Suitability of EC-VARMA Class models in FDI-Macroeconomic Determinants Nexus

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Declaration

I Thabang Cassius Nqume, hereby declare that this study "Suitability of EC-VARMA class of models in analysing FDI-Macroeconomic determinants nexus" is original and the results of my own work. It is further declared that all information used and quoted have been acknowledged by means of referencing, and that this dissertation was not previously in this entirely or partially submitted by me or any other person for degree purpose at this or any University.

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T.C. Nqume Date

Dedication

This Full dissertation is dedicated to my Mother Mpho Nqume and the memory of my late Father Daddy Nqume.

You will always be in our hearts.

Acknowledgement

To God, the son Jesus Christ and the Holly Sprit, through whom all things are possible.

I would like to pass my sincere gratitude gratitude to the following people:

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Abstract

The scarcity application of multivariate time series VARMA models using macroeconomic variables as compared to VAR application has raised a concern that led to further investigation about the EC-VARMA model approach in macroeconomic data. Therefore exploring the Multivariate Vector Autoregressive Integrated Moving Average (VARIMA) class of model build up using domestic Foreign Direct Investment (FDI) inflow and its determinants for the period spanning from 2002Q2 to 2016Q1was an attempt to identify factors which could explain VARMA (p,q) as compared to ability of VAR(p). The findings provides evidence that EC-VARMA (1, 1, 0) model has a better predictive power. The FDI determinants explain about 56% of total variation in suggested model. The error correction coefficient provides evidence that the system of FDI corrected itself at an adjustment speed of 14% per quarter in the short-run for the long-run. The technique further allows for information in related variables to be captured as indicated by the ARCH test results. Among the six variables, only the consumer price index (CPI) proved to be inadequate, other variables such as Gross Domestic Price (GDP), Labour productivity Index (LPI), Openness to Trade (OT) and domestic investment (GFCF) prove to be adequate as far as the prediction of FDI is concerned. Upon calculating the causal relationship between all the variables including the FDI, findings further reveal feedback relationship between all variables except CPI. This confirms that CPI may not be a very good measure of FDI in the context of South Africa.Recommendation for further studies and policy was formulated based on the findings.

Key Words: Cointegration, EC-VARMA, Forecasting, Time Series and VARMA Class of model.

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Acronyms

VARIMA	-	Vector Autoregressive Integrated Moving Averages
VAR	-	Vector Autoregressive
VMA	-	Vector Moving Average
EC	-	Error Correction
ARCH	-	Autoregressive Conditional Heteroscedasticity
ME	-	Mean Error
VECM	-	Vector Error Correction Model
BVAR	-	Bayesian vector auto regression
UVAR	-	unrestricted VAR
RMSE	-	Root Mean Square Error
QMLE	-	Quassi-Maximum Likelihood Estimator
ADF	-	Augmented Dickey Fuller
KPSS	-	Kwiatkowski-Philips-Schmidt-Shin
ACF	-	Autocorrelation Function
PACF	-	Partial Autocorrelation Function
AIC	-	Akaike Information Criterion
SBIC	-	Schwartz Bayesian information criterion
LR	-	Linear Regression
HQC	-	Hannan and Quinn Criterion
FDI	-	Foreign Direct Investment
GDP	-	Gross Domestic Investment
GFCF	-	Gross Fiscal Capital Formation
LP	-	Labor Productivity
OT	-	Openness to trade

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Chapter 1

Study Orientation

1.1 Background

This study explored the interrelatedness in multivariate time series using vector autoregressive integrated moving average (VARIMA) models. Lutkepohl, 2005) noted that the application of VARIMA models is more popular as far as econometrics and time series analysis are concerned. The author considers the VARMA models as multivariate time series. Luktepohl further posited that VARMA models are the natural extension of the univariate ARMA models. These models opened a floor for the discussion of univariate time series (Mainassara, 2009).

In this study, an overview of the relevant theory of VARMA models and a description of basic properties of linear multivariate time series analysis were considered. Lutkepohl (2004) suggested the utilisation of linear multivariate time series models in producing linear forecasts of time series variables. VARMA models came to being as a result of Wold decomposition theorem for multivariate stationary series. More detail about this decomposition can be found in Athanasopoulos et al. (2007).

According to Lutkepohl (2004), VARMA models have the advantage of being more parsimonious when linear transformations are imposed. When data is transformed to linearity, VARMA representation becomes predictable. As a result, this class of models provide a room to study linear aggregation issues. The use of VARMA models to represent multivariate time series is scarcely used (Dufour and Pelletier, 2002).

Recently, much attention has been paid to vector autoregressive (VAR) models because these models can be implemented with ease. The VAR models are estimated by least squares, while VARMA models have the ability to capture nonlinear structures in the data. Nonlinearity is usually present in economic and financial data. As such, the estimation of VARMA models becomes effective when estimated with maximum likelihood methods. From a theoretical perspective, VARMA models are preferable even though their implementation is made intricate by estimation difficulties. Typically, the estimation methods for VARMA models need to be optimised. This may be hampered as soon as the model involves a few time series due to an unexpected increase in the number of parameters (Dufour and Pelletier, 2011).

There are different ways that can be adopted to parameterise the same stochastic term of VARMA models. Since these models are developed for stationary time series, it is a requirement that prior to implementing this class, all variables involved are subjected to stationarity testing. Since economic variables applied to these models in the current study are generated by non-stationary processes, differencing was done to stabilise properties of the series data. Both variables were differenced d times until the stationarity was achieved.

This was done to allow the ease of application of cointegration methods as a main method of analysis to this study. Specifically, the study applied yet another multivariate technique vector error correction model (VECM) by Johansen (1991) to impose a balance between long-run and short-run dynamics. For cointegration methods to be effective, same order of integration should be implemented on the variables. To achieve this, the equilibrium correction (EC) was blended in the VARMA model (EC – VARMA) for the purpose of obtaining balance in interrelations between variables.

Furthermore, VARMA models are recommended to obtain forecasts for macroeconomic variables. Fackler and Krieger (1986) recommend the use of VARMA models for this purpose due to their ability to substantially outperform unrestricted vector autoregressive (VAR) models based on forecasting accuracy. According to literature, the multivariate VARMA class of models was developed upon a realisation of the successful performance of the univariate ARMA model in forecasting.

Forecasts produced from several number of interrelated variables are more precise compared to univariate case. It should however further be noted that, specification and estimation of multivariate models is also complicated than in a univariate analysis despite improvements made in computer softwares (Lutkepohl, 2004).

1.2 Study Problem

Multivariate time series analysis is not thoughtfully investigated in most cases due to the complex and challenging process it follows. If multivariate and univariate time series are compared, the likelihood that a model with a closer fit to the data, high predictive power and greater number of parameters is anticipated. A number of studies have in the past and recently been using the VAR model to model multivariate time series data. The VAR model has been reported to be unreliable with lower predictive power.

The VAR model only assumes that the variables used are linear and has a shortfall of handling the nonlinearity which is usually present in most data that is collected over time. Conversely, the application of ARMA class models is only suited to produce short term forecasts of a univariate time series. The successes in the application of this class of models have triggered interest in the extension of the model from univariate ARMA to multivariate VARMA. Due to an enormous attention drawing researchers to using the VAR model, the application of VARMA in modelling and forecasting is very scarce.

The VAR model is immensely used despite the fact that it is incapable of handling nonlinearity and it also yields insignificant conclusions to the data. There is a need for studies that apply relevant and more parsimonious models to be conducted, especially in case of the modelling and forecasting of multivariate time series data.

1.3 Research Aim and Objectives

The study aims to explore the predictive power of VARMA class of models in analysing multivariate time series data, in particular the FDI and related determinants. Specific objectives are to:

- fit VARMA models to multivariate time series data,

- select an optimal multivariate VARMA model for FDI and related variables,
- determine long-run and short-run cointegration relationships between variables with EC – VARMA model,
- determine the predictive power of EC-VARMA models,
- formulate suggestions for policy and further studies based on the study findings.

1.4 Study Significance

The study's intention was basically to trigger interest to scholars who analyse multivariate time series data to also consider the use of VARMA class models. The findings may be used as reference to students who intend to research on this area of time series analysis. There is scarce literature on the application of multivariate VARMA class of models. This study as a result, will contribute to the body of knowledge in this regard. The current study further intends to make a contribution to the subject on multivariate VARMA models by imposing an adjustment with an error correction factor, something that has not been considered by most studies. There is no evidence that similar studies have been conducted in South African context. The findings of this study may be used as a point of reference by policy makers on the FDI sector when embarking on relevant strategies and revising the policy.

1.5 Scope Limitations and Delimitations

The scope of this study is limited to multivariate error corrected VARMA class of models only. Other similar classes are not considered. VARMA class of models have not been recognised by many researchers and as a result the application of these models is scarce since its development by Tiao and Box in 1981. This may lead to citing sources older than ten years and as there are limited studies around the subject. On the same breadth, the study does not anticipate any delimitation.

1.6 Structure of the Research

Chapter 1 lists the objectives and sets the background against which the study is based upon. In Chapter 2, literature on the study is provides. This included definition, literature review on multivariate time series VARMA models, application process, comparison between VARMA and VAR model and the studies guiding the choice of variables. Chapter 3 describes the the proposed methods for building multivariate VARMA models such as the Error correction, Vector Error correction, cointegration, model diagnostics and causality tests. The data used is also described in this chapter. Chapter 4 provides and discusses the results of the VARMA models. Chapter 5 provides the discussion of the study findings, conclusions and recommendations.

1.7 Summary

The discussion in this chapter sheds a light that VARMA models are a natural addition to the ARMA model. It is also noted that VARMA class of models can become effective when estimated with maximum likelihood method. From the theoretical perspective, VARMA models are preferred even though their implementations are complicated by estimation difficulties. Among other things discussed are the problem statement, study objectives, study significance and the dissertation layout.

The next chapter discusses both the theoretical and empirical literature on the subject.

Chapter 2

Literature Review

2.1 Introduction

In Chapter 1, an overview of the relevant theory around the issue of multivariate time series VARMA model was established to provide introduction and background of VARMA models. Chapter 2 of this study reviews the literature of VARMA models from univariate to multivariate perspective. The literature reviewed describes the process of the multivariate time series VARMA model. The ARMA class is considered rational polynomial evaluation (Box and Jenkins, 1979). VARMA models require special consideration due to its complex process involved in application. Therefore the study utilizes VARMA process to the identification of the multivariate time series models. Also reviewed is literature on the cointegration analysis specifically where the application of VAR/VECM is concerned.

Literature is one of the important chapters of this study, because it forms the foundation of the study that contribute towards the examination of VARMA model. In this study it is very important to discuss the univariate process before discussing the multivariate time series model. Hence VAR and the VMA procedures are used in explaining the VARMA process. The study reviews literature on the subject and gather how other scholars have applied such models in their studies. This literature is also used as a basis for identifying some of the important multivariate time series VARMA class of model properties, process and drawbacks if any.

The remaining part of the chapter is organized as follows:

The chapter intend to follow the following blue print: In Section 2.2 the application of VARMA class of models and definition is provided; Section 2.3 describes the comparison of VAR to VARMA models; Section 2.4describes cointegration of VARMA models; Section 2.5 outlines forecasting method using VARMA models; Section 2.6 explain the chosen variables and lastly Section2.7 gives a brief summary of the chapter.

2.2 Application of the VARMA Class of Model

In the literature several methods to identify the VARMA model are discussed. Athanasopoulos and Vahid ($2008a$) identified two methodologies that can be applied to obtain a unique identification of the VARMA model. The authors made a comparison of the performance forecast made on VARMA models. Their methodology consisted of three stages. The first stage involves the identification of the scalar component of the model (SCM) achieved by applying canonical correlation test between different sets of variables. Secondly, the identification of the fundamental formula of the model is done. In the third step, model estimation is done using maximum likelihood (ML) method.

Tsay (1989; 2015) defines the multivariate time series (MTS) as a process of analysing multivariate linear time series data and the estimation of multivariate vitality models. The author recommends a MTS process in handling dynamic models, unnatural factor models as well as the asymptotic principal component model in econometrics and finance. Athanasopoulos and Vahid ($2008a$) define MTS as a statistical technique used specifically in time series to analyse two or more variables whose observations are arranged in chronological order with time.

According to Chatfield (2000), MTS is effective in modeling a number of interconnected factors. The author defines a time series multivariate VARIMA (p, d, q) model as a model which, when differenced once, gives a VARMA (p, 1) model, which is, of course, a VMA model of order 1. Dufour and Pelletier (2005) proposed a modification information criterion to determine VARMA orders. If the data set are analysed the number of parameters to be estimated are $(p + q + 3)d^2$ .Choosing a small $VARMA\ (p,q)$ order implies inconsistent estimators and if too large brings a decrease in forecast accuracy (Manasara,2010).

The finite order of the VAR model is preferred by most scholars to the VARMA model, since in the literature there information about the alternative use and identification of VAR. The VAR model is considered user friendly. Awokuse and Bessler (2002) (as cited by Cooley and Dawyer, 1998) argued that macroeconomic time series modeling using VAR model is not consistent with economic theories. Athanasopoulos, et al. (2014) showed that the forecasts based on VARMA models are better than those of VAR model. The difficulty of the VARMA methodology reflects on selection of the VAR model.

Lutkepohl (2004) defines VARMA model as a process appropriate to produce linear forecasts of a number of time series variables. VARMA models produce parsimonious depiction of linear data generation process. The process set up is when data is stationary and the variables are cointegrated of similar order. Furthermore, exceptional or recognized parameterisation on the basis of the level form is represented.

According to Macmillian (2001), using the multivariate linear time series analysis, an analyst is able to perform various tasks on the model. These tasks include the specification of the model, the estimation, performing a battery of diagnostic tests and forecasting. Multivariate model is a VAR with or without independent factors, the Moving averages(MA),ARMA including seasonal Vector ARMA(SVARMA) multivariate time series regression models modified VAR models, and the error corrected VAR (VECM). In specifying the model one could perform structural specification for physical specification to overcome problems associated with VARMA model identification. One could also consider applying the Kronecker Index and the scaler component models to perform structural specification if all else fails (Macmillian, 2001). To achieve the objectives, the current study explores the error corrected VARMA model using the FDI and selected macroeconomic determinants.

2.3. Comparing VARMA Model to VAR Model

According to Dufour and Pelletier (2011) VARMA models are scarcely used to represent MTS. However, due to the ease of application of the VAR models, most researchers have employed them in their studies. The difference between the VAR and VARMA is the estimation method they use. For instance, least square method are good estimators of VAR models while the VARMA models use the nonlinear approximation methods such as the maximum likelihood. The VAR models are easy to specify as the process involves a choice of only one lag order. Caution must be taken during this process, as there are some significant shortfalls. Two of these drawbacks are less parsimony in VAR compared to VARMA models and that VAR model family is not marginally and temporarily closed (Dufour and Pelletier, 2011). As suggested by the authors, sub vectors are not likely to satisfy the VAR but VARMA models if the Vector VAR satisfies the VAR model. Likewise, if a frequency is used in observing the VAR model, the result is not a VAR. Similarly, VARMA class model is closed under such process (Dufour and Pelletier, 2011).

