

The use of calculators during problem-solving activities in a Grade 9 mathematics classroom

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DECLARATION

I, the undersigned, hereby declare that the work contained in this dissertation is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

A handwritten signature in purple ink, reading "Lykanhalelo". The signature is written in a cursive style with a large initial 'L'.

Signature

28 February 2019

Date

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DEDICATION

This study is dedicated to my husband and best friend, Matti Tangeni Kanhalelo. His love, encouragement, support, and patience guided me throughout the completion of this study. To my four children who were always there for me and believed in me, they have been my inspiration throughout this difficult journey.

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SUMMARY

There is a growing concern in Namibia about the declining performance of learners in mathematics. Mathematics is most often taught through a traditional teaching approach which emphasises the memorisation of facts and allows little room to connect to real-life situations. The problem-solving approach allows learners to acquire information, develop knowledge, analyse and synthesise knowledge. Calculators promise to assist learners in problem-solving by improving their problem-solving skills, and also their higher-order thinking skills and conceptual understanding. This has potential to impact their achievement directly. The purpose of the study was to investigate the use of calculators in problem-solving activities in a grade 9 mathematics classroom. The population of the study consisted of all the grade 9 learners attending a rural school in Namibia. Purposive sampling was used in order to identify participants. This research employed a qualitative case study methodology since it aimed to develop explanations for social aspects of our world, and sought to determine participants' experiences of the problem-solving approach as well as the role that the calculator plays in problem-solving. Data were generated by giving participants various task-based interviews to complete.

These task-based interviews presented participants with a problem-solving task followed by interview questions that could provide insight into their experiences of the problem-solving process with the use of calculators. Data were further gathered through teacher reflections in which the teacher reflected on what happened in the classroom, which strategies were employed and what role the calculator played in the lessons. Data were analysed using content analysis in which themes were identified and discussed. The analysis was guided by Pólya's problem-solving model. All the performing groups attempted to find solutions to the problems by using various planned strategies. The most common reason for failure seemed to be an inability to understand both the question and the concept. The analysis of the task-based interviews indicated that: confidence has an influence on performance. Time allocation was a challenge and participants had to come to grips with the new problem-solving approach to learn mathematical concepts. The teacher acted as a mentor and guide throughout.

The research proved that with constant supervision, most learners found the usage of calculators positive and beneficial and they could further develop the skill of understanding when to use a calculator and when not to use a calculator for solving problems. Problem-solving activities especially in Grade 9 mathematics in Namibia is a novel concept to learners and sufficient time needs to be allocated in order to accommodate this. Participants improved their problem-solving strategies by working in groups where they could learn from each other, but they also needed to take individual responsibility.

KEYWORDS:

Mathematics; problem-solving; meaningful learning; calculators; problem-solving strategies; mathematics teaching and learning; mathematics curriculum; information technology.

OPSOMMING

Daar is groeiende kommer in Namibië oor die prestasie van leerlinge wat afneem. Wiskunde word dikwels aangebied d.m.v. 'n tradisionele benadering wat die klem op memorisering van feite plaas en wat min ruimte laat vir 'n skakel met die werklike lewe. Die probleemoplossingsbenadering tot wiskundeonderrig laat leerders toe om kennis op te doen en te ontwikkel, asook om hierdie kennis te analiseer en saam te voeg. Sakrekenaars bied die moontlikheid aan om leerders by te staan tydens probleemoplossing deurdat die gebruik daarvan hulle probleemoplossingsvaardighede verbeter en hulle hoërordedenke en konseptuelebegrip verbeter. Dit bied die potensiaal dat leerders se prestasie direk kan verbeter. Die doel van hierdie studie was om die gebruik van sakrekenaars tydens probleemoplossingsaktiwiteite in 'n Graad 9 Wiskunde-klas te ondersoek. Die deelnemers aan hierdie studie het bestaan uit al die Graad 9 leerders by 'n plattelandse skool in Namibië. Doelgerigte steekproewe is gebruik om die deelnemers te identifiseer.

Die studie het 'n kwalitatiewe gevallestudie metodologie gevolg aangesien dit gepoog het om verduidelikings te ontwikkel oor die sosiale aspekte van ons wêreld en dit ook gepoog het om deelnemers se perspektiewe van die probleemoplossingsbenadering te bepaal. Voorts het dit ook deelnemers se ervaring m.b.t. die gebruik van sakrekenaars tydens probleemoplossingsaktiwiteite ondersoek. Data is versamel d.m.v. die refleksie van die onderwyser m.b.t. wat in die klaskamer gebeur het en die rol wat die sakrekenaar hierin gespeel het, asook die strategieë wat gebruik is.

Pólya se probleemoplossingsmodel is gebruik as gids. Al die deelnemers moes strategieë vir probleemoplossings gevind en beoefen het. Die mees algemene rede vir mislukking, blyk die onvermoë om die vraag en die konsep te begryp. Die taakgerigte onderhoude wat met die deelnemers gedoen is, het aangedui dat selfvertroue 'n rol speel in probleemoplossing. Tydsbeperkings het ook 'n rol gespeel en deelnemers moes ook stoei met 'n nuwe benadering tot die hantering van Wiskundeprobleme. Die onderwyser het deurgaans as 'n gids en mentor opgetree. Die studie het aangedui dat met konstante toesig en begeleiding, leerders wel kan groei in hulle kennis t.o.v. die gebruik van sakrekenaars tydens probleemoplossing. Probleemoplossings aktiwiteite in Graad 9 Wiskunde in Namibië is 'n nuuttjie vir leerders, en hulle het genoegsame tyd nodig om dit onder die knie te kry. Groepswerk het ook die deelnemers se probleemoplossingstrategieë verbeter aangesien hulle by mekaar kon leer, maar dit is ook noodsaaklik dat hulle individuele verantwoordelikheid vir hulle vordering neem.

SLEUTELWOORDE:

Wiskunde; probleemoplossing; betekenisvolle leer; sakrekenaars; probleemoplossingstrategieë; wiskundeonderrig en -leer; wiskundekurrikulum; inligtingstegnologie.

LIST OF ACRONYMS AND ABBREVIATIONS

CAS	Computer Algebra Systems
CGI	Cognitive Guided Instruction
CK	Content Knowledge
CMS	Classroom Marks Schedule
ICT	Information and Communication Technology
IES	Institute for Education Sciences
JSP	Junior Secondary Phase
MBEC	Ministry of Basic Education and Culture
NIED	National Institute of Education Development
NWU	North-West University
RME	Realistic Mathematics Education
ZPD	Zone of Proximal Development

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CHAPTER 1 ORIENTATION AND PROGRAMME OF STUDY

1.1 Introduction and problem statement

There has been a growing concern in Namibia about the declining performance of learners in mathematics (Andima, 1992). Mathematics was taught with a strong emphasis on the memorisation of facts, with little connection to real life situations. Therefore, the Namibian Educational System was compelled to continually investigate the improvement of the quality of teaching and learning (Tjikuua, 2000). In 2001 the implementation of Information and Communication Technology (ICT) tools, specifically the use of calculators, in the teaching and learning of mathematics, formed part of the curriculum change (Isaacs, 2007).

In Namibia the main challenge facing the attainment of high performance in mathematics is on the level of the teaching and learning of mathematics ranging from inadequate teaching-learning resource materials to poor teaching-learning methods (Nambira *et al.*, 2014). Other contributing factors to poor performance are the shortage of mathematics teachers and the lack of teachers' competencies in mastering the curriculum content (Namupala, 2013; Courtney-Clarke & Wessels, 2014).

The National Institute of Education Development (NIED) is the only directorate in the Ministry of Basic Education and Culture (MBEC) responsible for designing and implementing curricula in all schools in Namibia. Although NIED made mathematics compulsory from Grades 1 - 12 from 2012, it has not had a great impact on improving the teaching and learning of mathematics (Angula, 2015).

The use of calculators in the teaching and learning of mathematics has the potential to enhance the understanding of mathematical concepts, and improve learners' attitudes towards mathematics (Moses, 2012). Clark (2011) points out that the use of calculators enables learners to investigate and explore concepts in a much more comprehensive way than when calculators are not used. The development of calculator skills encouraged by the mathematics curriculum for Junior Secondary Phase (JSP) (Grades 8-10) includes the ability of learners to appropriately choose and apply the correct calculations (Lupahla, 2014).

The compulsory implementation of the use of calculators in mathematics classrooms in Namibia, however, revealed the inadequacy of mathematics teachers' in presenting learners with appropriate calculator-based activities in the classroom (Moses, 2012). In other cases, mathematics teachers tend to present learners with activities that require calculator use, but these activities often lead to the inappropriate use of calculators because teachers tend to

neglect the monitoring of how learners acquire solutions for problem-solving activities (Sikukumwa, 2017).

Problem-solving in mathematics allows learners to acquire information, develop knowledge and understanding, and analyse, synthesise and evaluate the knowledge on their own level (Raoano, 2016; Schoenfeld, 2013). Problem-solving is an integral part of the mathematics curriculum in Namibia and requires teachers to develop learners' thinking by engaging them in problem-solving activities (Sikukumwa, 2017). Various studies (Close *et al.*, 2008; Hembree & Dessart, 1986; Mutsvangwa, 2016) have proved that where learners use calculators as a tool to assist in problem-solving, not only do their problem-solving skills improve, but also their higher order thinking skills and conceptual understanding, which in turn may directly impact their achievement. Therefore, the purpose of this study is to explore the use of calculators in problem-solving activities in a Grade 9 mathematics classroom.

1.2 Literature review

1.2.1 The learning of mathematics

Mathematics is a living subject which consists of patterns, experiments and observations in which the trained practitioner (learner) engages and understands the nature of numbers and symbols (Schoenfeld, 1992). Similarly, Nieuwoudt and Golightly (2006) view mathematics as a human invention in which various activities are undertaken during problem-solving, in order to come up with a fixed product such as a formula. Goldin (2002) defines mathematics as a powerful language, which provides access to viewing the world through numbers, shapes, measurements and statistics, which is useful and creative. Mathematics itself is a foundation of development and a key to open career opportunities for many learners (Muthomi *et al.*, 2012). Moses (2012) sees mathematics as a subject associated with rules and procedures to be followed in order to come up with a solution to a problem when it arises. For the purpose of this study, mathematics will be defined as a dynamic subject that provides the opportunity to access and engage with numbers, patterns, shapes, measurement and order, to solve problems.

Franke *et al.* (2007) urge that the learning of mathematics requires a teacher to transform the mathematical concepts effectively by teaching conceptual understanding, developing procedural fluency and accuracy, teaching strategies and providing opportunities for working mathematically. Learners learn better through different interactions, such as with other learners, the teacher, the mathematical content and the context.

Teachers as well as learners need to do different sorts of activities with different kinds of roles and responsibility (Hiebert & Grouws, 2007). This entails teachers creating an environment in

which learners interact and mentally engage in reflective thinking and internalising concepts (Van de Walle *et al.*, 2014). In their interactions, learners will adapt and expand on their existing network during classroom activities and engage with others working on the same idea.

It is known that, initially, most learners come to school as enthusiastic, curious thinkers, whose natural feeling is to try to make mathematical sense of the world around them (Ersoy & Güner, 2015). This curiosity can be encouraged within a problem-solving environment that nurtures learners' own ideas and methods.

The learning of mathematics through problem-solving helps learners to believe that they are capable of doing mathematics and that mathematics makes sense (Van de Walle *et al.*, 2014). Problem-solving enables learners to learn mathematics meaningfully. Meaningful mathematics learning involves learning that is active, constructive, intentional and cooperative (Hiebert & Carpenter, 1992:89). A mathematical idea, fact or procedure is understood if it is part of an internal network. The level of understanding is determined by the number and the strength of the connections in that network. Stylianides and Stylianides (2007:104) argue that when internal representations are constructed, they produce networks of knowledge. Mathematical understanding is built as new information is connected to existing networks (Van de Walle *et al.*, 2014). Understanding is a measure of the quantity and quality of connections that a new idea has with existing ideas. According to Zohar and Dori (2003), understanding increases as the network grows and as relationships within the network become stronger. As new relationships within the network are constructed, they replace existing connections in their relevant networks with new connections (Hiebert & Carpenter, 1992).

Teachers need to encourage social interactions in the classroom and introduce new methodology to allow teaching and learning situations, where learners are encouraged to challenge and question the teacher as well as other learners (Nickson, 1992). Meaningful learning can take place in a socio-cultural setting (Hiebert & Grouws, 2007). Learners' intellectual achievements are dependent on social interactions as well as their own efforts and innovations. The socio-cultural perspective implies that thinking and learning can be best understood within the specific context which is determined by the members of the community, the cultural tools that are used, the relationships that exist and the institution (such as a school) in which it exists (Mercer & Howe, 2012).

1.2.2 Problem-solving in mathematics

Problem-solving is defined by Pólya (1957) as the ability to identify and solve problems by applying appropriate skills in a systematic way. Pólya is often described as the pioneer of

problem-solving in mathematics– he was the first person to devise a problem-solving model which consists of four steps, which is widely adopted in problem-solving activities. These steps are described as: understanding the problem, devising a plan to solve the problem, carrying out the plan and looking back to determine if the plan will always work.

Schoenfeld (1992) views problem-solving as a process of challenging a novel situation, formulating connections between given ideas and exploring possible strategies for reaching the goal. The Glasgow City Council's Education Services (2006) defines problem-solving as trying to find a suitable action to reach a desired point but being unable to reach the expected end. Problem-solving is an on-going activity in which we use what we know to discover what we don't know (Avcu & Avcu, 2010).

Große (2014) suggests that mathematics teachers present learners with different methods to solve a specific problem. The use of multiple solution methods makes it possible to use different representation tools (e.g. calculators, computers). Mathematics teachers should include both routine and non-routine problems in problem-solving activities (IES, 2012). An example of a routine problem is as follows: 'Carlos has a cake recipe that calls for $2\frac{3}{4}$ cups of flour. He wants to make the recipe 3 times. How much flour does he need?' An example of a non-routine problem is: 'There are 20 people in a room, everybody high-fives with everybody else, how many high-fives occurred?'

Problem-solving activities help learners understand the meaning of a mathematical idea and develop learners' abilities to think mathematically (Pomerantz, 1999; Karatas & Baki, 2000). One important component in applying problem-solving activities is flexibility and knowing multiple approaches and methods to solve a specific problem (Star, 2008). Learners who benefit from sharing and comparing solution methods become better problem solvers and develop greater flexibility (Star, 2008; Malouff & Schutte, 2008). The teaching and learning of mathematics should portray an active and dynamic classroom with learners thinking, exploring and applying what they have learned (Liu *et al.*, 2011). Furthermore, technology tools are increasingly available to enhance and promote mathematical understanding (Admiraal *et al.*, 2011).

1.2.3 The use of calculators in the mathematics classroom

There are two schools of thought regarding the use of calculators in the mathematics classroom. On the one hand, there are those that believe that the use of the calculator is beneficial to learners, and on the other hand there are those that view calculator use as

negative. On the positive side, the use of calculators stimulates problem-solving, broadens learners' number sense and understanding of arithmetic operations (Pomerantz, 1997).

According to Brooks *et al.* (2003), using calculators effectively encourages learners to be inventive, develops their confidence and inspires their independence. Cavanagh and Mitchelmore (2003) are of the opinion that calculators are effective during problem-solving if learners gain confidence on how to justify their answers and link them to the mathematical concept they have learned. Ochanda and Indoshi (2011) assert that learners who use calculators possess better attitudes towards mathematics. In addition, the use of calculators improves problem-solving skills and promotes achievement (Muthomi *et al.*, 2011).

A calculator is a valuable educational tool that allows learners to attain a higher level of mathematical power and understanding (Lorch *et al.*, 2010). Calculators simplify tasks but they do not solve problems for the learners (Tajudin *et al.*, 2011). It is still up to the learner to read the problem, understand what is asked, determine an appropriate plan and implement the plan to solve the problem.

On the negative side, Surgenor *et al.* (2007:27-28) are of the opinion that learners with high mathematical abilities will lose basic computational skills through the use of calculators. The regular use of calculators will result in the weakening of basic facts and the decline in the use of paper-and-pencil algorithms for computations. Learners will become calculator dependent and become more likely to accept incorrect answers on the calculator (Suydam as cited by Masimura, 2016:10). It seems that teachers have become too reliant on calculators because of the pressure to advance in the yearly curriculum (Mbugua *et al.*, 2011).

1.3 Gaps in the literature

Many studies (El Sayed, 2002; Ye, 2009; Mbugua *et al.*, 2011; Karatas & Baki, 2013; Ng, 2013) relating to using calculators in problem-solving have recently emerged. These studies claim that using calculators is an effective way for learners to solve mathematical problems. This has been proved in several cases to be superior in terms of accuracy, performance and flexibility. Moreover, these studies encourage learners to use calculators in developing their mathematical thinking skills, so they can use this technology to generate new mathematical situations.

These studies collectively emphasise the importance of teachers' and parents' perceptions of the use of calculators in the mathematics classroom. Emphasis is placed on the advantages and disadvantages of using a calculator during problem-solving activities. There is however a gap in the literature in terms of the selective use of calculators in problem-solving activities as well as the development of a strategy on how or when to use calculators effectively. This study therefore endeavours to address this gap by investigating the use of calculators in problem-solving activities in a Grade 9 classroom environment.

1.4 Research questions

The central research question for this study was:

How can calculators be used in problem-solving activities in a Grade 9 mathematics classroom?

The above research question lead to the following sub-questions:

- *Which problem-solving strategies are used in Grade 9 mathematics classrooms?*
- *When is it considered best practice to use calculators in problem-solving activities?*
- *What should problem-solving activities look like in the context of a Grade 9 mathematics classroom?*

1.5 Aim and objectives of the study

1.5.1 Aim

The aim of the study was to investigate the use of calculators in problem-solving activities in a Grade 9 mathematics classroom.

1.5.2 Objectives

1. To identify which problem-solving strategies are used in a Grade 9 mathematics classroom.
2. To explore when it is best practice to use calculators in problem-solving activities.
3. To determine what problem-solving activities should look like in the context of a Grade 9 mathematics classroom.

1.6 Research design, methodology and approach

The research design is an overall plan for connecting the research problem(s) to appropriate and attainable empirical research (Leedy & Ormrod, 2005). This means that the research design determines what data is required, what methods are going to be used to generate and analyse the data, and addresses the ethical issues involved in producing the answers to the research question.

1.6.1 Philosophical framework

This research study was undertaken within the interpretive paradigm, based on a socio-constructivist perspective that seeks to understand the world in which participants live and work, in order to understand the social and cultural settings of the participants (Creswell, 2009). Creswell (2009) avers that the interpretive paradigm involves humans who engage with their world and make sense of it, based on their historical and social perspectives. This study wished to unpack the different levels of interactions taking place in mathematics classrooms as described by Nickson (1992). The mathematics teacher (the researcher) sought to understand how learners reason and apply different strategies during problem-solving, with or without the use of calculators.

1.6.2 Research methodology

This research employed a qualitative methodology, based on exploring and understanding the meaning individuals or groups ascribe to a social or human problem (Hancock *et al.*, 2009). A qualitative research method is appropriate for this study because it is concerned with developing explanations for social aspects of our world and seeks solutions on why people behave the way they do and how people are affected by the events that take place around them (Hancock *et al.*, 2009).

The research questions were addressed by employing a single instrumental case study (in a Grade 9 mathematics classroom) (Creswell, 2013). Merriam (1998:27) defines a case study as an empirical inquiry that investigates a contemporary phenomenon within its real-life context. Therefore, this research took place at a rural school in the northern part of Namibia. This rural school was selected because it was convenient for research purposes (the researcher was the only mathematics teacher in the only Grade 9 classroom).

1.6.3 Sampling strategy

The population of the study consisted of all the Grade 9 learners attending the above-mentioned rural school. Purposive sampling was used because the sampling was done with a specific purpose in mind (Maree & Pietersen, 2016). I used a Grade 9 mathematics classroom comprising 31 learners.

1.6.4 Data generation methods

The data were generated by using task-based interviews and teacher reflections. Participants were given freedom either to use or not to use calculators throughout the problem-solving activities. Moreover, I also reflected on what took place in the classroom in an attempt to understand how Grade 9 learners solve mathematical problems, what problem-solving strategies they use and whether they do this with or without the use of calculators.

1.6.4.1 Task-based interviews

I gathered data by using task-based interviews (see Addendum A) with twelve selected learners from the class group (four learners each from the low, middle and high performing groups). These interviews took place after school hours. Goldin (2000) defines task-based interviews as interviews in which a subject or group of subjects talks while working on a mathematical task. By using task-based interviews, I could make deductions about the mathematical thought processes of the learners while doing problem-solving tasks (Goldin, 2000). The focus was on the process of solving problems, rather than on the correctness of the solutions. Task-based interviews are relevant to this research because they involve learners reflecting on what they did to solve the problems, what strategies they used to solve the problems and whether they used calculators or not to solve a mathematical problem.

1.6.4.2 Teacher's reflections

Each learner completed problem-solving activities using their own problem-solving strategies and with their own choice of whether to use calculators or not. While learners were completing these activities, I observed them in order to reflect on the lessons afterwards. Teacher's reflections (see Addendum B) have the potential to increase the validity of the study since they may help me to have a better understanding of learners' behaviour during problem-solving activities.

1.6.5 Data analysis

Data were analysed as follows:

The selected learners' task-based problem-solving activities were analysed using a process of content analysis (see 4.8).

These activities were analysed by means of Creswell's (2009) approach:

- Read through the problem-solving tasks - to get an overall impression of learners' problem-solving skills.
- Break down the information into smaller meaningful parts (decoding).

The task-based interviews of the learners were then assessed by means of an assessment rubric (see 4.8.2). The learners' reflections relating to the process of problem-solving as well as their considerations concerning the use of calculators were also analysed by a process of thematic coding.

The teacher's reflections on the other hand were used to observe the learners learning behaviours (learning strategies and teacher's teaching strategies). The teacher's reflection was completed and analysed by making use of three aspects: the description (describe what happened in the classroom); interpretation (describing what teaching strategy I used in the lesson, the strategies the learners used to solve the problems and the calculator in the lesson) and lastly outcome (describing how successful were the learners in solving the problems, how I would change the lesson in future in case the lesson was unsuccessful) (Hampton, 2010).

In order to ensure trustworthiness and credibility in this qualitative study, as stipulated by Creswell (2009), certain measures were followed. Some of these measures included spending extensive time in the field (in the Grade 9 mathematics classroom), the use of thick description (content analysis) and feedback from others, for example the supervisors.

1.7 Ethical considerations

The North-West University (NWU) Ethical Application Form was completed and submitted to the Ethical Committee at the University. After ethical clearance had been obtained for the study, I requested permission (on the basis of informed consent) from the Namibian Ministry of Education, as well as from the management team of the participating school and the parents, of the participating learners, to conduct the study.

Participants' involvement was voluntary, and they could withdraw at any point. Participants and other stakeholders, like the Namibian Ministry of Education, the school management of the participating school and parents were informed about the aims and objectives of this study. The responses of the participants were treated as confidential and their identities would not be revealed during the research report writing or afterwards. The school's name was kept confidential in order to gain the participants' trust during the research process (Miles *et al.*, 2014).

Ethical details were clearly explained to each participant before the commencement of the study (see Addendum D). These include the following: every participant was provided with the informed consent letter to clearly inform them on what was expected of them during the completion of different activities. The letter gave various reasons why I embarked on researching the specific topic on the use of calculators in problem-solving activities in a Grade 9 mathematics classroom; participants were informed of their right to withdraw from the study at any time without penalty, were also informed that all collected data would remain confidential.

1.8 Outline of the dissertation

Chapter 1: Orientation and programme of the study

Chapter One contains the following aspects: introduction and problem statement of this study, literature review on the learning of mathematics, problem-solving in mathematics and the use of calculators, all in the mathematics classroom. This was followed by a research question and sub-questions and the aim and objectives of the study. The overview of the empirical study is also included where I discuss the research design, methods and methodology which include the philosophical framework, research methodology, sampling strategies, method of data analysis as well as the ethical considerations.

Chapter 2: The learning of mathematics

Chapter Two contains the literature review on the three aspects: the nature of mathematics, the learning of mathematics and the role of the mathematics teacher in the learning of mathematics. The first aspect focuses more on the definition of mathematics, different authors' views on mathematics and my own definition of what mathematics is. The second aspect is focused on the social-cultural aspect of the learning of mathematics, the influence of the classroom environment, teachers' views about the learning of mathematics and the use of technology in the classroom on the learning of mathematics.

Chapter 3: Problem-solving in mathematics

Chapter Three focuses on the definition and historical development of problem-solving in mathematics. There is also a discussion on the learning of mathematics and problem-solving which focuses on teachers' views/ beliefs with respect to problem-solving, the role of problem-solving in school mathematics, how to solve a mathematical problem and different problem-solving strategies. Lastly, this chapter concludes with the literature on the teaching of problem-solving, which focused on the teacher's role in problem-solving, planning of problem-solving lessons, different problem-solving tasks, the use of technology (the calculator and computer) in mathematics teaching and the language in mathematical teaching and learning.

Chapter 4: Research design and methods

Chapter Four discusses the research methods and methodology, which was introduced with the aim and objectives of the study from chapter one. Aspects such as philosophical framework, qualitative research design and methodology, sampling strategies and ethical consideration are then emphasised in depth in this chapter. Secondly, the chapter discusses the method of data generation (teacher's reflections and task-based interview), my role as a teacher and facilitator during teaching problem-solving. The chapter further emphasises the trustworthiness of the study, focusing more on the validity and reliability in qualitative research. Lastly, the chapter concludes with the methods of data analysis for the task-based interview and the teacher's reflections.

Chapter 5: The use of calculators in a Grade 9 mathematics classroom

Chapter Five comprises of the empirical investigation and discussion of data and information collected and analysed during the study. The discussion of data information on the empirical investigation took place by means of the implementation of analysing the task-based interviews which consisted of three aspects (the tasks on mensuration, geometry and algebra) focusing respectively on problem-solving, the use of a calculator, followed by the (learners and teachers) problem-solving strategies and learners perceptions. The teacher's reflections discuss the three main aspects such as *descriptions*, *interpretations* (Teaching strategies, learners' problem-solving strategies and the use of calculators) and *outcome* (Learners' success in solving the problems and suggestions for future improvements).

Chapter 6: Findings, conclusions, limitations and recommendations

The dissertation concludes by addressing the research questions and objectives of the study. The chapter further focuses on the limitations of the study, the contribution that the study will bring in the field of the global community, suggestions for future research as well as a description of my role as a researcher.

CHAPTER 2 THE LEARNING OF MATHEMATICS

2.1 Introduction

This chapter takes a close look at the studies published in the past ten years, exploring the aspects of the learning and teaching of mathematics. The first section consists of the description of what mathematics is and different views on mathematics and problem-solving. The second section consists of the description of how mathematics is learned in both social-cultural settings and in school mathematics. The last section consists of the role of mathematics teachers in the learning of mathematics exploring the classroom environment both socially and administratively; thereafter the teachers' views about the learning of mathematics and the use of calculator technology in the classroom. The chapter concludes with the summary of the important aspect of learning mathematics.

2.2 The nature of mathematics

2.2.1 What is mathematics?

Mathematics has been tremendously useful in every aspect of human life and it is an essential ingredient in the preparation of individuals for future challenges (Stewart & Tall, 1977). Therefore, the demands of the new century require that all learners acquire an understanding of concepts, proficiency, skills and a positive attitude in mathematics if they are to be successful in the future (Ernest, 2015). This section consists of researchers' different opinions on what mathematics really is.

The definition of mathematics has been a popular element of research in recent decades. Although different researchers use different definition, there is a common agreement that mathematics is a human endeavour and should be a part of everyone's basic knowledge which involves a wide variety of creativity, excitement and dynamic nature of mathematics (Reimer & Reimer, 1995; Wilson & Padron, 1994). Wilson and Padron (1994:51) have another similar definition, referring to mathematics as a human activity, seen as the creation of human mind, rather than something that exists somewhere to be discovered. Freudenthal's (1991:25) definition of mathematics is related to his conception of mathematics as human activity viewing the learning of mathematics as a way of organising mathematical activities by using horizontal and vertical mathematisation. In horizontal mathematisation, the learners use mathematical tools (calculators, computers etc.) to solve a problem of a real-life situation.

Vertical mathematisation is the process of organising, different strategies, ideas and facts within the mathematical problem (Freudenthal, 1991:25; Loc & Hao, 2016:20).

Secondly, mathematics is also described as a tool for problem-solving and a set of cultural understanding that arise out of a problem-solving activity (Stipek *et al.*, 2001). There are different types of tools in mathematics, such as language, calculators and computers. These are a few essential tools utilised in areas such as banking, engineering, manufacturing, medicine, social science, and physics (Dossey, 1992). These tools can also be used in different settings, for example classroom settings, where both learners and teachers use to employ effective teaching strategies in order to solve problems to better understand mathematical concepts (Schoenfeld, 2002). Teachers should have classroom practices that actively engage learners in activities that will assist them to construct mathematical concepts.

Thirdly, mathematics has been characterised as the science of pattern, and these patterns reside in the human mind and are influenced by the culture of the human behaviour (Schoenfeld, 1992). Wilson and Padron (1994) are of the opinion that mathematics also relies on logic and on investigating and discovering the reality of problem situations. Thompson (1992) describes mathematics as a kind of mental and social activity where the teachers are involved in the construction of conjecture, proofs and arguments. Mathematical conjecture, proofs and arguments involve making a series of logical statements from which only one conclusion can follow and once these proofs are constructed, they are always true (De Millo *et al.*, 1979:273). Historically, theorems such as Fermat's last theorem, the theorem of Pythagoras, and many more are the result of mathematicians' own proof of creations which are effectively proven to be true until today (Stipek *et al.*, 2001; Knuth, 2002). These proofs are derived by mathematicians by their own free will, trying to understand and realise new way of deriving new formulae of mathematics (Boaler, 2009). Asking learners to prove a theorem forces them to think logically by examining every statement thoroughly, and to also justify their explanations. Moreover, mathematics is far from being uninteresting and difficult, as it is so often depicted, it is full of creativity (Stipek *et al.*, 2001. Stipek *et al.* (2001) further suggest that teachers should engage in instructions that allow learners to create their own estimation related to understanding proofing theorems in mathematics. When school learners are given opportunities to formulate their own theories, they will feel that mathematics is a live subject, not something that has already been decided and just needs to be memorised (Boz, 2008).

Fourthly, several authors such as Bell (1978), Courant *et al.* (1996) and Hersh (1997) regard mathematics differently; they see mathematics as the study and classification of meaning or structures of the real world. These structures are sorted out in different relationships. According to Johnston-Wilder *et al.* (1999) these structures can be attached to rules and algorithms in mathematics and become valid only if they consist of concepts represented by physical objects. Freudenthal (1991:20) defines structure in mathematics as a means of organising mathematical objects in a form and content. For example, the relations in the system of addition structure are of the form $a + b = c$. The present one has some remarkable properties for instance, that $a + b$ is always the same as $b + a$.

The above definition is also similar to Stipek *et al.* (2001) who define a rational concept of mathematics as a conceptual structure that enables a learner to construct a strategy for a given task in order for them to become independent problem solvers. One can argue that mathematics can be related to numerical, spatial and relationships whereby learners use their knowledge to build and create mathematical objects such as whole number, the number line, geometrical shapes etc. These models are all mathematical objects which must be evaluated using different relationships (Stewart & Tall, 1977). Learners should develop and explore the mathematical object in depth and see that mathematics is an integrated whole of knowledge not merely an isolated piece of knowledge (Akinmola, 2014). Lenhard and Carrier (2015:14) also agree with the concept of structuring mathematical objects; they are of the opinion that mathematics deals with structures that can be found in creation of patterns in nature, such as spatial structural patterns, various geometrical shapes and growing patterns (e.g., 2, 4, 6, 8,). Mathematics is therefore responsible for bringing out patterns of any sort and creates a link between them to form a theory suitable for use in school mathematics (Ernest, 2015).

Lastly, mathematics is defined by Zarinnia and Romberg (1987) as a discipline that deals with integral and reciprocal relationships with other disciplines, especially science, social sciences and humanities. Hersh (1997) and Astuti (2013) share a similar definition, defining mathematics as one of the sciences that demand serious thinking, consistence and system in order to solve problems that require resolutions of concepts. It is against this background that mathematics has been introduced to humans since childhood, starting from known numbers and how to count until operating the complex numbers (Azram & Daoud, 2015). This resolution of new concept and skills should be understood through the creative process of accuracy, and through logical reasoning, which is the core fundamental mathematics (Ernest 2015).

2.2.2 Different perspectives on mathematics

There are four common views explored by several researchers in mathematics education, namely: the instrumentalist view, the constructivist view, the Platonist view and Freudenthal's view of what mathematics is.

Firstly, the instrumentalist view also known as the toolbox view, means that mathematics is seen as building up of facts, rules and skills to be used in the achievement of learning (Thompson, 1992). Thompson further suggests that this view of mathematics has an influence on how the teaching and learning of mathematics is seen because it is based on content-focused teaching. Halverscheid and Rolka (2007) suggest that the instrumentalist view not only actively involves the learners in the process of exploring and investigating ideas or either denies learners the opportunity to do real mathematics, but also misrepresents mathematics to the learners.

Secondly, the constructivist view of mathematics is based on knowledge that is actively created by the learners from their perceptions and experience, by making use of their existing knowledge (Clements & Battista, 1990). For example, a teacher can demonstrate how to add fractions; however, it is up to the learners to invent new ideas of understanding the concept. This is improved by engaging learners in explaining, evaluating and discussing classroom activities during teaching and learning (Hart, 1993). By doing so learners will hold a belief that they are constructing their own meaning without the help of the teacher. According to Roussouw (2002), when learners see their responsibilities in the mathematics classroom as completing assigned tasks and making sense of their own thinking and communicating about mathematics, it makes them feel independent. Such independent learners have the sense of themselves as controlling and creating mathematics. All that teachers need to do is to use a variety of resources to solve a mathematics problem and to construct explanations about the learning process by posing questions about the problem to clarify their solutions (Freudenthal, 1991). By doing this, mathematics is seen as knowledge constructed by learners through their interactions within the learning environment (Major & Mangope, 2012).

Thirdly, in the Platonist view, mathematics is characterised as a static subject, bound by a variety of information, interrelating in forms of structures that play an important role in doing mathematics (Dossey, 1992; Nickson, 1992; McLarty, 2005). Mathematical structures are seen to be real and exist independently of human, however they still have to discover these structures through rational activities (Nieuwoudt, 2006). Halverscheid and Rolka (2007:282) are of the opinion that learners utilising a Platonist view do not create mathematics by themselves. They view mathematics as static and related to historical truths.

Most mathematicians are Platonists, believing that the totality of their subject already exists, and it is the job of human investigators to discover it, rather than create it. Viholainen (2011) agrees with the above view that mathematics cannot be created but remains static and related to historical truths.

Aristotle's view of mathematics is based on experienced reality, whereby knowledge is obtained from experimentations, observations and abstractions (Dossey, 1992). This view supports the idea that one can construct a mathematical knowledge that has been in existence as a result of experience with objects (Barnes & Venter, 2008). Changing patterns in mathematics are example of this growth experience. One can explore patterns in nature from the growth of the area, circumference or perimeter of an object and changing it to a new theory (De Lange, 1999). Nieuwoudt (2006:108) describe this act better by stating that this is "a growth and change view, where knowledge is seen as resulting from competing theories that are tested against other theories and held to be true until falsified by better theories."

Lastly, Freudenthal (1991) views mathematics differently, he believes that mathematics is connected to reality and that it is a human activity. Realistic mathematics education has their root in Freudenthal's interpretation of mathematics as a human activity; an activity which he believed should consist of organising or mathematising subject matter taken from reality (Barnes & Venter, 2008). Learners should therefore learn mathematics by mathematising subject matter from real contexts and their own mathematical activity rather than from the traditional view of presenting (Barnes & Venter, 2008). Zulkardi (2010) suggests that the implications of his views for the teaching and learning of the subject are that mathematics must be close to learners' experiences and be relevant to their everyday life settings. By viewing mathematics as a human activity, influences the way mathematics education is organised. Learners should be given the opportunity to experience similar processes as the ones through which mathematics was invented (Phoshoko, 2013).

2.2.3 The view of mathematics as a problem-solving process

Problem-solving plays an imperative part in mathematics and ought to have a noticeable role in the mathematics education of learners (Wilson *et al.*, 1993). Fortunately, a considerable amount of research on problem-solving has been conducted during the past 40 years or so contesting the inclusion of problem-solving in mathematics education (Greeno, 1991; Szetela & Nicol, 1992; Wilson *et al.*, 1993; Perveen, 2010; Nickerson, 2010). Kolovou *et al.* (2008) view problem-solving in mathematics as a procedure to follow when approached with intellectual mathematical tasks that allow learners to think critically.

Schoenfeld (1992) identified three main views of mathematics problem-solving, which he considers relevant for mathematics education namely: problem-solving as a context, as a skill and as an art. The emphasis on problem-solving as a context is on finding interesting and engaging tasks that help unpack a mathematical concept by means of visualising strategies such as representations to solve problems (Hegarty & Kozhevnikov, 1999). A representation is defined as any arrangement of characters, images, concrete objects, etc., that can symbolise or represent something else (Brahier, 2013). For example, a teacher might present the concept of fractions assigning groups of learners the problem of dividing two pieces of pie so that each gets an equal share (Sajadi *et al.*, 2013). In this activity, the teacher's goals are to create opportunities for learners to make discoveries about mathematical concepts (fractions); to help make the concepts more concrete by means of practice; and to offer a rationale for learning about fractions by means of reasoning. Problem-solving as a context in mathematics means using reasoning, justification and practising during teaching and learning.

The second view outlined by Schoenfeld (1992) is problem-solving as a skill to be taught in the school curriculum. This view of problem-solving skills is taught as a separate topic in the curriculum as a means for developing conceptual understanding and basic skills (Szetela & Nicol, 1992). Learners are taught a set of rules to solve problems and given practice in using these rules to solve both routine and non-routine problems (Schroeder *et al.*, 1999). When problem-solving is viewed as a collection of skills, it means the skills are organised in such a way that learners are expected to first master the ability to solve routine problems before attempting non-routine problems (Schroeder *et al.*, 1999).

Consequently, when a problem solver knows how to go about solving a problem, the problem is routine. For example, two column multiplication problems, such as 5×2 are routine for most high school learners because they know the procedure (Yerushalmy *et al.*, 1999). A non-routine problem is when the problem solver does not initially know how to go about solving a problem. For example, the following problem is non-routine for most high-school learners: "If the area covered by water lilies in a lake doubles every 24 hours, and the entire lake is covered in 60 days, how long does it take to cover half the lake?" (Asman & Markovits, 1999:363). When defining the learning outcomes of a problem-solving activity, teachers should be aware of the difference between teaching problem-solving as a separate skill and infusing it within the content of the curriculum (McIntosh & Jarret, 2000).

The third view identified by Schoenfeld (1992:338) is problem-solving as art, which he describes as one important view that requires creativity in real-life problems. In his classic book *How to solve it*, George Pólya introduces the idea that problem-solving could be taught as a practical art (Pólya, 1945).

Pólya sees problem-solving as an act of discovery and introduces the term heuristics to describe the abilities needed to successfully investigate new problems. Foong (1991) defines his heuristic method as a path that learners should apply to different situations when approached with challenging problem-solving activities. Several researchers provide their own heuristics in mathematics problem-solving, Pólya being one of them. The first heuristic was introduced by John Dewey, who revealed a strategy of problem-solving in how people think. The second was introduced by George Pólya, whose technique was based on critical thinking in mathematics and the most recent one was created by Krulik and Rudnick (cited in Carson, 2007:7), in which they explain what, should happen in every phase of the problem-solving process.

By teaching problem-solving as art one can develop learners' abilities to become skilful and fervent problem solvers and assist them to become independent thinkers who are capable of dealing with difficult problems (Fernandez *et al.*, 1994). Problem-solving is the art of seeing the solution that is already there. The good problem solver then, is highly open to ideas that assist with creativity (Schoenfeld, 2002). Pólya's own view pertaining to problem-solving as art means that to solve a problem, the characteristics and properties of the problem should first be analysed. When the problem is understood then the learner can devise a plan, implement relevant strategies and reflect on the solution.

2.2.4 Mathematics as understood and defined for this study

As discussed above, mathematics is described by Schoenfeld (1992), Wilson and Padron (1994) and Thompson (1992) as the science of pattern and these patterns reside in the human mind. They are then, influenced by the culture of the human behaviour and most importantly, rely on logic as a mean of discovering the truth. The researcher's opinion on the definition of mathematics is parallel to Schoenfeld's notion of mathematics. Mathematics is seen as the language that describes patterns, both patterns in nature and patterns invented by the human mind. It can be patterns of shapes, patterns of numbers, etc. The idea is that nature is full of patterns, for example: the Fibonacci sequence which is described as a series of numbers in which each number is the sum of the previous two numbers. The flowering of an artichoke follows this sequence for example, with the distance between each petal and the next matching the ratio of the numbers in the sequence. These patterns can be real or imagined, visual or mental. They can arise from the world around us, either from the depth of space and time or from the inner workings of the human mind.

Secondly, mathematicians use different tools such as language to communicate ideas. Mathematic language is such a useful tool which is considered to be one of the basics in our formal educational system. Almost all activities that are carried out in the classroom should be translated into a common mathematical language. In general, everybody uses mathematics in various ways, whether they realise it or not. When people go shopping, cooking, building, travelling, fixing things, even playing games, they use mathematics as a tool to carry out daily activities. In addition, mathematics is also a tool in all branches of science; its knowledge is used to evaluate mostly equations, especially in chemistry to formulate new conjecture, formulae and other new mathematics discovery.

Lastly, mathematics is a way of interpreting the world. Learners who use mathematics, engage in mathematical performances and use different language in order to do something with mathematics. Learners should not merely memorise past methods; they need to engage through problem-solving. If they do not use mathematics throughout the learning process, they will find it difficult to apply it in other situations, including examinations.

2.3 The learning of mathematics

The learning of mathematics is the product of an interaction between what learners are taught and what they bring to any learning situation (Chapman, 2004). Assumptions are made that observation, listening to explanation from teachers, engaging in activities or practice with feedback will result in learning (Kennedy *et al.*, 2008). When learning is the goal, teachers and learners collaborate and provide relevant feedback to move learning forward. Assessment is also vital for learning and if frequently done, teachers can learn a great deal about their learners. They can gain an understanding of learners' existing beliefs and knowledge and can identify incomplete understandings. Teachers can observe and probe learners' thinking over time and can identify links between prior knowledge and new learning.

Learners should also be allowed to find their own levels of understanding activities and explore the paths leading with a little guidance as each particular case requires. There are pedagogical arguments which support this procedure (Pierson, 2008). The first one is that when knowledge and ability are acquired by one's own activity, they are more readily available than when imposed by others. Secondly, discovery learning can be enjoyable and motivating. In the third place, it fosters the attitude of experiencing mathematics as a human activity as described in the previous discussion.

Motivation is essential for the hard work of learning. The higher the motivation, the more time and energy a learner is willing to devote to any given task. Even when learners find the content

interesting and the activity enjoyable, learning requires sustained concentration and effort. To rephrase this idea, learners learn more and enjoy learning more when they are actively involved by discovering concepts on their own by means of researching in various books, making use of internet, and many more. The teacher is a designer and facilitator of this approach, permitting new life into the classroom and enabling learners to become more powerful problem solvers.

Learning is also enhanced when learners are encouraged to think about their own learning, to review their experiences of learning (What made sense and what didn't? How does this fit with what I already know, or think I know?), and to apply what they have learned to their future learning. When learners and teachers become comfortable with a continuous cycle of feedback and adjustment, learning becomes more efficient and they begin to internalise the process of standing outside their own learning and considering it against a range of criteria, not just the teacher's judgement about quality or accuracy of learning.

