THE INFLUENCE OF AN HOUR-GLASS MODEL OF COOPERATIVE LEARNING ON THE LEARNING AND ACHIEVEMENT OF GRADE 8 MATHEMATICS LEARNERS IN CROWDED CLASSROOMS

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2004

To whom it may concern

I hereby declare that I have edited the dissertation of

Mr Rantopo David Sekao.

I have made various suggestions re the use of language, which were attended to, and I am satisfied that the dissertation complies with the standard expected.

(Ms) J.A. Brönn, MA

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ABSTRACT

Cooperative learning has emerged to be a preferred teaching-learning model in South Africa since the inception of Curriculum 2005 (C2005) emphasising Outcomes-based education (OBE). However, the documented success rate of cooperative learning in mathematics was experienced in small group sizes (emanating from small class size) of about five learners. This study, therefore, aims at affording mathematics teachers and learners of crowded classes an opportunity to effectively use cooperative learning, namely the Hour-glass model in mathematics lessons. The prevalence of crowded classes in the majority of South African schools seems to inhibit the effectiveness of cooperative learning in mathematics. The big cooperative group size of about eight learners in South African context results in very complex lines of communication between learners. The teacher spends more time trying to manage off-task behaviour of learners instead of engaging them in active participation in the learning of mathematics.

The combined quantitative and qualitative research methods were used. For the former, the study orientation in mathematics (SOM) questionnaire and the mathematics academic achievement test were used to collect data with regard to the influence of the Hour-glass model on the learners' learning skills in mathematics, and on the mathematics academic achievement respectively. A specific true experimental design, namely, the Solomon Four-group design, was used because of a large sample size (n > 500), and its credited ability to control the sources of threats to internal validity. For the latter the lesson observation and interviews were conducted to collect information about the influence of the Hour-glass model on learners' social skills during cooperative learning in mathematics.

The groups that received the treatment (i.e. Hour-glass model) achieved higher scores of practical significance in mathematics academic achievement test than the groups that did not receive the treatment. The Hour-glass model also yielded positive social skills among learners during mathematics learning. The teachers who applied the Hour-glass model revealed that they coped easier with crowded mathematics classes when using cooperative small groups. However, the Hour-glass model did not significantly influence learners' learning skills in mathematics. Certain logistical and administrative limitations emerged with regard to the implementation of the Hour-glass model in the usual school setting.

Key words for indexing: cooperative learning, crowded classes, grade 8 mathematics, mathematics learning, mathematics teaching, mathematics achievement, Hour-glass model.

OPSOMMING

Die invloed van die uurglasmodel vir koöperatiewe leer op die leer en prestasie van graad 8-wiskundeleerders in groot klasse. Sedert die invoer van Kurrikulum 2005 (K2005) ter verwesenliking van Uitkomsgebaseerde Onderwys (UGO) is koöperatiewe leer n vookeur-onderrigleermodel in Suid-Afrikaanse skole. Gedokumenteerde sukses van koöperatiewe leer van wiskunde het in klein groepe van ongeveer vyf in klein klasse geskied. Hierdie ondersoek mik daarom om wiskundeonderwysers en -leerders in groot klasse geleentheid te bied om koöperatiewe leer doelmatig te gebruik, naamlik die gekonstrueerde "uurglasmodel". Dit wil voorkom as of die algemene voorkoms van groot klasse in Suid-Afrikaanse skole die gebruik van koöperatiewe leer beperk. Groot koöperatiewe groep-groottes van ongeveer agt leerders in die Suid-Afrikaanse konteks lei naamlik tot komplekse kommunikasiepatrone tussen die leerders. Die onderwyser spandeer ook n oormaat tyd aan die bestuur van nie-taakverwante gedrag van leerders, eerder as om hulle aktief in die leer van wiskunde te betrek.

"n Gekombineerde kwantitatiewe en kwalitatiewe ondersoekmetodologie is gebruik. Die Studieoriëntering in Wiskunde vraelys (SOW) en akademiese wiskundtoetse is gebruik om kwantitatiewe data in te samel ten einde die invloed van uurglasmodel op leerders se leervaardighede in wiskunde te bepaal. 'n Solomon-viergroepe eksperimentele opset is gebruik in die lig van die metode se bewese vermoë om vaktore wat interne geldigheid bedreig, te beheer, asook die beskikbaarheid van 'n groot steekproef (n>500) van wiskundeleerders. Kwantitatiewe data is by wyse van leswaarneming en onderhoude ingesamel ten einde die invloed van die uurglasmodel op leerders se sosiale vaardighede tydens koöperatiewe leer van wiskunde te bepaal.

Die groep wat aan die uurglasmodel blootgestel is, het akademies betekenisvol beter presteer as die kontrolegroepe. Die uurglasmodel het ook positiewe sosiale vaardighede tydens die leer van wiskunde by betrokke leerders tot gevolg gehad. Die onderwysers wat die uurglasmodel toegepas het, het aangedui dat hulle daardeur klein groepe beter in groot klasse kon hanteer. Die uurglasmodel het egter nie leerders se leervaardighede in wiskunde beduidend beïnvloed nie. Bepaalde logistieke en administratiewe beperkinge is ten opsigte die implementering van die uurglasmodel in die gewone skoolopset ondervind.

Sleutelwoorde vir indeksering: koöperatiewe leer; groot klasse; graad 8-wiskunde; wiskunde-leer; wiskundeonderrig; wiskundeprestasie; uurglasmodel.

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CHAPTER

ONE

Problem statement

and

Research programme

Evolution of constructivist approach to teaching resulted in the emergence of peer-based forms of learning in the classroom (Palincsar & Herrenkohl, 1999:152).

1.1 Introduction

Research has revealed numerous factors that impact on the teaching and learning of mathematics such as teachers' knowledge of mathematics content and representations, teaching and learning methods (Fennema & Franke, 1992:148), teachers' beliefs and conceptions (Thompson, 1992:127, Ashlock *et al.*, 1983:16), and assessment factors (Ashlock *et al.*, 1983:17; Department of Education, 2000a:42). However, this study focuses mainly on the impact of teaching and learning of mathematics with special emphasis on a cooperative teaching-learning strategy. This chapter provides a brief reflection on the ***basis*** and ***evolution*** of cooperative learning preceded by a background study on the "views on the nature of mathematics" (which have a bearing on the teaching and learning of mathematics).

The teaching and learning of mathematics in South Africa still needs urgent attention. This is evident in the continuous decline of the grade 12 national pass rate for mathematics from 1996 to 1998 (Department of Education, 1999:9). In 2000 and 2001 mathematics continued to be the science subject with the lowest pass rate compared to Biology and Physical Science (Department of Education, 2001a). In the North West Province, for instance, mathematics has recorded the lowest pass rate of all examined subjects in the past two years (Department of Education, 2001a). The complete blame cannot solely be levelled against grade 12 teachers, but should be shared by all mathematics teachers in the lower grades who have not contributed in equipping learners with adequate learning skills such as problem solving and critical thinking. The Third International Mathematics and Science Study (TIMSS-R) revealed that learners in grade 8 perform much lower than their international counterparts (Howie, 2001:18). As a way of attempting to cope with the situation, the majority of educators indicated their need to be trained in maths teaching methodologies such as small group teaching (Department of Education 2002:12).

The literature on the nature of mathematics provides a repertory from which numerous contrasting and/or interrelated rich mosaic of portrayals, views or convictions of mathematics are found (Dossey, 1992:39). Out of a number of possible variations, Curry (1983:202) presented a triad of the views on the nature of mathematics:

- **Realistic view** which contends that mathematical exactness exits in the physical environment around us;
- **Idealistic view** which departs from the notion that mathematical exactness emanates from the human intellect; and
- **Formalistic view** which, according to Brouwer (1983:77), contends that mathematical exactness exists on paper, that is, in the method of developing the series of relations and deducing other relations by fixed laws (algorithms).

Be as it may, the view, conception or portrayal of mathematics held by the teachers has an influence on the way in which they (teachers) approach the teaching of mathematics (Dossey, 1992:39; Thompson, 1992:127). For instance, teachers who hold a *formalistic view* about the nature of mathematics are likely to present mathematics content in a structural format (Dossey, 1992:42), using a product-directed traditional mathematics teaching approach which Nieuwoudt (2000:13) typified as follows:

"Mathematical learning proceeds algorithmically, rather than meaningfully. The automatisation of standard procedures and final techniques are the order of the day".

In contrast, teachers who hold a *realistic view* of mathematics are likely to employ a process-directed socio-constructivist teaching approach which is typified by the Department of Education (2000b:8) as follows:

> ..... it is also consistent with views of the world that recognise our interdependence on each other and the value of other persons. In a 'social-constructivist classroom' the teacher engages the learners in discourse that facilitates the actions of negotiation and interpretation. It is only through communication with others (written or verbal) that these subjective ideas in mathematics or science (or any other field) become candidates for subjective knowledge.

The evolution of the constructivist approach to teaching gave rise to the emergence of numerous peer-based forms of learning in the classroom such as collaborative learning (which subsumes cooperative learning) (Palincsar & Herrenkohl, 1999:152; Yager,1991:56). Constructivism, especially socio-constuctivism, and cooperative learning are characterised by the concomitant view that learning is more effective as individuals interpret their experiences through interaction with others (Department of Education, 2000b:8; Palincsar & Herrenkohl, 1999:152; Bitzer, 2001:99; Cooper, 1999:216).

According to the National Centre for Curriculum Research and Development (NCCRD), the pedagogical implications of [socio-] constructivism (from which cooperative learning evolved) began to dominate the mathematics education community in the late 1980s, and early 1990s in South Africa (Department of Education, 2000b:15). NCCRD revealed that since its inception, the socio-constructivist approach contributed to the improved positive attitudes and development of effective ways of learning mathematics. However, this was not the case with schools characterised by ***large classes*** and ***poor resources*** (Department of Eduaction 2000b:15). The situation pertaining to mathematics class size seems to be specifically problematic. This is evident from the TIMMS-R study which revealed that the average mathematics class size in South Africa is 50, which is much higher than the international average of 30 (Howie, 2001:100). The South African national average (learner-to-educator ratio for all the subjects) is approximately 34 in public schools (Department of Education, 2001b:4). While cooperative learning has proven to be effective in mathematics teaching, the context of crowding (large class sizes) in South Africa poses a hindrance to teaching and learning success.

1.2 Problem statement

Cooperative learning has emerged to be a preferred teaching-learning model in South African schools since the inception of Curriculum 2005 (C2005), emphasising outcome-based Education (OBE) (Bell *et al.,* 1999:268). While a good number of educators (65,3%) are willing to know and use cooperative learning methods in the teaching of mathematics (Department of Education, 2002:12), the process of applying these methods is made difficult by large group sizes (Orton, 1994:40) emanating from large class sizes.

There are numerous problems associated with the use of cooperative learning in large class sizes. In many schools in South Africa there are close to sixty learners in a class which translates to about twelve groups of five members each. Such a large group makes it difficult for a teacher to effectively or *successfully* monitor and assist all groups (Department of Education, 2001b:2), thus a warm, non-threatening climate in the classroom (Ashlock *et al.,* 1983:18) is unattainable.

The classroom size is also not adequate to accommodate so many groups. According to the information provided (through a telephonic interview) by the regional Department of Works (Ga-Rankuwa), the average surface area of a typical classroom in a public school in the Mabopane and Temba districts is 52,5m^2 (i.e. 7,5m x 7m). The space is made even more inadequate by the current use of desks which are not as effective for group work as tables and chairs. Johnson and Johnson

(1990:114) contend that some of the most common mistakes that teachers make are:

- placing learners at rectangular tables where they cannot have eye contact with all other members; and
- moving several tables together, which may place learners too far apart to communicate quietly with each other and share materials.

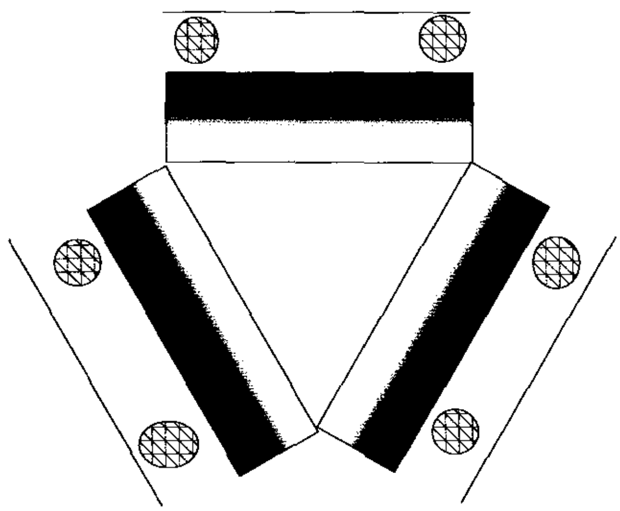
In figure 1.1 the two scenarios are illustrated with a group size of six members. The differences between the use of ***desks*** and ***tables and chairs*** in group work are summarized in table 1.1.

Other problems experienced by teachers when using cooperative learning strategies in the context of crowded classes are:

- The teacher spends more time trying to manage off-task behaviour than actual mathematical activities (Reymonds & Muijs, 1999) and therefore a thirty minute notional teaching-learning period (Department of Education, 2000c:7) will expire before all the groups are attended to. This may cause some group members to be discouraged (Bosch & Bowers, 1992:104); that is, they may develop an attitude of "*why try, the teacher will not attend to all of us today anyway*";
- Learners who are in dire need of assistant may not be identified and attended to immediately because of the large number of groups in a class (Department of Education, 2001b:2); and
- Teachers may resort to a traditionally teacher-fronted method of teaching while learners are in the so-called cooperative small-groups, euphemistically referred to by Bell *et al.*, (1999:269) as 'cluster-work' because learners are in groups but there is no learner-learner interaction (Taylor & Vinjevold, 1999:150). Subsequently the teacher may begin to give learners answers to the mathematics activity without explanation and this was found not to promote achievement (Gibbs & Orton, 1994:109).

Figure 1.1 Desks versus tables and chairs in group work

(a) Learners seated in desks during group work



(b) Learners seated on chairs during group work

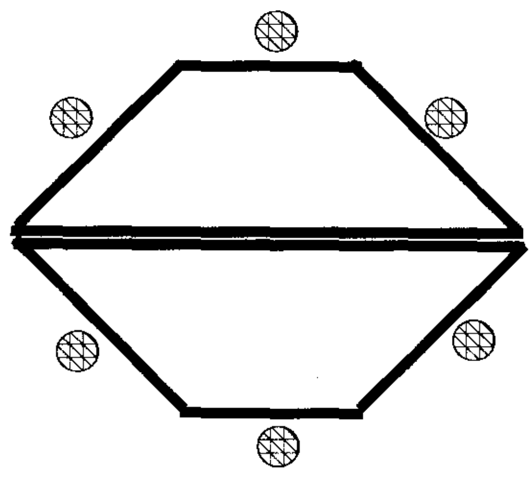


Table 1.1 The use of desks versus tables and chairs during cooperation

Desk (see figure 1.1(a))	***Table and Chairs (see figure 1.1(b))***
• More space is used for one group	• Very economic on space
• An *x* number of group members looks like half ***x-pairs*** if *x* in even, or ***half x + 1*** if x is odd. The probability of group coherence is low.	• Group is perceived holistically whether the number of group members is even or odd. The probability of group coherence is high.
• Dyads are far apart, hence positive interdependence is enhanced within dyads and not within the whole group.	• Individuals are close to one another, hence positive interdependence is enhanced within the whole group.
• A slant or oblique shape of the "desktop" promotes openness among dyads and not for all individual members of the group.	• A level or flat table top promotes openness among all members of the group.

The problems mentioned earlier call for a 'special' cooperative learning model, henceforth called the Hour-glass model. It is called the Hour-glass model due to its structural and functional resemblance to the hour-glass. The Hour-glass model is based on two fundamentally important determinants of effective mathematics instructional process, namely the ability to use cooperative small groups and coping with crowded (large) mainstream mathematics classes. Its point of departure is the use of small group investigation and later employs both peer assistance and group investigation in an integrated manner. Furthermore, for it to be used effectively in mathematics instruction, the use of heterogeneous work groups is preferred to homogeneous achievement groups.

1.3 Research questions

The central research question to be investigated in this study is:

What is the impact of the application of the Hour-glass model of cooperative learning on the learning of mathematics in crowded classrooms? In particular the following three questions will be investigated:

- How can the application of the Hour-glass model of cooperative learning be used to enhance the mathematics academic achievement of grade 8 learners, especially in the context of crowded classes?
- How does the application of the Hour-glass model of cooperative learning influence grade 8 learners' social skills (such as communication, conflict-management, decision-making, trust building, leadership)?
- How does the application of the Hour-glass model of cooperative learning influence grade 8 learners' learning skills, such as critical thinking, problem solving, mathematics anxiety and their attitudes about mathematics?

1.4 Purpose and aims of the research

This study has a dual purpose, namely to establish the potential usefulness of the Hour -glass model of cooperative learning to, firstly, learners of mathematics, and secondly, to teachers of mathematics. With regard to the former, the investigation is based on whether the Hour-glass model of cooperative learning enhances positive learning skills and academic achievement in mathematics. In relation to the latter, the investigation is based on whether the Hour-glass model enhances effective management of cooperative learning groups in large mathematics classes, and co-planning and co-teaching among mathematics teachers in the same grade.

The research aims at achieving the following:

- To enable mathematics teachers to apply a cooperative learning method without being hindered by the large class size;
- To enhance cooperative teaching among mathematics teachers;
- To enable mathematics teachers to manage cooperative small groups by continually keeping their learners on-task during mathematics lessons;
- To afford learners (in crowded classes) the opportunity to use social interaction as a tool for solving mathematics tasks; and
- To enable learners of diverse intellectual and cognitive mathematical backgrounds to work together and help one another in solving mathematics tasks.

1.5 Methods of investigation

1.5.1 Review of Literature

A thorough literature review was done by searches into the Nexus, Education Index, RSAT, EBSCOHost and DIALOG databases. The main intention of the literature review was to critically and objectively highlight the strengths and weaknesses of a cooperative learning strategy in the context of crowded classes and to pilot a model of cooperative learning that would eliminate some of the problems related to the teaching and learning of mathematics using cooperative small groups.

1.5.2 Empirical study

1.5.2.1 Quantitative research design

Since this study involves the application and assessment of the Hour-glass model and a large number of subjects ($n \geq 500$), a specific true experimental design namely **Solomon Four-group** design was used. A detailed discussion of the Solomon Four-group design follows in chapter 4.

1.5.2.2 Population and sample

The population consists of grade eight learners from Mabopane and Temba districts of the North West province. Schools with large mathematics classes (i.e. mathematics classes that ranged from fifty learners), were targeted (see § 4.2 for a detailed discussion)

1.5.2.3 Measuring Instruments

A quantitative research approach was adopted in order to establish the effectiveness of the Hour-glass model in the teaching and learning of mathematics. The Study Orientation in Mathematics questionnaire (SOM) (Maree, 1996) was used to obtain the quantitative data relating to the critical learning factors characterised by cooperative learning strategy, and a self-constructed mathematics academic achievement test was used to measure the mathematics academic achievement of learners.

The qualitative research approach was also adopted by:

- conducting an unstructured interview with the teachers of all the groups to establish the effectiveness of the treatment with regard to the critical elements of cooperative learning (particularly social skills), and
- observing the participants (especially the learners) during the actual mathematics lessons to establish the effects of interpersonal skills (if any) on the learning of mathematics.

1.5.3 Procedure

The Hour-glass model of cooperative learning was piloted in two schools (see § 4.3.1.1). The teachers in the two schools were trained about the application of the Hour-glass model as scheduled in table 4.4. Pre-tests (to gather quantitative data) were administered prior to the application of the Hour-glass model (referred to as the intervention or the treatment). The post-tests were administered after the treatment was introduced. Interviews and observations were also conducted to gather qualitative data. Chapter four provides a detailed discussion about the research procedure.

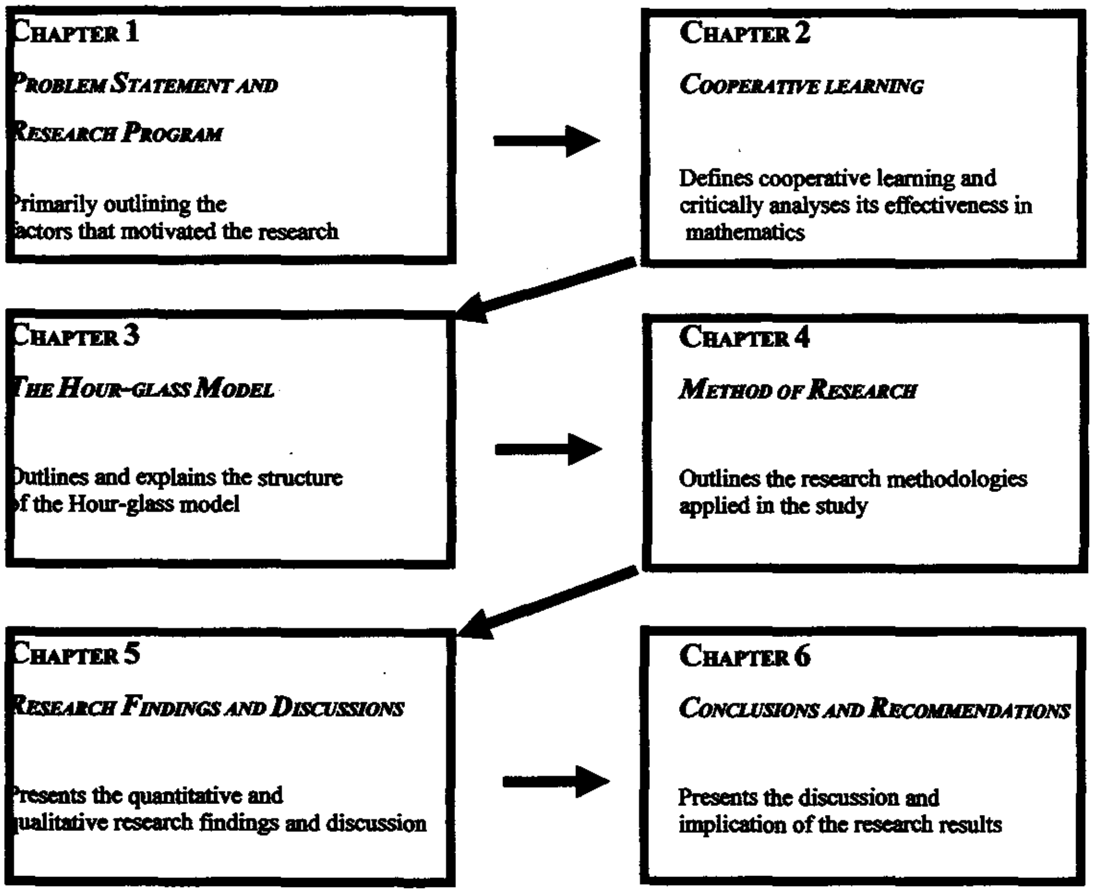
1.6 Related research in South Africa

The review conducted by means of Nexus Database Systems in March 2002 revealed four related studies. The principal aim of identifying the related researches was to establish the number of South African research community members who have investigated the effectiveness of cooperative learning methods (in South African context) in the teaching and learning of mathematics. This is against the background that crowded classes which characterise most of South African schools pose a hindrance towards the success of cooperative learning. From these studies it is evident that more research needs to be conducted to address the teaching and learning of mathematics using cooperative learning in the context of crowded mathematics classes.

1.7 Presentation of the research

The research is presented in six chapters as illustrated in figure 1.2.

Figure 1.2 Presentation of chapters



1.8 Conclusion

Chapter 1 presented an overview of what the study entails with regard to the rationale of the research (problem statement); the population dynamics from which the sample was drawn; research methods to be employed, and purpose ans aims. However a substantiated presentation of the chapters will be done next. In chapter two an attempt is made to critically and objectively highlight the strengths and weaknesses of cooperative learning methods (in the teaching and learning of mathematics) by reviewing the studies conducted by other researchers in and outside South Africa.

CHAPTER

TWO

COOPERATIVE LEARNING

Cooperative learning can benefit all students, even those who are low-achieving, gifted, or mainstreamed (Augustine *et al.,* 1989/90:4).

2.1. Introduction

Numerous studies have documented the effectiveness of cooperative learning in the classroom. This chapter attempts to provide a study review on the effects of cooperative learning in the teaching and learning of mathematics. This chapter encompasses the following six major areas: (a) what is cooperative learning? (b) the current situation: mathematics teaching and learning in South Africa, (c) the impact of cooperative learning on mathematics learning, (d) the effects of cooperative learning on mathematics learning skills, (e) the influence of cooperative learning on mathematics academic achievement and (f) context: group size.

2.2 What is cooperative learning?

2.2.1 An overview and definition

In the workplace power is frequently shared, collaboration encouraged and higher levels of thinking required (Adams *et al.,* 1990:5; Johnson *et al.*, 1991:120; De Villiers & Grobler, 1995:126). These two phenomena, namely power sharing and collaboration imply that, as the employer and the employee work together in an interactive manner, they learn from one another towards the attainment of a common goal - the increase in the effectiveness or quality production (Johnson *et al.*, 1991:11). In the process of power sharing and collaboration people differ in opinion and conflict is likely to arise, resulting in the collapse of the power sharing process and the collaboration system. To avoid the collapse of the system, all persons engaged in the process need to possess and practise social skills such as communication, conflict-management, informed decision-making, trust-building etc. The success of the working institution requires the realization among role players that employers and employees need one another (positive interdependence), that synergism is essential (individual accountability) and that regular monitoring of progress through face-to-face interactions and team processing is necessary. The principles of power sharing and collaboration during the performance of relevant task are not peculiar to the work place or mathematics classrooms. The application of cooperative learning methods in the classroom context enhances such principles in the form of positive interdependence and individual accountability (Johnson & Johnson, 1997:25). The social skills and collective problem solving behaviour acquired during cooperative learning in the classroom keep learners together (Slavin, 1988:31; Johnson & Johnson, 1990:108) as is the case with the society at large.

Outcomes-based education (OBE) strives to offer opportunities to learners at school level for the acquisition of the skills mentioned earlier. It promotes synchrony between educational social structure and social skills needed in the work place. In other words, the classroom should be a reflection of a society at large and be a stage on which real life learning is enacted. It is against this background that cooperative (small-group) teaching learning strategies form the cornerstone of OBE and Curriculum 2005 (Vermeulen, 1997:67; Department of Education, 1997:9). The workplace requirements (among other determinants such as the needs of learners, the nature of subject matter, *et cetera* (Vermeulen, 1997:70) dictate the appropriate instructional strategies (such as cooperative teaching-learning strategies) that can change the educational process, engage the minds of learners and connect schooling to the world of work (Abrami *et al.*, 1995:5).

