ANALYSIS OF COINTEGRATED MACROECONOMIC TIME SERIES

by

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CHAPTER 1

INTRODUCTION

1.1 Aim and Objectives

The aim of this study is to investigate mathematically formal tests for stationarity in macroeconomic univariate time series. As it is often the case, regressions involving one time series variable on one or more variables can give spurious results. One way of avoiding spurious regressions is to test for cointegration among time series data sets. This study we discusses extensively some formal tests for unit root and co-integration among macroeconomic time series data sets. Our major aim will be to investigate the efficiency of the Phillips-Ouliaris test for cointegration by employing the methods of ordinary least squares (OLS) and Yule-Walker. It has been established that the Johansen test for cointegration is seen to be the most powerful test among others in the literature. This study focuses on two of the simpler cointegration test procedures, namely, the Phillips-Ouliaris (PO) and the Cointegrating Regression Durbin-Watson (CRDW) tests. The two test procedures are compared using Monte Carlo methods. The results of this comparative study show that the PO test performs better than the CRDW test.

1.2 Research Methodology

Real-life data used in this study were obtained from the official publications of the South African Reserve Bank. Processing these data was carried out using two statistical packages, namely, *SAS* and *EViews*. Statistical methods employed in this dissertation are basically the simple and multiple regression analyses.

1.3 Theoretical Aspects and Project Outline

A time series is a sequence of observations of some variable taken at successive time points. The primary aim of the analysis of time series is to understand better the nature of the process that generated the past and present data and also to help make future projections based on the knowledge of past data. The empirical work based on time series data assumes that the underlying series is stationary, even though in practice most time series are non-stationary. By stationarity, we mean that the mean and variance for the time series $\{X_t\}_{t=1}^{T}$ are constant over time, and that the covariance between two time points depends only on the distance or lag between the two time points and not on the actual time point at which the covariance is computed. Mathematically,

$$E(X_t) = \mu \ , \tag{1-1}$$

$$\text{var}(X_t) = E(X_t - \mu)^2 = \sigma^2 \tag{1-2}$$

$$\text{cov}(X_t, X_{t-k}) = E[(X_t - \mu)(X_{t-k} - \mu) . \tag{1-3}$$

If X_t violates any of the conditions set above, it is said to be non-stationary. Quite recently, substantial research efforts have been shifted from traditional ways of analyzing time series to developing tests that can be used to differentiate the stationary from the non-stationary time series. Tests usually employed to differentiate these types of series boil down to the issue of checking whether or not a series in question should be modeled in levels or in first/second differences. In other words, the distinction between a stationary and a non-stationary time series is the distinction between some higher order distributed lag transformations of either levels or first/second differences. One formal way of testing for stationarity is based on testing for the presence of a unit root which relies heavily on asymptotic theory.

Unit root tests have become a standard tool in applied time series where test statistics are formulated to test for the need to difference a univariate time series, for example, Dickey and Fuller (1979, 1981) and Phillips and Perron (1988). Hasza and Fuller (1982), among others considered testing for a seasonal unit root in univariate time series. Dickey and Pantula (1987) extended the concept of testing for a unit root to the case where there are two unit roots in a univariate time series. The presence of a unit root or unit roots is an indicative that the series in question is integrated.

The extension of the concept of a unit root test by Engle and Granger (1987) arises in multivariate setting where it is pointed out that if individual time series are non-stationary, it is still possible that certain linear combinations of the time series in the system will be stationary. Such systems are referred to as co-integrated.

In Chapter 2, we discuss the concept of testing for a unit root in a non-seasonal univariate time series with the inclusion of an intercept and a trend in a given autoregressive model.

Chapter 3 discusses testing for a unit root in a seasonal univariate time series as discussed by Hylleburg et al (1990) and other researchers.

Chapter 4 presents the concept of cointegration. We pay particular attention to Engle-Granger, Engle-Yoo-Granger, the Cointegrating Regression Durbin-Watson, and the Phillips-Ouliaris cointegration test procedures. We also evaluate the efficiency of the Phillips-Ouliaris cointegration test via the OLS and Yule-Walker methods. To assess the efficiency of the Phillip-Ouliaris test we shall simulate two cointegrated time series each of size 100. Efficiency test will then be conducted via the two methods stated above. Chapter 5 discusses error correction models (ECM) and the Johansen cointegration test. Chapter 6 summarizes the whole study and recommends further research areas.

1.4 Motivation of the Study

Unit root tests have become a standard diagnostic tool in applied time series analysis. Test statistics have been developed to test the need to difference a univariate time series, for instance, Dickey and Fuller (1979, 1981) and Phillips and Perron (1988). An extension of the unit root concept has led to the concept of cointegration. In cointegration, it is pointed out that if two or more time series are integrated of order 1, then it is possible to have a linear combination of these series to be integrated of order zero. Several test procedures have been proposed in the literature to test for cointegration, among them, the Phillips-Ouliaris cointegration test. The application of the Yule-Walker method, as documented in most texts, is seen to be more efficient than the OLS method. We are motivated by this fact to investigate the efficiency of the Phillips-Ouliaris cointegration test procedure via these two methods.

1.5 Limitations

All interpretations of results will solely be based on those series analyzed in the study. Interpretations of these results are influenced strongly by the statistical packages used in this study namely, *SAS*, and *EViews*.

CHAPTER 2

NON-SEASONAL UNIT ROOT TESTS

2.1 Introduction

A non-stationary time series is said to contain a unit root if it is found that differencing the series once will induce stationarity. The implications of a unit root in macroeconomic time series are profound. It is profound in the sense that if a variable is truly difference-stationary (DS), then shocks to such a variable will have permanent effect. In such a situation, the advice is to reconsider the analysis of macroeconomic policy involving that variable.

The unit root tests developed in Fuller (1976) and, Dickey and Fuller (1979, 1981) have become the capstone in many subsequent publications regarding testing for the presence of a unit root. In the time-domain analysis of time series, extensions of the Dickey-Fuller methods that have been made to handle such problems as autocorrelated errors and heteroskedasticty include the Augmented Dickey-Fuller (ADF) and the Phillip-Perron (PP) tests. However, the literature is not without its sceptics. For example, Choi (1992), suggests that the formulation of the ADF test which assumes an autoregressive (AR) representation for the series would be highly distorted by the presence of a moving average (MA) component in the innovations. Another source of concern with regards to testing for a unit root using the ADF and PP test procedures is the possibility of structural breaks being in the time series. In the frequency-domain time series analysis, an approach due to Akdi and Dickey (1998) and Evans and Dickey (1998) has been found to be statistically powerful.

Talking about unit root tests should also raise the question of how these tests perform. Because of the process involved in data collection, time series data used in analysis and modelling are frequently obtained through temporal aggregation such as quarterly or monthly. It is of the view that aggregation should have an effect on such tests as the ADF and PP tests (Teles and Wei, 1998).

The chapter is organised as follows. Since the concept of cointegration depends heavily on unit root test, we discuss some three unit root tests commonly employed in practice in Section 2.2. In Section 2.3, we address the situation where there is more than one unit root. Section 2.4 studies some of the problems usually encountered in testing for unit roots. In Section 2.5, a unit root test as against an explosive unit root is discussed. Numerical examples are used in Section 2.6 to illustrate the concept of unit roots. Section 2.7 summarises the chapter.

2.2 Some Frequently Used Unit Root Tests

This section discusses some three unit root tests, namely, the Augmented Dickey-Fuller (ADF), the Phillips-Perron (PP), and the Frequency-Domain Regression (FDR) tests. For simplicity, we consider the case where the data-generating-process (dgp) of the series $\{X_t\}_{t=1}^{T}$ satisfies the AR(1) process

$$X_t = \rho X_{t-1} + a_t, \qquad (2\text{-}1)$$

where a_t is a white noise process with mean 0 and variance σ^2. Subtracting X_{t-1} from both sides of (2-1) gives the following equation

$$Y_t = (\rho - 1)X_{t-1} + a_t, \qquad (2\text{-}2)$$

where $Y_t = X_t - X_{t-1}$. From standard distribution theory, the likelihood function using T independent observations is

$$L = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\sum_{t=1}^{T} a_t^2\right] = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\sum_{t=1}^{T} (X_t - \rho X_{t-1})^2\right], \qquad (2\text{-}3)$$

since $a_t = X_t - \rho X_{t-1}$.

The log-likelihood function is

$$\ln L = -\left(\frac{T}{2}\right)\ln(2\pi) - \left(\frac{T}{2}\right)\ln\sigma^2 - \left(\frac{1}{2\sigma^2}\right)\sum_{t=1}^{T}(X_t - \rho X_{t-1})^2 . \tag{2-4}$$

Maximising (2-4) with respect to ρ and σ^2 gives

$$\frac{\partial}{\partial\rho}(\ln L) = \frac{1}{\sigma^2}\sum_{t=1}^{T}(X_t - \rho X_{t-1})X_{t-1} = \frac{1}{\sigma^2}\sum_{t=1}^{T}(X_t X_{t-1} - \rho X_{t-1}^2), \tag{2-5}$$

and

$$\frac{\partial}{\partial\sigma^2}(\ln L) = -\frac{T}{2}\ln(2\pi) + \frac{1}{2\sigma^4}\sum_{t=1}^{T}(X_t - \rho X_{t-1})^1 . \tag{2-6}$$

Setting the partial derivatives equal to zero and solving for ρ and σ^2 gives the OLS estimates

$$\hat{\rho} = \frac{\sum_{t=1}^{T} X_t X_{t-1}}{\sum_{t=1}^{T} X_{t-1}^2} = \frac{\sum_{t=1}^{T}(\rho X_{t-1} + a_t)X_{t-1}}{\sum_{t=1}^{T} X_{t-1}^2}$$

$$= \frac{\rho\sum_{t=1}^{T} X_{t-1}^2}{\sum_{t=1}^{T} X_{t-1}^2} + \frac{\sum_{t=1}^{T} X_{t-1}a_t}{\sum_{t=1}^{T} X_{t-1}^2}$$

$$\Rightarrow \qquad \hat{\rho} = \rho + \frac{\sum_{t=1}^{T} X_{t-1}a_t}{\sum_{t=1}^{T} X_{t-1}^2}$$

$$\Rightarrow \qquad \text{or} \qquad \hat{\rho} - \rho = \frac{\sum_{t=1}^{T} X_{t-1}a_t}{\sum_{t=1}^{T} X_{t-1}^2}, \tag{2-7}$$

and $$\hat{\sigma}^2 = \frac{\sum_{t=1}^{T} a_t^2}{T}. \qquad (2\text{-}8)$$

Thus, we can formulate our null hypothesis of a unit root as

$$H_0 : \rho = 1. \qquad (2\text{-}9)$$

It is readily seen that under the hypothesis of a unit root, (2-7) becomes

$$\hat{\rho} - 1 = \frac{\sum_{t=1}^{T} X_{t-1} a_t}{\sum_{t=1}^{T} X_{t-1}^2}, \qquad (2\text{-}10)$$

hence, the determination of $(\rho - 1)$ could be obtained from the autoregression (2-2)

$$Y_t = (\rho - 1) X_{t-1} + a_t$$

by regressing Y_t on X_{t-1}. If $(\rho - 1)$=0, then the series $\{X_t\}_{t=1}^{T}$ is said to contain a unit root. In this case, the autoregession (2-2) reduces to

$$Y_t = 0. \qquad (2\text{-}11)$$

2.2.1 The Augmented Dickey-Fuller (ADF) Unit Root Test

Using the simple AR(1) process, the ADF unit root test is based on the estimated autoregression

$$\hat{Y}_t = (\hat{\rho} - 1) X_{t-1}. \qquad (2\text{-}12)$$

Under the null hypothesis of a unit root, the test statistic associated with the OLS estimation of $(\rho-1)$ is

$$ADF^{*}=\frac{(\hat{\rho}-1)}{\sqrt{\operatorname{var}(\hat{\rho}-1)}}, \tag{2-13}$$

where $\operatorname{var}(\hat{\rho}-1)$ is the estimated variance of $(\rho-1)$. If $\hat{\rho}-1=0$, then $\hat{Y}_t=0$. Distributions of such estimates and their associated test statistics are non-standard. And so we cannot employ standard tables of critical values. Fortunately, Fuller (1976) has tabulated such critical values for this test statistic. For a given time series, the null hypothesis of a unit root is rejected if

$$ADF^{*}<(\text{critical value}). \tag{2-14}$$

Alternatively, if we are able to calculate the probability (prob-value) of observing the test statistic more extreme than X under the assumption that the null hypothesis is true, then we can reject this hypothesis when the prob-value is less than some nominal level, say 5%. Modifications can be made to the (2-1) to include a constant term and/or a time trend:

$$X_t=\alpha+\rho X_{t-1}+a_t, \tag{2-15}$$

$$X_t=(\alpha_0+\alpha_1 t+\alpha_2 t^2+\ldots+\alpha_n\alpha^n)+\rho X_{t-1}+a_t, \tag{2-16}$$

where α is a constant and $(\alpha_0+\alpha_1 t+\alpha_2 t^2+\ldots+\alpha_n t^n)$, an n-order polynomial in time t. In the absence of a clear trend, a constant term is included in the model if the absolute mean of the un-differenced series is greater than the corresponding standard deviation. Otherwise, a constant term is excluded in the autoregression. In this case, the modified autoregressions are, respectively, given by

$$Y_t=\alpha+(\rho-1)+\varepsilon_t. \tag{2-17}$$

and $$Y_t = (\alpha_0 + \alpha_1 t + \alpha_2 t^2 + \ldots + \alpha_n \alpha^n) + (\rho - 1)X_{t-1} + a_t. \tag{2-18}$$

The critical values employed in the autoregressions (2-17) and (2-18) take into consideration the inclusion of a constant term and a time trend. For higher-order AR(p) processes

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + a_t, \tag{2-19}$$

$$X_t = \alpha_0 + \sum_{i=1}^{p} \phi_i X_{t-i} + a_t, \tag{2-20}$$

$$X_t = (\alpha_0 + \alpha_1 t + \alpha_2 t^2 + \ldots + \alpha_n t^n) + \sum_{i=1}^{p} \phi_i X_{t-i} + a_t, \tag{2-21}$$

the appropriate autoregressions to fit are

$$Y_t = (\rho - 1)X_{t-1} + \sum_{i=1}^{p-1} \beta_i Y_{t-i} + a_t, \tag{2-22}$$

$$Y_t = \alpha_0 + (\rho - 1)X_{t-1} + \sum_{i=1}^{p-1} \beta_i Y_{t-i} + a_t, \tag{2-23}$$

$$Y_t = (\alpha_0 + \alpha_1 t + \alpha_2 t^2 + \ldots + \alpha_n t^n) + (\rho - 1)X_t + \sum_{i=1}^{p-1} \beta_i Y_{t-i} + a_t, \tag{2-24}$$

respectively. Again, the same critical values or prob-values employed in the autoregressions (2-2), (2-17) and (2-18) apply accordingly.

2.2.2 The Phillips-Perron (PP) Unit Root Test

The Phillips-Perron (PP) unit root test, based on the same estimated autoregression is overtly non-parametric. Whilst the ADF method assumes that the error terms follow a white noise process with mean 0 and variance σ^2, the PP test assumes that the error term are autocorrelated.

Based on the same autoregressions, Phillips and Perron (1988) showed that the test statistic

$$PP^{*} = \left(\frac{\hat{\gamma}_0}{\hat{\lambda}^2}\right)^{\frac{1}{2}}\left(\frac{\hat{\rho}-1}{\sqrt{\text{var}(\hat{\rho}-1)}}\right) - \frac{1}{2T}\left(\frac{1}{\sum_{t=2}^{T}(X_t-\bar{X})}\right)^{\frac{1}{2}}\left(\frac{\hat{\lambda}^2-\hat{\gamma}_0}{\hat{\lambda}}\right), \tag{2-25}$$

have the same asymptotic distribution as that of the ADF statistic in where (2-13). In (2-25), we have

$$\hat{\lambda}^2 = \hat{\gamma}_0 + 2\sum_{j-1}^{q}\left[1-\frac{j}{q+1}\right]\hat{\gamma}_j, \tag{2-26}$$

$$\hat{\gamma}_j = \tfrac{1}{T}\sum_{t=j+1}^{T}\hat{a}_t\hat{a}_{t-j}, \tag{2-27}$$

where $\hat{a}_t$ are the residuals from the least-squares fit of any of the autoregressions (2-22), (2-23), or (2-24). If the error terms are non-autocorrelated, then

$$\hat{\lambda}^2 = \hat{\gamma}_0 \tag{2-28}$$

so that (2-25) reduces to the ADF test statistic as

$$PP^{*} = \left(\frac{\hat{\rho}-1}{\sqrt{\text{var}(\hat{\rho}-1)}}\right) = ADF^{*}. \tag{2-29}$$

Here again, the null hypothesis of a unit root is rejected if

$$PP^{*} < (\text{critical value}). \tag{2-30}$$

The prob-value can also be used to reject/accept the hypothesis of a unit root. The null hypothesis of a unit is rejected if the prob-value is less some nominal level.

2.2.3 Frequency-Domain (FD) Unit Root Test

The FD unit root test is based on periodogram ordinates resulting from Fourier transform. Consider the AR(1) process with drift

$$X_t = \alpha + \rho X_{t-1} + a_t . \tag{2-31}$$

Akdi and Dickey (1998) examined the problem of testing for unit root in the frequency domain by calculating the moments of the Fourier coefficients of a unit root time series. Under the null hypothesis of a unit root, they proposed the test statistic

$$FD^* = \frac{4\pi^2}{\sigma^2\phi^2.T^2} I_X(w_1), \tag{2-32}$$

where

$$\sigma^2\phi^2 = \sum_{j=1}^{[\sqrt{T}]} \frac{I_Y(w_j)}{2[T]} . \tag{2-33}$$

In (2-32) and (2-33), $I_X(w_1)$ and $I_Y(w_j)$ are, respectively, the first periodogram ordinates of the undifferenced series and the j-th periodogram ordinate of the first-differnced series, $Y_t = X_t - X_{t-1}$. Akdi and Dickey (1998) showed that the inclusion of a polynomial time trend, $(\alpha_0 + \alpha_1 t + \alpha_2 t^2 + ... + \alpha_n t^n)$, does not affect the estimation. Furthermore, by applying Slutsky's Theorem, Evans and Dickey (1998) showed that

$$FD^* = \frac{4\pi^2}{\sigma^2\phi^2.T^2} I_X(w_1) \longrightarrow z_1^2 + 3z_2^2 , \tag{2-34}$$

where z_1 and z_2 are two independent standard normal variable. Akdi and Dickey (1998) have tabulated percentile values based on this test statistic. They were calculated by appealing to the mixture of chi-squares result from Johnson, Kotz, and Balakrishnan (1994).

Here again, the null hypothesis of a unit root is rejected if

$$FD^{*} < (\text{critical value}). \tag{2-35}$$

2.3 Multiple Unit Roots Tests

In this section, we briefly tackle the issue of multiple unit roots, something that is seldom in time series data. In their research, Hasza and Fuller (1979) developed a test for the null hypothesis that a time series has two characteristic roots equal to 1. The test is based on a statistic readily obtained by standard regression. Let the series assumes any of the representations

$$X_t = \rho_1 X_{t-1} + \rho_2 Y_{t-1} + \sum_{j=1}^{p-2} \beta_j (Y_t - Y_{t-j}) + \varepsilon_t \ , \tag{2-36}$$

$$X_t = \alpha_0 + \rho_1 X_{t-1} + \rho_2 Y_{t-1} + \sum_{j=1}^{p-2} \beta_j (Y_t - Y_{t-j}) + \varepsilon_t \ , \tag{2-37}$$

$$X_t = (\alpha_0 + \alpha_1 t + \alpha_2 t^2 + \ldots + \alpha_n t^2) + \rho_1 X_{t-1} + \rho_2 Y_{t-1} + \sum_{j=1}^{p-2} \beta_j (Y_t - Y_{t-j}) + \varepsilon_t \ , \tag{2-38}$$

where $Y_{t-j} = X_{t-j} - X_{t-j-1}$, $j = 0,1,2,\ldots$ Then, for instance, in (2-36), the characteristic equation of X_t

$$m^{p-2} - \sum_{j=1}^{p-2} \beta_j m^{p-2-j} = 0 \ , \tag{2-39}$$

will have two unit roots if and only if the true parameter values satisfy the null hypothesis

$$H_0^{(1)} : (\rho_1, \rho_2) = (1,1) \ . \tag{2-40}$$

Based on the autoregressions (2-36), (2-37) and (2-38), Hasza and Fuller (1979) proposed other test statistics for the following null hypotheses:

$$H_0^{(2)} : (\alpha_0, \rho_1, \rho_2) = (1,1,1), \tag{2-41}$$

$$H_0^{(3)} : (\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_n, \rho_1, \rho_2) = (0,0,0,\ldots,0,1,1). \tag{2-42}$$

Tables of critical values for this test have been given in Hasza and Fuller (1979). A null hypothesis of a unit root is rejected if the test statistic MADF is such that

$$MADF^* < (\text{critical value}). \tag{2-43}$$

One other convenient approach to testing for second differencing is by applying either the ADF or the PP unit test on the first-differenced series. In this case, the appropriate autoregressions that we consider are

$$(Y_t - Y_{t-1}) = (\rho^* - 1)X_{t-1} + \sum_{j=1}^{p-1} \beta_j^* (Y_{t-j} - Y_{t-j-1}) + a_t^*, \tag{2-44}$$

$$(Y_t - Y_{t-1}) = \alpha_0^* + (\rho^* - 1)X_{t-1} + \sum_{j=1}^{p-1} \beta_j^* (Y_{t-j} - Y_{t-j-1}) + a_t^*, \tag{2-45}$$

$$(Y_t - Y_{t-1}) = (\alpha_0^* + \alpha_1^* t + \alpha_2^* t^2 + \ldots + \alpha_n^* t^n) + (\rho^* - 1)X_{t-1} + \sum_{j=1}^{m} \beta_j^* (Y_{t-j} - Y_{t-j-1}) + a_t^*. \tag{2-46}$$

The applicable test statistic for the second-difference-series (SDS) is

$$SDS^* = \frac{\hat{\rho}^* - 1}{\sqrt{\operatorname{var}(\hat{\rho}^* - 1)}}. \tag{2-47}$$

The same percentile values used in the ADF and PP tests apply. The null hypothesis of a second unit root is rejected if

$$SDS^* < \text{(critical value)}. \tag{2-48}$$

2.4 Unit Root Tests and Temporal Aggregation

In the previous section, we have discussed some unit root tests commonly employed in practice in detail. Because of the process involved in collecting the data, time series data used in analysis and modelling are obtained through temporal aggregation, for instance, monthly or quarterly. Consequently, testing for unit root is often based on time series aggregates. To some extent, we can perceive the performance of a unit root test as being influenced by time aggregation. In this section, we briefly outline the effects of aggregation on the ADF unit root test as discussed by Teles and Wei (1998).

