CHAPTER 5

EMPIRICAL STUDY

5.1 Introduction

Indications are that first year students have problems with the transition from secondary to tertiary mathematics and that these students are under-prepared for tertiary mathematics education. This under-preparedness does not directly correspond to the final school examination results. Some students achieve good grades in secondary school but that does not show in their performance at university level. This study focused on the factors that are potential causes of insufficient mathematical performance of first year students. The literature study investigated the origin of this problem in terms of the nature and structure of mathematics. Chapter 2 aimed to establish how a worldview and current scientific and cultural paradigms influence a person's belief about the nature of mathematics and how this in turn has an effect on the teaching and learning practices of mathematics. Chapter 3 described the structure of mathematical understanding and the different processes through which mathematical understanding develops. A study of the cognitive development of mathematical thinking followed in chapter 4 in order to get insight into the stage of cognitive development of first year students.

The findings of the literature study acted as a guide for the empirical investigation. The first phase investigated the beliefs held by first year students regarding the nature of mathematics, as well as the teaching and learning of mathematics, and this was then compared to the beliefs of the mathematics lecturers. Phase 2 involved a determination of the learning preferences of the students and a comparison to how the lecturers perceived these preferences to be. This provided additional information on the beliefs held by first year students regarding the nature of mathematics, as well as the teaching and learning of mathematics, and to establish at what stage of cognitive development the students are at when they arrive at university. To further investigate the stage of development of the students the first mathematics test written by the students was analysed in phase 3 of the empirical investigation.

5.2 The research methodology

5.2.1 Research design

A cross-sectional survey design was used for this study. In a cross-sectional survey data are collected at one point in time from a sample selected to describe a larger population at that time (Babbie, 1990:56). The purpose of such a survey is not only to describe a population, but also to determine relationships between variables at the time of the study. A survey has a non-experimental, descriptive design and examines a situation as it is. It does not involve changing or modifying the situation under investigation and the intention is not to detect cause-and-effect relationships (Leedy & Ormrod, 2001:191). In this study a survey design was chosen to assess the current opinions and beliefs of first year students and lecturers regarding the learning of mathematics and what they believe the nature of mathematics to be at a specific time. The first test written by these students was used to determine their reasoning competencies at this stage.

5.2.2 Sampling

The study population for the empirical study consists of first year mathematics students, as well as mathematics lecturers from the North-West University at the Potchefstroom Campus. It was convenient for the researcher, who lectures at this university, to use the first year mathematics students enrolled for the mainstream mathematics module WISN111 in 2010 for this study. These are mostly Engineering students, students from the Centre of Business, Mathematics and Informatics (BMI) and a number of Economical Sciences and Natural Sciences students. The number of students who enrol for this module is normally about 500-600, but only bona fide first year students were asked to participate, and participation was on a voluntary basis. Answer sheets with more than one response to a question were disregarded. The final number of participants was 390 students. All the mathematics lecturers were asked to take part in the research, although not all of them teach the specific module, in order to determine the sense of mathematical thinking in the School for Computer, Statistical and Mathematical Sciences. Fourteen lecturers agreed to take part.

5.2.3 The collection of the data

The data from the students for phases 1 and 2:

Data from the students for phase 1 and phase 2 (see par 1.5.3) were collected on the same day. All the *students* present on 20 March 2010 in the class period of the module WISN111, were asked to take part in the research. The purpose of the research was explained to the students and also that it was important that a respondent had to decide which option fits him/her best. The fact that there should only be one response for each question was emphasized. It was communicated to the students that they could withdraw from the research at any stage. The two questionnaires were handed out and enough time was allocated for their completion. The data were captured in Excel files.

The data from the lecturers for phases 1 and 2 (see par 1.5.3):

The purpose of the research was communicated by an email sent to all *lecturers*. It was stressed that participation was voluntarily. Lecturers were asked to complete the two questionnaires, which were delivered to their offices. They had to hand in the completed questionnaires at a specified location away from the researcher's office to avoid recognition. The data for phases 1 and 2 were captured in Excel-files in a format as requested by the Statistical Consultation Services of the university in order to do the initial statistical analysis.

The data for phase 3:

The data for phase 3 *(see par 1.5.3)* of the empirical study was the answer sheets to the *first test* written in 2010 in the WISN111 module. The lecturers of WISN111 marked the answer sheets according to a set memorandum. The marks that the students obtained for each question and sub-question were recorded in an Excel-file. The averages obtained for each question and sub-question was determined.

5.3 The beliefs questionnaire (Annexure A)

5.3.1 Construction of the beliefs questionnaire

This questionnaire was designed by the researcher to determine the students' and lecturers' conceptions about the nature of mathematics. Relevant questions for this study were extracted from the questionnaire developed by Nieuwoudt (1998). The new questionnaire was peer reviewed by four experts in the field and adapted according to their recommendations. The instrument consisted of 30 questions and the intention of this questionnaire was to determine a person's specific belief and/or perception about the nature of mathematics, and the teaching and learning of mathematics.

A Likert-type questionnaire (Annexure A) was used with a possibility of four answers to choose from:

- Strongly disagree
- Disagree
- Agree
- Strongly agree

Each question could be linked to one of the three views of school mathematics (table 5.1) reported by Ernest (par. 2.5).

Formalist: passive reception of knowledge; eacher expert, learners follow instructions; focus is on product; computation of rules explained by teacher and practised by learners; task of teacher to see that learners master the subject; strict hierarchy; orderly and sequential;.

Dynamic: human activity; creativity and discovery; patterns are generated and distilled into knowledge; ordinary people of all ages construct concepts; not a finished product; focus is on process; learner-focused; teacher is facilitator and stimulator of learning; encourages "chaotic moments" so that critical thinking and reflective intuition will flourish.

Instrumentalist: Mathematics is a bag of tools made up of unrelated facts, rules and skills; mathematics has utility value; rules without reasons; in possession of rules and be able to use it; teacher demonstrates, explains in a way that learners understand; learners listen and respond to teacher; ability to get the correct answer is evidence of understanding.

TABLE 5.1: Questions linked to views of mathematics

	QUESTIONS	VIEW
1.	Lecturers should teach specific methods to solve problems.	Formalist
2.	Mathematics is primarily an abstract subject.	Formalist
3.	All people are capable of doing mathematics.	Dynamic
4.	Students can find a method to solve problems without the help of a lecturer.	Dynamic
5.	A good mathematics lecturer always demonstrates the correct method to solve problems.	Formalist
6.	Mathematics is an exact science.	Formalist
7.	The best way to teach mathematics is to show students how to solve problems.	Formalist
8.	It is essential that students often do drill exercises.	Instrumentalist
9.	To be successful in mathematics, a student needs to listen carefully to the lecturers' explanations.	Formalist
10.	The correctness of students' responses indicates how well they understand mathematics.	Instrumentalist
11.	New mathematics is expanded only by research at university level.	Formalist
12.	The most important thing in mathematics is not whether the answer of a problem is correct, but whether the students can explain their answers.	Dynamic
13.	Mathematics problems in a real-life context should be the central focus of the mathematics curriculum.	Dynamic
14.	Many things in mathematics simply have to be accepted and remembered, there is not really an explanation for it.	Formalist
15.	If students struggle to solve problems, it is usually because they don't know the correct rule or cannot remember the formula.	Instrumentalist
16.	Students should never leave the mathematics class with a sense of confusion.	Formalist
17.	Mathematics can be described best as a set of facts, rules and formulas that students have to learn.	Instrumentalist

	QUESTIONS	VIEW
18.	Mathematics is best taught if students are required to solve problems in a real-life context.	Dynamic
19.	The role of the mathematics lecturer is to convey knowledge to the student and to test whether it happened.	Formalist
20.	Mathematics is a practical structured guide to solve problems in real life.	Formalist
21.	Students first have to master basic mathematical facts, rules and procedures before they approach problems in a real-life context.	Formalist
22.	In the teaching of mathematics, lecturers should actively guide students to discover concepts.	Dynamic
23.	If students forget theorems or formulas in a test/exam while they have learned it, it means that they did not do enough exercises.	Instrumentalist
24.	If you use a calculator you are not doing mathematics.	Formalist
25.	Teachers should encourage students to find different ways to solve problems.	Dynamic
26.	Some students have a natural talent to do mathematics.	Formalist
27.	Lecturers should always be able to answer all the students' questions regarding mathematics.	Formalist
28.	Students should learn mathematical theorems and formulas until they know them by heart.	Formalist
29.	It is important to predict solutions in mathematics before the actual calculations are done.	Dynamic
30.	Mathematics problems given to students should easily be solved within the scope of the class time.	Formalist

For the validity of the classification four expert lecturers in the field were asked to pair a view with each question. The classification of the lecturers was compared and differences of opinion were discussed to reach a conclusion.

