THE EFFECT OF A DYNAMIC TECHNOLOGICAL LEARNING ENVIRONMENT ON THE GEOMETRY CONCEPTUALISATION OF PRE-SERVICE MATHEMATICS TEACHERS

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ABBREVIATIONS

GSP®	: Geometer's Sketchpad®
MA	: Mathematics anxiety
NRF	: National Research Foundation
PMTs	: Pre-service mathematics teachers
PSB	: Problem-solving behaviour
R	: Interviewer (Researcher)
RME	: Realistic mathematics education
S	: Interviewee (PMT)
SA	: Study attitude
SH	: Study habits
SM	: Study milieu
SOM	: Study Orientation in Mathematics
SOSI	: Spatial Orientation and Spatial Insight-Project
ZPD	: Zone of proximal development
SOW	: Studie Oriëntasie in Wiskunde
VWO's	: Voor-diens wiskunde onderwysers

SUMMARY

The effect of a dynamic technological learning environment on the geometry conceptualisation of pre-service mathematics teachers

Traditionally, geometry at school starts on a formal level, largely ignoring prerequisite skills needed for formal spatial reasoning. Ignoring that geometry conceptualisation has a sequential and hierarchical nature, causes ineffective teaching and learning with a long lasting inhibiting influence on spatial development and learning.

One of the current reform movements in mathematics education is the appropriate use of dynamic computer technology in the teaching and learning of mathematics. Concerning mathematics education, the lecturers may involve the introduction of both dynamic computer technology and mathematics in meaningful contexts that will enable interplay between the two. Pre-service mathematics teachers (PMTs) can be encouraged to become actively involved in their learning and, therefore, less frustrated in their study orientation in mathematics. Therefore, such learning environments may be essential to enhance the conceptual understanding of PMTs.

To be able to reach their eventual learners, PMTs' own conceptual understanding of geometry should be well developed. When PMTs have conceptual understanding of a mathematical procedure, they will perceive this procedure as a mathematical model of a problem situation, rather than just an algorithm.

This study aimed at investigating the effect of a technologically enhanced learning environment on PMTs' understanding of geometry concepts and their study orientation in mathematics, as prerequisite for deep conceptualisation.

A combined quantitative and qualitative research approach was used. The quantitative investigation employed a pre-experimental one-group pre-test post-test design. A Mayberry-type test was used to collect data with regard to PMTs' conceptualisation of geometry concepts, while the Study Orientation in Mathematics (SOM) questionnaire was used to collect data with regard their study orientation in mathematics. The qualitative investigation employed phenomenological interviews to collect supplementary information about the participating PMTs' experiences and assessment of the influence of the use of the dynamic software Geometer's Sketchpad (GSP) ® on their learning and conceptualisation of geometry concepts.

During post-testing the participating group of PMTs achieved practically significantly higher scores in the Mayberry-type test, as well as in all fields of the SOM questionnaire. Results seem to indicate that PMTs gained significantly in the expected high levels of conceptualisation, as well as high degrees of acquisition of those levels during the intervention programme. The main conclusion of the study is that a technologically enhanced learning environment (such as GSP) can be successfully utilised to significantly enhance PMTs' conceptualisation and study orientation, as prerequisite for deep conceptualisation, in geometry.

Key terms for indexing:
Mathematics and teaching; mathematics and technology; mathematics teacher; teacher education; dynamic software; computer technology; mathematics conceptualisation; Piaget; Vygotsky; Van Hiele; network theory, constructivism; behaviourism.

OPSOMMING

Die invloed van 'n dinamiese tegnologiese leeromgewing op die konseptualisering van voordiens-wiskunde-onderwysers

Tradisioneel begin meetkunde op skool op 'n formele vlak, wat die vereiste vaardighede nodig vir formele ruimtelike beredenering ignoreer. Die miskenning van die feit dat meetkunde konseptualisering 'n sekwensiële en hiërargiese aard het, veroorsaak oneffektiewe onderrig en leer met 'n langdurige stremmende invloed op ruimtelike ontwikkeling en leer.

Een van die huidige hervormingsbewegings in wiskunde onderrig is die gepaste gebruik van dinamiese rekenaartegnologie in die onderrig en leer van wiskunde. Rakende wiskunde onderrig, kan die dosente die bekendstelling van beide dinamiese rekenaartegnologie en wiskunde in betekenisvolle kontekste plaas wat wisselwerking bewerk. Voor-diens wiskunde onderwysers (VWO's) kan aangemoedig word om aktief betrokke te raak by hulle leer, en om sodoende minder gefrustreerd te wees in hulle studie orientasie in wiskunde. Daarom is sulke leeromgewings essensieel vir die bevordering van die konsepsuele begrip van VWO's.

VWO's se eie konseptuele begrip van meetkunde moet goed ontwikkel wees alvorens hulle hulle uiteindelike leerders kan bereik. Wanneer VWO's konsepsuele begrip het van 'n wiskundige prosedure, neem hulle die prosedure waar as 'n wiskundige model van 'n probleem situasie, eerder as net 'n algoritme.

Hierdie studie het gepoog om die effek van 'n tegnologies verrykde leeromgewing op VWO's se begrip van meetkunde konsepte en hulle studie oriëntasie in wiskunde, as voorvereiste vir diep konseptualisasie, te bestudeer.

'n Gekombineerde kwantitatiewe en kwalitatiewe benadering is gebruik. Die kwantitatiewe ondersoek het 'n pre-eksmerimentele een-groep voor-toets na-toets ontwerp gehad. 'n Mayberry-tipe toets is gebruik om data te versamel aangaande VWO's se konseptualisasie van meetkunde konsepte, terwyl die Studie Oriëntasie in Wiskunde (SOW) vraelys gebruik is om data te versamel met betrekking tot hulle studie orientasie in wiskunde. Die kwalitatiewe studie het gebruik gemaak van fenominalogiese onderhoude om bygaande inligting te versamel oor die deelnemende VWO's se ervarings. Dit het verder gedien as 'n evaluasie van die invloed van die gebruik van die dinamiese sagteware Geometer's Sketchpad (GSP) ® op hulle leer en konseptualisasie van meetkunde konsepte.

Gedurende die na-toetse het die deelnemende VWO's prakties veelseggende verbeterde punte behaal in die Mayberry-tipe toets, sowel as in al die velde van die SOW-vraelys. Die resultate dui aan dat VWO's beduidend gebaat het in die verwagte vlakke van konseptualisasie, sowel as in die vlakke van verwerwing gedurende die ingrypingsprogram. Die hoofkonklusie van die studie is dat 'n tegnologies verrykde leeromgewing, soos GSP, suksesvol gebruik kan word om VWO's se konseptualisasie en studie orientasie, as voorvereiste vir diep konseptualisasie in meetkunde, beduidend te verbeter.

Sleutelterme vir indeksering:
Wiskunde en onderrig; wiskunde en tegnologie; wiskunde onderwyser; onderwysersopleiding; dinamiese sagteware; rekenaartegnologie; wiskunde konseptualisasie; Piaget; Vygotsky; Van Hiele; netwerkteorie, konstruktivisme; behaviorisme.

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CHAPTER 1

INTRODUCTION AND PROBLEM STATEMENT

1.1 ORIENTATION

The fundamental characteristics of any teaching situation include the specific outcomes that the teacher aims to meet in order to attain. The teaching aim, the thoughts of the teacher and his beliefs are interwoven with each other (Steyn, 1988:160,161). Teachers should possess specific skills to be able to teach effectively, therefore, they need to have adequate skills regarding conceptual understanding (Nieuwoudt, 1998:169).

Korthagen and Kessels (1999:6) propose new ways of preparing pre-service mathematics teachers (PMTs) for their profession. The intended learning processes start from situated knowledge, developed in the interaction of the PMTs with realistic problem situations. The concrete situations thus remain the reference points during the learning process.

Mathematics education has changed considerably over the last twenty years, shifting from a mechanistic and structuralist approach to a realistic constructivist approach. The mechanistic point of view is that mathematics is a system of rules and algorithms. The emphasis is on verifying and applying these rules to problems that are similar to previous ones. In the structuralist view mathematics is an organised, deductive system and the learning process in mathematics education should be guided by the structure of this system (Korthagen & Kessels, 1999:6).

Realistic constructivist mathematics education of PMTs aims at the construction of their own mathematical knowledge by giving meaning to problems from realistic contexts. Many of these attempts can be characterised by an emphasis on reflective teaching, implying that pre-service mathematics teacher development is conceptualised as an ongoing process of experiencing practical teaching and learning situations. PMTs are challenged to develop their own strategies for solving such practical problems (Korthagen & Kessels, 1999:7).

One of the main premises of the current reform efforts in mathematics teacher education is that lecturers want to empower PMTs mathematically to ensure that they are confident and successful in exploring and engaging in significant mathematical situations (Allsopp, Lovin, Green & Savage-Davis, 2003:312). A study by Wilson (1993:247,248) revealed that teachers with higher levels of mathematical knowledge were more conceptual in their teaching than teachers with lower levels of knowledge. Teachers with lower levels of mathematical knowledge were more rules-based. Therefore, teachers must understand mathematical concepts well in order to teach them well.

According to Bright and Prokosch (1995:338) dynamic computer technology is useful in developing conceptual understanding. House (2002:113) said that computer-assisted instruction for mathematics learning can produce an effective learning situation. The effective environment for PMTs to learn mathematical concepts, to explore patterns and processes, and to solve problems, can be one in which they use dynamic computer technology (Fey, 1992:65).

The use of dynamic computer software allows PMTs to learn fundamental skills in new ways, so they do not have to relive experiences with frustration and failure (Reglin, 1990:405). According to Fey (1992:7,11,13), an environment where dynamic computer technology is available, results in the emphasis of mathematics teaching on meaningful concept development and problem solving, and not on computational procedures. Using dynamic computer technology, PMTs are able to discover those properties inductively and be able to make it their own. The use of dynamic computer technology must be connected to the broader objective – providing all PMTs access to a broad range of mathematical ideas.

The dynamic technology environment becomes a mathematics laboratory where PMTs may actively manipulate mathematical ideas as they construct their own concepts, where logic is established and they develop their reasoning skills (McCoy, 1996:439,440). McCoy (1996:446) found that the results of varied studies indicated that dynamic computer technology was effective in improving the PMTs intuitive understanding when compared to a control group, and the researchers also concluded that the computer-intensive group had develop clearer and deeper concepts.

According to Maree, Prinsloo and Claasen (1997:3,4) there is a significant relationship between study orientation in mathematics and mathematics achievement. Learners become frustrated when they do not understand mathematics. Learners' affective attitude influences their attitude towards mathematics. If mathematics does not make sense to learners, they become anxious and uncertain. When mathematics is presented in a too abstract manner (especially in the early stages), without learners being adequately exposed to enough concrete material, it leads to incomplete conceptualisation. Learners' attitude towards the solving of problems and their study environment forms an integrated part of their study orientation.

Maree (1997:3,4) highlights the following facets of study orientation in mathematics:

- The formation of basic concepts in mathematics is important and is an essential prerequisite for learning more advanced work in mathematics.
- The learners do not understand the relation between concepts when conceptualisation is incomplete, and therefore they will use theorems and formulas without thinking whether they are applicable to the situation at hand.

With this background in mind, the following questions can be asked:

- What will the effect of a dynamic technological learning environment be on the conceptual understanding of PMTs in geometry?
- How does the use of dynamic technology influence the conceptual understanding of PMTs in geometry?
- What will the effect of a dynamic technological learning environment be on the PMTs' study orientation?

1.2 AIMS OF THE RESEARCH

The aim of the research was to investigate the effect of a dynamic technological learning environment on the conceptualisation of PMTs. In particular, the research aimed to:

1.2.1 determine what effect a dynamic technological learning environment has on the conceptual understanding of PMTs in geometry.

1.2.2 determine how the use of a dynamic technological learning environment influences the conceptual understanding of PMTs in geometry.

1.2.3 determine what effect a dynamic technological learning environment has on the study orientation of PMTs in geometry.

1.3 RESEARCH DESIGN

1.3.1 Literature study

An intensive and comprehensive review of the relevant literature has been done. In A DIALOG search the following keywords was used: "mathematics and teaching", "mathematics and technology", "mathematics teacher", "teacher education", "dynamic software", "computer technology" and "mathematics conceptualisation".

1.3.2 Empirical Study

A combination of qualitative and quantitative research methods was employed (see § 5.3.1).

1.3.2.1 Quantitative design

Figure 1.1 depicts the pre-experimental design, namely the one-group pre-test/post-test design (Leedy & Ormrod, 2001:235) which was used with respect to research aim 1 and research aim 3.

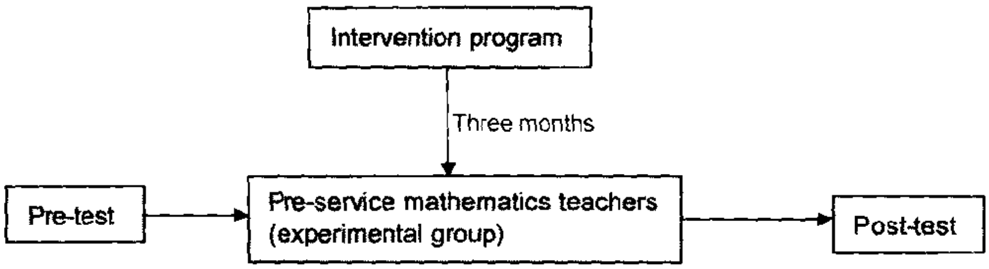


Figure 1.1: Experimental design

Population and sample

The study population consisted of 371 third year education students (in 6 classes) following the general mathematics module in geometry at the North-West University, Potchefstroom campus. A sample of 26 prospective mathematics teachers in one of the classes were chosen to take part in the experiment.

Instruments

The participants were presented with two questionnaires before intervention took place, as well as after the intervention took place. The Mayberry Type Test was conducted to determine if the intervention had any influence on the conceptualisation of PMTs. The SOM-questionnaire was distributed to determine if the intervention had any influence on the study orientation in mathematics, of the PMTs.

Statistical Analysis

Quantitative data analysis was done with the help of the Statistical Services of the North-West University, Potchefstroom campus.

Research Procedure

A literature review was done of related articles aimed at improving the conceptual knowledge of PMTs.

Only one group was used and there was a pre-test to test the conceptual understanding of PMTs before intervention and a post-test to evaluate the conceptual understanding of the PMTs after intervention.

Quantitative data analysis was done and will be discussed in more detail in chapter 5. Results were evaluated, analysed and interpreted as is reported in chapter 6. Final conclusions are given in chapter 7.

1.3.2.2 Qualitative design

The literature study forms the basis for the self-developed questionnaires, structured interviews and observation schedules (see Figure 1.2) used in the qualitative *phenomenological survey* with regard to research aim 2.

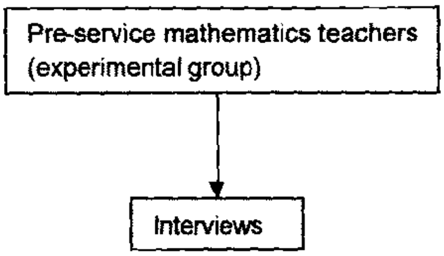


Figure 1.2: Qualitative design

Population and sample

A sample of 3 low and 4 top performers were identified to take part in the qualitative part of the research. The PMTs were selected on the basis of their profile as reflected by their examination in the geometry module.

Instruments

Self-constructed questionnaires and interview schedules were used to evaluate the PMTs with respect to the impact of the intervention.

Statistical Analysis

Qualitative data analysis was done (see § 5.3.2.4).

Research Procedure

Qualitative research was conducted over a period of three months. The goal was to determine whether and how the intervention program assisted in developing the conceptual understanding of the pre-service mathematics teachers.

Results were evaluated, analysed and interpreted and conclusions were made.

1.4 ETHICAL ASPECTS

A letter, requesting permission to use the above-mentioned study population, was sent to the Dean of the Faculty of Education Sciences of the North-West University, Potchefstroom campus. In addition, the relevant school director, subject head, lecturer and selected class of students were consulted to obtain their permission and full cooperation. The research project formed part of a bigger national project, sponsored by the National Research Foundation (NRF), and took place with full permission and cooperation of the Project Team.

1.5 STRUCTURE OF DISSERTATION

The research is presented in seven chapters as illustrated in Figure 1.3.

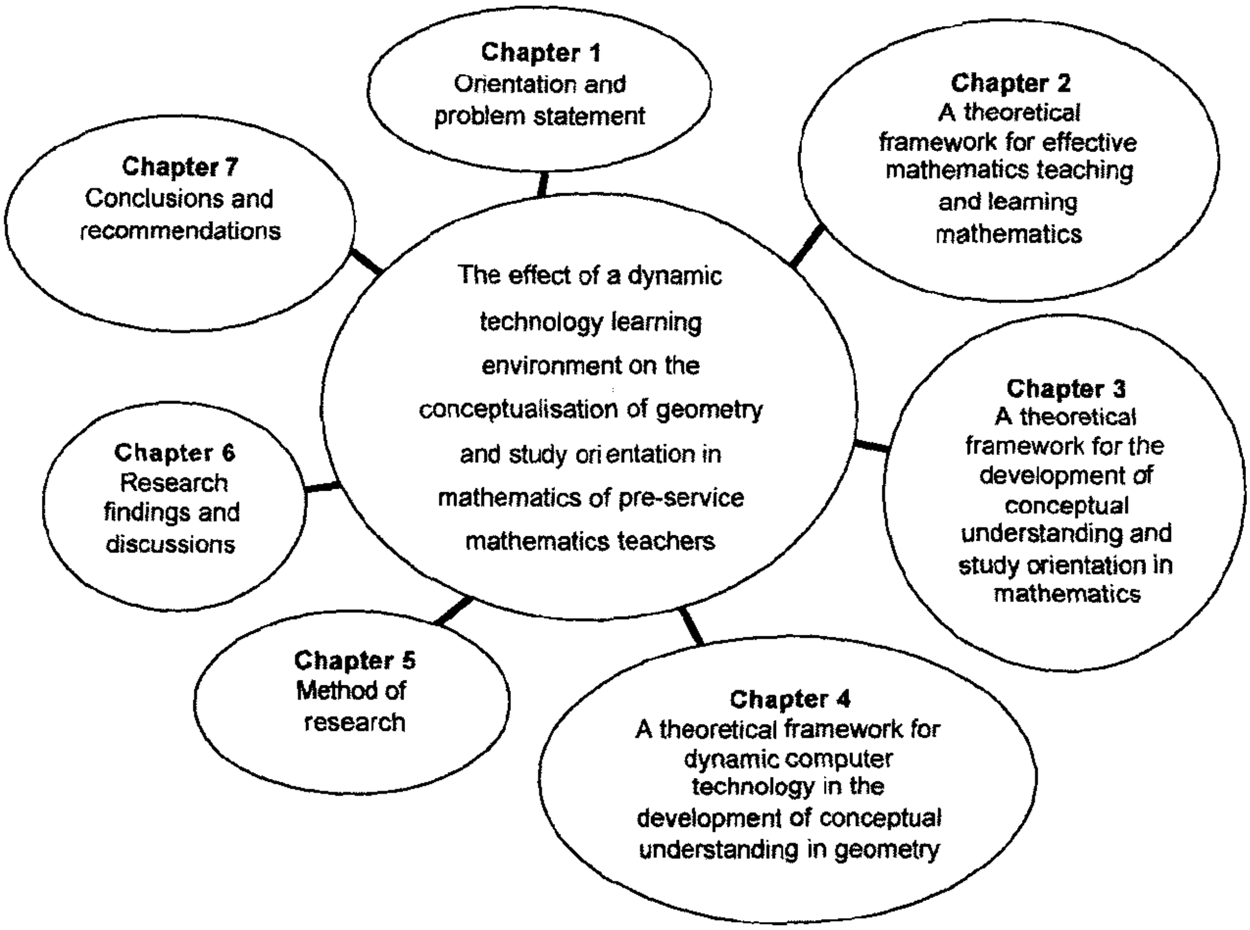


Figure 1.3: Presentation of chapters

CHAPTER 2

A THEORETICAL FRAMEWORK FOR EFFECTIVE MATHEMATICS TEACHING AND LEARNING

2.1 INTRODUCTION

According to Romberg and Kaput (1999:15,16), society's perception of the mathematical content that learners are expected to understand is changing, as is the field of mathematics itself. We can no longer assume that mathematics is a fixed body of concepts and skills to be mastered. The aims of mathematics teaching can be described as teaching learners to use mathematics to build and communicate ideas and to use it as a powerful analytic and problem-solving tool.

The aim of this chapter is to present a framework for effective mathematics teaching and learning. In this regard the views of Piaget and Vygotsky will be discussed. The effect of behaviourism and constructivism on mathematics teaching will also be discussed. Attention will be paid to Van Hiele's learning theory because one aspect of the theory deals with the belief that learners' geometric thinking skills develop in levels. The influence of process-product teaching, problem-based teaching and realistic mathematics education will also be discussed.

2.2 THEORETICAL PERSPECTIVES ON COGNITIVE DEVELOPMENT

2.2.1 Piaget

Jean Piaget spent much of his professional life listening to learners. He focused on universal learner development. Essentially Piaget's explanation of the development of intelligence postulates a series of stages according to which the learner functions in the world. Each preceding stage is a necessary condition for the subsequent stage. Piaget claims that development proceeds according to a series of transformations of one stage into another (Atkinson, 1983:13).

2.2.1.1 Piaget's theory of developmental stages

Piaget (1974:117) postulates four stages of mental development in which learners understand the world, namely the sensori-motor-, pre-operational-, concrete operational- and formal operational stages.

Sensori-motor Stage (birth to about 2 years)

This period is characterised by a number of performances such as the organisation of spatial relationships, the organisation of objects and a notion of their performance, and the organisation of casual relationships (Piaget, 1974:117).

According to Atkinson (1983:13-15) infants think and understand the world around them through their senses, using their eyes, ears, mouth and hands. At this level, infants develop their abilities through the coordination of sensations, their physical movements and actions in the environment. Learners use their senses and emerging motor skills to explore the environment. Verbal interaction and an object-rich setting are very important at this time.

Pre-operational Stage (about 2 to 7 years)

The learner is now able to have operational thought though symbolic function. The learner cannot perform referable internalised actions (Piaget, 1974:117).

Pre-school learners begin to represent the world with symbols. Learners at this stage have increased capacity for symbolic thinking and can go beyond their earlier sensori-motor discoveries through the use of language and images. The learner is perceptually bound and is unable to reason logically concerning concepts that are discrepant from visual clues (Atkinson, 1983:23,24).

Concrete-Operational Stage (about 7 to 11 years)

The learner is able to perform operations, internalised actions. These operations are concrete, for instance, the learner can classify concrete objects, establish correspondences between them or use numerical operations on them (Piaget, 1974:117).

According to Atkinson (1983:31-33) learners in this stage can think logically and are able to conserve, sedate, classify and organise objects into different sets. The learner is able to use this logic to analyse relationships and structure his environment into meaningful categories.

Formal Operational Stage (about 11 to adult)

This period can be characterised by formal or propositional operations. This means that the operations are no longer applied solely to the manipulation of concrete objects, but now cover hypotheses and propositions that the learner can use as abstract hypotheses and from which he can reach deductions through formal or logical means (Piaget, 1974:117).

Atkinson (1983:40-42) says that adolescents think in more logical and abstract ways. They can reason with symbols that are beyond the world of concrete experiences. They can imagine many possible combinations, separate real from possible, deal with hypothetical proportions and combine elements in a systematic way. They may pass into the period of formal operations and develop the ability to manipulate concepts abstractly through the use of propositions and hypotheses.

2.2.1.2 Piaget's intra-, inter- and trans-operational levels

According to Nixon (2005:23,47), Piaget and Garcia (1989), identified three levels in the development of thought, namely that of intra-operational or perceptual level, inter-operational or conceptual level and trans-operational or abstract level. These levels are not bound to learners' ages or fixed stages of development.

Intra-operational or perceptual level

The perceptual level may be related to Piaget's pre-operational level of thought. At these level relations appear in forms that might be isolated. In geometry, properties of individual figures are studied, but no consideration is given to space or to transformations of these figures. The intra-operational level applies to young learners, but could be applied to the introductory stage of the learning of any concept. Learners need to acquire an intuitive appreciation for concepts and be provided with examples, diagrams, pictures and illustrations that help them visualise or form mental pictures of concepts that have been introduced (Nixon, 2005:47,48,84).

Inter-operational or conceptual level

This conceptual level may be related to Piaget's concrete operation level of thought. It is characterised by efforts to find relationships. At this level learners are able to understand properties of figures and the learners are able to interrelate properties of figures and analyse specific cases. Whereas isolated forms are identified with perceptual levels, correspondences and transformations amongst these forms characterise the conceptual level (Nixon, 2005:85,102).

Rising up from the perceptual level to the conceptual level is an important step in the acquisition of knowledge, since it also forms a vital link between the perceptual level and the abstract level (Nixon, 2005:99).

Trans-operational or abstract level

This abstract level may be related to Piaget's original formal operation level of thought and involves definitions, proofs and theorems. At this level there are not only transformations, but also synthesis between them, which leads to the development of structures (Nixon, 2005:122,150).

Although any new topic needs to begin at the perceptual level and pass through the conceptual level, it is the attainment of the abstract level that is the ultimate aim in geometry (Nixon, 2005:161).

Encouraging learners to participate and pass through the perceptual level, the conceptual and abstract levels of learning help to establish a mode of investigation and a way of thought. These three levels of development can assist learners in developing mental structures to help them understand new learning material and integrate it with other material. Learners become accustomed to the processes involved, and therefore they could become independent in their study (Nixon, 2005:54,161,162).

2.2.1.3 Piaget's theory of cognitive development

For Piaget, there are four factors that determine cognitive development (Webb, 2001:93). Each is vital, as it is the interaction of these components that results in cognitive growth. Cognitive development includes:

- maturation of the nervous system, providing physical capabilities. Maturation refers to the onset of an ability. It occurs without previous training (Atkinson, 1983:154).
- social interaction that offers opportunities for the observation of a wide variety of behaviours.
- experiences based on interactions with the physical environment that leads to the discovery of the properties of objects and the development of organisational skills.
- an internal self-regulation mechanism that responds to environmental stimulation by constantly fitting new experiences into existing cognitive structures and revising these structures to fit the new data. A balance between the cognitive structures and new data maximises cognitive function.

2.2.1.4 Implications for teaching mathematics

Piaget proposes that cognitive development occurs in stages from birth to about adolescence. Thus, it seems appropriate that learning experiences should be organised and sequenced in terms of the PMTs developmental stage.

According to Piaget, secondary school learners are usually concretely and formally operational in terms of development. At this stage, learners demonstrate the beginning of logical thought. Although they are able to use certain logical operations, their thinking is concrete rather than abstract. Thus, in teaching geometry, learners should be provided with concrete objects to facilitate understanding. In teaching structural properties of, for example quadrilaterals, teachers should keep in mind that the learner is not proficient in stating generalisations (Wilson, 2001:85).

Wilson (2001:85) says that learners, who are formally operational, should be provided with the opportunity to develop relationships and think abstractly. There should be opportunities for these learners to solve problems by answering questions in a systematic way until reasonable conclusions are reached.

2.2.2 Vygotsky's Socio-cultural Theory

Vygotsky had a great influence on modern constructivism. A critical event in Vygotsky's life occurred in 1924 at the Second Psychoneurological Congress in Leningrad. Vygotsky contended that humans have the capacity to alter the environment for their own purposes. This adaptive capacity distinguishes humans from lower forms of life (Schunk, 1996:213,214).

2.2.2.1 Basic Principles

Schunk (1996:214-216) theorises that one of Vygotsky's central contributions to psychological thought was his emphasis on socially meaningful activity as an important influence on human consciousness. Rather than discarding consciousness or the role of the environment, he sought a middle ground of taking environmental influence into account through its effect on consciousness.

Vygotsky considered the social environment as critical for learning and thought the integration of social and personal factors produced learning. Social activity is a phenomenon that helps explain changes in consciousness and establishes a psychological theory that unifies behaviour and mind (Schunk, 1996:217).

According to Schunk (1996:217) the social environment influences cognition through its cultural objects, its language and social institutions. Cognitive change results from using cultural tools in social interactions and from internalising and mentally transforming these interactions. Vygotsky's position is an example of dialectical constructivism because it emphasises the interaction between persons and their environment.

Berger (2004:81) theorises that a learner uses a new mathematical sign (which may be in the form of symbols, graphs, diagrams or geometric shapes) both as an object with which to communicate (like a word). as an object on which to focus, and to organise his or her mathematical ideas (like a word). Through this sign usage, the mathematical concept evolves for the learner so that it eventually has personal meaning, like the meaning of a new word does for a child. Because the usage is socially regulated, the concept evolves for the learner so that its usage concurs with its usage in mathematical community.

An important concept in Vygotsky's theory is the zone of proximal development (ZPD) defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under guidance or in collaboration with more capable peers" (Vygotsky, 1978:6).

The ZPD represents the amount of learning possible by a learner given the proper instructional conditions. In the ZPD the learner and teacher work together on tasks that the learner could not perform independently because of the level of difficulty. As a result of pedagogical interventions within the ZPD of the learner, the learner does not remain inactive, but rather begins to use this mathematical sign (for example the properties of triangles) in communication with others and in mathematical activities. It is these functional usages of mathematical signs (like activities comprising manipulations, comparison and associations) that give an initial access point to the new object. Furthermore, this functional usage of the mathematical sign is mediated by the learner's knowledge of related signs (Berger, 2004:85).

According to Berger (2004:86) a learner starts to use a new mathematical sign in mathematical pursuits such as problem-solving, applications and proofs, before he or she fully understands how to use that mathematical sign in a culturally meaningful way. Through this use of the mathematical sign, the learner is able to engage with the mathematical object and to communicate with others about his or her developing mathematical ideas. On account of this functional use, the mathematical sign begins to acquire personal meaning for that learner and the learner begins to use the sign in mathematical discourse in a way that is compatible with its socially sanctioned meaning.

Vygotsky (1986:106) says that learners use words for communication purposes and for organising their own activities before they have a full understanding of what these words mean. It is a functional use of the word or any other sign that plays a central role in concept formation.

Cognitive change occurs in the ZPD as teacher and learner share cultural tools, and it is this culturally mediated interaction that produces cognitive change when it is internalised in the learner. Working in the ZPD requires a good deal of guided participation. However, learners do acquire cultural knowledge passively from these interactions. Rather, learners bring their own understandings to social interactions and construct meaning by integrating those understandings with their experiences in the context. During the interaction, the learner modifies his or her beliefs about working in the area based on present understandings and in light of new knowledge acquired from the teacher (Schunk, 1996:215,216).

2.2.2.2 Application

Vygotsky's ideas lend themselves to many educational applications. The field of self-regulation has been strongly influenced by theory.

According to Schunk (1996:216-218) a major application involves the concept of instructional scaffolding, which refers to the process of controlling task elements that are beyond the learner's capabilities so that the learner can focus on and master those features of the task that he or she can understand. Scaffolding has five major functions: to provide support, to function as a tool, to extend the range of the learner, to permit the attainment of tasks not otherwise possible and to use selectively only as needed.

2.2.3 The Van Hiele Theory

P.M. Van Hiele (1986:39) developed, in conjunction with his wife, D. Van Hiele–Geldof, the theory of cognitive levels in geometry. Van Hiele postulates that learners progress through these levels from a Gestalt-like visual level through increasingly sophisticated levels of description, analysis, abstraction and proof.

Van Hiele (1986:viii,5,6) acknowledges that his theory of cognitive levels originated with Piaget's theories, although he is critical of certain aspects of Piaget's theory. Van Hiele says that it is not necessary to refer to biological maturation to explain the development of logical thought, whereas Piaget (see § 2.2.1) suggests that the transition from one level to the next is a biological development rather than one stimulated by the learning process. The Van Hiele theory is based on the notion that learner-growth in geometry takes place in terms of identifiable levels of understanding and that the level of understanding of the learner is dependent on this experiences in geometry (Choi-Koh, 1999:301). In view of the analysis of Nixon's three levels in the development of thought (see §2.2.1.2), it becomes clear that Van Hiele's theory of cognitive levels in geometry follows the same trend.