2.4.1 The Aggregate Model

Define the r-nonoverlapping aggregates, W_t, of the series $\{X_t\}_{t=1}^{N}$ by

$$W_N = \sum_{t=r(N-1)+1}^{rN} X_t = \sum_{j=0}^{r-1} X_{rN-j}, \tag{2-49}$$

where r is a fixed order of aggregation and N, the aggregate time unit. Suppose that X_t assumes the random walk process

$$X_t = X_{t-1} + a_t, \qquad \text{where} \qquad a_t \sim WN(0, \sigma^2). \tag{2-50}$$

Then, according to Teles and Wei (1998), the aggregate time series given by (2-49) follows the ARIMA (0,1,1) process

$$W_N = W_{N-1} + a_N - \alpha . a_{N-1}, \quad \text{where} \quad a_N \sim WN(0, \sigma_\varepsilon^2). \tag{2-51}$$

Teles and Wei (1998) further showed that

$$\alpha = \begin{cases} 0 & , \quad r = 1 \\ -\dfrac{2r^2+1}{r^2-1} + \sqrt{\left(\dfrac{2r^2+1}{r^2-1}\right)^2 - 1} & , \quad r \geq 2 \end{cases} , \tag{2-52}$$

and

$$\sigma_\varepsilon^2 = \begin{cases} \sigma^2 & , \quad r = 1 \\ \sigma^2 \left[\dfrac{r(2r^2+1)}{3(r+\alpha^2)}\right] & , \quad r \geq 2 \end{cases} . \tag{2-53}$$

Our conclusion from this revelation is that the aggregate time series remains non-stationary even after temporal aggregation.

2.4.2 Effects of Temporal Aggregation on the ADF Test Statistic

Suppose that the aggregate time series $\{W_N\}_{N=1}^{W^*}$ assumes the following simple aggregate autoregressive model

$$W_N = \rho^{(N)} W_{N-1} + a_N, \qquad \text{where} \qquad a_N \sim WN(0, \sigma_\varepsilon^2), \tag{2-54}$$

where $W^* = T/r$. Then, OLS estimator of $\rho^{(N)}$, based on the Augmented Dickey-Fuller (ADF) unit root test is

$$\hat{\rho}^{(N)} = \frac{\sum_{N=2}^{W^*} W_{N-1} W_N}{\sum_{N=2}^{W^*} W_{N-1}^2} = \rho^{(N)} + \frac{\sum_{N=2}^{W^*} W_{N-1}^2 a_N}{\sum_{N=2}^{W^*} W_{N-1}^2}$$

or $$\hat{\rho}^{(N)} - \rho^{(N)} = \frac{\sum_{N=2}^{W^*} W_{N-1}^2 a_N}{\sum_{N=2}^{W^*} W_{N-1}^2}. \quad (2\text{-}55)$$

Under the null hypothesis of a unit root, $H_0^{(N)} : \rho^{(N)} = 1$, (2-55) becomes

$$\hat{\rho}^{(N)} - 1 = \frac{\sum_{N=2}^{W^*} W_{N-1}^2 a_N}{\sum_{N=2}^{W^*} W_{N-1}^2}. \quad (2\text{-}56)$$

The corresponding aggregate-ADF (AADF) test statistic is

$$AADF^* = \frac{(\hat{\rho}^{(N)} - 1)}{\sqrt{\text{var}(\hat{\rho}^{(N)} - 1)}}, \quad (2\text{-}57)$$

where $\text{var}(\hat{\rho}^{(N)} - 1)$ represents the estimated variance of $(\rho^{(N)} - 1)$. Teles and Wei (1998) have shown that

$$AADF^* \to \left(\sqrt{\frac{3r^2}{2r^2+1}}\right)\left(\frac{\frac{1}{2}\langle [P(1)^2 - 1\rangle + \frac{r^2-1}{6r^2}}{\sqrt{\int_0^1 \langle P(X)\rangle^2 dX}}\right), \quad (2\text{-}58)$$

where $P(.)$ is the standard Weiner process. We gather from (2-58) that the limiting distributions of the test statistic based on the aggregate time series depend on the order of aggregation, r.

Now, for the usual series $\{X_t\}_{t=1}^{T}$, the ADF test for a unit root is based on the test statistic, $AADF$, with a limiting distribution given by

$$AADF^{*} = \frac{(\hat{\rho}-1)}{\sqrt{\operatorname{var}(\hat{\rho}-1)}} \longrightarrow \frac{\frac{1}{2}\left\{[P(1)]^{2}-1\right\}}{\sqrt{\int_{0}^{1}[P(X)]^{2}\,dX}}. \qquad (2\text{-}59)$$

A comparison of (2-58) and (2-59) reveals that the distribution in an aggregate time series is shifted to the right. Therefore, the invalidity of the usual percentile values tabulated by Fuller (1976) may lead to wrong conclusion about a series.

2.5 Numerical Examples

In macroeconomics, a leading indicator is referred to a variable that consistently changes a few months in advance of changes in macroeconomic activity and thus can be used to forecast business cycle. Experience has it that depending on a single indicator is unwise, but combining several of them into a composite index yields much better forecasts for of business cycle. An example of such composite indices is the Index of Leading Indicators. In this section, we illustrate the unit root concept using the seasonally adjusted of Leading Indicators Indices (LII) for South Africa from September 1993 to June 2000.

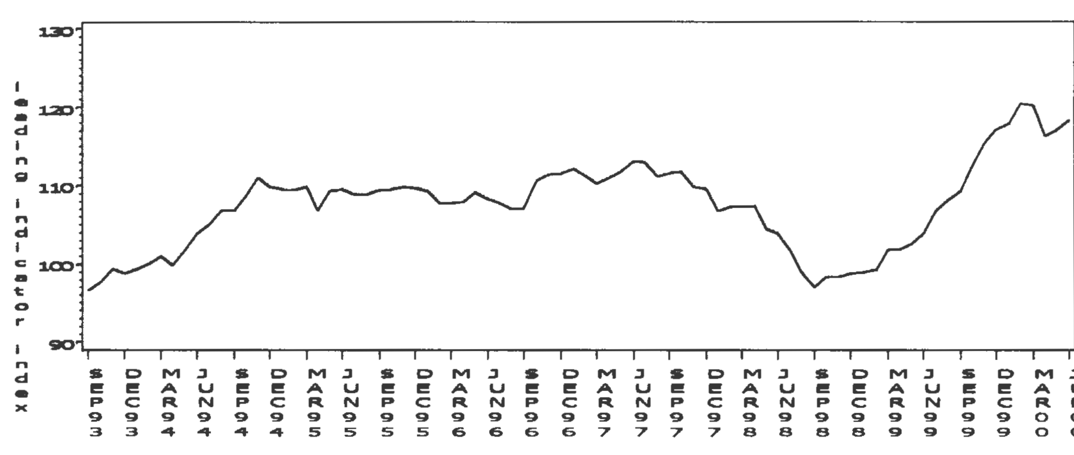


Exhibit 2.1: Leading Indicator Index (LII)

```
                      Name of variable         =       X.
                      Mean of working series = 107.5232
                      Standard deviation       = 5.602815
                      Number of observations =       82

                                    Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        Std
  1  28.860920    0.91939  |                .   |******************  |    0.110432
  2  26.074487    0.83062  |             .      |*****************   |    0.181139
  3  23.296967    0.74214  |           .        |***************     |    0.222798
  4  19.406750    0.61822  |          .         |************        |    0.251143
  5  14.969685    0.47687  |         .          |**********.         |    0.269062
  6  10.846014    0.34551  |         .          |*******   .         |    0.279179
  7   6.687859    0.21305  |         .          |****      .         |    0.284346
  8   2.289372    0.07293  |         .          |*         .         |    0.286286
  9  -1.675982   -0.05339  |         .         *|          .         |    0.286512
 10  -4.973233   -0.15843  |         .       ***|          .         |    0.286633
 11  -7.755944   -0.24707  |        .      *****|           .        |    0.287699
 12 -10.091102   -0.32146  |        .     ******|           .        |    0.290275
 13 -12.181498   -0.38805  |        .   ********|           .        |    0.294585
 14 -13.442850   -0.42823  |        .  *********|           .        |    0.300754
 15 -13.843601   -0.44100  |        .  *********|           .        |    0.308100
 16 -13.924934   -0.44359  |       .   *********|            .       |    0.315704
 17 -13.463428   -0.42889  |       .   *********|            .       |    0.323216
 18 -12.804108   -0.40788  |       .    ********|            .       |    0.330083
 19 -11.641017   -0.37083  |       .     *******|            .       |    0.336173
 20 -10.568211   -0.33666  |      .      *******|             .      |    0.341126
                                   "." marks two standard errors
```

Exhibit 2.2: Autocorrelation Functions for LII

```
                               Partial Autocorrelations
           Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.91939  |                .   |******************  |
             2    -0.09467  |                . **|   .                |
             3    -0.04420  |                .  *|   .                |
             4    -0.28270  |              ******|   .                |
             5    -0.18429  |                ****|   .                |
             6    -0.03673  |                .  *|   .                |
             7    -0.07047  |                .  *|   .                |
             8    -0.12123  |                . **|   .                |
             9    -0.04912  |                .  *|   .                |
            10     0.00107  |                .   |   .                |
            11     0.01472  |                .   |   .                |
            12    -0.03109  |                .  *|   .                |
            13    -0.12626  |                .***|   .                |
            14     0.01236  |                .   |   .                |
            15     0.05991  |                .   |*  .                |
            16    -0.01682  |                .   |   .                |
            17    -0.00594  |                .   |   .                |
            18    -0.11231  |                . **|   .                |
            19     0.03467  |                .   |*  .                |
            20    -0.07731  |                . **|   .                |
```

Exhibit 2.3: Partial Autocorrelation Functions for LII

Table 2.1: Unit Root Tests on LII

Augmented Dickey-Fuller Unit Root Tests

Type	Lags	RHO	Prob<RHO	T	Prob<T	F	Prob<F
Single Mean	0	-2.9180	0.6606	-1.2073	0.6683	2.0455	0.5550

Phillips-Perron Unit Root Tests

Type	Lags	RHO	Prob<RHO	T	Prob<T
Single Mean	0	-2.9180	0.6606	-1.2073	0.6683
	1	-3.6090	0.5761	-1.3428	0.6065
	2	-4.0768	0.5219	-1.4272	0.5654
	3	-4.6434	0.4607	-1.5232	0.5173

Table 2.2: Spectral Analysis of LII

OBS	FREQ	PERIOD	P_01	P_02	S_01	S_02
1	0.00000	.	1877577.48	11.5200	10.9001	1.37699
2	0.07757	81.0000	136.89	17.3053	15.1695	1.30327
3	0.15514	40.5000	430.33	12.2253	39.3248	1.12706
4	0.23271	27.0000	1073.60	17.7548	64.9900	1.15415
5	0.31028	20.2500	311.87	5.4972	33.5313	0.57290
6	0.38785	16.2000	149.68	2.5556	13.3153	0.22796
7	0.46542	13.5000	83.44	1.2954	6.7954	0.11598
8	0.54299	11.5714	27.75	0.9153	3.0093	0.15600
9	0.62056	10.1250	27.10	6.2499	1.7972	0.43566

P_01 = Periodogram Ordinates of the Undifferenced Series, X_t

P_02 = Periodogram Ordinates of the First-Differenced Series, $Y_t = X_t - X_{t-1}$

S_01 = Spectra of the Undifferenced Series

S_02 = Spectra of the First-Differenced Series

The base year is 1990=100. The series is given in Appendix A. We illustrate the ADF, the PP, and the FD unit root test procedures. The graphical representation of the series is given in Exhibit 2.1. Exhibit 2.2 and Exhibit 2.3 are, respectively, the autocovariance functions and partial autocovariance functions. Using a truncation lag of 3 (i.e. the required number of lags), the ADF^* and the PP^* test statistics are -1.2073 and -1.5232, respectively.

The corresponding probability values (prob-values), 0.6683 and 0.5173, are both greater than the nominal level of 0.05, i.e. 5%. Hence, the null hypothesis of unit root cannot be rejected. Therefore, the series is non-stationary. In the case of the FD test procedure, the required number of periodogram ordinates is the first

$$[\sqrt{T}] = [\sqrt{82}] = 9 \text{ periodogram ordinates.}$$

The first 9 periodogram ordinates obtained from the periodogram analysis of the series gives the following results in Table 3.2. From these results, we have

$$I_X(w_1) = 1877577.48$$

and

$$\sigma^2\phi^2 = \sum_{j=1}^{[\sqrt{T}]} \frac{I_Y(w_j)}{2[\sqrt{T}]} = \sum_{j=1}^{9} \frac{I_Y(w_j)}{2(9)} = \frac{1}{18}(11.5200 + 17.3053 + \ldots + 6.2499) = 4.1844$$

Therefore, the test statistic becomes

$$FD^* = \left(\frac{4\pi^2}{\sigma^2\phi^2.T^2} \right) I_X(w_1) = \frac{4\pi^2}{4.1844(82)^2}(1877577.48) = 2634.5\,.$$

At the nominal level of 0.05 (5%), the critical value is 0.178. Since the test statistic is greater than the nominal value 0.178, we cannot reject the null hypothesis of unit root. Therefore, the series is non-stationary.

2.6 Summary

In this chapter, we have discussed some unit root test procedures frequently employed in practice. It cannot be overemphasised that the importance of testing for the presence of a unit root in a given time series is quite profound. As we will see from Chapter 4, the unit root tests form the basis of cointegration analysis of time series. We have also been able to use a real-life data, the Leading Indicators Indices (LII) for South Africa, to illustrate the concept. The data was found to contain a unit root and hence is non-stationary.

CHAPTER 3

SEASONAL UNIT ROOTS TESTS

3.1 Introduction

Most time series data come in a seasonally unadjusted form and it has been advised that where necessary such series should be preferred to their seasonally adjusted ones. The reason being that filters usually employed in adjusting for seasonal patterns often distort the underlying properties of the series (Davidson and MacKinnon, 1993). Particularly, it is highly probable that the OLS estimator of ρ_4 in the ADF autoregression tends to be biased towards 1 when the series is seasonally adjusted, thus rejecting the null hypothesis of a unit root less often than it should (Harris, 1995).

Some variables exhibit strong seasonal patterns accounting for a greater percentage of the total variation in the series. Such patterns are usually a result from stationary seasonal processes, and are usually modelled using seasonal dummies that allow some variation but no persistent change in the seasonal pattern over time. Seasonal processes may also be non-stationary if there is a varying seasonal pattern over time. In this case, the series is seasonally differenced to induce stationarity. It is therefore important to consider testing for a seasonal unit root when a series exhibit a strong seasonal pattern. Several seasonal unit root test procedures have been proposed in the literature. Notable among them are due to Hylleberg et al (1990) and Dickey, Hasza and Fuller (1984). Saikkonen and Luukkonen (1993) and Tam and Reinsel (1997) have also considered seasonal moving average unit tests in Autoregressive Integrated Moving Average (ARIMA) processes. Monte Carlo studies have been conducted by some authors for instance Rodigues and Osborn (1999) to investigate the performance of seasonal unit root tests.

The remainder of the chapter is organised as follows. Since seasonal cointegration depends heavily on seasonal unit roots, we consider discussing some two seasonal unit roots tests in Section 3.2. Section 3.3 considers some seasonal unit root tests for monthly data. A general seasonal unit root test is discussed in Section 3.4. An illustrative example is considered in Section 3.5. Section 3.6 summarises the chapter.

3.2 Seasonal Unit Root Tests for Quarterly Data

In this section, we present two appealing test procedures, hitherto referred to as Test Procedure 1 and Test Procedure 2, for a seasonal unit root in a given quarterly time series. In sub-section 3.2.1, the series $\{X_t\}_{t=1}^{T}$ assumes the following representation

$$(1-\rho_4 B^4)X_t = a_t,$$

where a_t is a white noise process with mean 0 and variance σ^2.

3.2.1 Test Procedure 1

Let the univariate time series $\{X_t\}_{t=1}^{T}$ satisfy the model

$$(1-\rho_4 B^4)X_t = a_t, \qquad (3\text{-}1)$$

where a_t is a white noise process with mean 0 and variance σ^2. Under the null hypothesis of interest

$$H_0 : \rho_4 = 1 \qquad \text{against} \qquad H_1 : |\rho_4| < 1, \qquad (3\text{-}2)$$

the roots of (3-1) are the solutions of the characteristic equation (with $B = z$)

$$1 - z^4 = 0. \qquad (3\text{-}3)$$

To obtain the solution, we note that

$$1 - z^4 = 1^2 - (z^2)^2 = (1-z^2)(1+z^2) = (1-z^2)[1-(iz)^2], \quad \text{where } i = \sqrt{-1}. \qquad (3\text{-}4)$$

Thus,

$$1 - z^4 = (1-z)(1+z)(1-iz)(1+iz) = 0,$$

$$\Rightarrow \qquad z = \pm 1 \text{ and } \pm i. \qquad (3\text{-}5)$$

In the complex plane $z = \pm 1$ is a non-seasonal root whilst $z = -1, i, -i$ are seasonal roots. The roots $z = -1, i, -i$ correspond to long-run frequency 0, the semi-annual frequency π, and the annual frequency $\frac{\pi}{2}$, respectively. The test procedure to be discussed here, which is due to Dickey et al (1984), is just an extension of the procedure described in Section 2.3. From (3-1), we have

$$X_t = \rho_4 X_{t-4} + a_t. \tag{3-6}$$

The maximum likelihood estimate of ρ_4 in (3-6) which is the same as the OLS estimate is obtained in a fashion similar to the case of the non-seasonal AR(1) process in Chapter 2. The OLS estimator of ρ_4 is

$$\hat{\rho}_4 = \frac{\sum_{t=1}^{T} X_t X_{t-4}}{\sum_{t=1}^{T} X_{t-4}^2} = \frac{\sum_{t=1}^{T} (\rho_4 X_{t-4} + a_t) X_{t-4}}{\sum_{t=1}^{T} X_{t-4}^2} = \rho_4 + \frac{\sum_{t=1}^{T} X_{t-4} a_t}{\sum_{t=1}^{T} X_{t-4}^2}$$

$$\Rightarrow \qquad \hat{\rho}_4 - \rho_4 = \frac{\sum_{t=1}^{T} X_{t-4} a_t}{\sum_{t=1}^{T} X_{t-4}^2}. \tag{3-7}$$

Now, subtracting X_{t-4} from both sides of (3-6) yields

$$X_t - X_{t-4} = (\rho_4 - 1) X_{t-4} + a_t,$$

or

$$Y_t^* = (\rho_4 - 1) X_{t-4}, \tag{3-8}$$

where $Y_t^* = X_t - X_{t-4}$.

Under the null hypothesis $H_0 : \rho_4 = 1$, the Studentized Regression Statistic (SRS) associated with $(\rho_4 - 1)$ is

$$SRS^* = \frac{\hat{\rho}_4 - 1}{\sqrt{\operatorname{var}(\hat{\rho}_4 - 1)}}, \quad (3\text{-}9)$$

where

$$\sqrt{\operatorname{var}(\hat{\rho}_4 - 1)} = \sqrt{\frac{\sum_{t=1}^{T}(X_t - \hat{\rho}_4 X_{t-4})^2}{(T-1).\sum_{t=1}^{T} X_{t-4}^2}}, \quad (3\text{-}10)$$

is the standard error of $(\hat{\rho}_4 - 1)$. The statistics, $(\hat{\rho}_4 - 1)$ and SRS^* are standard output from the OLS regression of Y_t^* on X_{t-4}. If an intercept is included in the model, (3-8) becomes

$$Y_t^* = C + (\rho_4 - 1)X_{t-4}, \quad (3\text{-}11)$$

which suggests a regression of Y_t^* on a constant and X_{t-4}. Similarly, the inclusion of a time trend in (3-8) gives

$$X_t - (\alpha_0 + \alpha_1 t) = \rho_4\{X_{t-4} - [\alpha_0 + \alpha_1(t-4)]\} + a_t, \quad (3\text{-}12)$$

which simplifies to give

$$X_t = \beta_0 + \beta_1 t + \rho_4 X_{t-4} + a_t, \quad (3\text{-}13)$$

where $\beta_0 = (1-\rho_4)\alpha_0 + 4\alpha_1$ and $\beta_1 = (1-\rho_4)\alpha_1$. Subtracting X_{t-4} from both sides of (3-13) yields

$$X_t - X_{t-4} = \beta_0 + \beta_1 t + (\rho_4 - 1)X_{t-4} + a_t$$

or
$$Y_t^* = \beta_0 + \beta_1 t + (\rho_4 - 1)X_{t-4} + a_t, \quad (3\text{-}14)$$

where $Y_t^* = X_t - X_{t-4}$. Equation (3-14) suggests a regression of Y_t^* on a constant, *t*, and X_{t-4}. Percentiles of the finite-sample and limiting distribution of $(\hat{\rho}_4 - 1)$ for testing

$$H_0 : \rho_4 = 1$$

using the Monte Carlo method have been tabulated by Dickey et al (1984) and are given in the Appendix. The null hypothesis of a seasonal unit root is rejected if the test statistic is less than its corresponding critical value or when the probability value is small (significant) compared with a chosen nominal value.

3.2.2 Test Procedure 2

Given the univariate time series $\{X_t\}_{t=1}^T$, the seasonal unit root test procedure due to Hylleberg et al (1990) satisfies the process

$$(1 - \rho_4 B^4)X_t = a_t, \quad (3\text{-}15)$$

where a_t is a white noise process with mean 0 and variance σ^2. Let

$$Y_t^* = X_t - X_{t-4} = (1 - B^4)X_t. \quad (3\text{-}16)$$

Then, under the null hypothesis that $\rho_4 = 1$, the seasonal unit root autoregression we consider is

$$Y_t^* = \sum_{q=1}^{4} \beta_q X_{q,t} + \sum_{i=1}^{p} \alpha_i Y_{t-i}^* + a_t, \quad (3\text{-}17)$$

where

$$X_{1,t} = (1 + B + B^2 + B^3)X_t, \qquad (3\text{-}18a)$$

$$X_{2,t} = -(1 - B + B^2 - B^3)X_t, \qquad (3\text{-}18b)$$

$$X_{3,t} = -(1 - B^2)X_t, \qquad (3\text{-}18c)$$

$$X_{4,t} = -(1 - B^2)B^2 X_t. \qquad (3\text{-}18d)$$

Equation (3-17) suggests regressing Y_t^* on $X_{1,t}, X_{2,t}, X_{3,t}, X_{4,t},, Y_{t-1}^*, Y_{t-2}^*, ..., Y_{t-p}^*$. For the presence of a unit root, $\beta_1 = 0$. If there is a non-seasonal unit root at $z = +1$, $\beta_1 = 0$. If there is a seasonal unit root at $z = -1$, $\beta_2 = 0$. For complex seasonal roots at $z = \pm i$, $\beta_3 = \beta_4 = 0$. Testing the hypothesis for non-seasonal and seasonal roots employs a one-sided *t*-test. The alternatives are $\beta_1 < 0$, $\beta_2 < 0$. Testing for complex seasonal roots uses the joint F-test of $\beta_3 = \beta_4 = 0$ against the alternative, $\beta_3 \neq 0$ and/or $\beta_4 \neq 0$.

