

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

The arrival of students from the new school curriculum at universities directed the focus to the gap between the secondary school and tertiary institutions with regard to mathematics. It was initially speculated, in a simplistic way, that the students are not prepared for tertiary mathematics because of the lack of certain algorithmic knowledge due to the exclusion of several topics from the secondary curriculum. During the initial literature review for this study it became clear that the situation is more complex and that there is a gap by virtue of the beliefs about the nature of mathematics, as well as the way mathematics is learned.

6.2 Conclusions

6.2.1 The gap regarding the beliefs of the nature of mathematics

One of the aims of the new school curriculum is to encourage the relativist-dynamic view of mathematics and to use critical and creative thinking to solve problems. The intended curriculum gave the impression that learners from secondary schools should be prepared to work with others in groups, manage themselves, take responsibility for their activities, communicate effectively, collect, analyse and critically evaluate information, etc. However, the implemented curriculum does not provide the learners with these skills as was observed in the attained curriculum. In contrast, it unfortunately has the opposite effect. Students get an instrumentalist view of mathematics due to the pressure on learners to perform well in the final exam in order to be selected for university studies. The utility of mathematics as a means for university admission has become more important than to really understand mathematics, and therefore the learners see mathematics only as a tool to be used in pursuit of an end (par. 5.3.3.1). The learners' knowledge therefore tends to be

procedural or algorithmic in nature and their reasoning seems to be only on an imitative level of copying and following the steps in a similar example (par. 5.5.5).

The lecturers' view of mathematics is static-formalistic since they see the structure of mathematics as a fixed structure of axioms and theorems that must be learned (par. 5.3.3.2), but on the other hand they expect that their students should exhibit creative thinking skills (par.5.5.5). According to Plotz' model, understanding of mathematics entails more than just procedural fluency - students should be able to reason logically and deductively and should manage to transfer their knowledge to different problem contexts. This is in line with the relativistic-dynamic view of mathematics and with what Lithner classifies as global creative reasoning, but lacks in students' preparation for tertiary mathematics.

6.2.2 The gap regarding the learning of mathematics

The expectation of the students is that the learning of mathematics is the transfer of a pre-packed body of knowledge; explained carefully by the lecturers and absorbed by the students. Students consider their learning of mathematics as the responsibility of the lecturers. They expect that mathematics should be learned in the scope of a period and do not want to struggle with problems or try and develop their own method to solve a problem (par.5.3.3.1). The fact that students prefer courses with concrete facts and data that could be mastered in clear sequential linear steps with exact answers instead of those with open answers, indicate that they are not yet prepared for the learning of formal mathematics.

The expectation of the lecturers is that the students should be able to take responsibility for their own learning. They expect of the students to handle large volumes of information in a period and to be able to transfer their pure mathematical knowledge to various other domains (e.g. engineering and economics) themselves without any guidance. The lecturers believe that if the students practice enough in the form of drill exercises, they would be able to connect this knowledge to any other domain (par. 5.3.3.2).

6.2.3 The gap regarding the development from elementary to advanced mathematical thinking

Creative thinking and reasoning play a significant role in mathematics problem solving. Students go through different stages of cognitive development on their journey to learn mathematics with understanding. According to Piaget, Van Hiele and Tall understanding of mathematics starts with visualisation. Van Hiele and Tall agree that to be able to advance to creative thinking and conceptual understanding, one should proceed from the visual level (embodied world) to the descriptive level (symbolic world) onward to the formal level (formal world). The students' memorized and algorithmic reasoning skills are adequate, but they struggle when creative reasoning is required (table 5.15). At the stage that students come to university certain cognitive skills should have been developed. The ILS questionnaire showed that the students prefer that lecturers explain in words supplemented with pictures, diagrams and graphs (par. 5.4.5.2), which indicate that they are still on the visual level according to Van Hiele's theory. According to Tall's theory many students are still operating in the embodied world of pictures and real life situations, while their functioning in the world of procepts is based on imitative reasoning. The transition to the formal world builds on experiences with the embodied and the symbolic world and therefore new material should be presented as such. Students cannot make sense of the proceptual world if their understanding of symbols and relationships is not based on a concrete conceptual foundation (par. 4.6). According to Van Hiele (1986:50), students cannot progress from the visual to the descriptive level and beyond without the help of an instructor. They need good teaching that supports appropriate learning activities that result in deep learning. For each new topic or concept encountered at tertiary level students have to progress through this "learning cycle". E.g. to determine a derivative in calculus one can draw a graph with the action of moving a secant through a point on the graph towards a tangent at the point (embodied world). A student can "see" the changing slope of the curve and imagine the changing slope. Thereafter the slope itself can be computed numerically or symbolically (symbolic world). From there one can proceed to the formal world of defining and proof.