Traditionally, the use of small groups is commonly recommended to teachers by many authors as an alternative to ability grouping and as a way of involving learners in classroom activities than they would be involved through individual seat work. However, several researchers also oppose the idea of small groups because small-group work poses the danger that the work will be shifted to the group's most able learners, thereby allowing other learners to avoid doing their share of work; subsequently they do not learn (Secada, 1994). In my own opinion teachers should monitor small groups instead to ensure that everyone is in fact participating in and understanding the mathematics; these are the reasons why the group was formed in the first place. Perceptions of this nature do represent real progress in learning of mathematics. This section provides a description on how mathematics is learned and specifically the learning of school mathematics, the social-cultural aspect of mathematics, the learning of mathematics and the learning of school mathematics.

2.3.1 How is mathematics learned?

One of the most widely accepted ideas within the mathematics community is the idea that learners should understand mathematics by learning with understanding (Hiebert & Carpenter, 1992). Learning is a social process in which learners actively construct understanding, which means fitting the new information into what is already known. When learners acquire knowledge with understanding, they can apply that knowledge to learn new topics and solve new and unfamiliar problems. This approach is well known as a constructivist approach to learning. There is broad agreement among various researchers on the constructivist approach to learning. Constructivism is a theory of describing how learning happens, regardless of whether

learners are using their experiences to understand the problem (De Corte, 1995). The theory of constructivism suggests that learners construct knowledge out of their experiences.

An important aspect originating from the above definition is the fact that the teacher contributes to learning by ensuring that the learners are appropriately equipped with the necessary learning tasks in order to engage with the learning material through social interactions (Grösser, 2007). Researchers refer to learning as social interactions between a learner, teacher and environment engaging in a stimulating environment (Hurst *et al.*, 2013). Teachers play a vital role in imparting relevant knowledge and help shaping learners learning opportunities by stimulating an environment which make their learners to lead to better learning. Singh (2014:60) suggested that “instead of learners listening to the teacher’s explanations and trying to understand them, learners have to be engaged in active information processing before they interpret and relate new information to their prior knowledge”. Learners needs access to a variety of resources such as books, encyclopaedias, videos, etc. which they can manipulate, observe and experience on their own.

The goal of mathematics teaching is to transmit knowledge to learners so that they construct mathematical knowledge by organising their cognitive constructions through producing networks of knowledge (Hiebert & Carpenter, 1992). Hiebert and Carpenter (1992:67) define a network of knowledge as “a structure constructed to form up representations of ideas that has been in existence and building connections between different representations”. This idea is similar to Freudenthal’s view that he proposes that the actual activity of doing mathematics should mainly consist of organising subject matter taken from reality (Freudenthal, 1991). Learners should learn mathematics by mathematising subject matter from their own mathematical activity and real contexts rather than from the traditional view of presenting mathematics to them as a ready-made system with general applicability (Barnes & Venter, 2008). These real situations can include contextual problems for learners where they experience the problem presented as relevant and real.

Another approach to learning mathematics is by using realistic mathematics education, which is one of the learning approaches widely focusing on problem-solving and construction of meaningful learning (De Corte, 1995). This form of realistic mathematics education is also similar to the Freudenthal’s view on mathematics as explained above, of seeing mathematics as a human activity that needs to be connected to reality at all times. De Corte (2000:42) further explained that the first principle underlying realistic mathematics education is that “learners do not absorb concepts and procedures passively, but that they actively construct their mathematical knowledge and skills starting from the exploration of so-called context problems, using their own informal knowledge and working methods”.

2.3.2 The learning of school mathematics

Historically, the learning of school mathematics was the product of teachers' way of interpreting and understanding the principles, standard and curriculum during teaching and learning (Ernest, 1989). Now, school mathematics is viewed as a subject matter framework about how the ideas of standards should be interpreted, implemented and integrated in the classroom (Ferrini-Mundy, 2000). This framework consists of related portions of curriculum and evaluation standards for school mathematics, professional standards for teaching mathematics and assessment standards for school mathematics. The curriculum is viewed as a set of procedures, methods, skills, rules, algorithms and a collection of academic tasks (Bishop, 1993). Therefore, mathematics teachers should have the knowledge to represent the content in a variety of ways and understand the key aspects of each topic in the curriculum (Stein *et al.*, 1996).

School mathematics has a more combined organisation with almost all aspects, developing out of a foundation of whole number arithmetic and fractions (Van den Heuvel-Panhuizen, 2010). To represent this foundation, the teachers should have knowledge of mathematics and pedagogy to know mathematics so that they can be prepared to assess learners' learning through a variety of methods and make mathematical curricular decisions by understanding the mathematical content of state standards and grade-level expectations (Papick 2011; Ma, 2013). Teachers can then acquire the subject knowledge necessary for them to teach mathematics successfully.

Knowledge of mathematics refers to a teacher's way of understanding and thinking through acquiring content knowledge of the subject. Mathematical content is the foundation upon which knowledge is built (Harel, 2008). Pedagogical content knowledge is another vital component that includes knowledge of learner understanding of the ways to deal with learners during teaching and learning (Fennema & Franke, 1992). For example, every teacher of algebra will encounter learners who compute $(a + b)^2$ as $a^2 + b^2$. This mistake can be corrected if teachers come to recognise the different errors (e.g. procedural, conceptual, careless errors) frequently committed. This is done in order to help learners to see clearly what is wrong.

This kind of knowledge allows proficient teachers to prioritise and organise content so that learners are introduced to big ideas rather than getting lost in a mass of details (Schoenfeld & Kilpatrick, 2008).

Problem-solving must be the focus of school mathematics. Learners should be shown different techniques for learning and then given problems to practise on, until they have mastered the

technique (Schoenfeld, 1992). Consider the following problem taken from Schoenfeld and Kilpatrick (2008:324-325): Sera really likes cakes. She decides that a serving should be $\frac{3}{5}$ of a cake. If she has 4 cakes, how many servings does she have? The learners are encouraged to describe this problem in a way that makes sense to them. This can involve using their own self-invented symbols or pictures and identifying the central relations in the problem situation (Barnes & Venter, 2008). Teachers also need to encourage learners to build on one another's ideas and to participate in conversations about mathematics until they have constructed a shared understanding of a concept (Van den Heuvel-Panhuizen & Drijvers, 2014).

Mathematics uses its own specialised language that involves symbols and notations for describing numerical, geometric and graphical relationships that can be used during problem-solving (Schoenfeld, 1992). Anthony and Walshaw (2009) are of the opinion that if learners are to make sense of mathematical ideas, they need to understand the mathematical language used in the classroom. They use the example of words such as less than, more than, maybe and half which can have different meanings within different contexts. Learners can also be helped in grasping the underlying meaning through the use of words with the same mathematical meaning, for example, 'x' can mean multiply and times. In general, dealing with the written and spoken vocabulary of a concept is an essential part of its understanding that exceeds all aspects of school mathematics (Van De Walle, 2010). It is difficult to understand multiplication of fractions without knowing what a fraction is and what it looks like, and that multiplication is an operation which, given two numbers, produces a third (Papick, 2011).

Unfortunately, school mathematics is often presented as a static body of knowledge to be mastered for an exam, leaving it without character and supporting the erroneous view that mathematics is for the best few who are capable (Wilson & Padron, 1994). But even if that may be the case, the Namibian National Curriculum Framework acknowledges the fact that in Grade 9 there will be internal end-of-year examinations. As before, the purpose of these examinations is to focus on how well learners can demonstrate their thinking, communication, and problem-solving skills related to the areas of the syllabus, which are most essential for continuing in the next grade (MoE, 2010).

The most important aspect in this curriculum is that the kind of thinking that learners learn in mathematics is an ability to handle concepts and use different approaches to problem-solving by using mathematics language effectively (MoE, 2010). According to Papick (2011), the starting point for teaching and learning is the fact that the learner brings knowledge and social experiences to the classroom that they gained from the family, the community, and through interaction with the environment. Learning at school must involve building on, extending and challenging the learner's prior knowledge and experience. Teachers need to encourage

learners to build on one another's ideas and to participate in conversations about mathematics until they have constructed a shared understanding of a concept and solved problems situated to real life situations (Van den Heuvel-Panhuizen & Drijvers, 2014).

Context problems are mathematical problems that are presented within a broader framework of real-life situations with which learners are familiar (Dickinson *et al.*, 2010). Dickinson & Hough (2012) are of the opinion that using context problems in the teaching and learning of mathematics is allowing the subject to be made problematic. Therefore, the starting point of learning mathematics is to be engaged with solving real-world situations and contextual problems (Phoshoko, 2013).

2.4 The socio-cultural aspect of mathematics

It is apparent from the preceding discussion that in realistic mathematics education (RME), learning mathematics is an activity that is facilitated by a social context of the classroom. Van den Heuvel-Panhuizen (2003) is of opinion that social interaction is considered part of RME because of the importance towards learning by doing mathematics, exchanging and negotiating ideas, comparing solution methods and discussing arguments during teaching and learning. Consequently, in realistic mathematics education whole-class instruction and individual work are combined with cooperative learning in small groups and classroom discussion which is guided by the culture of the mathematics classroom (Phoshoko, 2013). This section briefly examines ways of how mathematics is learned in social-cultural settings, by making use of two important approaches to teaching and learning, such as the cultural inclusive education approach and the zone of proximal development approach.

Knowledge is constructed at the core of our interactions and is shaped by the skills and abilities valued in a particular culture of the mathematics classroom (Lampert, 1990; Nickson, 1992). Mathematics is viewed as a cultural element that is socially constructed by understanding the interactions between the development of mathematical thinking and the cultural context (Wilson & Padron 1994). Teaching is not only about presenting what is called "content", but it is also teaching learners useful tools that develop from culture, such as speech and writing to mediate the social environment (Lampert, 1990). The whole idea of a social culture of the mathematics classroom implies an acceptance of the idea that mathematics exerts a unique influence on the context of the classroom in which the subject is taught and learned (Nickson, 1992). Therefore from a socio-cultural perspective mathematics learning is a process where cognitive, affective, emotional and social aspects are deeply intertwined (Planas & Gorgorió, 2004).

Culture and language are key fundamentals of social activities because learners receive all knowledge of the world through language and other forms of communication (Lerman, 2002:88). The language helps learners be strategic, rather than precipitating, in their approach to complex problems and it also helps them to gain control over their own thinking and behaviour (Even & Tirosh, 2008; Pierson, 2008). Therefore, the language is considered the main tool that promotes thinking, develops reasoning, and is mostly used to externalise thinking so that our ideas become concrete and accessible to ourselves (Gutstein, 2003:40).

Another approach concerning the culture of the mathematics classroom is the notion of zone of proximal development (ZPD). Van de Walle *et al.* (2010:21) define ZPD as the “distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving”, but where learners can attain mastery if they are provided assistance and guidance by the adults or more able peers. A large body of researchers propose the ZPD approach to social learning (John-Steiner & Mahn, 1996; Palincsar, 1998; Hakkarainen & Bredikyte, 2008). They believe that learning takes place in a ZPD where learners are drawn beyond their current understanding by working with a teacher or a more competent peer. A teacher should collaborate with his/her learners in order to create meaning in ways that learners can make their own learning during the accomplishment of these tasks (Goos, 2004).

Secondly, social cultural approaches create the connection between social and individual progress in development of learning between the types of learning that teachers promote and the learning environment (John-Steiner & Mahn, 1996). Recent researchers of sociocultural approaches of teaching and learning of mathematics refer to this perception as drawn from the work of Vygotsky who is the legitimate founder of the social development theory (Chaiklin, 2003; Goos, 2008; Forrester & Chinnappan, 2010). Vygotsky was the first Russian psychologist who introduced social development theory and believed that child development is the process of social interaction. Learners’ understanding begins as a result of socialisation through participation in activities that require communicative meaning (Ewing, 2004). Learning in this manner is a dynamic, organised process through which the learners become active agents in their learning (Sharon, 2012; Kazak *et al.*, 2015).

There are various strategies which contribute to the access of ZPD, such as scaffolding, reciprocal teaching and cognitive guided instruction. Scaffolding is an action taken by the teacher to improve the performance of a learner (Bikmaz *et al.*, 2010). It provides individual support based on learners’ ZPD. This requires teachers to provide the opportunity to extend their current skills and knowledge by structuring tasks that engage learners’ interest. Reciprocal teaching allows the creation of dialogue between learner and teacher regarding segments of

text by using four strategies: summarizing, question generating, clarifying, and planning (Drake & Sherin, 2006). For example, the teacher asks learners to clarify words in a mathematics word problem, guides them in generating questions to identify the key parts of the word problem, summarises the purpose of the word problem, and constructs a plan with the solution steps (Pierson, 2008).

During the reciprocal process, learners are able to discuss their thinking with their group members, often ask the teacher for assistance, and mostly discuss problems with their peers (Quirk, 2010). Therefore, a reciprocal style approach provides learners with the opportunity to work in a social setting where they are comfortable asking questions and are planning to reach the end result (Quirk, 2010). Cognitive guided instruction (CGI) is another strategy for implementing the social development theory. CGI is an approach to teaching mathematics that builds on learners' natural problem-solving strategies (Bobis, 2010). This strategy involves the teacher and learner exploring mathematics problems and sharing their different solving strategies in an open dialogue.

2.5 The role of the mathematics teacher in the learning of mathematics

2.5.1 Social environment

Classroom social environment is an environment which is learner-centred and focuses on learners' learning rather than teachers' teaching (Stein *et al.*, 1996). This environment is democratic in nature where learners are free to share responsibilities and free to make decisions on their own. This environment is possible if the teacher has accomplished the role of developing relationships with learners in a way that supports opportunity for participation and developing classroom norms that support engagement during classroom activities (Driscoll, 2000). The classroom social environment comprises learners' and teachers' beliefs which tend to influence the nature of their engagement in academic tasks (Simon, 1986; Lampert, 1990).

Teacher support refers to learners' view that their teacher cares about them emotionally; when learners feel this way, they tend to engage more fully in academic tasks (Patrick *et al.*, 2007). Meanwhile, learner support is a way they view support from fellow classmates in terms of feeling cared about, with respect to their academic learning.

The role of the teacher in this instance is to support learners in believing that they are always cared for and valued and establish personal relationships with them (Irvin, 2008). Various researchers have found a good relation between the views of teacher support and learners' adaptive beliefs and engagement (Franke *et al.*, 2007; Bucholz & Sheffler, 2009; Rosa & Orey, 2011; Singh, 2014). When learners view their teacher as supportive, they report higher levels of interest, valuing, effort, mutual respect and enjoyment in their schoolwork (Hall & Sink, 2015).

Secondly, mutual respect in the classroom also plays an important role in social environment. It involves a perception that the teacher expects all learners to value one another and the contributions they make to classroom life and will not allow learners to make fun of others (Patrick *et al.*, 2007). This is because environments that are perceived as respectful are likely to be ones in which learners can focus on understanding tasks, without having their attention diverted by concern about what others might think or say if they are incorrect or experience difficulty. Subsequently, the role of the teacher in this aspect is to promote mutual respect in the classroom that contributes to learners' feelings of psychological safety and comfort, including low anxiety and low threat regarding making mistakes (Pierson, 2008).

Thirdly, the role of the teacher is to make decisions on how learners learn a particular content topic, by selecting quality activities that allow them to participate (Koehler & Grouws, 1992). Teachers need to be aware of the knowledge their learners have at various stages so that they can provide appropriate guidance to them when completing these activities. During classroom discourse learners should be encouraged to share their ideas and seek clarification until they understand (Wachira *et al.* as cited by Aineamani, 2017). To achieve this kind of classroom, teachers need to establish an atmosphere where learners are encouraged to interact and exchange ideas with each other during academic tasks. They have opportunities to ask or answer questions, make suggestions, give explanations, justify their reasoning, and participate in discussions (Schoenfeld, 2002; Patrick *et al.* 2007). In addition, Patrick *et al.* (2007) further emphasise that when teachers build such an environment, learners understand that it is acceptable to struggle with ideas, to make mistakes and to be uncertain.

This attitude encourages them to participate actively in trying to understand what they are asked to learn because they know that they will not be criticised personally, even if their mathematical thinking is criticised (Doyle, 2007). According to Anthony and Walshaw (2009) tasks should involve more than one taught procedure and provide opportunities for learners to struggle with important mathematical ideas.

According to Even and Tirosh (2008), teachers should pose tasks at an appropriate level of mathematical challenge, in order to promote learners' development and use of a wide range of mathematical thinking and reasoning activities. In this regard learners are encouraged to interact and exchange ideas with each other during small group activities (Valli & Buese, 2009).

2.5.2 Administrative environment

Teachers have the role of organising the physical environment where learning takes place, at the same time providing a learning environment that gathers information for planning purposes (Chifwepa, 2008). The classroom learning environment will then change learners' attitudes towards mathematics since the mathematics classroom learning environment is a major element of not only learners' cognitive outcomes but also their affective outcomes (Yang, 2013). This practice takes place in the administrative environment where the teacher's task is to convey mathematics to learners by making use of the curriculum, organised tasks and the best strategies to utilise maximum learning. Information collected from learners is meaningful and necessary for understanding the Namibian mathematics teaching culture and the effectiveness of the implementation of the mathematics curriculum.

The teacher has different tasks in yielding this type of environment. A very important part of the teacher's responsibility is to assign classroom and homework activities, relevant for the learning purposes, in order to find out the learner's progress in learning. The teacher should ensure that each lesson presented is organised and should use a lesson plan consisting of a clear outline indicating these lesson activities (Mercy, 2012). The lesson plan comprising the following components: the time frame, learning objectives, learners' and teachers' strategies, teaching materials and reflections. When a lesson is organised in this fashion, learners get the opportunity to practise and learn logically, and construct their own facts and ideas, so that they can be assessed better. Assessment of learners' learning is also vital to effective teaching and learning in most education systems (Nisbet & Warren, 2000).

Assessment has been found to be an effective method for the improvement of learners' learning in schools by providing opportunities for independent practice, and providing good foundation for self-assessment (Mercy, 2012). Therefore, the teachers are expected to use the assessment to guide effective decision-making, particularly with respect to on-going evaluation of learners. Dandis (2013) believes that assessment in the classroom includes considering learners as constructors of knowledge; teachers are then responsible for finding the reality in materials and activities, on-going evaluation tools and empowering them. These ideas once put into practice, can contribute to the choices learners make, vision, self-discipline, and trust in learners.

Teachers also need to know about their learners' problems and progress while learning, and the level of knowledge acquired so that they can adapt their teaching strategies to meet the learners' needs (De Lange, 1999). A teacher can find this information through observing the learners during teaching and learning. Lesson observation is a good mechanism and a suitable tool that helps teachers to further improve their teaching quality and effectiveness (Whitehurst *et al.*, 2014).

Another responsibility of the teacher is to keep record of the learner's existence in the school. This includes any written or recorded items that show the existence of a particular learner, such as the number of learners in the classroom, the number of desks acquired (Nakpodia, 2009). Record keeping is an essential administrative task and includes official documents, books, and files containing important information on the activities given in the classroom. Records are kept in the teacher's classroom for retrieval of information when needed (Regina, 2011).

Keeping important information and records of learners' learning helps teachers to make important decisions about their learning progress. Other record systems, such as learners' record reports, admission register, lesson plan, scheme of work, time-table, filing system, daily record, daily and weekly programme, samples of learners' work, and inventory sheets are also essential for organisation purposes (Gama, 2010; Singh, 2014). When fully implemented, the record management system can improve the administration environment, hence creating a good relationship between the learner and the teacher.

2.5.3 Teachers' views on the learning of mathematics

The teacher's views on how learning should take place in the classroom are strongly based on the teacher's understanding of the nature of mathematics (Thompson, 1992). Therefore, teachers should conceptualise the nature of mathematics. Teachers' views of mathematics are multi-layered and include how they personally perceive mathematics, how teachers view mathematics for their learners, how they view the mathematics curriculum (Renne, 1992). The following teachers' views are commonly held by the mathematics teachers as they give their perceptions on how they interpret themselves pertaining to the teaching and learning of mathematics.

Thompson (1992) identifies four dominant views on how mathematics is taught and learned, namely content focused view, content performance view, learner focused view and classroom focused view. Firstly, the content focused view which is based on mathematics teaching is guided by the content itself (Thompson, 1992). Content-oriented teachers emphasise activities

without connecting what is learned by simply just considering basic math facts, procedures for long division, or what unit to use for measurement (Rene, 1992).

Teachers follow the mathematics structures that consist of basic facts and procedures on how to convey content during teaching and learning (Wedegé, 1999; Stipek *et al.*, 2001). The most significant role of the teacher in this view is to explain, demonstrate, define the materials and present these in the expository manner (Danielson, 2007). The role of the learner is to listen and take part in the lesson by responding to teachers' questions and do exercises using procedures that have been modelled by the teacher.

Similarly, the second view is content focused with emphasis on performance in mathematics (Thompson, 1992). There are teachers who believe in teaching the subject for the knowledge and those who believe in teaching for developing learners (Rene, 1992). Collopy (2003) emphasises two types of teacher in the presentation of mathematics content. These are teachers who organise their lessons using the guidelines in the curriculum, be it the textbook, district curricular guides, state frameworks, all of which would be classified to impart knowledge to learners. On the other hand, there are also those teachers who plan their lessons by considering the learners' needs and interests, how the children learn, their unique backgrounds, etc.

These beliefs define the outcome of the learning of mathematics. The significance of the above views is crucial because the focus is on conceptual understanding and understanding based on learners' personal constructions of mathematical knowledge (Stipek *et al.*, 2001).

Thirdly, a learner focused view on mathematics teaching and learning is based on the constructivist view of mathematics learning (Cobb *et al.*, 1992). This view is based on the idea that for the learner to become a good problem solver he/she executes an act of knowledge discoveries. The knowledge is discovered through solving problems using strategies and appropriate tools such as a calculator and "having learners construct their own solutions leads to effective learning experiences" (Matlala, 2015:29). A constructivist view of mathematics learning typically centres on the learner's involvement in doing mathematics in terms of exploring and reinforcing ideas (Planas & Gorgorió, 2004). Teachers who advocate this view believes in mathematics as a useful subject when learners construct their own ideas during learning (Van de Walle *et al.*, 2010; Tirosh & Graeber, 2003). Teachers are seen as facilitators and stimulators of learners learning by posing interesting questions for investigation and challenging learners to think (Beswick, 2011).

Fourthly, the classroom focused view of learning is described as the teachers' views of connecting the content to the real world. The teacher's role is simply to assist learners to develop their own mathematical knowledge through creating learning environments posing problems, questioning learners about their problem solutions, and using learners' thinking to guide instructional decisions (Putnam *et al.*, 2000; Dossey, 1992).

Central to this view is the idea that classroom activities must be well structured. For example, the teacher should have a lesson plan which is organised around clear components such as: learning materials, teacher and learner activities and learning objectives (Remillard & Bryans, 2004).

In this classroom, the teacher is viewed as playing an active role in directing the classroom activities. Classroom activities can be such as presenting the teaching and learning materials, allowing learners to practise and identify what is asked for in the problem, discussing any unfamiliar terms in the problem and asking learners to restate the problem using their own words. On the other hand, the learners' role is to listen, cooperate with other peers, answer questions and complete tasks assigned by the teacher (Newman, 2002). This view is more teacher-centred, where teachers believe that mathematics is a set of real information that must be presented to learners. Phoshoko (2013:45) identifies this view as "problem-solving view, which is seen to be dynamic and a continually expanding field of human inquiry that is driven by problems".

2.6 The use of technology in the classroom

Through the years, technology tools relevant for mathematics education have greatly increased. Using technology materials should help learners deepen their understanding of key mathematical concepts (Kolovou *et al.*, 2008). Some of the commonly used technologies in education today are laptops, tablets, interactive display boards, digital and video cameras, document cameras, the Internet, computers, and calculators. All of these help to deliver learning materials and support learning processes in classrooms in order to improve academic learning goals (Van de Walle *et al.*, 2010). The advancements of technology started from the use of paper and pencil through to calculators, and then computers. But today, more technological tools are still emerging, impacting classrooms teaching and learning and readily available to enhance mathematical understanding (Tajudin *et al.*, 2011). This is a clear indication that mathematics is "a fundamental part of human knowledge and one of the central planks of the modern technological revolution" (Ernest, 2015:187).

According to Watson (2015:12) technology “is used appropriately when it is an active, integral part of the mathematical teaching process.” Certain technologies such as computers were commonly used in mathematics education with programs such as PowerPoint, an alternative for lesson presentation. Nowadays, computer technologies have advanced as a tool for investigative concepts in a multi-representational environment (Thomas *et al.*, 2007). Teachers are endeavouring to find more ways to integrate computer technologies into the classroom in order to increase learners’ engagement and achievement in mathematics.

Calculators are also one of the technologies widely used in solving mathematical calculations during teaching and learning. They serve as many purposes as other concrete materials in the teaching and learning of mathematics. Calculator technology can be used to facilitate problem-solving and to encourage learners’ interest in mathematics (Sheets, 2007). Several research studies have found that calculator use improves the problem-solving abilities of learners at all levels and at all grades (Surgenor *et al.*, 2007; Van de Walle *et al.*, 2010; Parrot & Leong, 2018). This is the reason why researchers describe the calculators as powerful tools in mathematics teaching and learning which can be used effectively to promote higher-order thinking and to help learners become flexible and resourceful problem solvers (Pomerantz, 1997; Koay, 2006). Moreover, calculators can even help learners discover new problem-solving strategies, which will allow them to learn basic mathematics concepts and increase positive attitude towards the use of calculators (Lin & Yuan, 2009).

Calculators are also thought to be useful in exploiting concepts and connections through multiple representations (Ruthven, 1994), and can effectively be used to improve conceptual understanding (Cavanagh & Mitchelmore, 2003). Consider the task of using a calculator to find a number that when multiplied by 5 will produce 25. In this situation, a learner can press $\sqrt{25}$ to get the square root of 25 which is 5. This activity serves as a meaningful and conceptual introduction to square roots (Van De Walle *et al.*, 2010). Idris (2006) agrees with Van De Walle *et al.* (2010), contemplating that a calculator will develop concepts when used effectively. For example, being asked to add 897 and 9874 could be efficient when using a calculator because it is quicker than using other methods such as mental computations or paper and pencil method and ensures a better chance of getting a correct answer. This is a way of using calculator when computing or manipulating numbers, especially in arithmetic (Pyke *et al.*, 2008). It is important to note that even the teachers acknowledge that learners still have to develop their computational skills, as well as know the principles for basic operations and the underlying concepts, the theories of the various algorithms and manipulative procedures (Mogari & Faleye, 2012). Effective use of calculators will help learners in developing the concept of number sense and computation skills in mathematics (Papadopoulos, 2013).

When technology such as calculators, is appropriately used, it may serve to improve learners' mathematics achievement as well as enhance the overall learning environment of the school (Pomerantz, 1997). This tool will support and extend learners' understanding of the pertinent concepts, processes, and themes, creating a powerful learning environment for learners. Waits and Demana (1998) suggest that teaching and learning mathematics should be happen in an active and dynamic classroom with learners thinking, exploring and applying what they have learned.

Warschauer *et al.* (2004) are of the opinion that the integration of calculators in learning mathematics may be beneficial in encouraging active learning with teacher instructional guidance and thus would help to improve learners' performance and free learners completely from tedious calculations and allow them to focus on problem-solving strategies (Warschauer *et al.*, 2004).

2.7 Conclusion

This chapter focused on the learning of mathematics, stating different definitions of what mathematics really is in order to get a better understanding and an overview of how mathematics is learned. Teachers' views, namely content views, constructivist views and more were described with specific emphasis on the importance of how individual teachers perceive and convey knowledge to learners using different disciplines. This drives the attitude of teaching mathematics with understanding. Several roles of the teacher in the learning of mathematics were emphasised, describing the social responsibility of the teacher during the learning environment up to the administrative environment. The administrative environment was discussed emphasising the organisational responsibility of the teacher. From this chapter it is clear that the role of the teacher is crucial. They have to understand and respect the learners' situation in each and every aspect of learning, and most importantly, teachers are encouraged to reflect on their own practices in the teaching of mathematics and to consider interpreting the mathematics curriculum based on standards and principles of school mathematics, so that they can design classroom experiments that produce a culture of mathematising amongst our learners.

It has been shown that technology can increase learners' achievement. Researchers have been examining the impact of calculator and computers use in classrooms for the past two decades, and the primary focus has been on using calculators and computers to solve problems.

CHAPTER 3 PROBLEM-SOLVING IN MATHEMATICS

3.1 Introduction

From the previous chapter, it should be evident that the major goal of the teaching of mathematics is to enable learners to apply what they have learned in the mathematics classroom to solve problems in familiar and unfamiliar situations or contexts. When learners learn mathematics with understanding, their thinking becomes more flexible and they are able to adapt to new situations during the solving of contextual problems (Elia *et al.*, 2009:605-606). Therefore, problem-solving is fundamental to mathematics education because teachers are interested in improving learners' ability to solve problems.

Mathematical problem-solving must deal with many aspects. This chapter deals with commonly known definitions of problem-solving; and the third section is founded on the overview of the historical development of the various definitions of problem-solving, based on authors' views with respect to the learning of mathematics. The definitions are followed by a brief overview of the role of problem-solving in the learning of school mathematics, focussing on mathematics teachers' views on problem-solving, as well as teachers' and learners' beliefs with respect to problem-solving. The teaching of problem-solving is discussed in detail, including the detailed planning of problem-solving lessons and examples of problem-solving tasks. The last section of this chapter is on the use of technology (calculators and computers) in the solving of mathematical problems.

3.2 Defining problem-solving in mathematics

Problem-solving in mathematics is a complex process which requires an individual to coordinate previously taught mathematical knowledge, understanding and perceptions, in order to gratify the demands of an unfamiliar situation (Kaur, 1997). Tripathi (2009) defines problem-solving as a thinking process directly connected to constructing, describing, explaining, manipulating, predicting, and finding possible solution paths. Solutions typically involve several modelling cycles in which descriptions, explanations, and calculations are gradually refined and elaborated (Nunes *et al.*, 2009). Schoenfeld (1992:353) refers to problem-solving "as a process where learners identify a problem for which they either have no immediate solution or a procedure that they can directly apply to get an answer". Problem-solving also refers to a process whereby a person encounters a problem which has no straightforward solution, nor a strategy that he/she can directly apply to get an answer (Schoenfeld, 1992).

Learners are required to read the problem carefully, analyse the information it contains, and examine their own mathematical knowledge to see if they can come up with a strategy that will help them find a solution. Mathematics teachers assist learners to construct new connections by asking questions to enable learners to review their knowledge (Karatas & Baki, 2013). When the new knowledge is embedded into existing cognitive frameworks, the result is an enrichment of the network of ideas through understanding (see 2.2.1).

3.3 Recent historical development of the definitions of problem-solving

Over time, mathematicians and mathematics education researchers have offered many definitions for mathematical problems and problem-solving in mathematics. Different opinions are reflected in the definitions concerning what constitutes a problem, while others simply refer to different ways of expressing ideas about what is important in mathematical problem-solving.

In the early 1940's, George Pólya changed the whole mathematics education world of that time by his book *How to solve it*. In his framework for problem-solving he introduces and explains that problem-solving in mathematics is about following four phases or steps, namely understanding the problem, devising a plan, carrying out the plan, and looking back (Pólya, 1945). Different problem-solving strategies are involved in each of these steps; for example, in order to understand the problem, learners have to apply their existing knowledge regarding the problem at hand; for learners to be able to devise/make a plan to solve the problem, they have different options like making a table, drawing a diagram, acting the problem out, and so forth. After learners have solved a given problem by following the mentioned steps, they should be able to solve other more complex problems related to the solved problem by working through the prescribed steps.

Lester's review of the literature on problem-solving revealed that despite the fact that Pólya's problem-solving strategies were useful, attempts to teach learners the use of general problem-solving strategies is not always successful (Schoenfeld, 2013). This is due to the fact that it requires a higher order thinking skill that should be at learners' disposal, in order for them to solve the problem alone. According to Limjap (2001), learners should be empowered to do mathematics in meaningful ways. The question is what can teachers do to help beginners gain intellectual empowerment? The goal of the four steps of Pólya's model is to create a set of practical reasoning which are only here as a set of direction for thinking along a technique most likely to lead to success of solving the problem (Rudd, 2010). Aydoğdu and Ayaz (2008) suggest that it may be possible for learners to solve complex problems by moving through them repeatedly to produce an answer. It is frequently the case that children move backwards and forwards between and across the steps in order to find a way to the solution.

After another extensive review of literature, more than a decade later, Lester and Kehle (2003) reported that little progress had been made in research on problem-solving, contributing little to school practice with respect to problem-solving. Laterell (2013) defines problem-solving as a process of reasoning which uses extensive prior knowledge to succeed in solving non-routine problems. He further defines problem-solving as a process of assessing possible strategies and abilities to apply them in reaching a solution, checking the results for accuracy, and writing out the solution in a clear technique. It requires learners to combine skills and concepts in order to deal with specific mathematical situations. If a learner knows his/her mathematics skills and concepts well, but he/she cannot put them together in a particular situation, then this learner will not be able to solve mathematical problems.

Lesh and Zawojewski (2007) define problem-solving as modelling: the problem solver will engage in mathematical thinking in order to deal with a real-life problem situation, while developing various strategies which can develop a sensible solution for the problem. For Schoenfeld (2013:25), "solving problems is part of the doing and sense-making of mathematics". In doing mathematics, learners investigate, make conjectures, and use problem-solving strategies to verify those conjectures; an act of interpreting, describing and explaining the nature of the task, not merely following procedures.

For the purpose of this study, problem-solving is referred to as a process where an individual completes a complex problem-solving task which he/she has no idea as to which procedure to follow. This task is organised in such a way that it requires a learner to think, explore and reason mathematically and to gain mathematical knowledge and skills while completing the task.

3.4 The learning of mathematics and problem-solving

3.4.1 Teachers' views on problem-solving

Historically, problem-solving was about demonstrating computational procedures without connections, expecting learners to solve problems by applying the knowledge acquired and procedures gained during a newly presented lesson (Hiebert *et al.*, 1996). In some classrooms, learners are taught about problem-solving and how to employ various methods as options when faced with a problem, but they are not given the opportunity to use their own methods/strategies and are inclined to be too dependent on their teachers.

Fortunately, in the twenty-first century school mathematics is viewed by some teachers as follows: school mathematics has to engage a learner in problem-solving, reasoning, developing problem-solving skills and fostering deep understanding (Limjap, 2001). Other teachers see

mathematics as constantly growing (Golafshani, 2002). A common argument for including problem-solving in a mathematics classroom is that problems have the potential to enhance the interest of learners and engage them in mathematical problems (Meletiou-Mavrotheris & Mavrotheris, 2012).

Consider the following problem: *It takes a dragonfly about 2 seconds to fly 18 metres. How long should it take to fly 110 metres?* Knowing that a learner has solved this problem using direct proportion, a teacher might adapt the task so that it is more likely to invite more reasoning: *How long should it take the dragonfly to fly 1100 metres? Or: How long should it take a dragonfly to fly 110 metres if it flies about 9 metres in 1 second?* (Anthony & Walshaw, 2009:152). When a problem-solving activity is complex and self-motivating, learners will actively direct their own participation to get deeply involved in the problem-solving task (Larson & Rusk, 2010).

3.4.2 Teachers' and learners' beliefs with respect to problem-solving

Beliefs about mathematics can determine how one (teacher/learner) mentally constructs mathematical ideas (Mkomange & Ajagbe, 2012). Teachers' beliefs about mathematical content and about how learners learn influence their (teachers') approaches to teaching. Furinghetti and Morselli (2009) categorise beliefs regarding mathematics education according to the object of the belief: beliefs about mathematics, beliefs about self, beliefs about mathematics teaching, and beliefs about contexts in which mathematics education occurs. For the purpose of this study, the focus is on beliefs concerning problem-solving in mathematics.

Teachers' beliefs regarding mathematical problem-solving are a dynamic factor for the success of creating a learning environment which determines the beneficial aspect for problem-solving (Memnun *et al.*, 2012). Lazim *et al.* (2004) list two structures of mathematical problem-solving beliefs about teaching and learning: teacher or subject-matter oriented beliefs and learner-oriented beliefs. The subject-matter oriented beliefs place a strong emphasis on teachers imparting subject-matter knowledge and its reproduction by learners (Bishaw, 2010). Teachers are largely responsible for the successfulness of the learners' learning process. Learner-oriented beliefs, on the other hand, focus on the supporting of learning, involving teaching learners how to learn. The emphasis is on the construction of knowledge (see 2.2.2). Learners are stimulated to take responsibility for their own learning processes and the regulation of these processes.

Beliefs and affective factors can assist or interfere with problem-solving. Learners' beliefs regarding problem-solving largely develop out of the experiences they have in mathematics classes and from the attitudes and beliefs passed on by their teachers. A learner's beliefs about mathematics can determine how the learner approaches a problem and how long and how hard he/she will work on it (De Corte & Op't Eynde, 2002). Therefore, many learners have certain beliefs about problem-solving that tend to impact their engagement with problem-solving negatively (Stylianides & Stylianides, 2011).

Learners' beliefs about themselves and their mathematical creativity, derived from their experiences with mathematics, shape the mathematical knowledge they draw upon during problem-solving and the ways they do or do not use that knowledge. These learner beliefs not only interfere with learners' ability to engage productively with problem-solving, but also often lead to low success rates and negative reactions to mathematics in general (Colby, 2007). For example, many learners believe that those who understand the content can solve assigned problems in five minutes or less (Colby, 2007). Learners with this belief tend to "give up on a problem after a few minutes of unsuccessful attempts, even though they might have solved it had they persevered" (Schoenfeld, 1992:359).

Most learners believe that all problems have an answer; that there is only one right answer and one correct solution method; and that slow learners are not expected to understand mathematics but can merely memorise and apply mathematical procedures in a mechanical way (Lazim *et al.*, 2004). Learners who believe that proofing mathematical facts (formulae) has nothing to do with discovering problems, make assumptions that contradict results they have just proven (Schoenfeld, 2015). Smith (2008) is of the opinion that learners need to justify their solutions. By checking the solution, they have obtained, they will better be able to determine that it works in the given situation. This is normally done in writing out the entire process of how to solve the problem.

3.4.3 The role of problem-solving in school mathematics

The mathematics teacher is the main source who creates an environment in which learners build their own new mathematical knowledge while they are solving mathematical problems (Rudd, 2010). Mathematics teachers' choices of problems and the way they describe and implement problem-solving into their lessons reveal their views and beliefs with respect to mathematical problem-solving (Van Bommel & Palmér, 2015).

Stanic and Kilpatrick (cited in Schoenfeld, 1992:338) identify three themes that have historically characterised the role of problem-solving in school mathematics: problem-solving as context, problem-solving as skill, and problem-solving as an activity.

The emphasis on using problem-solving as a context is on finding interesting and engaging problem-solving tasks (see 2.2.3) that help unpack a mathematical concept by means of visualising strategies such as representations to solve problems (Hegarty & Kozhevnikov, 1999). A representation is defined as any arrangement of characters, images, concrete objects, etc., that can symbolise or represent something else (Brahier, 2013). For example, a teacher might present the concept of fractions by assigning groups of learners the task to divide two pieces of pie among eight learners so that each learner gets an equal share (Van de Walle, 2010). By providing this problem-solving context, the teacher's goals are multiple: to create opportunities for learners to make discoveries about fraction concepts using familiar methods; to help make the concepts more concrete by means of practice; and to offer a rationale for learning about fractions by means of justification (Sajadi *et al.*, 2013).

According to the second theme, problem-solving is seen as a skill to be taught in the school curriculum. Here, problem-solving itself is the goal, part of the curriculum and added as one of a number of skills to be taught (Szetela & Nicol, 1992). Problem-solving skills are taught as a separate topic in the curriculum to develop basic skills and conceptual understanding (Van Bommel & Palmér, 2015). Learners are taught a set of rules to solve problems and given practice in using these rules to solve routine problems before they have opportunities to solve non-routine problems.

Routine problems are problems in which the path to the solution is readily known, and/or has only one solution and one method of reaching it. For example: *The product of two numbers is 64. If one number is 16, what is the other number?* is routine for most high school learners because they know the procedure (Yerushalmy *et al.*, 1999:233). In this problem, a true solution can be reached by solving mathematical operations in a systematic order (Bal, 2015). Therefore, problems require learners to use their analytical and thinking skills in combination with their conceptual and background knowledge (Bal, 2015).

On the other hand, a non-routine problem is one that has multiple solutions, or a single solution that can be reached using multiple paths to achieve it. These types of problems require learners to actively and consciously use their metacognitive abilities (Asman & Markovits, 1999). For example, the following problem is non-routine for most high-school learners: *If the area covered by water lilies in a lake doubles every 24 hours, and the entire lake is covered in 60 days, how long does it take to cover half the lake?* (Asman & Markovits, 1999:363). Therefore, non-routine

problems require learners to use their reasoning and thinking skills in combination with their conceptual and prior knowledge (Bal, 2015).

When teachers define the learning objectives of a problem-solving activity, they should be aware of the difference between teaching problem-solving as a separate skill and infusing problem-solving throughout the curriculum to develop conceptual understanding as well as basic problem-solving skills (McIntosh & Jarret, 2000).

Pólya introduces problem-solving as an activity (the third theme), referring to his view of problem-solving as the heart of mathematics (Schoenfeld, 1992). This view of problem-solving implies that the idea of merely being able to solve the problem is no longer central. Instead, the process of being able to solve a problem becomes the focal point. Pólya sees problem-solving as an act of discovery and introduces the term heuristics to describe the abilities learners should have when confronted with challenging problem-solving activities.

3.5 How to solve a mathematical problem

Pólya created a set of practical reasoning rules (heuristic reasoning) derived from his empirical, experimental method that directs thinking along the paths most likely to lead to success. The aim of heuristic reasoning is to study the methods and rules of discovery and invention (Rudd, 2010). It must be flexible to work well on a variety of problems and assist learners in directing their thinking along the paths that are most likely to lead to a successful solution.

The four basic phases or steps for solving problems (according to Pólya's problem-solving model) are to understand the problem, devise a plan, carry out the plan, and look back to examine the obtained solution (Aydoğdu & Ayaz, 2008). For learners to be successful in using the first phase of Pólya's problem-solving model, teachers need to teach learners to ask themselves comprehension and conceptual understanding questions (Rudd, 2010). This is done by stating the problem in your own words, pinpoint exactly what is being asked, identify the unknown and, most importantly, identify any information that is irrelevant to the problem (Arslan & Altun, 2007).

The second phase of the problem-solving process is to devise a plan to solve the problem. Now that the problem is understood, it can be solved step by step by choosing a relevant strategy to do so (El Sayed, 2002). If the solution cannot be found, the strategy can be changed (Windsor, 2010). In finding a solution for a problem it is sometimes possible that more than one strategy must be used. Learners need to collect possible strategies, for example, a table, a graph, or a diagram, and choose a suitable strategy (Bishaw, 2010:76).

The third phase/stage of Pólya's problem-solving model is to carry out the plan by implementing the chosen strategy. When a learner is carrying out the plan, he/she should keep a record of the steps while implementing the chosen strategy (Tyre *et al.*, 1995).

The final stage in Pólya's problem-solving model is to look back and examine the solution that has been obtained. (Rudd, 2010). Teachers have to ask learners to reflect on how they have arrived at the solution, and if there is another more efficient method or strategy they could have used. Another question to ask is: Does the answer make sense? To check an answer of problem-solving could imply that a learner has to solve the problem in another way to make sure he/she gets the same answer as before (Bishaw, 2016). Doing this will enable learners to predict which strategy to use to solve future problems.

