



Developing a neutronic core model of the NuScale Power Module using Serpent

KZ Madi

 **orcid.org/ 0009-0006-9334-8728**

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Supervisor:	Prof VV Naicker
Co-supervisor:	Miss GP Nyalunga

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DECLARATION

I, Kgomotso Ziphora Madi, hereby declare that the dissertation titled "Developing a neutronic core model of the NuScale power module using Serpent" is my own work and has not been submitted to any university before. I have adequately cited and referenced the original sources. I declare that I have adhered to all the university's policies on plagiarism and have not misrepresented or fabricated any work in my submission.

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ABSTRACT

In this study, the reactor core of the NuScale power module was simulated using the Serpent Monte Carlo code and KENO-VI from the SCALE code system, with the objective of analysing key core parameters (such as k-effective and neutron flux) as well as safety parameters (fuel temperature coefficient and moderator temperature coefficient). Neutronic calculations were performed for the first cycle of the reactor core (i.e., the fresh core). Fuel assembly models and a full-core model were developed, with detailed geometry and material specifications. NuScale Chapter 4 of the Safety Analysis Report served as the primary source for the specifications and key parameters used in this study. However, it is important to note that NuScale Chapter 4 did not provide all the necessary parameters for model development. Fuel enrichment and gadolinium content of the Fuel Assemblies (FAs) for the first cycle, the inner and outer radii of the pressure vessel, the nuclide composition of the spacer grids, and the chemical composition of the stainless steel for the Reactor Pressure Vessel (RPV) and the reflector were obtained from other sources, as acknowledged in this dissertation.

The results presented in this document indicate that the k-effective is higher in models that do not contain gadolinium or contain smaller amounts of it (2%,). Gadolinium acts as a burnable absorber and is commonly used for reactivity control in most light water reactors, due to the high neutron absorption cross-section of several gadolinium isotopes and its good solid solubility in UO_2 . The calculation results show that the criticality values for the full-core and fuel assembly are 1.17481 ± 0.00015 and 1.13991 ± 0.00014 , respectively, based on the core layout described in Section 3.2 and the layout of the fuel assembly in Section 3.2.1

The k-effective value for the full-core was found to be 3 490 pcm higher than that of the fuel assembly. This difference is attributed to the higher uranium enrichment in the full-core model. Additionally, the k-effective results for the full-core in this study deviated by 3 698 pcm from the first comparison model developed by other researchers. This discrepancy may be due to the inclusion of additional components in the full-core simulation, such as the radial stainless-steel reflector, bypass channels for coolant flow, the barrel, the thermal and biological shield, and the Reactor Pressure Vessel (RPV). Furthermore, the first comparison model did not specify the actual dimensions and positions of the bypass channels, which could have contributed to the discrepancy.

Further verification indicated that the introduction of coolant channels decreased the k-effective results. Additional verification was conducted by modifying the base full-core model to align with the second comparison model developed by other researchers and is different from the first comparison model. The first and second comparison models are discussed in detail in the

dissertation. The difference between the base model and the second comparison model was found to be 0.00088 (88 pcm).

The work proceeded chronologically, beginning with the first comparison model, followed by the second comparison model. However, the results of this study show better agreement with the second comparison model. Furthermore, the k-effective results for the full-core base model, calculated using the SCALE code, align with those obtained using Serpent, with a difference of 0.00075 (75 pcm). Thus, a successful code-to-code comparison has been achieved.

The distribution of the neutron flux in the full-core base model did not align with the expected cosine-like flux shapes, which are commonly observed in bare reactor cores. This deviation is due to the presence of different types of fuel assemblies in the reactor core, which introduce variations in the material composition of the fuel.

The moderator temperature coefficient falls within the typical values for a pressurized water reactor. The moderator temperature coefficient was found to be negative; more negative than the fuel temperature coefficient. Outside the scope of this study, it is worth noting that Boron concentration can influence the MTC. At the beginning of the cycle (BOC), higher Boron concentrations are typically used to control reactivity, and if not properly managed, could lead to a positive MTC. Burnable absorbers are commonly employed to mitigate this and ensure a stable, negative MTC throughout the reactor's operation."

KEYWORDS: *NuScale power module, small modular reactors, neutronics, criticality, Monte Carlo Method, serpent code, SCALE code.*

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ACRONYMS

BEAVRS:	Benchmark for Evaluation And Validation of Reactor Simulations
CDF:	Cumulative probability Distribution Function
CNEARG:	Computational Nuclear Engineering Analysis Research Group
CZP:	Cold Zero Power
FA:	Fuel Assembly
FC:	Full-Core
FTC:	Fuel Temperature Coefficient
GCR:	Gas Cooled Reactor
HFA:	Hot Fuel Assembly
HFP:	Hot Full Power
HMP:	High Mechanical Performance
HTP:	High Thermal Performance
HTR:	High Temperature Reactor
HWR:	Heavy Water Reactor
LWR:	Light Water Reactor
MLHGR:	Maximum Linear Heat Generating Rate
MSR:	Molten Salt Reactor
MTC:	Moderator Temperature Coefficient
NPM:	NuScale Power Module
NSSS:	Nuclear Steam Supply System
NWU:	North-West University
NWURCS:	North-West University Reactor Code Suite
PCM:	Per Cent Mille
PDF:	Probability Density Function
PPF:	Power Peaking Factor
PWR:	Pressurized Water Reactor
RPV:	Reactor Pressure Vessel
SAR:	Safety Analysis Report
SDM:	Shut Down Margin
SG:	Spacer Grids
SMR:	Small Modular Reactor
US-NRC:	Nuclear Regulatory Commission of the United States government
w/o:	weight percentage
WWER:	Water-Water Energetic Reactor

1. INTRODUCTION

This section outlines the aim of the research project, the research objectives, the problem statement, the scope of work, details regarding the basic configuration of the NuScale Power Module (NPM), a basic description of the nuclear engineering computer models used in this study, and finally, the structure of this dissertation.

1.1 Aim

The aim of this study is to develop a neutronic base model of the NuScale power module using NWURCS. Neutronic analyses are conducted using Serpent and KENO-VI from SCALE 6.2.4. The models are validated by comparing the results with findings from other researchers.

1.2 The Objectives of the Study

- To develop fuel assembly and full-core base input models, using the NWURCS (North-West University Reactor Code Suite);
- To verify models generated with NWURCS;
- To perform neutronic calculations of these models, using Serpent;
- To extract the neutronic results, such as the multiplication factor and the neutron flux, with appropriate methodology feedback given to the NWURCS developer for incorporation in NWURCS;
- To generate an equivalent full-core model, using KENO-VI from SCALE 6.2.4.
- To identify limiting safety parameters in terms of the modelling and analysis carried out; and
- To validate the model by comparing the results with those generated by other researchers (i.e. Sadegh-Noedoost *et al.* 2020 and Fridman *et al.* 2023)

1.3 Problem Statement

The central problem of this study is the need to develop an accurate neutronic base model of the NuScale power module to support its operational analysis. Given the importance of neutronics in reactor performance and safety. As a secondary outcome this study aims to support South Africa's preparatory process by contributing to the available expertise in the nuclear engineering discipline. This discipline encompasses areas such as neutronics, thermal hydraulics, nuclear safety, the nuclear fuel cycle, radioactive waste management, radiation protection, and more. The focus of this study will be on the neutronic analysis aspect of nuclear engineering, employing various computational tools.

The South African government has initiated the preparatory process for a nuclear build program with a capacity of 2 500 MW, proceeding at a pace and scale that the country can afford. The Department of Mineral Resources and Energy considers small nuclear units to be a more manageable investment, compared to a fleet-based approach (Department of Mineral Resources and Energy, 2019). Small modular reactors are anticipated to play a significant role in the near-term nuclear energy deployment mix.

As part of the preparatory process, South Africa requires expertise for the analysis of small modular reactors. According to the IAEA Safety Standards Series, Safety Guide No. NS-G-2.8, achieving and maintaining high levels of safety in nuclear power plants requires an adequate number of highly qualified and experienced personnel. For instance, chemistry or chemical engineering personnel should handle chemical and waste treatment functions; physics or nuclear engineering personnel should oversee reactor physics and fuel management functions; and physics or health physics personnel should be responsible for radiation protection functions, among others (IAEA, 2002).

1.4 Scope of Work

According to Haugh and Mohamed (2012), the neutronic parameters that are required to be validated can be parsed into two groups, i.e. global parameters and local parameters. This work focuses on global parameters. Global reactor parameters are those that are derived from examining the state of the entire reactor core, which include reactivity coefficients such as moderator temperature, fuel temperature (Doppler), and boron concentration, as well as the control rod's behaviour (rod worth). The combinations of these parameters define the operational characteristics of the reactor. Due to the scope, this study however, did not include the calculation of the boron reactivity coefficient and the reactivity associated with control rods. The study is carried out as a contribution to addressing key issues associated with SMRs. According to Haugh and Mohamed (2012), the development of SMRs presents unique issues for reactor vendors and potential operators. The lack of operating history creates a challenge in validating predictions of neutronic and other parameters, during the start-up of this new design.

1.5 Background: Current SMR Development

Currently, research is being conducted on SMRs for all principal reactor types, including light water reactors, heavy water reactors, molten salt reactors, and gas cooled reactors. Table 1.1 presents a list of light water SMRs that are under development worldwide. Each of the reactors listed in the table is distinguished by its design, safety features, power output, operating

conditions, and fuel characteristics. SMRs are expected to play a significant role in the future of near-term nuclear energy deployment.

Table 1.1: List of light water SMRs around the world (Dsouza, 2015).

Country	Reactor Name	Power Output (MWt)	Operating Conditions P (MPa)/T (°C)	Fuel Type/ Enrichment
USA	Westinghouse	800	15.5/310	UO ₂ / < 5 %
	NuScale	165	12.8/-	UO ₂ / < 4.95 %
	mPower	500	14.1/320	UO ₂ /5 %
European Consortium	IRIS	1 000	15.5/330	UO ₂ -MOX/4.95 %
Russia	WWER	850	16.2/325	UO ₂ /4.95 %
	KLT-30S	150	12.7/316	UO ₂ / < 20 %
Japan	IMR	1 000	15.5 345	UO ₂ /4.8 %
China	CNP-300	999	15.2/02	UO ₂ /2.4 – 3 %
	ACP 100	385	15/325	UO ₂ / < 4.95 %
Argentina	CAREM	100	12.25/326	UO ₂ /3.1 %

1.5.1 Overview of the NuScale power module

The NuScale Power Module (NPM) is an integral pressurized light water reactor consisting of 12 independent modules, each capable of producing an output of 45 MWe. It is also known by the acronym NPM-SMR. The preliminary designs for the SMR are in accordance with the Multi-Application Small Light Water Reactor (MASLWR), which was developed in 2003 by the Oregon State University in collaboration with the Idaho National Engineering Laboratory and Nexant Bechtel (Dsouza, 2015). The basic configuration of a single NPM is shown schematically in Figure 1.1 (NuScale Power LLC, 2022).

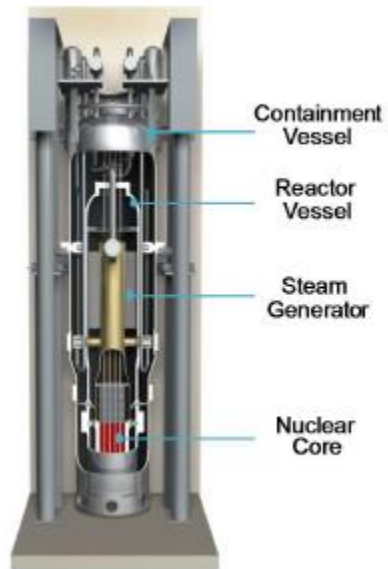


Figure 1.1: Schematics of a NuScale Module (NSSS inside containment).

The integrated nuclear reactor pressure vessel contains the nuclear core, a helical coil steam generator, and a pressurizer. It is approximately 20.0 m long by 2.8 m in diameter. The core active height is 2.0 m, which is approximately half the fuel length of the typical PWR fuel. The nuclear core consists of an array of reduced height LWR fuel assemblies at standard enrichments and control rod clusters. There are 37 fuel assemblies, 16 of which may have control rod clusters. The fuel assembly is of a 17×17 design. The fuel by design is uranium dioxide (UO_2) with gadolinium oxide (Gd_2O_3) as burnable absorber, mixed homogeneously in selected locations. This burnable absorber is used to control the excess reactivity at the beginning of the cycle (Pino-Medina & François, 2021).

The reactor uses three types of fuel, depending on the composition and enrichment. Type 1 is medium enriched, type 2 has the highest enrichment (4.55 w/o UO_2) and contains gadolinium (Gd_2O_3) and the type 3 fuel has the lowest enrichment of approximately 2.6 w/o UO_2 and appears in the centre of the reactor (Pino-Medina & François, 2021). There are 24 guide tubes for the control rod movements, and one central instrumentation tube in the fuel assembly. The assembly is supported by five spacer grids. The fuel cladding is M5 zirconium alloy, which is known for its improved corrosion resistance compared to traditional zirconium alloys. (NuScale Power LLC, 2020).

NuScale has been designed to operate in the natural circulation mode, as the main mechanism of primary coolant drive force. This has a significant role in the nuclear reactor design and the different operational and safety conditions. The helical once-through steam generator is located at a higher elevation than the reactor core, which provides the possibility of using natural circulation as the main flow driving force due to the interplay between fluid buoyancy and gravity. The natural circulation driven flow in the NuScale reactor is one of the important features that distinguish this reactor from the other SMR designs. Inside the reactor pressure vessel, the heated coolant water in the core outlet passes through the central riser channel and is cooled by the steam generator. The coolant returns to the core inlet through the down-comer after cooling through the steam generator (Hosseini *et al.* 2021). The entire nuclear steam supply system is enclosed in a steel containment that is 24.6 m long by 4.6 m in diameter (NuScale Power LLC, 2022) as shown in Figure 1.1

1.5.2 Modular configuration of NuScale power module

As can be seen in Figure 1.2, the NuScale module is in a pool of water. The water serves multiple purposes as listed below, including:

- It provides passive containment cooling and decay heat removal. The pool provides a heat sink with a capacity to absorb all the decay heat produced by a fully mature core for more than 30 days;
- It provides a means of dampening seismic events;
- It provides an additional fission product barrier;
- It provides radiation shielding to the outside containment; and
- The subterranean pool provides physical security (NuScale Power LLC, 2022).

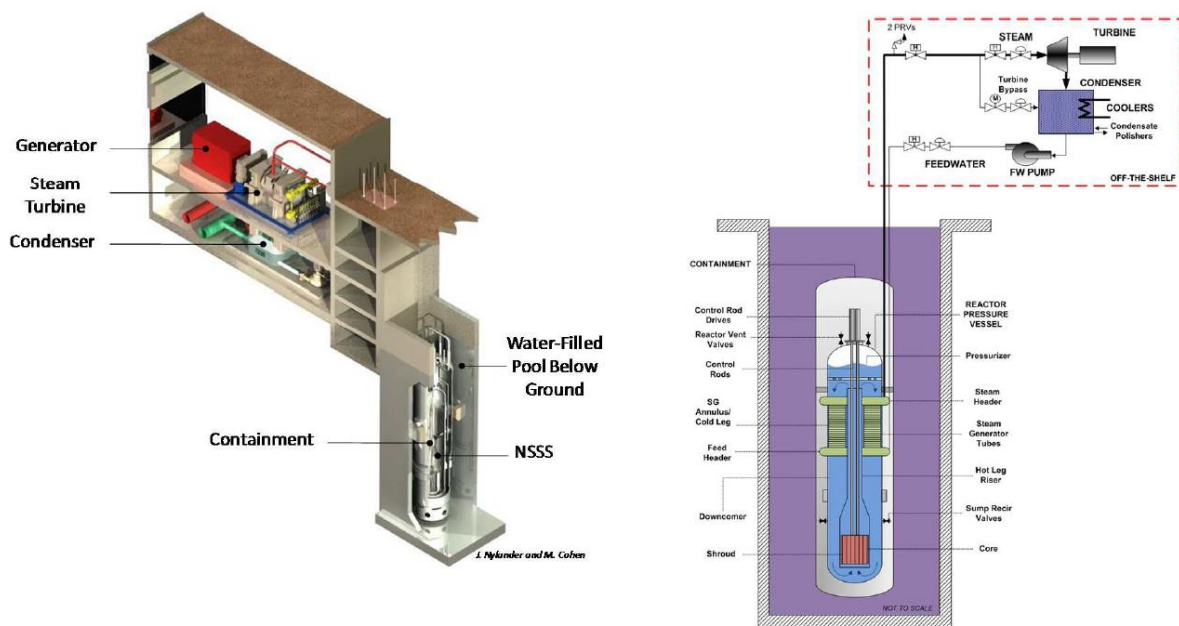


Figure 1.2: Schematics of a single NuScale power unit (NuScale Power LLC, 2022).

1.6 NuScale Licensing

The US-NRC staff reviewed the NuScale standard design and concluded that there is a reasonable assurance that the NuScale design can be constructed and operated without undue risk to the health and safety of the public (United States Nuclear Regulatory Commission, 2020). The US-NRC issued the document "Standard Design Approval for NPM" in September 2020, for the approval of the safety aspects of the NPM design. This means that customers can move forward with plans to procure NPMs (NuScale, 2020).

1.7 Nuclear Engineering Computer Models

The base model was developed for the Monte Carlo codes Serpent and KENO-VI from SCALE 6.2.4, where the geometry and materials of the reactor are defined in detail. This section provides a brief description of the codes used in this study. More detailed information about these codes can be found in the literature review section. The two codes used are the Monte Carlo codes Serpent and KENO-VI from SCALE 6.2.4.

Serpent is a multi-purpose, three-dimensional, continuous-energy neutron and photon transport code. It has been used to model various types of nuclear fission reactors, since the inception of its project. The standard geometry model is a universe-based Constructive Solid Geometry (CSG) type, which is adequate for most reactor types with regular geometries (Leppänen, 2015).

KENO-VI is a Monte Carlo radiation transport code within the SCALE (Standardized Computer Analysis for Licensing Evaluation) system. It provides a comprehensive, verified, and validated user-friendly toolset for criticality safety, reactor and lattice physics, radiation shielding, spent fuel and radioactive source term characterization, and sensitivity and uncertainty analysis (Oak Ridge National Laboratory, 2018).

1.8 Outline of the Dissertation

Chapter 1 of the dissertation is the introduction which includes the objectives and the main purpose of the study. Chapter 2 provides theory, and a literature review related to the existing SMR designs, neutronic analysis and an in-depth coverage of the nuclear engineering codes that are used in this study. Chapter 3 provides the methodology that was followed in the development of the base model and the neutronic analysis. In Chapter 4, the developed models, and results of the neutronic calculations, are given. Chapter 5 provides conclusions and recommendations.

2. LITERATURE REVIEW

2.1 Introduction

This chapter provides the background and an overview of the existing SMR designs, aspects of neutronic calculations such as the multiplication factor, neutron flux, and an interpretation of the criticality conditions. The chapter also provides the in-depth coverage of the fundamental methods used in modern Monte Carlo particle transport codes as applied in the nuclear industry and an overview of the key nuclear safety parameters according to the International Atomic Energy Agency reports.

2.2 Overview of Existing SMRs

Numerous concepts exist for SMRs, of which some are based on LWR technology. They enable the use of nuclear energy in locations unsuitable for large units and some designs can extend its use for non-electricity applications such as hydrogen production. Some existing SMR designs are based on HTRs; these designs are well suited to heat or co-generation applications. There are also other concepts for advanced SMR designs, including liquid metal-cooled fast reactors. Several designs are being promoted by nuclear industry companies, such as AREVA, Babcock & Wilcox (mPower), General Atomics, NuScale and Westinghouse (IRIS). Others are being developed by national research institutes in Argentina, China, Japan, Korea, and Russia (Sadegh-Noedoost *et al.* 2020).

NuScale is the USA's first small modular reactor which has been granted design certification by the USN-RC. It is considered to be one of the unique SMR reactors due to design advantages such as the immersion of the containment vessel in the reactor pool, a natural circulation cooling mechanism, an integrated loop, and flexibility of its safety functions for up to 72 hours after severe accidents (Kakaei *et al.* 2023).

2.3 NuScale Neutronic Calculations

Investigating the neutronic parameters of the NuScale reactor core is essential for reactor design and safety analysis. Neutronic parameters play a crucial role in the design process and safety assessment, making them a key concern for nuclear reactor designers (Kakaei *et al.* 2023). This study utilizes the Serpent and KENO-VI Monte Carlo codes (SCALE 6.2.4) to evaluate these parameters for the fresh core and fuel assemblies of the NuScale power module. The following subsections discuss some aspects of the neutronic calculations.

2.3.1 The multiplication factor

In a nuclear reactor, a neutron collides with a fissile nucleus, initiating fission and releasing energy, two light nuclei (fission products), and two to three neutrons. These neutrons can induce further fissions, creating a self-sustaining chain reaction necessary for energy production. This chain reaction is quantified by the multiplication factor k , defined as the ratio of fissions in one neutron generation to those in the preceding generation (Lamarsh & Baratta, 2001). The multiplication factor measures changes in the fission neutron population from one neutron generation to the following generation (Bickel, s.a.).