5.3.2 The reliability and validity of the beliefs questionnaire

In order to determine the construct validity of this questionnaire, an exploratory factor analysis was conducted. The factor analysis extracted 11 factors and with closer inspection these were reduced to 9 factors. Thereafter the Cronbach alpha coefficients were determined for these 9 factors. These scores were below the preferred scores of $\geq$ 0.5, and on the recommendation of the statistical advisor involved with this project, the students' and lecturers' results were compared on an item by item basis, taking in consideration the statistical significant difference between them. Only the items with a statistically and a practically significant difference were used to make conclusions regarding their views of mathematics.

An independent *t*-test was applied to compare the average values of the numeric outcomes between the two groups (students and lecturers). In the case of doubts about the suitability of the data for the requirements of a *t*-test, the Mann-Whitney test may be used instead (McElduff *et al*., 2010:128). The Mann-Whitney test is a non-parametric test to compare two unpaired groups. In the present study there was a great disparity in numbers between the two samples compared and therefore both these tests were used to compare the means of the two groups. The so-called *p*–values are used to indicate if the difference between two means is significant. If the p-value is small, one can reject the idea that the difference is coincidental, and can conclude that the populations have different means. A small p–value (<0.05) is considered as sufficient evidence that the result is statistically significant. Statistical significance does not necessarily imply that the result is important in practice, as these tests are dependent on sample size and have a tendency to yield small p-values as the size of the data set increases. A better measure is the effect size (so-called *d* value), which is independent of the sample size and is a measure of practical significance. It can be understood as a large enough effect to be important in practice and is described for differences in means (Ellis & Steyn, 2003:51). Cohen's d is a measure of effect size that provides an objective measure of the importance of an effect (Field, 2005:32). Cohen's d is given by the following formula: $d = \frac{|\bar{x}_1 - \bar{x}_2|}{s_{\max}}$, where $|\bar{x}_1 - \bar{x}_2|$ is the difference between $\bar{x}_1$ and $\bar{x}_2$ without taking the sign into consideration. When no control group exists, the division by $s_{\max}$ gives rise to a conservative effect size in the sense that a practically significant result will not be concluded too easily.

Cohen (quoted by Ellis & Steyn, 2003:51) gives the following guidelines for the interpretation of the effect size:

- Small effect: $0.2 \leq d < 0.5$
- Medium effect and possibly practically significant: $0.5 \leq d < 0.8$
- Large effect and practically significant: $d \geq 0.8$

5.3.3 Analysis of the beliefs questionnaire

Table 5.2 shows the percentages for the 30 questions of the beliefs questionnaire and indicates if the students or lecturers agreed or disagreed with the statements in the questions. The columns with the d-values for percentages indicate if there was a significant difference between the students' or the lecturers' preferences, i.e. if there was a large or a small difference in the responses.

TABLE 5.2 Student and lecturers percentages for the beliefs questionnaire

	STUDENTS			LECTURERS		
Question	Disagree (%)	Agree (%)	d-values	Disagree (%)	Agree (%)	d-values
1	12	88	1.5	29	71	0.8
2	42	58	0.3	50	50	0
3	55	45	0.2	57	43	0.3
4	73	27	0.9	43	57	0.3
5	9	91	1.6	29	71	0.8
6	19	81	1.2	14	86	1.4
7	13	87	1.5	36	64	0.6
8	7	93	1.7	7	93	1.7
9	5	95	1.8	36	64	0.6
10	27	73	0.9	29	71	0.8
11	36	64	0.6	71	29	0.8
12	37	63	0.5	43	57	0.3
13	25	75	1.0	79	21	1.2
14	66	34	0.6	93	7	1.7
15	31	69	0.8	71	29	0.8
16	16	84	1.4	43	57	0.3
17	49	51	0.0	93	7	1.7
18	21	79	1.2	50	50	0
19	9	91	1.6	21	79	1.2
20	23	77	1.1	29	71	0.8
21	11	89	1.6	7	93	1.7
22	7	93	1.7	7	93	1.7
23	39	61	0.4	57	43	0.3
24	84	15	1.4	71	29	0.8
25	12	88	1.5	0	100	2
26	4	96	1.8	0	100	2
27	8	92	1.7	71	29	0.8
28	43	57	0.3	43	57	0.3
29	41	59	0.4	50	50	0
30	24	76	1.0	71	29	0.8

5.3.3.1 Analysis of the students' beliefs

Table 5.3 shows the keywords of the statements that correspond with the questions with a large effect size and with which the students strongly agree or disagree ($d \geq 0.8$).

TABLE 5.3: Keywords about the nature of mathematics where students strongly agreed or disagreed ($d \geq 0.8$)

STUDENTS		
NATURE OF MATHEMATICS		
Question	**Agree**	**Disagree**
6: Exact science	X	
10: Correctness of answer indicates understanding	X	
13: Problems in real-life is central focus	X	
16: Never leave a class with a sense of confusion	X	
20: A structured guide to solve real-life problems	X	
24: Using a calculator is not mathematics		X
26: Natural talent to do mathematics	X	
27: Lecturers should be able to answer all questions	X	
TEACHING AND LEARNING OF MATHEMATICS		
Question	**Agree**	**Disagree**
1: Lecturer teaches	X	
4: Can solve problems without help of lecturer		X
5: Lecturer demonstrates	X	
7: Show students how	X	
8: Do drill exercises	X	
9: Student listens carefully	X	
15: Know the correct rule	X	
18: Mathematics best taught in real-life context	X	
19: Lecturer conveys knowledge	X	
20: A structured guide to solve real-life problems	X	
21: Master basic facts, rules, procedures	X	
22: Lecturer guide students	X	
30: Problems easily solved in class time	X	

Students' beliefs about the nature of mathematics

The keywords (about the nature of mathematics) in table 5.3 indicate that the students' beliefs about the nature of mathematics are typically static-formalist and/or instrumentalist (see par. 2.5.1 & 2.5.3 and table 5.1). Students believe that mathematics is an exact science and a practical structured guide to solve problems in real life. Students should never leave a class with a sense of confusion. To be successful in mathematics one must know the rules and formulas and thereafter applications in a real life context could be approached. The students believe that if their answers of problems are correct, one can conclude that they understand mathematics.

Students' beliefs about the teaching and learning of mathematics

The keywords (about the teaching and learning of mathematics) in table 5.3 indicate that students prefer that lecturers should teach specific methods and demonstrate to students specific methods to solve problems. The task of the lecturer is to convey knowledge to students and to test whether this has happened. Lecturers should actively guide students to discover concepts and find different ways to solve problems. Students should listen carefully to lecturers and do many drill exercises to master basic facts, rules and procedures. Mathematics teaching should occur in a real life context and therefore should be connected to the real world. The use of a calculator is an essential aid when doing mathematics. In the NCS for Mathematics (DoE, 2003:2) one of the critical outcomes is that learners should be able to use technology effectively – this includes the use of a pocket calculator. In secondary schools learners are sometimes encouraged to use their calculators to reduce the chances of calculation errors e.g. to add or subtract fractions. Students are very dependent on their calculators and when they are not allowed to use a calculator, even for very simple calculations, they get anxious about this.

In line with the instrumentalist view understanding of mathematics is reduced to recalling a rule or formula that will enable them to produce the correct answer. Typical of the static-formalist view, which is rooted in the mechanistic paradigm, the relationship between lecturer and student is a hierarchical relationship, where the lecturer is the specialist with all the knowledge who must teach, demonstrate, show how to and be able to answer all the questions of the students. The mathematics problems posed by the lecturers should be simple and clear-cut enough to allow completion within the time span of a class period. This correlates with the static-formalist view where knowledge is reduced to "chunks" transmitted by the lecturer as expert to students as novices. Students do not want to struggle with open-ended questions or questions that span more than one context.

These statements are in direct contrast to the aims of the National Curriculum Statement, which stresses that a learner-centred, activity-based approach should be followed in secondary schools (DoE, 2003:2). As explained in par. 2.6, this is a result of the mismatch between the intended curriculum and the implemented curriculum in secondary schools (see par. 2.6). In contrast to the intensions of the NCS, first year students prefer a lecturer-centred, passive approach. In this case "passive" indicates that the lecturer should be the one who must perform and the students will only do what the lecturer says.

