

MODELLING THE EFFECTIVE THERMAL CONDUCTIVITY IN THE NEAR-WALL REGION OF A PACKED PEBBLE BED

**Werner van Antwerpen
12279765**

Thesis submitted for the degree Doctor of Philosophy at the Potchefstroom campus of the
North-West University

Promoter: Prof. P.G. Rousseau
Co-promoter: Prof. C.G. du Toit

November 2009



ABSTRACT

Name: Werner van Antwerpen

Title: Modelling the effective thermal conductivity in the near-wall region of a packed pebble bed.

Date: November 2009

Inherent safety is claimed for gas-cooled pebble bed reactors, such as the South African Pebble Bed Modular Reactor (PBMR), as a result of its design characteristics, materials used, fuel type and physics involved. Therefore, a proper understanding of the mechanisms of heat transfer, fluid flow and pressure drop through a packed bed of spheres is of utmost importance in the design of a high temperature Pebble Bed Reactor (PBR). In this study, correlations describing the *effective thermal conductivity* through packed pebble beds are examined. The effective thermal conductivity is a term defined as representative of the overall radial heat transfer through such a packed bed of spheres, and is a summation of various components of the overall heat transfer.

This phenomenon is of importance because it forms an intricate part of the self-acting decay heat removal chain, which is directly related to the PBR safety case. In this study standard correlations generally employed by the thermal fluid design community for PBRs are investigated, giving particular attention to the applicability of the correlations when simulating the effective thermal conductivity in the near-wall region. Seven distinct components of heat transfer are examined namely: conduction through the solid, conduction through the contact area between spheres, conduction through the gas phase, radiation between solid surfaces, conduction between pebble and wall, conduction through the gas phase in the wall region, and radiation between the pebble and wall surface.

The effective thermal conductivity models are typically a function of porosity in order to account for the pebble bed packing structure. However, it is demonstrated in this study that porosity alone is insufficient to quantify the porous structure in a randomly packed bed. A new Multi-sphere Unit Cell Model is therefore developed, which accounts more accurately for the porous structure, especially in the near-wall region. Conclusions on the applicability of the model are derived by comparing the simulation results with measurements obtained from various experimental test facilities. This includes the PBMRs High Temperature Test Unit (HTTU) situated on the campus of the North-West University in Potchefstroom in South Africa. The Multi-sphere Unit Cell Model proves to encapsulate the impact of the packing structure in a more fundamental way and can therefore serve as the basis for further refinement of models to simulate the effective thermal conductivity.

Keywords: Effective thermal conductivity, conduction, radiation, randomly packed beds.



ACKNOWLEDGEMENTS

I would like to thank my two promoters Professor Pieter Rousseau and Professor Jat du Toit for many interesting discussions, as well as financial support; it is sincerely appreciated. Also, thank-you to my wife Leandré for giving me her love and understanding. Lastly, I wish to thank my Heavenly Father Jesus Christ for giving me the potential to add value in this research field.

*“If I have seen further it is only by standing on the shoulders of giants.”
Sir Isaac Newton*



TABLE OF CONTENTS

1. INTRODUCTION	1
1.1 BACKGROUND TO THE STUDY	1
1.2 RESEARCH PROBLEM STATEMENT	3
1.3 METHODOLOGY	5
1.4 CONTRIBUTIONS OF THIS STUDY	6
1.5 CHAPTER OUTLINE	7
2. POROUS STRUCTURE	8
2.1 INTRODUCTION	8
2.2 ANALYSING A RANDOMLY PACKED MONO-SIZED SPHERICAL PACKED BED	9
2.2.1 POROSITY	9
2.2.2 COORDINATION NUMBER	16
2.2.3 CONTACT ANGLES	20
2.2.4 RADIAL DISTRIBUTION FUNCTION	25
2.2.5 COORDINATION FLUX NUMBER	26
2.2.6 VORONOI POLYHEDRA	27
2.3 STRUCTURED (ORDERED) PACKINGS	29
2.4 LIMITATIONS ON DEFINING PACKING STRUCTURE WITH POROSITY ONLY	30
2.5 CONCLUSION	31
3. HEAT TRANSPORT IN A RANDOMLY PACKED BED	32
3.1 INTRODUCTION	32
3.2 DETERMINISTIC AND RESISTANCE MODELS	36
3.2.1 SOLID AND FLUID EFFECTIVE THERMAL CONDUCTIVITY	36
3.2.2 EFFECTIVE THERMAL CONDUCTIVITY DUE TO CONTACT AREA	62
3.2.3 EFFECTIVE THERMAL CONDUCTIVITY DUE TO RADIATION	73
3.2.4 EFFECTIVE THERMAL CONDUCTIVITY AT THE NEAR-WALL INTERFACE	85
3.3 STATISTICAL APPROACH	88
3.4 COMPARISON BETWEEN EFFECTIVE THERMAL CONDUCTIVITY MODELS ..	89
3.5 CONCLUSION	93
4. THE HTTU TEST FACILITY	95
4.1 INTRODUCTION	95
4.2 PLANT CONFIGURATION	96
4.3 TEST MATRIX	100
4.4 DERIVATION OF THE EFFECTIVE THERMAL CONDUCTIVITY FROM	



