

## Chapter 5

# Inferring Best-fit Parameters

This chapter illustrates the method used to obtain constraints on the viewing geometries of the six pulsars of interest. It also shows how a value for the flux correction factor  $f_\Omega$ , including errors, can be derived as a by-product of this approach. To do this, one of the pulsars (PSR J1509–5850) will be used as an example and best-fit solution contours in  $(\alpha, \zeta)$ -space, as well as an  $f_\Omega$  value with estimated errors, will be derived for it.

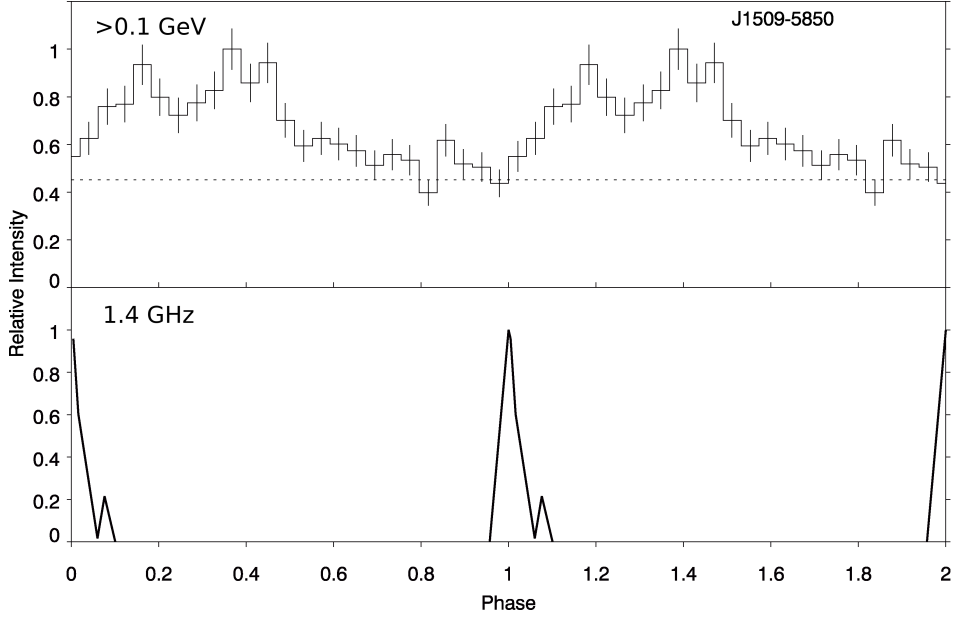
The atlases used in this section were generated using the period of the relevant pulsar. This means that the atlases in this section differ slightly from those generated in Section 4.1, where a period was chosen that would be roughly representative of all the pulsars in this study. In most cases, the most notable difference between the representative atlas and those generated for the individual pulsars is that the portion of  $(\alpha, \zeta)$ -space for which radio emission is visible depends on the period of the specific pulsar. Pulsars with shorter periods tend to have their radio emission visible over a larger portion of  $(\alpha, \zeta)$ -space (see Section 4.4).

The atlases shown in this section will be nine panes across and nine panes high, with each pane corresponding to a location in  $(\alpha, \zeta)$ -space. The  $(\alpha, \zeta)$  value a particular pane corresponds to is indicated in its top right corner. In each pane of the atlas the observed  $\gamma$ -ray (top) and radio (bottom) LCs are shown in black, with the corresponding predicted model LCs superimposed in colour, where blue denotes OG, red TPC, and green radio. In all the atlases in this section the radio LC is shown shifted by 0.5 in phase for clarity. At the locations where either the predicted radio emission or the predicted  $\gamma$ -ray emission is not visible, the pane will be left blank except for its location label.

### 5.1 The $(\alpha, \zeta)$ -contours of PSR J1509–5850

#### 5.1.1 The observed LCs

The radio profile of PSR J1509–5850 shown in Figure 5.1 has a single, roughly triangular peak at 1.4 GHz, as well as a small feature at the trailing edge of the main peak. The  $\gamma$ -ray profile has a more complex shape, and is reported by Weltevrede et al. (2010) as having a single broad peak with a radio-to- $\gamma$  phase lag of  $0.31 \pm 0.02$ . There is some evidence that the  $\gamma$ -ray LC of PSR J1509–5850 may consist of two overlapping components, but this evidence is not statistically significant given the current count statistics. This may be revisited upon the publication of the



**Figure 5.1:** Observed  $\gamma$ -ray and radio LCs of PSR J1509–5850.

*Second Pulsar Catalog* (Weltevrede et al., 2010).

### 5.1.2 The $10^\circ$ atlases

Figures 5.2 and 5.3 show the TPC and OG atlases with a resolution of  $10^\circ$  in  $\alpha$  and  $\zeta$  for PSR J1509–5850. Note that in the atlases in this section the  $\gamma$ -ray profiles are plotted over the phase range (0,1) while the radio profiles are plotted over the phase range (-0.5,0.5) for the sake of clarity. The radio-to- $\gamma$  phase lag is thus not the separation between the peaks drawn in these atlases, but rather the phase of the  $\gamma$ -ray peak itself, as the  $\gamma$ -ray profiles are aligned so that their zero corresponds to the actual position of the radio peak.