5.3.3.2 Analysis of lecturers' beliefs

Table 5.4 shows the keywords of the statements that correspond with the questions with a large effect size and with which the lecturers strongly agree or disagree ($d \geq 0.8$).

TABLE 5.4: Keywords about the nature of mathematics where lecturers strongly agreed or disagreed (d ≥ 0.8)

LECTURERS			
NATURE OF MATHEMATICS			
Question		**Agree**	**Disagree**
6:	Exact science	X	
10:	Correctness of answer indicates understanding	X	
11.	New mathematics expanded by research at university only		X
13:	Problems in real-life context central focus		X
14:	Things in mathematics should be accepted and remembered – no explanation		X
17:	Set of facts, rules and formulas		X
20:	Structured guide to solve real-life problems	X	
24:	Using a calculator is not mathematics		X
26:	Natural talent to do mathematics	X	
27:	Lecturers be able to answer all questions		X
30:	Problems easily solved in class time		X
TEACHING AND LEARNING OF MATHEMATICS			
Question		**Agree**	**Disagree**
1:	Lecturer teaches	X	
5:	Lecturer demonstrates	X	
8:	Do drill exercises	X	
15:	Know the correct rule		X
19:	Lecturer conveys knowledge	X	
21:	Master basic facts, rules, procedures	X	
22:	Lecturer guides students	X	
25:	Teacher encourages students to find different ways	X	

Lecturers' beliefs about the nature of mathematics

The keywords in table 5.4 give an indication of lecturers' beliefs about the nature of mathematics. Lecturers see mathematics as an exact science that is more than just facts, rules and formulas. Even though they do not consider that a real-life context should be the central focus of mathematics, they acknowledge that mathematics can be used as a structured guide to solve problems in real life. They regard the correctness of the answer as an indication of understanding. This indicates that they consider the product, and not the process to obtain the answer, as understanding. They do not think that everybody is able to do mathematics and they acknowledge the appropriateness of the use of a calculator when doing mathematics. The lecturers realise that creative reasoning is associated with long periods of work and reflection that can not be attained in the time span of a class period.

Lecturers' beliefs about the teaching and learning of mathematics

The lecturers' beliefs about the teaching and learning of mathematics can be related to the static-formalistic view of mathematics where the responsibility is that of the lecturer to transmit knowledge to the students. Lecturers should teach, demonstrate, guide, encourage, convey knowledge to students and drill them to master basic facts, rules and formulas before they approach real life problems. However, they indicated that mathematics is more than just knowledge of the correct rules and formulas, as they wanted students to understand concepts. This indicates that their views are more static-formalist than instrumentalist.

5.3.3.3 Comparison of students' and lecturers' beliefs

The statements listed in Table 5.5 were those with a statistically significant difference ($p < 0.05$), as well as a practically significant difference (effect size $\geq$ 0.5) between the answers of the lecturers and the students. Table 5.6 indicates the means, standard deviation, *p*-values of the *t*-test and the Mann-Whitney test, and effect sizes of the questions listed in table 5.5.

TABLE 5.5: Statements with a statistical and practical significant difference between the students and the lecturers on beliefs

1.	Lecturers should teach specific methods to solve problems.
4.	Students can find a method to solve problems without the help of a lecturer.
5.	A good mathematics lecturer always demonstrates the correct method to solve problems.
7.	The best way to teach mathematics is to show students how to solve problems.
9.	To be successful in mathematics, a student needs to listen carefully to the lecturers' explanations.
11.	New mathematics is expanded only by research at university level.
13.	Mathematics problems in a real-life context should be the central focus of the mathematics curriculum.
14.	Many things in mathematics simply have to be accepted and remembered, there is not really an explanation for it.
15.	If students struggle to solve problems, it is usually because they don't know the correct rule or cannot remember the formula.
17.	Mathematics can be described best as a set of facts, rules and formulas that students have to learn.
18.	Mathematics is best taught if students are required to solve problems in a real-life context.
27.	Lecturers should always be able to answer all the students' questions regarding mathematics.
30.	Mathematics problems given to students should easily be solved within the scope of the class time.

TABLE 5.6 Statistics for statements with $p < 0.05$ and $d \geq 0.5$ of the beliefs questionnaire

QUESTION	STUDENT		LECTURER		P-VALUE (T-TEST)	P-VALUE (MANN-WHITNEY TEST)	EFFECT SIZE (COHEN–D)
	Mean	Std Dev	Mean	Std Dev			
Q1	3.197	0.668	2.785	0.802	0.025	0.045	0.51
Q4	2.051	0.809	2.500	0.855	0.043	0.037	0.52
Q5	3.422	0.687	2.929	0.730	0.009	0.008	0.68
Q7	3.254	0.691	2.714	0.825	0.005	0.012	0.65
Q9	3.479	0.611	3.000	0.877	0.005	0.026	0.55
Q11	2.731	0.683	2.143	0.663	0.002	0.003	0.86
Q13	2.979	0.743	2.071	0.616	0.000	0.000	1.22
Q14	2.139	0.889	1.571	0.646	0.019	0.017	0.64
Q15	2.823	0.791	2.143	0.663	0.001	0.001	0.86
Q17	2.543	0.879	1.857	0.770	0.004	0.003	0.78
Q18	2.967	0.686	2.571	0.646	0.034	0.021	0.58
Q27	3.486	0.664	2.286	0.469	0.000	0.000	1.81
Q30	2.918	0.745	2.000	0.784	0.000	0.000	1.17

Despite displaying significant and practical differences, students and lecturers agreed with most of the statements, showing a tendency towards the static-formalist view of mathematics as discussed in par. 5.3.3.1. The exceptions are questions 13, 15, 27 and 30, where most of the lecturers are neutral regarding the statements while most of the students agree strongly (also see table 5.2). Questions 13 and 18 underline the importance of being able to solve real-life problems and is in accordance with one of the important aims of the NCS for mathematics (DoE, 2003:10): "*An important purpose of mathematics in the Further Education and Training band is the establishment of proper connections between mathematics as a discipline and the application of mathematics in the* *real-world context.*" This statement also correlates with the relativist-dynamic view that sees mathematics as a human endeavour applied to solve problems posed by the real world (par. 2.5.2). For the students it is important that there must be a connection between pure mathematics and the real world. Students need to see the relevance of mathematics for their every day world and they do not want to do mathematics for its inherent value. The lecturers were not convinced that mathematics should be taught mainly in a real-world context. This could also be seen as a confirmation that lecturers see mathematics in terms of a static-formalist view as an entity separated from its application, and when it is applied to a real-life context it is classified as something "different", such as for instance "applied mathematics" or "physics", in accordance with the reductionist nature of the mechanistic paradigm.

Question 4 is the only question where the lecturers' mean value was higher than that of the students. The lecturers indicated that they believe that students would be able to find solutions to problems on their own. This is indicative of lecturers expecting students to reason creatively. Students should be able to transfer the examples explained in class to other problems given as assignments. Students on the other hand, do not share the same view and do not believe that they can or want to cope without the help of their lecturers.

5.4 The Index of Learning Styles (ILS) questionnaire (Annexure B and C)

5.4.1 Background

Students entering university come from different social and knowledge backgrounds and therefore have different responses to specific classroom environments and instructional practices (Felder & Brent, 2005:57). A learning style is defined as the way in which an individual acquires, retains and retrieves information and is related to a person's cognitive, affective and psychological behaviours that serve as indicators of how students perceive, interact with and respond to the learning environment (Felder & Brent, 2005:58). Felder and Silverman (1988:674) developed a model (ILS) to describe engineering students' learning styles. This classification is based on the type of information received (sensory or intuitive), the modality through which information is perceived (visual and verbal), the manner in which information is processed (actively or reflectively) and the progression towards understanding (sequentially or globally).

The first application of the ILS is to provide guidance to lecturers on the diversity of learning styles in their classes and to help them design instruction that addresses the learning needs of all students (Felder & Spurlin, 2005:110) The second is to give students insight into their possible learning strengths and weaknesses. For the purpose of this research, the questionnaire was used to determine the preferences of the students regarding the learning of mathematics in order to understand their perception of the nature and learning of mathematics and to gain information about their cognitive level.