2.2.3.1 The Van Hiele levels of geometric thought

According to Van Hiele (1986:39–47) the most prominent feature of the model is a four–level hierarchy of ways of understanding spatial ideas. Van Hiele (1986) labels his levels as recognition (level 1), analysis (level 2), informal deduction (level 3) and formal deduction (level 4).

Level 1 recognition

According to Van Hiele (1986) learners recognise and name figures based on the global, visual characteristics of the figure. At this level the learners are able to make measurements and even talk about properties of shapes, but these properties are not abstracted from the shapes at hand. It is the appearance of the shape that defines it for the learners.

Learners at this level will sort and classify shapes based on their appearances. For example, learners will recognise quadrilaterals by their global appearance and they will learn the appropriate language concerning quadrilaterals. With a focus on appearances of shapes, learners are able to see how shapes are alike and different. As a result, learners can create and begin to understand classifications of shapes (Van de Walle, 2004:347).

Level 2-analysis

Van Hiele (1986) said that learners at the analysis level are able to consider all shapes within a class, rather than a single shape. By focusing on a class of shapes, learners are able to think about what makes a rectangle a rectangle. The irrelevant features fade into the background. At this level, learners begin to appreciate that a collection of shapes belong together because of properties.

Ideas about an individual shape can now be generalised to all shapes that fit the class. Learners operating on level 2 may be able to list all the properties of squares, rectangles and parallelograms, but can not see that they are subclasses of one another (Van de Walle, 2004:347).

As learners start to develop the ability to think about properties of geometric ideas without the constraints of a particular idea, they are able to develop relationships between these properties. Observation goes beyond properties themselves and begins to focus on logical arguments about the properties. Learners at level 2 will be able to follow and appreciate an informal deductive argument about shapes and their properties. Proofs may be more intuitive than rigorously deductive. However, there is an appreciation of the fact that a logical argument is compelling. An appreciation of the axiomatic structure of a formal deductive system remains under the surface (Van de Walle, 2004:348).

Level 3-informal deduction

At level 3 learners are able to examine more than just the properties of shapes. Their earlier thinking has produced conjectures concerning relationships among properties. Are these conjectures correct? Are they true? As this analysis of the informal arguments takes place, the structure of a system complete with axioms, definitions, corollaries and postulates begins to develop, and it can be appreciated as the necessary means of establishing geometric truth. Van Hiele stresses language appropriate to this level. Learners at this level are able to work with abstract statements about geometric properties. They can clearly observe that the diagonals of a rectangle bisect each other, just as a learner at a lower level of thought can. However, at level 3, there is an appreciation of the need to prove this from a series of deductive arguments (Van de Walle, 2004:348).

Level 4-formal deduction

Learners start developing longer sequences of statements and begin to understand the significance of deduction. They are able to devise a formal geometric proof and to understand the process employed. This is generally the level at which a PMT should understand geometry (Van de Walle, 2004, 348,349).

2.2.3.2 The Van Hiele phases between levels of geometric thought

Learners' progress from one level to the next is organised into five phases of sequenced activities that emphasise exploration, discussion and integration. Van Hiele's model postulates that these five phases of instruction are necessary to enable learners at a specific level to advance to a higher level of geometric thinking (Van Hiele, 1986:50,51).

Teppo (1991:210) says that during each phase learners investigate appropriate geometric figures, develop specific language related to these figures, and engage in interactive learning activities to help them to progress to the next level.

First phase: Information

The learners learn to recognise the field of investigation based on the material that is presented to them. This material causes the learners to discover a certain structure (Van Hiele, 1986:50).

The learners learn to recognise the field of investigation based on the material that is presented to them. This material causes the learners to discover a certain structure (Van Hiele, 1986:50).

Teppo (1991:212) suggests that when a teacher wants to develop the concept of symmetry, learners can demonstrate (at this phase) the reflection of a point A about the line L using a mirror and show how this reflection can be drawn using graph paper (see Figure 2.1).

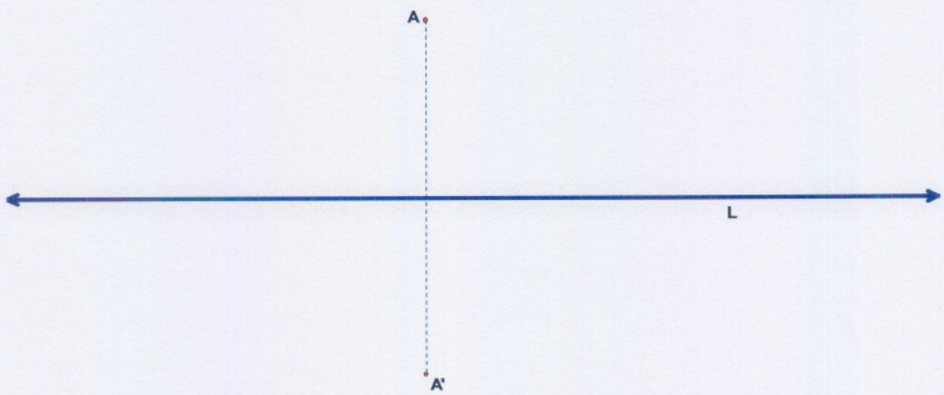


Figure 2.1: Demonstration of reflection of a point A about line L (Teppo, 1991:212)

Second phase: Directed orientation

Van Hiele (1986:50) says that learners explore the field of investigation through carefully guided, structured activities. The characteristic structures appear progressively.

According to Teppo (1991:212) learners can explore the field of inquiry through carefully guided activities, for example learners reflect the given line segments about the line L (see Figure 2.2) and determine the shape of the figure. After completing the reflections about L, they can make observations about the axes of symmetry:

a. What properties must the rhombus have to exhibit the axes of symmetry?

b. These axes are the diagonals of the figure. What observations can be made about the properties of the diagonals?

Third phase: Explication

The acquired experiences are linked to exact linguistic symbols. The customary terms are used in discussions. It is during the course of this third phase that the network of relations is partially formed (Van Hiele, 1986:51).

The learners and the teacher engage in discussions about the geometric figures, remembering to use the appropriate language (Teppo, 1991:212).

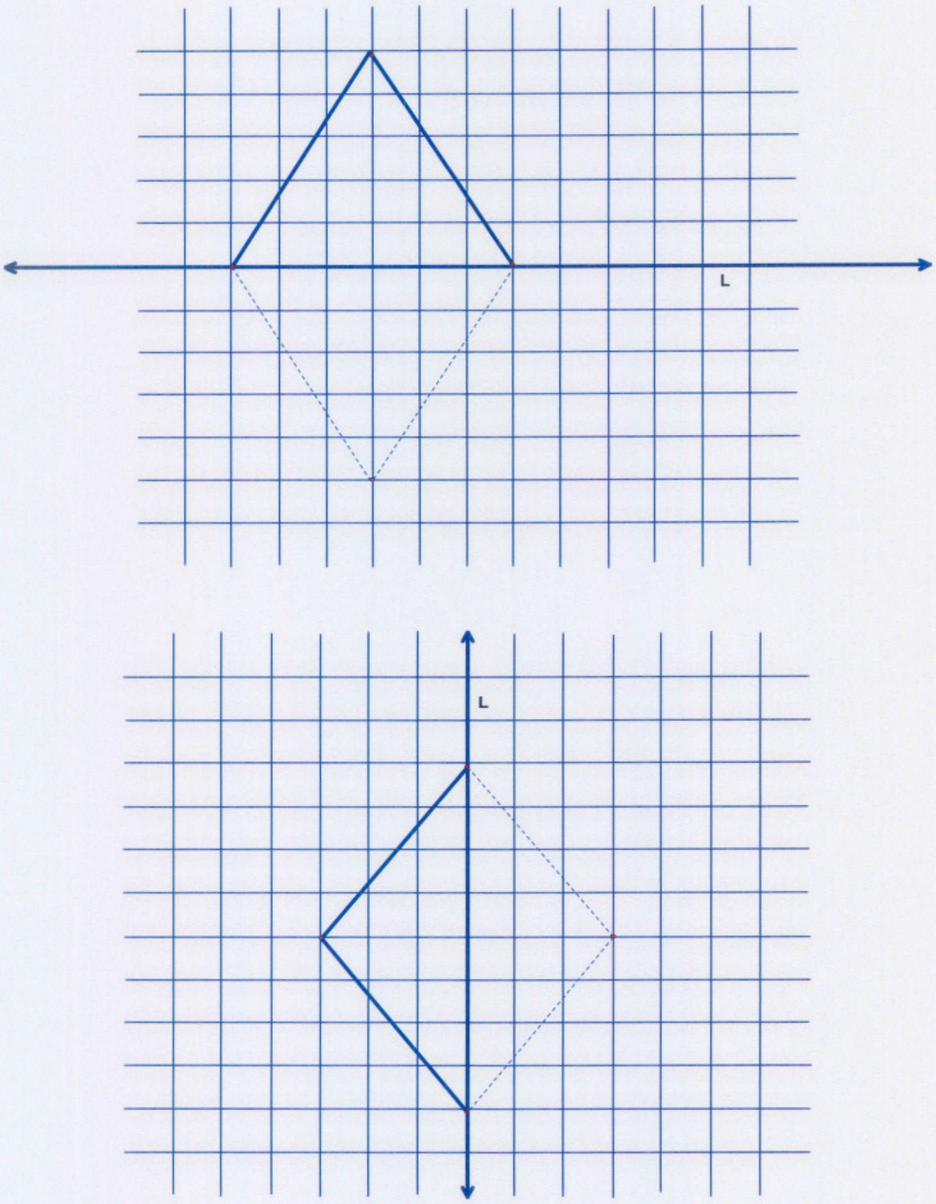


Figure 2.2: Reflect the given line segments about line L (Teppo, 1991:212)

Fourth phase: Free orientation

Learners must still find their way around this field, and this is achieved by assigning tasks that can be carried out in different ways. The learners engage in more open–ended activities that can be approached by several different types of solutions (Van Hiele, 1986:51).

Teppo (1991:212) suggests that learners can do the following activity at this level. Learners are given three vertices of isosceles trapezoid (see Figure 2.3) and are asked to find the fourth. They must explain what they did and why their procedure worked.

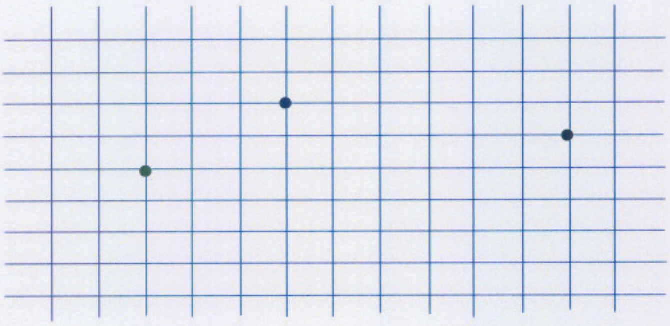


Figure 2.3: Given three vertices of an isosceles trapezoid (Teppo, 1991:212)

Fifth phase: Integration

The learners, according to Van Hiele (1986:51), still need to acquire an overview of the methods that are at their disposal. They then try to condense into a whole the domain which their thought has explored.

Learners summarise the characteristics of figures that have one or more axes of symmetry. The teacher can ask the learners how they will recognise a line of symmetry. Afterwards the learners can summarise the properties of a rhombus (Teppo, 1991:213).

During each phase learners investigate appropriate geometric figures, develop specific language related to these figures, and engage in interactive learning activities to enable them to progress to the next higher level of thinking (Teppo, 1991:210). The levels describe how learners think and what types of geometric ideas they think about.

Gutierrez, Jaime and Fortuny (1991:237-239) proposes a qualitative utilisation of the different ways in which learners reason for placement within a proposed range of 0 to 100, thus creating a scale of degrees of acquisition. Within this range, five stages of acquisition (see Table 2.1) are also identified.

Table 2.1: Degrees of acquisition of a Van Hiele level (Gutierrez et al., 1991:238)

NO ACQUISITION	LOW ACQUISITION	INTERMEDIATE ACQUISITION	HIGH ACQUISITION	COMPLETE ACQUISITION
0-15%	15-40%	40-60%	60-85%	85-100%

According to Van der Sandt (2003:34, following Gutierrez, 1991) answers are firstly classified according to the Van Hiele levels. Thereafter a numerical weight is assigned to each answer, weights of answers of a specific topic (e.g. right-angled triangles) leads to a classification of the degree of acquisition (see Table 2.1) for that specific topic (e.g. 77% average=high level of acquisition for right-angled triangles).

Table 2.2: Answer type and degree of acquisition (Van der Sandt, 2003:35, after Gutierrez, 1991)

Answer Type & Weight		**Description**	**Degree & Weight**		**Description**
0	0%	No reply, or answers that cannot be categorised.	No degree of acquisition	0-15%	Learners are not in need of or are not conscious of the existence of thinking methods specific to a new level.
1	0%	Answers that indicate that the learner has not reached the given level but has no knowledge of the lower level either.			
2	20%	Answers that contain incorrect and incomplete explanations, reasoning processes, or results.	Low degree of acquisition	15-40%	Learners are aware of methods of thinking, know their importance and try to use them. These learners make some attempts to work on a higher level, but have little or no success due to their lack of experience.
3	25%	Correct but insufficiently answered, indicating that the given level of reasoning has been achieved. Answers contain very few explanations as well as incoherent reasoning processes, or very incomplete results.			
4	50%	Correct and incorrect answers that clearly show characteristics of two consecutive Van Hiele levels. Answers contain clear reasoning processes and sufficient justifications.	Intermediate degree of acquisition	40-60%	Learners use methods of the higher level more often and with increasing accuracy, but still fall back on methods of a previous level. Typical reasoning is marked by frequent jumps between the two levels.
5	75%	Answers that represent reasoning processes that are complete but incorrect, or answers that reflect correct reasoning but that still do not lead to the solution.	High degree of acquisition	60-85%	Characterised by progressively strengthened reasoning that indicates that a learner is using a higher level of reasoning. Learners still make some mistakes or sometimes go back to the lower level.
6	80%	Correct answers that reflect the given level of reasoning that are complete or insufficiently justified.			
7	100%	Correct, complete and sufficiently justified answers that clearly reflect a given level of reasoning.	Complete	85-100%	Learners have completely mastered the new level of thinking and use it without difficulty

2.2.3.3 Characteristics of the Van Hiele levels of geometric thought

According to Van de Walle (2004:348) the products of thought at each level are the same as the ideas of thought at the next. The ideas must be created at one level so that the relationships among these ideas can become the focus of the next level. Van de Walle (2004:348) describes four characteristics of the Van Hiele levels:

- The levels are sequential. To arrive at any level above 0, learners must move through all prior levels. To move through a level means that one has experienced geometric thinking appropriate for that level and has created in one's own mind the types of ideas or relationships that are the focus of thought at the next level.
- The levels are not age–dependent in the sense of the development stages of Piaget. Some learners and adults may remain forever on level 0, and a significant number of adults may never reach level 2. Age is related to the amount and types of geometric experiences that learners have, but if they are not stimulated they will remain on a low level of acquisition.
- Geometric experience is the greatest single factor influencing advancement through the levels. Activities that permit learners to explore, talk about and interact with the content at the next level, while increasing their experiences at their current level, have the best change of advancing their level of thought.
- When instruction or language is a level higher than that of the learners, there will be a lack of communication and, hence, of understanding between the teacher and the learner. Learners required to wrestle with objects of thought that have not been constructed at the earlier level, may be forced into rote learning and achieve only temporary and superficial success.

2.3 LEARNING THEORIES

2.3.1 Behaviourism

Behaviourism is a psychological theory put forth by John Watson (1924) and then expounded upon by BF Skinner (1953). According to Bredo (1997:16) behaviourism was both the child of functionalism and empiricism.

According to Bredo (1997:17), Watson was concerned with the functions of behaviour, so Watson did not view learning as occurring through conscious thought, but through a process of conditioning.

For Skinner (1953:61) learning involved a change in response rate. Bredo (1997:19) says that Skinner defined learning as a change in response rate using many simple standardised responses by a single organism.

According to Skinner (1974:3,167,168) behaviourism is not the science of human behaviour, but it is the philosophy of the science of human behaviour. In a behavioural analysis a person is an organism that has acquired a repertoire of behaviour. A person remains unique and no one else will behave in precisely the same way.

Handal (2005) says that behaviourism focuses on the manipulation of the external conditions of the learner in order to modify behaviours that eventually lead to learning. In a behaviourist oriented environment completion of tasks is seen as ideal learning behaviour and mastering basic skills requires learners to move from basic tasks to more advanced tasks. In addition, learning is considered a function of rewarding and reinforcing learner learning.

Behaviourists saw the learner's affective domain as different from the cognitive domain. They categorised emotions "as imaginary constructs" that are causes of behaviour. Consequently, behaviourists assume that certain emotions and attitudes can influence behaviour, although, in general, affective issues are neglected (McLeod, 1992:586).

It has been said that behaviourism emphasises a process-product and teacher-centred model of instruction that have been prevalent in classroom teaching and in teacher education programs during the twentieth century (Marland, 1994:6179).

A behaviourist teaching style in mathematics education tends to rely on practices that emphasise rote learning and memorisation of formulas, one-way to solve problems, and adherence to procedures and drill. Repetition is seen as one of the greatest means to skill acquisition. Teaching is therefore a matter of transmission of knowledge and situated learning is given little value in instruction (Leder, 1994:41).

2.3.2 Constructivism

Jaworski (2005) believes that constructivism is a theory of knowledge acquisition. Knowledge is actively constructed by the learner, not passively received from the environment. Coming to know is a process of adaptation based on and constantly modified by the learner's experience of the world.

Constructivist theory has been prominent in research on mathematics education and has provided a basis for transforming mathematics teaching and learning. Learning is a constructive process that occurs while participating in and contributing to the practices of the local community (Cobb & Yackel, 1996:185).

Schunk (1996:208) is of opinion that different learning and teaching theories generally assume that:

- Thinking resides in the mind rather than in interaction with persons and situations.
- Processes of learning are relatively uniform across persons and some situations foster higher-order thinking better than others.
- Thinking derives from knowledge and skills develop in formal instructional settings more than on general conceptual competencies that result from ones experiences and abilities.

These assumptions are challenged by constructivist researchers who want cognitive accounts to address the full range of influences on learning, problem-solving and memory. Inherent in these views is the notion that thinking takes place in contexts and that cognition is largely structured by individuals as a function of their experiences in situations. These constructivist accounts highlight the contributions of individuals to what is learned. Social constructivist models further emphasise the importance of the individual's social interactions in acquisition of skills and knowledge (Schunk, 1996:208).

2.3.2.1 Perspectives on constructivism

Constructivism refers to a group of theories about learning that can in turn be used to guide teaching. Teachers who have adopted these theories believe that learners construct their own mathematical knowledge, rather than receiving it in finished form. So, rather than accepting new information, learners interpret what they see, hear or do in relation to what they already know (Carpenter, 2003:29).

Nieuwoudt (2000:1) says that the effectiveness of mathematics education depends on the degree to which teaching activities are linked to relevant and meaningful learning activities. According to Shuell (1988:277) cognitive conceptions of learning stress the active, constructive, cumulative, self-regulated and goal-orientated nature of learning. The learner must be actively involved in the learning process. The learner must construct his or her own knowledge because every learner perceives and interprets new information in a unique manner. Learning must be cumulative because new learning builds upon the learner's prior knowledge. The learner must be self-regulated because he must make decisions about what to do next. He or she must be goal-orientated because learning will be more meaningful if the learner has a general idea of the goal being pursued.

Clark (2000) theorises that constructivism places the emphasis on the learners rather than on the teacher. Teachers are seen as facilitators who assist learners in constructing their own conceptualisations and solutions to problems. Two schools of thought busy themselves with this theory namely social constructivism and cognitive constructivism:

Cognitive constructivism

Clark (2000) says that cognitive constructivism is based on the work of Jean Piaget (see § 2.2.1). Piaget's theory of cognitive development proposes that learners cannot be given information that they immediately understand and use. Instead, learners must construct their own knowledge. They build their knowledge through experiences. Cognitive constructivism is based on two different senses of construction (Clark, 2000):

- Learners learn by actively constructing new knowledge.
- Learners learn with particular effectiveness when they are engaged in constructing personally meaningful artefacts (e.g. dynamic computer programs).

Social constructivism

Lev Vygotsky (see § 2.2.2) is most often associated with social constructivism. He emphasises the influences of cultural and social contexts in learning and supports a discovery model of learning. This type of model places the teacher in an active role while the learners' mental abilities develop naturally through different paths of discovery (Clark, 2000).

According to Kim (2001) social constructivism emphasises the importance of culture and context in understanding what occurs in society and constructing knowledge based on this understanding. There are four general perspectives that inform how teachers can facilitate the learning within a framework of social constructivism (Kim, 2001):

- *Cognitive tools perspective*: It focuses on the learning of cognitive skills and strategies. Learners engage in those social learning activities that involved hands-on project-based methods.
- *Idea-based social constructivism:* It directs education's main aim at important concepts in the various disciplines (e.g. different types of triangles in geometry).
- *Pragmatic or emergent approach:* Social constructivists assert that the implementation of social constructivism in class should emerge as the need arises. Knowledge, meaning and

understanding of the world can be addressed in the classroom from both the view of the individual learner and the collective view of the entire class.

- *Situated cognitive perspectives:* This perspective focuses on the relationship between learners and their environment. When a mind operates, the learner interacts with the environment.

2.3.2.2 Assumptions of constructivism

According to Schunk (1996:211, 213) constructivist theories make various assumptions about human thought and actions in learning settings. He identifies two prominent assumptions that involve situated cognition and implicit theories:

- *Situated cognition* Situated cognition refers to the idea that thinking is situated in physical and social contexts. Cognitive processes, including thinking and learning, should be considered as involving relations between a person and a situation, rather than an activity that solely resides in a person's mind. The significance of these views is that they emphasise the construction of knowledge by people as they interact in situations. Situated cognition addresses the intuitive notion that many processes interact to produce learning.
- *Implicit theories*: A second assumption of constructivist theories is that people hold implicit theories about such issues as how we learn, what contributes to achievement, and how motivation affects performance. Learning and thinking occur in the context of individuals' beliefs about cognition. Implicit theories can also affect the way in which learners process information. Learners who believe that learning outcomes are under their control may expend greater mental effort, rehearse more, use organisational strategies and employ other tactics to improve learning, than learners who hold a fixed view of their abilities and may not expend the same effort.

2.3.2.3 Implications for teachers

Carpenter (2003:30) suggests that when a number of opportunities are provided for learners to represent their knowledge, teachers have to encourage learners to represent and construct their ideas. It is therefore important that discussions take place between the learners. This provides opportunities for the learners to indicate what they already know and understand about the topic, while it reveals any misconceptions that learners might have.

From a constructivist perspective, teachers do not teach in the traditional way. Rather, they use materials with which learners become actively involved through manipulation or social interaction. Activities stress learners' observance, collection of data, generation and testing of hypotheses and ability to collaborate with others. Learners are also taught to be more self-regulated and take a more-active role in their own learning by setting goals, monitoring and evaluating progress and going beyond basic requirements by exploring interests (Schunk, 1996:209).

Teachers can make use of concrete and manipulative materials to help learners as they become actively involved in the learning process. Open ended questions will encourage learners to investigate the activities and questions asked by the teacher and learners, will help the learners to construct their own ideas. When the teacher provides a number of opportunities for the learners to represent their knowledge, the teachers encourage the learners to represent and to construct their own ideas (Carpenter, 2003:30).

2.3.2.4 Implications for learners

Schunk (1996:208) states that constructivism is a psychological and philosophical perspective, and contends that learners form or construct much of what they learn and understand. It highlights the interaction of learners with situations in the acquisition and refinement of skills and knowledge. Constructivism places the locus of learning within the mind.

In a constructivist classroom, learners will be actively involved in their learning. They will be discussing ideas with other learners and representing these ideas in many different ways. Learners will also be involved in assessing their own work and would reflect on what they have learned. They will be actively involved in exploration, invention, discovery and application of, for example, the properties of quadrilaterals. They will be able to construct their own ideas of quadrilaterals and discuss, assess and reflect on what they have learned (Carpenter, 2003:31).

According to Schunk (1996:208) a basic assumption of constructivism is that people are active learners and must construct knowledge for themselves. To truly understand material, learners must rediscover for themselves the basic principles. Constructivism underlies the emphasis on integrated curricula where learners study the topic in various ways.

2.3.2.5 Contributions of constructivism

Learners need to be actively involved in their learning, and teachers need to provide experiences that challenge learners' thinking and force them to rearrange their beliefs (Schunk, 1996:210,211).

Alsup (2003:610) espouses a constructivist view of mathematics learning, saying that a lecturer cannot transmit mathematical knowledge directly to learners, but that the learners should construct their own solutions.

When organising mathematics teaching and learning with the help of technology, the theory of constructivism states that learners should be guided to discover the fundamental principles of a discipline and construct their own knowledge. Thus, constructivists believe that it is necessary to change the transmission approach of education (Pasqualotti & Freitas, 2002:410).

In a constructivist classroom, the emphasis is on learning rather than teaching. Learners are given the task of learning. The role of the teacher is to engage the learners by posing good problems and creating a classroom atmosphere of exploration and sense making (Van de Walle, 2004:32).

Social-constructivism, as opposed to behaviorist models of teaching and learning, claims that knowledge should not be transferred from one individual to another in educational environments. For constructivist educationalists, knowledge must be actively constructed by the learner because the learner is an entity with previous experiences that must be considered as a "knowing being". Learning is therefore seen as an adaptive and experiential process, rather than a knowledge transference activity (Jaworski, 2005).

2.4 TEACHING APPROACHES

2.4.1 Process-product teaching (traditional teaching)

Traditional teaching mostly used process-product teaching. For most learners mathematics is an endless sequence of memorising and forgetting facts and procedures that make little sense to them (Alsup, 2003:609). Gunter, Estes and Schwab (2003:66) states that the direct instructional method mostly used in traditional teaching, is most useful in teaching those skills that can be broken into small, discrete segments, with each segment building upon the prior one. The traditional approach to teacher education is characterised by relatively short instructional periods followed by practice until learning is achieved.

Alsup (2003:609, 610) also says that pre-service mathematics teachers have a weak, fragmented knowledge of mathematics, mostly acquired facts and memorised rules. They have rarely experienced a kind of teaching that focused on conceptual understanding. This can be ascribed to process-product teaching, which is in essence teacher-centred mathematics instruction that focuses on rules, formulas and answers. If they experience a mathematics course that is learner-centred, emphasises active-learning, communication and reasoning during their training, they will be able to teach mathematics effectively.

Wong (2002:211) says that learners often approach a mathematical problem by searching for a rule that identifies what is given, what is being asked and the category of the topic for the problem. This approach to mathematical problems is largely shaped by the way learners experience learning, their response to task demands, and the classroom environment. In other words, such a restricted conception of mathematics, which exists both within the learners and in the classroom culture, has led learners to tackle mathematical problems by searching for rules rather than approaching them through a conceptual understanding of the context.

Mok and Johnson (2000:553) suggest that possible problems in secondary school algebra may be due to the procedural paradigm orientation in the conventional style of teaching in the classroom, which do not provide sufficient opportunities for learners to express what they think and to develop conceptual understanding. Furthermore, Romberg and Kaput (1999:4) postulates that this traditional approach of teaching the basic skills and concepts isolates mathematics from its uses and from other disciplines. Thus, traditional school mathematics has failed to provide learners with any sense of its usefulness and has not enabled learners to learn mathematics with understanding.

Introduction of technology in lessons that embody a cognitive model in their design and delivery will provide a viable alternative for enhancing algebraic thinking.

2.4.2 Problem-solving based teaching

Cangelosi (2003:156) says that problem-solving means that learners engage in a task for which the strategies to solve the problem is not known in advance. In order to find a solution learners must draw on their knowledge and through this process, they will develop new mathematical understanding.

Shuell (1989:107) is convinced that problem-solving offers a productive way of thinking about classroom activities. The presence of an appropriate problem-solving attitude by both the teacher and the learner plays an important role in the way they view the teaching-learning situation.

Schoenfeld (1992:365) makes the following classroom recommendations for the teacher in a problem-solving environment:

- Model problem-solving behaviour whenever possible, for example exploring and experimenting along with the learners.
- Create a classroom atmosphere in which all learners feel comfortable to try out ideas.
- Invite learners to explain their thinking at all stages of problem solving.

- Allow learners to use their own strategies and approaches to solve problems.
- Present problem situations that closely resemble real situations so that the learners can use these experiences in real-life situations.

According to Cangelosi (2003:156) learners should have enough opportunity to formulate and solve complex problems, and the teacher should encourage them to reflect on their thinking.

By learning through problem-solving in mathematics, learners should acquire ways of thinking, habits of persistence and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. It can lead to great advantages in everyday life, and can help the learner to become a good problem solver (Cangelosi, 2003:156).

2.4.3 Realistic Mathematics Education

In realistic mathematics education (RME), context problems are intended for supporting a reinvention process that enables learners to understand mathematics. Context problems are defined as problems of which the situation is experientially real to the learners. In RME the point of departure is that context problems can function as anchoring points for the reinvention of mathematics by the learners themselves. Moreover, guided reinvention offers a way out of the generally perceived dilemma of how to bridge the gap between informal knowledge and formal mathematics (Gravemeijer & Doorman, 1999:112).

Freudenthal (1991:46) speaks of guided reinvention where the emphasis is on the character of the learning process rather than on invention as such. The idea is to allow learners to come to regard the knowledge they acquire as their own. This implies that certain norms must be in place, like you do not learn mathematics by guessing what the teacher has in mind, but by figuring things out for yourself.

According to Freudenthal (1991:30) mathematising may involve both mathematising everyday life subject matter and mathematising mathematical subject matter. Freudenthal does not see a fundamental difference between the two activities. Therefore, education might start with mathematising everyday life subject matter. However, reinvention demands that the learners mathematise their own mathematical activity as well. Therefore, for Freudenthal the core mathematical activity is mathematising.

Treffers (1987:348) discerns horizontal and vertical mathematisation. Horizontal mathematisation refers to the process of describing a context problem in mathematical terms, in other words to be able to solve the problem with mathematical means. Vertical mathematisation refers to mathematising ones own mathematical activity. Through vertical mathematisation the learners reach a higher level of mathematics. It is in the process of progressive mathematisation, which comprises both the horizontal and vertical component, that the learners construct new mathematics.

Mathematics should be taught as mathematising. For Freudenthal mathematics is a human activity. Mathematics as a human activity is an activity of solving problems and of looking for problems. Therefore, mathematising is an organising activity (Gravemeijer & Terwel, 2000:780,781).

Treffers (1987:337) explains mathematising according to the following strategies:

- *for generality*: for example looking for analogies, classifying and structuring.
- *for certainty*: for example using a systematic approach-reflecting, justifying and proving.
- *for exactness*: for example limiting interpretations and validity-modelling, symbolising and defining.
- *for brevity*: for example developing standard procedures and notations, symbolising and schematising.

Freudenthal (1991:41,42) distinguishes between horizontal and vertical mathematising in that horizontal mathematising leads from the world of life to the world of symbols. In vertical mathematising symbols are shaped, reshaped, and manipulated, mechanically, comprehensively and reflectively. In RME both horizontal and vertical mathematising are used to shape the long term learning process (Gravemeijer, 1994:1).

Gravemeijer and Doorman (1999:117) say that in RME, context problems are the basis for progressive mathematisation. The teacher tries to construe a set of contextual problems that can lead to a series of processes of horizontal and vertical mathematisation that together result in the reinvention of the mathematics that one is aiming for.

According to Gravemeijer and Doorman (1999:119) the goal is not only to help learners elaborate their informal understanding and informal solution strategies in such a manner that they can develop more formal mathematical insights and strategies. The objective is also to preserve the connection between the mathematical concepts and that which they describe. The learners' final understanding of the formal mathematics should remain connected with their understanding of these experientially real, everyday phenomena.