The requirement that both radio and  $\gamma$ -ray emission be *visible* provides the first constraint on the possible solution space. The radio visibility confines the possible solution space to a small range of  $|\beta|$  values, meaning that only a band of LCs along the diagonal of the atlas remains. The width of the band for PSR J1509–5850 is  $\sim 40^\circ$ , and is the same for the entire length of the diagonal. The requirement that  $\gamma$ -ray emission be visible is only relevant in the OG case as emission is visible throughout the entire solution space in the TPC case. In the OG case, the part of the diagonal closer to the origin than approximately  $[50^\circ, 50^\circ]$  is therefore excluded from the OG solution space. This difference between the OG and TPC cases can be accounted for by the absence of emission originating from below the null-charge surface in the OG case (see Section 3.2 for more detail).

The possible solution space can be further constrained using the  $\gamma$ -ray profile *shape*. In the region where the OG emission is not visible, the profiles predicted by the TPC model generally have a single broad peak at 0.2 in phase when discernible peaks are present. These profiles don't provide a very good fit to the data though, as the radio-to- $\gamma$  phase lags of these LCs are too low. They also underpredict the intensity of the emission between 0.3 and 0.5 in phase. For profiles lower on the diagonal than approximately  $[30^\circ, 30^\circ]$  no significant peaks can be identified, and thus they can be ruled out as possible solutions.

The profiles between approximately  $[50^\circ, 50^\circ]$  and  $[70^\circ, 70^\circ]$  in the TPC have widely varied shapes

and are generally more complex than those below  $[50^\circ, 50^\circ]$ . They do, however, generally display  $\gamma$ -ray emission profiles roughly comparable in width and position in phase with those constructed from the data. Most notably, promising fits are produced at  $[40^\circ, 60^\circ]$ ,  $[50^\circ, 60^\circ]$ ,  $[60^\circ, 50^\circ]$ , and  $[60^\circ, 40^\circ]$ .

For the OG case, relatively narrow profiles can be found at the lower left end of the visible diagonal band, with the width of the predicted  $\gamma$ -ray profiles generally increasing as one moves farther up along the diagonal. The most prominent constraining property here is the position of the second  $\gamma$ -ray peak on the predicted  $\gamma$ -ray profiles, which is present throughout the atlas. While there is quite a bit of variation in the visible profiles farther up along the diagonal than  $[60^\circ, 60^\circ]$ , it should be noted that this peak appears too late in phase in all of these profiles, excluding them as possible solutions.

For the TPC case the profiles beyond  $[60^\circ, 60^\circ]$  can also be excluded from the possible solution space on the same grounds. The profiles in this region also have a strong earlier peak which doesn't correspond to what can be seen in the data.

Taking these features into account, the narrowed down atlas can now be considered at an improved resolution of  $5^\circ$ .

### 5.1.3 The $5^\circ$ atlases

Figures 5.4 and 5.5 show the regions between  $[35^\circ, 35^\circ]$  and  $[75^\circ, 75^\circ]$  for the TPC model, and between  $[45^\circ, 45^\circ]$  and  $[85^\circ, 85^\circ]$  for the OG model, at a resolution of  $5^\circ$ . This finer resolution can now be employed to identify candidate LC fits which can then be used as starting points in the search for  $(\alpha, \zeta)$  solution contours.

The radio profile *peak multiplicity* is the next constraint that can be applied to eliminate some predicted LCs as best-fit candidates. From Figures 5.4 and 5.5, for  $|\beta| \lesssim 10^\circ$ , the radio model predicts a two-peak structure for the radio profile. Excluding this narrower band of predicted profiles, two diagonal bands of possible solutions are left, one at positive  $\beta$ , and another at negative  $\beta$ . These bands correspond to the two regions where  $10^\circ \lesssim |\beta| \lesssim 20^\circ$  on both the TPC and OG atlases.

