A COMPARATIVE STUDY OF THE VECM, GARCH AND MULTIVARIATE GARCH TECHNIQUES IN MODELLING EXTERNAL DEBT

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DECLARATION

I, Naledi Blessing Mokoena, hereby declare that this research report titled "A comparative study of the VECM, GARCH and multivariate GARCH techniques in modelling external debt" was composed by myself and all sources have been referenced and acknowledged, and that this document has not been submitted at any university in order to obtain any degree.

NB Mokoena **22 September 2020**

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LIST OF ACRONYMS

ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criteria
ARCH	Autoregressive Conditional Heteroscedasticity
ARDL	Autoregressive Distributed Lag
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
ATHEX	Athens Stock Exchange
BD	Breusch-Godfrey
BDS	Brock-Dechert-Scheinkman
BEKK	Baba Engle Kraft and Kroner
BET	Bucharest Exchange Trading
CCC	Constant Conditional Correlation
CF	Capital Fomation
CPI	Consumer Price Index
DCC	Dynamic Conditional Correlation
DF	Dickey-Fuller
DSI	Databank Stock Index
ECT	Error Correction Term
ED	External Debt
EGARCH	Exponential Generalized Autoregressive Conditional Heteroscedasticity
EXP	Exports
FDI	Foreign Direct Investment
FGLS	Feasible Generalized Least Square
FTSE	Financial Times Stock Exchange
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GARCH-M	GARCH-in Mean
GDP	Gross Domestic Product
GE	Government Expenditure
GJR-GARCH	Glosten Jagannathan and Runkle Generalized Autoregressive Conditional Heteroscedasticity
GMM	Generalized Methods of Moments
HAC	Heteroscedasticity and Autoregressive Consistent

HIPC	Highly Indebted Poor Countries
IGARCH	Integrated Generalized Autoregressive Conditional Heteroscedasticity
IMF	International Monetary Fund
JB	Jacque-Bera
KPSS	Kwiatkowski-Phillips-Schimdt-Shin
LCF	Logarithm of Capital Formation
LED	Logarithm of External Debt
LEXP	Logarithm of Exports
LGDP	Logarithm of Gross Domestic Product
LGE	Logarithm of Government Expenditure
LM	Lagrange Multiplier
MASE	Mean Absolute Scaled Error
MAPE	Mean Absolute Error
MGARCH	Multivariate Generalized Autoregressive Conditional Heteroscedasticity
MLE	Maximum Likelihood Estimator
MSE	Mean Squared Error
OLS	Ordinary Least Square
P-Value	Probability Value
PP	Phillips-Perron
QGARCH	Quadratic Generalized Autoregressive Conditional Heteroscedasticity
RIR	Real Interest Rate
RMSE	Root Mean Squared Error
S&P	Standard and Poor
SARB	South African Reserve Bank
SBIC	Schwartz Bayesian Information Criterion
TGARCH	Threshold Generalized Autoregressive Conditional Heteroscedasticity
TIC	Their Inequality Coefficient
UK	United Kingdom
USA	United States of America
VAR	Vector Autoregressive
VECM	Vector Error Correction Model

ABSTRACT

The study modelled the determinants of external debt using the VECM, GARCH and multivariate GARCH models with the intent to identify and recommend the most effective approach. The study employed quarterly time series data obtained from the South African Reserve Bank ranging from the second quarter of 1990 to the fourth quarter of 2018. The results of the VECM revealed that the long run relationship among the variables showed that LCF, LEXP, LGDP have a positive long run relationship with LED. The LGE has a negative long run relationship with LED. The results of the GARCH(1,1) model showed that the variance of the series is increasing over time since the sum of the ARCH and GARCH term is greater than one. Furthermore, the results of the GARCH(1,1) model revealed that a positive and statistically significant relationship exists between LEXP and LED; LGDP and LED. There is also a negative and statistically significant relationship found between LCF and LED; LGE and LED. The results of the BEKK model revealed that only one diagonal parameter B(5,5) was statistically insignificant which meant that past conditional volatility does not influence volatility in external debt. The results also revealed that there was a unidirectional volatility transmission between LGDP and LED; LCF and LED. The model evaluation results suggested that the GARCH(1,1) is more efficient in modelling financial data in South Africa as compared to the other two techniques. The recommendations for future studies were recommended based of the findings of the study.

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CHAPTER 1
OVERVIEW OF THE STUDY

1.1 INTRODUCTION

The study modelled the determinants of external debt using the vector error correction model (VECM), generalised autoregressive conditional heteroscedasticity (GARCH) and multivariate GARCH models. The issue of external debt in developing countries has been an ongoing concern ever since these countries achieved independence from their European colonial rulers. External debt has been identified as one of the major obstacles to the growth of the economy in most developing countries (Bader and Magableh, 2009). Some of the reasons external debt hinders the growth of an economy is due to the continual mismanagement of resources, unemployment rate, corrupt government, high rate of population growth and poverty rate.

The most frequently mentioned determinants of external debt in the 1970s and 1980s are the two oil price shocks and the subsequent recession in big industrialized countries, and the change on the global economic policy to name just a few. Awan, Anjum and Rahim (2015:383) are of the view that:-

> *"Corrupt governments are also one of the key factors that increase external debt burden. These governments spend money for their luxurious lifestyles instead of investing foreign borrowing to improve the lives of the poor and the state of the economy."*

Access to external debt is of importance to most developing countries because it can boost the economic growth of a country. External debt, depending on the way it is used, can impact the economy in both a positive and negative way; it impacts the economy positively when used for investment purposes and negatively when used for private and public consumption. In general, a lower rate of external debt affects the economy positively and vice versa (Dritsaki, 2013).

In order to reduce external debt, Economic and Financial Strategy consultant, Luüs (2012) believes that national government should reduce the national budget deficit, and government's overall tax expenditure policy should be conducted within the context of the imperative to limit interest payments in the budget and gradually reduce the proportion of the total government spending that is devoted to servicing the national debt. The Minister of Finance in South Africa during the period 1996-2009, Trevor Manuel, also intended to limit government consumption expenditure as a ratio of gross domestic product (GDP) but that was only in theory, and only practised during the period 2000-2003.

The running of fiscal surpluses in South Africa during 2006 and 2008 decreased dramatically in 2009 when government's counter-cycled fiscal policy saw the emergence of large deficits accompanied by the rise in its debt commitment (Luüs, 2012). In the period between 2011 and 2012, two of the largest international credit ratings changed their view on South Africa's risk ratings, from stable to negative. One of the reasons behind the downgrade was the financial problems fused in the course of the Marikana Massacre in August 2012. In 2011, Moody's commented that "there was a growing risk that the political commitment to low budget deficits and the ability to keep within current debt targets could be undermined by popular pressures". The relationship between external debt and economic growth has been one of the most debated topics in academia and policymakers (Eberhardt and Presbitero, 2015). The first element of the analysis computed by Eberhardt and Presbitero (2015) concerned the presence of a negative long-run relationship between debt and economic growth. An amount of literature exists motivating such a long run relationship between public debt and economic growth.

Fiscal rules are one of the policy measures found to be associated with a higher probability of stabilizing debt (Molnar, 2013). Molnar (2013) further states that reasons for fiscal rules not being followed are related to the reluctance of governments to keep to fiscal discipline, and also how governments abandon their announced plans before implementing them. Molnar (2013) also suggested that the only way government spending, spending of revenue windfalls and the budget balance or debt can be constrained is if fiscal rules are adhered to. These policies may reduce growth, especially if implemented during a recession (Perotti, 2012).

It is evident that over the past years, external debt has produced both an increase and a decrease in the growth of the economy. This proves that financial data is volatile. Volatility is defined as a statistical measure of the dispersion of returns for a given security or market index (Investopedia, 2019). It is important to note that there are two types of indicators; historical volatility and implied volatility. Historical volatility is backward-looking and implied volatility is forward-looking. Volatility models can be used to forecast volatility; this is a central requirement in almost all financial applications. There are different types of models that can be used to model volatility, with the most famous examples being the autoregressive conditional heteroscedasticity (ARCH) and GARCH models.

Many empirical studies modelled external debt assuming that external debt depends on fundamental variables. Various techniques by different researchers have been used to model such a relationship. Chiminya and Nicolaidou (2015), for example, investigated the factors affecting external debt making use of a dynamic panel data analysis using pooled ordinary least square (OLS) estimate and the fixed effect estimator. In a study by Awan, Anjum and Rahim (2015), determinants of external debt were examined by applying the autoregressive distributed lag (ARDL) model and the cointegration technique. It is important for policy makers to understand what drives external debt.

Like for other economies, a study of the behaviour of external debt and its determining factors is very important for South Africa. To the knowledge of this researcher, empirical studies on external debt in South Africa are limited. In this context, the main interest of this study is to examine the determinants of external debt in South Africa and the forecasting ability of the vector error correction (VEC), GARCH and multivariate GARCH models with intent to identify and recommend the best forecasting model.

1.2 BACKGROUND OF THE MODELS

The application and modelling of long run relationships in finance has attracted the attention of both academics and practitioners for many years (Guirguis, 2018). Among many of the academics are Johansen and Juselius (1990), Kao (1999), MacKinnon, Haug and Michelis (1999), Osterwald (1992), Pedroni (1999), White (1980), Lütkepohl (2005), and Engle and Granger (1987). Financial data is volatile in nature therefore it is important to determine the characteristics of the data prior to examining the relationship between variables. Test for stationarity is discussed in the following section.

1.2.1 Stationarity Test

A common belief in most time series techniques is that the data is stationary. A stationary time series is a series whose statistical properties are constant over time (Nason, 2006). There are two types of stationarity, namely, strong (or strict), and weak stationarity. A strong stationarity means that all moments of all degrees (expectations, variances, third order and higher) of the processes are the same at any point (Nason, 2006). With the definition of strong stationarity being too strict for everyday life, weak stationarity is used most of the time. Weak stationarity means that the mean and variance of a stochastic process do not depend on time t and the autocovariance between X_t and $X_{t+\lambda}$ can only depend on the lag λ (λ is an integer) (Nason, 2006).

Statistical tests of unit root hypothesis are of interest to economists and statisticians because they help in evaluating the nature of non-stationarity in macroeconomic data. In particular, these tests help in determining whether the trend is stochastic, through the presence of a unit root, or deterministic, through the presence of a polynomial time trend (Phillips and Perron, 1988). Popular tests for testing for a unit root in time series include the Phillips-Perron (PP) test, the Augmented Dickey Fuller (ADF) test, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test and the Zivot-Andrews test. The PP, ADF and Zivot-Andrews tests test the null hypothesis that the series contains a unit root, and the KPSS test, which was introduced by Kwiatkowski et al. (1992), tests the null hypothesis that the series is stationary. A few of the different types of unit root and stationarity tests are briefly discussed below.

i. Augmented Dickey Fuller (ADF) unit root test

The ADF test was named after American statisticians Dickey and Fuller. The ADF test is an augmented version of the Dickey-Fuller (DF) test which was originally developed in 1979 to test whether the presence of a unit root in an autoregressive model exists. The main objective of the DF test is to test the null hypothesis that $\theta = 1$ in

$$y_t = \theta y_{t-1} + u_t \tag{1.1}$$

against the alternative $\theta < 1$. The hypothesis of the DF to be tested is given as:

H_0: Series contains a unit root
H_1: Series is stationary

where $\mathrm{y_t}$ denotes the observed time series variable, θ is a coefficient, and $\mathrm{u_t}$ is the error term. With most economic and financial time series data having a more dynamic structure, the ADF was developed in 1984 to accommodate more complex models with unknown orders. The alternative model to equation (1.1) is now written as:

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-1} + u_t \qquad (1.2)$$

where Δ represents the change in variables, y_t denotes the observed time series variables, ψ and α_i are coefficients, and u_t is the error term.

ii. Phillips Perron (PP) unit root test

Phillips and Perron (1988) came up with unit root tests that became popular in analysing financial time series (Zivot and Wang, 2013). These tests are similar to the ADF tests and usually give the same conclusions as the ADF tests. The difference between the PP and ADF tests is mainly in how serial correlation and heteroscedasticity in errors are dealt with. The regression equation for the PP test is given by:

$$\Delta y_t = \beta' D_t + \pi y_{t-1} + u_t \qquad (1.3)$$

where Δ represents the first difference of the variables, y_t denotes the observed time series variables, β' and π are coefficients and u_t is I(0) and may also be heteroskedastic. The hypothesis to be tested is given as:

H_0: Series contains a unit root

H_1: Series is stationary

iii. Kwiatkowski-Phillips-Schmidt-Shin (KPSS) unit root test

The ADF and PP unit root tests test the null hypothesis that a time series y_t is I(1). Stationarity tests, however, are for the null hypothesis that y_t is I(0) (Zivot and Wang, 2013). Kwiatkowski et al. (1992) proposed a test of the null hypothesis of stationarity and it is the most commonly used stationarity test. A disadvantage of using the KPSS test is its high rate of Type 1 errors. It is advisable not to use the KPSS test alone but to combine it with either the ADF or PP test in order to confirm the results whether the time series is stationary or not. The KPSS test is given as:

$$KPSS = \frac{1}{T^2} \cdot \frac{\sum_{t=1}^{T} S_t^2}{\hat{\sigma}_\infty^2} \qquad (1.4)$$

where $S_t = \sum_{s=1}^{t} \hat{e}_s$ is a partial sum; $\hat{\sigma}_\infty^2$ is a Heteroskedasticity and Autocorrelation Consistent (HAC) estimator of the variance of $\hat{e}_t$. The hypothesis to be tested is given as:

H_0: Series is stationary

H_1: Series contains a unit root stationary

It is necessary to check whether a series is stationary because correlation could persist in non-stationary time series and may result in a spurious regression (Yule, 1926). Once the series is stationary, a cointegration technique may be applied and a vector VECM can be estimated to determine the short run relationship and long run relationship between variables of interest.

1.2.2 Vector Error Correction Model (VECM)

Stock and Watson (2001:101) stated that "macroeconometricians do four things: describe and summarize macroeconomic data, make macroeconomic forecasts, quantify what is known or unknown about the structure of the macroeconomy, and advise macroeconomic policymakers". In the 1970s, these four tasks were performed using various techniques. However, after the 1970s macroeconomic crisis, none of the techniques used was proved to be trustworthy. In a series of papers, Sims (1972, 1980a, 1980b) advocated the use of vector autoregressions (VARs), with the most significant of these papers being Sims (1980a), 'Macroeconomics and Reality' (Christiano, 2012). The VAR model is a general framework used to describe the dynamic interrelationship among stationary variables. The basic form of a VAR process is given by:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + u_t \quad (1.5)$$

where $y_t = (y_{1t}, \ldots, y_{nt})'$ denotes an $(n \times 1)$ vector of observed time series variables, β_i are $(n \times n)$ coefficient matrices, and u_t is an unobservable zero mean white noise error process with a time invariant covariance matrix.

The VAR processes are favoured in economics and other sciences because they are flexible and easy models to use for multivariate time series data, and are useful tools for forecasting (Lütkepohl, 2013). Estimation of the parameters of the VAR requires that no cointegration exists among the variables and is estimated using time series data that has been transformed. If some of the variables to be modelled with a VAR have stochastic trends, a more useful model setup may be used for analysing the properties of the variables (Lütkepohl, 2013). These stochastic trends are generated by models with unit roots; variables with such trends are non-stationary and can often be made stationary by differencing. They are called cointegrated if the stationary linear combinations exist. In the case of cointegrated variables, the standard VAR model is reparametrized to accommodate the cointegrating relations and this model is called the vector error correction model (VECM).

The basic VECM is a special case of the VAR for variables that are stationary in their differences. The VECM allows to test if there exists a long run and short run relationship between variables. The history of the error correction model (ECM) dates back to Sargan (1964), however, it was only integrated into modern time series econometrics in the mid-1970s from the publication of two significant papers; "The UK demand for broad money" Davidson, Hendry, Srba and Yeo (1978) and also "The UK demand for broad money" Hendry and Mizon (1978). Kanioura and Turner (2003:2) is of the view that "these papers were important because they emphasised the potential importance of levels terms within a time series regression framework as a means of capturing the equilibrium interaction between variables." The general form of a VECM used is:

$$\Delta y_t = \theta_1 + \beta_i \sum_{j=0}^{n} \Delta y_{t-j} + \alpha_i \sum_{j=0}^{n} \Delta x_{t-j} + \delta_i \sum_{j=0}^{n} ECT_{t-j} + u_t \qquad (1.6)$$

where Δ represents the change in variables, ECT_{t-1} is the time-lag error correction term, and the indices $(t, t-j)$ denote the time period of the variables (Engle and Granger, 1987).

With conditional heteroscedasticity being found everywhere in macroeconomic and financial time series, using the VECM can be a disadvantage. One of the disadvantages of using the VECM is the lack in modelling time series data that exhibit heteroscedasticity and volatility. A spurious relationship can also be found in a cointegration relationship due to a high degree of co-volatility. In order to avoid such problems Engle (1982) developed the ARCH model which was later generalized by Bollerslev (1986) in order to model and forecast volatility.

1.2.3 GARCH model

Conventional econometric models assume a constant one-period forecast variance. In the paper by Engle (1982), a new class of stochastic processes called the ARCH processes were introduced. These are zero mean, serially uncorrelated processes with non-constant variances conditional on the past, but constant unconditional variances (Engle, 1982). The ARCH (q) process is given by:

$$\sigma_t = \omega + \sum_{i=1}^{q} \alpha_i {\varepsilon^2}_{t-i} \qquad (1.7)$$

where the constants (ω, α_i) are estimated from empirical data and are nonnegative. The errors are given by:

$$\varepsilon_t = \sqrt{\sigma_t z_t} \quad (1.8)$$

where z terms are independent, standard normal variables (zero mean, unit variance, normal variables).

According to Silvennoinen and Teräsvirta (2009:2) "modelling volatility in financial time series has been the object of much attention ever since the introduction of the ARCH model in the seminal paper of Engle (1982)." Volatility is a key parameter used in most financial applications as it measures the size of the errors made in modelling financial variables. Since the introduction of the ARCH model, numerous extensions of the model have been proposed.

Liu, Erdem and Shi (2011:725) is of the view that "the GARCH model is a predominant approach for modelling and forecasting volatility." It built on Engle's ARCH model that assumed the variation of financial returns was not constant over time but is autocorrelated, or dependent on each other. In the ARCH process, the conditional variance is specified as a linear function of past sample variances only; the GARCH process on the other hand allows for lagged conditional variances as well (Bollerslev, 1986). The GARCH (p,q) process is given by:

$$\sigma_t = \omega + \sum_{i=1}^{q} \alpha_i {\varepsilon^2}_{t-i} + \sum_{i=1}^{p} \beta_i \sigma_{t-i} \quad (1.9)$$

where

$p \geq 0, \qquad q > 0, \qquad \omega > 0$

$\alpha_i > 0, \qquad i = 1, \ldots., q$

$\beta_i > 0, \qquad i = 1, \ldots, p$

In this notation the error term ε_t is said to follow a GARCH process of orders p and q. Note that this works if $\alpha + \beta < 1$, which means the weights estimated must be positive. If $p = 0$ then the model is said to be an ARCH (q) process.

The ARCH and GARCH models have become important tools in analysing time series data, more especially in the financial sector. Ever since the development of these two models, academic researchers have developed extensions to these models, such as the Threshold GARCH (TGARCH) by Zakoian (1994), Quadratic GARCH (QGARCH) by Sentana (1995), Exponential GARCH (EGARCH) by Nelson (1991), Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) by Glosten, Jagannathan and Runkle (1993) and the multivariate GARCH (MGARCH) model which is discussed below.

1.2.4 Multivariate GARCH model

The success of the ARCH and the GARCH model in capturing time-varying variances of economic data in univariate case has motivated many researchers to extend these models into multivariate dimensions (Minović and Simeunović, 2008). The invention of MGARCH models improved the possibilities to better model covariances between financial time series (Wilsum, 2013). One of the many extended multivariate GARCH models is the BEKK (named after Baba, Engle, Kraft and Kroner) model, proposed by Engle and Kroner (1995), which allows a greater range of interactions while also enforcing positive-definiteness. In economics and finance, after the BEKK model was developed, many empirical research used this MGARCH model. The BEKK(1,1,K) model is written as follows:

$$H_t = CC' + \sum_{k=1}^{K} A'_k \varepsilon_{t-1} \varepsilon'_{t-1} A_k + \sum_{k=1}^{K} B'_k H_{t-1} B_k \tag{1.10}$$

where C, A, B are $N \times N$ matrices and C is a triangular matrix. The BEKK(1,1,K) can be simplified as follows:

$$H_t = CC' + A' \varepsilon_{t-1} \varepsilon'_{t-1} A + B' H_{t-1} B \tag{1.11}$$

which is a multivariate equivalent of the GARCH(1,1) model. It should be noted that the parameters of the BEKK model are very large.