Given this, one equation for the multiplication factor is:

$$k = \frac{\text{number of fissions in one generation}}{\text{number of fissions in preceding generation}} \quad (2.1)$$

A multiplication factor is an expression which describes all the events in the life of a neutron and effectively describes the state of a reactor (critical, subcritical, or supercritical). If the multiplication factor for a reactor core is less than 1.0 (subcritical), then the system is decaying or dying out and not self-sustaining. With a multiplication factor greater than 1.0 (supercritical), a nuclear system is producing more neutrons than are being used by the system and is subjected to an increasing chain reaction that must be controlled by some exterior factor. A stable nuclear system occurs when the multiplication factor is equal to 1.0 (critical) (United States Nuclear Regulatory Commission Technical Training Center, s.a.).

In a system of infinite size, there can be no neutron leakage. For such a core the infinite multiplication factor, k_{∞} is used. In a system of finite size, however, some neutrons are lost by leaking out, and the multiplication factor is then called the effective multiplication factor, k_{eff} . The effective multiplication factor is the product of the non-leakage probability, P , and the infinite multiplication factor, k_{∞} (United States Nuclear Regulatory Commission Technical Training Center, s.a.)

$$k_{eff} = k_{\infty} P \quad (2.2)$$

According (Lamarsh & Baratta, 2001) k_{∞} can be expanded using the six-factor formula:

$$k_{\infty} = \eta \cdot f \cdot p \cdot \varepsilon \quad (2.3)$$

Where: η : reproduction factor, f : thermal utilization factor, p : resonance escape probability and ε : fast fission factor.

In the case of a system of infinite size the non-leakage probability is equal to 1.0 and k_{eff} is then k_{∞} . In reactor operation, k_{eff} is the most significant property regarding reactor control. At any

specific non-zero power level or operating condition of the reactor, k_{eff} is kept as near to the value of 1.0 as possible. At this point in operation, the neutron balance is kept to exactly one neutron completing the life cycle for each original neutron absorbed in the fuel (United States Nuclear Regulatory Commission Technical Training Center, s.a.).

2.3.2 Reactivity formulation

The time-dependent behaviour of a reactor is very sensitive to small deviations of the multiplication factor about 1.0. This behaviour is highlighted by writing the neutron kinetics equation in terms of the reactivity, defined as:

$$\rho = \frac{k-1}{k} \quad (2.4)$$

It is noted that with $\rho > 0$ the reactor will be supercritical and with $\rho < 0$, the reactor will be subcritical. The reactor will be critical when $\rho = 0$ (Lewis, 2008).

The reactivity coefficient for a parameter C is then defined as the partial derivative of reactivity with respect to that parameter:

$$\alpha_C = \frac{\partial \rho}{\partial C} \quad (2.5)$$

For practical purposes, this is often approximated using finite differences between two states (1 and 2):

$$\alpha_C = \frac{\rho_2 - \rho_1}{C_2 - C_1} \quad (2.6)$$

Using the relationship between reactivity and k-effective results in:

$$\alpha_C = \frac{1}{k_2 \cdot k_1} \cdot \frac{k_2 - k_1}{C_2 - C_1} \quad (2.7)$$

2.3.3 Prompt neutrons

Prompt neutrons are emitted directly from fission, and they are emitted within a very short time of approximately 10^{-14} seconds. Most of the neutrons produced in fission are prompt neutrons, approximately 99 %. For example, fission of ^{235}U by thermal neutron yields 2.43 neutrons, of which 2.42 neutrons are prompt neutrons, and 0.01585 neutrons are delayed neutrons. The production of prompt neutrons slightly increases with incident neutron energy. Almost all prompt fission neutrons have energies of between 0.1 MeV and 10 MeV (Nuclear Power, 2023).

2.3.4 Delayed neutrons

Delayed neutrons are emitted by neutron rich fission fragments that are called delayed neutron precursors. These precursors usually undergo beta decay, but a small fraction of them is excited enough to undergo neutron emission (Nuclear Power, 2023). The existence of the delayed neutrons is essential in nuclear reactors, because it controls the speed at which the reactor can increase its power. Without the delayed neutrons, the power of the reactor would increase to a high degree in a short period of time, leading to significant damage (Louis, 2021).

2.3.5 Interpretation of criticality conditions

In the equation discussed below, B_m^2 is material buckling, which is a measure of neutron production minus absorption and B_g^2 is geometric buckling, which is a measure of neutron leakage. Reactivity can be controlled with proper buckling incorporated in a reactor's design. For criticality, material buckling should be equal to geometrical buckling given as $B_m^2 = B_g^2$. This can be shown to yield:

$$1 = \frac{\nu\Sigma_f/\Sigma_a}{1+L^2 B_g^2} \equiv \frac{k_\infty}{1+L^2 B_g^2} = k_\infty P_{NL} = k_{eff} \quad (2.8)$$

where k_∞ is the infinite-medium multiplication constant and $P_{NL} = 1+L^2 B_g^2^{-1}$ is interpreted as the non-leakage probability. Eq. (2.8) shows how the fission cross-section, absorption cross-section and leakage of neutrons affect the neutron economy and subsequently affect the value of k_{eff} . If $\nu\Sigma_f$ increases, k_{eff} also increases, if Σ_a and $\frac{1}{1+L^2 B_g^2}$ increase, k_{eff} will decrease. $\frac{1}{1+L^2 B_g^2}$ is interpreted as non-leakage probability, $\nu\Sigma_f$ represents a fission cross-section and Σ_a is the absorption cross-section (Stacey, 2007).

2.3.6 Overview of neutron flux density and the reaction rate

2.3.6.1 Neutron flux density and cross-sections

The neutron flux density, n , is the number of neutrons per unit volume, per unit speed, per unit solid angle and at the instant considered (Reuss, 2008). The neutron flux density value is calculated as the neutron density (n) multiplied by the neutron velocity (v).

$$\Phi = n.v \quad (2.9)$$

where Φ is the neutron flux (neutrons.cm⁻².s⁻¹), n is the neutron density (neutrons.cm⁻³) and v is the neutron velocity (cm s⁻¹) (Nuclear Power, 2023).

Even though the neutron population is much diluted compared to the population of atoms, it is still very large. The neutron density could vary from one point to another and over time. It could also be possible to distinguish the speed of the neutrons in terms of their modulus and direction (Reuss, 2008).

The concept of cross-section is used to quantify the probability of an interaction. A cross-section can be microscopic, meaning that it is characteristic of an individual target or it can be macroscopic, meaning that it is characteristic of a material containing many targets. A neutron travels in a straight line at a constant speed until it hits a nucleus in its path. For a short path dx , this collision probability is infinitesimal and proportional to dx , this can be written as Σdx where Σ is the macroscopic cross-section (Reuss, 2008).

To understand the reaction rate, as a simplification, we assume n to be constant. Let v be the speed of the neutrons. During a time interval dt , each neutron travels a path $dx = v dt$ and therefore has a probability $\sigma dx = \sigma v dt$ of interacting with matter. Multiply this by $n(\vec{r})d^3r$, the number of neutrons present in the volume element d^3r , we obtain the number $n(\vec{r})\Sigma v d^3r dt$ of neutron-matter interactions in d^3r during dt . The parameter $R = n\Sigma v$, representing the number of interactions per unit volume and per unit time, is the reaction rate (Reuss, 2008).

2.3.6.2 Radial neutron flux profile

The neutron balance within the reactor core occurs as a result of the interplay between the neutron production due to fission, the neutron losses due to absorption and leakage, as mentioned above.

The leakage is the result of the neutron tendency to diffuse from the high concentrations region (i.e. the core centre) to the low concentrations region (i.e. the outer periphery of the core). The leakage is minimized by using reflectors, positioned outside the core (i.e. light water), the purpose of which is to reflect neutrons and thermalize the fast-moving neutrons. Figure 2.1 shows a sample radial neutron flux profile for reactor cores with reflectors (Dsouza, 2015).

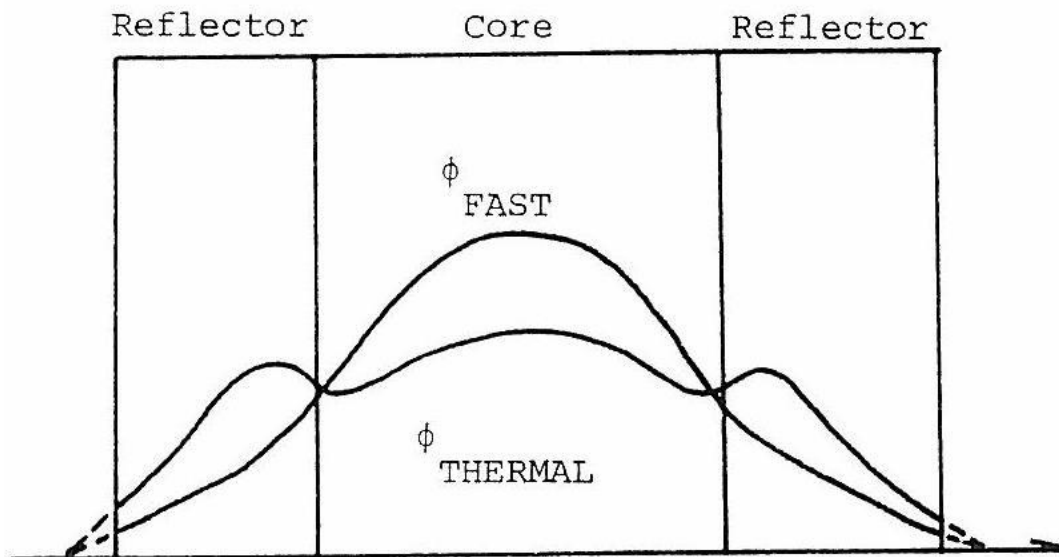


Figure 2.1: A sample reflector core radial flux profile (Dsouza, 2015).

2.3.6.3 Some aspects of neutron reactions

Neutrons are electrically neutral; therefore, they are not affected by the electrons in an atom or by the positive charge of the nucleus. Consequently, neutrons pass through the atomic electron cloud and interact directly with the nucleus (Lamarsh & Baratta, 2001). The cross-sections for these interactions influence the safety parameters. An introduction to the neutron interactions with matter is therefore given in this section, together with brief discussions on the cross-sections in the following sections.

Neutrons may interact with nuclei in one or more of the following ways:

- Elastic Scattering:** In this process neutrons strike the nucleus, which is almost always in its ground state. The neutron reappears, and the nucleus is left in its ground state. The neutron in this case is said to have been elastically scattered by the nucleus.
 Reaction example: $n + {}^1\text{H} \rightarrow n + {}^1\text{H}$
- Inelastic scattering:** This process is similar to elastic scattering except that the nucleus is left in an excited state. Because energy is retained by the nucleus, this is clearly an endothermic interaction. The excited nucleus decays by the emission of γ rays.
Reaction example: $n + {}^{56}\text{Fe} \rightarrow n + {}^{56}\text{Fe}^* \rightarrow n + {}^{56}\text{Fe} + \gamma$
- Radiative capture:** Here the neutron is captured by the nucleus, and one or more γ rays are emitted. This is an exothermic interaction. The neutron in this interaction is absorbed, this process is an example of a class of interactions known as absorption.
Reaction example: $n + {}^{238}\text{U} \rightarrow {}^{239}\text{U} + \gamma$

- **Charged-particle reactions:** Here the neutron may also disappear as a result of the absorption of the (n, α) and (n, p) types or similar. Such reactions may be either exothermic or endothermic.

Reaction Example (n, p) : $n + {}^{14}\text{N} \rightarrow {}^{14}\text{C} + p$

Reaction Example (n, α) : $n + {}^{10}\text{B} \rightarrow {}^7\text{Li} + \alpha$

- **Neutron-producing reactions:** This is a reaction of the type $(n, 2n)$ and $(n, 3n)$, occurring with energetic neutrons. These reactions are clearly endothermic since in the $(n, 2n)$ reaction one neutron and in the $(n, 3n)$ reaction two neutrons are extracted from the struck nucleus. The $(n, 2n)$ reaction is especially important in reactors containing heavy water or beryllium, since H^2 and B^9 have loosely bound neutrons that can easily be ejected.

Reaction Example $(n, 2n)$: $n + {}^9\text{Be} \rightarrow {}^8\text{Be} + 2n$

Reaction Example $(n, 3n)$: $n + {}^{27}\text{Al} \rightarrow {}^{25}\text{Al} + 3n$

- **Fission:** Neutrons colliding with certain nuclei may cause the nucleus to split apart to undergo fission. This reaction is the principal source of nuclear energy for practical applications.

Reaction Example: $n + {}^{235}\text{U} \rightarrow {}^{141}\text{Ba} + {}^{92}\text{Kr} + 3n + \text{Energy}$

Many of these reactions may be viewed as a two-step process, involving the formation of a compound nucleus. For example, the scattering reactions, both elastic and inelastic, may be thought of as a process in which the neutron is first absorbed by the target nucleus to form a new nucleus whose atomic number is unchanged but whose mass number is increased by one. Then depending on the specific process, the nucleus decays via neutron emission to produce the original nucleus plus a neutron. The product nucleus is left in the ground state or in an excited state according to the type of scattering reaction involved (Lamarsh & Baratta, 2001).

2.3.6.4 Fission cross-sections

The cross-sections of fissile nuclei for neutron-induced fission resemble radiative capture cross-sections in their dependence on the energy of the incident neutron. Fission cross-sections are denoted as σ_f . As seen in Figure 2.2, where σ_f is given for ${}^{235}\text{U}$, there are three distinct regions to the cross-section. At low energy, σ_f is $1/v$ or nearly so; this is followed by a region of resonances; finally, above the resonance region, σ_f is smooth and rolling. It should be noted that σ_f is especially large in the $1/v$ region. The fission cross-sections of fissionable but non-fissile nuclei, by contrast, are zero up to a threshold energy, which always occurs above the resonance

region. As a result, σ_f is relatively smooth at all energies. This is illustrated in Figure 2.3, where σ_f is shown for ^{238}U (Lamarsh & Baratta, 2001).

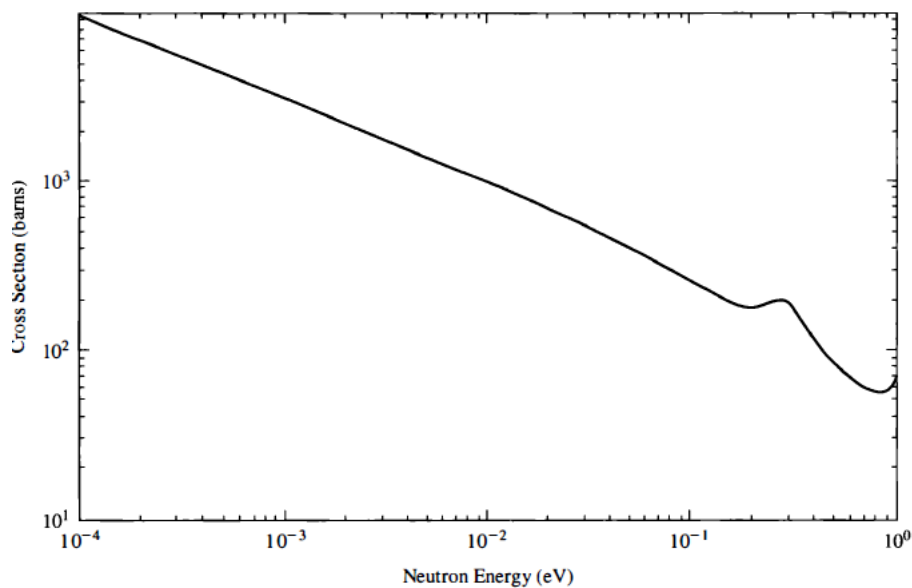


Figure 2.2: Fission cross-section of ^{235}U (Lamarsh & Baratta, 2001).

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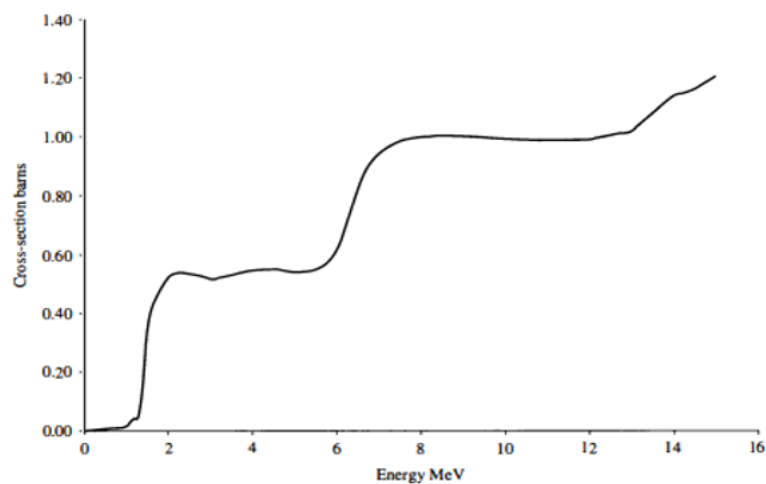


Figure 2.3: Fission cross-section of ^{238}U (Lamarsh & Baratta, 2001).

2.3.6.5 Elastic scattering cross-section

The elastic scattering cross-section is almost constant in energy below the neutron energies corresponding to the excited state of the compound nucleus. Figure 2.4 and Figure 2.5 show the elastic scattering cross-sections for two nuclides of interest in nuclear reactors. Elastic cross-sections are denoted as σ_e (Stacey, 2007). Hydrogen atoms (specifically protium, ^1H) are

excellent at slowing down (moderating) fast neutrons through elastic scattering. Oxygen is part of the coolant (H_2O) and forms uranium dioxide (UO_2), the most common fuel form in reactors.

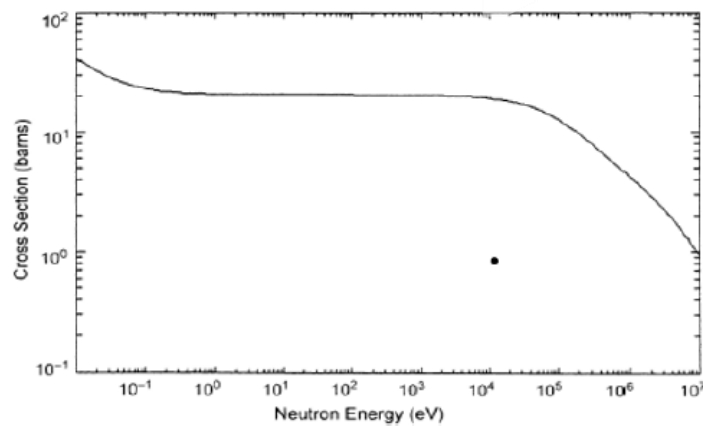


Figure 2.4: Elastic cross-section of hydrogen (Lamarsh & Baratta, 2001).

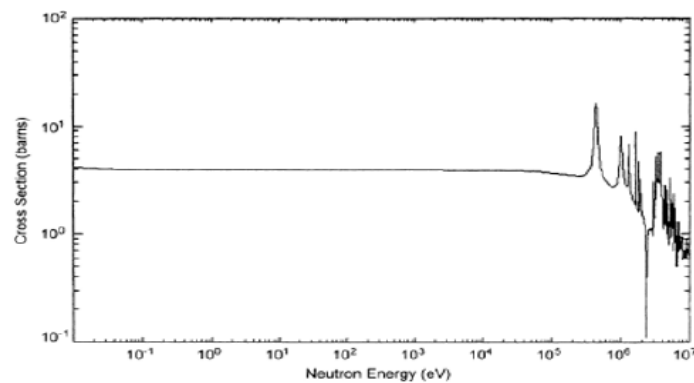


Figure 2.5: Elastic cross-section for oxygen (Lamarsh & Baratta, 2001).

2.3.6.6 Inelastic cross-section

Inelastic cross-sections are denoted as σ_{in} . The nucleus is left in an excited state, and the energy of the emitted neutron can be considerably less than the energy of the incident neutron. If the compound nucleus decays by the emission of two or more neutrons, the events are referred to as $n - 2n$, $n - 3n$, and so on, events, and the cross-sections are denoted as $\sigma_{in,2n}$, $\sigma_{in,3n}$, and so on. Increasingly higher incident neutron energies are required to provide enough excitation energy for single, double, triple, and so on, neutron emissions. Inelastic scattering is most important for neutrons 1 MeV and higher in energy (Stacey, 2007).

2.3.6.7 Radiative capture cross-section

Radiative capture cross-sections are denoted as σ_γ for some nuclei of interest in nuclear reactors as shown in Figures 2.6, 2.7 and 2.8. The resonance nature of the cross-sections over certain ranges correspond to the discrete excited states of the compound nucleus that is formed upon neutron capture. These excited states correspond to neutron energies in the range of a fraction of an eV to 10^3 eV for the fissile nuclides, and to neutron energies of 10 to 10^4 eV for even- A heavy mass nuclides. A notable exception is the thermal ^{240}Pu resonance which corresponds to much higher neutron energies than the lower mass nuclides (Stacey, 2007).