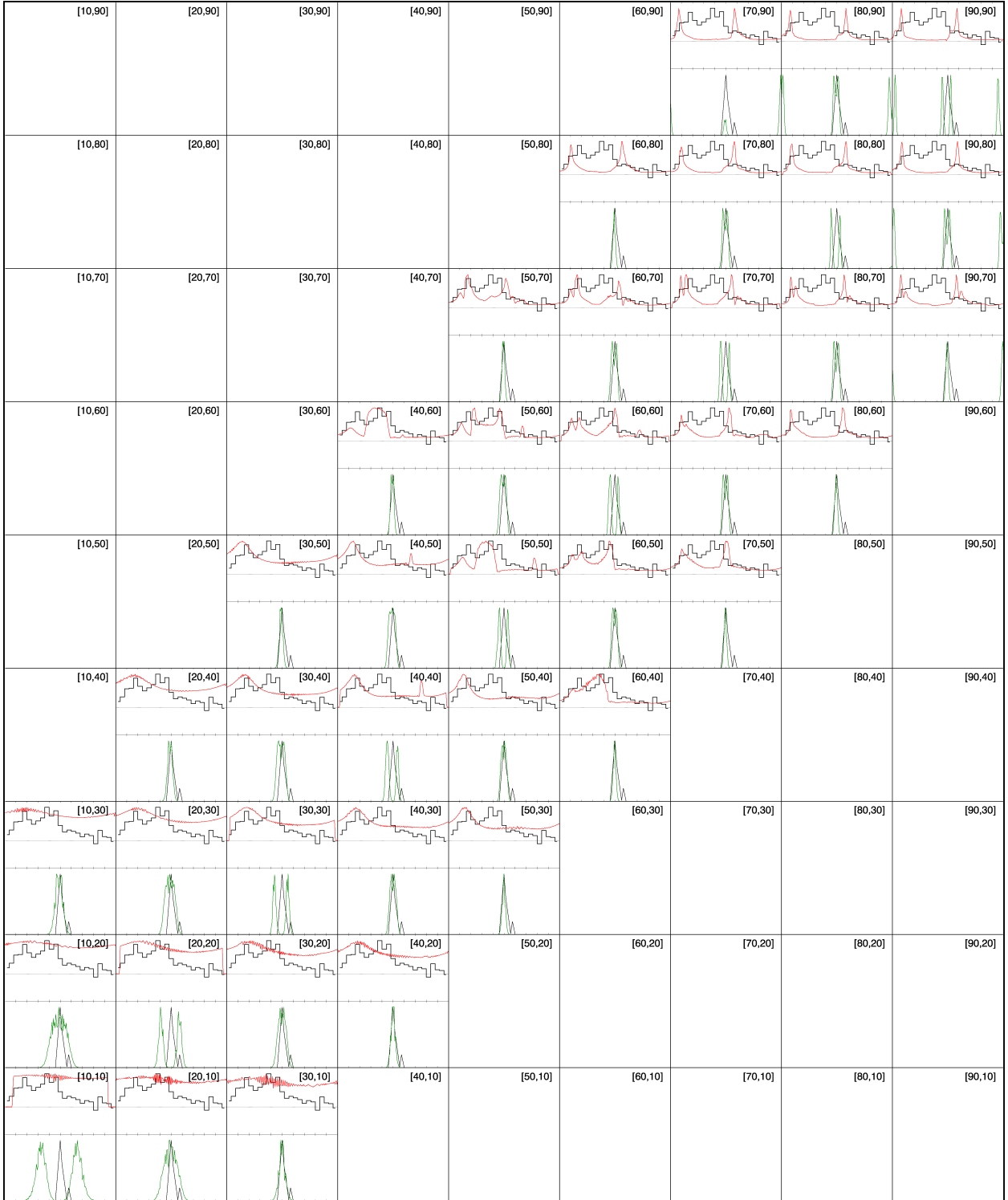
For the diagonal band at positive  $\beta$  (towards the upper left of the diagonal), profiles lower than approximately  $[40^\circ, 55^\circ]$  for the TPC case have already been excluded due to their incorrect radio-to- $\gamma$  phase lag, while the  $\gamma$ -ray profiles higher up than approximately  $[50^\circ, 65^\circ]$ , for both models, are excluded as being two-peaked (as discussed in Section 5.1.2).

For the diagonal band at negative  $\beta$ , TPC profiles below approximately  $[55^\circ, 40^\circ]$  can again be excluded due to their radio-to- $\gamma$  phase lag, while the profiles above approximately  $[65^\circ, 45^\circ]$  again classify as two-peak profiles in both cases.

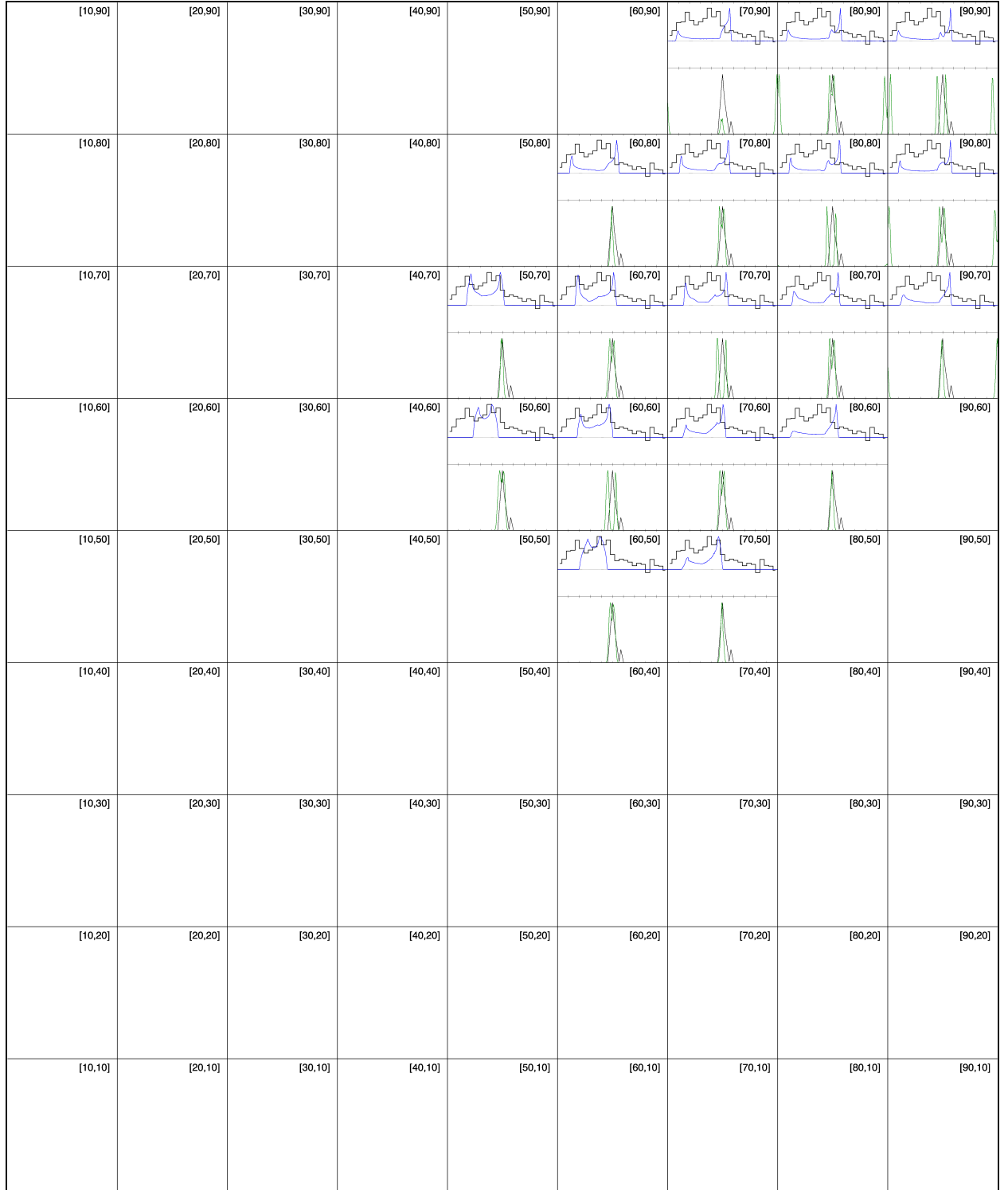
There are now two even more constrained regions within which the solutions could lie for both the TPC and OG cases. The profiles in these regions will now be considered at an even greater resolution, by an examination of the relevant  $1^\circ$  atlases.