Recently, the VAR model remained extensively used for modeling and analysing the monetary policy framework. For instance, Rhaghavan et al. (2013) investigated VARMA model versus VAR model by measuring the effect of Malaysian monetary policy. The authors used monthly data covering the period January 1974 from the International Financial Statistics (IFS). Seasonally adjusted and natural logarithms were implemented on the data. Stationary condition was confirmed after differencing using the Augmented Dickey Fuller and Philips-Perron unit root tests.

Using the Johansen's co-integration approach, long run relationship was confirmed between the seven variables. The test also provided suggestion of long-run associations between the seven variables. The current study is similar in part with Rhaghavan et al.'s study which was conducted in 2013. Similar stationary and cointegration methods are implemented.

Rhaghavan et al.'s (2013) study agrees with Ramaswamy and Sloke (1997) that VAR and VARMA with the variables in levels remain appropriate measures when the researcher desire to do correct identification of monetary effects shocks. The VARMA models are recommended for the modeling of financial data as opposed to the VAR models. Ramaswamy and Sloke (1997) compared the impulse responses generated by VARMA, VAR and Structural VAR for money, interest rate, exchange rate and foreign monetary shocks. Overall, the VARMA model performed much better than its counterparts as the impulse responses were found to be consistent with prior theoretical expectations particularly under different exchange rate regimes. However, the VARMA model is rarely employed for identifying the orthogonal monetary policy shocks, due to the difficulties associated with its use.

2.4 Cointegrated VARMA Models

Athanasopoulos et al. (2014) define cointegration as a process where numerous non stationary I(1) variables have a minimum of one joint stochastic trend. Kascha and Trenkler (2011) investigated the cointegration of VARMA models using the United States bond interest rate data of differing maturities and the United States Treasury bill. The authors proposed a relative specification and estimation strategy. According to the evaluation of the forecasts produced, the study revealed that a VARMA model provide good and reliable forecasts. The findings were in favor of a VARMA model instead of a pure VAR.

Kascha and Trenkler (2011) indicated that a number of variables produced by a VAR procedure is a classical process towards a VARMA, not by a VAR process. This implies that the variables of produced by a finite-order VARMA process also known as dynamic stochastic general equilibrium (DSGE). The results of Kascha and Trenkler's study favored of VARMA (1, 1,0) than pure VAR model. Also revealed by this study is the presence of final moving averages representation in cointegrated case which in most cases is simpler but lacks parsimony.

Literature purports that, cointegrated VAR or VARMA models are very advantageous. According to Luktepohl (2004), the VECM outperforms the VARMA model in most cases. Engle (2002) suggested the VARMA procedure with exogenous regressors in estimating the parameters of the model and production of forecasts. In a variety of financial and economic studies, response variables are influenced by variables outside the system under consideration. The VARMA procedure is in support of modeling the dynamic association between the endogenous variables and exogenous variables.

The procedure for examining stationary, specifications and estimation of the cointegrated VARMA model was investigated by Dufour et al. (1997). The authors used quarterly real money stock data, consisting of 136 observations. The data covered the period of 1954 Q1 to 1984 Q4. The authors proposed the modeling and estimation method which simplifies the use of VARMA models. The study identified VARMA the MA equation forms and the diagonal MA equation form. The two representations are typical extensions of the class of VAR models where MA operator is added, either on scalar or a diagonal operator.

An extension of the VAR with an MA produces more parsimonious depiction. However, simple form of the MA operators does not present unwarranted problems. In producing a simplified estimation, the study examined the problem of estimating VARMA models by modest technique requiring linear regressions. Considering a generalisation of the regression based estimation techniques suggested by Hannan and Rissanen (1982) for univariate ARMA models, the procedure was in three steps. In the first step a long-run VAR was fitted to the data.

In the first step, long-run VAR was fitted to the data. Secondly, the study replaced the lagged innovations in the VARMA model with the associated lagged residuals from the first step to produce a regression. The third step involves clarifying the data in the second step and producing another regression. Anticipated results from the third step is that estimators have similar asymptotic variance as their nonlinear counterpart.

One other recent study that adopted MTS approach was conducted by Simionescu (2013), who utilized the VARMA model to forecast the United States' macroeconomic indicators. The study used quarterly data collected from the United States' economy for the period Q1 1955 to Q4 2000 to build VAR and VARMA models. Predictions made were based on these models for the horizon Q1 of 2001 to Q2 of 2013. The study identified $VARMA$ (2, 1)and VAR (3).

The findings proved that the VARMA model provides forecasts with a higher amount of precision than VAR models. There was no evidence of structural shocks in the variables used in Simionescu's (2013) study. As a result, it is not surprising to gather that the forecasts based on VARMA models are better than those based on VAR models. This however creates contradiction since literature is more in favor of the VAR model than VARMA model. The current study verified this by applying these models to a quarterly South African data.

Bai et al. (2017) investigated the adoptive clustering and error correction methods for forecasting cyanobacteria blooms using VARMA models to improve the forecasting performance of multivariate time series. The author categorized EC-VARMA into some trends using Bayesian network to determine the relationship between the data trends of its corresponding VARMA error. Finally the estimated values of the VARMA errors from each trend obtained using the Bayesian network. The results indicated that the proposed model of VARMA (1, 1) can improve the prediction performance.

Dufour and Pelletier (2011) investigated the practical method for modeling weak VARMA process with macroeconomic data by following the Box-Jenkins ARIMA approach. In their study, the authors proposed modeling and estimation method which simplifies the use of VARMA models. The authors used Macmillian's (2001) data to fit VARMA and VAR model to the six macroeconomic data. The proposed models for this study were VARMA, MA and the diagonal MA forms. The last two representations were simply the extensions of the class of VAR models with the MA operator added on either a scalar or a diagonal operator. The motivation for adding the MA term to the model was to obtain more parsimonious representations since the easier form of the MA operators does not present unnecessary complication (Dufour and Pelletier, 2011).

Mainassara (2009; 2010) investigated the multivariate portmanteau test for structural VARMA model with uncorrelated but not exogenous error terms. The author firstly, identified the joint distribution of the quasi-maximum likelihood estimator (QMLE) or the least square estimator (LSE) and the noise empirical autocorrelations under weak assumption on the white noise. Furthermore, the author assumed the asymptotic distribution of the Ljung-Box portmanteau statistic for VARMA models with non-exogenous innovations.

The standard framework showed that asymptotically, the distribution associated with the modified Jung-Box's test is as a matter of fact for the weighted sum of independent chi-squared random variables Mainassara (2010).The author further cautioned of the difference the asymptotic distribution can produce when the assumption of independence assumption is protected. Accordingly, Mainassara hinted that traditional chi-squared distribution fails to provide sufficient estimation of the goodness-of-fit of Box-Pierce portmanteau tests. For this reason the author proposed technique which modifies the critical values for hybrid tests.

Park (1990) used a routine of challenging diagnostic techniques to evaluate the predictive performance of the five multivariate time-series models for the United States cattle sector. The study adopted the Root Mean Square Error criterion for model comparison. This criterion was used beside with an assessment of the rankings of predictive errors which reveal that the Bayesian vector autoregression (BVAR) and the unrestricted VAR (UVAR) models produce forecasts which are greater to both a restricted VAR (RVAR) and a VARMA model. To forecast direction of change, two methods were used. The findings reported that the BVAR and the UVAR models explicitly under performed as compared to the VARMA model in forecasting directional change, hence, the reason why in this study the EC was factored with the VARMA model to perform the analysis with the hope of strengthening the model.

Kascha and Trenkler (2011) investigated VARMA models using the United States' interest rate and cointegration. The authors acknowledged that there are very few studies on the performance evaluation of forecasts based on VARMA or cointegrated VARMA models. The authors combined current suggestion by literature on VARMA models to generate a moderately simple specification and estimation strategy for the cointegrated case. The study reported a simpler MA representation using the cointegrated case with fixed initial values. Furthermore, Kascha and Trenkler study confirmed that specification strategy is consistent also in the case of cointegrated series. Moreover, Poskitt (2003), Athanasopoulos and Vahid (2008*a*; 2008*b*) evaluated the accuracy of forecasts made using VARMA models and their study revealed a good performance, exceeding the one of VAR models.

Dufour and Stevonovic (2013) investigated the association between VARMA and factor representations of a vector stochastic procedure. Their study employed a VARMA model enhanced with error correction factor. This was done with a hope of substituting the model back to a standard VAR model. Firstly, the study reported that a vector of time series variables and their related variables diverge from a predetermined order of a VAR process. Special cases were an exception. It was observed that defining variables as linear combinations of observable series makes this observable series to follow a VARMA process but not a finite order VAR as classically expected. Secondly, as observed by the study, irrespective of whether the variables follow a finite order VAR model or not, a VARMA representation for the observable series is still necessary (Dufour and Stevonovic, 2013). In representing the dynamic interactions between several variables, the authors employed an integrated VARMA with factor analysis frameworks. This integration necessitated not only evaluating cointegration relationships, but also the dimension reduction in time series variables.

Applying the VARMA model to the out of sample forecasting of the United State and Canadian's monthly data, Dufour and Stevonovic's (2013) study proved that the VARMA model produce better forecast compared to the ordinary models. Lastly the study estimated the impact of monetary policy and the identification scheme of Bernanke et al. (2005). The results showed that impulse responses from a parsimonious 6-factor VARMA (2, 1) model give a precise and plausible picture of the effect and transmission of monetary policy in the United States. The current study enhanced the VARMA model with the error correction factor to allow not only depiction of long-run dynamics but also the short-run analyses. Factor analysis enhanced models are recommended when the study analyses a substantial number of variables which are assumed to be correlated and linearly related, hence it is not an option for the current study.

In the Dufour and Stevonovic (2013) study, the VARMA model required the estimation of 84 coefficients in order to represent the system dynamics, while the corresponding VAR model estimated510 VAR parameters. Even though the VARMA model is not easy to use, it has been proven by Dufour and Stevonovic (2013) that the model remains effective when integrated with factor models. Some author's content that the integration of VARMA class with other models could lead to more complications especially that more parameters are generated compared to when an ordinary VARMA was used. This study does not follow the approach taken by Dufour and Stevonovic (2013) illustrating integrated VARMA models with factor analysis.

2.5 Forecasting using VARMA Model

To evaluate the suitability and effectiveness of any model, one needs to produce the forecasts. This stage is preceded by subjecting the selected model to a battery of diagnostic tests. Just like the VECM model the VARMA model affords a researcher a chance to perform model diagnostics tests based on the estimated residuals.

Recent study by Athanasopouloul et al. (2016) investigated the dynamics of long-run and short run in EC-VARMA models using canonical correlation using simple coherent approach for identifying and estimating EC-VARMA models. Simplifying canonical correlation analysis the author implemented the cointegration rank and identified the short-run VARMA dynamics using scalar component methodology. The results revealed that EC-VARMA models generate significantly more accurate out of a sample forecast than Vector Error Correction Models (VECM) especially for short-run.

According to Lutkepohl (2004) when primary objective of a study is to forecast a set of variables, the researcher is advised to take into consideration the criteria for evaluating the forecasts performance. A good model is one that produces optimal forecasts according to the forecasts evaluation criteria. Moreover, it has been proven by many studies that VARMA procedures predominantly generate valuable forecasts with that minimized forecast mean squared error (MSE) (Lutkepohl, (2005). The author indicated that even though VMA have some theoretical interest, it is very uncommon to use such models in practically. He further cautioned that a VARMA process is regarded by some authors as a rational approximation to the infinite VMA process.

Lukepohl (2005) urged that linear transformations of VARMA process are regularly of interest, hence forecasts of transformed process are also of interest. Athanasopoulos et al., (2014) investigated forecasting with the EC-VARMA models. A complete technique for classifying and assessing EC-VARMA models was shadowed. The cointegrating rank was assessed in the first stage using an extension of the non-parametric method of Poskitt (2000).

Then, the structure of the VARMA model for variables in levels was identified using the scalar component model (SCM) methodology developed in Athanasopoulos and Vahid (2008), which lead to a uniquely identifiable VARMA model. In the last stage, the VARMA model was estimated in its error correction form. However, Monte Carlo simulation was directed using a 3-dimensional VARMA (1, 1,0) with cointegrating rank 1 so as to assess the forecasting performances of the EC-VARMA models. United States' interest rates was used as experimental unit for this study. The results revealed that the out-of-sample forecasts of the EC-VARMA (1,1,0) model are better than those produced by error correction vector auto-regressions (EC-VAR) of finite order, especially in short period (Athanasopoulos and Vahid, 2008).This finding concludes that irrespective of whether a simulated or original data is used, VARMA enhanced models remain effective when applied to time series data.

Aboagye-Sarfo et al. (2015) made a comparison of multivariate and univariate time series approaches to model and forecasting emergency department demand (ED) in Western Australia. The study focused on time series analysis using monthly emergency department (ED) demands in the public sector hospitals for the seven year period 2006/07 to 2012/13. The dependent variables in the study were the numbers of ED representations stratified by age group, place of treatment and triage category. VARMA models was used to develop multivariate time series models to forecast public hospital ED. The results showed that the descriptive analysis of all the dependent variables showed an increasing pattern of ED use with seasonal trends over time. The VARMA model provided a more precise and accurate forecast with smaller confidence interval and better measure of accuracy in predicting ED than with ARMA model and Winter's method.

2.6 Choice of variables

Macroeconomic variables have been used continuously in forecasting of univariate and the multivariate time series. According to Simionescu (2013), the VARMA model and the VAR model have been used in econometrics, particularly in the time series analysis to reveal the cross correlations between the series, exceeding the isolated analysis of the data series. Therefore literature reflect the use of macroeconomic variables in time series. According to Janicki and Wunnava (2004), FDI has a long and complex history in South Africa with unexpected and growing role. A recent study by Kiiru (2014) explored the South African determinants FDI.

According to Kiiru (2014), if South Africa wishes to compete for FDI with other countries, the country will have to at least offer attractive investments. The author suggested that FDI can offer domestic country with new marketplaces and marketing channels, cheaper production facilities, access to new technology, products, skills and financing. Additionally, it can provide new technology, capital, process, products, organisational technology and management skills, and as such can provide a strong encouragement to economic expansion. Durham (2004) define FDI as an establishment from one country making a physical investment into other countries.

The literature shows some of the relationship and effect the influencing factors might have on FDI inflow. Charkrabarti (2001) highlighted that there are diverse suggestions regarding the implication of determinants of FDI. For instance, the effect of openness of trade towards FDI is measured typically by the proportion of exports plus imports to GDP. Once the majority of investment projects are captivated by the tradable sector, it is given that the degree of openness to international trade for the country would be a relevant factor of choice Demirhan and Masca (2008).

According to Jordaan (2004), openness on FDI impact is on investment. This means that when investments are market seeking, trade restrictions and therefore less openness can consume a confident impact on FDI. Parletun (2008) reported that openness to trade has a positive and significant from towards FDI. The effect of this variable is expected not only to attract foreign capital to host country, but by also taking the competition between the foreign and domestic firms (Hoang et al. 2010). Therefore, GDP growth rate is expected to be positively affecting FDI.

The growth effect of FDI inflows is one of the most controversial issues in development economics (Kinoshita, 2003). Using a panel data for 25 conversion economies between 1990 and 1998, the study identified that the main determinants of inward FDI are foundations, agglomeration and trade openness. Ancharaz (2003) reported in his study a positive effect with lagged growth for the full sample and for the non-Sub-Saharan African countries. The study also reported and insignificant effect for the Sub-Saharan Africa. On the other hand Charkrabarti (2001) claims that wage as an indicator of labour cost has been the most contentious of all the potential determinants of FDI. Preferably, the importance of cheap labour in attracting multinationals is agreed upon by the advocates of the dependency hypothesis as well as those of the modernization hypothesis, though with very different insinuations. Hence, the FDI is expected to negatively affect labour cost.