If, however, one wishes to include an intercept, (3-17) becomes

$$Y_t^* = C + \sum_{q=1}^{4} \beta_q X_{q,t} + \sum_{i=1}^{p} \alpha_i Y_{t-i}^* + a_t, \qquad (3\text{-}19)$$

suggesting a regression of Y_t^* on a constant, $X_{1,t}, X_{2,t}, X_{3,t}, X_{4,t},, Y_{t-1}^*, Y_{t-2}^*, ..., Y_{t-p}^*$. In a similar fashion, if a linear time trend is included, (3-19) becomes

$$Y_t^* = (\alpha_0 + \alpha_1 t) + + \sum_{q=1}^{4} \beta_q X_{q,t} + \sum_{i=1}^{p} \alpha_i Y_{t-i}^* + a_t, \qquad (3\text{-}20)$$

a regression of Y_t^* on a constant, *t*, $X_{1,t}, X_{2,t}, X_{3,t}, X_{4,t},, Y_{t-1}^*, Y_{t-2}^*, ..., Y_{t-p}^*$. As in the non-seasonal case, the distribution of *t* and F-statistics are non-standard. The relevant tabulated critical values for the test provided by Hylleburg et al (1990) are given in the Appendix. The null hypothesis of a seasonal unit root is rejected if the test statistic is less than its corresponding critical value or when the probability value is small (significant) compared with a chosen nominal value.

3.3 Seasonal Unit Root Tests for Monthly Data

This section summarizes some three seasonal unit root test procedures in the context of monthly series. These tests are based on the need that the annual differencing filter, $(1-B^{12})$, is required to induce stationarity in the series at hand. In some instances, stationarity is attained by combining this annual differencing with the first-difference as $(1-B)(1-B^{12})$.

3.3.1 The Dickey-Hasza-Fuller (DHF) Seasonal Unit Root Test

The DHF seasonal unit root test for monthly data due to Dickey, Hasza, and Fuller (1984) assumes any of the following autoregressions:

$$Y_t = (\rho - 1)X_{t-12} + \sum_{i=1}^{p-1} \gamma_i Y_{t-i} + a_t \ , \tag{3.-21}$$

$$Y_t = C + (\rho - 1)X_{t-12} + \sum_{i=1}^{p-1} \gamma_i Y_{t-i} + a_t \ , \tag{3-22}$$

$$Y_t = \beta_0 + \beta_1 t + (\rho - 1)X_{t-12} + \sum_{i=i}^{p-1} \gamma_i Y_{t-i} + a_t \ , \tag{3-23}$$

where a_t is a white noise process with mean 0 and variance σ^2, and $Y_t = (1-B^{12})X_t$. The test is implemented by comparing the computed OLS test statistic

$$DHF^* = \frac{\hat{\rho} - 1}{\sqrt{\operatorname{var}(\hat{\rho} - 1)}} \tag{3-24}$$

against the critical values tabulated by DHF. The hypotheses we consider are

$$H_0 : \rho = 1 \qquad \text{against} \qquad H_1 : |\rho| < 1 . \tag{3-25}$$

The null hypothesis of a seasonal unit root is rejected if the test statistic is less than its corresponding critical value or when the probability value is small (significant) compared with a chosen nominal value.

3.3.2 The Osborn-Chui-Smith-Birchenhall (OCSB) Seasonal Unit Root Test

The OCSB seasonal unit root test for monthly data due to Osborn, Chui, Smith, and Birchenhall (1988) assumes any of the following autoregressions:

$$Y_t = (\rho_1 - 1)S(B)Z_{t-1} + \alpha Z_{t-12} + \sum_{i=1}^{p}\gamma_i Z_{t-i} + a_t\ , \tag{3-26}$$

$$Y_t = C + (\rho_1 - 1)S(B)Z_{t-1} + \alpha Z_{t-12} + \sum_{i=1}^{p}\gamma_i Z_{t-i} + a_t\ , \tag{3-27}$$

$$Y_t = \beta_0 + \beta_1 t + (\rho_1 - 1)S(B)Z_{t-1} + \alpha Z_{t-12} + \sum_{i=1}^{p}\gamma_i Z_{t-i} + a_t\ , \tag{3-28}$$

where $Y_t = (1-B^{12})Z_t$ and $Z_t = (1-B)X_t$. a_t is a white noise process with mean 0 and variance σ^2, and $S(B) = \sum_{i=1}^{11} B^i$. In the OCSB test, the filter $(1-B)$ in $Z_t = (1-B)X_t$ is assumed to be valid and that our concern is the validity of the filter $(1-B^{12})$ in $(1-B^{12})Z_t$.

Under the hypotheses

$$H_0 : \rho = 1 \qquad \text{vs} \qquad H_0 : |\rho| < 1\ , \tag{3-29}$$

the test is implemented by comparing the computed OLS test statistic

$$OCSB^* = \frac{\hat{\rho} - 1}{\sqrt{\operatorname{var}(\hat{\rho} - 1)}}\ , \tag{3-30}$$

against the critical values tabulated by OCSB. If the filter $(1-B)$ is found to be irrelevant, then the test is applied to the original data by setting Z_t to X_t. In this case, the autoregressions we consider are

$$Y_t = (\rho_1 - 1)S(B)X_{t-1} + \alpha X_{t-12} + \sum_{i=1}^{p} \gamma_i X_{t-i} + a_t \,, \tag{3-31}$$

$$Y_t = C + (\rho_1 - 1)S(B)X_{t-1} + \alpha X_{t-12} + \sum_{i=1}^{p} \gamma_i X_{t-i} + a_t \,, \tag{3-32}$$

$$Y_t = \beta_0 + \beta_1 t + (\rho_1 - 1)S(B)X_{t-1} + \alpha Y_{t-12} + \sum_{i=1}^{p} \gamma_i X_{t-i} + a_t \,, \tag{3-33}$$

where a_t is a white noise process with mean 0 and variance σ^2, and $Y_t = (1 - B^{12})X_t$. The null hypothesis of a seasonal unit root is rejected if the test statistic is less than its corresponding critical value or when the probability value is small (significant) compared with a chosen nominal value.

3.3.3 The Extended DF (EDF) and Extended ADF (EADF) Seasonal Unit Root Tests

It cannot be overemphasized that the Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) non-seasonal unit roots serve as platforms for most unit root tests in the literature, by far. In their paper, Ghysel et al (1994) were able to apply the DF and ADF tests to seasonal data with roots at seasonal frequencies, provided that the order of augmentation p is correctly specified. For any of the seasonal AR(1) processes

$$X_t = \rho X_{t-12} + a_t \,, \tag{3-34}$$

$$X_t = C + \rho X_{t-12} + a_t \,, \tag{3-35}$$

$$X_t = \beta_0 + \beta_1 t + (\rho - 1)X_{t-12} + a_t \,, \tag{3-36}$$

where a_t is a white noise process with mean 0 and variance σ^2. The EADF test is based, respectively, on the following autoregressions:

$$Y_t = (\rho - 1)X_{t-1} + \sum_{i=1}^{11} \gamma_i Z_{t-i} + a_t \,, \tag{3-37}$$

$$Y_t = C + (\rho - 1)X_{t-1} + \sum_{i=1}^{11} \gamma_i Z_{t-i} + a_t \,, \tag{3-38}$$

$$Y_t = \beta_0 + \beta_1 t + (\rho - 1)X_{t-1} + \sum_{i=1}^{11} \gamma_i Z_{t-i} + a_t \,, \tag{3-39}$$

where $Y_t = (1 - B)X_t$. Under the hypotheses

$$H_0 : \rho = 1 \tag{3-40a}$$

vs

$$H_0 : |\rho| < 1 \,, \tag{3-40b}$$

the test is implemented by comparing the computed OLS test statistic

$$EADF^* = \frac{\hat{\rho} - 1}{\sqrt{\text{var}(\hat{\rho} - 1)}} \,, \tag{3-41}$$

against the usual ADF critical values tabulated by Dickey and Fuller (1979).

For a higher order of augmentation, p, which allows for serial correlation, the modified EADF autoregressions we consider based on the same hypotheses and test statistic are

$$Y_t = (\rho - 1)X_{t-1} + \sum_{i=1}^{p+11} \gamma_i Z_{t-i} + a_t \,, \tag{3-42}$$

$$Y_t = C + (\rho - 1)X_{t-1} + \sum_{i=1}^{p+11} \gamma_i Z_{t-i} + a_t \,, \tag{3-43}$$

$$Y_t = \beta_0 + \beta_1 t + (\rho - 1)X_{t-1} + \sum_{i=1}^{p+11} \gamma_i Z_{t-i} + a_t \,, \tag{3-44}$$

where $Y_t = (1 - B)X_t$. The critical values employed here are the usual ADF critical values. The null hypothesis of a seasonal unit root is rejected if the test statistic is less than its corresponding critical value or when the probability value is small (significant) compared with a chosen nominal value.

3.4 A General Seasonal Unit Root Test

Hasza et al (1979) outline a seasonal unit root test procedure in a given time series $\{X_t\}_{t=1}^{T}$ satisfying the representation

$$(1-B)(1-\rho_d B^d)X_t = a_t, \tag{3-45}$$

where a_t is a white noise process with mean 0 and variance σ^2, and b = 2,4,6, or 12. If

- $d = 2$, the series is observed semi-annually,
- $d = 4$, the series is observed quarterly,
- $d = 6$, the series is observed bimonthly,
- $d = 12$, the series is observed monthly.

For a series with a seasonal unit root $\rho_d = 1$, and hence (3-45) becomes

$$(1-B)(1-B^d)X_t = a_t. \tag{3-46}$$

Testing of (3-46) is based on the autoregression

$$X_t = \alpha_1 X_{t-1} + \alpha_2 (X_{t-1} - X_{t-b-1}) + \alpha_3 (X_{t-b} - X_{t-b-1}) + \sum_{i=1}^{p} \gamma_i Y_{t-i} + a_t, \tag{3-47}$$

where

$$Y_t = (1-B)(1-B^d)X_t = (X_t - X_{t-d-1}) - (X_{t-1} - X_{t-d-1}). \tag{3-48}$$

Hasza (1979) indicated that testing of (3-46) is equivalent to testing the null hypothesis that

$$H_0 : (\alpha_1, \alpha_2, \alpha_3) = (1,0,1). \quad (3\text{-}49)$$

With the null hypothesis test given in (3-49), equation (3-47) becomes

$$X_t = X_{t-1} + (X_{t-d} - X_{t-d-1}) + \sum_{i=1}^{p} \gamma_i Y_{t-i} + a_t$$

or $$(X_t - X_{t-1}) + (X_{t-d} - X_{t-d-1}) = \sum_{i=1}^{p} \gamma_i Y_{t-i} + a_t . \quad (3\text{-}50)$$

Substituting (3-48) into (3-50) yields

$$Y_t = \sum_{i=1}^{p} \gamma_i Y_{t-i} + a_t$$

or $$\left(1 - \sum_{i=1}^{p} \gamma_i B^i\right) Y_t = a_t . \quad (3\text{-}51)$$

The roots $z_i \, (i = 1,2,3,\ldots,p)$ of the characteristic equation associated with (3-51)

$$\left(1 - \sum_{i=1}^{p} \gamma_i z^i\right) = 1 - \gamma_1 z - \gamma_2 z^2 - \ldots - \gamma_p z^p = 0 , \quad (3\text{-}52)$$

are such that $| z_i | < 1$. The parameters γ_i and the order p are left unspecified and hence have to be estimated and determined respectively. Equation (3-49) suggests regressing X_t on $X_{t-1}, (X_{t-1} - X_{t-d-1}), (X_{t-d} - X_{t-d-1}), Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}$. If a constant is included in the model, (3-49) becomes

$$X_t = C + \alpha_1 X_{t-1} + \alpha_2 (X_{t-1} - X_{t-d-1}) + \alpha_3 (X_{t-d} - X_{t-d-1}) + \sum_{i=1}^{p} \gamma_i Y_{t-i} + a_t , \quad (3\text{-}53)$$

which suggests a regression of X_t on a constant, $X_{t-1}, (X_{t-1} - X_{t-d-1}), (X_{t-d} - X_{t-d-1}), Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}$.

Percentiles of the test statistics for the finite-sample sizes using (3-46) and (3-51) are given in the Appendix. The null hypothesis of a seasonal unit root is rejected if the test statistic is less than its corresponding critical value or when the probability value is small (significant) compared with a chosen nominal value.

3.5 A Numerical Example

In this section, we apply a seasonal unit root test to 132 monthly observations on U.S. liquor sales (January 1970 – December 1980), given and analysed by Wei (1989). The series is contained in the Appendix. A graphical representation of the series is shown in Exhibit 3.1, which gives a clear evidence of seasonality as well as the series trending upward. The sample autocorrelations of the undifferenced series (Exhibit 3.2) die out only slowly at high lags, suggesting non-stationarity.

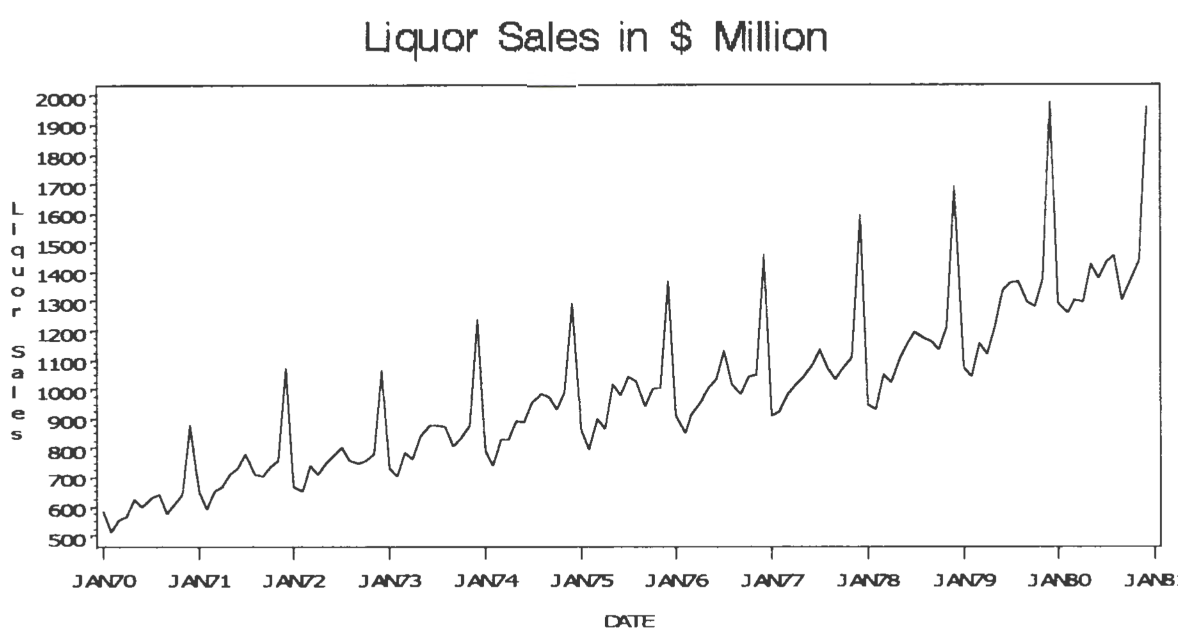
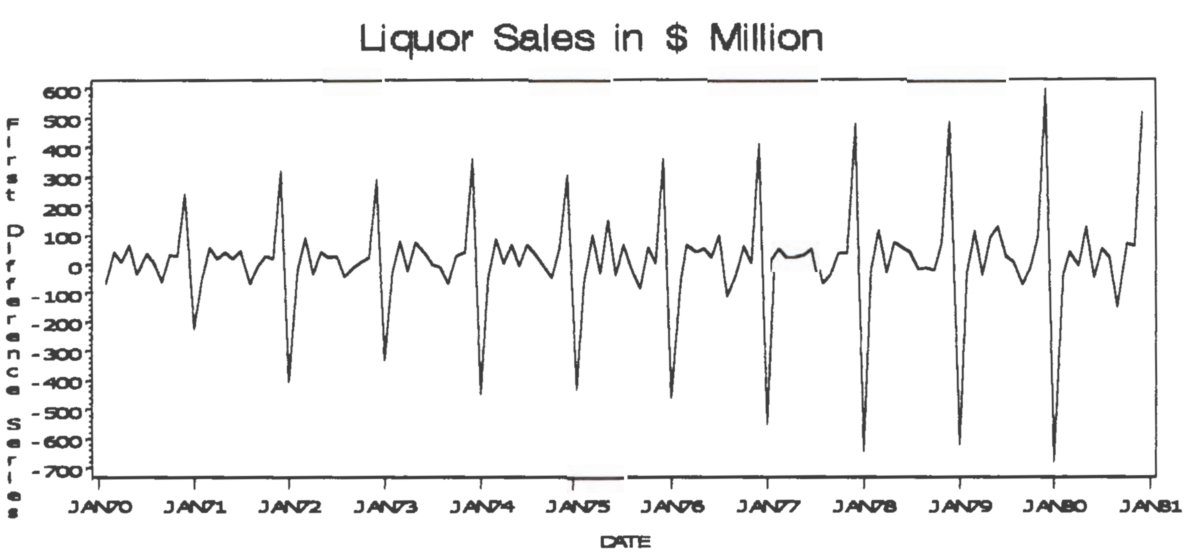


Exhibit 3.1: U.S. Liquor Sales (January 1970 – December 1980)

```
                         Name of variable          =        X.
                         Mean of working series = 985.6364
                         Standard deviation       = 276.2366
                         Number of observations =       132

                                   Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      Std
  1  54376.239   0.71260  |                 .  |**************      |  0.087039
  2  48900.632   0.64084  |               .    |*************       |  0.123571
  3  48463.044   0.63511  |              .     |*************       |  0.146602
  4  48479.041   0.63532  |             .      |*************       |  0.166144
  5  49805.829   0.65271  |             .      |*************       |  0.183628
  6  46996.379   0.61589  |            .       |************        |  0.200435
  7  46494.857   0.60932  |           .        |************        |  0.214293
  8  42184.587   0.55283  |           .        |***********         |  0.227039
  9  39591.727   0.51885  |           .        |**********          |  0.237018
 10  36816.225   0.48248  |          .         |**********          |  0.245471
 11  39630.332   0.51936  |          .         |**********          |  0.252553
 12  56000.273   0.73388  |          .         |***************     |  0.260519
 13  35880.984   0.47022  |         .          |********* .         |  0.275736
 14  30961.491   0.40575  |         .          |********  .         |  0.281746
 15  30393.347   0.39831  |         .          |********  .         |  0.286138
 16  29886.828   0.39167  |        .           |********   .        |  0.290308
 17  31228.059   0.40924  |        .           |********   .        |  0.294284
 18  28511.026   0.37364  |        .           |*******    .        |  0.298564
 19  27885.334   0.36544  |        .           |*******    .        |  0.302086
 20  24522.981   0.32137  |        .           |******     .        |  0.305417
 21  22406.121   0.29363  |        .           |******     .        |  0.307968
 22  19963.610   0.26162  |        .           |*****      .        |  0.310082
 23  23061.903   0.30223  |        .           |******     .        |  0.311749
 24  38317.602   0.50215  |       .            |**********  .       |  0.313961
                              "." marks two standard errors
```

Exhibit 3.2: Sample Autocorrelations (U.S. Liquor Sales - Undifferenced Series)



```
                      Name of variable          =        X
                      Period(s) of Differencing =        1
                      Mean of working series    = 10.51908
                      Standard deviation        =   188.71
                      Number of observations    =      131

                                   Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        Std
  1 -13007.563   -0.36526  |             *******|  .                  |  0.087370
  2  -4557.015   -0.12796  |               .***|   .                  |  0.098339
  3  40.037300    0.00112  |               .   |   .                  |  0.099602
  4  -2283.373   -0.06412  |               .  *|   .                  |  0.099602
  5   4171.430    0.11714  |               .   |** .                  |  0.099916
  6  -1900.490   -0.05337  |               .  *|   .                  |  0.100959
  7   3441.606    0.09664  |               .   |** .                  |  0.101174
  8  -1036.977   -0.02912  |               .  *|   .                  |  0.101877
  9    196.618    0.00552  |               .   |   .                  |  0.101940
 10  -5284.556   -0.14839  |               .***|   .                  |  0.101942
 11 -13238.700   -0.37175  |             *******|   .                 |  0.103578
 12  30907.236    0.86790  |              .    |*****************     |  0.113307
 13 -11116.770   -0.31217  |              ******|     .               |  0.156008
 14  -3590.937   -0.10084  |              .   **|     .               |  0.160705
 15   -173.452   -0.00487  |              .     |     .               |  0.161188
 16  -2318.809   -0.06511  |              .    *|     .               |  0.161189
 17   4142.591    0.11633  |              .     |**   .               |  0.161389
 18  -1825.457   -0.05126  |              .    *|     .               |  0.162028
 19   3421.959    0.09609  |              .     |**   .               |  0.162152
 20   -671.755   -0.01886  |             .      |      .              |  0.162586
                               "." marks two standard errors
```

Exhibit 3.4: Sample Autocorrelations (U.S. Liquor Sales - First-Differenced Series)

```
Name of variable          =        X         Standard deviation     = 51.77828
Period(s) of Differencing =     1,12         Number of observations =      119
Mean of working series    = -0.72269

                                   Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        Std
  1  -1187.085   -0.44278  |           *********|    .                |  0.091670
  2 -96.048464   -0.03583  |               .  *|    .                 |  0.108159
  3    284.633    0.10617  |               .   |** .                  |  0.108259
  4   -355.889   -0.13275  |               .***|    .                 |  0.109130
  5    270.787    0.10100  |               .   |** .                  |  0.110479
  6    306.323    0.11426  |               .   |** .                  |  0.111252
  7   -612.734   -0.22855  |              *****|    .                 |  0.112234
  8    180.383    0.06728  |               .   |*    .                |  0.116079
  9    313.789    0.11704  |               .   |**   .                |  0.116406
 10   -284.358   -0.10606  |               .  **|    .                |  0.117391
 11    451.188    0.16829  |               .   |***  .                |  0.118193
 12   -586.901   -0.21891  |              .****|     .                |  0.120190
 13  75.077822    0.02800  |               .   |*    .                |  0.123495
 14 -18.803059   -0.00701  |               .   |     .                |  0.123549
 15    170.450    0.06358  |               .   |*    .                |  0.123552
 16   -219.003   -0.08169  |               .  **|    .                |  0.123827
 17    137.000    0.05110  |               .   |*    .                |  0.124279
```

```
18 -33.382298   -0.01245 |                   .  |  .                   |  0.124455
19   -113.081   -0.04218 |                   . *|  .                   |  0.124465
20    245.109    0.09142 |                   .  |** .                  |  0.124586
                                   "." marks two standard errors
```

Exhibit 3.5: Sample Autocorrelations (U.S. Liquor Sales - Doubly-Differenced Series)

```
                         Partial Autocorrelations
Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    0.71260  |                    .  |**************       |
  2    0.27030  |                    .  |*****                |
  3    0.23949  |                    .  |*****                |
  4    0.19092  |                    .  |****                 |
  5    0.20221  |                    .  |****                 |
  6    0.05869  |                    .  |* .                  |
  7    0.08994  |                    .  |**.                  |
  8   -0.06367  |                    . *|  .                  |
  9   -0.05241  |                    . *|  .                  |
 10   -0.09295  |                    .**|  .                  |
 11    0.09988  |                    .  |**.                  |
 12    0.61016  |                    .  |************         |
 13   -0.61797  |         ************|  .                   |
 14   -0.13741  |                   ***|  .                   |
 15   -0.02014  |                    .  |  .                  |
 16   -0.06418  |                    . *|  .                  |
 17    0.03603  |                    .  |* .                  |
 18    0.04413  |                    .  |* .                  |
 19    0.01230  |                    .  |  .                  |
 20    0.08129  |                    .  |**.                  |
```

Exhibit 3.6: Sample Partial Autocorrelations (U.S. Liquor Sales - Undifferenced Series)

In order to detect seasonality, we take a look at the sample autocorrelation of the first-differenced series in Exhibit 3.4. For the first-differenced series, the very high sample autocorrelations at lags 12 and 24 indicate the desirability of seasonal differencing, a degree of 12. The sample autocorrelations of the series, $(1-B)^{12}X_t$ (Exhibit 3.5), appear to die out, but they do so rather slowly.