Although teachers have to teach learners mathematical concepts, theorems, formulae, and procedures for solving specific types of problems, it is not sufficient to ensure success in problem-solving. Learners do not only need to have mathematical knowledge and an understanding of mathematical concepts, they also have to be able to apply relevant knowledge and understand what they have done (Rudd, 2010).

3.5.1 Problem-solving strategies

The ability to solve problems is an important and integral reason for learning mathematics. Rudd (2010) further suggests that teaching learners to use heuristic problem-solving reasoning and strategies can help them become expert problem-solvers and assist them to apply their contextual knowledge to new problems and situations. The application of problem-solving teaching in mathematics mostly relies on two important skills such as contextualising the lesson and the applying of problem-solving strategies (Rudd, 2010). The teaching of general problem-solving strategies in the school classroom has the potential to enhance learners' problem-solving capacity in their daily life situations. To this effect Bishaw (2010) reports that seventh grade learners, who were taught problem-solving strategies, used each strategy and thus became better at solving problems. After a similar study, Hoon *et al.* (2013) report that learners who were taught using problem-solving strategies engaged profoundly in their learning. Such an active learning environment improved the learners' interest in solving mathematical problems and also enabled them to respond creatively to given problems.

Make a drawing

Drawings can help learners visualise problem situations (Taspinar & Bulut, 2012). This strategy is especially helpful when the problem solver wants to visualise representation of the problem and their relationship(s) (Posamenter *et al.*, 2010; Johnson & Herr, 2001). For example, in the

following task: *A school has 10 boxes of netball balls. Four boxes have 6 balls in each box. The other boxes have 5 balls in each box, how much balls are there altogether?*

Learners can make drawings representing four boxes containing 6 balls and the other 6 boxes containing 5 balls. They then add the number of netball balls together to visually show the total number of balls. The learners are then responsible to verbally come up with the drawings of boxes and give reasons why they think they are correct. "The need to recognize and utilize relationships in mathematics is vital to one's success in mathematics" (Blatto-Vallee *et al.*, 2007:433). When one talks about visual representation of the problem, it means representing the information in the problem, with a diagram.

Guess and check

The *guess and check* strategy for problem-solving can be helpful for many types of problems. The goal of this strategy is to get learners to solve mathematical problems by guessing the answer and then check if the features in the problem can be used to produce an effective answer (Chunlian, 2009). If the guess fits the conditions of the problem, then the solution can be worked out, but if the conditions are not satisfied then, the guess can be refined and another guess can be made (Rickard, 2005; Taspinar & Bulut, 2012). When using this strategy, learners are encouraged to make a reasonable guess, check the guess and revise the guess if necessary (Cechlárová, 2014). By repeating this process, a learner can arrive at a correct answer that has been checked. Using this strategy does not always yield a correct solution immediately, causing learners to persevere, but it provides information that can be used to better understand the problem and may suggest the use of another strategy. In Mrs Ingo's class, there are 24 learners. There are 6 more girls than boys. How many boys and girls are there? To solve this problem a learner can start by drawing a table, with the number of boys, girls and the total. Because we know there are six more girls than boys, we can guess the number of the boys and then calculate the number of girls and the total from there. From there, learners can incorporate their own guess, for example if the initial guess is getting 12 boys, then it is clear that there would be 18 girls, giving a total of 30 learners. The total, however, should be 24, which means this guess was too high. The number of boys is revisited until the result of 24 is met. Thus, learners are responsible to verbally come up with what the correct guess is and why they think they are correct.

Look for a pattern

The goal of the *look for a pattern* strategy is to get learners to solve mathematical problems by analysing patterns within the problem (Johnson & Herr, 2001). When learners use this strategy,

they are required to analyse patterns in data and make predictions and generalizations based on their analysis. Then, they have to check the generalization against the information in the problem and possibly make a prediction from, or extension of, the given information. A pattern is a regular, systematic repetition (Taspinar & Bulut, 2012).

Johnson and Herr (2001) are of the opinion that looking for a pattern is a very important strategy for problem-solving and is used to solve many kinds of problems. Making a number table often reveals patterns, and for this reason it is frequently used in conjunction with this strategy. Using this strategy with the example of the two odd integers adding up to 92, learners can start by making a table of possible ways to get an answer of 92. Using the table, they can look at the odd integers only and base their solution on those numbers.

Make an organised list

Much like looking for patterns in problems, the goal of the *make an organised list* strategy is to get learners to solve mathematical problems by analysing patterns within the problem and listing the information. Consider the following problem:

Kauna is on vacation and brought 3 pairs of pants (blue, black and white) and 3 shirts (pink, yellow and green). How many different outfit combinations can she make?

To solve this problem, learners should organise the information using a tree diagram. Learners can start by making an organised list of possible ways to making different colour combinations. For example:



From the list, they can then narrow their focus to the odd integers and thus find their solution.

Making an organised list is a problem-solving strategy which allows learners to organise the data and visually consider their options when answering a problem. This strategy also allows the learner to discover relationships and patterns among data. In the attempt to produce an organised list, learners will likely encounter frequent and repeated patterns (Posamenter *et al.*, 2010).

Solve a simpler problem

The problem-solving strategy to *solve a simpler problem* most often is used in conjunction with other strategies. Writing a simpler problem is one way of simplifying the problem-solving process.

For example: There will be 7 players in a tournament. Each player must play every other player once. How many games will take place in the tournament?

Game 1: The second person plays the first

Game 2: The 3rd person plays the first and second:

Game 3: The fourth person plays the first, second and third

The total number of games for five people is $1 + 2 + 3 + 4 = 10$ games. Therefore, six people would be $10 + 5 = 15$ games and seven people, would be $15 + 6 = 21$ games. The total number of games for 5 people = $1 + 2 + 3 + 4 = 10$ games. Therefore, for 6 people, it would be $10 + 5 = 15$ games and for 7 people, it would be $15 + 6 = 21$ games (FDE, 2000).

Re-wording the problem, using smaller numbers or using a more familiar problem setting may lead to an understanding of the solution strategy to be used (Johnson & Herr, 2001). Many problems may be divided into simpler problems to be combined to yield a solution. Sometimes a problem is too complex to solve in one step. When this happens, it is often useful to simplify the problem by dividing it into cases and solving each one separately.

Work backwards

“Among the range of problem-solving strategies in mathematics, working backwards is a particularly useful method in situations when the end result of a problem is known and one has to find the initial quantity” (Ramful, 2015:28). For example, given two problems:

Problem 1: A parking lot can contain a maximum of 45 cars. How many cars can $\frac{2}{3}$ of the parking lot contain?

Problem 2: There are 30 cars in a parking lot. This is $\frac{2}{3}$ of the number of cars that the parking lot can contain. How many cars can the parking lot contain?

To solve such problems, it is usually necessary to start with the answer and work methodically backwards to fill in the missing information. In the first problem, one has to find a part, from a given whole. In the second problem, the part is given and one has to construct the whole.

Thus, if $\frac{2}{3}$ of the number of cars represents 30, this means that $\frac{1}{2}$ of $\frac{2}{3}$ that is $\frac{1}{3}$, represents 15 cars. Therefore, one whole, which can be made from 3 one third units, corresponds to 3×15 cars = 45 cars. Clearly, the second problem creates more contexts for deep mathematical thinking.

This strategy is extremely useful in dealing with a situation or a sequence of events. Posamenter (2010) is of opinion that learners start working at the “end” of the problem situation and work through the process in reverse order to establish what happened in the original situation. The goal of the “work backwards” strategy is to get learners to undo key elements in the problem in order to find a solution (Johnson & Herr, 2001).

If the above-discussed strategies are carried out effectively, learners will become successful in handling a problem situation and the skills of learners with respect to problem-solving should improve (Limjap, 2001). These types of strategies can assist learners in solving problems in multiple formats and can be used in combination or separately (Rudd, 2010). In order to succeed in problem-solving, learners have to use their prior knowledge, apply acquired mathematical skills, understand the context of the problem situation, and choose the appropriate strategy to solve the problem (Karatas & Baki, 2013).

Drawing a graph

This strategy is especially helpful when the problem solver wants to visualise relationships. For example, you survey your friends to find the kind of movies they like watching best.

Comedy	Action	Romance	Drama	Sci-Fi
2	2	5	4	6

You can show the data on a pie chart, by first adding up the values to get the total. Divide each value by the total and multiply by 360° . The learners then are responsible to verbally come up with what the two odd integers are and why they think they are correct. The goal of the *make a graph* strategy is to get learners to create a graph to help them visualise the problem (Novotná *et al.*, 2014). Learners can use any type of graph to help them build a solution to their problem. Much like drawings, graphs can help learners visualise problem situations. This strategy incorporates the use of different types of graphs to represent the problem and solution.

3.6 The teaching of problem-solving

3.6.1 Introduction

Problem-solving is enhanced by a relaxed, even playful environment (Limjap, 2001:14). More importantly, learners who are engaged in creative problem-solving have to know that their ideas will be accepted. Learners who do well in tests on creative problem-solving appear to be less afraid of making mistakes or looking foolish than those who do not do that well. Successful problem solvers also seem to treat problem-solving situations more playfully. This implies that a relaxed, fun atmosphere is important while teaching problem-solving. Learners should certainly be encouraged to try different solutions and not be criticised for taking a wrong turn while solving a mathematics problem.

The teacher who wishes to develop the learners' ability to solve problems have to instil some interest in problems into their minds and give them plenty of opportunity for understanding and applying what they have learned (Pólya, 1945). Furthermore, teachers need to create a problem-solving environment that encourages learners to explore, take risks, share failures and successes, and challenge one another (Bishaw, 2010). In such supportive environments, learners develop the confidence that enables them to explore problems and the ability to make adjustments in their problem-solving strategies.

Teachers should focus their attention on their own abilities to become competent problem solvers and provide a learning environment for learners to explore problems on their own and to invent ways to solve the problems (see 2.2.3). For learners who are struggling to become better problem solvers the difficulty caused by the complexity of problem-solving is complicated by the fact that most of the learners do not receive adequate training in solving problems, either in quality or quantity (Ma, 2013).

In problem-solving, learners should be encouraged to consider all possibilities before trying out a solution. One specific method based on this principle is called brainstorming, in which two or more individuals suggest as many solutions to a problem as they can think of, no matter how farfetched these may seem (Sdouh, 2013). The point of brainstorming is to avoid focusing on one solution too early and in doing so ignoring better ways to proceed. After the brainstorming, learners should select one of the proposed strategies to solve the problem, estimating the answer to see if it seems reasonable. In addition, teachers have to decide which problems and problem-solving experiences to use, when to give problem-solving particular attention, how much guidance to give learners, and how to assess learners' progress (DBE, 2011).

3.6.2 The teacher's role in a problem-solving classroom

3.6.2.1 Planning a problem-solving lesson

In this sub-section a ten-step guide for the planning of a problem-solving lesson (Van de Walle *et al.*, 2014:68) is adapted for use in the Namibian school context. This is illustrated in Table 3-1. The topic of the Grade 9 lesson is: The perimeter and area of a trapezium.

Table 3-1 Problem-solving lesson plan

Steps	Description (Teachers' and learners' activities)
1. Determine the mathematical concepts and goals	Mathematics teachers are expected to identify the mathematics concepts (and related skills) in line with the Namibian Grade 9 Mathematics syllabus. In this case the learning objective is: By the end of the lesson learners should be able to Calculate the perimeter and area of a trapezium A possible goal for the lesson (with respect to the above objectives) is for learners to explore the relationship between perimeter and area of a trapezium. Johnson and Norris (2006:41) state that "The objective really sets the stage for the entire lesson". Therefore, it is important for the teacher to reveal the objectives to the learners in such a way that they realise the value of the lesson.
2. Consider the needs of the learners	At this stage the teacher will consider activating the prior knowledge of the learners. The knowledge that learners bring to the classroom is regarded as the most important factor influencing learning (Mochesela, 2007). In this case learners should have basic knowledge on how to find the area and perimeter of a rectangle and a triangle. The teacher's role is to assist learners in connecting new learning to previous learning, in order for learners to understand new knowledge. In addition, the teacher can recommend the use of calculators and other tools to support understanding.
3. Select a task on the perimeter and area of a trapezium.	The teacher selects or compiles a task that suits the learning objectives above. In this case the learners must complete the following task individually. <i>Onawa Supermarket is constructing a new building, an extension of the old building, to accommodate the amount of goods in the shop.</i>

Since finances are not sufficient to cover the whole extension, the shop owners want to put up a preliminary structure with two side walls, by using four poles (two front poles with the height of 30m and two back poles with the height of 14 m) and sink plates with the length of 15m. The distance at ground level between the two poles is 8 m.

Question: How many bricks will the owners have to buy to complete one side of the building?

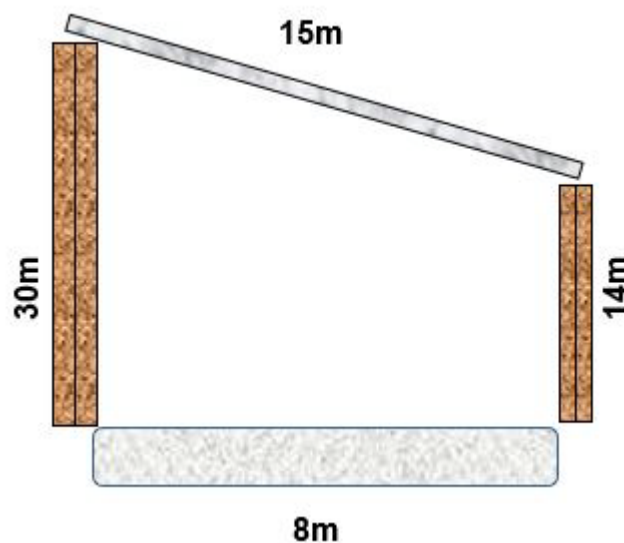


Figure 3-1. One side of the building

4.
**Design lesson
assessment**

The teacher has to decide what types of assessment will be used during the lesson. In this case the teacher will give a worksheet consisting of problem-solving questions for learners to complete on the area and perimeter of a trapezium. For example:

Objective 1: Learners will be able to calculate the perimeter and area of a trapezium.

Assessment The teacher will allow learners to complete the worksheet for finding the area and perimeter of a trapezium, and then ask individuals, if they are able to solve the problem, explain in their own words how to solve the problem, and describe the formula they used to solve the problem as well as the relationship they see between the perimeter and area of a trapezium.

5.
**Plan the
activity before
the task is
presented to**

Lester *et al.* (as cited by Schoenfeld, 1992:65) suggest that teachers give learners a short activity to activate their prior knowledge. This is done to ensure that learners realise that the task (see number 3 above) includes the area and perimeter of a rectangle and of a triangle.

Activity: Thomas has a garden, which he divides into different portions of grass, carrots and hay. Find the area of the whole garden and the

the learners.

distance around his garden.

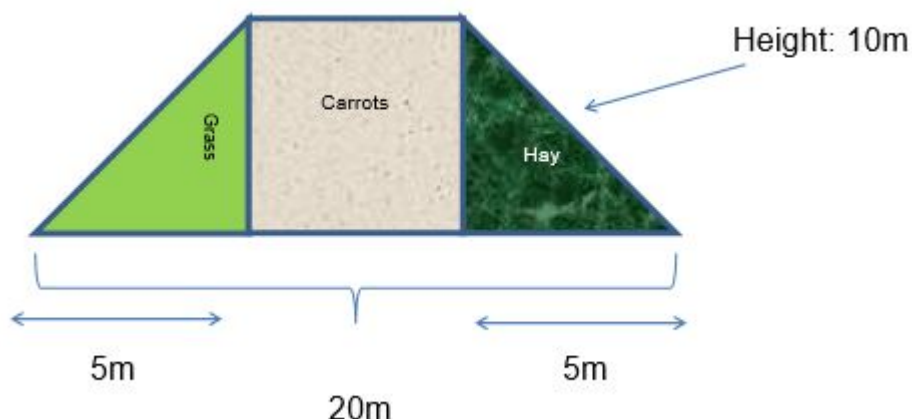


Figure 3-2 Thomas's garden

The learners are expected to find the perimeter and area of each shape in the garden in about 5 - 10 minutes. Give them freedom to use calculators. After the completion of the activity, 5 learners can be asked to give feedback on how they got their answer. The teacher has to lead the discussion and thus introduce the topic of the lesson.

6.

Plan the questions that will be asked during the completion of the task

At this stage the teacher will give the learners the opportunity to complete the task discussed in number 3 of this table. Teachers should make sure that learners understand the problem before they start working on it, in order to avoid misinterpretations and misconceptions. Encourage learners to solve the problem given in the task in a way different from how they have completed the activity (Alsawaie, 2003).

The teacher should ask learners appropriate questions to help them clarify the direction they are taking in solving the problem, and in the process providing hints to solve the problem. Questions like the following could be asked: Is that the correct formula that you are using? Do you think you can get the answer with those methods? What makes this problem difficult? Which strategy did you use? In doing this, the teacher is supporting the learners' thinking but is not telling them how to solve the problem.

7. Plan the discussion for after the learners have completed the task.	Firstly, the teacher will put up another (different) drawing about perimeter and area of a trapezium on the chalkboard. Secondly, the following questions may be asked: • What do you notice about the relationship between the perimeter and area of the trapezium on the chalkboard? • What steps did you take (in your calculations) to find the perimeter and area of the trapezium? • Is it helpful to use a calculator to find the area and perimeter of a trapezium? • Can you describe two different methods of finding the area of a trapezium? Hiebert and Grouws (2007) argue that good questions are motivational; they arouse the interest of learners and engage them in doing mathematics. Therefore, teachers can encourage learners to reflect on their solutions and on the method, they used by asking the above kind of questions. Afterwards, all the main points of the discussion are summarised by the learners in order to ensure that all the objectives of the lesson have been attained (Alsawaie, 2003).
8. Check that all aspects of the lesson are aligned with the objective of the lesson	The objective was used to plan the task as well as the activity that was supposed to activate prior knowledge, including all the questions set for both the task and the activity.
9. Anticipate learners' approaches to the task	Refer to the activity as presented in Figure 3-1 (used to check for prior knowledge). Learners might confuse the definition of area and perimeter. Examples: 1. A learner records the perimeter as 44m because these are the only sides that he/she can see on the diagram, not considering the other corresponding sides. One reason for this common misconception is when presenting a lesson with no visual objects to display. 2. Another misconception could be that a learner could give the area of the trapezium above as $352m^2$. In this case a learner omitted the factor $\frac{1}{2}$ in the formula, and therefore obtains the answer by determining $(30\text{ m} + 14\text{ m}) \times 8$. How to address the above misconceptions: Learners need to be guided through the process of the deduction of the

formula of the area of a trapezium in different ways. That should only happen after the learners have completed a task and an activity as the examples mentioned elsewhere in this table.

For example, the learners can deduce the area formula as indicated below.

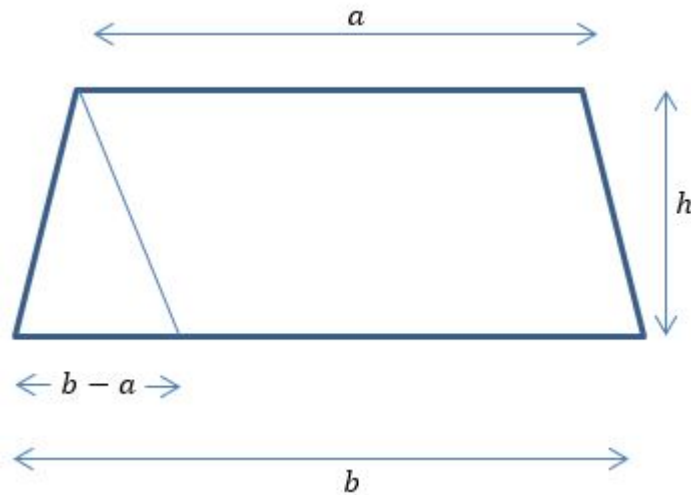


Figure 3-3 Illustration to determine the area formula

The area of parallelogram $a \times b$

The area of triangle $\frac{1}{2}(b - a) \times h$ so adding this together will give us:

Area of trapezium = $ah + \frac{1}{2}(b - a)h$ (remove the brackets)

$$= ah + \frac{1}{2}bh - \frac{1}{2}ah \text{ (combine like terms)}$$

$$= \frac{1}{2}ah + \frac{1}{2}bh \text{ (Finding common factor } \frac{1}{2} \text{ and } h)$$

$$= \frac{1}{2}(a + b)h$$

The area of the trapezium is $\frac{1}{2}(a + b)h$.

10. Identify essential questions in each phase.	During the lesson the teacher needs to ask learners questions based on the objectives. These questions should lead to deep understanding: <i>Questions asked before the task is presented to learners:</i> - Has anyone seen a problem like this before? - What is the problem asking you to find? - What strategies will you use to get started? - Can you guess the right answer? <i>Questions asked during the completion of the task:</i> - Tell me what you are doing? - Why are you doing this? - What will you do with the result of what you are doing? - Why is this idea better than that one? <i>Questions asked after the task is completed</i> - Have you considered all possible cases? - Have you checked your solutions? - Does your answer look reasonable? - Can you explain your solution to the rest of the class?
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3.6.2.2 Problem-solving tasks

In order for teachers to realise the role of problem-solving in the curriculum, they should be able to distinguish between the various types of problems and their roles (Yee, 2002). In addition, they (teachers) should also equip themselves with knowledge and understanding to be able to select and construct tasks for their learners that will promote different forms of thinking activities in a mathematics lesson. However, “one good way to encourage communication in the mathematics classroom is to provide the learners with a learning environment that arouses their active participation” (Viseu & Oliveira, 2012:290). It can be done by using mathematical tasks that challenge the learners and stimulate them to make connections (to think and to find meaning), to justify, to explain and to make new connections (see 2.2.1).

What learners learn is largely influenced by the tasks given to them. The tasks used in the classroom provide the starting point for their mathematical activities (Vale & Pimentel, 2011). Mathematical tasks include the problems and practical activities used by teachers in the mathematics classroom (Hsu, 2013). A mathematical task will take up a certain amount of time in the classroom, during which learners are expected to work hard at learning specific mathematical concepts.

Basically, most problems can be broadly classified as closed or open-ended in structure. Open-ended tasks will often be offered as a form of problem situation that contains many problems with different levels of difficulty. Ideally, they enable learners to demonstrate their thinking and understanding of mathematical concepts in a variety of ways (Yee, 2002).

On one hand, they offer more freedom to learners to think in the problem-solving phase. Most tasks in mathematics textbooks are closed problems, for example, a rectangle is $10m$ long and $5m$ wide. What is its perimeter and area? Closed problems are used to obtain knowledge or an understanding of facts and have only one correct answer (Yee, 2002). They provide little creativity and children are usually asked to find the “right” answer to discover their understanding of facts. Another example is: *If I have 3 dozen eggs, how many eggs do I have?* This question has 2 closed parts to it. Learners need to know the fact that there are 12 eggs in a dozen, and that 3 times 12 is 36. These are all facts and there is only one right answer (Al-Absi, 2012). In open tasks, learners may end up with different, but equally correct solutions; therefore, open tasks usually have several correct answers. Yeo and Yeap (2010:2) describe open-ended problems as “ill-structured” because they have no fixed procedures that guarantee a correct solution. In order to solve these problems, learners have to expand their existing knowledge to engage in the problem situations. Open-ended problem tasks can foster higher-order thinking and promote reflection.

Lee (2011) indicated that learners should be provided with problem-solving tasks for which they have no memorised rules. It is important to learn about alternative solutions in mathematical problem-solving because you can learn more from solving one problem in many different ways than you can learn from solving many different problems, each in only one way. Open problems provide more opportunities for varied ability learners to demonstrate their mathematical ability. Lee uses the example: *My mum gave me 28 lollies to share equally among myself and my friends. How many friends could I share my lollies with and how many would we each get?* This question has several possible correct answers, but not an infinite amount of correct responses. (Myself and 1 friend: 2×14 ; myself and 3 friends: 4×7 ; myself and 6 friends: 7×4). The focus of this open question is to assess learners’ ability to use division or multiplication accurately.

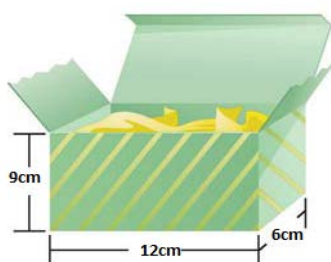
Therefore, a great way to engage learners in mathematical tasks, is to use open-ended questions in investigations and projects for them to explore and apply their knowledge. Being able to use processes or procedures taught in class is encouraged when learners respond to open-ended questions where their skills can also be assessed further.

Different types of mathematical tasks tend to lead to differences in teacher-learner interaction and in how the tasks are presented in the classroom (Hsu, 2013). Low cognitive demand mathematical tasks emphasise using memorised formulae or relationships to solve problems and require proficiency in computational procedures; thus, unidirectional or closed methods are the norm for teacher-learner interaction. High cognitive demand mathematical tasks involve more complex information and mental processing and thus require learners to create problem-solving methods and use a deep understanding of mathematical concepts during the process of thinking, discussing and exploring.

3.6.2.3 Examples of different types of problem-solving tasks

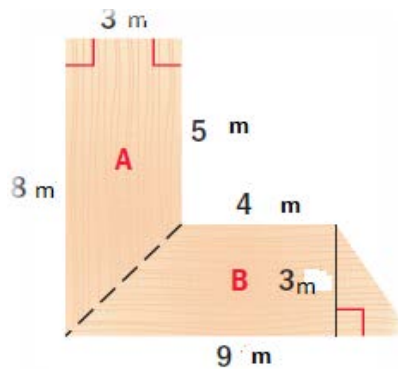
Teachers may design open-ended tasks that are challenging but not beyond learners' capabilities. In order to be able to solve open-ended problems, teachers need to plan various methods of training learners to be competent in solving problems. "Problem solving competency involves the ability to use the acquired knowledge in a new way, the ability to learn new things which are useful for the problem and to discover new methods for the solution. So the transfer of knowledge and skills to new situation (*sic*) is essential" (Marchis, 2012:50). Yeo (2007) notes that a teacher who does not know the differences between the types of mathematical tasks (Investigations, classwork, etc.) will not be able to use them to develop various aspects of the learners' mental structures since different tasks are used to cultivate different types of skills and thinking. The following tasks from different mathematical topics are examples of tasks for problem-solving:

Task 1: Pandu bought a small box in order to wrap in it the birthday gift for her friend. An open gift box is shown below. When the gift box is closed, it has a length of 12 cm, a width of 6 cm, and a height of 9 cm. She needs to buy wrapping paper that fits exactly on her box. What is the



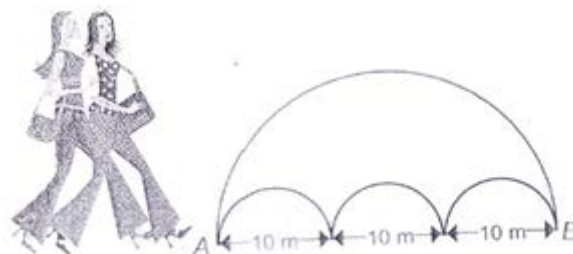
minimum amount of wrapping paper needed to cover the closed gift box?

Task 2: Joshua is constructing an L-shaped desk for his room. He has a design/outline and measurements with dimensions accurate for his table. What are the minimum materials to be bought in-order to complete his L –shaped table?



Task 3: John is t years old, his brother is 5 years younger and his sister is 11 years older than he is. His father's age is the same as his age squared and his mother is 3 years younger than his father. Write expressions for the age of his family members, if $t = 7$, work out all their ages.

Task 4: Two ladies walk at the same speed from A to B. Ariene takes the large semi-circular route while Daniela walks along the three small semicircles. Who arrives at B first?



The goal of this task is clearly defined: How much space of the box is empty? If a problem is a situation that is problematic to a learner, then they (learners) should have acquired the necessary knowledge and skills of reasoning and conjecturing between mathematical ideas. Knowing how to find out who of Daniela and Ariene arrives first requires application of formulae and solution procedures for the implementation of the problem. If problem-solving is viewed as an activity, then it includes both the problem and the process of solving it (Yeo, 2007). The teacher should pose a mathematical problem which requires from learners the use of inductive reasoning in order to reach a solution and make generalisations (Kolar *et al.*, 2011).

Once a teacher has selected valuable tasks, he/she must facilitate the tasks to the learners by first letting them work on a solution, and then give feedback as part of classroom discussion. As learners work on a solution (either individually or in small groups) the teacher must allow them to struggle with the mathematics (Selmer & Kale, 2013). Since problem-solving is at the heart of mathematics (see 3.4.3), learners should explore the mathematics using different technological tools such as calculators and computers in order to gain an understanding of skills in solving problems.

3.7 The use of technology in mathematical problem-solving

3.7.1 Introduction

Technology is essential in the teaching and learning of school mathematics because of the way it can influence learners' learning. A technologically rich learning environment influences the following important features of the classroom: the social culture of the classroom, the nature of classroom tasks, the role of the teacher, and the use of technology as learning support (Suh *et al.*, 2008). Technology is useful for assisting learners to look at mathematics not only as a set of procedures, but more as reasoning, exploring, solving problems and generating new information. For this study it is essential to know how technology can be used appropriately to enhance the teaching and learning of mathematics.

Modern education requires the usage of dynamic tools such as computers for effective learning. The computers in a mathematics classroom are used in the context of learning in different ways, mainly to motivate learners to improve understanding and investigating mathematical concepts (Rackov, 2011). Rackov avers that the role of teachers is to determine the most reliable and effective methods, the types of activities, tools, teaching resources and sources of knowledge. Because learning from various sources of knowledge is particularly important for active learning, the potential of digital technologies to enhance learners' mathematical learning is widely recognised and the use of computers and calculators is encouraged and required by secondary school mathematics curricula (Bennison & Goos, 2010).

3.7.2 The use of computers in the mathematics classroom

The role of the teacher in problem-solving is to organise the classroom in an environment with computers and incorporate various ways of using the computers in the teaching of mathematics. Several authors suggest different uses of computers in instructions: In his study on application of computers in initial teaching of mathematics, Rackov (2011) suggests that teachers use computers with software for creative activities. Software programmes are widely used by teachers to represent their content to learners during teaching and learning.

Software includes ready-made computer programmes, mostly Microsoft Office and Internet Explorer which are used in the teaching process in order to enable the learners to learn independently (Rackov, 2011). Such activities involve using computer programmes, designed for learning concepts such as equations, numbers, area and volume, size comparisons, as well as drawing and solving puzzles. Thomas *et al.* (2007) state that software such as Microsoft PowerPoint is an alternative tool for presenting lesson notes and to investigate concepts in a multi-representational environment.

The greatest use of computers is in teaching statistics, graphical work, geometry, algebra, trigonometry, or Pythagoras theorem. Aydin (2005) used a computer as a tool for educational computing using spread sheets such as Excel, computer algebra systems (CAS), the databases, communication facilities, and word processing. Databases are organisational structures which can be used to store and retrieve information. Computer spread sheets can be adapted for problem-solving, can assist in enhancing the learner's insight into the development and use of algorithms and models, spare learners from working with large numbers and lengthy manipulations, and allow them to see the progression of calculations on the screen as they are generated.

3.7.3 The use of calculators in the mathematics classroom

Calculator technology is one of the most significant technologies widely impacting mathematics classrooms and serves the same purpose as the use of other concrete materials in the teaching and learning of mathematics (Idris, 2004). Firstly, a calculator is used as a tool for computations; it is widely used for addition, subtraction, division and subtraction of numbers (Pyke *et al.*, 2008). Calculators allow learners to work with numbers in ways that they would not be able to do by using pencil and paper. Due to calculators being widely used, learners are urged to adapt to a suitable balance between technologies for calculation (mental, paper and calculator). Because calculators enable learners to work quickly and accurately, they can focus on the mathematical concepts underlying the computational task (Mogari & Faleye, 2012). When using calculators, learners begin to realise that doing mathematics is a dynamic activity.

Another reason for using calculators in a mathematics classroom is to assist learners in the understanding of when it is appropriate to use a calculator to find an answer and when it is not (Papadopoulos, 2013). It is the role of the teacher to help learners to become calculator aware. Consider the following task: *A school bus carries 46 children. If 500 children are going on a field trip, how many buses do you need?* A calculator in this case can be used to divide $500/46$ the answer will be approximately 10,87.

By using a calculator, learners are able to pay attention to the meaning of the remainder and make sense of how the remainder affects the number of buses rather than spending their mental energy on the computation. Learners then need to make sense of the answer the calculator provides. The main question will be: Do they need 10 buses or 11 buses? Using the knowledge that all learners must go on the field trip, they can use the remainder to determine that another bus is needed. Therefore, the answer is 11 buses.

The following activity also allows learners to explore real world activities that trigger their interest in learning mathematics (Papick, 2011).

Moreover, when using the calculator to solve the above complex problem, learners have to know how to use the calculator in order to solve the problem. After the answer appears on the screen of the calculator, the learner still has to interpret the answer in the context of the given problem (Banks, 2011). For example, consider the following task:

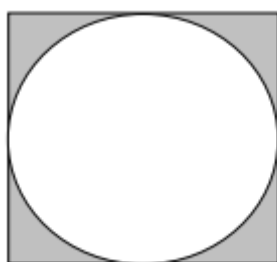


Figure 3-4 Example task

The length of the diameter of the circle is 8cm. The area of the circle can be determined by $A = \pi \times r^2$. Calculate the area of the shaded region. Use π on your calculator or take $\pi = 3.1415$. Give your answer correct to two decimal places.

Numbers such as π are complex to work with since they are difficult to do computations with. Many calculators are designed with the value of π pre-programmed. When using the calculator, the following computation is done: $(\pi \times 4^2 = 50.2654825)$ and the answer is then rounded off to two decimal place which gives 50.27 cm. According to Papadopoulos (2013) calculators can be used to enable learners to think about numbers in different ways. A typical example is the game *wipe-out* where the learners are instructed to enter a four-digit number, for example 8456, into the calculator, and then remove one of the digits by subtracting the appropriate amount. Removing a 4 requires subtracting by 400 and an understanding of arithmetic and place value. A calculator-aware learner is able to make an informed choice about using or not using a calculator for given calculations.

Effective teaching with calculators involves preventing learners from becoming over-reliant on calculators to the extent that they (calculators) are used for even simple calculations. Furthermore, calculators can be used to develop mathematical ideas and understand connections, relationships and patterns (Van de Walle *et al.*, 2010). An important aim of mathematics teaching is to assist learners to develop mathematical relationships with respect to number patterns. When whole numbers and decimal numbers are being multiplied or divided by 10 or powers of 10, the numbers follow a consistent pattern (Ochanda & Indoshi, 2011). Having an understanding of such relationships allows learners to develop a mental capacity for multiplying and dividing numbers by 10, 100, 1000 and so forth.

3.7.4 The use of calculators in the effective teaching and learning of mathematics

The integration of calculators in learning mathematics may have benefits with respect to encouraging active learning by freeing learners from tedious calculations and allowing them to focus on the use of problem-solving strategies (Warschauer *et al.*, 2004). In addition, the use of calculators will support and extend learners' understanding of specific mathematical concepts and thus create a powerful learning environment for learners (Ochanda & Indoshi, 2011).

For the past 20 years, the attention of teachers has been drawn away from the very important question of how calculators can be used effectively in the teaching of mathematics to whether to allow learners to use calculators (Johnston-Wilder *et al.*, 1999). However, the use of calculators in the classroom has been supported by many authors who urge that learners need to learn to use calculators effectively (Cooke, 2003:123; Ochanda & Indoshi, 2011:103; Mbugua *et al.*, 2011:132). Hitt (2011), however, suggests that the use of calculators should not replace the need for learners to develop proficiency in efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations. Mogari and Faleye (2012) have similar thoughts about the fact that learners should have full access to calculators when they are solving problems. Learners, however, should keep in mind that a calculator is merely an aid to assist in performing computations, in order to decrease the difficulty of the calculations. Therefore, it is the role of the mathematics teacher to ensure that calculators are used in such a way that the learners' development of mental arithmetic skills is neither compromised nor hampered (McCauliff, 2003).

Several research studies reveal that calculator use improves the problem-solving abilities of learners at all ability level at all grades (Mbugua *et al.*, 2011:132; Ochanda & Indoshi, 2011:103; Van de Walle *et al.*, 2010:107). That could be one of the reasons why calculators are described as powerful tools in mathematics teaching and learning. Calculators can be used to promote

higher-order thinking and thus assist learners to become flexible and resourceful problem solvers (Koay, 2006).

Moreover, calculators can enable learners to discover new problem-solving strategies, allowing them to learn basic mathematical concepts and increasing a positive attitude towards the use of calculators during problem-solving activities (Lin & Yuan, 2009).

3.8 The role of language in mathematical problem-solving

The language of mathematics is traditionally based on the ability to understand the text by reading, writing, and using arithmetic. Lailiyah (2017) attests that learners need to become mathematically literate in order to explore problem-solving situations, accommodate changing conditions, and actively create new knowledge in striving for self-fulfilment.

This requires experienced and competent literacy skills to become a proficient mathematical problem-solver (Doyle, 2005). Firdaus *et al.* (2017:214) define mathematical literacy as the willingness of a person (learner) to formulate, implement, and interpret mathematics in various situations, including the ability to perform reasoning mathematically and use the concepts, procedures, and facts to describe, explain, or predict a mathematical situation.

Literacy and language are central to learning mathematics. Literacy encompasses four roles such as code-breaking (how do I crack this), text-participant (what does this mean), text-user (what do I do with this here and now), and text-analyst (what does this do to me?) (Bernardo, 1993). Doyle (2005:3) further stated that “learners who have poor literacy skills inevitably have poor problem-solving skills” because problems require reading and understanding texts in order to gain necessary meaning from it. This does not necessarily imply that these learners are disadvantaged with respect to all mathematical problem-solving. It could be that they are competent in solving problems that do not require much text interpretation, such as numerical problems. Therefore, although learners are not necessarily poor at mathematics, their mathematical proficiency may be hindered by their lower literacy skills.

3.9 Conclusion

From this chapter it should be evident that problem-solving is the foundation to better understand mathematics. By exploring the history and importance of problem-solving it is evident that there is a need for teaching methods and strategies that will lead to learners' success with mathematical problem-solving. Mathematics teachers have to make a conscious and deliberate shift to teaching through problem-solving instead of teaching the knowledge and applying this knowledge to solve a mathematical problem.

This is done through designing coherent lesson planning and tasks that require learners' mathematical literacy and their ability to engage in mathematics applied in thinking and reasoning, discourse, mathematical tools, and attitudes and dispositions. In order to change teaching-learning processes, teachers' beliefs towards problem-solving teaching should be developed and changed. In the literature one finds numerous research reports on the requirements for changing and developing teachers' ways of presenting the content. However, none of the described intervention methods seem to be successful without involving a systematic approach to teaching that follows the following steps: understanding the problem, devising a plan, carrying out the plan, looking back. What is needed, therefore, is a new understanding of the problems of change and development in teachers' professional activities. Education is no longer about memorising facts and pictures, but rather, about learning where to find information. More importantly, it is about how and where the information which has been acquired can be used (Curri, 2012).

In addition, the section on the use of calculators and computers revealed that teachers have to develop methods that require appropriate uses of calculators and computers in problem-solving situations. Calculators and computers can be used for a variety of purposes: facilitating a search for patterns, creating problematic situations, supporting concept development, promoting number sense, and encouraging creativity and exploration.

CHAPTER 4 RESEARCH DESIGN AND METHODS

4.1 Introduction

This chapter describes the methodology that was used to collect and analyse the data from the participants. This includes the research design, methodology and methods, sampling strategies, methods of data generation, data analysis as well as ethical considerations.

4.2 Aims and objectives of the study

4.2.1 Aims

The aim of the study was to investigate the use of calculators in problem-solving activities in a Grade 9 mathematics classroom.

4.2.2 Objectives

4. To identify which problem-solving strategies are used in Grade 9 mathematics classrooms.
5. To explore when it is best practice to use calculators in problem-solving activities.
6. To determine what problem-solving activities should look like in the context of a Grade 9 mathematics classroom.

4.3 Research design, methodology and approach

4.3.1 Philosophical framework

Qualitative research starts with fundamental sets of beliefs, frameworks or mirrors that nurture the onset of the research (Creswell, 2013; Teherani *et al.*, 2015). This study is grounded in the interpretivism framework. The interpretivism framework underpins the view that the understanding of the phenomenon at the research site entirely depends on the participants' input on how they view the natural reality around the social world and from their own personal background (Creswell & Plano Clark, 2011). Interpretivism is understood (O'Donoghue cited in Punch, 2014:17; Nieuwenhuis, 2016) as existing ideas, knowledge and skills that participants bring to real situations and how they make sense of their world.

This reality is socially constructed. Merriam and Tisdell (2016) note that there is no single reality, but multiple realities for a single situation. Several researchers support the idea of multiple realism, indicating that interpretivism in qualitative research is simply an idea of constructing various natural realities that exist in a context of social interaction (Roller & Lavrakas, 2015, O'Neil & Koekemoer, 2016). These realities are constructed by an individual from his/her own personal background during interaction with the social world in order to understand the meaning of the situation at hand (Merriam & Tisdell, 2016). The researcher is an interpreter of the truth, interpreting, from the onset, the social behaviour of learners in order to be able to understand and describe their human nature (O'Neil & Koekemoer, 2016).

When I started teaching, I followed a more traditional teaching approach where I presented lessons and learners were expected to listen and do as I did. My journey as a researcher began at the combined school where I began to teach mathematics using a problem-solving approach, which I did not use before. I made use of collaboration, group and individual work, and I encouraged different problem-solving strategies and procedures. In this setting learners participated actively in groups, bringing in their own experiences, giving feedback and constructing their own knowledge in mathematics. Through group discussions I observed and realised what difficulties learners had and devised a plan on how to assist them into getting clear solutions to the problem. In this process I developed a desire to assist learners in their efforts to make mathematics more meaningful for themselves, and at the same time I gained valuable experience in teaching problem-solving in a Grade 9 mathematics classroom. The experience exceeded my greatest expectations; I found the journey of teaching problem-solving rewarding and enriching.

Interpretivism was selected for this study with the aim of understanding learners' experiences of real-life tasks. The interpretivism framework is thus relevant to this study, because it gave me an in-depth understanding of how participants interpret and construct their experience through solving different tasks, subsequently permitting me to explore participants' perceptions on how they view the real-life context of their social environment (Creswell & Plano Clark, 2011). I also considered how they felt about new experiences of solving real-life problems with calculators, the significance of this reality and whether the experience was meaningful to them.

4.3.2 Qualitative research methods

Qualitative research focuses on situations that occur in the real world and involve studying those situations in order to get an in-depth overview of its complexity (Leedy & Ormrod, 2008). Denzin and Lincoln (2011) define qualitative research as a unique field of inquiry, which comprise of systematic discipline and subject matter; it is however completed by using procedural approaches, methods and techniques to uncover a phenomenon.

Erickson (2011) indicates that qualitative research seeks to discover and describe the daily situations which occur in individual existence, looking at difference personality traits, interests and beliefs. Creswell (2013) defines qualitative research from a philosophical point of view, referring to qualitative research as an interpretive approach that guides the study of human problems. These problems are phenomena appearing in their natural settings. Recent studies define qualitative research as a complete body of knowledge with its own literature base, instruments of data collections, participants and implementation design (Merriam & Tisdell, 2016).

Bryman (2012) defines a qualitative method as a system of gathering data with different instruments such as questionnaires, interviews and observations. Qualitative methods involve the use of qualitative data, such as interviews, documents and observations, in order to understand and explain a social problem. Qualitative methods are based on exploring and understanding the meaning individuals or groups ascribe to a social or human problem (Hancock *et al.*, 2009). Hoepfl (1997) states that qualitative research methods are used to gain new perspectives on behaviours that are already known, or to gain more in-depth information that may be difficult to convey.