THE EXPERIMENTAL DATA	101
4.5 UNCERTAINTY ANALYSIS.....	103
4.6 EXPERIMENTAL RESULTS	104
4.7 CONCLUSION.....	105
5. MULTI-SPHERE UNIT CELL MODEL	106
5.1 INTRODUCTION	106
5.2 DEVELOPMENT OF THE MULTI-SPHERE UNIT CELL MODEL	107
5.1.1 CONDUCTION.....	107
5.1.2 RADIATION.....	117
5.1.3 SPHERE – WALL CONDUCTION.....	122
5.1.4 SPHERE–WALL AND WALL–SPHERE RADIATION.....	126
5.3 THE EFFECT OF SOLID CONDUCTIVITY ON THERMAL RADIATION	131
5.4 CONCLUSION.....	134
6. VALIDATION AND VERIFICATION.....	135
6.1 INTRODUCTION	135
6.2 VALIDATION OF MULTI-SPHERE UNIT CELL IN THE BULK REGION.....	136
6.3 VALIDATION OF MULTI-SPHERE UNIT CELL MODEL IN THE WALL REGION.....	146
6.4 VERIFICATION OF THE MULTI-SPHERE UNIT CELL MODEL IN A RANDOMLY PACKED BED.....	149
6.5 CONCLUSION.....	158
7. SUMMARY AND CONCLUSION	159
7.1 SUMMARY	159
7.2 CONCLUSION.....	170
7.3 RECOMMENDATIONS FOR FURTHER RESEARCH.....	170
APPENDIX A: SUMMARY OF RADIATION EXCHANGE FACTORS FOUND IN LITERATURE.....	159
APPENDIX B: SUMMARY OF CORRELATIONS FOUND IN LITERATURE.....	173
APPENDIX C: EXPERIMENTAL TEST FACILITIES FOUND IN LITERATURE	176
C.1 SPHERICAL-FLAT CONTACT EXPERIMENTAL TEST FACILITY.....	176
C.2 SPHERICAL-SPHERICAL CONTACT EXPERIMENTAL TEST FACILITY.....	178
C.3 HIGH TEMPERATURE OVEN EXPERIMENTAL TEST FACILITY	179
C.4 SANA-I EXPERIMENTAL TEST FACILITY	183
APPENDIX D: HTTU ANALYSIS	187
D.1 INSTRUMENT RANGE AND ACCURACY	187
D.2 RADIAL HEAT FLUX DISTRIBUTION	188



D.3	UNCERTAINTY ANALYSIS.....	190
D.4	TEMPERATURE AND EFFECTIVE THERMAL CONDUCTIVITY RESULTS.....	198
D.5	CONTACT FORCE DISTRIBUTION.....	204
APPENDIX E: INTEGRATION PROCESSES OF MULTI-SPHERE UNIT CELL		206
APPENDIX F: DEVELOPMENT OF THE NON-ISOTHERMAL CORRECTION FACTOR		212
APPENDIX G: COMPUTER CODES		221
G.1	COORDINATION AND CONTACT-ANGLE CALCULATION CODE	221
G.2	MULTI-SPHERE UNIT CELL CALCULATION CODE	223
REFERENCES.....		230

LIST OF FIGURES

Figure 1.1:	Pebble Bed Modular Reactor triso-coated fuel particles	2
Figure 1.2:	SANA-I experimental data 35 kW long heater with helium as the interstitial gas (see Table C.12)	4
Figure 2.1:	The various packing regions defined in this study	9
Figure 2.2:	Geometrical parameters for an annular packed bed	10
Figure 2.3:	Experimental and numerical results for an annular packed bed.....	11
Figure 2.4:	Experimental results displaying the thickness effect on bulk porosity for a given $d_p/D = 3/74$ ratio.....	12
Figure 2.5:	Comparison between radial oscillatory porosity correlations.....	14
Figure 2.6:	Comparison between radial exponential porosity correlations	16
Figure 2.7:	Comparison between various coordination number models (see Table 2.3)	18
Figure 2.8:	Average coordination number of the High Temperature Test Unit.....	19
Figure 2.9:	Contact angle between two spheres.....	20
Figure 2.10:	Average contact angles calculated in two radial directions for the High Temperature Test Unit	21
Figure 2.11:	Calculation of average total contact angles	22
Figure 2.12:	Average total contact angle calculated in the radial direction for the High Temperature Test Unit	22
Figure 2.13:	Average total contact angle versus average coordination number for data points at the same radial position in the High Temperature Test Unit.....	23
Figure 2.14:	Average total contact angle versus porosity for data points at the same radial position in the High Temperature Test Unit	24
Figure 2.15:	Calculated average total contact angle versus porosity with 1.5 mm radial slice thicknesses in all packing regions	24
Figure 2.16:	Two-dimensional radial distribution function for the High Temperature Test Unit ($\Delta r = 1.5\text{mm}$)	26
Figure 2.17:	Average coordination flux number calculated in two radial directions in the High Temperature Test Unit.....	27
Figure 2.18:	Schematic of a Voronoi polyhedron	28
Figure 2.19:	Schematic of Voronoi polyhedra for a binary packing of 1000 spheres	28
Figure 2.20:	Ordered packing structures.....	29
Figure 2.21:	Ordered packing structures.....	30
Figure 2.22:	Literature survey porous structure flowchart.....	31
Figure 3.1:	Heat transfer mechanisms through packed bed.....	33
Figure 3.2:	Heat transfer due to pressure dependence	35
Figure 3.3:	One-dimensional composite models.....	36
Figure 3.4:	Parallel and Series layers (boundaries), $\varepsilon = 0.36$	37
Figure 3.5:	Kunii & Smith heat transfer model near the contact points.....	38



