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FISCHER MATRICES OF DEMPWOLFF GROUP $2^{5\cdot}GL(5,2)$

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Abstract. In [U. Dempwolff, On extensions of elementary abelian groups of order 2^5 by $GL(5,2)$, *Rend. Sem. Mat. Univ. Padova*, **48** (1972), 359 - 364.] Dempwolff proved the existence of a group of the form $2^{5\cdot}GL(5,2)$ (a non split extension of the elementary abelian group 2^5 by the general linear group $GL(5,2)$). This group is the second largest maximal subgroup of the sporadic Thompson simple group Th. In this paper we calculate the Fischer matrices of Dempwolff group $\overline{G} = 2^{5\cdot}GL(5,2)$. The theory of projective characters is involved and we have computed the Schur multiplier together with a projective character table of an inertia factor group. The full character table of $\overline{G}$ is then can be calculated easily.

1. Introduction

It is well known (cf. Huppert [15]) that $H^2(GL(n,q),V(n,q)) = 0,\ \forall\ q > 2$, where $H^2(K,M)$ is the second cohomology group of a group K with coefficients in M. Hence a non split extension of the form $V(n,q)^{\cdot}GL(n,q),\ q > 2$ does not exist. In the case $q = 2$, Dempwolff proved in [8] that apart from the case $n = 5$, then

$$\dim_{\mathbb{F}_2} H^2(GL(n,2),\mathbb{F}_2^n) = \begin{cases} 1 & \text{if } n = 3 \text{ or } 4,\\ 0 & \text{otherwise.}\end{cases}$$

Hence non split extensions of the forms $2^{3\cdot}GL(3,2)$ and $2^{4\cdot}GL(4,2) \cong 2^{4\cdot}S_6$ exist and they are unique up to isomorphisms. In fact the group $2^{3\cdot}GL(3,2)$ is a maximal subgroup of the Chevalley group $G_2(3)$, while $2^{4\cdot}GL(4,2)$ sits maximally in the sporadic Conway group Co_3 (see $\mathbb{ATLAS}$ [6]). Later on in 1972, Dempwolff proved in [9] that a non split extension of the form $2^{5\cdot}GL(5,2)$ does exist and it is unique up to isomorphism. This group, which is known as *Dempwolff group* and we denote it in this section by $\overline{G}$, is the second largest maximal subgroup of the sporadic Thompson group Th (see $\mathbb{ATLAS}$). In the unique

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class of involutions $2A$ of Thompson group Th, there are 5 commuting involutions, say n_1, n_2, n_3, n_4 and n_5. The group $\langle n_1, n_2, n_3, n_4, n_5\rangle := K$ is an elementary abelian 2−group of order 32 and $N_{\mathrm{Th}}(K) \cong 2^{5\cdot}GL(5,2) = \overline{G}$. Clearly the group $\overline{G}$ has order $32 \times 9\ 999\ 360 = 319\ 979\ 520$ and index 283 599 225 in Th. In this paper we are interested in the Fischer matrices of $\overline{G}$ and hence the re-computation of the character table of $\overline{G}$ using Clifford-Fischer theory.

Let x and y be in the 248−dimensional matrix group over $\mathbb{F}_2$, that are generators of Th given by the electronic ATLAS of Wilson. Using MAGMA [5] it is possible to construct $\overline{G}$ inside Th since $\overline{G}$ is generated as follows

$$\overline{G} = \left\langle x, ((xyyxy)^2)^{((xy)^{15}(xyy)^9(xy)^{12}(xyy)^{16}(xy)^{17})} \right\rangle.$$

However $\overline{G} \leq GL(69,2)$ and generators a and b of $\overline{G}$ in terms of 69×69 matrices over $\mathbb{F}_2$ are also given by the electronic ATLAS of Wilson. Using this 69−dimensional representation over $\mathbb{F}_2$, one can obtain the 5 generators n_1, n_2, n_3, n_4 and n_5 of the normal subgroup N of $\overline{G}$ using Magma or GAP [13]. The commands "Complements($\overline{G}$, N)" and "Complementclasses($\overline{G}$, N)" of Magma and GAP respectively reveal the complements of N in $\overline{G}$, where in our case, an empty set will be returned confirming that the extension is a non-split.

2. The Conjugacy Classes of $\overline{G} = 2^{5\cdot}GL(5,2)$

We have used the method of the *coset analysis* (Moori [23] and [24]) to calculate the conjugacy classes of $\overline{G} = 2^{5\cdot}GL(5,2)$. This method have been used by various authors such as Barraclough [3] and in particular by several MSc and PhD students, such as Mpono [26], Rodrigues [29], Whitely [31] and in [18], [19], [20], [21] and [22] by the authors of this paper. Thus in this paper we only explain the notations used in Table 1 and interested readers can refer to any of the above mentioned references for a detailed description on how the coset analysis can be used to determine the conjugacy classes of any group extension.

In Table 1,

- g_i is the i^{th} conjugacy class of $G = GL(5,2)$ as listed in the order given by the $\mathbb{ATLAS}$.
- k_i is the number of orbits $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$ on the action of N on the coset $N\overline{g}_i = 2^5\overline{g}_i$, where $\overline{g}_i$ is a pre-image of g_i under the natural epimorphism $\pi : \overline{G} \longrightarrow G$. In particular, the action of N on the identity coset N produces 32 orbits each consists of singleton. Thus $k_1 = 32$.
- f_{ij} is the number of orbits fused together under the action of $\overline{G}$ on $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$. In particular, the action of $\overline{G}$ on the orbits $Q_{11}, Q_{12}, \cdots, Q_{1k_1}$ leaves invariant the orbit Q_{11} (this orbit assumed to consist of the identity element of N), while fuse the other 31 orbits (consisting of the involutions of N) into a single orbit. Thus $f_{11} = 1$ and $f_{12} = 31$.
- m_{ij}'s are weights (attached to each class of $\overline{G}$) that will be used later in computing the Fischer matrices of $\overline{G}$. These weights are computed through the formula

$$m_{ij} = [N_{\overline{G}}(N\overline{g}_i) : C_{\overline{G}}(g_{ij})] = |N|\frac{|C_G(g_i)|}{|C_{\overline{G}}(g_{ij})|}. \tag{2.1}$$

- g_{ij} is a representative of conjugacy class of $\overline{G}$ that correspond to the class g_i of G. In particular the action of $\overline{G}$ on N produces two conjugacy classes in $\overline{G}$ represented by $g_{11} = 1_{\overline{G}} = 1_N$ and g_{12} and these two classes have sizes 1 and 31 respectively.

TABLE 1. The conjugacy classes of $\overline{G} = 2^{5\cdot}GL(5,2)$

$[g]_{GL(5,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert C_{\overline{G}}(g_{ij})\rvert$	$\lvert [g_{ij}]_{\overline{G}}\rvert$
$g_1 = 1A$	$k_1 = 32$	$f_{11} = 1$	$m_{11} = 1$	g_{11}	1	319979520	1
		$f_{12} = 31$	$m_{12} = 31$	g_{12}	2	10321920	31
$g_2 = 2A$	$k_2 = 16$	$f_{21} = 8$	$m_{21} = 16$	g_{21}	4	43008	7440
		$f_{22} = 8$	$m_{22} = 16$	g_{22}	2	43008	7440
$g_3 = 2B$	$k_3 = 8$	$f_{31} = 8$	$m_{31} = 32$	g_{31}	4	1536	208320
$g_4 = 3A$	$k_4 = 8$	$f_{41} = 1$	$m_{41} = 4$	g_{41}	3	4032	79360
		$f_{42} = 7$	$m_{42} = 28$	g_{42}	6	576	555520
$g_5 = 3B$	$k_5 = 2$	$f_{51} = 1$	$m_{51} = 16$	g_{51}	6	360	888832
		$f_{52} = 1$	$m_{52} = 16$	g_{52}	3	360	888832
$g_6 = 4A$	$k_6 = 8$	$f_{61} = 8$	$m_{61} = 32$	g_{61}	8	384	833280
$g_7 = 4B$	$k_7 = 4$	$f_{71} = 4$	$m_{71} = 32$	g_{71}	4	128	2499840
$g_8 = 4C$	$k_8 = 4$	$f_{81} = 4$	$m_{81} = 32$	g_{81}	8	32	9999360
$g_9 = 5A$	$k_9 = 2$	$f_{91} = 1$	$m_{91} = 16$	g_{91}	10	30	10665984
		$f_{92} = 1$	$m_{92} = 16$	g_{92}	5	30	10665984
		$f_{10,1} = 1$	$m_{10,1} = 8$	$g_{10,1}$	12	96	3333120
$g_{10} = 6A$	$k_{10} = 4$	$f_{10,2} = 1$	$m_{10,2} = 8$	$g_{10,2}$	12	96	3333120
		$f_{10,3} = 2$	$m_{10,3} = 16$	$g_{10,3}$	6	48	6666240
$g_{11} = 6B$	$k_{11} = 2$	$f_{11,1} = 2$	$m_{11,1} = 32$	$g_{11,1}$	12	12	26664960
$g_{12} = 7A$	$k_{12} = 4$	$f_{12,1} = 1$	$m_{12,1} = 8$	$g_{12,1}$	7	168	1904640
		$f_{12,2} = 3$	$m_{12,2} = 24$	$g_{12,2}$	14	56	5713920
$g_{13} = 7B$	$k_{13} = 4$	$f_{13,1} = 1$	$m_{13,1} = 8$	$g_{13,1}$	7	168	1904640
		$f_{13,2} = 3$	$m_{13,2} = 24$	$g_{13,2}$	14	56	5713920
$g_{14} = 8A$	$k_{14} = 2$	$f_{14,1} = 2$	$m_{14,1} = 32$	$g_{14,1}$	8	16	19998720
$g_{15} = 12A$	$k_{15} = 2$	$f_{15,1} = 1$	$m_{15,1} = 16$	$g_{15,1}$	24	24	13332480
		$f_{15,2} = 1$	$m_{15,2} = 16$	$g_{15,2}$	24	24	13332480
$g_{16} = 14A$	$k_{16} = 2$	$f_{16,1} = 1$	$m_{16,1} = 16$	$g_{16,1}$	28	28	11427840
		$f_{16,2} = 1$	$m_{16,2} = 16$	$g_{16,2}$	14	28	11427840

