CHAPTER 2

THE GAP BETWEEN SECONDARY AND TERTIARY LEVEL CONCERNING BELIEFS ON THE NATURE AND LEARNING OF MATHEMATICS

2.1 Introduction

Students' beliefs are shaped by their experiences, the impact of continuous perceptions from the world around them, the present dominant paradigm, as well as the beliefs of their teachers. As indicated by Coulter (1993:51) there is a gap between the beliefs of lecturers and students due to differences in the worldviews held by students and lecturers.

This chapter offers an overview of the nature of mathematics, how changes in beliefs are influenced by changes in the general worldview, and the impact that beliefs have on the gap between secondary and tertiary mathematics.

2.2 The influence of worldviews and paradigm shifts on beliefs

A person's worldview is a set of beliefs about fundamental aspects of life and the universe that influences and affects the person's thinking, knowing and doing. It is a mental model of reality – a framework of ideas and attitudes about the world with answers for a wide range of questions, such as the origin of life and the universe. It informs the basic way of interpreting things and becomes a person's concept of reality – what is good, what is important, what is sacred, what is real. A worldview has a personal and a group character, as beliefs held by a person are influenced by the current paradigm.

A paradigm is a set of assumptions, concepts, values and practices determining the way of life and the thoughts of a community over a certain period of time. It refers to the system of thought in a society that is most standard and widely held at a given time. Dominant paradigms are shaped by the community's cultural background and by the context of the historical moment. It is the overall perspective from which one sees and interprets the world. Paradigms provide the foundation for knowledge, understanding and solutions to problems (Sönge & Arjun, 1996:88). A paradigm shift involves a rethinking of beliefs and structures in human consciousness (Slattery, 2006:19).

Beliefs include ontological beliefs about the nature of reality (Schraw & Olafson, 2008:25), as well as epistemological beliefs of what knowledge entails and how knowing and learning are realized (Ertekin *et al*., 2010:631). Op‘t Einde *et al.* (2002:23) state that beliefs refer to what “I” believe to be true, regardless of the fact if others agree with me or not. Pehkonen (2001:13-14) concludes that an individual’s belief system is a compound of one’s conscious and sub-conscious beliefs, hypotheses or expectations. Beliefs are organized in clusters around specific situations and contexts that form an individual’s entire belief system (Op’t Einde *et al*., 2002:25). They are formed and changed in the social environment as individuals compare their beliefs with their experiences and the beliefs of other individuals they come across, thus beliefs are under continuous evaluation and change according to the context.

2.3 The influence of the current paradigm shift on education in general

The current worldview also influences the way in which education is perceived. It has an influence on teaching and teaching methods, as well as on the way that students learn. In recent western civilisation three macro-paradigms can be identified: the pre-modern era where the meta-physical was dominant, the modern era where science and the human mind (rationality) determined our destination, and the post-modern era, which is characterized by fast-changing and cyclical concepts of time with diverse cultures and many genres of expressions (Slattery, 2006:19).

We are in the midst of a paradigm shift from a modern, scientific worldview with its technical rationality, to a post-modern worldview with an eclectic perspective (Söhnge & Arjun, 1996:88). Education in a modern worldview can be thought of as a closed system (Slattery, 1995:623) with the emphasis on the product of education, whereas education in a post-modern worldview is an open system where the process of teaching and learning has inherent meaning (Parker, 1997:11). In the modern paradigm the concepts of time management and discipline are of great importance (Slattery, 2006:273), but in the post-modern paradigm moments of chaos and uncertainty are encouraged and no strict schedules should be followed, as real-life is unpredictable and complex in nature. Typical of the modern paradigm is rote memorization, conformity and control that restrict learning to a one-dimensional level imposed upon students. However, in the post-modern view learning does not occur in a linear sequence, but is a cyclic process (Badenhorst & Claassen, 1995:10). Critical thinking, reflective intuition and global problem solving will flourish as these are seen as stimulants for learners to learn, investigate and experiment (Söhnge, 1994:9-11). Beliefs on the structure of knowledge have changed from compartmentalized, consisting of unrelated bits, to viewing knowledge as interrelated and interwoven. The view of the source of knowledge as being handed down by someone with authority has been replaced by a view of knowledge that is reasoned out through objective and subjective means (Mercan, 2012:1413). In the post-modern paradigm knowledge is characterized by its utility and need to be functional: we learn things not just to know them, but to be able to use that knowledge. There is an increased emphasis on technology and on practical applications and therefore knowledge has turned from abstraction to representation. Computers are transforming the way we discover and prove ideas (Moslehian, 2005:102-104). Slattery (2006:19) indicates that the post-modern paradigm shift is sometimes called the "global information revolution".