An adapted version of the ILS was implemented to analyse the lecturers' preferred teaching styles and how they prefer their students to learn mathematics (Visser *et al*., 2006). This questionnaire contained questions such as:

When I am teaching something new, I would let my students:

a. Talk about it; or

b. Think about it.

The corresponding question in the original ILS for students is:

When I am learning something new, it helps to:

a. Talk about it; or

b. Think about it.

The results of these two questionnaires were used to determine how students prefer to learn mathematics and how the lecturers prefer to teach mathematics.

5.4.2 The four dimensions of learning styles

The ILS is an instrument that assesses preferences on four dimensions of the learning styles of students. The four dimensions are sensing/intuitive, visual/verbal, active/reflective and sequential/global (Felder & Silverman, 1988).

Sensing vs intuitive learners:

Sensing involves observing or gathering data through the senses. Sensors tend to be concrete and methodical. They like to learn facts, data and do experiments. They often prefer solving problems using set standard procedures. Sensors tend to be good at memorising facts and do not like courses that have no connection to the real world. Thus they are concrete thinkers, practical and oriented towards facts and procedures.

Intuition involves indirect perception by way of the subconscious (memory, speculating, imagining). Intuitors tend to be abstract and imaginative and therefore will deal better with principles, concepts and theories. They often prefer discovering possibilities and relationships and do not like repetition. They are often more comfortable with abstractions and mathematical formulations. They tend to work fast and do not like courses with much memorization and routine calculations. They are abstract thinkers, innovative and oriented towards theories and underlying meanings. These learners would be more likely to relate to the abstract nature of formal mathematics.

Visual vs verbal learners:

Visual learners prefer information to be presented visually in pictures, diagrams, time lines, films and demonstrations. If something is simply said to them, they will probably forget it.

Verbal learners get more out of written or spoken words and prefer verbal explanation and discussion.

Active vs reflective learners:

Active learners tend to retain and understand information best by discussing it, applying it or explaining it to others. They do not learn much in situations that require them to be passive. They want to do something physical and sitting through a lecture just taking notes is hard for them. They want to try something out to see how it works. They tend to like working in groups.

Reflective learners prefer to think quietly about new information first. They do not learn much in situations that provide no opportunities to think about the information being presented. They want to think information through first and mostly prefer working on their own or with a single familiar partner.

Sequential vs global learners:

Sequential learners tend to gain understanding of material in small connected chunks. They absorb information in linear steps, with each step following logically on the previous one. They may be strong in convergent thinking and analysis and may learn best when material is presented in a steady progression from simple and easy to complex and difficult. Sequential learners can function with incomplete understanding of material being presented, but they may lack grasping the bigger body of knowledge and its interrelationships with other subjects and disciplines.

Global learners tend to learn in large jumps, absorbing material almost randomly without seeing connections and then suddenly “getting it”. They may be able to solve problems quickly, but may have difficulty explaining how they did it. They may appear to be slow and can do poorly on tests until they grasp the total picture. Global learners sometimes do better by jumping directly to more complex and difficult material. They are the synthesizers, the multidisciplinary researchers, the system thinkers, the ones who see the connections no one else sees.

5.4.3 Scoring of the ILS questionnaire

The ILS consists of 44 questions in total, of which groups of 11 relate to each of the four dimensions discussed in the previous paragraph. Each question has an a) and b) option to choose from. The prescribed method of recording the score (see table 5.7) is to get the difference between the total of all the a) and all the b) scores. For example, if the total of the a) scores of the dimension active/reflective is 5 and the total of the b) scores are 6, the final score will be 1b). It means that there is a difference of 1 between the number of a) scores and the number of b) scores with the most b) scores. A score of 1 or 3 indicates that a person is well balanced between the two dimensions of that scale. A score of 5 or 7 means that you have a moderate preference for one dimension and will learn more easily in a teaching environment that favours that dimension. If the score is 9 or 11 a person has a very strong preference for one dimension and may find it difficult to learn in an environment that does not support that preference. See a typical example of a score sheet in table 5.7.

TABLE 5.7: An example of a scoresheet of the ILS questionnaire

ACT/REF			SNS/INT			VIS/VRB			SEQ/GLO		
Q	**a**	**b**	**Q**	**a**	**b**	**Q**	**a**	**b**	**Q**	**a**	**b**
1		x	2	x		3		x	4	x	
5		x	6	x		7		x	8	x	
9		x	10		x	11		x	12	x	
13	x		14	x		15		x	16	x	
17		x	18	x		19		x	20	x	
21		x	22	x		23		x	24	x	
25	x		26	x		27		x	28	x	
29	x		30	x		31		x	32	x	
33		x	34		x	35		x	36	x	
37	x		38	x		39	x		40	x	
41	x		42	x		43	x		44	x	
TOTAL: SUM OF X'S IN EACH COLUMN											
	ACT/REF			**SNS/INT**			**VIS/VRB**			**SEQ/GLO**	
	a	b		a	b		a	b		a	b
	5	6		9	2		2	9		11	0
(Larger – smaller) + letter of larger											
	1b			7a			7b			11a	

Figure 5.1 shows a summary of the scoresheet in table 5.7. This person has a well balanced preference for the "Active/reflective" dimension, a moderate preference for the "Visual/verbal" and the "Sensing/intuitive" dimensions and a very strong preference for the "Sequential" dimension.

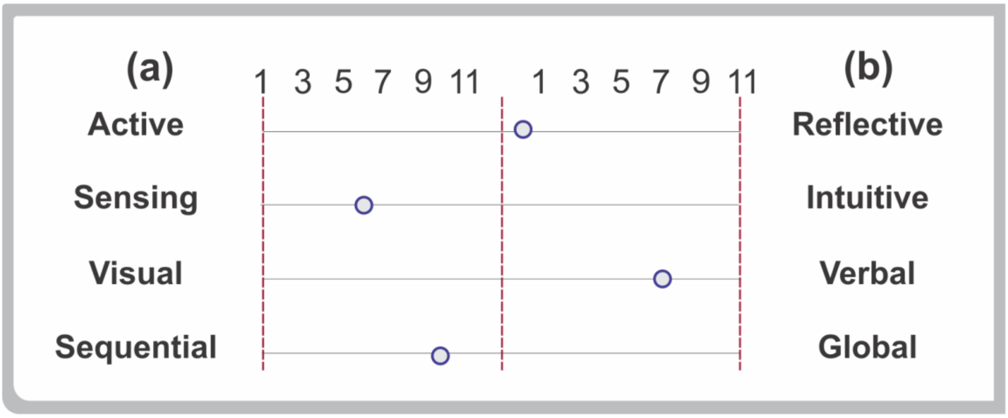

Fig. 5.1: An example of a summary of a scoresheet with learning preferences

The questionnaire for the lecturers' teaching styles was analysed in the same way as the students' questionnaire.

5.4.4 The reliability of the ILS

The reliability of an instrument refers to the extent to which that instrument is repeatable and consistent (Pietersen & Maree, 2010:215). There are different types of reliability that could be used, but internal reliability was used in this instance. When a number of items are formulated to measure a certain construct, there should be a high degree of similarity among them because they measure a common construct. The internal reliability of this instrument was measured by Cronbachs' alpha coefficient, which is based on inter-item correlations. If the items are closely related, the alpha coefficient will be close to one, but on the other hand if the alpha coefficient is close to zero, the items do not correspond closely. Different degrees of internal reliability are required, depending on what an instrument is used for. If an instrument measures achievement, a Cronbach alfa of 0.75 will be acceptable and for attitude and learning style assessments a value of 0.5 will be an acceptable indication of reliability (Felder & Spurlin, 2005:107).

Table 5.8 lists the Cronbach alpha coefficients of the four dimensions of this study. The Visual/Verbal dimension showed an acceptable Cronbach alpha and could be used as a valid group dimension in the discussion of students' preferences and in the comparison between the lecturers and the students.

However, the scores of the Active/Reflective, Sensing/Intuitive and the Sequential/Global dimensions were below 0.5, which indicated that the reliability of the ILS questionnaire is not sufficient for these dimensions.

TABLE 5.8: Cronbach alpha coefficients of the dimensions of the ILS

	CRONBACH ALPHA	CRONBACH ALPHA IF ITEMS DELETED	ITEMS DELETED
Active/Reflective	0.463	0.474	Item 29
Sensing/Intuitive	0.471	0.493	Item 18
Visual/Verbal	0.690	-	-
Sequential/Global	0.468	0.480	Item 16

5.4.5 Analysis of the ILS data

The analysis of the ILS is twofold: First to compare preferences between lecturers and students to get insight into beliefs, then to attempt to get a deeper insight into the transformation from secondary to tertiary mathematics.

