

**Design of effective decoding techniques in network
coding networks**

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Abstract

Random linear network coding is widely proposed as the solution for practical network coding applications due to the robustness to random packet loss, packet delays as well as network topology and capacity changes. In order to implement random linear network coding in practical scenarios where the encoding and decoding methods perform efficiently, the computational complex coding algorithms associated with random linear network coding must be overcome.

This research contributes to the field of practical random linear network coding by presenting new, low complexity coding algorithms with low decoding delay. In this thesis we contribute to this research field by building on the current solutions available in the literature through the utilisation of familiar coding schemes combined with methods from other research areas, as well as developing innovative coding methods.

We show that by transmitting source symbols in predetermined and constrained patterns from the source node, the causality of the random linear network coding network can be used to create structure at the receiver nodes. This structure enables us to introduce an innovative decoding scheme of low decoding delay. This decoding method also proves to be resilient to the effects of packet loss on the structure of the received packets. This decoding method shows a low decoding delay and resilience to packet erasures, that makes it an attractive option for use in multimedia multicasting.

We show that fountain codes can be implemented in RLNC networks without changing the complete coding structure of RLNC networks. By implementing an adapted encoding algorithm at strategic intermediate nodes in the network, the receiver nodes can obtain encoded packets that approximate the degree distribution of encoded packets required for successful belief propagation decoding.

Previous work done showed that the redundant packets generated by RLNC networks can be used for error detection at the receiver nodes. This error detection method can be implemented without implementing an outer code; thus, it does not require any additional network resources. We analyse this method and show that this method is only effective for single error detection, not correction.

In this thesis the current body of knowledge and technology in practical random linear network coding is extended through the contribution of effective decoding techniques in practical network coding networks. We present both analytical and simulation results to show that the developed techniques can render low complexity coding algorithms with low decoding delay in RLNC networks.

Keywords: Error detection, Earliest decoding, Fountain codes, Luby Transform codes, Practical network coding, Random linear network coding.

Opsomming

Willekeurige lineêre netwerk kodering word in die literatuur voorgestel as die praktiese oplossing vir netwerk kodering in praktiese netwerke. Willekeurige lineêre netwerk kodering bied robuustheid teen data verlies, transmissie vertraging asook verandering in netwerk topologie. Om willekeurige lineêre netwerk kodering effektief in praktiese netwerke te kan implementeer, moet die komplekse berekeninge geassosieer met willekeurige lineêre netwerk kodering voorkom word.

Ons wys dat wanneer brondata in sekere patrone versend word, die netwerk gebruik kan word om struktuur in die data by die ontvanger nodes voort te bring. Hierdie struktuur laat ons toe om 'n dekoderingsmetode met lae kompleksiteit, lae vertraging en robuustheid teen data verlies te gebruik. Hierdie dekoderingsmetode kan gebruik word in multimedia kommunikasie omgewings aangesien dit lei tot lae dekoderingsvertraging en robuustheid teen data verlies.

Ons wys dat Fonteinkodes saam met willekeurige lineêre netwerk kodering geïmplementeer kan word sonder om die totale koderingstruktuur van die netwerk te verander. Deur die enkoderingsalgoritmes by die netwerknodes aan te pas, kan die ontvanger nodes data ontvang in 'n spesifieke struktuur sodat kodering met lae kompleksiteit en lae vertraging kan plaasvind.

Laastens wys ons dat addisionele data wat in die willekeurige lineêre netwerk kodering netwerk gegenereer word, gebruik kan word om foute in die data op te spoor. Hierdie foutopsporingsalgoritme kan in die netwerk geïmplementeer word sonder die implementering van 'n addisionele foutkorreksiekode.

In hierdie tesis word die huidige kennis en tegnologie van praktiese willekeurige lineêre netwerk kodering uitgebrei deur 'n bydrae te lewer op grond van praktiese koderingsmetodes. Simulasie- en analitiese resultate wys dat die ontwikkelde koderingsmetodes geïmplementeer kan word in willekeurige lineêre netwerk koderingsnetwerke en sodoende lae kompleksiteit en lae dekoderingsvertraging lewer.

Sleutelwoorde: Fonteinkodes, Foutopsporing, Luby Transform kodes, Praktiese netwerk kodering, Willekeurige lineêre netwerk kodering.

*Trust in the Lord with all your heart
and lean not on your own understanding;
in all your ways acknowledge Him
and He will make your paths straight.*

Proverbs 3:5-6

Declaration

I, **Suné von Solms**, declare herewith that this dissertation entitled “**Design of effective decoding techniques in network coding networks**”, which I herewith submit to the North-West University in partial fulfilment of the requirements for the **Doctor of Philosophy** degree, is my own work and has not already been submitted to any other university.

I understand and accept that the copies submitted for examination are the property of the North-West University.



A handwritten signature in cursive script, appearing to read 'S. von Solms', is written over a horizontal line.

University number: 12987611

Signed at: **Potchefstroom** on this 11th day of **March 2013**.

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List of Abbreviations

BP	Belief Propagation
ED	Earliest Decoding
EH-LTNC	Enhanced Hybrid-Luby Transform Network Codes
FEC	Forward Error Correction
GE	Gaussian Elimination
H-LTNC	Hybrid- Luby Transform Network Codes
LT	Luby Transform
LTNC	Luby Transform Network Codes
MED	Modified Earliest Decoding
NC	Network Coding
RGG	Random Geometric Graph
ER	Érdos-Rényi
RLNC	Random Linear Network Coding
RS	Robust Soliton
XOR	exclusive or

List of Symbols

\mathcal{G}	directed graph
\mathcal{V}	set of nodes
R	number of nodes in network
\mathcal{E}	set of edges
s	source node
Z	set of sink nodes
z	sink node
u, v, w	nodes in the network
e	incoming edge in the network
e'	outgoing edge in the network
C	capacity
f	flow
σ	network min-cut
\mathbb{F}	Finite field
q	size of Finite field
r	achievable rate
X	set of source symbols
x	source symbol
m	size of source symbol
Y	set of encoded packets
y	encoded packets
$\beta(e)$	local encoding vector
$g(e)$	global encoding vector
G	global encoding matrix / generator matrix
n	number of source symbols
k	number of undecoded source symbols
h	number of generations
N	number of received packets
C	forward error correction code
U	set of encoded symbols
u	encoded symbol
$E(e)$	error packet
H	parity check matrix
z	syndrome vector
t	error correction capability
ζ	number of additional received packets
d_{min}	minimum distance
d	degree of packet

c	positive constant for RS distribution
δ	decoding failure probability for RS distribution
b	buffer size
d_T	target degree
t_s	time step
d_H	Hamming distance
w_H	Hamming weight
l	communication radius
α	alpha block
β	betha block
φ	number of entries in betha block
κ	decoded symbols before erasure
ℓ	remaining generation size after erasure
γ	gamma block
θ	number of entries in gamma block