



# **Mathematics teachers' metacognitive skills and mathematical language in the teaching-learning of trigonometric functions in township schools**

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degree Philosophiae Doctor in Mathematics Education  
at the Potchefstroom Campus of the North-West University**

Promotor: **Prof. M.S. van der Walt**

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# DECLARATION

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NORTH-WEST UNIVERSITY<sup>®</sup>  
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POTCHEFSTROOM CAMPUS

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## SOLEMN DECLARATION

### 1. Solemn declaration by student

I, **JOHANNA SANDRA FRANSMAN**,

declare herewith that the thesis entitled

### **MATHEMATICS TEACHERS' METACOGNITIVE SKILLS AND MATHEMATICAL LANGUAGE IN THE TEACHING-LEARNING OF TRIGONOMETRIC FUNCTIONS IN TOWNSHIP SCHOOLS**

which I herewith submit to the North-West University, Potchefstroom Campus, in compliance with the requirements set for the **PhD (Mathematics Education)** degree, is my own work, has been language edited and has not already been submitted to any other university.

I understand and accept that the copies that are submitted for examination are the property of the University.

Signature of student \_\_\_\_\_ University number **10691847**

Signed at **Potchefstroom** this \_\_\_\_\_ day of \_\_\_\_\_ 2014

## DEDICATIONS

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### **I dedicate this PhD**

#### **To my parents:**

Willem and Eva Beukes: Thank you for all the years of love and support. I am truly blessed that you are still in my life.

#### **To my late parents-in-law:**

Aubrey and Maria Fransman: I miss you.

#### **To my late brothers:**

Kobus, Derrick, Wiam and Abie Beukes: Each one of you left a big empty space in my life.

#### **To my husband and children:**

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## SUMMARY

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Metacognition is commonly understood in the context of the learners and not their teachers. Extant literature focusing on how Mathematics teachers apply their metacognitive skills in the classroom, clearly distinguishes between *teaching with metacognition* (TwM) referring to teachers thinking about their own thinking and *teaching for metacognition* (TfM) which refers to teachers creating opportunities for learners to reflect on their thinking. However, in both of these cases, thinking requires a language, in particular appropriate mathematical language to communicate the thinking by both teacher and learners in the Mathematics classroom. In this qualitative study, which forms part of a bigger project within SANPAD (South Africa Netherlands Research Programme on Alternatives in Development), the metacognitive skills and mathematical language used by Mathematics teachers who teach at two township schools were interrogated using the design-based research approach with lesson study. Data collection instruments included individual interviews and a trigonometric assessment task. Lessons were also observed and video-taped to be viewed and discussed during focus group discussions in which the teachers, together with five Mathematics lecturers, participated. The merging of the design-based research approach with lesson study brought about teacher-lecturer collaboration, referred to in this study as the Mathematics Educators' Reflective Inquiry (ME'RI) group, and enabled the design of a hypothetical teaching and learning trajectory (HTLT) for the teaching of trigonometric functions. A metacognitive performance profile for the two grade 10 teachers was also developed. The Framework for Analysing Mathematics Teaching for the Advancement of Metacognition (FAMTAM) from Ader (2013) and the Teacher Metacognitive Framework (TMF) from Artzt and Armour-Thomas (2002) were adjusted and merged to develop a new framework, the Metacognitive Teaching for Metacognition Framework (MTMF) to analyse the metacognitive skills used by mathematics teachers TwM as well as TfM. Without oversimplifying the magnitude of these concepts, the findings suggest a simple mathematical equation: **metacognitive skills + enhanced mathematical language = conceptualization skills**. The findings also suggest that both TwM and TfM are required for effective mathematics instruction. Lastly the findings suggest that the ME'RI group holds promise to enhance the use of the metacognitive skills and mathematical language of Mathematics teachers in Mathematics classrooms.