In this study, the focus is on modelling the relationship between External Debt (ED), Gross Domestic Product (GDP), Exports (EXP), Government Expenditure (GE) and Capital Formation (CF). The study will fit a VECM, GARCH and BEKK model and examine the forecasting ability of these models using the error measurement approaches such as MSE, RMSE and MAPE.

1.3 PROBLEM STATEMENT

External debt has been identified as one of the major obstacles to the growth of an economy in most developing countries. Some of the reasons for external debt hindering the growth of the economy are the continual mismanagement of resources, corrupt governments, unemployment rates, high rates of population growth, and poverty rates. The most frequently mentioned determinants of external debt in the 1970s and 1980s are the two-oil price shocks, subsequent recession in big industrialized countries, and the change in the global economic policy, to name just a few.

Most studies in the past have used different techniques to model the determinants of external debt, such as Akram (2015), who made use of the ARDL technique, Cholifani (2008) and Bader and Magableh (2009), who applied cointegration analysis, Dritsaki (2013), who used the VECM approach and Waheed (2017), who employed the panel least square method, to mention just a few. There are very few studies that model external debt with GARCH and BEKK models and this study seeks to fill this gap.

Cointegration is an important issue in time series, and conditional heteroscedasticity seems to be found everywhere in macro and financial time series (Lee and Tse, 1996). A study by Franses, Kofman and Moser (1994) found that there can be empirical occasions in which a detected cointegration relationship is spurious due to a high degree of co-volatility. Earlier studies indicate that the Johansen trace test is not robust in presence of heteroscedasticity, and tests based on resampling methods have been proposed to solve this problem. Some of the tests include the non-parametric bootstrap test and two different types of wild bootstrap tests (Englund, 2013). One of the advantages of modelling financial data with GARCH and BEKK models are their ability to model and forecast volatility. The GARCH models and their extensions have shown to be a better fit for modelling time series data that exhibit heteroscedasticity and volatility.

This study aims to fill the gap in existing literature on the determinants of external debt in South Africa. This study seeks to examine the determinants of external debt and the forecasting ability of the VECM, GARCH and BEKK models using the error measurement approaches such as MSE, RMSE and MAPE to determine the ability of the models to forecast future values.

1.4 RESEARCH AIMS AND OBJECTIVES

The aim of this study is to model the determinants of external debt using the VECM, GARCH and BEKK models with the intent to identify and recommend the most effective approach. The objectives of the study are:

- To examine the relationship between External Debt and Exports, GDP, Government Expenditure and Capital Formation in South Africa.
- To fit VEC, GARCH and BEKK models to the abovementioned variables.
- To examine the forecasting ability of the 3 models.
- To determine the best method that can be used to model external debt.

1.5 RESEARCH QUESTIONS

The following research questions were formulated from the research objectives:

- Is there a relationship between External Debt and Exports, GDP, Government Expenditure and Capital Formation?
- Can VEC, GARCH and BEKK be fitted to model the abovementioned variables?
- What is the forecasting ability of the three models?
- Which model is best in modelling the relationship between External Debt and Exports, GDP, Government Expenditure and Capital Formation in South Africa?

1.6 SIGNIFICANCE OF THE STUDY

The significance of this study is to model external debt in South Africa with different types of time series models. The results of this study will give a clear view to further researchers on which models can be used to model external debt in South Africa. The study will show how the VECM, GARCH and BEKK can be fitted using time series in order to forecast external debt. The study will help outline the types of trends that can be noticed when modelling external debt. The findings of this paper will be of importance for making economic decisions and hopefully be beneficial to scholars who are interested in doing research in the area of modelling financial data.

1.7 RESEARCH OUTLINE

The study is divided into five main chapters:

Chapter 1 provides a brief overview of the study. It outlines the introduction and background of the study, problem statement, the aim and objectives of the study, research questions as well as the significance of the study. Chapter 2 delivers the literature review of the different statistical methods used as well as the review of financial literature. Chapter 3 discusses the detailed research methodology and gives a theoretical discussion of the techniques to be used. Chapter 4 provides the data analysis and interpretation of the results. Lastly, Chapter 5 provides the conclusion and the recommendations of the study.

1.8 CONCLUSION

This chapter provided a brief introduction and background of the study. The problem statement was identified and from there, the research aims and objectives, as well as the research questions were clearly explained in detail. This chapter highlighted the importance of modelling external debt using different time series models. Finally, the structure of the research paper was also outlined. The next chapter provides a review of the literature related to the study.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

The current chapter reviews the theoretical and empirical literature of different studies related to the models used in the study. The intention of the study is to identify any gaps in the literature on modelling the data using VECM, GARCH, BEKK models and any other related models. The determinants of external debt in developing countries have generated a lot of interest among scholars and policy makers in the past years (Cholifihani, 2008). Different researchers have used various techniques to analyse the determinants of external debts, most of them being time series, panel or cross-section techniques. This chapter discusses the literature on financial data. It starts with the theoretical literature and also reviews the literature on factors that influence financial data as well as empirical studies on financial data.

2.2 THEORETICAL LITERATURE

Ahmad, Taskaya-Temizel and Ahmad (2004:1) state that "most financial time series processes are non-stationary and their frequency characteristics are time-dependent." Autoregressive (AR) analysis of time series has been carried out over the past few years and the results are encouraging. Ahmad, Taskaya-Temizel and Ahmad (2004:1) also state that "techniques for time series analysis have ranged from machine learning approaches, which use artificial neural networks for prediction of stock time series, to genetic algorithms that learn to predict." The relevant aspects that form part of the theoretical review of this study are characteristics of the datasets, stationarity, cointegration, VECM, GARCH models and BEKK models.

2.2.1 Stationarity

According to Ssekuma (2011:5), "time series data consist of observations that are considered as a realisation of random variables that can be described by some stochastic processes." Stationarity is a property of an underlying stochastic process and not of observed data. A stationary time series is defined by Nason (2006) as a series whose statistical properties are constant over time. There are two types of stationarity, namely, strict stationarity and weak stationarity. Let $X_t; t \in \mathbb{Z}$ be a time series. A process is said to be strictly stationary if

$$F_{t1+k,t2+k,\dots,ts+k}(b_1, b_2, \dots, b_s) = F_{t1,t2,\dots,ts}(b_1, b_2, \dots, b_s)$$

for any finite set of indices $\{t_1, t_2, \dots, t_s\} \subset \mathbb{Z}$ with $s \in \mathbb{Z}^+$ and any $k \in \mathbb{Z}$.

Therefore, a process $\{X_t; t \in \mathbb{Z}\}$ is strictly stationary if the joint distribution function of the vector $(X_{t_{1+k}}, X_{t_2+k}, \dots, X_{t_{s+k}}) = (X_{t_1}, X_{t_2}, \dots, X_{t_s})$ for any finite set of indices $\{t_1, t_2, \dots, t_s\} \subset \mathbb{Z}$ with $s \in \mathbb{Z}^+$, and any $k \in \mathbb{Z}$. Strong stationarity is defined by Nason (2006:3) as "all moments of all degrees (expectations, variances, third order and higher) of the process anywhere are the same."

A weak stationarity means that the mean and variance of a stochastic process are constant, and the autocovariance between X_t and $X_{t+\lambda}$ can only depend on the lag λ (Nason, 2006). By contrast, non-stationarity can be defined as processes that are not stationary and that have statistical properties that are inevitable functions of time.

Standard unit root tests fail to reject the null hypothesis of a unit root for many economic time series. This was first argued in an influential paper by Nelson and Plosser (1982), who applied Dickey-Fuller type tests to 14 annual United States (US) time series data and failed to reject the hypothesis of a unit root in all but one of the series (Kwiatkowski et al., 1992). The empirical evidence concluded that many or most economic time series contain a unit root. A basic model that may contain a unit root is the AR(1) model.

Consider the AR(1) below,

$$y_t = \emptyset y_{t-1} + u_t \qquad (2.1)$$

where y_t denotes the observed time series variable, $\emptyset$ is a coefficient, and u_t is the error term. If $\emptyset = 1$, equation 2.1 is said to be a nonstationary process and contains a unit root. Popular tests for testing for a unit root in time series include the PP test, KPSS test, the ADF and the Zivot-Andrews test. The ADF and PP unit root tests are the commonly used tests for stationarity. The hypotheses for the ADF and PP tests are given as:

H_0: Series contains a unit root
H_1: Series is stationary

The ADF test is an augmented version of the DF test. The DF test was augmented because in practice, the error of the term in the DF test usually shows evidence of serial correlation. The ADF equation is written as:

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-1} + u_t \tag{2.2}$$

where Δ represents the first difference of the variables, $\mathrm{y_t}$ denotes the observed time series variables, ψ and α_i are coefficients, and $\mathrm{u_t}$ is the error term. The PP unit root tests differ from the ADF tests in how they deal with heteroscedasticity and serial correlation. The regression equation for the PP test is given by:

$$\Delta y_t = \beta' D_t + \pi y_{t-1} + u_t \tag{2.3}$$

where Δ represents the first difference of the variables, $\mathrm{y_t}$ denotes the observed time series variables, β' and π are coefficients and u_t is I(0) and may also be heteroskedastic. The ADF and PP tests usually give the same conclusions, with the PP having an advantage over the ADF because of its ability to correct for heteroscedasticity and serial correlation in the error terms. According to Shin and Schmidt (1992:1), "it is thought to be important for both economic and statistical reasons to be able to distinguish whether a series, after removal of deterministic trend, has a unit root ['is I(1)'] or whether it is stationary ['is I(0)']." Kwiatkowski et al. (1992) proposed a test of the null hypothesis of stationarity. An important thing to note against the use of tests for the null hypothesis of stationarity is the difficulty to control their size when the process is stationary, but highly autoregressive. The KPSS, on the other hand, is oversized in that case. The KPSS test is probably the best-known test for stationarity in econometrics because it rejects the true hypothesis of stationarity too often (Hobijn, Franses and Ooms, 2004). The hypothesis for the KPSS test is given as:

H_0: Series is stationary
H_1: Series contains a unit root

The KPSS test is derived from the following model:

$$y_t = \varphi t + \rho t + \varepsilon_t \tag{2.4}$$

$$\rho_t = \rho_{t-1} + \mu_t \tag{2.5}$$

where deterministic trend is denoted by t with coefficient y and ρ. The error process, ε_t, is an I(1) process from equation (2.4), and μ_t is the error process of equation (2.5). Once the variables have been tested for stationarity, cointegration technique may be applied.

2.2.2 Cointegration

Ssekuma (2011:8) is of the view that "if a group of variables are individually integrated of the same order and there is at least one linear combination of these variables that is stationary, then the variables are said to be cointegrated." It is important to test for the existence of cointegration between variables. A cointegration test is used to establish if there is a correlation between several time series in the long term.

Before cointegration tests were introduced, economists relied on linear regressions to find the relationship between several time series processes. However, Granger and Newbold (1974) argued that linear regression was not a correct approach for analysing time series due to the possibility of a spurious correlation. The concept of cointegration was first recognized by the international community after publication of the seminal article by Engle and Granger in 1987 (Meuriot, 2015). Since the seminal paper by Granger, the cointegration concept has undergone an impressive evolution. Many new methods and concepts have been developed, such as stochastic and deterministic integration and cointegration, polynomial cointegration, with time-varying parameters, for fractional integrated and cointegrated time series in nonlinear framework, and so on (Syczewska, 2011). There are three main methods of testing for cointegration. These methods are used to identify the long-term relationship between two or more sets of variables. These methods are discussed in the following subsections.

2.2.2.1 Engle-Granger Two Step Method

The Engle-Granger two step method creates residuals based on the static regression and then tests the residuals for the presence of a unit root. If the time series is cointegrated, the Engle-Granger method will show the stationarity of the residuals. One of the limitations when it comes to the Engle-Granger method is that if there are two or more variables, the method may show more than two cointegrating relationships. Another limitation with the Engle-Granger method is that it is a single equation model. However, these issues have been addressed in cointegration tests like the Phillips-Ouliaris and Johansen's test.

2.2.2.2 Phillips-Ouliaris test

Phillips and Ouliaris (1990) showed that under the null hypothesis of no cointegration, the ADF and PP unit root tests applied to the estimated cointegrating residuals do not have the DF distributions. Due to the spurious regression phenomenon under the H_0, the distribution of the

ADF and PP unit root tests have asymptotic distribution that are functions of Wiener processes that depends on:

- The number of deterministic trend terms
- The number of variables in which cointegration is being tested.

These distributions are known as the Phillips-Outliaris distributions and their critical values are tabulated.

2.2.2.3 Johansen test

The Johansen multivariate test was derived by Johansen (1991) in order to test for cointegration in multivariate time series (Englund, 2013). Johansen's method builds cointegrated variables directly on maximum likelihood (ML) estimation rather than relying on OLS estimation. An advantage of the Johansen approach is that if the data set contains two or more time series, it can estimate more than one cointegrating relationship. The Johansen cointegration method has two different likelihood ratio tests of significance, and they are the trace test and the maximum eigenvalue test. These two test statistics are shown in equations 2.6 and 2.7.

$$\lambda_{trace} = -T \sum_{i=r+1} log\,(1-\lambda) \tag{2.6}$$

$$\lambda_{max} = -T log(1-\lambda_{r+1}) \tag{2.7}$$

where r is the number of separate series, T is the number of usable observations and λ is the estimated eigenvalues. "The trace test tests the null hypothesis of r cointegrating vectors against the alternative hypothesis of n cointegrating vectors. The maximum eigenvalue test, on the other hand, tests the null hypothesis of r cointegrating vectors against the alternative hypothesis of r + 1 cointegrating vectors" (Osterholm and Hjalmarsson, 2007:5).

2.2.3 Vector Error Correction Model (VECM)

The VECM is a special case of the VAR for variables that are stationary in the differences. It allows to test if a long and short run relationship exists between variables. If cointegration has been detected between series, then there exists a long-term equilibrium relationship between them. The VECM is applied "in order to evaluate the short run properties of the cointegrated series" (Asari et al., 2011:51). The general form of a VECM model is given by:

$$\Delta y_t = \theta_1 + \beta_i \sum_{i=0}^{n} \Delta y_{t-i} + \alpha_i \sum_{i=0}^{n} \Delta x_{t-i} + \delta_i \sum_{i=0}^{n} ECT_{t-i} + u_t \tag{2.8}$$

where Δ represents the first difference of the variables, ECT_{t-1} is the time-lag error correction term and the indices $(t, t-i)$ denote the time period of the variables (Engle and Granger, 1987). Equation (2.8) represents the error correction model, which contains the long-run cointegration relationship in the form of the lagged residuals obtained from the estimated long-run cointegration equation (also known as the error correction mechanism) as well as the short-run dynamic structure allowing for adjustment towards equilibrium. The coefficients in the VECM describe how deviations from the long-run relationship affect the changes in them in the next period.

2.2.4 GARCH model

Song (2009:3) is of the view that "in the last few decades, so many volatility models have been put forward. The most popular and successful models among them are the autoregressive conditional heteroscedasticity (ARCH) and extended to generalized ARCH (GARCH) model by Bollerslev (1986)." After ARCH and GARCH models were introduced, many researchers proposed the extensions to the models and alternative specifications to the models such as Asymmetric GARCH model (Engle (1990)), GARCH-M, IGARCH, EGARCH (Nelson, 1991), Threshold GARCH (Glosten et al., 1993) and Fractionally Integrated model (Baillie et al., 1996) to name a few. The development of these alternative models was to try and improve the GARCH model in capturing the characteristics of return series (Lim and Sek, 2013). Frimpong and Oteng-Abayie (2006:7) indicated that "the EGARCH and the TGARCH models are specifically designed to capture the asymmetry shock to the conditional variance. In the EGARCH model, the natural logarithm of the conditional variance is allowed to vary over time as a function of the lagged error terms rather than lagged squared errors." The EGARCH model can be written as:

$$r_t = \mu + \phi r_{t-1} + \varepsilon_t \; ; \; \varepsilon_t \, / \, \Phi_{t-1} \sim N(0, h_t) \tag{2.9}$$

$$lnh_t{}^2 = \varpi + \alpha \left| \frac{\varepsilon_{t-1}}{h_{t-1}} \right| + \gamma \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta lnh_{t-1}{}^2 \tag{2.10}$$

The exponential nature of the EGARCH ensures that the conditional variance is always positive even if the parameter values are negative, therefore there is no need for parameter restrictions to impose non-negativity; γ captures the asymmetric effect.

The TGARCH modifies the original GARCH specification using a dummy variable. The TGARCH is based on the assumption that unexpected changes in the market returns have different effects on the conditional variance of the returns (Frimpong and Oteng-Abayie, 2006:8). The TGARCH model is written as:

$$r_t = \mu + \phi r_{t-1} + \varepsilon_t \; ; \; \varepsilon_t \,/\, \Phi_{t-1} \sim N(0, h_t) \tag{2.11}$$

$$h_t = \varpi + \alpha {\varepsilon_{t-1}}^2 + \gamma {\varepsilon_{t-1}}^2 \zeta_{t-1} + \beta h_{t-1} \tag{2.12}$$

where ζ_{t-1} =1 if $\varepsilon_{t-1} < 0$ and ζ_{t-1} = 0 if $\varepsilon_{t-1} > 0$.

The MGARCH models include the BEKK model, Constant Correlation Model (CCC) (Bollerslev, 1990), the VEC model (Bollerslev, Engle and Wooldridge, 1988) and Dynamic Conditional Correlation Model (DCC) (Tse and Tsui, 2002, and Engle, 2002). The MGARCH models specify equations for how the variances and covariances move over time. The disadvantage of the multivariate approach is that the number of parameters to be estimated in the GARCH equation increases at a great rate, which limits the number of assets that can be included (Minović and Simeunović, 2008). The CCC model is an MGARCH model in which all conditional correlations are constant and the conditional variances are modelled by univariate GARCH models. The CCC model is given by.

$$h_t = c + \sum_{j=1}^{q} A_j {\varepsilon_{t-j}}^2 + \sum_{j=1}^{p} B_j h_{t-j} \tag{2.13}$$

where c is $n \times 1$ vector, A_j and B_j are diagonal $n \times n$ matrices, and ${\varepsilon_{t-j}}^2 = \varepsilon_{t-j} \odot \varepsilon_{t-j}$ is the element-wise product. H_t is ensured positive definite when the elements of c and A_j and B_j are positive. There also exists a CCC model for which A_j and B_j are not diagonal.

For N-dimensional returns, Tse and Tsui (2002) assume that the conditional matrix follows the model. The correlation structure can be extended to the general DCC-GARCH model given as:

$$\rho_t = (1 - \theta_1 - \theta_2)\rho + \theta_1 \Psi_{t-1} + \theta_2 \rho_{t-1} \tag{2.14}$$

$$\psi_{ij,t-1} = \frac{\sum_{m-1}^{M} e_{i,j-m} e_{j,t-m}}{\sqrt{(\sum_{m=1}^{M} {e_{i,t-m}}^2)(\sum_{m=1}^{M} {e_{j,t-m}}^2)}} \, , \; e_{it} = \frac{e_{it}}{\sqrt{\sigma_{iit}}} \tag{2.15}$$

where θ_1 and θ_2 are scalar parameters, $\theta_1, \theta_2 > 0$ and $\theta_1 + \theta_2 < 1$. ψ_{t-1} is the N×N sample correlation matrix using shocks. If $\theta_1 = \theta_2 = 0$, the CCC model is obtained. The estimation of the two scalar parameters θ_1 and θ_2 requires special constraints to ensure positive definiteness of the correlation matrix.

The GARCH model is a powerful approach for modelling and forecasting volatility. While ARCH models incorporate the feature of autocorrelation observed in return volatility of most financial assets, GARCH improves ARCH by adding a more general feature of conditional heteroscedasticity (Matei, 2009). Frimpong and Oteng-Abayie (2006:6) state that “the GARCH (1,1) model is based on the assumption that forecasts of time varying variance depend on the lagged variance of the asset. An unexpected increase or decrease in returns at time t will generate an increase in the expected variability in the next period.” The GARCH (p,q) model can be expressed as:

$$\sigma_t = \omega + \sum_{i=1}^{q} \alpha_i {\varepsilon^2}_{t-i} + \sum_{i=1}^{p} \beta_i \sigma_{t-i} \tag{2.16}$$

where $p \geq 0,\ q > 0,\ \omega > 0$

$\alpha_i > 0, \qquad i = 1, \dots, q$

$\beta_i > 0, \qquad i = 1, \dots, p$

the error term ε_t is said to follow a GARCH process of orders p and q.

