)]} \text{ for } \alpha < 0 \quad (2-71)$$

$$B_u(H) = B_i(H) - [F(H)]^2 \quad (2-72)$$

$$B_d(H) = B_u(H) + 2F(H)F(-H) \quad (2-73)$$

$$B_i(H) = [F(-H) - F(H)]^2 \quad (2-74)$$

where

- H_n = the magnetic field of the last reversal point;
- $B(H_n)$ = the flux density of last reversal point;
- $B_u(\alpha)$ = the magnetic flux density at field α on the rising leg of the major hysteresis loop, and
- $B_d(\beta)$ = the magnetic flux density at field β on the falling leg of the major hysteresis loop.

From these equations, it becomes clear that if one has both legs of the limiting major hysteresis loop and a program, which stores the reversal points, then the Preisach model can be implemented. Simulation results presented in [160] compare very well to experimental measurements for two different types of cores. However, it should be noted that the fundamental frequency in [160] is 1 Hz, and since hysteresis is frequency dependent, it is the opinion of the author of this report that further investigation at higher frequencies should still be done.

Several modifications to the classical Preisach model have been proposed in the literature. Rezaei-Zare *et al.* make reference to a moving Preisach model in [28], [52], [102], [109] and [167]. In this moving Preisach model, they propose the use of a moving coefficient k_B such that an effective magnetic field intensity can be defined as:

$$H_e = H + k_B \cdot B_h(H_e) \quad (2-75)$$

where B_h represents the hysteretic part of magnetic flux density where the core has not yet gone into saturation. A moving Preisach model is proposed and discussed extensively by Mayergoyz [129]. However, this moving Preisach model presented by Mayergoyz [129] presents an alternative geometrical interpretation for the classical Preisach model and the derived equations are equivalent to those seen in (2-65).

Mayergoyz [129] also presents a nonlinear model, a restricted model and a dynamic Preisach model. These models are presented as additions to the classical Preisach theory. They are motivated by the following five shortcomings of the classical Preisach model, as pointed out by Mayergoyz [129]:

- a The congruency of minor loops described by the classical model does not correlate to experimental data;
- b The classical model does not account for dynamic effects, which can be essential in the case of fast input variations;
- c Loop formation may be prefaced by a stabilisation process, which the classical model does not take into account;
- d The classical Preisach model does not consider material stress, which may also contribute to hysteresis formation, and
- e The classical Preisach model is a scalar model. Vector hysteresis may also be encountered and the properties may then be different.

In the nonlinear model, the distribution function μ becomes dependent on the current value of the input. In other words, if the input to the model is the magnetic field intensity, and is given by $H(t)$, then the distribution function becomes $\mu(\alpha, \beta, H(t))$.

The restricted model presented by Mayergoyz [129] makes the distribution function μ dependent on the past global maxima and minima. If the distribution function μ is only dependent on one maximum or minimum, then it is referred to as a first-order restricted Preisach model [129]. If the model is dependent on both the past global maximum and global minimum, then it becomes a second-order restricted model and may be given by $\mu(\alpha, \beta, M_1, m_1)$, where M_1 represents a global maximum and m_1 global minimum. Higher order models can be defined, which are associated with increased model accuracy. However, these modifications have the drawback of increased complexity and an increase in the amount of experimental data required for the model. For this reason, Mayergoyz [129] says that models higher than the second-order are unattractive and unnecessary. Mayergoyz [129] also points out that a second-order restricted Preisach model is better at modelling second-order reversal curves when compared to the classical Preisach model. The model also presents a more accurate depiction of the congruency property of minor loops, which was briefly discussed in section 2.10.1. For more information in this regard, the reader is referred to [129].

The dynamic Preisach model presented by Mayergoyz [129] serves as an extension to the classical, nonlinear and restricted Preisach models. It proposes that the distribution function μ also be made dependent on the rate at which the input is changing. In other words, for the classical Preisach model with magnetic field $H(t)$ as input, the distribution function becomes

$\mu\left(\alpha, \beta, \frac{dH}{dt}\right)$. The dynamic model can also be applied to the nonlinear and the restricted model. One would expect this dynamic Preisach model to be the most accurate of the models presented thus far since hysteresis is frequency-dependent. However, no reference was made to this dynamic model in the literature studied by the author of this report.

Further extensions addressing the shortcomings of the classical Preisach model are presented by Mayergoyz [129]. These include a vector Preisach hysteresis model. However, those elaborations are considered beyond the scope of this report.

A few final notes can be made on the Preisach model. Firstly, Ribas *et al.* [51] point out that the Preisach model is efficient. However, its numerical implementation is complex and it is also complex for material characterisation. For this reason, Tygai and Besarab [55] state that the Preisach model is mostly used for the development of new ferromagnetic materials since the model requires a lot of parameters. Note, however, that this is in contrast to the claims of Hui and Zhu [160]. Finally, in the introduction to hysteresis models in general in section 2.10.2, it was stated that the ideal hysteresis model has four properties. Mousavi and Engdahl [110] point out that the Preisach model only has the first one.

2.10.19 Jiles-Atherton Model

Theory

The Jiles-Atherton (JA) model is the last of the four classical models mentioned by Chandrasena *et al.* [81] and Liorzou, Phelps and Atherton [127]. It is also the final model to be discussed in this report. The Preisach and the JA models are the most frequently referenced models in the literature.

Chandrasena [169] states that the basis of the JA theory is the existence of the anhysteretic magnetization, which is the global minimum energy state. For any given field, if the magnetization is greater than the anhysteretic magnetization, the domain walls of the material will experience a force that attempts to reduce the magnetization. If, however, the magnetization is less than the anhysteretic magnetization, the domain walls will experience a force that tends to increase the magnetization.

In developing the JA theory, Jiles and Atherton [43] used soft magnetic materials. The saturation characteristics were then modelled with the help of a Langevin function. The Takács model in section 2.10.9 and the Harrison model in section 2.10.10 both also make use of Langevin functions. Annakkage *et al.* [168] state that the Langevin function is used to obtain a sigmoid type

curve for the anhysteretic magnetization. The modified Langevin function originally proposed by Jiles and Atherton [43] took the form:

$$M_{an}(H_e) = M_s \left(\coth\left(\frac{H_e}{a}\right) - \left(\frac{a}{H_e}\right) \right) \quad (2-76)$$

where M_s represents the saturation magnetization. Parameter a has the dimensions of a magnetic field and characterises the shape of the anhysteretic magnetization curve. The parameter H_e represents the effective field, which will be discussed shortly.

Despite the use of the Langevin function, Jiles and Atherton [43] first introduced anhysteretic magnetization as $M_{an} = M_s f(H_e)$, where $f(H_e)$ is an arbitrary function. Jiles and Atherton [43] only chose the Langevin function for modelling. A polynomial alternative was suggested by Annakkage *et al.* [168]. This polynomial approach will also be discussed in Chapter 3.

One of the most prominent features of the JA model is that it traditionally expresses a relationship between magnetization M and magnetic field intensity H . The equations may be modified to express other relationships, such as the relationship between flux and current, but the original model proposed by Jiles and Atherton in [43] referred to magnetization and field intensity. Conversion between magnetic flux density B and magnetization M is not difficult. Equation (2-67) provides an equation for this conversion.

The JA also makes use of the concept of an effective field. The effective field is given by:

$$H_e = H + \alpha M \quad (2-77)$$

where

- H_e = the effective field;
- M = the magnetization, and
- α = the mean-field parameter representing interdomain coupling.

The JA model breaks the magnetization down into two components as follows [168]:

$$M = M_{irr} + M_{rev} \quad (2-78)$$

M_{irr} represents the irreversible component of magnetization and is a function of the pinning of magnetic domains due to discontinuities in the material structure [168], [170]. Annakkage *et al.* [168] state that it can be likened to the friction effect. M_{rev} , on the other hand, represents the reversible component of magnetization. Annakkage *et al.* [168] state that this is due to elastic

domain wall bending. This is confirmed by Lederer *et al.* [170]. The Globus model of section 2.10.17 also made reference to domain wall bending.

The original model proposed by Jiles and Atherton [43], following some derivations, assumed the form:

$$\frac{dM}{dH} = \frac{1}{(1+c)} \frac{1}{\frac{\delta k}{\mu_0} - \alpha(M_{an} - M)} (M_{an} - M) + \frac{c}{(1+c)} \frac{dM_{an}}{dH} \quad (2-79)$$

where

$$\begin{aligned} c &= \left(\frac{m\pi y^4}{4E} \right) C' = \text{the coefficient of proportionality;} \\ E &= \frac{1}{2} (M - M_{an})^2 = \text{the effective magnetostatic energy;} \\ C' &= \text{a constant;} \\ k &= \frac{n \langle \varepsilon_\pi \rangle}{2m}; \\ n &= \text{the average density of pinning sites in a solid;} \\ \langle \varepsilon_\pi \rangle &= \text{the average pinning energy of pinning sites;} \\ m &= \text{the magnetic moment per unit volume;} \\ \delta &= \text{sign} \left(\frac{dH}{dt} \right); \\ \alpha &= \text{the mean-field parameter, and} \\ M_{an} &= \text{the anhysteretic magnetization.} \end{aligned}$$

Though the definition of the parameter c is complex, Jiles and Atherton [43] point out that it is determined experimentally by the ratio of initial differential susceptibilities of the normal and anhysteretic magnetization curves. It may also be noted that Annakkage *et al.* [168] present a slightly modified equation. In their discussion of the parameters, they claim that α, c and k are constants for the material being used. It is also assumed that the parameter μ_0 represents the vacuum permeability, though it is not expressly defined by Jiles and Atherton [43]. An expression for $\frac{dM_{an}}{dH_e}$ is derived by Halim *et al.* [80] based on (2-76):

$$\frac{dM_{an}}{dH_e} = \frac{M_s}{a} \left[1 - \frac{1}{\tanh^2 \left(\frac{H_e}{a} \right)} + \frac{1}{\left(\frac{H_e}{a} \right)^2} \right] \quad (2-80)$$

Solutions to (2-79) for a monotonically increasing magnetic field yields the initial magnetization curve of the hysteresis loop [43]. It is also expressly pointed out by Jiles and Atherton [43] that the initial magnetization curve of a ferromagnet lies below the anhysteretic magnetization curve.

However, when the magnetic field intensity becomes sufficiently high, the initial magnetization curve approaches the anhysteretic curve. Therefore, (2-76) can model the initial magnetization curve under high magnetic field intensities.

Several permutations to the original *JA* model exist. Although not all of these permutations will be given in this report, one particular alternative of note is that presented by Jiles, Thoeke and Devine [173]. It is interesting for two reasons. Firstly, most other modifications are made by alternative authors in an attempt to extend the original model. This model involves the work of one of the original authors. Secondly, the literature in [173] is particularly focused on determining the parameters associated with the *JA* model through experimentation. It is the intent of the author of this report to refer to some of the principles highlighted by [173] and hence it is worth noting their approach to the *JA* model. According to Jiles, Thoeke and Devine [173], the *JA* model is given by:

$$\frac{dM}{dH} = (1 - c) \frac{M_{an} - M_{irr}}{k\delta - \alpha(M_{an} - M_{irr})} + c \frac{dM_{an}}{dH} \quad (2-81)$$

Another important component of [173] is its interpretation of the parameters c and k . Jiles, Thoeke and Devine [173] claim that c represents the reversible wall motion and k determines the hysteresis losses. This provides the reader with a better understanding of the parameters of the *JA* model than does the original discussion of Jiles and Atherton [43]. Also note that in the discussion of Jiles, Thoeke and Devine [173], k is defined as $k = \frac{n\langle\varepsilon_\pi\rangle}{2m\mu_0}$ so that it has units of $A \cdot m^{-1}$ instead of Tesla.

Chiesa and Høidalen [134] present alternative equations which they say are the basic *JA* equations. They then also present a way to implement them in EMTP using the trapezoidal rule. However, this includes twelve different equations, which will not be presented here. Interested readers may refer to their work for more details. Several other sources of literature also reference various permutations to the original *JA* model. One of these is the work of Annakkage *et al.* [168]. These equations were also employed by references [80] and [172]. Other alternatives were employed by [171], [174], [175], [176] and [177]. It may also be noted that Ribas *et al.* [51] expressed the model in terms of flux ϕ and current i and references [51], [178], [179], [180] and [181] express the model in terms of magnetization M and flux density B . These alternatives are discussed as extensions to the original *JA* model in this section of the report.

Kis and Iványi [171] present a table to summarize the physical properties represented by the *JA* parameters. This table is shown in Table 2-11 and supplemented by further descriptions of the parameters, which can also be found in [127], [173] and [175].

Table 2-11: A summary of the Jiles-Atherton model parameter properties

Parameter	Physical Property
M_s	Saturation Magnetization.
α	Domain interaction. Influences effective field.
a	Shape parameter of M_{an} . Deals with thermal aspect.
c	Reversibility coefficient. Determines hysteresis loop width.
k	Linked to hysteresis loss. Determines pinning site density.

Fulginei and Salvini [176] investigated the influence of the various parameters on the saturated hysteresis loop when all the other parameters were held constant. They found that the domain interaction coefficient α mainly controlled the derivative of $\frac{dB}{dH}$ in regions around the coercive point. This derivative was also controlled by the saturation magnetization M_s [176]. The shape parameter a controls the knee area of the hysteresis loop, as well as the remanence value and $\frac{dB}{dH}$ around remanence. Parameters c and k controlled the width of the loop, the coercive field value and the derivative $\frac{dB}{dH}$ close to the coercive point. Saturation magnetization M_s naturally also controlled the maximum magnetization of the loop [176].

Chiesa and Høidalen [134] say that the JA parameters are obtained via a fitting procedure and that standard data is sufficient. It involves two steps: first, the anhysteretic curve is fit by looking at the magnetization characteristic without the losses. This yields the values of the anhysteretic parameters. Note, however, that standard data rarely exceeds 110% of rated values, in which case the transformer may not be fully saturated. This means that extrapolation is required to estimate the saturation characteristics, which can be inaccurate. Curve fitting is a potential solution, but accuracy cannot be guaranteed. The second step is to add the losses and find the remaining parameters.

The articles written by Jiles, Thoeke and Devine [173], [182] are dedicated to the determination of the JA model parameters. They claim that this is difficult since there is no clear definition of the fixed points to be used for parameter determination. However, they state that in their experience, the following parameters should be measured and used for parameter determination:

- a initial normal susceptibility χ'_{in} ;
- b initial anhysteretic susceptibility χ'_{an} ;
- c coercivity H_c ;

- d differential susceptibility at the coercive point χ'_{H_c} ;
- e remanence M_r ;
- f differential susceptibility at remanence χ'_M ;
- g coordinates of loop tip (H_m, M_m) , and
- h differential susceptibility of initial magnetization curve at loop tip χ'_m .

Susceptibility χ is defined as the ratio of the differential magnetization to the differential field $\frac{dM}{dH}$.

In other words, the initial normal susceptibility is defined as $\chi'_{in} = \left(\frac{dM}{dH}\right)_{H=0, M=0}$. Before discussing the parameter determination technique further, it is worth noting the definition of initial susceptibility as provided by Takács [146]. He claims that for low fields, the susceptibility can be given by

$$\chi_{in} = \frac{\mu_0 m M_0}{k_B (T - T_c)} \quad (2-82)$$

where

- $T_c = \frac{\mu_0 m M_0 N_w}{k_B}$ = the Curie temperature;
- m = the magnetic moment;
- k_B = the Boltzmann constant;
- M_0 = the maximum amplitude of magnetization, and
- N_w = the mean-field interaction constant.

Jiles, Thoeke and Devine [173] derive the various equations and provide explanations as to why the various parameters can be calculated from the measured values. These discussions are omitted in this report and only the critical equations are discussed. Jiles, Thoeke and Devine [173] also point out that some of the JA parameters are defined in terms of others. Hence, they propose an algorithm whereby the parameters can be determined by successive iteration. It is the opinion of the author of this report that there is an error in logic in the process defined by [173]. This error is pointed out following the discussion of the algorithm itself.

The first step, according to Jiles, Thoeke and Devine [173], is to calculate the reversibility coefficient c . This coefficient is dependent on the initial normal susceptibility χ'_{in} and is given by

$$c = \frac{3a\chi'_{in}}{M_s} \quad (2-83)$$

which [173] defines as a direct relationship between c and χ'_{in} .

The next step is to calculate parameters α , a and k , which are implicitly defined in terms of each other. The four important equations here are:

$$a = \frac{M_s}{3} \left(\frac{1}{\chi'_{an}} + \alpha \right) \quad (2-84)$$

$$k = \frac{M_{an}(H_c)}{1-c} \cdot \left\{ \alpha + \frac{1}{\left(\frac{1}{1-c} \right) \chi'_{max} - \left(\frac{c}{1-c} \right) \frac{dM_{an}(H_c)}{dH}} \right\} \quad (2-85)$$

$$M_r = M_{an}(M_r) + \frac{k}{\left(\frac{\alpha}{1-c} \right) + \frac{1}{\chi'_r - c \frac{dM_{an}(M_r)}{dH}}} \quad (2-86)$$

$$M_m = M_{an}(H_m) - \frac{(1-c)k\chi'_m}{\alpha\chi'_m + 1} \quad (2-87)$$

Having obtained a value for c from (2-83), a seed value of α is chosen and inserted into (2-84). This yields an estimate for a . From the seed value of α , k can also be determined from (2-85). Based on the first iteration values of a and k , α can be calculated using (2-86). Finally, using the calculated values of α and k , a new value of a can be calculated using (2-87). Jiles, Thoeke and Devine [173] then state that the process is repeated for k , α and a .

The problem with this argument lies in the calculation of c . Jiles, Thoeke and Devine [173] state that (2-83) is a direct relationship between c and χ'_{in} . However, it is clear that a is also a part of (2-83), which is still an unknown parameter. Furthermore, knowledge of c is required to solve (2-84) through (2-87). Therefore, it is clear that a guess value of a is required when c is determined. This means that the iterative procedure involves the calculation of all four parameters, rather than only α , a and k as pointed out by [173].

Chwastek and Szczyglowski [183] list several weak points of this approach to JA parameter determination:

- a The altered formula for $\frac{dM}{dH}$ presented in [173] and [182] can violate the law of thermodynamics and present ill-conditioned formulations to hysteresis;
- b The sequence of the proposed algorithm is left with no proof of its correctness;
- c The authors make use of the slopes $\frac{dM}{dH}$ and $\frac{dM_{an}}{dH}$ at the origin twice and the singularity of the modified Langevin function at zero remains unnoticed;
- d The domain wall displacement is not the prevailing phenomenon responsible for hysteresis in the low and high field regions, whereas the JA model was aimed at the

description of hysteresis as the effect of irreversible domain translation through pinning sites, and

- e No measurement noise is taken into account in the measurements.

Leite *et al.* [180] elaborate on the work of [173] and [182] since they claim that the derived parameters are not sufficiently accurate. They make use of a mean square error (*MSE*) criterion to vary one parameter at a time while keeping the other parameters constant. This is motivated by the fact that one of the disadvantages of the *JA* model is its inability to close minor loops, as pointed out by Lederer *et al.* [170]. Liorzou, Phelps and Atherton [127] introduce the use of a scaling factor to fix this problem. However, Leite *et al.* [180] claim that the need for a scaling parameter is eliminated by their *MSE* approach. The results presented by Leite *et al.* [180] show significant improvement over the accuracy of the scaling approach, especially with regards to minor loop simulation.

Kis and Iványi [171] present an alternative parameter determination technique. Their technique makes use of a fitting procedure, but with the addition of a nonlinear least-squares criterion function to estimate the value of the parameters which give the best fit to the hysteresis loop. The criterion function requires an initial guess of the parameter values. Based on the error generated by the criterion function, the parameters are adjusted. The speed of the algorithm depends on the accuracy of the initial guess values, but Kis and Iványi [171] state that the algorithm usually converges within 50 iterations.

Toman, Stumberger and Dolinar [175] also discuss the use of an optimisation algorithm to determine the *JA* parameters. The parameters are varied and the error is calculated using a root mean squared error approach.

Salvini [176], Leite *et al.* [178], [181] and Wilson, Ross and Brown [184] all discuss the use of a genetic algorithm to optimise the *JA* parameters. A detailed discussion of genetic algorithms is considered beyond the scope of this report. However, some of the advantages of this approach are:

- a It is good at finding a general area of the solution;
- b It is tolerant of local maxima and minima;
- c It is suitable for global multi-objective optimisation;
- d It presents good performance when the guess values are far from the optimum solution;
- e It optimises with continuous or discrete parameters;
- f It does not require information about gradients;
- g It is not significantly influenced by discontinuities;
- h It is suitable for numerically generated data, experimental data and analytical data, and

- i It does not require a lot of computational effort.

One of the drawbacks of the genetic algorithm is that it becomes increasingly slow as the algorithm converges. To this end, Wilson, Ross and Brown [184] suggest supplementing the genetic algorithm with a simulated annealing algorithm. The simulated annealing algorithm is slow to converge when the initial guess values are poorly chosen but quick to converge accurately when the guess values are close to the correct solution. Wilson, Ross and Brown [184] found that the combination of these two algorithms presented better results than either algorithm individually.

Lederer *et al.* [170] claim that the method used to determine the parameters proposed by Jiles, Thoeke and Devine [173] is dependent on the initial guess values and the order in which the equations are evaluated. As such, they [170] claim that the process often does not converge. Lederer *et al.* [170] propose only using equations (2-83) through (2-87) as a first iteration approach to parameter determination. Thereafter, the simulated annealing algorithm should be used for convergence. The criterion function suggested by Lederer *et al.* [170] is the *RMS* difference between the measured and calculated B-H loops.

The *JA* model presents several advantages. The following is a summarized list of the model's advantages based on references [51], [134], [170], [181] and [185]:

- a It predicts residual flux in the core;
- b It represents saturation and air-core inductance;
- c Excessive losses and eddy current losses can be taken into account by the model (as is done by [134]);
- d It requires low computational effort and has a simple numerical implementation;
- e Parameter characterisation is quite simple. Only 5 parameters are required, which can be obtained from a single experimental loop running deep into saturation;
- f Additional loss components and remnant magnetic flux can be introduced in circuit analysis, which has a significant influence on ferroresonance behaviour, and
- g Its formulation is in terms of a first-order differential equation.

Despite the claimed ability of the *JA* model to predict residual flux, Chiesa and Høidalen [134] claim that this is one of the areas where the *JA* model can be improved. The *JA* model also has a few other disadvantages, as pointed out by [134], [170], [176], [179] and [180]. These disadvantages are summarised below:

- a Poor fitting of the loss characteristic over a wide range of excitation;
- b Difficult parameter identification process (claimed by [170]);
- c Unphysical behaviour of model near loop tips;
- d Hysteresis loops are unsymmetrical and may not close if saturation is not reached, and

- e Is accurate at saturation, but becomes increasingly inaccurate as loops become smaller.

There is also a consensus between [176], [179] and [180] that the JA model is not excellent at modelling minor loops. Fulginei and Salvini [176] claim that this is one of the reasons that the Preisach model is better than the JA model. This problem is solved by Liorzou, Phelps and Atherton [127] through the use of a scaling factor n . Leite *et al.* [180] claim that their *MSE* approach to parameter determination eliminates the need for a scaling factor. They also claim that their *MSE* magnetization approach improves the fitting property of the model over a wider range of excitation.

Parameter determination is listed as both an advantage and a disadvantage of the model. These are contradictory statements between various sources of literature. It is the opinion of the author of this report that parameter determination for the JA model is relatively simple, especially in comparison to other models such as the Preisach model. All of the parameters of the JA are measurable. However, the parameters are not determined from standard transformer tests, which can complicate the process. For instance, the method proposed by Jiles, Thoeke and Devine [173] requires measuring the anhysteretic magnetization curve.

Model Extensions

Some of the disadvantages listed above are addressed by various extensions to the model found in the literature and include altering the equation for the anhysteretic curve and adding the effects of eddy currents. The ensuing paragraphs are focused on the discussion of these extensions.

Perhaps the most widely referenced JA extension is the inverse JA model introduced by Sadowski *et al.* [179]. Recall that the traditional model uses magnetic field intensity H as input and yields magnetization M as output. The model proposed by Sadowski *et al.* [179] is referred to as an inverse model since it uses magnetic induction B as an input to calculate the magnetization. The equations associated with this model are [179]:

$$\frac{dM}{dB} = \frac{(1-c) \frac{dM_{ir}}{dB_e} + \left(\frac{c}{\mu_0}\right) \frac{dM_{an}}{dH_e}}{1 + \mu_0(1-c)(1-\alpha) \frac{dM_{ir}}{dB_e} + c(1-\alpha) \frac{dM_{an}}{dH_e}} \quad (2-88)$$

$$M_{ir} = \frac{M - cM_{an}}{(1-c)} \quad (2-89)$$

$$B = \mu_0(H + M) \quad (2-90)$$

$$B_e = \mu_0 H_e \quad (2-91)$$

$$\frac{dM_{ir}}{dB_e} = \frac{M_{an} - M_{ir}}{\mu_0 k \delta} \quad (2-92)$$

where

- μ_0 = the vacuum permeability;
- M_s = a saturation magnetization;
- a = a model parameter;
- c = a model parameter;
- α = a model parameter;
- k = a model parameter;
- δ = a directional parameter related to $\frac{dB}{dt}$. Assumes value of +1 for $\frac{dB}{dt} > 0$ and -1 for $\frac{dB}{dt} < 0$;
- H_e = the effective magnetic field, and
- B_e = the effective magnetic induction.

In addition to this inverse model, Sadowski *et al.* [179] also present a detailed algorithm on how (2-88) can be used to obtain both magnetic field H and magnetization M using numerical methods.

The inverse JA model holds a couple of advantages. Since magnetic induction B is obtained through integration, it is naturally filtered. This means that the magnetic induction waveform has fewer oscillations than the magnetic field waveform [178], [181]. The added noise introduced by the magnetic field presents additional difficulties to parameter identification. It is the experience of Leite *et al.* [180] that parameter identification processes for the standard JA model rarely converge. Convergence is more readily attained with the inverse model. Furthermore, the inverse model retains all of the advantages of the original model [178]. It can also be used directly in finite element stepping calculations.

Ribas *et al.* [51] elaborate on this inverse model even further and makes it specific to ferroresonance simulation. Following some derivations detailed in [51], Ribas *et al.* modify the inverse model to present a flux-current approach to the JA model. The final equation is given by

$$\frac{di}{d\phi} = \frac{1}{\varepsilon} \left(\frac{1}{\mu_0} - \frac{dM}{dB} \right) = \frac{1}{\varepsilon} \left(\frac{1}{\mu_0} - S \frac{dM}{d\phi} \right) \quad (2-93)$$

where

$$\varepsilon = \frac{N}{l_m S} = \text{scaling factor from B-H to } \phi - i \text{ model};$$

$$\begin{aligned}
l_m &= \text{the magnetic mean length;} \\
N &= \text{the number of turns, and} \\
S &= \frac{\phi}{B}.
\end{aligned}$$

Lederer *et al.* [170] make two suggestions to improve the inaccuracy of the JA model for smaller loops. The first entails a process whereby the model parameters are allowed to change based on the size of the loop. This means that an array of parameters is created and the parameters are chosen based on the size of the loop. The drawback of such an approach is that each set of parameters needs to be determined, which means that multiple hysteresis loop measurements are required.

The second suggestion made by Lederer *et al.* [170] is to scale the hysteresis loop relative to the saturation magnetization. This means that only a single hysteresis loop is required for parameter determination. However, it was still concluded by Lederer *et al.* [170] that the model does not show good agreement for smaller hysteresis loops. Yet Lederer *et al.* [170] conclude that this may have an insignificant effect, which begs the question of whether improvements to the original model are required.

The IEEE Power System Relaying Committee [158] present two alternatives for the magnetization curve, as opposed to that seen in (2-76). These are

$$M_{an} = M_s \frac{M(H)}{|M(H)| + 1} \quad (2-94)$$

and

$$M_{an} = M_s (\tanh(M(H))) \quad (2-95)$$

These models are referred to as level 2 models. Models making use of (2-76) are referred to as level 1 models. Level 2 models do not consider interdomain coupling, but are better suited to ferrite and powdered iron cores and are also more efficient.

Szewczyk [186] and Biedrzycki *et al.* [177] both refer to the use of a Ramesh extension for a better depiction of anhysteretic magnetization. They claim that the original Langevin approach employed by Jiles and Atherton is accurate for isotropic materials. However, for anisotropic materials, the Ramesh extension is suggested [177]. The Ramesh extension, according to [177], is given by:

$$M_{ah\_aniso} = M_s \left[\frac{\int_0^\pi e^{0.5(E(1)+E(2))} \sin \theta \cdot \cos \theta \cdot d\theta}{\int_0^\pi e^{0.5(E(1)+E(2))} \sin \theta \cdot d\theta} \right] \quad (2-96)$$

where

$$E(1) = \frac{H_e}{a} \cos \theta - \frac{K_{an}}{M_s \cdot \mu_0 \cdot a} \sin^2(\psi - \theta);$$

$$E(2) = \frac{H_e}{a} \cos \theta - \frac{K_{an}}{M_s \cdot \mu_0 \cdot a} \sin^2(\psi + \theta);$$

K_{an} = the average energy density, and

ψ = the angle between the magnetising field and the easy axis of magnetization.

Biedrzycki *et al.* [177] does not say anything else about the other parameters. One is therefore left to assume that the rest of the parameters are the same as in the original JA model. They also suggest the use of the Gauss-Kronrod approximation for the anhysteretic function and a 4th order Runge-Kutta (RK) algorithm for the solving of the differential equations.

Annakkage *et al.* [168] presented another approach to anhysteretic magnetization modelling, which involved two polynomials. This approach presents the advantage of greater flexibility of the overall B-H curve shape when compared to the original Langevin approach. Furthermore, Annakkage *et al.* [168] also claim that the modelling of the shoulder region is improved with the new approach.

Toman, Stumberger and Dolinar [175] say that after detailed analysis, they found that the pinning parameter k is dependent on the magnetic field intensity. This is confirmed by Wilson, Ross and Brown [184]. The original JA model assumes that k is constant. Both [175] and [184] suggest the use of a Gaussian function as an improvement to make the pinning parameter k dependent on H :

$$k = k_0 + k_1 e^{-\frac{H^2}{2\sigma^2}} \quad (2-97)$$

where k_0 and k_1 are constant values and σ is the standard deviation of the Gaussian function. This means that three extra parameters are introduced to the model, bringing the total number of parameters to be determined up to seven. Wilson, Ross and Brown [184] point out that the advantage of this approach is that there is no discontinuity around zero when the magnetic field changes polarity. This, in turn, improves simulation convergence.

Biedrzycki *et al.* [177] confirm the claim made by Toman, Stumberger and Dolinar [175] that k is not constant. However, they propose that k be modelled with:

$$k = k_0 + \frac{e^{k_2 \cdot (1 - |M|/M_s)} - 1}{e^{k_2} - 1} \cdot (k_1 - k_0) \quad (2-98)$$

where M_s represents the saturation magnetization and k_0 , k_1 and k_2 are constants to define the function.

Annakkage *et al.* [168] make use of a variable k parameter to change the width of the B-H loop in the shoulder area. They suggest modifying k using:

$$k_{modified} = k \left[1 - \beta \left(\frac{M}{M_s} \right)^2 \right] \quad (2-99)$$

However, Annakkage *et al.* [168] claim that this had little effect on secondary current. They also conclude that the slope of the loop in the shoulder area is more important than the width of the hysteresis loop.

Kis and Iványi [171] consider additional losses due to higher frequency components, which are not considered by the original JA model. These higher frequency components include eddy current losses and anomalous losses, which cannot be ignored at higher frequencies. To include these effects into the JA model, Kis and Iványi add the terms

$$k_1 \frac{dM dM}{dt dH_e} \quad (2-100)$$

and

$$k_2 \left(\frac{dM}{dt} \right)^{\frac{1}{2}} \frac{dM}{dH_e} \quad (2-101)$$

to the original JA model, where (2-100) represents the eddy current losses and (2-101) represents the anomalous losses. Kis and Iványi [171] then proceed to present a new differential equation for $\frac{dM}{dH_e}$, which takes the anomalous losses and eddy currents into account in [171, eq. (9)]. However, the original JA equations presented by [171] differ from those presented by Jiles and Atherton [43]. Therefore, interested readers are referred to the work of Kis and Iványi [171] for a more thorough discussion of the associated equations. Simulations conducted by Kis and Iványi [171] showed significant improvements when the eddy current and anomalous losses were included, but the loop tips were not very accurate at saturation. Kis and Iványi [171] claim that this is caused by the directional parameter δ_M . However, ignoring this parameter caused nonphysical solutions in the simulation.

Sadowski *et al.* [179] propose a modification of the *JA* model to determine differential reluctivity. The differential reluctivity is defined as

$$v_d = \frac{H(t + \Delta t) - H(t)}{B(t + \Delta t) - B(t)} \quad (2-102)$$

This differential reluctivity approach allows for the easier addition of eddy current and anomalous losses. The addition of eddy current and anomalous losses, in turn, allows for a more accurate simulation as shown in [179]. However, the approach requires the user to know the thickness and conductivity of the core. 2D Finite element analyses are also required to solve some of the equations associated with this approach.

2.10.20 Model Comparisons

This section of the report summarizes some of the comparisons which have been made between the various hysteresis models in the literature. Most of the comparisons exist between the *JA* or Preisach model and some other models.

JA Model vs. Harrison Model

Based on [159], the following differences between the *JA* and Harrison models can be identified:

Table 2-12: The differences between the *JA* and Harrison models

JA Model	Harrison Model
Focuses on <i>domains</i> .	Focuses on <i>magnetons</i> .
Domains described using classical physics – uses Langevin function.	Magnetons described quantum-mechanically – uses tanh-function.
Equations take the form of a DE. Requires numerical integration.	The equation is algebraic – no integration is required.
Can model minor loops and initial magnetization.	Cannot model minor loops or initial magnetization.
Restricted to narrow loop characteristics of “soft” magnetic materials.	Not restricted to certain magnetic materials.
Does not have a domain coefficient term.	Has a domain coefficient term.

The following similarities between the JA and Harrison models can be identified based on [159]:

- a Both make use of the concept of an effective field, and
- b The basic forms of both models contain feedback properties, which allow for hysteresis loops with vertical jumps.

Jiles-Atherton vs. Takács

The JA model and the Takács model are similar in many ways. From literature and sections 2.10.9 and 2.10.18 it can firstly be seen that both of these models make use of the concept of an effective field. They also both use the tanh-function, though the equations are not identical. Furthermore, where other models generally relate the magnetic flux density (B) to the magnetic field intensity (H), both of these models make use of the concept of magnetization (M) to describe hysteresis. Finally, both these models also have inverse models found in literature, where the magnetization (M) or the magnetic flux density (B) are defined as the independent parameter. This makes the magnetic field intensity (H) the dependent parameter.

Despite the similarities in their equations, the parameters used to define the two models differ vastly. The only parameter used by both models is the domain interaction parameter α , which is used to define the effective field.

JA, Preisach, Stoner-Wohlfarth and Globus Models

The JA, Preisach, Stoner-Wohlfarth and Globus models are the four classical hysteresis models according to Liorzou, Phelps and Atherton [127]. In their discussion of these four models, they present the following useful breakdown of the important aspects of these four models:

Table 2-13: A comparison of the JA, Preisach, Stoner-Wohlfarth and Globus models as seen in [127]

Models Characteristics	Extended Stoner-Wohlfarth	Jiles-Atherton	Modified Globus	Preisach
Mechanism	Rotation	Not specified	Wall motion	Not specified
Anisotropy	Uniaxial	Multi-axis	Multi-axis	Not specified
Interaction	Yes	Yes	No	Moving Model
Pinning	Yes	Yes	Yes	Not specified
Texture	Anisotropic or isotropic	Isotropic	Uniaxial (180° walls)	Not specified

Wall Energy	No	No	Yes	No
Reversibility	Yes	Additional model	Yes	Additional Model
Minor Loops	Yes	Approximation	N/A	Yes
Demagnetization	N/A	Yes	N/A	Yes
Anhysteretic	Yes	Yes	Yes	Yes
Parameters²¹	M_s^* k_u^* η^* σ α p	M_s^* c^* a^* α^* k^*	M_s^* γ^* R^* f^*	M_s^* h_c σ_m σ_c α
Grains	Single domain	Multi-domain	Bi-domain	Not specified
Computational time	+++	++	+	++++
Materials	Hard magnetic materials	-Bulk materials -Medium ferrites	Soft ferrites	-Magnetic recording -Thin films

Various sources of literature, including [52], [74], [127], [134], [185] and [187], have also listed some of the advantages of the Preisach and JA models over each other. These advantages are listed in Table 2-14.

According to [188], the Preisach model has an advantage over the Stoner-Wohlfarth model concerning parameter identification and numerical implementation. It has also widely been used in the modelling of two-dimensional vector hysteresis characteristics. On the other hand, Atherton and Beattie [149] state the inherently three-dimensional nature of the Stoner-Wohlfarth is an advantage over the Preisach and Jiles-Atherton model, which are only made three-dimensional with considerable effort.

²¹ Parameters marked with a * are measurable

Table 2-14: A list of advantages of the Preisach and JA models when compared to each other

Preisach Model	JA Model
Provides better fit for major loops.	Simpler implementation.
Requires extensive measurements but almost no fitting.	Requires few measurements but extensive fitting procedures.
Correspondence between measurements and simulations is generally better, especially for higher-order loops and for large excitation range.	Ten times more computationally efficient.
Widely recognized as the best model for static scalar hysteresis, especially accurate prediction of minor loops.	Good for investigating the effect of temperature and stress on the material.
	Parameters easier to obtain.

Annakkage *et al.* [168] say that the reversible component of magnetization in the JA model is caused by domain wall bending. The Globus model also referred to domain wall bending for the reversible component in section 2.10.5. In the introduction to the JA model, Jiles and Atherton [43] also said that they agree with the conclusion of the Globus model that irreversible changes in magnetization are caused during wall displacement by the pinning process. However, they [43] also add that they believe the Globus model is restricted in its interpretation of where domain pinning may occur, as they [43] believe pinning can also be caused by inhomogeneities. Furthermore, Jiles and Atherton [43] also believe that domain walls are planar on the anhysteretic magnetization curve – not at remanence as suggested by the Globus model.

Chiesa [74] states that the JA and Preisach models are generally preferred above other models since they can represent both minor loops and de-energization transients. This improves their ability to estimate remnant magnetization in the iron core, which is particularly important for inrush currents and ferroresonance.

Despite the various advantages of the other models, Fang [189] says that the JA model is widely regarded as the most suitable and practical model for engineering applications and time-domain simulations. Reasons for this claim include:

- a It is recognized for proven validity in representing ferromagnetic materials;
- b It is capable of remanence retention;

- c Its parameters are physically significant and can be identified and optimized with quasi-static hysteresis measurements;
- d It considers energy dissipation from the magnetic wall displacement;
- e It can incorporate frequency-dependent losses, and
- f It has favourable computational efficiency.

Finally, a very strong argument can be made in favour of the *JA* model based on [190], where they proved that the *JA* model works well for the simulation of ferroresonance.

Other Models

The remaining models, which have not yet been discussed in this comparison section, are the Zou model, the Rayleigh model, the Chua-Type model, various numerical approximations and piecewise-linear approximations.

This report has already claimed that piecewise-linear approximations are insufficient for ferroresonance modelling. The Zou model is not officially recognized – rather it was labelled the Zou model in this report based on the last name of the first author. Hence, there is not a lot of literature available on this model at this stage. The Rayleigh model was only briefly discussed in section 2.10.12 since it is not accurate for deep saturation modelling. The Chua-type model is closely related to the Preisach and Rayleigh models. Therefore, these models will not be compared to the more complex and classical models, such as the *JA* model and the Preisach model.

2.10.21 Hysteresis Model Extensions

Section 2.11 discusses frequency-dependent components, which includes a section on the modelling of eddy currents. However, several literature sources on hysteresis also discuss a hysteresis model extension, which can be used to consider the effect of eddy currents. The main idea of the extension is to modify the calculated magnetic field intensity to take eddy current losses and anomalous losses into consideration. It is based on the assumption that the total power losses in the core can be broken down into three components:

$$P_{total} = P_{hys} + P_{cls} + P_{exc} \quad (2-103)$$

where

P_{total} = the total power loss in the core;

P_{hys} = the hysteresis loss;

P_{cls} = the classical eddy current loss assuming uniform magnetization, and
 P_{exc} = the excessive/anomalous loss.

The concept of anomalous loss originates from the fact that when the hysteresis and eddy current losses are added, their sum is significantly less than the measured total loss. Classical eddy current losses are calculated based on the assumption that the flux pattern in the magnetic material is smooth, which is not necessarily accurate. Ferromagnetic materials are made up of self-saturated domains [81], which prevents the microscopic flux pattern from being smooth and continuous, especially if the domains are relatively large [81].

Based on the separation of losses described in (2-103), the magnetic field intensity can be defined as:

$$H_{tot} = H_{hys} + H_{cls} + H_{exc} \quad (2-104)$$

The hysteretic component of the magnetic field intensity, H_{hyst} , is calculated using a hysteresis model, such as the Jiles-Atherton model. The classical power loss due to eddy currents is proportional to the rate of change of the magnetic flux [81]. Therefore, the magnetic field intensity component caused by the classical eddy currents can be equated to an equivalent magnetic field proportional to dB/dt . The anomalous loss can be represented by an equivalent magnetic field proportional to $\left(\frac{dB}{dt}\right)^{\frac{1}{2}}$. The total magnetic field is then given by:

$$H_{tot} = H_{hyst} + k_1 \frac{dB}{dt} + k_2 \left(\frac{dB}{dt}\right)^{\frac{1}{2}} \quad (2-105)$$

where k_1 and k_2 are constants for a given transformer. With this extension, the eddy current losses and the anomalous losses can be included in the hysteresis model.

2.11 Frequency-Dependent Components

The importance of frequency-dependent components is a contradictory subject in literature. Chiesa [74] states that for inrush current and ferroresonance modelling, no major improvement is observed when the frequency-dependent effect of the core is represented. More importantly, Chiesa [74] states that the conventional nonlinear inductor in parallel with a constant resistance should be sufficient for a frequency range up to 3 kHz. The reader may recall from section 2.2.1 that the frequency spectrum of ferroresonance rarely exceeds 2 kHz.

Zhu, Hui and Ramsden [191] claim that two things are needed to model transformer transients accurately: the hysteresis effect of the inductance and the domain wall motion effect on eddy

current loss, which causes anomalous losses. This is an interesting observation, since the Globus model, the Harrison model and the Jiles-Atherton hysteresis model all refer to domain wall motion as a hysteresis phenomenon.

Cigre [9] claims that taking the frequency-dependent behaviour into consideration results in higher damping for higher frequency components and reduced *DC* losses. Furthermore, if the frequency dependency is neglected, the duration, if not the amplitude, of the resulting overvoltages will be overestimated. Chandrasena *et al.* [135] state that the modelling of eddy current losses in a transformer is at least as important as the modelling of hysteresis.

2.11.1 Eddy Currents

Short [3] describes eddy currents as small circulating currents in the core material. These circulating currents cause eddy current losses, which are a function of the core volume, the frequency, the flux density, the lamination thickness and the core material resistivity. This can be expressed mathematically as

$$P_e \propto \frac{V_e B^2 f^2 t^2}{r} \quad (2-106)$$

where

- V_e = the volume of the core;
- B = the maximum flux density;
- f = the frequency;
- t = the thickness of the laminations, and
- r = the resistivity of the core.

This relation also implies that eddy current losses are very low for amorphous cores since the resistivity of an amorphous core is very low and the laminations are very thin. It should also be noted that Cigre [9] reiterates that eddy current losses are proportional to the square of the flux and the frequency. However, their [9] relation is only expressed as $P_e \propto B_{max}^2 f^2$, which is not as elaborate as that of Short [3].

Section 2.10.21 discussed an extension, which can be applied to hysteresis models to take eddy currents into account. Chandrasena *et al.* [81] also investigated the use of a constant resistor to match the rated losses at 60 Hz. This approach tends to be more accurate than the $k_1 \frac{dB}{dt}$ extension at rated frequency but becomes increasingly inaccurate when there is a variation in frequency.

It should also be noted that eddy currents can be induced in the windings of the transformer under high frequencies [192]. These eddy currents in the windings manifest themselves as an increase in the effective resistance and winding losses [79]. Jazebi, de León and Vahidi [192] claim that accurate representation of low-frequency transients such as ferroresonance require accurate models of frequency-dependent effects in both the windings and the core. However, in [91], Jazebi *et al.* conclude that for thin wires with a diameter of less than 3 mm, low-frequency transient losses in the windings are adequately represented by a constant resistor. If a constant resistor is not sufficient, several analytical expressions also exist for calculating eddy current effects. However, Foster circuits have also been used to model eddy current effects in windings.

An alternative to the Foster circuit approach is presented by Fuchs, Yildirim and Grady [193], where an analytical expression is given for the variation in the resistance of windings due to eddy currents. The effective resistance of the windings can be given by

$$R_{AC} = R_{DC} + \Delta R_{AC} \cdot \left(\frac{f}{f_0}\right)^m \quad (2-107)$$

where

- R_{DC} = the DC resistance of the windings;
- ΔR_{AC} = the per-unit winding eddy-current loss at rated load and frequency;
- f = the frequency;
- f_0 = the rated frequency, and
- m = a factor between 1.2 and 2 [56].

Nevertheless, Martinez *et al.* [56] still suggest the use of Foster circuits for low- and mid-frequency transients. Rezaei-Zare, Iravani and Sanaye-Pasand [102], however, claim that since ferroresonance is a low-frequency phenomenon, it is sufficient to represent eddy current losses with a constant resistance.

2.11.2 Skin and Proximity Effects

The skin and proximity effect are two other factors influencing the frequency behaviour of the conductors. The skin effect is caused by current flowing on the surface of the conductor at higher frequencies [86]. This results in increased effective resistance. The proximity effect is due to external magnetic fields being generated by the current flowing through conductors close to the conductor being considered. The skin effect tends to dominate the coil losses up to a certain frequency, after which the proximity effect also becomes apparent [86]. A graphic relation between the skin effect losses and the proximity losses is shown in [197, Fig. 2].

Khelil and Elleuch [194] provide a more elaborate description of the skin and proximity effect. An alternating current in a conductor causes a magnetic flux, which in turn generates an eddy current on the surface of the conductor loop. The eddy current opposes the original current and causes a reduction in current at the centre of the conductor. On the surface of the conductor, however, the current is in the same direction as the original current, which leads to an increase in the total current. Therefore, the current density is non-uniform across the cross-section of the conductor. This is called the skin effect [194]. The proximity effect is similar but is caused by alternating current in other conductors close to the conductor being considered. Semlyen and de León [195] claim that the combination of these two effects is often referred to as the “eddy current effect in the windings.”

Skin effect losses remain constant at their *DC* value as long as the penetration depth is greater than the radius of the conductor. When this condition no longer holds, the skin effect increases with the square root of the frequency [197]. Khelil and Elleuch [194] claim that the *DC* value of the skin effect loss will be maintained up to a frequency of 10 kHz since the current density remains approximately uniform. Greene and Gross [196] verify this claim where they studied the skin effect up to a frequency of 3 kHz and found it to be negligible.

Proximity losses initially rise with the square of the frequency. However, there comes a point where this relation no longer holds, after which the proximity losses increase with the square root of the frequency [197].

2.11.3 Foster Circuits

Foster circuits are used to represent the frequency dependency of windings and were mentioned briefly in section 2.11.1. Foster circuits are described by de León and Semlyen [195] as “circuits obtained from the partial fraction expansion of impedance equations into partial fractions which are fitted by canonical R, L, C circuits.” Foster circuits can be in parallel (Fig. 2-63) or series (Fig. 2-64).

The series Foster circuit is more suitable for low-frequency studies. Semlyen and de León [195] claim that a series foster circuit of the second-order (series resistance plus two foster blocks) are sufficient for a frequency range up to 1 MHz. Fitting procedures for Foster circuits can be found in [195]

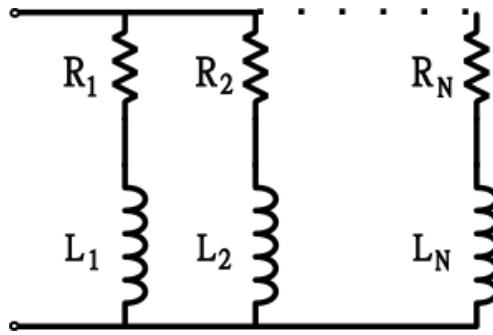


Fig. 2-63: The parallel Foster circuit

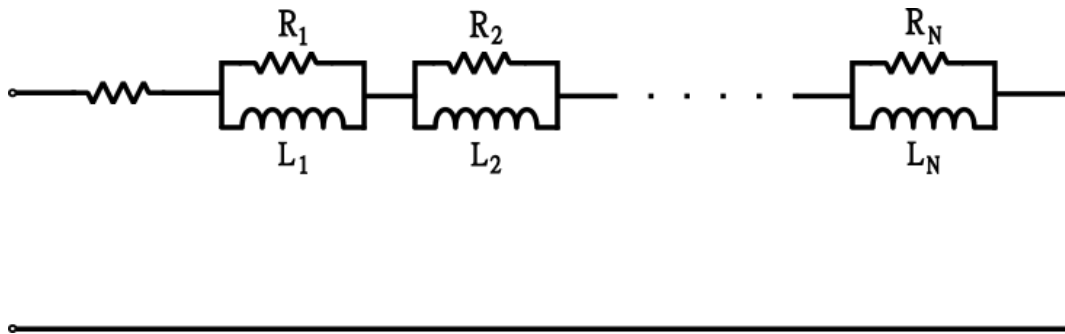


Fig. 2-64: The series Foster circuit

2.11.4 Cauer Circuits

Tarasiewicz, Morched, Narang and Dick [198] propose the use of a continued fraction model rather than a partial fraction expansion model. This resulting circuit is shown in Fig. 2-65. References [79], [82], [195] and [199] refer to Fig. 2-65 as the standard Cauer circuit. Tarasiewicz, Morched, Narang and Dick [198] claim that four terms are required for frequencies up to 200 kHz. Fig. 2-66 shows the Dual Cauer circuit, which has shunt resistances and series inductors. Four terms are also required for the Dual Cauer circuit at frequencies of 200 kHz [79]. Hysteresis loss and saturation can be considered by making the inductors nonlinear [198].

Cauer circuits are used to represent the frequency behaviour of the core. This is the main difference between Foster and Cauer circuits. The Dual Cauer circuit is the best representation of eddy currents in core laminations. The Cauer circuit also has a more physical meaning, but Foster circuits hold a computational advantage.

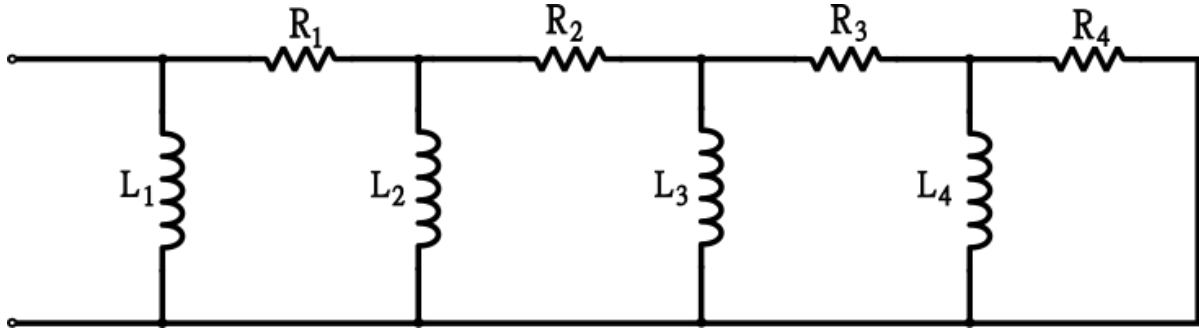


Fig. 2-65: The continued fraction model circuit

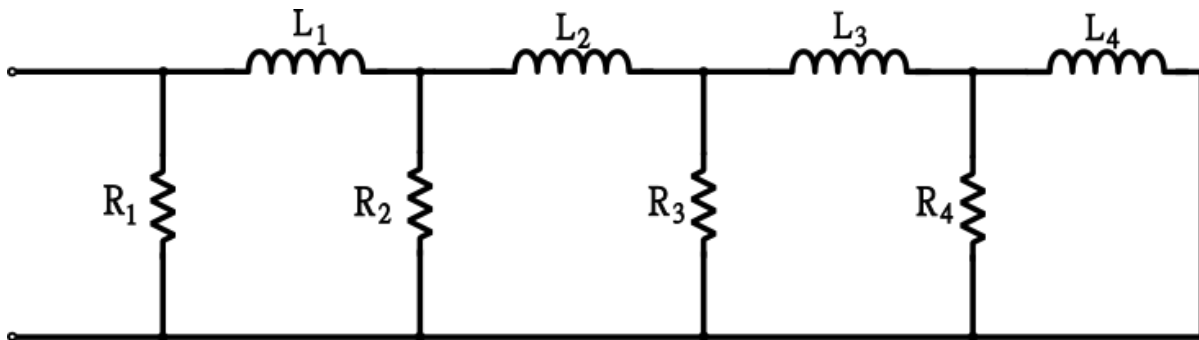


Fig. 2-66: The dual Cauer circuit

2.12 Inrush Currents

The concept of inrush current is another transient associated with the switching of transformers. Modelling inrush currents also requires an advanced and accurate transformer model [200], which is why inrush currents are also being discussed in this literature review.

According to Cigre [9], inrush currents are also associated with the saturation of a transformer. The transformer may become saturated due to a sudden change in applied voltage. This change in applied voltage may be caused by switching transients, out-of-phase synchronization of a generator, external faults or fault clearance. When the transformer is saturated, it absorbs a magnetization current, which is also referred to as inrush current. The flux in the transformer core can reach a theoretical value of 2-3 times the rated peak flux.

It was stated earlier that using amorphous cores increases the likelihood of ferroresonance. This principle also holds for inrush currents. Sima *et al.* [200] claim that the peak inrush current for amorphous alloy cores is about 10 times higher than those for grain-oriented silicon steel. The inrush current for amorphous cores is also higher than the inrush currents for non-oriented silicon steel [200]. This is because when the permeability of the core is low after saturation, a marginal increase in flux can lead to a magnification of the inrush current [9].

Section 2.6.3 discussed the π -transformer model and its benefits. The π -model has also proven to be much better than the T and T-equivalent models for inrush current modelling. Sima *et al.* [200] claim that this is because the parameters of the π -model have a direct relationship with the physical components of the transformer. Nevertheless, the π -model still requires improvement because flux-current curves of both the T-model and π -model are rarely known beyond the knee of the saturation curves. Therefore, even for inrush current modelling, detailed knowledge of the saturation process is required, which reinforces the need for accurate hysteresis models as discussed in Section 2.10.

Sima *et al.* [200] claim that four major factors affect inrush current:

- a the remanence in the transformer;
- b the applied voltage waveform at the transformer terminal;
- c the source impedance and transformer leakage impedance, and
- d the magnetizing inductance.

Chiesa, Mork and Høidalen [201] reaffirm point (c) made by Sima *et al.* [200]. They [201] claim that a detailed leakage inductance model is required if energization inrush current transients are to be modelled. This is because the leakage inductance and air-core paths determine the final core inductance when the core enters deep saturation. However, one's ability to accurately measure and estimate these parameters limit the detail level of the model.

Chiesa, Mork and Høidalen [201] elaborate on the list of parameters influencing inrush current, by stating that the shape, magnitude and duration of inrush current depends on several factors, including network topology, transformer construction, winding coupling, residual fluxes and circuit breaker current-chopping characteristics. The highest calculated current peak based on the combination of all of these parameters is defined as the works-case inrush current. This worst-case current peak occurs in the first period after the energization of the transformer. Despite all of these important parameters, Chiesa Mork and Høidalen [201] also claim that the slope of the magnetization curve at extreme saturation is the most influential parameter of all. Zirka *et al.* [94] state that the most severe underestimation of inrush current peaks is due to incorrect extrapolation of the saturated region of the magnetization curve. Cigre [9] Chiesa *et al.* [185] and León *et al.* [92] affirm the importance of the air-core inductance for inrush currents. It is also interesting to note that Chiesa [74] claims that the unsaturated magnetizing inductance does not influence the inrush current peak.

Residual flux was regularly discussed in Section 2.10, where hysteresis was covered in detail. Hysteresis does not add noticeable damping to inrush currents [112], but it does affect the value of the residual flux. It is, therefore, clear that residual flux is important to both ferroresonance and

inrush current. The ringdown transient determines the residual flux. The ringdown transient, in turn, is a function of the opening time (phase angle of the voltage) of the circuit breaker [202].

The influence of residual flux is also determined by the instant of energization (or the closing time of the circuit breaker). This means that the initial conditions of the transformer and the magnitude of the inrush current can be changed by varying the opening and closing times of the circuit breakers. The maximum inrush current will be found when the energization takes place at the zero voltage crossing [88]. Cigre [9] states that when the switch is closed at the zero-voltage crossing, the flux-linkage can theoretically reach a value of 2 p.u. plus the residual flux. When the energization instant takes place at the peak of the voltage waveform, there is no DC offset and the flux-linkage is only determined by the residual flux.

Zirka *et al.* [94] add that the air gap in the transformer core and the transformer capacitance indirectly influence inrush currents since they have a crucial impact on the residual flux.

Based on this foregoing discussion, Chiesa [74] compiled the following list of inrush current modelling challenges:

- a Correct representation of core behaviour in deep saturation;
- b Modelling of a nonlinear, hysteretic core;
- c Representation of frequency-dependent core and winding losses;
- d Determination of the initial conditions, particularly residual flux;
- e Description of the concurrent magnetic and electric coupling in a multi-phase transformer;
- f Estimation of the model parameters, and
- g Collection of data measurements for verification of a model.

Chiesa [74] makes an important statement regarding the relationship between power ratings and inrush current. A lower power rating (in VA) gives a higher ratio of inrush current to the rated load current. This statement was first made by ABB [203] in 1964. ABB [203] also presents an equation to approximate the magnitude of the inrush currents. Chiesa [74] claims that for distribution size transformers, the peak inrush current can exceed the short-circuit current. However, in [202], Chiesa and Høidalen claim that peak inrush currents are not a problem for small transformers – they are only important for large and medium power transformers.

It is also important to distinguish between the worst-case inrush current and the maximum overvoltage during an inrush current transient. The worst-case inrush current appears during the first period following transformer energization. However, Cigre [9] states that the maximum overvoltage often occurs during the inrush current decay, which is not immediately after the

energization transient. This is due to the variation of individual harmonic currents, which may increase with inrush current decay [9].

The effect of winding resistance on inrush current should also not be underestimated. De León, Jazebi and Farazmand [92] state that winding resistance is one of the most important factors that influence inrush current magnitude. Chiesa, Mork and Høidalen present a very interesting conclusion in this regard in [185] and [201]. Winding resistance is usually divided on a per unit basis between the primary and secondary sides. However, Chiesa, Mork and Høidalen [201] divided the winding resistance based on the *DC* resistance, which resulted in a lower value of winding resistance on the primary side. This did not affect the first peak of the inrush current, but the damping of the inrush transient was considerably slower.

2.13 Shunt Compensation

Shunt compensation is used to compensate for the effect that distributed line capacitance can have on a power system [6]. Cigre [6] states that shunt reactors are usually required for *EHV* overhead lines that are longer than 200 km. The degree of compensation in a line is quantified as a degree of positive sequence susceptance in the circuit. Cigre [6] provides an equation for calculating the degree of shunt compensation:

$$k[\%] = \frac{1}{\omega_s^2 \cdot L_+ \cdot C_+} \quad (2-108)$$

where

- k = degree of shunt compensation;
- ω_s = system angular frequency;
- L_+ = positive sequence shunt reactor inductance per phase, and
- C_+ = positive sequence line capacitance.

Shunt compensation plays a very important role in determining the residual flux of the system. This was noted in Section 2.5, where it was stated that shunt compensation can increase the duration of the ringdown transient. The ringdown transient is defined by [73] as “a natural LC response that appears as the stored energy dissipates whenever a transformer is de-energized.” The duration of the ringdown transient is increased because the damping of the circuit is reduced by the high capacitance. The ringdown transient determines the residual flux. According to Chiesa, Høidalen and Ischenko [73], the maximum residual flux decreases approximately proportional to the logarithm of the shunt capacitances. Chiesa, Høidalen and Ischenko [73] give an example where the maximum residual flux in a transformer was reduced by one third when a shunt capacitance of 1 μF was attached to the transformer terminals.

Despite the possible advantages of shunt compensation, it should be noted that Cigre [6] warns against the use of excessive shunt compensation. According to Cigre [6], the most common cases of power frequency resonance in transmission circuits is related to the use of shunt compensation. As a general rule, shunt compensation above 70% can lead to high temporary overvoltages during single-phase switching and circuit breaker malfunctions.

2.14 Line and Cable Modelling

Overhead lines and underground cables are used to connect transformers to a power system. It has also been pointed out throughout this literature review that lines and cables are a source of capacitance that needs to be considered when modelling a power system.

It is firstly important to note that Iravani *et al.* [87] state that underground cables have a large shunt capacitance compared to overhead lines, so ferroresonance usually involves underground cables. However, they [87] also make it clear that even in the absence of underground cables, the winding capacitance of a transformer may be sufficient.

2.14.1 Overhead Lines

The most important phenomena to represent in an overhead line model is the line asymmetry due to untransposed lines and circuit coupling, and the frequency dependency of the ground mode [9]. The corona effect and surge arresters are also important, but the corona effect is very difficult to model and often neglected. Surge arresters may influence the damping of transients, but their damping effect is often limited. Therefore, the main concern with surge arrester modelling is whether or not they will fail.

Cigre [9] recommends that distributed parameter models be used for transient studies. These distributed parameter models should also be able to reproduce the frequency-dependent behaviour of the cable if accuracy is desired at various frequencies. The exact pi model is not recommended for transient studies such as transformer energization. Exact pi models are only suggested for frequency scans.

An example of a distributed parameter model at constant frequency is the Bergeron model. This is one of three standard models discussed by Cigre [9]. This model is only suggested for transient studies at a single frequency, such as the power frequency or very low harmonics. The Bergeron model is also known as the “CP Model”. It takes into account the distributed nature of the line parameters. Cigre [9] claims that the Bergeron model is more suitable than the PI model for the calculation of switching overvoltage. The parameters of the Bergeron model are constants, which

means that the model should be used when frequency dependency is not a concern. It is also sensitive to the model frequency specified by the user.

Another model discussed by Cigre [9] is the cascaded-PI model. This is also the typical transmission line model discussed in power system textbooks such as [204]. The cascaded-PI model does not provide frequency dependency. It involves several PI-circuit connected in series to represent the total length of the line. The PI-circuits are calculated at fixed frequencies. There is also an equation dictating the maximum length to be modelled by a single PI-circuit. This is given by:

$$L_{max} = \frac{v}{5 \cdot f_{max}} \quad (2-109)$$

where

- L_{max} = the maximum line length to be represented by a single PI-section;
- v = the propagation speed of the electromagnetic wave (~300 km/ms), and
- f_{max} = the maximum frequency of simulated phenomena.

Cigre [9] makes an important note regarding cascaded-PI circuits in time-domain and frequency-domain simulations. In the frequency domain, electromagnetic transient programs calculate the parameters of the PI-circuits at each frequency. This means that frequency dependency is taken into account, but in the time domain, the π -circuit constants are kept constant and frequency dependency cannot be maintained. For this reason, PI-circuits are called “exact-PI” circuits in the frequency domain and “nominal-PI” circuits in the time domain.

The final model discussed by Cigre [9] is the JMarti model – also known as the FD model. The parameters of the JMarti model are both distributed and frequency-dependent. Model parameters for the JMarti model are typically calculated from the geometrical coordinates of the conductors. In such cases, it is very important to also consider shield wires and to set them to zero voltage.

All three models (cascaded-PI, Bergeron and JMarti) take ground resistivity as an input parameter. This is a very uncertain parameter, but [205] claims that impedance sensitivity to ground resistivity is very low below 5 kHz.

Cigre [9] lastly cautions that for frequencies below 1 kHz, the Bergeron and JMarti models are very good for single-circuit lines, but not good for double or triple-circuit lines.

2.14.2 Underground Cables

Underground cables can also be used to feed transformers. Aloui, Amar and Abdallah [206] state the underground cables have several advantages over overhead lines, including reduced maintenance and being immune to unfavourable climatic conditions. Underground cables also have the advantage of having small inductances [207]. However, the drawback of underground cables is that they have larger capacitances [207]. It is also difficult to locate and repair a fault in underground cables.

Cigre [9] claims that there are two major difficulties involved with modelling underground cables, i.e. the frequency dependency of the cables due to the skin effect and the ground or earth return path. However, this frequency dependency is more pronounced in overhead lines.

The frequency of interest for transformer energization studies is usually limited to 1 kHz. The phenomenon of highest concern in cable circuits is normally harmonic resonance due to inrush currents. For this reason, the frequency of interest when modelling cables is usually restricted to the lower harmonic frequency range.