While cooperative learning has many definitions, what remains a common factor is that it is an instructional strategy that encourages and promotes positive interdependence among learners, social interaction between learner and learner and learners and teachers, and classroom discourse through carefully designed small groups (Abrami *et al.,* 1995:1; Artzt & Newman, 1990b:448; Leikin & Zaslavsky, 1999:240; Chang & Mao, 1999:374; Jacobsen *et al.,* 1999:227; Van de Walle, 1997:35). Put synoptically, cooperative learning employs the principle of synergy, that is, the effect of combined action is greater than the sum of the individual actions. While the emphasis is on learner-centeredness, cooperative learning is not merely another name for group work where learners are left alone in their groups. By contrast teachers have to exert concerted efforts to help learners develop social skills required for the effectiveness of a group and guide them on task related activities.

2.2.2 Critical elements of cooperative learning

Numerous studies (Johnson *et al.*, 1991:16; Sutton, 1992:63; Johnson & Johnson, 1995:349 & 1999:70; Bitzer, 2001:99) contend that, in order for the lesson to be effectively cooperative, it has to be characterized by the following carefully structured critical elements: positive interdependence, individual accountability, face-to-face interaction, group processing and social skills.

2.2.2.1 Positive interdependence.

Positive interdependence is based on the premise that learners feel linked to other learners to the extent that they need one another for their success (Johnson & Johnson, 1997:24). Learners have to realise that they cannot and will never know everything as individuals, therefore they have to complement one another with their unique contributions in a mutual manner.

Positive interdependence can be structured in a variety of ways:

- Positive goal interdependence: The goal of a learning task serves as a focal point for all group members; therefore a shared goal holds individual learners together as a group (Johnson & Johnson, 1997:25).

- Positive reward interdependence: Each group member receives the same reward if the group achieves its goal.
- Positive resource interdependence: Each member is given a distinct but interconnected portion of resources, information or material necessary for the completion of the learning task. Learners therefore need each other and each other's portion of resource to attain the goal of the learning task.

- Positive role interdependence: Each group member is assigned a distinct but interconnected role such as reader, group leader, "elaborator", recorder, "encourager" etc. If one member does not carry out his/her responsibility, the whole group will be in disarray. They therefore need one another for effective cooperation.

2.2.2.2 Individual accountability

The basic purpose of cooperative learning groups is to make each member of a group a stronger individual who possesses a repertoire of cognitive, social and affective skills of learning gained from his/her group mates. Each learner within the group must be held accountable or personally responsible for the mastery of the instruction presented to the group. They work together without hitchhiking on the work of others so that they can perform higher as individuals. Jacobsen *et al.*, (1999:229) assert that individual accountability can be enhanced by rewarding the group based on the individual member's average score. Johnson and Johnson (1999:71) further suggest the following ways to structure individual accountability:

- Learners should study in groups but given individual tests;
- The teacher may randomly select one student's work to represent the entire group;
- Each learner in a group may be asked to explain what they have learnt; and

- Group members can teach and assess one another in order to ascertain that each group member can independently show mastery of whatever the group is studying.

2.2.2.3 Face-to-face interaction

Learners verbally assist, encourage, guide and support one another in their endeavour to learn the given task. In the process they orally explain, elaborate and argue about the given learning tasks in order to establish connections between present and previous knowledge. Non-verbal responses also provide some very important information to other group members. For instance, the silence of a learner who is not contributing to the learning process may probably be an indication that s/he does not understand the task to be learned. Johnson and Johnson (1999:71) assert that the size of the group needs to be small (2 to 4 members) in order to enhance meaningful face-to-face interaction.

2.2.2.4 Group-processing

Johnson and Johnson (1997:29) refer to group processing as "reflecting on a group session to (a) describe what member actions were helpful or unhelpful in achieving the group's goals and maintaining effective working relationships and (b) make decisions about what actions should be continued or changed". Groups should frequently assess how well (i.e. the process) they are functioning together and what needs to be done to improve their effective functioning. When members of cooperative small groups invest quality time in processing their group functions they are likely to attain higher academic achievement and acquire more group cohesion (Johnson & Johnson, 1997:29) than group members who do not engage in group processing. According to Johnson and Johnson (1991:51), group processing has the following benefits:

- it enables learners to focus on maintaining good working relationships;
- it facilitates the learning of cooperative skills;
- it ensures that members receive feedback on their participation; and
- it provides the means to celebrate the success of the group and reinforce positive behaviour among the group members.

Group processing takes place at two levels, *viz.* small-group processing and whole-class processing. The former is a self-assessment that helps the group to identify their weaknesses and/or strengths, subsequently decide on which behaviour need to change or be enhanced especially when there are divisive behavioural patterns among members. Whole-class processing is carried out by the teacher during group cooperation when s/he systematically moves from group to group and observes them at work, gathers data on how different groups work, summarises his/her observations and gives the rest of the class feedback on his/her observations (positive and/or negative observations for encouragement and improvement).

2.2.2.5 Social skills

The broad spectrum of research in cooperative learning seems to regard the enhancement of social skills, i.e. leadership, decision-making, trust-building, accurate and unambiguous communication, conflict-management, tolerating others and negotiating skills as the foundation or basis of cooperation (Slavin, 1988:31, Gunter *et al.*,1999:281; Johnson *et al.*, 1991:11) especially in the mainstream classes. However, the social skills must be well structured and be taught to learners in the same way as the teaching of academic skills (Johnson & Johnson, 1995:349) because "placing socially unskilled learners in a group and telling them to cooperate does not guarantee that they will be able to do so effectively" (Johnson & Johnson, 1999:71). It is therefore absolutely essential that the teaching and learning of social skills be integrated with the teaching and learning of academic skills - reserving time exclusively for teaching social skills may be a laborious and time-consuming exercise for the teacher.

2.2.3 Group structuring

There are two methods of structuring cooperative learning groups, *viz.* **homogeneous grouping** and **heterogeneous grouping**. However, there is substantial research evidence that heterogeneous grouping is preferred over homogeneous grouping (Slavin, 1995:139; Linchevski & Kutscher, 1998:534; Sutton, 1992:64; Boaler *et al.,* 2000:643; Serra, 1989:16; Van de Walle, 1997:489; Gunter *et al.*, 1999:281). Homogeneous grouping in terms of ability has been widely criticized for promoting class polarization, i.e. low-achievers on the one end and high-achievers on the other (Van de Walle 1997:489; Boaler *et al.,* 2000:643) especially in mathematics classes. Concerns such as "high achievers will be held back by low achievers" if the group is heterogeneous has never received any research support (Slavin, 1995:142). For the purpose of this study the researcher prefers heterogeneous group structuring, i.e. mixed in ability, gender, ethnicity, race and so forth. Groups have to be changed regularly "to avoid cliques, allowing many students to get to know and like others as they study together" (Gunter *et al.*, 1999:281).

2.3 Current situation: Mathematics teaching and learning in South Africa

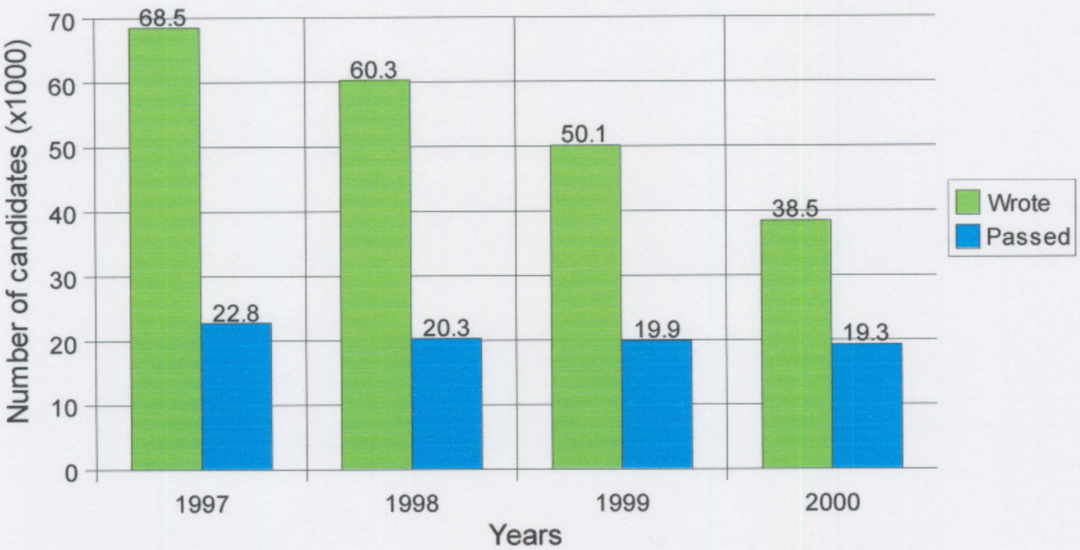
There seems to be an improvement, as yet insignificant, in the general achievement in mathematics world-wide (Howie, 2001:71); however, mathematics teaching and learning in South Africa still need urgent attention (Taylor & Vinjevold, 1999:131). South African learners are still under-performing/achieving in the subject compared to their international counterparts (Howie, 2001:17).

In a broader sense mathematics teaching (and learning) approaches are classified in two categories, namely product-directed and process-directed approaches (Nieuwoudt, 2000:11). The former are predominantly traditional in nature in that "mathematics is viewed as a static and bounded discipline to be taught and studied within the boundaries of the discipline" (Nieuwoudt, 2000:12), and is also characterised by teacher-centeredness where a complete focus is on getting correct answers as directed by the teacher (Van de Walle, 1997:10). Learnt procedures and computations with countless sets of rules are to be practiced and executed accurately in the forthcoming test for a learner to proceed to the next grade. Contrary to these, process-directed approaches primarily depart from the context of learner-centeredness where teaching and learning of mathematics are regarded as a process, that is, from the planning of lessons by a teacher to the demonstration of acquired skills and mathematical proficiencies.

While there is a significant change in how mathematics is taught in the USA, for instance (Van de Walle, 1997:3), mathematics in South Africa is still taught and learnt in a product-oriented manner in the majority of schools (Rossouw *et al.*, 1999:322). This is attributed to, among other things, an insufficient number of qualified teachers (Paras, 2001:66; Department of Education, 2001c:12; Mokoka, 1998:18) and inadequate interaction strategies for the promotion of effective learning and lack of co-planning or communication among teachers (Paras, 2001:68-71). As a result of these and other attributes, there are poor enrolment and poor performance in mathematics. The following scores for national enrolment and performance for higher grade mathematics (see Figure 2.1) serve as a proxy for the effectiveness in mathematics instructional strategies currently used.

According to the Department of Education (2001a) enrolment for mathematics in the North-West province has dropped by 9,8% from 2000 to 2001. This revelation may warrant, without disregarding other factors mentioned earlier, a radical shift from teacher-centered approaches that normally emphasise product, to more learner-centered approaches aligned to C2005 (Rossouw *et al.*, 1999:321) that predominantly emphasise the process of teaching and learning mathematics of which cooperative learning is an example.

Figure 2.1 Higher grade Senior Certificate mathematics participation and performance (Department of Education, 2001c:9**)**



2.4 Impact of cooperative learning on mathematics instruction

Cooperative learning continues to prove its effectiveness in many facets of mathematics education (Shaw & Chambless, 1997) but this should not be misconstrued as implying that cooperative learning is absolute. Numerous studies have documented the advantages and disadvantages (henceforth referred to as **positive** and **negative** impact respectively) of cooperative learning without which further research in the subject would be unnecessary.

2.4.1 Positive impact

Whicker *et al.*, (1997:43) have identified the following positive results emanating from the use of cooperative learning:

- **Increased social skills**: In mathematics students' problem solving abilities are enhanced by engaging in social collaboration practices such as communicating (that is, meaningful classroom discourse) with one another, listening, engaging in constructive argument and reaching consensus about each other's inputs or opinions. Duncan and Dick (2000:365) assert that integrating these social skills into mathematics learning improves academic achievement. The interaction between the learner and the teacher enhances mathematical meaning-making or conceptualization without compromising intellectual autonomy (Cobb, 1988:88) - as learners and teachers share ideas on mathematical problem-solving the learners' self-confidence and motivation to initiate new methods of problem-solving are enhanced. Learners and teachers, therefore, tend to respect and value each other's role during instruction.

- **Cognitive benefits:** Critical thinking and high level thinking are promoted, subsequently problem-solving ability is improved (Artzt & Newman, 1990b:450). As they work together, learners are able to analyse, elaborate, apply and reflect on their learning.

- **Behavioural benefits:** Time on task is improved; as a result learners are motivated to do more mathematics. Furthermore, classroom discourse is enhanced (Artzt, 1999:11), that is, a platform is created where learners can read, talk, write about and explain mathematics ideas collaboratively. This is against the background that traditionally teachers dominated the instructional process while learners were passive recipients of information which did not contribute towards the effective learning of mathematics.

- **Affective benefits:** As learners give and receive help and explanation to and from their group members, they develop a sense of belonging, especially adolescents who want to belong and be recognised. As a result they become motivated to do more mathematics, their self-confidence in mathematics improves and mathematics anxiety gradually diminishes.

2.4.2 Negative impact

Low achieving learners tend to become passive because they are dominated by high achieving students (Corno, 1988:184; Whicker *et al.,* 1997:43) or because they perceive themselves as having little to contribute, and even if they can contribute their contributions are not valued (Reynolds & Muijs, 1999). The passiveness exhibited by less-able learners may result in the free-rider effect (Johnson & Johnson, 1990:105; Abrami *et al*, 1995:23). Learners may be tempted to engage in off-task social interactions especially when the teacher is busy helping other groups (Reynolds & Muijs, 1999). This is more prevalent in the large class-size. As a result widespread adamant behaviour and resistance of teachers towards the use of cooperative learning may surface. This is also attributed to unacceptable learner behaviour such as off-task engagement that it (cooperative learning) is perceived (by teachers) to promote.

Some groups need more time to form a cohesive bond to work effectively and this impedes progress (Whicker *et al.*, 1997:42). If this is the case, it contradicts the assertion that group members should be rotated at regular intervals to give learners the opportunity to work with different people and learn new problem solving strategies from new group members (Serra, 1989:16; Gunter *et al.*, 1999:281). While the former assertion is acceptable (with some reservations), the latter assertion acquires more credit because demonstrating the ability to adapt to and work with different people is a very important interpersonal or social skill in mathematics problem solving that has to be acquired by learners. The process of forming a cohesive bond during cooperative learning is in itself an important social skill for the learner and should not be perceived as 'impeding progress' as if 'progress' refers to the mastery of mathematics content only.

2.5 Effects of cooperative learning on mathematics learning skills

Mathematics learning skills or mathematics study orientation is the result or the prevalence of the acquired behaviour that falls outside the cognitive field, which condition can either promote or inhibit mathematical cognitive achievement amongst the learners (Maree *et al.*, 1997:1). Such acquired behaviour, assert Maree *et al.* (1997), may be affective, social and/or psychological as it covers a broad range of aspects such as study attitudes, study habits, mathematical anxiety, motivation, problem-solving behaviour, study environment, etc. These factors may sometimes cause learners with high mathematical ability (high-achievers) to under achieve in the learning area (mathematics) or learners with low mathematical ability (low-achievers) to achieve higher in the subject; therefore learners are said to possess negative and positive study orientation respectively in mathematics.

The following learning skills will be addressed in the next sections of this chapter: attitudes; mathematics anxiety; motivation and problem-solving behaviour. The main focus will be on whether research has revealed any effects of cooperative learning on the above-mentioned learning skills in mathematics.

2.5.1 Attitudes about mathematics

Student attitudes towards mathematics have been associated with peer group influence and intelligence (Dungan & Thurlow, 1989:10). This finding suggests that high achievers have a higher interest in and get more enjoyment (positive attitudes) from mathematics than low achievers and they (high achievers) can transfer these attitudes to their peers. This seemingly forms the basis of cooperative small group work which, according to Artzt and Newman (1990b:448) capitalises on the powerful influence of peer relationships. In order for students to acquire positive attitudes towards mathematics (and influence other students positively) they have to be encouraged by their teachers, and subsequently their academic performance will improve (Dungan & Thurlow, 1989:10). This finding supports the notion that, while cooperative learning is learner-centred, teachers have a mammoth task of facilitating, guiding, assisting and encouraging learners through the whole cooperative learning process, instead of abrogating their responsibility in the teaching-learning process. The Hour-glass model aims at offering this kind of support and encouragement as will be explained in the chapter 3.

2.5.2 Mathematics anxiety

Documented studies have indicated that some students have a tendency to panic, being helpless and having mental disorganisation when confronted with a mathematics problem (Dungan & Thurlow, 1989:9; Costello, 1991:122). One of the sources of mathematics anxiety is the requirement to learn mathematics at an inappropriate fast pace especially in ability group setting where students in high ability groups are expected to work at an abnormally fast pace (Boaler *et al.*, 2000:633). For this reason, heterogeneous grouping seems to work more effectively than homogeneous grouping when employing cooperative learning strategy. Learning cooperatively in small (heterogeneous) groups reduces the pressure experienced by low achievers in mathematics (King, 1993:412; Newstead, 1998:55) because their immediate learning resource is no longer a teacher but other students with whom they can associate in a relatively relaxed manner. In their small cooperative groups learners are able to talk freely, listen and be listened to, challenge and defend a point of view, freely share ideas, feel safe to make and learn from mistakes (Duran & Cherrington, 1992:80). In other words, they do not feel the anxiety they normally have for not understanding the work in mathematics (Artzt & Newman, 1990b:452) because they support and depend on one another.

2.5.3 Motivation

Student attitudes towards mathematics are related to achievement and teachers must therefore be concerned about their students' motivation to be successful in mathematics (Higbee & Thomas, 1999). Wienschenberg (1994) further accentuates that lack of desire (motivation) to learn mathematics results in failure,and subsequently in a hostile attitude towards mathematics. Cooperative learning has a motivating impact, especially intrinsically, on learners during mathematics learning (Artzt & Newman, 1990b:452). This is evident in the learners' persistent manner of tackling mathematics problems when working in cooperative small groups compared to those working independently who tend to give up quickly if they do not get the solution immediately (Duran & Cherrington, 1992:82). Furthermore, individualized mathematics learning reduces motivation by requiring extensive interaction between the student and written material (Corno, 1988:201).

2.5.4 Problem-solving behaviour

A fundamental principle of the constructivist view of mathematics learning is that students learn mathematics best through problem-solving (Van de Walle, 1997:39) by engaging in small group learning with little teacher intervention (Yager, 1991:55). Invoking cooperative group work in mathematics problem-solving is a confirmation that cooperative learning strategies are credited with improved problem-solving abilities (Artzt & Newman, 1990b:450). Johnson and Johnson (1990:108) typify the enhancement of mathematics problem-solving behaviour using cooperative learning as follows:

> Mathematical problem-solving is an interpersonal enterprise. Talking through mathematics problems with classmates helps students understand how to solve problems correctly. Explaining reasoning strategies and analysing of problems to classmates... result in discovering insights, using higher-level reasoning... most students are more comfortable speculating, questioning and explaining concepts in order to clarify their thinking in small groups.

Problem-solving skill is characterized by critical, analytic and high-level thinking. These are evident in learners' ability to engage in brainstorming a problem before engaging in the actual solution thereof. They share their own way of thinking and reflect on them and on thinking and ideas of others, and this is tantamount to providing support and assistance to one another (Reynolds & Muijs, 1999). By so doing, students tend to be accommodative of the opinions of others, and learn that others possess both strengths and weaknesses. Students who are less-able 'problem solvers' can therefore overcome their insecurity about problem-solving because they realize that more-able students are also struggling over difficult problems in mathematics. As a result all learners will probably be able to choose and apply their knowledge of facts, concepts, formulas and procedures effectively in solving problems (Muth, 1997).

2.6 Influence of cooperative learning on mathematics achievement

Research findings have generally revealed that cooperative learning enhances academic achievement in mathematics (Abrami *et al.* 1995:196; Bennett & Dunne, 1994:29; Brush, 1997:52; Ma, 1996:379). Put more precisely, there has been a growing consensus that cooperative learning promotes student achievement (Mevarech, 1999:195; Slavin, 1989/90:52) especially those cooperative learning models that emphasise group goals and individual accountability (Slavin, 1988:31; 1989/90:52). Group goals enhance motivation while individual accountability reduces passivity during mathematics instruction (Reynolds & Muijs, 1999) and both build up towards high academic performance in mathematics.

Enhancement of achievement is evident from the assertion that low achievers benefit from cooperative learning by having mathematics content explained to them by high achievers in a manner that is more comprehensible to them than the teachers' explanation, while they (high achievers) benefit by consolidating their own learning by teaching others (Mulryan, 1994:283). However, cooperative learning models, especially peer tutoring models of cooperative learning, tend to emphasise the benefits attained by both high and low achievers disregarding average-achievers who normally constitute a significant percentage of mathematics class membership. This polarization, therefore, does not reflect the reality of our classrooms. As a result this is a shortfall for cooperative learning models especially those that advocate peer tutoring, i.e. help-giving and help-seeking by high achievers and low achievers respectively. The flexible nature of an Hour-glass model, especially when deciding which learners should be included in an 'interclass', can successfully address this shortfall (*see* chapter three for a more detailed discussion of the Hour-glass model).

Johnson and Johnson (1990:107) cite the following influences of cooperative learning on mathematical thinking, understanding of the connections among various mathematics facets and procedures, and applying formal mathematical knowledge flexibly and meaningfully:

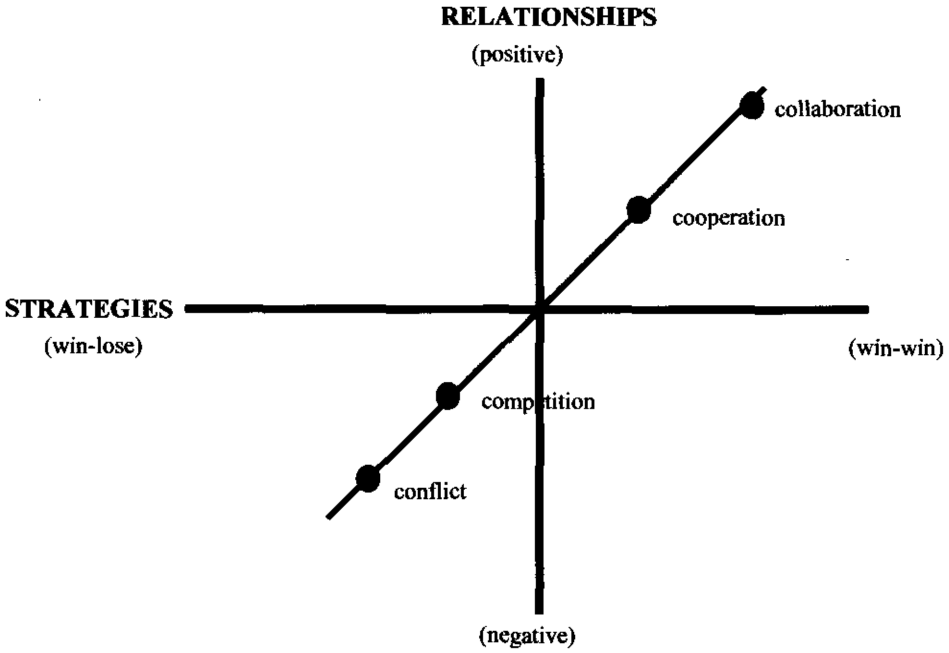
- Cooperative learning promotes higher achievement in mathematics class than do competition and individualistic efforts. When learners learn mathematics in cooperative conditions they are more likely to perform much higher than their group's average when exposed to individual or competitive conditions (Johnson & Johnson, 1990:107). This suggests that learners in cooperative conditions frequently discover and use high quality reasoning strategies; master and retain mathematics facts and principles; generate new ideas and solutions; and transfer all the mathematics strategies and facts learned within the group to problems considered individually. Hence high achievement gains in mathematics exhibited by learners from cooperative to individualistic conditions are mainly attributed to the small group effect and not necessarily to the individual. This is a very significant effect of cooperative learning on individual mathematics learners in that learners may learn mathematics and be assessed cooperatively but examination is normally written individually.

- Mathematical concepts and skills are best learned as part of a dynamic process with active engagement on the part of learners (Johnson & Johnson, 1990:107). During cooperative learning mathematics is learned actively and not passively as was/is the case with traditional mathematics instruction (Johnson & Johnson, 1990:107). When learners do mathematical tasks actively in cooperative small groups they become curious about each other's inputs, consequently they are likely to discover new methods of doing mathematics.

- Johnson and Johnson (1990:108) contend that mathematics problem-solving is an interpersonal enterprise. As learners talk and relate with one another through mathematics problems they learn and understand how to solve the problems correctly because they are likely to correct or guide and be corrected or guided by others. This suggests that positive relationships within a cooperative group will lay a foundation for effective cooperation where learners can explain reasoning strategies and analyses of mathematical problems to their group mates. Learners in a cooperative small group will all emerge winners because their individual contributions (right or wrong) lead to collective problem-solving (Askew & Carnell, 1998:41). In contrast, when learners solve mathematics problems individually to achieve success, a win-loose situation (competition) arises and a negative relationship among mathematics learners is enhanced (*see* Figure 2.2). Effective collaboration and cooperation are characterised by positive relationships and a win-win situation, while competition is characterised by a win-lose situation and negative relationships among learners.

- Mathematically aligned career choices are commonly influenced by peers. This is no exception when learning mathematics cooperatively as opposed to individualistic or competitive mathematics learning (Johnson & Johnson, 1990:109). "If one's peers perceive certain classes (or fields of study) as being inappropriate ... there is considerable resistance to taking them. Within cooperative learning situations ... students tend to like and enjoy mathematics more ... and be motivated to learn more about it continually" (Johnson & Johnson, 1990:109). This implies that when learners are exposed to learning mathematics in cooperative small groups where they share ideas and make collective decisions about their solutions, they are also likely to influence one another about taking mathematically aligned careers. Johnson and Johnson (1990:109) further accentuate that learners receive considerable encouragement and support in their efforts to learn mathematics processes, strategies and concepts. Through this interaction learners gain confidence in their individual mathematics ability.

Figure 2.2 The conflict-collaboration continuum (Askew & Carnell, 1998:41)



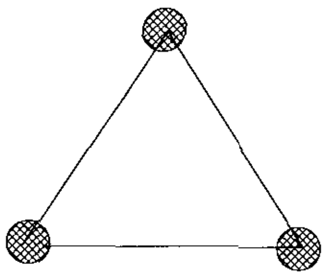
In essence, studies have documented a relationship between academic achievement and attitudes (Dungan & Thurlow, 1989:10), anxiety, motivation, and problem solving behaviour in mathematics. By implication learners who have positive attitudes towards mathematics, a low (or non-existence) level of mathematics anxiety, and a high desire (or motivation) to do mathematics and possess problem-solving skills are likely to perform well in mathematics. All or some of these factors, therefore, contribute to the attainment of success in mathematics.