These considerations suggest examining the doubly-differenced series $(1-B)(1-B^{12})X_t$. A constant term is required in the unit root autoregression because the mean of the undifferenced series is greater than its standard deviation. The sample partial autocorrelations for the series in Exhibit 3.6 are all very small after the first 4, compared with the interval

$$\pm 2/\sqrt{N} = \pm 2/\sqrt{131} = \pm 0.1747.$$

This suggests the order of augmentation, *p*, to be 5.

Therefore, the seasonal unit root autoregression we consider is the DHF autoregession

$$Z_t = C + (\rho - 1)X_{t-12} + \sum_{i=1}^{4} \gamma_i Z_{t-i} + a_t$$

where $Z_t = (1 - B^{12})X_t$. The DHF seasonal unit root test results using the U.S. liquor series are given in Table 3.1.

Table 3.1: DHF Seasonal Unit Root Test – U.S Liquor Sales

Seasonal Augmented Dickey-Fuller Unit Root Tests

Type	Lags	RHO	Prob<RHO	T	Prob<T
Zero Mean	4	5.4802	0.9014	1.8659	0.9744
Single Mean	4	4.2050	0.8792	1.4006	0.9530

The results in Table 3.1 clearly show that we cannot reject the null hypothesis of a seasonal unit root since the prob-value of 0.9530 is greater the nominal size of 0.05. We conclude that a seasonal differencing is required to induce stationarity in the U.S. liquor series.

3.6 Summary

In this chapter, we have considered some unit root tests applied to induce stationarity in seasonal time series. A unit root test applied to monthly U.S. liquor consumption series strongly supports the null hypothesis of the presence of a unit root in the underlying data-generating process.

CHAPTER 4

COINTEGRATION AND EFFICIENCY OF THE PHILLIPS-OULIARIS TEST

4.1 Introduction

Cointegration has become a widely used concept to analyse relations among non-stationary time series. The concept is based on the null hypothesis that all linear combinations of variables in levels (undifferenced series) are non-stationary. Since in practice most time series require one differencing to induce stationarity, the null hypothesis of non-cointegration is that all linear combinations of variables are integrated of order 1, i.e. $I(1)$. The alternative hypothesis of cointegration is that at least one of the linear combinations of variables is stationary, i.e. $I(0)$. Quite a number of cointegration test procedures have been cited in the literature. These tests can be grouped into two, namely, regression-based tests and vector-autoregression tests. The regression-based tests are the most commonly applied in practice, however.

Given a set of n macroeconomic time series, $\{X_{1,t}, X_{2,t}, \ldots, X_{n,t}\}_{t=1}^{T}$, one can assume the presence of non-seasonal or seasonal unit roots. Dickey and Fuller (1979, 1981) and Phillips and Perron (1988), among several others, have proposed formal tests to investigate non-seasonal units in univariate time series. These test procedures have been discussed in Chapter 2. Likewise, Hylleberg et al (1990), among others, have proposed formal tests to investigate seasonal unit roots in univariate time series. These test procedures are also discussed in Chapter 3. Thus, a usual next step in analysing a set of time series involves testing for cointegration in the context of non-seasonality and seasonality.

The outline of this chapter is as follows. In Section 4.2, we start with the Engle-Granger test for cointegration. Section 4.3 discusses the Engle-Yoo-Granger cointegration test, while Section 4.4 handles the Cointegrating Regression Durbin-Watson cointegration test procedure. In Section 4.5, the Phillips-Ouliaris cointegration test is discussed.

Seasonal Cointegration is discussed in Section 4.6. Section 4.7 discusses the problem of spurious regression. Section 4.8 assesses the efficiency of the Phillips-Ouliaris cointegration test via the methods of OLS and Yule-Walker. In Section 4.9, a numerical example is used to illustrate the concept of cointegration. Section 4.10 summarises the chapter.

4.2 The Engle-Granger (EG) Cointegration Test

The Engle-Granger (EG) cointegration test procedure is perhaps the most commonly used test for cointegration. For simplicity, let's consider the bivariate time series

$$\{X_{1,t}, X_{2,t}\}_{t=1}^{T}. \tag{4-1}$$

The EG test assumes that $X_{1,t} \sim I(1)$ and $X_{2,t} \sim I(1)$. Under the null hypothesis of non-cointegration, all their linear combinations are also $I(1)$. Under the alternative hypothesis of cointegration, at least one $I(0)$ linear combination exists. If the two time series are $X_{1,t} \sim I(1)$ and $X_{2,t} \sim I(1)$, then the static regression of $X_{1,t}$ on $X_{2,t}$ is

$$X_{1,t} = \rho X_{2,t} + a_t, \tag{4-2}$$

where

$$a_t \sim i.i.d. N(0,\sigma^2) \qquad \text{and so} \qquad a_t \sim I(0). \tag{4-3}$$

A practical interpretation of (4-2) and (4-3) is that if the two series are both $I(0)$, then their linear combination

$$a_t = X_{1,t} - \rho X_{2,t} \sim I(0). \tag{4-4}$$

This means that whereas the two series may have such components as trends, cyclical or seasonal variations, the movements in one are matched by the movements in the other and so the scaled difference should not show such components but should instead show stationarity. The parameter, ρ, is consistently estimated by OLS estimation as

$$\hat{\rho} = \frac{\sum_{t=1}^{T} X_{1,t} X_{2,t}}{\sum_{t=1}^{T} X_{2,t}^2} = \rho + \frac{\sum_{t=1}^{T} X_{2,t} a_t}{\sum_{t=1}^{T} X_{2,t}^2}. \tag{4-5}$$

Rearranging (4-5) and introducing T in the result simplifies to give

$$\hat{\rho} - \rho = \frac{\sum_{t=1}^{T} X_{2,t} a_t}{\sum_{t=1}^{T} X_{2,t}^2}, \tag{4-6}$$

or

$$T(\hat{\rho} - \rho) = \frac{T^{-1} \sum_{t=1}^{T} X_{2,t} a_t}{T^{-2} \sum_{t=1}^{T} X_{2,t}^2}. \tag{4-7}$$

Now, since $a_t \sim I(0)$ and $X_{2,t} \sim I(1)$,

$$T^{-1}.\sum_{t=1}^{T} X_{2,t}^2 \sim O_p(T), \tag{4-8}$$

whereas

$$T^{-1}.\sum_{t=1}^{T} X_{2,t} a_t \sim O_p(1). \tag{4-9}$$

Therefore, (4-7) becomes

$$T(\hat{\rho} - \rho) = \frac{T^{-1} \sum_{t=1}^{T} X_{2,t} a_t}{T^{-2} \sum_{t=1}^{T} X_{2,t}^2} \sim \frac{O_p(1)}{T^{-1}.O_p(T)} = O_p(1), \tag{4-10}$$

which means that

$$(\hat{\rho} - \rho) \sim O_p(T^{-1}). \tag{4-11}$$

Equation (4-11) shows that $\hat{\rho}$ converges to ρ at a rate of $O_p(T)$ which is rapid asymptotically compared with the usual rate of $O_p(T^{1/2})$. In the framework of Engle and Granger (1987), testing the hypotheses that $X_{1,t}$ and $X_{2,t}$ are non-cointegrated or cointegrated amounts to directly testing the hypotheses

$$H_0 : \rho = 1 \text{ or } a_t \sim I(1) \qquad \text{(non-cointegration)} \qquad (4\text{-}12a)$$

against

$$H_1 : |\rho| < 1 \text{ or } a_t \sim I(0) \qquad \text{(cointegration).} \qquad (4\text{-}12b)$$

The ADF and PP unit root tests discussed at length in Chapter 2, using the residuals as a variable, are used to test for cointegration. However, a seminal work by Haug (1996) suggests that the ADF test perform better than the PP test. Thus, the EG cointegration test involves fitting the ADF autoregression

$$\hat{a}_t^* = (\hat{\rho} - 1)\hat{a}_{t-1} + \sum_{t=1}^{p-1} \beta_t . \hat{a}_{t-1}^* + u_t , \qquad u_t \sim i.i.d.N(0, \sigma_u^2) \qquad (4\text{-}13)$$

where the $\hat{a}_t$'s are obtained from the estimating (4-2), while $\hat{a}_{t-i}^* = \hat{a}_{t-i} - \hat{a}_{t-i-1}$ for $i = 0,1,2,..., p$. If an intercept or a time trend is included in (4-2), we have

$$X_{1,t} = C + \rho X_{2,t} + a_t , \qquad (4\text{-}14)$$

$$X_{1,t} = (\alpha_0 + \alpha_1 t + \alpha_2 t^2 + ... + \alpha_n t^n) + \rho X_{2,t} + a_t . \qquad (4\text{-}15)$$

In such cases, tests for cointegration are based on the ADF autoregressions

$$\hat{a}_t^* = C + (\hat{\rho} - 1)\hat{a}_{t-1} + \sum_{t=1}^{p-1} \beta_i \hat{a}_{t-i}^* + u_t , \qquad (4\text{-}16)$$

$$\hat{a}_t^* = (\alpha_0 + \alpha_1 t + \alpha_2 t^2 + ... + \alpha_n t^n) + (\hat{\rho} - 1)\hat{a}_{t-1} + \sum_{t=1}^{p-1} \beta_i \hat{a}_{t-i}^* + u_t . \qquad (4\text{-}17)$$

However, it is important to include an intercept if the alternative hypothesis of cointegration allows a non-zero mean for $\hat{a}_t = X_{1,t} - \hat{\rho}X_{2,t}$. A trend is included if the alternative hypothesis allows a non-zero deterministic time trend for $\hat{a}_t$. The null hypothesis of non-cointegration can be tested by the *t*-ratio of $(\hat{\rho}-1)$ in (4-13), (4-16) or (4-17) although it does not have a limiting normal distribution. Here, the standard ADF tables of critical values cannot be applied because of two main reasons. (1) The OLS estimator selects the residuals in the autoregressions (4-13), (4-16) and (4-17) to have the smallest sample variance, even though $X_{1,t}$ and $X_{2,t}$ may not necessarily be cointegrated. In this case, the standard ADF critical values would tend to over-reject the null hypothesis of non-cointegration.

(2) Under the null hypothesis of non-cointegration, the distribution of the test statistic is highly influenced by the number of exogenous variable that may be present in (4-13), (4-16) and (4-17). This means that different critical values are required as the number of explanatory variables, *m*, changes. Engle and Granger (1987), Engle, Yoo and Granger (1997) and Phillips and Ouliaris (1990) have tabulated critical values for all the tests. MacKinnon (1991), however, found the values to be relatively inaccurate and tabulated reasonably accurate critical values. The null hypothesis of non-cointegration is rejected if the test statistic

$$(ADF)_{res} = \frac{\hat{\rho}-1}{\sqrt{\text{var}(\hat{\rho}-1)}} \tag{4-18}$$

is smaller than the appropriate critical value.

4.3 The Engle-Yoo-Granger (EYG) Cointegration Test

The Engle-Yoo-Granger (EYG) test procedure is an improvement over the EG test. The procedure seeks to solve the problems of the residuals having a non-normal distribution by proposing a third step. Non-normality means that the standard *t*-ratios cannot be used to test the hypothesis concerning ρ.

The correction ensures that the estimate of ρ has a normal distribution. For simplicity, let's consider the static equation (4-2)

$$X_{1,t} = \rho X_{2,t} + a_t . \tag{4-19}$$

If $p = 0$, the EG cointegration test equation is

$$\hat{a}_t^{*} = (\hat{\rho} - 1)\hat{a}_{t-1} + u_t$$

or $$\hat{a}_t - \hat{a}_{t-1} = (\hat{\rho} - 1)\hat{a}_{t-1} + u_t . \tag{4-20}$$

From (4-19), we have

$$\hat{a}_t = X_{1,t} - \hat{\rho} X_{2,t}, \tag{4-21a}$$

$$\Rightarrow \quad \hat{a}_{t-1} = X_{1,t-1} - \hat{\rho} X_{2,t-1} . \tag{4-21b}$$

Substituting $\hat{a}_t$ from (4-21) into (4-20) gives

$$\hat{a}_t - \hat{a}_{t-1} = (\hat{\rho} - 1)\hat{a}_{t-1} + u_t$$

$$\Rightarrow \quad (X_{1,t} - \hat{\rho} X_{2,t}) - (X_{1,t-1} - \hat{\rho} X_{2,t-1}) = (\hat{\rho} - 1)(X_{1,t-1} - \hat{\rho} X_{2,t-1}) + u_t$$

$$\Rightarrow \quad (X_{1,t} - X_{1,t-1}) - \hat{\rho}(X_{2,t} - X_{2,t-1}) = (\hat{\rho} - 1)(X_{1,t-1} - \hat{\rho} X_{2,t-1}) + u_t$$

or $$X_{1,t}^{*} = \hat{\rho} X_{2,t}^{*} + (\hat{\rho} - 1)(X_{1,t-1} - \hat{\rho} X_{2,t-1}) + u_t , \tag{4-22a}$$

or $$X_{1,t}^{*} = \hat{\rho} X_{2,t}^{*} + (\hat{\rho} - 1)\hat{a}_{t-1} + u_t , \tag{4-22b}$$

where $X_{1,t}^{*} = X_{1,t} - X_{1,t-1}$, $X_{2,t}^{*} = X_{2,t} - X_{2,t-1}$ and $\hat{a}_{t-1} = X_{1,t-1} - \hat{\rho} X_{2,t-1}$.

The set of residuals, $\hat{u}_t$, from (4.22) are then used in the third-step

$$\hat{u}_t = -c(\hat{\rho}-1).X_{1,t-1} + v_t, \tag{4-23}$$

where c is a constant and $v_t \sim i.i.d.N(0,\sigma_v^2)$. The estimate of c obtained, together with its standard deviation, which provides the correct standard deviation, is used to correct the first-step estimates

$$\hat{\rho}^* = \hat{c} + \hat{\rho}. \tag{4.24}$$

The two-step EG test discussed in Section 4.2 then proceeds as usual with $\hat{\rho}$ replaced by $\hat{\rho}^*$.

4.4 The Cointegrating Regression Durbin-Watson (CRDW) Method

The Cointegrating Regression Durbin-Watson (CRDW) test procedure due to Sargan and Bhargava (1983) is based on the standard Durban-Watson statistic obtained from any of the regression models with *m* explanatory variables

$$X_{1,t} = \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \ldots + \alpha_m X_{m,t} + a_t, \tag{4-25}$$

$$X_{1,t} = C + \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \ldots + \alpha_m X_{m,t} + a_t, \tag{4-26}$$

$$X_{1,t} = (c_0 + c_1 t + c_2 t^2 + \ldots + c_n t^n) + \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \ldots + \alpha_m X_{m,t} + a_t, \tag{4-27}$$

where $c_0, c_1, \ldots, c_n, \alpha_1, \alpha_2, \ldots, \alpha_m$ are unknown parameters. The CRDW test procedureis known to be uniformly most powerful invariant test, i.e. it is not affected by the inclusion of an intercept or a trend term.

The test strictly applies to the case where the a_t's are first-order autoregressive with autocorrelation ρ

$$a_t = \rho . a_{t-1} + u_t , \qquad u_t \sim i.i.d. N(0, \sigma_u^2) , \tag{4-28}$$

then the least-squares estimator of ρ is

$$\hat{\rho} = \frac{\sum_{t=2}^{T} a_t a_{t-1}}{\sum_{t=2}^{T} a_{t-1}^2} . \tag{4-29}$$

The CRDW test statistic is

$$CRDW^* = \frac{\sum_{t=2}^{T} (a_t - a_{t-1})^2}{\sum_{t=1}^{T} a_t^2} . \tag{4-30}$$

Expanding the CRDW statistic, we obtain

$$CRDW^* = \frac{\sum_{t=2}^{T} (a_t^2 - 2a_t a_{t-1} + a_{t-1}^2)}{\sum_{t=1}^{T} a_t^2} = \frac{\sum_{t=2}^{T} a_t^2 - 2\sum_{t=2}^{T} a_t a_{t-1} + \sum_{t=2}^{T} a_{t-1}^2}{\sum_{t=1}^{T} a_t^2} . \tag{4-31}$$

For large samples

$$\sum_{t=2}^{T} a_t^2 \cong \sum_{t=2}^{T} a_{t-1}^2 \cong \sum_{t=1}^{T} a_{t=1}^2 . \tag{4-32}$$

Therefore, (4-31) becomes

$$DW^* = \frac{2\sum_{t=2}^{T} a_{t-1}^2 - 2\sum_{t=2}^{T} a_t a_{t-1}}{\sum_{t=2}^{T} a_{t-1}^2} = 2 - \frac{2\sum_{t=2}^{T} a_t a_{t-1}}{\sum_{t=2}^{T} a_{t-1}^2} . \tag{4-33}$$

Substituting (4-98) into (4-33) yields

$$DW^* \cong 2 - 2\hat{\rho} = 2(1-\hat{\rho}). \qquad (4\text{-}334)$$

In (4-34), testing the null hypothesis that a_t contains a unit root (i.e. a_t follows a random walk process)

$$H_0 : \rho = 1 \qquad (4\text{-}35a)$$

against $$H_1 : |\rho| < 1, \qquad (4\text{-}35b)$$

is equivalent to testing the null hypothesis

$H_0 : X_{1,t}$ and $X_{2,t}$ are not cointegrated, (4-36a)

against $H_1 : X_{1,t}$ and $X_{2,t}$ are cointegrated. (4-36b)

Failure to reject (4-36a) means that

$$CRDW^* \cong 0 \quad \text{since } \hat{\rho} = 1. \qquad (4\text{-}37)$$

This means that the null hypothesis is rejected in favour of the alternative if

$$CRDW^* > \text{Critical Value}. \qquad (4\text{-}38)$$

4.5 The Phillips-Ouliaris (PO) Cointegration Test

The starting point for the Phillips-Ouliaris (PO) test for cointegration employs either of the cointegration regressions (4-25), (4.26) or (4.27)

$$X_{1,t} = \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \ldots + \alpha_m X_{m,t} + a_t, \tag{4-39}$$

$$X_{1,t} = C + \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \ldots + \alpha_m X_{m,t} + a_t, \tag{4-40}$$

$$X_{1,t} = (c_0 + c_1 t + c_2 t^2 + \ldots + c_n t^n) + \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \ldots + \alpha_m X_{m,t} + a_t. \tag{4-41}$$

As in (4-28), the estimated residuals $\hat{a}_t$ in (4-39), (4-40) or (4-41) enter in the AR(1) process

$$\hat{a}_t = \rho.\hat{a}_{t-1} + u_t, \tag{4-42}$$

estimated using the least squares method. The null hypothesis of non-cointegration, formulated in terms of the correlation coefficient ρ, is

$$H_0 : \rho = 1 \quad \text{(non-cointegration)}. \tag{4-43}$$

The Phillips-Ouliaris (PO) test statistic associated with the residual autoregression (4-42) is given by

$$PO^* = \left(\frac{\hat{\gamma}_0}{\hat{\lambda}^2}\right)^{\frac{1}{2}} . \left(\frac{\hat{\rho} - 1}{\sqrt{\text{var}(\hat{\rho} - 1)}}\right) - \left(\frac{T-1}{2}\right)\left(\frac{\sqrt{\text{var}(\hat{\rho})}}{\sqrt{\text{var}(u)}}\right)\left(\frac{\hat{\lambda}^2 - \hat{\gamma}_0}{\hat{\lambda}}\right), \tag{4-44}$$

where $\text{var}(u)$ is the OLS estimate of the variance of u_t for the regression (4-42), and is given by

$$\text{var}(u) = \frac{\sum_{t=1}^{T} u_t^2}{T-2} = \frac{\sum_{t=1}^{T} (\hat{a}_t - \hat{\rho}.a_{t-1})^2}{T-2}. \tag{4-45}$$

In (4-44), $\text{var}(\hat{\rho})$ is the OLS estimator of the variance of ρ in (4-42), and is given by

$$\text{var}(\hat{\rho}) = \frac{\text{var}(u)}{\sum_{t=2}^{T} \hat{a}_{t-1}^2} . \qquad (4\text{-}46)$$

An alternative approach is to include the lagged changes of the residuals in (4-42) as in the ADF test without an intercept or a time trend. That is

$$\hat{a}_t^* = (\rho - 1).\hat{a}_{t-1} + \sum_{i=1}^{p-1} b_i.\hat{a}^*_{t-i} + u_t , \qquad (4\text{-}47)$$

where $\hat{a}_{t-i}^* = \hat{a}_{t-i} - \hat{a}_{t-i-1}$ for $i = 0,1,2,\ldots, p-1$. The null hypothesis of non-cointegration (4-43) is rejected if the probability value (prob value) of the t-ratio is sufficiently large in absolute value (i.e. H_0 is rejected if the prob value of the t-ratio is greater than a known level of significance, say 0.05).