In the original questionnaire the questions that were grouped together for the different dimensions were:

Active/Reflective: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41
Sensing/Intuitive: 2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42
Visual/Verbal: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43
Sequential/Global: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44

5.4.5.1 A comparison between the lecturers' and the students' preferences of the visual/verbal dimension

As indicated in table 5.8, only the visual/verbal dimension yielded an acceptable Cronbach alpha with a value $\geq$ 0.5. On advice from the Statistical Consultation Services the different questions were compared separately to determine which of the questions showed a significant difference between the lecturers and the students. Chi-square tests were used to determine the *p*-values for each question in this dimension to evaluate if there are significant differences between the lecturers and the students. The Phi coefficients were determined to evaluate the effect sizes of the differences. Only Question 15 showed an effect size that can be interpreted as a medium effect in practice (see table 5.9). The effect sizes in this case were considered:

- Small if $0.1 \leq \varphi < 0.3$
- Medium and possibly practically significant if $0.3 \leq \varphi < 0.5$
- Large and practically significant if $\varphi \geq 0.5$

TABLE 5.9: Questions in the visual/verbal dimension that showed statistically significant differences

	VISUAL (A)		VERBAL (B)			
	Lecturers %	**Students %**	**Lecturers %**	**Students %**	**p-values**	**Phi coefficients**
Question 11	50	74.3	50	25.7	0.044	0.107
Question 15	21.4	76.6	78.6	23.4	0.000	0.247
Question 39	28.6	69.2	71.4	30.8	0.001	0.170

There was only one question in the visual/verbal dimension (Question 15) that yielded a medium effect size and which could therefore possibly show a medium effect in practice.

Question 15:

Student question: I like lecturers who a) explain by using a lot of diagrams (sketches, graphs etc.) b) explain a lot, using words.

Lecturer question: As lecturer I do like to a) put a lot of diagrams on the board b) spend a lot of time explaining.

The students prefer that the lecturers should explain by using diagrams like sketches and graphs and not just explaining with words only, while the lecturers had a strong preference to spend much time explaining.

The information acquired from the above was not adequate. Therefore the preferences of the students on their own, as well as those of the lecturers on their own, were investigated to get a deeper insight regarding their teaching/learning preferences.

5.4.5.2 Discussion of the preferences of the students and the lecturers on their own in terms of the different dimensions

The different percentages, as well as the d-values for percentages was determined for each question in the different dimensions for the students and the lecturers and are summarized in tables 5.10, 5.11, 5.12 and 5.13. The d-values indicate whether there is a significant difference between the students who chose option a) and those who chose option b) and the lecturers who chose option a) and those who chose option b). Effect size magnitudes vary across different research areas. E.g. in a controlled laboratory an effect size of 0.8 could be considered not large enough, whereas in a study where attitudes are measured an effect size of 0.7 can be considered large enough (Rencher, 2002). According to Thompson (2001) it would be “stupid” to interpret effect sizes as rigid as we do with statistical significance. Therefore for this specific case the effect sizes are considered:

- Small effect: $0.2 \leq d < 0.5$
- Medium effect and possibly practically significant: $0.5 \leq d < 0.8$
- Large effect and practically significant: $d \geq 0.8$

The active/reflective dimension

Table 5.10 shows a summary of the mean percentages, as well as the d-values of the lecturers and the students for each of the 11 questions for the active/reflective dimension.

TABLE 5.10: Summary of the mean percentages and d-values for the questions in the active/reflective dimension

	STUDENTS			LECTURERS		
Question	Active (a)	Reflective (b)%	d-values	Active (a)	Reflective (b)%	d-values
1	**83.7**	16.3	1.4	**92.9**	7.1	**1.7**
5	44.1	55.9	0.2	44.9	57.1	0.2
9	44.1	55.6	0.2	**100**	0	2.0
13	53.0	47	0.1	28.6	**71.4**	**0.9**
17	45.0	55	0.2	7.1	**92.9**	**1.7**
21	30.2	**69.8**	**0.8**	57.1	42.9	0.3
25	41.4	58.6	0.3	57.1	42.9	0.3
29	**79.3**	20.7	**1.2**	**85.7**	14.3	**1.4**
33	44.4	55.6	0.2	57.1	42.9	0.3
37	49.4	50.6	0.0	64.3	35.7	0.6
41	37.9	62.1	0.5	**71.4**	28.6	**0.9**

The questions that yielded a large effect size ($d \geq 0.8$) for the students are shown below. The underlined options indicate the majority choice.

Question 1: I understand something better after I a) have tried it out b) have thought it through

Question 21: I prefer to study a) In a study group b) Alone

Question 29: I more easily remember a) something I have done b) something I have given a lot of thought

The questions that yielded a large effect size ($d \geq 0.8$) for the lecturers are shown below. The underlined options indicate the majority choice.

Question 1: My students will understand something better if I let them a) try it out b) think it through

Question 13: In classes I am teaching a) I usually know many of the students b) I rarely know any of the students

Question 17: When I give a problem/case study to my students I want them to a) start working on the solution immediately b) try to fully understand the problem first

Question 29:	My students will more easily remember a) <u>something they have done</u> b) something they have thought a lot about
Question 41:	The idea of a study group, where a group of students work together on an assignment with one grade for the entire group a) <u>Appeals to me</u> b) Does not appeal to me

The lecturers want their students to be active in the sense that they indicated that they learn better in situations that enable them to do something physical in class while the lecturer explains. They retain and understand information best when they work with it or discuss or apply it. This has implications in the class situation. Students should be busy doing something instead of just listening. Students who prefer an active learning style should be keen to work in groups, but the students in this study prefer to work on their own instead. They also prefer not to get one mark for the entire group who worked together on an assignment, while the lecturers do. Johnson *et al*. as cited by Felder and Henriques (1995:25) point out that students often respond negatively to group work at first, but the benefits of this approach become clear only after they realized the appropriate uses of teamwork. According to the NCS (DoE, 2003:10) one of the skills learners from secondary school should have, is to be able to work effectively in groups to enhance mathematical understanding.

The sensing/intuitive dimension

Table 5.11 shows a summary of the mean percentages, as well as the d-values of the lecturers and the students for each of the 11 questions for the sensing/intuitive dimension.

TABLE 5.11: Summary of mean percentages and d-values for the questions in the sensing/intuitive dimension

	STUDENTS			LECTURERS		
Question	**Sensing (a)%**	**Intuitive (b)%**	**d-values**	**Sensing (a)%**	**Intuitive (b)%**	**d-values**
2	**77.2**	22.8	**1.1**	50.0	50.0	0.0
6	**72.5**	27.5	**0.9**	57.1	42.9	0.3
10	29.6	**73.1**	**0.9**	35.7	64.3	0.6
14	43.3	56.7	0.3	64.3	35.7	0.6
18	68.0	32.0	0.7	**71.4**	28.6	**0.9**
22	52.4	47.6	0.1	50.0	50.0	0.0
26	43.5	56.5	0.3	**71.4**	28.6	**0.9**
30	61.2	38.8	0.5	50.0	50.0	0.0
34	**71.0**	29.0	**0.8**	50.0	50.0	0.0
38	**77.8**	22.2	**1.1**	50.0	50.0	0.0
42	62.7	37.3	0.5	57.1	42.9	0.3

The questions that yielded a large effect size ($d \geq 0.8$) for the students are shown below. The underlined options indicate the majority choice.

Question 2: I would rather be considered a) realistic b) innovative

Question 6: If I were a lecturer I would rather teach a course a) that deals with facts and real life situations b) that deals with ideas and theories

Question 10: I find it easier a) to memorize facts b) to understand concepts

Question 34: I consider it higher praise to call someone a) sensible b) imaginative

Question 38: I prefer courses that emphasize a) concrete material (facts, data) b) abstract material (concepts, theories)

The questions that yielded a large effect size ($d \geq 0.8$) for the lecturers are shown below. The underlined options indicate the majority choice.

Question 18:	I prefer to teach the idea of a) <u>certainty</u> b) theory.
Question 26:	When I am teaching I like to a) <u>say clearly what I mean</u> b) say things in creative, interesting ways.