### 5.1.4 The $1^\circ$ atlases

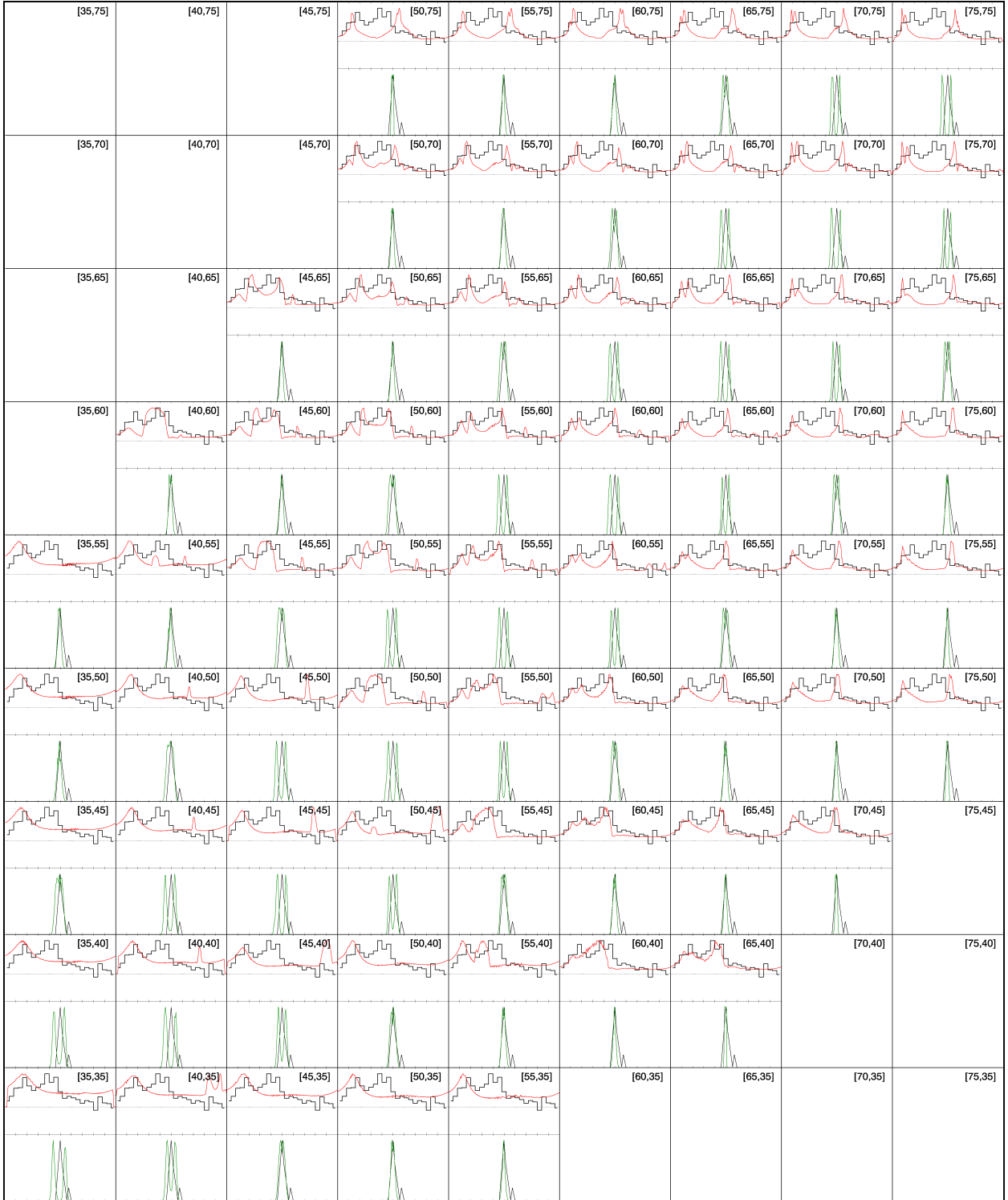
At this point the size of the possible solution space has been narrowed down sufficiently to allow for a manual search for the boundaries of the contours at a  $1^\circ$  resolution. To find the contour