4.6 Seasonal Cointegration

A strong seasonal pattern in a time series is indicative of seasonal unit roots (see Chapter 3). Consequently, any potential cointegration is bound to occur at seasonal frequencies as well as at the zero frequency. Let $\{X_{1,t}, X_{2,t}, \ldots, X_{n,t}\}_{t=1}^{T}$ be an $n \times 1$ vector of quarterly time series, each of which potentially has both non-seasonal and seasonal unit roots. Consider a quarterly time series $\{X_t\}_{t=1}^{T}$ satisfying the process

$$(1 - B^4)X_t = a_t \qquad \text{where} \qquad a_t \sim i.i.d.N(0, \sigma^2) . \qquad (4\text{-}48)$$

If our aim is to test the null hypothesis that the roots of $(1 - B^4)$ lie outside the unit circle, then we can define three positive integers, r_1, r_2, r_3, such that

$$(1-B^4)=(1-r_1B)(1+r_2B)(1+r_3B^2), \quad (4\text{-}49)$$

where B is the back-shift operator. If $r_j \to 1$, we obtain the seasonal filters (Banerjee et al, 1993, pp 122-123):

$$S_1 = 1+B+B^2+B^3, \quad (4\text{-}50a)$$

$$S_2 = -(1-B+B^2-B^3), \quad (4\text{-}50b)$$

and $$S_3 = -(1-B^2), \quad (4\text{-}50c)$$

Let all series being analysed be $X_{i,t} \sim I(1,1)$ for $i = 1,2,\ldots,m$, where $I(1,1)$ implies each series requires one non-seasonal differencing and one seasonal differencing to induce stationarity. For brevity, let's consider two of such series, $X_{1,t}$ and $X_{2,t}$:

$$X_{1,t} = \rho.X_{2,t} + a_t. \quad (4\text{-}51)$$

The static model test for non-seasonal cointegration (at the zero frequency) between $X_{1,t}$ and $X_{2,t}$ is

$$(S_1.X_{1,t}) = \rho(S_1.X_{2,t}) + d_t. \quad (4\text{-}52)$$

Testing the null hypothesis of non-cointegration against the alternative of cointegration uses the residuals from (4-52) in any of the ADF autoregressions ($\hat{a}_t$ is replaced by $\hat{d}_t$):

$$\hat{d}^*_t = (\hat{\rho}-1)\hat{d}_{t-1} + \sum_{t=1}^{p-1} \beta_i.\hat{d}^*_{t-i} + u_t, \quad (4\text{-}53)$$

$$\hat{d}^*_t = C + (\hat{\rho}-1)\hat{d}_{t-1} + \sum_{t=1}^{p-1} \beta.\hat{d}^*_{t-i} + u_t, \quad (4\text{-}54)$$

$$\hat{d}^*_t = (\alpha_0 + \alpha_1 t + \alpha_2 t^2 + ... + \alpha_n t^n) + (\hat{\rho} - 1)\hat{d}_{t-1} + \sum_{i=1}^{p-1} \beta.\hat{d}^*_{t-i} + u_t \; . \qquad (4\text{-}55)$$

where $u_t \sim i.i.d.N(0, \sigma_u^2)$ and $\hat{d}^*_{t-i} = \hat{d}_{t-i} - \hat{d}_{t-i-1}$ for $i = 0,1,2,..., p-1$. The hypotheses to test are

$$H_0 : \rho = 1 \qquad \text{(non-cointegration)} \qquad (4\text{-}56a)$$

against

$$H_1 : | \rho | < 1 \qquad \text{(cointegration).} \qquad (4\text{-}56b)$$

The test statistic is the t-ratio of ρ which is compared an appropriate critical value given by MacKinnon (1991).

A test for half-year seasonal cointegration involves multiplying the static equation by S_2:

$$(S_2.X_{1,t}) = \rho.(S_2 X_{2,t}) + e_t \, , \qquad (4\text{-}57)$$

where $S_2 = -(1 - B + B^2 - B^3)$. Testing for non-cointegration uses the residuals in (4-57) and the modified version of the ADF test

$$(\hat{e}_t + \hat{e}_{t-1}) = \left(\alpha_0 + \sum_{q=1}^{3} \alpha_q.D_{q,t}\right) + \rho.\hat{e}_{t-1} + \sum_{i=1}^{p-1} \beta_i.(\hat{e}_{t-i} + \hat{e}_{t-i-1}) + f_t \, , \qquad (4\text{-}58)$$

where $f_t \sim i.i.d.N(0, \sigma_f^t)$ and $D_{q,t}$, the 0-1 dummy corresponding to quarter q. The hypotheses in (4-56) and the same MacKinnon table of critical values apply.

Lastly, testing for annual seasonal cointegration, the cointegrating regression (4-51) is multipliedl by S_3:

$$(S_3.X_{1,t}) = \rho.(S_3.X_{2,t}) + g_t \, , \qquad (4\text{-}59)$$

and using the residuals $\hat{g}_t$ from (4-59) in the following ADF test

$$(\hat{g}_t + \hat{g}_{t-2}) = \left(\alpha_0 + \sum_{q=1}^{3} \alpha_q . D_{q,t}\right) + \delta_1.(-\hat{g}_{t-1}) + \delta_2(-\hat{g}_{t-2}) + \sum_{i=1}^{p-1} \beta_i.(\hat{g}_{t-i} + \hat{g}_{t-i-1}) + u_t \tag{4-60}$$

The test of the null hypothesis on non-cointegration requires a joint F-test:

$$H_0 : \delta_1 = \delta_2 = 0 \qquad \text{(non-cointegration)} \tag{4-61}$$

Critical values for this test when there are two variables in the cointegration regression is contained in the Appendix.

4.7 Spurious Regression and Cointegration Regression

Consider the bivariate process $\{(X_{1,t}, X_{2,t})\}$ with

$$\{(a_{1,t}, a_{2,t})\} \sim i.i.d.N(0, \Sigma), \tag{4-62}$$

where 0 and Σ are, respectively, the zero mean vector and covariance matrix of the process. Let $X_{1,t}$ be generated by two alternative regression models

$$X_{1,t} = \rho X_{2,t} + a_{1,t} \tag{4-63a}$$

and

$$X_{1,t} = \rho X_{2,t} + c_{1,t}, \tag{4-63b}$$

where $c_{1,t} = \sum_{k=1}^{t} a_{1,k}$, $c_{2,t} = \sum_{k=1}^{t} a_{2,k}$ and $X_{2,t} = c_{2,t}$.

Then, clearly in $X_{1,t}$ and $X_{2,t}$ are cointegrated in (4-63a) but are not cointegrated in (4-63b). Regression between non-cointegrated variables is said to be *spurious*. Discriminating between a spurious regression and cointegration regression involves checking whether R^2 is greater or less than the Durban-Watson statistic. A regression model is spurious if

$$R^2 > \text{Durban-Watson Statistic.} \qquad (4\text{-}64)$$

4.8 Efficiency Assessment of the Phillips-Ouliaris Test

In the Phillips-Ouliaris cointegration test, the residuals in the estimated cointegration regression are assumed to be first-order autocorrelated. When first-order autocorrelation exists in the OLS residuals, though the OLS estimator is unbiased, it is not efficient. In such a situation, the sampling variances are under-estimated rendering invalid inferences from t-test and F-test.

One approach of making up for the autocorrelated residuals is the application of the Yule-Walker method. The method estimates the autoregressive form of the error term and then estimates the coefficients using the method of generalised least squares. In this section, we investigate the gain in efficiency of the Phillips-Ouliaris cointegration test via simulation using two methods, namely, the OLS and Yule-Walker methods.

4.8.1 Simulation Methods

The first step in our investigation involves simulating two cointegrated time series $X_{1,t}$ and $X_{2,t}$ each with size 100. $X_{1,t}$ and $X_{2,t}$ are both $I(1)$ and so we expect their linear combination $\hat{a}_t = X_{1,t} - \rho X_{2,t}$ to be $I(0)$. By setting $\rho = 1.00$, a program used to obtain the two cointegrated time series can be found in the following SAS program. Complete results from this program are also found in the Appendix. Exhibit 4.1, a graphical representation of the two series, clearly shows that the two series are cointegrated.

```
data simdat;
   c1 = 0.65;
   c2 = 0.80;
 phi1 = 1.00;
 phi2 = 1.00;
   e1 = sqrt(1)*rannor(0);
   e2 = sqrt(1)*rannor(0);
 do t = 1 to 120;
   x1 = c1 + phi1*e1 + sqrt(1)*rannor(0);
   x2 = c2 + phi2*e2 + sqrt(1)*rannor(0);
   e1 = x1;
   e2 = x2;
   keep t x1 x2;
   label x1 = 'series1'
         x2 = 'series2';
 if t > 20 then output;
 end;

 proc gplot data=simdat;
    plot (x1 x2)*t / overlay frame legend;
    symbol1 l=1 i=join v=none;
    symbol2 l=2 i=join v=star;
    symbol1 l=1 i=join c=none;
    symbol2 l=2 i=join c=star;
 run;

 proc autoreg data=simdat;
    model x1 = x2 / normal stationarity=(pp);
    output out=results residual=r;
 run;

 proc print data=results noobs;
 run;

/* efficiency comparison */
data assess;
    do samp = 1 to 1000;

    rho = 1.0;

     /* first residual term r1 */
     r1 = sqrt(1)*rannor(0);

    do x2 = 1 to 100;
       x1 = -7.451560 + 0.943278*x2 + r;
        r = rho*r1 + rannor(0);
       r1 = r;
        output;
     end;
 end;

 /* Phillips-Ouliaris via OLS Method */
 proc autoreg data=assess outest=b1;
    by samp;
    model x1 = x2 / normal;
 run;
```

```
data c1;
   set b1;
   rename x2 = x21;
   rename intercep=int1;
run;

proc univariate data=c1;
   var int1 x21;
run;


/* Phillips-Ouliaris via Yule-Walker Method */
proc autoreg data=assess outest=b2;
   by samp;
   model x1=x2 / nlag=1 normal;
run;

data c2;
   set b2;
   rename x2 = x22;
   rename intercep=int2;
run;

proc univariate data=c2;
   var int2 x22;
run;
```

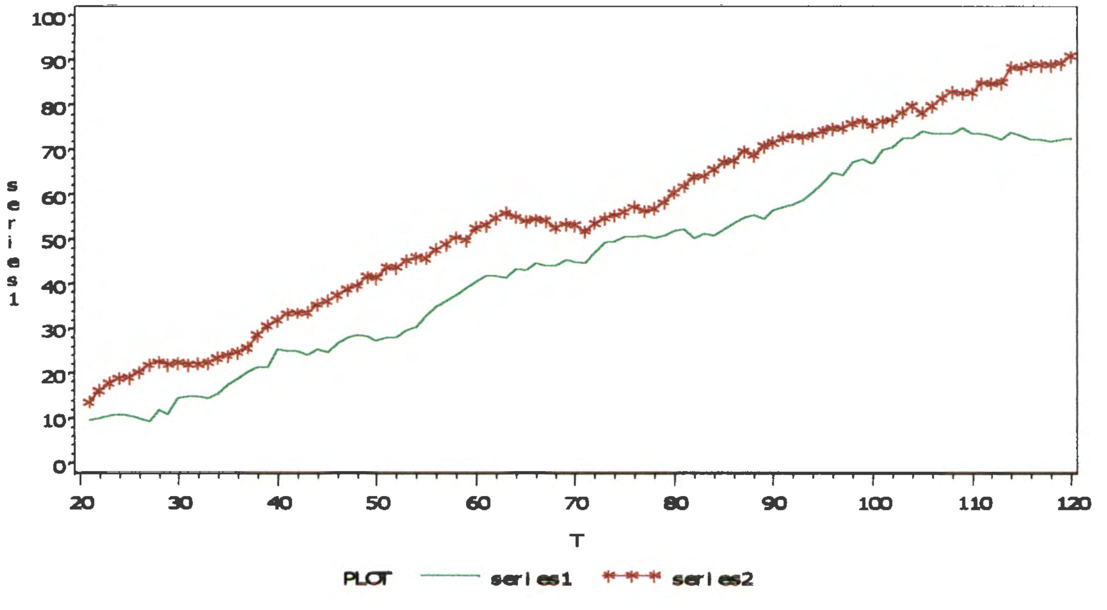


Exhibit 4.1: Simulated Bivariate Time Series ($X_{1,t}$ = Series1, $X_{2,t}$ = Series2)

Table 4.1: Phillips-Ouliaris Cointegration Test of $X_{1,t}$ and $X_{2,t}$

```
Dependent Variable = X1         series1

                    Ordinary Least Squares Estimates

              SSE            1080.912    DFE               98
              MSE            11.02971    Root MSE    3.321101
              SBC             531.037    AIC         525.8267
              Reg Rsq          0.9753    Total Rsq     0.9753
              Normal Test      3.5140    Prob>Chi-Sq   0.1726
              Durbin-Watson    0.1786

                     Phillips-Ouliaris Cointegration Test

                         Lags          RHO            T

                            2      -9.4942      -2.1187

   Variable      DF       B Value   Std Error   t Ratio Approx Prob  Variable Label

   Intercept      1     -7.451560      0.8974    -8.304      0.0001
   X2             1      0.943278      0.0152    62.202      0.0001  series2
```

Table 4.1 contains the Phillips-Ouliaris cointegration test of $X_{1,t}$ and $X_{2,t}$. The cointegration regression is

$$X_{1,t} = -7.451560 + 0.943278 X_{1,t} + a_t , \qquad (4\text{-}65)$$

$$\hat{a}_t = \hat{a}_{t-1} + u_t , \qquad \text{where} \qquad u_t \sim i.i.d.N(0,1) . \qquad (4\text{-}66)$$

Next, using (4-65) and (4-66), a Monte Carlo experiment on the Phillips-Ouliaris cointegration test with 1000 replications via the OLS and Yule-Walker methods gave the results in Table 4.2 and Table 4.3, respectively. The mean value of the intercept via the OLS and Yule-Walker methods are respectively given by –7.40589 and –7.41486. These values are very close to –7.451560, the true value. The respective variances are 14.23508 and 12.47977.

Table 4.2: Efficiency Test for the Phillips-Ouliaris Cointegration Test by Simulation (OLS)

Method = OLS

Univariate Procedure

Variable=INT1 Intercept Parameter

Moments				Quantiles(Def=5)			
N	1000	Sum Wgts	1000	100% Max	4.365921	99%	1.976837
Mean	-7.40589	Sum	-7405.89	75% Q3	-4.80798	95%	-1.37699
Std Dev	3.772941	Variance	14.23508	50% Med	-7.45225	90%	-2.49303
Skewness	0.047356	Kurtosis	-0.08944	25% Q1	-10.0192	10%	-12.3241
USS	69067.99	CSS	14220.85	0% Min	-18.2971	5%	-13.5177
CV	-50.9452	Std Mean	0.119311			1%	-16.13
T:Mean=0	-62.0722	Pr>|T|	0.0001	Range	22.66299		
Num ^= 0	1000	Num > 0	25	Q3-Q1	5.21121		
M(Sign)	-475	Pr>=|M|	0.0001	Mode	-18.2971		
Sgn Rank	-248778	Pr>=|S|	0.0001				

Extremes

Lowest	Obs	Highest	Obs
-18.2971(	964)	2.697867(	744)
-17.8608(	946)	2.75428(	732)
-17.8104(	885)	2.840098(	70)
-17.464(	345)	3.403264(	922)
-17.4269(	279)	4.365921(	539)

Univariate Procedure

Variable=X21 Parameter Estimate for X2

Moments				Quantiles(Def=5)			
N	1000	Sum Wgts	1000	100% Max	1.312503	99%	1.201625
Mean	0.940482	Sum	940.4818	75% Q3	1.013701	95%	1.117052
Std Dev	0.110203	Variance	0.012145	50% Med	0.941743	90%	1.079694
Skewness	0.018469	Kurtosis	0.060283	25% Q1	0.86574	10%	0.800138
USS	896.6387	CSS	12.13265	0% Min	0.579242	5%	0.75994
CV	11.71776	Std Mean	0.003485			1%	0.67939
T:Mean=0	269.8704	Pr>|T|	0.0001	Range	0.733261		
Num ^= 0	1000	Num > 0	1000	Q3-Q1	0.147962		
M(Sign)	500	Pr>=|M|	0.0001	Mode	0.579242		
Sgn Rank	250250	Pr>=|S|	0.0001				

Extremes

Lowest	Obs	Highest	Obs
0.579242(	915)	1.240036(	492)
0.614888(	601)	1.270217(	437)
0.640234(	644)	1.284876(	139)
0.642085(	746)	1.287489(	193)
0.644941(	276)	1.312503(	589)

Table 4.3: Efficiency Test for the Phillips-Ouliaris Cointegration Test by Simulation (Y-Walker)

Method = Yule-Walker

Univariate Procedure

Variable=INT2 Intercept Parameter

Moments

N	1000	Sum Wgts	1000
Mean	-7.41486	Sum	-7414.86
Std Dev	3.532671	Variance	12.47977
Skewness	0.160338	Kurtosis	0.012223
USS	67447.42	CSS	12467.29
CV	-47.6431	Std Mean	0.111713
T:Mean=0	-66.3743	Pr>|T|	0.0001
Num ^= 0	1000	Num > 0	25
M(Sign)	-475	Pr>=|M|	0.0001
Sgn Rank	-249094	Pr>=|S|	0.0001

Quantiles(Def=5)

100% Max	4.272431	99%	1.82901
75% Q3	-5.02964	95%	-1.72283
50% Med	-7.43225	90%	-3.02661
25% Q1	-9.93282	10%	-12.0238
0% Min	-18.1427	5%	-12.9809
		1%	-15.0887
Range	22.41511		
Q3-Q1	4.903185		
Mode	-18.1427		

Extremes

Lowest	Obs	Highest	Obs
-18.1427(	279)	2.755802(	744)
-17.5281(	349)	3.389003(	908)
-15.9231(	652)	3.602884(	222)
-15.9112(	345)	3.666019(	901)
-15.8915(	964)	4.272431(	741)

Univariate Procedure

Variable=X22 Parameter Estimate for X2

Moments

N	1000	Sum Wgts	1000
Mean	0.940894	Sum	940.8936
Std Dev	0.107827	Variance	0.011627
Skewness	0.011871	Kurtosis	0.00935
USS	896.8958	CSS	11.61513
CV	11.46011	Std Mean	0.00341
T:Mean=0	275.9378	Pr>|T|	0.0001
Num ^= 0	1000	Num > 0	1000
M(Sign)	500	Pr>=|M|	0.0001
Sgn Rank	250250	Pr>=|S|	0.0001

Quantiles(Def=5)

100% Max	1.289633	99%	1.19878
75% Q3	1.015327	95%	1.113263
50% Med	0.943314	90%	1.079239
25% Q1	0.866112	10%	0.802215
0% Min	0.589405	5%	0.762735
		1%	0.696592
Range	0.700228		
Q3-Q1	0.149215		
Mode	0.589405		

Extremes

Lowest	Obs	Highest	Obs
0.589405(	915)	1.231686(	193)
0.630832(	746)	1.238169(	492)
0.642181(	644)	1.280546(	589)
0.65512(	377)	1.28117(	437)
0.656221(	341)	1.289633(	139)

Similarly, the mean of the coefficient of $X_{2,t}$ via the OLS and Yule-Walker methods are 0.940482 and 0.940894. The respective variances are 0.012145 and 0.011627. In all cases, we notice that the variances from the Yule-Walker method are less than that from the OLS method. In addition, an examination of the quantiles shows a less dispersion via the Yule-Walker method than the OLS method. This means that, the Phillips-Ouliaris test for cointegration using the Yule-Walker method is slightly more efficient than using the OLS method.

To test whether the variances of the coefficients of $X_{2,t}$ differ significantly when the methods are used, we calculate the F-ratio

$$F = \frac{\text{variance of coefficent of } X_{2,t} \text{ via OLS}}{\text{variance of coefficient of } X_{2,t} \text{ via Yule-Walker}} = \frac{0.012145}{0.011627} = 1.0446 \qquad (4\text{-}67)$$

using the following SAS program:

```
data;
   p = 1 - probf(1.044551475, 999, 999);
   put p=;
   proc print;
run;
```

With 1000-1=999 degrees of freedom for both the numerator and the denominator, the corresponding probability value (p-value) is

$$\text{p-value} = 1 - \text{prob}F(1.044551475, 999, 999) = 0.24553. \qquad (4\text{-}69)$$

The program used to obtain the p-value is also found in the Appendix. Since the p-value is greater than the conventional levels of 1%, 5%, and 10%, we conclude that the efficiency of the Phillips-Ouliaris cointegration test via the two methods does not differ significantly.

However, the fact that standard deviations of the distribution of the coefficient of $X_{2,t}$ from the two methods, $\sigma_{ols} = 0.110203$ and $\sigma_{yw} = 0.107827$, are such that

$$\sigma_{yw} < \sigma_{ols} \tag{4-70}$$

is a convincing proof of greater efficiency of the Phillips-Ouliaris cointegration test via the Yule-Walker method.

4.9 A Numerical Example

We now proceed to the EG, CRDW, and the Phillips-Ouliaris cointegration tests using data on money supply (M1) and amount of coin and banknotes in circulation in South Africa. These are monthly data from October 1990 to September 2000 and can be found in the Appendix.

