A network traffic model for wireless mesh networks

Dissertation submitted in fulfilment of the requirements for the degree
***Master of Engineering* in Computer Engineering at the Potchefstroom campus of the North-West University**

Z.S. van der Merwe

20749864

Supervisor: Prof. A.S.J. Helberg

November 2012

Declaration

I, Zuann Stephanus van der Merwe hereby declare that the thesis entitled "A network traffic model for wireless mesh networks" is my own original work and has not already been submitted to any other university or institution for examination.

Z.S. van der Merwe

Student number: 20749864

Signed on the 20th day of November 2012 at Potchefstroom.

Acknowledgements

Thank you to my prof. for all of his helpful insights.

Thank you to my parents for all their love and encouragement.

Abstract

Design and management decisions require an accurate prediction of the performance of the network. Network performance estimation techniques require accurate network traffic models. In this thesis we are concerned with the modelling of network traffic for the wireless mesh network (WMN) environment. Queueing theory has been used in the past to model the WMN environment and we found in this study that queueing theory was used in two main methods to model WMNs. The first method is to consider each node in the network in terms of the number of hops it is away from the gateway. Each node is then considered as a queueing station and the parameters for the station is derived from the number of hops each node is away from the gateway. These topologies can be very limiting in terms of the number of physical topologies they can model due to the fact that their parameters are only dependent on the number of hop-counts each node is away from the gateway. The second method is to consider a fixed topology with no gateways. This method simplifies analysis but once again is very limiting. In this dissertation we propose a queueing based network traffic model that uses a connection matrix to define the topology of the network. We then derive the parameters for our model from the connection matrix. The connection matrix allows us to model a wider variety of topologies without modifying our model. We verify our model by comparing results from our model to results from a discrete event simulator and we validate our model by comparing results from our model to results from models previously proposed by other authors. By comparing results from our model to results of other models we show that our model is indeed capable of modelling a wider variety of topologies.

Keywords: *Network traffic model, Queueing Theory, Wireless Mesh Networks (WMN).*

Contents

List of Figures **ix**

List of Tables **xi**

List of Acronyms **xii**

List of Symbols & Subscripts **xiii**

1 Introduction **1**

1.1 Relevant Background 1

1.1.1 Wireless Mesh Networks 1

1.1.2 Queueing Theory 3

1.2 Related Work 5

1.3 Problem statement 6

1.4 Methodology 7

1.5 Chapter Breakdown 8

2 Literature Study **9**

2.1 Introduction to Network Traffic Modelling 9

2.2 The problem entity: The WMN Environment 12

2.2.1 Architecture of the WMN Environment 12

2.2.2 Network Capacity 13
2.2.3 OSI Layers of the WMN environment 13
2.3 Commonly Used Traffic Models 16
2.3.1 Stochastic Processes 16
2.3.2 Renewal Traffic Models 17
2.3.3 Markov Processes and Markov chains 19
2.3.4 Fluid Traffic Models 21
2.3.5 Queueing Based Models 21
2.4 Background on Queueing Theory 22
2.4.1 General Results for $G/G/c$ queueing systems 22
2.4.2 Little's Formulas 23
2.4.3 Simple Markov Queueing Models 24
2.4.4 Queueing Networks 27
2.5 Queueing Based Models for Wireless Mesh Networks 31
2.5.1 Feng et al. 31
2.5.2 Wu et al. 37
2.5.3 Bisnik and Abouzeid 38
2.6 Conclusion 40

3 Conceptual Model **41**
3.1 Network Model 41
3.2 Mathematical Model 42
3.3 Performance Parameters 45
3.4 Summary 47

4 The Computerized Model **48**

4.1 Implementing the Conceptual Model 48
4.2 Verifying the Computerized model 49
4.2.1 Mathematical Model Set up 51
4.2.2 Simulation Results 55

5 Validating the Computerized Model 60

5.1 Wu et al. 60
5.1.1 Model Setup: Wu et al. 61
5.1.2 Model Setup: QBM 62
5.1.3 Results 63
5.2 Feng et al. 64
5.2.1 Simulation Set up: NS2 64
5.2.2 Model Set up: Feng et al. 64
5.2.3 Model Set up: QBM 65
5.2.4 Results 66
5.3 Bisnik and Abouzeid 68
5.3.1 Model Set up: Bisnik and Abouzeid 69
5.3.2 Model Set up: QBM 69
5.3.3 Results 70
5.4 Conclusion 70

6 Conclusion 72

6.1 Summary of thesis 72
6.2 Conclusion 73
6.2.1 Connection Matrix 73
6.2.2 Abstraction of the Routing Protocols 74

6.2.3 Abstraction of the MAC Protocols 74

6.2.4 Focus of this study 75

6.3 Future Work 75

6.3.1 Assumptions of the gateways 75

6.3.2 Multiple Traffic Classes 76

Bibliography **77**

List of Figures

1.1 WMN Architecture . . . 2

1.2 Validation and Verification in the modelling process . . . 8

2.1 The flow of a discrete-event simulation . . . 11

2.2 Exposed Node Problem . . . 14

2.3 Hidden Node Problem . . . 15

2.4 State transition diagram for a birth-death process . . . 25

2.5 Example of WMN with up-streams for Feng et al. [1] Model . . . 33

4.1 Flow diagram of model implemented in Scilab . . . 49

4.2 Topology A . . . 50

4.3 Topology B . . . 51

4.4 Topology A - Mean Number of Customers . . . 55

4.5 Topology A - Mean Response Time . . . 56

4.6 Topology A - Utilization . . . 56

4.7 Topology B - Mean Number of Customers . . . 57

4.8 Topology B - Mean Response Time . . . 58

4.9 Topology B - Utilization . . . 58

5.1 Wu et al.: Linear Topology . . . 61

5.2 Wu et al.: Average waiting time in queue 63
5.3 Feng et al.: Topology: L1 65
5.4 Feng et al.: Topology: L2 65
5.5 Feng et al.: Network Throughput 67
5.6 Feng et al.: Effective output of each hop 68
5.7 Bisnik and Abouzeid: Topology 69
5.8 Bisnik and Abouzeid: Average end-to-end delay in the network 70

List of Tables

1.1 Kendall 4

2.1 Classification of Markov processes 20

2.2 Summary of symbols 24

2.3 Summary of general results for $G/G/c$ queues 24

2.4 Types of nodes in a BCMP network 31

4.1 Summary of parameters 52

List of Acronyms

DES Discrete-Event Simulation

FCFS First Come First Served

IS Infinite Server

JMT Java Modelling Tools

LCFS-PR Last Come First Served with Pre-emptive Resume

MAC Media Access Control

MMPP Markov Modulated Poisson Process

PS Processor Sharing

RNG Random Number Generation

TDMA Time Division Multiple Access

WMN Wireless Mesh Network

List of Symbols & Subscripts

List of Symbols

B	Bandwidth of the network
d	Elements of the connection matrix
D	Connection Matrix
γ	Arrival rates of traffic generated by mesh clients at each node
G	Number of gateways in our model
$G/G/1$	General distribution for inter-arrival and inter-service times with one server
h_s	Number of nodes in each hop-count
λ	Effective arrival rate for each node
L	Mean number of customers in the system
Lq	Mean number of customers in queue
μ	Service rate of each node
$M/D/1$	Markovian inter-arrival times and deterministic inter-service times with one server
$M/M/c$	Markovian inter-arrival and inter-service times with c parallel servers
$M/M/c/k$	Markovian inter-arrival and inter-service times with c parallel servers and queue length k
N	Number of pure mesh routers and gateways in our model
π	Steady state probability
$p(s_i)$	Channel access probability
P	Average packet size

ρ	Utilization
r	Elements of the routing probability matrix
R	Routing probability matrix
s	Hop-count from the gateway
s_i	Numbering of nodes where s is the hop-count and i is the number of the node in that hop-count
W	Mean waiting time in the system
Wq	Mean waiting time in queue

Chapter 1

Introduction

Network performance estimation is very important for network planning. Techniques that are used to estimate the performance of the network include analytical techniques, simulation and also experimentation [2]. These techniques rely heavily on network traffic models. It is thus very important that the traffic models are reliable and accurate. In this thesis we make use of queueing theory to create a network traffic model for the Wireless Mesh Network (WMN) environment. The model extends on the work done by Feng et al. [1] and Bisnik and Abouzeid [3]. In this chapter we present brief background information to the problem followed by a discussion of related work. We then give the problem statement as well as the objective of this study. Finally we discuss the methodology of this dissertation followed by a chapter breakdown.

1.1 Relevant Background

1.1.1 Wireless Mesh Networks

Nodes in a WMN dynamically organize and configure themselves to establish a network [4]. There are two types of nodes in a WMN namely *mesh routers* and *mesh clients*.

Mesh routers are mostly stationary and form the backbone of the wireless network. Mesh clients connect to the mesh routers in order to gain access to the network. Mesh clients can also act as mesh routers but their hardware and software requirements are much less than that of an actual mesh router. Mesh routers usually support multiple wireless interfaces and thus their hardware and software requirements are greater. Mesh routers that are connected to the internet are called gateways. Most network traffic in a WMN is usually directed to gateways because most clients usually connect in order to gain access to the internet. This is especially true in the case where WMNs are used as a last mile technology for internet service providers [5]. Multi-hop communication is used in a WMN to achieve the same network coverage with a much lower transmission power. Figure 1.1 depicts the architecture of a WMN.

Note: In this dissertation we refer to a pure mesh router as a mesh router, and we refer to a mesh router that acts as a gateway as a gateway.

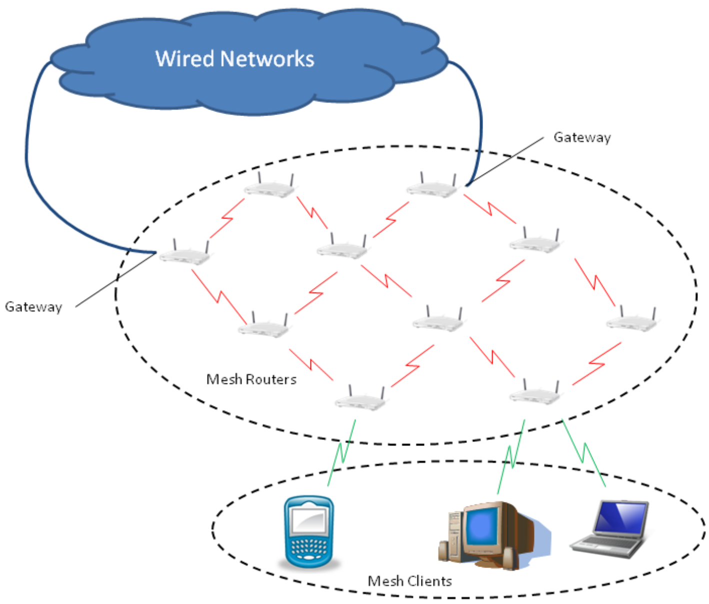

Figure 1.1: WMN Architecture

1.1.2 Queueing Theory

Queueing theory is the mathematical study of waiting lines. A queueing system can be described as customers arriving for service, waiting for service if the customer can not be serviced immediately, and then leaving the system after being serviced. In most cases, six basic characteristics of queueing processes provide an adequate description of a queueing system: (1) arrival pattern of customers, (2) service pattern of servers, (3) queue discipline, (4) system capacity, (5) number of service channels, and (6) number of service stages [6]. Arrival and service patterns are usually defined by stochastic processes where the stochastic process usually defines the inter arrival and service times. The queue discipline refers to the manner in which customers are selected from a queue to be serviced. The most commonly used discipline in queueing situations is the First Come First Served (FCFS) discipline. The system capacity refers to the amount of waiting room available for customers, in other words the maximum length of the queue. The number of service channels refers to the number of service stations that can serve customers simultaneously. A queueing system may also have multiple stages of service. Stochastic processes, such as the Poisson process, are used to describe the queueing system mathematically.

Kendall's Notation

To describe queueing systems in this thesis we will use Kendall's notation [6]. Kendall's notation has the form $A/B/X/Y/Z$, where A indicates the arrival process, B the inter-service time distribution, X the number of parallel servers, Y the queue length and Z the queue discipline. Often the notation is abbreviated by leaving out the Y and Z parameters. When the Y and Z parameters are omitted the queue length is assumed to be infinite and the service discipline is assumed to be FCFS. Table 1.1 shows all of the queueing systems used in this thesis.

Table 1.1: Kendall

$M/M/c$	Markovian inter-arrival and inter-service times with c parallel servers.
$M/M/c/k$	Markovian inter-arrival and inter-service times with c parallel servers and queue length k.
$G/G/1$	General distribution for inter-arrival and inter-service times with one server.
$M/D/1$	Markovian inter-arrival times and deterministic inter-service times with one server.

Queueing Networks

There are two main types of queueing networks, namely *open* and *closed* queueing networks. An open queueing network is one where customers are allowed to enter from outside the system. A closed queueing network does not allow for arrivals from outside the system and the number of customers in the system is always constant.

One of the first persons to make a major breakthrough in queueing networks was James R. Jackson. He studied the waiting lines of open queueing networks [7] and proved that under certain conditions the local and global balance equations are satisfied. One of the important conditions for Jackson queueing networks is that the arrivals for each node occur according to a Poisson process and the inter-service times be exponentially distributed. In a Jackson queueing network the effective arrival rate for each node is first calculated using the balance equations and after that each node can be studied as an independent $M/M/c$ queue.

Gordon and Newell [8] extended the work of Jackson by considering closed queueing networks. The work done by Jackson and Gordon and Newell were then extended even further by Baskett, Chandy, Muntz and Palacios in [9]. BCMP (named after the authors) networks include queueing networks with more than one customer class, different queueing strategies, and generally distributed service times. BCMP networks can be open, closed or mixed.

The results of the queueing networks discussed give us a means to calculate the steady state probabilities of a network from which the network can be statistically analysed.

1.2 Related Work

The literature reviews by Adas [10] and Frost and Melamed [2] discuss various network traffic models that are applicable to a wide variety of networks. Most of the models discussed in the literature reviews are mathematically relatively simple and are aimed at modelling a single arrival or service pattern of network traffic at a node. A key point that the literature reviews highlight is that these relatively simple mathematical models usually form part of an analytical model or is used to drive a discrete event simulator.

Queueing theory has been used in the past to model the wireless network environment. For example Ali and Gu [11] have used Jackson queueing networks to model a wireless sensor network with Time Division Multiple Access (TDMA) media access protocol with slot reuse. Ashtiani et al. [12] have also used Jackson queueing networks to create a mobility model for wireless multimedia networks.

Feng et al. [1] have created a queueing based network traffic model for wireless mesh networks where they assume that gateways are the destination of all network traffic and that the routing paths to the gateways are known. The reason they assume that gateways are the destination of all network traffic is because WMNs are commonly used as a last mile technology for internet service providers [5]. This means that most mesh clients will connect to the network in order to gain access to the internet, i.e. the gateways. Feng et al. model each node in terms of the number of hop-counts each node is away from a gateway. The queuing based model regards gateways and the most outward nodes as infinite queuing systems and regards the inner mesh nodes as finite queuing systems.

In [3] Bisnik and Abouzeid characterizes the average delay and capacity of a random access MAC based WMN. They model residential area WMNs as $G/G/1$ queueing systems. To avoid complexity in their model they assume that mesh routers are placed in uniformly apart block areas, and that mesh clients who wish to gain access to the network are distributed uniformly between the mesh routers. Network traffic arriving at a node either travel to one of the nodes adjacent to it with equal probability or the traffic leaves the system with a given probability. Their model takes into account the density of the network, the random packet arrival process as well as the collision avoidance mechanism of random access MAC. Their model does not account for the presence of gateways in the network.

In this study we aim to extend the work done by Feng et al. [1] and Bisnik and Abouzeid [3]. Feng et al. have created a model that accounts for the gateways in the system but their model's ability to model the physical topology of the network is very limited. Their model is only dependent on the number of nodes in a given hop-count from the gateway and is not dependent on the actual location of the nodes. Bisnik and Abouzeid have created a model that accurately models the physical properties of the MAC layer of a WMN within their assumptions, but their model does not take into account the effect of gateways and their model also has a fixed topology. In this study we aim to create a queueing based traffic model that takes into account the effect of gateways and is able to model a wide variety of topologies while still being able to model the physical properties of the MAC layer.

1.3 Problem statement

A need has been identified for an analytical model, that is capable of modelling the individual scenarios proposed by *Feng et al.* and *Bisnik and Abouzeid*.

1.4 Methodology

In this section we briefly describe the methodology that will be followed to achieve the objectives stated in the previous section. The methodology used is based on the model development process proposed by Sargent [13]. Figure 1.2 depicts the model development process and the role of validation and verification in the model development process. The *problem entity* refers to the problem or system that needs to be modelled which in this case is the WMN environment. The *conceptual model* is a mathematical or logical representation of the problem entity and the *computerized model* is an implementation of the conceptual model on a computer. The conceptual model is developed through an *analysis and modelling phase* and the computerized model is developed through a *computer programming and implementation* phase. The last phase is the *experimentation phase* where conclusions can be drawn on the problem entity by conducting experiments on the computerized model.

Validation and verification are performed throughout each phase of the model development process. In the analysis and modelling phase *conceptual model validation* is performed to ensure that the theories and assumptions underlying the conceptual model are correct and that the model representation of the problem entity can fulfil the purpose of the model. *Computerized model verification* is performed in the computer programming and implementation phase to ensure that the conceptual model was implemented correctly. *Operational validation* is performed in the experimentation phase to ensure that the output of the model is reasonably accurate for the intended purpose of the model. *Data validity* is performed to ensure that the data used to build the conceptual model, and the data used to test the computerized model are correct and accurate. The model development process is iterated until a satisfactory model is achieved and validation and verification processes are performed during each iteration.