The sign of the coefficient for Human capital is expected to be positive. Saunders (1982) points that researches have shown that a further sophisticated labour force is likely to adopt new technologies faster and at lower training cost. For this reason, an indicator of the common level of education can be included among the independent variables to capture this effect according to the author. Moreover, the specifics of workers also affect the quality of production as well as saving more money and time for training and production running. Zekiwos (2012) used CPI as a proxy for inflation and found that theoretically the sign of the coefficient for CPI is expected to be negative.

Jasperson et al. (2000) investigated the relationship between FDI and its determinants. The authors argued that low inflation rate is reflected to be a sign of internal economic stability in the host country. The high rates of inflation symbolises inability of the government to balance its budget and disappointment of the central bank in conducting suitable economic policy. The study by Hosein (2011) investigated the effect of FDI and other foreign capital inflows on growth and investment in developing economies. The author used the Gross Fiscal Capital Formation (GFCF) as proxy for total domestic investment. Theoretically the sign of the coefficient for GFCF is expected to be positive.

Enoma and Mustaph (2010), studied the FDI inflow in Africa. Time Series Econometric method was used to compromise a dynamic neoclassical investment purpose built on a system of comprehensive technique of moments estimation by Karima and Sainib (2012), Principal Component Analysis by Enoma and Mustaph (2010) and Tsadu and Gunu (2011) and multi-criteria decision making procedure of analytic hierarchy process by Newell and Searook (2006).

Sechei and Kinyodo (2012) mentioned that FDI refers to all activities which involve the use of resources to produce goods and service in the country. On the contrary, Jackson (1995) reasoned that FDI can be many things such as machinery, building, facilities and computers; operating expenditure on training, education and research can also be regarded as investment to another country. Consequently, physical investment is regarded as the most obvious as it involves constructing of new buildings, roads and facilities (Jackson, 1995).

Consulting Ajayi (2006) refers to FDI as being significant to a domestic nation for various reasons. It carries investable monetary capitals, delivers new technologies and might improve the competence of existing technologies. FDI could enable right of entry to export markets, thereby singing an imperative role in strengthening the export proficiencies of the domestic economy. It may also improve skills and management techniques, and provide cleaner technologies and contemporary environment management systems.

According to Hussain and Kimuli (2012), FDI is particularly a type of foreign capital as opposed to domestic investment. This compromises activities that are controlled and organised by firms or group of firms outside the country on which they are based and where their principal decisions are located. This type of investment is considered as a major component of capital flow for emerging market, its influence towards economic growth is broadly contended.

Samad (2008) argued that in current years, certain rapid growth and change in worldwide investment designs, the definition of FDI has been widened to include the achievement of a lasting management interest in a company or enterprise outside the investing firm's home country. In place of such, it may yield many systems, such as direct achievement of foreign firm, construction of a facility, or investment in a joint venture or strategic association with a local firm with attendant input of technology, licensing of intellectual property.

Moosa (2005) define FDI as an incorporated or unincorporated initiative in which a foreign investor owns 10 per cent or more of the ordinary shares of combined initiative or the similarity of an unincorporated enterprise. Similarly, Criscuolo (2005) defines FDI as an initiative in the financial or non-financial business sectors of the economy in which a non-resident investor owns 10 per cent or more of the voting power of an incorporated enterprise or has the equivalent ownership in an enterprise operating under another legal structure. According to Javorcik (2008) FDI can be classified as a group of investment that imitates the objective of establishing a lasting interest by a resident enterprise in one economy in an enterprise that is resident in an economy other than that of the direct investor.

Contessi and Weinberg (2009) define FDI as the investment completed to obtain a permanent interest in or actual control over an enterprise functioning outside the economy of the investor. FDI net inflows are the significance of inward direct investment made by non-resident investors in the reporting economy, including reinvested earnings and company loans, net of deportation of capital and repayment of loans. Moreover, FDI net outflows are the value of outward direct investment made by the residents of the reporting economy to external economies, including reinvested earnings and company loans, net of receipts from the removal of capital and repayment of loans. These series are expressed as shares of GDP (Contessi and Weinberg, 2009).

2.7 Summary

The VARMA model is set to be a better tool for forecasting compared to VAR. Therefore the utilization of VARMA model is effective in forecasting using macroeconomic indicators. Special attention has been given on the process of a class of VARMA models and its ability to forecast with linearly transformed and aggregated process. It is mostly and notably identified that forecasting with VARMA is better than with VAR. Literature identified appropriate models for forecasting a specific set of time series. However it is essential to use relevant information to specify and estimate an appropriate model from the VARMA class of models. According to Luktepohl (2014), considering many series in one system is not a good strategy when modeling VARMA class of model. The increase in estimation and specification ambiguity may offset the advantages of using additional information.

VARMA models appear to be most useful for analysing small set of time series according to information provided. Furthermore choosing the best set of variables for a particular forecast can be quite tricky. The process of VARMA parameters requires development on efficient methods which builds on the estimation procedure described for VARMA models. In conclusion, although VARMA class of models are important especially as a tool for forecasting, they have some limitations to a certain extent like any other method. Hence this study aims to fill the gap of modelling using VARMA class of models enhanced with an error correction term as suggested by several authors. This is done to simplify the complicated estimation process in a simple VARMA model.

Moreover, literature has made emphasis on the modelled effect and relationship regarding determinants of FDI. FDI can be classified as a category of investment that reflects the objective of establishing growth. Modelling and consolidation of the important FDI factors assist in finding solutions and recommendations towards growth or development in terms of technology improvement and sustainability in the country as stipulated by the literature. Therefore, the role of FDI can be regarded sum of the few elements that are set as priority in developing a country like South Africa.

Chapter 3 gives a review of the methodology followed by the study with reference to the objectives outlined in Chapter 1.

Chapter 3

Research Data and Methodology

3.1 Introduction

This chapter provides a description of multivariate time series VARMA model and how it rules to Box-Jenkins univariate ARMA class models. Moreover, the description of models is made with reference to the literature. Nevertheless, as most statistical activities, MTS analysis and forecasting frequently involve discovering an appropriate model for a set of data. This section reviews the methods proposed for the study and the criteria for choosing optimal model. It is relatively easy to look at the theoretical properties of different models, but it can be challenging to decide which model is appropriate for a given set of data, especially in instances where MTS is concerned (Athanasopoulos, et al., 2007).

Furthermore, this chapter give a description of the data used. The exposure of this study is limited to the background exposed by the literature in Chapter 2, and the problem defined in Chapter 1. Notice that, this study explores multivariate VARMA class of models using the South African domestic FDI inflow data and its determinants.

Time series analysis assumes that the actual values of a random series are influenced by a variety of environmental forces operating over time. There are four underlying forces, individually and collectively determining the reliability and robustness of the data in time series, namely, Trend, Cyclical, Seasonal and Irregular. Trend and seasonal components account for a significant proportion of the actual values in a time series. Hence, this chapter is organised on some basic (Box-Jenkins, 1970) ARMA approaches to MTS VARMA model. The discussion in this chapter is also informed by Dufour and Pillitier (2011), Dufour and Stevanovic (2013), Dufour (2006) and Lutkepohl, et al. (2006) among others.

3.2 Data Description

The study used quarterly time series data sourced from the South African Reserve Bank database covering the period 2002 second quarter to 2016 first quarter. The variables used in this study include foreign direct investment (FDI) as dependent variable.Independent variablesare Openness to trade, (OT), Gross Domestic Product (GDP), and Labour Productivity (LP). Literature was used to decide on the appropriate determinants of FDI. More information can be obtained in Chapter 2. Search engines such as google and yahoo were used to obtain more valuable information. The research refereed to articles in google scholar and was very selective in identifying published research. Time series data was obtained from the South African Revenue Bank, World Bank and other publishers of economic data.

Charkrabarti (2001), Jordaan (2004), Parletun (2008) and Kinoshita (2003) investigated the effect of the factors of foreign direct investment on the FDI inflow in South Africa. The variable FDI showed to be positively affected by OT, GDP and domestic investment (GFCF). However, two variables in the literature showed that FDI can be negatively affected by LP which may demand high wage and unstable Inflation rate (CPI) which causes unstable economy in the country (Ancharaz, 2003; Charkrabarti, 2001; Saunders, 1982; Zekiwos, 2012 and Phil, 2014). FDI and domestic investment are measured in millions of rands, LP is an index and GDP, CPI and OT are in percentages.

VARMA model is a process guided by the ARMA model approach by Box-Jenkins method and the autoregressive moving average (VAR) procedure. According to Chatfield (2000) the importance of starting the initial inspection of the time series data is to describe behaviors of the series. These plots reveal either the presence of a trend, seasonality, outliers and discontinuity in some or all of the series. These features provide a guide to the selection of a suitable transformation. A total of 51 observations for each variable are used with the aid of some statistical software such as SAS 9.3 and Eviews 8 to execute the analysis.

3.3 Methodology

The study follows both the preliminary and primary methods to analyse the data. This helps in obtaining optimal results. This is also done in an endeavour to obtain best linear unbiased estimates (BLUE) and robust results.

3.4. Preliminary Data Analysis

Time series data were first plotted to establish if the data is stationary. There are different methods that can be applied to make the data stationary, if it is found to be non-stationary. Economic variables are known to be generated by non-stationary processes. In some cases log transformations was used depending on the unit of measurement for a certain variable. Otherwise, differencing was enforced to stabilize the stochastic properties of the series. Both variables were differenced d times until the stationarity is achieved.This is so because this class was solely developed for stationary data. The study applied the two most recommended tests, augmented Dickey-Fuller (ADF) (1979) and the KPSS to confirm stationarity of the data. Depending on what the data offers, seasonal differencing might be applied. A subsequent section provides a discussion for data transformation to logs and with differencing.

3.4.1 Data Transformation

Any time series data can be thought of as being produced by stochastic or random procedure (Box and Jenkins, 1970). This process has drawn a great deal of attention in time series data analysis. A series is said to be stationary if its stochastic properties such as the mean and the variance remain unchanged.

It is important to remove the irregularities present in the data prior to subjecting it to transformations (Sadowski, 2010). This initial transformation of data may also help in avoiding the violation of basic assumptions such as normality and heteroscedasticity. The transformation method is applied if the properties of the series are time invariant.

Transforming the time series can suppress large fluctuations. The most standard transformation is the Box log transformation defined as follows:

$$y_t = \begin{pmatrix} \frac{x^{\lambda}-1}{\lambda}, \lambda \neq 0 \\ Inx, \lambda = 0 \end{pmatrix} [3.1]$$

The logarithm in a Box transformation is always in a natural form, thus $\boldsymbol{\lambda} = \mathbf{0}$is a natural transformation and$\boldsymbol{\lambda} \neq 0$ is a power transformation. Macroeconomic variables are used in this study, therefore the Box-Cox log transformation was multiplied by 100% excluding for variables that are expressed as percentages (Box and Cox, 1964). The following formula was used:

$$y_t = \begin{pmatrix} ({X^{\lambda}}_{t-1})\lambda * 100, & \lambda \neq 0 \\ lnx_t * 100, & \lambda = 0 \end{pmatrix} [3.2]$$

If the data shows a variation that increases or decreases with the level of the series, transformation can be useful. Adjusting the historical data can often lead to simpler forecasting model as suggested by Sadowski (2010), hence this transformation simplified the pattern in the historical data.

3.4.2 Stationarity Testing

Two stationarity tests namely the Augmented Dickey and Fuller (ADF) and the Kwiatkowski-Philips-Schmidt-Shin (KPSS) tests were used in test for stationary. The data for this study was collected over time and this is one of the causes of unit root in economic and financial data. Depending on what the data offers, if not regular, seasonal differencing was applied. The first step in the analysis of the time series data prior to applying formal test is to provide a plot of the series. Time series plots provide initial clue about the nature of the series and the model properties. Unit root tests are used to determine stationarity properties of the data, *i.e* to assess if the mean is equal to a unit and that the variance is constant.

Time series data tend to fluctuate around the mean independent of time and the variance over time. Stationarity is evaluated using a time series plot that depicts no changes in the mean over time and no noticeable change in the variance over time. Before identifying the pattern of the model, time series values $y_1, y_2, \ldots, y_n$ must be stationary where the mean and the variance are stationary through time. If a series has unit root, and unless it combined with other unit root series to form stationary cointegration association, then the regression concerning the series can cause spurious regression (Yu, 2012). Discussed below is the ADF unit root and the KPSS stationarity tests.

3.4.3.1TheAugmented Dickey Fuller Test

The ADF test is used to investigate the hypothesis that all the variables have a unit root, in the level of variables as well as in their differences depending which stage the data becomes stationary. The expectation is to obtain a constant mean and variance over time,$iid \sim 0; \delta^2$ (Dickey and Fuller, 1979). ADF test has three possible types of models such as:

$$\Delta y_t = \alpha + \delta y_{t-1} + \varepsilon_t \qquad [3.3]$$

$$\Delta y_t = \alpha + \delta y_{t-1} + \varepsilon_t \text{ , and} \qquad [3.4]$$

$$\Delta y_t = \delta y_{t-1} + BT + \varepsilon_t \qquad [3.5]$$

where 3.3 denotes a series without a constant and trend,3.4 denotes a series with a constant and 3.5 denotes a series with constant and trend

The calculation of unit root test requires an identification of the correct model and estimation of the parameters (Moroke et al., 2014). For all three equations [3.3], [3.4] and [3.5], the unit root test is given as:

$$\tau_{ADF} = \frac{\emptyset_1 - 1}{se(\widehat{\emptyset_1})} \qquad [3.6]$$

The stationary hypothesis takes the following form:

$$H_0: \delta_0 = 0 \ There\ is\ a\ unit\ root$$
$$H_1: \delta_0 < 0\ Stationary$$

If $t^* > ADF$ critical values, this follows non rejection of null hypothesis that unit root exists and if $t^* < ADF$ critical value. This implies lack of unit root in the series. The null hypothesis postulates that the series contains unit root (non-stationary process) versus the alternative of stationary process (Hosein, 2011). To test the null hypothesis, (3.6) was compared with the corresponding critical value at a conventional significant level. To perform (ADF) test, the study perform all three series 3.3, 3,4 and 3.5 which include a constant, a constant and a n the test regression guided by the features of time series plots. One approach could be to run the test with both constant and a linear trend. Inclusion of unnecessary regressors in the model could lead to misleading conclusion about the null hypothesis (Hosein, 2011).

To overcome the problem of non-stationarity, the form of test regression is based upon the graphical inspection of a series (Verbeek, 2004). If the plot of the data does not start from the origin, then the estimation equation should include a constant. If the plot of the data indicates an upward or downward moving trend, then the trend component should be contained in the regression. The main criticism of ADF test is that the power of test can be very low if the process is weakly stationary. Alternatively, the process becomes stationary but with a root close to the non-stationary boundary (Brooks 2002). If a non-stationary time series y_t has to be differenced d times to induce its stationarity, then y_t is said to contain d unit root. It is customary to denote $y_t \sim I(d)$ which reads y_t is integrated of order "d".