Continued on next page

Table 1 (continued)

$[g]_{GL(5,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert C_{\overline{G}}(g_{ij})\rvert$	$\lvert[g_{ij}]_{\overline{G}}\rvert$
$g_{17} = 14B$	$k_{17} = 2$	$f_{17,1} = 1$	$m_{17,1} = 16$	$g_{17,1}$	28	28	11427840
		$f_{17,2} = 1$	$m_{17,2} = 16$	$g_{17,2}$	14	28	11427840
$g_{18} = 15A$	$k_{18} = 2$	$f_{18,1} = 1$	$m_{18,1} = 16$	$g_{18,1}$	15	30	10665984
		$f_{18,2} = 1$	$m_{18,2} = 16$	$g_{18,2}$	30	30	10665984
$g_{19} = 15B$	$k_{19} = 2$	$f_{19,1} = 1$	$m_{19,1} = 16$	$g_{19,1}$	15	30	10665984
		$f_{19,2} = 1$	$m_{19,2} = 16$	$g_{19,2}$	30	30	10665984
$g_{20} = 21A$	$k_{20} = 1$	$f_{20,1} = 1$	$m_{20,1} = 32$	$g_{20,1}$	21	21	15237120
$g_{21} = 21B$	$k_{21} = 1$	$f_{21,1} = 1$	$m_{21,1} = 32$	$g_{21,1}$	21	21	15237120
$g_{22} = 31A$	$k_{22} = 1$	$f_{22,1} = 1$	$m_{22,1} = 32$	$g_{22,1}$	31	31	10321920
$g_{23} = 31B$	$k_{23} = 1$	$f_{23,1} = 1$	$m_{23,1} = 32$	$g_{23,1}$	31	31	10321920
$g_{24} = 31C$	$k_{24} = 1$	$f_{24,1} = 1$	$m_{24,1} = 32$	$g_{24,1}$	31	31	10321920
$g_{25} = 31D$	$k_{25} = 1$	$f_{25,1} = 1$	$m_{25,1} = 32$	$g_{25,1}$	31	31	10321920
$g_{26} = 31E$	$k_{26} = 1$	$f_{26,1} = 1$	$m_{26,1} = 32$	$g_{26,1}$	31	31	10321920
$g_{27} = 31F$	$k_{27} = 1$	$f_{27,1} = 1$	$m_{27,1} = 32$	$g_{27,1}$	31	31	10321920

3. **The Theory of Clifford-Fischer Matrices**

Let $\overline{G} = N{\cdot}G$, where $N \lhd \overline{G}$ and $\overline{G}/N \cong G$, be a group extension. To construct the character table of $\overline{G}$ we need to have

- the character tables (ordinary or projective) of the inertia factor groups,
- the fusions of classes of the inertia factors into classes of G,
- the Fischer matrices of $\overline{G} = N{\cdot}G$.

The theory of Clifford-Fischer matrices, which is based on Clifford Theory (see Clifford [7]), was developed by B. Fischer ([10], [11] and [12]). This technique has also been discussed and applied to both split and non-split extension in several publications, for example see Ali and Moori [2], Barraclough [3], Fischer [12], Moori [23], Moori and Basheer [21], [22], Moori and Mpono [26], Pahlings [27], Rodrigues [29], Whitely [31] and in a recent book by K. Lux and H. Pahlings [28].

Let $\overline{H} \trianglelefteq \overline{G}$ and let $\phi \in \mathrm{Irr}(\overline{H})$. For $\overline{g} \in \overline{G}$, define $\phi^{\overline{g}}$ by $\phi^{\overline{g}}(h) = \phi(\overline{g}h\overline{g}^{-1}),\ \forall\ h \in \overline{H}$. It follows that $\overline{G}$ acts on $\mathrm{Irr}(\overline{H})$ by conjugation and we define the *inertia group* of ϕ in $\overline{G}$ by $\overline{H}_\phi = \{\overline{g} \in \overline{G} \mid \phi^{\overline{g}} = \phi\}$. Also for a finite group $\mathcal{K}$, we let $\mathrm{IrrProj}(\mathcal{K}, \alpha^{-1})$ denotes the set of irreducible projective characters of $\mathcal{K}$ with factor set α^{-1}.

Theorem 3.1 (Clifford Theorem). *Let $\chi \in \mathrm{Irr}(\overline{G})$ and let $\theta_1, \theta_2, \cdots, \theta_t$ be representatives of orbits of $\overline{G}$ on $\mathrm{Irr}(N)$. For $k \in \{1, 2, \cdots, t\}$, let $\theta_k^{\overline{G}} = \{\theta_k = \theta_{k1}, \theta_{k2}, \cdots, \theta_{ks_k}\}$ and let $\overline{H}_k$ be the inertia group in $\overline{G}$ of θ_k. Then*

$$\chi{\downarrow}_N^{\overline{G}} = \sum_{k=1}^{t} e_k \sum_{u=1}^{s_k} \theta_{ku}, \quad \textit{where} \quad e_k = \left\langle \chi{\downarrow}_N^{\overline{G}}, \theta_k \right\rangle.$$

Moreover, for fixed k

$$\mathrm{Irr}(\overline{H}_k, \theta_k) := \left\{\psi_k \in \mathrm{Irr}(\overline{H}_k) |\ \left\langle \psi_k{\downarrow}_N^{\overline{H}_k}, \theta_k \right\rangle \neq 0\right\} \longleftrightarrow \left\{\chi \in \mathrm{Irr}(\overline{G}) |\ \left\langle \chi{\downarrow}_N^{\overline{G}}, \theta_k \right\rangle \neq 0\right\} := \mathrm{Irr}(\overline{G}, \theta_k)$$

under the map $\psi_k \longmapsto \psi_k{\uparrow}_{\overline{H}_k}^{\overline{G}}$.

Proof. See Theorems 4.1.5 and 4.1.7 of Ali [1] with the difference in notations. □

Theorem 3.2. *Further to the settings of* Theorem 3.1, *assume that for $k \in \{1, 2, \cdots, t\}$, there exists $\psi_k \in \mathrm{Irr}(\overline{H}_k, \theta_k)$. Then*

$$\mathrm{Irr}(\overline{G}) = \bigcup_{k=1}^{t} \left\{(\psi_k \inf(\zeta)){\uparrow}_{\overline{H}_k}^{\overline{G}} |\ \zeta \in \mathrm{Irr}(\overline{H}_k/N)\right\}. \tag{3.1}$$

Proof. See Ali [1] or Whitley [31]. □

Remark 3.3. *It is by no means necessarily the case that there exists an extension ψ_k of θ_k to the inertia group (that is the case $\mathrm{Irr}(\overline{H}_k, \theta_k) = \emptyset$, the empty set, is feasible). However, there is always a projective extension $\widetilde{\psi}_k \in \mathrm{IrrProj}(\overline{H}_k, \overline{\alpha}_k^{-1})$ for some factor set $\overline{\alpha}_k$ of the Schur multiplier of $\overline{H}_k$. Thus the more proper formula for Equation* (3.1) *is (see Remark 4.2.7 of Ali* [1]*)*

$$\mathrm{Irr}(\overline{G}) = \bigcup_{k=1}^{t} \left\{(\widetilde{\psi}_k \inf(\zeta)){\uparrow}_{\overline{H}_k}^{\overline{G}} |\ \widetilde{\psi}_k \in \mathrm{IrrProj}(\overline{H}_k, \overline{\alpha}_k^{-1}),\ \zeta \in \mathrm{IrrProj}(\overline{H}_k/N, \alpha_k^{-1})\right\}, \tag{3.2}$$

where the factor set α_k is obtained from $\overline{\alpha}_k$ as described in Corollary 7.3.3 of Whitely [31]. *Hence the character table of $\overline{G}$ is partitioned into t blocks $\mathcal{K}_1, \mathcal{K}_2, \cdots, \mathcal{K}_t$, where each block $\mathcal{K}_k$ of characters (ordinary or projective) is produced from the inertia subgroup $\overline{H}_k$.*

Note 3.4. *Observe that if $\alpha_k \sim [1]$ in Equation* (3.2)*, then we get Equation* (3.1)*. That is* $\mathrm{IrrProj}(\overline{H}_k, \overline{1}) = \mathrm{Irr}(\overline{H}_k)$ *and* $\mathrm{IrrProj}(H_k, 1) = \mathrm{Irr}(H_k)$.

By convention we take $\theta_1 = \mathbf{1}_N$, the trivial character of N. Thus $\overline{H}_{\theta_1} = \overline{H}_1 = \overline{G}$ and thus $\overline{H}_1/N \cong G$. Since $\{\mathbf{1}_{\overline{G}}\} \subseteq \mathrm{Irr}(\overline{G}, \mathbf{1}_N)$ and such that $\mathbf{1}_{\overline{G}}{\downarrow}_N^{\overline{G}} = \mathbf{1}_N$, the block $\mathcal{K}_1$ will consists only of the ordinary irreducible characters of G.

We now fix some notations for the conjugacy classes.

