

The relationship between problem solving and self-directed learning in Grade 7 mathematics classrooms

Shain Jurie Hofmeyer
Student no: 12895911

Dissertation submitted in the fulfillment of the requirements for
the degree *Magister Educationis* in *Mathematics Education* in
the Faculty of Education Sciences at the Potchefstroom Campus
of the North-West University

Supervisor: **Dr SM Nieuwoudt**

Potchefstroom
May 2016



NORTH-WEST UNIVERSITY
YUNIBESITHI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT

Private Bag X6001, Potchefstroom
South Africa 2520

Tel: (018) 299-4900
Faks: (018) 299-4910
Web: <http://www.nwu.ac.za>

Ethics Committee

Tel +27 18 299 4850
Fax +27 18 293 5329
Email Ethics@nwu.ac.za

7 March 2013

ETHICS APPROVAL OF PROJECT

- ☐ The North-West University Ethics Committee (NWU-EC) hereby approves your project as indicated below. This implies that the NWU-EC grants its permission that, provided the special conditions specified below are met and pending any other authorisation that may be necessary, the project may be initiated, using the ethics number below.

Project title: The relationship between problem solving and self-directed learning in grade 7 mathematics classrooms

Project Leader: Dr. Nieuwoudt

Ethics number:

N	W	U	-	0	0	1	2	4	-	1	2	-	A	2
Institution			Project Number						Year			Status		

Status: S = Submission; R = Re-Submission; P = Provisional Authorisation; A = Authorisation

Approval date: 2013/03/07

Expiry date: 2018/03/06

Special conditions of the approval (if any): None

General conditions:

While this ethics approval is subject to all declarations, undertakings and agreements incorporated and signed in the application form, please note the following:

- The project leader (principal investigator) must report in the prescribed format to the NWU-EC:
 - annually (or as otherwise requested) on the progress of the project,
 - without any delay in case of any adverse event (or any matter that interrupts sound ethical principles) during the course of the project.
- The approval applies strictly to the protocol as stipulated in the application form. Would any changes to the protocol be deemed necessary during the course of the project, the project leader must apply for approval of these changes at the NWU-EC. Would there be deviation from the project protocol without the necessary approval of such changes, the ethics approval is immediately and automatically forfeited.
- The date of approval indicates the first date that the project may be started. Would the project have to continue after the expiry date, a new application must be made to the NWU-EC and new approval received before or on the expiry date.
- In the interest of ethical responsibility the NWU-EC retains the right to:
 - request access to any information or data at any time during the course or after completion of the project;
 - withdraw or postpone approval if:
 - any unethical principles or practices of the project are revealed or suspected,
 - it becomes apparent that any relevant information was withheld from the NWU-EC or that information has been false or misrepresented,
 - the required annual report and reporting of adverse events was not done timely and accurately,
 - new institutional rules, national legislation or international conventions deem it necessary.

The Ethics Committee would like to remain at your service as scientist and researcher, and wishes you well with your project. Please do not hesitate to contact the Ethics Committee for any further enquiries or requests for assistance.

Yours sincerely

Prof Amanda Lourens
(chair NWU Ethics Committee)

DECLARATION

I, SHAIN JURIE HOFMEYER, declare that **“The relationship between problem solving and self-directed learning in Grade 7 mathematics classrooms”**, is my own work and that all the sources I have used or quoted have been indicated and acknowledged by means of complete references.

Signature: _____

Date: _____

Potchefstroom

DEDICATION

This dissertation is dedicated to my wife and three sons who always believe in me and who are my inspiration in everything I do.

ACKNOWLEDGEMENTS

My sincere gratitude to the following people who contributed immensely to the successful completion of this study:

- My supervisor Dr S.M. Nieuwoudt for her wisdom, sharpness, patience, encouragement, expertise, constructive criticism and motivation throughout this study.
- The North-West Department of Education for permission granted to access primary schools to conduct this research.
- The principals and educators of the participating schools, for their mutual cooperation, respect and assistance in completing research questionnaires.
- The North-West University for granting me funding through the *Queen Mother Semane Molotlegi Bursary* to undertake this study.
- Professor Faans Steyn from the statistical Consultation Services of North West University for his expert advice and statistical analysis of the data.
- All participating Grade 7 Mathematics learners from both the experimental group and control group for their co-operation and contributions during collection of data and task-based interviews.
- Mrs Cecilia Van Der Walt for language editing in such a professional manner.
- My wife, Alwin, and children Kerwyn, Tristan and Chad for the sacrifices they have made to make this study possible.
- My colleague for showing compassion and support throughout this study

I also want to acknowledge God's amazing grace by which grace everything is possible. May this study in some way be used by others, and in so doing bring glory and honour to God's name.

ABSTRACT

THE RELATIONSHIP BETWEEN PROBLEM SOLVING AND SELF-DIRECTED LEARNING IN GRADE 7 MATHEMATICS CLASSROOMS

The learners in the South African school system did not perform well in international assessments such as the Trends in Mathematics and Science Studies (TIMSS) in 1995, 1999 and 2003, respectively. Because of these below par achievements, problem areas were identified in terms of literacy and numeracy. In 2011 the Department of Basic Education implemented the Annual National Assessments in an attempt to achieve the goals set by the Department. These goals included, amongst others, to improve learners' reading and writing skills, equip them to think critically and to solve mathematical problems, in order for learners to become productive and meaningful citizens (DBE, 2011:8).

The primary goal of this study was to investigate the relationship between problem solving and self-directed learning in Grade 7 Mathematics classrooms. In this study a sequential explanatory mixed-method research design was used (Ivankova *et al.*, 2006:4) in order to determine the influence of problem solving activities on Grade 7 learners' self-directed learning abilities, and to determine whether self-directed learning, through problem solving, has an influence on learners' mathematical achievement.

During the quantitative investigation, learners from the experimental as well as the control groups completed questionnaires to determine their self-directed learning ability. A self-directed learning instrument (SDLI) and selected fields of the LASSI(HS) were used in both pre- and post-tests. In addition, learners' March as well as their November report results were also taken into consideration.

The qualitative investigation included task-based activities conducted with selected learners from the experimental group. These learners had to solve mathematical problems with respect to topics from each learning content area in grade 7 Mathematics. The qualitative investigation was based on Polya's model of problem-solving, where learners had to implement the four suggested phases, namely; (1) understand the problem, (2) make a plan to solve the problem, (3) carry out the plan and (4) look back or reflect on the solution. The learners had to make predictions regarding the solutions to given mathematical problems. However, the learners in general over-estimated their problem-solving abilities, and their predictions were lower than their actual abilities.

The findings did not give a clear indication with respect to whether or not a relationship exists between self-directed learning, problem-solving and mathematical achievement. Although it is not

clear whether self-directed learning or problem-solving activities or both had an influence on learners' mathematical achievement, there was an improvement in the experimental group's average mathematical achievement after the intervention through problem-solving activities.

KEYWORDS: Learning and teaching of mathematics; Mathematics; mathematical learning strategies; mathematical problem solving; self-directed learning; self-directed learning ability

OPSOMMING

DIE VERBAND TUSSEN PROBLEEMOPLOSSING EN SELFGERIGTE LEER IN GRAAD 7 WISKUNDEKLASKAMERS

Die leerders in die Suid-Afrikaanse skolestelsel het tydens die internasionale assesserings soos die *Trends in Mathematics and Science Studies (TIMSS)* in 1995, 1999 en 2003 nie goed gevaar nie. As gevolg van swak uitslae, is probleemareas geïdentifiseer ten opsigte van geletterdheid en gesyferdheid. In 2011 het die Departement van Basiese Onderwys die Jaarlikse Nasionale Assessering ingestel in 'n poging om doelwitte te bereik wat deur die Departement gestel is. Hierdie doelwitte sluit onder andere die verbetering van leerders se lees- en skryfvermoë, kritiese denke en vermoë om wiskundeprobleme op te los, sowel as om hulle toe te rus om produktiewe en sinvolle burgers te word, in (DBE, 2011:8).

Die primêre doel van hierdie studie was om die verband tussen probleemoplossing en selfgerigte leer in graad 7 wiskundeklaskamers te ondersoek. In hierdie studie is 'n opeenvolgende verduidelikende gemengde navorsingsmetode gebruik (Ivankova *et al.*, 2006:4) om die invloed van probleemoplossingsaktiwiteite op graad 7 wiskundeleerders se selfgerigte leervermoëns te bepaal, asook om te bepaal of selfgerigte leer deur middel van probleemoplossing 'n invloed op leerders se wiskundeprestasie het.

Tydens die kwantitatiewe ondersoek, het leerders van die eksperimentele sowel as die kontrolegroepe vraelyste ingevul om hulle selfgerigte vermoëns te bepaal. 'n Selfgerigte leer-instrument (SDLI) en geselekteerde velde van die LASSI(HS) is in beide voor- en natoetse gebruik. Bykomend is leerders se Maart sowel as hulle November rapportpunte in ag geneem.

Die kwalitatiewe ondersoek het die voltooiing van taakgebaseerde aktiwiteite met geselekteerde leerders van die eksperimentele groep ingesluit. Hierdie leerders moes wiskundeprobleme met betrekking tot die onderwerpe van die leerinhoud in graad 7 Wiskunde oplos. Die kwalitatiewe ondersoek was gebaseer op Polya se probleemoplossingsmodel, waartydens leerders die vier voorgestelde fases moes implementeer, naamlik (1) verstaan die probleem, (2) maak 'n plan om die probleem op te los, (3) voer die plan uit en (4) kyk terug of dink na oor die oplossing. Die leerders moes ook voorspellings maak met betrekking tot die oplossing van die gegewe

wiskundeprobleme. Oor die algemeen het die leerders hul probleemoplossingsvermoë oorskat, en was hul voorspellings laer as hul werklike vermoë.

Die bevindinge het nie 'n duidelike aanduiding gegee ten opsigte van of daar 'n verband bestaan of nie tussen selfgerigte leer, probleemoplossing en wiskundeprestasie nie. Alhoewel dit nie duidelik is of selfgerigte leer of probleemoplossingsaktiwiteite of beide leerders se wiskundeprestasie beïnvloed het nie, was daar 'n verbetering in die eksperimentele groep se gemiddelde wiskundeprestasie na die intervensie deur middel van probleemoplossingsaktiwiteite.

SLEUTELWOORDE: Leer en onderrig van Wiskunde; wiskunde leerstrategieë; selfgerigte leer; selfgerigte leervermoë; wiskunde; wiskundige probleemoplossing.

TABLE OF CONTENTS

ETHICAL APPROVAL OF PROJECT.....	i
DECLARATION	ii
DEDICATION.....	iii
ACKNOWLEDGEMENTS	iv
ABSTRACT	v
OPSOMMING	vii
LIST OF FIGURES.....	xvii
LIST OF TABLES	xx
<u>CHAPTER 1:</u> ORIENTATION AND PROGRAM OF STUDY	1
1.1 Introduction and problem statement.....	1
1.2 Research questions	3
1.2.1 What is the influence of problem-solving activities on Grade 7 learners' self-directed learning abilities?	3
1.2.2 What is the influence of self-directed learning through problem-solving on Grade 7 learners' achievement in Mathematics?	3
1.3 Literature study	3
1.4 Research design and methodology	4
1.4.1 Quantitative research method.....	5
1.4.1.1 Study population and sampling.....	6
1.4.1.2 Measuring instruments and data collection	7
1.4.1.3 Data analysis.....	8
1.4.2 Qualitative research method	8

1.4.2.1	Data generation and participants	9
1.4.2.2	Data analysis	9
1.5	Procedure of the empirical investigation.....	9
1.6	Contribution of this study	11
1.7	Limitations of the research study.....	11
1.8	Ethical aspects of the research	11
1.9	Structure of the dissertation and overview	12
<u>CHAPTER 2:</u>	SELF-DIRECTED LEARNING IN THE MATHEMATICS	
	CLASSROOM.....	14
2.1	Introduction	14
2.2	Approaches to learning Mathematics	14
2.2.1	The behaviouristic approach.....	14
2.2.2	The cognitive approach	16
2.2.3	The constructivist approach.....	18
2.3	Brief history of self-directed learning	19
2.3.1	Definition of self-directed learning (SDL)	21
2.3.2	Factors influencing self-directed learning.....	21
2.3.2.1	External factors.....	23
2.3.2.1.1	The social setting / learning environment.....	23
2.3.2.1.2	The Mathematics curriculum.....	24
2.3.2.1.3	The Mathematics teacher	24

2.3.2.1.4	The use of resources in the classroom	25
2.3.2.1.5	The use of mathematical tasks	26
2.3.2.2	Internal factors.....	27
2.3.2.2.1	Personal characteristics of a mathematics learner.....	27
a)	Learner motivation.....	27
b)	Goal orientation	28
c)	Self-efficacy.....	28
d)	Locus of control	29
2.3.2.2.2	The use of learning strategies	29
a)	Cognitive learning strategies	29
b)	Metacognitive learning strategies	31
c)	Affective learning strategies.....	32
2.3.2.2.3	The application of skills.....	32
2.4	Self-directed learning ability.....	33
2.4.1	Learning motivation	34
2.4.1.1	Intrinsic motivation.....	34
2.4.1.2	Extrinsic Motivation.....	34
2.4.2	Planning and implementing	35
2.4.3	Interpersonal communication.....	36
2.4.4	Self-monitoring	36
2.5	Other factors influencing self-directed learning ability	36
2.5.1	Concentration (CON).....	37

2.5.2	Time management (TMT)	37
2.5.3	Self-testing (SFT)	38
2.5.4	Motivation (MOT).....	38
2.5.5	Using Study Aids (STA)	38
2.6	The mathematics teachers' role during the facilitation of SDL	39
2.7	Advantages of self-directed learning	44
2.8	Conclusion.....	44
<u>CHAPTER 3:</u>	PROBLEM-SOLVING IN THE MATHEMATICS CLASSROOM.....	45
3.1	Introduction	45
3.2	Definitions of problem-solving	45
3.3	Models of problem-solving	47
3.4	Characteristics of a mathematical problem solver.....	52
3.5	The mathematics teachers' role in teaching problem-solving	54
3.5.1	Planning for problem-solving	54
3.5.2	Teachers' actions during problem-solving sessions	60
3.5.2.1	Understanding the problem (Phase 1)	61
3.5.2.2	Making a plan and carrying it out (Phases 2 and 3)	61
3.5.2.3	Reflecting on the solution (Phase 4)	61
3.6	Factors influencing mathematical problem-solving.....	62
3.6.1	Attitudes of teachers and learners towards Mathematics	62
3.6.2	Emotions of learners.....	63
3.6.3	Metacognition	63

3.6.4	The reading and comprehending ability of learners during problem-solving	64
a)	General structure features of a problem	64
b)	The semantic structure of a problem.....	65
c)	The problem-solving process.....	65
3.7	Theoretical relationship between SDL and problem solving.....	65
3.8	Problem solving: Implications for teachers and learners.....	66
3.9	Conclusion.....	67
<u>CHAPTER 4:</u>	THE INFLUENCE OF PROBLEM-SOLVING ACTIVITIES ON GRADE 7 MATHEMATICS LEARNERS' SELF- DIRECTED LEARNING ABILITIES AND THEIR MATHEMATICAL ACHIEVEMENT	
4.1	Introduction	68
4.2	Purpose and aims of the research	68
4.2.1	To determine the influence of problem-solving activities on Grade 7 Mathematics learners' self-directed learning abilities.	68
4.2.2	To determine the influence of self-directed learning through problem-solving on learners' mathematical achievement	68
4.3	Empirical investigation	68
4.3.1	The quantitative investigation	69
4.3.1.1	Experimental design and procedure	69
4.3.1.2	Study population.....	70
4.3.1.3	Measuring instruments	70
4.3.1.4	Validity and reliability of the instruments and data	73
4.3.2	Discussion of the results of the quantitative investigation	75
4.3.2.1	The influence of problem solving activities on grade 7 learners'	

	self-directed learning ability	75
4.3.2.1.1	Learners' SDL-ability prior to intervention of problem-solving activities....	76
4.3.2.1.2	Learners' self-directed learning ability after the intervention of problem solving	77
4.3.2.2	The relation between factors influencing self-directed learning and problem-solving activities.....	79
4.3.2.2.1	Factors influencing self-directed learning (prior to the intervention of problem-solving activities).....	79
4.3.2.2.2	Factors influencing self-directed learning (after the intervention of problem-solving activities).....	82
4.3.2.2.3	Factors influencing self-directed learning (comparing the pre- and post-test of the experimental group)	83
4.3.2.3	The influence of problem-solving activities on the mathematical achievement of the Grade 7 learners.....	84
4.3.2.4	Summary of the quantitative results.....	86
4.3.3	The qualitative investigation.....	86
4.3.3.1	Aim and motivation	86
4.3.3.2	Participants.....	87
4.3.3.3	Data collection procedures	87
4.3.3.4	Trustworthiness	89
4.3.3.5	Qualitative results	90
4.3.3.5.1	Analysis of the task-based activities.....	90
4.3.3.5.2	Discussion of the results of the task-based activities	91
4.3.3.5.3	Summary of the findings of the qualitative investigation	114
<u>CHAPTER 5:</u>	SUMMARY, CONCLUSION AND RECOMMENDATIONS	115

5.1	Introduction	115
5.2	Overview of the investigation	115
5.3	Findings of the investigation	116
5.3.1	Theoretical findings.....	116
5.3.1.1	Self-directed learning in the Mathematics classroom	116
5.3.1.2	Problem-solving in the Mathematics classroom	117
5.3.2	Empirical findings	118
5.3.2.1	The influence of problem-solving activities on Grade 7 learners' self-directed learning abilities.....	118
5.3.2.2	The influence of self-directed learning through problem-solving on grade 7 learners' achievement in Mathematics	118
5.4	Conclusions.....	119
5.5	Limitations of the study	119
5.6	Recommendations	120
5.7	Final word	120
REFERENCES.....		122
APPENDIX A:	Permission from the North West Department of Education	144
	Permission letter to the Principal (Experimental Group)	145
	Permission letter to the Principal (Control Group)	146
	Permission letter to Parents (Experimental Group)	147
	Permission letter to Parents (Control Group)	148
	Permission from Learners/Participants (Experimental Group and Control Group).....	149

APPENDIX B:	Adapted LASSI questionnaire Afrikaans	150
APPENDIX C:	Adapted LASSI questionnaire English	153
APPENDIX D:	Self-Directed Learning instrument (SDLI)	156
APPENDIX E:	An example of a task-based activity.....	157
APPENDIX F:	Checking of Bibliography	158
APPENDIX G:	Language-editing declaration	159
APPENDIX H:	Permission to conduct research.....	160
APPENDIX I:	Statistical consultation	161

LIST OF FIGURES

Figure 1.1:	A mixed-method sequential explanatory design (Ivankova, Creswell & Stick, 2006:16).....	5
Figure 1.2:	Procedure of the empirical investigation.....	10
Diagram 1	Stimulus and response diagram.....	15
Figure 2.1:	The Information Processing Model (adapted from AeU, 2011:113)	17
Figure 2.2:	A model for self-directed learning in primary school learners (adapted from Van Deur & Murray-Harvey, 2005:168)	22
Figure 2.3:	The mathematical tasks framework (Adapted from Stein & Smith, 1998: 270).	27
Figure 3.1:	A linear model of problem-solving as illustrated in most grade 7 Mathematics textbooks (Wilson, Fernandez & Hadaway, 1994)	48
Figure 3.2:	A cyclic illustration of the problem-solving model of Polya (Wilson, Fernandez & Hadaway, 1994).....	50
Figure 3.3:	The mathematical problem-solving process (adapted from Rigelman, 2007:313)	51
Figure 4.1:	Differences between the domains of SDL-domains of the experimental and control groups prior to intervention.....	77
Figure 4.2:	Differences between the SDL-domains of the experimental and control groups after intervention.....	78
Figure 4.3:	Comparison of the SDL-domains of the experimental group	79
Figure 4.4:	Comparison of the SDL-domains of the control group.....	80
Figure 4.5:	Comparison between the experimental and the control groups with respect to the factors influencing self-directed learning (prior to intervention).....	82

Figure 4.6:	Comparison between the experimental and the control groups with respect to the factors influencing self-directed learning (after the intervention).....	83
Figure 4.7:	Comparison between the domains of SDLA of the experimental group (pre- and post-test)	84
Figure 4.8:	An outline of qualitative trustworthiness (Adapted from Lincoln <i>et al.</i> 1985:290; Markula <i>et al.</i> , 2011:205).....	89
Figure 4.9:	Comparison of low level SDL ability learners' prediction and actual achievement in Question 1.....	95
Figure 4.10:	Comparison of medium level SDL ability learners' prediction and actual achievement in Question 1.....	95
Figure 4.11:	Comparison of high level SDL ability learners' prediction and actual achievement in Question 1.....	96
Figure 4.12:	Comparison of low-level SDL ability learners' prediction and actual achievement in Question 2.....	99
Figure 4.13:	Comparison of medium-level SDL-ability learners' prediction and actual achievement in Question 2.....	100
Figure 4.14:	Comparison of high-level SDL-ability learners' prediction and actual achievement for Question 2.....	100
Figure 4.15:	Comparison of low-level SDL-ability learners' prediction and actual achievement in Question 3.....	104
Figure 4.16:	Comparison of high-level SDL-ability learners' prediction and actual achievement in Question 3.....	105
Figure 4.17:	Comparison of high-level SDL-ability learners' prediction and actual achievement in Question 3.....	105
Figure 4.18:	Comparison of medium-level SDL-ability learners' prediction and actual achievement in Question 4.....	108
Figure 4.19:	Comparison of medium-level SDL-ability learners' prediction and actual achievement in Question 4.....	109

Figure 4.20:	Comparison of high-level SDL-ability learners' prediction and actual achievement in Question 4.....	109
Figure 4.21:	Comparison of high-level SDL-ability learners' prediction and actual achievement in Question 5.....	112
Figure 4.22:	Comparison of low-level SDL-ability learners' prediction and actual achievement in Question 5.....	113
Figure 4.23:	Comparison of high level SDL ability learners' prediction and actual achievement in Question 5.....	113

LIST OF TABLES

Table 1.1:	The experimental design	6
Table 2.1:	Principles for Strengthening and Weakening Behaviour (Surgenor., 2010)	16
Table 2.2:	Integrate Staged Self-Directed Learning Model (SSDLM) (Adapted from Grow, 1991:129) & Self-directed Learning Skills (Adapted from Francom, 2010:30-45)	41
Table 3.1:	Interrelationships among Teachers' Goals, Beliefs, and Actions Regarding Problems and Problem-Solving (Rigelman, 2007:313).....	60
Table 4.1:	The experimental design	69
Table 4.2:	Examples of items from the SDLI-questionnaires.....	71
Table 4.3:	Examples of items from the adapted LASSI-HS questionnaire	72
Table 4.4:	Reliability-coefficients (Cronbach's Alpha) for the four domains of the SDLI	74
Table 4.5:	Reliability coefficients (Cronbach's Alpha) for the five fields of the LASSI-HS	75
Table 4.6:	Differences between the domains of SDL-ability of the experimental and control groups prior to the intervention	76
Table 4.7:	Differences between the SDL-ability domains of the experimental and control groups after the intervention	77
Table 4.8:	Comparison of the self-directed learning abilities of the experimental group	79
Table 4.9:	Comparison of the self-directed learning abilities of the control group	80

Table 4.10:	Comparison between the selected LASSI-HS (factors influencing self-directed learning) of the experimental and the control groups prior to intervention.....	81
Table 4.11:	Comparison between selected LASSI-HS fields (factors influencing self-directed learning) of the experimental and the control groups after the intervention.....	82
Table 4.12:	Comparison of the pre- and post-tests of the experimental group with respect to the factors influencing self-directed learning	84
Table 4.13:	Mathematical achievements of the experimental and control groups.	85
Table 4.14:	Changes in the mathematical achievement of the experimental- and control groups.....	85
Table 4.15:	Selection of participants for the task-based activities	87
Table 4.16:	Summary of the categories of the task based activities.....	90
Table 4.17:	Participants' predictions to Question 1 in the task-based activities.....	93
Table 4.18:	Responses of participants to number 3 for Question 1	93
Table 4.19:	Participants' predictions to Question 2 of the task-based activities	98
Table 4.20:	Responses of participants to number 3 for Question 2	98
Table 4.21:	Participants' predictions to Question 3 in the task-based activities.....	103
Table 4.22:	Responses of participants to number 3 for Question 3	103
Table 4.23:	Participants' predictions to Question 4 in the task-based activities	107
Table 4.24:	Responses of participants to number 3 for Question 4	107
Table 4.25:	Participants' predictions to Question 5 of the task-based activities	111
Table 4.26:	Responses of participants to number 3 for Question 5	111

CHAPTER 1:

ORIENTATION AND PROGRAM OF STUDY

1.1 Introduction and problem statement

South Africa participated in the Trends in Mathematics and Science Study (TIMSS) in 1995, 1999 and 2003, but not in the study conducted in 2007 (Long, 2008:1). Sceptics accused the then Minister of Education of opting out of the study in 2007 owing to consecutive poor results obtained during previous studies. The value or purpose of TIMSS for Mathematics education is that teachers can learn from best international practices that may lead to the improvement of teaching and learning of Mathematics (Unal & Jakubowski, 2007:62; Long, 2008:2). A further purpose of an external assessment is to identify problem areas so that resources and funds can be distributed to these areas (Long, 2008:2). Problem areas identified in South African schools were reading and numerical skills, or rather the lack thereof (DoE, 2011:6). Therefore, the Department of Education implemented Foundations for Learning in 2008, which involves getting more learning and teaching materials to schools and instructing teachers how to implement the curriculum (DoE, 2011:10).

However, there was and still is a belief regarding the South African schooling system that indicates that learners, including Mathematics learners, perform well below expected levels or potential (DoE, 2011:8). Hence, the government introduced Annual National Assessment (ANA) in 2011 to assist schools and school districts to achieve the goals set by the National Department of Education. Learners should be better prepared by schools to read, write, think critically and solve numerical problems, because these skills are important foundations on which further studies, job satisfaction, productivity and meaningful citizenship are based (DoE, 2011:8).

For purposes of this study the researcher of this dissertation focused only on the Grade 3 and Grade 6 Mathematics results released by the Assessment Drive of 2011. The Mathematics external paper in these 2 grades focused on three cognitive areas, namely: knowledge of basic concepts (20%), non-routine problem-solving (20%) and application of concepts (60%). The results indicated that the South African schooling system is really in dire straits, the average Grade 3 result for numeracy was 28% and for Grade 6 was 30% (Department of Basic Education, 2011:13). The DoE is aiming for a 60% average by the year 2014 in these particular grades (DoE, 2011:5). In order for the DoE to achieve this percentage, the mandate for teachers is to improve critical thinking and solving of numerical problem skills needed by learners to improve their overall performance in Mathematics.

From the above arguments it should be clear that the same difficulties experienced by Grades 3 and 6 will be experienced by the Grade 7 learners. Over the last eighteen years the researcher has experienced, in Grade 7 Mathematics classes, that learners are struggling with self-directed learning aspects such as self-monitoring, self-management, self-discipline and self-confidence (Garrison, 1997:18) and successful completion of problem-solving. That led me to the question: How can I assist the learners in my class to overcome this situation?

Self-directed learning (SDL) is defined as a lifelong learning experience, by means of which the learner takes control over and assumes responsibility for his/her own learning and learning experiences (Knowles, 1975:16). Knowles (1975:17) states that self-directed learning is the ability of humans to learn on their own. This construct (SDL) of Knowles (1975) has become the foundation of research with respect to self-directed learning. Hiemstra (1994) views self-directed learning as any form of study in which an individual assumes primary responsibility for planning, implementing and evaluating a learning activity.

Self-directed learning (Garrison, 1997:20) invokes both cognitive (independent and critical thinking) and social issues, which lead to “self-direction” and “learning” respectively. Independent thinking involved in SDL (Garrison, 1997:18) relates to the fact that a good deal of independence in thinking is required in deciding what to learn and how to approach the learning task. The critical thinking construct reflects the complex cognitive processes associated with constructing personal meaning and worthwhile knowledge through understanding (Garrison, 1997:21). Self-directed learning is also a collaborative approach to construct and confirm meaningful (cognitive) and worthwhile (social) learning, where the individual assumes responsibility for constructing his/her meaning to knowledge and to make the constructed knowledge meaningful and worthwhile. Due to the fundamental features of Mathematics, learners are required to think more independently and/or critically to enhance Mathematics learning (Cheng, 2011:78). In addition, Mathematics learners need to improve their problem-solving skills to improve their achievement in Mathematics (DoBE, 2011:8).

Han and Teng (2005:1) as well as Kirk (2003:923) state that the rapid social changes and development in the world have resulted in an increasingly higher demand for more effective teaching approaches and methodologies. In a search for more effective teaching methods, problem-solving has emerged as a strategy that promotes self-directed learning (Han & Teng, 2005:1).

Since Polya formulated his problem-solving framework in 1957, research on mathematical problem-solving has largely been based on this framework (Lesh & Zawojewski, 2007:763). Mayer (1983:3) defines problem-solving as a multiple-step process by means of which the learner or problem solver has to find a correlation between past experiences and the given problem, in an attempt to find a solution. Schoenfeld (1992:352-353) refers to problem-solving in Mathematics as follows: Students have to solve unfamiliar problems by reading through the problem, analysing the problem, and examining and evaluating their own mathematical knowledge in order to come up with a solution or answer. With the assistance of the teacher who asks meta-cognitive questions, for example “Why are you doing this? How will it help you?”, the student is able to make new connections, to reorganise existing knowledge and to construct new knowledge.

According to Kirkley (2003:2-3), problem -solving skills depend on mastering basic literacy skills and mathematical concepts. Learners often learn facts, concepts and rote procedures with few connections and applications to knowledge or tasks. A task that can be a simple exercise for one person may prove to be much more complicated and testing for another (Goos *et al.*, 2000:2). Therefore the problem does not exist in the task itself, but in the relationship between the task and the problem solver (Smith & Confrey, 1991). For problem-solving to succeed, learners have to be willing to solve problems, as well as believe that they can solve unfamiliar problems (Kirkley, 2003:7). Aspects such as effort (self-discipline), confidence, persistence, and knowledge of the self are important qualities for the problem-solving process (Kirkley, 2003:7). Therefore, a learner has to be self-directed in his/her learning to be a successful problemsolver.

1.2 Research questions

The central research question of this study was: What is the relationship between problem- solving and self-directed learning in Grade 7 Mathematics classrooms? In an attempt to answer the research question, I explored the following research questions:

1.2.1 What is the influence of problem-solving activities on Grade 7 Mathematics learners’ self- directed learning abilities?

1.2.2 What is the influence of self-directed learning, through problem-solving, on learners’ mathematical achievement?

1.3 Literature study

The exposition of the literature study is done in Chapters 2 and 3. In Chapter 2 the focus is on self-directed learning, beginning with approaches to learning Mathematics, followed by a brief

history on self-directed learning; then factors that influence self-directed learning; next self-directed learning ability is defined, components of self-directed learning are discussed, as well as the Mathematics teachers' role during self-directed learning and advantages of self-directed learning, and finally it concludes with self-directed learning strategies.

In Chapter 3 the emphasis is specifically on mathematical problem-solving, beginning with definitions of problem-solving by different authors, discussing models of problem-solving, identifying characteristics of a mathematical problem solver, describing the role of a teacher in teaching mathematical problem solving and identifying and discussing factors influencing mathematical problem-solving. In conclusion, a theoretical relationship between SDL and problem-solving is established and the implications of mathematical problem-solving for both the teacher and the learner are discussed.

1.4 Research design and methodology

Paradigms are seen as opposing world views or belief systems that are a reflection of and guide the decisions that researchers make (Tashakkori & Teddlie, 1998). The nature of the study called for a pragmatic approach to be used. Pragmatism evolved due to disputes between two different paradigms, namely the positivist and the interpretivist paradigms (Johnson & Onwuegbuzie, 2004:18).

Positivism is associated with a quantitative approach (Fraenkel & Wallen, 2008:423). Positivism suggests that the "positive stage" of knowledge is reached when people (researchers) rely on empirical data, reason, and the development of scientific laws to explain phenomena. Positivists believe that the researcher should be objective and should not become involved in the phenomenon being studied. Interpretivism is associated with qualitative approaches; it is well-suited for the social sciences because of the emphasis on human actions and experiences (Fossey *et al.*, 2002:718).

For purposes of this study a sequential explanatory mixed-method research design was used (Ivankova *et al.*, 2006:4) (see Figure 1.1). The reason for the selection of this research method was to use the qualitative findings to assist in contextualising or interpreting the quantitative results (Ivankova *et al.*, 2006:3). During the first phase (pre-test) the researcher collected and analysed quantitative data; during the second phase an alternative teaching method was introduced (problem-solving teaching) and qualitative data were collected and analysed, and during the last phase (post-test) quantitative data were collected and analysed.

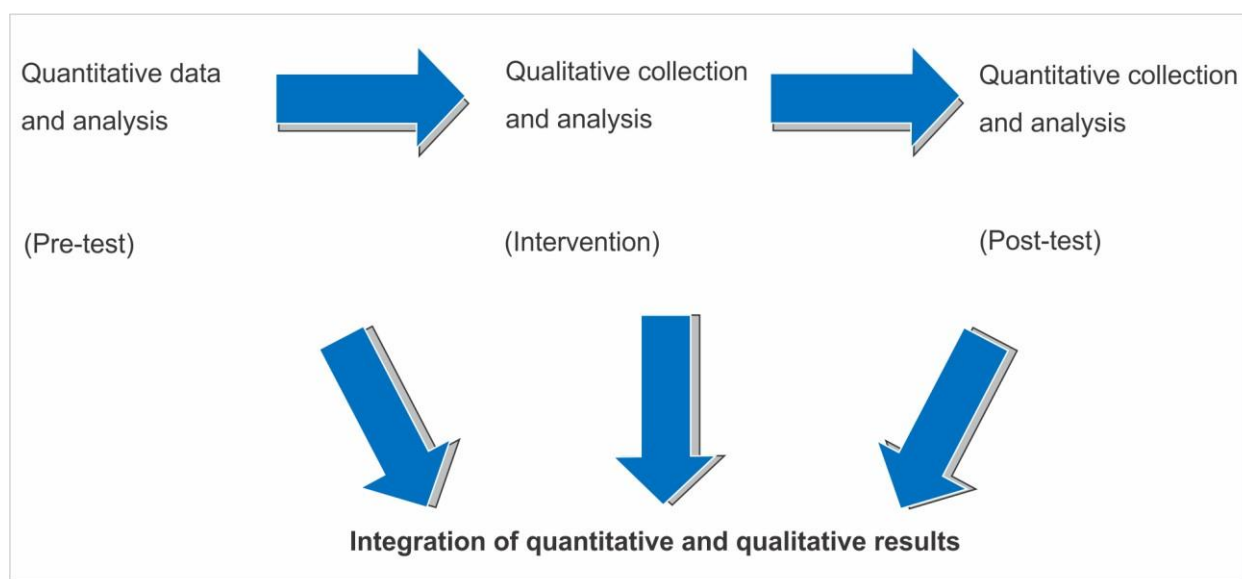


Figure 1.1: A mixed-method sequential explanatory design (Ivankova, Creswell & Stick, 2006:16)

1.4.1 Quantitative research method

Quantitative research is defined by Maree (2007:145) as a systematic and objective process in which numerical data of a selected group is used to make the results specific for that selected group.

The self-directed learning ability (see 2.3) of Grade 7 Mathematics learners was measured at the beginning of the study and after the intervention (a problem-solving approach to teaching and learning), using a self-directed learning instrument (SDLI) (see Appendix D). The March report mark of both the experimental and control groups were used as a pre-test score. Because the mark was compiled by a combination of continuous assessments, it could be used as a reliable source of the learners' mathematical achievement before the intervention. A change in mathematical achievement could be detected by comparing the November (after the intervention) report marks with the March report marks.

The rationale for the pre-test, post-test experimental design (Leedy & Ormrod, 2005:225) (see Table 1.1) was to determine the effect of a problem-solving approach to teaching and learning (the intervention) on Grade 7 learners' self-directed learning ability as well as on their mathematical achievement. The experimental design is represented in **Table 1.1**, where T_x refers to the intervention period (implementation of problem-solving) and (-) refers to the period during which no intervention took place.

Table 1.1: The experimental design

	Pre-test	Intervention	Post-test
Experimental group:	1) Self-Directed Learning Instrument (SDLI)	T_x	1) Self-directed Learning Instrument
School A	2) Selected fields of the LASSI (HS)	Implementation of a problem-solving approach to teaching and learning	2) Selected fields of the LASSI (HS)
(n=163)	3) March report results (Mathematics)		3) November report results
Control group:	1) Self-Directed Learning Instrument (SDLI)	No intervention	1) Self-directed Learning Instruments (SDLI)
School B	2) Selected fields of the LASSI (HS)		2) Selected fields of the LASSI (HS)
(n=154)	3) March report results		3) November report results

The independent variable for this study was a problem-solving approach to teaching and learning, while the dependent variables were self-directed learning abilities (see 2.4), factors influencing self-directed ability (see 2.3.2) and mathematical achievement of the Grade 7 learners. Other factors that could have influenced the study were the application of teaching strategies, the mathematical content, and the degree of difficulty of the mathematical tasks.

Any changes in the dependent variables could be attributed to the independent variable (Lincoln & Guba, 1985:290). The internal validity of the study was ensured by controlling the variables as far as possible. However, different Mathematics teachers, as well as the degree of the difficulty of tasks are relative variables that could not be controlled.

1.4.1.1 Study population and sampling

The population group was all the grade 7 mathematics classes in a city in the North-West Province, while the research study took place in two selected primary schools because of its convenience and accessibility. Marshall (1996:523) described convenience sampling as the least rigorous

sampling technique, which involves the most accessible subjects.

There were four Grade 7 classes at both schools. Both schools are bilingual and one quarter of the learners in both schools is Afrikaans speaking, while three quarters of the learners have English as first language at school (home languages: Setswana; isiXhosa and isiZulu). This could have an influence on the completion of the self-directed learning instrument (SDLI) questionnaire (see Appendix D); learners completed the questionnaire in English (learners with English as Home Language classes) and Afrikaans (learners with Afrikaans as Home Language) as well as the completion of the LASSI (HS) (only selected fields) (see Appendices B and C).