Cigre [9] claims that the complex geometry of underground cables makes finite element methods the best modelling method for cables. However, electromagnetic transient programs do not work with finite element analyses. Distributed parameters models work best for underground cables unless the cables are very short. For accurate results, the model must also be able to include the frequency behaviour of the cable. Exact pi models are only good for frequency scans – they should not be used to model transients with multiple frequency components [9]

Frequency dependant modal domain models are very accurate for simple cable systems. Under the assumption that the transformation matrix can remain constant, these types of models provide very little error and provide for efficient simulation times. An example of such a model is the LMARTI model discussed in [9].

When complex cable configurations and large frequency ranges are being considered, Cigre [9] suggest the use of frequency-dependent phase domain models such as the ARMA, z-cable and Universal Line model

The three most important parameters for cable modelling are the conductor radius, the permittivity of the cable insulation and the thickness of the main insulation layer of the cable. Other less important parameters include conductor resistivity, sheath thickness and resistivity, inner semi-conductive layers and the resistivity of the earth.

2.14.3 Critical Cable Length

Several literature sources refer to a critical cable length related to ferroresonance. This relates to the maximum length of cable that can be connected to a lightly loaded transformer to avoid ferroresonance.

More recently, Baitch [5] defined an equation titled the “Baitch Ferroresonance Critical Cable Length Formula”:

$$L_{crit} = \frac{0.6 \cdot I_{mag}\% \cdot kVA_R \cdot 1000}{\left(1.58 + \frac{C'_{CC}}{C'_{CS}}\right) \cdot 62.8 \cdot (kV_R)^2 \cdot C'_{CS}} \quad (2-110)$$

where

- L_{crit} = the critical cable length in meters;
- $I_{mag}\%$ = the transformer magnetising current as a percentage;
- kVA_R = the rated transformer power in kVA;
- kV_R = the rated transformer voltage in kV;
- C'_{CC} = the specific cable core-to-core capacitance in $\mu\text{F}/\text{km}$, and
- C'_{CS} = the specific cable core-to-sheath capacitance in $\mu\text{F}/\text{km}$.

Walling *et al.* [38] caution that critical cable capacitance is related to the no-load losses of distribution transformers. Furthermore, since the no-load losses of distribution transformers have been reduced considerably, critical cable lengths for modern transformers are considerably lower than predicted by long-standing ferroresonance-avoidance guidelines. Walling *et al.* [38] made this claim in 1994. The Baitch Ferroresonance Critical Cable Length Formula was published in 2000. One might argue that Baitch’s formula should, therefore, take precedence over the claim made by Walling *et al.* [38]. However, the critical cable length formula does not consider no-load losses, nor does [5] acknowledge the claims made by Walling *et al.* [38].

The critical cable length was more recently estimated using the ATP-EMTP environment by Abdi, Walker and Djeumen [208]. In some cases, the critical cable length was as little as 6 m. This goes to show that limiting transmission lines to critical cable lengths is not a viable solution for ferroresonance avoidance. In such cases, ferroresonance mitigation techniques, as discussed in Section 2.5, should be considered.

2.15 Sweep Frequency Response Analysis

The sweep frequency response analysis (SFRA) is a potential tool that can be used to test the accuracy of the transformer simulation model. In general, the SFRA is used to monitor the

condition of the transformer. However, the literature on the SFRA indicates the output of the SFRA is unique to the geometry of the transformer. Consequently, its frequency response should be unique and can be compared to the frequency response of the simulation model. Frequency response analysis (FRA) plots the ratio of the transmitted voltage to the applied voltage in dB. The impedance of the transformer attenuates the input voltage.

It is important to note that the SFRA is not the only type of FRA. Tiberg *et al.* [209] describe the comparison of a transformer simulation with measurements for lightning impulse tests. This comparison is done in both the time domain and the frequency domain. It is referred to as an impulse frequency response analysis.

Cigre [210] states that the main purpose of the SFRA is to detect winding displacements after over-currents caused by faults. Other applications include mechanical condition assessment of the transformer after transportation and the detection of any other faults. These faults are detected by changes to the capacitance and the inductance distribution of the system. FRA results and high-frequency modelling are also used to understand and predict the interactions that transformers may have with the network to which they are connected. This includes investigating potential internal resonances caused by remote switching. The FRA has therefore been used for modelling, though literature in this regard is scarce. The literature on the topic of SFRA is generally more focused on the impact that different types of faults can have on the overall SFRA shape. This literature review will focus on the literature which applies to transformer modelling, such as literature that explains the frequency bands of the SFRA and the various impacts that different factors have on the waveforms. The focus of this report is not to investigate transformer faults and, therefore, the literature focused on this topic will not be discussed.

The literature on the topic of SFRA has tried to identify the parameters that dominate various frequency ranges. However, there is some discrepancy in different sources of literature in this regard – particularly when trying to define specific frequencies. Pleite *et al.* [211] claim that the core behaviour dominates at low frequencies. This includes the magnetizing inductance, the core power losses and the parasitic capacitance. Winding effects, such as copper losses and leakage inductance, can be neglected at these frequencies. Moodley and de Klerk [212] agree by saying that at low frequencies, the response is inductive and the capacitance of the transformer can be neglected. This response is dominated by the inductance of the magnetic circuit.

One interesting source of literature regarding frequency ranges is [214], which claims that the influence of winding inductance is predominant in a frequency up to 10 kHz. This seems to contradict other sources of literature, such as Moodley and de Klerk [213] which claim that the winding inductance plays a role at higher frequencies. Furthermore, all other sources of literature

point out that magnetizing inductance dominates the low-frequency range. Bagheri *et al.* [7] also speak about the effect of winding inductance at low frequencies. They claim that this includes winding self-inductance, which is a function of the magnetic core reluctance and the number of turns. One has to assume that this is what [214] also referred to when they spoke about winding inductance. Consequently, it would appear as though the nomenclature used in the literature to refer to the inductance of the transformer core can be misleading. Nevertheless, the transformer core dominates the *SFRA* behaviour at low frequencies. As the frequency increases, the self-inductance will assume values that are large enough to be neglected [7].

Bagheri, Phung, Blackburn and Naderian [7] provide an equation for the calculation of the frequency of the anti-resonance point at low frequencies:

$$f_{r(low-frequency)} = \frac{1}{2\pi\sqrt{L_m C}} \quad (2-111)$$

where

- L_m = the winding self-inductance, and
- C = the sum of series and shunt capacitance.

Bagheri, Phung, Blackburn and Naderian [7] claim that the mutual inductance reactance becomes more significant in the mid-frequency range. Moodley and de Klerk [212] agree by stating that at higher frequencies, the *SFRA* behaviour becomes a little more complex due to multiple resonant frequencies. They [212] claim that the magnetizing inductance is effectively screened by the winding inductance at these frequencies. Bagheri, Phung, Blackburn and Naderian [7] state that the fluctuations seen in this mid-frequency range are influenced by the winding structure, which influences the series and shunt capacitances and the inductances. Series capacitances, according to [7], refer to turn-to-turn and disc-to-disc capacitances. Shunt capacitances refer to the capacitance of the winding to the core and the tank. The resonance frequencies of the mid-frequency range are given by:

$$f_{r(mid-frequency)} = \frac{k}{2\lambda} \left(\frac{1}{\sqrt{(c_g + k^2 \pi^2 (\lambda^2)^{-1} c_s) l}} \right) \quad (2-112)$$

where

- c_g = the shunt capacitance per unit conductor length;
- c_s = the series capacitance per conductor length;
- l = the inductance per unit conductor length;

- λ = the winding mean turn length, and
- k = the variable coefficient ($k=1, 2, 3, \dots$)

Bagheri, Phung, Blackburn and Naderian [7] point out that it can be deduced from (2-112) that high values of c_g increase the number of resonant frequencies. On the other hand, high values of c_s reduce the number of resonant frequencies.

Sano and Miyagi [214] also claim that in frequency ranges around 30 kHz and from 60 kHz to 100 kHz, the capacitance to the ground of the HV winding and localized resonance of the interwinding affect the *FRA* response. This is confirmed by Bagheri, Phung, Blackburn and Naderian [7].

At very high frequencies, the winding inductance becomes screened by the capacitance of the transformer. Brandt, Peniak and Makarovič [215] as well as Singh, Verma and Jarial [216] both present the same table which summarized some of the causes of faults in various frequency ranges. This table is also shown in Table 2-15. Cigre [210] confirms the data in Table 2-15 by stating that the shunt capacitance of the windings and the bulk movement of the winding or bushing capacitance influences the intermediate frequency range between 2 kHz and 20 kHz. This causes a characteristic second maximum (in dB) in the magnitude of the impedance.

The literature on the topic of the *SFRA* regularly mentions the effect that residual magnetism can have on the measurements. The IEEE [213] indicates that residual magnetism can shift the first peak of the impedance graph to the left or the right. Abeywickrama, Serdyuk and Gubanski [217] confirm this claim and add that this can be prevalent up to a frequency of 10 kHz. Therefore, *FRA* diagnostics should never be performed directly after disconnection from the power system. Instead, an idle period of 24 hours should be allowed. Cigre [210] on the other hand claim that residual magnetism should only have an effect up to about 5 kHz. They claim that this is because, at higher frequencies, eddy currents prevent penetration of the magnetic field into the laminations.

This also means that if eddy currents are not considered for the modelling of the transformer, one can expect deviations to occur from 5 kHz. It is also interesting to note that Table 2-15 limits the effect of residual flux to 2 kHz. Therefore, it is clear that there is some discrepancy in the literature regarding the frequency range affected by residual magnetism.

Another factor that can influence the *SFRA* output is the cable connected to the transformer for the test. The IEEE [213] indicates that cables limit the area of reproducibility of the plots to around 100 kHz to 150 kHz. Furthermore, above 40 kHz there are significant changes to the plots. This is important to consider since simulations typically do not take cables into account. Consequently, one can conclude that a simulation *SFRA* will become very inaccurate beyond 40 kHz. Sano and Miyagi [214] claim that the characteristics below 10 kHz are virtually unaffected by the length of

Table 2-15: The frequency bands and possible sources of variation as presented in [215, 216]

Band	Likely Causes of Variation
<2 kHz	Core Deformation, Open Circuits, Shorted Turns and Residual Magnetism.
2 kHz – 20 kHz	Bulk Winding Movement Relative to Each Other, clamping structure.
20 kHz – 400 kHz	Deformation Within the main tap windings.
400 kHz – 2 MHz	Movement of main and tap winding Leads; axial shift.

the measurement cables. However, characteristics after the first resonant frequency, which are in the range where capacitance is dominant, are changed by the length of the measurement cables. They [214] hypothesize that this is because the inductance of the transformer is much larger than the inductance of the cables. Secondly, the capacitance of the cables is equal to or greater than the capacitance of the transformer. Therefore, the effect that cables have on *SFRA* results is mainly determined by their capacitance to ground [214].

Cigre [210] also state that large test setups, such as long cables and extension leads, show significant interaction with the transformer in the typical interpretation range of the *SFRA*. They [210] also proceed to discuss the effect that the resistance of cables can have on the interpretation of results. The comparison of only the phase or only the amplitude is typically sufficient for *SFRA* analysis. However, test connections with a high contact resistance can significantly influence the resonance amplitude without affecting the phase. Therefore, considering both these parameters in *SFRA* analysis provides more certainty of accuracy. Secue and Mombello [218] also conclude that to consider the complex nature of the *SFRA*, both the phase and the amplitude need to be taken into consideration.

Some other factors that can influence the *SFRA* characteristic, according to [214], are the presence of test bushings, the presence of insulating oil and the tap connection. Cigre [210] elaborates on the effect of oil by stating that the high permittivity of the oil increases the capacitance. This reduces the resonance frequencies by essentially shifting the entire graph to the left.

Moodley and de Klerk [212] discusses the interpretation of the *SFRA* on a fundamental level. They highlight that the impedance at different frequencies is fundamentally related to the

inductance, capacitance and resistance of the transformer. The resistance is related to the physical construction of the transformer and tends to shift the entire waveform vertically. The capacitance and inductance are related to the geometry of the windings and result in a horizontal shift of the *SFRA* plot. Cigre [210] states that the interpretation of *FRA* results is generally done by visual comparison to previous measurements of the same transformer. If this is not possible, it is compared to an identical transformer or one of the other phases of a three-phase transformer.

Cigre [210] also discusses the potential of comparing measured *SFRA* plots to simulation data. They caution that the complexity of real winding assemblies and their mechanical tolerances makes this comparison very difficult. Mathematical models based on pure geometric data tend to show significant differences to the measured data. Cigre [210] also says that even in the case of simplified test geometries, good agreement is difficult to obtain. A different Cigre working group elaborated on the simulation of *SFRA* waveforms in [219]. They claim that when the transformer model is to be used in the MHz frequency range, then every turn of the winding needs to be represented along with all mutual, inductive and capacitive couplings. Furthermore, to extend the application range of the RLCG lumped model to a few MHz, the model needs to use a turn-to-turn modelling procedure rather than a disk-to-disk modelling procedure. Martinez *et al.* [56] confirm that capacitances are generally very important for high-frequency behaviour.

Cigre [219] also discuss a black-box model that can be used to replicate the results from a measured *SFRA* plot. This black box model is a combination of several damped series resonant circuits connected in parallel. The main inductance is obtained from the low-frequency range, where the phase plot indicates that the transformer is purely inductive. This is below 300 Hz in their example. The surge capacitance is identified in a high-frequency range (10 kHz to 200 kHz in their example) where the phase plot indicates that the transformer is showing capacitive behaviour. The first resonance point is used to determine the terminal resistance. Cigre [219] proceeds to elaborate on this black box model to improve the fit. However, this simplified approach already shows good agreement up to 100 kHz and could be useful for simulation purposes.

2.16 Summary

Chapter 2 presented an extensive critical review of the literature relevant to this study. The Chapter started by introducing the concept of resonance and looked at how resonance can be analysed and mitigated.

The problem statement in Chapter 1 mentioned that this study is conducted under the assumption that ferroresonance is the cause of overvoltages. Therefore, a lot of emphases was placed on

ferroresonance within this Chapter. Ferroresonance modelling requires a sufficiently accurate transformer model. For this reason, Section 2.6 looked at the various approaches to transformer modelling which can be found in the literature. It also focused on some of the existing transformer models, which have already been developed.

The literature on ferroresonance proved that ferroresonance modelling is highly dependent on several factors. Two of these factors include hysteresis minor loops and, more importantly, remnant flux. The modelling of remnant flux requires an accurate hysteresis model. The literature on ferroresonance also proved that accurate hysteresis modelling can have a significant impact on ferroresonance simulation results. Consequently, section 2.10 was dedicated to the discussion of hysteresis. The field of hysteresis warrants a study in itself. The discussions in section 2.10 are, therefore, not exhaustive. However, several hysteresis models are briefly discussed along with a summary of the advantages and disadvantages of those models. The Jiles-Atherton, Preisach and Tellinen models receive special emphasis.

Traditional transformer models do not take the frequency-dependency of transformers into account. Factors that influence frequency dependency include the skin effect and proximity effect. Foster and Cauer circuits are used to make transformer models more accurate by taking these phenomena into account. These aspects of transformer modelling are discussed in section 2.11.

Other topics discussed in this literature study include inrush currents, shunt compensation, geomagnetic induced currents, distribution systems, the modelling of overhead lines and underground cables and the sweep frequency response analysis of a transformer. All of these topics contribute to the development of an accurate transformer model, which is the focus of the next Chapter.

CHAPTER 3 DERIVATION OF TRANSFORMER MODEL

Following the detailed breakdown of literature on the topic of transformer modelling found in Chapter 2, the next step is to develop an accurate model of the transformer in question. This chapter is dedicated to this derivation.

Even though the literature may provide a breakdown of transformer models and their accuracies, it is also important to ensure that the results of a model found in the literature can be replicated. This verifies that one has a proper understanding of the model. In some cases, the author of a source of literature may omit important assumptions which he took for granted. In such cases, it is very difficult to replicate the results. Therefore, successful replication of a model found in literature serves as an added form of verification of its accuracy. This chapter will start with such a form of verification, where a π -model found in literature is successfully replicated.

The chapter will then proceed to build on this verified model by modifying the parameters to reflect the transformer being investigated in this report. The parameters for this model are based on the OC and SC tests. The results of these tests are also presented in this Chapter.

The chapter also develops two hysteresis models – the Jiles-Atherton model and the Tellinen model. These hysteresis models are implemented in a circuit in MATLAB and are used to replicate magnetization current results found in the literature. Two optimization algorithms for the determination of hysteresis parameters are presented along with a discussion of their results and shortcomings. However, problems with measurement prohibited the author from developing models which accurately reflect the saturated behaviour of the distribution transformer being studied in this report. These problems are discussed in section 3.4.3.

Having concluded that it is not currently possible to develop a hysteresis model of the transformer, the accuracy of the developed π -model is verified. This is done by replicating the results of the OC and SC tests. An SFRA test was also done on the transformer. The chapter concludes by showing the SFRA results of the simulated transformer model.

3.1 Literature Verification

Before a model of the transformer in question can be developed, two important steps first need to take place: a decision must be made on the type of model to be used and the accuracy of the proposed model found in the literature needs to be verified. This verification provides the user with confidence in his understanding of the model and its operation. It is hindered by the fact that authors of academic journals often neglect to mention important assumptions which were taken

for granted. Furthermore, sources of literature also often neglect to provide the reader with sufficient data, such as the data points used for nonlinear inductors. In such cases, it is nearly impossible to replicate the results of the paper.

The first step in developing an accurate transformer model is deciding on the modelling approach. Section 2.6.3 mentioned five mainstream approaches to transformer modelling: modelling based on self and mutual inductances, modelling based on leakage inductances, modelling based on the principle of duality, modelling based on measurements and analyses based on electromagnetic fields. It was made clear by [93] that models based on the principle of self and mutual inductance become ill-conditioned when the core is taken into consideration. The solution to this problem makes the model identical to the model based on leakage inductances. The model based on leakage inductances, in turn, still presents a poor representation of the core of the transformer, though the leakage inductances are accurately represented. Models based on the principle of duality present an accurate model of the core at the cost of leakage inductance accuracy. Models based on measurements can be classified as black-box approaches and analyses based on electromagnetic fields require finite-element analyses.

The focus of this study is to model ferroresonance. It should also be clear that to study ferroresonance, the modelling of the core is very important. This is why hysteresis models were discussed extensively in Chapter 2. Based on this premise, it can be concluded that models based on the principle of duality should be preferred above models based on accurate leakage inductance modelling. It has also become evident through the course of this study that black-box approaches to transformer modelling are insufficient for ferroresonance studies since ferroresonance studies require the transformer to be saturated. This can be difficult to achieve during measurements and consequently, the accuracy of the model is affected. This will be proven at the end of this chapter when the *SFRA* results of the transformer are discussed. Finally, analyses based on electromagnetic fields require detailed knowledge of the construction of the transformer. This information was not available to the author during this study. It should then be clear that models based on the principle of duality yield the most applicable transformer models for ferroresonance studies.

The most prominent model based on the principle of duality is the renowned π -model. Circuit diagrams of the π -model can be found in Fig. 2-22 and Fig. 2-23. Furthermore, Jazebi *et al.* [95] compared the standard T-model and the π -model for the purpose of ferroresonance simulation. They [95] conclude that the T-model fails to reproduce ferroresonance measurements but the π -model predicts the measurements accurately in all tested cases. The reader should also recall that the literature study stated that the π -model is a physically accurate representation of the

transformer windings. Based on these advantages associated with the principle of duality and the π -model, the rest of this report will make use of the π -model for simulation.

Jazebi *et al.* [95] make use of nonlinear inductors to simulate ferroresonance. This is important since transformer saturation is one of the requirements of ferroresonance. Ferroresonance cannot be attained with constant inductors, even if the value of those inductors are given by the saturated inductance of the transformer core. However, Jazebi *et al.* [95] do not provide the user with the nonlinear data points of their inductors. This makes verification and reproducing the results of [95] particularly difficult. Fortunately, Jazebi *et al.* conducted another experiment on the T- and π -models in [88], where the nonlinear data points of the inductors are provided. Furthermore, since the other electrical parameters such as the winding resistance and leakage inductance are identical in both [88] and [95], one can conclude that the same data points were used in both simulations. If these data points can be used to replicate the results of [95], then one can have confidence in the accuracy of the model. This verification process, therefore, serves as a baseline for establishing the model to be used in the ferroresonance study of this report.

Replication of the results of [95] involves the open-circuit simulation of a T-model and a π -model with a series capacitance of 20 μ F. The electrical parameters of the transformer are: $R_1 = 0.306 \Omega$, $R_2 = 0.305 \Omega$, $R_m = 1074.7 \Omega$ and $L_s = 1.6696$ H. The circuit diagrams of the T-model and π -model are shown in Fig. 3-1. The applied source has a magnitude of 120 V and a frequency of 60 Hz. The secondary open-circuit waveforms presented by [95] are shown in Fig. 3-2. In some cases it may be necessary to reference the secondary winding resistance R_2 to the primary side. However, in this case, the winding ratio is 1:1.

Modelling the nonlinear inductor proved more challenging than initially anticipated. The data points in [88] relate the flux (in Wb) in the core to the current (in A) flowing through the core. The Specialized Power Systems environment in MATLAB requires data points that relate the flux-linkage (in Wb-turns) to the current. This also holds for the ATP-Draw environment. Flux linkage is related to flux by the following relation:

$$\lambda = \phi \cdot N \quad (3-1)$$

where λ represents the flux linkage, ϕ represents the flux and N represents the number of turns of the winding. Fortunately, [88, Table VIII] indicates that the transformer winding consists of 196 turns. Therefore, it is possible to convert the flux/current data of [88] to flux linkage/current data. The data being considered is for the transformer with enlarged leakage inductance as reflected in the final columns of [88, TABLE VI] and [88, TABLE VII].

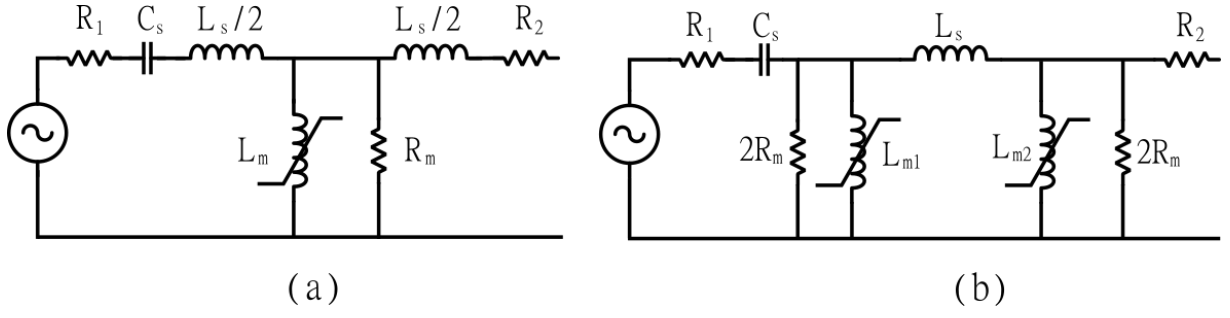


Fig. 3-1: The circuit diagram for the ferroresonance replication of [95] a) T-model and b) π -model

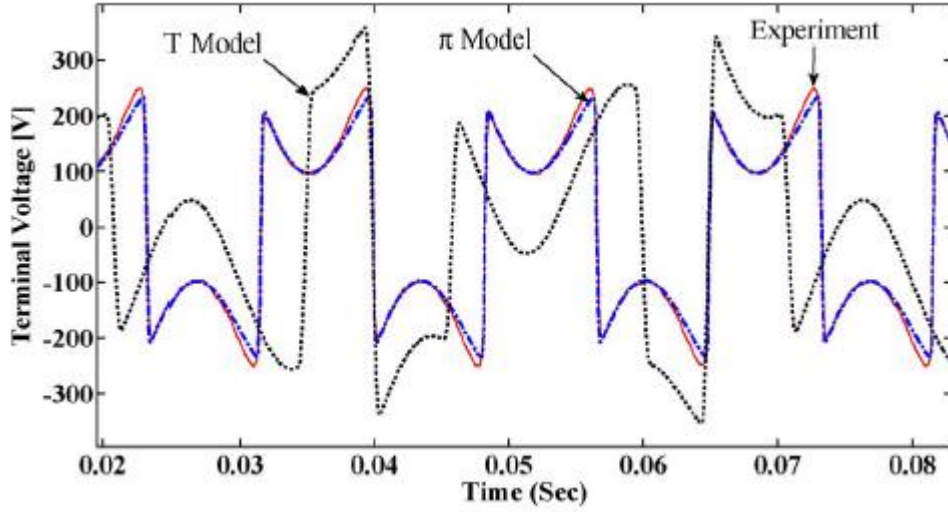


Fig. 3-2: The ferroresonant waveforms from [95, Fig. 2] to be replicated

The resulting waveforms from simulations in MATLAB with the adjusted data are shown in Fig. 3-3. There is a significant difference between these simulation results and Fig. 3-2. Most notable is the fact that the π -model and T-model results are identical and that ferroresonance has not taken place. This indicates that the transformer has not entered saturation.

However, if the flux data seen in [88] is imported directly into the MATLAB Specialized Power Systems Environment, the results change to those seen in Fig. 3-4. Comparing the results of Fig. 3-4 to the results of Fig. 3-2 shows an almost perfect correlation. Small differences include the x-axis, which reflects the time. However, [95] does not indicate the initial phase of the source during simulation. This will impact the time at which a peak voltage is reached and could cause a shift of the waveforms on the time axis.

The only other difference is the location of the T-model waveform relative to the π -model waveform. In Fig. 3-2, the descending leg of the T-model waveform lies on the descending leg of the π -model waveform. This is not the case in Fig. 3-4, where there is a slight offset between the two descending legs of the waveforms. However, the general shape of the waveforms is identical,

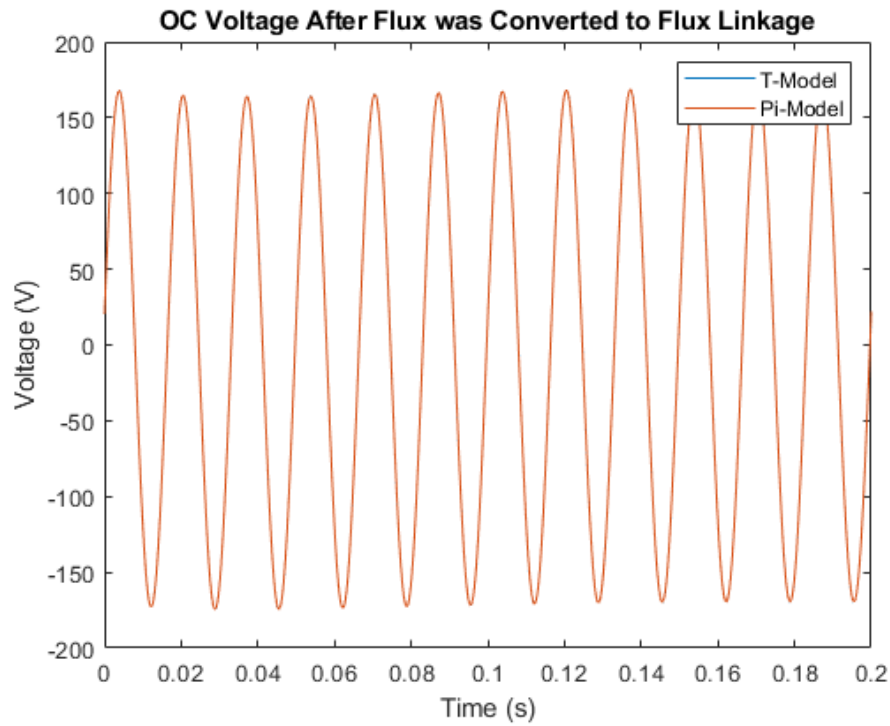


Fig. 3-3: The MATLAB simulation results when [88] flux data is converted to flux linkage

which indicates the mode of ferroresonance is the same. The peaks of the waveforms are also the same. This provides confidence in MATLAB's ability to model ferroresonance using a nonlinear inductor.

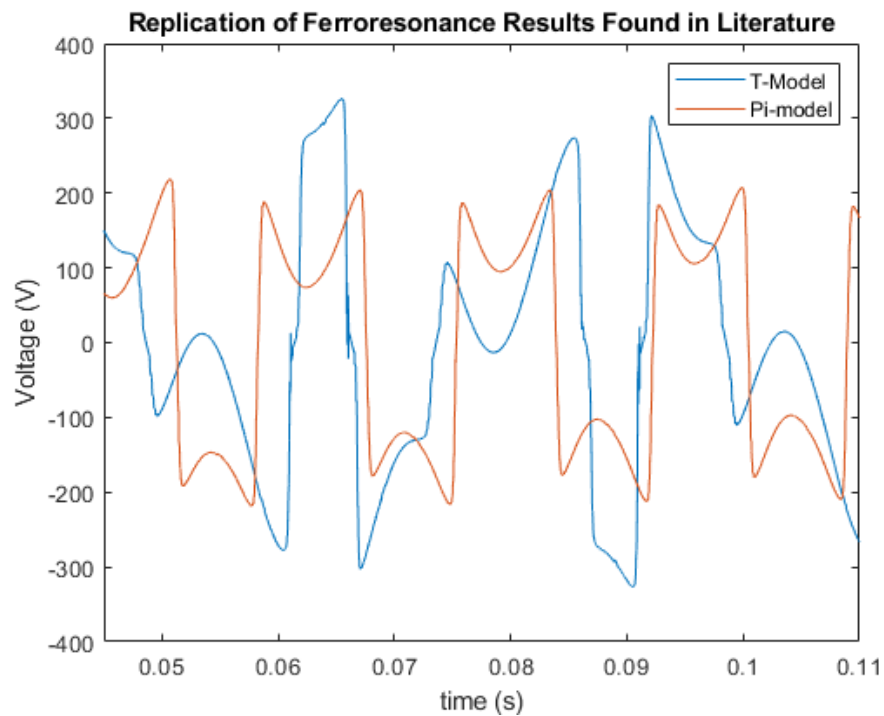


Fig. 3-4: The MATLAB simulation results when the flux data of [88] is imported as flux linkage

These results were also replicated in the ATPDraw environment, where the results were identical. Therefore, the results of [95] were replicated across two different simulation environments and both environments yielded the same output. This serves as a form of verification of the accuracy of the π -model. Consequently, the π -model can now be modified to reflect the behaviour of the transformer being investigated in this report and one can have confidence in its output.

The author would like to caution readers who attempt to replicate this output in a simulation environment to ensure that they fully understand the environment being worked in. The ATPDraw environment replicates the positive saturation data to the negative side. This means that it is only necessary to enter the positive half of the saturation data. MATLAB, on the other hand, does not always do this. Consequently, the nonlinear inductor in the Specialized Power Systems environment in MATLAB requires the user to enter the negative saturation data and the positive saturation data. Failure to do this means that MATLAB will only recognize the saturation of the inductor during every positive cycle, which also impacts the accuracy of the waveforms. Furthermore, the nonlinear inductor in the Specialized Power Systems environment in MATLAB takes flux linkage as input. The nonlinear inductor in the Simscape Electrical environment of MATLAB takes flux as input. The documentation of the nonlinear inductor in the Simscape Electrical environment states that MATLAB will rotate the data about (0,0), provided that the vector starts at 0.

It is important to note the difference between Fig. 3-3 and Fig. 3-4 and the conditions which lead to their outputs. Jazebi *et al.* [88] indicate that the data points in [88, TABLE VI] and [88, TABLE VII] reflect the relationship between current and flux. However, when the flux data is modified according to the relationship between flux and flux linkage, the results are no longer correct. Furthermore, the documentation of the nonlinear inductors in the Specialized Power System environment of MATLAB indicates that the input should be a flux linkage input with units of Wb-turns. This leads to the inevitable conclusion that a mistake is made in [88] – the headings of [88, TABLE VI] and [88, TABLE VII] should indicate that the data points reflect the flux linkage in Wb-turns. To reaffirm this claim, a nonlinear inductor was also built from scratch using fundamental components in the MATLAB environment. These fundamental components are: a voltage measurement block, a controlled current source, a lookup table and the appropriate mathematical blocks to convert voltage to flux linkage. This fundamental model is explained in detail in APPENDIX B. The simulations of [88] are replicated with this newly constructed nonlinear inductor and the results agree with those presented in [88].

3.2 Developed π -Model

Having established that the π -model used in [95] is accurate and can model ferroresonance, the next step is to modify the model to reflect the behaviour of the transformer being studied in this report. This involves conducting SC and OC tests on the transformer. These tests provide the parameters to be used in the π -model.

When the purpose of a simulation is to model steady-state behaviour, the OC and SC tests are conducted at rated values and their output yields constant linear parameters. However, since this report considers saturation and nonlinear behaviour of the core, the OC test needs to be conducted across a wide range of voltage values. Chapter 2 also discussed frequency-dependent components which also need to be considered. Consequently, this discussion of parameters is broken down into three sub-sections. The first subsection discusses the linear parameters, which can be obtained from the SC and OC tests. The second subsection discusses the frequency-dependency of some of these parameters. The final subsection discusses the nonlinear behaviour of the core.

3.2.1 Linear Parameters

This subsection will focus on the linear parameters which can be derived from the SC and OC tests. These include the winding resistances, leakage inductances, core loss resistance and magnetizing inductance.

The winding resistance and leakage inductance are obtained from the SC test. Since the SC test requires the rated current to flow through the winding, it is conducted on the primary side where the rated current is lower. However, limitations on the voltage source in the laboratory restricted this test. The rated current for a 32 kVA, 22 kV transformer on the HV side is 1.45 A. For the SC test conducted on this transformer, the current was limited to 1.197 A. In the case of the SC test, however, this is not critical as the SC test does not display saturation. Therefore, a current of 1.197 A was considered close enough to measure the winding resistance and leakage reactance. Both the SC and OC tests were conducted with a Newtons4th Ltd (N4L) PPA5500-TE Precision Power Analyzer. The N4L has a 0.07% nominal power accuracy, a 0.55% power accuracy at power factor levels of 0.1 and a 0.003° phase accuracy. Furthermore, it indicates the apparent and reactive power, which eliminates human error when trying to estimate the power factor from waveforms. The results of the SC are shown in Table 3-1.

The winding resistance was also tested separately using a 4-wire method with two multimeters. Each winding was measured two times and the average of the measurements was taken to represent the DC winding resistance. The results are shown in Table 3-2. Recall that Chapter 1

Table 3-1: The SC and OC test results

	Voltage (V)	Current (A)	Power Factor	Apparent Power (VA)	Active Power (W)	Reactive Power (var)
SC Test	735.60	1.1970	0.3430	880.49	302.03	-827.07
OC Test	242.56	0.46656	0.7642	113.17	86.48	-72.996

mentioned that this distribution transformer has a centre tap on the *LV* winding. Therefore, the resistance between the outside terminal and the centre tap was also measured. The centre tap is designated with the symbol n in Table 3-2, e.g. winding a2- n represents the winding resistance between terminal 2 and the centre tap. The resistance between winding a2-a1 represents the total winding resistance of the *LV* winding and is almost equal to the sum of the a1- n and a2- n resistances.

The *OC* test is conducted on the *LV* side since the laboratory at the North-West University does not have a voltage source that can supply 22 kV. Furthermore, the test is conducted between one terminal and the centre tap, since the second terminal of the *LV* winding is not required to establish flux through the core. This further reduces the requirements on the source. The *OC* test is conducted on a wide range of voltages from 2.5% to 120% of rated voltage. The rated voltage on the *LV* winding between one terminal and the centre tap is 242 V. The range of *OC* data points will be used to define the nonlinear inductor in section 3.2.3. However, if the aim was to develop a steady-state model, the data collected at rated voltage would be used to define the core parameters of the model. Therefore, the rated voltage test results are also shown in Table 3-1.

The equations used to convert the results of the *OC* and *SC* to transformer parameters will not be repeated in this section of the report. Interested readers are referred to the work of Guru [84]. The transformer parameters are summarized in Table 3-3, where:

Table 3-2: The 4-wire method winding resistance test results

	Average Voltage (V)	Average Current (A)	Average Resistance
HV Winding Tap 3	15.36	0.138	111.304
LV winding a2-n	0.0515	2.808	0.018
LV Winding a1-n	0.0465	2.807	0.0165
LV Winding a2-a1	0.0955	2.727	0.035

- R_H = the HV winding resistance;
- R_L = the LV winding resistance;
- L_H = the HV winding leakage reactance;
- L_L = the LV winding leakage reactance;
- R_{cH} = the core loss resistance referenced to HV side, and
- L_{mH} = the magnetizing inductance referenced to the HV side.

There is only a single leakage reactance defined for the π -model. This is based on the definition of the π -model, which only has a single inductance between the two legs representing the core. This leakage inductance is equal to the sum of the *HV* and *LV* leakage inductances and is referenced to the *HV* side of the transformer. The core loss resistance and the magnetizing inductance of the π -model are twice as large as those for the T-model. This is based on the definition of the π -model. Refer to Fig. 3-1 where it is also indicated that the core loss resistance is double the value used for the T-model. In this case, the multiplication has already been done and the user can simply import these values directly. The magnetizing inductance parameter is indicated in Table 3-3, but will not be used in the simulation since the core inductance is defined by a nonlinear relationship based on the OC test results.

Table 3-3: A summary of the transformer parameters from the OC and SC tests

	R_H (Ω)	R_L (Ω)	L_H (H)	L_L (H)	R_{cH} (Ω)	L_{mH} (H)
T-Model	111.304	0.035	1.130	0.00036	5 622 368.13	21 204.6266
π-Model	111.304	0.035	1.865		11 244 736	42 409.253

3.2.2 Frequency-Dependent Components

Section 2.11 spoke about frequency-dependent components, such as eddy currents in the core and the windings, the proximity effect and the skin effect. It also discussed the use of Foster and Cauer circuits to model these phenomena.

However, the question is whether the increase in accuracy presented by these circuits warrants the added complexity introduced to the modelling process. It is the opinion of the author that the core does not need to be modelled this accurately. This is because both Chiesa [74] and Rezaei-Zare, Iravani and Sanaye-Pasand [102] agree that modelling the frequency dependency of the core does not have a significant impact on ferroresonance simulations. Chiesa [74] states that the conventional nonlinear inductor in series with a constant resistor is sufficient up to frequencies of 3 kHz. The literature study indicated that ferroresonance is a low-frequency phenomenon that

does not exceed 2 kHz. The only contradictory source of literature in this regard is Chandrasena *et al.* [135], who claims that the modelling of eddy currents is at least as important as hysteresis. However, the focus of the article written by Chandrasena *et al.* [135] is not on ferroresonance, but overvoltages in general. Chiesa [74] and Rezaei-Zare, Iravani and Sanaye-Pasand [102] specifically speak about ferroresonance. Therefore, eddy currents in the core will be neglected in this simulation model under the assumption that the constant resistor calculated in Table 3-3 is sufficient.

With regards to the skin effect, the literature study in section 2.11.2 states that the skin effect will maintain a constant DC value up to a frequency of 10 kHz. It also emphasises that the skin effect dominates the losses up to a certain frequency. Consequently, if it can be proven that the skin effect can be ignored, then one can conclude that the proximity effect will also be negligible. Greene and Gross [196] claim that their studies show that the skin effect is negligible up to a frequency of 3 kHz. Consequently, one can conclude that the skin effect, and therefore the proximity effect, can be neglected for ferroresonance studies.

The final aspect to consider is that of eddy currents in the windings. Section 2.11.1 states that eddy current effects in the winding can be neglected if the winding has a diameter of less than 3 mm. A good rule of thumb for defining the thickness of wires used in transformer windings is that the current density should not exceed 3 A/mm². On the primary side of the transformer in question, this is not a concern. The rated current for a 32 kVA, 22 kV transformer is 1.45 A. The thickness of the wire used for the primary winding is determined by the mechanical integrity of the wire, rather than its electrical properties since the current density is so low.

On the secondary side, however, the rated current is 66.11 A. This means that the area of the wire needs to be at least 22.03 mm². If a round wire is used on this secondary winding, then the radius of the wire needs to be 2.64 mm, which means that the diameter is 5.29 mm. However, even though the manufacturer of the transformer did not define the exact dimensions of the wire used for the secondary winding, he did confirm that strip wire is used on the secondary side, which is rectangular. This means that if a wire of dimension 8 mm x 3 mm was used for the secondary winding, the current density would be below the 3 A/mm² threshold. One can then also assume that since the wire is so thin, eddy current effects in the wire can also be ignored.

Based on all of these assumptions, the decision was therefore made to ignore frequency-dependent components for both the core and the windings. This is substantiated by the background given in Chapter 2 and simplifies the transformer model considerably.

3.2.3 Transformer Capacitance

Section 2.6.3 concluded the discussion on transformer models found in the literature by stating that the circuit diagram seen in Fig. 2-25 is the most physical model of a transformer to date. It contained Foster and Cauer circuits to model frequency-dependent components, but more importantly, it contained the capacitances between the HV and LV winding, the HV winding and ground and the LV winding and ground. The importance of these capacitances was highlighted by Geldenhuys and Padayachee [1], which also gave rise to this study.

A tan-delta test was performed on the distribution transformer to measure these capacitances. The tan-delta test is typically used as an indicator of how dry the insulation paper of the windings is. However, it also yields the above-mentioned capacitances as output. Horak [10] confirms that transformer insulation testing is the best likely source of transformer phase-to-ground and winding-to-winding capacitance. The results of the tan-delta test are shown in Table 3-4. The “% PF Correction” is an indication of the tan-delta output and is used to measure how well the transformer insulation paper has been dried out. In this case, the results indicate that the insulation is not sufficiently dry. However, this is not regulated by the SANS 780 standard at this stage. The more important column is the capacitance, which is yielded directly from the test.

Two important notes should be made regarding how these capacitances are represented in the π -model. Fig. 2-25 connects a capacitance across the primary and secondary winding but does not explicitly say what these capacitances represent. The figure does indicate that capacitance C_{12} is connected between the HV and LV winding. This correlates to C_{HV-LV} in Table 3-4. One might then be tempted to conclude that these capacitances connected across the windings represent C_{HV-GND} and C_{LV-GND} . This is confirmed by [98] and contradicted by [79]. The author believes that C_{HV-GND} and C_{LV-GND} should not be represented by capacitances between the terminals of the windings as seen in Fig. 2-25. Rather, these capacitances across the windings in Fig. 2-25 represent the turn-to-turn and disc-to-disc capacitances. This can be confirmed based on the iron core model seen in [79, Fig. 6], where Martinez and Mork present a capacitor in parallel with the core loss resistance and the magnetization inductance. They [79] state that this capacitance represents the turn-to-turn capacitance. This would then imply that Fig. 2-25 does not take C_{HV-GND} and C_{LV-GND} into account. Geldenhuys and Padayachee [1] indicate how C_{HV-GND} can be represented in the classical T-model in Fig. 3-5. This representation, where the capacitor is connected between the winding and a ground reference, makes more sense than connecting the capacitance between the terminals of the corresponding winding. Fig. 3.5 does not model the C_{LV-GND} capacitance but it can be assumed that it will be represented similarly to C_{HV-GND} .

Table 3-4: The distribution transformer capacitances from the tan-delta test

	% PF Correction	Capacitance (pF)
C_{HV-GND}	2.191	462.2
C_{LV-GND}	1.923	704.8
C_{HV-LV}	0.7334	449.3

The second important note regarding Fig. 2-25 and Fig. 3.5 is how the capacitance between the HV and LV winding is represented. Fig. 2-25 connects it between the two windings. This approach is also employed by [10] and [98]. Fig. 3.5 connects it between the HV winding and ground. The author of this report believes that connecting the capacitance between the two windings is more accurate. Consequently, this is the approach that will be employed for the simulation model.

An example of how the capacitances are connected in the final transformer model is shown in Fig. 3-14 and Fig. 3-15 in section 3.2.5. The reader may note that there are capacitances connected to the top and the bottom terminals of both the *HV* and *LV* windings. This is also consistent with the model seen in Fig. 2-25. This is important for accurate switching simulations, which will become relevant in Chapter 4. The value of the capacitances shown in Table 3-4 is divided by two to ensure that the total capacitance remains constant. This means half of C_{HV-GND} is connected to the top terminal of the transformer and the other half is connected to the bottom terminal. This also holds for C_{HV-LV} and C_{LV-GND} .

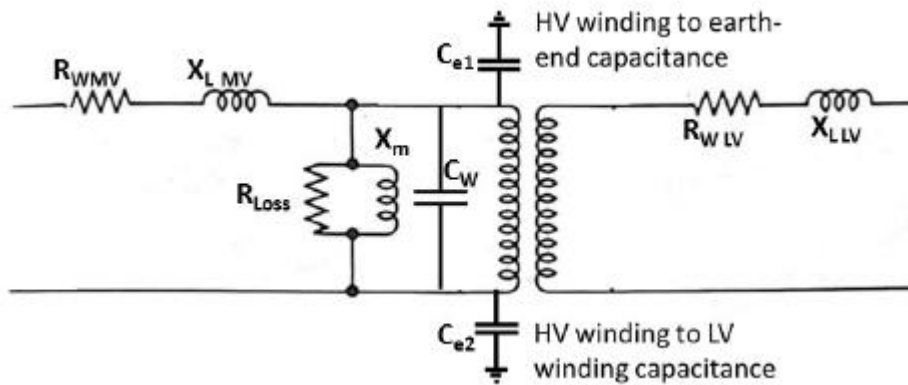


Fig. 3-5: A T-Model with the capacitance representation as seen in [1, Figure 5]

An additional capacitance, C_w , is represented in Fig. 3-5. Geldenhuys and Padayachee [1] describe this as the winding self-capacitance and claim that it should also be taken into consideration. However, this capacitance cannot be measured with the Tan-delta test. Instead, it needs to be calculated based on the geometric data of the transformer, which was not available to the author. A potential solution to this problem is presented in section 0.

3.2.4 SFRA Black Box Model

Before the nonlinear inductor of the core is discussed, it is worth reflecting on a potential model briefly mentioned in Chapter 2. In the discussion of the literature surrounding the *SFRA* test, it was mentioned that Cigre [219] developed a black-box approach to transformer modelling that entailed the parallel connection of series resonant circuits. This configuration poses some interesting prospects for transformer modelling.

Fig. 3-6 shows the example of the black box model in [219]. This shows the first iteration of the black box model – it is improved by adding additional components. However, the red graph shows the amplitude of the impedance as a function of frequency, while the green graph shows the measured *SFRA*. This shows a good fit. The important thing to note, however, is the simplicity of the model. It is a parallel connection of a resistor, a capacitor and an inductor. The inductor is calculated at low frequencies since the phase plot shows that the behaviour is inductive. The peak impedance yields the value of the resistor and the capacitor is calculated at a frequency of 100 kHz, where the phase plot shows that the behaviour is capacitive.

The interesting prospect of this approach is that it can indicate the turn-to-turn and disc-to-disc capacitance of the transformer. Recall that Fig. 2-25 presented a capacitor that is connected across the windings of the transformer. Section 3.2.3 stated that this represents the disc-to-disc capacitance, but is not measurable with the Tan-delta test. More importantly, knowledge of the geometry of the transformer is required to calculate the value of this capacitor. However, the capacitor given by this black box approach can fulfil the same function. At low frequencies, the core of the transformer dominates the *SFRA* behaviour. This means that the inductor and the resistor in Fig. 3-6 can be interpreted as the magnetization inductance and the core loss resistance respectively. Given this assumption, the capacitance then indicates the turn-to-turn and disc-to-disc capacitance which could not be measured in section 3.2.3.

The *SFRA* of the distribution transformer being investigated in this report is shown in Fig. 3-7. The corresponding phase angle is shown in Fig. 3-8. These two figures can be used to calculate a black box core model of the transformer using the approach given by [219].

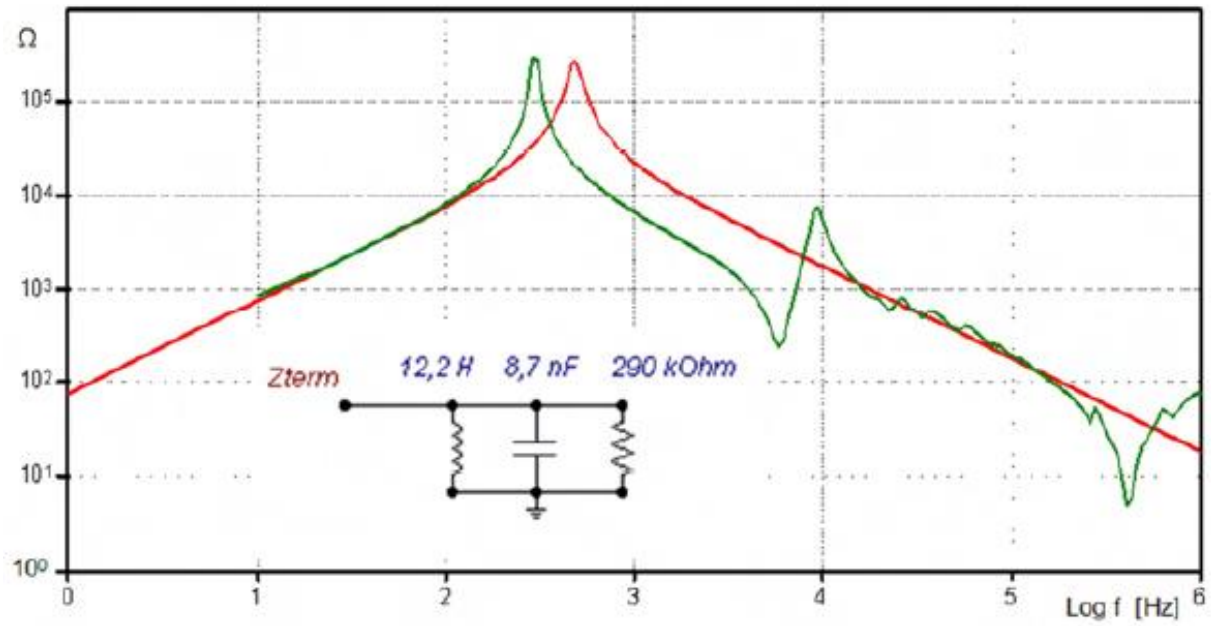


Fig. 3-6: An example of a black box model from the SFRA seen in [219]

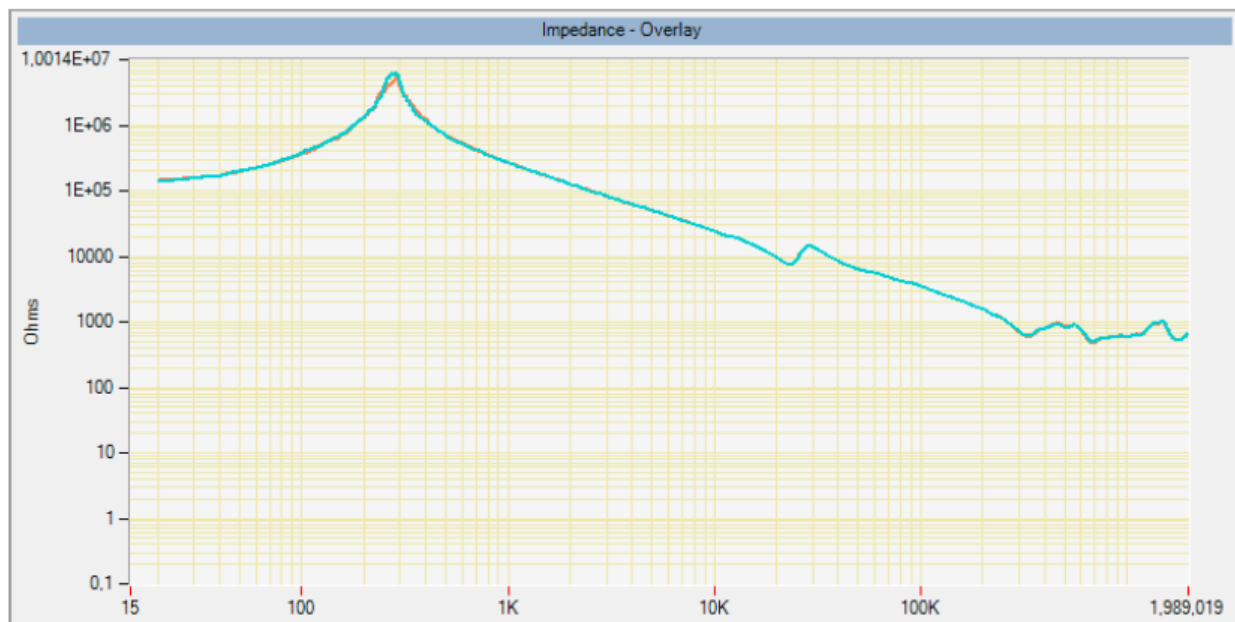


Fig. 3-7: The impedance amplitude of the SFRA done on the transformer in question with the LV winding open

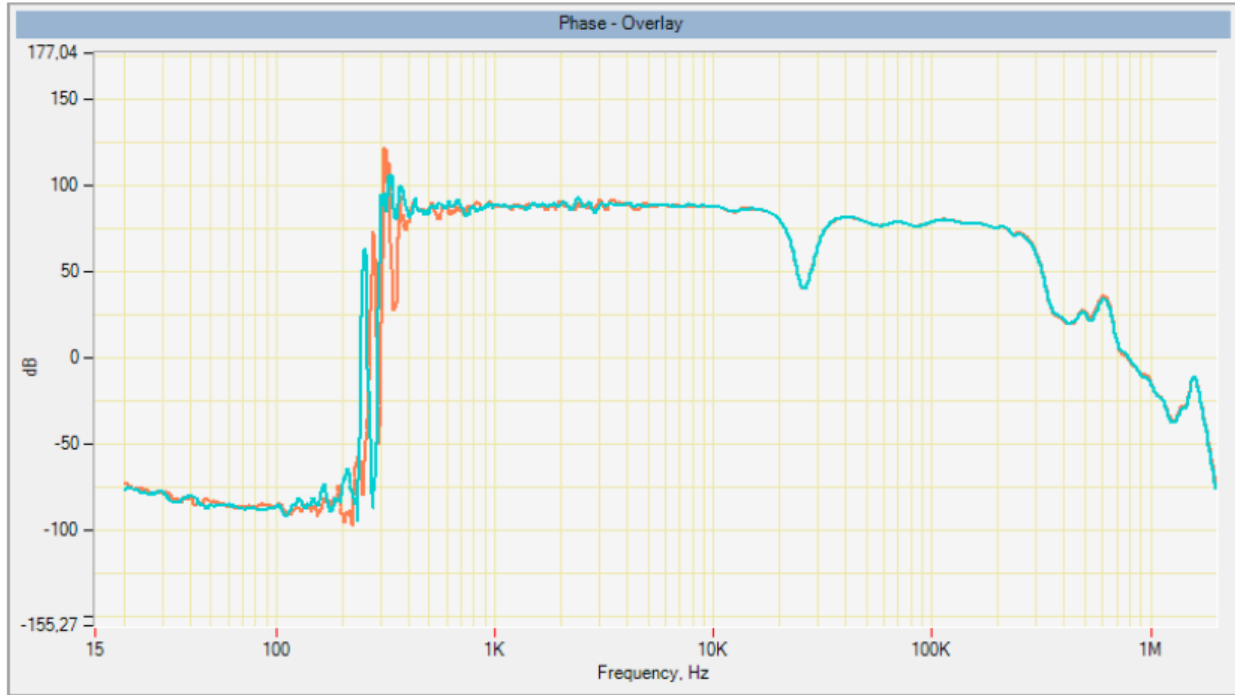


Fig. 3-8: The phase angle of the SFRA done on the transformer in question with the LV winding open

It is clear from Fig. 3-8 that at low frequencies below 100 Hz, the behaviour of the *SFRA* is inductive. Therefore, the inductance of the black-box model is calculated at a frequency of 50 Hz. Note that Fig. 3-7 and Fig. 3-8 were given to the author in PDF format – the data points used to generate these graphs were not available. Consequently, the data points used for the black-box calculations require the user to read the data of a logarithmically scaled figure and are subject to human error. From the graphs, it was determined that the impedance at 50 Hz is given by:

$$\begin{aligned}
 Z &= 2 \times 10^5 \, \Omega \\
 \therefore \omega L &= 2 \times 10^5 \\
 \therefore 2\pi(50)L &= 2 \times 10^5 \\
 \therefore L &= 636.6 \, H
 \end{aligned}$$

The phase plot in Fig. 3-8 shows that the phase is zero at the resonant peak seen in Fig. 3-7. This was also the case in [219]. This is at a frequency of 300 Hz. At this frequency, the impedance is purely resistive and therefore the value of the core loss resistance is calculated from this reading. At 300 Hz, the impedance is given by:

$$\begin{aligned}
 Z &= 6 \times 10^6 \\
 \therefore R &= 6 \times 10^6 \, \Omega
 \end{aligned}$$

The phase angle in Fig. 3-8 has already reached a 90° angle at a frequency of 1 kHz. However, manual transcription of the graph crossings at these frequencies lead to two drastically different answers. It was assumed that attempting to model the capacitance at a higher frequency will lead to better accuracy for a wider frequency range. Therefore, the decision was made to choose a frequency of 10 kHz to model the capacitor. The impedance is then given by:

$$\begin{aligned} Z &= 25000 \, \Omega \\ \therefore \frac{1}{2\pi f C} &= 25000 \\ \therefore C &= 636.6 \, pF \end{aligned}$$

The core loss resistance of the black-box model is relatively close to the core loss resistance calculated from the OC test seen in Table 3-3. The traditional approach to parameter determination yielded a core loss resistance of $5.622 \times 10^6 \, \Omega$. The black box model approach yielded a value of $6 \times 10^6 \, \Omega$. This is a difference of 6.3%. This is very small when one considers that the plot in Fig. 3-7 contains two tests, with overlapping lines which makes interpretation of the graph difficult. Furthermore, the thickness of the lines is significant compared to the logarithmic scale being used on the y-axis. Consequently, the interpretation of the graph is very prone to human error and a 6.4% inaccuracy is very small.

The concern with this black box model should rather be the small value of magnetizing inductance yielded at 50 Hz. The magnetizing inductance of the black-box model is 636 H compared to the 21 204.6 H of the OC test. A possible explanation for this difference is the low voltage value used for the SFRA test. The Doble M5400 Sweep Frequency Response Analyzer used for the SFRA test applies a peak-to-peak voltage of 20 V. This low voltage applied to the HV winding is only 0.03% of the rated voltage. The inductance in Table 3-3 is calculated based on rated voltage. This could have a significant impact on the value of the inductance of the core.

A naming convention is now established whereby the model based on this SFRA black-box approach will be termed the black-box model. The transformer model based on the traditional OC and SC tests will be termed the traditional model.

3.2.5 Nonlinear Core Parameters

In section 2.11 it was mentioned that Chiesa [74] stated that the conventional constant resistor in parallel with the nonlinear inductor is sufficient for transients up to a frequency of 3 kHz. Up to this stage, this report has defined all of the conventional components of the π -model, except for the nonlinear inductor.

The definition of the nonlinear inductor, however, is not as straightforward as the other linear components of the model. Chiesa and Høidalen [134] caution the reader on the potential inaccuracy of OC test results. The accuracy of linear extrapolation of the data points of the OC test become doubtful when the test was only conducted at 110% of rated excitation levels. This is because the transformer has not yet been fully saturated and a linear extension of the curve results in a severe underestimation of the current required for extreme levels of saturation. Chiesa and Høidalen [134] advise that curve fitting is a useful improvement to the nonlinear characteristic of a piecewise nonlinear inductor. To this end, they [134] make use of a modified version of the Frolich equation:

$$\lambda = \frac{i}{a + b \cdot |i| + c \cdot \sqrt{|i|}} \quad (3-2)$$

where

λ = the flux linkage, and
 i = the current.

This modified Frolich equation differs from the equation used by Mork *et al.* [86] and the IEEE Power System Relaying Committee [158]. The author of this thesis decided to use the modified approach presented by Chiesa and Høidalen [134] since it was described in greater detail. However, the necessity thereof will be discussed after the modelling of the nonlinear inductor is complete.

Chiesa and Høidalen [134] do not explicitly define the current being used for the modified Frolich equation. There are three potential interpretations to be investigated. The first potential interpretation is to define the flux linkage and the current relationship based on the measured voltage and current from the OC test without modification. However, the author believes this is inherently faulty since the current measured with the OC test does not only represent the current through the magnetization inductance. Rather, it represents the current through the entire magnetization branch. This includes both the core loss resistor and magnetization inductor. Therefore, if one does not subtract the current which flows through the core loss resistance, the data points defined for the nonlinear inductor will be inaccurate. Therefore, the current measured in the OC test must first be modified to reflect the true current flowing through the nonlinear inductor.

The second potential interpretation relates to the use of *RMS* values as opposed to peak values. [134, Figure 4] indicates that Chiesa and Høidalen relate the flux linkage to the peak current.

However, it does not indicate whether this flux linkage is a peak flux linkage value or an *RMS* value. The relationship between flux linkage and voltage is given by Guru [84, eq. 4.7]:

$$\tilde{E}_1 = \frac{1}{\sqrt{2}} N_1 \omega \phi_m \angle 0^\circ = 4.44 f N_1 \phi_m \angle 0^\circ \quad (3-3)$$

where

- $\tilde{E}_1$ = the RMS voltage applied across winding 1 in V;
- N_1 = the number of turns of winding 1;
- ω = the angular frequency in rad/s;
- ϕ_m = the peak magnetic flux in Wb, and
- f = the frequency in Hz.

Furthermore, the relationship between the flux and the flux linkage is given by $\lambda_m = N \phi_m$, which means that the flux linkage can be given by:

$$\lambda_m \angle 0^\circ = \frac{\tilde{E}_1}{4.44 f} \quad (3-4)$$

This relates the peak flux linkage to the RMS voltage. One must then be careful to ensure that the peak flux linkage is related to the peak current.

The problem with the peak current, however, is that the current of the OC test contains a significant amount of harmonics. Moreover, the harmonic content of this distribution transformer is abnormally high. An example of the current at rated voltage is shown in Fig. 3-9. This abnormally high harmonic content is problematic for hysteresis modelling and will be discussed in greater detail in section 3.3. This high harmonic content implies that it becomes difficult to convert the measured current to a peak value. The N4L oscilloscope presents the user with the *RMS* current. If the waveform was purely sinusoidal, as is the case for the voltage, then the peak current is related to the *RMS* current by a factor of $\sqrt{2}$. In this case, however, such an approach would be inaccurate.

A potential solution to this problem could be to define a relationship between the *RMS* flux linkage and the *RMS* current. This approach can be justified based on the assumption that when a steady-state model with a constant inductor is defined, the *RMS* values for voltage and current are used to define the inductor. However, one should also be aware that, since the inductance is based on the ratio of flux linkage to current, *RMS* values for current and voltage will result in a higher inductance value. This will be particularly prominent in the saturated region where the peaks of the current are high. The equation to calculate the *RMS* flux linkage can be derived from (3-3).