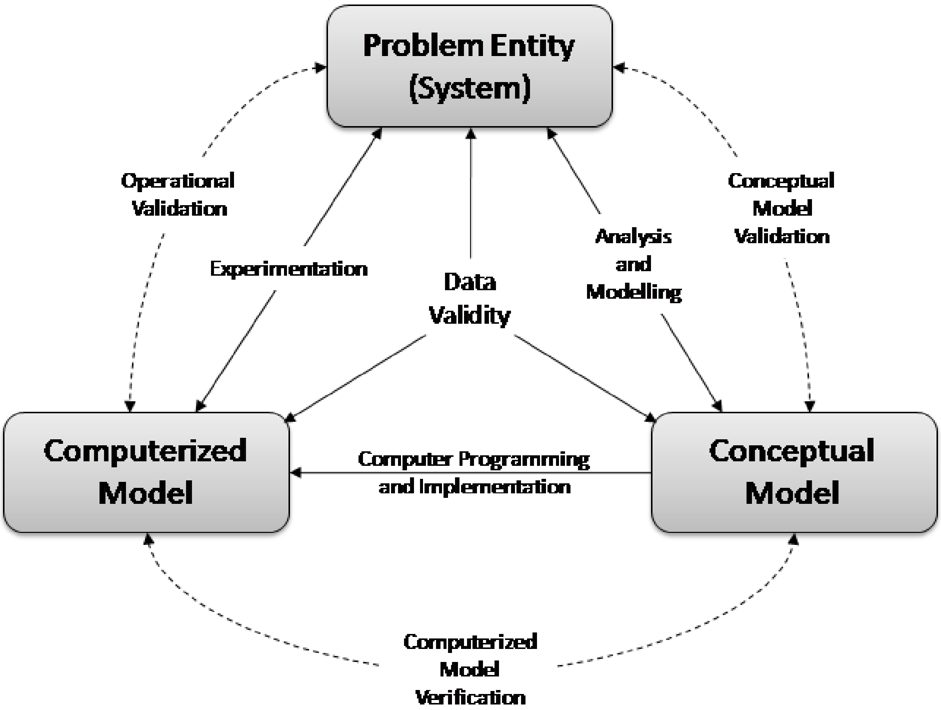

Figure 1.2: Validation and Verification in the modelling process

1.5 Chapter Breakdown

In chapter two we discuss the literature used for this study in detail which is followed by chapter three the conceptual model. In chapter four we implement and verify a computerized model from the conceptual model. We implement our model in Scilab which is a freeware alternative to Matlab and we verify our model by comparing results from the mathematical model to the results from a discrete event simulator. In chapter five we validate our model by comparing results from our model to results obtained from the models on which we aim to extend our model. This will ensure that our model is indeed capable of modelling the scenarios proposed by the authors of the papers on which we aim to extend our model. Finally in chapter six we conclude this thesis by giving a summary of this study and drawing final conclusions.

Chapter 2

Literature Study

Before we can construct a more general mathematical model for wireless mesh networks we have to take a look at what has previously been done. In this chapter we take a look at what network traffic modelling entails. Next we discuss some of the key elements of the WMN environment that need to be modelled. After that we discuss general network traffic modelling techniques and how they have been used in the past to model network traffic. From this we will see that queueing theory is a good solution to modelling the WMN environment. This is followed by a discussion of the basics of queueing theory so that we are able to understand the principles surrounding it. Finally we discuss a few network traffic models that use queueing theory to model the wireless mesh network and we also discuss how we plan to extend on these models.

2.1 Introduction to Network Traffic Modelling

In chapter one we discussed that there are three main ways to estimate the performance of a network namely analytical techniques, simulation and experimentation. Experiments that can give accurate performance estimation can be costly, especially for large scale networks. Experiments also need to be performed multiple times for extended periods of time to acquire enough data to accurately calculate the performance statistics of the network. Due to the high costs associated with experimentation it makes sense to rather want to use analytical techniques or simulation if they give rise to results that are accurate enough.

When it comes to computer simulation one of the performance prediction tools used widely in science and engineering is *Monte Carlo* simulation programs [2]. The name Monte Carlo arises from the fact that this method uses random numbers similar to those coming out of roulette games [14]. The Monte Carlo method is basically a computer program that uses random number generators to simulate a system under study. Running such a simulation is analogous to conducting an experiment involving randomness and thus the outputs of the simulation should be treated as random observations. Frost states in [2] that developing a simulation program for communication networks requires the following:

- Modelling random user demands for network resources.
- Characterizing network resources needed for processing those demands.
- Estimating system performance based on output data generated by the simulation.

The modelling of user demands and the processing of those demands in communication networks are easily encapsulated by events. A method that is ideal for simulating these event driven systems is called Discrete-Event Simulation (DES) [15]. DES models keep time via simulation clocks and the events are ordered in an event list according to the time they need to be executed. The event list is then used to determine the next event that needs to be executed and the simulation clock is then forwarded to the time of the next event. The execution of the event may change the state of the system and may also add or remove events from the event list. Figure 2.1 depicts the flow of a discrete-event simulation.

Analytical techniques consist of mathematical models that usually make many as-

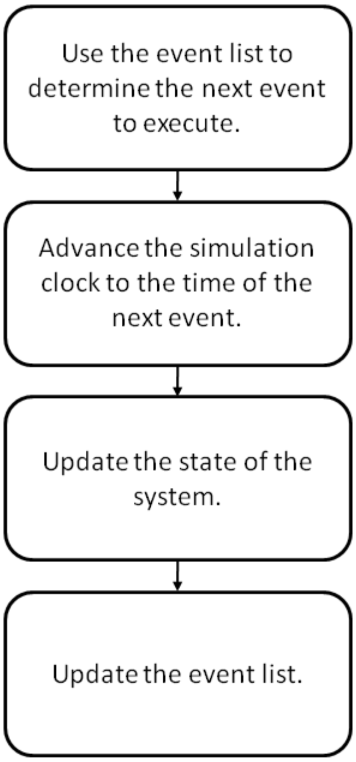

Figure 2.1: The flow of a discrete-event simulation

sumptions in order to keep the problem analytically tractable. This is because complex mathematical problems require a lot of computational resources in order to solve the problem and in many cases a mathematical problem that is too complex cannot be solved. The trade off between analytical models and Monte Carlo models is that although analytical models make many assumptions, they solve relatively fast and give exact answers. Monte Carlo simulations may be allowed to be more complex but it is important to remember that a complex system could still take a long time to simulate, even on more advanced modern computers. The trade off between the two methods becomes more important when it comes to rare event estimation. Statistical models can predict the chance of a rare event but again these type of analytical models usually make many assumptions in order to keep the model analytical tractable. Monte Carlo simulation programs need to be run for a very long time in order to predict the chance of a rare event with a given confidence interval [2].

Whether an analytical model or a simulation program is used, the quality of the model is very important and is determined by four main criteria namely: goodness-of-fit, number of parameters, parameter estimation, and analytical tractability. Goodness of fit refers to how suitable the model is to the scenario that needs to be modelled. Goodness-of-fit is also directly related to the performance measures that need to be predicted, i.e. the model should be able to predict the required performance measures. All models take input in the form of parameters to set up the model. Models that have a high number of parameters can be difficult to work with especially if those parameters are difficult to estimate. It is also important that the parameters can be accurately estimated as inaccurately estimated parameters will give rise to inaccurate performance predictions. Analytical tractability means that the model is mathematically easy to work with, i.e. it is easy to solve. Models that are not analytical tractable are hard to solve and often require a lot more resources to solve. It is difficult to put a quantitative value in terms of these criteria on a model. These criteria are highly application related. This means that the quality of the model is also highly application related.

2.2 The problem entity: The WMN Environment

In order to create a network traffic model for the WMN environment we first have to take a look at how the WMN environment functions and how the WMN environment is different from other networks.

2.2.1 Architecture of the WMN Environment

The architecture of WMNs can be classified into three main groups, namely *infrastructure* based WMN, *client* based WMN, and a *hybrid* WMN [4]. In an *infrastructure* based WMN the mesh routers form the backbone of the mesh network and mesh routers usually have multiple network interfaces to which clients can connect using wired or wireless technologies. Gateways are connected to the wired network and provide internet access to mesh clients. Figure 1.1 depicts an *infrastructure* based WMN. In a *client* based WMN the clients act as the routers to create a peer-to-peer network between mesh clients. A *client* based WMN does not require a mesh router, however hardware and software requirements for end-user devices increase due to the fact that the clients need to perform extra tasks such as routing. A *hybrid* architecture is a combination between an *infrastructure* based WMN and a *client* based WMN. Mesh routers create a backbone for the network and gateways provide access to the internet, but clients can also mesh with other mesh clients directly which increases network coverage.

2.2.2 Network Capacity

In [16] Gupta and Kumar derive the upper and lower bounds for ad hoc wireless networks. They point out in their paper that in order to increase network capacity nodes should only communicate with nearby nodes, i.e. relaying nodes should be used to forward network traffic and nodes should use a shorter transmission range. In [17] a scheme is proposed that increases network capacity by exploiting node mobility. The research results in [16,17] have inspired many other works [18–20] that study the trade-off between delay and throughput in an ad hoc wireless network. However most of these works focus on the asymptotic case and very few research works are dedicated to focussing on the statistical modelling of location dependant throughput and delay in a WMN [21]. The models described surrounding our problem statement in chapter one focus on the statistical modelling of WMNs and are not concerned with the asymptotic case.

2.2.3 OSI Layers of the WMN environment

In this section we briefly discuss the different layers in the OSI stack with regards to the modelling of a WMN. It is important to understand how each of the layers for a WMN function differently from a normal wireless network in order to accurately capture those characteristics in the network traffic model.

Physical Layer

At the physical layer it is important to remember that in wireless, communication nodes share the same transmission media. This means that nodes can interfere with the communication of other nodes. Nodes that are within each others' transmission area and that are on the same channel cannot transmit at the same time. There are two problems that can occur in a wireless communication environment, i.e. the exposed node problem and the hidden node problem. The exposed node problem occurs when node A wants to transmit to node B but can't because node C is busy transmitting next to it. The hidden node problem occurs when node A is able to transmit to node B and node C is able to transmit to node B, but node A and C are unable to see each other. Node A and C might start to transmit to node B at the same time. Figure 2.2 and 2.3 depict the two problems.

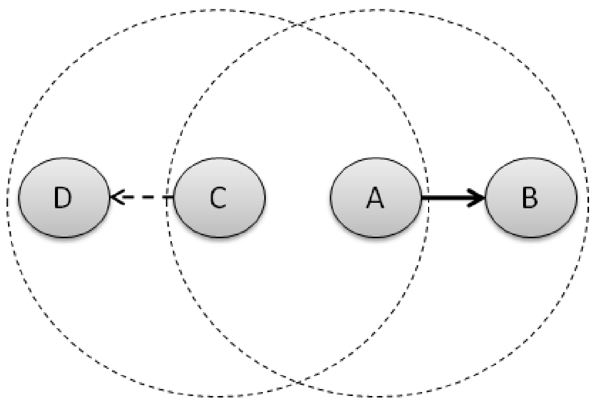

Figure 2.2: Exposed Node Problem

Data Link and Network Layer

Different Media Access Control (MAC) mechanisms and routing protocols significantly impact the capacity of a wireless network [4]. In order to create a model that is applica-

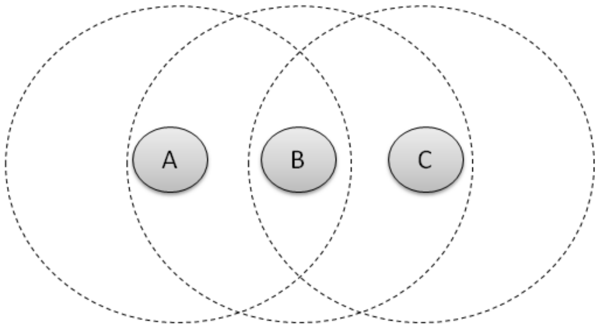

Figure 2.3: Hidden Node Problem

ble to a wider range of scenarios it is best to abstract the MAC mechanisms and routing protocols from the main model. If the model is implemented correctly then a wide variety of MAC and routing protocols can be used without affecting the mechanics of the main model. In this thesis we do not focus on the creation or improvement of MAC or routing protocols, but it is important that our model will be able to use different MAC and routing protocols which means that abstraction of these protocols in our model is very important.

Other Layers

The higher layers are more difficult to model in an analytical model when one is concerned with modelling the whole network. At the application layer there exists many models that model the source of specific applications such as voice, email and video [2, 10]. These types of models are typically used to drive a discrete event simulation. These models can also be used as part of an analytical model but complex source models can become difficult to impossible to work with in larger models that concern themselves with modelling the whole network. In this thesis we do not focus on specific source models and only use general source models for our network traffic model.

2.3 Commonly Used Traffic Models

In this section we first give a brief introduction to stochastic processes and then we discuss commonly used traffic models that can either be used as part of an analytical model or used to drive a discrete-event simulation. We state some of the advantages and disadvantages of each and we discuss how the models could be applicable to WMN.

2.3.1 Stochastic Processes

A stochastic process is a random process whose outcome is governed by probabilistic laws. From a mathematical point of view a stochastic process can be described as a family of random variables, $\{X(t), t \in T\}$, defined over the parameter space T. $X(t)$ denotes the state of the stochastic process at time t. A stochastic process can be classified as a discrete-parameter or a continuous-parameter process. If T is a discrete sequence, then the stochastic process $\{X(t), t \in T\}$ is said to be a discrete-parameter process defined on T. If T is a continuous interval or a combination of continuous intervals, then the stochastic process $\{X(t), t \in T\}$ is said to be a continuous-parameter process defined on T. [6]

Stochastic processes are ideal for modelling the arrival and service of network traffic at a specific node in the network. In [22], [23], [10] and [2] many examples of network traffic sources that are modelled with stochastic processes are given. The arrival, departure, or service of network traffic at a node can be accurately modelled by stochastic processes if sufficient real world network traffic data is available to accurately estimate the parameters of the stochastic process.

2.3.2 Renewal Traffic Models

Renewal models are mathematically relatively simple. Because of this they are very popular [2]. In a renewal traffic process, the inter arrival time process, which is a non-negative random sequence $\{A_n\}$, the A_n are independent, identically distributed (IID), but their distribution is allowed to be general. One drawback of renewal models is that the superposition of independent renewal processes does not necessarily yield a renewal process [2]. Another drawback of renewal models is that they do not accurately capture the autocorrelation of $\{A_n\}$. The autocorrelation function serves as a statistical proxy for temporal dependence and traffic models with a positive autocorrelation have the ability to capture the effect of traffic bursts, that is $\{A_n\}$ tends to give rise to relatively short inter arrival times followed by relatively long inter arrival times.

Poisson Process and the Exponential Process

One of the most commonly used stochastic processes is the Poisson process. A Poison process can be characterized as a renewal process whose inter arrival times are exponentially distributed with rate λ [2]. This means that a process which assumes an exponential distribution for the inter-arrival times is equivalent to a process which assumes an Poisson distribution for the arrival rate.

Let Ω be a sample space and P a probability measure on it. In this section an arrival process refers to the stochastic process $N = \{N_t; t \geq 0\}$ defined on Ω such that for any $\omega \in \Omega$, the mapping $t \longrightarrow N_t(\omega)$ is non-decreasing, increases by jumps only, is right continuous, and has $N_0(\omega) = 0$.

Definition: An arrival process $N = \{N(t); t \geq 0\}$ is called a Poisson process provided that the following axioms hold [24]:
(a) for all ω, each jump of $t \longrightarrow N_t(\omega)$ is of unit magnitude;
(b) for any $t, s \geq 0$, $N_{t+s} - N_t$ is independent of $\{N_u; u \leq t\}$;
(c) for any $t, s \geq 0$, the distribution of $N_{t+s} - N_t$ is independent of t.

The probability of n arrivals in a time interval of length t is given by

$$p_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, n = 0, 1, 2, ... \tag{2.1}$$

which is the well known formula for the Poisson probability distribution.

An important advantage of Poisson processes is that the superposition of independent Poisson processes results in a Poisson process with rate equal to the sum of the independent rates. Poisson processes are also memoryless which means that the current output does not depend on past input. Another great advantage of Poisson processes is Palm's Theorem [25] which roughly states that a large number of not necessarily Poisson renewal processes combined will have Poisson properties. This is why it is common practice to assume a Poisson process in traffic applications that physically comprise a large number of independent traffic streams, each of which may have a general distribution.

Bernoulli Process

Bernoulli processes can be seen as the discrete-time counter part of Poisson processes [2]. A Bernoulli process defines the arrival probability at any discrete-time instance as p. Let Ω be a sample space and P a probability measure on Ω. Let $\{X_n; n = 1, 2, ...\}$ be a sequence of random variables defined on Ω and taking only the two values 0 and 1.

Definition: The stochastic process $\{X_n; n = 1, 2, ...\}$ is called a *Bernoulli process* with success probability p provided that [24]:
(a)$X_1, X_2, ...$ are independent, and
(b)$P\{X_n = 1\} = p, P\{X_n = 0\} = q = 1 - p$ for all n.