1.4.1.2 Measuring instruments and data collection

Learners' self-directed abilities were measured with an adapted self-directed learning instrument of Cheng *et al.*, (2010:1157). The instrument consists of 20 questions distributed over four domains, namely learning motivation (Questions 1 to 6), planning and implementation (Questions 7 to 12), self-monitoring (Questions 13 to 16), and interpersonal communication (Questions 17 to 20) (Cheng *et al.*, 2010:1156). A detailed discussion on the four domains of the self-directed learning instrument (SDLI) (see Appendix D) is done in Chapter 2 (see 2.4).

Selected fields of the LASSI-HS, namely motivation, concentration, the use of study aids (resources), time-management and self-testing strategies (see Appendices B and C) were also used as a pre-test as well as a post-test instrument. The mentioned fields of the LASSI correspond to a great extent with the components of self-directed learning as defined by Cheng *et al.* (2010:1153). The rationale for using only specific selected fields will become clear in Chapter 2 (see 2.4).

For any data-collection procedure to be effective, it is important that it should comply with the principles of reliability and validity. Reliability refers to the degree of consistency of the instrument - the results have to be similar when the instrument is used under similar circumstances (Joppe, 2000:1). Validity refers to the strength of the conclusions, inferences and propositions. Validity is the extent to which the instrument measures what it intends to measure (Joppe, 2000:1)

Continuous assessment and examination papers moderated by the Mathematics subject specialists were used as measuring instruments for learners' mathematical achievement (see Table 1.1). The March report marks of both schools consist of the continuous assessment of the first quarter as well as the March examination (control tests) mark. The March report mark is a reliable reflection of the learners' Mathematics achievements and was used as pre-test. The November report marks of both schools consisted of the continuous assessment of the last

quarter as well as the November examination mark. The November report mark was a reliable reflection of the learners' Mathematics achievements and was used as a post-test. Grade 7 learners from both schools (experimental and control groups) wrote the same examination papers.

1.4.1.3 Data analysis

The data of the questionnaires was analysed by the Statistical Consultation Services of the North-West University (Potchefstroom).

Descriptive statistics: Calculations of averages, standard deviations, frequencies and percentages were done for both the pre- and post-tests. Descriptive statistics makes it possible for the researcher to use the information that occurs in the data-sets in a meaningful way by calculating averages, standard deviations, frequencies and percentages (Creswell, 2009:152). Practical meaningfulness (d-values) and statistical meaningfulness (p-values) were calculated to determine possible differences between the groups in the study population (see Table 4.6).

Factor-analysis: It was done in order to establish the construct validity of the measuring instruments (the SDLI and LASSI-HS) as well as the construct validity of the SDLI components and of the selected fields of LASSI-HS (see Table 4.7). A correlation was established between the different measurable variables.

Reliability coefficient (Cronbach Alpha coefficient): It was used to determine the internal consistency of the items in each component of the SDLI and selected fields of the LASSI-HS respectively (see 4.3.1.4).

1.4.2 Qualitative research method

A qualitative investigation formed the second phase of the empirical study, determining Grade 7 learners' experiences during problem-solving activities (the intervention).

Lincoln and Guba (2000:3) see qualitative research as naturalistic and interpretive, in the sense that studies are done in natural settings, attempting to make sense of or interpret phenomena in terms of the meanings people attach to them. The aims of qualitative research are to describe and understand human behaviour (Babbie & Mouton, 2009:53). In this study results from the problem-solving activities completed by selected learners, were supposed to provide insight into the results from the quantitative relationship between problem solving and self-directed learning (see 1.2.1).

1.4.2.1 Data generation and participants

Data from learners' experiences during the use of a problem-solving approach to teaching and learning, was collected by means of structured task-based activities (see Appendix I) completed by individual learners from the experimental group. Learners were grouped into three categories according to their SDLI ability, namely (a) high level of ability, (b) medium level of ability and (c) low level of ability. Five learners from each of these groups were randomly selected to participate in the completion of the mentioned activities.

In the task-based activities (see Appendix E), the selected learners completed mathematical problem-solving tasks. The structured tasks were supplemented with open-ended questions that give possibilities for expansion or feedback. The activities provided the researcher with the opportunity to focus on how learners solved mathematical problems (Evens & Houssart, 2007:20). The aims of the open-ended questions were to grant the learners the opportunity to put into words their solutions to the questions and to reflect on the possible solutions they have provided.

1.4.2.2 Data analysis

Task-based problem-solving activities were conducted. Five (5) learners from each of the selected groups per class of the experimental group were selected. The questionnaire has two multi-choice questions, namely number 2 (understanding the problem) and number 5 (looking back). Number 3 (making a plan) asked the learners to give an explanation of what he or she intended to do concerning the stated problem (number 1 – understanding the problem). In number 4 (solve the problem) learners' were expected to solve the problem. Number 6 (looking back) referred to strategies that learners' have used to solve the problem. Number 7 (looking back) asks learners what type of mistakes they made in solving a similar problem as stated in number 1.

The different categories were compared with one another regarding the responses of the learners. Polya's model of problem-solving was used as a basis for the analysis of the task-based activities (see 4.3.3.5.1).

1.5 Procedure of the empirical investigation

After the learners have completed the questionnaires (see SDLI and LASSI-HS) during the **first phase** (see Figure 1.2) of the investigation, five learners on each ability level of the experimental group are selected as follows: a) high level of SDLI ability, b) average level of SDLI ability and c) low level of SDLI ability.

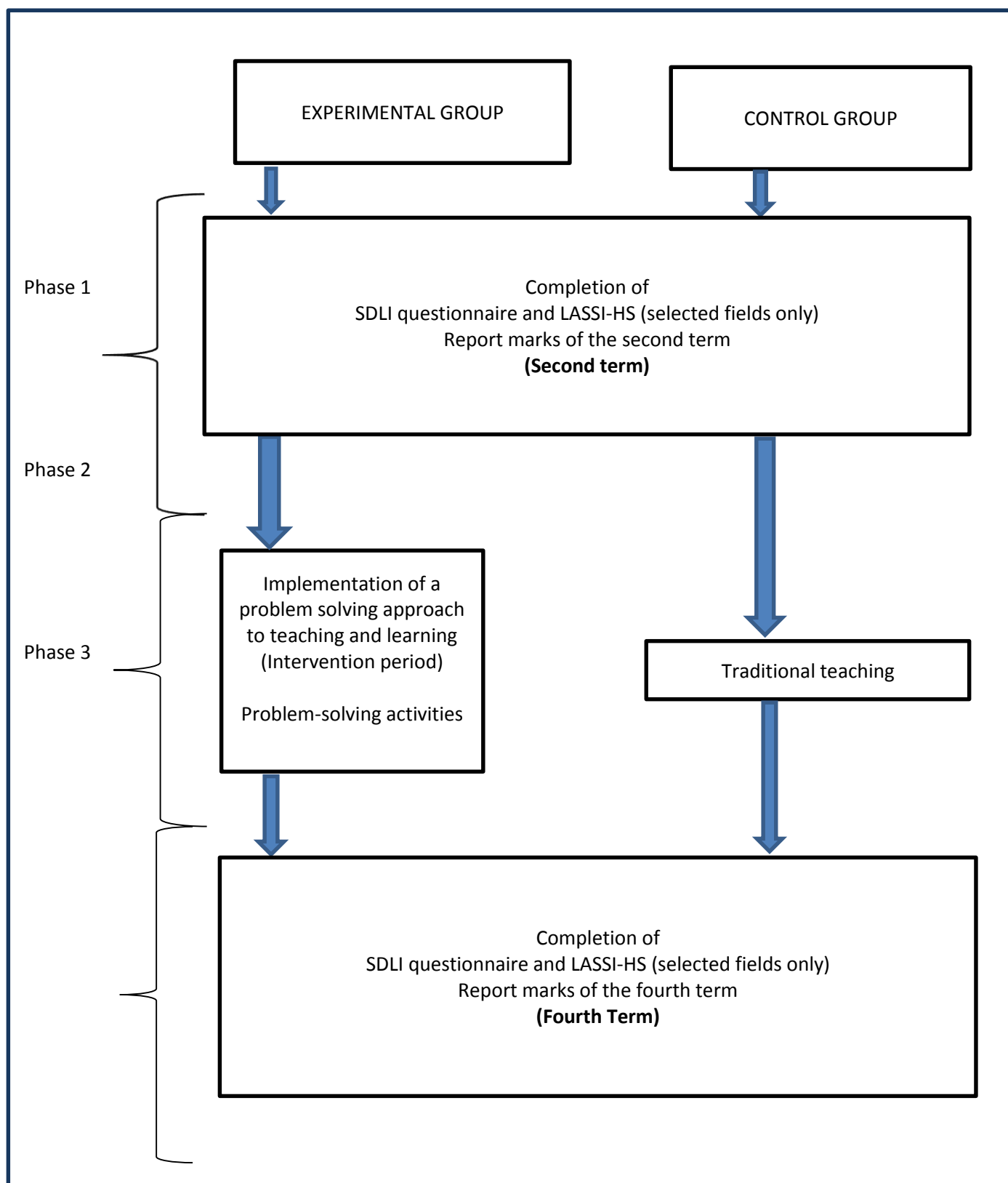


Figure 1.2: Procedure of the empirical investigation

The **second phase** of the investigation is the intervention (the use of a problem-solving approach to the teaching and learning of mathematics), while the control group is taught in a traditional way.

Mathematics content is handled in large group presentations during single periods, and double periods were used for teaching mathematical problem-solving. During double periods the learners were asked to work individually on solutions (understanding the problem, making a plan, carrying out the plan, looking back) to the mathematical problems (peer assistance was encouraged for learners who were struggling). At the end of the second phase learners completed the task-based activities and the data were manually analysed.

During **phase three** (see Figure 1.2) both the control and experimental groups completed the second set of questionnaires (post-test) in the fourth term. Both groups also completed a common Mathematics examination paper that was set up by the SES (Subject Education Specialist). Final Mathematics report results were analysed to establish whether the intervention in the experimental group had led to greater or better achievement than the control group.

1.6 Contribution of this study

After this study the researcher hopes:

- to add to the local and international literature, with regard to the self-directed learning ability and problem-solving strategies implemented by Grade 7 Mathematics learners; and
- to identify, analyse and describe learners' self-directedness and problem-solving strategies in an attempt to provide educators with guidelines to assist learners improving their mathematical achievements.

1.7 Limitations of the research study

This study was conducted in two schools in one suburb in the North West Province; hence the conclusion cannot be generalised. The study was done in a limited time period with 163 Grade 7 Mathematics learners. The research results were interpreted against the background of the research theme.

Different researchers can approach the same study from different perspectives and interpretation and might draw different conclusions. On the flip side, different researchers can use different research tools to investigate the same theme and might record different findings or draw different conclusions.

1.8 Ethical aspects of the research

The NWU Ethics application form was completed and submitted to the Ethics Committee at the

University. After ethical clearance had been obtained for the study, the researcher requested permission for conducting this study from the North West District Education Office of the Department of Education, as well as from the management teams of the participating schools and the parents of the participating learners. Participants' involvement was voluntary and they could withdraw at any stage. Participants and other stakeholders, such as the DoE, school managements of participating schools and parents were informed about the aims and objectives of this study (Cohen & Manion, 1994:89). The responses of the participants were treated as confidential and their identities were not revealed during the investigation or the report writing. The schools' names were also kept confidential to ensure participants' trust not to be lost during the research process (Cohen *et al.*, 2007).

1.9 Structure of the dissertation and overview

Chapter 1: Orientation and program of study.

Chapter 1 contains the introduction as well as the problem statement of this study, followed by research questions. An overview of the empirical study is also included where I discussed the research design and methodology, as well as the procedure of the empirical investigation.

Chapter 2: Self-directed learning in the Mathematics classroom.

Chapter 2 contains a brief history of SDL in Mathematics, definitions of SDL, characteristics of a self-directed learner, factors influencing self-directed learning, components of self-directed learning, the roles of the Mathematics teacher and of the learners during facilitation of SDL.

Chapter 3: Problem-solving in the Mathematics classroom

Problem-solving and the learning and teaching of Mathematics with specific reference to different definitions from literature of problem-solving as well as different models of problem-solving are discussed. There is also a discussion on characteristics of a problem solver and factors influencing problem-solving, the role of the Mathematics teacher during problem-solving, theoretical relationship between problem-solving and self-directed learning, and finally implications of problem-solving for teachers and learners.

Chapter 4: The influence of teaching of problem-solving activities on Grade 7 Mathematics learners' self-directed learning abilities and their mathematical achievement

Chapter 4 comprises the empirical investigation and discussion of data and information collected and analysed during the study. The discussion of data and information on the empirical

investigation took place by means of the implementation of a consecutive declaratory method-research-design, which is a combination of quantitative and qualitative research methods.

Chapter 5: Summary, conclusion and recommendations

In the last chapter the findings of the study are summarised, conclusions are drawn concerning the investigation and recommendations are made for further studies.

CHAPTER 2:

SELF-DIRECTED LEARNING IN THE MATHEMATICS CLASSROOM

2.1 Introduction

Chapter 1 was dedicated to the problem statement and research procedures regarding self-directed learning and problem-solving in Mathematics classrooms. In Chapter 2 the focus is on approaches to the learning of Mathematics. A definition is given of self-directed learning, factors influencing self-directed learning, components of self-directed learning, as well as the role of the teacher and of the learners during facilitation of self-directed learning and self-directed learning strategies.

2.2 Approaches to learning Mathematics.

The notion of *what is learning?* should first be understood. Learning is characterised by Jonassen (2000:2) as an activity within intentional and interconnected activity systems. Conscious learning and activity (action) are interdependent and interactive, which means one cannot act without thinking or think without acting (Jonassen, 2000:2). Therefore, in order to think and learn, it is necessary to act. Teachers need to model learning techniques such as predicting, questioning, clarifying and summarizing in order for learners to develop the ability of following these techniques or strategies on their own. Learners need to be allowed to approach tasks in different ways using different strategies (Many, *et al.*, 1996).

Learning approaches are behaviours or thoughts that affect the way in which learners select, acquire, organise and integrate new knowledge. Graaff (2005:12) and AeU (2011) state that the learning approaches that have the most influence on successful Mathematics learning are the behaviouristic, cognitive and the constructivist approach.

2.2.1 The behaviouristic approach

The importance of quantifiable and observable performance and the influence of the learning environment form the basis of the behaviouristic approach to learning. According to the behaviourist approach, learning is a persistent change in performance or performance potential which is the consequence of interacting with and experiencing the world (Driscoll, 2000:3). This approach implies that certain behavioural responses are related to specific environmental stimuli (concept of association). A stimulus is anything that can directly influence behaviour, and the stimulus produces a response.

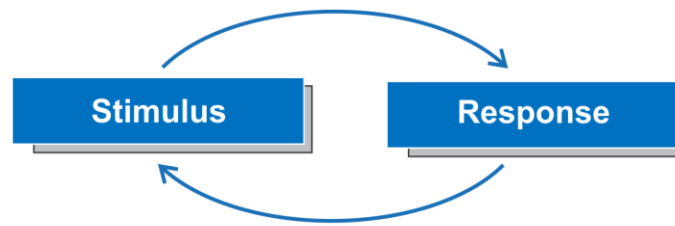


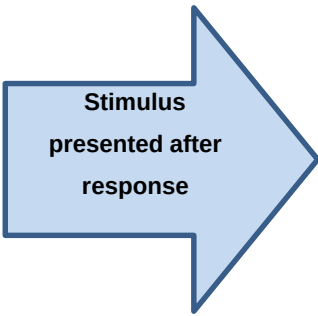
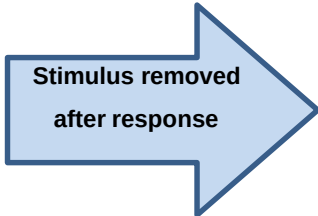
Figure 1.1 Stimulus and response diagram

Behaviourists see learners as passive individuals who respond to stimuli. According to them, learners start as a clean slate (*tabula rasa*) and the behaviour of the learners is shaped by reinforcement (Learning-Theories Knowledgebase, 2011).

Slavin (2006) extends this concept of association of a stimulus and response in the sense that he argues that behaviours are more likely to re-occur when they are reinforced or rewarded. Morris and Maisto (2001) point out that behaviour which is reinforced is fitting to be performed again, whereas behaviour that brings about punishment is bound to be blocked. If a learner, for example, often encounters unpleasant stimuli in the Mathematics class, such as unfriendly teachers, difficult questions and a great deal of homework, that learner may learn to dislike Mathematics.

Behaviourist techniques can be used to strengthen or weaken behaviour, as well as to maintain or teach new behaviour. The information presented below, based on Driscoll (2004:3), outlines the principles associated with each.

Table 2.1: Principles for Strengthening and Weakening Behaviour (Surgenor., 2010:6)

	Strengthening Behaviour	Weakening Behaviour
	<u>Positive Reinforcement</u> Following up on the occurrence of the appropriate behaviour the person receives reinforcement which results in the strengthening of that behaviour. E.g. Learners receive a high grade for effort in Mathematics assessment	<u>Punishment</u> The presentation of a negative stimulus following on an unwanted behaviour. E.g. learners fail an assessment task for not putting in enough effort
	<u>Negative Reinforcement</u> This refers to an individual working to avoid the occurrence of a negative stimulus (often confused with punishment). E.g. Learner is rewarded for continuous excellent work in Mathematics	<u>Reinforcement Removal</u> Reducing the frequency of behaviour by removing reinforcement when it occurs. <i>E.g. Percentage of grade allocation removed for poor spelling or grammar</i>

This approach is still applicable to Grade 7 Mathematics with respect to the learning of addition and multiplication facts. In schools, this approach is viewed as rote learning and repetition. For example, Grade 7 learners must be able to recall the multiple tables of twelve without hesitation.

2.2.2 The cognitive approach

The cognitive approach was a response to the behaviourist learning approach. The cognitive approach focuses on how people think, understand and know things; this approach emphasises learning on how people grasp and embody the outside world within themselves and how their ways of thinking influence their behaviour (AeU, 2011:106). According to the cognitive learning approach, learning occurs when new knowledge is learned or current knowledge is altered by experience. The teacher can apply a multitude of techniques such as insightful learning, meaningful learning, scaffolding, and techniques of memorising devices such as mind- mapping and reminders as means to assist learners (AeU, 2011:107).

The process of how people think, know and understand is captured in an Information Processing Theory (IPT) (AeU, 2011:113). Information processing theorists propose a human mind as a system that processes information through the application of rational rules and strategies. The mind has a limited capacity for the amount and nature of the information it can process. Most IPM (Information Processing Model) theorists see the computer as only a metaphor for human mental activity. The Information Processing Model as illustrated in Figure 2 emphasises the significance of “encoding” (input) of information, the “storage” of information, and the “retrieval” (access) of information which is a “powerful” analogy between the working of the mind and how computers work (Neisser in Tan *et al.*, 2003).

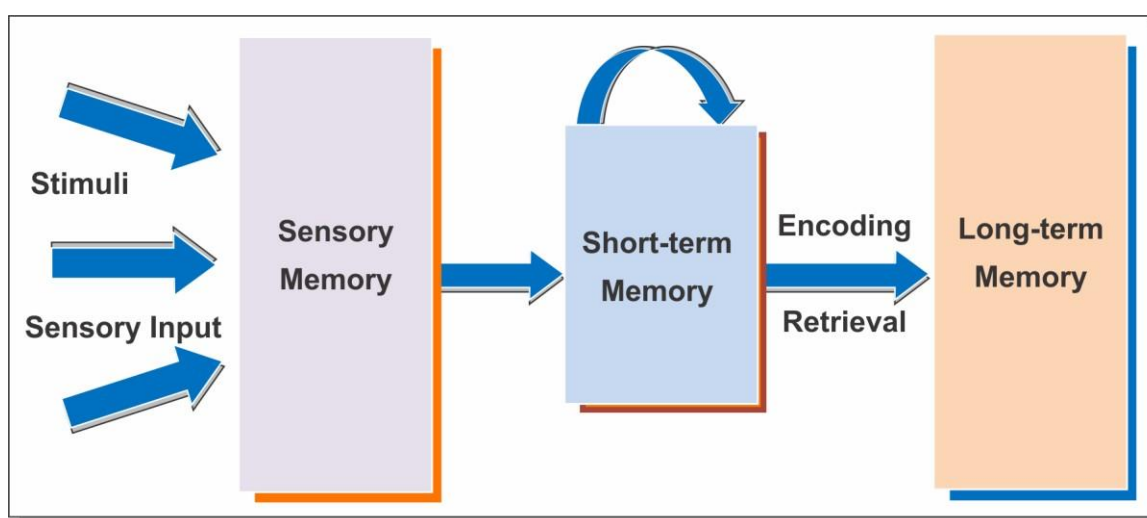


Figure 2.1: The Information Processing Model (adapted from AeU, 2011:113).

“Just as the computer can be made into a better information processor by changing its hardware and its software (programming), so do learners who become more sophisticated thinkers through changes in their brains and sensory systems (hardware) and in the rules and strategies (software) that they learn.” (AeU, 2011:113).

The cognitive approach proposes that the human memory involves three processes, which are sensory memory, short-term memory and long-term memory. During the first process the stimuli enter our sensory memory, containing receptors that hold on to information which enters through our senses for a short while (AeU, 2011:116). For example, the graphical system holds the iconic memory for graphic stimuli such as mathematical shapes, colour and location. The second process is the short-term memory, which can be seen as a temporary storage system. Short-term memory is created by paying attention to external stimuli and internal thinking, or both. Short-term memory relates to things we are thinking about at any given moment; it is also called the working memory. Long-term memory is the third process during which information can be stored for a few

minutes, up to a lifetime. The long-term memory has a limitless amount of space for storing information (AeU, 2011:116).

2.2.3 The constructivist approach

Constructivists give precedence to individual learners' sensory-motor and conceptual activity referring to learners' experiences and valuing their interests. This implies a focus on learners' qualitative interpretations and personal goals which they pursue in the classroom. It further implies mathematical learning as an active construction of knowledge (Cobb, 1994:14).

Cobb and Yackel (1996:178) state that the constructivist approach cannot be divorced from the social interaction that learners experience in the Mathematics classroom. The teacher is seen as the giver of instructions or classroom authority and the learners are seen as contributing to the social norm of the Mathematics classroom.

The constructivist approach encourages the use of appropriate learning strategies (see 2.3.2.2.2) as a major factor in effective learning (Anthony, 1996:23). Unlike the cognitive approach where the focus is only on cognitive involvement during learning, the constructivist approach refers to an active learning process in which learning is understood as a self-directed process of resolving inner conflict (Anthony, 1996:23).

Constructivism is a psychological and philosophical perspective, illustrating what the individual constructs of what he/she learns and understands (Brunning *et al.*, 2004). For many Mathematics teachers the constructivist approach captures the essence of learning. (Alenezi, 2008:18). Van de Walle *et al.* (2013:22-23) recommend that learners need to be actively involved in their own learning experiences, for learning takes place in a specific context and the learners' cognition is formed by experiences gained in that specific context. According to the constructivist approach, knowledge has to be constructed by learners in their own minds, implicating that teachers cannot simply transmit or give learners new knowledge.

For purposes of this study learning is seen as knowledge which learners construct through problem-solving as well as the application of knowledge in real-life situations, enabling life-long learning. Applying learning strategies enables learners to assume more responsibility for their own learning, and to become lifelong learners (Weinstein *et al.*, 2011:45).

2.3 Brief history of self-directed learning

Self-directed learning (SDL) is defined as a lifelong learning experience, during which the learner takes control over and assumes responsibility for his/her own learning and learning experiences (Knowles, 1975:16). Knowles (1975:17) states that self-directed learning is the ability of humans to learn on their own. This construct (SDL) of Knowles (1975) has become the foundation of research with respect to self-directed learning.

Knowles (1975:21) lists two types of learning, namely teacher-directed learning as “pedagogy” and self-directed learning as “andragogy”. A self-directed learning (andragogy) approach assumes that learners do not need an instructor to instruct on how and what to learn; the learners are able to set learning goals for themselves by negotiation. It is also assumed that self-directed learners are mature in their learning and their experiences ensure or provide a basis for learning. Self-directed learners’ keenness for learning is a result of how they approach the problems they encounter in their individual lives. Self-directed learners also view learning from a task-orientated or problem-orientated perspective. These learners prefer to learn through problem-solving (see 3.2) rather than through subject content. Self-directed learners are motivated by internal incentives such as the desire to achieve, the satisfaction of accomplishments and the need for specific knowledge.

Teacher-centred learners, on the other hand, depend more on text books; they rely on different levels of maturation for learning. These learners furthermore demonstrate a preference for sequence and structure and are motivated by external rewards such as awards, grades and degrees. According to Knowles (1975:21), self-directed learning or teacher-centred learning does not necessarily have to be good or bad. Some learners have a preference for some aspects of teacher-centred learning and others for some aspects of self-directed learning.

Knowles (1975: 18) further contends that self-directed learning is a process in which individuals take initiative, diagnose their own learning needs, formulate goals, identify resources, choose and implement appropriate learning strategies (see 2.3.2.2.2), and evaluate their learning outcomes; with or without the help of others (including teachers). Knowles (1975, 1990) also suggests that learning does not take place in isolation, but in association with others such as teachers, tutors and peers. Therefore learning can be placed on a scale ranging from teacher- directed learning or others on the one end, to self-directed learning on the other. In SDL, control steadily shifts from the teacher to the learner; learners apply a great deal of independence in setting learning goals and deciding what is valuable to learn as well as how to approach the learning task (Morrow, Sharkley & Firestone, 1993).

Guglielmino (1977:73) describes a self-directed learner as someone who exhibits initiative, independence and persistence in learning. A self-directed learner is goal-orientated, assumes responsibility for his/her own learning and views problems as challenges. In addition, a self-directed learner is self-disciplined, self-confident, displays a high degree of curiosity, and has a desire to learn. Furthermore, a self-directed learner manages time successfully, uses basic study skills, and sets an appropriate pace for learning.

Pintrich and De Groot (1990:33) see self-directed learning as a combination of cognitive activities based on beliefs. Learners who are more self-directed have the tendency to use more cognitive strategies and are more likely to persist in completing a task or assignment than learners who do not believe in themselves regarding the task at hand (Pintrich & De Groot, 1990:34). Therefore learners with a low level of self-belief will put in less effort and less energy and will not experience a high success rate in completing tasks (see 2.3.2.2.1). In today's sophisticated world, however, learners have to become self-reliant, self-confident and self-directed in their own learning (Guglielmino & Long, 2011:1).

Hiemstra (1994) regards self-directed learning as any form of study in which an individual assumes primary responsibility for planning, implementing and evaluating a learning activity. Self-directed learning (Garrison, 1997:20) invokes both cognitive (independent and critical thinking) and social issues, which leads to "self-direction" and "learning" respectively. Independent thinking involved in SDL (Garrison, 1997:18) relates to the fact that a good deal of independence in thinking is required in deciding what to learn and how to approach the learning task (see 2.3.2.2.1). The critical thinking construct reflects the complex cognitive processes associated with constructing personal meaning and worthwhile knowledge through understanding (Garrison, 1997:21). Self-directed learning is also a collaborative approach to construct and confirm meaningful (cognitive) and worthwhile (social) learning, where the individual assumes responsibility for constructing his/her meaning to knowledge and to make the constructed knowledge meaningful and worthwhile.

Fisher *et al.* (2001:516) state that the assumption is made that adults are inherently self-directed. Adults are expected to know or have an idea of what they would like or want to achieve and will therefore find ways and means (strategies) to achieve their goals. Self-directed learning is a method of instruction that is increasingly used in tertiary education (Murray *et al.*, 2007:516). Self-directed learning is defined as lifelong learning, therefore, even primary school Mathematics learners can be taught how to implement self-directed learning in an effort to improve their Mathematics achievement as well as to prepare them for life after school.

Loyens *et al* (2008: 414) argue that self-directed learning promotes individual freedom, responsibilities and personal views. This implies that learning should empower a learner to become a free, mature and authentic person (Savin-Baden & Major, 2004:14).

2.3.1 Definition of self-directed learning (SDL)

For purposes of this study self-directed learning is conceptualised as an approach where learners assume responsibility for their own learning experiences, are persistent in their learning, have a desire to learn, take initiative for learning and implement learning strategies with or without the assistance of the teacher, peers, parents, or other people (Knowles, 1975:16). These learners are able to set their own goals without guidance from the Mathematics teacher.

Most research regarding SDL was done in either adult learning centres or in tertiary institutions. However, Williamson (2007:68) declares that all individuals are capable of self-directed learning, but the degree of development differs due to individual differences. A self-directed learner is equipped with specific features such as assuming responsibility for his / her learning experiences, is persistent in learning and the completion of tasks, has a desire to learn and is internally motivated to achieve, takes initiative in learning and implements what has been learned, and has a need for specific knowledge. Furthermore, a self-directed learner is self- disciplined, self-confident, independent, displays a high degree of curiosity, manages time efficiently and effectively, and uses basic study skills and a variety of resources or study aids (Guglielmino, 1977:73; Pintrich & De Groot, 1990:33; Guglielmino & Long, 2011:1).

2.3.2 Factors influencing self-directed learning

Self-directed learning is influenced by internal as well as external factors (Kek & Huijser, 2011:186). Internal factors comprise personal characteristics (learner motivation, goal orientation, self-efficacy and locus of control) of the learner (see 2.3.2.2.1), which influences the way learners approach a task as well as their persistence and initiative in completing the task (Van Deur & Murray-Harvey, 2005: 167). Internal factors also include learning strategies: cognitive, metacognitive and affective learning strategies. External factors include the learning environment / social settings, resources used in the classroom as well as the role of the teacher, the Mathematics curriculum, the availability of resources and the use of mathematical tasks (see Figure 2.2).

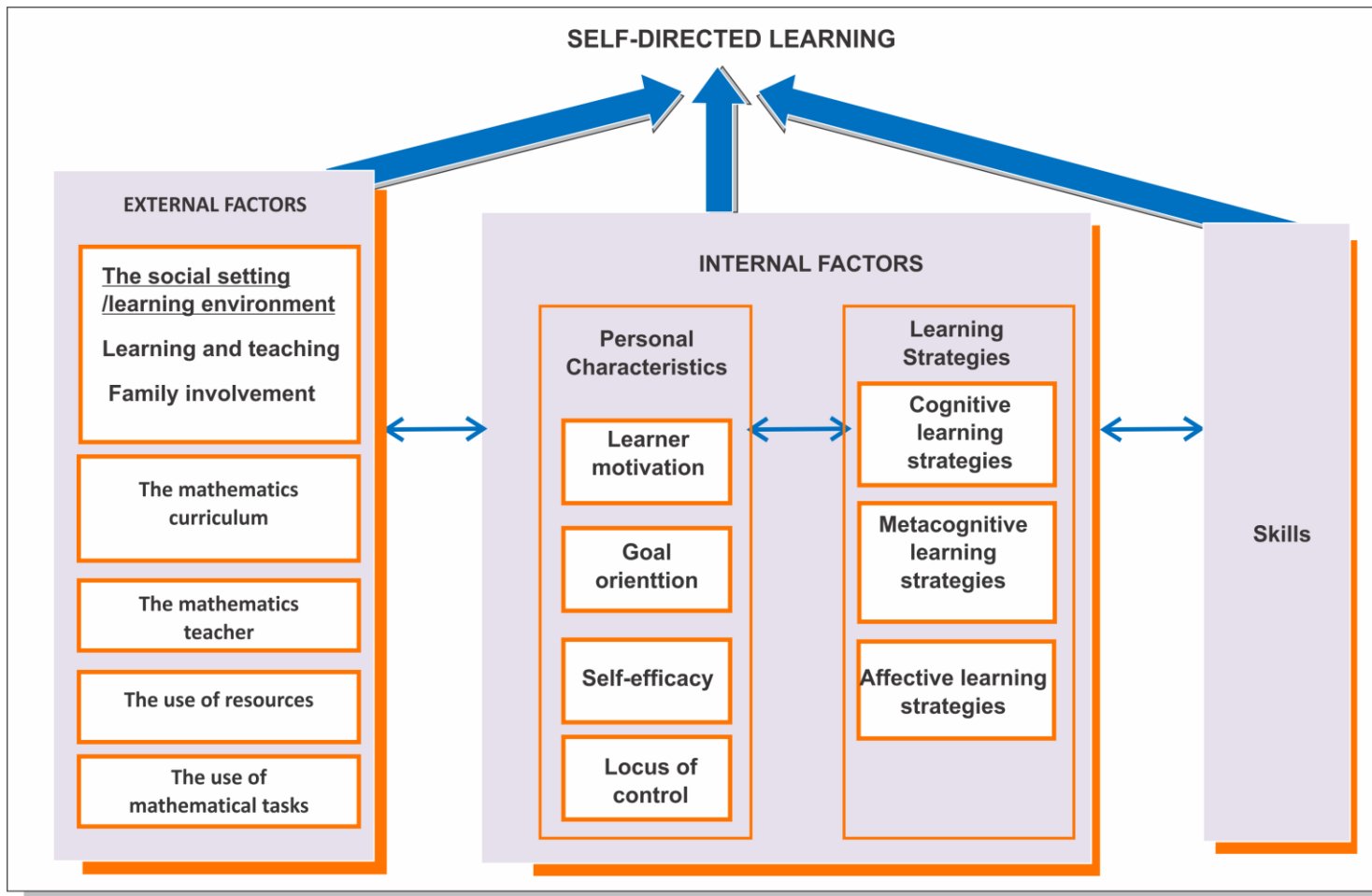


Figure 2.2: A model for self-directed learning in primary school learners (adapted from Van Deur & Murray-Harvey, 2005:168).

2.3.2.1 External factors

2.3.2.1.1 The social setting / learning environment

The learning environment refers to the social and psychological contexts of learning and the determinants of learning that affect a learner's achievement and attitudes (Fraser, 1998a:1). Many of the studies that were conducted have established that learners' learning is affected by social and psychological climate. It is not only affected by the learning perception of the learner, but also by other persons involved in education such as teachers, parents and administrators (Walberg, 1982).

With regard to learning environments, many studies have researched the family environment as a social setting that influences learners' self-directed learning. These studies have examined the relationship between family and learner outcomes and focused on primary and secondary school contexts (Kek & Huijser, 2011:188). Marjoribanks (1995:91) in particular shows how the home or family and the school environment interact and co-determine learners' Mathematics achievement. Marjoribanks (2002:1) claims that if parents are positively involved in the schooling activities of their children, the Mathematics achievement of those children should improve.

Rutter and Maughan (2002:455) focus on the quality of teaching experienced by learners in the primary school classroom as a factor influencing SDL. Contractual factors such as school organization and management, group management in the classroom, and the teaching qualities of the teacher have an influence on learners' behaviour. Classrooms have to be designed in such a way that understanding is the primary goal of teaching (Rhine, 1998:29). In the Mathematics classroom learners interact with their peers and teachers, and these interactions have a major role in the self-directed learning ability (see 2.4) of the learners (Wigfield, Eccles & Rodriguez, 1998:83).

Learners' approaches to learning develop in response to the learning environment, which implies that it is important to change the existing learning environment of the learners in order to help them develop in different ways of thinking (Chan, 2001). Chan emphasises the importance of bridging a learning gap with direct teaching. Westwood (1997) argues that teaching learners explicitly about SDL-skills reduces the likelihood that learning self-directed skills is left to chance. Teaching learners SDL skills involves the analysing of tasks into easy steps, the teaching of task-approach learning strategies (see 2.3.2.2.1), frequently revising previously taught skills and maximising time on tasks (Westwood, 1997).

2.3.2.1.2 The Mathematics curriculum

Little work has been published on the influence or development of an SDL curriculum (Thornton, 2013:144) and even less work or literature is available in terms of primary school learners. The Senior Phase (Grades 7 – 9) Mathematics curriculum emphasises the following skills to be developed:

- the correct use of mathematical language;
- number vocabulary, number concept, calculation and application skills;
- learn to listen, think, communicate, reason logically and apply the mathematical knowledge gained;
- learn to investigate, analyse, represent and interpret information;
- learn to pose and solve mathematical problems, and
- build an awareness of the important role Mathematics plays in real-life situations, including the personal development of the learner (DBE, 2011:8-9).

In the South African Public School context, where class sizes are very large, it tends to make curriculum delivery challenging. Not all Mathematics teachers, especially in primary public schools, are professionally qualified to teach Mathematics.

2.3.2.1.3 The Mathematics teacher

Engagement and support for self-directed learning is critical when learning becomes an integral part of life; driven by a desire and need to understand something, or to get something done instead of merely solving a problem given in a classroom setting. Self-directed learning de-emphasises teaching as a process in which a teacher tells something to a passive learner. It focuses on mutual dialogues and joint knowledge construction, enhanced by the creation, discussion, and evolution of mathematical knowledge. Therefore Mathematics teachers should understand their roles not only as educators but also as coaches (Fischer & Sugimoto, 2006: 7). The role of the teacher is discussed in more detail later on.

2.3.2.1.4 The use of resources in the classroom

Mathematical resources is defined by Drews (2007:21) as any form of specific mathematical apparatus or manipulatives (structured or unstructured) such as mathematical games, worksheets and textbooks or everyday material which can be used to provide mathematical teaching or learning support. Manipulatives or apparatus are objects designed to represent clearly and concretely mathematical ideas that are abstract (Moyer, 2001:176). Through manipulation of the apparatus learners reflect the equivalent manipulations within the mathematical structure (Drews, 2007:21).

Mathematical games are usually activities that are highly motivating to learners and encourage concentration and engagement with Mathematics. Mathematical games can be used to consolidate mathematical learning to practise mathematical skills, to explore mathematical relationships and to develop problem-solving strategies. Games put learners in a situation where they are in control of their own learning; there is often no specific way to win a game or solve a problem. Such control encourages flexibility of thinking and mental fluency (Drews, 2007:22-24).