In the modernist or traditional classroom the teacher–student bond is similar to that of a master–apprentice relationship. The teacher is in possession of all the knowledge and the student the passive receiver. The teacher defines the path by which students learn and if they are not able to do this, they are labelled inadequate (Coulter, 1993:53). Effective learning is based on quantity, the more students are taught the better. In the post-modern classroom there is an increasing freedom of instruction: "*When lifelong learners bring their life experience to class, teachers can become students and students can become teachers*" (Coulter, 1993:57). Thus in the post-modern classroom, the instructor is freed from being expected to know all and there may even be a collaborative relationship between students and lecturers toward shared goals where students are engaged as full partners.

Coulter (1993:51) makes an assertion that many of the educators standing in front of the classroom hold a modern worldview, while students/learners sitting in front of them come from a post-modern era. Educators and students may therefore have different views of teaching and learning. These differences will be more radical and difficult to understand for persons from different eras than for persons of the same era (Vorster, 1999:107). In order to deal with this situation and for successful teaching and learning to take place, educators have to adapt to a new post-modern vision and attempt to understand the differences between their own view and their students' view (Coulter, 1993:51). This brings into focus the need for a re-evaluation of the differences in which mathematics and its learning and teaching are perceived.

2.4 The influence of the current paradigm shift on mathematics education

There are many different views of the nature of mathematics. These include ontological beliefs of what mathematics entail, epistemological beliefs about finding the truth and views on the teaching and learning of mathematics. Schoenfeld (1985:45) perceives a person's mathematical belief system as an individual's perspective on how one engages in mathematical tasks and pedagogical practices. This is referred to as someone's *mathematical worldview*. When trying to understand someone's mathematical behaviour, a person's *mathematical beliefs* play a key role. Mathematical beliefs are considered personal philosophies or conceptions about the nature and structure of mathematics, as well as about the teaching and learning of mathematics (Thompson, 1992:130-132). Thus mathematics related beliefs are often divided into subgroups e.g. beliefs about the nature and structure of mathematics, beliefs about mathematics teaching, and beliefs about the learning of mathematics. These different beliefs are connected with each other and form a belief system. Such a structure of an individual's mathematics belief system, which is a wide spectrum of beliefs and conceptions, is called the person's view of mathematics (Pehkonen, 2001:14). Ernest (1988:1) states that these views held by someone may be implicitly held philosophies that the person is not consciously aware of. These belief systems are dynamic in nature and can change and undergo restructuring as individuals evaluate their beliefs against their experiences (Thompson, 1992:130).

To be able to understand someone's mathematical beliefs, one must take a closer look at the influence of current paradigms on a person's mathematical worldview. In the mechanistic (or modern) paradigm, mathematics is viewed as bits of information sequences, explicitly taught and then practiced with much repetition for learning to occur. In order to facilitate such learning, teachers must deliver carefully sequenced bits of mathematics to students through explanation and demonstration. The ability of students to reason about mathematical concepts is mostly ignored. During the last decades of the 20th century a growing awareness of and interest in the construction of meaningful mathematical understanding appeared. A broad recognition is emerging that teaching mathematics requires something other than just the transmission of objective, disembodied content. The ways in which students experience mathematical ideas and concepts and how this is connected to their experiences, is critical to the learning process.

2.5 Current views of school mathematics

Ernest (1988) distinguishes three views of school mathematics, namely the Platonist view, the dynamic problem-solving view and the instrumentalist view. He notes that educators in practice might even combine elements from these different views, but the weight of the importance of the different elements may vary from person to person.