## KEY WORDS

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Cognition

Complexity

Knowledge

Lesson planning

Lesson study

Mathematical language

Mathematics teaching

Metacognition

Metacognitive skills

Professional development

Reflection

Teacher change

Teacher learning

Trigonometry teaching

## OPSOMMING

---

Metakognisie word gewoonlik gebruik in die konteks van die leerders en nie hul onderwysers nie. Bestaande literatuur wat fokus op die wyse waarop Wiskunde onderwysers hul metakognitiewe vaardighede in die klaskamer toepas, onderskei duidelik tussen *onderrig met metakognisie* (TwM) wat verwys na onderwysers se denke oor hul onderrig, en *onderrig vir metakognisie* (TfM) wat verwys na onderwysers wat geleentehede skep vir leerders om na te dink oor hulle denke. In albei hierdie gevalle vereis denke 'n taal, in die besonder 'n gepaste wiskundige taal, om die denke deur leerders sowel as onderwysers in die Wiskunde klaskamer uit te druk. In hierdie kwalitatiewe studie, wat deel vorm van 'n groter projek van SANPAD (South Africa Netherlands Research Programme on Alternatives in Development), word die metakognitiewe vaardighede en wiskundige taal gebruik deur Wiskunde onderwysers wat onderrig gee by twee skole in 'n voorheen benadeelde woonbuurt, ondersoek deur die ontwerp-gebaseerde navorsingsbenadering aan te wend saam met les studie. Data-insamelingsinstrumente het ingesluit individuele onderhoude en 'n trigonometrie assesseringstaak. Lesse is waargeneem, en video-opnames is daarvan gemaak om te bestudeer en te bespreek tydens fokusgroep-besprekings waaraan die onderwysers en vyf Wiskunde dosente deelgeneem het. Die samevoeging van die ontwerp-gebaseerde navorsingsbenadering met lesstudie het samewerking tussen dosente en onderwysers meegebring, waarna in hierdie studie verwys word as die ME'RI-groep (Mathematics Educators' Research Inquiry group). Dit het die ontwerp van 'n hipotetiese onderrig-en-leertrajek vir die onderrig van trigonometriese funksies funksies moontlik gemaak. 'n Metakognitiewe prestasieprofiel vir die twee graad 10-onderwysers is ook ontwikkel. Ader (2013) se FAMTAM (Framework for Analysing Mathematics Teaching for the Advancement of Metacognition) en Artzt en Armour-Thomas (2002) se TMF (Teacher Metacognitive Framework) is aangepas en saamgevoeg om 'n nuwe raamwerk te ontwikkel, die MTMF (Metacognitive Teaching for Metacognition Framework) om die metakognitiewe vaardighede te verstaan wat deur Wiskunde-onderwysers aangewend word by beide TwM en TfM. Sonder om afbreek te doen aan die omvang van hierdie konsepte, dui die bevindings van die studie op 'n baie eenvoudige wiskundige vergelyking: **metakognitiewe vaardighede + verbeterde wiskundige taal = konseptualiseringsvaardighede**. Verder dui die bevindings ook daarop dat beide TwM en TfM vereis word vir effektiewe Wiskunde-onderrig. Laastens dui die bevindings daarop dat die ME'RI-groep belofte inhou vir die verryking van die metakognitiewe vaardighede en wiskundige taal van Wiskunde onderwysers in Wiskunde klaskamers.

## SLEUTELWOORDE

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Kognisie

Kompleksiteit

Kennis

Lesbeplanning

Lesstudie

Wiskundige taal

Wiskunde onderrig

Metakognisie

Metakognitiewe vaardighede

Professionele ontwikkeling

Refleksie

Onderwyser-verandering

Onderwyser-leer

Trigonometrie onderrig

## ABREVIATIONS

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CAPS: Curriculum and Assessment Policy Statement

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DBE: Department of Basic Education

DHE: Department of Higher Education

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FAMTAM: A framework for Analysing Mathematics Teaching for the Advancement of Metacognition

---

HI: Hierarchic Interactionalism

HLT: Hypothetical Learning Trajectory

HTLT: Hypothetical Teaching and Learning Trajectory

---

LTT: Learning Through Teaching

---

ME'RI: Mathematics Educators' Reflective Inquiry

MKT: Mathematical Knowledge for Teaching

---

PCK: Pedagogical Content Knowledge

---

RME: Realistic Mathematics Education

---

SANPAD: South Africa Netherlands Research Programme on Alternatives in  
Development

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TfM: Teaching for Metacognition

TMF: Teacher Metacognitive Framework

TwM: Teaching with Metacognition

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## QUOTE

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*“Sen, what are you doing?” the teacher asked.*

*“Thinking”, he answered.*

*“What are you supposed to be doing?”*

*“Working,” Sen sighed, shifting back in his seat and picking up his pencil.*

(Davis, Sumara, & Luce-Kapler, 2008, p. 65)

This episode that was recorded during a mathematics lesson surely raises the question whether thinking is an appropriate activity in the mathematics classroom...

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