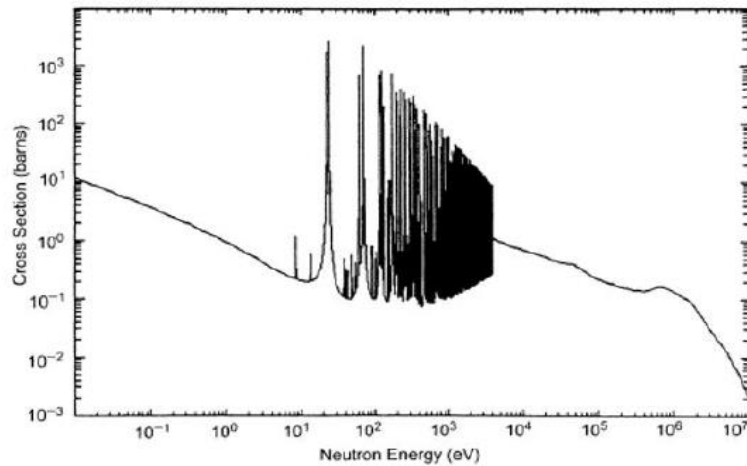


Figure 2.6: Radiative capture cross-section for ^{232}Th (Lamarsh & Baratta, 2001).

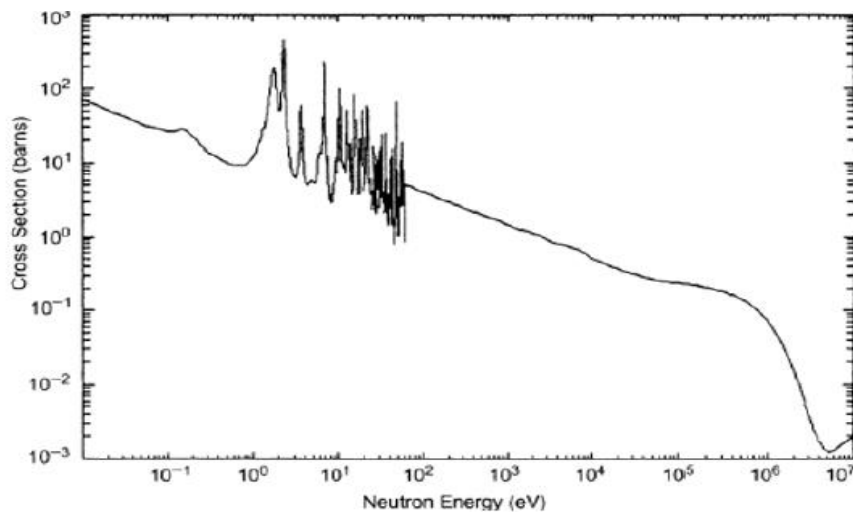


Figure 2.7: Radiative capture cross-section of ^{233}U (Lamarsh & Baratta, 2001).

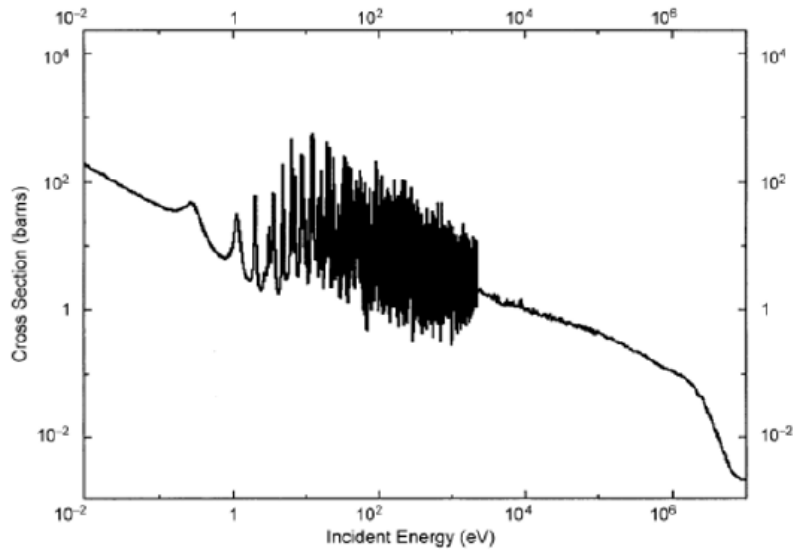


Figure 2.8: Radiative capture cross-section of ^{235}U (Lamarsh & Baratta, 2001).

2.4 Key Nuclear Safety Parameters

Reactivity coefficients are the important parameters which influence the safety and controllability of the nuclear reactors. Increasing temperature will create feedback mechanisms in the reactor; for example, Doppler broadening, thermal expansion and density changes which will induce spectral shifts. These changes will influence the reactivity, thus causing transients. Fuel Temperature Coefficient (FTC) is a major parameter in the assessment of a nuclear reactor core (Louis, 2021). Reactivity coefficients are intrinsic to the reactor design and the reactor should be designed to guarantee overall negative reactivity feedback from the reactor core during transients/accidents (Pazdera & Belac, 2003).

For western reactors, the general criterion is that the total of all reactivity coefficients be negative when the reactor is critical. An individual reactivity coefficient may be positive; however, the effect of such positive feedback must be inconsequential. For WWERs in Russia, each reactivity coefficient must be negative under all power conditions. This Russian requirement is also accepted in the Ukraine and Bulgaria for WWER1000 reactors. In the Czech Republic, the requirement is identical to that for western reactors; for the Temelin reactor there is an additional licensing criterion, namely that the moderator temperature coefficient must be positive under all power conditions (Pazdera & Belac, 2003). According to the status report for the NuScale SMR (NuScale Power, LLC, 2020) by the United States of America, the temperature coefficient of reactivity of the core is also negative.

Below are some of the typical nuclear key safety parameters, according to the International Atomic Energy Agency.

- The temperature coefficients of reactivity for the fuel and the moderator;
- The boron reactivity coefficient and concentration (for pressurized water reactors);
- The shutdown margin;
- The maximum reactivity insertion rate;
- The control rod worth and control bank worth;
- The maximum linear heat generation rate; and
- The void coefficient of reactivity (IAEA, 2019).

Any intentional or unintentional sudden increase in reactivity may result in a very rapid rise in the reactor power, followed by a loss of control over reactor operation. To compensate for the effects of rapid change in reactivity and in turn, reactor power, several safety features need to be included in a nuclear reactor. For proper implementation of the safety features, reactivity of the reactor core needs to be monitored continuously. However, reactivity can't be measured instantaneously during reactor operation without a reactivity meter. As reactivity meters are not available in many facilities, instead of trying to limit the reactivity of a reactor, the rate of reactor power rise is limited. In other words, the reactor period is controlled. The smaller the value of the reactor period, the more rapid the change in reactor power (Hossain *et al.* 2018). Some key safety parameters of significance to this study are defined in the following subsections, however the reactivity formulation is addressed prior to that.

2.4.1 Moderator temperature coefficient of reactivity

According to Edenius (1976), one important quantity in determining the operating characteristics and safety of nuclear reactors, is the temperature coefficient of reactivity. According to Da Silva *et al.* (2017), the moderator temperature coefficient of the reactivity of water moderated reactors, is an important operational parameter that is strongly associated with safety issues of the nuclear installation. In general, a reactor is designed to have $\alpha_T^M < 0$, (α_T^M is the Moderator Temperature Coefficient of Reactivity), thus ensuring that negative reactivity feedback will be performed in the event of a power increase. However, accidents may happen if the value is excessively negative due to the cooling system, especially in PWRs. The MTC is defined as the change of reactivity per degree change of the core-averaged moderator temperature. As a rule, a reactor core is designed such that the MTC has a negative value. This ensures that negative reactivity feedback will be provided in the event of a power excursion. However, the value of MTC should not be too negative because there exist certain off-normal sequences, particularly some cool-down

accidents in PWRs, which are aggravated by a large negative MTC. Therefore, in power PWRs, limits are established on how negative MTC may become during the fuel cycle, and surveillance tests are performed during the fuel cycle to determine if the MTC value complies with the specified limits (Mourtzanos *et al.* 2001).

In the expression for the moderator temperature coefficient of reactivity, a_c : the term $\frac{\delta \rho}{\delta T_M}$ represents the change in reactivity ρ with respect to a change in moderator temperature T_M . The alternative form expresses a_c as $\frac{1}{k_2 \cdot k_1} \cdot \frac{k_2 - k_1}{T_{M2} - T_{M1}}$, where k_1 and k_2 are the neutron multiplication factors at moderator temperatures T_{M1} and T_{M2} respectively. The equation used in the calculation is:

$$a_c = \frac{\delta \rho}{\delta T_M} \quad (2.10)$$

$$a_c = \frac{1}{k_2 \cdot k_1} \cdot \frac{k_2 - k_1}{T_{M2} - T_{M1}} \quad (2.11)$$

(Hafez *et al.* (2021)

2.4.2 Doppler coefficient

The interaction probability of an incident neutron with nuclear fuel, depends on the relative velocity between the neutron and the target nuclei. However, since the nuclei are in continual thermal motion, this relative velocity can be changed to be faster or slower than the neutron speed, and their thermal motion is mainly influenced by a change in fuel temperature (Lamarsh & Baratta, 2001). In a nuclear reactor, the cross-sections (discussed in Section 2.3.6) of neutrons are a function of the temperature. Thermal motion of nuclei alters because of fuel temperature fluctuation. It has been observed that as the fuel temperature rises, the cross-sections in resonance regions widen while their magnitude of the peak reduce. This event is called the Doppler effect. The Fuel Temperature Coefficient (FTC) of reactivity, or the Doppler coefficient, is an essential parameter in a nuclear reactor core assessment. The FTC assessment contains the reactivity change, with fuel temperature change, by keeping other parameters such as burnup, moderator density, boron concentration and moderator temperature fixed (Akyurek, 2018). The Doppler coefficient is a very strong contributor for the safety and stability of nuclear reactors during operation (Hafez *et al.* (2021).

The Doppler coefficient is estimated as the change in reactivity per degree change in fuel temperature using Eq. (2.12) and is expressed in percent mille k_{eff} (pcm/K). $k_{eff}^{T_2}$ and $k_{eff}^{T_1}$ are the effective multiplication factors corresponding to T_2 and T_1 temperature conditions.

In the expression for the Doppler coefficient of reactivity, denoted as D_C the term $\frac{\Delta\rho}{\Delta T}$ represents the change in reactivity ρ with respect to a change in fuel temperature (T_F). k_1 and k_2 are the effective neutron multiplication factors at fuel temperatures T_{F1} and T_{F2} respectively. T_{F1} and T_{F2} are the two different fuel temperatures. The equations that are used for calculating the Doppler reactivity coefficient in this work are found below:

$$D_C = \frac{\Delta\rho}{\Delta T} \quad (2.12)$$

$$= \frac{k_{eff}^{T_2}-1}{k_{eff}^{T_2}} - \frac{k_{eff}^{T_1}-1}{k_{eff}^{T_1}} = \frac{k_{eff}^{T_2}-k_{eff}^{T_1}}{k_{eff}^{T_2} \times k_{eff}^{T_1}}$$

Which yields

$$D_C = \frac{1}{K_2.K_1} \cdot \frac{K_2-K_1}{T_{F2}-T_{F1}} \quad (2.13)$$

(Hafez *et al.* (2021).

2.4.3 Boron reactivity coefficient

Boron is a neutron absorber, the change in boron concentration has an influence on the reactivity of the reactor. Boron concentration adjustment is necessary in the operation of any reactor moderated/cooled with purified water which contains boric acid. The regulatory concerns are usually during reactor start-up, load change, fuel burnup, cold shutdown, and boron dilution events. The effect of boron concentration can be monitored by the boron coefficient α_B , which is defined as a change in reactivity per the change in the boron concentration (Hafez *et al.* (2021).

The effective multiplication factor of the reactor core must be calculated without boric acid (purified water) and calculated again after the introduction of the boric acid at certain concentration. In the expression for boron reactivity coefficient, denoted as α_B , the term $\frac{d\rho}{dC}$ represents the rate of change in reactivity ρ with respect to the boron concentration C . The boron coefficient α_B , is calculated using the equations below:

$$\alpha_B = \frac{d\rho}{dC} \quad (2.14)$$

where C is the boric acid concentration in wt% ($\text{H}_3\text{BO}_3/\text{H}_2\text{O}$) (Hafez *et al.* (2021).

2.4.4 Void coefficient

Consider a liquid-moderated and/or liquid-cooled reactor. At a point where the liquid boils, the volume occupied by the vapour is essentially a void, since the density of the vapour is so much less than that of the liquid. The void fraction, χ , is defined as a fraction of a given volume, occupied by voids. For instance, if 25 % of a volume is occupied by vapour and the remainder is liquid, then $\chi = 0.25$. Changing the void fraction in the reactor normally affects the reactivity of the system. This phenomenon is described in terms of the void coefficient, α_V , which is defined as:

$$\begin{aligned}\alpha_V &= \frac{d\rho}{d\chi} \\ &= \frac{1}{k_2 \cdot k_1} \cdot \frac{k_2 - k_1}{x_2 - x_1}\end{aligned}\tag{2.15}$$

where ρ is the reactivity and χ is the void fraction (Lamarsh & Baratta, 2001). In the expression above void coefficient, denoted as α_V , the term $\frac{d\rho}{d\chi}$ represents the rate of change in reactivity ρ with respect to the void fraction χ , which is the proportion of voids present in the moderator or coolant.

2.4.5 The control rod worth and control bank worth

The change in reactivity, caused by control rod motion, is referred to as control rod worth. For a reactor with a single control rod, the control rod worth has a maximum effect if placed in the centre of a reactor core having maximum flux. The worth of a control rod directly relates to its effect on reactivity, and it has usually the same unit as the reactivity of the core. The effectiveness, or worth, of a control rod, depends mostly on the value of the neutron flux at the location of the rod (Surbakti *et al.* 2018).

The effect of an individual control rod, and groups of such rods, on reactivity, can be determined in two ways: the integral and the differential rod worth. The integral rod worth is the total reactivity change, due to movement of the control rod to the reactor core. But the differential rod worth is the reactivity change per unit movement of the control rod into the core. The integral and differential worth of the control rod are obtained by the reactivity change, due to the insertion of a partial length of the control rod into the core. It is shown in Eqs. 2.16 and 2.17 respectively:

$$\Delta\rho_i = \sum_{j=1}^i \frac{k_j - k_{j-1}}{k_j k_{j-1}}, i = 1, 2, \dots, N\tag{2.16}$$

$$\frac{\delta \rho_l}{\delta H} = \frac{1}{\Delta H} \frac{k_j - k_{j-1}}{k_j k_{j-1}}, j = 1, 2, \dots, N \quad (2.17)$$

Where ρ is reactivity, H is the height of the control rod and k_j is the effective multiplication factor after the movement of the control rod at the j th step. k_{j-1} is the effective multiplication factor before the movement of the control rod at the j th step, which is 1. N is the number of steps and ΔH is the height change of the control rod before and after insertions (Saadatian-derakhshandeh *et al.* 2014).

2.4.6 Shut down margin

Shut Down Margin (SDM) is defined as the amount of reactivity that can cause a reactor to become subcritical within a state where regulators use SDM as a safety requirement. The SDM is the surplus negative reactivity supplied by all the control rod devices after full insertion. To calculate the minimum SDM, we must know the control rod worth curves (integral and/or differential worth) discussed in Section 2.4.5 (Faghihi & Mirvakili, 2011). An applied relationship for calculating SDM is shown in the following equation:

$$SDM = 0.9 \times (\rho_{ACRD} - \rho_{MCRD}) - \rho_{HFP-HZP} - \rho_{IA} \quad (2.18)$$

where ρ_{ACRD} represents the reactivity of all the Control Rod Devices (CRDs) and ρ_{MCRD} represents the reactivity of the most reactive CRD. $\rho_{HFP-HZP}$ denotes the difference between the core reactivity at Hot Full Power (HFP) and Hot Zero Power (HZP), often referred to as the power defect and finally, ρ_{IA} indicates the rod insertion allowance. The term $0.9 \times (\rho_{ACRD} - \rho_{MCRD})$ introduces a negative reactivity, while the last term introduces a positive reactivity $\rho_{HFP-HZP} - \rho_{IA}$ (Faghihi & Mirvakili, 2011).

2.4.7 Core power distribution

The connection of neutron physics with heat transport requirements may be understood in broad terms as follows. Let P signify the reactor power and V the core volume. Then

$$\overline{P'''} = \frac{P}{V} \quad (2.19)$$

defines the core-averaged power density. The ratio of maximum to average power density is the power peaking factor:

$$F_a = \frac{P'''_{max}}{P'''} \quad (2.20)$$

Eliminating $\overline{P'''} between these definitions, offers some insight into the interdisciplinary nature of reactor core design:$

$$P = \frac{P'''_{max}}{\overline{F_a}} V \quad (2.21)$$

Reactors are normally designed to produce a specified amount of power, and with other variables held constant, the cost of construction rises dramatically with the core volume. Thus, to minimize the volume, maximizing the ratio $\frac{P'''_{max}}{\overline{F_a}}$ is a central optimization problem of core design. The achievable maximum power density is primarily dependent on material properties and the temperatures and pressures that can be tolerated by fuel, coolant, and other core constituents. Minimizing the peaking factor falls much more into the domain of reactor physics, because non-uniform distributions of fuel enrichment, the positioning of control rods and other neutron poisons, as well as other neutronic considerations, largely determine the value of $\overline{F_a}$ (Lewis, 2008).

2.5 Monte Carlo Particle Transport Codes

Monte Carlo codes and methods are used extensively in the modelling of nuclear systems and provide some of the most accurate analyses currently available. However, the use of such codes requires the operator to have a high level of understanding of both the physics of the problem as well as the computational methods used to solve the problem correctly. Knowing that the source distribution of neutrons of a model has fully converged, before beginning to collect results, is critical to ensuring that those results are accurate (Macdonald, 2012).

Two basic ways to approach the use of Monte Carlo methods for solving the transport equation are:

- Mathematical technique for numerical integration; and
- Computer simulation of a physical process.

The mathematical approach is useful for importance sampling, convergence, variance reduction, random sampling techniques, and eigenvalue calculation schemes. The simulation approach is useful for: collision physics, tracking, and tallying (Brown, 2005).

2.5.1 Serpent

Serpent is a three-dimensional continuous-energy Monte Carlo reactor physics code. The code is used for a multitude of applications in reactor physics. Spatial homogenization based on the Monte Carlo method was the original incentive for starting the work. A significant advantage is

that Monte Carlo lattice physics codes not only allow performing spatial homogenization at the fuel assembly level, but can be also used for running transport simulations for the full-scale heterogeneous system (Leppänen *et al.* 2016).

Serpent has a built-in burnup calculation capability. It has been developed at the VTT Technical Research Centre of Finland since 2004 and has been publicly distributed by the OECD/NEA Data Bank and RSICC since 2009. Recently, the code has gained the international community's attention; being already used by 43 organizations in 20 countries around the world. From a geometrical point of view, Serpent is like other Monte Carlo codes (e.g. MCNP and KENO) as it uses a universe-based geometry model that allows the description of complicated three-dimensional reactor geometries (Piovezan *et al.* 2011).

Serpent uses the collision estimate of neutron flux for calculating user-defined reaction rates integrated over space and energy:

$$R = \frac{1}{V} \int_V \int_{E_{i+1}}^{E_i} f(\mathbf{r}, E) \Phi(\mathbf{r}, E) d^3r dE \quad (2.22)$$

The response function $f(\mathbf{r}, E)$ and the spatial and energy domains of the integration are set by the detector parameters. f is an arbitrary response function, Φ is neutron flux and the integration is carried over the phase-space variables (position $\mathbf{r}$ and energy E) (Leppänen, 2015).

2.5.2 SCALE 6.2.4

SCALE provides a comprehensive, verified and validated, user-friendly tool set for criticality safety analysis, reactor and lattice physics calculations, radiation shielding calculations, spent fuel and radioactive source term characterization, and sensitivity and uncertainty analysis. Since 1980, regulators, licensees, and research institutions around the world have used SCALE for safety analysis and design. SCALE provides an integrated framework with dozens of computational modules, including three deterministic and three Monte Carlo radiation transport solvers that are selected, based on the desired solution strategy. SCALE includes current nuclear data libraries (i.e. ENDF/B-VII.1 Cross-Section Libraries), problem-dependent processing tools for Continuous-Energy (CE) and Multi-Group (MG) neutronics and coupled neutron-gamma calculations, as well as activation, depletion, and decay calculations. SCALE includes unique capabilities for automated variance reduction for shielding calculations, as well as sensitivity and uncertainty analysis. SCALE's graphical user interfaces assist with accurate system modelling, visualization of nuclear data, and convenient access to desired results (Oak Ridge National Laboratory, 2018).