- With π being the natural epimorphism from $\overline{G}$ onto G, we use the notation $U = \pi(\overline{U})$ for any subset $\overline{U} \subseteq \overline{G}$. We have seen from Section 2 that $\pi^{-1}([g_i]_G) = \bigcup_{j=1}^{c(g_i)} [g_{ij}]_{\overline{G}}$ for any $1 \leq i \leq r$. Let us assume that $\pi(g_{ij}) = g_i$ and by convention we may take $g_{11} = 1_{\overline{G}}$. Note that $c(g_1)$ is the number of $\overline{G}-$conjugacy classes obtained from N.

- $[g_{ij}]_{\overline{G}} \bigcap \overline{H}_k = \bigcup_{n=1}^{c(g_{ijk})} [g_{ijkn}]_{\overline{H}_k}$, where $g_{ijkn} \in \overline{H}_k$ and by $c(g_{ijk})$ we mean the number of $\overline{H}_k-$conjugacy classes that form a partition for $[g_{ij}]_{\overline{G}}$. Since $g_{11} = 1_{\overline{G}}$, we have $g_{11k1} = 1_{\overline{G}}$ and thus $c(g_{11k1}) = 1$ for all $1 \leq k \leq t$.
- $[g_i]_G \bigcap H_k = \bigcup_{m=1}^{c(g_{ik})} [g_{ikm}]_{H_k}$, where $g_{ikm} \in H_k$ and by $c(g_{ik})$ we mean the number of H_k-conjugacy classes that form a partition for $[g_i]_G$. Since $g_1 = 1_G$, we have $g_{1k1} = 1_G$ and thus $c(g_{1k1}) = 1$ for all $1 \leq k \leq t$. Also $\pi(g_{ijkn}) = g_{ikm}$ for some $m = f(j, n)$.

Proposition 3.5. *With the notations of* Theorem 3.2 *and the above settings, we have*

$$(\widetilde{\psi}_k \inf(\zeta))\uparrow_{\overline{H}_k}^{\overline{G}}(g_{ij}) = \sum_{m=1}^{c(g_{ik})} \zeta(g_{ikm}) \sum_{n=1}^{c(g_{ijk})} \frac{|C_{\overline{G}}(g_{ij})|}{|C_{\overline{H}_k}(g_{ijkn})|} \widetilde{\psi}_k(g_{ijkn}).$$

Proof. See Ali [1] or Barraclough [3]. □

We proceed to define the Fischer matrix $\mathcal{F}_i$ corresponds to the conjugacy class $[g_i]_G$. We label the columns of $\mathcal{F}_i$ by the representatives of $[g_{ij}]_{\overline{G}}$, $1 \leq j \leq c(g_i)$ obtained by the coset analysis and below each g_{ij} we put $|C_{\overline{G}}(g_{ij})|$. Thus there are $c(g_i)$ columns. To label the rows of $\mathcal{F}_i$ we define the set $\overline{J}_i$ to be (this equivalent to the notation $R(g)$ used by Ali [1] (page 49), where g is a representative for a conjugacy class of G)

$$\overline{J}_i = \{(k, g_{ikm})|\ 1 \leq k \leq t,\ 1 \leq m \leq c(g_{ik}),\ g_{ikm} \text{ is } \alpha_k^{-1} - \text{regular class}\},$$

or for more brevity we let

$$J_i = \{(k, m)|\ 1 \leq k \leq t,\ 1 \leq m \leq c(g_{ik}),\ g_{ikm} \text{ is } \alpha_k^{-1} - \text{regular class}\}. \tag{3.3}$$

Then each row of $\mathcal{F}_i$ is indexed by a pair $(k, g_{ikm}) \in \overline{J}_i$ or $(k, m) \in J_i$. For fixed $1 \leq k \leq t$, we let $\mathcal{F}_{ik}$ be a sub-matrix of $\mathcal{F}_i$ with rows correspond to the pairs $(k, g_{ik1}), (k, g_{ik2}), \cdots, (k, g_{ikr_{ik}})$ or for brevity $(k, 1), (k, 2), \cdots, (k, r_k)$. Now let

$$a_{ij}^{(k,m)} := \sum_{n=1}^{c(g_{ijk})} \frac{|C_{\overline{G}}(g_{ij})|}{|C_{\overline{H}_k}(g_{ijkn})|} \widetilde{\psi}_k(g_{ijkn}) \tag{3.4}$$

(for which $\pi(g_{ijkn}) = g_{ikm}$). For each i, corresponding to the conjugacy class $[g_i]_G$, we define the Fischer matrix $\mathcal{F}_i = \left(a_{ij}^{(k,m)}\right)$, where $1 \leq k \leq t,\ 1 \leq m \leq c(g_{ik}),\ 1 \leq j \leq c(g_i)$. The Fischer matrix $\mathcal{F}_i$,

$$\mathcal{F}_i = \left(a_{ij}^{(k,m)}\right) = \begin{pmatrix} \mathcal{F}_{i1} \\ \hline \mathcal{F}_{i2} \\ \hline \vdots \\ \hline \mathcal{F}_{it} \end{pmatrix}$$

together with additional information required for their definition are presented as follows:

$\mathcal{F}_i$

g_i		g_{i1}	g_{i2}	$\cdots$	$g_{ic(g_i)}$
$\lvert C_{\overline{G}}(g_{ij})\rvert$		$\lvert C_{\overline{G}}(g_{i1})\rvert$	$\lvert C_{\overline{G}}(g_{i2})\rvert$	$\cdots$	$\lvert C_{\overline{G}}(g_{ic(g_i)})\rvert$
(k,m)	$\lvert C_{H_k}(g_{ikm})\rvert$				
$(1,1)$	$\lvert C_G(g_i)\rvert$	$a_{i1}^{(1,1)}$	$a_{i2}^{(1,1)}$	$\cdots$	$a_{ic(g_i)}^{(1,1)}$
$(2,1)$	$\lvert C_{H_2}(g_{i21})\rvert$	$a_{i1}^{(2,1)}$	$a_{i2}^{(2,1)}$	$\cdots$	$a_{ic(g_i)}^{(2,1)}$
$(2,2)$	$\lvert C_{H_2}(g_{i22})\rvert$	$a_{i1}^{(2,2)}$	$a_{i2}^{(2,2)}$	$\cdots$	$a_{ic(g_i)}^{(2,2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(2,r_2)$	$\lvert C_{H_2}(g_{i2r_{i2}})\rvert$	$a_{i1}^{(2,r_2)}$	$a_{i2}^{(2,r_2)}$	$\cdots$	$a_{ic(g_i)}^{(2,r_2)}$
$(u,1)$	$\lvert C_{H_u}(g_{iu1})\rvert$	$a_{i1}^{(u,1)}$	$a_{i2}^{(u,1)}$	$\cdots$	$a_{ic(g_i)}^{(u,1)}$
$(u,2)$	$\lvert C_{H_u}(g_{iu2})\rvert$	$a_{i1}^{(u,2)}$	$a_{i2}^{(u,2)}$	$\cdots$	$a_{ic(g_i)}^{(u,2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(u,r_u)	$\lvert C_{H_u}(g_{iur_{iu}})\rvert$	$a_{i1}^{(u,r_u)}$	$a_{i2}^{(u,r_u)}$	$\cdots$	$a_{ic(g_i)}^{(u,r_u)}$
$(t,1)$	$\lvert C_{H_t}(g_{it1})\rvert$	$a_{i1}^{(t,1)}$	$a_{i2}^{(t,1)}$	$\cdots$	$a_{ic(g_i)}^{(t,1)}$
$(t,2)$	$\lvert C_{H_t}(g_{it2})\rvert$	$a_{i1}^{(t,2)}$	$a_{i2}^{(t,2)}$	$\cdots$	$a_{ic(g_i)}^{(t,2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(t,r_t)	$\lvert C_{H_t}(g_{itr_{it}})\rvert$	$a_{i1}^{(t,r_t)}$	$a_{i2}^{(t,r_t)}$	$\cdots$	$a_{ic(g_i)}^{(t,r_t)}$
m_{ij}		m_{i1}	m_{i2}	$\cdots$	$m_{ic(g_i)}$

In the above the last entries give the weights m_{ij} as defined by Equation (2.1). These weights are required for computing the entries of $\mathcal{F}_i$ (see Proposition 3.6).

Fischer matrices satisfy some interesting properties, which help in computations of their entries. We gather these properties in the following Proposition.

Proposition 3.6. (i) $\displaystyle\sum_{k=1}^{t} c(g_{ik}) = c(g_i)$,

(ii) *$\mathcal{F}_i$ is non-singular for each i,*

(iii) $a_{ij}^{(1,1)} = 1,\ \forall\ 1 \leq j \leq c(g_i)$,

(iv) *If $N\overline{g}_i$ is a split coset, then $a_{i1}^{(k,m)} = \frac{|C_G(g_i)|}{|C_{H_k}(g_{ikm})|},\ \forall i \in \{1,2,\cdots,r\}$. In particular for the identity coset we have $a_{11}^{(k,m)} = [G:H_k]\theta_k(1_N),\ \forall\ (k,m) \in J_1$,*

(v) *If $N\overline{g}_i$ is a split coset, then $|a_{ij}^{(k,m)}| \leq |a_{i1}^{(k,m)}|$ for all $1 \leq j \leq c(g_i)$. Moreover if $|N| = p^{\alpha}$, for some prime p, then $a_{ij}^{(k,m)} \equiv a_{i1}^{(k,m)} (\mathrm{mod}\ p)$,*

(vi) *For each $1 \leq i \leq r$, the weights m_{ij} satisfy the relation $\displaystyle\sum_{j=1}^{c(g_i)} m_{ij} = |N|$,*

(vii) ***Column Orthogonality Relation:***

$$\sum_{(k,m)\in J_i} |C_{H_k}(g_{ikm})| a_{ij}^{(k,m)} \overline{a_{ij'}^{(k,m)}} = \delta_{jj'} |C_{\overline{G}}(g_{ij})|,$$