2.5.1 Platonist (static-formalist) view

In this view, mathematics is seen as a static, but unified body of knowledge consisting of a logical and meaningful network of inter-related structures and truths bounded together by logic and meaning. This view stems from Plato's learning's according to which the origin of mathematics is outside the individual in the "external world" (Dossey, 1992:40). In this view mathematics is discovered by man rather than created. Nieuwoudt (1998:69) labels this the [traditional] static-formalist view of mathematics. The learning of mathematics is seen as a passive reception of knowledge with the teacher the possessor and explainer. In the traditional classroom the teacher was the one that knows all the mathematics to be learned. Lessons started with a review of the material covered the previous day, homework is checked and then learners followed the instructions of the teacher to do the assigned exercises. The curriculum consisted largely of computation with rules presented in bite-sized chunks that were explained briefly and practised extensively. Nieuwoudt (1998:357) quoted the following statements made about the nature of mathematics by [gifted] learners (emphasis by Nieuwoudt):

> *"The thing I liked most about math was that there is* ***no ambiguity*** *about it. It's* ***just numbers*** *and* ***a formula*** *and there's* ***always a right answer and a wrong answer with no grey area in between****."*
>
> *"It is so* ***logical*** *..."*
>
> *"It is a challenging subject, to some extent, but I enjoyed it because it was* ***precise*** *with* ***no exceptions to any rules****, like grammar or spelling, etc."*

The impact of this view on the teaching and learning of mathematics is the assumption that the teacher can gradually unfold this body of knowledge in neat chunks and transfer it to the learner. The task of the teacher is thus to direct all classroom activities, to present the material to the class and to provide opportunities for students to practice individually. The teacher should see that students master the curriculum by explaining and rehearsing subject matter while students practice using rote computations, sometimes without understanding. From this perspective, effective teachers can skilfully explain, assign tasks, monitor learners' work and provide feedback to learners while the role of the learner is to listen attentively and cooperate by doing what the teacher instructs (Kuhs & Ball, as quoted by Thompson, 1992:137). However, Nieuwoudt (1998:77) contends that a person cannot receive such a logical structure of mathematics from someone else.

This view of mathematics is an example of a typically modernist worldview with a strict hierarchical structure where the emphasis is on the product of education. Every teaching moment is planned for and predictable and teachers are the specialists who convey their knowledge to the learners as novices.

2.5.2 Dynamic, problem-solving (constructivist) view

In contrast to the above rigid view, mathematics can also be viewed from a relativist-dynamic perspective. In this view mathematics is seen as a continually expanding field of human activity, creativity and discovery, in which patterns are generated and then distilled into knowledge. Mathematics is seen as a "*human endeavour in which otherwise ordinary people of all ages construct concepts, discover relationships, invent methods, execute algorithms, communicate, and solve problems posed by their own real worlds*" (Cangelosi, 2003: v). Furthermore, mathematics is seen as an internal act of investigation, of searching to "know what to do and why" and of meaningful extension of knowledge. It is not a finished product, its results remain open to revision. This view is "problem-driven" and accordingly mathematics is viewed from a "change and grow" perspective.

A constructivist view typically underlies a learner-focused view for the teaching and learning of mathematics because this view centres on a learner's active involvement in constructing meaning from experiences by doing mathematics (Thompson, 1992:136). This kind of active involvement is based on a learner making small or great discoveries and is not promoted by just listening to or informed by a teacher on the topic (Gavalcovà, 2008:118). From a learner-focused perspective of teaching, the teacher is viewed as facilitator and stimulator of learning, posing interesting questions and situations for investigation, challenging learners to think and provide guidelines to arrive at their own conclusions by transforming information into knowledge. The teacher is in continuous dialogue with the learners. Constructivists believe that for higher levels of cognition to occur, learners must build their own knowledge through activities that engage them in active learning. Learning in an active way leads to engagement with a topic, to think it through and observe connections between related topics (Gavalcovà, 2008:118).

This view supports the post-modernist worldview which encourages “chaos” so that critical thinking and reflective intuition will flourish in search of deeper understanding. Mathematics is seen as a part of human experience and interaction that allow learners to meaningfully communicate and form connections between important mathematical concepts and ideas. This implies a need for flexibility in how teaching is approached and how learning is evaluated (Ellis & Berry, 2005:12).