The probability of n arrivals at discrete interval k is binomial and is given as:

$$Pr\{N_k = n\} = \binom{k}{n} p^n (1-p)^{k-n}, n = 0, 1, 2, ..., n. \tag{2.2}$$

The time between intervals is geometric with parameter p and is given as:

$$Pr\{X = k\} = (1-p)^{k-1} p, k = 1, 2, 3, ... \tag{2.3}$$

Phase-type Renewal Processes

The phase-type renewal process is very popular due to the fact that they are relatively tractable and can approximate any inter-arrival distribution arbitrary closely [2]. Phase-type processes are also very popular in single node queueing systems where they are used to model the inter-arrival and/or service distributions [26]. Phase-type processes model inter-arrival times as the time to absorption in a continuous time Markov chain $C = \{C_t; t \geq 0\}$ with state space $\{0, 1, 2, ..., m\}$. State zero is the absorbing state and absorption is guaranteed. To determine the inter-arrival time, A_n, the process is started with some initial distribution π and A_n is then calculated as the time until absorption occurs. The process is then restarted.

2.3.3 Markov Processes and Markov chains

Markov and Markov-renewal traffic models introduce dependence into the random sequence A_n by allowing each state of the Markov traffic model to control the parameters of the traffic model [2]. This means that Markov traffic models could possibly capture the effect of traffic bursts.

A Markov process is a stochastic process whose future state of the process is independent of the past. The definition of a Markov process is given as follows according to [24]:

Definition: The stochastic process $Y = \{Y_t; t \geq 0\}$ is said to be a *Markov process* with state space E provided that for any $t, s \geq 0$ and $j \in E$,

$$Pr\{Y_{t+s} = j | Y_u; u \leq t\} = Pr\{Y_{t+s} = j | Y_t\}$$

Classification of Markov Processes

A Markov process is classified according to its parameter set (discrete or continuous) and its state space (discrete or continuous) [6]. A Markov process with a discrete parameter set and a discrete state space is generally referred to as a Markov chain. In this literature study we will refer to a Markov process with a discrete parameter set and a discrete state space as a Markov chain, and we will refer to a Markov process with a continuous parameter set and a discrete state space as a continuous-parameter Markov chain. We will refer to a Markov process with a discrete parameter set and a continuous state space as a discrete-parameter Markov process, and we will refer to a Markov process with a continuous parameter set and a continuous state space as a continuous-parameter Markov process. Table 2.1 shows a summary of the classification of Markov Processes.

Table 2.1: Classification of Markov processes

State Space	Type of parameter	
	Discrete	Continuous
Discrete	(Discrete-Parameter) Markov Chain	Continuous-Parameter Markov Chain
Continuous	Discrete-Parameter Markov Process	Continuous-Parameter Markov Process

Markov Modulated Traffic Models

Markov modulated traffic models consist of Markov models of which the state of the Markov model controls the parameters of a stochastic process [10]. One example of this is the Markov Modulated Poisson Process (MMPP) [27]. An MMPP consists of a continuous-time Markov chain of which the state of the Markov chain, s_k, controls the rate, λ_k, of a Poisson process. A MMPP is referred to as a double stochastic process.

2.3.4 Fluid Traffic Models

Fluid models do not concern themselves with the arrival of individual traffic units but instead model the traffic as a continuous stream with the flow rate as a parameter of the model [10]. The greatest advantage of fluid models is that flow changes occur much less frequently than the arrival of individual packets which means that when a fluid model is implemented in a computer simulation the model uses a lot less computing and memory resources.

2.3.5 Queueing Based Models

Queueing theory is well suited for modelling the WMN environment. The arrival process can be used to model the arrival of network traffic. The service process can be used to model the time a node takes to process a packet as well as the time a node needs to send a packet. The probability of the channel being available for a node to transmit can also be incorporated into the service process. The routing protocol of the network can be incorporated into the routing probability matrix of the queueing network. Queueing theory has been used in the past in similar ways to model a WMN such as the model in [1,3,21,28]. There are two fundamental ways in which queueing theory is used to model a WMN. The first is to consider each node in terms of the hop-count it is away from the gateway. Each node in the network is then analysed as a queueing system and the parameters for the queueing system is dependent on the number of nodes in each hop-count. This approach was followed by [1,21,28]. The second method is to consider a WMN with uniformly distributed nodes and no gateways such as in [3]. The network becomes easier to analyse since it looks the same from all perspectives but it is also very limiting in terms of model flexibility.

A method which we have not yet encountered in the literature with regards to queueing based network traffic models is the use of a connection matrix. A connection matrix can be used to define the topology of the network and the parameters of the model can then be dependent on the connection matrix. In this study we aim to use a connection matrix to define the topology of the network and then create a queueing based network traffic model which derives its parameters from the connection matrix. In the next section we take a more in-depth look at the basics of queueing theory and queueing networks after which we discuss some queueing based network traffic models on which we aim to extend to create our own network traffic model.

2.4 Background on Queueing Theory

In this section we discuss the basics of queueing theory. We start by discussing general results for a $G/G/c$ queueing system. This is followed by discussing results for the $M/M/1$, $M/M/c$, and $M/M/c/K$ queueing systems. We then discuss results for queueing networks. The results given in this section are very important tools which we will use to build our network traffic model.

2.4.1 General Results for $G/G/c$ queueing systems

We consider a $G/G/c$ queueing system to which customers arrive according to a general arrival process with rate λ and customers are serviced according to a general service process with rate μ. The queueing system has c number of servers and has an infinite buffer size. The traffic congestion, also referred to as traffic intensity, ρ, is given as:

$$\rho \equiv \lambda/c\mu \tag{2.4}$$

For the system to have steady state results, ρ must be less than one [6]. The probability distribution of the total number of customers in the system, $N(t)$, at time t, consists of the customers waiting in queue, $N_q(t)$, and the customers being serviced, $N_s(t)$. The mean number of customers in the system is given by:

$$L = E[N] = \sum_{n=0}^{\infty} np_n \tag{2.5}$$

where $p_n(t) = \Pr\{N(t) = n\}$, and $p_n = \Pr\{N = n\}$ in the steady state. The expected number of customers in queue is given by:

$$L_q = E[N_q] = \sum_{n=c+1}^{\infty} (n-c)p_n \tag{2.6}$$

2.4.2 Little's Formulas

In the early 1960's John D.C. Little developed a relationship in queueing theory between the steady-state mean system sizes and the steady-state average customer waiting times. Refer to [29] for the full proof of the formulas.

Let T_q be the time a customer spends in queue prior to entering service, S the time a customer spends in service, and $T = T_q + S$ the total time a customer spends in the system. Little's formulae are given as follows:

$$L = \lambda W \tag{2.7}$$

and

$$L_q = \lambda W_q \tag{2.8}$$

where $W = E[T]$ is the mean waiting time for a customer in the system, and $W_q = E[T_q]$ is the mean waiting time for a customer in queue [6]. If we take the mean of the total time a customer spends in the system we get $E[T] = E[T_q] + E[S]$, which can also be written as $W = W_q + 1/\mu$. If we subtract equation 2.8 from equation 2.7 we get:

$$L - L_q = \lambda(W - W_q) = \lambda(1/\mu) = \lambda/\mu \tag{2.9}$$

But we already know that $L - L_q = E[N] - E[N_q] = E[N - N_q] = E[N_s]$, where $E[N_s]$ is the expected number of customers in service in the steady state. Thus the expected number of customers in service denoted by τ, is equal to λ/μ.

In the single server situation ($c = 1$), if we subtract equation 2.6 from equation 2.5 and use algebra to simplify we get:

$$L - L_q = \sum_{n=0}^{\infty} np_n - \sum_{n=2}^{\infty} (n-1)p_n = \sum_{n=1}^{\infty} p_n = 1 - p_0 \tag{2.10}$$

For a single server system $\tau = \rho$, and since $L - L_q = \lambda/\mu = \tau$, we can see that the probability that a $G/G/1$ system is empty is $p_0 = 1 - \rho$.

Table 2.2: Summary of symbols

L	Mean number of customers in the system
L_q	Mean number of customers in queue
S	Service time
T	Total time a customer spends in the system
T_q	Time a customer waits in queue prior to entering service
W	Mean waiting time in the system
W_q	Mean waiting time in queue
λ	Mean customer arrival rate
μ	Mean customer service rate
τ	Mean number of customers in service

Table 2.3: Summary of general results for $G/G/c$ queues

$\rho = \lambda/c\mu$	Traffic congestion / Traffic intensity
$L = \lambda W$	Little's formula
$L_q = \lambda W_q$	Little's formula
$W = W_q + 1/\mu$	Expected-value argument
$p_b = \lambda/c\mu = \rho$	Busy probability for an arbitrary server
$\tau = \lambda/\mu$	Expected number of customers in service
$L = L_q + \tau$	Combined result of Little's formulas
$p_0 = 1 - \rho$	$G/G/1$ empty-system probability
$L = L_q + (1 - p_0)$	Combined result for $G/G/1$

2.4.3 Simple Markov Queueing Models

Figure 2.4 depicts a specific type of continuous-time Markov chain called a birth-death process. The states of the Markov chain denote the population of the system. With an arrival (birth) the system moves from state n to state $n + 1$ and with a departure (death) the system moves from state n to state $n - 1$. Queues that can be modelled with birth-death processes include $M/M/1$, $M/M/c$, $M/M/c/K$, $M/M/\infty$, and variations of these queues with state-dependant arrival and service rates [6]. The following subsections briefly discusses different types of queues that are modelled according to a birth-death process.

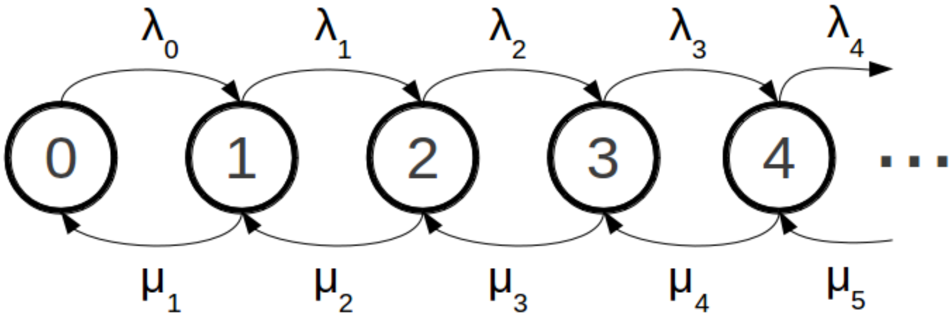

Figure 2.4: State transition diagram for a birth-death process

Single-Server Queues - $M/M/1$

In this section we discuss single-server queues as given in [6]. Consider a single-server $M/M/1$ queue where the arrivals are Poisson with rate λ, and the service-times are exponentially distributed with mean $1/\mu$. Let p_n denote the probability that the system is in state n (has n customers in the system). For a $M/M/1$ queue, p_n is given by:

$$p_n = (1-\rho)\rho^n \qquad (\rho = \lambda/\mu < 1) \tag{2.11}$$

It can be shown that the mean number of customers in the system is then given by

$$L = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}, \tag{2.12}$$

and the mean number of customers in queue is given by:

$$L_q = \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)} \tag{2.13}$$

Applying Little's formulas to equations 2.12 and 2.13 then gives:

$$W = \frac{L}{\lambda} = \frac{\rho}{\lambda(1-\rho)} = \frac{1}{\mu-\lambda} \tag{2.14}$$

$$W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu(1-\rho)} = \frac{\rho}{\mu-\lambda} \tag{2.15}$$

Multi-Server Queues - $M/M/c$

Next we will discuss multi-server queues as given in [6]. Consider a multi-server $M/M/c$ queue where the arrivals are Poisson with rate λ, and the service-times of each server is independently and identically distributed according to an exponential distribution with mean $1/\mu$. For this section we let $\tau = \lambda/\mu$ and $\rho = \tau/c = \lambda/c\mu$. The probability that the system is in state n is given by

$$p_n = \begin{cases} \frac{\lambda^n}{n!\mu^n} p_0 & (0 \le n < c), \\ \frac{\lambda^n}{c^{n-c}c!\mu^n} p_0 & (c \le n) \end{cases} \tag{2.16}$$

where

$$p_0 = \left(\frac{\tau^c}{c!(1-\rho)} + \sum_{n=0}^{c-1} \frac{\tau^n}{n!} \right)^{-1} \qquad (\tau/c = \rho < 1). \tag{2.17}$$

The mean number of customers and waiting times are then given by:

$$L_q = \left(\frac{\tau^c \rho}{c!(1-\rho)^2} \right) p_0 \tag{2.18}$$

$$W_q = \frac{L_q}{\lambda} = \left(\frac{\tau^c}{c!(c\mu)(1-\rho)^2} \right) p_0 \tag{2.19}$$

$$W = \frac{1}{\mu} + \left(\frac{\tau^c}{c!(c\mu)(1-\rho)^2} \right) p_0 \tag{2.20}$$

$$L = \tau + \left(\frac{\tau^c \rho}{c!(1-\rho)^2} \right) p_0 \tag{2.21}$$

Queues with truncation - $M/M/c/K$

Finally we discuss queues with truncation as given in [6]. For queues with truncation the same assumptions are made as those for $M/M/c$ queues, except now λ_n must be equal to 0 whenever $n \ge K$. The probability that the system is in state n is then given by

$$p_n = \begin{cases} \frac{\lambda^n}{n!\mu^n} p_0 & (0 \le n < c), \\ \frac{\lambda^n}{c^{n-c}c!\mu^n} p_0 & (c \le n \le K) \end{cases} \tag{2.22}$$

where

$$p_0 = \begin{cases} \left[\frac{\tau^c}{c!} \left(\frac{1-\rho^{K-c+1}}{1-\rho} \right) + \sum_{n=0}^{c-1} \frac{\tau^n}{n!} \right]^{-1} & (\rho \ne 1), \\ \left[\frac{\tau^c}{c!} (K-c+1) + \sum_{n=0}^{c-1} \frac{\tau^n}{n!} \right]^{-1} & (\rho = 1). \end{cases} \tag{2.23}$$

The mean number of customers in queue are given by:

$$L_q = \frac{p_0 \tau^c \rho}{c!(1-\rho)^2}[1 - \rho^{K-c+1} - (1-\rho)(K-c+1)\rho^{K-c}] \tag{2.24}$$

Remember that $L = L_q + \tau$. The result needs to be adjusted since a fraction p_K of the arrivals do not join the system because they are dropped if they arrive when the queue is full. The arrival rate needs to be adjusted and we denote the adjusted arrival rate as λ_{eff}. The effective arrival rate is given as $\lambda_{eff} = \lambda(1 - p_K)$, which means that the mean number of customers in the system is then given as:

$$L = L_q + \frac{\lambda_{eff}}{\mu} = L_q + \frac{\lambda(1-p_K)}{\mu} = L_q + \tau(1-p_K). \tag{2.25}$$

The waiting times are then given as:

$$W = \frac{L}{\lambda_{eff}} = \frac{L}{\lambda(1-p_K)} \tag{2.26}$$

$$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda_{eff}} \tag{2.27}$$

2.4.4 Queueing Networks

In this section we consider queueing networks. We only take a look at a special type of queueing networks, namely networks that have product-form. The advantage of queueing networks with product-from is that their solutions can be obtained without generating their underlining state-space [30]. This makes solving large networks of queues relatively easy.

Jackson Queueing Networks

Jackson queueing networks were first created by James R. Jackson in [7]. Jackson networks consist of an open network of waiting lines where each waiting line can be considered as an independent $M/M/c$ queue. The assumptions for these types of networks are as follow:

- There are $i = 1, 2, ..., N$ nodes in the system and each node has c_i identical servers.

- There is only one customer class in the system and the overall number of customers in the network is unlimited.

- Customers arriving from outside the system will arrive at node i with a Poisson process with rate γ_i.

- Service times at node i are independent and identically distributed according to an exponential distribution with rate μ_i (a node's service rate may be allowed to be dependent on its queue length).

- The probability that a serviced customer at node i will go next to node j (routing probability) is given by τ_{ij}, where $i = 1, 2, ..., k$ and $j = 0, 1, 2, ..., k$. The routing probability is independent of the state of the system, and τ_{i0} indicates the probability that a customer will leave the system at node i.

A routing probability matrix, $\mathbf{R}$, is created by placing each routing probability τ_{ij} in the i'th row and the j'th column of the matrix. The effective arrival rate λ_i is then calculated using the following formula:

$$\boldsymbol{\lambda} = \boldsymbol{\gamma} \cdot (\mathbf{I} - \mathbf{R})^{-1}, \tag{2.28}$$

where $\mathbf{I}$ is the identity matrix. Each node i can then be analysed as if it was an independent $M/M/c$ queue with arrivals following a Poisson process with rate λ_i and service times following an exponential distribution with rate μ_i. Jackson's Theorem states that if the overall ergodicity ($\lambda_i \leq \mu_i c_i$) holds for the network then the steady state probabilities are given as:

$$\pi(k_1, k_2, ..., k_n) = \pi_1(k_1)\pi_2(k_2)...\pi_N(k_N), \tag{2.29}$$

where $\pi_i(k_i)$ is the probability that there are k_i customers at node i.