Based on the above definitions, qualitative researchers design their studies based on steps in research process. Various researchers proposed characteristics that are relevant for qualitative studies (Farber, 2006; Creswell, 2013; Fraenkel *et al.*, 2012). Creswell (2013:45) and Fraenkel *et al.* (2012:427) present common characteristics of qualitative research, which are supported by several research studies.

Some of these characteristics are listed below.

- Natural settings - qualitative researchers collect data in the field where participants experience issues. They believe that behaviours of human beings can be best understood in actual settings such as schools (Fraenkel *et al.*, 2012). Researchers spend a lot of time at school observing the general behaviour of learners in their classrooms by interviewing, observing videotaping, etc.
- The researcher is a key mechanism and role-player - qualitative researchers collect data themselves to observe the behaviours of learners (Merriam & Tisdell, 2016). During the process, they record their findings on note pads, take pictures or examine written work as evidence of data. According to Creswell (2014) researchers should first organise all collected data, make sense of the reviewed data, arrange reviewed data into themes and create multiple realities.
- Complex reasoning through inductive and deductive judgement – Qualitative researchers build their own themes from the reviewed data (Creswell, 2014). Inductive judgement implies that the researcher characterises and builds themes as they arise in the data (Fraenkel *et al.*, 2002) and deductive judgement means the researcher identifies themes that are consistently reviewed against data.

I acted as the researcher in this study; I collected, analysed and interpreted the data, which made me the key role-player. The study took place in a natural setting, my Grade 9 mathematics classroom. After collecting and reviewing the data I created themes to interpret the said data. A qualitative researcher should exhibit a set of characteristics as suggested by Merriam and Tisdell (2016:18-19), namely having a questioning stance, being highly tolerant and consistent, being a careful observer, asking good questions, thinking inductively and finding comfort within writing. During the research process I strived to adhere to the characteristics as mentioned above in order to comply with the expectations set in the literature for a qualitative researcher.

The purpose of conducting qualitative studies is mainly to understand the social meaning of the participants, whether it be their behaviour, culture, situations, or experience (Creswell, 2013). Qualitative research is conducted because a problem has arisen and needs to be solved and needs an in-depth understanding of the phenomenon (Creswell, 2013). The initial problem that was identified in this research was that learners are most often taught in a traditional manner which allows little room for them to think and reason, resulting in poor performance. The study

explored the problem-solving approach, particularly with the use of calculators, as a means to address this issue.

The focus of this study was on exploring the experiences of Grade 9 mathematics learners with regard to problem-solving and the use of calculators in problem-solving.

Creswell (2013) identifies some of the reasons for conducting qualitative research. These include to empower individuals to reveal their hidden truth and create a strong tie between the researcher and a participant; to present the written report in a fashion that sends a message to academics; to understand the setting of the participants and problems arising within that setting; and to discover new theory and build on the existing body of knowledge. Qualitative researchers are guided by these objectives to study the behaviour and experience within which the participant acts. As mentioned above, the research aimed to reveal some truths about Grade 9 learners' experiences of the problem-solving approach with and without the use of calculators. The hope is that these results as presented in this research can be useful to academics, curriculum designers, subject advisors, teachers and learners.

Qualitative methods should be selected based on the situation at hand. Any good researcher will know which method is appropriate for the phenomenon (Silverman, 2013). A case study is concerned with developing explanations for social aspects of our world and seeks solutions on how learners behave towards problem-solving tasks and how they are affected by a new teaching and learning strategy (to solve problem-solving tasks using problem-solving strategies) in a mathematics classroom. Qualitative research methods are associated with a case study research design.

A qualitative case study was used to help the researcher understand learners' social behaviours towards real-life problem-solving tasks in a Grade 9 mathematics classroom. The classroom environment where the data were collected presented a dynamic situation. This is because people (the teacher and learners) were unique individuals with varying attitudes and behaviours towards the problem-solving tasks (Kapenda, 2008; Kielmann *et al.*, 2017).

4.3.2.1 Qualitative case study design

A research design is a structure for the collection and analysis of data, which describes a group of participants, the nature of their behaviour in social context and the connection within the behaviour (Bryman, 2012). A case study is suitable for any qualitative research to uncover unknown situations or events occurring (Leedy & Ormrod, 2005). To understand the behaviours of learners towards real-life problem-solving tasks, I employed a qualitative case study strategy.

A qualitative case study is a method whereby a group of learners are studied in-depth for a defined period of time (Leedy & Ormrod, 2005). This group can be in the form of a social unit (a classroom), a cultural group or even the entire community (Kothari, 2004; Maree, 2007). Others, like Gerring (2004:342), see a case study differently, describing it as a study of “a single unit of learners for a purpose of understanding a larger class of similar units”. The numbers of selected units of the study would then be limited and reviewed based on given criteria (Zainal, 2007). A case study is also a strategy of investigating in-depth interviews (Henning *et al.*, 2004; Creswell, 2009) which should take place in a real-world context, especially in a case whereby the boundaries between phenomenon and context are not evident (Creswell, 2013). A case study can make use of a wide range of data collection methods which can best answer the research questions (Newby, 2014).

In this study I conducted an in-depth study over an extended period of time of a specific unit, in this case a class of Grade 9 mathematics learners. I did task-based interviews with the participants to explore their experiences of problem-solving with and without the use of calculators.

This included investigating the problem-solving strategies used during solving problems; the correct use of a calculator on particular tasks, and the confidence and interest in the problems. I also used different methods of data collection: task-based interviews as well as teacher reflections. An intensive investigation was employed in a Grade 9 mathematics classroom. I spent extensive time in the field studying learners' abilities in-depth with regard to applying problem-solving strategies, while doing problem-solving tasks, and collected extensive data. This was done with the aim of obtaining enough information for drawing correct findings on: identifying problem-solving strategies used in the mathematics classroom; when it is best to use a calculator during solving problem-solving tasks, and the type of problem-solving activities suitable for Grade 9 mathematics classrooms.

4.4 Sampling strategy

Purposive sampling is one of the techniques widely used to identify and select information-rich cases best producing understanding of the social problem (Palinkas *et al.*, 2002). Purposive sampling is defined by Creswell (2013) as a sampling method used to decide on who should be selected to participate in the study, together with the sampling strategies and the size of the sample to be studied. Maxwell (cited by Gray 2014:217) and Kumar (2011) hold a similar opinion to Creswell (2013) by indicating that purposive sampling is a priority when certain people are selected because they are known to provide important information that could not be gained from other sampling designs.

Etikan *et al.* (2016:2) argue that participants are not just selected in a vacuum but also categorised by looking at the qualities they possess; they refer to this choice as “judgement sampling”. Purposive sampling is commonly preferred in the field of education because it is wide-ranging, and more narrative in terms of screening all potentially relevant data materials (Benoot *et al.*, 2016). This means the interest is not in seeking a single correct answer, but rather in examining the complexity of different views of people. In this case the researcher provides a clear judgment about who will provide the best perspective on the phenomenon of interest because she knows the participants at hand (Gray, 2014; Kumar, 2011). The characteristics of individuals are used as primary selections chosen to reflect the variety of the sample population (Wilmot, 2005) and where sampling is done with a detailed purpose in mind (Maree & Pietersen, 2016).

Purposive sampling enabled the researcher to purposefully select twelve participants from a rural school in Namibia, with the following distinct characteristics: learners with different performance abilities, characteristics and achievement during the first school term. Participants were identified during the first term, by employing a Classroom Marks Schedule (CMS). The CMS used for this selection is a tool that consists of a class-list, schedules, grading scale and grade descriptor per learner for each term.

The CMS was used as a tool to determine the number of categories of participants. The participants were selected by making use of the CMS as follows: from the class of twenty-seven learners I identified three groups of participants as illustrated in Table 4-1 below.

Table 4-1 Selection of participants

Number of groups	Categories	Symbol	Percentage score (%)	Number of participants
1	Low performing group	G – U	0—29	4
2	Middle performing group	F—E	30—49	4
3	High performing group	C—A	50—80+	4
Total				12

The first group consisted of learners who obtained 0-29% in the first term. The second group consisted of learners who obtained 30-49% for the first term and the last group consisted of learners who obtained 50-80%+ in the first school term, respectively. These three main selections emerged into three categories namely Low, Middle, and High performing groups; each group consisted of four participants making up a population of twelve participants. The participants were rearranged as follows: The low performing group comprised participants 1, 2, 3, and 12; the middle performing group comprised participants 4, 5, 6, and 11 and the high performing group comprised participants 7, 8, 9, and 10.

I am a Grade 9 mathematics teacher, who possesses knowledge and know-how of the participants' performance from previous grades and the current grade. During the selection process, I did not only look at the performance in terms of scores and symbols, but I also looked at the participants' interest, characteristics and ability to solve problems. I used purposive sampling mainly to identify learners from all three performance levels (low, middle and high) in order to get a better understanding of the reactions of Grade 9 learners across a wide spectrum when dealing with problem-solving in a mathematics classroom.

4.5 Data generation methods

Qualitative data are any kind of data which convey an extremely rich variety of cultural and social aspects of human or object situations that convey meaningful information (Dey, 1993). In qualitative research, data generation is a collection of any form of data (interviews, observations, written document etc.) put together to form a unit of interpretations and findings (Leedy & Ormrod, 2005). Interpretation in qualitative research means the researcher scrutinises the analysed data and forms a comprehensive meaning about the phenomenon based on personal views and comparisons within themes (Creswell, 2014). Findings are discussions that recap the major findings commonly as detected from the description of individual responses (Creswell, 2014).

The data were collected and compiled during the second and third school trimesters, by making use of two instruments, namely: task-based interviews and teacher reflections. Teacher reflections (see Addendum B) were compiled after the presentation of each of the classroom lessons by the teacher and the task-based interview (see Addendum A) data were generated in the Grade 9 mathematics classrooms, during the afternoon session. Each learner present at the ten afternoon session completed a task-based interview (all learners were not present at each session) and the teacher completed ten reflection schedules after the ten lessons. Details of the teacher's reflections and task-based interviews are discussed in more detail below.

Investigations are activities that assess the ability of learners to think and reason independently without the teacher's intervention (MoE, 2010). They can be used to discover rules or concepts and may involve inductive reasoning, identifying, testing patterns, relationships and drawing conclusions (DBE, 2011). Investigation work provides rich experience of the processes involved in mathematics, a way in which a teacher can learn more about the way learners' minds work (Backhouse, 1992).

The teaching approach in my classroom was adapted in order to carry out this study. Instead of using a traditional teaching approach as I had used in the past, I made use of a problem-solving teaching approach. This meant that participants were given real-life problem-solving activities to do in the classroom that required of them to use different problem-solving strategies to solve the problem (see Chapter 3). Learners completed these classroom activities (in groups and individual) using their own problem-solving strategies, and with their own choice of whether to use calculators or not. I walked around the classroom, observing, assisting and giving hints on how to solve the problem. Some activities such as classwork and investigations were given during the lessons, either for introductory purposes or discovery purposes. At the end of the lesson I gave homework based on the content learned. The aim of selecting these tasks was mainly to examine how learners applied the gained problem-solving skills from the classroom. The tasks were selected because their real-world context was familiar to the learners.

According to the Institute for Education Sciences (IES, 2012), learners normally solve problems successfully when they are familiar with the content in a problem. Apart from the normal daily classroom activities as described above, I made use of task-based interviews to collect data for this study.

4.5.1 Task-based interviews

Task-based interviews can be useful tools for helping teachers assess the mathematical thinking of learners, particularly when mathematical concepts are provided in real-life contexts (Hurst, 2008). Task-based interviews also provide opportunities to assess learners' theoretical knowledge, but they provide opportunities to extend learners understanding on different problem-solving activities in mathematics (Assad, 2015).

The task-based interviews were relevant for this study, because they involved learners reflecting on the strategy used to solve the problems, whether the planned strategy was implemented or not in the calculations, and whether they used calculators to solve the mathematical problem correctly (see Addendum A).

The task-based interviews were used as one of the data-gathering strategies because they provided the best context in assessing individual problem-solving strategies and mathematical understanding. The twelve participants who agreed to take part in the study met in my classroom on certain specified afternoons to complete the task-based interviews.

These task-based interviews consisted of a problem-solving task which was followed by seven questions relating to the strategies they planned to use; the actual solution of the problem; the strategies that they did use; their confidence in their problem-solving ability, as well as if, when and where they used the calculator. The tasks required the usage of various problem-solving strategies and varied in difficulty. My role was to give the learners the tasks, assist in the classroom as they completed these tasks and then collect the tasks individually. Before the collection of the tasks, I verified if the learners' codes were correctly written on the task-based interview and if all spaces provided on the task-based interview worksheet were completely filled. Although the attendance at all these sessions was very good, all participants were not present for all ten the task-based interviews.

The theoretical framework for this study is based on Pólya's four problem-solving steps. The same model was used in setting appropriate questions for the task-based interviews. These steps entail: understanding the problem, devising a plan, carrying out the plan and looking back or checking the answer.

A brief explanation of what is expected of the learners at each of these steps follows:

- **Understand the problem** – they read the problem carefully a number of times until they fully understand what is wanted. What is the problem asking me to do, what information is relevant and necessary for solving the problem? Underline any unfamiliar words and find out their meanings. They should select the information they know and decide what is unknown or needs to be discovered. They should see if there is any unnecessary information. A sketch of the problem often helps their understanding (Taspinar & Bulut, 2012).
- **Devise a plan** – Learners should decide how they will solve the problem by thinking about the different strategies that could be used. They should always think in terms of how this problem relates to other problems that they have solved. Some strategies widely used by various researchers are: drawing a sketch, graph or table; acting out situations or using concrete materials; organising a list; identifying a pattern and extending it; guessing and checking; working backwards; using simpler numbers to solve the problem; writing a number sentence; using logic and clues; and breaking the problem into smaller parts (Taspinar & Bulut, 2012; Posamenter *et al.*, 2010).

- **Carry out the plan** – Learners should write down their ideas as they work so that they do not forget how they approached the problem. Their approach should be systematic. If stuck, learners should reread the problem and rethink their strategies. Learners should be given the opportunity to orally demonstrate or explain how they reached an answer.
- **Looking back** – In this case the teacher provides the opportunity for learners to examine the solution through activities such as checking and verifying the result; checking the argument; deriving the result differently; using the result or the method for some other problem; reinterpreting the problem; interpreting the results or stating a new problem to solve. Despite being a very important part of problem-solving, it is often neglected by teachers because of the entrenched belief that problem-solving is about only reaching an answer. Research reports have shown that *looking back* is hard to accomplish and found little evidence among learners of looking back (Aydoğdu & Ayaz, 2008).

Pólya's model for problem-solving was incorporated in open-ended questions in the task-based interview (see Addendum A). Open-ended questions indicate the area to be explored without suggesting to the participant how it should be explored (Fraenkel *et al.*, 2012).

Therefore the aims of the open-ended questions were specifically designed to test the learners' ability in solving the problem (questions 1 and 4) and to grant the learners the opportunity to interpret the task in written words either by means of solving (question 3) or by means of explaining (questions 2, 5, 6, 7), since these were the main areas the study wanted to explore.

4.5.2 Teacher reflection

Reflection is a process of self-examination and self-evaluation in which teachers regularly engage to improve their professional practices (Shandomo, 2010). Cohen-Sayag and Fischl (2012) define reflective writing in teacher education as an on-going and developmental process performed after the teaching process. Hampton (2010) categorises reflections in three ways: describing what happened in the classroom, interpreting that which took place and thinking about what the situation meant to your teaching and growth as a practical profession. According to Hampton (2010), reflective writing often involves disclosing weaknesses as well as strengths. If the lesson was successful, the reflector thinks of ways to sustain the strength; if a weakness is identified, possible improvement strategies should be implemented.

Mostly teachers gain power to remember, recall, recreate and represent what they learn from their teaching, when they write about their experiences using the above recommended reflection format in classroom.

By reflecting, a teacher integrates theory and practice (Shandomo, 2010). Reflective writing also helps teachers become thoughtful practitioners in interpreting written reflection about lesson plans. Teachers' reflections have the potential to increase the validity of a study because it provides formative feedback to the researcher that helps to improve the practice (Whitehurst et al., 2014). In this study, reflecting has helped me to analyse the problem-solving process in my classroom and it has assisted in understanding the learners' behaviour during problem-solving activities.

Several researchers formulate a series of steps to follow when writing a comprehensive teacher reflection. Burton (2009:7-8) suggests four steps which involve writing responses to a short series of essential questions; they are *What happened?*, *How did it happen?*, *Why did it happen?* and *What does it mean?* Hampton (2010) mentions three steps. They are descriptions, interpretations and outcome. For this study, a teacher reflection schedule (see Addendum B) was adopted from Hampton's (2010) three steps of reflective writing. Table 4-2 below indicates the overview of the reflective writing that I used in this study.

Table 4-2 Teacher reflective templates

Type	Phase	Descriptions
1	Description	What happened in a classroom during teaching and learning discourse?
2	Interpretation	What teaching strategy did you use in the lesson? What strategies did the learners use to solve the problems? Describe the learners' use of the calculator in the lesson.
3	Outcome	Assess the outcome/results of the lesson. How successful were the learners in solving the problems? How would you change the lesson in future?

For the purpose of this study I used the reflective schedule in Table 4-2 to collect data. I prepared ten lessons based on the Grade 9 mathematics curriculum (see 3.6.2.1) and did self-reflection for each lesson using the above structure (Shandomo, 2010). The learners were previously taught how to solve mathematical tasks using a traditional method of teaching and learning. I, however, specifically used a problem-solving method which is constructivist in nature and based on Pólya's model which encompasses the following aspects: understanding the problem, devising a plan, carrying out the plan, looking back.

According to Khalid and Azeem (2012), traditional teaching focuses more on rote memorisation, where learners are expected to imitate the teacher's demonstration. In constructivist teaching, the learner acts as a constructor of knowledge based on his/her personal experiences and how he/she views the learning environment. My reflection involved my experience of gauging if the learners gained new problem-solving skills and knowledge to solve the selected activities because of the teaching of this new method. My reflections were done following the classroom activities that were based on the problem-solving teaching approach.

Classroom activities are activities designed for every learning lesson, for the purpose of facilitating readiness for learning (Al-Qaisi, 2010). Giving classroom activities is an appropriate way to engage learners in problem-based activities in practice (Van de Walle 2010). I designed different tasks of varied cognitive abilities, such as classwork and investigations. These tasks were given to learners during teaching and learning in a Grade 9 mathematics classroom. During the teaching and learning process, several characteristics were noted: the behaviours of learners towards problem-solving, the problem-solving strategies used, as well as the task itself.

At the end of the lesson I wrote down what had happened in the classroom using the notes taken during the lessons. Each reflection was attached to the lesson plan and kept in a file for data analysis purposes.

4.6 Role of the researcher

According to Kincheloe *et al.* (2011:166), a researcher's critical role is to be able to access clear competency, to explore and interpret learning processes during teaching and learning. The researcher should also create collaboration among participants in order to collect data that are valid and reliable with the aim of creating understanding (Jansen, 2016). In the field, the researcher's role is to apply research instruments and gather documents that can help answer the research question (Barrett, 2007). Patton and Cochran (2002) suggest that researchers have responsibilities towards their participants; therefore, they should involve themselves in studying the participants in order to understand the phenomenon under study.

As researcher I had two distinct roles in this study, namely acting as a teacher in which I facilitated lessons and being a researcher. I am a mathematics teacher at a combined school, and I work specifically with the Grade 9 mathematics learners. I selected participants from a wide range of grade levels (low, middle and high achieving learners).

As researcher it was my responsibility to acquire permission to conduct the research at the combined school from the relevant stakeholders such as the North-West University Research Ethics Committee, the Director of Education, the school principal, parents and learners. This process was done by giving all the stakeholders consent letters and by explaining the main motive behind the research study.

Furthermore, I had to collect relevant research instruments (in this case task-based interviews, and teacher reflections). When learners had completed a task, I facilitated the completion of task-based interviews during the afternoon sessions. This was done by presenting the tasks to all participants, leading them to the understanding of a problem, observing the process and collecting the tasks for filing. After collection of the task-based interviews I compiled a file which contains the task-based interview tasks and ethical consent letters. Additionally, I also filed the relevant classroom activities and lesson plans which form part of my regular pedagogical duties. The researcher is then responsible to store the files in a strong-room at a combined school for safe keeping.

My second role focused on being a facilitator or guide to the learners. Johnson and Herr (2001:12) describe the role of a teacher as “guide”. His/her role is simply to keep learners on task, to guide them by keeping them on track and to ask prompting questions. The teacher simply interprets the lesson in such fashion that learners construct their own meaning and understanding so that they can gain ownership of the mathematical topics (Johnson & Herr, 2001). One of the primary responsibilities of teachers is to select and develop worthwhile tasks (Jones & Pepin, 2016) which are based on knowledge of the mathematical goals of the lesson and the mathematics embedded in the tasks themselves as stipulated in chapter two, and arranging classroom dialogue focusing on mathematical thinking, reasoning and communication. Backhouse (1992) specifies four roles of a teacher such as: knowing what learners understand, introducing work that should enhance learner’s prior knowledge, allowing learners to use a method they feel will work and encouraging learners to take responsibility for their progress.

The teacher who integrates this teaching strategy can be a teacher and a facilitator as well. The difference between a teacher and a facilitator is that the teacher transfers the information to learners through explaining and clarifying errors, explaining and describing the content, whereas a facilitator guides, drives, advises learners who discover their own theories or conclusion (Posamenter, 2010). When learners are given an opportunity to be actively involved in the learning process, they become more eager to develop their own understanding of mathematics concepts and ideas (Posamenter, 2010) (see 2.2.2). The goal of learning in the

mathematics classroom is to develop learners' relationships with the discipline of mathematics (Grootenboer & Zevenbergen, 2008).

In this regard, I acted as a teacher and a facilitator at the same time. The main teacher/facilitator role was to set up plan designed in a problem-solving manner, with clear basic competencies from the curriculum. I also had to complete the teacher reflections in writing (see 5.3.1). The second role of teacher/facilitator was to create real-life problem-solving activities such as classwork, topic tasks, investigations and homework that are in line with the curriculum and are appropriate for the Grade 9 mathematics classroom. In the beginning phase of a lesson I introduced the new topic by activating prior knowledge in the before phase. During classroom discourse learners share mathematical strategies, and teachers ask probing questions and listen to the learners' thinking (Neumann, 2014). During the after phase I engaged the class in discussions, allowing learners to defend their answers (Intaros *et al.*, 20073) (see 3.6.2.1 for full lesson). The third role as a teacher and facilitator was to facilitate continuous assessment. Learners were assessed based on each topic covered. Written tests, topic tasks, and investigations were collected for marking. This process gave an overview of learners' performance on specific topics.

4.7 Reliability and trustworthiness

Two of the most important criteria for assessing trustworthiness in qualitative research are reliability and validity (Bryman, 2016; Nieuwenhuis, 2016). Several researchers draw inference on validity and reliability in a different manner, focusing the idea that the data obtained from instruments should be appropriate, credible and backed up by evidence (Brink, 1993; Bannigan & Watson, 2009; Fraenkel *et al.*, 2012). Validity and reliability have been identified in the history of research as fundamental aspects in assessing the quality of qualitative research (Brink, 1993). Reliability and validity in qualitative research are central concerns of all measurement and they are components designed to establish the truth and credibility of findings (Neumann, 2014).

Reliability is the degree of consistency in the process of the study, which includes the researcher, research methods, research method biases that may have influenced the outcome, and different logical procedures (Brown & Uhde, 2001; Neumann, 2014). Bryman (2012) defines reliability as a question of whether the outcome of a study is repeatable. Vogt *et al.* (2012) refer to reliability as a type of measurement in making subjective judgments. Similarly, reliability is defined by Merriam (1998) as a strategy of obtaining consistency and dependable data as well as data that are corresponding in reality. It ought to be noted that the purpose behind qualitative research does not aim to create outcomes that can be compared to other different

circumstances, but instead to clarify phenomena being examined by the researcher who is included in the phenomena (Brown & Uhde, 2001).

Validity is when a researcher uses certain procedures to check for the accuracy of the research findings. These findings should reflect the data and must hold true value by nature. Neumann (2014) refers to validity as “truthfulness” in the data instrument and researcher judgement on the material. Qualitative research does not set out to demonstrate or refute theories; rather it centres on inductive examination, whereby the information provides an uncovering of understandings and connections (Brown & Uhde, 2001).

Researchers such as Creswell (2013) and Fraenkel *et al.* (2012) list some validity and reliability strategies widely used by many researchers to measure trustworthiness, such as: prolonged engagement in the field which implies spending time in the field with participants; triangulation in which multiple and different methods are used; peer review and debriefing which relies on support from other professionals willing to provide scholarly guidance; and rich thick description which enables judgments about how well the research context fits other contexts. Some of the reliability strategies suggested by Anney (2014) are: the use of purposive sampling in which a selection of knowledgeable participants under investigation; transferability, which implies judgment by a potential user through thick description; and purposeful sampling.

To ensure validity and reliability of data inquiry for this study I used the following validity and reliability strategies to ensure trustworthiness as suggested by Creswell (2013): I utilised direct responses obtained from the learner's task-based interviews and teacher reflection as a source of support for the findings and I used multiple data sources, spent a prolonged time in the field and compiled thick, rich descriptions pertaining to the data in order to establish confidence in the findings of the study.

Prolonged time in the field experience – Prolonged engagement in the field means that the researcher is creating trust with the participants (Creswell, 2013). I stayed in the field for a lengthy time, approximately eight months. In the first place to build a trustful relationship with the participants, and to gain understanding of participants' social cultural behaviour by giving participants several problem-solving activities in the mathematics classroom and observing any development in their ability to solve problems. Trustworthiness in qualitative research is enhanced when there is clear evidence that a length of time has been spent in the field and there are multiple forms of fixed materials that can help to collaborate explanations (Butler-Kisber, 2010).

Multiple data sources which are collected from a number of different instruments enhance validity (Fraenkel *et al.*, 2012). The use of multiple data collection strategies allows for examination of the data from different viewpoints, and aids in constructing a better understanding of and insight into the complexity of learners' development of mathematical concepts (Brown & Uhde, 2001).

I used two different instruments – task-based interviews and teacher reflections which assisted me in examining the data from different viewpoints. It also helped to identify, where applicable, common themes which were evident in the different data. Thick, rich description refers to a technique which ensures reliability in qualitative research. (Creswell, 2013). I captured the learners' responses (as provided by means of the task-based interviews) and discussed these in detail. Similarly, I reported the teacher reflections thoroughly and throughout described the common themes that arose in both the task-based interviews and the teacher reflections.

4.8 Data analysis methods

Qualitative data need to be interpreted in order to be analysed (Dey, 1993:100). To analyse literally means to take apart words, problem activities, and sentences in order to be able to interpret the data (Boeije, 2010, Fraenkel *et al.*, 2012). Lacey and Luff (2009) refer to qualitative data analysis as the amount of generated data which need to be described and summarised into some form of themes.

Flick (2014) defines qualitative data analysis as an interpretation of visual materials such as documents and interviews with the aim of obtaining relevant meaning in the material and the content/text underlying in it. Henning *et al.* (2004) state that this process requires the ability to understand the situation by putting it in narrative writing. Moreover, the “aim of analysis is to describe both the data and the object to which the data refer” (Henning *et al.*, 2004:128). These data provide natural occurring information that allows the researchers to increase their understanding of phenomena. Flick (2013) also outlines the aim of analysing data, such as to describe different cases in detail, e.g. a group and individual and how they link to each other; to identify the conditions on which these links are based; to develop a model of the phenomenon under study from the analysis of practical material and to reduce large data sets to decompose them into smaller pieces of data by adding extensive interpretations.

There are different types of qualitative data analysis. Miles *et al.* (2014) note that drawing and constructing a cognitive map is an example of an analysis. A researcher can make use of any tools at his/her disposal for the interpretation of the situation. This study will make use of qualitative content analysis which is a set of systematic procedures that can be applied to any

material (activities, interviews, text, video recordings) in order to know what is being communicated (Newby, 2014). During the data analysis of these materials, the data are examined with the aim of arriving at a valid finding through the application of categorising (Henning *et al.*, 2004). The data are analysed into categories in order to capture the main components of social action of participants and then noticing and documenting how these categories interconnect (Dey, 1993).

4.8.1 Analysis of task-based interviews

The first part of the task-based interview was analysed using Creswell (2014:162-165) and Leedy and Ormrod's (2005:143) approach of data analysis via the following steps:

- The researcher identifies the specific materials to be studied.
- The researcher defines the features to be examined in greater depth in order to create themes and categories.
- If the materials involve complex and lengthy items, the researcher breaks down each item into a manageable segment that is analysed separately.
- The researcher scrutinises the features in all material to understand the clear meaning of the data.

The specific materials that the participants had to engage with consisted of various tasks (Table 5-1) based on a range of mathematical concepts as determined by the curriculum (algebra, mensuration and geometry). At each of these tasks the same seven task-based interview questions were asked. The participants were given the tasks and the questions simultaneously to complete these at the same time.

Two sets of data were gathered and analysed: the task-based interviews which learners completed and the teacher's reflections (see 4.8.2). The interpretation and evaluation of the data was done by comparing how each participant from each of the three different performance groups (low, middle and high) approached and applied problem-solving strategies when doing the mathematical tasks. In addition, each participant's use of a calculator during these tasks was also interpreted and evaluated.

At Question 3 of the task-based interview an assessment rubric was used to evaluate the learners' solutions which led to the emergence of certain themes. At the other questions themes and categories were recorded by closely analysing each participant's work – the actual solving of the problem (or attempt to solve it) and the participants' responses to the questions they were asked in the task-based interview. The data that were gathered from the teacher's reflection were obtained by analysing the reflection schedule (see 4.5.2).

4.8.2 Assessment rubric

An assessment rubric is a set of scoring procedures for evaluating learners' work (Egodawatte, 2010). An assessment rubric is an appropriate tool to examine learners' ability to solve mathematical problems (Rosli *et al.*, 2013). This tool provides learners with expectations about what will be assessed, by looking at the information on the standards that need to be met (Egodawatte, 2010). Teachers are able to compare learners' performance across the problems if a rubric is applied consistently in rating performance (Beckmann *et al.*, 2010; Egodawatte, 2010). Beckmann *et al.* (2010) are of the opinion that teachers should have an approach to assess the learners' problem-solving tasks, firstly by creating an item-specific rubric, looking at the specific context of the problem and outlining the type of thinking about the problem that results in each score level. Levels of achievement determine the degree of performance which has been met and will provide for consistent objectives. These levels tell learners what they are expected to do.

For the purpose of this study the assessment rubric was developed for each performance task (Table 5-1). The structure of the assessment rubric consists of the following components: 1) Six relevant criteria for assessing the performance of a task namely:

1. Problem-solving strategy used and applied – This indicates if a reasonable planned strategy was used and developed.
2. The use of content knowledge – This provides evidence of using prior knowledge on a given task.
3. Solving the problem – This indicates whether problems were solved correctly with an indication of diagrams and labelled work to support the strategy.
4. The use of calculation – There is correct use of calculations accompanied by labelled work to support the strategy, in a calculation.
5. The use of mathematical language – The correct mathematical language is used to communicate the reasoning and mathematical ideas.
6. Connection to other mathematics – The mathematics connections are present and used to extend the solution to other mathematics in solving the problem.

Learners' performances are evaluated on the four levels of achievement (one to four points) as indicated below:

- Level 1:* Insufficient; shows little or no understanding of the problem
- Level 2:* Low; shows limited understanding of the problem
- Level 3:* Proficient; shows substantial understanding of the problem
- Level 4:* Achieved; beyond shows rigorous understanding of the problem

The rubric was used to determine the level of achievement of each participant. As each participant's tasks were assessed by using the rubric, I categorised learners as insufficient, low, proficient, achieved beyond or successful, depending on their level of achievement. The rubric was used for all the mathematical tasks for the low, middle and high performing groups. The data from the rubric were incorporated in the themes that arose.

4.8.3 Analysis of teacher's reflections

The teacher's reflections were analysed in three parts using Creswell (2014) and Leedy and Ormrod's (2005) approaches of data analysis and are described in Table 4-2. For each lesson given, the teacher described the lesson, interpreted what happened in the classroom and evaluated the outcome of the lesson. The aim was to observe the problem-solving process as well as the learning strategies used and how participants used calculators during problem-solving.

4.9 Ethical considerations

Merriam (1998:29) states that "a good qualitative study is one that has been conducted in an ethical manner". Therefore, a researcher should "consider moral accuracy of the research activities in relation to people they meet, such as participants" (Boeije, 2010:44). Ethical considerations in qualitative research refer to conducting the study in a fashion that does not result in harming the participants and, if possible, to produce some gain for the participants (Neumann, 2014; Walliman, 2017).

The researcher needs to plan how ethical issues will be addressed because research in general is a human practice in which social values and ethical consideration apply. Creswell (2013) suggests that validity and reliability of a study depend upon the ethical consideration of the researcher. One needs to foresee ethical issues that may arise during the research study by means of informed consent (Boeije, 2010). Kielmann *et al.* (2012) define

informed consent as the act of reliable participants who agree to take part in the study, with full understanding of the research activities and any risks or benefits.

The participants must agree voluntarily to participate without any emotional pressure (Neumann, 2014; Miles *et al.*, 2014). Important ethical considerations are based on protecting participants; this includes obtaining letters of consent (Jansen, 2016). Informed consent is grounded by the theory that qualitative research participants have the right to be informed about the nature of the study in which they are involved (Christian, 2011). Informed consent can be given in the form of a letter indicating ethical behaviours as demonstrated by Salkind (2014).

It is crucial to acquire approval from the university's ethical committee for the data collection process to be facilitated (Creswell, 2013). The following procedures were used in order to gain permission to carry out research at a rural school. Firstly, the North-West University ethical application form was completed and submitted to the EMHS-REC ethical committee at the North-West University. When the committee approved the application, I was issued with an ethical certificate which allowed me to commence with the study.

After ethical clearance had been approved and given for the study, I first obtained permission from the Ministry of Education in Namibia to conduct the research. Thereafter I also obtained permission from the school principal, the parents and the learners to continue with the research. This was done in the form of a letter of consent which each stakeholder completed (see Addendum D). The consent letters comprised various reasons why I embarked on researching the specific topic ensuring that their participation is entirely voluntary. Participants were ensured of the following aspects: that they were free to withdraw from the study at any point in time; they were informed about what the research involved; why the participant was invited to take part in the study; if there were any gains or risks in participating; how I ensured that the information remained confidential; what would happen with the findings; how the participant would know about the results; and whether he or she would be compensated for taking part in the study. All collected data were kept in various files and kept in the strong room at a rural school.

4.10 Conclusion

This chapter discussed the research methodology for qualitative research, data collection instruments and how data were gathered and analysed. A full explanation of the research instruments (task-based interviews and teacher's reflection) and their application and administration was given. It further indicated how issues of validity and reliability were addressed through the use of several data gathering methods and research inquiry. Ethical considerations were also explained.

CHAPTER 5 DATA ANALYSIS AND DISCUSSION OF FINDINGS ON THE USE OF CALCULATORS IN PROBLEM-SOLVING ACTIVITIES.

5.1 Introduction

Chapter four described the processes and descriptions of data generation from the task-based interviews and teacher's reflections. This chapter describes the analysis of empirical data of the task-based interviews, teacher's reflections and the discussion of the research findings. The findings are related to the research goal that guides the study namely *to investigate the use of calculators in problem-solving activities in a Grade 9 mathematics classroom*. The data generated in the task-based interviews and teacher's reflections were analysed in two sections. The first section describes the task-based interviews which are analysed based on seven open-ended questions and the second section comprises the teacher's reflections, which are analysed in three categories namely descriptions, interpretations and outcome.

5.2 Task-based interviews

Ten task-based interviews (see Addendum A) were completed by the 12 participants. This implies that there were ten occasions where all the available learners (maximum 12) each completed a task-based interview. These interviews were selected from three mathematics topics, namely mensuration, geometry and algebra according to the Grade 9 mathematics curriculum. The discussion for each of these topics will be done according to the dominant themes in the study, namely problem-solving, the use of calculators, problem-solving strategies and learner perceptions about problem-solving. The task-based interview consisted of a mathematics task as well as seven questions relating to the task (see Addendum A). The questions provide insight into the relevant themes as follows: problem-solving (Questions 2, 3, 4, and 6); the use of calculators (Question 5); problem-solving strategies (Question 7); and learner perceptions (Questions 1 and 4). Reference will be made throughout the chapter to the three performance groups, namely low performers, middle performers and high performers.

Generally, low performing learners are described as those learners who perform below average in school achievement (Hassan & Sylaja, 2013). They understand the concept, but the main difference between them and other learners is that it takes them a lengthy period to understand the concepts (Mandima, 2015). In this case, their academic performance ranges between 0-29

percent (see 4.4). Most of them have limitations, such as a lower capacity of abstract thinking and inability to correlate prior experience, lack of determination, motivation, reasoning ability and self-confidence (Hassan & Sylaja, 2013).

Middle performing learners are described as average learners in school achievement, who have generally positive attitudes toward solving tasks (Abu-Hamour & Al-Hmouz, 2013). In this case, in terms of academic performance they range between 30-49% (see 4.4) in their academic performance. High performing learners are described as those learners who are above average in school achievement (Hassan & Sylaja, 2013). In terms of performance they range between 50-80% (see 4.4) in their academic performance.

The questions in the task-based interview which were given to low, middle and high performing groups are listed below:

1. Do you think you can solve the problem correctly?
2. Explain in your own words how you would solve the problem.
3. You may solve the problem now. Show all your steps.
4. Do you think that you have solved the problem correctly?
5. Did you need to use a calculator for this task? Where in the task did you use a calculator? Why did you use the calculator there/then?
6. What do you think may be the reason(s) for the mistakes you made in solving the problem?
7. Describe the strategies or plans you used to solve the problem.

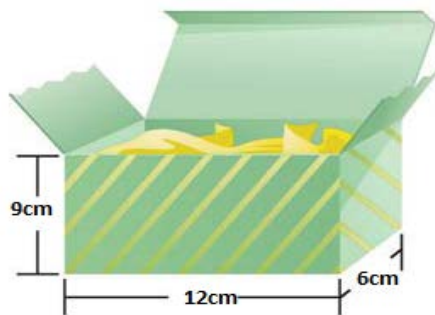
5.2.1 The problem-solving tasks

The tasks given to the participants are presented on the following page in Table 5-1.

Table 5-1 Problem-solving tasks

Task 1: Surface area of a cuboid

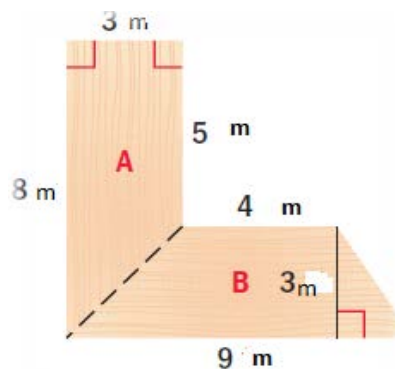
Pandu bought a small box in order to wrap a birthday gift for her friend. An open gift box is shown below. When the gift box is closed, it has a length of 12 cm, a width of 6 cm, and a height of 9 cm. She needs to buy wrapping paper that fits on her box exactly. What is the



minimum amount of wrapping paper needed to cover the closed gift box?

Task 2: Area of a trapezium

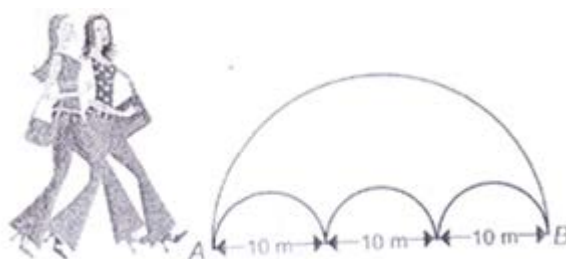
Joshua is constructing an L-shaped desk for his room. He has designed the table and indicates the measurements of the table. What are the minimum materials to be bought in



order to build his L-shaped table?

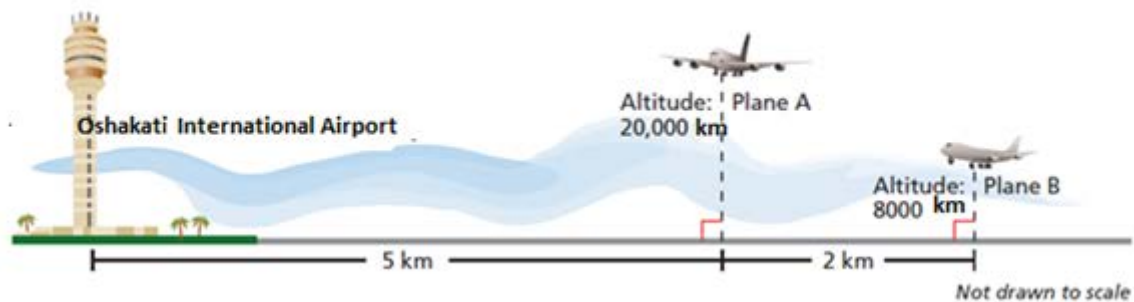
Task 3: Circumference of a circle

Two ladies walk at the same speed from A to B. Ariene walks along the large semi-circular



route while Daniela walks along the three small semicircles. Who arrives at B first?

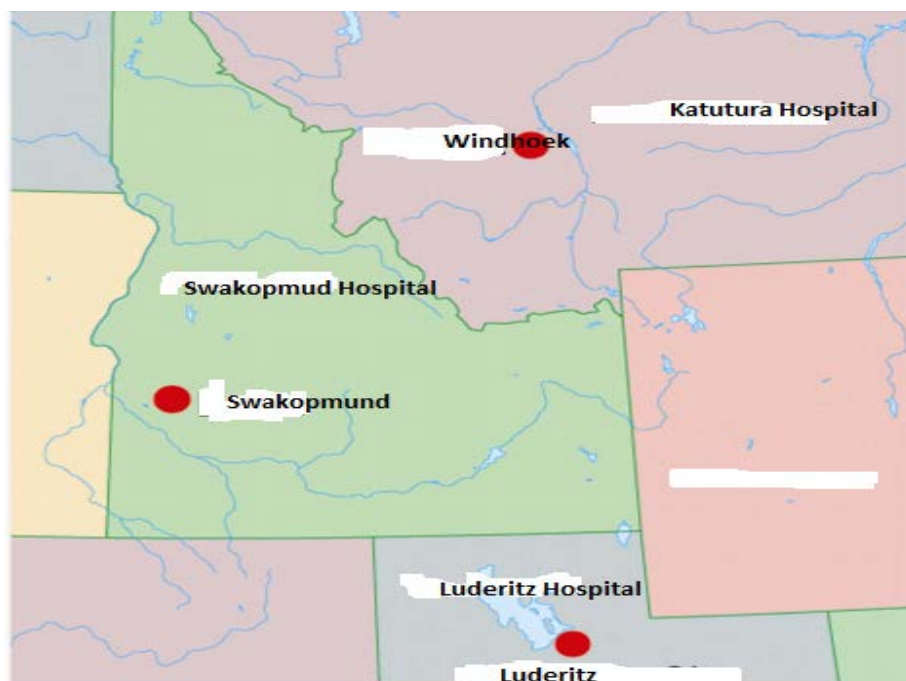
Task 4: Pythagoras theorem



Two planes are heading to Oshakati airport. Which plane is closer to the tower? Explain.