Gravemeijer and Doorman (1999:126) says that the RME approach tries to transcend the dichotomy between informal and formal knowledge, by designing a hypothetical learning trajectory along which the students can reinvent formal mathematics. The actual learning trajectory unfolds in such a manner that the formal mathematics emerges in the mathematical activity of the learners. This is connected to Freudenthal's (1991:4) contention that mathematics should start and stay within common sense. Common sense evolves in the course of the learning process and it is not static.

Gravemeijer and Terwel (2000:786) says that learners must be allowed to regard the knowledge they acquire as their own, personal knowledge, knowledge for which they themselves are responsible. Learners must also be given the opportunity to build their own mathematical knowledge base.

2.5 CONCLUSION

The current global movement in the reform of mathematics education seems to focus on a number of new ideas, including standards, quality and teacher preparation. As is said, the best conceived programme in the world, can easily come to naught if those who are to implement the programme at classroom level are not confident to carry out their tasks efficiently and effectively (Jegede, Taplin & Chan, 2000:288).

Carpenter (2003:32) says that the teacher is responsible for determining what ideas learners have about a particular subject so that new material can be introduced and related to learners' experiences.

Mathematics learning is a deep, rich process, emphasising conceptual understanding, reasoning, communication and problem-solving (Alsup, 2003:615).

When learners do not understand what they have learned, they perceive each topic as an isolated skill, they cannot apply their skills to solve problems nor extend their learning to new topics (Romberg & Kaput, 1999:19).

CHAPTER 3
3 A THEORETICAL FRAMEWORK FOR THE DEVELOPMENT OF CONCEPTUAL UNDERSTANDING AND STUDY ORIENTATION IN MATHEMATICS

3.1 INTRODUCTION

To keep up with the demands of life and work in the twenty first century, pre-service mathematics teachers are expected to learn to teach mathematics in ways that advance conceptual understanding. The PMTs must act as learning facilitators, providing engaging settings for the construction of knowledge and proposing challenges that encourage mathematical constructions (Klein, 2004:35, 36).

The aim of this chapter is to provide a framework for conceptualisation, study orientation in mathematics, and the learning of mathematics.

3.2 CONCEPTUAL UNDERSTANDING OF MATHEMATICS

3.2.1 Orientation

Schunk (1996:218) says that constructing concepts in our minds enables us to extend what we understand beyond the specific situations we have experienced in the past. Concepts are the building blocks of mathematical knowledge, but it is not the only type of mathematical content included in curricula. There are also discoverable relationships, conventions and algorithms (Cangelosi, 2003:177).

To construct a concept, learners use inductive reasoning, distinguishing examples of the concept from non-examples. Inductive reasoning generalises from broad encounters and moves towards specifics. It is the cognitive process through which people discover commonalities among specific examples, thus leading them to formulate abstract categories, concepts, or discover abstract relationships (Cangelosi, 2003:177).

Much learning involves concepts. Concepts are labelled sets of objects, symbols or events that share common characteristics or attributes. A concept is a mental construct or representation of a category that allows one to identify examples and non-examples of the category. Concept learning refers to forming representations to identify attributes, generalising them to new examples and discriminating examples from non-examples. Concepts may involve concrete objects or abstract ideas (Schunk, 1996:218).

Romberg and Kaput (1999:30) say that the development of understanding is an ongoing and continuous process and one that should pervade everything that happens in mathematics classrooms. When learners learn skills without understanding, the rote application of the traditional approach to teacher education, often interferes with a learner's subsequent attempts to develop understanding. When learners learn skills in relation to developing and understanding, however, not only does understanding develop, but mastery of skills is also facilitated.

It is more appropriate to think of understanding as emerging or developing rather than presuming that someone either does or does not understand a given topic, idea or process. Romberg and Kaput characterise understanding in terms of mental activity that contributes to the development of understanding rather than as a static attribute of an individual's knowledge (Romberg & Kaput, 1999:20).

Porter and Masingila (2000:165) suggest that learners' difficulties in doing mathematics can be related to their views of mathematics. It is not unusual to find learners who use mathematical procedures with little or no understanding of the concepts behind these procedures.
Some learners are not even aware that there are concepts underlying the procedures they use. Such learners do not realise that there is meaning in mathematics. They believe that doing mathematics means performing pointless operations on meaningless symbols and that everyone learns mathematics by memorisation (Porter & Masingila, 2000:165).

3.2.2 The nature of concepts

Two distinct views have emerged concerning the nature of concepts. The classical theory postulates that concepts involve definitions that define the critical characteristics, the intrinsic attributes, of the concept. A second view is the prototype theory. A prototype is a generalised image of the concept, which may include only some of the concept's defining characteristics, for example right-angled triangles (Schunk, 1996:219, 220).

Schunk (1996:218) distinguishes between three different types of concepts:

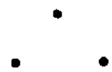
- *Conjunctive concepts*: These are represented by two or more characteristics. Other characteristics of that concept are not relevant, for example two blue rectangles.
- *Disjunctive concepts*: These are represented by one of two or more characteristics of a specific concept for example two rectangles of any colour or one blue rectangle.
- *Relational concepts*: These specify a relationship between characteristics that must be present in the concept, for example the number of objects in the figure must outnumber the number of borders. The type of object and colour are unimportant.

Cangelosi (2003:176) theorises that whether a specific is an example of a particular concept or not, depends on whether that specific possesses the defining attributes of the concept. A concept attribute is a characteristic common to all examples of a particular concept, for example a right-angled triangle has three sides, one angle of ninety degrees, and is a closed figure. A concept attribute is a necessary requirement for a specific to be subsumed within a concept. A set of attributes define the concept, also called the critical or intrinsic attributes.

According to Cangelosi (2003:173-175) researchers categorise and sub-categorise specifics according to certain attributes. The categories provide a mental filing system for storing, retrieving and thinking about information. The process (see Table 3.1) through which a person groups specifics to construct a mental category, is referred to as conceptualisation, the category itself is a concept. A specific is a unique entity, something that is not abstract. Cangelosi defines a concept is a category people mentally construct by creating a class of specifics possessing a common set of characteristics. In other words, a concept is an abstraction. Concepts can relate to one another, with broader concepts including narrower sub-concepts.

Table 3. 1: Concept formation (Cangelosi, 2003:174)

CONCEPTS AND SUBCONCEPTS	
DEFINITIONS	**EXAMPLES**
A polygon is a concept, it is a set with more than one element.	**polygon**
Each of the following concepts is a subconcept of a polygon: triangle, quadrilateral, pentagon, hexagon, heptagon.	**Triangle**

Special types of triangles (e.g., isosceles), quadrilaterals (e.g., rectangles), and pentagons (e.g., regular) are subconcepts of triangles, quadrilaterals and pentagons respectively (i.e., subsets of subsets).	**isosceles triangle**
A specific example of a concept is not a concept but a constant; it is the specified element of a set.	The unique isosceles triangle determined by the following three points: 

Most concepts can be represented in a hierarchy with superordinate and subordinate concepts. For any given concept, similar concepts may be at the same level in the hierarchy. These are known as coordinate concepts, for example, the concept "equilateral triangle" has "triangles" and "polygons" as superordinate concepts, the different categories of triangles (isosceles triangle, equilateral triangle, obtuse triangle, scalene triangle) as subordinate concepts and the other categories of polygons (quadrilateral, pentagon, hexagon, heptagon) as coordinate concepts. There are critical attributes (e.g., all sides are congruent) and variable attributes (e.g., three sides, three angles) (Schunk, 1996:221).

3.2.3 Teaching of concepts

3.2.3.1 Concept Attainment

Romberg and Kaput (1999:20) propose five forms of mental activity from which conceptualisation emerges:

- Constructing relationships.
- Extending and applying mathematical knowledge.
- Reflecting about experiences.
- Articulating what one knows.
- Making mathematical knowledge one's own.

Gunter, Estes and Schwab (2003:82, 83) suggests that concepts are the ideas that are formed as a result of categorising data from a number of observations. Learners form concepts and give them names in order to make sense of all the various stimuli. Many concepts used are abstract and have many interpretations.

According to Cangelosi (2003:178,179) the objective for learners to use inductive reasoning to distinguish between examples and non-examples of a mathematical concept, is at the construct-a-concept level. Teachers must make sure that the choice examples, non-examples, problems and leading questions that they choose, will stimulate learners to use inductive reasoning to form concepts.

Schunk (1996:222) advises that a concept should be defined with its critical attributes before examples and non-examples are given. Presenting a definition does not ensure students will learn the concept. Examples should differ widely in variable attributes and non-examples should differ from examples in a small number of critical attributes at once. This will prevent learners from overgeneralising, classifying non-examples as examples, and undergeneralising, classifying examples as non-examples. Pointing out relationships between examples is an effective way to arrange sets to foster generalisation.

Schunk (1996:222) says that it is helpful to present examples that differ in optional attributes, but have relevant attributes in common so that the latter can be clearly pointed out. Cangelosi (2003:177) calls these optional attributes example noise and defines it as any characteristic of an example of a concept that is not an attribute of that concept, for example two sides of the right-angled triangle are equal. The noise in the examples a teacher uses in lessons for leading learners to construct mathematical concepts, plays a key role in how well they conceptualise. To foster concept discrimination, teachers should present negative examples that clearly differ from positive examples. As learners' skills develop, they can be taught to make finer discriminations (see Table 3.2).

Table 3.2: Steps for generalising and discriminating between concepts (Schunk, 1996:222)

STEP	**EXAMPLE**
Name concept	Triangle
Define concept	Three non-collinear points joined by three straight lines and a close figure.
Give relevant attributes	Three sides, three angles and a close figure.
Give irrelevant attributes (example noise)	Angle of 90°, two / three equal sides.
Give examples	Triangle ABC with two sides=4cm and an angle=90°.
Give non-examples	 

3.2.3.2 Teaching models

Cangelosi (2003:179) says that inquiry learning activities stimulating learners to reason inductively to form a concept, can be embedded in a lesson with four stages:

- *Sorting and categorising*: Present learners with a task requiring them to sort and categorise specifics.
- Reflecting *and explaining*: Learners explain their rationales for categorising the specifics as they did. The teacher raises leading questions, stimulates thought and clarifies learners' expressions.
- *Generalising and articulating*: Learners describe the concept in terms of attributes. They may also develop a definition for the concept; however, it is not necessary for the conventional name of the concept to be used.
- *Verifying and refining*: The definition is tested with additional specifics and non-examples. The definition of the concept is modified in light of the outcome of the tests.

Gunter, et al. (2003: 82, 83) proposes a model, the concept attainment model, whereby learners construct a concept by extracting critical attributes form examples and non-examples.
In preparing to use the concept attainment model, a teacher must determine the following basic elements of the concept to be learned:

- Name of the concept.
- Concept definition or rule.
- Conceptual attributes.
- Examples of concept.
- Relationship of the concept to other concepts.

3.2.4 Conclusion

We can conclude with the following words of Cangelosi, (2003:179): "Students' conceptualizations provide the basis for subsequent meaningful learning of mathematics...... The failures of many students to develop healthy attitudes about mathematics, algorithmic skills, comprehension and communicating skills with mathematics, and application-level abilities to do mathematics to solve problems is well publised. Many of these failures can be traced to conceptual gaps in their learning

However, when learners acquire knowledge with understanding it generates new knowledge because they can apply that knowledge to new topics and solve new and unfamiliar problems (Romberg and Kaput, 1999:19).

3.3 NETWORK THEORY

3.3.1 Orientation

Van de Walle (2004:22,23) says that the tools that learners use to build understanding include the learners' existing ideas and the knowledge that they already have. Therefore, integrated networks are the way learners use their existing knowledge to understand and to integrate new knowledge. The diagram in Figure 3.1 is meant as a metaphor for the constructions of ideas. The small dots represent existing ideas. The lines joining the ideas represent the logical connections that have developed between the ideas. The large dot is an emerging idea. Whichever existing ideas are used in the construction will necessarily be connected to the new idea, because those were the ideas that gave meaning to the new idea.

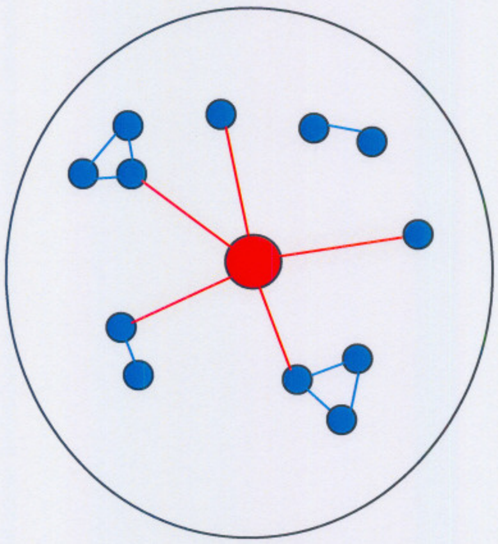


Figure 3.1: Learners use the ideas they already have (small dots) to construct a new idea (large dot), in the process developing a network of connections between ideas (Van de Walle, 2004:23)

Hiebert and Carpenter (1992:66, 67, 69) indicate that a mathematical idea or procedure is understood if it is part of an internal network, in other words the mathematics is understood if its mental representation is part of a network of representations. Thus, knowledge can be represented internally and these internal representations are structured. Understanding mathematics can be described in terms of the way in which an individual's internal representation is structured and the degree of understanding is determined by the number and the strength of the internal connections.

Understanding can be defined as a measure of the quality and quantity of connections that an idea has with existing ideas. Understanding depends on the existence of appropriate ideas and on the development of new connections. Therefore, the greater the number of connections to a network of ideas, the better the understanding (Van de Walle, 2004:24, 25).

To think about mathematical ideas, learners need to represent them internally in such a way that it allows the mind to operate on them. Learners need to recognise relationships between pieces of information and then understanding will occur as representations get connected into increasingly structured and cohesive networks (Hiebert & Carpenter, 1992:66, 67, 69).

Gunter, et al. (2003:279,280) theorises that new information can be retained and accessed more readily if the learners are able to link already familiar information to new data. It is also effective to link familiar knowledge to new data as an aid to remembering key concepts.

According to Hiebert and Carpenter (1992:68) there are different kinds of connections to construct and create mental networks:

- One kind of relationship is based on similarities and differences. When learners think about the similarities and differences between triangles, they can construct relationships between these representations.
- A second kind is based on inclusion. Inclusion is when one mathematical procedure is seen as a special case of another.

From their prior experiences learners are likely to have an internal network connected to their mental representations.

3.3.2 Building internal network representations and understanding of concepts

Even and Lappan (1994:136) say that learners cannot understand a mathematical concept in isolation. Connections to other concepts, procedures and pieces of information deepen and broaden their knowledge. Two important aspects of connections include the use of different representations and applications both within mathematics and between mathematics and other subjects.

According to Hiebert and Carpenter (1992:69) networks of mental representations are built gradually as new information is connected to existing networks or new relationships are constructed between previously disconnected information. Understanding grows as the networks become larger and more organised. Understanding can be rather limited if only some of the mental representations of potentially related ideas are connected or if the connections are weak. Connections that are weak may be useless when the students are confronted with conflicting or non-supportive situations.

Even and Lappan (1994:136) theorises that if learners represent ideas and problems in different ways, for instance geometrically, verbally, numerically and algebraically, it allows them to see how different representations give different insights into problem situations.

According Hiebert and Carpenter (1992:69) growth of networks may occur in several ways. Growth can be characterised as changes in networks as well as additions to networks. Learners build their understanding sporadically, rather than through smooth, monotonic increase. Changes in networks can be described as reorganisations. Representations are rearranged, new connections are formed and old connections may be modified or abandoned. The construction of new relationships may force a reconfiguration of affected networks. Therefore, internal networks are better thought of as dynamic instead of as static, because networks are constantly undergoing realignment and configuration as new relationships are constructed.

According to Lesh and Carmona (2003:71) the conceptual models that learners develop can be thought of as having both internal and external components. The internal components are often referred to as constructs or conceptual systems and the external components are often referred to as either artifacts or representations (see Figure 3.2).

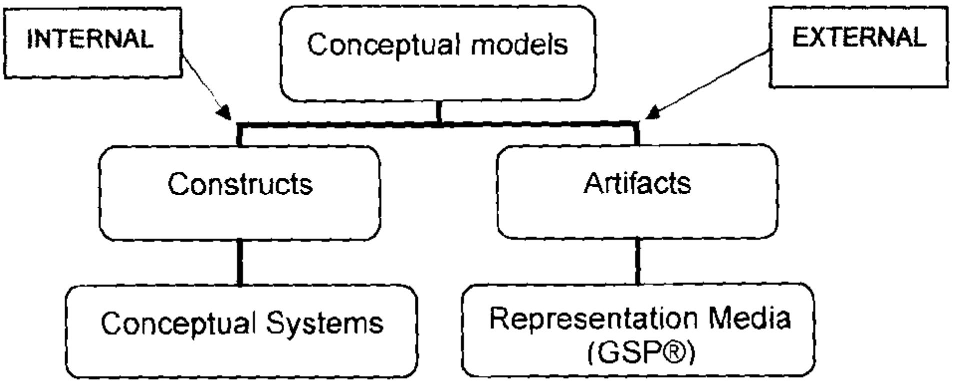


Figure 3.2: Mathematical models have internal and external components (Lesh and Carmona, 2003:71)

Gunter, et al. (2003:86,87) said that concept attainment is the process of defining concepts by attending to those attributes that are absolutely essential to the meaning, and disregarding those that are not. It also involves learning to discriminate between what is and is not an example of the concept. Using concept attainment in the classroom is aimed at helping learners attain the meaning of concepts through the inductive process of comparing examples and non-examples of the concept until the learner derives a definition. In taking ownership of concepts which they have a part in developing, learners can become authorities in what they are taught (see § 3.2)

3.3.3 Consequences of understanding mathematics

Hiebert and Carpenter (1992:74-77) identify the following consequences of understanding mathematics:

- *Understanding is generative*: Learners create their own internal representations of their interactions and build their own networks of representations. A crucial aspect of learners' constructive processes is their inventiveness. Learners continually invent ways of dealing with the world. Learners' inventions can lead to productive mathematics if the arguments of their inventions are parts of well-connected networks. If the mental representations are enriched by being connected within a network, then the inventions are stimulated, guided and monitored by much related knowledge.
- *Understanding promotes recall*: Memory is a constructive or reconstructive process, rather than a passive activity of storage. One advantage of the inclination to create connections between new and existing knowledge is that well-connected knowledge is remembered better. An entire network of information is less likely to deteriorate than a piece of information, and retrieval of knowledge is enhanced if it is connected to a larger network.
- *Understanding reduces the amount that must be recalled*: A consequence of understanding related to enhanced memory pertains to what must be remembered. If something is understood, it is represented in a way that connects it to a network. The more structured the network, the fewer individual pieces need to be retrieved separately. Memory of any single part of the network comes with memory of the network as a whole, reducing the number of items that must be remembered.
- *Understanding enhances transfer*: Transfer is essential because new problems need to be solved using previously learned strategies. It would be impossible to become competent if a separate strategy would need to be learned for every problem. Therefore, learners should be able to make connections between existing knowledge and newly learned knowledge.

- *Understanding influences beliefs*: Understanding yields affective consequences as well. Learners' beliefs about mathematics influence their growth in understanding. It is also plausible that the process of building understanding influences learners' beliefs about mathematics. The kind of work learners do determines how they think about a particular domain and what they believe about the nature of the subject.

Learners' understanding can be associated with many other existing ideas in a meaningful network of concepts. This network of concepts can be referred to as "webs" of interrelated ideas. A clear example of the potential for rich relational understanding is found in the many ideas that can be associated with the concept of "ratio" (Van de Walle, 2004:25) (see Figure 3.3).

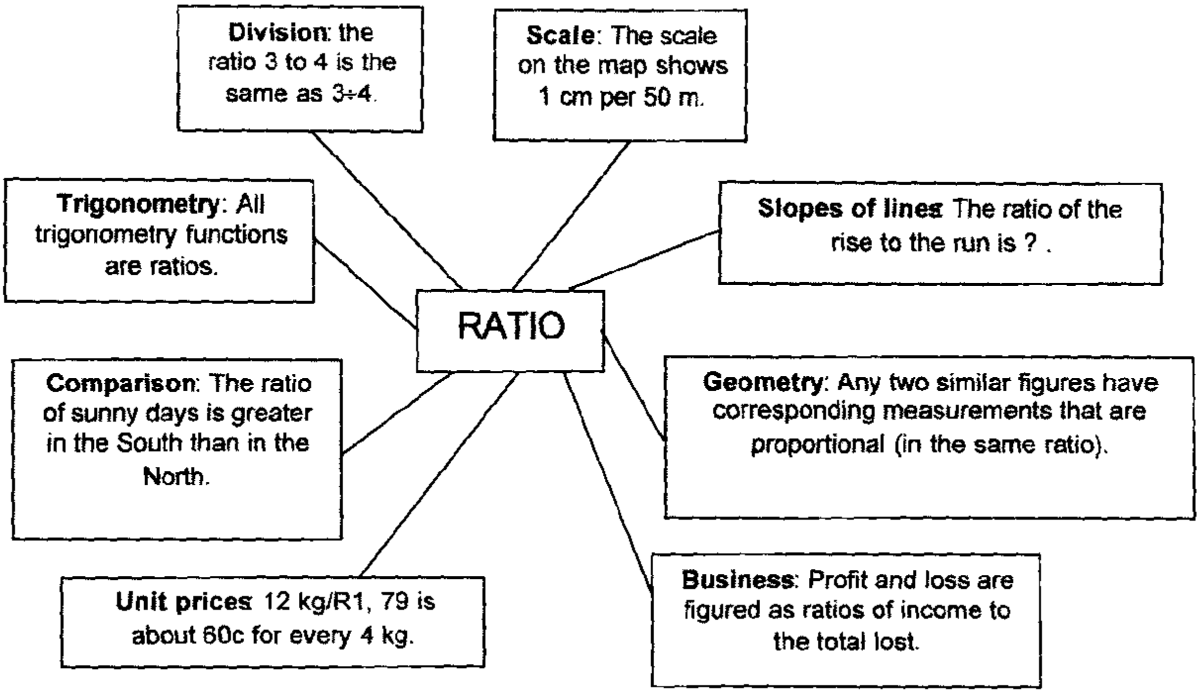


Figure 3.3: Potential web of associations that could contribute to the understanding of "ratio" (Van de Walle, 2004:25)

3.4 STUDY ORIENTATION

3.4.1 Introduction

According to Maree (1997:3,4) the formation of basic concepts in mathematics is very important. This concept of acquisition is an essential prerequisite for learning more advanced work in mathematics. Learners display a specific study attitude towards mathematics. This includes matters like learners' views of the nature of mathematics and the nature of learning mathematics. When learning mathematical content do not link up with the learners' level of knowledge and thinking, it leads to frustration.

When conceptualisation (see § 3.2.6) is incomplete, problem solving in mathematics is inhibited. Therefore learners do not easily understand the relation between concepts. Under such circumstances learners will use theorems and formulas without thinking whether they are applicable to the specific problem or not.

Learners' engagement in mathematics refers to their motivation to learn mathematics, their confidence in their ability to succeed in mathematics, and their emotions about mathematics. Learner engagement in mathematics plays a key role in the acquisition of mathematics skills and knowledge. Learners who are engaged in the learning process will tend to learn more (Maree, 1997:4). Yates (2000:77) theorises that the performance of learners is related to their prior achievement, attitudes towards specific aspects of learning and to motivational factors like self-efficacy, self-regulation and self-determination.

Hassan (2004:64) says that study orientation in mathematics focuses on the approaches, the practice of how, what where and when of learning (see Figure 3.4).

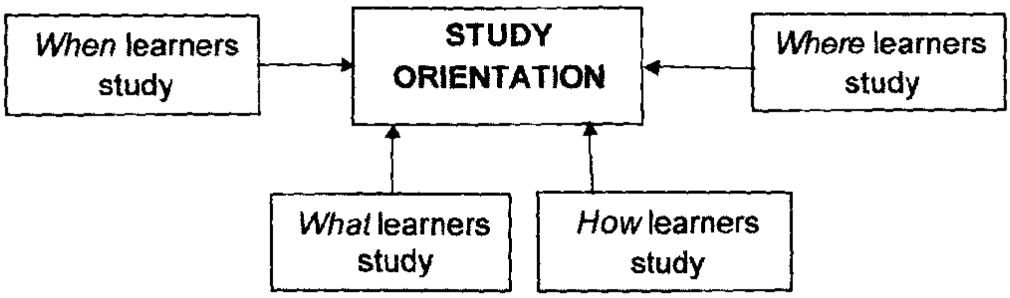


Figure 3.4: Conceptual understanding of study orientation (after Hassan, 2004:64)

Attitudes are internal believes that influence personal actions and that reflect such characteristics as generosity, honesty and commitment to healthy living (Schunk, 1996:392).

Mathematics anxiety is a complex and subtle problem with no simple solutions. Perry (2004) says that a common occurrence in mathematics is that learners have a superficial understanding of mathematics limited to computational skills, with little conceptual understanding (see § 3.2) and hence no framework within which to organise his/her knowledge. As a result, this type of learner forgets what he/she learns very quickly, and experiences frustration.

Mathematics anxiety reduces the storage and processing capacity of the memory system involved with task performance, and increases the amount of on-task effort required to maintain the performance (Hopko, 2002:164). When the body is tense, the mind cannot function.

According to Hopko (2002:157) mathematics anxiety is defined as feelings of tension and apprehension surrounding the manipulation of numbers and the solving of mathematical problems in academic, private and social settings.

Wilson, Fernandez and Hadaway (s.a.) says that problem solving has a special importance in the study of mathematics. A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problems.

Maree (1997:4) says that learners' study habits in mathematics are important in terms of the practising of insights. The execution of assignments in mathematics and the consistent practising of mathematics concepts form an important part of the learners' study orientation in mathematics.

3.4.2 The study Orientation in Mathema tics (SOM) Questionnaire

The need to measure learners' attitudes towards study in mathematics is based on the premise that mathematics is particularly vulnerable to poor teaching, and very little attention is paid to learners' orientation towards studying mathematics (Maree, 1997:1).

The SOM questionnaire consists of five fields, including 76 statements that relate to how learners feel or act regarding aspects of their achievement in mathematics. The SOM was developed for high school learners, but the scope of the questions is also applicable to tertiary students.

The five fields of the SOM can be summarised as follows (Maree, 1997:7,8,9):

- *Study attitude (SA) in mathematics:* This field has a bearing on feelings and attitudes towards mathematics and aspects of mathematics. This affects the learners' motivation and expectations with regard to mathematics. Attitudes include various factors like enjoyment of the subject, self-confidence and the challenge that Mathematics offers.
- *Mathematics anxiety (MA):* Learners' motivation in mathematics is affected negatively when they are emotionally disturbed. When learners have not adequately mastered the concepts and technical language of mathematics, their mathematics anxiety is increased.

- *Study habits (SH) in mathematics:* Study habits address the displaying of acquired, consistent and effective study methods and habits like planning time, preparation, working through previous tests, working through problems as well as following up problems in mathematics. It also includes how often they do their assignments, keep homework up to date and how much time they spend on doing mathematics.
- *Problem-solving behaviour (PSB) in mathematics:* It includes planning, self-monitoring, self-evaluation, self-regulation and decision making during the process of problem solving in mathematics. Problem–solving behaviour can also be described as thinking about thinking in mathematics.
- *Study milieu (SM) in mathematics:* Learners come from different environments. Therefore, study environment includes factors relating to social, physical and experiential environment. Milieu disadvantages often lead to mathematics anxiety and undermine learners' self-confidence.

According to Maree (1997:5), the following features were taken into consideration when compiling the SOM:

- The content had to be meaningful to the testee.
- The questionnaire had to have diagnostic value.
- Item biases towards language, race, gender and socio-economic environment had to be limited.
- Mark allocation had to be objective.

The aims of SOM were (Maree, 1997:5):

- To identify learners' negative study orientation in mathematics.
- To analyse the data obtained that could help counsellors and mathematics educators to obtain a better understanding of learners' poor achievement in mathematics.
- To use the information gained from the results,to help learners improve their study orientation in mathematics.

The SOM could be used as (Maree *et al.*, 1997:5,6):

- A diagnostic tool for identifying those learners who need support, remediation and counselling.
- An aid to make a systematic analysis of a number of important background particulars, feelings, attitudes, habits and customs with regard to the learners' academic orientation in mathematics.
- A study guideline in mathematics to familiarise learners with basic principles of effective studying in mathematics.

3.5 EFFECTIVE MATHEMATICS TEACHING AND LEARNING

Klein (2004:36) says that the pedagogic emphasis must move away from a sole preoccupation with transmission of the content to a concentration on PMTs' acfve participation in the learning processes. Therefore teacher education programs must be so that the emphasis is on active PMTs' participation, engaged thought and the investigation of mathematical and pedagogical ideas. PMTs must be able to transmit conceptual ideas and recognise the learner's active part in the learning process. In teacher education, the assumption must be that the PMTs' involvement in these learning processes will lead to re-conceptualisations of what mathematics is and how it is learned and taught.

Effective mathematics teaching must promote an understanding of concepts, relationships, and processes that will lead to a better understanding of mathematics (Wilson,1993:5). Learners should be given the opportunity to construct their own representations of mathematical concepts, rules, and relationships. Learners who construct their own knowledge focus on the underlying structure of problems (Wilson,1993:7,8). Human interaction and physical manipulation in the acquisition of knowledge is therefore very important.

Tytler (1999:193) correctly sees that the central goal of professional training should be the elaboration and expansion of PMTs' knowledge base. In reforming their educational practice, they must acquire richer knowledge of subject matter.

According to Cooney and Shealy (1998:308) PMTs' belief structures and their orientation towards context are central to their learning how to teach mathematics, and they should be reflective and adaptive agents. PMTs want to know what principles underlie instructional systems and what these foundations imply for their classroom practice (Lindschitl, 1999:190).

Shuell and Moran (1994:3341) states that a big difference between meaningful, cognitive learning and simpler forms of learning is that the former is usually concerned with understanding, while the latter is usually concerned with behavioural change. Knowledge must be structured and organised to be meaningful. Another difference is that meaningful learning involves the acquisition of a complex body of knowledge while simpler forms of learning involve a collection of separate, isolated facts.

3.6 CONCLUSION

Concept learning involves higher order processes of forming mental representations of critical attributes of categories. Current theories of concept learning emphasise the analyses of characteristics and formation of hypotheses about concepts, characteristic analysis, as well as forming generalised images of concepts that include only some defining of concepts, characteristics, and prototypes (Schunk, 1996:232).

Knowledge structures are addressed at two levels: generally with respect to conceptual-procedural properties; and with a specific mathematical focus. Conceptual knowledge is shaped by the construction of new relationships between existing information, or through linking existing knowledge to some new information. Conceptual knowledge is stored as a linked network of units, where the more elaborate the network, the more nodes there are for activation to be initiated. Inadequate conceptual knowledge means that a needed piece of information will not be retrieved when required, or that some incomplete or inaccurate version will be acted on (Galbraith & Haines, 2000:652).

Learners' beliefs should be the key to understanding their actions. Their failure to solve mathematical problems is can be directly attributed to their less powerful beliefs about the nature of mathematics and mathematics problem solving (Wong, 2002:15).

Porter and Masingila (2000:166) say that a rote conception of mathematics can interfere with learners' procedural ability. It can also prevent them from gaining an understanding of mathematical concepts. Both procedural ability and conceptual understanding are necessary for success in mathematics.

CHAPTER 4

4 A THEORETICAL FRAMEWORK FOR DYNAMIC COMPUTER TECHNOLOGY IN THE DEVELOPMENT OF CONCEPTUAL UNDERSTANDING IN GEOMETRY

4.1 INTRODUCTION

Diagrams are one way to represent geometric figures. These visual representations have a powerful influence on PMTs' development of geometric concepts. Dynamic computer technology such as Geometer's sketchpad® (GSP®) has the potential to provide such a rich visual learning environment. By designing a rich visual learning environment in conjunction with GSP®, PMTs can overcome the visual obstacles imposed by their limited perception (Contreras, 2005).