2.7 Context: group size

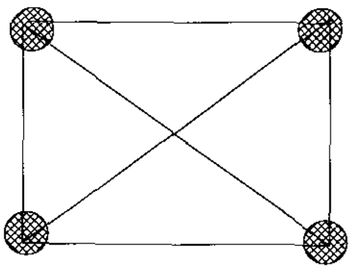
Positive effects produced by the use of cooperative learning are attainable in the small group context. This is evident in the consensus reached by numerous studies that five is the optimum group size for effective cooperation during teaching and learning (Bruffee, 1993:32; Bennettee & Dunne, 1994:114; Artzt & Newman, 1990b:449; Leikin & Zaslavsky, 1999:244; Artzt, 1999:12; Serra, 1989:16; Abrami *et al.*, 1995:60). As the class size increases it is more difficult for the teacher to maintain order, the learner attention rate decreases and the teacher morale becomes low (Glass *et al.*, 1982:68). This, therefore, raises a very important aspect, namely the effectiveness of cooperative learning in the context of overcrowded or large class sizes, subsequently large group sizes. Most of South African public schools have large class sizes which therefore leave cooperative learning as an idealistic approach in mathematics teaching and learning as opposed to a practical and user friendly approach based on the aforementioned assertion. In the following paragraphs a concurrent reflection is carried out on the two classroom contexts, namely small class size context (which is idealistic) and large class size context (which is practical and realistic) in most of South African public schools.

Cooperative learning models seem to have favoured and put more emphasis on small class sizes (hence small group sizes) (Johnson & Johnson, 1999:71) and discourage large groups for its successful application in mathematics classes. Large class sizes, hence large group sizes, make teaching, consequently learning, difficult because teachers have little time, if any, for discussion and learners' active participation (Papo, 1998: 187) since the size of a group determines its ability to be productive (Artzt & Newman, 1990a:15; 1990b:449; Bruffee, 1993:32). This suggests that as the group size increases, the lines of communication become more complex (Bennette & Dunne, 1994:114) and the more difficult it becomes to promote equal participation and possibly learning (Abrami *et al.* 1995:60). This complexity of lines of communication emanating from the increase in group size is illustrated in figure 2.3 by Bennette and Dunne (1994:115).

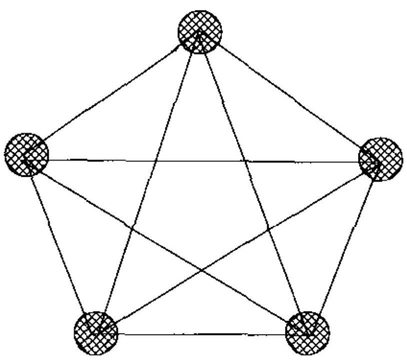
Figure 2.3 Lines of communication in groups of three to five



Two lines of communications per learner



Three lines of communications per learner



Four lines of communications per learner

Further Jonson & Johnson (1997:225) and Abrami *et al.*, (1995:23) identify the following negative results of applying cooperative learning in large groups:

- ***Social loafing*** (hiding in a crowd) becomes more prevalent in additive mathematics tasks (the summing together of individual group member's input to maximise the group product). Less-able learners in particular tend to capitalise on the large group size and the fact that the teacher may not be able to notice them.

- Less-able learners are likely to develop a ***free-rider effect*** (getting something for nothing). The free-rider effect is more prevalent in disjunctive mathematics tasks (if one person does it, everyone receives the benefit).

- More-able learners are likely to become less motivated due to perceived inquiry resulting from the free-rider effect and social loafing. They may expend less effort to avoid the ***sucker effect***.

- If the more-able learners do not avoid the sucker effect they are likely to take all the responsibilities of the group and the result will be the ***rich-get-richer effect***.

In reality, the situation in schools in the Mabopane and Temba districts is that a group is constituted by six or more learners (derived from about sixty learners per mathematics classroom). This is supported by the actual number of learners in grade eight classes that was collected from eight schools in the Mabopane and Temba districts recently as illustrated in table 2.1. At school A, for instance, the class size is eighty four (84), two times more than the international mathematics class average of 30 (Howie, 2001:100) and the national and provincial average (inclusive of all schools and learning areas) of 34 and 30 respectively (Department of Education, 1999:2). The class-size of school A may be translated to ten (10) groups of about eight (8) members each. In such a mathematics class where the notional teaching learning time is about thirty minutes per period (13% of the total teaching-learning time for mathematics), it becomes practically difficult for a teacher to successfully and effectively monitor and assist all groups (Department of Education, 1999:2).This may cause some learners to be discouraged (Bosch & Bowers, 1992:104), that is, they may develop a feeling of "*why try, the teacher will not attend to all of us today anyway?*". The surface area or floor area of a classroom is also not adequate to accommodate (with adequate space between the groups for movement) so many groups. Thus, for cooperative learning to be effectively implemented in the teaching and learning of mathematics, group sizes should be small, which in essence requires small class sizes.

Table 2.1 Typical grade 8 average class sizes in Mabopane and Temba districts

School	*Grade 8 enrolment*	*Classes*	*Average class size*
A	251	3	84
B	148	2	74
C	215	4	54
D	313	5	63
E	310	5	62
F	342	6	57
G	369	6	62
H	532	7	65

2.8 Conclusion.

Slavin (1988:31 & 1989/90:52) accentuates two essential conditions, namely group goal and individual accountability for cooperative learning to realize achievement goals. However, there exists substantial evidence that large group size does not enhance effective cooperation. The researcher regards the size of the group (in terms of the numerical quantity and not merely large or small) as the third essential condition for cooperative learning to realize achievement goals. It is essential that a particular cooperative learning model should specify the group and/or class size in which its effectiveness in mathematics teaching can be attained because not all (or any) cooperative learning model can be used effectively in large class size context. Finally, while cooperative learning has proven its enormous effectiveness in mathematics teaching and learning, it should not be regarded as panacea.

CHAPTER

THREE

THE HOUR-GLASS MODEL

> **When a student assistance (staffed by both peer and adult tutors) is established, students can seek assistance [in mathematics] before and after school as well as during school day (Blum-Anderson, 1992:435).**

3.1 Introduction

Due to the difficulty of employing cooperative learning in, and to effectively curb problems posed by large class sizes, the Hour-glass model of cooperative learning seeks to offer solutions. However, this model is not only tailor-made for large class sizes, but can even be used in small class size context.

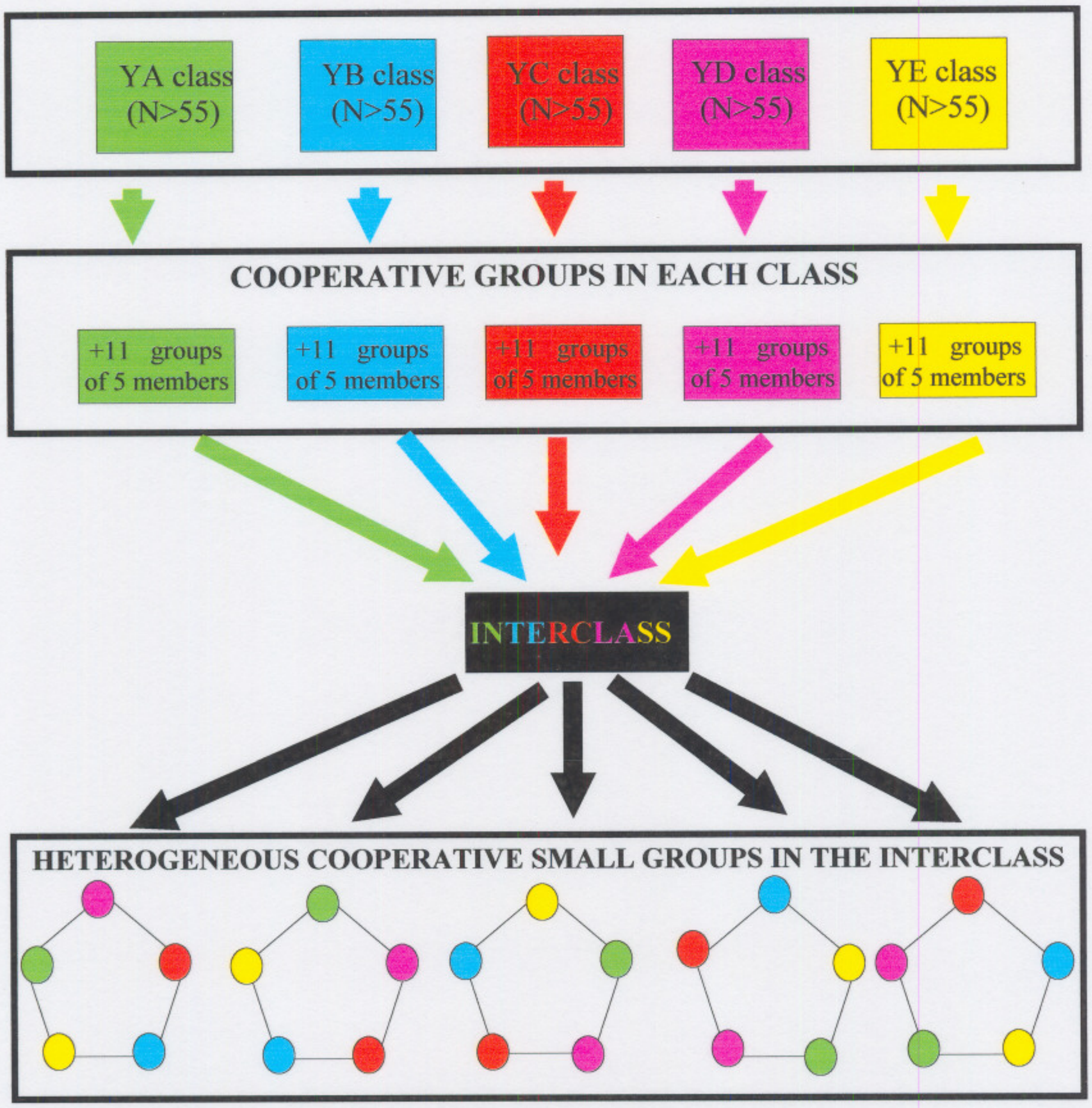
The Hour-glass model is based on two fundamentally important determinants of effective mathematics instructional process, namely the ability to use cooperative small groups and coping with crowded (large) mainstream mathematics classes. Its point of departure is the use of small group investigation and later employs both peer tutoring and group investigation (Lambdin-Kroll *et al.*, 1992:620) in an integrated manner. Furthermore, for it to be used effectively in mathematics instruction, the use of heterogeneous work groups is preferred to homogeneous achievement groups (Good *et al.*, 1989/90:56-57).

3.2 The general structure of the Hour-glass model

Figure 3.1 provides a diagrammatic illustration (flow-chart) of the general structure of the Hour-glass model of cooperative learning in mathematics. A synoptic reflection on the structure of the Hour-glass model will be made. However, the criterion for identifying learners who have to be included in the interclass will be discussed in detail in a later section of this chapter.

Suppose grade Y (Y = any one grade) in a particular school has classes YA to YE with the large class-size (for instance N>55, i.e. number above the national mathematics class average of 50). Learners in each class are arranged in heterogeneous cooperative small groups - either fewer groups per class with big group sizes or many groups per class with small group sizes. The two scenarios are the attributes of large class size and they both impact negatively on cooperative learning in mathematics. The latter scenario is used to illustrate the general structure of the Hour-glass model in figure 3.1. Each teacher identifies learners (see § 3.3.1.2)

Figure 3.1 The general structure of the Hour-glass model



(not more than one learner per group) to be included in the interclass. It should be emphasised that it is not compulsory for all the groups to have a representative in the interclass. It is, however, of utmost importance that the total number of members of the interclass should not exceed half the average class size in a particular grade. The interclass size of between twenty-five and thirty-five learners is generally convenient to work with. The number of interclass members (derived from the average class size) can therefore give an indication (to the teacher) of how many learners should be identified per class. Learners in the interclass are arranged into small cooperative learning groups. Either one or both teachers can facilitate the lesson to the interclass but in both options the teachers involved in teaching mathematics in that grade must have planned the lesson together. Finally, all learners and the teacher(s) in the interclass reflect on the lesson, i.e. group-processing by learners and whole-class processing by the teacher(s). The following day learners from the interclass rejoin their classes and their groups to offer assistance and guidance to their group-mates when the same lesson as the one offered to the inter-class is newly taught to the rest of the class.

3.3 Steps in the Hour-glass model

The following case scenario will be used to illustrate each step of the Hour-glass model.

Case scenario

Middle school X is situated in a township about 30 km away from a big city. The total learner enrolment is about 630 with 18 teachers (one principal; one deputy-principal; three heads of department and thirteen educators). The school has eleven classrooms, three of *which are occupied by grade 8 learners (average of 65 learners per class) and the other three are occupied by grade 7 learners (average of 70 per class). Grade 9 learners are much better distributed in the remaining five classes (average of 46 learners per class). There are five mathematics educators in the whole school (four are fully qualified mathematics teachers while one is under-qualified). Two educators, Mr A and Ms B, share the three grade 8 mathematics classes. Educators are very keen to implement the OBE-approach and the school management team (SMT) is very supportive in this regard - they (SMT) do their best to provide educators with the necessary OBE tools the school can afford. However the seating pattern of learners in all the classes is not the same:*

- *grade 9 learners (whose class-size average is about 46) are seated in small cooperative groups around two jointed tables per group,*
- *grade 7 and 8 learners (whose class average is about 70 and 65 respectively) are seated in rows and columns.*

In all three grades the instructional process is predominantly teacher-centred. When learners of middle school X are compared to learners of other schools in the township they are relatively well disciplined. However, grade 7 and 8 learners in school X are not as disciplined as their grade 9 counterparts:

- *they frequently go out during the lessons (especially mathematics lessons);*
- *they regularly engage in academically unproductive noise even when the teacher is in the class;*

- *they like engaging in idle-chatter with their 'neighbours' (in rows and columns).*

There is a variety of factors that may affect the general running of school X, but for the purpose of this research specific attention is only given to factors that particularly affect the instructional process of mathematics. The following possibilities and their associated attributes relative to the use of cooperative learning in mathematics instruction were identified:

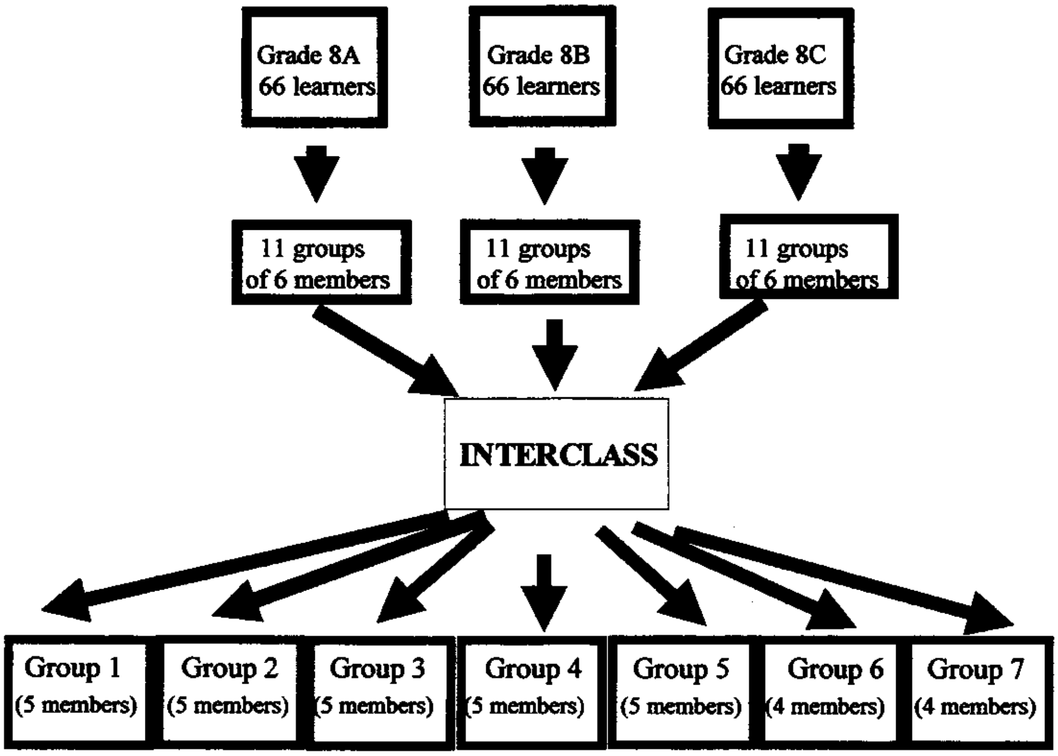
- The prevalence of teacher-centred instruction even when the learners are seated in groups and the seating pattern of grade 7 and 8 learners (rows and columns) may be attributed to the lack of knowledge of cooperative learning method(s);
- Grade 7 and 8 learners cannot be seated in cooperative small groups because of the large classes, i.e. group sizes will also be big subsequently cooperation will be ineffective;
- The regular ill-disciplined behavioural pattern exhibited by particularly grade 7 and 8 learners may be an indication that teachers are unable to keep almost all learners on task for the duration of the lesson especially during mathematics lessons; and
- Regular avoidance of mathematics lessons may be an indication that high-achievers get bored or low achievers have a negative attitude towards mathematics and lack motivation to do more mathematics exercises.

3.3.1 Assemble a mathematics interclass

Before embarking on the purpose and the criteria for assembling the interclass, it is important to emphasise that the structure of the Hour-glass model that will suit the context depicted in the case scenario will resemble the general structure (see Figure

3.1), except that three grade 8 classes will be used instead of five. Figure 3.2 illustrates the Hour-glass model in the context of the previous case scenario.

Figure 3.2 The structure of the Hour-glass model as per case scenario



3.3.1.1 The purpose of the mathematics interclass

The mathematics interclass is characterised by very important purposes:

- Creation of small manageable mathematics class of about thirty learners (Howie, 2001:100) from which cooperative groups of about five learners can be formed (Webb & Farivar, 1999:147);
- Creation of high level of heterogeneous groups based on different classes (in the same grade), academic achievement in mathematics, gender, etc.;

- Enhancement of cooperative teaching where more than one teachers share mathematics classes in the same grade; and
- Attending to the academic, psychological and/or environmental mathematical needs of learners by providing a warm and non-threatening classroom climate (Ashlock *et al.* 1983:18) in which all learners can be attended to by their teacher(s).

3.3.1.2 Criteria for the selection of learners

Deciding on which learners should constitute an interclass at any given instance depends solely on what the teacher has identified to be the need(s) of learners to learn mathematics effectively or what factor(s) impact negatively on the learning of mathematics. Therefore, the fulfilment of such learners' needs becomes the goal of the teacher about the mathematics lesson(s). For instance, if there are learners who are almost always performing poorly in mathematics, the teacher may decide to include all such learners in the interclass mixed with few high-achievers, or if there are high-achievers or gifted learners who, as a result of giftedness, tend to become bored and disruptive during mathematics lessons, they may be assembled into the interclass (mixed with few under-achievers). These guiding factors or needs include, but are not limited to the following:

- mathematics anxiety;
- poor mathematics academic achievement;
- lack of or poor mathematics problem-solving skills;
- passivity during group work;
- giftedness;
- negative attitude towards mathematics;
- off-task indulgence during mathematics learning; and
- lack of motivation to do more mathematics.

As mentioned earlier, it appears as if Mr A and Ms B's grade eight learners (high-achievers) are bored, hence they spend a significant time off-task or do not attend the mathematics classes. The low-achievers lack a positive attitude to mathematics and are highly unmotivated to do more mathematics. The need for the high-achievers is therefore to be kept on-task and be engaged in more challenging mathematics tasks, while the low-achievers need motivation and a positive attitude about mathematics. Based on this, Mr A and Ms B's instructional goal is to help both the high and low achievers fulfil their needs. The identification and selection process will be based on the attitude about mathematics, time on-task during mathematics lessons and motivation to do more mathematics.

Since the average class-size is sixty-six, the interclass should consist of about thirty-three learners, i.e. the interclass size should not exceed half the average class-size (66) as explained earlier. This suggests that if each class has eleven groups of six members each on average (see Figure 3.3), teacher A and teacher B may identify and select one learner from each group of the three classes based on the criterion explained earlier. It is, however, important to high-light that:

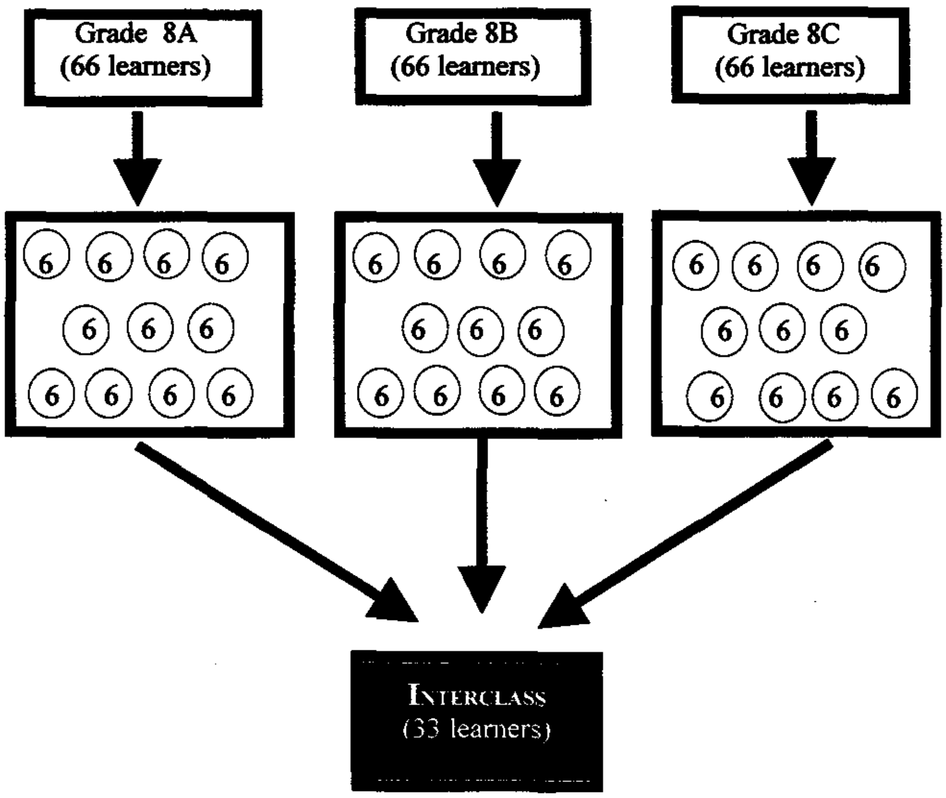
- the number of learners selected from each class should not necessarily be equal;
- the assembled interclass should not necessarily be divisible by any specific number; and
- each cooperative group should not necessarily have a representative in the interclass because the teacher will be available to assist groups which were not represented.

The main idea is to have a manageable interclass size, and subsequently a manageable interclass cooperative group size (5 members per group).

Implications for teachers and learners

- Teachers have to know their learner, i.e. their strengths and/or weaknesses in mathematics;
- Teachers have to be knowledgeable about factors affecting mathematics learning such as mathematics anxiety, attitudes about mathematics, problem-solving behaviour, beliefs about mathematics etc. They must also possess the ability (skill) to identify or diagnose each of them. Teaching mathematics is not only about going to class and teach, but also about being able to identify each method

Figure 3.3 Assembly process of the interclass



of teaching that does not lead to effective mathematics learning, and what factors (especially psychological and cognitive factors) pose hindrance to mathematics teaching and learning;

- In this case there are two teachers who teach mathematics in the same grade, therefore they have to work collaboratively. This means that they have to help each other to select learners, and prepare and facilitate a lesson to the interclass. Co-planning and co-teaching is enhanced among teachers across classes in the same grade, hence cooperation (Paras, 2001:70);
- Learners, on the other hand, will interact with learners from other classes in the same grade and probably be taught by and interact with a different teacher. By so doing they give and gain new ideas of doing mathematics. They must therefore strive towards improving their social skills such as tolerance, communication, conflict-management, trust-building, leadership etc.;
- The teacher must keep a record of all learners who were in the interclass and the purpose or the need they had in order to avoid selecting same learners at all times; and
- The teacher should monitor the progress of learners (who attended the interclass) to ascertain if their problems with mathematics were mitigated.

3.3.2. Explication

The purpose of an interclass (*see* § 3.3.1.1) is explained to learners and a ***pep talk*** is given to them in order to build their self-esteem and for them to gain willingness to do mathematics cooperatively (Weinschenberg, 1994). The teacher explains to learners that:

- They are about to be exposed to effective cooperative group work in a conducive classroom environment;
- They should work together on the given mathematics learning tasks by sharing

ideas - each input is valuable;

- They should draw the attention of the teacher to what they do not understand because they (interclass learners) will in turn have to assist other learners the following day in their classes;
- They have to depend on one another in their groups but at the same time they have to be individually accountable to the success or failure of their groups;
- If the teacher poses a question, group members must collectively find the solution before any member can raise his/her hand because a raised hand in a group represents the feeling of all group members. Thus the teacher may point at any member of that group to give an answer including those whose hands were not raised. It is therefore the responsibility of individual members of a group to ensure that all members have a common understanding of the answer they will give. The idea is that the raised hand implies **we know** and not **I know**;
- The presence of more than one teacher in the interclass should not frighten the learners - s/he is there to help and learn from all group members as they (learners) will be expected to help and learn from other learners the following day in their respective classes; and
- They must strive to attain the goals of the lesson because they are going to assist other learners. They will also have to transfer the skills acquired from the use of the Hour-glass model of cooperative learning to their class/group mates.

Caution should be taken not to explain the criterion used to select learners because this may exacerbate their weaknesses and obliterate even the lowest self-confidence they have about mathematics. Negative statements are very detrimental especially to students who have doubts about their abilities in mathematics (Blum-Anderson, 1992:433). The aim, one should bear in mind, is to motivate them (learners) by explaining what they will be doing after step five is completed. For instance, by telling a learner that s/he will be helping other learners after attending an interclass may promote his/her self-confidence about mathematics.

Implications for the teachers

- Teachers have to thoroughly understand the critical elements of the Hour-glass model because primarily this step is about explaining it. This should not be misconstrued as implying that learners have to know the name and steps of the model, but the application thereof; and
- Teachers should be well conversant with the critical elements of cooperative learning (and their significance in mathematics learning) such as positive interdependence, individual accountability, social skills, face-to-face interaction and group processing because the 'explication step' of the Hour-glass model revolves primarily around them.

3.3.3 Cooperative small-group formation

This is fundamentally a very important step that to a large extent determines the success or effectiveness of each group. As said earlier, groups should be as heterogeneous as possible, that is in terms of gender, achievement ability and so on. Cooperative small groups should be formed by teachers (and not learners) because it is expected that they are accustomed to the principles that underlie group dynamics such as homogeneous versus heterogeneous grouping and their associated merits and demerits. Terwel (1990:240) alludes to the fact that if cooperative small groups are formed by teachers the following mistakes (that are likely to be committed by learners if they are given the freedom to form their own groups) may be avoided:

- leaving out certain learners;
- developing cliques among learners in the groups;
- forming homogeneous groups; and
- developing static communication patterns.