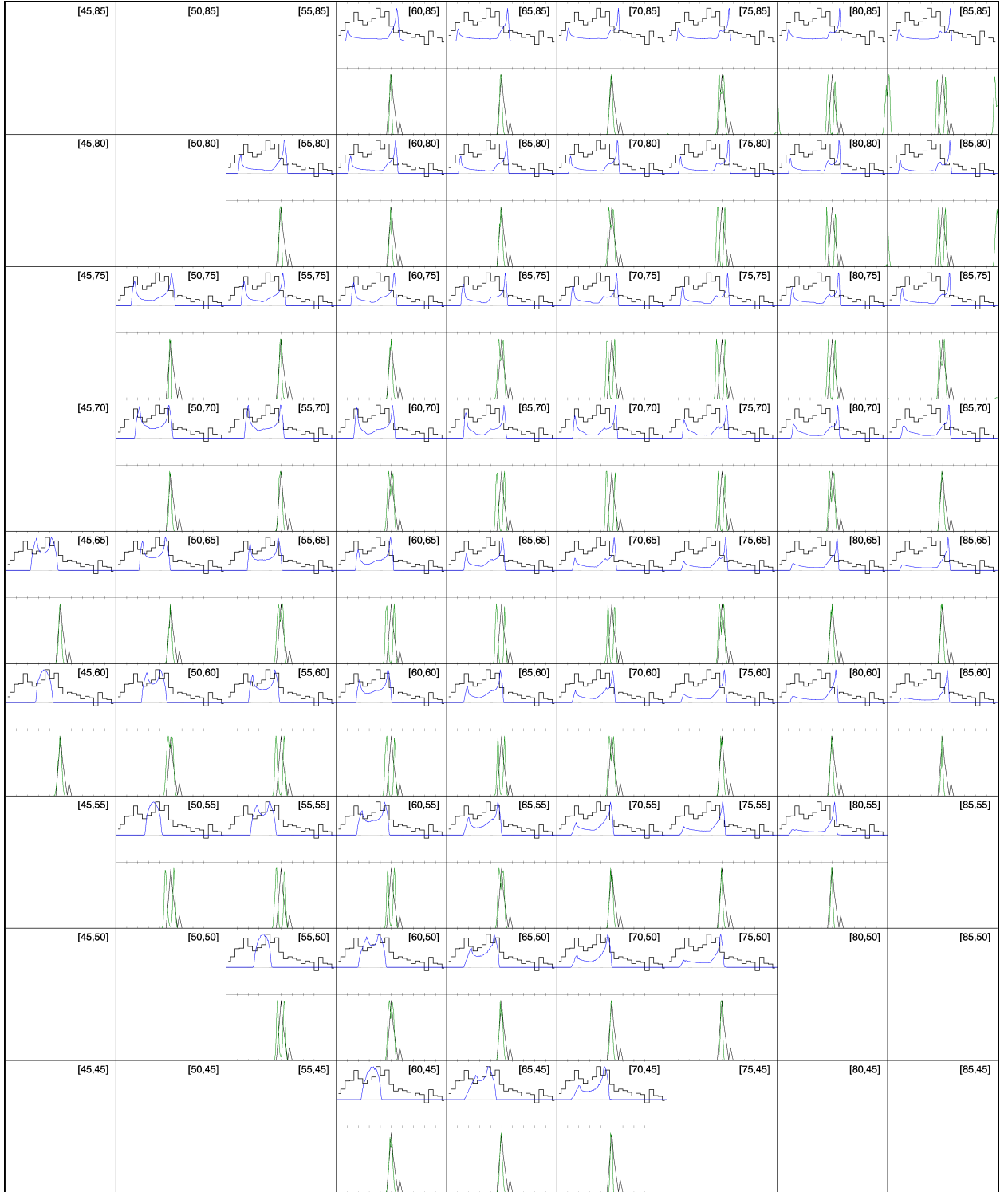
**Figure 5.2:** TPC atlas for PSR J1509–5850 with a 10° resolution.