If the ADF test is not significant, differencing transformation are required as suggested by Bowerman, at el. (2005).Other graphical methods such as the autocorrelation function (ACF) and partial autocorrelation function (PACF) also give a visibility of the data. Removing non-stationary in the data can be achieved by:

First Difference: $z_t = y_t - y_{t-1}$where $t = 1,2,3, \dots, n$ [3.7]

Second Difference:

$z_t = (y_t - y_{t-1}) - (y_{t-1} - y_{t-2})\ where\ t = 3,4, \dots, n$ [3.8]

Equation [3.4] can also be presented as:

$z_t = y_t - 2y_{t-1} + y_{t-2}$ [3.9]

3.4.3.2 The KPSS Stationarity Test

The most commonly used stationarity test is by Kwiatkowski, et al. (1992). The authors derived the test from the model:

$y_t = \beta' D_t + \mu_t + e_t$ [3.10]

$$\mu_t = \mu_{t-1} + e_t, e_t \sim WN(0, \sigma_\epsilon{}^2)$$

where D_t contains a deterministic component, e_t is a pure random walk with variance $\sigma_\epsilon{}^2$ and that μ_t is $I(0)$ and may be heteroskedastic. The null hypothesis that y_t is $I(0)$ is formulated as:

$h_0: \sigma_\epsilon{}^2 = 0$ and the alternative hypothesis

$$h_1: \sigma_\epsilon{}^2 > 0$$

The null hypothesis implies that μ_t is $I(0)$ and $\mu_t$is a constant.The KPSS test statistic for this hypothesis is based on the Lagrange Multiplier approach represented as:

$KPSS = (T^{-2} \sum_{t=1}^{T} S_t{}^2)/\lambda^2.$ [3.11]

$\hat{S}_t = \sum_{j=1}^{t} \hat{u}_j, \hat{u}_j$ is the residual of a regression of y_t on D_t and λ^2 is a consistent estimate of the long run variance of μ_t, using$\hat{u}_t$.The null hypothesis is rejected when the [3.11] is in excess of the critical value from the KPSS table, providing concrete suggestion that the series wander from its mean.

3.5 Primary Data Analysis

To help achieve the objectives set for this study, quantitative methods were adopted. The extended Box-Jenkins (1976) approach to multivariate time series by Tiao and Box (1981) were adopted as methodological framework. This framework follows exactly same principle of univariate time series analysis. The study started by identifying the appropriate models. In the next step, estimation procedure was reviewed and finally the models were subjected to diagnostic testing before proceeding with forecasting (Chatfield, 2000).

3.5.1 Model Specification

The review of the VARMA class of models was guided by Dufour and Stevanovic (2013), Lutkepohl (2004) and Chatfield (2000). Upon specification of the model, maximum likelihood method was used to estimate the parameters. As highlighted in previous sections, this parameter estimation method is capable of accommodating nonlinearity in the data as opposed to the ordinary least squares estimation method. Any changes to the data alter the originality of the model.The following multivariate model was suggested by the study:

$$FDI_t = \beta_0 + \beta_1 GDP_{t-i} - \beta_2 CPI_{t-i} + \beta_3 TTrade_{t-i} - \beta_4\, LPI_{t-i} + \beta_5\, GFCF_{t-i} + \varepsilon_t \quad [3.12]$$

Alternatively, [3.12] can be summarised as:

$$FDI = f\ (GDP, CPI, Open, LPI, GFCF, e_t), [3.13]$$

where GDP = Economic Growth

CPI = proxy of inflation (Consumer Price Index)

OT = Openness to trade

LPI = labour Productivity Index

$GFCF$ = proxy for Domestic Investment (Gross Fiscal Capital Formation)

The variables, CPI, GDP, GFCF OT and LC represent the determinants of FDI inflow. It is similarly common in the literature of underdeveloped or developed economies to replace the domestic country GFCF (Gross Fiscal Capital Formation) with domestic investment (Kiiru, 2014).

3.5.2 Optimal Lag Length Selection

To select the optimal lag length, information criteria such as the AIC and SBIC are used. The two criteria are discussed in subsequent sections. The best model is the one that maximizes linear regression (LR) or minimizes the information criterion. If AIC and SBIC suggest the contradictive lag length, SBIC criterion is preferred according to literature. The reason is that SBIC delivers the correct model with fewer lags, while on average AIC will choose a model with too many lag orders. Otherwise the Hannan Quinn criterion is used to make final decision.

The Akaike Information Criterion is computed using the formula:

$$AIC = 2k - 2In(\hat{L}) \quad [3.14]$$

Bayesian Information Criterion is calculated as:

$$SBIC = -2In(\hat{L}) + K(ln)(n) \quad [3.15]$$

Hannan and Quinn Criterion is calculated as:

$$HQC = nlog(\hat{\sigma}^2{}_{\epsilon}) + 2klog(n) \quad [3.16]$$

where $ln = log$ of the likelihood function, $T =$ number of observations, n is the sample size K the number of repressors including intercept and $\hat{L}$ is the maximum value of the likelihood functions of the model.The goal is to select a model with the least associated criterion.

3.5.3 Model Identification

Upon specification of the model, maximum likelihood method is used to estimate the parameters. As highlighted in previous sections, this parameter estimation method is capable of accommodating nonlinearity in the data as opposed to the ordinary least squares estimation method. According to Dufour and Stevanovic (2013), the computation of the VARMA model is demanding, especially when the VAR structure is simple. The authors cautioned about the complications associated with the MA part to represent procedure that makes this task even more difficult, for approximating VARMA models. The following section describes the procedure used for estimating the *p* and *of* the ACF and the PACF:

$$ACF(s,t) = \frac{E(x_t - u_t)(x_s - u_s)}{\sigma_t \sigma_s}, \quad [3.17]$$

$$PACF = \frac{Covariance\ (x_t, x_{t-k} | x_{t-k-1})}{\sqrt{variance\ (x_t | x_{t-k-1}) variance\ (x_t | x_{t-k-i|} x_{t-k-1})}}, \quad [3.18]$$

where *s* and *t* are values of the process correlating at different times as a function of the two times or the time lag, where E is the expected value operator. Theoretically, the value of ACF lies between -1 and -1 with 1 representing a significant association and -1 indicating anti-autocorrelation. The value of a PACF is indicated otherwise. The ACF and the PACF are used in the identification of the model as being seasonal or non-seasonal..

The identification and selection of the models was completed using both the graphical measures ACF and PACF plots to select the values of *d* and then *p* and *q* in the VARIMA (*p, d, q*) model and information criteria. The reader may refer to Dufour and Pelletier (2013) for a clear identification of the model. One could identify the scalar component with a view of examining embedded structures fundamental to a VARMA (p,q) process. For instance, suppose K dimensional VARMA (p,q) process exists, Dufour and Pelletier (2013) modelled the VARMA (p,q) as follows:

$$\emptyset_0 y_t = \emptyset_1 y_{t-1} + \cdots + \emptyset_p y_{t-p} + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots \theta_q \varepsilon_{t-q} + \varepsilon_t (\varepsilon_t) \sim WN(0, \varepsilon_0) \quad [3.19]$$

with$\varepsilon_t$denotes a white noise process which is assumed to be normally distributed, $\varepsilon_t \sim N(0, \sum \varepsilon_t)$. As Lutkepoh (2005) states, on a given a sample$y_1, \ldots, y_t$and pre sample values$y_0, \ldots, y_{p-1}, \varepsilon_0, \ldots, \varepsilon_{q-1}$ the log likelihood function of VARMA model is given as a:

$$l(\vartheta) = \sum_{t=1}^{T} l_t(\vartheta), \qquad [3.20]$$

where ϑ represents vector of all parameters to be estimated, hence:

$$l_t(\vartheta) = -\frac{K}{2}\log 2\pi - \frac{1}{2}\log det \sum \varepsilon_t - \frac{1}{2}\varepsilon_t^{\prime \Sigma \varepsilon^{-1}{}_t \varepsilon_t}, \qquad [3.21]$$

$$\varepsilon_t = \theta^{-1}{}_0(\emptyset_0 y_t - \emptyset_1 y_{t-1} - \cdots - \emptyset_p y_{t-p} - \theta_0 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q}). \qquad [3.22]$$

Luktepoh (2005) imposed a unique restriction of a VARMA process whereϑ contain free varying parameters only. The author assumed that the initial parameters are fixed. A replacement with zero may be done provided$\varepsilon_t(t \leq 0)$are not available. This could be done without affecting the asymptotic properties of the estimators. If$l(\vartheta)$ is maximised, the results are optimised non-linearly and this is complicated by the indifference in the constraints that insure inevitability of the MA process. The first step entails the unrestricted VAR model of the order$h_T$with the ordinary least squares estimation. Denoting the estimated residualby$\widehat{\varepsilon_t}$, the VARMA model form can be estimated when all laggedε_t's are replaced by estimating residual from the long VAR. The resulting parameter estimates can be used as a starting values for an interactive algorithm.

3.5.4 VARMA Model Selection

The identification and selection of the models was done using both the graphical measures and information criteria.This section is preceded by the estimation of model parameters discussed above. The study used the ACF and PACF to help identify the competing models. Theoretical characteristics of the ACF and PACF were referred to when making a decision about the appropriate model(s) to fit.

The choice was between $VARIMA\ (p, d, q)$, $VAR\ (p, d, q)$ and $VMA\ (p, d, q)$ class models. The values of p, d and $q$werereplaced with the lag length selected and number of differencing imposed on the series. Furthermore, minimum information criteria discussed in Section 3.5.2 were used to select the appropriate model with least number of lags.

3.5.5 VARMA Model Parameter Estimation

As highlighted in previous sections, this parameter estimation is capable of accommodating nonlinearity in the data as opposed to the ordinary least squares estimation method. According to Dufour and Stevanovic (2013), the computation of a VARMA model is problematic more specifically when the variables have a simple VAR structure. A VARMA Model process is written as follows:

$$Y_t = \delta + \sum_{i=1}^{p} \emptyset_t y_{t-1} + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-1} \quad [3.23]$$

Or $\emptyset(B)y_t = \delta + \theta(B)\varepsilon_i$,

where $\emptyset(B) = \iota_k - \sum_{i=1}^{p} \phi_i B^i$ and $\theta(B) = \iota_k - \sum_{i=1}^{q} \theta_i B^i$.For Stationary and inevitability of VARMA process, the roots of$|\theta(z)| = 0$ and $|\phi(z)| = 0$ are outside the unit circle. Given normality assumption, the ε_t have a mean vector and non-singular covariance matrix $\sum$, considering conditional (estimated log likelihood function of a VARMA (p, q) model with mean zero. defined as follows:

$Y = (y_t, \dots, y_T)'$ and $E = (\epsilon_t, \dots, \epsilon_T)'$ with $B'Y = (y_{t-1}, \dots, y_{T-1})$ and [3.24]

$B'E = (\epsilon_{t-1}, \dots, \epsilon_{T-1})$ define $Y = vec(y')$ and $e = vec(E')$ [3.25]

$$Y = \sum_{i=1}^{p} (\iota_T * \phi_i) B^i Y = e - \sum_{i=1}^{q} (\iota_T * \theta_i) B \quad [3.26]$$

where $B'Y = vec[(B^iY)']$ and $B'e = vec[(B^iE)']$

3.6 Cointegration Test

The analysis was progressed with the Johansen (1991) cointegration method. Due to the nature of the study, for cointegration methods to be effective, same order of integration must be evident in the variables. Johansen's (1988) test procedure exploits this relationship for identifying the number of parallel association between the variables. It is advisable to evaluate whether a collection of a time series have same order of integration. The expectation is that at least one stationary linear combination of these series exist (Dickey and Fuller, 1979). Testing for cointegration implies investigating the possibility of a long run association among time series variables.

3.6.1 The Johansen Cointegration Method

As mentioned before, this study used the Johansen and Juselius (1990) cointegration as one method for data analysis. In principle, the Johansen framework for co-integration is a multivariate unit root test which estimates the contegration rank r given several variables. Johansen test further evaluate the links and coefficients of the variables (Udah, 2012).

To achieve the objective of determining cointegration between variables, the error correction factor was factored in the VARMA model. In particular, EC-VARMA was be used in determining the amount of discrepancies between the long-run and short-run relationships. Non-stationary variables were tested for the number of significantly nonzero eigenvalues of the $(m\ x\ m)$ matrix in π in:

$$\Delta x_t = \pi_0 + \pi x_{t-1} + \textstyle\sum_{i=1}^{p} \pi_i \Delta x_{t-1} + \varepsilon_t \qquad [3.27]$$

The Johansen cointegration test statistic include the trace and the maximum eigenvalue statistic defined as:

$$\lambda_{trace}(r) = -T \textstyle\sum_{i=1}^{m} \log(1 - \hat{\lambda}_i), \text{ and} \qquad [3.28]$$

$$\lambda_{max}(r, r+1) = -Tlog(1 - \hat{\lambda}_{r+1}), [3.29]$$

where T is the number of practical observations and $\hat{\lambda}_i$ are the approximated characteristics roots obtained from the estimated π matrix in decreasing order. The trace statistic tests the hypothesis of less or equal to r distinct cointegrating vectors versus$m$ cointegrating relations that is stationary in levels. Conversely, the maximum eigenvalue test evaluate the null hypothesis r cointegrating vectors in favor of the alternative hypothesis $r + 1$ cointegrating vectors. Neither of these test statistics follow a chi-square distribution in general. The following are statements representing the said hypothesis:

$H_0 : \hat{\lambda}_i = 0$ meaning there is no cointegrating equation

$H_1 : \hat{\lambda}_i \neq 0$ meaning there is at-least one cointegrating equation

The estimated characteristics roots are from zero. The more negative is $\log(1 - \hat{\lambda}_i)$, the larger is the trace statistics. According to Hjalmarsson and Osterholm (2007), even though the Johansen's test confirms that all variables in the system are I(1), using stationary factors in the structure is hypothetically not a concern. On the other hand, Johansen (1995) hinted that there is no prerequisite to pretest the factors in the system to establish their order of integration. Johansen hinted that if a single variable is I(0) instead of I(1), this will reveal itself through a cointegrating vector which is determined by only stationary variables in the model. Thus, if the Maximum Eigenvalue and the Trace statistic values are less than the 5% critical value, the study concludes that there occurs a long-run association between the factors.