PMTs will embrace dynamic computer technology since it is the medium of the time. Teachers need to make this medium their own too. It has been recognised that learning doesn't stop at school, but has become a life-long process.

The purpose of this study was to investigate the conceptualisation of geometric learning of PMTs during instruction with the aid of dynamic computer technology, with special reference to GSP® (refer to § 1.2).

4.2 AN OVERVIEW

In this day of rapidly changing technologies, we cannot anticipate all the skills that learners will need in their lifetime or the problems they will encounter. We need to prepare learners to learn new skills and knowledge and to adapt their knowledge to solve new problems (Romberg & Kaput, 1999:19, 20).

Information and communication technology play an increasing and significant role in all facets of society. Teachers want their learners to be leaders in the field, to set the values and agendas of the corporate world. Teachers need to equip their learners to think and work in a creative and connected, lifelong learning environment.

Learners are exposed to direct sources of information like never before. They have to learn to be critical of information, developing skills to analyse and sort relevant material, skills that were not needed when the textbook was the main source of information. Such analytical skills can be learned and actively pursued with the potential of dynamic computer technology. Given the challenge, it is an exciting way to learn how to be responsible and discerning in the quest for knowledge (De Villiers, 2004:703).

Learners can take more initiative in, and responsibility for their learning with dynamic computer technology. When learners do that, they experience real excitement.
Therefore it will be important to:

- Create a learning environment that can be further explored as the learners grow in understanding, skills and knowledge;
- Change the classroom from a static environment, where the teacher dictates (see § 2.3.1), to a more active, engaging and collaborative environment.

4.3 DYNAMIC COMPUTER TECHNOLOGY

4.3.1 Orientation

It becomes clear in research done by Van der Sandt (2003:83) that learners leave school with higher levels and degrees of geometric acquisition than the levels and degrees of acquisition attained by the PMTs who were exposed to three years of academic and mathematical methodology training (see § 6.2.1.3).

With this background in mind, dynamic computer technology can be effectively used to facilitate the PMTs conceptual understanding of, and study orientation towards mathematics (Jiang, 2005).

4.3.2 Dynamic computer technology as a tool for teaching and learning

According to Clements and Battista (2000:761,764) there is little doubt that dynamic computer technology will have a major impact on the teaching and learning of mathematics. The complexity of dynamic computer technology teaching and learning, includes both the processes and the products of leaning. Reflecting on the actions and activities that are enabled by dynamic computer technology can catalyse a reconceptualisation of the nature and the content of the mathematics that could be learned. The flexibility of dynamic computer technology allows the creation of a vision less hampered by the limitations of traditional materials and pedagogical approaches.

According to Becker (2000) a number of research studies have indicated that dynamic computer technology can play a positive role in academic achievement. Dynamic computer technology is most effective when:

- It is combined with instructional strategies that actively involve learners in learning intellectually complex work that demands higher order thinking and problem solving skills.
- Teachers have the necessary professional development.

Becker's (2000) research notes that dynamic computer technology is a strong tool for supportive, active, inquiry based learning. Becker argues that the kind of active learning necessary to master principles and concepts is easier to implement in a technology-rich environment where learners have a rich array of information to work with. Dynamic computer technology seems to be associated with significant gains in mathematics achievement when it is used to facilitate the construction of higher order concepts and when teachers are proficient enough in the use of dynamic computer technology (Wenglinsky, 1998:32).

If learning were viewed merely as an increase in knowledge, active participation on the part of the learner would not be so important. However, if one accepts Piaget's (see § 2.2.4) view that learning involves a restructuring of the learner's cognitive schemata, learner involvement becomes mandatory (Webb, 2001:96).

Teachers ought to introduce their learners to the art of problem posing early and allow sufficient opportunity for exploring, conjecturing, reformulating and explaining. However, if teachers themselves have never been exposed to such approaches in their own learning of mathematics, it is hardly likely that they would attempt to implement it in their own classrooms. It is therefore important in mathematics teacher education to devise ways of expanding learners' views of proof and to allow sufficient opportunity for exploring, conjecturing and explaining (De Villiers, 2004:704).

Olive (2000) theorises that dynamic geometry turns mathematics into a laboratory science rather than a game of mental gymnastics, dominated by computation and symbolic manipulations. Mathematics becomes an investigation of interesting phenomena and the role of the students becomes that of a scientist, observing, recording, manipulating, predicting, conjecturing and testing. Students also develop theories as explanations for phenomena.

To illustrate this, Olive (2000) uses dynamic computer technology namely GSP®. Geometric figures can be constructed by connecting components, for example a triangle can be constructed by connecting three line segments. This triangle, however, is not a single static instance of a triangle that would be the result of drawing three line segments on paper. By grasping a vertex of this triangle and moving it with the mouse, the length and orientation of the two sides of the triangle meeting at the vertex will change continuously. The use of the dynamic drag feature of this type of computer tool, illustrates how such dynamic manipulations of geometric shapes can help learners to abstract the essence of a shape from seeing what remains the same as they change the shape. In the case of the triangle, the learners can abstract the basic definition of a triangle as a closed figure with three straight sides. Length and orientation of those sides are irrelevant as the shape remains a triangle, no matter how they changed these aspects of the figure. Such dynamic manipulations help in the transition from the first to the second van Hiele level. From the recognition of shape to the awareness of the properties of a shape.

Ben-Zvi (2000:139) adopted an approach based on empirical research and theoretical analysis that views computers as cognitive tools. A cognitive tool can be described as dynamic computer technology that helps transcend the limitations of the mind.
This approach is based on a specific conception of the human cognition, of which the following are key aspects:

- Cognitive processes have a concrete and imagistic base and are not organised by formal or general rules.
- Cognition depends on available dynamic computer technology. Cognitive development is understood not merely as development of the individual mind, but also as a social development of available dynamic computer technology.
- Cognition tends to be context-bound (Ben-Zvi, 2000:139).

This conception of cognition leads to specific ways of using dynamic computer technology and how dynamic computer technology lends itself to supporting cognitive activities:

The amplifier metaphor:
In environments that are not based on dynamic computer technology, representations produced and used during classroom activities are limited in number. Instruction often concentrates simply on

translation skills between representations, and mastery of skills tends to become the central goal of teaching. The use of dynamic computer technology turns many of the manipulations of representations into automatic operations. Many more calculations and representations can be carried out (Ben-Zvi, 2000:140).

The reorganisation metaphor:

An appropriate usage of dynamic computer technology has the potential to bring about structural changes in the learners' cognitive activities. Dynamic computer technology brings about the re-organisation of physical or mental work in at least the following ways:

- Shifting the activity to a higher cognitive level.
- Changing the objects of the activity.
- Focusing the activity on transforming and analysing representations.
- Supporting the situated cognition mode of thinking and problem solving.
- Accessing mathematical conceptions.
- Constructing meaning of conceptions (Ben-Zvi, 2000:140-143).

It is therefore clear that dynamic computer technology is an essential tool for teaching, learning and doing mathematics. It furnishes visual images of mathematical ideas. It facilitates organising and analysing data and it can be used to compute efficiently and accurately. It can support investigations by learners in every area of mathematics, including geometry, statistics algebra and measurement. When using dynamic computer technology learners can focus on decision making, reflection, reasoning and problem-solving. Dynamic computer technology is now available on which activities with hands-on, physical manipulatives can be simulated on a computer (Cangelosi, 2003:147).

Learners can learn mathematics more intensively with the appropriate use of technology. Technology should not be used as a replacement for basic understanding and intuitions, it can rather be used to foster those understandings and intuitions. Learners' engagement with abstract mathematical ideas can be fostered through dynamic computer technology. It enriches the range and quality of investigations by providing a means of viewing mathematical ideas from multiple perspectives (Cangelosi, 2003:147).

As illustrated by Olive (2000) many examples of a phenomenon can be created quickly and easily. This makes it easier for the teacher to facilitate inductive thinking. Inductive reasoning is in essence the skill of making connections, which promotes creative thinking.

The curriculum for learners should include many opportunities to learn how to use dynamic computer technology to understand mathematics better themselves and how to promote learners' learning of mathematical concepts. The availability of dynamic computer technology has forced mathematicians to rethink the way they teach mathematics. Learners need to develop critical thinking skills, to understand the main concepts and to be able to apply them in a wide variety of situations (Pecuch-Herrero, 2000:181). Dynamic computer technology has become essential.

According to Drier (2001:70,71) the majority of mathematics courses in PMTs' backgrounds are taught using traditional instructional methods with little use of dynamic computer technology. The use of dynamic computer technology can provide students with a deeper understanding of concepts embedded in a problem. Lecturers are challenged with the task of preparing students who can utilise dynamic computer technology as an essential tool in developing a deep understanding of mathematics, for themselves as well as for their students. Students should learn how to use dynamic computer technology as a conceptual teaching and learning tool.

It is believed that dynamic computer technology provides a promising environment for developing understanding of difficult symbolic ideas and techniques. According to Hennessy *et al.* (2001:282) findings confirm the implications that portable graphing technologies present a unique opportunity to help learners develop concepts and skills in traditionally difficult curriculum areas.

According to Schwars and Hershkowitz (2001:260) research has indicated that PMTs who have engaged in dynamic geometry tasks, are able to capitalise on the ambiguity of figures in the learning of geometrical concepts. Funkhouser (2003:165) reported that learners who used computer-augmented methods as a supplement to traditional instruction were better able than a control group to visualise and describe angles and polygons. Dynamic computer technology (e.g., geometric supposer) promoted the development of geometry concepts and positive attitudes toward mathematics. Ng and Teong (2003:5) says that the use of dynamic computer technology enables learners to model and have an interactive experience with a large variety of two-dimensional shapes.

According to Ben-Zvi (2000:128) dynamic computer technology has been developed to support the following:

- Learners' active knowledge construction, by doing and seeing mathematics.
- Opportunities for learners to reflect on observed phenomena.

- The development of learners' metacognitive capabilities.
- The renewal of instruction and curriculum.

Jiang (2005) believes that the integration of dynamic computer technology will effectively help PMTs reach a better understanding of mathematical concepts and develop stronger problem solving abilities.

4.4 GEOMETER'S SKETCHPAD (GSP®)

4.4.1 Origin of Geometer's Sketchpad®

The GSP® was developed as part of the Visual Geometry Project, a National Science Foundation project. GSP® creator and programmer, Nicholas Jackiw, developed GSP® first versions in an open, academic environment in which teachers and researchers provided design input. Key Curriculum Press continues to study how GSP® can be most effectively used (Bennett, 1997:viii).

4.4.2 Different uses of Geometer's Sketchpad

The following uses of GSP® were documented during research done by the Visual Geometry Foundation (Bennett, 1997:ix):

- GSP®'s power enable PMTs to create figures of arbitrarily great complexity, but PMTs who are beginners at using GSP® grasp concepts best when their thinking is directed toward relationships and simple constructions.
- GSP® can integrate different geometry topics in ways textbooks can not. For example in a GSP® triangle investigation, PMTs might investigate line and angle relationships, area, transformations, symmetry and coordinate geometry.
- Opportunities for PMTs insight come in many places throughout the course of an investigation, not just from dragging a completed construction. For this reason, PMTs are explicitly asked to drag parts of their figures during the course of construction and leading questions are interspersed throughout the activities (see Appendix B).

According to De Villiers (2004:703) GSP® can be used to develop PMTs' understanding of other functions of proof than just the traditional function of verification. These other functions are explanation, discovery, intellectual change and systematisation. However, proof has many other functions within mathematics, which are of greater importance than mere verification. Some of these functions are:

- *Explanation*: providing insight into why a statement is true.

- *Discovery*: the discovery or invention of new results.
- *Intellectual challenge*: the self–realisation derived from constructing a proof.
- *Systematisation*: the organisation of various results into a deductive system of axioms, concepts and theorems.

Bennett (1997:xiv) says that PMTs can construct translations, reflections, rotations, dilations and iterations with GSP®. PMTs can create animations that trace sine waves, explore other trigonometric identities and they can encapsulate complex geometric constructions in single steps.

GSP® can also be used to enhance the teaching and learning of geometrical concepts and relationships of the PMTs (Ng & Teong, 2003:5). According to Garofalo and Bell (2004:233) GSP® is a tool to facilitate PMTs' visualisation and exploration of mathematics concepts. The dynamic features of GSP® can support the conceptual development of PMTs.

4.4.3 Rationale for using GSP® as a learning environment

Wilson (s.a.) states that dynamic geometry programs such as GSP® provide exploration tools with rich potential for all ages. The tools can be used to explore relationships of and among geometric objects in a plane.

PMTs can construct an object and then explore its mathematical properties by dragging the object with the mouse. All mathematical relationships are preserved, allowing the PMTs to examine an entire set of similar cases in a matter of seconds. This leads by natural course to generalisation. GSP® encourages a process of discovery in which PMTs first visualise and analyse a problem, and then make conjectures, before attempting a proof (Bennett, 1997:vii).

4.4.4 A framework for teaching geometry with GSP®

Choi-Koh (1999:302) says that active visualisation is the process of forming and interpreting geometric representations of mathematical concepts, principles or problems, within dynamic computer technology (GSP®) across all levels of geometric thought.

The traditional approach to geometry focuses mostly on developing the ability of making deductive proofs, especially for riders. It is aimed at (De Villiers, 2004:710):

- Providing the PMTs with exemplars of how geometric content could be organised in learning activities corresponding to the Van Hiele levels.
- Developing understanding of varied meanings of proof at the different levels.

- Actively engaging the PMTs in the process of defining in order that they may realise:
  - That different, alternative definitions for the same concept are possible.
  - That definitions may be uneconomical or economical.
  - That some economical definitions lead to shorter, easier proofs of properties.
- Developing the PMTs' ability to construct formal, economical definitions for geometrical concepts.

Webb (2001:96) suggests the following rationale for selecting worthwhile activities:

- Each learner's success must be measured in terms of bettering his/her own performance. Motivation (see § 3.4) is hard to maintain if the learner repeatedly fails.
- Avoid activities that are so structured that there is only one correct way to respond.
- Provide activities that are challenging.
- Most of the learners' time must be focused on the activities and not on the teacher (see § 2.3.1).
- Provide individual activities to be accomplished in the company of peers. While individual effort is necessary for cognitive growth, peer interaction provides encouragement and assistance.

GSP® presents the possibility of new kinds of tasks and new ways of looking at old tasks. In teacher education lecturers can use GSP® to enhance their teaching abilities, and it has the potential to consolidate the lecturer's task in geometrical concepts, providing informal proofs of conjectures and easing PMTs' thinking in problem solving activities. Table 4.1 illustrates a suggested framework where some GSP® tasks of different emphases can be crafted in the mathematics classroom. These levels are not fixed (Ng & Teong, 2003:5).

Table 4.1: A framework for teaching geometry with GSP® (Ng & Teong, 2003:5)

<table>
<tr><th>LEVELS</th><th>PURPOSE OF INSTRUCTION</th><th>TEACH CONCEPT</th><th>CONSOLIDATE CONCEPT</th><th>INFORMAL PROOF</th><th>PROBLEM SOLVING</th></tr>
<tr><td>1</td><td>Teacher demonstration</td><td colspan="2" rowspan="2">See Figure 4.1</td><td rowspan="2"></td><td rowspan="2">See Figure 4.2</td></tr>
<tr><td>2</td><td>Templates/pre-made sketches</td></tr>
<tr><td>3</td><td>Guided exploration /construction tasks</td><td colspan="3">See Figure 4.3</td><td></td></tr>
<tr><td>4</td><td>Black box tasks</td><td></td><td>See Figure 4.4</td><td></td><td></td></tr>
</table>

In the following activity the PMTs can experiment with drawing, dragging, measuring and labelling points segments, rays and lines according to the Van Hiele levels of geometric thought (see §

2.2.3.1). These objects, along with the circle, are the building blocks of most geometric constructions.

Levels 1 and 2: demonstration of sketches or templates/pre-made sketches

The PMTs can make use of demonstration sketches or pre-made sketches (See Figure 4.1 and Figure 4.2). The difference between a demonstration sketch and a pre-made sketch is that in the former, the lecturer shows the GSP® sketch to his/her class and leads the PMTs to an understanding of the key concepts presented by the sketch through careful oral questioning and guided observation. The latter, on the other hand, involves PMTs exploring the sketches prepared by the lecturer, following certain guidelines. PMTs can drag the items around, add new constructions into the existing templates, make appropriate measurements and answer questions accompanying the pre-made sketch. In this way they are steered into making conjectures based on their observations and then testing them on the templates using the features of GSP®. A more challenging task with a problem solving focus can also be presented using pre-made sketches (Ng & Teong, 2003:6).

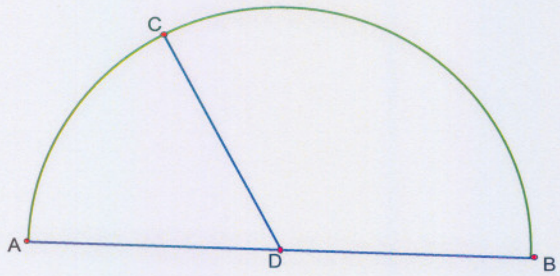


Figure 4.1: Pre-made sketch of adjacent angles (Ng & Teong, 2003, 6)

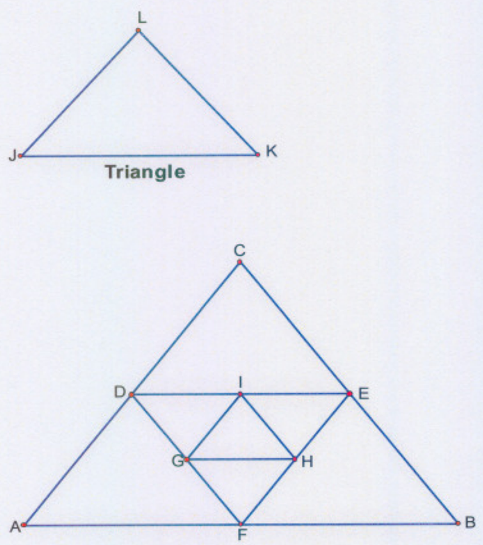


Figure 4.2: Different triangles created in different layers of triangles (Ng & Teong, 2003, 6)

Level 3: guided explorations/ construction tasks

According to Ng and Teong (2003:6) the PMTs can also make use of guided explorations or construction tasks (see Figure 4.3), where they are provided with simple construction steps for constructing certain figures. No pre-made sketches are used. PMTs construct the GSP® sketches from scratch and are directed to make discoveries about geometrical concepts explored through lecturer questioning. These activities are usually given to PMTs who are already familiar with the basic features of GSP® so that time is not wasted on the technicalities of how to use GSP®.

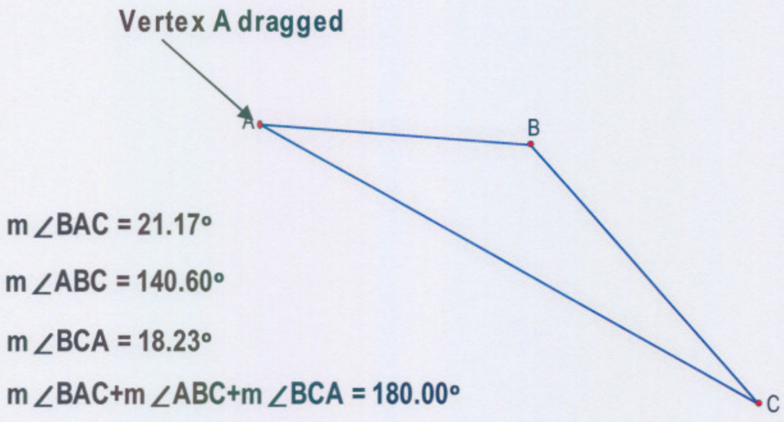


Figure 4.3: Explore the sum of angles in triangles (Ng & Teong, 2003, 7)

Level 4: Black box tasks

Ng and Teong (2003:8) suggest that with GSPs® reliable accuracy in constructing basic objects, construction tasks in GSP® can involve the creation of more complicated figures for fun or for verifying a geometrical phenomenon. Black box tasks entail a certain degree of freedom on constructions. Room can be made for different approaches to constructing the same figure. PMTs are tested based on their abilities to construct geometrically sound figures so that correct conclusions can be drawn from their sketches. In doing so, PMTs are made to realise that their sketches should possess certain geometric properties and understand that these geometric properties are required to construct such objects. In the process, pre-service mathematics teachers investigate the underlying geometrical relationships between the objects in the construction and also make use of their problem solving skills, such as working backwards and making deductions.

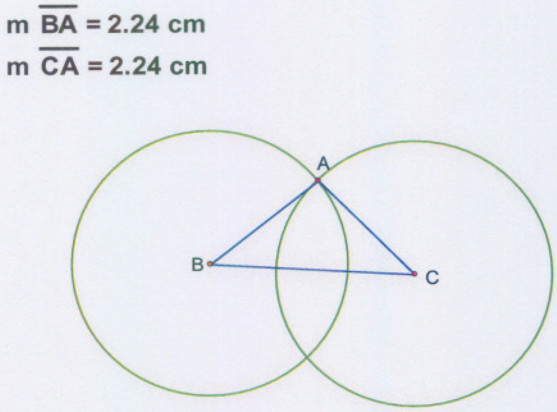


Figure 4.4: Identify an isosceles triangle (Ng & Teong, 2003, 8)

GSP(®) can also be used to solve a more conceptual problem, for example:

The power plant problem

A power plant (see CD) has to be built to serve the needs of three cities. Where should the power plant be located to use the least amount of high-voltage cable that will feed electricity to the three cities? If the three cities are represented by the vertices of ΔABC, then this problem can be solved by finding a point with minimum sum of distances to all three cities. In exploring this situation in GSP®, PMTs can measure the three distances from an arbitrary point P and the three vertices A, B and C of the triangle (see Figure 4.5). They can then sum these distances and move P around to find a location with minimum sum. When such a location has been found, PMTs can make conjectures concerning relations among P and the three vertices.

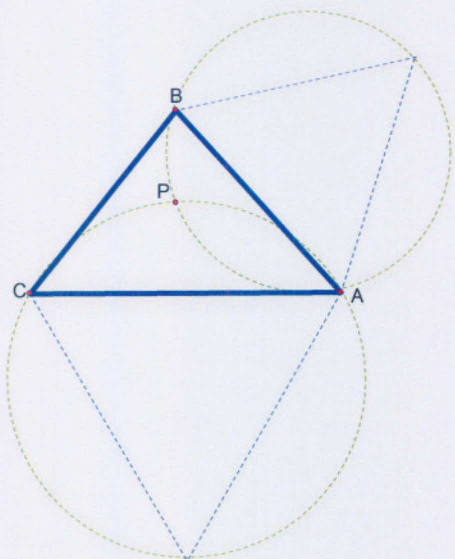


Figure 4.5: Constructing the location of the power plant P

After the PMTs have successfully located the position of the power plant and found a way of constructing that position, they have to explain why this point provides the minimum sum of distances to each vertex of the triangle formed by the three cities. This question challenges them to find a way of proving that their constructed point P must be the minimum point.

4.5 CONCLUSION

Over the last decade, there were huge progress in dynamic computer technology that have become more powerful, flexible and efficient. Despite all this progress, the penetration of these technologies in educational practice proves to be very slow. The shortage of technology in schools is one reason. However, the limited commitment of lecturers and curriculum developers and the great ignorance about teaching and learning in computer-based environments are important factors contributing to the scarcity of actual implementation in the classrooms. Educators should be encouraged to view dynamic computer technology as legitimate extensions of cognitive systems (Ben-Zvi, 2000:149).

Teachers are a fascinating, intelligent, and somewhat eclectic group of people. Yet some teachers resist change and nobly cling to traditional teaching models; spurning the integration of dynamic computer technology. In doing so they deprive their learners and themselves of a creative and exiting learning environment.

Whether we like it or not, information and dynamic computer technology are the learning and teaching media of our time.

CHAPTER 5

METHOD OF RESEARCH

5

5.1 INTRODUCTION

In this chapter the empirical investigation is described and motivated. The nature of the research is both quantitative and qualitative and multiple methods of data collection were employed with a view to increase the reliability of the results.

The layout of this chapter covers the aim, experimental setting, tasks, the population and sample, the instruments used, the variables used, the method of research, the statistical techniques, which were used to analyse the data and the conclusion.

5.2 AIM OF INVESTIGATION

The aim of the investigation was to gain more knowledge with regard to the relationship between a dynamic technological learning environment and the conceptual understanding of PMTs in geometry, as well as the relationship between a dynamic technological learning environment and the study orientation of PMTs (see § 1.2).

The quantitative research served to enable the researcher to determine the effect of a dynamic technological learning environment, firstly on the conceptual understanding and secondly on the study orientation of PMTs in geometry.

Mathematics education lecturers in the faculty Education Sciences, of the North-West University, Potchefstroom Campus, decided that all the PMTs in their third year (2005) had to participate in the dynamic computer programme. This arrangement excluded the possibility of selecting a control group. A pre-experimental research design was employed (Leedy & Ormrod, 2001:229). To strenghten the research, the researcher also did a qualitative research (interviews). The qualitative research provided an opportunity to interact face-to-face with the PMTs through interviews in order to gain more insight in how the PMTs experienced GSP®.

Furthermore, this study was both operational and applied in nature as it was carried out in the PMTs' learning environment.

Both qualitative and quantitative research were conducted (see Figure 5.1).

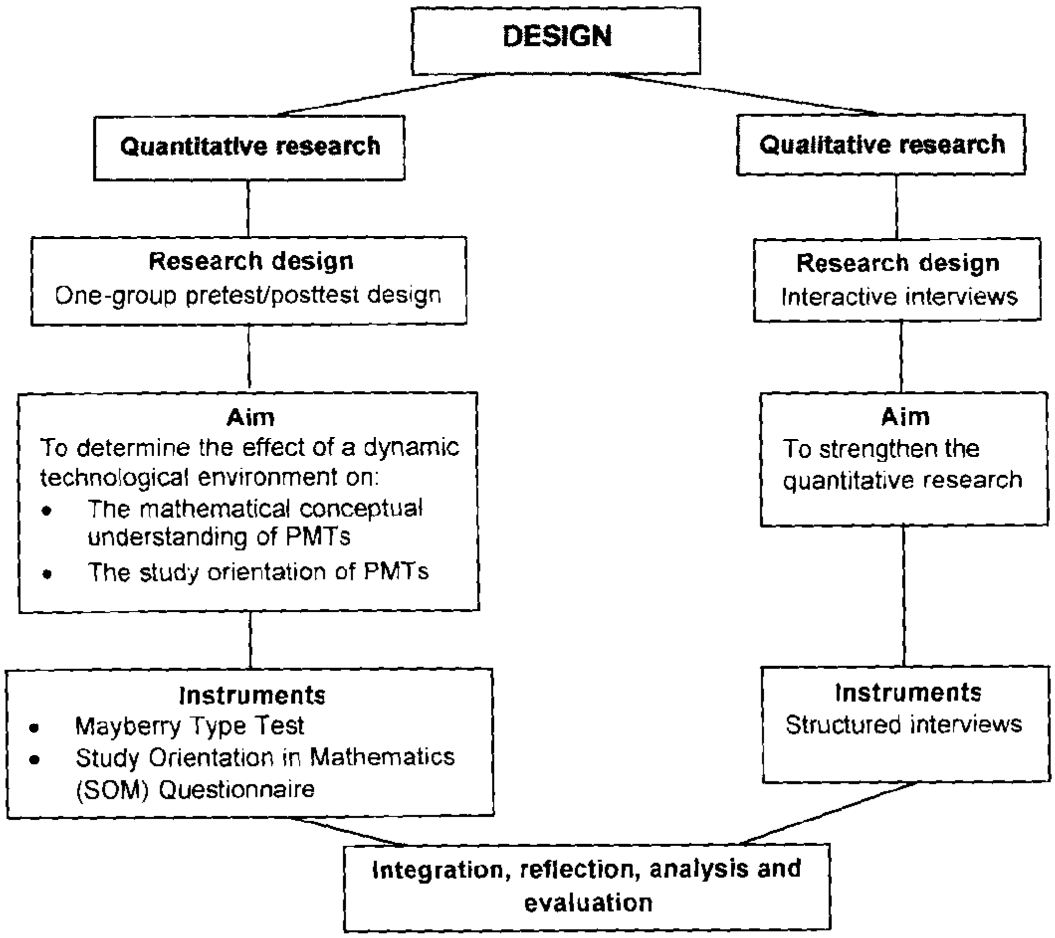


Figure 5.1: Combined research method

5.3 METHODOLOGY

5.3.1 Quantitative design

The pre-experimental design namely the one-group pre-test/post-test design (see Table 5.1) was used. In a one-group pre-test/post-test design, a single group (a) has a pre-experimental evaluation, then (b) is administered the experimental treatment, and finally (c) is evaluated after the experiment (Leedy & Ormrod, 2001: 235).

Table 5.1: The one-group pretest-posttest design (Leedy & Ormrod, 2001:235)

GROUP	TIME ?						
Group	Pre-test	Obs	Obs	Tx	Obs	Obs	Post-test

Where:

- Obs: indicates that observations, reflecting on the dependent variable, are made.
- Tx: indicates that a treatment, reflecting the independent variable, is presented.

The researcher did a pilot study in 2004 to gain some experience of the GSP® and to test the chosen procedures and materials.

The pre-test (SOM and Mayberry Type Tests) was administered during February 2005. The PMTs were assured of the confidentiality of the results. The answers to the questionnaires were anonymous and identification numbers were allocated to each PMT (McMillan & Schumacher, 2001:198). The same identification numbers were used during the administration of the pre-test and the post-test to make comparison possible.

There were no right or wrong answers to the SOM questionnaire and PMTs were encouraged to give honest answers. The answer sheets were collected immediately afterwards. The PMTs wrote the Mayberry Type Test (see Appendix A) and the SOM Test on the same day.

After the intervention programme had been completed the post-tests were administered. The researcher marked the Mayberry Type Test as well as the SOM questionnaire and the marks were moderated by a specialist on this field. The scores for the Mayberry Type Test and the SOM test were submitted to Statistical Consultation Services of North-West University, Potchefstroom Campus, for processing the data.

5.3.1.1 Intervention

The activities were designed according to the reconstructive approach and were structured in accordance with the Van Hiele theory of learning geometry.

The activities (see Appendix B and CD) that the participating lecturer developed, focused on the development of understanding the concepts to be covered during the course.

In Activity 1 the PMTs learned how to draw a triangle with GSP®. Then, in Activity 2 they developed the concepts of midpoint, median and ratio (assisting in the development of the concept of similarity). During Activity 3 they developed the concepts of perpendicular bisector, circumcircle and right-angled triangle. Activity 4 aimed to develop the concepts of altitudes, acute triangle, obtuse triangle and right-angled triangle. After that they developed the concepts of angle bisectors and inscribed circles in Activity 5. Finally, in activity 6, the concepts of radius, isosceles triangle and base angles of an isosceles triangle were reinforced.

The language of instruction was Afrikaans and the PMTs were Afrikaans speaking students. Completion of the activities occurred through medium Afrikaans. The current dissertation is presented in English and for the benefit of the reader the activities were translated into English.

5.3.1.2 Variables

The following dependent and independent variables have been used in this study:

Independent variables

A dynamic technological learning environment in the form of a Van Hiele based learning programme, employing a software package namely Geometer's Sketchpad® 4 (GSP®) in a problem solving context.

Dependent variables

The following dependent variables were used:

- Study Orientation in Mathematics (SOM) Questionnaire that consists of the following fields: Study attitudes, mathematics anxiety, study habits, problem-solving behaviour and study milieu.
- Conceptual understanding of triangles.