Task 5: Constructions

Windhoek, Swakopmund and Lüderitz are three large cities in the southern part of Namibia. Although each city has a local hospital for minor needs and emergencies, an advanced medical facility is needed for organ transplants. Imagine the benefit that we will receive if heart and kidney transplants are shared by these three cities and their surrounding communities. You have been hired to determine the best location for these facilities. Use a straight edge and compass, and on this map to show where you think the medical centre



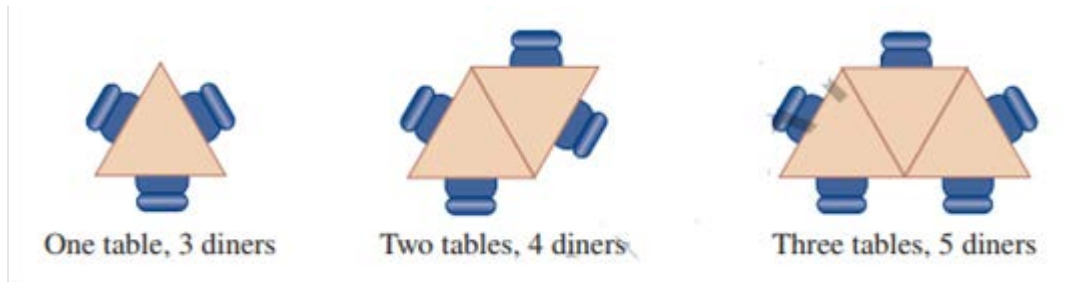
should be.

Task 6: Factorising

Farmer John and farmer Jane are planning their fruit orchards. Farmer John is planting orange trees and farmer Jane is planting cherry trees. Farmer John has 30 orange trees to plant and farmer Jane has 24 cherry trees to plant. They want to plant the trees so that each row has the same number of trees. What is the greatest number of trees each row can have?

Task 7: Arithmetic rule

The dining tables at a theatre restaurant are triangular in shape. Diners are seated at the



tables in the arrangements shown below:

The manager of the restaurant has received a booking from a large party. She sets out a row of 24 tables. How many diners can be seated at this row of tables?

Task 8: Substitution

Consider the following problem:

Fysal Fresh Fruits received crates of bananas, for new stock. Each crate of bananas contains



n bananas. Two bananas are removed from each crate.

If there are 7 crates, how many bananas are there in total?

Task 9: Linear inequalities

A company wants to order t-shirts for their 34 employees. Each t-shirt's cost varies by design, but shipping is a flat rate of N\$12.99. The company's budget for t-shirts is N\$400. Which design(s) can the company afford?

- Design A- Black and white: N\$ 8.50/t-shirt
- Design B- Colour: N\$11.25/t-shirt

- Design C-Extra Comfort: N\$12.50/t-shirt

Task 10: Age problem

John is t years old, his brother is 5 years younger and his sister is 11 years older than he is. His father's age is the same as his age squared and his mother is 3 years younger than his father. Write expressions for the age of his family members and if $t = 7$, work out all their ages.

5.2.2 Analysing the problem-solving tasks

Since the study explored the use of calculators in problem-solving activities, problem-solving features as one of the main themes of the study. The term “problem-solving” is defined by many authors as a process by which an individual attempts to identify solutions for solving a variety of real-life problems (see 3.2). The focus of this study considers problem-solving as a cognitive activity that entails strategic thinking, and that includes more than just performing calculations.

The aim of problem-solving is to find a useful strategy, solutions and answer to a given problem (Aydoğdu & Ayaz 2008). A strategy is a procedure designed to solve a problem. Therefore, learners can write down the facts they need to answer a question beforehand, and then verify their computations against their written facts. This helps them see different ways to arrive at an answer (Urquhart, 2009).

On the other hand, a solution is the whole process of solving a problem, including the procedure of obtaining an answer and the answer itself (Kolovou *et al.*, 2008). The goal for searching for a solution is to regain previously presented information rather than rely on one's own perception. This may limit the development of learners' abilities to think logically in searching for the answer to the problem (Limjap, 2001).

In this study each participant had an opportunity to obtain his/her unique solutions. Participants also had more opportunities to make comprehensive use of their mathematical knowledge and skills to solve the problem using suitable strategies to find the final answers the problems.

Problem-solving is about choosing learning strategies and applying appropriate strategies to solve the problem-solving tasks where the initial state, strategy and the solution of problem-solving tasks are unknown (see 3.2). It is expected that the tasks given to learners provide some degree of challenges, address important mathematical ideas and foster communication and reasoning. It is only tasks with such features that can stimulate learners to engage in creating knowledge for themselves (see 3.6.2.2). A major characteristic of these tasks is that they do not have straightforward solutions but require a good understanding and modelling of the situation

(see 3.2). In'am (2014:151) indicates that "the process of problem-solving needs organised activities with logical planning, including appropriate strategies and methods in the implementation".

Different non-routine tasks with the above features were given to participants with the aim of their solving those using different problem-solving strategies. Non-routine problems are problems that are unfamiliar to learners (see 2.2.3). They demand effective thinking required from classroom activities, and remain firmly challenging, even when the knowledge and skills required for their solution have been learned (Schloeglmann, 2004). They consist of problems which have more than one correct answer and more than one strategy to obtain this answer (Pelfrey, 2000).

Learners were expected to explain their solutions and/or strategies which should lead them to an answer. The aim of giving these types of tasks was to determine the type of problem-solving activities suitable for the context of a Grade 9 mathematics classroom which is the main objective of this study. Each of these tasks addresses a specific aspect of the Grade 9 curriculum, ranging respectively from calculating area and perimeter of different shapes (e.g. area and perimeter of a trapezium, circle, and cuboid. Problem-solving tasks were formulated specifically to test problem-solving skills with the use of technology (in this case a calculator) as an aid in solving different problems.

The theoretical framework for this study is based on Pólya's model for problem-solving, of which the steps are: understand the problem, devise a plan, carry out the plan and look back or check the answer (see 1.2.2). The data within the dominant themes will be analysed within the framework of Pólya's Heuristics. The first four questions in the task-based interview are based on the four steps as suggested by Pólya.

Table 5-2 depicts the first four questions of the task-based interviews, which are based on Pólya's model for problem-solving.

Table 5-2 Pólya's problem-solving model compared to questions in the task-based interviews

Pólya's four steps for solving problems (Pólya, 1945:7-11)	Questions from the task-based interviews
1. Understanding a given problem	1. Do you think you can solve the problem correctly?
2. Planning suitable strategies for a given problem	2. Explain in your own words how you would solve the problem.
3. Carrying out the plan	3. You may solve the problem now. Show all your steps. 4. Describe the strategies or plans you used to solve the problem.
4. Looking back/reflecting on the problem	5. Do you think that you have solved the problem correctly? 6. Did you need to use a calculator for this task? Where in the task did you use a calculator? Why did you use the calculator there/then? 7. What do you think may be the reason(s) for the mistakes you made in solving the problem?

As depicted above, the task-based interviews will be analysed using Pólya's problem-solving model.

5.2.2.1 Understanding a given problem (Perceptions of whether they understand the problem)

In'am (2014) defines problem identification as a process of understanding and knowing certain features exist in the problem. Each problem listed above possesses important features that make mathematical concepts clear. It is necessary for participants to identify these features, as

a step to determine an appropriate approach, method and strategy to solve the problems. The first step they were expected to do, was to read the statement of the problem carefully to make sure they understand the information given and understand the problem being asked. To help in understanding the problem learners were expected to draw a diagram.

According to In'am (2014), understanding is an action that should be done by participants before they attempt to solve any problem. This stage also is intended to obtain information, including materials to use (e.g. calculators, pen and pencil) and facts dealing with the problem. The majority of the low, middle and high performers seemed confident that they had a clear understanding of solving problems (their features, characteristics, strategies as well as how to implement them). They were either sure or fairly sure they would solve mensuration problems correctly, especially in algebra tasks. However, some low performance learners indicated that they were not sure they could solve the problem, fearing that they would make mistakes. In some instances, they could not measure their own understanding of the problem. Instead they solved the problem without knowing what they were really looking for. A detailed discussion about the learners' confidence in their own problem-solving ability will be given at 5.2.4.1 and in Table 5-14.

There could be various reasons why this was the case. They might lack the necessary knowledge of problem-solving strategies and how to apply them in a meaningful way. They might not have understood the question correctly (Tambychik & Meerah, 2010) or they might not have understood the mathematical concept (Limjap, 2001). In some instances, they opted not to make any selection from the given predictions (see Question 1, Addendum A). This means that they did not trust they could solve the mensuration problems correctly. Aydoğdu and Keşan (2014) indicate that the initial question to be asked when trying to solve a problem is: *Do I think I can solve this problem correctly?* According to Aydoğdu and Keşan (2014), this question shows learners what they can do. When we begin to ask this question in all problems, we are stimulating the learners' reflection on what they are capable of doing.

I noticed from the onset that many participants struggled to understand the mensuration and geometry problems. To understand the geometry problem, learners were expected to construct, restate the problems in their own words, list important details and look for connections. However, the majority of them did not use relevant tools to solve the problem. Instead of using a compass for the constructions, they started using a ruler. The calculator was used at this stage to obtain information while doing the construction.

There was a similar case with the mensuration problems. The first problem-solving tasks were from the content area of mensuration. Learners were confident but confused due to the new

teaching approach and the unfamiliar and complex tasks. Generally, learners had difficulty in reading and understanding what the question required. For example, when the problem asked to find the volume of a certain figure, they rather opted to work with surface area. These were common misunderstandings among all participants.

The last topic of algebra was treated differently by all participants. Despite the complexity of the algebra problems, learners were able to grasp the knowledge of problem-solving and find answers to the problems. This might be because they were used to working with problem-solving tasks by the time, they did the algebra tasks. Jameson (2000) suggests that a critical aspect of how to solve algebra problems correctly lies in the mathematical language. The language of algebra involves more than the use of a letter to represent a number.

When participants were approached with the tasks, they were left to wonder and struggle with the problems without the teacher's help. The initial stage of solving a problem was to read the problem and interpret it in a conventional way. This involves perceptions, attention, and understanding representations (Nunes *et al.*, 2009). In general, solving algebra problems requires learners to write down what is known in the problem faced (Widodo *et al.*, 2017). It was difficult for me to ascertain whether these learners understood the algebra problems or not, because the learners did not write down what was known and what was being asked in a problem. It appeared that that learners were not in the habit of writing down what is known and what is being asked in understanding the problem. Nevertheless, half of the responses with regard to the strategies used in solving the task (30 out of 60) implied that participants were certain about solving the algebra problems correctly when asked if they thought they could solve the problem (see Table 5-4).

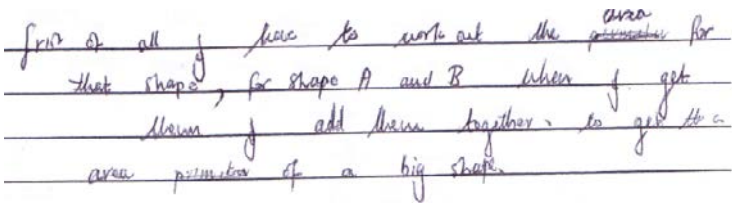
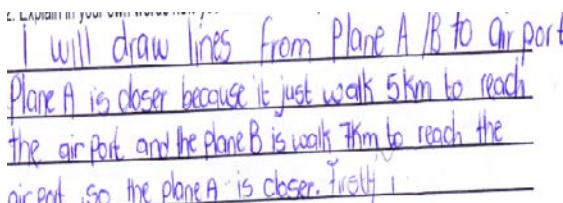
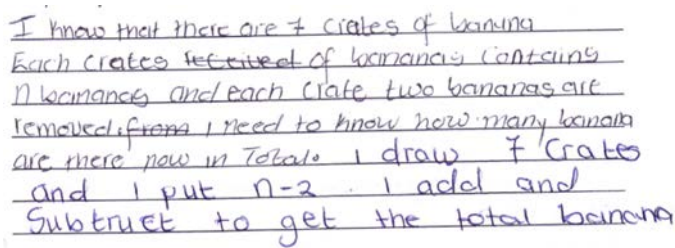
5.2.2.2 Planning suitable strategies for a given problem (Explain in your own words how you would solve the problem (see Addendum A, question 2)

Before attempting to solve the problem, a participant has to find a plan (Aydoğdu & Keşan, 2014). A planned strategy is a strategy that the participants aim to use to solve the problem and is dependent on the nature of the problem (e.g. formulae, areas, perimeters). As discussed in chapter three, various authors discussed other problem-solving strategies suitable for solving different tasks. These strategies include drawing pictures; making charts; working backwards; guessing and checking; trial-and-error and creating equations (see 3.5.1).

When asked to explain in their own words how they would solve their problems (before actually solving the problem), the participants' responses indicated that all performing groups used different planned strategies for solving mensuration, geometry and algebra problems. Table 5-3

depicts the planned strategies used by low performance, middle performance and high-performance groups.

Table 5-3 Responses on planned strategies used on tasks

Performing groups	Planned strategies
Low performing group	Planned strategies: (Task 2 Mensuration) Using a formula and solving a simpler problem (Posamenter <i>et al.</i>, 2010:113) 
	Planned strategies: (Task 5 Geometry) Using a drawing (Arslan & Altun, 2007:54) 
	Planned strategies: (Task 7 Algebra) Using a drawing (Arslan & Altun, 2007:54) 

Planned strategies: (Task 3 Mensuration)

Using a formula and solving a simpler problem (Posamenter *et al.*, 2010:113)

First I find area of big half circle. Using area - then I find the area of all three circle add them together to find the total area of working of Danie/a

Planned strategies: (Task 5 Geometry)

Comparing (Yew & Zamri, 2016:20)

Plane A is closer to the tower because it is higher than plane B and can easily step on the tower. I know just by looking. But I can also add the total squares on both sides to compare and see which plane is closer.

Planned strategies: (Task 7 Algebra)

Using a formula/rule (Posamenter *et al.*, 2010:113)

I have to find the difference of diners and then I use the formula $2n + 2$ that will help me to find the diners those can be seated at this row of table and I have to use the table to make sure that everything is in order.

Planned strategies: (Task 1 Mensuration)

Using a formula and solving a simpler problem (Posamenter *et al.*, 2010:113)

I have to find the TSA of a gift box
 If I find the height of a box is 9cm the length is 12cm the width is 6cm.
 My formula $2(l \times b) + 2(b \times h) + 2(h \times l)$ for me to get answer and then the total of $2(l \times b) +$ the total of $2(b \times h) +$ the total of $2(h \times l)$ and get my final answer.

Planned strategies: (Task 6 Geometry)

Using tools (Lingefjård & Holmquist, 2003:120)

I need to constructing the line through black dots and the line make up a triangle. And then I started need a pair of straight edge and a compass in order to make a new medical centre.

Planned strategies: (Task 8 Algebra)

Making a table and formula (Posamenter *et al.*, 2010:113)

table	1	2	3	4	24
dinner	3	4	5	6	?

$$\begin{aligned}
 T_n &= (1+n) + 1 \\
 &= (1+1) + 1 \\
 &= (1+2) + 1 \\
 &= (1+3) + 1 \\
 &= (1+4) + 1 \\
 &= (1+24) + 1 \\
 &= 26 \text{ dinner}
 \end{aligned}$$

Table 5-3 illustrates some of the responses of low, middle and high performing groups concerning the different strategies that they planned to use in solving different tasks. There was clear evidence that all performing groups attempted to find/explore solutions to the problems by

formulating various planned strategies that should be implemented in their calculations. The strategies of using a formula and solving one part at a time were commonly used concurrently in solving most of the mensuration tasks.

Pólya's first step for solving problems as stated earlier is to read and understand the nature of the problem, then devise a suitable strategy to solve the problem (see 3.5). Using a formula and solving a simpler problem were used concurrently in solving mensuration problems.

In the geometry tasks, the participants used the following strategies: making drawing using tools, using theorems and solving a simpler problem (see 3.5.1). In the algebra tasks, the participants used drawings, formulae, rules and tables. One important aspect which is sought to improve struggling learners' learning of algebra is the practice of comparing multiple solution strategies for mathematics problems, which may often involve the use of multiple representations such as symbols, tables, and graphs (Lynch & Star, 2014).

The strategies that participants used to solve the tasks are represented in Table 5-4 on the following page.

Table 5-4 Problem-solving strategies used in solving the tasks

No.	Problem-solving strategies	Low	Middle	High	Total
1	Making a diagram (Van Garderen & Scheuermann, 2014)	mensuration=0 geometry=3 algebra=7	mensuration=0 geometry=1 algebra=8	mensuration=0 geometry=4 algebra=8	mensuration=0 geometry=6 algebra=23
2	Comparing (Yew & Zamri, 2016)	mensuration=1 geometry=0 algebra=0	mensuration=0 geometry=0 algebra=0	mensuration=2 geometry=0 algebra=0	mensuration=3 geometry=0 algebra=0
3	Use a formula or rule (Posamenter <i>et al.</i> , 2010)	mensuration=8 geometry=0 algebra=7	mensuration=6 geometry=0 algebra=4	mensuration=11 geometry=0 algebra=5	mensuration=25 geometry=0 algebra=16
4	Using a table (Posamenter <i>et al.</i> , 2010)	mensuration=0 geometry=0 algebra=0	mensuration=0 geometry=0 algebra=4	mensuration=0 geometry=0 algebra=4	mensuration=0 geometry=0 algebra=8

5	Solving a simpler problem (Posamenter <i>et al.</i> , 2010)	mensuration=7 geometry=0 algebra=5	mensuration=8 geometry=0 algebra=4	mensuration=9 geometry=0 algebra=4	mensuration=24 geometry=0 algebra=13
6	Use tools (Lingefjärd & Holmquist (2003)	mensuration=0 geometry=3 algebra=0	mensuration=0 geometry=3 algebra=0	mensuration=0 geometry=1 algebra=0	mensuration=0 geometry=7 algebra=0
7	Using a theorem (Lingefjärd & Holmquist (2003) and then solving a simpler problem (Posamenter <i>et al.</i> , 2010)	mensuration=3 geometry=3 algebra=0	mensuration=4 geometry=4 algebra=0	mensuration=4 geometry=3 algebra=0	mensuration=3 geometry=10 algebra=0
Total		mensuration=19 geometry=9 algebra=19	mensuration=18 geometry=8 algebra=23	mensuration=26 geometry=8 algebra=17	mensuration=63 geometry=24 algebra=60

Table 5-4 illustrates different problem-solving strategies used by low, middle and high performing groups on the tasks. Participants raised a number of problem-solving strategies related to problem-solving in grade 9 mathematics classrooms. They planned to solve the problem using the following strategies: drawing a diagram; comparing, using a formula/rule; using a table, solving a simpler problem, using tools and using a theorem.

a) Making a drawing

The strategy of drawing a diagram is a problem-solving technique in which learners make a visual representation of a problem. This helps learners to organise the information presented in the problem so that they can use another strategy to reach the solution (see 3.5.1). For example, in task six (see Figure 5-1), the learner first found the highest common factor of 30 and 24 which is six. Then drew six squares where in each of the squares there are five rows of oranges (O) and four rows of cherries (C).

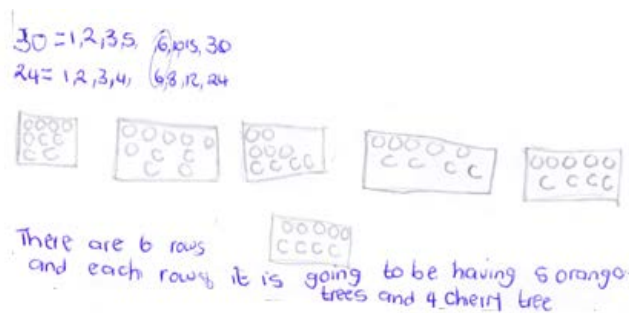


Figure 5-1: Example of making a drawing

b) Comparing

The comparing problem-solving strategy involves comparing the structure of one problem to that of another or comparing the solution strategy used for one problem to that used for another (Hattikudur & Alibali, 2011), for example Figure 5-2 (see Task 3).

Daniela $A = \frac{1}{2} \pi r^2$ the radius is $10 \div 2 = 5$ $\frac{1}{2} \times 3.14 \times 5 \times 5$ 39.3 m^2 $\therefore 39.3 + 39.3 + 39.3$ $= 117.9 \text{ m}^2$	Ariane the radius is $30 \div 2 = 15$ $A = \frac{1}{2} \times \pi \times r^2$ $\frac{1}{2} \times 3.14 \times 15 \times 15$ $= 353.3 \text{ m}^2$ $117.9 \text{ m}^2 < 353.3 \text{ m}^2$
---	--

Then I realise that Ariane come first.

Figure 5-2 Example of comparing

The participant first worked out the total distance walked by both Ariene and Daniela as 117.9 m^2 and 353.3 m^2 and then compared which distance was shorter or longer than the other.

c) Using a formula

Using a formula is a problem-solving strategy that learners can use to solve problems involving any content area in mathematics, such as geometry, algebra, or mensuration. In order to solve some problems successfully one has to choose an appropriate formula and substitute the information into the correct places in a formula. Figure 5-3 illustrates this strategy. The following

I firstly should find a total surface area
of a cuboid - firstly I identify all ~~phase~~ b phase
① phases $2(l \times b)$
② phases $2(l \times h)$
③ phases $2(b \times h)$
And I add the total surface together.

Figure 5-3 Example of using a formula

problem (see Task 1) would be best solved using a formula.

According to the Department of Basic Education, to solve mensuration problems, participants must choose “the appropriate formula and substitute data in the correct places of a formula” (MoE, 2010:47). Obviously, using a formula alone, does not yield a final answer, it is just a model with some of the important features of the problem.

d) Using a table

This strategy involves drawing a table and writing information in a more organised manner and looking critically at the data to find the pattern and develop a solution. This problem-solving strategy allows learners to discover relationships and patterns. An example of where participants used a table to solve the problem is given in Figure 5-4.

The rule is $(n+1)+1$

T	1	2	3	24
D	3	4	5	x

$T_n = (i+n)+1 = (1+1)+1 = 3$
 $= (1+n)+1 = (1+2)+1 = 4$
 $= (1+n)+1 = (1+3)+1 = 5$
 $= (1+n)+1 = (1+24)+1 = 26$
 $= 26 \text{ dinners}$
 so, my x represent 26 dinners

Figure 5-4 Example of using a table

e) Solving a simpler problem

Algebra problems can be very complicated for the learners, so they need to be broken down before they can be solved. Solving a simpler problem is a useful strategy when you have a problem that cannot be solved in one step. Divide it into parts and solve each part separately.

f) Using tools

This strategy involves using tools, including pencil and paper, concrete models, a ruler, a protractor, a calculator, a spread sheet, a computer algebra system, a statistical package, or dynamic geometry software. Learners have access to these tools to curb long and tedious calculations (Cooper, 2011). The application of tools such as calculators is significant for aiding with complex calculations and the understanding of ways providing the solution to the answer (see 3.7.4). For example, task three is one of the tasks that consist of complex calculations. In the example presented in Figure 5-5, this learner used a calculator to multiply $\pi \times d$ in order to get the answer easily and quickly.

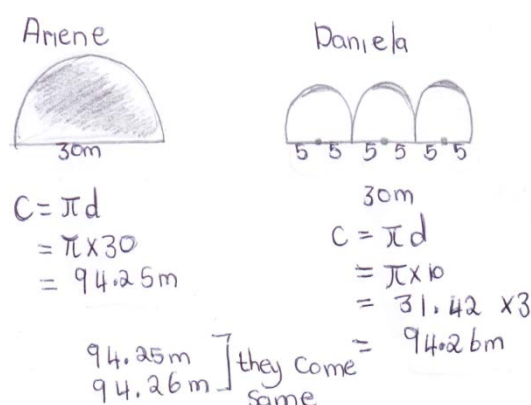


Figure 5-5 Example of using tools

Learners were not always familiar with the calculator and did not always have a sound knowledge of when and how to use it. They were not always able to recognise the insight to be gained when correctly used and their limitations where necessary.

g) Use a theorem and solving a simpler problem

These strategies involve using a formula or formulating your own formula based on the problem situation. Figure 5-6 presents an illustration of how a learner formulated the Pythagoras theorem ($A^2 + B^2 = C^2$) based on a given situation (see Task 4).

The image shows a student's handwritten work on a piece of paper, divided into two columns for 'Plane A' and 'Plane B'. The student is using the Pythagorean theorem to find the distance from a tower to each plane.

Plane A:

$$A^2 + B^2 = C^2$$

$$= 20\,000\text{km}^2 + 5\text{km}^2 = C^2$$

$$= 400\,000\,000 + 25\text{km} = C^2$$

$$= \sqrt{400\,000\,000} + \sqrt{25\text{km}} = \sqrt{C^2}$$

$$= 20\,000\text{km} + 5\text{km} = C$$

$$C^2 = 80005$$

Plane A is 20005km

Plane B:

$$A^2 + B^2 = C^2$$

$$= 8000\text{km}^2 + 2\text{km}^2 = C^2$$

$$= 64\,000\,000 + 4\text{km} = C^2$$

$$\sqrt{64\,000\,000} + \sqrt{4\text{km}} = \sqrt{C^2}$$

$$= 8000\text{km} + 2\text{km} = C$$

$$= C^2 = 8002^2$$

So plane A is closer to tower

Plane B is 8002 km

Figure 5-6 Example of using a theorem

By first labelling the sides, the learners identified the formula for finding the distance travelled by plane A and plane B. S/he broke up the problem into simpler problems and used the appropriate formula ($A^2 + B^2 = C^2$) based on the task. He/she then, substituted in the values and used a calculator to find the answer. The above strategies were frequently used for solving especially the geometry problems.

According to Schoenfeld (2015), learners identify their own strategies themselves through experience. Effective problem solvers plan properly, keep track of their progress and consistently re-evaluate their progress (Schoenfeld, 2015). Once the problem solver has selected relevant strategies, the second stage is to see whether any of the strategies can work or if it is necessary to use a calculation (Aydoğdu & Ayaz, 2008). The planned strategy is significant in that, if well implemented, it can yield the correct answer. The idea was that what learners planned to do in their descriptions (how they would solve the problem) should be implemented in their calculations. In this case implementing a planned strategy is the process that turns strategies and plans into action in order to accomplish a strategic goal.

5.2.2.3 Carrying out the plan

The third question of the task-based interview was: You may solve the problem now. Show all your steps. Once the strategy has been identified, the implementation of that strategy means working out a solution to the problems using the information obtained in the first step (problem identification) (ME, 2005).

The Canadian Ministry of Education (ME, 2005:53) lists five stages to implement the solution:

1. Solve the problem using your plan.
2. Be sure to double check each step.
3. If the plan is not working after a few attempts, try a different plan.
4. Allow for mistakes (remember the plan may need some revision).
5. Check your answer.

This corresponds with Pólya's model of problem-solving, whereby one has to (1) understand the problem, (2) make a plan to solve the problem, (3) carry out the plan and (4) look back or reflect on the solution in an attempt to solve the problem. This strategy was positively implemented by learners during the task-based interviews.

Each available participant completed the various tasks using different strategies and approaches in their calculations. I assessed each participant's task with a rubric (see Addendum C). A rubric is designed as a tool for assessing learner's performance, giving standard expectations or criteria for the task. A rubric was designed to identify different levels of learners' achievement. The rubric consisted of four performance levels – insufficient, low, proficient and achieved beyond, and six assessment categories.

Each calculation was assessed by using the following categories: problem-solving strategies used and developed, the use of content knowledge, solving the problem, the use of a calculator, the use of mathematical language and the connection to other mathematics. The marks were allocated on the appropriate level of achievement, depending on the number of participants, located/recorded on each level. The marks were compiled and recorded for each performing group. For the purpose of this study, all the tasks, were selected for this discussion in order to compare differences in performance between low, middle and high performing groups. The following categories will be discussed in more detail: problem-solving strategies used and developed, solving the problem, and the use of a calculator.

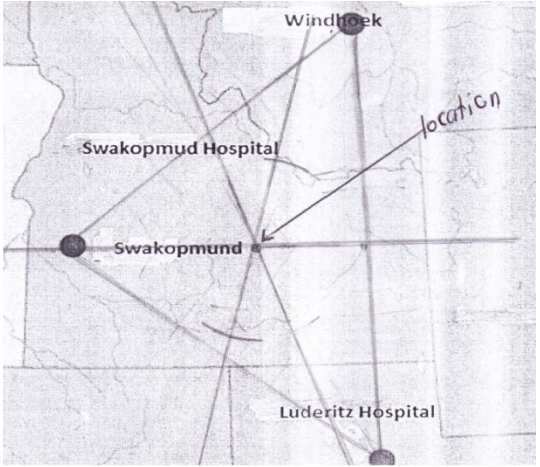
5.2.2.4 Understanding the problem

The fourth question on the task-based interview was: Do you think that you have solved the problem correctly? This was guided by Pólya's fourth step of the problem-solving model which is to reflect on the problem, to then evaluate their solutions, to see if the answer is reasonable or not and whether there is another way to find the solution (Arslan & Altun, 2007). Once the solution has been obtained, the learner should look back and check to see if the plan worked. It is, however, helpful to start asking yourself guiding questions: 1. Did I answer the question? 2. Is my outcome reasonable? 3. Did I double check to make sure that all of the conditions related to the problem are satisfied? 4. Did I double check any computations involved in finding the solution? If you find that your solution does not work, there may be errors in the calculation. It is advisable to start fixing the errors made until a tentative solution is met (see 3.5).

Szetela and Nicol (1992) state that performance in problem-solving is to obtain a general impression about the quality of a solution while scanning learners' work. These general impressions are strongly influenced by the proximity of the correctness of the answer. In this case the study will look at the reasons for the mistakes made. Apart from asking the participants whether they thought they had solved the problem correctly, the solutions which were obtained from question three of the task-based interviews (see Addendum A) by the participants were examined and assessed for accuracy. At question six on the task-based interview, participants were asked to provide possible reasons for their mistakes. These responses will also be discussed in this section. When asked to state the reasons for making a mistake, participant responses were as follows:

Table 5-5 Reasons for making mistakes in solving tasks and actual solutions

PG	Reasons for making a mistake	Actual solutions Correct (C) or Incorrect (I)															
Low	Task 4 (Geometry): Not understanding the mathematics problem: a) I know I have the correct sides 20000^2 and 5^2 / 8000^2 and 2^2 but I don't know if or whether I am correct to add to get a correct answer. concept and the question	Incorrect Plane A $A^2 + B^2 \neq C^2$ $= 20000\text{km}^2 + 5\text{km}^2 = C^2$ $= 400000000 + 25\text{km}^2 = C^2$ $= \sqrt{400000000} + \sqrt{25\text{km}^2} = C^2$ $= 20000\text{km} + 5\text{km} = C^2$ $C^2 = 20005$ Plane A is 20005km^2 Plane B $A^2 + B^2 \neq C^2$ $= 8000\text{km}^2 + 2\text{km}^2 = C^2$ $= 64000000 + 4\text{km}^2 = C^2$ $\sqrt{64000000} + \sqrt{4\text{km}^2} = C^2$ $= 8000\text{km} + 2\text{km} = C^2$ $= C^2 = 8002$ Plane B is 8002km^2 closer to tower															
	Task 8 (Algebra): Not understanding the mathematics concept problem: • I still have a problem with working with letters and numbers • working with patterns • the rules of subtraction in algebra	Incorrect The pattern is <table> <tr> <td>n</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> </tr> <tr> <td>T_n</td> <td>1</td> <td>6</td> <td>10</td> <td>14</td> <td>18</td> <td>22</td> <td>26</td> </tr> </table> n = unknown $T_n = 7(n-2)$ $= 7 - 14$ $= -7$ There are -7 bananas in each crates.	n	1	2	3	4	5	6	7	T _n	1	6	10	14	18	22
n	1	2	3	4	5	6	7										
T _n	1	6	10	14	18	22	26										

Middle	Task 3 (Mensuration): Not understanding the mathematics concept I know that I have made a mistake by using the Formular of surface area. I don't know the formula.	Incorrect $2l \times 2b \times 2h$ $SA = 2(12) \times 2(6) \times 2(9)$ $SA = 24 \times 12 \times 18$ $SA = 5184 \text{ cm}^2$ the minimum wrapping paper needed is 5184 cm^2
	Task 5 (Geometry): Not understanding the mathematics concept I am think I made some mistake because maybe am not understanding the question well especially to direct, I try	 Incorrect
	Task 9 (Algebra): Not understanding the question I am not sure if I am correct the question I don't know whether to use inequality method or linear equation	Incorrect 34 employee need T-shirt for $400 \div 34 = \$11.76$ So, 1 t-shirt is \$11.76 + \$12.99 is \$24.75 per person So All design the compney can afford

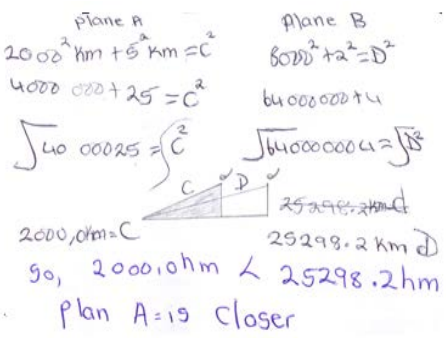
High	Task 4 (Geometry): Not understanding the question Sometimes I did not understand what is a question asked. I confused me. I did not find plane B's longer side seems like they are equal.	 Incorrect
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Table 5-5 shows different reasons for participants to make a mistake in solving the tasks, taken randomly from their actual solution. Problem-solving is not just about the abilities to use strategies, methods and skills in a good way, but also about the abilities to look back at a situation and evaluate its validity.

A problem exists when a learner tries to resolve a goal and their initial attempts prove unsuccessful (Shuell, 1990). There is no chance of being able to solve a problem unless you can understand it first. This process requires knowing what you have to find first and the key pieces of information that need to be put together to obtain the answer (Aydoğdu & Ayaz, 2008). According to In'am (2014), not understanding the question means the learner will not be able to: 1) Identify the unknown variables in a problem; 2) Establish the relationship between the unknown and what has been determined and 3) Identify the procedure needed to yield a correct answer.

According to Ali and Reid (2012), not understanding the mathematics concept means not being able to do the following: 1) Explain mathematical concepts and facts; 2) Make logical connections between different facts and concepts; 3) Recognise the connection when you encounter something new that is close to the mathematics you understand; and 4) Identify the features in the given piece of mathematics that make the procedure work. Learners need to learn mathematics with understanding, because when learners learn with understanding they become flexible, and adapt to new situations during solving contextual problems (see 2.2.1).

The following categories detailing the various reasons as supplied by the participants were evident:

- (a) Not understanding the question
- (b) Not understanding the mathematics concept
- (c) Not understanding both the mathematics concept and the question

Some participants did not understand both the mathematics concept as well as the problem. This means that they don't know both what mathematics exists within a problem and also do not know the procedures to follow in solving the solutions to a given problem (Stylianou, 2013:24). The data were summarised into the three categories discussed above. The number of participants' reasons for making a mistake were grouped in these categories according to the content topics, and recorded per performing group. This was done by stating the number of participants whose solutions were incorrect as follows:

Table 5-6 A summary of participants' reasons for mistakes made in solving all tasks

Categories	Low			Middle			High			Total
Not understanding the mathematics concept	Mensuration	Geometry	Algebra	Mensuration	Geometry	Algebra	Mensuration	Geometry	Algebra	
	1	2	4	2	1	4	3	2	0	19
Not understanding the question	1	0	3	1	2	5	0	2	0	14
Not understanding mathematics concept and the question	2	3	2	2	0	0	0	0	4	13
Total	4	5	9	5	3	9	3	4	4	46

Table 5-6 illustrates the different responses of the participants which indicate the reasons for their mistakes in attempting to solve the different tasks. The table has proven that in the 46 tasks that were solved, 19 of those that were not solved correctly, were due to participants not understanding the mathematics concepts. Fourteen of the tasks were incorrect because participants did not understand the question and 13 were incorrect because participants did not understand both the mathematics concept and the question. This seems to indicate that participants struggle more to understand the mathematics concepts than the actual question.

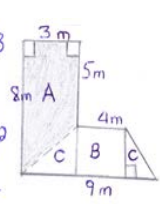
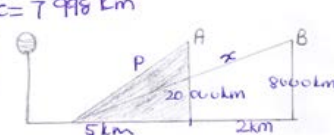
5.2.2.5 Problem-solving strategy planned and implemented

The main idea in this category is to discuss the problem-solving strategy that the participants described and used. As is represented on the rubric (see Addendum C), the rating criteria for this category are listed below together with their level of achievement:

- The strategy selected cannot lead to the solution: “sufficient”.
- A correct strategy is selected for solving only part of the problem: “low”. A correct strategy is selected based on a mathematical situation: “proficient”.
- A reasonable strategy was selected and developed, “achieved beyond”.

Table 5-7 below illustrates examples of the planned strategy and implemented strategy for some of the mensuration, algebra and geometry tasks.

Table 5-7 **Planned strategy and solution for tasks**

Planned strategy	Implemented strategy															
Task 2 (Mensuration) Using a formula and solving simpler problem firstly I cut my shape B in 3 parts than label missing part like of than I find area B than Area c and Area A. Than I add my area A+c+B+c to gether to get the area of the table	Using a formula and solving simpler problem Area A $\frac{1}{2} \times (5+8) \times 3$$= \frac{1}{2} \times 13 \times 3$$= 84.5 \text{ cm}^2$ Area C $\frac{1}{2} \times 5 \times 3$$= 7.5 \text{ m}^2$ Area B $l \times b$$= 9 \times 3$$= 27 \text{ m}^2$ Answer $84.5 \text{ cm}^2 + 7.5 \text{ m}^2 + 27 \text{ m}^2$$= 119 \text{ cm}^2$ 															
Task 5 (Geometry) Solve one part at a time and using a theorem problem-solving strategy I would solve the problem when because p as follows: first finding Side p by using 20000m² and 5km² Secondly finding Side x by using 80000m² and 2km² Go counter to the tower than I know that plane that is closer to the tower	Solve one part at a time, using theorem and drawing problem-solving strategies Plan A 20000 $20000^2 - 5^2$$400000000 - 25$$= 20000 - 5$$p = 19995 \text{ km}$ Plan B $8000^2 - 2^2$$= 64000000 - 4$$= 8000 - 2$$x = 7998 \text{ km}$ 															
Task 10 (Algebra) Using a formula 2. Explain in your own words... firstly I have to write an expression for all the ages of the family than I will find the age of John's brother because I have given John's age already than I will find the age of his sister and find the age of his mother father and than lastly I will find the age of his mother.	Using a formula/expression John's expression of age of family Brother = $t-5$ Sister = $t+1$ Father = t^2 Mother = $t-3$ Then age <table><tr><td>John</td><td>t</td><td>$t=7$</td></tr><tr><td>Brother</td><td>$t-5$</td><td>$7-5=2 \text{ yrs}$</td></tr><tr><td>Sister</td><td>$t+1$</td><td>$7+1=8$</td></tr><tr><td>Father</td><td>t^2</td><td>$49+3=52$</td></tr><tr><td>Mother</td><td>$t-3$</td><td>$7-3=4$</td></tr></table> than Substitute	John	t	$t=7$	Brother	$t-5$	$7-5=2 \text{ yrs}$	Sister	$t+1$	$7+1=8$	Father	t^2	$49+3=52$	Mother	$t-3$	$7-3=4$
John	t	$t=7$														
Brother	$t-5$	$7-5=2 \text{ yrs}$														
Sister	$t+1$	$7+1=8$														
Father	t^2	$49+3=52$														
Mother	$t-3$	$7-3=4$														

Most participants planned to use a formula and solving a simpler problem when solving mensuration tasks as their implemented strategies. In many cases, the planned strategy and the implemented strategy were the same, however there were cases where they were different. For example, as illustrated above in task two, learners were able to find area A correctly, using a correct formula, substituting in correctly and find $84.5 m^2$. However, they failed to get the correct answer for area B and area C due to incorrect substitution. It is clear that only part of the problem was solved correctly here. Learners failed to attend to relevant features of a problem while problem-solving; in this case choosing which values to use in a formula, the type of shape they are dealing with. This links to what In'am (2014) indicates, which is that understanding a problem and creating a plan to solve the problem will not be useful if it has not been implemented. An effort to show that the problem-solving is suitable for solving the problem is by implementing the problem-solving in line with the chosen strategy or model.

Pertaining to Task Five (see Table 5-8), learners planned to find p and x by solving a simpler problem and then solved the actual problem using both solving a simpler problem and making a drawing, problem-solving strategies. It is clear that their planned strategy was entirely the same as the implemented strategy – when solving the problem; they implemented additional strategies in order to find the solution. At this task many low and middle performing learners implemented strategies that they had not planned. Their initial strategies could not lead them to the solution which meant that they had to adapt and use a different strategy. Low performing learners regularly see mathematics as simply a collection of formulae, they need to retain; rather than create ideas, and they have to understand. Freudenthal defines mathematics as a human activity; he urges that mathematics should never be presented to the learners as readily-made product. The learners should re-invent mathematics (see 2.2.3).

With regard to task ten, there is a clear indication that the participants solved task ten correctly by first writing down the expression that represents that age of all family members. Thereafter, they substituted into the expressions to find the age of each individual. Most of the participants managed to solve this algebra problem correctly, despite the complexity within the concepts of algebra. Low performers specifically, were able to recall the facts of algebra, solving the problem by first writing an expression, and then making meaning out of the given situation. The majority of the middle and high performers described and used problem-solving strategies which were rated as “Achieved beyond”. This implies that they had selected and developed reasonable strategies in solving problems, as is illustrated below.

Table 5-8 Planned strategy and solution for Task 3 (Middle and High performers)

Planned strategies	Implemented strategies
Task 3 Using a formula and solving simpler problem I find the distance walks by Ariane by using circumference. Then I find the distance walked by Daniela, then I divide the circumference by 2, then I know and compare the distance of the two girls.	Using a formula and solving simpler problem Ariane $C = \pi d$ $= 3.14 \times 30$ $= \frac{94,2}{2} = 47,1 \text{ m}$ Daniela $C = \pi d$ $= 3.14 \times 30$ $= \frac{94,2}{2}$ $= 47,1 \text{ m}$ They arrived at the same time.

The majority of the middle and high performers' strategies in task three were based on finding the distance (perimeter) around the semi-circles. These learners planned to use perimeter because it implies the concept of length and not area. Therefore, they opted to use the circumference formula ($C = \pi d$) which most low performers couldn't identify. In most case low performers planned to solve this problem using area formulae as illustrated below:

Table 5-9 Planned strategy and solution for Task 3 (Low performers)

Planned strategies	Implemented strategies
Task 3 first of all we work out a semi-circle for a big top large circular and when we work out using area method then work out the area for Daniels the two area will deter mine who come first	Ariane $C = \pi r^2$ $d = 30 \div 2 = 15$ $= \pi \times 15^2$ $= \pi \times 225$ $= 706 \text{ m}^2$ Ariane $r = 15 \text{ m}$ Daniela $r = 5 \text{ m}$ So, $78.5 + 78.5 + 78.5 = 235.5 \text{ m}^2$ So Ariane Come first

Solving a mathematical problem requires more than understanding a description of the problem (D'Zurilla et al., 2004). It requires determining what mathematical facts to use and when and how to use those facts to develop a solution to the problem.

A specific goal of problem-solving in mathematics is to improve learners' abilities to plan and implement problem-solving strategies accurately (Aydoğdu & Ayaz, 2008). Attempting to solve a problem with a wrong strategy may lead to work without producing a correct solution. One of the aims of this study is to determine what problem-solving activities should look like in the context of a Grade 9 mathematics classroom. Task three is one of the tasks that require conceptual and procedural understanding for learners to solve it correctly as well as mathematical knowledge and skills in critical thinking. The middle and high performers were able to devise effective plans and strategies after understanding the problem, which seems to indicate that they not only understood the problem, but also chose the appropriate problem-solving strategy, while the low performing group was not able to do this.