Worksheets and textbooks as resources are main features in many Mathematics classrooms. Even though worksheets and textbooks play an important role in mathematical learning, Liebeck (1984:16) notes that the mentioned resources focus primarily on pictures and symbols rather than on concrete experiences and language. This notion, however, is problematic for Atkinson (1992:13) who sees meaningful Mathematics as rooted in action, meaning learning through doing. A teaching approach that predominately makes use of worksheets and textbooks for Mathematics can produce difficulties for learners, such as those with visual learning styles, who struggle with a print-based curriculum (Clausen-May, 2005). There are semantic levels (see 3.6.4) of reading and interpretation which can lead to confusion (Santos & Bernard, 1997 in Harries & Spooner, 2000). Mathematical work completed on worksheets does not always reflect an accurate view of what learners are capable of accomplishing (Drews, 2007:24).

Everyday materials that can be used in the Mathematics class to assist the learners making sense of Mathematics as a real-life activity are endless. Examples can include timetables for receipts, catalogues, and any form of container or measuring device. Manipulating familiar objects (everyday materials), enables learners to rationalise their mathematical experiences (Drews, 2007:25).

Black and Atkin (1996) argue that the most critical resources required for implementing innovation and change in the Mathematics classroom are human resource -, for example, qualified Mathematics teachers. Innovation in education requires more people who are willing and able to go the extra mile in overcoming inadequate resources to support educational change. Human resources in the Mathematics classroom, however, are insufficient; Mathematics teachers have a continued need for more visual resources. Visual resources refer to visual tools such as diagrams, pictures, transparencies, use of colour and textbooks as well as information technology (examples; computers and internet access) (Naidoo,2012:2). The use of visual aids and colour creates an interesting and exciting Mathematics classroom. More visual resources, however, do not necessarily lead to better practices in the Mathematics classroom. In learning environments where visual resources are used, learners are able to interact more easily with abstract mathematical concepts (Naidoo, 2012:2).

The use of resources in the Mathematics classroom has the potential of motivating learners, providing a variety of teaching and learning experiences, connecting classroom tasks to the real world and supporting the understanding of mathematical ideas, and helping learners to build up a repertoire of mathematical knowledge and encouraging mathematical communication (Drews, 2007:25-26).

2.3.2.1.5 The use of mathematical tasks

Tasks that learners perform in the Mathematics classroom form the basis of learners' learning (Stein & Smith, 1998:268). Routine tasks focused on memorised answers in a routine manner lead to one type of thinking opportunity for the learner, while tasks that require learners to think conceptually and to make connections between different tasks and previous knowledge, lead to a different set of opportunities for learners' thinking. The daily completion of mathematical tasks, as well as interacting with these tasks, develops learners' ideas about the nature of Mathematics. How tasks are perceived and implemented by learners is also imperative in the completion of the tasks. The nature of tasks often changes as it passes through the different phases, indicated by the mathematical tasks framework (Figure 2.3). A task that appears in the curriculum or instructional material is not always identical to the task that is set up by the teacher and is not always the same task that the learners actually do or have to complete (Stein & Smith, 1998: 268-270).

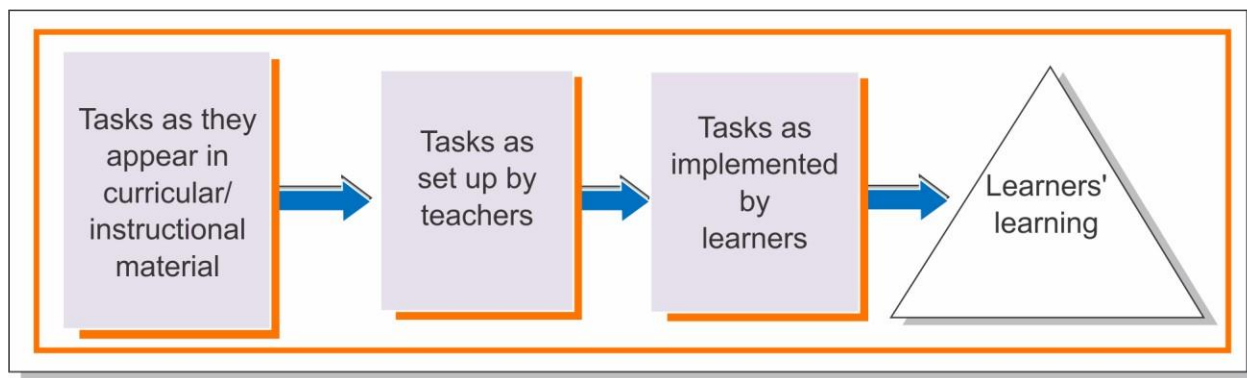


Figure 2.3: The mathematical tasks framework (Adapted from Stein & Smith, 1998: 270).

Tasks, however, should provide opportunities for learners to struggle with ideas and to develop and implement an increasingly sophisticated range of mathematical ideas such as justification, abstraction and generalisation (Anthony & Walshaw, 2007:13).

Self-directed learners are able to implement multiple learning across mathematical learning tasks in order to attain their learning goals and are able to adjust their learning strategies to facilitate their progress (Paris & Paris, 2001:90).

2.3.2.2 Internal factors

2.3.2.2.1 Personal characteristics of a Mathematics learner

Nelson and Conner (2008) note that learner motivation, goal orientation, self-efficacy, locus of control and self-regulation (see 2.3.2.2.2 (b)) are characteristics of self-directed learners which learners, parents as well as teachers should take note of. They should assist learners to develop these characteristics. These characteristics provide a framework for helping learners to gain an understanding of themselves as learners and ways in which they can improve their self-directed ability.

a) Learner motivation

Learner motivation deals with the desire of a learner to actively participate in the learning process. It also focuses on the reasons that underpin a person's involvement or non-involvement in mathematical activities. Learners' sources of motivation may differ, which is one of the main concerns in determining motivational levels of individual learners. Although learner motivation is affected by the intrinsic motivation (see 2.5.1.1) of a learner, there are many extrinsic (see 2.4.1.2) factors that can possibly influence the development of learners' motivation (Shannon, 2008:18).

Learners become motivated when they use one or more learning strategies to keep themselves on-track towards a set of learning goals (Zumbrunn *et al.*, 2011:10). Learner motivation occurs in the absence of external rewards or incentives, indicating that a learner is becoming self-directed in his or her own learning process (Zimmerman, 2004:155). By finding motivation from within and establishing their own learning goals, learners are more than likely to persist through difficult Mathematics tasks and will find the learning process gratifying. (Wolters, 2003:193). A positive, motivated learner initiates an effort to apply learning strategies (see 2.3.2.2.2), to find resources and to persist when running into difficulties. Motivation is also increasingly recognised as being dependent on external social factors such as the way in which the classroom is organised (Pintrich & De Groot, 1990:36). In the classroom learners interact with their peers and teachers and these interactions influence the learners' motivation (Van Deur & Murray-Harvey, 2005:167).

b) Goal orientation

Goal orientation can be seen as the aims that regulate the learners' actions (Schunk, 2001). In the Mathematics classroom, goals may be as simple, such as getting a good grade on a task, assignment or examination, or gaining a broader understanding of concepts such as addition of fractions or decimal fractions. The Mathematics teacher should, however, encourage learners to set short-term goals in order to track their learning progress (Zimmerman, 2004:141).

Goal orientation is also defined by Caraway *et al.* (2003:418) as the learners' ability to make plans and set goals to increase their motivation. The majority of literature related to goal orientation tends to support the notion that goal orientation is conducive to positive behaviours, resulting in achievements in Mathematics (Shannon, 2008:16).

c) Self-efficacy

Perceived self-efficacy is defined as learners' beliefs about their abilities to produce levels of performance that exercise influence over events that affect their lives (Bandura, 1994). Self-efficacy is a personal belief of ability, rather than emotional reaction to an actual accomplishment (Nelson & Conner, 2008). Learners with greater self-efficacy in their abilities approach mathematical tasks as challenges to be mastered rather than tasks to be avoided. Individual learners will set challenging goals for themselves and maintain strong commitment in achieving them. The most effective way of creating a strong sense of self-efficacy is through the mastery of experiences (Shannon, 2008:16-17).

d) Locus of control

Learners must be able to control their attention in order to become self-directed (Winne, 1995:175). Attention control is a cognitive process that requires significant self-motivation and self-monitoring (see 2.5.4) (Harnishferger, 1995:184). This cognitive process requires clearing the mind of distracting thoughts, as well as looking for suitable environments that are conducive to learning (Winne, 1995:175). Mathematics teachers can assist learners by removing stimuli that may cause distractions, and giving learners regular breaks to help them in extending their attention span (Zumbrunn *et al.*, 2011:10).

Locus of control is defined as the tendency learners have to attribute achievements and failures to either internal factors which they can control or external factors beyond their control (Miller *et al.*, 2003:549). A self-directed learner is described as a learner having a greater internal than external locus of control (Shannon, 2008:16-17).

2.3.2.2.2 The use of learning strategies

Learning strategies refer to a sequence of acts influencing the way in which a learner selects, acquires, organises and integrates new knowledge in order to achieve a certain goal (Weinstein & Meyer, 1991:16). Learning strategies refer to methods and techniques used by a learner to improve learning (Rachal *et al.*, 2007:191).

According to Van Den Broek *et al.* (2001:1084), most learners in the primary school do not have a large repertoire of learning strategies at their disposal; therefore it takes time for these learners to learn and become comfortable using different learning strategies. Through modelling and providing appropriate scaffolding on how to use different strategies, teachers can enable learners to become independent strategy appliers (Van Den Broek *et al.*, 2001:1084).

a) Cognitive learning strategies

Cognitive learning refers to a person's ability to process information, reason, relate and remember. It revolves around many factors that include problem-solving, memory retention, thinking skills, how learned material is perceived and the understanding of language (AeU, 2011:106-107).

Cognitive learning strategies include rehearsal strategies, elaboration strategies and organisational strategies (Weinstein, *et al.*, 2011:47). Rehearsal strategies use a repetitive method to what learners are trying to learn. Examples may include repeating mathematical definitions over and over, flash cards and listening to recordings. Not all rehearsal strategies are effective or efficient for learning or integrating new knowledge and skills. There is a distinct difference between passive and active rehearsal strategies. Passive rehearsal strategies promote simple repetition, do not

involve much cognitive processing and do not result in meaningful learning over a period of time. Active rehearsal strategies involve more cognitive processing and meaningful learning that will stand over a period of time, and active rehearsal strategies create opportunities for understanding and learning to take place.

Elaboration learning strategies involve adding to and modifying the material to be learned to make it more memorable and meaningful (Weinstein *et al.*, 2011:48). Elaboration learning strategies can take different forms such as; summarizing, paraphrasing or rephrasing, teaching to someone else and setting up and answering possible mathematical questions. These forms of elaboration include some level of cognitive processing (Weinstein *et al.*, 2011:48). However, more complex forms of elaboration involve greater cognitive processing effort and more complex thinking which lead to better understanding and can be recalled for lower-order thinking as well as higher-order thinking, such as mathematical problem-solving (Weinstein *et al.*, 2006).

Organisation strategies focus on reorganising and elaborating new material. Examples include creating cause-effect diagrams, mind maps and relation diagrams that learners can use to help them to create meaning to new material that they are learning in Mathematics. These organisers require active and complex cognitive processes.

Weinstein *et al.* (2011:49) note that the way a specific strategy is used is not as important as the cognitive processes involved. It is imperative for learners to have a repertoire of strategies, because different strategies will fit different learning situations and the usefulness of a specific strategy is also dependent on the cognitive level of individual learners.

Above all, cognitive learning strategies include problem-solving. When learners are confronted by a mathematical problem, seemingly without the resources to solve the problem, self-directed learning is initiated. As learners identify gaps in their understanding, they set goals for learning and put strategies in place to solve the problem (Hmelo & Cotè, 1996:421). *Problem-solving* can be conceptualised as the quest to attain a goal where the path to that goal is uncertain (Martinez, 2006:697). It loosely relates to what you do if you don't know what you are doing. Problem-solving is an activity that is being exercised daily, especially in the Mathematics classroom, where following established mathematical rules and concepts is not enough to experience success. For example, learners are solving mathematical equations on a daily basis. For some learners a simple equation ($2x - 4 = 10$) can be challenging and can therefore become a problem to solve. An example of a problem is: Mr Williams' piece of ground is 18m long and 9m wide. He builds a fence around it but leaves an opening of 3,5m for a gate. How long is the fence? According to Martinez (2006:697), a prerequisite for problem-solving requires cognition, because one has

to continuously be busy generating possibilities, weighing options, exploring subsets of options and evaluating the final results in an attempt to solve the problem.

b) Metacognitive learning strategies

Metacognition can be defined as thinking about one's own thinking (Weinstein *et al.*, 2011:48). Metacognitive strategies are methods learners use to understand the way they learn; in other words, it is a process learners go through to think about their own thinking. Metacognitive strategies are helpful strategies for mathematical problem-solving (Shannon, 2008:18).

Metacognition is further an appreciation of what one already knows, combined with the strategies to apply appropriate mathematical knowledge to a particular situation, in an efficient and effective manner (Peirce, 2003:2). Learners can be encouraged to develop a sense of their own knowledge by asking questions such as "What do I know?", and "What do I need to know?" These types of questions assist learners in becoming more self-aware and making real-world connections to the mathematical information they are learning (Shannon, 2008:18). In order to help learners develop metacognitive strategies, Mathematics teachers can incorporate learners' active reflection throughout the learning process.

Effective metacognitive strategies include the following:

- (1) Evaluating work – learners / teachers determine where their strengths and weaknesses lie by reviewing their work.
- (2) Questioning by the teacher – "What are you working on?", "Why are you working on it?", and "How does it help you?" are questions that the Mathematics teacher can ask while learners are working on mathematical problem-solving.
- (3) Self-assessment – learners reflect on their learning and determine how well they have learned something.
- (4) Selecting strategies – learners decide which learning strategies are useful for a given mathematical task.
- (5) Using directed thinking – learners choose to follow a specific line of thinking.
- (6) Using discourse – learners discuss ideas with each other and with the Mathematics teacher.
- (7) Critiquing – learners provide feedback to other learners about their work in a positive, constructive manner.

- (8) Revising – learners revise their work after they have received feedback from the Mathematics teacher (Darling-Hammond *et al.*, 2008).

c) Affective learning strategies

Learning strategies are thoughts and behaviours that affect learners' affective state and the way learners select, acquire, organise and integrate new knowledge (Anthony, 1996:1). Affective learning strategies can be defined as the development of learners' specific attitude towards Mathematics (Cangelosi, 2003:417). The Mathematics teacher has a critical role to play in developing positive attitudes towards Mathematics. An affective learning strategy further implies that the learners develop a sense of appreciation, understanding and curiosity to solve mathematical problems (Cangelosi, 2003:168).

2.3.2.2.3 The application of skills

Skills can often be divided into domain-specific and domain-general skills.

- Domain-general skills may for example refer to skills required to be a self-directed learner, such as:
 - Goal setting skills.
 - Information processing skills.
 - Other cognitive skills.
 - Decision-making skills and self-awareness (Long, 2005:1).

Domain-general skills can be used to solve any mathematical problem.

- Domain-specific skills refer to memorised information or knowledge that can lead to solving a specific mathematical task or problem. For example, using the rules from order of operations in the Grade 7 classroom is the application of memorised rules or steps for specific calculations.

Ability can be seen as a learned skill to carry out a task with pre-determined goals often within a given length of time.

Goal-setting skills: Many learners have not learned the skill or ability to determine what is important and how to select it from the possible options. The learners have become used to questions and problems identified for them rather than having the ability to engage in problem posing and problem-solving. Therefore, teachers have to assist these learners in developing skills

in order to become self-directed learners.

Information processing skills: A strong reading ability is generally and often identified as a characteristic of a successful self-directed learner. However, there are other skills that are also useful processing skills, such as listening, observing, as well as seeing and translating information.

Cognitive skills: The following cognitive skills appear to be associated with self-directed learning success; the ability to select from multiple sensory information (sensory skills), the working memory is important before distributing information into the long term memory (memory skills), and problem posing and problem solving skills.

Decision making skills: The self-directed learner must be able to develop the ability to develop skills to identify, prioritise, select, validate, evaluate and interpret information or knowledge. Learners who are unable to differentiate between useful and less useful information for a given situation are unlikely to be self-directed learners. A self-directed learner develops the ability or skill to determine and evaluate the sources of information as well as the reliability, validity and meaning of information.

Self-awareness: The ability to be self-aware is an attribute of a successful self-directed learner. This attribute enables learners to be aware of their learning processes, to know which approach to use, to know what is distracting in their learning environment, to know when they need assistance, to know their strengths and weaknesses and to have a realistic expectation in the ability to reach a learning goal.

2.4 Self-directed learning ability

For learning to take place, some form of teaching has to be applied. Teaching is more commonly viewed or accepted as helping someone else to learn (Towle & Cottrell, 1996:357). SDL in Mathematics in the senior primary class can therefore be taught to learners. Literature provides guidance as to who will facilitate self-directed learning as well as assistance with preparing for critical thinking skills for lifelong learners (Towle & Cottrell, 1996:357).

Self-directed learning ability is defined by Cheng (2011:11) as domains, namely learning motivation, planning and implementing, self-monitoring and interpersonal communication (Cheng *et al.*, 2010:1153).

For purposes of this investigation self-directed learning ability (SDLA) refers to the ability of learners to be motivated, to plan, to implement, to monitor and to have interpersonal communication skills during their own learning.

2.4.1 Learning motivation

Motivation (see 2.3.2.2.1) is a key element of self-directed behaviour such as learning and is dependent on learners' believing in what they are able to learn. Motivation influences their learning by determining the tasks that learners select to do, their attitudes while working on a task and their industriousness when struggling to do a task or solve a problem (Van Deur, 2004:64). Two specific types of learning motivation are identified by most literature, namely intrinsic and extrinsic motivation.

2.4.1.1 Intrinsic motivation

Intrinsic motivation is seen as the drive or desire that learners have to engage in academic (Mathematics) tasks for the enjoyment of doing it or for "own sake" (Middleton & Spanias, 1999:66). These learners feel that learning is important to their self-image. They look at learning activities (Middleton, 1992) for the enjoyment thereof, and their motivation is focused on a learning goal such as comprehending and control of mathematical concepts (Ames & Archer, 1988; Duda & Nicholls, 1992). Motivation is also related to the learners' sense of their mathematical ability; be they motivated by grades or curiosity. Intrinsic motivation is more complex than the effects of achievement, ability, and perceived competence even though these domains contribute to learners' desire to learn Mathematics. Hence, learners, who view themselves as capable of doing well in Mathematics, tend to value Mathematics more than learners who do not. (Eccles, Wigfield, Reuman & Maciver, 1987; Midgley, Feldlaufer, & Eccles, 1989). It is therefore likely that, before intrinsic motivation can be developed, learners should feel comfortable with Mathematics, they must be challenged to achieve and must anticipate experiencing success in Mathematics (Middleton & Spanias, 1999:67). Intrinsic motivation is considered by teachers to be more desirable than extrinsic motivation. Amongst other things, it results in better learning outcomes (Deci *et al.*, 1999). Intrinsic motivation is often contrasted by researchers with extrinsic motivation, which is motivation governed by strengthened possibilities.

2.4.1.2 Extrinsic Motivation

Extrinsic motivation has been defined as a pale and pauperized form of motivation, which, according to Ryan and Deci (2000: 55), is in contrast with intrinsic motivation. Intrinsic motivation results in high-quality learning. Extrinsically motivated actions can be performed by learners with resentment, resistance, disinterest or alternatively with an attitude of willingness which reflects an acceptance of the value of the task at hand. Extrinsic motivation is therefore propelled by external factors such as approval of parents or teachers, avoidance of negative consequences, such as

poor results, and acceptance and approval of friends.

Many of the tasks learners need to complete at school are not inherently enjoyable or interesting (intrinsically motivated) and thus learners need to be motivated extrinsically. It is therefore pertinent that Mathematics teachers should be able or in a position to promote more active and “free-will” forms of extrinsic motivation to affect successful learning in the Mathematics classroom and elsewhere (Ryan & Deci, 2000: 56-57).

Intrinsically motivated action becomes increasingly reduced by social demands and roles that require learners to assume responsibility for non-intrinsically interesting tasks (Ryan & Deci, 2000: 60). For example, in schools it seems that intrinsic motivation becomes weaker with each grade. Learners need to find, assisted by the Mathematics teacher, activities that are inherently pleasurable while at the same time paying attention to the extrinsic consequences of those activities in any specific context.

South African schools are driven by *achievement*, particularly in Mathematics and literature in the North West Provincial Assessments (NWAPA) and Annual National Assessment (ANA), strategies that are used by the Department of Basic Education to improve teaching and learning in South African schools (DoE, 2012:4). Hence, learners are compelled to complete tasks and problem-solving activities in these particular assessment activities. Subsequently there is little room for learners to explore or implement intrinsic motivation; extrinsic motivation thus becomes a persistent factor in the Mathematics classroom. A particular concern for teachers in Grade 7 Mathematics classrooms is that research has shown that intrinsic motivation declines with age (Ryan & Deci, 2000: 60). The decrease in motivation is coherent with prior research that suggests that positive academic beliefs and behaviours progressively erode or diminish as children progress through the school system (e.g. Nicholls, 1978; Anderman & Maehr, 1994; Sansone & Morgan, 1992; Lepper *et al.*, 2005).

2.4.2 Planning and implementing

Planning is similar to goal setting (see 2.3.2.2.3) in the sense that planning can help learners to self-direct their learning before engaging in the learning task (Zumbrunn *et al.* 2011:10). Goal setting and planning are complementary processes: planning assists learners in formulating well thought-out goals and strategies; planning occurs in three stages: firstly, goal setting for a Mathematics learning task, secondly, ways in achieving those goals, and finally determining how much time and resources will be needed to achieve the set goal (Schunk, 2001).

Learners plan involvement in and commitment to self-chosen goals. These goals include their

ability to define ongoing and upcoming activities against their own wishes, needs and expectations. Learners' ability to protect their own goals from conflicting alternatives will have an influence on their self-directed ability (Boekaert, 1999:451).

With a planned curriculum in Mathematics which Grade 7 learners have to cover, their self-chosen goals and objectives should be in line with the goals and objectives of the Mathematics teacher/curriculum.

2.4.3 Interpersonal communication

In its broadest sense, interpersonal communication is a process involving the accidental or deliberate transfer of meaning and or knowledge. Whenever you are observing or giving meaning to behaviour, communication is taking place. Interpersonal communication can also be defined as a distinct type of interaction between two people (Wood, 2010:19) and the ability of learners to interact with others to promote their own learning (Shen *et al.*, 2014:2).

2.4.4 Self-monitoring

Self-monitoring refers to the learners' reflection and evaluation of the strategies, methods and outcome of learning. Self-monitoring is critical for self-directed learning to take place (Mok *et al.*, 2007:855). Self-directed learners need to assume ownership for their learning and achievement by monitoring their progress towards the learning goals (Kirstner *et al.*, 2010:164). In order for learners to self-monitor progress, they need to set goals (see 2.3.2.2.3), plan, autonomously motivate themselves, focus their attention on the task and use learning strategies (see 2.3.2.2.2) to facilitate their understanding of the material or concepts (Zimmerman, 2004). By encouraging learners to keep record of how many times they worked on a particular mathematical task, the strategies they used and the time spent on these tasks, learners will be able to visualise their progress and make changes if needed.

2.5 Other factors influencing self-directed learning ability

In order to obtain a broader understanding of self-directed learning ability or influences of self-directed learning, factors pertaining to it are subsequently discussed. These factors are seen by Van Deur (2004:64) as critical factors influencing the self-directed learning ability in primary school learners, and they are: concentration, time management, self-testing, motivation and using of study aids.

2.5.1 Concentration (CON)

Concentration or attention in learning can be conceptualised as a meta-cognitive component of self-direction and the interaction with other components of self-directed learning (Beronia, 2008:3). Concentration refers to the way learners are focused and committed to attempt and complete a learning task in a given context. Concentration is paying attention to the right things at the right time. It is the ability to attend to relevant factors and disregard irrelevant factors. Concentration is defined as the ability to give something our undivided attention to the exclusion of other distractions (Beronia, 2008:3). Concentration is essential for success within teaching and learning. It is about being totally wrapped up in the here and now. The past and/or future is not important. The major component of concentration is the ability to focus one's attention on the task at hand and thereby not be disturbed or affected by irrelevant external and internal stimuli (Wilson *et al.*, 2006:404). There are two strategies to help keep and maintain concentration:

- Learn to increase concentration to relevant stimuli: This involves training to focus on something specific, for example, the attention learners are paying to their work, whether they are working on mathematical tasks, or writing a test, or completing an assignment. However, the learner may first practise this with something simple, for example, focusing on a lit candle, or your breathing, for increasing periods of time.
- Learn to decrease concentration to irrelevant stimuli: This involves training to shut out anything that may hinder concentration, for example, the noise from another classroom (Castle & Buckler, 2009).

2.5.2 Time management (TMT)

Effective time management in the classroom is critical to ensure that both teacher and learner are able to optimise the given time period for learning to take place (Van Deur, 2004:70). This may be done through:

- Precise planning of teacher time and learner time for group work, individual work and whole-class work. This plays an important role in nurturing active learner engagement during activities and smooth transitions between the mathematical activities or dealing with concepts.
- Organising a variety of creative and purposeful activities so that each learner is occupied meaningfully all the time, while minimising disruptions due to boredom or distraction.

Self-directed learners have to plan how they will use the time they have to find out about a topic or solve a problem (Van Deur, 2004:70).

2.5.3 Self-testing (SFT)

An activity in which you test yourself and check your own answers is defined as self-testing (Sexton, 2011:1). It is a way of detecting what you have learned or not learned when a test takes place. Reviewing the material using self-testing helps you to set apart what you know or don't know so that you can focus on learning the missing pieces of mathematical knowledge. It helps you get prepared on paper, which leads to making connections that explain the material at a deeper level. Self-testing is also a good way to improve your memory because when you practise retrieving knowledge, it pushes your learning. Self-testing improves overall learning and improves your ability to remember the information you learn on a long-term basis. It is an opportunity to exercise your critical thinking skills and it promotes lifelong learning (Sexton, 2011:1). Self-testing is necessary to check understanding, consolidate knowledge, integrate related information, and identify whether additional studying should be done.

2.5.4 Motivation (MOT)

Motivation is a fundamental component in self-directed behaviour such as learning, and depends on learners believing that they are capable of learning. Motivational knowledge possessed by learners influences their learning by determining the tasks they select to do, their disposition while working on tasks and their persistence when striking difficulty (Winne, 1991). Motivation is accepted as important through the processes of self-efficacy which is the decision made about one's capabilities to work on a task, and vital attributions which explain who or what was responsible for success. Motivation for self-directed learning would enable a learner to make an effort to carry out self-directed learning strategies to find resources and to persist when running into difficulties (Van Deur, 2004:65). Also see 2.4.1 on learning motivation.

2.5.5 Using Study Aids (STA)

Self-directed learning action involves inquiry. A self-directed learner needs to think and ask questions and needs to use resources such as the computer; encyclopaedia, reading books and looking for information on the Internet (Van Deur, 2004:70) (see 2.3.2.1.4). Learners also need to know how to use study aids created by others, as well as how to create their own study aids. Authors and/or publishers use special techniques such as headings, special markings, summaries and statements and objectives to highlight important material to assist learners in learning. Unless learners learn how to identify or recognise these clues and study aids they will not profit from it. Learners also need to create their own study aids by methods such as underlining, creating diagrams, creating summary sheets, or text-marking main concepts. Using and creating study

aids improves both the effectiveness and efficiency of learning (Weinstein & Palmer, 1990:16)

2.6 The Mathematics teachers' role during the facilitation of SDL

Teachers who want to encourage SDL must free themselves from the fixation of tracking and correcting errors (Guthrie *et al.*, 1996). Teachers need to bring real-life problems into the Mathematics classroom for learners to work on (Temple & Rodero, 1995). Cobb (1994) emphasises the importance of sociocultural or real-life Mathematics teaching and learning. He referred to examples such as learners completing worksheets in school. Therefore teachers need to plan for SDL teaching mathematical tasks or activities.

Planning is a cardinal and often underappreciated aspect of teaching practice. From tasks to activities to instructional practices used during lessons, teachers need to consider a variety of aspects even before the learners enter the classroom. Effective teachers understand that teaching requires a considerable effort for planning, which many teachers consider to be the core of teaching. Teachers first consider the learning activity that takes into account the learners' interest and abilities or skills, then the learning goal and the objectives of the lesson, and finally the evaluation and assessment techniques to be used (Superfine, 2008:11). Additional factors, such as teachers' experience and conceptions of Mathematics teaching and learning, also influence the ways in which they plan their lessons (Kilpatrick *et al.*, 2001:337). Implementing what is planned is as important as that which the teacher has planned to do.

Teachers can assist learners in improving their self-directed abilities (see 2.3.2.1.3) by following Grow's Staged Self-Directed Learning Model (SSDLM) (see Table 2.2). According to the model the teacher's role is to match the learner's stage of self-direction and prepare the learner to advance to higher stages. Grow states that the SSDLM is focused on the teacher / learner relationship. He suggests that learners develop through the stages of increasing self-direction and that teachers can play a role in developing or hindering that development (Grow, 1991:125). His model contains four stages of self-direction, ranging from dependent to self-directed. At each stage, the role of the teacher and teaching methods that are best suited to help the learner, are described.

Francom developed four stages of self-directed learning skills (SDLS) which can be integrated in the SSDLM. The Mathematics teacher has an active role to play in teaching self-directed learning skills (see 2.3.2.2.3), increasing the capacity of learners to direct their own learning (Costa & Kallick, 2004:99; Merriam *et al.*, 2007). According to Francom (2010: 29) there are four main principles for instilling learners' self-directed learning skills in formal education, namely: (1) match the level of SDL required in learning activities or tasks to student readiness; (2) progress from

teacher to learner direction of learning; (3) support the acquisition of subject knowledge and self-directed learning skills together; and (4) learners practise self-directed learning in the context of learning tasks. For Mathematics learners to become more self-directed in their learning, teachers have an active role to play in designing tasks.

Table 2.2: Integrate the Staged Self-Directed Learning Model (SSDLM) (Adapted from Grow, 1991:129) & Self-directed Learning Skills (Adapted from Francom, 2010:30-45)

Stage	Learner & Teachers roles	Teaching strategies	Examples of tasks (Addition of fractions)	Progression ability
Stage 1	Learner is dependent. Learners receive explicit directions on what to do, how to do it and when.	The teacher is the expert or authority. Methods: large group teaching strategies, highlighting specific tasks	<i>Large group presentation. Coaching with immediate feedback. Drill. Informational “lecture”. Overcoming deficiencies and resistance.</i> Example: Concept of addition of fractions with different Denominators $\left(2\frac{1}{2} + 2\frac{1}{4}\right)$ Rules: • Group the whole numbers and fractions separately. • The denominators must be the same. • If they are not the same use LCM. • Then add the whole numbers and fractions separately. • Lastly add whole numbers with fraction. Possible deficiencies: ✓ Be careful with addition, not to add fractions with different denominators	Match the level of SDL required in learning activities to Grade 7 Mathematics’ learners’ abilities.

Stage	Learner & Teachers roles	Teaching strategies	Examples of tasks (Addition of fractions)	Progression ability
Stage 2	The learner is interested.	Motivator / Guide Teacher provides direction and assistance. Encourages learners to build confidence and skills. Explains learning tasks, demonstrate practices, supervises projects and provides feedback in large group presentations. Inspiring lesson presentation plus guided discussion. Assists learners in goal-setting and presents/provides learning strategies.	Example: Concept of addition of fractions with different denominators $\left(2\frac{1}{2} + 2\frac{1}{4}\right)$. Rules: - No rules are given. - Learners have to make an hypothesis, based on prior knowledge	Progression from teacher to learner direction over time
Stage 3	The learner is involved Learners learn more about how they learn. Work on group projects.	Facilitator Teacher participates in learning experience. The teacher uses open-ended tasks in lesson presentation. Teacher facilitates discussion as an equal class member.	Example: Addition of fractions with different denominators. (Chad works on his homework for $2\frac{1}{2}$ hours and Tristan worked on his homework for $2\frac{1}{4}$ hours. How long did they work all together in total?).	Support the acquisition of subject matter knowledge and skills

Stage	Learner & Teachers roles	Teaching strategies	Examples of tasks (Addition of fractions)	Progression ability
Stage 4	Learner is more self-directed. Learners are able and willing to assume responsibility for own learning. Work on group projects.	Mentor/ Facilitator Lesson is learner-centred. Discussion facilitated by teacher who participates as equal. Teacher facilitates the lesson and is a mentor to the learners.	Example: Concept of addition of fractions with different denominators. (Tristan walks $2\frac{1}{2} \text{ km}$ to the taxi rank. He then travels $8\frac{1}{2} \text{ km}$ to work.). How long is the journey?	Learners practice SDL in the context of learning tasks

2.7 Advantages of self-directed learning

Advantages of self-directed learning are best described by the kind of learners developed through self-directed learning. Self-directed learners demonstrate a greater awareness of their responsibility by making learning meaningful to them (Garrison, 1997). These learners are curious and willing to try new things; view problems as challenges; desire change; enjoy learning; are motivated; persistent, independent, self-disciplined and goal-orientated (Taylor, 1995).

Creating and maintaining a classroom that is conducive to SDL hold advantages for both teacher and learners. A self-directed learner is a good role model for learners who are weaker or less self-directed. By enhancing learners' self-direction, teachers can spend more time with learners who need additional support. A further advantage of SDL is that encouraging learners to have greater control over their learning, improves their beliefs about their effectiveness and increases their motivation to learn (Van de Walle, 2013:7).

2.8 Conclusion

Although different approaches to the learning of Mathematics were discussed, namely the behaviourist, the cognitive and the constructivist approach to Mathematics learning, it seemed from the discussion that the constructivist approach is best suited for enhancing self-directed learning in primary school learners. The history of self-directed learning definitions was discussed to give the reader direction to the notion that learning is a life-long process. Specific influencing factors (internal and external) which have an impact on how learners become self-directed in their own learning with reference to Mathematics education were also discussed in depth. . These factors have an impact on the self-directed abilities of the learners, which ultimately have an impact or influence on learner achievement in the Mathematics classrooms.

The chapter was concluded with the advantages of self-directed learning for both the Mathematics teacher and the learner.

CHAPTER 3:

PROBLEM SOLVING IN THE MATHEMATICS CLASSROOM

3.1 Introduction

A principal goal of Mathematics teaching and learning is to cultivate the ability to solve a wide variety of intricate Mathematics problems (Wilson, Fernandez & Hadaway, 1993). Polya formulated his problem-solving model in 1945; and research on mathematical problem-solving has largely been based on this model (Lesh & Zawojewski, 2007:763).

There are three parts to learning Mathematics (Giganti, 2007:15): skills, concepts and problem-solving. Skills are seen as the tools of Mathematics, such as adding two numbers to get a correct answer. Concepts are the ideas in Mathematics which we need to understand before we can do Mathematics, while **problem-solving** is the ability to apply and combine the skills and concepts learned in Mathematics in specific mathematical situations. Mathematical concepts and skills are best taught through problem-solving (Van de Walle, Karp & Bay-Williams, 2013:56).

In this chapter the focus is on different definitions for problem-solving from literature, characteristics of a problem solver, the teacher's role during problem-solving, problem-solving implications for both teacher and learner, factors influencing problem-solving and the relationship between self-directed learning (SDL) and problem-solving.

3.2 Definitions of problem-solving

Reitman (1965) defines a problem as a situation in which a person has been given a Mathematics problem but does not yet have anything to satisfy or solve the problem. Reitman further describes a problem solver as an individual that recognises and accepts the objective without an immediate means of reaching that objective.

Mayer (1983:3) defines problem-solving as a multiple-step process during which the learner or problem solver has to find a correlation between past experiences and the given problem, in an attempt to find a solution. Schoenfeld (1985) emphasises that what a problem is, is relative to the individual learner; for what a Mathematics problem is for one learner may not be the same for another learner. For example "The product of two numbers is 48, if one number is equal to 16, what is the other number?" One learner may be able to solve the problem immediately by applying multiplication tables. Another learner may solve this particular problem by using the inverse of multiplication, namely division.

Mathematical problem-solving is referred to by Schoenfeld (1992: 352-353) as follows: learners have to solve unfamiliar problems by reading through the problem, analysing the problem, examining and evaluating their own mathematical knowledge in order to come up with a solution or answer. With the assistance of the teacher who asks meta-cognitive questions, for example “Why are you doing this? How will it help you?” the learner is able to make new connections, to re-organise existing knowledge and to construct new knowledge.

Unfamiliar mathematical word problems can be seen as situations the learners do not encounter in their day to day lives such as: *Mr Williams's piece of ground is 18m long and 9m wide. He builds a fence around it but leaves an opening of 3,5 m for a gate. How long is the fence?* Grade 7 learners do not have to deal with area or perimeter on a regular or day to day basis. However, it is important for future learning, whilst familiar problems can be seen as situations that are familiar to the learners, for example: *Paul loses $\frac{2}{3}$ of his 63 marbles. How many marbles did he lose?*

Mathematics problems, be they familiar or unfamiliar, are dependent on the view of the learner; for one learner the solution to the problem can be straightforward, and on the other hand the same problem can be challenging for another Schoenfeld (1985). Learners therefore have to be equipped with skills and strategies to solve mathematical problems.

Kirkley (2003:2-3) states that problem-solving skills depend on mastering mathematical concepts and basic literacy skills (reading and comprehending the problem, see 3.7.4). Learners often learn facts, concepts and rote procedures with few connections and applications to knowledge or tasks. A task that can be an exercise for one person may prove to be much more complicated and testing for someone else (Goos *et al.*, 2000:2). Therefore, the problem does not exist in the task itself, but in the relationship between the task and the problem solver (Smith & Confrey, 1991). For problem-solving to succeed, learners have to be willing to solve problems, as well as believe that they can solve unfamiliar problems (Kirkley, 2003:7). Aspects such as effort (self-discipline), confidence, persistence, and knowledge of the self are important qualities for the problem-solving process (Kirkley, 2003:7). Therefore, a learner has to be self-directed in his/her learning to be a successful problem solver (see characteristics of a self-directed learner in Chapter 2).

Lesh and Zawojewski (2007:782) describe problem-solving as a process during which the problem solver needs to develop a more productive mathematical way of thinking about the given situation. The problem solver must participate in a task in a process of interpreting the given problematic situation (Lesh & Zawojewski, 2007:782). The work of Lesh and Zawojewski

(2007) focuses on the development of problem-solving models (way of thinking) rather than the problem solver (what to do) capabilities.