5.3.1.3 Study population and sample

The study population consisted of 371 third-year education students (in 6 classes) following the general mathematics module in geometry at the North-West University, Potchefstroom Campus. A sample of 26 prospective mathematics teachers from one of the classgroups took part in the experiment.

5.3.1.4 Instruments

For the purposes of a quantitative research two questionnaires were completed by the participants. The first questionnaire was the Mayberry Type Test (see Appendix A) to determine the influence of a dynamic computer technology programme on the conceptual development of geometric thought levels of PMTs (see § 3.2). The second questionnaire was the Study Orientation in Mathematics (SOM) Questionnaire (see § 3.3.1) to determine the influence of a dynamic computer technology programme on the study orientation of PMTs.

Mayberry Type Test

The Mayberry Type test (Lewin & Pegg Version as published by Lawrie, 1998) includes 40 items each with up to 5 sub-items on a variety of geometric concepts. This questionnaire was used to assess the concepts included in the activities over the first four Van Hiele levels (see § 2.2.3). The answers to the items were assessed and scored according to the acquisition scales of Gutiérrez *et al.* (1999) (see § 2.2.3.2).

Study Orientation in Mathematics (SOM) Questionnaire

The Study Orientation in Mathematics (SOM) Questionnaire (Maree *et al.*,1997) was used. According to Maree *et al.* (1997:26) the level of reliability in terms of Cronbach Coefficient Alpha (grade 8 and grade 9 learners), for the SOM questionnaire as a whole ranges from α=0,89 to α=0,95. Steyn and Maree (2002:13) report on an action research done during 2000-2001, involving first-year engineering students in an extended study programme of the School of Engineering at the University of Pretoria. These two researchers posit that SOM fields, study attitude, mathematics anxiety, study habits, problem-solving behaviour and study milieu, could be regarded as significant predictors of performance in mathematics at university level. The SOM questionnaire comprises seventy six questions covering the five fields of learning skills (see Table 5.2).

Table 5.2: Number of items per SOM fields (Maree et.al., 1997:7-9)

	SOM FIELD	**NUMBER OF ITEMS**
1	Study attitudes (SA)	14
2	Mathematics anxiety (MA)	14
3	Study habits (SH)	17
4	Problem-solving behaviour (PBS)	18
5	Study milieu (SM)	13
		76

The rationale for using the SOM questionnaire includes the following:

- To measure (before and after intervention) the influence of a dynamic technological learning environment on the study orientation of PMTs.
- To ascertain whether or not the intervention had any effect on improving the study orientation in mathematics of PMTs.

The questionnaire could be completed by making use of a five-point scale (see Table 5.3) according to which the PMTs could estimate their response ratings about the five fields of learning skills.

Table 5.3: The five-point scale of the SOM questionnaire (Maree, 1996:1)

Rarely	Sometimes	Frequently	Generally	Almost always
0 to 15 %	16 to 35%	36 to 65%	66 to 85%	86 to 100%

5.3.1.5 Data analysis

The Mayberry Type Test was moderated and the Gutierrez, Jaime and Fortuny (1991:237-239) method (see § 2.2.3.2), to evaluate those answers that denote a possible transition between the levels, was used. The SOM questionnaire was also moderated to ensure reliability and content validity.

For both the Mayberry Type Test and the Study Orientation in Mathematics (SOM) Questionnaire the following statistical procedures and techniques were applied:

- The assistance of the Statistical Consultation Services of the North-West University, Potchefstroom Campus, was sought to analyse the quantitative data. Descriptive statistical techniques, means and standard deviation, were used to describe changes in the group.
- Cronbach Coefficient Alpha was employed to determine the reliability of the Mayberry Type Test and was also used to estimate the reliability of the SOM Test that had no right or wrong answers or scores and whose test items had more than two scores.
- The paired t-Test was used to compare the mean difference within the group as an indication of statistically significant differences. When $p<0,05$, a statistically significant difference between the groups exists (Gall *et al.*,1996:391).
- Effect-size (Steyn, 1999:3) was used to determine whether the statistically significant differences between the pre-test and post-test results were of practical significance.

The following formula was used to determine the effect-size:

$$d = \left| \frac{\Delta x}{\sigma} \right|$$

with d=effect size, Δx the mean of differences and **s**=standard deviation of the group. If $0{,}2 \leq d < 0{,}5$ it indicates a small effect, if $0{,}5 \leq d < 0{,}8$ it indicates a medium effect, if $d \geq 0{,}8$ it indicates a large effect Only if $d \geq 0{,}8$ is there a practical significant difference between the groups and the levels, although an effect size of $0{,}5 \leq d < 0{,}8$ indicates a possible practical significant difference.

5.3.2 Qualitative design

The PMTs made use of a dynamic computer programme namely GSP ® (see § 4.4). Initially the PMTs identified and operated on shapes according to their appearance. They recognised figures as visual "gestalts". Thereafter, they started to recognise and characterise shapes by their properties. Later on during the programme, some of the PMTs managed to form abstract definitions and distinguish between necessary and sufficient sets of conditions to develop specific concepts.

Phenomenological interviews were held after the PMTs had completed the dynamic computer technology programme. A phenomenological interview is regarded as a special type of in-depth interview used to study the meanings of a lived experience among selected participants (see § 5.3.2.2).

Responses, as captured on tape during the interviews, were transcribed. The transcriptions were translated into English because this dissertation is presented in English. All the information gained from the interviews was interpreted and documented (see § 6.2.2.3). The researcher discussed the findings with the participating lecturer.

5.3.2.1 Study population and sample

In executing the qualitative research, interviews (see Appendix C) were held. Britten (1995:256) says qualitative interviewing is a flexible and powerful method. According to Leedy and Ormrod (2001:159) interviews can yield a great deal of useful information.

Seven PMTs were selected for interviews as sample for the qualitative study. The PMTs were selected on the basis of their profile as reflected by their mathematics marks of the June 2005 examination in the geometry module. Therefore 3 low and 4 top performers were identified to take part in the qualitative part of the research.

5.3.2.2 Data generation

McMillan and Schumacher (2001:444) identify three types of interviews, namely interview guide (semi-structured interview), informal conversation (unstructured interview) and standardised interview (structured interview). See Table 5.4.

Table 5.4: Types of interviews (McMillan & Schumacher, 2001:444)

TYPES OF INTERVIEWS	
Informal conversation	Questions emerge from the immediate context.
Interview guide	Topics are outlined in advanced
Standardised interview	The exact words and sequence of questions are predetermined

Qualitative studies often require planned interviews so that the researcher can design the format in advance. The researcher used phenomenological interviews which is a specific type of in-depth interview. Phenomenological interviews investigate what was experienced, how it was experienced and finally the meanings that the interviewees assigned to the experience. The experience studied, was whether a dynamic computer technology programme such as GSP®, affected the mathematical conceptual understanding and study orientation in mathematics of PMTs (McMillan & Schumacher, 2001:445).

This method, the in-depth interview, is one of the most powerful tools in qualitative research. When selecting in-depth interviews for a qualitative study, the researcher assumes that the perspectives of the interviewees, who have personal experiences of the issue under investigation, form a vital source of information. In a study that uses the in-depth interview, the researcher decides in advance to which extent the interview will be structured. Most in-depth interviews use at least some predetermined system that places them between the structured and unstructured interviews (McMillan & Schumacher, 2001:446).

In a phenomenological interview the aim is to discover the interviewee's own framework of meanings and the research task is to avoid imposing the researcher's structures and assumptions (Britten, 1995:252).

Kvale (1996:32,33) says that the interviewees describe as precisely as possible what they experience and feel and how they act. The primary task for both the interviewer and the interviewee, however, remains that of obtaining descriptions so that the researcher will have relevant and precise material from which to draw interpretations.

5.3.2.3 Data analysis

Phenomenological interviews were held with the PMTs, after the intervention had taken place because the PMTs were then able to reflect on the activities that they had performed with GSP®. Responses to the interviews were recorded manually in the field book shortly after the interviews. It was done in this manner for the following reasons:

- To avoid distracting the interviewer's concentration if notes were to be taken during the interview.
- To avoid making the interviewee nervous by writing down his/her words during the interview.

The PMTs were interviewed to

- collect information about the positive and negative influences that the dynamic technological learning environment (GSP®) had exercised on their understanding of mathematics and all the variables.
- identify improvements, if any, related to the utilisation of the dynamic technological learning programme.
- gain additional information about the activities that were used during the lessons.

The duration of the interviews ranged between twenty and forty minutes per interview.

The data obtained through interviews were analysed in a narrative manner. In the case of the interviews the actual words of the interviewees were quoted as recorded in the interviewer's field notes and inductively interpreted in a narrative manner.

5.4 ETHICAL ASPECTS

A letter, requesting permission to use the afore mentioned study population, was sent to the Dean of the Education Department of Science, of the North-West University Potchefstroom Campus (see Appendix D).

Meetings were held with the mathematics subject chairman and the participating lecturer in order to explain the research aims, role of the participating lecturer as well as the support and commitment from them.

Furthermore, the details of the research were fully discussed with the mathematics subject chairman as well as the participating lecturer and the researcher also negotiated the procedure of how the classes would be conducted. As far as possible, the answering of the tests and questionnaires by the PMTs were conducted in such a way as to minimise disruption of lectures.

The lecturer involved, as well as the researcher, discussed the experimental programme with the PMTs and obtained their cooperation as participants in the research.

5.5 CONCLUSION

The use of a dual research approach, i.e. quantitative and qualitative research approaches afforded the researcher the opportunity to critically discuss and identify attributes about the statistical data.

Chapter 6 presents the research findings of the data gathered quantitatively and qualitatively and the statistics will be analysed and the results interpreted.

CHAPTER 6

RESEARCH FINDINGS AND DISCUSSION

6.1 INTRODUCTION

The aim of this research (see § 1.2) was to investigate whether and how a dynamic technological learning environment will influence the conceptual knowledge of PMTs. The purpose of this chapter is to present and discuss the research findings.

The descriptive statistical results with regard to the quantitative research approach are discussed in paragraph 6.2, while the qualitative results are described in paragraph 6.3, followed by a discussion of the quantitative as well as the qualitative research findings in paragraph 6.4. Thereafter a conclusion has been made in paragraph 6.5.

6.2 RESULTS

6.2.1 Quantitative results

6.2.1.1 Reliability and validity of instruments

Conceptual understanding of geometry concepts

The Mayberry Type Test (see § 5.3.1.4) was the instrument that the researcher used to determine whether or not the dynamic computer technology had an influence on the conceptual understanding of the PMTs over the first four Van Hiele levels (see § 2.2.3). The answers to the items were quantified according to the acquisition scales of Gutiérrez *et al.* (1991) (see § 2.2.3.2).

Cronbach Coefficient Alpha is a test for a model or survey's internal consistency (Gall *et al.*, 1996:256,257). It assesses the reliability of a rating which measures some underlying factor. A score is computed from each test item and the overall rating, called a 'scale' is defined by the sum of these scores over all the test items. The reliability of scores should be established before the research is undertaken and a reliability evaluation for the post-test has to be performed as well.

Cronbach Coefficient Alpha (see Table 6.1) was employed to estimate the reliability of the Mayberry Type Test.

Table 6.1: Level of reliability of Mayberry Type Test (Cronbach Coefficient Alpha)

	GEOMETRIC FIGURES	PRE-TEST ALPHA VALUES
1	**Squares**	0,50
2	**Right-angled triangle**	0,59
3	**Isosceles triangle**	0,67
4	**Congruency**	0,50
5	**Similarity**	0,50

Computation of the Cronbach Coefficient Alpha had a moderate alpha coefficient reading (≥0,5), indicating sufficient internal consistency and reliability.

Study Orientation in Mathematics

The SOM questionnaire was the instrument that the researcher used to determine whether or not the dynamic computer technology had an influence on the study orientation of the PMTs. Gay (2000:174) and McMillan and Schummacher (2001:246) contend that the Cronbach Coefficient Alpha is also used to estimate reliability of a test that has no right or wrong answers or scores and whose test items have more than two scores. As explained earlier in chapter 5 (§ 5.3.1), the items of the SOM questionnaire have no right or wrong answers. Instead, the PMTs were required to choose the option that best suited them from the five-point given scale. For these reasons Cronbach Coefficient Alpha (see Table 6.2) was also employed to estimate the reliability of the SOM questionnaire as well as the reliability of the post-test.

According to Maree *et al.* (1997:26) the estimated reliability level of the SOM questionnaire ranges from 0,74 to 0,95 for Afrikaans language speakers.

Table 6.2: Level of reliability of SOM fields (Cronbach Coefficient Alpha)

	SOM FIELDS	PRE-TEST ALPHA VALUES
1	**Study attitudes (SA)**	0,83
2	**Mathematics anxiety (MA)**	0,77
3	**Study habits (SH)**	0,80
4	**Problem solving behaviour (PSB)**	0,81
5	**Study milieu (SM)**	0,77

Computation of the Cronbach Coefficient Alpha had a high alpha coefficient reading (≥0,8), indicating high internal consistency and reliability.

Maree *et al.* (1997:7) explain that, in terms of construct validity, the SOM questionnaire aims at measuring the study attitudes, mathematics anxiety, study habits, problem-solving behaviour and the study milieu of learners when learning mathematics. Each of these phenomena was clearly defined in chapter 3 (see § 3.3.1).

6.2.1.2 Significance of difference

A paired t-test was conducted to establish the mean difference between the pre-test and post-test results within the experimental group that received the treatment. The mean difference within the experimental group was practically significant (see § 5.3.1.5) for both the Mayberry Type Test (see Tables 6.3-6.7) and the SOM test (see Table 6.8).

It was desired that the mean difference within the experimental group should differ practically significantly in order to accurately measure the influence of the treatment on the experimental group. The Cohen effect size (d), Cohen's category (Cohen, 1988:222), t-value and p-value were used as an indication of practical or meaningful difference (see § 5.3.1.5). It is clear, from the results, that the dynamic computer programme has a positive influence on the conceptualisation of PMTs A synopsis of the t-test results is provided in Tables 6.3-6.8.

Table 6.3: t-Test, conceptualisation of squares

SQUARES	n	MEAN DIFFERENCE (Δx)	STANDARD-DEVIATION (σ)	STATISTICAL SIGNIFICANCE ($p < 0,05$)	EFFECT SIZE ($d = \frac{\Delta x}{\sigma}$)
Level 1	26	0,08	0,25	0,14 (>0,05)	-
Level 2	26	0,20	0,31	0,003	0,65**
Level 3	26	0,38	0,23	<0,0001	1,64***
Level 4	26	0,32	0,32	<0,0001	1,00***

** medium effect, *** large effect (practically significant)

The test results (see Table 6.3) revealed that there is not a statistically significant difference (p=0,14) at level 1, indicating that it is not practically significant. A possible reason why there is not a statistically significant difference at level 1 is that the PMTs had already acquired a high level of conceptual understanding before intervention. The test results (see Table 6.3) revealed a statistically significant difference between levels 2 to 4 (p<0,01), the effect size of level 2 was medium and the effect sizes of levels 3 and 4 indicated a large effect.

Table 6.4: t-Test, conceptualisation of right-angled triangles

RIGHT ANGLED TRIANGLE	n	MEAN DIFFERENCE (Δx)	STADARD-DEVIATION (σ)	STATISTICAL SIGNIFICANCE ($p < 0,05$)	EFFECT SIZE ($d = \frac{\Delta x}{\sigma}$)
Level 1	26	0,35	0,27	<0,0001	1,32***
Level 2	26	0,31	0,33	<0,0001	0,92***
Level 3	26	0,22	0,24	<0,0001	0,92***
Level 4	26	0,23	0,23	<0,0001	1,02***

*** large effect (practically significant)

The test results (see Table 6.4) revealed a statistically significant difference at the four levels (p<0,01), the effect sizes of all the levels were large and therefore of practical significance.

Table 6.5: t-Test, conceptualisation of isosceles triangles

ISOSCELES TRIANGLES	n	MEAN DIFFERENCE (Δx)	STANDARD-DEVIATION (σ)	STATISTICAL SIGNIFICANCE $(p < 0,05)$	EFFECT SIZE $\left(d = \frac{\Delta x}{\sigma}\right)$
Level 1	26	0,21	0,28	0,001	0,72**
Level 2	26	0,37	0,44	0,003	0,84***
Level 3	26	0,24	0,17	<0,0001	1,42***
Level 4	26	0,21	0,32	0,003	0,66**

** medium effect, *** large effect (practically significant)

The test results (see Table 6.5) revealed a statistically significant difference at the four levels (p<0,01), the effect sizes of levels 1 and 4 were medium and the effect sizes of levels 2 and 3 were large and therefore of practical significance.

Table 6.6: t-Test, conceptualisation of congruency

CONGRUENCY	n	MEAN DIFFERENCE (Δx)	STANDARD-DEVIATION (σ)	STATISTICAL SIGNIFICANCE $(p < 0,05)$	EFFECT SIZE $\left(d = \frac{\Delta x}{\sigma}\right)$
Level 1	26	0,15	0,29	0,011	0,54**
Level 2	26	0,21	0,49	0,042	0,43*
Level 3	26	0,30	0,19	<0,0001	1,60***
Level 4	26	0,69	0,47	<0,0001	1,47***

* small effect, ** medium effect and *** large effect (practically significant)

The test results (see Table 6.6) revealed a statistically significant difference at the four levels (p<0,01), the effect size of level 1 was medium, the effect size of level 2 was small and the effect sizes of levels 3 and 4 were large, indicating an effect size of practical significance.

Table 6.7: t-Test, conceptualisation of similarity

SIMILARITY	n	MEAN DIFFERENCE (Δx)	STANDARD-DEVIATION (σ)	STATISTICAL SIGNIFICANCE $(p < 0{,}05)$	EFFECT SIZE $\left(d = \frac{\Delta x}{\sigma}\right)$
Level 1	26	0,31	0,35	0,0001	0,88***
Level 2	26	0,33	0,31	<0,0001	1,07***
Level 3	26	0,35	0,19	<0,0001	1,83***
Level 4	26	0,29	0,45	0,003	0,64**

** medium effect, *** large effect (practically significant)

The test results (see Table 6.7) revealed a statistically significant difference between the groups (p<0,01). The effect size of level 4 was medium and the effect sizes of levels 1 to 3 were large and practically significant.

Study Orientation in Mathematics

Table 6.8: t-Test: Study Orientation in Mathematics (SOM) Questionnaire

SOM FIELDS	MEAN DIFFERENCE (Δx)	STANDARD -DEVIATION (σ)	STATISTICAL SIGNIFICANCE $(p < 0{,}05)$	EFFECT SIZE $\left(d = \frac{\Delta x}{\sigma}\right)$
Study attitudes (SA)	9,00	4,72	<0,0001	1,91***
Mathematics anxiety (MA)	9,08	4,62	<0,0001	1,97***
Study habits (SH)	8,87	3,78	<0,0001	2,35***
Problem solving behaviour (PSB)	10,52	4,61	<0,0001	2,28***
Study milieu (SM)	4,85	4,50	<0,0001	1,08***

*** large effect (practically significant)

The test results (see Table 6.8) revealed a statistically significant difference between the SOM fields (p<0,01). The effect sizes of all the levels were of practical significance. It is clear that a dynamic computer technology programme such as GSP®, results in a marginally significant difference.

6.2.1.3 General acquisition of conceptual understanding and study orientation in mathematics

Conceptual understanding of geometry concepts

Research done by Van der Sandt (2003:83) revealed that learners leave school with higher levels and degrees of geometric acquisition than the levels and degrees of acquisition attained by the PMTs who had been exposed to three years of academic and mathematical methodology training. It seems that the geometry acquisition that learners obtained during school, decays significantly within only three years, to levels far below those levels expected from teachers. This decay could point to ineffectiveness of the educational paradigm followed at school level where rote learning is encouraged. The emphasis is on memorisation without conceptual understanding of mathematics. This pattern continues into the PMTs' pre-service training, especially considering the low degree of geometry acquisition shown by PMTs.

This problem can be addressed by integrating a dynamic computer programme into the curriculum for teacher education.

The PMTs consistently achieved higher levels of acquisition after they had followed the intervention programme, leading to the conclusion that the intervention programme did have a positive effect on the acquisition of higher levels of geometric thought. KL is the cut off line above which the degrees of acquisition are high and the dotted line MN is the cut off line below which the degrees of acquisition are low. Between the two lines the degrees of acquisition are intermediate (see Figures 6.1 to 6.5).

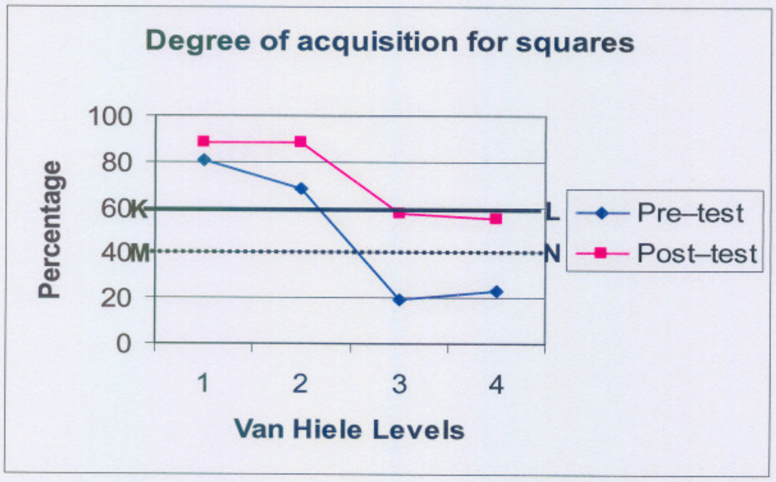


Figure 6.1: Degree of acquisition for squares

The graph compares the degree of acquisition for squares (see Figure 6.1) before intervention and thereafter. The post-test reveals that the PMTs are on a higher degree of acquisition after the intervention programme. According to Gutierrez *et al*. (1991:238), the PMTs achieved a high degree of acquisition in levels 1 and 2 and their degree of acquisition for levels 3 and 4 is intermediate (see Table 2.1).

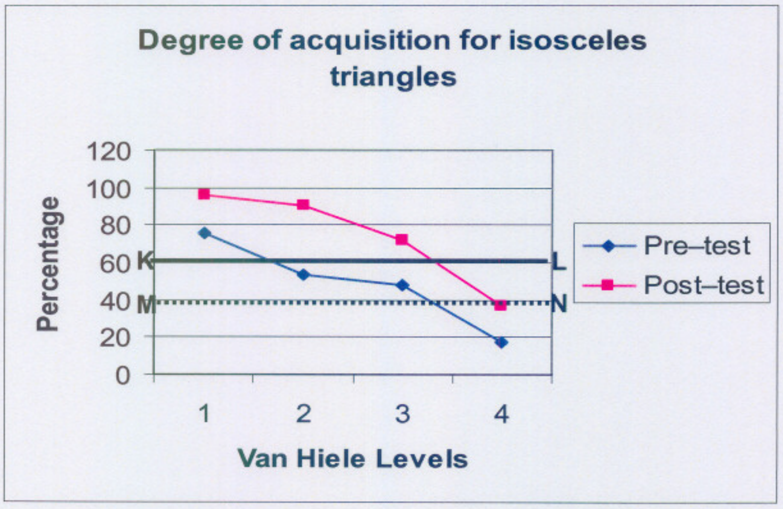


Figure 6.2: Degree of acquisition for isosceles triangle

The graph compares the degree of acquisition (see Figure 6.2) for isosceles triangles before intervention and thereafter. According to Gutierrez *et al*. (1991:238), the PMTs' degree of acquisition for levels 1, 2 and 3 is high after having followed the intervention programme, but they reached a low degree of acquisition for level 4 (see Table 2.1).

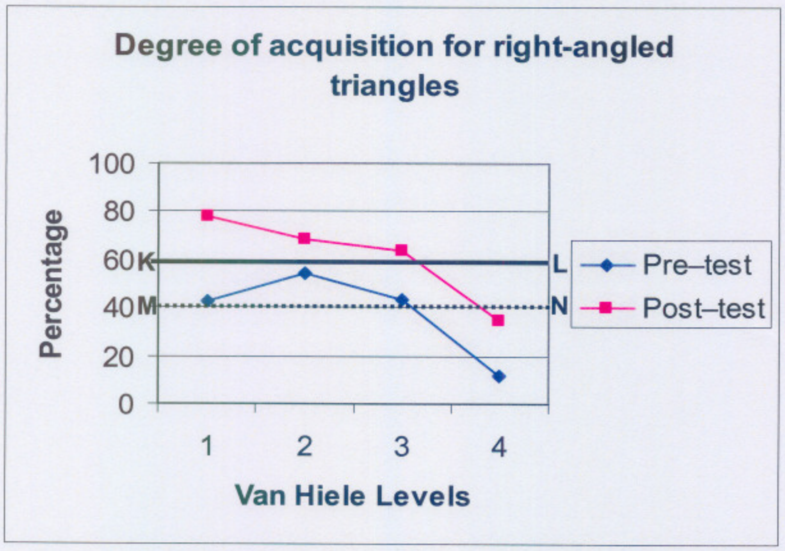


Figure 6.3: Degree of acquisition for right-angled triangle

The graph compares the degree of acquisition (see Figure 6.3) for right-angled triangles before intervention and thereafter. According to Gutierrez *et al.* (1991:238), the PMTs' degree of acquisition for levels 1, 2 and 3 is high after following the intervention programme, but they reached a low degree of acquisition for level 4 (see Table 2.1).

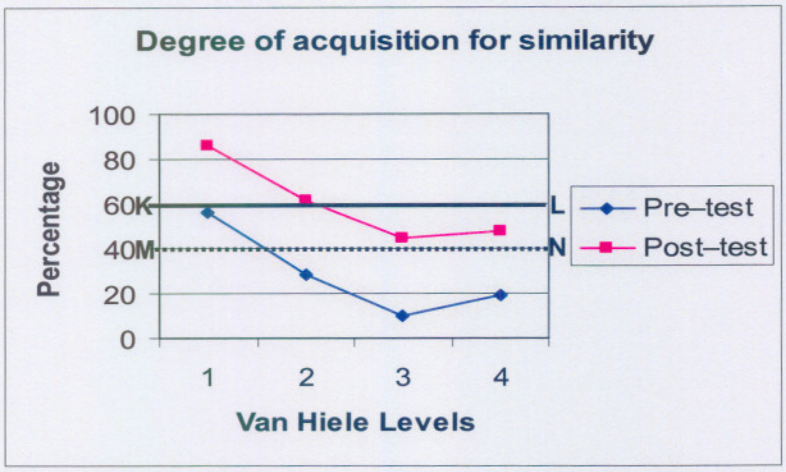


Figure 6.4 Degree of acquisition for similarity

The graph compares the degree of acquisition for similarity (see Figure 6.4) before intervention and thereafter. According to Gutierrez *et al.* (1991:238), the PMTs' degree of acquisition for levels 1 and 2 is high after having followed the intervention programme, but they reached an intermediate degree of acquisition for levels 3 and 4 (see Table 2.1).

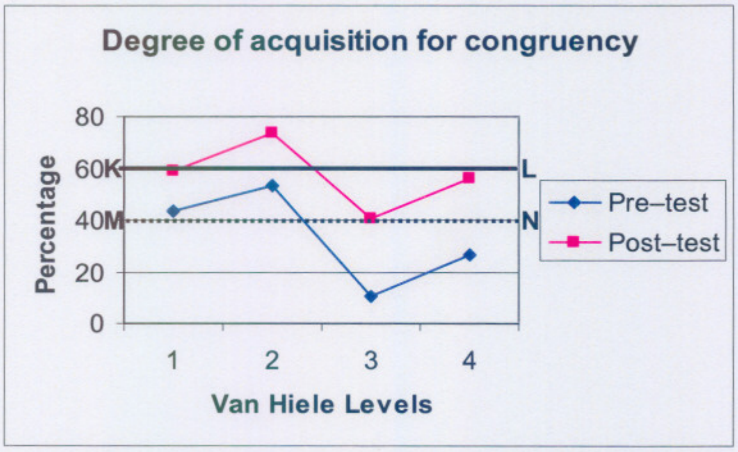


Figure 6.5: Degree of acquisition for congruency

The graph compares the degree of acquisition for congruency (see Figure 6.5) before the PMTs followed the intervention programme and thereafter. According to Gutierrez *et al.* (1991:238), the

PMTs' degree of acquisition for levels 1 and 2 is high after having followed the intervention programme, but they reached an intermediate degree of acquisition for levels 3 and 4 (see Table 2.1).

Geometry is a practical and structured guide for approaching situations and it is evident that for PMTs to do well in mathematics they have to understand geometric principles and concepts (see Figures 6.1 to 6.5). Geometry should not be learned as a set of rules but it should rather be understood and the PMTs should know how to use and apply geometry.

Study Orientation in Mathematics

A high percentile rank indicates a positive study orientation while a low score indicates a negative study orientation in mathematics. A high percentile ranking for mathematics anxiety indicates that the PMTs have a relatively low level of anxiety for mathematics (Maree *et al.*, 1997:15).

According to Maree *et al.* (see Table 6.9) the following data can be used as a guideline for the interpretation of the scores:

Table 6.9: A guideline for the interpretation of the percentile scores (Maree et al., 1997:15)

PERCENTILE SCORE	INTERPRETATION
70-100%	Positive study orientation
40-69%	Neutral: can contribute to positive or negative study orientation
0-39%	Negative study orientation

The raw mean scores, of the PMTs, were converted to percentile ranks (Maree *et al.*, 1997:14) and showed that the level of study orientation of the PMTs is significantly higher (see Figure 6.6) after they have completed the dynamic computer technology programme (GSP®). The line at 70% indicates that the different fields above that line have a positive study orientation.

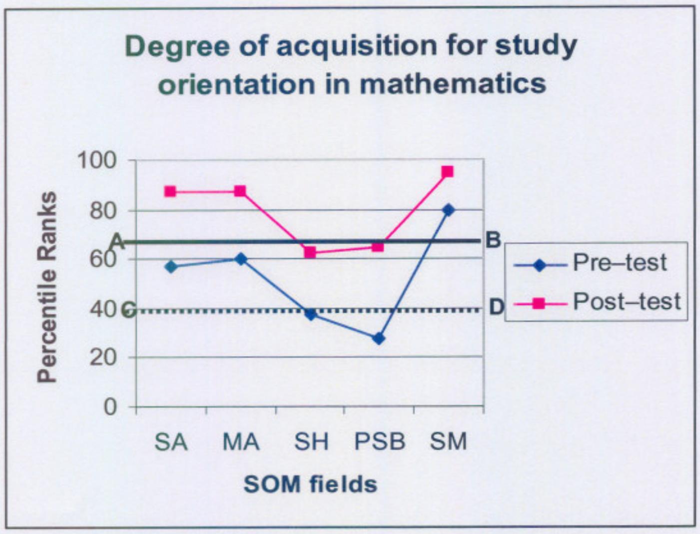


Figure 6.6: Degree of acquisition for study orientation in mathematics

The graph compares the level of acquisition for study orientation in mathematics (see Figure 6.6) before intervention and thereafter. The PMTs consistently achieved higher levels of acquisition, after they had followed the intervention programme, leading to the conclusion that the intervention programme did have a positive effect on the acquisition of higher levels of geometric thought.

AB is the cut off line above which the study orientation in mathematics (or specific SOM fields) is positive and line CD is the cut off line below which the study orientation in mathematics is negative. Between the two lines the study orientation in mathematics is neutral. The level of acquisition for SA, MA and SM is above 70%, which indicates that the PMTs clearly have a positive study orientation in these three fields. The level of acquisition for SH and PSB is less than 70% and therefore the PMTs' study orientation in these two fields is neutral (see Table 6.9).

6.2.2 Qualitative results

6.2.2.1 Reliability and validity

According to Cowger and Menon (2001:477) triangulation is the process of incorporating multiple viewpoints of the same phenomenon so as to provide greater validity to the research endeavour. It provides additional evidence of what the PMTs were observing. To ensure the internal validity of the interviews, the techniques of triangulation were used (see Figure 6.7).