Figure 3.6:	Zehner & Schlünder unit cell model.....	40
Figure 3.7:	Zehner & Schlünder curve fit through diffusivity experiments	41
Figure 3.8:	Okazaki et al. unit cell model of packed bed	43
Figure 3.9:	(a) An array of touching square cylinders (above) and (b) its unit cell (below)	45
Figure 3.10:	(a) An array of touching circular cylinders (above) and (b) the unit cell (below)	47
Figure 3.11:	(a) In-line touching cubes (left) and (b) Hsu <i>et al.</i> 's unit cell (right)	50
Figure 3.12:	Connection models between two neighbouring Voronoi polyhedra	51
Figure 3.13:	Various contact conditions between two spheres.....	51
Figure 3.14:	Heat conduction model between two neighbouring Voronoi polyhedra	53
Figure 3.15:	Sectional view showing the thermal path through sphere "C" in the (a) SC, (b) BCC and (c) FCC unit cells.....	55
Figure 3.16:	Schematic showing the angle of the thermal path in the (a) SC, (b) BCC and (c) FCC unit cells	57
Figure 3.17:	Unit cell in the FCC fraction (a) top view and (b) side view	58
Figure 3.18:	Model for two contacting spheres: (a) geometrical parameters (b) conductances	59
Figure 3.19:	Schematic model of heat transfer through a contact area in a powder bed	61
Figure 3.20:	Bauer & Schlünder unit cell model with contact area	63
Figure 3.21:	Hsu et al. modifications to the Zehner & Schlünder unit cell	66
Figure 3.22:	Contact of rough spheres with presence of interstitial gas	67
Figure 3.23:	Thermal resistance network, spherical rough contacts in presence of gas... ..	68
Figure 3.24:	Schematic of pebble roughness and slope.....	69
Figure 3.25:	Summary of geometrical modelling	71
Figure 3.26:	Diffuse and specular reflection	74
Figure 3.27:	Radiation principles.....	74
Figure 3.28:	Radiant properties of a packed bed of 5 mm glass spheres	77
Figure 3.29:	Layer model of Vortmeyer (for demonstrating radiation fluxes the layers are shown separated)	78
Figure 3.30:	Radiation transmission number B.....	79
Figure 3.31:	Added parameters by Robold to Vortmeyer's model.....	81
Figure 3.32:	Transmission number versus (a) porosity and (b) emissivity	82
Figure 3.33:	Radiation exchange factor versus dimensionless solid conductivity	84
Figure 3.34:	Connecting models between two neighbouring Voronoi polyhedra	85
Figure 3.35:	Effective thermal conductivity, k_e^g , models at $\varepsilon = 0.36$ versus experimental data	90
Figure 3.36:	Effective thermal conductivity $k_e^{g,c}$ models at $\varepsilon = 0.36$ versus	



	experimental data	91
Figure 3.37:	Effective thermal conductivity models (contact area included, no radiation) versus porosity variation, ($\kappa = 488.5$)	92
Figure 3.38:	Comparison of F_E and F_E^* models with emissivity with $T = 1067^\circ\text{C}$, $\varepsilon = 0.43$ and $k_s = 30[\text{W/mK}]$	93
Figure 3.39:	Literature survey heat transport in randomly packed beds flowchart	94
Figure 4.1:	High Temperature Test Unit.....	96
Figure 4.2:	High Temperature Test Unit vessel	97
Figure 4.3:	High Temperature Test Unit internals.....	98
Figure 4.4:	High Temperature Test Unit.....	99
Figure 4.5:	Thermocouple levels in the High Temperature Test Unit	99
Figure 4.6:	Thermocouple levels in the High Temperature Test Unit	100
Figure 4.7:	Test matrix for Nitrogen High Temperature Near-Vacuum tests	100
Figure 4.8:	Isothermal temperature distribution of level C (1200°C steady-state)	102
Figure 4.9:	HTTU experimental results effective thermal conductivity against temperature.....	104
Figure 4.10:	HTTU experimental results effective thermal conductivity against sphere diameter from inner wall level C (1200°C steady-state)	105
Figure 5.1:	Multi-sphere Unit Cell Model (conduction).....	108
Figure 5.2:	Interstitial gas conduction incorporating the Smoluchowski effect (middle region).....	113
Figure 5.3:	Pebble orientation for outer solid thermal resistance derivation.....	116
Figure 5.4:	Decreasing long-range diffuse view factor in the bulk region based on surface area of a full sphere	120
Figure 5.5:	Long-range diffuse view factor in the bulk region (Pitso, 2009).....	121
Figure 5.6:	Multi-sphere Unit Cell Model (sphere-wall conduction)	122
Figure 5.7:	Interstitial gas conduction incorporating the Smoluchowski effect	124
Figure 5.8:	Long-range diffuse view factor for thermal radiation from spheres to the wall	129
Figure 5.9:	Long-range diffuse view factor for thermal radiation from wall to spheres ..	131
Figure 5.10:	Thermal radiation heat transfer between two hemispheres.....	132
Figure 5.11:	Non-isothermal correction factor.....	133
Figure 6.1:	Comparison between Multi-sphere Unit Cell Model and Simple Cubic experimental data	138
Figure 6.2:	Comparison between Multi-sphere Unit Cell Model and Simple Cubic experimental data	139
Figure 6.3:	Face-centred Cubic conduction area.....	140
Figure 6.4:	Comparison between Multi-sphere Unit Cell Model and Face-centred Cubic experimental data	141