(viii) ***Row Orthogonality Relation:***

$$\sum_{j=1}^{c(g_i)} m_{ij} a_{ij}^{(k,m)} \overline{a_{ij}^{(k',m')}} = \delta_{(k,m)(k',m')} a_{i1}^{(k,m)} |N|.$$

Proof. Proofs for many assertions of Proposition 3.6 can be founded in Moori's students theses, for example see Ali [1] or Mpono [25] and some other assertions are provided in Schiffer [30] as well as in Moori and Basheer [22] and Lux and Pahlings [28]. □

3.1. **Character Table of $\overline{G}$.** For fixed $1 \leq k \leq t$ and $1 \leq i \leq r$, let $\mathcal{K}_{ik}$ be the fragment of the projective character table of H_k, with factor set α_k^{-1}, consisting of columns correspond to the conjugacy classes $g_{ik1}, g_{ik2}, \cdots, g_{ikr_{ik}}$ of H_k (those are the α_k^{-1}−regular classes of H_k that fuse to $[g_i]_G$ and thus $r_{ik} = c(g_{ik})$). Then the characters of $\overline{G}$ on the classes $[g_{ij}]_{\overline{G}}$, $1 \leq j \leq c(g_i)$, is given by the matrix $\mathcal{K}_{ik}\mathcal{F}_{ik}$, where $\mathcal{F}_{ik}$ is the sub-matrix of $\mathcal{F}_i$ defined previously with rows correspond to the pairs $(k, g_{ik1}), (k, g_{ik2}), \cdots, (k, g_{ikr_{ik}})$. Note that the size of $\mathcal{K}_{ik}$ is $|\mathrm{IrrProj}(H_k, \alpha_k^{-1})| \times r_{ik}$ and the size of $\mathcal{F}_{ik}$ is $r_{ik} \times c(g_i)$. Therefore the character table of $\overline{G}$ will have the form

	g_1				g_2				$\cdots$	g_r			
	g_{11}	g_{12}	$\cdots$	$g_{1c(g_1)}$	g_{21}	g_{22}	$\cdots$	$g_{2c(g_2)}$	$\cdots$	g_{r1}	g_{r2}	$\cdots$	$g_{rc(g_r)}$
$\mathcal{K}_1$	$\mathcal{K}_{11}\mathcal{F}_{11}$				$\mathcal{K}_{12}\mathcal{F}_{12}$				$\cdots$	$\mathcal{K}_{1r}\mathcal{F}_{1r}$			
$\mathcal{K}_2$	$\mathcal{K}_{21}\mathcal{F}_{21}$				$\mathcal{K}_{22}\mathcal{F}_{22}$				$\cdots$	$\mathcal{K}_{2r}\mathcal{F}_{2r}$			
$\vdots$	$\vdots$				$\vdots$				$\ddots$	$\vdots$			
$\mathcal{K}_t$	$\mathcal{K}_{t1}\mathcal{F}_{t1}$				$\mathcal{K}_{t2}\mathcal{F}_{t2}$				$\cdots$	$\mathcal{K}_{tr}\mathcal{F}_{tr}$			

Note 3.7. *Observe that characters of $\overline{G}$ consisted in $\mathcal{K}_1$ are just $\mathrm{Irr}(G)$ and therefore the size of $\mathcal{K}_{1i}\mathcal{F}_{1i}$, for each $1 \leq i \leq r$, is $|\mathrm{Irr}(G)| \times c(g_i)$. In particular, columns of $\mathcal{K}_{11}\mathcal{F}_{11}$ are the degrees of irreducible characters of G repeated themselves $c(g_1)$ times, where we know that $c(g_1)$ is number of $\overline{G}$−conjugacy classes obtained from the normal subgroup N.*

4. The Inertia Groups of $\overline{G} = 2^{5\cdot}GL(5,2)$

We have seen in Section 2 that the action of $\overline{G} = 2^{5\cdot}GL(5,2)$ on $N = 2^5$ produced two orbits with lengths 1 and 31. By Brauer Theorem (Lemma 4.5.2 of Gorenstein [14]), it follows that the action of $\overline{G}$ on $\mathrm{Irr}(N)$ will also produce two orbits. These two orbits must necessarily have lengths 1 and 31 and the first orbit consists of the identity character $\mathbf{1}_N$ while the other orbit consists of the other non-trivial linear characters of N. Thus the corresponding inertia factor groups H_1 and H_2 have indices 1 and 31 respectively in $GL(5,2)$. By looking at the maximal subgroups of $GL(5,2)$ (see 𝔸𝕋𝕃𝔸𝕊), it is readily verified that $H_1 = GL(5,2)$ and $H_2 = 2^4{:}GL(4,2)$ (note that there are two isomorphic non-conjugate subgroups of $GL(5,2)$ of the form $2^4{:}GL(4,2)$). Also note that the corresponding inertia groups are non-split extension groups of the forms $\overline{H}_1 = \overline{G} = 2^{5\cdot}GL(5,2)$ and $\overline{H}_2 = 2^5{\cdot}(2^4{:}GL(4,2))$. In this section we show that we need to use projective characters, we also calculate the Schur multiplier of H_2 and supply the projective character table of H_2 with factor set α, $\alpha \sim [2]$.

As a subgroup of $GL(5,2)$, the group $H_2 = 2^4{:}GL(4,2)$ is generated by the following two elements

$$h_1 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad h_2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The following theorem characterizes the type of character table (ordinary or projective) of H_2 that we will use to construct the character table of the Dempwolff group $\overline{G}$.

Theorem 4.1. *The character $\sum_{i=2}^{32} \theta_i$, where $\theta_2, \theta_3, \cdots, \theta_{32}$ are the non-trivial linear characters of $N = 2^5$, is not extendable to a character of its inertia group $\overline{H}_2 = 2^5{\cdot}(2^4{:}GL(4,2))$.*

Proof. We recall from Table 1 that the number of conjugacy classes of $\overline{G}$ is 41. Thus we have to find 41 irreducible characters. From Subsection 3.1, these 41 characters are distributed into two blocks $\mathcal{K}_1$ and $\mathcal{K}_2$ corresponding to the inertia factor groups H_1 and H_2 respectively. From Note 3.7 we see that H_1 contributes with 27 characters to the character table of $\overline{G}$ (these 27 characters are the ordinary irreducible characters of $G = GL(5,2)$). If $\sum_{i=2}^{32} \theta_i$ is extendable to an ordinary character of $\overline{H}_2$, then we will use the ordinary character table of H_2 to construct the character table of $\overline{G}$. The two matrices h_1 and h_2 can be used to generate H_2 in either Magma or GAP and then one can ask for the character table of H_2, where we get $|\mathrm{Irr}(H_2)| = 25$. Using this, we obtain that $|\mathrm{Irr}(\overline{G})| = |\mathrm{Irr}(H_1)| + |\mathrm{Irr}(H_2)| = 27 + 25 = 52$, contradicting the fact that $|\mathrm{Irr}(\overline{G})| = 41$. Therefore $\sum_{i=2}^{32} \theta_i$ is not extendable to an ordinary character of its inertia group $\overline{H}_2$. $\square$

Remark 4.2. *Note that* Theorem 4.1 *asserts* $\mathrm{Irr}(\overline{H}_2, \sum_{i=2}^{32} \theta_i) = \emptyset$ *and hence we have to make the use of projective representations. Thus the projective character table of H_2 with factor set α^{-1}, that we will use to construct the ordinary character table of $\overline{G}$, must have 14 irreducible characters.*

We note that the group $H_2 = 2^4{:}GL(4,2)$ has order $2^{10} \times 3^2 \times 5 \times 7$. To find the Schur Multiplier $M(H_2)$ of H_2, we have to find the Multiplier of each Sylow $p-$subgroup of H_2, where p is a prime dividing $|H_2|$. To find these multipliers in Magma, we firstly convert the matrix group $H_2 = \langle h_1, h_2 \rangle$ into a permutation group. The group H_2 can be constructed in terms of permutations of a set of cardinality 32. The following two permutations $\widetilde{h}_1$ and $\widetilde{h}_2$ generate the group H_2 :

$$\begin{aligned} \widetilde{h}_1 &= (3\ 28\ 12\ 26\ 17\ 19\ 10)(4\ 27\ 11\ 25\ 18\ 20\ 9)(5\ 22\ 7\ 15 \\ &\quad 14\ 24\ 30\ 6\ 21\ 8\ 16\ 13\ 23\ 29)(31\ 32), \\ \widetilde{h}_2 &= (3\ 19\ 30\ 22\ 4\ 20\ 29\ 21)(5\ 14\ 27\ 12\ 6\ 13\ 28\ 11)(7\ 32 \\ &\quad 8\ 31)(9\ 23\ 17\ 16)(10\ 24\ 18\ 15)(25\ 26). \end{aligned}$$

The following sequence of Magma commands will produce the Schur multipliers of the Sylow $p-$subgroups for $p \in \{2,3,5,7\}$. Also we use the command “pCove” to construct the central extension $p{\cdot}H_2$, but the above command works only for finitely presented groups and thus we used the command "FPGroup" to convert our permutation group H_2 into a finitely presented group.

```
H:= PermutationGroup< 32 | (3,28,12,26,17,19,10)(4,27,11,25,18,20,9)
                     (5,22,7,15,14,24,30,6,21,8,16,13,23,29)(31,32),
                     (3,19,30,22,4,20,29,21) (5,14,27,12,6,13,28,11)
                     (7,32,8,31)(9,23,17,16)(10,24,18,15)(25,26)>;
> pMultiplicator(H,2);
[ 2 ]
> pMultiplicator(H,3);
[ 1 ]
> pMultiplicator(H,5);
[ 1 ]
> pMultiplicator(H,7);
[ 1 ]
> F := FPGroup(H);
> F2 := pCover(H, F, 2);
> Order(F2);
645120
> p, H:= CosetAction(F2, sub<F2|>);
> s:= SylowSubgroup(H, 3);
> p2, H1:= CosetAction(H, s);
> Order(H1);
645120
> ct:= CharacterTable(H1);
> ct;
```