5.2.3 Analysing the use of calculators

5.2.3.1 The use of calculators

As discussed in chapter three there are two schools of thought when it comes to the use of a calculator. One viewpoint suggests that in order to develop a rich sense of numbers and fluency, learners should not be allowed to use calculators, but rather be able to do operations mentally, without the calculator (Asare-Inkoom *et al.*, 2007). The other viewpoint suggests that calculators can support children in developing higher-order mathematical thinking (Banks, 2011; Denton, 1992; Clark, 2011; Sheets, 2007). Learners should be taught when to use a calculator and when mental computing is more effective or appropriate.

The calculator can assist in the development of mathematical content ideas such as: multiplication, repeated addition and the learning of basic facts (Sparrow *et al.*, 1994). One of the important uses of a calculator is to check the validity of the solution (Rohrbaugh & Cooper, 2016). Without calculators, it can be difficult to address more complex mathematical materials (Miles, 2008), so calculators can be very useful when dealing with mathematical problems. However, then they must be used correctly and appropriately.

The discussion in this category will be based on whether the calculator was used correctly or incorrectly in the calculation. The following table illustrates the use of calculators by low, middle and high performers.

Table 5-10 The use of calculations in problem-solving

Category on rubric	Low achievers			Middle Achievers			High Achievers			Totals
	Mensuration	Geometry	Algebra	Mensuration	Geometry	Algebra	Mensuration	Geometry	Algebra	
Insufficient	5	2	2	5	1	0	2	0	0	17
Low	6	5	10	5	2	11	6	2	1	49
Proficient	2	0	10	2	4	4	2	4	17	37
Achieved Beyond	3	1	2	4	1	9	6	1	6	41
Totals	16	8	24	16	8	24	16	8	24	143

As is represented on the rubric (Addendum C), the rating criteria for this category are listed below together with their level of achievement:

- Calculations are completely incorrect, leading to an incorrect answer: “insufficient”
- Calculations contain major errors; calculator was used incorrectly: “low”
- Calculations are completely correct, calculator was used randomly: “proficient”
- Calculations are completely correct and answers properly labelled, calculator was used appropriately: “achieved beyond”

Approximately a third of the responses by the participants (49 out of 143) were rated “low” in this category. This implies that most of their calculations contained errors. The calculator was used correctly during solving these problems, but only as a tool for computation. This is common especially in the algebra and mensuration problems where approximately a quarter of the responses (41 out of 143) from the low, middle and high performers indicated that the participants were able to use the calculator correctly in their calculations, but unable to solve the problems correctly due to incorrect use of mathematical procedures. The following table shows some of the low, middle and high performers’ calculations.

Table 5-11 Calculations of participants in low, middle and high performing groups

Low performing Group

Task 2 (Mensuration)

Shape A Shape B

$$A = \frac{1}{2}hb + ab$$

$$= \frac{1}{2} \times 3 \times 3 + 5 \times 8$$

$$44.5m^2$$

$$A = \frac{1}{2}hb + ab$$

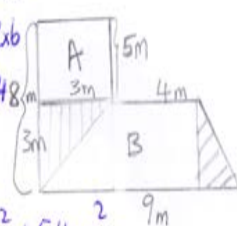
$$= \frac{1}{2} \times 3 \times 9 + 4 \times 8$$

$$54m^2$$

$$44.5m^2 + 54m^2$$

$$98.5m^2$$

The materials will going to be bought there are $98.5m^2$



Task 8 (Algebra)

The pattern is

n	1	2	3	4	5	6	7
Tn	1	6	10	14	18	22	26

n = unknown

$$T_n = 7(n-2)$$

$$= 7 - 14$$

$$= -7$$

There are -7 bananas in each Crates.

Middle Performing Group

Task 2 (Mensuration)

Area of shape A

$$A = \frac{1}{2}h(a+b)$$

$$= \frac{1}{2} \times 3m(8m+3m)$$

$$= \frac{1}{2} \times 3m \times 11m$$

$$= 19.5m^2$$

Area of shape B

$$A = \frac{1}{2}h(a+b)$$

$$= \frac{1}{2} \times 3m(4m+9m)$$

$$= 0.5 \times 3m \times 13m$$

$$= 19.5m^2$$

than $19.5m^2 + 19.5m^2$

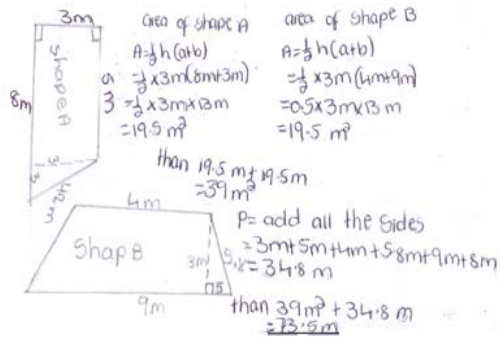
$$= 39m^2$$

P = add all the sides

$$= 3m + 5m + 4m + 5.8m + 9m + 8m$$

$$= 34.8m$$

than $39m^2 + 34.8m$

$$= 73.8m$$


Task 7 (Algebra)

The rule is $(n+1)+1$

T	1	2	3	4
D	3	4	5	x

$$T_n = (1+n)+1 = (1+1)+1 = 3$$

$$= (1+n)+1 = (1+2)+1 = 4$$

$$= (1+n)+1 = (1+3)+1 = 5$$

$$= (1+n)+1 = (1+24)+1 = 26$$

= 26 dinners

so, my x represent 26 dinners

High Performing Group	
Task 1 (Mensuration) $= 2(l \times b) + 2(b \times h) + 2(l \times h)$ $= 2(120\text{cm} \times 6\text{cm}) + 2(6\text{cm} \times 9\text{cm}) + 2(120\text{cm} \times 9\text{cm})$ $= 2(72) + 2(54) + 2(108)$ $= 144\text{cm} + 108\text{cm} + 216\text{cm}$ $= 468\text{cm}^2$ So the minimum wrapping paper is 468 cm²	Task 4 (Geometry) Plane A: $2000^2\text{km} + 5^2\text{km} = C^2$ $4000000 + 25 = C^2$ $\sqrt{4000025} = C$ $2000.01\text{km} = C$ Plane B: $8000^2 + 2^2 = D^2$ $64000000 + 4 = D^2$ $\sqrt{64000004} = D$ $25298.2\text{km} = D$ 90, 2000.01km < 25298.2km Plan A is closer

Table 5-11 shows that low and middle performers' calculations contain major errors. This is especially the case at task two, where the learners were able to formulate a correct formula; however, they failed to substitute into the formula correctly. The calculator in this regard was used to work out $\frac{1}{2} \times 3 \times 3 + 5 \times 8 = 44.5\text{m}^2$ correctly. As far as the calculator is concerned, the answer is solved correctly, but in fact this procedure would not lead to a correct solution because the numbers substituted in the formula are incorrect. The calculator was used in this calculation as an aid of computation: to multiply and add the numbers together. However, it was wrongly applied on the mathematical situation.

For instance, in task eight, the formula used in the calculation is not relevant for the mathematical situation. The second step in this calculation represents the problem situation; however, the learners used a wrong procedure by using the calculator to subtract ($7 - 14$). In fact, the calculator was used as an aid of computation. This subtraction operation is correct, but it did not answer the problem. In response to this, if learners do not know whether the operation selected in applying their mathematical knowledge to is incorrect when solving a problem, the calculator by itself is not going to provide a correct solution (Sparrow *et al.*, 1994).

The second error commonly committed by low performers was conceptual and factual errors. Factual errors are mistakes that learners make when they cannot recall a fact required to solve a problem or if they have not mastered basic facts; and conceptual errors are mistakes made when learners misunderstand the concept (Lestiana, 2017). The errors committed on tasks one and three are commonly based on the application of a wrong formula in a calculation.

Similarly, for task three a learner used the area formula ($A = \pi r^2$) instead of using the circumference formula ($C = \pi \times \text{diameter}$). There is no doubt that this learner used a calculator accurately especially with multiplying ($\pi \times 15^2 = 706\text{m}^2$) and for the repeated addition ($78.5 + 78.5 + 78.5 = 235.5\text{m}^2$). It appears to be a similar case as on task one, where a calculator was used correctly. It is true $12 \times 6 \times 9 = 648$. However, this is not really the sole solution to the problem. Again, the learner was supposed to use the surface area formula but not the volume formula. Ultimately, in this case it is clear that learners feel comfortable using a calculator only to calculate answers, yet sometimes show a lack of understanding of the mathematical concepts involved in the problem.

The outcome of these tasks was not determined by correct/incorrect use of a calculator but by the misunderstanding/misconception of some of the concepts of mensuration. These errors/mistakes are caused by failure to make connections with what learners already know about the topic (Sarwadi & Shahrill, 2014), not necessarily the mistake from a calculator.

Regarding the middle and high performers, the majority of the calculations were rated as “Achieved beyond”. This implies that most of their calculations were completely correct. Their answers were properly labelled, and the calculator was used appropriately throughout their calculations, as illustrated below:

Table 5-12 Example of correct calculations

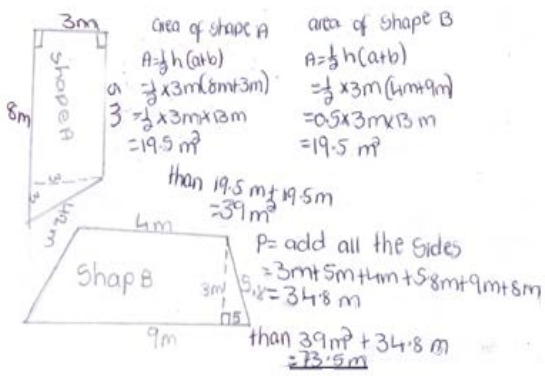
Middle performer	High performer
Task 2 (Mensuration)  Area of shape A $A = \frac{1}{2}h(a+b)$ $= \frac{1}{2} \times 3m(3m+5m)$ $= \frac{1}{2} \times 3m \times 8m$ $= 12m^2$ Area of shape B $A = \frac{1}{2}h(a+b)$ $= \frac{1}{2} \times 3m(4m+9m)$ $= \frac{1}{2} \times 3m \times 13m$ $= 19.5m^2$ than $12m^2 + 19.5m^2$ $= 31.5m^2$ P = add all the sides $= 3m + 5m + 4m + 5m + 9m + 8m$ $= 34m$ than $31.5m^2 + 34m$ $= 65.5m$	Task 1 (Mensuration) $= 2(l \times b) + 2(b \times h) + 2(l \times h)$ $= 2(12cm \times 6cm) + 2(6cm \times 9cm) + 2(12cm \times 9cm)$ $= 2(72) + 2(54) + 2(108)$ $= 144cm + 108cm + 216cm$ $= 468cm^2$ So the minimum wrapping paper is $468cm^2$

Table 5-12 shows an illustration of how the calculator was used in a calculation to solve mensuration, geometry and algebra problems. Unlike the low performers, who used a calculator correctly but produced incorrect solutions due to using incorrect procedures, middle and high performers were able to find a connection between mathematical ideas, recalling their formulae and entering clear data in a calculator in order to produce an accurate answer (e.g. in task two they entered $\frac{1}{2} \times 3(8 + 3) = 19.5m^2$ and $\frac{1}{2} \times 3(5 + 9) = 19.5m^2$ to get $39m^2$ which is the correct answer.

In task one, they used a calculator correctly by entering $2(12 \times 6) + 2(6 \times 9) + 2(2 \times 9) = 468cm^2$. A lengthy calculation as in this case requires time to compute mentally. A calculator in this instance allows learners to spend less time on tedious calculations and more time on understanding and solving problems (Asare-Inkoom *et al.*, 2007). In task four, the calculator was used to aid with complex calculations (e.g. $\sqrt[2]{40\ 000\ 25} = 2000.0\ km.$) to find using mental calculation. This implies that their calculations were completely correct; the calculator was randomly used in the solutions to aid with complex calculations such as on square roots, for checking answers, and exploration of number concepts.

In task seven, a calculator was used for addition operations in a particular rule or pattern. This means that the calculator is used to make connections among mathematical ideas. With this connection they were able to get the desired answer. Several authors revealed that calculators make learners actively involved in their own learning, enhance learners' achievement in problem-solving, and assess their computational and numerical skills. Learners using calculators effectively exhibit greater self-confidence and generate more enthusiasm about mathematics (Asare-Inkoom *et al.*, 2007; Mereku *et al.*, 2007).

5.2.3.2 Reasons for calculator use

Some tasks required the use of technology such as using a calculator to assist with challenging problems. Throughout the task-based interviews, each participant was in possession of a calculator. Access to calculators does not negate the need for learners to develop paper-and-pencil and mental methods. Rather, when used appropriately, calculators play a key role in developing learners' fluency with numbers and strategies and estimation skills, improve learners' computational skills, influence their mathematical achievement positively and help learners by stimulating their problem-solving thinking (Papadopoulos, 2013). The grade 9 mathematics curriculum also encourages the use of calculators in the grade 9 mathematics

classrooms, urging that calculators must be integrated in a meaningful way while promoting mental computation, basic computational skills, estimation, and problem-solving (MoE, 2010).

Knowing how to use the calculator is one of the most basic skills to have in the mathematics classroom (Lin & Yuan, 2013). Learners seemed to benefit from the calculator use, due to its availability. It can also be inferred that if the calculator had not been available it would have been difficult for them to calculate the correct answer. Calculators help learners visualise problems and instantly check the validity of their answers and explore different ways of solving problems (Pomerantz, 1997). Calculators can also help learners to work faster, gain mathematical insight and value mathematics, and subsequently encourage mathematical understanding.

If a calculator is permitted, then learners must rely on their skills in entering arithmetic calculations into the calculator to answer the items correctly (MoE, 2010). This explains why it is vital for learners to be afforded the opportunity to explore and utilise a calculator during problem-solving. When used appropriately it may result in a better attitude towards problem-solving. It is useful to let learners with low confidence or low ability in mathematics use calculators to check their answers as the immediate feedback encourages them to check the accuracy of their computations frequently (Masimura, 2011).

In order to determine the best practices for calculator use, participants had to answer three questions from the task-based interview (see question 5 and Addendum A) as stated below:

- (a) Did you need to use a calculator for this task?
- (b) Where in the task did you use it?
- (c) Why did you use the calculator?

In this section the data on the calculator use will be discussed by describing where and why they needed to use a calculator. When asked if they needed to use a calculator, where and why they needed to use a calculator in solving mensuration tasks, participants' responses were the following:

Table 5-13 Reasons reported by participants for using a calculator

Low performing groups	Task 1 (Mensuration) Yes, I need a calculator so that I can find the area for the closed box which is 468 cm. To put my 3 sides together e.g. multiply $2(a+b)$; $2(9 \times 6)$; $2(12 \times 6)$ I use a calculator to find the correct answer.
	- Yes, I use the calculator - I use it in powers, when I add the answers together and I also use it to find the answer using a square root - I use it to get the correct answers Task 4 (Geometry)
	Task 6 (Algebra) Yes, I need the calculator to find the answer number of cherries and the number of oranges. I used it to find the highest common factor so that I can have the greatest number. e.g. dividing 30 by 2, 15 by 3, 5 by 5 or 24 by 2, 12 by 2, 6 by 3 and 3 by 3. Just so that I can get my answer fast and quickly
Middle performing groups	Task 1 (Mensuration) - Yes I need to use a calculator where I was calculating the total perimeter - I use it to get the correct answer easily
	Task 4 (Geometry) Yes, I used the calculator in this task. too I use the calculator to find the to subtract from distance to distance e.g. $20000^2 - 5^2$ I use the calculator to find the answer easily because the numbers are too big to work mentally e.g. 8000^2 and $20\ 000^2$ are too big

High performing groups	
	Task 10 (Algebra) Yes, I used the calculator in this task. to i I use the calculator to find the to Subracket from distance to distance e.g $20000^2 - 5^2$ I use the calculator to find the answer easily because the numbers are too big to work mentally e.g 8000^2 and $20\ 000^2$ are too big
	Task 3 (Mensuration) Yes, I need to use a calculator. I used it to multiply half πr^2 or $\frac{1}{2}\pi r^2$ and also to add $39.3 + 39.3 + 39.3$. That is to make my calculation easier.
	Task 4 (Geometry) yes, I ~~not~~ use a calculator. I use a square root. I don't subtract. because I want to get a correct answer because I sure I am not going to get a correct answer without a calculator
	Task 9 (Algebra) yes. to multiply the price of t-shirt with people number. also to subtract budget fee and ship fee $400 - 12.99$. It easy to calculate decimal number with calculator. for getting answer quickly.

All participants who indicated that they had used the calculator indicated that they had done so during mensuration, geometry and algebra problems. At algebra, some of the low, middle and high performers opted not to not always use a calculator, especially on tasks ten and eight. Some used a calculator in all tasks.

When asked to describe exactly where in the task they used a calculator, participants' common responses were to add, subtract and multiply numbers. When asked to explain why they used a calculator, participants' responses were categorised as follows: to assist with the calculation of large numbers; to get the correct answer; to get the answer easily and quickly; and to reduce time on spent on tedious parts of the task.

Most of the common reasons given by the participants on reasons for calculator use were that the calculator can give accurate answers quickly and easily. Other reasons such as to reduce tedious time spent on tasks and to aid with complex calculations are also valid reasons for using a calculator. A calculator was mainly used by the higher performance group in this study as a tool for computations because of its ability to deal easily and quickly with large numbers and complex data. This group often used the calculator in the geometry tasks, to aid with big numbers that are too complex to compute mentally (see middle performer example of task four). In the tasks related to mensuration, participants used a calculator to add simple calculations such as in the low performing participants' example of task one (mensuration). In general, however, the majority of participants used the calculator in the algebra tasks to get the answer easily and quickly.

The Grade 9 mathematics curriculum stipulates that the calculator should be introduced as a tool to deal with more complex calculations as well as irrational numbers, numbers in standard form, the value of trigonometric ratios and π . As illustrated in Table 5-13, at task three (illustrated under the high performing column), the calculator was used appropriately, especially when working with complex numbers such as π .

However, based on my observation, it appears that learners sometimes did not scrutinise the nature of the problem before they used a calculator, and this meant their problem-solving strategies were not always correct.

In my research most learners used a calculator for simple addition and subtraction tasks, and to get accurate answers quickly and easily and so reduce time. They did not always properly investigate the problem before they started using the calculator and did not always keep in mind that a calculator cannot replace the understanding and application of sound mathematical concepts and formulae. Unfortunately, the lower performance group struggled to use a calculator correctly, and also struggled at times to differentiate when to use it.

5.2.4 Participants' perceptions

5.2.4.1 Participants' perceptions of their ability before solving the problem

Choy and Cheah (2009) state that a perception depends on individuals' experience. Perceptions help learners build up their own knowledge and skills in order to help anticipate future happenings and deal with them appropriately. Attitudes towards solving problems can also influence the participation rate of learners (Farooq & Shah 2008). According to Gursen Otacioglu (2008:916) when "learners have self-confidence, they feel better within the process of learning and higher level of learning is accomplished". Confidence in this case means having the self-belief and determination to be able to solve problems. For the purpose of this study, I used the term "ability" to mean the capacity to solve unfamiliar challenging problems.

Participants were asked to predict how confident they felt about solving the problem after having seen it (Question 1) The problem was presented to the participants and thereafter the first question in the task-based interview was *Do you think that you can solve the problem correctly?* (see Addendum A). The responses to this question provide information about the participants' perceptions of their own ability in solving the problems. For each problem, the participants made a prediction regarding their possible ability in solving the problem by selecting an appropriate option from the following perceptions:

- (a) I am sure that I can solve the problem correctly
- (b) I am fairly sure I can solve the problem correctly
- (c) I am not sure how correctly I can solve the problem
- (d) I am not sure I can solve the problem. I think that I might make a mistake
- (e) I know that I will make a mistake in solving the problem
- (f) No selection

For the purpose of this discussion, the above perceptions were divided into four categories:

- (a) *I am sure that I can solve the problem correctly and I am fairly sure I can solve the problem correctly* - the classification will be referred to as: Confidence in own problem-solving ability.
- (b) *I am not sure how correctly I can solve the problem and I am not sure I can solve the problem. I think that I might make a mistake* - the classification will be referred to as Partial confidence in own problem-solving ability.
- (c) *I Know that I will make a mistake in solving the problem* will be classified as Low confidence in own problem-solving ability.
- (d) *No selection* will be recorded on its own.

All the perceptions for low, middle and high performing groups were combined and placed in the appropriate category as illustrated on Table 5-14 below:

Table 5-14 Learners' confidence in their own ability

Categories	Perception of own problem-solving ability	Low			Middle			High			
		Mensuration	Geometry	Algebra	Mensuration	Geometry	Algebra	Mensuration	Geometry	Algebra	Total
Confidence in own problem-solving ability	(a) I am sure that I can solve the problem correctly	5	2	6	7	3	10	6	3	17	77
	(b) I am fairly sure I can solve the problem correctly	3	1	4	2	0	1	4	2	1	
Partial confidence in own problem-solving ability	(c) I am not sure how correctly I can solve the problem	1	3	3	2	2	4	2	2	2	37
	(d) I am not sure I can solve the problem. I think that I might make a mistake	2	1	3	1	3	5	0	1	0	
Low confidence in own problem-solving ability	(e) I know that I will make a mistake in solving the problem	0	1	0	0	0	0	0	0	0	1
No selection made	(f) No selection	1	8	0	0	8	0	0	8	0	25
Total		12	16	16	12	16	20	12	16	20	140

Table 5-14 illustrates the number of responses by the different participants (low, middle and high performing groups, placed under appropriate categories; high, partial and low confidence in their own ability in solving problems. For each problem, the learners made a prediction regarding their possible achievement.

There were common perceptions among low, middle and high performing groups towards solving the mensuration problems. The participants in general were confident about solving the problems correctly. If one considers Table 5-15 which shows how many answers the participants actually did get right (I=Incorrect C=Correct) it suggest that the high performing group was fairly high on confidence regarding the belief that they can solve the problem, and they also performed best. Some responses (37 out of 140) indicated that certain participants had partial confidence in their own ability in solving mathematical problems. This implies that they were not sure if they could solve the problem. They also feared making mistakes during problem-solving. This could have contributed to the fact they were not as successful as they could have been. The participants who exhibited low confidence levels also did not do well.

5.2.4.2 Participants' reflection on their own ability after completion of the tasks

After the participants were given an opportunity to solve the problem, they were asked *Do you think that you have solved the problem correctly?* (see Addendum A). Five different options were given to the participants:

- (a) I am sure I solved the problem correctly
- (b) I am fairly sure I solved the problem correctly
- (c) I am not sure I solved the problem correctly
- (d) I am not sure that I solved the problem correctly. I think I may have made a mistake
- (e) I know I made a mistake

The options were divided into three categories:

- (a) *I am sure I solved the problem correctly* and *I am fairly sure I solved the problem correctly* – were classified as: Confidence in own problem-solving ability.
- (b) *I am not sure I solved the problem correctly* and *I am not sure that I solved the problem correctly. I think I may have made a mistake* – were classified as: Partially confident about own problem-solving ability.
- (c) *I know I made a mistake* was referred to as: No confidence in own problem-solving ability.

This question (question four) ties in with the first question, since the latter asked the participant how confident they were in solving a problem before they actually attempted to solve the problem. Question four asked the participants to indicate how confident they were that they had indeed solved the problem. The level of success in solving the problem is reflected by Table 5-15. The latter shows the comparison of learners with high, medium and low performing learners' abilities' with their actual achievement. Actual achievement in this case refers to whether the task was correct (C) or incorrect (I).

Table 5-15 Confidence in their own ability in solving tasks following their completion

Categories	Confidence in own ability	Low			Middle			High			Total		
		Mensuration	Geometry	Algebra	Mensuration	Geometry	Algebra	Mensuration	Geometry	Algebra	Mensuration	Geometry	Algebra
Confidence in own problem-solving ability	(a) I am sure I solved the problem correctly	C=3 I=1	C=1 I=2	C=7 I=2	C=4 I=0	C=2 I=0	C=8 I=0	C=5 I=0	C=1 I=0	C=15 I=4	C=12 I=1	C=4 I=2	C=30 I=6
	(b) I am fairly sure I solved the problem correctly	C=2 I=1	C=0 I=2	C=3 I=4	C=3 I=0	C=0 I=0	C=0 I=3	C=2 I=3	C=1 I=0	C=1 I=0	C=7 I=4	C=1 I=2	C=4 I=6
Partially confident in own problem-solving ability	(c) I am not sure I solved the problem correctly.	C=1 I=1	C=1 I=0	C=2 I=0	C=1 I=0	C=2 I=1	C=3 I=3	C=1 I=0	C=3 I=1	C=0 I=0	C=3 I=1	C=6 I=2	C=5 I=3

	(d) I am not sure I solved the problem correctly. I think that I may have made a mistake	C=0 I=2	C=1 I=1	C=1 I=1	C=0 I=3	C=1 I=1	C=1 I=2	C=1 I=0	C=1 I=1	C=0 I=0	C=1 I=5	C=3 I=3	C=2 I=3
Not confident in own problem-solving ability	(e) I know that I made a mistake	C=0 I=1	C=0 I=0	C=0 I=0	C=0 I=1	C=1 I=0	C=0 I=0	C=0 I=0	C=0 I=0	C=0 I=0	C=0 I=2	C=0 I=1	C=0 I=0
Total		C=6 I=6	C=3 I=5	C=13 I=7	C=8 I=4	C=5 I=2	C=12 I=8	C=9 I=3	C=6 I=2	C=0 I=0	C=23 I=13	C=14 I=10	C=41 I=19
Key: Correct (C): Incorrect (I).													

The data indicate that participants who showed that they were confident about their problem-solving ability generally got more tasks correct than incorrect. For the low and middle achievers who indicated that they were confident, algebra proved to be challenging since seven tasks were recorded as incorrect and only three were correct. For the participants who indicated that they were partially confident, there seemed to be a balance between tasks that were correct and incorrect across the achievement levels. For participants who were not confident about their own ability, more tasks were incorrect than correct.

The following figures (Figures 5-7 and 5-8) illustrate the participants' predictions about whether they thought they could do the task as well as the actual achievement.

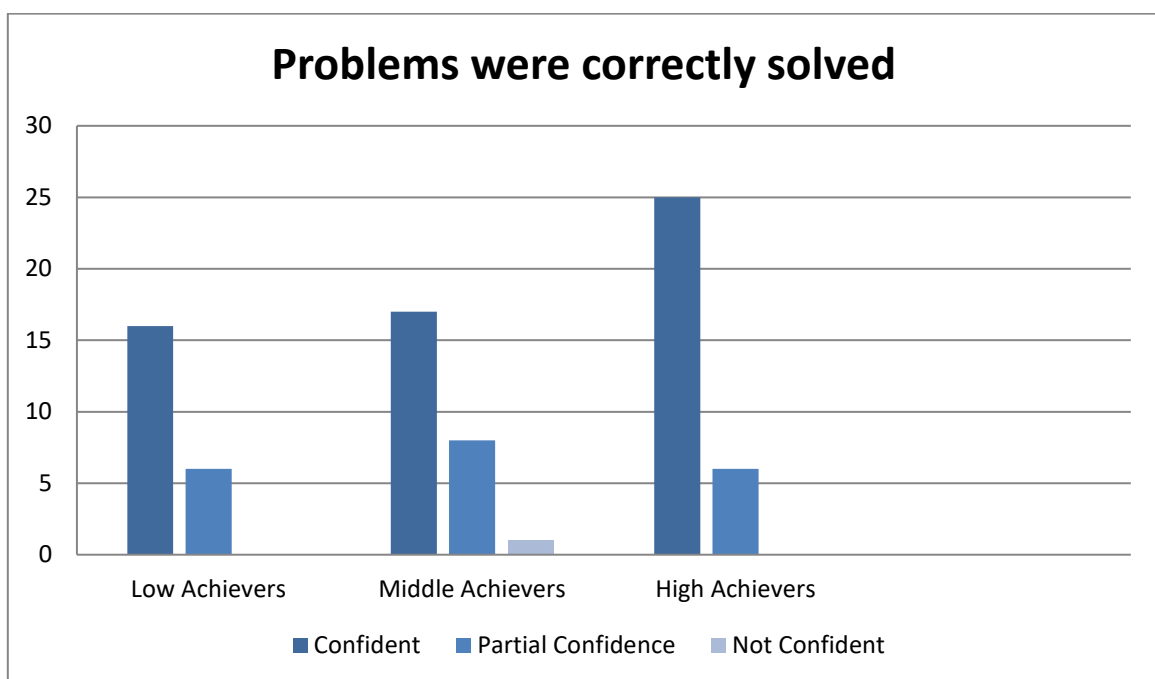


Figure 5-7 Comparison of low, middle and high performers' prediction and actual achievement

Figure 5-7 illustrates that the majority of participants in all performing groups (low, middle and high) who solved the problems correctly, were confident about solving mensuration, geometry and algebra tasks. This implies that the majority of low, middle and high performers who solved the problems correctly have high perceived confidence towards solving algebra, mensuration, and geometry tasks. Confidence is an individual ability which generally enhances motivation, making it a valuable weapon for learners who lack power to solve problems (Bénabou & Tirole, 2002).

This shows that there is a strong link between confidence and performance. In their study O'Shea *et al.* (2010) found that self-confidence plays a crucial role in learners' mathematics achievement and also influences their persistence at difficult mathematical tasks, and this also is the case in this study. Figure 5-8 indicates the spread of learners who could not solve the problems correctly.

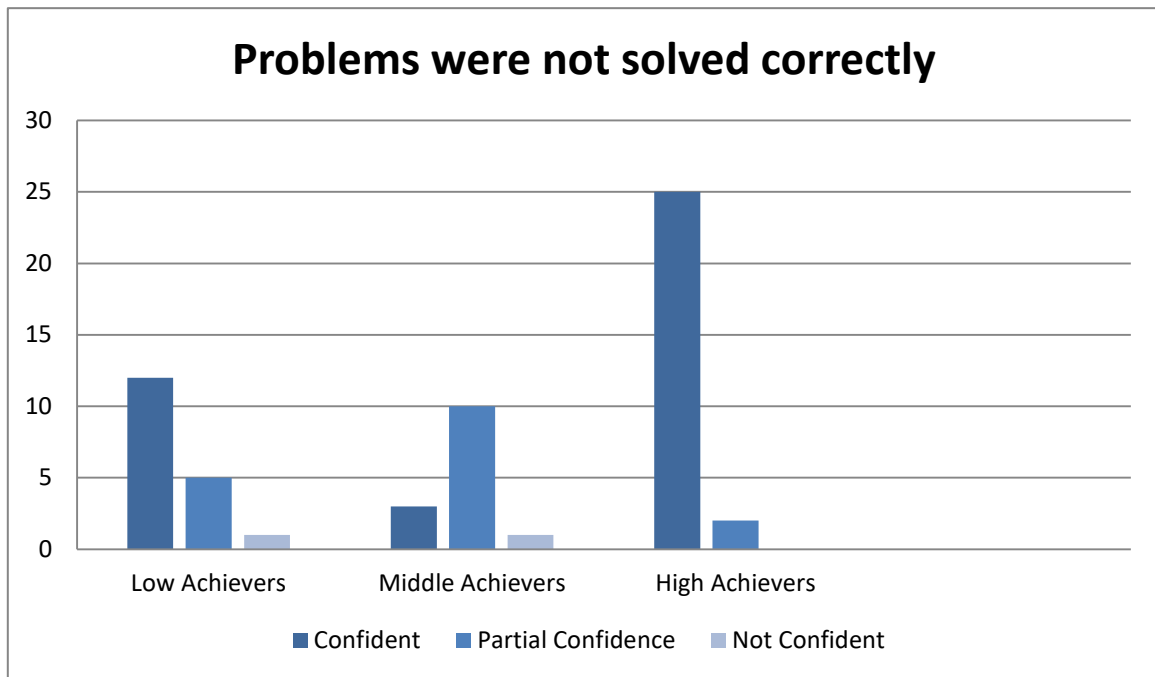


Figure 5-8 Comparison of low, middle and high performers' prediction and actual achievement

Conversely, Figure 5-8 gives us a different side of the coin since it focuses on the incorrect answers supplied by participants. Participants from the high performing group showed the greatest gap between being confident in solving the problem and actually being able to solve it. There were 25 responses indicating confidence by the high performing group as opposed to 10 by the middle performing group and just over 20 by the lower performing group in the segment that reflects the incorrect answers. So, some participants in the higher performing group had more than twice as much confidence in their abilities when compared to the other two groups, yet still made a fair number of mistakes when trying to solve the tasks.

5.2.5 Comparison of the groups

The comparisons between low, middle and high performing groups will be based on the problem-solving strategies used in mensuration, geometry and algebra tasks and the use of the calculators in the calculations.

5.2.5.1 Problem-solving strategies

Problem-solving is a matter of solving unfamiliar problems which have no immediate solution (see 3.3). The first step in Pólya's model regarding problem-solving, deals with understanding the problem. It became clear that at times some participants either understood the question but not the mathematical concept, or they did not understand the mathematical concept and/or the question, and this had a direct influence on their performance. It meant that some participants had no immediate solution nor a procedure that they could directly apply to get an answer.

If one considers Table 5-4 which shows the different strategies that the participants had used, it becomes clear that the most common strategies selected were using a formula or a rule, solving a simpler problem and making a diagram. The selection of strategies obviously played a part in whether a participant solved a problem or did not solve it. As Aydoğdu and Keşan (2014:54) state: "Problem-solving is the work of establishing a connection between what is given and requested in a problem". Establishing this connection correctly occurs with the help of strategies. This is reflected in the research since low performers often made use of irrelevant mathematical procedures and committed computational errors in solving problems (see Table 5-5, task eight). Some middle performers on the other hand, could not solve some of the problems correctly; due to not being able to recall the correct formulae (see Table 5-5). This implies that often the low and middle performers could not solve problems because the strategies that they selected did not lead them to the solution.

On the other hand, the majority of the high performers' levels of achievement (see Table 5-11), show that they were able to identify suitable problem-solving strategies for solving the problems correctly. Strategies such as using a formula and using tools and tables were commonly used by high performers effectively to solve problems. Although not all of their problems were solved correctly, there were positive signs in the data that they were able to solve most problems given to them with confidence.

5.2.5.2 The use of calculators

The calculator was commonly used by low, middle and high performer to aid with complex calculations such as square roots and Pi (π) which appear mostly in mensuration and geometry. Most of the complex calculations in mensuration and geometry required learners to use a calculator because the numbers were too large and complex to compute mentally. Certain formulae such as the surface area of a cuboid, circumference and area of a circle, and the Pythagoras theorem were a few of the formulae calculated using a calculator. The calculator was used for checking answers and exploring number concepts in geometry and algebra topics.

Unlike the mensuration and geometry tasks, the algebra tasks did not really consist of complex calculations. Participants had varied reactions towards calculator use in algebra. Some of the low, middle and high performers opted not to use a calculator. Participants were able to make a distinct difference between using and not using a calculator on some specific tasks. Most participants (low and middle) however opted to use a calculator in solving the algebra problems. The calculators did not hinder the learners' skills in arithmetic. Rather, most of the participants were positive about using it, and mostly explored different mathematical ideas to support their planned strategies.

5.3 Teacher's reflections

A teacher reflection is defined as the process of thinking about what happened during teaching and learning, and what might need improvement to make the lesson more effective (see 4.5.2). Teacher's reflections are a means of evaluating what a teacher has done in the classroom during teaching and learning. It is about thinking of reasons why certain situations happened or did not happen. Teachers' reflection also involves analysing the classroom situation by thinking in depth, from different perspectives, on what really happened during teaching and trying to explain in writing, often with reference to the lesson taught (Hampton, 2010).

I evaluated each of the ten lessons by using the teachers' reflection schedule (see Addendum B). This reflection schedule is based on the work by Hampton (2010) who suggests that there are three stages of reflective writing namely the description, interpretation and outcome phases (see 4.5.2). "Description" is where I explain what really happened during teaching and learning, and also explain the situations that I am evaluating. "Interpretation" is where I explain/list the different strategies used by both the learners and teachers in solving different tasks, and also ways of how learners use a calculator to solve the problem. "Outcome" is where I state whether the lesson was successful for the learners and suggest changes that could be made for future

lessons (see 4.5.2). I reflected on each lesson (geometry, mensuration and algebra) by writing my observations concerning the three structures as suggested by Hampton (2010).

The data will be organised within the lessons on the three content areas: mensuration, geometry and algebra. I completed the ten lessons after school; reflecting mainly on the lesson and the task-based interviews focusing on my teaching strategies, problem-solving strategies used by the learners in solving problems, how successful the learners were in solving the problem, use of the calculator in the problem-solving process and suggestions for future improvement.

5.3.1 Description

The first phase of the teacher reflections is based on describing what happened in the classroom/lesson during teaching and learning (see Addendum B). The description phase of the lesson involves the teaching aids and resources used, the duration of the lesson, the learning objectives, the themes and topics covered. During this phase, I was expected to write down in detail, what happened during the teaching and learning process and what was being examined (Hampton, 2010). It is important to consider the evaluation of what actually happens in class for future improvement and enhancement of the lesson.

The discussion in this category will be based on the description of the lesson (duration, topics involved, and nature of the tasks, teaching aids, classroom arrangements and the learning objectives of the teacher's lesson. Table 5-16 presents a general structure that was used on all ten lessons.

Table 5-16 General structure used in lessons

Component	Description
Time	40 minutes
Topic and theme	Mensuration (Area of a trapezium)
Teaching aids	Worksheet, calculators, chalkboard, compass, ruler, protractors
Classroom arrangement	Group work and individual

The topics for the mensuration lessons were: surface area of a cuboid, area of a trapezium and circumference of a circle. The learning materials which the learners used were calculators, rulers, protractors and worksheets. For all lessons, learners worked in groups during classroom activities and completed the task-based interviews individually. I performed this arrangement in order to establish how learners approach different problem-solving task with the use of a calculator, and the type of problem-solving strategies they use in solving problems when working in groups and individually.

I identified that the classroom environment also plays a major role in the way learners solve problems. This includes the physical environment of the class, the type of activities, the time, classroom connection and behaviour management. Time was a factor; each lesson was allocated only 40 minutes. Although I was aware of the recommendations to teach problem-solving, it was difficult to use the exact time frame for the lessons, due to the fact that it was the first time that I implemented problem-solving in a Grade 9 mathematics classroom. When I noticed that most learners took a lot of time on completing tasks, I decided to shift some of the lessons to the learners' study time, where we had enough time to complete some of the lessons.

The common classroom arrangement for all mensuration, geometry and algebra lessons was based on grouping learners in (two or three groups). Later in the afternoon I then gave them a task which was related to what they had learned that particular day. I did this since I believe group work helps to develop personal, social and communication skills, and also helps learners to learn from each other.

5.3.1.1 Mensuration

Before the commencement of the mensuration lessons, I rearranged the classroom in groups of two to three learners. The aim of grouping learners was to promote collaborative learning among learners. Also, problem-solving is constructed at the core of the interactions of the teachers and learners and is shaped by the skills and abilities valued in a particular culture of the mathematics classroom (see 2.2.2). As explained earlier, in chapter two, mathematics is viewed as a cultural element that is socially constructed by understanding the interactions between the development of mathematical thinking and the cultural context in which this development occurs (see 2.4); I segmented my teaching based on this theory of interaction in order to promote a classroom learning environment with understanding through the social interaction of mathematical facts and ideas.

Three mensuration lessons were presented to learners with the following learning objectives: The learners should be able to calculate the area of a trapezium; the surface area of the cuboid, and the circumference of a circle using formulae (MoE, 2010). Most of the mensuration classroom tasks were based on formula discovery. The tasks related to mensuration which I reflected on were tasks one, two and three (see Table 5-1).

When completing these tasks, the learners were given the opportunity to use any problem-solving strategy of their choice and, also, any mathematical tool which includes a calculator in order to solve the problem.

The tasks consisted of various questions guiding learners to the discovery of the solution by making use of area or circumference formulae. Instructions involved an explicit request for showing the solution strategy. Initially, learners worked individually and explored the problems by stating the relevant features and then solving each problem.

The following discussion refers to task one, where I describe what happened in the lesson/classroom before I gave them task one to solve. I rearranged the classroom in groups of two/three. Each group was provided with a cuboid box. I instructed each group to study the features of the box and asked questions (*What happens if you open up the box, Can you find the area of each rectangle on the box?*) to guide them into discovering the cuboid formula. Learners gave feedback on their discovery. Each group then received a worksheet on a task, based on finding the surface area of a cuboid. Later in the afternoon I gave them task one to complete individually in order to evaluate whether they were able to apply the strategies they had acquired during the lesson, with the help of the calculator to solve the problem.

The learners did not only work in groups; there was also an opportunity for learners to work alone. I noticed that when learners started working alone, they gradually started developing self-confidence, thinking on their own and experimenting without much help from other learners or the teacher. Hence, learners need to work individually and collaboratively. Problem-solving in mathematics is a complex process which requires an individual to coordinate previously taught mathematical knowledge, understanding and perceptions, in order to gratify the demands of an unfamiliar situation (see 3.2).

There were also factors that hindered learners' learning during problem-solving in the mensuration lessons. The first factor was the time allocation for the lesson. In most cases learners were unable to complete the task within the minimum range of time given per lesson (40 minutes). This was mainly caused by the lengthy investigatory tasks that required learners to discover/construct their own meaning in mathematics.

Time management is considered to be a necessary tool for educational improvement, especially in problem-solving. In order to deal with the problem, I decided to teach after school or during the weekend in order to fulfil the basic competency. Tools such as calculators were used widely to speed up learners' thinking and reasoning ability to complete the work and were also mainly used as an aid to complex and complicated calculations.

5.3.1.2 Geometry

Similarly, before the commencement of the geometry lessons, the teacher still rearranged the classroom in groups of two to three learners. Two geometry lessons were presented to learners with the following learning objectives: The learners should be able to "use a straight edge and a pair of compasses only to construct the perpendicular bisector of a line segment; apply the theorem of Pythagoras to prove that an angle is a right angle and calculate the third side of a right-angled triangle, if two sides are given" (MoE, 2010:16-17). The tasks related to geometry which I reflected on were tasks four and five (see Table 5-1).

These problems do not have a straightforward solution but require a good understanding and modelling of the situation that is, recognizing the strategy; tools, knowledge and skills to use in solving the problem (see 3.2). Learners explored the problem differently by drawing on the relevant features. They used tools (e.g. a ruler and a thread were used to assist with measuring the distance to determine the answer. Calculators were also used as the main tool to add and subtract numbers. Moreover, learners used the calculator to assist with calculating the square root of numbers when solving problems that involve the Pythagoras theorem. Other tools such as models were central to most of the geometry lessons. According to Kelly (2006), models such as rulers and threads must be demonstrated directly by the teachers in order to help learners see their significance and usefulness in problem-solving and communicating mathematically and should be continuously included as a part of an exploratory workstation or work time once open explorations have been completed.

One of the geometry lessons was based on discovering the theorem of Pythagoras and another on constructions of various shapes using real life situations. For example, to prove the theorem of Pythagoras (in one of the classroom activities) learners were instructed to measure the surroundings of their classroom by using a thread and a ruler, including the diagonal side of their own classroom to determine whether the classroom was square or not. They would apply the theorem of Pythagoras to prove that they apply the formulae to real life situations. In contrast with construction tasks, learners have constructed various lines and tried to connect them in order to determine a certain centre/location they are looking for before I give them (task five) to solve individually.

These activities were necessary for the learners to invite and activate the day to day knowledge and experiences during the teaching and learning, since the tasks were related to daily life occurrences.

5.3.1.3 Algebra

Algebra is one of the topics in mathematics that present a challenge to many learners (IES, 2015). The subject can be particularly challenging, not only because it introduces more abstract representations and more complex relationships between quantities (which require extensive abstract thinking) but also because it can extend the misconceptions which appear right from the onset before solving the problem (Booth *et al.*, 2014). Five algebra lessons were prepared, which required learners to explore the complex relationships between numbers and variables. I presented an approach for easy learning to solve algebra word problems by including word problems in order for learners to grasp the content easily. Tasks six, seven, eight, nine and ten (see Table 5-1) were the word problems that were presented to the learners.