Gordon/Newell Networks

Gordon and Newell [8] extended the work on queueing networks by considering closed queueing networks, i.e. networks where no arrivals can occur from outside. The number of customers in a closed network, K, is always constant and is given by:

$$K = \sum_{i=1}^{N} k_i. \tag{2.30}$$

The number of states in a Gordon/Newell network is given by the binomial coefficient:

$$\binom{N+K-1}{N-1}, \tag{2.31}$$

which gives the number of ways to distribute K customers between N nodes. The Gordon/Newell Theorem gives the probability of the network state as follows:

$$\pi(k_1, k_2, ..., K_n) = \frac{1}{G(K)} \prod_{i=1}^{N} F_i(k_i), \tag{2.32}$$

where $G(K)$ is a normalization constant and is given by:

$$G(K) = \sum_{\sum_{i=1}^{N} k_i = K} \prod_{i=1}^{N} F_i(k_i). \tag{2.33}$$

The $F_i(k_i)$ function corresponds to the state probabilities of the i'th node and is given by:

$$F_i(k_i) = \left(\frac{e_i}{\mu_i}\right)^{k_i} \frac{1}{\beta_i(k_i)}, \tag{2.34}$$

where e_i is the visit ratio of node i and $\beta_i(k_i)$ is given by:

$$\beta_i(k_i) = \begin{cases} k_i! & k_i \leq c_i \\ c_i! c_i^{k_i - m_i} & k_i > c_i \\ 1 & c_i = 1 \end{cases} \tag{2.35}$$

BCMP Queueing Networks

Baskett, Chandy, Muntz and Palacios extended the work done by Jackson, Gordon and Newell in [9]. Their queueing networks are referred to as BCMP networks and include queueing networks with more than one customer class, different queueing strategies, and generally distributed service times. BCMP networks can be open, closed or mixed. The assumptions for a BCMP network are as follow:

- The queueing discipline can be FCFS, Processor Sharing (PS), Last Come First Served with Pre-emptive Resume (LCFS-PR) or Infinite Server (IS).
- The service times of a node with FCFS service discipline must be exponentially distributed and customer class independent. For a node with a PS, LCFS-PR or IS service discipline the service times can be any distribution with a relational Laplace transform. The mean service times of the last three mentioned service disciplines may differ for different customer classes.
- The service rate of node i may be allowed to be dependant on the number of customers at node i if the service discipline of node i is FCFS. If the service discipline of node i is PS, LCFS-PR or IS, then the service rate for customer class m may also be allowed to be dependant on the number of customers of class m at node i
- The arrival process for a BCMP network is defined for two different scenarios for open networks:

  Scenario 1: The network contains one source with a Poisson arrival process a with rate λ, where λ may be allowed to be dependant on the number of customers in the network. If the network contains N nodes and M number of customer classes, then the arriving customers are distributed across the network with probability $r_{0,im}$ where:

  $$\sum_{i=1}^{N}\sum_{m=1}^{M} r_{0,im} = 1 \tag{2.36}$$

  Scenario 2: The network contains M sources, each corresponding to a customer class, with arrivals happening at each source according to an independent Poisson process with rate λ_m. The arrival rate λ_m from the m'th source may be allowed to be load dependent. The arriving customers are then distributed across the network with probability $r_{0,im}$ where:

$$\sum_{i=1}^{N} r_{0,im} = 1, \quad m = 1, 2, ..., M \tag{2.37}$$

The assumptions for BCMP networks lead to four node types given in table 2.4. The notation $-/M/c$ is used because the arrival process to a node in a BCMP network is in general not Poisson distributed. For an open queueing network with independent arrival and service rates the steady state probabilities are then given by:

$$\pi(k_1, k_2, ..., k_n) = \pi_1(k_1)\pi_2(k_2)...\pi_N(k_N), \tag{2.38}$$

with:

$$\pi_i(k_i) = \begin{cases} (1-\rho_i)\rho_i^{k_i}, & Type - 1, 2, 4(c_i = 1), \\ e^{-\rho_i}\frac{\rho_i^{k_i}}{k_i!}, & Type - 3. \end{cases} \tag{2.39}$$

The equations for the steady state probabilities for the other types of BCMP networks are more complex and are not discussed here, but can be found in [9].

Table 2.4: Types of nodes in a BCMP network

Type-1: $-/M/c$ - FCFS	Type-2: $-/G/1$ - PS
Type-3: $-/G/\inf$ - IS	Type-4: $-/G/1$ - LCFS-PR

2.5 Queueing Based Models for Wireless Mesh Networks

The previous sections in this chapter gave us a good idea about what network traffic modelling entails. In this section we discuss network traffic models that make use of queueing theory to model a WMN. The models discussed in this section are very important to this study as these are the models that we plan to expand upon.

2.5.1 Feng et al.

The model proposed by Feng et al. [1] models a WMN in terms of hop count a given node is away from a gateway. They refer to a node that is s number of hops away from a gateway as an s-hop node. They then model each s-hop layer of nodes as one queueing system.

Network Model

Feng et al. considers a WMN of N mesh routers and C gateways. They assume that the gateways are uniformly distributed across the network and that all nodes use the same channel to communicate with each other. They also assume that the routing paths between mesh routers and gateways are known and they define their model according to the number of hops a mesh router is away from a gateway. A mesh router is referred to as an s-hop mesh node where $s, s \geq 0$, is the number of hops it is away from a router. When $s = 0$ the mesh node is a gateway. Let $N(s)$ denote the number of s-hop nodes and $r(s)$ denote the ratio between $N(s)$ and N. We then have:

$$\sum_{s=1}^{S} r(s) = 1 \tag{2.40}$$

and

$$N(s) = \begin{cases} C, & s = 0 \\ N \cdot r(s), & 1 \leq s \leq S \end{cases} \tag{2.41}$$

Where S is the hop count of the maximum routing path. The function $r(s)$ is used to describe the topology of the network, i.e. how many nodes are s-hops away from the gateway. Feng et al. assumes that $r(s)$ is known. One s-hop node may have more than one $(s-1)$-hop nodes to which it can relay network traffic due to the nature of the wireless medium. For simplicity Feng et al. assumes that s-hop nodes do not forward loads for other nodes of the same hop count. Let $N(s, S)$ be the number of mesh nodes between the s-hop nodes and the S-hop nodes. $N(s, S)$ is then given by:

$$N(s, S) = \sum_{r=s}^{S} N(r) \tag{2.42}$$

Feng et al. only analyses traffic flows between mesh clients and the gateways since most mesh clients connect to the network in order to gain access to the internet which is accessed via the gateways. To simplify their calculations they only analyse traffic in one direction, namely from the mesh clients to the gateways. Figure 2.5 depicts the described network under the given assumptions. The first number in each hop count depicts its hop count. It is assumed that gateways do not communicate with mesh clients directly, i.e. they do not generate network traffic. It is assumed that the amount of traffic generated by mesh clients in the area of each mesh router are equal. Let λ be the ideal number of packets loaded to every mesh router from its mesh clients and let $\lambda(s)$ be the total number of packets that arrive at s-hop nodes. $\lambda(s)$ is then given by:

$$\lambda(s) = \begin{cases} \lambda(1) & s = 0 \\ \lambda \cdot N(s) + \lambda(s+1) & 1 < s < S \\ \lambda \cdot N(s) & s = S \end{cases} \tag{2.43}$$

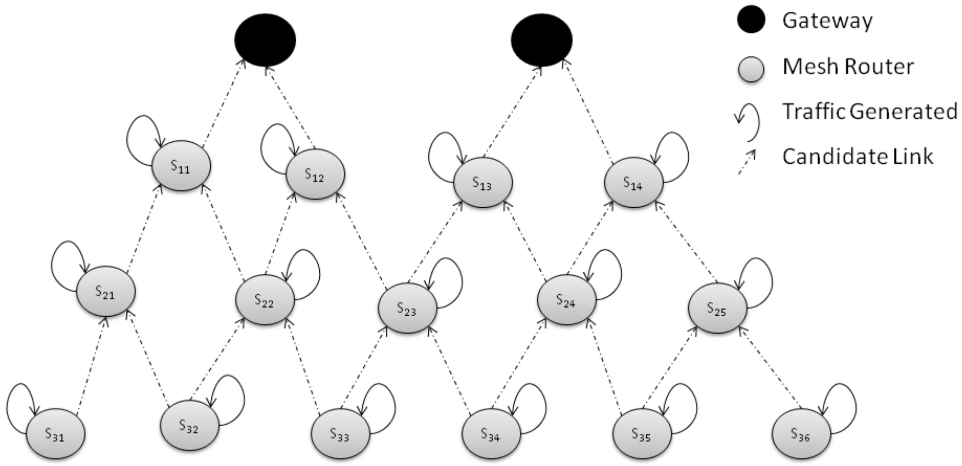

Figure 2.5: Example of WMN with up-streams for Feng et al. [1] Model

Mathematical Model

Feng et al. models the WMN as a series of queueing systems where each mesh router is considered a service station. Let μ denote the mean packet processing rate of every mesh router and let λ denote the mean packet generation rate for network traffic generated by mesh clients at every mesh router. It is assumed that λ and μ follow an independent and identically distributed Poisson process.

Define the utilization factor, ρ_s, as of the s-hop queueing system as the ratio between the packet arrival rate and the packet processing rate of the s-hop queueing system. ρ_s is then given as:

$$\rho_s = \tfrac{N(s,S)}{N(s)}\delta \quad 0 \leq s \leq S, \tag{2.44}$$

where δ is the relationship between λ and μ. Both δ and ρ_s should be less than one for the system to be in balance, i.e. for the system to be able to reach a steady state.

S-hop nodes only receive packets from mesh clients that are within their transmission range and from nowhere else. The traffic loads on S-hop nodes are thus independent from each other and because of this Feng et al. models the S-hop nodes as a queueing system with a single server. The traffic loads on the S-hop nodes are the lowest in the system and thus it can be assumed that the packet arrival rate at each S-hop node is much less than the packet processing rate of that node and thus the buffer length of the S-hop nodes can be modelled as infinite. The S-hop nodes are thus modelled as a $M/M/1/\infty$ queueing system.

The traffic loads of s-hop nodes $(0 \leq s < S)$ may choose one of several $(s-1)$-hop nodes as the next destination. Feng et al. models the s-hop nodes $(0 < s < S)$ as queueing systems with multiple servers. Feng et al. further debates that because the bandwidth links of the wireless nodes are very limited that the buffer length can be modelled as finite.

Let k denote the buffer capacity of every mesh router in the system. The buffer capacity of s-hop nodes, $K(s)$, is then given by $k \cdot N(s)$. The s-hop nodes $(0 < s < S)$ are then modelled as a $M/M/N(s)/K(s)$ queueing system.

The gateways $(s = 0)$ are modelled as $M/M/C/\infty$ queueing systems. The gateways have C parallel servers since there are C gateways and the buffer length is infinite since the bandwidth of the wired links is much more than the bandwidth of the wireless links, which implies that the packet processing rate of the gateways is much higher than the packet arrival rate.

Let $M(s)$ be the traffic model of the s-hop nodes in the WMN. $M(s)$ is then given as:

$$M(s) = \begin{cases} M/M/C/\infty & (s = 0) \\ M/M/N(s)/K(s) & (0 < s < S) \\ M/M/1/\infty & (s = S) \end{cases} \tag{2.45}$$

Performance Parameters

Feng et al. uses basic queueing theory to derive expressions for the probability, p_k, that k packets arrive at a queueing system for each queueing system in their model. They then continue to calculate the throughput of the network. We present the following analysis as derived in [1]. To calculate the throughput of the network they first calculate the output of each s-hop layer. Denote $p_{K(s)}$ as the packet loss probability of the s-hop nodes. The $M/M/N(s)/K(s)$ queue has a possibility for packet loss because it has a finite buffer. Packets arriving when the buffer is full will be dropped. The probability of an arriving packet being dropped, $p_{K(s)}$, is thus given as:

$$p_{K(s)} = \frac{N(s,S)^{K(s)}\delta^{K(s)}}{N(s)!} p_0 \quad (0 < s < S). \tag{2.46}$$

Let λ_s^e denote the efficient output of the s-hop layer. λ_s^e is then given as:

$$\lambda_s^e = (N(s)\lambda + \lambda_{s+1}^e)(1 - p_{K(s)}) \quad (0 < s < S). \tag{2.47}$$

Gateways have no packet loss according to the proposed model and because gateways have no traffic that is directly generated by mesh clients their output is that of the 1-hop nodes. Thus $\lambda_0^e = \lambda_1^e$. Since the S-hop nodes also have no packet loss their effective output is equal to that of the traffic generated by the mesh clients within their range. Thus $\lambda_S^e = \lambda N(s)$. To sum up, the efficient output of each s-hop layer is then given as:

$$\lambda_s^e = \begin{cases} (N(1)\lambda + \lambda_2^e)(1 - p_{K(1)}) & (s = 0) \\ (N(s)\lambda + \lambda_{s+1}^e)(1 - p_{K(s)}) & (0 < s < S) \\ \lambda N(s) & (s = S) \end{cases} \tag{2.48}$$

Define θ as the maximum achievable throughput of the network. Due to the assumptions of the network flows in the network the throughput of the network is given as the output of the gateways. Thus the throughput of the network is given as:

$$\theta = \lambda_0^e = \lambda_1^e = (N(1)\lambda + \lambda_2^e)(1 - p_{K(1)}) \tag{2.49}$$

From the equation it is clear that one will have to go through an iterative process from λ_S^e in order to calculate $\lambda_0^e = \lambda_1^e$.

Using the principles of queueing theory discussed in section 2.4 the expected waiting time, W_s, for packets in the system and the expected number of customers, L_s, in the system can also be calculated.

In their paper Feng et al. mention that their model does not take into account wireless interference. To amend this problem they modified the service rate of each node. They calculate the service rate of each node as follows:

$$\mu_i = \frac{B - I(i)}{P}, \tag{2.50}$$

where B is the bandwidth in Mbps, P is the packet size in bytes and $I(i)$ is the function used to take into account wireless interference. Feng et al. defines $I(i)$ as the bandwidth that cannot be used due to all of the interfering signals. Feng et al. does not however give an indication in their paper on how they obtain $I(i)$.

Analysis

From the above presentation of the work done by Feng et al. in [1], we can draw the following conclusions. Modelling the network in terms of the number of hops that nodes are away from the gateway gives a good idea of the effect that gateways have in the network. However modelling each s-hop layer as a single queueing system means that the model has very little input in terms of the topology of the network. The model only accounts for a symmetrical network where the only input into the model is the number of nodes in each s-hop layer. This makes the identification of bottlenecks in the network very difficult.

2.5.2 Wu et al.

Wu et al. [28] used a much simpler aproach to modelling a WMN. Instead of modelling each node in the mesh network as a queueing system, they only model the gateways as queueing systems.

Network Model

Wu et al. considers a WMN with N mesh routers and M gateways. Wu et al. assumes that gateways are always connected to the wired network and thus the bandwidth of the wired links are much larger than the bandwidth of the wireless links. Because of this Wu et al. assumes that the service rate of gateways are constant and that they have an infinite buffer size. They then model the gateways as $M/D/1$ queueing systems. The arrival rate of the gateways is assumed to be Poisson and is calculated as a function of the number of mesh routers and gateways there are in the network. They derive expressions for a linear mesh topology and a grid mesh topology.

Performance Parameters

Wu et al. uses the general queueing results of a $M/D/1$ queueing system to calculate the average delay and average queue length of the gateway.

Analysis

From the above presentation of the work done by Wu et al. [28], we can draw the following conclusions. The model proposed by Wu et al. is a very simplistic model that can only model the load on the gateways. The number of topologies that the model can accommodate is also very limited and the model does not account for any wireless interference or routing protocols.

2.5.3 Bisnik and Abouzeid

Bisnik and Abouzeid [3] characterises the average delay and throughput in a random access MAC based WMN. They model residential area WMNs as open $G/G/1$ queueing systems. They also presented their model in [31] where they give more in-depth simulation results. Unfortunately their model does not account for the effect of gateways in the network.

Network Model

Bisnik and Abouzeid considers a network of n mesh clients distributed uniformly over an area that consists of non-overlapping zones of area $a(n)$. Each zone contains one mesh router and two mesh routers are said to be neighbours if their zones share a common point. The set of neighbours of router i is denoted by $N(i)$ and the number of neighbours that node i has equals K. Bisnik and Abouzeid assumes that client-router and router-router communication takes place on different channels. Each mesh client may be a source or a destination for network traffic. Bisnik and Abouzeid assumes that packets of size L bits are generated by each mesh client according to an independent and identically distributed Poisson process with rate λ. Generated packets are directly transmitted to the mesh router of the zone it is in. The mesh routers then relay the packets until the packets reach the zone containing the destination mesh client. The probability that a packet received by a mesh router is destined to one of the mesh clients in its zone is given as $p(n)$. If a packet is not destined for the zone of the mesh router then the packet is forwarded to one of the neighbours of the mesh routers with equal probability.

Mathematical Model

Bisnik and Abouzeid model each mesh router in the network as a $G/G/1$ queueing system. They use routing probabilities to characterize the flow of packets in the network. The routing probability, $p_{ij}(n)$, that a packet is forwarded from node i to one of its neighbouring nodes j is given as:

$$p_{ij}(n) = \begin{cases} \frac{1-p(n)}{K} & j \in N(i) \\ 0 & otherwise \end{cases} \tag{2.51}$$

Bisnik and Abouzeid uses a random access MAC model that is able to account for wireless interference from other wireless nodes. A wireless transmission from one mesh router to another will only be successful if none of the neighbour nodes are transmitting at the same time. By allowing the service time of the queueing system to be dependent on the number of interfering neighbours Bisnik and Abouzeid are able to account for the effect of wireless interference.

Performance Parameters

Bisnik and Abouzeid use the diffusion approximation technique [30] to obtain closed from expressions for the end-to-end delay and per node throughput. Bisnik and Abouzeid only derive expressions for queueing systems with exponential inter-arrival times and service times.