2.2.5 Multivariate GARCH-BEKK model

The ARCH and GARCH models have generated a large spectrum of models, which have been applied and tested in many areas (Song, 2009). As mentioned in chapter 1, the success of the ARCH and GARCH models in modelling and forecasting time-varying variances of economic data in univariate cases has motivated many academic researchers to extend these models into multivariate dimensions (Minović and Simeunović, 2008). These models allow for time-varying conditional variances as well as covariances (De Goeij and Marquering, 2004). The BEKK model of Engle and Kroner (1995) was developed to ensure positive definiteness. The model “achieves positive definiteness of the conditional covariance by formulating the model in a way that this property is implied by the model structure” (Su and Huang, 2010:7). The BEKK model is given by:

$$H_t = CC' + \sum_{k=1}^{K} {A'}_k \varepsilon_{t-1} {\varepsilon'}_{t-1} A_k + \sum_{k=1}^{K} {B'}_k H_{t-1} B_k \tag{2.17}$$

where C, A, B are $N \times N$ matrices and C is a triangular matrix. According to Su and Huang (2010:8), “the purpose of decomposing the constant term into a product of two triangular matrices is to guarantee the positive semi-definiteness of H_t.” The first order BEKK model is simplified as follows:

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \quad (2.18)$$

The BEKK model also has its diagonal form by assuming A_k and B_k matrices are diagonal. Estimation of a BEKK model bears large computations due to several matrix transpositions (Su and Huang, 2010).

2.3 EMPIRICAL LITERATURE

In the year 2017, the South African government maintained a wise approach to its borrowing requirement and higher financial cost. The response to major fiscal decisions resulted in the medium-term borrowing requirement declining significantly from projections made in the 2017 Medium Term Budget Policy Statement. Investor confidence improved as well following the political developments in December 2017 which led to a stronger exchange rate and lower government funding costs (National Treasury, 2018). Reports by the National Treasury (2018) also state that the economy benefited from strong growth in higher commodity prices, agriculture, and improving investor sentiment. The world economic growth has been at its highest since 2014 and is continually growing. The International Monetary Fund (IMF) forecasted global growth of 3.7% in 2017 and 3.9% in 2018.

The Supply-leading financial development hypothesis that states that financial development leads to economic growth has been supported by many researchers such as King and Levine (1993), who showed that countries that have a less developed financial system grow much slower than countries that have a more developed financial system; also the study by Rajan and Zingales (1998) concluded that the industries that are more dependent on the financial sector grow at a much higher rate in countries that have a more developed financial system (Songul, 2011). These results are just some examples of the fact that causality moves from financial development to economic growth. The demand-following hypothesis, on the other hand, assumes a causal relationship from economic growth to financial development. This means that a growth in the economy increases the demand for financial services, and as a result, financial development leads to economic growth (Songul, 2011).

Chiminya and Nicolaidou (2015) investigated the economic factors affecting external debt and also incorporated political factors for the period from 1975 to 2012. Chiminya and Nicolaidou (2015) focused on sub-Saharan Africa for a number of reasons, one of them being that the region has gone through political transitions, from dictatorships to democratic regimes. The study made use of dynamic panel data analysis using pooled OLS estimate and the fixed effect estimator. The system GMM was also used for robustness check. The results showed that growth estimates are negative and statistically significant, which emphasises the significance of economic activity in reducing debt in the region. More importantly, the results gave evidence on the role that political factors play; it showed that governments which are not constrained accumulate more debt and that democratic governments accumulate more debt than autocratic regimes.

Awan, Anjum and Rahim (2015) used the annual data for the period 1976 to 2010 to investigate the macroeconomic determinants of external debt in Pakistan. Cointegration technique was used to analyse the long run equilibrium relationship and ECM was used to find the short run dynamics. The ARDL model was also applied. The study found that exchange rate, fiscal deficit, and trade openness are statistically significant determinants of external debt. The study also found evidence of a positive long run association between fiscal deficit and external debt, nominal exchange rate and the external debt burden of Pakistan. The study by Al-Fawwaz (2016) also applied the ARDL model to investigate the impact that trade openness, terms of trade, exchange rate and GDP per capita have on external debt in Jordan for the period 1990 to 2014. The study showed that there is a positive statistically significant effect that the terms of trade variable has on external debt in the long run, and a negative statistically significant effect for the GDP per capita variable on external debt.

Akram (2015) examined the implications of external debt for economic growth and investment for the Philippines for the period 1975 to 2010. The study also used the ARDL technique to model the data. The results revealed that external debt has a negative and significant relationship with economic growth and investment. It also revealed domestic debt to have a negative relationship with investment and a positive relationship with economic growth. For economic growth to increase in developing countries, Akram (2015) suggested that these countries should put in place policies that will help them reduce their debt burden and not to allow the debt to reach an unsustainable level.

Since the Old Order Indonesia used foreign borrowing to finance development, the country became largely indebted by 1966, because in the mid-1960s, foreign borrowing was greater than export earnings (Cholifihani, 2008). The paper by Cholifihani (2008:1) analysed the "long term and short term relationships between public debt service and GDP by applying cointegration analysis of time series model from 1980 to 2005.” The results showed that GDP, capital stock, debt service, labour force and human capital inputs have a long run equilibrium relationship. The ratio between external debt services and GDP showed a significant negative relationship in the long run.

The study by Bader and Magableh (2009) also used cointegration analysis to model the determinants of external debt in Jordan for the period 1980 to 2005. The study aimed to examine how external debt and domestic debt respond to changes in some explanatory variables. The variables included savings gaps, size of foreign aid, chronic government budget deficit and real exchange rate. The results revealed that real exchange rate is the most effective and significant variable in affecting the outstanding balance of external debt, followed by the financial position of the government and the foreign aid. Additionally, the results showed that chronic government deficit not only decreases the government’s ability to pay the debt service of outstanding loans but also creates a demand for new public loans.

Dritsaki (2013) investigated how exports affect the growth of the economy and also included government debt as a third variable for the period 1960 to 2011. Exports have been regarded as a determinant which promotes economic growth and therefore increases the rewards of factors of production. The VECM and the Granger Causality technique were used to explore the presence of causality among the variables. The results revealed that a long and short run relationship exists among these variables. The results also showed that a unilateral causal relationship in the short run exists between exports and economic growth, and from economic growth to external debt, but there was no causal relationship between exports and external debt.

The study by Tiruneh (2004) analysed poverty, income stability and outside factors such as debt service payments and capital flight to be the main causes of external debt in developing countries. The study used random and fixed effects models and cross-section pooled time series. The results showed that debt service payments, the imports-to-GDP ratio, capital flight, income per capita, and the growth rate of GDP to be the main determinants of the increasing demand of external debt.

In 2013, Baum, Checherita-Westphal and Rother used a dynamic threshold panel model to examine the non-linear impact of public debt-to-GDP ratios on GDP growth on 12 euro-area countries for the period 1990 to 2010. The results suggested “that the short-run impact of debt on GDP growth is positive and highly statistically significant, but it decreases to approximately zero and loses significance beyond public debt-to-GDP ratios of around 67%" (Baum, Checherita-Westphal and Rother 2013:1). For high debt-to-GDP ratios (above 95%), debt has a negative impact on economic activity.

The study by Imimole, Imoughele and Okhuese (2014) applied the Johansen cointegration and ECM to analyse the determinants of public external debt and its sustainability in the economy of Nigeria for the period from 1986 to 2010. The study found that a long run relationship exists between external debt and its determinants. Imimole, Imoughele and Okhuese (2014:211) "observed that Nigeria's external debt is not sustainable in terms of willingness and ability to pay." To reduce the effect of external debt on the economy and make it sustainable, the study made a few recommendations: - firstly, the use of external borrowed funds for government projects must be monitored very closely in order to ensure that they are applied effectively and efficiently, and secondly, government should restructure its revenue base to finance fiscal deficit expansion instead of embarking on external borrowing.

Waheed (2017) employed the panel least square method to examine the macroeconomic determinants of external debt in oil and gas importing and exporting countries covering the period 2004 to 2013. The study focused on oil and gas exporting and importing countries for a number of reasons such as:- firstly, exporting countries mostly generate a lot of income while importing countries are mostly low-income indebted countries; secondly, exporting countries have an excess in their current account while importing countries have a deficit in their account, and thirdly, countries in both classifications suffer badly from high external debt, and many of them are classified as highly indebted poor countries by the World Bank. The results of the panel data analysis showed that the determinants of external debt and their effects differ between exporting and importing countries. The results of the panel data for exporting countries showed that increased economic growth, foreign exchange reserves, general government revenue, price of oil and domestic investment are essential factors in reducing external debt, while the results of the importing countries differed slightly. The results further showed that the increase in economic growth, general government revenue and gross domestic savings are essential factors in reducing external debt.

Shiddiqui and Malik (2001) studied the growth experience of selected South Asian countries for the period 1975 to 1998 using OLS and Fixed Effects models. According to Shiddiqui and

Malik (2001), macro imbalances, mismanagement of resources, the role of political interest groups and loss of competitiveness in the international market has worsened the debt burden in Pakistan. The fixed effect model results showed that all the indicators of debt burden included in the study (Investment-to-GDP ratio, deficit-to-GDP ratio, openness, population growth, debt servicing-to-export ratio, debt-to-GDP ratio and debt-to-export ratio) somehow indicated how important improving the economic management is.

The study by Barusman et al. (2018) used the GARCH model to analyse weekly closing share price data of JAPFA Comfeed Indonesia over the period June 2015 to October 2016. The conditional mean model ARIMA(0,1,2) and conditional variance model GARCH(1,1) were found to be the best models. The results showed that the application of the two models for forecasting the share price data for the following 5-week period were very close to the real values, and that the forecast values were within the 95% confidence interval.

Malik (2017) attempted to present a basic method of time series analysis, modelling and forecasting performance of ARIMA, GARCH(1,1) and ARIMA-GARCH(1,1) models using daily closing price for the GE company in the USA over the period 2001 to 2004. The results showed that the 95% confidence interval of the ARIMA(2,1,2) is wider than that of the combined model ARIMA(2,0,2)-ARCH(1,1). The ARIMA-GARCH forecasting shows that the share price moves between 22.19 and 34.04.

Lim and Sek (2013) used symmetric and asymmetric GARCH models to model volatility of the Malaysian stock market. The data used in the study ranged from January 1990 to December 2010 and was divided into three-time frames, namely, pre-1997 crisis, during the crisis, and post-1997 crisis. Three statistical error measures, the MSE, RMSE and MAPE were used to compare the performances of the GARCH type models for in-sample and out-of-sample analysis. The results showed that symmetric and asymmetric GARCH models perform differently in the different time frames. For the pre- and post-crisis periods, the results showed that the symmetric GARCH model performs better than the asymmetric GARCH model, but for the crisis period the asymmetric GARCH model performed better. The results also show that exchange rate and crude oil price have significant impacts on the Malaysian stock market volatility in the normal period (pre- and post-crisis) but were not significant in the crisis period.

In the study by Gabriel (2012), daily stock index return data from Romania, covering the period 3 September 2001 to 29 February 2012, was used to analyse the forecasting performance of GARCH-type models in terms of in-sample and out-of-sample forecasting accuracy. The forecasting accuracy of each model was measured by the RMSE, MAE, MAPE, and TIC. Based on the results obtained from the error measures it was concluded that the TGARCH model is the most successful model in forecasting volatility of the BET index.

Franses and Van Dijk (1996) studied the forecasting performance of the GARCH, Quadratic GARCH, and the GJR GARCH models to forecast weekly stock market volatility. The data covered the period from 1986 to 1989. The results showed that the QGARCH model is able to significantly improve on the linear GARCH model when extreme observations such as the 1987 stock market crash are not included in the estimation sample. The results also show that the GJR GARCH model cannot be recommended for forecasting.

The study by Frimpong and Oteng-Abayie (2006) models and forecasts volatility on the Ghana Stock Exchange using random walk, GARCH(1,1), EGARCH(1,1) and TGARCH(1,1) models. The sample of the data used was the daily closing prices of the Ghana Stock Exchange Databank Stock Index (DSI) over the period 15 June 1994 to 28 April 2004. The volatility models were estimated, their specification and forecasting performance were compared using LL information, AIC and BDS nonlinearity diagnostic checks. The forecasting performance of the models was evaluated using RMSE, MAE, MAPE and TIC. The random walk hypothesis was rejected for the DSI, the GARCH(1,1) model outperformed the other models and the EGARCH(1,1) performed the least well in forecasting the conditional volatility of the DSI returns.

The study by Duasa (2007:1) had two main objectives: - firstly "to examine the casual relationship between foreign direct investment and economic growth in Malaysia and second, to look at the impact of FDI on the stability of economic growth and the impact of growth on stability of FDI." Quarterly data that covered the period from the first quarter of 1990 to the last quarter of 2002, making a total of 52 observations, was used. The Toda and Yamamoto's (1995) non-causality test was used to establish the direction of causation between the two variables, and the impact of growth on stability of FDI was tested using the GARCH model. The results of the Toda and Yamamoto test found no solid evidence of causal relationship between FDI and economic growth. The GARCH model results showed that in the case of

Malaysia, FDI does not cause economic growth and vice versa, but FDI does contribute to stability of economic growth, as economic growth contributes to stability of FDI.

Fountas, Karanasos and Mendoza (2004) examined the empirical relationship between output variability and output growth for the Japanese economy using quarterly data from 1961 to 2000. The Bollerslev's model, Taylor/Schwet's model and Nelson's EGARCH model were used. The results show that the "in-mean" coefficient is not statistically significant. The findings support various theories of economic fluctuations, that output variability does not affect output growth. The results also reveal no evidence of asymmetry between output variability and growth.

Chen and Zapata (2015) examined hog price linkages between the USA and China from June 1996 to December 2013. The MGARCH-BEKK model was used to model volatility and spillover effects. The results reveal that volatility in Chinese hog prices was explained by own-price volatility and past unexpected events, and the American hog price volatility was mostly explained by its own past events.

Au-Yeung and Gannon (2002) used the BEKK model with multiple switch points in the variance equations, and also captured the structural changes that took place in the Hong Kong markets. The results found that the BEKK(1,1) model with 3 switching points in the variance equation was better than the models with fewer switching points. These results showed that there are significant impacts on the informational efficiency in the stock and futures market following policy changes.

The study by Sadorsky (2012:1) used MGARCH models to model "conditional correlations and to analyse the volatility spillovers between oil prices and the stock prices of clean energy companies and technology companies." The data covered the period from 1 January 2001 to 31 December 2010. The BEKK, DVECH, CCC and DCC models were used. The DCC model was preferred over the other models, with the BEKK model being a second choice. The BEKK model also produced more evidence of volatility spillovers than the DCC did.

Mohammadi and Tan (2015) analysed the dynamic relationships between the two stock exchanges of mainland China, Hong Kong and the USA. Daily data from 2 January 2001 to 8 February 2013 was used. Different time series models were used and the results showed that:

- The VAR model suggested unidirectional spillovers in returns from the US to the other three markets, with the returns of mainland China and Hong Kong having no influence on each other;
- The BEKK model suggested unidirectional volatility spillovers from the US to the other three markets.
- The CCC model suggested that mainland China's two markets were highly correlated, but by contrast, correlations between China's two markets and Hong Kong and the US were low.

Lastly, the patterns of the DCC model suggested moderate increase in conditional correlation between China and other stock markets since the 2007 financial crisis. Li and Giles (2013:1) also examined "linkages of stock markets across the US, Japan and six Asian developing countries over the period January 1993 to December 2012. The volatility spillover was modelled with an asymmetric MGARCH model." The results of the asymmetric BEKK model revealed a significant unidirectional shock and volatility spillovers from the US market to both Japanese and the Asian emerging markets. The results also revealed that volatility spillovers between the US market and the Asian markets were stronger and bidirectional during the Asian financial crisis.

The study of Nortey et al. (2015) investigated the volatility and conditional relationship among inflation rates, exchange rates and interest rates, and also constructed a model using BEKK and DCC models with data from Ghana from January 1990 to December 2013. The results revealed that the BEKK model was robust to model and forecast volatility of exchange rates, inflation rates and interest rates. The results of the DCC model revealed that the DCC model is robust to model the conditional and unconditional correlation among exchange rates, inflation rates and interest rates.

The study of Su and Huang (2010) focused on the diagnostics of the BEKK and DCC models. The results of the MAE showed that the fitting performance of the BEKK-GARCH form is better than DCC-GARCH. The difference between the two was due to the number of parameters of the BEKK model were more; so the BEKK model had a better capability in explaining the information hidden in the historical data.

Bala and Takimoto (2017) examined stock returns volatility spillovers in emerging and developed markets during the financial crisis (2007-2009) using MGARCH models and their variants. The different MGARCH models used in the study were the DVECH, CCC, CCC-VARMA-(A), CCC-VAR-EGARCH, BEKK-MGARCH, DCC-MGARCH (with Gaussian and *t* densities) and DCC-with-skewed-*t* density models. The results revealed that in terms of selecting the best MGARCH model for analysing volatility interactions and spillover, the DCC-MGARCH-with-skewed-*t* was recommended because it was more suited than its competitors when skewness and fat tails are present in the data.

Asteriou and Price (2000) examined the influence of political instability on the economic growth of the UK between 1961 and 1997. Six variables that quantify political instability were constructed and their effect on economic growth was examined using GARCH models. The results revealed that there is a strong link between political instability and growth of UK GDP per capita. The results from simple linear regressions and GARCH models show that political instability affects GDP growth negatively. The GARCH-M models were estimated, and revealed negative effects of instability on growth, and positive effects on growth uncertainty.

The study by Lee (2010) used a dynamic panel GARCH model for G7 countries over the period from 1965 to 2007 to examine the link between output growth and volatility. Estimation results show that higher output growth is associated with higher volatility of the innovations to growth, but higher growth does not lead to more economic uncertainty.

Tabassaum, Hashmi and Rehman (2016) investigated how political instability affects the economic growth of Pakistan using annual data for the period 1988 to 2010. The variables used in the study were GDP as the dependant variable, and Elections, Terrorism, Regimes and Strikes as explanatory variables. ARCH and GARCH models were used to examine the outcome of political uncertainty on the economic growth. The GARCH (1,1) model found that only terrorism has a negative significant effect on the mean equation of the dependent variable. The variance equation results show that elections and regimes have significant negative effect on volatility of GDP.

Setiawan and Maekawa (2014) extended the VECM assumption of the iid normal distribution of disturbance term in the model by including the GARCH error process. By extending the VECM model, the parameters in the VEC-GARCH model are large and the ML method is computationally demanding. The study searched for alternative estimation methods to overcome computational difficulties and compared them by Monte Carlo simulation. To overcome the computational burden of ML estimator, a feasible generalized least square (FGLS) estimator was considered. The results of the feasible generalized least square (FGLS) estimator showed comparable performance to ML estimator.

The study by Hansson, Andersson and Holmberg (2015) compared ARMA and GARCH models for three metal commodities. The data of silver price covered the period between 1980 and 2013, while the data for nickel and copper price covered the period between June 2004 and June 2014. The models forecasted values and running standard deviations were cross-validated using MASE, MAPE and correct pairs of sign measure. For forecast values ARMA and GARCH were equal in accuracy. However, the GARCH was found to be more efficient at forecasting the running standard deviation.

The study by Anorou and Braha (2008) employed the cointegration analysis and GARCH enhanced VECM to examine the relationship between housing and stock market returns for the US. The study used the quarterly time series data on housing and the S&P 500 indexes covering the first quarter of 1975 to the fourth quarter of 2005. The results from the GARCH enhanced VECM suggested that housing market returns affected the stock market returns through the statistically significant error correction term. The results also showed that the housing and stock markets are integrated rather than segmented.

Kavussanos, Visvikis and Alexakis (2008) analysed the lead-lag relationships in daily returns and volatilities between cash and future price series in the FTSE/ATHEX and FTSE-ATHEX Mid-40 markets. The study made use of Granger causality, VEC-GARCH model and GIR analysis to model the data. The Augmented Bivariate VEC-GARCH-X models were also used to examine the volatility spillovers. The results showed that there are bi-directional spillover effects between cash and future markets. Both the lagged squared disequilibrium ECTs and lagged futures volumes were found to be significant determinants of conditional volatilities in cash and future markets. Another study by Kavussanos, Visvikis and Dimitrakopoulos (2010) used the Granger causality and VEC-GARCH model approach to examine return and volatility spillover effects between Panamax freight and commodity derivatives markets. The dataset consists of daily Panamax Baltic Forward and commodity future prices series covering the period September 2005 to December 2008. The results showed that there were significant spillover effects between freight and commodity derivatives markets.