3.6.2 Long-run relation

The review of the VARMA class model was guided by Dufour and Stevanovic (2013), Lutkepohl (2004) and Chatfield (2000). As discussed earlier, any change in the data may also alter the changes in the model. Adopting these authors suggestion the following multivariate model was suggested:

$$FDI_t = \beta_0 + \beta_1 GDP_{t-i} + \beta_2 CPI_{t-i} + \beta_3 TTrade_{t-i} + \beta_4\, LPI_{t-i} + \beta_5\, GFCF_{t-i} + \varepsilon_t \quad [3.30]$$

$$logFDI_{t-i} = \beta_0 + \beta_1 GDP_{t-i} + \beta_2 CPI_{t-i} + \beta_3 logTTrade_{t-i} + \beta_4\, logLPI_{t-i} + \beta_5\, logGFCF_{t-i} + \varepsilon_t \quad [3.31]$$

3.6.3 Short- run relation

Specifically, the study applied a multivariate technique EC-VARMA model to impose a balance between long-run and short-run dynamics. The equilibrium error correction (EC) factor was blended with the VARMA model (EC-VARMA) for the purpose of obtaining balance in interrelations between variables. The variation in the related variables characterises short-term elasticity, whereas the coefficient of the error correction term (ECT) signifies the speed of adjustment back to the long-run association among the variables. The EC-VARMA is shown below:

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2 (y_{t-1} - y_{xt-1}) + \mu_t \qquad [3.32]$$

The ECT is given by$y_{t-1} - y_{xt-1}$. The implied coefficient on x_{t-1} suggest a proportional long run relationship between$x \; and \; y$, where y is supposed to change between $t-1$ and t as a results of change in the values of the explanatory variables x. The ECT would appear without any lag for this would imply that y changes between $t-1$ and t response to a disequilibrium at time. γ Defines the long run relationship between $x \; and \; y$. β_2 describes the short run relationship between changes in $x \; and \; chages \; in \; y$ while β_1 describes the speed of adjustment back to equilibrium, and its strict definition is that it measures the proportion of the last period equilibrium error that is corrected. To achieve the objective of determining cointegration between variables, the error correction factor was factored in the VARMA model. In this instance, EC-VARMA is used in determining the amount of discrepancies between the long-run and short-run relationships. The study obtain the EC-VARMA form representation from VARMA equation (3.15) as follows:

$$\emptyset_0 \Delta y_t = \pi y_{t-1} + \omega_1 \Delta y_{t-1} + \cdots + \omega_p \Delta y_{t-p} + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots \theta_q \varepsilon_{t-q} + \varepsilon_t, [3.33]$$

where $\pi = -(\theta_0 - \theta_1 - \cdots - \theta_q), and \; \omega_i = \emptyset_{i+1} + \cdots + \emptyset_p) \; for \; i = 1 \ldots p-1$

3.7 Diagnostic Testing

In this section the best model was subjected to a battery of diagnostic tests. The assumptions associated with the residuals of the error term of the models were assessed for autocorrelation using covariance matrix and stability cumulative sum (CUSUM) respectively. Positive results from this section might imply that the model(s) is/are good and may be used for further analysis.

3.7.1CUSUM Test

In this study the CUSUM test was used to test the stability of the model in the regression. The CUSUM test was used as follows:

$$C_i = \sum_{j=1}^{i} \bar{x}_j - T. \qquad [3.34]$$

The appropriate hypotheses state as:

H_0: The model is not stable

H_a: The model is stable

The CUSUM test is classified into three limits, the center line, the upper control limit and the lower control limit defined as follows:

Upper control limit

$$C_i^{+} = \max[0, X_i - (T + K) + C_{i-1}^{+} \qquad [3.35]$$

Lower Control Limit

$$C_i^{-} = \max[0, (T + K) - X_i + C_{i-1}^{-}] \qquad [3.36]$$

If the line goes beyond any of the limits, the model is declared unstable.

3.7.2 Normality Test

To confirm assumptions that each variable in the series is randomly draw from a population with zero mean and constant variance, the study used the Jacque-Bera (JB) test. This was also done to confirm that model residuals are normally distributed. The JB test checks if the sample data have the skewness and kurtosis matching the normal distribution. The hypotheses and test are defined as follows:

$$H_O: The\ data\ is\ normally\ distributed$$
$$H_1: The\ data\ is\ not\ normally\ distributed$$

$$JB = \frac{n}{6\left(S^2+\frac{K^2}{4}\right)}, \qquad [3.37]$$

where n is the sample, S is the sample skewness, C is the sample kurtosis and K is the number of repressors:

$$JB \sim {X_a}^2(n), \qquad [3.38]$$

${X_a}^2(n)$ is the chi-square probability distribution function and α is the degree of freedom of the chi-square distribution. The JB test has an asymptotic chi-square distribution with two degrees of freedom. This normality test was followed by the autoregressive conditional heteroscedasticity (ARCH) test.

The two tests are used to check if the identified model with suggested variables fit significantly better with more or less predictor variables. In econometrics, ARCH test is used to model observed time series data. In particular, the ARCH test assumes the variance of the current error or innovation to be a function of the actual size of the previous time periods error terms. The following hypotheses statements are formulated and tested:

$$H_{0:}\rho_1 = \rho_2 = \cdots \rho_m = 0$$

$$H_1: \rho_k \neq 0 1 \leq k \leq m$$

In the above hypothesis statements, ρ is the population autocorrelation for the squared time series (i.e $y_t = x_t^2$) and m is the maximum quantity of lags included in the ARCH effect. The ARCH test is used to characterise and model observed time series. The study aimed to identify the impact of the current time series on future time series values using an ARCH process. The ε_t denotes the error terms (return residual) with respect to a mean process) *i.e* the series terms. The ε_t are divided into a stochastic piece z_t and a time dependent standard deviation σ_t characterising the typical size of the terms so that $\varepsilon_t = \sigma_t z_{t.}$
The random variable $z_t$follows astrong white noise process. The series $\sigma^2{}_t$ is modelled by:

$$\sigma^2{}_t = \beta_0 + \beta_1 \varepsilon^2{}_{t-1} +, \ldots, + \beta_3 \varepsilon^2{}_{t-q} = \beta_0 + \sum_{i=1}^{q} \beta_i \varepsilon^2{}_{t-i}, \quad [3.39]$$

where $\beta_1 > 0$ *and* $\beta_i \geq 0, i > 0$. An ARCH (q) model can be estimated using ordinary least squares. A methodology to test for the lag length of ARCH errors using the Lagrange multiplier test was proposed by Engle (1982).

3.7.3 Portmanteau Test

The portmanteau Q statistic was used in this study to determine the adequacy of the model. The following hypotheses were tested:

$$H_o: The\ residuals\ are\ uncorrelated$$
$$H_o: The\ residuals\ are\ correlated$$

where $C_\varepsilon(\iota)$ be residual cross – covariance matrices and$\widehat{p_\varepsilon}(\iota)$ be residual cross correlation matrices as:

$$C_\varepsilon(\iota) = T^{-1} \sum_{i=1}^{T-1} \varepsilon_t \varepsilon'{}_{t+\iota} \text{ and} \quad [3.40]$$

$$\widehat{p_\varepsilon}(\iota) = V^{-\frac{1}{2}}{}_\varepsilon C\varepsilon(\iota) \widehat{V_\varepsilon}^{-1/2} \text{ and } \widehat{p_\varepsilon}(-\iota) = \widehat{p_\varepsilon}(\iota) \quad [3.41]$$

$\widehat{V_\varepsilon} = Diag\left(\widehat{\vartheta_{ll}}^2, \dots, \widehat{\vartheta_{kk}}^2\right)$ and $\widehat{\vartheta_{ll}}^2$ are the diagonal elements of $\widehat{\Sigma}$

The Portmanteau test defined in Hosking (1980) is:

$$Q_s = T^2 \sum_{i=1}^{s} (T-1)^2 tr\,\{p_\varepsilon(0)^{-1} p_\varepsilon(0)^{-1}\} \quad [3.42]$$

The statistic Q_s has roughly the chi-square distribution with $k^2(S-P-q)$degree of freedom. If the decision rule is set as 5% significant level, and the probability value is greater than 0.05, the model is considered adequate.

3.8 Causality Test

Toda-Yamamoto causality test process delivers the likelihood of testing for causality amongst combined variables based on asymptotic theory. Toda and Yomamoto (1995) augmented Granger causality test method is based on the following equation:

$$Y_t = a + \sum_{i=1}^{h+d} B_i Y_{t-1} + \sum_{j=1}^{k+d} \gamma_i X_{t-j} + u_{yt} \quad [3.44]$$

$$X_t = a + \sum_{i=1}^{h+d} \theta_i X_{t-i} + \sum_{j=1}^{k+d} \delta_i Y_{t-j} + u_{xt} \quad [3.45]$$

where d is the maximal order of integration of the variables in the system , h and k are the optimal lag length of Y_t and X_t, and $u_{yt}$are error terms that are assumed to follow a white noise with zero mean, constant variance and no autocorrelation. Thus by determining the maximum order of integration d, which occur in the model and the constructed model in levels.

$$H_{0:} \sum_{j=1}^{k} \gamma_i = 0 \;\; or\; X_t\; does\; not\; cause\; Y_t$$

$$H_{1:} \sum_{j=1}^{k} \gamma_i \neq 0 \text{ or } X_t \text{ cause } Y_t$$

$$F = \frac{(RSS_{RX} - RSS_{UY})/k}{RSS_{UX}/(N-K)}, \qquad [3.46]$$

where k is the number of estimated coefficients, when the added$F$ value the critical F value, reject the null hypothesis and the conclusion is that X_t causes Y_t, or Y_t causes X_t.

3.9 Forecasting

Forecasts are generated from competing models and an optimal model is selected based on the amount of forecast errors generated. The three forecast error measures are used to determine the predictive influence of the models. The model whose associated error metric is less than others is recommended. The suggested minimum forecast error measures for this study are Mean Percentage Error (MPE), Mean Squared Error (MSE) and the Sum of Squared Error (SSE).

The estimated model was used to obtain forecasts and the confidence limits of the forecast. After obtaining the estimated VARMA model (3.11), forecasting was done. Following Lutkepohl (2004, 2005), forecasts of the VARMA process in (3.11) are obtained from pure VAR form:

$$\hat{y}_{t+h|t} = \sum_{i-1}^{h-1} \theta_i \hat{y}_{t+h-i|t} + \sum_{i=1}^{\infty} y_{t+h-i} \emptyset_i \qquad [3.47]$$

where $\hat{y}_{t+h|t}$ is the optimum prediction based on known coefficient and the two terms on the right side of the equation uncorrelated up to period t are used for approximation.

According to Chatfield (2000), the error forecasting metric, minimum mean square error (MMSE) is willingly be calculated for VAR and VARMA models by a natural extension of methods employed for univariate. Krieger (1986) recommended the use of VARMA models in forecasting due to its ability to substantially outperform unrestricted vector autoregressive (VAR) models based on forecasting accuracy. There are number of forecasting measures that entail how accurately a forecast is for a particular model that were used, these include:

The mean error is given by:

$$ME = \frac{1}{n}\sum_{i=1}^{n} \hat{e}_t \qquad [3.48]$$

The Mean Absolute Deviation is given by:

$$MAD = \frac{1}{n}\sum_{i=1}^{n} \hat{e}_t \, |Yi - \ddot{Y}| \qquad [3.49]$$

The Mean Square Error is given by:

$$MSE = \frac{1}{n}\sum_{i=1}^{n} \widehat{e^2}_t \qquad [3.50]$$

The Mean Percentage Error is given by:

$$MPE = \left(\frac{\frac{1}{n}\sum_{i=1}^{n}|\hat{e}_t|}{y_t}\right) * 100 \qquad [3.51]$$

The mean absolute percentage error is given by:

$$MAPE = \frac{\frac{1}{n}\sum_{i=1}^{n}\hat{e}_t}{y_t} * 100 \qquad [3.52]$$

These measures the overall forecast t bias and is one component of the accuracy. It is worth checking the bias for selected subsets of the data, as well as for all the data. For example, with FDI data, it is worth checking to see if there is a bias of one sign during economic expansion and a bias of the opposite sign during economic contraction. The multivariate VARMA class of models was developed upon realisation of the successful performance of the univariate ARMA model in forecasting. Forecasts produced from several number of interrelated variables are more precise compared to univariate case (Fackler and Krieger, 1986). The MSE approach to forecasting (MMSE) in multivariate time series model is denoted by:

$$\hat{X}_N(h) = \emptyset_1{}^{h-1}\hat{X}_N(p)[3.53]$$

$$\hat{X}_N(p) = \emptyset_1\hat{X}_N + \theta_1 z_N, \text{for } h=2, 3... \qquad [3.54]$$

The presence of these formulae undertake comprehensive information of the model, plus the values of the model parameters, and also undertake that the white noise series is known precisely (Chatfield, 2000). The model parameters thus have to be estimated and the white noise process has to be concluded from the forecast errors. Forecasts are generated from competing models and an optimal model is selected based on the amount of forecast errors generated.

3.10 Summary

The study applies the proposed methods in classifying and approximating the VARMA models and further diagnose the adequacy of the identified class of models using the portmanteau test ARCH test and the CUSUM test. In this study, the EC-VARMA models and VECM estimated. The Johansen cointegration framework is adopted. The study applied the Box-Jenkins methodology which follows four steps into modelling the time series VARMA model. The following steps are followed; model identification, estimation, diagnostics checking, and forecasting. This study subsidises to the increasing body of literature on the identification and estimation of VARMA models. The models are likely to be used more broadly in macroeconomic modelling and forecasting. The next chapter provides the results produced from the methodology as discussed.

Chapter 4

Data Analysis and Results

4.1. Introduction

This chapter presents data analysis with the aid of statistical software such as SAS 9.4 and Eviews 8 in order to execute the analysis. The first section outlines the descriptive statistics, Section 4.1outlinesthe stationarity process. This section further discuss the preliminary data analysis results through plots and other time series tests. The main section of this Chapter discusses the results from the VARMA, EC-VARMA model and diagnostic test results. These results are presented as figures and tables. The results from the data analysis are intended to achieve the objectives stated in section 1.3.

The reminder of this chapter is rearranged as follows: Section 4.2 provides the results for preliminary data analysis and Section 4.3 discusses the primary data analysis results. Section 4.4 gives and discusses the diagnostic tests and the forecasts are presented in Section 4.5.

4.2. Preliminary Data Analysis

This section describes the preliminary data analysis through describing the time series data and preparing the data for main analysis.

4.2.1 Data Description

Table 4.1 reports the summary statistics of the variables used in the study. The time series data used in this study contains a set of six variables each with56 quarterly observations collected from South African Reserve Bank (SARB). The data was subjected to logarithm transformation to obtain the descriptive statistics summarised in Table 4.1.

Table 4.1 Descriptive Statistics

Variable	N	Mean	Standard Deviation	Min	Max
FDI	56	8.75652	2.68879	5.18180	11.61000
CPI	56	7.24643	6.16763	-1.30000	20.50000
GDP	56	24.12143	2.23994	20.50000	29.30000
LP	56	4.58251	0.10058	4.39938	4.72206
OT	56	10.78534	0.35210	10.12280	11.34920
GFCF	56	13.11826	0.23512	12.58630	13.37360

Source: Authors' own calculation

Table 4.1 contains information that is useful in understanding the nature of the data and descriptive qualities. The number of cases is recorded on Row N showing 56 observation. The information about the range of variables is contained in the Minimum and Maximum columns showing that FDI range from 5.18% to 11.61% where CPI range from -1.3% to 20% between the year 2002 and 2016 on quarterly basis. The variability is assessed by examining the values of the standard deviation column. The standard deviation measures the amount of variability in the distribution of the variables which differ from the mean. Thus the more the individual data points differ from each other, the larger the standard deviation is and vice versa. Thus, FDI value of 2.68% is one standard deviation bellow the mean 8.75% and GFCF is one standard deviation 0.24% below mean of 13.11%. Nevertheless, examining the difference in variability can be useful in making further analysis. Therefore it is clear that there is much greater variability in GFCF compared to FDI.