The initial step for each algebra lesson was to devise a word problem. I asked different questions on the word problem and each learner responded. I later divided learners into groups of three to four, whereby each learner in the group received a worksheet consisting of the similar word problems. We (the learners and I) read the problem together in order to clarify all possible features of the question. Learners were given an opportunity to discuss the problem. As learners worked, I roamed around the classroom and gave hints, after which each group then gave feedback.

The algebra lessons were presented to learners with the following learning objectives: The learners should be able to: simplify expressions by applying the four rules of operations and the index rules for multiplication and division; solve simple linear equations where the unknown appears on both sides of the equation; construct and solve simple linear inequalities; and factorise expressions by taking out a common factor (MoE, 2010).

I grouped learners in pairs as usual. In order to acquaint the learners with the topic of algebra, I provided them with different algebra word problems on the chalkboard. Learners copied the problem from the chalkboard and read the problem within their groups. I asked learners to discuss, try to understand and write down what the problem was asking. Learners then responded to the teacher's questions in preparation for the main question. In addition, two problems related to the given story (word problem) were given. They worked together in pairs and gave feedback on the two problems.

Most of the tasks on algebra were based on solving initial problems based on a given story, in order to prepare learners for the main task. This is referred to by Van den Heuvel-Panhuizen and Drijvers (2014) as enhancement of learners' prior knowledge, which the teacher needs to do in order to be able to encourage learners to build on one another's ideas and to participate in conversations about mathematics. If well planned and implemented then, it may contribute successfully to solving problems relating to real life situations (see 2.2.4). I previously did do this, but this study has made me realise that I need to do much more.

5.3.2 Interpretation

The second phase of the teacher's reflection was based on reflecting on teaching strategies that I used, and on the strategies that learners used in solving the problems, and the description of how the learners used the calculator in the lesson. The questions that I answered in my reflection were:

- What teaching strategy did you use in the lesson?
- What strategies did the learners use to solve the problems?
- Describe the learners' use of the calculator in the lesson (see Addendum B).

One strategy that I used in all topics (mensuration, geometry and algebra) was to teach learners to solve problems using the four steps developed by Pólya. 1) Understand the problem, 2) devise a plan, 3) carry out the plan, and 4) look back (see 3.2). I reflected that in all topics I presented to learners, it appeared that learners were eager to learn a new method of problem-solving. There were challenges towards solving the problems - learners complained that the problems were difficult, *I have no idea on how to solve this problem, how do I come up with a strategy?* These were some of the learners' statements and questions when they received the tasks for the first time. However, as I started explaining the first four steps of Pólya and other strategies (drawing, constructions, solving a simpler problem-solving strategies) learners started becoming confident about solving problems. It seemed that the learners appreciated the fact that they had some framework or model to use to guide them. They felt that it gave direction and more clarity and offered some sort of formula or strategy which helped them during the problem-solving activities.

5.3.2.1 Teaching strategies

Mensuration

The initial strategy I have undertaken in the first lesson of mensuration was to teach learners the tools, processes, and strategies (four steps of Pólya) needed to solve any mathematical problem. Learners had not been exposed to solving problems. The Grade 9 learners are novices to problem-solving, and to help them become better problem solvers, I had to develop a stronger base of their conceptual and procedural knowledge, as Kirkley (2003) suggests, in order to assist with problem-solving (see 3.4.2). Pólya (cited by Breen & O'Shea, 2010:44) believes that if a teacher “challenges the curiosity of his learners by setting them problems equivalent to their knowledge and helps them to solve their problems with stimulating questions, he may give them some means of independent thinking”. I aimed to do that by teaching in such a way that learners felt more confident with using strategies and methods of their choice as well as using calculators that helped them to be more successful mathematical problem solvers and the self-confidence within, which acts as a flame to problem-solving.

My main role in teaching mensuration lessons was to give hints of what to do next, by asking prompting questions. Mensuration is one of the topics in mathematics that require numerous formula discoveries. I was consistently involved in asking questions (e.g. *Did you read the problem? What is your plan in tackling the problem? How will you solve the problem? How do you know that the solution is correct?*). I reflected that the strategy that I used in teaching mensuration problems was to explain different steps on how to solve the problem: Understand the problem, devise the plan, carry out the plan and look back. Once learners were comfortable using this strategy, I kept asking prompting questions in order for the learners to realise their own path to the problem. What is needed, therefore, is a new understanding of the problems of change and development in teachers' professional activities. The new shift in education is no longer about memorising facts and pictures, but rather, it is about learning where to find information (see 3.8).

Another strategy I used was the collaborative teaching strategy, whereby learners were asked to work on the problem in groups. In this case I kept circulating among the learners, listening carefully to the way they communicated in their discussions; observing the group collaboration and questioning the group about the strategy they were using.

Geometry

Teaching geometry well involves knowing how to recognise interesting geometrical problems and theorems. Jones (2003:125) is of the opinion that “geometry contributes to helping learners develop the skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof”. Most of the geometry lessons given to the learners were practical activities which required of them to prove and make conjectures about mathematical ideas. During teaching and learning, I introduced different teaching strategies to assist in solving geometry problems.

Common problem-solving strategies were, for example, using equipment (ruler, compass) and solving a simpler problem. I kept using different strategies of guiding and observing learners as they did their constructions by. I asked questions, made suggestions, and clarified the vocabulary in order to lead them into realising steps forward without giving them an answer, and observing their method of constructions and the use of tools (compass, ruler).

When describing the type of teaching strategies, I used in teaching geometry lessons, I noted the following: I used the strategy of guiding the learners through the constructions and instructing them what to do. Different tools such as compass, rulers and set squares were used to demonstrate and illustrate the constructions of various objects. The second strategy I used was collaboration among learners, whereby learners was given an opportunity to discuss the solutions and gave feedback to the whole class.

My role was that of a mentor, a guide, observing learners work by moving around to see whether learners were recording clear information correctly, assisting the struggling learners, asking prompting questions and leading them to give feedback. When a learner is solving a problem, he/she must undertake a reasoning process that involves understanding the problem statement, making a plan for the solution, applying the plan and making an evaluation (Akgün et al., 2012).

Algebra

My role here too, was that of a facilitator, who asked varied questions based on the situation. For example, in lesson seven in order for learners to get an overall idea of how to solve the problem, I asked prompting questions like: *Which strategies are suitable for this problem? Do you need a calculator to answer this question?* Questions of this nature are important in this regard because they develop a productive classroom conversation among learners themselves and the teacher (Ulleberg & Solem, 2018). When I pose a question, learners always try to give the answer; I then evaluate the answer, in order to give clear guidance to the learners.

My second strategy was using collaborative teaching which was based on the collaboration between me and the learners, and the learners with one another. My main role was to observe and guide learners into getting the correct solution to the problem. It is through this method that I got to know how learners cooperated to use their own knowledge to construct meaning.

5.3.2.2 Participants' problem-solving strategies

Mensuration

In most of the mensuration lessons, learners were arranged in groups of three to four. This arrangement provides learners with the opportunity to work in a social setting where they are comfortable to ask questions and can plan the solution of the problem (see 2.2.2). My goal was to maximise their understanding of the problem by involving myself in their learning.

As they were working on the tasks (task one, task two and task three), I observed that learners used the following problem-solving strategies to solve the problem: solving a simpler problem and using a formula to solve the problem. When learners received a problem, they first wrote a formula, then substituted in the missing variables, broke the formula into parts and found the solution. Formulae such as $A = \pi r^2$; $C = 2\pi r$ or $C = 2d$; $SA = 2(lb) + 2(bh) + 2(lh)$, were used to solve area related questions. For example, in task one I noticed that most of the learners did not use a correct formula for the problem. Learners were expected to find the amount of wrapping paper needed to cover a closed gift box, but they did not notice that they were dealing with finding the surface area; instead they used a volume formula which is $V = l \times b \times h$. The correct surface area of a gift box is $SA = 2(lb) + 2(bh) + 2(lh)$. I think this is one of the misconceptions underlying the concept of mensuration, since I noticed that learners were unable to link concepts with each other, for example volume and surface area. In order to curb this problem, I designed activities that engaged learners in deriving these formulae on their own through investigatory tasks.

Geometry

The activities that I prepared for the geometry lessons required learners to work together by moving around the classroom, measuring, recording what they measured, calculating and directing one another. I divided learners in groups of three; each member of the group was assigned a different responsibility in solving the problem. The problem required learners to find out whether their classroom corners were built in the form of a square or not. Some group members were responsible for measuring, some for recording, and the rest for calculating.

I assigned different tasks to each member, therefore each member had a role to play in solving the geometry problem. I noticed that each individual was contemplating on different responsibilities within their groups. In previous lessons learners did not do that, the activities that I had designed required them to sit, discuss and give feedback. I am of the opinion that learners were not active and successful in solving especially mensuration tasks because they were not involved in guiding and assisting one another in coming up with the solution to the problem.

When I reflected on the learners' problem-solving strategies that they used in solving geometry tasks (task four and task five), I felt that the learners were at the centre of their own learning, formulating their own strategies such as using a drawing, constructing and solving a simpler problem strategy and using tools. This is because during group work, they worked together in order to discover the theorem of Pythagoras. In most cases they were motivated by the activity itself, which prompted them to solve their problems correctly.

During the geometry tasks the driving force of the group was their collaboration, motivation and perseverance. In order for learners to realise their full potential in solving problems, teachers need to assist them in constructing new connections between mathematical ideas by asking questions, assigning responsibility to enable learners to review their knowledge (see 3.2).

Algebra

When I reflected on which strategies learners used to solve the algebra problems, I realised that learners used readily available formulae, especially in substitutions (see task eight) which they used to replace an unknown value in order to get the solution. Learners also constructed/formulated their own formula from problem statements and applied the derived formula to solve problems (see task nine and task ten). Some tasks required visual representations (see task six); this task required learners to take out a common factor. The question is based on finding the greatest number of trees each row can have. Most learners answered this problem correctly using visual representation. Figure 5-9 presents how one learner represented the situation.

Firstly, they found the highest common factor of 30 and 24 which is six. Divide each number (30 & 24) by six. Then make a drawing of six squares. In each square they represented five oranges and four cherries. I was not expecting this type of results, due to the fact that learners generally find algebra challenging. However, most of the lessons I presented in algebra involved representing a word problem which they were required to solve using a drawing to represent the information.

By then most learners were used to problem-solving, I am of the opinion that with enough knowledge on problem-solving it became easier for them to produce a correct solution. Lastly, they also used a table to represent the information, especially in task seven.

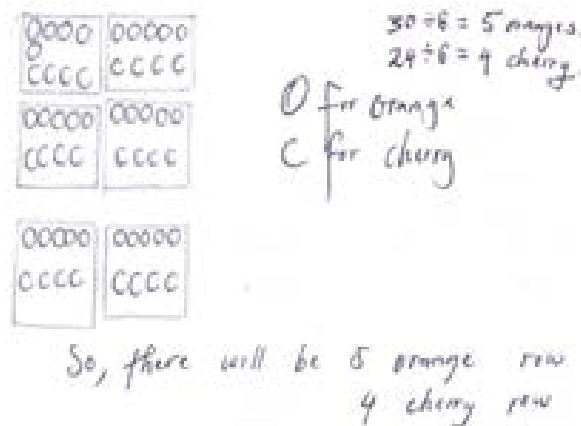


Figure 5-9 Example of learner's work

5.3.2.3 The use of calculators in the lessons

Mensuration

Throughout the process of teaching mensuration tasks, learners used calculators.

Calculators were designed to support learners in solving problems of different levels of complexity, including much more powerful ways than merely providing numerical answers to arithmetic questions. To date, calculators continue to be one of the few technologies designed particularly for problem-solving purposes (Kissane & Kemp, 2012).

Most of the tasks on mensuration that I designed consisted of larger calculations, which were too complex for learners to work out mentally. For example, complex calculations were involved especially in finding the relationship between the circumference and diameter of the circle (see task three). Learners used the circumference formula ($C = \pi d$) (see 5.2.2.2, Figure 5-5) substituted in a place of the diameter, and used a calculator to calculate Pi with the diameter, so they also used the calculator during complex calculations. However, the calculator was mainly used to add, subtract and divide numbers.

When reflecting on learners' doing task two, I realised that the calculator was mainly used to add, divide and multiply numbers (see Table 5-11). A learner breaks the shape into two parts with the aim of finding the total area of two shapes (Area A + Area B), they use a calculator to add the values together. Some learners used the calculator successfully and got the answer quicker and more easily.

Other learners could not interpret the results on the calculator correctly and misinterpreted the answer. This is problematic to learners' thinking ability since they see any answer produced by the calculator as final and correct.

The calculator is a tool to do calculations and so are the human minds. Learners who are taught the appropriate way to use a calculator are more effective in solving problems.

Geometry

The two topics that were presented in geometry were based on the theorem of Pythagoras and constructions. When I reflected on the learners' use of the calculator, I realised that the calculator was not needed in the construction tasks, especially in tasks that involved constructions of lines and figures (see task 5, Table 5-1). However, the learners used the compass, rulers, protractors and pencils to bisect lines and different figures. The calculator was widely used in the topic of the Pythagoras theorem, randomly to aid with square roots of numbers.

Learners should be aware of how and when it is necessary to use a calculator for given calculations (see 3.7.3). Topics such as constructions, does not require the use of a calculator, but rather the use of equipment such as rulers, compass, etc. to solve them. In Pythagoras' theorem the calculator was an effective tool because it was mostly used to verify and connect mathematics to real-life context. Throughout the experience of teaching using a calculator, I noticed that learners who used the calculator appropriately considered whether it was necessary to use the calculator or rather to use mental calculations. Learners who provided answers quickly made a quick preliminary estimate of the answer, to check their work and sometimes they were able to interpret symbols such as square roots accurately.

Algebra

With regard to the algebra, I realised that some learners did not use a calculator in some of the algebra tasks, especially in task seven and task eight (see Table 5-11). Perhaps this was the case because task eight and task ten required learners to list the variables or expressions involved, and link them together to find the solution. I noticed that it was easy for the learners to produce an expression, especially in task ten, where they are required to produce an expression and substitute. For example, subtracting $7 - 5 = 2$; $7 + 11 = 18$; $7 \times 7 = 49$; $49 - 3 = 46$ is some of the simple arithmetic calculations that learners worked on mentally. There are also some learners who worked mentally and verified their answers with the calculator. This is also a good practice since the calculator is effective when used in verifying the answer.

I also realised that the learners who used a calculator on these tasks did not produce a correct answer especially on task eight. The correct answer for task eight is $7(n - 2) = 7n - 14$, but some learners went further to subtract $7n - 14$ with a calculator and got $-7n$. I concluded that this type of error is caused by lack of procedural and conceptual skills. Learners did not recall the rules of algebra that state that unlike terms cannot be subtracted in algebra. It is clear that in algebra problem-solving, learners tend to use a calculator without looking back at the procedure. For the rest of the tasks the calculator was used throughout algebra tasks to put numbers together effectively to promote learning and problem-solving.

5.3.3 Outcome

5.3.3.1 Learners' success in solving the problems

Mensuration

Surprisingly, not all learners managed to grasp the concept of area right away in most mensuration tasks. Weak learners specifically struggled to realise which strategies were appropriate for the applicable problem. The most significant idea through these lessons of discovery is that some learners were able to see the connection between and within the formula and the application of this formula in real world context. However, there were learners who had difficulties understanding and retrieving concepts, formulae, facts and procedures and lacked the ability to visualise the mathematics problems. When I reflected on how successful the learners were in solving the problems, I thought that only some learners were successful in using the four steps of Pólya in solving problems. In some cases, they spent time listing the problem features as instructed, but they were not able to connect what they had discovered to produce a meaningful solution. They would look at their solution and ask themselves questions to verify whether they had solved the problem accordingly.

For example, the majority of the low performing group were unable to discover the area and circumference formula, some were not able to apply the discovered formula and apply it to real life problems. The common contributing factor of this failure was the inability to read and understand the problem. Problem-solving is a difficult process, having learners do it for the first time was difficult for them to do. Learners were uncertain and did not always know what to do next, and in many cases I heard learners say from one group to another: *this is difficult, is it possible to solve this problem? How am I supposed to get the answer?* This required them to read the problem numerous times. I had to intervene in most of the mensuration tasks by giving clear feedback, and the learners then realised what they could have done to avoid their errors.

Geometry

Learners had already been exposed to solving problems by the time they did the geometry problems. In the geometry lessons, I paid particular attention to creating a supportive environment whereby learners were successful, in the sense that some of them were able to recall and apply the Pythagoras theorem in the real-life situations provided to them. They could read the problem on their own, understand the problem features and create appropriate mathematical connections. However, there were a few cases of some learners who were confused about finding the short side of the right-angled triangle (see task four). When I reflected on the outcome of the lesson, I thought that the learners were successful on the one side and also unsuccessful on the other. Only a few learners solved the geometry problems incorrectly because they were confused about the application of squares on the shorter and longer sides of the right-angled triangle. In the previous lesson, learners were not able to recall the formulae and apply them in real life situations correctly.

For example, one common strategy learners had for solving problems involving the Pythagoras theorem was that if they wanted to find a longer side of the right-angled triangle they needed to add their two short sides $a^2 + b^2 = c^2$; but again if they were asked to find a short side, they would still add instead of subtracting ($c^2 + b^2 = a^2$). This might imply that learners who have better reasoning skills have more opportunities to think independently and are able to become better problem solvers (Kelly, 2006).

Algebra

Learners were fairly successful in solving algebra tasks, especially tasks that involved visual representations such as drawings. As I circulated during the lesson, I noticed that most learners made use of drawings in order to represent the information in a problem. For example, when learners solved task six, they represented the problem in many ways, firstly by means of drawing and secondly by solving a simpler problem. This is a clear indication that learners are able to use two or more strategies to get to the same answer. Therefore, it concluded that the reason for most of the success in solving algebra problem was the learners' ability to apply the knowledge that they had discovered. Some factors were that learners used correct strategies and procedures that lead to successful situations.

There were some specific topics in algebra that learners found difficult. In algebra learners found it difficult to work with variables. The difficulty of understanding the use of letters to indicate unknowns and the complexity involved in transforming the verbal formulation into an equation was problematic for learners.

For example, learners would solve the equation $2x = 5 + 4x$ by adding the x values (like terms) together $2x + 4x = 2x$, forgetting that when putting like terms together the sign changes from positive to negative. So too did learners make errors when working with inequalities - they did not understand the sign change when working with inequalities. I therefore gave additional homework in this regard for learners to practise and understand the relations in different mathematical ideas and situations.

5.3.3.2 Suggestions for future improvements

Mensuration

As I monitored the classroom, I noticed that some problems on mensuration tasks were not solved correctly due to a misconception between using the area and circumference formulae. However, I also noticed that those that used a correct formula knew how to solve the mathematics problems, but they solved the problem incorrectly because they did not know how to use their calculators properly. The way they displayed the data on the calculator, enabled them to get an incorrect answer in most cases. For example, a learner would type in $\pi 9^2$ instead of $\pi(9^2)$ or $\pi \times 9 \times 9$. Clark (2011) reports that if learners are allowed to use a calculator without developing an understanding of the mathematical calculations of the topic, they will produce incorrect solutions to the problem. The main concern is that both of these concepts were explored by learners through investigatory activities.

In future I will still give the same task (especially tasks based on discovering formulae) but with different questioning techniques that allow learners to grapple with the content in quest of the solution. Having analysed learners' pace and capability of learning, I noticed that when learners engaged with the content more, they remembered that content better when applying it to the future problems. Lambdin (2003) is of the opinion that if learners are given freedom to discover their own learning, then they will be able to use the same ideas on any unfamiliar mathematical problems, but if those ideas are poorly understood, then they will be incorrectly applied and lead to incorrect solutions. I noticed that mostly learners are prone to making mistakes owing to the lack of practice on previous material. Homework can be an essential form of practice which allows learners to have an opportunity to practice and review materials that were presented in class (see 2.5.2). I will consider giving relevant homework activities in order to pace the understanding of learners on the taught concepts. Furthermore, I have learned that larger groups promote a culture of social learning whereby weak learners also get a chance to contribute and learn from their mistakes. I will however keep mixing and matching the learners based on their ability and determine the correct size and arrangement of the groups.

Geometry

The previous lessons proved that groups are effective in the sense that, when learners collaborate in groups, then they achieve better than those who work individually. But I also noticed that when learners worked individually, they were forced to think for themselves and when they solved a problem correctly, they felt it is their own efforts and confidence. I personally think that, when a learner is allowed to grapple with the content without consistent input of the teacher or other learners than, (in other words the learner is also allowed to work independently at times); he/she can easily remember the same content when approached with the same problem in the future.

The majority of learners came to appreciate problem-solving when they were given the responsibility of solving problems with a specific task in mind. Problem-solving is a task that is difficult for a learner to solve alone, without guidance. Teachers should then allow learners to work in groups during problem-solving situations because it helps them to evaluate situations, to ask questions, check for accurate results and use different strategies (Atteh *et al.*, 2012). In future I will keep assigning tasks that involve both groups and individuals.

Algebra

In future I will keep giving algebra word problems, but will consider the possible misconceptions first before planning a lesson. Learners' inability to recall algebra rules were of great concern in my first attempts of the lessons. Misconceptions such as misinterpretation of the change of negative sign to positive sign, inability to recall algebra rules, failure in identifying similar terms are some of the few I noticed in learners' work. Schwieger (2003) suggests that the primary sources of difficulty in solving problems in mathematics are mainly caused by the complexity of the algebra language and terminology, textual and written materials, and learners' attitudes and expectations towards learning.

Secondly, I will also acquire necessary knowledge (teaching strategies, procedures, subject knowledge) required for effective learning. I have noticed that algebra is a topic that requires not only rules and comprehensive procedures in order to solve them correctly, but also requires comprehensive mathematical content knowledge. In order for me to progress in teaching problem-solving successfully through a variety of methods, I should acquire additional teaching pedagogical and content knowledge (CK) of the Grade 9 mathematics content knowledge. CK refers to the basic mathematics knowledge possessed by the teacher (see 2.2.4). This knowledge determines the teacher's mathematical literacy. Researchers emphasise that content

knowledge in mathematics is an important construct that can either support or hinder progress toward exemplary classroom instruction (Harel, 2008).

Lack of understanding in mathematical areas such as algebra is a result of teachers' lack of content knowledge within this subject area. This implies that teachers who do not acquaint themselves with mathematical content are not likely to have the knowledge they need to help learners succeed in problem-solving. In future I will engage in designing and creating tasks that are based on practising the rules of algebra. Homework is also essential, in this case for learners, to practise learned skills and become better problem solvers. Central to effective improvement in the life of the teacher is the development of consistent mathematical content knowledge which ranges from the content (including procedure and technique, representations and connections) to thinking processes (learners' problem-solving strategies; mathematical point of view; circumstances of performance and pedagogical knowledge).

5.4 Conclusion

The analysis of the two sets of data, the problem-based interviews and the teacher's reflection, offered valuable insight into the research questions that this study aimed to answer. It shed light on how participants approached problem-solving strategies, which strategies they used and also what part confidence plays in the whole process of problem-solving activities and strategies. It also dealt with the participants' use of calculators when it comes to problem-solving strategies. The teacher's reflection was also very useful and highlighted the need for group work and individual work as well the role of the teacher as guide and facilitator. The data and the consequent analysis of the data also helped to point towards future practices and areas that can be improved. The following chapter presents a summary of the study, conclusions and recommendations.

CHAPTER 6 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

This last chapter summarises the findings of the study. An overview of the investigation is provided and the research questions are addressed. Conclusions for the study will be presented and the contributions and limitations are discussed. The chapter concludes with suggestions for future research and a brief discussion on the role of the researcher.

6.2 Overview

Chapter One presented an overview of the study. It presented the problem statement, and the research questions and the aims for the study were formulated. A brief literature review was presented to contextualise the study within the existing research, and the gaps in the literature were pointed out. I presented details regarding the research design, methodology and methods that would be followed in the study. In this section I provided more information about the philosophical framework, the research methodology, the sampling strategy, methods of data analysis and the ethical considerations.

Chapter Two and chapter Three presented a review of the literature. Chapter two contained a description of the learning of mathematics which focused on the following essential components of the study: the nature of mathematics, the learning of mathematics and the role of the mathematics teacher in the learning of mathematics. The nature of mathematics is based on various definitions of mathematics, different views about mathematics and views of mathematics as problem-solving. Secondly, a brief discussion was presented on the learning of mathematics, based on how mathematics is learned, the social-cultural aspect of mathematics, the learning of mathematics and the learning of school mathematics. Thirdly, the literature further explored the role of the mathematics teacher in the learning of mathematics which includes social environment, administrative environment, teachers' views about the learning of mathematics and the use of technology in the classroom.

Chapter Three continued the discussion of the literature on problem-solving in mathematics. This included a discussion on the definition of problem-solving in mathematics and started by exploring the historical development of the definitions of problem-solving. This was followed by a discussion on teachers' views on problem-solving, teachers' and learners' beliefs with respect to problem-solving, the role of problem-solving in school mathematics, how to solve a mathematical problem, and problem-solving strategies.

I also explored the literature regarding the teaching of problem-solving, discussed the teachers' role in the problem-solving classroom, and investigated problem-solving tasks. In this chapter I also looked at the use of technology in mathematical problem-solving. I reviewed literature concerning the use of computers and calculators in the mathematics classroom and briefly considered the role of language in mathematical problem-solving.

Chapter Four detailed the research design and methodology for this study. The aims and objectives of the study were given, followed by a detailed description of the research design and methodology. In this section I discussed the philosophical framework, gave an explanation of qualitative research methods and discussed the case study design. I also explained the sampling strategy that I used as well as the method of data generation. This included an explanation of how data were generated through task-based interviews as well as teacher reflections. I explained my role as researcher within the qualitative design. I addressed issues of trustworthiness and described how the data were analysed. I also discussed the ethical considerations for the study.

Chapter Five presented the analysis of the data. This analysis focused on the task-based interviews and the teacher's reflections. When discussing the task-based interviews, I started by presenting the tasks used in the task-based interviews. This was followed by the analysis of the tasks based on Pólya's model. I explored the problem-solving strategies that the participants planned and implemented, the reasons for making mistakes as well as the use of the calculator throughout this process. This was followed by a discussion of the participants' perceptions of and reflections about their problem-solving ability. The final section in the discussion of the task-based interviews presented a comparison of the different performance groups in terms of their problem-solving strategies, and their use of calculators. The chapter continued with the analysis of the teacher's reflections which was analysed under the following themes: description of the lesson, the teacher's interpretation of what happened in the classroom and finally a discussion of the outcome of the lessons.

6.3 Addressing the research questions

From the results of the investigation the following findings were reported with regard to the use of calculators in problem-solving activities in a Grade 9 mathematics classroom. The findings were synthesised in order to answer the following research sub-questions.

6.3.1 Which problem-solving strategies are used in Grade 9 mathematics classrooms?

6.3.1.1 Task-based interviews

Pólya's model was used to analyse and interpret the task-based interviews. This model consists of four steps: understanding the problem, planning a suitable strategy, carrying out the plan and looking back or reflecting on the problem. Participants were given ten mathematical problems to solve in algebra, mensuration and geometry. They were also given seven task-based interview questions to answer for each of those tasks.

Question one of the task-based interview focused on the first step, question two focused on the second step. Questions three and seven focused on the third step and questions four, five and six focused on the last step.

The participants first perused the problem and then had to answer the first question and indicate their level of confidence with regard to solving the problem. Most participants seemed confident that they could solve the problem and successfully complete the tasks, especially when it came to algebra (see Table 5-14). This suggests that they felt they understood the problem. With the mensuration tasks some participants were confident, but some confusion also surfaced because they found the new problem-solving teaching approach difficult to come to grips with, and they also found some of the tasks complex. Table 5-14 details the participants' responses.

Table 5-5 illustrates the different responses from the participants with regard to the reasons for the mistakes they made in the process of trying to solve the mathematical problems. A large percentage of the participants failed to solve the problem because they failed to understand the mathematical concept that the question contained. This goes straight back to the first step in Pólya's model – understanding the problem; most participants who made mistakes struggled with understanding the mathematical concept found in the question.

Question two of the task-based interview addressed the planning of a strategy to solve the mathematical problem. An important part of having a proper strategy is to first determine what you know and what you don't know about the problem, and this is often done by writing this down. This is especially helpful in algebra. However, most of the participants did not do this.

It was encouraging to see that all the performing groups attempted to find solutions to the problems by using various planned strategies. Participants described the strategies that they planned to use and all the performing groups were able to do this. When it came to planned strategies at mensuration, for instance, all the participants indicated they would either use a formula or solve a simpler problem. However, when it came to the actual implementation, participants also used other strategies such as comparing and using a theorem (see Table 5-4). Using a formula or solving a simpler problem remained the most popular options, but not the only options that were eventually used. It does, however, indicate that the majority of the participants followed through with their initial strategies.

Questions three and seven of the task-based interview, dealt with the actual carrying out of the plan and the actual application of the problem-solving strategies. Table 5-4 gives a very clear picture of which strategies the participants used. For algebra the most popular problem-solving strategies were: making a diagram, followed by using a formula or rule and solving a simpler problem. Comparing and using tools were not utilised at all as problem-solving strategies in algebra. At mensuration the most popular strategy seemed to be using a formula or rule, with solving a simpler problem a close second. Using tools and drawing a diagram were not used at all. In the geometry section, using a theorem and making a diagram proved to be popular, with comparing or using a formula or rule and solving a simpler problem not being used.

A rubric was drawn up to measure how successful the participants had been in solving the mathematical problems (see Addendum C). Table 5-15 reflects whether the participants successfully completed the tasks or not. Participants in the high performing group on average did better than the participants in the middle and low performing groups. This group seemed to find the mensuration tasks to be the most challenging. The middle performing group was fairly successful, but struggled a bit with algebra, while, contrary to popular belief, geometry seemed to be the area that they did the best in. The low performing group found geometry and mensuration difficult and coped better with algebra, but in general did not do that well.

Questions four, five, six and seven of the task-based interview focused on reflection by the participants regarding the problem-solving strategies they used. Question four asked the participants to indicate, after they had done the tasks, how confident they were in their belief that they had indeed completed the tasks successfully and thus solved the problems. Table 5-15 indicated the confidence levels of the participants as well as if they had indeed used the correct strategies at the respective questions. Across all three performing groups (low, middle and high), participants who felt confident that they could solve the problem (their answer to question one) did indeed manage to use the correct strategy (see Table 5-14).

The group that indicated they were partially confident did fairly well, but not as well as the confident group, and the group that was not confident at all did the worst (see Table 5-14). This could be an indication that confidence boosts learners' problem-solving abilities; it could also indicate that learners know and understand if they grasp a problem, and their confidence is directly linked to this.

Question six specifically asked the participants to indicate reasons why they did not complete the mathematical task correctly, and their answers were classified into three categories, they did not understand the question, or did not understand the mathematical concept, or both. (see Table 5-6). The most common reason for failure seemed to be an inability to understand both the question and the concept. The lower performing group also had the highest number of failures with regard to solving the problem, with the higher performing group being the most successful in solving problems.

Question seven of the task-based interview asked the participants to explain which strategies they actually used. This seems to suggest that they had chosen and implemented reasonable strategies in solving problems; however, there were exceptions. At task five for instance it was clear that many participants did not use the initial strategy that they had planned, but devised additional strategies to solve the problems. Question five of the task-based interview asked the participants about their use of a calculator; when and where specifically they did use the calculator. The discussion of their responses we dealt with in 6.3.2.

The analysis of the task-based interviews seemed to indicate the following that confidence has an influence on performance, and that most learners plan a strategy before attempting a problem; however, in an area such as algebra (where the actual writing down of the strategy is very helpful), this does not always happen. Some strategies are more popular than others when it comes to the different areas of mathematics, and some strategies are not used at all in certain areas. Different performing groups seemed to find different mathematical areas difficult, and the high performing group on average did best of all the performing groups. Reflection is useful and helps participants to think about reasons why they were not successful in performing certain tasks. The main reason for the latter seemed to be a lack of understanding of the question and the mathematical concept. Finally, most participants used the actual strategies that they had planned to use, with minor exceptions at certain tasks.

6.3.1.2 Teacher's reflection

The second research instrument that I used was the teacher's reflection. This involved the process of reflecting on (thinking about) what occurred during teaching and learning, looking at possible improvements and ways in which to increase effectivity (see 4.5.2). Basically, this comes down to three different stages of reflective writing, namely the description (explanation of what happened), interpretation (explanation of different strategies by the teacher and learners), and outcome phase (a statement of success of the lesson and possible future changes) (see 4.5.2). I did this at each lesson in geometry, mensuration and algebra.

Description

I introduced an approach in my classroom that I had not used before - the problem-solving approach. I used Pólya's model (which consist of four steps) to do this. I let participants work as individuals and in groups. I used different strategies and played an active part as facilitator, guide and teacher. I also paid attention to the strategies that the learners used, and gained some valuable insight in this regard.

I noticed that the classroom environment also plays a significant role in the way learners solve problems. Aspects such as the physical environment of the class, the type of activities, the time, classroom connection and behaviour management all form part of this. Time management was a challenge since it was the first time, I introduced problem-solving, and it took longer than I had anticipated, and additional time was needed. There were challenges: time allocation (as mentioned before), and participants had to come to grips with a new approach (problem-solving) to learn mathematical concepts. During all the lessons with all the different topics I tried to act as mentor and guide, and as teacher, alternating between guiding and teaching.

Interpretation

The second phase of the teacher's reflection was based on reflecting on teaching strategies that I used and the strategies that learners used in solving the problems. The description of how the learners used the calculator in the lesson will be discussed at the second sub-research question.

One strategy that I used at all topics (mensuration, geometry and algebra) was to encourage learners to solve problems using the four-step model developed by Pólya. 1) understand the problem, 2) devise a plan, 3) carry out the plan, and 4) look back (see 1.2.2). I grouped learners in two to three different groups, but at other times they also worked on their own. When they

worked on their own, it led to increased self-confidence and they managed to do more work on their own, while group work helped them with exchanging ideas and learning from each other.

At one of the geometry lessons, discovering the theorem of Pythagoras, I realised that the learners found it very helpful if I activated their everyday knowledge (real-life knowledge) and experiences when teaching certain mathematical concepts. During the algebra lessons I devised real-life problems; I then asked different questions related to the problem, and asked learners to respond individually. They also had to discuss and write down what they thought the problem entailed. They found this very helpful.

During the mensuration lessons I decided to first teach learners the tools, processes, and strategies (the four steps of Pólya) which were needed to solve any mathematical problem. I did this because learners at that stage had not been exposed to problem-solving, and this would help them to become better problem solvers. It would also assist in developing a stronger base for their conceptual and procedural knowledge and help them to use calculators more effectively. When doing mensuration, I also often used the strategy of asking prompting questions in order to supply them with some hints, since mensuration involves the usage of the correct formulae. If I saw that they had used the wrong formula I tried to design an activity that would guide them back and make them investigate again.

During the mensuration lessons I noticed learners first wrote down a formula, then substituted in the missing variables, and then went ahead and broke the formula up into different parts in order to find a solution. Here, at the mensuration task, I also had to intervene quite often in order to supply feedback and guidance.

At geometry I sensed that having a supportive environment really assisted the learners in solving the mathematical problems. At task four and five (geometry) specifically, I saw that learners formulated their own strategies by working together and trying to find solutions together. During the geometry lesson the group's collaboration and motivation helped them to find solutions, while in algebra they tended to use readily available formulae. This made me understand that in certain tasks, group work might be more preferable and in others individual work was the best. I came to the realisation that I should encourage learners more to build on each other's ideas and link these ideas to their prior knowledge.

Outcome

Learners struggled to understand the concept of area in mensuration, but many could see the link between the application of the formula and the real-world context. Some learners had issues understanding and recalling concepts, formulae, facts and procedures and could also not visualise the mathematical problems. Some learners were not successful at using the four steps of Pólya in solving the problems.

They could list the problems but struggled to connect the correct strategies to the respective problems; this seemed to occur more often during the mensuration tasks than at the other tasks.

When it came to geometry, learners had already been exposed to problem-solving techniques which made them more comfortable with the approach. It also helped with their confidence. When they did algebra, they often used drawings to try and understand a problem, and with the geometry, they seemed to apply their knowledge more effectively to solve problems.

It became clear to me that in future I will have to look at different questioning techniques in order to assist learners to properly grapple with the content on their way to reaching a solution. I also realised that I would need to ensure that I give the learners relevant homework activities. The second research sub-question will be discussed in the next section.

6.3.2 When is it best practice to use calculators in problem-solving activities?

This question will be answered by summarising the findings in terms of the task-based interviews as well as the teacher reflections. The second aim was to explore when it is best practice to use calculators in problem-solving activities. The root of the problem was that “mathematics teachers tend to present learners with activities that require calculator use, but these activities often lead to the inappropriate use of calculators because teachers tend to neglect the monitoring of how learners acquire solutions for problem-solving activities” (see 1.1). The finding relating to the task-based interviews revealed the following information:

Task-based interviews

Table 5-10 indicates how successful the participants were in using calculators. The low and middle performing groups most often used the calculator incorrectly during algebra tasks, while the high performing group used it most effectively in geometry tasks and were also the most successful performing group in terms of calculator usage in general.

When I studied the reasons why the participants used the calculator, it showed that some participants sometimes did not properly consider the problem before using the calculator, which led to the incorrect use of the calculator. Those that did use it successfully did so when using it for complex calculations and for the addition of large numbers. In general, it did not seem that using a calculator caused a deterioration of participants' arithmetic skills. The research proved that with constant supervision, most learners found the use of calculators positive and beneficial, and they could further develop the skill of understanding when to use a calculator and when not to use a calculator for solving problems (see Table 5-10).

Teacher's reflection on the use of calculators in the lesson

Participants used calculators throughout the mensuration tasks, especially when doing larger calculations which were too difficult to do without a calculator (see task three). At task two participants used the calculator mainly to add, divide and multiply numbers (see Table 5-11). The use of the calculator helped some to get the answer quicker, while others misinterpreted the answers they received from the calculator. This could prove to be a problem, since often learners see any answer produced by the calculator as final and correct. This means they must be taught how to use the calculator correctly, which calls for constant supervision and exposure to the use of a calculator.

In geometry I realised that the calculator was not necessary when doing construction tasks. Also, in algebra at tasks seven and eight, most participants did not use calculators where it was not necessary, and did not get the correct answer. These tasks involved applying the correct algebraic formulae, and not the use of a calculator. This ties in with the idea that learners must be taught when and how to use a calculator, and the most successful learners were the ones that understood and applied this.

6.3.3 What should problem-solving activities look like in the context of a Grade 9 mathematics classroom?

The third aim was to determine what problem-solving activities should look like in the context of a Grade 9 mathematics classroom. The findings indicated a few aspects relating to this aim. Problem-solving activities are needed in Grade 9 mathematics in order to assist learners to grasp mathematical concepts. As pointed out at 1.1, problem-solving is part of the curriculum in Namibia. Problem-solving in mathematics assists learners to gain information, develop understanding, analyse and synthesise as well as evaluate knowledge. Even more important is that learners can do this at their own level (Raoano, 2016; Schoenfeld, 2013).

Additionally, research seems to indicate that the use of calculators is a useful tool in problem-solving (see 1.1; Close *et al.*, 2008:46; Hembree & Dessart, 1986; Mutsvangwa, 2016).

Schoenfeld (1992) (see 1.2.2) states that problem-solving challenges learners to confront a new situation and then to create connections between supplied ideas and exploring possible strategies to reach a goal or solution. This is exactly what happened in this study. The learners were confronted with ten mathematical problems and then had to find solutions to these problems. In order to do this they had to consider and implement strategies. One of the main strategies was the use of a calculator. The latter featured in especially mensuration and algebra, and less so in geometry. It also became clear that at certain tasks a calculator could help to solve complex problems (see 5.2.1).

However, the study also showed that learners did not always understand and appreciate the proper use of a calculator and because of that, problem-solving suffered. It is very important that learners appreciate when and how to use calculators, and the guidance and mentorship of the teacher is vital in this process. Furthermore, problem-solving activities should also be afforded enough time when being introduced to learners. Problem-solving activities, especially in Grade 9 mathematics in Namibia, are a novel concept to learners, and I realised that I had not allocated sufficient time initially to accommodate this; I had to create extra opportunities in order to implement the approach. It became clear that participants improved their problem-solving strategies by working in groups where they could learn from one another, but they also needed to take individual responsibility.

The study also underlined the importance of a theoretical framework for the implementation of problem-solving strategies. Pólya's model offered this. The four steps that the model suggested worked very well and were used to explain and implement problem-solving strategies. It guided the participants when they had to develop problem-solving strategies, sometimes even without them fully realising it, and it also helped me to understand the study and the concept of problem-solving strategies far better.

6.3.4 Addressing the main research question

The central research question for this study was: How can calculators be used in problem-solving activities in a Grade 9 mathematics classroom?

Problem-solving activities form a vital part of Grade 9 mathematics in Namibia and the usage of a calculator is one of the main strategies that learners use. Although calculators form a crucial part of problem-solving in Grade 9 mathematics, learners need to be guided and educated as to the proper use of calculators.

The efficiency of the usage of calculators will suffer if learners do not know how and when to use them; even so will learners' mathematical computation skills deteriorate through the incorrect use of calculators.

The "how" involves understanding that a calculator can be used for more than just addition, multiplication and computation. It also involves that you should first understand the problem (first step in Pólya's model) before you utilise the calculator, and here the teacher's guidance and facilitation will be essential. Table 5-10 reflects this, especially when it came to algebra and mensuration where nearly a quarter of all participants used the calculator correctly in their calculations, but still could not solve the problems owing to incorrect use of mathematical procedures. It furthermore means that learners should realise a calculator is not a magic wand, and it does not replace either mathematical knowledge or the proper understanding of mathematical formulae.

The "when" involves grasping which problems need to be solved with the aid of a calculator. This also involves understanding the problem as well as understanding which tools to use in order to craft a solution. For instance, in geometry with regard to construction questions, the use of a calculator is not needed. These are essential skills in mathematics that learners need to develop and foster, and the proper and effective usage of calculators will help with this. Calculators are, and should be, used in problem-solving in mathematics, but it should be done in a sensible, effective manner which ensures optimal efficiency. A calculator can also not replace understanding of sound mathematical principles and the correct application of mathematical formulae.

6.4 Contribution of the study

This study can assist in giving more clarity with regard to the implementation of problem-solving strategies in a Grade 9 mathematics class. It can assist with developing and using a theoretical framework to guide the practical implementation of problem-solving in mathematics in Grade 9. The study also explored different problem-solving strategies and tasks based on how the problem-solving approach could be successfully facilitated by technology such as the calculator, to improve teaching and learning practice.

It may also reveal useful information regarding the use of a calculator to support purposeful teaching and meaningful learning of school mathematics. The contribution of this study can help to develop and implement the use of the calculator in a Grade 9 mathematics classroom.

The study contributed towards the subject area and discipline of mathematics by providing insight into the needs of mathematics teachers in terms of making mathematics more constructive for the learners. The findings of this study will assist curriculum developers and teachers in the design and adaptation of study material that is more relevant for teaching problem-solving. It could also assist teachers as well as learners by making the mathematical content more relevant and meaningful for them.

6.5 Limitations of the study

One of the limitations that I faced in this study was the availability of time allocated to study the situations of the learners during problem-solving. The results of this study would have been fascinating if I could have taken a year to study the nature of the problem, ranging from knowing the weaknesses and strengths of each individual to being afforded more time to ensure the optimal implementation of problem-solving strategies. This I believe would have ensured improved performances from the participants.