2.6 NWURCS

NWURCS is a code wrapper, currently under development at the North-West University, Faculty of Engineering, Unit for Energy and Technology Systems (UETS), by the Computational Nuclear Engineering Analysis Research Group (CNEARG). The code can currently create input files for MCNP6, NEWT, KENO-VI, Serpent and NESTLE embedded in RELAP5-3D. Coupling between the neutronic and thermal hydraulic codes were carried out in Version 1 and is currently being coded into Version 3. The coupling (with input generation) will soon be extended to the fuel performance codes. NWURCS is being developed for two main reasons. The first is so that consistent base models can be given to users, from which further models can be developed. The second is to couple neutronic and thermal hydraulic codes (Naicker, 2021).

2.7 The Analogue Monte Carlo Method

A user specifies the fission source distribution and an estimated value for k-effective. Random walks of a generation of neutrons are carried out to estimate the fission source distribution of the next generation. Each set of random walk calculations for a given generation is called a cycle and a number of cycles are used, the iterative process is repeated for each cycle until the fission source distribution has converged after a number of cycles. Results obtained before this point are discarded (set as inactive cycles). Data accumulation will then begin after the source has converged; the cycles after the source has converged are called active cycles. Cycles are continued until statistical uncertainties are acceptably small. That is why there is a need to divide the cycles into two, these being the inactive and active cycles. The inactive cycles are before convergence and active cycles are post convergence of the fission source distribution (Brown, 2005).

In the Serpent Monte Carlo code, the Shannon entropy is used as a metric to evaluate the uniformity of the fission source distribution across the modelled geometry. This provides insight into whether the source distribution has stabilized, indicating convergence to a steady state. In the first several cycles, the spatial dependence of the placement of the neutrons generally varies, as the neutron source shifts towards the locations in which the highest number of neutrons are produced in the material. This variation is quantified, using the fission source entropy (Shannon entropy). When it becomes statistically constant, this is then an indication that the source distribution has stabilized (converged). Until convergence of the fission source is achieved, it is important that the results of the eigenvalue and other calculations are discarded (Brown, 2005).

By tracing the path of an individual neutron as it traverses matter and considering the various processes that may determine its history, we can understand how a Monte Carlo calculation

simulates the stochastic nature of neutron transport through matter. We begin with the source of neutrons in a nuclear reactor, which is predominantly, if not entirely, the fission source. The fission source has a distribution in space, a distribution in energy given by the fission spectrum, and a distribution in direction that is isotropic. Each of these distributions may be characterized by a pdf and a cdf. Generating a random number and selecting it from the cdf for the spatial fission distribution, defines a location in space for the source particle. Generating another random number and selecting it from the cdf for the fission spectrum, determines the energy of the source particle. Generating third and fourth random numbers and selecting them from the cdfs for the two independent angular variables (say $\mu = \cos \theta$ and ϕ) defines the direction of the source neutron. Once launched, the source neutron will travel in a straight line until it has a collision. The probability that a neutron has a collision at a distance s along the flight path is:

$$T(s) = \Sigma_t(s) \exp\left[-\int_0^s \Sigma_t(s') ds'\right] \quad (2.23)$$

which is the pdf for the collision distance s . Generating a random number λ and selecting s from the cdf: $\Sigma_t(s)$ is the Macroscopic total cross section as a function of distance along the flight path.

$$-\ln \lambda = \int_0^s \Sigma_t(s') ds' \quad (2.24)$$

locates the position of the first collision, in principle. In fact, the process is considerably complicated by the non-uniform geometry. It is necessary to know the composition at the point of the first collision. $\Sigma_t(s')$ is the macroscopic total cross section at position s' along the neutron's flight path. (Stacey, 2007). At the collision point, another random number is used to determine the type of collision. If scattering, then the path of the neutron is tracked using (2.23) and the process is repeated. Account is also taken to determine whether the scattering is elastic or in-elastic. Should the collision result in an absorption, another random number is used to determine whether the reaction would be fission or otherwise. If fission, then further random numbers are used to determine the number of neutrons produced by fission, and the energy of the fission neutrons. The original neutron's history is then terminated.

3. METHODOLOGY: MODEL DEVELOPMENT

3.1 Introduction

This chapter summarizes the method used to develop the fresh core and the fuel assembly models, detailing the reactor core design, core geometry, and fuel assembly configurations. The reactor's geometry and material are defined, down to the level of individual fuel pins. Additionally, the physical verification process and the methodology for performing neutronic calculations are described.

Chapter 4 of the NuScale Safety Analysis Report (NuScale Power LLC, 2020) serves as the primary source for the specifications and key parameters used in this study. These specifications and parameters were extracted and incorporated into NWURCS. Additional parameters, such as fuel enrichment for the first cycle, Reactor Pressure Vessel (RPV) inner and outer radii, the chemical composition of the stainless steel for the RPV and reflector, the specifications of the dashpot for guide tubes, and the nuclide composition of the spacer grids, were sourced from other references because NuScale Chapter 4 did not provide all the necessary parameters for model development. Table 3.1 lists the detailed key parameters modelled in this study.

The fuel enrichment and gadolinium content of the fuel assemblies for the first cycle were adopted from Sadegh-Noedoost *et al.* (2020). The RPV inner and outer radii were obtained from Dsouza (2015), while the nuclide composition of the spacer grids (comprising zircaloy-4, inconel, and stainless steel) was acquired from Fridman *et al.* (2023). The fuel assemblies include two types of spacer grids: the majority are HTP spacer grids made of zircaloy-4, while the bottom-most HMP spacer grid is made of alloy 718 (inconel). The chemical composition of the stainless steel used in the RPV, and the reflector was sourced from Horelik *et al.* (2018).

Both the full-core and fuel assembly models were developed using NWURCS, and the model was subsequently verified. Key parameters essential for developing the base models include the active core height, top and bottom non-active fuel assembly heights, fuel rod pitch, fuel pellet radius, gap thickness, cladding thickness, RPV inner and outer radii, inner and outer dashpot radii for the guide tubes, dashpot height, the chemical compositions of the fissile material and the materials used in the cladding, reflector, and core barrel. The chemical compositions for the cladding, reflector, spacer grid, and fissile material are detailed in Appendices A, B, and C.

3.2 Core Geometry as Modelled in this Study

The reactor core developed in this study is a small, light-water-cooled, Pressurized Water Reactor (PWR) with an active core height of 2.0 meters. Figure 3.1 shows the loading patterns for the reference equilibrium core, as excerpted from NuScale Power LLC (2020). The core structure, including the fuel assemblies depicted in Figure 3.1, had previously undergone irradiation for one, two, or three cycles. The fuel pellet density used in this study is 10.53 g/cm^3 (96 % of the theoretical density), as provided in data released by NuScale Power LLC.

Although the same core dimensions, including radii and active length, were simulated in this study as specified by NuScale Power LLC (2020), the enrichment of the fuel assemblies differs because NuScale has not published the fuel enrichment for the first cycle. This study adopts the enrichment values proposed by Sadegh-Noedoost *et al.* (2020). These values include four types of fuel assemblies: one type without gadolinium content located in the outer region of the core, and three types with gadolinium enrichment levels of 2 %, 6 %, and 8 % Gd_2O_3 , as illustrated in Figure 3.2. The isotopic composition of the fuel material is detailed in Appendix C. The stainless steel reflector, including the bypass channels for coolant flow, the core barrel, and the Reactor Pressure Vessel (RPV), were also simulated in this study.

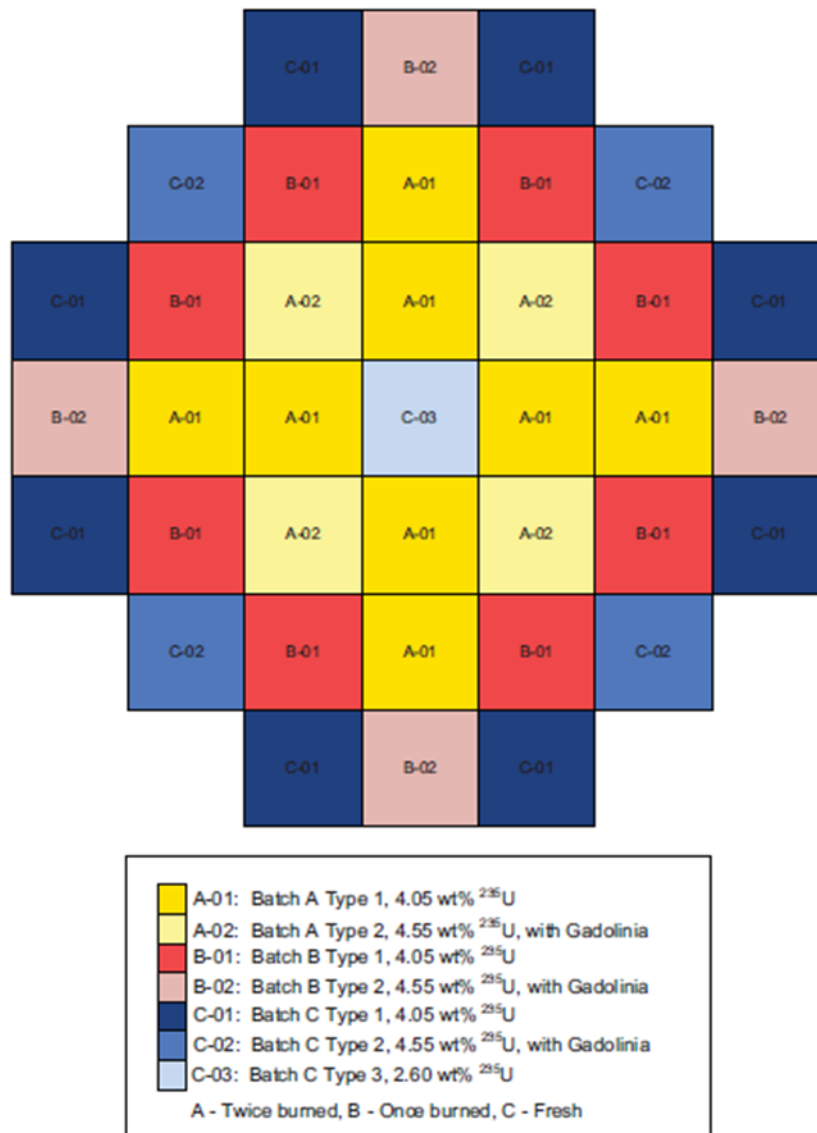


Figure 3.1: Loading pattern for the reference equilibrium core, four types of radiated FAs are shown (NuScale Power LLC, 2020).

The fuel cladding material used in this study is M5, an advanced zirconium alloy developed and implemented by AREVA to enhance corrosion resistance, minimize hydrogen uptake, and reduce irradiation growth (NuScale Power LLC, 2020). The density of M5 is 6.50 g/cm^3 , following the modelling approach outlined in the COPENIC code (FRAMATOME, 2004). The isotopic composition of M5 is provided in Appendix A. The BEAVRS benchmark was utilized to model the dashpot for the guide tubes, with the calculations detailed in Section 4.2 (see Table 4.2 and Table 4.3). The material for the instrumentation pin and guide tubes is zircaloy-4, as specified in the BEAVRS benchmark. The isotopic composition of zircaloy-4 is also listed in Appendix A.

16	16	100	200	100	16	16
16	100	200	300	200	100	16
100	200	300	300	300	200	100
200	300	300	400	300	300	200
100	200	300	300	300	200	100
16	100	200	300	200	100	16
16	16	100	200	100	16	16
	4.33	0.0	100	type 1		
	4.32	2.0	200	type 2		
	4.30	6.0	300	type 2		
	4.29	8.0	400	type 2		

Figure 3.2: First loading pattern for NuScale.

Table 3.1: Key parameters of the NuScale model used in this study.

Parameter	Value
Height of active core	200 cm
Core barrel radius	93.98 cm
Gap radius	0.41402 cm
FA pitch	21.5036 cm
Fuel rod pitch	1.2598 cm
Cladding radius outside	0.47498 cm
Fuel pellet radius	0.405765 cm
Fuel density	10, 53 g/cm ³
RPV inner radius	115 cm
RPV outer radius	120 cm

Appendix A provides the isotopic compositions for the cladding, reflector, instrumentation tubes, guide tubes, and Reactor Pressure Vessel (RPV). Appendix B details the isotopic compositions of the spacer grids as modelled in this study, while Appendix C presents the isotopic compositions of the fuel material (Sadegh-Noedoost *et al.* 2020). Approximately five full-core models were developed in this study. The first four models (labelled B003, C001, D001, and E001) differ slightly in terms of whether spacer grids and coolant channels are included. Otherwise, all other parameters remain consistent. The fifth and final full-core model was adjusted to align closely with the model described in Fridman *et al.* (2023). Table 3.2 summarizes the key parameters of the Emil Fridman model. The material specifications for Emil Fridman's model are as follows: The core is loaded with 37 all-fresh, no-Xenon UO_2 fuel assemblies at six different ^{235}U enrichment levels, adjusted to approximate batch-wise ^{235}U content at the beginning of the equilibrium fuel cycle. Four fuel assemblies include 16 fuel pins containing Gd_2O_3 burnable poison (Figure 3.3, left). In total, there are seven distinct fuel assembly types. The core is regulated and controlled by 16 control rod assemblies, divided into two regulating banks and two shutdown banks, as illustrated in the right panel of Figure 3.3. Surrounding the core is a heavy steel reflector, enclosed by a cylindrical core barrel. The inner and outer radii of the core barrel are also depicted in the right panel of Figure 3.3. The radial heavy reflector is modelled in a simplified manner as a homogeneous mixture of steel and water (Fridman *et al.* 2023).

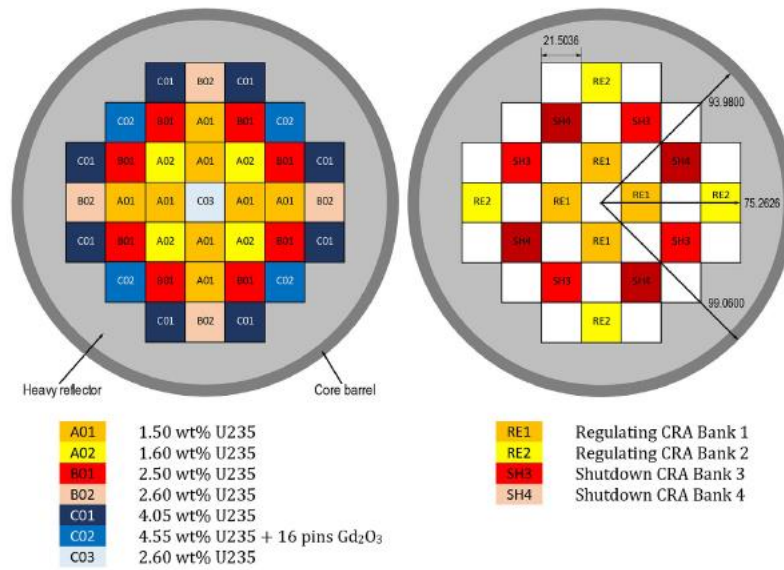


Figure 3.3: Core loading pattern and control rod assembly locations
All dimensions are in cm (Fridman *et al.* 2023).

The first four models are described as follows:

B003- with SG and coolant channels in the reflector;

C001- with SG and without coolant channels in the reflector;

D001- without SG, with coolant channels in the reflector; and

E001- without SG, without coolant channels in the reflector

Table 3.2 Key parameters of Emil Fridman's model

Parameter	Value
Height of active core	2.0m
Core barrel radius	93.98 cm
Cladding radius	0.4750 cm
Fuel radius	0.4058 cm
Gap radius	0.4140 cm
FA pitch	21.5036 cm
Fuel rod pitch	1.2598 cm
Fuel density	10, 53 g/cm ³
RPV inner radius	93.980000 cm
RPV outer radius	99.060000 cm

3.2.1 Fuel assembly configurations

This section presents the fuel assembly configurations of the NuScale power module as modelled in this study. The fuel assembly is arranged in a 17×17 square lattice with a pitch of 21.50364 cm. Each fuel assembly is supported by five spacer grids (the axial calculation for the spacer grids is provided in Table 3.3, and their isotopic composition is detailed in Appendix B), 24 guide tubes, and top and bottom nozzles, which together form the structural framework for 264 fuel rods. Each fuel assembly also includes a central instrumentation tube. The NuScale core contains a total of 37 fuel assemblies. Among these, 25 FAs consist of a homogeneous fuel mixture of UO₂ and

Gd₂O₃ with gadolinium serving as a burnable absorber. The remaining 12 FAs contain only UO₂ fuel (NuScale Power LLC, 2020).

Table 3.3 Spacer grid axial dimensions

No	Bottom Edge (cm)	Height (cm)	Top Edge (cm)
SG1	0.0000	3.7230	3.7230
	3.7230	45.0524	48.7754
SG2	48.7754	4.1148	52.8902
	52.8902	45.0524	97.9426
SG3	97.9426	4.1148	102.0574
	102.0574	45.0524	147.1098
SG4	147.1098	4.1148	151.2246
	151.2246	45.0524	196.2770
SG5	196.2770	3.7230	200.0000

3.2.2 Fuel rod configurations

Two types of fuel rods were modelled in this study: Type 1 fuel rods contain UO₂ with varying enrichment but no Gd₂O₃, while Type 2 fuel rods contain UO₂ with varying enrichment and Gd₂O₃ as a burnable absorber. The total fuel rod length is 216 cm, with an active length of 200 cm. Notably, Chapter 4 of the NuScale Safety Analysis Report (SAR) does not provide an axial view of these two types of fuel rods. Therefore, the axial configurations used in this study were sourced from (Sadegh-Noedoost *et al.* 2020). Figure 3.4 illustrates the axial details of the two fuel rod types, highlighting the presence or absence of Gd₂O₃ and differences in the upper and lower material distributions. Additionally, the rods include 8 cm upper and lower blankets made of stainless steel, though these are not depicted in Figure 3.4.

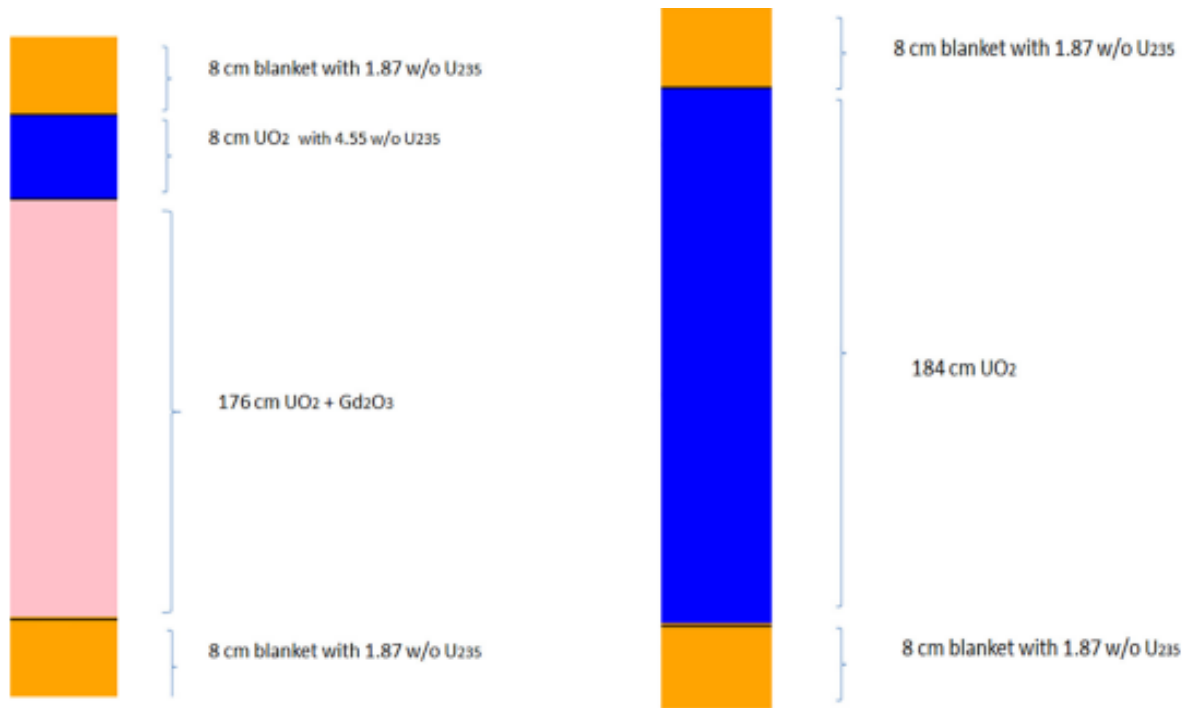


Figure 3.4: Axial schemes of two types of fuel rods (Sadegh-Noedoost et al. 2020).