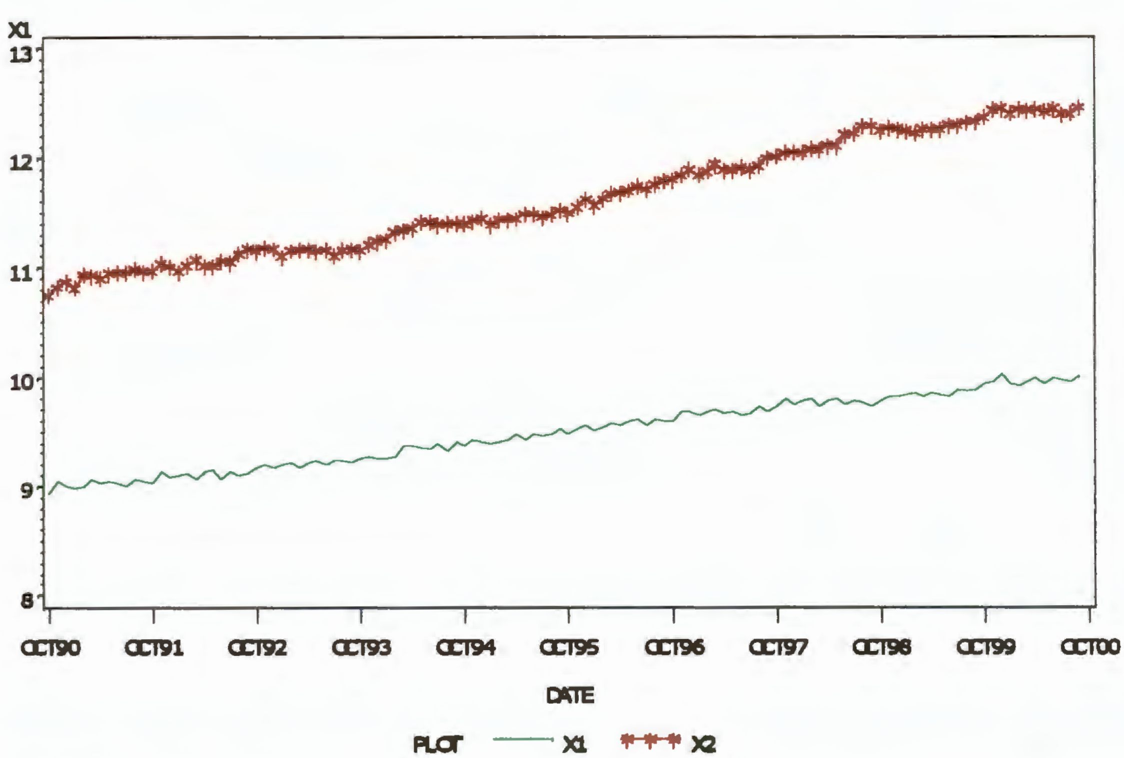


Exhibit 4.2: ln(M1) and ln(coin and banknotes)

Table 4.4: Phillips-Perron Unit Root Tests 1

Phillips-Perron Unit Root Tests on $X_{1,t}$

Type	Lags	RHO	Prob<RHO	T	Prob<T
Zero Mean	4	0.1127	0.7074	4.7178	0.9999
Single Mean	4	-0.5698	0.9207	-0.7280	0.8348

Phillips-Perron Unit Root Tests on $X_{2,t}$

Type	Lags	RHO	Prob<RHO	T	Prob<T
Zero Mean	4	0.1469	0.7156	5.3039	0.9999
Single Mean	4	-0.3282	0.9375	-0.5146	0.8834

Table 4.5: Phillips-Perron Unit Root Tests 2

Phillips-Perron Unit Root Tests on First-Differenced $X_{1,t}$

Type	Lags	RHO	Prob<RHO	T	Prob<T
Zero Mean	4	-161.510	0.0001	-18.2420	0.0001
Single Mean	4	-150.422	0.0001	-24.8187	0.0001

Phillips-Perron Unit Root Tests on First-Differenced $X_{2,t}$

Type	Lags	RHO	Prob<RHO	T	Prob<T
Zero Mean	4	-150.965	0.0001	-12.1627	0.0001
Single Mean	4	-142.806	0.0001	-14.6358	0.0001

Our cointegration analysis will involve logarithms of these time series data, where

$$X_{1,t} = \ln(\text{Money Supply}) = \ln(\text{M1}),$$

while $$X_{2,t} = \ln(\text{Coin and Banknotes in Circulation}).$$

Essentially, we investigate whether there is any long-run equilibrium in existence over time between the money supply ($X_{1,t}$) and coin and banknotes in circulation in South Africa. Exhibit 4.2 is a representation of $X_{1,t}$ and $X_{2,t}$. Clearly the two series appear to trend upwards together. The Phillips-Perron unit root tests on the two series are given in Tables 4.4 and 4.5. In both the undifferenced series and first-differenced series, the required Newey-West truncation lag is 4.

Results in Table 4.4 show that at lag 4 the probability values, (prob<T), of the Phillips-Perron test statistics are all greater than the conventional level of 0.01, 0.05, 0.10. We therefore cannot reject the null hypothesis of unit root and conclude that the undifferenced are non-stationary, a conclusion that suggests differencing each series once.

Table 4.6: Cointegration Regression

Dependent Variable = X1

Ordinary Least Squares Estimates

SSE	0.287755	DFE	118
MSE	0.002439	Root MSE	0.049382
SBC	-373.856	AIC	-379.431
Reg Rsq	0.9751	Total Rsq	0.9751
Normal Test	3.3689	Prob>Chi-Sq	0.1855
Durbin-Watson	0.8300		

Phillips-Ouliaris Cointegration Test

Lags	RHO	T
2	-41.4413	-5.2526

Variable	DF	B Value	Std Error	t Ratio	Approx Prob
Intercept	1	2.570246	0.1020	25.198	0.0001
X2	1	0.594910	0.00876	67.914	0.0001

By a similar argument, results from the Phillips-Perron unit root tests on the differenced series show that the differenced series are stationary, i.e. the prob-values at lag 4 are all less than the conventional level of 0.01, 0.05, or 0.10. We therefore conclude that that

$$X_{1,t} \sim I(1) \quad \text{and} \quad X_{2,t} \sim I(1). \tag{4-71}$$

Hence, the first step involved in cointegration test has been satisfied.

Next, we test for cointegrating relationship between $X_{1,t}$ and $X_{2,t}$ using the EG, CRDW and the Phillips-Ouliaris methods. A regression of $X_{1,t}$ on $X_{2,t}$ yields the following results in Table 4.6. The cointegration regression is

$$X_{1,t} = 2.5702 + 0.5949 X_{2,t} + a_t. \tag{4-72}$$

A series made up of the estimated residuals $\hat{a}_t$ are contained in the Appendix. Plots of the autocorrelation and partial autocorrelations of the residuals are shown in Exhibit 4.3 and Exhibit 4.4. An examination of the Exhibit 4.5 suggests that the Engle-Granger test for cointegration (i.e. the ADF unit root test on the residuals) has order $p = 3$. That is

$$\hat{a}_t^* = c + (\rho - 1).\hat{a}_{t-1} + \sum_{i=1}^{3-1} \beta_i.\hat{a}_{t-i}^* + u_t, \tag{4-73}$$

where $\hat{a}_t^* = \hat{a}_t - \hat{a}_{t-1}$ and $u_t \sim i.i.d.N(0, \sigma_u^2)$.

The Engle-Granger cointegration test results are given in Table 4.7. At lag $p - 1 = 3 - 1 = 2$, the probability value of the ADF test statistic is 0.0299. Since this value is less than the conventional level of 0.05, we reject the null hypothesis of a unit root in the residuals. We therefore accept the alternative of stationarity and conclude that $X_{1,t}$ and $X_{2,t}$ are cointegrated.

```
                    Name of variable         =          R          /* R=Residuals */
                    Mean of working series = 1.97E-15
                    Standard deviation       = 0.048969
                    Number of observations =       120

                                   Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1          Std
  1  0.0013958     0.58208 |                .   |************        |   0.091287
  2  0.0014059     0.58629 |               .    |************        |   0.118238
  3  0.0015267     0.63665 |              .     |*************       |   0.140389
  4  0.0010257     0.42774 |             .      |*********           |   0.162679
  5  0.0010998     0.45864 |             .      |*********           |   0.171796
  6  0.0012180     0.50792 |             .      |**********          |   0.181713
  7 0.00072499     0.30234 |            .       |****** .            |   0.193182
  8 0.00088614     0.36954 |            .       |*******.            |   0.197086
  9 0.00099508     0.41497 |            .       |********            |   0.202778
 10 0.00053884     0.22471 |            .       |****   .            |   0.209735
 11 0.00072624     0.30286 |            .       |****** .            |   0.211732
 12 0.00084981     0.35439 |           .        |******* .           |   0.215312
 13 0.00044277     0.18465 |           .        |****    .           |   0.220119
 14 0.00070395     0.29356 |           .        |******  .           |   0.221406
 15 0.00053201     0.22186 |           .        |****    .           |   0.224626
 16 0.00029073     0.12124 |           .        |**      .           |   0.226445
 17 0.00053379     0.22260 |           .        |****    .           |   0.226985
 18 0.00045569     0.19003 |           .        |****    .           |   0.228797
 19 0.00007839     0.03269 |           .        |*       .           |   0.230109
 20 0.00043659     0.18207 |           .        |****    .           |   0.230147
 21  0.0001284     0.05355 |           .        |*       .           |   0.231345
 22 -0.0000218    -0.00910 |           .        |        .           |   0.231448
```

Exhibit 4.3: Sample Autocorrelations of the Residuals

```
                              Partial Autocorrelations
           Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.58208 |                .   |************        |
             2     0.37429 |                .   |*******             |
             3     0.36101 |                .   |*******             |
             4    -0.14279 |                .***|   .                |
             5     0.02936 |                .   |*  .                |
             6     0.18498 |                .   |****                |
             7    -0.16108 |                .***|   .                |
             8     0.01538 |                .   |   .                |
             9     0.16247 |                .   |***.                |
            10    -0.11879 |                . **|   .                |
            11    -0.02520 |                .  *|   .                |
            12     0.17081 |                .   |***.                |
            13    -0.05977 |                .  *|   .                |
            14     0.00013 |                .   |   .                |
            15    -0.11065 |                . **|   .                |
            16     0.00366 |                .   |   .                |
            17     0.03676 |                .   |*  .                |
            18     0.03724 |                .   |*  .                |
            19    -0.16910 |                .***|   .                |
            20     0.08737 |                .   |** .                |
            21    -0.10728 |                . **|   .                |
            22    -0.04062 |                .  *|   .                |
            23     0.00320 |                .   |   .                |
            24    -0.08725 |                . **|   .                |
```

Exhibit 4.4: Sample Autocorrelations of the Residuals

Table 4.7: Engle-Granger Cointegration Test (ADF Unit Root Test on the Residuals)

Augmented Dickey-Fuller Unit Root Tests

Type	Lags	RHO	Prob<RHO	T	Prob<T	F	Prob<F
Zero Mean	0	-49.6814	0.0001	-5.5980	0.0001	--	--
	1	-22.1910	0.0006	-3.3037	0.0011	--	--
	2	-9.9104	0.0271	-2.1633	0.0299	--	--

In the case of the Phillips-Ouliaris cointegration test, the test statistic (see Table 4.6) is

$$PO^{*} = -41.4413. \tag{4-74}$$

With one explanatory variable in the cointegration equation, the critical value from Phillips and Ouliaris (1990) at 5% level of significance is –20.4935. Since the test statistic is less than the critical value, we reject the null hypothesis of non-cointegration and conclude that $X_{1,t}$ and $X_{2,t}$ are cointegrated. Lastly, at 5% level of significance, the CRDW critical value is 0.386 (Engle and Granger, 1987). Also, from Table 4.6, the Durbin-Watson test statistic is $CRDW^{*} = 0.8300$. Since the test statistic is greater than the critical value, we reject the null hypothesis of non-cointegration and conclude that $X_{1,t}$ and $X_{2,t}$ are cointegrated.

4.10 Summary

In this chapter we have extensively discussed some commonly used univariate cointegration tests in the literature. My personal investigation into the efficiency of the Phillips-Ouliaris cointegration test via the OLS and Yule-Walker method revealed a greater efficiency of the test via the Yule-Walker method. It is therefore recommended that the use of this cointegration test procedure should be via the Yule-Walker method. Lastly, an application of the Engle-Granger, Cointegrating Regression Durban-Watson, and Phillips-Ouliaris methods using two time series, money supply (M1) and amount of coin and banknotes in circulation in South Africa, showed that the two series indeed cointegrated. This means that there is long-run equilibrium relationship between these variables.

CHAPTER 5

ERROR CORRECTION MODELS AND MULTIVARIATE COINTEGRATION

5.1 Introduction

The cointegration concept and the error correction models (ECM) have come to play a vital role in the analysis of macroeconomic time series. If two or more time series are cointegrated, then there exists a common stochastic trend that tends to move them together through time following a long-run equilibrium. However, this long-run equilibrium is disturbed by random shocks that are temporary, since the series eventually adjust for these. The process of adjusting for such shocks is what is referred Engle and Granger (1987) as error correction mechanism. In their paper, Engle and Granger (1987) showed that if two or more time series are cointegrated, then an error correction model (ECM) exists that is capable of capturing both short-term departures from the long-run equilibrium and the long-run equilibrium in the process.

While the Engle-Granger cointegration procedure discussed in Chapter 4 is most appropriate for systems of only two variables with one and only one possible cointegrating vector, it becomes inappropriate when there are several variables to consider. In this case, the Johansen cointegration test (to be discussed in Section 5.4) is preferred.

In the next section, we consider some theories. Section 5.3 discusses the ECM concept. The Johansen cointegration test is thoroughly discussed in Section 5.4. In Section 5.5, we consider the estimation of cointegrating vector and other forms of the vector error correction model (VECM). Section 5.6 presents a numerical example, while Section 5.5 provides a summary of the chapter.

5.2 Some Theoretical Considerations

Let us consider the n-dimensional vector $\mathbf{X}_t = \{X_{1,t}, X_{2,t}, \ldots, X_{n,t}\}_{t=1}^{T}$, where $X_{k,t} \sim I(1)$ for $k = 1,2,\ldots,n$ such that the vector $\mathbf{Y}_t = (\mathbf{X}_t - \mathbf{X}_{t-1}) \sim \mathbf{I(0)}$.

That is, the first-differenced vector has a zero mean and purely non-deterministic stationary. Then, employing the Wold decomposition theorem, we can write

$$(1-B)\mathbf{X}_t = \mathbf{\Pi}(B)\mathbf{A}_t = \left(\sum_{k=0}^{\infty}\mathbf{\Pi}_k B^k\right)\mathbf{A}_k = \sum_{k=0}^{\infty}\mathbf{\Pi}_k\mathbf{A}_{t-k}\,, \tag{5-1}$$

where B is the backshift operator and $\mathbf{A}_t$, an n-variate white-noise process with

$$E(\mathbf{A}_t) = 0\,, \tag{5-2}$$

and $$E(\mathbf{A}_t\mathbf{A}'_{t-k}) = \begin{cases} \mathbf{\Omega} & , \quad k=0 \\ \mathbf{0} & , \quad k=1,2,\ldots \end{cases} \tag{5-3}$$

The coefficients matrices $\mathbf{\Pi}_k$ are defined such that

$$\sum_{k=0}^{\infty}\mathbf{\Pi}_k\mathbf{\Pi}'_k \tag{5-4}$$

exists and hence

$$\mathbf{\Pi}(1) = \sum_{k=0}^{\infty}\mathbf{\Pi}_k \tag{5-5}$$

also exists. If we write $\mathbf{R}(B) = \mathbf{\Pi}(B) - \mathbf{\Pi}(1)$, then $\mathbf{R}(1) = \mathbf{0}$, and hence all the elements of $\mathbf{R}(B)$ have a unit root. Thus, we can write

$$\mathbf{\Pi}(B) = \mathbf{\Pi}(1) + (1-B)\mathbf{\Pi}^*(B)\,. \tag{5-6}$$

Multiplying through (5-6) by $\mathbf{A}_t$ yields

$$\mathbf{\Pi}(B)\mathbf{A}_t = \mathbf{\Pi}(1)\mathbf{A}_t + (1-B)\mathbf{\Pi}^*(B)\mathbf{A}_t\,, \tag{5-7}$$

or $$\mathbf{Y}_t = (1-B)\mathbf{X}_t = \mathbf{\Pi}(1)\mathbf{A}_t + (1-B)\mathbf{\Pi}^*(B)\mathbf{A}_t\,. \tag{5-8}$$

Following Granger (1986), the vector $\mathbf{X}_t$ is cointegrated if there an $n \times r$ vector $\boldsymbol{\alpha}$ (a cointegrating matrix) such that

$$\mathbf{Z}_t = \boldsymbol{\alpha}'\mathbf{X}_t. \tag{5-9}$$

Multiplying through (5-9) by $(1-B)$ and using (5-8) yields

$$(1-B)\mathbf{Z}_t = \boldsymbol{\alpha}'(1-B)\mathbf{X}_t = \boldsymbol{\alpha}'\left[\boldsymbol{\Pi}(1)\mathbf{A}_t + (1-B)\boldsymbol{\Pi}_t^*(B)\mathbf{A}_t\right],$$

or $$(1-B)\mathbf{Z}_t = (1-B)\boldsymbol{\alpha}'\boldsymbol{\Pi}^*(B)\mathbf{A}_t$$

$\Rightarrow$ $$\mathbf{Z}_t = \boldsymbol{\alpha}'\boldsymbol{\Pi}^*(B)\mathbf{A}_t. \tag{5-10}$$

Equation (5-10) means that $\mathbf{Z}_t$ is stationary if and only if $\boldsymbol{\Pi}^*(B)\mathbf{A}_t$ is stationary. A sufficient condition for $\boldsymbol{\Pi}^*(B)\mathbf{A}_t$ to be stationary is that the elements of $\boldsymbol{\Pi}^*(B)$ are rational polynomials. Hence, a sufficient condition is that the elements of $\boldsymbol{\Pi}(B)$ are rational polynomials, i.e.

$$\boldsymbol{\Pi}(B) = \frac{\boldsymbol{\Theta}(B)}{\boldsymbol{\Phi}(B)}, \tag{5-11}$$

where $\boldsymbol{\Theta}$ and $\boldsymbol{\Phi}$ are finite-lag polynomials with roots lying outside the unit circle. For n jointly determined variables, the cointegrating matrix will be of dimension $n \times n$, but rank $r \leq n-1$, where r is the number of linearly independent cointegrating equations. For instance, if $n = 3$ then $r = 2$, implying that there are *two* equilibrium relationships among the *three* variables. Estimation of the two cointegrating parameters may be obtained using the multivariate Engle-Granger two-step estimator or the Johansen maximum-likelihood estimator.

5.2.1 Autoregressive Distributed Lag (ARDL) Models

For simplicity, let us consider the case where $\mathbf{X}_t = \{X_{1,t}, X_{2,t}\}_{t=1}^{T}$. Then the autoregressive distributed lag (ARDL) model is given

$$\Phi(B)X_{1,t} = c + \Theta(B)X_{2,t} + a_t \tag{5-12}$$

where c is a constant, while $\Theta(B)$ and $\Phi(B)$ are finite-lag polynomials given by

$$\Theta(B) = \theta_0 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q B^q \tag{5-13a}$$

and
$$\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p . \tag{5-13b}$$

If both $\Theta(B)$ and $\Phi(B)$ are linear, then $p = q = 1$ and so (5-12) becomes

$$X_{1,t} = c + \phi_1 X_{1,t-1} + \theta_0 X_{2,t} + \theta_1 X_{2,t-1} + a_t . \tag{5-14}$$

5.3 Error Correction Models (ECM)

The ECM can be interpreted as a reparameterization of the general ARDL model given in (5-12) where a_t is the residual such that $a_t \sim i.i.d.N(0,\sigma^2)$. For instance, if both $\Theta(B)$ and $\Phi(B)$ are linear, then (5-14) can be rewritten as

$$X_{1,t} - X_{1,t-1} = c^* + \phi_1^* X_{1,t-1} + \theta_0^*(X_{2,t} - X_{2,t-1}) + \theta_1^* X_{2,t-1} + a_t , \tag{5-15}$$

$$\Rightarrow \qquad Y_{1,t} = c^* + \phi_1^* X_{1,t-1} + \theta_0^* Y_{2,t} + \theta_1^* X_{2,t-1} + a_t , \tag{5-16}$$

where $c^* = c$, $\phi_1^* = \phi_1 - 1$, $\theta_0^* = \theta_0$, and $\theta_1^* = \theta_0 + \theta_1$.

Alternatively, we can write (5-15) as

$$Y_{1,t} = \theta_0^* Y_{2,t} - \kappa.(X_{1,t-1} - b_0 - b_1 X_{2,t-1}), \tag{5-17}$$

where $\kappa = -\phi_1^*$, $b_0 = c^*/\phi_1^*$, $b_1 = (\theta_0 + \theta_1)/(1-\phi_1)$. In (5-17), the parameters, θ_0^*, κ, and b_1 interpreted as impact effects, a scalar adjustment coefficient, and long-run effects, respectively. For some specific applications, for instance, variables in logarithm form, we employ the restriction $b_1 = 0$ to ensure that all relevant ratios are constant in the long run. In this case, (5-17) becomes

$$Y_{1,t} = \alpha_0 + \theta_0^* Y_{2,t} - \kappa.(X_{1,t-1} - X_{2,t-1}), \tag{5-18}$$

where $\alpha_0 = \kappa b_0$. Substituting the relations $X_{1,t} = X_{1,t-1} = X_1$ and $X_{2,t} = X_{2,t-1} = X_2$ in (5-18) yields the static long-run solution

$$X_{1,t} = \frac{\alpha_0}{\kappa} + X_{2,t}. \tag{5-19}$$

In their paper, Davidson et al (1978) interpreted $\exp(\alpha_0/\kappa)$ as an estimate of the long-run average propensity to consume in a consumption function postulating proportionality between consumption expenditure and income.

5.3.1 The Engle-Granger Error Correction Equation

Equation (5-18) has been a bone of contention among time series statisticians. A question that arises from this equation is "how could a stationary time series $Y_{1,t}$ be explained by two non-stationary time series, $X_{1,t}$ and $X_{2,t}$. Simply put, since the two sides of the equation have different order of integration, (5-18) becomes questionable, unless the linear combination $(X_{1,t} - \theta_0^* X_{2,t})$ is $I(0)$.

This has been the motivation for the error correction equation. The vector $\mathbf{X}_t = \{X_{1,t}, X_{2,t},..., X_{n,t}\}_{t=1}^{T}$ which is $I(1)$ for each $r = 1,2,...,n$, has an error correction representation

$$\mathbf{P}(B)(1-B)\mathbf{X}_t = -\boldsymbol{\gamma}.\mathbf{Z}_{t-1} + \mathbf{A}_t \qquad \text{or} \qquad \mathbf{P}(B)\mathbf{Y}_t = -\boldsymbol{\gamma}.\mathbf{Z}_{t-1} + \mathbf{A}_t, \qquad (5\text{-}20)$$

where B is the backshift operator, γ is a matrix of coefficients $(n \times r)$ of rank r. $\mathbf{Z}_t = \boldsymbol{\alpha}'\mathbf{X}_t$ is an $(r \times 1)$ matrix, while $\mathbf{A}_t$ is a stationary multivariate disturbance term. The equilibrium is interpreted as $\mathbf{Z}_t = \mathbf{0}$. Hence $\mathbf{Z}_t$ is a measure of deviation from equilibrium.