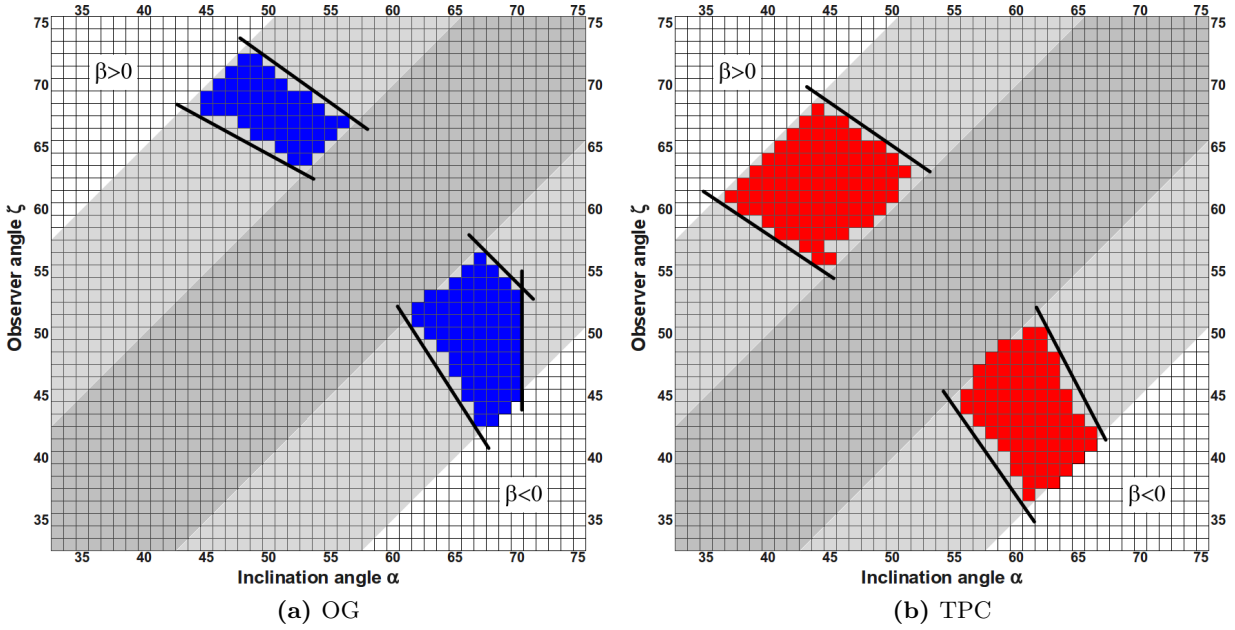
**Figure 5.3:** OG atlas for PSR J1509–5850 with a  $10^\circ$  resolution.



**Figure 5.4:** TPC atlas for PSR J1509–5850 with a  $5^\circ$  resolution.



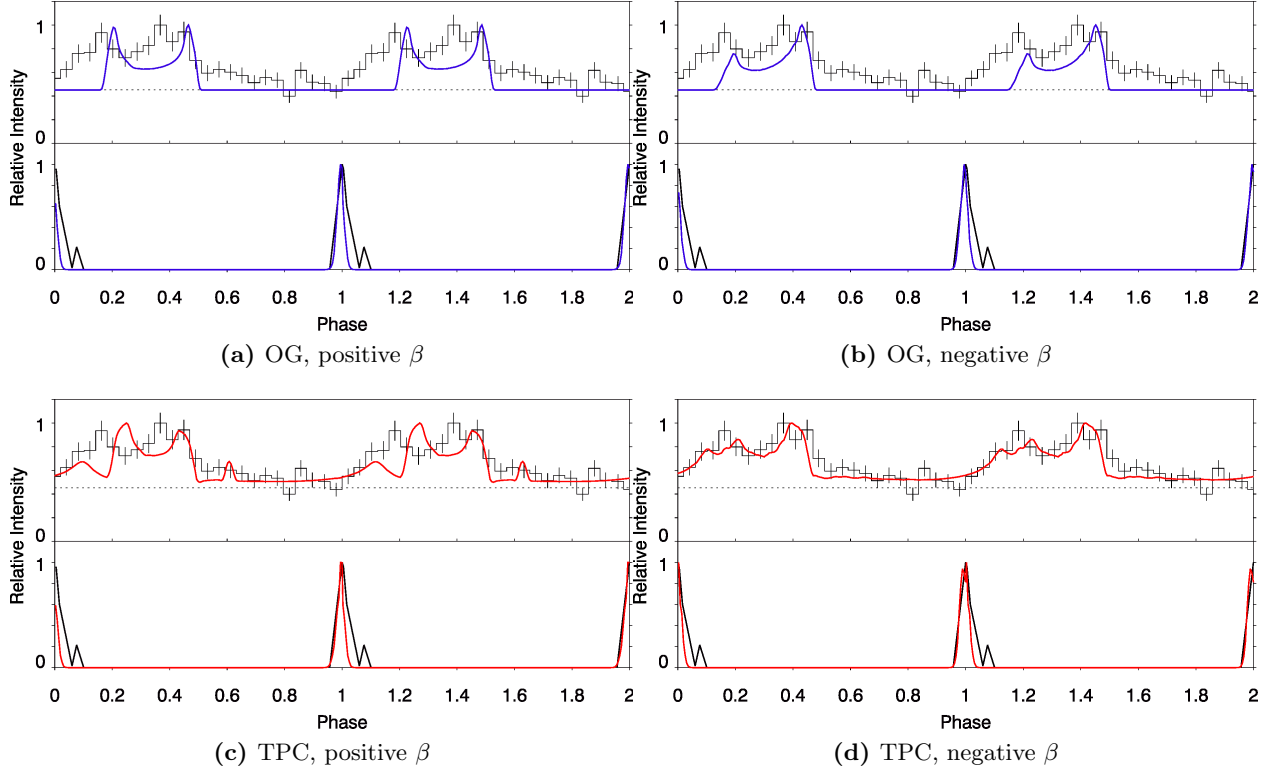
**Figure 5.5:** OG atlas for PSR J1509–5850 with a  $5^\circ$  resolution.