Analysis

The model of Bisnik and Abouzeid proposes a good way to account for wireless interference. By allowing the service rate of each node to be dependent on the number of interfering nodes all the nodes will have equal probability to transmit. Unfortunately the model does not account for the effect of gateways in the network and is also limited to one fixed topology.

2.6 Conclusion

In this chapter we first presented an introduction to network traffic modelling in general. We then discussed the problem entity of our modelling problem namely the WMN environment. This was followed by a discussion of commonly used traffic models. We discussed some of the advantages and disadvantages of these models. At the end of this we found that queueing theory seems to give an adequate solution to modelling the WMN environment. We then gave a brief background on how queueing theory and queueing networks work which was followed by a discussion of queueing based network traffic models of WMNs. We discussed the shortcomings as well as the strong points of the models.

We now aim to extend on these models by creating a model that uses a connection matrix to define the topology of the network. The parameters of the model can then be calculated from the connection matrix.

Chapter 3

Conceptual Model

In the previous chapter we discussed various queueing based network traffic models for WMNs. We saw that there are two main methods of modelling a WMN using queueing theory. One is to model the nodes in terms of the number of hops they are away from the gateways. The second is to use a routing probability matrix to define the network. We will now attempt to extend on these models. We do this through the use of a connection matrix. Through the use of a connection matrix we aim to model the network in terms of the number of hops the nodes are away from the gateways without losing any information on the topology of the network.

3.1 Network Model

We consider a WMN with N mesh routers of which G are gateways. Mesh clients connect to the mesh routers to gain access to the network and we assume that router-router and router-client communication takes place on different channels. We define each mesh router in terms of the number of hops, s, a mesh client connecting to it would be away from the closest gateway with $s = 1, 2, ..., S$. When s is equal to 1 the mesh router is considered a gateway. We define h_s as the number of mesh routers that are s hops away from a gateway which gives $h_1 = G$ and:

$$\sum_{s=1}^{S} h_s = N \tag{3.1}$$

We label each node as s_i, which means that it is the i'th node that is s hops away from the closest gateway with $0 \leq i \leq h_s$.

To describe the topology of the network we define the connection matrix, $\mathbf{D}$, with elements as follows:

$$d_{s_i,u_j} = \begin{cases} 1, & \text{if node } s_i \text{ is able to transmit to node } u_j, \\ 0, & \text{otherwise.} \end{cases} \tag{3.2}$$

To clarify, since nodes do not transmit to themselves the value of d_{s_i,s_i} is set to zero. The elements of the connection matrix are ordered first according to s and then according to i. Equation 3.3 gives an example of how the elements in the matrix are ordered. For the example $h_s = 2$ for all s and $S = 3$.

$$\mathbf{D} = \begin{vmatrix} d_{1_1,1_1} & d_{1_1,1_2} & d_{1_1,2_1} & d_{1_1,2_2} & d_{1_1,3_1} & d_{1_1,3_2} \\ d_{1_2,1_1} & d_{1_2,1_2} & d_{1_2,2_1} & d_{1_2,2_2} & d_{1_2,3_1} & d_{1_2,3_2} \\ d_{2_1,1_1} & d_{2_1,1_2} & d_{2_1,2_1} & d_{2_1,2_2} & d_{2_1,3_1} & d_{2_1,3_2} \\ d_{2_2,1_1} & d_{2_2,1_2} & d_{2_2,2_1} & d_{2_2,2_2} & d_{2_2,3_1} & d_{2_2,3_2} \\ d_{3_1,1_1} & d_{3_1,1_2} & d_{3_1,2_1} & d_{3_1,2_2} & d_{3_1,3_1} & d_{3_1,3_2} \\ d_{3_2,1_1} & d_{3_2,1_2} & d_{3_2,2_1} & d_{3_2,2_2} & d_{3_2,3_1} & d_{3_2,3_2} \end{vmatrix} \tag{3.3}$$

We assume that mesh clients generate traffic to each mesh router s_i according to a Poisson process with rate γ_{s_i}. We also assume that the size of packets that are generated to be exponentially distributed with an average packet size of P bytes.

3.2 Mathematical Model

Because the inter-arrival and inter-service times are exponentially distributed, we model the WMN as a Jackson queueing network. Each mesh router in the network is modelled as a $M/M/1$ queueing system. We define the routing probability matrix, $\mathbf{R}$, of which its elements, r_{s_i,u_j}, give the probability that packets will go to node u_j when leaving node s_i. The sum of the rows of the probability matrix will always be less or equal to one. When the sum is less than one the remaining probability is the probability that a packet will leave the system. The probability that a packet will leave the system, $r_{s_i,0}$, at node s_i is thus given as:

$$r_{s_i,0} = 1 - \sum_{u=1}^{S} \sum_{j=1}^{h_u} r_{s_i,u_j}. \tag{3.4}$$

We will discuss the routing protocol we use for our model later on. We construct the vector γ which contains the elements γ_{s_i}. Once again the elements are ordered first according to s and then according to i. Using the results of Jackson queueing networks given in section 2.4 we calculate the effective arrival rate for each node as:

$$\boldsymbol{\lambda} = \boldsymbol{\gamma} \cdot (\mathbf{I} - \mathbf{R})^{-1}, \tag{3.5}$$

where the element λ_{s_i} of the vector $\boldsymbol{\lambda}$ gives the effective arrival rate for node s_i.

The service rate of the queueing model represents the time it takes the node to transmit a packet. Because the size of packets is exponentially distributed the service times will also be exponentially distributed. If the maximum bandwidth in bps is given by B, and the average packet size in bytes is given by P, then the service rate, μ_{s_i}, for node s_i in packets per second is calculated as:

$$\mu_{s_i} = \frac{B}{8P} \cdot p(s_i), \tag{3.6}$$

where $p(s_i)$ is defined as the channel access probability. We use the channel access probability to model the MAC mechanisms of the WMN.

It is important to note at this stage that our model only has queues with infinite buffers. This is because there do not exist queueing networks with product form for queues with truncation. Queueing networks with truncation are referred to as blocking queueing networks [30]. The only way to obtain exact results for general queueing networks with blocking is by generating and numerically solving the underlining continuous time Markov chain. This is possible for medium sized networks. For small networks closed form results may be derived but either way it is time consuming. Other techniques that are used to analyse queueing networks with blocking are approximate techniques. In section 5.2 we use a similar method used by Feng et al. [1] to approximate the results of queues with finite buffers.

Routing protocol and MAC mechanism

The calculation of the routing probability matrix, **R**, and the channel access probability, $p(s_i)$, gives us a means to abstract the modelling of the routing protocol and the MAC mechanisms. We present here a basic routing protocol and MAC mechanism for our model but in chapter 5 when we validate our model we show that by just changing the way we calculate $p(s_i)$ and **R** we are able to model all of the queueing based models for WMNs presented in chapter 2 in a similar way. The routing protocol we use for our basic model is a basic spanning tree routing protocol. We assume that the destination of all network traffic is to the gateways in the network. In this scenario there are two types of traffic that mesh clients can generate, namely upload traffic and download traffic. For our spanning tree protocol the packets will always follow the shortest path to the destination. Due to the nature of this protocol the paths for upload traffic and download traffic will be the same, only in opposite directions. The results of the analysis of the queueing network is not dependent on the direction of the traffic and because of this we only have to analyse the traffic in one direction. We analyse the upload traffic and note that the arrival rate, γ_{s_i}, for node s_i now consists of two elements namely the upload traffic, $\gamma^u_{s_i}$, and the download traffic, $\gamma^d_{s_i}$.

The routing protocol works as follows: Because it is a basic spanning-tree protocol we assume that each node that is s hops away from a gateway will transmit a packet to a node that is $s - 1$ hops away from a gateway. Each node is only able to transmit to other nodes that are within its transmission range, i.e. connected in the connection matrix. We assume that if there is more than one possible node to which it can transmit then the packet is forwarded to one of the possible nodes with equal probability. When the packets arrive at a gateway, i.e. $s = 1$, the packets leave the system. The routing probabilities are then calculated as follows:

$$r_{s_i,(s-1)_j} = \begin{cases} \left[\sum_{k=1}^{h_{s-1}} d_{s_i,(s-1)_k}\right]^{-1}, & 1 < s \leq S, \ \ d_{s_i,(s-1)_j} = 1 \\ 0, & \text{elsewhere.} \end{cases} \tag{3.7}$$

The MAC mechanism our model uses works as follows: We assume that no two mesh routers that are in range of each other can transmit at the same time due to them sharing the same channel. We assume that nodes within each other's range have equal opportunity to transmit. We then calculate the channel access probability as:

$$p(s_i) = \left[1 + \sum_{u=1}^{S} \sum_{j=1}^{h_u} d_{s_i,u_j}\right]^{-1}. \tag{3.8}$$

The simplified MAC and routing protocol used by our model are adequate to describe the operation of the network and allow the traffic model to be constructed and analysed. We remind the reader that the focus of this study is not to create or improve MAC and/or routing protocols, it is to create a model that is not limited to one topology and/or scenario. Since the calculation of the channel access probabilities and the routing probabilities are abstracted from the main model it gives us a means to implement other MAC and routing protocols without affecting the main model.

3.3 Performance Parameters

Now that we have the effective arrival rate and the service rate of each node we can calculate the performance measures of the network. We use the results from section 2.4 and analyse each node as an independent $M/M/1$ queue. The utilization, ρ_{s_i} of node s_i is given by:

$$\rho_{s_i} = \frac{\lambda_{s_i}}{\mu_{s_i}} \tag{3.9}$$

The utilization gives the probability that a node is busy. For every node s_i the utilization needs to be less than one, i.e. $\rho_{s_i} < 1$, for the network to be stable. The mean number of customers, L_{s_i}, at node s_i is given by:

$$L_{s_i} = \frac{\rho_{s_i}}{1 - \rho_{s_i}} = \frac{\lambda_{s_i}}{\mu_{s_i} - \lambda_{s_i}}, \tag{3.10}$$

and the mean number of customers in queue, Lq_{s_i}, at node s_i is given by:

$$Lq_{s_i} = \frac{\rho_{s_i}^2}{1 - \rho_{s_i}} = \frac{\lambda_{s_i}^2}{\mu_{s_i}(\mu_{s_i} - \lambda_{s_i})}. \tag{3.11}$$

We can obtain the average waiting time of a packet at a node, W_{s_i}, as well as the average waiting time of a packet in queue, Wq_{s_i}, by applying Little's formulae:

$$W_{s_i} = \frac{L_{s_i}}{\lambda_{s_i}} = \frac{1}{\mu_{s_i} - \lambda_{s_i}}, \tag{3.12}$$

$$Wq_{s_i} = \frac{Lq_{s_i}}{\lambda_{s_i}} = \frac{\rho_{s_i}}{\mu_{s_i} - \lambda_{s_i}}. \tag{3.13}$$

The overall throughput of the network is defined as the rate at which packets successfully leave the network. For our model which does not allow dropped packets, the throughput of the network is equal to the arrival rate of the network. The throughput of the network, λ, is thus calculated as:

$$\lambda = \sum_{s=1}^{S} \sum_{i=1}^{h_s} \lambda_{s_i}, \tag{3.14}$$

and the average number of packets in the network, L, is given by:

$$L = \sum_{s=1}^{S} \sum_{i=1}^{h_s} L_{s_i}. \tag{3.15}$$

We can then calculate the response time of the network, W, as follows:

$$W = \frac{L}{\lambda}. \tag{3.16}$$

The steady-state probability, $\pi_{s_i}(k)$, that there are k packets at node s_i is given by:

$$\pi_{s_i}(k) = (1 - \rho)\rho^k, \tag{3.17}$$

and the steady-state probability of the network is given by:

$$\pi(k_{1_1}, k_{1_2}, ..., k_{S_{h_S}}) = \sum_{s=1}^{S} \sum_{i=1}^{h_s} \pi_{s_i}(k_{s_i}). \tag{3.18}$$

3.4 Summary

In this chapter we presented our network traffic model that uses the connection matrix to define the topology of the network. We model a WMN as a Jackson queueing network and we abstracted the implementation of the MAC and routing protocols. Finally we derived the performance parameters of our model using the results from section 2.4. In the next chapter we discuss the implemented model.

Chapter 4

The Computerized Model

In this chapter we discuss the implemented model. The model was implemented in Scilab which is a free ware alternative to Matlab. Scilab does not have all of the advanced features that Matlab offers but it does have all of the necessary mathematical capabilities to implement our conceptual model.

4.1 Implementing the Conceptual Model

To implement our conceptual model we only need software that is capable to solve the mathematical equations presented in chapter 3. To do this we use the software package Scilab [32]. Scilab is a free open source numerical computation software package that is aimed at engineering and scientific applications. Scilab is ideal for our model because it is free and it has all of the mathematical capabilities needed by our model.

Our model was implemented as follows: We use a main scripting file which Scilab executes. The main scripting file then calls other scripting files to do the various steps that need to be executed. First a scripting file is called which declares all of the parameter inputs used by our model. This scripting file is basically used to set up our model. Then a scripting file is called which calculates the routing probabilities followed by a scripting file which calculates the channel access probabilities. A scripting file is then called to calculate the arrival rates and service rates for each node and lastly a scripting file is called which calculates the performance parameters for each node. Figure 4.1 depicts a flow diagram of our model implemented in Scilab. The scripting files can be located on the attached CD.

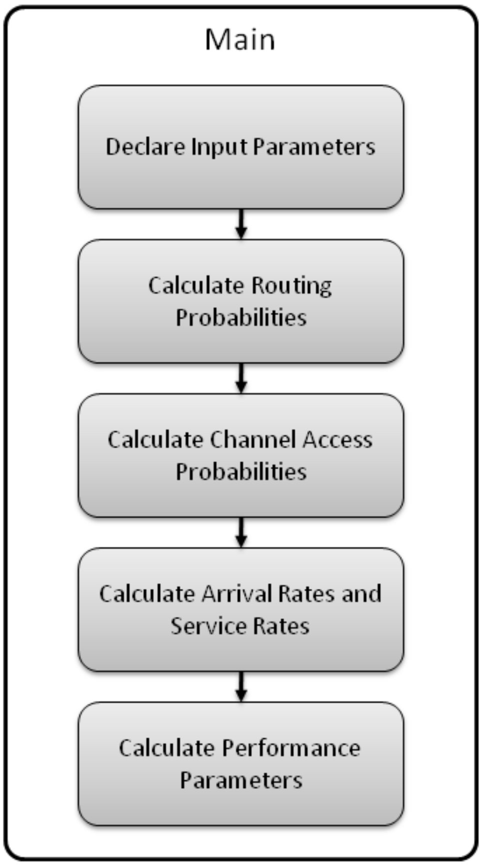

Figure 4.1: Flow diagram of model implemented in Scilab

4.2 Verifying the Computerized model

In this section we aim to verify our implemented mathematical model by comparing the results from our computerized model to that of a discrete event simulator. The discrete event simulator we use is called Java Modelling Tools (JMT) and was created by Bertoli, Casale and Serazze [33]. JMT contains a discrete event simulation program to model queueing networks and is ideal for verifying our model. JMT uses Random Number Generation (RNG) algorithms to generate customers according to a given distribution and then collects statistics as the customers travel through the simulated queueing network. Since JMT is a queueing network modeller and not a wireless network modeller we set the value of the channel access probability, $p(s_i)$, equal to one for the verification of the model. We do this because JMT does not take into account wireless interference.

We test our model using the two main topologies we discussed in the introduction. The first topology routes traffic to adjacent nodes with equal probability and does not take into account the effect of gateways in the network. The second topology uses the routing strategy proposed by us for our basic model in chapter 3. Figure 4.2 and 4.3 depicts the two topologies used in the test respectfully.

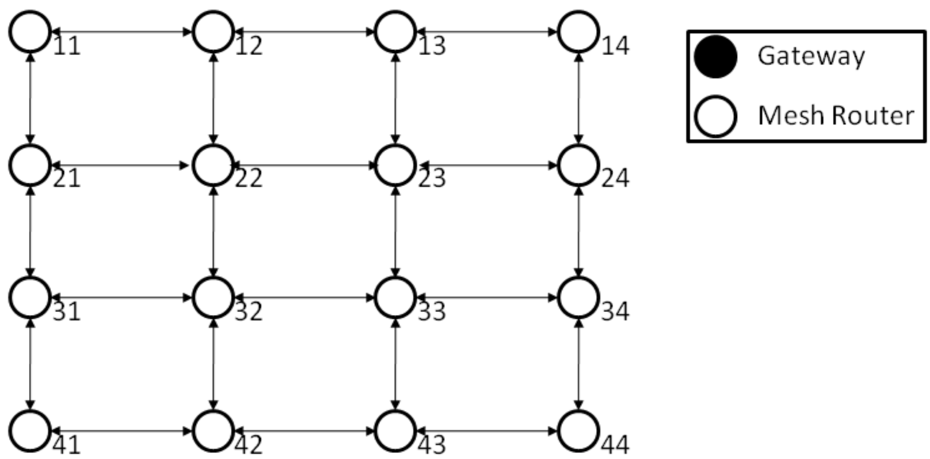

Figure 4.2: Topology A

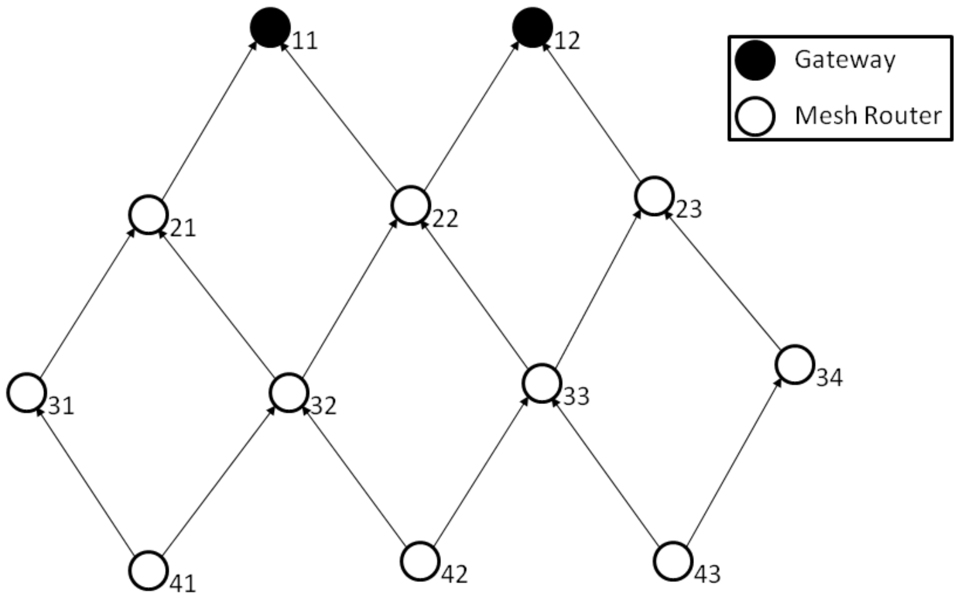

Figure 4.3: Topology B

4.2.1 Mathematical Model Set up

For both topologies the bandwidth, B, was set to 11 Mbps and the average packet size, P, was set to 1200 bytes. This leads to the service rate, μ_{s_i}, for all nodes in both topologies being given as:

$$\mu_{s_i} = \frac{B}{8P} \cdot p(s_i) = \frac{11 \cdot 1024 \cdot 1024}{1200 \cdot 8} \cdot 1 = 1201.49. \quad (4.1)$$

The arrival rates for both topologies were randomly chosen with consideration that ρ_{s_i} should be less than one for the system to be stable. The arrival rates are given in table 4.1.