4.2.2 Seasonality Test Results

This section presents the ACF and PACF plots to determine the presence or absence of seasonality. The results are summarised on Figure 4.2

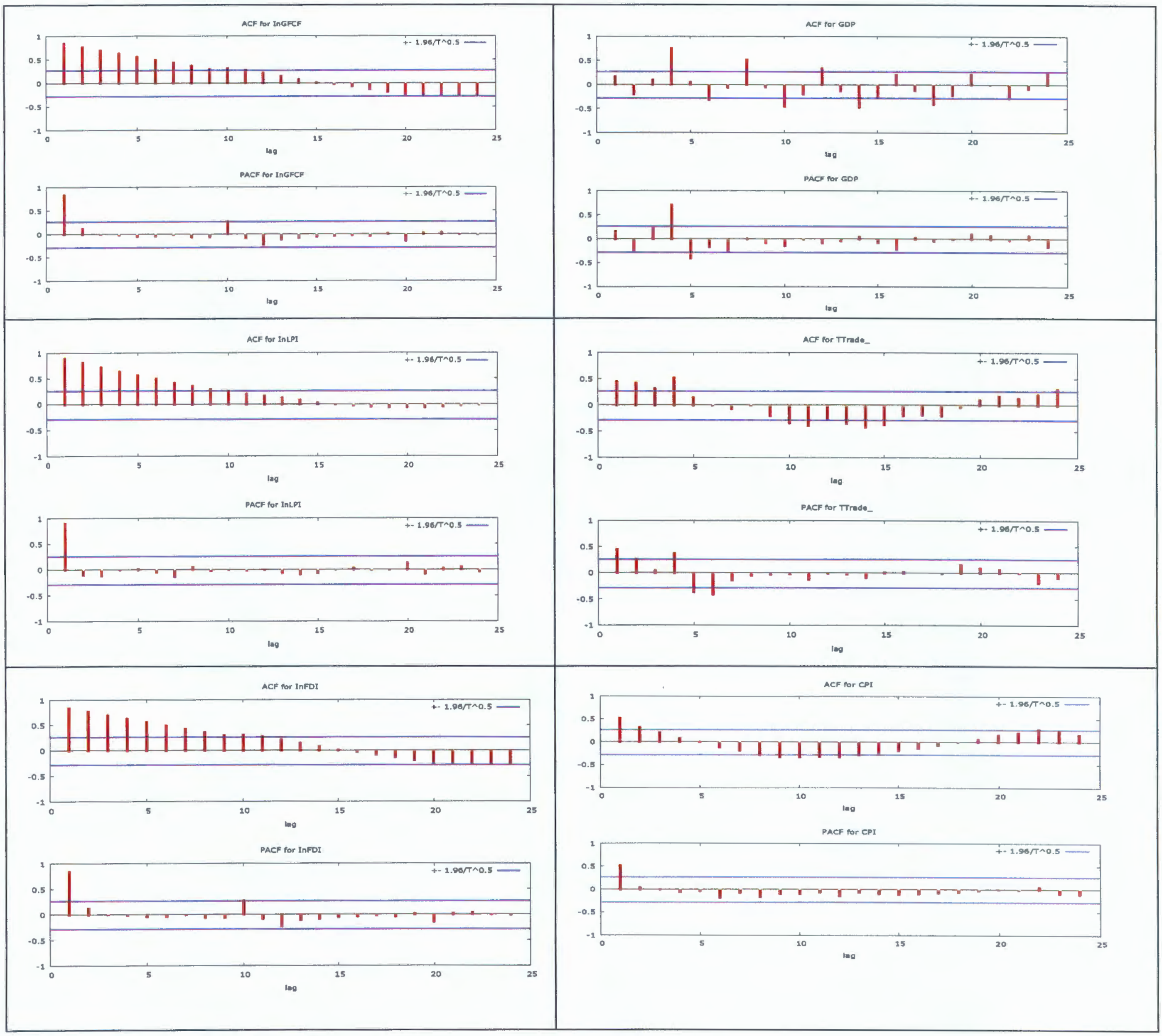

Figure 4.1 Seasonality test of six variables

Figure 4.1 is a depiction of the correlogram of the six variables. The series depicts non seasonality in the data from lag 4 to 16 as suggested by Box and Jenkins (1979). Non-seasonality is illustrated by up and down movement in the ACF with 24 time lags (t). Hence, the variables are deemed non-seasonal with time. Although the sample autocorrelations contain random fluctuations for moderate sample sizes, they are fairly accurate in signaling the order of the ARIMA model. This random movements are also an indication of the presence of unit root. According to Box and Jenkins (1979), the series is considered stationary if it cuts off fairly quickly at the non-seasonal level or near non-seasonal lags, otherwise they are considered non stationary.

4.2.3 Stationarity test results

The next step is to plot the series and confirm stationarity or lack-of-it thereof. The ADF and KPSS tests were calculated and the findings are summarised on Figure 4.2.

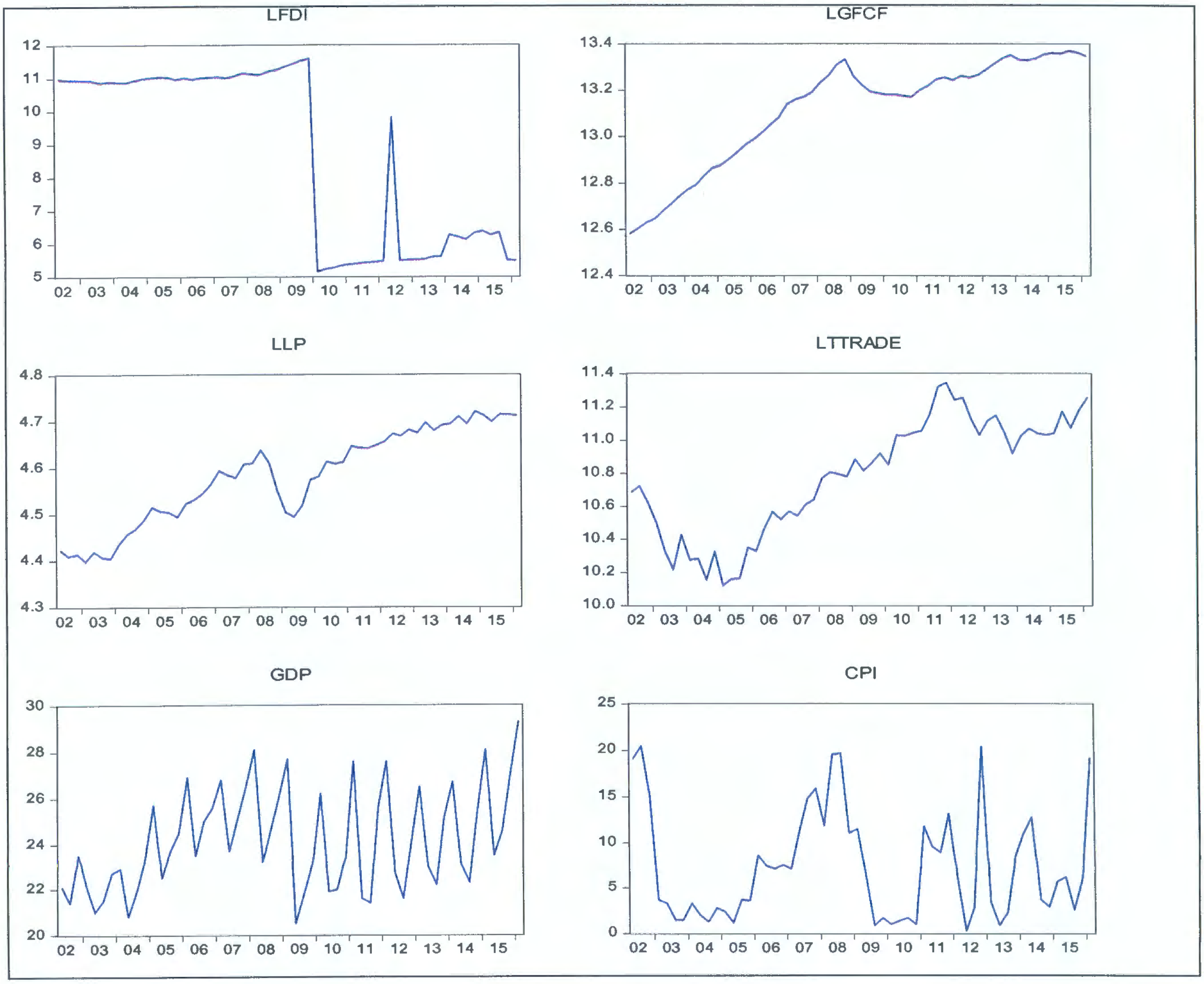

Figure 4.2 Time series plots at Level

The results in Figure 4.2 show a fluctuation in the CPI. This movements could be due to volatility associated with this sector .It is also noticed that GDP has some time epochs for each year. This sector was at its lowest in the year 2009 according to the results. Accordingly, FDI was high between 2002 and 2008 and a sudden drop was observed from 2009 just few months after the global financial crisis of the USA. South Africa experienced recession in 2009 which saw the rest of the country losing reputation compelling most investment companies to withdraw from entering into investment contracts. As a result, the country was unable to take advantage of opportunities to trade with other countries. The movement of GFCF might be due to high risks associated with investments, high interest rates and excessive supply of capital goods. Furthermore, GDP was moving at a decreasing rate for the selected period, which might be caused by low performance of the related macroeconomic indicators such as those mentioned above (Charkrabarti, 2001).

Labour productivity and OT experienced similar movements that are also non-stationary. Moreover, the movement in LP might be due to a decrease in wage, technological change (machines and materials), cheap labour, less protection to worker because of policies. The movement of this series might be due to the country's economic policy that restrict trade between South Africa and other countries (Parletun, 2008).

As Figure 4.1 stands, there is no sign of stationarity with constant mean and variance over time. Therefore, as time series rule dictates, the data are unpredictable and cannot be modelled, with the proposed framework. This limits the applicability of the VARMA class models, hence the data was subjected to first difference transformation. The results of the transformed data are summarised in the form of unit root test in Table 4.2.

Table 4.2. The ADF Test

Variables	Equation	t Statistics	1%	5%	10%	Durbin Watson	Decision
CPI	Trend and Intercept	-6.84	-4.14	-3.50	-3.18	2.00	Reject null hypothesis
GDP	Trend and Intercept	-10.30	-4.14	-3.50	-3.18	2.00	Reject null hypothesis
FDI	No Trend & Intercept	-6.04	-4.14	-3.50	-3.18	2.01	Reject null hypothesis
OT	Trend and Intercept	-4.85	-4.14	-3.50	-3.18	2.00	Reject null hypothesis
GFCF	No Trend & Intercept	-3.85	-4.14	-3.50	-3.18	2.00	Reject null hypothesis
LPI	No Trend & Intercept	-4.31	-4.14	-3.50	-3.18	2.00	Reject null hypothesis

Source: Authors' own calculations

The depiction of the ADF test results in Table 4.2 rejected the null hypothesis of unit root for all the time series at their first differences. The observed t-statistics are lesser than 1%, 5% and 10% levels of significance. The KPSS test results in Table 4.3 and time series plots in Figure 4.3 agrees with this decision. Consequently, all the variables are stationary and integrated of the same order, i.e., *I* (1). The Durbin Watson test also shows that there is no problem of autocorrelation in the series. These results allow the study to proceed to primary data analysis where the VARMA class of models are estimated and the cointegration association amongst the variables was established.

Table 4.3 KPSS Stationarity Test

Variables	Equation	t Statistics	1%	5%	10%
CPI	Trend and Intercept	0.05	0.22	0.15	0.12
FDI	No Trend and Intercept	0.05	0.22	0.15	0.12
GDP	No Trend & Intercept	0.02	0.22	0.15	0.12
GFCF	Trend and Intercept	0.10	0.22	0.15	0.12
LP	No Trend & Intercept	0.04	0.22	0.15	0.12
OT	No Trend & Intercept	0.14	0.22	0.15	0.12

Source: Authors own calculations

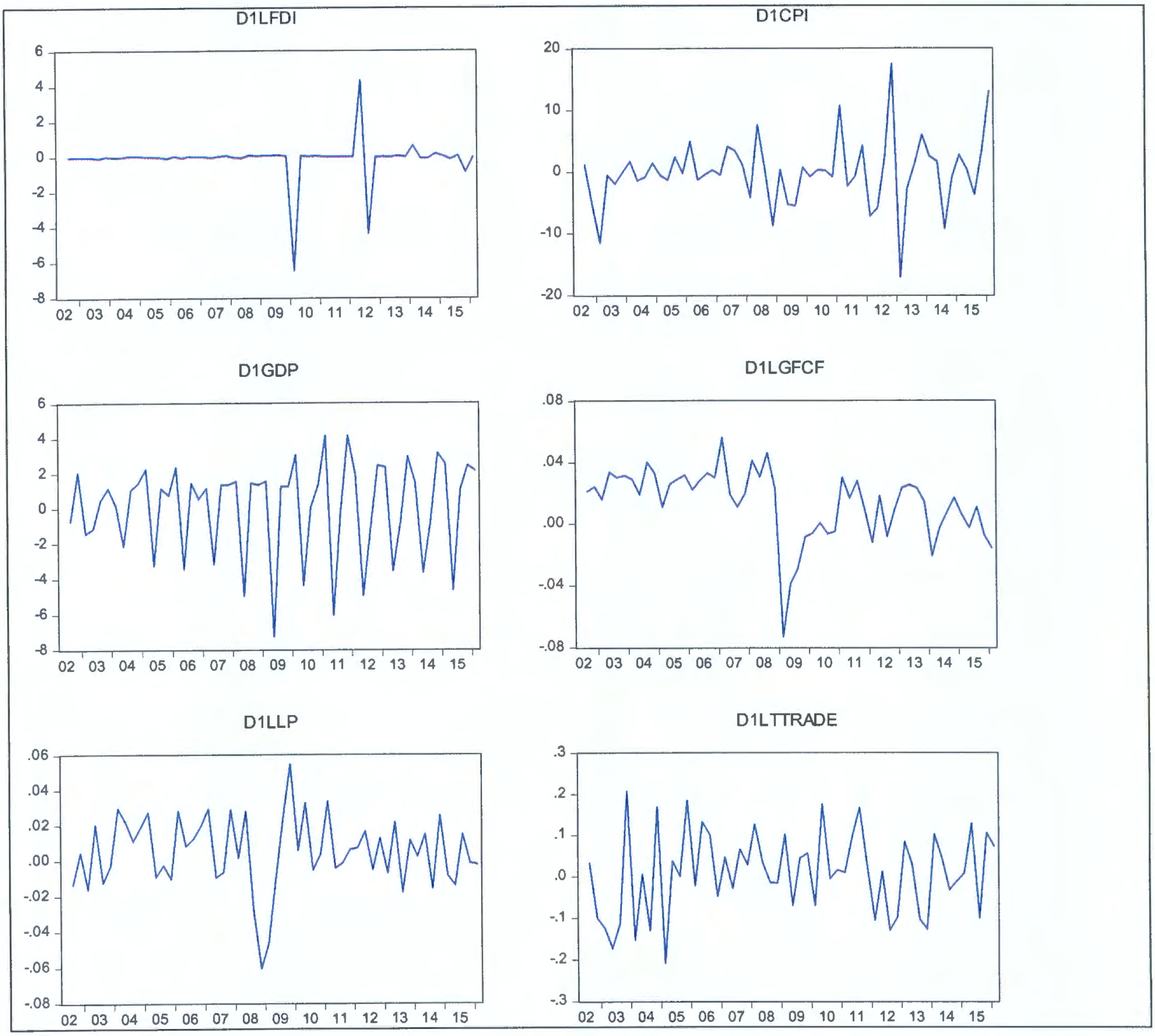

Figure 4.3 Time Series Plots First Difference

4.3 Empirical Findings

This section performs data analysis with respect to study objectives. The four stages of Box-Jenkins procedure are followed in building VARMA class of models. Cointegration methods are also used in this section .Forecasts from the selected model are provided in the final section.