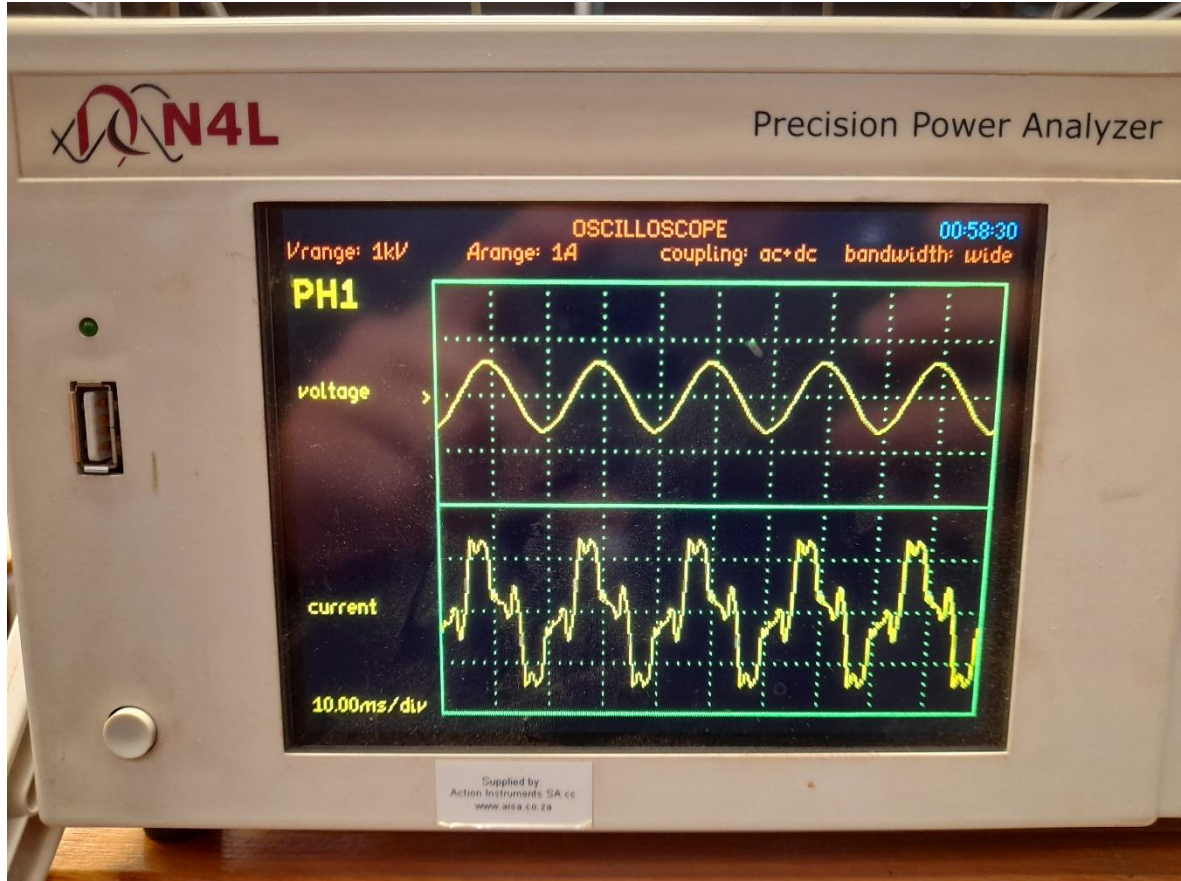


Fig. 3-9: The OC test voltage and current waveforms at rated voltage

The flux is also a sinusoidal waveform, which means that the *RMS* value of ϕ_m is given by $\phi_{RMS} = \frac{\phi_m}{\sqrt{2}}$. This then means that the *RMS* flux linkage is given by:

$$\tilde{E}_1 = \omega \lambda_{RMS} \angle 0^\circ = 2\pi f \lambda_{RMS} \angle 0^\circ \quad (3-5)$$

The main argument against the use of *RMS* flux linkage and current values, however, is based on the internal operation of nonlinear inductors in the MATLAB environment. MATLAB makes use of a controlled current source to model the nonlinear inductor²². The current source is controlled by a one-dimensional lookup table of current values and the corresponding flux linkage, but MATLAB requires the user to define peak voltage for the voltage source. Consequently, when it measures the voltage across the inductor to determine the flux linkage and the current, it measures a peak value and not an *RMS* value. Therefore, if *RMS* data points are defined in the lookup table, MATLAB will search in the wrong interval and the inductance will be drastically lower than it should be for the given voltage. This mistake was realized when a peak current of 1500 A was drawn during the OC test of the simulation model at rated voltage. The measured data from

²² Recall that a nonlinear inductor was also built from first principles by the author of this report and is presented in APPENDIX B.

Table 3-1 shows an *RMS* current of 0.466 A. Fig. 3-9 shows that the peak current is slightly above 1 A.

Following an extensive investigation into the abnormally high current produced by simulation, it was discovered that the problem was the *RMS* definition of the data points. When the data points are defined in terms of *RMS* quantities, the nonlinear inductor tends to saturate a lot earlier because MATLAB interpolates in the wrong interval. Moreover, if the data points are defined in terms of *RMS* quantities, the maximum flux linkage in the lookup table is below the rated peak flux linkage. For instance, rated peak voltage on the primary side is $V_{peak} = 31.11 \text{ kV}$. If this voltage is mistakenly interpreted as an *RMS* voltage rather than a peak voltage, it would correspond to an *RMS* flux linkage of $\lambda = 99.03 \text{ Wb-turns}$, which is beyond the limit of what an accurate Frolich equation approximation could reach. This means that the flux linkage being calculated by MATLAB is outside of the limits of the one-dimensional lookup table. MATLAB then attempts to linearly extrapolate the flux linkage and the current. The resulting inductance has a very small value, which explains the significant current. Therefore, the internal operation of the nonlinear inductor forces the user to define peak quantities for flux linkage and current.

The challenge of defining peak data points is to define the peak values of the current. The high harmonic content means that the $\sqrt{2}$ relation between *RMS* and peak values is no longer valid. Some of the waveforms of the OC test were exported since they are required for hysteresis modelling. However, the OC test was conducted for the following voltage levels as a percentage of rated voltage: 2.5%, 5%, 10%, 20%, 30%, 40%, 50%, 70%, 80%, 85%, 90%, 95%, 100%, 105%, 110%, 115%, 118% and 120%. Exporting all of these waveforms was not justified for hysteresis modelling, since the parameters of the Jiles-Atherton hysteresis model are only calculated for heavily saturated conditions. The Jiles-Atherton parameters do not vary with voltage as the nonlinear inductor data points do. Consequently, waveforms were only exported for the following voltage levels as a percentage of rated voltage: 70%, 80%, 90%, 100%, 110%, 120% and 130%.

Furthermore, recall that the accurate N4L oscilloscope could not interface with the computer used in the laboratory, which means that the waveforms were exported with a different oscilloscope. The oscilloscope used to export these data points has reduced functionality and is not capable of indicating the power factor at a given voltage level. One might argue that one could calculate the power factor from the waveforms themselves but the oscilloscope struggled to trigger the waveforms. Therefore, the author does not have confidence in the phase shift between the waveforms, which means that the waveforms for the 130% data cannot be used. This is because

one needs to know the power factor to subtract the core loss resistance current from the total current.

A potential solution to this problem could be to accept the inaccuracy that the $\sqrt{2}$ factor would introduce to the peak current values. A set of data points giving the peak current based purely on a $\sqrt{2}$ relation should yield better results than a set of data points defined based on *RMS* values. This approach was attempted and section 3.5 will show the results of the OC test conducted using this approach. However, the inaccuracy introduced by this approach is too significant to be ignored. Table 3-5 shows a comparison of the *RMS* and peak currents from these two sources. The *RMS* value of the exported waveform is calculated based on the definition of *RMS* values – all of the exported current data points are squared and the square root of the mean of these data points yield the *RMS* current.

The author would like to point out two important notes regarding Table 3-5. Firstly, the *RMS* values of both the N4L and the less accurate oscilloscope used for exporting the waveforms are very similar. This means that even though the exporting oscilloscope could not trigger the waveforms properly to yield the power factor, the data points can be deemed accurate as the *RMS* current is comparable to that of the N4L oscilloscope, which is a very accurate measuring instrument. This also provides a form of verification of both of these measuring instruments since their results are so similar. Subtle differences can be allocated to slight differences in voltage levels since the measurements were not taken simultaneously. Furthermore, the difference will become more prominent as the voltage level increases since saturation of the transformer plays a significant role.

Table 3-5: A comparison between the RMS and peak values for the exported vs the N4L OC test results

Voltage Level	RMS Current (A)		Peak Current (A)	
	N4L	Exported	N4L* $\sqrt{2}$	Exported
80%	0.332	0.330	0.4698	0.5493
90%	0.378	0.375	0.5340	0.6344
100%	0.423	0.419	0.5976	0.7597
110%	0.467	0.468	0.6598	0.9070
120%	1.622	1.976	2.2931	5.3451

The more prominent feature of Table 3-5, however, is the significant difference in the peak current. At 120% of rated voltage, the difference between the $\sqrt{2}$ relation and the true peak current is 133%. The deviations in peak current can be seen more clearly in Fig. 3-10. It is also interesting to note the impact that the extra data points have on Fig. 3-10. For the N4L data points which are multiplied by $\sqrt{2}$, there are two additional data points at 115% and 118% of rated voltage. These two extra data points provide a much better indication of the shoulder region of the saturation curve. More importantly, these extra data points in the shoulder region determine the parameters of the Frolich equation. The Frolich equation, in turn, determines all data points beyond 120% of the rated voltage. This leads to the important conclusion that not only does the modelling of the Frolich equation require data points up to 120% of rated voltage but the data points should be measured in intervals smaller than 10%.

Another important feature of Fig. 3-10 is that the graphs tend to converge at lower voltage levels. This means that even though the peak current can not be determined from the exported waveforms below 90% of rated voltage, it can be assumed that the $\sqrt{2}$ relation between RMS current and peak current holds for voltage levels below 90%.

Since the data points at 115% and 118% of rated voltage severely impact the shoulder area of the curves in Fig. 3-10 additional data points for the exported peak waveform between 110% and 120% of rated voltage would assist in more accurate Frolich parameters. The y-coordinates of the corresponding data points are known – they are equal to the peak voltage data points

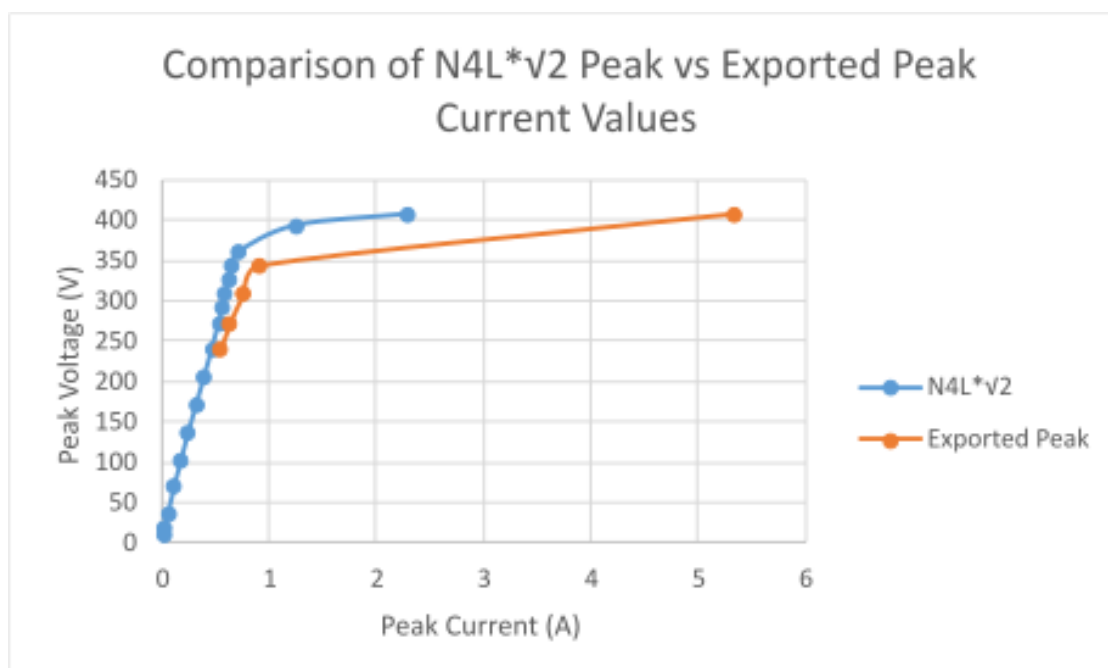


Fig. 3-10: A comparison of the $N4L \cdot \sqrt{2}$ and exported peak currents

measured by the N4L oscilloscope. Fictitious data points can be defined such that the overall shape of the shoulder area remains the same. This is done by calculating the difference in current between 110% and 120% of rated voltage for both waveforms. In the case of the N4L data points, this difference is 1.744 A. In the case of the exported peak data points, this difference is 4.472 A. The peak current at 115% of rated voltage is then given by:

$$I_{exported,115\%} = \frac{I_{N4L,115\%} - I_{N4L,110\%}}{I_{N4L,120\%} - I_{N4L,110\%}} \cdot (I_{exported,120\%} - I_{exported,110\%}) + I_{exported,110\%}$$

$$\therefore I_{exported,115\%} = \frac{0.467 - 0.426}{1.744} \cdot 4.472 + 0.585$$

$$\therefore I_{exported,115\%} = 0.690 [A]$$

A similar process can be followed for the other missing data points. The resulting graph of inductor current with the fictitious data points is shown in Fig. 3-11. This figure shows that the shoulder area of the saturation curve maintains its shape.

Following this elaborate discussion of the nature of the data points, the determination of the parameters of the Frolich equation can now be discussed. This is the final step of the modelling of the nonlinear inductor.

The equation of the modified Frolich equation was presented in (3-2). Equation (3-2) can also be used to relate voltage to current rather than flux linkage to current. This equation requires the optimisation of three parameters. A Particle Swarm Optimisation (PSO) has been written to optimise the Jiles-Atherton parameters. However, the *PSO* algorithm can also be used to optimize the three Frolich equation parameters using an *MSE* cost function. This yields the optimal parameters for the Frolich equation within a few hundred iterations.

The problem with the *MSE* cost function, however, is that the data points at low voltage levels can cause problems for the *PSO* algorithm. If all of the data points are taken into account, from 5% of rated voltage to 120% of rated voltage, the *PSO* algorithm yields parameters of $a=0.0007271$, $b=0.002169$ and $c=1e-09$. This yields the curve seen in Fig. 3-12. It can be seen that although the relationship between current and voltage is well-represented at low voltage levels, the shoulder region of the curve is poorly represented. Furthermore, the gradient of the curve at high voltage levels is higher in the Frolich approximation than for the actual data. This means that the Frolich approximation will overestimate the value of the magnetizing inductance. This defeats the purpose of the Frolich approximation since the Frolich equation is used to extend the saturated data of the transformer – not replicate the linear data.

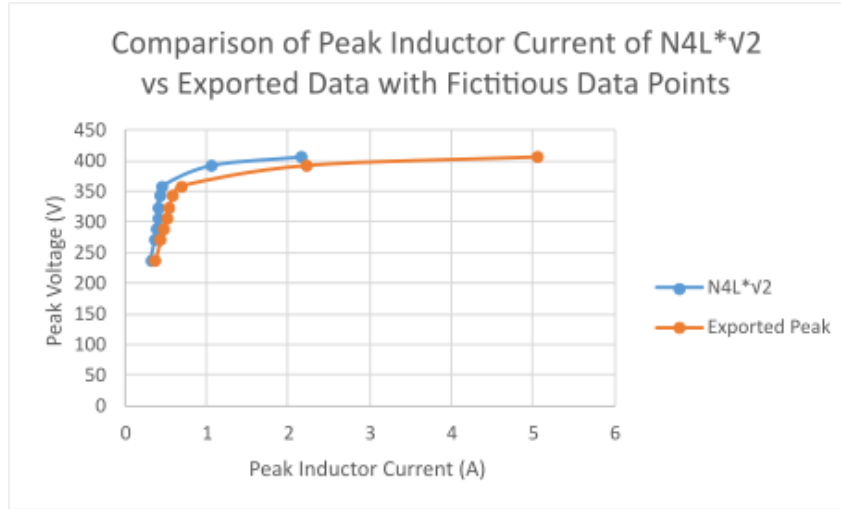


Fig. 3-11: The peak inductor current with fictitious data points to improve the shoulder region

The solution to this problem is to limit the data points such that the linear region of the curve does not skew the *MSE* cost function. If only the last three data points in Fig. 3-11 are used for the *PSO* optimisation algorithm, the best solution is found to be: $a=6.889e-05$, $b=0.002307$ and $c=3.12179e-04$. Fig. 3-13 shows how this approximation relates to the actual data. In this case, it can be seen that the Frolich approximation is not a good representation of the data points at low voltage values. However, the shoulder region of the curve shows an almost perfect correlation to the actual data. This is considered to be a better set of parameters since the Frolich equation can now be used to extend the voltage and current data points beyond 120% of rated voltage with good accuracy. Furthermore, the actual data points for low voltage values are available to the user. The best solution is, therefore, obtained by keeping the actual data points up to a voltage

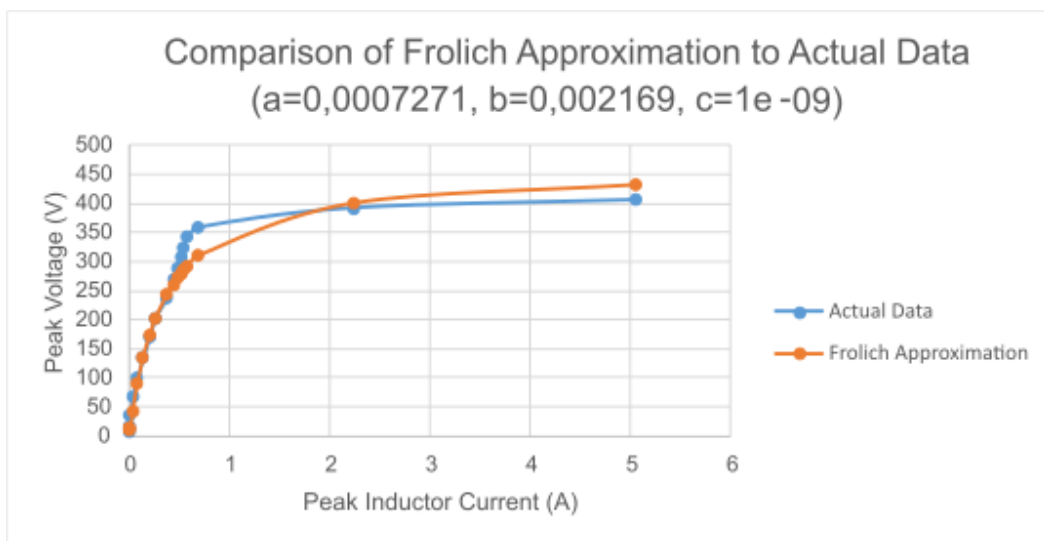


Fig. 3-12: The frolich approximation of the actual data when all data points are considered for the *PSO* algorithm

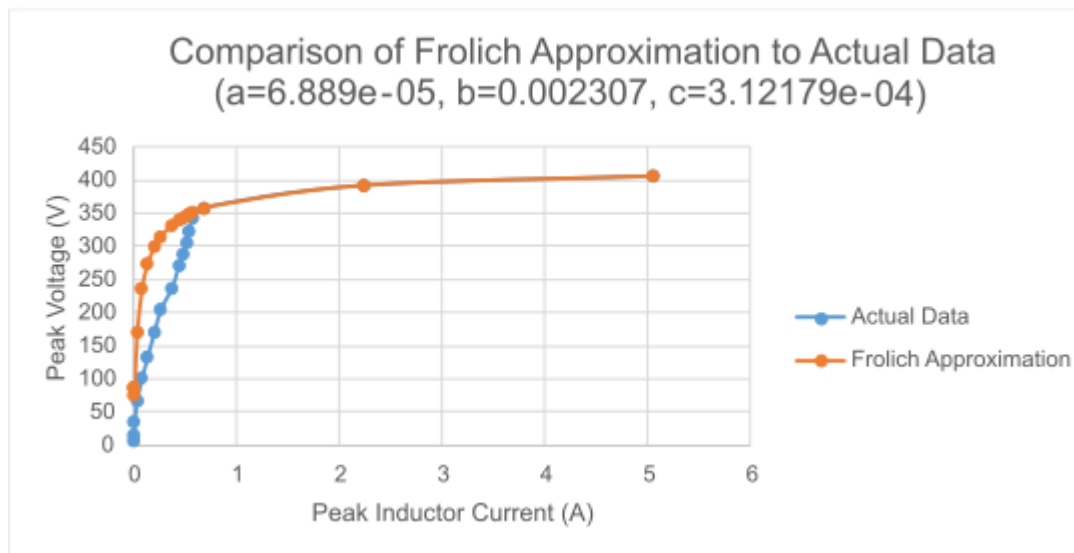


Fig. 3-13: The frolich approximation of the actual data when only saturated data points are considered for the PSO algorithm

level of 120% and then using these Frolich parameters to extend the data beyond 120% of the rated voltage.

The modelling of the nonlinear inductor of the transformer core is not a straightforward process and can be summarized as follows: firstly, the current flowing through the core loss resistor needs to be deducted from the measured current. The current needs to be converted to a peak value. The high harmonic content of the current means that the *RMS* current cannot just be multiplied by $\sqrt{2}$ to yield the peak value. Rather, the current waveform needs to be exported and the true peak value of the current needs to be determined from the waveform itself. The measurements should be done up to a voltage level of at least 120% of the rated voltage and measurements should be taken in small increments. Once this has been done, the parameters of the Frolich equation can be calculated using an optimisation algorithm like the *PSO* algorithm. However, taking all of the data points into consideration yields a poor representation of the shoulder area of the curve. Therefore, only the data points in the saturated region of the curve should be considered for the Frolich parameters. Thereafter, the actual data points should still be used to define the nonlinear inductor and the Frolich approximation should only be used to extrapolate the data points beyond 120% of the rated voltage.

If the black-box approach is used to model the transformer, the calculation of the inductor current becomes even more complex. This is because the magnetisation branch of the transformer now consists of three parts: the core loss resistor, the magnetisation inductance and the disc-to-disc parallel capacitance. The inductor current is then given by the difference between the total current and the current flowing through both the resistor and the capacitor.

One final note should be made regarding the accuracy of the modified Frolich equation as opposed to the traditional Frolich equation presented by [86]. It is the opinion of the author that the added complexity of the modified Frolich equation is not justified. When the Frolich parameters are calculated using the *PSO* algorithm, the user needs to define upper and lower limits for the three parameters. Whenever the algorithm was run to calculate the Frolich parameters, one of the three parameters would almost always get stuck at the lower limit of $1\text{e-}09$. When one of the parameters has such a small value, it no longer influences the equation. In all cases, either the a parameter or the c parameter would converge to this minimum bound. Furthermore, the cost function was always better when c converged to $1\text{e-}09$ than when a converged. The traditional Frolich equation does not have a c parameter. Therefore, it can be concluded that the original Frolich equation would be capable of yielding the same results. In very rare instances, such as the case represented by Fig. 3-13, the *PSO* algorithm will calculate a value for all three parameters which is within the bounds of the algorithm. However, the difference in accuracy is very small.

Frolich approximations were calculated for four transformer models: a) the traditional π -model without the parallel capacitance where the peak current values were calculated by multiplying by $\sqrt{2}$. b) The black box π -model with the parallel capacitance where the peak current values were calculated by multiplying by $\sqrt{2}$. c) The traditional π -model without the parallel capacitance where the peak currents were extracted from the exported waveforms. d) The black box π -model with the parallel capacitance where the peak currents were extracted from the exported waveforms. The flux linkage/current relationships for these four different models are presented in APPENDIX C. The final circuit diagram for model 1 and model 3 is shown in Fig. 3-14. The final circuit diagram for model 2 and model 4 is shown in Fig. 3-15. Table 3-6 summarizes the parameters of the four transformer models.

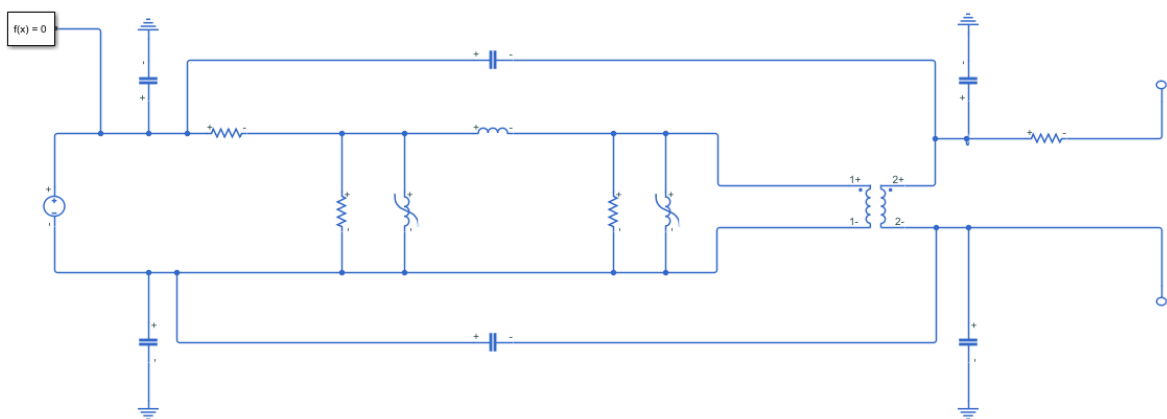


Fig. 3-14: The final circuit diagram for Models 1 and 3

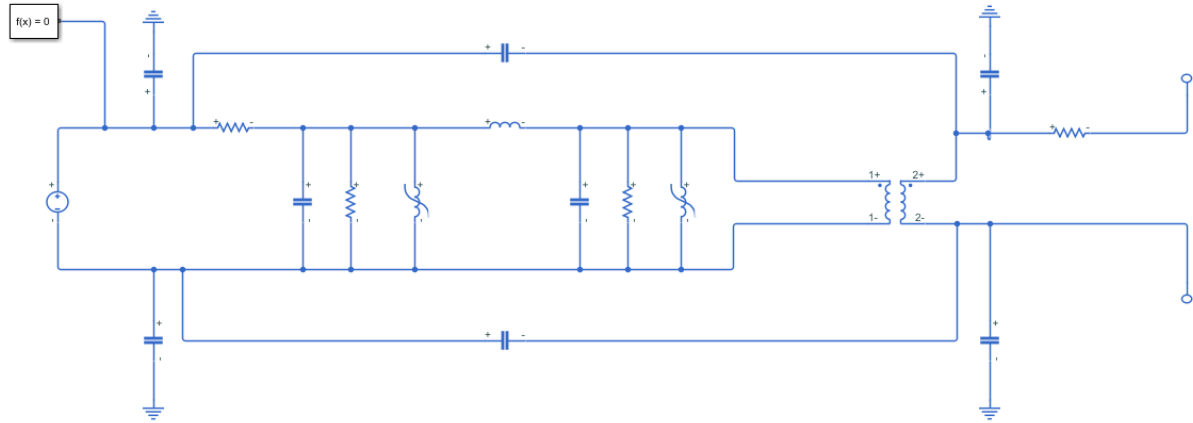


Fig. 3-15: The final circuit diagram for Models 2 and 4

Table 3-6: A summary of the model parameters for the four transformer models

	$R_{HV} (\Omega)$	$R_{Core} (M\Omega)$	$C_{turn-to-turn} (pF)$	$L_{Leakage} (H)$	$R_{LV} (\Omega)$
Models 1 and 3	111.304	11.24	N/A	1.865	0.035
Models 2 and 4	111.304	12	318.3	1.865	0.035

The next step in the modelling process is to add a hysteresis model to the transformer. This is the topic of the next section.

3.3 Hysteresis Model

From the literature study of this report, it should be apparent that there are contradictory views on the importance of a hysteresis model. Some sources claim that piecewise linear models are sufficient for ferroresonance modelling. Other sources claim that ferroresonance modelling requires a detailed hysteresis model capable of retaining remnant flux upon de-energization. There are also contradictory claims regarding which hysteresis models are sufficient. Some authors claim that the Jiles-Atherton model is used most often. Other authors claim that it cannot accurately model minor loops, which are incredibly important.

Nevertheless, despite the various contradictions in the literature regarding the hysteresis models in existence, it is widely recognized that the Jiles-Atherton and the Preisach hysteresis models are most commonly used. They are also considered the most accurate. Between these two models, the literature discussed in section 2.10 seems to indicate that the Preisach model is superior. However, it is also more complex. From Table 2-13 it can be seen that all of the JA parameters are measurable. This is not the case for the Preisach model. The JA model is also

more computationally efficient, as is evident from both Table 2-13 and Table 2-14. Furthermore, the statistical nature of the Preisach model makes its implementation difficult, where the *JA* model only requires solving a differential equation. Hui and Zhu [160] attempted to present the equations of the Preisach model more understandably. However, even though their discussion allows for easy numerical implantation, it was found by the author of this report that their equations present logical errors and will not be sufficient for hysteresis modelling. A more elaborate discussion in this regard is presented in Appendix A.

Based on this numerical advantage of the *JA* model, as well as the various claims in the literature regarding its accuracy and applicability for ferroresonance modelling, the *JA* model will be used to model hysteresis in this report. One of the most important properties influencing this selection is the ability of the *JA* model to retain residual flux. Its computational efficiency and easy parameter determination are added advantages.

Since some sources in the literature also claim that advanced hysteresis models are unnecessary for ferroresonance simulation, a simplified hysteresis model will also be developed. To this end, the Tellinen model presents very simple equations for hysteresis modelling. The model has also been used to model ferroresonance in [33]. The author of this report hopes that using both these models will yield insight into whether an advanced hysteresis model is truly necessary for ferroresonance simulation.

3.3.1 Jiles-Atherton Model

Section 2.10.19 made it clear that several permutations to the original *JA* model have been developed. However, unless the permutations are expressly compared to each other and to experimental data within the article itself, it is difficult to evaluate which equation shows the best accuracy. Furthermore, it is beyond the scope of this study to implement all of these solutions and evaluate them.

A different criterion, therefore, needs to be used to determine which equation will be used for the hysteresis model in this simulation. Parameter determination is a good option in this regard. Leite *et al.* [180] present an algorithm to optimise the *JA* model parameters based on an *MSE* approach. This algorithm eliminates the need for complex measurements as initially pointed out by Jiles, Thoeleke and Devine [173]. It also holds the added advantage of eliminating the need for a scaling factor for improved minor loop simulation. Furthermore, the differential equations employed by Leite *et al.* [180] take the form $\frac{dM}{dB}$. This means that the input parameter is the magnetic flux density (*B*). As mentioned in Section 2.10.19, this means that the input is naturally filtered by integration. It is also the experience of the author that it is easier to extract the voltage across the windings of

the transformer in simulation than it is to extract the current through the transformer. For the sake of clarity, *JA DEs* expressed in terms of $\frac{dM}{dH}$ will be referred to as the forward model in this report since the original *DE* was expressed in this form. *DEs* of the form $\frac{dM}{dB}$ will be referred to as the inverse *JA* model according to the naming convention established by [179]. The equations employed by Leite *et al.* [180] were initially derived by Sadowski *et al.* [179]. These equations were presented in Section 2.10.19 in (2-88) through (2-92).

A fourth-order *RK* algorithm is used to solve the *DE* seen in (2-88). A MATLAB script showing the implementation is presented in APPENDIX D. However, the article written by Sadowski *et al.* [179] does not use the *RK* algorithm. Rather, it employs the algorithm seen in [179, Fig. 1]. A MATLAB script using this algorithm is also presented in APPENDIX D. In this regard, the author would like to recognize the contributions and assistance of Professor Jean Leite [220], who assisted in fixing a few numerical errors with regards to the effective field H_e . It should be noted that the algorithm presented by [179] and the *RK* algorithm does not yield identical results. An in-depth analysis of the differences for various parameter sets was not conducted, but the difference did appear to be small for the set of parameters that were considered while developing the MATLAB scripts.

The reader may recall that Annakkage *et al.* [81], [168] presented an alternative equation for the anhysteretic magnetization, which involved two polynomials. They [168] claim that this polynomial approach holds several advantages over the original Langevin function. However, following several attempts, it was found that the algorithm presented in APPENDIX D failed whenever such an approach was used. This includes attempts using the coefficients presented in [81]. The *DE* employed by Annakkage *et al.* [81], [168] is expressed in terms of $\frac{dM}{dH}$ and they do not employ a *RK* algorithm to solve the *DE*. Nevertheless, one would expect that if it was established that the *RK* algorithm could solve the *JA DE*, then a change from the Langevin function to a polynomial function would only improve the accuracy. This was not the case. For this reason, the Langevin function is used for the anhysteretic magnetization in this model. The Langevin function was presented in (2-76).

Another strong argument for the use of the *JA* model can be made based on [221], where Jayasinghe and McLaren prove that conventional CT models cannot retain residual flux upon de-energization. The *JA* model can. Chiesa, Mork and Høidalen [185] confirm this ability of the *JA* model. The importance of residual magnetism was emphasised in Chapter 2 since it can affect the saturation of the transformer.

Section 2.10.19 discussed methods to determine the parameters of the *JA* model. Jiles and Thoeke [173] presented a range of tests that can be performed to determine the parameters.

However, this approach will not be employed since the tests are not straightforward and the equations contain a logic error, as proven in section 2.10.19. Moreover, Chwastek and Szczyglowski [183] list five problems with this approach to JA parameter determination. The literature also discusses the need for a scaling factor since the original model does not always close minor loops.

However, the parameter optimisation algorithm presented by Leite *et al.* [180] eliminates the need for the scaling factor. It also eliminates the need for complex measurements. It only requires data points on the hysteresis loop, which can be measured with relative ease. Given its numerous advantages and easy measurements, an attempt was made to employ this algorithm for JA parameter determination in this study. This algorithm makes use of several function files and is therefore not attached in an appendix.

3.3.2 Tellinen Model

The Tellinen model is one of the most elementary models discussed in section 2.10. Its equations are easy to implement in a simulation model and the model has also been used to model ferroresonance.

Section 2.10.14 mentioned the use of a nonlinear function to define the ascending and descending legs of the hysteresis loop. The equation used by [33], given in (2-56), is a single-valued function. The ascending and descending legs are then obtained by shifting this single-valued function to the left and the right. To this end, it can also be argued that the Langevin function is a good alternative for the single-valued function since the JA model so heavily relies on it to model the anhysteretic magnetization.

The Langevin function used by the JA model is shown in (2-76). However, one should also note that the original JA model makes use of magnetization (M) rather than magnetic flux density (B). Therefore, the saturation magnetization (M_{sat}) in (2-76) should be replaced with saturation magnetic flux density (B_{sat}). Furthermore, the Langevin function employed by the original JA model makes use of the concept of an effective magnetic field (H_e), which requires the definition of a mean-field parameter (α). A simpler approach entails only using the magnetic field (H) itself. Taking these modifications into account, a modified Langevin function for the Tellinen model is shown in (3-6).

$$B = B_{sat} * \left(\coth\left(\frac{H}{a}\right) - \left(\frac{a}{H}\right) \right) \quad (3-6)$$

The anhysteretic shape parameter (a) is still evident in (3-6). This means that a Tellinen model employing the Langevin function has three parameters, which need to be defined: B_{sat} , a and σ . Refer to section 2.10.14 for a discussion of σ . A MATLAB script showing the implementation of such an approach is shown in APPENDIX E. Since the equations of the Tellinen model are also in the form of a DE as seen in (2-52) and (2-53), the RK algorithm is used to solve the equation.

The inverse Tellinen model was presented in (2-53). As was the case with the JA model, the convention will be used whereby the term “inverse model” refers to a model which uses magnetic field intensity (H) as input and yields flux density (B) as output. Equation. (2-53) can be rewritten to

$$\begin{aligned} \frac{dh}{db} &= \frac{1}{\mu_0 + \frac{b_{h-}(h) - b}{b_{h-}(h) - b_{h+}(h)} \left(\frac{db_{h+}}{dh}(h) - \mu_0 \right)}, \frac{db}{dt} > 0 \\ \frac{dh}{db} &= \frac{1}{\mu_0 + \frac{b - b_{h+}(h)}{b_{h-}(h) - b_{h+}(h)} \left(\frac{db_{h-}}{dh}(h) - \mu_0 \right)}, \frac{db}{dt} < 0 \end{aligned} \quad (3-7)$$

A MATLAB script showing the implementation of the inverse Tellinen model is given in APPENDIX E.

An alternative approach to the Tellinen model involves the use of a lookup table. The nature of equations (2-52) and (3-7) allows one to evaluate $\frac{db}{dh}$ or $\frac{dh}{db}$ given that the ascending and descending legs of the hysteresis loops are known. This means that if the data points of the major hysteresis loop are known, then the differential can be calculated with ease. This also eliminates the need for a Langevin function to describe the relationship between B and H . Furthermore, when the Langevin function is no longer needed, then there also isn't a need to determine parameters B_{sat} , α and σ . A MATLAB script showing the implementation of an inverse Tellinen model with a lookup table is shown in APPENDIX E.

3.4 Hysteretic Transformer Model

3.4.1 Literature Verification

This report has thoroughly discussed the importance of hysteresis. Section 2.10 presented a detailed breakdown of the theory of hysteresis and the various models available for modelling this phenomenon. However, the discussions have mostly been confined to the magnetic domain and the effect it has on parameters such as flux density, magnetization and magnetic field intensity. Manitoba Hydro International [222] present a very good description of the effect that core

saturation has on the electrical quantities of the power system. They [222] claim that transformers are limited by the magnetic flux limitations of the core. Excessive overvoltage on the primary winding may cause the core to saturate during peak moments of the sinusoidal cycle. In such a case, the voltage induced on the secondary will no longer match the waveform of the primary coil. The overloaded transformer will distort the waveform between the primary and secondary coil, which leads to harmonics on the secondary winding's output.

The JA and Tellinen models described in section 3.1 provide a method to calculate the magnetic flux density or the magnetic field intensity. However, these magnetic quantities need to be converted to electric quantities if they are to be used in electrical simulation models. The magnetic flux density is related to magnetic flux by

$$B = \frac{\Phi}{A} \quad (3-8)$$

where Φ is the magnetic flux and A is the cross-sectional area through which the flux moves. The flux, in turn, is related to the voltage across the transformer winding by Faraday's Law:

$$\begin{aligned} v &= N \frac{d\Phi}{dt} \\ \therefore \Phi &= \frac{1}{N} \int v dt \end{aligned} \quad (3-9)$$

where N represents the number of turns of the transformer winding and v represents the instantaneous voltage across the transformer winding.

The magnetic field intensity (H) is related to the current by

$$H = \frac{NI}{l} \quad (3-10)$$

where I represents the current through the winding and l represents the magnetic path length travelled by the flux.

From equations (3-8), (3-9) and (3-10) it can be seen that if the number of turns of the winding, the magnetic path length and the cross-sectional area of the core is known, then the output of the hysteresis models can be converted to electrical quantities. It should be noted, however, that these quantities are rarely known unless the design data of the transformer can be obtained. This complicates the modelling process.

Abdull Halim, Dao, Phung and Fletcher [80] present an approach to hysteresis modelling, which incorporates the equations presented above. The first step involves measuring the branch

voltage. The branch voltage is then integrated to yield the flux linkage (λ). The flux linkage is divided by the cross-sectional area (A) and the number of turns (N) to yield the magnetic flux density. The JA model is used to calculate the magnetic field intensity, which is then multiplied by the path length and divided by the number of turns to yield the magnetization current.

Chandrasena *et al.* [81] make use of a controlled current source to inject current across one of the windings of the transformer. In this manner, the effect of hysteresis can be taken into account. The PSCAD simulation program developed by Manitoba Hydro International also uses this approach to model hysteresis [222]. They [222] point out that the controlled current source should be attached to the winding closest to the core. This is not expressly pointed out by Chandrasena *et al.* [81]. They [81] attach a controlled current source to both windings of the transformer without explaining. This can be seen in [81, Fig. 5]. Based on the discussion provided by Manitoba Hydro International [222], it is the opinion of the author that attaching two controlled current sources to the transformer model is incorrect. A circuit diagram showing a controlled current source across the primary winding of a transformer is shown in Fig. 3-16. This is similar to the circuit shown in [81, Fig. 5].

Attaching a controlled current source to the transformer allows for a controlled current injection across the winding based on the voltage measured across the winding. Halim, Dao and Phung [80] and the PSCAD simulation program [222] apply such a method in conjunction with the forward JA model. However, because the integration of the voltage across the winding yields the magnetic flux (B) and the forward model requires magnetic field intensity (H) as input, Manitoba Hydro International [222] states that the JA model equations are solved iteratively to obtain H and M for each time step.

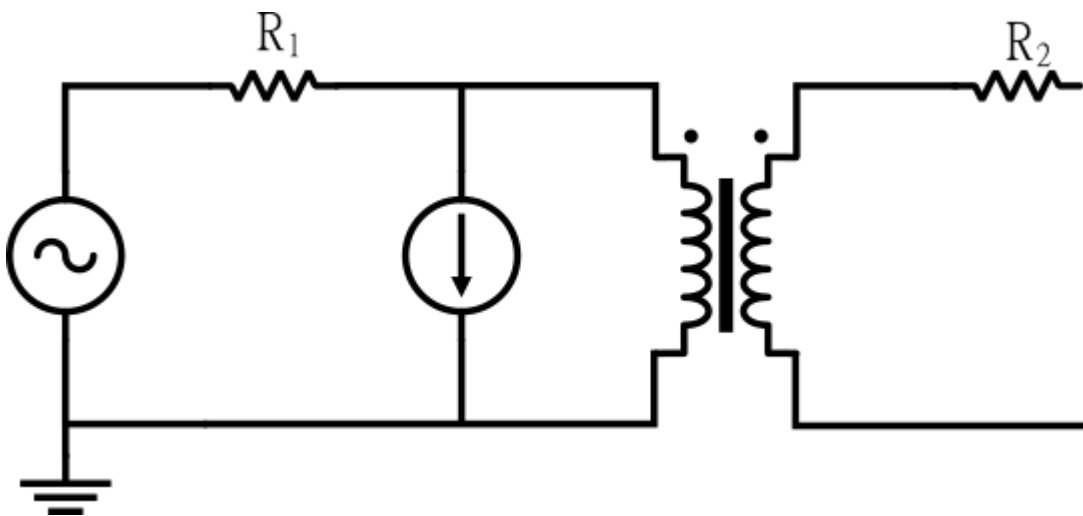


Fig. 3-16: A circuit diagram showing a controlled current source for modelling hysteresis

Chandrasena *et al.* [81] also use the forward *JA* model in conjunction with a controlled current source. However, Chandrasena *et al.* [81] make use of the polynomial approach to anhysteretic magnetization discussed in section and 3.3.1. Recall that it is the experience of the author that this approach tends to fail with both the *RK* algorithm and the algorithm presented by Sadowski *et al.* [179]. Since Chandrasena *et al.* [81] make use of the forward model, they also make use of an iterative algorithm to solve for magnetization (*M*). A breakdown of the equations they use in their algorithm is presented in [81].

Chandrasena *et al.* [81] elaborate on one of the advantages of the controlled current source – namely the ability to add the effects of eddy currents and anomalous losses. Equation (2-105) presented such an extension to the *JA* model. Chandrasena *et al.* [81] state that this can be used to modify the magnetic field intensity (*H*) to take eddy current losses and anomalous losses into account. Alternatively, a constant resistor can be used to represent the eddy currents [81].

The advantages of this controlled current source, as well as its recognition by programs such as PSCAD, makes it a very strong contender for the simulation of hysteresis in the model employed in this study. However, the inverse hysteresis models (inverse *JA* and inverse Tellinen) will be used in conjunction with the controlled current source. This eliminates the need for an iterative process since the voltage across the winding can be used as direct input to the model. Fig. 3-17 shows the Simulink subsystem used to implement this controlled current source into a transformer model. The voltage across the transformer winding is used as input. This input is converted to magnetic flux density using the process described above. The inverse *JA* algorithm is implemented using a MATLAB function block. The output of the function block yields the magnetic field intensity (*H*), which is then converted back to current. This current is the output of the subsystem used to control the current source.

A similar subsystem has also been developed for the inverse Tellinen model. However, the inputs to the inverse Tellinen subsystem are not constants as was the case for the inverse *JA* model. Rather, the inputs to the subsystem are the ascending and descending legs of measured B-H curves since the Tellinen model makes use of interpolation between known data points. The MATLAB function block performs the interpolation and yields the magnetic field intensity (*H*), which is converted back to current as output. The experimental data used for the interpolation is stored in an excel file. The excel file is opened and its data is assigned to variables using PreLoad Callback functions defined in the Model Properties of the Simulink model. The inverse Tellinen subsystem as a whole is shown in Fig. 3-18.

The Tellinen or *JA* hysteretic subsystem is incorporated into a transformer model as seen in Fig. 3-19. The Specialized Power Systems Library of Simscape is used to develop this transformer

model since the model is then easily interfaced with power system components such as transmission lines.

The reader should note a few things regarding this transformer model. Firstly, the model is developed in an attempt to replicate the waveforms seen in [81]. This will serve as verification of the accuracy and applicability of the hysteretic transformer model. Once the model presented by [81] has been verified, it can be modified to reflect the behaviour of the distribution transformer being investigated in this report. Since [81, Fig. 5] does not have a secondary winding resistance, this model also doesn't have a secondary winding resistance. Furthermore, the secondary winding of [81, Fig. 5] is left open, which explains why the circuit in Fig. 3-19 is also open-circuit. The elements connected to the secondary winding are current and voltage measurement probes, whose outputs are connected to scopes to view the waveforms of the circuit. It is also important to note that the initial phase of the voltage source is 90° . If the phase is not set to 90° , the integration functionality of the hysteresis subsystem will cause an offset on the calculation of the flux, which will immediately drive the transformer into saturation.

To replicate the results of [81], one needs information on the design of the transformer. More specifically, it is necessary to know the number of turns, the cross-sectional area of the core, the magnetic path length and the resistor used to model eddy currents. The lack of this data is often the challenge in replicating the results found in literature and [81] is no exception. The area of the core and the magnetic path length is given. However, the number of turns is not given. Furthermore, the units of the area and the magnetic length are also not given. [81, Appendix A] states that the maximum magnetic flux density is given as $B_{\max}=1.6$ T. This begs the question – does $B_{\max}$ refer to the maximum allowable flux density in the core or does it represent the peak magnetic flux density when rated voltage is applied to the transformer? Furthermore, if it refers to the peak magnetic flux density at rated voltage, is the peak voltage applied to the transformer given by $V_{peak} = 115$ V or $V_{peak} = 115 \cdot \sqrt{2}$ V? These uncertainties make replicating the data presented in [81] particularly difficult.

Following a test of a couple of permutations, it can be assumed that the parameters have the units presented in Table 3-7. Furthermore, it must then be assumed that the peak voltage applied to the transformer in [81, Fig. 5] is $V_{peak} = 115$ V. The reader may also note that a magnetic path length of 0.0717 m is not sensible, since this would imply that the total path length is only 7 cm.

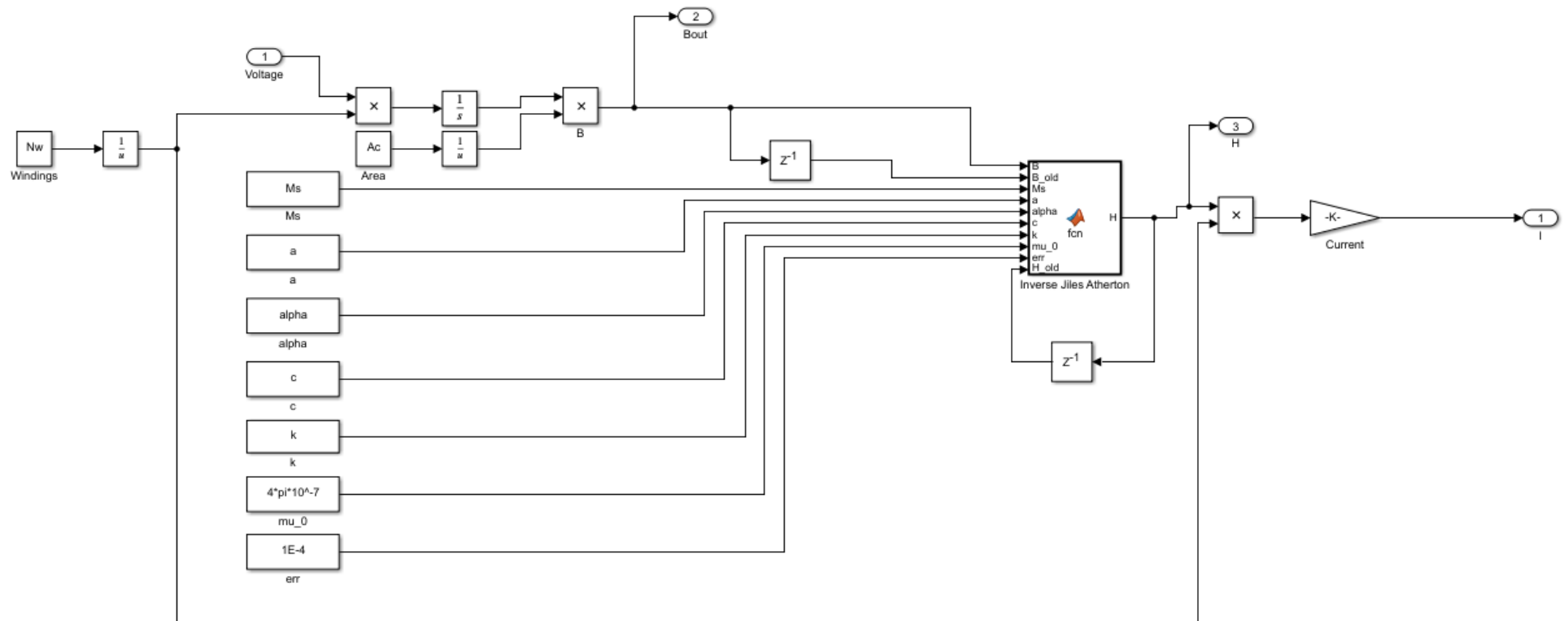


Fig. 3-17: The Simulink subsystem for a controlled current source using the JA hysteresis model

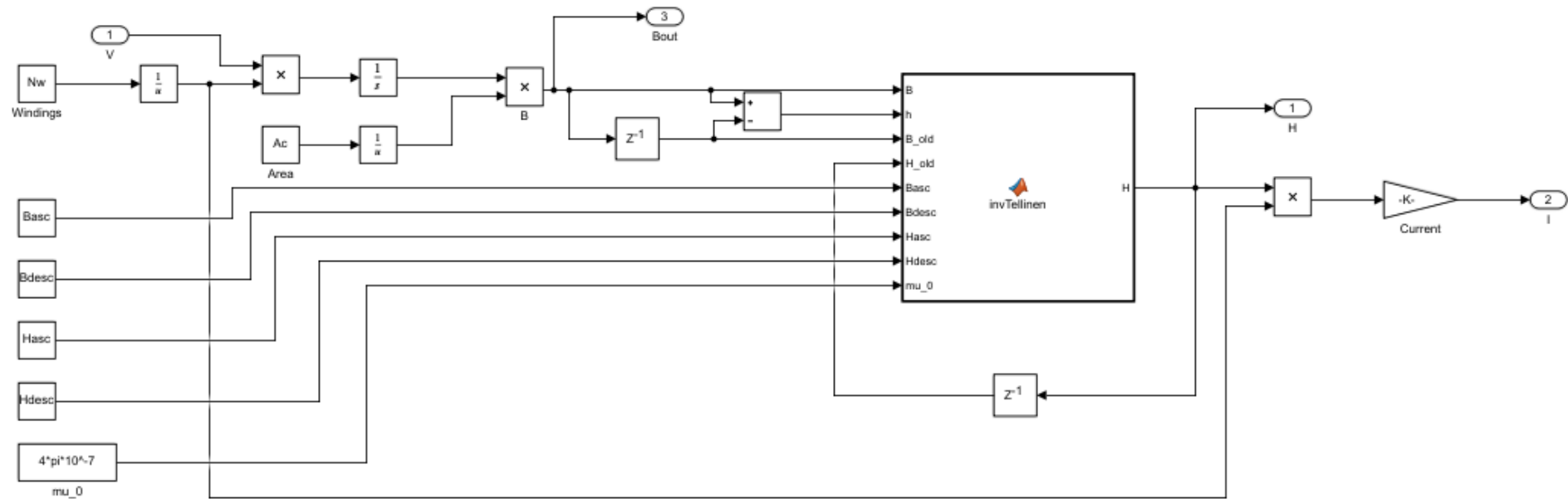


Fig. 3-18: The Simulink subsystem for a controlled current source using the Tellinen hysteresis model

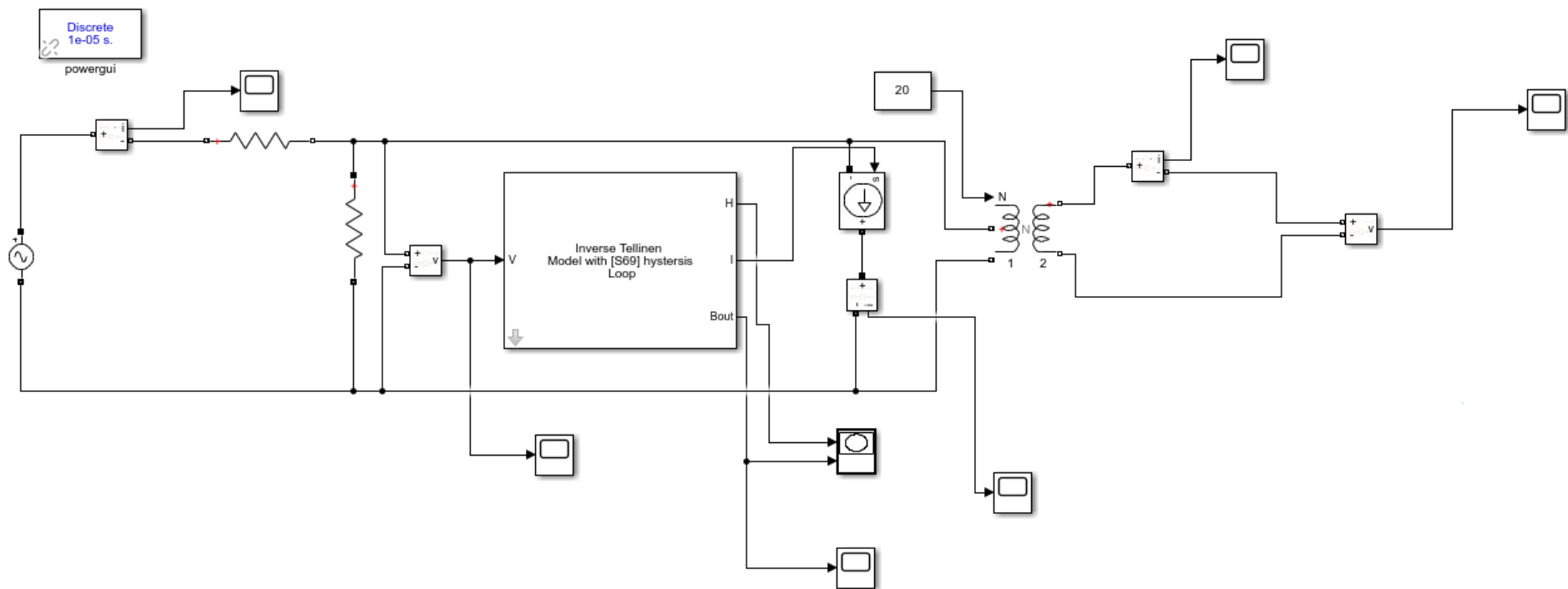


Fig. 3-19: A transformer model with the inverse JA hysteresis subsystem to replicate the results in [81]

Table 3-7: The transformer constants for the replication of the hysteresis results of [81]

B_{max}	1.65 T
Magnetic path length	0.0717 m
Cross sectional area of core	0.0227 m ²
R_{eddy}	548 Ω

The area of the core is chosen as $A=0.0227 \text{ m}^2$ from [81]. Therefore, the maximum flux of the core is:

$$\begin{aligned}\phi_{max} &= B_{max}A \\ \therefore \phi &= 1.6 \cdot 0.0227 \\ \therefore \phi &= 0.0363 \text{ Wb}\end{aligned}$$

Furthermore, from (3-3) it is known that

$$\begin{aligned}E_{1,max} &= \omega N_1 \phi_{max} \\ \therefore N_1 &= \frac{115}{2 \cdot \pi \cdot 60 \cdot 0.03632} \\ \therefore N_1 &= 8.399 \approx 8 \text{ turns}\end{aligned}$$

The next step in the replication of the waveforms of [81] is to obtain the hysteresis loop of the core. The authors of [81] do not provide data points to be used. However, they do provide a graph of the hysteresis loop. This graph is shown in Fig. 3-20. Data points can be extracted manually from this figure. The data points obtained from this graph are presented in APPENDIX F for readers who would like to replicate the results. The data points of the 60 Hz loop are shown in Fig. 3-21, which shows a good correlation to Fig. 3-20.

The author would like to caution readers that to use these data points for the Tellinen model, the data points need to be strictly monotonic. The lookup table of MATLAB fails if this is not the case. Furthermore, the ascending and descending legs need to be symmetric for the Tellinen model. If this condition is not met, the magnetic field intensity (H) tends to drift with each voltage cycle and the hysteresis loops do not close onto each other.

Chandrasena *et al.* [81] use the forward JA model to simulate their transformer. As such, they provide parameters for the JA model in [81, Appendix A]. However, the forward and inverse JA

models yield different results for the same parameter set. Since the transformer model in Fig. 3-19 makes use of the inverse *JA* model, a new set of parameters needs to be determined to yield the graph seen in Fig. 3-21. A modification to the Leite *et al.* optimisation algorithm presented in APPENDIX D yields the parameter set given in Table 3-8. This modification will be discussed in section 3.4.2.

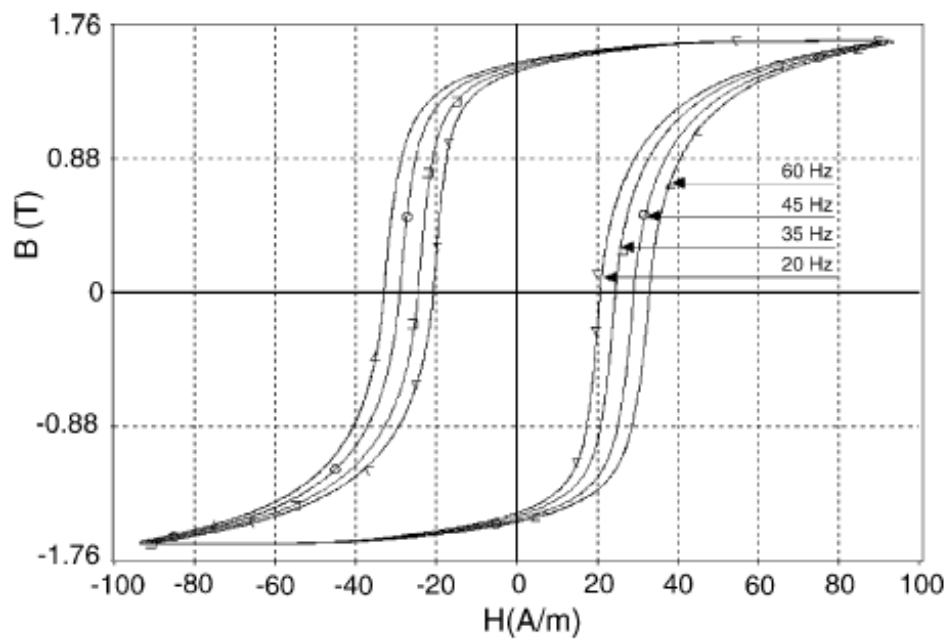


Fig. 3-20: The hysteresis loop found in [81, Fig. 12]

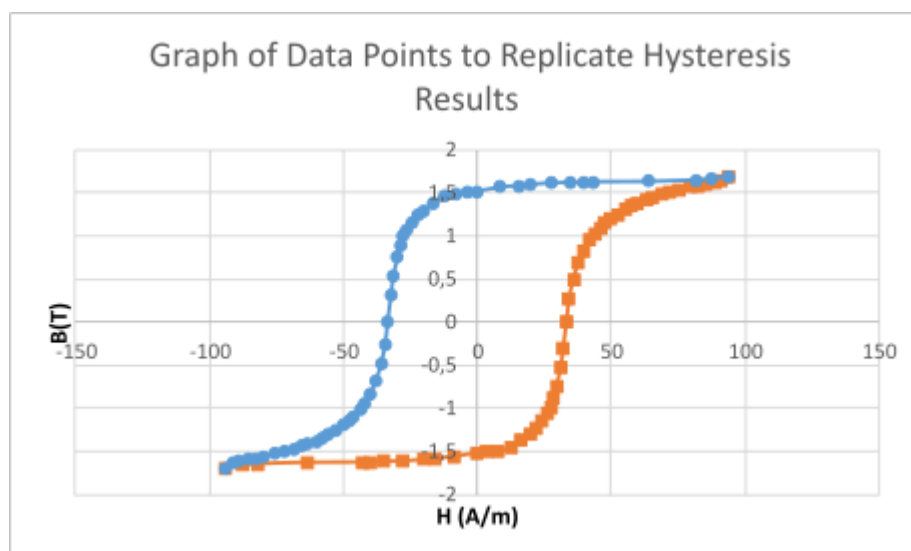


Fig. 3-21: A manual replication of the 60 Hz hysteresis loop of Fig. 3-20

Fig. 3-22 shows the magnetization current simulated by [81] at rated voltage. Fig. 3-23 shows the replication of this magnetization current using the inverse *JA* model. Fig. 3-24 shows the replication of this current using the inverse Tellinen model and Fig. 3-25 shows the comparison of the inverse *JA* and inverse Tellinen models.

A couple of notes can be made based on the results of Fig. 3-22 through Fig. 3-25. The first important thing to note is that both the inverse Tellinen and the inverse *JA* show a very good correlation to the results presented in [81], especially considering the lack of transformer data in [81]. The shape of the waveforms is identical and the peak current value simulated by the models is only slightly lower than the peak current in Fig. 3-22. The difference can be allocated to manual transcription of the data in the hysteresis loop of Fig. 3-21 and the assumptions presented earlier. This serves as verification that both hysteresis models presented thus far are accurate and capable of simulating hysteresis. The strong correlation between these results also confirms an argument made earlier by the author of this report – namely that the injection of current based on the hysteresis model should only be made on one side of the transformer. Chandrasena *et al.* [81] inject current on both sides of the ideal transformer.

A second important note relates to the comparison of the results of the inverse *JA* model and the inverse Tellinen model found in Fig. 3-25. The results are very similar. However, the inverse *JA* model yields a smoother graph. This is because the Tellinen model interpolates between numerical values which have been read off a graph. Manual transcription can therefore cause small deviations in the calculations. The inverse *JA* model, on the other hand, is an analytical solution of a *DE* with constant parameters. One would, therefore, expect the inverse *JA* model to be more consistent. Fig. 3-25 also shows that the inverse *JA* model simulates a higher peak current than the inverse Tellinen model. This can be caused by non-ideal parameters determined by the optimization program. The *MSE* for the set of parameters presented in Table 3-8 was 20.98.