Song (2009:1) reviewed "multivariate GARCH models using the daily data of the four Great China Region stock markets, namely Hong Kong, Shanghai, Shenzhen and Singapore, and data of Japan as one exogenous variable to investigate the volatility and shocks spillover behaviour and also to establish the market linkage among the four markets. The results showed that there is volatility spillover between Shanghai and Shenzhen and that correlation between the two regions is very high, ranging from 0.75 to 0.98. Hong Kong and Singapore also present a mildly high correlation ranging from 0.25 to 0.9. The results presented evidence that Chinese stock markets are more integrated with the global markets, and the Great China Region markets are more integrated with one another.

2.4 CONCLUSION

The purpose of this chapter was to review the theoretical and empirical literature on financial data and factors influencing external debt. There seem to be many factors that drive external debt such as GDP, savings gaps, chronic government budget deficit, size of foreign aid and real exchange rate, to name just a few. The empirical literature revealed that different time series models have been used to model financial data in the past; for example, Waheed (2017) employed the panel least square method to examine the macroeconomic determinants of external debt in oil and gas exporting countries, Akram (2015) examined the implications of public debt for economic growth and investment for the Philippines by using the ARDL technique. Cholifihani (2008) analysed the long- and short-term relationships between public debt service and GDP by applying cointegration analysis.

The VECM was used in many studies to model financial data. The study by Awan, Anjum and Rahim (2015) examined the macroeconomic determinants of external debt in Pakistan. Dritsaki (2013) also used the VECM to investigate how exports affect the growth of the economy, with the inclusion of government debt as a third variable. The study by Imimole, Imoughele and Okhuese (2014) also applied the VECM to examine the determinants of public debt. The results of the abovementioned studies revealed that there is a relationship between external debt and its determinants.

In the studies of Su and Huang (2010), Bala and Takimoto (2017), and Sadorsky (2012), the results revealed that the DCC model performed better than the BEKK model in modelling financial data. Hansson, Andersson and Holmberg (2015), Barusman et al. (2018), Lim and Sek (2013), and Frimpong and Oteng-Abayie (2006) revealed that the GARCH(1,1) model performed better than other models when it came to modelling financial data.

The empirical models to be estimated later in this study are expected to show a relationship between External Debt and Exports, GDP, Government Expenditure and Capital Formation in South Africa. The next chapter discusses the research methodology of the study.

CHAPTER 3
RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter outlines the research methodology applied to model the South African financial data. The methods used in the data analysis will be discussed in this chapter. The chapter describes the data and data sources, stationarity test, cointegration test, GARCH model, BEKK model, model diagnostic and the evaluation of the models. The ethical consideration is also presented in the chapter.

The rest of the chapter is organised as follows: Section 3.2 outlines the ethical considerations, section 3.3 discusses the data description and source, Section 3.4 outlines the model specification, Section 3.5 is the test for stationarity. Section 3.6 presents the cointegration test, Section 3.7 is the vector error correction model, Section 3.8 is the GARCH model followed by Section 3.9 which is the BEKK model. Section 3.10 outlines the model diagnostic; Section 3.11 outlines the model evaluation and Section 3.10 is the conclusion.

3.2 ETHICAL CONSIDERATIONS

This study will not include human and animal participation; it only focuses on the analysis of the secondary data obtained from the South African Reserve Bank (SARB) using statistical techniques. The study will cite and reference each and every article that will be reviewed, and interpret the information with clarity, transparency and honesty. The postgraduate manual was referred to when conducting this study to ensure that the rules and regulations of the university are adhered to. Permission to conduct the study was obtained from the university by applying to the research ethics committee through the supervisor after the approval of the proposal.

3.3 DATA DESCRIPTION AND SOURCES

The empirical analysis of this study employs quarterly time series data for South Africa that ranges from the second quarter of 1990 to the last quarter of 2018. The data consists of 115 observations for each variable. The data was extracted from the SARB website. Many variables fail to meet the assumptions of parametric statistical tests, they are either not normally distributed or the standard deviations are not homogeneous, or both, and so using statistical tests (such as ANOVA or linear regression) on such data may give a misleading result. It is for this reason that data is transformed, so that the data will make it fit the assumptions better (McDonald, 2009).

The variables in this study are all expressed in their logarithmic transformation. Log transformation can be used to reduce skewness of the distribution. This can be valuable both for making patterns in the data more interpretable and for helping to meet the assumptions of inferential statistics (Lane et al., 2017). The EViews 10 Student Version, RStudio version 1.1.463 and WinRATS Trail 10.0 software was used to run the analysis. In this study, three different models were used for the analysis, namely the VECM, GARCH and BEKK models. The underlying models were generated by External Debt (ED) being the dependent variable as a function of Gross Domestic Product (GDP), Exports (EXPORTS), Government Expenditure (GE) and Capital Formation (CF). The models are briefly discussed in the following subsections.

3.4 MODEL SPECIFICATION

The underlying models were generated with ED being the dependent variable as a function of EXP, GDP, GE and CF. This can be written as follows:

$$ED_Q = f\left(GDP_Q, EXP_Q, GE_Q, CF_Q\right) \tag{3.1}$$

where

ED_Q= External Debt in quarter Q

GDP_Q= Gross Domestic Product in quarter Q

EXP_Q= Exports in quarter Q

GE_Q= Government Expenditure in quarter Q

CF_Q= Capital Formation in quarter Q.

The next Section, 3.5, discusses the stationarity tests applied in the study.

3.5 UNIT ROOT TESTS

Macroeconomic variables are in most cases non-stationary in their nature, and non-stationary time series may produce spurious results. To avoid spurious results, it is important to test whether time series variables are stationary or non-stationary. Nason (2006:3) defined stationary process as “one whose statistical properties do not change over time.” There are many approaches that can be performed to detect the stationarity of a time series. The most popular stationarity tests are the ADF test, PP test and KPSS test. However, in this study, ADF by Dickey and Fuller (1984) and the PP by Phillips and Perron (1988) unit root tests will be used to test the presence of unit root.

The ADF and PP unit root tests test the null hypothesis that a time series y_t is, *I(1)*, (Zivot and Wang, 2001). A major field where the hypothesis of a unit root has important implications is in economics (Phillips and Perron, 1988). The unit root tests are discussed in the following subsections.

3.5.1 Augmented Dickey-Fuller (ADF) Test

The ADF test is an augmented version of the Dickey-Fuller (DF) test, which was developed originally by American statisticians Dickey and Fuller in 1979 to test the presence of a unit root in an autoregressive model. The ADF unit root test is based on the null hypothesis that there exists a unit root in the time series variables. The ADF test is given as:

$$\Delta y_t = \alpha + \beta t + \delta y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-1} + u_t \tag{3.2}$$

where Δ denotes the first difference of the variables, $\mathrm{y_t}$ denotes the observed time series variables and α is a constant. β, δ, and α_i are coefficients, and $\mathrm{u_t}$ is the error term. The coefficients of δ and γ_i are computed as follows:

$$\delta = -1\left(1 - \sum_{i=1}^{p} \alpha_i\right) \tag{3.3}$$

and

$$\gamma_i = -\sum_{\boldsymbol{j=i}}^{\boldsymbol{p}} \alpha_j \tag{3.4}$$

If $\alpha = 0$ and $\beta = 0$, then equation (3.2) models a random walk. If $\alpha \neq 0$ and $\beta \neq 0$, then equation (3.2) models a random walk with a drift. The parameters in equation (3.2) are estimated using the OLS regression. The hypothesis for the ADF test are given as:

H_0: $\gamma = 0$ (Series contains a unit root)
H_1: $\gamma < 0$ (Series is stationary)

If the p-value of the test statistic is less than 1%, 5% or 10% level of significance, then the null hypothesis of a unit root is rejected, and it is concluded that the series is stationary. An important part of the ADF test is the specification of the lag length p. In this study, the Akaike Information Criteria (AIC) and the Schwarz Bayesian Information Criterion (SBIC) are used to determine the optimal length (p).

3.5.2 Phillips-Perron (PP) Test

Phillips and Perron (1988) came up with an extensive theory of unit root non-stationarity. The tests are similar to the ADF test, but they incorporate a correction to the DF procedure to allow for autocorrelated residuals (Brooks, 2019). The PP test also differs from the ADF test in how they deal with heteroscedasticity in the errors. This test usually gives the same conclusions as the ADF tests. The PP test is given as:

$$\Delta y_t = \theta + \delta y_{t-1} + \varepsilon_t \qquad (3.5)$$

where Δ represents the first difference of the variables, y_t denotes the observed time series variables, θ is a constant and δ is a non-parametric correction to the t-statistic which makes it robust to the presence of serial correlation and heteroscedasticity, ε_t is *I(0)* and may also be heteroscedastic. The modified statistics denoted Z_t and Z_δ are given as:

$$Z_\delta = n\hat{\delta} - \frac{1}{2}\frac{n^2\left(s.e(\hat{\delta})\right)}{\hat{\sigma}^2}\left(\hat{\lambda}^2 - \hat{\sigma}^2\right) \qquad (3.6)$$

$$Z_t = \sqrt{\frac{\hat{\sigma}^2}{\hat{\lambda}^2}}\, t_{\hat{\delta}} - \frac{1}{2}\left(\frac{\hat{\lambda}^2 - \hat{\sigma}^2}{\hat{\lambda}^2}\right)\left(\frac{n\left(s.e(\hat{\delta})\right)}{\hat{\sigma}^2}\right) \qquad (3.7)$$

where $\hat{\sigma}^2$ and $\hat{\lambda}^2$ are consistent estimates of the variance parameters. The hypothesis to be tested is formulated as follows:

H_0: $\delta = 0$ (Series contains a unit root)

H_1: $\delta < 0$ (Series is stationary)

If the p-value of the test statistic is less than 1%, 5% or 10% level of significance, then the null hypothesis of a unit root is rejected and it is concluded that the series is stationary. Section 3.6 discusses the cointegration test.

3.6 COINTEGRATION TEST

Ssekuma (2011:8) is of the view that "if a group of variables are individually integrated of the same order and there exists at least one linear combination of these variables that is stationary, then the variables are said to be cointegrated." In practice, economic time series which contain unit roots move together over time; although the variables under study may drift away from equilibrium for a while, there exist some forces on the series that make them converge upon some long-run value (Xu, 2012). In this study, the Johansen cointegration approach that was proposed by Johansen (1988) and Johansen and Juselius (1990) will be used to test for the presence of cointegration. The Johansen cointegration test is discussed in subsection 3.6.1.

3.6.1 Johansen Cointegration Test

The first step in the Johansen cointegration test is to determine the lag length where a selection of lag length is made. The study used the AIC and SBIC to select the optimal lag length. The formulae for selecting the optimal length p are given as:

$$AIC = ln\left(\hat{\sigma}_p{}^2\right) + \frac{2p}{T} \qquad (3.8)$$

$$SIC = ln\left(\hat{\sigma}_p{}^2\right) + \frac{pln(T)}{T} \qquad (3.9)$$

where T is the sample size and $\hat{\sigma}_p{}^2$ represent the error variance. The error variance is computed using the following equation:

$$\hat{\sigma}_p{}^2 = \frac{\sum_{t=p}^{T} \hat{\mu}_t{}^2}{T-p-1} \qquad (3.10)$$

where μ_t is the error term.

After determining the lag length, the next step will be to compute the Johansen cointegration technique. The Johansen cointegration procedure uses the maximum eigenvalue test and the trace test to determine the number of cointegration vectors (Banumathy and Azhagaiah, 2015). The maximum eigenvalue statistic tests the null hypothesis of r cointegrating relations against the alternative of r + 1 cointegrating relations $\forall r = 0,1,2 \ldots, n-1$. The trace statistic tests the null hypothesis of r cointegrating relations against the alternative of n cointegrating relations, where n is the number of variables in the system for r = 0,1,2,…,n-1. The trace and maximum eigenvalue test statistic will determine the number of cointegrating vectors. The trace test and the maximum eigenvalue test statistic equations are computed using:

$$\lambda_{trace} = -T \sum_{i=r+1} ln\,(1-\lambda) \quad (3.1)$$

$$\lambda_{max} = -T ln(1-\lambda_{r+1}) \quad (3.2)$$

where r is the number of cointegrating vectors, T is the number of usable observations and λ is the estimated eigenvalues. The hypothesis to be tested is given as:

H_0: $r = 0$ (There is no cointegration between the variables)
H_1: $r > 0$ (There is cointegration between the variables)

The H_0 shows that "the number of distinct cointegrating vector is less than or equal to *r* against the alternative that it is greater than *r"* (Banumathy and Azhagaiah, 2015:250). If the test statistics computed is greater than the table value, reject the H_0 that there is no cointegration between variables. Section 3.7 discusses the VEC model.

3.7 VECTOR ERROR CORRECTION MODEL (VECM)

If the presence of cointegration exists, a VECM can be estimated. A VECM is a restricted VAR designed for use with non-stationary series that are said to be cointegrated (Songul, 2011). Banumathy and Azhagaiah (2015:250) state that "the VECM has cointegration relations built into the specification so that it restricts the long-run behaviour of the endogenous variables to converge to their cointegrating relationships while allowing for short-run adjustment dynamics." The cointegration term is known as the error correction term because the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments (Banumathy and Azhagaiah, 2015). If the variables are cointegrated of the same order, then an error correction model exists between the variables and two equations arise:

a) The long run equation:

$$ED_t = \beta_0 + \beta_1 GDP_t + \beta_2 EXP_t + \beta_3 GE_t + \beta_4 CF_t + u_t \quad (3.3)$$

where ED, GDP, EXP, GE and CF represent external debt, gross domestic product, exports, government expenditure and capital formation respectively. The u_t represent the stochastic error term with mean zero and constant variance.

b) The VEC equation:

$$\Delta ED_t = \alpha_1 + \sum_{i=1}^{m} \beta_{1i}\, \Delta GDP_{t-i} + \sum_{i=1}^{m} \theta_{1i}\, \Delta EXP_{t-i} + \sum_{i=1}^{m} \delta_{1i}\, \Delta GE_{t-i} + \sum_{i=1}^{m} \phi_{1i}\, \Delta CF_{t-i} + \varphi_1 ECT_{t-1} + u_t \qquad (3.4)$$

where Δ represents the difference operator, m is the number of lags, ECT is the error correction term and u_t is the stochastic error term with mean zero and constant variance. Section 3.8 discusses the GARCH model.

3.8 GENERALISED AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY (GARCH) MODEL

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model is the leading approach for modelling and forecasting volatility. It was developed by Bollerslev (1986) to generalize the ARCH model proposed by Engle in 1982 (Liu, Erdem and Shi, 2010). The GARCH(1,1) model is the most used GARCH process. In order to estimate a GARCH model, the presence of heteroscedasticity in the series needs to be checked first. A time series exhibiting heteroscedasticity or autocorrelation in the squared series is said to have ARCH effects. Engle's ARCH test is a Lagrange Multiplier test to assess the presence of ARCH effects in a series. The hypothesis for the ARCH test is formulated as:

H_0: There is no heteroscedasticity among residuals

H_1: There is heteroscedasticity among residuals

The test statistic for Engle's ARCH test is the F statistic for the regression on squared residuals. The F statistic follows a χ^2 distribution under the null hypothesis. If the p-value of the χ^2 is less than 5% then the null hypothesis is rejected and it can be concluded that the residuals exhibit the presence of ARCH effects. The GARCH model can now be estimated. The GARCH(1,1) model, is based on the assumption that forecasts of time-varying variance depend on the lagged variance. A sudden or unexpected increase or decrease in the financial data at time *t* will generate an increase in the expected variability in the next period. The GARCH processes have zero mean and are serially uncorrelated with non-constant variances conditional on the past, but with constant unconditional variances. The GARCH (p,q) process is given by:

$$\sigma_t^2 = \lambda + \sum_{i=1}^{q} \alpha_i \varepsilon^2{}_{t-i} + \sum_{i=1}^{p} \beta_i \sigma^2{}_{t-i} \qquad (3.15)$$

where

$p \geq 0, \qquad q > 0, \qquad \omega > 0$

$\alpha_i > 0, \qquad i = 1, \dots, q$

$\beta_i > 0, \qquad i = 1, \dots, p$

The conditional variance,σ_t, is a weighted function of its long-run value (dependent on λ), information about volatility during previous periods, $\alpha_i {\varepsilon_{t-i}}^2$ (the ARCH term), and the fitted variance from previous periods, $\beta_i {\sigma^2}_{t-i}$ (the GARCH term). The model tells us that tomorrow's variance is a function of today's squared innovations, today's variance, and the weighted average long-term variance (Ding, 2018). The model is subject to non-negativity constraints (as shown above) to ensure that the variance is strictly positive. The stationary condition of $\alpha + \beta < 1$ should hold to ensure weak stationarity of the GARCH process (Lim and Sek, 2013). The error term can be specified as:

$$\varepsilon_t = y_t - E\{y_t | \psi_{t-1}\} \tag{3.16}$$

where ε_t is a random, unobservable variable with mean and variance conditional on ψ. The GARCH model for ε_t has $E\{\varepsilon_t | \psi_{t-1}\} = 0$ and $\sigma_t = E\{{\varepsilon_t}^2 | \psi_{t-1}\}$ and is decomposed as:

$$\varepsilon_t = z_t {h_t}^{1/2} \tag{3.17}$$

The sequence $\{z_t\}$ is an iid sequence of random variables with mean zero and unit variance. The next Section, 3.9, presents the multivariate GARCH model.

3.9 MULTIVARIATE GARCH MODEL

The success of the ARCH and GARCH models in modelling and forecasting time varying variances of economic data in univariate cases has motivated many academic researchers to extend these models into multivariate dimensions (Minović and Simeunović, 2008). The multivariate BEKK-GARCH model of Engle and Kroner (1995), which is seen as another restricted version of the VEC model, can be written as:

$$H_t = CC' + \sum_{k=1}^{K} A'_k \varepsilon_{t-1} \varepsilon'_{t-1} A_k + \sum_{k=1}^{K} B'_k H_{t-1} B_k \tag{3.18}$$

where C, A, B are $N \times N$ matrices and C is a triangular matrix. The above equation guarantees all positive definite representations. "It achieves positive definiteness of the conditional covariance by formulating the model in a way that this property is implied by the model structure" (Su and Huang, 2010:7). In this study, the lag length is set to one, which results in a parsimonious specification of the BEKK model which can be written as:

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \tag{3.19}$$

As stated in chapter 2, the BEKK model bears large computation due to several matrix transpositions. The number of parameters of the BEKK model are too large. Su and Huang (2010:8) stated that "the BEKK form is not linear in parameters, which makes the convergence of the model difficult. However, the strong point lies in that the model structure automatically guarantees the positive definiteness of H_t." Section 3.10 discusses the model diagnostics.

3.10 MODEL DIAGNOSTICS

A model diagnostic is one of a set of procedures that are available for regression analysis that seek to examine the validity of a model in different types of ways. It also helps in assessing if a model satisfies the assumptions of classical normal regression. The assumptions of a normal regression include:

- Serial correlation
- Normality
- Heteroscedasticity

This study employs a number of diagnostic tests to test whether the selected model is adequate and efficient by testing the residuals of the models. The different types of diagnostic tests are discussed in the following subsections.

3.10.1 Serial correlation test

According to (Williams, 2015), when error terms from different time periods are correlated, the error terms are said to be serially correlated. Furthermore, Williams (2015:1) indicated that "serial correlation happens when the errors associated with a given time period carry over into future time periods." In this study, the Breusch-Godfrey (BG) (1980), Lagrange Multiplier (LM) test and the Ljung Box Q-Test is used to test for serial correlation.

3.10.1.1 Breusch-Godfrey LM test

The BG LM test is used to test the presence of serial correlation. The BG LM test is based on the following auxiliary regression model:

$$\hat{u}_t = \alpha_0 + \alpha_1 X_{t,1} + \alpha_2 X_{t,2} + \rho_1 \hat{u}_{t-1} \tag{3.20}$$

The test statistic is given as:

$$LM_h = T\left[K - tr\left(\widetilde{\sum R} \widetilde{\sum e}\right)\right] \tag{3.21}$$

where $\widetilde{\sum R}$ and $\widetilde{\sum e}$ are the residual covariance matrix of the restricted and unrestricted models respectively. The test statistic LM_h is distributed as chi square χ^2(hK^2). The BG test hypothesis is formulated as:

H0: $\rho_i = 0\ for\ all\ i$ (There is no serial correlation)
H1: $\rho_i \neq 0\ for\ all\ i$ (There is serial correlation)

The null hypothesis is rejected if the probability in the BG test statistic is less than 5% level of significance.