Most importantly, no learners from the same class should be allocated to the same cooperative small group if possible. Homogeneous grouping should be avoided because research has shown that it promotes or creates polarisation (Abrami *et al.*, 1995:64; Boaler *et al.*, 2000:642). This suggests that if homogeneity is informed by achievement, hence achievement grouping (Good *et al.*,1989/90:56), high-ability groups regard themselves (and are regarded by low ability groups) as 'mini-mathematicians' who can work at a very fast pace, whereas students in low ability groups regard themselves (and are regarded by high ability groups) as failures in mathematics who can only cope at a very slow pace. Based on this and other tacit problems posed by homogeneous grouping, an Hour-glass model employs heterogeneous grouping, especially heterogeneous-ability grouping (Abrami *et al.*, 1995:65) for its success in mathematics teaching and learning. The main aim is to minimise competition and promote cooperation in which learners can share ideas with one another.

The following guidelines may be of assistance during interclass group formation:

- Group-size should not exceed five for it to be manageable;
- Determine the number of groups by dividing the interclass-size by five. Mr A and Ms B (*see* case scenario) will therefore have five groups of five members each and two groups of four members each (see *F*igure 3.4).
- Members of groups that have four members should be seated in such a manner that one learner occupies one side of a rectangular table(see Figure 3.5(a)) and not two learners per side (see Figure 3.5(b)). The aim is, firstly, to enhance or encourage group cooperation among all the group members rather than enhancing pair cooperation (which is tantamount to group polarization that may result in competition between pairs in the same group). Secondly to encourage each learner to have an eye contact (vision lines) with all the group members; and
- Group the learners heterogeneously (to avoid class polarisation) in terms of

gender and identified needs. The needs of learners identified by Mr A and Ms B are off-task behaviour, academic achievement problem and attitudes. Heterogeneous groups will therefore have to reflect this diversity in their composition (*see* Figure 3.6).

Figure 3.4 Interclass cooperative group formation

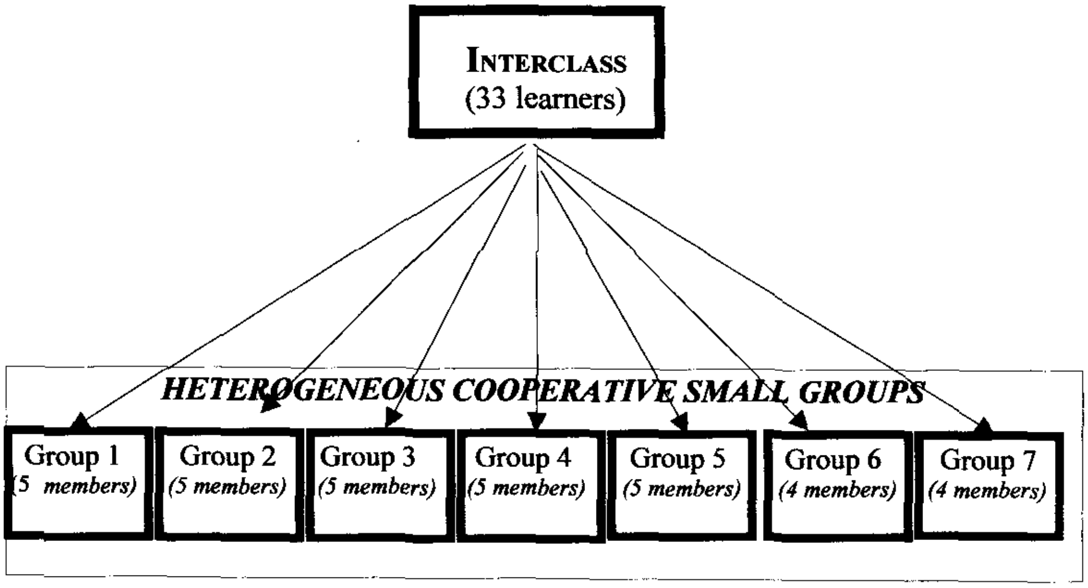


Figure 3.5 Seating pattern during cooperative group work

Figure 3.5(a) One learner per side of the table

Figure 3..5(b) Two learners per side of the table

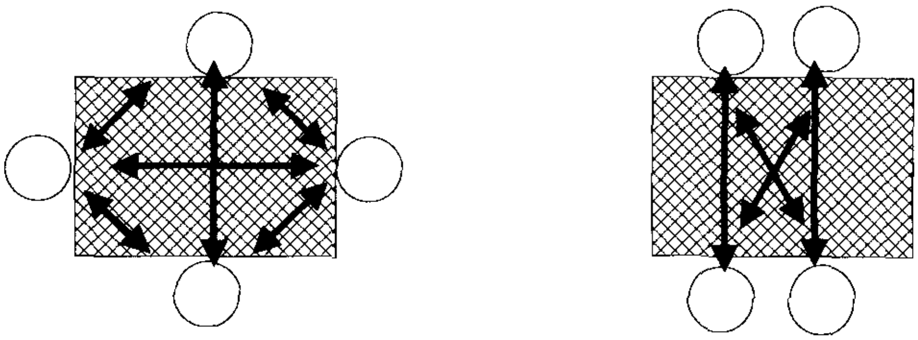
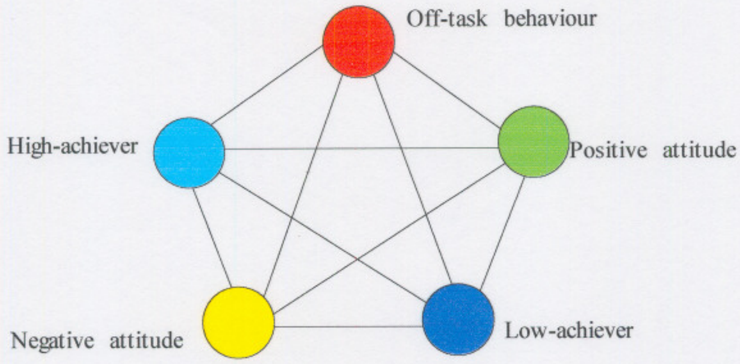


Figure 3.6 Heterogeneous grouping and lines of communication



Implications for the teachers and learners

- Teachers have to acquaint themselves with the guidelines of group formation as suggested earlier and gather as much information as possible about what cooperative learning is all about, i.e. merely placing learners in groups is not and will never be equated to cooperative learning;
- It is absolutely important for the teacher to know his learners and what their strengths and weaknesses are because this is a requisite for grouping them with relevant group members; and
- Learners should view cooperative small groups as a platform for them to help and be helped by others in doing mathematics and not a platform for perpetuating *idle chatter*.

3.3.4 Lesson facilitation in the interclass

Teachers may never understand the realities of cooperative learning if they do not put the theory of cooperation into practice for them to explore its (cooperative learning's) effects - hence the need for cooperative teaching. Blythman and Macleod

(1990:853) typify cooperative teaching as follows:

> Cooperative teaching provides the opportunity for two [or more] teachers with different backgrounds of training and experience to develop common understandings, shared meanings and the will to explore teaching and learning within the habits, customs,...and bureaucratic routines of schools and teachers in order to improve the quality of teaching and learning (of mathematics) which goes on in the classroom.

The mathematics lesson to be facilitated should have been planned by teacher A and teacher B (*see* Case scenario) respectively. Even if they may be trained and qualified to teach mathematics, Mr A and Ms B do not necessarily have the same understanding of how to approach the teaching of the same mathematics lesson. It is therefore critically important that they engage in a process of establishing a common understanding of the mathematics learning task (in which learners will engage) and shared mathematical meanings before the lesson facilitation process ensues. As they engage in cooperative lesson preparation, the two teachers will have to agree on *inter alia:*

- What learning tasks will be given to the cooperative learning groups;
- How the lesson will be planned to promote positive interdependence, i.e. through positive goal, resources, reward and/or role interdependence (*see* § 2.2.2.1);
- How assessment will be structured to promote individual accountability, i.e. through individual test-writing, random selection of one learner from a group to represent his/her group in oral presentation to explain mathematics concepts and/or through learners' self-assessment;
- How the lesson and social skills will be integrated, i.e. through assigning roles such as a group leader, scribe, 'motivator' etc. (*see* § 2.2.2.5); and
- Lastly they must clarify each other's role during the process of lesson facilitation -

for instance teacher A may engage in leading the actual lesson facilitation and teacher B may assist and guide the groups that encounter problems relating to the mathematics learning task.

The guidelines for the mathematics lesson facilitation involving two teachers are suggested in table 3.1 and illustrated in figure 3.7. Teacher A leads the lesson introduction and in the process teacher B quietly offers clarity of instructions to individual groups that need such assistance (see Figure 3.7.1). During the actual lesson presentation (where learners are actively engaged in solving mathematical tasks) both teacher A and teacher B offer assistance to different groups (see Figure 3.7.2). However, if there is only one teacher involved in teaching mathematics in the whole grade, the interclass lesson facilitation will resemble the lesson facilitation of an ordinary class during a mathematics period as informed by the school timetable or activity plan.

During mathematics lesson facilitation teacher(s) should not lose focus of engaging learners in the learning process to maximise meaningful learning. This is attained by *inter alia* employing a learner-centred approach during cooperation, using correct mathematical language and using authentic and practical mathematical examples or illustrations. The three concepts are discussed in detail in the next sections.

3.3.4.1 Learner-centred approach

Nieuwoudt (2000:21) typifies this approach as follows:

> Learners do practical activities and talk for their own **understanding,** linking their thinking aloud to action. The teacher introduces the necessary language to enable them to talk about what they are doing, and encourages them to explain in their own words.

The role of a teacher should be to guide learners as they 'battle' to comprehend mathematics they are doing in order to attain the set outcomes. Research has proven that learners do not prefer a teacher-centred instruction because lack of interaction does not promote learning (Meloth & Deering, 1999:236; Paras, 2001:68). So, if teachers adopt a transmission view of mathematics instruction (Cobb, 1988:88), subsequently imposing mathematical meaning of concepts (inherent in his/her words and actions) or methods of solving mathematical problems on learners, the learners are likely to regurgitate it without attaching any meaningful understanding to it.

3.3.4.2 Mathematics language

It is absolutely important for the teachers to inculcate correct mathematics language to learners (Setati, cited by Tailor & Vinjevold, 1999:139) in order to prevent the confusion that normally arises later in their scholastic career. This should be done without compromising the use of their own words to express their understanding. The use of correct mathematics language when defining notation and concepts enables learners to read, write and interpret mathematics symbols when they do mathematics on their own (at home for instance) (Paras, 2001:69). Misconceptions caused by the incorrect use of mathematics language have been cited as one of the hindrances towards effective learning of mathematics (Cangelosi, 1996:13).

Table 3.1. Schedule of the steps of the Hour-glass model.

Step	***Activity***	***Time/Period***
1. Assemble a mathematics inter-class.	Learners are identified according to their needs and the goal of the teacher.	***Identification***: during the lessons (mathematics periods as per time table). ***Assembly:*** after notional teaching-learning time.
2. Explication.	Explain the purpose, i.e. ***what*** and ***how*** they will learn.	After teaching-learning time, i.e. period normally reserved for either extra classes or extra-curricular activities.
3. Formation of cooperative small groups.	Formation of heterogeneous cooperative small groups of not more than five members each.	After teaching-learning time, i.e. period normally reserved for either extra classes or extra-curricular activities.
4. Lesson facilitation.	Learner-centered approach with the intention of promoting discourse and interactive learning among learners.	After teaching-learning time, i.e. period normally reserved for either extra classes or extra-curricular activities.
5. Feedback.	Learners and teacher(s) engage in group-processing exercise and reflect on the facilitated lesson.	After teaching-learning time, i.e. period normally reserved for either extra classes or extra-curricular activities.
6. Invert the Hour-glass.	Learners rejoin their classes and groups. They assist their group-mates by offering help as the teacher repeats the same lesson to the rest of the class.	The following day during the mathematics lesson as per maths period on the time table.

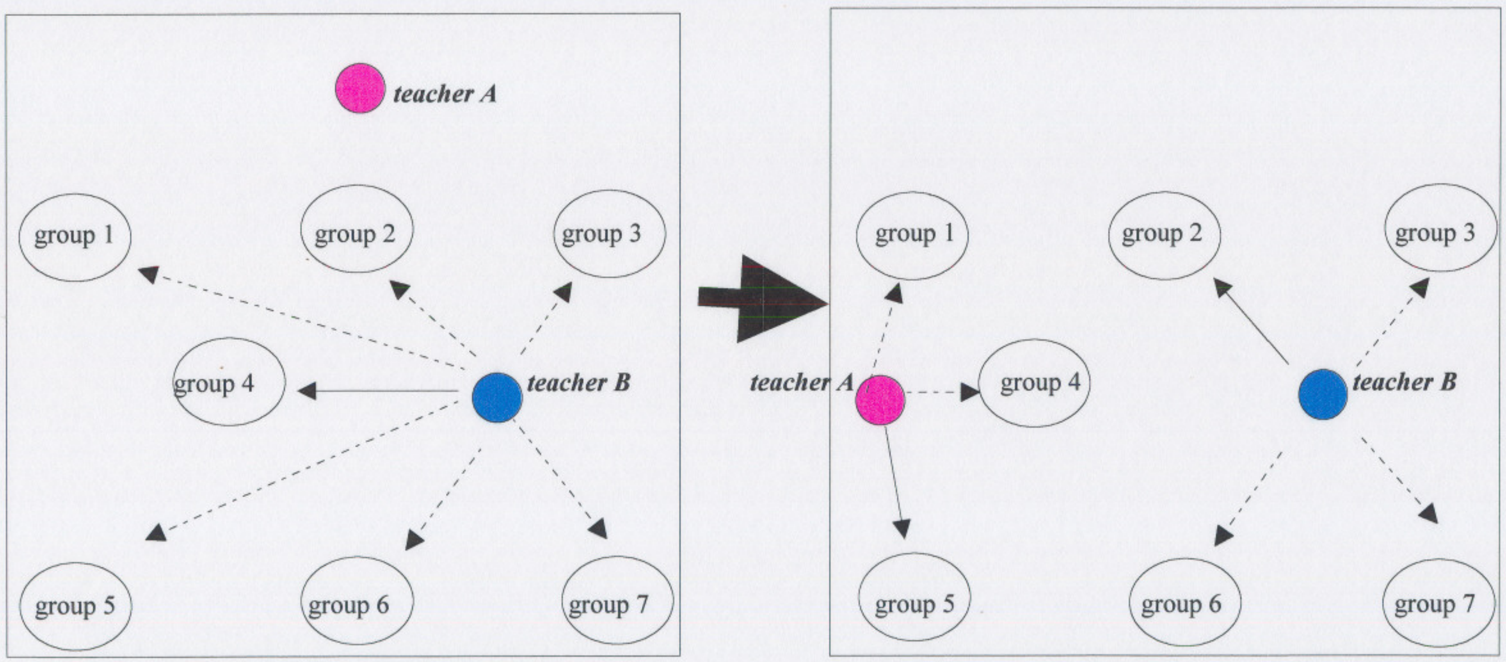
Figure 3.7 Lesson facilitation involving two teachers

3.7.1 Introduction of a lesson

(Teacher A leads facilitation while teacher B attends to groups that needs assistance)

3.7.2 Process of group cooperation

(Teachers A and B are busy assisting individual groups during cooperative group work)



3.3.4.3 Authentic and practical mathematics

Kahn and Volmink (2000:11) contend that 'school-taught' (mathematical) knowledge, if done in abstraction, is of little use". Students feel extremely alienated from the subject with its associated mystification, rigidity and irrelevance. Learners should instead be allowed to investigate mathematical problem situations in a realistic and authentic context (Nieuwoudt, 2000:32). Mathematics learnt in a classroom context has to be perceived to have a relation with what takes place outside the classroom. This connection, especially between mathematics and career opportunities, motivates students to persist in the study of mathematics (Blum-Anderson, 1992:435).

The main emphasis in step four is that learners should be exposed to the use of cooperative learning in small and manageable groups/classes group setting where all of them can freely express their feelings and promptly receive assistance from their group members and/or their teacher(s).

Implications for the teachers and learners

- Teachers should be open-minded and be prepared to learn from learners or even engage in constructive argument with learners. This is against the background that learner-centred approaches (including cooperative learning approaches) equip learners with skills such as information seeking, problem solving, analysis of information, etc. By so doing learners may, in some instances, possess more information than teachers;
- Teachers should (seek to) possess skills of developing mathematics learning programmes at micro-level. This will enable them to carefully identify and structure mathematics content and translate it into authentic and practical activities with which learners can identify. This is a laborious and sometimes a

daunting task that requires critical thinking and creativity among teachers, and willingness to consult other mathematics teachers to solicit ideas on what content should be taught and how to teach such mathematics content;

- Learners, on the other hand, have to shift **from** being passive recipients of information that is later regurgitated when the teacher so demands, **to** active information seekers who are able to analyse, interpret, comprehend, share ideas and present their view-points freely about what they have learnt in mathematics; and
- If there are two or more teachers who teach mathematics in the same grade they should frequently engage in cooperative teaching - this will give them a feeling of what mathematics learning experiences (especially social skills) learners acquire during cooperative learning.

3.3.5 Feedback

Learners are allowed to reflect on their lesson (that is the process of teaching and learning). They are allowed to express their feelings about the lesson, what new things they have learnt and discovered; what their group did right or wrong and how they intend improving their weaknesses and/or enhancing their strengths (**group processing**); what their general impressions were/are about the assistance they gave and received from one another and the facilitating role displayed by their teacher(s). In turn the teacher gives feedback about the lesson and what learners need to improve on if necessary (**whole-class processing**). The main emphasis should, however, be on the positive cooperative learning skills learners have demonstrated and positive behavioural patterns they have exhibited because step six requires them to be confident and rearing to move on. If negative experiences are emphasised (though necessary) these negative experiences may enhance negative sentiments about mathematics (Blum-Anderson, 1992:433) and about the Hour-glass model of cooperative learning - learners' problems about mathematics may be exacerbated. Learners may even be reluctant to engage in peer tutoring and/or assisting their group mates in their classes the following day due to lack of self-confidence about mathematics.

Lastly, teachers A and B must round off the lesson by reminding the learners (in the interclass) that they are going to help their class/group mates the following day. All critical points on how to carry on during cooperation (see Step 2 - Explication) are reiterated because that is how different groups will be expected to conduct themselves in their actual classes. They must further be reminded that the aim is to help other learners solve mathematics tasks through cooperative group-work and learn mathematics effectively rather than to '*spoon-feed*' them.

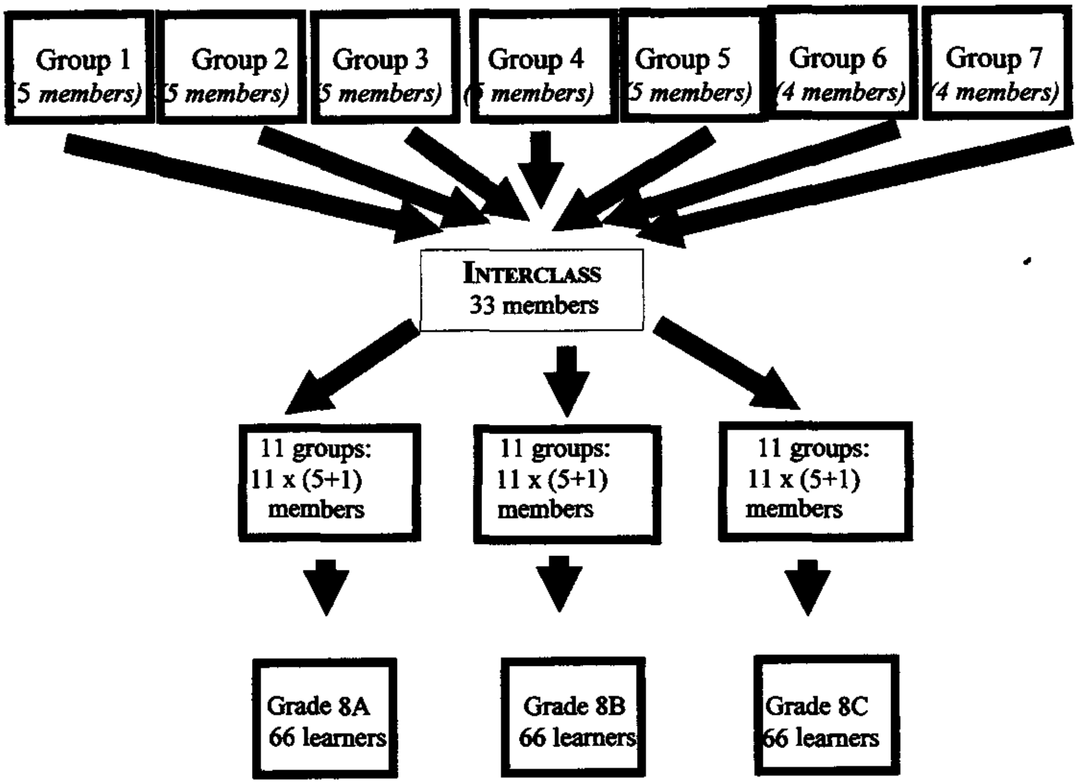
Implications for the teachers and learners

- From the feedback the teacher is able to make judgements whether or not learners are ready to help others with the same lesson in the class; and
- Learners should be (encouraged to be) truthful about their experiences, else they will find it a daunting task to help other learners in class as explained in step six.

3.3.6 Invert the Hour-glass

At this stage learners from the inter-class are well versed in the lesson to be taught in their respective classes the following day, and as a result they have to plough back. Each learner rejoins his/her class (*see* Figure 3.8) the following day to engage in peer tutoring. For instance, class 8A in the case scenario has 66 learners which translates to 11 groups of 6 members each (including learners from the interclass). Under normal circumstances 11 groups are too many to manage. The total number

Figure 3.8 Inverted Hour-glass



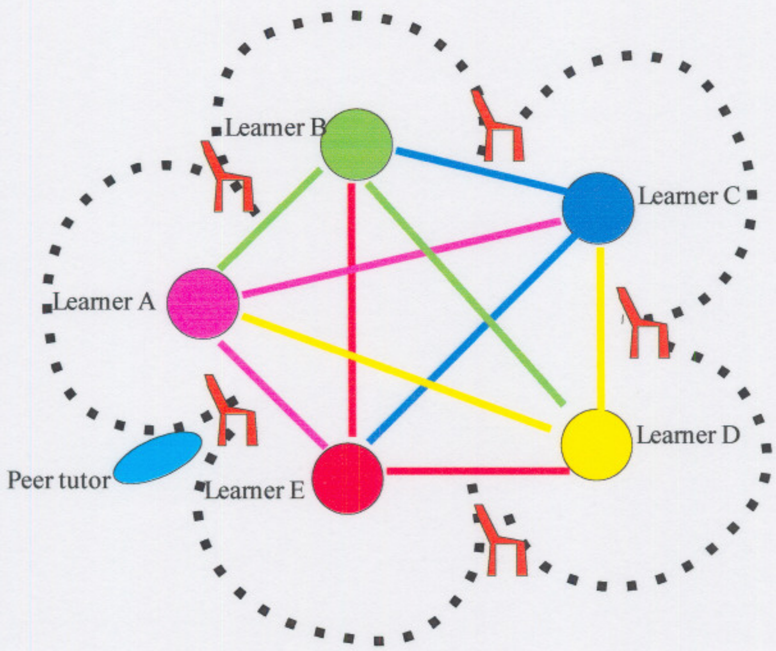
of the groups remains the same, i.e. 11, but the group size is technically reduced to 5 members because the sixth member is a peer-tutor. Learners from the interclass are "technically" not counted as "group members"; instead, they are there to assist other learners because they "already know" what will transpire during the lesson. At any given moment of lesson facilitation in class each group has an assistant. Figure 3.9 illustrates how the group of 6 members, including the assistant or peer-tutor, looks like. The symbols used denote the following:

- **turquoise oval** denotes an assistant or peer tutor;
- the **broken line** shows the peer tutor's movement during help-giving;
- the **orange chair** represents the peer-tutor's position as s/he moves around the

group during help-giving;

- the differently **coloured circles** represent heterogeneity among learners in the cooperative small group; and
- the differently **coloured solid straight lines** show the four lines of communication among the group members during cooperation.

Figure 3.9 Peer assistance during cooperative learning



The assistance offered by learners from the interclass should not be misconstrued as implying that the teacher should not help groups which are assisted by learners from interclass; instead, a teacher remains an overall monitor of the learning process. The intention is to afford learners an abundance of help (from their teacher and their peers) during mathematics lesson in class, during school day outside the classroom (by learners from interclass) when the teacher is busy with other work-related matters (Blum-Anderson, 1992:435). Research has proven beyond any doubt that most learners learn more effectively when they are being tutored by their peers than by adults. The advantage for the Hour-glass model is that it is quite probable that learners offering assistance will do so correctly because they have already received tutoring from an interclass on the same lesson. They are sure of what they explain to their peers.

Implication for the teachers and learners

- Peer tutoring can be conducted by any learner, not only by high-achievers as is the case with other cooperative learning models, but also by low-achievers and/or those learners who were identified as suffering from mathematics anxiety, have negative attitudes towards mathematics, have low self-confidence in mathematics, and so on;
- Teachers are able to use cooperative learning in mathematics teaching and learning in a large class-size without hindrances that were mentioned earlier on; and
- Learners from an interclass should be taught not to spoon-feed their peers, but to help them in their endeavour to comprehend mathematics content, else the lesson will resemble a teacher-centred approach of teaching and learning mathematics. The lesson must always be characterised by cooperation.

3.4 Conclusion

While an Hour-glass model is a brain-child and a product of a large class context, it can also be used to address the plight of learners with mathematics difficulty as explained earlier. This is done in the form of offering 'extra' classes to such learners in a 'disguised' manner. This 'disguised extra class' is evident in the distinguishing characteristic of an hour-glass model in that a new lesson is first taught to such learners before the same lesson is taught to the rest of the class rather than the other way round as it is normally done. This mitigates and/or removes the stigma associated with remedial mathematics classes where a lesson is taught to the rest of the class and later repeated to the 'mathematically incapable' ones - this has a negative affective and psychological impact as learners are labelled 'low-achievers' in mathematics. An Hour-glass model instils a feeling of self-worth among learners and they perceive themselves as 'helpers in mathematics' rather than 'mathematically incapable'.