3.3 Serpent Input: Build-up of the Geometry in the Input File

At the foundation of the geometry is the cell, which is a spatial region defined by simple boundary surfaces. Each cell is filled with a homogeneous material, void, or another universe. Cells are defined as intersections of spatial regions shaped by various surfaces. Each surface splits space into two regions, marked as positive and negative, with the negative regions lying inside a closed surface. Once the cells are defined, they are grouped into universes, which represent specific components of the reactor. At the highest level, the geometry comprises fuel pins, where fuel pellets are enclosed by cladding and surrounded by coolant. Each pin type is defined as its own universe. The next level is the fuel assembly, where the pin universes are organized into a regular lattice. The assembly may also include moderator channels or support structures. At the subsequent geometry level, these assembly universes are arranged into another lattice to form the core layout. This layout is typically surrounded by radial and axial reflectors and, finally, the reactor pressure vessel wall (Leppänen, 2015).

3.3.1 Universes

After modelling the cells described in the preceding section, each cell was assigned to a specific universe. Universes are logical containers used to organize and manage groups of cells systematically. Cells were assigned to a universe by specifying the universe number in their definition. In the model, only Universe 0 is directly reflected, representing the highest-level

structure, while all other universes are virtual. These virtual universes serve as components that are nested within other universes to build the overall geometry. The universes are organized in a branch-like structure, all converging into Universe 0 at the top level. Each universe is defined independently, and their relationships are managed using the fill command, which replaces a material name in a cell definition with a reference to another universe (Leppänen, 2015).

3.3.2 Lattices

Lattices are specialized universes, designed to be filled with a regular, repeating structure of other universes. The Serpent code supports eight types of lattices: square lattice, two hexagonal lattices, circular cluster array, three infinite 3D lattices filled with a single universe, and the vertical stack. To define a lattice, the lattice type is specified, followed by its dimensions and pitch. Universes are then assigned to individual lattice elements, creating a structured and organized representation of the geometry.

3.4 Source Convergence

The method described in this section was used to determine the parameters for the criticality source calculation (Set pop) necessary for the convergence of the source point distribution. Initially, a trial run was conducted, where an initial estimate of the number of source neutrons per cycle and the number of active and inactive cycles was provided. According to the Serpent manual, lattice calculations typically converge within the first 20 cycles, so this was used as the initial estimate for the number of cycles required for convergence. However, full-core calculations may require more cycles for source convergence, so 100 inactive cycles were selected. Additionally, 300 active cycles were chosen to allow the calculation to progress beyond the inactive cycles. After the trial run, the results were analysed to determine whether the source point distribution had converged. This was done by plotting Shannon entropy against the number of cycles and visually inspecting the trend. If convergence had not yet been achieved, another trial run was conducted with an increased number of inactive cycles. Once source convergence was confirmed, the number of active cycles was increased to ensure that the statistical uncertainty of the multiplication factor was reduced to ≤ 0.00015 .

3.5 Criticality Calculations

The calculations in the Serpent code for determining essential parameters, such as k-effective and neutron flux were performed, using 100 000 neutron sources per cycle, 100 inactive cycles, and 800 active cycles, resulting in a total of 80 million neutron histories. The standard deviation of the criticality calculation was ± 0.00015 . The fuel, helium, M5 cladding, and water moderator

were modelled at 300K, representing an "isothermal" state. This condition corresponds to the reactor's state just before start-up, assuming an ambient temperature of 300K. Criticality calculations were performed for both the full-core and all four fuel assembly types using the Serpent code. To validate the results obtained from Serpent, k-effective calculations were also conducted using KENO-VI from the SCALE 6.2. The comparison between the two sets of calculations helps assess the consistency of the results. The method involved performing both simulations under similar conditions,

3.6 Calculation of Safety Parameters

This section describes the method used to determine the safety level of the NuScale core by calculating the moderator temperature coefficient and the Doppler reactivity coefficient.

3.6.1 Doppler reactivity coefficient

The reactivity change was investigated at various fuel temperatures (301K, 302K, 305K, 310K, and 315K), while the temperatures of the moderator, gap, and cladding were maintained at 300K. The calculation of k-effective was conducted using Serpent. The equations used to calculate the reactivity change ($\Delta\rho$) and the Doppler coefficient (D_c) are adopted from (Tiep *et al.* 2015). The equation for the Doppler coefficient is provided in the preceding chapter as (Eq. 2.12).

3.6.2 Moderator temperature coefficient

The Moderator Temperature Coefficient (MTC) was calculated at five different temperatures of the moderator (301K, 302K, 305K, 310K and 315K), while the temperature of the fuel, gap and cladding was left at 300K. The value of MTC, or a_c , was determined from its definition, which involves dividing the change in reactivity due to a change δT_M in moderator temperature T_M . The equation used in the calculation can be found in the preceding chapter (Eq. 2.10).

3.7 Method of Recording the Neutron Flux

For the neutron flux calculation, a detector name and detector parameters were defined in the Serpent input. A three-dimensional mesh was used for the thermal flux distribution with detector parameters set for dx , dy , and dz (which defines the x -mesh, y -mesh, and z -mesh). Reactions were scored in each element of the mesh. To achieve a 1:1 mapping for flux results and the core lattice, the neutron flux was calculated for 9 bins in both the x and y directions, as the lattice of the modelled reactor core is 9×9 , including water nodes. The output from all detectors was printed in the MATLAB m-file format in a single file. Verification of the neutron flux calculations was

necessary, because the full-core contains heterogeneous fuel material, and theoretically, a typical symmetric plot can only be obtained with a homogeneous core. To further verify the neutron flux results for the full-core model, the calculations were also performed with a single fuel assembly type throughout the core to obtain a typical symmetric plot in the radial plane. The single fuel assembly used in the verification process contained 4.33 wt% enrichment for UO₂ with no gadolinium content.

3.8 Verification of the Model

One of the objectives of this study is to verify the core model developed using NWURCS. Verification of the model was essential to ensure its correctness. Verification serves as a quality assurance tool, confirming that the NWURCS generated model complies with specifications and that no errors were introduced in the input files generated by NWURCS. The results of the verification process are presented in the subsequent chapter.

The Serpent input file was inspected using a text editor to ensure that all data entered in the input files was correctly reflected in the Serpent input file. The model was verified by comparing the Serpent input generated by NWURCS with data provided by the NuScale Safety Assessment Report, Chapter 4, and with a model simulated by Sadegh-Noedoost *et al.* (2020). The user (student) edited a set of input files, including the unit files and lattice files required for the level scheme approach. Additionally, material definitions were contained in a file named "inputmat", and the materials specified in the unit files were indexed correctly to match the material IDs in the "inputmat" file. The temperatures were also specified in a file named "f005.i"; these temperature values were properly indexed in the unit files.

NWURCS is essential because it significantly reduces the time needed to develop a model and is crucial for standardizing the model. The models generated are considered "high-fidelity" models due to the detailed geometry represented. For the research goals of CNEARG at NWU, these models are used in uncertainty analysis and code coupling research, which are outside the scope of this project. Given the time constraints of a Master's project, developing such models "from scratch" is not feasible, so users are provided with so-called base models. Users are then required to verify these models and further manipulate them for various numerical modelling tests, such as calculating reactivity coefficients, which are investigated in this work. Ensuring that the results follow physical principles and logical arguments contribute to the robustness of the base models.

It is also important to note that while NWURCS generates the input file for Serpent, the preparation of the dataset required to generate this input file must be carried out by the user. This was the case in this project for NuScale. Additionally, users of the code must first demonstrate proficiency

in developing simpler models for Serpent calculations, before being allowed to use NWURCS. This process facilitates both learning and model assurance in terms of verification and validation efforts.

4. RESULTS AND DISCUSSIONS

4.1 Introduction

The results and discussions of this study are presented in this chapter. Section 4.2 outlines the results of the verification process, highlighting the verification of the developed fresh core model against Chapter 4 of the Safety Analysis Report issued by NuScale and the model developed by Sadegh-Noedoost *et al.* (2020). The results of the convergence investigation, neutronic calculations (k-effective and neutron flux), and safety parameters (moderator temperature coefficient and fuel temperature coefficient) are also presented in the subsequent sections.

4.2 Verification of the Fresh Core Geometry

The model was generated using NWURCS, as described in Section 3.1. NWURCS produced the input file for Serpent, using data from the input files outlined in Section 3.8, along with additional required files. Verification of the fuel and non-fuel components of the model was performed, using a text editor to ensure that all data entered in the input files were accurately transferred to the Serpent input file and aligned with the data in Chapter 4 of the NuScale Safety Analysis Report. Any discrepancies identified during this process were promptly corrected. The material compositions were confirmed to be accurate. During the verification process, it was observed that the length of the dashpot for guide tubes was not specified in Chapter 4 of the NuScale Safety Analysis Report. Consequently, the BEAVRS benchmark was utilized for this parameter (refer to Tables 4.2, and 4.3). Table 4.1 presents the radii as excerpted from Chapter 4 of the NuScale Safety Analysis Report.

Table 4.1 Radii as excerpted from the SAR

Guide tube ID above dashpot	0.5715 cm
Guide tube ID dashpot	0.50419 cm
Guide tube OD	0.61214 cm, assumed to be constant throughout

Table 4.2 Data used from BEAVRS (Except for the active length of NuScale fuel rod)

Column Heading	Column Heading
Length of dashpot from bottom of fuel rod	$98.025 - 35.0 = 63.025 \text{ cm}$
Length of guide tube above dashpot to top of fuel rod	$423.049 - 98.025 = 325.024 \text{ cm}$
Total height of fuel rod	$423.049 - 35.0 = 388.049 \text{ cm}$
Fraction of dashpot to fuel rod length	$63.025/388.049 = 0.162415 \text{ cm}$
Active length of NuScale fuel rod	200.00 cm
Length of dashpot	$0.162415 \times 200 = 32.48301 \text{ cm}$

Table 4.3 Empty guide tube pincell axial specification as excerpted from BEAVRS benchmark (Horelik et al. 2018)

	Source Reference	Elevation (cm)	Description
46	Water		
46	Nozzle/support plate borated water	431.876	Top of upper nozzle
44	Guide tube pincell	423.049	Bottom of Upper Nozzle
36	Guide tube pincell w/grid	415.164	Grid 8 top
44	Guide tube pincell	411.806	Grid 8 bottom
36	Guide tube pincell w/grid	364.725	Grid 7 top
44	Guide tube pincell	359.01	Grid 7 bottom
36	Guide tube pincell w/grid	312.528	Grid 6 top
44	Guide tube pincell	306.813	Grid 6 bottom
36	Guide tube pincell w/grid	260.311	Grid 5 top
44	Guide tube pincell	254.6161	Grid 5 bottom
36	Guide tube pincell w/grid	208.134	Grid 4 top

	Source Reference	Elevation (cm)	Description
44	Guide tube pincell	202.419	Grid 4 bottom
36	Guide tube pincell w/grid	155.222	Grid 3 top
44	Guide tube pincell	150.222	Grid 3 bottom
36	Guide tube pincell w/grid	103.74	Grid 2 top
44	Guide tube pincell	98.025	Grid 2 bottom
44	Guide tube pincell w/grid	40.52	Grid 1 top
36	Dashpot guide tube w/grid	39.958	Control rod step 0
44	Dashpot guide tube w/grid	37.1621	Grid 1 bottom
66	Nozzle/support plate borated water	35	Bottom of fuel rod
	Water	20	Bottom of support plate
		0	Lowest extent

A second type of verification involved visually inspecting the geometry plots generated by Serpent. In this process, the geometry was verified visually, with materials correctly identified using colour codes. The verification confirmed, through Serpent output plots, that the core configuration consists of 37 fuel assemblies, 16 control rods, and a fuel assembly design of 17×17 . Four types of fuel assemblies were considered for this model (see Table 4.4). It is important to note that Chapter 4 of the NuScale Safety Analysis Report does not specify the four types of fuel assemblies for the first cycle. The enrichment levels of the fuel assemblies used in this study were obtained from Sadegh-Noedoost *et al.* (2020). Detailed information about the four fuel assembly types, as well as the fuel pellet density and core barrel inner and outer diameters, is presented in Chapter 3.

Table 4.4 Four types of fuel assemblies that are considered for this model

Fuel Assembly	Gd ₂ O ₃ Enrichment (W/O)	²³⁵ U Enrichment (W/O)
Centre 12 FAs	2	4.32
12 FAs	6	4.30
1 FA	8	4.29
12 FAs	0	4.33

Figure 4.1(A), generated by Serpent, illustrates the core layout modelled in this study, while Figure 4.1(B) presents the corresponding model from Sadegh-Noedoost *et al.* (2020). Both models depict the Fuel Assemblies (FAs), the radial stainless steel reflector with bypass channels for coolant flow, the core barrel, and the Reactor Pressure Vessel (RPV). Figure 4.2 presents the fuel assembly design, which includes two pin types (representing fuel pins with gadolinium and one without gadolinium). Additionally, the design comprises 24 guide tubes and one instrumentation pin. The fuel assembly layout follows a 17 × 17 configuration, consisting of 264 fuel rods.

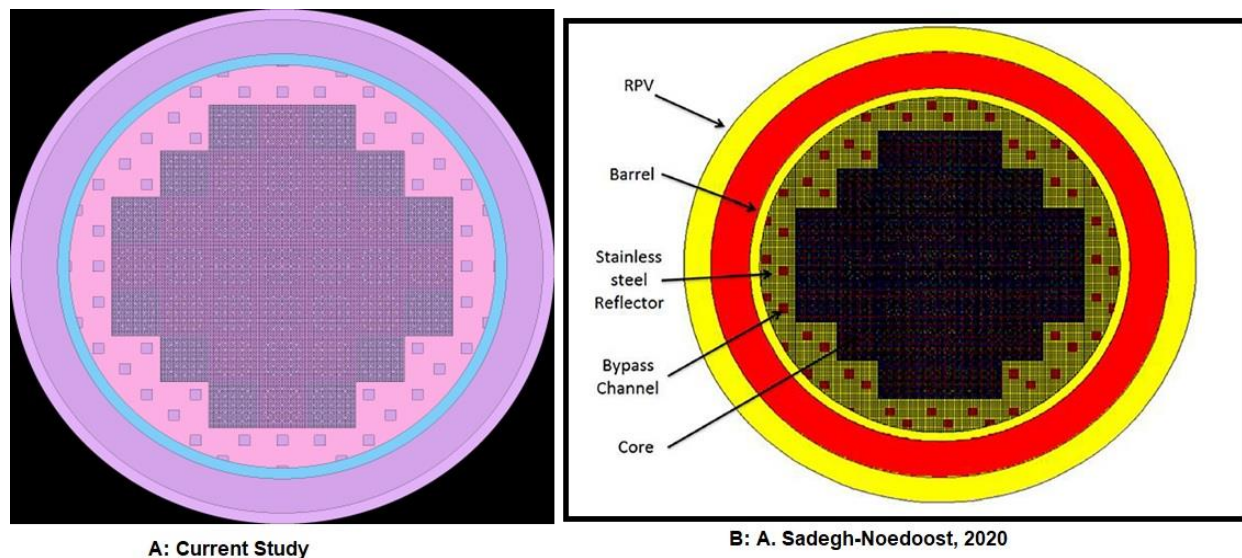


Figure 4.1: Top view of the NuScale full-core modelled using the Serpent code (A), and Top view of the simulated core using the MCNP5 code (B) (Sadegh-Noedoost et al. 2020).

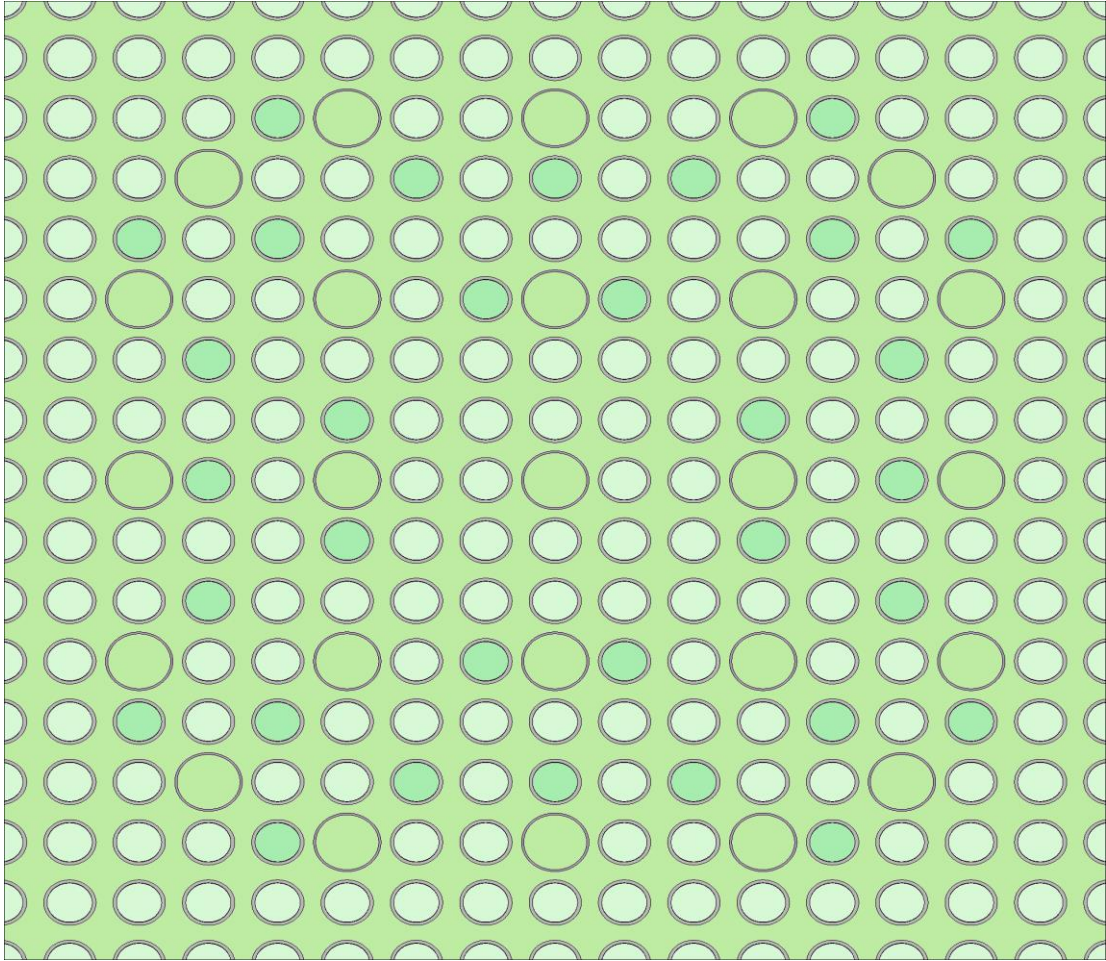


Figure 4.2: Fuel assembly structure excerpt from Serpent.

4.3 Convergence

This section presents the convergence results for the developed fresh core model. The Shannon entropy is used to assess convergence, with a detailed explanation of this concept provided in an earlier section (Section 2.7). A specific number of inactive cycles is determined, prior to performing calculations, such as the multiplication factor and neutron fluxes. As discussed in Section 2.7, Monte Carlo codes track the history of neutron origins and place them at these locations in subsequent cycles. In Serpent, source convergence data is generated by enabling the batch history recording option. When this option is activated, Serpent collects batch-wise data on parameters such as k-effective and fission source entropy, producing a separate output file. This allows for examining the fission source entropy as a function of the cycle. It is critical to ensure that the source distribution of the model has fully converged before beginning to collect results. Failure to achieve convergence can lead to inaccurate criticality results, with the k-effective value potentially being overestimated or underestimated.

The results obtained from the investigation of Shannon entropy convergence for the developed fresh core model are presented below. In this study, Monte Carlo methods, as implemented in the Serpent code, are used to calculate the reactor's criticality and neutron flux distribution. Consequently, the output parameters are not represented by a single real number but rather by a statistical distribution, which can be analysed to determine whether the output parameter has reached a "constant" state. In this context, as the source distribution approaches a statistically stable distribution, the Shannon entropy converges to a steady value, indicating convergence. A sufficient number of inactive cycles is required to ensure the initial fission source distribution has fully converged before collecting results.

Table 4.5: Starting parameters used to study source convergence (statistical uncertainty was ± 0.00024).

Number of neutron sources	100 000
Number of inactive cycles	100
Number of active cycles	300

Figure 4.3 was produced using the results of Shannon entropy versus the number of cycles. As shown in the figure, the Shannon entropy value decreased from 0.85 to approximately 0.81, and then became statistically constant after cycle 100. From point A onwards, as indicated in Figure 4.3, the curve suggests that the Shannon entropy has reached a statistically constant value (i.e. it has converged). To reach point A, Serpent runs 100 000 source points over 100 cycles, totalling 10 000 000 histories. This means that the first 100 cycles were discarded as inactive.