5.4 The Johansen Cointegration Test

Given a group of time series, $\mathbf{X}_t = \{X_{1,t}, X_{2,t},..., X_{n,t}\}_{t=1}^{T}$ where $X_{k,t} \sim I(1)$ for $k = 1,2,...,n$, the Johansen cointegration approach to testing for cointegration tests the restrictions imposed by cointegration on the unrestricted vector autoregressive (VAR) process

$$\mathbf{X}_t = \mathbf{A}_1\mathbf{X}_{t-1} + \mathbf{A}_2\mathbf{X}_{t-2} + ... + \mathbf{A}_p\mathbf{X}_{t-p} + \mathbf{D}.\mathbf{T}_t + \mathbf{a}_t, \qquad (5\text{-}21)$$

where $\mathbf{T}_t$ is a vector of deterministic variables, and $\mathbf{a}_t$ is a vector of disturbances. $\mathbf{A}_1, \mathbf{A}_2,..., \mathbf{A}_p$ and $\mathbf{D}$ are matrices of coefficients to be estimated. Let rewrite (5-21) as

$$(\mathbf{I} - \mathbf{A}_1 B - \mathbf{A}_2 B^2 - ... - \mathbf{A}_p B^p)\mathbf{X}_t = \mathbf{D}.\mathbf{T}_t + \mathbf{a}_t, \qquad (5\text{-}22a)$$

$$\text{or} \qquad [\mathbf{I} - \mathbf{A}(B)]\mathbf{X}_t = \mathbf{D}.\mathbf{T}_t + \mathbf{a}_t, \qquad (5\text{-}22b)$$

where $\mathbf{A}(B) = \mathbf{A}_1 B - \mathbf{A}_2 B^2 - ... - \mathbf{A}_p B^p$.

Let

$$\boldsymbol{\rho} = \sum_{i=1}^{p} \mathbf{A}_i \qquad \text{and} \qquad \boldsymbol{\Gamma}_i = -\sum_{j=i+1}^{p} \mathbf{A}_i \,, \quad i = 1,2,\ldots, p-1 \,. \tag{5-23}$$

Then

$$(\mathbf{I} - \boldsymbol{\rho}B) - (\mathbf{A}_1 B + \mathbf{A}_2 B^2 + \ldots + \mathbf{A}_{p-1} B^{p-1})$$
$$= \mathbf{I} - \boldsymbol{\rho}B - (\mathbf{A}_1 B + \mathbf{A}_2 B^2 + \ldots + \mathbf{A}_{p-1} B^{p-1} - \mathbf{A}_1 B^2 + \mathbf{A}_2 B^3 + \ldots + \mathbf{A}_{p-1} B^p)$$

or

$$(\mathbf{I} - \boldsymbol{\rho}B) - \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i B^i (1 - B) = \mathbf{I} - (\boldsymbol{\rho} + \boldsymbol{\Gamma}_1) B - (\mathbf{A}_2 - \mathbf{A}_1) B^2 - (\mathbf{A}_3 - \mathbf{A}_2) B^3 - \ldots$$
$$\ldots. - (\mathbf{A}_{p-1} - \mathbf{A}_{p-2}) - (-\mathbf{A}_{p-1}) B^p \tag{5-24}$$

Also, we have

$$\boldsymbol{\rho} + \boldsymbol{\Gamma}_1 = \sum_{i=1}^{p} \mathbf{A}_i - \sum_{i=2}^{p} \mathbf{A}_i = \mathbf{A}_1 + \sum_{i=2}^{p} \mathbf{A}_i - \sum_{i=2}^{p} \mathbf{A}_i = \mathbf{A}_1$$

$$\boldsymbol{\Gamma}_2 - \boldsymbol{\Gamma}_1 = -\sum_{i=3}^{p} \mathbf{A}_i - \left(- \sum_{i=2}^{p} \mathbf{A}_i \right) = -\sum_{i=3}^{p} \mathbf{A}_i + \mathbf{A}_2 + \sum_{i=3}^{p} \mathbf{A}_i = \mathbf{A}_2 \,,$$

$$\boldsymbol{\Gamma}_3 - \boldsymbol{\Gamma}_2 = -\sum_{i=4}^{p} \mathbf{A}_i - \left(- \sum_{i=3}^{p} \mathbf{A}_i \right) = -\sum_{i=4}^{p} \mathbf{A}_i + \mathbf{A}_3 + \sum_{i=4}^{p} \mathbf{A}_i = \mathbf{A}_3 \,,$$

$$\begin{matrix} . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{matrix}$$

$$\boldsymbol{\Gamma}_{p-1} - \boldsymbol{\Gamma}_{p-2} = -\sum_{i=p}^{p} \mathbf{A}_i - \left(- \sum_{i=p-1}^{p} \mathbf{A}_i \right) = -\sum_{i=p}^{p} \mathbf{A}_i + \mathbf{A}_{p-1} + \sum_{i=p}^{p} \mathbf{A}_i = \mathbf{A}_{p-1} \,.$$

Employing the results above in (5-24) yields

$$(\mathbf{I}-\boldsymbol{\rho}B)-\sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i B^i(1-B)=\mathbf{I}-\mathbf{A}_1B-\mathbf{A}_2B^2-\ldots-\mathbf{A}_{p-1}B^{p-1}-\mathbf{A}_pB^p\,, \tag{5-25}$$

where we have used the result, $-\boldsymbol{\Gamma}_{p-1}=\mathbf{A}_p$. Substituting (5-25) in (5-22) gives

$$[(\mathbf{I}-\boldsymbol{\rho}B)-\sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i B^i(1-B)]\mathbf{X}_t=\mathbf{D}.\mathbf{T}_t+\mathbf{a}_t$$

$$(\mathbf{I}-\boldsymbol{\rho}B)\mathbf{X}_t=\sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i B^i(1-B)\mathbf{X}_t+\mathbf{D}.\mathbf{T}_t+\mathbf{a}_t$$

$$\mathbf{X}_t=\boldsymbol{\rho}B\mathbf{X}_t+\sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i(B^i\mathbf{X}_t-B^{i+1}\mathbf{X}_t)+\mathbf{D}.\mathbf{T}_t+\mathbf{a}_t$$

$$\Rightarrow \qquad \mathbf{X}_t=\boldsymbol{\rho}\mathbf{X}_{t-1}+\sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i(\mathbf{X}_{t-i}-\mathbf{X}_{t-i-1})+\mathbf{D}.\mathbf{T}_t+\mathbf{a}_t\,. \tag{5-26}$$

Subtracting $\mathbf{X}_{t-1}$ from both sides of (5-26) yields

$$\mathbf{X}_t-\mathbf{X}_{t-1}=\boldsymbol{\rho}\mathbf{X}_{t-1}-\mathbf{X}_t+\sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i(\mathbf{X}_{t-i}-\mathbf{X}_{t-i-1})+\mathbf{D}.\mathbf{T}_t+\mathbf{a}_t$$

$$\mathbf{Y}_t=(\boldsymbol{\rho}-\mathbf{I})\mathbf{X}_{t-1}+\sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\mathbf{Y}_{t-i}+\mathbf{D}.\mathbf{T}_t+\mathbf{a}_t\,, \tag{5-27}$$

$$\mathbf{Y}_t=\boldsymbol{\Pi}\mathbf{X}_{t-1}+\sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\mathbf{Y}_{t-i}+\mathbf{D}.\mathbf{T}_t+\mathbf{a}_t\,, \qquad \mathbf{a}_t\sim i.i.d.N(\mathbf{0},\boldsymbol{\Omega}) \tag{5-28}$$

where $\mathbf{Y}_{t-i}=\mathbf{X}_{t-i}-\mathbf{X}_{t-i-1}$, $i=0,1,2,\ldots,p-1$, and $\boldsymbol{\Pi}=\boldsymbol{\rho}-\mathbf{I}$ is a $n\times n$ matrix. $\mathbf{I}$ is the identity matrix. Equation (5-28) is the *vector* ECM (or VECM) form VAR given in (5-21).

5.5 Estimation of Cointegrating Equation and Other Forms of the VECM

If $\mathbf{\Pi}$ is of full rank n, then $\mathbf{X}_t$ is stationary. If the rank of $\mathbf{\Pi}$ is zero (i.e. $\mathbf{\Pi}$ is the null matrix), then the undifferenced $\mathbf{X}_t$ has no effect and a model in first differences is appropriate. If the rank of $\mathbf{\Pi}$ is $r \in (0,n)$, then there are $n \times r$ matrices, $\boldsymbol{\gamma}$ and $\boldsymbol{\alpha}$, such that

$$\mathbf{\Pi} = \boldsymbol{\gamma}.\boldsymbol{\alpha}'. \tag{5-29}$$

In that case, there r cointegrating vectors and (5-28) becomes

$$\mathbf{Y}_t = \boldsymbol{\gamma}\boldsymbol{\alpha}'.\mathbf{X}_{t-1} + \sum_{i=1}^{p-1} \mathbf{\Gamma}_i \mathbf{Y}_{t-i} + \mathbf{D}.\mathbf{T}_t + \mathbf{a}_t . \tag{5-30}$$

Following Johansen and Juselius (1990), the estimation procedure first concentrates the likelihood to remove the effects of the right-hand side differences by regressing $\mathbf{Y}_t$ and $\mathbf{X}_{t-1}$ on $\mathbf{Y}_{t-i}$ for $i = 1,2,..., p-1$. These regressions produce $n \times 1$ residual vectors $\mathbf{R}_{0,t}$, from regressing $\mathbf{Y}_t$ on $\mathbf{Y}_{t-i}$, and $\mathbf{R}_{1,t}$, from $\mathbf{X}_{t-1}$ on $\mathbf{Y}_{t-i}$, with covariance matrix

$$\mathbf{S}_{ij} = \tfrac{1}{T}\sum_{t=1}^{T} \mathbf{R}_{i,t}\mathbf{R}'_{j,t}, \qquad i,j = 0,1. \tag{5-31}$$

The roots, $\boldsymbol{\lambda} = (\lambda_1, \lambda_2,..., \lambda_r, \lambda_{r+1},...\lambda_n)$, of the characteristic equation,

$$|\boldsymbol{\lambda}\mathbf{S}_{11} - \mathbf{S}_{10}\mathbf{S}_{00}^{-1}\mathbf{S}_{01}| = 0 \tag{5-32}$$

give the eigenvalues $1 > \lambda_1 > \lambda_2 > .. > \lambda_r > \lambda_{r+1} > ... > \lambda_n > 0$ and $\lambda_{n+1} = 0$. The corresponding eigenvectors $\mathbf{v} = (v_1, v_2,..., v_{n+1})$ are normalised as

$$\mathbf{v}'\mathbf{S}_{11}\mathbf{v} = \mathbf{I}. \tag{5-33}$$

The eigenvalues $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_r, \lambda_{r+1}, \ldots \lambda_n)$ are used in many ways. It is used for determining the number of cointegrating equations, r. Testing the null hypothesis that the distinct number of cointegrating equations employs the *trace test* statistic

$$\Psi(r) = -T \sum_{i=r+1}^{n} \ln(1-\lambda_i), \qquad r = 0,1,2,\ldots,n-2,n-1. \qquad (5\text{-}34)$$

Johansen (1988, 1989) suggested a second statistic (the eigen-statistic) for testing the null hypothesis that the number of cointegrating equations is r against the alternative of $r+1$:

$$\Psi_{\max}(r+1) = -T.\ln(1-\lambda_{r+1}), \quad r = 0,1,2,\ldots,n-2,n-1. \qquad (5\text{-}35)$$

Critical values for these statistics are given in Johansen (1988, 1989) and Hall (1989). Using r cointegration equations, Johansen (1995) further gave the following possible forms for (5-28):

Form 5.1: If $X_{1,t}, X_{2,t}, \ldots, X_{n,t}$ has no deterministic trends and the cointegrating equations do not have intercepts, then

$$\mathbf{\Pi}.\mathbf{X}_{t-1} + \mathbf{D}.\mathbf{T}_t = \boldsymbol{\alpha\beta}'.\mathbf{X}_{t-1}. \qquad (5\text{-}36)$$

Form 5.2: If $X_{1,t}, X_{2,t}, \ldots, X_{n,t}$ have no deterministic trends and the cointegrating equations have intercepts, then

$$\mathbf{\Pi}.\mathbf{X}_{t-1} + \mathbf{D}.\mathbf{T}_t = \boldsymbol{\alpha}(\boldsymbol{\beta}'.\mathbf{X}_{t-1} + \boldsymbol{\rho}_0). \qquad (5\text{-}37)$$

Form 5.3: If $X_{1,t}, X_{2,t}, \ldots, X_{n,t}$ have linear time trends and the cointegrating equations have only intercepts, then

$$\mathbf{\Pi}.\mathbf{X}_{t-1} + \mathbf{D}.\mathbf{T}_t = \boldsymbol{\alpha}(\boldsymbol{\beta}'.\mathbf{X}_{t-1} + \boldsymbol{\rho}_0) + \boldsymbol{\alpha}_{\perp}\boldsymbol{\gamma}_0. \qquad (5\text{-}38)$$

Form 5.4: If $X_{1,t}, X_{2,t}, \ldots, X_{n,t}$ and the cointegrating equations have linear time trends, then

$$\mathbf{\Pi}.\mathbf{X}_{t-1} + \mathbf{D}.\mathbf{T}_t = \boldsymbol{\alpha}(\boldsymbol{\beta}'.\mathbf{X}_{t-1} + \boldsymbol{\rho}_0 + \boldsymbol{\rho}_1 t) + \boldsymbol{\alpha}_\perp \boldsymbol{\gamma}_0. \tag{5-39}$$

Form 5.5: If $X_{1,t}, X_{2,t}, \ldots, X_{n,t}$ have quadratic time trends and the cointegrating regressions have linear time trends, then

$$\mathbf{\Pi}.\mathbf{X}_{t-1} + \mathbf{D}.\mathbf{T}_t = \boldsymbol{\alpha}(\boldsymbol{\beta}'.\mathbf{X}_{t-1} + \boldsymbol{\rho}_0 + \boldsymbol{\rho}_1 t) + \boldsymbol{\alpha}_\perp (\boldsymbol{\gamma}_0 + \boldsymbol{\gamma}_1 t). \tag{5-40}$$

In (5-36)-(5-40), $\boldsymbol{\alpha}_\perp$ is the $n \times (n-r)$ matrix such that $\boldsymbol{\alpha}'\boldsymbol{\alpha}_\perp = 0$ and $\text{rank}([\boldsymbol{\alpha} \mid \boldsymbol{\alpha}_\perp]) = n$.

5.6 Multivariate AIC and SBC Criteria and Selection of Optimal Lag Length

An optimal lag length, p, can be determined by using the multivariate generalizations of the Akaike Information Criterion (AIC) and Schwarz Bayesian Criterion (SBC) given by

$$\text{AIC} = -\frac{2L}{T} + \frac{2n^*}{T} \quad \text{and} \quad \text{SBC} = -\frac{2L}{T} + \frac{n^* \ln T}{T}, \tag{5-41}$$

where n^* is the total number of the estimated parameters in the VAR model. L is the log-likelihood value computed assuming multivariate normal distribution:

$$L = -\frac{nT}{2}[1 + \ln(2\pi)] - \frac{T}{2}\ln|\hat{\boldsymbol{\Sigma}}|, \tag{5-42}$$

where $|\hat{\boldsymbol{\Sigma}}|$ is the determinant of the residual covariance given by

$$|\hat{\boldsymbol{\Sigma}}| = \det\left(\sum_{t=1}^{T} \frac{\hat{\mathbf{a}}_t \hat{\mathbf{a}}_t'}{T}\right). \tag{5-43}$$

5.7 A Numerical Example

In this section, we illustrate the Johansen methodology using two time series data. The two series are monthly exchange rate of the South African rand (in cents) to the British pound and the U.S. dollar, spanning the period January 1992 to October 2000. The data are contained in the Appendix. Exhibit 5.1 is a representation of the two series.

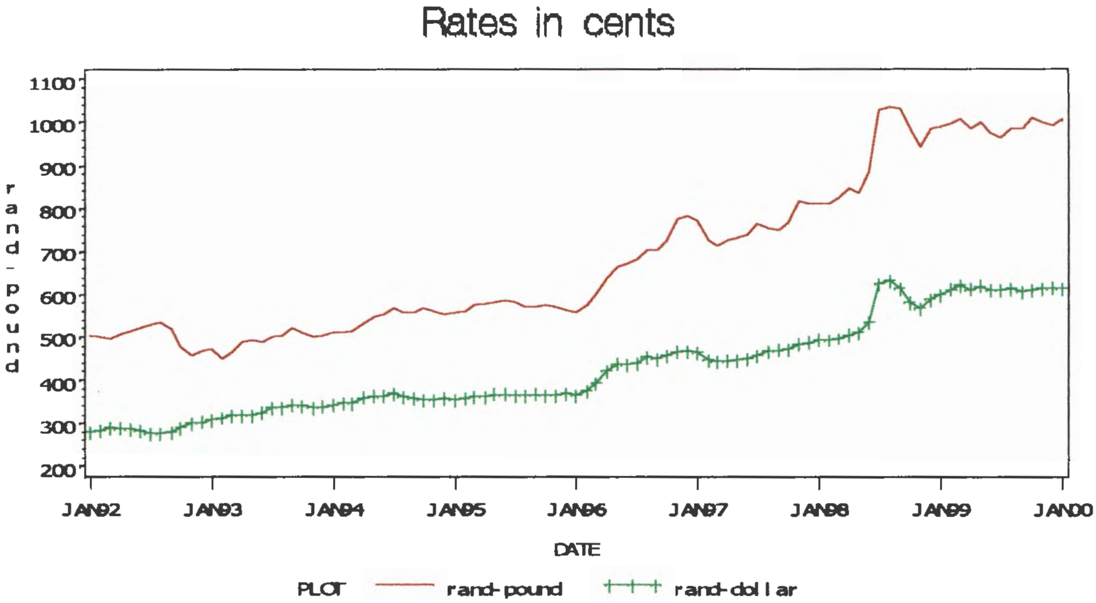


Exhibit 5.1: Rand-Pound and Rand-Dollar Exchange Rates

Clearly, the two series exhibit a drift, hence we allow a drift term $\mathbf{c} = (c_1, c_2, ..., c_n)'$ in all our equations. Therefore, we consider determining the optimal system lag length p for the following VAR(p) process in levels:

$$\mathbf{X}_t = \mathbf{c} + \mathbf{X}_{t-1} + \mathbf{X}_{t-2} + ... + \mathbf{X}_{t-p} + \mathbf{a}_t . \qquad (5\text{-}41)$$

Next, we employ the multivariate generalizations of the AIC or SBC to determine the optimal lag length p. Beginning with lag length of $p = 6$ and moving to shorter lengths, the VAR(p) estimation output for $p = 6,5,4,..,1$ are given in Table 5.1 (see Appendix for complete results).

Table 5.1: Estimation

	p					
	1	2	3	4	5	6
Determinant Residual Covariance	32588.48	29000.42	25976.16	24856.63	23344.25	21047.33
Log Likelihood	-843.5421	-829.4430	-815.7960	-805.6280	-794.5600	-781.5140
Akaike Information Criterion	16.18175	16.14313	16.11253	16.14958	16.16950	16.15028
Schwarz Criterion	16.33341	16.39740	16.47065	16.61281	16.73913	16.82763

Using the AIC criterion, the optimal lag length is $p = 3$. We therefore estimate the following models.

VAR: $$\mathbf{X}_t = \mathbf{c} + \mathbf{A}_1\mathbf{X}_{t-1} + \mathbf{A}_2\mathbf{X}_{t-2} + \mathbf{A}_3\mathbf{X}_{t-3} + \mathbf{a}_t \quad (5\text{-}42)$$

VECM: $$\mathbf{Y}_t = \mathbf{c} + \mathbf{\Pi}.\mathbf{X}_{t-1} + \sum_{i=1}^{3-1}\Gamma_i.\mathbf{Y}_{t-i} + \mathbf{a}_t$$

or $$\mathbf{Y}_t = \mathbf{c} + \mathbf{\Pi}.\mathbf{X}_{t-1} + \Gamma_1\mathbf{Y}_{t-1} + \Gamma_2\mathbf{Y}_{t-2} + \mathbf{a}_t. \quad (5\text{-}43)$$

Outputs for VAR and VECM estimation are given in Table 5.2 and Table 5.3, respectively. The estimated VAR(3) model is

$$X_{1,t} = 1.326 + 1.107X_{1,t-1} - 0.494X_{1,t-2} + 0.295X_{1,t-3} + 0.361X_{2,t-1} + 0.022X_{2,t-2} - 0.230X_{2,t-3}$$

$$X_{2,t} = -1.234 + 1.433X_{2,t-1} - 0.524X_{2,t-2} + 0.136X_{2,t-3} - 0.020X_{1,t-1} - 0.091X_{1,t-2} + 0.090X_{1,t-3}.$$

The estimated VECM model is

$$Y_{1,t} = -0.008X_{1,t} + 0.152Y_{1,t-1} + 0.290Y_{2,t-1} - 0.345Y_{1,t-2} + 0.314Y_{2,t-2}$$

$$Y_{2,t} = -0.008X_{2,t} + 0.401Y_{2,t-1} - 0.007Y_{1,t-1} - 0.123Y_{2,t-2} - 0.098Y_{1,t-2}.$$

Table 5.2: VAR(3) Estimation

Sample(adjusted): 1992:04 - 2000:10
Included observations: 103 after adjusting endpoints
Standard errors & t-statistics in parentheses

	POUND	DOLLAR
POUND(-1)	1.107083	-0.020481
	(0.15127)	(0.08367)
	(7.31848)	(-0.24478)
POUND(-2)	-0.494362	-0.090900
	(0.23067)	(0.12759)
	(-2.14313)	(-0.71245)
POUND(-3)	0.295393	0.090326
	(0.15844)	(0.08764)
	(1.86434)	(1.03068)
DOLLAR(-1)	0.361400	1.432674
	(0.26716)	(0.14777)
	(1.35276)	(9.69540)
DOLLAR(-2)	0.022476	-0.524333
	(0.42292)	(0.23392)
	(0.05314)	(-2.24147)
DOLLAR(-3)	-0.230319	0.136590
	(0.28573)	(0.15804)
	(-0.80607)	(0.86427)
C	1.326068	-1.234057
	(8.06350)	(4.46002)
	(0.16445)	(-0.27669)
R-squared	0.989338	0.991655
Adj. R-squared	0.988672	0.991133
Sum sq. resids	45327.12	13867.08
S.E. equation	21.72919	12.01868
F-statistic	1484.703	1901.228
Log likelihood	-459.6277	-398.6317
Akaike AIC	9.060731	7.876343
Schwarz SC	9.239790	8.055403
Mean dependent	720.1323	451.9971
S.D. dependent	204.1580	127.6349

Table 5.3: VECM Estimates

Sample(adjusted): 1992:04 - 2000:10
Included observations: 103 after adjusting endpoints
Standard errors & t-statistics in parentheses

Cointegrating Eq:	CointEq1	
POUND(-1)	1.000000	
DOLLAR(-1)	-3.141453	
	(6.95342)	
	(-0.45179)	
C	217.5799	
	(1097.00)	
	(0.19834)	
Error Correction:	**D(POUND)**	**D(DOLLAR)**
CointEq1	-0.008516	-0.007966
	(0.00466)	(0.00256)
	(-1.82580)	(-3.11207)
D(POUND(-1))	0.151784	-0.006834
	(0.14862)	(0.08156)
	(1.02131)	(-0.08379)
D(POUND(-2))	-0.345406	-0.098178
	(0.15279)	(0.08385)
	(-2.26067)	(-1.17088)
D(DOLLAR(-1))	0.290056	0.400650
	(0.26726)	(0.14667)
	(1.08529)	(2.73163)
D(DOLLAR(-2))	0.314276	-0.123409
	(0.27589)	(0.15141)
	(1.13912)	(-0.81508)
R-squared	0.124589	0.183714
Adj. R-squared	0.088858	0.150397
Sum sq. resids	46107.80	13886.32
S.E. equation	21.69073	11.90366
F-statistic	3.486843	5.514008
Log likelihood	-460.5071	-398.7031
Akaike AIC	9.038973	7.838895
Schwarz SC	9.166872	7.966795
Mean dependent	5.704078	4.452718
S.D. dependent	22.72380	12.91435
Determinant Residual Covariance		26644.70
Log Likelihood		-817.1041
Akaike Information Criteria		16.11853
Schwarz Criteria		16.45107

Table 5.4: Johansen Cointegration Test

```
Sample: 1992:01 - 2000:10
Included observations: 103
Test assumption: No deterministic trend in the data
Series: DOLLAR  POUND
Exogenous series: C
Warning: Critical values were derived assuming no exogenous series
Lags interval: 1 to 2
```

Eigenvalue	Likelihood Ratio	5 Percent Critical Value	1 Percent Critical Value	Hypothesized No. of CE(s)
0.094698	12.86445	19.96	24.60	None
0.025091	2.617342	9.24	12.97	At most 1

*(**) denotes rejection of the hypothesis at 5%(1%) significance level
L.R. rejects any cointegration at 5% significance level

Unnormalized Cointegrating Coefficients:

DOLLAR	POUND	C
-0.000675	0.000215	0.046785
0.004538	-0.002984	0.113772

Normalized Cointegrating Coefficients: 1 Cointegrating Equation(s)

	DOLLAR	POUND	C
	1.000000	-0.318324	-69.26091
		(0.70459)	(211.182)
Log likelihood		-817.1041	

From the results in Table 5.4, the trace statistic is the likelihood ratio. Testing the null hypothesis of non-cointegration is equivalent to testing the trace statistic $\Psi(r)$ with no cointegrating equations, i.e. when the number of *hypothesized CE's* is *none*. In this case

$$H_0 : \Psi_1(r = 0) = 12.86445 \quad \text{non-cointegration.}$$

At 5% level of significance, the critical value is 19.96. Since the trace statistic of 12.86445 is less than the critical value of 19.96, we cannot reject the null hypothesis of non-cointegration.