The students tend to be more sensing than intuitive. This implies that they prefer concrete and problems with a fixed or prescribed method. The students are oriented towards concrete facts and data with exact answers and not towards abstract concepts and theories. They do not prefer to think up answers or procedures themselves (to be innovative) or to answer open ended questions. They prefer to *understand* concepts rather than to *memorize* facts. Lecturers do not prefer to teach theory and prefer to express themselves clearly when explaining. In five of the questions the lecturers showed a 50:50 preference for the sensing and the intuitive dimensions. This is an indication that they have a fairly balanced preference for the sensing/intuitive dimension.

The visual/verbal dimension

Table 5.12 shows a summary of the mean percentages as well as the d-values of the lecturers and the students for each of the 11 questions for the visual/verbal dimension.

TABLE 5.12: Summary of the mean percentages and d-values for the questions in the visual/verbal dimension

	STUDENTS			LECTURERS		
Question	**Visual (a) %**	**Verbal (b) %**	**d-values**	**Visual (a) %**	**Verbal (b) %**	**d-values**
3	**74.6**	25.4	**1.0**	64.3	35.7	0.6
7	60.4	39.6	0.4	**78.6**	21.4	**1.1**
11	**74.3**	25.7	**1.0**	50.0	50.0	0.0
15	**76.6**	23.4	**1.1**	21.4	**78.6**	**1.1**
19	**78.1**	21.9	**1.1**	**71.4**	28.6	**0.9**
23	66	34.0	0.6	64.3	35.7	0.6
27	64.5	35.5	0.6	**71.4**	28.6	**0.9**

	STUDENTS			LECTURERS		
Question	**Visual (a) %**	**Verbal (b) %**	**d-values**	**Visual (a) %**	**Verbal (b) %**	**d-values**
31	55.6	44.4	0.2	57.1	42.9	0.3
35	66.3	33.7	0.7	42.9	57.1	0.3
39	**69.2**	30.8	**0.8**	28.6	**71.4**	**0.9**
43	**74.9**	25.1	**1.0**	**92.9**	7.1	**1.7**

The questions that yielded a large effect size ($d \geq 0.8$) for the students are shown below. The underlined options indicate the majority choice.

Question 3: When I think about what I did yesterday, I am most likely to think of a) a picture b) words.

Question 11: In a book with lots of pictures and charts, I am likely to a) focus on the pictures and charts b) focus on the written text.

Question 15: I like lecturers who a) explain by using a lot of diagrams (sketches, graphs etc.) b) explain a lot, using words.

Question 19: I remember best a) what I see b) what I hear.

Question 39: For entertainment, I would rather a) watch television b) read a book.

Question 43: I tend to picture places I have been a) easily and fairly accurately b) with difficulty and without much detail.

The questions that yielded a large effect size ($d \geq 0.8$) for the lecturers are shown below. The underlined options indicate the majority choice.

Question 7: I prefer to teach new information using a) pictures, diagrams, graphs, or maps b) written directions or verbal information.

Question 15: As lecturer I do like to a) put a lot of diagrams on the board b) spend a lot of time explaining.

Question 19: In my classes my students will learn best a) what they see b) when talking about it.

Question 27:	When I see a diagram or sketch in a text book, I am most likely to a) show the picture to the students b) tell my students what it is about.
Question 39:	For entertainment, I would rather want my students to a) watch a television programme about their subject b) read a book about their subject.
Question 43:	I tend to picture the material I have to teach a) easily and fairly accurately b) with difficulty and without much detail.

The students tend to have a strong preference for the visual dimension. In all the questions with a large effect size their indication was that they prefer the visual option (table 5.12). In contrast with this the lecturers had a mixed preference. In many questions they also preferred the visual option, but it is interesting that they indicated in question 15 that they spend much time explaining. Students' responses to the belief questionnaire indicated that they want the lecturers to explain a lot, but the responses to the ILS questionnaire indicate that they do not prefer that the lecturers only do much explaining, but they rather prefer explanations assisted with diagrams and visual aids. The lecturers prefer explanations in words above drawing diagrams on the board. The students indicated that they remember more easily when they see than when they hear. Diagrams, graphs and demonstrations could enable students to clarify their thinking processes and help to organise, prioritise and connect new information to existing concepts, which could stimulate creative thinking.

The sequential/global dimension

Table 5.13 shows a summary of the mean percentages, as well as the d-values of the lecturers and the students for each of the 11 questions for the sequential/global dimension.

TABLE 5.13: Summary of mean percentages and d-values for the questions in the sequential/global dimension

	STUDENTS			LECTURERS		
Question	**Sequential (a)%**	**Global (b)%**	**d-values**	**Sequential (a)%**	**Global (b)%**	**d-values**
4	43.2	56.8	0.3	50	50	0.0
8	57.7	42.3	0.3	42.9	57.1	0.3
12	**79.0**	21	**1.1**	**71.4**	28.6	**0.9**
16	61.5	38.5	0.5	57.1	42.9	0.3
20	**81.1**	18.9	**1.2**	**78.9**	21.4	**1.2**
24	57.7	42.3	0.3	**92.9**	7.1	**1.7**
28	41.1	58.9	0.4	14.3	**85.7**	1.4
32	**69.2**	30.8	**0.8**	50	50	0.0
36	55.6	44.4	0.2	57.1	42.9	0.3
40	46.4	53.6	0.1	14.3	**85.7**	**1.4**
44	58.3	41.7	0.3	50	50	0.0

The questions that yielded a large effect size ($d \geq 0.8$) for the students are shown below. The underlined options indicate the majority choice.

Question 12: When I solve math problems a) I usually work my way to the solutions one step at a time b) I often just see the solutions, but then have to struggle to figure out the steps to arrive at a solution.

Question 20: It is more important to me that a lecturer a) lays out the material in clear sequential steps b) gives me an overall picture and relate the material to other subjects.

Question 32: When writing a paper, I am more likely to a) work on (think about or write) the beginning of the paper and progress forward b) work on (think about or write) different parts of the paper and then order them.

The questions that yielded a large effect size ($d \geq 0.8$) for the lecturers are shown below. The underlined options indicate the majority choice.

Question 12:	When I teach math problems I expect from my students to a) work their way to the solutions one step at a time b) guess the solutions but then let them struggle to figure out the steps to get to them
Question 20:	As lecturer it is more important to me to a) lay out the material in clear sequential steps b) give an overall picture and relate the material to other subjects.
Question 24:	I teach a) at a fairly regular pace. If my students put in their best, they'll have success b) in fits and starts and at first it may seem that I am totally confused, but then suddenly it all falls into the structure.
Question 28:	When teaching a body of information, I am more likely to a) focus on details and miss the big picture b) try to explain the big picture before getting into the details.
Question 40:	Some lecturers start their lectures with an outline of what they will cover. I am of opinion that students can benefit a) somewhat from that b) very much from that.

Students prefer that lecturers explain in clear sequential steps. They also prefer to solve mathematics problems using steps sequentially. They don't want lecturers to give an overall picture and relate to other subjects. Sequential learners can function with incomplete understanding of the course material and may lack a grasp of the broad body of knowledge and its interrelationships with other subjects and disciplines (Felder & Henriques, 1995:25). These learners may have trouble relating specific aspects to different situations. Creative thinking requires from students to connect existing knowledge to different new situations.

The lecturers prefer at large to teach sequentially, but can see the relevance of giving the bigger picture before going into the detail.

5.5 Analysis of the first mathematics test written at tertiary level (Annexure D)

5.5.1 Introduction

This section describes the empirical investigation into the reasoning abilities of the first year mathematics students by analysis of the first test of the module WISN111 that was written in 2010. Chapter 4 showed that reasoning is an important component of mathematical knowledge and that the journey from elementary to advanced thinking entails a gear shift in reasoning. After investigating different types of reasoning and after an initial analysis of the first mathematics test written by the sample group, Lithner's (2006) categorization in terms of imitative and creative reasoning seemed to be the most suitable to investigate the reasoning abilities of students at entrance level (see par. 5.5.2.1).

5.5.2 Framework for classification of reasoning

The framework of Lithner (2006), used by Palm *et al*. (2011:229–237) was applied for the classification and the analysis of the test. The idea is to introduce a formal procedure for evaluating test questions in terms of imitative or creative reasoning.

5.5.2.1 Classification categories

The imitative/creative classification is subdivided into four levels, namely familiar algorithmic reasoning (FAR), guided algorithmic reasoning (GAR), memorized reasoning (MR) and creative mathematical reasoning (CMR).