Table 3-8: The Inverse *JA* model parameters for the replication of the hysteresis loop seen in Fig. 3-21

M_{sat}	1.8208e06
k	56.2231
c	0.3090
α	5.9100e-05
a	29.1461

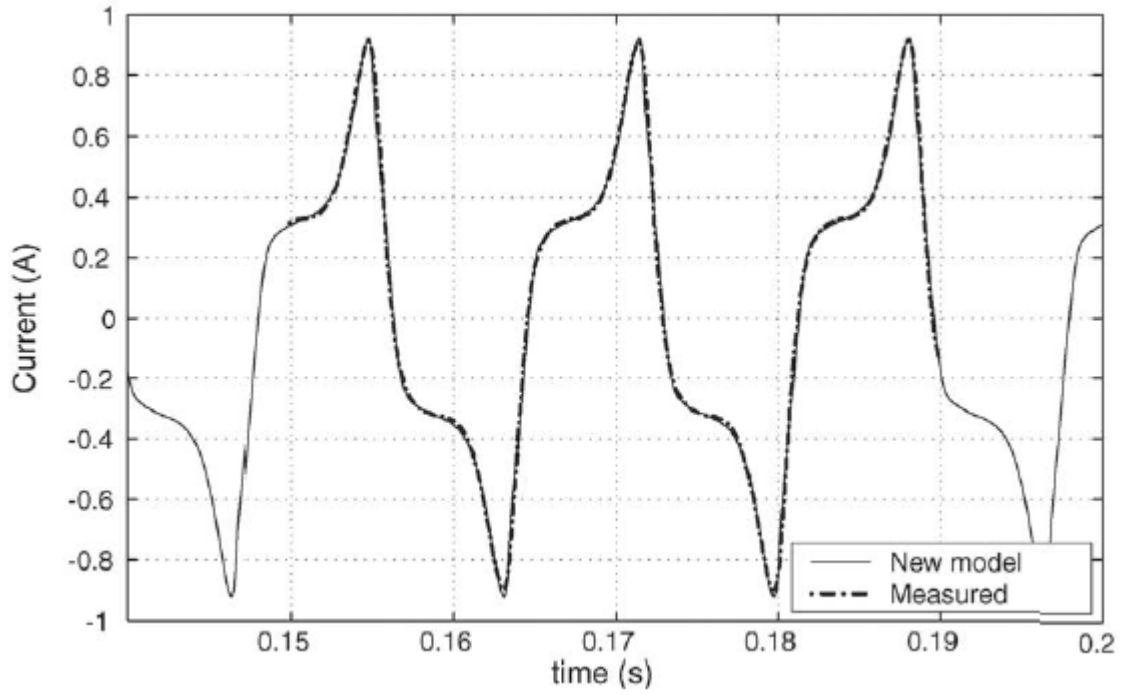


Fig. 3-22: The magnetization current at rated voltage as seen in [81, Fig. 6]

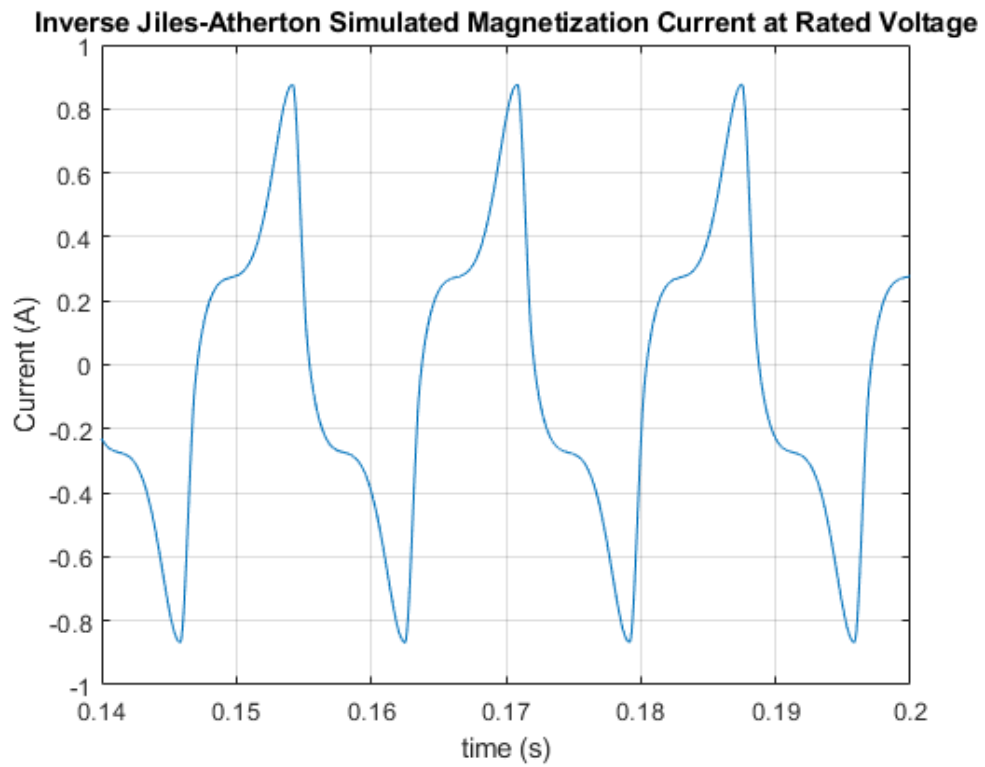


Fig. 3-23: The magnetization current replication of [81] using the Inverse Jiles-Atherton model at rated voltage

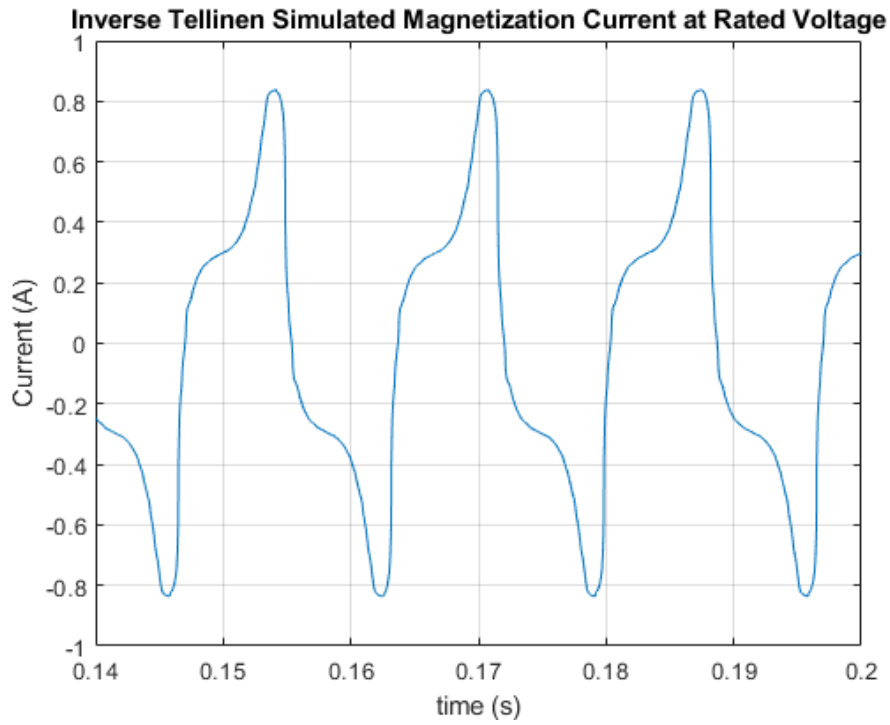


Fig. 3-24: The magnetization current replication of [81] using the inverse Tellinen model at rated voltage

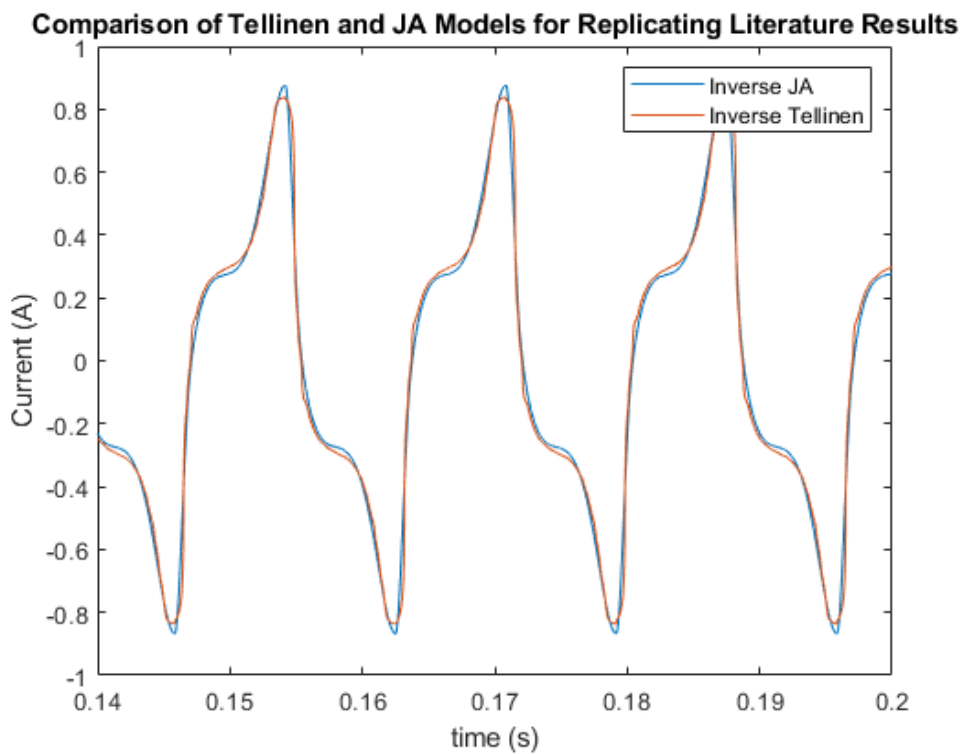


Fig. 3-25: A comparison of the inverse Tellinen and inverse JA models in the replication of [81]

An interesting observation can be made at this stage regarding the ability of these two models to retain residual flux. Retention of residual flux is one of the main advantages of the JA model and

an important criterion for its selection. To test this functionality, a circuit breaker can be connected between the source and the winding resistance in Fig. 3-19. When the circuit breaker is opened, the flux (or the flux density) should then settle on some value determined by the decay of the voltage. For this experiment, the circuit breaker was opened at a time of 0.0542 s and closed again at a time of 0.1 s. A switching time of 0.0542 s correlates to a zero-crossing of the voltage. The flux lags the voltage by 90° and therefore the flux is maximum at this point, which correlates to the worst-case switching scenario. A closing time of 0.1 s correlates to a maximum point on the voltage waveform and a zero-crossing of the flux.

The magnetic flux density of the inverse *JA* model following these switching procedures is shown in Fig. 3-26. The magnetic flux density generated by the inverse Tellinen model is shown in Fig. 3-27. The vertical lines in both these figures are used to indicate when switching took place.

It is clear from Fig. 3-27 that the inverse Tellinen model cannot retain magnetic flux. The magnetic flux is directly related to the magnetic flux density, which immediately drifts after switching. Furthermore, once the switch is closed again, the graph shows sinusoidal behaviour with a significant offset.

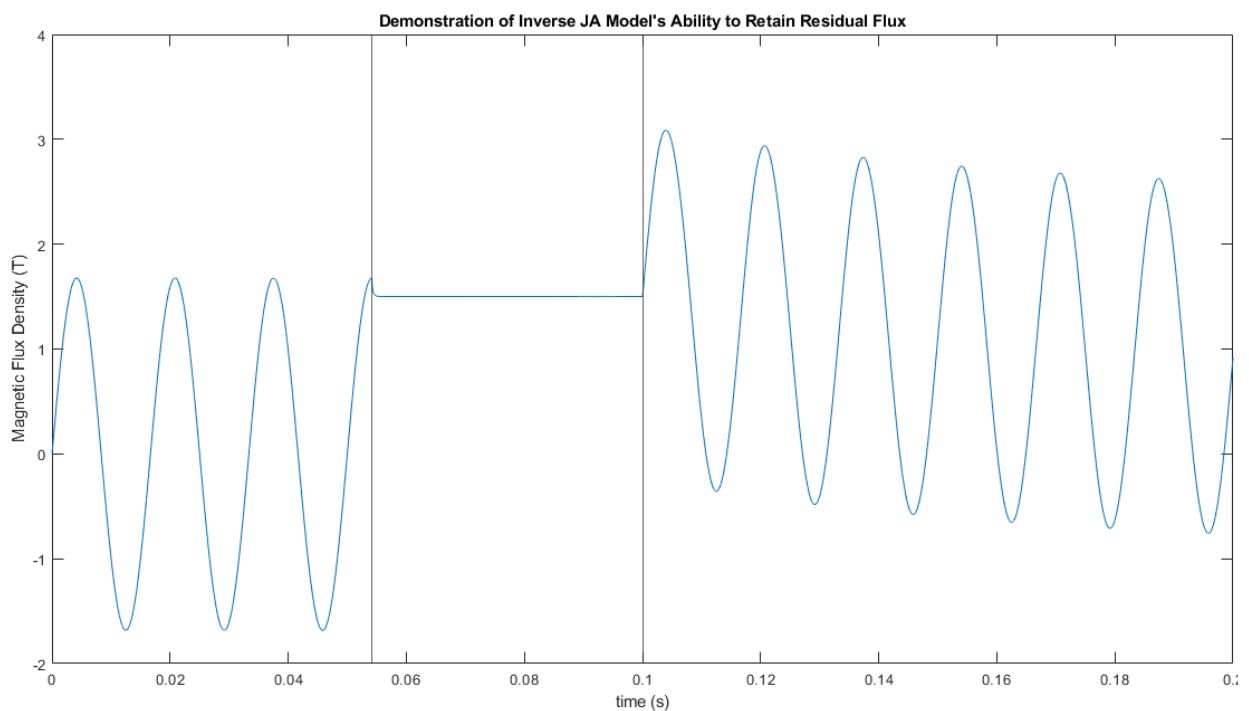


Fig. 3-26: The magnetic flux density generated by the inverse *JA* model following switching

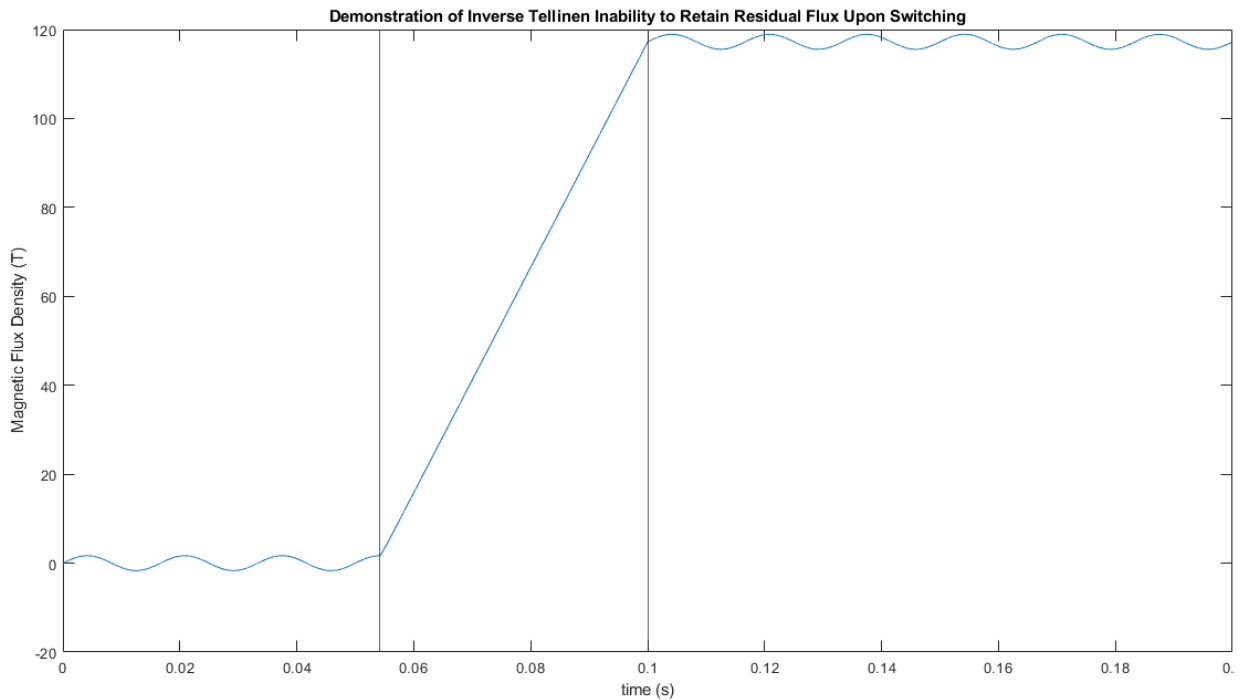


Fig. 3-27: The magnetic flux density generated by the inverse Tellinen model following switching

The JA model, on the other hand, does seem to retain the magnetic flux in Fig. 3-26. The accuracy of this retained flux cannot be verified with experimental data at this stage but it can be concluded that the flux does not drift. It is also interesting to note that the flux does not remain at the steady-state value established by the time of switching. Rather, there is an exponential decay of the flux after switching. This exponential decay is shown more clearly in Fig. 3-28.

Another interesting phenomenon in Fig. 3-26 is the decay of the sinusoidal waveform after the switch is closed again. This is not evident for the inverse Tellinen model in Fig. 3-27. It would, therefore, appear as though the offset caused by residual flux will be eliminated after a sufficient time has passed.

Finally, it is also worth looking at the magnetization current generated by the inverse JA model after switching takes place. The magnetization current is shown in Fig. 3-29. After the switch is closed, as indicated by the second vertical line, it is clear that the transformer draws a significant inrush current. This is to be expected since the transformer is driven into heavy saturation by the residual flux. The decay of this inrush current also explains the decay of the sinusoidal magnetic flux density in Fig. 3-26. The magnetization current before switching is identical to the current shown in Fig. 3-23, but cannot be seen since the y-axis is scaled to show the inrush current. The magnetization current when the switch is open is zero as expected.

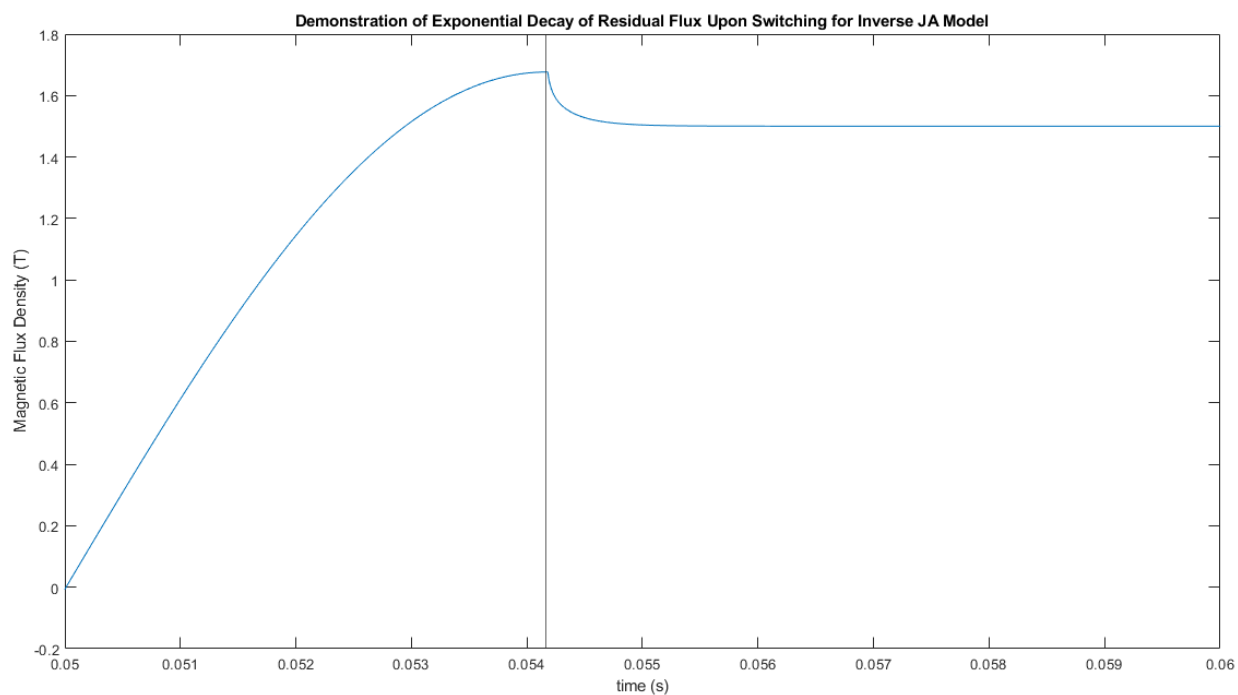


Fig. 3-28: The exponential decay of magnetic flux density after switching the inverse JA hysteresis model

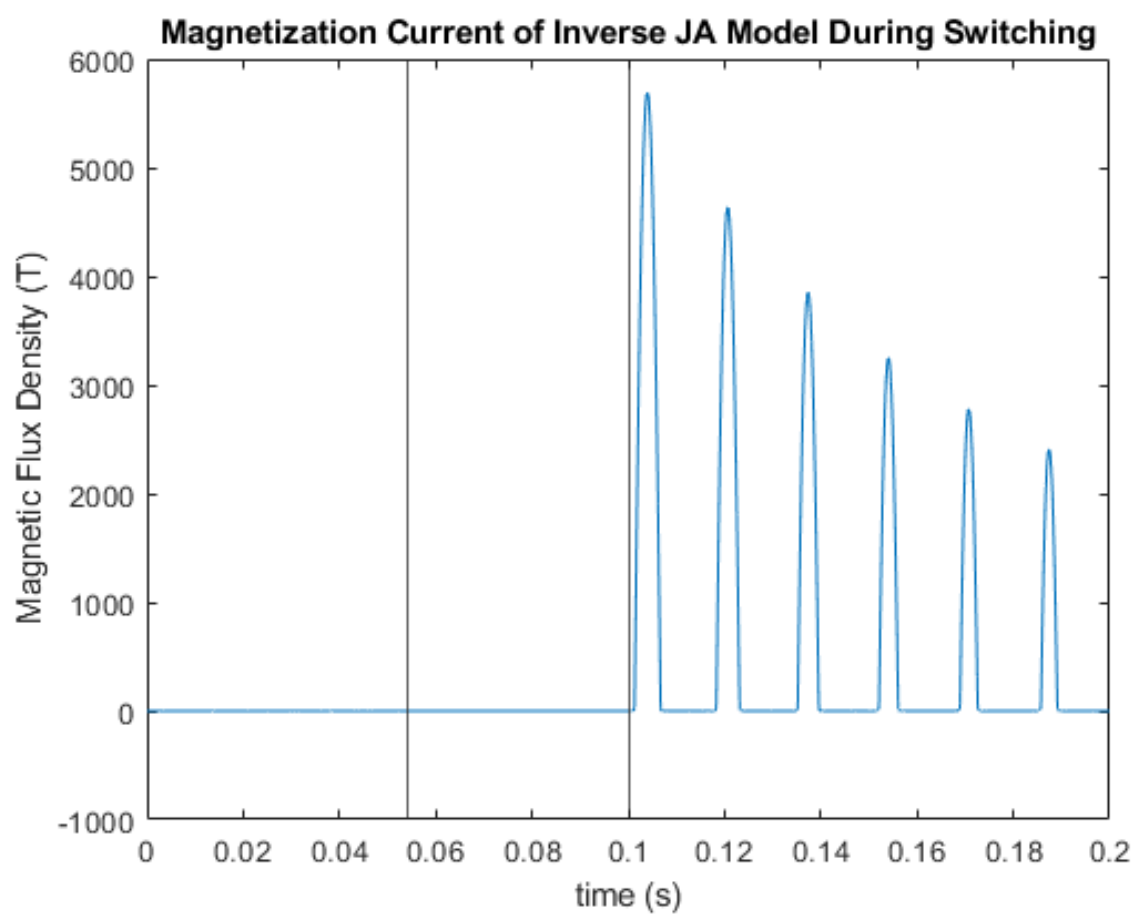


Fig. 3-29: The magnetization current of the inverse JA model during switching

The inevitable conclusion of this residual flux experiment is that the inverse Tellinen model is not sufficient in this regard. Any attempt to model ferroresonance during switching transients will be highly inaccurate if the Tellinen model is used. The inverse JA model, on the other hand, is likely to be more accurate than the nonlinear inductor model of section 3.2, which is also not capable of retaining residual flux. This also verifies the literature which claimed that one of the advantages of the JA is its ability to retain residual flux.

The hysteresis model presented thus far can be interpreted as the parallel combination of the core loss resistance and the magnetization inductance. Under this assumption, it should make sense that the model presented by [81] is an example of a T-model, as described in section 2.6. Based on the results of section 3.1, it can be argued that a π -model using hysteresis models should also be more accurate than the classical T-model. The first step, however, is to verify that a hysteretic T-model and a hysteretic π -model yield the same steady-state results. To convert the hysteresis T-model to a hysteresis π -model, the current output needs to be divided by two. This is how de León, Farazmand and Joseph [88] modified the nonlinear inductor for the π -model.

Another important factor to take into consideration for the π -model is the leakage inductance. The transformer simulated by [81] does not contain any leakage inductances. For the π -model, however, the leakage inductance separates the magnetization branches. Without a leakage inductance to separate these branches, the π -model becomes identical to the T-model. For the comparison of the hysteretic T- and π -model, the leakage inductances from Table 3-3 will be used. This means that the leakage inductance on the 2.3 kV side of the transformer in Fig. 3-19 will be given a value of 1.13 H, while the LV leakage inductance will be given a value of 0.00036 H. This also means that the equivalent leakage inductance of the π -model, referenced to the LV side, is 3.185 mH. A circuit diagram of the hysteretic π -model in MATLAB is shown in Fig. 3-31. The comparison of the input current for the T-model and the π -model is shown in Fig. 3-30. This figure shows that the current flowing through the LV winding is identical and unless the current of the π -model is limited to a time of 0.05 s, it would not be possible to see both waveforms because the two waveforms would be on top of each other. Therefore, it can be concluded that the principle of the π -model also applies to hysteretic models of the core.

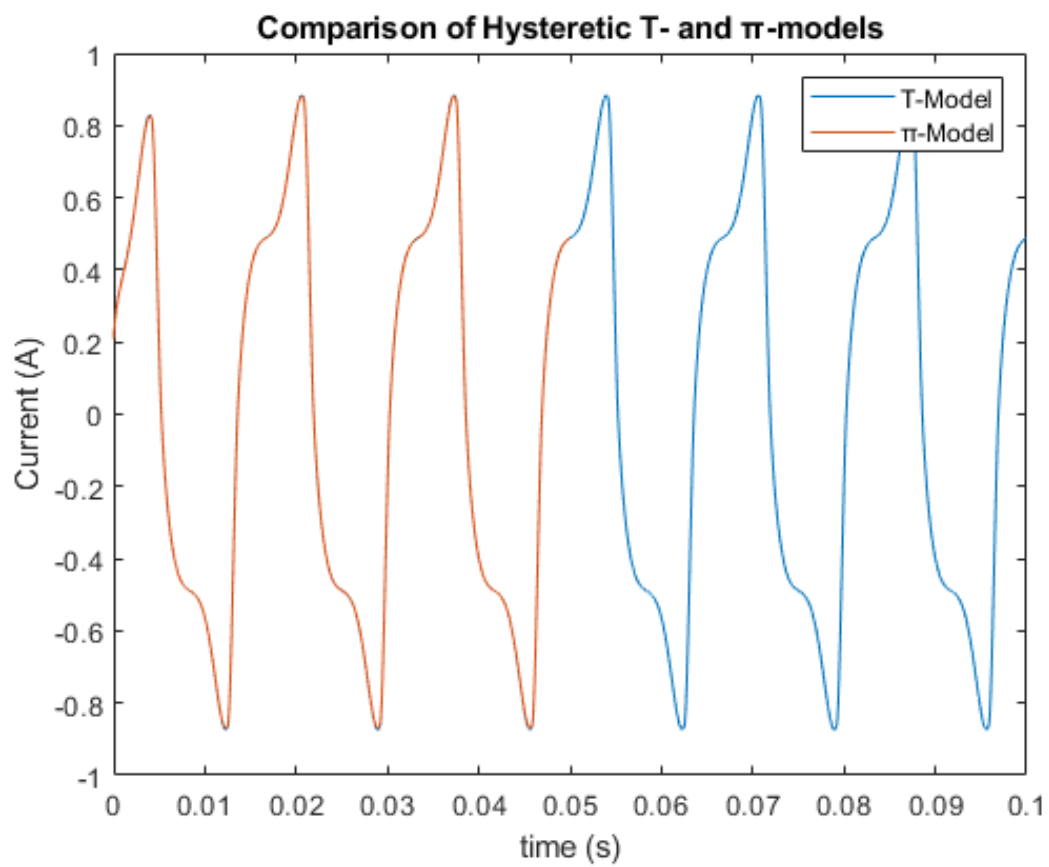


Fig. 3-30: A comparison of the input current of the hysteretic T- and π -models

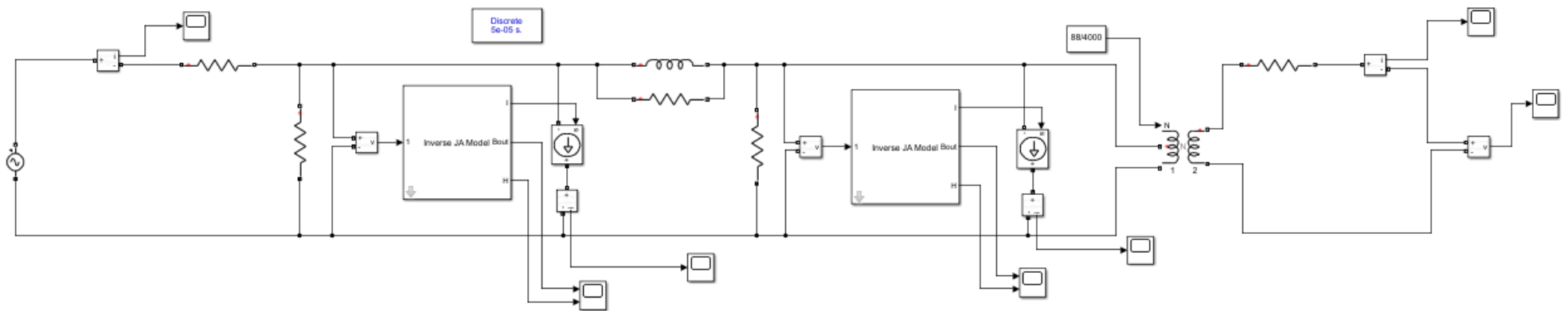


Fig. 3-31: The MATLAB circuit of a hysteretic π -model

3.4.2 Particle Swarm Optimisation

In section 3.4.1 it was mentioned that the inverse JA parameter optimization algorithm presented by Leite *et al.* [180] had to be modified to yield parameters that could replicate the hysteresis loop of [81]. This is because the algorithm was not capable of sufficiently optimizing the parameters. It has been found by the author of this report that this algorithm is susceptible to local minima and maxima of the parameters.

Table 3-9 shows a set of parameters obtained by Leite *et al.* [180] using their optimization algorithm. Fig. 3-32 shows the hysteresis loop generated by the parameters in the “Optimized Values” column. To verify the ability of the optimization algorithm programmed by the author of this report, an attempt was made to replicate these results. A reference loop can be generated by using the “Optimized Values” column to generate data points. These data points can then be saved and used for the optimization algorithm. If the parameters in the “First Try” column of Table 3-9 are used as the initial input to the optimization algorithm, then one would expect the algorithm to converge to the parameters in the “Optimized Values” column. This would serve as sufficient proof that the algorithm is functional.

Unfortunately, the Leite optimization algorithm was not capable of yielding these results. The best attempts are summarized in Table 3-10. Based on the *MSE* result of these two attempts, one might be tempted to conclude that the algorithm works relatively well. However, one would expect the algorithm to find parameters that yield an *MSE* of zero since the reference loop is not a set of experimental data but was generated by the inverse JA hysteresis model.

It is also interesting to note in Table 3-10 that even though the *MSE* is lower for attempt 2, only M_{sat} is closer to the target value. For all the other parameters, attempt 1 yielded parameters that are closer to the “Optimized Values” column of Table 3-10. This leads to two important conclusions:

Table 3-9: The JA parameters for the verification of Leite optimization algorithm as seen in [183. Table III]

Parameter	First Try	Optimized Values
M_s	1.5870×10^6	1.4552×10^6
k	51.319	72.312
c	134.29×10^{-3}	350.49×10^{-3}
a	129.16	88.498
α	220.78×10^{-6}	177.05×10^{-6}

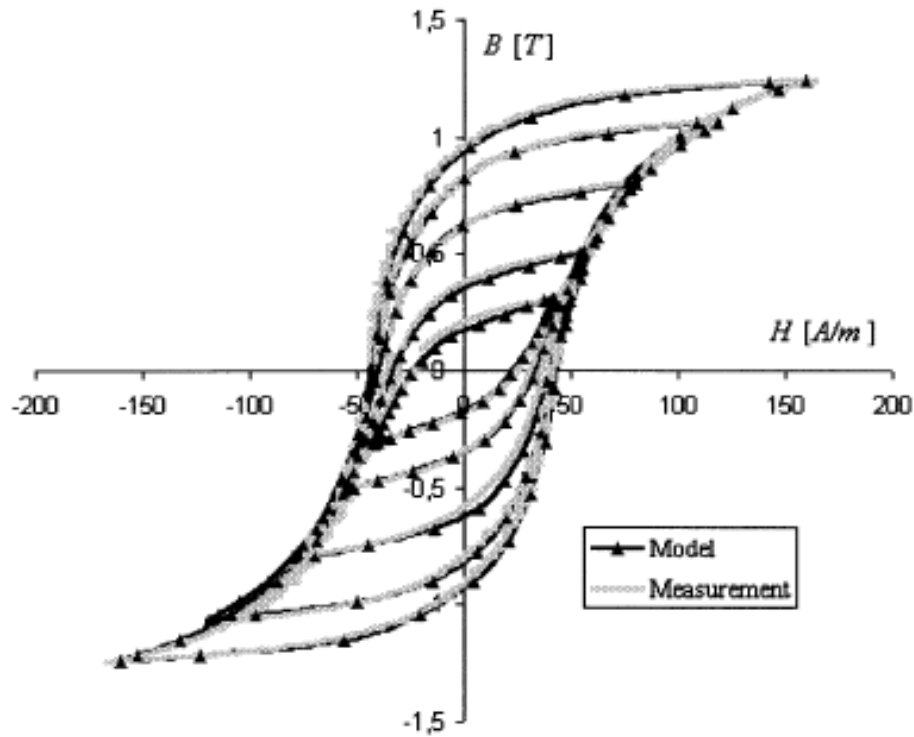


Fig. 3-32: The hysteresis loop for the parameters shown in Table 3-9²³

Table 3-10: The inverse JA parameters determined by the Leite optimization algorithm to match Fig. 3-32

Parameter	First Attempt (MSE = 5.004)	Second Attempt (MSE=4.1940)
M_s	1.5362×10^6	1.5518×10^6
k	69.4751	66.6142
c	0.3249	0.3086
a	111.39595	125.3049
α	206.97×10^{-6}	0.23484

M_{sat} plays a more dominant role than the other parameters and 2) The parameters of the JA model are interdependent. In other words, the parameters cannot be optimized in isolation.

²³ This figure was taken from [180, Fig. 7]. The quality of the figures within the article is poor and could not be improved for this report.

The poor performance of the Leite optimization algorithm became more evident when an attempt was made to use the algorithm to match the loop presented in Fig. 3-20 in section 3.4.1. There it was discovered that the optimization algorithm yielded parameters that were not sensible. The MSE could be reduced to 93.25, but the shape of the hysteresis graph assumed the form shown in Fig. 3-33, where it can be seen that the slope of the loop becomes negative. Padilha *et al.* [223] refer to this negative slope of the hysteresis loop as non-physical behaviour.

The shortcomings of the algorithm were fixed by sweeping across two parameters at a time. Initially, a sweep was only done for each parameter independently, which did show an improvement in the MSE from 93 to 58. However, when two parameters were swept at a time, the MSE was further improved to 20.97. This reaffirms the claim made earlier that the parameters cannot be optimized in isolation as they are highly dependent on each other. This also means that even when a sweep is done across two parameters at a time, three other parameters are still kept constant. If two parameters are so interdependent upon one another, then one must assume that all five parameters are interdependent. A potential solution, then, would be to perform a sweep across all five parameters using five nested for-loops. However, the time associated with such an exercise makes it impractical. If 20 iterations are tested for each parameter, this is the equivalent of testing 3.2 million different combinations of parameters.

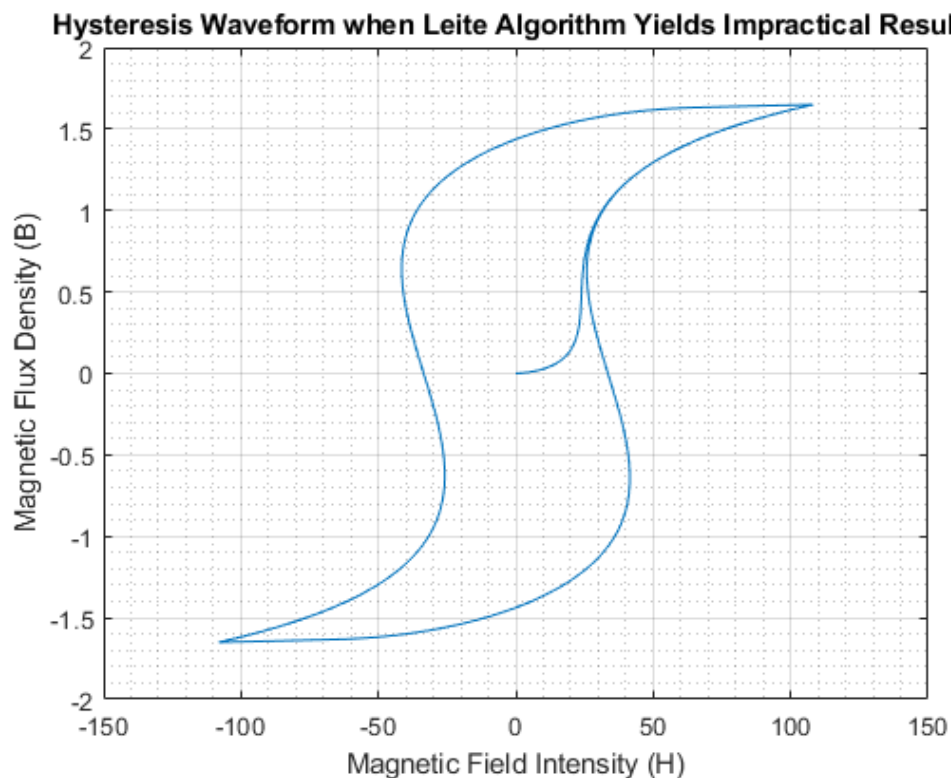


Fig. 3-33: The non-physical hysteresis loop generated by the Leite optimization algorithm in an attempt to replicate [Fig. 3-20]

Another potential solution to this problem was found in [224], which discusses the *PSO* algorithm. *PSO* is an evolutionary computation technique similar to the concept of genetic algorithms discussed in section 2.10.19. However, Marion *et al.* [224] prove that *PSO* is more likely to converge than genetic algorithms. *PSO* was first developed by Kennedy and Eberhart in [225] and [226]. It is based on the initialization of a population of random solutions. These solutions are referred to as particles. Each particle has an associated velocity and the particles then fly across the search space. The velocities are dynamically adjusted based on the best solution of the iteration and the overall best solution. This forces the swarm to fly towards an optimal solution.

The *PSO* algorithm written by the author contains several function files and is too elaborate to add in an appendix. However, the verification of the optimization ability of the algorithm is two-fold. Firstly, the algorithm was tested on the hysteresis loop shown in Fig. 3-32. Recall that when the Leite algorithm was used to optimize the parameters, a result of $MSE = 5.004$ was the best result. The *PSO* algorithm managed to find a solution with $MSE = 1.5561 \times 10^{-7}$ within a few hours. Furthermore, the optimization algorithm did not require any manual intervention. When two parameters are swept to improve the Leite algorithm, the user is required to make an educated guess on the potential range of the parameters to reduce the simulation time. The *PSO* algorithm does not require an educated guess from the user – it only requires the user to define a range that is wide enough to include all potential solutions. The parameters calculated by the *PSO* algorithm are shown in Table 3-11. When these parameters are compared to the “Optimized Values” column of Table 3-9, it can be seen that the *PSO* algorithm nearly replicated the original results. Consequently, it can be concluded that *PSO* is much more efficient and accurate than the Leite algorithm.

The second verification of the ability of the *PSO* algorithm is related to the calculation of harmonic current phase angles. Section 3.2 spoke about the significant harmonic currents in the waveforms of the OC test. This will also be discussed in section 3.4.3. The N4L oscilloscope used to perform

Table 3-11: The Inverse JA parameters calculated by PSO to match Fig. 3-32

Parameter	Value
M_s	1.455167×10^6
k	72.280333
c	0.350227
a	88.466177
α	1.795480×10^{-4}

the OC test provides a breakdown of the amplitudes of the first 20 harmonics. An *FFT* can also be done on the exported waveforms, which shows that the exported waveforms have magnitudes similar to those measured by the more accurate N4L oscilloscope. This provides confidence in the accuracy of the exported waveforms, which will be important for the discussion of section 3.4.3.

The N4L oscilloscope, however, does not indicate the phase shift of the harmonic currents relative to the voltage. It only indicates the amplitudes. An example of the output of the N4L oscilloscope at 125% of rated voltage is shown in Fig. 3-34, where harmonic number 1 refers to the fundamental waveform of 50 Hz. The current exported at this voltage level is shown in Fig. 3-35.

The harmonic amplitudes shown in Fig. 3-34 can be used to generate a waveform based on the principle of superposition. Note, however, that the harmonic currents indicated in Fig. 3-34 are *RMS* values and need to be multiplied by $\sqrt{2}$ to show a current waveform similar to that seen in Fig. 3-35. If one assumes that the phase angles of these harmonics are all 0°, then the principle of superposition yields the waveform seen in Fig. 3-36.

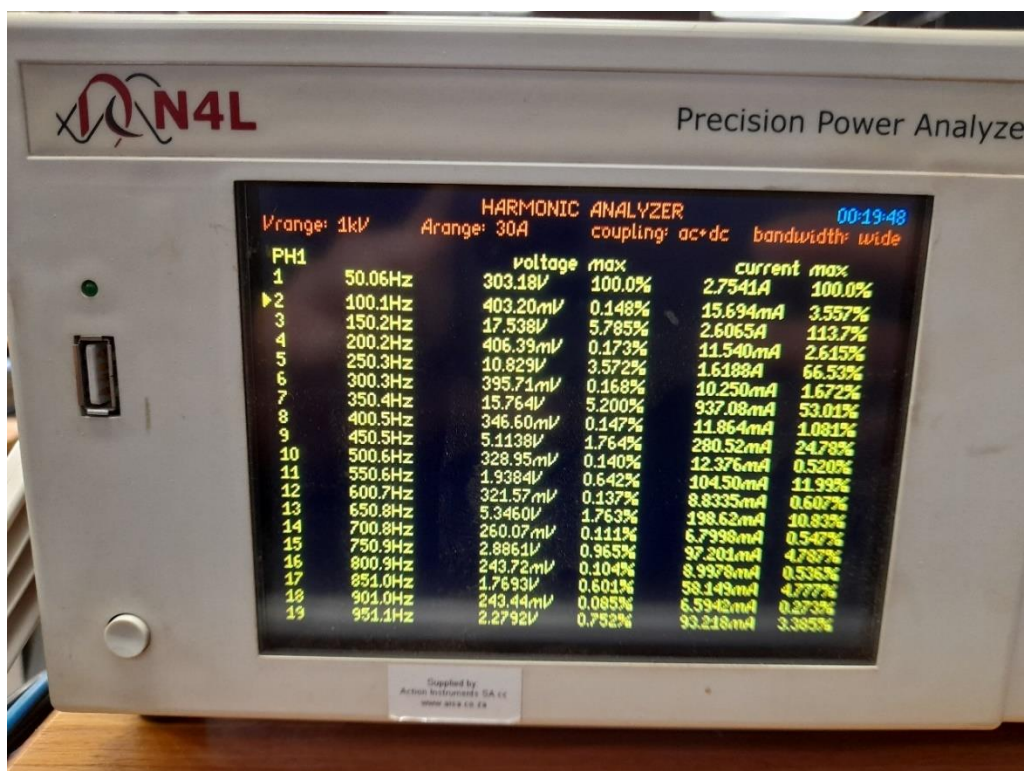


Fig. 3-34: An example of the harmonic current breakdown by the N4L during the OC test at 125% of rated voltage

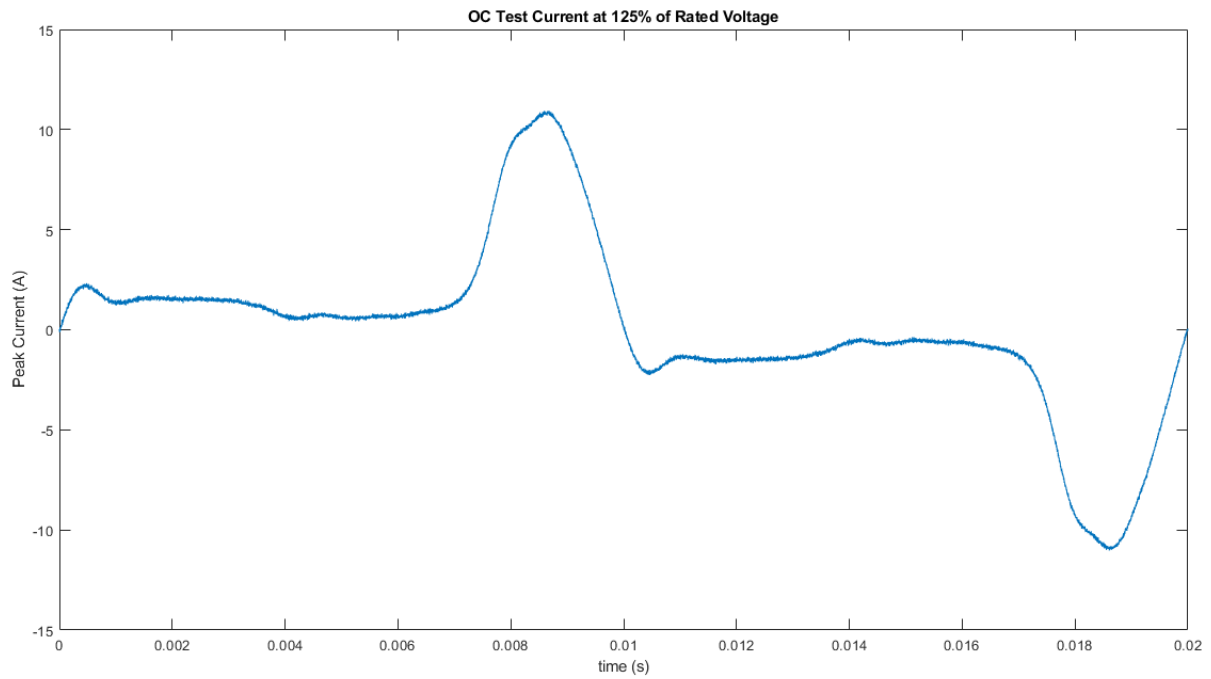


Fig. 3-35: The measured OC test current at 125% of rated voltage

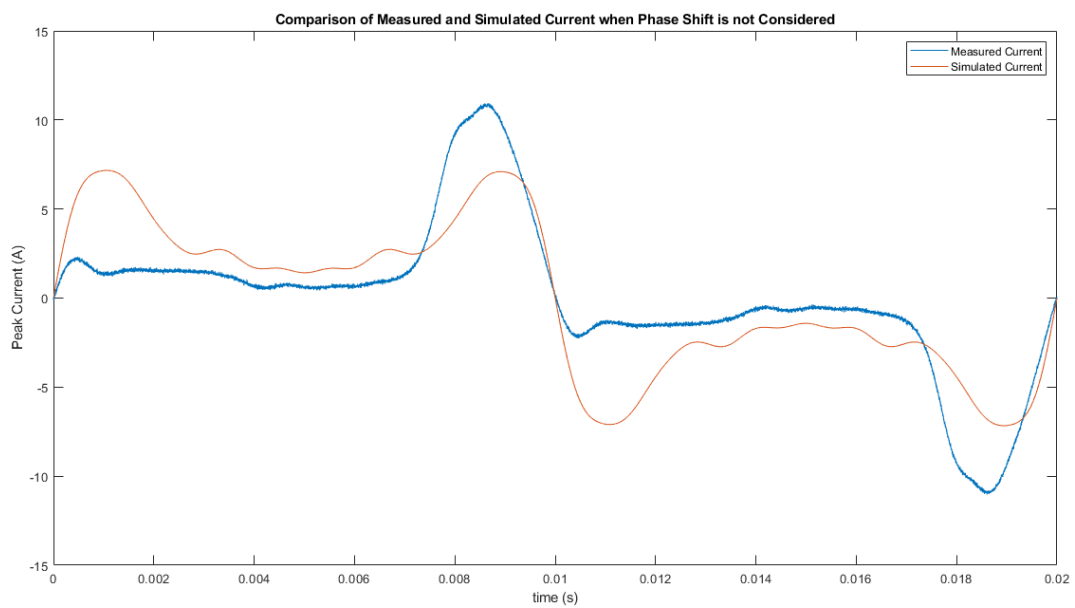


Fig. 3-36: A comparison of the measured and generated harmonic current waveforms when phase angles are assumed to be 0°

To improve the fit of the simulated waveform, the *PSO* algorithm was used to optimize the phase angles of the 20 harmonic amplitudes. The measured current waveform was used in the cost function. The limits for the phase angle were set between -180° and 180° . The resulting waveform is shown in Fig. 3-37, which shows a significant improvement. This serves as proof that the *PSO*

algorithm is capable of optimizing multiple parameters simultaneously across a wide search space at an exceptional rate.

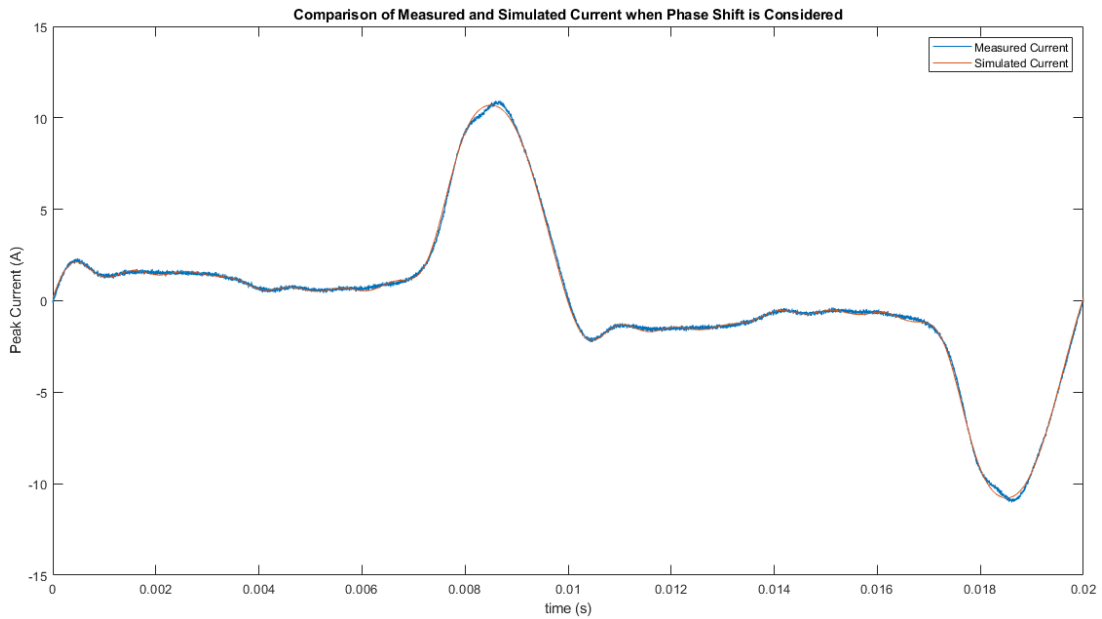


Fig. 3-37: The measured and generated harmonic current waveforms after phase angles were optimized using PSO

3.4.3 Harmonic Currents and Hysteresis

The extensive discussions presented in sections 3.4.1 and 3.4.2 served several purposes. They firstly verified the accuracy of the inverse *JA* and inverse Tellinen models and their ability to model magnetization currents. They also proved that the inverse *JA* model can retain residual flux upon switching but that the inverse Tellinen model fails in this regard. Section 3.4.2 proved that the Leite optimization algorithm is not sufficient for estimating *JA* parameters and that the *PSO* algorithm is a good replacement. These discussions were provided as a build-up to the development of a hysteresis model that reflects the behaviour of the distribution transformer being studied in this report.

However, the introduction to Chapter 3 should already have provided the reader with an indication that this was not possible. The high harmonic content of the *OC* current made the modelling of a hysteresis loop impossible. This section of the report discusses the effect that the high harmonic current has on hysteresis modelling.

The main problem with the high harmonic content of the current is that the current is no longer monotonic. A lack of monotonic current means that the gradient of the hysteresis loop becomes negative, which Padilha *et al.* [223] described as non-physical behaviour. The voltage and current waveforms at 100%, 110% and 120% of rated voltage are shown in Fig. 3-38, Fig. 3-39 and Fig.

3-40 respectively. It should be clear that the current waveforms are abnormal, even for rated conditions. The dips in the current at the peak of the waveforms are particularly concerning and are caused by the harmonics.

Consider the magnetization current simulated by [81], which was shown in Fig. 3-22. It can be seen that even though the current is no longer sinusoidal, it remains monotonic when it increases or decreases. Fig. 3-41 shows an *FFT* of this current. This shows that the odd harmonics tend to dissipate quickly.

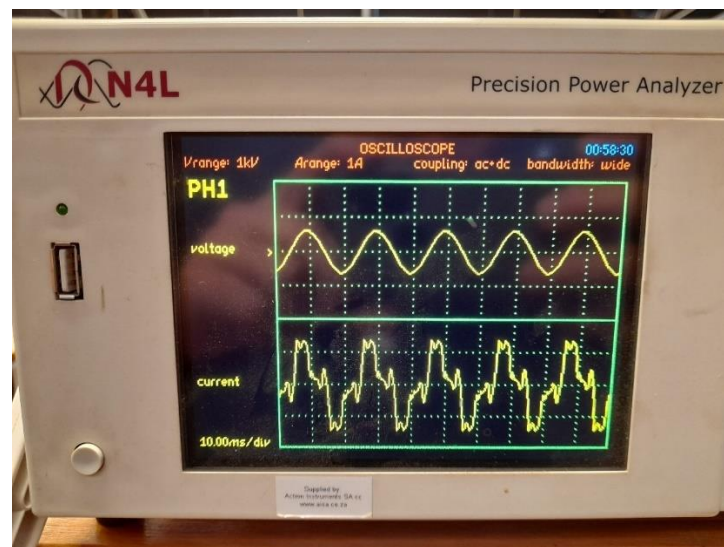


Fig. 3-38: The OC test voltage and current waveforms at rated voltage

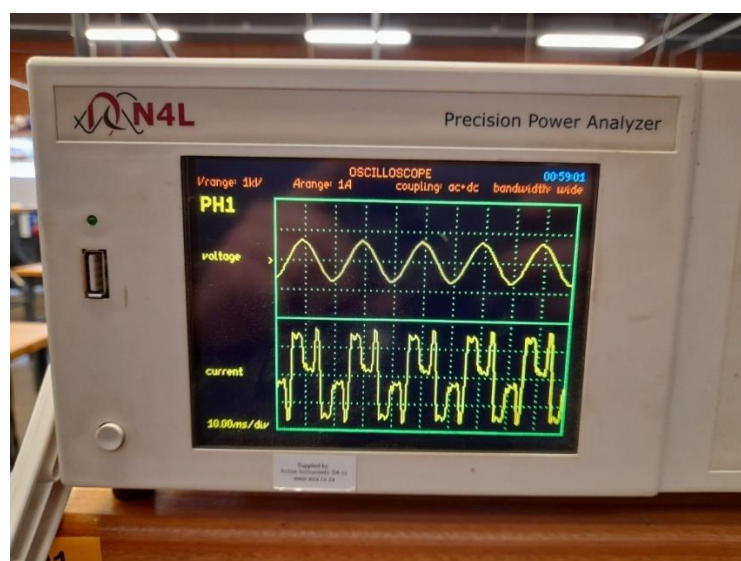


Fig. 3-39: The OC test voltage and current waveforms at 110% of rated voltage

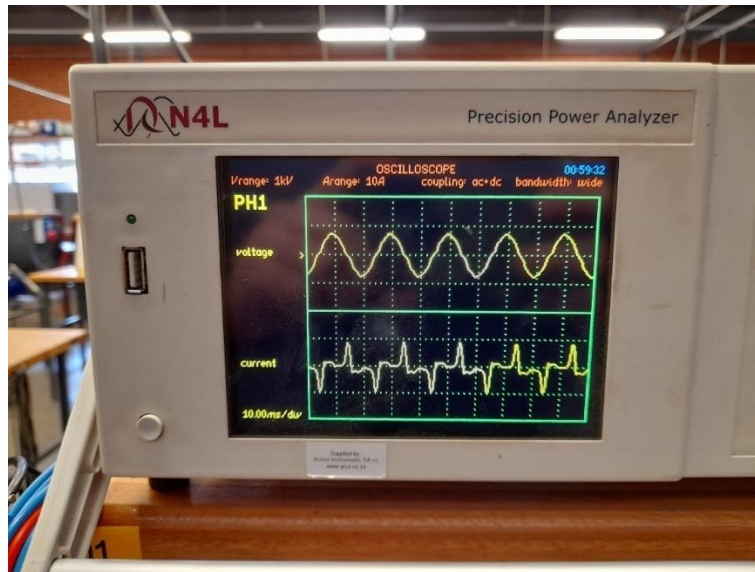


Fig. 3-40: The OC Test voltage and current waveforms at 120% of rated voltage

Fig. 3-42 shows the *FFT* of the measured current at rated voltage. It is interesting to note that the third harmonic is only 43% of the fundamental component. This is similar to the results seen in Fig. 3-41, which shows the *FFT* results of the magnetization currents studied in section 3.4.1. However, the most prominent feature of Fig. 3-42 is that the 7th harmonic is larger than the 5th harmonic. This is not true for the monotonic current data seen in Fig. 3-22. It is also interesting to note that the magnetization current of Fig. 3-41 contains a DC component, where the measured data of the transformer being studied does not. Finally, the even harmonics of Fig. 3-41 are noticeable. In the case of the transformer being investigated in this report, the even harmonics are insignificant in comparison to the magnitudes of the odd harmonics.

An *FFT* was also done on the measured OC current at 110%, 120% and 125% of rated voltage. These graphs are shown in Fig. 3-43, Fig. 3-44 and Fig. 3-45. These graphs show a trend whereby the magnitude of the odd harmonic increase relative to the fundamental component. In Fig. 3-44, where 120% of rated voltage was applied to the transformer, the 3rd harmonic has a larger amplitude than the fundamental component.

These abnormal current harmonics have a significant impact on the shape of the hysteresis graph. The measured hysteresis graph at 125% of rated voltage is shown in Fig. 3-46. Throughout the detailed literature study conducted for section 2.10, not a single hysteresis graph was found which replicates the form of Fig. 3-46. The non-physical, negative slope of the graph should also be apparent.

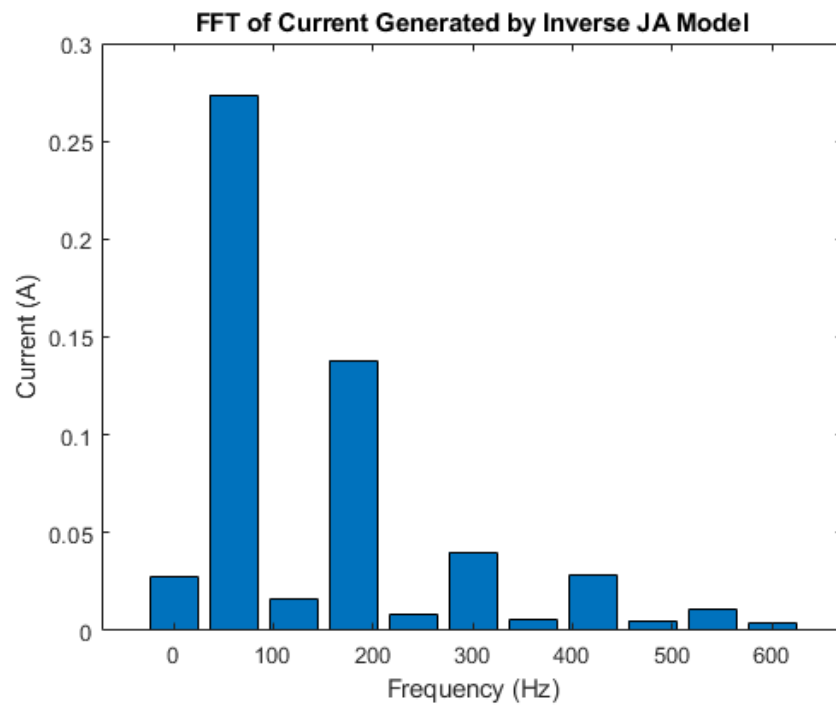


Fig. 3-41: The FFT of the magnetization current seen in Fig. 3-22

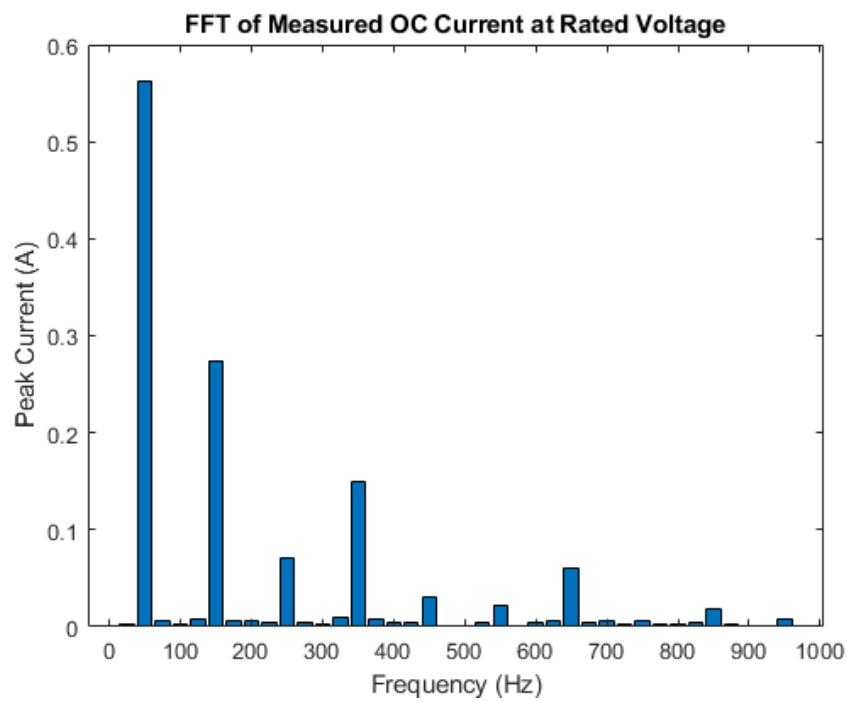


Fig. 3-42: The FFT of the measured OC current at rated voltage

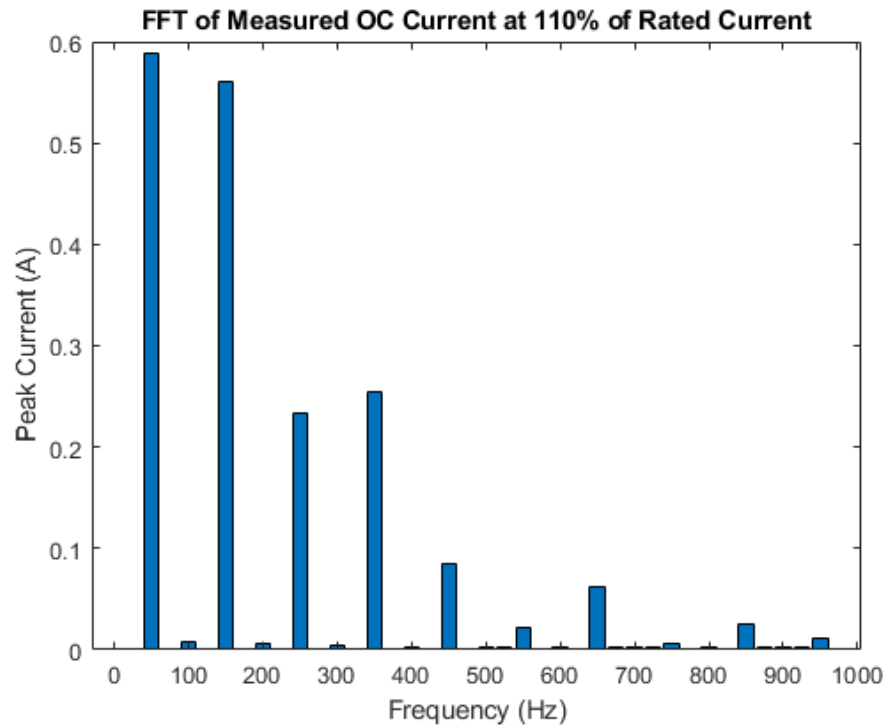


Fig. 3-43: The FFT of the measured OC current at 110% of rated voltage

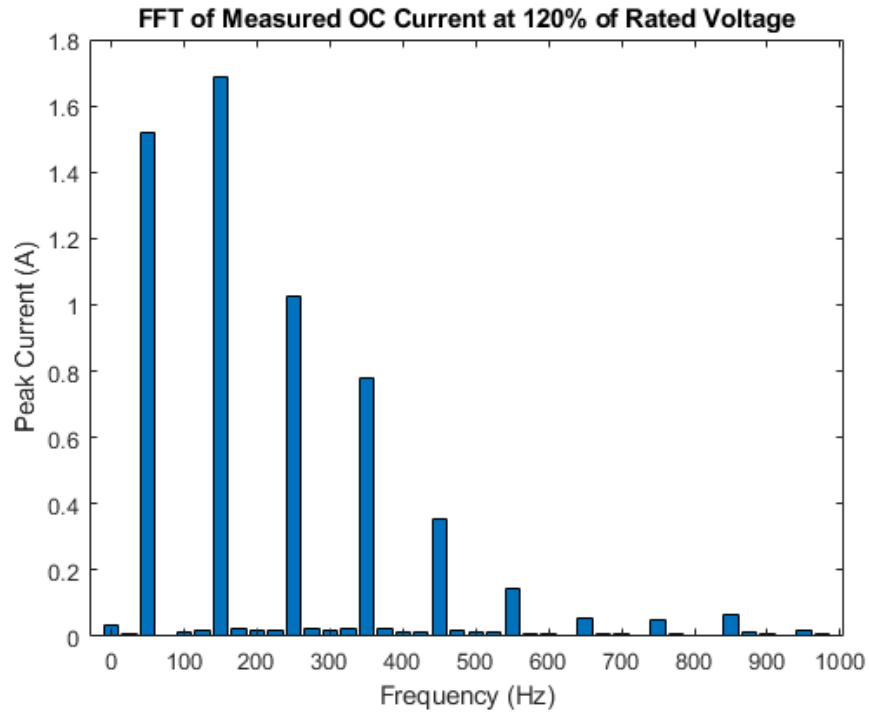


Fig. 3-44: The FFT of the measured OC current at 120% of rated voltage

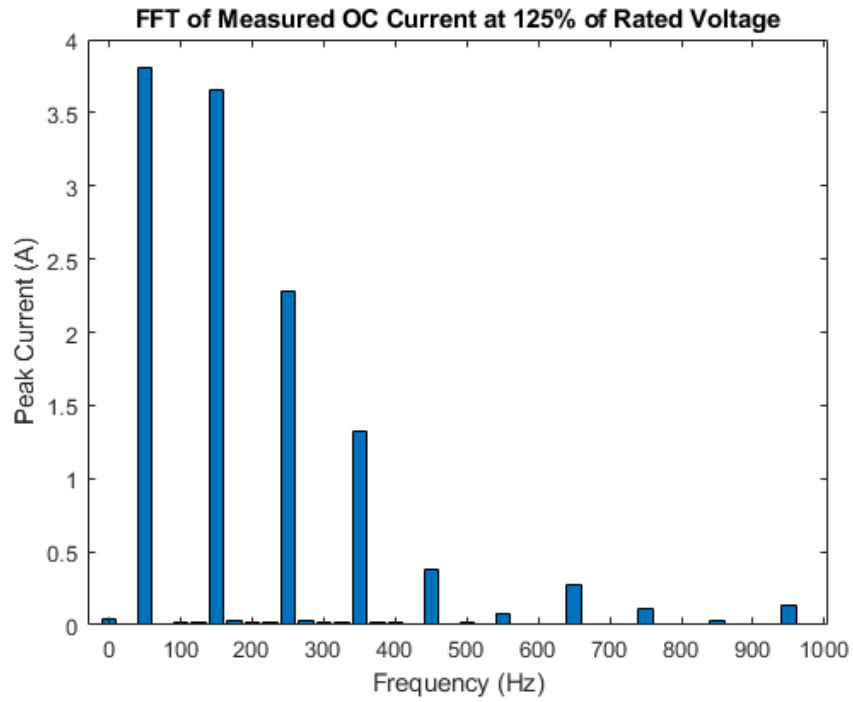


Fig. 3-45: The FFT of the measured OC current at 125% of rated voltage

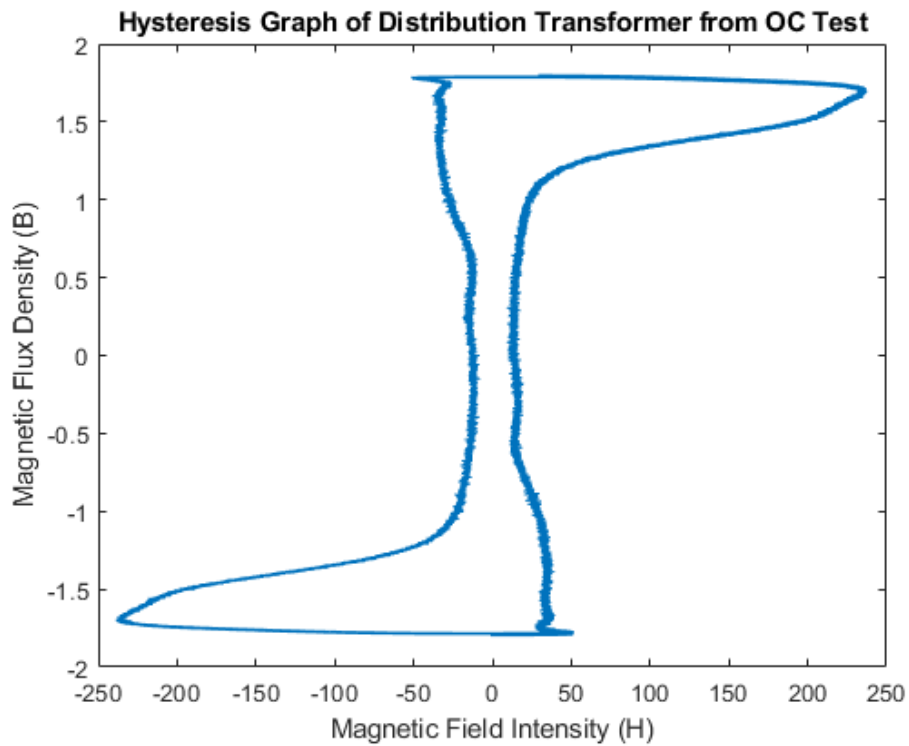


Fig. 3-46: The hysteresis graph of the transformer in question at 125% of rated voltage

As further proof that this hysteresis waveform is odd and difficult to replicate, consider Fig. 3-47. This figure shows the first quarter of the hysteresis loop seen in Fig. 3-46. One would expect the

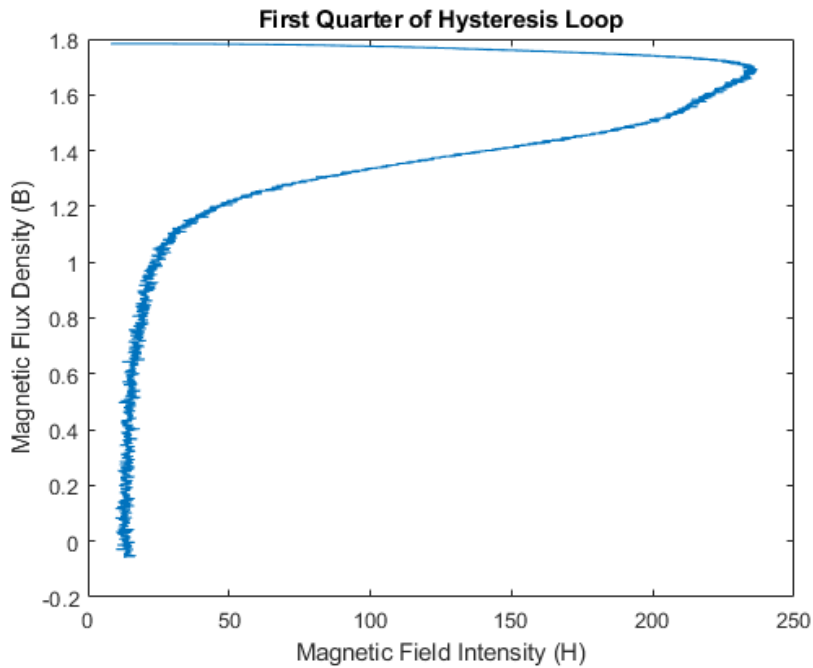


Fig. 3-47: The first quarter of the measured hysteresis loop

first quarter of the waveform to end at the tip of the hysteresis loop. Instead, Fig. 3-47 shows that the hysteresis loop is already returning to what can be considered as part of the descending hysteresis loop. This is another element of the loop's behaviour that has not been encountered in literature before.