Table 4.1: Summary of parameters

Parameter	Topology A	Topology B
λ_{1_1}	160	150
λ_{1_2}	150	160
λ_{1_3}	160	-
λ_{1_4}	140	-
λ_{2_1}	160	140
λ_{2_2}	160	170
λ_{2_3}	170	120
λ_{2_4}	150	-
λ_{3_1}	150	160
λ_{3_2}	160	150
λ_{3_3}	170	140
λ_{3_4}	140	150
λ_{4_1}	150	130
λ_{4_2}	170	120
λ_{4_3}	160	150
λ_{4_4}	150	-
μ_{s_i}	1201.49	1201.49
h_1	4	2
h_2	4	3
h_3	4	4
h_4	4	3

Topology A

For topology A the connection matrix, **D**, is given as:

$$\mathbf{D} = \begin{matrix}
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0
\end{matrix} \tag{4.2}$$

Topology A uses a routing strategy where each node transmits its traffic to one of its adjacent nodes with equal probability. The probability that a packet leaves the system, $r_{s_i,0}$, at node s_i was randomly selected as 0.2. It was important that $r_{s_i,0}$ was not zero to ensure that packets do leave the system at some point. The routing probabilities are then given as:

$$r_{s_i,u_j} = \begin{cases} (1 - r_{s_i,0}) \cdot \left[\sum_{u=1}^{S} \sum_{j=1}^{h_u} d_{s_i,u_j} \right]^{-1}, & d_{s_i,u_j} = 1 \\ 0, & \text{elsewhere.} \end{cases} \tag{4.3}$$

Topology A is a good example of how we were able to easily implement another routing strategy without affecting our main model.

Topology B

For topology B the connection matrix, **D**, is given as:

$$\mathbf{D} = \begin{matrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{matrix} \tag{4.4}$$

Topology B uses the routing strategy proposed in chapter 3 and for that strategy the routing probabilities are given as:

$$r_{s_i,(s-1)_j} = \begin{cases} \left[\sum_{k=1}^{h_{s-1}} d_{s_i,(s-1)_k} \right]^{-1}, & 1 < s \leq S, \quad d_{s_i,(s-1)_j} = 1 \\ 0, & \text{elsewhere.} \end{cases} \tag{4.5}$$

JMT Set up

Two separate simulations were created using JMT, one for topology A and one for topology B. In both simulations the service times of each node was set to be exponentially distributed with mean 1/1201.5. Arrivals at each node was set to occur according to a Poisson process with the rates for each node given in table 4.1. Each node was set to have one server with an infinite buffer size. The routing probabilities for each node was set according to those given by equations 4.3 and 4.5. JMT was set to obtain results with a confidence interval of 0.99 and a maximum relative error of 0.03.

4.2.2 Simulation Results

Topology A

The results for topology A are given in figures 4.4, 4.5 and 4.6. Figure 4.4 depicts the mean number of customers for each node. Figure 4.5 depicts the mean response time for each node and figure 4.6 depicts the utilization of each node. As can be seen from the figures the simulated results from JMT match very closely to the analytical results from our model.

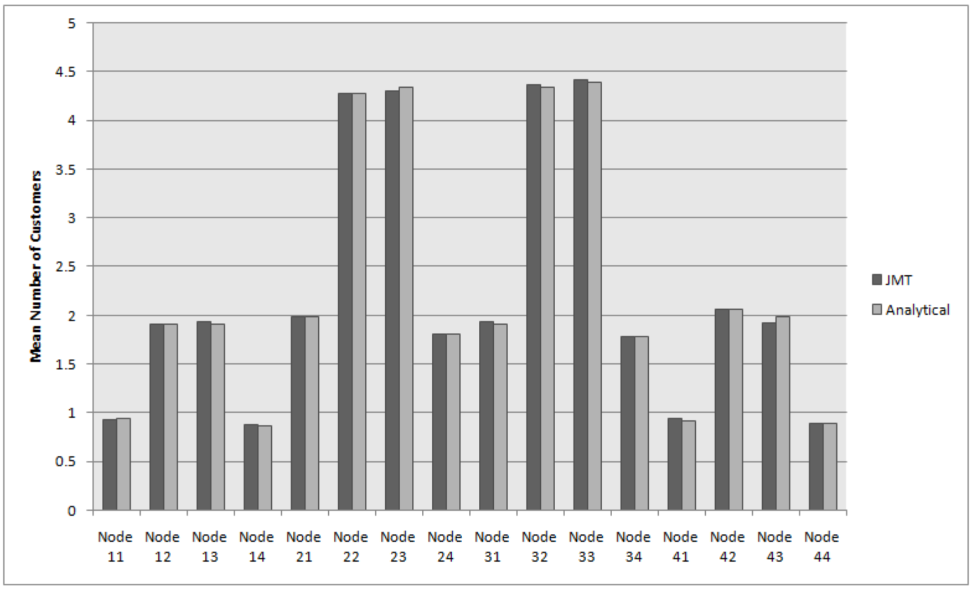

Figure 4.4: Topology A - Mean Number of Customers

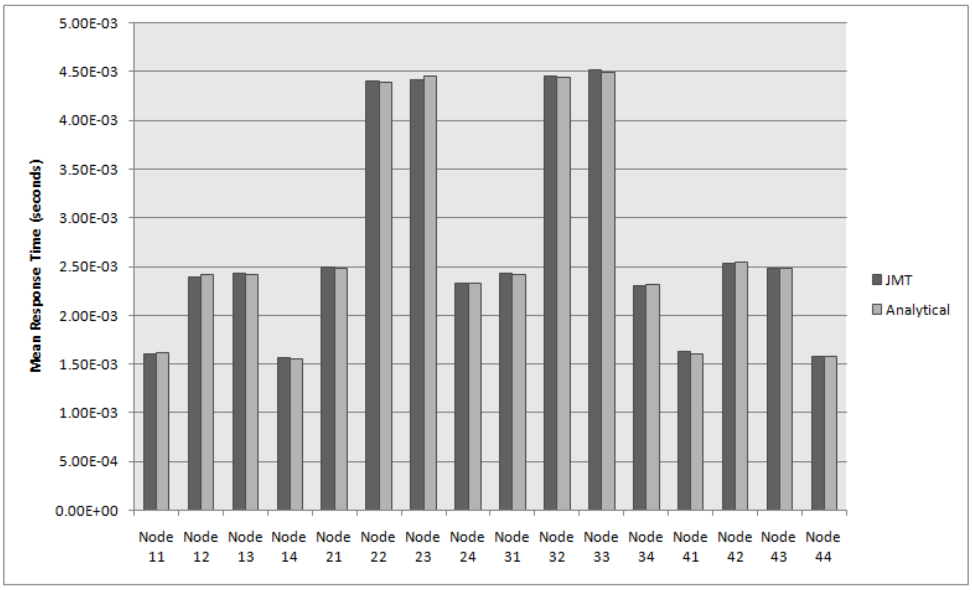

Figure 4.5: Topology A - Mean Response Time

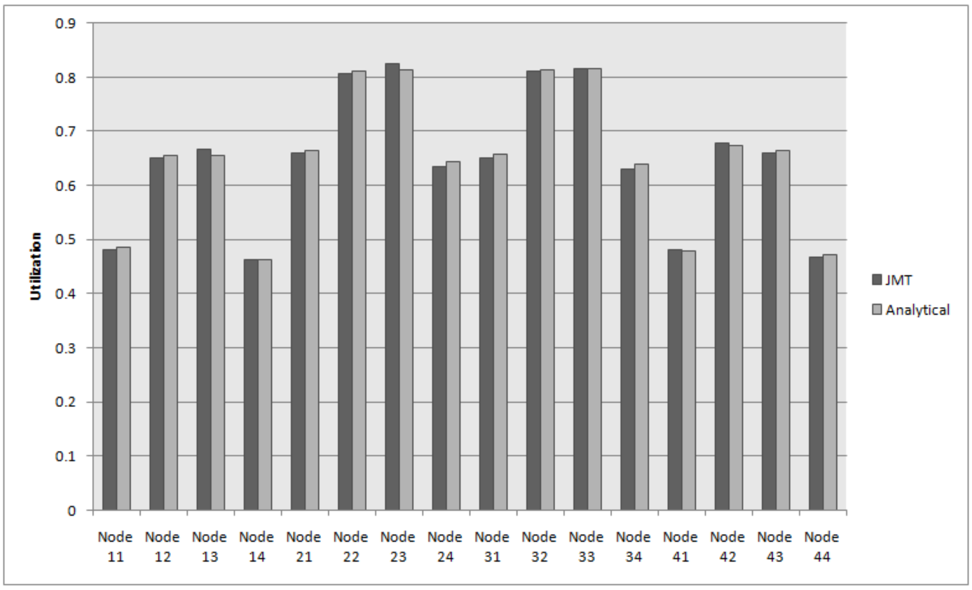

Figure 4.6: Topology A - Utilization

Topology B

The results for topology B are given in figures 4.7, 4.8 and 4.9. Once again these figures depict the mean number of customers, mean response time and utilization for each node respectively. As can be seen from the figures the simulated results and the analytical results are close to each other.

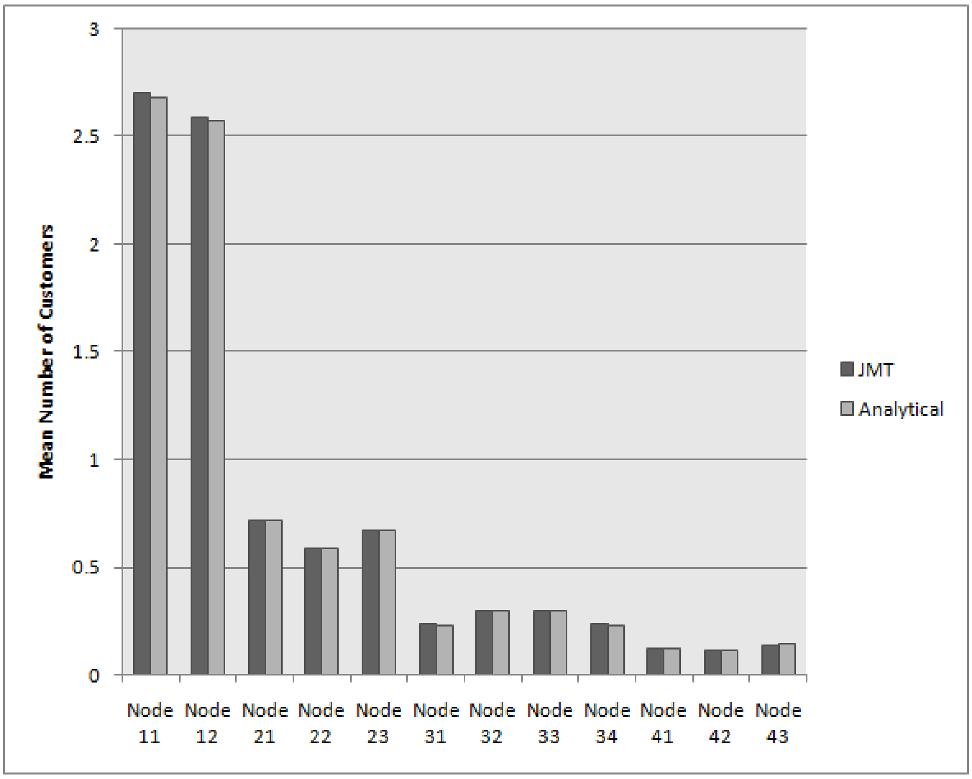

Figure 4.7: Topology B - Mean Number of Customers

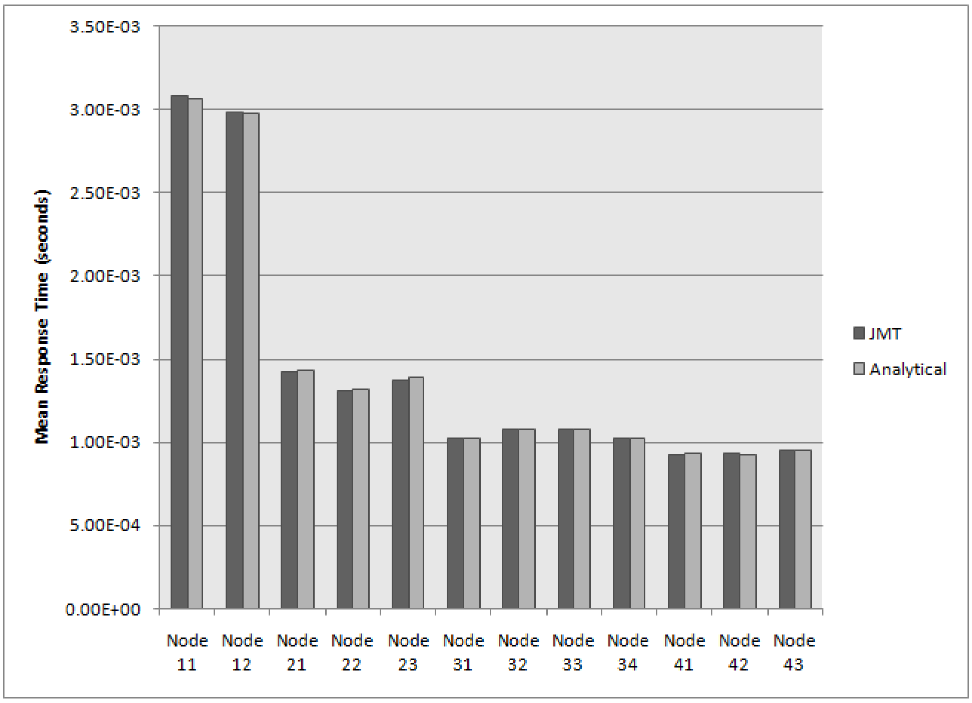

Figure 4.8: Topology B - Mean Response Time

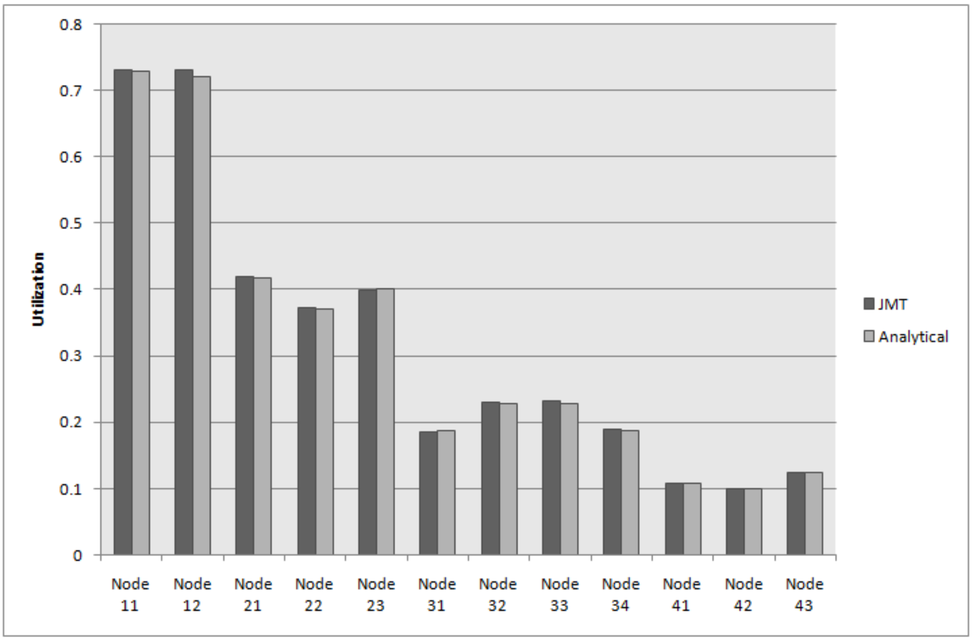

Figure 4.9: Topology B - Utilization

Conclusion

We were able to predict performance measures for each node in the network that matched closely to the simulation results. We can conclude from this that the mathematics in our model is accurate when the network holds to the assumptions made and that our mathematical model is verified.