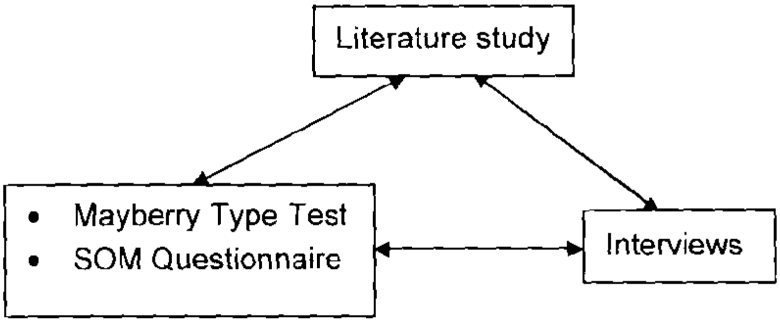


Figure 6.7: Triangulation of perceptions

In this study, the triangulation technique involved looking for common perceptions from different sources and /or statements that appeared in the interviews.

6.2.2.2 Phenomenological interviews

The basic purpose of the phenomenological interviews (see § 5.3.2.3) was to gather additional information of how the PMTs felt about the dynamic technological programme (GSP®).

6.2.2.3 Discussion of the qualitative research findings

The questions set to the participants (S) were taken from a prepared interview schedule (see Appendix C) and the researcher (R) posed further questions as these came up in the spontaneous development of the interaction between the interviewer (R) and the interviewee (S). The core responses that were received are noted below (Table 6.10):

Table 6.10: Core responses

		RESPONSES	
	QUESTION	**LOW ACHIEVERS**	**HIGH ACHIEVERS**
1	On a scale of 1 to 10, 10 being the highest, how much do you like mathematics?	6	9
2	Do you always do your assignments? "How much time do you spend on assignments?"	Most of them. It depends on how much other work I have to do.	I am doing all my assignments. I try to do more than necessary

3	In general, what influences you to work hard in mathematics? (evidence of task orientation, ability orientation, social orientation). "Is there anything that causes you to work harder?"	If I understand the work better it will influence me to work harder. I don't think so	Good results. If my results are not so good it will inspire me to work harder. The stimulation when my marks are good
4	How good are you at mathematics? "How do you know?"	Not so good. I struggle to do all my assignments.	I think I am fairly good. I understand the work quite quickly.
5	Are you better at some sections of mathematics than other sections? For example, are you better at algebra than geometry? Why?	I don't like geometry at all. I do not understand the work, I cannot do the work.	I am better at algebra and trigonometry than at geometry. I didn't understand it when I was at school but now I understand it better.
6	Do you think it takes special talent to do well in mathematics? Do you have such talent? Can people do well in mathematics even without special talent?	Yes, I think so. No, my marks are not so good. Not really.	I think so, but the talent develops over time. I think so, my marks are good. I don't think so.

	Why?	You have to understand the work.	You have to know what is going on.
7	How important is memorisation in mathematics?	It is very important.	It is not only a subject that you must understand, you have to learn as well. There is a difference between memorisation and learn. If I am talking about learning then I know what is going on, if you memorise then you don't know what is going on, you have to understand what is going on.
	Can someone who is not very good at memorisation be good at mathematics?	People with talent do not have to memorise that much.	Yes, when you have talent you can still do well.
8	How often do you do the least amount of work you can to get by? (look for evidence of work avoidant-orientation).	If I do not know what is going on then I will not do my work.	It depends on how much work I have to do, if I have a lot of work in my other subjects then I will spend less time on mathematics but if I have enough time then I will do more than expected from me.
9	How do you study for a test in mathematics?	I revise the problems that we have done in class and then I will do a lot of problems again and again.	I do the problems that I have difficulty with first of all and then I will do revision.

	Has anyone taught you special skills for studying mathematics?	No.	Not really.
10	Give examples of the activities you have done in mathematics.	Cubes and GSP.	GSP which is very interesting and cubes.
11	Of the activities that you have mentioned in your response to the last question, are there any that were particularly enjoyable or interesting?	GSP helps me to understand mathematics better and therefore it is easier to learn.	When you have to do geometry it is very easy to draw the sketches and you can see the results very quickly. I can see gradually what happened and then it is easier to understand the work
12	Are there problems in mathematics that can be solved in more than one way? Give examples	Yes, I think so.	Yes, especially with OBE where the students have to use their own way of solving a problem.
13	Does GSP influence your ability to understand mathematics in a better way?	Yes, it helps me to understand the work.	Definitely, I enjoy it very much and I understand the work quicker.
14	Is there anything that you think is important about learning mathematics that you haven't said?	No I think I have said everything.	I can just say again that GSP is a nice tool to have in hand.

The interviews conducted with the PMTs provided some insight into the nature of the influence of a dynamic computer programme on the geometry conceptualisation of PMTs. Firstly, in a dynamic technologically enhanced learning environment a positive correlation seems to exist between PMTs that prefer mathematics as a subject and their orientation to complete geometry assignments, whereas PMTs that do not prefer mathematics as a subject show a tendency to often not complete their geometry assignments. Secondly, in the particular learning environment, PMTs that prefer mathematics as a subject seem to think that they are expert in doing mathematics (particularly geometry), whereas PMTs who do not prefer mathematics as a subject seem to think that they do not have the ability to do mathematics (particularly geometry). Thirdly, PMTs report that their attitudes towards mathematics (geometry) have changed positively after they started to use GSP®, and that they have a better understanding of mathematics, particularly certain geometry concepts, after using GSP® during the course of the module. The PMTs' seem confident that GSP® can be used to explain some of the work so that they will be able to understand the work better. Overall, the interviewed PMTs are convinced that a dynamic technologically enhanced learning environment does help to improve their execution of geometry tasks and their learning of geometry concepts, as well as to promote positive dispositions relating to the subject taught and learned in the module.

It is, therefore, the contention of the author that GSP®, in particular, and dynamic computer technology, in general, should no longer be seen as an option. Rather it should be viewed as an essential part of the professional development of the PMTs. The utilisation of dynamic computer technology should be seen as a powerful opportunity, albeit it challenging, to invigorate learning environments for PMTs.

According to Reed (1995:241), any professional programme in teacher education should be dedicated to the idea of excellence teaching and dynamic computer technology should be part of it.

6.3 CONCLUSION

The aim of this chapter was to investigate whether and how a dynamic computer technology programme influenced learning strategies as well as the development of conceptual knowledge.

The PMTs consistently achieved significantly higher levels of conceptual knowledge.

With regard to the identification of triangles it can be said that the PMTs who followed the dynamic computer technology programme, were able to correctly identify triangles and gave fairly complete answers to substantiate their answers. These PMTs did not show the tendency to confuse the different types of triangles with each other.

Combining a dynamic computer technology programme with a constructivist-centred teaching approach can deliver results that would be better than the results obtained by means of a conventional process-product teaching approach.

Chapter 7 presents the conclusions and the recommendations.

CHAPTER 7

7

CONCLUSIONS AND RECOMMENDATIONS

7.1 INTRODUCTION

This chapter provides an overall summary of the research, by firstly giving a synopsis of the research and secondly the general conclusions and recommendations about the study. The synopsis will include an overview of the literature review regarding the theoretical framework; the impact of the methods of the research employed in the study the implications of the research findings in the teaching and learning of mathematics. The second part, will present the limitations of the study, recommendations for future research and general concluding remarks.

7.2 PROBLEM STATEMENT

Mathematics problem solving is the core of functionality within mathematics. Appropriate uses of dynamic computer technology can enhance mathematics learning and teaching and support conceptual development of mathematics as well as study orientation in mathematics. The growing availability of dynamic computer technology provides an opportunity to assist teachers in teaching well and in improving the mathematics experiences of PMTs (Wilson, 2001).

The aim of the research was to determine how the implementation of a Van Hiele based dynamic computer technology programme influences the following aspects:

- determining what effect a dynamic technological learning environment has on the conceptual understanding of PMTs in geometry.
- determining how the use of a dynamic technological learning environment influences the conceptual understanding of PMTs in geometry.
- determining what effect a dynamic technological learning environment has on the study orientation of PMTs.

7.3 REVIEW OF LITERATURE

The literature review was done to critically and objectively highlight the strength and weaknesses of a dynamic computer technology programme in the context to enhance conceptual understanding of mathematics and study orientation in mathematics. The theoretical framework has the implications for teaching and learning of PMTs that it is essential to find appropriate methods and environments to improve the conceptual understanding (see § 3.2) of PMTs.

Piaget (see § 2.2.1) says that the development of learners proceeds according b a series of transformations from one stage to another and therefore it seems appropriate that learning experiences should be organised in terms of the learner's developmental stage. According to Nixon (2005:23,47), Piaget and Garcia (1989) identified three levels in the development of thought and they state that these levels are not bound to learners' ages or fixed stages of development. In view of the analysis of Nixon's three levels in the development of thought (see § 2.2.1.2), it becomes clear that Van Hiele's theory of cognitive levels in geometry follows the same trend. Van Hiele (see § 2.2.3) developed a theory that dealt with the belief that learners' thinking skills develop in levels and these levels represent a hierarchy of growth.

Vygotsky (see § 2.2.2) had a great influence on constructivism (see § 2.3.2) and is most often associated with the social constructivism. The influence of Vygotsky's ideas, together with the constructivist theory, lend themselves to many educational applications and have provided a basis for transforming mathematics teaching and learning. Learning is a constructive process that occurs while participating in and contributing to the practices of the local community. From this perspective, lecturers can use a dynamic computer technology programme to enable PMTs to construct their own knowledge.

One of the current reform movements in mathematics education is the appropriate use of dynamic computer technology in the teaching and learning of mathematics. Concerning mathematics education, the lecturers may involve the introduction of both dynamic computer technology and mathematics in meaningful contexts that will enable interplay between the two (Abramovich & Strock, 2002:184). PMTs will be actively involved in their learning (see § 2.3.2) and they will therefore be less frustrated in their study orientation in mathematics (Maree *et al.*, 1997:1,2) (see § 3.4).

To be able to reach learners, PMTs' own conceptual understanding (see § 3.2) should be developed. When PMTs have conceptual understanding of a mathematical procedure, they will perceive this procedure as a mathematical model of a problem situation, rather than just an algorithm (Abramovich & Strock, 2002:173). PMTs will be less frustrated and their study orientation in mathematics will also improve if they do understand mathematics (Maree *et al.*, 1997:1,2).

7.4 EMPIRICAL INVESTIGATION

7.4.1 Design

This study aimed to investigate the effect of a technologically enhanced learning environment on PMTs' understanding of geometry concepts and their study orientation in mathematics, as prerequisite for deep conceptualisation.

A combined quantitative and qualitative research approach was used. The quantitative investigation employed a pre-experimental one-group pre-test post-test design. A Mayberry-type test was used to collect data with regard to PMTs' conceptualisation of geometry concepts, while the Study Orientation in Mathematics (SOM) questionnaire was used to collect data regarding their study orientation in mathematics. The qualitative investigation employed phenomenological interviews to collect supplementary information about the participating PMTs' experiences and assessment of the influence of the use of the dynamic software Geometer's Sketchpad (GSP®) on their learning and conceptualisation of geometry concepts (see chapter 5).

7.4.2 Results

During post-testing the participating group of PMTs achieved practically significantly higher scores in the Mayberry-type test, as well as in all fields of the SOM questionnaire. Results seem to indicate that PMTs gained significantly in the expected high levels of conceptualisation, as well as high degrees of acquisition of those levels during the intervention programme.

The responses during the interviews were overwhelmingly positive (see § 6.2.2.3). The results attested to an acknowledgement of the success of the use of GSP® as a dynamic computer technology programme, to enhance the PMTs' geometry conceptualisation and study orientation in geometry.

7.5 GENERAL CONCLUSIONS AND RECOMMENDATIONS

7.5.1 Limitations of the study

The study might have suffered because of the following limitations:

- It was a limited, local study, and the findings reported have limited value for generalisaion.
- All the third year education students followed the general mathematics module in geometry in conjunction with a dynamic computer technology programme, and therefore the researcher has to use the pre-experimental design (see § 1.3.2) with no control group.
- The SOM questionnaire is a questionnaire that is developed for use of grade 8 to grade 12 learners, but as mentioned before, Steyn and Maree (see § 5.3.1.4) used the questionnaire involving first-year engineering students and found that it could be regarded as significant predictors of performance in mathematics at university level. Questionnaires developed and standardised for tertiary levels were not available, and therefore no alternative existed other than to use the above mentioned questionnaire.
- The interruption in the middle of the program because of the University's holiday as well as semester tests could have negatively influenced the results. Another factor that could have influenced the results negatively is the fact that the lecturer could not continue with the classes (near the end of the program) and a substitute lecturer was used.

7.5.2 Main conclusion

The main conclusion of the study is that a technologically enhanced learning environment (such as GSP®) can be successfully utilised to significantly enhance PMTs' conceptualisation and study orientation, as prerequisite for deep conceptualisation, in geometry.

From this study, the importance of study orientation for conceptualisation in geometry, becomes clear.

7.5.3 Recommendations for future research

It is recommended that:

Mathematics education

- GSP® can be used to develop the PMTs' conceptual understanding of geometry.
- GSP® can be used to enhance the PMTs' study orientation in geometry.
- The PMTs can use the experiences that they have gained from GSP® to facilitate their own learners' conceptualisation, when they start teaching.

Future research

- Comparative studies in a bigger study population should be undertaken. Several factors have limited the generalisation of the results of this research (see § 7.5.1).
- Longitudinal studies should be undertaken (pre-and post-tests), to determine whether the results of the dynamic computer technology programme such as GSP® and the study orientation in geometry, are permanent over a longer period of time.

7.6 VALUE

It is essential to find methods to improve the conceptual understanding of PMTs. The results in this study pointed to the use of dynamic computer technology in the training of PMTs, which may provide a valuable and practical contribution to help the development of conceptual understanding of PMTs.

7.7 FINAL REMARKS

In this research the effect of an integrated use of a dynamic computer technology programme in a problem solving context, was analysed. The implementation of a dynamic computer technology programme had a positive effect not only on the conceptualisation and levels of geometric thought of the PMTs in the sample, but also on their level of study orientation. These PMTs were more confident in doing geometry than previously and their way of studying geometry improved as well.

In summary, it can be stated that a dynamic computer technology programme in combination with a problem solving environment appears to be a potentially useful strategy to facilitate optimal achievements and conceptualisation in geometry.

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APPENDIX A

THE MAYBERRY-TYPE VAN HIELE TEST

Name/Naam:.. ....

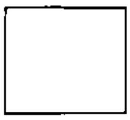
Institute/Instelling: ....

Sex/Geslag: ....

Age/Ouderdom:.................................... ....

1. This figure is which of the following?

Watter een van die volgende is hierdie figuur?



a) triangle/driehoek

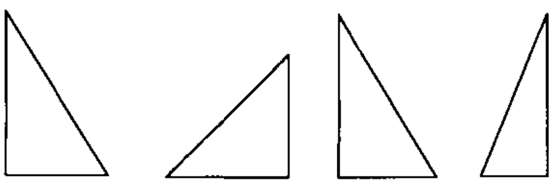
b) quadrilateral/vierhoek

c) square/vierkant

d) parallelogram

e) rectangle/reghoek

2.



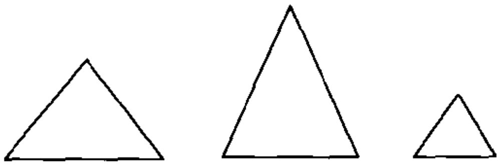
Are all of these triangles? YES/NO, Explain:

Is al hierdie figure driehoeke? JA/NEE, Verduidelik:

Do they appear to be a special kind of triangle? If so what kind?

Lyk dit of hulle spesiale soort driehoeke is? Indien, watter soort?

3.



These appear to be what kind of triangles?

Watter soort driehoek is hierdie driehoeke?

4.

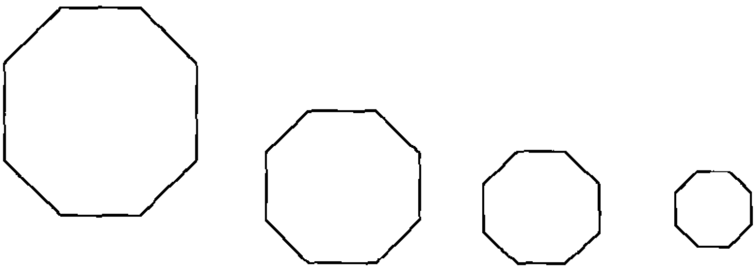
What is true of A and B? What is true of C and D?

Wat is waar van A en B? Wat is waar van C en D?

What word describes this?

Watter woord beskryf die verskynsel die beste?

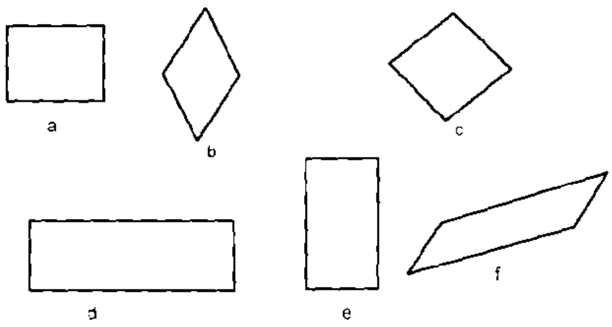
5.



Are these figures alike in any way? YES/NO/, What word describes this?

Is hierdie figure in enige opsig dieselfde? JA/NEE, Watter woord beskryf die verkynsel die beste?

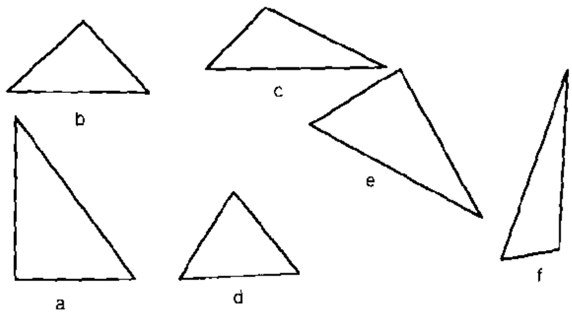
6.



Which of these figures are squares?

Watter van hierdie figure is vierkante?

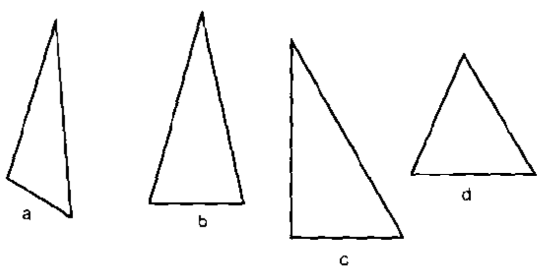
7.



Which of these appear to be right-angled triangles?

Watter van hierdie figure lyk soos reghoekige driehoeke?

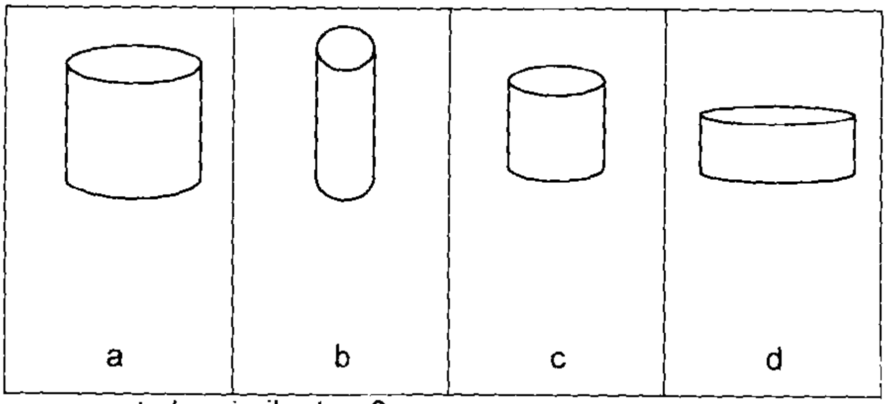
8.



Which of these figures appear to be isosceles triangles?

Watter van hierdie driehoeke lyk soos gelykbenige driehoeke?

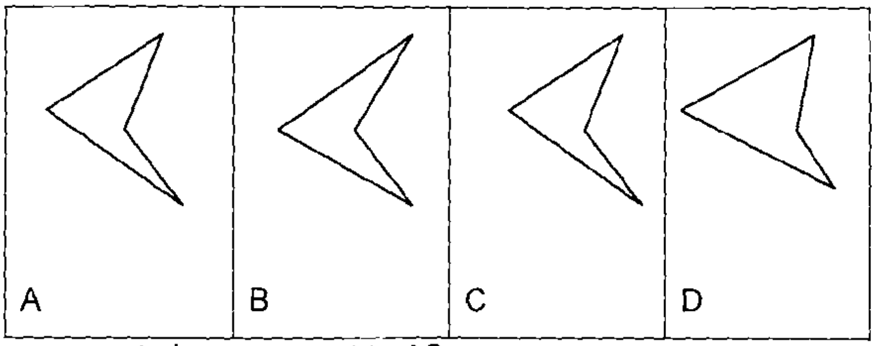
9.



Which figure appears to be similar to a?

Watter figuur lyk of dit gelykvormig kan wees aan a?

10.



Which figure appears to be congruent to A?

Watter figuur lyk of dit kongruent kan wees aan A?

11. Draw a square/Teken 'n vierkant.

a. What must be true about the sides?/Wat moet waar wees van die sye?

b. What must be true about the angles?/Wat moet waar wees van die hoeke?

12. Does a right-angled triangle always have a long side? If so, which one?

Het 'n reghoekige driehoek altyd 'n langer sy? Indien wel, watter een?

Does a right-angled triangle always have a largest angle? If so, which one?

Het 'n reghoekige driehoek altyd 'n hoek wat die grootste is? Indien wel, water een?

---

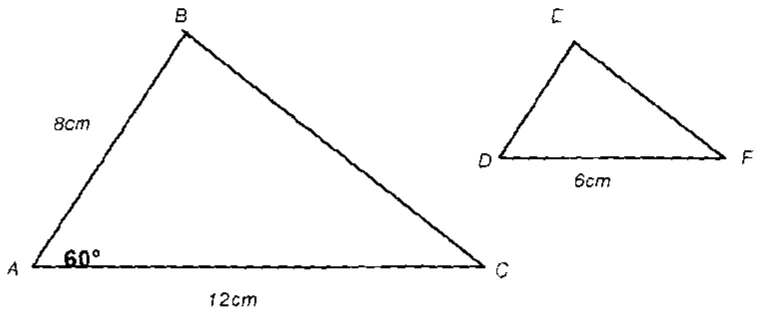
13. What can you tell me about the sides of an isosceles triangle?

Wat weet jy van die sye van 'n gelykbenige driehoek?

What can you tell me about the angles of an isosceles triangle?

Wat weet jy van die hoeke van 'n gelykbeninge driehoek?

14.



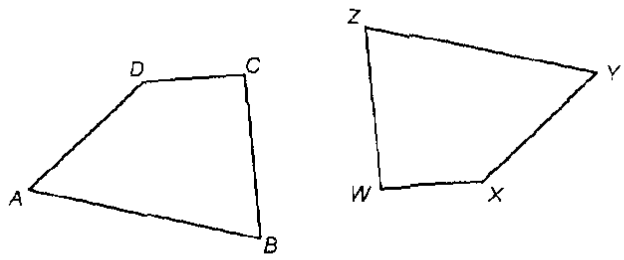
Triangle ABC is similar to triangle DEF. How long is ED? How do you know?

Driehoek ABC is gelykvormig aan driehoek DEF driehoek. Hoe lank is ED? Hoe weet jy?

What is the size of $\angle$EDF? How do you know?

Wat is die grootte van $\angle$EDF Hoe weet jy?

15.



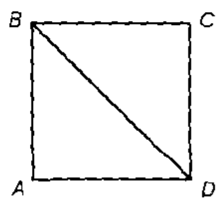
These are congruent figures. What is true about their sides? AD = ___

Hierdie is kongruente figure. Wat is waar omtrent hul sye? AD = ___

What is true about their angles? ∠B =___

Wat is waar omtrent hul hoeke? ∠B =___

16.



ABCD is a square, BD is a diagonal. Name an angle congruent to ∠ABD.

ABCD is 'n vierkant, BD is 'n diagonaal (hoeklyn). Benoem 'n hoek wat kongruent is aan ∠ABD.

How do you know?/Hoe weet jy?

---

17. Circle the smallest combination of the following which guarantees a figure to be a square.

Omkring die kleinste kombinasie wat sal verseker dat die figuur 'n vierkant is.

(a) It is a parallelogram. /Dit is 'n parallelogram.

(b) It is a rectangle. /Dit is 'n reghoek.

(c) It has right angles. /Dit het 'n regte hoek.

(d) Opposite sides are parallel. /Teenoorstaande sye is parallel.

(e) Adjacent sides are equal in length. /Aangrensende sye is ewe lank.

(f) Opposite sides are equal in length. /Teenoorstaande sye is ewe lank.

18. Name some ways in which squares and rectangles are alike?

Noem 'n paar ooreenkomste tussen vierkante en reghoeke.

____

Are all squares also rectangles? Why?

Is alle vierkante reghoeke? Hoekom?

____

____

19. Circle the smallest combination of the following which guarantees a triangle to be a right triangle?

Omkring die kleinste kombinasie van idie volgende wat verseker dat die driehoek 'n reghoekige driehoek is.

(a) It has two acute angles./Dit het twee skerphoekige hoeke.

(b) The measures of the angles add up to 180°/Die som van die binnehoeke is 180°.

(c) An altitude is also a side./'n Hoogtelyn is ook 'n sy.

(d) The measures of two angles add up to 90°./Die som van twee hoeke is 90°.

20. QAB is a triangle. Suppose angle Q is a right angle. Does that tell you anything about angles A and B?

QAB is 'n driehoek. Veronderstel hoek Q is 'n regte hoek. Vertel dit enigiets vir jou in verband met hoeke A en B?

___

If so, what?/Indien wel, wat?

___

Suppose angle Q is less than 90°. Could the triangle be a right-angled triangle? Why?

Veronderstel hoek Q is minder as 90°. Kan die driehoek 'n reghoekige driehoek wees? Hoekom?

___

___

Suppose angle Q is more than 90°. Could the triangle be a right-angled triangle? Why?

Veronderstel hoek Q is meer as 90°. Kan die driehoek 'n reghoekige driehoek wees? Hoekom?

___

___

21. Circle the smallest combination of the following which guarantees a triangle to be isosceles.

Omkring die kleinste kombinasie wat sal verseker dat die driehoek 'n gelykbenige driehoek is.

(a) It has two congruent angles./Dit het twee kongruente hoeke.

(b) It has two congruent sides./ Dit het twee kongruente sye.

(c) An altitude bisects the opposite side./'n Hoogtelyn halveer die teenoorstaande sy.

(d) The measure of the angles add up to 180°./Die som van die hoeke is 180°.

22. Suppose all we know about ΔMNP is that $\angle$M is the same as $\angle$N. What do you know about sides MP and NP?

Veronderstel dat al wat ons weet in verband met ΔMNP is dat $\angle$M dieselfde is as $\angle$N. Wat weet ons van die sye MP en NP?

Suppose $\angle$M is larger than $\angle$N. What do you know about MP and NP?

Veronderstel dat $\angle$M groter is $\angle$N. Wat weet ons van MP en NP?

Could ΔMNP be isosceles?

Kan ΔMNP 'n gelykbenige driehoek wees?

---

23. Triangle DEF has three congruent sides. It is an isosceles triangle. Why or why not?

Driehoek DEF het drie kongruente sye. Dit is 'n gelykbenige driehoek. Hoekom of hoekom nie?

Is the following true or false? All equilateral triangles are isosceles.

Is die volgende waar of vals? Alle gelyksydige driehoeke is gelykbenige driehoeke.

---

24. Which are true? Give reasons:

Wat is waar? Gee redes:

All isosceles triangles are right triangles.

Alle gelykbenige driehoeke is reghoekige driehoeke.

Some right-angled triangles are isosceles triangles.

Sommige reghoekige driehoeke is gelykbenige driehoeke.

---

25. What does it mean to say that two figures are similar?

Wat beteken dit om te sê dat twee figure gelyksoortig is?

---

26. Triangle ABC is similar to triangle DEF (in that order). Are the following

(a) certain (b) possible, or (c) impossible? Give reasons for your answers.

Driehoek ABC is gelyksoortig aan driehoek DEF (in die volgorde) Is die volgende

(a) seker (b) moontlik, of (c) onmoontlik? Gee redes vir jou antwoord.

AB = DE

AB > DE

$\angle A = \angle E$

$\angle A > \angle E$

AB = EF

$\angle A > \angle D$

---

27. Will figures A and B be similar?

I – always II – sometimes or III – never/nooit? Give reasons for your answers.

Sal figure A en B gelyksoortig wees?

I – altyd II – soms of III – nooit? Gee redes vir jou antwoord.

A		B	
(a)	a square/'n vierkant	(a)	a square/'n vierkant

____

____

(b)	an isosceles triangle Gelykbenige driehoek	(b)	an isosceles triangle Gelykbenige driehoek

____

____

(c)	a Δ congruent to B 'n Δ kongruent aan B	(c)	a Δ congruent to A 'n Δ kongruent aan A

____

____

(d)	a rectangle/'n reghoek	(d)	a square/'n vierkant

____

____

(e)	a rectangle/'n reghoek	(e)	a triangle/'n driehoek

____

____

28. Δ ABC is congruent to Δ DEF (in that order).

(a) certain (b) possible, or (c) impossible? Give reasons for your answers.

Δ ABC is kongruent aan Δ DEF (in daardie volgorde)

(a) seker (b) moontlik, of (c) onmoontlik? Gee redes vir jou antwoord.

(a) AB = DE

(b) $\angle A = \angle E$

(c) $\angle A < \angle D$

(d) AB = EF

---

29. Will figures A and B be congruent?

I – always II – sometimes or III – never? Give reasons for your answers./

Sal figuurA en B kongruent wees?

I – altyd II – soms of III – nooit? Gee redes vir jou antwoord.

A	B
(a) a square/'n vierkant	(a) a triangle/'n driehoek

(b)	a square with a 10cm side 'n vierkant met 'n sy van 10cm	(b)	a square with a 10cm side 'n vierkant met 'n sy van 10cm
(c)	a right-angled triangle with a 10cm hypotenuse 'n reghoekige driehoek met 'n skuinssy van 10cm	(c)	a right-angled triangle with a 10cm hypotenuse 'n reghoekige driehoek met 'n skuinssy van 10cm
(d)	a circle with 10cm chord	(d)	'n sirkel met 'n koord van 10cm
(e)	a Δ similar to B 'n Δ gelykvormig aan B	(e)	a Δ similar to B 'n Δ gelykvormig aan B

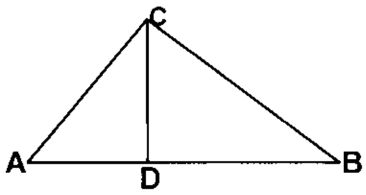
30. ABCD is a four sided figure. Suppose we know that opposite sides are parallel. What are the fewest facts necessary to prove that ABCD is a square?
ABCD is 'n figuur met vier sye. Veronderstel ons weet dat die teenoorstaande sye parallel is. Wat is die minste feite nodig om te bewys dat ABCD 'n vierkant is?

---

31. Figure ABCD is a parallelogram, AB $\equiv$ BC and $\angle$ABC is a right angle. Is ABCD a square?
Figuur ABCD is parallelogram, AB $\equiv$ BC en $\angle$ABC is 'n regte hoek. Is ABCD 'n vierkant?

Prove your answer./Bewys jou antwoord

32.



CD is perpendicular to AB. ∠C is a right angle. If you measure ∠ACD and ∠B, you find that they have the same measure. Would this equality be true for all right-angled triangles? Why or why not?

CD is loodreg op AB. ∠C is 'n regte hoek. As jy ∠ACD en ∠B meet, sal jy sien dat hulle ewe groot is. Is hierdie gelykheid waar vir alle reghoekige driehoeke? Hoekom of hoekom nie?