Next, we test the null hypothesis of one cointegrating equation, i.e. $r = 1$. When there is at most 1 hypothesised number of CE's, the corresponding trace value $\Psi(1)$ is 2.617342. We therefore test the null hypothesis

$$H_0 : \Psi(r = 1) = 2.617342 \qquad \text{(1 cointegrating equation).}$$

At 5% level of significance, the critical value is 9.24. Since trace value is less than the critical value, we cannot reject the null hypothesis that there is one cointegrating equation. The same conclusions can be drawn at the 1% level of significance. Next, we test the null hypothesis that there is more than one cointegrating equation, i.e. $r \geq 1$. $r = 1$. Moreover, when there is 1 hypothesised CE, the maximum eigen-statistic is

$$\Psi_{\max}(r = 1) = -103\ln(1 - 0.025091) = 2.617348$$

At the 5% level of significance, the critical value from Enders (1995) adopted from Osterwald-Lenum (1992) is 9.094. Since 2.617348 is less than the critical value of 9.094, the null hypothesis that $r = 1$ cannot be rejected, at 5% significance level. This conclusion also holds at the 1% level of significance. Lastly with $r = 1$, the cointegrating equation is

$$(\text{Dollar} - 0.31824\text{Pound} - 69.26091).$$

5.8 Summary

From time series analysis and economic theory, a linear combination of certain non-stationary variables are stationary. Such variables are referred to as cointegrated variables. In this chapter, we have discussed cointegration in the multivariate context with particular attention to the Johansen methodology. We also considered the error correction representation of cointegrated variables. An application of the methodology to two time series data, monthly exchange rate the rand to the British pound and U.S. dollar showed that the two series are cointegrated and that there exists only one cointegrating equation.

CHAPTER 6

SUMMARY AND SUGGESTIONS FOR FURTHER RESEARCH

6.1 Summary

The tradition of using the sample autocorrelation and partial autocorrelation functions to determine whether a series is stationary or otherwise, has become a thing of the past. In this modern era, testing for a unit root has become a common starting point in applied time series analysis. The consequences of ignoring this procedure have been documented in the literature. Various unit root estimation techniques have been discussed several authors. Most commonly applied unit root test procedures have been the Augmented Dickey-Fuller (ADF) and Phillips-Perron tests due to Dickey and Fuller (1979) and Phillips and Perron (1988). A frequency domain approach due Akdi and Dickey (1998) and Evans and Dickey (1998) have also been found to be effective. An application of a unit root test using local data on Leading Indicator Indices (LII) for South Africa established that the series contains a unit root and hence is non-stationary.

We have also considered the situation where the series might be seasonally non-stationary. Seasonal unit root tests were discussed with special attention to quarterly data. Again, an application of this concept using monthly U.S. liquor consumption data confirmed the presence of a seasonal unit root. Unit root tests form the bedrock of the concept of cointegration. The concept of cointegration and error correction model (ECM) has come to play an important role in much of macroeconomic time series analysis in recent years. Financial statisticians as well as econometricians, in much of their work, may want to find out whether there is a long-run relationship among variables. Some tests have been developed in the literature and some of these have been discussed in this study. The Engle-Granger method is preferable when the analysis involves two time series data.

Our aim to conduct an empirical research into the efficiency of the Phillips-Ouliaris cointegrating test via the OLS and Yule-Walker methods also revealed that the test performs better when the Yule-Walker method is used instead of the OLS method. A numerical example of the Engle-Granger, Cointegrating Regression Durban-Watson, and Phillips-Ouliaris methods using data on money supply (M1) and amount of coin and banknotes in circulation in South Africa showed that the two series are indeed cointegrated, hence the existence of a long-run equilibrium relationship between these variables. In the multivariate context, the Johansen cointegrating test is most appropriate. An application of the Johansen cointegration test using monthly data on exchange rates of the rand to the British pound and the U.S. dollar established a long-run relationship between these variables.

6.2 Suggestions for Further Research

We suggest the use of the Bootstrap methods to investigate the performance of any of the cointegration tests discussed in this study, more importantly to investigate the performance of the Phillips-Ouliaris cointegration test via the OLS and Yule-Walker methods. Results can then be compared with those obtained using Monte Carlo methods.

Leading Indicator Indices (Seasonally Adjusted: 1990=100)

	1993	1994	1995	1996	1997	1998	1999	2000
Jan	*	99.3	109.5	109.3	112.2	106.7	98.9	117.9
Feb	*	100.1	109.4	107.9	111.3	107.2	99.2	120.5
Mar	*	101	109.9	107.7	110.3	107.2	101.8	120.2
Apr	*	99.8	106.8	108	111	107.4	101.8	116.3
May	*	101.8	109.3	109.1	111.8	104.4	102.5	117
Jun	*	103.8	109.6	108.3	113.1	103.8	103.8	118.3
Jul	*	105.1	108.9	107.9	113	101.7	106.8	
Aug	*	106.8	108.9	107	111.2	99	108.1	
Sep	96.7	106.8	109.4	107.1	111.6	97	109.3	
Oct	97.7	108.6	109.5	110.7	111.9	98.4	112.4	
Nov	99.3	111	109.8	111.4	109.8	98.3	115.4	
Dec	98.8	109.8	109.7	111.5	109.5	98.8	117.1	

U.S. Liquor Sales (in millions of dollars)

	1970	1971	1972	1973	1974	1975
Jan	580	650	669	734	789	860
Feb	514	594	652	707	744	799
Mar	555	650	743	785	827	899
Apr	563	668	709	762	831	866
May	627	712	751	838	895	1016
Jun	596	731	774	876	889	978
Jul	632	779	803	878	955	1042
Aug	639	712	760	871	983	1026
Sep	577	708	749	807	976	944
Oct	611	738	757	834	929	1002
Nov	639	758	779	877	989	1009
Dec	875	1073	1066	1236	1294	1368

	1976	1977	1978	1979	1980
Jan	908	910	950	1071	1294
Feb	849	927	932	1044	1258
Mar	916	981	1049	1158	1301
Apr	958	1011	1021	1122	1297
May	1008	1041	1097	1209	1425
Jun	1033	1080	1151	1334	1378
Jul	1129	1138	1194	1360	1429
Aug	1019	1072	1174	1368	1452
Sep	984	1033	1160	1297	1305
Oct	1045	1072	1135	1283	1377
Nov	1049	1111	1209	1375	1439
Dec	1459	1591	1692	1974	1958

Money Supply M1 and Amount of Coin and Banknotes in Circulation in South Africa (in millions of rands)

	coinote	M1		coinote	M1		coinote	M1
Oct-90	7553	46828	Feb-94	10721	83180	Jun-97	15535	150270
Nov-90	8446	50315	Mar-94	11858	84606	Jul-97	15641	146147
Dec-90	8064	53048	Apr-94	11688	86038	Aug-97	16793	152896
Jan-91	7972	49952	May-94	11627	91411	Sep-97	15983	164340
Feb-91	8121	56666	Jun-94	11359	91011	Oct-97	16804	166127
Mar-91	8588	56221	Jul-94	11997	88986	Nov-97	18017	172057
Apr-91	8309	54589	Aug-94	11270	89588	Dec-97	17308	173057
May-91	8562	57014	Sep-94	12135	90158	Jan-98	17660	171943
Jun-91	8410	57561	Oct-94	11759	89644	Feb-98	17939	178516
Jul-91	8151	57873	Nov-94	12458	92033	Mar-98	16784	177163
Aug-91	8612	59434	Dec-94	12237	94538	Apr-98	17754	183365
Sep-91	8453	58074	Jan-95	12005	88549	May-98	17978	183147
Oct-91	8381	58465	Feb-95	12241	93353	Jun-98	17114	203346
Nov-91	9280	62476	Mar-95	12473	93387	Jul-98	17666	205516
Dec-91	8834	60910	Apr-95	13099	94149	Aug-98	17356	219645
Jan-92	8893	58098	May-95	12351	98090	Sep-98	16882	219582
Feb-92	9104	61644	Jun-95	12989	98088	Oct-98	17912	210298
Mar-92	8645	64192	Jul-95	12821	95363	Nov-98	18409	216128
Apr-92	9249	61058	Aug-95	13064	97799	Dec-98	18505	213921
May-92	9415	60758	Sep-95	13725	101604	Jan-99	18642	209814
Jun-92	8733	63946	Oct-95	12977	98338	Feb-99	18985	205021
Jul-92	9336	63087	Nov-95	13846	104613	Mar-99	18258	213116
Aug-92	8976	68303	Dec-95	14331	112745	Apr-99	19195	212422
Sep-92	9126	71520	Jan-96	13546	105904	May-99	18758	213332
Oct-92	9522	70252	Feb-96	13992	112480	Jun-99	18545	220402
Nov-92	9851	72255	Mar-96	14594	118571	Jul-99	19856	220745
Dec-92	9535	71571	Apr-96	14183	119453	Aug-99	19382	227461
Jan-93	9939	66466	May-96	14818	121754	Sep-99	19730	228899
Feb-93	10022	70355	Jun-96	15111	126196	Oct-99	20917	238211
Mar-93	9561	70843	Jul-96	14326	122752	Nov-99	20943	255230
Apr-93	10037	71248	Aug-96	15010	128655	Dec-99	22660	258284
May-93	10331	70314	Sep-96	14762	132844	Jan-00	20830	246137
Jun-93	10016	70995	Oct-96	14815	134880	Feb-00	20486	256900
Jul-93	10337	67075	Nov-96	16101	140160	Mar-00	20945	252871
Aug-93	10244	70730	Dec-96	15938	147580	Apr-00	21874	256265
Sep-93	10038	71503	Jan-97	15552	139956	May-00	20714	251623
Oct-93	10429	69928	Feb-97	15943	145335	Jun-00	21726	257606
Nov-93	10550	74185	Mar-97	16392	154227	Jul-00	21467	245023
Dec-93	10482	76398	Apr-97	15764	146411	Aug-00	20929	246519
Jan-94	10516	77830	May-97	15979	146954	Sep-00	22139	261145

Exchange Rates of the South African Rand to the British Pound and U.S Dollar (in cents)

	pound	dollar		pound	dollar		pound	dollar
Jan-92	503.78	277.93	Jan-95	557.02	353.8	Dec-97	810.22	487.07
Feb-92	501	281.53	Feb-95	559.62	355.93	Jan-98	807.75	493.91
Mar-92	496.77	288.1	Mar-95	576.12	359.92	Feb-98	809.05	493.57
Apr-92	505.31	287.85	Apr-95	579.25	360.06	Mar-98	825.43	497.11
May-92	515.73	284.74	May-95	581.21	365.8	Apr-98	843.73	504.61
Jun-92	520.29	280.98	Jun-95	583.86	366.19	May-98	833.45	509.18
Jul-92	528.54	275.33	Jul-95	580.65	364.07	Jun-98	883.96	536.09
Aug-92	535.91	276.32	Aug-95	570.55	364.08	Jul-98	1025.55	623.86
Sep-92	517.6	279.8	Sep-95	570.44	366.2	Aug-98	1032.5	632.26
Oct-92	479.16	288.35	Oct-95	576.22	365.09	Sep-98	1028.47	612.15
Nov-92	457.67	299.58	Nov-95	570.18	364.79	Oct-98	983.08	580.71
Dec-92	467.29	301.39	Dec-95	563.99	366.5	Nov-98	940.41	565.95
Jan-93	471.48	306.84	Jan-96	557.02	364.1	Dec-98	982.53	588.57
Feb-93	449.44	312.01	Feb-96	574.38	374	Jan-99	987.9	598.35
Mar-93	463.82	317.86	Mar-96	599.61	392.82	Feb-99	995.2	611.07
Apr-93	489.86	316.76	Apr-96	637.38	420.57	Mar-99	1005.57	620.9
May-93	492.44	317.56	May-96	662.33	437.27	Apr-99	984.91	611.33
Jun-93	488.71	323.38	Jun-96	670.93	435.02	May-99	997.86	618.15
Jul-93	500.97	335.01	Jul-96	681.77	438.88	Jun-99	971.94	608.83
Aug-93	503.38	336.46	Aug-96	701.22	452.4	Jul-99	961.16	610.6
Sep-93	519.44	340.72	Sep-96	701.1	449.68	Aug-99	984.5	612.95
Oct-93	510.7	339.51	Oct-96	724.33	457.26	Sep-99	982.73	605.93
Nov-93	498.51	336.42	Nov-96	774.1	465.56	Oct-99	1010.04	609.28
Dec-93	503.3	337.5	Dec-96	779.31	468.16	Nov-99	996.6	613.74
Jan-94	508.7	340.93	Jan-97	771.62	464.42	Dec-99	992.1	614.6
Feb-94	510.56	345.03	Feb-97	723.47	445.27	Jan-00	1004.51	611.94
Mar-94	514.97	345.35	Mar-97	712.83	443.61	Feb-00	1011.94	631.56
Apr-94	530.6	358.52	Apr-97	723.84	444.12	Mar-00	1021.15	645.97
May-94	545.5	362.54	May-97	729.42	446.83	Apr-00	1047.41	661.2
Jun-94	552.85	362.53	Jun-97	739.41	449.81	May-00	1058.76	702.05
Jul-94	566.75	366.82	Jul-97	761.86	455.74	Jun-00	1045.26	692.74
Aug-94	555.35	359.94	Aug-97	751.01	468.41	Jul-00	1037.58	687.62
Sep-94	555.82	355.64	Sep-97	750.29	469.01	Aug-00	1034.87	695.14
Oct-94	568.58	353.87	Oct-97	768.16	470.9	Sep-00	1026.22	716.14
Nov-94	560.56	352.41	Nov-97	816.08	483.61	Oct-00	1084.29	746.73
Dec-94	554.63	356.01						

PROGRAMS

Leading Indicator Indices (Seasonally Adjusted: 1990=100)

```
data;
   input x @@;
   y = dif(x);
   date = intnx( 'month', '31aug1993'd, _n_ );
   format date monyy.;
   label  x = 'leading indicator index';
cards;
 96.7  97.7  99.3  98.8  99.3 100.1
101.0  99.8 101.8 103.8 105.1 106.8
106.8 108.6 111.0 109.8 109.5 109.4
109.9 106.8 109.3 109.6 108.9 108.9
109.4 109.5 109.8 109.7 109.3 107.9
107.7 108.0 109.1 108.3 107.9 107.0
107.1 110.7 111.4 111.5 112.2 111.3
110.3 111.0 111.8 113.1 113.0 111.2
111.6 111.9 109.8 109.5 106.7 107.2
107.2 107.4 104.4 103.8 101.7  99.0
 97.0  98.4  98.3  98.8  98.9  99.2
101.8 101.8 102.5 103.8 106.8 108.1
109.3 112.4 115.4 117.1 117.9 120.5
120.2 116.3 117.0 118.3
;

proc gplot;
   plot x*date = 1 / frame haxis= '1sep1993'd to '1jun2000'd by qtr;
   symbol1 i=join;
run;

proc arima;
   identify var=x stationarity=(adf=0);
   identify var=x stationarity=(pp=3);
run;

proc spectra out=b p s;
    var x y;
    weights parzen;

proc print data=b(obs=9);
run;
```

U.S. Liquor Sales (in millions of dollars)

```
data sales;
   input x @@;
   y = dif(x);
   date = intnx( 'month', '31dec1969'd, _n_ );
   format date monyy.;
   label x = 'Liquor Sales'
         y = 'First Difference Series';
cards;
 580      514      555      563      627      596
 632      639      577      611      639      875
 650      594      650      668      712      731
 779      712      708      738      758     1073
 669      652      743      709      751      774
 803      760      749      757      779     1066
 734      707      785      762      838      876
 878      871      807      834      877     1236
 789      744      827      831      895      889
 955      983      976      929      989     1294
 860      799      899      866     1016      978
1042     1026      944     1002     1009     1368
 908      849      916      958     1008     1033
1129     1019      984     1045     1049     1459
 910      927      981     1011     1041     1080
1138     1072     1033     1072     1111     1591
 950      932     1049     1021     1097     1151
1194     1174     1160     1135     1209     1692
1071     1044     1158     1122     1209     1334
1360     1368     1297     1283     1375     1974
1294     1258     1301     1297     1425     1378
1429     1452     1305     1377     1439     1958


title'Liquor Sales in $ Million';
proc gplot data=sales;
   plot (x y)*date = 1 / frame haxis='1jan1970'd to '1jan1981'd by year;
   symbol1 i=join;
run;

proc arima data=sales;
   identify var=x;
   identify var=x(1);
   identify var=x(1,12);
run;
```

Money Supply M1 and Amount of Coin and Banknotes in Circulation in South Africa (in millions of rands)

```
/* r= exchange rate of the rate to the pound, m=M1 */
data coint;
   input cb m1 @@;
   x1 = log(cb);
   x2 = log(m1);
   date = intnx( 'month', '30sep1990'd, _n_ );
   format date monyy.;
   label m1 = 'M1'
         cb = 'coin and banknotes';
cards;
    7553   46828   10037    71248   12977    98338   17754   183365
    8446   50315   10331    70314   13846   104613   17978   183147
    8064   53048   10016    70995   14331   112745   17114   203346
    7972   49952   10337    67075   13546   105904   17666   205516
    8121   56666   10244    70730   13992   112480   17356   219645
    8588   56221   10038    71503   14594   118571   16882   219582
    8309   54589   10429    69928   14183   119453   17912   210298
    8562   57014   10550    74185   14818   121754   18409   216128
    8410   57561   10482    76398   15111   126196   18505   213921
    8151   57873   10516    77830   14326   122752   18642   209814
    8612   59434   10721    83180   15010   128655   18985   205021
    8453   58074   11858    84606   14762   132844   18258   213116
    8381   58465   11688    86038   14815   134880   19195   212422
    9280   62476   11627    91411   16101   140160   18758   213332
    8834   60910   11359    91011   15938   147580   18545   220402
    8893   58098   11997    88986   15552   139956   19856   220745
    9104   61644   11270    89588   15943   145335   19382   227461
    8645   64192   12135    90158   16392   154227   19730   228899
    9249   61058   11759    89644   15764   146411   20917   238211
    9415   60758   12458    92033   15979   146954   20943   255230
    8733   63946   12237    94538   15535   150270   22660   258284
    9336   63087   12005    88549   15641   146147   20830   246137
    8976   68303   12241    93353   16793   152896   20486   256900
    9126   71520   12473    93387   15983   164340   20945   252871
    9522   70252   13099    94149   16804   166127   21874   256265
    9851   72255   12351    98090   18017   172057   20714   251623
    9535   71571   12989    98088   17308   173057   21726   257606
    9939   66466   12821    95363   17660   171943   21467   245023
   10022   70355   13064    97799   17939   178516   20929   246519
    9561   70843   13725   101604   16784   177163   22139   261145
;

proc gplot data=coint;
   plot (x1 x2)*date / overlay frame haxis='1oct1990'd to '01jan2001'd by year legend;
   symbol1 l=1 i=join v=none;
   symbol2 l=2 i=join v=star;
   symbol1 l=1 i=join c=none;
   symbol2 l=2 i=join c=star;
run;
```

```
proc arima data=coint;
   identify var=x1 stationarity=(pp=4);
   identify var=x2 stationarity=(pp=4);
run;

proc arima data=coint;
   identify var=x1(1) stationarity=(pp=4);
   identify var=x2(1) stationarity=(pp=4);
run;

proc reg data=coint;
   model x1 = x2 / dw;                         /* gives the CRDW statistic */
   output out=rino residual=r;                 /* produces the residual for the EG Test
*/
   keep r;
run;

proc print noobs;
run;

proc autoreg data=coint;
   model x1 =x2 / normal stationarity=(pp);    /* performs the Phillips-Ouliaris Test */
run;
```

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