3.10.1.2 Ljung-Box test

In financial time series, serial correlation and the volatility clustering effect of asset returns are commonly checked by Ljung-Box Q-test. The test statistic for the Ljung Box test is given as:

$$Q_{LB} = T(T+2)\sum_{j=1}^{k} \frac{{r_j}^2}{T-j} \tag{3.22}$$

where T is the number of observations, and k is the highest order of autocorrelation for which to test and ${r_j}^2$ and j^{th} autocorrelation. The hypothesis for the Q_{LB}test is formulated as:

H0: $\rho_i = 0\ for\ all\ i$ (There is no serial correlation)
H1: $\rho_i \neq 0\ for\ all\ i$ (There is serial correlation)

The null hypothesis is rejected if the probability in the BG test statistic is less than 1%, 5% or 10% level of significance

3.10.2 Heteroscedasticity test

Error terms are said to be heteroscedastic when they do not have a constant variance. The study will employ White's General test and the LM to test for the existence of heteroscedasticity.

3.10.2.1 White's test

The White's test is a special case of the Breusch-Pagan test, which is a test for linear forms of heteroscedasticity (Williams, 2015). The White's test for heteroscedasticity does not rely on the normality assumption and it is easy to apply. To test the assumption of heteroscedasticity, consider the following auxiliary equation:

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X^2{}_1 + \beta_4 X^2{}_2 + \beta_5 X_1 X_2 \qquad (3.23)$$

The test statistic is given by:

$$nR^2 \sim \chi^2{}_{df}$$

where n is the number of observations, and R^2 is the coefficient of determination of the regression. The hypothesis for the White's test is given as:

H_0: Residuals are homoscedastic
H_1: Residuals are heteroscedastic

The null hypothesis is rejected if the probability of the test statistic is less than 5% level of significance and it is concluded that the residuals are heteroscedastic.

3.10.2.2 Lagrange Multiplier (LM) test

The LM test proposed by Engle (1982) is a test for autoregressive conditional heteroscedasticity (ARCH) in the residuals. The test statistic is given by:

$$LM_E = nR^2$$

where n is the number of observations, and R^2 is the coefficient of determination of the regression. The hypothesis for the LM test is given as:

H_0: Residuals are homoscedastic
H_1: Residuals are heteroscedastic

The null hypothesis is rejected if the probability of the test statistic is less than 5% level of significance and it is concluded that the residuals are heteroscedastic.

3.10.3 Normality test

The Jacque-Bera (JB) test will be used to determine whether the models are normally distributed. The JB test "measures the difference in kurtosis and skewness of a variable compared to those of the normal distribution" (Ssekuma, 2011:19). To test the presence of normality in the data set, the following hypothesis was formulated:

H_0: Residuals are normally distributed

H_1: Residuals are not normally distributed

The test statistic is given as

$$JB = \frac{N-k}{6}\left[S^2 + \frac{(K-3)^2}{4}\right] \qquad (3.24)$$

where N is the number of observations, k is the number of estimated parameters, S is the skewness of a variable and K is the kurtosis of a variable (Ssekuma, 2011). The formula for skewness and kurtosis are given as:

$$s = \frac{\sum_{t=1}^{T}(x_t - \bar{x})^3 \big/ T-1}{s^3} \qquad (3.25)$$

and

$$k = \frac{\sum_{t=1}^{T}(x_t - \bar{x})^4 \big/ T-1}{s^4} \qquad (3.26)$$

The null hypothesis is rejected if the p-value is less than 1%, 5% or 10% level of significance.

3.11 MODEL EVALUATION

Model evaluation is an important part of developing a model. It helps to find the best model that will represent the data well. The forecasting ability of the models will be examined by three different error measures. The error measures are discussed in the following subsections.

3.11.1 Means Squared Error (MSE)

The MSE is mostly used for comparing a model's forecasting ability. It is considered as the most appropriate error measure to determine which methods avoid large errors. The MSE is given by:

$$MSE = \sum_t^n \frac{{e_t}^2}{n} \qquad (3.27)$$

where $e_t = y_t - \hat{y}_t$, where y_t is actual observed value in time t and $\hat{y}_t$ is the fitted value in time.

3.11.2 Root Mean Square Error (RMSE)

The RMSE is a frequently used measure of the differences between values predicted by a model and actual values. The RMSE is computed using:

$$RMSE = \sqrt{\sum_t^n \frac{{e_t}^2}{n}} \qquad (3.28)$$

where $e_t = y_t - \hat{y}_t$, y_t is actual observed value in time t and $\hat{y}_t$ is the fitted value in time.

3.11.3 Mean Absolute Percentage Error (MAPE)

The MAPE is another measure of prediction accuracy of a forecasting and it is computed using the following equation:

$$MAPE = \sum_{t=1}^n \frac{\left|\left(\frac{e_t}{y_t}\right)\right| * 100}{n} \qquad (3.29)$$

where n is the number of fitted points and $\left|\left(\frac{e_t}{y_t}\right)\right| * 100$ is defined as the absolute percentage error calculated on the fitted values for a particular forecasting method.

3.12 CONCLUSION

This chapter discussed the research methodology, variable analysis and the estimation techniques that will be used to model financial data in South Africa. The Johansen Cointegration, VECM, GARCH and BEKK frameworks were presented as the estimation methods that will be employed in this study. This was followed by a discussion of the diagnostic tests. The chapter ended with a discussion on the model evaluation. The contents of this chapter provide a basis for the actual estimations for the study, which will be portrayed in the next chapter.

CHAPTER 4
DATA ANALYSIS AND INTERPRETATION OF RESULTS

4.1 INTRODUCTION

This chapter reports the results and interpretation of estimating the VEC, GARCH and BEKK models using financial data in South Africa. All empirical work is carried out using RStudio version 1.1.463, WinRATS Trial 10.0 and EViews 10 Student Version. Graphical representations of the variables are presented, and they reveal interesting patterns about the variables under study.

The following concepts are covered in this chapter: preliminary data analysis, followed by the cointegration analysis, the VECM and the diagnostic tests. The study further modelled the series using the standard GARCH model and the multivariate GARCH-BEKK models and the diagnostics of the models. The last section of the analysis presents the conclusion of the chapter.

4.2 PRELIMINARY DATA ANALYSIS

The preliminary data analysis comprises the descriptive statistics, graphical presentation of the series, unit root testing using the ADF and PP test. The financial series are transformed into logarithms so that the series can be unit free. Section 4.2.1 presents the descriptive statistics of the series.

4.2.1 Descriptive Statistics

Descriptive statistics are useful for describing the features of the data. The descriptive statistics used in the study are the mean, median, maximum and minimum value, standard deviation, skewness, kurtosis, JB test with their p-values. The descriptive statistics of the financial data are given in Table 4.1 below.

Table 4.1: Descriptive statistics for log transformed financial data

Measures	LED	LGDP	LEXP	LGE	LCF
Mean	10.5668	5.8771	4.0869	11.4542	9.3032
Median	11.2400	5.9245	4.0910	11.4393	9.1838
Maximum	12.5690	7.1166	5.0814	12.9071	10.5520
Minimum	7.4339	4.3268	2.9549	9.7741	8.1236
Std.Dev	1.4794	0.8459	0.6827	0.8961	0.8106
Skewness	-0.7746	-0.1903	-0.1447	-0.0580	0.1321
Kurtosis	2.3791	1.7645	1.6403	1.7676	1.5128
JB test Probability	13.3467 0.0013	8.0090 0.0182	9.2601 0.0098	7.3422 0.0254	10.9328 0.0042

Table 4.1 presents the descriptive statistics of the financial series of the study. The financial data used are LED, LGDP, LEXP, LGE and LCF, where LED is the logarithm of External Debt, LGDP is the logarithm of Gross Domestic Product, LEXP is the logarithm of Exports, LGE is the logarithm of Government Expenditure and LCF is the logarithm of Capital Formation.

The means and the medians of all the five variables are within the same range and not far away from each other, so this could indicate that the series in the data are slightly symmetric. The standard deviations of the variables are all small which means that the data are more concentrated around the mean. The results reveal that LED, LGDP, LEXP and LGE are negatively skewed. This means that most of the financial data are concentrated to the right of the mean. The LCF is found to be positively skewed, which indicates that the data is concentrated to the left. Overall, it can be concluded that the data are moderately skewed since the skewness lies between -1 and 1. The kurtosis values for all five variables are less than 3, which means that the distribution is platykurtic. The probabilities of the variables are all significant and therefore it can be concluded that these variables are not normally distributed. We reject the null hypothesis of normality.

4.2.2 Graphical presentation of the series

The graphical presentation of the series at level are presented in Figure 4.1.

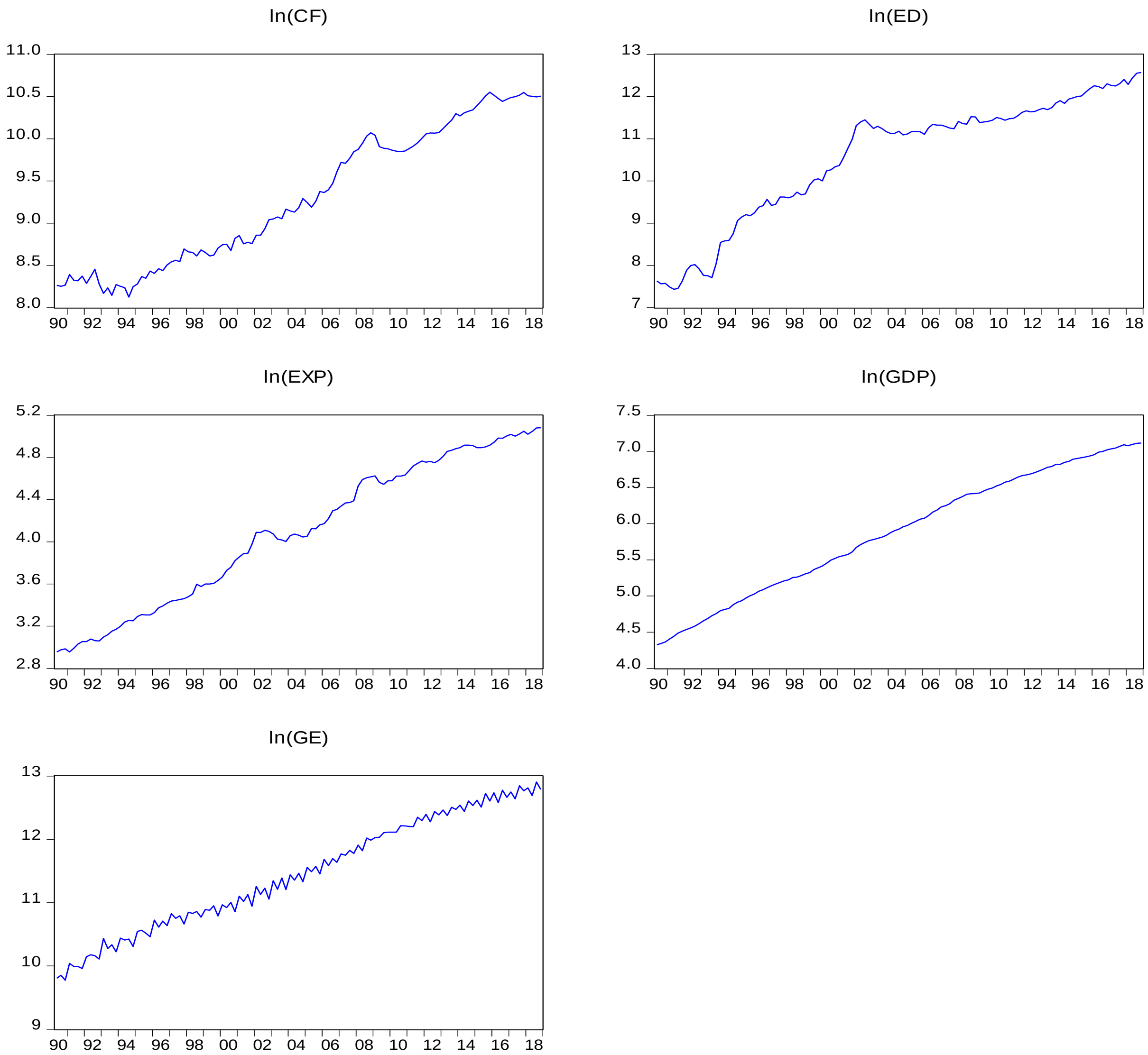

Figure 4.1: Plots of financial data at level

Figure 4.1 presents the graphical presentation of the financial series. The series of LCF, LED, LEXP, LGDP and LGE show a strong increasing trend while the series of LGE shows both a strong increasing trend and evidence of seasonality. By eye inspection, one could conclude that all the variables seem to be non-stationary. There is no evidence of any cyclic behaviour. Formal tests for stationarity were computed to verify the visual inspection and the results are presented in subsection 4.2.3.

4.2.3 Unit Root Tests

Most of the financial data is non-stationary in nature. The ADF and PP unit root tests are used to determine whether the series is stationary or not. The unit root test results are summarised and presented in Table 4.2.

Table 4.2: Unit Root Tests

Variables	ADF				PP			
	Level		1st Difference		Level		1st Difference	
	Test statistic	P-value	Test statistic	P-value	Test statistic	P-value	Test statistic	P-value
LED	-1.9798	0.2954	-7.2642	0.0000	-1.8095	0.3743	-7.9213	0.0000
LCF	-0.0521	0.9510	-4.4572	0.0004	-0.0114	0.9549	-11.2820	0.0000
LEXP	-0.8613	0.7971	-8.6325	0.0000	-0.8792	0.7917	-8.6226	0.0000
LGDP	-4.7185	0.0002	-8.0680	0.0000	-4.5182	0.0003	-8.2546	0.0000
LGE	-1.5597	0.4996	-3.0021	0.0379	-1.3476	0.6054	-51.1657	0.0001

Note: ADF and PP significant at 5% level of significance.

Table 4.2 shows the results of the ADF and PP unit root test of each variable at level and at first difference. From the results, it is evident that the p-values of the test statistic for LED, LCF, LEXP and LGE at level for both ADF and PP tests are greater than 5% level of significance. The null hypothesis that the series contains a unit root could not be rejected. Therefore, the variables are non-stationary at level. The LGDP series is stationary at level since the p-value of the test statistic for both ADF and PP test is less than 5% level of significance. The ADF and PP tests show that all the variables become stationary after first difference. It can be concluded that all variables are integrated of order 1, I(1). The next step is to compute the cointegration test to determine the long run relationship among the series. The cointegration test is presented in Section 4.3.

4.3 COINTEGRATION ANALYSIS

The cointegration test used in the study is the Johansen cointegration test. For the Johansen test, it is essential to determine the optimal lag length, which removes serial correlation in the residuals as well as determining the deterministic trend for the VAR model. For choosing the lag order for the VAR, the lag order selection criteria is applied. In this study, the lag order selection is computed using a maximum of 8 lags. The study determines the lag length based on the AIC and SBIC. Table 4.3 shows the lag lengths selected by the different types of information criteria.

Table 4.3: VAR Lag Order Selection Criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	203.5740	NA	1.68e-08	-3.7117	-3.5868	-3.6610
1	983.9308	1473.197	1.24e-14	-17.8305	-17.0811*	-17.5267
2	1015.425	56.5133	1.10e-14	-17.9519	-16.5780	-17.3949
3	1032.440	28.9407	1.29e-14	-17.8026	-15.8042	-16.9925
4	1117.907	137.3864	4.23e-15*	-18.9328*	-16.3100	-17.8696*
5	1133.557	23.6939	5.16e-15	-18.7581	-15.5101	-17.4416
6	1152.578	27.0206	5.98e-15	-18.6463	-14.7745	-17.0767
7	1182.921	40.2694*	5.69e-15	-18.7462	-14.2499	-16.9234
8	1201.471	22.8835	6.89e-15	-18.6256	-13.5048	-16.5497

Note: * indicates lag order selected by the criterion

LR: sequential modified LR test statistic (each test at 5% level)

FPE: Final prediction error

AIC: Akaike information criterion

SC: Schwarz information criterion

HQ: Hannan-Quinn information criterion

The results in Table 4.3 presents the VAR lag length selection criterion. According to the results in Table 4.3, both the SIC and AIC suggest a different number of lag order to be selected. The SIC selected the lag order of 1 while the AIC selected the lag order of 4. In this case, the lag order having a minimum AIC or SIC value will be selected. The AIC is chosen to determine the optimal lag length and a decision to adopt 4 lags is made.

4.3.1 Johansen Cointegration Analysis

The cointegrating relationship between variables is tested using the Johansen cointegration technique. The Johansen cointegration is based on two likelihood ratio tests, namely the trace and maximum eigenvalue test. The results of the trace and maximum eigenvalue test are discussed in subsection 4.3.1.1 and subsection 4.3.1.2 below.

4.3.1.1 The trace test

The trace test tests the null hypothesis of r cointegrating vectors against the alternative hypothesis on n cointegrating vectors. The results are given in Table 4.4 below.

Table 4.4: The trace test

Hypothesized no. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None*	0.3347	98.2727	69.8189	0.0001
At most 1*	0.1941	53.4424	47.8561	0.0136
At most 2	0.1311	29.7003	29.7971	0.0513
At most 3	0.0665	14.2430	15.4947	0.0765
At most 4*	0.0589	6.6774	3.8415	0.0098
Note: Trace test indicates 2 cointegration eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level ** MacKinnon-Haug-Michelis (1999) p-values				

The results of the trace test presented in Table 4.4 indicates that there are two cointegrating equations at 5% level of significance.

4.3.1.2 The maximum eigenvalue test

In the maximum eigenvalue test, the null hypothesis of r cointegrating vectors against the alternative hypothesis of $r+1$ cointegrating vectors is tested. The results are presented in Table 4.5 below.

Table 4.5: The maximum eigenvalue test

Hypothesized no. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None*	0.3347	44.8303	33.8769	0.0017
At most 1	0.1941	23.7420	27.5843	0.1440
At most 2	0.1311	15.4573	21.1316	0.2581
At most 3	0.0665	7.5657	14.2646	0.4246
At most 4*	0.0589	6.6774	3.8415	0.0098
Note: Max-eigenvalue test indicates 1 cointegration eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level ** MacKinnon-Haug-Michelis (1999) p-values				

The results of the maximum eigenvalue test are summarised in Table 4.5. The test indicates that one cointegrating equation exists at 5% level of significance.

The trace statistics revealed that there are two cointegrating vectors, while the maximum eigenvalue indicates that there is only one cointegrating vector. Lütkepohl, Saikkonen and Trenker (2001) suggest that the power of the trace tests is in some situations superior to that of the maximum eigenvalue tests. In this case the trace test is preferred to the maximum eigenvalue test. It can be concluded that there are more than one cointegration equations in the model. It can be concluded that there is an existence of a long run relationship among the variables and a VECM can be used to analyse the short-term dynamics of the cointegrated series. The VECM is discussed in Section 4.4.

4.4 VECTOR ERROR CORRECTION MODEL (VECM)

Given the findings that LED, LCF, LEXP, LGDP, LGE are cointegrated in the long run, the cointegration vector is used to construct the VECM. The results of the VEC estimates are summarised in Table 4.6.

Table 4.6: VEC estimates

Cointegrating Eq:	**CointEq1**
LED(-1)	1.0000
LCF(-1)	-0.0555 (0.5219) [-01064]
LEXP(-1)	-2.9671 (1.3891) [-2.1359]
LGDP(-1)	-5.1747 (1.7784) [-2.9098]
LGE(-1)	6.2925 (1.2823) [4.90725]
C	-39.6334

Note: Standard errors in () & t-statistics in []

Table 4.6 presents the results of the VECM estimates. The long run relationship among the variables in Table 4.6 is shown by estimating the following equation:

$$LED = 39.6334 + 0.0555LCF + 2.9671EXP + 5.1747LGDP - 6.2925LGE \qquad (4.1)$$

The equation 4.1 shows that LCF, LEXP, LGDP have a positive long run relationship with LED, and LGE has a negative long run relationship with LED. This can be interpreted by saying that a unit increase in LCF, LEXP and LGDP results in an increase for LED and a unit increase for LGE results in a decrease in LED. The results of the VECM are given in Table 4.7 below.