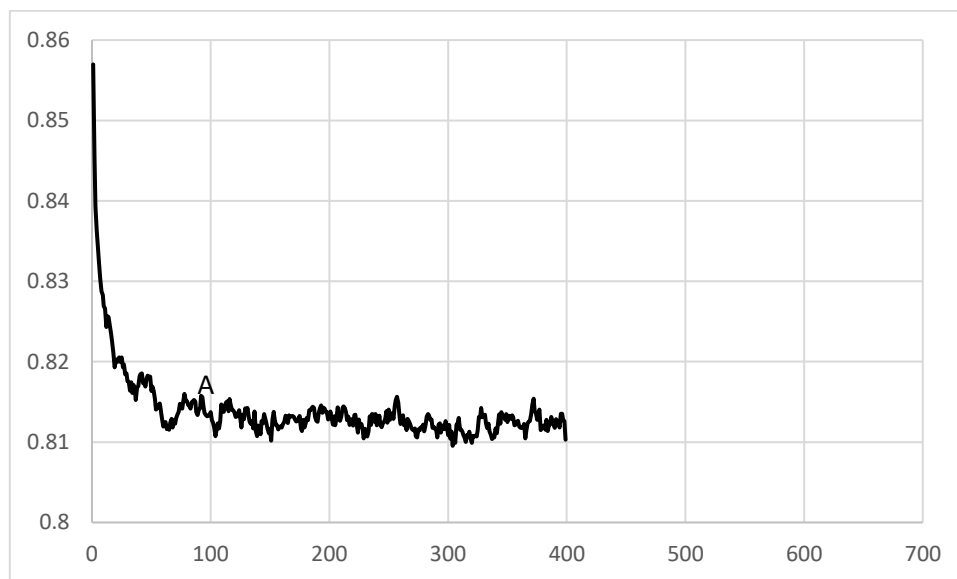


Figure 4.3: Shannon entropy vs. number of cycles.

After establishing source convergence in the initial case, further analysis was conducted to reduce the statistical uncertainty of the multiplication factor to ≤ 0.00015 . To achieve this, the number of active cycles was increased, as shown in Table 4.6.

Table 4.6: Second case (statistical uncertainty was ± 0.00017).

Number of neutron sources	100 000
Number of inactive cycles	100
Number of active cycles	600

Figure 4.4 was produced using the results of Shannon entropy versus the number of cycles. The convergence in this case is like that of the previous case; however, the statistical uncertainty of ≤ 0.00015 was still not achieved. At that time, the statistical uncertainty was ± 0.00017 , prompting further investigation, Table 4.7 provides data for this investigation, in which the statistical uncertainty of the multiplication factor was reduced to ± 0.00014 , which is considered acceptable.

Table 4.7: Third case (statistical uncertainty was ± 0.00014).

Number of neutron sources	100 000
Number of inactive cycles	100
Number of active cycles	800

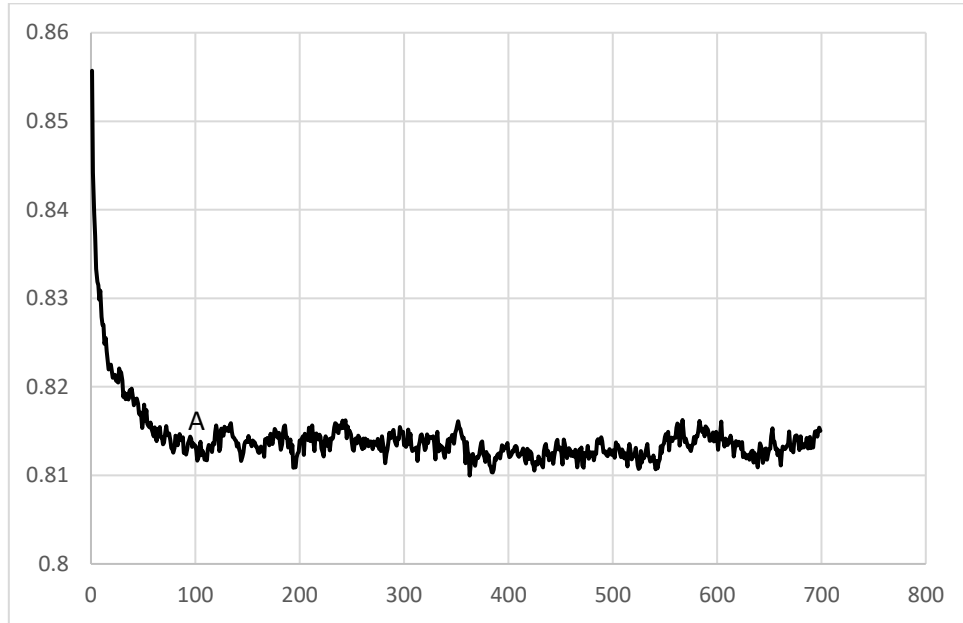


Figure 4.4: Shannon entropy vs. number of cycles.

Although the desired statistical uncertainty of less than 0.00015 was achieved, it was possible to reduce it further by increasing the number of active cycles. While this increased the computational time, it becomes crucial when comparing cases where the differences in the multiplication factor are around ± 0.00015 . A lower statistical error enables a more conclusive analysis. Therefore, k -effective values for the full-core and fuel assembly are also reported with a statistical uncertainty of ± 0.00014 , and at a much lower statistical uncertainty of ± 0.00009 as the cases warrant. Table 4.8 provides data for the investigation, in which the statistical uncertainty of the multiplication factor was reduced to ± 0.00009 .

Table 4.8: Data for statistical uncertainty of ± 0.00009 .

Number of neutron sources	100 000
Number of inactive cycles	100
Number of active cycles	2000

4.4 Criticality Results

Criticality calculations (i.e. the multiplication factor described as k -effective) for the NuScale full-core and fuel assemblies were conducted using Serpent. The Full-Core (FC) and Fuel Assembly (FA) models were developed without control rods, meaning that the control rods were withdrawn from the models. The core temperature was set at 300K, and the fuel was assumed to be fresh.

This section presents a comparison of the criticality results. Initially, as part of the verification process, FA results are compared among themselves, followed by a comparison of the FA results with those of the FC. Table 4.9 shows the k-effective results for the fuel assemblies across different enrichments.

Table 4.9: Fuel assembly calculations.

Calculation Number	FA Enrichment	k-eff
1	4.29 w/o (Gd_2O_3) content is 8 %	1.13991 ± 0.00014
2	4.30 w/o (Gd_2O_3) content is 6 %	1.15028 ± 0.00014
3	4.32 w/o (Gd_2O_3) content is 2 %	1.18845 ± 0.00014
4	4.33 w/o	1.46381 ± 0.00014

Looking at the results presented in Table 4.9, it is evident that the k-effective is higher in models that do not contain gadolinium or contain lower amounts of gadolinium. Gadolinium acts as a burnable absorber and is commonly used for reactivity control in light water reactors, due to the high neutron absorption cross-section of several gadolinium isotopes and its good solid solubility in UO_2 . Zuhair *et al.* (2021) demonstrated that fuel pins with pure UO_2 , achieve the highest criticality. In Zuhair's work, the study focused on the criticality and neutronic safety parameters of the NuScale fuel assembly, using the Monte Carlo code MCNP6 and the ENDF/B-VII library. The calculation results showed that the criticality of fuel assembly type D is the highest among other assembly types because it contains a fuel pin with pure UO_2 , without the addition of Gd_2O_3 . Fuel assembly type D contains UO_2 -fuel pins of 4.33 % ^{235}U enrichment in the middle zone. Higher fuel enrichment also contributes to a larger k-effective, which is why the FA k-effective results behaved as expected.

When comparing the k-effective values for the full-core and the chosen fuel assembly (calculation number 1), as shown in Table 4.10, it was noted that although the fuel assembly calculation uses a reflective boundary condition and is therefore considered infinite (based on the assumption that no neutrons leak out of the reactor), the k-effective value for the full-core was found to be 3 490 pcm higher than that of the fuel assembly.

Table 4.10: Fuel assembly vs. full core.

Model Description	k-eff
Base case model (full-core)	1.17481±0.00015
Fuel assembly, calculation 1	1.13991±0.00014

The k-effective for the full-core could be higher than that of the fuel assembly (Calculation number 1) due to differences in uranium enrichment. Specifically, the full-core contains heterogeneous fuel material with uranium enrichments of 4.29 %, 4.30 %, 4.32 %, and 4.33 % w/o, and varying gadolinium oxide (Gd_2O_3) content of 8 %, 6 % and 2 %. In contrast, the enrichment of the fuel assembly material (Calculation number 1) is 4.29 w/o, and the (Gd_2O_3) content is 8 %.

4.5 Base Case Compared with a Model Developed by Sadegh-Noedoost *et al.* (2020)

The base case is the model developed in this study, from which all other comparative models were derived. As part of the model verification, the temperature of all the materials and the density of the moderator/coolant were adjusted to match those used by Sadegh-Noedoost *et al.* (2020) (referred to as the first comparison model in the abstract). The temperatures in the base case and the first comparison model were found to be nearly identical. This exercise was conducted to determine if the k-effective results from this study would match those of Sadegh-Noedoost *et al.* (2020). The k-effective results of this study was found to differ by 3 698 pcm, as shown in Table 4.11. This was a significant difference, which warranted further explanation.

Upon reviewing Figure 4.1B in Section 4.2, it was found that, in addition to the fuel assemblies, Sadegh-Noedoost *et al.* (2020) also simulated the radial stainless steel reflector, including bypass channels for coolant flow, the core barrel, thermal and biological shields, and the Reactor Pressure Vessel (RPV). However, the actual dimensions and positions of the bypass channels were not provided in Sadegh-Noedoost *et al.* (2020) and had to be estimated from Figure 4.1B. Additionally, while spacer grids were mentioned, their dimensions and chemical composition were not specified. When comparing the k-effective results before and after adding the coolant channels and spacer grids (see Table 4.12), it is evident that these additions lowered the k-effective by 2 196 pcm. This provides a possible explanation for the discrepancy between the results. Further investigation into the effects of the spacer grids, coolant channels with reflector, core barrel, water in the down-comer, and RPV will be presented later, to provide additional insight.

It is also possible that the model in Sadegh-Noedoost *et al.* (2020) was not fully interpreted correctly, particularly regarding the geometry and fuel enrichment allocations, which may have contributed to the variation in results. However, this possibility was thoroughly examined by reviewing the paper in detail to ensure no such oversight.

A second comparison was carried out whereby the model developed in this study was amended to suit the one found in Fridman *et al.* (2023). The full-core neutronic simulation of the second comparison was executed, using Serpent 2.2.0 and the ENDF/B-VII.1 nuclear data library. The difference between the base model and the model simulated by Fridman *et al.* (2023) is as documented in Table 4.13. The difference is 0.00088 (88 pcm), which was very good, and therefore leads to confidence in the model generated, using NWURCS.

Table 4.11: Comparison of base case model and model developed by Sadegh-Noedoost *et al.* (2020).

	Base Case (Serpent Model)	Sadegh-Noedoost Model	Standard Deviation
k-effective	1.17481±0.00015	1.21152±0.00007	0.03671≈3 671 pcm
Temperature	300k	299.85k	

Table 4.12: k-effective results for different core models.

Model Description	k-eff
B003 – with SG and coolant channels in reflector	1.17481±0.00015
C001 – with SG, without coolant channel in the reflector	1.19434±0.00014
D001 – without SG, with coolant channels in the reflector	1.18276±0.00013
E001- without SG, without coolant channels in the reflector	1.19677±0.00014

Table 4.13: Comparison of base case model with model developed by Fridman et al. (2023).

	Base Case	Emil Fridman	Standard Deviation
k-effective value	1.02856±0.00014	1.02768±0.00001	0.00088≈88 pcm

4.6 Comparison of KENO-VI from SCALE 6.2.4 and Serpent

The k-effective results obtained using KENO-VI from SCALE 6.2.4 agree with the calculations conducted using Serpent, with a difference of 0.00075 (75 pcm), as shown in Table 4.14. Both input models were generated using NWURCS. While there are differences in the input file syntax for the two codes, these are appropriately handled by NWURCS. The good agreement between the two codes further strengthens the verification of NWURCS and contributes to the validation of Serpent and KENO-VI through a code-to-code comparison.

Table 4.14: Serpent vs. KENO-VI from SCALE 6.2.4.

	Base Case Serpent	Base Case KENO-VI	Standard Deviation
k-effective value	1.17481±0.00015	1.17556E+00	0.00075 ≈ 75

4.7 Effect of the Material in the Core Barrel, RPV, and Down-comer

Since the difference with the Sadegh model was substantial, various aspects of the model were examined to identify which components could have led to the significant changes mentioned in Section 4.5. The Core Barrel (CB), Reactor Pressure Vessel (RPV), and Down-Comer (DC) surround the reactor core and can influence neutron transport in terms of transmission, absorption, and reflection. To isolate their effects, as a first step, the materials in these components were modelled as a single material. The selected materials were stainless steel, water, and helium, with their densities adjusted to observe their effect on k-effective.

For calculations i080, i070, i040, and i020, the material in the core barrel, RPV, and down-comer was water at different densities, as specified in Table 4.15. The results in Table 4.15 are plotted in Figure 4.5. For calculation i101, the material in these reactor components was helium at a density of 0.001 g/cm³. For calculations i220, i240, and i280, the material was stainless steel at varying densities. The results show that the presence of water or helium in the core barrel, down-comer, and RPV does not significantly influence the multiplication factor. However, the effect of

different stainless steel densities shows a difference of 49 pcm between the first and last densities chosen. It is important to note that this difference is much smaller than the substantial difference observed between the base model and the Sadegh reported value. Therefore, while the specifications of the core barrel, down-comer, and RPV in Sadegh's model may have differed from those used in this study, this discrepancy does not appear to account for the large differences observed.

Table 4.15: Effect of the material in the core barrel, RPV and down-comer.

Calculation Numbers	Material	Density (g/cm ³)	k-effective
i080	H ₂ O	0.10062	1.17386±0.00009
i070	H ₂ O	0.07500	1.17380±0.00009
i040	H ₂ O	0.05031	1.17381±0.00009
i020	H ₂ O	0.02500	1.17389±0.00009
i101	He	0.001	1.17368±0.00009
i210	SS	0.01100	1.17418±0.00009
i220	SS	0.02199	1.17431±0.00009
i240	SS	0.04397	1.17466±0.00009
i280	SS	0.08795	1.17467±0.00009

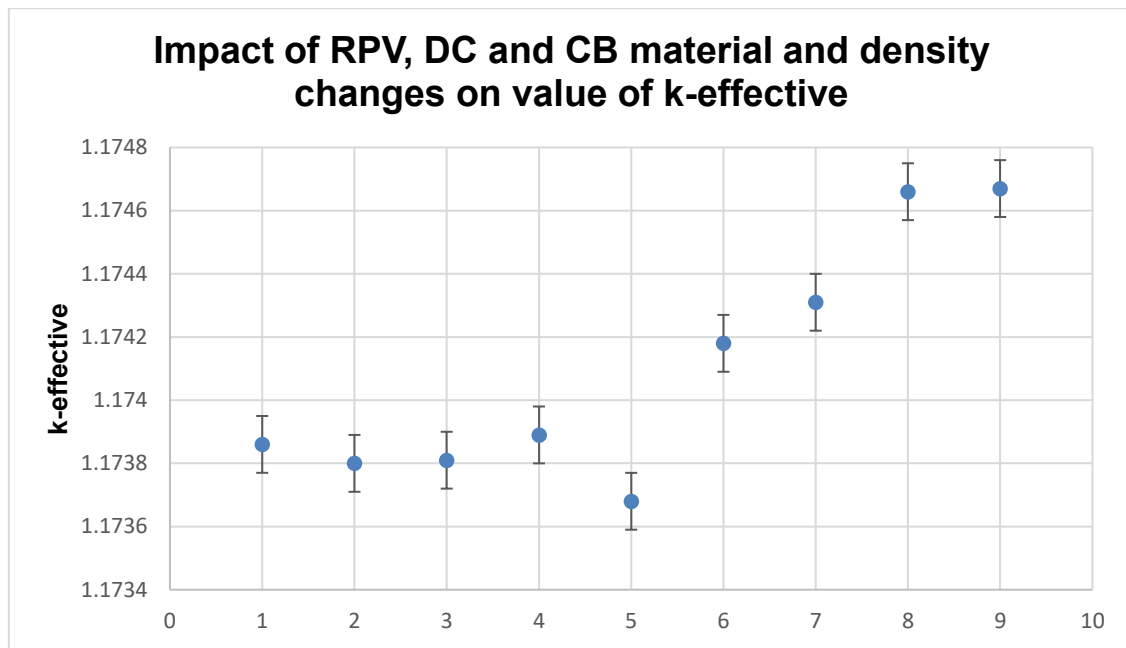


Figure 4.5: Effects of the reactor pressure vessel, down-comer and core barrel.

4.8 Effect of the Material in the Reflector

The reflector is the material positioned between the fuel assemblies and the core barrel, as shown in Figure 4.1B. This reflector is made of stainless steel and contains water channels to cool the reflector material. An exercise was carried out in which the materials in the water channels were changed in terms of density to observe its effect on k-effective. The material in the reflector was stainless steel, and its density was kept constant, while the density of the water was varied. The results are shown in Table 4.16.

One calculation was performed with water replaced by helium gas, and the last two calculations were performed with the water channels replaced by stainless steel, effectively making the reflector solid. This exercise was conducted to assess the effects of the water channels on k-effective. The k-effective results for calculations j020, j101, j240, and j280 show that decreasing the density of the water in the coolant channels, increases the value of k-effective, with a difference of 679 pcm between the lowest and highest densities tested. The test using helium (J101) is equivalent to having a void present in the coolant channel, as helium, with its low density, does not significantly influence k-effective in this model.

As the water density decreases from models J080 to J020, it simulates the state where there is a void in the channels. The behaviour of the multiplication factor shows a smooth trend in Figure 4.6, from the high-density water (J080, point 1 in the figure) to the low-density water (J020, point 4 in the figure) and to the void case (J101, point 5 in the figure). It is important to note that helium was used for case J101, because the code would not work unless a material was specified (true voids cannot be modelled in Serpent). Neutrons in water can undergo scattering or absorption with hydrogen (Lamarsh & Baratta, 2001). k-effective will increase if scattering dominates, as more moderation of the neutrons will occur, while k-effective will decrease if absorption dominates. There is, therefore, competition between elastic scattering and absorption of neutrons by hydrogen. Since k-effective increases in Figure 4.6 as the water density decreases, it can be inferred that elastic scattering dominates as the water density decreases.

An important implication of these results should be considered in terms of an accident scenario where the moderator temperature increases, leading to a decrease in density. In such accidents, the void fraction, rather than the density of water, should be considered, since the water (fluid) would be in a "two-phase" state. As the temperature increases, the amount of vapour will increase, resulting in an increase in the void fraction and a decrease in the mixture density of water. The neutronic calculation only requires the density of the water. However, it should also be considered that the Doppler coefficient would be negative when the core temperature rises (the Doppler coefficient is a reactivity parameter related to an increase in fuel temperature, as

discussed in Section 2.5.2). Therefore, a coupled neutronic-thermal hydraulic calculation should be performed to obtain the net effect of the reactivity coefficient. Such an analysis is recommended for further study, as the scope of the current study focuses only on the neutronic analysis.

In terms of using stainless steel in the coolant channels, it can be assumed that the effect is not significant for the different densities chosen, as the differences between the values fall within the statistical errors shown in Figure 4.6. It is noted that the difference in the multiplication factor between the highest water density (J080, point 1 in Figure 4.6) and the highest stainless steel density (J280, point 7 in Figure 4.6) is 1 218 pcm, which is significant. This is less than half the difference between the base model and the Sadegh model, which is 3 671 pcm. Therefore, the differences in the material specifications of the coolant channels in the models could contribute to the variation in the multiplication factor.

Table 4.16: k-effective acquired from changing material/density of the reflector.

Calculation Numbers	Material	Density (g/cm ³)	K-effective
J080	Water	0.10062	1.17383±0.00009
J070	Water	0.07500	1.17565±0.00009
J040	Water	0.05031	1.17772±0.00009
J020	Water	0.02516	1.18062±0.00009
J101	HE	0.00001	1.18588±0.00009
J240	SS	0.04397	1.18514±0.00009
J280	SS	0.08794	1.18601±0.00009

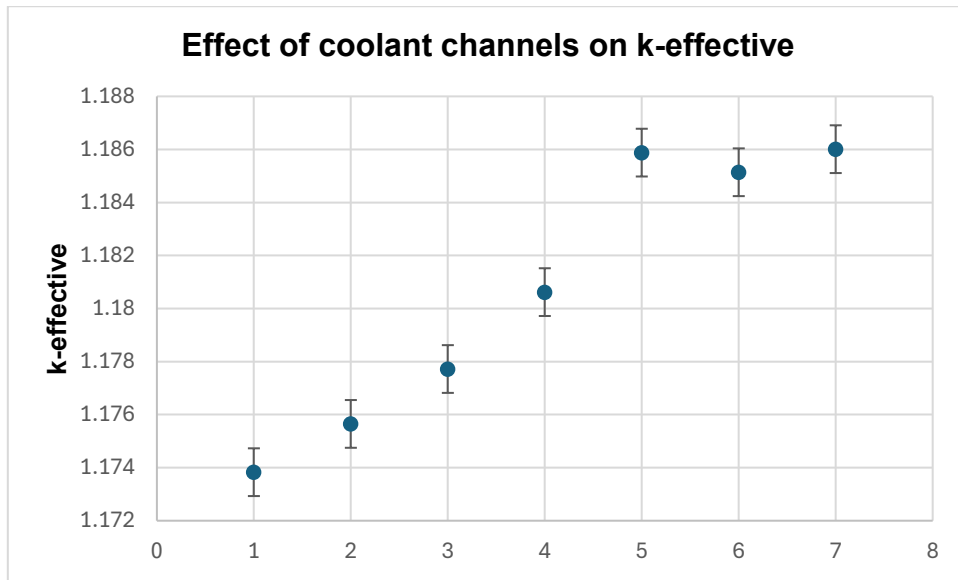


Figure 4.6: Effects of coolant channels on k-effective.