4.3.1 Lag Length selection results

Firstly, the results of the lag length selection criteria discussed in Chapter 3 are interpreted. This enable us to decide on the type of VARMA and EC-VARMA models to estimate. The results of lag length selection criteria are summarised in Table 4.

Table 4.4 Lag length selection

Lag	AIC	SC	HQ
0	8.916351	9.145794	9.003725
1	1.223921	2.830021*	1.835533
2	0.807799	3.790555	1.94365
3	0.636841	4.996254	2.296931
4	-1.156204	4.579865	1.028124
5	-2.339539	4.773187	0.369028
6	-5.132718*	3.356664	-1.899913*

Source: Authors own calculations

As shown in Table 4.4, the SBIC and HQ select lag 1 as optimal lag length and AIC is in favor of lag 6. According to theory, the SBIC overrules the AIC and as a result lag 1 is chosen and considered in this study. This lag length together with the order of differencing are later used in the construction of the VARMA and EC-VARMA in preceding section.

4.3.2. VARMA Class of Models Identification

The results in Table 4.5 are for the three models fitted with lag length 1 and first order of differencing. It is evident that VARMA (1,1,0) is an optimal model for the data. This means that the output in preceding sections are based on this model

Table 4.5 VARMA class model of identification

Model	HQC	AIC	SBIC
VARMA(1,1,0)	0.57	0.48	0.70**
VARMA(1,1,1)	0.69	0.60	0.82
VARMA (0,1,1)	0.75	0.65	0.91
VARMA(2,1,0)	0.70	0.59	0.88
VARMA (0,1,2)	0.72	0.61	0.90

Source: Authors own calculations

The parameters of this model are estimated and provided as a summary in Table 4.5. A constant is included in the model and no seasonal variation is accommodated according to Figure 4.2.

4.3.3 Number of cointegrating equations

The study follows Johansen (1988) multivariate cointegration to determine the number of cointegrating vectors. The Maximum Eigenvalue and Trace test are used to accomplish this objective. The Johansen's (1988) test procedure exploits the relationship of FDI by identifying the number of cointegrating associations among the variables. It is crucial to test whether or not the variables are exclusively integrated of the same order and that at least one linear grouping of these variables that is stationary is present (Dickey-Fuller, 1979). Cointegration has a precondition which states that variables must be at levels or non-stationary. In other words cointegration analysis is conducted when the variables are all differenced in the same order. The results for Trace and Maximum eigenvalue are depicted in Table 4.8.

Table 4.6 Cointegration Results

Cointegration Rank Results					
H0:Rank=r	**H1:Rank>r**	**Trace**	**5% Critical Value**	**Max Eigenvalue**	**5% Critical Value**
0	0	160.9804	93.92	182.5451	101.84
1	1	100.1905	68.68	111.4279	75.74
2	2	58.6269	47.21	69.6339	53.42
3	3	26.1508	29.38	32.1744	34.8
4	4	10.9631	15.34	15.37	19.99
5	5	1.6696	3.84	4.8359	9.13

Source: Authors own calculations from FDI and the determinants data

The two test results are less than the related critical values from Rank 3 suggesting the rejection of the hypothesis of no cointegrating association. It is therefore decided that there are at least three cointegrating vectors, proving a long run relationship among FDI and its factors. Based on this outcome, the analysis continues by fitting EC-VARMA model that may be used for development of longer term forecasting of FDI and its determinants.

4.3.4. VARMA Class of Model Results

After cointegration has been determined between the series, it is evident that a long term equilibrium association among the variables is present. An EC-VARMA model was established in this study to assess the speed of adjustment at which FDI return to equilibrium after alteration in the related factors (Moroke, et al., 2014). A natural development from VARMA representation is the EC-VARMA model, thus cointegration rank was tested using the methodology by Johansen (1988). The EC-VARMA model illustrates the variation in the related variables and the characteristics in the short-term resistance, whereas the coefficient of the ECT explains the speed of adjustment in the short-run for the long-run.

The study further employs the VECM to assess the short-run properties of the cointegrated series. The existence of cointegration among the variables suggests their long-term. Table 4.7 gives a summary long-run beta estimates with FDI normalised.

Table 4.7VARMA (1,1,0) Model Long Run Results

Long-Run Parameter Beta Estimates When RANK=3			
Variable	**1**	**2**	**3**
FDI	1.00000	1.00000	1.00000
CPI	-0.00607	0.04011	0.01837
GDP	-0.03086	-0.15916	-0.03374
LP	0.03690	0.04640	-0.08062
OT	-3.35177	-2.74543	-0.97875
GFCF	0.22309	-1.09199	-0.37945
1	-0.82723	11.25708	-2.49539

Source: Authors own calculations

From Table 4.7 the signs of beta parameters estimates of Rank 2 have signs (reversed) consistent with economic theory discussed in Chapter 2 and a specified model in equation (3.11). Therefore this equation is interpreted as it makes more economic sense compared to others. The estimated equation is expressed as follows:

$$InFDI_t = -11.2 + 0.159InGDP_{t-i} - 0.04InCPI_{t-1} + 2.745OTe_{t-i} - 0.046LPI_{t-i} + 1.092GFCF_{t-i}. \quad [4.1]$$

Holding other independent variables (CPI, OT, LPI and GDP) constant, if domestic investment (GFCF), it expected that quarterly foreign direct investment (FDI) would increase by 1.092%. According to the findings, OT and GFCF play positive significant role as far as FDI is concerned. Though GDP was anticipated to positively affect FDI in the long run, this effect is significant. It should also be noted that CPI and LPI are expected to have almost similar negative effect on FDI growth in the long-run. Having obtained this information, it was imperative to examine the effect of the five variables on FDI in the short-run and to also calculate the speed of adjustment due to changes in the system of FDI. The results for EC-VARMA (1,1,0) are summarised in Table 4.8.

Table 4.8EC-VARMA (1,1,0) Model Results

Model Parameter Estimates						
Equation	**Parameter**	**Estimate**	**Standard Error**	**t Value**	**Pr > |t|**	**Variable**
D_FDI	**CONST1**	0.29513	0.04874			1
	XL0_1_1	0.00033	0.00039	0.85	0.3986	CPI(t)
	XL0_1_2	0.00344	0.00098	3.52	0.0010	GDP(t)
	XL0_1_3	-0.00024	0.00110	-0.22	0.8262	LP(t)
	XL0_1_4	0.37669	0.04854	7.76	0.0001	OT(t)
	XL0_1_5	-0.01723	0.01237	-1.39	0.1704	GFCF(t)
	EC1_1_1	-0.14097	0.02328			FDI(t-1)

Source: Authors own calculations

The EC-VARMA (1,1,0) model results proved that GFCF, CPI and LPI have no significant effect on FDI in the short-run. The conclusion was reached due to the insignificant-values corresponding to these variables. The coefficient of the ECT was negative (-0.14097) as expected indicating the speed of adjustment in the short-run for long-run. However, GDP and OT prove to be significant determinants of FDI presently. The results imply that the system corrected its previous disequilibrium period due to its shock in one period at an adjustment speed of about 14.1% per quarter. According to Yafee and MacGee, (2000), model parameter estimates must not be close to 1 to be rendered significant. Judging from the results in Table 4.8, none of the parameters are close to or equal to 1. This implies that the stability condition for EC-VARMA (1,1,0) was satisfied and encouraged the researcher to conclude that this model provided a better fit to the data.

4.4 Diagnostic Test Results

As discussed in Chapter 3, the model should be tested for stability and proper specification prior to using it for further analysis. In this section the residuals of EC-VARMA (1,1,0) were tested for normality, serial correlation and heteroscedasticity. The model was also tested for overall significance prior to causality analysis.

4.4.1 Stability Test Results

The CUSUM plot was used for this purpose and the results are summarised in Figure 4.3

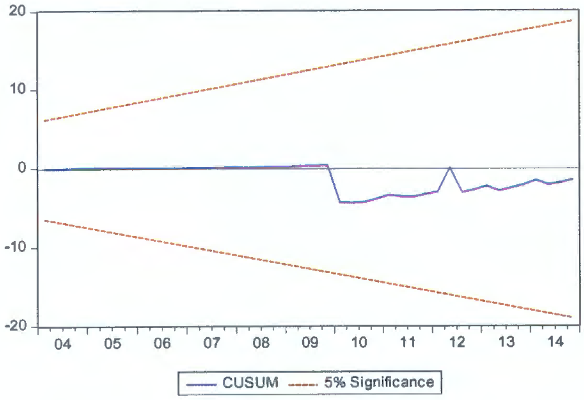

Figure 4.4CUSUM Test

The CUSUM plot illustrates the stability of the EC-VARMA (1, 1, 0) model. The CUSUM line shown in blue colour lie within the dotted 5% significance line in red indicating that stability condition was not violated. Hence the model is considered stable and may be suggested for further analysis. This finding is in support of the results in Table 4.8.

Table 4.9 Serial correlation, normality and homoscedasticity test results

Model White Noise Diagnostics					
Variable	**Durbin Watson**	**Normality**		**ARCH**	
		Chi-Square	**Pr > ChiSq**	**F Value**	**Pr > F**
FDI	2.53110	0.93	0.6275	6.62	0.0130

Source: Authors own calculations

Table 4.9summarises the results indicating how well each univariate equation fits the data. The FDI model residual is not off from normality and also has an ARCH effect. The residuals of this model are not serially correlated according to the Durbin-Watson test which is 2.5311 shows no sign of autocorrelation. The results confirm that EC-VARMA (1,1,0) is a correct fitted model for the data and that this model may be used for further analyses.

4.4.2 Portmanteau Test Results

A multivariate portmanteau test results summarised in Table 4.10 confirm the suitability of the model. The chi-square test for residual cross correlations provide an indication that FDI model with AR parameters of up to lag 2 or even more could be adequate using 10% level of significance.

Table 4.10 Portmanteau Test

Portmanteau Test for Cross Correlations of Residuals			
Up To Lag	**DF**	**Chi-Square**	**Pr > ChiSq**
2	36	107.54	<.0001
3	72	139.16	<.0001

Source: Authors own calculations

Table 4.11 Overall significance test results

Univariate Model ANOVA Diagnostics				
Variable	**R-Square**	**Standard Deviation**	**F Value**	**Pr > F**
FDI	0.5627	0.01473	10.08	<.0001

Source: Authors own calculation

Judging from the value of the R-square of the model, it could be concluded that globally the model is significant and may be recommended to produce forecasts. The observed probability is less than 1%, 5% and 10% significance levels. The results of this model are also not spurious since the R-square is reasonably high. Some of the t-tests are also significant supporting this judgement (Gujarati and Porter, 2010). The model is also proven to be free from spurious correlation judging from the diagnostics performed.

4.5 Causality Test

The results in this section provide an indication of the type of relationship between the variables. The researcher sought to determine if FDI have causal relationship with other variables. This helped in deciding which variable are good predictors of the dependent variable.

Table 4.12 Granger Causality Wald Test Results

Test	Group 1 Variables	Group 2 Variables	DF	Chi-Square	Pr>ChiSqr
1	FDI	GFCF GDP LP OT CPI	5	35.12	<.0001
2	GFCF	FDI GDP LP OT CPI	5	23.35	0.0003
3	GDP	GFCF FDI GDP LP OT CPI	5	27.92	<.0001
4	LP	GFCF GDP FDI OT CPI	5	16.20	0.0063
5	OT	GFCF GDP LP FDI CPI	5	14.27	0.0140
6	CPI	GFCF GDP LP OT FDI	5	3.08	0.6873

Source: Authors own calculations

Toda-Yamamoto Granger causality outcomes presented in Table 4.12 revealed feedback relationship between all variables except for the CPI. A bi-causal association was revealed successively among all variables. This decision was based on the observed probabilities compared with 1% 5% and 10% significance levels. The results implied that all the independent variables are found not to be weakly exogenous in the system of FDI excluding the CPI. This confirms that CPI may not be a significant measure of FDI in the context of South Africa.

4.6 Forecasting EC-VARMA (1,1,0) Model

This section provides the forecasts of FDI from VARMA (1,1,0) model. The results are summarised as Figure 4.4.

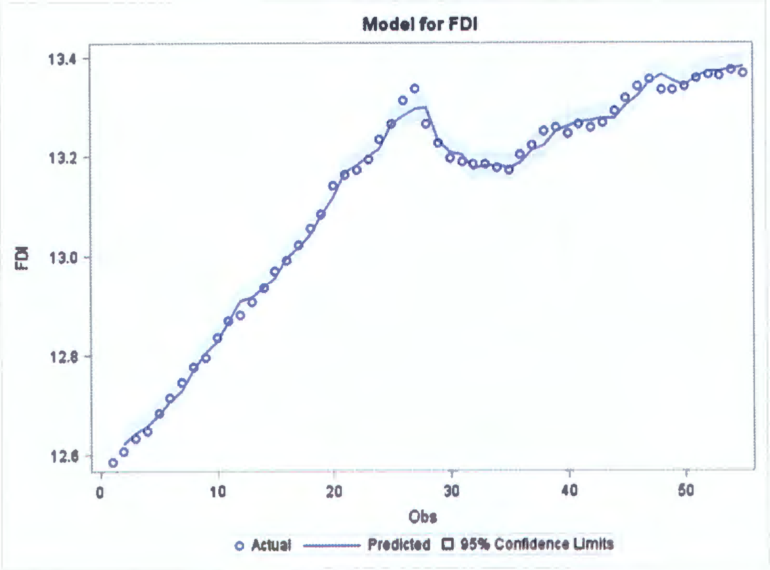

Figure 4.5 Model and forecast for FDI

As illustrated in the figure, the forecasted values moved along with the actual values with time. A conclusion was reached that EC-VARMA models are good for producing forecasts using a set of time series variables. To evaluate the forecast ability of the model, distribution of forecast were plotted. This also helped in investigating the distance between the original data and the errors committed by the model.

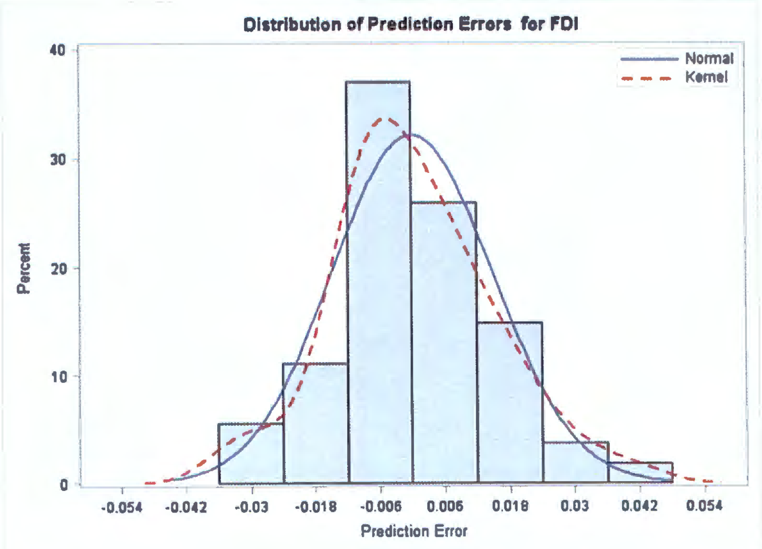

Figure 4.6 Distribution of prediction Errors for FDI perceived

As shown on Figure 4.6, the errors generated by EC-VARMA (1,1,0) model are approximately distributed just like the original series. These are also closer to the original values of FDI confirming the effectiveness and suitability of this model in forecasting FDI.