**Figure 5.6:** Solution contours for the OG and TPC models at  $1^\circ$  resolution. The grey bands denote single-peaked (light grey) and double-peaked (dark grey) radio profiles, while the blue and red regions indicate the possible solution space where best-fits to the data can be obtained.

boundaries, some location within the region identified using the  $5^\circ$  atlas is used as a starting point. Moving away from this location incrementally while keeping  $\alpha$  constant, both in the direction of increasing  $\zeta$  and decreasing  $\zeta$ , each successive location is either included or excluded from the solution contour based on the quality (as judged by eye) of the LC fit at that location. This process is continued until the upper and lower boundaries of the solution contour at that specific  $\alpha$  are found. Next, this procedure is repeated for each successive value of  $\alpha$  as it is similarly incremented and decremented, until the left-most and right-most boundaries of the solution contour are found. Once that contour has been found, a survey of the profiles exterior to it is done to ensure that there isn't a second solution contour in the same region. Note that as it is impractical to include the  $1^\circ$  atlases in this text, only the  $2^\circ$  atlases are shown. These atlases serve well enough to illustrate the process used to obtain the constraints.

In most cases, the constraints affecting the boundaries of the contours are merely those identifiable on the  $5^\circ$  atlas. This means that the procedure described above simply resolves their position more sharply. In some cases, however, further constraints become apparent when considering the regions identified using the  $5^\circ$  atlas. In the case of PSR J1509–5850, three of the four regions identified using the  $5^\circ$  atlas contain solution contours bounded only by the constraints already applied to the  $5^\circ$  atlas. The solution contour in the fourth region, at negative  $\beta$  on the OG atlas, has an additional boundary which is not readily apparent when considering the  $5^\circ$  atlas. This boundary excludes profiles with  $\alpha > 70^\circ$  from the solution contour, as they sufficiently underpredict the observed emission between the main peak (around 0.4 in phase) and the small secondary peak (around 0.2 in phase) to justify their exclusion from the solution contour.



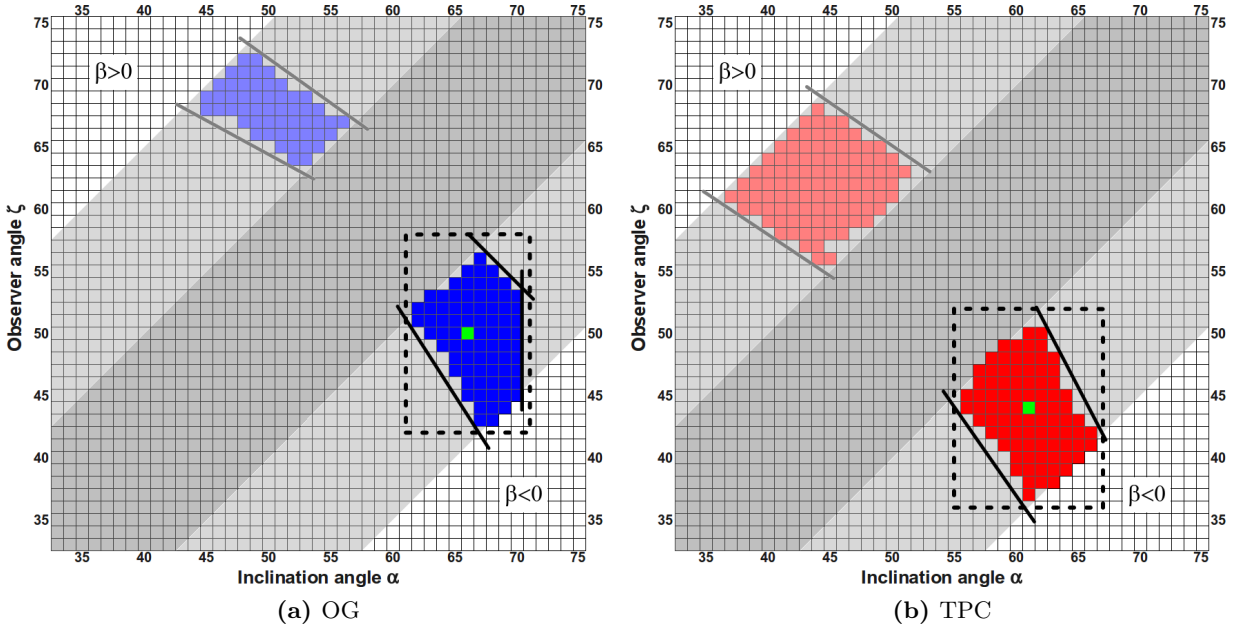
**Figure 5.7:** Representative LC fits for PSR J1509–5850, taken from the resulting solution contours for both the OG (top) and TPC (bottom) models.

### 5.1.5 The resulting contours

The procedure outlined in the previous subsection yields the solution contours shown in Figure 5.6. For both the OG and TPC cases, two possible solution contours for PSR J1509–5850 are left. To further constrain the possible solution space, it is now useful to directly compare the properties of the profiles lying within the distinct contours for each case. Figure 5.7 shows representative profiles from each of the four possible solution contours derived for PSR J1509–5850: two for the OG model, and two for the TPC model. For the TPC case a direct comparison of the profiles produced in the two regions easily disqualifies the region at positive  $\beta$ , as the profiles at negative  $\beta$  fit the data remarkably well. For the OG case, the difference between the profiles predicted in the two regions is less extreme. In both regions the predicted profiles display a significant peak around 0.4 in phase, with a secondary peak around 0.2 in phase. For the profiles predicted in the region at positive  $\beta$ , however, this secondary peak is more comparable in intensity to the peak around 0.4 in phase, so that the profiles can more readily be described as two-peaked profiles. This means that the contour at positive  $\beta$  can be discarded in favour of the contour at negative  $\beta$ .