From the above, we deduce that the Schur Multiplier of H_2 is $\mathbb{Z}_2 \times \mathbb{Z}_1 \times \mathbb{Z}_1 \times \mathbb{Z}_1 \cong \mathbb{Z}_2$, abbreviated to be just 2. Thus the only factor set α that $M(H_2)$ contains is $\alpha \sim [2]$. The *covering group* $M(H_2){\cdot}H_2$[1] (central extension of $M(H_2)$ by H_2) is isomorphic to $2{\cdot}(2^4{:}GL(4,2))$. To find the projective character table of H_2 with factor set α (since $\alpha \sim [2],\ \alpha$ and α^{-1} are identical), it is sufficient to find the ordinary character table of the double cover group $2{\cdot}H_2$ of H_2 (in this situation, the 2-fold cover $2{\cdot}H_2$ is the full covering group $M(H_2){\cdot}H_2 \cong 2{\cdot}(2^4{:}GL(4,2))$). From the above sequence of Magma commands, one can see that the group F2 is the double cover group $2{\cdot}H_2$ and it is clear that $|\mathrm{F2}| = 645120 = 2 \times 322560 = 2 \times |H_2|$. Thus one can proceed computationally to obtain the character table of $2{\cdot}H_2$. Alternatively since $M(H_2){\cdot}H_2$ is of extension type, the Clifford-Fischer theory can be applied recursively to obtain its character table. In fact the character table of this group is partitioned into two blocks $\widetilde{\mathcal{K}}_1$ and $\widetilde{\mathcal{K}}_2$, where $\widetilde{\mathcal{K}}_1$ consists of the 25 ordinary irreducible characters of H_2, while $\widetilde{\mathcal{K}}_2$ consists of a 14 irreducible characters (that is $|\mathrm{Irr}(2{\cdot}(2^4{:}GL(4,2)))| = 39$). We are interested in these 14 characters contained in $\widetilde{\mathcal{K}}_2$, where these characters represent the projective character table of H_2 with factor set $\alpha,\ \alpha \sim [2]$. Note that $|\mathrm{Irr}(H_1)| + |\mathrm{IrrProj}(H_2, 2)| = 27 + 14 = 41 = |\mathrm{Irr}(\overline{G})|$. In Table 2 we list the projective character table of H_2 with factor set $\alpha,\ \alpha \sim [2]$.

[1]some authors refer to this group as the *representation group*.

TABLE 2. Projective characters of $H_2 = 2^4{:}GL(4,2)$ with factor set α, $\alpha \sim [2]$

$[g_{ikm}]_{H_2}$	$1a$	$2a$	$2b$	$2c$	$4a$	$2d$	$4b$	$3a$	$3b$	$6a$	$4c$	$4d$	$4e$
$[g_{ikm}]_{H_2}$	g_{121}	g_{221}	g_{222}	g_{321}	g_{621}	g_{322}	g_{721}	g_{521}	g_{421}	$g_{10,21}$	g_{622}	g_{722}	g_{821}
$\lvert C_{H_2}(g_{ikm})\rvert$	322560	21504	1536	512	384	384	128	180	72	24	64	64	32
ψ_1	8	8	0	0	0	0	0	-4	2	2	0	0	0
ψ_2	24	24	0	0	0	0	0	-6	0	0	0	0	0
ψ_3	24	24	0	0	0	0	0	-6	0	0	0	0	0
ψ_4	48	48	0	0	0	0	0	6	0	0	0	0	0
ψ_5	56	56	0	0	0	0	0	-4	-1	-1	0	0	0
ψ_6	56	56	0	0	0	0	0	-4	-1	-1	0	0	0
ψ_7	56	56	0	0	0	0	0	2	2	2	0	0	0
ψ_8	56	56	0	0	0	0	0	2	2	2	0	0	0
ψ_9	64	64	0	0	0	0	0	4	-2	-2	0	0	0
ψ_{10}	120	-8	0	0	0	0	0	0	6	-2	0	0	0
ψ_{11}	120	-8	0	0	0	0	0	0	-3	1	0	0	0
ψ_{12}	120	-8	0	0	0	0	0	0	-3	1	0	0	0
ψ_{13}	360	-24	0	0	0	0	0	0	0	0	0	0	0
ψ_{14}	360	-24	0	0	0	0	0	0	0	0	0	0	0

Table 2 (continued)

$[g_{ikm}]_{H_2}$	$4f$	$8a$	$5a$	$6b$	$6c$	$12a$	$7a$	$14a$	$7b$	$14b$	$15a$	$15b$
$[g_{ikm}]_{H_2}$	g_{822}	$g_{14,21}$	g_{921}	$g_{11,21}$	$g_{10,22}$	$g_{15,21}$	$g_{12,21}$	$g_{16,21}$	$g_{13,21}$	$g_{17,21}$	$g_{18,21}$	$g_{19,21}$
$\lvert C_{H_2}(g_{ikm})\rvert$	16	16	15	12	12	12	14	14	14	14	15	15
ψ_1	0	0	2	0	0	0	1	1	1	1	1	1
ψ_2	0	0	1	0	0	0	A	A	$\overline{A}$	$\overline{A}$	-1	-1
ψ_3	0	0	1	0	0	0	$\overline{A}$	$\overline{A}$	A	A	-1	-1
ψ_4	0	0	2	0	0	0	-1	-1	-1	-1	1	1
ψ_5	0	0	-1	0	$-i\sqrt{3}$	$i\sqrt{3}$	0	0	0	0	1	1
ψ_6	0	0	-1	0	$i\sqrt{3}$	$-i\sqrt{3}$	0	0	0	0	1	1
ψ_7	0	0	-1	0	0	0	0	0	0	0	B	$\overline{B}$
ψ_8	0	0	-1	0	0	0	0	0	0	0	$\overline{B}$	B
ψ_9	0	0	1	0	0	0	1	1	1	1	-1	-1
ψ_{10}	0	0	0	0	0	0	1	-1	1	-1	0	0
ψ_{11}	0	0	0	0	$-i\sqrt{3}$	$-i\sqrt{3}$	1	-1	1	-1	0	0
ψ_{12}	0	0	0	0	$i\sqrt{3}$	$i\sqrt{3}$	1	-1	1	-1	0	0
ψ_{13}	0	0	0	0	0	0	A	$-A$	$\overline{A}$	$-\overline{A}$	0	0
ψ_{14}	0	0	0	0	0	0	$\overline{A}$	$-\overline{A}$	A	$-A$	0	0

where in Table 2, $A = -\frac{1}{2}\left(1 + i\sqrt{7}\right)$ and $B = -\frac{1}{2}\left(1 + i\sqrt{15}\right)$.

Table 2 indicates that classes $2b$, $2c$, $2d$, $4a$, $4b$, $4c$, $4d$, $4e$, $4f$, $6b$ and $8a$ are $\alpha-$irregular classes of H_2 by Theorem 7.2.1(3) of Whitely [31]. Thus H_2 has fourteen $\alpha-$regular classes and 14 characters with factor set α, as required by Theorem 7.2.1(1) of Whitely [31].

5. Fusion of Classes of H_2 into Classes of $GL(5,2)$

The permutation character $\chi(G|H_2)$ of $GL(5,2)$ on H_2 is of degree 31. From the $\mathbb{ATLAS}$, we see that $\chi(G|H_2)$ decomposes into the form **1a**+**31a**, where **1a** is the identity character of $GL(5,2)$ and **31a** is the irreducible character of $GL(5,2)$ of degree 31. With the aid of the permutation character and centralizer sizes, we were able to determine the fusion of classes of H_2 into classes of $GL(5,2)$. We list this fusion in Table 3.

TABLE 3. The fusion of $H_2 = 2^4{:}GL(4,2)$ into $G = GL(5,2)$

Inertia Factor Group H_2	Class of H_2	$\hookrightarrow$	Class of $GL(5,2)$	Class of H_2	$\hookrightarrow$	Class of $GL(5,2)$
$H_2 = 2^4{:}GL(4,2)$	$1a = g_{121}$		$1A$	$5a = g_{921}$		$5A$
	$2a = g_{221}$		$2A$	$6a = g_{10,21}$		$6A$
	$2b = g_{222}$		$2A$	$6b = g_{11,21}$		$6B$
	$2c = g_{321}$		$2B$	$6c = g_{10,22}$		$6A$
	$2d = g_{322}$		$2B$	$7a = g_{12,21}$		$7A$
	$3a = g_{521}$		$3B$	$7b = g_{13,21}$		$7B$
	$3b = g_{421}$		$3A$	$8a = g_{14,21}$		$8A$
	$4a = g_{621}$		$4A$	$12a = g_{15,21}$		$12A$
	$4b = g_{721}$		$4B$	$14a = g_{16,21}$		$14A$
	$4c = g_{622}$		$4A$	$14b = g_{17,21}$		$14B$
	$4d = g_{722}$		$4B$	$15a = g_{18,21}$		$15A$
	$4e = g_{821}$		$4C$	$15b = g_{19,21}$		$15B$
	$4f = g_{822}$		$4C$			

6. Fischer Matrices of $2^{5\cdot}GL(5,2)$

We recall that we label the top and bottom of the columns of the Fischer matrix $\mathcal{F}_i$, corresponding to g_i, by the sizes of the centralizers of $g_{ij},\ 1 \le j \le c(g_i)$ in $\overline{G}$ and m_{ij} respectively. In Table 1 we supplied $|C_{\overline{G}}(g_{ij})|$ and $m_{ij},\ 1 \le i \le 27,\ 1 \le j \le c(g_i)$. Also having obtained the fusion of the inertia factor group H_2 into $GL(5,2)$, we are able to label the rows of the Fischer matrices as described in Subsection 3.1. Since the size of the Fischer matrix $\mathcal{F}_i$ is $c(g_i)$, it follows from Table 1 that the sizes of the Fischer matrices of $\overline{G} = 2^{5\cdot}GL(5,2)$ range between 1 and 3 for every $i \in \{1,2,\cdots,27\}$.

