

A practical implementation of the higher-order transverse-integrated nodal diffusion method



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This work is dedicated to the act of dedication, motivated by the need to keep moving and justified by the dream of becoming. It is not for us to measure how we will be measured, to define how we will be judged or to value what we have created. We may humbly give it life and watch it grow. As such and without expectation, I allow myself a modicum of vanity to hope that this work somehow contributes to the people I work with, the company I work for, the family I care for and the body of knowledge which remains long after all of these reach their respective destinations.

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Samevatting

Transversaal geïntegreerde nodale diffusie metodes verteenwoordig steeds die standaard in reaktor berekeninge. Die primêre tekortkoming in hierdie benadering is die gebruik van die sogenaamde kwadratiese transversale lekkasie aanname. Hierdie aanname word algemeen gebruik in die berekening van ligte water reaktore, maar is sonder teoretiese grondslag. Dit is nie direk afleibaar van die diffusie oplossing nie en kan akkuraatheids- en konvergensie probleme tot gevolg hê. In hierdie werk word 'n verbeterde, konsekwente hoër-orde lekkasie aanname geformuleer. Die kritiese suksesfaktore in so 'n metode is gekoppel aan beide akkuraatheid en effektiwiteit (berekenningskoste), en gevolglik word 'n reeks iterasie metodes verder ontwikkel om die voorgestelde oplossing van praktiese waarde te maak. Die mees belowende van hierdie skemas gebruik die hoër-orde lekkasie aanname om korreksiefaktore vir die standaard kwadratiese transversale lekkasie aanname te bereken. Numeriese resultate word produseer aan die hand van 'n reeks standaard toetsprobleme. Verder word die toepassing van die metode ook demonstree op 'n stel realistiese SAFARI-1 reaktor berekeninge. Die uiteindelige voorgestelde oplossing is geïmplimenter in a losstaande FORTRAN-90 module wat naatloos aan bestaande nodale kodes gekoppel kan word. Ter illustrasie word die module ook aan die OSCAR-4 kodesistiem gekoppel, wat oor dertig jaar by Necsa ontwikkel is en wat as primêre berekeningskode vir 'n aantal internasionale navorsingsreaktore gebruik word.

Abstract

Transverse-integrated nodal diffusion methods currently represent the standard in full core neutronic simulation. The primary shortcoming of this approach is the utilization of the quadratic transverse leakage approximation. This approach, although proven to work well for typical LWR problems, is not consistent with the formulation of nodal methods and can cause accuracy and convergence problems. In this work, an improved, consistent quadratic leakage approximation is formulated, which derives from the class of higher-order nodal methods developed some years ago. In this thesis a number of iteration schemes are developed around this consistent quadratic leakage approximation which yields accurate node average results in much improved calculational times. The most promising of these iteration schemes results from utilizing the consistent leakage approximation as a correction method to the standard quadratic leakage approximation. Numerical results are demonstrated on a set of benchmark problems and further applied to realistic reactor problems for particularly the SAFARI-1 reactor operating at Necsa, South Africa. The final optimal solution strategy is packaged into a standalone module which may be simply coupled to existing nodal diffusion codes, illustrated via coupling of the module to the OSCAR-4 code system developed at Necsa and utilized for the calculational support of a number of operating research reactors around the world.

keywords: transverse leakage, nodal diffusion, higher order methods

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Nomenclature

ANM Analytic Nodal Method

ANOVA Analysis of Variance

BOC Beginning of Cycle

CCSI Chebyshev Cyclic Semi-Iterative

CLA Consistent (Transverse) Leakage Approximation

CQLA Consistent Quadratic Leakage Approximation

EOC End of Cycle

FHO Full Higher-Order

HOTR Higher-Order Transverse Leakage and Reconstruction (code module)

MANM Multi-group Analytic Nodal Method

MGRAC Multi-group Reactor Analysis Code

MR (Higher-order) Model Reduction

NEM Nodal Expansion Method

OSCAR Overall System for the Calculation of Reactors

PLC Partial Leakage Convergence

QLAC (Standard) Quadratic Leakage Approximation Correction

RLCS Reduced Leakage Correction Scheme

SANS Standard Analytic Nodal Solver

SFSIM Standard Fission Source Iterative Method

SQLA Standard Quadratic Leakage Approximation

TLSIM Transverse Leakage Source Iterative Method