A potential cause of Fig. 3-47 could be that the oscilloscope used to export the voltage and current waveforms did not trigger the waveforms correctly. The voltage and current waveforms are connected on two different channels. Furthermore, since the current is not sinusoidal, the oscilloscope struggled to trigger the current waveform. This difficulty implies that the phase shifts of the voltage and the current can be incorrect. From Fig. 3-47 it can be seen that the peak of the magnetic flux density and the peak of the magnetic field intensity does not correlate. This can be fixed manually by introducing a phase shift into the current waveform. If this is done, the hysteresis waveform changes to Fig. 3-48. This already seems more sensible since the width of the hysteresis loop is more consistent. Furthermore, the peak of the magnetic flux density and magnetic field intensity now correlate. However, it is still evident that the harmonics in the current are causing non-physical slopes. These slopes are most prominent in the shoulder areas of the second and fourth quadrant.

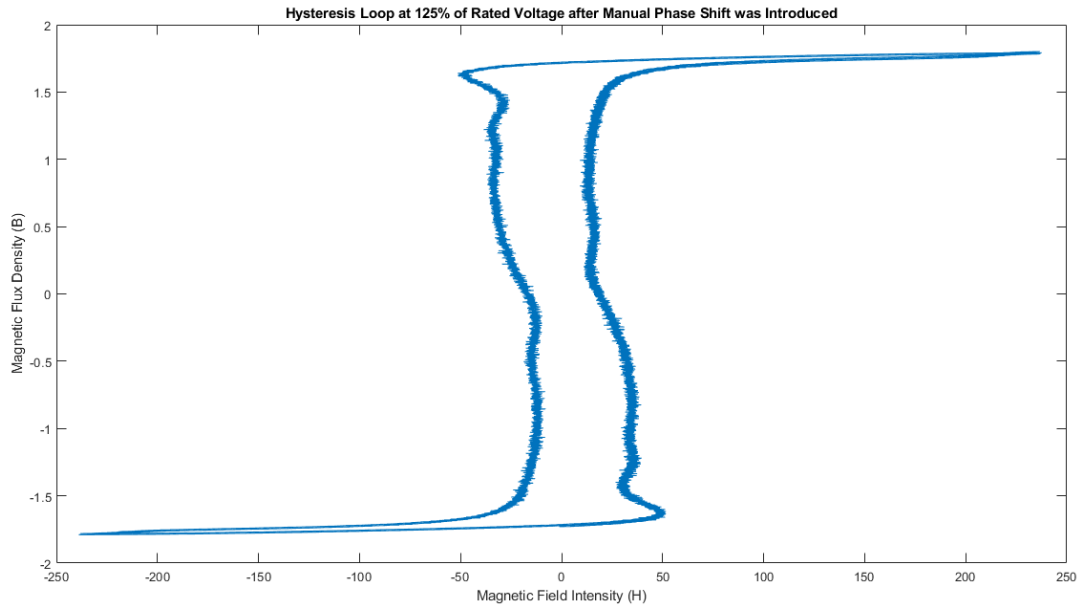


Fig. 3-48: The hysteresis graph after manual phase shift was applied

Given that the phase shift determined by the oscilloscope used to export the data is inaccurate, one might be tempted to conclude that the harmonics in the current are also a function of measurement inaccuracy. However, there are two reasons why this is not the case. Firstly, the *FFT* of the current performed by the N4L and the *FFT* performed on the exported data show a very good correlation. Secondly, Fig. 3-49 shows the voltage and current waveforms measured by the more accurate N4L oscilloscope. When this is compared to Fig. 3-50, which shows the exported waveforms, it can be seen that the shape of the waveforms is very similar. The non-physical behaviour in the shoulder areas of Fig. 3-48 is therefore not caused by measurement error.

An attempt was made to calculate parameters for the inverse *JA* model based on Fig. 3-48 using *PSO*. However, the non-physical data points tend to be problematic for the *MSE* cost function and the resulting parameters also yield a non-physical hysteresis graph.

The non-physical behaviour of Fig. 3-48 is also problematic for the inverse Tellinen model. Recall that the inverse Tellinen makes use of a one-dimensional lookup table to generate the hysteresis loop. It can then be argued that the Tellinen model could replicate the odd behaviour of Fig. 3-48, even if the *JA* algorithm cannot. However, the lookup table in MATLAB requires strictly monotonic data points. To this end, the shoulder regions in quadrants 2 and 4 still present problems.

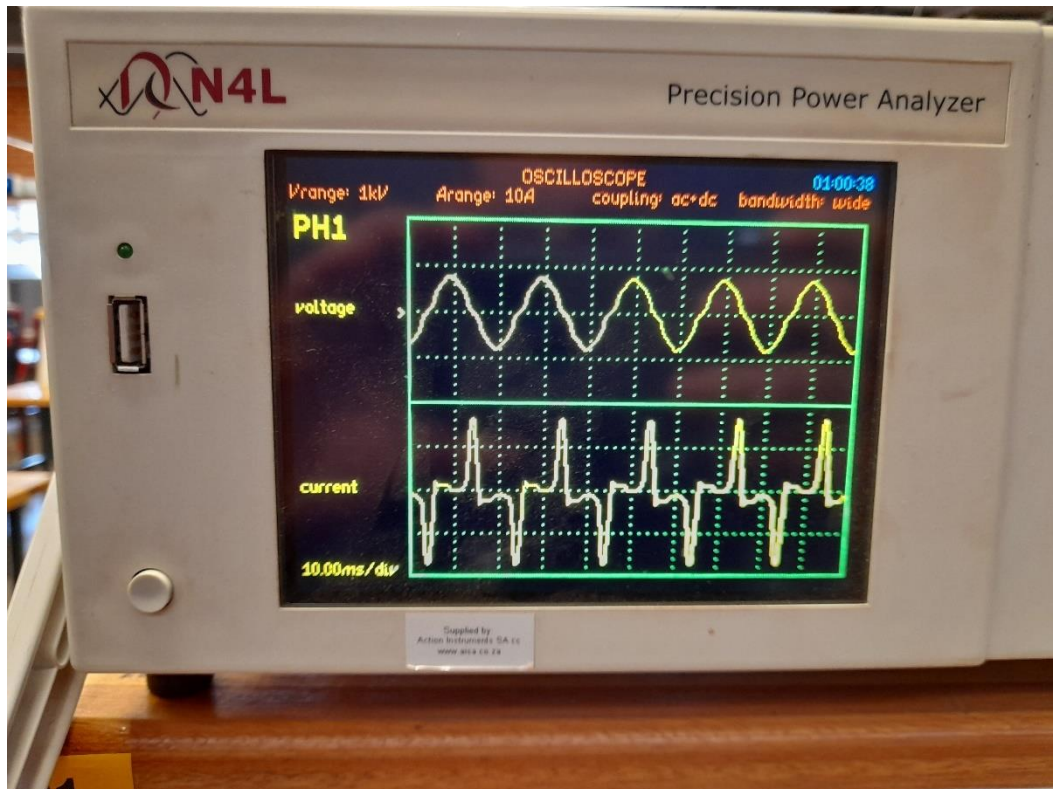


Fig. 3-49: The N4L voltage and current waveforms at 125% of rated voltage

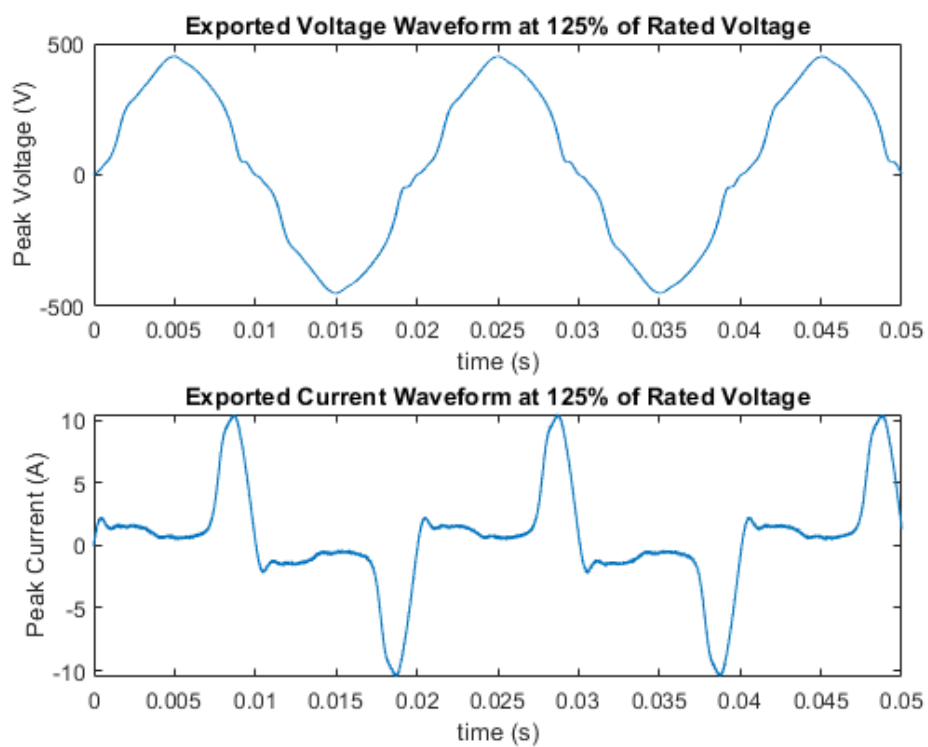


Fig. 3-50: The exported voltage and current waveforms at 125% of rated voltage

A potential solution to this non-monotonic data point problem is to smooth the shoulder regions of the hysteresis loop. The shoulder regions in the first and third quadrant of Fig. 3-48 are monotonic – harmonic current effects are not evident in these regions. Furthermore, the hysteresis loops found in literature generally tend to be symmetrical in the sense that the shoulder areas demonstrate very similar curves. If the shoulder areas in the third and third quadrant can be used to redefine the shoulder areas in the second and fourth quadrants, then a new hysteresis loop with monotonic data points can be created.

The entire process of replacing the data points in the shoulder region of the curve will not be discussed in detail. There are two important steps involved in this process: redefining the problematic shoulder areas and removing non-monotonic data points. The noise induced by the measuring tools also plays a role in the monotonicity of the data. Removing the non-monotonic data results in a significant loss of data points. However, the resulting hysteresis loop is shown in Fig. 3-51. This hysteresis loop looks more similar to the loops encountered in the literature thus far. One would therefore expect that the JA algorithm should be able to replicate this waveform. Furthermore, since the data points are monotonic, the inverse Tellinen model should be able to use this data in its lookup table.

Despite the familiar shape of Fig. 3-51, neither the inverse JA model nor the inverse Tellinen model could replicate these results. The hysteresis loop was used as input to the PSO algorithm but the algorithm was not able to produce sensible results for the five parameters of the model. The resulting hysteresis waveform is shown in Fig. 3-52. The initial magnetization is the first indicator that this is not a sensible combination of parameters. Furthermore, the non-physical negative slope of the loop, in general, is also problematic.

Fig. 3-53 shows the waveforms generated by the inverse Tellinen model. This result is even worse than the inverse JA model presented in Fig. 3-52. The magnetic field intensity (H) waveform does not show any sign of a periodic waveform and the resulting plot of B vs. H does not yield a hysteretic graph.

Two conclusions can be drawn based on the poor fit generated by Fig. 3-52 and Fig. 3-53. Firstly, even though the PSO algorithm managed to reproduce the results of [180] and was proven to be superior to the Leite algorithm, the cost function of the PSO algorithm still requires improvement. Secondly, the logic of the inverse Tellinen model program has an error that is causing numerical instability for the data which was measured on the transformer of interest. This numerical instability was not evident when the results of [81] were being replicated.

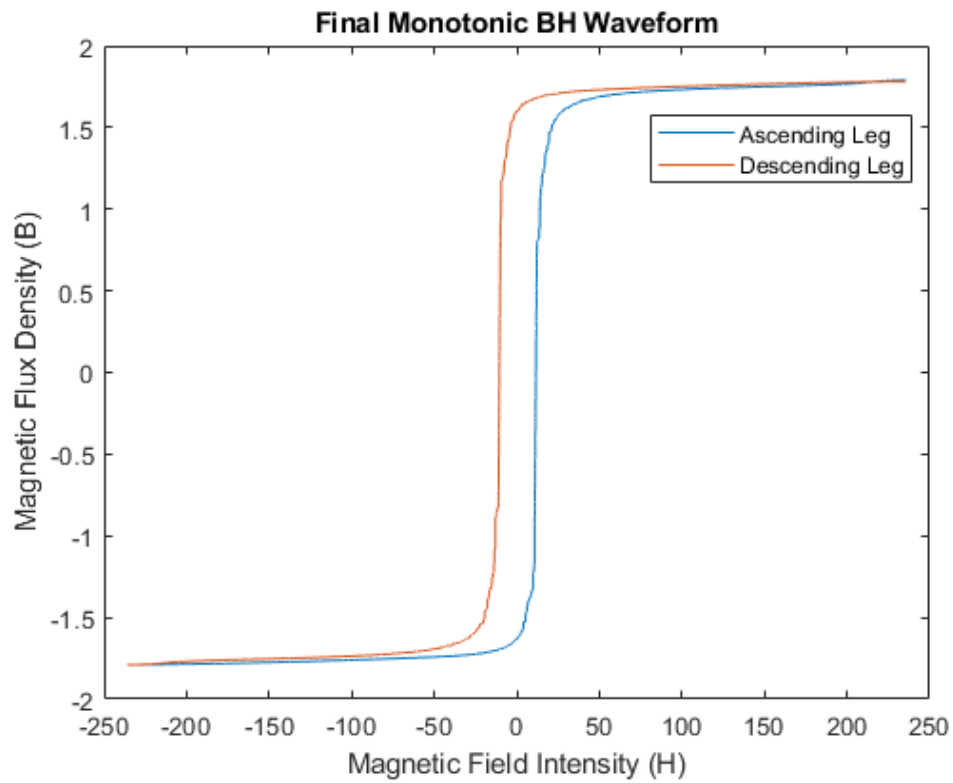


Fig. 3-51: The measured hysteresis loop after monotonicity data point improvement

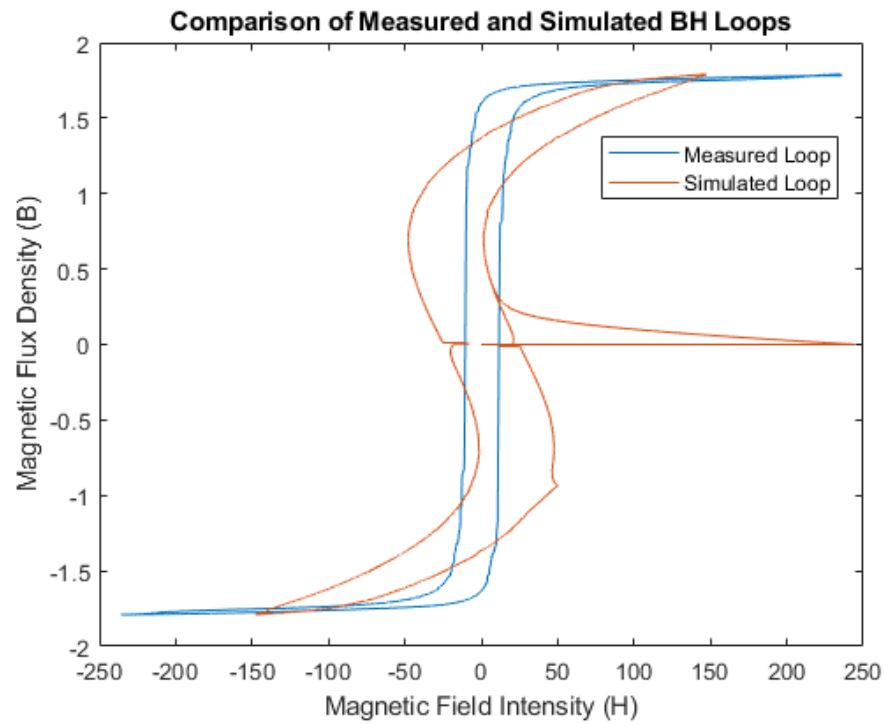


Fig. 3-52: The poor JA hysteresis loop determined using PSO optimization

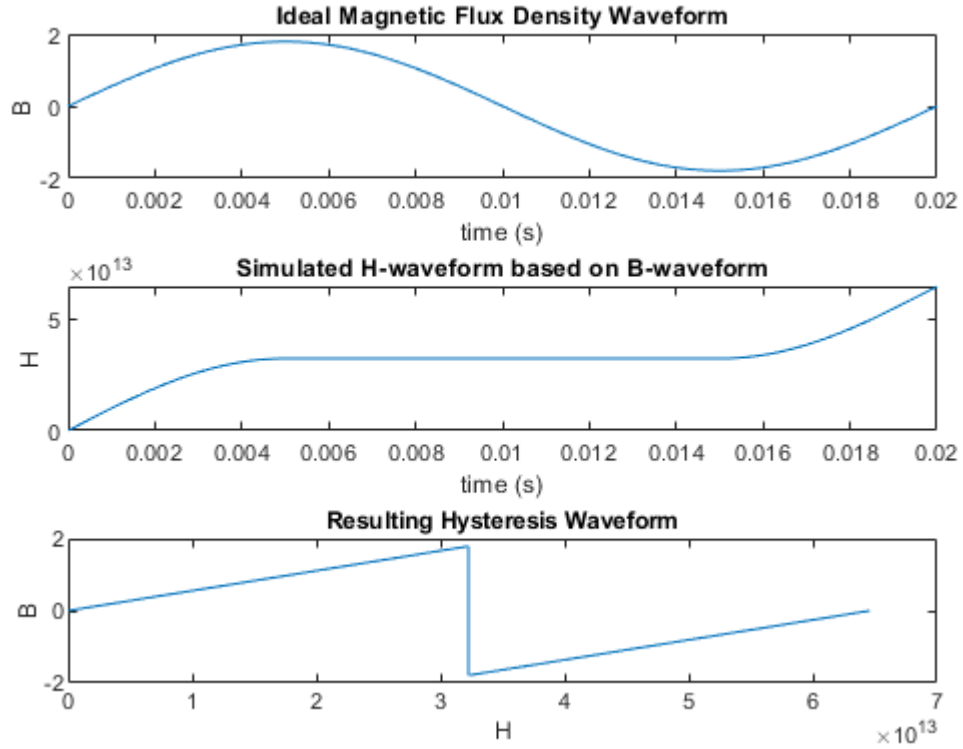


Fig. 3-53: The inverse Tellinen waveforms and hysteresis loop

Time constraints prohibited the author of this report from further investigating these problems with the *PSO* algorithm and the inverse Tellinen model. The author has a couple of hypotheses on alternatives that can be investigated. These hypotheses will be discussed as potential improvements in Chapter 5. Since neither of the hysteresis models could replicate the results measured in the laboratory, a hysteretic transformer model could not be developed. Consequently, the rest of this report will discuss the results and simulations of the π -model developed in section 3.2. One must then bear in mind that the nonlinear inductor π -model could potentially be less accurate than a functional hysteresis model. The effects of residual flux can also not be considered with the nonlinear inductor approach.

3.5 Simulation of OC and SC Results

An important form of verification of the transformer model is its ability to replicate the standard tests conducted on the physical transformer. These include the OC and SC tests. These tests have been discussed extensively in section 3.2 since they were used to develop the π -model. This section of the report will discuss the ability of the developed π -model to replicate the measured results.

3.5.1 OC Test

The OC test is more involved than the SC test and will therefore be discussed first. This is because the OC involves the nonlinear behaviour of the core. This means that the OC test needs to be conducted across a wide range of voltage levels.

An important component of the OC test is the power factor. However, calculating the power factor is not straightforward. Traditionally, the power factor is calculated based on the phase shift between the voltage and the current. Guru [84, Fig. 1.3] shows that this is measured by measuring the time difference between the zero-crossing of the two waveforms. Under the assumption that the waveforms are both sinusoidal, it then stands to reason that this should be equal to the time difference between the peaks of the waveforms.

However, this correlation no longer holds for waveforms that are not sinusoidal. This makes the high harmonic content discussed in section 3.4.3 problematic. Consider, for instance, the voltage and current waveforms shown in Fig. 3-54. These waveforms represent the OC test of one of the permutations of the π -model at 120% of rated voltage. The time difference between the zero-crossing of the voltage and the zero-crossing of the current is 0.0025 s. This correlates to a power factor of 0.7071. The time difference between the peaks of the two waveforms is 0.0049 s. This correlates to a power factor of 0.0314.

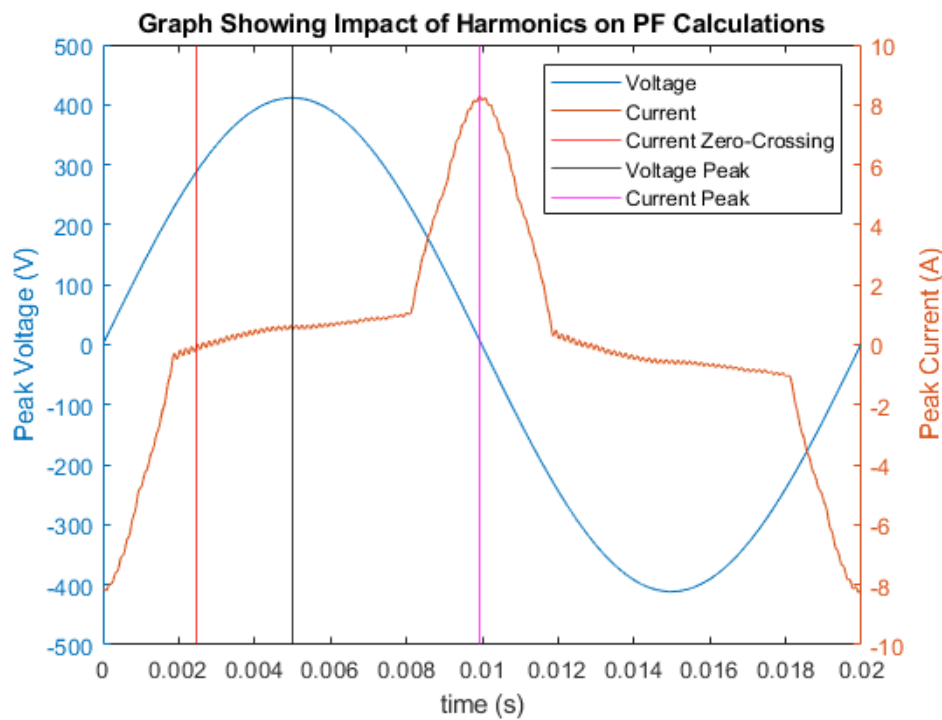


Fig. 3-54: A demonstration of the effect of current harmonics on phase shift interpretation

The power factor is also related to the ratio of the real and apparent power. This means that the real and apparent power can potentially be used to define the power factor. However, the calculation of real power should also be considered carefully. Real power is traditionally related to *RMS* quantities. However, this originates from a mathematical relation based on purely sinusoidal waveforms. Real power can also be calculated as the product of the instantaneous voltage and current values:

$$p(t) = v(t) \cdot i(t) \quad (3-11)$$

This approach should rather be employed, considering the harmonic content of the current. The average real power is then given by the average of the sampled power data points.

The apparent power is traditionally related to the product of the *RMS* voltage and current. This begs the question: does the high harmonic content of the current also influence the apparent power? This uncertainty means that it might be a more realistic approach to calculate the real power based on (3-11) and use the phase shift to define the power factor. The power factor can then be used to calculate the apparent power based on the following relation:

$$pf = \frac{P}{S} \quad (3-12)$$

where

- pf = the power factor;
- P = the real power, and
- S = the apparent power.

It can also be argued that if the high harmonic content of the current does not influence the apparent power calculation, then the apparent power can be calculated based on the product of the *RMS* voltage and current and (3-12) can be used to define the power factor. This power factor can potentially also differ from the phase shift approach discussed earlier. This means that there are three potential interpretations of the power factor.

The *OC* test simulation results need to be compared to the results measured using the N4L oscilloscope. The N4L oscilloscope provides a breakdown of the *RMS* voltage, *RMS* current, apparent power, active power, reactive power and power factor. The approach to power calculations in general, therefore, should be determined by the user manual of the N4L oscilloscope [227]. The user manual [227] indicates that the real power is calculated according to the following relation:

$$P = \frac{1}{n} \sum_{i=0}^{i=n-1} v[i] \cdot a[i] \quad (3-13)$$

where

- P = the real power;
- n = the number of samples;
- v = the voltage, and
- a = the current.

The *RMS* voltage is given by:

$$RMS = \sqrt{\left[\frac{1}{n} \sum_{i=0}^{i=n-1} v^2[i] \right]} \quad (3-14)$$

This relation also holds for the current. The apparent power is then given by

$$S = V_{rms} \times I_{rms} \quad (3-15)$$

and the power factor is defined based on (3-12). The reactive power is given by

$$Q = \sqrt{S^2 - P^2} \quad (3-16)$$

For the sake of consistency, these are the equations used to calculate the results of the simulated *OC* tests. Table 3-12 shows the results of the *OC* tests compared to the measured results. The table shows the results of 4 different transformer models. This is based on the lengthy discussion presented in section 3.2 on the approach to peak current definition e.g. based on exported data versus a $\sqrt{2}$ relation. Section 0 also discussed a black box model which incorporates a parallel capacitance across the magnetization leg based on the *SFRA* response. For clarity and easy reference, the following naming convention is defined for the four different approaches to modelling: Model 1 is the traditional transformer model without the black box capacitance where the peak current of the nonlinear inductor was defined based on a $\sqrt{2}$ relation to the measured *RMS* current. Model 2 is a black box transformer model based on the *SFRA* results where the nonlinear inductor was defined based on a $\sqrt{2}$ relation to the *RMS* current. Model 3 is the traditional transformer model (without capacitance) where the nonlinear inductor peak current was defined based on the peak current of the exported data. Model 4 is the black box transformer model where the nonlinear inductor peak current is defined based on the exported data.

Readers interested in replicating these results should take note of a couple of factors: the initial phase of the voltage source needs to be set to -90° for the integration process of the nonlinear inductor. Furthermore, section 3.2 mentioned that the OC test is conducted between the centre tap and one of the windings on the LV side since this reduced the burden on the source. This means that the rated voltage for the OC test is 242 V.

Table 3-12: The simulated OC test results

		RMS Voltage (V)	RMS Current (A)	Apparent Power (VA)	Real Power (W)	Imaginary Power (var)	Power factor
2.5%	Measured	6.042	0.013602	0.072303	0.082189	-0.03908	0.8797
	Model 1	6.042	0.0111	0.0668	0.0543	-0.0390	0.8121
	Model 2	6.042	0.0106	0.0640	0.0509	-0.0389	0.7947
	Model 3	6.042	0.0111	0.0668	0.0543	-0.0390	0.8119
	Model 4	6.042	0.0106	0.0641	0.0509	-0.0389	0.7943
5%	Measured	12.696	0.024605	0.31239	0.29392	-0.10581	0.9409
	Model 1	12.695	0.0211	0.2677	0.2396	-0.1195	0.8948
	Model 2	12.695	0.0201	0.2548	0.2246	-0.1202	0.8818
	Model 3	12.695	0.0211	0.2677	0.2396	-0.1196	0.8947
	Model 4	12.695	0.0201	0.2549	0.2246	-0.1204	0.8814
10%	Measured	25.549	0.045099	1.1522	1.1328	-0.21051	0.9832
	Model 1	25.547	0.0394	1.0078	0.9701	-0.2731	0.9626
	Model 2	25.547	0.0372	0.9495	0.9097	-0.2720	0.9581
	Model 3	25.547	0.0394	1.0078	0.9701	-0.2731	0.9626
	Model 4	25.547	0.0372	0.9497	0.9097	0.2727	0.9579
20%	Measured	48.636	0.085342	4.1507	3.9528	-1.2662	0.9523
	Model 1	48.636	0.0762	3.7066	3.5160	-1.1734	0.9486
	Model 2	48.636	0.0719	3.4960	3.2972	-1.1621	0.9431
	Model 3	48.636	0.0762	3.7070	3.5160	-1.1745	0.9485

	Model 4	48.636	0.0719	3.4972	3.2972	1.1657	0.9428
30%	Measured	72.587	0.13075	9.4907	8.4256	4.3684	0.8878
	Model 1	72.586	0.1201	8.8295	7.8315	4.0778	0.8870
	Model 2	72.587	0.1139	8.2696	7.3441	3.8014	0.8881
	Model 3	72.586	0.1201	8.7171	7.8315	3.8282	0.8984
	Model 4	72.586	0.1140	8.2742	7.3441	3.8115	0.8876
40%	Measured	95.926	0.17779	17.055	14.143	9.5301	0.8293
	Model 1	95.920	0.1675	16.0694	13.6756	8.4384	0.8510
	Model 2	95.920	0.1598	15.3284	12.8244	8.3961	0.8366
	Model 3	95.920	0.1676	16.0744	13.6755	8.4478	0.8508
	Model 4	95.920	0.1599	15.3389	12.8244	8.4153	0.8361
50%	Measured	121.30	0.23098	28.018	21.848	17.541	0.7798
	Model 1	121.29	0.2224	26.9720	21.8650	15.7926	0.8107
	Model 2	121.29	0.2129	25.8275	20.5042	15.7047	0.7939
	Model 3	121.29	0.2225	26.9815	21.8650	15.8089	0.8104
	Model 4	121.29	0.2131	25.8469	20.5042	15.7366	0.7933
60%	Measured	145.52	0.28353	41.259	30.744	27.516	0.7452
	Model 1	145.51	0.2767	40.2611	31.4725	25.1085	0.7817
	Model 2	145.51	0.2658	38.6838	29.5137	25.0075	0.7629
	Model 3	145.51	0.2770	40.3033	31.4724	25.1763	0.7809
	Model 4	145.51	0.2663	38.7503	29.5137	25.1103	0.7616
70%	Measured	168.76	0.33218	56.058	40.734	38.513	0.7266
	Model 1	168.75	0.3287	55.4701	42.3285	35.8502	0.7631
	Model 2	168.75	0.3164	53.3899	39.6941	35.7052	0.7435
	Model 3	168.75	0.3408	57.5165	42.3285	38.9417	0.7359
	Model 4	168.75	0.3288	55.4891	39.6941	35.7740	0.7153
80%	Measured	191.81	0.37756	72.418	52.285	50.106	0.7220

	Model 1	191.79	0.3785	72.5887	54.6722	47.7501	0.7532
	Model 2	191.79	0.3645	69.9139	51.2696	47.5330	0.7333
	Model 3	191.79	0.4018	77.0682	54.6722	54.3182	0.7094
	Model 4	191.79	0.3883	74.4736	51.2696	54.0161	0.6884
85%	Measured	205.35	0.40225	82.602	59.983	56.790	0.7262
	Model 1	205.34	0.4063	83.4290	62.6743	55.0667	0.7512
	Model 2	205.34	0.3912	80.3222	58.7737	54.7477	0.7317
	Model 3	205.34	0.4373	89.7889	62.6742	64.2961	0.6980
	Model 4	205.34	0.4223	86.7210	58.7736	63.7667	0.6777
90%	Measured	217.51	0.42255	91.907	67.638	62.225	0.7359
	Model 1	217.49	0.4300	93.5218	70.3108	61.6662	0.7518
	Model 2	217.49	0.4142	90.0786	65.9350	61.3737	0.7320
	Model 3	217.49	0.4683	101.8537	70.3108	73.6924	0.6903
	Model 4	217.49	0.4623	100.5384	65.9350	75.8982	0.6558
95%	Measured	230.19	0.44276	101.92	76.550	67.288	0.7511
	Model 1	230.17	0.4530	104.2634	78.7443	68.3387	0.7552
	Model 2	230.17	0.4361	100.3847	73.8436	68.0017	0.7356
	Model 3	230.17	0.4991	114.8860	78.7442	83.6549	0.6854
	Model 4	230.17	0.5019	115.5237	73.8436	88.8418	0.6392
100%	Measured	242.56	0.46656	113.17	86.48	72.996	0.7642
	Model 1	242.56	0.4748	115.1487	87.4304	74.9344	0.7593
	Model 2	242.53	0.4570	110.8444	81.9891	74.5941	0.7397
	Model 3	242.53	0.5303	128.6061	87.4303	94.3159	0.6798
	Model 4	242.53	0.5538	134.3093	81.9891	106.3803	0.6105
105%	Measured	254.22	0.50638	128.73	97.467	84.097	0.7571
	Model 1	254.21	0.4981	126.619	96.0499	82.5033	0.7586
	Model 2	254.21	0.4793	121.8429	90.0721	82.0530	0.7392

	Model 3	254.21	0.5694	144.7407	96.0498	108.2788	0.6636
	Model 4	254.21	0.9196	233.7644	90.0772	215.7125	0.3853
110%	Measured	266.93	0.61684	164.65	112.51	120.22	0.6833
	Model 1	266.92	0.5633	150.3506	105.8960	106.7303	0.7043
	Model 2	266.91	0.5487	146.4509	99.3029	107.6420	0.6781
	Model 3	266.91	0.7627	203.5848	105.8946	173.8767	0.5201
	Model 4	266.91	1.6168	431.5534	99.3294	419.9667	0.2302
115%	Measured	279.44	0.98976	276.58	133.37	242.30	0.4822
	Model 1	279.41	0.6818	190.4982	116.0392	151.0777	0.6091
	Model 2	279.41	0.6661	186.1045	108.8175	150.9756	0.5847
	Model 3	279.41	1.1147	311.4536	116.0463	289.0270	0.3726
	Model 4	279.41	2.3588	659.0738	108.8829	650.0175	0.1652
118%	Measured	287.48	1.6215	466.14	151.00	441.00	0.3239
	Model 1	287.47	0.9242	265.6914	122.8391	235.5896	0.4623
	Model 2	287.47	0.9121	262.2000	115.1946	235.5399	0.4393
	Model 3	287.4730	1.8464	530.7800	122.8708	516.3625	0.2315
	Model 4	287.47	3.0968	890.2600	115.3146	882.7601	0.1295
120%	Measured	290.90	2.0135	585.72	160.17	563.40	0.2735
	Model 1	290.87	1.0893	316.8336	125.7646	290.8037	0.3969
	Model 2	290.87	1.0789	313.8243	117.9384	290.8199	0.3758
	Model 3	290.87	2.4783	720.8646	125.8315	709.7974	0.1746
	Model 4	290.87	3.4869	1014.2	118.0922	1007.3	0.1164

It is difficult to draw a concrete conclusion of the best model from Table 3-12. This would depend on the parameter being used as criteria for selection. One could argue that power factor and current provide good indicators of the suitability of the model. However, even based on these criteria, various models tend to suit the experimental data better at different voltage levels. At low voltage levels, Model 1 and Model 3 tend to show the best correlation for power factor and *RMS*

current amplitude. In the medium voltage levels, model 2 and model 4 show a better correlation to the power factor but model 1 and model 3 show a good correlation to the *RMS* current. At rated voltage, Model 1 shows the best behaviour. At 105% and 110% of rated voltage, Model 2 shows the best correlation. Finally, at deep levels of saturation, as seen at 115% to 120% of rated voltage, Model 3 shows a good correlation to the experimental data.

One model, however, does seem to indicate a generally poor fit to experimental data. Model 4 starts to underestimate the power factor at a voltage level of 80% of the rated voltage. This continues to be the case for all voltage levels up to 120% of rated voltage. At 120% of rated voltage, Model 4 severely underestimates the power factor and overestimates the current. Consequently, the losses are also poorly reflected by Model 4. It can therefore be concluded with certainty that Model 4 should not be used for further simulations.

A peculiar observation can be made based on the correlation of Model 1 vs. Model 2 and Model 3 vs. Model 4. In the case of the first two models, the peak current is defined based on an $\sqrt{2}$ relationship to the *RMS* current. In the case of Model 3 and Model 4, the peak current is based on experimental data. One would therefore expect that similar trends should be seen in Model 1 vs. Model 2 and Model 3 vs. Model 4. This is not the case, as significant deviations are seen between Model 3 and Model 4. For instance, one would expect that if Model 4 severely overestimates the current, Model 3 should also overestimate the current. The difference, however, is remarkable. It is even more interesting to note that while Model 2 generally shows a lower current than Model 1, Model 4 generally shows a higher current than Model 3. This deviation from expected behaviour is a potential indicator that the nonlinear inductor data points are incorrect in one of the models. The author believes this to be true but has not been able to locate the error.

One might initially be tempted to argue that the error must be in the data of Model 4 based on its peculiar behaviour and abnormally high current simulation. However, the author hypothesises that the problem might be in Model 2 based on the following argument: when the capacitor is considered for the model, the calculated inductor current is higher than when the capacitor is ignored. This holds for Model 1 vs Model 2 and Model 3 vs Model 4. Consider for instance Model 3 vs Model 4: at a peak voltage of 406.56 V, Model 3 draws a peak inductor current of 5.06 A. Model 4 draws a peak inductor current of 5.73 A. However, the Frolich equation calculates the voltage based on the current. Consequently, the current data points are generally the same for the two models while the flux or flux linkage data points tend to differ. It then stands to reason that if Model 4 draws a higher current at the same voltage level, then for a given current level, Model 4 should show a lower voltage than Model 3. This is the case for Model 3 vs Model 4. However, this is not the case for Model 1 vs Model 2. At the maximum current defined for the Frolich equation, Model 2 shows a higher voltage than Model 1. The author believes that this is a mistake.

However, despite several attempts to redefine the Frolich parameters for both Model 1 and Model 2, this error could not be fixed. Furthermore, Models 1, 2 and 3 generally tend to fit the experimental data quite well. Therefore, their parameters will not be changed. Rather, the frequency behaviour of these models discussed in section 3.6 should hopefully give a better indication of the most suitable model. Model 4, however, will not be considered further in this report.

3.5.2 SC Test

For the SC test, it is not necessary to evaluate all the models. Guru [84, Fig. 4.25] indicates that the only factors that influence the SC test are the leakage inductance and winding resistance. The voltage is so small that the magnetization branch can be ignored. Since the various models discussed for the OC test all have the same winding resistance and leakage inductance, this means that only one of them need to be tested. To this end, Model 2 will be used for the SC test.

The SC test is conducted from the *HV* side of the transformer. Power constraints of the source limited the current to below-rated current. However, since the SC test is not characterized by nonlinear behaviour, the results are sufficient. Table 3-13 shows the results of the simulated SC test and how they compare to the experimental results.

Table 3-13 shows two different simulation results. The first of these is based on the value of the *DC* resistance test conducted using a 4-wire method. These results were shown in Table 3-2. Note, however, that the power factor and the real power is incorrect.

Traditionally, the winding resistance is calculated based on the results of the OC test when resistance tests cannot be performed on the windings. The winding resistance is then given by:

$$R_{eH} = \frac{P_{sc}}{I_{sc}^2} \quad (3-17)$$

where

- R_{eH} = the equivalent winding resistance referenced to the HV side;
- P_{sc} = the short circuit active power, and
- I_{sc} = the short circuit current.

This approach yields a resistance value that is higher than the results of Table 3-2. This then also means that the leakage inductance needs to be recalculated. Using this approach, it can be found that $R_{eH} = 210.79 \Omega$ and $L_{eH} = 1.836 H$. If these values are used in the SC test, the results shown in the final row of Table 3-13 are obtained. Even so, this approach is deemed to be less accurate

since it overestimates the winding resistance. Consequently, the 4-wire method results will still be used for future simulations.

Table 3-13: A comparison of the simulated SC test results to the experimental results

	RMS Voltage (V)	RMS Current (A)	Apparent Power (VA)	Real Power (W)	Imaginary Power (var)	Power factor
Measured	735.60	1.1970	880.49	302.03	-827.07	0.3430
4-wire Resistance Approach	735.59	1.1970	880.5304	263.8073	-840.08	0.2996
SC Test Results Approach	735.59	1.1970	880.53	302.2767	-827.0187	0.3433

3.6 Simulation SFRA Result

The concept of the *SFRA* has been introduced several times in this report. The results of the *SFRA* have also been briefly discussed in section 0 where the black box model was developed. This section of the report serves to focus more on the measured *SFRA* results and compare them to the simulated *SFRA* results of the various models developed thus far. The discussion will be broken down into *OC* and *SC SFRA* results.

3.6.1 OC SFRA

The measured *OC SFRA* was presented in Fig. 3-7 when the black box transformer model was developed. However, this section of the report will regularly refer to this figure. Therefore, for the sake of easy reference and accessibility, the *SFRA graph* is presented again in Fig. 3-55.

A couple of important notes should be made regarding the *SFRA*. The Doble M5400 analyzer used to perform the test applies a peak-to-peak voltage of 20 V to the transformer [228]. It can measure in a frequency range of 10 Hz to 25 MHz. Fig. 3-55 is limited to a frequency range of 2 MHz. It defaults to plotting 1000 points at a 1.2% logarithmic spacing of the data points. For this analysis, however, the author is not interested in the behaviour of the transformer at frequencies as high as 25 MHz. It has been repeatedly emphasized that ferroresonance is a low-frequency phenomenon below 2 kHz. Consequently, the simulation *SFRA* will be limited to a frequency



Fig. 3-55: The impedance amplitude of the SFRA done on the transformer in question with the LV winding open

range of 10 kHz, which is the first decade above 2 kHz. Furthermore, simulating 1000 different data points has a significant impact on simulation time. MATLAB defines a default of 50 data points when the *logspace* instruction is used. It was found that 50 points are sufficient to indicate the shape of the *SFRA* plot without suffering from extreme simulation times.

The simulated *SFRA* is a reflection of the total impedance as seen from the source. To determine the impedance in simulation, the input voltage and current are measured. The peak value of the voltage divided by the peak value of the current indicates the impedance. The *SFRA* simulation is conducted in the Simscape Specialized Power Systems environment rather than the Simscape Electrical environment. This is because the Specialized Power Systems environment holds a significant simulation speed advantage. The *SFRA* for Model 1 was conducted in both environments and it was found that the results are identical. It must then also be noted that the nonlinear inductor in the Simscape Specialized Power Systems environment takes flux linkage and current as input rather than flux and current. The reader should also note that the variable ratio transformer in the Simscape Specialized Power Systems environment causes problems for the *SFRA*. The variable-ratio transformer is modelled as a current source, which cannot be connected in parallel to the capacitor of Model 2. The solution to this problem is to connect a resistor in series with the capacitor. However, this has a significant impact on the *SFRA*. For simulation purposes, the variable-ratio transformer should therefore be removed and all parameters should be referred to the HV side. The *OC SFRA* is conducted from the *HV* side with the *LV* winding open. A comparison of all the simulated *SFRA* graphs is shown in Fig. 3-56.

The *SFRA* of the pure black-box model is also shown in Fig. 3-56. This model contains a constant inductor, capacitor and resistor with the parameters determined in section 0. This black box model can be considered to be the reference for the comparison of the other three models since the black-box model shows the best correlation to Fig. 3-55.

Fig. 3-56 shows a plot of the black-box model with and without the leakage inductance of the transformer. These are shown by the solid blue and dotted red lines respectively. The behaviour of the models is identical up to a frequency of 2 kHz. Thereafter the black-box model with the leakage inductance (blue line) shows a sharp dip followed by a sharp rise in the impedance. It can therefore be concluded that this dip and rise is caused by the leakage inductance. It also leads to the conclusion that the leakage inductance does not play a role in the frequency behaviour below a frequency of 2 kHz in this case. It can also be noted that Fig. 3-55 shows similar behaviour, though at a much higher frequency.

The most prominent conclusion from Fig. 3-56 is that Model 2 shows the best *SFRA* behaviour. Models 1 and 3 do not remotely resemble the frequency behaviour measured on the actual transformer. This leads to an important conclusion – the parallel capacitance which models the turn-to-turn and disc-to-disc capacitance is critical to the frequency behaviour of the transformer at low frequencies. This also means that based on the combined results of the *OC* test in section 3.5.1 and the *OC SFRA*, Model 2 is undoubtedly the best reflection of the actual transformer. It

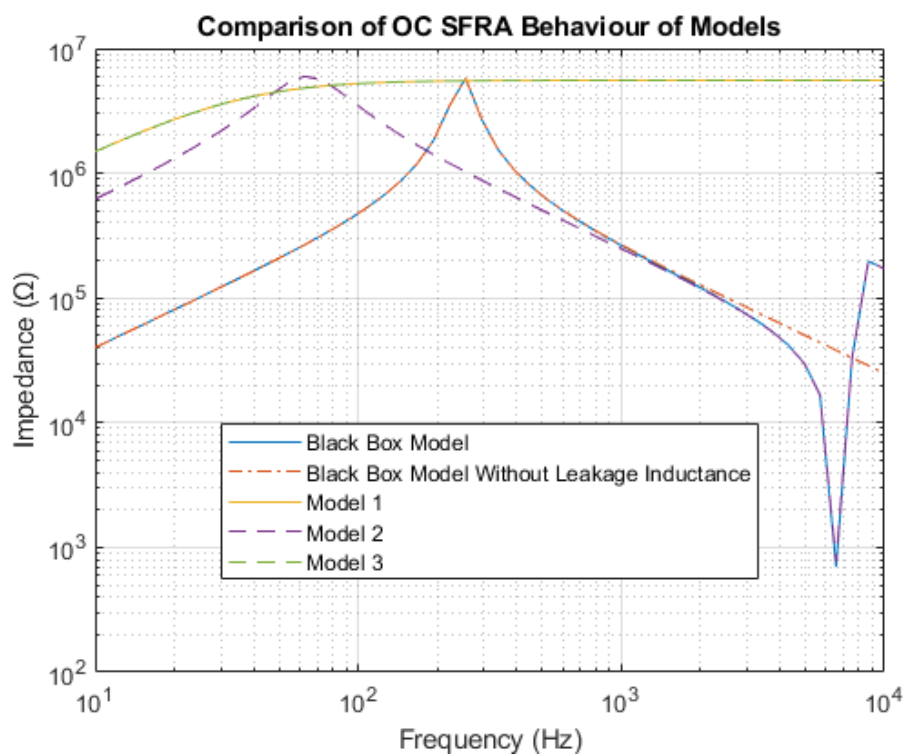


Fig. 3-56: A comparison of the *OC SFRA* plots of Models 1-4

can be argued that the black-box model is even better than Model 2, however, Chapter 4 will prove that this is not true for ferroresonance studies.

This conclusion on the effect of disc-to-disc and turn-to-turn capacitance is also important concerning claims made by literature. Section 2.15 stated that the literature claims that the magnetization inductance dominates the frequency behaviour at low frequencies. Fig. 3-56 proves that this is not entirely true since the disc-to-disc and winding-to-winding capacitance cannot be ignored.

Model 1 and Model 3 show identical behaviour up to a frequency of 10 kHz. This is to be expected since the applied voltage of the *SFRA* is so low. Recall from the discussion in section 3.5.1 that Model 1 and Model 3 should show similar behaviour at voltages below 70% of rated voltage since the current harmonics are not significant.

Despite showing better results than Model 1 and Model 3, Model 2 still shows some deviation from the experimental results – particularly with regards to the frequency of the peak of the impedance. However, it should be recalled that the literature study claimed that residual magnetism can have an influence on the *SFRA* at frequencies up to 5 kHz. The residual magnetism tends to shift the waveform to the left or the right. Residual magnetism was not considered in the simulation.

Recall from section 2.15 that Cigre [210] stated that the complex assembly of the windings and mechanical tolerances makes the comparison of simulated *SFRA* plots to measured plots very difficult. Based on this claim it can be concluded that the results of Model 2 are very good.

Fig. 3-57 shows a comparison of the *SFRA* of Model 2 with and without the core loss resistance. This figure shows that the core loss resistance does not influence the location of the resonance and anti-resonance frequencies – it only influences the value of the impedance amplitude at those frequencies.

Section 3.2.3 discussed the capacitances between the HV and LV windings and ground. The *SFRA* behaviour of the transformers is not influenced by any of these capacitances up to a frequency of 10 kHz. If the capacitances are removed altogether, the shape of Fig. 3-56 does not change. However, these capacitances still play a very important role, as will become evident in Chapter 4.

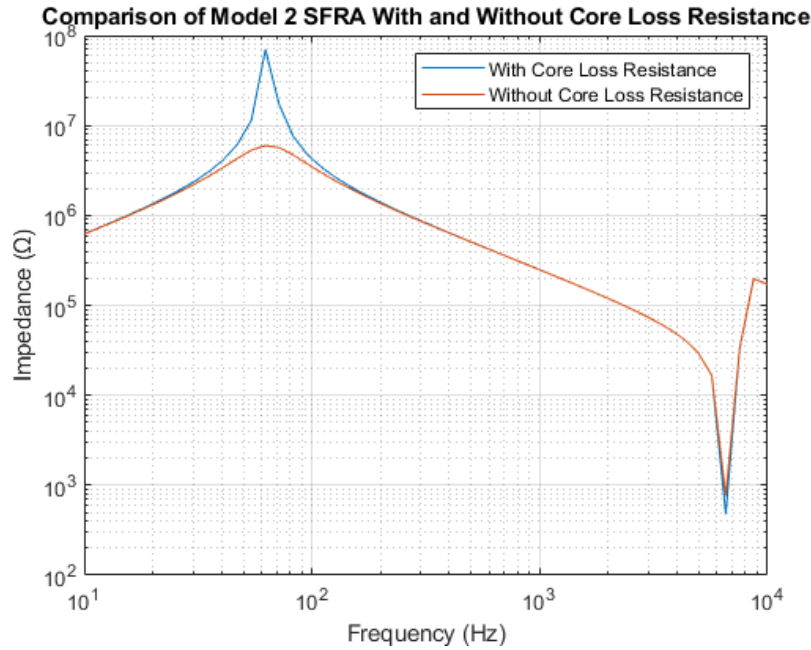


Fig. 3-57: The influence of core loss resistance on the OC SFRA behaviour

3.6.2 SC SFRA

The SC SFRA is also done from the HV winding with a short-circuit applied across the LV winding. The results of the SC SFRA done on the physical transformer are shown in Fig. 3-58. A comparison of the simulated SC SFRA for Models 1, 2 and 3 is shown in Fig. 2-59.

The results of Fig. 3-59 are interesting and unexpected. Recall that for the SC test in section 3.5.2, it was not necessary to compare the three models. This is because Model 1, Model 2 and Model 3 only differ in their core characterization. The winding resistance and the leakage inductance of the three models are the same. Fig. 3-59 proves that even though this assumption is true at 50 Hz, it does not hold for high frequencies. This then also leads to the conclusion that the core cannot be ignored for the SC characterization at high frequencies. Since the main difference between Model 2 and the other models is the parallel capacitor across the core, one must assume that this parallel capacitor is the cause of the deviation seen in Fig. 3-59. As was the case with the OC SFRA, the results of Model 1 and Model 3 are identical.

When Fig. 3-59 is compared to Fig. 3-58, it is clear that Model 2 is the most accurate in this case as well. Model 2 shows a very good correlation to the experimental results of Fig. 3-58. All three models show a good correlation to the experimental results up to a frequency of 1 kHz. From 1 kHz, Model 2 starts to deviate from Models 1 and 3 and shows a sharper rise in amplitude than Fig. 3-58. There is a discrepancy in the frequency and magnitude of the peak impedance. Model 2 reaches its peak value at a frequency slightly above 5 kHz. The actual peak in Fig. 3-58 is closer

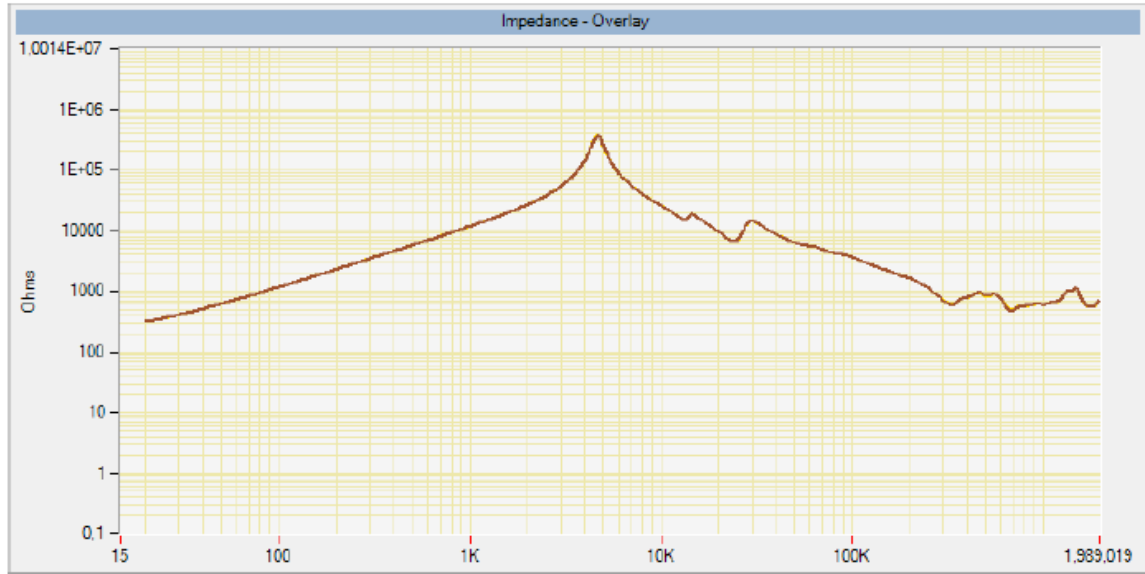


Fig. 3-58: The impedance amplitude of the SFRA done on the transformer in question with the LV winding short-circuited

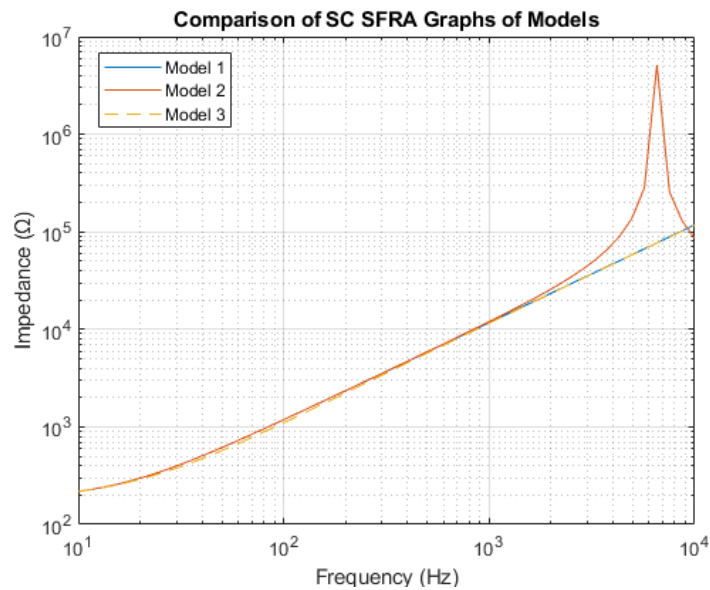


Fig. 3-59: A comparison of the simulated SC SFRA results for Models 1, 2 and 3

to 4 kHz. However, it must be remembered that frequency-dependent components of the windings were ignored for reasons given in section 0. If Foster and Cauer circuits were used to represent those frequency-dependent components, Model 2 could show a better correlation to Fig. 3-58 in both frequency and amplitude.

It is interesting to note that for the *SC SFRA* simulation, the position of the winding inductance and leakage reactance within the circuit does not matter. The winding resistance can be lumped together and the results will remain the same. In other words, the secondary winding resistance

does not have to be located after the magnetization branch. The leakage inductance for the π -model is already lumped together.

As was the case for the OC SFRA, the capacitance between the HV and LV winding does not affect the SC SFRA. This is also true for the capacitance between the windings and ground.

A final interesting note can be made on the effect that the leakage inductance and winding resistance have on the SC SFRA. Fig. 3-60 shows the impact that variation in the leakage inductance can have on the SC SFRA. It shows that an increase in the leakage inductance tends to increase the maximum impedance and shift the peak to lower frequencies. This is not ideal since the graph of Model 2 needs to be shifted to the left and needs to decrease. In other words, changing the leakage inductance of Model 2 will have contradicting effects concerning the target frequency and target impedance magnitude. A change in the winding inductance tends to shift the peak magnitude up or down without changing the frequency of the peak amplitude. However, it is not possible to find a combination of winding resistance and leakage inductance that will change the behaviour of Model 2 to that seen in Fig. 3-58. Since the core influences these high frequencies, it can be concluded that it may be necessary to edit the core parameters to obtain an optimal solution. This optimization process, however, will not be investigated further in this report.

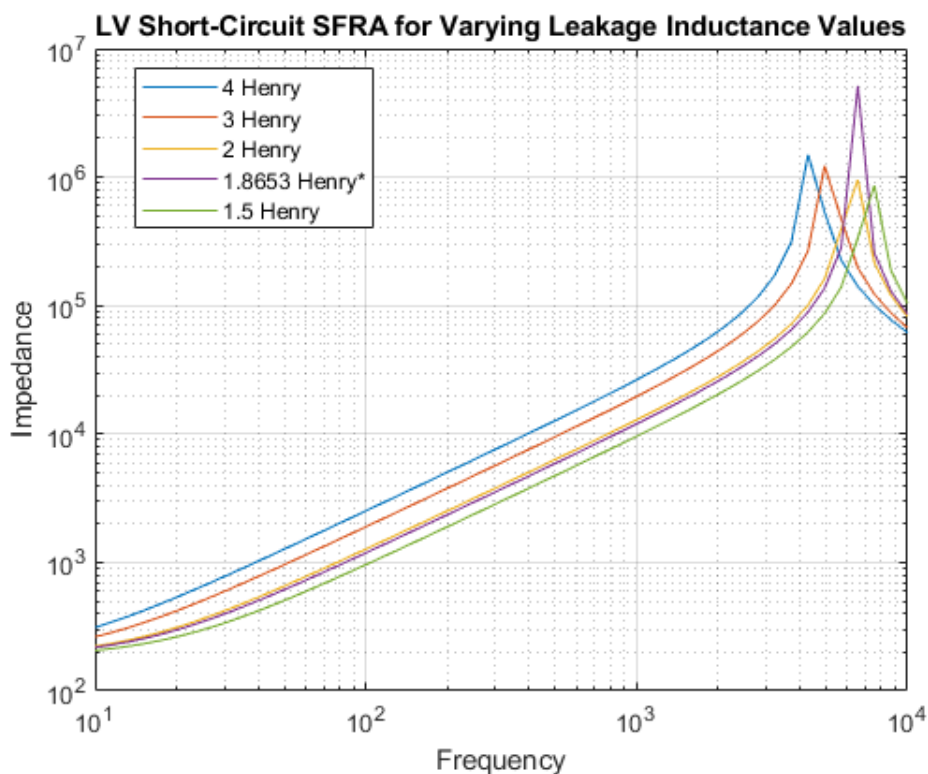


Fig. 3-60: A comparison of the SC SFRA results for Model 2 with varying values of leakage inductance

One other interesting observation can be made from Fig. 3-60. The purple graph shows the *SFRA* of the actual leakage inductance calculated for Model 2. It is also marked by a star in the legend. It can be seen that this value of leakage inductance does not follow the general trend discussed in the previous paragraph. It shows a significantly higher impedance than when the leakage inductance is 2 H. This potentially means that the calculated leakage inductance from the SC test shows anti-resonance behaviour with the rest of the components in the transformer model.

3.7 Power System Model

Thus far this Chapter has discussed transformer modelling in detail. However, it is also important to reflect on the power system which will be used to simulate transients. A power system has been constructed which is a good representation of the typical power system where 32 kVA distribution transformers are used. Fig. 3-61 shows this power system in the Simscape Electrical environment.