Chapter 5

Validating the Computerized Model

In chapter 3 we presented our conceptual model and in chapter 4 we implemented a computerized model from the conceptual model and verified it. In this chapter we will validate our model, i.e. does our model solve the problem given in the problem statement in chapter 1. The problem statement stated that a need was identified for an analytical model, that extends on the models proposed by Feng et al. and Bisnik and Abouzeid, that is capable of modelling both the scenarios proposed by the fore mentioned models and more. To prove that our model is indeed capable of modelling these different scenarios we will model each of the queueing based network traffic models for WMNs discussed in chapter 2. We will show that our model is indeed capable of modelling the different scenarios presented by the aforementioned models.

5.1 Wu et al.

We start with the simplest model of the four, namely that of Wu et al. [28]. We will compare results from our model to the results Wu et al. obtained from their model. For ease of explanation we will refer to our model as QBM (Queueing Based Model) in this chapter.

5.1.1 Model Setup: Wu et al.

Wu et al. derived expressions for two topologies, namely a linear topology and a grid topology. The linear topology is depicted in figure 5.1. Wu et al. only models the gateways in the network. For the linear topology Wu et al. assumes the arrival rate to the gateway to be equal to $N\lambda$, where N is the number of mesh routers in the network and λ is the arrival rate for each mesh router. For the grid topology Wu et al. assumes that the traffic of all nodes are distributed equally among the gateways. Wu et al. assumes that the arrival rate to the gateways for the grid topology to be equal to $N\lambda/M$, where M is the number of gateways in the network. The grid network is thus analysed as a series of linear networks. Because of this we only compare results of the linear topology.

Figure 5.1: Wu et al.: Linear Topology

For the results we are comparing with Wu et al. allowed the service rate to be equal to $1/0.01$ and the number of mesh routers in the network to be equal to 10. They then varied λ.

5.1.2 Model Setup: QBM

The routing strategy we used for this scenario is the basic spanning tree protocol we proposed in chapter 3. The **D** matrix is given as:

$$\mathbf{D} = \begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{matrix} \tag{5.1}$$

The arrival rate for all nodes, except the gateway, was set to be the same and equal to λ. The arrival rate for the gateway was set to zero.

Due to the routing protocol, there are no traffic flows from the gateway to the other nodes which means the arrival rate of the other nodes are not dependent on the arrival rate of the gateways. This means we are allowed to choose any service process for the gateway without affecting the other nodes. We set the service rate of the gateway to be constant, the same as Wu et al.'s model, with rate $\mu_{s_i} = 1/0.01$. We will compare the average waiting time of a packet in queue. The average waiting time of a packet in queue for a $M/D/1$ system is given as [6]:

$$Wq_{s_i} = \frac{2 - \rho_{s_i}}{2\mu_{s_i}(1 - \rho_{s_i})} \tag{5.2}$$

5.1.3 Results

The results are depicted in figure 5.2. The results show how the response time of the gateway increases as the arrival rate of the mesh routers increase. Both of the models go to infinity as they approach an arrival rate of ten. This is because the system is not stable when $\lambda > \mu$. As can be seen from the figure, the results from our model matches We et al.'s model exactly. The reason is that in both our model and Wu et al.'s model the gateway is represented as a $M/D/1$ queue with the same parameters. The advantage of our model is that we are able to calculate the performance parameters of the other nodes in the network as well. Another advantage of our model is that we only have to redefine the connection matrix to model other topologies. Wu et al. need to derive the performance parameters each time they change the topology. We conclude that our model is valid for this scenario.

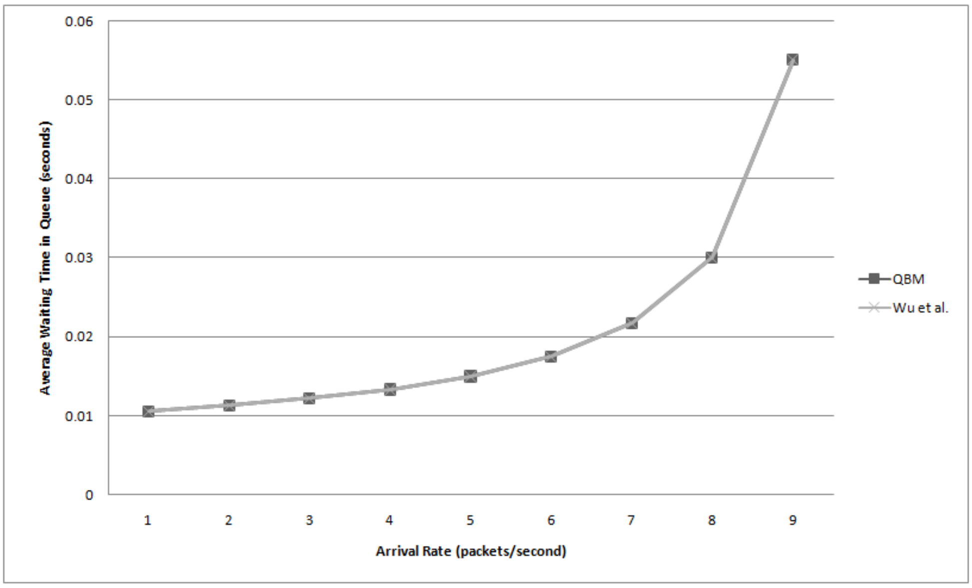

Figure 5.2: Wu et al.: Average waiting time in queue

5.2 Feng et al.

The main advantage of Feng et al.'s model is that it takes into account queues with finite buffers. Queues with truncation are not applicable to queueing networks with product form because the input of a truncated queue is actually not Poisson Distributed [6]. However, we can still use our model and then analyse each node as a $M/M/1/k$ queueing system to approximate the packet loss probability. Feng et al. analysed their model in a similar way.

Feng et al. compared results from their model to results from simulations run in NS2. In this section we will be comparing results from our model to the NS2 results as well as the results from Feng et al.'s model.

5.2.1 Simulation Set up: NS2

The simulation settings Feng et al. used for the simulation were as follows: All nodes in the network are distributed in a $1500m \times 1500m$ grid space. The buffer length of each mesh node was set to 50. The transmission range of every mesh node was set to $140m$ and the interference range was set to $280m$. The flow of all network traffic was set to be from the mesh nodes to the gateways. Packet size was set to 1500 bytes and the network adopts 802.11b as its MAC layer protocol.

5.2.2 Model Set up: Feng et al.

Feng et al. used the topologies depicted in figures 5.3 and 5.4 which they denote as L1 and L2. Feng et al. chose the bandwidth of every flow as 0.412 Mbps and the packet size as 1500 bytes. Each topology contains one gateway and the maximum hop count was set to six for both topologies (figures 5.3 and 5.4 only depict up to 3 hop counts). This means that L1 contains 18 nodes (excluding the gateway) and L2 contains 127 nodes. The queue length of each node was set to 50.

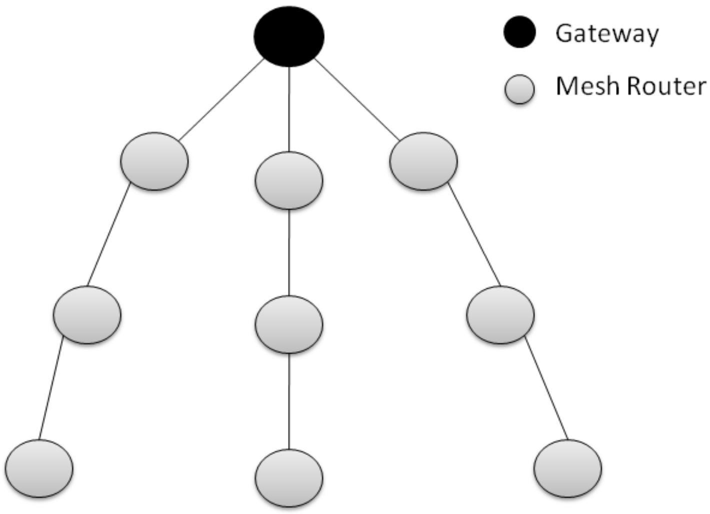

Figure 5.3: Feng et al.: Topology: L1

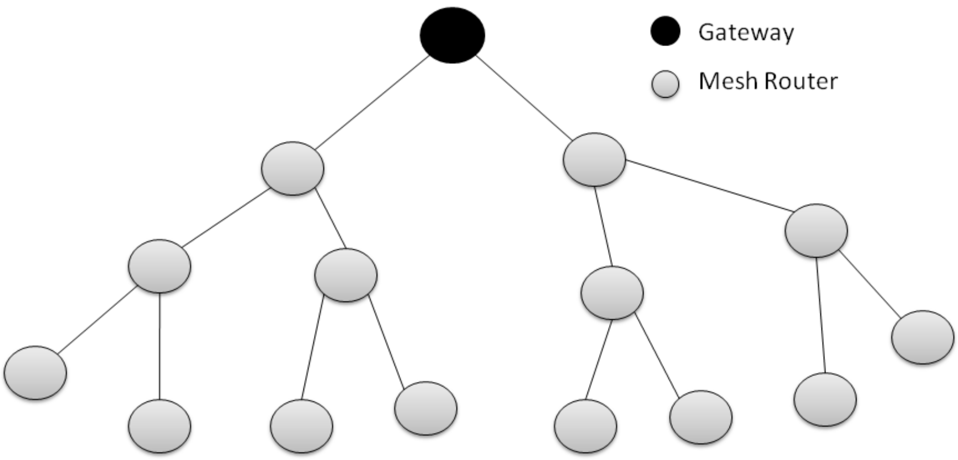

Figure 5.4: Feng et al.: Topology: L2

5.2.3 Model Set up: QBM

We will be using our model with the routing protocol and channel access probability scheme proposed in chapter 3. To account for the effect of finite buffers, we will approximate the packet loss rate by analysing the nodes in our model as $M/M/1/k$ queueing systems. We will then subtract the approximated packet loss rate from the arrival rate at each node to account for the dropped packets. The probability that the buffer will be full for a $M/M/1/k$ queueing system is given by:

$$p_{K(s_i)} = \frac{(1-\rho_{s_i})\rho_{s_i}^K}{1-\rho_{s_i}^{K+1}} \tag{5.3}$$

The effective arrival rate, $\lambda_{s_i}^{(eff)}$, for node s_i is then given as:

$$\lambda_{s_i}^{(eff)} = \lambda_{s_i}(1 - p_{K(s_i)}) \tag{5.4}$$

The arrival rate, λ_{s_i}, for node s_i needs to be calculated from the effective rates that flow into node s_i. The arrival rate is thus given as:

$$\begin{aligned} \lambda_{s_i} &= \gamma_{s_i} + \sum_{j=1}^{h_{(s+1)}} \lambda_{(s+1)_j}^{(eff)} \cdot r_{(s+1)_j, s_i} \quad && 1 \leq s < S \\ \lambda_{s_i} &= \gamma_{s_i} && s = S \end{aligned} \tag{5.5}$$

We analyse each node as a $M/M/1/k$ queueing system and we calculate L_{s_i}, Lq_{s_i}, W_{s_i} and Wq_{s_i} according to the formulas given in section 2.4. The rest of the parameters for our model are set as follows: The bandwidth, B, is set to 11 Mbps and the average packet size, P, is set to 1500 bytes. The arrival rate, γ_{s_i}, for each node is set to 36 packets per second which relates to 0.412 Mbps.

5.2.4 Results

Figure 5.5 depicts the network throughput for each topology. Traffic flows only from mesh routers to gateways and the gateways are set to have a much higher processing rate than the other nodes. Because of this the throughput of the network is given by the effective output of the 1-hop nodes. As can be seen from the figure our model's results closely match the simulation results and the results from Feng et al.'s model for both topologies.

Figure 5.6 depicts the effective output of each hop. For topology L1 our results closely match the results of Feng et al. However, for topology L2 our results differ from Feng et al's at hop two and three. For topology L2 the arrival rate for the nodes closer to the gateway is much larger than the process rate of the nodes. The amount of packets dropped for topology L2 is highly dependent on the process rate. Feng et al. uses the function $I(i)$ in their model to account for wireless interference where we use the channel access probability $p(s_i)$. However Feng et al. never gives any indication in their paper on how they obtain $I(i)$, whether it was calculated or assumed to be known from simulations. If more was known about $I(i)$ we could have calculated $p(s_i)$ in a similar way. Our focus in the study is the topology of the network and not the MAC model. Since we were able to model the scenarios proposed by Feng et al. in [1], we can conclude that our model is valid for this scenario.

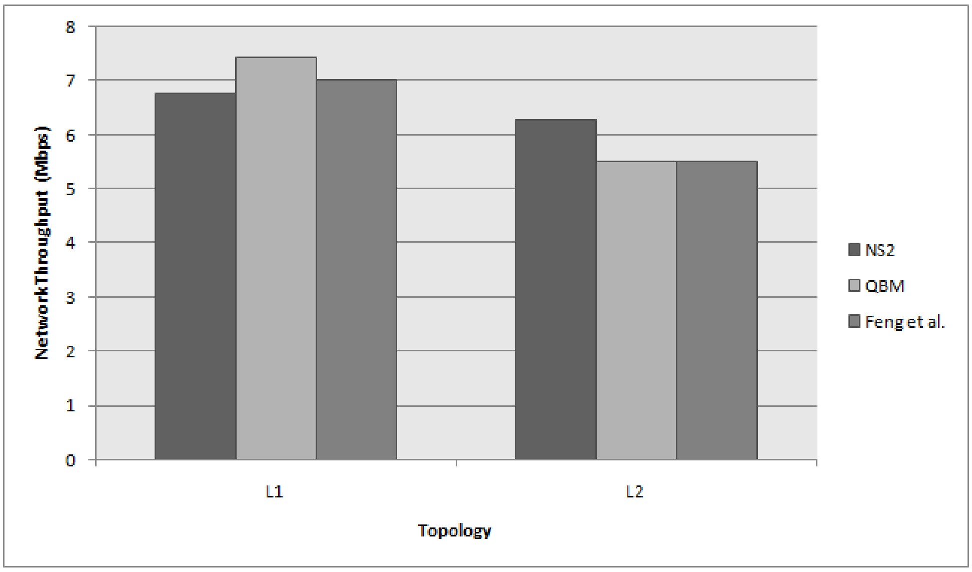

Figure 5.5: Feng et al.: Network Throughput

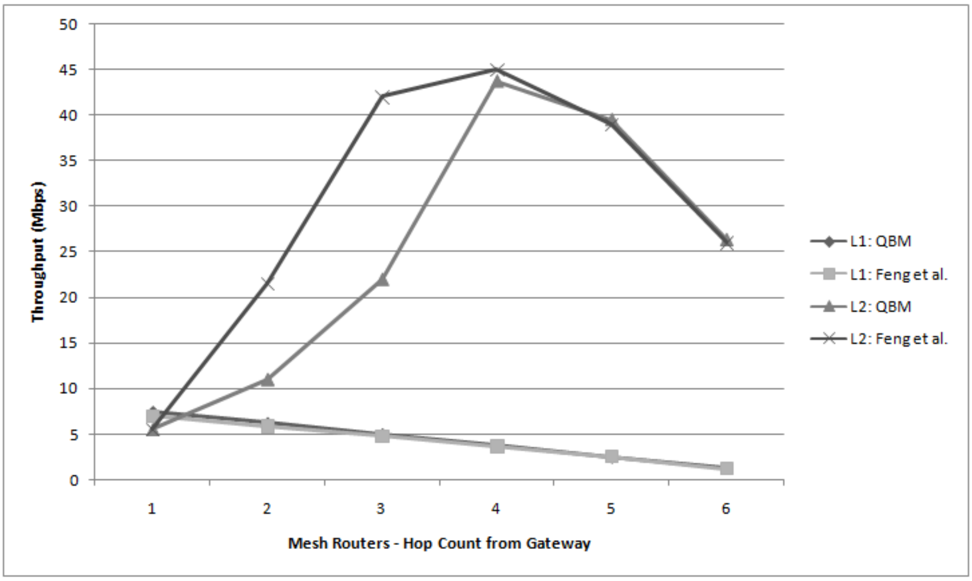

Figure 5.6: Feng et al.: Effective output of each hop

5.3 Bisnik and Abouzeid

Lastly we compare results from Bisnik and Abouzeid to our model. Bisnik and Abouzeid [3, 31] created a very complicated MAC model for their network traffic model. Their MAC model uses a random back-off timer before a node starts transmitting. If another node is detected transmitting, the back-off timer is paused until the other node has finished transmitting. Bisnik and Abouzeid implemented the back-off timer in their model by allowing the mean service time to consist of: (i) the mean random back-off period, (ii) the mean time other nodes are busy transmitting, and (iii) the mean time it takes to transmit a packet. The mean random back-off period is dependent on among others the packet size, the bandwidth and the number of nodes in the network. In this section we compare our model to the results Bisnik and Abouzeid presented in [31]

5.3.1 Model Set up: Bisnik and Abouzeid

Bisnik and Abouzeid set the packet size for this simulation to 1 kbits and the bandwidth to 10^6 bps. The probability that a packet leaves the system at a node is set to $\sqrt{\log n/n}$, where n is the number of nodes in the network which is set to 400. The topology used by Bisnik and Abouzeid is depicted in figure 5.7.