6.2.4 Final conclusions on the gap

These ideas are summarised in the schematic representation of the learning of mathematics with understanding in fig. 6.1.

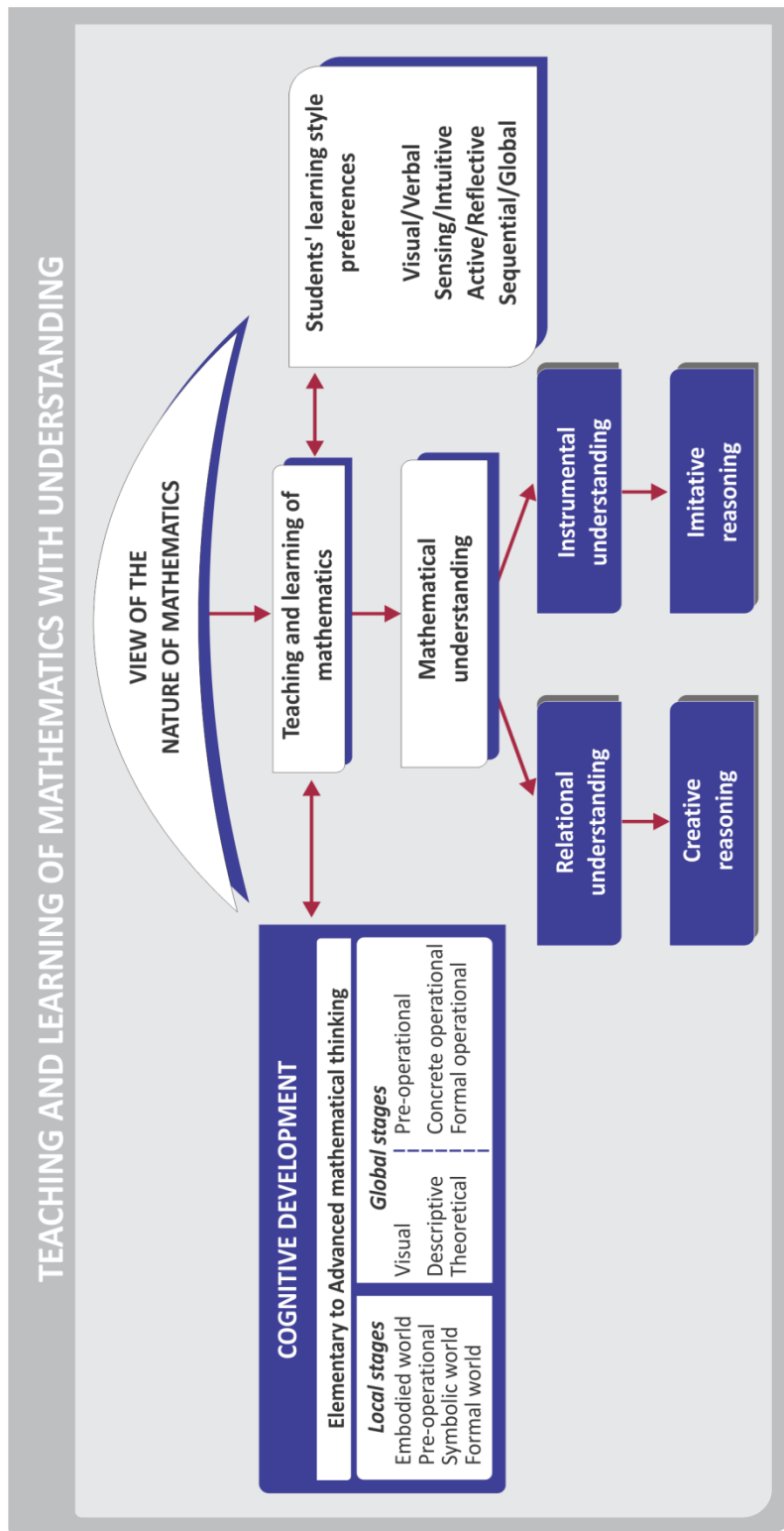


Fig 6.1: Schematic representation of the teaching and learning of mathematics with understanding

Someone's view of the nature of mathematics has an influence on how mathematics is taught, learned and understood and whether this understanding is relational or instrumental. If a person has a relational understanding of mathematics it will allow him/her to reason creatively, while an instrumental understanding of mathematics can only help someone to develop imitative reasoning skills. The views of mathematics can therefore be seen as an umbrella for the teaching, learning and understanding of mathematics.

A student's learning style preferences, as well as their cognitive development do not fall under the umbrella because someone's view of mathematics do not have an influence on these aspects. A student's learning style preference has an influence on the learning of mathematics and therefore also on the preferred teaching of mathematics. However, the way mathematics is taught also has an influence on a student's learning style preference. There are different levels of cognitive development; these levels should be an indication for a teacher or a lecturer of the appropriate teaching style.

6.3 Recommendations

6.3.1 Recommendations for lecturers

Based on the literature study and the empirical results the following recommendations can be useful for the lecturers:

- Educators at universities need to be aware of the mental processes needed by students to understand the abstract concepts, master the formal language and do rigorous reasoning *en route* to mathematical maturity.
- Dealing with formal proof before a student is prepared for this may lead to rote learning.
- The transition to the formal world builds on experiences with the embodied and the symbolic world. New material should be presented as such.
- Students learn mathematics better when visual representations are used during verbal explanations. If the focus is on symbolism and not on related

embodiments then the vision of the students will be limited to performing a procedure, but they will not be able to compress it into thinkable concepts to be used flexibly for more sophisticated thinking.