4.7 Summary

This chapter used methods discussed in Chapter 3 to achieve the intended objectives. Preliminary analyses were done to prepare the data for primary analysis. The data was log transformed in order to stabilise stochastic properties and also to ensure that all the variables are measured in the same scale. The Box-Jenkins methodology was adopted and an enhanced VARMA model was used. The study followed the Johansen cointegration method to investigate relationships between the variables. The Toda-Yamamoto causality test successfully determined which of the variables good predictors of FDI are and which ones are not. Forecast were generated using the selected model.

The next chapter presents the findings to formulate suggestions for policy and further studies based on the findings of the study.

Chapter 5

Summary and Recommendations

5.1 Introduction

The study explored the multivariate VARMA class of models using FDI data South Africa as an experimental unit. Multivariate methods were applied to time series data ranging from 2002 Q2 to 2016Q1. Chapter 5 summarises the findings of this study in line with the objectives. The statistical methods were applied and have provided a better understanding of the data and the models. In particular this study's uniqueness was set by exploring the enhanced multivariate VARMA class of models using FDI and determining factors. For instance the use of Lutkepohl (2004) framework have impacted this study significantly.

Chapter 5 is structured as follows: Section 5.2 provides a small summary of the previous four Chapters; Section 5.3 provides findings of the study with regards to the research objectives; Section 5.4 provides recommendations based on the findings; Section 5.5 provides limitations that were encountered and finally Section 5.6 provides the general conclusion.

5.2 Findings with regard to research objectives

The primary objective of this study was to explore VARMA class of models in multivariate time series data.

Objective one: to fit VARMA models to multiple time series data.

Conclusion one

Six economic and financial indicators obtained from the South African Revenue Serve Bank ranging from 2002 Q2 to 2016 Q1 were used to identify and estimate the VARMA model. iI was found that among the candidates models, the suitable model was VARMA(1,1,0) according to the three information criteria. The ACFs and PAFCs Box-Jenkins framework were used to determine the lags of the VARMA model. All preliminary tests were conducted and the data was prepared for primary analyses. The analyses was continued using VARMA (1,1,0) model.

Objective two: to select an optimal multivariate VARMA model for FDI and the determining factors.

Conclusion two

To achieve this objective the original data needed to be transformed to induce stationarity. This necessitated the selection of optimal lag length. The SBIC and HQ selected lag 1 as optimal and AIC was in favor of lag 6. According to theory, the SBIC overrules the AIC and as a result lag 1 was chosen and considered in this study. This lag length together with the order of differencing were used in the construction of the VARMA and EC-VARMA in the preceding section. This enabled the researcher to decide on the type of VARMA and EC-VARMA models to estimate. The finding supports the authors such as Dufour and Stevonovic (2013) who suggested the integrated framework of the VARMA model with others so as to ease the complications when analysing multivariate data.

Objective three: to determine the cointegration relationships between variables.

Conclusion three

To achieve this objective, the researcher followed the Johansen (1988) multivariate cointegration to decide on the number of cointegrating vectors. The Maximum Eigenvalue and Trace test were used to attain this objective. The Johansen's (1988) test procedure exploited the relationship of FDI by identifying the number of cointegrating relations between the variables. It is crucial to test whether or not the collection of variables are exclusively combined of the similar order and that at least one linear grouping of these variables that is stationary is present (Dickey Fuller, 1979). Cointegration has a precondition which states that variables must be at levels or non-stationary when cointegration is conducted, unless they are integrated at the same order. In other words, cointegration is done when the variables are all differenced in the same order.

The Trace test and Maximum eigenvalue proved to be less than the related critical values from Rank 3 suggesting the rejection of the hypothesis no cointegrating association. Therefore it was concluded that there are at least three cointegrating vectors given that evidence of long-run association amongst FDI and its factors.

The finding provided evidence that the variables move together in the long run. From the three ranks generated, the second one provided the sign that were in accordance with economic theory as far as literature around FDI is concerned. The results of this rank proved that among the determinants selected, only two negatively affected FDI in the long run. In the short-run, the model showed that GFCF (-0.01723) that is injected into the country has a negative relationship towards FDI. This relationship might have been caused by foreign investors who decided not to do investments with South Africa. Thus, this is also shown by the negative impact of labour productivity (-0.00024) on foreign investment.

EC-VARMA (1,1,0) model results showed that GFCF, LP and CPI do not impact significantly on FDI in the long-run. The deduction is grounded on the insignificant observed probabilities. The coefficient of the EC-VARMA (1,1,0) model is negative as expected indicating the error correction term of 14.1%. The finding implies that the system of FDI corrected itself back to equilibrium at a speed of 14.1% in the short-run to maintain equilibrium in the long-run. It is evident that according to the identified model, some of the parameters are insignificant at 10% significance level but in essence the model appears to be a good estimator of FDI. The findings are similar to those reported by Kascha and Trenkler (2011) who identified the EC-VARMA (1,1,0) as the most appropriate model using the United States bond interest rate of differing maturities and the United States Treasury bill data. The study by Athanasopoulos, et al. (2014) also reported EC-VARMA (1,1,0) as being more powerful than the EC-VAR model.

Objective four: to determine the predictive power of different VARMA models.

Conclusion four

To achieve this objective the study used the selected models from a class of VARMA. The selected model was exposed to a string of diagnostic tests prior to implementing it for forecasting. The residuals of this model were confirmed to be normally distributed, homoscedastic and lacks serial correlation. In overall, the model was confirmed to be significant according to the R-squared test. The selected determinants according to the R-squared coefficient explained about 56% of total variance in the estimation of FDI model. This is reasonably good statistic to conclude that the model is significant. The forecasts generated by the EC-VARMA (1,1,0) model moved along with the actual values confirming that the model is suitable and effective in producing the forecast. The forecast showed that FDI will be increasing in the next six quarters. The findings are in accordance with Simionescu, (2013).

5.3 Recommendations

This section formulates recommendations for future studies on the basis of the findings. The following recommendations are formulated, in order to advance the application of VARMA class of model:

- This study enhanced the VARMA model with the EC factor to simplify the complications associated with this model. Other studies could use other existing enhancers to the model and make comparison with the results of the current study. Data generating process suggested by Simionescu (2013) may also be considered to obtain the data.
- The study recommends the use other variables that could not be included in the analysis due so as to confirm the suitability of EC-VARMA model. Proxies of other factors not included in the analysis may be used in further studies.

- It may be worthwhile to explore the multivariate VARMA class of model using Bayesian technique.

- The study accurately modelled the VARMA and EC-VARMA models using FDI and the determining factors. The study recommend the use financial and economic variables with greater number of observation in the future.

- For policy, the study recommends that authorities may refer to this findings when making decisions about investments.

5.4 Limitations

Although the research was carefully prepared, some unavoidable limitations and shortcomings were identified. First of all the study was limited to few time series data sources that consist of relevant data, hence certain variables were used as proxies to cover for other important factors. Secondly the sample period of the data is small and only five independent variables where used. Thirdly, not much literature was reviewed due to dearth of studies on the topic.

5.5 Summary

The goal of this study was to determine the suitable VARMA class of model using FDI and its determinants. The first objective was to fit the VARMA model using time series data. The fitted model was identified as VARMA (1,1,0) which was later successfully enhanced with the error correction term to ease the complications when estimating the model parameters. The model passed all the diagnostic tests and was later used to produce the forecasts of FDI.

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7 Annexure

Annexure 1: Original Data

Original Data						
Date	OT	LP	CPI	GDP	GFCF	FDI
2002/02	43900	83.4	19.2	22.1	292527	59622
2002/03	45475	82.3	20.5	21.4	298925	57948
2002/04	41202	82.7	15.1	23.5	306437	57646
2003/01	36442	81.4	3.7	22.1	311520	57067
2003/02	30703	83.1	3.3	21	322397	56212
2003/03	27449	82.1	1.5	21.5	332473	53016
2003/04	33830	81.9	1.5	22.7	343360	54720
2004/01	29106	84.4	3.3	22.9	353674	53883
2004/02	29300	86.3	2	20.8	360668	53376
2004/03	25787	87.3	-1.3	21.9	375645	56858
2004/04	30599	89	2.8	23.4	388418	60474
2005/01	24904	91.5	2.4	25.7	392814	61930
2005/02	25896	90.7	1.2	22.5	403360	63199
2005/03	25980	90.5	3.7	23.7	415474	62974
2005/04	31312	89.6	3.6	24.5	429140	58852
2006/01	30696	92.2	8.6	26.9	438994	61554
2006/02	35103	93	7.4	23.5	451850	59200
2006/03	38909	94.2	7.1	25	467348	61530
2006/04	37172	96.1	7.5	25.6	481864	62813
2007/01	39000	99	7.1	26.8	510060	63644
2007/02	38000	98.1	11.3	23.7	520181	61759
2007/03	40662	97.5	14.8	25.1	526153	65063
2007/04	41930	100.4	15.9	26.5	536830	71139
2008/01	47676	100.6	11.9	28.1	559722	69557
2008/02	49451	103.5	19.6	23.2	577411	67042
2008/03	48846	100.7	19.7	24.7	604986	74431
2008/04	48163	94.8	11.1	26.1	619509	78593
2009/01	53438	90.5	11.5	27.7	576222	85869
2009/02	49924	89.6	6.3	20.5	554497	93630
2009/03	52296	91.9	-0.9	21.8	538820	104436
2009/04	55446	97.1	1.7	23.1	534409	110192
2010/01	51782	97.7	1	26.2	531278	178
2010/02	61841	101	1.4	21.9	531834	193
2010/03	61642	100.5	1.7	22	528591	200
2010/04	62731	100.9	1	23.4	526021	216

2011/01	63461	104.4	11.8	27.6	542401	222
2011/02	69995	104	9.6	21.6	551872	229
2011/03	82836	103.9	8.9	21.4	567786	234
2011/04	84900	104.6	13.2	25.6	572961	239
2012/01	76509	105.4	6.1	27.6	566251	245
2012/02	77537	107.2	-0.3	22.7	576797	18738
2012/03	68228	106.7	2.8	21.6	572294	251
2012/04	61926	108.1	20.4	24.1	577894	252
2013/01	67468	107.4	3.4	26.5	591948	255
2013/02	69674	109.8	0.9	23	607469	257
2013/03	62911	107.9	2.3	22.2	622073	277
2013/04	55495	109.2	8.4	25.2	631302	280
2014/01	61584	109.5	11	26.7	618633	541
2014/02	64474	111.2	12.8	23.1	617212	506
2014/03	62575	109.5	3.7	22.3	621952	467
2014/04	61987	112.4	2.9	25.5	632847	569
2015/01	62605	111.5	5.7	28.1	636940	606
2015/02	71329	110	6.2	23.5	635557	537
2015/03	64688	111.7	2.6	24.6	642798	584
2015/04	72026	111.6	6	27.1	638285	250
2016/01	77637	111.4	19.2	29.3	628530	248

Annexure2: Log Transformed Data

Date	LFDI	CPI	GDP	LGFCF	LLP	LTTRADE
2002Q2	11.00	19.20	22.10	12.59	4.42	10.69
2002Q3	10.97	20.50	21.40	12.61	4.41	10.72
2002Q4	10.96	15.10	23.50	12.63	4.42	10.63
2003Q1	10.95	3.70	22.10	12.65	4.40	10.50
2003Q2	10.94	3.30	21.00	12.68	4.42	10.33
2003Q3	10.88	1.50	21.50	12.71	4.41	10.22
2003Q4	10.91	1.50	22.70	12.75	4.41	10.43
2004Q1	10.89	3.30	22.90	12.78	4.44	10.28
2004Q2	10.89	2.00	20.80	12.80	4.46	10.29
2004Q3	10.95	1.30	21.90	12.84	4.47	10.16
2004Q4	11.01	2.80	23.40	12.87	4.49	10.33
2005Q1	11.03	2.40	25.70	12.88	4.52	10.12
2005Q2	11.05	1.20	22.50	12.91	4.51	10.16
2005Q3	11.05	3.70	23.70	12.94	4.51	10.17
2005Q4	10.98	3.60	24.50	12.97	4.50	10.35
2006Q1	11.03	8.60	26.90	12.99	4.52	10.33
2006Q2	10.99	7.40	23.50	13.02	4.53	10.47

2006Q3	11.03	7.10	25.00	13.05	4.55	10.57
2006Q4	11.05	7.50	25.60	13.09	4.57	10.52
2007Q1	11.06	7.10	26.80	13.14	4.60	10.57
2007Q2	11.03	11.30	23.70	13.16	4.59	10.55
2007Q3	11.08	14.80	25.10	13.17	4.58	10.61
2007Q4	11.17	15.90	26.50	13.19	4.61	10.64
2008Q1	11.15	11.90	28.10	13.24	4.61	10.77
2008Q2	11.11	19.60	23.20	13.27	4.64	10.81
2008Q3	11.22	19.70	24.70	13.31	4.61	10.80
2008Q4	11.27	11.10	26.10	13.34	4.55	10.78
2009Q1	11.36	11.50	27.70	13.26	4.51	10.89
2009Q2	11.45	6.30	20.50	13.23	4.50	10.82
2009Q3	11.56	0.90	21.80	13.20	4.52	10.86
2009Q4	11.61	1.70	23.10	13.19	4.58	10.92
2010Q1	5.18	1.00	26.20	13.18	4.58	10.85
2010Q2	5.26	1.40	21.90	13.18	4.62	11.03
2010Q3	5.30	1.70	22.00	13.18	4.61	11.03
2010Q4	5.38	1.00	23.40	13.17	4.61	11.05
2011Q1	5.40	11.80	27.60	13.20	4.65	11.06
2011Q2	5.43	9.60	21.60	13.22	4.64	11.16
2011Q3	5.46	8.90	21.40	13.25	4.64	11.32
2011Q4	5.48	13.20	25.60	13.26	4.65	11.35
2012Q1	5.50	6.10	27.60	13.25	4.66	11.25
2012Q2	9.84	0.30	22.70	13.27	4.67	11.26
2012Q3	5.53	2.80	21.60	13.26	4.67	11.13
2012Q4	5.53	20.40	24.10	13.27	4.68	11.03
2013Q1	5.54	3.40	26.50	13.29	4.68	11.12
2013Q2	5.55	0.90	23.00	13.32	4.70	11.15
2013Q3	5.62	2.30	22.20	13.34	4.68	11.05
2013Q4	5.63	8.40	25.20	13.36	4.69	10.92
2014Q1	6.29	11.00	26.70	13.34	4.70	11.03
2014Q2	6.23	12.80	23.10	13.33	4.71	11.07
2014Q3	6.15	3.70	22.30	13.34	4.70	11.04
2014Q4	6.34	2.90	25.50	13.36	4.72	11.03
2015Q1	6.41	5.70	28.10	13.36	4.71	11.04
2015Q2	6.29	6.20	23.50	13.36	4.70	11.18
2015Q3	6.37	2.60	24.60	13.37	4.72	11.08
2015Q4	5.52	6.00	27.10	13.37	4.71	11.18
2016Q1	5.51	19.20	29.30	13.35	4.71	11.26