We have used the arithmetical properties of Fischer matrices, given in Proposition 3.6, to calculate some of the entries of the Fischer matrices and also to build an algebraic system of equations. With the help of the symbolic mathematical package Maxima [16], we were able to solve these systems of equations and hence we have computed all the Fischer matrices of $\overline{G}$, which we list below.

$\mathcal{F}_1$

g_1		g_{11}	g_{12}
$o(g_{1j})$		1	2
$\|C_{\overline{G}}(g_{1j})\|$		319979520	10321920
(k,m)	$\|C_{H_k}(g_{1km})\|$		
$(1,1)$	9999360	1	1
$(2,1)$	322560	31	-1
m_{1j}		1	31

$\mathcal{F}_2$

g_2		g_{21}	g_{22}
$o(g_{2j})$		4	2
$\|C_{\overline{G}}(g_{2j})\|$		43008	43008
(k,m)	$\|C_{H_k}(g_{1km})\|$		
$(1,1)$	21504	1	1
$(2,1)$	21504	1	-1
m_{2j}		16	16

$\mathcal{F}_3$

g_3		g_{31}
$o(g_{3j})$		4
$\lvert C_{\overline{G}}(g_{3j})\rvert$		1536
(k,m)	$\lvert C_{H_k}(g_{3km})\rvert$	
$(1,1)$	1536	1
m_{3j}		32

$\mathcal{F}_4$

g_4		g_{41}	g_{42}
$o(g_{4j})$		3	6
$\lvert C_{\overline{G}}(g_{4j})\rvert$		4032	576
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$		
$(1,1)$	504	1	1
$(2,1)$	72	7	-1
m_{4j}		4	28

$\mathcal{F}_5$

g_5		g_{51}	g_{52}
$o(g_{5j})$		6	3
$\lvert C_{\overline{G}}(g_{5j})\rvert$		360	360
(k,m)	$\lvert C_{H_k}(g_{5km})\rvert$		
$(1,1)$	180	1	1
$(2,1)$	180	1	-1
m_{5j}		16	16

$\mathcal{F}_6$

g_6		g_{61}
$o(g_{6j})$		8
$\lvert C_{\overline{G}}(g_{6j})\rvert$		384
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$	
$(1,1)$	384	1
m_{6j}		32

$\mathcal{F}_7$

g_7		g_{71}
$o(g_{7j})$		4
$\lvert C_{\overline{G}}(g_{7j})\rvert$		128
(k,m)	$\lvert C_{H_k}(g_{7km})\rvert$	
$(1,1)$	128	1
m_{7j}		32

$\mathcal{F}_8$

g_8		g_{81}
$o(g_{8j})$		8
$\lvert C_{\overline{G}}(g_{8j})\rvert$		32
(k,m)	$\lvert C_{H_k}(g_{8km})\rvert$	
$(1,1)$	32	1
m_{8j}		32

$\mathcal{F}_9$

g_9		g_{91}	g_{92}
$o(g_{9j})$		10	5
$\lvert C_{\overline{G}}(g_{9j})\rvert$		30	30
(k,m)	$\lvert C_{H_k}(g_{9km})\rvert$		
$(1,1)$	15	1	1
$(2,1)$	15	1	-1
m_{9j}		16	16

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$	$g_{10,2}$	$g_{10,3}$
$o(g_{10j})$		12	12	6
$\lvert C_{\overline{G}}(g_{10j})\rvert$		96	96	48
(k,m)	$\lvert C_{H_k}(g_{10km})\rvert$			
$(1,1)$	24	1	1	1
$(2,1)$	24	1	1	-1
$(2,2)$	12	2	-2	0
m_{10j}		8	8	16

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$
$o(g_{11j})$		12
$\lvert C_{\overline{G}}(g_{11j})\rvert$		12
(k,m)	$\lvert C_{H_k}(g_{11km})\rvert$	
$(1,1)$	12	1
m_{11j}		32

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$	$g_{12,2}$
$o(g_{12j})$		7	14
$\lvert C_{\overline{G}}(g_{12j})\rvert$		168	56
(k,m)	$\lvert C_{H_k}(g_{12km})\rvert$		
$(1,1)$	42	1	1
$(2,1)$	14	3	-1
m_{12j}		8	24

$\mathcal{F}_{13}$

g_{13}		$g_{13,1}$	$g_{13,2}$
$o(g_{13j})$		7	14
$\lvert C_{\overline{G}}(g_{13j})\rvert$		168	56
(k,m)	$\lvert C_{H_k}(g_{13km})\rvert$		
$(1,1)$	42	1	1
$(2,1)$	14	3	-1
m_{13j}		8	24

$\mathcal{F}_{14}$

g_{14}		$g_{14,1}$
$o(g_{14j})$		8
$\lvert C_{\overline{G}}(g_{14j})\rvert$		16
(k,m)	$\lvert C_{H_k}(g_{14km})\rvert$	
$(1,1)$	16	1
m_{14j}		32

$\mathcal{F}_{15}$

g_{15}		$g_{15,1}$	$g_{15,2}$
$o(g_{15j})$		24	24
$\lvert C_{\overline{G}}(g_{15j})\rvert$		24	24
(k,m)	$\lvert C_{H_k}(g_{15km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{15j}		16	16

$\mathcal{F}_{16}$

g_{16}		$g_{16,1}$	$g_{16,2}$
$o(g_{16j})$		28	14
$\lvert C_{\overline{G}}(g_{16j})\rvert$		28	28
(k,m)	$\lvert C_{H_k}(g_{16km})\rvert$		
$(1,1)$	28	1	1
$(2,1)$	28	1	-1
m_{16j}		16	16

$\mathcal{F}_{17}$

g_{17}		$g_{17,1}$	$g_{17,2}$
$o(g_{17j})$		28	14
$\lvert C_{\overline{G}}(g_{17j})\rvert$		28	28
(k,m)	$\lvert C_{H_k}(g_{17km})\rvert$		
$(1,1)$	28	1	1
$(2,1)$	28	1	-1
m_{17j}		16	16

$\mathcal{F}_{18}$

g_{18}		$g_{18,1}$	$g_{18,2}$
$o(g_{18j})$		15	30
$\lvert C_{\overline{G}}(g_{18j})\rvert$		30	30
(k,m)	$\lvert C_{H_k}(g_{18km})\rvert$		
$(1,1)$	30	1	1
$(2,1)$	30	1	-1
m_{18j}		16	16

$\mathcal{F}_{19}$

g_{19}		$g_{19,1}$	$g_{19,2}$
$o(g_{19j})$		15	30
$\lvert C_{\overline{G}}(g_{19j})\rvert$		30	30
(k,m)	$\lvert C_{H_k}(g_{19km})\rvert$		
$(1,1)$	30	1	1
$(2,1)$	30	1	-1
m_{19j}		16	16

$\mathcal{F}_{20}$

g_{20}		$g_{20,1}$
$o(g_{20j})$		21
$\lvert C_{\overline{G}}(g_{20j})\rvert$		21
(k,m)	$\lvert C_{H_k}(g_{20km})\rvert$	
$(1,1)$	21	1
m_{20j}		32

$\mathcal{F}_{21}$

g_{21}		$g_{21,1}$
$o(g_{21j})$		21
$\lvert C_{\overline{G}}(g_{21j})\rvert$		21
(k,m)	$\lvert C_{H_k}(g_{21km})\rvert$	
$(1,1)$	21	1
m_{21j}		32

$\mathcal{F}_{22}$

g_{22}		$g_{22,1}$
$o(g_{22j})$		31
$\lvert C_{\overline{G}}(g_{22j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{22km})\rvert$	
$(1,1)$	31	1
m_{22j}		32

$\mathcal{F}_{23}$

g_{23}		$g_{23,1}$
$o(g_{23j})$		31
$\lvert C_{\overline{G}}(g_{23j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{23km})\rvert$	
$(1,1)$	31	1
m_{23j}		32

$\mathcal{F}_{24}$

g_{24}		$g_{24,1}$
$o(g_{24j})$		31
$\lvert C_{\overline{G}}(g_{24j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{24km})\rvert$	
$(1,1)$	31	1
m_{24j}		32

$\mathcal{F}_{25}$

g_{25}		$g_{25,1}$
$o(g_{25j})$		31
$\lvert C_{\overline{G}}(g_{25j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{25km})\rvert$	
$(1,1)$	31	1
m_{25j}		32

$\mathcal{F}_{26}$

g_{26}		$g_{26,1}$
$o(g_{26j})$		31
$\lvert C_{\overline{G}}(g_{26j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{26km})\rvert$	
$(1,1)$	31	1
m_{26j}		32

$\mathcal{F}_{27}$

g_{27}		$g_{27,1}$
$o(g_{27j})$		31
$\lvert C_{\overline{G}}(g_{27j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{27km})\rvert$	
$(1,1)$	31	1
m_{27j}		32

7. The Character Table of the Dempwolff Group $\overline{G} = 2^{5\cdot}GL(5,2)$

Now we are in the position to construct the character table of Dempwolff group $\overline{G} = 2^{5\cdot}GL(5,2)$. In the previous sections we have found

- the conjugacy classes of $\overline{G} = 2^{5\cdot}GL(5,2)$ (Table 1),
- the projective character table of the inertia factor $H_2 = 2^4{:}GL(4,2)$ with factor set α, $\alpha \sim [2]$ (Table 2),
- the fusion of classes of the inertia factor H_2 into classes of $GL(5,2)$ (Table 3),
- the Fischer matrices of $\overline{G}$ (Section 6).