The source on the left-hand side of Fig. 3-61 is not ideal. The short-circuit power level of the transformer is defined as 667.087 MVA and the X/R ratio is defined as 3. This gives a short-circuit fault level of 1 kA and an X/R ratio of 3 at the terminals of the single-phase distribution transformer. The source operates at an *RMS* line-to-line voltage level of 132 kV. A 10 MVA three-phase transformer steps the voltage down from 132 kV to 24 kV. The secondary voltage of the transformer is chosen as 24 kV to compensate for voltage drops across the power system. This gives a rated voltage of 22 kV at the terminals of the distribution transformer, which is the main focus of the study. The parameters of the three-phase transformer are shown in Table 3-14.

The three-phase transformer is followed by a 9 km overhead line with parameters as seen in Table 3-15. The origin of these parameters will be discussed shortly. A series three-phase load is connected to ground after this line, which draws an *RMS* current of 140 A. This is nearly 50% of the capacity of a Hare conductor. The load consists of a resistor with a value of 73.33 Ω and an inductor of 134.75 mH. This gives a phase shift of approximately 30°.

After the series load, another 1 km overhead line is connected to the system. The parameters of this overhead line are identical to the 9 km section before the load, as shown in Table 3-15. The π transformer model is connected between phases A and B. Phase C is left open for simplification. The secondary of the transformer is left open since this represents the worst-case scenario for ferroresonance analysis. A switch is connected to phase A and phase B before the π -transformer model.

The parameters in Table 3-15 are based on Eskom guidelines for the geometry of overhead lines at 22 kV. Table 3-16 shows an extract of the minimum clearances dictated by Eskom [229]. The

worst-case scenario is represented by lines which are above roads and railway lines. Therefore, the assumption is made that the clearance from the ground for the overhead line is 6.4 meters and that the lines are 0.9 meters apart.

Table 3-14: The parameters of the three-phase transformer seen in Fig. 3-61

Rated Power	10 MVA
Rated Electrical Frequency	50 Hz
Winding 1 Connection Type	Delta 1 o'clock
Winding 2 Connection Type	Wye with neutral port
Primary Rated Voltage	132 kV
Secondary Rated Voltage	24 kV
Core Type	Three-phase three-limb
Primary Winding Resistance	0.00242 pu
Primary Leakage Reactance	0.04 pu
Secondary Winding Resistance	0.00242 pu
Secondary Leakage Reactance	0.04 pu
Shunt Magnetizing Resistance	1128 pu
Shunt Magnetizing Reactance	182 pu
Zero Sequence Reactance	0.068 pu
LV Grounding Resistor	38.5 Ω

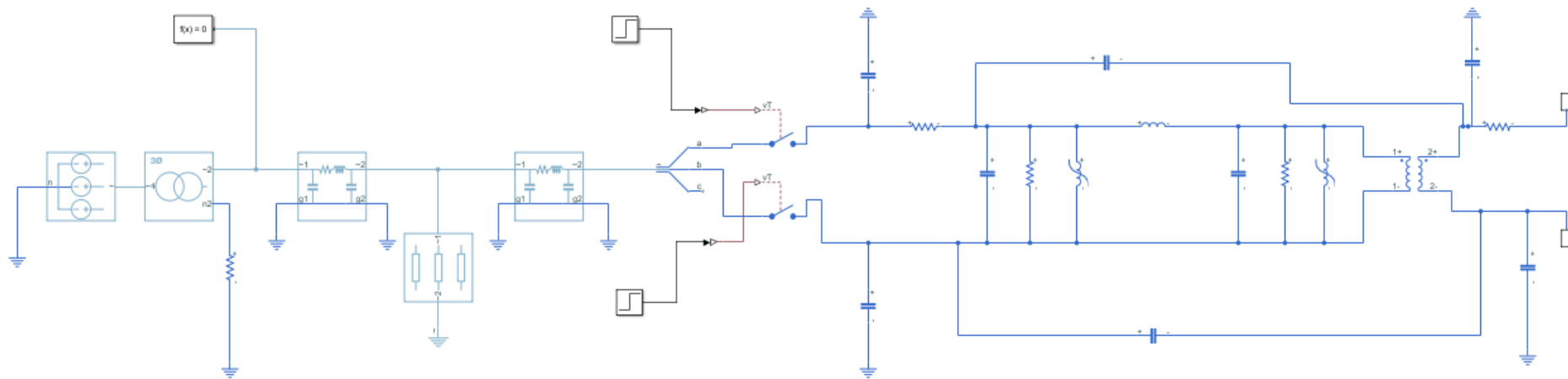


Fig. 3-61: The circuit diagram of the power system used for simulating transients

Table 3-15: The parameters for the overhead lines in Fig. 3-61

Resistance	0.335573 Ω/km
Inductance	2.406666 mH/km
Mutual Inductance	1.326666 mH/km
Line-line Capacitance	2.1865 pF/m
Line-ground Capacitance	4.5106 pF/m
Mutual Resistance	0.05826 Ω/km
The number of segments:	1

The Specialized Power Systems environment in Simscape has a power line parameter calculation tool. This tool can be used to calculate the 3x3 RLC matrices for power lines based on geometric input. The accuracy of this built-in library was verified in two ways. Firstly, the output was compared to the parameters calculated by ATP and the results matched perfectly. Secondly, the capacitance matrix was calculated by hand based on the method of images as described in [204] and the results were also identical. It can then be assumed that the inductance and resistance matrices are also correct.

Table 3-17 shows the input used for the Power Line Parameters tool of MATLAB. The ground resistivity is left at a default value of 100 ($\Omega\cdot\text{m}$) since [205] claimed that power line parameters are not sensitive to ground resistivity below 5 kHz. The X-coordinates and Y-coordinates of the conductors are based on Table 3-16. The outside diameter of the bundle and the DC resistance is based on the standard data for Hare lines. The Geomagnetic Mean Radius of Hare lines was calculated to be 0.51365 by [230].

Table 3-16: The Eskom guidelines for overhead line clearances for Hare/Oak conductors

Maximum rated phase to phase voltage (kV r.m.s)	Clearance				
	To ground outside townships	To ground inside townships	Above roads and railway lines	To communication, power lines and other lines	To buildings and structures not forming part of power lines
24	5.2	5.5	6.4	0.9	3.0

Table 3-17: The input for the power line parameters tool in MATLAB

Units	Metric
Ground Resistivity	100 (Ω .m)
Nominal Frequency	50 (Hz)
Phase Conductors (bundles)	3
X(m)	[-0.9, 0, 0.9]
Y tower (m)	[6.4, 6.4, 6.4]
Y min (m)	[6.4 6.4 6.4]
Ground Conductors	0
Conductor types	1
Internal conductor inductance evaluated from:	Geomagnetic Mean Radius
Conductor Outside Diameter	1.416 cm
T/D ratio	Not Relevant. Left Standard
GMR	0.51365 cm
DC Resistance	0.27777 (Ω /km)
Number of conductors per bundle	1
Bundle Diameter	Not relevant. Set to 1.416 cm
Angle	Not relevant. Left at 45°
Frequency Range (Hz)	[-2, 5, 141]
Line Length	10 km

The Power Line Parameters Tool gives the RLC matrices and the positive- and zero-sequence components as output. The Fortesque transform was applied to the RLC matrices to verify the sequence component values. Finally, the parameters seen in Table 3-15 are calculated from the positive and zero sequence components based on the MATLAB documentation of the three-phase transmission line in the Simscape Electrical environment

3.8 Conclusions

This Chapter was focused on four main points of discussion: 1) Literature Verification, 2) Transformer Model Development, 3) Transformer Model Verification and 4) Power System Modelling.

The verification of literature is often hindered by a lack of data in academic journals. Two different sources of literature were verified in this Chapter. Section 3.1 verified the results of [95] and proved that the π -model is superior to the T-model in ferroresonance modelling. Section 3.4.1 verified the results of [81] and proved the ability of both the inverse JA and inverse Tellinen models to replicate hysteresis loops found in the literature.

Transformer model development involved an extensive discussion of both the π -model and hysteresis. The π -model found in [95] was modified to reflect the behaviour of the transformer being studied in this report. Capacitances were added to the model to make it more accurate at higher frequencies. Four different models were developed based on varying approaches to the definition of nonlinear inductor data points. Two of these models also contained an additional parallel capacitor across the core of the transformer model based on the results of an *SFRA* test. This modelling approach can be considered to be a black-box approach. Transformer models based on hysteresis were also developed in section 3.4. However, high harmonic content proved to be problematic for the determination of hysteresis loop parameters. A particle swarm optimization algorithm was written and implemented to improve parameter estimation, but the cost function used in the algorithm was insufficient to overcome the harmonic obstacles. The hysteretic transformer model was then discarded.

Sections 3.5 and 3.6 focused on verifying the accuracy of the four developed π -models. The results of the *OC* test, *SC* test and experimental *SFRA* plots were used as criteria. Section 3.5 conclusively proved that Model 4 was inaccurate in replicating the measured *OC* test results. Section 3.6 proved that Model 2, the modified black-box model, showed the best frequency behaviour.

Section 3.7 discussed a power system which is a good reflection of the typical conditions being studied in this report. A detailed breakdown of the source, the transformers and the transmission lines and their parameters were presented.

The reader should now have confidence in the ability of Model 2 to replicate the behaviour of the actual distribution transformer. Model 2 performed well in the *OC* test as well as the *SFRA* test. Furthermore, Model 2 is based on the ferroresonant π -model presented in [95] and is therefore

capable of modelling ferroresonance. This model can now be used to investigate the resonant overvoltages introduced in Chapter 1, which is the focus of the next Chapter.

CHAPTER 4 RESULTS

Chapter 3 developed a π -model with a nonlinear inductor based on literature where ferroresonance was simulated with a similar model. This Chapter aims to verify whether ferroresonance is a potential cause of the unexpected overvoltages reported in Chapter 1.

Before the transformer is inserted into the power system discussed at the end of Chapter 3, it is first necessary to verify whether the transformer is susceptible to ferroresonance. To verify this, a simulation similar to that found in [95] is performed. This simulation indicates that there are certain values of series capacitance that will initiate ferroresonance.

Having verified that the transformer is susceptible to ferroresonance, the Chapter proceeds to investigate the range of capacitance values that cause ferroresonance. A bifurcation diagram is drawn to provide a range of capacitance values that can be dangerous.

The simulation that proves that the transformer is susceptible to ferroresonance, is an open-circuit simulation. This is because an open LV winding results in the worst-case scenario for ferroresonance. This Chapter also investigates the load that is required on the secondary winding to avoid ferroresonance.

Sections 4.4 and 4.5 also investigate the influence that core losses and source voltage have on ferroresonance initiation. This is based on the literature study of Chapter 2, which specifically mentioned that these are two factors that can influence ferroresonance.

Following these detailed simulations, the transformer is inserted into the power system discussed at the end of Chapter 3. This simulation replicates the results initially introduced in Chapter 1 – namely that overvoltages are reported on the secondary of the transformer when only one of the phases of the three-phase system has been closed. This resonance path is investigated further in this Chapter so that conclusions can be drawn on this resonance phenomenon in Chapter 5.

4.1 Transformer Ferroresonance

Chapter 3 established that Model 2 shows the best overall correlation to the measurements taken on the actual transformer. It does not represent the OC results at low voltage levels as well as Models 1 and 3 did. However, it does show good results at rated voltage and it undoubtedly shows the best frequency behaviour.

Chapter 3 did not verify whether Model 2 is susceptible to ferroresonance. This is the first question that needs to be answered before the model is inserted into a power system. To test its

ferroresonance susceptibility, a simulation is done which is similar to that found in [95] and replicated in section 3.1.

Fig. 4-1 shows a MATLAB circuit where Model 2 is connected in series with a 20 nF capacitor. Fig. 4-2 shows the voltage waveform of the open-circuited secondary winding at rated voltage. Recall that MATLAB requires peak voltage as input. This means that the input voltage on the *HV* side is $V_{in} = 22 \cdot \sqrt{2} = 31.11 \text{ kV}$. The initial phase angle of the source is 0° . The maximum voltage measured on the *LV* side is $V_{out} = 1.685 \text{ kV}$. This is an overvoltage of 2.46 pu.

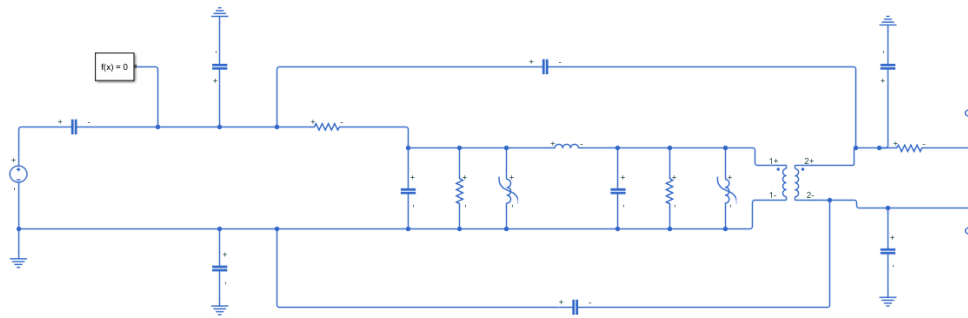


Fig. 4-1: Model 2 with a 20 nF Series Capacitor to model Ferroresonance

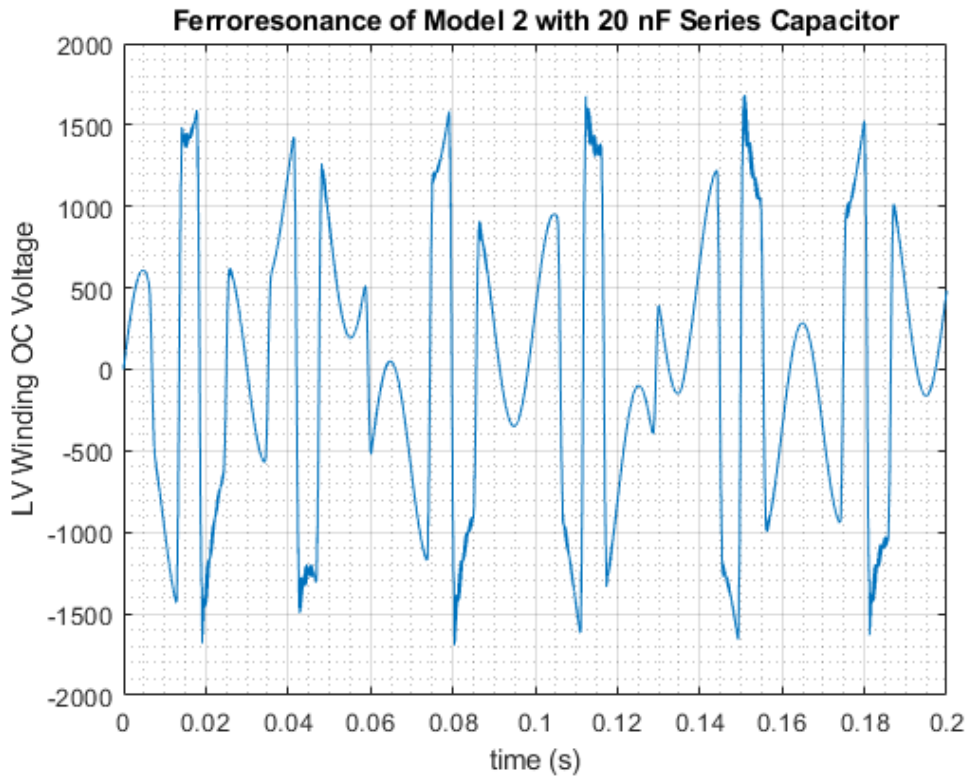


Fig. 4-2: The open-circuit secondary voltage waveform of Fig. 4-1 at rated voltage

If the capacitance in Fig. 4-1 is changed to 20 μF , which is the value initially employed by [95], then the waveforms change to that seen in Fig. 4-3, where the maximum overvoltage is only 1.086 pu.

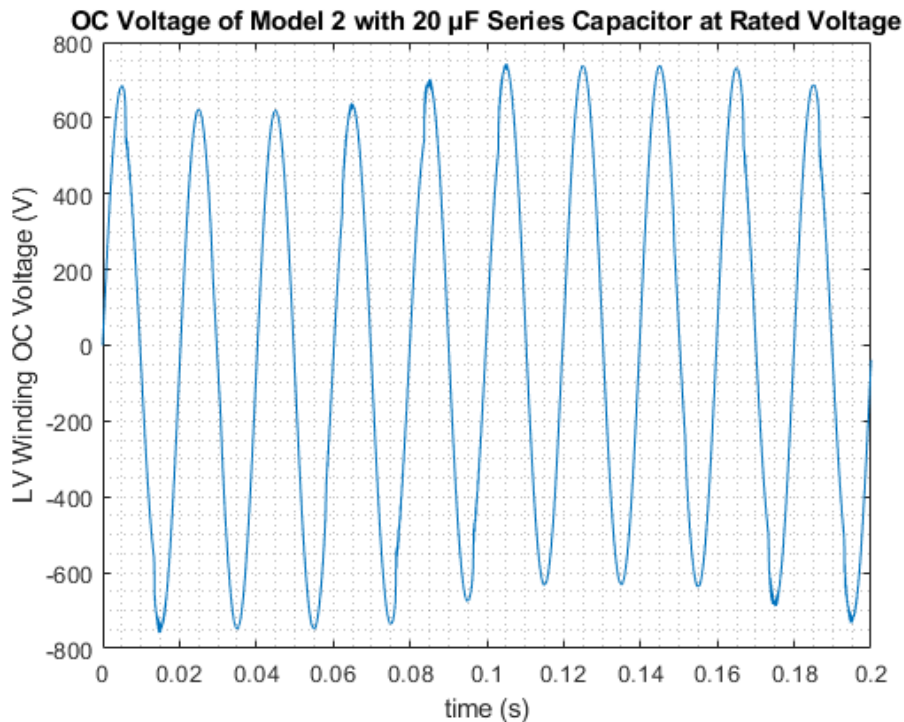


Fig. 4-3: The open-circuit secondary voltage at rated voltage when the capacitor in Fig. 4-1 is 20 μF

4.2 Influence of Capacitance on Ferroresonance

Based on the results of the previous section, it can be concluded that there is a range of capacitance values for which the transformer is susceptible to ferroresonance. At low capacitance values, the transformer shows definite ferroresonance and high overvoltages. However, Fig. 4-3 could potentially indicate that as the capacitance increases, the overvoltages are reduced.

Section 2.7.3 spoke about bifurcation diagrams, which can be a very useful tool to determine the range of dangerous series capacitance values. A bifurcation sweep across a capacitance range can indicate ferroresonance and dangerous overvoltages.

Fig. 4-4 shows the bifurcation diagram for a variation in capacitance. A detailed discussion of the results of this bifurcation diagram will be given in Chapter 5, along with its potential implication. However, it is clear from Fig. 4-4 that there is a range of capacitance values that drives the transformer into ferroresonance. It is also clear that the overvoltages in some cases can reach extreme values – almost 10 pu.

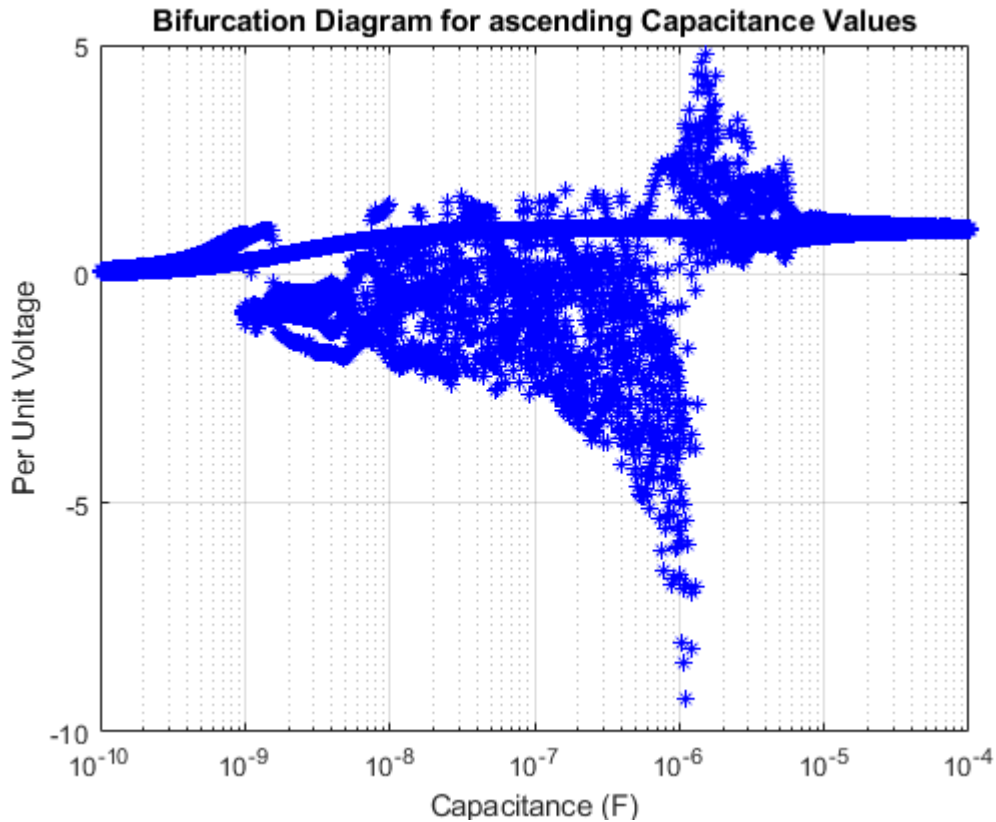


Fig. 4-4: The ferroresonance bifurcation diagram for a variation of series capacitance

4.3 Influence of Load on Ferroresonance

The ferroresonance simulations in sections 4.1 and 4.2 are done with an open *LV* winding. Open circuit conditions represent the worst-case scenario for ferroresonance studies. This begs the question: how much load is required on the secondary of the transformer to eliminate ferroresonance?

The answer to this question depends on how extreme the ferroresonance is. Consider Fig. 4-2, which shows the ferroresonance waveforms when a 20 nF capacitor is connected in series with the transformer. If a purely resistive load is connected across the *LV* winding, which draws 4% of rated current, then the waveforms change to that seen in Fig. 4-5. However, if one considers a series capacitor of 1 μ F, which represents some of the worst-case overvoltages in Fig. 4-4, then a load of 30% is not sufficient to dampen out the overvoltages.

A bifurcation diagram of the overvoltages as a function of the *LV* winding load is shown in Fig. 4-6. This bifurcation diagram is drawn for a series capacitance of 1 μ F, which induced the worst overvoltages in Fig. 4-4. Conclusions based on Fig. 4-6 will be made in Chapter 5.

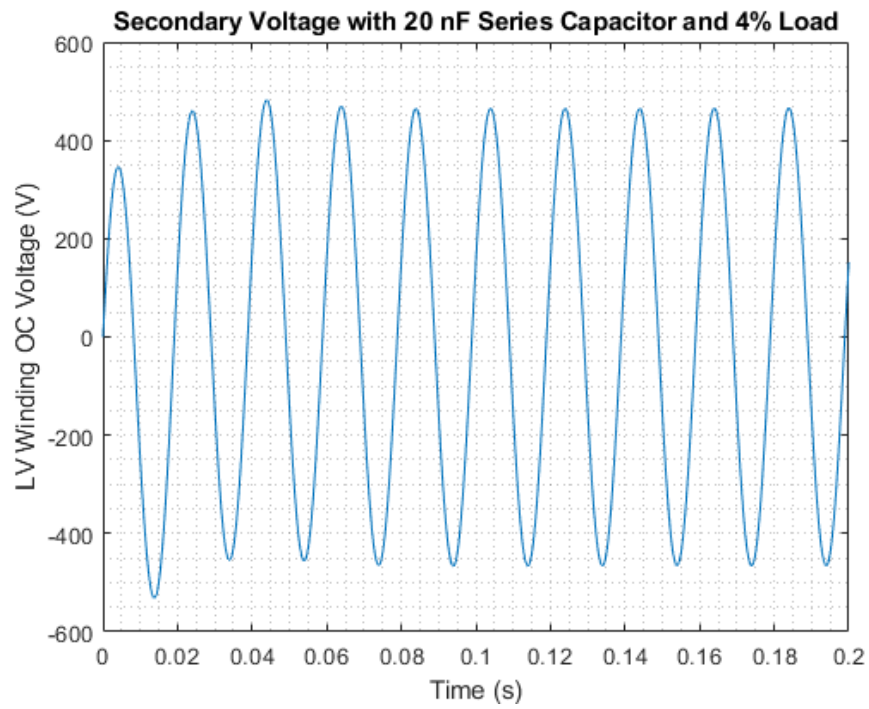


Fig. 4-5: The OC secondary waveform with a 20 nF series capacitor and 4% of rated load

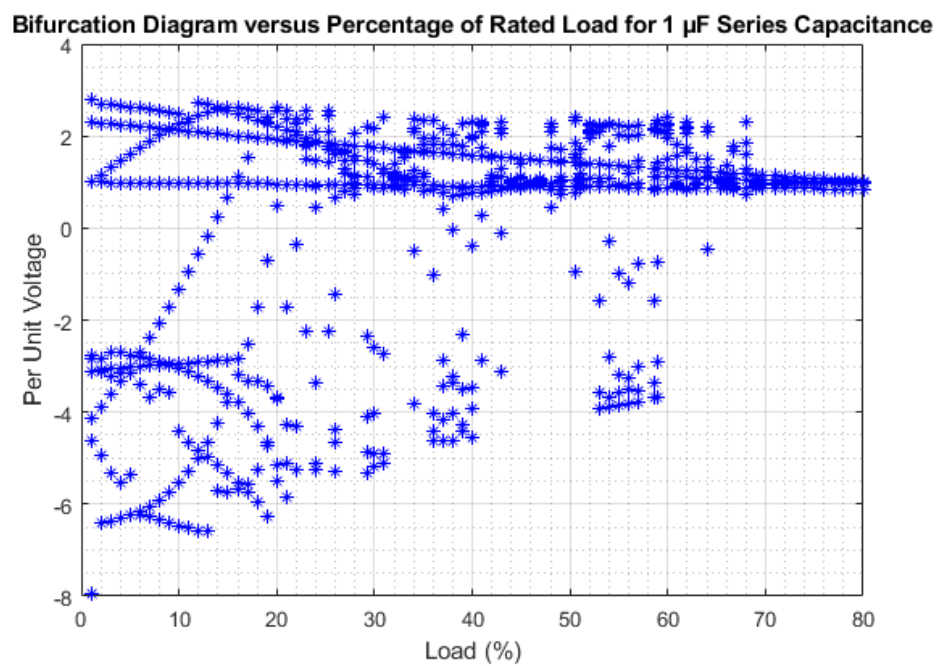


Fig. 4-6: The bifurcation diagram of loads connected to the secondary winding for a 1 μ F series capacitor

4.4 Impact of Core Losses on Ferroresonance

Chapter 1 introduced the new changes to the SANS 780 standard and specifically emphasised the reduction of core losses. The literature study in Chapter 2 discussed how ferroresonance is strongly related to the core losses of the transformer and Section 4.1 has now proven that the transformer is susceptible to ferroresonance. However, it must still be verified whether the reduced core losses necessitated by the new SANS 780 standard is the cause of the overvoltages discussed in Chapter 1.

The recent changes to the SANS 780 [2] standard dictate that the core losses of the transformer need to be 76 W with a +15% tolerance. The maximum permitted core losses are 87.4 W. This is the equivalent of a 5.53 M Ω core loss resistor in the traditional T-transformer model. Chapter 5 will discuss how the transformer in question complies with this new standard. Nevertheless, Revision 4 of the SANS 780 standard permitted nominal core losses of 130.8 W. This is the equivalent of a 3.7 M Ω resistor in the traditional T-transformer model.

Fig. 4-7 shows the OC secondary waveforms when the 3.7 M Ω resistor is used in the π -model and a series capacitor of 20 nF is connected to the transformer. These waveforms are identical to those seen in Fig. 4-2. It would, therefore, seem as though the reduced core losses necessitated by the new SANS 780 standard is not the cause of ferroresonant resurgence. This is confirmed by Fig. 4-8, which shows the bifurcation diagram of a transformer with increased core

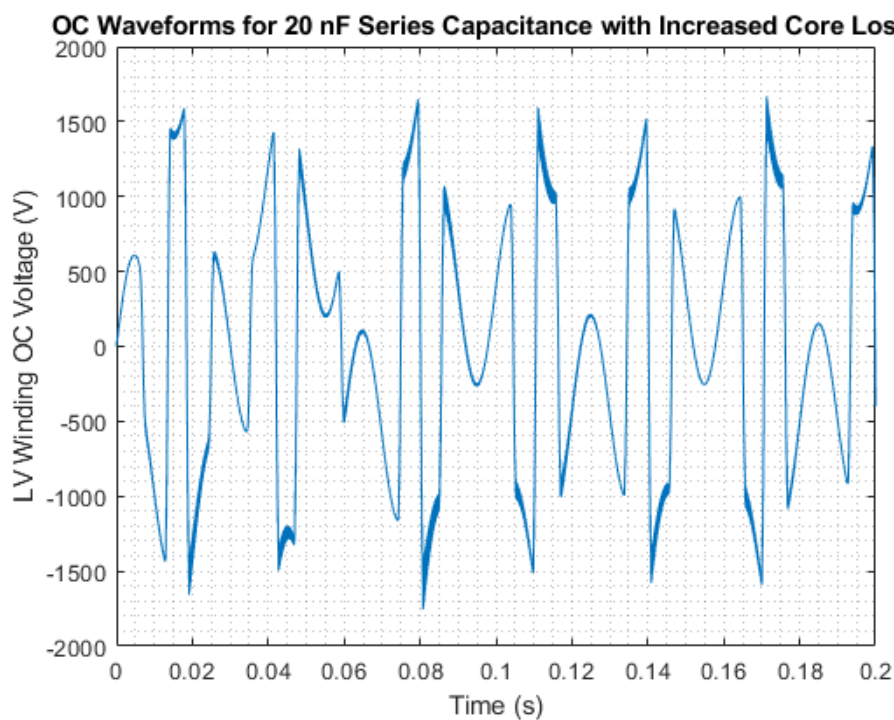


Fig. 4-7: The ferroresonant waveform for a 20 nF series capacitor with increased core losses

losses. A detailed comparison of Fig. 4-8 to Fig. 4-4 may show small differences, but the two bifurcation diagrams are mostly identical despite the change in core loss resistance.

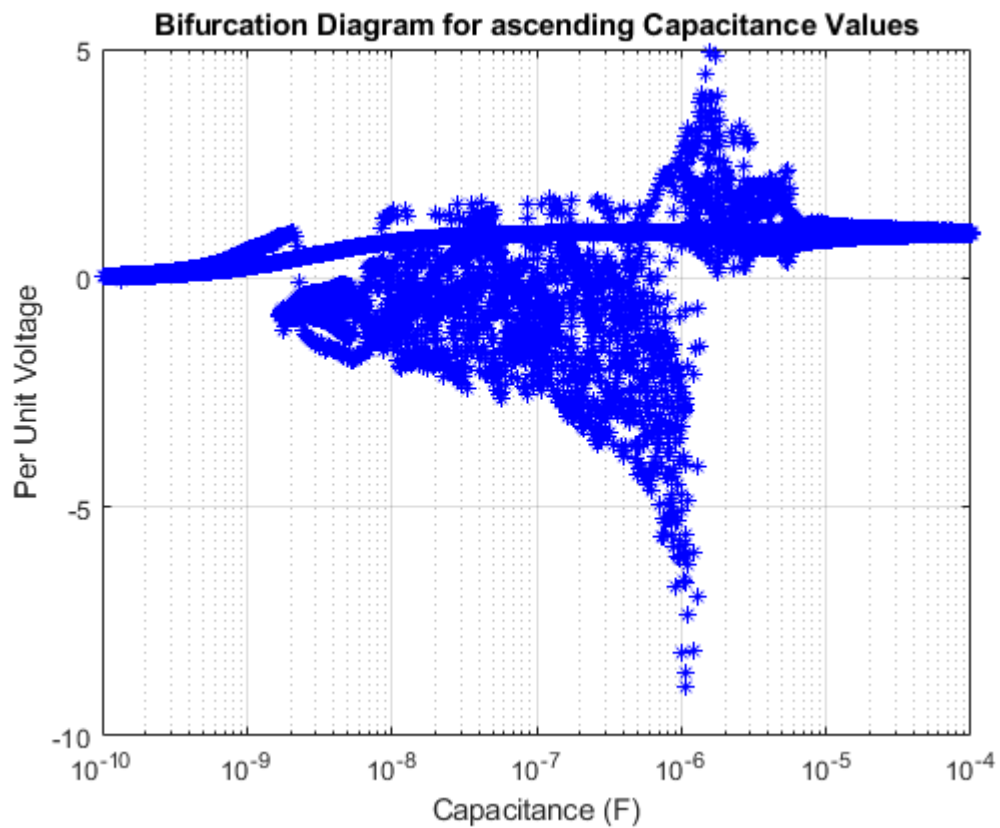


Fig. 4-8: The capacitance bifurcation diagram for a transformer with increased core losses

4.5 Impact of Source Voltage on Ferroresonance

Chapter 2 discussed the effect that source voltage can have on ferroresonance. More specifically, Fig. 2-7 showed that an increase in the voltage increases the likelihood of ferroresonance. Consequently, a bifurcation diagram of the overvoltages on the secondary winding as a function of the source voltage can yield valuable insights into the occurrence of ferroresonance.

Fig. 4-9 shows a bifurcation diagram where the source voltage is varied between 17 kV and 27 kV. Note that a peak voltage of 27 kV is 86.78% of the rated voltage. Fig. 4-4 proved that ferroresonance will occur for a 20 nF capacitor at rated voltage. Fig. 4-9 proves that ferroresonance will also be initiated for voltage levels below rated voltage.

Fig. 4-4 also showed that the worst case of overvoltages can be seen for a 1 μ F series capacitor. Consequently, Fig. 4-10 shows a bifurcation diagram of source voltage for a constant series capacitor of 1 μ F. Chapter 5 will discuss the implications of these bifurcation diagrams.

Bifurcation Diagram Showing Impact of Source Voltage for 20 nF Series Capacitor

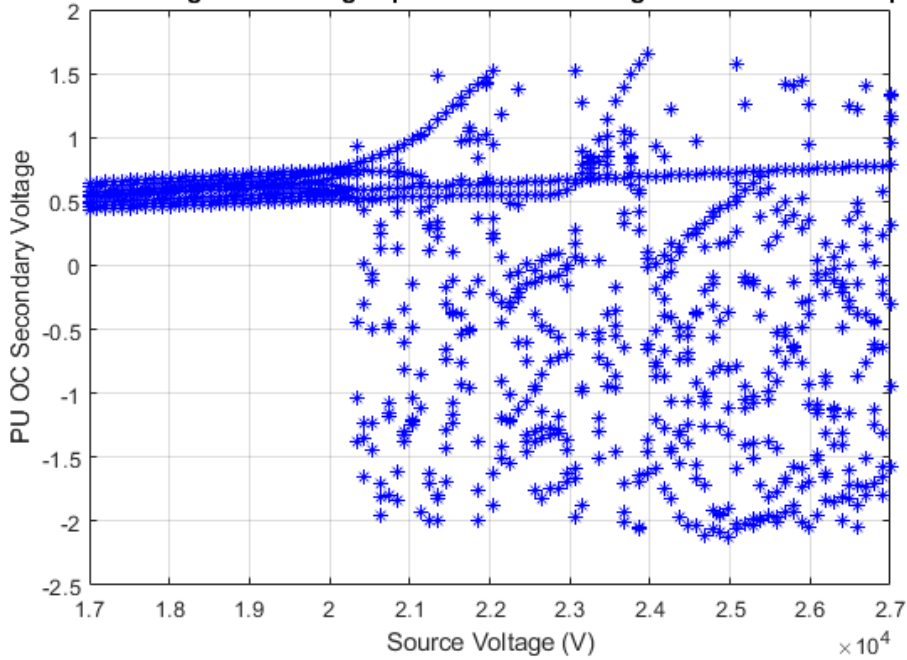


Fig. 4-9: The bifurcation diagram of source voltage for a 20 nF capacitor

Bifurcation Diagram Showing Impact of Source Voltage for 1 μ F Series Capacitor

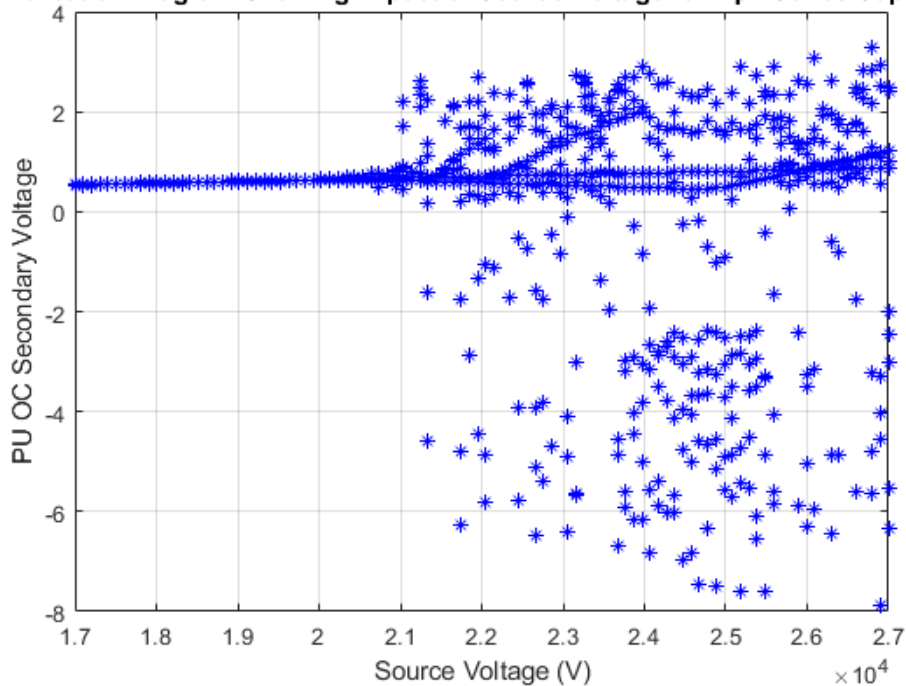


Fig. 4-10: The bifurcation diagram of source voltage for a 1 μ F capacitor

4.6 Single-Phase Switching in Power System

Thus far, Chapter 4 has looked at the ferroresonance of the distribution transformer in isolation. However, Chapter 1 pointed out that the overvoltages measured in practice resulted from single-

phase switching of the transformer during commissioning. Replication of these overvoltages requires the distribution transformer to be connected to a power system.

Fig. 3-61 shows a power system in the MATLAB environment which is a good representation of a typical power system where a 32 kVA distribution transformer is used. Fig. 3-61 shows two switches – one on phase A and one on phase B, which can be switched independently. The literature study in Chapter 2 pointed out that closing a single-phase at a time makes ferroresonance more likely. The South African power system employs such switching protocols.

Fig. 4-11 shows the voltage waveform on the secondary side of the transformer when the two switches in Fig. 3-61 are switched independently. Phase A is switched first after 0.1 s and phase B is switched after 0.3 s. Note that the waveforms in Fig. 4-11 are peak voltage values.

There are two important things to note from Fig. 4-11. Firstly, when only a single phase is closed, a voltage is measured on the secondary of the transformer. This is an unexpected and undesirable result in practice but correlates to the overvoltages reported by Geldenhuys and Padayachee in [1]. This indicates that even when one of the phases are open, there is a path for the current which allows for inductive coupling of the windings of the transformer. Further investigation shows that the current flows through the transformer capacitances to ground, as shown in Fig. 4-12. This correlates to the original hypothesis presented by Geldenhuys and Padayachee [1] and reaffirms the accuracy of this simulation model.

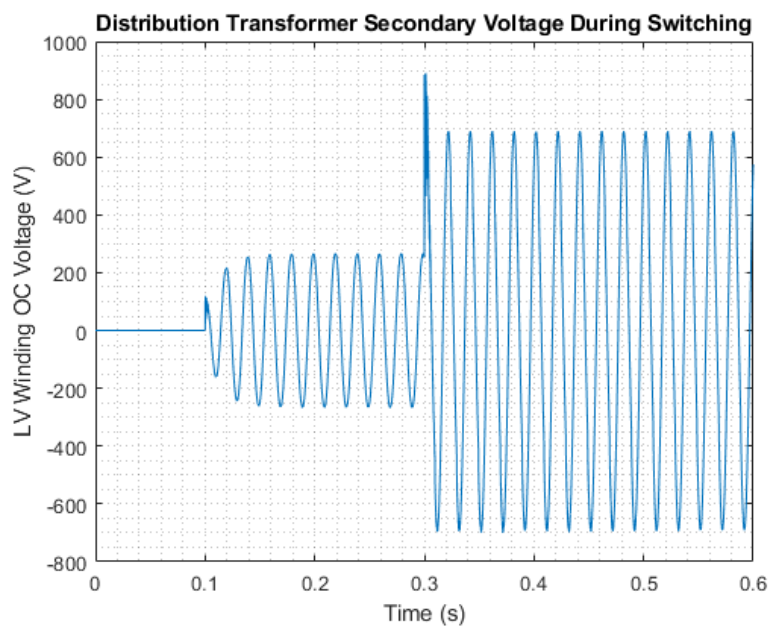


Fig. 4-11: Single-phase switching of the distribution transformer at 22 kV

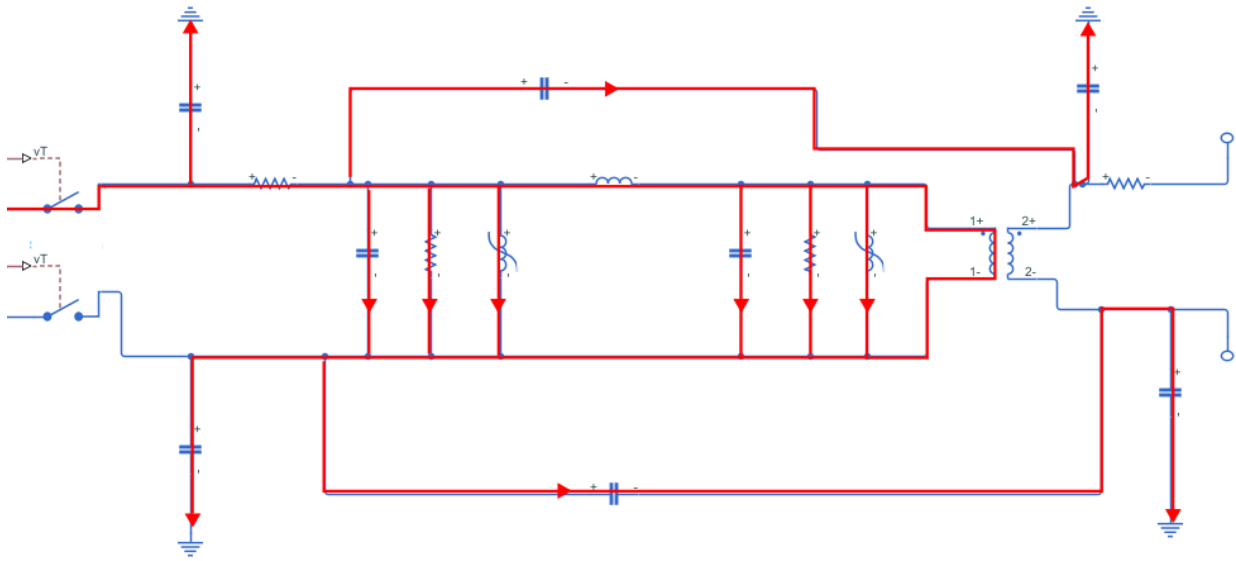


Fig. 4-12: The path of the current during single-phase switching of the distribution transformer

The second important point to note from Fig. 4-11 is that the waveforms are not ferroresonant. An example of the four modes of ferroresonance was shown in Fig. 2-11. Fig. 4-2 also shows an example of typical ferroresonant waveforms. The waveforms in Fig. 4-11 do not show any resemblance to the examples of ferroresonant waveforms seen thus far. These waveforms are sinusoidal and do not show any indication of overvoltage. When only a single switch is closed, the measured voltage is 266 Volt, where the expected peak line-to-ground voltage is 395 Volt. The measured voltage when both switches are closed is 689 Volt, where the rated peak voltage is 684 Volt. Therefore, it is clear that ferroresonance is not the cause of the overvoltages during single-phase switching.

This conclusion, however, does not eliminate ferroresonance as a potential catastrophe. It only indicates that for the topology being simulated in this case, ferroresonance does not cause the overvoltages measured by Geldenhuys and Padayachee in [1] when only a single switch is closed.

A couple of permutations of the power system can also be investigated. These permutations yield valuable insights into the behaviour of the single-phase transformer under different conditions. Consider, for example, the case where the capacitance between the *HV* winding and the *LV* winding, C_{HV-LV} , is removed. The power system is then represented by Fig. 4-13. The switching voltages for this configuration are shown in Fig. 4-14.

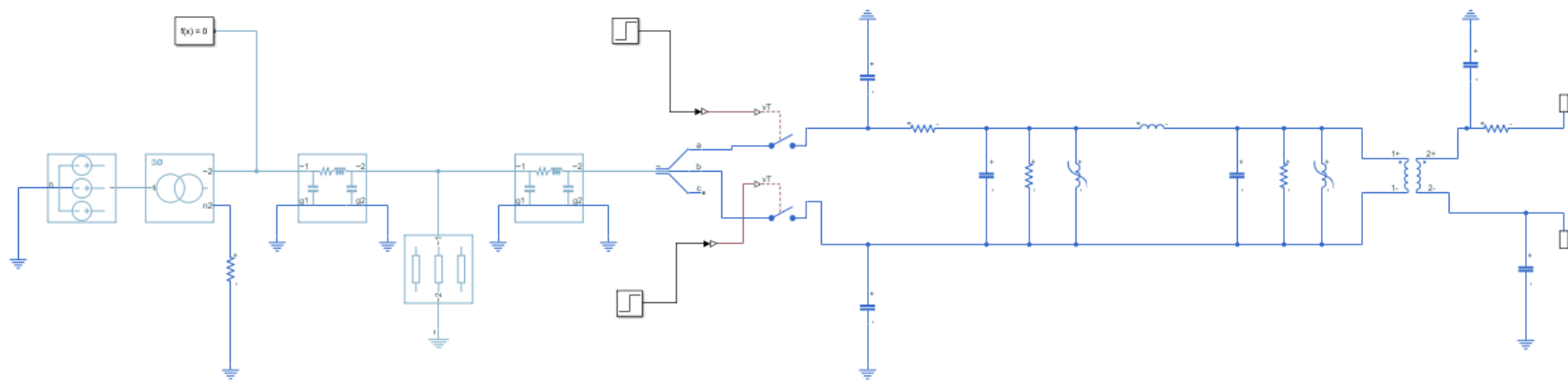


Fig. 4-13: The power system when C_{HV-LV} is removed

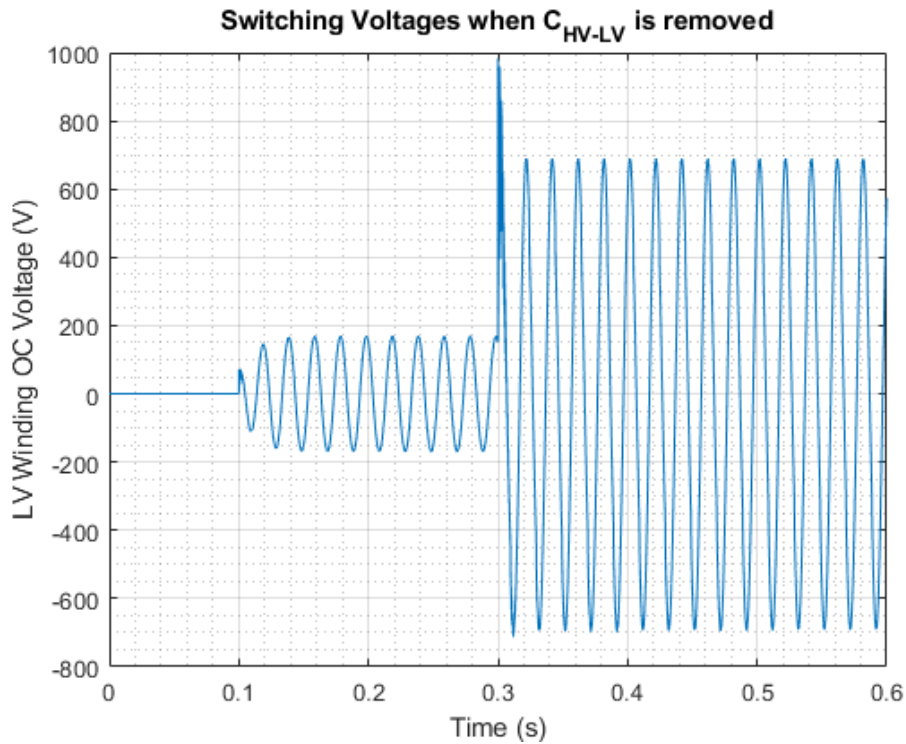


Fig. 4-14: The secondary switching voltages when C_{HV-LV} is removed

Two features can be noted when Fig. 4-14 is compared to Fig. 4-11. Firstly, the open-circuit voltage on the *LV* winding is lower in Fig. 4-14. When capacitors are connected in parallel, the total capacitance increases. Removing the C_{HV-LV} capacitance, therefore, decreases the total capacitance. This shows that a decrease in the capacitance through which the current can flow results in a lower voltage on the *LV* side. This is because a lower capacitance yields a larger impedance. If the impedance is sufficiently large, no current will flow and the overvoltage on the *LV* winding will disappear altogether.

The second prominent feature of Fig. 4-14 is that the voltage spike when the second switch is closed is more prominent. This indicates a trade-off between the *LV* winding voltage when only a single switch is closed and the voltage spike when the second switch is closed.

Another permutation of the power system involves an increase of the transformer capacitances. The measured C_{HV-GND} capacitance, as shown in Table 3-4, is 462.2 pF. Fig. 4-3 shows that ferroresonance only becomes prominent when the capacitance reaches the nF range. This indicates that the internal capacitance of the transformer is currently in a safe range. However, design changes can influence this capacitance and an investigation into the influence of increased capacitance is warranted.

Consider the example where the C_{HV-GND} capacitance of Fig. 4-13²⁴ is increased from 462.2 pF to 20 nF. Fig. 4-2 has shown that a 20 nF capacitance is likely to result in a ferroresonant condition. The switching voltages for this configuration are shown in Fig. 4-15. This figure shows that even for a 20 nF capacitor, ferroresonance is not initiated when only one switch is closed.

However, consider the case where the secondary voltage of the 10 MVA three-phase transformer is changed from 24 kV to 34 kV. This is the equivalent of applying a peak line-to-ground voltage of 24.52 kV across the transformer when only one switch is closed. The corresponding voltages are shown in Fig. 4-16, which clearly shows ferroresonant overvoltages when only one switch is closed. It can also be seen in Fig. 4-16 that the voltages being measured on the secondary side are larger between a time of 0.1 s and 0.3 s than between a time of 0.3 s and 0.6 s. In all the other cases analysed thus far, this was not the case. This shows the impact that ferroresonance can have on the output voltage. Fig. 4-16 also proves beyond any doubt that the transformer internal capacitance can potentially cause ferroresonance during single-phase switching. This will be discussed in greater detail in Chapter 5.

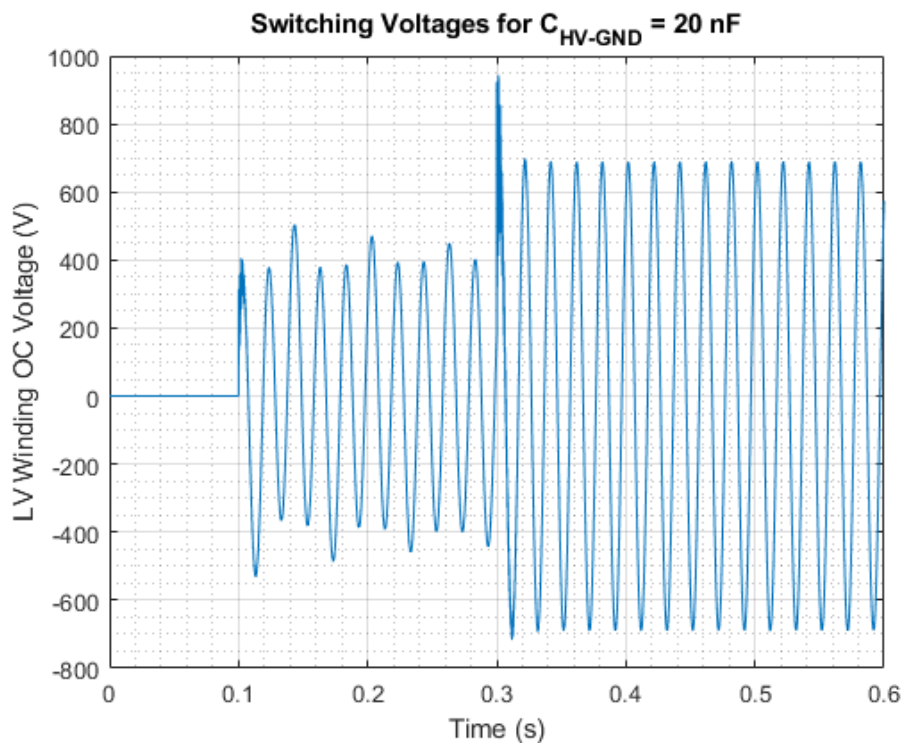


Fig. 4-15: The switching voltages when C_{HV-LV} is removed and $C_{HV-GND}=20$ nF

²⁴ Recall that for Fig. 4-14, the C_{HV-LV} capacitance has been removed.

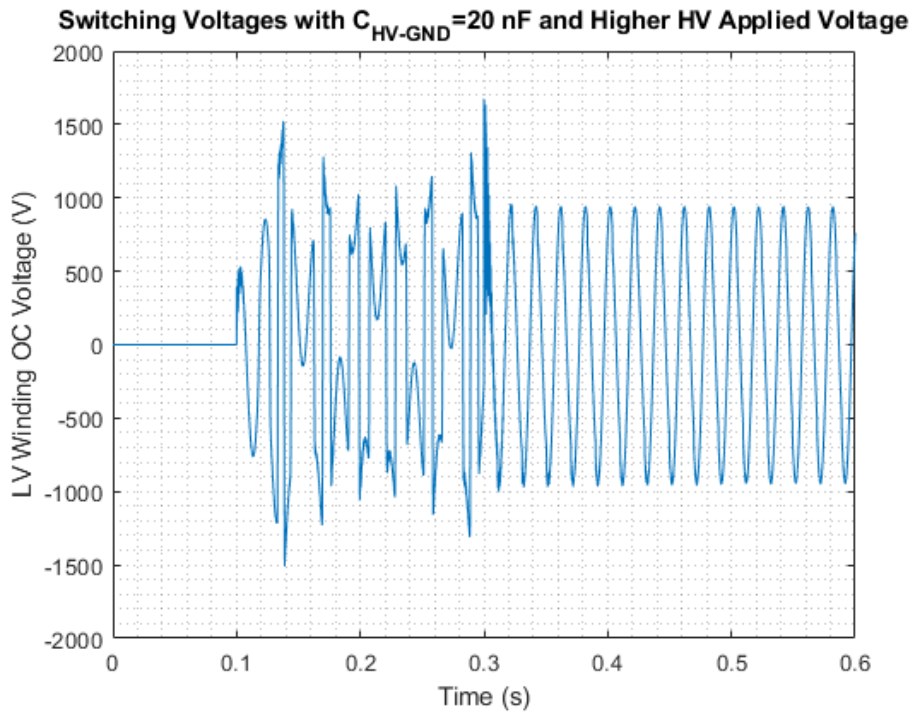


Fig. 4-16: The switching voltages when $C_{HV-GND} = 20 \text{ nF}$ and the HV voltage is increased

Another interesting note regarding Fig. 4-16 relates to the voltage required to initiate ferroresonance. Ferroresonance was not initiated in the transformer for voltage levels below 24.52 kV. Fig. 4-9 showed that for a 20 nF series capacitor, which was also used in Fig. 4-16, a voltage of 20 kV was sufficient to initiate ferroresonance. This provides for an interesting discussion on the effect of dividing the transformer capacitance between the top and bottom terminals and the accuracy of such a model. This will be discussed in greater detail in Chapter 5.

4.7 Centre Tapped Distribution Transformer

Another permutation worth discussing is the addition of the centre tap into the distribution transformer. Chapter 1 discussed the basic construction of the distribution transformer, where it was mentioned that a neutral point is created between the two terminals of the LV winding. The neutral winding is grounded, which allows the 484 V LV winding to feed two 242 V networks. Since the neutral winding is at the centre of the LV winding, the phase shift between the two outputs is 180°.

This configuration results in an additional capacitance between the two terminals of the LV winding and the neutral point, which is connected to ground. If the wire connecting the neutral terminal to ground is stolen, which is a likely scenario in South Africa, the transformer

representation seen in Fig. 3-61 is correct. The analysis presented in Section 4.6 is therefore warranted. Nevertheless, it is also worth considering the behaviour of the transformer when the neutral winding is grounded.

Fig. 4-17 shows the circuit diagram of the centre tapped distribution transformer. The MATLAB Simscape environment does not present an ideal centre tapped transformer component. Consequently, it is necessary to use two ideal transformers to represent this configuration. The winding ratio of both ideal transformers is $a = \frac{22000}{242}$ (as opposed to $a = \frac{22000}{484}$) to ensure that the voltage between the outer terminal and the centre winding is correct. On the *HV* side, the ideal transformers are connected in parallel. On the *LV* side, the ideal transformers are connected in series and the series connection is grounded. Two additional capacitors are connected between the outer windings of each ideal transformer and the grounded centre tap. These capacitances have not been measured. For the sake of simulation, they are estimated as 10 pF for a lack of better data. All other parameters of the transformer are identical to the parameters derived in Chapter 3. This centre tapped transformer is connected to a power system identical to the one described in section 3.7.

Fig. 4-18 shows the switching voltages of the centre tapped transformer. In this case, the voltages are measured between the outer windings and the grounded centre tap. For this reason, the magnitudes of the voltages seen in Fig. 4-18 are half of those seen in Fig. 4-11. Fig. 4-18 also demonstrates the 180° phase shift between the two terminal voltages.

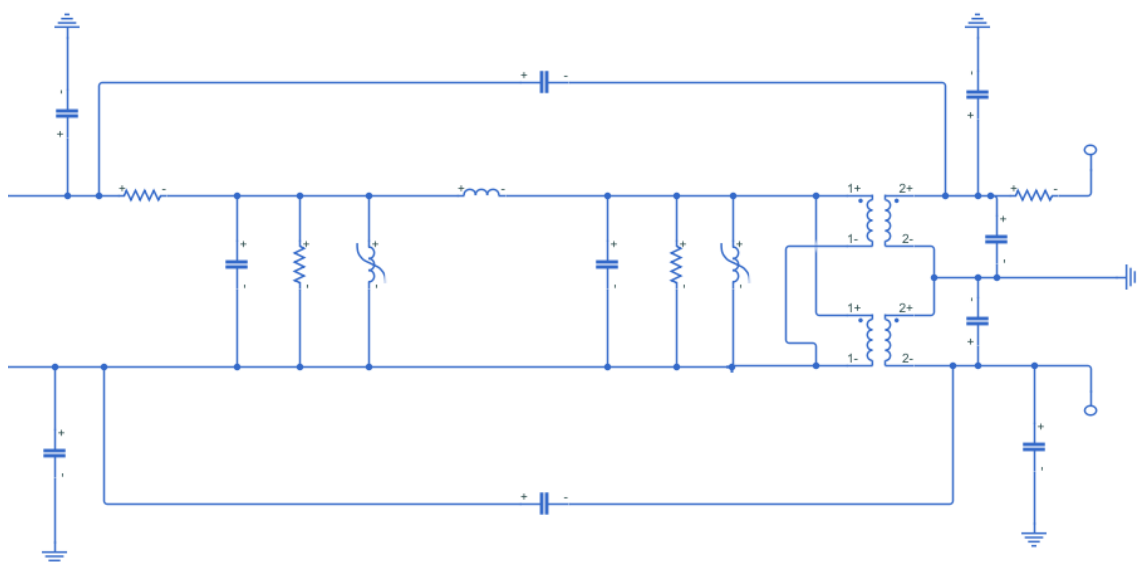


Fig. 4-17: The circuit diagram of the distribution transformer with a grounded centre tap

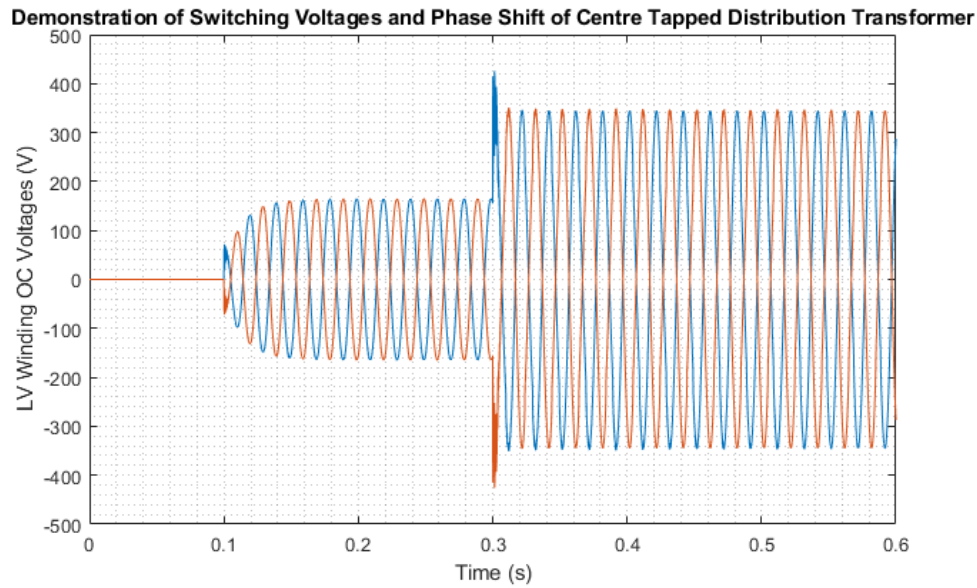


Fig. 4-18: The switching voltages of a centre tapped transformer showing the phase shift between terminal voltages

Fig. 4-19 shows a comparison of the voltage between the outer terminals of the distribution transformer with and without the centre tap. The orange waveform shows the measured voltage when the centre tap is present. The blue waveform is identical to the graph seen in Fig. 4-11. Fig. 4-19 shows that the additional capacitances introduced by the grounded centre tap have a minor influence on the secondary voltages. Furthermore, the difference is only notable when only a single switch is closed. When both switches are closed, the behaviour of the two transformer models is identical. Based on the minor difference between the centre tapped model and the

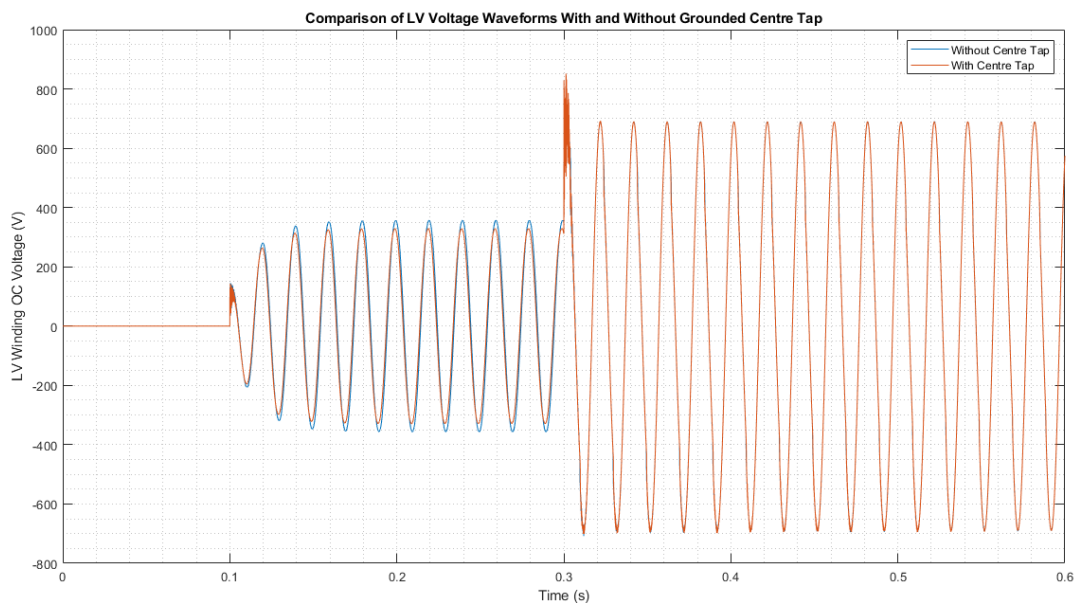


Fig. 4-19: A comparison of the voltage between the outer terminals of a distribution transformer with and without a centre tap

model shown in Fig. 3-61, it can be concluded that the model of Fig. 3-61 can be used for further investigations in Chapter 5.