2.5.3 Instrumentalist view

From an instrumentalist point of view mathematics is reduced to a bag of tools made up of an accumulation of unrelated facts, rules and skills that need to be memorized and used in the pursuance of some external end. This view is sometimes called the engineers’ view with an overemphasis of the utility value of mathematics. The feeling is that there is a job to do, never mind the theorems (Gibson, 2005:153). Skemp (1978:9) describes this instrumental understanding of mathematics as “rules without reasons” and realizes that some learners’ and educators’ idea of understanding mathematics is only to be in possession of a rule and be able to use it to produce the correct answer. In this view the role of the teacher is to demonstrate, explain and define the material and present it in a way that learners can understand it. Accordingly the role of the learners is to listen, respond to the teacher’s questions, and do exercises or problems using procedures that have been modelled by the teacher (Kuhs & Ball, as quoted by Thompson, 1992:136). This model of mathematics teaching often produces learners who are capable of performing operations with symbols, but who may not be able to connect the formal manipulation procedures with the real world (Schoenfeld, 1988:150). This way of teaching elicits much criticism because the ability to get the correct answer, performing algorithms and state definitions cannot be evidence of “knowing” mathematics. These objections are based on reports of studies documenting that learners who perform adequately on routine mathematical tasks often have impoverished conceptions and significant misunderstanding of mathematical ideas in those tasks (Thompson, 1992:136). This view has elements of both the Platonist and the dynamic view: the product of the formalist view and the utility of the dynamic view.

2.6 Views of mathematics implicit and explicit to the present school curriculum (NCS)

After the change of government in South Africa in 1994, the whole education system was modified to be learner centred and an activity-based approach was introduced where the teacher is the facilitator and the learners take responsibility for their own learning (DoE, 2003:2). The new vision of mathematics is that learners must be engaged in the science of pattern and order to "make sense of" and to "figure out". If they are engaged in activities such as to explore, investigate, explain, predict, discover, describe, verify, etc., it is virtually impossible to be passive observers and they will be actively doing mathematics (Van de Walle, 2007:12-14).

According to the NCS (DoE, 2003:2-5) the intention of the curriculum is that learners from secondary schools in South Africa should be able to demonstrate an ability to think logically and analytically and be able to transfer skills from familiar to unfamiliar situations. They should be able to work effectively with others in groups and communicate effectively using visual, symbolic and language skills in various modes. They should also be able to collect, analyse, organise and critically evaluate information and demonstrate an understanding of the world as a set of related systems by recognising that problem solving does not exist in isolation.

Change is a multidimensional action (Fullan, 2001:39). The use of new instructional materials, new teaching strategies and the possible alteration of beliefs are three dimensions at stake in implementing a new policy or program and change has to occur along all three dimensions in order to affect the outcome. Change can also be cosmetic, i.e. an educator may use new resources or modify teaching practices without changing their internal beliefs. This indicates a change on the surface and not a deeper change concerning new approaches to teaching and learning.

The new education system required from teachers to change their beliefs of how mathematics is taught and learned and they had to adapt to a new way of teaching. However, as noted by Dossey (1992:42), change in the teaching of mathematics is a slow and difficult process. If teachers hold beliefs compatible with the change, then acceptance will be more likely to occur. However, if they hold opposing beliefs then reform will not be likely to follow (Handal & Herrington, 2003:60-61). A mismatch between curriculum goals and teacher belief systems affects curriculum change in mathematics education. It is apparent that any successful change will need to take into account the beliefs of the teachers. Teachers' approaches to mathematics teaching depend on their system of beliefs and in particular on their conceptions of the nature of mathematics and on their mental models of the teaching and learning of mathematics (Ernest, 1988:1). The manner in which mathematics is presented is an indication of what a person believes to be most essential (Thompson, 1992:135). Teachers are the organizers of the learning environment and therefore their beliefs affect many aspects of their professional work that are essential for the success and quality of teaching and learning.