Figure 6.5:	Comparison between Multi-sphere Unit Cell Model and Face-centred Cubic experimental data	142
Figure 6.6:	Thermal resistance parametrical study in the bulk region	143
Figure 6.7:	Comparison between Multi-sphere Unit Cell Model, Breitbach & Barthels and Robold correlations with experimental data of the High Temperature Oven with graphite spheres at vacuum conditions	144
Figure 6.8:	Comparison between Multi-sphere Unit Cell Model and the proposed International Atomic Energy Agency correlation with experimental data of the High Temperature Oven, ($\bar{n}_{long} = 4.7$)	145
Figure 6.9:	Comparison between Multi-sphere Unit Cell Model and Kitscha & Yovanovich experimental data for the wall region.....	147
Figure 6.10:	Comparison between Multi-sphere Unit Cell Model and experimental data in the wall region at vacuum conditions	148
Figure 6.11:	Comparison between Multi-sphere Unit Cell Model and experimental data in the wall region at elevated pressure saturated with Helium	148
Figure 6.12:	Comparison between various correlations and SANA-I experimental data versus temperature saturated with helium.....	150
Figure 6.13:	Comparison between the Multi-sphere Unit Cell Model components and SANA-I effective thermal conductivity experiment, experimental results	151
Figure 6.14:	Comparison between effective thermal conductivity correlations and experimental results of the SANA-I experimental test facility for the 10 kW steady-state	152
Figure 6.15:	Comparison between effective thermal conductivity correlations and experimental results of the SANA-I experimental test facility for the 35 kW steady-state	152
Figure 6.16:	Comparison between effective thermal conductivity correlations (components) and experimental results of the SANA-I experimental test facility for the 35 kW steady-state.....	153
Figure 6.17:	Packing structure in the inner region of the SANA-I experimental test facility	154
Figure 6.18:	Comparison between effective thermal conductivity correlations and experimental results of the High Temperature Test Unit experimental test facility for the 20 kW steady-state, Test 1.....	155
Figure 6.19:	Comparison between effective thermal conductivity correlations and experimental results of the High Temperature Test Unit experimental test facility for the 20 kW steady-state, Test 2.....	155
Figure 6.20:	Comparison between effective thermal conductivity correlations and experimental results of the High Temperature Test Unit experimental test facility for the 82.7 kW steady-state, Test 1.....	157

Figure 6.21:	Comparison between effective thermal conductivity correlations and experimental results of the High Temperature Test Unit experimental test facility for the 82.7 kW steady-state, Test 2.....	157
Figure 6.22:	Comparison between effective thermal conductivity correlations (components) and experimental results of the High Temperature Test Unit experimental test facility for the 82.7 kW steady-state, Test 1	158
Figure 7.1:	Flowchart of Multi-sphere Unit Cell Model.....	161
Figure C.1:	Sphere-flat contact experimental setup	176
Figure C.2:	Buonanno et al. experimental apparatus for a Simple Cubic packing.....	178
Figure C.3:	High Temperature Oven	180
Figure C.4:	SANA-I experimental test facility	183
Figure C.5:	Isothermal temperature distribution of SANA-I 35 kW helium steady-state	184
Figure C.6:	Effective thermal conductivity results and other parameter for the 10 kW long heater element helium steady-state (SANA-I).....	186
Figure C.7:	Effective thermal conductivity results and other parameters for the 35 kW long heater element helium steady-state (SANA-I).....	186
Figure D.1:	Insulation discretisation of top insulation	188
Figure D.2:	Temperature curve fits for Level A and Level E (82.7 kW, Test 1).....	189
Figure D.3:	Heat loss in each increment as a function of radial position.....	189
Figure D.4:	Temperature profile of level C (20 kW steady-state)	203
Figure D.5:	Temperature profile of level C (82.7 kW steady-state)	203
Figure D.6:	Contact force.....	204
Figure D.7:	Contact force radial distribution for different height portions	205
Figure D.8:	Contact force height distribution	205
Figure F.1:	Comparison between proposed correlation and Computational Fluid Dynamics results.....	213
Figure F.2:	Comparison between proposed correlation and Computational Fluid Dynamics results.....	214
Figure F.3:	Non-isothermal correction factor.....	220



LIST OF TABLES

Table 2.1:	Summary of the oscillatory porosity correlations in the radial direction.....	13
Table 2.2:	Summary of the exponential porosity correlations in the radial direction	15
Table 2.3:	Equations for relation between average coordination number and porosity .	17
Table 2.4:	The comparison of coordination number and porosity with different arrangements	29
Table 3.1:	Areas of solid and macro-void part at various porosities.....	43
Table 3.2:	Suitable packing arrangements for different porosity range	56
Table 3.3:	Magnitude of structural parameters for different close packings of spheres .	67
Table 3.4:	Correlations for m , Gaussian surfaces	70
Table 3.5:	Constants for the radiation exchange factor Eq. (3.166).....	83
Table 3.6:	Contact area constants for models evaluated in Figure 3.36	91
Table 6.1:	Experimental tests used for the validation of the Multi-sphere Unit Cell Model in the bulk region.....	135
Table 6.2:	Experimental tests used for the validation of the Multi-sphere Unit Cell Model in the wall region	136
Table 6.3:	Experimental tests used for the verification of the Multi-sphere Unit Cell Model in the wall, near-wall and bulk regions	136
Table 7.1:	Summary of Multi-sphere Unit Cell Model	162
Table A.1:	Summary of radiation exchange factors	172
Table B.1:	Summary of correlations found in literature.....	173
Table C.1:	Physical and thermal properties of test specimens	176
Table C.2:	Experimental results for argon tests	177
Table C.3:	Gas parameters	178
Table C.4:	Experimental and simulation results for Buonanno et al. experimental tests.....	179
Table C.5:	Test summary	180
Table C.6:	Material property summary	181
Table C.7:	Experimental test and simulation results for bulk region, $\bar{\varepsilon} = 0.39$, graphite spheres conducted in the High Temperature Oven.....	181
Table C.8:	Experimental test results for bulk region, $\bar{\varepsilon} = 0.39$, graphite spheres conducted in the High Temperature Oven.....	182
Table C.9:	Experimental test and simulation results for wall region, graphite spheres conducted in the High Temperature Oven.....	183
Table C.10:	Temperature measurements of the SANA-I experimental test facility (10kW long heater)	184
Table C.11:	Temperature measurements of the SANA-I experimental test facility (35kW long heater)	184
Table C.12:	Calculated effective thermal conductivity results for the 10 kW long	