## 5.2 Extracting $\alpha$ , $\zeta$ , and $f_\Omega$ values and errors from the solution contours

The awkward shapes of the solution contours shown in Figure 5.6 make it difficult to naturally relate the results in terms of  $\alpha$  and  $\zeta$  values with errors. In order to do this, a simple convention

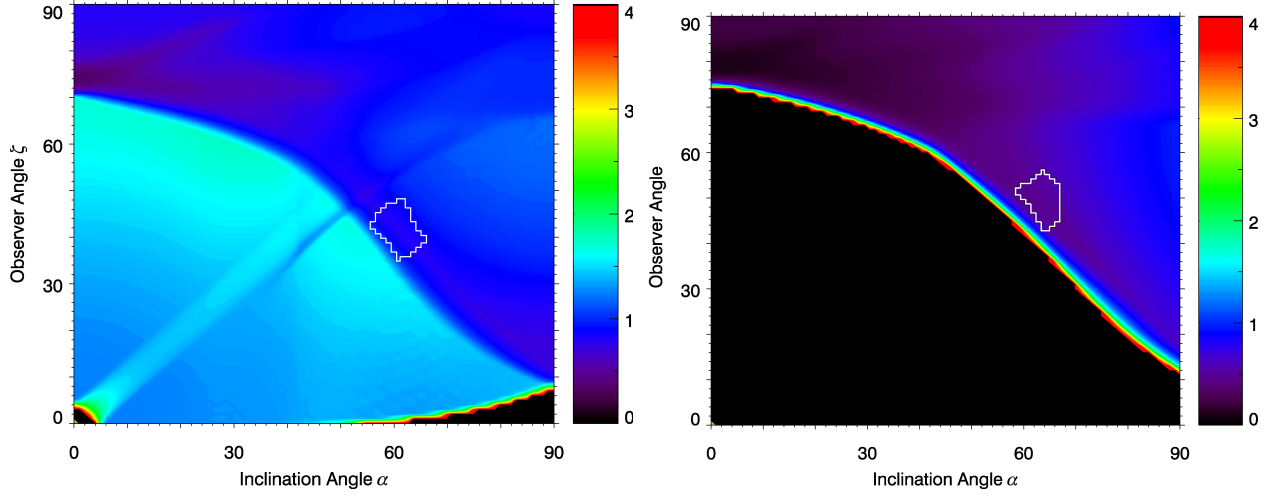


**Figure 5.8:** The resulting solution contours for both the OG and TPC models. The faded contours have been disqualified in favour of the other contours. The green squares indicate the locations of the reported best-fit solutions, while the dashed rectangles indicate the region of  $(\alpha, \zeta)$ -space bounded by the reported errors.

for characterising the contours is implemented. Each contour is associated with an  $\alpha$  and  $\zeta$  value, with conservative errors on both, chosen such that the specific contours are entirely encompassed by the corresponding region in  $(\alpha, \zeta)$  space. Figure 5.8 illustrates this principle graphically for the two remaining solution contours of PSR J1509–5850.

For PSR J1509–5850, the best-fit values found in the TPC case are  $\alpha = 61^\circ \pm 5^\circ$  and  $\zeta = 44^\circ \pm 7^\circ$ , corresponding to the region bounded by the dashed box in Figure 5.8 (b), centred on the location indicated in green. This characterisation of the solution contours results in the inclusion of a significant number of previously excluded locations. A more stringent characterisation would consider the range of values included along the diagonal and “anti-diagonal”. The first implies a constraint on  $\beta = \zeta - \alpha$ , similar to what is found in the radio polarisation fits of Weltevrede et al. (2010) (see Table 7.1), while the second implies a constraint on  $\zeta + \alpha$ , which is not a commonly used quantity.

These contours also allow for the derivation of a predicted value for  $f_\Omega$  (see Section 3.5) corresponding to each remaining solution contour. This is accomplished by computing the range of values  $f_\Omega$  assumes inside the solution contour and considering this range in a way similar to that for the ranges for  $\alpha$  and  $\zeta$ . This means that the value for  $f_\Omega$  reported is the value at the centre of this range, while the error on it is equal to half the size of the range. This procedure is quite robust, as the solution contours tend to lie in regions where the gradient in  $f_\Omega$  is relatively small. Figure 5.9 shows the remaining solution contours for both the OG and TPC case superimposed onto contour plots of  $f_\Omega$ . For PSR J1509–5850 the value derived in this way is  $f_\Omega = 0.77 \pm 0.11$  for the OG case, and  $0.89 \pm 0.10$  for the TPC case.



**Figure 5.9:** Contour plot of  $f_{\Omega}$  as function of  $\alpha$  and  $\zeta$  yielded by the TPC model (*left*) and the OG model (*right*) for PSR J1509-5850. The *red* and *white* areas indicate the best-fit  $(\alpha, \zeta)$ -contours for the radio and  $\gamma$ -ray LCs. The corresponding values of  $f_{\Omega}$  are  $0.77 \pm 0.11$  for the OG case, and  $0.89 \pm 0.10$  for the TPC case. These values are both less than unity, which means that the profiles in this region underrepresent the pulsar's true luminosity.