Table 4.7: VECM Equation Coefficients

	Coefficient	**Std. Error**	**t-Statistic**	**Prob.**
C(1)	-0.0426	0.0197	-2.1617	0.0332
C(2)	0.3100	0.1029	3.0116	0.0033
C(3)	-0.1755	0.1047	-1.6759	0.0971
C(4)	-0.0019	0.1010	-0.0193	0.9847
C(5)	-0.2536	0.1689	-1.5012	0.1366
C(6)	-0.1359	0.1693	-0.8030	0.4240
C(7)	-0.0592	0.1689	-0.3507	0.7266
C(8)	-0.1870	0.3766	-0.4966	0.6206
C(9)	0.8291	0.3782	2.1922	0.0308
C(10)	0.3501	0.3798	0.9217	0.3590
C(11)	-0.3916	1.0507	-0.3727	0.7102
C(12)	-0.8143	1.0613	-0.7673	0.4448
C(13)	-0.4406	1.0645	-0.4139	0.6799
C(14)	-0.0101	0.1511	-0.0669	0.9468
C(15)	-0.4052	0.1745	-2.3213	0.0224
C(16)	-0.2585	0.1274	-2.0299	0.452
C(17)	0.0887	0.0409	2.168329	0.0327
Determinant residual covariance	0.0079			

According to the results presented in Table 4.7, the VECM equation is given as:

$$\Delta LED = -0.042630 * (LED_{t-1} - 0.0555 * LCF_{t-1} - 2.9671 * LEXP_{t-1} - 5.1747 * LGDP_{t-1} - 6.2925 * LGE_{t-1} - 39.6333) + 0.31 * LED_{t-1} - 01755 * LED_{t-2} - 0.0019LED_{t-3} - 0.2536 * LCF_{t-1} - 0.1359 * LCF_{t-2} - 0.0592LCF_{t-3} - 0.187LEXP_{t-1} + 0.8291LEXP_{t-2} + 0.3501LEXP_{t-3} - 0.3916LGDP_{t-1} - 0.8143LGDP_{t-2} - 0.4406LGDP_{t-3} - 0.0101LGE_{t-1} - 0.4052LGE_{t-2} - 0.2585LGE_{t-3} + 0.0887 \quad (4.2)$$

where $-(LED_{t-1} - 0.0555 * LCF_{t-1} - 2.9671 * LEXP_{t-1} - 5.1747 * LGDP_{t-1} - 6.2925 * LGE_{t-1} - 39.6333)$ is the error correction term. Table 4.8 gives the summary of the ECM's.

Table 4.8: Summary of ECM's

Error Correction	**D(LED)**	**D(LCF)**	**D(LEXP)**	**D(LGDP)**	**D(LGE)**
CointEq1	-0.0426 (0.0197) [-2.1617]	0.0386 (0.0114) [3.3967]	-0.0008 (0.0060) [-0.1283]	-0.0047 (0.0021) [-2.2971]	-0.0401 (0.0081) [-4.9323]
Note: Standard errors in () & t-statistics in []					

The error correction term which measures the speed of adjustment towards equilibrium is negative which shows convergence towards equilibrium level in the long run. The magnitude of these coefficients suggests that approximately 4.26% 0.08%, 0.47% and 4.01% of the disequilibrium is corrected for LED, LEXP, LGDP and LGE respectively. The coefficient of D(LCF) is positive, which suggests that a percentage change in the cointegrating equation is associated with a 3.86% increase in LCF in the short run. The diagnostic tests of the model are presented in Section 4.5.

4.5 VEC DIAGNOSTIC TESTS

This section of the analysis presents the diagnostic tests applied. The diagnostic tests computed in the study are the test for serial correlation using the LM test, test for normality using the JB test, and test for heteroscedasticity using the White's test. Section 4.5.1 presents the results of the LM test.

4.5.1 Serial Correlation

The LM test is computed to determine whether there is a presence of serial correlation or not. The results of the LM test are summarised in Table 4.9.

Table 4.9: VEC Residual Serial Correlation LM Tests

Lag	LRE* stat	Df	Prob.	Rao F-stat	Df	Prob.
1	18.7239	25	0.8102	0.7437	(25,317.3)	0.8105
2	24.2642	25	0.5042	0.9720	(25,317.3)	0.5048
3	35.3883	25	0.0814	1.4422	(25,317.3)	0.0817
4	24.2739	25	0.5036	0.9724	(25,317.3)	0.5042

The results in Table 4.9 revealed that there is no serial correlation among the residuals since the p-values of the LM test are more than 5% level of significance. Section 4.5.2 presents the test for normality.

4.5.2 Normality

The test for normality is computed using the JB test. The results of the JB test are summarised in Table 4.10 below.

Table 4.10: JB Test for normality

Component	Jarque-Bera	Df	Prob.
1	2.3237	2	0.3129
2	1.4586	2	0.4822
3	6.6077	2	0.0367
4	5.3561	2	0.0687
5	0.9701	2	0.6157
Joint	16.7162	10	0.0809

In Table 4.10, the JB test was used to test for normality in the model. The results reveal that the residuals are normally distributed since most of the p-values of the JB test are more than 5% level of significance. The joint p-value is also found to be more than 5% level of significance. Therefore, it is concluded that the residuals are normally distributed. The test for presence of heteroscedasticity is computed in Section 4.5.3.

4.5.3 Heteroscedasticity

The presence of heteroscedasticity is tested using the White’s test. The results are presented in Table 4.11.

Table 4.11: White's Test for heteroscedasticity

Joint test:		
Chi-sq	**Df**	**Prob.**
547.6993	480	0.0174

According to the results in Table 4.11, there is the presence of heteroscedasticity among the residuals since the p-value of the Chi-square test is less than 5% level of significance. Therefore, it is concluded that the residuals are heteroscedastic.

The VECM passes most of the diagnostic tests computed. According to the results, there is the presence of heteroscedasticity among residuals, which is not desirable. The series is then modelled using the GARCH model. The GARCH model is discussed in Section 4.6 below.

4.6 GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY (GARCH) MODEL

As mentioned before, the GARCH model is a predominant approach for modelling and forecasting volatility. For a GARCH model to be estimated, the presence of heteroscedasticity should be checked first. Once the presence of heteroscedasticity is determined, a GARCH model can be estimated for better results. In this study, the ARCH LM test is used to check for the presence of heteroscedasticity. The results of the ARCH LM test are presented in Table 4.12.

Table 4.12: Heteroscedasticity Test

F-statistic	45.1896	**Prob.F(1,112)**	0.0000
Obs*R-squared	32.7732	**Prob.Chi-Square(1)**	0.0000

The results of the ARCH LM test reveal that there is the presence of heteroscedasticity among the residuals since the p-value is less than 5% level of significance. Since the p-value = 0.0000, the residuals exhibit the presence of ARCH effects in the model. Therefore, GARCH model can be estimated. The results for GARCH test are summarised in Table 4.13.

Table 4.13: GARCH(1,1) model estimates

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	10.0260	0.5591	17.9330	0.0000
LCF	-1.6624	0.0799	-20.8158	0.0000
LEXP	0.7185	0.2797	2.5685	0.0102
LGDP	3.0579	0.2524	12.1163	0.0000
LGE	-0.4267	0.1250	-3.4139	0.0006
Variance Equation				
C	0.0087	0.0026	3.4069	0.0007
RESID(-1)^2	1.1262	0.3242	3.4733	0.0005
GARCH(-1)	-0.0659	0.0463	-1.4250	0.1542

According to the results presented in Table 4.13, the mean equation is given as:

$$\widehat{led}_t = 10.0260 - 1.6624\widehat{lcf} + 0.7185\widehat{lexp} + 3.0579\widehat{lgdp} - 0.4267\widehat{lge} \quad (4.3)$$

LED is 10.0260 and the coefficients of LEXP and LGDP are positive and significantly predict the series by 0.7185 and 3.0579 respectively. The coefficients of LCF and LGE are negative and significantly predict the series by -1.6624 and -0.4267 respectively. The coefficients of all the variables are statistically significant at 5% level of significance. A positive and statistically significant relationship exists between LEXP and LED and between LGDP and LED, implying that higher volatility in LEXP and LGDP increases the LED volatility. A negative and statistically significant relationship exists between LCF and LED and LGE and LED, this means that lower volatility in LCF and LGE decreases the LED volatility.

The variance equation is given as:

$$\widehat{h_t} = 0.0087 - 0.0659\hat{h}_{t-1} + 1.1262\hat{u}^2{}_{t-1} \quad (4.4)$$

The coefficients of the constant variance term and the ARCH term are positive and are both statistically significant. The GARCH term is negative and statistically insignificant at 5% level of significance. The results of the GARCH model includes the time-varying volatility constant (0.0087), plus its past ($-0.0659\hat{h}_{t-1}$) and a component which depends on past errors ($1.126\hat{u}^2{}_{t-1}$). The sum of the ARCH and GARCH terms is greater than 1. This means that the variance is increasing over time. The model is then tested for adequacy and the results are presented in Section 4.7.

4.7 GARCH DIAGNOSTIC TESTS

This section of the analysis presents the diagnostic tests applied to the GARCH model. The diagnostic tests computed in the study are the test for serial correlation using the LM test, test for normality using the JB test and test for heteroscedasticity using White's test. Section 4.7.1 presents the results of the LM test.

4.7.1 Serial Correlation

The Q-Statistic is used determine whether there is a presence of serial correlation. The results are presented in the following Table 4.14.

Table 4.14: VEC Residual Heteroscedasticity Tests

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.012	-0.012	0.0173	0.895
		2	0.175	0.175	3.6729	0.159
		3	-0.123	-0.122	5.4765	0.140
		4	0.019	-0.012	5.5204	0.238
		5	0.051	0.098	5.8326	0.323
		6	0.033	0.016	5.9677	0.427
		7	-0.025	-0.054	6.0456	0.534
		8	-0.083	-0.077	6.9139	0.546
		9	-0.097	-0.081	8.1155	0.523
		10	-0.027	-0.012	8.2082	0.609
		11	-0.142	-0.143	10.831	0.457
		12	0.010	-0.002	10.844	0.542
		13	-0.066	-0.007	11.417	0.576
		14	-0.055	-0.085	11.825	0.620
		15	-0.030	-0.016	11.948	0.683
		16	-0.067	-0.048	12.550	0.705
		17	0.049	0.032	12.875	0.744
		18	-0.086	-0.094	13.894	0.736
		19	0.054	0.007	14.296	0.766
		20	-0.040	-0.018	14.525	0.803
		21	0.024	-0.019	14.609	0.842
		22	-0.027	-0.057	14.712	0.874
		23	-0.072	-0.100	15.460	0.877
		24	-0.029	-0.046	15.588	0.902
		25	-0.079	-0.101	16.519	0.898
		26	-0.015	-0.057	16.553	0.922
		27	0.047	0.038	16.893	0.934
		28	0.061	0.064	17.468	0.939
		29	-0.030	-0.099	17.613	0.952
		30	0.021	0.011	17.681	0.963
		31	0.002	0.011	17.681	0.973
		32	-0.016	-0.094	17.725	0.981
		33	-0.055	-0.129	18.228	0.983
		34	0.062	0.027	18.870	0.983
		35	0.122	0.160	21.392	0.966
		36	0.097	0.017	22.989	0.954

*Probabilities may not be valid for this equation specification.

The ACF and PACF lies within their confidence interval and the p-values of the Q-statistics are well above 5% level of significance, which means that there is no serial correlation among the residuals.

4.7.2 Normality

The test for normality is computed using the JB test. The results for JB test are presented in Table 4.15.

Table 4.15: Normality test for GARCH model

Test	Statistic	Probability
Jarque-Bera	1.4691	0.4797

Table 4.15 shows results of the JB for the GARCH model. The JB test reveals that the residuals are normally distributed since the p-value of the JB test is more than 5% level of significance.

4.7.3 Heteroscedasticity

The presence of heteroscedasticity is tested using White's test. The results are summarised in Table 4.16.

Table 4.16: GARCH Heteroscedasticity Tests

F-statistic	1.9854	**Prob.F(4,110)**	0.1017
Obs*R-squared	7.7436	**Prob.Chi-Square(4)**	0.1014
Scaled explained SS	1073.302	**Prob.Chi-Square(4)**	0.0000

The results of the White's test in Table 4.16 reveal that there is no presence of heteroscedasticity among the residuals since the p-value is greater than 5% level of significance. Since the p-value = 0.1014, it is clear the residuals of the GARCH(1,1) are not heteroscedastic.

The results of the diagnostic test shows that the GARCH(1,1) model passes all of the diagnostic test and that it is adequate. The results of the multivariate GARCH model is discussed in Section 4.8 below.

4.8 BABA, ENGLE, KRAFT AND KRONER (BEKK) MODEL

In this section, the patterns of conditional volatility in returns and the possibility of volatility in the variables are examined. Parameter estimates of the BEKK model are provided in Table 4.17. All the subscripts involving the estimated coefficients below are assigned as follows: if the coefficient is a_{ij}, for i,j=1,2,3,4,5, then LGDP =1, LEXP = 2, LED = 3, LGE = 4, LCF = 5.

Here, A$(.,i)$ and B$(.,i)$ are the corresponding ARCH and GARCH parameters associated with variable i. The squared ARCH parameters $[A(.,i)]^2$ capture the responses of volatility in variable i to squared standardized innovations in each of the five variables. For example, the estimated ARCH response for LED ($i = 3$) to its own innovations, $[A(3,3)]^2$, is $(0.957)^2$, to innovations in LEXP is $[A(1,3)]^2$, is $(-0.089)^2$, to innovations in LED is $[A(2,3)]^2$, is $(-0.125)^2$, to innovations in LGE is $[A(4,3)]^2$, is $(0.132)^2$ and to innovations in LGE is $[A(5,3)]^2$, is $(0.099)^2$. Thus, the volatility of LED responds significantly to past squared shocks in its own innovation, as well as in the LGE and LCF innovations.

The BEKK form of multivariate GARCH models takes the following form:

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \quad (4.5)$$

Table 4.17: Results from BEKK model

Triangular matrix of constant				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C(1,1)	0.0462	0.0060	7.6687	0.0000
C(2,1)	0.0394	0.0051	7.7805	0.0000
C(2,2)	0.0146	0.0022	6.5744	0.0000
C(3,1)	0.0470	0.0089	5.2791	0.0000
C(3,2)	0.0232	0.0072	3.2232	0.0014
C(3,3)	0.0368	0.0047	7.7750	0.0000
C(4,1)	0.0510	0.0119	4.2878	0.0000
C(4,2)	0.0096	0.0071	1.3595	0.1740
C(4,3)	0.0650	0.0046	14.2965	0.0000
C(4,4)	0.0060	0.0146	0.4107	0.6813
C(5,1)	0.0745	0.0083	8.9596	0.0000
C(5,2)	-0.0187	0.0050	-3.7542	0.0002
C(5,3)	-0.0028	0.0044	-0.6325	0.5271
C(5,4)	-0.0045	0.0113	-0.3992	0.6897

Triangular matrix of constant				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C(5,5)	0.0003	0.0066	0.0459	0.9634

Table 4.18: Results from BEKK model

ARCH effect				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
A(1,1)	0.5070	0.0146	34.8380	0.0000
A(1,2)	-0.2116	0.0312	-6.7876	0.0000
A(1,3)	-0.0889	0.0861	-1.0323	0.3019
A(1,4)	1.1529	0.0696	16.5622	0.0000
A(1,5)	-0.6634	0.0355	-18.7035	0.0000
A(2,1)	0.0470	0.0209	2.2439	0.0248
A(2,2)	0.9201	0.0395	23.3146	0.0000
A(2,3)	-0.1246	0.0777	-1.6032	0.1089
A(2,4)	-0.1270	0.0472	-2.6873	0.0072
A(2,5)	0.2140	0.0518	4.1282	0.0000
A(3,1)	0.0628	0.0079	7.9305	0.0000
A(3,2)	0.0370	0.0088	4.1943	0.0000
A(3,3)	0.9566	0.0394	24.3024	0.0000
A(3,4)	-0.0648	0.0159	-4.0849	0.0000
A(3,5)	0.0659	0.0120	5.4864	0.0000
A(4,1)	0.1452	0.0159	9.1105	0.0000
A(4,2)	0.0531	0.0229	2.3142	0.0207
A(4,3)	0.1321	0.0529	2.4965	0.0125
A(4,4)	-0.1033	0.0331	-3.1174	0.0018
A(4,5)	0.2117	0.0270	7.8495	0.0000
A(5,1)	0.2064	0.0123	16.8402	0.0000
A(5,2)	0.1683	0.0191	8.8030	0.0000
A(5,3)	0.0993	0.0490	2.0247	0.0429
A(5,4)	0.2367	0.0258	9.1669	0.0000
A(5,5)	1.1759	0.0481	24.4270	0.0000

Table 4.19: Results from BEKK model

GARCH effect				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
B(1,1)	0.3207	0.0166	19.3388	0.0000
B(1,2)	0.3465	0.0440	7.8797	0.0000
B(1,3)	-0.1935	0.1120	-1.7276	0.0841
B(1,4)	0.0593	0.0237	2.5006	0.0125
B(1,5)	0.6281	0.0460	13.6476	0.0000
B(2,1)	-0.1265	0.0186	-6.7960	0.0000
B(2,2)	0.1836	0.0355	5.1706	0.0000
B(2,3)	0.5285	0.1074	4.9202	0.0000
B(2,4)	-0.1745	0.0584	-2.9872	0.0028
B(2,5)	-0.3574	0.0509	-7.0245	0.0000
B(3,1)	-0.1994	0.0135	-14.8210	0.0000
B(3,2)	-0.1904	0.0117	-16.3331	0.0000
B(3,3)	-0.2445	0.0345	-7.0890	0.0000
B(3,4)	-0.2101	0.0185	-11.3449	0.0000
B(3,5)	-0.1292	0.0218	-5.9145	0.0000
B(4,1)	0.1005	0.0166	6.0479	0.0000
B(4,2)	-0.0735	0.0223	-3.2916	0.0010
B(4,3)	-0.1715	0.0677	-2.5329	0.0113
B(4,4)	0.5214	0.0132	39.3540	0.0000
B(4,5)	-0.2548	0.0227	-11.2467	0.0000
B(5,1)	-0.1485	0.0199	-7.4441	0.0000
B(5,2)	-0.2302	0.0299	-7.6908	0.0000
B(5,3)	0.0229	0.0679	0.3379	0.7354
B(5,4)	-0.2767	0.0382	-7.2505	0.0000
B(5,5)	0.0348	0.0413	0.8417	0.4000

Most variables estimated on Table 4.18 and Table 4.19 are statistically significant. The estimated BEKK-GARCH model can be obtained by substituting the following matrices into equation (4.5).

$$A = \begin{pmatrix} 0.5070 & \cdots & -0.6634 \\ \vdots & \ddots & \vdots \\ 0.2064 & \cdots & 1.1759 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.3207 & \cdots & 0.6281 \\ \vdots & \ddots & \vdots \\ -0.1485 & \cdots & 0.0348 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.0462 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0.0745 & \cdots & 0.0003 \end{pmatrix}$$

The ARCH effects demonstrate a unique pattern: all diagonal elements A(1,1), A(2,2), A(3,3), A(4,4) and A(5,5) are statistically significant, suggesting that each conditional variance depends on its own past conditional volatility. A(5,5) has the largest own ARCH effect with the coefficient value of 1.1759^2, while A(4,4) has the smallest own ARCH effect with the value of -0.1033^2.

The GARCH effects show four out of five of the conditional variances depend on their own history; this is evident in the estimated B(1,1), B(2,2), B(3,3) and B(4,4). The B(5,5) is insignificant at 5% level of significance, suggesting that the past conditional volatility does not influence volatility in external debt.

Most of the off-diagonal elements of matrix A are statistically significant at 5% level of significance. This suggests that most of the parameters affect each other. Only ($A_{1,3}$ and $A_{2,3}$) of the off-diagonal parameters are statistically insignificant, meaning that there is no effect between LGDP and LED; LEXP and LED.

The off-diagonal elements of matrix B capture the volatility transmission. Most of the off-diagonal parameters of matrix B are statistically significant at 5% level of significance, which means that there is a bidirectional volatility transmission between the parameters. The coefficients of $B_{3,1}, B_{3,5}$ are statistically significant, whereas their counterparts $B_{1,3}, B_{5,3}$ are statistically insignificant. This means that there is a unidirectional volatility transmission between LGDP and LED; LCF and LED. Section 4.9 presents the diagnostic tests for the BEKK model.

4.9 BEKK DIAGNOSTIC TESTS

This section of the analysis presents the diagnostic tests applied to the BEKK model. The diagnostic tests computed in the study are the test for serial correlation using the Q-Statistic test and the test for heteroscedasticity using ARCH LM test. Section 4.9.1 presents the results of the Q-Statistic for serial correlation.

4.9.1 Serial Correlation

The Q-Statistic is used to determine whether there is a presence of serial correlation. The results are summarised in the following Table 4.20.