By Section 3, it follows that the full character table of $\overline{G}$ can be constructed easily. We give an example on how to construct the character table of $\overline{G}$, which is partitioned into 54 blocks corresponding to the 27 cosets and the two inertia factor groups. As an example we construct the parts $\mathcal{K}_{10,1}\mathcal{F}_{10,1}$ and $\mathcal{K}_{10,2}\mathcal{F}_{10,2}$ of the character table of $\overline{G}$ (this means that we are listing the values of all the irreducible characters of $\overline{G}$ on the classes $g_{10,1}$, $g_{10,2}$ and $g_{10,3}$ of $\overline{G}$, which correspond to the conjugacy class $6A$ of $GL(5,2)$). The two parts $\mathcal{K}_{10,1}\mathcal{F}_{10,1}$ and $\mathcal{K}_{10,2}\mathcal{F}_{10,2}$ can be derived as follows: From Table 3 we can see that there are two $\alpha-$regular classes, namely $6a = g_{10,21}$ and $6c = g_{10,22}$ of H_2 that fuse into the class $g_6 = [6A]_{GL(5,2)}$. To construct the part $\mathcal{K}_{10,1}\mathcal{F}_{10,1}$, we multiply the column of the character table of $H_1 = GL(5,2)$ corresponds to the class $6A$ of $GL(5,2)$ (see the $\mathbb{ATLAS}$), by the first row of $\mathcal{F}_{10}$, namely $(1 \quad 1 \quad 1)$ and thus the part $\mathcal{K}_{10,1}\mathcal{F}_{10,1}$ of size 27×3, consists of the column of the character table of $GL(5,2)$ corresponds to the class $6A$ repeated 3 times. To construct the part $\mathcal{K}_{10,2}\mathcal{F}_{10,2}$, select the two columns of the projective character table of $H_2 = 2^4{:}GL(4,2)$, with factor set α, $\alpha \sim [2]$, correspond to the classes $6a$ and $6c$ of H_2 (see Table 2) and multiply these two columns by the two rows of $\mathcal{F}_{10}$ correspond to the pair $(2,1)$ and $(2,2)$. Thus we get a block in the character table of $\overline{G}$ of size 14×3. The above two parts have the following form:

$$\mathcal{K}_{10,1}\mathcal{F}_{10,1} = \begin{pmatrix} 1\\ 2\\ 1\\ 0\\ -1\\ -1\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ -1\\ -1\\ -1\\ -1\\ 0\\ 0\\ 0\\ 0\\ -1\\ 2\\ 1\\ 1\\ -2\\ 0\\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} = \begin{array}{c} \begin{array}{ccc} g_{10,1} & g_{10,2} & g_{10,3} \end{array} \\ \begin{pmatrix} 1 & 1 & 1\\ 2 & 2 & 2\\ 1 & 1 & 1\\ 0 & 0 & 0\\ -1 & -1 & -1\\ -1 & -1 & -1\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ -1 & -1 & -1\\ -1 & -1 & -1\\ -1 & -1 & -1\\ -1 & -1 & -1\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ -1 & -1 & -1\\ 2 & 2 & 2\\ 1 & 1 & 1\\ 1 & 1 & 1\\ -2 & -2 & -2\\ 0 & 0 & 0\\ 1 & 1 & 1 \end{pmatrix} \end{array},$$

$$\mathcal{K}_{10,2}\mathcal{F}_{10,2} = \begin{pmatrix} 2 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ -1 & -i\sqrt{3}\\ -1 & i\sqrt{3}\\ 2 & 0\\ 2 & 0\\ -2 & 0\\ -2 & 0\\ 1 & -i\sqrt{3}\\ 1 & i\sqrt{3}\\ 0 & 0\\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1\\ 2 & -2 & 0 \end{pmatrix} = \begin{array}{c} \begin{array}{ccc} g_{10,1} & g_{10,2} & g_{10,3} \end{array} \\ \begin{pmatrix} 2 & 2 & 2\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ -1-2i\sqrt{3} & -1+2i\sqrt{3} & 1\\ -1+2i\sqrt{3} & -1-2i\sqrt{3} & 1\\ 2 & 2 & -2\\ 2 & 2 & -2\\ -2 & -2 & 2\\ -2 & -2 & 2\\ 1-2i\sqrt{3} & 1+2i\sqrt{3} & -1\\ 1+2i\sqrt{3} & 1-2i\sqrt{3} & -1\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{pmatrix} \end{array}.$$

Similarly one can construct all the other 52 parts $\mathcal{K}_{ik}\mathcal{F}_{ik},\ k \in \{1,2\},\ i \in \{1,2,\cdots,27\} \setminus \{10\}$. The full character table of $\overline{G} = 2^{5\cdot}GL(5,2)$ is available in many sources such as Magma, GAP or the book of G. Michler [17]. In Table 4 we supply the character table of $\overline{G}$ in the format of Clifford-Fischer theory (characters are organized in blocks corresponding to the inertia factors and the conjugacy classes of $\overline{G}$, where the conjugacy classes are arranged in blocks corresponding to the cosets $2^5\overline{g}_i$, where $\overline{g}_i$ is a pre-image of a representative g_i of a conjugacy class of $G = GL(5,2)$).

Table 4. The character table of Dempwolff group $\overline{G} = 2^{5\cdot}GL(5,2)$

$[g_i]_G$		$1A$		$2A$		$2B$	$3A$		$3B$		$4A$	$4B$	$4C$	$5A$		$6A$			$6B$
$[g_{ij}]_{\overline{G}}$		$1a$	$2a$	$2b$	$4a$	$4b$	$3a$	$6a$	$6b$	$3b$	$8a$	$4c$	$8b$	$10a$	$5a$	$12a$	$12b$	$6c$	$12c$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	2	15	15	11	11	9	6	6	3	3	7	7	5	1	1	5	5	4	2
	3	2	2	1	1	1	2	2	2	2	1	0	0	1	1	1	1	1	1
	5	1	1	0	0	0	0	0	1	1	0	0	0	1	1	0	0	0	0
	7	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	31	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		30	30	14	14	6	6	6	0	0	6	2	2	0	0	2	2	2	0
χ_3		124	124	28	28	12	1	1	4	4	4	4	0	-1	-1	1	1	1	0
χ_4		155	155	27	27	-5	8	8	5	5	3	-5	-1	0	0	0	0	0	1
χ_5		217	217	-7	-7	9	7	7	4	4	-7	1	1	2	2	-1	-1	-1	0
χ_6		280	280	56	56	8	7	7	-5	-5	8	0	0	0	0	-1	-1	-1	-1
χ_7		315	315	-21	-21	3	0	0	0	0	3	-1	-1	0	0	0	0	0	0
χ_8		315	315	-21	-21	3	0	0	0	0	3	-1	-1	0	0	0	0	0	0
χ_9		315	315	-21	-21	3	0	0	0	0	3	-1	-1	0	0	0	0	0	0
χ_{10}		315	315	-21	-21	3	0	0	0	0	3	-1	-1	0	0	0	0	0	0
χ_{11}		315	315	-21	-21	3	0	0	0	0	3	-1	-1	0	0	0	0	0	0
χ_{12}		315	315	-21	-21	3	0	0	0	0	3	-1	-1	0	0	0	0	0	0
χ_{13}		465	465	17	17	-15	3	3	0	0	1	1	1	0	0	-1	-1	-1	0
χ_{14}		465	465	17	17	-15	3	3	0	0	1	1	1	0	0	-1	-1	-1	0
χ_{15}		465	465	-31	-31	9	3	3	0	0	1	-3	1	0	0	-1	-1	-1	0
χ_{16}		465	465	-31	-31	9	3	3	0	0	1	-3	1	0	0	-1	-1	-1	0
χ_{17}		496	496	48	48	16	-8	-8	1	1	0	0	0	1	1	0	0	0	1
χ_{18}		651	651	-21	-21	-5	0	0	6	6	3	3	-1	1	1	0	0	0	-2
χ_{19}		651	651	-21	-21	-5	0	0	-3	-3	3	3	-1	1	1	0	0	0	1
χ_{20}		651	651	-21	-21	-5	0	0	-3	-3	3	3	-1	1	1	0	0	0	1
χ_{21}		868	868	-28	-28	4	7	7	1	1	-4	4	0	-2	-2	-1	-1	-1	1
χ_{22}		930	930	50	50	-6	6	6	0	0	-6	-2	-2	0	0	2	2	2	0
χ_{23}		930	930	-14	-14	-6	-3	-3	0	0	2	-2	2	0	0	1	1	1	0
χ_{24}		930	930	-14	-14	-6	-3	-3	0	0	2	-2	2	0	0	1	1	1	0
χ_{25}		960	960	64	64	0	-6	-6	0	0	0	0	0	0	0	-2	-2	-2	0
χ_{26}		1024	1024	0	0	0	-8	-8	4	4	0	0	0	-1	-1	0	0	0	0
χ_{27}		1240	1240	-8	-8	8	1	1	-5	-5	-8	0	0	0	0	1	1	1	-1
χ_{28}		248	-8	-8	8	0	14	-2	4	-4	0	0	0	2	-2	2	2	-2	0
χ_{29}		744	-24	-24	24	0	0	0	6	-6	0	0	0	1	-1	0	0	0	0
χ_{30}		744	-24	-24	24	0	0	0	6	-6	0	0	0	1	-1	0	0	0	0
χ_{31}		1488	-48	-48	48	0	0	0	-6	6	0	0	0	2	-2	0	0	0	0
χ_{32}		1736	-56	-56	56	0	14	-2	-2	2	0	0	0	-1	1	2	2	-2	0
χ_{33}		1736	-56	-56	56	0	14	-2	-2	2	0	0	0	-1	1	2	2	-2	0
χ_{34}		1736	-56	-56	56	0	-7	1	4	-4	0	0	0	-1	1	$-\overline{A}$	$-A$	1	0
χ_{35}		1736	-56	-56	56	0	-7	1	4	-4	0	0	0	-1	1	$-A$	$-\overline{A}$	1	0
χ_{36}		1984	-64	-64	64	0	-14	2	-4	4	0	0	0	1	-1	-2	-2	2	0
χ_{37}		3720	-120	8	-8	0	42	-6	0	0	0	0	0	0	0	-2	-2	2	0
χ_{38}		3720	-120	8	-8	0	-21	3	0	0	0	0	0	0	0	A	$\overline{A}$	-1	0
χ_{39}		3720	-120	8	-8	0	-21	3	0	0	0	0	0	0	0	$\overline{A}$	A	-1	0
χ_{40}		11160	-360	24	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}		11160	-360	24	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Continued on next page