CHAPTER

FOUR

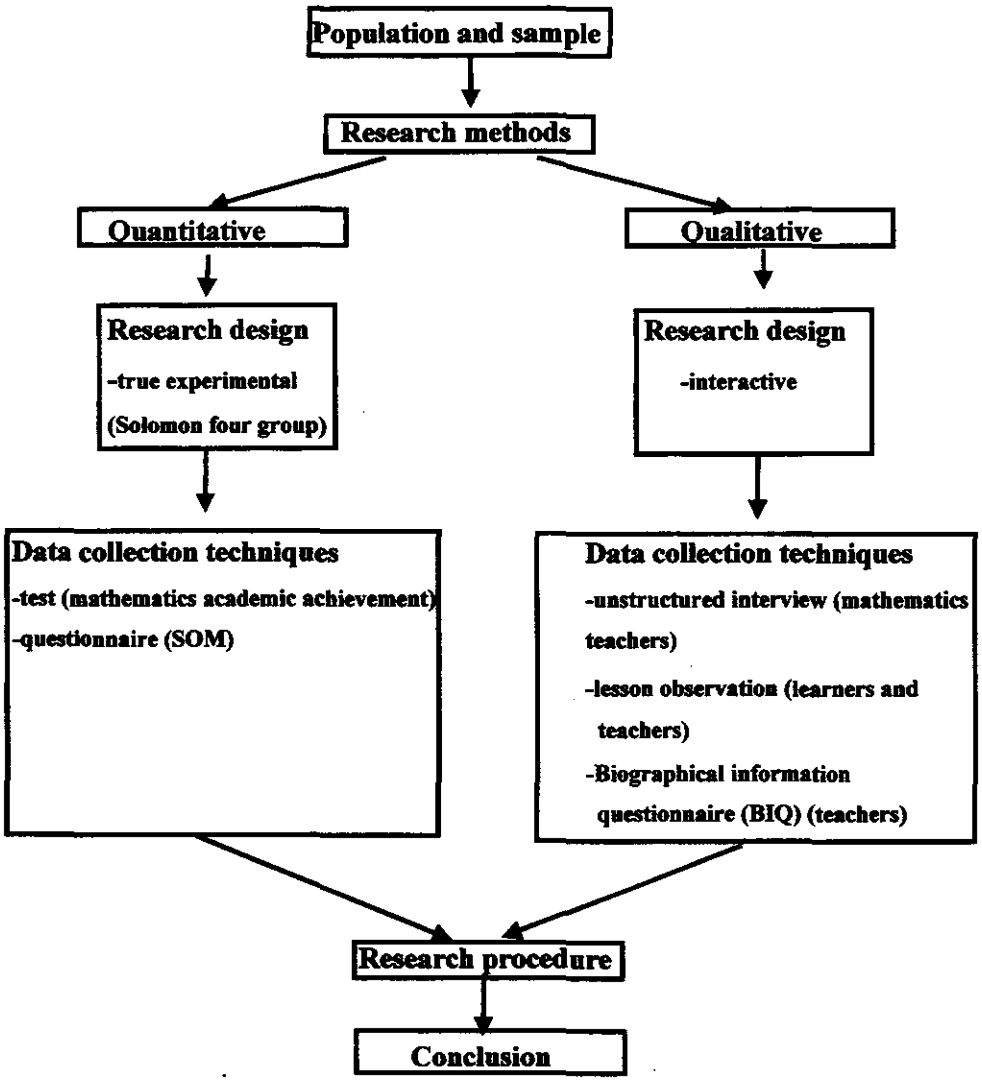
METHOD OF RESEARCH

The larger the group size, the more chance there is that some members of the group will play little or no part (Orton, 1994:40).

4.1 Introduction

This chapter endeavours to give a detailed description of the methodology employed in this project. The nature of this research is both quantitative and qualitative and multiple methods of data collection techniques (see Figure 4.1) were employed with a view to increasing the reliability of the results (De Vos, 2002:365).

Figure 4.1 Combined research method



The quantitative approach aims to enable the researcher to test the theory, adopt a detached view in the process of research and use a large representative sample (Leedy & Ormrod, 2001:102). In order to draw a cause-and-effect conclusion at the end of the research process, the experimental (research) method was employed (Sprinthall *et al,* 1991:52, Leedy & Ormrod, 2001:229). The qualitative research approach affords the researcher an opportunity to interact face-to-face with the learners and teachers (McMillan & Schumacher, 2001:395) through observation and interview respectively during the data collection process. Furthermore, this study is both operational and applied in nature (Banerjee, 1993:48) as it is carried out in the researcher's field of occupation and aims at solving emergent teaching and learning problems in mathematics education (Sprinthall *et al.,* 1991:163).

The layout of this chapter covers topics as illustrated in figure 4.1. However, the quantitative research method (with its associated sub-topics) will precede the qualitative research method (and its associated sub-topics).

4.2 Population and sample

The population in this study consists of grade 8 mathematics learners in the Mabopane and Temba districts of North West province. Schools with large mathematics class-sizes ($n \geq 50$) were targeted. Sample frames (Gall *et al.*, 1996:222; Ross & Rust, 1997:428) were obtained from the Mabopane (21 middle schools) and Temba (41 middle schools) districts (*see* Appendix A) in order to draw a sample comprising two schools from each district using a simple random sampling (Leedy & Ormrod, 2001:214). RE and RC_2 were sampled from the Temba district while RC_1 and RC_3 were sampled from the Mabopane district in order to eliminate the possibility of treatment diffusion due to close physical proximity (McMillan & Schumacher, 2001:338). All grade 8 learners of the four schools were used which gave rise to $n \geq 500$ (where n is the actual number of learners). The large sample was preferred because it is a determinant of the accuracy and precision of results (McMillan & Schumacher, 2001:180, Banerjee, 1993:81); it is more representative of the population (to enhance external validity) and it offers reliable conclusions and produces statistical power (Sprinthall *et al.*, 1991:87).

Grade 8 mathematics teachers of the four schools automatically constituted the teacher population of the research. Table 4.1 illustrates the number of teachers and their respective number of classes and learners for each school.

Table 4.1 Teacher population

School	*Teachers (n=8)*	*Classes (n=12)*	*Learners (n=641)*
RE	1	2	146
RC_1	2	2	98
RC_2	2	4	205
RC_3	3	4	204

4.3 Research methods

4.3.1 Quantitative approach

4.3.1.1 Quantitative research design

The True experimental design, namely the Solomon four-group design (see Figure 4.2) was used because of the following two primary reasons:

- a considerably large sample size (n>500) (Leedy & Ormrod, 2001:237); and

- its credited ability to control the sources of threats to internal validity such as selection, statistical regression, pre-testing and maturation (McMillan & Schumacher, 2001:337).

However, the principal value of the Solomon four-group design is in its unsurpassed ability to eliminate the pre-test influence (Leedy & Ormrod, 2001:237).

Figure 4.2 The Solomon Four-group design

Group	*Pretest*	*Independent variable*	**Posttest**
RE	Y_1	X	Y_2
RC_1	Y_1	-	Y_2
RC_2	-	X	Y_2
RC_3	-	-	Y_2

With:

- RE the randomly assigned experimental group (n = 146, two classes of 73 each);
- RC_1, RC_2, RC_3 the randomly assigned control groups (respectively $n_1 = 98$, two classes of 49; $n_2 = 205$, four classes of 51; $n_3 = 204$, four classes of 51);
- Y_1, Y_2 the dependent variable (learning and social skills, mathematics academic achievement) before and after the intervention respectively; and
- X the independent experimental variable (Hour-glass model).

The strength of the Solomon four-group design is explicitly typified by Ary *et al.* (1990:330) as follows:

> If the post-test mean of the experimental group (RE) is significantly

greater than the mean of the first control group (RC_1) and if the post-test mean of the second control group (RC_2) is significantly greater than that of RC_3 (the third control group), we have evidence for the effectiveness of the experimental treatment ... If the average differences between post-test scores, i.e. RE - RC_1 and RC_2 - RC_3 are about the same, then the experiment must have had comparable effect on post-test and unpre-tested groups".

4.3.1.2 Data collection techniques

Mathematics academic achievement test

Ary *et al.* (1990:227) and Thomas (1998:149) identify the following two types of tests: *published standardised tests* and *researcher created tests*. For the purpose of this study the latter was used in order to ensure content validity and to measure academic performance of the learners before and after the treatment (Hour-glass model). Two equivalent or alternate forms (Thomas, 1998:153; Gay, 2000:138) of mathematics academic achievement tests were compiled - one test (see Appendix B) was administered as a pretest earlier in the year when the current grade 8 learners had just graduated from grade 7 (content validity had to be ensured); the second (equivalent) test (see Appendix C) was mainly drawn from grade 8 mathematics content and was administered as a posttest later in the year. Each test comprised thirty questions (each with four multiple choice answer options) with one mark allocated to each question. The two tests were moderated by three practising mathematics teachers/educators in the senior phase of the general education and training band (GET) to ensure test reliability and content validity. Two teachers each were from the Mabopane and Temba districts and the third was from the school outside Mabopane and Temba ('external moderator').

Study Orientation in Mathematics (SOM) Questionnaire

The study orientation in mathematics (SOM) questionnaire (Maree, 1996) was used. The SOM questionnaire comprises seventy six questions covering the five fields or learning skills (see Table 4.2).

Table 4.2 Number of items per SOM fields

SOM field	***Number of items***
1. Study attitudes (SA)	14
2. Mathematics anxiety (MA)	14
3. Study habits (SH)	17
4. Problem-solving behaviour (PSB)	18
5. Study millieu (SM)	13
	76

The rationale for using the SOM questionnaire is three-fold:

- to measure (before and after the treatment) the learning skills that characterise cooperative learning strategies, namely improved problem-solving behaviour, mathematics anxiety, attitudes about mathematics, study habits and study environment;
- to ascertain whether or not the treatment (Hour-glass model) had any effect on improving these learning skills in the learning of grade 8 mathematics; and
- thirdly it can easily be applied to the large sample size (Ary *et al.*, 1990:421) and it is economical with regard to time in terms of administering and marking (Maree *et al.*, 1997:3).

The questionnaire was answered using a five-point scale (see Table 4.3) according to which the learners estimated their response ratings (in terms of duration) about the five fields or learning skills.

Table 4.3 The five-point scale of the SOM questionnaire

Rarely	Sometimes	Frequently	Generally	Almost always
R	**S**	**F**	**G**	**A**

According to Maree *et al.* (1997:26), the level of reliability of the SOM questionnaire as a whole ranges from 0,89 to 0,95 (which is presumably a reliable instrument to measure the learning skills in this investigation).

4.3.2 Qualitative approach

4.3.2.1 Qualitative research design

McMillan and Schumacher (2001:31) distinguish between interactive and non-interactive modes of enquiry in qualitative research. The former refers to the face-to-face data collection activity from the people in their natural settings by the researcher, and the latter refers to the investigation of concepts and/or events through an analysis of documents (McMillan & Schumacher, 2001:35). The researcher used the former (interactive qualitative research mode) for two reasons, namely:

- The school (classroom) is a natural setting or environment in which formal learning takes place. The researcher visited learners (and their teachers) in their classrooms to observe the process of learning (and teaching) mathematics. This is a face-to-face activity that provided the researcher with first-hand information about what transpired during mathematics lessons.

- The researcher wanted to have a face-to-face conversation with mathematics teachers in order to acquire information about the mathematics instructional process.

4.3.2.2 Data collection techniques

Interview

McMillan and Schumacher (2001:444) identify three types of interviews, namely interview guide (or semi-structured interview); informal conversation (or unstructured interview) and standardised interview (or structured interview). The researcher used the unstructured interview because it provides a greater breadth of data than the structured and the semi-structured interviews (Fontana & Fray, 2000:652). The strength of the informal conversation or the unstructured interview resides in the opportunities to individual differences and situational changes (Patton, 2002:343).

The main purpose of conducting the interview was to acquire information about:

- why teachers prefer to use (or not to use) cooperative learning in mathematics, i.e. the general advantages and disadvantages of cooperative learning methods in mathematics;
- the effectiveness (or not) of the Hour-glass model of cooperative learning (only teachers in RE and RC_2); and
- various phenomena that the researcher observed in class during the mathematics lesson that needed more clarity (Berg, 2001:70).

Lesson observation

Cohen, Manion and Morrison (2000:306) distinguish between structured and unstructured observation (see Table 4.4). This study adopted unstructured observation primarily because it "operates within the agenda of the participants, i.e. it is responsive to what it finds and therefore, by definition, is honest to the situation which it finds" (Cohen, Manion & Morrison, 2000:306).

Table 4.4 Comparison between structured and unstructured observation

Structured observation	***Unstructured observation***
• Takes much time to prepare	• Quicker to prepare
• Data analysis is rapid	• Data analysis takes longer time
• Operates within the agenda of the researcher	• Operates within the agenda of the participants
• Key issues emerge from the researcher	• Key issues emerge from and follow from the observation

Gathering research data by observation in the classroom context involves watching and/or listening to the teaching-learning events (Thomas, 1998:136). The researcher engaged in a field observation where direct observation was conducted without interaction (McMillan & Schumacher, 2001:437). This means that the researcher observed the actual mathematics lessons but did not make comments or inputs even though learners and teachers were conscious of the researcher's presence. The researcher intended to observe the following:

- ***physical setting*** (position of table for the teacher; whether desks or tables and chairs were used for learners; arrangement of tables/desks for learners i.e. groups versus rows and columns);

- ***learner seating*** (organisation of learners; nature of groups - heterogeneous versus homogeneous in terms of gender; nature of pairs in columns/rows i.e. homogeneous or heterogeneous pairs in terms of gender; number of learners in the class - crowded or not);
- ***interactional setting*** (formal versus informal interactions; communication or conversation skills); and
- ***program setting*** (use of resources; teaching-learning styles or methods; correct application of the Hour-glass model by RE and RC_2 teachers).

Biographical information questionnaire (BIQ)

The BIQ (see Appendix D) did not form an integral part of the data needed to test a particular hypothesis; however it was used to control factors relating to the teacher profile that might have had an influence in learner performance. The principal purpose of the BIQ was to ascertain whether or not the grade 8 mathematics teachers of the schools that participated in the research were significantly homogeneous in terms of their teaching experience; qualifications; methods they predominantly use in the teaching of mathematics; knowledge of the leaning skills and their impressions of cooperative teaching-learning methods. This is against the background that factors such as the ones mentioned above may influence learners' academic achievement in mathematics if they are not controlled. If the teachers' biographical backgrounds are significantly the same the research results of learners will be attributed to the treatment with a significant degree of confidence. However, if their biographical backgrounds are significantly different the learners research results may be adversely affected and the results may be misleading.

The BIQ was completed by the teachers from RE and RC_2 schools before they received training about the use of the Hour-glass model of cooperative learning (receiving training about cooperative learning before completing BIQ would influence the answers they would give for questions 2.2; 2.3 and 5, (see Appendix D). In contrast, RC_1 and RC_3 teachers completed the BIQ towards the end of the fourth month of the treatment (if administered earlier some questions could sensitise them about the approaches to mathematics teaching, especially cooperative learning (see question 5 of Appendix D). Teachers were requested not to use the dictionary (to refer to the meaning of concepts listed under 'learning skills') when answering questions 3 and 4 because the two questions required teachers' current knowledge and/or understanding of those concepts.

4.4 Research procedure

4.4.1 Permission from the department of education

Letters requesting permission to use the aforementioned study population were sent to the Mabopane and Temba district managers (*see* Appendix E) and principals of the four schools (research sites) used in this study as advocated by Maruyama and Deno (1992:10). Meetings were held with the teachers and/or principals concerned in order to explain the research aims, roles of the teachers as well as to solicit support and commitment from them (Maruyama & Deno, 1992:18). As far as possible, meetings held with the teachers and the answering of tests and questionnaires by learners were conducted in such a manner as to minimise disruption of classes.

4.4.2 Training of teachers

A period of four days (towards the end of February) was reserved to train teachers in RE and RC_2 about the application of the Hour-glass model of cooperative learning. Teachers were trained on aspects that include critical elements of cooperative learning; ways to identify learners and assemble the interclass; formation of cooperative small groups in the interclass; lesson facilitation during cooperative group work and peer-tutoring. The training scheduled is presented in table 4.5.

The teachers were also trained on how to assess learners, i.e. giving mathematics

Table 4.5 Schedule of training program of teachers (RE and RC_2)

Topic	*Duration*
1.1. Overview of cooperative learning (Hour-glass model) 1.2. Critical elements of cooperative learning	40 minutes
2.1. Identification and assembly process of the interclass 2.2. Cooperative group formation	30 minutes
3.1. Cooperative teaching and lesson facilitation 3.2. Feedback (group processing) and assessment	40 minutes
4. Peer-tutoring (help-giving by learners from the interclass)	30 minutes

exercises to be written in groups (one answer sheet per group) to enhance positive interdependence or giving mathematics exercises to be written by individual group members to enhance individual accountability.

4.4.3 Administration of the tests

The pre-tests (SOM and mathematics academic achievement tests) were administered during March. Learners were assured of the confidentiality of the results (especially the SOM questionnaire). If learners' test results are disclosed to the teacher, s/he (the teacher) may have expectations that can influence her/his future behaviour towards the learners (Gall *et al.*, 1996:91). Further, learners did not use their real names (to ensure confidentiality); instead, identification numbers were allocated to them (McMilan & Schumacher, 2001:198; Sammons, 1989:45). The same identification numbers were used during the administration of the pre-test and post-test. The identification numbers were designed in such a manner that they differentiated between the schools, between classes within the same school, and between learners within the classes. For instance, schools RE, RC_1, RC_2, and RC_3 were allocated letters A, B, C, and D respectively. If the learner's identification number is BA01 (see Appendix F), it refers to the learner in school RC_1 (B in BA01); grade 8A (A in BA01) and learner number one (01 in BA01). The identification numbers of grade 8B learners of the same school would therefore start with BB.

There were no right or wrong answers for the SOM questionnaire and learners were encouraged to give honest answers. A blank sheet was provided to learners for doing calculations when answering the academic achievement test. The assistance of the teachers during the administration of the tests was sought and the answer sheets were immediately collected by the researcher after being written. Each school wrote the two tests on two successive days after the formal teaching-learning time (see Table 4.6)

The posttests were administered to the four schools as explained earlier. The academic achievement pretest and posttest were marked by the researcher and the marks were moderated by one practising teacher from each district. The marks (scores) for academic achievement test were converted into percentages and recorded in the mark sheet (see Appendix G). Data capturing of the SOM questionnaire was also done by the researcher. The scores for both tests were submitted to Statistical Consultation Services of PU for CHE for analysis.

Table 4.6 Schedule of test administration

School	*Day*	*Pre/Post test*
RE	1	SOM
	2	Mathematics academic achievement test
RC_1	1	SOM
	2	Mathematics academic achievement test
RC_2	1	SOM
	2	Mathematics academic achievement test
RC_3	1	SOM
	2	Mathematics academic achievement test

4.4.4 Visit to schools

4.4.4.1 Lesson observation

The RE and RC_2 schools were visited for three consecutive days per month for the observation of mathematics lessons (30 minutes per teaching-learning period). The correct application of the Hour-glass model of cooperative learning was also monitored during the lesson observation process. The RC_1 and RC_3 schools were also visited during March and May for three consecutive days to observe the teaching and learning methods in mathematics.

Table 4.7 illustrates the months and purposes of the lesson observation for each school. The main aim of observing the learners and teachers (especially RE and RC_2) on repeated occasions was to evaluate the changes in their (teachers' and learners') behaviour in terms of teaching and learning to obtain an indication in success of the use of the Hour-glass model (Strydom, 2002:285). The observed classroom environment of each school, applied teaching-learning methods, and the change in teaching-learning behaviours were recorded in the field note book (Strydom, 2002:285).

4.4.4.2 Unstructured interview

The unstructured interviews were held with the grade 8 mathematics teachers whose learners participated in the research. Responses to the interviews were recorded manually in the field note book shortly after the interview for two reasons (Gay, 2000:293): firstly to avoid distracting the interviewer's concentration if notes were to be taken during the interview; and secondly to avoid making the interviewee nervous by writing down his/her words (when the interview is in process). However, the teachers in schools RE and RC_2 were interviewed for reasons not very similar to those of RC_1 and RC_3. The former were interviewed to:

- collect information about the positive influences (or advantages) and negative influences (or disadvantages) of the Hour-glass model;
- gather inputs (if any) towards the improvement of the Hour-glass model for future use in mathematics teaching and learning; and
- gain additional information or clarity about various classroom activities that were observed during the lesson observation process (Berg, 2001:70).

RC_1 and RC_3 teachers were interviewed predominantly to:

- gather information about their preferences regarding the methods of teaching and learning mathematics and the rationale for their preferences; and
- gain additional information or clarity about various classroom activities and/or phenomena that were observed during the lesson observation process.

Table 4.7 Schedule of lesson observation

School	***Visit 1***	***Purpose***	***Visit 2***	***Purpose***	***Visit 3***	***Purpose***	***Visit 4***	***Purpose***
RE	Three consecutive days in March	Observation of: • teaching-learning methods prior the treatment and pretest; • classroom environment such as seating pattern.	Three consecutive days in April	Day 1: observing the mathematics lesson in the interclass; Days 2 & 3: • observing the application of the Hour-glass model during mathematics lesson in the actual crowded classroom; • noting the observable changes in teaching-learning behaviours	Three consecutive days in May	Day 1: observing the mathematics lesson in the interclass; Days 2 & 3: • observing the application of the Hour-glass model during mathematics lesson in the actual crowded classroom; • noting the observable changes in teaching-learning behaviours	Three consecutive days in July	Day 1: observing the mathematics lesson in the interclass; Days 2 & 3: • observing the application of the Hour-glass model during mathematics lesson in the actual crowded classroom; • noting the observable changes in teaching-learning behaviours
RC_1	Three consecutive days in March	Observation of: • teaching-learning methods prior the pretest; • classroom environment such as seating pattern.			Three consecutive days in May	• Observing the teaching-learning methods; • Noting observable changes (if any) in teaching-learning behaviours that compare and/or contrast those of RE		
RC_2	Three consecutive days in March	Observation of: • teaching-learning methods prior the treatment; • classroom environment such as seating pattern.		Day 1: observing the mathematics lesson in the interclass; Days 2 & 3: • observing the application of the Hour-glass model during mathematics lesson in the actual crowded classroom; • noting the observable changes in teaching-learning behaviours	Three consecutive days in May	Day 1: observing the mathematics lesson in the interclass; Days 2 & 3: • observing the application of the Hour-glass model during mathematics lesson in the actual crowded classroom; • noting the observable changes in teaching-learning behaviours	Three consecutive days in July	Day 1: observing the mathematics lesson in the interclass; Days 2 & 3: • observing the application of the Hour-glass model during mathematics lesson in the actual crowded classroom; • noting the observable changes in teaching-learning behaviours
RC_3	Three consecutive days in March	Observation of: • teaching-learning methods; • classroom environment such as seating pattern.			Three consecutive days in May	• Observing the teaching-learning methods; • Noting observable changes (if any) in teaching-learning behaviours that compare and/or contrast those of RC2		

The interviews were conducted after each lesson was observed (see Table 4.6) during any "free" time identified by the teacher(s). The duration of the interview differed from teacher to teacher because it was dependent upon:

- the instant and prompt dissemination of information by the interviewee required by the interviewer. When the interviewee did not respond to the question in a "direct-to-the-point" manner, the interview was prolonged.
- the minimal use of probing questions - when probing questions were used less frequently to extract information from the interviewee, the duration of the interview tended to become shorter.

However, the duration of the interviews ranged between twenty and forty minutes per interview. The field notes were taken during the interview.

4.5 Hypotheses tested in the research

The following research hypotheses were tested:

- H_{01} -The application of the Hour-glass model influences the mathematics academic achievement of grade 8 learners in crowded classes.
- H_{02} -The application of the Hour-glass model influences grade 8 mathematics learners' social skills.
- H_{03} -The application of the Hour-glass model influences the learning skills of grade 8 mathematics learners.

4.6 Methods of data analysis

4.6.1 Quantitative data analysis

The assistance of the Statistical Consultation Services of PU for CHE was sought in the analysis of quantitative data. Descriptive statistical techniques (means and standard deviations) were used to describe changes within the groups. The *t*-Test was used to compare the pre-test means of RE and RC_1 with regard to mathematics academic achievement test and SOM questionnaire. Anova was used to compare the post-test means of RE, RC_1, RC_2 and RC_3 with regard to the mathematics academic achievement test and SOM questionnaire. The paired *t*-Test was also used to compare the mean difference within the RE group and within the RC_1 group (i.e. groups that took the pre-test and the post-test).

4.6.2 Qualitative data analysis

McMillan and Schumacher (2001:487) typified the analysis of qualitative data as follows:

"The hallmark of most qualitative research is the narrative presentation of data ... Data are presented as quotations of participants' language, ...".

The data obtained through interviews and observations were analysed in a narrative manner. In the case of interviews the actual words of the interviewees were quoted as recorded in the interviewer's field notes and inductively interpreted in a narrative manner. With regard to observations the researcher interpreted what was observed and how such observations impacted on the teaching and learning of mathematics.

4.7 Conclusion

The use of a dual research approach, i.e. quantitative and qualitative research approaches afforded the researcher the opportunity to critically discuss and identify attributes about the statistical data. For instance, the numerical value of the test result may be well interpreted by attributing it to the observed (behavioural) changes of learners and/or teachers. Further, the three research hypotheses may not all be tested effectively or accurately by using either the quantitative research approach or the qualitative research approach only. H_{01} and H_{03} may best be tested by using the quantitative approach while H_{02} may be accurately tested by the qualitative research methods. The next chapter presents the research findings of the data gathered quantitatively and qualitatively.