The fact that the beliefs of teachers influence what happens in classrooms, what teachers communicate to learners and what learners eventually learn, is important and cannot be ignored (Wilson & Cooney, 2002:144). The conception of mathematics held by the teacher has a strong impact on the way in which mathematics is approached in the classroom (Cooney, 1985: 325; Schoenfeld, 1988:161). If a teacher expects and assess for an error-free mechanical performance from the learner, they will believe that this is what mathematics is all about. Someone who views mathematics as a collection of facts and rules to be memorized and applied will more likely teach in a prescriptive manner, emphasizing rules and procedures (Thompson, 1992:127). With a focus on good results in the grade 12 examinations, teachers tend to coach learners procedurally with little or no conceptual understanding. Even though some learners are good at learning rules and do get good grades, this rewards procedural learning without actual thinking and offers little opportunity to actually learn mathematics with understanding.

Literature does report inconsistencies where teachers' conception of what mathematics is does not relate to their instructional practices (Thompson, 1992:138). Practices are not only dependent on beliefs, but also on other sources such as the political climate, the curriculum, the philosophical view of the educational system at large, the learners, the fellow teachers, the textbooks that they use, etc. All of these may have an effect on the teachers' practice without influencing their beliefs. Ernest (1988:3) noted that the effect of the context is so powerful that despite having different beliefs about mathematics and its teaching, teachers in the same school often adopt similar classroom practices. The degree of change in many South African schools was limited due to the fact that the beliefs about mathematics underlying the innovation did not match the teachers' beliefs (Webb & Webb, 2008:47). In the privacy of their classrooms educators tend to fall back to traditional mathematics teaching as innovations bring additional burdens to them despite the merits and advantages that the change might potentially bring (Skemp, 1978:13).

In reality different curricula were enacted in mathematical classrooms across South Africa. According to Sigthorsson (2008:49–52) there are three temporal phases of a curriculum in the education system, namely the intended, implemented and attained curriculum. The *intended curriculum* comprises the formal, approved guidelines that provide goals of teaching a subject, as well as the depth and breadth of the content that has to be covered (Nyaumwe *et al*., 2010:63). It is the backbone of the national curriculum issued by the educational authorities. It covers the information of "why", "when", "how" and "what" should be taught. This intended curriculum is implemented by the teachers and shaped by their beliefs, as well as prior understandings about teaching and learning, the purpose of learning and what the outcomes of learning should be. Therefore teachers need to understand the goals in depth before the intended curriculum can become a reality because they transform it into a curriculum that they believe will be workable in the classroom (Stein *et al*., 2007:321). The views of the teachers greatly determine how the implementation is effected (see par. 2.6).

The *implemented curriculum* is what is actually taught by the teachers and how it is presented to the students. It emanates from the actual process of teaching in the classroom as a result of the teachers' interpretations and their perceived understanding of the curriculum. Within the implementation phase the teacher and the learner, in interaction with each other, bring the curriculum to life and create something different than what could exist on the pages of the book (Stein *et al.*, 2007:321). Thus it is possible that the same curriculum can be interpreted and implemented differently by individual teachers. The teachers' conceptions of mathematics instruction play an important role in how they implement the curriculum (Nyaumwe *et al*., 2010:65).

The *attained curriculum* is the part of the intended and the implemented curriculum that the learners learn and are able to demonstrate. In other words, it is what the learners have actually learned. It refers to knowledge, skills and competencies gained by learners as a result of the teaching and learning process (Mucavele, 2008:28). The variations in the implementation of the curriculum resulted in the attained curricula varying from one teacher to another, as well as from school to school.

In the South African context the intended curriculum required from teachers to teach differently from how they taught traditionally (DoE, 2003). Teaching mathematics entailed creation of learning environments where learners could use their intuitions to experiment, search for patterns, reason logically, generalize and make conjectures on their own or together with others. However, many teachers do not believe in the new way of teaching and do not see the relevance of the changes made to the curriculum and keep using the traditional approaches. Other teachers tried to change, but when they experience difficulties they tend to fall back in their old ways of teaching (Webb & Webb, 2008:43). If there is a gap between the intended and the implemented curriculum, it will have an effect on the attained curriculum and first year students from secondary schools will not exhibit the characteristics described in the NCS (see par. 2.6).