As seen above, quite a number of authors have their own ideas or opinions on how to define mathematical problem-solving. Given the many definitions on problem-solving already in use, Grugnetti and Jaquet (2005:374) suggest that a common definition cannot be provided on mathematical problem-solving. Some even argue that some definitions on problem-solving are out-dated (Lesh, 2003; Rosenstein, 2006).

However, for purposes of this study, mathematical problem-solving will be defined as a process during which the learner has to find answers or solutions to a mathematical problem where there might not be straightforward answers or solutions. A mathematical problem is a situation in which one looks at the “problem” and does not immediately know what to do or how to solve it. A “good” problem provides one with a situation in which one has to pause, ponder and scratch one’s head before even putting pen to paper. Unfortunately, a person does not become a good problem solver over night; it is a process that should be developed every day from early childhood to adulthood (Giganti, 2007).

3.3 Models of problem-solving

Polya’s phases of Mathematics problem-solving are depicted as linear steps in most of the Grade 7 Mathematics textbooks, illustrated in Figure 3.1. However Wilson, Fernandez and Hadaway (1994) state that Polya’s phases (1. understanding the problem, 2. making a plan, 3. carrying out the plan and 4. looking back), are more flexible, because they teach learners to think. Genuine enquiry and problem-solving in Mathematics are led by “How to think”. Care must also be taken during the teaching of mathematical problem-solving not to confuse “how to think”, with “what to think” or “what to do”. “What to do”, reflects the idea that problem-solving is a linear procedure or process, which is inconsistent with genuine mathematical problem-solving (Wilson, Fernandez & Hadaway, 1994).

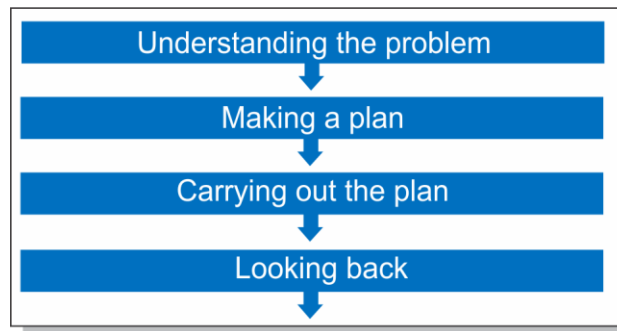


Figure 3.1: A linear model of problem-solving as illustrated in most grade 7 Mathematics textbooks (Wilson, Fernandez & Hadaway, 1994).

In understanding this model one needs to get a better understanding of the four phases of problem-solving:

1. Understanding the problem.

Even though this seems so obvious that it is not even mentioned; a lot of learners find it difficult to solve problems because they do not understand the problem. Polya, in his book “How to solve it”, teaches teachers how to ask questions in an attempt to assist learners in understanding the problem, such as: Do you understand all the words in the stated problem? What are you asked to find, discover or show? Can you state the problem in your own words? Can you think of a picture or diagram that might help you to comprehend the problem? Is there enough information to allow you to find a solution?

Polya (1957) also gives advice to learners on how to tackle the different phases of problem-solving. In the first phase his advice is to: Firstly, understand the problem by determining what is known, what the data are, and under what conditions the mathematical problem should be solved. Then, the learner should also determine whether it is possible to satisfy the stated condition and whether the condition is sufficient (or insufficient or redundant or contradictory) to solve the problem. If possible, learners should draw a picture or figure to have a better understanding of the problem as well as to identify and separate the various parts of the stated conditions of the mathematical problem.

2. Making a plan

The skill of choosing an appropriate strategy or plan to solve the problem is best learned by solving many problems. By solving many problems, learners will find it increasingly easier to choose a strategy to solve mathematical problems, such as: guess and check, look for a pattern, draw a picture or diagram, eliminate possibilities, solve a similar problem, use a model or formula,

consider special cases, solve an equation, work backwards, be innovative or resourceful and use direct reasoning.

In the second phase his advice is to: Find the connection between the data and the unknown. The learner may consider a similar problem if an immediate connection cannot be found and eventually devise a plan to solve the mathematical problem. The learner should restate the problem in own words and, if necessary, go back to definitions. If the learner cannot find a solution to the problem, he or she should try to solve another related problem that is more accessible. The learner can then try to solve part of the problem, and in doing so determine how far the unknown determined is and whether there is new data that could be abstracted or used. And, if necessary, change the unknown or the data or both, in an attempt to draw the unknown and the new data closer to each other. The learners also need to remember to ensure that all conditions of the problem are taken into account.

3. Carrying out the plan

This phase is usually easier than devising or making a plan. This phase entails that the learners must have the necessary skills and need to enforce care, patience and persistence to carry out the plan. If the plan does not work, the learner must choose another path or plan in order to solve the mathematical problem.

In the third phase his advice is to: Carry out the devised plan. Check each step. See that each step is correct and prove that each step is mathematically correct.

4. Looking back

Much can be gained by looking back and reflecting on what was done, what worked and did not work. Doing this will enable a learner to predict what strategy to use to solve future problems (Polya: 1957).

In the fourth phase his advice is to: Examine the solution obtained, check the result obtained and see whether you can reach the same solution under different mathematical conditions.

Wilson, Fernandez and Hadaway (1994) developed a framework to illustrate the dynamic and cyclic nature of problem-solving. A learner starts with a problem and engages in thought and activity in order to understand the problem. The learner's next step may then be to devise a plan to solve the mathematical problem and in the process make sure he/she understands the problem, or discover that the devised plan cannot be implemented. The next step is to develop a new plan or to go back and develop a new understanding of the plan, or pose a new related

problem to work on. The framework in Figure 3.2 is an illustration of the dynamic cyclic *interpretation* of Polya's (1973) phases of problem-solving.

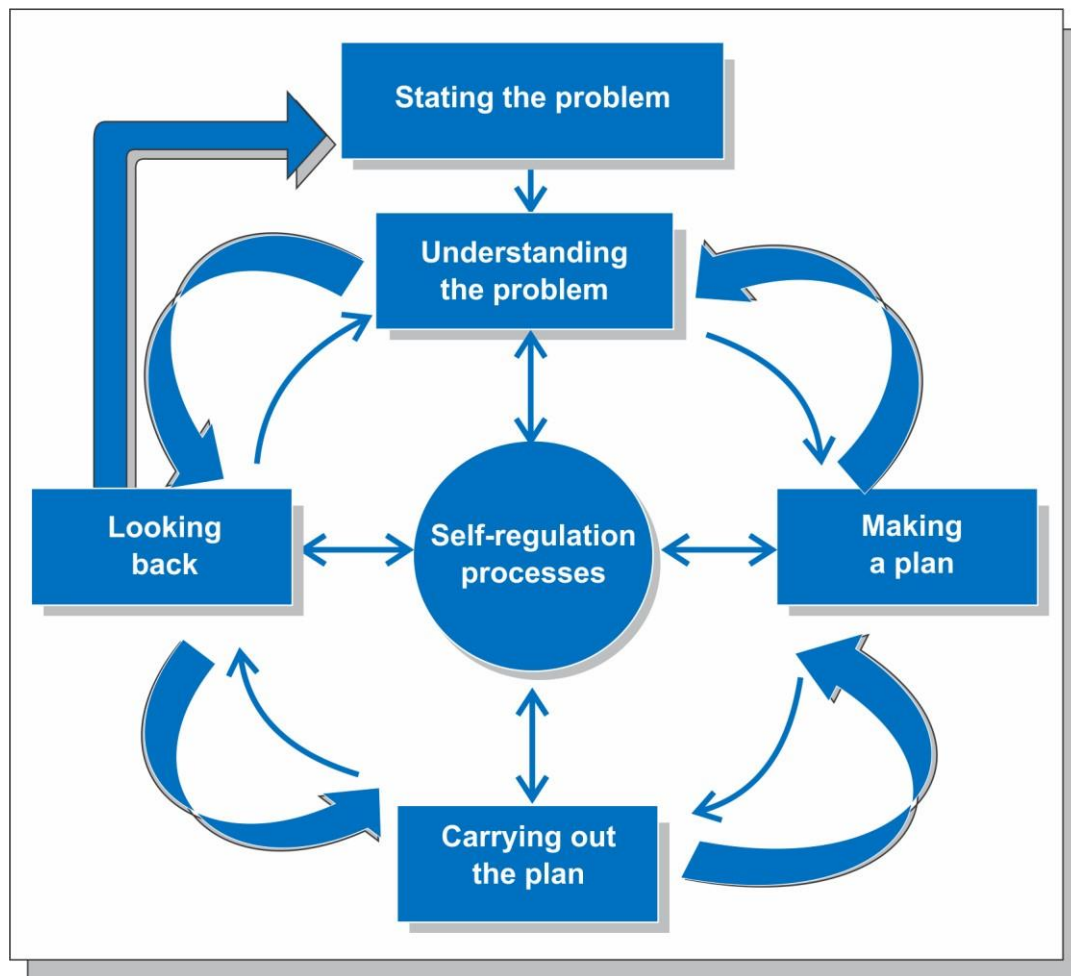


Figure 3.2: A cyclic illustration of the problem-solving model of Polya (Wilson, Fernandez & Hadaway, 1994)

Genuine problem-solving experiences in Mathematics can therefore not be described or captured by one-dimensional arrows alone (Wilson, Fernandez & Hadaway, 1994), which is in contrast with Polya's original linear problem-solving model.

As learners participate in problem-solving activities in the learning of mathematical content and concepts, they explore the problem, and from this exploration they develop methods and problem-solving modelling. From this problem-solving modelling and methods learners develop their reasoning and prove their thinking skills. During this process learners discuss and validate their reasoning and their solutions to the mathematical problems they have solved. Rigelman (2007:313) shows the process of mathematical problem-solving as a cyclical model (Figure 3.3).

The model indicates that this process is unending and may not be brought to completion before engaging in the next action and that some actions will occur simultaneously. In addition, teachers are also actively involved in this process; they ask questions to probe learners' thinking, encourage proof for reasoning, make sense of multiple approaches and reflect for the purpose of making instructional decisions. The mathematical problem-solving process developed by Rigelman in 2007 has similarities to Polya's four phases of problem-solving in the sense that it implies that learners as well as teachers have an active role to play during problem-solving.

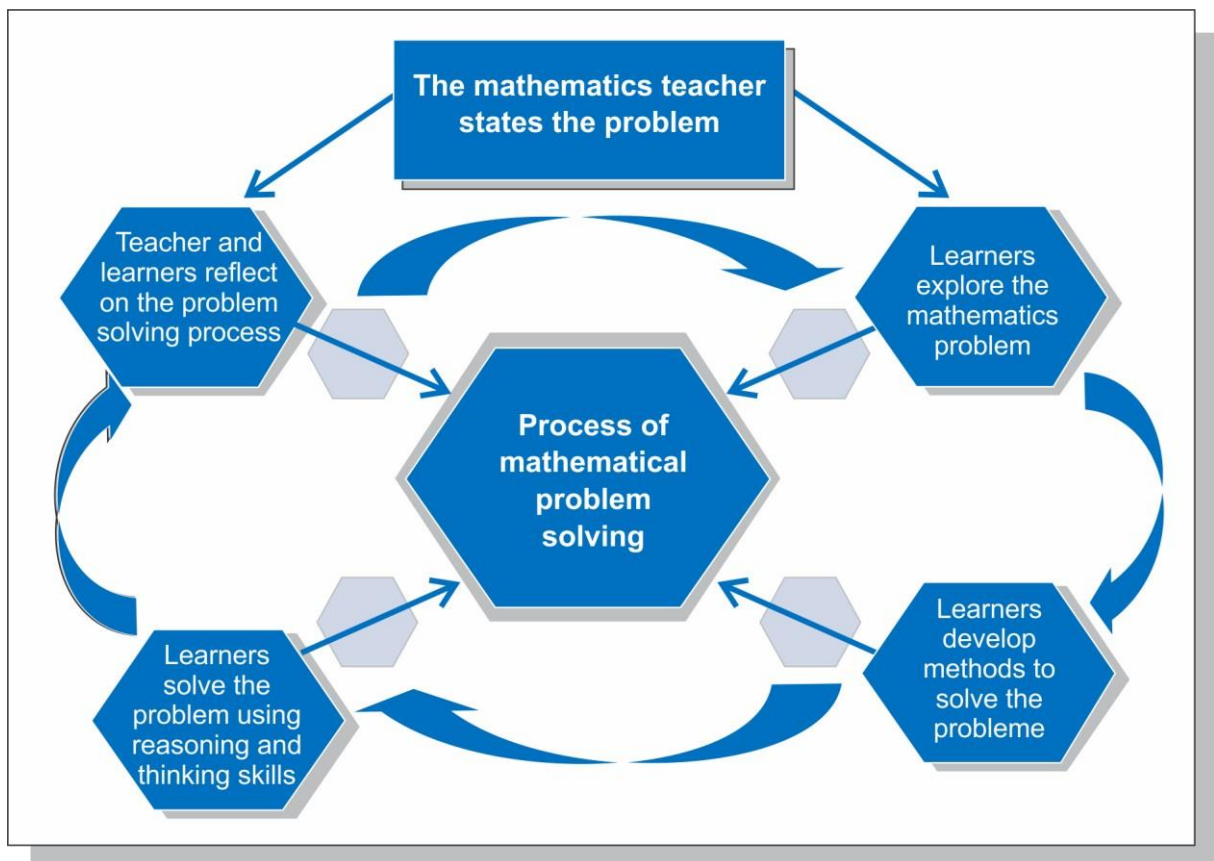


Figure 3.3: The mathematical problem-solving process (adapted from Rigelman, 2007:313)

Badger *et al.* (2012:18) imply that Mathematics is concerned with finding solutions to problems, whether related to real-world problems or to Mathematics. In undertaking problem-solving, learners need to develop both intellectual and temperamental qualities. In developing temperamental qualities, learners need to accept the challenge and take responsibility for finding a solution; work out a strategy/plan; carry out the plan using a sequence of linked steps, know when to turn back and try a different approach; organise, present and defend a solution; and submit a solution for scrutiny by the teacher and their peers; explore the consequences of the

solution and try out generalisation. (Badger *et al.*, 2012:18-19).

The development of intellectual qualities can be explained in reference to the following example from a Grade 7 text book:

Example: *The product of two numbers is 48, if one number is 16, what is the other number?*

The learners must know that product means the answer or result when two numbers are multiplied. The learners can use this information either to get to the solution through multiplying, or use the inverse of multiplication and divide the product with the number that is given. By understanding multiplication the learner may calculate the answer accurately or make another plan and use the inverse method. The learner can then organise and present the solution and subsequently submit it for scrutiny by the peers and/or teacher. The learner can lastly develop a new similar problem and see if the same procedure can be used and processed.

For purposes of this study the model of Polya (1957) was re-implemented. As indicated earlier in this chapter, most Grade 7 text books and resources that deal with mathematical problem- solving are based on this model. Grade 7 learners therefore have already experienced the use of the model indirectly.

3.4 Characteristics of a mathematical problem solver

Characteristics of a mathematical problem solver is summarised by different authors as to indicate the similarities or relationships that have been established over a period of time in different studies related to the characteristics of a mathematical problem solver.

Schoenfeld (1985) offers a framework for analysing how and why learners (or people) are successful (or not) when engaging in problem-solving. He argues that four factors are necessary and sufficient for understanding the quality and success of problem-solving. They are: (i) the knowledge base, (ii) problem-solving strategies, (iii) control: monitoring and self-regulation, or metacognition, and (iv) beliefs and the practice that gives rise to them. Lester (1994:663) agrees with the argument of Schoenfeld that “good” mathematical problem solvers are characterised as possessing more knowledge (deep subject knowledge), control, beliefs and added socio-cultural contexts.

Drawing from the available literature related to problem-solving, Carlson and Bloom (2005) developed a broad grouping to characterise problem-solving attributes that are relevant to problem-solving success. They are the following:

Resources: The conceptual understandings, knowledge, facts and procedures used during

problem-solving.

Control: It includes the selection and implementation of resources and strategies, as well as behaviours that determine the effectiveness with which facts, techniques, and strategies are exploited, such as planning, monitoring, decision-making and conscious metacognitive acts.

Methods: The general strategies used when working on a mathematical problem, such as constructing new ideas and statements, carrying out computations and accessing resources.

Affect: Includes attitudes (enjoyment, motivation, and interest), beliefs (self-confidence, pride, and persistence), emotions (joy, frustration and impatience) and values or ethics (mathematical intimacy and integrity).

Wilson, Fernandez and Hadaway (1994) also argue that to become a good problem solver in Mathematics, one must possess or have to develop a good base of mathematical knowledge, which is in agreement with what Kirkley (2003:7) says about mastering basic mathematical concepts and skills. The ability to organise mathematical knowledge obtained or developed is of paramount importance in contributing to successful problem-solving (Wilson, Fernandez & Hadaway, 1994). Good mathematical problem solvers demonstrate the ability to identify relevant information and ignore irrelevant information (Souviney, 1994). This ability allows capable problem solvers to see a problem as one example of a whole group of similar problems, a critical quality that seems to distinguish good problem solvers from their less skilful peers (Souviney, 1994).

Problem-solving is seen as an important life skill that involves a range of processes including analysing, interpreting, reasoning, predicting, evaluating and reflecting (Anderson, 2009:1).

Developing successful mathematical problem solvers is a complex task (Stacey, 2005). Learners require deep mathematical knowledge (knowledge of concepts, rules,) and general reasoning abilities as well as investigative strategies for solving non-routine problems. Learners should also have beliefs and personal attributes for organising and directing the efforts in solving mathematical problems, as well as good communication skills (Anderson, 2009).

Good and effective mathematical problem-solvers are flexible and fluent thinkers, confident in the use of knowledge, willing to take on a challenge and persevere in their quest to make sense of a situation and solve a problem (Rigelman, 2007:308). These individuals are also curious, seek patterns and connections and are reflective in their thinking. Characteristics discussed by Rigelman in 2007, are not limited to problem solvers in Mathematics, but are characteristics that are desired for all individuals in both their professional and personal lives. Possessing these characteristics is useful to individuals, not only to learn new things, but also to make sense of

existing knowledge. Problem-solving habits prepare individuals for real problems; prepare the individual for situations that require effort and thought, as well as situations lacking immediate solution or straightforward answers (Rigelman, 2007:308).

3.5 The Mathematics teacher's role in teaching problem-solving

Most teachers seem to teach problem-solving as a series of steps in a linear fashion as demonstrated by the model illustrated in Figure 3.1. Most learners need more than the linear steps; they need a full collection of on-going, supported opportunities to inherently and indirectly develop problem-solving techniques (Kelly, 2006:187).

3.5.1 Planning for problem-solving

Planning for the demand and challenges of the teaching of mathematics and specifically mathematical problem-solving, teachers are required to engage in a planning process that contains the development of a skeletal framework rather than complete scripts for lessons (Rosebery, 2005:301). Teaching mathematical problem-solving includes not only the physical action of teaching, but also the time teachers spend planning for classroom interactions (Superfine, 2008:13).

Planning is a very important part of teaching. Planning refers to the time teachers spend preparing and designing activities for learners; from tasks or activities to instructional practices that will be used during the lesson (Superfine, 2008:11). During this process teachers make decisions about a variety of teaching aspects that will ultimately shape learners' opportunities to learn. In the South African educational context, the prescribed Mathematics curriculum plays a pivotal part in the planning of Mathematics lessons, teaching and activities or tasks.

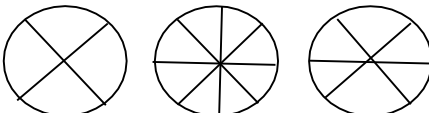
During the planning process, teachers need to plan for questions they will ask in guiding the learners to think about the content or solution to the problem being offered without giving too much information, at the same time encouraging learners to validate their ideas (Moyer & Milewicz, 2002:295). Teachers also need to anticipate potential errors or misunderstandings in order to respond appropriately and assist learners in learning from incorrect solutions. Therefore teachers need to plan for difficulties that learners may experience during problem-solving, and be prepared to change or modify the task in ways that preserve the task's complexity and assist learners to learn from working on the tasks. Planning for problems can be considered to be the anticipation of instructional problems (Superfine, 2008:14).

Using the framework for mathematical lesson planning set out by Van De Walle *et al.* (2013), the following is an example of how planning for mathematical problem-solving can be done:

Mathematics Content	Teacher Activities	Learner Activities
Topic: Comparing fractions 1. Aims: To recognise, classify and represent fractions in order to describe and compare them. 2. Against the background of: Recognises and uses equivalent forms of fractions. Estimates and calculates by selecting and using operations appropriate to solving word problems that involve: equivalent fractions. Uses a range of techniques to perform calculations. Recognises, describes and uses: the commutative, associative and distributive properties (the expectation is that learners should be able to use these properties and not necessarily to know the names of the properties).	The teacher should bear in mind the following: Pre-knowledge such as concepts of common and equivalent fractions, which had been introduced to learners in previous grades, which can be tapped into. Most learners are not taught in their mother tongue. For those learners reading and comprehending what they have read might be a challenge.	

Mathematics Content	Teacher Activities	Learner Activities
3. Select, design or adapt a task or Activity	Teacher designs a task to reach the aim of the lesson, taking into account learners' previous experiences. Uses different resources such as black board, textbooks or any relevant innovation.	The following task is given to the learners: <i>Mary eats $\frac{3}{4}$ of her pizza and Rendani eats $\frac{5}{8}$ of his pizza. Who has eaten the most pizza?</i> Learners can also present it mathematically by using prior knowledge of common fractions, which will include the concept of equivalent fractions: $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8} \qquad \frac{5}{8} \times \frac{1}{1} = \frac{5}{8}$ Therefore Mary ate $\frac{6}{8}$ of her pizza. Therefore Rendani ate $\frac{5}{8}$ of his pizza.

Mathematics Content	Teacher Activities	Learner Activities
4. Design of lesson assessment	Formative assessment: The teacher will play the role as the facilitator and will ask probing questions, like: How do you know which fraction is bigger? What does the denominator tell us about the fraction? What does the numerator tell us about the fraction?	Summative: Learners will solve the problem of equivalent fractions in groups. Identify essential questions or difficulties.
5. Planned activities before the problem are presented to the learners.	A short question and answer session on prior knowledge to establish what the learners' knowledge is regarding comparing fractions, as an activity before the problem is presented to the learners. Teacher shows learners a picture of a pizza with 8 pieces. Asks learners how many pieces they see. Explains to learners that because there are 8 pieces, each piece is an eighth of the whole pizza. Ask learners how many pieces will be left over if 6 pieces are eaten.	Actively participate in question and answer session.

Mathematics Content	Teacher Activities	Learner Activities
	Teacher explains that 6 pieces will be $\frac{6}{8}$ and the 2 pieces left will be $\frac{2}{8}$.	
6. Activities that will be done while learners are solving the problem	Learners will be advised to draw for example a picture of how they see the problem. They must verbalise their ideas or solution. Teacher facilitates.	Learners work on activity (draw a picture or use innovative ideas to solve the problem).
7. Activities after the problem has been solved	Teacher will facilitate what learners are doing and make sure that learners understand the instruction and offer assistance where required.	Learners will do a short activity by shading in the given fractions. Learners will complete the following by looking at the given pizzas.  Write the equivalent fraction e.g.: $\frac{1}{4} = \frac{\quad}{8}, \quad \frac{1}{2} = \frac{\quad}{6} = \frac{\quad}{8}$

Mathematics Content	Teacher Activities	Learner Activities
8. Report back	Give individual learners the opportunity of explaining what reasoning they have used to solve the above-mentioned mathematical problem	Individual learners explain their reasoning on how they have solved the above-mentioned mathematical problem
9. Check for alignment within the lesson	Discussions on fractions by using pizza pieces, will lead to pre-knowledge stimulation, the main activity and discussion of equivalent fractions. Learners are able to identify equivalent fractions as well as how to do mathematical calculations on the presented problem.	Learners participate in the discussion and if necessary complete examples of equivalent fractions on the blackboard.
10. Anticipate learners approaches	Learners' approach might differ according to what / how they understand fractions. The teacher will provide guidance if necessary. Some learners might find it difficult to understand the concept of fractions. The teacher will then have to explain and give more examples.	Learners do similar problem-solving activities under the facilitation of the teacher.

The in depth development of mathematical problem-solving strategies does not indicate full understanding of the mathematical task at hand, nor does it imply that problem-solving is done in isolation. It is rather usually accomplished through engaging problems in which learners can make connection with new and previous information as well as linking the problem with everyday situations the learners might encounter (Kelly, 2006:187). The development of mathematical strategies in itself does not direct learners to a solution, but the application of how the learners understand these strategies indicates understanding or not.

3.5.2 Teachers' actions during problem-solving sessions

Teachers' actions and decisions related to problem-solving (see no 5 to no 7 in the Mathematics lesson plan) often differ and are influenced by the teacher's beliefs about Mathematics and about problem-solving as well as learners' abilities in the Mathematics classroom. Effective Mathematics teachers base their actions and decisions during the planning of problem-solving teaching and activities or tasks on main goals. These goals assist learners in developing (a) flexible understanding of mathematical concepts; (b) confidence and eagerness to approach the unknown situation; (c) metacognitive skills; (d) verbal and written communication skills and (e) acceptance and exploration of multiple solution strategies. Rigelman (2007:313) developed a model (see Table 3.1) that further explains the link or interrelationship between teachers' goals for teaching, teachers' beliefs about problem-solving opportunities and teachers' actions (Rigelman, 2007:312).

Table 3.1: Interrelationships among teachers' goals, beliefs and actions regarding problems and problem-solving (Rigelman, 2007:313)

Teachers' Goals for problem-solving teaching	Teachers' related actions	Teachers' beliefs about opportunities afforded by problem-solving
1. Help learners in developing flexible understanding of mathematical concepts. 2. Foster learners' confidence and eagerness to approach an unknown situation.	1. Set problems and ask questions. 2. Encourage reasoning and proof. 3. Encourage reflection 4. Have learners discuss strategies, share ideas and	1. Learners learn concepts and apply existing understanding. 2. Learners observe, discover, estimate and generalise. 3. Learners see multiple approaches

3. Help learners in developing their metacognitive skills. 4. Help learners develop their verbal and written communication skills. Foster learners' acceptance and exploration of multiple solution strategies.	collaborate towards solutions. 5. Encourage multiple approaches.	
--	---	--



Teachers play a cardinal role during problem-solving. Alsawaie (2003:38) describes the role of the teacher during the following three stages of problem-solving (see 3.5).

3.5.2.1 Understanding the problem (Phase 1)

The learners should understand a problem and determine what is expected from them. This implies reading the problem carefully or listening to someone who is doing the reading. Therefore, teachers should emphasise the importance of reading coupled with understanding. To ensure that learners understand the problem teachers may ask series of questions, for example: what is asked for in the problem?, rewrite the problem in your own words. Teachers should also clarify the task or problem at hand.

3.5.2.2 Making a plan and carrying it out (Phases 2 and 3)

While learners are working on the plan, the teacher should provide hints to those learners who need it. The teacher should listen, observe and question the strategies being used as well as the solutions. Learners that solve their problems first, should be encouraged to solve it in a different way as well. It is also important for teachers to encourage learners to make extensions or generalise their solutions.

3.5.2.3 Reflecting on the solution (Phase 4)

After having solved the problem, the Mathematics teacher should encourage learners to reflect on the process they have used and on their solutions. Learners have to justify their solutions

verbally or in written form. The teacher should ask learners to think about the problem they have solved, and compare the problem with other problems (similar or different) they solved before. Reflection and discussion of the main aspects of the problem should follow.

3.6 Factors influencing mathematical problem-solving

Teachers seem to be more often concerned with the procedures and methods regarding mathematical problem-solving than with the kind of problems which teaching in itself may bring. Learners, for example, work on the mathematical problems on the writing board and are helped by their peers and teacher if help is needed. Thus for many learners it gives the impression that mathematical problems can be solved in just a few minutes – if they knew the correct procedures. That is because mathematical problem-solving steps or strategies are taught in a linear fashion and learners are drilled into memorising them (Kontra, 2001:4). However, this way of teaching mathematical problem-solving does not capture the dynamic enquiry involved in problem-solving. Areas that should be key are the metacognitive components (Kontra, 2001:4) which include areas such as learners' attitudes towards Mathematics and the emotional status of learners while completing mathematical tasks or activities.

3.6.1 Attitudes of teachers and learners towards Mathematics

Attitudes can be seen as personal intentions and behaviours towards certain tendencies, according to Blanco *et al.* (2013:339). What teachers and learners think and believe about Mathematics influences their feelings towards Mathematics as a subject and their tendency to act out the consequence thereof (Blanco *et al.*, 2013:339). If learners have negative attitudes towards mathematics, they will tend to show negative feelings towards the mathematical problem solving tasks (Blanco *et al.*, 2013:339). Blanco *et al.* (2013:339) distinguish between Mathematics attitudes and attitudes towards Mathematics as a subject. Mathematics attitudes relate to a cognitive component and generally relates to cognitive skills that are important to solve mathematical problems. In attitudes towards Mathematics as a subject, the affective component dominates. The affective component refers to the manifestation of interest, satisfaction and curiosity, or on the other hand it can refer to rejection, denial, frustration and avoidance of mathematical problem-solving tasks. Cornoldi (1998:143) emphasises the role of learners' beliefs about thinking and point out that if learners feel confident that they can solve problems they tend to do better. However, Hidalgo *et al.* (2008) in Blanco *et al.* (2013:340) suggest that interest in Mathematics in general declines with age, especially during secondary education.

3.6.2 Emotions of learners

Emotions appear in response to an internal or external event which has a positive or negative influence on a person. Mathematical problem-solving tasks bring about emotions in learners that are characterised by high intensity and physiological arousal, reflecting a positive or negative connotation attached to the task in question (Blanco *et al.*, 2013:340). A learner may thus encounter difficulties which lead to frustration and anxiety with regard to their personal expectations in completing a problem-solving task (Blanco *et al.*, 2013:340). Quite a few authors like DeBellis & Goldin (2006) and Zakaria and Nordim (2008) agree that anxiety or fear interacts negatively with cognitive and motivational processes, therefore interacts negatively with learners' overall performance. Learners with more anxiety towards Mathematics have less confidence in their Mathematical abilities, less confidence as learners of mathematics and therefore less confidence in solving mathematical problems (Gil, Blanco & Guerrero, 2006a).

3.6.3 Metacognition

Metacognition can be defined as 'thinking about thinking', (Bogdan, 2000; Metcalfe, 2000; Flavell, 1999). Metacognition also involves knowing how to reflect and analyse thought, how to draw conclusions from the analysis and how to implement what has been learned. Learners need to understand how they perform important cognitive tasks such as remembering, learning and problem-solving (Downing, *et al.*, 2009:610). Hacker (1998) points out the difference between cognitive tasks (remembering things learned earlier that might help with current tasks or problems) and metacognitive tasks (monitoring and directing the process of problem-solving), emphasising the importance of learning about thinking. Although there is a variety of learning strategies, they all require the successful learner to monitor and direct the process of problem-solving; bringing memory of concepts and processes learned earlier to solve the current or stated problem. Therefore, problem-solving is ideally tailored for the development of metacognition in learners (Downing *et al.*, 2009:611).

In a study done by Tambychik and Meerah (2010:150) on "Learners' difficulties in Mathematics problem-solving", they conclude that learners are faced with many difficulties in mathematical problem-solving due to lack of acquiring mathematical skills and lack of cognitive abilities. They state that transferring knowledge is a critical mathematical skill, without which learners are unable to understand or make effective connections between the information given in mathematical problems. Inadequate language skills, information skills and mastery of the number fact skills inhibit the problem-solving process and lead to errors in mathematical problem-solving. The inability to concentrate during mathematical problem-solving may also lead to further challenges

during problem-solving. Identifying essential mathematical skills is of cardinal importance to respond to the difficulties learners face during mathematical problem-solving. The mentioned abilities could help learners to be motivated in trying to improve their mathematical problem-solving skills (Tambychik & Meerah, 2010:150).

3.6.4 The reading and comprehending ability of learners during problem-solving

Mathematical problem-solving is an important part of a learner's mathematical development (Bernando, 1999:152). In Mathematics, reading demands are high (Lopez, 2008:20). The amount of reading can be overwhelming for learners (Borasi *et al.*, 1998:297). While solving mathematical problems, learners are required to view written text as a set of small units that become meaningful in combination with one another. This means that learners should use their metacognitive skills in mathematical problem-solving (think about thinking). The inability to perform such tasks implies that learners lack not only problem-solving skills but also reading skills as well. Another problem associated with learning to read mathematical word problems is more of an interpretive nature. Learning the relationship between the sentences involves metacognition. Metacognition refers to theory and research of learners' knowledge and use of their own thinking ability or cognitive resources. Meta-comprehension involves the abilities required to understand written and oral texts (Lopez, 2008:24), which can be linked to components of self-directed learning.

The following factors influence the reading and comprehending ability of the learner during mathematical problem-solving: the influence of general structure features of the problem (how a problem is presented or formulated), semantic structure of the problem (meaning of statements in the problems and their relationships) and the problem-solving process (the order and method with which the information is presented can make the problem more or less difficult to comprehend) (Bernando, 1999:154).

a) General structure features of a problem

General structure features refer to how a word problem is formulated or presented (Lopez, 2008:18). The influence of general structure features includes the word length of the problem stated, the number of arithmetic operations, the number of sentences in the stated problem, the average words in each sentence and the frequency of nouns, verbs and conjunctions (Bernando, 1999:154). While solving word problems, learners experience difficulties and are not able to comprehend or solve these mathematical problems, because the problem statements are too long and may require multiple operations (Lopez, 2008:18). Due to the high demand of vocabulary words in a word problem, many learners become afraid of reading the word problem and therefore have difficulty in finding a possible solution to the mathematical word problem. Different structure

features predict problem-solving difficulties and learners may struggle to solve the word problem, or even not attempt to solve the word problem (Lopez, 2008:18).

b) The semantic structure of a problem

The semantic structure of mathematical word problems refers to meanings of the statements in the problems and their relationships (Lopez, 2008:19). Learners have a difficult time solving problems according to Lopez (2008:19), because of the differences in the language (for example the word *product* in Mathematics means the answer or result of a multiplication sum, while in economics the same word has a different meaning). These differences in meaning can influence how the learners interpret and represent word problems. These differences influence the conceptual understanding of what is being asked as well as what strategies to use to solve the problem (Lopez, 2008:19). The semantic structure of mathematical word problems has a substantial influence on the type of strategy which learners use in problem-solving (Lopez, 2008:20).

c) The problem-solving process

The problem-solving process consists of four stages: problem translation and integration (learner's representation of the problem), solution planning and solution implementation which is in agreement with the phases of Polya (1957). The way in which learners interpret problems depends on how well the problem is structured and presented. Thus, the order and method according to which the information is presented can make the problem more or less difficult to comprehend. The problem-solving process requires learners to first understand the problem and implications of the information; next the learner develops an appropriate representation of the problem. Finally the learners connect this representation to the best strategy or method to solve the problem (Lopez, 2008:20).

As with most human activities, effortful practice under appropriate help and guidance builds confidence and breeds success. Employers value mathematicians because of their ability to solve problems (Badger *et al.*, 2012:19).

3.7 Relationship between SDL and problem solving

Self-directed learning is a fundamental part of problem solving; in fact, problem solving cannot happen without SDL (Branda, 1990:548). Problem solving can be seen as one way of organising learning that allows for learner responsibility and control. Problem solving can also be seen as facilitating the development of SDL abilities during each phase of the problem-solving process

(Williams, 2001:95).

In the first phase of problem-solving learners need to understand the given problem, by taking responsibility for their own learning experiences, establishing the educational goals intended, identifying what they do and do not know, restating the problem in a way that it is understood and making a plan to meet those goals. With coaching or facilitation from the teacher, these activities help the learner to develop self-monitoring skills needed to identify learning needs by revealing their internal thinking process (Williams, 2001: 95).

During the second phase of problem-solving learners are making a plan to solve the given mathematical problem. Learners determine or select a strategy (see page 4) on how to approach the given problem, and thus determine what skills, resources and knowledge they have identified will be necessary in order to solve the mathematical problem. By solving many problems it becomes easier to choose a “correct” strategy and in the process assist learners in developing the self-directed learning skills which are critical components of problem-solving and life-long learning (Williams, 2001: 95).

During the third phase of problem-solving learners carry out or execute the plan to solve the mathematical problem. Carrying out the plan requires the learner to be self-motivated to solve the problem, to concentrate by using prior knowledge of the selected strategy and to make full use of the available resources or information. By evaluating the strategy and resources, learners gain insight into which strategies are successful and why, what resources were effective and why, and what could be changed in future with similar problem-solving activities (Williams, 2001: 95).

During the fourth and final phase learners look back and consciously reflect on what they have done and what they have learned and integrated into their existing cognitive structures. The process of situational learning needs motivation, application of knowledge, evaluation of resources, and reflection on what was learned during a problem-solving activity and develops the learners to be self-directed in their learning. Problem solving engages learners to reveal their thinking processes in order to monitor the effectiveness of their ability to reason and acquire knowledge and enables them to assume responsibility for and control of their own learning (Williams, 2001: 95).

3.8 Problem solving: Implications for teachers and learners.

There is a common belief in society that suggests that a teacher that knows Mathematics very well is the best person to teach Mathematics. The question is, however, does that teacher “know how to teach Mathematics?” (Fennema & Franke, 1992).

In classes where the focus is on problem-solving, the role of the teacher as well as the role of the learner changes from the traditional teacher-learner relationship, where the teacher is the holder of all knowledge and the learner is an empty vessel that needs to be filled with the teacher's knowledge. Instead of focusing on helping learners to find answers, the teacher is prepared to see where the observations and questions of the learners lead them. Instead of providing answers or solutions to questions or problems, the teacher encourages multiple solution strategies and allows learners time to communicate and reflect on those strategies. Instead of expecting specific responses, the teacher is asking questions to uncover learners' thinking and insist on learners reasoning behind the thinking processes (Rigelman, 2007:312).

3.9 Conclusion

In this chapter the focus was on problem-solving in the Mathematics classroom, by stating different definitions of problem-solving to get a better understanding of mathematical problem-solving for the purposes of this study. A model of problem-solving (Polya, 1957) was discussed with specific emphasis on the importance of problem-solving during the learning and teaching of Mathematics, as well as the roles of both the teacher and the learners during problem-solving activities.