CHAPTER

FIVE

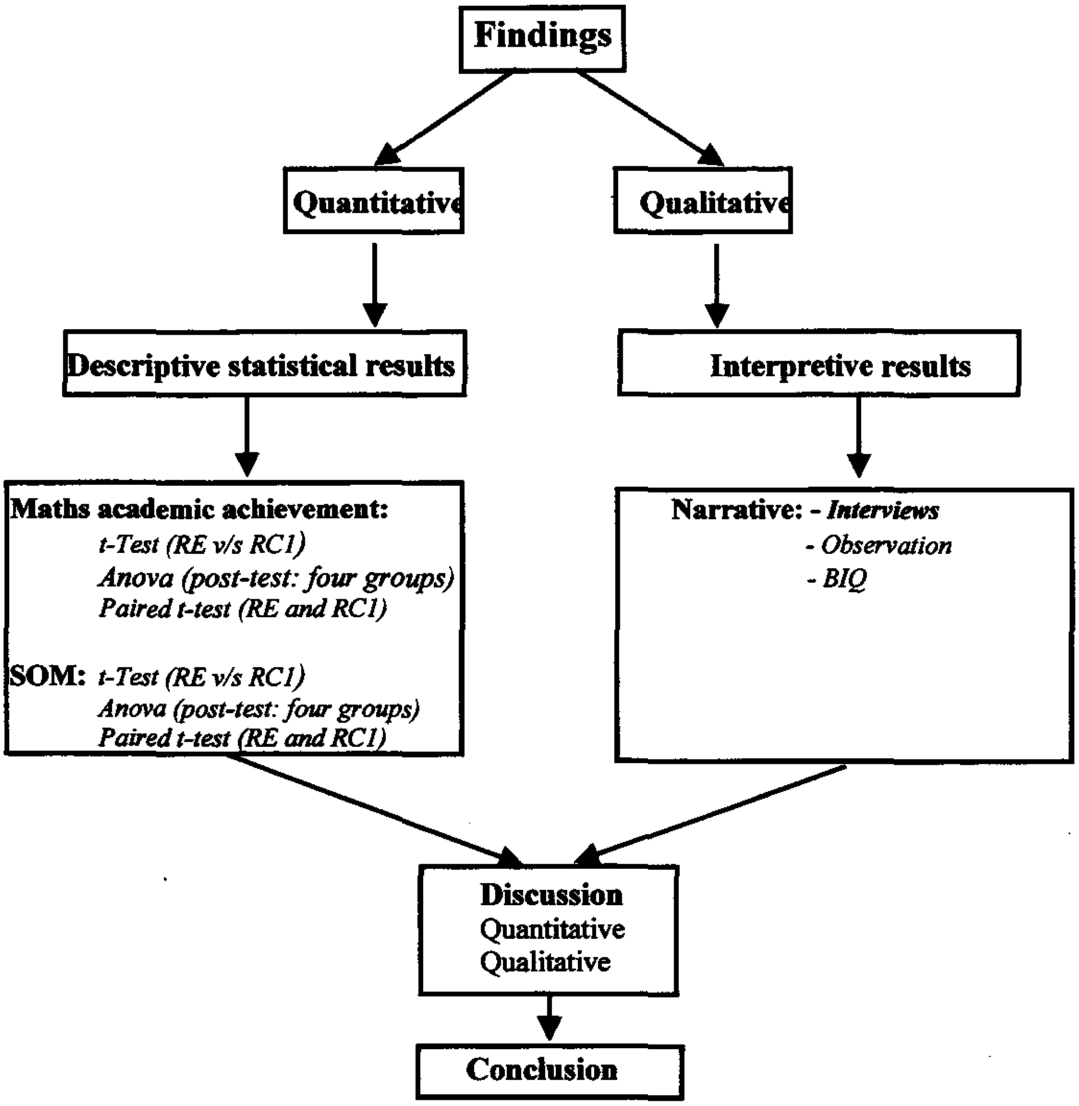
RESEARCH FINDINGS AND DISCUSSIONS

Classrooms with group arrangements of seats, as compared to those in rows, were characterised by being more innovative and having more teacher support for students (Lambert 1995:197).

5.1 Introduction

The purpose of this chapter is to present the research findings and the discussion thereof as laid out in figure 5.1.

Figure 5.1 Layout of chapter 5



The chapter is essentially divided into three main sections, namely:

- The actual ***descriptive statistical results*** with regard to the quantitative research approach. This section will give the analysis of the data gathered through the Study Orientation questionnaire in mathematics and the mathematics academic achievement test. The *t*-test was used to compare the means of the two pre-tested groups. The analysis of variance (Anova) was used to compare the means of post-tested groups because four groups (and not two) wrote the post-test. Tables and graphs will be used to present the scores of the statistical findings obtained from all the research groups.

- The ***interpretive results*** with regard to the qualitative research approach. This section will give the analysis of data collected through the unstructured interviews; the class observations and brief results of the biographical information questionnaire.

- The ***discussion section*** which mainly addresses the implication of the results on mathematics teaching and learning, and whether or not to accept the three research hypotheses. While the discussion of the research results will be classified as quantitative and qualitative, the integrated discussion will be adopted because of the nature of the research method, namely, the combined quantitative-qualitative research approach.

5.2 Descriptive statistical results

5.2.1 Mathematics academic achievement test

5.2.1.1 *t*-Test between the groups: Pre-mathematics achievement test

A *t*-test was used to determine whether the means of RE and RC_1 differed significantly with regard to the mathematics academic achievement. It was desired that the two means should not differ practically significant in order to accurately measure the influence of the treatment on RE. The Cohen effect sizes (d), Cohen's category (Cohen, 1988:222), t-value and p-value were used as an indication of practical or meaningful difference. A synopsis of the *t*-test results is provided in table 5.1.

Table 5.1 *t*-Test: pre-maths academic achievement test

Group	*N*	*variable*	$\bar{x}$	*SD*	*t*	$P > \|t\|$	*Effect size (d)*	*Cohen's category*
RE	148	Pre-math test	29.52	9.55	–3.52	0.0005*	0.44	medium
RC_1	98	Pre-math test	34.03	10.23				

* significant at 1% level

The t-test results revealed a statistically significant difference between the groups ($p<0.01$), the effect size was medium ($d=0.44$) which is not practically significant.

5.2.1.2 Anova between the groups: Post-mathematics achievement test

Analysis of variance (Anova) was used to compare the means of all four schools (i.e. those that received the treatment, namely RE and RC_2 and those that did not receive the treatment, namely RC_1 and RC_3) for the pre-test. Table 5.2 shows the d-value comparison of the four schools. Anova revealed a comparison of no statistical significance between RE and RC_2 (d=0.38), and between RC_1 and RC_3 (d=0.27). However, RE and RC_1 (d=1.38), and RE and RC_3 (d=1.64) yielded a comparison of practical significance (Cohen's effect size = large). Similarly when RC_2 and RC_1, and RC_2 and RC_3 were compared they yielded a difference of practical significance (d=1.15 and d=1.41 respectively) with large Cohen's effect sizes.

Table 5.2 Anova: Effect sizes (d-values)

	RE	***RC_1***	***RC_2***	***RC_3***
RE	-	1.38***	0.23*	1.64***
RC_1	1.38***	-	1.15***	0.27*
RC_2	0.23*	1.15***	-	1.41***
RC_3	1.64***	0.27*	1.41***	-

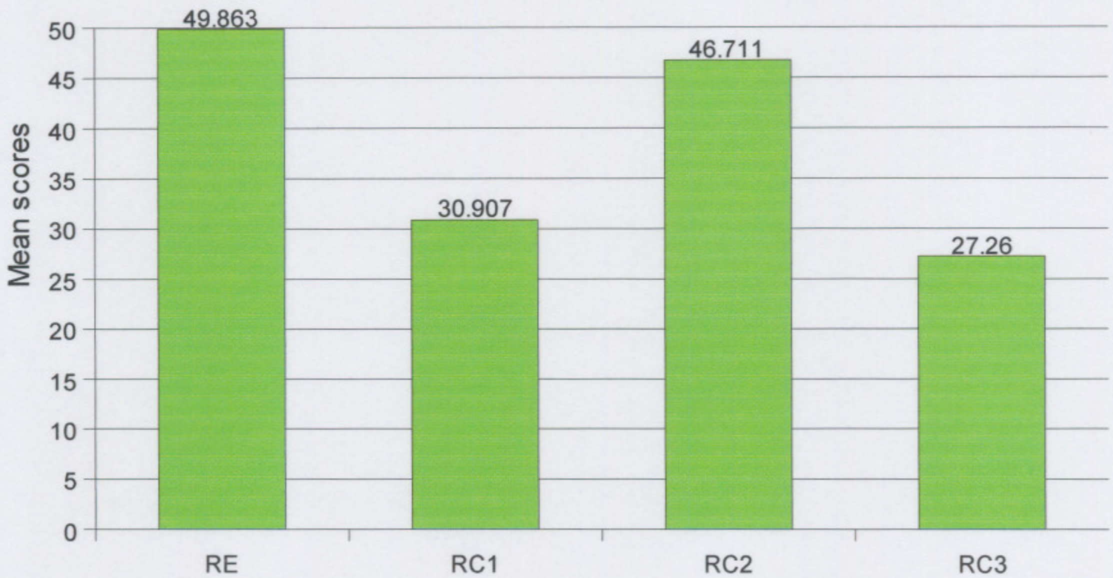
*d < 0.3 = small

**d ≥ 0.3=medium

***d ≥ 0.8=large (practically significant)

This, therefore, suggests that the means of the groups that received the treatment differ practically significant relative to the means of the groups that did not receive the treatment (see Figure 5.2). However, the groups that received the treatment (RE and RC_2) did not differ statistically significant according to their means (which implies that they were relatively similar). The same pattern applies to the means of the groups that did not receive the treatment namely, RC_1 and RC_3.

Figure 5.2 Group means: Post-Maths academic achievement test



5.2.1.3 Paired *t*-test: mathematics academic achievement test (RE and RC_1)

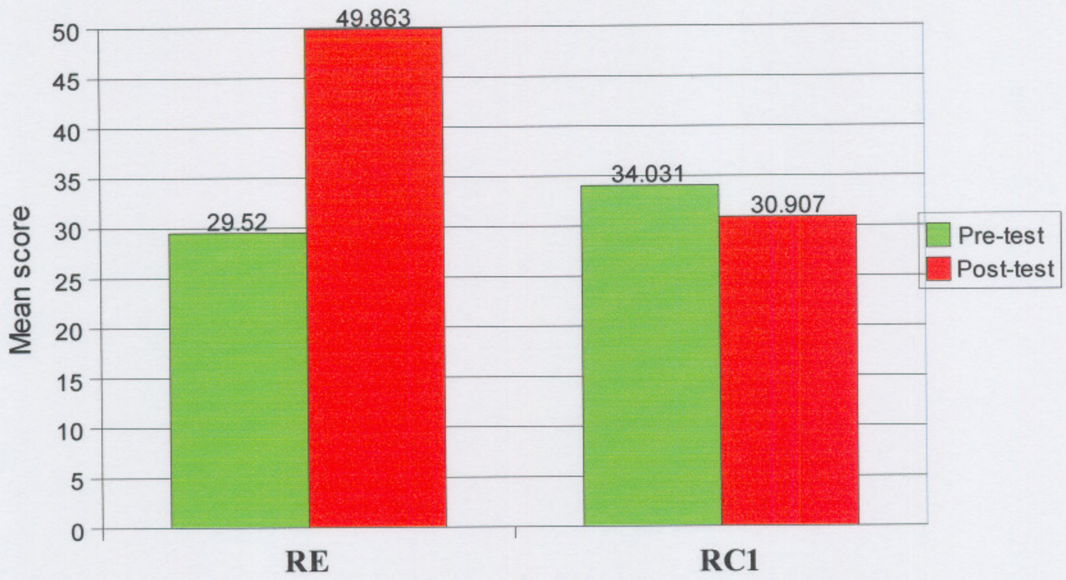
A paired t-test was conducted to establish the mean difference between the posttest and pretest within the groups that received the treatment. The mean difference within the experimental group (RE) was practically significant (d = 1.15 with large effect size) and for the control group (RC_1) the mean difference was statistically significant (d = 0.33 with small effect size) (*see* Table 5.3).

Table 5.3 Paired *t*-test academic achievement (RE versus RC_1)

Group	*N*	*Mean difference*	*SE*	*d*	*Cohen's effect size*
RE	146	20.35	1.46	1.15	large
RC_1	86	-4.41	1.43	0.33	small

The respective means for RE and RC_1 were 29.52 and 34.03 for the pretest and 49.86 and 30.91 for the posttest (*see* Figure 5.3).

Figure 5.3 Mean scores: Maths academic achievement test (RE & RC_1)



5.2.2 Study Orientation in Mathematics questionnaire.

5.2.2.1 Reliability and validity

Gay (2000:174), and McMillan and Schummacher (2001:246) contend that the Cronbach alpha is used to estimate the reliability of the test items of the test that has no right or wrong answers or scores, and whose test items have more than two scores. As explained earlier in Chapter 4, the items of the SOM questionnaire have no right or wrong answers. Instead, learners were required to choose the option that suits them from the five given scales. For these reasons Cronbach alpha was employed to estimate the reliability of the SOM questionnaire. According to Maree *et al.* (1997:26), the estimated reliability level of the SOM questionnaire ranges from 0.63 to 0.77 for African language speakers. However, this study revealed that the Cronbach alpha reliability coefficient of the different fields (see Table 5.4) of the SOM questionnaire ranges from 0.599 to 0.684 for the pre-test and 0.611 to 0.720 for the post-test (see Table 5.4).

Table 5.4 Level of reliability of SOM fields (Cronbach alpha)

SOM field	*Pre-test (alpha value)*	*Post-test (alpha value)*
1. Study attitude (SA)	0.599	0.667
2. Mathematics anxiety (MA)	0.684	0.660
3. Study habits (SH)	0.614	0.699
4. Problem solving behavior (PSB)	0.684	0.720
5. Study milieu (SM)	0.633	0.611

Maree *et al.* (1997:27) accentuate the fact that the content validity of a test is based on the logical analysis of the content and objectives of the measuring instruments and it is not expressed in terms of quantitative indices. The following steps were therefore taken to ensure the content validity (Maree et al., 1997:27):

- extensive literature study was undertaken;
- phrasing and placement of the items in fields were checked by various experts;
- item fields were checked; and
- it was ensured that the most important facets of different fields were accounted for.

In terms of the construct validity the SOM questionnaire aims at measuring the study attitudes, mathematics anxiety, study habits, problem-solving behaviour, and the study milieu of learners when learning mathematics (Maree *et al.,* 1997:7). Each of these phenomena was clearly defined (Maree *et al.,* 1997:7). Similarly this study aimed at measuring the extent to which the Hour-glass model could enhance the aforementioned constructs among learners of mathematics.

5.2.2.2 *t*-Test between the groups: Pre-SOM questionnaire

The *t*-test was used to establish the difference between the means. It revealed that RE and RC_1 did not show any practically significant difference between the means for all the fields of the SOM questionnaire when it was administered as a pre-test. There was a statistical difference in the means of study attitude ($p < 0.10$, but with small effect size, $d = 0.253$) and also for problem-solving behaviour ($p < 0.05$, with medium effect size, $d = 0.307$). The differences, however, were not practically significant, hence in general the means of RE and RC_1 were not different (*see* Table 5.5). This suggests that the RE and RC_1 groups did not differ according to their pre-SOM test. The means of all the SOM fields for RE and RC_1 were further translated into percentile ranks (Maree *et al.,* 1997:14) to establish their level of study orientation in mathematics.

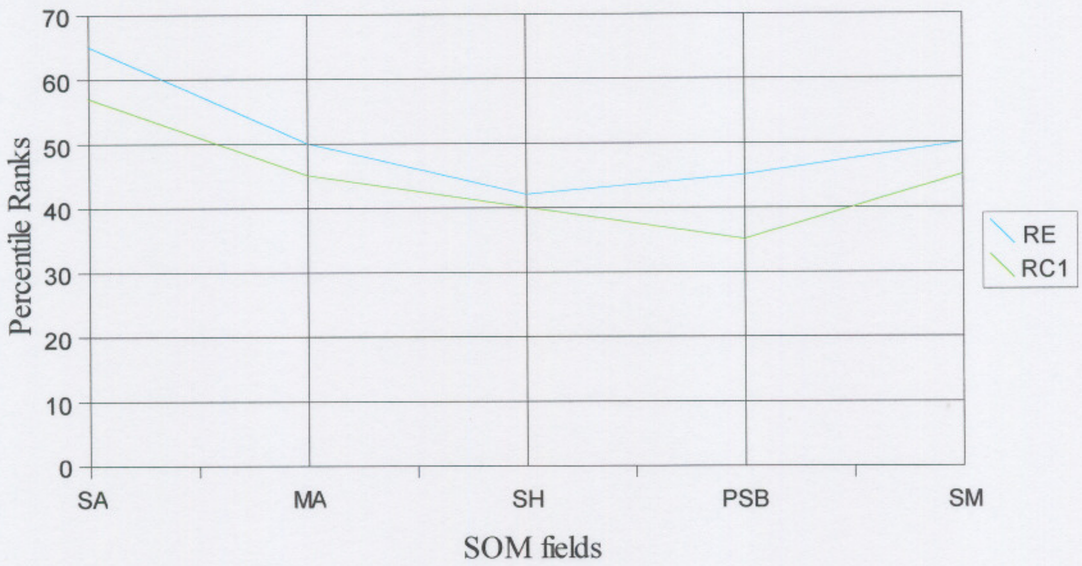
Table 5.5 *t*-Test results for pre-SOM

Variable	$\bar{x}_{RE}(n=145)$	*SD*	$\bar{x}_{RC1}(n=98)$	*SD*	*P*	*d*	*Cohen's category*
1. SA	38.924	8.227	36.847	8.210	0.054***	0.253	small
2. MA	35.683	8.780	35.398	9.052	0.807	0.032	small
3. SH	37.103	8.059	36.265	8.425	0.436	0.099	small
4. PSB	36.372	9.015	33.367	9.788	0.015**	0.307	medium
5. SM	33.586	7.577	32.735	7.127	0.380	0.119	small

*significant at 1% level
**significant at 5% level
***significant at 10% level

According to figure 5.4 learners in RE and RC_1 have a neutral (between 40% and 69%) study orientation in mathematics which suggests that the learners' mathematical achievement can either be according to their abilities or below their abilities at school (Maree *et al.,* 1997:15).

Figure 5.4 Level of study orientation in mathematics for RE and RC_1 (Pre-test)



5.2.2.3 Anova between the groups for post-SOM questionnaire

Figure 5.5 presents anova between the groups, namely RE, RC_1, RC_2 and RC_3 for post-SOM test. The raw mean scores were converted to percentile ranks (Maree, 1997:14) and showed that the level of study orientation of the subjects is still neutral. There was a statistically significant difference between RC_1 and RC_3 ($p = 0.044$; $d = 0.36$ with medium effect size) and between RC_2 and RC_3 ($p = 0.0121$; $d = 0.32$ with medium effect size) for study milieu. These differences were, however, of no practical significance.

Table 5.6 Paired *t* -test for RC_1

Variable	*N*	*Mean*	*SE*	*p*	*d*	*Cohen's category*
SA	78	0.603	0.820	0.465	0.083	small
MA	78	1.244	0.895	0.169	0.157	small
SH	78	-0.295	0.958	0.759	0.035	small
PSB	78	0.859	0.982	0.385	0.099	small
SM	78	2.000	0.730	0.008	0.310	medium

Table 5.7 Paired *t*-test for RE

Variable	*N*	*Mean*	*SE*	*p*	*d*	*Cohen's category*
SA	146	-1.616	0.754	0.034	1.978	small
MA	146	0.788	0.808	0.331	0.810	small
SH	146	-1.363	0.785	0.085	0.144	small
PSB	146	-2.692	0.868	0.002	0.257	small
SM	146	0.459	0.598	0.441	0.064	small

The comparison between the paired *t*-test results for the difference between means of RC_1 and RE for the SOM pre and post-test revealed no practical significant influence of the Hour-glass model on the five learning skills.

5.2.2.5 Inter-correlation of fields between groups (Pre and Post-SOM)

The Pearson correlation coefficient (r) was used to establish the degree of correlation of fields in RE and RC_1 during the pre-test. According to McMillan and Schumacher (2001:230); Ary *et al*. (1996:146), and Gall *et al*. (1996:412), the respective implications of $r = -1$; $r = 0$ and $r = 1$ are that the relationship is strongly negative; there is no relationship; and the relationship is strongly positive. In a general sense when $r \geq 0.5$, the relationship is strongly positive and when $r < 0.5$ the relationship is moderately positive. Similarly, when $r \leq -0.5$ the relationship is strongly negative and when $r > -0.5$ the relationship is moderately negative.

The statistical data in table 5.8 and table 5.9 have a dual purpose: firstly to compare the inter-correlation of pre-SOM and post-SOM fields for the following groups - within RE for pre-SOM; within RC_1 for pre-SOM; and between RE versus RC_1 for pre-SOM. Secondly, it compares the inter-correlation of post-SOM fields between the four groups.

From table 5.8, RE and RC_1 revealed stronger correlation or relationships between SA and SH; SA and PSB; MA and SM; and SH and PSB for the pre-SOM test. The same correlation pattern was revealed by the four groups, namely RE, RC_1, RC_2 and RC_3 for the post-SOM test (see Table 2.9).

Table 5.8 Inter-correlation between the SOM fields for pre-SOM (RE & RC_1)

	SA	*MA*	*SH*	*PSB*	*SM*
SA	–				
MA	0.17 (RE) 0.31 (RC_1)	–			
SH	0.59 (RE) 0.66 (RC_1)	0.17 (RE) 0.11 (RC_1)	–		
PSB	0.53 (RE) 0.48 (RC_1)	0.04 (RE) -0.03 (RC_1)	0.56 (RE) 0.67 (RC_1)	–	
SM	0.23 (RE) 0.36 (RC_1)	0.6 (RE) 0.57 (RC_1)	0.22 (RE) 0.21 (RC_1)	0.12 (RE) 0.06 (RC_1)	–

Table 5.9 Post-SOM fields correlation between the groups

	SA	*MA*	*SH*	*PSB*	*SM*
SA	-				
MA	0.28 (RE) 0.29 (RC_1) 0.23 (RC_2) 0.1 (RC_3)	-			
SH	0.72 (RE) 0.8 (RC_1) 0.68 (RC_2) 0.62 (RC_3)	0.18 (RE) 0.22 (RC_1) 0.14 (RC_2) -0.08 (RC_3)	-		
PSB	0.66 (RE) 0.76 (RC_1) 0.62 (RC_2) 0.65 (RC_3)	-0.05 (RE) 0.07 (RC_1) 0.04 (RC_2) -0.20 (RC_3)	0.75 (RE) 0.77 (RC_1) 0.68 (RC_2) 0.69 (RC_3)	-	-
SM	0.31 (RE) 0.3 (RC_1) 0.44 (RC_2) 0.16 (RC_3)	0.64 (RE) 0.56 (RC_1) 0.55 (RC_2) 0.55 (RC_3)	0.32 (RE) 0.27 (RC_1) 0.31 (RC_2) -0.06 (RC_3)	0.08 (RE) 0.22 (RC_1) 0.21 (RC_2) -0.06 (RC_3)	-

5.2.2.6 Inter-correlation within the groups (pre and post-SOM)

Tables 5.8 and 5.9 compare the correlation of SOM fields between combined groups that were pre-tested and those that were post-tested respectively. The pattern in the two groups is the same except for a prevalence of a weak negative correlation between MA and SH ($r = -0.04$) for post-test. Stronger correlation is found between SA and SH, SA and PSB, MA and SM, SA and PSB in both the pre-tested and post-tested groups.

Table 5.10 Inter-correlation: SOM fields within the pre-tested groups (n = 243)

Fields	*SA*	*MA*	*SH*	*PSB*	*SM*
SA	1.000				
MA	0.227	1.000			
SH	0.617	0.147	1.000		
PSB	0.518	0.119	0.609	1.000	
SM	0.292	0.590	0.220	0.100	1.000

Table 5.11 Inter-correlation of SOM fields within a post-tested group (n = 587)

Fields	*SA*	*MA*	*SH*	*PSB*	*SM*
SA	1.000				
MA	0.221	1.000			
SH	0.693	0.104	1.000		
PSB	0.657	-0.041	0.712	1.000	
SM	0.314	0.574	0.192	0.093	1.000

5.3 Qualitative results

5.3.1 Teaching-learning observation

This section will present the observed findings in the four schools. The findings will be presented according to the following sub-categories: **physical settings; learner seating; interaction settings and program settings.** The observed findings of the four schools will be presented simultaneously under each sub-category. However, it should be emphasised that the findings presented according to the aforementioned sub-categories will firstly be inclusive of the observation of RE and RC_2 before the learners in both schools were re-arranged to suit cooperative learning method (i.e. the Hour-glass model). The findings regarding the observation of RE and RC_2 during the application of the Hour-glass model will be presented under the sub-category entitled **"Observation: application of the Hour-glass model - RE & RC_2".**

5.3.1.1 Physical setting

The observed physical setting is summarised in table 5.10 for all four schools RE, RC_1, RC_2 and RC_3. The arrangement of desks for school RE and the two classes of school RC_3 were the same i.e. rows and columns. School RC_2 had five grade 8 classes, but only four were available for research. All the four classrooms are approximately equal in terms of surface area (approximately 53 m^2).

Table 5.12 Physical setting of the classrooms

	Number of classes	***Floor area***	***Class arrangement***	***Distance for mobility***
RE	2	Approximately $53m^2$	Rows and columns but later groups	Approximately 1,5m from the chalkboard wall
RC_1	2	Approximately $53m^2$	groups	Enough for the teacher to move between the groups
RC_2	4	Approximately $53m^2$	groups	Enough for the teacher to move between the groups
RC_3	4	Approximately $53m^2$	2 classes rows 2 classes groups	Enough for the teacher to move between the groups and between the rows

5.3.1.2 Learner seating

Table 5.13 presents a summary of observed learner seating pattern. Learners who were seated in rows and columns in school RE and RC_3 were accommodated in pairs and they were all facing the chalkboard. The average class-size exceeds the international mathematics class average of 30 (Howie, 2001:100). The worst case scenario is found in school RE with the average class size of seventy-four. Learners are generally seated in homogeneous groups or pairs in terms of gender.

Table 5.13 Learner seating

	Number of learners	*Class average*	*Class description*	*Organisation of learners*		
				rows/columns	*Average group size*	*homogeneous/heterogeneous*
RE	148	74	over crowded	*rows	**8	Mostly homogeneous pairs in terms of gender
RC_1	98	49	crowded	groups	7	Predominant homogeneous groups in terms of gender
RC_2	205	51	crowded	groups	7	Predominant homogeneous groups in terms of gender
RC_3	204	51	crowded	groups	7	Homogeneous groups
				rows		pairs in terms of gender

*Seating pattern before learners were rearranged for the purpose of the application of the Hour-glass model
**Group size after learners were rearranged for the purpose of the application of the Hour-glass model

5.3.1.3 Interaction setting

Learners in school RE and two classes from RC_3 were predominantly interacting in pairs (with each other in the same desk) and with their teacher when answering the question. However, the pair interaction (i.e. learner-learner interaction) seemed predominantly non-verbal (when one learner raises his/her hand to answer the question the other partner seemed to be needing his/her partner's encouragement to raise his/hers). The teachers dominated the conversation where learners seemed to be doing more "listening" while the teacher did more "talking". The teacher-learner and the learner-learner interaction was predominantly formal. The interaction pattern in RC_1, RC_2 and the two classes (seated in groups) in RC_3 was the same as in RE and RC_3 (seated in rows). Learners were seated in groups but individual learners interacted with the teacher and predominantly non-verbally with each other.

5.3.1.4 Program setting

The teachers of all the four schools used the question and answer method but the mathematics lesson was teacher-centered. There was a noticeable pattern of beginning the lesson by doing the corrections of the previous day's homework or class-work. The lessons of the teachers in RE, RC_2 and one teacher in RC_3 were dominated by a cliché statement: "Do you understand?" and learners responded by saying "Yes Sir". The group arrangement of learners seemed not to influence the teaching style or teaching method employed by the teachers, i.e. even when learners were seated in groups, teachers in RC_1 and RC_2 continued to dominate the mathematics lesson. The human resource, which is the most abundant resource in every mathematics class, was not adequately used, i.e. learners helping one another during mathematics learning.

The teachers in RC_2 and RC_1 were frequently switching from English to Setswana when explaining mathematics concepts during the mathematics lessons as compared to the RE and RC_3 teachers.

5.3.1.5 Observation during the application of the Hour-glass model

After the first observation and the training of teachers in RE and RC_2, learners in RE and RC_2 were arranged in groups that were heterogeneous. The number of groups of each RE class was nine with an average group size of eight. RC_2 classes comprised eight groups with an average group size of seven.