4.9 Neutron Flux Results

The results obtained from the neutron flux calculations are discussed in this section. The calculations were performed for both a full-core and a single fuel assembly (for verification purposes), as outlined in Section 3.7. The neutron flux plots for the verification process and the full-core are shown in Figures 4.7 and 4.8, respectively. The verification plot is used to justify the shape of the neutron flux distribution observed in the full core results. Figure 4.7 illustrates a flux distribution in the z -direction, characterized by a single peak at the centre ($z = 5$) and a lower flux near the edges. This distribution aligns with the expected cosine-like flux shapes, which are commonly observed in bare reactor cores. These shapes arise due to boundary conditions, with zero flux at the edges and maximum flux at the centre, following a cosine function mathematically. The shape of the neutron flux plot reflects the loss of neutrons at the edge of the reactor, which significantly influences the shape of the distribution. As neutrons approach the reactor boundary, they interact with the materials at the edge and are either absorbed or escape the system entirely. This results in a sharp decrease in neutron flux near the boundaries, as neutrons that leave the core do not return.

The flux shape in Figure 4.7 is comparable to the flux distribution in the region without control rods, as depicted in Figure 4.9. In Figure 4.9, the region without control rods ($h = H$) exhibits a cosine-like flux distribution. The flux peaks near the middle of this region, where neutron moderation and fission occur most effectively, as there are fewer neutron-absorbing materials present.

The neutron flux results shown in Figure 4.8 were extracted at the centre of the reactor core, along the x -direction, with $y = 4$ and $z = 10$. The x - y plane represents the radial plane, while z denotes the axial direction. In homogeneous reactors that are just critical, the neutron flux typically exhibits the cosine-like shape, shown in Figure 4.10. Figure 4.10 (adapted from Lamarsh & Baratta (2001)) shows the flux in radial plane. The case with "rod out" shows the expected sinusoidal shape, however, this was not the case for the neutron flux of the base model due to the presence of different types of fuel assemblies in the reactor core. This variation can be attributed to differences in the material composition of the fuel, as detailed in Section 3.4. Although not as severe as the case of "rod in" of Figure 4.10, the curve corresponding to "rod in" can be used to interpret the flux shape of Figure 4.8. It is noted however, that control rods and burnable poisons have different functions in a reactor core.

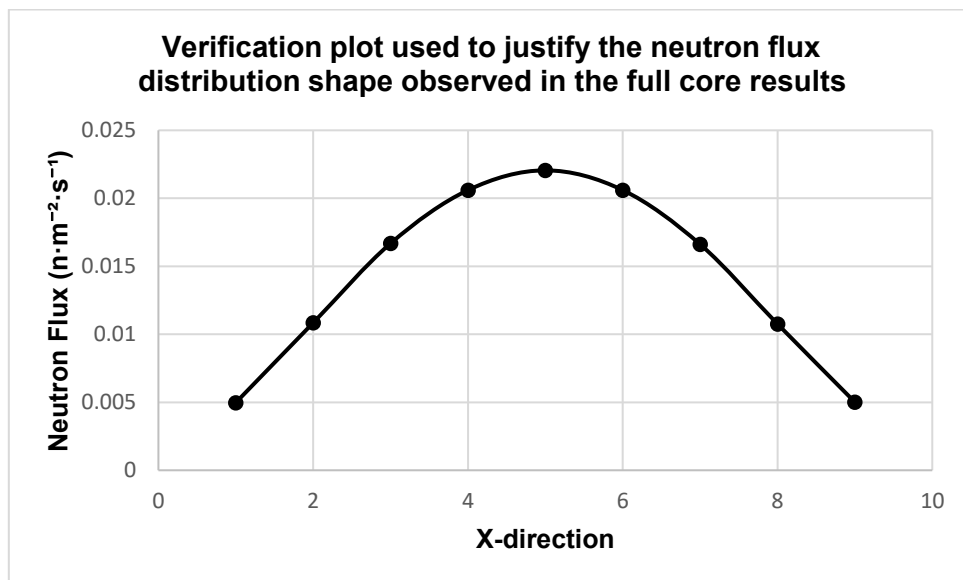


Figure 4.7: Verification of neutron flux

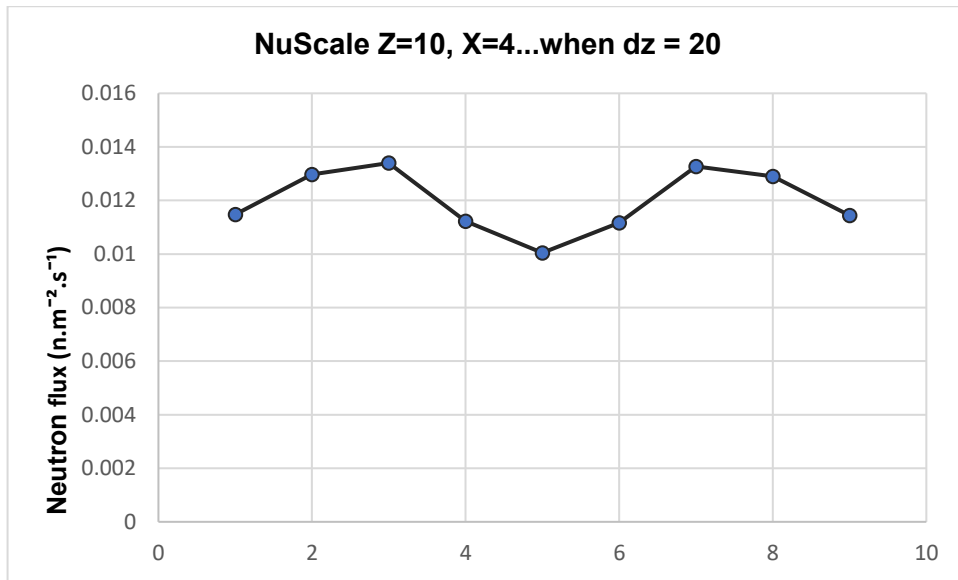


Figure 4.8: Neutron flux for full core.

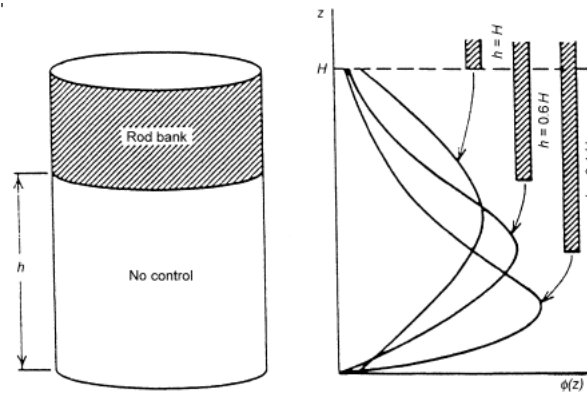


Figure 4.9: Insertion of control rod bank in a bare cylindrical core (Stacey, 2007).

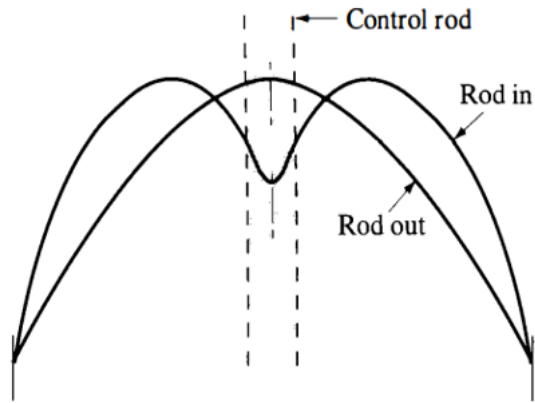


Figure 4.10: Flux in a bare reactor with and without a central control rod (Lamarsh & Baratta, 2001).

4.10 Results of the Safety Parameters

The safety parameters addressed in this study, are the moderator temperature coefficient and the fuel temperature coefficient. Figure 4.11 presents a plot of k -effective values for six different moderator temperatures, and the steps for calculating the moderator temperature coefficient and its associated error bars are detailed in Appendices D and F, respectively. The results indicate that the moderator temperature coefficient is negative for all tested temperatures. Additionally, the magnitude of these negative values increases with higher moderator temperatures, meaning that as the moderator temperature rises, the coefficient becomes more negative, but not excessively so. According to the primary criteria for PWR core design, as outlined in Table 2 of Sadegh-Noedoost *et al.* (2020), the moderator temperature coefficient should be less than zero and more negative than the Doppler coefficient, particularly at hot full power. The results confirm that the moderator temperature coefficient remains below zero, across all six temperatures and is indeed more negative than the Doppler coefficient, as shown in Tables 4.17 and 4.18.

Table 4.17: k-effective at different moderator temperatures.

Calculation Number	Temperature of the Moderator	k-effective	Moderator Temperature Coefficient (pcm/°C)	Error Bar
1	300K	1.17447±0.00009		
2	301K	1.17429±0.00009	-13,051	1.847
3	302K	1.17402±0.00009	-16,318	1.847
4	305K	1.17392±0.00009	-7,9783	1.847
5	310K	1.17353±0.00009	-6.820	1.847
6	315K	1.17318±0.00009	-6.2415	1.847

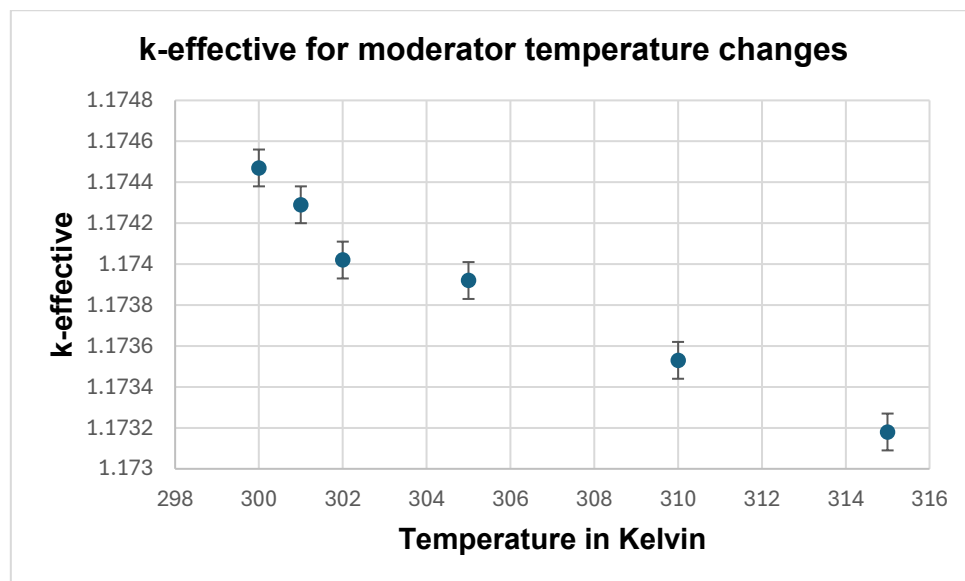


Figure 4.11: Relation between moderator temperature and k-effective.

In comparison to the design values of the AP1000 reactor, which range from 0 pcm/°C to -72 pcm/°C for the moderator temperature coefficient (Pinem *et al.* 2018), all calculation results from this study fall within the AP1000 design range. According to Da Silva *et al.* (2017) and

Edenius (1976), typical values of the moderator temperature coefficient for pressurized water reactors range between -5 and -30 pcm/°C. Additionally, for Cycle 3 of the Indian Point Nuclear Power Plant, the moderator temperature coefficient ranged from -35 to 0 pcm/°F (Skaritka, 1982). This confirms that the calculated moderator temperature coefficients in this study are within the typical range for a PWR.

Furthermore, the moderator temperature coefficient for the end of cycle life, with all control rods withdrawn or operating at rated thermal power, for the R.E. Ginna Nuclear Power Plant core is required to be less negative than -42.9 pcm/°F, as specified in the Core Operating Limits Report (COLR) for Cycle 43, Revision 1. All calculations from this study comply with this specification.

Similarly to the moderator temperature coefficient, the Doppler coefficient is also expressed in units of pcm/°C. The Doppler coefficient is critical for reactor stability and is considered even more important than the moderator temperature coefficient. In the event of Reactivity-Initiated Accidents (RIAs), the fuel temperature coefficient is the first and most crucial feedback mechanism, compensating for the insertion of positive reactivity almost instantaneously. While heat transfer to the moderator occurs over seconds, the fuel temperature coefficient acts immediately. As stated by nuclear-power.com (n.d.), in power reactors using low-enriched fuel (e.g. PWRs and BWRs with 3 – 5 % ^{235}U enrichment), the Doppler coefficient is always negative. The k-effective results presented in Table 4.18 confirm that k-effective decreases as the fuel temperature increases (see Figure 4.12). The methodology for calculating the fuel temperature coefficient and associated error bars is detailed in Appendices E and G, respectively.

According to Rokhmadi, *et al.* (2016), this decrease is due to the increased resonance absorption in the slowing-down region. Akyurek (2018) also notes that ^{238}U resonances broaden with increasing temperature, resulting in greater neutron absorption. As Light Water Reactors (LWRs) typically use low-enriched uranium fuel containing a significant amount of ^{238}U , a higher degree of Doppler broadening is expected. The negative Doppler coefficient ensures safe reactor operation. For the base model, the fuel temperature coefficient remains negative throughout (as shown in Table 23), leading to a reduction in reactivity and contributing to the overall safety of the core.

Table 4.18: k-effective at different fuel temperatures.

Calculation Number	Temperature of the Fuel	K-effective	Fuel Temperature Coefficient (pcm/°C)	Error Bars
1	300k	1.17501±0.00009		
2	301K	1.17499±0.00009	-1.4486	1.847
3	302K	1.17478±0.00009	-1.0867	1.847
4	305K	1.17467±0.00009	-4.9266	1.847
5	310K	1.17439±0.00009	-4.493	1.847
6	315K	1.17437±0.00009	-3.092	1.847

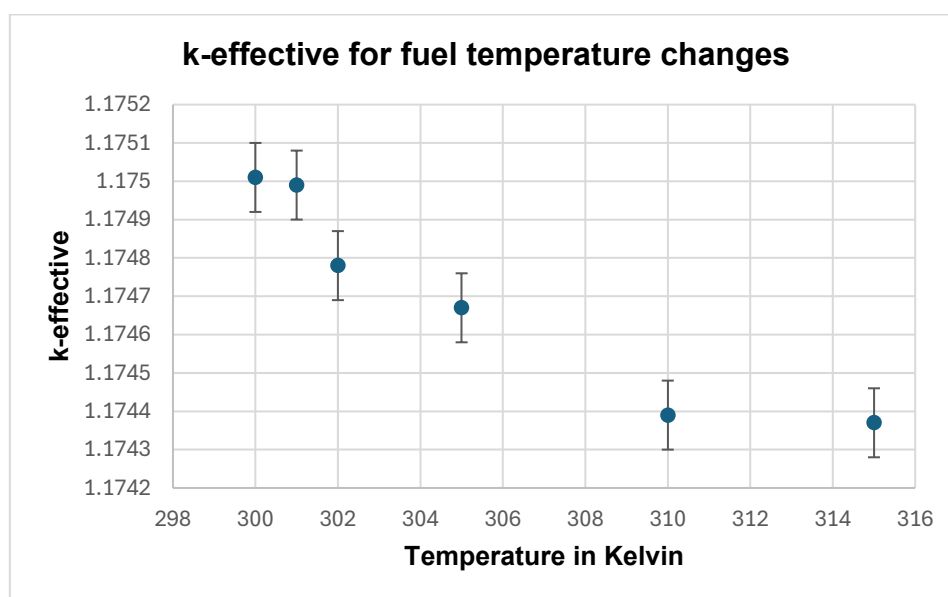


Figure 4.12: Relation between fuel temperature and k-effective.

5. CONCLUSIONS

This work focused on the neutronic analysis (criticality calculation and two safety parameters) of the NuScale full-core and fuel assembly model using Serpent, SCALE, and the ENDF/B-VII library. The results show that the multiplication factors for the full-core and fuel assembly are 1.17481 ± 0.00015 and 1.13991 ± 0.00014 , respectively. It can be concluded that reflective boundary conditions, gadolinium content (which acts as a burnable absorber), and higher fuel enrichment affect the value of k-effective as expected. This helps verify the model generated by NWURCS.

The study also demonstrated that a decrease in water density in the coolant channels increases the k-effective. This could occur under accident conditions when the void coefficient increases (density decreases). At first glance, this reactor state may seem undesirable. However, one must also consider the Doppler coefficient and the thermal feedback from the fuel to the coolant channel in the reflector. Such neutronic-thermal hydraulics coupled studies were beyond the scope of this work and are reserved for future research.

The criticality results obtained, using SCALE, agree with those obtained using Serpent, confirming a successful code-to-code comparison. The model created in this study also aligns with the model by Fridman *et al.* (2023). The analysis performed in this study positively contributes to the verification of NWURCS.

The distribution of the neutron flux in the full core did not align with the expected cosine-like flux shapes, which are commonly observed in bare reactor cores. This deviation is due to the presence of different types of fuel assemblies in the reactor core, which introduce variations in the material composition of the fuel

The moderator temperature coefficient falls within the typical values (range between -5 and -30 pcm/°C) for a pressurized water reactor. The moderator temperature coefficient was found to be negative; more negative than the Doppler coefficient.

In addition to the study on the void coefficient associated with the coolant channels in the reflector, further studies are recommended to analyse the remaining safety parameters for the NuScale power module, such as:

- The boron reactivity coefficient and concentration (for pressurized water reactors);
- The shutdown margin;
- The maximum reactivity insertion rate;

- The control rod worth and control bank worth;
- The radial and axial power peaking factors, including allowance for xenon-induced oscillations;
- The maximum linear heat generation rate; and
- The void coefficient of reactivity in general (not specific to that of the coolant channels in the reflector).