- Active engagement in the learning process is essential for constructing knowledge with understanding. Students indicated that they learn better when they are physically busy during class periods.
- Students indicated that they learn mathematics better when lecturers explain in clear sequential steps. However, for the acquisition of problem solving skills based on creative thinking, students should also be exposed to problems that require multi-step solutions.
- By choosing appropriate tasks, lecturers should guide students to:
 - Use different representations in various situations;
 - Use appropriate reasoning styles;
 - Make connections with prior knowledge;
 - Apply their knowledge to problems in different contexts.

6.3.2 Recommendations for students

Based on the literature study and the empirical results the following recommendations can be useful for the students:

- Students should consciously pay attention and reflect on the meaning of the procedures and algorithms that they use in order to develop a sense for creativity and a deeper and flexible understanding.
- To be successful in mathematics students should link concepts and processes with existing networks of relationships.
- Students should take responsibility for their own learning and not only rely on the lecturers' input.
- If a student is a visual learner and the presentations in class are more verbal, the student should try to find diagrams, sketches or videotapes of the class material on the internet. The student can prepare a concept map

and draw arrows between concepts to show connections or colour-code the notes with a highlighter.

- Understanding of mathematics entails more than just obtaining a correct answer. Students should make sure that they understand why certain procedures are appropriate and should be used.

6.4 A final note

The transition from secondary to tertiary mathematics is complex and varies according to several factors that should receive attention. The level of cognitive development of the students in this study showed that they are not yet prepared for the formal world of axioms, definitions and proof. Therefore the task of the lecturers is to guide the students through the worlds of embodiment and symbolism to eventually arrive in the world of formalism.

Confucius rightly said:

“I hear and I forget, I see and I remember, I do and I understand”