Learners in RE seemed to have adapted quicker to cooperative group work than their RC_2 counterparts. They asked questions and the teacher re-directed the questions to other groups. The most distinguishable characteristic of learners in RE was that they predominantly used English to express themselves during mathematics lessons. Learners in RC_2 seemed to be very shy and reluctant to help others during cooperation. This was confirmed by the continuous and frequent encouragement by the teacher. However, in both RE and RC_2 learners had begun to interact verbally with one another. The learner-learner and learner-teacher interactions in the two schools gradually moved from formal to informal (constructively noisy mathematics classroom) and from teacher-centered to learner-centered during mathematics lessons.

The teachers in the two schools gradually shifted from teaching mathematics in an abstract manner to a more practical mathematics. For instance, one teacher from RC_2 used a weather chart to teach positive and negative integers. Temperatures below zero represented negative integers while temperatures above zero represented positive integers. In RE the teacher used the idea of a see-saw to illustrate the concept "equations" to learners. Initially the majority of learners did not know what a see-saw was but after the teacher requested one learner who knew what it was to explain it to the rest of the class, it (see-saw) was comprehensively equated to the concept "equations" in mathematics by learners.

5.3.2 Unstructured interview

The basic purpose of the unstructured interview was to gather additional information or to seek clarity about the observed classroom activities. This suggests that the interview questions would be dependent on the unclear observed classroom activities and would therefore differ from school to school and from one mathematics teacher to the other in the same school. The unstructured interview was held on the same day the lesson was observed.

5.3.2.1 Interview follow-up regarding the physical setting

The teacher in RE was asked why he preferred the rows and columns arrangement (before the class was rearranged for the application of the Hour-glass model). He responded that there are many learners per class and "group work is impossible to manage". The similar question was posed to the teachers of RC_1, RC_2 and RC_3 about their class arrangement and they respectively said:

- RC_1 teacher - "It is prescribed by OBE approach."
- RC_2 teacher - "We are required to group them because OBE is all about group work."
- RC_3 teacher A (grouped learners) - "To encourage mathematics learners to solve mathematics problems together."
- RC_3 teacher B (ungrouped learners) - "Learners placed in groups do not think for themselves, they depend on other learners and most of them simply copy mathematics solutions from other learners."

5.3.2.2. Interview follow-up regarding learner seating pattern

Teachers in the four schools were asked why their learners were predominantly placed in homogeneous pairs (RE learners and two of RC_3 classes) and in predominantly homogeneous groups (RC_1, RC_2 and two of RC_3 classes). A similar response was received from the four school teachers. They indicated that learners were given a freedom to sit with whoever they wanted to sit with. Their partners or positions in class were not prescribed by the teachers.

5.3.2.3 Interview follow-up regarding the interaction and program setting

Teachers in RC_2 and RC_1 used code switching more frequently than their RC_3 and RE counterparts. RC_2 and RC_1 teachers were requested to respond on the rationale for frequent code switching in mathematics lessons:

- RC_1 - "Learners in Mabopane speak predominantly Setswana. They do not understand English very well and I have to translate almost every mathematics task for them to understand what they must do."
- RC_2 - "These learners do not understand English. They tend to understand mathematics if I mix the two languages. If you ask them whether they understand you, they will respond by saying "Yes Sir" but they do not really mean it.

5.3.2.4 Interview follow-up regarding the application of the Hour-glass model

Interviews were conducted with RE and RC_2 teachers with the intention of obtaining a feedback on the evaluation of the effectiveness of the Hour-glass model. A teacher from RE highlighted that he stopped using cooperative small groups in mathematics instruction about two years ago due to the large class sizes in his school (n = 73 per class). He maintained that since he used the Hour-glass model he was inspired to use cooperative learning regularly because of the high level of social interaction among learners and improved classroom behaviour (discipline improved because learners' time on-task has improved). He contemplated using the Hour-glass model in grade 9 "because I can cope easier with group work and once learners are selected to the interclass they do not want to give way to other learners". The same experience was reiterated by teachers in RC_2. One teacher said: "This model enhances a high level of discourse among learners and they seem to understand one another better during peer tutoring than they do with teachers." They (RC_2 teachers) further highlighted that the Hour-glass model became even more effective towards encouraging learners to do more mathematics when learners from the interclass are interchanged, not only across the groups but across the classes, i.e. learners from 8A assist learners in 8B and *vice versa*. They added that the model also benefits teachers as it encourages them to work together 'because not all of us understand the same mathematics section or topic the same way, so sharing ideas before teaching a lesson makes us more confident'. The following are the summarised advantages of the Hour-glass model as presented by RE and RC_2 teachers:

- they adapted to large class sizes;
- it improves social interaction and high level mathematics discourse;
- it promotes time on-task and class/group discipline;
- it motivates or encourages learners to do more mathematics;
- it enhances peer tutoring; and it enhances cooperative teaching among teachers.

The two groups of teachers (RE and RC_2) revealed the following disadvantages or concerns about the Hour-glass model:

- After investing quality time in selecting and teaching the interclass, some learners (constituting a small population of the class) become shy to help others while some become unwilling to help others seemingly because of selfishness;

- Creating time for an interclass in the afternoon may be problematic when extra-curricular activities dominate the school schedule especially in the first six months of the year;
- Some learners tend to forget what they did in the interclass the previous day when they have to help others the next day during mathematics lesson; and
- If learners are interchanged across the classes, those from the interclass will miss out on other lessons because mathematics periods in all classes do not coincide.

5.3.3 Biographical Information Questionnaire

The purpose of the BIQ was outlined in Chapter 4 (see § 4.3.2.2). The BIQ was given to eight teachers to complete but only seven returned the completed questionnaires. The results of the BIQ with regard to the teachers' qualifications and teaching experience are summarised in table 5.14.

Table 5.14 Teacher qualifications and teaching experience

Group	*Teacher*	*Qualification*			*Teaching experience*
		Highest standard	*Tertiary*	*Current studies*	
RE	A	10	UDE	FDE	12
RC_1	A	10	STD; HED	BCom	14
	B	10	UDE	Not studying	3
RC_2	A	10	UDE; HED	Not studying	12
	B	10	STD	Not studying	4
RC_3	A	10	JSTD	Not studying	Did not provide
	B	10	UDE; FDE	BEd	11

The findings seem to reveal that the qualifications of the teachers did not differ significantly. The teacher with the highest teaching experience was found in RC_1.

However, this did not constitute a threat to the validity of the results because learners in school RC_1 did not achieve high marks in the mathematics academic achievement test. They were outperformed by the other three groups whose teachers had a low teaching experience. Therefore, neither the teacher qualifications nor their teaching experience had an influence on the results.

Teachers were also requested to tick either YES or NO as an indication of whether or not they have a knowledge about mathematics learning skills. Their responses are summarised as follows:

Table 5.15 Teachers' knowledge of learning skills

Group	***Teacher***	***Learning skills***				
		SA	***MA***	***PSB***	***SH***	***SM***
RE	A	Yes	Yes	Yes	No	No
RC_1	A	Yes	No	Yes	No	No
	B	Yes	No	Yes	No	No
RC_2	A	Yes	No	No	No	No
	B	Yes	Yes	Yes	No	No
RC_3	A	Yes	Yes	Yes	Yes	No
	B	Yes	Yes	No	No	No

Question 5 of the BIQ aims at ascertaining whether or not mathematics teachers find cooperative learning methods effective. A synoptic version of their responses is that "it encourages learners to discover things on their own; learners learn from one another; learners get to know each other easily; it promotes leadership skills and discipline". The two teachers in RC_3 seemed not to have knowledge about cooperative learning. One teacher did not respond while the other attributed the integration of learning areas to the use of cooperative learning which, in essence, is not correct.

5.4 Discussions

5.4.1 Discussion of the quantitative research findings

5.4.1.1 Mathematics academic achievement test

In chapter one (see § 1.2) and chapter two (see § 2.7) various (yet interrelated) factors that hinder the effectiveness of cooperative learning methods in the teaching and learning of mathematics in large group sizes were outlined. The consensus reached by numerous researchers was that cooperative small groups should be kept very small (maximum of five members) in order to enhance academic success. This, therefore, suggests that there is an inverse proportion between the group size and mathematics academic achievement, i.e. when the group size increases the academic achievement becomes less and vice versa.

The quantitative findings of this research have revealed that the large class size (hence large group size) does not hinder the effectiveness of a particular cooperative learning method namely the Hour-glass model. The high achievement of practical significance exhibited by RE and RC_2 (in contrast to RC_1 and RC_3) is an indication that mathematics learners in large classes can also benefit from using the cooperative learning method. The high mathematics academic achievement attained by learners in RE and RC_2 (groups that applied the Hour-glass model) may be attributed to the following main features that characterise the Hour-glass model:

- **Peer-tutoring or peer-assistance:** when learners are engaged in solving mathematics tasks cooperatively they spend most of their time on-task. They do not have to wait for the teacher (who might be busy helping other groups) to attend to them immediately when they need help. This, therefore, addresses the concern of the Department of Education (2001b:2) that when the class is large learners do not get assistance immediately when they need it. Instead one of the peers in the group is a help-giver or a guide. This confirms the earlier research findings that learners seem to grasp mathematical concepts much easier when such mathematical concepts are explained by their peers than when the explanation is done by adults. However the success of the Hour-glass model relies largely on the teacher and it should not be misconstrued as implying that the whole responsibility should be left to the peer-tutors.

- **Disguised "remediation"**: learners who were identified to join the interclass have a particular mathematical need such as the need to achieve higher in the subject or the need to be motivated to do more mathematics. However, "remediation" is done in the form of an "extra class" before the lesson is taught to the rest of the other learners in their actual mathematics classes. When, for instance, the rest of the learners are "peer-assisted" by learners who are generally regarded as low-achievers in mathematics, they are likely to develop an attitude of "if s/he is able to do it, I can do it twice as good." In essence this improves their academic achievement in mathematics and they become eager to join the interclass.

The trend of using an extra class as a means of "remediation" is that of firstly offering a mathematics lesson during the day and all learners with mathematical needs remain in one class while the rest of the learners are released to go home. If a mathematics extra class is offered (for remedial purposes) after the same lesson was offered to the rest of the class, learners who have certain mathematical needs may develop a negative attitude towards the same class whose intention was to help them. The learners in the extra class see themselves as "mathematical failures" who have been grouped together in an afternoon class in order to place them on par with the "mathematically capable ones" who were allowed to go home at that instant. The Hour-glass model employs the technique of mixing learners of diverse mathematical abilities and needs who are grouped in the interclass (euphemistically "extra class") in order to de-stigmatise the "mathematics extra class" or " mathematics remedial class". Learners from the interclass are being recognised by allowing them to help their teacher in the form of peer-tutoring or peer help-givers. This approach encourages learners to want to do more mathematics, take part in the activities of the interclass, learn mathematics cooperatively and demonstrate a high level of social and cognitive skills.

The findings of this research with regard to mathematics academic achievement suggest or imply that the Hour-glass model of cooperative learning enhances high mathematics academic achievement among learners in large mathematics classes. Therefore, the large group size with its associated complex lines of communications (Bennette & Dunne, 1994:115) is no longer a hindrance when mathematics is taught and learnt using the Hour-glass model of cooperative learning. The findings of this research with regard to the mathematics academic achievement supports what was earlier hypothesised (H_{01}), namely, that the application of the Hour-glass model positively influences the mathematics academic achievement of grade 8 learners in crowded classes.

5.4.1.2 Effects of the Hour-glass model on mathematics learning skills

Relevant research (Artzt & Newman, 1990b:450; King, 1993:412; Newstead, 1998:55) revealed that when learners learn mathematics in cooperative small groups their problem-solving skills, attitude towards mathematics, motivation to do more mathematics, and their anxiety about mathematics improve. However, the above assertion was based on small class context whose group size was about five. The statistical research findings in this study revealed a lack of practically significant influence of the Hour-glass model in the aforementioned mathematics learning skills. The lack of influence of practical significance necessitated the rejection of H_{03}, namely, that the application of the Hour-glass model of cooperative learning influences the learning skills of grade 8 mathematics learners. It is, however, worth highlighting the lack of consistency between the research findings of RE-RC_1 and RC_2-RC_3 (see Figure 5.4). According to the Solomon Four-group design (see § 4.3.1.1) it was expected that the pattern of the difference between the means of RE versus RC_1 would be similar or equivalent to that of RC_2 versus RC_3 with regard to the learning skills as it was with regard to the mathematics academic achievement. The inconsistency in the research results regarding the learning skills may be attributed to following two factors:

- A loss of 20,4% of the subjects from RC_1 during the SOM posttest might have had a significant effect on the post-SOM mean score (Ary *et al.,* 1990:314; Tuckman, 1994:124). This differential loss of subjects (possibly attributed to the difficult language of the SOM questionnaire) might have also contributed to higher RC_1 mean scores relative to RE. As a result RE and RC_1 do not reflect the same mean score difference pattern as RC_2 and RC_3 (RC_2 scores were higher than RC_3 scores in all five fields of SOM). It therefore seems that the SOM pretest had a negative influence (subject attrition) on RC_1 and subsequently led to the pretest-treatment interaction (Leedy & Ormrod, 2001:237; Ary *et al.*, 1990:333; McMillan & Schumacher, 2001:379; Gall *et al.,* 1996:517). The H_{03} cannot be rejected with absolute confidence relative to the findings displayed by RE-RC_1 comparison (which differed significantly from the RC_2-RC_3 comparison).

- Learners in the four research groups experienced difficulty in understanding the language used in the SOM questionnaire. This was evident in the questions they asked about the meanings of numerous words in the questionnaire during the test administration. According to Maree *et al.* (1997:26), the fields of the SOM questionnaire were significantly reliable for grade 8 and 9 learners who speak African languages. It is, however, not known whether it referred to "African language speakers" who attended the previously model C or private schools (probably taught in English by English language speakers); or "African language speakers" in "exclusively" African public schools (probably taught in English by African language speakers). It stands to reason that the English language proficiency of African language speakers from the previously model C schools or private schools is generally higher than that of their counterparts in the then African public schools taught by African teachers. This confirms the findings of TIMSS (Gray, 1997:109) that a large number of problems in the teaching and learning of mathematics emanate primarily from the basic language problems of both teachers and learners.

Based on the two above-mentioned attributes, an H_{03} cannot be rejected with absolute confidence relative to the findings displayed by RE-RC_1-comparison which differed significantly from the RC_2-RC_3 comparison.

The SA-SH correlation suggests that learners with positive attitudes towards mathematics are likely to employ effective study methods in mathematics. Further, learners are more likely to display positive PSB and are prepared to engage in social interaction in mathematics class (Maree *et al.,* 1997:29). Cooperative learning methods, including the Hour-glass model, have been credited with the promotion of social interactions. As a result there is a high probability that a well structured cooperative learning method will enhance a positive study attitude and positive study habits and positive problem-solving behaviour during the learning of mathematics. The prevalence of these correlations in large mathematics classes where the Hour-glass model was used is an indication that the model is an effective way of teaching mathematics in large classes through cooperation.

The SH-PSB correlation is also an indication that the two fields are interdependent. Learners who have a positive study habits in mathematics are likely to display a positive or effective problem-solving behaviour in mathematics. Further the MA-SM correlation suggests that a warm, stimulating environment in which mathematics teachers are accessible to learners for help-giving tends to enhance a non-threatening and a relaxed feeling within learners when learning mathematics. The Hour-glass model provided an environment where learners could access help from their peers and their teachers about mathematics problem solving. Learners gained more confidence about mathematics and their mathematics academic achievement was enhanced.

5.4.2 Discussion of the qualitative research findings

Previous research on cooperative learning methods emphasised the effectiveness of the methods with small groups of not more than five. The findings of this study have proven that learners in large mathematics classes can no longer be deprived of the mathematics achievement gains experienced by learners in small classes when learning mathematics cooperatively. The complex lines of communication (associated with large classes, hence large group sizes) that promoted group management problems (Bennette & Dunne, 1994:115) is mitigated by the presence of the peer-tutor in the groups when applying the Hour-glass model.

The physical classroom seating pattern has an influence on the learning of mathematics. When learners are arranged in rows and columns, a very tense and formal environment exists and learners become passive recipients of mathematics "knowledge". When learners are arranged in cooperative small groups, they tend to interact with one another, their teacher or peer-assistant and engage in mathematical discourse. They tend to spend more time on-task in the cooperative setting than in the whole-class setting as confirmed by Mulryan (1994:282). However, it was evident in this study that when learners are merely placed in groups without structuring cooperation, there is no guarantee that they will cooperate. This finding is supported by Johnson and Johnson (1995:349) and it was evident in RC_2 learners who could not engage in cooperative work before the introduction of the Hour-glass model even though they were seated in groups. The teachers' perceptions (and knowledge) and subsequently the purpose of using the cooperative group is a determining factor influencing the success or failure of the cooperative method in mathematics. This was again evident in RC_2 learners who were placed in small groups and their performance in mathematics began to improve after their teachers introduced the Hour-glass model probably because they (teachers) began to perceive cooperative learning methods differently as a result of the training they received.

Learners in RE and RC_2 were taught in a small group setting where they solved most of the tasks together. However, when they were tested (post-test) they wrote the test individually and scored much higher than their counterparts in RC_1 and RC_3. This supports the assertion by Johnson and Johnson (1990:107) that the individual performance of learners who worked in small groups in mathematics is higher than the average group performance.

Students receive encouragement and support in their efforts to learn mathematical processes, strategies and concepts. In the interclass learners are initiated in a "smaller platform" where they can express themselves mathematically. As they work cooperatively within the mathematics interclass, learners gain confidence in their individual mathematics abilities and in time they become very confident to help other learners.

It is generally not easy for teachers to understand the benefits associated with cooperative learning in mathematics if they (teachers) cannot cooperate i.e. engage in cooperative teaching. The Hour-glass model encourages teachers to work together (if they teach mathematics in the same grade) and share ideas about the mathematics lesson they are about to teach.

The formation of cooperative small groups during cooperation should not be the responsibility of learners. When learners choose their own groups they are more likely to form homogenous groups. The observed homogenous groups and homogenous pairs (in RC_1, RC_2 and RC_3) in terms of gender bear testimony to the previous assertion and it supports the finding by Terwel (1990:240).

The high academic achievement in mathematics in RE and RC_2 is generally attributed to improved social skills among learners. Mathematics, with its perceived difficulty, is effectively learnt and taught when learners:

- share ideas about mathematics (positive interdependence);
- engage in mathematical discourse (face-to-face interaction);
- solve mathematical problems collectively (positive interdependence);
- engage in heterogeneous groups;
- interact with one another and with their teacher; and
- help other learners.

All the aforementioned aspects are the characteristics of the use of positive social skills in mathematics learning. In the process of sharing ideas about mathematics, engaging in mathematical discourse and collective mathematical problem-solving activity, there is a likelihood that learners will argue and debate and sometimes conflict may arise. If in the process learners are about to reach one conclusion about the answer to the mathematics problem, or to lead other learners during mathematics lessons (peer-tutoring) and in the process of peer-tutoring are able to accurately and unambiguously communicate with other learners for them to understand a particular mathematics concept, then such learners possess positive social skills. The Hour-glass model seems to have enhanced such skills among learners through the interclass. Such social skills were positively correlated with mathematics academic achievement. It is against this background that H_{02} is accepted.

5.5 Conclusion

The Hour-glass model of cooperative learning seems to have offered a breakthrough towards the implementation of cooperative learning in crowded classes which characterise the majority of South African schools. However, the application of the Hour-glass model has revealed other aspects of mathematics teaching and learning that call for more research. The next chapter will address the shortcomings of this research and outline associated aspects that require further research.

CHAPTER

SIX

CONCLUSIONS

AND

RECOMMENDATIONS

The knowledge of a particular approach to teaching and learning is a pre-requisite to the implementation of such approach (Mahlobo, 2000:122).

6.1 Introduction

This chapter aims to address two main sections, namely the **synopsis of the research** and the **general conclusions and recommendations** about the study. The former will present a synoptic discussion about the literature review regarding the research on cooperative learning; the impact of the methods of research employed in the study; and the implications of the research findings in the teaching and learning of mathematics. The latter will present the limitations of the study, recommendations for future research, and general concluding remarks.

6.2 Synopsis of the research

6.2.1 Literature review

The literature review was thoroughly done to critically and objectively highlight the strengths and weaknesses of the cooperative learning method in the context of crowded mathematics classes. Numerous researchers agree on two fundamental aspects that characterise effective cooperation, namely

- the prevalence of the five principles (positive interdependence, face-to-face interaction, social skills, individual accountability, group processing) that form the cornerstone of "good" cooperation (see and § 2.2.2).
- the consensus about a five-member group to maximise productivity during mathematics learning (see § 2.7).

However, Slavin.(1988:31 & 1989/90:52) identifies two main requisites (without undermining the previously mentioned five principles) that are of utmost importance for achieving gains during cooperation: **individual accountability** and **group goals**. The emphasis is that if individual accountability and group goals are well structured, the effective cooperation will prevail and achievement gains (in mathematics) are guaranteed.

Group structuring is also a determinant of successful cooperation. There is substantial research evidence that heterogeneous grouping is more advantageous than homogeneous grouping (see § 2.2.3). Heterogeneous grouping during cooperative work enhances sharing of knowledge, help-giving and help-receiving, interpersonal and social skills, and on-task behaviour during mathematics learning. In contrast, homogeneous grouping promotes competition and class polarization in which some learners regard themselves (or are regarded by others) as being "good" in mathematics proficiency while others regard themselves (or are regarded by others) as being "poor" in mathematical proficiency.

6.2.2 Problem statement

Cooperative learning has emerged to be a preferred teaching-learning model in South African schools since the inception of Curriculum 2005 (C2005) emphasising outcome-based Education (OBE) (Bell *et al.,* 1999:268). While a good number of educators (65,3%) are willing to know and use comparative learning methods in the teaching of mathematics (Department of Education, 2002:12), the process of applying these method(s) is made difficult by large group sizes (Orton, 1994:40) emanating from large class sizes. The majority of South African schools have class sizes of up to seventy learners which translates to about ten groups of seven members in the average class area of 52,5m^2. It is for this reason that the preferred teaching-learning method i.e. cooperative learning method is incompatible with C2005 and OBE in the majority of South African schools.

The consensus reached by researchers about five as the maximum group size for effective cooperation suggests that if the class size (hence the group size) is big (about 50 or more), the use of cooperative learning method is virtually impossible. Large group sizes (emanating from large class sizes) are characterised by complex lines of communication (Bennettee & Dunne, 1994:115). The complex lines of communication promote a situation where learners cannot listen to one another. All learners in a group want to talk at the same time and each one wants to be listened to and groups become very difficult for the teacher to manage.

6.2.3 Research method

The Hour-glass model of cooperative learning in mathematics was identified as an attempt to address the problem of using cooperative learning method in large class sizes in South Africa. In order to ensure its (Hour-glass model's) effectiveness in the teaching and learning of mathematics, it was tested (using the Solomon four-group design) in two schools and the combined qualitative-quantitative research method was used. Data were collected using a SOM questionnaire, the academic achievement tests (quantitative data) and unstructured observations and interviews (qualitative data).

In this study the random sampling was done to obtain a sample of four schools (with grade 8) from the population of schools in the Mabopane and Temba districts (two schools from each district). A specific True-experimental design, namely Solomon four-group design was used for primarily two reasons: firstly for its ability to control factors that may pose a threat to internal validity, and secondly for its adapted nature to use large sample size (n>500). The two schools (RE and RC_2) from the Temba district received the treatment (Hour-glass model) while the other two schools from Mabopane (RC_1 and RC_3) did not receive the treatment. However, one school (RE) from the Temba district and one school (RC_1) from the Mabopane district received both the pretest and the posttest. The remaining two schools (RC_2 and RC_3 from the Temba and Mabopane districts respectively) received the posttest only. Schools RE and RC_2 from the Temba district received the treatment, i.e. the Hour-glass model

6.2.4 Research findings

Before the treatment was administered the t-Test was conducted to ascertain whether the means of RE and RC_1 were similar. The t-Test revealed that the two groups did not differ practically significantly. However, after the treatment was introduced on RE and RC_2 anova was conducted, it revealed that there was a practically significant difference (with regard to the academic achievement test) between the groups that took the treatment and those that did not. There was, however, no practically significant difference between the groups with regard to the mathematics learning skills.

The observations and interviews that were conducted further revealed that learners of the groups that received the treatment engaged in higher social interactions than their counterparts in the groups that did not receive the treatment. Learners in the interclass did not want to make way for other learners and this was indicative of the motivation and encouragement associated with the Hour-glass model when learning mathematics. This revelation suggests that learners in crowded classes can no longer be deprived of the mathematical achievement gains associated with cooperative learning methods. The two hypotheses, namely H_{01} and H_{03} were accepted while H_{02} was rejected (without absolute confidence because of the loss of about 20% of some learners in RC_1 during the post-SOM test).

6.3 General conclusions and recommendations

6.3.1 Limitations of the study

- It was assumed that teachers knew the learning factors such as mathematics anxiety, attitudes about mathematics, positive/negative problem-solving behaviour in mathematics, and high/low-achievers in mathematics. These factors (and other related factors that the teacher may have identified) are used when identifying learners to be included in the interclass and in the formation of cooperative small groups to promote heterogeneity. However, teachers tend to equate low mathematics academic achievement with either mathematics anxiety, negative attitudes towards mathematics, or negative problem-solving behaviour in mathematics.

  Further, the choice of learners for the interclass based on low or high-achieving learners in mathematics may be misleading because the diagnosis of such is subjective, i.e. based on the particular teacher's point of reference. Teachers normally conclude that a learner is a low-achiever or high-achiever in mathematics based on few class tests or class-works which might have not been moderated by another teacher. The study could have gained more authenticity if a diagnostic instrument was used to identify the mathematics learning needs of all learners who took part in the research. Teachers would, therefore, have taken an informed decision about the mathematical learning needs of their learners and a more authentic decision about who to include in the interclass. Based on this, the Hour-glass model could have been used successfully and effectively as a "remedial teaching-learning method" in mathematics. It is therefore assumed that the Hour-glass model could have had a positive impact on the RE and RC_2 learners with regard to the five fields of the SOM questionnaire.

- While the main purposes of qualitative methods, i.e. observation and unstructured interviews are outlined (see § 4.3.2.2), it could have provided more useful findings if the learners were also interviewed. The learners' interview could have probably provided information about some of the dynamics of mathematics learning enhanced by the Hour-glass model. This is against the background that learners are the actual "consumers" of the Hour-glass model. It is, therefore, on the basis of this that the limitation of the qualitative aspect of this research is recognised.

- After investing quality time in selecting and teaching the interclass some learners (constituting a small population of the class) become shy to help other learners while others become unwilling to help their peers seemingly because of selfishness;

- Creating time for an interclass in the afternoon may be problematic when extra-curricular activities dominate the school schedule especially in the first six months of the year. Schools at which the research was conducted normally become engaged in activities such as athletics and music during the first semester. Such activities usually take place in the afternoon after the formal teaching and learning has ended. It is during this time when the interclass is taught in preparation of the following lesson (the next day) that will take place in the actual grade 8 class.