CHAPTER 4:

THE INFLUENCE OF PROBLEM-SOLVING ACTIVITIES ON GRADE 7 MATHEMATICS LEARNERS' SDL-ABILITIES AND ON THEIR MATHEMATICAL ACHIEVEMENT

4.1 Introduction

The empirical investigation was conducted against the theoretical framework established in Chapters 2 (Self-directed learning in the Mathematics classroom) and 3 (Problem-solving in the Mathematics classroom)

The aim of this chapter is to report the empirical investigation regarding the influence of problem-solving activities on Grade 7 Mathematics learners' self-directed learning abilities and on their achievement in Mathematics.

4.2 Purpose and aims of the research

The purpose of this study was to determine the relationship between problem solving and self-directed learning in Grade 7 Mathematics classrooms.

In an attempt to achieve the purpose, the following aims were set for this study:

4.2.1 To determine the influence of problem-solving activities on Grade 7 Mathematics learners' self-directed learning abilities.

4.2.2 To determine whether self-directed learning, through problem-solving, has an influence on learners' mathematical achievement.

4.3 Empirical investigation

For the purpose of this study a sequential explanatory mixed method research design was used (see 1.4). The quantitative and qualitative data collection and analysis took place in three consecutive phases (Ivankova *et al.*, 2006:4). During the first and third phases, data with respect to Grade 7 learners' self-directed learning ability and the factors influencing their self-directed learning ability were collected and analysed. A qualitative investigation formed the second phase of the empirical study, determining Grade 7 learners' experiences during problem-solving activities (the intervention).

4.3.1 The quantitative investigation

4.3.1.1 Experimental design and procedure

A pre-test, post-test experimental design was used (Leedy & Ormrod, 2005:225) (see Figure 1.1). The experimental group was exposed to intervention (the teaching of problem-solving), while the control group was exposed to traditional teaching methods. The teachers of the control group did not teach Mathematics through problem-solving, but rather used problem-solving as an activity done after concepts had been taught. The experimental design and research procedure of this study are set out in Table 4.1 (see 1.4.1.1).

Table 4.1: The experimental design

	Pre-test	Intervention	Post-test
Experimental group: School A (n=163)	1) Self-directed learning Instrument (SDLI) 2) Selected fields of the LASSI-HS 3) March report results	T_x (A problem-solving approach to teaching and learning)	1) Self-directed Learning Instrument (SDLI) 2) Selected fields of the LASSI-HS 3) November report results
Control group: School B (n=154)	1) Self-directed learning instrument (SDLI) 2) Selected fields of the LASSI-HS 3) March report results	(-) (No intervention)	1) Self-directed learning instrument (SDLI) 2) Selected fields of the LASSI-HS 3) November report results

The experimental and control groups' self-directed learning abilities had been measured prior to the intervention (problem-solving activities). At the end of the intervention, which took two terms to complete, the self-directed learning abilities of both groups were measured applying the adapted SDLI and LASSI-HS (factors influencing self-directed learning abilities). The change in mathematics

achievement could be established by comparing the November report results with the March report results (see Table 4.13). The pre- and post-test control design provided a basis from which a conclusion on cause-and-effect relations could be drawn. A variable that could have an influence on the self-directed learning ability of learners and their mathematical achievement could be that the learners of the experimental group and the control group, respectively, were from different schools (see 1.4.1.1).

4.3.1.2 Study population

The study population comprised Grade 7 learners from two schools in the North West Province of South Africa. These schools were selected based on convenience. The learners of the experimental group were taught Mathematics by the researcher, while the learners of the control group were from a school nearby and taught by a different Mathematics teacher (see 4.2.1.1). School A (the experimental group) was a mixed school where the LOLT was English (3 classes) and Afrikaans (1 Class) respectively. The Grade 7 classes from School B had the same composition as those of school A (see 1.4.1.1).

4.3.1.3 Measuring instruments

An adapted SDLI (Self-directed Learning Instrument) (see Appendix D) was used to measure the self-directed learning ability of learners in the Grade 7 Mathematics classes.

The questionnaires provided information with respect to the self-directed learning ability of Grade 7 learners in Mathematics. Analysis of the questionnaires can:

- Provide reasons why certain learners are more self-directed and others less self-directed in Mathematics.
- Give an overall picture enabling a teacher to evaluate learners' self-directed abilities and give guidance on how to enhance this ability.

The SDLI consists of twenty items/questions distributed over four domains related to learners' self-directed ability, namely: learning motivation, planning and implementing, self-monitoring and interpersonal communication (see 2.4). With respect to the domains of the SDLI, learners had to evaluate themselves regarding their perspectives on their own self-directed learning ability. All items are positively stated on a five-point Likert scale, anchored at extreme poles ranging from 1 to 5 as

follows:

1 = never, 2 = seldom, 3 = sometimes, 4 = often and 5 = always.

A few examples of the items in each domain are displayed in Table 4.2.

Table 4.2: Examples of items from the SDLI-questionnaires

NR	Domains	ITEMS
1	Learning motivation	I identify my own learning needs
3	Learning motivation	My inner drive directs me towards further development and improvement in my learning.
7	Planning and implementation	I am able to set and plan my own learning goals.
10	Planning and implementation	I am able to select the best method for my own learning.
15	Self-monitoring	I am able to monitor my learning progress.
19	Interpersonal communication	I am successful in communicating verbally.

Only selected fields of the adapted LASSI-HS were used, because those fields seemed to be factors influencing self-directed learning ability (see Section 2.5). The adapted LASSI-HS questionnaire consists of 39 items/questions distributed over five fields relating to learners' self-directed ability, namely:

- motivation – addresses learners' diligence, self-discipline and willingness to work hard;
- concentration – focuses on learners' ability to pay close attention;
- self-testing strategies – focus on reviewing and preparing for classes, tests and assignments;
- use of study aids – examines the degree to which learners create or use support techniques or material to help them learn and remember new information, and
- time management – examines learners' use of time management principles for academic work (Weinstein & Palmer, 1990:4-5) (see 2.4).

With respect to the fields of the LASSI-HS (factors influencing self-directed learning ability) (see 2.5), learners had to evaluate themselves regarding their perspectives on their own learning and study habits. Items are positively and negatively stated on a five point Likert scale. Items that were positively stated were scored from 1 to 5 and items that were negatively stated were scored vice-versa, in other words, from 5 to 1 (Weinstein & Palmer, 1990). They had to choose from the following responses:

1 = not at all like me, 2 = not very much like me, 3 = somewhat like me, 4 = fairly much like me and 5 = very much like me.

Table 4.3: Examples of items from the adapted LASSI-HS questionnaire

NR	FIELDS	ITEMS
1.	Time management (TMT)	I find it difficult to stick to a study timetable for Mathematics
3.	Concentration (CON)	I think of other things during the Mathematics lesson and don't really listen to what is being said in class.
6	Motivation (MOT)	Problems outside of school such as financial problems, conflict with parents, dating (being in love), etc. cause me to not do my Mathematics.
11	Use of study aids (STA)	I try to think of possible test questions when studying work done in the Mathematics class.
19	Self-testing strategies (SFT)	I check to see if I understand what my teacher is saying during a Mathematics lesson
27	Use of study aids (STA)	I make drawings or sketches to help me understand the Mathematics I am studying.

The measurement instruments for determining the learners' mathematical achievement included continuous assessment and examination question papers (see 1.4.1.2).

4.3.1.4 Validity and reliability of the instruments and data

The validity of a measuring instrument entails the extent to which the instrument measures what it

intends to measure (see 1.4.1.2) (Joppe, 2000:1). Content validity is based on the logical analysis of the content and the aims of the measuring instruments. The following steps were followed to ensure the content validity of the SDLI:

- A comprehensive literature study was performed on the domains.
- The questions of the different domains were adapted for Grade 7 learners.

Cheng *et al.* (2010:1156) indicated the inter-correlation of the different domains of the SDLI questionnaire. For the purpose of this study, the construct validity test indicated that the SDLI questionnaire is appropriate for determining Grade 7 Mathematics learners' self-directed learning abilities.

The reliability coefficients of each of the self-directed learning ability domains (learning motivation, planning and implementation, self-monitoring and interpersonal communication skills) of the SDLI were determined for the specific Grade 7 learners (see Table 4.4).

It is essential to calculate and report on the reliability coefficients (*Cronbach's Alpha*) with respect to the domains of a measuring instrument such as the SDLI (see Table 4.4). *Cronbach's Alpha* (α) is the average value of the reliability coefficients that could be obtained for all possible combinations of splitting up the items into two groups. It was done by using the formula below (Gliem & Gliem, 2003:84,87; Avry *et al.*, 2006:264):

$$\alpha = \left(\frac{K}{K-1} \right) \left(\frac{s_x^2 - \sum s_i^2}{s_x^2} \right)$$

where K = number of items in the questionnaire;

$\sum s_i^2$ = sum of the variances of the item counts;

s_x^2 = variance of the total count of the scale (all K items), $0 < \alpha < 1$.

The reliability coefficients for the domains of the SDLI are compared with the results of the original study, to determine the reliability of the data of the current study (see table 4.4).

The reason for the lower *Cronbach Alpha* (α) (see Table 4.4) could be that the original study was executed on tertiary level (nursing students) where students are expected to be more self-directed in their own learning.

Table 4.4: Reliability-coefficients (Cronbach's α) for the four domains of the SDLI.

SDLI – DOMAINS		CRONBACH ALPHA (α)	
		Original Study (Cheng, et al., 2010:1152)	This study (Grade 7 mathematics learners)
1)	LM – Learning motivation	0.80	0.52
2)	PI – Planning and implementation	0.86	0.62
3)	SM – Self-monitoring	0.79	0.60
4)	IC – Interpersonal communication	0.77	0.50

The content and construct validity of the selected fields of the adapted LASSI -HS questionnaire (see Appendices B and C) were determined. The following steps were taken to establish the content validity of the LASSI- HS questionnaire:

- A comprehensive literature study was performed on the respective fields;
- The item correlation was checked by the Statistical Consultation Services of the North- West University (Potchefstroom Campus).

The reliability of the LASSI-HS fields (motivation, concentration, self-testing strategies, use of study aids and time management) was checked for the specific Grade 7 learners by determining reliability coefficients for each of the fields.

With respect to the fields of a measuring instrument such as the LASSI-HS, it is essential to calculate and report on the reliability coefficients (*Cronbach's Alpha*) of each of the fields in order to check for consistency of the measuring instrument and data. Due to the low Cronbach Alpha value, time management (TMT) (see Table 4.5) was omitted from further analysis and discussion of data.

Table 4.5: Reliability coefficients (Cronbach's Alpha) for the five fields of the LASSI-HS

SELECTED LASSI-HS FIELDS	GROUPING OF ITEMS	RELIABILITY COEFFICIENTS (α)	
		Original Study (Weinstein & Palmer, 1990)	This study (based on standardised items)
1) MOT – Motivation	5, 7, 8, 15, 17, 20, 30	0.78	0.54
2) CON – Concentration	3, 6, 18, 22, 24, 29, 32, 36	0.82	0.75
3) SFT – Self-testing Strategies	2, 9, 11, 14, 16, 19, 34, 37	0.81	0.71
4) TMT – Time management	1, 12, 21, 25, 31, 35, 39	0.77	0.30
5) STA – Use of study aids	4, 10, 13, 23, 27, 28, 33, 38	0.68	0.64

The item grouping of the selected fields of the LASSI-HS was done for high school learners in the original study. During this study all selected fields of LASSI-HS scored much lower than in the original study. The reliability coefficients were determined by the Statistical Consultation Services of the North-West University (Potchefstroom campus). There could be many reasons for the lower values scored for this study (see 1.4.1.1).

4.3.2 Discussion of the results of the quantitative investigation

The influence of mathematical problem-solving activities on learners' self-directed abilities and on their Mathematics achievement is discussed separately.

4.3.2.1 The influence of problem solving on grade 7 learners' self-directed learning ability

4.3.2.1.1 Learners' SDL-ability before the intervention of problem-solving

Learners from the experimental and control groups were compared with respect to their self-directed ability before the use of a problem-solving approach to teaching and learning (see Table 4.6). Because the investigation was based on convenience sampling, the two groups were viewed

as sub-populations, and thus the *t*-test for independent sampling was not performed. Instead, effect sizes were calculated by using the following formula:

$$d = \frac{\bar{x}_1 - \bar{x}_2}{s_{\max}} \text{ with } d = \text{an estimation of Cohen's effect size for unequal standard deviations;}$$

$\bar{x}_1$ = average of the experimental group;

$\bar{x}_2$ = average of the control group; and

$s_{\max}$ = experimental and control groups (Thalheimer & Cook, 2002:4).

In the case where the means of the post-test are compared with those of the pre-test within a group, the denominator of the formula above is taken to be the standard deviation of the pre-test. The *d* values are interpreted as follows: If $|d| = 0.2$ (small effect), $|d| = 0.5$ (medium effect - possible meaningful difference), or $|d| = 0.8$ (large effect - practical meaningful difference) (Steyn, 2005:20-24).

Table 4.6: Differences between the domains of SDL-ability of the experimental and control groups before the intervention

Domains of SDL ability	EXPERIMENTAL GROUP (n = 163)		CONTROL GROUP (n = 154)		Effect size (<i>d</i>)
	Average	Standard deviation	Average	Standard deviation	
LM (learning motivation)	3.83	0.55	3.80	0.60	0.05
PI (planning and implementation)	3.97	0.58	3.95	0.60	0.03
SM (self-monitoring)	3.69	0.73	3.70	0.71	0.02
IC (interpersonal communication)	3.73	0.70	3.73	0.68	0.001
Average SDL ability	3.81	0.64	3.80	0.65	0.02

The average effect size of the SDL- domains is 0.02 which indicates a small effect. Thus, there were no meaningful differences between the SDL-domains of the experimental and control groups before the intervention of a problem solving approach to the teaching and learning of mathematics (see Figure 4.1).

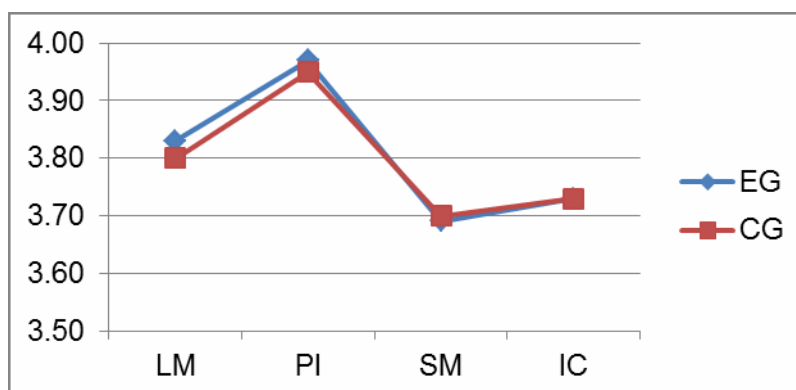


Figure 4.1: Differences between the domains of SDL-domains of the experimental and control groups before the intervention

4.3.2.1.2 Learners' self-directed ability after the intervention of problem solving

Learners of the experimental (EG) and control groups (CG) were compared with respect to their SDL-ability domains after the intervention. The effect sizes of the domains are given in Table 4.7.

Table 4.7: Differences between the SDL-ability domains of the experimental and control groups after the intervention

Domains of self-directed learning ability	Experimental group (n = 163)		Control group (n = 154)		Effect size (d)
	Average	Standard deviation	Average	Standard deviation	
LM (learning motivation)	3.57	0.62	4.0	0.63	0.68
PI (planning and implementation)	3.77	0.58	4.0	0.59	0.40
SM (self-monitoring)	3.51	0.70	3.8	0.66	0.41

IC (interpersonal communication)	3.52	0.70	3.9	0.66	0.54
Average SDL-ability	3.59	0.65	3.92	0.64	0.51

The average effect size of the SDL-ability domains is 0.51, which indicates a medium effect. There could be many reasons for the differences between the SDL-domains of the experimental and control groups after the intervention of problem solving. Some of the reasons are explained in the discussion of the results of tables 4.8 and 4.9 later in this section. The differences between the SDL-abilities of the experimental and control groups are also reflected in the graph in Figure 4.2.

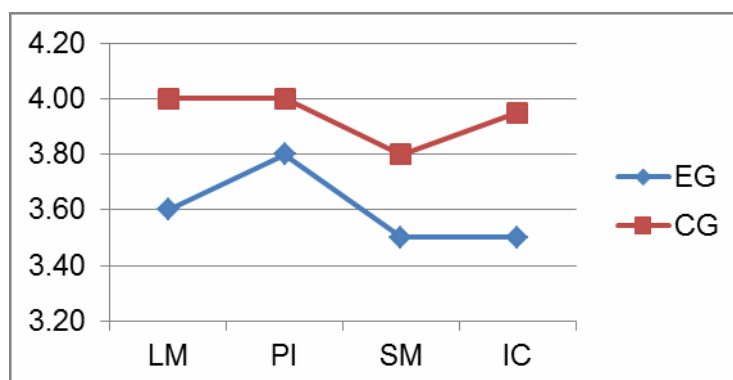


Figure 4.2: Differences between the SDL-domains of the experimental and control groups after intervention

The differences between the self-directed learning abilities of the experimental group when the pre- and post-test results are compared appear in Table 4.8. The average effect size is 0.32 which shows a relatively small difference or effect between the pre- and post-tests. There is a decline in the self-directed learning ability of the experimental group. The average SDL-ability of the experimental group shows a decline from 3.81 to 3.59, evident in all domains of their self-directed learning ability. This decline may be attributed to the fact that learners over-rated their self-directed ability during the pre-test, or to the intervention of problem-solving teaching. Because learners were not used to the teaching and learning of mathematics through problem solving, it could have had a negative impact on their self-directed learning abilities.

Table 4.8: Comparison of the self-directed learning abilities of the experimental group

Domains of self-directed learning ability	EXPERIMENTAL GROUP (n = 163) Pre-test		EXPERIMENTAL GROUP (n = 163) Post-test		Effect size (d)
	Average	Standard deviation	Average	Standard deviation	
LM (learning motivation)	3.83	0.55	3.57	0.62	0.41
PI (planning implementation) and	3.97	0.58	3.77	0.58	0.34
SM (self-monitoring)	3.69	0.73	3.51	0.70	0.24
IC (interpersonal communication)	3.73	0.70	3.52	0.70	0.30
Average SDL-ability	3.81	0.64	3.59	0.65	0.32

Figure 4.3 shows a graph comparing the SDL-domains of the experimental group before and after the intervention of a problem-solving approach to teaching and learning.

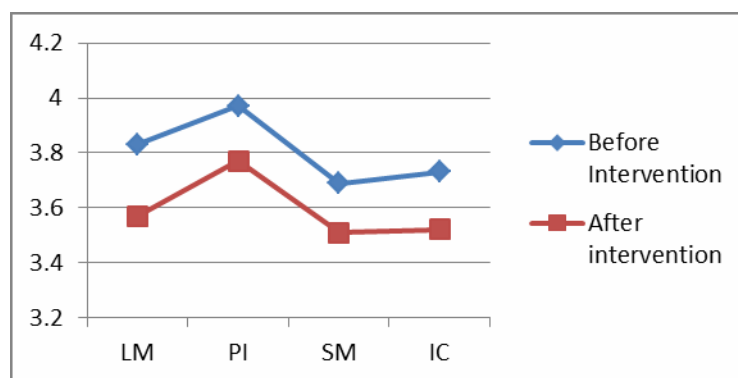


Figure 4.3: Comparison of the SDL-domains of the experimental group

When comparing the pre- and post-tests with respect to the SDL abilities of the control group

(see Table 4.9), the average SDL ability is 0.19, which indicates a negligible effect. The control group, however, shows a small increase in their self-directed learning abilities. This increase can be attributed to the fact that no intervention occurred and that learners were used to the teaching methods of their teachers (see 1.4.1.2). Figure 4.4 graphically reflects the comparison of the SDL-domains for the pre- and post-test results of the control groups.

Table 4.9: Comparison of the self-directed learning abilities of the control group

Domains of self-directed learning ability	CONTROL GROUP (n = 154) Pre-test		CONTROL GROUP (n = 154) Post-test		Effect size (d)
	Average	Standard deviation	Average	Standard deviation	
LM (learning motivation)	3.80	0.60	4.0	0.63	0.32
PI (planning implementation) and	3.95	0.60	4.0	0.59	0.08
SM (self-monitoring)	3.70	0.71	3.8	0.66	0.14
IC (interpersonal communication)	3.73	0.68	3.9	0.66	0.25
Average SDL-ability	3.80	0.65	3.92	0.64	0.19

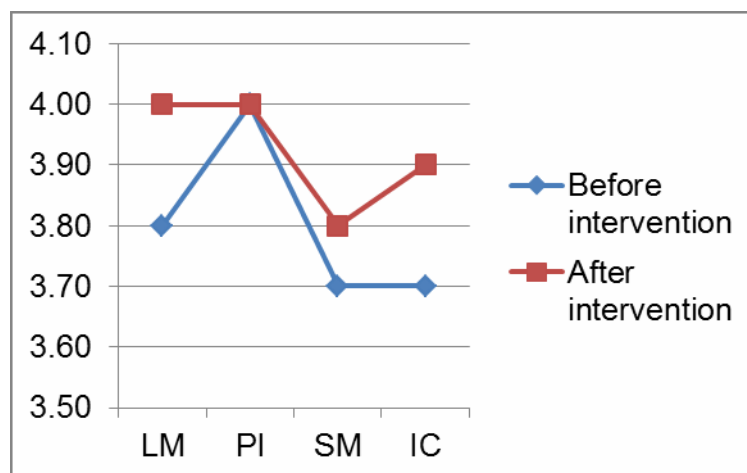


Figure 4.4: Comparison of the SDL-domains of the control group

4.3.2.2 The relationship between factors influencing self-directed learning and problem-solving activities

4.3.2.2.1 Factors influencing self-directed learning (prior to the intervention of problem-solving activities)

After completing the LASSI-HS questionnaire (selected fields only), the experimental and control groups were compared with respect to the factors influencing self-directed learning (see 2.5). With respect to the factors influencing SDL, there are no meaningful differences between the experimental and the control groups before the intervention of problem solving. From the results in table 4.10 it is evident that the averages of the selected fields of the LASSI-HS (factors influencing SDL) for the control group are higher than those for the experimental group. Figure 4.5 is a graphical representation of the comparison between the selected LASSI-HS of the experimental and control groups prior to intervention of problem-solving activities.

Table 4.10: Comparison between the selected LASSI-HS (factors influencing self-directed learning) of the experimental and the control groups prior to intervention

Factors influencing SDL	EXPERIMENTAL GROUP (n = 163) Pre-test		CONTROL GROUP (n = 154) Pre-test		Effect size (d)
	Average	Standard deviation	Average	Standard deviation	
MOT (Motivation)	3.83	0.70	3.88	0.60	0.07
CON (Concentration)	3.14	0.56	3.31	0.60	0.28
STA (Use of study aids)	3.53	0.72	3.52	0.86	0.01
SFT (Self-testing strategies)	3.34	0.70	3.48	0.60	0.20

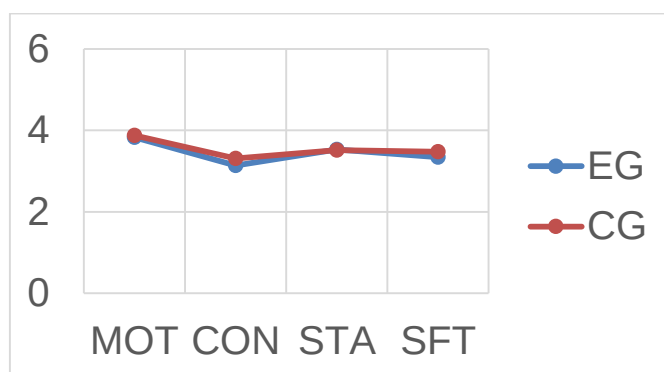


Figure 4.5: Comparison between the experimental and the control groups with respect to the factors influencing self-directed learning (before the intervention)

4.3.2.2.2 Factors influencing self-directed learning (after the intervention of problem-solving activities)

After the intervention of problem-solving activities, the experimental and control groups were compared with respect to the factors influencing self-directed learning. An ANCOVA-analysis was conducted after the post-test to statistically rectify the differences in the pre-test results of the respective fields of the LASSI-HS (Leedy & Ormrod, 2005:274). From the small values of the effect sizes (see table 4.11) it is clear that there were no meaningful differences between the experimental and control groups regarding the following four factors influencing self-directed learning, namely motivation, concentration, the use of study aids, and the use of self -test strategies (see 2.6.4)

The graph in Figure 4.6 also illustrates the almost insignificant differences between the experimental and control groups for three of the four factors, as well as a small difference for the fourth factor, namely motivation.

Table 4.11: Comparison between selected LASSI-HS fields (factors influencing self- directed learning) of the experimental and the control groups after the intervention

Factors influencing self-directed learning	EXPERIMENTAL GROUP (n = 163) Post-test		CONTROL GROUP (n = 154) Post-test		
	Average	Standard deviation	Average	Standard deviation	Effect size (d)
MOT (Motivation)	3.78	0.72	3.95	0.61	0.23
CON (Concentration)	3.43	0.61	3.48	0.64	0.08
STA (Use of study aids)	3.38	0.68	3.28	0.70	0.14
SFT (Self-testing strategies)	3.37	0.59	3.40	0.52	0.05

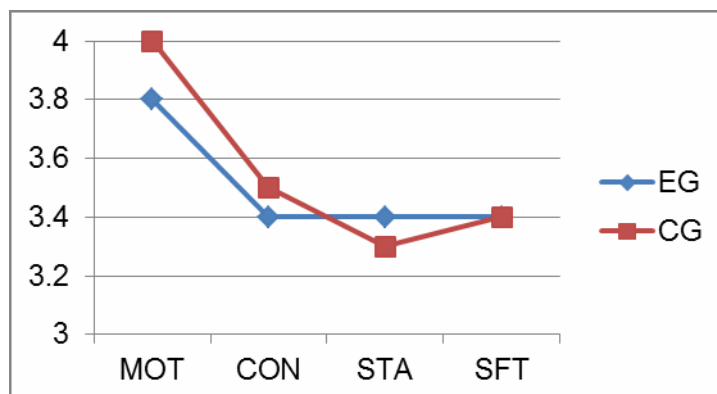


Figure 4.6: Comparison between the experimental and the control groups with respect to the factors influencing self-directed learning (after the intervention)

4.3.2.2.3 Factors influencing self-directed learning (comparing the pre- and post-test of the experimental group)

The experimental group's results for the pre- and post-tests with respect to the factors influencing self-directed learning are indicated in Table 4.12. The following is evident from the table: Concentration is the only factor that shows an increase from the pre-test to the post-test with an effect size of 0.45, which indicates a possibly meaningful difference. This may be attributed to the fact that learners from the experimental group had to concentrate when they were confronted with solving mathematical problems, comparing to "normal" situations where

they had to execute mathematical operations and calculations. Figure 4.7 shows a graph of the comparison between the domains of SDLA of the experimental group (pre- and post-test).

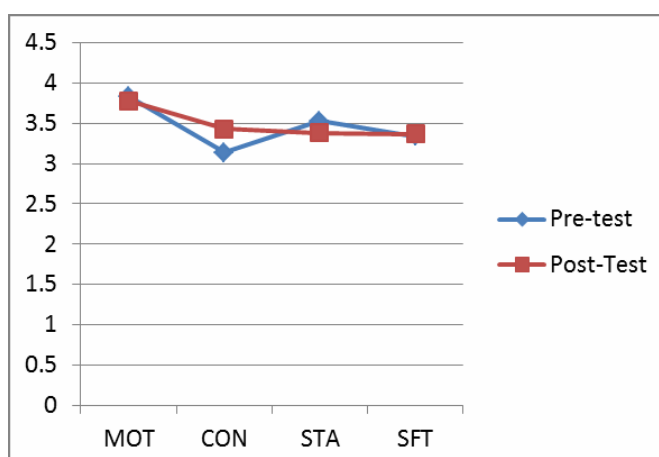


Figure 4.7: Comparison between the domains of SDLA of the experimental group (pre- and post-tests)

Table 4.12: Comparison of the pre- and post-tests of the experimental group with respect to the factors influencing self-directed learning

Factors influencing self-directed learning	Experimental group (n = 163) Pre-test		Experimental group (n = 163) Post-test		Effect size (d)
	Average	Standard deviation	Average	Standard deviation	
MOT (Motivation)	3.83	0.70	3.78	0.72	0.07
CON (Concentration)	3.14	0.56	3.43	0.61	0.45
STA (Use of study aids)	3.53	0.72	3.38	0.68	0.20
SFT (Self-testing strategies)	3.34	0.70	3.37	0.59	0.04

4.3.2.3 The influence of self-directed learning, through problem-solving, on the Grade 7 learners' mathematical achievement

The average mark for Mathematics of the experimental and the control groups, respectively, is compared in Table 4.13.

Table 4.13: Mathematical achievement of the experimental and control groups.

Mathematical achievement	Average percentage	
	EG (n = 163)	CG (n = 154)
March report results	42	52
November exam results	51	38

To study the change in mathematical achievement for each group, respectively, the effect size for average increase is obtained by using the following formula:

$$d = \frac{\bar{x}_{diff}}{s_{diff}}; \text{ with } \bar{x}_{diff} = \text{the average increase in mathematical achievement; and}$$

s_{diff} = standard deviation of the changes.

The average difference is determined between the March and the November report results (Ary *et al.*, 2006:194-195). The effect sizes (Cohen's d) are calculated to determine if there are statistical differences between the experimental and control groups' mathematical achievement.

The experimental and control groups' change in mathematical achievement is compared in Table 4.14. The results show a significant difference ($d = 2.0$) in the average achievement of the experimental group. The control group, however, also shows a significant difference ($d = -2$) in its mathematical achievement when the November examination results are compared with the March report results. Both groups wrote the same tests and November examination question paper (externally set and moderated) (see 1.5). Although it seemed that the experimental group did not improve with respect to their self-directed abilities, their average mathematical achievement increased significantly.

Table 4.14: Changes in the mathematical achievement of the experimental- and control groups

Differences between report results	EXPERIMENTAL GROUP (n = 163)			CONTROL GROUP (n = 154)		
	Average increase	Standard deviation	Effect size for average increase (d)	Average increase	Standard deviation	Effect size for average increase (d)
November – March	9	4.5	2	-14	7	-2

4.3.2.4 Summary of the quantitative results

The first research aim was to determine the influence of problem-solving activities on Grade 7 Mathematics learners' self-directed learning abilities. This aim was attained empirically by using both the SDLI and selected fields of the LASSI-HS questionnaires (see 4.3.2.1.2). From the interpretation and discussion of the results of the mentioned questionnaires, it became apparent that there were no practically significant differences between the experimental and control groups for the pre-tests. The results for the post-tests, however, revealed a practical difference of medium effect ($d=0.51$) between the average self-directed learning abilities of the control and empirical groups (see Table 4.7).

The second research aim was to determine the influence of self-directed learning, through problem solving, on the mathematical achievement of Grade 7 Mathematics learners. The mathematical achievement of the experimental learners showed a significant improvement after the intervention, whilst the control group learners' mathematical achievement decreased significantly over the same period (March to November) (see Table 4.14).

4.3.3 The qualitative investigation

4.3.3.1 Aim and motivation

The aim of the qualitative investigation was to provide information regarding the Grade 7 learners' problem-solving experiences in Mathematics (Evens & Houssart, 2007:20) (see 3.3), as well as a possible explanation for the quantitative relationship between problem-solving (see 3.7) and self-directed learning abilities (see 4.3.2.1).

4.3.3.2 Participants

The participants of the task-based activities were selected at random from the experimental group according to their self-directed learning ability levels. Participants were selected from all four participating class groups, representing all three levels of self-directed learning ability (see Table 4.15). Although a total of 60 learners (15 per class group) were originally selected to participate, only 32 learners participated in the investigation.

Table 4.15: Selection of participants for the task-based activities

Self-directed learning ability levels	Number of learners
High level	11
Medium level	11
Low level	10

4.3.3.3 Data collection and analysis

The task-based activities provided the researcher with the opportunity to focus on how learners solve mathematical problems (Evens & Houssart, 2007:20). The following is an example of a problem-solving activity.

1. Study the following problem:

The soccer field is 50 m long and 30 m wide. The ground-man needs to draw a line to indicate the perimeter of the soccer field. How long will this line be?

2. Do you think you can solve the problem correctly? Circle your choice.

a.	I am confident I can solve the problem correctly.
b.	I am reasonably sure I can solve the problem correctly.
c.	I am not sure how correct I can solve the problem.
d.	I am not sure how correct I can solve the problem. I think I might make a
e.	I know that I will make a mistake in solving the problem.

3. Explain in words how you would solve the problem.

4. Solve the problem now. Show all your steps.

5. Do you think you have solved the problem correctly? **Circle your choice.**

a.	I am confident I have solved the problem correctly.
b.	I am reasonably sure I have solved the problem correctly.
c.	I am not sure I have solved the problem correctly.
d.	I am not sure how correctly I have solved the problem. I think that I might have made a mistake.
e.	I know I have made a mistake.

6. Do you think about what you have done? Describe the strategies or plans you used when solving the problem.

7. Which type of mistakes do you make when solving similar problems?

The mathematical problems in the task-based activities were compiled from random Mathematics Grade 7 textbooks. The problem-solving activities included structured as well as open-ended questions (see Appendix C), to afford learners the opportunity of explaining how they would solve a particular mathematical problem. The participants read number 1 (the problem) until he/she

understands the problem, then they answer number 2 and 3 before solving the problem (number 4). The participant's solution to the problem is then assessed by the teacher by using a rubric. The analysis of the problem-solving activities was conducted by the researcher by using a rubric. The analysis is based on Creswell's (2009:151-191) view, namely:

- Collect data from interviews/activities;
- Read through the data and analyse by using a generalised overview of the information;
- Use a coding process (breaking down information into smaller parts in order to give more structure to the data) for detailed analysis of the information.

4.3.3.4 Trustworthiness

The trustworthiness of the qualitative investigation was ensured by using the criteria set by Lincoln and Guba (1985:290, 305). The criteria are a compilation of four components, namely: credibility, portability, reliability and confirmability (see Figure 4.8). The criteria provide an evaluation of the trustworthiness of the qualitative data and are similar to the conventional criteria of internal and external validity, reliability and objectivity (Fossey *et al.*, 2002:723; Nieuwenhuis, 2007:80; Leedy & Ormrod *et al.*, 2005:262).

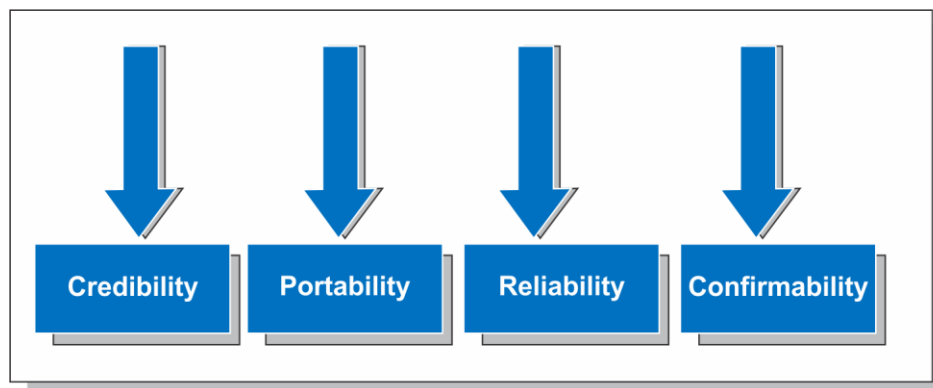


Figure 4.8: An outline of qualitative trustworthiness (Adapted from Lincoln *et al.* 1985:290; Markula *et al.*, 2011:205)

The first criterion deals with the credibility of the researcher's confidence in the accuracy of the empirical investigation and the credibility of the data collected during the empirical investigation (Lincoln & Guba, 1985:305). In this study the information of the qualitative phase was verified and combined with the information collected during the quantitative phase.

The second criterion is portability which refers to the applicability or relevance of the empirical findings in different situations or different contexts. Unique findings come to the fore in qualitative research therefore the findings of this investigation cannot be generalised.

The third criterion is reliability which refers to the consistency of the findings when the empirical investigation is to be repeated in similar circumstances or different situations (Lincoln & Guba, 1985:293). Other researchers need to take the procedure followed in this investigation, into consideration when a planning a similar investigation.

The last criterion is confirmability which refers to objectivity of the empirical investigation where the findings are not the perspective of the researcher, but the perspectives of the participants (Lincoln & Guba, 1985:290-317; Krefting, 1991:214-221). For the purpose of this investigation, the researcher ensured that the findings of the qualitative investigation reflected the participants' problem-solving experiences.

4.3.3.5 Qualitative results

4.3.3.5.1 Analysis of the task-based (problem-solving) activities

The participants completed five (5) tasks based on the content areas of the Senior Phase Mathematics Curriculum. Cresswell's method was used during the analysis of the task-based activities (see 4.3.3.3). In addition, the task-based activities were also analysed by using the ability of the learners to predict their achievement in each question against their actual achievement. The following categories were used during the analysis:

Table 4.16: Summary of the categories of the task based activities

Categories	Prediction	Problem Solving (Out of 5)
Totally (Good)	a & b	4 & 5
Partially (Average)	c	3
Not at all (Poor)	d & e	1 & 2

For each problem, the learners made a prediction regarding their possible achievement (see 4.3.3.3, question 2). When learners predicted that they were confident or reasonably confident that they could solve the given problem, the prediction was listed in the category "totally"; when the learners predicted that they were not sure, the prediction fitted into the category "partially";

and lastly, when the learners predicted that they would make a mistake in solving the problem, the prediction fitted into the category “not at all”. The learners’ actual problem-solving efforts were listed in the last column of the table (see Table 4.17 for an example).

4.3.3.5.2 Discussion of the results of the task-based activities

Learners’ responses to the questions or statements in the task-based activities are discussed as follows:

Question 1 (Numbers, Operations and Relationships)

1. Study the following problem:

Siphiwe walks $\frac{4}{5}$ km to school. Rachel walks $\frac{5}{6}$ km to school. Who walks the longest distance?