4.8 Conclusions

This chapter was built on the π -model developed in Chapter 3. It started by proving that the distribution transformer being investigated in this report is susceptible to ferroresonance when connected to a series capacitor. Section 4.2 investigated the impact of capacitance on ferroresonance overvoltages and showed that there is a range of capacitance values that results in ferroresonance.

Chapter 2 mentioned that OC conditions result in the worst-case ferroresonant overvoltages. Consequently, adding a load to the transformer can potentially mitigate the ferroresonant overvoltages. Section 4.3 presented a bifurcation diagram on the load required to mitigate ferroresonance. However, it was seen that the load is dependent on the series capacitor giving rise to the ferroresonant condition.

Chapter 2 also spoke about the effect that core losses have on ferroresonance. Section 4.4 compared the ferroresonant overvoltages of transformers complying with the new SANS 780 standard to those of the older standard. It was seen that the bifurcation diagram of a transformer with increased losses is identical to the bifurcation diagram of a transformer with reduced losses.

Fig. 2-7 in Chapter 2 showed that ferroresonance becomes increasingly likely with increased levels of the source voltage. Section 4.5 looked at the minimum voltage required for ferroresonance to be initiated for a given series capacitance. The required voltage is once again related to the magnitude of the capacitance. In this case, only a 20 nF and 1 μ F capacitor were used for the investigations. The results of these investigations provide interesting conclusions in Chapter 5.

Following the abovementioned investigations into the various factors influencing ferroresonance, section 4.6 looked at the interaction of the modelled distribution transformer with a real power system. Two switches were closed independently and overvoltages similar to those reported by Geldenhuys and Padayachee [1] were simulated. The resonant path is discussed in section 4.6. Preliminary results indicate that the overvoltages measured by Geldenhuys and Padayachee [1] are not caused by ferroresonance. However, section 4.6 also proves that ferroresonance cannot be eliminated as a dangerous phenomenon for the transformer.

The final discussion of Chapter 4 focused on the centre tap of the distribution transformer and the effect that it can have on capacitance distribution. A new transformer model taking the centre tap

into account was developed and simulated in the power system. The results are nearly identical to the transformer model without the centre tap and it can be concluded that the centre tap does not improve or significantly influence the simulations.

The aim of Chapter 4 was to provide the reader with the data required to make conclusions on the nature of the overvoltages reported by [1]. The intent was not to discuss the results of the simulations. This discussion of the results and the subsequent conclusions is the topic of the next chapter.

CHAPTER 5 CONCLUSION AND RECOMMENDATIONS

The purpose of Chapter 5 is two-fold: it discusses the conclusions that can be made from the simulations of Chapters 3 and 4 and it presents recommendations for future improvements.

The Chapter starts by discussing the difficulties encountered in Chapter 3 regarding the Tellinen and Jiles-Atherton hysteresis model. The numerical instability of the Tellinen model is solved and alternative cost functions for the PSO algorithm are presented to improve Jiles-Atherton parameter estimation. A limit is also presented which should be implemented into the algorithm to avoid non-physical behaviour.

The Chapter then proceeds to discuss the resonance of the transformer. It is concluded that the overvoltages measured by Geldenhuys and Padayachee [1] are not caused by ferroresonance due to low capacitance and low voltage. Multiple mitigation strategies are discussed in an attempt to reduce the measured overvoltages.

Even though the overvoltages are not caused by ferroresonance, Chapter 4 has proven that the distribution transformer is favourable to ferroresonance under certain conditions. These conditions are discussed in Section 5.4.

Section 5.5 reflects on the accuracy of the simulation model. This includes a discussion of the frequency behaviour of Model 2, the representation of the internal capacitances and a discussion of the inter-turn and inter-layer capacitance determined from the SFRA.

Finally, recommendations on future work are made in Section 5.6. Most of these recommendations are focused on improving the Jiles-Atherton hysteresis model to take remnant flux into account in future simulations.

5.1 Hysteresis

Section 3.3 and 3.4 focused on the development of the hysteresis model. The inverse JA and inverse Tellinen models were both used to verify and replicate results found in the literature. However, despite their ability to match results found in literature, neither of these models could replicate the behaviour of the distribution transformer being studied in this report.

The harmonic content of the distribution transformer OC test results is the main challenge for the inverse Tellinen model. The inverse Tellinen model makes use of one-dimensional lookup tables to define the hysteresis loop. If the data points of the lookup tables are not monotonic, then the algorithm fails. The hysteresis loop shown in Fig. 3-46 is not monotonic. Furthermore, even if

potential phase shift inaccuracies introduced by measurement are fixed, the data points of Fig. 3-48 are still not monotonic.

Fig. 3-51 showed the hysteresis loop after the shoulder regions of quadrant 1 and quadrant 3 were used to improve the shoulder regions of quadrant 2 and quadrant 4. However, manipulating the measured data in this manner results in a significant loss of data points. The author of this report would like to hypothesize that this loss of data can significantly influence the accuracy of the inverse Tellinen model. This is evident in the area of the hysteresis loop when the data points are edited. Numerical integration of the original hysteresis loop seen in Fig. 3-46 indicates that the hysteresis loss of the transformer is 310.56 W at 125% of rated voltage. The power loss for Fig. 3-48 after a manual phase shift is introduced to the current waveform is 173.4 W. When the non-monotonic data points are removed and the shoulder areas are improved, as seen in Fig. 3-51, the power loss is reduced to 89.15 W. The power loss at 120% of rated voltage, as measured by the N4L oscilloscope, is 160.17 W. Consequently, one can assume that a power loss of 173.4 W at 125% of rated voltage will be the closest to the correct answer. This means that the monotonic hysteresis loop of Fig. 3-51 severely underestimates the power loss in the core of the transformer.

The author would like to note that the numerical problems of the inverse Tellinen model shown in Fig. 3-53 were solved during the closing phases of this study. The inverse Tellinen model requires the calculation of a differential $\frac{db}{dh}$. Numerical calculation of this differential involves interpolation between data points for very small changes in h . In the original inverse Tellinen model, which replicated the results of [81], the change in h used to calculate $\frac{db}{dh}$ was 1×10^{-6} . However, this small step in h was too small for the data points derived from Fig. 3-51. When the step size of h is changed to 2, the inverse Tellinen model replicates the hysteresis loop of Fig. 3-51 very well. The correlation between Fig. 3-51 and the inverse Tellinen results is shown in Fig. 5-1. An updated MATLAB program of the inverse Tellinen model is shown in APPENDIX E.

Time constraints prevented the author from implementing this inverse Tellinen model in the power system studies of Chapter 4. However, the severe underestimation of the core losses by the hysteresis loop suggests that this model will still not be a good representation of the transformer. Furthermore, one of the most important arguments for the use of a hysteresis model is that it should retain residual flux, which can play an important role in ferroresonance studies. Section 3.4.1 proved that the inverse Tellinen model cannot retain residual flux and that switching transients will introduce additional inaccuracies. Consequently, future work should rather be focused on improving the inverse JA model and its parameter estimation.

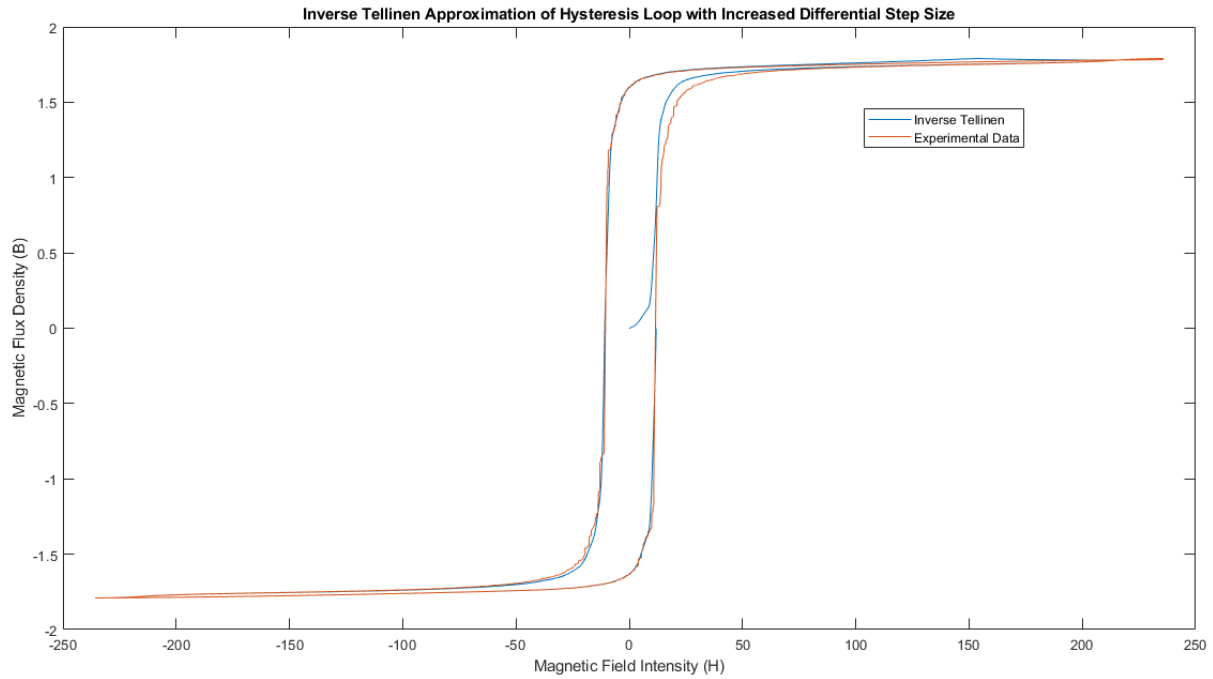


Fig. 5-1: A comparison of the inverse Tellinen model to the monotonic hysteresis loop when numerical errors are fixed

The author believes that improvements for the inverse *JA* model should be focused on better parameter determination. To this end, there are a few alternatives that can be investigated to obtain better parameters from the *PSO* algorithm.

There are two main problems related to the *PSO* algorithm at this stage. The first is the poor cost function which is not capable of yielding a good *MSE*. The second problem is the nonphysical behaviour of the hysteresis loop for the parameters which have been determined thus far. Solving problem 1 can potentially also save problem 2. However, some limits can be introduced into the algorithm which could also help with problem 2.

The cost function used for the *PSO* algorithm at this stage is purely related to the *MSE* difference between the measured and simulated data points. Knypinski, Nowak, Sujka and Radziuk [231] propose an alternative cost function where the *MSE*, the area of the loop and the maximum value of *H* is taken into consideration. They assign a weight of 10% to the *MSE* and 45% to the area and the maximum value of *H*. An attempt was made to implement this cost function but the results were worse than those reported in Fig. 3-52. Furthermore, based on the work of Marion *et al.* [224] it can be argued that it is not possible to optimize both the *MSE* of the simulated data points and the area of the loop. Instead, Marion *et al.* [224] state that considering both the *MSE* and the area of the loop presents a Pareto front. It then becomes impossible to define a set of parameters that satisfies both cost functions since they present a trade-off between each other. The core of

the *PSO* algorithm needs to be modified since the concept of a global best no longer exists. Future work on the optimization of *JA* parameters should be focused on this revised Pareto cost function.

The second problem with the calculated inverse *JA* parameters is the nonphysical behaviour of the loop. The *PSO* algorithm does not check for physical behaviour. Consequently, a set of parameters resulting in the non-physical behaviour of the model can present the best *MSE* between the measured and simulated data points.

Marion *et al.* present a range for each of the five *JA* parameters in [224, Table I]. The *PSO* algorithm requires limits for the search space of each of the parameters. However, even when the limits presented in [224, Table I] are used as input to the *PSO* algorithm, the interdependency of the five parameters can still result in non-physical behaviour.

A potential solution to this problem is found in [223], where Padilha *et al.* derived an analytical restriction to the parameters of the *JA* algorithm. This restriction is given by the following equation:

$$1 - \alpha c \frac{M_s}{3a} - F \cdot \alpha(1 - c) \frac{M_s}{k} > 0 \quad (5-1)$$

where $0 < F < 1$. The challenge of this equation is that the user cannot know which parameter is causing the problem. Consequently, it may be necessary to keep four of the parameters constant and calculate a new value for the fifth parameter. This can be done for all five parameters and the cost of the five permutations can be calculated using the existing cost function of the *PSO* algorithm. The best combination of the five permutations should then be used for that iteration of calculations. This check for non-physical parameter combinations should be the first improvement added to future versions of the *PSO* algorithm.

In summary, future improvements on the estimation of *JA* parameters should be focused on the Pareto front of the cost function discussed in [224] and the parameter limits presented in (5-1).

5.2 Transformer Resonance

This study was initiated with the intent of investigating the resonance in a distribution transformer when only one of the phases feeding the transformer is closed. Single-phase distribution transformers are connected between two lines of a three-phase system. When the single-phase distribution transformer is commissioned, the circuit breakers of the three-phase system are closed one at a time. Geldenhuys and Padayachee [1] reported that unexpected overvoltages were measured between the closing of the first circuit breaker and the second circuit breaker. This study was conducted under the hypothesis that ferroresonance is the cause of these overvoltages.

Chapter 4 unequivocally proved two important points: 1) the distribution transformer is susceptible to ferroresonance and 2) ferroresonance is not the cause of the overvoltages measured by Geldenhuys and Padayachee [1]. Chapter 4 also provided data for the discussion of potential mitigation strategies, such as loads on the secondary and a variation of transformer internal capacitances.

There are two reasons for the exclusion of ferroresonance as the cause of the overvoltages in the power system. Firstly, the voltage applied to the transformer is too small to initiate ferroresonance. When only a single switch is closed, the voltage applied across the transformer is equal to the line-to-ground voltage of the power system. If the RMS rated line-to-line voltage of the power system is 22 kV, then the RMS line-to-ground voltage is 12.70 kV. The peak line-to-ground voltage is 17.96 kV.

Fig. 5-2 shows the bifurcation diagram of capacitance for a voltage level of 17.96 kV. For capacitance values below 200 pF, the impedance is so large that the voltage on the LV winding is below 0.1 pu. The multiple bifurcation points between 200 pf and 500 nF could create the perception that ferroresonance is possible in this range. However, consider Fig. 5-3, which shows the LV winding voltage at 17.96 kV with a 20 nF series capacitance. The black vertical lines show the sampling points for the bifurcation diagram of Fig. 5-2. Fig. 5-3 shows that there is a 12.5 Hz oscillation of the fundamental waveform, which results in varying voltages at the sampling points. However, the overvoltages caused by this oscillation are not extreme and consequently, not dangerous. It is the experience of the author that ferroresonance simulations are often preceded by these types of oscillations. However, this indicates that the bifurcations seen in Fig. 5-2 are not a cause for concern.

The implication of Fig. 5-2, therefore, is that ferroresonance is unlikely at a peak voltage of 17.96 kV because the voltage is too low to drive the transformer into saturation. Consequently, even a variation of the transformer internal capacitance should not result in ferroresonance during switching of the transformer.

The second reason for the exclusion of ferroresonance as the cause of the overvoltages measured by Geldenhuys and Padayachee [1] is the low internal capacitance of the transformer. Fig. 4-4 showed the bifurcation diagram of a variation of series capacitance at rated voltage, where there was a clear indication of ferroresonance. The first capacitance value which yields a voltage above 1.1 pu was 1.1723 nF. Lower capacitance values already started to show typical ferroresonant waveforms but the overvoltages were less than 1.1 pu. Table 3-4 showed the internal capacitances of the transformer. The series ferroresonant path during single-phase switching is completed by the internal capacitances of the transformer, as was shown in Fig. 4-12.

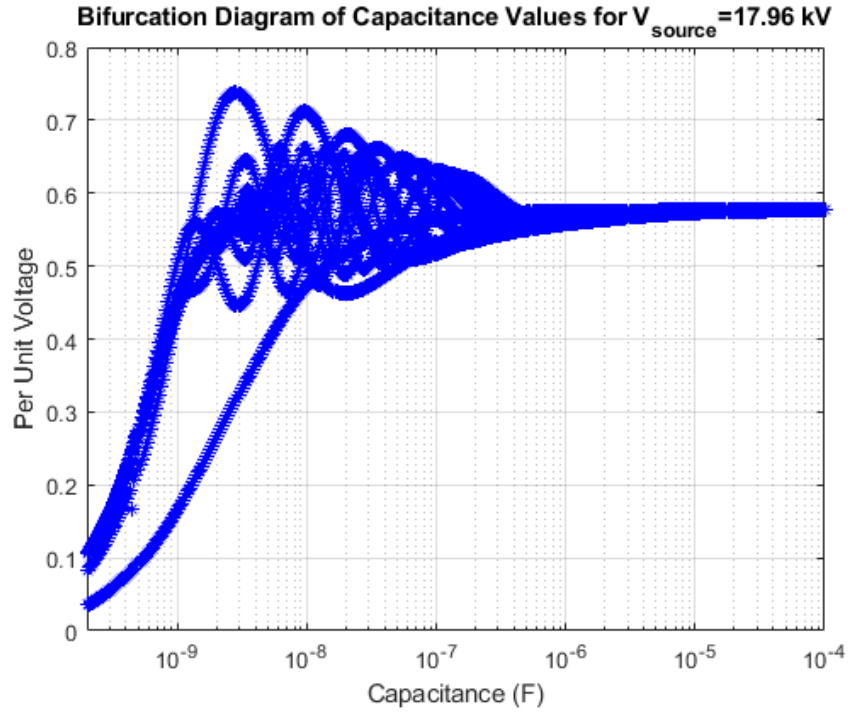


Fig. 5-2: The bifurcation diagram of capacitances for $V_{peak}=17.96$ kV

C_{HV-LV} is in series with C_{LV-GND} and this series combination is connected in parallel with C_{HV-GND} . The equivalent capacitance of this series and parallel combination yields $C_{total} = 736.96$ pF. This means that even if rated voltage was applied across the transformer when one of the switches are closed, the internal capacitances are too small to initiate ferroresonance.

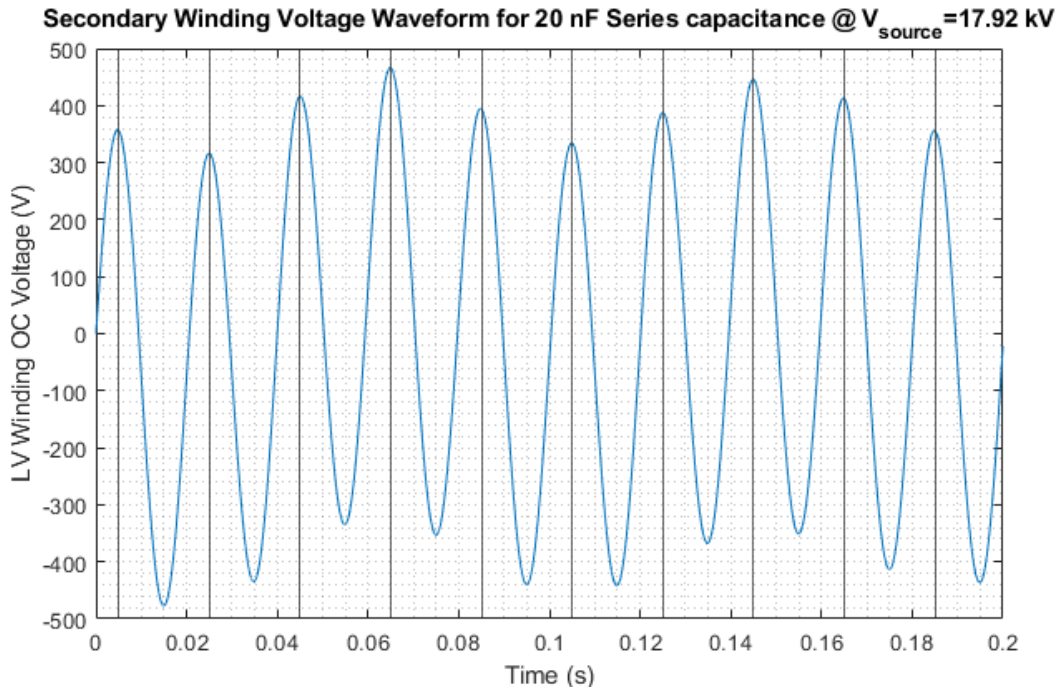


Fig. 5-3: The secondary winding voltages for a 20 nF series capacitance and 17.96 kV peak source voltage

5.3 Resonance Mitigation

Section 5.2 established that the overvoltages measured by Geldenhuys and Padayachee [1] are not caused by ferroresonance. However, the overvoltages during switching still need to be mitigated. This section will look at three potential mitigation strategies: 1) a load on the secondary winding during switching 2) the elimination of C_{HV-LV} and 3) a parallel capacitor connected across the *HV* terminals.

5.3.1 LV Winding Load

Section 4.3 looked at the influence that a load can have on ferroresonant overvoltages. It was shown adding a load can eliminate ferroresonance. Furthermore, the load required to eliminate ferroresonance is influenced by the nature of the overvoltages.

Even though the overvoltages being investigated are not caused by ferroresonance, a load on the secondary winding still helps to mitigate the magnitude of the overvoltages. A load that drew 1% of the rated current reduced the overvoltages from 266 V to 69.10 V. A load of 2% reduced the voltage during switching to 38.04 V, which is less than 10% of the original voltage.

The drawback of this type of mitigation strategy is that it negates the purpose of the change in the SANS 780 standard. The changes were introduced into the standard to reduce the losses in the distribution system. If a load is added to the winding, more losses are introduced into the system.

The load can potentially be removed once both switches have been closed. However, the literature study of Chapter 2 cautioned that removing loads can reintroduce ferroresonance. Furthermore, since the load will be removed after both switches have been closed, the voltage level will be high enough to initiate ferroresonance. On the other hand, if both switches have been closed, the series path established by the internal capacitance of the transformer has been removed. Section 5.4 will talk about parallel ferroresonance and show that parallel ferroresonance is not likely in this power system. Without the internal series capacitance of the transformer, ferroresonance will not be initiated. Therefore, attaching a load to the secondary winding during commissioning and then removing that load when both switches have been closed can reduce the magnitude of the overvoltages. However, this requires an expensive automated system.

5.3.2 Elimination of C_{HV-LV}

Section 4.6 investigated some permutations to the power system, including the effect of removing the C_{HV-LV} capacitance. Since capacitances connected in series increases the total capacitance, a removal of C_{HV-LV} reduces the total capacitance. This reduces the magnitude of the overvoltages measured on the secondary winding.

This solution can be achieved by adding a screen between the *HV* and *LV* winding and connecting the screen to ground. This will increase the C_{HV-GND} capacitance but will eliminate the C_{HV-LV} capacitance.

An increase in the C_{HV-GND} capacitance is an unwanted result. This can increase the overvoltages on the secondary side again. The question, then, is whether this increase in capacitance is greater than the series combination of C_{HV-LV} and C_{LV-GND} in the original transformer. If the resulting C_{HV-GND} capacitance is greater than 736 pF, the overvoltages will not be reduced. Moreover, if the resulting C_{HV-GND} becomes greater than 1.712 nF, then ferroresonance can be initiated if the voltage becomes sufficiently large. The effect of this screen should therefore be considered carefully.

The magnitude of C_{HV-GND} in the presence of a screen can be approximated based on the design data of the transformer. Nevertheless, one must bear in mind that the screen between the *HV* and *LV* winding is guaranteed to increase the C_{HV-GND} capacitance. If the C_{HV-GND} capacitance is left unchanged and the C_{HV-LV} capacitance is removed, Fig. 4-14 demonstrated that the peak secondary voltages will still be 169 V. The screen between the *HV* and *LV* winding will not show improvements beyond this voltage. Consequently, even though the screen is a simple and cost-efficient solution to reduce the capacitance, the designer will have to decide on the acceptable levels of overvoltage during single-phase switching.

Fig. 4-4 showed that if the series capacitance connected to the distribution transformer becomes sufficiently large, ferroresonance can disappear. It then stands to reason that perhaps an increase in the value of C_{HV-GND} can also eliminate or reduce the overvoltages on the secondary winding. This hypothesis does not hold. An increase in the internal capacitance of the transformer does reduce the secondary transient voltages. However, it also decreases the steady-state secondary voltage when both switches are closed. In essence, an increase in the capacitance results in larger voltage drops across the internal capacitance of the transformer and a smaller voltage drop across the transformer during both transients and steady-state conditions. To maintain rated voltage during steady-state conditions, the voltage of the three-phase transformer higher up in the power system can be increased. The measured voltage during switching will then return to those indicated by Fig. 4-11 and the effect of increased internal capacitance will be negated.

5.3.3 HV Winding Capacitance

Geldenhuys and Padayachee [1] mention the possibility of adding a 20 μ F capacitor across the *LV* winding to detune the resonance from 50 Hz but claim that this is unacceptable for several

reasons. They do not elaborate on these reasons, which leads to the assumption that their reasons are related to a resonance phenomenon with the leakage inductance of the transformer.

The author of this report believes that if a capacitor is added to the terminals of the transformer, it should rather be added to the *HV* terminals. This allows for a smaller capacitor to be attached to the transformer terminals, which will yield the same impedance. Table 5-1 shows the influence that a parallel capacitor can have on the secondary voltages. A 20 μF capacitor on the *LV* winding, as suggested by [1], is the equivalent of a 9.68 nF capacitor across the terminals of the *HV* winding. Table 5-1 shows that a 10 nF across the terminals of the *HV* winding reduces the overvoltages to 15.7 V.

Consider the case where a resonant path can be formed between this 10 nF capacitor and the equivalent leakage inductance referenced to the *HV* side. The equivalent leakage inductance on the *HV* side is 1.865 H, as shown in Table 3-3. The resonant frequency is given by (2-1) and yields

$$f_{resonant} = \frac{1}{2\pi\sqrt{1.865 \cdot 10 \times 10^{-9}}} = 1165.4\text{Hz} \quad (5-2)$$

Switching transients can potentially initiate parallel resonance in the circuit if a 10 nF capacitor is connected to the *HV* winding.

Table 5-1: The influence of a parallel capacitor connected across the HV winding on the secondary overvoltages during single-phase switching

Capacitance	Peak OC Secondary Voltage
1 pF	265.92 V
10 pF	265.27 V
100 pF	256.82 V
1 nF	108.73 V
10 nF	15.678
100 nF	1.634 V
1 μF	141.18 mV
10 μF	14.587 mV

It is also possible to connect larger capacitances across the *HV* winding. A 10 μF capacitor will practically eliminate the overvoltages on the secondary winding and bring the resonant frequency down to 36.8 Hz. A 20 μF capacitor connected across the *HV* winding shifts the resonant frequency to 26.06 Hz. Both these frequencies are below the fundamental frequency.

An added advantage of the parallel capacitor across the terminals of the *HV* winding is the effect that it can have on the ringdown transient. The literature study of Chapter 2 mentioned that parallel capacitors across transformer windings prolong the ringdown transient during switching. This has the effect of decreasing the residual magnetism in the transformer, which could potentially still affect ferroresonance.

However, a parallel capacitor connected across the terminals of the transformer is a more expensive mitigation technique.

5.4 Transformer Ferroresonance

Section 5.2 concluded that ferroresonance is not the cause of the overvoltages measured by Geldenhuys and Padayachee [1]. However, Chapter 4 also showed that the distribution transformer is susceptible to ferroresonance. This subsection will discuss some of the interesting phenomena seen in the ferroresonance simulations of Chapter 4.

5.4.1 Parallel Ferroresonance

Section 2.1 spoke about parallel and series resonance. The simulations of Chapter 4 were all focused on series ferroresonance. However, one would expect that parallel ferroresonance is also a possibility.

Parallel ferroresonance, however, is rarely mentioned in literature. Ferracci [26] briefly touches on the topic when speaking about voltage transformers connected to isolated neutral systems and three-phase transformers connected to sources with low short-circuit power. Tanaka *et al.* managed to simulate parallel ferroresonance with the Chua-type magnetization model in [232] and [233]. However, other sources of literature which specifically demonstrate parallel ferroresonance in transformers are scarce.

A bifurcation sweep of voltages for varying values of parallel capacitance was also done on the transformer model. The sweep did not show any sign of ferroresonance. This is a positive outcome since overhead lines and underground cables can provide the necessary capacitance for parallel ferroresonance. If parallel ferroresonance was detected in the bifurcation sweep, it could indicate that parallel ferroresonance can occur under steady-state conditions when rated voltage is being applied across the transformer.

5.4.2 Series Ferroresonance

Chapter 4 proved that ferroresonance is possible if the voltage is sufficiently high and there is a series capacitance connected to the transformer. Consequently, it can be concluded that the transformer is susceptible to series ferroresonance but parallel ferroresonance is unlikely.

The source of the series capacitance is something to consider. Throughout literature, the source of series capacitance is often provided by circuit breaker capacitances. However, the low voltage level of a distribution transformer means that circuit breaker capacitances are not justified. Furthermore, the South African power grid does not employ series capacitances to reduce transmission line impedance. This means that distribution grids will rarely have sources of series capacitance in the system.

The exception to this has been demonstrated in Fig. 4-12, where it was shown that the transformer internal capacitance is in series with the transformer magnetization branch during single-phase switching. Consequently, if the internal capacitance is sufficiently large and the voltage is high enough to drive the transformer into saturation, ferroresonance can occur when only one of the switches are closed. However, when both switches are closed, the series connection between the internal capacitance and the transformer is removed. This explains why the ferroresonant voltages disappeared in Fig. 4-16 when both switches were closed.

5.4.3 Ferroresonant Capacitance Range

Fig. 4-4 shows the bifurcation diagram for a variation in series capacitance at rated voltage. For low values of capacitance, there is a negligible voltage on the secondary winding. A series capacitance of 560 pF is the first capacitance value where the secondary voltage rises above 0.5 pu. Bifurcations start to become evident at 945 pF.

The secondary voltage starts to rise above 1.1 pu at a capacitance of 1.172 nF. The secondary voltage only drops below 1.1 pu for capacitance values beyond 14.84 μ F. This bifurcation diagram implies that series capacitance values between 1.172 nF and 14.84 μ F should be avoided at all costs. This includes the internal capacitance of the transformer for the likely case that the line-to-ground voltage in the power system could rise to an RMS value of 22 kV.

5.4.4 Ferroresonant Voltage Range

Fig. 4-9 and Fig. 4-10 showed bifurcation diagrams for a variation in the source voltage at a series capacitance of 20 nF and 1 μ F respectively. A series capacitance of 1 μ F resulted in the largest overvoltage at the rated voltage in Fig. 4-4. However, it is interesting to note that a larger voltage is required to initiate ferroresonance for a 1 μ F capacitor than for a 20 nF capacitor. For the 20

nF capacitor, ferroresonance was first initiated at a peak source voltage of 20.33 kV. For a 1 μ F capacitor, ferroresonance was initiated at a peak source voltage of 21.04 kV.

The two bifurcation diagrams imply that the voltage required to initiate ferroresonance depends on the series capacitance. Therefore, a fixed guideline cannot be given for voltage levels that are susceptible to ferroresonance. A new bifurcation diagram should be generated for the given series capacitance value whenever the designer wants to determine whether ferroresonance is likely.

It is worth reflecting on the voltage required to initiate ferroresonance relative to the shoulder region of the saturation curve. Fig. 5-4 shows the saturation curve of the transformer based on the OC test. It is interesting to note that voltages of 20.333 kV and 21.04 kV still appear to be in the linear region. One would expect that the transformer would only start to show saturation, and consequently ferroresonance, at voltage levels somewhere above 30 kV. This could mean that the transformer does not even have to enter into the shoulder region of the saturation curve for ferroresonance to be initiated.

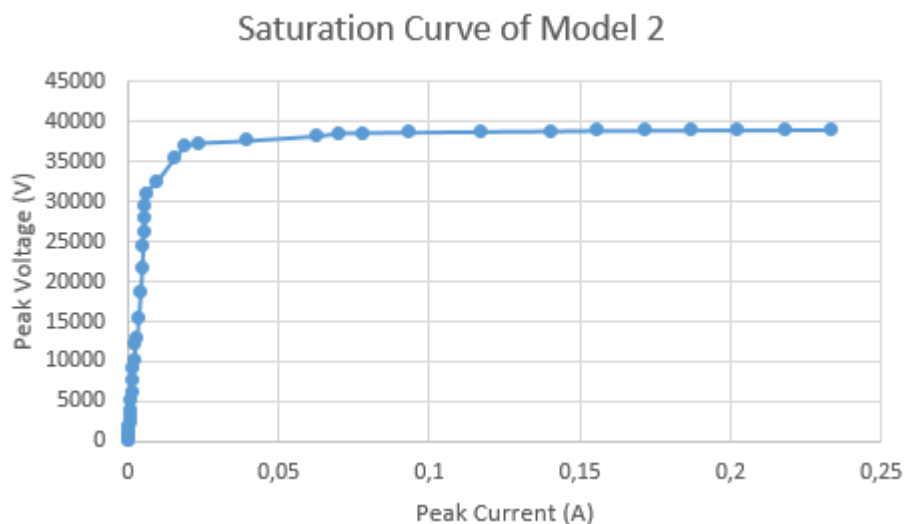


Fig. 5-4: The saturation curve of Model 2 showing peak voltage and peak current data points

5.4.5 Influence of Core Losses

Fig. 4-8 showed the bifurcation diagram of a variation of capacitance at rated voltage when the core losses of the transformer are increased. The core loss resistor was calculated based on the previous SANS 780 standard. It was interesting to note that this bifurcation diagram with increased core losses was identical to the original bifurcation diagram seen in Fig. 4-4. Further investigation shows that for a series capacitance of 20 nF, the ferroresonant voltages of Fig. 4-2

and Fig. 4-7 will be maintained for core loss resistors greater than 520 k Ω . This is the equivalent of 930 W of power loss in the transformer core.

This observation leads to the conclusion that the recent change in the SANS 780 standard does not make the distribution transformer more susceptible to ferroresonance. Even under the previous SANS 780 guidelines, the distribution transformer would have been driven into ferroresonance if a sufficient series capacitance was added to the power system or the line-to-ground voltage was sufficiently large.

5.4.6 Remnant Flux

This report has not investigated the effect that remnant flux has on ferroresonance. This is because the JA model could not be modified to represent the behaviour of the distribution transformer. Furthermore, Section 3.4.1 also proved that the Tellinen model is not capable of retaining remnant flux after switching. Therefore, even if the modifications of Section 5.1 were to be implemented to the Tellinen model, this would still not be sufficient for remnant flux analysis.

The nonlinear inductor of the Simscape Electrical environment allows the user to define an initial condition for the inductor. This initial condition can either be a current or a flux value. Hypothetically, this could be the equivalent of analysing remnant flux in the inductor. However, time constraints did not allow the author to investigate this functionality and its effect on ferroresonance in detail. A brief simulation showed that introducing a remnant flux of 0.3 Wb into the power system model of Fig. 3-61 was still not sufficient to initiate ferroresonance in the transformer during single-phase switching. It would therefore appear as though ferroresonance during single-phase switching is very unlikely, even when the remnant flux is taken into consideration.

5.4.7 Ferroresonance Mitigation

Fig. 4-6 showed the effect that a load on the LV winding has on ferroresonant overvoltages for a 1 μ F series capacitor at rated voltage. The bifurcation diagram showed that a load drawing 73% of the rated current is required to mitigate the secondary voltage below 1.1 pu. Section 4.3 also mentioned that a load of 4% is required if the series capacitance has a value of 20 nF.

This observation indicates that the load required to mitigate ferroresonance is dependent on the series capacitance initiating the ferroresonant overvoltages. It can also be hypothesized that the load required to mitigate these voltages will also be influenced by the source voltage.

5.5 Transformer Model Accuracy

It is important to reflect on the accuracy of the models which have been used for the simulations conducted in this study. This section aims to discuss the accuracy of the transformer models developed in Chapter 3.

5.5.1 SFRA Accuracy

The *SFRA* was an integral part of the transformer model development of Chapter 3. Not only did it indicate the inter-turn and inter-layer capacitance, but it also provided a benchmark for selecting Model 2 above the other models in Section 3.6.

Fig. 3-56 presented the simulated *OC SFRA* results compared to the ideal black-box model. This figure indicated that Model 2 was far superior to Model 1 and Model 3 in its frequency behaviour. However, there is still a significant difference between the frequency behaviour of Model 2 and the black-box model.

The discrepancy in Fig. 3-56 is caused by a lack of data points at low voltages. The *OC* test was conducted on the *LV* side of the transformer, where the voltage was applied between one terminal and the centre tap of the *LV* winding. The minimum *RMS* voltage applied between these two terminals was 6.042 V. When converted to the *HV* side, this is the equivalent of 549 V applied across the primary winding. The measured *SFRA* of Fig. 3-7 was conducted at a peak voltage of 20 V across the primary winding.

Fig. 5-5 shows how the inductance of the transformer varies based on the voltage being applied to the transformer on the *HV* side. The most interesting feature is the sudden decrease of the inductance below 3000 V. The nonlinear inductor of MATLAB interpolates between the defined data points. If positive and negative data points are used as input, the inductance will remain constant between 776 V and -776 V. This inductance has a value of 9656 H²⁵. The black box model employs an inductance of 636 H. This explains the deviation of Model 2 from the ideal black box model in Fig. 3-56.

Based on the trend shown at low voltages in Fig. 5-5, it was hypothesized that at a voltage as low as 20 V, the inductance of the nonlinear inductor could be as low as 636 H. To confirm this hypothesis, three additional measurements were taken on the transformer. These measurements were at an *RMS* voltage of 5.095 V, 2.497 V and 1.0591 V between one terminal and the neutral tap of the *LV* winding. After these measurements were taken into account, the graph of inductance

²⁵ Note that this inductance is for the traditional T-model. For the π -model, the inductance should be multiplied by 2.

vs *HV* peak voltage changed to that seen in Fig. 5-6. This clearly shows that the inductance continues to decrease as the voltage decreases. Furthermore, Fig. 5-7 shows the effect that the additional data points have on the *SFRA* of Model 2. The additional data points, and consequently the lower inductance, shift the peak of the *SFRA* curve to the right, which is closer to the ideal black-box model curve.

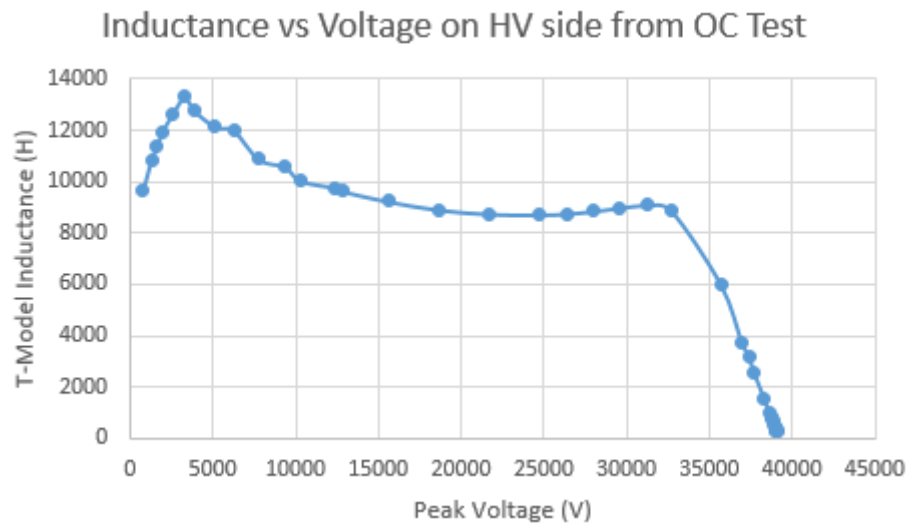


Fig. 5-5: A graph of the inductance of the transformer for a variation in the peak applied HV voltage

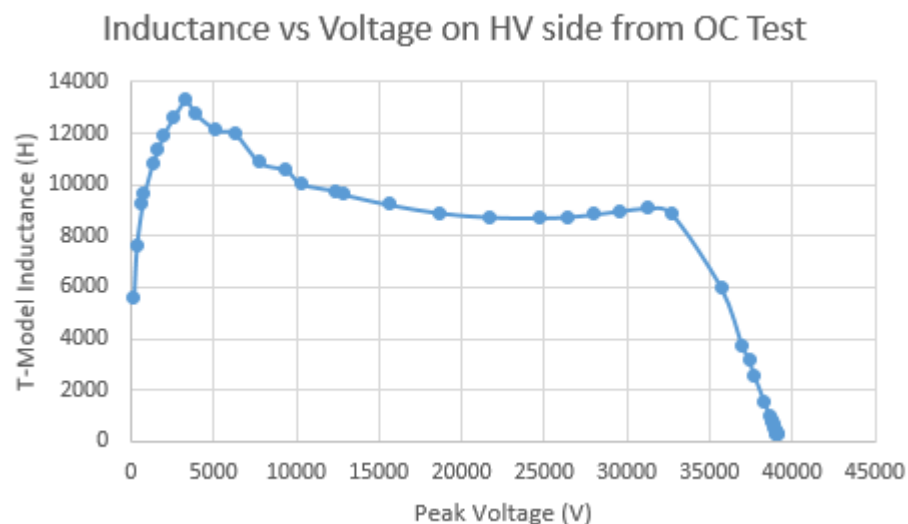


Fig. 5-6: The extended graph of the inductance of the transformer for a variation in peak applied HV voltage

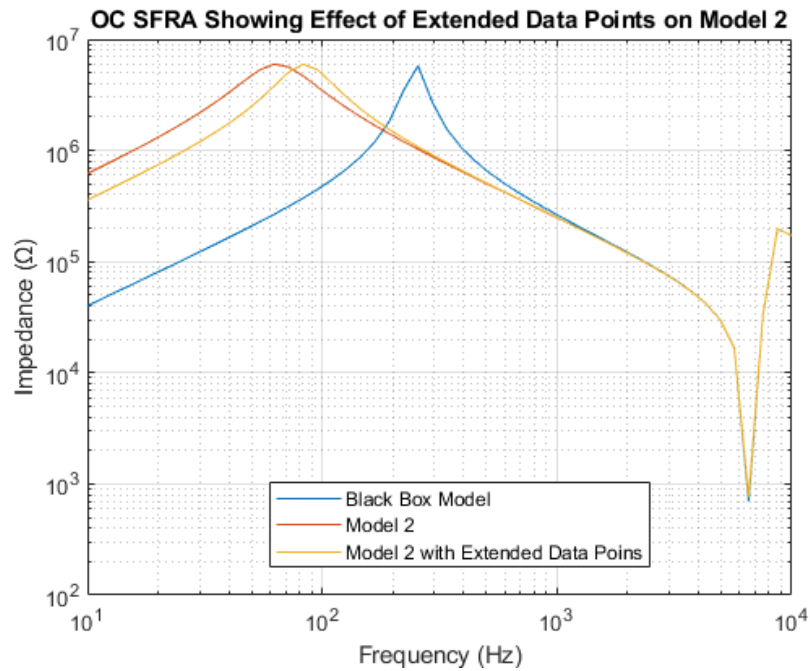


Fig. 5-7: A graph showing the effect of extended data points on the SFRA of Model 2

The inductance at voltages below 3.5 kV is shown more clearly in Fig. 5-8. A logarithmic trendline fits this data well and is shown in Fig. 5-8. Using this trendline, it can be determined that the transformer should have an inductance of 636 H at a peak voltage of 16.89 V. This is very close to the 20 V at which the *SFRA* is performed. It can then be concluded that Model 2 shows very good frequency behaviour when compared to the measured *SFRA*. This provides added confidence in the accuracy of Model 2. All simulations of Chapter 4 were done with the extended data points shown in Fig. 5-6.

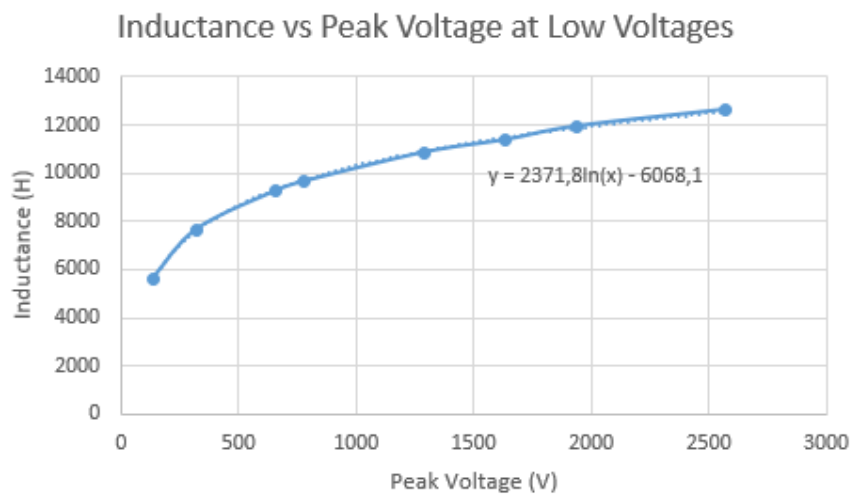


Fig. 5-8: The inductance at low voltage levels with a logarithmic trendline

5.5.2 Transformer Internal Capacitance

Fig. 3-15 showed that the transformer internal capacitances are divided between the top and bottom terminals of the transformer. This is based on the model presented by [82], which is also shown in Fig. 2-25. However, dividing the capacitance between the two terminals does not yield the same behaviour as a single capacitor lumped at the bottom terminal.

Initial simulations seemed to indicate that the behaviour of the transformer is the same in both cases. However, a difference was detected when series ferroresonance was simulated with a 20 nF capacitor. The waveforms of the two cases are shown in Fig. 5-9.

The difference between the lumped and the split approach is more significant than simply just a variation in the shape of the waveforms. Section 5.4.5 spoke about the core losses required to eliminate ferroresonance. It was found that a 520 k Ω core loss resistor eliminated the ferroresonant waveform. This was for the case where the capacitances were split between the terminals. If the capacitance is to be lumped at the bottom terminal, the core loss resistor needs to be decreased to 500 k Ω . In other words, core losses need to be higher for the lumped case than the split case to eliminate ferroresonant overvoltages.

Section 5.4.4 mentioned that if a 20 nF series capacitance is connected to the transformer, ferroresonance was initiated at 20.33 kV. This was for the case where the internal capacitances were split between the terminals. If the capacitances were lumped at the bottom terminal, ferroresonance would be initiated at 19.92 kV.

Based on these observations, it can be concluded that lumping the internal capacitances results in less favourable ferroresonant conditions. However, if the transformer capacitances are lumped at a terminal, the order of switching in power system simulations becomes important. Consider the case where the capacitances are lumped at the bottom terminal. If the switch at phase A in Fig. 3-61 is closed first, the internal capacitance is in series with the transformer magnetization branch and a series ferroresonant path is created. If, however, phase B is closed first, the current flows directly to ground as shown in Fig. 5-10. This eliminates the voltages measured on the secondary winding. This does not seem realistic since this would mean that the order in which the phases are switched will determine if an overvoltage is measured on the secondary winding.

A more realistic approach could be to lump the C_{HV-GND} capacitance on one terminal and lump the C_{HV-LV} capacitance on the other terminal. The connection of the capacitances could then be determined by the physical attributes of the transformer itself. The C_{HV-GND} capacitance should be

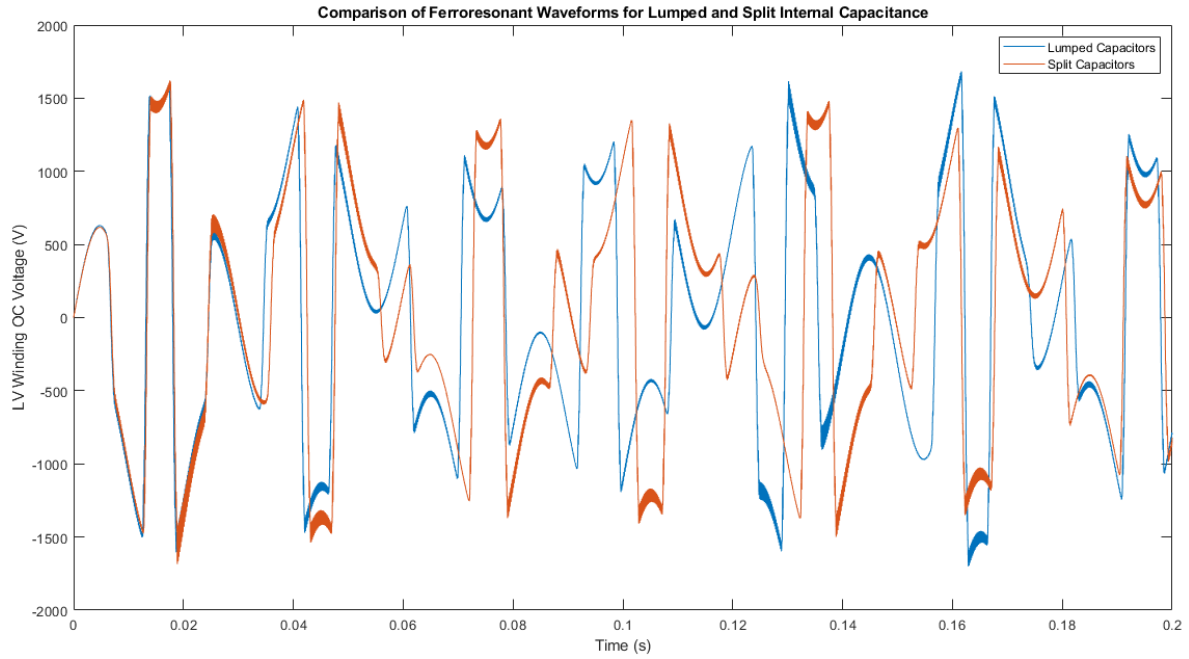


Fig. 5-9: A comparison of ferroresonant waveforms for split and lumped transformer internal capacitances

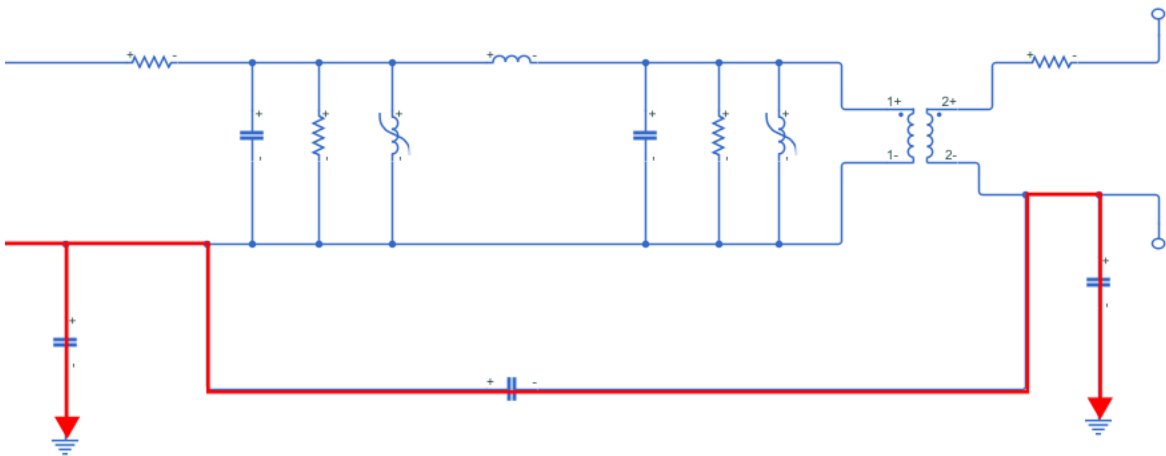


Fig. 5-10: The path of current if internal capacitances are lumped and phase B closes first

connected to the terminal closest to the tank of the transformer. The C_{HV-LV} capacitance should be connected to the terminal closest to the LV winding.

Experimental verification will be required to make a definite conclusion on the correct topology. Fig. 5-11 shows the waveforms of the three different topologies when ferroresonance is initiated with a 20 nF series capacitance. Verification of these waveforms involves connecting a 20 nF series capacitance to the actual transformer and applying an RMS voltage of 22 kV to the HV terminals.

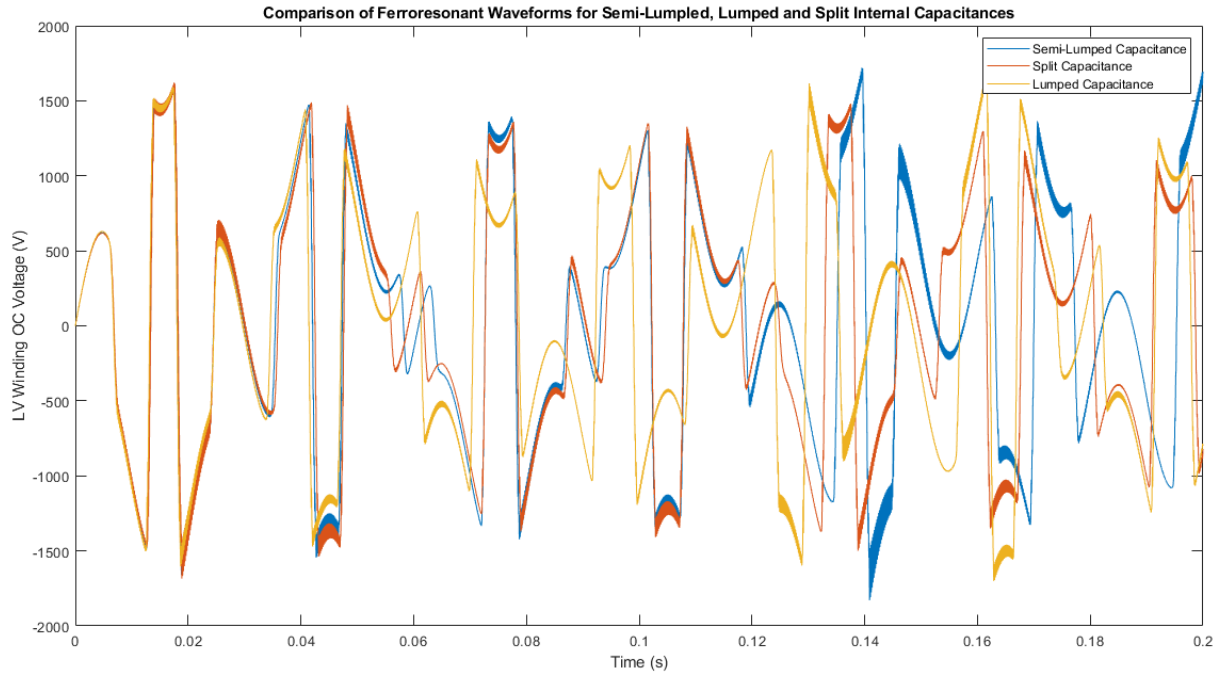


Fig. 5-11: A comparison of semi-lumped, lumped and split internal capacitance ferroresonant waveforms

5.5.3 Inter-turn and Inter-Layer Capacitance

The black box modelling approach discussed in Section 3.2.4 provided a means to calculate the inter-turn and inter-layer capacitance of the transformer. The black box model does not expressly define the capacitance as an inter-turn or inter-layer capacitance. However, the capacitance calculated from the *SFRA* can be interpreted as this capacitance based on the literature.

It should be noted, however, that both the *HV* and the *LV* winding have inter-turn and inter-layer capacitances. At this stage, Model 2 only connects a capacitance across the magnetization branch located on the *HV* side of the model. For the π -model, this capacitance is divided by two and connected across each magnetization branch. The important thing to note is that a capacitor is not connected across the two terminals of the *LV* winding where the *LV* inter-turn and inter-layer capacitance would typically be connected.

This lack of an *LV* capacitance can be determined as a physical inconsistency based on the construction of the transformer. However, connecting a capacitor across the *LV* winding has a significant impact on both the frequency behaviour of the transformer and the OC results of Model 2. The conclusion is made that the capacitance determined from the *HV SFRA* is a lumped capacitance that represents both the *HV* and *LV* windings' inter-turn and inter-layer capacitance. Therefore, even though the circuit diagram of Model 2 in Fig. 3-15 is not entirely physically accurate, it is a better representation of the transformer than the alternative.

5.6 Contributions and Future Work

The studies conducted in this report were focused on the analysis of ferroresonance. This report has made a couple of meaningful contributions to the topics of ferroresonance and transformer modelling, which are briefly summarized in the ensuing paragraphs.

Firstly, the study has developed an inverse Jiles Atherton hysteresis model in the MATLAB environment. This model has been interfaced with a circuit diagram in Simulink and has proven to be capable of replicating results found in the literature.

Secondly, the study has developed an inverse Tellinen hysteresis model, which has also proven to be capable of replicating results found in the literature. Furthermore, the study confirmed the literature which claims that the *JA* model is capable of retaining residual flux, while the Tellinen model is not.

Thirdly, the study has developed a Particle Swarm Optimization algorithm that can optimize the five Jiles-Atherton parameters. However, this algorithm is not complete and should be refined further.

Fourthly, the study has developed a π -transformer model and replicated results found in the literature which claim that the π -model is superior to the traditional T-model. The study has also elaborated on the π -model by taking the *SFRA* test into account for modelling purposes. This allows for the definition of a lumped capacitor across the magnetization branch which represents the *HV* and *LV* inter-turn and inter-layer capacitance.

The fifth important contribution of the study is proof that the overvoltages measured by Geldenhuys and Padayachee [1] are not caused by ferroresonance. Moreover, the study proves that ferroresonance is highly unlikely during the switching transients due to the low internal capacitance and the low applied voltage across the transformer during switching. However, the internal capacitance of the transformer is in series with the magnetization branch during switching, which can lead to ferroresonance under more favourable conditions.

Finally, the study has looked into several mitigation strategies which can be used to reduce the overvoltages on the secondary winding. The practicality of these suggested strategies is also discussed.

Future work to further this study should be focused on the improvement of the hysteresis models. Section 5.1 discussed the Pareto front of the *PSO* algorithm which is created when multi-objective cost functions are used. Alternative cost functions need to be investigated since the current cost

functions are not sufficient. Equation (5-1) should be added to the algorithm in an attempt to limit the non-physical behaviour of the loop.

A better understanding of the high harmonic content of the distribution transformer is also required. This may require a comparison of the new transformer to transformers complying with the older SANS 780 standard. It would also be advantageous if the behaviour of the studied transformer could be compared to transformers from other manufacturers. The high harmonic content measured during the OC test of the transformer has not been encountered anywhere else in the literature.

Once a better understanding of the harmonic content has been obtained and the hysteresis models have been improved, the switching studies should be performed with hysteresis taken into account. This will allow for ferroresonance studies that take remnant flux into account. Remnant flux can also be taken into account by defining the initial conditions of the nonlinear inductor in the Simscape Electrical environment.

Future work should also attempt to better define how transformer internal capacitances should be connected to the transformer model. Section 5.5.2 spoke about the effect that splitting transformer internal capacitances, as opposed to lumping them at one terminal, can have on ferroresonant studies. Experimental measurements are required to conclude the best approach.

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APPENDIX A: PROOF OF ERROR IN [160]

Hui and Zhu [160] attempted to simplify the Preisach model in such a manner that only the major hysteresis loop is required for the model. The equations derived by [160] are presented in (2-68) through (2-74). However, there is an error in their equations, which casts doubt on the accuracy of the overall model.

The error is specifically related to (2-74), which is the equation for the initial magnetic flux density. The equation takes the form $B_i(H) = [F(-H) - F(H)]^2$. The squared term will always result in a positive initial magnetic flux density. This causes problems when the initial magnetic field intensity is negative.

The Preisach model developed by [160] calculates the magnetic flux density from the magnetic field intensity. The Tellinen and Jiles-Atherton models developed in Chapter 3 calculated the magnetic field intensity from the magnetic flux density. Fig A-1 shows the Preisach hysteresis loop generated when the magnetic field intensity starts at zero and is initially increasing.

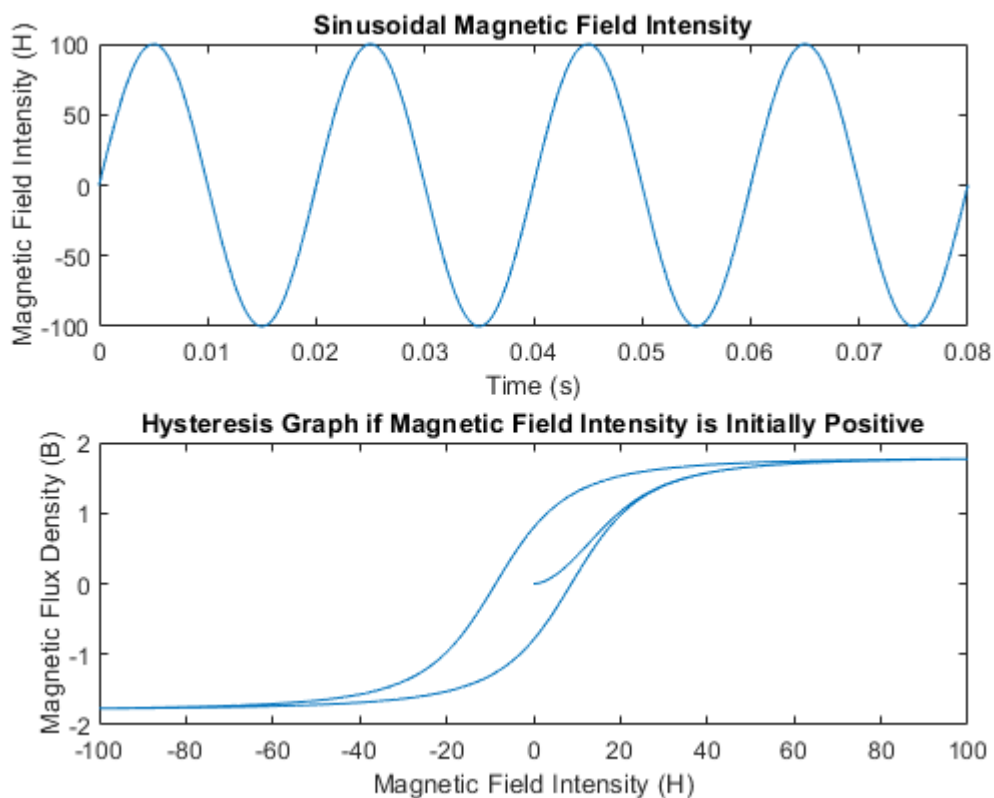


Fig A-1: The Preisach hysteresis loop when the magnetic field intensity is initially positive

Fig. A-2 shows a similar hysteresis graph. In this case, however, the magnetic field intensity starts at zero and is initially negative. This is shown in the top graph of Fig. A-2. The hysteresis graph shows that the initial magnetic flux density is still positive, even though the magnetic field intensity is descending. This is caused by the squared term mentioned earlier. This positive initial magnetic flux density shifts the entire hysteresis loop upward by a factor of nearly 2. For this reason, the Preisach model developed by Hui and Zhu [160] cannot be used, especially for simulations where switching plays an important role. It cannot be guaranteed that switches will close on positive cycles and all subsequent simulations will be significantly influenced by incorrect magnetic flux densities.

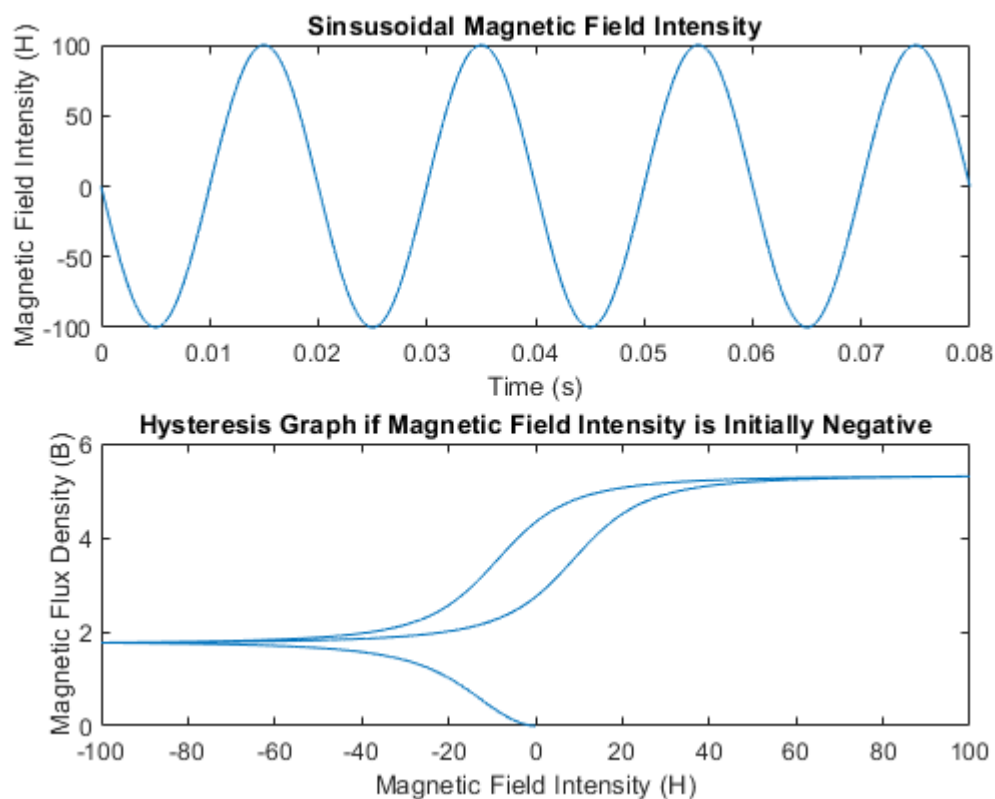


Fig. A-2: The Preisach hysteresis loop when the magnetic field intensity is initially negative

APPENDIX B: NONLINEAR INDUCTOR FROM FIRST PRINCIPLES

Section 3.1 discussed the replication of results found in [95]. In an attempt to better understand the operation of the nonlinear inductor in the MATLAB environment, a nonlinear inductor was built from first principles. The subsystem of this first-principles nonlinear inductor is shown in Fig B-1. A mask is created for this subsystem and a traditional T-transformer model showing the incorporation of the subsystem is shown in Fig. B-2. The combination of the masked subsystem and the controlled current source to its right can be interpreted as the nonlinear inductor of the transformer. The voltage measurement tool to the left of the masked subsystem provides the V_{in} input to Fig B-1.