6. REFERENCES

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**APPENDIX A: ISOTOPIC COMPOSITION FOR CLADDING,
REFLECTOR, INSTRUMENTATION TUBES, GUIDE
TUBES, AND RPV**

Cladding: M5	
Nuclide	Number Density (Unit)
8016	7.0873E-03
8017	2.6942E-06
40090	5.0583E-01
40091	1.1031E-01
40092	1.6861E-01
40094	1.7087E-01
40096	2.7528E-02
41093	9.7630E-03
Reflector /RPV: SS_304	
Nuclide	ND
14028	1.0836E-02
14029	5.5048E-04
14030	3.6331E-04
24050	8.7317E-03
24052	1.6838E-01
24053	1.9093E-02
24054	4.7527E-03
25055	2.0020E-02
26054	3.9370E-02
26056	6.1803E-01
26057	1.4273E-02
26058	1.8995E-03
28058	6.3788E-02
28060	2.4571E-02
28061	1.0681E-03

28062	3.4055E-03
28064	8.6728E-04
Instrumentation Tubes/Guide Tubes: Zircaloy-4	
Nuclide	ND
8016	7.0773E-03
8017	2.6904E-06
24050	7.5864E-05
24052	1.4630E-03
24053	1.6589E-04
24054	4.1293E-05
26054	1.9961E-04
26056	3.1334E-03
26057	7.2364E-05
26058	9.6303E-06
40090	5.0250E-01
40091	1.0958E-01
40092	1.6750E-01
40094	1.6974E-01
40096	2.7347E-02
50112	1.0758E-04
50114	7.3201E-05
50115	3.7709E-05
50116	1.6126E-03
50117	8.5179E-04
50118	2.6862E-03
50119	9.5272E-04
50120	3.6135E-03
50122	5.1351E-04
50124	6.4217E-04

APPENDIX B: ISOTOPIC COMPOSITION OF THE SPACER GRIDS AS MODELLED IN THIS STUDY

HMP_Inconel	
Nuclide	ND (Unit)
14028	6.4648E-03
14029	3.2842E-04
14030	2.1675E-04
24050	8.9124E-03
24052	1.7187E-01
24053	1.9488E-02
24054	4.8510E-03
25055	8.9099E-03
26054	1.6856E-02
26056	2.6460E-01
26057	6.1107E-03
26058	8.1322E-04
28058	3.3398E-01
28060	1.2865E-01
28061	5.5922E-03
28062	1.7830E-02
28064	4.5409E-03
Zircaloy-4	
Nuclide	ND
8016	7.0773E-03
8017	2.6904E-06
24050	7.5864E-05
24052	1.4630E-03
24053	1.6589E-04
24054	4.1293E-05
26054	1.9961E-04

26056	3.1334E-03
26057	7.2364E-05
26058	9.6303E-06
40090	5.0250E-01
40091	1.0958E-01
40092	1.6750E-01
40094	1.6974E-01
40096	2.7347E-02
50112	1.0758E-04
50114	7.3201E-05
50115	3.7709E-05
50116	1.6126E-03
50117	8.5179E-04
50118	2.6862E-03
50119	9.5272E-04
50120	3.6135E-03
50122	5.1351E-04
50124	6.4217E-04

**APPENDIX C: ISOTOPIC COMPOSITION FOR THE FUEL MATERIAL
(SOURCE (SADEGH-NOEDOOST *et al.* 2020))**

UO ₂ , w/o 1.87	
Nuclide	ND (Unit)
8016	66.3116E-03
8017	2.5333E-04
92235	6.3116E-03
92238	3.2702E-01
UO ₂ , w/o 4.33	
Nuclide	ND
8016	6.6641E-01
8017	2.5333E-04
92235	1.4610E-02
92238	3.1872E-01
UO ₂ , w/o 4.55	
Nuclide	ND
8016	6.6641E-01
8017	2.5333E-04
92235	1.5318E-02
92238	3.1802E-01
UO ₂ , w/o 4.32 and 2 % Gadolinium	
Nuclide	ND
8016	6.6477E-01
8017	2.5271E-04
64152	1.9761E-05
64154	2.1539E-04
64155	1.4623E-03
64156	2.0225E-03

64157	1.5463E-03
64158	2.4543E-03
64160	2.1598E-03
92235	1.4216E-02
92238	3.1088E-01
UO₂, w/o 4.30 and 6 % Gadolinium	
Nuclide	ND
8016	6.6152E-01
8017	2.5147E-04
64152	5.8719E-05
64154	6.4004E-04
64155	4.3452E-03
64156	6.0099E-03
64157	4.5948E-03
64158	7.2929E-03
64160	6.4180E-03
92235	1.3444E-02
92238	2.9542E-01
UO₂, w/o 4.29 and 8 % Gadolinium	
Nuclide	ND
8016	6.5992E-01
8017	2.5087E-04
64152	7.7922E-05
64154	8.4935E-04
64155	5.7662E-03
64156	7.9753E-03
64157	6.0974E-03
64158	9.6779E-03
64160	8.5169E-03
92235	1.3065E-02
92238	2.8780E-01

UO ₂ , w/o 4.55	
Nuclide	ND
8016	6.6641E-01
8017	2.5333E-04
92235	1.5352E-02
92238	3.1798E-01

APPENDIX D: CALCULATIONS FOR THE MODERATOR TEMPERATURE COEFFICIENT

The equation used in this calculation is:

$$1. \quad a_c = \frac{1}{K_2 \cdot K_1} \cdot \frac{K_2 - K_1}{T_{M2} - T_{M1}} \quad (2.11)$$

$$a_c = \frac{1}{1.17429 \times 1.17447} \cdot \frac{1.17429 - 1.17447}{301 - 300}$$

$$a_c = \frac{1}{1.17429 \times 1.17447} \cdot \frac{1.17429 - 1.17447}{301 - 300}$$

$$= 0.72507 \times (-0.00018)$$

$$= -0.00013051$$

$$= -1,3051E - 04$$

$$= \mathbf{-13,051 \text{ PCM}/^{\circ}\text{C}}$$

$$2. \quad a_c = \frac{1}{1.17402 \times 1.17447} \cdot \frac{1.17402 - 1.17447}{302 - 300}$$

$$a_c = \frac{1}{1.17402 \times 1.17447} \cdot \frac{1.17402 - 1.17447}{302 - 300}$$

$$= 0.72524 \times (-0.000225)$$

$$= -0.00016318$$

$$= -16,318E - 05$$

$$= \mathbf{-16,318 \text{ PCM}/^{\circ}\text{C}}$$

$$3. \quad a_c = \frac{1}{1.17392 \times 1.17447} \cdot \frac{1.17392 - 1.17447}{305 - 300}$$

$$a_c = \frac{1}{1.17392 \times 1.17447} \cdot \frac{1.17392 - 1.17447}{305 - 300}$$

$$= 0.72530 \times (-0.00011)$$

$$= -0.000079783$$

$$= -79,783E - 06$$

$$= \mathbf{-7,9783 \text{ PCM}/^{\circ}\text{C}}$$

$$4. \quad a_c = \frac{1}{1.17353 \times 1.17447} \cdot \frac{1.17353 - 1.17447}{310 - 300}$$

$$a_c = \frac{1}{1.17353 \times 1.17447} \cdot \frac{1.17353 - 1.17447}{310 - 300}$$

$$= 0.72554 \times (-0.000094)$$

$$= -0.000068200$$

$$= -68.200E - 06$$

$$= \mathbf{-6.820 \text{ PCM}/^{\circ}\text{C}}$$

5.

$$\begin{aligned}a_c &= \frac{1}{1.17318 \times 1.17447} \cdot \frac{1.17318 - 1.17447}{315 - 300} \\a_c &= \frac{1}{1.17318 \times 1.17447} \cdot \frac{1.17318 - 1.17447}{315 - 300} \\&= 0.72576 \times (-0.000086) \\&= -0.000062415 \\&= -62.415E - 06 \\&= \mathbf{-6.2415 \text{ PCM/}^\circ\text{C}}\end{aligned}$$

APPENDIX E: CALCULATIONS FOR FUEL TEMPERATURE COEFFICIENT

The equation used in these calculations is:

(2.13)

$$D_c = \frac{1}{K_2 \cdot K_1} \cdot \frac{K_2 - K_1}{T_{F2} - T_{F1}}$$

$$\begin{aligned} 1. \quad D_c &= \frac{1}{1.17499 \times 1.17501} \cdot \frac{1.17499 - 1.17501}{301 - 300} \\ &= 0.72431 \times -0.00002 \\ &= -0.000014486 \\ &= -1.4486E - 5 \\ &= \mathbf{-1.4486 \text{ PCM}/^{\circ}\text{C}} \end{aligned}$$

$$\begin{aligned} 2. \quad D_c &= \frac{1}{1.17478 \times 1.17501} \cdot \frac{1.17478 - 1.17501}{302 - 300} \\ &= 0.72444 \times -0.00015 \\ &= -0.000108666 \\ &= -1.08666E - 4 \\ &= \mathbf{-1.08666 \text{ PCM}/^{\circ}\text{C}} \end{aligned}$$

$$\begin{aligned} 3. \quad D_c &= \frac{1}{1.17467 \times 1.17501} \cdot \frac{1.17467 - 1.17501}{305 - 300} \\ &= 0.7245069 \times -0.000068 \\ &= -0.0000492664 \\ &= -4.92664E - 4 \\ &= \mathbf{-4.92664 \text{ PCM}/^{\circ}\text{C}} \end{aligned}$$

$$\begin{aligned}
4. \quad D_c &= \frac{1}{1.17439 \times 1.17501} \cdot \frac{1.17439 - 1.17501}{310 - 300} \\
&= 0.7246796 \times -0.000062 \\
&= -0.00004493 \\
&= -4.493E - 5 \\
&= \mathbf{-4.493 \text{ PCM}/^{\circ}\text{C}}
\end{aligned}$$

$$\begin{aligned}
5. \quad D_c &= \frac{1}{1.17437 \times 1.17501} \cdot \frac{1.17437 - 1.17501}{315 - 300} \\
&= 0.7246920 \times -0.00004266666 \\
&= -0.000030920 \\
&= -3.0920E - 5 \\
&= \mathbf{-3.0920 \text{ PCM}/^{\circ}\text{C}}
\end{aligned}$$

APPENDIX F: ERROR BAR CALCULATIONS FOR THE MODERATOR TEMPERATURE COEFFICIENTS

Three general error formulae are used (Taylor, Introduction to Error Analysis)

$$A = B \pm C \rightarrow \sigma A = \sqrt{(\sigma B)^2 + (\sigma C)^2} \quad (1)$$

$$A = BC \rightarrow \frac{\sigma A}{A} = \sqrt{\left(\frac{\sigma B}{B}\right)^2 + \left(\frac{\sigma C}{C}\right)^2} \quad (2)$$

$$A = \frac{BD}{CE} \rightarrow \frac{\sigma A}{A} = \sqrt{\left(\frac{\sigma B}{B}\right)^2 + \left(\frac{\sigma D}{D}\right)^2 + \left(\frac{\sigma C}{C}\right)^2 + \left(\frac{\sigma E}{E}\right)^2} \quad (3)$$

We are given

$$D_c = \frac{1}{K_2 \cdot K_1} \cdot \frac{K_2 - K_1}{T_{M2} - T_{M1}} \quad (4)$$

with:

$$K_2 = 1.00561 \pm 0.00031$$

$$K_1 = 1.00089 \pm 0.00050$$

$$TF_2 = 300 \pm 0.1$$

$$TF_1 = 305 \pm 0.1$$

All of the above are given.

The error in the temperature is estimated, and the error in K is from the Serpent calculation.

$$\text{Let } E = \frac{1}{K_2 \cdot K_1} \quad (5)$$

$$\text{And } F = \frac{K_2 - K_1}{T_{M2} - T_{M1}} \quad (6)$$

Then Eq. (4) becomes:

$$D = E F \quad (7)$$

We use Eq. (2) to get

$$\frac{\sigma D}{D} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \quad (8)$$

In Eq. (7), we can calculate D , E , and F . We want σD .

But we don't have σE and σF .

From Eq. (5):

$$E = \frac{1}{K_2 K_1} \quad (5)$$

Using Eq. (3), the error of Eq. (5) becomes

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{\sigma K_2}{K_2}\right)^2 + \left(\frac{\sigma K_1}{K_1}\right)^2} \quad (9)$$

Consider now F :

$$F = \frac{K_2 - K_1}{T_{M2} - T_{M1}}$$

This can be written as

$$F = \frac{G}{H} \quad (10)$$

where:

$$G = K_2 - K_1 \quad (11)$$

$$H = T_{M2} - T_{M1}$$

Therefore, using Eq. (3) we get:

$$\frac{\sigma F}{F} = \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2} \quad (13)$$

We can calculate G and H , but we need σG and σH .

Using Eq. (1), the error equation for 11 is

$$\sigma G = \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2} \quad (14)$$

Similarly, we get:

$$\sigma H = \sqrt{(\sigma T_{M2})^2 + (\sigma T_{M1})^2} \quad (15)$$

Summary:

1. Calculate $\sigma G, \sigma H$ using (14,15);
2. Substitute into Eq. (13) to get $\frac{\sigma F}{F}$;
3. Calculate $\frac{\sigma E}{E}$ using Eq. (9); and
4. Substitute these two into 8, to get $\frac{\sigma D}{D}$.

Note that D, E, F, G and H are calculated from the K 's and T 's.

$$\begin{aligned}
 1. \quad \sigma G &= \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2} \\
 &= \sqrt{(1.17429)^2 + (1.17447)^2} \\
 &= \mathbf{1.598}
 \end{aligned}$$

$$\begin{aligned}
 \sigma H &= \sqrt{(\sigma T_{F2})^2 + (\sigma T_{F1})^2} \\
 &= \sqrt{(301)^2 + (300)^2} \\
 &= \mathbf{301.50}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\sigma F}{F} &= \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2} \tag{13} \\
 &= \sqrt{\left(\frac{1.598}{1.598}\right)^2 + \left(\frac{301.50}{301.50}\right)^2} \\
 &= \mathbf{1.414}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\sigma E}{E} &= \sqrt{\left(\frac{1.17429}{1.17429}\right)^2 + \left(\frac{1.17447}{1.17447}\right)^2} \tag{9} \\
 &= \mathbf{1.414}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\sigma D}{D} &= \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \tag{8} \\
 &= \mathbf{1.847}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \sigma G &= \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2} \\
 &= \sqrt{(1.17402)^2 + (1.17447)^2} \\
 &= \mathbf{1.598}
 \end{aligned}$$

$$\sigma H = \sqrt{(\sigma T_{F2})^2 + (\sigma T_{F1})^2}$$

$$= \sqrt{(302)^2 + (300)^2}$$

$$= \mathbf{302.50}$$

$$\frac{\sigma F}{F} = \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2}$$

$$= \sqrt{\left(\frac{1.598}{1.598}\right)^2 + \left(\frac{302.50}{302.50}\right)^2}$$

$$= \mathbf{1.414}$$

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{1.17402}{1.17402}\right)^2 + \left(\frac{1.17447}{1.17447}\right)^2} \quad (9)$$

$$= \mathbf{1.414}$$

$$\frac{\sigma D}{D} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \quad (8)$$

$$= \mathbf{1.847}$$

$$3. \quad \sigma G = \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2}$$

$$= \sqrt{(1.17392)^2 + (1.17447)^2}$$

$$= \mathbf{1.598}$$

$$\sigma H = \sqrt{(\sigma T_{F2})^2 + (\sigma T_{F1})^2}$$

$$= \sqrt{(305)^2 + (300)^2}$$

$$= \mathbf{305.50}$$

$$\frac{\sigma F}{F} = \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2}$$

$$= \sqrt{\left(\frac{1.598}{1.598}\right)^2 + \left(\frac{305.50}{305.50}\right)^2}$$

$$= \mathbf{1.414}$$

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{1.17392}{1.17392}\right)^2 + \left(\frac{1.17447}{1.17447}\right)^2} \quad (9)$$

$$= \mathbf{1.414}$$

$$\frac{\sigma D}{D} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \quad (8)$$

$$= \mathbf{1.847}$$

$$\begin{aligned} 4. \quad \sigma G &= \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2} \\ &= \sqrt{(1.17353)^2 + (1.17447)^2} \\ &= \mathbf{1.598} \end{aligned}$$

$$\begin{aligned} \sigma H &= \sqrt{(\sigma T_{F2})^2 + (\sigma T_{F1})^2} \\ &= \sqrt{(310)^2 + (300)^2} \\ &= \mathbf{310.48} \end{aligned}$$

$$\begin{aligned} \frac{\sigma F}{F} &= \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2} \\ &= \sqrt{\left(\frac{1.598}{1.598}\right)^2 + \left(\frac{310.48}{310.48}\right)^2} \\ &= \mathbf{1.414} \end{aligned}$$

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{1.17353}{1.17353}\right)^2 + \left(\frac{1.17447}{1.17447}\right)^2} \quad (9)$$

$$= \mathbf{1.414}$$

$$\frac{\sigma D}{D} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \quad (8)$$

$$= \mathbf{1.847}$$

$$\begin{aligned} 5. \quad \sigma G &= \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2} \\ &= \sqrt{(1.17318)^2 + (1.17447)^2} \\ &= \mathbf{1.598} \end{aligned}$$

$$\begin{aligned} \sigma H &= \sqrt{(\sigma T_{F2})^2 + (\sigma T_{F1})^2} \\ &= \sqrt{(315)^2 + (300)^2} \end{aligned}$$

$$= \mathbf{315.48}$$

$$\frac{\sigma F}{F} = \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2}$$

$$= \sqrt{\left(\frac{1.598}{1.598}\right)^2 + \left(\frac{315.48}{315.48}\right)^2}$$

$$= \mathbf{1.414}$$

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{1.17318}{1.17318}\right)^2 + \left(\frac{1.17447}{1.17447}\right)^2} \quad (9)$$

$$= \mathbf{1.414}$$

$$\frac{\sigma D}{D} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \quad (8)$$

$$= \mathbf{1.847}$$

APPENDIX G: ERROR BAR CALCULATION FOR THE FUEL TEMPERATURE COEFFICIENTS

We are given:

$$a_c = \frac{1}{K_2 \cdot K_1} \cdot \frac{K_2 - K_1}{T_{F2} - T_{F1}}$$

Using the equation already stated in Appendix E

1. The following equations were derived:

$$\sigma G = \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2}$$

$$\frac{\sigma F}{F} = \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2}$$

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{\sigma K_2}{K_2}\right)^2 + \left(\frac{\sigma K_1}{K_1}\right)^2}$$

$$\frac{\sigma a}{a} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2}$$

$$\begin{aligned} 1. \quad \sigma G &= \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2} \\ &= \sqrt{(1.17499)^2 + (1.17501)^2} \\ &= \mathbf{1.598} \end{aligned}$$

$$\begin{aligned} \sigma H &= \sqrt{(\sigma T_{F2})^2 + (\sigma T_{F1})^2} \\ &= \sqrt{(301)^2 + (300)^2} \\ &= \mathbf{301.48} \end{aligned}$$

$$\begin{aligned} \frac{\sigma F}{F} &= \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2} \\ &= \sqrt{\left(\frac{1.598}{1.598}\right)^2 + \left(\frac{301.48}{301.48}\right)^2} \\ &= \mathbf{1.414} \end{aligned}$$

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{1.17499}{1.17499}\right)^2 + \left(\frac{1.17501}{1.17501}\right)^2} \quad (9)$$

$$= \mathbf{1.414}$$

$$\frac{\sigma a}{a} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \quad (8)$$

$$= \mathbf{1.847}$$

$$2. \quad \sigma G = \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2}$$

$$= \sqrt{(1.17478)^2 + (1.17501)^2}$$

$$= \mathbf{1.598}$$

$$\sigma H = \sqrt{(\sigma T_{F2})^2 + (\sigma T_{F1})^2}$$

$$= \sqrt{(302)^2 + (300)^2}$$

$$= \mathbf{302.48}$$

$$\frac{\sigma F}{F} = \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2}$$

$$= \sqrt{\left(\frac{1.598}{1.598}\right)^2 + \left(\frac{302.48}{302.48}\right)^2}$$

$$= \mathbf{1.414}$$

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{1.17478}{1.17478}\right)^2 + \left(\frac{1.17501}{1.17501}\right)^2} \quad (9)$$

$$= \mathbf{1.414}$$

$$\frac{\sigma a}{a} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \quad (8)$$

$$= \mathbf{1.847}$$

$$3. \quad \sigma G = \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2}$$

$$= \sqrt{(1.17467)^2 + (1.17501)^2}$$

$$= \mathbf{1.598}$$

$$\sigma H = \sqrt{(\sigma T_{F2})^2 + (\sigma T_{F1})^2}$$

$$= \sqrt{(305)^2 + (300)^2}$$

$$= \mathbf{305.49}$$

$$\frac{\sigma F}{F} = \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2}$$

$$= \sqrt{\left(\frac{1.598}{1.598}\right)^2 + \left(\frac{305.49}{305.49}\right)^2}$$

$$= \mathbf{1.414}$$

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{1.17467}{1.17467}\right)^2 + \left(\frac{1.17501}{1.17501}\right)^2} \quad (9)$$

$$= \mathbf{1.414}$$

$$\frac{\sigma a}{a} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \quad (8)$$

$$= \mathbf{1.847}$$

$$4. \quad \sigma G = \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2}$$

$$= \sqrt{(1.17439)^2 + (1.17501)^2}$$

$$= \mathbf{1.598}$$

$$\sigma H = \sqrt{(\sigma T_{F2})^2 + (\sigma T_{F1})^2}$$

$$= \sqrt{(310)^2 + (300)^2}$$

$$= \mathbf{310.48}$$

$$\frac{\sigma F}{F} = \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2}$$

$$= \sqrt{\left(\frac{1.598}{1.598}\right)^2 + \left(\frac{310.48}{310.48}\right)^2}$$

$$= \mathbf{1.414}$$

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{1.17439}{1.17439}\right)^2 + \left(\frac{1.17501}{1.17501}\right)^2} \quad (9)$$

$$= \mathbf{1.414}$$

$$\frac{\sigma a}{a} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \quad (8)$$

$$= \mathbf{1.847}$$

$$\begin{aligned} 5. \sigma G &= \sqrt{(\sigma K_2)^2 + (\sigma K_1)^2} \\ &= \sqrt{(1.17437)^2 + (1.17501)^2} \\ &= \mathbf{1.598} \end{aligned}$$

$$\begin{aligned} \sigma H &= \sqrt{(\sigma T_{F2})^2 + (\sigma T_{F1})^2} \\ &= \sqrt{(315)^2 + (300)^2} \\ &= \mathbf{315.48} \end{aligned}$$

$$\begin{aligned} \frac{\sigma F}{F} &= \sqrt{\left(\frac{\sigma G}{G}\right)^2 + \left(\frac{\sigma H}{H}\right)^2} \\ &= \sqrt{\left(\frac{1.598}{1.598}\right)^2 + \left(\frac{310.48}{310.48}\right)^2} \\ &= \mathbf{1.414} \end{aligned}$$

$$\frac{\sigma E}{E} = \sqrt{\left(\frac{1.17437}{1.17437}\right)^2 + \left(\frac{1.17501}{1.17501}\right)^2} \quad (9)$$

$$= \mathbf{1.414}$$

$$\frac{\sigma a}{a} = \sqrt{\left(\frac{\sigma E}{E}\right)^2 + \left(\frac{\sigma F}{F}\right)^2} \quad (8)$$

$$= \mathbf{1.847}$$