____

____

33.

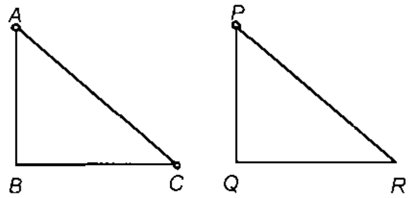


Figure ABC and PQR are right-angled isosceles triangle with angles B and Q being right angles. Prove that ∠A = ∠P and ∠C = ∠R.

Figuur ABC en PQR is reghoekige, gelykbenige driehoeke met hoeke B en Q regte hoeke. Bewys dat ∠A = ∠P en ∠C = ∠R.

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____

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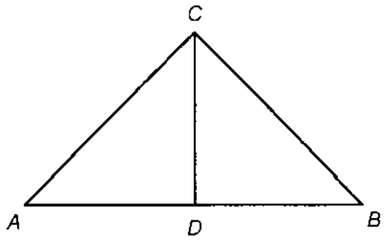
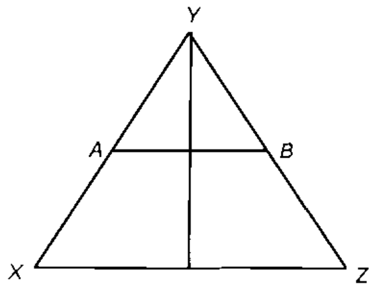
34.



ABC is a triangle. $\Delta ADC \equiv \Delta BDC$. What kind of triangle is ΔABC? Why?/

ABC is 'n driehoek. $\Delta ADC \equiv \Delta BDC$. Watter soort driehoek is ΔABC? Hoekom?

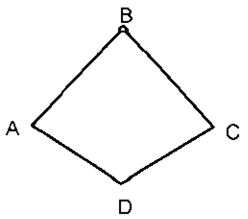
35.



AB is the line segment with A and B the midpoints of the equal sides of the isosceles triangle XYZ. AY = BY and ΔAYB is similar to ΔXYZ so that $\angle A = \angle X$. AB is parallel to XZ. What have we proved?

AB is die lynsegment met A en B die middelpunte van die gelyke sye van die gelykbenige driehoek XYZ. AY = BY en ΔAYB is gelyksoortig aan ΔXYZ sodat $\angle A = \angle X$. AB parallel is aan XZ. Wat het ons bewys?

36.



In this figure AB and CB are the same length. AD and CD are the same length. Will $\angle A$ and $\angle C$ be the same size? Why or why not?

In hierdie figuur is AB en CB ewe lank. AD en CD is ewe lank. Sal $\angle A$ en $\angle C$ ewe groot wees? Hoekom of hoekom nie?

____

____

____

____

37. What is the least additional information needed to ensure that a pair of right-angled triangles are similar?

Wat is die minste inligting nodig om te verseker dat 'n paar reghoekige driehoeke gelyksoortig is?

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____

____

____

38.

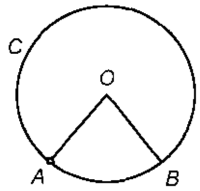
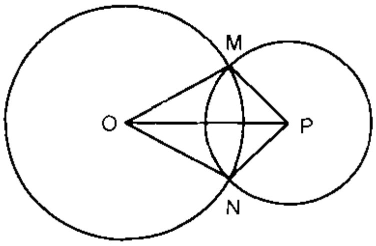


Figure C is a circle. O is the centre. Prove that Δ AOB is isosceles.

Figuur C is 'n sirkel. O is die middelpunt van die sirkel. Bewys dat Δ AOB 'n gelykbenige driehoek is.

____

39.



These circles with centres O and P intersect at M and N. Prove: Δ OMP $\equiv$ Δ ONP.

Hierdie sirkels met middelpunte O en P sny mekaar in M en N. Bewys dat: Δ OMP $\equiv$ Δ ONP.

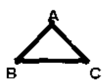
40. Prove that the perpendicular from a point (not on the line) to a line is the shortest line segment that can be drawn from the point to the line.

Bewys dat die loodregte lyn van 'n punt (wat nie op die lyn lê nie) na 'n lyn, die kortste segment is wat na die lyn geteken kan word.

APPENDIX B

The PMTs must construct a triangle before they can do the following activities. (The activities, except activity 1, are based on ideas in Bennett, D. Exploring geometry with the geometer's sketchpad. Emeryville, Calif. : Key Curriculum Press. 285 p.).

Activity 1: Construct a triangle

Inleiding:	**Introduction:**
1. Maak GSP® oop en kostrueer driehoek ABC deur van die "**segment tool**" gebruik te maak.	1. Open GSP® and construct a triangle ABC by making use of the **segment tool**.
2. Maak gebruik van die "**text tool**" om die driehoek te benoem.	2. Make use of the **text tool** to appoint the triangle.
	
Nota: Die sye van die driehoek kan verander word na "thin, thick" en "dashed" deur die sye van die driehoek te kies deur gebruik te maak van die "**selection arrow**" en dan daarna die "**display**" funksie te gebruik.	**Note**: The line width of the triangle can be changed by first selecting the sides of the triangle by making use of the **selection arrow** and then thereafter going to the **display function** and choose line width. The sides of the triangle can then be changed to thin, thick or dashed.

Activity 2: Medians in a triangle

A median in a triangle is a straight line drawn from the vertex of a triangle to the midpoint of the opposite side (Laridon *et al.*, 1995:310).

Swaartelyne:	**Median:**
1. Konstrueer driehoek ABC.	1. Construct triangle ABC.
2. Konstrueer die middelpunte van die driehoek. (gaan na die "construct menu" en kies "midpoint").	2. Construct the midpoints of the sides (go to the construct menu and choose midpoint).
3. Herhaal die stap 2 met die ander twee sye.	3. Repeat step 2 with the other two sides as well.

4. Verbind die hoekpunt met die middelpunt van die teenoorstaande sy.	4. Connect the vertex with the midpoint of its opposite side.
5. Herhaal die stap 4 met die ander twee sye en hoekpunte.	5. Repeat step 4 with the other two vertices and sides as well.
6. Wat neem jy waar ten opsigte van die hoogtelyne? Skryf dit neer.	6. What do you notice about the medians? Write it down.
7. Konstrueer die punt waar die swaartelyne mekaar sny. Die punt waar die swaartelyne mekaar sny word die swaartepunt genoem.	7. Construct the point of intersection of the medians. The point where the medians intersect is called the centroid.
8. Meet die afstand (maak gebruik van die **"measure menu"** en kies "distance") vanaf die hoekpunt na die swaartepunt en dan weer van die swaartepunt na die middelpunt van die sy.	8. Measure the distance (make use of the **measure menu** and choose distance) from the vertex to the centroid and then the distance from the centroid to the midpoint of the side.
9. Bereken hierdie verhouding.	9. Calculate these ratio.
10. Herhaal die stap 8 en 9 met die ander twee hoekpunte en middelpunte van die sye.	10. Repeat step 8 and 9 with the other two vertices and midpoint of the sides as well.
11. Skryf 'n veronderstelling neer oor hoe die swaartepunt elke swaartelyn van van 'n driehoek verdeel.	11. Write a conjecture about the way the centroid divides each median in a triangle.

Activity 3: Perpendicular bisector

A perpendicular bisector of a side of a triangle is a line drawn perpendicular to the side that it bisects (Laridon *et al.*, 1995:310).

Middelloodlyne:	**Perpendicular bisector:**
1. Konstrueer driehoek ABC.	1. Construct triangle ABC.
2. Kies 'n sy van die en konstrueer die middelpunt.	2. Choose a side of the triangle, and construct the midpoint.
3. Kies hierdie punt asook die sy en konstrueer die middelloodlyn (gaan na die "construct menu" en kies "perpendicular line").	3. Select this point as well as the side and construct a perpendicular bisector (go to the construct menu an choose perpendicular line).

4. Herhaal die stap 3 met die ander twee sye en middelpunte.	4. Repeat step 3 with the other two sides and midpoints.
5. Wat neem jy waar ten opsigte van die middelloodlyne? Skryf dit neer.	5. What do you notice about the perpendicular bisectors? Write it down.
6. Konstrueer die punt waar die middellyne mekaar sny. Die punt waar die middellyne mekaar sny word die middelpunt van die omgeskrewe sirkel (ommiddelpunt) genoem.	6. Construct the point of intersection of the perpendicular bisectors. The point where the perpendicular bisectors intersect is called the circumcenter.
7. Meet die afstand vanaf die middelpunt van die omgeskrewe sirkel na elk van die hoekpunte.	7. Measure the distance from the circumcenter to each of the three vertices.
8. Wat merk jy op ten opsigte van die afstand vanaf die middelpunt van die omgeskrewe sirkel na die drie hoekpunte van die driehoek?	8. What do you notice about the distance from the circumcenter to the three vertices of the triangle?
9. Trek een van die hoekpunte totdat die middelpunt van die omgeskrewe sirkel op 'n sy van die driehoek lê. Watter tipe driehoek is dit? Waar presie lê die middelpunt van die omgeskrewe sirkel?	9. Drag a vertex until the circumcenter falls on a side of a triangle. What kind of triangle is this? Where exactly does the circumcenter lie?

Activity 4: Altitudes in a triangle

An altitude of a triangle is measured by the length of a line drawn from a vertex of a triangle perpendicular to the opposite side, or an extension of the side called the base (Laridon *et al.*, 1995:310).

Hoogtelyne:	**Altitudes:**
1. Konstrueer driehoek ABC.	1. Construct triangle ABC.
2. Kies 'n hoekpunt en die teenoorgestelde sy, dan, in die "construct menu", kies "perpendicular line" om 'n loodregte lyn deur die sy te konstrueer.	2. Select a vertex and the opposite side , then, in the construct menu, choose perpendicular line to construct a line perpendicular to the side.
3. Herhaal die stap 2 met die ander twee sye en hoekpunte.	3. Repeat step 2 with the other two sides and vertices.

4. Solank as wat jou driehoek 'n skerphoekige driehoek is, behoort die hoogtelyn die sy van die driehoek te sny. Trek nou die hoekpunt sodat die hoogtelyn buite die driehoek lê. Watter tipe driehoek is nou?	4. As long as your triangle is acute, this perpendicular line should intersect a side of the triangle. Drag the vertex so that the line falls outside the triangle. Now what kind of triangle is it?
5. Trek jou driehoek sodat dit weer 'n skerphoekige drehoek is. Konstrueer die punt waar die hoogtelyne mekaar sny. Die punt waar die hoogtlyne mekaar sny word die ortosnypunt genoem.	5. Drag your triangle so that it is acute again. Construct the point of intersection of the altitudes. The point where the altitudes intersect is called the orthocenter.
6. Wat neem jy waar ten opsigte van die hoogtelyne as die driehoek 'n skerphoekige driehoek is? Skryf dit neer.	6. What do you notice about the altitudes if the triangle is acute? Write it down.
7. Trek die hoekpunte van die driehoek en neem waar wat met die hoogtelyn gebeur.	7. Drag vertices of the triangle and observe how your altitude behaves.
8. Waar lê jou hoogtelyn wanneer een van die hoeke van die driehoek 'n regte hoek is?	8. Where is your altitude when one of the angles of the triangle is a right angle?
9. Konstrueer 'n sirkel deur die ortosnypunt en die hoekpunte van die driehoek. Wat noem ons hierdie sirkel?	9. Construct a circle through the orthocenter and the vertices of the triangle. What do we call this type of circle?

Activity 5: Angle bisectors in triangles

A bisector of an angle is a line that bisects an angle of a triangle (Laridon *et al.*, 1995:311).

Halveerlyne:	**Angle bisectors**
1. Konstrueer driehoek ABC.	1. Construct triangle ABC.
2. Konstrueer die halveerlyn van die hoeke van die driehoek deur drie punte te kies met die hoekpunt wat jy wil halveer in die middel. Kies dan in die "construct menu, angle bisector".	2. Construct the bisector of the angles of the triangle by selecting three points, with the vertex your middle selection. Then, in the construct menu, choose angle bisector.
3. Konstrueer die punt waar die halveerlyne mekaar sny. Die punt waar die	3. Construct the point of intersection of the angle bisectors. The point where the angle

halveerlyne mekaar sny word die middelpunt van die ingeskrewe sirkel (inmiddelpunt) genoem.	bisectors intersect is called the incenter.
4. Wanneer jy ∠A klaar gehalveer het, meet die twee hoeke wat by ∠A gevorm is (in die "measure menu", kies "angle").	4. When you have bisect ∠A, measure the two angles which is formed at ∠A (in the measure menu, choose angle).
5. Konstrueer 'n sirkel deur die middelpunt van die ingeskrewe sirkel en die hoekpunte van die driehoek. Wat noem ons hierdie sirkel?	5. Construct a circle through the incenter and the vertices of the triangle. What is the name of this circle?

Activity 6: Isosceles triangles

A triangle is an isosceles triangle when two sides of the triangle are equal (Loots *et al.*, 2000:154).

Gelykbenige driehoeke:	**Isosceles triangles:**
1. Konstrueer 'n sirkel met middelpunt A en radiuspunt B.	1. Construct a circle with center A and radius point B.
2. Konstrueer radius AB.	2. Construct radius AB.
3. Konstrueer radius AC. Trek punt C om seker te maak dat die radius aan die sirkel raak.	3. Construct radius AC. Drag point C to make sure the radius is attached to the circle.
4. Konstrueer $\overline{BC}$.	4. Construct $\overline{BC}$.
5. Trek elke hoekpunt van jou driehoek en kyk wat gebeur. Verduidelik hoekom die driehoek altyd 'n gelykbenige driehoek is.	5. Drag each vertex of your triangle to see how it behaves. Explain why the triangle is always isosceles.
6. "Hide" sirkel AB.	6. Hide circle AB.
7. Meet die drie hoeke in die driehoek.	7. Measure the three angles in the triangle.
8. Trek die hoekpunte van jou driehoek en kyk wat gebeur met die hoeke wat jy gemeet het. Wat neem jy waar in verband met die basishoeke en die oorblywende hoek van die driehoek?	8. Drag the vertices of your triangle and observe the angles measures. What do you observe about the measures of the base angles and the remaining angle of the triangle?

APPENDIX C

Onderhoud 1	Interview 1
1.	1.
R Op 'n skaal van 1 tot 10, met 10 die hoogste, hoe baie hou jy van wiskunde?	R On a scale of 1 to 10, with ten being the highest, how much do you like mathematics?
S1 5	S1 5
R Jy hou dus nie so baie van wiskunde nie	R You don't like mathematics that much?
S1 nee, nie so baie nie	S1 No, not so much.
R Hoekom nie?	R Why not?
S1 Ek sukkel met wiskunde.	S1 I struggle with mathematics.
2.	2.
R Doen jy altyd jou werksopdragte?	R Do you always complete your assignments?
S1 Ek probeer om my werksopdragte almal te doen, maar dis nie so maklik nie.	S1 I try to do all the assignments, but it is not that easy.
R hoeveel tyd spandeer jy om jou werksopdragte te doen?	R How much time do you spend on doing your assignments?
S1 Dit verskil van hoe moeilik dit is, as dit baie moeilik is sal ek dit dalk nie doen nie.	S1 It depends on how difficult it is. If it is very difficult, I might not do it.
R Hm.	R Hm.
3.	3.
R In die algemeen, wat beïnvloed jou om hard te werk?	R In general, what influences you to work hard?
S1 Dit is lekker om 'n goeie punt te kry.	S1 It is nice to get good marks.
4.	4.
R Is daar enigiets wat veroorsaak dat jy nog harder sal werk?	R Is there anything that causes you to work harder?
S1 Nee.	S1 No.
R OK.	R OK.
5.	5.
R Is jy beter in sekere afdelings van wiskunde? Byvoorbeeld, is jy beter in algebra as in meetkunde?	R Are you better in certain sections of mathematics? For instance, are you better in algebra than in geometry?
S1 Ek hou nie van trig en meetkunde nie. Ek kry dit nie reg nie.	S1 I don't really like trig and geometry. I can't do it.

6. R Dink jy iemand het spesiale talent nodig om goed te doen in wiskunde? S1 Ek dink mens moet 'n talent hê maar ek dink waar 'n probleem opduik is by die onderwyser by wie jy wiskunde kry. Die onderwyser kan 'n baie groot rol speel of mens die werk verstaan of nie.	6. R Do you think that a person needs a special talent to be good in mathematics? S1 I think a person should have an aptitude, but I think the problem comes in with the teacher that teaches you mathematics. The teacher plays an important role in whether you understand the work or not.
7. R Hoe belangrik is dit om te memoriseer in wiskunde? S1 Ek moet goed uit my kop leer om somme in die eksamen te kan doen R Dink jy dat iemand wat nie goed is om te memoriseer nie, goed doen in wiskunde? S1 Ja die mense wat die werk maklik regkry.	7. R How important is it to memorise in mathematics? S1 I must learn sums by heart to be able to do it in the exam. R Do you think that someone who is not good in memorising can do well in mathematics? S1 Yes, those who manage the work easily.
8. R Hoe gereeld doen jy so min as moontlik werk net om by te bly? S1 Dit hang af hoe besig ek is met my ander vakke. R OK.	8. R How often do you do as little as possible just to keep up? S1 It depends on how busy I am in my other subjects. R OK.
9. R Hoe studeer jy vir 'n wiskunde toets? S1 Ek doen eers die somme waarmee ek gesukkel het op papier en dan sal ek die res hersien. R OK.	9. R How do you study for a mathematics test? S1 I first do the sums that I struggled with on paper and then I will revise the rest. R OK.
10. R Gee voorbeelde van die aktiwiteite wat jy in wiskunde doen. S1 Blokkies, geo-stroke en GSP. R Hm.	10. R Give examples of the activities that you do in mathematics. S1 Blocks, geo-strips and GSP. R Hm.
11. R Van hierdie aktiwiteite wat jy nou net opgenoem het, is daar enige van die aktiwiteite wat jy baie geniet en interessant vind? Van hierdie aktiwiteite wat jy nou net opgenoem het, is daar enige van die aktiwiteite wat jy baie geniet en	11. R Of these activities that you have just mentioned, are there any that you enjoy and that you find interesting?

interessant vind? S1 GSP is baie interessant. R Hoekom? S1 Ek geniet dit om te kan sien waarmee ek werk en dit help my om in te sien wat die dosent vir my verduidelik. 12. R Is daar probleme in wiskunde wat op meer as een manier opgelos kan word? S1 Ek dink elke liewe som kan anders opgelos word. R Goed so. 13. R Hou jy van GSP? S1 Ja soos ek reeds gesê het, ek verstaan die werk makliker en dit is interessant om die probleme met GSP te doen. 14 R Is daar enigiets waaraan jy kan dink wat belangrik is om wiskunde te leer wat jy nie genoem het nie. S1 Nee.	S1 GSP is very interesting. R Why? S1 I enjoy being able to see what I am working with and it helps me to see what the lecturer is explaining to me. 12. R Are there problems in mathematics that can be solved in more than one way? S1 I think each and every sum can be done in different ways. R Alright. 13. R Do you like GSP? S1 Yes, as I have already said, I understand the work better and it is interesting to do the problems better with GSP. 14 R Is there anything that is important to learn in mathematics that you have not mentioned? S1 No.
Onderhoud 2 1. R Op 'n skaal van 1 tot 10, met 10 die hoogste, hoe baie hou jy van wiskunde? S2 10 2. R Doen jy altyd jou werksopdragte? S2 Ja. R En hoeveel tyd spandeer jy om jou werksopdragte te doen? S2 dit hang net af hoe groot is dit. As dit byvoorbeeld 20 somme is sal ek meer tyd spandeer as wat dit net 6 is. Ek hou daarvan om al die somme te doen.	**Interview 2** 1. R On a scale of 1 to 10, with ten being the highest, how much do you like mathematics? S2 10 2. R Do you always complete your assignments? S2 Yes. R And how much time do you spend on doing your assignments? S2 It depends on how much it is. If it is for instance 20 sums, I would spend more time that if it were 6. I like doing all the sums.

R OK.	R OK.
3.	3.
R In die algemeen, wat beïnvloed jou om hard te werk?	R In general, what influences you to work hard?
S2 Goeie punte. Dit is vir my 'n uitdaging om al my somme reg te kry. As ek sukkel met 'n som sal ek aanhou en aanhou tot ek hom regkry. As ek die som dan reggekry het sal ek nog 'n soortgelyke een doen om seker te maak ek verstaan die som.	S2 Good marks. For me it is a challenge to have all my sums correct. If I struggle I will keep going until I get it right. If I have succeeded in doing the sum I will do another similar one to make sure I understand the sum.
R Mooi. Is daar ander enigiets anders wat jou beïnvloed om harder te werk?	R Good, is there anything else that influences you to work harder?
S2 Ek werk altyd konstant hard.	S2 I constantly work hard.
R OK.	R OK.
4.	4.
R Is daar enigiets wat veroorsaak dat jy harder werk?	R Is there anything that causes you to work harder?
S2 Ek dink ek is redelik goed.	S2 I think I am reasonably good.
R Hoe weet jy dit?	R How do you know that?
S2 Ek kry goeie punte.	S2 I get good marks.
5.	5.
R Is jy beter in sekere afdelings van wiskunde? Byvoorbeeld, is jy beter in algebra as in meetkunde?	R Are you better in certain sections of mathematics? For instance, are you better in algebra than in geometry?
S2 Ek het nou-nou gesê ek hou van wiskunde, maar ek het vergeet van die meetkunde ek haat meetkunde ek leer al die bewyse soos 'n papegaai.	S2 I said that I like mathematics, but I forgot about geometry. I hate geometry and I learn all the proofs by heart.
R Goed so.	R Good.
6.	6.
R Dink jy iemand het spesiale talent nodig om goed te doen in wiskunde?	R Do you think that a person needs a special talent to be good in mathematics?
S2 Ek dink so ek dink jy kan as jy wil nog steeds goed doen in wiskunde al is jy nie aangelê vir wiskunde nie maar ek dink party mense is net doodeenvoudig goed in wiskunde jy kan eenkeer na 'n som kyk en jy sal weet wat daar aangaan.	S2 I think so, and I think if you want to you can still do well in mathematics if you do not have aptitude for it, but I think some people are simply just good in mathematics and you can look at a sum once and know what is going on there.
R Het jy sulke talent?	R Do you have such talent?
S2 Ja ek dink so.	S2 Yes, I think so.
R Hoekom dink jy so?	R Why do you think so?

S2 My punte is goed.	S2 My marks are good.
7.	7.
R Hoe belangrik is dit om te memoriseer in wiskunde?	R How important is it to memorise in mathematics?
S2 Op Universiteit het ek eers besef hoe belangrik dit is om nie goed te memoriseer nie maar om te kyk waar kom dit vandaan en hoekom is dit so om dit regtig te kan verstaan. As jy dit verstaan dan hoef jy nie eers rêrig te leer nie want as jy dit eenkeer verstaan sal jy dit altyd verstaan. As jy die werk memoriseer dan vergeet jy dit weer en dan moet jy weer van voor af leer.	S2 I only realised at university that it is important not to memorise, but rather to look at where something comes from, why it is so, in order to really understand it. If you understand you don't even really have to study, because if you understand it once you will always understand it. If you memorise the work, you forget it again and then you have to learn it all over again.
R Dink jy dat iemand wat nie goed is om te memoriseer nie, goed doen in wiskunde?	R Do you think that someone who is not good in memorising can do well in mathematics?
S2 Ja jy moet net die werk verstaan dan hoef jy dit nie te memoriseer nie.	S2 Yes, you should just understand the work, then you don't have to memorise.
8.	8.
R Hoe gereeld doen jy so min as moontlik werk net om by te bly? Hoe gereeld doen jy so min aas moontlik werk net om by te bly?	R How often do you do as little as possible just to keep up?
S2 Dit hang net af hoeveel werk word van my vereis, as ek baie werk in my ander vakke het sal ek minder tyd spandeer aan my wiskunde maar as ek genoeg tyd het spandeer ek baie tyd aan wiskunde dan sal ek meer doen as wat van my verwag word.	S2 It just depends on how much work I have. If I have much work in my other subjects I will spend less time on my mathematics, but if I have enough time I spend much time on mathematics, and I do more that what is expected of me.
R hm.	R Hm.
9.	9.
R Hoe studeer jy vir 'n wiskunde toets?	R How do you study for a mathematics test?
S2 Ek werk gewoonlik konstant baie hard en dan sal ek die probleme wat ons gedoen het deurlees en seker maak dat ek almal verstaan en weet hoe om dit te doen. Ek doen nie weer al die somme nie want ek het deur die jaar gesorg dat ek almal kan doen. Ek sê basies die stappe vir myself op. Op hierdie manier kan ek my werk meer keer hersien.	S2 I understand all of the sums and know how to do them. I don't do all the sums again because through the year I saw to it that I can do all of them. I repeat the basic steps for myself. In this way I can revise the work for myself more times.
R Goed so.	R Alright.

10.	10.
R Gee voorbeelde van die aktiwiteite wat jy in wiskunde doen.	R Give examples of the activities that you do in mathematics.
S2 Geo-stroke, GSP wat nogal help om die werk in te sien, unfix-blokkies	S2 Geo-strips, GSP, which helps to see the work, unfix-blocks.
R Hm.	R Hm.
11.	11.
R Van hierdie aktiwiteite wat jy nou net opgenoem het, is daar enige van die aktiwiteite wat jy baie geniet en interessant vind?	R Of these activities that you have just mentioned, are there any that you enjoy and that you find interesting?
S2 Ek dink dat ek GSP kan gebruik om beter te verstaan ek hou nie van geo-stroke en sulke goed nie, dit verveel my.	S2 I think that I can use GSP to understand better, but I don't like geo-strips and such things, it bores me.
12.	12.
R Is daar probleme in wiskunde wat op meer as een manier opgelos kan word?	R Are there problems in mathematics that can be solved in more than one way?
S2 Ja.	S2 Yes.
R Hoekom?	R Why?
S2 Elke liewe persoon dink anders en sal 'n ander manier gebruik om by die regte antwoord uit te kom.	S2 Each and every person thinks differently and will use a different way to get to the right answer.
13.	13.
R Hou jy van GSP?	R Do you like GSP?
S2 Ja, dit is baie interessant en dit is baie makliker om meetkunde daarop te doen. Vandat ek meetkundige sketse op GSP doen geniet ek die meetkunde meer en ek verstaan meer van die meetkunde ek kan die uitwerking dadelik sien bv as ons gelykbenige die hoeke moet teken en die hoeke meet, kan ek dadelik siek die basishoeke is gelyk.	S2 Yes, it is very interesting and it is much easier to do mathematics on it. Since I do geometrical sketches on GSP I enjoy geometry more and I understand more of the geometry. I can see immediately, for instance if we do isosceles triangles and we have to measure the angles, I can immediately see that the base angles are equal.
14	14
R Is daar enigiets waaraan jy kan dink wat belangrik is om wiskunde te leer wat jy nie genoem het nie.	R Is there anything that is important to learn in mathematics that you have not mentioned?
S2 Ek dink ek het alles gesê waaraan ek kan dink.	S2 I think I have said everything that I could think of.
R OK.	R OK.

Onderhoud 3	**Interview-3**
1.	1.
R Op 'n skaal van 1 tot 10, met 10 die hoogste, hoe baie hou jy van wiskunde?	R On a scale of 1 to 10, with ten being the highest, how much do you like mathematics?
S3 10	S3 10
R So, jy hou baie van wiskunde.	R So. you like mathematics very much.
S3 Ja.	S3 Yes.
2.	2.
R Doen jy altyd jou werksopdragte?	R Do you always complete your assignments?
S3 ek doen al my opdragte.	S3 I always do all my assignments.
R Hoeveel tyd spandeer jy om jou werksopdragte te doen?	R How much time do you spend on doing your assignments?
S3 dit hang af hoeveel somme ek moet doen. Ek sal nie my werk alleen doen nie, ek sal my antwoorde met iemand anders sin vergelyk om seker te maak dat my antwoorde reg is.	S3 It depends on how many sums I have to do. I will not do my work alone; I will compare my answers with someone else's to make sure that my answers are correct.
3.	3.
R In die algemeen, wat beïnvloed jou om hard te werk?	R In general, what influences you to work hard?
S3 Kom ons sê ek het 'n toets geskryf en my punte was nie so goed nie, sal dit my klaar inspireer om harder te werk en dan, hm, sal ek somme doen tot ek dit reg kry of ek sal hulp vra.	S3 Let's say I wrote a test and my marks are not so good, it will already inspire me to work harder and then, hm, I will do sums until I get them right or I will ask help.
R Mooi. Is daar ander enigiets anders wat jou beïnvloed om harder te werk?	R Good. Is there anything else that influences you to work harder?
S3 Dis lekker om te weet wat aangaan.	S3 It is good to know what is going on.
R Goed so.	R Good.
4.	4.
R Is daar enigiets wat veroorsaak dat jy harder werk?	R Is there anything that causes you to work harder?
S3 Uhm, ek dink ek is redelik goed in wiskunde maar ek moet sê ek leer ook baie hard, behalwe as ek die werk gesnap het.	S3 Uhm, I think I am reasonably good in mathematics, but I must say I work hard, except if I have grasped it.
R Wat bedoel jy met gesnap?	R What do you mean with grasped?
S3 Ek verstaan die werk, ek weet wat daar aangaan ek kan vir myself vertel wat daar aangaan.	S3 I understood the work, I know what is going on there and I can tell myself what is going on there.
R Mooi so.	R Good.
5.	