2. Do you think you can solve the problem correctly? Circle your choice.

a.	I am confident I can solve the problem correctly.
b.	I am reasonably sure I can solve the problem correctly
c.	I am not sure how correctly I can solve the problem
d.	I am not sure how correctly I can solve the problem. I think I might make a mistake.
e.	I know that I will make a mistake in solving the problem

3. Explain in words how you would solve the problem.

4. Solve the problem. Show all your steps.

5. Do you think you have solved the problem correctly? Circle your choice.

a.	I am confident I have solved the problem correctly.
b.	I am reasonably sure I have solved the problem correctly.
c.	I am not sure I have solved the problem correctly.
d.	I am not sure how correctly I have solved the problem. I might have made a mistake.
e.	I know I have made a mistake

6. Do you think about what you have done? Describe the strategies or plans you used when solving the problem.

7. Which type of mistakes do you make when solving similar problems?

The participants read number 1 until they understood the problem, then answered numbers 2 and 3 before solving the problem in number 4. Learners' predictions pertaining to number two in the questionnaire are in Table 4.17:

Table 4.17: Participants' predictions to Question 1 in the task-based activities

Groups	Prediction of learners		
	Totally	Partially	Not at all
Learners with high level SDL-ability	3	4	4
Learners with medium level SDL-ability	2	4	5
Learners with low level SDL-ability	4	1	5

Most of the learners in the high- and medium-level self-directed learning ability groups selected either "partially" or "not at all" to number 2, meaning that they were either sure they would make a mistake in solving the problem or they were not sure they could solve the problem successfully or correctly. Quite a few (4) learners of the low-level self-directed learning ability, however, predicted that they were either sure or reasonably sure that they would solve the problem correctly, whereas five (5) of these learners were sure they would make a mistake or not be able to solve the problem at all. Some of the learners' responses to number 3 (explain in words how you would work to solve the problem) are given in Table 4.18:

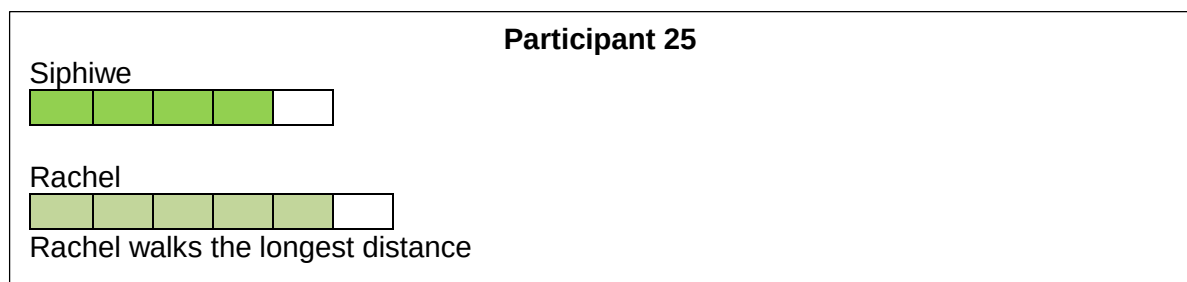
Table 4.18 Responses of participants to number 3 for Question 1

Participant no	Responses	SDL-ability level
14	I am going to use two methods to solve the problem. I am going to draw blocks so I can understand what I am doing and do it again mathematical.	High
19	 am going to make the denominator to have the same number by multiplying it.	High
21	I would work by multiplying.	Low
22	Simphiwe walks only $\frac{4}{5} km$ to school, but Rachel walks $\frac{5}{6} km$ to school	Medium

23	I did multiply the two numbers with the number that can go into the two numbers without a remainder except 1	Medium
24	I would work by multiplying. First I have to make the denominator the same so I can multiply correctly	Low
25	I would draw squares to show which one walks the longest distance	High
27	I would multiply $\frac{4}{5} km$ and $\frac{5}{6} km$	Medium
30	By working on my words	Low
32	I will try my best to solve the problem	Medium

Although most of the learners interpreted the question correctly by identifying the given problem, it is apparent from their responses that most of them, irrespective of their SDL-ability level, had no idea how to explain the possible solution to the problem in words. Participants 24 and 25 interpreted the problem correctly and gave meaningful explanations on how the problem would be solved.

After answering numbers 2 and 3 of the task-based activity, the selected learners were required to solve the problem (number 4) given in number 1. Learners were allowed to use any method to solve the problem. Participant 25 used a practical approach by drawing 2 horizontal blocks and divided them into 5 and 6 equal parts respectively, but made a mistake in not making the blocks' length the same.



Most of the learners were not able to solve the problem correctly. In the following graph (Figure

4.9) the comparison of learners with high, medium and low levels of self-directed learning abilities' prediction with their actual achievement is illustrated.

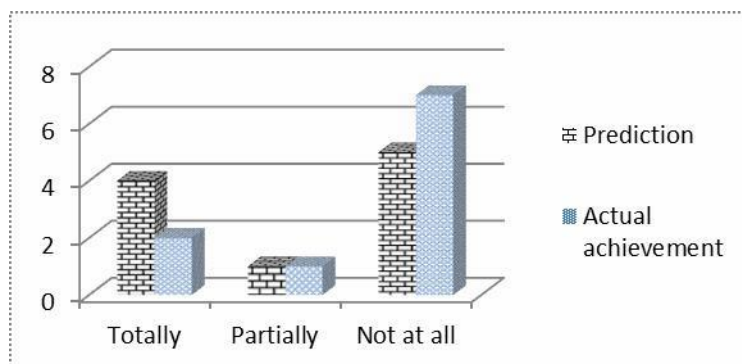


Figure 4.9: Comparison of low level SDL ability learners' prediction and actual achievement in Question 1

Figure 4.9 illustrates that participants with low-level SDL-ability's predictions did not correspond with the actual problem-solving results. The number of participants who predicted that they were confident or reasonably confident that they could solve the problem correctly ("totally") was twice the number that actually solved the problem correctly. The number of learners who predicted "partially" was the same as the number of the learners who succeeded in solving the problem. The number of participants that predicted "not at all" was less than the actual result. Most participants with low SDL-ability seemed to be over-confident in their prediction of their ability to solve the given problem.

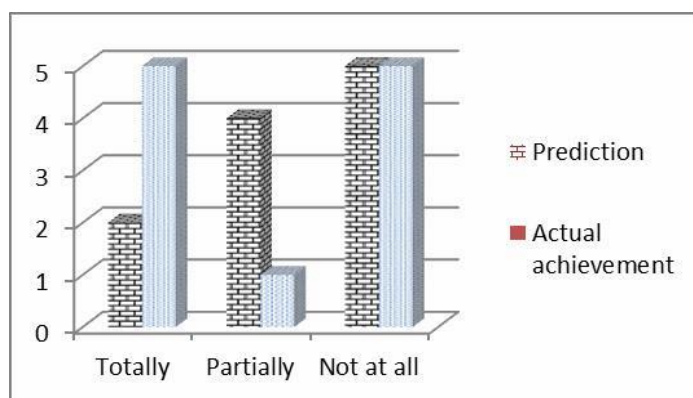


Figure 4.10: Comparison of medium level SDL ability learners' prediction and actual achievement in Question 1

Figure 4.10 illustrates that those participants with medium-level SDL-ability had a clear

understanding of their problem-solving abilities. The column of “totally” actually illustrates that these learners under-estimated their ability, while the column of “not at all” illustrates that prediction and actual achievement were the same. The number of learners who predicted “partially” seemed to select a so-called safe prediction, because only about one third actually predicted correctly.

Figure 4.11 illustrates that participants with high-level SDL-ability in the “totally” column, underestimated themselves and did better than predicted. The “not at all” column illustrates that these learners had a clearer understanding of their problem-solving abilities

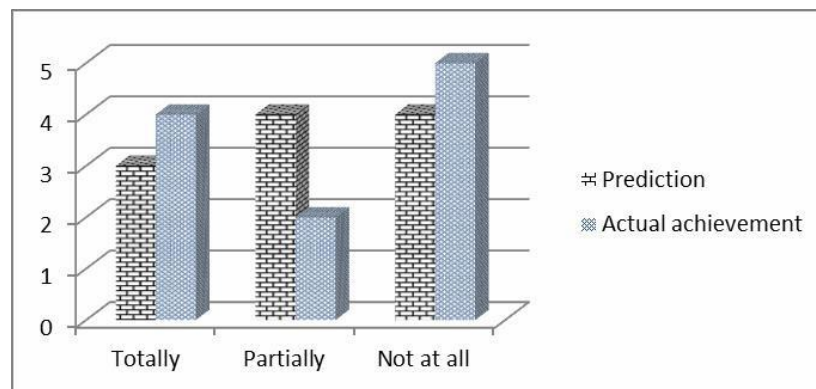


Figure 4.11: Comparison of high level SDL ability learners’ prediction and actual achievement in Question 1

Question 2 (Measurement)

1. Study the following problem:

Mr. Williams' piece of land is 18m long and 9m wide. He builds a fence around it but leaves an opening of 3,5m for a gate. What is the length of the fence?

2. Do you think you can solve the problem correctly? Circle your choice.

a.	I am confident I can solve the problem correctly
b.	I am reasonably sure I can solve the problem correctly.
c.	I am not sure how correctly I can solve the problem.
d.	I am not sure how correctly I can solve the problem. I think that I might make a mistake.
e.	I know that I will make a mistake in solving the problem

3. Explain in words how you would solve the problem.

4. Solve the problem. Show all your steps

5. Do you think you have solved the problem correctly? Circle your choice.

a.	I am confident I have solved the problem correctly.
b.	I am reasonably sure I have solved the problem correctly
c.	I am not sure I have solved the problem correctly.
d.	I am not sure how correctly I have solved the problem. I think I might have made a mistake.
e.	I know I have made a mistake.

6. Do you think about what you have done? Describe the strategies or plans you used when solving the problem.

7. Which type of mistake do you make when solving similar problems?

With respect to Question 2 (measurement), the learners read number 1 (the problem), until they understood what was asked or what the problem was that needed to be solved. Secondly, they had to answer number 2 and 3, before they attempted to solve the problem (question 4). After the participants had solved the problem, they had to answer number 5 to 7. The responses of the learners to number 2 in the task-based activity were the following:

Table 4.19: Participants’ predictions to Question 2 in the task-based activity

Groups	Prediction of learners		
	Totally	Partially	Not at all
Learners with high-level SDL-ability	5	1	5
Learners with medium-level SDL-ability	5	2	4
Learners with low-level SDL-ability	2	5	3

According to Table 4.19, twelve (12) learners indicated they were sure that they would make a mistake or are reasonably sure they would make a mistake, while another twelve (12) learners indicated that they were sure or reasonably sure they would solve the problem correctly. Only 8 learners were not sure or were partially sure they would solve the given problem correctly. In number 3 of the task-based activity learners had to explain in words how they would solve the problem. In Table 4.20 are some of the participants’ responses:

Table 4.20: Responses of participants to number 3 for Question 2

Participant No.	Responses	SDL-ability level
3	I will use the area to work out how long is the fence	Medium
6	I look at distance around the triangle	Medium
7	I have no idea to work out the sum , that mean I don’t understand.	High
8	By thinking how you are doing Maths ...	Low
9	I have no idea how I will work it out	High

11	Am gona subtract 3,5 and plus 18 with 18 and 9 with 9	Low
12	Will subtract because if multiplying 18×9 it will be $162m^2$	High
15	Me would work as in number 4	Medium
17	I multiply the distance around the ground which is me area = $l \times b$, when I get my answer I subtract it by $3,5m$, and I get my answer in m	Medium
20	 Area. Multiply it by $l \times b$ and add it to the answer	Low
23	I am going to use a formula	Medium
31	This was at first confusing but now it is really easy. I just have to work with premier so I have to check the distance	Low

Participant 31 understood the problem and had an idea how to solve the problem. The participant wrote “premier” instead of “Perimeter”. This participant, however, did not take into consideration the length of the gate that needed to be subtracted. Participant 17 seemed to understand the stated problem, however got confused with area and perimeter. This participant took the gap or length of the gate into consideration. Participant 5 added the lengths and two breadths and subtracted the length of the gate. It gives the idea that the learner understood the problem and had a plan how to solve it.

Most of the learners were not able to solve the problem stated in task-based activity correctly. In the following graphs (Figures 4.12 – 4.14). The comparisons of learners with high, medium and low levels of self-directed abilities’ predictions are portrayed along with the actual achievement.

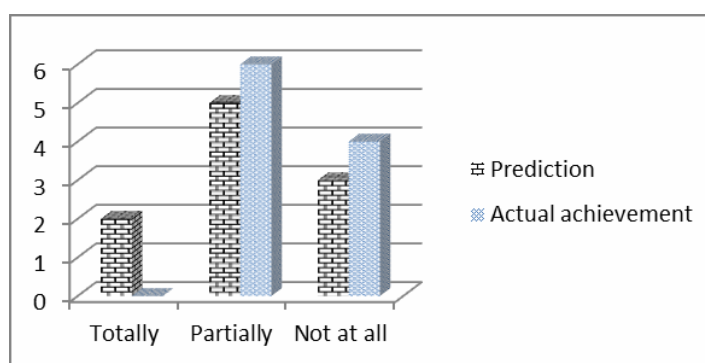


Figure 4.12: Comparison of low-level SDL ability learners’ prediction and actual achievement in Question 2.

Figure 4.12 illustrates that participants with low-level SDL-ability have an unclear understanding of their problem-solving ability. None of the participants that predicted “totally” actually solved the problem correctly. In the columns of “partially” as well as “not at all” the predictions were less than the actual achievements.

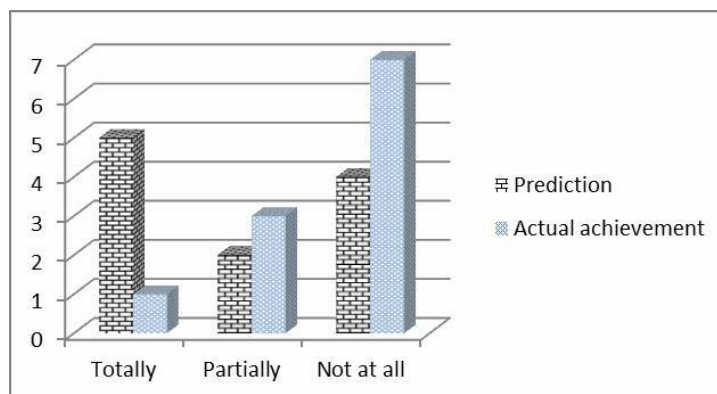


Figure 4.13: Comparison of medium-level SDL-ability learners’ prediction and actual achievement in Question 2

Figure 4.13 illustrates that five (5) participants predicted that they would solve the problem correctly. However, only one participant was successful. It seems that these participants over-estimated their problem-solving ability in Question 2. For the columns “partially” and “not at all” the prediction was less than the actual achievement. Notable is the fact that the number of participants who predicted “not at all” also is smaller than the actual achievers, which indicates that these learners also over-estimated their problem-solving abilities. From figures 4.12 to 4.14 a conclusion can be drawn that the participants struggled to apply their self -directed learning skills in solving a given problem (see 2.3.2.2.3).

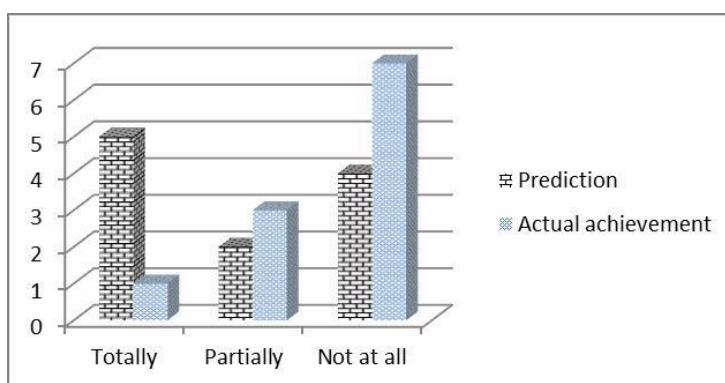


Figure 4.14: Comparison of high-level SDL-ability learners’ prediction and actual achievement for Question 2.

Figure 4.14 illustrates that in the “totally” column five (5) participants predicted they would solve the problem correctly. However, only one participant was successful. It seems that these participants over-estimated their problem-solving ability. For the columns “partially” and “not at all” the prediction was less than the actual achievement. Notable is the fact that the participants who predicted “not at all” are also fewer than the participants who were actually successful (“actual achievement”), which shows that these learners also over-estimated their problem-solving abilities.

Question 3

1. Study the following problem:

A long jump athlete jumps $5\frac{1}{4}$ metres . Her next jump is $5\frac{1}{8}$ metres . How far has she jumped altogether? What is the difference between the two jumps?

2. Do you think you can solve the problem correctly? Circle your choice.

a.	I am confident I can solve the problem correctly.
b.	I am reasonably sure I can solve the problem correctly
c.	I am not sure how correctly I can solve the problem.
d.	I am not sure how correctly I can solve the problem. I think I might make a mistake.
e.	I know I will make a mistake in solving the problem.

3. Explain in words how you would work to solve the problem.

4. Solve the problem. Show all your steps

5. Do you think you have solved the problem correctly? Circle your choice.

a.	I am confident I have solved the problem correctly.
b.	I am reasonably sure I have solved the problem correctly.
c.	I am not sure I have solved the problem correctly.
d.	I am not sure how correctly I have solved the problem. I might have made a mistake.
e.	I know I have made a mistake

6. Do you think about what you have done? Describe the strategies or plans you have used when solving the problem.

7. Which types of mistakes do you make when solving similar problems?

Table 4.21: Participants' predictions to Question 3 of the task-based activities

Groups	Prediction of learners		
	Totally	Partially	Not at all
Learners with high-level SDL-ability	6	3	2
Learners with medium-level SDL-ability	4	5	2
Learners with low-level SDL-ability	4	4	2

From Table 4.21 it is clear that most learners made the prediction that they were sure or reasonably sure that they could solve the given problem correctly. However, it is also apparent that some of the learners did not understand the problem (see the responses of the learners in the next table). The given problem expects two answers from the learners. Firstly, they had to calculate the total distance the athletes jumped altogether, and secondly, the difference between the lengths of the jumps. Most learners only attempted the first part of the problem and paid no attention to the second part of the question. The learners might have experienced difficulties in reading and understanding the mathematical problem (see 3.6.4).

In the following table are some of the responses from the participants to Question 3:

Table 4.22 Responses of participants to number 3 for Question 3

Participant No	Responses	SDL-ability
3	I can use addition and brackets to solve the problem	Medium
6	I would multiply the first sum first than the second one	Medium
7	 I will need to add the fraction and make the denominators the same.	High
9	I think I will need to add the fraction and make the denominators the same	High
11	I think am gone multiply the numbers and 5's am gone plus them	Low
14	I am going to add them together and	High

Participant No	Responses	SDL-ability
19	I am going to do LCM so that I can be able to add and subtract.	High
21	By multiplying 5×5 and multiplying $\frac{1}{4}$ of $\frac{1}{8}$	Low
30	By doing my steps down	Low
32	I will solve my problem by work on my sums	Medium

The participants had basic understanding of the addition of fractions (which only is the first part of solving the problem), which demonstrates a partial understanding of the given problem. There were, however, learners who understood the problem and tried to answer both questions. Learner 19, for example, knew how to solve the problem by using or remembering previous work that was done in class (see 3.6.3)

Figure 4.15 illustrates that four (4) participants predicted that they would solve the problem reasonably correctly. However, their actual achievement shows that none (0) of them solved the problem correctly. The column illustrating “not at all”, shows that only two (2) out of a total of nine (9) participants predicted they would not be able to solve the problem and seven participants actually could not solve the problem at all.

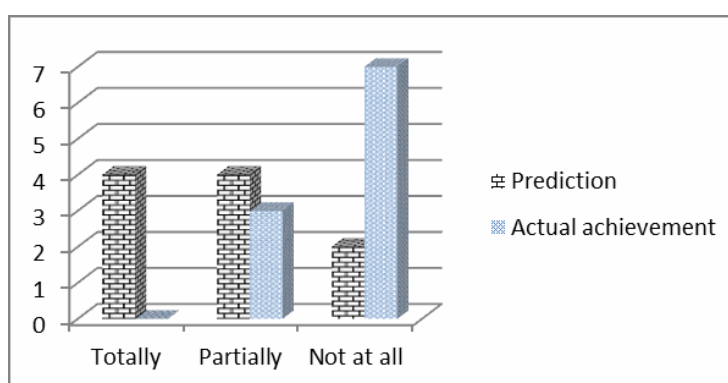


Figure 4.15: Comparison of low-level SDL-ability learners' prediction and actual achievement in Question 3.

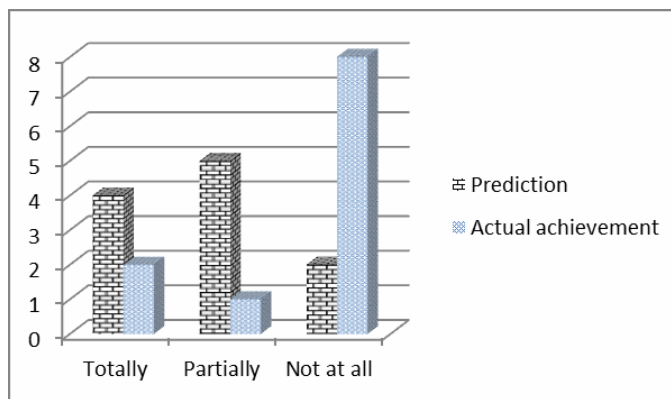


Figure 4.16: Comparison of medium-level SDL-ability learners' prediction and actual achievement in Question 3.

The graph in Figure 4.16 illustrates that most learners did not understand the given problem or over-estimated their problem-solving ability for Question 3. Only two (2) participants solved the problem correctly, and most of the learners could not solve the problem at all. It could be that the participants had trouble understanding the given problem (see 3.6.2).

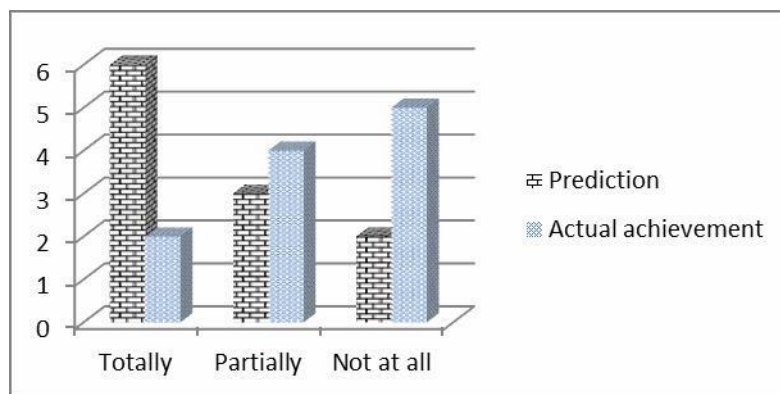


Figure 4.17: Comparison of high-level SDL-ability learners' prediction and actual achievement in Question 3.

Figure 4.17 illustrates that most (six) participants predicted that they could solve the problem correctly, however only two (2) learners actually achieved success in solving the problem. In the "not at all" column fewer participants predicted that they would not be able to solve the problem than the number of learners who were actually able to solve the problem

Question 4

1. Study the following problem:

Paul loses $\frac{2}{7}$ of 63 marbles. How many marbles did he lose?

2. Do you think you can solve the problem correctly? Circle your choice.

a.	I am confident I can solve the problem correctly.
b.	I am reasonably sure I can solve the problem correctly
c.	I am not sure how correctly I can solve the problem.
d.	I am not sure how correctly I can solve the problem. I think that I might make a mistake.
e.	I know that I will make a mistake in solving the problem.

3. Explain in words how you would work to solve the problem.

4. Solve the problem. Show all your steps.

5. Do you think you have solved the problem correctly? Circle your choice.

a.	I am confident I have solved the problem correctly.
b.	I am reasonably sure I have solved the problem correctly.
c.	I am not sure I have solved the problem correctly.
d.	I am not sure how correctly I have solved the problem. I think I might make a mistake.
e.	I know I have made a mistake

6. Do you think about what you have done? Describe the strategies or plans you have used when solving the problem.

7. Which type of mistakes do you make when solving similar problems?

Table 4.23: Participants' predictions to Question 4 in the task-based activities

Groups	Prediction of learners		
	Totally	Partially	Not at all
Learners with high-level SDL-ability	4	2	5
Learners with medium-level SDL-ability	3	6	2
Learners with low-level SDL-ability	2	5	3

From table 4.23 it is evident that most of the learners predicted that they were not sure how correctly they would solve the problem and therefore selected “partially”. The number of learners that indicated “totally” or “not at all” is almost the same. It seems most learners regarded “partially” as a safe prediction. One reason for choosing that option could be that the given problem looked familiar, but the learners were not sure they remembered how to solve such a problem. Learners did not know how to apply skills they had learned earlier to solve the given problem (see 3.6.3). The following are some of the participants' responses to Question 4:

Table 4.24 Responses of participants to number 3 for Question 4

Participant No.	Responses	SDL-ability
6	I would have to keep the value to get the correct answer	Medium
10	 I will sit down.....	Low
13	I made that “of” a multiplication and I multiply them. I multiplied with 2 with 63 and 7×1 that make it cosy for me to understand it.	High
14	I knew that a percentage is out of a 100 so it's about percentage so I did it like in class.	High
17	I take the whole number and make it a fraction. And multiply the two fractions	Medium
18	I will divide $\frac{2}{7}$ with $\frac{63}{1}$ to solve the problem	High

Participant No.	Responses	SDL-ability
19	To make 63 marbles to keeps its value. I'm going to write it as 63 over 1 and multiply to get the answer	High
21	Fraction sums and multiplying (2×63) and (7×100)	Low
24	I think I'll try and solve it by subtracting and multiplying	Low

Participant 18 had no idea of how to solve the given mathematical problem. It seems this participant did not understand the given problem, the participant's reading and comprehending ability might have been a challenge (see 3.6.4). Participants 23 and 14 had a misunderstanding of what was asked to solve. They tried to calculate percentage, whereas the given problem asked "How many marbles did he lose?" and not the percentage of marbles being lost. Participant 19 had an understanding of what was asked and how to solve the given problem.

In the following figures (Figures 4.18 – 4.20) learners with high- , medium- and low-levels of self-directed learning abilities' prediction are compared with the actual achievement:

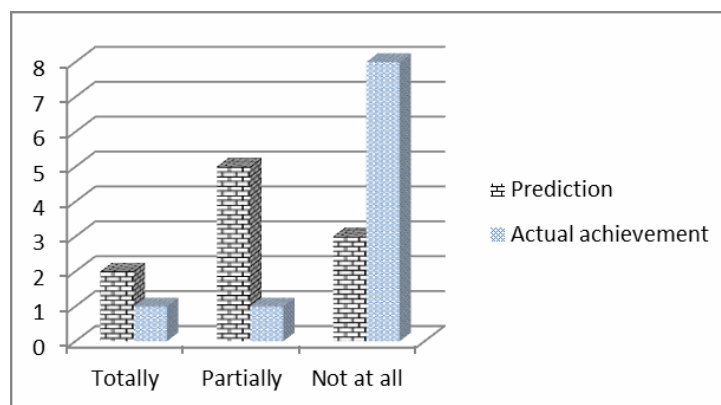


Figure 4.18: Comparison of low-level SDL-ability learners' prediction and actual achievement in Question 4

Figure 4.18 illustrates that none of the participants solved the given problem successfully, though three learners predicted that they would be able to solve the problem successfully. Although only three low-level SDL ability participants indicated that they were sure they would not be able to solve the problem correctly, eight of them were not able to solve the problem at all. Therefore, the participants with low-level SDL-ability over-estimated their problem-solving ability.

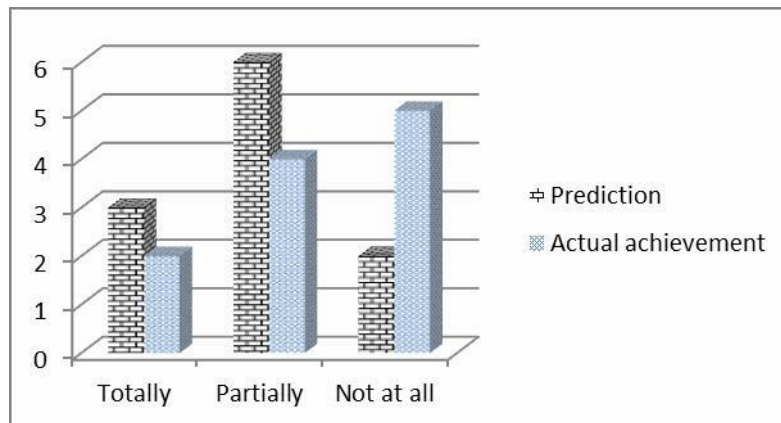


Figure 4.19: Comparison of medium-level SDL-ability learners' prediction and actual achievement in Question 4

From figure 4.19 illustrated that 3 participants predicted that they would solve the problem correctly and 2 participants predicted that they would not solve the problem at all. However, 2 participants completed the problem correctly and 5 participants could not solve the problem at all. Although 6 participants indicated that they would solve the problem partially, only 4 learners could solve the problem partially. It seems as if participants with medium-level SDL-ability also over-estimated themselves with respect to their problem-solving abilities in all three categories.

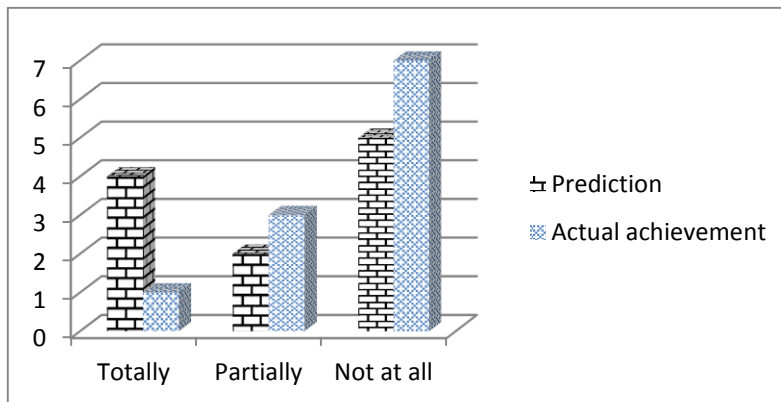


Figure 4.20: Comparison of high-level SDL-ability learners' prediction and actual achievement in Question 4

From Figure 4.20 it seems that most participants with high-level SDL-ability over-estimated their problem-solving abilities in attempting to solve the problem in Question 4. The number of participants who predicted "totally" was much higher than the actual achievement.

Question 5 (Number, Operations and Relationships)

1. Study the following problem:

There are 40 learners in a class. 15% of the learners wear glasses. How many learners wear glasses?

2. Do you think you can solve the problem correctly? Circle your choice.

a.	I am confident I can solve the problem correctly.
b.	I am reasonably sure I can solve the problem correctly
c.	I am not sure how correctly I can solve the problem.
d.	I am not sure how correctly I can solve the problem. I think I might make a mistake.
e.	I know that I will make a mistake in solving the problem.

- 3.. Explain in words how you would solve the problem.

4. Solve the problem. Show all your steps.

5. Do you think you have solved the problem correctly? Circle your choice.

a.	I am confident I have solved the problem correctly.
b.	I am reasonably sure I have solved the problem correctly.
c.	I am not sure I have solved the problem correctly.
d.	I am not sure how correctly I have solved the problem. I think I might make a mistake.
e.	I know I have made a mistake

6. Do you think about what you have done? Describe the strategies or plans you have used when solving the problem.

7. Which types of mistake do you make when solving similar problems?

Table 4.25: Participants' predictions to Question 5 of the task-based activities

Groups	Prediction of learners		
	Totally	Partially	Not at all
Learners with high-level SDL-ability	5	2	4
Learners with medium-level SDL-ability	2	5	4
Learners with low-level SDL-ability	3	4	3

From Table 4.25 it is evident that the predictions of the participants are spread evenly over the three categories. Five (5) participants with a high-level of SDL-ability were sure or reasonably sure they would solve the problem correctly. These learners showed confidence in their ability to solve the given problem. Five (5) learners with a medium-level of SDL-ability selected “partially”, which indicates that they were not sure whether they would be able to solve the given problem correctly. Three (3) learners with a low-level of SDL-ability selected “totally” and “not at all” respectively. Some of the participants' responses to Question 5 are illustrated in the following table:

Table 4.26 Responses of participants to number 3 for Question 5

Participant No	Responses	SDL-ability level
6	A percentage is out of 100 you need to find the percentage	Medium
7	I have no idea to work it out	High
11	I will work it out like I work out percentage in the classroom. But I don't think it fits	Low
17	I take 15 and make it a fraction, but over 100 and I make 40 a fraction over 1 and I subtract it to get my answer. First it will be a fraction out of 100 and I'll put into percentage	Medium
19	I will subtract it all up	High
20	 And I will make 40 and 15 and make it a fraction	Low

Participant No	Responses	SDL-ability level
23	I will see what can go into 40 and 15 without a remainder	Medium
28	I take 15 and make it a fraction, but over 100 and I make 40 a fraction over 1 and I subtract it to get my answer. First it will be a fraction out of 100 and I'll put into percentage	High

Participants 7 and 28 had no idea how to solve the given problem. The explanation given by participant 28 does not even make sense. Participant 19 showed some understanding, but did not give an explanation of how the problem would be solved. From the first part of the explanation given by participant 17, it seemed the learner understood the problem, but the second part revealed that the learner did not fully comprehend the problem.

Most of the learners in all the task-based activities interpreted the problem incorrectly, or did not understand what was asked or were incoherent in their responses as to how they would solve the problem (see 3.6.4).

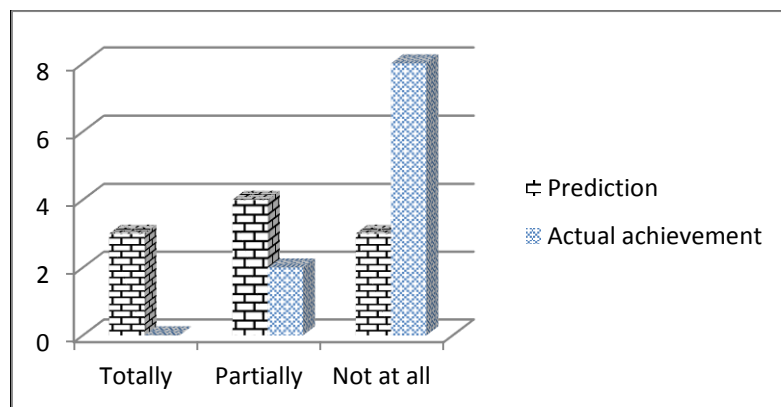


Figure 4.21: Comparison of low-level SDL-ability learners' prediction and actual achievement in Question 5

Figure 4.21 illustrates that participants with low-level SDL-ability have an unclear understanding of their problem-solving ability. None of the participants that predicted “totally” have actually solved the problem correctly. In the columns of “partially” as well as “not at all” the predictions were less than the actual achievements.

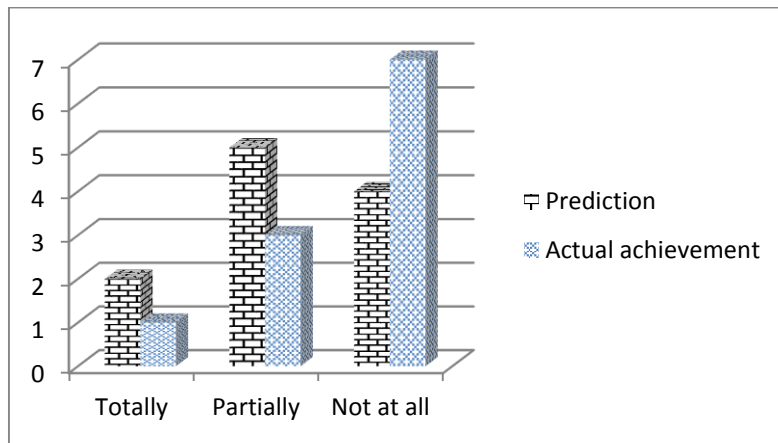


Figure 4.22: Comparison of medium-level SDL-ability learners' prediction and actual achievement in Question 5

Figure 4.22 illustrated that 2 participants predicted that they would solve the problem correctly. However, only one participant was successful. It seems that these participants over-estimated their problem-solving ability in Question 5. For the columns "partially" the prediction was less than the actual achievement. Notable is the fact that the number of participants who predicted "not at all" also is fewer than the actual achievers, which indicates that these learners also over-estimated their problem-solving abilities.

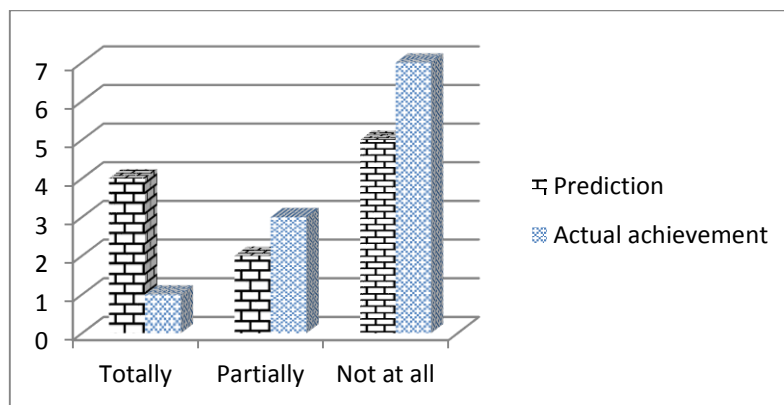


Figure 4.23: Comparison of high level SDL ability learners' prediction and actual achievement in Question 5.

From Figure 4.23 it is evident that the participants in the high-level SDL-ability category either did not understand the given problem or over-estimated their problem-solving abilities. Only one (1) participant solved the problem given in Question 5 correctly. From Figure 4.23 it seems that most participants with high-level SDL-ability over-estimated their problem-solving abilities in attempting to solve the problem in Question 5.