	heater element helium steady-state (SANA-I)	185
Table C.13:	Calculated effective thermal conductivity results for the 35 kW long heater element helium steady-state (SANA-I)	185
Table C.14:	Effective thermal conductivity data versus temperature (SANA-I) (Helium)	185
Table D.1:	Overview of category A ranges and accuracies	187
Table D.2:	Heat flux distributions for various steady-states	190
Table D.3:	Temperature measurements for both tests on Level C for the 20 kW steady-state	198
Table D.4:	Temperature measurements for both tests on Level C for the 82.7 kW steady-state	199
Table D.5:	Effective thermal conductivity extracted values for Test 1 on Level C for the 20 kW steady-state	200
Table D.6:	Effective thermal conductivity extracted values for Test 1 on Level D for the 20 kW steady-state	201
Table D.7:	Effective thermal conductivity extracted values for Test 2 on Level C for the 20 kW steady-state	201
Table D.8:	Effective thermal conductivity extracted values for Test 1 on Level C for the 82.7 kW steady-state	202
Table D.9:	Effective thermal conductivity extracted values for Test 2 on Level C for the 82.7 kW steady-state	202
Table F.1:	Empirical constants for the non-isothermal correction factor	212
Table F.2:	Derivation to calculate the non-isothermal correction factor for $\varepsilon_r = 1.0$	215
Table F.3:	Derivation to calculate the non-isothermal correction factor for $\varepsilon_r = 0.8$	216
Table F.4:	Derivation to calculate the non-isothermal correction factor for $\varepsilon_r = 0.6$	217
Table F.5:	Derivation to calculate the non-isothermal correction factor for $\varepsilon_r = 0.4$	218
Table F.6:	Derivation to calculate the non-isothermal correction factor for $\varepsilon_r = 0.2$	219



NOMENCLATURE

ABBREVIATIONS	
BCC	Body-centred Cubic packing
BHN	Brinell Hardness Number
CFD	Computational Fluid Dynamics
DEM	Discrete Element Method
FCC	Face-centred Cubic packing
HCN	Hertzian Contact Network
HPTU	High Pressure Test Unit
HTO	High Temperature Oven
HTR	High Temperature Reactor
HTTU	High Temperature Test Unit
KTA	Kerntechnischer Ausschuss
OIBC	Oxford International Biomedical Centre
PBMR	Pebble Bed Modular Reactor
PBR	Pebble Bed Reactor
PFC 3D	Particle flow code three-dimensional
PWR	Pressurised Water Reactor
RCN	Rough Contact Network
RDF	Radial Distribution Function
RMS	Root Mean Square
RTC	Radiative Transfer Coefficient
RTE	Radiative Transfer Equation
SC	Simple Cubic packing
SEE	Standard Estimate Error
ZBS	Zehner, Bauer, Schlünder
VARIABLES	
A_{Solid}	Area of solids at a specific radial position, m^2
A_{Total}	Total area at a specific radial position, m^2
A	Area, m^2
a_T	Thermal accommodation coefficient
a_H	$\equiv r_c$, Hertzian contact area radius Bahrami <i>et al.</i> (2006:3691) and Kaviany (1991:2582)
a_L	$\equiv r_a$, Radius of macro-contact, m
a	Empirical parameter, Mueller (1992:269) Effective length defined by Hsu <i>et al.</i> (1995:264)

	Absorption coefficient
	Contact area radius between two cylinders representing two spheres defined by Shapiro <i>et al.</i> (2004:268)
$a_1 - a_4$	Empirical constants
b	Empirical parameter, Mueller (1992:269) Empirical parameter, Suzuki <i>et al.</i> (1981:482) Scattering coefficient
b_L	Specimen radius defined by Bahrami <i>et al.</i> (2006:3691)
B	Empirical deformation parameter Radiation transmission number, $B = f(\varepsilon, \varepsilon_r)$
B_0	Radiation transmission number, $B_0 = f(\varepsilon, 0)$
c_p	Specific heat constant pressure, $\text{kJ kg}^{-1} \text{K}^{-1}$
c_v	Specific heat constant volume, $\text{kJ (m}^3\text{)}^{-1} \text{K}^{-1}$
C	Constant parameter in oscillatory porosity correlation, Martin (1978:913) Constant parameter in exponential porosity correlations, Zehner & Schlünder (1970:933)
c	Contact area/plate thickness defined by Hsu <i>et al.</i> (1995:264)
c_1	Vickers microhardness coefficient, Pa Polynomial coefficients for heat flux distributions $c_{0...3}$
c_2	Vickers microhardness coefficient
D	Diameter of cylinder, m
D_e	Diffusivity of a fluid/gas-saturated packed bed
D_f	Diffusivity of a fluid or gas phase
D_m	Molecular diameter, m
D_{tot}	Total geometrical distance between two spheres, m
$D(r)$	Geometrical distance between two surfaces as a function of radial position, m
d_p	Diameter of sphere, m
d	Distance between sphere centres, m Geometric dimension of a certain void space, m
d^e	Transformed pebble diameter, m
d_f^e	Fluid gap width, m
d_F	Distance between two pebbles with applied force, m