	5.
R Is jy beter in sekere afdelings van wiskunde? Byvoorbeeld, is jy beter in algebra as in meetkunde?	R Are you better in certain sections of mathematics? For instance, are you better in algebra than in geometry?
S3 OK ek hou baie van algebra en trigonometrie maar ek hou niks van meetkunde nie ek het gesukkel met meetkunde. As 'n nuwe stelling gedoen is dan word dit dadelik toegepas op 'n moeilike probleem miskien moes ons dit eers op 'n eenvoudig, baie eenvoudige probleem te doen net om eers die beginsel vas te lê.	S3 OK, I like algebra and trigonometry, but I don't like geometry at all. I struggled with geometry. When a new theorem has been done, it is immediately applied to a difficult problem. Maybe we should first do a simple, a very simple problem just to first establish the principles.
6.	6.
R Dink jy iemand het spesiale talent nodig om goed te doen in wiskunde?	R Do you think that a person needs a special talent to be good in mathematics?
S3 Ek dink die gesindheid waarmee jy dinge doen in die lewe bepaal baie. So as jy met 'n negatiewe gesindheid in die wiskunde klas instap gaan jy nie baie leer nie. Ek dink daar is mense wat talent het maar ek dink gesindheid speel 'n groot rol. Ek dink ook dat onderwysers 'n invloed het op 'n mens of jy die werk verstaan of nie ek het ondervind toe ek geproef het, dat 'n juffrou wat vir graad 5 wiskunde gegee het nie daarvoor opgelê is nie en die kinders het dan alreeds in die Laerskool 'n agterstand omdat die juffrou nie mooi weet hoe om die werk te verduidelik nie. As die werk in die grondslag fase nie goed vasgelê is nie is daar al klaar 'n probleem, so ek glo nie dis net talent nie maar ook hoe die werk verduidelik is.	S3 I think the attitude with which you do things in life determines much. So if you have a negative attitude in the mathematics class, you won't learn much. I think there are people with aptitude, but I think attitude plays an important role. I also think that teachers have an influence on whether one understands the work or not, and I found that when I did practicals that the teacher teaching grade 5 was not trained for it. The children then have a backlog in primary school already because the teacher does not know how to explain the work. If the work in the first phase is not well understood, there is a problem already, so I believe that it is not only talent, but also how the work has been explained.
R hm. Dink jy, jy het talent?	R Hm. Do you think you have talent?
S3 Ek het seker talent maar ek werk baie hard.	S3 I guess so, but I work very hard.
7.	7.
R Hoe belangrik is dit om te memoriseer in wiskunde?	R How important is it to memorise in mathematics?
S3 Dit is nie net 'n verstaan vak nie daar is bietjie leerwerk aan betrokke. Daar is 'n verskil tussen memoriseer en leer. As ek praat van leer dan weet ek wat daar aangaan. As jy memoriseer dan weet jy nie wat daar aangaan nie en vind leer glad nie plaas nie. Jy moet verstaan wat aangaan.	S3 It is not just a subject that involves understanding, there is a little studying. There is a difference between memorising and studying. If I speak of studying I know what is going on there. If you memorise then you don't know what is going on and learning does not take place. You have to understand what is going on.
R Dink jy dat iemand wat nie goed is om te	

memoriseer nie, goed doen in wiskunde? S3 Ja ek dink so.	R Do you think that someone who is not good in memorising can do well in mathematics? S3 Yes I think so.
8. R Hoe gereeld doen jy so min aas moontlik werk net om by te bly? S3 As ek iets nie verstaan nie sal ek moeite doen om uit te vind hoe dit gedoen moet word anders sal ek sorg dat my werk op datum is. R Goed so.	8. R How often do you do as little as possible just to keep up? S3 If I don't understand something I will go to trouble to find out how it should be done, otherwise I see to it that my work is up to date. R Good.
9. R Hoe studeer jy vir 'n wiskunde toets? S3 Ek doen somme oor en oor en maak seker dat ek almal verstaan. R Het iemand jou spesiale vaardighede geleer om vir wiskunde te leer? S3 Nee.	9. R How do you study for a mathematics test? S3 I do sums over and over and make sure that I understand all of them. R Did someone teach you special skills to study for mathematics? S3 No.
10. R Gee voorbeelde van die aktiwiteite wat jy in wiskunde doen. S3 GSP, wat baie interessant is, en geo-stroke. R OK.	10. R Give examples of the activities that you do in mathematics. S3 GSP, which is very interesting, and geo-strips. R OK.
11. R Van hierdie aktiwiteite wat jy nou net opgenoem het, is daar enige van die aktiwiteite wat jy baie geniet en interessant vind? S3 Ek hou baie van GSP en dit het my gehelp om die werk beter te snap, te verstaan. R Hm. S3 Wanneer meetkunde gedoen moet word is dit maklik en vinnig om die skets op GSP te teken en jy kan dadelik sien wat gebeur. Ek kan stelselmatig sien wat gebeur en dit maak dit makliker om die werk te verstaan. Jy kan onmiddellik sien wat gebeur en wat die effek is wanneer ek byvoorbeeld kyk na gelykbenige driehoeke dan kan ek sien dat die basishoeke ewegroot is. Jy kan nie op die bord so vinnig verduidelik soos met die tegnologie nie.	11. R Of these activities that you have just mentioned, are there any that you enjoy and that you find interesting? S3 I like GSP and it helped me to grasp the work, to understand it better. R Hm. S3 When geometry is being done it is fast and easy to draw the sketch on GSP and you can see what happens immediately. I can systematically see what happens and that makes it easier to understand the work. You can see what happens immediately when I for instance look at isosceles triangles. Then I can see that the base angles are equal. You can not explain as fast on the board as you can with

	technology.
12. R Is daar probleme in wiskunde wat op meer as een manier opgelos kan word? S3 Ja veral nou met OBE moet kinders toegelaat word om hul eie manier op te los.	12. R Are there problems in mathematics that can be solved in more than one way? S3 Yes, especially with OBE children should be allowed to solve things in their own way.
13. R Is daar enigiets waaraan jy kan dink wat belangrik is om wiskunde te leer wat jy nie genoem het nie. S3 Nee wat, ek dink ek het alles gesê.	13. R Is there anything that is important to learn in mathematics that you have not mentioned? S3 No, I think I have said everything.
Onderhoud 4	**Interview 4**
1. R Op 'n skaal van 1 tot 10, met 10 die hoogste, hoe baie hou jy van wiskunde? S4 6	1. R On a scale of 1 to 10, with ten being the highest. how much do you like mathematics? S4 6
2. R Doen jy altyd jou werksopdragte? S4 Die meeste. R Hoe besluit jy watter gaan jy doen en watter nie? S4 Wanneer ek te veel sukkel om die probleme te doen sal ek daardie opdrag los. R Hoeveel tyd spandeer jy om jou werksopdragte te doen? S4 As ek weet wat aangaan dan sal ek meer tyd spandeer om die opdrag te voltooi. R Goed.	2. R Do you always complete your assignments? S4 Most of them. R How do you decide which you are going to do and which not? S4 When I struggle too much with the problems I will leave that assignment. R How much time do you spend on doing your assignments? S4 If I know what is going on I will spend more time to complete the assignment. R Good.
3. R In die algemeen, wat beïnvloed jou om hard te werk? S4 As ek die werk verstaan sal ek harder werk. R Is daar enigiets wat veroorsaak dat jy harder werk? S4 Dit sal lekker wees om beter punte te kry R OK.	3. R In general, what influences you to work hard? S4 If I understand the work I will work harder. R Is there anything that causes you to work harder? S4 It will be good to have better marks. R OK.

4.	4.
R Hoe goed is jy in wiskunde?	R How good are you in mathematics?
S4 Nie goed nie.	S4 Not good...
R Hoe weet jy dit?	R How do you know that?
S4 Ek sukkel om party van die opdragte te doen.	S4 I struggle to do some of the assignments.
R Goed.	R Alright.
5.	5.
R Is jy beter in sekere afdelings van wiskunde? Byvoorbeeld, is jy beter in algebra as in meetkunde?	R Are you better in certain sections of mathematics? For instance, are you better in algebra than in geometry?
S4 Ek hou niks van meetkunde nie.	S4 I don't like geometry at all.
R Hoekom nie?	R Why not?
S4 Ek verstaan dit nie	S4 I don't understand it.
6.	6.
R Dink jy iemand het spesiale talent nodig om goed te doen in wiskunde?	R Do you think that a person needs a special talent to be good in mathematics?
S4 Ja.	S4 Yes.
R Het jy sulke talent?	R Do you have such talent?
S4 Nee glad nie.	S4 No, not at all.
R Hoekom dink jy dat jy nie talent het nie?	R Why do you think you don't have aptitude?
S4 Ek sukkel met die wiskunde	S4 I struggle with the mathematics.
7.	7.
R Hoe belangrik is dit om te memoriseer in wiskunde?	R How important is it to memorise in mathematics?
S4 Ek leer altyd rympies en stappe.	S4 I always learn rhymes and steps.
R Dink jy dat iemand wat nie goed is om te memoriseer nie, goed doen in wiskunde?	R Do you think that someone who is not good in memorising can do well in mathematics?
S4 Ja, die persone wat talent het kan goed doen sonder om goed uit hul kop te leer.	S4 Yes, the persons who have aptitude can do well without learning things by heart.
8.	8.
R Hoe gereeld doen jy so min as moontlik werk net om by te bly?	R How often do you do as little as possible just to keep up?
S4 Nie so gereeld nie, ek probeer om by te bly met my werk.	S4 Not often, I try to keep up.
R Hm.	R Hm.

9.	9.
R Hoe studeer jy vir 'n wiskunde toets?	R How do you study for a mathematics test?
S4 Ek doen die opdragte en probleme wat ons in die klas gedoen oor en oor.	S4 I do the assignments and problems that we did in class over and over.
R Is daar 'n ander manier wat jy ook gebruik om vir wiskunde te leer	R Is there another method that you use to study mathematics?
S4 Nee.	S4 No.
10.	10.
R Gee voorbeelde van die aktiwiteite wat jy in wiskunde doen.	R Give examples of the activities that you do in mathematics.
S4 Geo-stroke, GSP en blokkies	S4 Geo-strips, GSP and blocks
R Goed.	R Good.
11.	11.
R Van hierdie aktiwiteite wat jy nou net opgenoem het, is daar enige van die aktiwiteite wat jy baie geniet en interessant vind?	R Of these activities that you have just mentioned, are there any that you enjoy and that you find interesting?
S4 GSP, dit het my gehelp om die werk beter te verstaan.	S4 GSP. It helped me understand the work better.
12.	12.
R Is daar probleme in wiskunde wat op meer as een manier opgelos kan word?	R Are there problems in mathematics that can be solved in more than one way?
S4 Ja, verskillende persone sal byvoorbeeld probleme op verskillende maniere uitwerk	S4 Yes. Different people will for instance work out problems in different ways.
13	13
R Hoe hou jy van GSP?	R Do you like GSP?
S1 Ja ek geniet dit om die sketse te teken en dit maak dat ek meer van wiskunde hou, meer van meetkunde hou	S1 Yes, I enjoy drawing the sketches and it makes me like mathematics more and like geometry more.
14.	14.
R Is daar enigiets waaraan jy kan dink wat belangrik is om wiskunde te leer wat jy nie genoem het nie.	R Is there anything that is important to learn in mathematics that you have not mentioned?
S4 Ek dink nie so nie.	S4 I don't think so.
Onderhoud 5	**Interview-5**
1.	1.
R Op 'n skaal van 1 tot 10, met 10 die hoogste,	R On a scale of 1 to 10, with ten being the

hoe baie hou jy van wiskunde?	highest, how much do you like mathematics?
S5 6	S5 6
2.	2.
R Doen jy altyd jou werksopdragte?	R Do you always complete your assignments?
S5 Nee, nie altyd nie.	S5 No, not always.
R Hoe besluit jy watter gaan jy doen en watter nie?	R How do you decide which to do and which not?
S5 Wanneer ek begin met 'n opdrag en ek verstaan nie en ek het nie baie tyd nie, sal ek dit nie doen nie.	S5 When I start with an assignment and I don't understand and I don't have much time, I will not do it.
R Hoeveel tyd spandeer jy om jou werksopdragte te doen?	R How much time do you spend on doing your assignments?
S5 Dit hang af hoeveel ander werk ek het om te doen en weer of ek die werk verstaan of nie.	S5 It depends on how much work I have to do and whether I understand the work or not.
R Goed.	R Good.
3.	3.
R In die algemeen, wat beïnvloed jou om hard te werk?	R In general, what influences you to work hard?
S5 As ek weet wat aangaan sal ek meer doen en meer aandag aan my werksopdragte gee.	S5 If I know what is going on I will do more and pay more attention to my assignments.
R Is daar enigiets wat veroorsaak dat jy harder werk?	R Is there anything that causes you to work harder?
S5 Ek dink nie so nie.	S5 I don't think so.
R OK.	R OK.
4.	4.
R Hoe goed is jy in wiskunde?	R How good are you in mathematics?
S5 Nie so goed nie.	S5 Not so good.
R Hoe weet jy dit?	R How do you know that?
S5 Ek sukkel om die werk te doen en my punte lyk nie so goed nie.	S5 I struggle to do the work and my marks are not so good.
R OK.	R OK.
5.	5.
R Is jy beter in sekere afdelings van wiskunde? Byvoorbeeld, is jy beter in algebra as in meetkunde?	R Are you better in certain sections of mathematics? For instance, are you better in algebra than in geometry?
S5 Ek hou niks van meetkunde nie.	S5 I don't like geometry at all.
R Hoekom nie?	R Why not?
S5 Ek kry dit nie reg nie.	S5 I can not do it.

6.	6.
R Dink jy iemand het spesiale talent nodig om goed te doen in wiskunde?	R Do you think that a person needs a special talent to be good in mathematics?
S5 Ja.	S5 Yes
R Het jy sulke talent?	R Do you have such talent?
S5 Ek dink nie so nie.	S5 I don't think so.
R Hoekom dink jy dat jy nie talent het nie?	R Why do you think that you don't have aptitude?
S5 Omdat ek met die werk sukkel.	S5 Because I struggle with the work.
7.	7.
R Hoe belangrik is dit om te memoriseer in wiskunde?	R How important is it to memorise in mathematics?
S5 Ek leer die meeste van die werk uit my kop uit.	S5 I learn most of the work by heart.
R Dink jy dat iemand wat nie goed is om te memoriseer nie, goed doen in wiskunde?	R Do you think that someone who is not good in memorising can do well in mathematics?
S5 Daar is mense wat nie hard leer nie en goed doen, hulle weet gewoonlik wat in die klas aangaan.	S5 There are people who don't study hard and do well; they usually know what is going on in class.
8.	8.
R Hoe gereeld doen jy so min as moontlik werk net om by te bly?	R How often do you do as little as possible just to keep up?
S5 Gereeld.	S5 Often.
R Hoekom?	R Why?
S5 Wanneer ek met die werk sukkel gebruik dit baie van my tyd wat ek nie altyd het nie.	S5 When I struggle with the work it takes much of my time, which I do not always have.
9.	9.
R Hoe studeer jy vir 'n wiskunde toets?	R How do you study for a mathematics test?
S5 Ek doen die somme wat ons in die klas gedoen het weer.	S5 I do the sums that we did in class again.
R Het iemand jou 'n spesiale vaardighede geleer om vir wiskunde te leer?	R Did someone teach you special skills to study for mathematics?
S5 Nee.	S5 No.
10.	10.
R Gee voorbeelde van die aktiwiteite wat jy in wiskunde doen.	R Give examples of the activities that you do in mathematics.
S5 Ek kan op die oomblik net aan geo-stroke dink	S5 At the moment I can only think of geo-strips

R Goed so.	R Good.
11.	11.
R Is daar probleme in wiskunde wat op meer as een manier opgelos kan word?	R Are there problems in mathematics that can be solved in more than one way?
S5 Ja.	S5 Yes.
R Kan jy voorbeelde gee van sulke probleme.	R Can you give me examples of such problems?
S5 As mens byvoorbeeld goed moet bymekaar tel sal verskillende mense verskillende metodes gebruik om die antwoord te kry.	S5 If one for instance has to add things together, different people would use different methods to find the answer.
12.	12.
R Het jy al met GSP gewerk?	R Have you worked with GSP?
S5 Ja in die wiskundeklas	S5 Yes, in the mathematics class.
13.	13.
R Hou jy van GSP?	R Do you like GSP?
S5 Ja dit is vir my lekker.	S5 Yes, I enjoy it.
14	14
R Het GSP enige invloed op jou verstaan van wiskunde?	R Does GSP have any influence on you understanding of mathematics?
S5 Ja, ek verstaan meer wat in die klas aangaan en dit is lekker om met GSP te werk, dit maak die werk interessant.	S5 Yes, I understand better what is going on in class and it is enjoyable to work with GSP. It makes the work interesting.
15.	15.
R Is daar enigiets waaraan jy kan dink wat belangrik is om wiskunde te leer wat jy nie genoem het nie.	R Is there anything that is important to learn in mathematics that you have not mentioned?
S5 Nee ek kan nie aan nog iets dink nie.	S5 No I can not think of anything else.
Onderhoud 6	**Interview-6**
1.	1.
R Op 'n skaal van 1 tot 10, met 10 die hoogste, hoe baie hou jy van wiskunde?	R On a scale of 1 to 10, with ten being the highest, how much do you like mathematics?
S6 8, 9	S6 8, 9
2.	2.
R Doen jy altyd jou werksopdragte?	R Do you always complete your assignments?
S6 Ek doen soms my werksopdragte ek doen al	S6 I sometimes do my assignments. I do all the

die opdragte waarvoor ons punte kry	assignments for which we receive marks.
R en hoeveel tyd spandeer jy om jou werksopdragte te doen?	R And how much time do you spend on doing your assignments?
S6 Die opdragte is gewoonlik maklik en dan sal ek nie so baie tyd spandeer om dit te doen nie	S6 The assignments are usually easy and then I will not use so much time on doing it
R OK.	R OK.
3.	3.
R In die algemeen, wat beïnvloed jou om hard te werk?	R In general, what influences you to work hard?
S6 Vir my is die werk lekker maklik, dit is pret.	S6 For me the work is easy, it is fun.
R Mooi. Is daar ander enigiets anders wat jou beïnvloed om harder te werk?	R Good. Is there anything that causes you to work harder?
S6 Die werk is vir my interessant en daarom sal ek harder werk.	S6 The work is interesting and therefore I will work harder.
R OK.	R OK.
4.	4.
R Hoe goed is jy in wiskunde?	R How good are you in mathematics?
S6 Ek dink ek is nogal goed in wiskunde.	S6 I think I am rather good in mathematics.
R Hoe weet jy dit?	R How do you know it?
S6 My uitslae wys dat ek goed is.	S6 My results show that I am good.
5.	5.
R Is jy beter in sekere afdelings van wiskunde? Byvoorbeeld, is jy beter in algebra as in meetkunde?	R Are you better in certain sections of mathematics? For instance, are you better in algebra than in geometry?
S6 Ek is ewe goed in algebra en trig maar nie so goed in meetkunde nie.	S6 I am equally good in algebra and trig, but not so good in mathematics.
R Goed so.	R Alright.
6.	6.
R Dink jy iemand het spesiale talent nodig om goed te doen in wiskunde?	R Do you think that a person needs a special talent to be good in mathematics?
S6 Ek sal dit nie 'n talent noem nie, ek sal 'n aangelegdheid noem. Iemand wat aangelê is in wiskunde doen makliker goed in wiskunde	S6 I will not call it talent; I will rather call it aptitude. Someone who has aptitude in mathematics do well in mathematics easily.
R Het jy sulke talent?	R Do you have aptitude?
S6 Ek dink ek is aangelê in wiskunde.	S6 I think I have aptitude for mathematics.
R OK.	R OK.

7. R Hoe belangrik is dit om te memoriseer in wiskunde? S6 Sekere goed moet gememoriseer word maar as jy nie goed is in memoriseer nie is dit beter om te leer hoe om die goed self af te lei. R Dink jy dat iemand wat nie goed is om te memoriseer nie, goed doen in wiskunde? S6 Ja, as jy die werk verstaan kan jy goed doen.	7. R How important is it to memorise in mathematics? S6 Certain things should be memorised, but if you are not good in memorising it is better to learn to deduct things by yourself. R Do you think that someone who is not good in memorising can do well in mathematics? S6 Yes, if you understand the work you can do well.
8. R Hoe gereeld doen jy so min as moontlik werk net om by te bly? Hoe gereeld doen jy so min aas moontlik werk net om by te bly? S6 Dit hang net af hoeveel werk word van my vereis, as ek baie werk in my ander vakke het sal ek minder tyd spandeer aan my wiskunde maar as ek genoeg tyd het spandeer ek baie tyd aan wiskunde dan sal ek meer doen as wat van my verwag word. R Hm.	8. R How often do you do as little as possible just to keep up? S6 It depends on how much work is expected of me. If I have much work in my other subjects I will spend less time on my mathematics, but if I have enough time I spend much time on mathematics and then I will do more than expected. R Hm.
9. R Hoe studeer jy vir 'n wiskunde toets? S6 Ek gaan die studiegids deur en kyk wat is die opskrifte en dan sal ek seker maak dat ek daardie werk waaroor die opskrifte gaan kan doen en dat ek dit goed verstaan en dan sal ek so een of twee voorbeelde doen R Goed so.	9. R How do you study for a mathematics test? S6 I go through the study guide and look at the headings. Then I will make sure that I can do the work that the heading deals with and that I understand it well, and then I will do one or two examples. R Alright.
10. R Gee voorbeelde van die aktiwiteite wat jy in wiskunde doen. S6 Elke probleem skep sy eie tipe hulpbron wat gebruik kan word om dit beter te verstaan, tydens fasilitering was daar 'n probleem waarmee die studente gesukkel het en ek het toe GSP gebruik om die probleem op te los R Hm.	10. R Give examples of the activities that you do in mathematics. S6 Each problem creates its own type of resources that can be used to understand it better. During facilitation there was a problem with which the students struggled and I used GSP to solve the problem. R Hm.

11. R Van hierdie hulpbronne wat jy al gebruik het, is daar enige een wat jy baie geniet en interessant vind? S6 Ek dink dat ek GSP kan gebruik om van die werk duideliker te maak.	11. R Of these activities that you have just mentioned, are there any that you enjoy and that you find interesting? S6 I think GSP can be used to clarify the work.
12. R Is daar probleme in wiskunde wat op meer as een manier opgelos kan word? S6 Ja. R Hoekom? S6 Almal dink nie dieselfde nie en een persoon sal met 'n ander oplossing kom as iemand anders.	12. R Are there problems in mathematics that can be solved in more than one way? S6 Yes. R Why? S6 All people do not think the same, and one person will come with a solution different from someone else's.
13. R Is daar enigiets waaraan jy kan dink wat belangrik is om wiskunde te leer wat jy nie genoem het nie. S6 Ek dink ek het alles gesê waaraan ek kan dink. R OK.	13. R Is there anything that is important to learn in mathematics that you have not mentioned? S6 I think I have said everything that I can think of. R OK.
14 R Hou jy van GSP? S6 Ja, GSP kan op verskillende maniere gebruik word soos byvoorbeeld om die ooreenkomste en verskille van verskillende soorte driehoeke. Jy kan dadelik verwantskappe sien en dit help jou om die werk beter te verstaan.	14 R Do you like GSP? S6 Yes, GSP can be used in different ways, like for instance with the similarities and differences between different types of triangles. You can immediately see relationships and it helps you to understand the work better.
Onderhoud 7 1. R Op 'n skaal van 1 tot 10, met 10 die hoogste, hoe baie hou jy van wiskunde? S7 7 na 8 R OK. 2. R Doen jy altyd jou werksopdragte? S7 ja op 'n skaal van 1 tot 10, so 9.	**Interview 7** 1. R On a scale of 1 to 10, with ten being the highest, how much do you like mathematics? S7 7 to 8 R OK. 2. R Do you always complete your assignments? S7 Yes, on a scale of 1 to 10, about 9.

R en hoeveel tyd spandeer jy om jou werksopdragte te doen?	R And how much time do you spend on doing your assignments?
S7 Ek sal sê bietjie meer as die gemiddeld wat nodig is om te doen omdat ek bietjie meer te spandeer om die beginsel te verstaan en dan doen ek meer somme	S7 I use more than average because I do more than what is necessary because I spend more time on understanding the principle, and then I do more sums.
R OK.	R OK.
3.	3.
R In die algemeen, wat beïnvloed jou om hard te werk?	R In general, what influences you to work hard?
S7 Wiskunde is 'n vak wat soos die Bybel sê wat jy saai sal jy maai, dit is direk eweredig hoe harder jy werk hoe meer resultate sier jy dit is nie soos tale wat oor 'n lang tydperk vorm nie.	S7 Mathematics is a subject that as the Bible says, you reap what you sow. It is directly related to how hard you work. The more you work the better you do. It is not like language that forms over a long period of time.
R Mooi. Is daar ander enigiets anders wat jou beïnvloed om harder te werk?	R Good. Is there *anything that* causes you to work harder?
S7 Die stimulasie van jy sien die produk op jou harde werk.	S7 The stimulation of seeing the product of your hard work.
R OK.	R OK.
4.	4.
R Hoe goed is jy in wiskunde?	R How good are you in mathematics?
S7 Ek dink ek sal net eers onderskei tussen skoolvlak wiskunde en ingenieurs wiskunde. T tot op skoolvlak wiskunde en eerstejaars wiskunde is ek baie sterk	S7 I think I will first distinguish between school mathematics and engineering mathematics. I am strong in school mathematics up to first year mathematics.
R Hoe weet jy dit?	R How do you know?
S7 Ek dink ek snap redelik vinnig as ek my meet aan ander studente dan snap ek v nniger as die meeste van hulle.	S7 I *think* I grasp it quickly and if I measure myself according to other students, I grasp things quicker than most.
5.	5.
R Is jy beter in sekere afdelings van wiskunde? Byvoorbeeld, is jy beter in algebra as in meetkunde?	R Are you better in certain sections of mathematics? For instance, are you better in algebra than in geometry?
S7 My algebra was nog altyd die sterker een gewees. My meetkunde het egter beter geword vandat ek op Universiteit is.	S7 My algebra has always been the stronger one. My geometry has improved since I have been at University.
R Kan jy enige rede gee hoekom dit so is?	R Can you give *a reason* for why this is so?
S7 Ja ek dink ek begin meetkunde beter	S7 Yes, I think I am starting to understand it

verstaan, ek snap dit makliker, ek sien dit meer in sy geheel.	better, I grasp it quicker, I see the whole better.
R Hm.	R Hm.
6.	6.
R Dink jy iemand het spesiale talent nodig om goed te doen in wiskunde?	R Do you think that a person needs a special talent to be good in mathematics?
S7 Ja ek dink so, dit kan ontwikkel word ek het egter van graad 1 'n 1 vir wiskunde gekry. Vir my gaan dit om redenasie vermoë, om dit te kan insien	S7 Yes I think so. It can be developed, but I had a 1 for mathematics since grade 1. For me it is about an ability to argue, to see things.
R Het jy sulke talent?	R Do you have such talent?
S7 Ja in 'n mate.	S7 Yes, to a certain extent.
R OK.	R OK.
7.	7.
R Hoe belangrik is dit om te memoriseer in wiskunde?	R How important is it to memorise in mathematics?
S7 Daar is sekere dinge wat gememoriseer moet word. Daar is meetkunde stellings wat geleer moet word maar ek dink as jy die vermoë het om iets uit te redeneer dan is dit belangrik om net die stellings te leer. As jy te goed memoriseer sal jy dalk lui wees om te probeer verstaan wat aangaan, insig het in dit wat aangaan.	S7 There are certain things that have to be memorised. There are geometry theorems that have to be studied, but I think if you have the ability to argue something it is important to just study the theorems. If you are good at memorising, you might be lazy to try to understand what is going on and to have insight.
R Dink jy dat iemand wat nie goed is om te memoriseer nie, goed doen in wiskunde?	R Do you think that someone who is not good in memorising can do well in mathematics?
S7 Ja die hele vak bestaan nie uit memoriseer werk nie, dit is in 'n minder mate, jy moet beginsels verstaan.	S7 Yes, the subject does not consist of memorising. Memorising is to a lesser extent, you must understand the principles.
8.	8.
R Hoe gereeld doen jy so min as moontlik werk net om by te bly? Hoe gereeld doen jy so min aas moontlik werk net om by te bly?	R How often do you do as little as possible just to keep up?
S7 Wiskunde eis sy pond vleis, as jy net genoeg doen om by te bly, eis dit nog steeds van jou 24 uur dag 'n uur of twee. Die minimum om by te bly in wiskunde is dalk die maksimum van 'n ander vak om by te bly	S7 Mathematics takes its toll, if you only do enough to keep up, it still takes an hour or two of a 24 hour day. The minimum in mathematics might be the maximum in another subject.
R Hm.	R Hm.

9. R Hoe studeer jy vir 'n wiskunde toets? S7 As die dosent iets belangriks sê merk ek dit af of omkring dit met 'n kleur en voor 'n toets sal ek dit eers deurgaan,die werk wat ek nie verstaan nie sal ek ook eers deurgaan en seker maak ek verstaan dit voor ek gaan skryf. R Goed so.	9. R How do you study for a mathematics test? S7 If the lecturer says something important I mark it or circle it with a colour and then I will go through it before a test. I also go through the work that I don't understand and make sure I understand it before I take the test. R Alright.
10. R Gee voorbeelde van die aktiwiteite wat jy in wiskunde doen. S7 Ek verstaan wiskunde makliker as daar 'n praktiese som is wat gedoen kan word. R Hm.	10. R Give examples of the activities that you do in mathematics. S7 I understand mathematics more easily if there is a practical problem that can be solved. R Hm.
11. R Van hierdie aktiwiteite wat jy nou net opgenoem het, is daar enige van die aktiwiteite wat jy baie geniet en interessant vind? S7 Ek dink dat ek GSP kan gebruik om beter te verstaan ek hou nie van geo-stroke en sulke goed nie, dit verveel my.	11. R Of these activities that you have just mentioned, are there any that you enjoy and that you find interesting? S7 I think that I can use GSP to understand better, and I do not like geo-strips and such things, it bores me.
12. R Is daar probleme in wiskunde wat op meer as een manier opgelos kan word? S7 Ja. R Hoekom? S7 Elke persoon dink anders.	12. R Are there problems in mathematics that can be solved in more than one way? S7 Yes. R Why? S7 Every person thinks differently.
13. R Is daar enigiets waaraan jy kan dink wat belangrik is om wiskunde te leer wat jy nie genoem het nie. S7 Ek dink ek het alles gesê waaraan ek kan dink. R OK	13. R Is there anything that is important to learn in mathematics that you have not mentioned? S7 I think I have said everything that I can think of. R OK.
14 R Hou jy van GSP? S7 Ja, dit is baie interessant en dit is baie	14 R Do you like GSP? S7 Yes, it is very interesting and it is easier to

makliker om meetkunde daarop te doen. Vandat ek meetkundige sketse op GSP doen geniet ek die meetkunde meer en ek verstaan meer van die meetkunde ek kan die uitwerking dadelik sien bv as ons gelykbenige driehoeke moet teken en die hoeke meet, kan ek dadelik siek die basishoeke is gelyk.	do geometry on it. Since I have been doing geometry sketches on GSP I enjoy geometry more and I understand more of the geometry. I can immediately see the effect, for instance if we do isosceles triangles, I will immediately see that the basis angles are equal.

APPENDIX D

YUNIBESITI YA BOKONE-BOPHIRIMA
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17 Mei 2004

Dekaan Opvoedkunde
Potchefstroom Kampus
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Geagte Prof. HJ Steyn

Ek is op die oomblik besig met my M.Ed verhandeling (wiskunde-onderwys) en die fokus van ondersoek is: "The effect of a dynamic technological learning environment on the geometry conceptualisation of pre-service mathematics teachers". Verwagte uitkoms van die projek is 'n betekenisvolle bydrae tot die herontwerp van bestaande voordiensopleidingsprogramme vir wiskundeonderwysers om hulle vlak van relevante konseptuele wiskundekennis tot verwagte ontwikkelingsvlakke te verhoog. Die projek vorm deel van die NRF-ondersteunde SOSI-Projek, met spanlede proff. Dirk Wessels (Unisa), Michael de Villiers (UKZN) en Hercules Nieuwoudt (Potchefstroomkampus, UNW).

Ek vra asseblief toestemming om die projek ooreenkomstig my goedgekeurde voorlegging binne die Fakulteit Opvoedingswetenskappe aan te pak en welmet die hulp van die derde-jaarsgroep wat algemene wiskunde (**WSGK 311**) neem. Die navorsing sal plaasvind in deurlopende medewerking met die verantwoordelike personeellid, mev. Annalie Roux, en onder toesig van die studieleiers, prof. Hercules Nieuwoudt en mev. Mariana Plotz. Die betrokke dosent sal te alle tye teenwoordig wees. Die bestaande studiegids en module-uitkomste bly op alle **WSGK 311**-groepe van toepassing en dieselfde werkkaarte sal te alle tye deur alle groepe gebruik word, ongeag of hulle deel van die navorsingsgroep uitmaak. Alle werk (leertake) wat een groep doen sal die navorsingsgroep ook doen, en omgekeerd. Die enigste wesenlike verskil is die hulpmiddels wat die groepe gaan gebruik. en die konteks waarbinne hulle leer. Die navorsingsgroep gaan die dinamiese sagtewarepakket Geometer's Sketchpad gebruik as hulpmiddel en leerkonteks, terwyl die ander groepe byvoorbeeld geo-stroke in die gebruiklike klassikale opset gaan gebruik. Die doelwitte en leeruitkomste van alle groepe is dieselfde.

Die navorsing geskied ook met die medewete van die wiskunde–vakvoorsitter: Dr. Susan Nieuwoudt, en die saak is reeds deur die studieleier met die betrokke skooldirekteure, prof. Barry Richter en dr. Elsa Mentz bespreek.

By voorbaat dankie vir u gunstige oorweging van die aansoek.

Jeannette Kotzé
Lektor: SNWTO
M.Ed.-student (studente no: 10117784)

Hierdie aansoek word ten volle ondersteun en daar word onderneem om by alle ooreengekome vereistes wat gestel is of mag word te hou.

Hercules Nieuwoudt
Medeprofessor: NSO
Studieleier

Mariana Plotz
Lektor: SNWTO
Medeleier

By voorbaat dankie.

Jeannette Kotzé