- Some learners tend to forget what they did in the interclass the previous day when they have to help others the next day during mathematics lesson; and

- If learners are interchanged across the classes, those from the interclass will miss out on other lessons because mathematics periods do not occur at the same time in all classes.

6.3.2 Recommendations for future research

The study has revealed that a particular cooperative learning model namely the Hour-glass model can be employed in crowded mathematics class settings to enhance academic achievement and social skills in mathematics. However future research is recommended on:

- How the Hour-glass model can be structured to enhance mathematics learning skills such as positive attitudes about mathematics and improved mathematics anxiety; and
- The effects of teachers' knowledge of mathematics learning skills on mathematics academic achievement.

The two recommendations emanate from the background that cooperative learning methods enhance such learning skills in small classes but it seemed not to be the case in crowded classes.

6.3.3 Conclusions

The Hour-glass model has offered a breakthrough to both learners and teachers of crowded mathematics classes. Mathematics can be learnt cooperatively by all learners irrespective of the class-size or the group size. In the current South African situation with numerous problems (including crowding in classes) facing education, the only readily available resource teachers can use to address the problem of crowded mathematics classes is 'learners'. Engaging learners in peer tutoring or peer assistance activity during mathematics lessons has been proven effective during cooperation. Peer assistance minimises the teacher's work (especially the work associated with maintaining learner discipline emanating from engaging in off-task behaviour) and enables him/her to cope with large mathematics class.

The superiority of cooperative over competitive and individualistic mathematics learning increases as the mathematics task becomes more conceptual, requires more problem solving, necessitates more higher-level reasoning and critical thinking, needs more creative answers, and requires more application of what is learned. Cooperative learning offers learners an opportunity to talk and share ideas about mathematics. It promotes a win-win situation as learners interact with one another and with the mathematics they are learning i.e. low achieving learners receive help from high achieving learners, and high achieving learners in turn tend to internalise the learnt mathematical concepts as they repeatedly explain them to other learners. As learners contribute their perceptions and/or ideas about the mathematics task during cooperation learning, they tend to gain confidence about the subject. This unique feature of the Hour glass-model offers all learners an opportunity to offer mathematical help to their peers. As a result low mathematical achievers are not dominated by their high achieving counterparts. It should, however, be accentuated that for cooperative learning, including the Hour-glass model to be effective in mathematics teaching and learning, the following five critical elements underpinning effective cooperation must be well structured:

- face-to-face interaction;
- social skills;
- individual accountability;
- group processing; and
- positive interdependence.

While the Hour-glass model has revealed effectiveness in the teaching and learning of mathematics, its success is not tailor-made only for mathematics classes. It can also be effectively applied in other learning areas as well, especially in the General Education and Training band (where integration of learning areas is emphasised), in schools where large class sizes pose a hindrance to the use of other cooperative learning methods.

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Appendix A: Letter requesting school list

P.O.Box 11917
Tramshed
0126
04 March 2002

The District Manager

....

Re: Request for the school list for research purposes

I humbly request your office to furnish me with a list of all middle schools in your district for research purposes as communicated to you in my earlier correspondence.

I appreciate your usual assistance towards the success of my research project.

Yours faithfully

R.D.Sekao (Mr)
Cell:
Student Number:

Appendix B: Mathematics academic achievement test (Pre-test)

Mathematics Test

for

Research Purpose

(2002)

Instructions to candidates

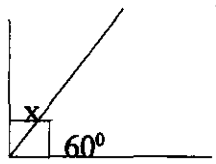
1. *Answer all questions in 60 minutes.*
2. *All questions are multiple choice questions and should be answered on the answer sheet provided.*
3. *Choose only **ONE** answer from the four given answers. Please make a cross on the symbol of a question you have chosen, e.g. if you choose answer B in question 1, do this: 1. A ~~B~~ C D*
4. *Use strictly a pencil so that you can rub it out if you want to make changes.*
5. *Do **NOT write or make marks** on the question paper. Both the question paper and answer sheet must be given to the tester before leaving the testing room.*
6. *A blank sheet is provided for rough work.*
7. *Fill in your particulars on the answer sheet.*

1. How many millimeters make 30 cm?
    A. 30
    B. 300
    C. 0,3
    D. 3000

2. What is the answer: 3 + 5 x 2 - 7
    A. 9
    B. 6
    C. -22
    D. -9

3. In $12 \div 6 = 2$, 6 is called a_____________of 12.
    A. variable
    B. multiple
    C. factor
    D. sum

4. What is the size of angle **x** in the following diagram?



    A. 30^0
    B. 20^0
    C. 90^0
    D. 60^0

5. John has scored 15 out of 20 in a maths test. What percentage has he obtained?
    A. 15%
    B. 65%
    C. 20%
    D. 75%

6. A rectangular garden measures 25m by 10m. Your daddy wants to buy a fence for this garden. How long should the fence be?
    A. 35m
    B. 250m
    C. 70m
    D. 15m

7. Study the following number pattern and fill in the missing numbers represented by question mark (?).

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 ? 10 ? 5 1
1 6 15 20 15 6 1

A. 4 & 6
B. 1 & 4
C. 5 & 10
D. 6 & 10

Use the following information to answer question 8; 9; 10 :
During a very cold winter day temperatures in three South African cities were recorded as follows: Johannesburg (-1 ^{0}C); Bloemfontein (-4 ^{0}C); Pretoria (0 ^{0}C).

8. Which city is the coldest?
   A. Johannesburg
   B. Bloemfontein
   C. Pretoria
   D. none

9. Which city is warmer than others?
   A. Pretoria
   B. Johannesburg
   C. Bloemfontein
   D. none

10. What is the temperature difference between Pretoria and Johannesburg?
   A. -1 ^{0}C
   B. 0 ^{0}C
   C. -4 ^{0}C
   D. 1 ^{0}C

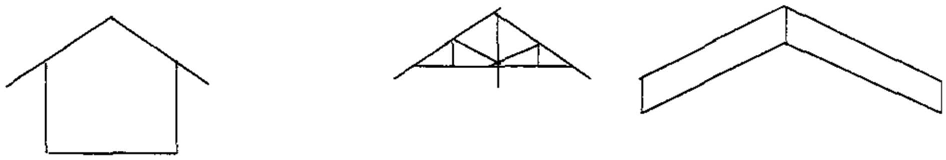
11. An instrument used to measure temperature is called_______________
A. thermometer
B. barometer
C. manometer
D. massmeter

12. A normal body temperature of a human being is almost_______________
A. 37°C
B. 27°C
C. 45 °C
D. 47 °C

13. A car travels at a speed of 120 km/h. This means that it will cover a distance of _____in_______hour(s).

A. 120 km in 120 hours
B. 60 km in 60 hours
C. 1 km in 120 hours
D. 120 km in 1 hour

14. Which roof truss is very rigid (ie. very strong or firm)



(1) (2) (3)

A. (1)
B. (3)
C. (2)
D. all of them

15. A reason for my answer in 14 above is that:
A. pentagons are the most rigid polygons even without support.
B. triangles are the most rigid polygons even without support.
C. parallelograms are the most rigid polygons even without support.
D. All of them are very rigid even without support.

16. A box contains tiles that can cover an area of $1{,}5m^2$.If an area of a room to be tiled is$15m^2$, how many boxes of tiles are needed?

A. 15
B. 1,5
C. 15,5
D. 10

17.A soccer ball costing R120 is discounted by 10%. How much will you pay for that soccer ball?

A. R108
B. R110
C. R130
D. R112

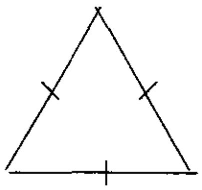
18.Write the following sentence as an equation: The sum of three and another number is twenty-one.

A. $3 - y = 21$
B. $3 \times y = 21$
C. $3 + y = 21$
D. $3 \div y = 21$

19. What is the value of x in the following equation: $2x + 15 = 23$.

A. 2
B. 4
C. 3
D. 6

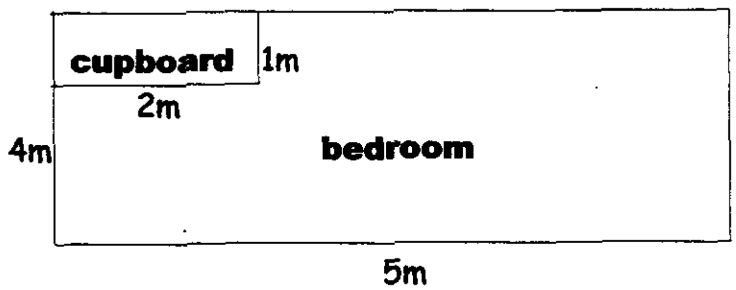
20.The following triangle is called an equilateral triangle.



What is the size of each angle?

A. 45^0
B. 20^0
C. 90^0
D. 60^0

21.The following diagram shows a rectangular bedroom with a built-in cupboard. The cupboard measures 2m by 1m and the total bedroom floor measures 5m by 4m. What is the area of a floor to be covered by a carpet



A. $18m^2$
B. $20m^2$
C. $22m^2$
D. $12m^2$

22. Write ¼ as a decimal fraction.
A. 0,25
B. 1,4
C. 4,1
D. 2,5

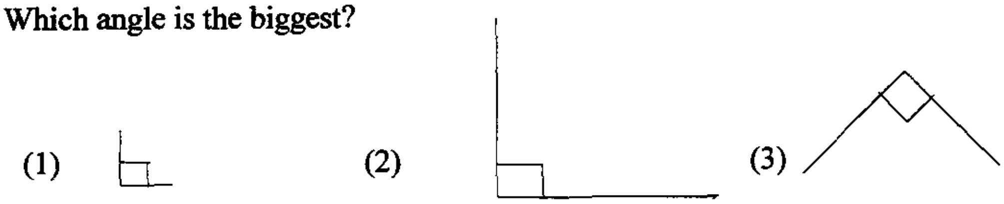
23.A 14% VAT is charged when buying a R50 **'bafana-bafana'** hat. How much VAT is charged?
A. R50
B. R14
C. R5
D. R7

24. What does VAT stand for?
A. variable and tax.
B. values added together.
C. value added tax.
D. value and tax

25. Work out the following: 20 x 0,5 x 0 =
   A. 25
   B. 0
   C. 10
   D. 20,5

26. Which numbers are multiples of three: 1; 3; 5; 12; 4.
   A. 4 & 5
   B. 1 & 3
   C. 3 & 12
   D. 4 & 12

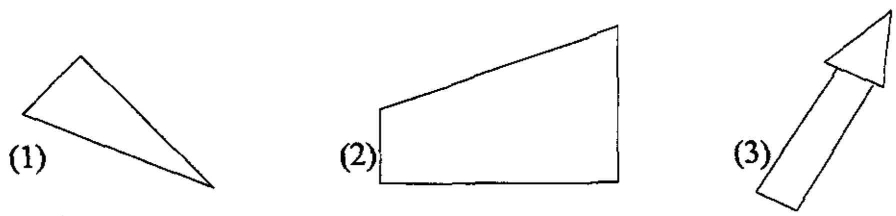
27. Which angle is the biggest?



   A. (2)
   B. (3)
   C. (1)
   D. none

28. There are 600 learners in your school. 2/3 of them voted for Mandla as the president of Representative Council of Learners (RCL). How many votes did Mandla get?
   A. 400
   B. 200
   C. 300
   D. 230

29. Which of the following figures are symmetrical?



   A. (1) & (2)
   B. (1) & (3)
   C. (2) & (3)
   D. none

30. In the following illustration matchsticks were used to construct triangles:

A rule for working out number of matchsticks from number of triangles is T x 2 + 1, where T stands for number of Triangles. Use this rule to work out the number of matchsticks needed to make six triangles.

A. 12
B. 9
C. 13
D. 7

Appendix C Mathematics academic achievement test (Posttest)

Mathematics Test

for

Research Purpose

(2002)

POSTTEST

Instructions to candidates

1. *Answer all questions in 60 minutes.*
2. *All questions are multiple choice questions and should be answered on the answer sheet provided.*
3. *Choose only **ONE** answer from the four given answers. Please make a cross on the symbol of the answer you have chosen, eg if you choose answer B in question 1, do this: 1. A ~~B~~ C D.*
4. *Use strictly a pencil so that you can rub it out if you want to make changes.*
5. *Do **NOT write** or **make marks** on the question paper. Both the question paper and answer sheet must be given to the tester before leaving the testing room.*
6. *A blank sheet is provided for rough work.*
7. *Please fill in your particulars on the answer sheet.*

Use the following information to answer question 1; 2; and 3 :

During a very cold winter day temperatures in three South African cities were recorded as follows: Johannesburg (-1 ^{0}C); Bloemfontein (-4 ^{0}C); Pretoria (0 ^{0}C).

1. Which city was the coldest?

    A. Johannesburg
    B. Bloemfontein
    C. Pretoria
    D. none

2. Which city was warmer than others?

    A. Johannesburg
    B. Pretoria
    C. Bloemfontein
    D. none

3. What was the temperature difference between Pretoria and Johannesburg?

    A. -1^{0}C
    B. 0^{0}C
    C. 1 ^{0}C
    D. -4^{0}C

4. A normal body temperature of a human being is almost________________

    A. 37 ^{0}C
    B. 27 ^{0}C
    C. 45 ^{0}C
    D. 47 ^{0}C

5. Many learners in Mabopane and Temba eat "sephatlo" during break. What fraction of a loaf of bread is "sephatlo"?

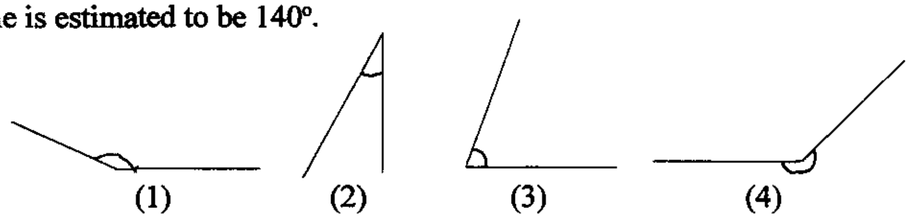
    A. one-third.
    B. half
    C. three-quarter
    D. quarter

6. If $a = 2$ and $b = 3$, $a^2 + b^2$ will be equal to.............

A. 5
B. 9
C. 13
D. 4

7. Which angle is estimated to be 140°.

A. (3)
B. (4)
C. (1)
D. (2)



8. Simplify: 2/3 x 3/4

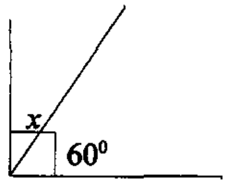
A. 5/12
B. 1/2
C. 6/12
D. 5/7

9. Study the following number pattern carefully and identify the missing numbers represented by a star (*).

1; 1; 2; 3; 5; 8; *; 21; *; 55

A. 13 and 34
B. 11 and 24
C. 13 and 23
D. 11 and 34

10. What is the size of angle x in the following diagram?



A. 60°
B. 20°
C. 90°
D. 30°

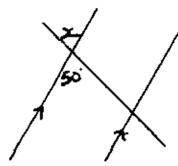
11. What is the answer: 3 + 5 x 2 - 7

A. 6
B. 9
C. -22
D. -9

12.A price of petrol is about R4,16 per litre in North West province. How much does a motorist pay to fill his 60-litre tank?

A. R249,60
B. R41,60
C. R416,60
D. R260,60

13. Angle x is equal to .... because ....



A. 50° because it is vertically opposite to the given angle
B. 50° because it is an alternate angle to the given angle.
C. 130° because it forms part of a straight angle.
D. 50° because it is a corresponding angle to the given angle.

14.A car travels at a speed of 120 km/h. This means that it will cover a distance of _____in________hour(s).

A. 120 km in 120 hours
B. 60 km in 60 hours
C. 120 km in 1 hour
D. 1 km in 120 hours

15. Work out the following: 20 x 0,5 x 0 = ...

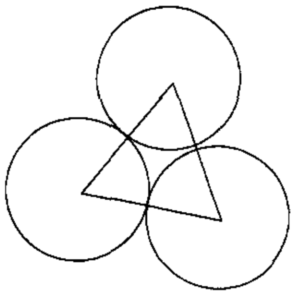
A. 25
B. 0
C. 10
D. 20,5

16. In $2x$, 2 is called.......... and x is called.............

A. Numerical coefficient and multiple
B. Variable and numerical coefficient
C. Multiple and factor.
D. Numerical coefficient and variable

17. The following three circles are equal, this means that their circumferences and radii are equal. What is the name of the triangle formed by joining the radii as shown in the diagram below?

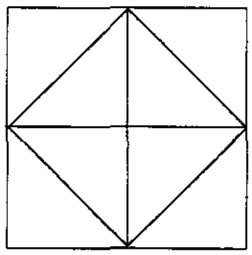
A. equilateral triangle.
B. Isosceles triangle.
C. Scalene triangle.
D. right-angled triangle.



18. Solve the following equation and find the value of x: $2(2x + 1) = 10$

A. $x = 4$
B. $x = -2$
C. $x = 2$
D. $x = -4$

19. How many squares are in the following diagram?



A. 2
B. 6
C. 4
D. 5

20. 2^3 is the same as

A. 6
B. 23
C. 32
D. 8

21.A 14% VAT is charged when buying a R50 'bafana-bafana' hat. How much VAT is charged?

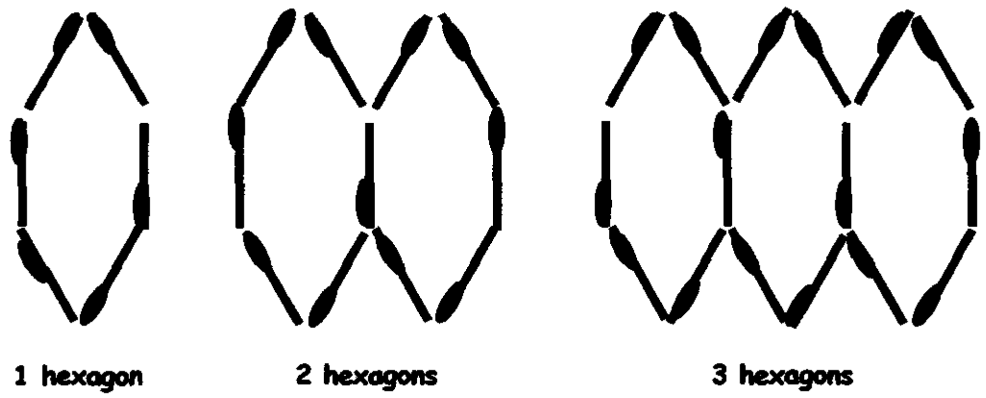
A. R7
B. R14
C. R5
D. R50

22.What does VAT stand for?

A. value added tax.
B. values added together.
C. variable and tax.
D. value and tax

The following information will guide you to answer questions 23; 24 and 25.

Matchsticks were used to construct hexagons as follows:



This information is summarised in the following table.

Number of hexagons	Number of matchsticks used
1	(5 x 1) + 1 = 6 matchsticks
2	(5 x 2) + 1 = 11 matchsticks
3	(5 x 3) + 1 = 16 matchsticks
4	* = * matchsticks
t	* = * matchsticks

23. What is a hexagon?

A. three-sided figure
B. six-sided figure
C. four-sided figure
D. five-sided figure

24. If you use the above table, how many matchsticks do you need to construct 4 hexagons?

A. 17 matchsticks
B. 20 matchsticks
C. 21 matchsticks
D. 18 matchsticks

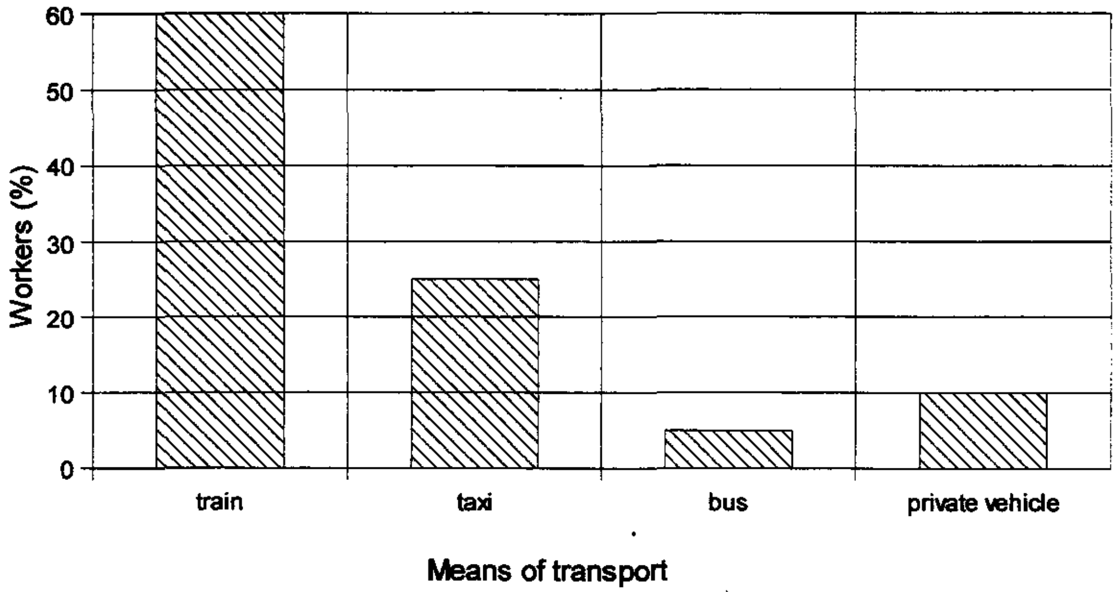
25.How many matchsticks do you need to construct *t* hexagons?

A. $(5 + t) + 1$ matchsticks
B. $5t + 1$ matchsticks
C. $t + 1$ matchsticks
D. $5 \times t$ matchsticks

26. Do you think $\sqrt{9} + \sqrt{16}$ is the same as $\sqrt{9 + 16}$?

A. No because when I simplify $\sqrt{9} + \sqrt{16}$ I get 5 and $\sqrt{9 + 16}$ gives 7
B. Yes because when I simplify $\sqrt{9} + \sqrt{16}$ I get 25 and $\sqrt{9 + 16}$ gives 25
C. No because when I simplify $\sqrt{9} + \sqrt{16}$ I get 7 and $\sqrt{9 + 16}$ gives 5
D. Yes because when I simplify $\sqrt{9} + \sqrt{16}$ I get 7 and $\sqrt{9 + 16}$ gives 7

27. Workers travelling from Mabopane to Pretoria use different means of transport daily as shown in the diagram below. Which of the following statements is true?



A. less than half of workers use trains.
B. buses are used more than private vehicles.
C. more than half of workers use taxis.
D. the majority of workers use train to travel to work.

28.John has scored 15 out of 20 in a maths test. What percentage has he obtained?

A. 75%
B. 65%
C. 20%
D. 15%

29.Rands are used in South Africa and dollars are used in United States of America for buying and selling goods. If R10,50 is equal to 1 dollar, how much (in rands) will you pay for a book that costs 5 dollars in the USA?

A. R5,00
B. R52,50
C. R5,50
D. R15,25

30. If you solve this equation $3(x + 2) = x + 8$, the value of x will be..........

A. $x = 3$
B. $x = 2$
C. $x = 1$
D. $x = -1$

Appendix D: Biographical information questionnaire

BIOGRAPHICAL INFORMATION QUESTIONNAIRE

(FOR RESEARCH PURPOSES)

NB: Please complete this questionnaire as honestly as possible. The information you provide will only be used for the purpose of research and will remain confidential.

1. Qualifications:
   - 1.1. Highest standard passed..
   - 1.2. Professional qualifications....................................
   - 1.3. Academic qualifications.......................................
   - 1.4. Current studies...
2. Teaching experience:
   - 2.1. How long have you been teaching mathematics?......................
   - 2.2. Which teaching and/or learning method(s) do you predominantly use in your mathematics lessons?..
     ....
     ....
     ....
     ....
   - 2.3. Briefly explain the rationale for using the methods you mentioned in 2.2 (Give your answer(s) in point form)
     ....
     ....
     ....
     ....
     ....
     ....
3. The following are some of the learning skills that can either enhance or inhibit learning of mathematics. Do you have the capability to identify the effects of these learning skills in learners? (Tick either YES or NO in the following table).

Learning skill	*YES*	*NO*
1. Study Attitudes		
2. Mathematics Anxiety		
3. Problem Solving Behavior		
4. Study Habits		
5. Study Milieu		

4. Give a brief description of the characteristics associated with the learning skill you ticked **YES.**
   - 4.1. Study Attitudes..
   - 4.2. Mathematics Anxiety...
   - 4.3. Problem Solving Behavior..
   - 4.4. Study Habits...
   - 4.5. Study Milieu...
5. Since the dawn of the new curriculum in South Africa, cooperative teaching/learning method(s) are encouraged. Do you find this method effective or not effective in Mathematics teaching/learning? (Support your answer - Give your answer(s) in a point form point)

....
....
....
....

(IF YOU NEED MORE SPACE, USE THE REVERSE SIDE OF THIS QUESTIONNAIRE)

Appendix E: Letter of permission to conduct research

P. O. Box 11917
Tramshed
0126
07 February 2002

The District Manager
....

Re:**<u>REQUEST:PERMISSION FOR RESEARCH WORK - M.Ed (Maths Education)</u>**

I am a PU for CHE student doing an M.Ed (Maths Education). I intend to conduct a research on: **The influence of an Hour-Glass model of cooperative learning on Mathematics academic achievement of middle school learners in the context of crowded classes.**

I therefore ask for a permission to conduct such an investigation in two of your middle schools (Grade 8 classes).

I appreciate your cooperation in this regard.

Yours faithfully

R.D.Sekao (Mr)
Student Number:
Cell Number:

Appendix F: Score sheet for mathematics academic achievement test

PRE/POST-TEST: Academic Achievement Test: Score sheet

Learner Number	***Raw Score***	**%**
BA01		
BA02		
BA03		
BA04		
BA05		
BA06		
BA07		
BA08		
BA09		
BA10		
BA11		
BA12		
BA13		
BA14		
BA15		
BA16		
BA17		
BA18		
BA19		
BA20		
BA21		
BA22		
BA23		
BA24		
BA25		
BA26		

Appendix G Answer sheet: Mathematics academic achievement test

ANSWER SHEET:PRE/POST-TEST ACADEMIC ACHIEVEMENT

Learner Number:.............................

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D
11	A	B	C	D
12	A	B	C	D
13	A	B	C	D
14	A	B	C	D
15	A	B	C	D
16	A	B	C	D
17	A	B	C	D
18	A	B	C	D
19	A	B	C	D
20	A	B	C	D
21	A	B	C	D
22	A	B	C	D
23	A	B	C	D
24	A	B	C	D
25	A	B	C	D
26	A	B	C	D
27	A	B	C	D
28	A	B	C	D
29	A	B	C	D
30	A	B	C	D