4.3.3.5.3 Summary of the findings of the qualitative investigation

The second research aim was attained with the aid of the completion of the quantitative investigation as well as the qualitative investigation. The second research question refers to the influence of learners' self-directed ability, through problem solving, on learners mathematical achievement (see 4.3.2.4).

Participants with a low score of self-directed learning ability seemed not to be able to solve the given mathematical problems as described in Chapter 3. The problem-solving process refers to the use of certain problem-solving steps, namely (1) learners have to understand the problem, (2) plan to solve the problem, (3) solve the problem, and (4) look back (reflecting on the solution) (see 3.3). The learners with low SDL-abilities seemed not to understand a given problem in the task-based interviews. They could not explain in words how they would solve the problem or give an account of how they solved the problem eventually (see 3.6.4). Therefore, most of these learners did not solve the given mathematical problems correctly. The same can be said of the participants with medium or high levels of SDL-ability. Although more participants with a high SDL-ability did actually understand a given problem, only a few of these learners could explain how they would solve the problem, and solved the problem correctly (see 3.6).

CHAPTER 5:

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This last chapter is an attempt to summarise both the theoretical and the empirical findings of the study. Firstly, an overview of the investigation will be provided. Secondly, the main findings will be summarised. Thirdly, conclusions will be drawn regarding the study and the limitations thereof will be discussed. Lastly, recommendations will be made regarding the study.

5.2 Overview of the investigation

In Chapter 1 a problem statement, as well as the research questions and aims, were formulated, based on a brief literature review. Once the research design and methodology had been briefly explained, an overview of the study followed. To attain the research aims, a literature study and an empirical investigation were undertaken.

The first chapter of the literature study (Chapter 2) concentrated on one element of the study, namely *self-directed learning in the Mathematics classroom*. Approaches to the learning of mathematics (see 2.2), a brief discussion on the history of SDL (see 2.3), self-directed learning ability (2.4), other factors influencing self-directed learning ability (see 2.5), components of self-directed learning, as well as the roles of the Mathematics teacher and the learners regarding SDL, respectively, attributed to the mentioned attempt.

In Chapter 3 problem-solving in the Mathematics classroom was investigated as follows: Definitions of problem-solving (see 3.2) were discussed in general, leading to problem-solving models based on the model of Polya (1957) (see 3.3). Then, the characteristics of a mathematics problem solver (see 3.4) were outlined. The role of the Mathematics teacher in a problem-solving classroom (see 3.5) was discussed with respect to the planning for mathematical problem-solving (see 3.5.1), as well as the teacher's actions during a problem-solving session (see 3.5.2). Factors influencing mathematical problem-solving (see 3.6), and the implications thereof for both the Mathematics teacher and the learners were explained (see 3.8). The theoretical relationship between self-directed learning and problem-solving was investigated briefly (see 3.7).

The empirical investigation was reported in Chapter 4. A sequential explanatory mixed method research design was employed, involving quantitative and qualitative investigation methods (see 4.3). The quantitative investigation (see 4.2.1) was undertaken in order to (1) determine the

influence of problem-solving activities on grade 7 Mathematics learners' self-directed learning abilities, and (2) to determine whether self-directed learning, through problem-solving, has an influence on learners' mathematical achievement. The qualitative investigation (see 4.3.3.5) was conducted to provide more insight into the quantitative findings regarding the influence of self-directed learning on mathematical problem-solving in grade 7. The qualitative investigation was done by following a problem-solving approach to the teaching and learning of mathematics (see Appendix I). Data on learners' self-directed learning ability and problem-solving in Mathematics were obtained by means of open-ended questions answered by participants while they were solving mathematical problems (see 4.3.3.3).

5.3 Findings of the investigation

5.3.1 Theoretical findings

The theoretical findings were made regarding the influence of self-directed learning on mathematical problem-solving.

5.3.1.1 Self-directed learning in the Mathematics classroom

In a comprehensive literature study in chapter two, an investigation into self-directed learning in the Mathematics classroom started with a discussion on approaches to the learning of Mathematics (see 2.2). The learning of Mathematics was regarded as knowledge constructed by learners through problem-solving (see 2.2.3). A brief discussion followed on the history of self-directed learning (see 2.3) with the emphasis on the idea or definition that self-directed learning is an approach assuming that learners are responsible for their own learning experiences, are persistent in their learning, have a desire to learn, take initiative for learning and implement learning strategies with or without the assistance of the teacher, peers, parents, or other people (Knowles, 1975:16) (see 2.3.1). Factors that influence self-directed learning (see 2.3.2) are categorised into external (see 2.3.2.1) and internal factors (see 2.3.2.2). One of the main aims of teaching and learning is to develop the ability of learners to solve a wide spectrum of mathematical problems (see 2.3.2.1.2). In order for learning to take place, learners need to be self-directed in their learning.

Mathematics learners' self-directed learning ability (see 2.4) during problem-solving was briefly discussed. Self-directed learning ability (SDLA) refers to the ability of learners to be motivated, to plan, to implement, to monitor and display interpersonal skills during their own learning (see 2.4). Other factors influencing learners' self-directed learning ability (see 2.5) which include (1) concentration, (2) time management, (3) self-testing, (4) motivation and (5) the use of study aids,

are critical for primary school learners (see 2.5). The mathematics teacher's role as a facilitator of self-directed learning (see 2.6), however, cannot be neglected. Moreover, self-directed learning holds advantages for both the Mathematics teacher and the Mathematics learner (see 2.7). It became clear that self-directed learning forms the foundation of mathematical problem-solving.

5.3.1.2 Problem-solving in the Mathematics classroom

Problem-solving in the Mathematics classroom was discussed in detail in Chapter 3 (see 3.2). Although various definitions of problem-solving were looked into, the focus was on mathematical problem-solving. For the purpose of this study, mathematical problem-solving was defined as a process during which the learner has to find answers or solutions to mathematical problems, where the possibly for straight-forward answers does not exist. Problem-solving is a process that needs to be developed on a daily basis to enable learners to become "good" problem solvers. Models of problem-solving (see 3.3) were discussed, based on the model of Polya with its different problem-solving phases, namely: (1) understanding the problem, (2) making a plan to solve the problem, (3) carrying out the plan to solve the problem, and (4) looking back in order to reflect on the solution. These phases, however, should not be carried out in a linear manner, but have to interact with one another. This implies that learners may work forward or backward, depending on the specific problem they are trying to solve (see 3.3).

Learners have to develop a set of characteristics (see 3.4) to enable them to become mathematical problem solvers. These characteristics (subject knowledge, problem-solving skills, mathematical concepts, reasoning abilities, interpreting, predicting, analysing and reflecting) develop over time, preparing a Mathematics learner for solving real-world problems. Mathematics learners should have a sound Mathematics knowledge base in order to become good problem solvers. Therefore, the Mathematics teacher has an important role to play in teaching problem-solving (see 3.5), ranging from planning for problem-solving (see 3.5.1) to the teacher's actions during the problem-solving activities (see 3.5.2). The following factors, namely attitude, emotions, metacognitive skills or ability and reading and comprehending ability of learners, have an influence on both the Mathematics teacher's and learners' problem-solving abilities (see 3.6).

Since self-directed learning forms a fundamental part of problem-solving, a specific theoretical relationship exists between self-directed learning and problem-solving (see 3.7). This relationship requires both the Mathematics teacher and the learners to change from the traditional teacher-learner relationship, where the teacher is the main provider of all knowledge and the learner an empty vessel that needs to be filled up with the teacher's knowledge, to assisting learners in such

a manner that they can construct their own mathematical knowledge.

5.3.2 Empirical findings

From the results of the empirical investigation the following findings were reported with regard to (1) the influence of problem-solving activities on Grade 7 Mathematics learners' self-directed learning abilities; and (2) the influence of self-directed learning through problem-solving on learners' mathematical achievement.

5.3.2.1 The influence of problem-solving activities on Grade 7 learners' self-directed learning abilities

The first aim of the empirical investigation was to determine the influence of problem-solving activities on grade 7 learners' self-directed learning abilities. From the quantitative results it was evident that no meaningful differences existed between the experimental and the control groups (see 4.3.2.4).

For all the SDL-ability domains (learning motivation, planning and implementation, self-monitoring and interpersonal communication), the experimental group's overall score after the intervention with problem solving, was lower than that of the control group (see Table 4.7). Learners from the experimental group, however, showed a decrease in the average scores for their SDL-ability, when comparing post-test with pre-test (see Table 4.8).

With respect to the results of the factors influencing self-directed learning, namely motivation, concentration, the use of study aids and self-testing strategies, the experimental group's average score for "the use of study aids" was higher than that of the control group (see Table 4.11). When comparing the experimental group's pre- and post-test results, the learners showed a remarkable increase in the average score on the learners' concentration level, as well as a minimal increase in their self-testing strategies (see Table 4.12).

5.3.2.2 The influence of self-directed learning through problem-solving on grade 7 learners' achievement in Mathematics

The second aim of the empirical investigation was to determine the influence of self-directed learning through problem-solving on grade 7 learners' achievement in Mathematics. The results of the quantitative data (see Table 4.14), showed a slight increase of 9% in the average mathematical achievement of the experimental group, while that of the control group showed a remarkable decrease of 14%.

From the individual task-based interviews no real differences could be found between the low-, medium- and high-level achieving learners regarding their problem-solving predictions and their actual achievement in the problem solving activities (see 4.3.3.5). The low-level achieving, as well as the medium-level achieving learners, over-estimated their mathematical problem-solving abilities by predicting their achievement higher than their actual achievement (see 4.3.3.5.2). Learners from the high-level category seemed to under-estimate their problem-solving abilities by predicting that they would not be successful, comparing with the actual number of learners who were successful (see 4.3.3.5.2). Most learners from the low- and medium-level categories also struggled to explain their possible solutions in a meaningful manner. This could be attributed to their reading and comprehension abilities regarding the given problem, or a general challenge in reading with understanding (see 3.6.4). A few learners from the high-level category also struggled to give meaningful explanations with respect to how they had intended to solve a given mathematics problem.

5.4 Conclusions

The following conclusions can be drawn from this study:

Grade 7 Mathematics learners' self-directed learning abilities were not influenced positively or negatively by the intervention of problem-solving activities. However, learners' abilities to read and understand a mathematical problem are of cardinal importance in their abilities to solve mathematical problems (see 3.6.4). In Mathematics the reading demands are high (Lopez, 2008:20), and can be overwhelming for learners (Borasi *et al.*, 1998:297). Although it is not clear whether self-directed learning or problem-solving activities or both had an influence on learners' mathematical achievement, there was an improvement in the control group's average mathematical achievement after the intervention with problem solving activities.

5.5 Limitations of the study

Although the researcher was the Mathematics teacher for the experimental class groups, the research was limited to the grade 7 learners of the specific school (see 1.4.1.1), while the control class groups were taught by different Mathematics teachers at a neighbouring school. The time constraints during the application of problem-solving in the Mathematics classrooms of the experimental group, as well as the completion of the problem solving activities after school hours, could have placed the learners under pressure to complete the activities in a short time span.

Since it was not evident up front to what extent learners had been exposed to the solving of non-

routine mathematical problems previously, the introduction of mathematical problem-solving activities could have been viewed by some learners as something separate, and not as part of the learning of mathematical concepts. It is possible that the mentioned aspect could have had a negative influence on learners' self-directed learning abilities.

5.6 Recommendations

To establish a worthwhile relationship between problem-solving and self-directed learning in grade 7 Mathematics classrooms, the following aspects need to be considered:

- The Mathematics teacher has an active role to play in mathematical problem-solving in the classroom;
- The Mathematics teacher has to be a facilitator during problem-solving activities, assisting and guiding where and when needed;
- Learners have to be actively involved in the teaching and learning process and need to know exactly what is expected from them;
- To be able to solve routine and non-routine mathematical problems, learners need to be continuously exposed to problem-solving activities;
- Mathematical problem-solving has to be integrated into the teaching and learning of Mathematics;
- Learners need to take responsibility for their own learning and have to be guided by the Mathematics teacher in this regard;
- The Mathematics teacher has to acknowledge the relationship between problem solving and self-directed learning, and prepare the learners accordingly.

5.7 Final word

In this study the researcher attempted to answer the following research question:

“What is the relationship between *problem-solving* and *self-directed learning* in grade 7 Mathematics classrooms?”

From the research results it seemed that there were no significant differences between the experimental and control groups regarding their self-directed learning abilities. The experimental

group, however, showed a minimal increase in their mathematical achievement after the intervention with problem-solving, while the control group's achievement showed a remarkable decline.

REFERENCES

Abdullah, M.H. 2001. Self-directed learning. Bloomington, Ind.: ERIC Clearinghouse on Reading, English and communication. (ERIC Document Reproductive Service No. ED459458,)

Adams, D. & Hamm, M. 1994. New designs for teaching and learning: promoting active learning in tomorrow's school. San Francisco, Calif.: Jossey-Bass.

Adler, J. 2000. Conceptualizing resources as a theme for teacher education. *Journal of mathematics teacher education*, 3(3):205-224.

Alenezi, D.F. 2008. A study of learning mathematics related to some cognitive factors and attitudes. Glasgow: University of Glasgow. (Thesis - PhD.)

Allen, K. 2003. Explaining Cronbach's alpha. Norman, Okla.: University of Oklahoma. Department of Industrial Engineering.

Alsawaie, O.N. 2003. What roles do mathematics teachers play when their students solve problems? (Proceedings of the Decidable and the Undecidable in Mathematics Education Conference, Brno, Czech Republic.)

Ames, C. & Archer, J. 1988. Achievement goals in the classroom: students' learning strategies and motivation processes. *Journal of educational psychology*, 80(3):260-267.

Anderman, E.M. & Maehr, M.L. 1994. Motivation and schooling in the middle grades. *Review of educational research*, 64(2):287-309.

Anderson, J. 2009. Mathematics curriculum development and the role of problem solving. (Proceedings of 2009 Australian Curriculum Studies Association National Biennial Conference. Curriculum: a national conversation. p. 1-8.)

Anthony, G. 1996. When mathematics students fail to use appropriate learning strategies. *Mathematics research journal*, 8(1):23-37.

Anthony, G. & Walshaw, M. 2007. Effective pedagogy in mathematics/Pāngarau: best evidence synthesis iteration (BES). Wellington, NZ: Ministry of Education.

Avry, D., Jacobs, L.C., Razavieh, A. & Sorensen, C. 2006. Introduction to research in education. 7th ed. Belmont, Calif.: ThomsonWadsworth.

Asia e University. 2011. Learning theories: cognitive learning theories. Asia e University. Knowledge Center. http://peoplelearn.homestead.com/BEduc/Chapter_5.pdf Date of access: 11 November 2013.

Atkinson, S., ed. 1992. Mathematics with reason: the emergent approach to primary maths. London: Hodder & Stoughton.

Babbie, E. & Mouton, J. 2009. The practice of social research. Cape Town: Oxford University Press.

Badger, M.S., Hawkes, T.O. & Sangwin, C.J. 2012. Background to problem-solving in undergraduate mathematics. (*In* Badger, M.S., Sangwin, C.J., Hawkes, T.O., Burn, R.P., Mason, J. & Pope, S., eds. Teaching problem-solving in undergraduate mathematics. [Coventry, England]: Coventry University. p. 9-32.)

Bandura, A. 1994. Self-efficacy. (*In* Ramachaudrana, V.S., ed. Encyclopedia of human behavior. New York: Academic Press. v. 4:71-81.) (Reprinted in Friedman, H., ed. 1998. Encyclopedia of mental health. San Diego, Calif.: Academic Press.)

Barrows, H. 1983. Problem based, self-directed learning. *Journal of the American Medical Association*, 250(22):3077-3080.

Berlyne, D.E. 1960. Conflict, arousal, and curiosity. New York: McGraw-Hill.

Bernando, A.B.I. 1999. Overcoming obstacles to understanding and solving word problems in mathematics. *Educational psychology*, 19(2):149-164.

Beronia, G. 2008. "Arobase": a model of self-learning. Roma: Learning Community.

Biryucov, P. 2004. Metacognitive aspects of solving combinatorics problems. *International journal for mathematics teaching and learning*. <http://www.cimt.plymouth.ac.uk/journal/biryukov.pdf>. Date of access: 10 August 2012.

Black, P.J. & Atkin, J.M. 1996. Changing the subject: innovations in science, mathematics and technology education. London: Routledge.

Blanco, L.J., Barona, E.G. & Carrasco, A.C. 2013. Cognition and affect in mathematics problem solving with prospective teachers. *The mathematics enthusiast (TME)*, 10(1&2):335- 365.

Boekaert, M. 1999. Self-regulated learning: where we are today. *International journal of educational research*, 31(6):445-457.

Bogdan, R.J. 2000. Minding minds: evolving a reflexive mind by interpreting others. Cambridge, Mass.: MIT Press.

Borasi, R., Siegel, M., Fonzi, J. & Smith, C.F. 1998. Using transactional reading strategies to support sense-making and discussion in mathematics classrooms: an exploratory study. *Journal for research in mathematics education*, 29(3):275-305.

Borko, H. & Shavelson, R. 1990. Teachers' decision making. (In Jones, B. & Idol, L., eds. Dimensions of thinking and cognitive instruction. Hillsdale, N.J.: Erlbaum. p.311-346.)

Branda, L. 1990. Implementing problem based learning. *Journal of dental education*, 54:548- 549.

Broussard, S.C. & Garrison, M.E.B. 2004. The relationship between classroom motivation and academic achievement in elementary school-aged children. *Family and consumer sciences research journal*, 33(2):106-120.

Brunning, R.H., Schraw, G.J., Norby, M.M. & Ronning, R.R. 2004. Cognitive psychological instruction. 4th ed. Upper Saddle River, N.J.: Merrill/Prentice Hall.

Calan, V.G., Rubin, R. & Morgan, B.M. 2004. Cooperative learning, mathematical problem solving, and Latinos. (Annual Meeting of the American Educational Research Association, San Diego, Calif. p. 1-19.)

Cangalosi, J.S. 2003. Teaching mathematics in secondary and middle school: an interpretative approach. Upper Saddle River, N.J.: Prentice Hall.

Caraway, K., Tucker, C.M., Reinke, W.M. & Hall, C. 2003. Self-efficacy, goal orientation, and fear of failure as predictors of school engagements in high school students. *Psychology in the schools*, 40(4):417-427.

Carlson, M. & Bloom, I. 2005. The cyclic nature of problem solving: an emergent problem- solving framework. *Educational studies in mathematics*, 58(1):45-75.

Castle, P.C. & Buckler, S. 2009. How to be a successful teacher: strategies for personal and professional development. London: Sage.

Chan, C. 2001. Promoting learning and understanding through constructivist approaches for Chinese learners. (In Watkins, D.A. & Biggs, J.B., eds. Teaching the Chinese learner: psychological perspectives. Hong Kong/Melbourne: Comparative Education Research Centre, University of Hong Kong/Australian Council for Educational Research. p.181-204.)

Cheng, H. 2011. A case study of cooperative learning in mathematics: middle school course design. *Journal of mathematics education*, 4(1):75-88

Cheng, S., Kuo, C., Lin, K. & Lee-Hsieh, J. 2010. Development and preliminary testing of a self-rating instrument to measure self-directed learning ability of nursing students. *International journal of nursing studies*, 47(9):1152-1158.

Clausen-May, T. 2005. Teaching maths to pupils with different learning styles. London: Paul Chapman.

Cobb, P. 1994. Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational researcher*, 23(7):13-20.

Cobb, P. & Yackel, E. 1996. Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational psychologist*, 31(3/4):175-190.

Cohen, L. & Manion, L. 1994. Research methods in education. London. Croom Helm.

Cohen, L., Manion, L. & Morrison, K. 2007. Research methods in education. 6th ed. London: Routledge.

Cornoldi, C. 1998. The impact of metacognitive reflection on cognitive control. (*In* Mazzoni, G. & Nelson, T., eds. Metacognition and cognitive neuropsychology. Mahwah, N.J.: Erlbaum. p. 139-159.)

Costa, A.L. & Kallick, B. 2004. The teacher's role in self-directed learning: assessment strategies for self-directed learning. Thousand Oaks, Calif.: Corwin Press.

Cranton, P. 2001a. Adult learning: theory principles and applications. University of Phoenix.

Cranton, P. 2001b. Strategies for self-directed learning. (*In* Merriam S.B., Caffarella, R.S., Wlodkowski, R.J. & Cranton, P. Adult learning: theory principles and applications. Phoenix, Ariz.: University of Phoenix. p. 45-63.)

Creswell, J.W. 2005. Research design: qualitative, quantitative and mixed-methods approaches. 2nd ed. Thousand Oaks, Calif.: Sage.

Creswell, J.W. 2009. Research design: qualitative, quantitative and mixed-methods approaches. 3rd ed. Thousand Oaks, Calif.: Sage.

Darling-Hammond, L., Austin, K., Cheung, M. & Martin, D. 2008. Thinking about thinking: metacognition. <http://www.learner.org/resources/series172.html> Date of access: 6 October 2014.

DeBellis, V.A. & Goldin, G.A. 2006. Affect and meta-affect in mathematical problem solving: a representational perspective. *Educational studies in mathematics*, 6(2):131-147.

Deci, E.L., Koestner, R. & Ryan, R.M. 1999. A meta-analytical review of experiments examining the effects of extrinsic rewards on intrinsic motivation. *Psychological bulletin*, 125(6):627-668.

Downing, K., Kwong, T., Chan, S., Lam, T. & Downing, W. 2009. Problem-based learning and the development of metacognition. *Higher education*, 57(5):609-621.

- Drews, D. 2007. Do resources matter in primary mathematics teaching and learning? (*In* Drews, D. & Hansen, A., eds. Using resources to support mathematical thinking, primary and early years. Exeter: Learning Matters. p. 19-31.)
- Driscoll, M.P. 2000. Psychology of learning for instruction. 2nd ed. Boston, Mass.: Allyn & Bacon.
- Driscoll, M.P. 2004. Psychology of learning for instruction. 3rd ed. Boston, Mass.: Allyn & Bacon.
- Duda, J.L. & Nicholls, J.G. 1992. Dimensions of achievement motivation in schoolwork and sport. *Journal of educational psychology*, 84(3):290-299.
- Eccles, J., Wigfield, A., Reuman, D. & MacIver, D. 1987. Changes in self-perceptions and values at early adolescence. (Paper presented at the Annual Meeting of the American Educational Research Association, Washington.)
- Elia, I. & Philippou, G. 2004. The functions of pictures in problem solving. (Paper presented at the 28th Conference of the International Group for the Psychology of Mathematics Education, Bergen.)
- Ellis, S.M. & Steyn, H.S. 2003. Practical significance (effect sizes) versus or in combination with statistical significance (p-values). *Management dynamics*, 12(4):51-53.
- Evens, H. & Houssart, J. 2007. Paired interviews in mathematics education. (Proceedings of the British Society for Research into Learning Mathematics. Open University, Institute of Education. 27(2):19-24.)
- Fennema, E. & Franke, M. 1992. Teachers' knowledge and its impact. (*In* Gouws, D.A., ed. Handbook of research mathematics teaching and learning. New York: Macmillan Publishing. p. 147-164.)
- Ferrance, E. 2000. Themes in education: action research. Providence, R.I.: Laboratory at Brown.
- Fischer, G. & Sugimoto, M. 2006. Supporting self-directed learners and learning communities with sociotechnical environments. *Research and practice in technology enhancing learning (RPTEL)*, 1(1):1-27.

- Fisher, M., King, J. & Tague G. 2001. Development of a self-directed learning readiness scale for nursing education. *Nurse education today*, 21(7):516-525.
- Flavell, J.H. 1999. Cognitive development: children's knowledge about the mind. *Annual review of psychology*, 50(1):21-45.
- Fossey, E., Harvey, C., McDermott, F. & Davidson, L. 2002. Understanding and evaluating qualitative research. *Australian and New Zealand journal of psychiatry*, 36(6):717-732.
- Fraenkel, J.R. & Wallen, N.E. 2008. How to design and evaluate research in education. 7th ed. New York: McGraw-Hill.
- Francom, G.M. 2010. Teach me how to learn: principles for fostering students' self-directed learning skills. *International journal of self-directed learning*, 7(1):29-44.
- Fraser, B.J. 1998a. The birth of a new journal: editor's instruction. *Learning environments research*, 1(1):1-5.
- Garrison, D.R. 1997. Self-directed learning: toward a comprehensive model. *Adult education quarterly*, 48(1):18-33.
- Giganti, P. Jr. 2007. Why teach problem solving, part 1: The world needs good problem solvers! *CMC ComMuniCator*, 31(4):15-16.
- Gil, N., Blanco, L.J. & Guerrero, E. 2006a. The affective domain in mathematics learning. *International electronic journal of mathematics education (IEJME)*, 1(1):16-32.
- Gliem, J.A. & Gliem, R.R. 2003. Calculating, interpreting, and reporting Cronbach's alpha reliability coefficient for Likert-type scales. (Paper presented at the Midwest Research-to- Practice Conference in Adult, Continuing, and Community Education, Ohio State University, Columbus, OH., October 8-10, 2003. p. 82-88.)
- Goos, M., Galbraith, P. & Renshaw, P. 2000. A money problem: a source of insight into problem solving action. *International journal for mathematics teaching and learning*, 80:1-21.

- Gottfried, A.E. 1990. Academic intrinsic motivation in young elementary school children. *Journal of educational psychology*, 82(3):525-538.
- Graaff, M. 2005. Probleemoplossing en die onderrig en leer van wiskunde in graad 4. Potchefstroom: North-West University. (Thesis - PhD.)
- Grow, G.O. 1991. Teaching learners to be self-directed. *Adult education quarterly*, 41(3):125-149.
- Grugnetti, L. & Jaquet, F. 2005. A mathematical competition as a problem solving and a mathematical education experience. *Journal of mathematical behavior*, 24(3):373-384.
- Guay, F., Chanak, J., Ratelle, C.F., March, H.W., Larose, S. & Boivin, M. 2010. Intrinsic, identified, and controlled types of motivation for school subjects in young elementary school children. *British journal of educational psychology*, 80(4):711-735.
- Guba, E.G. & Lincoln, Y. 1988. Do inquiry paradigms imply methodologies? (In Fetterman, D.M., ed. *Qualitative approaches to evaluation in education*. New York: Praeger. p.89-115.)
- Guglielmino, L.M. 1977. Development of self-directed learning readiness scale. Atlanta, Ga.: University of Georgia. (Thesis - PhD.)
- Guglielmino, L.M. & Long, H.B. 2011. The international self-directed learning symposium: a 25 year perspective. *International journal of self-directed learning*, 8(1):1-7.
- Guthrie, J.T., Van Meter, P., McCann, A., Wigfield, A., Bennett, L. & Poundstone, C. 1996. Growth of literature engagement: changes in motivations and strategies during concept- orientated reading instruction. *Reading research quarterly*, 31(3):306-332.
- Hacker, D.J. 1998. Metacognition: definitions and empirical foundations. (In Hacker, D.J., Dunlosky, J. & Graesser, A.C., eds. *Metacognition in educational theory and practice*. Mahwah, N.J.: Erlbaum. p. 1-23.)

Han, L.C. & Teng, N.H. 2005. Effects of problem-based learning on learners' self-directed learning behaviours in mathematics. "Book effects of problem-based learning on students self- directed learning behaviours in mathematics." Singapore: Centre for Educational Development. Republic Polytechnic.

Harnishferger, K.K. 1995. The development of cognitive inhibition: theories, definitions, research. (*In* Dempster, F.N. & Bainerd, C.J., eds. Inference and inhibition in cognition. San Diego, Calif.: Academic Press. p. 1176-206.)

Harries, T. & Spooner, M. 2000. Mental mathematics for the numeracy hour. London: David Fulton.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A. & Wearne, D. 1996. Problem solving as a basis for reform in curriculum and instruction: the case of mathematics. *Educational researcher*, 25(4):12-21.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K.C., Wearne, D. & Murray, H. 1997. Making sense: teaching and learning mathematics with understanding. Portsmouth, N.H.: Heinemann.

Hiemstra, R. 1994. Self-directed learning. (*In* Husen, T. & Postlethwaite T.N., eds. The International encyclopedia of education. 2nd ed. Oxford: Pergamon Press. p. 9-20.)

Hmelo, C.E. & Coté, N.C. 1996. The development of self-directed learning strategies in problem-based learning. (Proceedings of the 1996 International Conference on Learning Science. Grandville, Mich.: International Society of the Learning Sciences. p. 421-426.)

Hunt, J.M.V. 1965. Intrinsic motivation and its role in psychological development. (*In* Levine, D., ed. Nebraska Symposium on Motivation. Lincoln: University of Nebraska Press. 13:189- 282.)

Ivankova, N.V., Creswell, J.W. & Plano Clark, V.L. 2007. Foundations and approaches to mixed method research. (*In* Maree, K., ed. First steps in research. Pretoria: Van Schaik Publishers. p. 253-282.)

Ivankova, N.V., Creswell, J.W. & Stick, S.L. 2006. Using mixed-method sequential explanatory design: from theory to practice. *Field methods*, 18(1):3-20.

Johnson, B. & Christensen, L. 2004. Educational research: quantitative, qualitative, and mixed approaches. New York: Pearson.

Johnson, R.B. & Onwuegbuzie, A.J. 2004. Mixed method research: a research paradigm whose time has come. *Educational researcher*, 33(7):14-26.

Jonassen, D. 2000. Learning as activity. (Paper presented at the Meaning of Learning Project, Learning Development Institute. Presidential Session at AECT Denver. University of Missouri. October 25-28.)

Joppe, M. 2000. The research process. <http://www.ryerson.ca/~mjoppe/rp.htm> Date of access: 22 December 2014.

Kek, M. & Huijser, H. 2011. Exploring the combined relationship of student and teacher factors on learning approaches and self-directed learning readiness at a Malaysian university. *Studies in higher education*, 36(2):185-208.

Kelly, C.A. 2006. Using manipulatives in mathematics problem solving: a performance based analysis. *Mathematics enthusiast*, 3(2):184-193.

Kilpatrick, J., Swafford, J. & Findell, B., eds. 2001. Adding it up: helping children learn mathematics. Washington D.C.: National Academy Press.

Kirk, G. 2003. Teacher education institutions. (In Bryce, T. & Humes, W., eds. Scottish education. 2nd ed. Edinburgh: University Press. p.921-933.)

Kirkley, J. 2003. Principles for teaching problem solving. Bloomington, Ind.: Indiana University. PLATO Learning. (Technical paper 4.)

Kirstner, S., Rakoczy, K. & Otto, B. 2010. Promotion of self-regulated learning in classrooms: investigating frequency, quality, and consequences for student performance. *Metacognition and learning*, 5(2):157-171.

Knowles, M.S. 1975. Self-directed learning: a guide for learners and teachers. New York: Associated Press.

Knowles, M.S. 1990. The adult learner: a neglected species. 4th ed. Houston, Tex.: Gulf Publishing.

Kontra, J. 2001. Factors influencing problem solving and successfulness in mathematics. Szeged, Hungary: University of Szeged. (Thesis - PhD.)

Krefting, L. 1991. Rigor in quality research: the assessment of trustworthiness. *American journal of occupational therapy*, 45(3):214-222.

Learning Theories Knowledgebase. 2011. Behaviorism at learning. <http://www.learning-theories.com/behaviorism.html>. Date of access: 28 September 2014.

Lee, S. 2011. The effect of alternative solutions on problem solving performance. *International journal for mathematics teaching and learning*. <http://www.cimt.plymouth.ac.uk/journal/lee.pdf>. Date of access: 10 August 2012.

Leedy, P.D. & Ormrod, J.E. 2005. Practical research: planning and design. 8th ed. Upper Saddle River, N.J.: Prentice Hall.

Lepper, M.R., Corpus, J.H. & Iyengar, S.S. 2005. Intrinsic and extrinsic motivational orientation in the classroom: age differences and academic correlates. *Journal of educational psychology*, 97(2):184-196.

Lesh, R. & Zawojewski, J.S. 2007. Problem solving and modeling. (In Lester, F., ed. Second handbook of research on mathematics teaching and learning. Greenwich, Conn.: Information Age Publishing. p. 763-804.)

Lesh, R.A. 2003. A models and modeling perspective on problem solving. (In Lesh, R. & Doerr, H., eds. Beyond constructivism: models and modeling perspectives on mathematics problem solving, learning, and teaching. Mahwah, N.J.: Erlbaum. p. 317-336.)

Lester, F.K. 1994. Musings about mathematical problem-solving research: 1970-1994. *Journal of research in mathematics education*, 25:660-675.

Liebeck, P. 1984. How children learn mathematics. London: Penguin Books.

Lincoln, Y. & Guba, E. 1985. Naturalistic enquiry. Beverley Hills, Calif.: Sage.

Lincoln, Y. & Guba, E. 2000. Paradigmatic controversies, contradictions and emerging confluences. (In Denzin, N. & Lincoln, Y., ed. Handbook of qualitative research. London: Sage. p. 97-128.)

Lipman, M. 1998. Critical thinking: what can it be? (In Behar-Horenstein, L.S. & Ornstein, A.C., eds. Contemporary issues in curriculum. Boston, Mass.: Allyn & Bacon. p. 145-153.)

Livingston, C. & Borko, H. 1989. Expert-novice differences in teaching: a cognitive analysis and implications for teacher education. *Journal of teacher education*, 40(4):36-42.

Livingston, C. & Borko, H. 1990. High school mathematics review lesson: expert-novice distinctions. *Journal for research in mathematics education*, 21:372-387.

Long, C. 2008. What can we learn from TIMSS 2003? Pretoria: University of Pretoria. Centre for Evaluation and Assessment.

Long, H.B. 2005. Skills for self-directed learning. <http://faculty-staff.ou.edu/L/Huey.B.Long-1/Articles/sd/selfdirected.html>. Date of access: 30 July 2014.

Lopez, L. 2008. Helping at-risk students solve mathematical word problems through the use of direct instruction and problem solving strategies. Orlando, Fla.: University of Central Florida. (Thesis - PhD.)

Loyens, S.M.M., Magda J. & Rikers R.M.J.P. 2008. Self-directed learning in problem-based learning and its relationship with self-regulated learning. *Educational psychology review*, 20(4):411-427.

Many, J.E., Fyfe, R., Lewis, G. & Mitchell, E. 1996. Traversing the tropical landscape: exploring students' self-directed reading-writing-research process. *Reading research quarterly*, 31(1):12-35.

Maree, J.G. & Schoeman, W.J. 1997. Ontwikkeling van 'n studie-oriëntasievraelys in wiskunde. *SA tydskrif vir natuurwetenskap en tegnologie*, 16(3):127-134.

Maree, K. 2007. first steps in research. Pretoria: Van Schaik Publishers.

Marjoribanks, K. 1995. Factors affecting the learning environments and school related outcomes of Australian adolescents. *Journal of social psychology*, 135(1):89-95.

Marjoribanks, K. 2002. Family and school capital: towards a context and theory of students' school outcomes. Dordrecht: Kluwer.

Marshall, M.N. 1996. Sampling for qualitative research. *Family practice*, 13(6):522-526.

Martinez, M.E. 2006. What is metacognition? *Phi Delta Kappan*, 87(9):696-699.

Mayer, R. 1983. Thinking, problem solving, cognition. New York: Freeman.

McCutcheon, G. 1981. Elementary school teachers' planning for social studies and other subjects. *Theory and research in social education*, 9(1):45-66.

McMillan, J.H. & Schumacher, S. 2010. Research in education: evidence-based inquiry. 7th ed. Boston, Mass.: Pearson.

Merriam, S.B., Caffarella, R.S. & Baumgartner, L.M., eds. 2007. Learning in adulthood: a comprehensive guide. 3rd ed. San Francisco, Calif.: Wiley.

Metcalfe, J. 2000. Metamemory: theory and data. (In Tulving, E. & Craik, F.I., eds. The Oxford handbook of memory. New York: Oxford University Press. p. 197-211.)

Middleton, J.A. 1992. An analysis of the congruence of teachers' and students' personal constructs regarding intrinsic motivation in the mathematics classroom. Madison, Wis.: University of Wisconsin-Madison. (Thesis - PhD.)

Middleton, J.A. & Spanias, P.A. 1999. Motivation for achievement in mathematics: findings, generalization, and criticisms of the research. *Journal for research in mathematics education*, 30(1):66-88.

Midgley, C., Feldlaufer, H. & Eccles, J.S. 1989. Student/teacher relations and attitudes toward mathematics before and after transition to junior high school. *Child development*, 60:981-992.

Miller, C.A., Fitch, T. & Marshall, J.L. 2003. Locus of control and at-risk youth: a comparison of regular education high school students and students in alternative schools. *Education*, 123(3):548-552.

Ming-Tien, T. & Nia-Chang, C. 2005. The effect of self-efficacy and self-regulation on knowledge sharing behavior under social cognitive perspective. Tainan, Taiwan: National Cheng Kung University.

Mok, M.M., Cheng, Y., Leung, S., Shan, S.W., Moore, P. & Kennedy, K. 2007. Self-directed learning as a key approach to effectiveness in education: a comparison among mainland China, Hong Kong, Macau, and Taiwan. (In Townsend, T., ed. International handbook of school effectiveness and school improvement. Dordrecht/Boston: Springer. p. 839-858.)

Morris, C.G. & Maisto, A. 2001. Understanding psychology. 5th ed. Englewood Cliff, N.J.: Prentice Hall.

Morrow, L.M., et al. 1993. Promoting independent reading and writing through self-directed literacy activities in a collaborative setting. College Park, Md.: National Reading Research Center. (Reading research report no. 2.) (ED 356 455.)

Moyer, P. 2001. Are we having fun yet? How teachers use manipulatives to teach. *Education studies in mathematics*, 47(2):175-197.

Moyer, P. & Milewicz, E. 2002. Learning to question: categories of questions used by preservice teachers during diagnostic mathematics interviews. *Journal of mathematics teacher education*, 5(4):293-315.

Naidoo, J. 2011. Exploring master teachers' use of visuals as tools in mathematics classrooms. Durban: University of KwaZulu-Natal. (Thesis - PhD.)

Naidoo, J. 2012. Teacher reflection: the use of visual tools in mathematics classrooms. *Pythagoras*, 33(1):1-9.

National Council for Teachers of Mathematics (NCTM). 2000a. Professional standards for teaching mathematics. Reston, Va.: NCTM.