Table 4 (continued)

$[g_i]_G$		$7A$		$7B$		$8A$	$12A$		$14A$		$14B$		$15A$		$15B$		$21A$	$21B$
$[g_{ij}]_{\overline{G}}$		$7a$	$14a$	$7b$	$14b$	$8b$	$24a$	$24b$	$28a$	$14c$	$28b$	$14d$	$15a$	$30a$	$15b$	$30b$	$21a$	$21b$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	2	3	3	3	3	4	3	3	2	2	2	2	1	1	1	1	0	0
	3	1	0	1	0	0	1	1	0	0	0	0	1	1	1	1	1	1
	5	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
	7	1	1	1	1	0	0	0	1	1	1	1	0	0	0	0	1	1
	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	-1	-1
χ_3		-2	-2	-2	-2	0	1	1	0	0	0	0	-1	-1	-1	-1	1	1
χ_4		1	1	1	1	-1	0	0	-1	-1	-1	-1	0	0	0	0	1	1
χ_5		0	0	0	0	1	-1	-1	0	0	0	0	-1	-1	-1	-1	0	0
χ_6		0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_7		0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_8		0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_9		0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{10}		0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{11}		0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{12}		0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}		B	B	$\overline{B}$	$\overline{B}$	1	1	1	B	B	$\overline{B}$	$\overline{B}$	0	0	0	0	B	$\overline{B}$
χ_{14}		$\overline{B}$	$\overline{B}$	B	B	1	1	1	$\overline{B}$	$\overline{B}$	B	B	0	0	0	0	$\overline{B}$	B
χ_{15}		B	B	$\overline{B}$	$\overline{B}$	-1	1	1	$-B$	$-B$	$-\overline{B}$	$-\overline{B}$	0	0	0	0	B	$\overline{B}$
χ_{16}		$\overline{B}$	$\overline{B}$	B	B	-1	1	1	$-\overline{B}$	$-\overline{B}$	$-B$	$-B$	0	0	0	0	$\overline{B}$	B
χ_{17}		-1	-1	-1	-1	0	0	0	-1	-1	-1	-1	1	1	1	1	-1	-1
χ_{18}		0	0	0	0	-1	0	0	0	0	0	0	1	1	1	1	0	0
χ_{19}		0	0	0	0	-1	0	0	0	0	0	0	C	C	$\overline{C}$	$\overline{C}$	0	0
χ_{20}		0	0	0	0	-1	0	0	0	0	0	0	$\overline{C}$	$\overline{C}$	C	C	0	0
χ_{21}		0	0	0	0	0	-1	-1	0	0	0	0	1	1	1	1	0	0
χ_{22}		-1	-1	-1	-1	0	0	0	1	1	1	1	0	0	0	0	-1	-1
χ_{23}		D	D	$\overline{D}$	$\overline{D}$	0	-1	-1	0	0	0	0	0	0	0	0	$-B$	$-\overline{B}$
χ_{24}		$\overline{D}$	$\overline{D}$	D	D	0	-1	-1	0	0	0	0	0	0	0	0	$-\overline{B}$	$-B$
χ_{25}		1	1	1	1	0	0	0	1	1	1	1	0	0	0	0	1	1
χ_{26}		2	2	2	2	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{27}		1	1	1	1	0	1	1	-1	-1	-1	-1	0	0	0	0	1	1
χ_{28}		3	-1	3	-1	0	0	0	1	-1	1	-1	1	-1	1	-1	0	0
χ_{29}		E	$-B$	$\overline{E}$	$-\overline{B}$	0	0	0	B	$-B$	$\overline{B}$	$-\overline{B}$	-1	1	-1	1	0	0
χ_{30}		$\overline{E}$	$-\overline{B}$	E	$-B$	0	0	0	$\overline{B}$	$-\overline{B}$	B	$-B$	-1	1	-1	1	0	0
χ_{31}		-3	1	-3	1	0	0	0	-1	1	-1	1	1	-1	1	-1	0	0
χ_{32}		0	0	0	0	0	0	0	0	0	0	0	$\overline{C}$	$-\overline{C}$	C	$-C$	0	0
χ_{33}		0	0	0	0	0	0	0	0	0	0	0	C	$-C$	$\overline{C}$	$-\overline{C}$	0	0
χ_{34}		0	0	0	0	0	$-F$	F	0	0	0	0	1	-1	1	-1	0	0
χ_{35}		0	0	0	0	0	F	$-F$	0	0	0	0	1	-1	1	-1	0	0
χ_{36}		3	-1	3	-1	0	0	0	1	-1	1	-1	-1	1	-1	1	0	0
χ_{37}		3	-1	3	-1	0	0	0	-1	1	-1	1	0	0	0	0	0	0
χ_{38}		3	-1	3	-1	0	I	$-I$	-1	1	-1	1	0	0	0	0	0	0
χ_{39}		3	-1	3	-1	0	$-I$	I	-1	1	-1	1	0	0	0	0	0	0
χ_{40}		E	$-B$	$\overline{E}$	$-\overline{B}$	0	0	0	$-B$	B	$-\overline{B}$	$\overline{B}$	0	0	0	0	0	0
χ_{41}		$\overline{E}$	$-\overline{B}$	E	$-B$	0	0	0	$-\overline{B}$	$\overline{B}$	$-B$	B	0	0	0	0	0	0

Continued on next page

Table 4 (continued)

$[g_i]_G$		$31A$	$31B$	$31C$	$31D$	$31E$	$31F$
$[g_{ij}]_{\overline{G}}$		$31a$	$31b$	$31c$	$31d$	$31e$	$31f$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	2	0	0	0	0	0	0
	3	0	0	0	0	0	0
	5	0	0	0	0	0	0
	7	0	0	0	0	0	0
	31	1	1	1	1	1	1
χ_1		1	1	1	1	1	1
χ_2		-1	-1	-1	-1	-1	-1
χ_3		0	0	0	0	0	0
χ_4		0	0	0	0	0	0
χ_5		0	0	0	0	0	0
χ_6		1	1	1	1	1	1
χ_7		G	H	I	$\overline{H}$	$\overline{I}$	$\overline{G}$
χ_8		H	I	$\overline{G}$	$\overline{I}$	G	$\overline{H}$
χ_9		I	$\overline{G}$	$\overline{H}$	G	H	$\overline{I}$
χ_{10}		$\overline{G}$	$\overline{H}$	$\overline{I}$	H	I	G
χ_{11}		$\overline{H}$	$\overline{I}$	G	I	$\overline{G}$	H
χ_{12}		$\overline{I}$	G	H	$\overline{G}$	$\overline{H}$	I
χ_{13}		0	0	0	0	0	0
χ_{14}		0	0	0	0	0	0
χ_{15}		0	0	0	0	0	0
χ_{16}		0	0	0	0	0	0
χ_{17}		0	0	0	0	0	0
χ_{18}		0	0	0	0	0	0
χ_{19}		0	0	0	0	0	0
χ_{20}		0	0	0	0	0	0
χ_{21}		0	0	0	0	0	0
χ_{22}		0	0	0	0	0	0
χ_{23}		0	0	0	0	0	0
χ_{24}		0	0	0	0	0	0
χ_{25}		-1	-1	-1	-1	-1	-1
χ_{26}		1	1	1	1	1	1
χ_{27}		0	0	0	0	0	0
χ_{28}		0	0	0	0	0	0
χ_{29}		0	0	0	0	0	0
χ_{30}		0	0	0	0	0	0
χ_{31}		0	0	0	0	0	0
χ_{32}		0	0	0	0	0	0
χ_{33}		0	0	0	0	0	0
χ_{34}		0	0	0	0	0	0
χ_{35}		0	0	0	0	0	0
χ_{36}		0	0	0	0	0	0
χ_{37}		0	0	0	0	0	0
χ_{38}		0	0	0	0	0	0
χ_{39}		0	0	0	0	0	0
χ_{40}		0	0	0	0	0	0
χ_{41}		0	0	0	0	0	0

where in Table 4,

- $A = 1 + 2\sqrt{3}i$, • $B = -\frac{1}{2} + \frac{\sqrt{7}}{2}i = b7$, • $C = -\frac{1}{2} + \frac{\sqrt{15}}{2}i = b15$, • $D = -1 + \sqrt{7}i = 2b7$,
- $E = -\frac{3}{2} + \frac{3\sqrt{7}}{2}i = 3b7$, • $F = \sqrt{3}i$, • $G = E(31)^3 + E(31)^6 + E(31)^{12} + E(31)^{17} + E(31)^{24}$,
- $H = E(31)^5 + E(31)^9 + E(31)^{10} + E(31)^{18} + E(31)^{20}$,
- $I = E(31)^{15} + E(31)^{23} + E(31)^{27} + E(31)^{29} + E(31)^{30}$.

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