d_{ij}	Distance between centres of spheres i and j [m] (Cheng <i>et al.</i> , 1999:4199), m
d_s^e	Solid thickness parameter, m
E_p	Young modulus, Pa
E'	Effective elastic Young's modulus, Pa
F_{1-2}	Diffuse view factor between two surfaces
F_{1-2}^L	Long-range diffuse view factor
$F_{1-2,avg}^L$	Average long-range diffuse view factor
f_k	Dimensionless non-isothermal correction factor
F	Collinear force, N
F_E	Radiation exchange factor, $F_E = f(\varepsilon, \varepsilon_r)$
F_E^*	Radiation exchange factor, $F_E^* = f(\Lambda_s, \varepsilon, \varepsilon_r)$
G	Conductance parameter
g_{3D}	Radial distribution function in a three-dimensional system
g_{2D}	Radial distribution function in a two-dimensional system
h_r	Average height of surface roughness, m
H_B, BHN	Brinell hardness, GPa and Brinell number
H_{BGM}	Hardness constant $H_{BGM} = 3.178 GPa$
H^*	$= c_1 (\sigma' / m_{RMS})^{c_2}, Pa$
h^*	Contact thickness height Hsu <i>et al.</i> (1995:264)
H_{mic}	Vickers microhardness
h	Parameter defined by Cheng <i>et al.</i> (1999:4199), $h = (d_{ij} - 2R_p) / 2$
H	Height of a packed bed, m
H_{1r}, H_{2r}, H_{3r}	Geometrical parameters defined by Cheng <i>et al.</i> (1999:4199)
I_j, i^+	Forward radiation flux
J_0	Bessel function of the first kind
j	The j -th sphere centre Temperature jump parameter, m
K_j, i^-	Backward radiation flux
k	Conductivity



	Boltzmann's constant $1.3806505 \times 10^{-23} [J/K]$ Spring stiffness, kN/m
k_{es}	Apparent effective thermal conductivity of solid part, Okazaki <i>et al.</i> (1977:164), $Wm^{-1}K^{-1}$
k_{rs}	Thermal conductivity due to radiation from a solid to a solid, Kunii & Smith (1960:71) & Yagi & Kunii (1957:373), $Wm^{-1}K^{-1}$
k_{rv}	Thermal conductivity due to radiation from a void to a void, Kunii & Smith (1960:71) & Yagi & Kunii (1957:373), $Wm^{-1}K^{-1}$
k_{sf}	Effective thermal conduction of the inner cylinder of Zehner & Schlünder (1970:933) model, $Wm^{-1}K^{-1}$ Effective thermal conductivity of composite layers defined by Hsu <i>et al.</i> (1995:264), $Wm^{-1}K^{-1}$
k_{bed}	Total effective thermal conductivity in a packed bed due to k_{eff} , $k_{f,eff}$ and $k_{s,eff}$, $Wm^{-1}K^{-1}$
k_{eff}	Effective thermal conductivity due to thermal conduction and radiation, $Wm^{-1}K^{-1}$
k_{ins}	Thermal conductivity through insulation, $Wm^{-1}K^{-1}$
$k_{r,eff}$	Effective thermal conduction due to fluid mixing (braiding effect), $Wm^{-1}K^{-1}$
$k_{s,eff}$	Effective thermal conduction due to solid pebble movement, $Wm^{-1}K^{-1}$
k_f, k_g	Thermal conductivity of fluid or gas phase, $Wm^{-1}K^{-1}$
k_G	Thermal conductivity with variance in gas pressure, $Wm^{-1}K^{-1}$
k_δ	Thermal conductivity of a thin gap defined by Shapiro <i>et al.</i> (2004:268)
k_a	Thermal conductivity of a contact point defined by Shapiro <i>et al.</i> (2004:268)
k_e^r	Effective thermal conductivity due to radiation, $Wm^{-1}K^{-1}$
$k_e^{r,S}$	Effective thermal conductivity due to short-range radiation, $Wm^{-1}K^{-1}$
$k_e^{r,S}$	Effective thermal conductivity due to short-range radiation vector, $Wm^{-1}K^{-1}$
$k_e^{r,L}$	Effective thermal conductivity due to long-range radiation, $Wm^{-1}K^{-1}$
k_e^c	Effective thermal conductivity through contact area, $Wm^{-1}K^{-1}$