National Council of Teachers of Mathematics (NCTM). 2000b. Principles and standards for school mathematics. Reston, Va.: NCTM.

Nelson, S. & Conner, C. 2008. Developing self-directed learners. <http://www.nwrel.org/planning/reports/self-direct/self.pdf> Date of access: 6 October 2014.

New Zealand. Ministry of Education. 1992. Mathematics in the New Zealand curriculum. Wellington: Learning Media.

Nicholls, J.G. 1978. The development of the concepts of effort and ability, perception of academic attainment and the understanding that difficult tasks require more ability. *Child development*, 49:800-814.

Nieuwoudt, S.M. 2003. Strategiese onderrig en leer van skoolwiskunde in 'n videoklasstelsel. Potchefstroom: Potchefstroomse Universiteit vir CHO. (Thesis - PhD.)

Nieuwenhuis, J. 2007. Introducing qualitative research. (*In* Maree, K., ed. First steps in research. Pretoria: Van Schaik Publishers. p. 47-66.)

Paris, S.G. & Paris, A.H. 2001. Classroom applications of research on self-regulated learning. *Educational psychologist*, 36(2):89-101.

Peirce, W. 2003. Metacognition: study strategies, monitoring, and motivation. <http://academic.pg.cc.md.us/~wpeirce/MCCCTR/metacognition.htm> Date of access: 8 October 2014.

Pfeiffer, J. 2006. Self-directed learning in the middle school classroom. Action research projects. Math in the middle institute partnership. Lincoln, ebr.: University of Nebraska-Lincoln. (Paper 47.) <http://digitalcommons.unl.edu/mathmidactionresearch/47> Date of access: 1 February 2014.

Pintrich, P.R & De Groot, E. 1990. Motivational and self-regulated learning components of classroom academic performance. *Journal of educational psychology*, 82(1):33-40.

Polya, G. 1957. How to solve it: a new aspect of mathematical method. 2nd ed. Princeton, N.J.: Princeton University Press.

Polya, G. 1973. How to solve it. Princeton, N.J.: Princeton University Press.

Rachal, K.C., Daigle, S. & Rachal, W.S. 2007. Learning problems by college students: are they using learning strategies? *Journal of instructional psychology*, 34(4):191-199.

Reitman, W.R. 1965. Cognition and thought: an information processing approach. New York: Willey.

Rhine, S. 1998. The role of research and the teachers' knowledge base in professional development. *Educational researcher*, 27-31.

Rigelman, N. 2007. Fostering mathematical thinking and problem solving: the teacher's role. *Teaching children mathematics*, 13(6):308-314.

Rosebery, A. 2005. What are we going to do next? Lesson planning as a resource for teaching. (In Nemirovsky, R., Rosebery, A., Solomon, J. & Warren, B., eds. *Everyday matters in science and mathematics*. Mahwah, N.J.: Earlbau. p. 299-327.)

Rosenstein, J.G. 2006. Discrete mathematics in 21st century education: an opportunity to retreat from the rush to calculus. (In Lesh, R., Hamilton, E. & Kaput, J., eds. *Models and modeling as foundations for the future in mathematics education*. Mahwah, N.J.: Erlbaum. p. 58-79.)

Rutter, M. & Maughan, B. 2002. School effectiveness findings 1979-2002. *Journal of school psychology*, 40(6):451-475.

Ryan, R.M. & Deci, E.L. 2000. Intrinsic and extrinsic motivations: classic definitions and new directions. *Contemporary educational psychology*, 25(1):54-67.

Sansone, C. & Morgan, C. 1992. Intrinsic motivation and education: competence in context. *Motivation and emotion*, 16(3):249-270.

Savin-Baden, M. & Major, C.H. 2004. Foundations of problem-based learning. Maidenhead, Berkshire: SRHE & Open University Press.

Schoenfeld, A.H. 1992. Learning to think mathematically: *problem solving, metacognition, and sense making in mathematics*. (In Gouws, D., ed. Handbook of research on mathematics teaching and learning. New York: McMillan. p. 334-370.)

Schoenfeld, A.H. 1985. Mathematical problem solving. Orlando, Fla.: Academic Press.

Schunk, D.H. 2001. Social cognitive theory and self-regulated learning. (In Zimmerman, B.J. & Schunk, D.H., eds. Self-regulated learning and academic achievement: theoretical perspectives. Mahwah, N.J.: Erlbaum. p. 125-151.)

Seng, T.O., Parson, R.D., Hinson, S.L. & Sardo-Brown, D. 2003. Educational psychology. Singapore: Thomson Learning.

Sexton, S. 2011. What is self-testing and why should I do it? <https://www.roanestate.edu/webfolders/hillm/study-help/What%20is%20self-testing%20and%20why%20should%20I%20do%20it.pdf>.
Date of access: 1 February 2014.

Shank, G. 2002. Qualitative research: a personal skills approach. Upper Saddle River, N.J.: Merrill Prentice Hall.

Shannon, S.V. 2008. Using metacognitive strategies and learning styles to create self-directed learners. *Institute for Learning Styles journal*, 1:14-28

Shen, W.-Q., Chen, H.-L. & Hu, Y. 2014. The validity and reliability of the self-directed learning instrument (SDLI) in mainland of Chinese nursing students. *BMC medical education*, 14(1):108.

Skinner, B.F. 1938. The behavior of organisms. New York: Appleton-Century-Crofts.

Slavin, R.E. 2006. Educational psychology: theory and practice. Boston, Mass.: Pearson/Allyn & Bacon.

Smith, E. & Confrey, J. 1991. Understanding collaborative learning: small group work on contextual problems using a multi-representational software tool. (Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, April 3-7.)

Snowman, J. 1993. Psychology applied to teaching. 7th ed. Boston, Mass.: Houghton Mifflin.

South Africa. Department of Basic Education. 2011a. Report on the Annual National Assessment of 2011. Pretoria: Department of Education.

South Africa. Department of Basic Education. 2011b. Curriculum and assessment policy statement (CAPS). Mathematics: senior phase, final draft. Pretoria: Department of Basic Education.

South Africa. Department of Education. 2008. National curriculum statement grade 10-12 (General). Pretoria: Department of Education.

South Africa. Department of Education. 2012. Annual National Assessments 2012: a guide for the interpretation and use of ANA results. Pretoria: Department of Education

Souviney, R.J. 1994. Learning to teach mathematics. 2nd ed. New York: Merrill.

Stacey, K. 2005. The place of problem solving in contemporary mathematics curriculum documents. *Journal of mathematical behavior*, 24(3):341-350.

Stein, M.K. & Smith, M.S. 1998. Mathematical tasks as a framework for reflection: from research to practice. *Mathematics teaching in the middle school*, 3(4):267-275.

Steyn, H.S. 2005. Handleiding vir die bepaling van effektegrootte-indeksie en praktiese betekenisvolheid. Potchefstroom: Potchefstroomse Universiteit vir CHO. <http://www.puk.ac.za/fakalteite/natuur/skd/indeks.html>. Date of access: 01 March 2015.

Stokes, S. 2000. Visual literacy in teaching and learning: a literature perspective. *Electronic journal for the integration of technology in education*, 1(1):10-19.

Superfine, A.C. 2008. Planning for mathematics instruction: a model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics educator*, 18(2):11-22.

Surgenor, P. 2010. Teaching toolkit: how students learn: effect of assessment on learning. Dublin: University College Dublin.

Tambychik, T. & Meerah, T.S. 2010. Students' difficulties in mathematics problem solving: what do they say? (International Conference on Mathematics Education Research 2010.) *Procedia - social and behavioral sciences*, 8:142-151

Tashakkori, A. & Teddlie, C. 1998. Mixed methodology: combining qualitative and quantitative approaches. London: Sage.

Taylor, B. 1995. Self-directed learning: revisiting an idea most appropriate for middle school students. (Paper presented at the Combined Meeting of the Great Lakes and Southeast International Reading Association, Nashville, Tenn., Nov 11-15.) [ED 395 287.]

Temple, C. & Roderio, M.L. 1995. Active learning in a democratic classroom: the "pedagogical invariants": of Celestin Freinet (Reading around the world). *Reading teacher*, 49(2):164-167.

Thalheimer, W. & Cook, S. 2002. How to calculate effect sizes from published research articles: a simplified methodology. *Work-learning research*: 1-9. http://work-learning.com/effect_sizes.htm. Date of access: 2 March 2015.

Thornton, K. 2013. A framework for curriculum reform: re-designing a curriculum for self-directed language learning. *Studies in self-access learning journal*, 4(2):142-153.

Towle, A. & Cottrell, D. 1996. Self-directed learning. *Archives of disease in childhood*, 74(4):357-359.

Unal, H. & Jakubowski, E. 2007. Middle and secondary preservice mathematics teachers' comparative analysis of TIMSS videotape lesson study. *Turkish online journal of educational technology (TOJET)*, 6(3).

Van de Walle, J.A. 2006. Elementary and middle school mathematics: teaching developmentally. 5th ed. Boston, Mass.: Pearson/Allyn & Bacon.

- Van de Walle, J.A., Karp, K.S. & Bay-Williams, J.M. 2013. Elementary and middle school mathematics: teaching developmentally. Upper Saddle River, N.J.: Pearson Education.
- Van den Broek, P., Lorch, R.R., Linderholm, T. & Gustafson, M. 2001. The effects of readers' goals on inference generation and memory for texts. *Memory & cognition*, 29(8):1081-1087.
- Van Deur, P. 2004. Gifted primary students' knowledge of self-directed learning. (Education Research Conference 2003. Special issue.) *International education journal*, 4(4):64-74.
- Van Deur, P. & Murray-Harvey, R. 2005. The inquiry nature of primary schools and students' self-directed learning knowledge. *International education journal*, 5(5):166-177.
- Walberg, H.J. 1982. Educational climates. (In Walberg, H.J., ed. Improving educational standards and productivity: the research basis for policy. Berkeley, Calif.: McCuthan. p. 289-303.)
- Weinstein, C.E., et al. 2006. Teaching students how to become more strategic and self-regulated learners. (In McKeachie, W.J. & Svinicki, M., eds. Teaching tips: strategies, research, and theory for college and university teachers. 12th ed. Lexington, Mass.: Houghton Mifflin. p. 300-317.)
- Weinstein, C.E., Acee, T.W. & Jung, J. 2011. Self-regulation and learning strategies. *New directions for teaching and learning*, 126:45-53.
- Weinstein, C.E. & Meyer, D.K. 1991. Cognitive learning strategies. *New directions for teaching and learning*, 45:15-26.
- Weinstein, C.E. & Palmer, D.R. 1990. Learning and study strategies inventory: high school version: user's manual. Clearwater, Fla.: H & H Publishing Company.
- Westwood, P. 1997. Commonsense methods for children with special needs. 3rd ed. London: Prentice Hall.
- White, R.W. 1959. Motivation reconsidered: the concept of competence. *Psychological review*, 66(5):297-333.

Wigfield, A., Eccles, J.S. & Rodriguez, D. 1998. The development of children's motivation in school contexts. (*In* Pearson, D.P. & Iran-Nejad, A. Review of research in education. Washington, D.C.: American Educational Research Association. p. 73-118.)

Williams, B. 2001. The theoretical links between problem-based learning and self-directed learning for continuing professional nursing education. *Teaching in higher education*, 6(1):85- 98.

Williamson, S.N. 2007. Development of a self-rating scale of self-directed learning. *Nurses researcher*, 14(2):66-83.

Wilson, J.W., Fernandez, M.L. & Hadaway, N. 1993. Mathematical problem solving. (*In* Wilson, P.S., ed. Research ideas for the classroom: high school mathematics. New York: MacMillan. p. 57-78.).

Wilson, V.E., Peper, E. & Schmid, A. 2006. Training strategies for concentration. (*In* Williams, J.N., ed. Applied sport psychology: personal growth to peak performance. 5th ed. Boston, Mass.: McGraw-Hill. p. 404-422.)

Winne, P. 1991. Motivation and teaching. (*In* Waxman, H. & Walberg, H., eds. Effective teaching: current research. New York: McCutchan. p. 295-314.)

Winne, P.H. 1995. Inherent details in self-regulated learning. *Educational psychologist*, 30(4):173-188.

Winne, P.H. & Hadwin, A.F. 1998. Studying as self-regulated learning. (*In* Hacker, D.J. & Dunlosky, J., eds. Metacognition in educational theory and practice. Mahwah, N.J.: Earlbaum. p. 27-30.).

Wolters, C.A. 2003. Regulation of motivation: evaluating an underemphasized aspect of self-regulated learning. *Educational psychologist*, 38(4):189-205.

Wood, J.T. 2010. Interpersonal Communication: everyday encounters. 6th ed. Boston, Mass.: Wadsworth, Cengage Learning.

Zakaria, E.Y. & Nordin, N.M. 2008. The effects of mathematics anxiety on matriculation students as related to motivation and achievement. *Eurasia journal of mathematics, science & technology education*, 4(1):27-30.

Zimmerman, B.J. 2001. Theories of self-regulated learning and academic achievement: An overview and analysis. (In Zimmerman, B. & Schunk, D., ed. *Self-regulated learning and academic achievement: theoretical perspectives*. Mahwah, N.J.: Erlbaum. p.1-65.)

Zimmerman, B.J. 2004. Sociocultural influence and students' development of academic self-regulation: A social-cognitive perspective. (In McInerney, D.M. & Van Etten, S., eds. *Big theories revisited*. Greenwich, Conn.: Information Age Publishing. p.139-164.)

Zumbrunn, S., Tadlock, J. & Roberts, E.D. 2011. Encouraging self-regulated learning in the classroom: a review of the literature. Richmond, VA.: Virginia Commonwealth University. Metropolitan Educational Research Consortium (MERC).

APPENDIX A:

PERMISSION FROM THE NORTH WEST DEPARTMENT OF EDUCATION

(Date)

The Superintendent General

North West Department of Education

PERMISSION TO CONDUCT RESEARCH at ALABAMA PRIMARY SCHOOL and GOUE AREND

My name is Shain Jurie Hofmeyer, and I am a student at the North West University at Potchefstroom Campus. The research I wish to conduct for my Master's dissertation involves "the influence of mathematical problem solving on self-directed learning in a grade 7 mathematics classroom". This research project will be conducted under the supervision of Dr. S.M. Nieuwoudt (NWU, South Africa).

I am hereby seeking your consent to approach Alabama Primary School as well as Goue Arend Primary School in the Dr. Kenneth Kaunda District to provide participants for this research project.

I am currently a teacher at Alabama Primary School and have been teaching Mathematics for the past eighteen years at the same institution. I have experienced that learners are struggling with self-directed learning aspects such as self-monitoring, self-management, self-discipline and self-confidence and successful completion of problem solving. That led me to the question: How can I assist the learners in my class to overcome this situation and hopefully assist other educators that experience the same difficulties in their mathematics classes? That gave me motivation to embark on this specific research project.

I have provided you with a copy of my dissertation proposal which includes copies of the measure, consent and assent forms to be used in the research process, as well as a copy of the approval letter which I received from the NWU Research Ethics Committee.

Upon completion of the research study, I undertake to provide the Northwest Department of Education with a bounded copy of the full research report. If you require any information, please do not hesitate to contact me at sjhofmeyer@webmail.co.za or on 083 278 7685.

Thank you for your time and consideration in this matter. Yours
sincerely

Shain J. Hofmeyer

North West University Potchefstroom Campus

PERMISSION LETTER TO THE PRINCIPAL (EXPERIMENTAL GROUP)

(Date)

Dear Mr O.J.P. Hennicks

I, Mr Shain J. Hofmeyer, hereby ask permission to do research for my Master's degree in Mathematics teaching at your school. I have already applied for permission and was granted by the North West Department of Education.

The research project entails "problem solving and self-directed learning in the grade 7 mathematics classroom". I, the researcher, am currently a teacher at Alabama Primary School and have been teaching Mathematics for the past eighteen years at the same institution. I have experienced that learners are struggling with self-directed learning aspects such as self-monitoring, self-management, self-discipline and self-confidence and successful completion of problem solving. That led me to the question: How can I assist the learners in my class to overcome this situation and hopefully assist other educators that experience the same difficulties in their mathematics classes? That gave me motivation to embark on this specific research project.

The actual research includes answering of questionnaires (pre- and post-test) as well as semi-structured interviews with selected participants. There will also be an intervention period where I will introduce problem solving as a teaching strategy in an attempt to improve the self-directedness of the learners. The research project will start at the beginning of the second school term and will be concluded at the end of the fourth school term.

I assure you that all information will be handled with utmost confidentiality and that neither names of participants nor the name of your school will be mentioned in the analyzing and publication of the data.

I intend to get permission from the parents/guardians as well as from the learners before the research project will commence.

I will appreciate it if you could inform me as soon as possible regarding your decision.

Regards

Shain J. Hofmeyer

PERMISSION LETTER TO THE PRINCIPAL (CONTROL GROUP)

(Date)

Dear (Principal)

I, Shain J. Hofmeyer, hereby ask permission to do research for my Master's degree in Mathematics teaching at your school. I have already applied for permission and was granted by the North West Department of Education.

The research project entails "problem solving and self-directed learning in the grade 7 mathematics classroom". I, the researcher, am currently a teacher at Alabama Primary School and have been teaching Mathematics for the past eighteen years at the same institution. I have experienced that learners are struggling with self-directed learning aspects such as self-monitoring, self-management, self-discipline and self-confidence and successful completion of problem solving. That led me to the question: How can I assist the learners in my class to overcome this situation and hopefully assist other educators that experience the same difficulties in their mathematics classes? That gave me motivation to embark on this specific research project.

With your permission, I will utilise your school (.....) as the control group of the research project. Where, the grade 7 mathematics learners will only be asked to complete pre- and post-test questionnaires. There will also be a request for your grade 7 final results over the past three (3) years to enable me to make an informed comparison regarding the research project.

I assure you that all information will be handled with utmost confidentiality and that neither names of participants nor the name of your school will be mentioned in the analyzing and publication of the data.

I intend to get permission from the parents/guardians as well as from the learners before the research project will commence.

I will appreciate it if you could inform me as soon as possible regarding your decision.

Regards

Shain J. Hofmeyer

PERMISSION LETTER TO PARENTS (EXPERIMENTAL GROUP)

(Date)

Dear Parents / Guardians

Re: Invitation to your child to be part of a research project

I am currently an educator at ALABAMA PRIMARY SCHOOL and am busy with research aimed at establishing the influence of problem solving on self-directed learning in the mathematics grade 7 classrooms.

Self-directed learning is being defined as a lifelong learning experience, where the learner takes control and responsibility for his/her own learning and learning experiences. Mathematics as a subject are seen as very important in the South African context, however the results nationwide does not speak to the importance of mathematics as a subject. Mathematics problem solving and self-directed learning needs to be addressed according to my experience as a mathematics educator, in an effort to address the dismal mathematics results.

The learners that participate in the project will be required to complete two sets of questionnaires (one at the beginning of the research and one at the end of the research project). A random selection of learners will take place to participate in semi-structured interviews based on tasks done. No learners names will be mentioned in the study and confidentiality of the participants will be protected throughout the research study. This research study will take place during the second and third school term of 2013. Participation in the research project is voluntary. Any learner can withdraw from the project at any time if he/she decides to do so, without having to give a reason.

If you as a parent / guardian or the learner have any questions regarding the research project at any time, feel free to contact me. Your permission is therefore necessary for your child to participate in this research project, because lawfully, your child is still a minor.

PERMISSION: I have read the above information and understand the nature of the research study. I accept that the researcher will not undermine my child's human rights in any way. I understand that I can contact the researcher at any time during the research period. I thus give permission for my child to participate in the research study.

Participant's Name and Surname:	_____	Grade 7	_____
Parent/Guardian Name and Surname:	_____		
Parent Telephone Number:	_____		
Parent / Guardian Signature	_____	Date:	_____
Researcher's Signature	_____	Date:	_____

Sincerely Yours

S.J. Hofmeyer (Researcher)

PERMISSION LETTER TO PARENTS (CONTROL GROUP)

(Date)

Dear Parents / Guardians of Primary

Re: Invitation to your child to be part of a research project

I am currently an educator at ALABAMA PRIMARY SCHOOL and am busy with research aimed at establishing the influence of problem solving on self-directed learning in the mathematics grade 7 classrooms.

Self-directed learning is being defined as a lifelong learning experience, where the learner takes control and responsibility for his/her own learning and learning experiences. Mathematics as a subject are seen as very important in the South African context, however the results nationwide does not speak to the importance of mathematics as a subject. Mathematics problem solving and self-directed learning needs to be addressed according to my experience as a mathematics educator, in an effort to address the dismal mathematics results.

The learners that participate in the project will be required to complete two sets of questionnaires (one at the beginning of the research and one at the end of the research project). No learners names will be mentioned in the study and confidentiality of the participants will be protected throughout the research study. This research study will take place during the second and third school term of 2013. Participation in the research project is voluntary. Any learner can withdraw from the project at any time if he/she decides to do so, without having to give a reason.

If you as a parent / guardian or the learner have any questions regarding the research project at any time, feel free to contact me. Your permission is therefore necessary for your child to participate in this research project, because lawfully, your child is still a minor.

PERMISSION: I have read the above information and understand the nature of the research study. I accept that the researcher will not undermine my child's human rights in any way. I understand that I can contact the researcher at any time during the research period. I thus give permission for my child to participate in the research study.

Participants Name and Surname: _____ Grade 7 _____

Parent/Guardian Name and Surname: _____

Parent Telephone Number: _____

Parent / Guardian Signature _____ Date: _____

Researcher's Signature _____ Date: _____

Sincerely Yours
S.J. Hofmeyer (Researcher)

PERMISSION FROM LEARNERS/PARTICIPANTS (EXPERIMENTAL AND CONTROL GROUPS)

(Date)

Dear Participant

Permission to participate in research study

I hereby agree to participate in the research study that involves “problems solving and self-directed learning” in the grade 7 mathematics classroom. I understand that the research study will be conducted from the beginning of the second term to the end of the third term.

My participation in the study will involve completing two questionnaires and possible participation in a task orientated semi-structured interviews.

I understand that my identity and opinions will be kept in confidence throughout the research study. I also understand that all questionnaires will be kept in a safe place for a period of time and data or information collected will be used in the research study.

I confirm that I understand the aims and objectives of the study and understand that my participation is completely voluntary and can withdraw my participation at any time without any consequences.

I hereby give informed permission for the use of opinions in connection with problem solving and self-directed learning in the grade 7 mathematics classroom.

Participants Name and Surname: _____

Grade 7: _____

Date: _____

School: _____

APPENDIX B:

ADAPTED LASSI QUESTIONNAIRE AFRIKAANS

LEER- EN STUDIESTRATEGIE VRAELYS - LSV (SKOOL)

Die LSV (skool) is die Afrikaanse weergawe van die LASSI-HS
(Learning And Study Strategies Inventory - High School Version)

wat ontwikkel is deur: Clair E Weinstein & David R Palmer
Department of Educational Psychology, University of Texas at Austin

Die LASSI-HS is in Afrikaans vertaal en vir Suid-Afrikaanse Wiskundeleerders aangepas deur: JL deK Monteith, HD Nieuwoudt en SM Nieuwoudt (NWU, Potchefstroom)

INSTRUKSIES

NOMMER VAN VRAELYS

--	--	--

Die LSV (skool) is ontwerp om vas te stel hoe jy leer en studeer en om jou houding teenoor leer en studeer te bepaal. Op die volgende bladsye is 76 bewerings wat met die leer of studeer van wiskunde te doen het. Lees elke bewering en merk dan een van die volgende beskrywings.

1. Glad nie kenmerkend van my nie	2. Nie baie kenmerkend van my nie	3. In 'n mate kenmerkend van my	4. Redelik/taamlik kenmerkend van my	5. Baie kenmerkend van my
-----------------------------------	-----------------------------------	---------------------------------	--------------------------------------	---------------------------

Probeer antwoord in terme van hoe goed die bewering jou beskryf en nie in terme van hoe jy dink jy behoort te wees of van wat ander doen nie. Daar is geen regte of verkeerde antwoorde op hierdie bewerings nie. Werk asseblief so vinnig as moontlik, sonder om agterlosig te wees en voltooi al die items.

Stelling	1	2	3	4	5
1. Dit is vir my moeilik om by 'n studierooster vir wiskunde te hou.					
2. Na 'n wiskundeklas gaan ek weer deur die werk wat ons in die klas gedoen het om my te help om dit beter te verstaan.					
3. Gedurende 'n wiskundeles dink ek aan ander dinge en luister nie regtig na wat in die klas gesê word nie.					
4. Ek gebruik spesiale studiehulpmiddels soos hoofopskrifte en skuins- of vetgedrukte woorde wat in my wiskundehandboek voorkom.					
5. Ek is op datum met my wiskunde-opdragte					
6. Probleme buite skoolverband soos finansiële probleme, konflik met ouers, verliefdheid, ensovoorts, veroorsaak dat ek nie my wiskunde doen nie.					
7. Al is gedeeltes van die wiskunde wat ons doen, eentonig en oninteressant, kry ek dit reg om daarmee aan te hou totdat ek klaar is.					
8. Ek kom onvoorbereid na 'n wiskundeklas.					
9. Wanneer ek vir 'n wiskundetoets of -eksamen voorberei, dink ek na oor vrae wat ek dink moontlik gevra kan word.					
10. Die aantekeninge wat ek maak terwyl ek in my wiskundehandboek lees, maak dit vir my maklik om die werk te hersien.					
11. Ek probeer om aan moontlike toetsvrae te dink terwyl ek die werk wat in die wiskundeklas gedoen is, deurgaen.					
12. Ek studeer (leer) wiskunde slegs wanneer ek 'n toets moet skryf.					
13. Ek vergelyk my wiskunde klaswerk/huiswerk met dié van ander leerlinge om seker te maak dat dit korrek is.					
14. Ek hersien die wiskunde van die vorige les voor die volgende les.					
15. Ek werk hard om goeie punte te behaal, al hou ek nie van die wiskunde wat gedoen word nie.					
16. Terwyl ek besig is om wiskunde te doen, stop ek kort-kort en dink daaroor na of hersien dit wat ek gedoen het.					
17. Wanneer ek nie 'n huiswerkopdrag gedoen het nie, oortuig ek myself dat ek 'n verskoning daarvoor het.					

1. Glad nie kenmerkend van my nie	2. Nie baie kenmerkend van my nie	3. In 'n mate kenmerkend van my	4. Redelik/taamlik kenmerkend van my	5. Baie kenmerken d van my
18. Dit is vir my soms moeilik om op my wiskundewerk te konsentreer omdat ek rusteloos of buiering is.				
19. Ek toets myself tydens 'n wiskundeles om te sien of ek verstaan wat die onderwyser sê.				
20. Ek stel hoë eise of mikpunte aan myself in wiskunde.				
21. Dit gebeur dikwels dat ek voor 'n wiskundetoets baie kennis in 'n kort tyd in my kop moet probeer inprop.				
22. Dit is vir my moeilik om gedurende 'n wiskundeles te konsentreer				
23. Wanneer ek my wiskundehandboek lees, gee ek veral aandag aan die eerste en/of laaste sin van die meeste paragrawe.				
24. My aandag word maklik van die wiskunde wat ek doen, afgetrek.				
25. Ek maak goed gebruik van studietye na skool om ook wiskunde te doen.				
26. Wanneer ek wiskunde doen wat vir my moeilik is, doen ek net die maklike gedeeltes of los dit sommer heeltemal uit.				
27. Ek maak diagramme of sketse om my te help om die wiskunde wat ek leer, te verstaan.				
28. Ek gebruik simbole, sleutelwoorde, diagramme of tabelle om my wiskunde op te som.				
29. Ek verstaan sommige gedeeltes van die wiskunde swak omdat ek nie deeglik luister nie.				
30. Ek gebruik my wiskundehandboek om werksopdragte uit te voer.				
31. Wanneer ek besluit om wiskundehuiswerk te doen, sit ek 'n bepaalde hoeveelheid tyd daarvoor opsy en hou dan daarby.				
32. Ek konsentreer ten volle wanneer ek wiskunde leer.				
33. Ek gebruik die opskrifte van paragrawe en die gedeeltes binne-in raampies om my te help om belangrike gedeeltes in my wiskundehandboek raak te sien.				
34. Ek toets myself om seker te maak dat ek die wiskunde wat ek geleer het, ken.				
35. Ek stel die wiskunde wat ek behoort te doen, te dikwels uit.				
36. My gedagtes dwaal baie terwyl ek wiskunde doen.				
37. Wanneer ek wiskunde hersien, gaan ek ook my huiswerkopdragte na.				
38. As ekstra wiskundeklasse of hersieningsessies aangebied word, woon ek dit by.				
39. Ek bring soveel tyd saam met my vriende deur dat my wiskunde daaronder ly.				

APPENDIX C:

ADAPTED LASSI QUESTIONNAIRE

LEARNING AND STUDY STRATEGIES INVENTORY – HIGH SCHOOL VERSION

by Claire E. Weinstein & David Palmer, Department of Educational Psychology, University of Texas at Austin.

Adapted for South African Mathematics Learners by JL de K. Monteith, HD Nieuwoudt and S.M. Nieuwoudt (NWU, Potchefstroom)

--	--	--

NUMBER OF QUESTIONNAIRE

The Learning and Study Strategies Inventory (LASSI) is designed to find out how you learn, how you study, and how you feel about learning and studying. On the following pages you will find 39 statements about the learning and studying of mathematics.

Read each statement and then mark one of these choices.

1. Not at all like me	2. Not very much like me	3. Somewhat like me	4. Fairly much like me	5. Very much like me
------------------------------	---------------------------------	----------------------------	-------------------------------	-----------------------------

Try to answer according to how well the statement describes you, not how you think you should be or what others do. There are no right or wrong answers to these statements. Please work as quickly as you can without being careless and please answer all the items.

Statements	1	2	3	4	5
1. I find it difficult to stick to a study timetable for maths.					
2. After a maths class, I look over the work we did in class to help me better understand it.					
3. I think of other things during the maths lesson and don't really listen to what is being said in class.					
4. I use special study aids, such as main headings and words printed in italics or bold face, that are in my maths textbook.					
5. I am up-to-date in my maths assignments.					

6. Problems outside of school such as financial problems, conflict with parents, dating (being in love), etc. cause me to not do my maths.					
7. Even when some parts of the maths are dull and not interesting, I manage to keep working until I finish.					
8. I come to a maths class unprepared.					
9. When studying for a maths test or exam, I think of questions that I think might be asked.					
10. The notes I take as I read my maths textbook are helpful when I review the work.					
11. I try to think of possible test questions when studying work done in the maths class.					
12. I only study maths when I have to write a test.					
13. I compare my maths class work / homework with other students to make sure it is correct.					
14. I review the maths of the previous lesson before the next lesson.					
15. I work hard to get good marks in maths, even when I don't like the maths being done.					
16. I stop often while doing maths and think over or review what I have been doing.					

Statements	1	2	3	4	5
17. I talk myself into believing some excuse for not doing a homework assignment in maths.					
18. I am sometimes unable to keep my mind on my maths work because I am restless or moody.					
19. I check to see if I understand what my teacher is saying during a maths lesson					
20. I set high standards or goals for myself in maths.					
21. I end up "cramming" (learning a lot of maths in a very short period) for almost every test.					
22. I find it hard to pay attention during a maths lesson.					
23. I pay special attention to the first and/or last parts of most paragraphs when reading my maths textbook.					
24. I am very easily distracted from the maths I'm doing.					

25.	I make good use of study hours after school to also study maths.					
26.	When doing maths which is difficult for me I either give up or study only the easy parts.					
27.	I make drawings or sketches to help me understand the maths I am studying.					
28.	I use symbols, key words, diagrams, or tables in summarising my maths.					
29.	I don't understand some sections of maths because I do not listen carefully.					
30.	I use my maths textbook to prepare assignments.					
31.	When I decide to do my maths homework, I set aside a certain amount of time and stick with it.					
32.	I pay attention fully when studying maths.					
33.	I use the headings of paragraphs and sections inscribed in blocks as guidelines for important ideas in my maths textbook.					
34.	I test myself to be sure I know the maths I have been studying.					
35.	I put off the maths I'm supposed to do more than I should.					
36.	My mind wanders a lot when I do maths.					
37.	I go over homework assignments when reviewing maths done in class.					
38.	When they are available, I go to review sessions or extra classes in maths.					
39.	I spend so much time with my friends that my maths suffers.					

APPENDIX D

The **SELF-DIRECTED LEARNING INSTRUMENT (SDLI)** is developed as a self-rating instrument to measure the self-directed learning ability of students.

Directions:

Number of questionnaire

--	--	--

Please read each statement and make an (x) in the block that best describes how you think and feel about your own learning. There is no right or wrong answer.

1. Never	2. Seldom	3. Sometimes	4. Often	5. Always
----------	-----------	--------------	----------	-----------

Statements	1	2	3	4	5
1. I identify my own learning needs.					
2. I am able to stay self-motivated.					
3. My inner drive directs me towards further development and improvement in my learning.					
4. I find that both success and failure inspire me to further learning.					
5. I regard problems as challenges.					
6. I will not give up learning because I face some difficulties.					
7. I am able to set and plan my own learning goals.					
8. I can decide my own learning strategies.					
9. I am responsible for my own learning.					
10. I am able to select the best method for my own learning.					
11. I am good at planning and managing my own learning time.					
12. I know how to find resources for my learning.					
13. I am able to connect new knowledge with my own personal experience.					
14. I am able to identify the strengths and weaknesses of my learning.					
15. I am able to monitor my learning progress.					
16. I monitor whether I have accomplished my learning goals.					
17. My interaction with others helps me to plan for further learning.					
18. I intend to learn more about other cultures and languages I am frequently exposed to.					
19. I am successful in communicating verbally.					
20. I am able to express my ideas effectively in writing.					

APPENDIX E:

EXAMPLE OF TASK BASED ACTIVITY

1. Study the following problem:

The soccer field is 50m long and 30m wide. The ground-man needs to draw a line to indicate the perimeter of the soccer field. How long will this line be?

2. Do you think you can solve the problem correctly? **Circle your choice.**

a.	I am confident I can solve the problem correctly.
b.	I am reasonably sure I can solve the problem correctly.
c.	I am not sure how correct I can solve the problem.
d.	I am not sure how correct I can solve the problem. I think I might make a mistake.
e.	I know that I will make a mistake in solving the problem.

3. Explain in words how you would solve the problem.

4. Solve the problem now. Show all your steps.

5. Do you think you have solved the problem correctly? **Circle your choice.**

a.	I am confident I have solved the problem correctly.
b.	I am reasonably sure I have solved the problem correctly.
c.	I am not sure I have solved the problem correctly.
d.	I am not sure how correctly I have solved the problem. I think that I might have made a mistake.
e.	I know I have made a mistake.

6. Do you think about what you have done? Describe the strategies or plans you used when solving the problem.

7. Which type of mistake do you make when solving similar problems?

APPENDIX F:

CHECKING OF BIBLIOGRAPHY

Gerrit Dekker Street
POTCHEFSTROOM
2531
7 December 2015

Mr Shain Hofmeyer
NWU (Potchefstroom Campus)
POTCHEFSTROOM

CHECKING OF BIBLIOGRAPHY

Hereby I declare that I have checked the technical correctness of the Bibliography of Ms Shain Hofmeyer according to the prescribed format of the Senate of the North-West University.

Yours sincerely

A handwritten signature in black ink, appearing to read 'CJH Lessing', is written on a light blue grid background.

Prof CJH LESSING

APPENDIX G: LANGUAGE DECLARATION



6 December 2015

I, Ms Cecilia van der Walt, hereby confirm that I took care of the editing of the thesis of Mr Shain Hofmeyer titled *THE RELATIONSHIP BETWEEN PROBLEM-SOLVING AND SELF-DIRECTED LEARNING IN GRADE 7 MATHEMATICS CLASSROOMS*.

MS CECILIA VAN DER WALT

BA (*Cum Laude*)

HOD (*Cum Laude*),

Plus Language editing and translation at Honours level (*Cum Laude*),

Plus Accreditation with SATI for Afrikaans and translation

Registration number with SATI: 1000228

Email address: ceciliavdw@lantic.net

Mobile: 072 616 4943

Fax: 086 578 1425

APPENDIX H:

PERMISSION TO CONDUCT RESEARCH



education

Lefapha la Thuto
Onderwys Departement
Department of Education
NORTH WEST PROVINCE

Teemane Building
8 OR Tambo Street
Private Bag X1256
POTCHEFSTROOM 2520
Tel: (018) 299 8299
Fax: (018) 299 8290
Enquiries: Ms Carabilla Mokubung
E-mail: emokubung@nwpg.gov.za

DR KENNETH KAUNDA DISTRICT

OFFICE OF THE CHIEF DIRECTOR

To : S.J Hofmeyer

From : Dr S.H Mvula - Chief Director

Date : 06 May 2013

PERMISSION TO CONDUCT RESEARCH

We hereby acknowledge receipt of your letter dated 30 April 2013. Please note that your request to the above subject has been granted under the following provisions:

1. The activities you undertake at the two schools (Alabama Primary and Goue Arend Primary) should not tamper with the normal working process;
2. You inform the Area Manager, Circuit Manager and the School of your impending visit and activity; and
3. You obtain prior permission from this office before availing your findings for public or media consumption.

Wishing you well in your endeavour.

Thanking you

DR S.H MVULA
CHIEF DIRECTOR

APPENDIX I:

STATISTIC CONSULTATION



Privaatsak X6001 Potchefstroom 2520
Tel (018) 299 1111 Faks (018) 299 2799
<http://www.puk.ac.za>

Statistiese Konsultasiediens

Tel: (018) 299 2017

Faks: (018) 299 2557

4 Desember 2015

Re: M. Ed. verhandeling: Mnr. S.J. Hofmeyer, studentenommer 12895911

Hiermee word bevestig dat Statistiese Konsultasiediens die data verwerk het en ook betrokke was by die interpretasie van die resultate. Enige opinie, bevinding of aanbeveling uitgespreek in die dokument is egter dié van die outeur en Statistiese Konsultasiediens van NWU (Potchefstroomkampus) neem nie verantwoordelikheid vir die statistiese korrektheid van die gerapporteerde data nie.

Vriendelike groete

A handwritten signature in black ink, appearing to read 'H.S. Steyn'.

Prof. H.S. Steyn (Pr. Sci. Nat.)
Statistiese Konsultant