Table 4.20: Multivariate Q-Statistic test

Test	Statistic	Significant
Multivariate Q-Test	651.5699	0.0000

The results in Table 4.20 reveal that there is serial correlation in the mean since the p-values of the LM test are less than 5% level of significance.

4.9.2 Heteroscedasticity

The presence of heteroscedasticity is tested using ARCH LM test. The results of the ARCH LM test are summarised in Table 4.21.

Table 4.21: Multivariate ARCH test

Test	Statistic	Significant
Multivariate ARCH Test	4037.57	0.0000

The results of the multivariate ARCH test in table 4.21 reveal that there is the presence of heteroscedasticity among the residuals since the p-value is less than 5% level of significance. The results of the diagnostic tests for BEKK reveal that the model estimated is inadequate.

4.10 MODEL EVALUATION

The next and final step in the estimation of the results is evaluating the performance of the VECM, GARCH and BEKK models using the following error measure approaches: MSE, RMSE and MAPE. The results are summarised in Table 4.22.

Table 4.22: Error measures for the VECM, GARCH and BEKK model.

	VECM	**GARCH(1,1)**	**BEKK**
MSE	0.7889	0.0834	1526741007.748
RMSE	0.8882	0.2888	39073.533
MAPE	205.5909	2.1069	4.083

The results in Table 4.22 show low values for the GARCH(1,1) model compared to the VECM and BEKK models for all the three error measures (MSE, RMSE and MAPE). This strongly suggests that the GARCH(1,1) model is more efficient than the VECM and BEKK models for the selected time series. The results of the error measures can also be supported by the results of the diagnostic tests. The results of the diagnostic tests revealed that the GARCH(1,1) model is the only model that passed all of its diagnostic tests and was proven to be adequate.

4.11 CONCLUSION

This chapter estimated the VECM, GARCH and BEKK models using financial data in South Africa. The first section of the chapter covered the unit root testing using the ADF and PP test and it was concluded that the variables are I(1). The AIC was then chosen to determine the optimal lag length, and the decision to adopt 4 lags was made. Given the findings that LED, LCF, LEXP, LGDP and LGE were cointegrated in the long run, the cointegration vector was used to construct the VECM. The VECM, GARCH and BEKK models were estimated together with their diagnostic tests. The VECM passed two out of three tests, which is not desirable. Although the GARCH estimates showed that the model is explosive, the model passed all of its diagnostic tests and it was concluded that the GARCH(1,1) model is adequate to model external debt in South Africa. The BEKK model diagnostics showed that the model was inadequate. Finally, the forecasting ability for the three models was estimated and the results strongly suggested that the GARCH(1,1) was more efficient than the VECM and BEKK models. The next chapter presents the summary of the study, discussions of the results and recommendations.

CHAPTER 5
CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter presents a summary of the study, study limitations, draws conclusions in respect of the objectives and gives recommendations. The remainder of this chapter is structured as follows: Section 5.2 discusses the results of the study according to the objectives. Section 5.3 discusses the limitations of the study. Section 5.4 discusses the recommendations for further study. Section 5.5 is the summary of the study.

5.2 DISCUSSION OF RESULTS

This section of the chapter presents the discussion of the findings of the study with reference to the objectives stated in chapter one. The aim of this study was to model the determinants of external debt using the VECM, GARCH and BEKK models with the intention of identifying and recommending the most effective approach. The empirical analysis of this study employed quarterly time series data for South Africa that ranged from the second quarter of 1990 to the last quarter of 2018. The variables used were External Debt, Capital Formation, Exports, Gross Domestic Product and Government Expenditure.

The key findings are discussed to address the objectives of the study as follows:

Objective 1:
To examine the relationship between External Debt and Exports, GDP, Government Expenditure and Capital Formation in South Africa.

Conclusion 1:

In this study the results of the Johansen cointegration test concluded that there is the existence of a long run relationship among the variables. The results showed that LCF, LEXP, LGDP have a positive long run relationship with LED, and LGE has a negative long run relationship with LED. These results are in contrary with the findings by Chiminya and Nicolaidou (2015), and also by Cholifani (2008).

Objective 2:
To fit VEC, GARCH and BEKK models to the abovementioned variables.

Conclusion 2:

- **Vector Error Correction Model (VECM)**

Given that the series was cointegrated in the long run, the cointegration vector was used to construct the VECM. The long run relationship among the variables showed that LCF, LEXP, LGDP have a positive long run relationship with LED, and LGE has a negative long run relationship with LED. Diagnostics for the VECM were performed and the VECM passed most of the diagnostic tests. The results are supported by the studies of Awan, Anjum and Rahim (2015) and Dritsaki (2013).

- **Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model**

The ARCH LM test was first used to determine whether there was the presence of heteroscedasticity. The results of the ARCH LM test revealed that the residuals exhibit the presence of ARCH effects. The coefficients of the constant term and ARCH terms were positive and both statistically significant; the GARCH term however was negative and statistically insignificant at 5% level of significance. The sum of the ARCH and GARCH terms was greater than 1, which means that the variance is increasing over time. Diagnostics for the GARCH model were performed and the results showed that the GARCH(1,1) model passed all the diagnostics. The study by Malik (2017) supports these results by showing that a GARCH(1,1) model can be fitted using historical stock price data.

- **BABA, ENGLE, KRAFT AND KRONER (BEKK) Model**

The results of the BEKK-GARCH model showed that most of the variables were statistically significant. The estimates of the diagonal parameters show that only B(5,5) was statistically insignificant, suggesting that the past conditional volatility does not influence volatility in external debt. Only (A_{13} and A_{23}) of the off-diagonal parameters are statistically insignificant, meaning that there is no effect between LGDP and LED, and LEXP and LED. Most of the off-diagonal parameters of matrix B are statistically significant at 5% level of significance, which means that there is a bidirectional volatility transmission between the parameters. There was a unidirectional volatility transmission between LGDP and LED, and LCF and LED.

The determinants of external debt in developing countries have generated a lot of interest among scholars and policy-makers in recent years (Cholifihani, 2008). Different researchers have used various techniques to analyse the determinants of external debts, most of them being time series, panel or cross section techniques.

Objective 3:
To examine the forecasting ability of the 3 models.

Conclusion 3:

The results of the error measures showed that the GARCH(1,1) model had the lowest values for all the three error measures, followed by the VECM. The BEKK had the highest error measure values of the three models. The GARCH model has a better ability to forecast financial data compared to the VECM and BEKK model. This was also supported by the diagnostic tests of the models; the GARCH(1,1) model was the only model that passed all of the diagnostic tests. The results were supported by Frimpong and Oteng-Abayie (2006).

Objective 4:
To determine the best model that can be used to model financial data

Conclusion 4:

In this study, the results showed low values for the GARCH(1,1) model compared to the VECM and BEKK model for all the three error measures (MSE, RMSE and MAPE). This strongly suggested that the GARCH(1,1) model is more efficient than the VECM and BEKK models for the selected time series. The results were again supported by Frimpong and Oteng-Abayie (2006).

5.3 CONCLUSION

The study modelled the determinants of external debt using the VECM, GARCH and BEKK models with the intention of identifying and recommending the most effective approach. The analysis revealed the GARCH model to be more desirable and adequate according to the diagnostic and the forecast evaluation results. According to the mean equation results, the coefficients of LEXP and LGDP are positive and predict the series by 0.7185 and 3.0579 respectively. The coefficients of LCF and LGE are negative and predict the series by -1.6624 and -0.4267 respectively. The variance equation results revealed a positive and significant ARCH term and a negative and insignificant term. The results of the GARCH model revealed that the variance is increasing over time. It can be concluded that the most effective approach based on the results of this study is the GARCH model.

5.4 LIMITATIONS

The data used in this study is secondary data of external debt, capital formation, exports, government expenditure and gross domestic product from the second quarter of 1990 to the last quarter of 2018. The focus is on South African financial data. No surveys, interviews or questionnaires were conducted in this study. No other financial variables other than those relevant for the study were covered. The results of this study cannot be generalized as they apply only to the variables used in the study.

5.5 RECOMMENDATIONS

Based on the discussion of the findings, the study recommends the following areas for further studies.

- A comparative study can be done comparing different time series models such as the DCC-GARCH, VEC-GARCH and CCC-GARCH. Using different models may help in narrowing and suggesting more adequate models.
- A similar study can also be done using the same models but different financial data. This will help in confirming the results of this study.
- Another study can be done using the same models and the same variables from a different country to see if the same conclusion as in the study will be reached.
- The study also suggests that the South African government should implement effective external debt management strategies in safeguarding the financial stability of the country.
- The government could also achieve a high level of exports and GDP in order to decrease the external debt of the country and to have sustainable economic growth.
- Furthermore, government could also assist researchers financially to help in finding solutions to reduce external debt in South Africa for the country to have sustainable economic growth

5.5 SUMMARY OF THE STUDY

This study was divided into five chapters in order to address the objectives mentioned in chapter one. Chapter 1 provided a brief introduction and background of the study. The problem statement was identified and from there, the research aims and objectives, as well as the research questions were clearly explained in detail. Chapter 1 also highlighted the importance of modelling external debt using different time series models. Chapter 2 reviewed the theoretical and empirical literature on financial data and factors influencing financial data. The theoretical framework identified that there were many factors that drive financial data such as GDP, savings gaps, chronic government budget deficit, size of foreign aid and real exchange rate. Chapter 3 discussed the research methodology, variable analysis and the estimation techniques that were used to model financial data in South Africa, and chapter 4 presented the data analysis and interpretation of the results. Chapter 5 presented the discussion of the results, drew conclusions in respect of the objectives of the study and made recommendations for future study.

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APPENDICES

Appendix A1.1 Augmented Dickey Fuller test for Capital Formation at level

Null Hypothesis: LCF has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.052072	0.9510
Test critical values:	1% level	-3.488585	
	5% level	-2.886959	
	10% level	-2.580402	

*MacKinnon (1996) one-sided p-values.

Appendix A1.2 Augmented Dickey Fuller test for External Debt at level

Null Hypothesis: LED has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.979750	0.2954
Test critical values:	1% level	-3.489659	
	5% level	-2.887425	
	10% level	-2.580651	

*MacKinnon (1996) one-sided p-values.

Appendix A1.3 Augmented Dickey Fuller test for Exports at level

Null Hypothesis: LEXP has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.861314	0.7971
Test critical values:	1% level	-3.489117	
	5% level	-2.887190	
	10% level	-2.580525	

*MacKinnon (1996) one-sided p-values.

Appendix A1.4 Augmented Dickey Fuller test for Gross Domestic Product at level

Null Hypothesis: LGDP has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.718498	0.0002
Test critical values:	1% level	-3.488585	
	5% level	-2.886959	
	10% level	-2.580402	

*MacKinnon (1996) one-sided p-values.

Appendix A1.5 Augmented Dickey Fuller test for Government Expenditure at level

Null Hypothesis: LGE has a unit root
Exogenous: Constant
Lag Length: 7 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.559684	0.4996
Test critical values:	1% level	-3.492523	
	5% level	-2.888669	
	10% level	-2.581313	

*MacKinnon (1996) one-sided p-values.

Appendix A2.1 Augmented Dickey Fuller test for Capital Formation at first difference

Null Hypothesis: D(LCF) has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.457214	0.0004
Test critical values:	1% level	-3.490772	
	5% level	-2.887909	
	10% level	-2.580908	

Appendix A2.2 Augmented Dickey Fuller test for External Debt at first difference

Null Hypothesis: D(LED) has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.264228	0.0000
Test critical values:	1% level	-3.489659	
	5% level	-2.887425	
	10% level	-2.580651	

*MacKinnon (1996) one-sided p-values.

Appendix A2.3 Augmented Dickey Fuller test for External Debt at first difference

Null Hypothesis: D(LEXP) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.632545	0.0000
Test critical values:	1% level	-3.489117	
	5% level	-2.887190	
	10% level	-2.580525	

*MacKinnon (1996) one-sided p-values.

Appendix A2.4 Augmented Dickey Fuller test for Gross Domestic Product at first difference

Null Hypothesis: D(LGDP) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.068030	0.0000
Test critical values:	1% level	-3.489117	
	5% level	-2.887190	
	10% level	-2.580525	

*MacKinnon (1996) one-sided p-values.

Appendix A2.5 Augmented Dickey Fuller test for Government Expenditure at first difference

Null Hypothesis: D(LGE) has a unit root
Exogenous: Constant
Lag Length: 6 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.002092	0.0379
Test critical values:	1% level	-3.492523	
	5% level	-2.888669	
	10% level	-2.581313	

*MacKinnon (1996) one-sided p-values.

Appendix A3.1 Phillips Perron test for Capital Formation at level

Null Hypothesis: LCF has a unit root
Exogenous: Constant
Bandwidth: 5 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-0.011423	0.9549
Test critical values:	1% level	-3.488585	
	5% level	-2.886959	
	10% level	-2.580402	

*MacKinnon (1996) one-sided p-values.

Appendix A3.2 Phillips Perron test for External Debt at level

Null Hypothesis: LED has a unit root
Exogenous: Constant
Bandwidth: 0 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-1.809484	0.3743
Test critical values:	1% level	-3.488585	
	5% level	-2.886959	
	10% level	-2.580402	

*MacKinnon (1996) one-sided p-values.

Appendix A3.3 Phillips Perron test for Exports at level

Null Hypothesis: LEXP has a unit root
Exogenous: Constant
Bandwidth: 2 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-0.879236	0.7917
Test critical values:	1% level	-3.488585	
	5% level	-2.886959	
	10% level	-2.580402	

*MacKinnon (1996) one-sided p-values.

Appendix A3.4 Phillips Perron test for Gross Domestic Product at level

Null Hypothesis: LGDP has a unit root
Exogenous: Constant
Bandwidth: 5 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-4.518209	0.0003
Test critical values:	1% level	-3.488585	
	5% level	-2.886959	
	10% level	-2.580402	

*MacKinnon (1996) one-sided p-values.

Appendix A3.5 Phillips Perron test for Government Expenditure at level

Null Hypothesis: LGE has a unit root
Exogenous: Constant
Bandwidth: 13 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-1.347582	0.6054
Test critical values:	1% level	-3.488585	
	5% level	-2.886959	
	10% level	-2.580402	

*MacKinnon (1996) one-sided p-values.

Appendix A4.1 Phillips Perron test for Capital Formation at first difference

Null Hypothesis: D(LCF) has a unit root
Exogenous: Constant
Bandwidth: 5 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-11.28197	0.0000
Test critical values:	1% level	-3.489117	
	5% level	-2.887190	
	10% level	-2.580525	

*MacKinnon (1996) one-sided p-values.

Appendix A4.2 Phillips Perron test for External Debt at first difference

Null Hypothesis: D(LED) has a unit root
Exogenous: Constant
Bandwidth: 2 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-7.921281	0.0000
Test critical values:	1% level	-3.489117	
	5% level	-2.887190	
	10% level	-2.580525	

*MacKinnon (1996) one-sided p-values.

Appendix A4.3 Phillips Perron test for Exports at first difference

Null Hypothesis: D(LEXP) has a unit root
Exogenous: Constant
Bandwidth: 3 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-8.622566	0.0000
Test critical values:	1% level	-3.489117	
	5% level	-2.887190	
	10% level	-2.580525	

*MacKinnon (1996) one-sided p-values.

Appendix A4.4 Phillips Perron test for Gross Domestic Product at first difference

Null Hypothesis: D(LGDP) has a unit root
Exogenous: Constant
Bandwidth: 4 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-8.284587	0.0000
Test critical values:	1% level	-3.489117	
	5% level	-2.887190	
	10% level	-2.580525	

*MacKinnon (1996) one-sided p-values.

Appendix A4.5 Phillips Perron test for Government Expenditure at first difference

Null Hypothesis: D(LGE) has a unit root
Exogenous: Constant
Bandwidth: 11 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-51.16570	0.0001
Test critical values:	1% level	-3.489117	
	5% level	-2.887190	
	10% level	-2.580525	

*MacKinnon (1996) one-sided p-values.

Appendix B1 VAR lag order selection estimates

Lag	LogL	LR	FPE	AIC	SC	HQ
0	203.5740	NA	1.68e-08	-3.711663	-3.586764	-3.661031
1	983.9308	1473.197	1.24e-14	-17.83048	-17.08109*	-17.52669
2	1015.425	56.51332	1.10e-14	-17.95187	-16.57799	-17.39492
3	1032.440	28.94068	1.29e-14	-17.80261	-15.80424	-16.99250
4	1117.907	137.3864	4.23e-15*	-18.93284*	-16.30997	-17.86956*
5	1133.557	23.69389	5.16e-15	-18.75807	-15.51070	-17.44163
6	1152.578	27.02055	5.98e-15	-18.64631	-14.77446	-17.07671
7	1182.921	40.26936*	5.69e-15	-18.74619	-14.24985	-16.92344
8	1201.471	22.88346	6.89e-15	-18.62562	-13.50478	-16.54970

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Appendix B2.1 Trace test estimates

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.334721	98.27267	69.81889	0.0001
At most 1 *	0.194133	53.44235	47.85613	0.0136
At most 2	0.131095	29.70033	29.79707	0.0513
At most 3	0.066467	14.24303	15.49471	0.0765
At most 4 *	0.058898	6.677361	3.841466	0.0098

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Appendix B2.2 Maximum eigenvalue estimates

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.334721	44.83032	33.87687	0.0017
At most 1	0.194133	23.74202	27.58434	0.1440
At most 2	0.131095	15.45730	21.13162	0.2581
At most 3	0.066467	7.565665	14.26460	0.4246
At most 4 *	0.058898	6.677361	3.841466	0.0098

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Appendix B3.1 Vector error correction estimates

Vector Error Correction Estimates
Date: 05/18/20 Time: 14:29
Sample (adjusted): 1991Q2 2018Q4
Included observations: 111 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
LED(-1)	1.000000
LCF(-1)	-0.055519
	(0.52192)
	[-0.10638]
LEXP(-1)	-2.967074
	(1.38912)
	[-2.13593]
LGDP(-1)	-5.174740
	(1.77840)
	[-2.90978]
LGE(-1)	6.292528
	(1.28229)
	[4.90725]
C	-39.63335

Error Correction:	D(LED)	D(LCF)	D(LEXP)	D(LGDP)	D(LGE)
CointEq1	-0.042630	0.038642	-0.000772	-0.004716	-0.040075
	(0.01972)	(0.01138)	(0.00602)	(0.00205)	(0.00813)
	[-2.16167]	[3.39653]	[-0.12830]	[-2.29708]	[-4.93227]
D(LED(-1))	0.309950	0.014537	0.026303	0.017860	-0.055232
	(0.10292)	(0.05937)	(0.03139)	(0.01071)	(0.04240)
	[3.01162]	[0.24484]	[0.83785]	[1.66700]	[-1.30253]
D(LED(-2))	-0.175540	-0.014106	-0.034860	-0.028585	0.025732
	(0.10475)	(0.06043)	(0.03195)	(0.01090)	(0.04316)
	[-1.67586]	[-0.23344]	[-1.09102]	[-2.62146]	[0.59625]
D(LED(-3))	-0.001945	-0.065538	0.013441	0.040745	-0.023467
	(0.10104)	(0.05829)	(0.03082)	(0.01052)	(0.04163)
	[-0.01925]	[-1.12432]	[0.43609]	[3.87359]	[-0.56370]
D(LCF(-1))	-0.253594	-0.157099	-0.021401	-0.024266	0.052950
	(0.16892)	(0.09745)	(0.05153)	(0.01759)	(0.06960)
	[-1.50123]	[-1.61204]	[-0.41532]	[-1.37988]	[0.76080]
D(LCF(-2))	-0.135932	-0.175485	0.013986	-0.007810	0.172945
	(0.16927)	(0.09765)	(0.05163)	(0.01762)	(0.06974)
	[-0.80304]	[-1.79701]	[0.27087]	[-0.44323]	[2.47980]
D(LCF(-3))	-0.059230	0.038418	-0.018001	-0.011099	0.087331
	(0.16887)	(0.09742)	(0.05151)	(0.01758)	(0.06958)
	[-0.35074]	[0.39435]	[-0.34946]	[-0.63137]	[1.25518]
D(LEXP(-1))	-0.187034	0.135328	0.072567	-0.018899	0.041586
	(0.37664)	(0.21728)	(0.11489)	(0.03921)	(0.15518)
	[-0.49659]	[0.62282]	[0.63164]	[-0.48201]	[0.26799]
D(LEXP(-2))	0.829095	0.067431	0.057730	0.072774	-0.119730
	(0.37821)	(0.21819)	(0.11537)	(0.03937)	(0.15583)
	[2.19216]	[0.30905]	[0.50040]	[1.84837]	[-0.76835]
D(LEXP(-3))	0.350064	0.285124	-0.065717	-0.131313	0.127509
	(0.37980)	(0.21911)	(0.11585)	(0.03954)	(0.15648)
	[0.92171]	[1.30129]	[-0.56724]	[-3.32121]	[0.81485]
D(LGDP(-1))	-0.391565	1.019377	0.684261	0.181684	0.461938
	(1.05070)	(0.60616)	(0.32050)	(0.10938)	(0.43290)
	[-0.37267]	[1.68170]	[2.13495]	[1.66103]	[1.06707]
D(LGDP(-2))	-0.814319	1.676902	-0.037996	0.010325	-0.670252
	(1.06130)	(0.61227)	(0.32374)	(0.11048)	(0.43727)
	[-0.76728]	[2.73883]	[-0.11737]	[0.09346]	[-1.53282]
D(LGDP(-3))	-0.440628	1.085129	-0.156263	0.079851	-0.603790
	(1.06451)	(0.61412)	(0.32472)	(0.11082)	(0.43859)
	[-0.41392]	[1.76695]	[-0.48123]	[0.72056]	[-1.37665]
D(LGE(-1))	-0.010118	-0.195473	-0.026019	0.016170	-0.808118
	(0.15114)	(0.08719)	(0.04610)	(0.01573)	(0.06227)
	[-0.06695]	[-2.24189]	[-0.56437]	[1.02771]	[-12.9777]
D(LGE(-2))	-0.405162	-0.115354	-0.044492	0.005927	-0.716793
	(0.17454)	(0.10069)	(0.05324)	(0.01817)	(0.07191)
	[-2.32130]	[-1.14560]	[-0.83567]	[0.32618]	[-9.96756]
D(LGE(-3))	-0.258537	0.093816	-0.027532	0.014804	-0.757943
	(0.12736)	(0.07348)	(0.03885)	(0.01326)	(0.05248)
	[-2.02990]	[1.27681]	[-0.70867]	[1.11655]	[-14.4438]
C	0.088724	-0.069256	0.008725	0.017734	0.101661
	(0.04092)	(0.02361)	(0.01248)	(0.00426)	(0.01686)
	[2.16833]	[-2.93384]	[0.69906]	[4.16330]	[6.03017]