$\vec{k}_e^{g,c}$	Effective thermal conductivity point and contact area vector, $Wm^{-1}K^{-1}$
k_e^g	Effective thermal conductivity through fluid/gas and point contact, $Wm^{-1}K^{-1}$
$k_e^{g,c}$	Combination of effective thermal conductivity k_e^g and k_e^c , $Wm^{-1}K^{-1}$
$k_e^{g,c,W}$	Effective thermal conduction in wall region, $Wm^{-1}K^{-1}$
$k_e^{r,W}$	Effective thermal conductivity due to radiation in wall region, $Wm^{-1}K^{-1}$
K	The number of coefficients in the polynomial equation.
k_{eff}^W	Effective thermal conductivity due to thermal conduction and radiation in wall region, $Wm^{-1}K^{-1}$
k_s	Thermal conductivity of solid phase, $Wm^{-1}K^{-1}$
Kn	$\equiv \lambda/d$ Knudsen number
L	Length of packed bed Length of unit cell Length of cylinder defined by Shapiro <i>et al.</i> (2004:268)
L_{ins}	Height of insulation material, m
l	Gap between spheres at the point of interest Modified free path of gas molecules
L_r	Radiation distance
$L_{r,avg}$	Average radiation distance
l_v, l_s	Effective lengths
m	Empirical parameter, Zehner & Schlünder (1970:933)
\dot{m}	Mass flow, kg/s
m_{RMS}	Combined root mean squared surface slope
M_g	Molecular mass of gas, $kg kmol^{-1}$
M_s	Molecular mass of solid surface, $kg kmol^{-1}$
M^*	Effective molecular mass, $kg kmol^{-1}$
N	Parameter in exponential porosity correlations Parameter, ZBS Number data points in an experimental data set
\bar{N}_c	Average coordination number
N_A	Number of spheres per unit area



N_L	Number of spheres per unit length
n	Coordination flux number
n_m	Number of gas molecules per unit volume
n_s	Number of spheres in a unit cell defined by Siu <i>et al.</i> (2000:3917)
$n(r)$	Number of spheres in a considered radial slice
\bar{n}	Average coordination flux number
\bar{n}_{long}	Effective long-range coordination flux number
P_F	Pressure due to an external force, Pa
P_g	Gas pressure, Pa
P_0	Atmospheric gas pressure
$P_{0,c}$	Maximum contact pressure, Pa
$P_{0,H}$	Hertzian contact pressure, Pa
Pr	Prandtl number
Q_{ij}	Heat flux through the i -th and j -th sphere, W (Cheng <i>et al.</i> , 1999:4199)
Q_{bed}	Extracted heat flux through HTTU packed bed, W
Q_G	Heat transfer through interstitial gas within macro-gap (Bahrami <i>et al.</i> , 2006:3691)
Q_s	Heat transfer through micro-contacts (Bahrami <i>et al.</i> , 2006:3691)
$Q_{i,loss}$	Heat flux lost in the axial direction of the i th increment, W
Q_{wj}	Heat flux extracted by water jacket, W
Q_{in}	Heat flux through bed at r_{in} , W
$Q_{total,loss}$	The summation of all the heat fluxes lost in the axial direction, W
q_g''	Heat flux through gas, W/m^2
q_{fm}	Heat flux through gas in free-molecule region, W/m^2
q_{tr}	Heat flux through gas in transition region, W/m^2
q_{sl}	Heat flux through gas in slip region, W/m^2
q_{cont}	Heat flux through gas in continuum region, W/m^2
R	Outer radius of cylindrical packed bed, m Unknown radial parameter Zehner & Schlünder (1970:933)



	Thermal resistance
	Radiation reflection number
R_i	Inner radius of annulus, m
R_o	Outer radius of annulus, m
R_{ij}	Parameter defined by Cheng <i>et al.</i> (1999:4199)
R_j	Thermal resistance of joint, K/W
$R_{in,1,2}$	Inner solid material resistance, K/W
$R_{L,1,2}$	Macro-contact constriction/spreading resistance, K/W
R_s	Micro-contact constriction/spreading resistance, K/W
R_g	Resistance of the interstitial gas in the micro-gap, K/W
$R_{mid,1,2}$	Middle solid material resistance, K/W
R_λ	Resistance of the interstitial gas in the Knudsen regime (Smoluchowski effect) of the macro-gap, K/W
$R_{out,1,2}$	Outer solid material resistance, K/W
R_G	Resistance of the interstitial gas in the macro-gap, K/W
$R_{HERTZ,1,2}$	Hertzian microcontact, K/W
$r_{0,j}$	Distance in contact angle calculations, m
r	Radial coordinate, Radius m
r_{in}	Radius at inner annulus $r_{in} = 0.3m$
r_{out}	Radius at outer reflector $r_{out} = 1.15m$
Δr	Radial difference, m
r_p	Radius sphere, m
r_c	Hertzian radius of the contact area, m
r_a	$\equiv a_L$, Radius of the contact area with consideration of surface roughness, m
$r_{p,eq}$	Equivalent pebble radius, m
r_λ	Mean free-path radius between two spheres, m
r_{sf}	Parameter defined by Cheng <i>et al.</i> (1999:4199)
S	Packing variable
S_F	Packing variable
s	Contact area parameter, Robold (1982:102)
T	Temperature, K