Although I am an experienced Grade 9 mathematics teacher, and I did my utmost to guide the participants, and facilitated the whole project or study; it appeared to have been difficult for learners to complete their tasks in a given period of time, i.e. 40 minutes per period. Due to the fact that it was the first time that learners were engaged in problem-solving, they needed time to think and apply new skills and ideas on how to go about solving a new problem. The time constraints during the application of problem-solving in the mathematics classroom as well as the completion of the problem-solving activities after school hours, placed the learners under pressure to complete the activities in a short time span. It is possible that the mentioned aspects could have had a negative influence on learners' problem-solving abilities.

6.6 Suggestions for future research

This study has indicated that learners learn mathematics with understanding and meaning through the construction of their own mathematical ideas during problem-solving. I found the journey of doing qualitative research interesting, rewarding and enriching.

I would encourage any qualitative researcher to take the opportunity and do more research especially in the following future research questions:

- 1) What are the most important factors that hinder learners from implementing technology (calculator and computer) in mathematics learning?
- 2) What are the factors contributing to the difficulties of solving complex mathematical tasks through problem-solving?
- 3) What effect do learners have on the implementation of problem-solving in their learning with or without the use of calculators?
- 4) How does group work interaction among learners influence problem-solving in mathematics?

6.7 My role as researcher

As a researcher, my first experience with research was employing qualitative research. I am a practical person, who likes to work with different methods or situations in human lives, and I am a mathematics teacher, exploring numbers every day, therefore my confidence is steered by the love of working with numbers and statistics. Subsequently, quantitative research would have been an apparent choice. However, being a person who likes to be challenged, I decided that I wanted to learn more about qualitative research. The research question for my MEd study steered me in the direction of qualitative research and gave me the opportunity to explore my interest with using the calculator during problem-solving in a Grade 9 mathematics classroom. I spent time in the field, interacting, facilitating, mentoring and guiding the learners on how best they can use a calculator in solving complex tasks. I enjoyed doing qualitative research and found it interesting and rewarding.

Experiencing the feedback from the learners after introducing them to the new problem-solving strategies was a fulfilling experience, because I am fully aware of their circumstances and contexts, being able to provide them with problem-solving tools (strategies and a calculator) that can assist them during problem-solving. Their reaction to the new teaching and learning was positive, and they too could see the potential thereof for solving problems out of the ordinary. They felt responsible for their solutions and persevered throughout their learning. It was also satisfying to notice a positive change in some of the low performers. They had high expectations with regard to their own ability to solve problems. Even if the outcome was below average, it was good to find out that slow performing learners can also succeed as long as they have self-confidence and persevere in learning.

Throughout my research journey I had two roles to play: one as researcher and one as teacher. Doing a literature review on the learning of mathematics and problem-solving was a fascinating process. That which appeared to be unclear in the beginning, turned out to be a highly structured framework which I could use to analyse the rest of the data in the study. This structure that the literature review provided was familiar and comforting to me as a mathematician. I found the idea of having to do task-based interviews and a teacher's reflection satisfying, since I had enough experience in that field.

The research involved the presentation of lessons based on the Grade 9 mathematics curriculum. These lessons were integrated with different complex tasks. During classroom teaching and learning I presented worksheets that required learners to explore, investigate and grasp knowledge for them. My role was that of a facilitator, guide, and mentor who was responsible for monitoring the way learners think and reason during group discussions and individual work. The main idea was to keep asking prompting questions that steered the learners in the right direction of learning.

Both as teacher and researcher I learned much, improved my skills and gained useful knowledge with regard to problem-solving and the use of calculators in a Grade 9 mathematics classroom. I also had to grapple with the research questions and structure a study rooted in a qualitative framework with interpretivism as a starting point. It was also very exciting to implement the theory of problem-solving in an actual Grade 9 mathematics classroom, to see the results and get the feedback from the participants.

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ADDENDUM A

TASK BASED INTERVIEW

Code of learner.....

Consider the following problem:

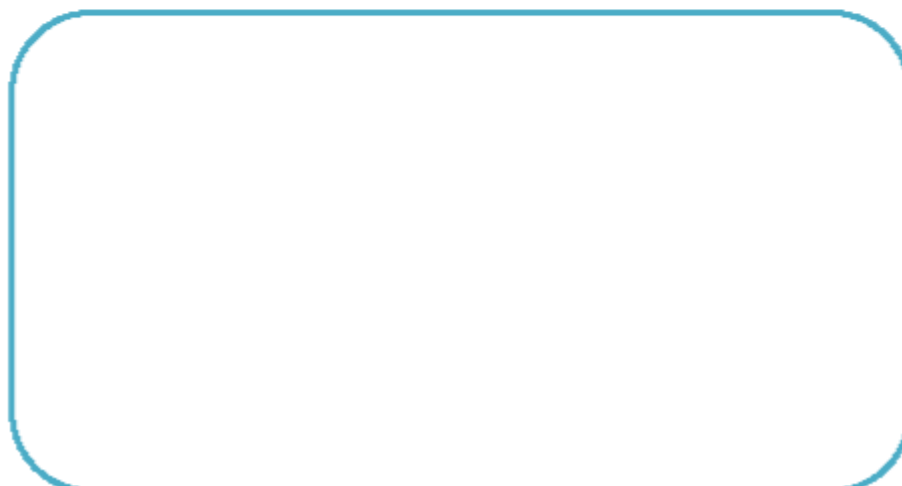
An orange farmer from Assenkehr in Namibia has 120 trees in his orange grove. Each orange tree normally produces 650 oranges. The farmer is interested in raising his orange production and knows that because of lost space and sunlight, every additional tree he plants will cause a reduction of 5 oranges from each tree. What is the maximum number of oranges that he will be able to produce in his orange grove. How many trees will have to be planted to reach this maximum? (10)

1. Do you think you can solve the problem correctly? Circle your choice.

a.	I am sure that I can solve the problem correctly.
b.	I am fairly sure I can solve the problem correctly.
c.	I am not sure how correctly I can solve the problem.
d.	I am not sure I can solve the problem. I think that I might make a mistake.
e.	I know that I will make a mistake in solving the problem.

2. Explain in your own words how you would solve the problem.

3. You may solve the problem now. Show all your steps.



4. Do you think that you have solved the problem correctly? Circle your choice.

a.	I am sure I solved the problem correctly.
b.	I am fairly sure I solved the problem correctly.
c.	I am not sure I solved the problem correctly.
d.	I am not sure that I solved the problem correctly. I think that I may have made a mistake.
e.	I know I made a mistake.

5. Did you need to use a calculator for this task? Where in the task did you use a calculator? Why did you use the calculator there/then?

6. What do you think may be the reason(s) for the mistakes you made in solving the problem?

7. Describe the strategies or plans you used to solve the problem.

ADDENDUM B

Reflection on lessons

Date: _____

Lesson Topic: _____

1. Description

What happened in the lesson/classroom?

2. Interpretation

What teaching strategy did you use in the lesson? What strategies did the learners use to solve the problems? Describe the learners' use of the calculator in the lesson.

3. Outcome

Assess the outcome/results of the lesson. How successful were the learners in solving the problems? How would you change the lesson in future?

ADDENDUM C

Rubric: Problem solving tasks

Name.....

Date.....

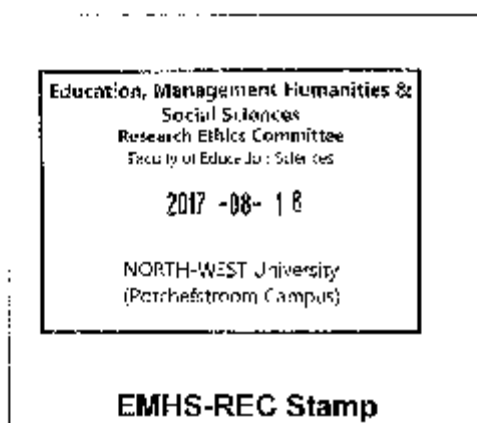
	Levels of achievement				
Criteria	1 (Insufficient)	2 (Low)	3 (Proficient)	4 (Achieved beyond)	Comments
Problem solving strategy used and applied	The strategy that has been selected cannot lead to the solution.	A correct strategy for only solving part of the problem was chosen.	A correct strategy is chosen based on the mathematical situation in the task.	Reasonable strategy selected and developed.	
The use of content knowledge	No evidence of the use of prior knowledge	Evidence of drawing on some relevant previous knowledge is present, showing some relevant engagement in the task.	Evidence of solidifying prior knowledge and applying it to the problem-solving situation	Evidence of analysing the situation in mathematical terms, using content knowledge correctly.	
Solving the problem	Attempted to solve the problem. A limited amount of work shown.	Work is partially shown. The problem was not solved.	The problem was solved correctly, but the learner was unable to explain the strategy used.	Calculated the correct answer. Work shown is logical. Diagrams or labeled work support the strategy.	
The use of calculations	Calculations are completely incorrect, leading to an incorrect answer. No sign of the use of calculators	Calculations contain major errors. Incorrect use of a calculator	Calculations are completely correct. Calculator was randomly used.	Calculations are completely correct and answers properly labeled. Appropriate use of a calculator.	
The use of mathematical language	The use of mathematical language and of reasoning are inappropriate	The use of mathematical language is evident through verbal/written accounts and explanation.	Formal mathematical language and symbols are used to share and clarify ideas.	Formal mathematical language and symbolic notation is used to consolidate math thinking and to communicate the reasoning and mathematical ideas.	
Connections to other mathematics	No connections are made. Connections are contextually irrelevant.	A mathematical connection is attempted but is partially incorrect	A mathematical connection is made. Proper contexts are identified that link both the mathematics and the situation. Exploration of mathematical situations in the context of the broader topic in which the problem is situated.	Mathematical connections are used to extend the solution to other mathematics or to deeper understanding of the mathematics in the problem-solving task.	

ADDENDUM D

PERMISSION LETTER TO THE DIRECTOR



The Faculty of Health Sciences Ethics Office of the North-West University is acknowledged for the use of their document with minor adjustments by the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) of the North-West University.



INFORMED CONSENT FOR Mr Shapange (Director of Education)

TITLE OF THE RESEARCH STUDY: *The use of calculators in problem-solving activities in a Grade 9 mathematics classroom in Namibia*

ETHICS REFERENCE NUMBERS: NWU-HS-2016-0115

PRINCIPAL INVESTIGATOR: Dr SM Nieuwoudt

POST GRADUATE STUDENT: Mrs G Amutlanya

ADDRESS: Epsto Combined school

CONTACT NUMBER: 0814781799 gisberthakapandu@gmail.com

The Grade 9 learners at Epsto Combined School are being invited to take part in a **research study** that forms part of my MEd study. Please take some time to read the information presented here, which will explain the details of this study. Please ask the researcher or person explaining the research to you any questions about any part of this study that you do not fully understand. It is very important that you are fully satisfied that you clearly understand what this research is about and how you might be involved. Also, the learners' participation

is **entirely voluntary** and he/she is free to say no to participate. If he/she says no, this will not affect him/her negatively in any way whatsoever. They are also free to withdraw from the study at any point, even if you do agree to let them take part now.

This study has been approved by the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) of the Faculty of Education Sciences of the North-West University (NWU-HS-2016-0115) and will be conducted according to the ethical guidelines and principles of Ethics in Education Research: Principles, Processes and Structures (DoH, 2015) and other international ethical guidelines applicable to this study. It might be necessary for the research ethics committee members or other relevant people to inspect the research records.

1.1 What is this research study all about?

- This study will be conducted in Okahao at Epato Combined School, in my (Mrs Amutenya's) classroom and will involve a different teaching approach (problem-based learning using calculators) which will be used in the classroom on a daily basis for all learners as well as about 12 problem solving activities that will be done in the afternoons for those learners who would like to join me. This teaching approach and the activities will be done with an experienced teacher who has been teaching Mathematics for 9 years. Approximately 30 participants will be included in this study.
- We plan to use a different teaching approach (problem-based learning using calculators) which is not normally used in the classroom, on a daily basis for all learners. This will provide all learners with the opportunity to have a greater insight and knowledge into the topics discussed. For those learners who would like to join me in the afternoons, I plan to give them about 12 problem solving activities to do after school, from July 2017 to November 2017. In this time I will discuss with them how they go to work solving the problems.

1.2 Why have the learners been invited to participate?

- The learners have been invited to be part of this research because they are in Grade 9 and I would like them to be part of the problem-based learning approach as it will assist them in the understanding of mathematics.
- The learners are also suitable for this research because I would like to teach them new skills regarding the learning of Mathematics.

1.3 What will be expected of you?

- The learners will be expected to be part of the daily Mathematics lessons in which I will be using a problem-based approach to the teaching of Mathematics. He or she will be invited to take part in afternoon activities where we will do specific tasks based on the syllabus and where I will discuss with them, the different strategies that they used to do the tasks and also whether they made use of calculators to do the tasks or not.

1.4 Will the learners gain anything from taking part in this research?

- The gains for the learners if they take part in this study will be to gain mathematical knowledge through using problem-based tasks.
- The other gain of the study for the learners is to understand better when it is necessary to use a calculator in mathematics activities or not.

1.5 Are there risks involved in the learners taking part in this research and what will be done to prevent them?

- We do not foresee any risks for the learners in this study.

1.6 How will we protect the learners' confidentiality and who will see the findings?

- Anonymity of the findings will be protected by myself and the study leaders (the research team). The participants will not be identified according to their names, I will make use of codes to which only I have access. The learners' privacy will be respected by the research team. The results will be kept confidential by the research team. Only the researchers will be able to look at the results. Findings will be kept safe by locking hard copies in the strong room of Epato Combined School and electronic data will be password protected. Data will be stored for 7 years.

1.7 What will happen with the findings or samples?

- *The findings of this study will only be used for this study.*

1.8 How will the learners know about the results of this research?

- *I will tell the learners the results of this research by arranging an information session with them before the examinations start at the end of the year. This session will take place in the afternoon after school in my classroom. In this session I will discuss the findings of the study: I will discuss different problem-solving strategies that the participants used in the research and also the role of the calculator in the activities that they did.*

1.9 Will the learners be paid to take part in this study and are there any costs for you?

This study is funded by the North-West University. No, the learners will not be paid to take part in the study because they will be learning a new way of doing mathematics which will benefit them in their further studies of mathematics. There will be no costs involved for the learners if they do take part in this study.

1.10 Is there anything else that you should know or do?

- *You can contact Ms Gisbertha Amutenya at 0814781799 or gisberthakapandu@gmail.com, the researcher or Dr Susan Nieuwoudt (the study leader) at 018 2991912 or Susan.Nieuwoudt@nwu.ac.za if you have any further questions or have any problems.*
- *You can also contact the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) via Miss Jessica Mabaso at 018 299 4707 or Ethics-EMHS@nwu.ac.za if you have any concerns that were not answered about the research or if you have complaints about the research.*
- *You will receive a copy of this information and consent form for your own purposes.*

Declaration by participant

By signing below, I agree to allow the Grade 9 learners of Epato Combined School to take part in the research study titled: *The use of calculators in problem-solving activities in a Grade 9 mathematics classroom in Namibia*

I declare that:

- I have read this information/it was explained to me by a trusted person in a language with which I am fluent and comfortable.
- The research was clearly explained to me.
- I have had a chance to ask questions to both the person getting the consent from me, as well as the researcher and all my questions have been answered.
- I understand that taking part in this study is **voluntary** for the learners and that they have not been pressurised to take part.
- I understand that the learners may choose to leave the study at any time and will not be handled in a negative way if he/she did so.

Signed at (place) on (date) 20....

Signature of participant

Signature of witness

1.11.1 Declaration by person obtaining consent

I (name) declare that:

- I clearly and in detail explained the information in this document to
- I did/did not use an interpreter.
- I encouraged him/her to ask questions and took adequate time to answer them.
- I am satisfied that he/she adequately understands all aspects of the research, as discussed above
- I gave him/her time to discuss it with others if he/she wished to do so.

Signed at (place) on (date) 20....

.....
Signature of person obtaining consent

.....
Signature of witness

1.11.2 Declaration by researcher

I (name) declare that:

- I explained the information in this document to or I had it explained by who I trained for this purpose.
- I did/did not use an interpreter
- I encouraged him/her to ask questions and took adequate time to answer them or I was available should he/she want to ask any further questions.
- The informed consent was obtained by an independent person.
- I am satisfied that he/she adequately understands all aspects of the research, as described above.
- I am satisfied that he/she had time to discuss it with others if he/she wished to do so.

Signed at (place) on (date) 20....

.....
Signature of researcher

.....
Signature of witness

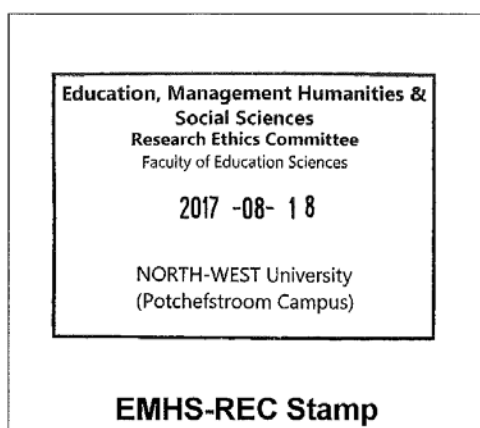
PERMISSION LETTER TO THE PRINCIPAL



NORTH-WEST UNIVERSITY
YUNIBESITHI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOM CAMPUS

Faculty of Education Sciences

The Faculty of Health Sciences Ethics Office of the North-West University is acknowledged for the use of their document with minor adjustments by the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) of the North-West University.



INFORMED CONSENT FOR Mrs Nakambale (Principal of Epato Combined School)

TITLE OF THE RESEARCH STUDY: *The use of calculators in problem-solving activities in a Grade 9 mathematics classroom in Namibia*

ETHICS REFERENCE NUMBERS: NWU-HS-2016-0115

PRINCIPAL INVESTIGATOR: Dr SM Nieuwoudt

POST GRADUATE STUDENT: Mrs G Amutenya

ADDRESS: Epato Combined school

CONTACT NUMBER: 0814781799 gisberthakapandu@gmail.com

The Grade 9 learners at your school are being invited to take part in a **research study** that forms part of my MEd study. Please take some time to read the information presented here, which will explain the details of this study. Please ask the researcher or person explaining the research to you any questions about any part of this study that you do not fully understand. It is very important that you are fully satisfied that you clearly understand what this research is about and how you might be involved. Also, the learners' participation is

entirely voluntary and he/she is free to say no to participate. If he/she says no, this will not affect him/her negatively in any way whatsoever. They are also free to withdraw from the study at any point, even if you do agree to let them take part now.

This study has been approved by the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) of the Faculty of Education Sciences of the North-West University (NWU-HS-2016-0115) and will be conducted according to the ethical guidelines and principles of Ethics in Education Research: Principles, Processes and Structures (DoH, 2015) and other international ethical guidelines applicable to this study. It might be necessary for the research ethics committee members or other relevant people to inspect the research records.

1.1 What is this research study all about?

- This study will be conducted in Okahao at Epato Combined School, in my (Mrs Amutenya's) classroom and will involve a different teaching approach (problem-based learning using calculators) which will be used in the classroom on a daily basis for all learners as well as about 12 problem solving activities that will be done in the afternoons for those learners who would like to join me. This teaching approach and the activities will be done with an experienced teacher who has been teaching Mathematics for 9 years. Approximately 30 participants will be included in this study.
- We plan to use a different teaching approach (problem-based learning using calculators) which is not normally used in the classroom, on a daily basis for all learners. This will provide all learners with the opportunity to have a greater insight and knowledge into the topics discussed. For those learners who would like to join me in the afternoons, I plan to give them about 12 problem solving activities to do after school, from July 2017 to November 2017. In this time I will discuss with them how they go to work solving the problems.

1.2 Why have the learners been invited to participate?

- The learners have been invited to be part of this research because they are in Grade 9 and I would like them to be part of the problem-based learning approach as it will assist them in the understanding of mathematics.
- The learners are also suitable for this research because I would like to teach them new skills regarding the learning of Mathematics.

1.3 What will be expected of you?

- The learners will be expected to be part of the daily Mathematics lessons in which I will be using a problem-based approach to the teaching of Mathematics. He or she will be invited to take part in afternoon activities where we will do specific tasks based on the syllabus and where I will discuss with them, the different strategies that they used to do the tasks and also whether they made use of calculators to do the tasks or not.

1.4 Will the learners gain anything from taking part in this research?

- The gains for the learners if they take part in this study will be to gain mathematical knowledge through using problem-based tasks.
- The other gain of the study for the learners is to understand better when it is necessary to use a calculator in mathematics activities or not.

1.5 Are there risks involved in the learners taking part in this research and what will be done to prevent them?

- We do not foresee any risks for the learners in this study.

1.6 How will we protect the learners' confidentiality and who will see the findings?

- Anonymity of the findings will be protected by myself and the study leaders (the research team). The participants will not be identified according to their names, I will make use of codes to which only I have access. The learners' privacy will be respected by the research team. The results will be kept confidential by the research team. Only the researchers will be able to look at the results. Findings will be kept safe by locking hard copies in the strong room of Epato Combined School and electronic data will be password protected. Data will be stored for 7 years.

1.7 What will happen with the findings or samples?

- *The findings of this study will only be used for this study.*

1.8 How will the learners know about the results of this research?

- *I will give the learners the results of this research by arranging an information session with them before the examinations start at the end of the year. This session will take place in the afternoon after school in my classroom. In this session I will discuss the findings of the study: I will discuss different problem-solving strategies that the participants used in the research and also the role of the calculator in the activities that they did.*

1.9 Will the learners be paid to take part in this study and are there any costs for you?

This study is funded by the North-West University. No, the learners will not be paid to take part in the study because they will be learning a new way of doing mathematics which will benefit them in their further studies of mathematics. There will be no costs involved for the learners if they do take part in this study.

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- *You can contact Ms Gisbertha Amutenya at 0814781799 or gisberthakapandu@gmail.com, the researcher or Dr Susan Nieuwoudt (the study leader) at 018 2991912 or Susan.Nieuwoudt@nwu.ac.za if you have any further questions or have any problems.*
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- *You will receive a copy of this information and consent form for your own purposes.*

Declaration by participant

By signing below, I agree to allow the Grade 9 learners to take part in the research study titled: *The use of calculators in problem-solving activities in a Grade 9 mathematics classroom in Namibia*

I declare that:

- I have read this information/it was explained to me by a trusted person in a language with which I am fluent and comfortable.
- The research was clearly explained to me.
- I have had a chance to ask questions to both the person getting the consent from me, as well as the researcher and all my questions have been answered.
- I understand that taking part in this study is **voluntary** for the learners and that they have not been pressurised to take part.
- I understand that the learners may choose to leave the study at any time and will not be handled in a negative way if he/she did so.

Signed at (place) on (date) 20....

Signature of participant

Signature of witness

1.11.1 Declaration by person obtaining consent

I (name) declare that:

- I clearly and in detail explained the information in this document to
- I did/did not use an interpreter.
- I encouraged him/her to ask questions and took adequate time to answer them.
- I am satisfied that he/she adequately understands all aspects of the research, as discussed above
- I gave him/her time to discuss it with others if he/she wished to do so.

Signed at (place) on (date) 20....

.....
Signature of person obtaining consent

.....
Signature of witness

1.11.2 Declaration by researcher

I (name) declare that:

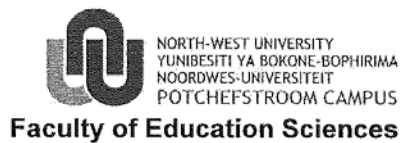
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- I encouraged him/her to ask questions and took adequate time to answer them or I was available should he/she want to ask any further questions.
- The informed consent was obtained by an independent person.
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Signed at (place) on (date) 20....

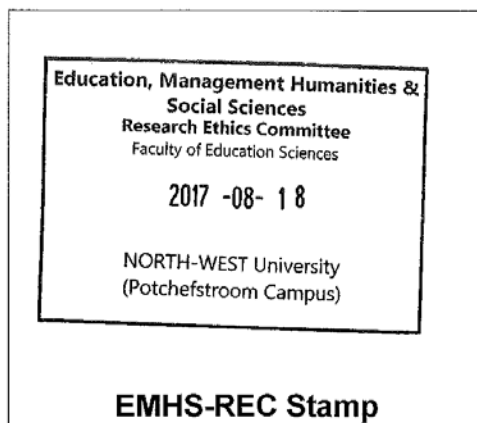
.....
Signature of researcher

.....
Signature of witness

PERMISSION LETTER TO THE PARENTS



The Faculty of Health Sciences Ethics Office of the North-West University is acknowledged for the use of their document with minor adjustments by the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) of the North-West University.



INFORMED CONSENT FOR(name will be inserted here)...(parent)

TITLE OF THE RESEARCH STUDY: *The use of calculators in problem-solving activities in a Grade 9 mathematics classroom in Namibia*

ETHICS REFERENCE NUMBERS: NWU-HS-2016-0115

PRINCIPAL INVESTIGATOR: Dr SM Nieuwoudt

POST GRADUATE STUDENT: Mrs G Amutenya

ADDRESS: Epato Combined school

CONTACT NUMBER: 0814781799 gisberthakapandu@gmail.com

Your child is being invited to take part in a **research study** that forms part of my MEd study. Please take some time to read the information presented here, which will explain the details of this study. Please ask the researcher or person explaining the research to you any questions about any part of this study that you do not fully understand. It is very important that you are fully satisfied that you clearly understand what this research is about and how you might be involved. Also, your child's participation is **entirely voluntary** and he/she is

free to say no to participate. If he/she says no, this will not affect him/her negatively in any way whatsoever. They are also free to withdraw from the study at any point, even if you do agree to let them take part now.

This study has been approved by the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) of the Faculty of Education Sciences of the North-West University (NWU-HS-2016-0115) and will be conducted according to the ethical guidelines and principles of Ethics in Education Research: Principles, Processes and Structures (DoH, 2015) and other international ethical guidelines applicable to this study. It might be necessary for the research ethics committee members or other relevant people to inspect the research records.

1.1 What is this research study all about?

- *This study will be conducted in Okahao at Epato Combined School, in my (Mrs Amutenya's) classroom and will involve a different teaching approach (problem-based learning using calculators) which will be used in the classroom on a daily basis for all learners as well as about 12 problem solving activities that will be done in the afternoons for those learners who would like to join me. This teaching approach and the activities will be done with an experienced teacher who has been teaching Mathematics for 9 years. Approximately 30 participants will be included in this study.*
- *We plan to use a different teaching approach (problem-based learning using calculators) which is not normally used in the classroom, on a daily basis for all learners. This will provide all learners with the opportunity to have a greater insight and knowledge into the topics discussed. For those learners who would like to join me in the afternoons, I plan to give them about 12 problem solving activities to do after school, from July 2017 to November 2017. In this time I will discuss with them how they go to work solving the problems.*

1.2 Why has your child been invited to participate?

- *Your child has been invited to be part of this research because he/she is in Grade 9 and I would like them to be part of the problem-based learning approach as it will assist them in the understanding of mathematics.*
- *Your child is also suitable for this research because I would like to teach him/her new skills regarding the learning of Mathematics.*

1.3 What will be expected of you?

- *Your child will be expected to be part of the daily Mathematics lessons in which I will be using a problem-based approach to the teaching of Mathematics. He or she will be invited to take part in afternoon activities where we will do specific tasks based on the syllabus and where I will discuss with them, the different strategies that they used to do the tasks and also whether they made use of calculators to do the tasks or not.*

1.4 Will your child gain anything from taking part in this research?

- *The gains for your child if he or she takes part in this study will be to gain mathematical knowledge through using problem-based tasks.*
- *The other gain of the study for your child is to understand better when it is necessary to use a calculator in mathematics activities or not.*

1.5 Are there risks involved in your child taking part in this research and what will be done to prevent them?

- *We do not foresee any risks for your child in this study.*

1.6 How will we protect your child's confidentiality and who will see the findings?

- *Anonymity of the findings will be protected by myself and the study leaders (the research team). The participants will not be identified according to their names, I will make use of codes to which only I have access. Your child's privacy will be respected by the research team. The results will be kept confidential by the research team. Only the researchers will be able to look at the results. Findings will be kept safe by locking hard copies in the strong room of Epato Combined School and electronic data will be password protected. Data will be stored for 7 years.*

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- *I will give your child the results of this research by arranging an information session with them before the examinations start at the end of the year. This session will take place in the afternoon after school in my classroom. In this session I will discuss the findings of the study; I will discuss different problem-solving strategies that the participants used in the research and also the role of the calculator in the activities that they did.*

1.9 Will your child be paid to take part in this study and are there any costs for you?

This study is funded by the North-West University. No, your child will not be paid to take part in the study because they will be learning a new way of doing mathematics which will benefit them in their further studies of mathematics. There will be no costs involved for you or your child, if they do take part in this study.

1.10 Is there anything else that you should know or do?

- *You can contact Ms Gisbertha Amutenya at 0814781799 or gisberthakapandu@gmail.com, the researcher or Dr Susan Nieuwoudt (the study leader) at 018 2991912 or Susan.Nieuwoudt@nwu.ac.za if you have any further questions or have any problems.*
- *You can also contact the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) via Miss Jessica Mabaso at 018 299 4707 or Ethics-EMHS@nwu.ac.za if you have any concerns that were not answered about the research or if you have complaints about the research.*
- *You will receive a copy of this information and consent form for your own purposes.*

Declaration by participant

By signing below, I agree to allow my child to take part in the research study titled: *The use of calculators in problem-solving activities in a Grade 9 mathematics classroom in Namibia*

I declare that:

- I have read this information/it was explained to me by a trusted person in a language with which I am fluent and comfortable.
- The research was clearly explained to me.
- I have had a chance to ask questions to both the person getting the consent from me, as well as the researcher and all my questions have been answered.
- I understand that taking part in this study is **voluntary** for my child and that they have not been pressurised to take part.
- I understand that my child may choose to leave the study at any time and will not be handled in a negative way if he/she did so.

Signed at (place) on (date) 20....

.....
Signature of participant

.....
Signature of witness

1.11.1 Declaration by person obtaining consent

I (name) declare that:

- I clearly and in detail explained the information in this document to
- I did/did not use an interpreter.
- I encouraged him/her to ask questions and took adequate time to answer them.
- I am satisfied that he/she adequately understands all aspects of the research, as discussed above
- I gave him/her time to discuss it with others if he/she wished to do so.

Signed at (place) on (date) 20....

.....
Signature of person obtaining consent

.....
Signature of witness

1.11.2 Declaration by researcher

I (name) declare that:

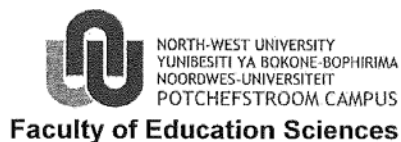
- I explained the information in this document to or I had it explained by who I trained for this purpose.
- I did/did not use an interpreter
- I encouraged him/her to ask questions and took adequate time to answer them or I was available should he/she want to ask any further questions.
- The informed consent was obtained by an independent person.
- I am satisfied that he/she adequately understands all aspects of the research, as described above.
- I am satisfied that he/she had time to discuss it with others if he/she wished to do so.

Signed at (place) on (date) 20....

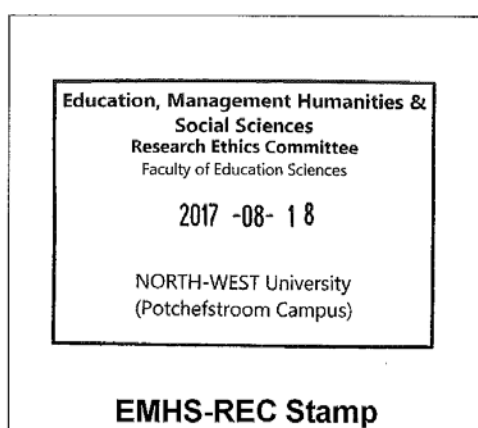
.....
Signature of researcher

.....
Signature of witness

PERMISSION LETTER TO THE PARTICIPANTS (LEARNERS)



The Faculty of Health Sciences Ethics Office of the North-West University is acknowledged for the use of their document with minor adjustments by the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) of the North-West University.



INFORMED ASSENT FOR(name will be inserted here)...(learner)

TITLE OF THE RESEARCH STUDY: *The use of calculators in problem-solving activities in a Grade 9 mathematics classroom in Namibia*

ETHICS REFERENCE NUMBERS: NWU-HS-2016-0115

PRINCIPAL INVESTIGATOR: Dr SM Nieuwoudt

POST GRADUATE STUDENT: Mrs G Amutenya

ADDRESS: Epato Combined school

CONTACT NUMBER: 0814781799 gisberthakapandu@gmail.com

You are invited to take part in a **research study** that forms part of my MEd study. Please take some time to read the information presented here, which will explain the details of this study. Please ask the researcher or person explaining the research to you any questions about any part of this study that you do not fully understand. It is very important that you are fully satisfied that you clearly understand what this research is about and how you might be involved. Also, your participation is **entirely voluntary** and you are free to say no

to participate. If you say no, this will not affect you negatively in any way whatsoever. You are also free to withdraw from the study at any point, even if you do agree to take part now.

This study has been approved by the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) of the Faculty of Education Sciences of the North-West University (NWU-HS-2016-0115) and will be conducted according to the ethical guidelines and principles of Ethics in Education Research: Principles, Processes and Structures (DoH, 2015) and other international ethical guidelines applicable to this study. It might be necessary for the research ethics committee members or other relevant people to inspect the research records.

1.1 What is this research study all about?

- *This study will be conducted in Okahao at Epato Combined School, in my (Mrs Amutenya's) classroom and will involve a different teaching approach (problem-based learning using calculators) which will be used in the classroom on a daily basis for all learners as well as about 12 problem solving activities that will be done in the afternoons for those learners who would like to join me. This teaching approach and the activities will be done with an experienced teacher who has been teaching Mathematics for 9 years. Approximately 30 participants will be included in this study.*
- *We plan to use a different teaching approach (problem-based learning using calculators) which is not normally used in the classroom, on a daily basis for all learners. This will provide all learners with the opportunity to have a greater insight and knowledge into the topics discussed. For those learners who would like to join me in the afternoons, I plan to give them about 12 problem solving activities to do after school, from July 2017 to November 2017. In this time I will discuss with them how they go to work solving the problems.*

1.2 Why have you been invited to participate?

- *You have been invited to be part of this research because you are in Grade 9 and I would like you to be part of the problem-based learning approach as it will assist you in the understanding of mathematics.*
- *You are also suitable for this research because I would like to teach you new skills regarding the learning of Mathematics.*

1.3 What will be expected of you?

- *You will be expected to be part of the daily Mathematics lessons in which I will be using a problem-based approach to the teaching of Mathematics. You will be invited to take part in afternoon activities where we will do specific tasks based on the syllabus and where I will discuss with you, the different strategies that you used to do the tasks and also whether you made use of calculators to do the tasks or not.*

1.4 Will you gain anything from taking part in this research?

- *Your gains for taking part in this study will be to gain mathematical knowledge through using problem-based tasks.*
- *The other gain of the study for you is to understand better when it is necessary to use a calculator in mathematics activities or not.*

1.5 Are there risks involved in your taking part in this research and what will be done to prevent them?

- *We do not foresee any risks for you in this study.*

1.6 How will we protect your confidentiality and who will see the findings?

- *Anonymity of the findings will be protected by myself and the study leaders (the research team). The participants will not be identified according to their names, I will make use of codes to which only I have access. Your privacy will be respected by the research team. The results will be kept confidential by the research team. Only the researchers will be able to look at the results. Findings will be kept safe by locking hard copies in the strong room of Epato Combined School and electronic data will be password protected. Data will be stored for 7 years.*

1.7 What will happen with the findings or samples?

- *The findings of this study will only be used for this study.*

1.8 How will you know about the results of this research?

- *I will give you the results of this research by arranging an information session with the participants before the examinations start at the end of the year. This session will take place in the afternoon after school in my classroom. In this session I will discuss the findings of the study; I will discuss different problem-solving strategies that the participants used in the research and also the role of the calculator in the activities that they did.*

1.9 Will you be paid to take part in this study and are there any costs for you?

This study is funded by the North-West University. No, you will not be paid to take part in the study because you will be learning a new way of doing mathematics which will benefit you in your further studies of mathematics. There will be no costs involved for you, if you do take part in this study.

1.10 Is there anything else that you should know or do?

- *You can contact Ms Gisbertha Amutenya at 0814781799 or gisberthakapandu@gmail.com, the researcher or Dr Susan Nieuwoudt (the study leader) at 018 2991912 or Susan.Nieuwoudt@nwu.ac.za if you have any further questions or have any problems.*
- *You can also contact the Education, Management, Humanities and Social Sciences Research Ethics Committee (EMHS-REC) via Miss Jessica Mabaso at 018 299 4707 or Ethics-EMHS@nwu.ac.za if you have any concerns that were not answered about the research or if you have complaints about the research.*
- *You will receive a copy of this information and consent form for your own purposes.*

Declaration by participant

By signing below, I agree to take part in the research study titled:
The use of calculators in problem-solving activities in a Grade 9 mathematics classroom in Namibia

I declare that:

- *I have read this information/it was explained to me by a trusted person in a language with which I am fluent and comfortable.*
- *The research was clearly explained to me.*
- *I have had a chance to ask questions to both the person getting the consent from me, as well as the researcher and all my questions have been answered.*
- *I understand that taking part in this study is **voluntary** and that I have not been pressurised to take part.*
- *I understand that I may choose to leave the study at any time and will not be handled in a negative way if I did so.*

Signed at (place) on (date) 20....

.....
Signature of participant

.....
Signature of witness

1.11.1 Declaration by person obtaining consent

I (name) declare that:

- I clearly and in detail explained the information in this document to
- I did/did not use an interpreter.
- I encouraged him/her to ask questions and took adequate time to answer them.
- I am satisfied that he/she adequately understands all aspects of the research, as discussed above
- I gave him/her time to discuss it with others if he/she wished to do so.

Signed at (place) on (date) 20....

.....
Signature of person obtaining consent

.....
Signature of witness

1.11.2 Declaration by researcher

I (name) declare that:

- I explained the information in this document to or I had it explained by who I trained for this purpose.
- I did/did not use an interpreter
- I encouraged him/her to ask questions and took adequate time to answer them or I was available should he/she want to ask any further questions.
- The informed consent was obtained by an independent person.
- I am satisfied that he/she adequately understands all aspects of the research, as described above.
- I am satisfied that he/she had time to discuss it with others if he/she wished to do so.

Signed at (place) on (date) 20....

.....
Signature of researcher

.....
Signature of witness

ADDENDUM E



Private Bag X6001, Potchefstroom,
South Africa, 2520

Tel: (018) 259-4900
Fax: (018) 259-4910
Web: <http://www.nwu.ac.za>

Institutional Research Ethics Regulatory Committee

Tel: +27 18 259 4949

Email: Ethics@nwu.ac.za

ETHICS APPROVAL CERTIFICATE OF STUDY

Based on approval by Education Management Humanities and Social Sciences Research Ethics Committee (EMHS-REC) on 16/08/2017 after being reviewed at the meeting held on 17/07/2017, the North-West University Institutional Research Ethics Regulatory Committee (NMU-IRERC) hereby conditionally approves your study as indicated below. This implies that the NMU-IRERC grants its permission that provided the special conditions specified below are met and pending any other authorisation that may be necessary, the study may be initiated, using the ethics number below.

Study title: The use of calculators in problem-solving activities in a Grade 9 mathematics classroom																																	
Study Leader/Supervisor: Doctor Susan Nieuwoudt																																	
Student: Ms Gibertha Ndiapandula Amutenya																																	
Ethics number:	<table border="1"> <tr> <td>N</td><td>W</td><td>U</td><td>-</td><td>H</td><td>S</td><td>-</td><td>2</td><td>0</td><td>1</td><td>6</td><td>-</td><td>0</td><td>0</td><td>1</td><td>5</td> </tr> <tr> <td colspan="4">Institution</td> <td colspan="4">Study Number</td> <td colspan="4">Year</td> <td colspan="4">Ethics</td> </tr> </table>	N	W	U	-	H	S	-	2	0	1	6	-	0	0	1	5	Institution				Study Number				Year				Ethics			
N	W	U	-	H	S	-	2	0	1	6	-	0	0	1	5																		
Institution				Study Number				Year				Ethics																					
Application Type: Single study																																	
Commencement date: 2017-08-16	Risk: Minimal																																
Continuation of the study is dependent on receipt of the six months monitoring report and the concomitant issuing of a letter of continuation.																																	

Special conditions of the approval:

- Translation of the informed consent document to the languages applicable to the study participants should be submitted to the EMHS-REC (if applicable).
- Approval from the Department of Education in Namibia

General conditions:

While this ethics approval is subject to all declarations, undertakings and agreements incorporated and signed in the application form, please note the following:

- The study leader (principle investigator) must report in the prescribed format to the NWU-IRERC via EMHS-REC:
 - annually (or as otherwise requested) on the monitoring of the study, and upon completion of the study
 - without any delay in case of any adverse event or incident (or any matter that interrupts sound ethical principles) during the course of the study.
- Annually a number of studies may be randomly selected for an external audit.
- The approval applies strictly to the proposal as stipulated in the application form. Would any changes to the proposal be deemed necessary during the course of the study, the study leader must apply for approval of these amendments at the EMHS-REC, prior to implementation. Would there be deviation from the study proposal without the necessary approval of such amendments, the ethics approval is immediately and automatically forfeited.
- The date of approval indicates the first date that the study may be started.
- In the interest of ethical responsibility the NWU-IRERC and EMHS-REC retains the right to:
 - request access to any information or data at any time during the course or after completion of the study;
 - to ask further questions, seek additional information, require further modification or monitor the conduct of your research or the informed consent process.
 - withdraw or postpone approval if:
 - any unethical principles or practices of the study are revealed or suspected;
 - it becomes apparent that any relevant information was withheld from the EMHS-REC or that information has been false or misrepresented;
 - the required amendments, annual (or otherwise stipulated) report and reporting of adverse events or incidents was not done in a timely manner and accurately;
 - new institutional rules, national legislation or international conventions deem it necessary.
- EMHS-REC can be contacted for further information or any report templates via Ethics-EMHS@nwu.ac.za or 018 259 4707.

The IRERC would like to remain at your service as a scientist and researcher, and wishes you well with your study. Please do not hesitate to contact the IRERC or EMHS-REC for any further enquiries or requests for assistance.

Yours sincerely

Linda Du

Plessis

Prof Linda du Plessis

Chair NWU Institutional Research Ethics Regulatory Committee (IRERC)

Digitally signed by Linda Du Plessis
DN: cn=Linda Du Plessis, o=North-West University, ou=Institutional Research Ethics Regulatory Committee, email=Linda.DuPlessis@nwu.ac.za

ADDENDUM F

H C Sieberhagen
SATI no 1001489
hettiesieb@gmail.com

Translator and Editor
082 3359846
021 829 7015

CERTIFICATE OF LANGUAGE EDITING

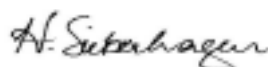
ISSUED ON 10 DECEMBER 2018

This is to certify that I have edited the language of the dissertation

**The use of calculators
during problem-solving activities
in a Grade 9 mathematics classroom**
by

GNA Kanhalelo

submitted in fulfilment of the requirements for the degree
Masters of Education in Mathematics Education
at the North West University



H C Sieberhagen

SATI no 1001489
20 December 2018

ADDENDUM G



To whom it may concern

This letter is to confirm that Ms Gisbertha Kanhalelo (Student no.: 22897666) submitted her Master's dissertation to me for bibliographic control, as well as for the technical formatting of the document according to the prerequisites of the North-West University. I hereby confirm that the final reference list meets the requirements of the NWU Harvard style.

Regards

A handwritten signature in dark ink, appearing to read "Kirchner van Deventer", is written above a horizontal line.

Kirchner van Deventer

21 February 2019