- *Familiar algorithmic reasoning (FAR)*

  The task is very similar to at least three of the exercises or examples in the textbook in terms of the task variables. Students can relate to the tasks in the textbook and apply the algorithms used in those tasks. The choice of three as the required number of similar tasks is based on the idea that students need to encounter not only one task, but a few before they can recall them. This was validated by another study (Palm *et al*., 2011:230).

- *Guided algorithmic reasoning (GAR)*

The test conditions include a textbook or a formula sheet that can be used to copy a described procedure. Students don't need to recall the exercise or example, and therefore only one such exercise or example is enough to classify an assessment task as solvable by GAR.

- *Memorized reasoning (MR)*

For this classification at least three answers in the textbook that solve the assessment task are required.

- *Creative mathematical reasoning (CMR)*

If a task cannot be classified as algorithmic reasoning (AR) or memorized reasoning (MR), the task has the potential to be classified as "being possible" to solve using creative reasoning. The qualification of "possible" means that the content knowledge needed to solve the problem must be included in the textbook (or notes) and that the required steps are within the reach of at least a few of the students. Reasoning that is mainly based on MR or AR, but contains minor local elements of CMR are referred to as local CMR while reasoning with large CMR elements are referred to as global CMR.

5.5.2.2 Classification procedure:

The main classification involves a distinction between algorithmic or memorized reasoning on the one hand, and whether creative reasoning is required on the other hand. Algorithmic and memorized reasoning is sufficient for solving a task if students can identify the task type and carry out the imitation. This is reasonable if:

- the students have encountered an applicable algorithm or fact in the textbook on several occasions (so that they could have the opportunity to learn the algorithm or fact by rote);
- the characteristics of the assessment task are such that the students can relate this task to tasks or examples in the textbook that can be solved with the same algorithm.

Palm *et al*. (2011) decided to use the textbook as a reference to what students have had an opportunity to learn. They acknowledged that one can never access students' complete thinking processes, but the textbook may represent the actual curriculum that represents the students' learning experience. In the case of this research the students' study guide as well as assignments given before the test will also be taken into consideration.

5.5.2.3 The steps to analyse the task

The classification procedure consists of five steps that have to be followed by the reviewer.

I. *Analysis of the assessment task – answers and solutions*: Identify the possible answers (for MR) or algorithms (for AR) that can solve the task.

II. *Analysis of the assessment task – other task characteristics*: Description of the task by means of the task variables.

   Task variables:

   - A*ssignment*: Students use the similarity of tasks to relate one task to another. If a student recognizes the assignment from the textbook they can recall a certain algorithm or fact to use to solve the task.
   - *Explicit information given*: The task may suggest a solution method or give extra information that students can use to relate the task to other tasks in the textbook.
   - *Representations*: The main mathematical components of the situation described in the task can be conveyed to the students by different types of representations such as pictures, symbols, text, tables or graphs.
   - *Linguistic features*: A task includes words that may work as verbal clues to a situation. Key words can influence the solution method chosen by students e.g. if most of the tasks in the textbook that include the word "maximum" can be solved by differentiation and setting the derivative equal to zero, then this

word will function as a trigger for applying the algorithm in the test/task.

- *Explicitly formulated hints*: An example of such a hint is: "use derivatives to…..".
- *Response format*: The format of the required answer is a characteristic that may make task relatedness more or less visible.

III. *Analysis of the textbook – answers and solutions*:

- Search for exercises and examples in the textbook that can be solved with the same answer or algorithm as in the task. Make sure of the equipment (e.g. calculators) available when solving the task.
- Search the rest of the textbook for parts that include the answer or algorithm. This can be rules, theorems, described facts, etc.

IV. *Analysis of the textbook – other task characteristics*:

- Search for exercises and examples in the textbook that are similar to the task with respect to the task variables described above.
- Search in the rest of the textbook for information that seems closely related to the task with respect to the task variables described above.

V. *Conclusion about and criteria for a required reasoning type*: This step comprises an argument and backs up a conclusion that it is possible and reasonable for students to solve the task with one of the imitative reasoning types or that creative mathematical reasoning is required to solve the task.

Table 5.14: Summary of the criteria for the classification of reasoning (Palm *et al*., 2011:231)

REASONING TYPE	FAR	GAR	MR	CMR
Argument	Assessment task is similar to at least three other instances in the textbook where the same solution algorithm is applicable	Assessment task is similar to at least one instance in the textbook where the same solution algorithm is applicable. This algorithm is available during the assessment.	Assessment task is similar to at least three instances in the textbook with the same answer.	AR and MR are not possible. CMR is possible.

5.5.3 The validity and reliability of the task classification

The criteria used for the appropriateness of the procedure used for the task classification and the set of task variables used in the task classification is part of the validation of the analysis (Palm *et al*., 2011:238). Palm *et al*. indicate that the establishment of reasoning requirements in this way can provide meaningful results and could be a valuable tool for the evaluation of assessments. All classifications were separately made by five mathematics lecturers in order to establish high reliability. Three of them are not currently lecturing this module and two of them do lecture the module. All initial classifications that differed were discussed by all five the lecturers to come to consensus. The lecturers could not reach consensus about question 2.3, but the researcher concluded with the decision of the majority.

5.5.4 The classification of the questions in the test

Question 1.1
Complete the following:
$\lvert x \rvert = \begin{cases} \ldots.\text{if } x \geq 0 \\ \ldots..\text{if } x < 0 \end{cases}$

Classification: MR

Explanation: There are no difficult or uncommon linguistic features. The student just has to fill in two values. The property is written out in the textbook on several places, as well as in the study guide.

Question 1.2
Solve the following inequality and give your answer in interval notation*:*
$\lvert 5x - 2 \rvert < 6$

Classification: FAR

Explanation: Question 1.1 gave an indication of the property that should be used in the question that follows, which is an explicit hint for this question. A step-by-step explanation of how to get the solution of similar problems is described in the textbook. It is also shown in several examples. There are more than three textbook tasks that can be solved with the same algorithm as the question in the test and that possess similar characteristics regarding the set of task variables used in the analysis.

Question 1.3
Graph the following function:
$p(x) = \dfrac{2x + \lvert x \rvert}{x}$

Classification: Local CMR

Explanation: The questions preceding this one gave an indication of the property that should be used, which is an explicit hint for this question. No examples or exercises that are similar to this one occur in the textbook. There is not sufficient information in the textbook, study guide or assignments given to classify the task as solvable by imitative reasoning. However, the mathematical content needed to solve the task is included in the textbook. Some of the students should be able to carry out the creative steps required to solve the task.

Question 2.1
Complete:
2.1.1 $-70° = \cdots \ldots \ldots \text{rad}$
2.1.2 $\frac{2\pi}{3} = \cdots \ldots \ldots \ldots \ldots .°$

Classification: FAR

Explanation: The response format is a short answer in radians and an answer in degrees. Although there is not a specific example with these exact values in the textbook or in the study guide, there are several examples and many exercises to convert degrees to radians and radians to degrees. The same algorithm could be used as in the textbook.

Question 2.2
Prove the following identity:
$\cot^2\theta + \sec^2\theta = \tan^2\theta + \csc^2\theta$

Classification: Local CMR

Explanation: The task should be solved by using the square identities. The task does not contain explicitly formulated hints. There are no examples similar to this one in the textbook, but there are several exercises in the textbook that require from students to prove identities. Students had to practice certain problems in the textbook, but the ones similar to the question in the test were not part of those. To prove identities is part of the grade 12 curriculum, but the inverse trigonometric functions are not part of the grade 12 curriculum and therefore students are not familiar with these. There is not a specific exercise or example exactly like this one in the textbook and in the study guide and therefore not sufficient information to classify this task as fully or largely solvable by imitative reasoning. However, the mathematical content to prove the identity is available in the textbook.

Question 2.3

Find all the values of θ which satisfy the following equation:

$$\csc\theta = -2, \qquad \theta \in [0, 2\pi]$$

Classification: Local CMR.

Explanation: The student should solve a trigonometric equation, without using a calculator. To be able to do that the student should use a sketch and from that extract information to be able to solve the trigonometric equation. There are no examples in the textbook, but there are several exercises that require from students to solve trigonometric equations of this type, although these exercises were not explicitly given to students to practice. There is not a specific exercise exactly like this one in the textbook, study guide or assignments. The mathematical content needed to prove the identity is available in the textbook. The group of lecturers who classified the questions could not reach agreement with regard to this question. Two of the lecturers concluded that this needs familiar algorithmic reasoning, but the remaining three concluded that this should be classified as local CMR because there is not sufficient information in the textbook, study guide or assignments to classify this task as fully or largely solvable by imitative reasoning. I conclude with the majority and therefore classify this question as local CMR.