T_{we}	Exit temperature of water jacket, °C
T_{wi}	Inlet temperature of water jacket, °C
ΔT_{wj}	Difference between outlet and inlet temperatures of water jacket, °C
$T_{i,exp}$	The i -th measurement in the experimental data set, °C
$T_{i,poly}$	The calculated value at r_i from the polynomial curve fitted, °C
T_{env}	Temperature in the environment or near environment above or below thermal insulation, K
T_{bed}	Temperature at top/bottom of bed near thermal insulation, K
T_s	Solid surface temperature, K
T_0	$\equiv 273 K$
\bar{T}	Average temperature, K
ΔT	Temperature difference, K
$u(T)$	Uncertainty of temperature measurements, °C
$u(k_{eff})$	Uncertainty of effective thermal conductivity, $Wm^{-1}K^{-1}$
$u(TE745K)$ $u(TE750K)$	Uncertainties of thermocouples in top insulation, °C
$u(Q_{bed})$	Uncertainty of heat flux through the bed, W
$u(dT/dr)$	Uncertainty of the polynomial curve fit slope, °C/m
$u(Q_{wj})$	Uncertainty of heat flux extracted by water jacket, W
$u(Q_{total,loss})$	Uncertainty of total heat flux through top and bottom insulation, W
$u(\dot{m})$	Uncertainty on mass flow through water jacket, kg/s
$u(T_{wi})$	Uncertainty of water jacket inlet temperature, °C
$u(T_{we})$	Uncertainty of water jacket exit temperature, °C
$u(T_{bed,statistical})$	Maximum statistical variance of the top (level E) and bottom (level A) temperature measurements, °C
$u(T_{bed,instrument})$	Maximum instrument uncertainty of the top (level E) and bottom (level A) temperature measurements, °C
$u(SEE)$	Standard estimate error, °C
$u(x_{i,instrument})$	The instrument uncertainty for a specific instrument obtained from the manufacturer, °C

$u(x_{i,drift})$	Uncertainty obtained from difference between measurements and measurements from a secondary standard instrument after each major calibration, °C
$u(x_{i,statistical})$	Statistical variance in a specific data set, °C
$u(k_{eff})$	Uncertainty of effective thermal conductivity, $Wm^{-1}K^{-1}$
V_{Void}	Void volume, m^3
V_{Total}	Total volume, m^3
V	Volume, m^3
W	$\equiv (R_o - R_i)$ Width of annular packed bed, m
x	Dimensionless distance Coordinate in the x direction
y	Coordinate in y direction
z	Pebble diameters from a wall Coordinate in z direction
GREEK SYMBOLS	
α	$\equiv 90^\circ - \phi_c$
α_0	Deformation factor defined by Hsu <i>et al.</i> (1994:2751)
α	Geometrical correction factors defined by Slavin <i>et al.</i> (2002:4151) Non-dimensional parameter defined by Bahrami <i>et al.</i> (2006:3691) Absorptivity of radiation Contact point parameter defined by Shapiro <i>et al.</i> (2004:269)
β	Empirical parameter, Kunii & Smith (1960:72) Contact angle defined by Siu <i>et al.</i> (2000:3917)
ψ	Radiation reflection and transmission parameter ratio
ψ_t	Heat transfer parameter, Kunii & Smith (1960:72)
ψ_1	Heat transfer parameter, Kunii & Smith (1960:72), calculated by ψ_{1or2} with $n = 1.5$
ψ_2	Heat transfer parameter, Kunii & Smith (1960:72), calculated by ψ_{1or2} with $n = 4\sqrt{3}$
ψ_{1or2}	Heat transfer parameter, Kunii & Smith (1960:72)
ε	Porosity or void fraction
ε^*	Porosity correction factor
ε_{min}	Minimum porosity in near-wall region



ε_b	Bulk porosity
ε_0	Reference porosity
$\bar{\varepsilon}$	Average porosity
ε_∞	Porosity of infinite bed
ε_r	Emissivity
ε_{wall}	Constant porosity in the near-wall region
π	Pi
$\bar{\phi}_c$	Average contact angle in the radial position, deg
σ	Stephan–Boltzmann constant $\sigma = 5.67 \times 10^{-8}, Wm^{-2}K^{-4}$
σ_{RMS}	Combined root mean squared surface roughness, m
θ_0	Angle corresponding to boundary of heat flow area for one contact point [radians]
θ_λ	Polar angle defined by Slavin <i>et al.</i> (2002:4151)
θ_c	Contact angle [radians], Hsu <i>et al.</i> (1995:264)
Λ_s	Dimensionless solid conductivity
μ_m	$\equiv M_g/M_s$ Molecular mass ratio between gas and solid surface
μ_p	Poisson ratio
κ	$\equiv k_s/k_f$, Dimensionless parameter
κ_r	Radiation ratio parameter
κ_G	Gas conduction ratio in Knudsen regime parameter
κ_p	Non-dimensional parameter defined by Bahrami <i>et al.</i> (2006:3691)
Δ_0	Dimensionless weighting function defined by Robold (1982:129)
λ	Mean free path of gas molecules, m Radiation wavelength
δ	Packing density Contact area between two spheres Contact thickness between two cylinders representing two spheres defined by Shapiro <i>et al.</i> (2004:268)
ω_0	Deformation depth at origin, m
γ	Parameter defined by Kunii & Smith (1960:72) Specific heat ration = c_p/c_v Contact radius ratio Siu & Lee (2000:3920)
γ_a	Effective length ratio defined by Hsu <i>et al.</i> (1995:264) = a/L



γ_c	Effective length ratio defined by Hsu <i>et al.</i> (1995:264) = c/a
ϕ	Surface fraction parameter
\bar{v}	Average molecular velocity
χ	Deformation ratio Radiation exchange ratio Robold (1982:39)
SUBSCRIPTS	
$c, cont$	Contact
cp	Closed-packed region of a unit cell
gv	Gas conduction across void
i	Pebble i , Inner gap, Increment
j	Pebble j
o	Outer gap
p	Pebble
r	Radiation
rv	Radiation across void
RMS	Root Mean Square
s	Solid
sf	Solid fluid region
v	Void
1	Pebble number one
2	Pebble number two
SUPERSCRIPTS	
L	Long-range
W	Wall