R-squared	0.280914	0.276458	0.128370	0.346331	0.917603
Adj. R-squared	0.158516	0.153302	-0.019992	0.235068	0.903578
Sum sq. resids	0.875725	0.291459	0.081484	0.009490	0.148657
S.E. equation	0.096521	0.055683	0.029442	0.010048	0.039768
F-statistic	2.295090	2.244776	0.865247	3.112726	65.42622
Log likelihood	111.2418	172.3003	243.0344	362.3667	209.6661
Akaike AIC	-1.698050	-2.798203	-4.072693	-6.222823	-3.471461
Schwarz SC	-1.283077	-2.383230	-3.657720	-5.807850	-3.056488
Mean dependent	0.045755	0.019027	0.019158	0.024418	0.024746
S.D. dependent	0.105220	0.060515	0.029152	0.011489	0.128068

Determinant resid covariance (dof adj.)	2.53E-15
Determinant resid covariance	1.10E-15
Log likelihood	1123.955
Akaike information criterion	-18.62982
Schwarz criterion	-16.43290
Number of coefficients	90

Appendix B3.2 Vector error correction estimates

System: UNTITLED
Estimation Method: Least Squares
Date: 12/09/19 Time: 10:28
Sample: 1991Q2 2018Q4
Included observations: 111
Total system (balanced) observations 111

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.042630	0.019721	-2.161670	0.0332
C(2)	0.309950	0.102918	3.011616	0.0033
C(3)	-0.175540	0.104746	-1.675863	0.0971
C(4)	-0.001945	0.101041	-0.019254	0.9847
C(5)	-0.253594	0.168924	-1.501232	0.1366
C(6)	-0.135932	0.169271	-0.803040	0.4240
C(7)	-0.059230	0.168870	-0.350742	0.7266
C(8)	-0.187034	0.376635	-0.496593	0.6206
C(9)	0.829095	0.378209	2.192160	0.0308
C(10)	0.350064	0.379800	0.921705	0.3590
C(11)	-0.391565	1.050705	-0.372668	0.7102
C(12)	-0.814319	1.061300	-0.767284	0.4448
C(13)	-0.440628	1.064515	-0.413924	0.6799
C(14)	-0.010118	0.151136	-0.066949	0.9468
C(15)	-0.405162	0.174540	-2.321305	0.0224
C(16)	-0.258537	0.127364	-2.029903	0.0452
C(17)	0.088724	0.040918	2.168329	0.0327

Determinant residual covariance	0.007889

Equation: D(LED) = C(1)*(LED(-1) - 0.0555194989558*LCF(-1) - 2.96707366506*LEXP(-1) - 5.17474036122*LGDP(-1) + 6.29252781407*LGE(-1) - 39.6333484638) + C(2)*D(LED(-1)) + C(3) *D(LED(-2)) + C(4)*D(LED(-3)) + C(5)*D(LCF(-1)) + C(6)*D(LCF(-2)) + C(7)*D(LCF(-3)) + C(8)*D(LEXP(-1)) + C(9)*D(LEXP(-2)) + C(10) *D(LEXP(-3)) + C(11)*D(LGDP(-1)) + C(12)*D(LGDP(-2)) + C(13) *D(LGDP(-3)) + C(14)*D(LGE(-1)) + C(15)*D(LGE(-2)) + C(16)*D(LGE(-3)) + C(17)

Observations: 111

R-squared	0.280914	Mean dependent var	0.045755
Adjusted R-squared	0.158516	S.D. dependent var	0.105220
S.E. of regression	0.096521	Sum squared resid	0.875725
Durbin-Watson stat	2.056063		

Appendix B4.1 Heteroscedasticity estimates for GARCH(1,1)

Heteroskedasticity Test: ARCH

F-statistic	45.18958	Prob. F(1,112)	0.0000
Obs*R-squared	32.77324	Prob. Chi-Square(1)	0.0000

Appendix B4.2 GARCH(1,1) estimates

Dependent Variable: LED
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 12/09/19 Time: 13:38
Sample: 1990Q2 2018Q4
Included observations: 115
Convergence achieved after 72 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	10.02603	0.559083	17.93301	0.0000
LCF	-1.662384	0.079862	-20.81584	0.0000
LEXP	0.718473	0.279726	2.568491	0.0102
LGDP	3.057859	0.252375	12.11634	0.0000
LGE	-0.426719	0.124995	-3.413878	0.0006
Variance Equation				
C	0.008713	0.002558	3.406943	0.0007
RESID(-1)^2	1.126152	0.324230	3.473313	0.0005
GARCH(-1)	-0.065948	0.046279	-1.425020	0.1542

R-squared	0.963704	Mean dependent var	10.56676
Adjusted R-squared	0.962384	S.D. dependent var	1.479440
S.E. of regression	0.286935	Akaike info criterion	-0.366499
Sum squared resid	9.056495	Schwarz criterion	-0.175547
Log likelihood	29.07370	Hannan-Quinn criter.	-0.288993
Durbin-Watson stat	0.267016		

Appendix B5 BEKK estimates

Variable	Coeff	Std Error	T-Stat	Signif
1. Mean(LNGDP)	6.01499437	0.01716593	350.40303	0.00000000
2. Mean(LNEXP)	4.13731466	0.01379770	299.85544	0.00000000
3. Mean(LNED)	11.20282527	0.01933528	579.39828	0.00000000
4. Mean(LNGE)	11.56341461	0.02223571	520.03809	0.00000000
5. Mean(LNCF)	9.22925519	0.02812857	328.10971	0.00000000
6. C(1,1)	0.04618627	0.00602273	7.66866	0.00000000
7. C(2,1)	0.03944208	0.00506933	7.78053	0.00000000

8. C(2,2)	0.01463829	0.00222656	6.57441	0.00000000
9. C(3,1)	0.04695602	0.00889476	5.27907	0.00000013
10. C(3,2)	0.02319667	0.00719684	3.22317	0.00126779
11. C(3,3)	0.03683464	0.00473759	7.77497	0.00000000
12. C(4,1)	0.05098930	0.01189161	4.28784	0.00001804
13. C(4,2)	0.00962450	0.00707937	1.35951	0.17398410
14. C(4,3)	0.06504920	0.00455002	14.29647	0.00000000
15. C(4,4)	0.00598527	0.01457300	0.41071	0.68128550
16. C(5,1)	0.07452314	0.00831774	8.95955	0.00000000
17. C(5,2)	-0.01865438	0.00496894	-3.75420	0.00017390
18. C(5,3)	-0.00279125	0.00441299	-0.63251	0.52705579
19. C(5,4)	-0.00450944	0.01129505	-0.39924	0.68971613
20. C(5,5)	0.00030145	0.00656580	0.04591	0.96338032
21. A(1,1)	0.50703016	0.01455395	34.83797	0.00000000
22. A(1,2)	-0.21160108	0.03117470	-6.78759	0.00000000
23. A(1,3)	-0.08888142	0.08609775	-1.03233	0.30191684
24. A(1,4)	1.15287677	0.06960901	16.56218	0.00000000
25. A(1,5)	-0.66337284	0.03546783	-18.70351	0.00000000
26. A(2,1)	0.04698010	0.02093641	2.24394	0.02483606
27. A(2,2)	0.92006861	0.03946313	23.31464	0.00000000
28. A(2,3)	-0.12457856	0.07770577	-1.60321	0.10888860
29. A(2,4)	-0.12696116	0.04724523	-2.68728	0.00720365
30. A(2,5)	0.21399208	0.05183709	4.12817	0.00003657
31. A(3,1)	0.06283457	0.00792315	7.93050	0.00000000
32. A(3,2)	0.03701066	0.00882411	4.19427	0.00002738
33. A(3,3)	0.95655976	0.03936073	24.30239	0.00000000
34. A(3,4)	-0.06479133	0.01586117	-4.08490	0.00004410
35. A(3,5)	0.06591189	0.01201366	5.48641	0.00000004
36. A(4,1)	0.14515070	0.01593229	9.11047	0.00000000
37. A(4,2)	0.05307129	0.02293290	2.31420	0.02065679
38. A(4,3)	0.13214845	0.05293331	2.49651	0.01254226

39. A(4,4)	-0.10331387	0.03314158	-3.11735	0.00182485
40. A(4,5)	0.21172866	0.02697361	7.84947	0.00000000
41. A(5,1)	0.20637765	0.01225506	16.84020	0.00000000
42. A(5,2)	0.16828976	0.01911733	8.80300	0.00000000
43. A(5,3)	0.09928444	0.04903736	2.02467	0.04290135
44. A(5,4)	0.23666463	0.02581721	9.16693	0.00000000
45. A(5,5)	1.17591454	0.04813997	24.42699	0.00000000
46. B(1,1)	0.32073505	0.01658506	19.33879	0.00000000
47. B(1,2)	0.34649998	0.04397380	7.87969	0.00000000
48. B(1,3)	-0.19351001	0.11200898	-1.72763	0.08405468
49. B(1,4)	0.05927923	0.02370561	2.50064	0.01239686
50. B(1,5)	0.62805815	0.04601977	13.64757	0.00000000
51. B(2,1)	-0.12653989	0.01861971	-6.79602	0.00000000
52. B(2,2)	0.18363558	0.03551571	5.17055	0.00000023
53. B(2,3)	0.52853616	0.10742129	4.92022	0.00000086
54. B(2,4)	-0.17446942	0.05840565	-2.98720	0.00281544
55. B(2,5)	-0.35744665	0.05088597	-7.02446	0.00000000
56. B(3,1)	-0.19943526	0.01345627	-14.82099	0.00000000
57. B(3,2)	-0.19039500	0.01165698	-16.33313	0.00000000
58. B(3,3)	-0.24452529	0.03449373	-7.08898	0.00000000
59. B(3,4)	-0.21007216	0.01851691	-11.34488	0.00000000
60. B(3,5)	-0.12917402	0.02184038	-5.91446	0.00000000
61. B(4,1)	0.10047844	0.01661372	6.04792	0.00000000
62. B(4,2)	-0.07351777	0.02233507	-3.29158	0.00099625
63. B(4,3)	-0.17149684	0.06770704	-2.53292	0.01131152
64. B(4,4)	0.52139494	0.01324885	39.35398	0.00000000
65. B(4,5)	-0.25483116	0.02265837	-11.24667	0.00000000
66. B(5,1)	-0.14848448	0.01994657	-7.44411	0.00000000
67. B(5,2)	-0.23023827	0.02993672	-7.69083	0.00000000
68. B(5,3)	0.02292667	0.06785110	0.33790	0.73544095
69. B(5,4)	-0.27665582	0.03815676	-7.25051	0.00000000

70. B(5,5) 0.03475389 0.04129221 0.84166 0.39997981

Appendix C1.1 Serial correlation estimates for VECM

VEC Residual Serial Correlation LM Tests
Date: 11/27/19 Time: 12:39
Sample: 1990Q2 2018Q4
Included observations: 111

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	18.72385	25	0.8102	0.743734	(25, 317.3)	0.8105
2	24.26424	25	0.5042	0.972049	(25, 317.3)	0.5048
3	35.38833	25	0.0814	1.442248	(25, 317.3)	0.0817
4	24.27393	25	0.5036	0.972452	(25, 317.3)	0.5042

Appendix C1.2 Normality estimates for VECM

Component	Jarque-Bera	df	Prob.
1	2.323686	2	0.3129
2	1.458595	2	0.4822
3	6.607749	2	0.0367
4	5.356078	2	0.0687
5	0.970108	2	0.6157
Joint	16.71622	10	0.0809

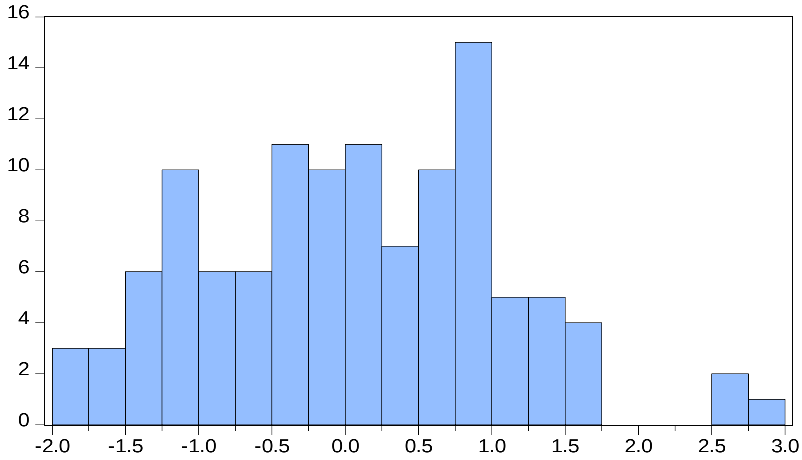
Appendix C1.3 Heteroscedasticity estimates for VECM

VEC Residual Heteroskedasticity Tests (Levels and Squares)
Date: 11/27/19 Time: 12:42
Sample: 1990Q2 2018Q4
Included observations: 111

Joint test:

Chi-sq	df	Prob.
547.6993	480	0.0174

Appendix C2.1 Normality estimates for GARCH(1,1)



Series: Standardized Residuals
Sample 1990Q2 2018Q4
Observations 115

Mean	0.056574
Median	0.015302
Maximum	2.900309
Minimum	-1.859194
Std. Dev.	1.002464
Skewness	0.262021
Kurtosis	2.821199
Jarque-Bera	1.469072
Probability	0.479728

Appendix C2.2 Heteroscedasticity estimates for GARCH(1,1)

Heteroskedasticity Test: White
Null hypothesis: Homoskedasticity

F-statistic	1.985417	Prob. F(4,110)	0.1017
Obs*R-squared	7.743588	Prob. Chi-Square(4)	0.1014
Scaled explained SS	1073.302	Prob. Chi-Square(4)	0.0000

Appendix C3.1 Serial correlation estimates for BEKK

Multivariate Q Test

Test Run Over 1990:02 to 2018:04

Lags Tested 5

Degrees of Freedom 125

Q Statistic 651.5699

Signif Level 0.0000

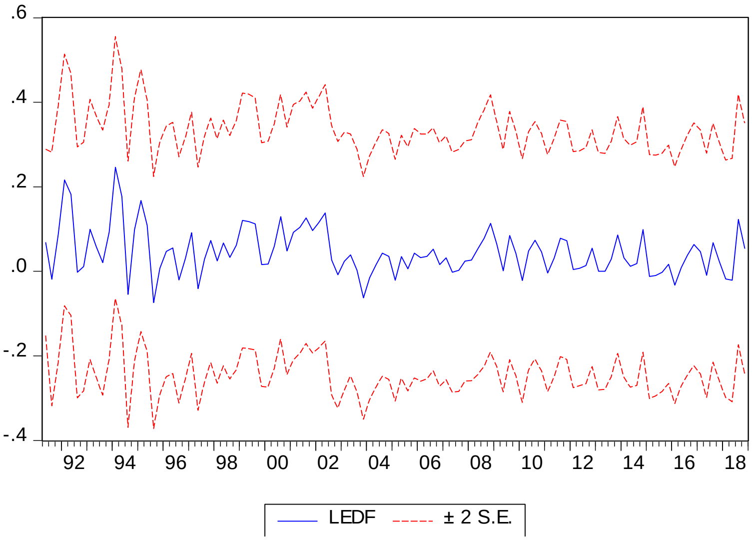
Appendix C3.2 Heteroscedasticity estimates for BEKK

Multivariate ARCH Test

Statistic Degrees Signif

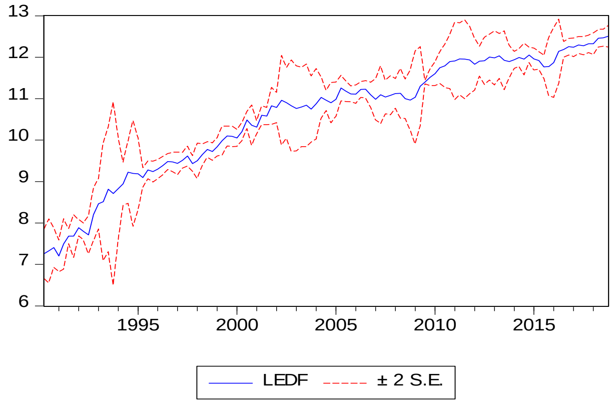
4037.57 900 0.00000

Appendix D1 Model evaluation estimates for VECM

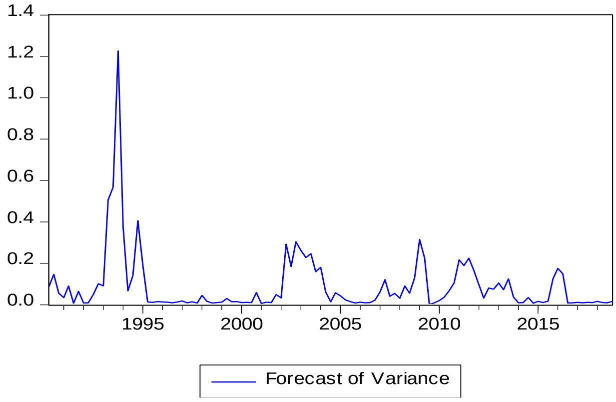


Forecast: LEDF	
Actual: D(LED)	
Forecast sample: 1990Q2 2018Q4	
Adjusted sample: 1991Q2 2018Q4	
Included observations: 111	
Root Mean Squared Error	0.088822
Mean Absolute Error	0.069900
Mean Abs. Percent Error	205.5909
Theil Inequality Coef.	0.476915
Bias Proportion	0.000000
Variance Proportion	0.307179
Covariance Proportion	0.692821
Theil U2 Coefficient	1.063689
Symmetric MAPE	130.2673

Appendix D2 Model evaluation etsimates for GARCH(1,1) model



Forecast: LEDF	
Actual: LED	
Forecast sample: 1990Q2 2018Q4	
Included observations: 115	
Root Mean Squared Error	0.288847
Mean Absolute Error	0.211454
Mean Abs. Percent Error	2.106894
Theil Inequality Coef.	0.013542
Bias Proportion	0.000001
Variance Proportion	0.036109
Covariance Proportion	0.963890
Theil U2 Coefficient	2.482838
Symmetric MAPE	2.092800



Appendix D3 Model evaluation for BEKK model

Forecast Analysis for H
From 1990:02 to 2018:04

Mean Error	-3893.500
Mean Absolute Error	3893.500
Root Mean Square Error	39073.533

Mean Square Error	1526741007.748451
Theil's U	2.336045
Mean Pct Error	-4.082529
Mean Abs Pct Error	4.082529
Root Mean Square Pct Error	4.434276
Theil's Relative U	2.808334