In conclusion, beliefs will shape teachers' decisions about what knowledge is relevant, what teaching routines are appropriate, what goals should be accomplished and what the important features are of the social context of the classroom (Speer, 2005:365). This will filter through to learners who develop their understanding of mathematics from their classroom experiences of it (Schoenfeld, 1988:161).

2.7 The gap between beliefs of mathematics at secondary and tertiary level

Previously, students enrolling for university mathematics courses tended to fit a particular profile and university lecturers could assume that all students have obtained the same level of skills at secondary level (Engelbrecht & Harding, 2008:57). This situation has changed, as curricular changes in secondary schools have an influence on the characteristics of first year students at university (Kajander & Lovric, 2005:150). The changes in the curriculum in secondary schools brought uncertainty at tertiary level because lecturers may be unaware of the exact changes in the curriculum. As a result of the gap between the intended and the implemented curriculum, first year students now have a wide range of mathematical skills and have varying competencies and knowledge bases. They have different backgrounds, interests, ambitions, levels of motivation and approaches to studying, which means that their learning needs will also vary. Although not all incoming students' worldviews are the same, most of them are from a post-modern era and their worldview may differ from the lecturers' view (Coulter, 1993:51). Many of the lecturers at universities come from the modernist era and have their own view of what mathematics is according to their worldview and how it should be taught. These different views may lead to a mismatch in ideas on the teaching and learning of mathematics (Felder & Silverman, 1988: 674).

The university curriculum is usually developed from the knowledge base of the lecturers and from the strategic requirements of the university and the demands of relevant industries (Petocz & Reid, 2005:91). Lecturers of mathematics make decisions on the content of the curriculum based on their own experiences of being students and mathematicians. The learning material is presented in a strict logical sequence and the emphasis is on the correct representation of definitions, wording and proof of theorems (Hong *et al*., 2009:881). Formal definitions and rigorous proofs are given precedent over its intuitive nature and application of mathematics to real-life situations is reserved for a separate subject called "applied mathematics". The main goal is that students must be capable of producing rigorous proofs (Weber, 2004:116). In this sense, mathematics at tertiary level corresponds to the static-formalist view of school mathematics. It seems as if first year students are not prepared for the rigour and precision of university mathematics. This could be attributed to the change in the intended school curriculum from a static-formalist to a relativist-dynamic view.

Any approach to the teaching of mathematics is closely related to the question of what mathematics is perceived to be. In a study done by Hong *et al*. (2009:881) in New Zealand, which is part of a larger research project entitled “Analysing the transition from secondary to tertiary education in mathematics”, researchers observed that teaching at tertiary level is more teacher-centred than at secondary level and that the interaction between lecturers and students is not sufficient. Large numbers of students in a class, which is typical at universities, prevent an interactive, investigative approach to teaching. In these classes students find it difficult to ask questions and classes consist of passive lecturing instead of active learning.

At tertiary level the pace of material covered in a period is much faster than in secondary school. In a lecture at university the information is extensive, consequently students just copy instead of trying to understand what is being taught (Hong *et al*., 2009:886). Lecturers only provide a framework with some particulars to guide the students in the process of learning the concepts and methods of the course (Zucker, 1996:864-865) and lecturers do not expect their students to understand everything in the classroom. It requires time and effort to attain the level of understanding one would like to achieve and most of this takes place outside of the classroom. It is the responsibility of the student to learn the material on their own as lecturing time should be used efficiently and everything cannot be taught in the classroom. After students have attended a class they are supposed to study the lectured content by reading notes, working through the textbook and doing some exercises on their own. Alsina (2001:5) indicates that lecturing takes place in the classroom while “*learning is an individual after-class activity of assimilating results and practising techniques*”. Students exposed to a learner-centred approach at school level may find it difficult to adapt to this new learning environment.

From the literature investigated in this chapter one can conclude that there are gaps in the transition from secondary school and tertiary institutions regarding the beliefs of the nature of mathematics, as well as beliefs concerning the teaching and learning of mathematics. The empirical study (Chapter 5) reports on an empirical investigation to determine what the beliefs of the first year students and the lecturers at the selected institution are on the nature of mathematics and the teaching of mathematics. The gap between secondary and tertiary mathematics concerning the beliefs on the structure of mathematics will be investigated in the next chapter.