The subsystem requires three inputs: the number of windings, the current data points and the flux data points. Flux-linkage can also be used in this subsystem by setting the number of windings to one. The flux-linkage data points should then replace the flux data points and the results will remain identical.

The transformer model in Fig. B-2 contains a 20 μF series capacitor, which can be used to initiate ferroresonance. The lookup table in Fig B-1 is defined from the final two columns of [88, Table VI]. This is the same data that was used to replicate the T-model ferroresonance results in Section 3.1. The winding resistance, leakage inductance and core loss resistance of Fig. B-2 are determined from the final column of [88, Table II].

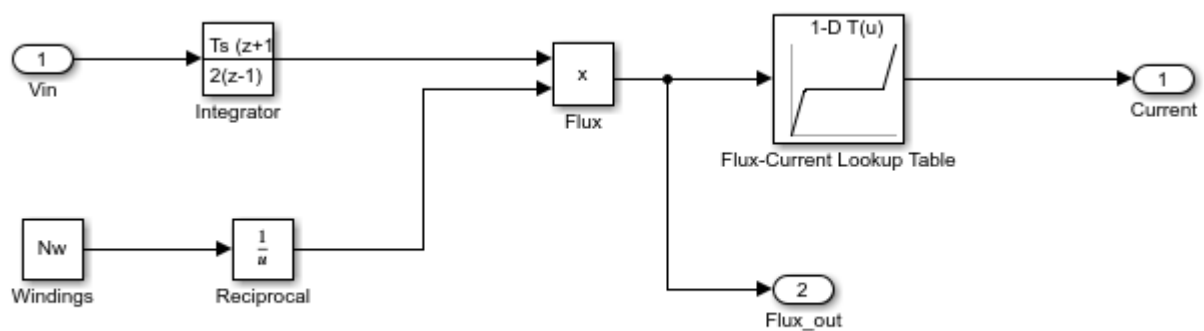


Fig B-1: A first-principles nonlinear inductor from fundamental components

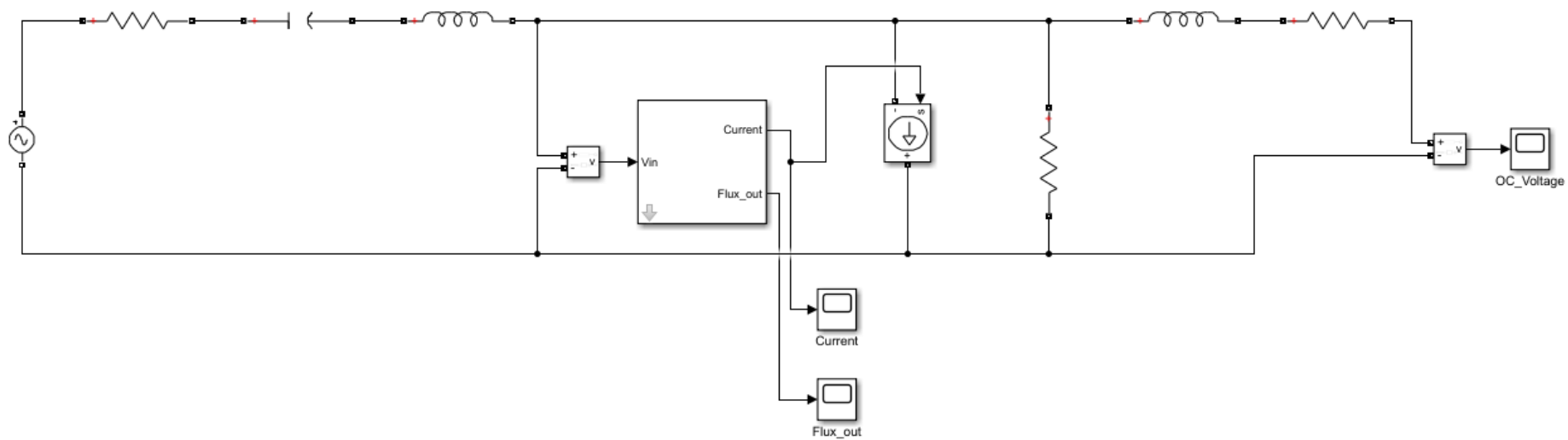


Fig. B-2: The T-Transformer model with a masked subsystem of the nonlinear inductor

Fig. B-3 shows two sets of ferroresonant waveforms generated when the circuit shown in Fig. B-2 is fed by a source with an initial phase angle of -90° and a magnitude of $V_{peak} = \sqrt{2} \cdot 120 [V]$. The top waveform shows the case where the nonlinear inductor in the MATLAB Specialized Power System environment is used. The bottom figure shows the waveform where the subsystem of Fig B-1 is used. If the two waveforms are placed on the same figure, they cannot be distinguished from each other. For this reason, they are shown in two different figures. This shows that the self-made nonlinear inductor of Fig B-1 designed from first principles is identical to the nonlinear inductor found in MATLAB.

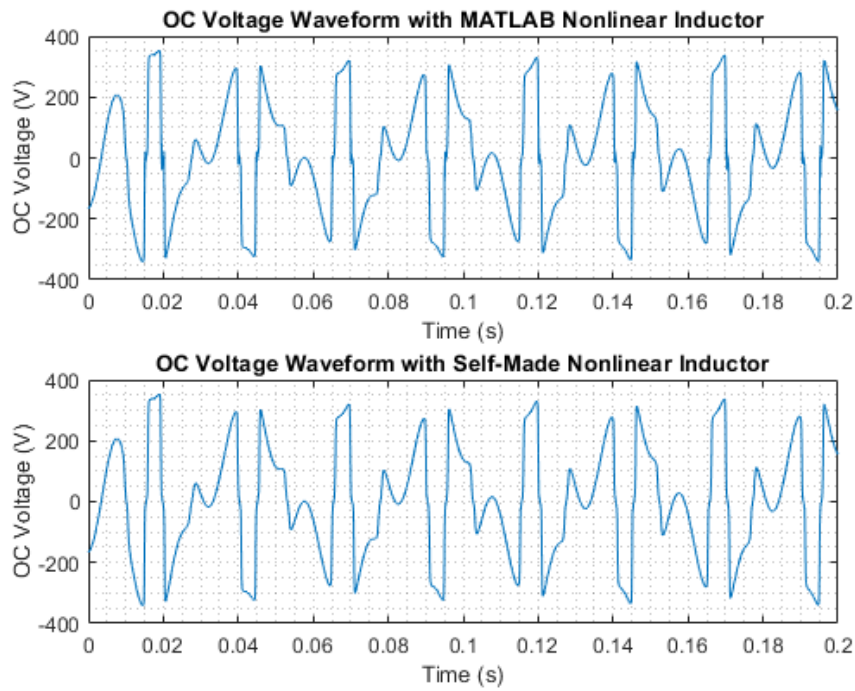


Fig. B-3: A comparison of the MATLAB nonlinear inductor and the self-made nonlinear inductor ferroresonant waveforms

APPENDIX C: FLUX LINKAGE/CURRENT DATA POINTS

This section presents the data points used for the four models discussed in Sections 3.2.5, 3.5 and 3.6. The data points are labelled Model 1 – Model 4 according to the convention established in Section 3.2.5. The reader is reminded that Model 2 showed the overall best behaviour when the *OC* test and frequency behaviour were taken into account.

The data points are presented as flux linkage/current data points. For the simulations of Chapters 3, 4 and 5, the author used flux/current data points. However, to convert flux linkage to flux, the user needs to divide the flux linkage by the number of windings. The number of windings was known to the author but is considered proprietary and cannot be given in this report. Consequently, the flux linkage data points are given here since these data points are purely related to voltage measurements and are not proprietary.

Certain simulation environments, such as ATPDraw and the Specialized Power Systems environment of MATLAB, require flux linkage as input. A value for the flux can easily be calculated by dividing the flux linkage by an arbitrary number of windings. Nonlinear inductors in the Simscape Electrical environment require the number of windings as an input and the author has verified that as long as the flux data points and the number of windings are consistent, the results will remain the same irrespective of the number of windings used. Moreover, if the number of windings in the Simscape Electrical environment is set to one, then the flux linkage data points presented in the following table will yield the correct results. It is, therefore, easy to convert flux linkage data points to flux data points. The contrary is not true – it is not easy to convert flux data points to flux linkage data points without knowing the exact number of windings of the transformer. The flux linkage data points presented in the following table should, therefore, be more useful while also protecting the design data of the manufacturer.

The reader is also reminded to carefully consider the environment being used for the simulations. The Specialized Power Systems environment in MATLAB requires the user to enter positive and negative saturation data. The Simscape Electrical environment can operate on only the positive saturation data, provided that the first data point is (0;0). The ATPDraw environment also requires only the positive saturation data.

The data in Table C-1 is referenced to the *HV* side. Both the current and flux linkage data reflect peak data points, as opposed to *RMS* data points. Chapter 3 contains a detailed motivation for the use of peak data points.

Table C-1: The nonlinear inductor data points of Models 1 Through 4 for Chapter 3 simulations

Model 1		Model 2		Model 3		Model 4	
<i>Flux Linkage (Wb-turns)</i>	<i>Current (A)</i>	<i>Flux Linkage (Wb-turns)</i>	<i>Current (A)</i>	<i>Flux Linkage (Wb-turns)</i>	<i>Current (A)</i>	<i>Flux Linkage (Wb-turns)</i>	<i>Current (A)</i>
0	0	0	0	0	0	0	0
2.4742	5.031e-5	0.4337	3.856e-5	2.4726	5.03e-5	2.4726	1.281e-4
5.1990	6.394e-5	1.0225	6.670e-5	5.1956	6.394e-5	5.1956	2.282e-4
10.4623	6.402e-5	2.0864	1.128e-4	10.4555	6.401e-5	10.4555	3.930e-4
19.9165	2.025e-4	2.4742	1.281e-4	19.9035	2.025e-4	19.9035	8.293e-4
29.7244	4.681e-4	4.1065	1.889e-4	29.7051	4.681e-4	29.7051	1.403e-3
39.2817	7.737e-4	5.1990	2.282e-4	39.2562	7.737e-4	39.2562	2.009e-3
49.6724	1.125e-3	6.1724	2.581e-4	49.6402	1.125e-3	49.6402	2.689e-3
59.5905	1.471e-3	8.1929	3.237e-4	59.4209	1.471e-3	59.4209	3.335e-3
69.1073	1.775e-3	10.4623	3.930e-4	69.0624	2.076e-3	69.0625	4.245e-3
78.5422	2.032e-3	12.2867	4.801e-4	78.4912	2.414e-3	78.4912	4.880e-3
84.0909	2.151e-3	16.3890	6.746e-4	84.0363	2.670e-3	84.0363	5.299e-3
89.0704	2.225e-3	19.9165	8.293e-4	89.0126	2.829e-3	89.0126	5.626e-3
94.2629	2.274e-3	24.7150	1.135e-3	94.2017	2.916e-3	94.2017	5.969e-3
99.3284	2.341e-3	29.7244	1.403e-3	99.2640	3.217e-3	99.2640	6.336e-3
104.1032	2.573e-3	32.7875	1.632e-3	104.036	3.814e-3	104.0356	7.287e-3
113.6896	5.877e-3	39.2817	2.009e-3	113.616	0.01229	113.6158	0.0162
117.7232	0.0119	41.0278	2.131e-3	117.647	0.02781	117.6469	0.0315
119.1237	0.0150	49.6724	2.688e-3	119.269	0.044	119.0578	0.044
120.1350	0.0233	59.5905	3.347e-3	119.628	0.04950	119.4370	0.0495
121.0603	0.0389	69.1073	3.951e-3	119.931	0.055	119.7421	0.055
121.5871	0.0622	78.5422	4.505e-3	120.417	0.066	120.2028	0.066
121.6852	0.070	84.0909	4.801e-3	120.954	0.0825	120.6669	0.0825

121.7637	0.0778	89.0704	5.031e-3	121.349	0.099	120.9784	0.099
121.8817	0.0933	94.2629	5.243e-3	121.561	0.11	121.1347	0.11
121.9999	0.1167	99.3284	5.468e-3	121.743	0.121	121.2629	0.121
122.0789	0.1400	104.1032	5.853e-3	121.902	0.132	121.3700	0.132
122.1184	0.1556	113.6896	9.457e-3	122.042	0.143	121.4607	0.143
122.1508	0.1711	117.7232	0.0156	122.166	0.154	121.5385	0.154
122.1777	0.1867	119.1237	0.0188	122.278	0.165	121.6061	0.165
122.2006	0.2022	120.4025	0.0233				
122.2202	0.2178	123.3805	0.0389				
122.2371	0.2333	125.6085	0.0622				
		126.0992	0.0700				
		126.5171	0.0778				
		127.1956	0.0933				
		127.9538	0.1167				
		128.5189	0.1400				
		128.8246	0.1556				
		129.0887	0.1711				
		129.3199	0.1867				
		129.5244	0.2022				
		129.7072	0.2178				
		129.8717	0.2333				

APPENDIX D: MATLAB PROGRAM OF JA MODEL

Forward Model

```
%The purpose of this program is to try to solve the Jiles-Atherton DE using
%the 4th order Runge-Kutta numerical method.

%For this model, reference to "Theory of Ferromagnetic Hysteresis -
%D.C. Jiles & D.L. Atherton" will be termed [S168]. The equations of
%"Numerical Determination of Hysteresis Parameters for the Modeling of
%Magnetic Properties Using the Theory of Ferromagnetic Hysteresis - D.C.
%Jiles, J.B. Thoelke & M.K. Devine" will be used, which will be referred to
%as [S66].

%EDIT: Having spoken to other authors who also implemented the algorithm,
%it has come to my attention that the presence of  $\mu_0$  does not matter. It
%only influences the parameter k. Therefore, if  $\mu_0$  is present, then the
%value of k should just be different. But I haven't tried to play around
%with k yet. I just left  $\mu_0$  out.

%S.J. Kruger
%Initiated 2020/11/23
%Updated: 2021/11/16

close all
clear all
clc

%Define the JA constants that we will be using

%The JA parameters used below are based on [S164, Table III]

alpha = 177.05*10^-6;
Msat = 1.4552*10^6;
k = 72.312;
c = 350.49*10^-3;
a = 88.498;

mu_0 = 4*pi*10^-7;      %Permeability of air.

%Move all of the constants into an array that can be passed to functions

constants.alpha = alpha;
constants.Msat = Msat;
constants.k = k;
constants.c = c;
constants.a = a;
constants.mu_0 = mu_0;

%Create some time values
fs = 100000;             %sample frequency [Hz]
timestep = 1/fs;
t=0:timestep:0.03;
constants.timestep = timestep;
```

```

% %Now create an array of sinusoidal H values

%For my experiment to see how the inverse and forward models compare, I am
%going to limit H to H = 182.3609

Hmax = 182.3609;

for i=1:max(size(t))
    H(i) = Hmax*sin(2*pi*60*t(i));
end

M(1) = 0.1;
%I found that if I use M(1)=0, then there is a discontinuous jump for the
first iteration of calculations. But if I use 0.1, then it is fixed, and in
comparison to the peak value of  $1.5480 \times 10^6$ , 0.1 is basically nothing.

%%I am not sure if this next line of code is necessary when dealing with my
%%own generated sinusoidal waveform. But when I import data that starts at
%%the tip of the major loop, and M(1) != 0.1, then it is important to also
define B(1), otherwise, the first point is very wonky.

B(1) = mu_0*(M(1) + H(1));

%Now implement the RK4 method for all the H values:
for i=2:max(size(H))
    h = H(i)-H(i-1);
    He_old = H(i-1) + alpha*M(i-1);
    constants.He_old = He_old;
    constants.h = h;
    K1 = h*jilesAtherton(constants, H(i-1), M(i-1));
    K2 = h*jilesAtherton(constants, H(i-1)+h/2, M(i-1)+K1/2);
    K3 = h*jilesAtherton(constants, H(i-1)+h/2, M(i-1)+K2/2);
    K4 = h*jilesAtherton(constants, H(i-1)+h, M(i-1)+K3);
    M(i) = M(i-1) + K1/6 + K2/3 + K3/3 + K4/6;
    B(i) = mu_0*(H(i)+M(i));
end

function JA = jilesAtherton(constants, H, M)
    Man = anhMagnetization(constants, H, M);
    He = H + constants.alpha*M;
    delta = sign(constants.h);
    term1 = (1-constants.c);
    term2 = (Man-irrMagnet(constants, M, Man))/(constants.k*delta -
constants.alpha*(Man-irrMagnet(constants, M, Man)));
    term3 = constants.c*(anhMagnetization(constants, He+1e-6, M) -
anhMagnetization(constants, He-1e-6, M))./2e-6;
    JA = term1*term2+term3;
end

function dMandH = differential(constants, H, M)
    if(H==0 && M==0) %This should take care of the singularities
        dMandH = 0;
        return;
    end
    dMandH = constants.Msat*((-
1/constants.a)*csch((H+constants.alpha*M)/constants.a) +
constants.a*(H+constants.alpha*M)^-2);
end

```

```

function Man = anhMagnetization(constants,H,M)
    if (H==0 && M==0)    %This should take care of the singularities
        Man = 0;
        return;
    end
    Man = constants.Msat*(coth((H+constants.alpha*M)/(constants.a))-
    (constants.a)/(H+constants.alpha*M));
end

function Mirr = irrMagnet(constants, M, Man)
    Mirr = (1/(1-constants.c))*(M-constants.c*Man);
end

```

Inverse Model

```

%For this model, reference to "An Inverse Jiles-Atherton Model to Take Into
%Account Hysteresis in Time-Stepping Finite Element Calculations - N.
%Sadowski, N.J. Batistela, J.P.A. Bastos and M. Lajoie-Mazenc" will be
%termed [S156]. Furthermore, reference to [S164] refers to "The Inverse
%Jiles-Atherton Model Parameters Identification - J.V. Leite, N. Sadowski,
%P. Kuo-Peng, N.J. Batistela & J.P.A. Bastos"

```

```

%I would also like to specifically recognize Professor J.V. Leite, whose
%assistance was invaluable in getting the program to work.

```

```

%%NOTE TO SELF: If it should ever happen that I want to use experimental
%%data to run the algorithm, then there should be no value of B that
%%repeats itself, because if that happens, then delta becomes zero, and
%%then the algorithm stops working.

```

```

%S.J. Kruger
%Initiated 2020/12/31
%Updated: 2021/11/16

```

```

close all
clear all
clc

```

```

%Define the JA constants that we will be using

```

```

%The JA parameters used below are the optimized values from [S164, Table III]

```

```

alpha = 177.05*10^-6;
Msat = 1.4552*10^6;
k = 72.312;
c = 350.49*10^-3;
a = 88.498;

```

```

mu_0 = 4*pi*10^-7;    %Permeability of air.

```

```

%Move all of the constants into a struct that can be passed to functions

```

```

constants.alpha = alpha;
constants.Msat = Msat;
constants.k = k;
constants.c = c;

```

```

constants.a = a;
constants.mu_0 = mu_0;

%Create some time values
fs = 100000;           %sample frequency [Hz]
timestep = 1/fs;
t=0:timestep:0.04;
constants.timestep = timestep;

%Now create an array of sinusoidal B values. The array will give a sine
%wave with a frequency of 60 Hz and a peak amplitude of Bmax.
Bmax = 1.3;           %Based on [S164, Fig. 7]

for i=1:max(size(t))
    B(i) = Bmax*sin(2*pi*60*t(i));
end

H(1) = 0;           %Assume that the initial magnetic field is 0

%Now implement the numerical procedure seen in [S156, fig. 1]

err = 1E-4; % Tolerance for He

for i=2:max(size(B))
    deltaB = B(i) - B(i-1);
    delta = sign(deltaB);
    M(i-1) = B(i-1)/mu_0 - H(i-1);
    He(i-1) = H(i-1) + alpha*M(i-1);

    %==== Avoid overflow for zero division =====
    if abs(He(i-1))<err
        if sign(He(i-1))~=0
            He(i-1)=sign(He(i-1))*err;
        else He(i-1)=err;
        end
    end
    %=====

    Man(i-1) = Msat*(coth(He(i-1)/a) - a/He(i-1));

    dMandHe = (Msat/a)*(1-(coth(He(i-1)/a))^2+(a/He(i-1))^2);

    Mirr(i-1) = (M(i-1)-c*Man(i-1))/(1-c);

    dMirrdBe = (Man(i-1)-Mirr(i-1))/(mu_0*k*delta);

    %==== Avoid unphysical behavior =====
    if dMirrdBe < 0
        dMirrdBe = 0;
    end
    %=====

    num = (1-c)*dMirrdBe+(c/mu_0)*dMandHe; %This is the numerator of dM/dB
    denom = 1+mu_0*(1-alpha)*(1-c)*dMirrdBe + c*(1-alpha)*dMandHe;
    dMdB = num/denom;
    M(i) = M(i-1) + dMdB*deltaB;
    H(i) = B(i)/mu_0 - M(i);
end

```

APPENDIX E: MATLAB PROGRAM OF TELLINEN MODEL

Forward Model

```
%In this model, I am trying to implement the Tellinen model. The Tellinen
%model is described by "A Simple Scalar Model for Magnetic Hysteresis" by
%Juhani Tellinen (from hereon referred to as [S32]), and is also
%implemented by "Experimental and Simulation Analysis of Ferroresonance in
%Single-phase Transformers Considering Magnetic Hysteresis Effects" by P.S.
%Moses and M.A.S. Masoum (from hereon referred to as [S27])

%S.J. Kruger
%Initiated 2020/11/27
%Updated: 2021/11/16
close all
clear all
clc

%These next two sigma constants are based on concepts from [S27], where
%sigma was used to shift the main function up or down.

sigma_plus = 50; %The chosen value of 50 was just based on trial
and error to find a graph that looked decent.
sigma_minus = 50;

mu_0 = 4*pi*10^-7; %Permeability of free space

%Now let us move all the constants into a single struct that can be passed
%to functions

constants.sigma_plus = sigma_plus;
constants.sigma_minus = sigma_minus;
constants.mu_0 = mu_0;

%Create some time values
fs = 10000000;
%fs = sample frequency [Hz]. This has to be quite high to ensure our time
steps are small enough for the algorithm to converge
timestep = 1/fs;
t=0:timestep:0.08;
constants.timestep = timestep;

%Now create an array of H values
Hmax = 600;

for i=1:max(size(t))
    H(i) = Hmax*sin(2*pi*50*t(i));
end

%Define some JA-type parameters for the Langevin function

Bsat = 1.5; %Bsat takes the place of Msat
a = 50; %'a' is required for the langevin function
constants.Bsat = Bsat;
constants.a = a;
```

```

B(1) = 0;          %Assume that the initial flux density is 0

%Now for the main for-loop where we calculate flux density
for i=2:max(size(H))
    h = H(i) - H(i-1);
    if(h>=0)
        sign = 1;
    else
        sign = -1;
    end
    K1 = h*Tellinen(constants, H(i-1),B(i-1), sign);
    K2 = h*Tellinen(constants, H(i-1) + h/2, B(i-1)+K1/2, sign);
    K3 = h*Tellinen(constants, H(i-1) + h/2, B(i-1)+K2/2, sign);
    K4 = h*Tellinen(constants, H(i-1) + h, B(i-1) + K3, sign);
    B(i) = B(i-1) + K1/6 + K2/3 + K3/3 + K4/6;

end

function T = Tellinen(constants, H, B, sign)
    %I will now do a bunch of calculations which will eventually be used to
    %calculate the final term in the algorithm.

    bhn = Langevin(constants.Bsat, constants.a, H + constants.sigma_minus);
    %bhn is the b_h-(h) term in [S32, eq. 6]
    bhp = Langevin(constants.Bsat, constants.a, H - constants.sigma_plus);
    %bhp is the b_h+(h) term in [S32, eq. 6]

    if(sign == 1) %If dh>0
        dbdh = (Langevin(constants.Bsat, constants.a, H -
constants.sigma_plus + 1e-6)-Langevin(constants.Bsat, constants.a, H -
constants.sigma_plus - 1e-6))./2e-6; %This is the dbh+/dh term of [S32,eq 6a]
        T = constants.mu_0 + ((bhn-B)/(bhn-bhp))*(dbdh - constants.mu_0);
    %This is [S32, eq. 6a]
    end
    if(sign == -1) %If dh<0
        dbdh = (Langevin(constants.Bsat, constants.a, H +
constants.sigma_minus + 1e-6) - Langevin(constants.Bsat, constants.a, H +
constants.sigma_minus - 1e-6))./2e-6; % dbh-/dh term of [S32, eq. 6b]
        T = constants.mu_0 + ((B-bhp)/(bhn-bhp))*(dbdh - constants.mu_0);
    %This is [S32, eq. 6b]
    end
end

function B = Langevin(Bsat, a, H)
    if(H == 0)
        B = 0;
    else
        B = Bsat*(coth(H/a)-(a/H));
    end
end

```

Inverse Model

```
%In this model, I am trying to implement the inverse Tellinen model. The
%Tellinen model is described by "A Simple Scalar Model for Magnetic
%Hysteresis" by Juhani Tellinen (from hereon referred to as [S32]) and is
%also implemented by "Experimental and Simulation Analysis of Ferroresonance
%in Single-phase Transformers Considering Magnetic Hysteresis Effects" by
%P.S. Moses and M.A.S. Masoum (from here on referred to as [S27])
```

```
%The inverse Tellinen model is given in [S32, eq. 7]. This basically means
%that I want to have B as my input and calculate H as an output. Note,
%however, that I am going to try to implement the algorithm as dh/db, so
%that I can lose the dh/dt and db/dt terms.
```

```
%%RESULTS: This seems to have worked, but there is one VERY IMPORTANT
%%criteria that needs to be met. The maximum value of B (the input) needs
%%to be smaller than the parameter Bsat. Bsat is used in the Langevin
%%function. If the maximum value of B (defined as Bmax here, but if it
%%is an input from a circuit, one needs to take careful note of its peak
%%value) exceeds Bsat, then the algorithm will spit out garbage.
%%Also note that if there are harmonics on B, then it could increase the
%%peak value, which could once again exceed Bsat, and the algorithm will
%%then no longer work.
```

```
%S.J. Kruger
%Initiated 2020/12/30
%Updated: 2021/11/16
```

```
close all
clear all
clc
```

```
%These next two sigma constants are based on concepts from [S27], where
%sigma was used to shift the main function up or down.
```

```
sigma_plus = 50; %The chosen value of 50 was just based on trial
and error to find a graph that looked decent.
sigma_minus = 50;
```

```
mu_0 = 4*pi*10^-7; %Permeability of free space
```

```
%Now let us move all the constants into a single struct that can be passed
%to functions
```

```
constants.sigma_plus = sigma_plus;
constants.sigma_minus = sigma_minus;
constants.mu_0 = mu_0;
```

```
%Create some time values
```

```
fs = 1000000;
%fs = sample frequency [Hz]. This has to be quite high to ensure our
timesteps are small enough for the algorithm to converge
timestep = 1/fs;
t=0:timestep:0.08;
constants.timestep = timestep;
```

```
%Now create an array of B values
```

```
Bmax = 1.5; %Let us assume that B only goes up to 1.5
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%NB!!!%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Bmax needs to be smaller than Bsath (defined a little later), otherwise, the
%algorithm will spit out garbage. It seems to me it can be by the smallest
%of margins (e.g. Bmax = 1.5 & Bsath = 1.51), but it must be smaller.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%NB!!!%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for i=1:max(size(t))
    B(i) = Bmax*sin(2*pi*50*t(i));
end

%Define some JA-type parameters for the Langevin function

Bsath = 2.3;          %NB!!! Bsath needs to be greater than Bmax, otherwise the
algorithm will spit out garbage.
a = 50;              %'a' is required for the langevin function
constants.Bsath = Bsath;
constants.a = a;

H(1) = 0;           %Assume that the initial field intensity is 0

%Now for the main for-loop where we calculate magnetic field intensity
for i=2:max(size(B))
    h = B(i)- B(i-1);
    if(h>=0)
        sign = 1;
    else
        sign = -1;
    end
    K1 = h*Tellinen(constants, B(i-1),H(i-1), sign);
    K2 = h*Tellinen(constants, B(i-1) + h/2, H(i-1)+K1/2, sign);
    K3 = h*Tellinen(constants, B(i-1) + h/2, H(i-1)+K2/2, sign);
    K4 = h*Tellinen(constants, B(i-1) + h, H(i-1) + K3, sign);
    H(i) = H(i-1) + K1/6 + K2/3 + K3/3 + K4/6;
end

function T = Tellinen(constants, B, H, sign)
    %I will now do a bunch of calculations which will eventually be used to
    %calculate the final term in the algorithm.

    bhn = Langevin(constants.Bsath, constants.a, H + constants.sigma_minus);
    %bhn is the b_h-(h) term in [S32, eq. 6]
    bhp = Langevin(constants.Bsath, constants.a, H - constants.sigma_plus);
    %bhp is the b_h+(h) term in [S32, eq. 6]

    if(sign == 1)    %If dB>0
        dbdh = (Langevin(constants.Bsath, constants.a, H -
constants.sigma_plus + 1e-6)-Langevin(constants.Bsath, constants.a, H -
constants.sigma_plus - 1e-6))./2e-6; %This is the dbh+/dh term of [S32,eq 7a]
        T = 1/(constants.mu_0+((bhn-B)/(bhn-bhp))*(dbdh-constants.mu_0));
        %This is [S32, eq. 7a]
    end
    if(sign == -1)    %If dh<0
        dbdh = (Langevin(constants.Bsath, constants.a, H +
constants.sigma_minus + 1e-6) - Langevin(constants.Bsath, constants.a, H +
constants.sigma_minus - 1e-6))./2e-6; % dbh-/dh term of [S32, eq. 6b]
        T = 1/(constants.mu_0+((B-bhp)/(bhn-bhp))*(dbdh-constants.mu_0));
    end
end
end

```



```

function B = Langevin(Bsat, a, H)
    if(H == 0)
        B = 0;
    else
        B = Bsat*(coth(H/a) - (a/H));
    end
end

```

Inverse Tellinen Model using Interpolation

```

%In this revision of the model, I am trying to take experimental data and
%interpolate that data for the ascending and descending legs. The data
%being imported must be in the form of four columns - column 1 must contain
%the ascending H-values, column 2 must contain the descending H-values,
%column 3 must contain the corresponding ascending B-values and column 4
%must contain the corresponding descending B-values. Also, note that the
%data for each column must be sorted in ascending order for the MATLAB
%interpolate function to work.

```

```

%%RESULTS: This seems to work, but one must be VERY CAREFUL that the
%%experimental data being imported is exactly symmetrical!!! In other words,
%%one must really just take data for one leg of the loop and invert it.
%%Otherwise human error will cause small deviations in the calculation of
%%H, which then compounds and eventually breaks the program.

```

```

%%Reference to [S69] refers to the article entitled "Simulation of hysteresis
%%and eddy current effects in a power transformer - W. Chandrasena, P.G.
%%McLaren, U.D. Annakkage, R.P. Jayasinghe, D. Muthumuni, E. Dirks"

```

```

%The Tellinen model is described by "A Simple Scalar Model for Magnetic
%Hysteresis" by Juhani Tellinen (from here on referred to as [S32]) and is
%also implemented by "Experimental and Simulation Analysis of Ferroresonance
%in Single-phase Transformers Considering Magnetic Hysteresis Effects" by
%P.S. Moses and M.A.S. Masoum (from here on referred to as [S27])

```

```

%S.J. Kruger
%Initiated 2020/12/30
%Updated: 2021/11/16

```

```

close all
clear all
clc

```

```

%%I want to import some data from excel which I read off the hysteresis
%%graph of [S69].

```

```

ExpData = xlsread('S69_Hysteresis_Loop_Data_Without_Repetitions_Rev3.xlsx');
Basc = ExpData(:,3);
Bdesc = ExpData(:,4);
Hasc = ExpData(:,1);
Hdesc = ExpData(:,2);

```

```

%%Create sinusoidal B values that we can use to calculate H through
%%interpolation of our experimental data.

```

```

%Create some time values
fs = 100000; %sample frequency [Hz]. This has to be quite high to
ensure our timesteps are small enough for the algorithm to converge
timestep = 1/fs;
t=0:timestep:0.02;
constants.timestep = timestep;

%Now create an array of B values
Bmax = 1.6923; %Let us assume that B only goes up to 1.6923

for i=1:max(size(t))
    B(i) = Bmax*sin(2*pi*60*t(i));
%Note that the frequency is 60 Hz, since the experimental
%loop from which we obtained our data is 60 Hz.
end

mu_0 = 4*pi*10^-7; %Permeability of free space

%Now let us move all the constants into a single struct that can be passed
%to functions

constants.mu_0 = mu_0;

H(1) = 0; %Assume the initial magnetic field is 0

%Now for the main for-loop where we calculate magnetic field intensity
for i=2:max(size(B))
    h = B(i) - B(i-1);
    if(h>=0)
        delta = 1;
    else
        delta = -1;
    end
    K1 = h*Tellinen(constants, Basc, Bdesc, Hasc, Hdesc, B(i-1),H(i-1),
delta);
    K2 = h*Tellinen(constants, Basc, Bdesc, Hasc, Hdesc, B(i-1) + h/2, H(i-
1)+K1/2, delta);
    K3 = h*Tellinen(constants, Basc, Bdesc, Hasc, Hdesc, B(i-1) + h/2, H(i-
1)+K2/2, delta);
    K4 = h*Tellinen(constants, Basc, Bdesc, Hasc, Hdesc, B(i-1) + h, H(i-1) +
K3, delta);
    H(i) = H(i-1) + K1/6 + K2/3 + K3/3 + K4/6;
end

function T = Tellinen(constants, Basc, Bdesc, Hasc, Hdesc, B, H, sign)
    %I will now do a bunch of calculations which will eventually be used to
    %calculate the final term in the algorithm.

    bhn = interp1(Hdesc, Bdesc, H, 'pchip');
    %bhn = b_h-(h) term in [S32, eq. 6]
    bhp = interp1(Hasc, Basc, H, 'pchip');
    %This is the b_h+(h) term in [S32, eq. 6]

    if(sign == 1) %If dB>0
        term1 = interp1(Hasc,Basc,H+1e-06,'pchip');
        term2 = interp1(Hasc,Basc,H-1e-06,'pchip');
        dbdh = (term1-term2)./2e-06; %This is the dbh+/dh term of [S32,eq 7a]
        T = 1/(constants.mu_0+( (bhn-B) / (bhn-bhp) ) * (dbdh-constants.mu_0));
    %This is [S32, eq. 7a]

```

```

end
if(sign == -1) %If dh<0
    dbdh = (interp1(Hdesc,Bdesc,H+1e-06,'pchip')-interp1(Hdesc,Bdesc,H-
1e-06,'pchip'))./2e-06; %This is the dbh-/dh term of [S32, eq. 6b]
    T = 1/(constants.mu_0+((B-bhp)/(bhn-bhp))*(dbdh-constants.mu_0));
end
end
end

```

Inverse Model with Numerical Error Correction as discussed in Chapter 5²⁶

%In this revision of the model, I am trying to take experimental data and %interpolate that data for the ascending and descending legs. The program was %modified by changing the step size of interpolation because the small step %size caused numerical problems. The experimental data used for the %interpolation is contained in a MATLAB data file. The data file contains 4 %fields: Basc, Bdesc, Hasc and Hdesc

%%RESULTS: This seems to work, but one must be VERY CAREFUL that the %%experimental data being imported is exactly symmetrical!!! In other words, %%one must really just take data for one leg of the loop and invert it. %%Otherwise human error will cause small deviations in the calculation of %%H, which then compounds and eventually breaks the program.

%%Reference to [S69] refers to the article entitled "Simulation of hysteresis %%and eddy current effects in a power transformer - W. Chandrasena, P.G. %%McLaren, U.D. Annakkage, R.P. Jayasinghe, D. Muthumuni, E. Dirks"

%The Tellinen model is described by "A Simple Scalar Model for Magnetic %Hysteresis" by Juhani Tellinen (from here on referred to as [S32]) and is %also implemented by "Experimental and Simulation Analysis of Ferroresonance %in Single-phase Transformers Considering Magnetic Hysteresis Effects" by %P.S. Moses and M.A.S. Masoum (from here on referred to as [S27])

%S.J. Kruger
 %Initiated 2020/12/30
 %Updated: 2021/03/03

```

close all
clear all
clc

```

```

Data = load('D:\Documents\MATLAB\NWU Masters\Experimental Data Programs\Paddy
Transformer\Paddy_Transformer_BH_Data_Tellinen_Monotonic_Fixed.mat');
Basc = Data.ascending.B;
Bdesc = Data.descending.B;
Hasc = Data.ascending.H;
Hdesc = Data.descending.H;

```

%%Create sinusoidal B values that we can use to calculate H through %interpolation of our experimental data.

```

%Create some time values
fs = 100000; %sample frequency [Hz]. This has to be quite high to
ensure our timesteps are small enough for the algorithm to converge
timestep = 1/fs;

```

²⁶ The color of the text has been made red where changes were implemented to correct the numerical errors

```

t=0:timestep:0.02;
constants.timestep = timestep;

%Now create an array of B values
Bmax = 1.7904; %Let us assume that B only goes up to 1.7904

for i=1:max(size(t))
    B(i) = Bmax*sin(2*pi*50*t(i));
%Note that the frequency is 50 Hz, since the experimental loop from which we
obtained our data is 50 Hz.
end

mu_0 = 4*pi*10^-7; %Permeability of free space

%Now let us move all the constants into a single struct that can be passed
%to functions

constants.mu_0 = mu_0;

H(1) = 0; %Assume the initial magnetic field is 0

%Now for the main for-loop where we calculate magnetic field intensity
for i=2:max(size(B))
    h = B(i) - B(i-1);
    if(h>=0)
        delta = 1;
    else
        delta = -1;
    end
    K1 = h*Tellinen(constants, Basc, Bdesc, Hasc, Hdesc, B(i-1),H(i-1),
delta);
    K2 = h*Tellinen(constants, Basc, Bdesc, Hasc, Hdesc, B(i-1) + h/2, H(i-
1)+K1/2, delta);
    K3 = h*Tellinen(constants, Basc, Bdesc, Hasc, Hdesc, B(i-1) + h/2, H(i-
1)+K2/2, delta);
    K4 = h*Tellinen(constants, Basc, Bdesc, Hasc, Hdesc, B(i-1) + h, H(i-1) +
K3, delta);
    H(i) = H(i-1) + K1/6 + K2/3 + K3/3 + K4/6;
end

function T = Tellinen(constants, Basc, Bdesc, Hasc, Hdesc, B, H, sign)
    %I will now do a bunch of calculations which will eventually be used to
    %calculate the final term in the algorithm.

    bhn = interp1(Hdesc, Bdesc, H, 'pchip');
    %This is the b_h-(h) term in [S32, eq. 6]
    bhp = interp1(Hasc, Basc, H, 'pchip');
    %This is the b_h+(h) term in [S32, eq. 6]

    if(sign == 1) %If dB>0
        term1 = interp1(Hasc,Basc,H+2,'pchip');
        term2 = interp1(Hasc,Basc,H-2,'pchip');
        dbdh = (term1-term2)./4; %This is the dbh+/dh term of [S32,eq 7a]
        T = 1/(constants.mu_0+((bhn-B)/(bhn-bhp))*(dbdh-constants.mu_0));
%This is [S32, eq. 7a]
    end
    if(sign == -1) %If dh<0
        dbdh = (interp1(Hdesc,Bdesc,H+2,'pchip')-interp1(Hdesc,Bdesc,H-
2,'pchip'))./4; %This is the dbh-/dh term of [S32, eq. 6b]
    end
end

```

```
        T = 1/(constants.mu_0+((B-bhp)/(bhn-bhp))*(dbdh-constants.mu_0));  
    end  
end
```

APPENDIX F: DATA POINTS EXTRACTED FROM [81, FIG. 12]

Section 3.4.1 replicated the hysteresis loop and magnetization current found in [81]. Chandrasena *et al.* [81] did not provide data points that could be used for the Tellinen and the Jiles-Atherton model. Therefore, data points had to be manually extracted from the hysteresis loop seen in [81, Fig. 12]. The extracted data points are given in Table F-1. The data points are given in four different columns which represent the ascending and descending legs of the hysteresis loop respectively. The hysteresis loop generated by these data points is shown in Fig. 3-21.

Table F-1: The ascending and descending hysteresis loop data from [81, Fig. 12]

H_{ascending} (A/m)	B_{ascending} (T)	H_{descending} (A/m)	B_{descending} (T)
-93.75	-1,69231	-93,75	-1,69231
-87.5	-1,66222	-91,25	-1,62963
-81.875	-1,64593	-88,75	-1,61333
-63.75	-1,62963	-85,625	-1,59704
-43.125	-1,62637	-82,5	-1,58074
-40	-1,62311	-80	-1,56444
-35	-1,61985	-75,625	-1,5237
-27.5	-1,61333	-71,875	-1,49926
-20	-1,59704	-68,75	-1,47481
-15.625	-1,58074	-65,625	-1,44222
-8.75	-1,56444	-63,125	-1,40963
0	-1,51556	-60	-1,38519
3.75	-1,49926	-57,5	-1,35259
8.125	-1,49111	-55,3125	-1,3037
12.5	-1,46667	-52,5	-1,24667
16.25	-1,36889	-50	-1,18963
20	-1,29556	-47,8125	-1,14889
21.875	-1,23852	-46,25	-1,09185

24.375	-1,15704	-43,75	-1,01037
26.25	-1,05926	-41,875	-0,94519
27.5	-0,99407	-40	-0,83111
28.75	-0,88	-37,8125	-0,68444
29.6875	-0,74963	-35,9375	-0,48889
31.25	-0,53778	-34,375	-0,26074
32.1875	-0,30963	-33,4375	0
33.4375	0	-32,1875	0,30963
34.375	0,260741	-31,25	0,537777
35.9375	0,488889	-29,6875	0,74963
37.8125	0,684444	-28,75	0,88
40	0,831111	-27,5	0,994074
41.875	0,945185	-26,25	1,059259
43.75	1,01037	-24,375	1,157037
46.25	1,091852	-21,875	1,238519
47.8125	1,148888	-20	1,295555
50	1,18963	-16,25	1,368888
52.5	1,246667	-12,5	1,466667
55.31254	1,303704	-8,125	1,491111
57.5	1,352593	-3,75	1,499259
60	1,385185	0	1,515555
63.125	1,40963	8,75	1,564444
65.625	1,442222	15,625	1,580741
68.75	1,474815	20	1,597037
71.875	1,499259	27,5	1,613333
75.625	1,523704	35	1,619852
80	1,564444	40	1,623111

82.5	1,580741	43,125	1,62637
85.625	1,597037	63,75	1,62963
88.75	1,613333	81,875	1,645926
91.25	1,62963	87,5	1,66222
93.75	1,69231	93,75	1,69231

APPENDIX G: SAUPEC 2021 REVIEW ARTICLE

The following article was submitted to the SAIEE South African Universities Power Engineering Conference in 2021. The article was also presented virtually due to Covid-19 restrictions. The purpose of the article was to summarize some of the existing literature on the topic of ferroresonance. The article does not cover all of the literature on ferroresonance and specifically does not look at mitigation techniques. The article can be considered to be a summary of the literature seen in Section 2.2.1.

Ferroresonance: A Review of the Phenomenon and Its Effects

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Abstract: This paper aims to provide a review of the current literature on the topic of ferroresonance. The definition of ferroresonance and the requirements for transients to be classified as such are discussed and then the paper identifies the various factors which influence ferroresonance and its initiation. The four main modes of ferroresonance are identified, along with the criteria for their classification. Finally, the paper discusses the effects that ferroresonance can have on the power system. The paper does not discuss ferroresonance analysis or mitigation strategies.

Keywords: ferroresonance, magnetic saturation, core losses, distribution transformers.

1. INTRODUCTION

Ferroresonance is a nonlinear form of resonance that was first identified and described in 1914 [1]. Owing to its nonlinear nature, it is still a poorly understood phenomenon, despite extensive research having been conducted in this regard.

A recent change to the SANS 780 standard [2] has resulted in a decrease in core losses of distribution transformers in South Africa. Ferroresonance is strongly related to the core loss of a transformer. Therefore, a resurgence of this phenomenon is to be expected in new distribution transformers in the South African grid. Unexpected overvoltages have recently been measured on distribution transformers in the Eskom grid [3] and ferroresonance is the suspected cause.

With an expected increase in ferroresonance, this paper aims to summarize the current literature on the topic. Section 2 discusses the theory of ferroresonance and the requirements for its initiation. Section 3 discusses ferroresonance dependencies. Section 4 discusses the four modes of ferroresonance and Section 5 provides an overview of the effects that ferroresonance can have on a transformer.

2. DEFINITION AND REQUIREMENTS

2.1 Definition

Ferroresonance, also referred to as “Neutral Instability” or “Voltage Displacement” by [4], is a nonlinear oscillation between an iron-core and a capacitor. The iron-core is nonlinear, since its inductance is dependent on the magnetic flux in the transformer. Horak [5] elaborates by adding the iron-core to the inductor as it oscillates in and out of its saturation region. This can result in severe overvoltages as well as overcurrents. With sufficient energy supplied to the system, these oscillations can be maintained indefinitely.

Horak [5] also states that ferroresonance can describe a system where there is an increase in current and voltage even when the transformer does not enter saturation. However, this seems to constitute the definition of resonance, and based on [1], [6]-[12], it can be concluded that ferroresonance requires the presence of a saturable inductor in the system.

Short [13] represents the magnetizing reactance of the transformer as a switched inductance. When the core enters saturation and the inductance drastically decreases, the switch is closed and the charge in the capacitor is dumped into the system. This leads to voltage overshoot. Every half cycle, the transformer moves out of saturation and a charge of the opposite polarity is stored in the capacitor, unless there is sufficient load in the system to drain the capacitor charge. This analogy is also used by Walling *et. al* [14].

A main feature of ferroresonance is that there is more than one stable steady-state response for the circuit [7]. This multiplicity of solutions is defined by Kieny [15] as a requirement for transients to be classified as ferroresonance. It becomes particularly evident when one evaluates the Kirchhoff’s Voltage Law (KVL) equation of a simplified ferroresonant circuit, as seen in Fig. 1. A graphical solution to this KVL equation is shown in Fig. 2.

The $V_M = X_M I_M$ graph represents the magnetisation characteristic of the transformer, along with its saturation. From Fig. 2 one can see that there are three solutions which satisfy the KVL equation [1]. Solution 1 represents a stable non-ferroresonant state [19]. Solution 2 represents a stable ferroresonant state since the transformer has entered into saturation. Solution 3 is also a ferroresonant state, but it will never be realised. Price [19] presents a detailed explanation as to why solution 3 is not attainable. At point 1 and point 2, a small change in voltage is in opposition to the resulting change in current. In other words, the change in current forces the operation back to operating point number 1 or 2. However, a small change in voltage at point

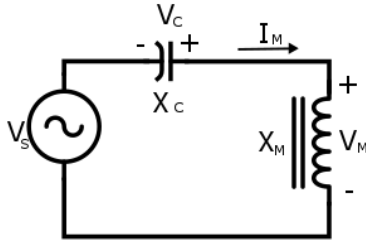


Fig. 1: Elementary Ferroresonant Circuit

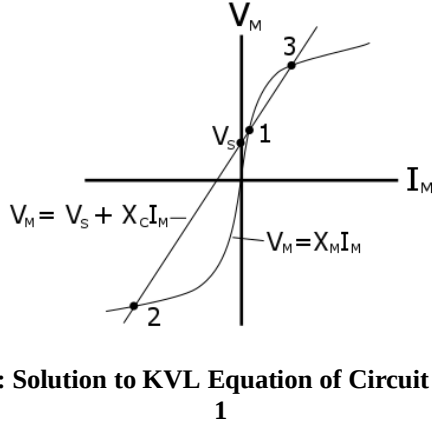


Fig. 2: Solution to KVL Equation of Circuit in Fig. 1

Point 3 creates an additional current in the direction of the voltage change. This causes the current to avalanche away from the operating point. Tsao and Ning [17] describe this as a violation of the equilibrium theory. Nevertheless, it is possible that the system may pass through point 3 briefly during a transient [16].

An increase in the magnetic flux of the transformer will lead to operation in the saturated region, which in turn may cause a sudden transition from solution 1 to solution 2. This sudden transition, also referred to as “jump resonance” allows for multiple solutions, as mentioned by Kieny [15]. Erbay [18] says “Jump resonance refers to a condition in a sinusoidally excited system: if an incremental change in amplitude or frequency of the input to the system or in the magnitude of one of the parameters of the system causes a sudden jump in signal amplitude somewhere in the system, then jump resonance is said to have occurred.” Santoso, Dugan and Nedwick [16] also point out that the current resulting from a jump to solution number 2 may be so great that the voltage drops again. The result is an oscillation between solution 1 and solution 2. Such instances have been reported in actual systems [16].

At point 1 in Fig. 2, the largest voltage drop is found across the inductor. This voltage drop is equal to the sum of the source voltage and the capacitor voltage. However, at point 2, the largest voltage drop is found across the capacitor. This in turn implies a danger to the electrical equipment [1].

Another important characteristic of solution 2 is that it can be self-sustaining. Horak [5] claims that once solution point 2 is reached, a reduction in voltage across the transformer will not necessarily lead to a transition back to

solution point 1 in Fig. 2. This is confirmed by [26] where it was found that a ferroresonant jump was initiated at a voltage of 0.8 pu. However, when the system voltage was decreased, the system did not move out of ferroresonance before the voltage reached 0.1 pu. Even when the source voltage is completely removed, there is still a voltage drop across the capacitor, which may be enough to maintain the ferroresonant state [4]. The system’s ability to move out of ferroresonance is related to the core losses of the transformer [5].

2.2 Requirements

Based on the discussions of [1], [6], [7], [12], [13], [15] and [17], the following requirements for ferroresonance can be identified:

- A saturable inductor
- System capacitance
- A voltage source
- Low losses in the inductor core
- Multiplicity of solutions.

The saturable inductor was motivated by the definition of ferroresonance and is supplied by the transformer. The system capacitance can be provided by long transmission lines, capacitor banks and circuit breaker capacitances [1]. The voltage source provides the energy required for the oscillations. The effect of low losses will be discussed in Section 3.

2.3 Causes

Ferroresonance is strongly associated with the saturation of a transformer. Transformers are not designed to operate in the saturated region under steady-state conditions. As such, the transition into the saturated region needs to be initiated. Ferracci [7] and Erbay [18] identify the following list of causes, which may lead to ferroresonance: transients, lightning overvoltages, energizing and de-energizing of transformers, phase-to-ground faults and phase-to-phase faults. The most common causes of ferroresonance initiation discussed in the literature are the opening of a circuit breaker and the clearance of a fault.

3. FERRORESONANCE DEPENDENCIES

Ferroresonance is strongly dependent on the capacitance of the system and the magnitude of the source voltage. Changes in these parameters can influence the ferroresonance likelihood. This is made clear in Fig. 3.

Fig. 3a highlights the impact of an increase in system voltage on the ferroresonant points. A change in system voltage allows for the $V_M = V_s + X_c I_M$ line to move parallel to itself. When the system voltage is sufficiently increased, the stable non-ferroresonant solution point disappears and ferroresonance is guaranteed, as depicted by the V_s'' line. To this end, [4], [6], and [12] claim that an increase in system voltage level increases ferroresonance likelihood. However, ferroresonance has been reported for

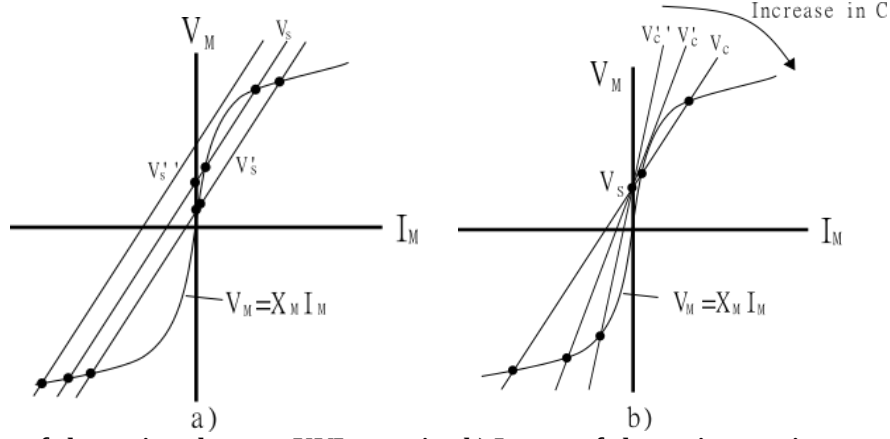


Fig. 3: a) Impact of change in voltage on KVL equation b) Impact of change in capacitance on KVL equation

voltage levels as low as 12 kV [13]. Fig. 3b shows the impact of a change in capacitance on the ferroresonant states. An increase in the capacitance, leads to a decrease in the gradient of the $V_M = V_s + X_C I_M$ line, with the rotation being about the y-intercept of the line. This implies that if the capacitance of the system is sufficiently increased, there will be only a single solution to the system, which will be non-ferroresonant.

However, one should also note that when the capacitance is decreased and the gradient of the line increases, the stable non-ferroresonant solution may disappear altogether. This implies only a single, ferroresonant solution to the KVL equation, as pointed out by [4] and [19].

Valverde *et.al* [20] also points out, with reference to Fig. 3b, that there is a range of capacitance values for which ferroresonance is possible. This is also confirmed by [7]. This distinguishes ferroresonance from resonance. However, the existence of a ferroresonant solution to the KVL equation does not guarantee ferroresonance when there is also a non-ferroresonant solution. In such cases, ferroresonance will be determined by the initial conditions.

The reader should note that the circuit in Fig. 1 and consequently the graphs in Fig. 3, does not take losses into account. However, ferroresonance is dependent on the losses. In fact, [21] claims that the critical capacitance for ferroresonance initiation is directly related to the core loss of the transformer. When resistance is added in series to Fig. 1, the resulting KVL equation becomes [4]:

$$U_s = R \cdot I_M + j(X_M(I_M) - X_C) \cdot I_M \quad (1)$$

and rearranging yields:

$$|V_M(I_M) - X_C \cdot I_M| = \sqrt{V_s^2 - (R \cdot I_M)^2} \quad (2)$$

The resulting plots of eq. (2) are shown in Fig. 4. Fig. 4a shows the effect of reduced resistance on the ferroresonant solutions of the system. In this case, there are still three

possible solutions to the KVL equation. From left to right, the solution points are solution 1, solution 3 and solution 2. This is also indicated in the legend. The solution points are numbered in this order so that solution point number 3 is still the unstable ferroresonant point, as was established in Fig. 2. Solution 1 is still the non-ferroresonant solution and solution 2 is the stable ferroresonant point.

Fig. 4b shows the effect of increasing the losses in the circuit. In this case, there is only a single solution to the KVL equation, which is the stable non-ferroresonant solution. Therefore, a reduction in circuit losses makes ferroresonance more likely. It is for this reason that section 2 defined low losses as one of the requirements of ferroresonance. Ribas *et. al* [22] claims that transformer cores with lower core losses are more susceptible to chaotic ferroresonance. Walling *et. al* [21] states that core losses are the critical factor influencing ferroresonance susceptibility in grounded wye-wye transformers with five-limb cores.

In addition to the system capacitance, system voltage and system losses, there are several other factors upon which ferroresonance onset is dependent, these include [4], [6], [12] and [23]:

- The initial charge of the capacitor
- The source voltage frequency
- The initial magnetic flux in the transformer core
- The time of switching
- The saturation levels in the transformer
- The source impedance, and
- The type of capacitance

In the abovementioned list, the type of capacitance has an odd effect. Mork and Steuhm [23] found that transformers connected in the field tended to make more noise than transformers tested in a laboratory despite showing similar waveforms. They also found that dampening ferroresonance in field based transformers required larger loads. Since system capacitance was supplied by underground cables in the field, as opposed to capacitors in the lab, Mork and Steuhm [23] conclude that the type of capacitance has an effect on ferroresonance.

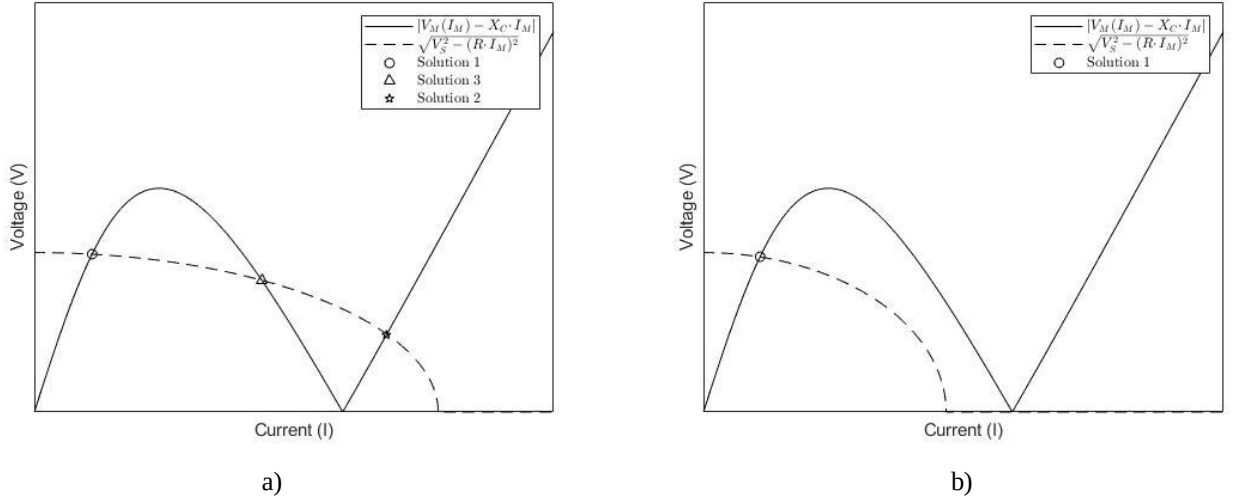


Fig. 4: Effect of Resistance on Ferroresonant Points: a) Reduced Losses b) Increased Losses

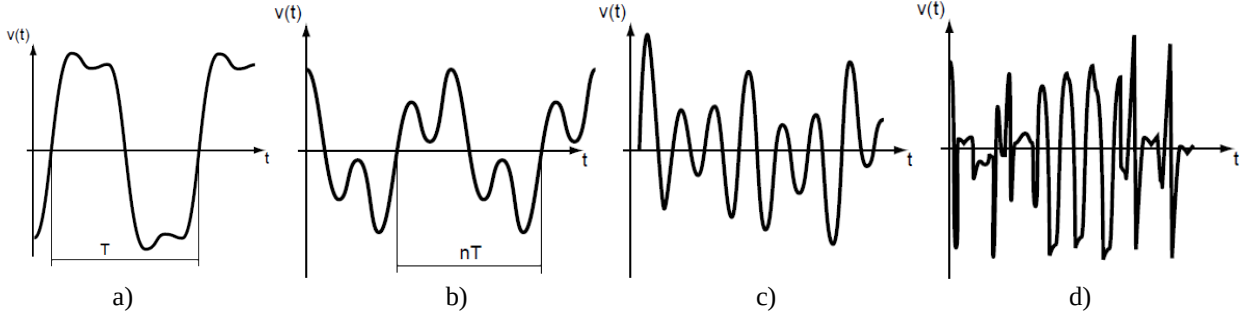


Fig. 5: Ferroresonance Modes as seen in [7, Fig. 4]: a) Fundamental b) Subharmonic c) Quasi-periodic d) Chaotic

4. FERRORESONANCE MODES AND FREQUENCY

Ferracci [7] was the first to point out that there are four different ferroresonance modes. This classification was then accepted by various other authors, including [1], [4], [24] and [25]. Abdi, Abbasi and Moradi [26] presented a more elaborate list of modes, but this is very uncommon. The four main modes of ferroresonance are: fundamental, subharmonic, quasi-periodic and chaotic.

The classification of the ferroresonant mode is based on the waveform. The waveforms of the four different ferroresonant modes are shown in Fig. 5.

Cigre [4] classifies fundamental and subharmonic ferroresonance as a form of periodic ferroresonance, since there is an indication of periodicity. Fundamental ferroresonance is characterized by a frequency equal to that of the source. Subharmonic ferroresonance assumes a frequency equal to the source frequency divided by some integer n . The integer is used to refer to the subharmonic degree [1].

Quasi-periodic ferroresonance has a discontinuous spectrum of frequencies [1], [4], where the waveforms seem to transition from one frequency to another without stabilizing at a specific one. Chaotic ferroresonance is highly irregular and unpredictable, as also seen in Fig. 5d.

Fundamental and subharmonic ferroresonance are the most likely to occur. Cazacu *et. al* [25] only predicts these two ferroresonance modes in low voltage systems due to reduced voltage and capacitance values.

Cigre [4] claims that chaotic ferroresonance has never been reported in extra high voltage (EHV) substations to date. Radmanesh, Rostami and Abassi [27] have predicted chaotic ferroresonance in voltage transformers, but only under unrealistic conditions. However, they [27] did find that quasi-periodic ferroresonance was possible in a VT for realistic system parameters.

Ferroresonance is classified as a low-frequency phenomenon since the frequency spectrum rarely exceeds 2 kHz. Cigre [4] presents a formula to calculate the ferroresonant frequency:

$$T_{ferro} = 4 \cdot \frac{\phi_{sat}}{U_0} + 2\pi \sqrt{L_{sat} \cdot C} \quad (3)$$

where T_{ferro} is the ferroresonant period, ϕ_{sat} is the saturation magnetic flux, U_0 is the initial capacitor charge, L_{sat} is the saturated inductance and C is the system capacitance.

According to [4], if the frequency calculated using the inverse of eq. (3) is greater than the source frequency, then the frequency will “lock” onto the source frequency, resulting in fundamental ferroresonance. If, however, the ferroresonant frequency is less than the source frequency, sub-harmonic ferroresonance will result at an odd sub-multiple of the source frequency.

5. FERRORESONANCE EFFECTS

Moses and Masoum [9] claim that ferroresonance is a phenomenon with a low probability of occurrence. However, the effects of ferroresonance can be catastrophic.

There is some discrepancy in literature regarding the effect that ferroresonance can have on transformers. Walling *et. al* [21] reports no transformer damage to their transformer units despite severe ferroresonance exposure. Mork [28] reports the failure of 72 voltage transformers after three minutes of ferroresonance exposure. Cigre [4] claims that no power transformers have failed due to ferroresonance, but that sustained ferroresonance contributes to accelerated aging.

Despite the contradictions regarding transformer failures, the following list of effects due to ferroresonance can be identified based on [6], [10], [18], [21] and [24]:

- Random overvoltages for short periods
- Significant continuous overvoltages and overcurrents
- High and sustained oscillating phase-to-phase and phase-to-ground overvoltages
- Sustained levels of distortion to both current and voltage waveforms
- Transformer heating
- Loud noise due to magnetostriction
- Electrical equipment damage
- Thermal/insulation breakdown
- Maloperation of protective devices
- Flicker
- Displacement of neutral point voltage
- Zero-sequence overcurrent
- Charring and bubbling of transformer paint

6. CONCLUSION

This article served as a summary of the current literature on ferroresonance. It is motivated by the potential resurgence of ferroresonance necessitated by a recent change to the SANS 780 standard [2].

Ferroresonance is highly nonlinear since it depends on the magnetization characteristic of the transformer. Furthermore, even though the possibility of ferroresonance exists for a given set of parameters, ferroresonance is not guaranteed, since it also depends on a range of other initial conditions. Fig. 3b showed that there is a range of capacitance values for which ferroresonance can be initiated. This makes ferroresonance avoidance

particularly difficult since the resonant frequency can change. Fig. 4 showed that a decrease in system losses can cause ferroresonance at lower voltage levels.

Section 3 mentioned that decreasing the capacitance of the circuit could potentially eliminate the stable non-ferroresonant point. However, the authors of this paper would like to argue that if the capacitance is sufficiently decreased and the gradient of the line in Fig. 3b becomes sufficiently large, the single operation point may move back into the linear region of the magnetization curve and ferroresonance may be avoided altogether.

Though there is a discrepancy in the literature regarding whether ferroresonance can cause transformer failure, the list of side-effects highlighted in Section 5 are still harmful and should be avoided wherever possible.

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