Question 3.1

Define the concept function.

Classification: MR

Explanation: The student just has to write down a definition. There are at least three instances where this definition is written out in the textbook, as well as in the study guide.

Question 3.2

Given:

$$f(x) = \begin{cases} x + 1 & \text{if } x < -2 \\ x & \text{if } x \geq -2 \end{cases}$$

Determine:

3.2.1 $f(-2) = \cdots \ldots \ldots ....$

3.2.2 $f(-4) = \cdots \ldots \ldots ..$

Classification: FAR

Explanation: The student should determine the values of the function. Although there are no difficult or uncommon linguistic features, mathematical language in the form of mathematical symbols is used to describe a given function with certain conditions. To evaluate a function is described in the textbook and there are more than three textbook tasks similar to these questions that can be solved with the same algorithm.

Question 4.1

Define the following concepts:

4.1.1 $(f \text{ o } g)(x) = \cdots \ldots \ldots \ldots \ldots ..$

Classification: MR

Explanation: The student should write down a definition of a composite function. Mathematical language in the form of mathematical symbols is used to describe the given composite functions. There are at least three instances where this definition is written out in the textbook, as well as in the study guide.

Question 4.1

4.1.2

$(f \text{ o } f^{-1})(x) = \ldots \ldots \ldots \ldots ..$

Classification: Local CMR

Explanation: The student should determine the value of a composite function containing the function, as well as its inverse. Mathematical language in the form of mathematical symbols is used to describe the given composite functions. Question 4.1.1 is the definition of a compound function and should be used in this case. This is an explicit hint for answering this question. There is not a specific exercise or example similar to this one in the textbook or in the study guide. Therefore the information in the textbook, study guide and assignments is not sufficient to classify this task as fully or largely solvable by imitative reasoning. The mathematical content necessary to answer this question is available in the textbook.

Question 4.2

Given:

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \frac{x}{x+1}$$

Determine:

4.2.1 $D_f = \cdots \ldots \ldots ..$

4.2.2 $D_g = \cdots \ldots \ldots ..$

Classification: FAR

Explanation: The student should write down the domain of the functions f and g.

Mathematical language in the form of mathematical symbols is used to describe the given functions, as well as the questions. There are more than three textbook tasks that can be solved with the same algorithm as the question in the test and possess similar characteristics so that the same algorithm could be used.

4.2.3 $g \circ f = \cdots \ldots \ldots$

Classification: FAR

Explanation: The answer should be a function composed of the two given functions. Question 4.1.1 gave an indication of the property that should be used in the question that follows. It is an explicit hint for this question. Mathematical language in the form of mathematical symbols is used to describe the given functions, as well as the question. There are more than three textbook tasks that can be solved with the same algorithm as the question in the test and possess similar characteristics in the textbook and in the study guide. The same algorithm could be used as in the textbook.

4.2.4	This question was excluded from the classification process because of an error in the memorandum.

Question 5

Solve x:

$$\ln\ (x+1)\ +\ \ln\ (x-1) = 1$$

Classification: local CMR

Explanation: The student should solve the equation using the rules of logarithms. The abbreviation ln is new to the students – they are used to log. The students should go through different steps, using the laws of logarithms to solve the equation. This requires a multi-step answer. There are some examples and exercises to solve logarithmic equations, but in the study guide no reference is made to these specific examples and exercises and it was not expected of the students to practice these exercises. The mathematical content to prove the identity is available in the textbook.

5.2 Determine the inverse of $f(x)$ (SHOW ALL YOUR STEPS!!):

$$f\,(x) = \frac{x+1}{2x+1}$$

Classification: Local CMR

Explanation: A mathematical function in the form of mathematical symbols is given. The students should go through different steps to find the inverse of the function. There are some examples and exercises to determine the inverse of a function, but the functions are not fractions. The textbook, study guide and assignments do not provide enough information to classify this task as fully or largely solvable by imitative reasoning. The mathematical content to prove the identity is available in the textbook.

Question 6
6.1 Sketch the inverse of sin $x,\ x \in [-1, 1]$

Classification: MR

Explanation: The student should sketch the inverse of the given function. A mathematical function that is restricted to a certain domain is given. There are three exactly similar examples with the precise values in the textbook and the study guide.

6.2 Find the value of the following expression:
$\cos^{-1}(-\frac{1}{\sqrt{2}})$

Classification: FAR

Explanation: The student should determine the value of the expression given without using a calculator. There is not a specific example with these exact values in the textbook or in the study guide, but the same algorithm could be used as in the textbook. There are more than three textbook tasks that can be solved with the same algorithm as the question in the test.

In the end the questions could be divided into three subgroups, namely MR, FAR and Local CMR. No cases of global creative reasoning could be traced.

5.5.5 Discussion of results

Regarding the reasoning types needed for the different questions, it is apparent that students struggle to answer the questions where local creative reasoning is required (table 5.15). Local creative reasoning contains only minor elements of creative reasoning and is mainly based on memorized or algorithmic reasoning. For the parts of the problems that needed creative reasoning, no complete solution schemes were available and the reasoning had to be constructed by the students themselves. This kind of reasoning requires a deep, flexible knowledge and a student should be able to connect new ideas with existing ideas. It seems as if there was no difficulty with the questions that required familiar or memorised algorithmic reasoning. In many countries in the world the problem seems to be that secondary schools put an emphasis on procedural skills, which bring about that students that arrive at university are confident if the focus is on procedures and symbol manipulation instead of a deeper conceptual thinking (Engelbrecht *et al*., 2009:927). But learners "*learn what they are given the opportunity to learn*" (Hiebert, 1999:12) and if learners have more opportunities to learn simple calculations and procedures and are seldom engaged in mathematical processes other than calculation and memorization, they cannot be proficient in mathematics because for a deeper understanding more than just a reproduction of facts and routine algorithms are needed.

TABLE 5.15: Average percentages for reasoning types

CLASSIFICATION OF REASONING	QUESTION	AVERAGE	AVERAGE OF MR/FAR/CMR
MR	1.1	87.6	69.9
	3.1	61.8	
	4.1	83.5	
	6.1	46.8	
FAR	1.2	67.7	66.0
	2.1.1 & 2.1.2	70.7	
	3.2	77.5	
	4.2.1	67.2	
	4.2.2	57.9	
	4.2.3	81.8	
	6.2	39.4	
Local CMR	1.3	40.2	33.3
	2.2	26.7	
	2.3	30.9	
	5.1	22.2	
	5.2	46.6	

5.6 Conclusion

The empirical study investigated the views of the students and lecturers regarding the nature of mathematics, the preferences of the students regarding the learning of mathematics compared to how the lecturers prefer students should learn and the reasoning abilities of the students.

It is apparent that the students in this study see mathematics as static-formalist and/or instrumentalist. They are extremely dependant on the lecturer for their learning and for them the lecturer is the one in possession of all the knowledge that has to be transferred to the students. Knowing the correct rule and to be procedurally fluent is important for these students and for them this is what mathematics is all about. Their preferences for learning mathematics tend to be more visual than verbal and they prefer graphs, pictures and diagrams together with verbal explanations above written or verbal explanations alone. They prefer to learn concrete facts and data with exact answers and not open ended questions where they should be innovative to reach the answer.

The lecturers' beliefs about the teaching and learning of mathematics can be related to the static-formalist view of mathematics. They see themselves as responsible to transmit knowledge to the students using a lot of explaining but also indicated that they prefer that their students should be engaged in the lesson by means of some physical activities.

Certain topics were taken out of the school curriculum and for this reason students lack some skills (for example certain reasoning skills) that had to be developed during secondary school. At the stage that students come to university particular cognitive skills should have been developed. The students further prefer to learn in sequential steps where everything is done step-by-step from beginning to the end. They also struggled with questions that required local creative reasoning in their first test. It is clear that the level of cognitive development of the students in this study seemed to be on a visual and theoretical level, which is an indication that they are not yet prepared for the formal world of axioms, definitions and proof.

The next chapter will present the findings and the conclusions of the study, as well as how lecturers and students could be assisted to simplify the transition form secondary to tertiary mathematics.