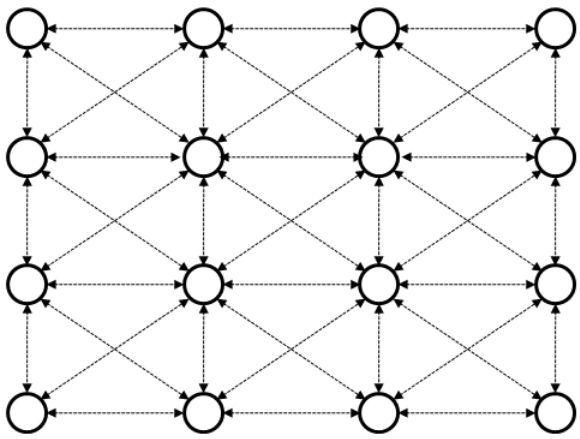

Figure 5.7: Bisnik and Abouzeid: Topology

5.3.2 Model Set up: QBM

For our model we use our basic channel access probability scheme proposed in chapter 3. The routing scheme we use is the one we proposed in section 4.2.1 for topology A. The routing probabilities are given by equation 4.3. We set $B = 10^6$ bps, $P = 1000/8 = 125$ bytes and $r_{s_i} = \sqrt{\log n/n}$, where n is set to 400. The average end-to-end delay is given as:

$$W_{end} = \frac{W_{avg}}{r_{s_i}}, \tag{5.6}$$

where W_{avg} is the average delay of all the nodes.

5.3.3 Results

The results are depicted in figure 5.8. As can be seen from the figure our results closely match the results of Bisnik and Abouzeid. The reason Bisnik and Abouzeid's average end-to-end delay increases faster as λ increases is due to the random back-off timer they implemented. The back-off timer is executed before each packet is transmitted, which means the more packets there are in the system the more back-off timers are executed and the higher the average end-to-end delay. Once again, our focus in this study is the topology of the network and not the MAC model.

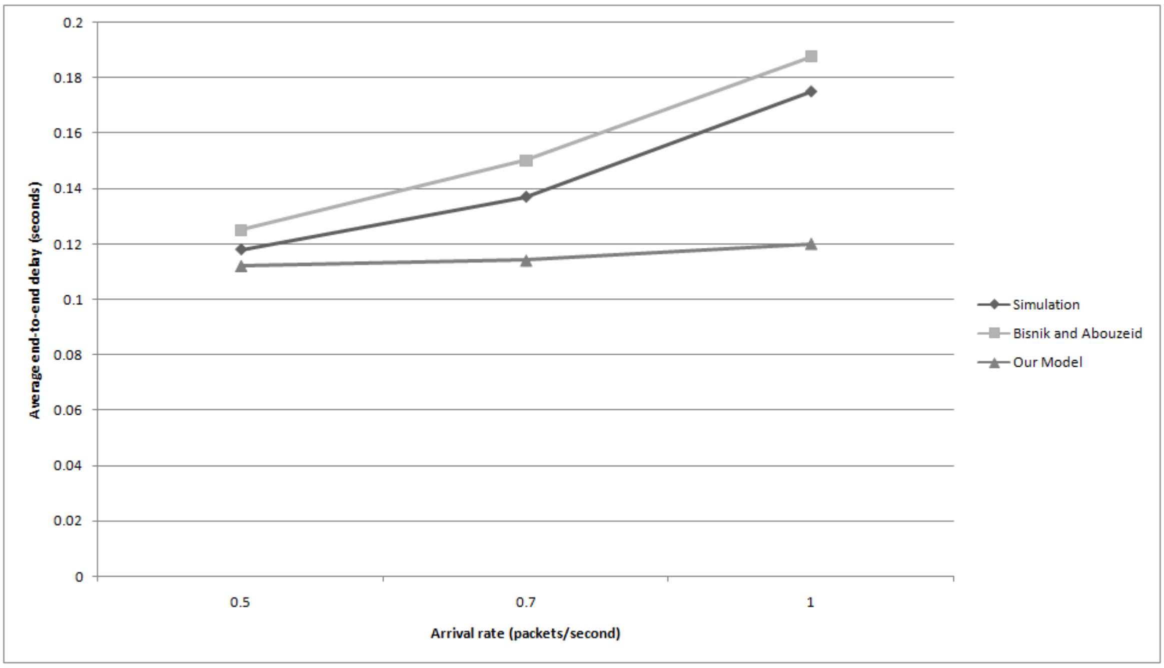

Figure 5.8: Bisnik and Abouzeid: Average end-to-end delay in the network

5.4 Conclusion

In this chapter we used our model to model various scenarios presented by other models. Our model was capable of modelling the scenarios presented by the models of *Feng et al.* [1], *Wu et al.* [28] and *Bisnik and Abouzeid* [31] within known and adjustable parameters. We can conclude from this that our model is valid. All of the presented models derived performance parameters based on assumptions surrounding the topology. Our model derived the performance parameters from the connection matrix and there were no limits on how the connection matrix was set up. This allowed us to model all of the presented scenarios with ease.

Chapter 6

Conclusion

In this chapter we give a summary of this thesis and we draw final conclusions.

6.1 Summary of thesis

In chapter 1 we presented a brief background on our problem which stated that a need has been identified for an analytical model, that is capable of modelling both the scenarios presented by *Feng et al.* and *Bisnik and Abouzeid*. This led to the objective of this study which is to use queueing theory to create a single analytical model that is capable of modelling the scenarios presented by the models from both *Feng et al.* and *Bisnik and Abouzeid*.

A literature study is given in chapter 2 where we first introduced network traffic modelling. This was followed by a discussion of the WMN environment. We then discussed commonly used traffic models and found that queueing theory is a good solution to modelling the WMN environment. This was followed by a background on queueing theory. Finally we discussed various queueing based network traffic models from which we concluded that an optimal way to define the topology of the network, that has not been used before as far as we know, is to use a connection matrix. The performance parameters of our model can then be dependent on the connection matrix.

After the literature study we presented our conceptual model in chapter 3 which was created during the *analysis and modelling* phase of our study. In chapter 4 we implemented a computerized version of our model in Scilab which is a freeware alternative to Matlab. This was done during the *computer programming and implementation* phase of our study. We verified the implementation of our model by comparing results from our model to results from a discrete event simulator. From the results we concluded that the mathematics in our model is accurate under the assumptions that were made.

Finally in chapter 5 we validated our model by comparing results from our model to results from the queueing based models we discussed in chapter 2. Our combined model allowed us to model the individual scenarios presented by Wu et al. [28], Feng et al. [1] and Bisnik and Abouzeid [31]. We conclude from this that the objective of this study was obtained.

6.2 Conclusion

In this section we draw final concluding remarks and recommendations.

6.2.1 Connection Matrix

A novel contribution of this thesis is the use of the connection matrix. To our knowledge a connection matrix has not been previously used coupled with queueing theory to model a WMN. By deriving the parameters of our model from the connection matrix instead of deriving them based on assumptions of the topology we are able to model a wider variety of topologies. We defined the connection matrix in such a way that we are also able to locate the gateways in the network which means that our model is able to account for the effect of gateways in the network.

6.2.2 Abstraction of the Routing Protocols

By abstracting the calculation of the routing protocols from the main model we are able to use different routing schemes with our model without affecting the main model. Different routing schemes are implemented in our model by just calculating the routing probabilities differently. In this thesis we used two routing schemes with our model namely the basic spanning-tree protocol proposed in chapter 3 and the random routing protocol proposed in section 4.2.1. The basic spanning tree routing protocol directs network traffic to the gateways. The random routing protocol directs traffic with equal probability to one of the neighbour nodes with each node having a probability of being a destination of a packet. We conclude from this that our model is able to use different routing schemes without affecting the calculations of the main model.

6.2.3 Abstraction of the MAC Protocols

We defined the channel access probability in the calculation of the service rate of each node for our model in order to abstract the implementation of the MAC protocols from the main model. The channel access probability allows us to implement a variety of MAC models without affecting the calculations of the main model. In this thesis we only used a simplistic MAC model. The MAC model simply stated that all nodes within each other's range have equal probability to transmit. For the most part this model was sufficient but when we compared results from our model to results from Bisnik and Abouzeid's model [3] we observed slight differences. This was due to the fact that Bisnik and Abouzeid implemented a much more complex MAC model. By redefining our channel access probability we would however be able to implement a similar MAC model but the creation and/or improvement of MAC models was not the focus of this study.

6.2.4 Focus of this study

In this study we focussed on the creation of an analytical network traffic model for WMNs. We did not focus on the creation and/or improvement of MAC and routing protocols. The abstraction of the implementation of the MAC and routing schemes in our model has allowed us to be able to implement a variety of MAC and routing schemes without affecting the calculations of the main model. We can conclude from this that we have accomplished the objective of this study. Our model has created a good platform to analytically analyse and compare different MAC and routing protocols and our future recommendation is to use our model for this purpose.

6.3 Future Work

6.3.1 Assumptions of the gateways

Wu et al. [28] and Feng et al. [1] made the assumption that traffic is destined for the gateways. Wu et al. further assumed that because the bandwidth of the wired links is much greater than the bandwidth of the wireless links, the inter-service times of the gateways can be constant. Feng et al. made a similar assumption only they assumed that the buffer of the gateway will be infinite. These assumptions are valid when the traffic only flows to the gateways. However when traffic flows from the gateways to the nodes these assumptions are not valid any more since the gateways will now have to transmit their load over the wireless medium. The assumption of the destination of all traffic being the gateway was based upon the fact that WMNs are commonly used as a last mile technology for internet service providers. When it comes to using the internet, the amount of download traffic is usually much greater then the amount of upload traffic. This leads to the open question if it is really viable to model the gateways with increased service rates and/or buffer sizes? Future work can include using our model to study a suitable way to analytically model the gateways within a

WMN.

6.3.2 Multiple Traffic Classes

Our model is based upon a Jackson queueing network which is capable of using multiple traffic classes as long as each traffic class uses the same service rate. If different traffic classes are used within our model then each traffic class can use its own routing scheme. Each traffic class is analysed separately and the results are then the sum of the separate results of each traffic class. This means that our model can be used to model for instance a network where most of the traffic is destined to the gateways but also has traffic that is destined for other nodes in the network. Future work that can be done is to analyse various networks and associated traffic classes using our queueing based model.

Bibliography

[1] Y. Feng, X. Shen, Z. Gao, and G. Dai, "Queuing based traffic model for wireless mesh networks," in *15th International Conference on Parallel and Distributed Systems (ICPADS), 2009 DOI - 10.1109/ICPADS.2009.68*, 2009, pp. 648–654.

[2] V. Frost and B. Melamed, "Traffic modeling for telecommunications networks," *IEEE Communications Magazine DOI - 10.1109/35.267444*, vol. 32, no. 3, pp. 70–81, 1994.

[3] N. Bisnik and A. Abouzeid, "Delay and throughput in random access wireless mesh networks," in *IEEE International Conference on Communications, 2006. ICC '06. DOI - 10.1109/ICC.2006.254761*, vol. 1, 2006, pp. 403–408.

[4] I. Akyildiz and X. Wang, "A survey on wireless mesh networks," *IEEE Communications Magazine DOI - 10.1109/MCOM.2005.1509968*, vol. 43, pp. S23–S30, 2005.

[5] J. Jun and M. Sichitiu, "The nominal capacity of wireless mesh networks," *IEEE Wireless Communications*, vol. 10, pp. 8 – 14, 2003.

[6] D. Gross, J. F. Shortle, J. M. Thompson, and C. M. Harris, *Fundamentals of Queueing Theory*, 4th ed., D. Balding, N. Cressie, G. Fitzmaurice, I. Johnstone, G. Molenberghs, D. Scott, A. Smith, R. Tsay, and S. Weisberg, Eds. John Wiley & Sons, 2008.

[7] J. R. Jackson, "Networks of waiting lines," *Operations Research*, vol. 5, no. 4, pp. pp. 518–521, 1957. [Online]. Available: http://www.jstor.org/stable/167249

[8] W. J. Gordon and G. F. Newell, “Closed queuing systems with exponential servers,” *Operations Research*, vol. 15, no. 2, pp. 254–265, 1967. [Online]. Available: http://www.jstor.org/stable/168557

[9] F. Baskett, K. M. Chandy, R. R. Muntz, and F. G. Palacios, “Open, closed, and mixed networks of queues with different classes of customers,” *Journal of the ACM*, vol. 22, pp. 248–260, 1975. [Online]. Available: http://doi.acm.org/10.1145/321879.321887

[10] A. Adas, “Traffic models in broadband networks,” *IEEECommunications Magazine DOI - 10.1109/35.601746*, vol. 35, pp. 82–89, 1997.

[11] M. Ali and H. Gu, “Performance analysis of a wireless sensor network,” in *IEEE Wireless Communications and Networking Conference, 2006. WCNC 2006.*, vol. 2, 2006, pp. 1166 –1171.

[12] F. Ashtiani, J. Salehi, and M. Aref, “Mobility modeling and analytical solution for spatial traffic distribution in wireless multimedia networks,” *IEEE Journal on Selected Areas in Communications*, vol. 21, pp. 1699 – 1709, 2003.

[13] R. Sargent, “Validation and verification of simulation models,” in *Proceedings of the 2004 Winter Simulation Conference, 2004. DOI - 10.1109/WSC.2004.1371298*, vol. 1, 2004, pp. 22–28.

[14] K. Binder and A. Baumgrtner, *Applications of the Monte Carlo method in statistical physics*, K. Binder, Ed. Springer-Verlag, 1987. [Online]. Available: http://books.google.co.za/books?id=EKjvAAAAMAAJ

[15] G. Fishman, *Principles of discrete event simulation*, ser. Wiley series on systems engineering and analysis, J. Wiley and S. Inc, Eds. Wiley, 1978. [Online]. Available: http://books.google.co.za/books?id=qndRAAAAMAAJ

[16] P. Gupta and P. R. Kumar, “The capacity of wireless networks,” *IEEE Transactions On Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[17] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad-hoc wireless networks," in *INFOCOM 2001. Twentieth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 3, 2001, pp. 1360 –1369 vol.3.

[18] A. Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay trade-off in wireless networks," in *INFOCOM 2004. Twenty-third Annual Joint Conference of the IEEE Computer and Communications Societies*, 2004.

[19] N. Bansal and Z. Liu, "Capacity, delay and mobility in wireless ad-hoc networks," in *INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications. IEEE Societies*, vol. 2, 2003.

[20] B. Liu, Z. Liu, and D. Towsley, "On the capacity of hybrid wireless networks," in *INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications. IEEE Societies*, vol. 2, 2003.

[21] T. Liu and W. Liao, "Location-dependent throughput and delay in wireless mesh networks," *IEEE Transactions on Vehicular Technology*, vol. 57, no. 2, pp. 1188 –1198, march 2008.

[22] P. Tran-Gia, D. Staehle, and K. Leibnitz, "Source traffic modeling of wireless applications," *AEU - International Journal of Electronics and Communications*, vol. 55, no. 1, pp. 27–36, 2001. [Online]. Available: http://www.sciencedirect.com/science/article/B7GWW-4DPSMDX-6R/2/5d1625a9d96531353ee54d715e70e0b5

[23] M. Menth, A. Binzenhöfer, and S. Mühleck, "Source models for speech traffic revisited," *IEEE/ACM Transactions on Networking*, vol. 17, pp. 1042–1051, 2009. [Online]. Available: http://dx.doi.org/10.1109/TNET.2008.2006222

[24] E. Cinlar, *Introduction to Stochastic Processes*, E. Cinlar, Ed. Prentice-Hall: New Jersey, 1975.

[25] H. Larson and B. Shubert, *Probabilistic Models in Engineering Sciences*, H. Larson, Ed. New York: John Wiley and Sons, 1979.

[26] A. Alfa, *Queueing Theory for Telecommunications: Discrete Time Modelling of a Single Node System*, Springer, Ed. Springer, 2010. [Online]. Available: http://books.google.co.za/books?id=24ZjEdFbm_wC

[27] H. Heffes and D. Lucantoni, "A markov modulated characterization of packetized voice and data traffic and related statistical multiplexer performance," *IEEE Journal on Selected Areas in Communications*, vol. 4, no. 6, pp. 856 – 868, 1986.

[28] X. Wu, J. Liu, and G. Chen, "Analysis of bottleneck delay and throughput in wireless mesh networks," in *IEEE International Conference on Mobile Adhoc and Sensor Systems (MASS)*, 2006, pp. 765 –770.

[29] J. D. C. Little, "A proof for the queuing formula: L= xw," *Operations Research*, vol. 9, no. 3, pp. 383–387, 1961. [Online]. Available: http://www.jstor.org/stable/167570

[30] G. Bolch, *Queueing networks and Markov chains: modeling and performance evaluation with computer science applications*, ser. Wiley-Interscience Publication, G. Bolch, Ed. Wiley, 1998. [Online]. Available: http://books.google.co.za/books?id=puFQAAAAMAAJ

[31] N. Bisnik and A. Abouzeid, "Queuing delay and achievable throughput in random access wireless ad hoc networks," in *3rd Annual IEEE Communications Society on Sensor and Ad Hoc Communications and Networks.*, vol. 3, 2006, pp. 874 –880.

[32] Scilab. [Online]. Available: http://www.scilab.org/

[33] M. Bertoli, G. Casale, and G. Serazzi, "Jmt: performance engineering tools for system modeling," *SIGMETRICS Performance Evaluation Review*, vol. 36, no. 4, pp. 10–15, 2009.